Modeling spatial continuity

Modeling Uncertainty in the Earth Sciences

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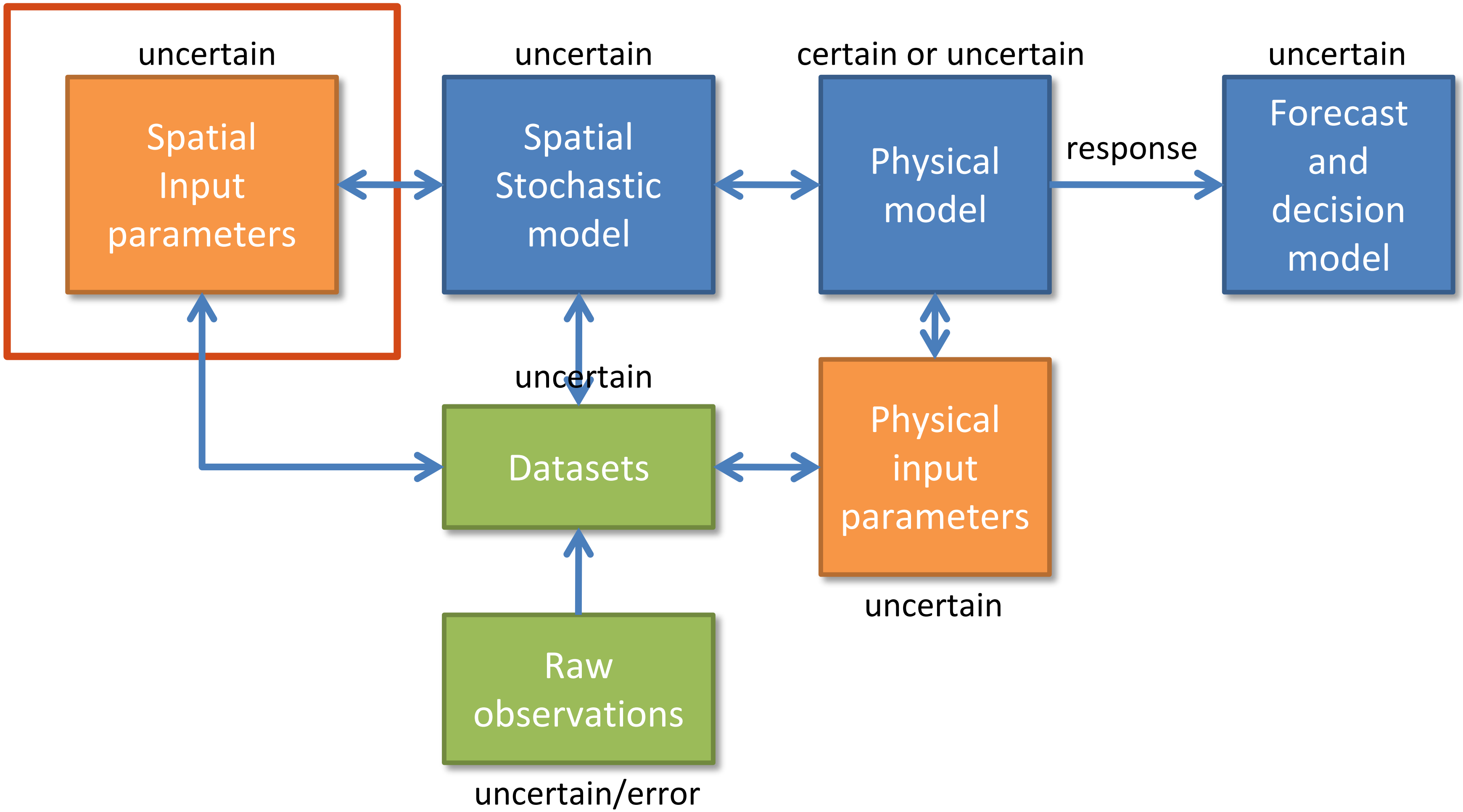
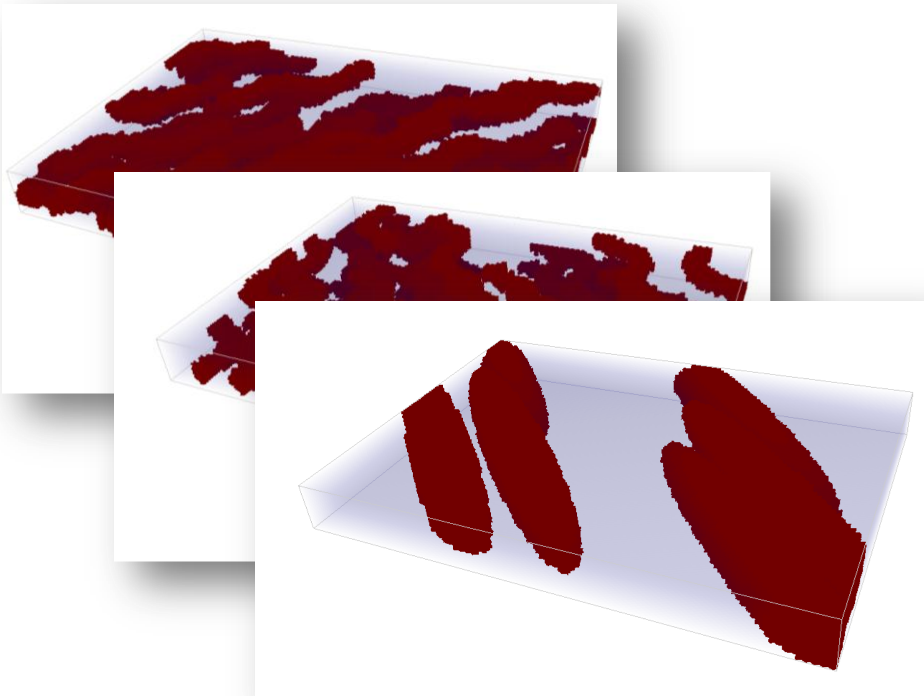
Motivation

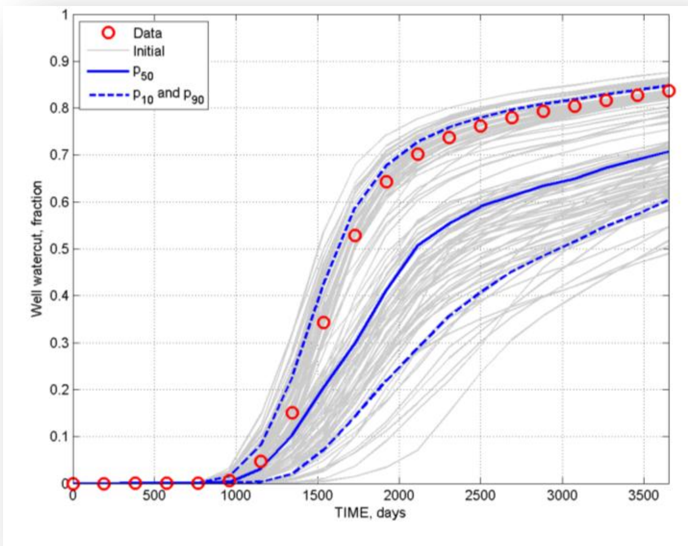

uncertain
Spatial Input parameters
uncertain
Spatial Stochastic model
certain or uncertain
Physical model
response
uncertain
Forecast and decision model
uncertain
Datasets
Physical input parameters
uncertain
Raw observations
uncertain/error

Motivation



Calculate expected costs / profits

Build models

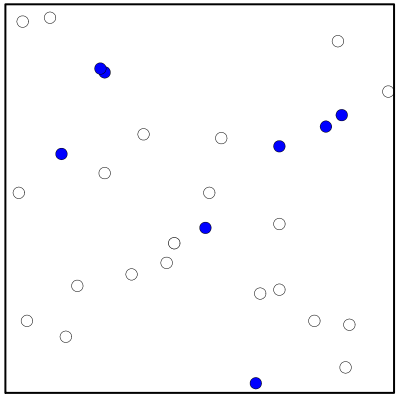


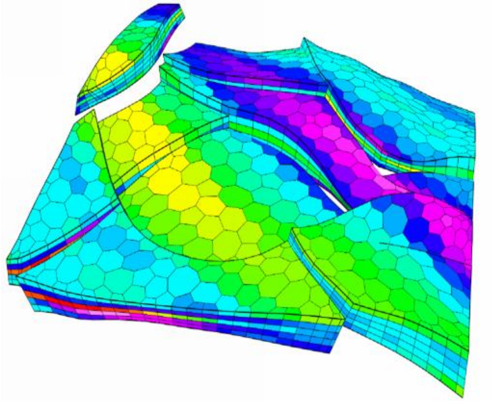
Get responses

Motivation

- Earth phenomena are not randomly distributed in space and time: this makes them predictable !
- Surface and subsurface modeling: a medium exists that has been created by processes (geological, morphological etc…)
- A form of "continuity" exists
  - "discontinuity" (e.g. faults) is a specific form of such continuity

Why modeling spatial continuity?



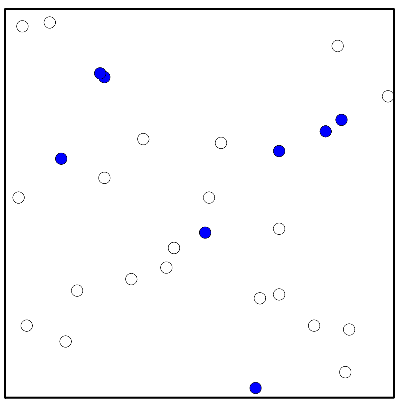


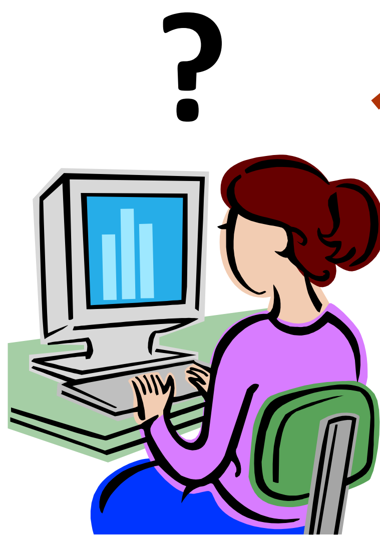
What are mathematical or computer-based models that describe the spatial distribution of properties observed in these (complete or incomplete) datasets

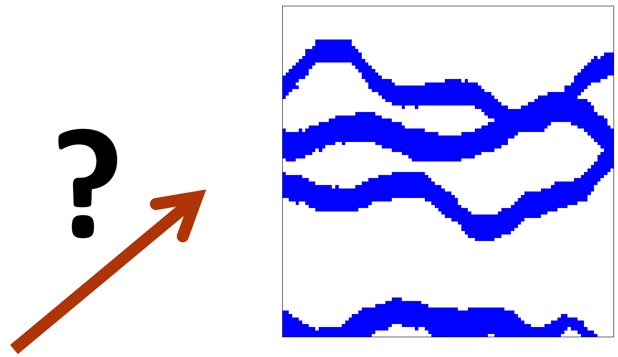
Why modeling spatial continuity?

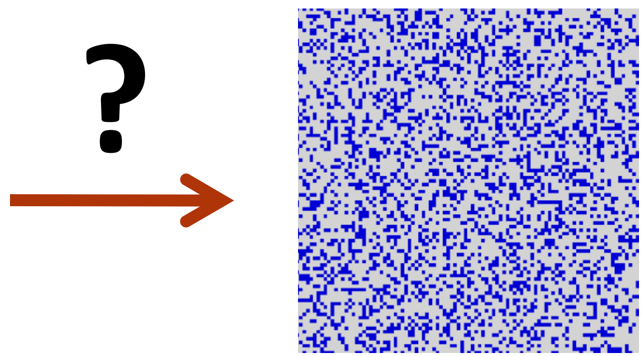
What is the spatial variation like ?

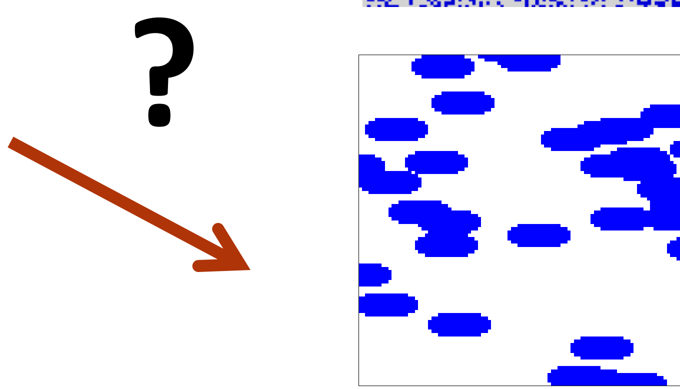
Data





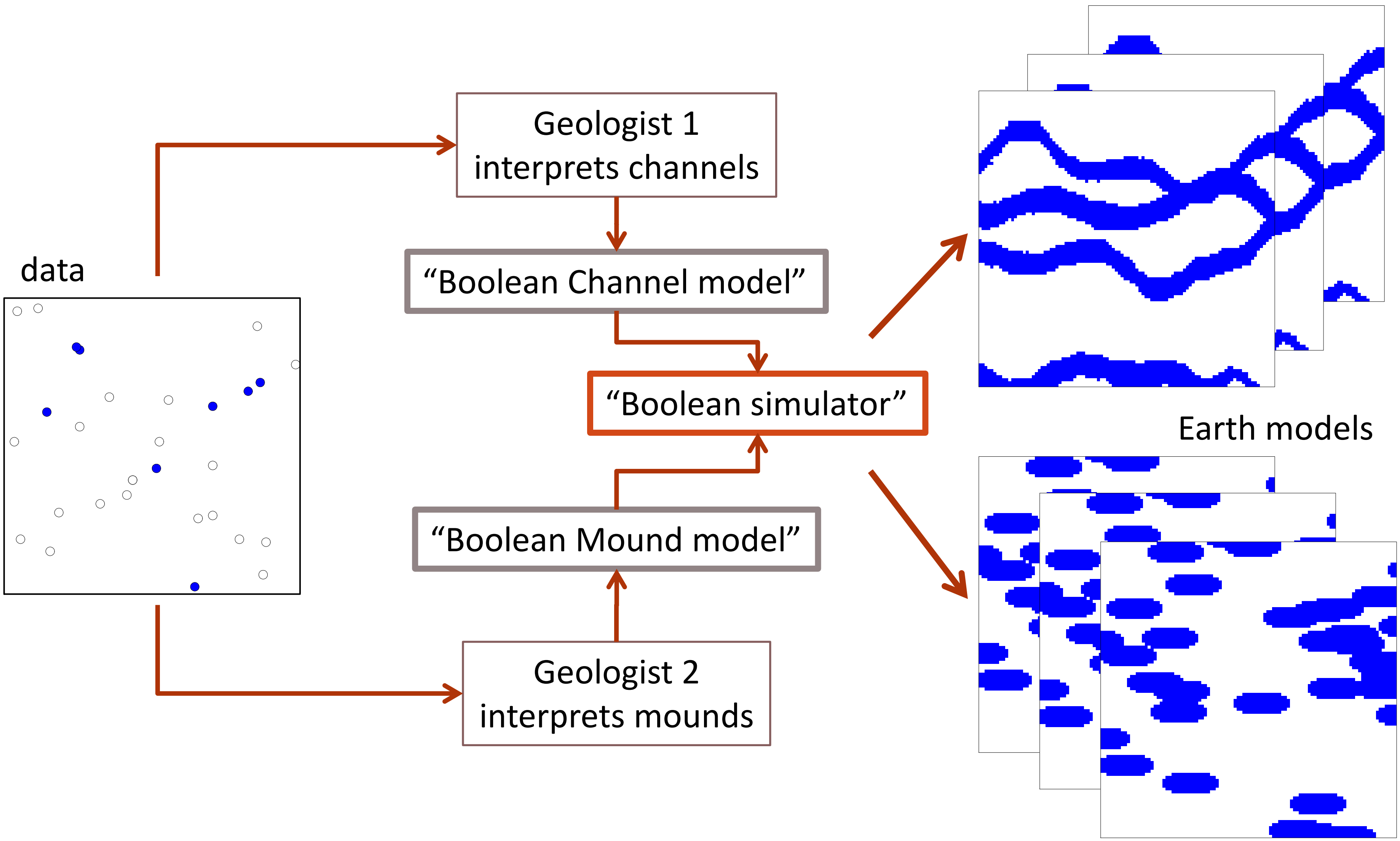






How does one build a model that looks like what I think is there and constrained to data?

Why modeling spatial continuity?



Why modeling spatial continuity?

- A model allows “filtering” and “exporting” the spatial variation seen in the dataset
- Allows building “Earth models” with similar spatial variation, but possibly constrained to data
- Allows randomizing the spatial variation and represent “spatial uncertainty”

Most common type models

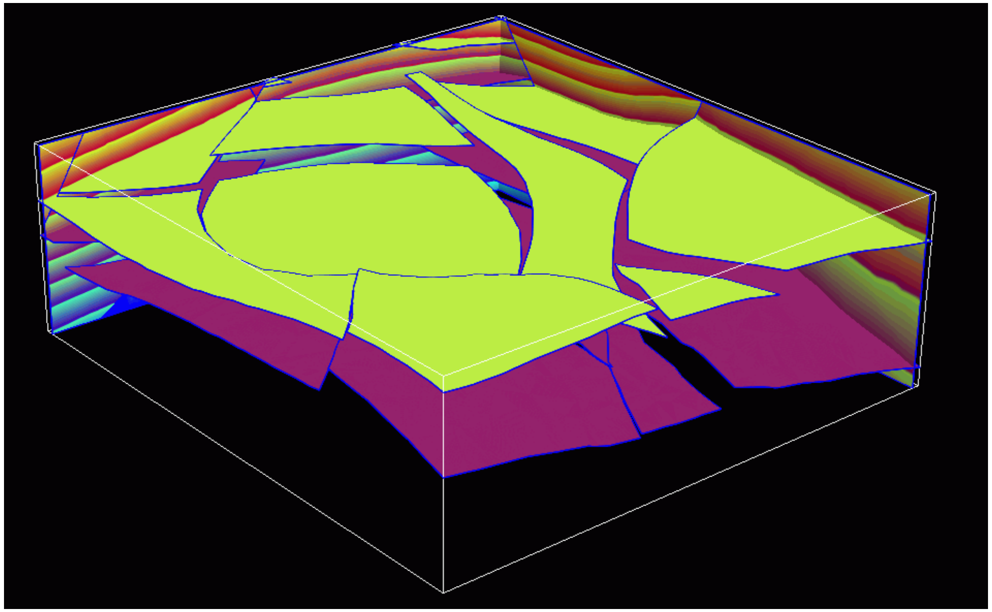
- Variogram-based models
  - Simple, few parameters
  - Limited modeling capabilities

- Boolean (or object-based) models
  - More realistic
  - Difficult to constrain

- Training image-based models
  - Realistic
  - Easy to constrain

Limitations of these methods

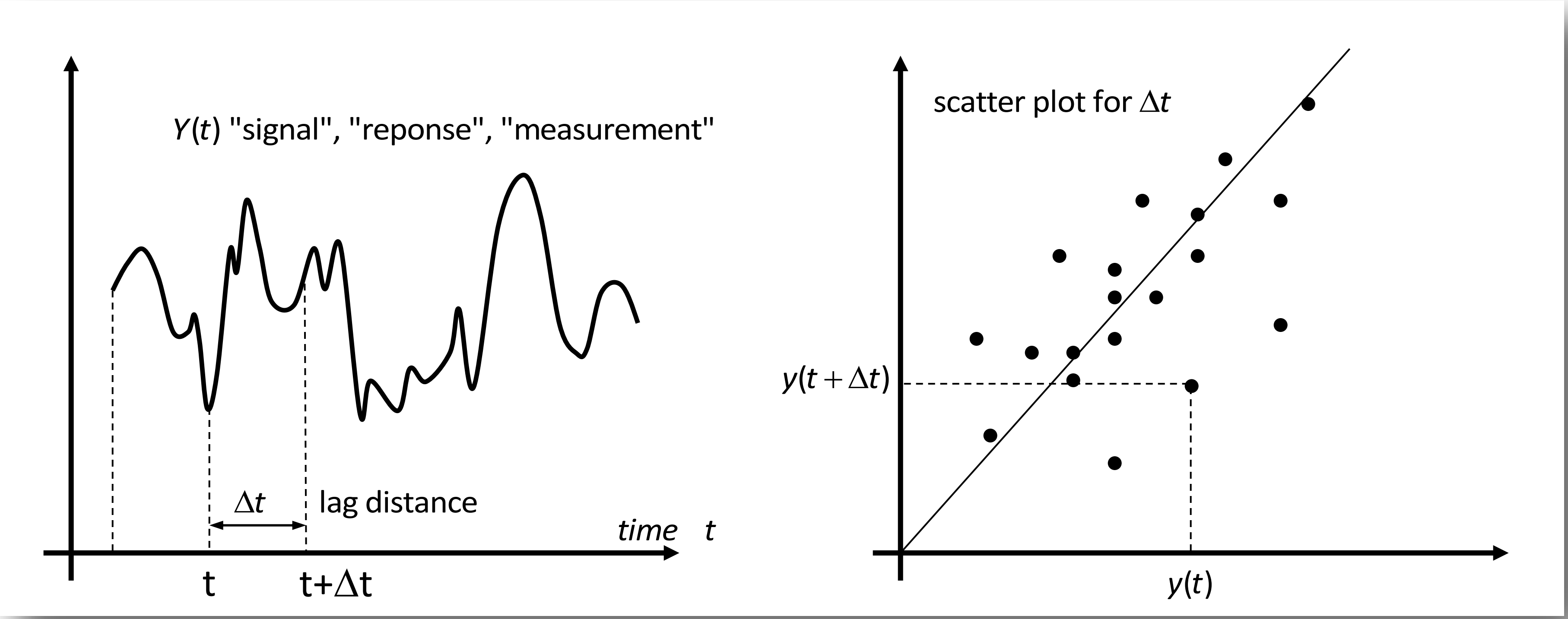
Not applicable to modeling “structures”



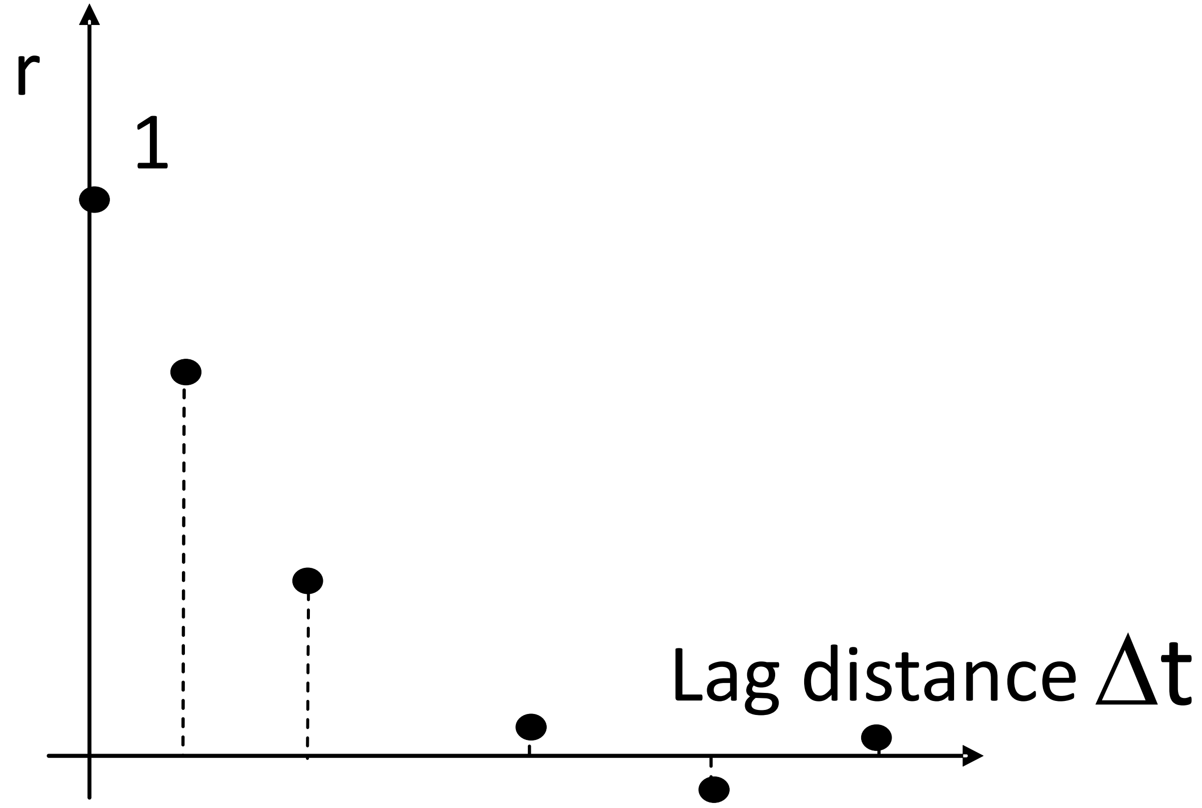
The variogram

Modeling spatial continuity

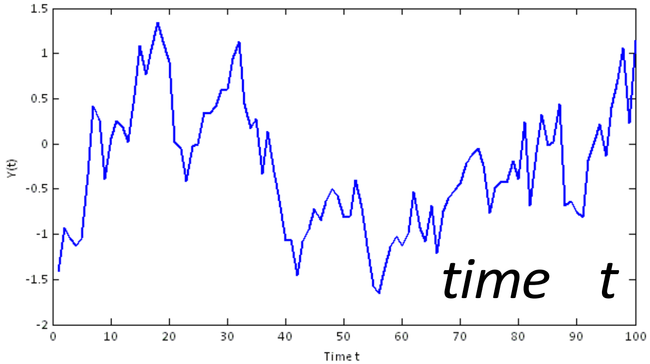
Autocorrelation



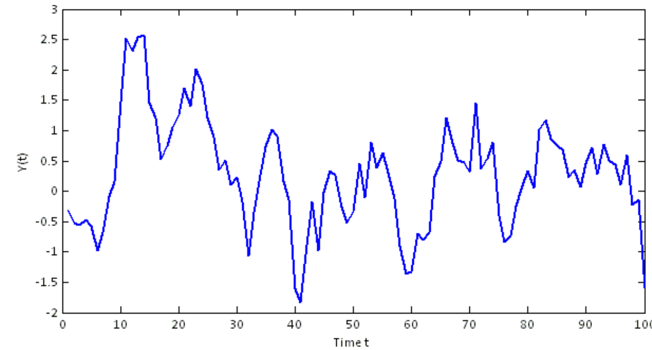
Autocorrelation



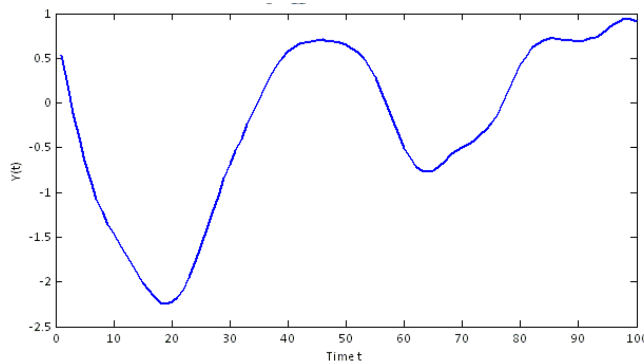
Case 1



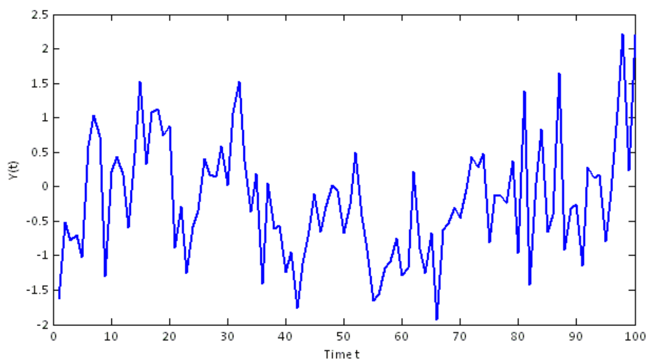
Case 2



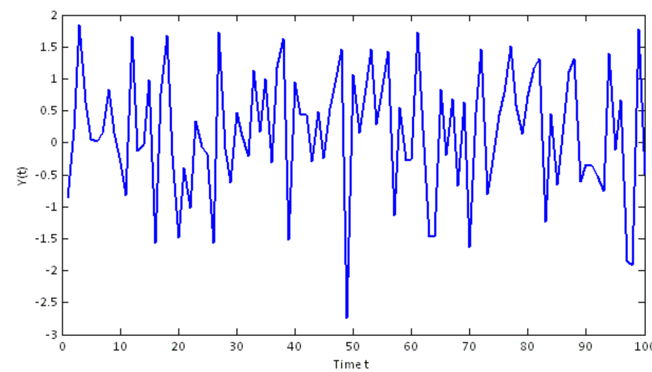
Case 3

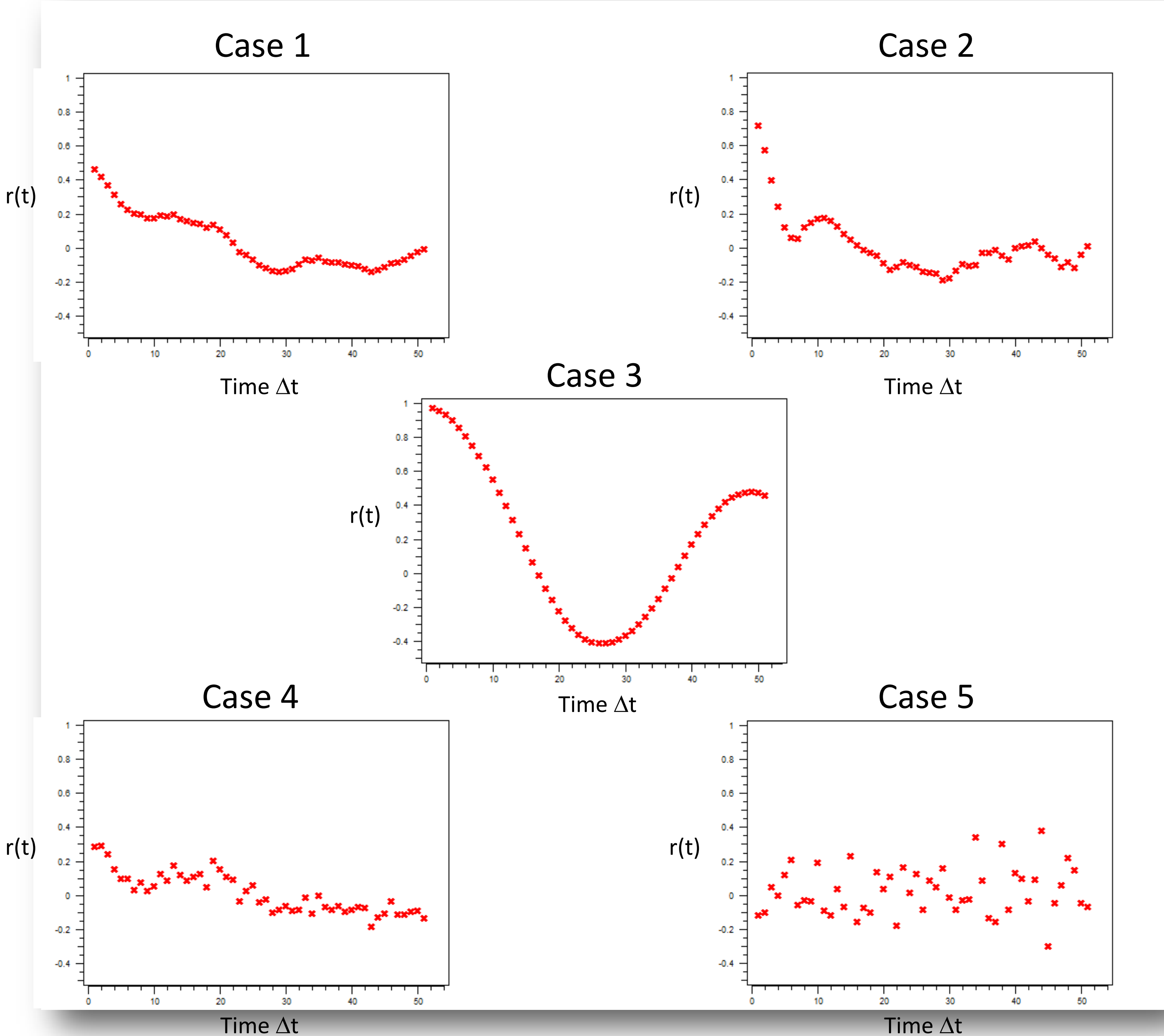


Case 4

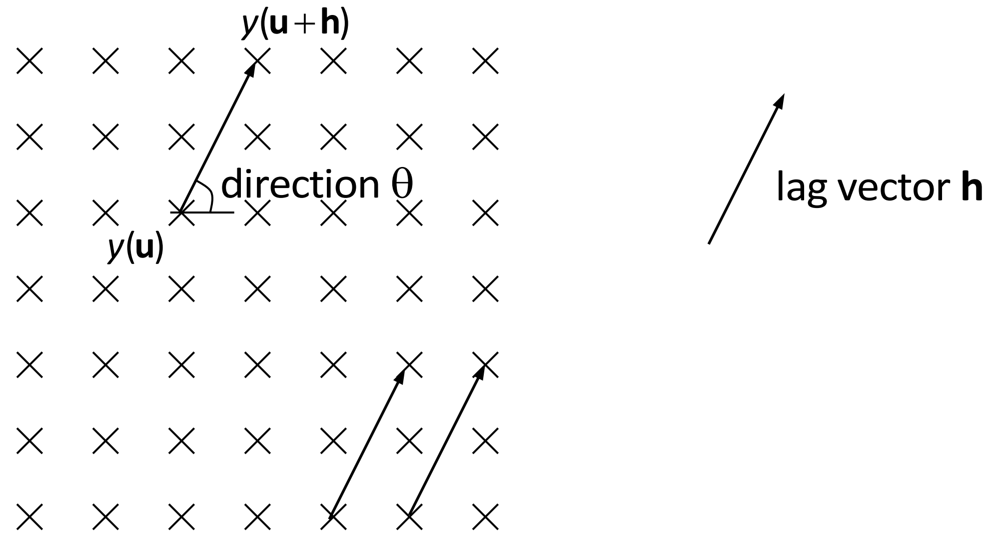


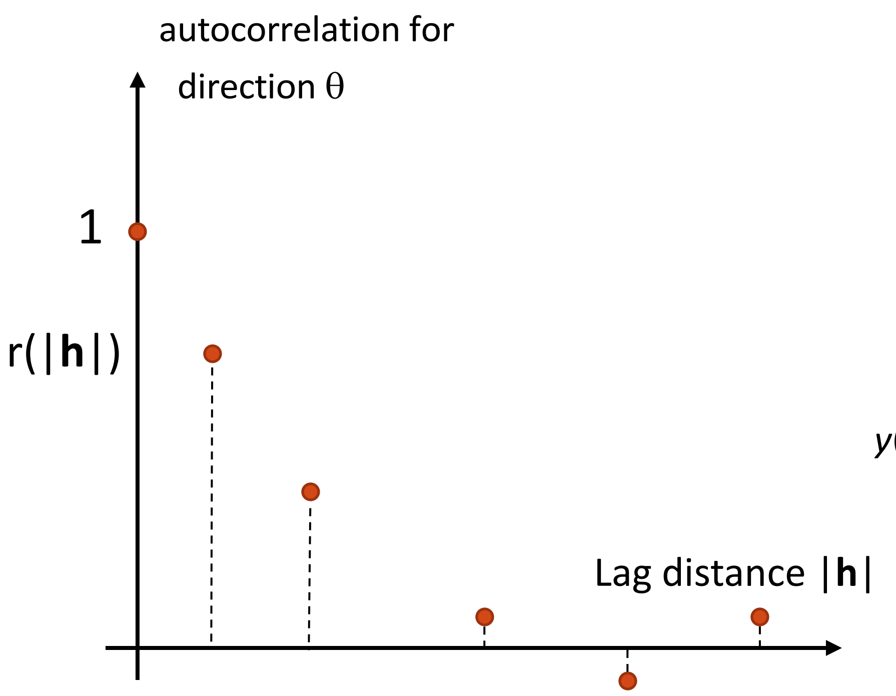
Case 5

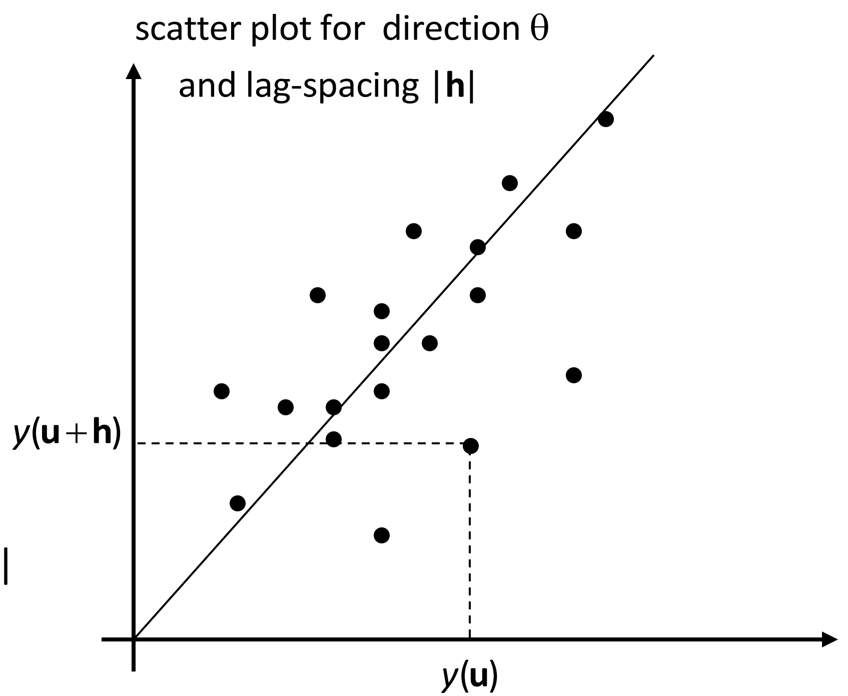



Case 1
r(t)
Time Δt
Case 2
r(t)
Time Δt
Case 3
r(t)
Time Δt
Case 4
r(t)
Time Δt
Case 5
r(t)
Time Δt

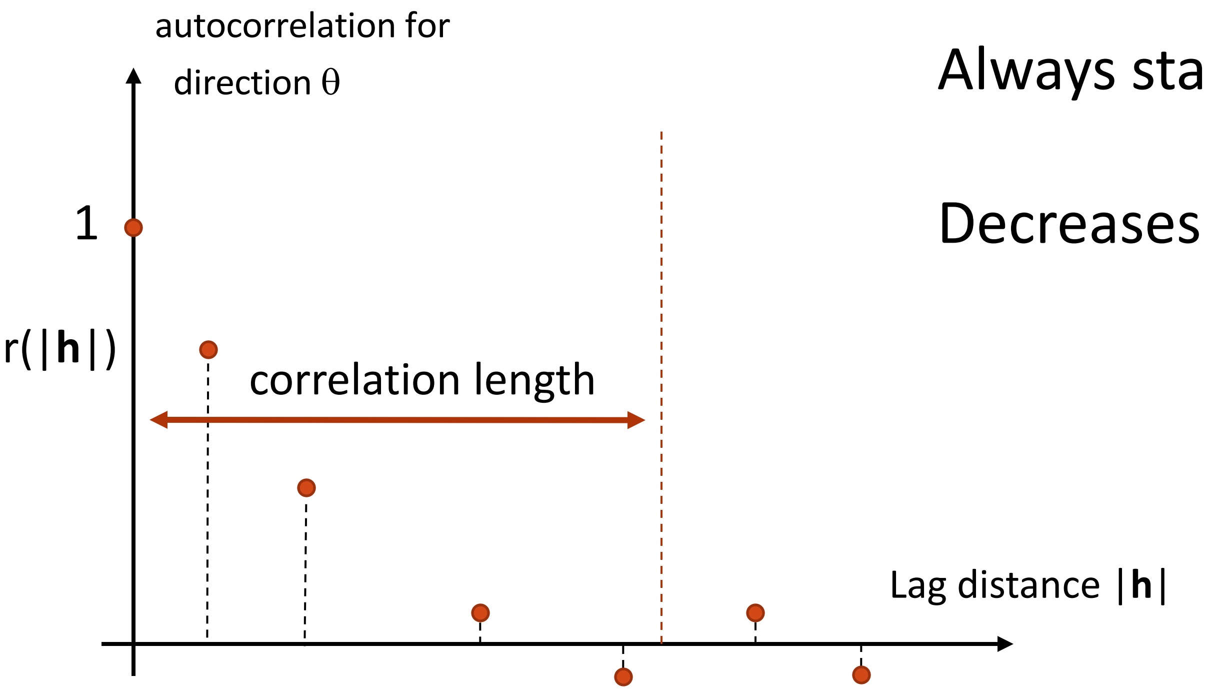
Autocorrelation in 2D







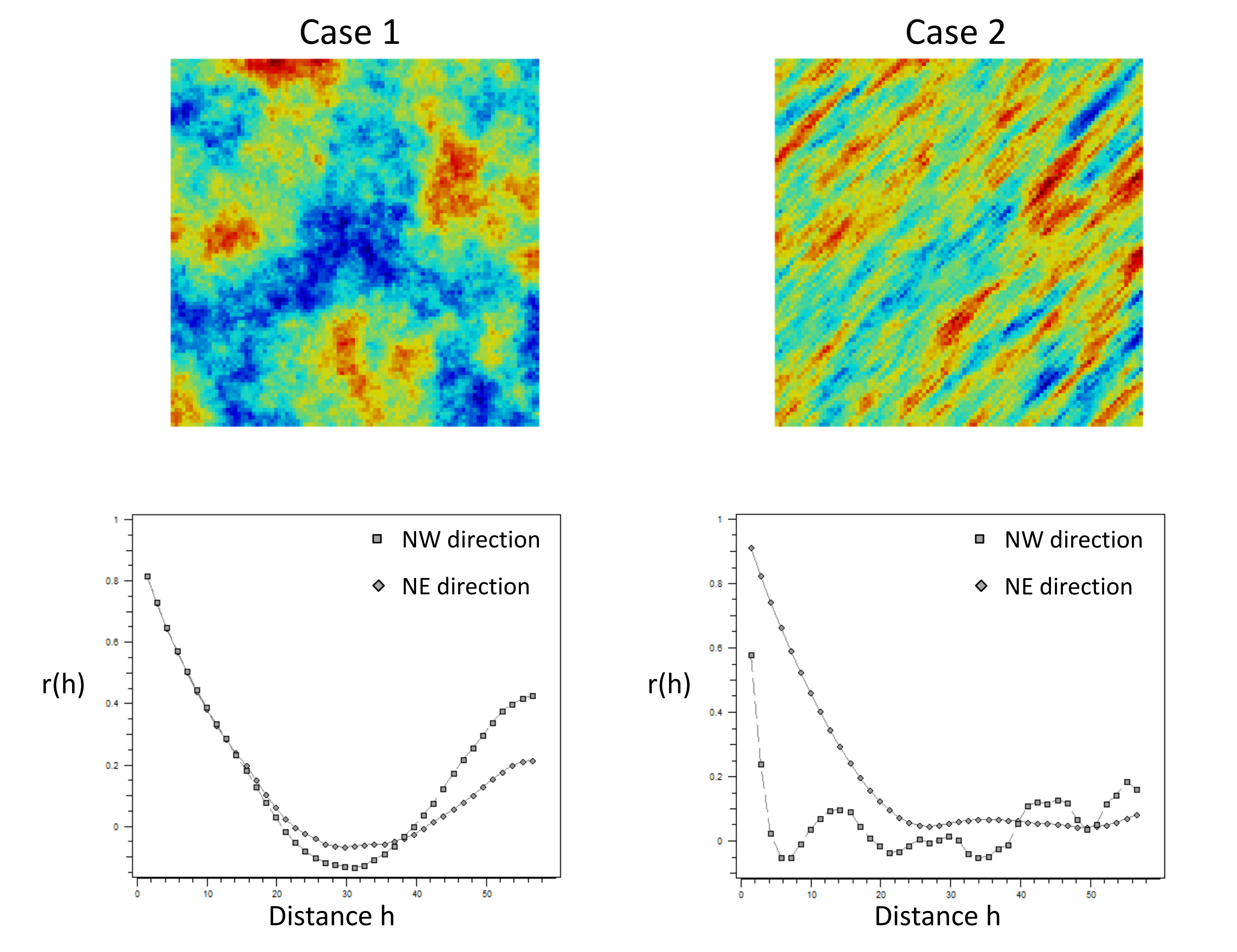
Properties of the correlogram



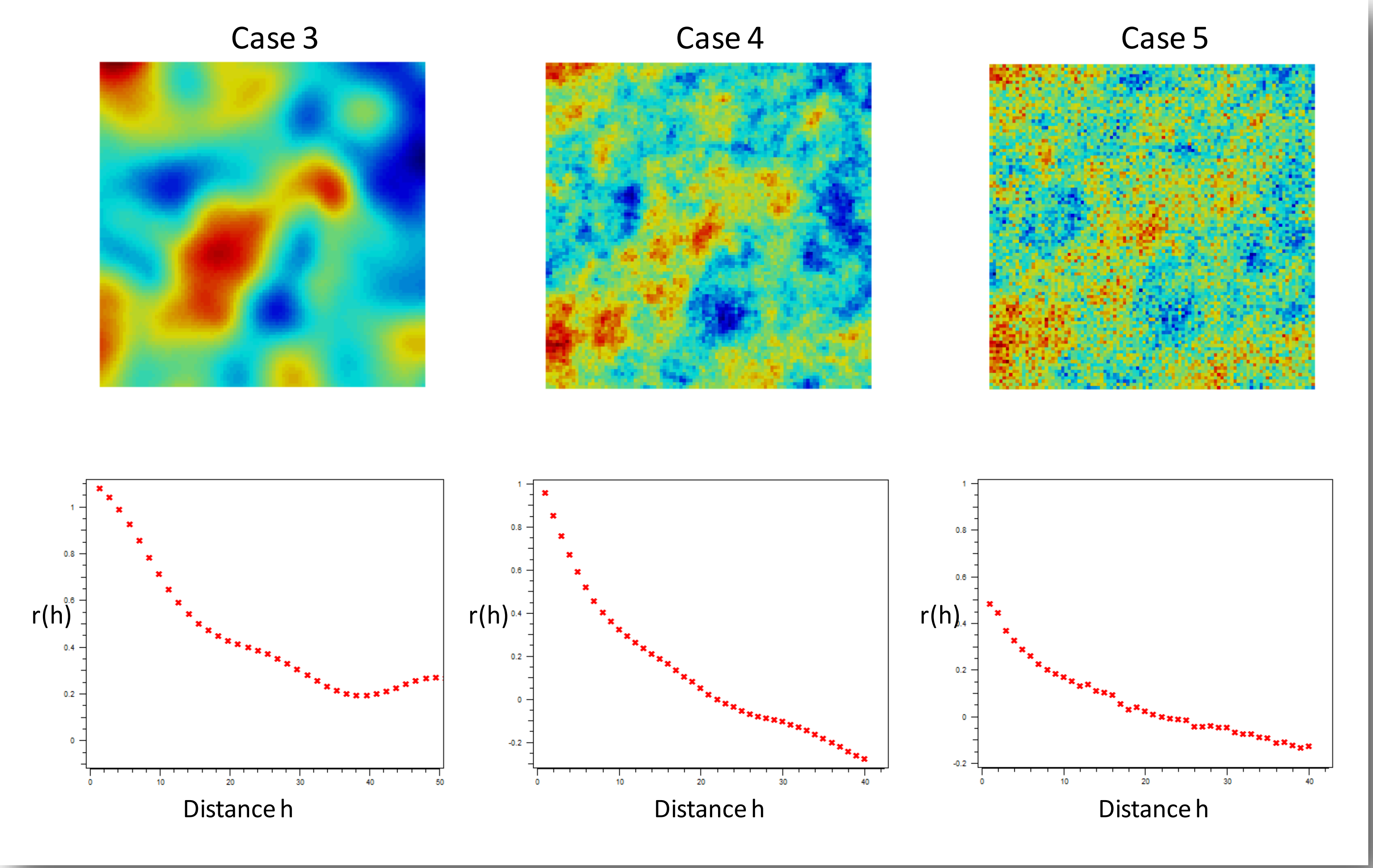
Always starts at 1

Decreases to zero

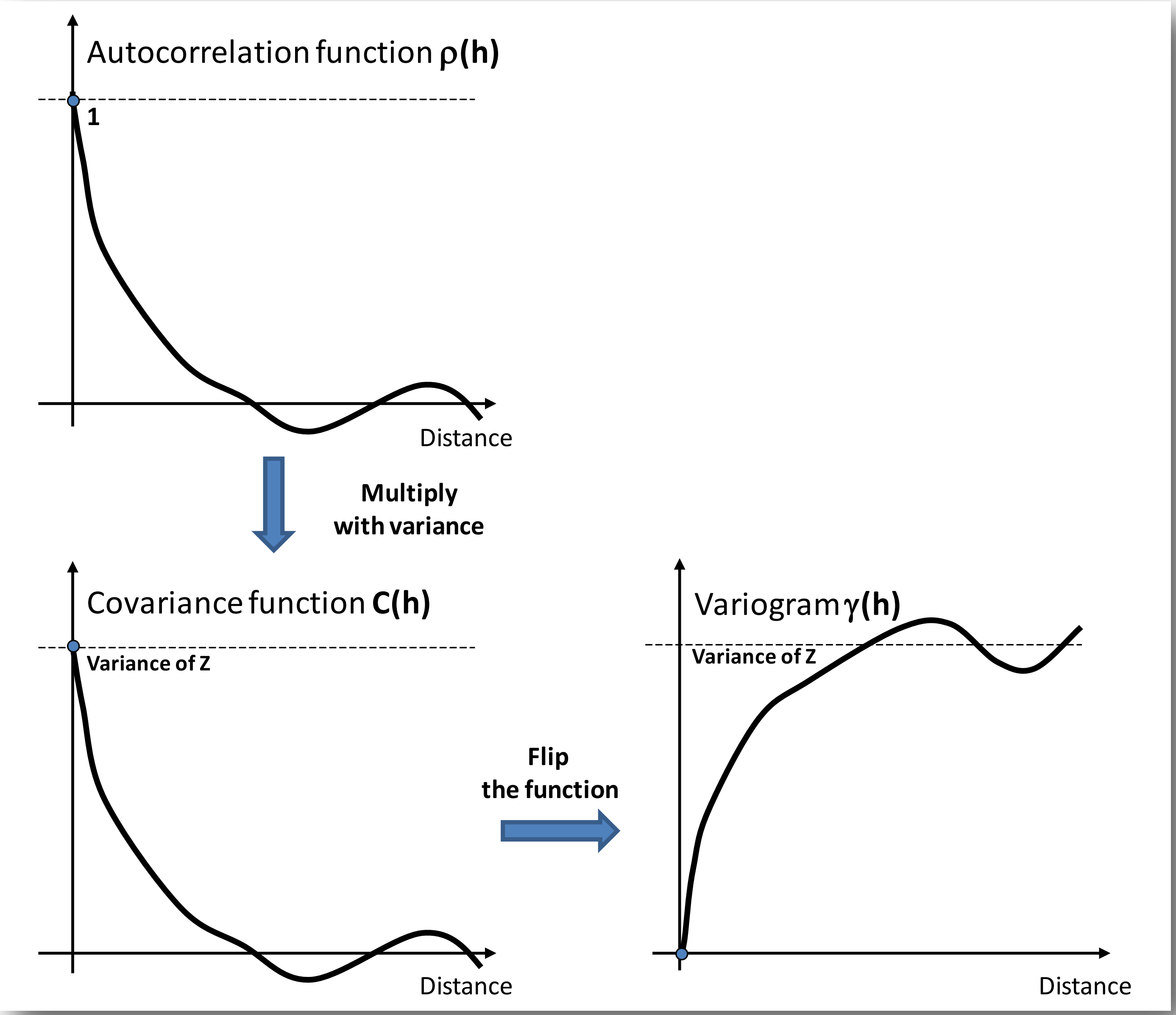
Examples



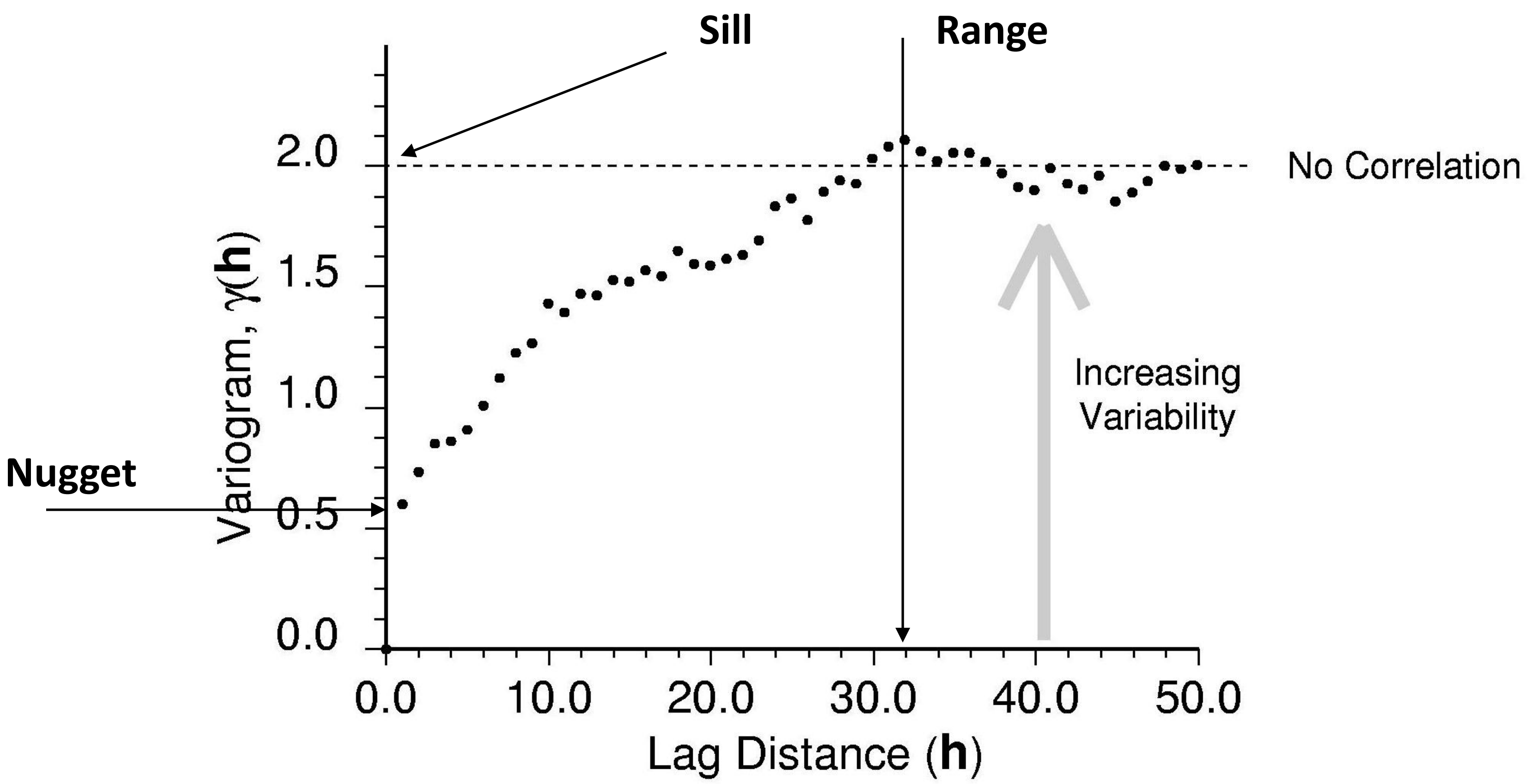
Examples



Other representations



Typical experimental (semi)-variogram



Summarizing variograms

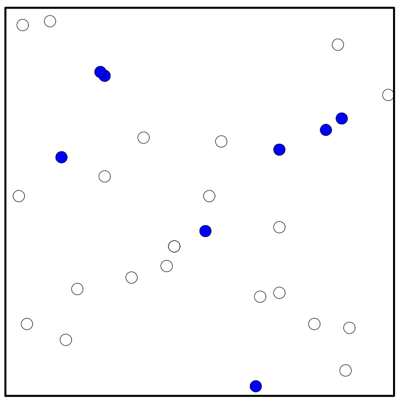
- What is the range and how does it vary with direction
- What is the nugget effect
- What is the behavior at the origin
- What is the sill value

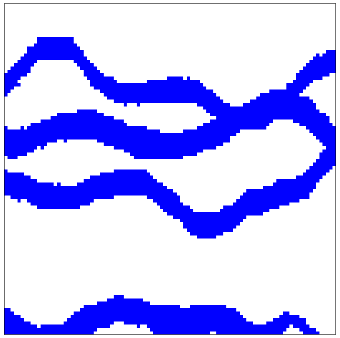
These four elements constitute a model,
i.e. you summarized a complex spatial variation
with a limited set of parameters

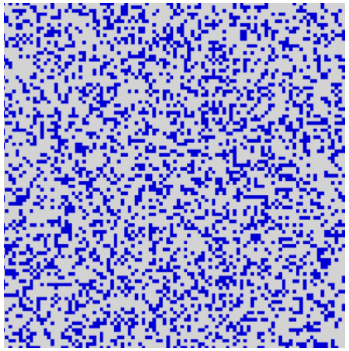
Why modeling spatial continuity?

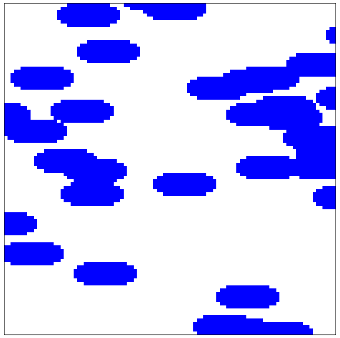
What is the spatial variation like ?

Data





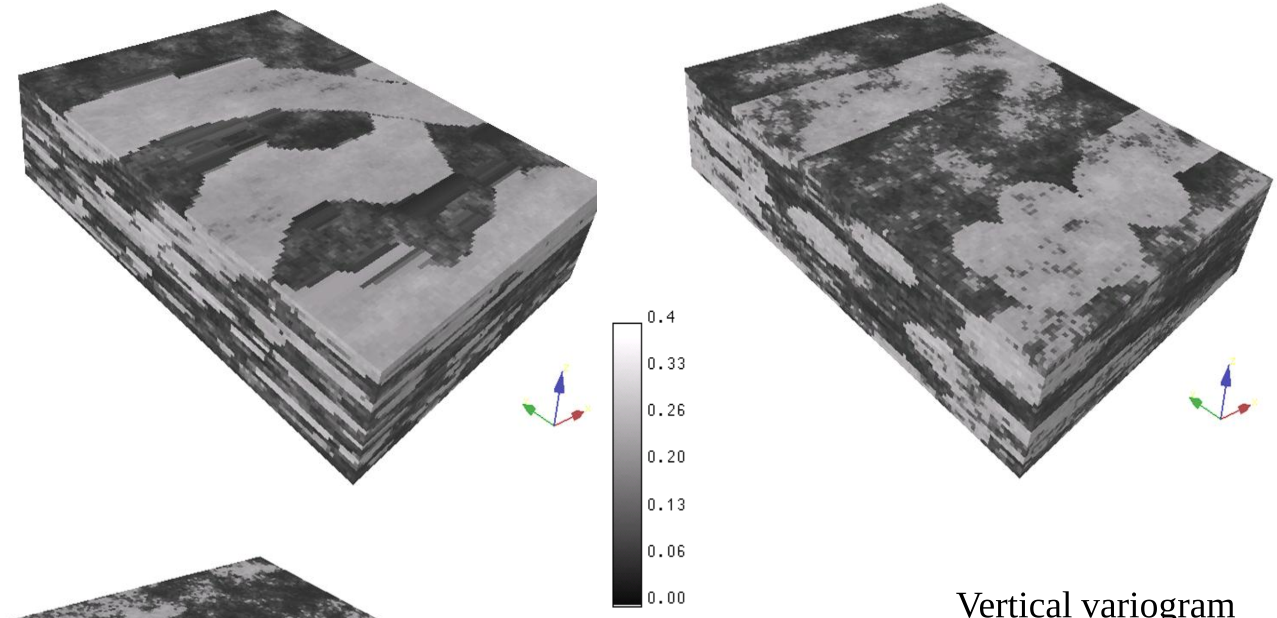


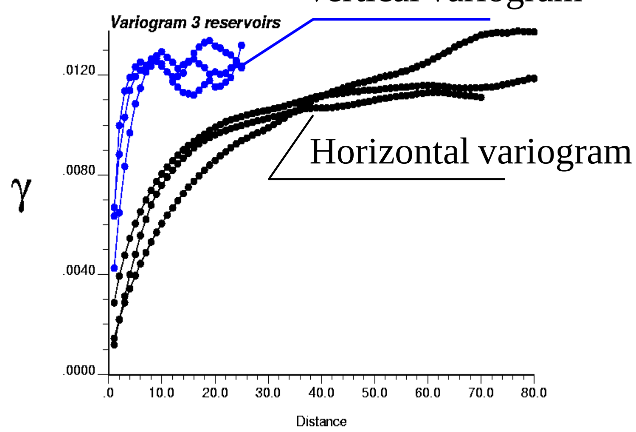


How does one build a model that looks like what I think is there and constrained to data?

Limitations of variograms

Variograms: modeling "homogeneous heterogeneity"
for modeling properties within major layers or facies

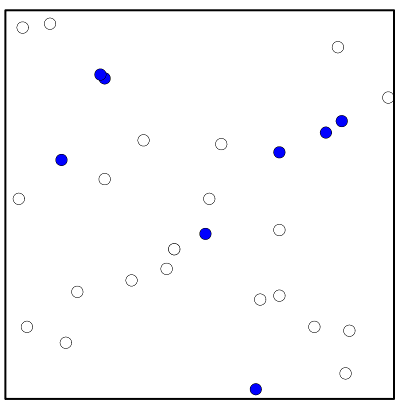


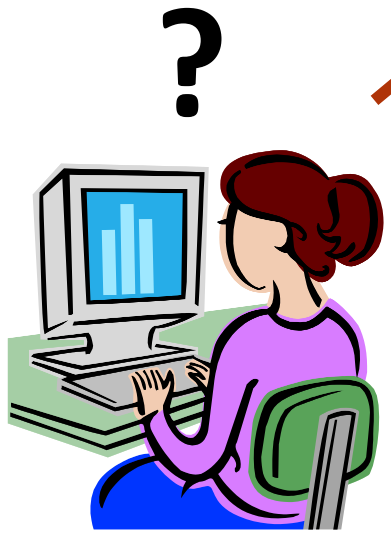


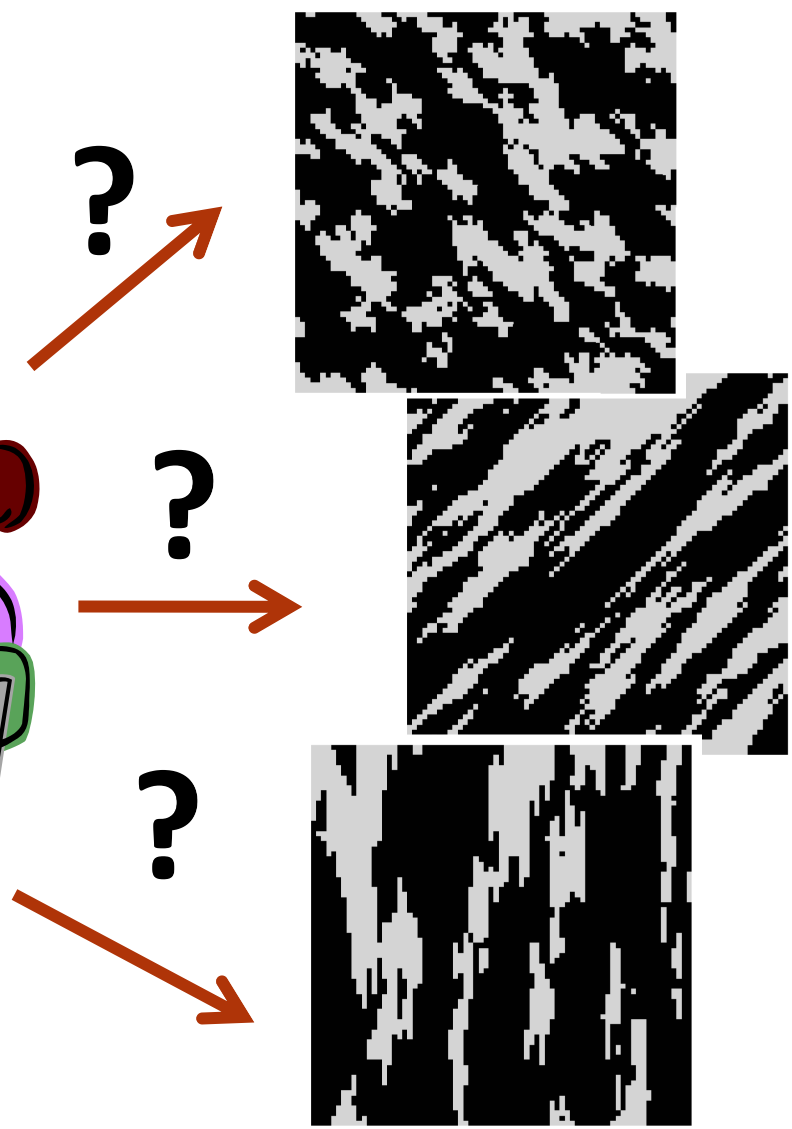
Limitations of the variogram

What is the
spatial variation like ?

Data







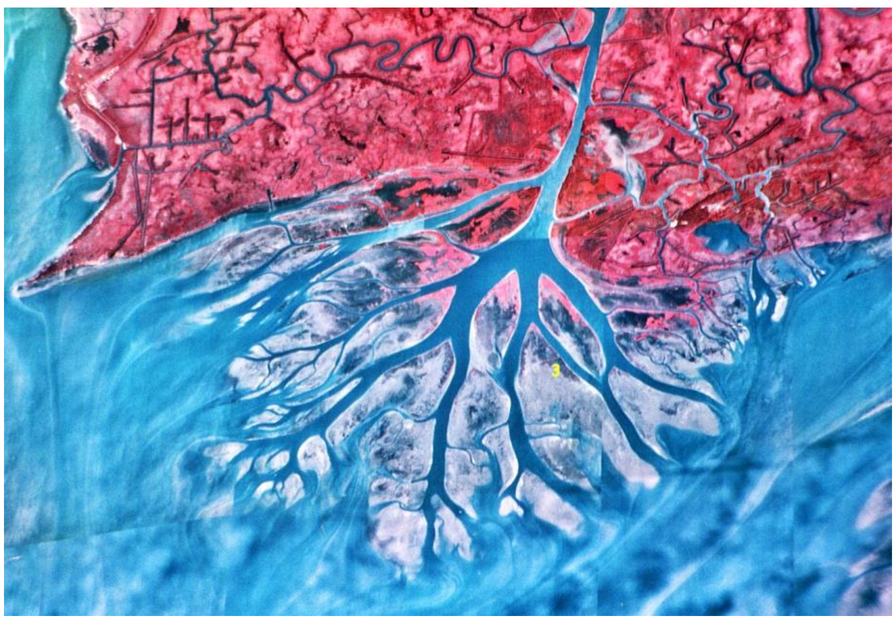
How does one build a model
that looks like what I think is there
and constrained to data?

What does the Earth really look like?

Tidal sand bars

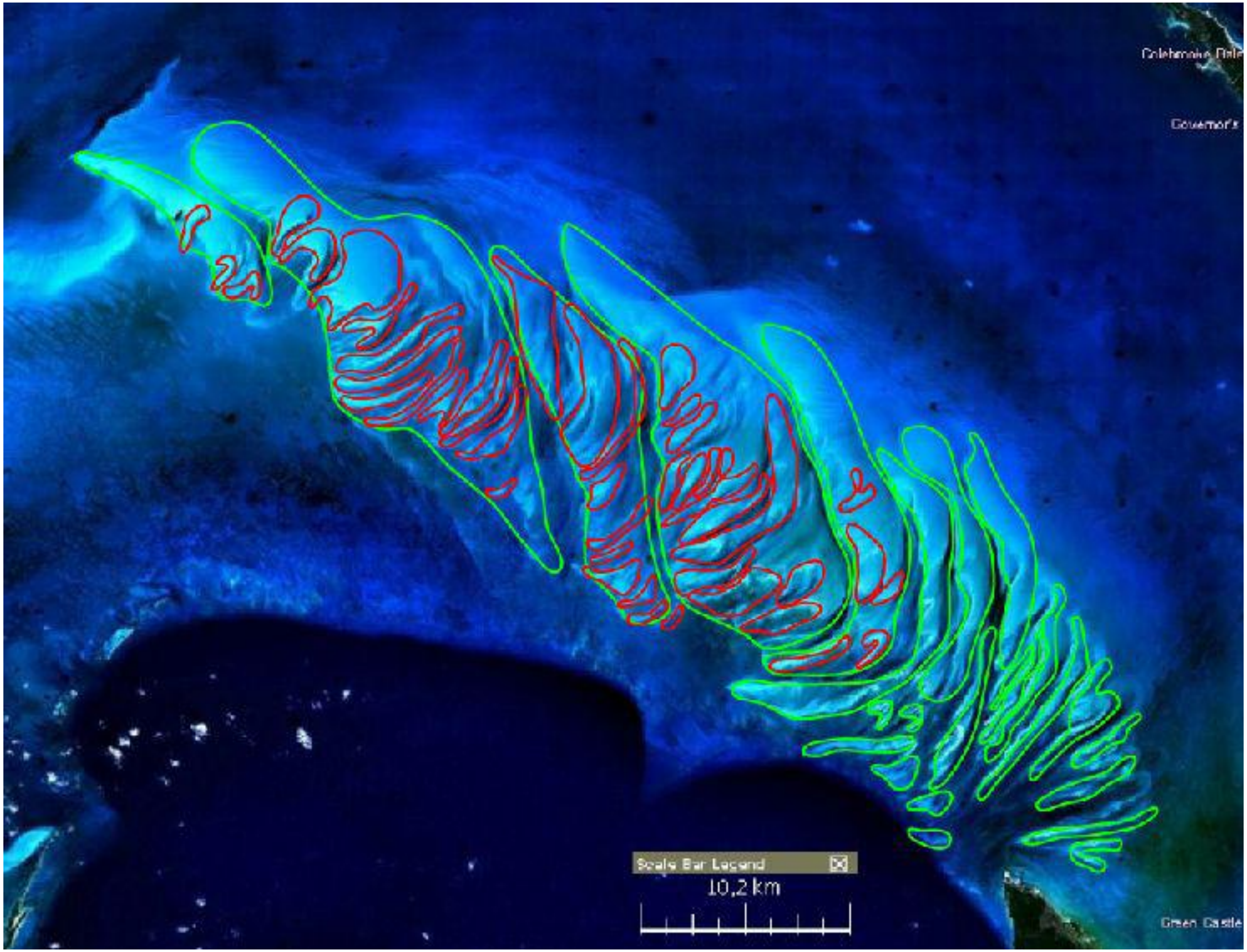
Meandering rivers

Deltas



Craters

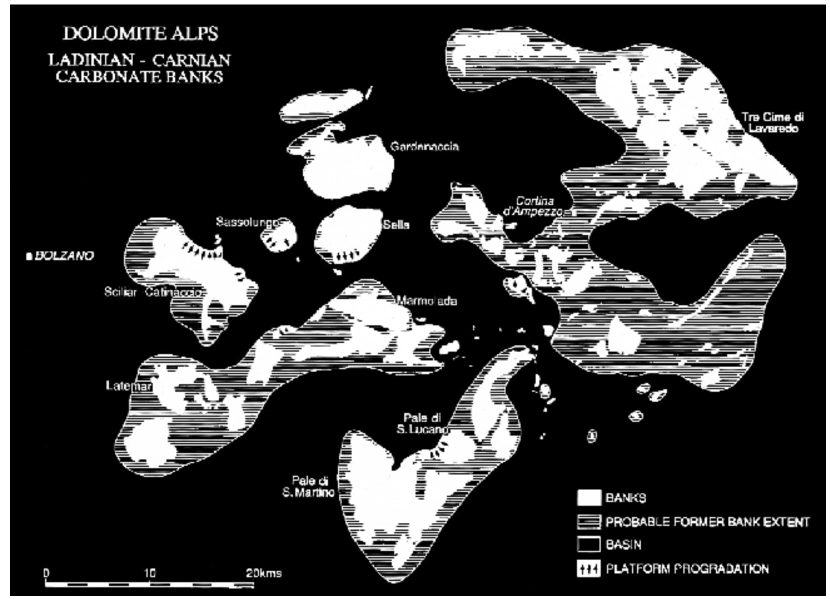
Carbonate Reefs (today)



Carbonate Mounds (paleo)

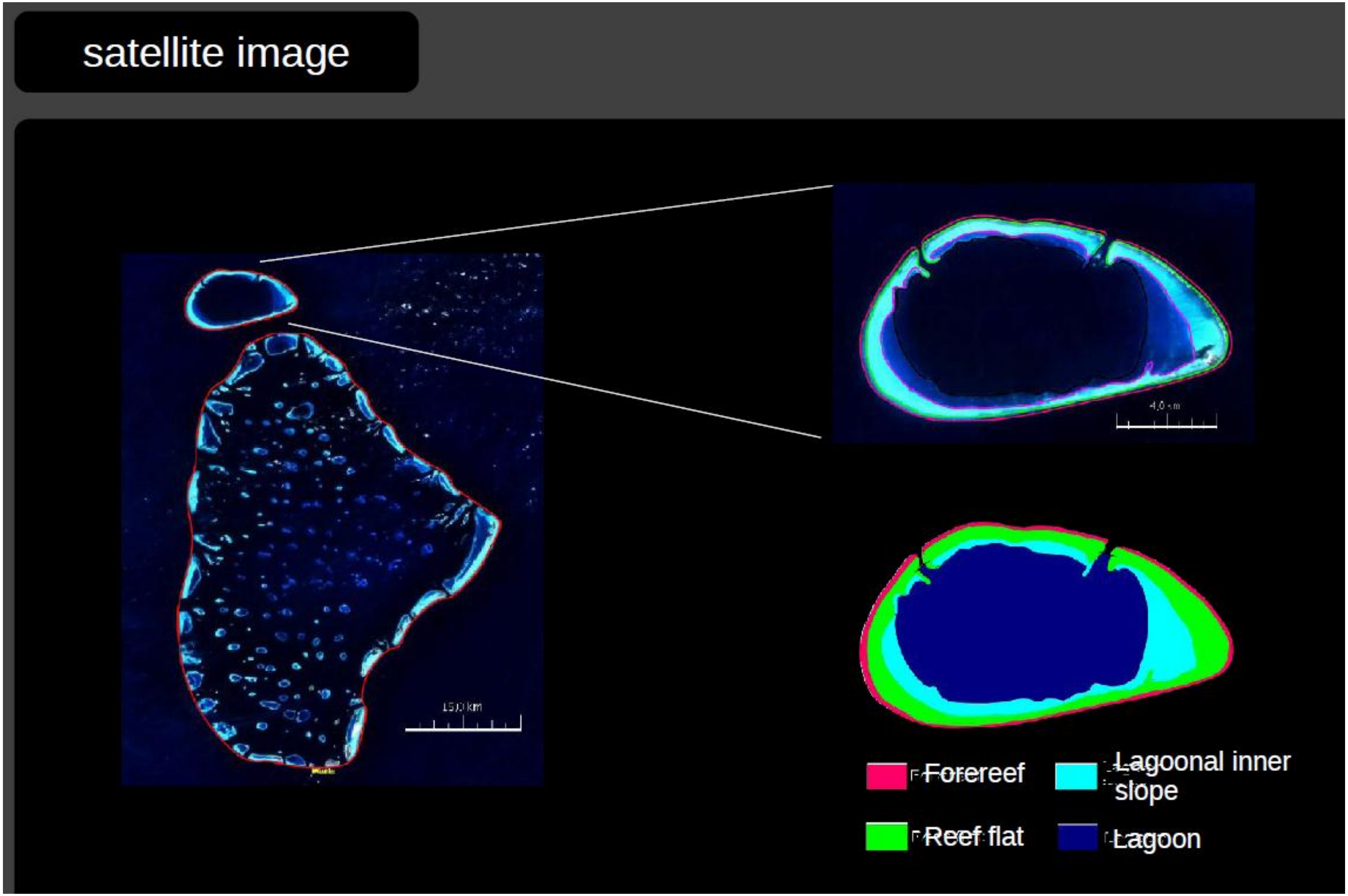
Atol (today)

Atol (paleo)



Spatial distribution of Atols

Inner architecture of an Atol

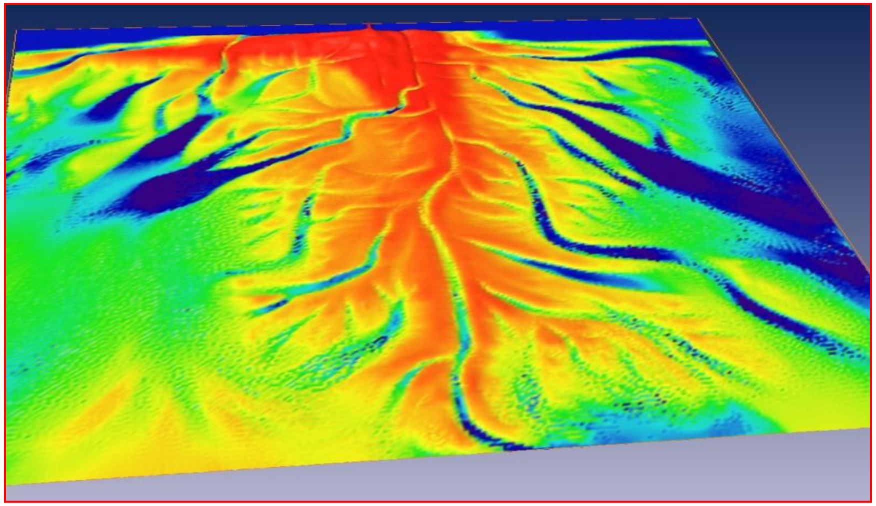


How to create Earth models that represent this observation ?

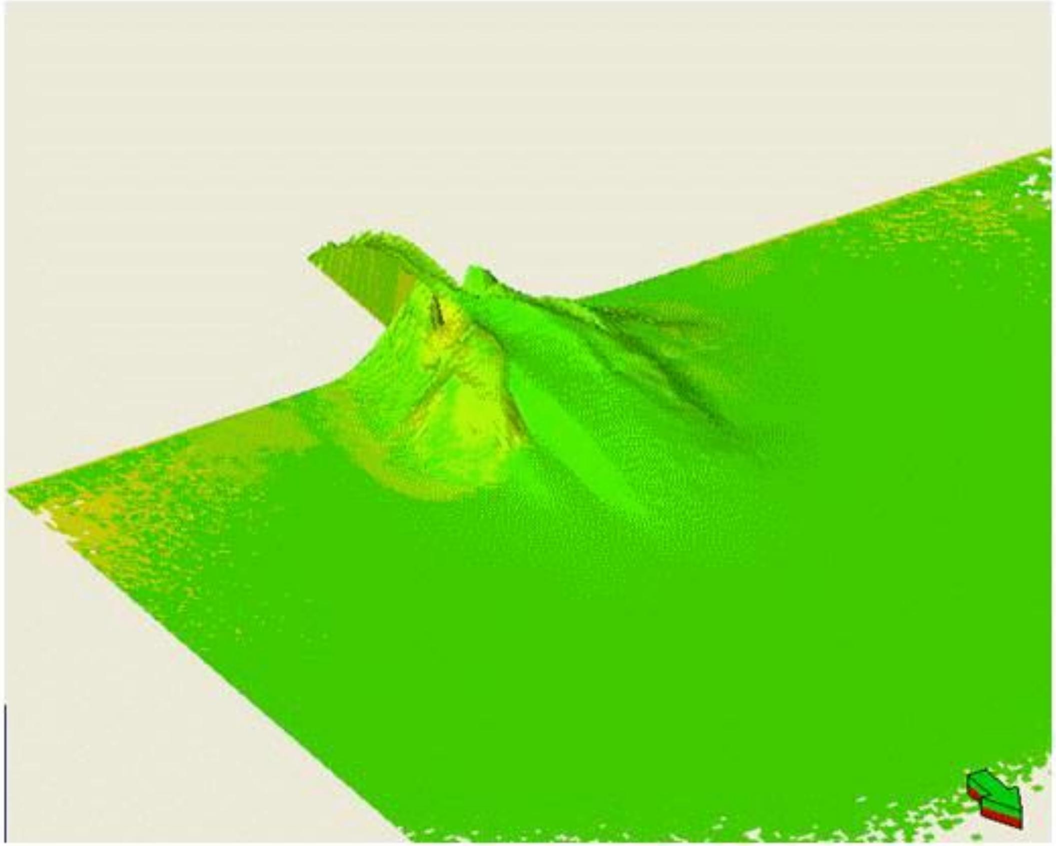
"Simulate" the physical processes of deposition on a computer

Observed

Simulated



Physical-process models

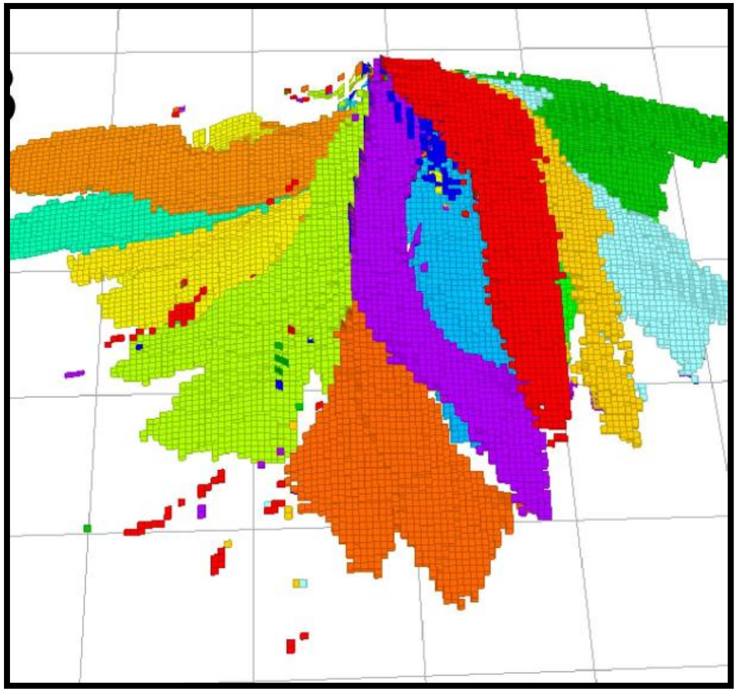


Physical process models

- Take weeks to run on a computer
- Results are deterministic: one computer run = one model => NO UNCERTAINTY

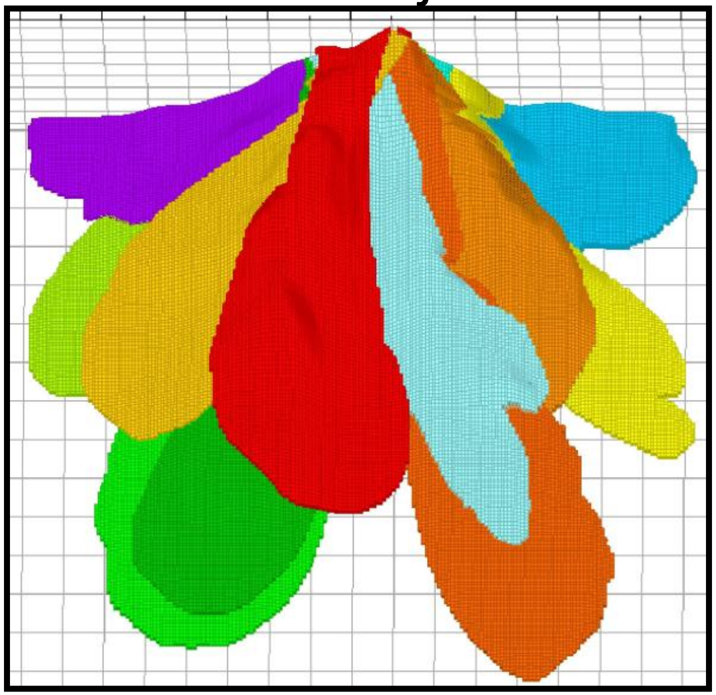
Idea: mimic the physical process with a "statistical process"

Process model



Weeks

Boolean or object model



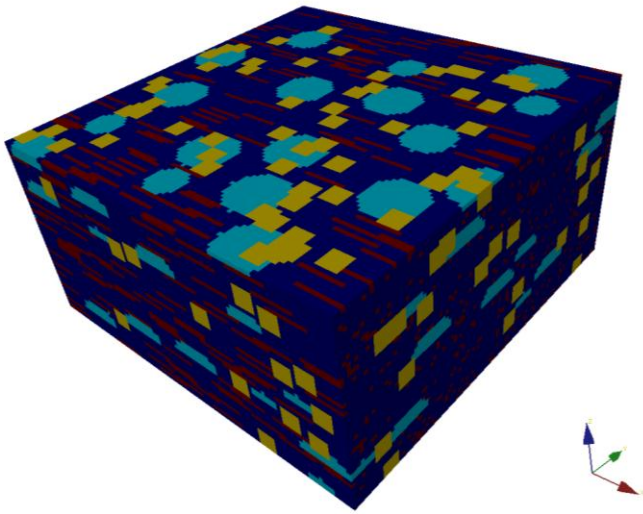
Seconds

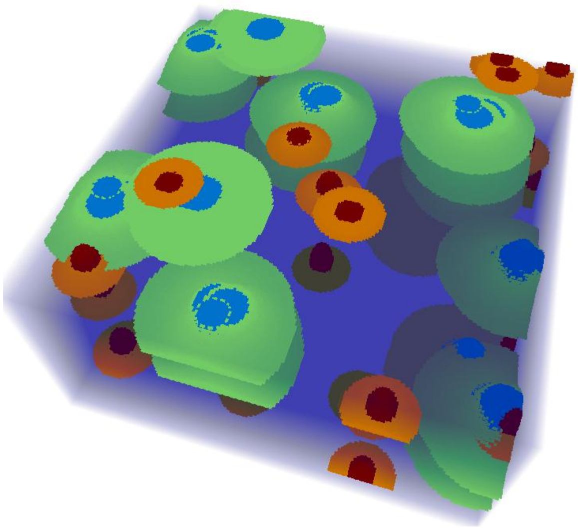
(Quantifying uncertainty is possible)

The object-based or Boolean model

Modeling spatial continuity

Object (Boolean) model



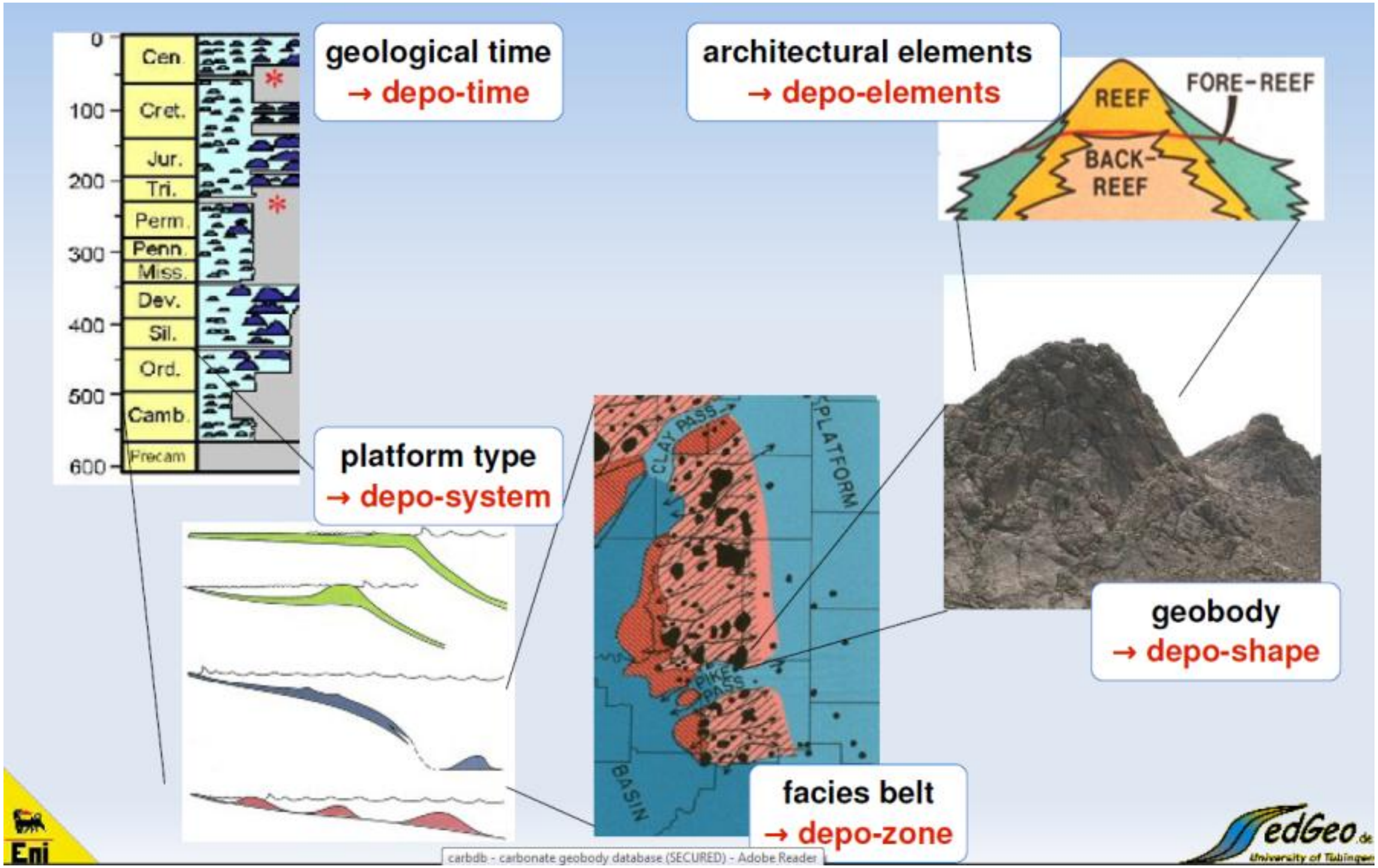


Define spatial variation as a set of objects, each type of object defined using a limited set of parameters

Define spatial placement of an object and interaction between objects

We can raster the objects on a grid

Building a Boolean model



Carbonate Geology

Hierarchy

- **Depo-Time**: era of deposition
- **Depo-system**: deepwater, fluvial, deltaic…
- **Depo-zones**: regions with similar depo-shapes
- **Depo-shapes**: basic geometries, geobodies
- **Depo-elements**: internal architectures
- **Depo-facies**: lithologies, associations

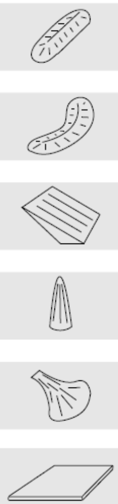
Constructing a Boolean model

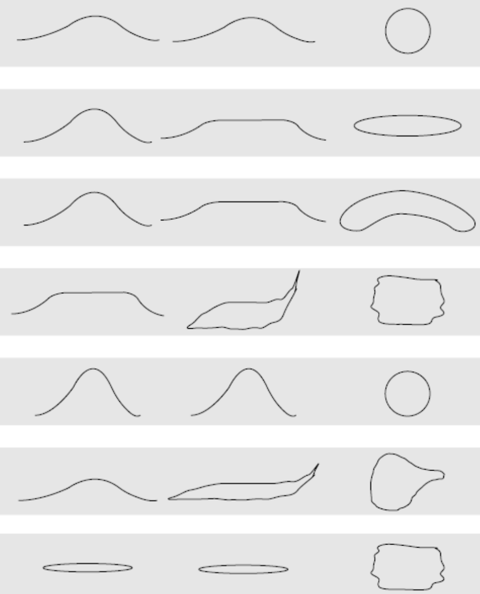
- Define a hierarchy of objects
- Define object geometry
- Define internal “architecture” of the object
- Define placement of object spatially
- Define interaction between objects

Geometries/dimensions

Example

type (class)	examples	x-section	y-section	z-section
mound	mud mounds, rudist mounds			
bars	shoals, barrier reefs, tidal-, channel bars			
bows	atoll, fringing reef, tidal-, channel bars			
wedges	reef debris, aprons, progradational wedges			
pinnacle	pinnacle reef, knob			
fans	fans, reef debris, spill-over lobes			
sheets	biostromes, tempestites, mud flats			





Internal parameters of the object

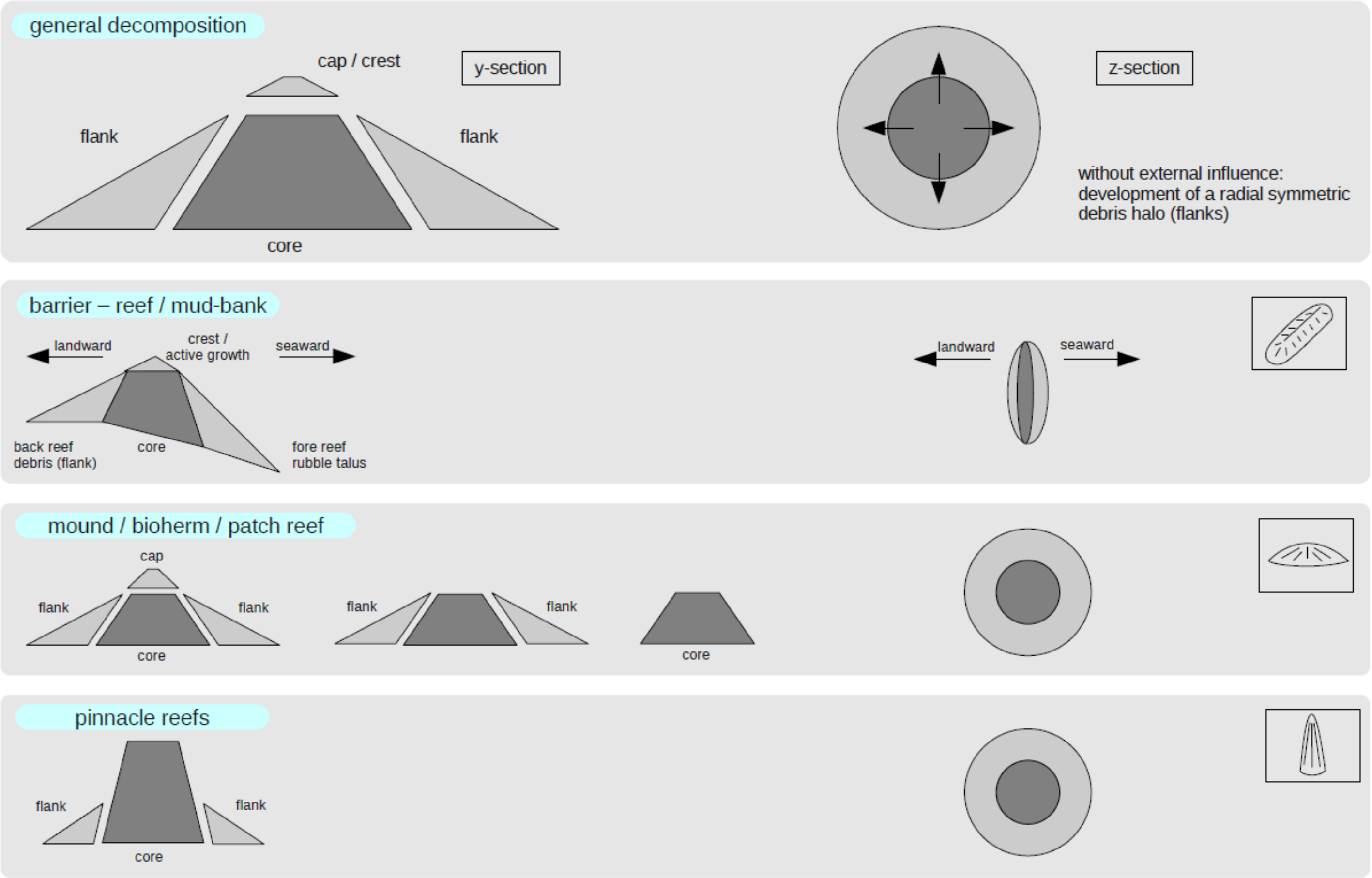
those parameters defining geometries (e.g. width, length, orientation)

External variables controlling the shape

spatial properties such as topography, water depth that control shape

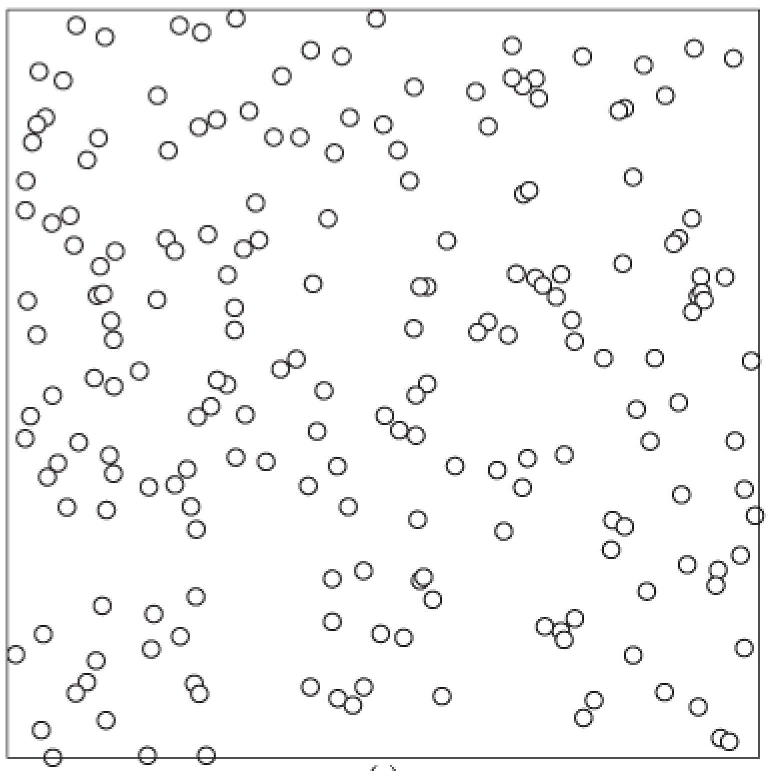
Architectural elements

<u>Example</u>



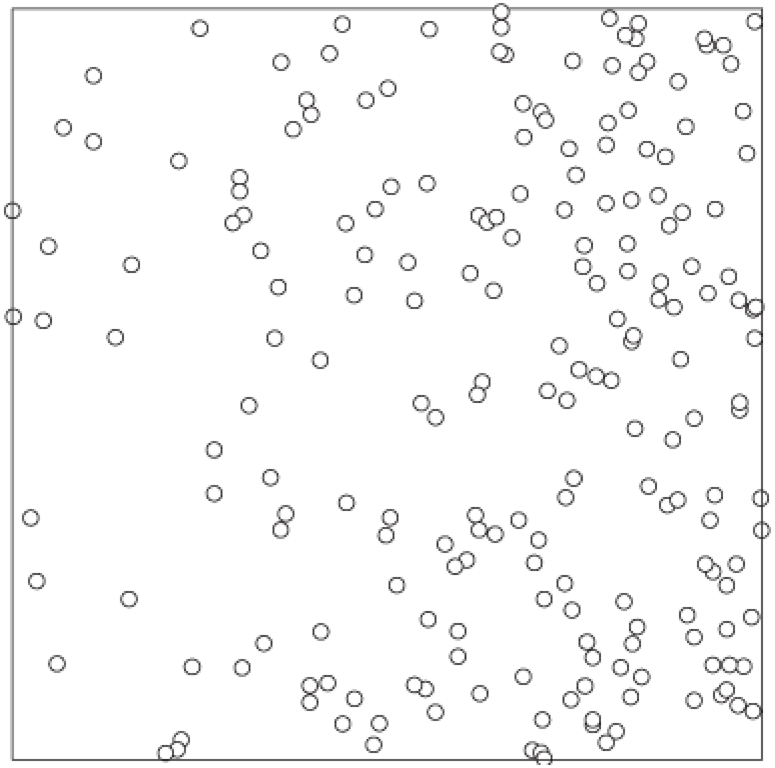
Spatial distribution

Most basic statistical process
= Poisson Process

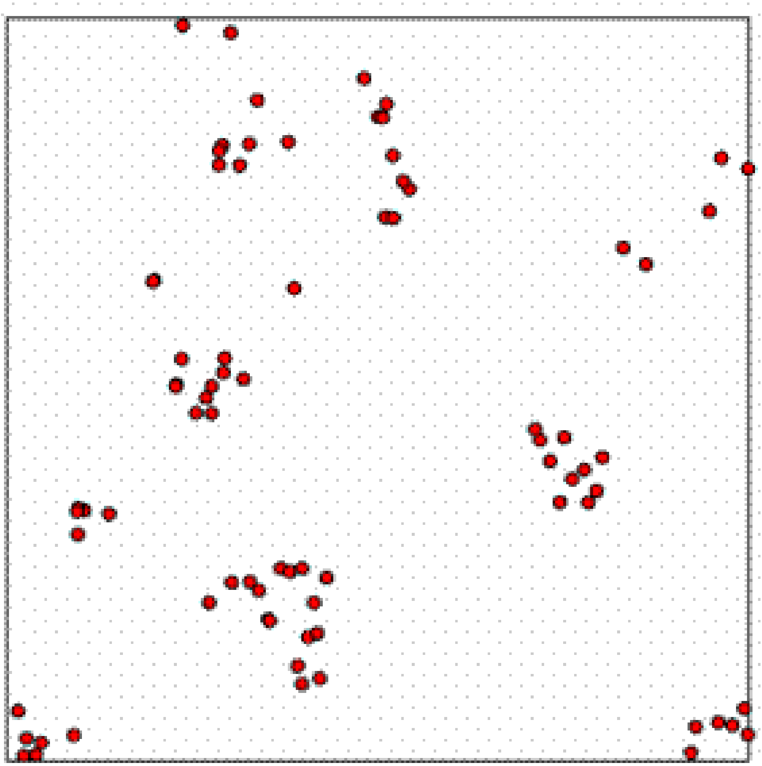


Extensions of Poisson

Poisson process with spatially varying intensity (density of points)



Cluster Process



Marked Poisson process

Each poisson point gets a "mark" which could be an object with varying size

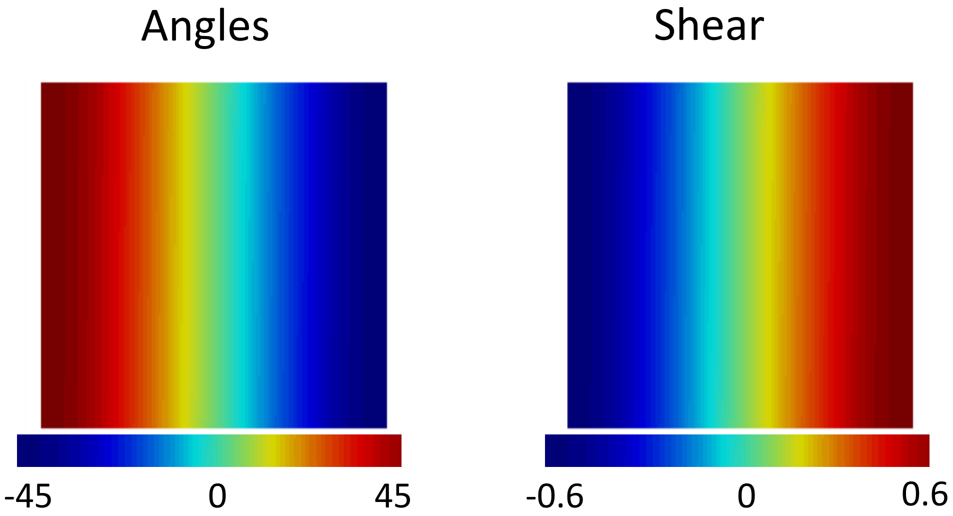
Rules

- Spatial distribution of depo-shapes: default = Poisson process
- Interaction between depo-shapes (overlap and erosion)
- Rules are parameterized with internal parameters(e.g. Poisson intensity) that may be controlled by external variables (e.g. topography)

Parameterization of the object

Every parameter can be defined as constant or a distribution

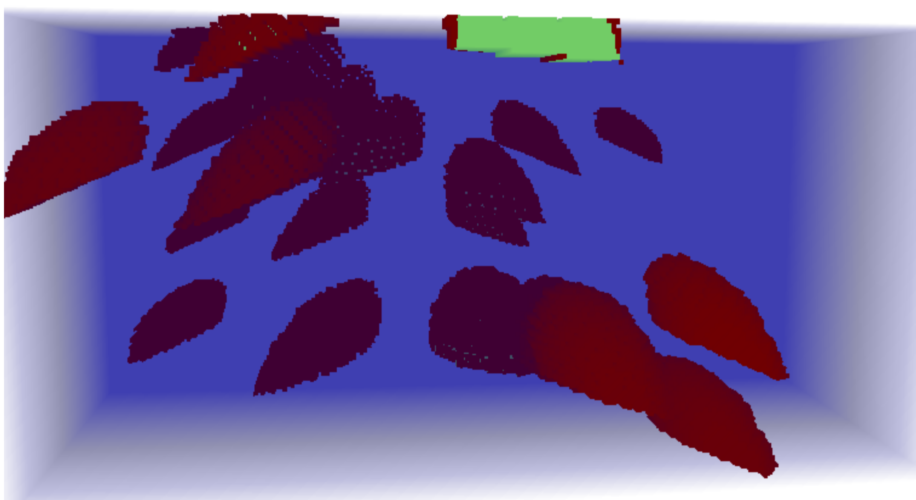
Parameter values can be either constant or following an intensity function (locally varying property)



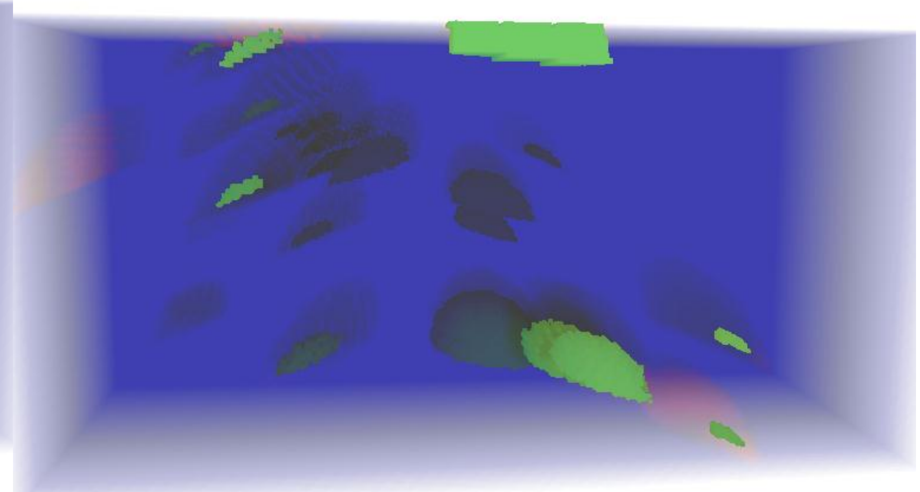
Carbonate mounds on a anticline

Increased shearing
Decreasing outer envelope
Rapidly decreasing core size

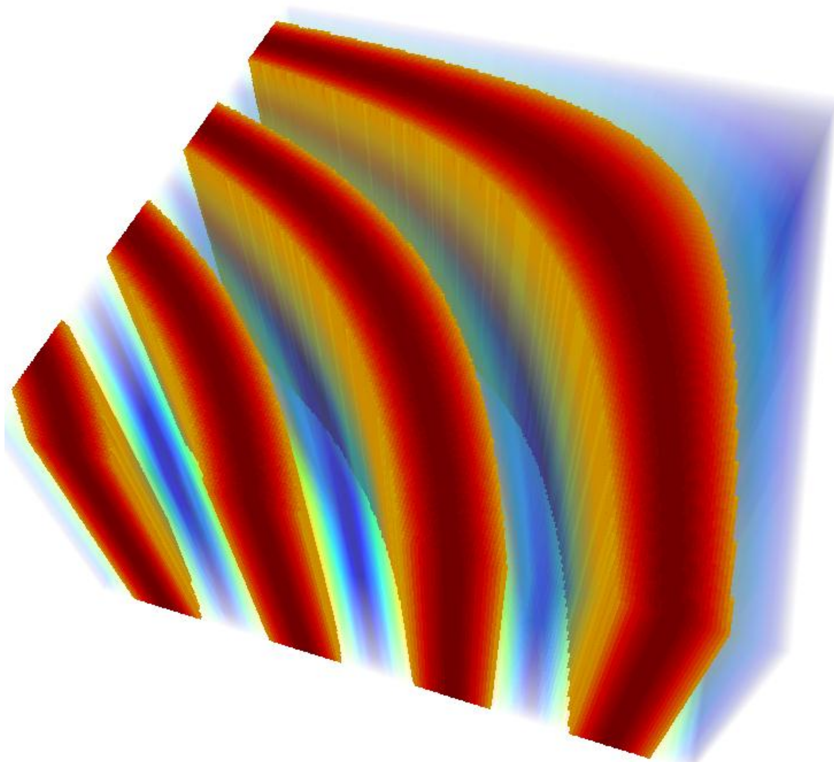
Mound



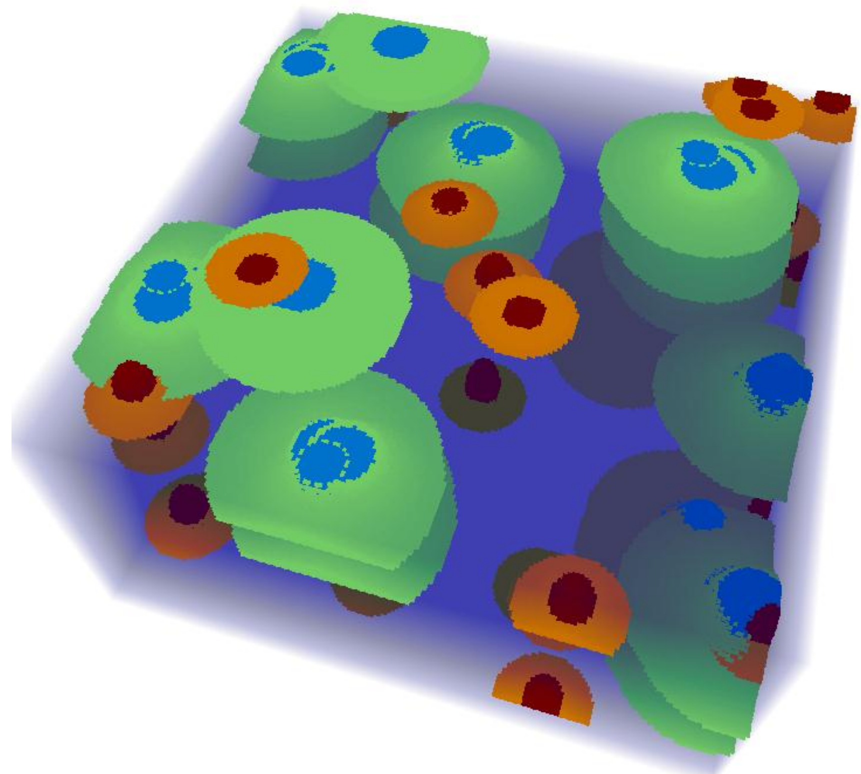
Inner part function of the slope



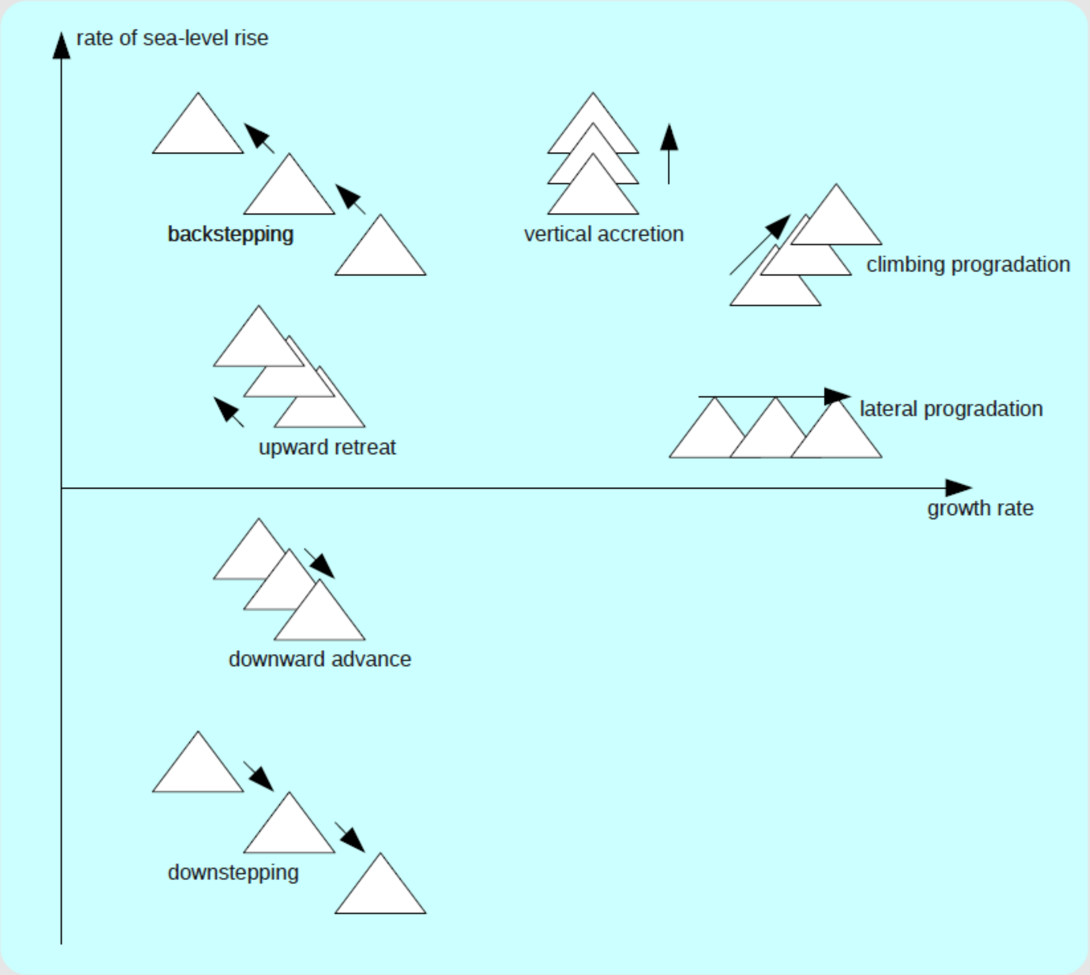
Positioning



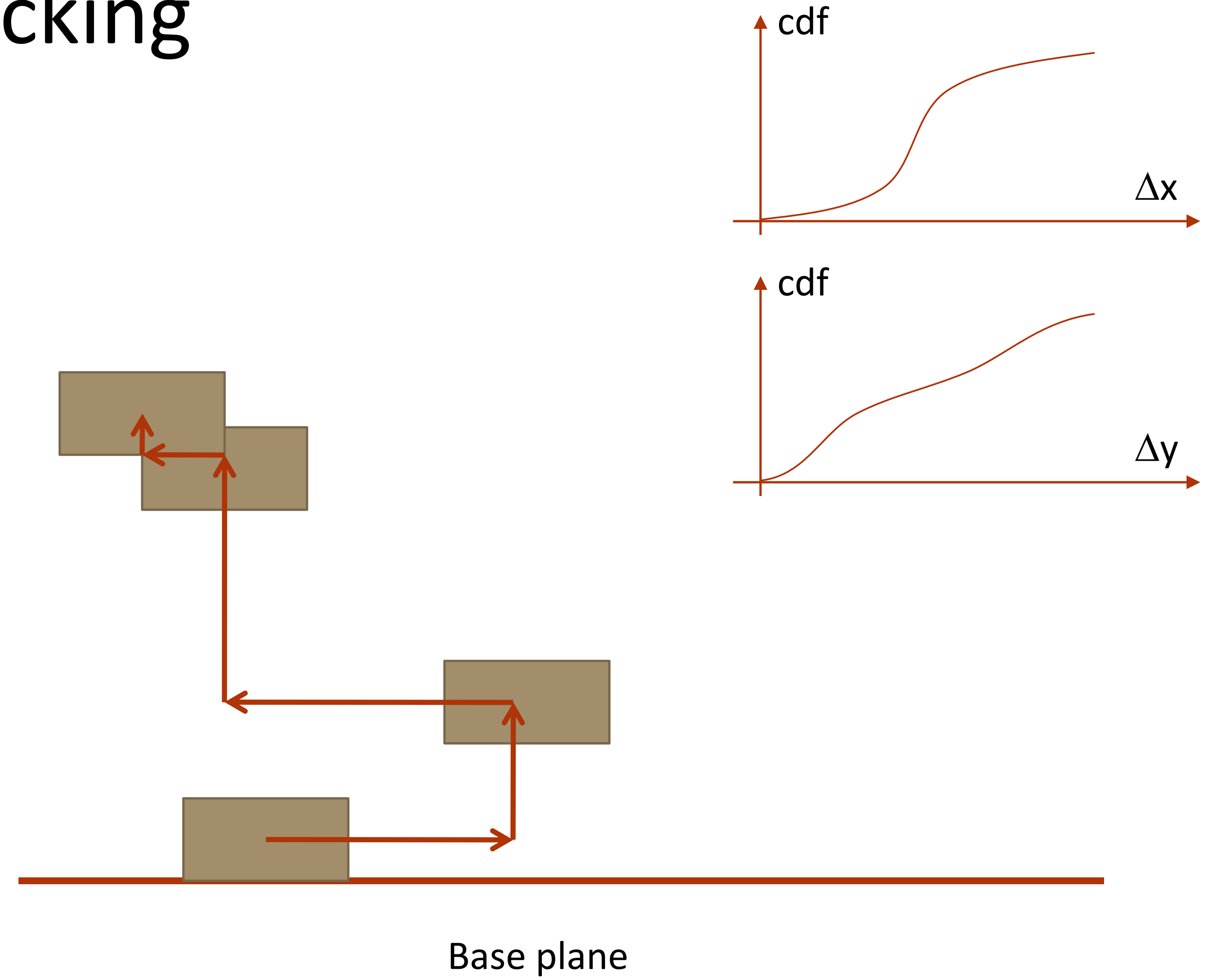
Intensity Field



Stacking


rate of sea-level rise
backstepping
vertical accretion
climbing progradation
upward retreat
lateral progradation
growth rate
downward advance
downstepping

Stacking



Interaction between objects

- Hierarchy by the order of definition
  - First defined object erodes the second one etc…
- Overlap rules
  - No overlap
  - Full overlap
  - Attach