H.W. -11

Path Integrals and Classical Action

Questions

(1) Consider a free particle of mass m moving in 1D from $x(0) = 0$ to $x(T) = 0$. The Lagrangian is

$$L = \frac{1}{2}m\dot{x}^2.$$

(a) Write down the classical path $x_{\rm cl}(t)$ that satisfies the boundary conditions and calculate the action $S[x_{\rm cl}]$.

(b) Consider a variation path

$$X(t) = \alpha \sin\left(\frac{\pi t}{T}\right),$$

where α is a small constant representing the deviation magnitude. Calculate $S[X]$ for this path.

(c) Show that $S[X] \geq S[x_{\rm cl}]$ for any $\alpha \neq 0$.

(2) Given the free–particle propagator $K_{\rm free}(x_f, T; x_i, 0)$, consider a particle moving in a constant potential $V(x) = V_0$.

(a) Write down the relationship between the new Lagrangian L' and the free–particle Lagrangian L.

(b) Using the path–integral representation

$$K = \int \mathcal{D}x \, \exp\left(\frac{i}{\hbar}\int L\, dt\right),$$

determine the exact relation between the propagator K_{V_0} and $K_{\rm free}$.

(c) Interpret the physical meaning of the extra term in the exponent.