STAT1012 Statistics for Life Sciences

Quick Revision Notes

Fall, 2019

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(Reference: lecture and tutorial notes)

Contents

I) Descriptive Statistics

Data type: Qualitative (Special: Categorical), Quantitative (Discrete, Continuous)

Population: the whole set of entities of interest

Sample: a subset of the population

Central tendency

Sample mean: $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$

Sequential update property: $\bar{X}_n = \frac{1}{n}[(n-1)\bar{X}_{n-1} + X_n]$

Mode: the value which has the greatest number of occurrence (may not be unique)

Median: the "middle" value, or the average of the two values closest to "middle" after sorting

Percentile: the p-th percentile ($V_{\frac{p}{100}}$) is a value such that p% of the data are less than or equal to $V_{\frac{p}{100}}$. In particular, upper quantile = $V_{0.75}$, median = $V_{0.5}$, lower quantile = $V_{0.25}$.

Denote the sorted data by $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ where $X_{(1)} \le X_{(2)} \le \cdots \le X_{(n)}$. This is equivalent to saying that $X_{(1)}$ is the smallest, $X_{(2)}$ is the second smallest etc.

Median: $V_{0.5} = X_{\left(\frac{n+1}{2}\right)}$ if n is odd or $\frac{1}{2}\left[X_{\left(\frac{n}{2}\right)} + X_{\left(\frac{n}{2}+1\right)}\right]$ if n is even

Percentile: $V_{\frac{p}{100}} = X_{(k)}$ where $k = roundUp\left(\frac{np}{100}\right)$ if $\frac{np}{100}$ is not an integer.

Otherwise, $V_{\frac{p}{100}} = \frac{1}{2}\left[X_{\left(\frac{np}{100}\right)} + X_{\left(\frac{np}{100}+1\right)}\right]$

Dispersion

Symmetric: the left hand side of the distribution mirrors the right hand side

Unimodal: the mode is unique

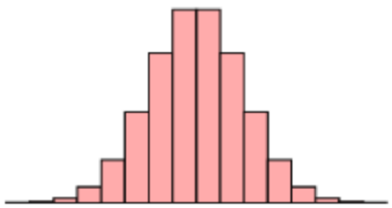
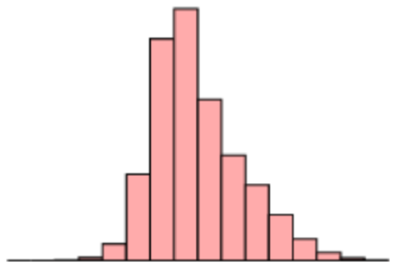
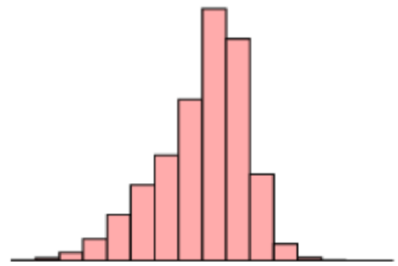
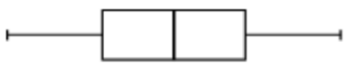
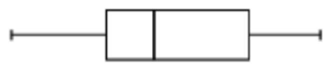
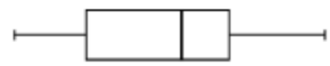
Skewness: measure of asymmetry

Left-skewed (negatively skewed): mean < median, have a few extreme small values

Right-skewed (positively skewed): mean > median, have a few extreme large values

Symmetric → mean = median (converse not true)

Symmetric + unimodal → mean = median = mode (converse not true)

Symmetric	Skewed right (positive)	Skewed left (negative)
		
		
Q_1 and Q_3 should be approximately equally spaced from the median (Q_2).	Q_3 is farther from the median(Q_2) than Q_1	Q_1 is farther from the median(Q_2) than Q_3

Range: maximum – minimum ($X_{(n)} - X_{(1)}$)

Interquartile range: $V_{0.75} - V_{0.25}$

Sample variance: $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ or $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i^2 - n\bar{X}^2)$

Sample standard deviation: $SD = \sqrt{S^2}$

Graphical methods

Bar graph: use for categorical data, show the number of observations in each category

Histogram: use for quantitative data, showing the number of observations in each range

Stem-and-leaf plot: ordered the data into a tree-like structure

Boxplot: show 5 numbers (min, Q1, median, Q3, max), help locate outliers (As a rule of thumb, some people define outliers as values > Q3 + 1.5*IQR or < Q1 – 1.5*IQR)

II) Probability

Notations

Sample space: the set of all possible outcomes, often denoted as Ω

Outcome: a possible type of occurrence

Event: any set of outcomes of interest, can be denoted as $E \subset \Omega$

Probability (of an event): denoted by $P(E)$, always lies between 0 and 1 (both inclusive)

$$P(E) = \frac{\#\ of\ outcomes\ in\ E}{\#\ of\ outcomes\ in\ \Omega}$$

Union: either A or B occurs, or they both occurs, denoted by $A \cup B$ (logically equivalent to OR)

Intersection: both A and B occur, denoted by $A \cap B$ (logically equivalent to AND)

Complement: A does not occur, denoted by A^C (logically equivalent to NOT)

DeMorgan's laws: $(A \cup B)^C = A^C \cap B^C, (A \cap B)^C = A^C \cup B^C$

Probability theory

Mutually exclusive: A and B are mutually exclusive if $P(A \cap B) = 0$ (cannot co-occur)

Independence: $P(A \cap B) = P(A)P(B)$ iff A and B are independent. Their complements (A and B^C; A^C and B; A^C and B^C) will be pairwise independent as well

Addition law: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Multiplication law: if $A_1, \ldots, A_k$ are mutually independent, then $P(A_! \cap \ldots \cap A_k) = P(A_1) \times \ldots \times P(A_k)$

Conditional probability

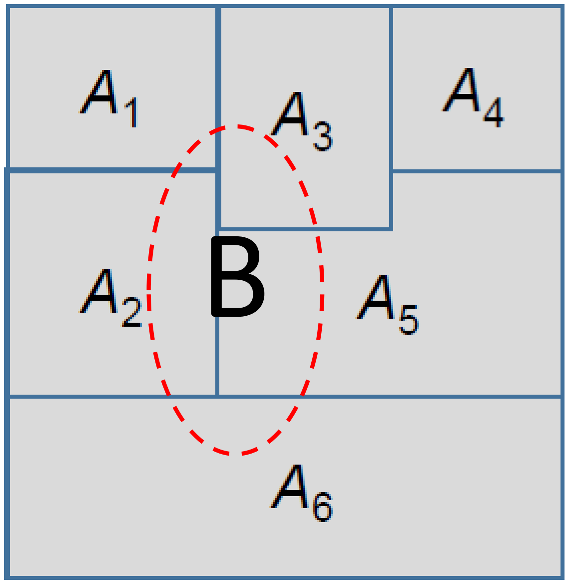
Conditional probability: $P(B|A) = \frac{P(A \cap B)}{P(A)}$, if $P(B|A) = P(B)$, then A and B are independent

Relative risk: $RR(B|A) = \frac{P(B|A)}{P(B|A^C)}$

Total probability rule: $P(B) = P(B|A)P(A) + P(B|A^C)P(A^C)$

Exhaustive: if $A_1, \ldots, A_k$ are exhaustive, then $A_1 \cup \ldots \cup A_k = \Omega$ (at least one of them must occur)

Generalized total probability rule: let $A_1, \dots, A_k$ be mutually exclusive and exhaustive events. For any event B, we have $P(B) = \sum_{i=1}^{k} P(B|A_i)P(A_i)$



Bayes' theorem: conditional probability + generalized total probability rule. let $A_1, \dots, A_k$ be mutually exclusive and exhaustive events. For any event B,

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(A_i)P(B|A_i)}{P(B)} = \frac{P(A_i)P(B|A_i)}{\sum_{j=1}^{k} P\left(B\middle|A_j\right)P(A_j)}$$

III) Discrete Probability Distributions

Random variables: numeric quantities that take different values with specified probabilities

Discrete random variable: a R.V. that takes value from a discrete set of numbers

Continuous random variable: a R.V. that takes value over an interval of numbers

Discrete random variables

Probability mass function: a pmf assigns a probability to each possible value x of the discrete random variable X, denoted by $f(x) = P(X = x)$

$\sum_{i=1}^{n} f(x_i) = 1$ (total probability rule)

Cumulative distribution function: a cdf gives the probability that X is less than or equal to the value x, denoted by $F(x) = P(X \leq x)$

Expected value: $\mu = E(X) = \sum_{i=1}^{n} x_i P(X = x_i)$ (the idea is "probability weighted average")

Variance: $\sigma^2 = Var(X) = \sum_{i=1}^{n} (x_i - \mu)^2 P(X = x_i)$, alternatively $Var(X) = E(X^2) - [E(X)]^2$

Translation/rescale: $E(aX + b) = aE(X) + b$, $Var(aX + b) = a^2 Var(X)$

Linearity of expectation: $E(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} E(X_i)$

Binomial distribution

Factorial: $n! = n \times (n-1) \times \ldots \times 1$, note that $0! = 1$

Permutation (order is important): $P_k^n = \frac{n!}{(n-k)!}$

Combination (order is not important): $C_k^n = \frac{n!}{k!(n-k)!}$, also denoted as $\binom{n}{k}$

Binomial distribution: probability distribution on the number of successes X in n independent experiments, each experiment has a probability of success p, then $X \sim B(n, p)$

Pmf: $P(X = x) = \binom{n}{k} p^x (1-p)^{1-x}$ for $x = 0, 1, 2, \ldots, n$

Mean: $E(X) = np$

Variance: $Var(X) = np(1-p)$

Skewness: right-skewed if p<0.5, symmetric if p=0.5, left-skewed if p>0.5

Poisson distribution

Poisson distribution: probability distribution on the number of occurrence X (usually of a rare event) over a period of time or space with rate μ, then $X \sim Po(\mu)$

Pmf: $P(X = x) = \frac{e^{-\mu}\mu^k}{k!}$ for $k = 0, 1, 2, \ldots$

Mean: $E(X) = \mu$

Variance: $Var(X) = \mu$

Skewness: right-skewed

Poisson limit theorem (poisson approximation to binomial): if $X \sim B(n, p)$ where $n \geq 20,\ p < 0.1$ and $np < 5$, then $X \approx Y \sim Po(\mu)$ where $\mu = np$

IV) Continuous Probability Distribution

Continuous random variable

Probability density function: a pdf specifies the probability of the random variable falling within a particular range of values, denoted by $f(x)$

$P(a \le X \le b) = \int_a^b f(x)dx$, which is the area under the curve from a to b

$P(X = a) = \int_a^a f(x)dx = 0$ for all a

$\int_{-\infty}^{\infty} f(x)dx = 1$ (total probability rule)

Cumulative distribution function: a cdf gives the probability that X is less than or equal to the value x, denoted by $F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)dt$

Expected value: $\mu = E(X) = \int_{-\infty}^{\infty} xf(x)dx$

Variance: $\sigma^2 = Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$

Normal distribution

Normal distribution: if X follows normal distribution with mean μ and variance σ^2, then $X \sim N(\mu, \sigma^2)$, often used to represent continuous random variable with unknown distributions

Pdf: $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$ for $-\infty < x < \infty$

Shape: bell-shape, symmetric about the mean, unimodal

Standard normal distribution: $Z \sim N(0,1)$

Cdf of standard normal: denoted as $\Phi(z) = P(Z \le z)$

$P(a \le Z \le b) = P(Z \le b) - P(Z \le a) = \Phi(b) - \Phi(a)$

$\Phi(-z) = 1 - \Phi(z)$ by symmetric property

Percentile of standard normal: $\Phi(1.645) = 0.95, \Phi(1.96) = 0.975$

Standardization: if $X \sim N(\mu, \sigma^2)$, then $\frac{X-\mu}{\sigma} \sim N(0,1)$

$P(a < X < b) = P\left(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$

De Moivre–Laplace theorem (normal approximation to binomial): if $X \sim B(n,p)$, $P(a < X < b) \approx P(a + 0.5 \le Y \le b - 0.5)$ where $Y \sim N(np, np(1-p))$. The 0.5s are continuity correction.

V) Point Estimation