Assignment 1

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Course Title: Digital Signal Processing

Course Code: EEE321

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$\therefore a = 2$

$b = 10$

1.a) $H(z) = \dfrac{az}{z-b}$

$H(z) = \dfrac{2z}{z-10}$

Given,

$x(n) = u(n)$

$X(z) = Z[u(n)] = \dfrac{z}{z-1}$

We know,

$Y(z) = H(z) \cdot X(z)$

$= \dfrac{2z}{z-10} \times \dfrac{z}{z-1}$

$= \dfrac{2z^2}{(z-10)(z-1)}$

$\therefore$ The system response in z-domain,

$Y(z) = \dfrac{2z^2}{(z-10)(z-1)}$

b) $y(n) = Z^{-1}[Y(z)]$

$$= Z^{-1}\left[\frac{2z^2}{(z-10)(z-1)}\right]$$

$$= Z^{-1}\left[2 + \frac{80}{3(z-10)} - \frac{14}{3(z-1)}\right]$$

$$= 2\delta(n) + \frac{80}{3}(10)^{n-1}u(n-1) - \frac{14}{3}(1)^{n-1}u(n-1) \quad \text{(Ans:)}$$

$$z^2 - 11z - 10 \,\overline{)\, 2z^2 \,}\; 2$$

$$\frac{-\; 2z^2 - 22z - 20}{22z + 20}$$

$$= 2 + \frac{22z+20}{z^2 - 11z - 10}$$

$$= 2 + \frac{22z+20}{(z-10)(z-1)}$$

By Partial Fraction:

$$\frac{22z+20}{(z-10)(z-1)} \equiv \frac{A}{(z-10)} + \frac{B}{(z-1)}$$

$$22z + 20 \equiv A(z-1) + B(z-10)$$

Let $z = 1$

$42 = -9B$

$B = -\frac{14}{3}$

Let $z = 10$

$240 = 9A$

$A = \frac{80}{3}$

$$2 + \frac{80}{3(z-10)} - \frac{14}{3(z-1)}$$

∴ The system response in time domain,

$$y(n) = 2\delta(n) + \frac{80}{3}(10)^{n-1}u(n-1) - \frac{14}{3}(1)^{n-1}u(n-1)$$

(Ans:)

c) $x(n) = 2^n u(n-1)$

$$X(z) = Z[2^n u(n-1)]$$

$$= \frac{2}{z-2}$$

$$Z[u(n)] = \frac{z}{z-1}$$

$$Z[u(n-1)] = z^{-1} \frac{z}{z-1}$$ [Sample Shifting property]

$$= \frac{1}{z-1}$$

$$Z[2^n u(n-1)] = \frac{1}{\frac{z}{2} - 1}$$ [Frequency Shifting property]

$$= \frac{2}{z-2}$$

$$Y(z) = H(z) . X(z)$$

$$= \frac{2z}{z-10} \times \frac{2}{z-2}$$

$$= \frac{4z}{(z-10)(z-2)}$$

The output of response, in z domain,

$$Y(z) = \frac{4z}{(z-10)(z-2)}$$

$$\frac{4z}{(z-10)(z-2)} = \frac{A}{z-10} + \frac{B}{z-2}$$

$$4z = A(z-2) + B(z-10)$$

Let $z=2$: $8 = -8B$, $B = -1$

Let $z=10$: $40 = 8A$, $A = 5$

$$= \frac{5}{z-10} - \frac{1}{z-2}$$

In Time domain,

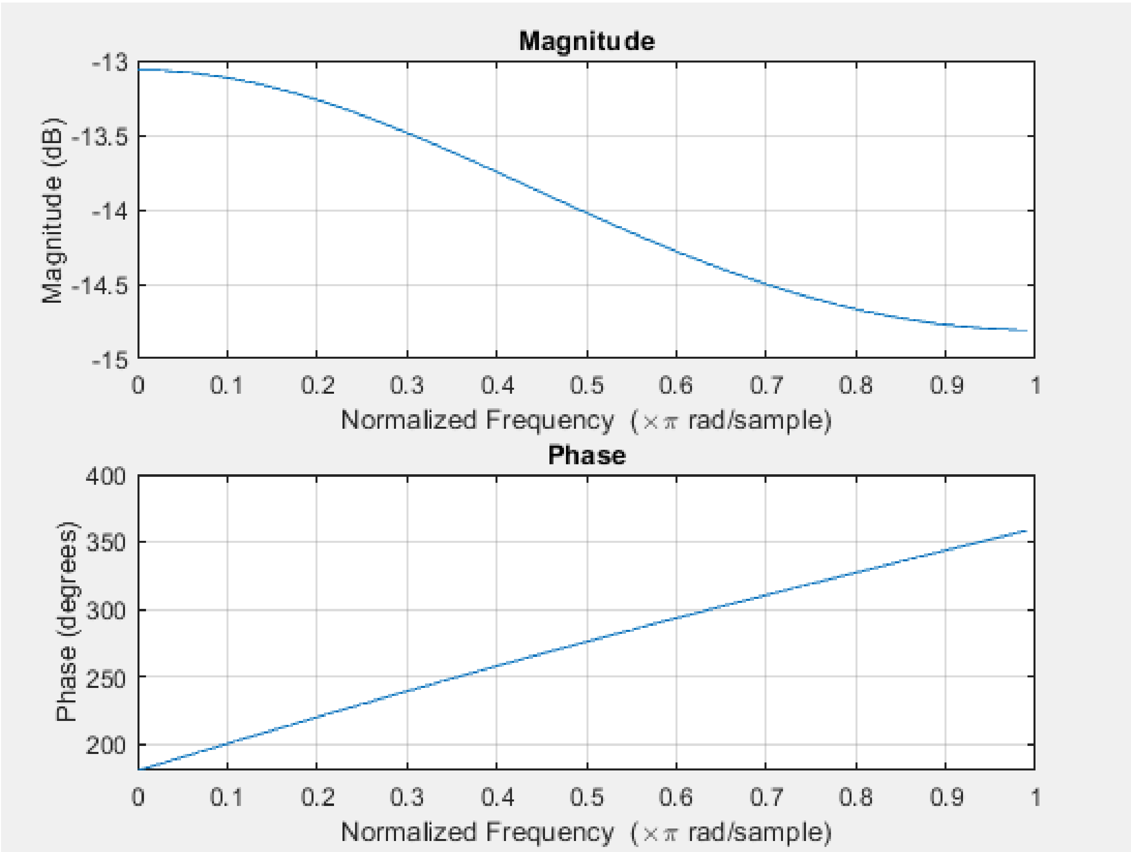
$$y(n) = Z^{-1}\left[\frac{4z}{(z-10)(z-2)}\right]$$

$$= Z^{-1}\left[\frac{5}{z-10} - \frac{1}{z-2}\right]$$

$$= 5.(10)^{n-1} u(n-1) - (2)^{n-1} u(n-1)$$

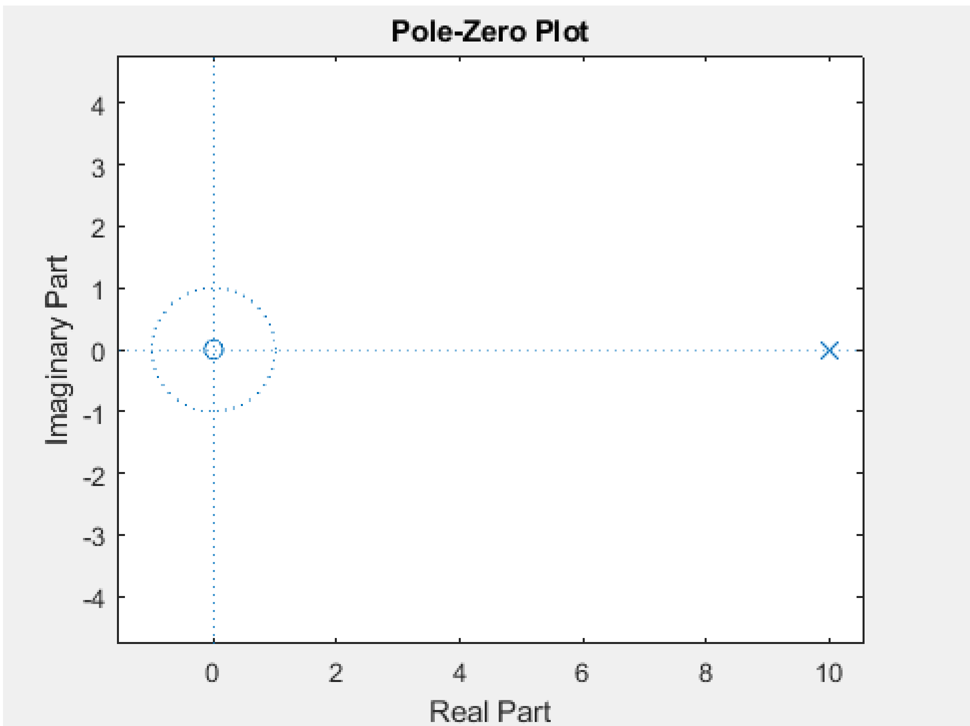
d) Plot the frequency response of the LTI system using MATLAB simulation.

```
%frequency plot of the LTI system
% H(z) = 2z/(z-10)
a=[1,-10];
b=[2, 0];
freqz(b,a,100);
```



e) Plot the pole-zero plot on a complex z-plane for the system using MATLAB. Comment on the stability of the system?

```
%Pole-zero plot
% H(z) = 2z/(z-10)
a=[1,-10];
b=[2,0];
zplane(b,a)
```



Since the pole of the z-transfer function is outside of the unit circle on the z-plane pole-zero plot, the system is unstable.

2\. a) $x(n) = 5u(n-1) - 2\delta(n-1)$

$\therefore X(z) = Z[5u(n-1) - 2\delta(n-1)]$

$= Z[5u(n-1)] - Z[2\delta(n-1)]$

$= \frac{5}{z-1} - \frac{2}{z}$

$= \frac{5z - 2(z-1)}{z(z-1)}$

$= \frac{5z - 2z + 2}{z(z-1)}$

$= \frac{3z+2}{z(z-1)}$

$(ROC_1)\ z \neq 0 \quad (ROC_2)\ : z > 1$

$\therefore ROC = (ROC_1) \cap (ROC_2)$

$\boxed{ROC = z > 1}$

$Z[u(n)] = \frac{z}{z-1}$

$Z[u(n-1)] = z^{-1} \frac{z}{z-1}$

$= \frac{1}{z-1}$

$Z[\delta(n)] = 1$

$Z[\delta(n-1)] = z^{-1} = \frac{1}{z}$

b) $Z\{n[5u(n-1)-2\delta(n-1)]\}$

$$-z \cdot \frac{d}{dz}\left(\frac{3z+2}{z^2-z}\right)$$

$$-z \cdot \frac{z^2-z\,\frac{d}{dz}(3z+2)-(3z+2)\frac{d}{dz}(z^2-z)}{(z^2-z)^2}$$

$$-z \cdot \frac{(z^2-z).3-(3z+2)(2z-1)}{(z^2-z)^2}$$

$$-z \cdot \frac{3(z^2-z)-(6z^2-3z+4z-2)}{(z^2-z)^2}$$

$$-z \cdot \frac{-3z^2-4z+2}{z^2(z-1)^2}$$

$$= \frac{3z^2+4z-2}{z(z-1)^2}$$

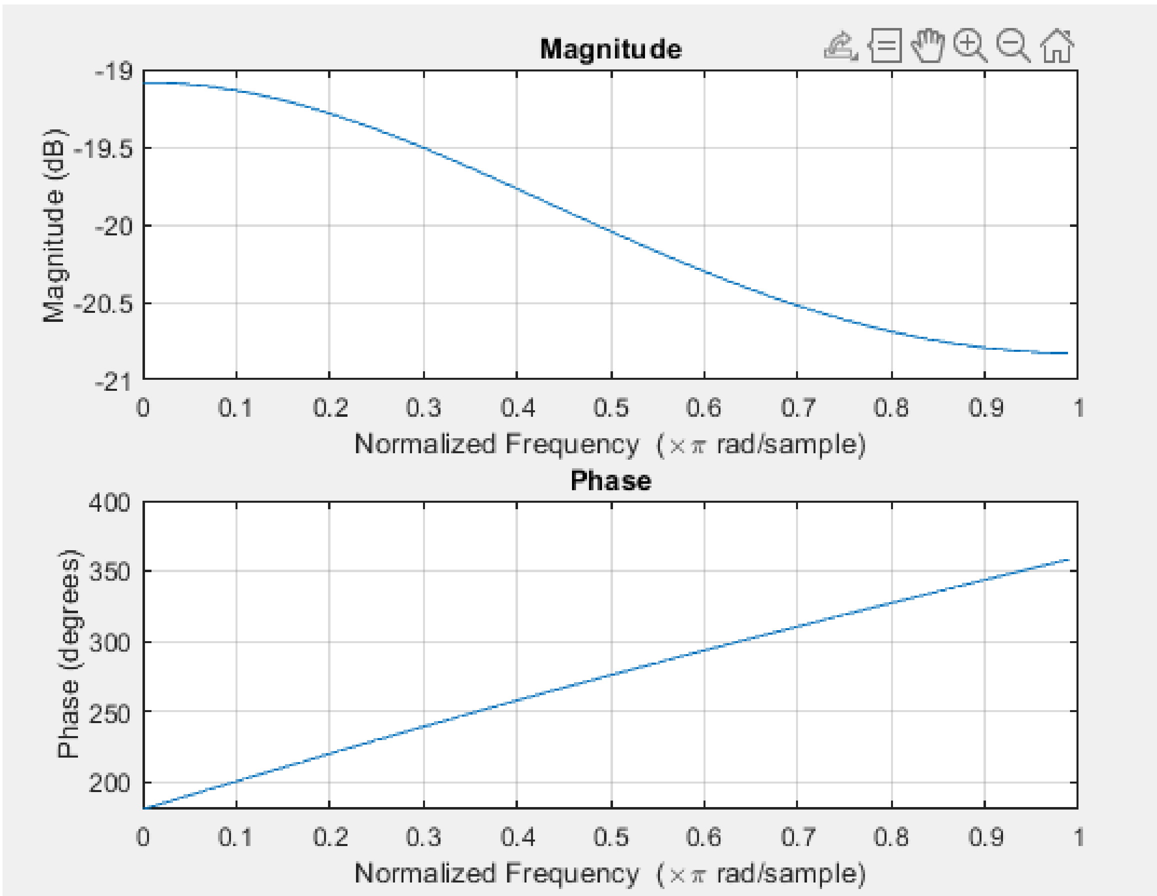
$\downarrow$ $\downarrow$

$(ROC_1)\ z \neq 0$ $(ROC_2)\ z > 1$

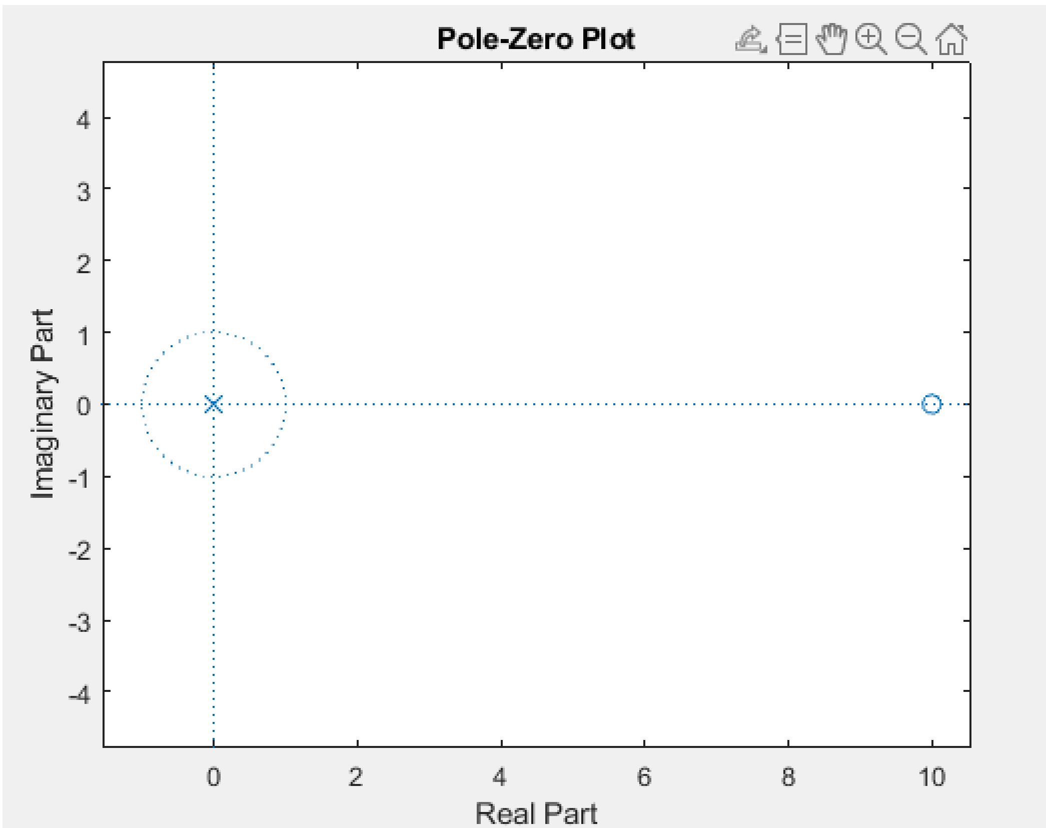
$ROC = (ROC_1) \cap (ROC_2)$

$$\boxed{ROC = z > 1}$$

```
%frequency plot of the LTI system
% H(z) = z/(z-10)
a=[1,-10];
b=[1, 0];
freqz(b,a,100);
```



```
%Pole-zero plot
% H(z) = z/(z-10)
b=[1,-10];
a=[1,0];
zplane(b,a)
```



Since the pole of the z-transfer function is outside of the unit circle on the z-plane pole-zero plot, the system is unstable.

c) $H(z) = \dfrac{z}{z-10}$

$H(z) = \dfrac{1}{1-10z^{-1}}$, Here,

$b_0 = 1$, $b_1 = 0$

$a_0 = 1$, $a_1 = -10$

$\therefore$ The difference equation is,

$y(n) - 10y(n-1) = x(n)$ (Ans:)

d) The impulse response $h(n)$ of the system is given.

$h(n) = b^n u(n)$

$h(n) = 10^n u(n)$

```
%Name: Hemal Sharma
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function [x,n] = stepseq(n0, n1, n2)
    n = n1:n2
    x = (n>=n0)
    stem(n,x)

end
```

```
%h(n) = 10^nu(n)
n = 0:10;
x = (10.^n).*stepseq(0,0,10);
stem(n,x)
```