Grand-reaction ensemble.

Grand canonical ensemble

The free energy of the grand-canonical ensemble for single particle type

$$\Omega = E - TS + \mu N$$

The entropy S expands via Boltzmann formula

$$S = k_B \ln \frac{V^N}{N!}$$

$$\Omega = E - k_B T \ln \frac{V^N}{N!} + \mu N$$

The change of free energy associated with a sngle particle exchange is

$$\Delta\Omega = k_B T \ln \left(V^{\xi} \frac{N!}{(N+\xi)!} \right) + \xi \mu + \Delta E$$

accept if $\mathcal{R}^{\xi} < e^{\Delta\Omega/k_BT}$

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