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Course: EEE 321

Assignment 02

1. Given,

$b_0 = 1, \quad b_1 = 2, \quad b_2 = 3, \quad b_3 = 1.5$

Order, $M = 3$

Difference Equation

$$y(n) = \sum_{m=0}^{3} b_m x(n-m)$$

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + b_3 x(n-3)$$

$$y(n) = x(n) + 2x(n-1) + 3x(n-2) + 1.5x(n-3)$$

System function

$$H(z) = \sum_{m=0}^{3} b_m z^{-m}$$

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}$$

$$H(z) = 1 + 2z^{-1} + 3z^{-2} + 1.5z^{-3}$$

Impulse Response

For FIR; $h(n) = b_m \; ; \quad 0 \le m \le M$

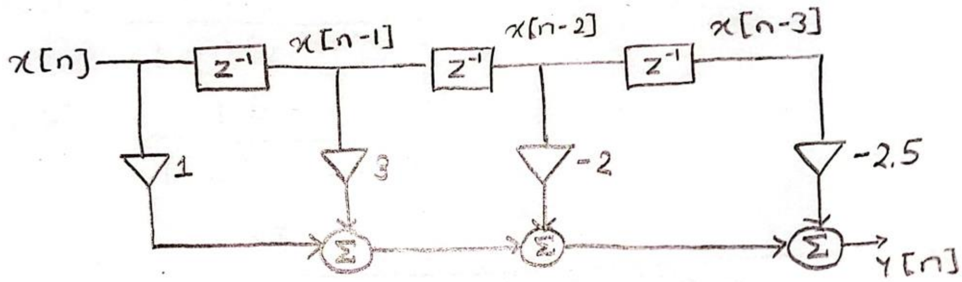
$$\therefore h(n) = \{ \underset{\uparrow}{1}, 2, 3, 1.5 \}$$

2a) $y(n) = x(n) + 3x(n-1) - 2x(n-2) - 2.5\,x(n-3)$

$$y(n) = \sum_{m=0}^{M} b_m\, x(n-m)$$

This system is an FIR Filter since it has no a_k coefficients or feedback. The output of the filter depends on past and present inputs.

b)



c)

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b = [1,3,-2,-2.5];
a = [1,0,0,0];
zplane(b,a)
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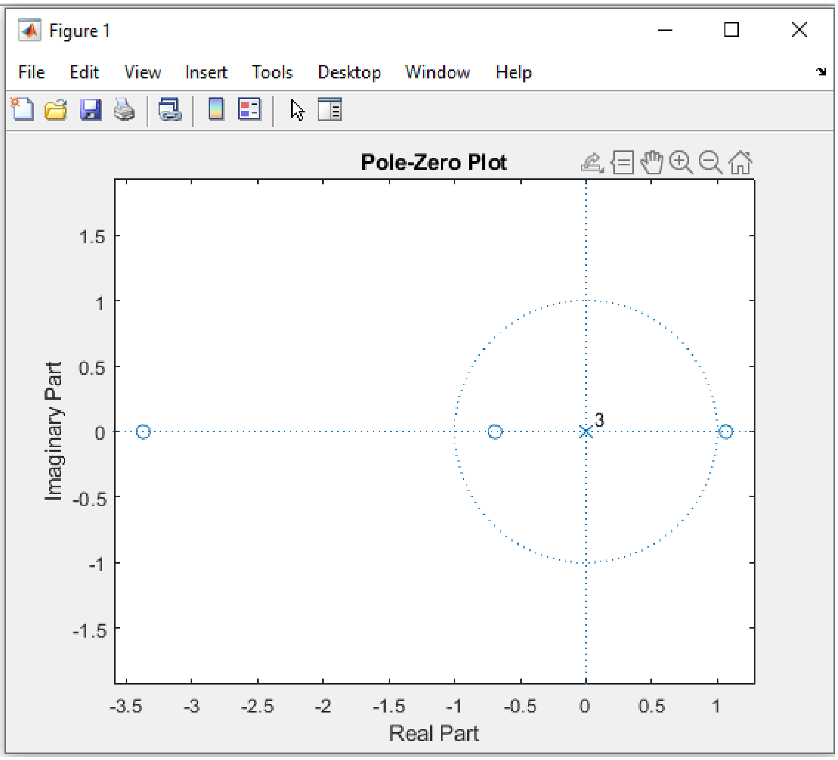

The system is stable since there is no left side pole.

3. a)

$$y(n) = 0.5y(n-1) - 3y(n-3) + 1.5x(n)$$

$$y(n) - 0.5y(n-1) + 3y(n-3) = 1.5x(n)$$

$$a_0 = 1,\ a_1 = -0.5,\ a_3 = 3,\ b_0 = 1.5$$

$$H(z) = \frac{\sum_{m=0}^{M} b_m z^{-m}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

$$H(z) = \frac{1.5}{1 - 0.5z^{-1} + 3z^{-3}}$$

$$H(\omega) = \frac{1.5}{1 - 0.5e^{-j\omega} + 3e^{-3j\omega}}$$

$$= \frac{1.5}{1 - 0.5(\cos\omega - j\sin\omega) + 3(\cos 3\omega - j\sin 3\omega)}$$

$$= \frac{1.5}{(1 - 0.5\cos\omega + 3\cos 3\omega) + j(0.5\sin\omega - 3\sin 3\omega)}$$

Magnitude, $|H(\omega)| = \dfrac{1.5}{\sqrt{(1-0.5\cos\omega + 3\cos 3\omega)^2 + (0.5\sin\omega - 3\sin 3\omega)^2}}$

Phase Response,

$$\angle H(\omega) = -\tan^{-1}\left(\frac{0.5\sin\omega - 3\sin 3\omega}{1 - 0.5\cos\omega + 3\cos 3\omega}\right)$$

b) $H_{HP}(\omega) = H_{LP}(\omega - \pi)$

$$H_{HP}(\omega) = \frac{\sum_{m=0}^{M} b_m (-1)^m e^{-j\omega m}}{1 + \sum_{k=1}^{N} a_k (-1)^k e^{-j\omega k}}$$

$$H_{HP}(\omega) = \frac{1.5}{1+0.5e^{-j\omega} - 3e^{-3j\omega}}$$

or

$$H(z) = \frac{1.5}{1+0.5z^{-1} - 3z^{-3}}$$

c) $$H(z) = \frac{1.5}{1-0.5z^{-1}+3z^{-3}}$$

$$H(z) = \frac{1.5z^3}{z^3-0.5z^2+3}$$

$$h_{lp}(n) = Z^{-1}\left[\frac{1.5}{1-0.5z^{-1}+3z^{-3}}\right]$$

For high pass filter,

$$h_{hp}(n) = (-1)^n h_{lp}(n)$$

$$h_{hp}(n) = Z^{-1}\left[\frac{1.5}{1+0.5z^{-1}-3z^{-3}}\right]$$

d) The difference equation for high pass filter

$$y(n) + \sum_{k=1}^{N} (-1)^k y(n-k) = \sum_{m=0}^{M} (-1)^m b_m x(n-m)$$

$$y(n) + 0.5y(n-1) - 3y(n-3) = 1.5x(n)$$

$$y(n) = -0.5y(n-1) + 3y(n-3) + 1.5x(n)$$

```
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%For Low Pass Filter
b = [1.5,0,0,0];
a = [1,-0.5,0,3];
freqz(b,a,500)
```

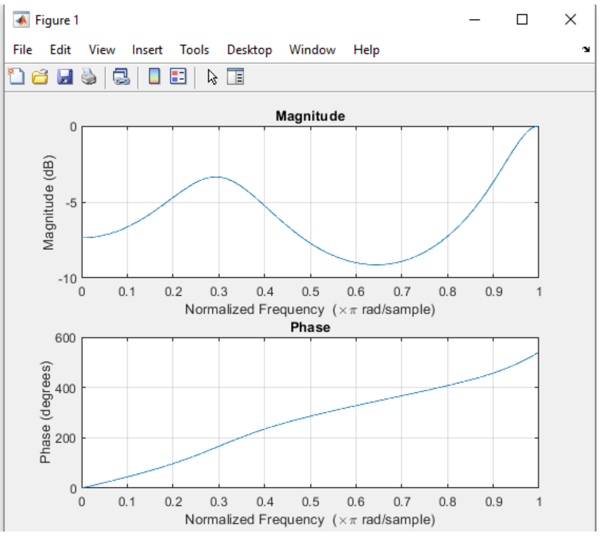


```
%Hemal Sharma
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%For High Pass Filter
b = [1.5,0,0,0];
a = [1,0.5,0,-3];
freqz(b,a,500)
```

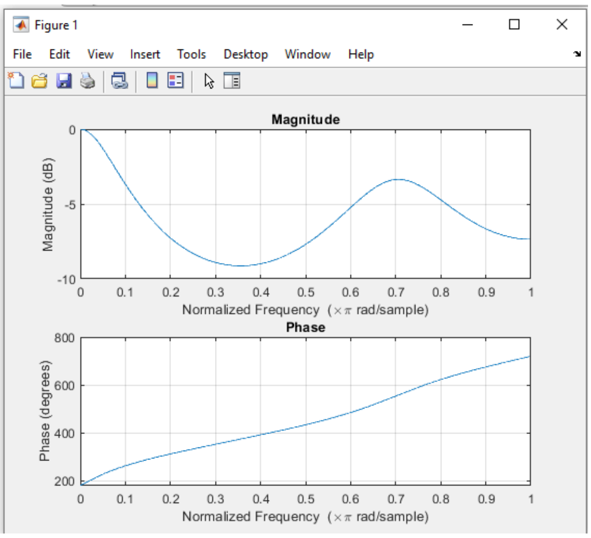

4. Given,

$x_1(n) = \{1, -2, 3\}$

$x_2(n) = \{4, -3, 2\}$

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n) e^{-j\frac{2\pi nk}{N}}$$

$$= \sum_{n=0}^{2} x_1(n)\, e^{-j2\pi nk/3}$$

$$\therefore X_1(k) = x_1(0) e^{-j0} + x_1(1) e^{-j\frac{2\pi k}{3}} + x_1(2) e^{-j\frac{4\pi k}{3}} \quad \text{—(i)}$$

$$X_2(k) = \sum_{n=0}^{2} x_2(n) e^{-j\frac{2\pi nk}{3}}$$

$$X_2(k) = x_2(0) e^{-j(0)} + x_2(1) e^{-j\frac{2\pi k}{3}} + x_2(2) e^{-j\frac{4\pi k}{3}} \quad \text{—(ii)}$$

We know,

$$X_3(k) = X_1(k).X_2(k)$$

$$= \left[1 - 2e^{-jk(\frac{2\pi}{3})} + 3e^{-jk(\frac{4\pi}{3})}\right]\left[4 - 3e^{-jk(\frac{2\pi}{3})} + 2e^{-jk(\frac{4\pi}{3})}\right]$$

$$= 4 - 3e^{-jk(\frac{2\pi}{3})} + 2e^{-jk(\frac{4\pi}{3})} - 8e^{-jk(\frac{2\pi}{3})} + 6e^{-jk(\frac{2\pi}{3})} - 4e^{-jk(\frac{6\pi}{3})}$$
$$+ 12e^{-jk(\frac{4\pi}{3})} - 9e^{-jk(\frac{6\pi}{3})} + 6e^{-jk(\frac{4\pi}{3})} \quad \text{—(iii)}$$

$$= -9 - 5e^{-jk(\frac{2\pi}{3})} + 20e^{-jk(\frac{4\pi}{3})}$$

$$\therefore x_3(n) = \frac{1}{N} \sum_{k=0}^{N-1} x_3(k)\; e^{j\frac{2\pi nk}{N}}$$

$$= \frac{1}{3}\left[x_3(0)e^0 + x_3(1)\, e^{j(\frac{2\pi}{3})n} + x_3(2)\, e^{j(\frac{2\pi}{3})n}\right] \quad \text{—(iv)}$$

From (iii) and (iv)

$x_3(0) = -9$

$x_3(1) = -5$

$x_3(2) = 20$

$\therefore$ Circular Convolution,

$x_3(n) = \{-9, -5, 20\}$

Command Window

```
>> %Hemal Sharma
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x1 = [1, -2, 3];
x2 = [4, -3, 2];
x3 = cconv(x1,x2,3)

x3 =

   -9.0000   -5.0000   20.0000
```

5.a) $h(n) = \{1, 2, 3, 0, -3, -2, -1\}$

Since, $h(n)$ is a finite duration impulse response, it is a FIR filter. This filter does not have any feedback. The output of the filter depends on past and present inputs.

b) Transfer function,

$$H(z) = \sum_{n=0}^{6} h(n) z^{-n}$$

$$H(z) = h(0)z^{0} + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}$$

$$H(z) = 1 + 2z^{-1} + 3z^{-2} - 3z^{-4} - 2z^{-5} - z^{-6}$$

$$H(\omega) = H(z)\Big|_{z=e^{j\omega}}$$

$$= 1 + 2e^{-j\omega} + 3e^{-2j\omega} - 3e^{-4j\omega} - 2e^{-5j\omega} - e^{-6j\omega}$$

c) <u>Difference Equation</u> :

$$y(n) = 1 + 2x(n-1) + 3x(n-2) - 3x(n-4) - 2x(n-5) - x(n-6)$$

(Ans:)

d)

```
%Hemal Sharma
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b = [1,2,3,0,-3,-2,-1];
a = [1,0,0,0,0,0,0];
freqz(b,a,500)
```
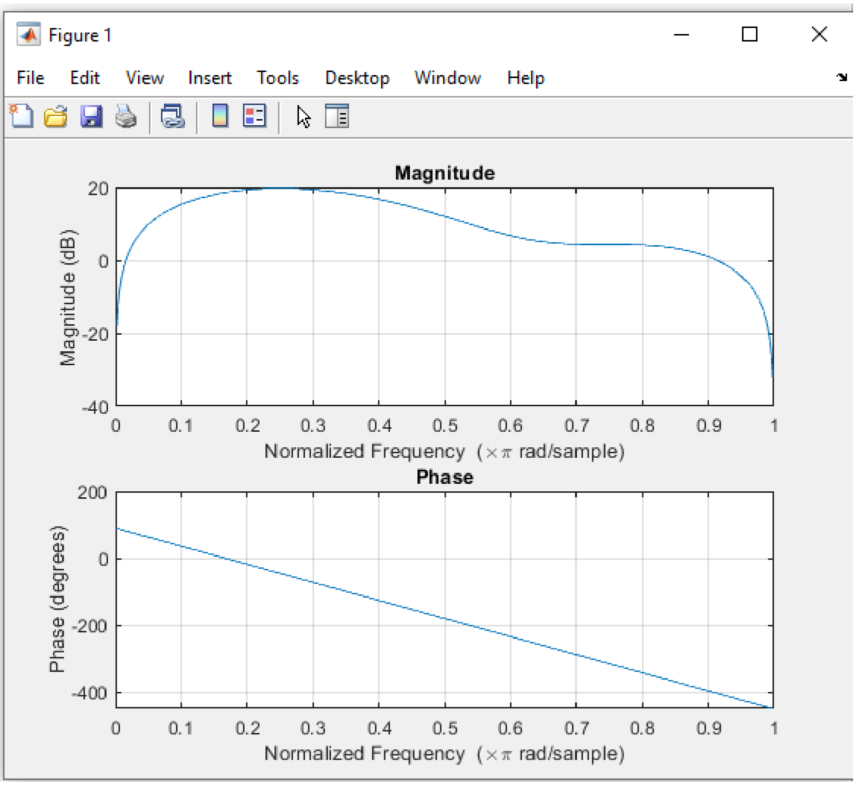



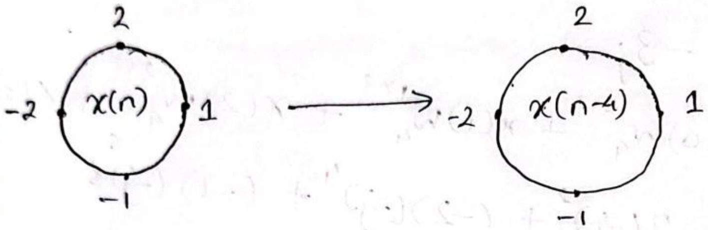
The Phase response is linear.

6. $x(n) = \{1, 2, -2, -1\}$

a) $(x(n-4))_N$

$N=4$

Shifted by 4



$\therefore (x(n-4))_4 = \{1, 2, -2, -1\}$

b) <u>DFT</u>: $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}$ $\qquad W_N = e^{-j\frac{2\pi}{N}}$

$N=4$

$W_4 = e^{-j\frac{2\pi}{4}} = \cos\left(\frac{2\pi}{4}\right) - j\sin\left(\frac{2\pi}{4}\right)$

$= -j$

$\therefore X(k) = \sum_{n=0}^{3} x(n) W_4^{nk}$

$W_4^0 = W_4^4 = W_4^8 = 1 \qquad W_4^1 = W_4^5 = W_4^9 = -j$

$W_4^2 = W_4^6 = -1 \qquad W_4^3 = W_4^7 = j$

$X(0) = x(0) W_4^0 + x(1) W_4^{0\times 1} + x(2) W_4^{0\times 2} + x(3) W_4^{0\times 3}$

$= 1 + 2 + (-2)(1) + (-1)(1)$

$= 0$

$X(1) = x(0)W_4^{0\times1} + x(1)W_4^{1\times1} + x(2)W_4^{2\times1} + x(3)W_4^{3\times1}$

$= 1 + 2(-j) + (-2)(-1) + (-1)j$

$= 3 - 3j$

$X(2) = x(0)W_4^{0\times2} + x(1)W_4^{1\times2} + x(2)W_4^{2\times2} + x(3)W_4^{3\times2}$

$= 1 + 2(-j)^2 + (-2)(-j)^4 + (-1)(-j)^6$

$= -2$

$X(3) = x(0)W_4^{0\times3} + x(1)W_4^{1\times3} + x(2)W_4^{2\times3} + x(3)W_4^{3\times3}$

$= 1 + 2j + (-2)(-1) + (-1)(-j)$

$= 3 + 3j$

$\therefore X(k) = \{0, 3-3j, -2, 3+3j\}$

```
>> %Hemal Sharma
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x = [1,2,-2,-1];
fft(x,4)

ans =

   0.0000 + 0.0000i   3.0000 - 3.0000i  -2.0000 + 0.0000i   3.0000 + 3.0000i
```

c) No. of complex adders $= N(N-1)$

$= 4(4-1)$

$= 12$

No. of complex multipliers $= N^2$

$= 4^2$

$= 16$

d) For FFT method

$N=4$

No. of complex adders $= N \log_2 N$

$= 4 \log_2 4$

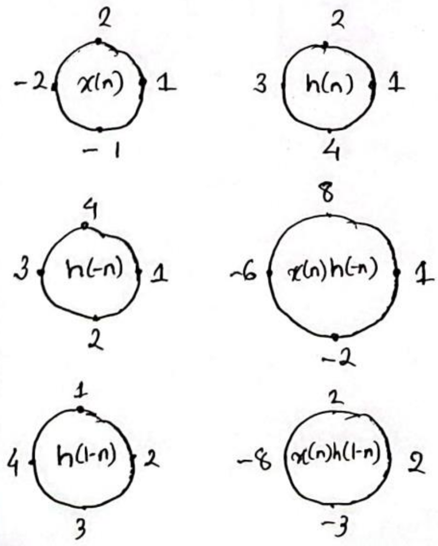
$= 8$

No. of complex multipliers $= \frac{N}{2} \log_2 N$

$= \frac{4}{2} \log_2 4$

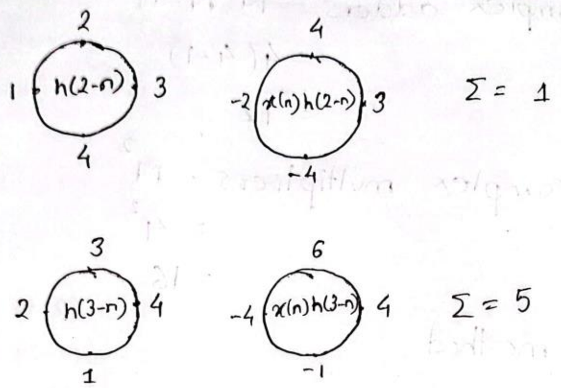
$= 4$

e) $h(n) = \{1, 2, 3, 4\}$, $x(n) = \{1, 2, -2, -1\}$



$\Sigma = 1$

$\Sigma = -7$



$\therefore y(n) = \{1, -7, 1, 5\}$

```
>> %Hemal Sharma
%ID: 2221855
x = [1, 2, 3, 4];
h = [1, 2, -2, -1];
y = cconv(x,h,4)

y =

     1    -7     1     5
```

f) From b,

$X(k) = \{0, 3-3j, -2, 3+3j\}$

$$\therefore H(k) = \sum_{n=0}^{3} h(n) e^{-j\frac{2\pi nk}{4}}$$

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1+2+3+4 \\ 1-2j-3+4j \\ 1-2+3-4 \\ 1+2j-3-4j \end{bmatrix}$$

$$H(k) = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$Y(k) = X(k) \cdot H(k)$$

$$= \begin{bmatrix} 0 \\ 3-3j \\ -2 \\ 3+3j \end{bmatrix} \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} = \begin{bmatrix} 0 \\ 12j \\ 4 \\ -12j \end{bmatrix}$$

$$y(n) = \frac{1}{N} \sum_{k=0}^{N-1} Y(k) W_N^{-nk}$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 0 \\ 12j \\ 4 \\ -12j \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 0 + 12j + 4 - 12j \\ 0 - 12 - 4 - 12 \\ 0 - 12j + 4 + 12j \\ 0 + 12 - 4 + 12 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -7 \\ 1 \\ 5 \end{bmatrix}$$

$y(n) = \{1, -7, 1, 5\}$

$\therefore \left[x(n) * h(n)\right] = X(k).H(k)$ [Proved]