

GATE/ESE NOTES for
ELECTRIC CIRCUITS (EEE)
NETWORK THEORY (ECE)
BASICS OF CIRCUITS (INST)

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Networks

What to study to get Rank.

1. Notes of Every Teacher Volume-1 classwork Volume-2 H/W +100% Attendance.
2. collect and study all pervious year IES objective of EEE and E&T(ECE) prelims
3. collect & study all previous year Gate solution bits of EEE & ECE

Instrumentation Common Subjects.

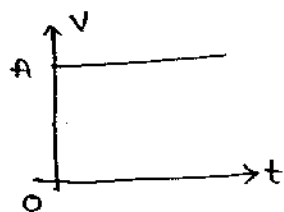
4. Text Book bits \rightarrow standard.

Ex: CL Wadhwa, B.S. Bimbhra, CM/c and power Electronics) + GenCO Transco papers + UPSC Civil Services, ISRO, DRDO, BARC.

5. Write lot of online & offline Exams. and Take self feedback.

Syllabus:

first three chapters are of DC circuits

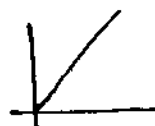
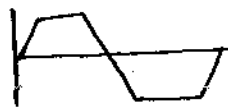
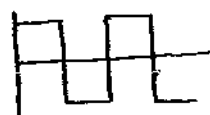


$R \rightarrow$ Homogeneous & proportional

$L \rightarrow$ S.C

$C \rightarrow$ O.C

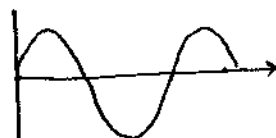
4. chapter Basics of Time Variance.



$R \rightarrow$ Homogeneous & proportional

$\left. \begin{matrix} L \\ C \end{matrix} \right\}$ will Respond.

Chaper : 5, 6, 7, 8, 9, 10, 11
AC circuits

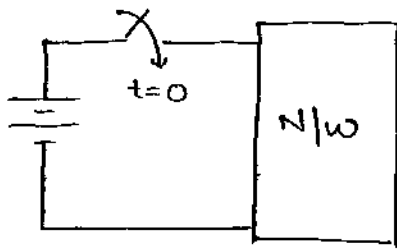


$R \rightarrow$ Homogeneous & proportional (in phase elements)

$L \rightarrow$ Lagging

$C \rightarrow$ Leading.

12. } Transient Response
13. }



Level 1 \rightarrow 1-4 chapters.

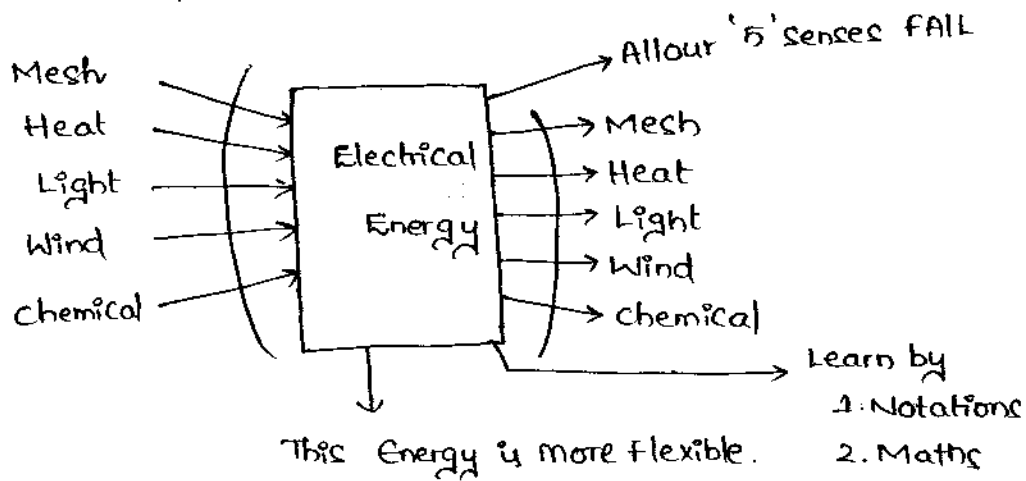
Level 2 \rightarrow 5-10 chapters.

Level 3 \rightarrow 11-15 chapters.

Electrical Energy: Dominant & Most Superior form of Energy by

1. Interconvertability
2. Transportation.

2



2. Generate & Transport

- Bulk Quantities
- Larger distances
- Most Economically way

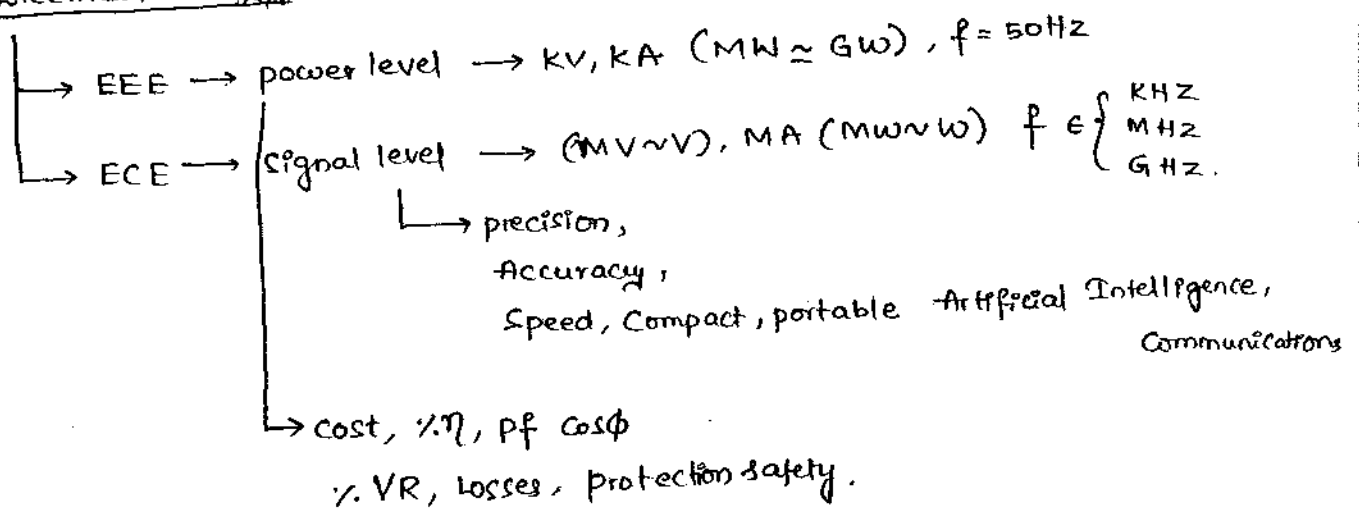
Ex: 1. unit of Electrical Energy = 1 kWh

$$= 1000 \times 1 \text{ hr}$$

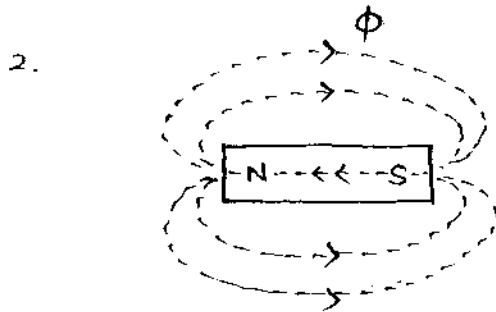
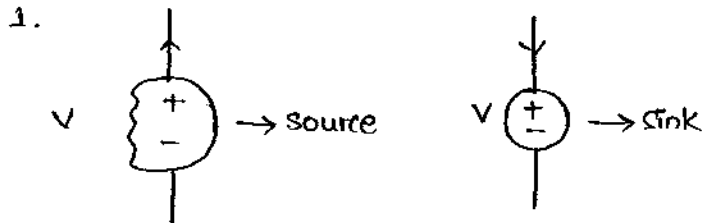
RS 3/- per unit

C > 70% population in India.

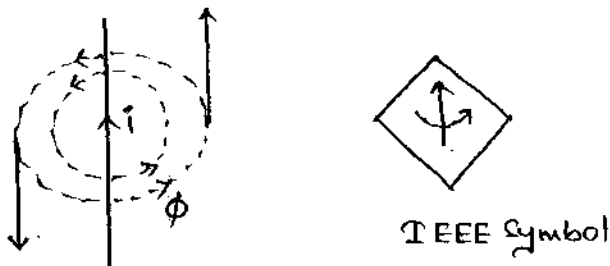
Electrical Energy



Notations :



→ Ampere Right Hand Thumb Rule :



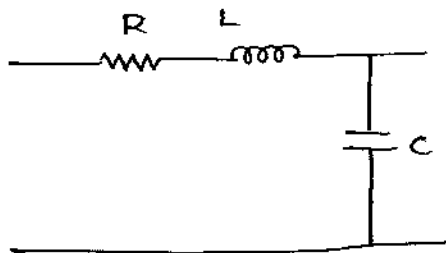
Any material

Resistor → Represents Linear Time invariant Electrical property of Matter.

Inductor → Electro Magnetic property of Matter.

Capacitor → Electrostatic property of Matter.

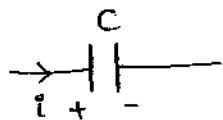
Ex:1:



* greater than (or) Equal to 10mA of Current flows through Human Body
the Human can die.

* 2mA Current Can effect body of paralysis

Capacitor:

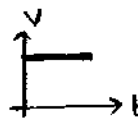


from Ohm's Law

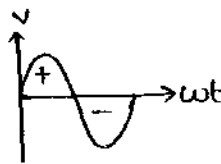
$$i = C \frac{dv}{dt}$$

→ DC

→ AC

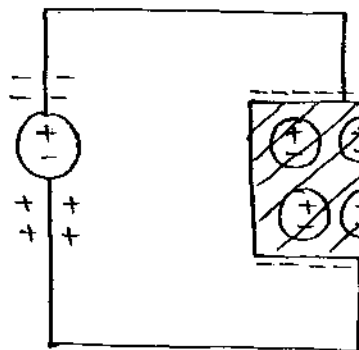


③ $i_C = 0 \Rightarrow$ Capacitor acts as o.c



$$i_C = I_m \sin(\omega t + 90^\circ)$$

↓
Leading.

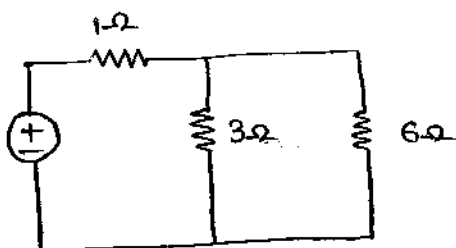


charge is a function (displacement current)

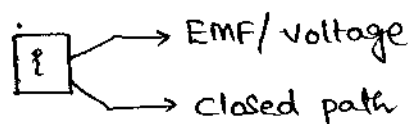
dielectric exhibits polarization

Note: Engineering is Application of Science & Analysing by mathematical modelling
DC Generator is introduced by Tesla in 1902

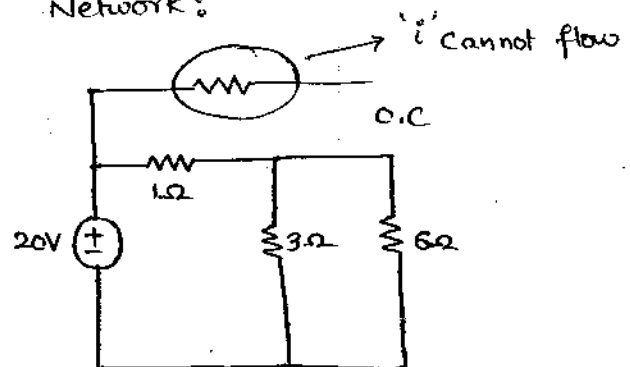
Circuit:



⇒ Current 'i' is intended to flow through all the Components.



Network:



here there is no closed path for the flow of Current.

* All our practical big interconnected systems are networks but we do circuits analysis to those parts where current is flowing.

* circuits are Building Blocks of Networks.

Ex: Our power system is a Big Network But a motor running in it is ckt at power level.

Ex: our Communication System is a big Network But a Transmitter working in it is a ckt at signal level.

EEE = power level = circuits

Ex: fault Analysis, power Electronics, T&D

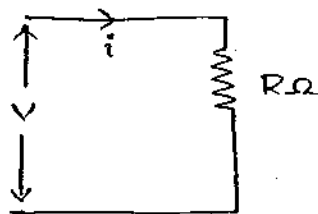
ECE/Instruments = signal level = Network.

Ex: Mobile Communication, WiFi, Bluetooth, Microwave Engineering,

Network Components (or) Elements:

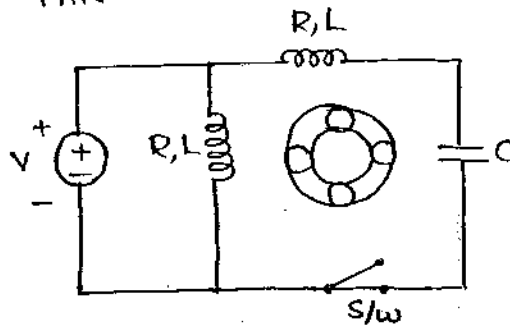
⇒ All our Application in Electrical Engineering are our Components But when this applications are ^{mathematically} modelled as ckt or N/w we use fundamental N/w Components such as V, I, R, L, C etc to model them

Ex: Heater

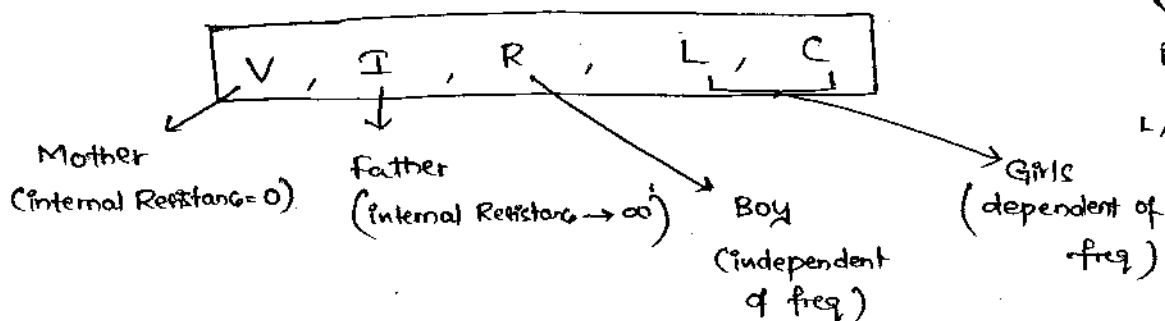
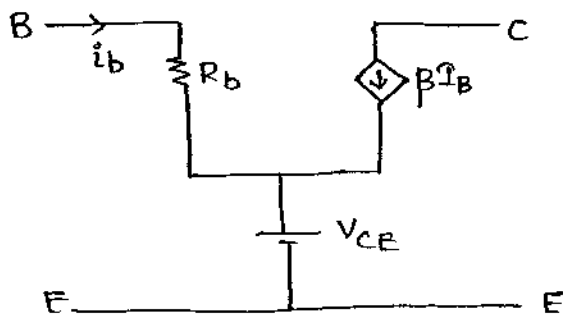


$$P_{\text{abs}} = i^2 R = \frac{V^2}{R}$$

FAN



Ex: BJT



$$V = L \frac{di}{dt}$$

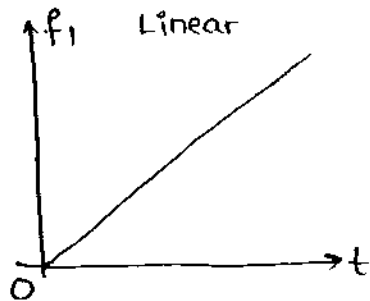
$$I = C \frac{dV}{dt}$$

$L, C \rightarrow$ Reactive power.

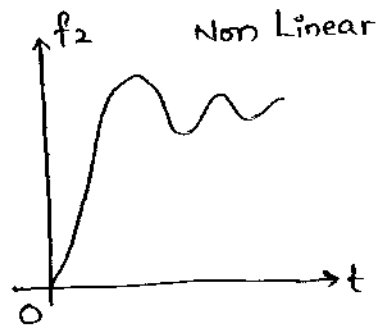
Linear Components & Linear Networks:

(4)

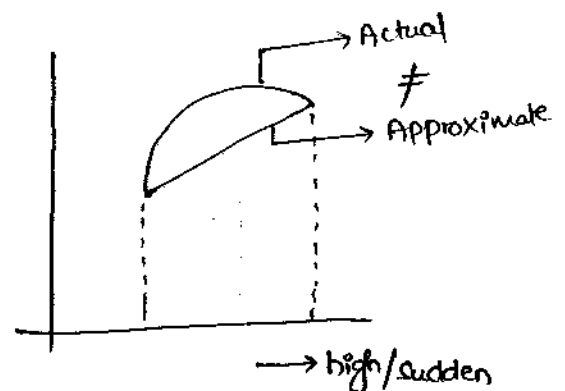
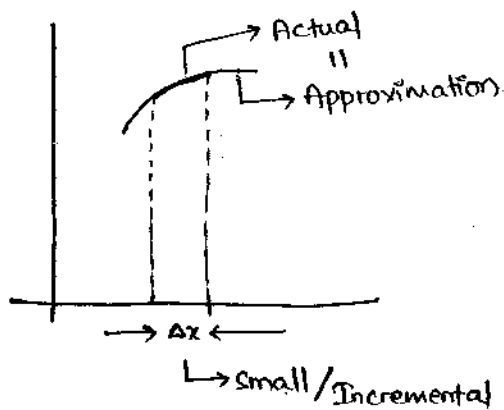
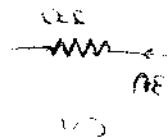
* All our practical Components are originally non Linear in Nature. But Any non Linear System Can be Linearised by small Incremental change in time. But the Same System For the Sudden big changes undergoes non Linear Mode of operation.



→ it is a function of Homogeneous
→ it Can be Modelled in maths

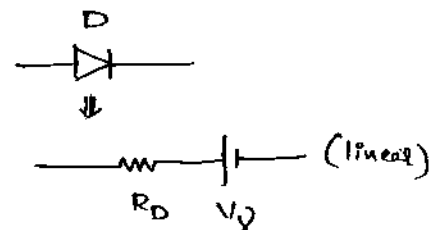
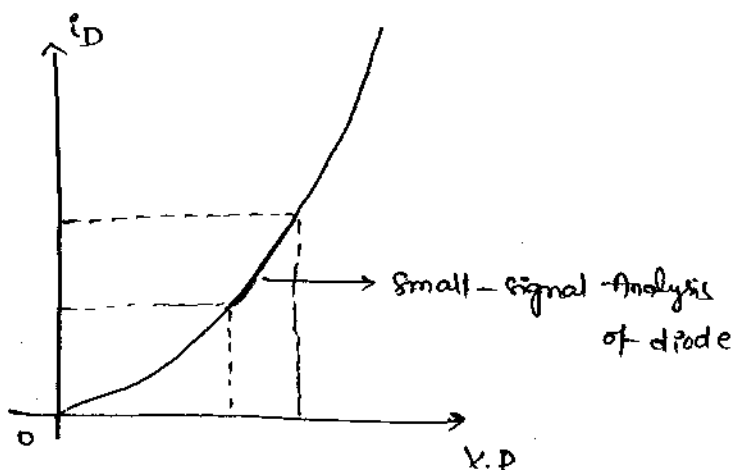


→ Non Homogeneous
→ Can't be modelled in maths



* All our Engineering Components & Design have Rating / Specifications.

Ex: Diode:

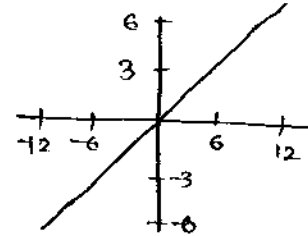
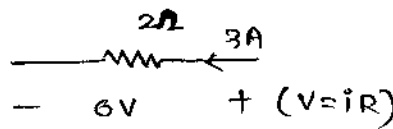
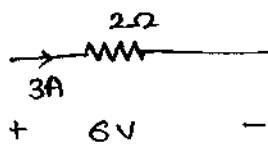


* in electrical ckt analysis if every Component is designed & operated within the Ratings is said to be linear. Where it obeys ohm's law kirchoff's Law, Super position Theorem etc

Bilateral Component:

$$I = III \rightarrow \text{Symmetrical.}$$

Ex: Resistor.



$$P_{abs} = 18W = \frac{V^2}{R} = \frac{36}{2}$$

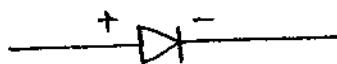
$$P_{abs} = \frac{V^2}{R} = \frac{6^2}{2} = \frac{36}{2} = 18W$$

⇒ The properties & characteristics of such elements are independent to ckt Condition

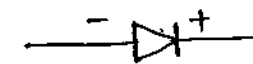
Ex: R, L, C are Bilateral.

Unilateral Components:

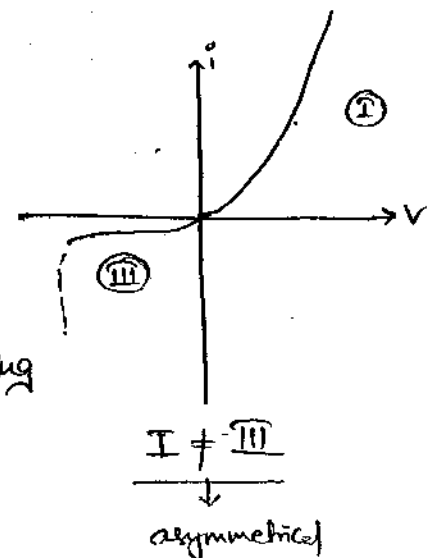
Ex: Diode



- F.B
- will Conduct
- $R_{ON} = 0\Omega$



- R.B
- Not Conducting
- $R_{OFF} = \infty\Omega$

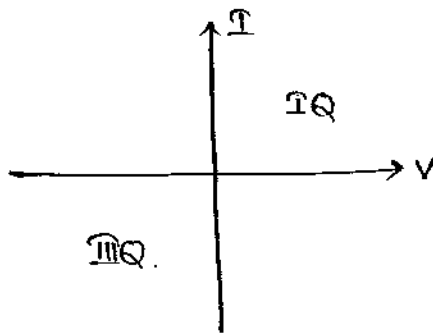


→ The properties & characteristics of such Components depend upon ckt Condition

Ex: Diode, BJT, MOSFET etc are Unilateral.

Based on static V-I characteristics:

(5)

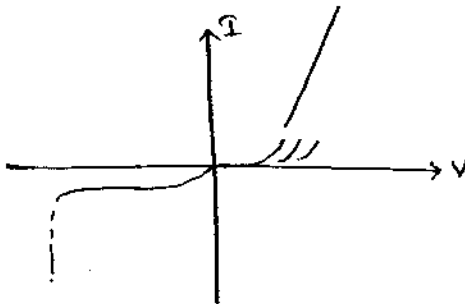


if $I_Q = III_Q \Rightarrow \text{Symmetrical} \Rightarrow \text{Bilateral}$

$I_Q \neq III_Q \Rightarrow \text{asymmetrical} \Rightarrow \text{Unilateral}$

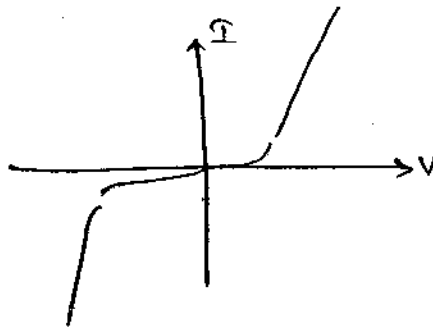
Examples:

SCR



$I_Q \neq III_Q$
unilateral switch

TRIAC



$I_Q = III_Q$
Bilateral switch

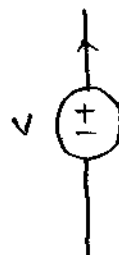
Active elements: (source)

Any element can

- Generate
- give out
- drive
- Energise
- deliver

externally
E.E.

Ex: V, I



→ Source

$$P_{del} = +V \cdot I \text{ W}$$

$$P_{abs} = -V \cdot I \text{ W}$$

passive elements (components):

Any element can

- Absorb
 - store
 - Convert
 - dissipate
 - Waste
- } Electric Energy
in any form

ex: R, L, C.

\boxed{R} → dissipate.

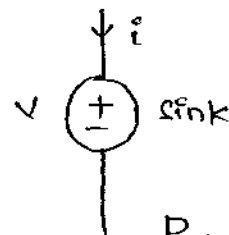
E.E → Heat.

\boxed{L} → store

E.E → E.M Form (flux)

\boxed{C} → store

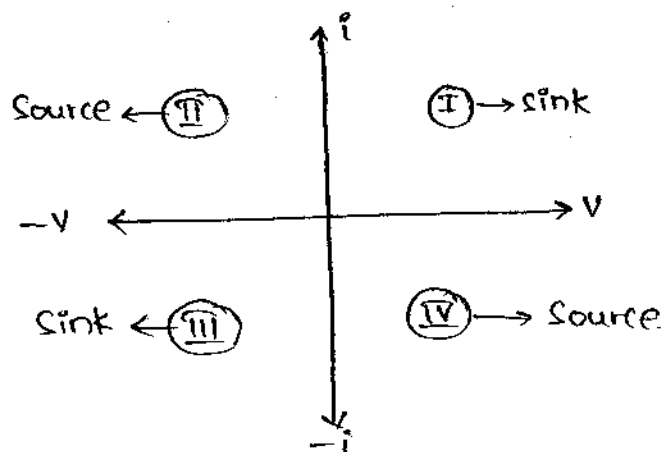
E.E → E.S form (charge)



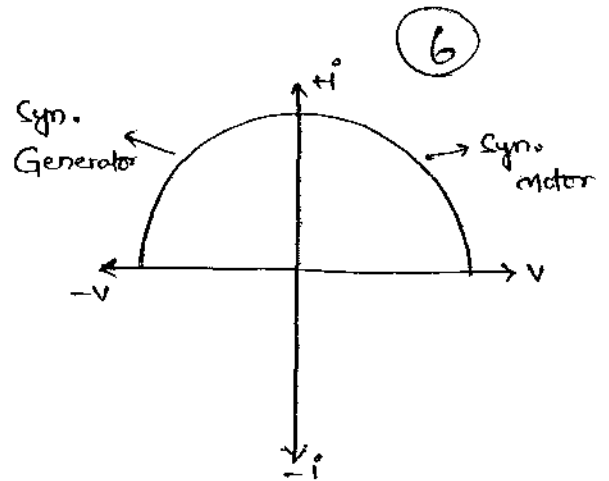
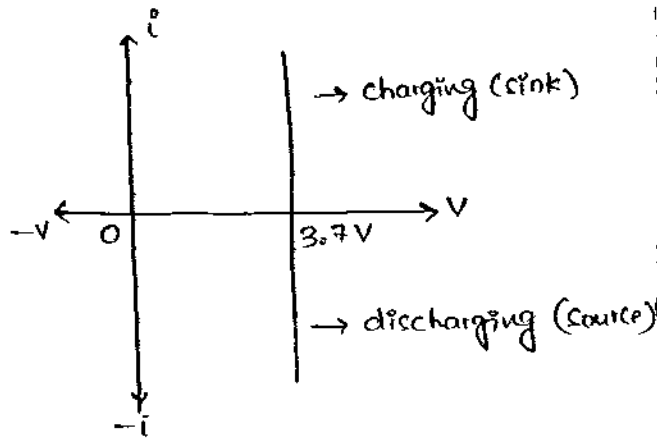
$$P_{abs} = +v.i \text{ W}$$

$$P_{del} = -v.i \text{ W}$$

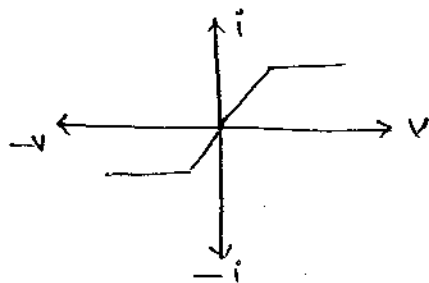
Based on static V-I characteristics:



Ex: Rechargeable Battery

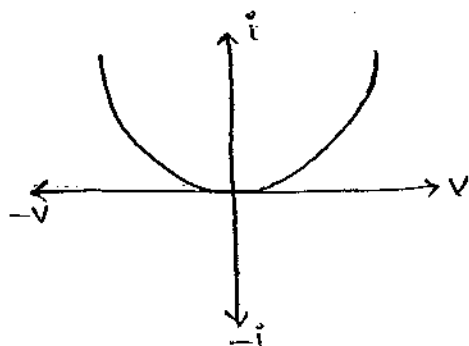


* The static V-I characteristics of a particular Component are shown below then the Component is NLPBi



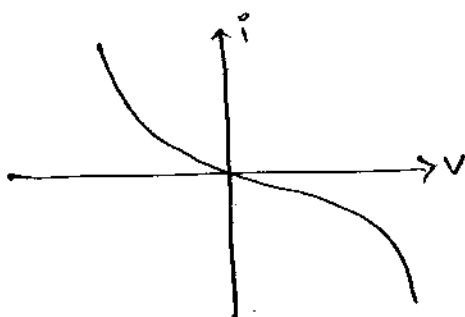
NonLinear passive Bilateral

* Static V-I char. of particular Component are shown below then the Component is



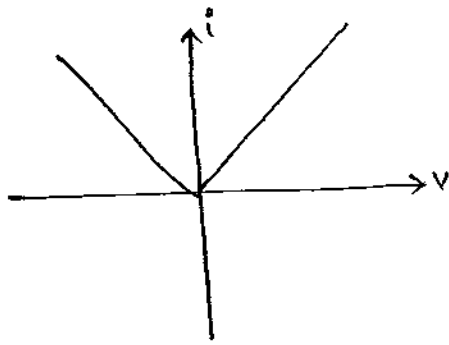
Non Linear Both Active & passive
Non Linear globally Active unilateral

AS → Nonlinear Active unilateral



Non linear Active Bilateral

gate
*



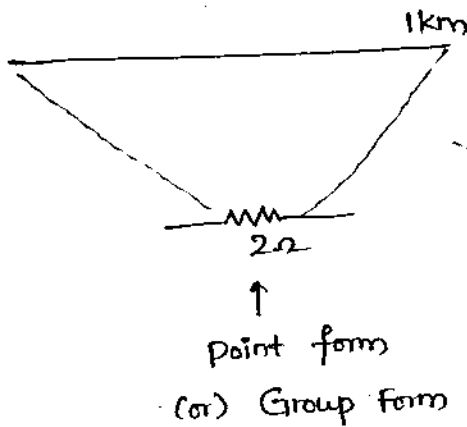
→ Non Linear Both passive & Active unilaterally
ans: Non linear Active unilaterally

Note:

- (i) A Transistor BJT is globally passive but locally as an Amplifier is said to be Active
- (ii) Most of Semiconductor devices when Biased with internal cut in voltages & leakage currents are said to be Active.
- (iii) Inductors & Capacitors are globally passive But locally during Transient When they release energy due to initial conditions are said to be Active.
- (iv) Op-amp is globally non linear. But locally below saturation is said to be linear.
- (v) The time duration of each operating mode of the device must be Consider first & then decide its property.

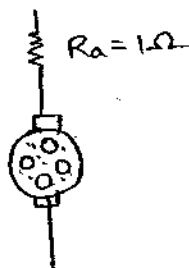
Lumped parameters: → (Abstract Model)

↓
Used in Electric circuit Analysis (or) Networks.



We neglect any parametric change in length, Area, Temp, shape etc.

Ex: PCB Components



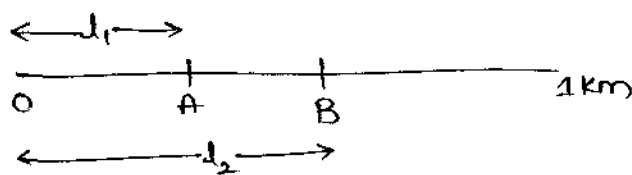
Ex: Model of Medium distance Transmission lines as T, π Networks etc.

Lumped parameters are

1. easy to Represent
2. Simple maths (Linear Algebra, matrices etc)
3. Solutions are faster
4. But Approximate answers.

Distributed parameters: (Absolute models)

→ used in EMF.



$$R = \frac{\rho l}{a} \Omega$$

⇒ We consider parametric changes in Modelling N/w Components.

$$R_{\text{actual}} = \frac{\partial R}{\partial l}$$

Ex: Model of Long distance T-lines [Rigorous solution]

⇒ EMF Concepts

⇒ Antenna

⇒ Wave guide principles etc.

Distributed parameters

- * Complex to Represent
- * Advanced Maths (Transform Theory, D.E, calculus etc)
- * Solutions are Tedious
- * But very accurate answer

} principles.

* The relationship $V = \lambda$ is valid for distributed parameters

IES

(a) Lumped only

(b) distributed only

(c) Both lumped & distributed

(d) None.

Topological definitions of Network:

node (n): a node is a point of interconnection or junction b/w two or more components.

Branch (b): Branch is an elemental connection b/w two nodes.

Degree of a node (δ): The no. of Branches in sit at any node represents its degree. δ

if $\delta > 2 \Rightarrow$ principle node (n_p)

$\delta = 2 \Rightarrow$ simple node (n_s)

Note: for any ckt or network

$$\sum_{i=1}^n \delta_i = 2(b)$$

mesh (m): mesh is a closed path of ckt or n/w. it should not have further closed paths in it.

Loop (l): Loops are all possible closed paths of ckt or n/w

Note:

\Rightarrow for any ckt or n/w

$$m = (b - n + 1)$$

\Rightarrow The minimum number of KVL Equations^{required} to solve ckt or n/w is

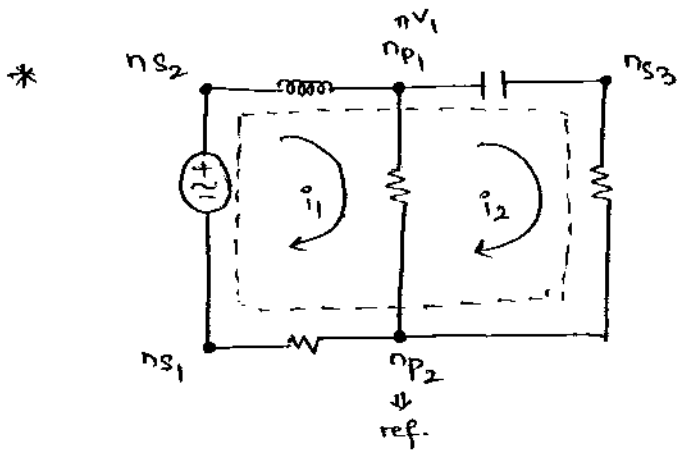
$$m = (b - n + 1)$$

\Rightarrow meshes are specifically known as independent loops.

\Rightarrow all meshes are by default loops. but all loops are not meshes.

\Rightarrow in nodal analysis we may neglect simple nodes and one of the principle node is taken as reference. So the minimum number of KCL Equations required to solve ckt or n/w is $= (n_p - 1)$

$$= (n_p - 1)$$



$$n = \underline{5} \quad b = \underline{6}$$

$$m = \underline{2} = [6 - 5 + 1]$$

8

$$l = \underline{2+1 = 3}$$

$$\sum \delta_i = 2 + 2 + 2 + 3 + 3$$

$$= 12$$

$$= 2(6)$$

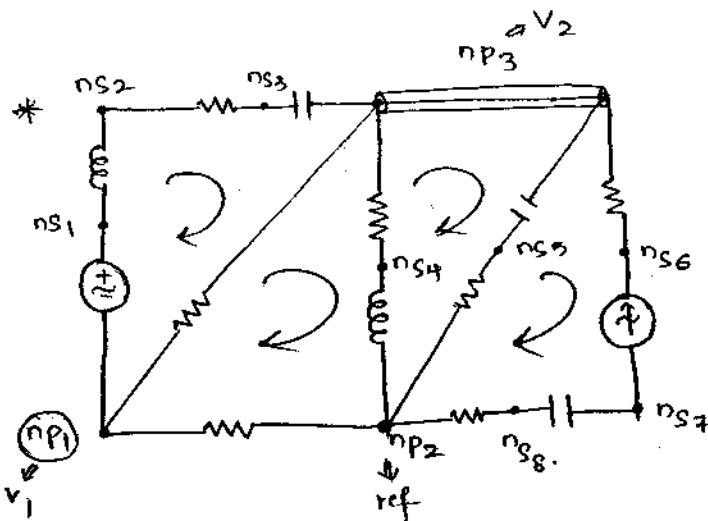
$$\sum \delta_i = 2(b)$$

Mesh

$$\rightarrow \text{KVL} \Rightarrow m = 2 \quad i_1 \quad i_2$$

Nodal

$$\rightarrow \text{KCL} \Rightarrow (n_p - 1) = (2 - 1) \rightarrow V_1 = 1$$



$$n = \underline{11}, \quad b = \underline{14}$$

$$m = \underline{4} = [14 - 11 + 1]$$

$$l = \underline{4 + 3 + 2 + 1 = 10}$$

Mesh

$$\rightarrow \text{KVL} \Rightarrow m = 4 \rightarrow i_1, i_2, i_3, i_4$$

Nodal

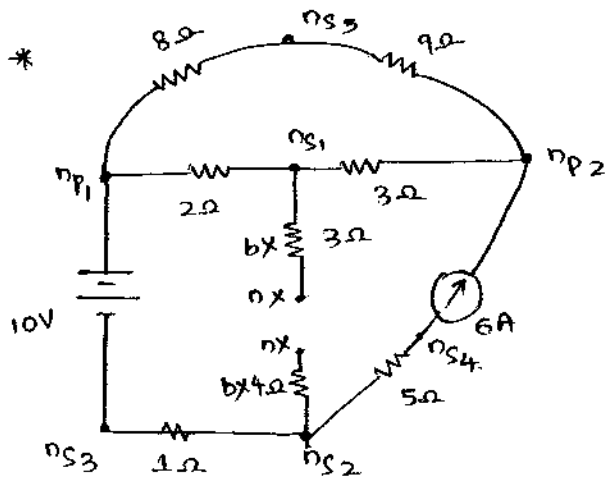
$$\rightarrow \text{KCL} \Rightarrow (n_p - 1) = (3 - 1) = 2 \rightarrow V_1, V_2$$

$$\sum \delta_i = 8(2) + 3 + 4 + 5 = 28$$

\uparrow 8 nodes of deg 2 (ns1 to ns8)
 \uparrow 3rd deg node (np1)
 \uparrow 4th deg node (np3)

$$= 2(14)$$

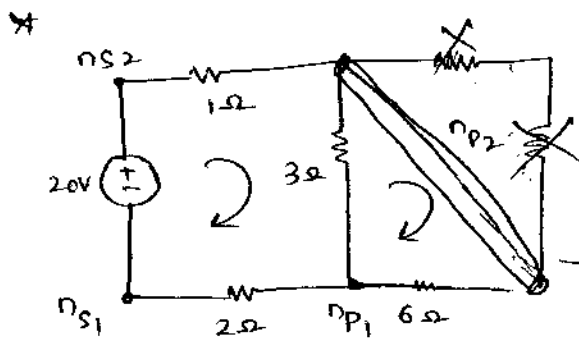
$$= 2(b)$$



$$n = 7$$

$$b = 8$$

$$m = 2 = (8 - 7 + 1)$$



$$n = 4$$

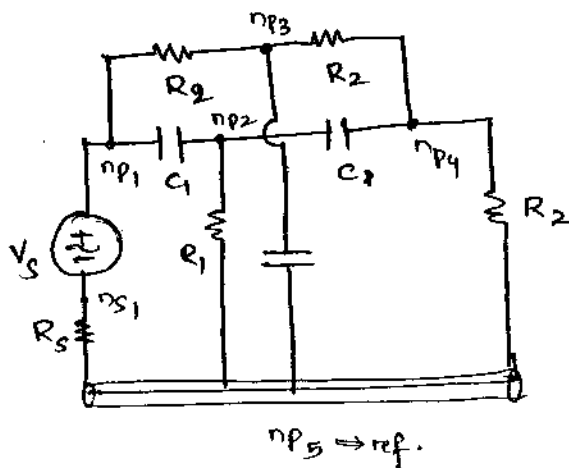
$$b = 5$$

$$m = 2 = [5 - 4 + 1]$$

elements parallel to s.c path are redundant neglect
(if $\bar{y} = N/w$)
not ckt

gate
* The min, no. of eqns required to solve the given ckt below is

- (a) 2 (b) 3 ~~(c) 4~~ (d) 5



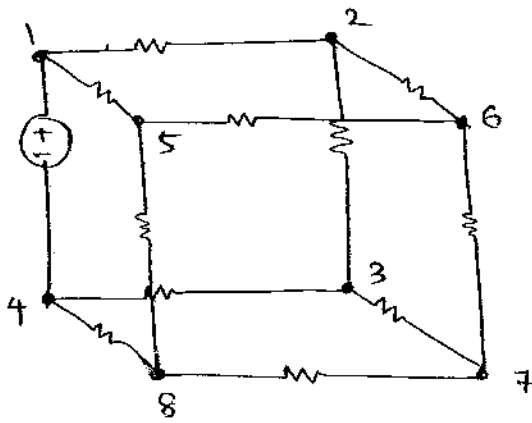
sol mesh \rightarrow KVL $\Rightarrow m = [b - n + 1]$

$$= 9 - 6 + 1$$

$$m = 4$$

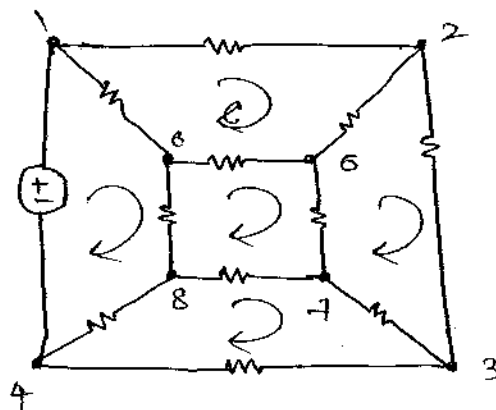
Nodal \rightarrow KCL $\Rightarrow (n_p - 1) = (5 - 1) = 4$

\rightarrow is this ckt PLANAR or Not? \Rightarrow planar
(without any overlap) \Rightarrow if we represent on 2-D plane without any overlap it
called planar



is it PLANAR or Not?

9



$$n = 8$$

$$b = 12$$

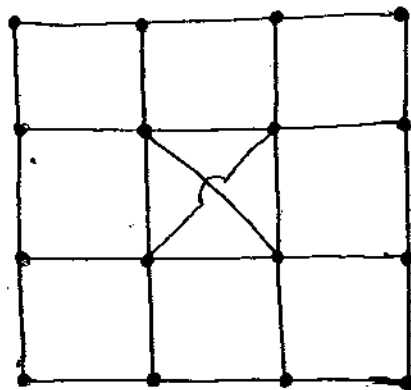
$$m = 5$$

$$= [12 - 8 + 1] = 5.$$

⇒ it is PLANAR.

→ Mesh analysis requires PLANAR arrangement in the N/w.

Example for Non planar graph: (graph & skeleton of N/w)



* for Non-planar N/w
better to go for
Nodal analysis
(mesh analysis requires
PLANAR N/w)

a. Nlw has n - principles nodes, b - no. of branches

if mesh analysis simpler than Nodal analysis then n is greater than —

(a) $b-1$

$m = (b-n+1)$

(b) $b+1$

$\text{nodal} = (n_p - 1) = (n - 1)$

(c) $\frac{b}{2} - 1$

$\text{mesh} < \text{nodal}$

~~(d)~~ $\frac{b}{2} + 1$

$b - n + 1 < n - 1 \Rightarrow b + 2 < 2n$

~~$b < 2n + 2$~~

$2n > b + 2$

$n > \frac{b}{2} + 1$

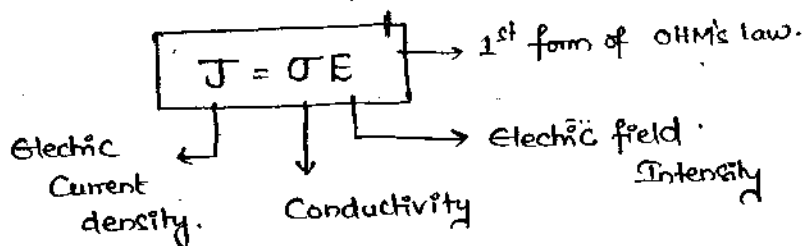
* Generally or most of the time nodal is faster than mesh.
 but when $n > \frac{b}{2} + 1 \Rightarrow$ then mesh analysis is faster than nodal.

OHM's Law:

I. LTI Domain:

→ Assume Temp is Constant

→ Assume Uniform Cross-Sectional area of material.



$\frac{I}{a} = \sigma \cdot \frac{V}{l} \Rightarrow V = \left[\frac{l}{\sigma a} \right] \cdot I$

$V = \left[\frac{\rho l}{a} \right] \cdot I$

$\left(\because \frac{l}{\sigma} = \rho \right)$

↓
Resistivity.

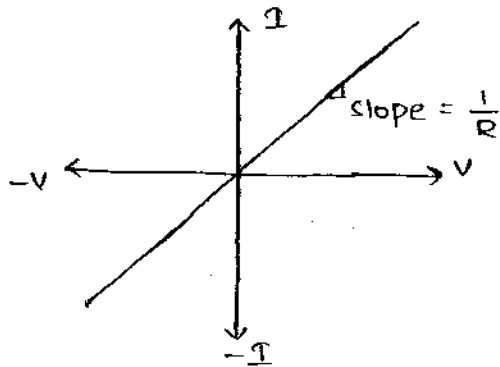
$V = [R] I$ → 2nd form of OHM's law.

↓
Resistance

units : ohms (Ω)

Graphically.

$$I = \left[\frac{1}{R} \right] \cdot V$$



- ⇒ Linear
- ⇒ passive
- ⇒ Bilateral [Symmetry]

10

if permeability is uniform
Inductance is constant.

II. Electromagnetic Domain:

$$\boxed{\psi = L i} \rightarrow 3^{rd} \text{ form of OHM's law}$$

since $\psi = N \cdot \phi$

↳ flux linkages (wb-T)

$$N \cdot \phi = L i$$

↓
we can't represent ϕ in lumped form

∴ to represent it is linear we consider small differential length (small incremental length)

$$\underline{N \cdot \frac{d\phi}{dt} + \phi \cdot \frac{dN}{dt} = L \frac{di}{dt} + i \frac{dL}{dt}}$$

∴ turns const

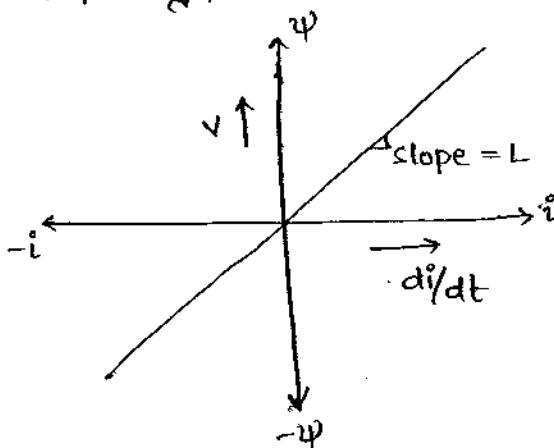
∴ Inductance = const

(∵ we assume permeability is uniform)

$$\boxed{V = L \cdot \frac{di}{dt}} \rightarrow 4^{th} \text{ form of OHM's law}$$

Inductance
units: Henry.

Graphically,



III. Electrostatic Domain:

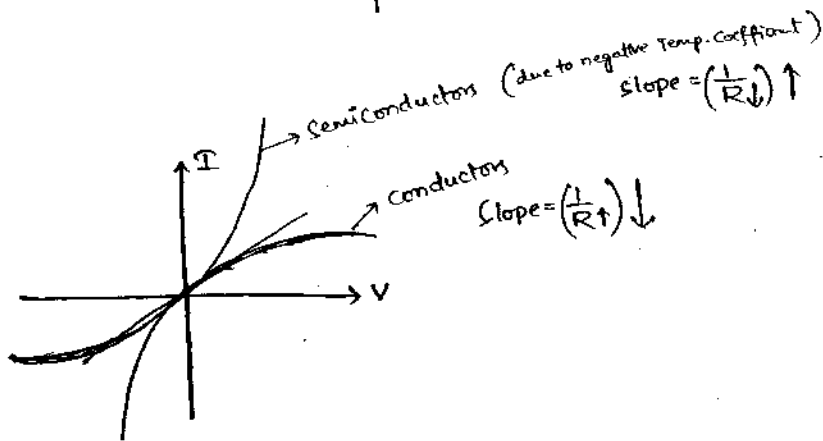
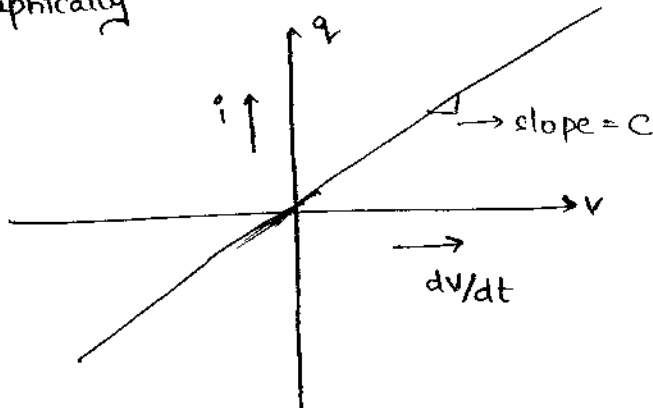
$$\boxed{q = CV} \quad \text{5th form of Ohm's law}$$

$$\frac{dq}{dt} = C \frac{dv}{dt} + \cancel{v \frac{dC}{dt} = 0}$$

$$\boxed{i = C \frac{dv}{dt}} \quad \text{6th form of Ohm's law}$$

→ capacitance units: Farads.

Graphically



* properties of DC Supply Systems:

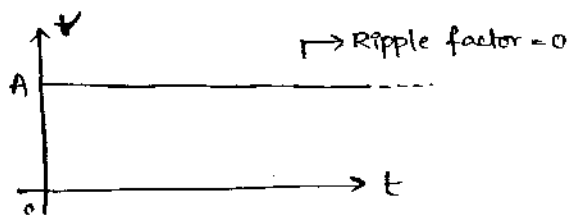
⇒ They are unipolar in voltage,
unidirectional in current,
no change in phase,
power frequency is zero (= 0 Hz)

⇒ They are used in small Independent Isolated power supply systems,
where elec. Energy can be stored in small Capacities.

Ex: Batteries in chemical form.

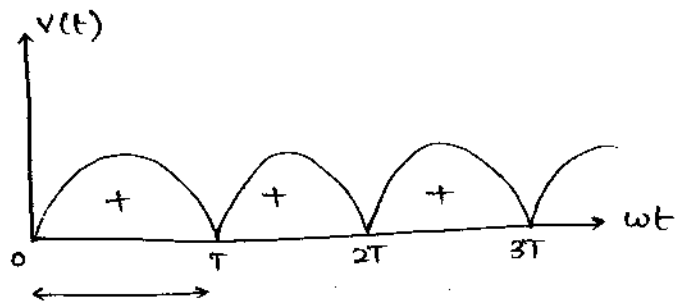
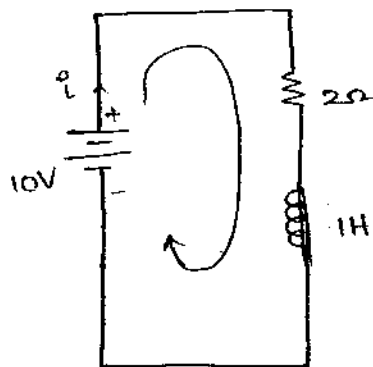
⇒ DC Supply Driven Systems are Superior with respect to precision, Accuracy, Speed, portable, Compact & easyⁿ Control

⇒ Ideal DC Waveform look like



⇒ Example applications include toys, Cellphones, wireless Communication System, electronic Board operators like PCs, laptops, DSP Drives etc, Automobiles, machine tools, medical Instrumentation etc.

* Conventional Current is opposite to e^- flow.



$$f = \frac{1}{T}$$

→ freq. where magnitude changes (no phase change)

∴ it is called ripple frequency.

⇒ By Increasing Ripple freq, Ripple factor Increases.

⇒ for Ideal DC, Ripple factor = 0 (∴ no filter Required)

⇒ Solar, Newly purchased battery, precision volt. regulators
pure DC

→ as no. of pulses less ⇒ requires bulky filter ckts.

* properties of AC Supply system:

They are

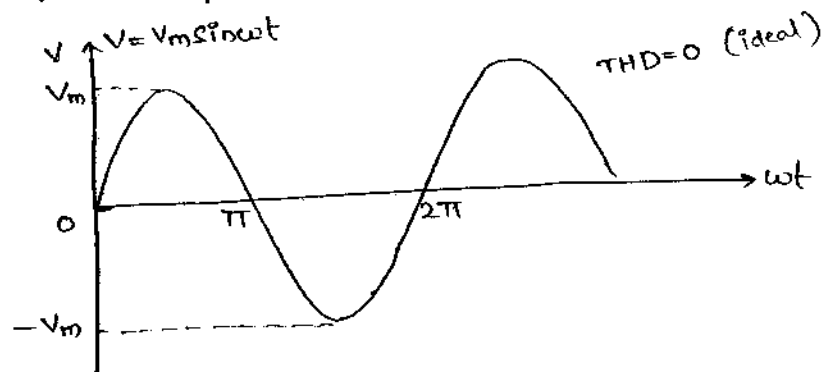
- Bipolar in Voltage
- Bidirectional in Current
- Definite change in phase
- Power freq. Exists.
- in INDIA = 50 Hz

⇒ They are used in large, Bulk, Continuously driven power supply systems.

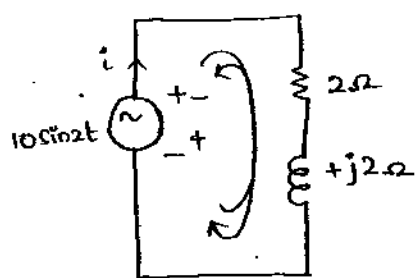
⇒ Where electrical Energy cannot be stored.

⇒ AC Supply driven systems are Superior w.r.t large power ratings, Robust design, Continuously duty cycle operation, High efficiency and less maintenance.

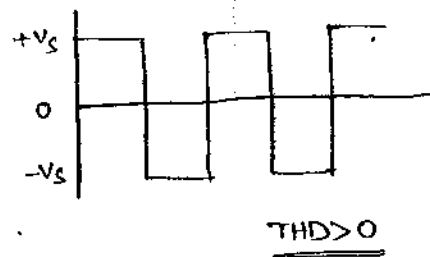
⇒ ideal AC waveform is a Sinusoid.



⇒ Example applications include generation, Transmission, Distribution, Utilisation of Elec. Energy at power level is through sinusoidal wave-form that to 3φ and then 1φ.

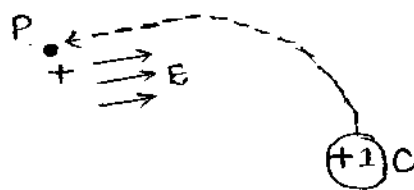


$$\left. \begin{aligned} Z &= 2 + j2\Omega \\ \phi &= \tan^{-1}\left(\frac{2}{2}\right) \\ &= 45^\circ \end{aligned} \right\}$$



1. Voltage : it is defined as the amount of work done in moving one unit positive charge from "∞" to that "point" against the intensity of electric field.

$$V = \frac{dW}{dq}$$



$\frac{-}{\equiv}$ (ground or neutral or reference)

⇒ it is like a force [EMF], which can drive charge.

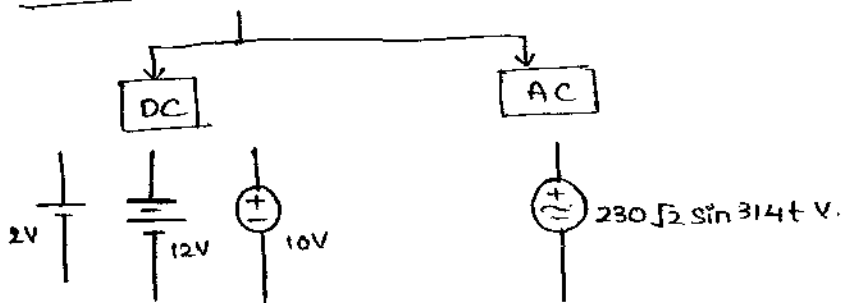
Units: Volts (or) $\frac{J}{C}$

Range: mV, V, kV, MV

12

Symbols: → v → to represent instantaneous value
 V → DC steady state
 \bar{V} → RMS of sinusoid
 $V(t)$ → for any function.

Circuit Symbols:



DC: cell, Battery, Fuel cell, p-v solar cell, Rectifiers, DC-DC Converters, SMPS, DC Gen motors

AC: Alternators [S.G],
 UPS,
 Inverters

II. Current: it is rate of flow of charge in any material.

$$i = \frac{dq}{dt}$$

$$I = \frac{Q}{t}$$

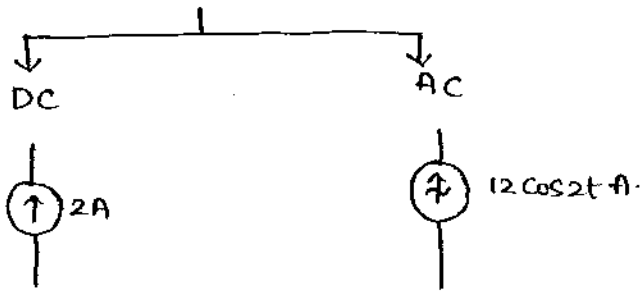
⇒ Units: Ampere (or) C/sec.

Range: mA, A, kV → fault analysis.

⇒ Symbols: i → Instantaneous
 I → DC
 \bar{I} → Sinusoidal or RMS
 $i(t)$ → any function.

* 1 MW Gen & Tran Cost
 ↓
 21 crores

Circuit Symbols



DC: BJT & dependent
Current Source

DC Series Generator

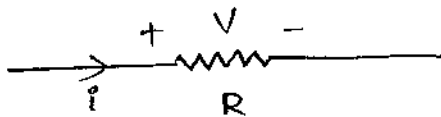
AC: Feeder line in P.S
CST Drive in P.E

III. Resistance :

it is L.T.I electrical property of matter.

Resistor: is a Component to model & design this property.

it is classified based on material.



- Nichrome } (Heater coils)
- Tungsten } High melting & Boiling points.
- Carbon }
- Cu, Al.

units : Ohms (Ω), $\frac{V}{A}$

Range : $\mu\Omega$, $m\Omega$, Ω , $k\Omega$, $M\Omega$, $G\Omega$

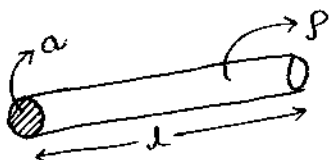
OHM's Law :

$$V = IR$$

$$I = \frac{V}{R}$$

Basic Formulae :

$$R = \frac{\rho l}{a} = \frac{l}{\sigma \cdot a}$$



$\rho \rightarrow$ Specific Resistance (or) Resistivity ($\Omega\text{-m}$)

when V is applied to R
charges scatter & collide
elec. Energy conv. Heat Energy
 \therefore the symbol is
when V is applied to Conductor
charges smoothly Travel

→ practically Resistance depends upon Temperature.

$$R_t = R_0 [1 + \alpha t]$$

(or)

$$R_2 = R_1 [1 + \alpha (t_2 - t_1)]$$

13

' α ' is temperature Coeff. of resistance

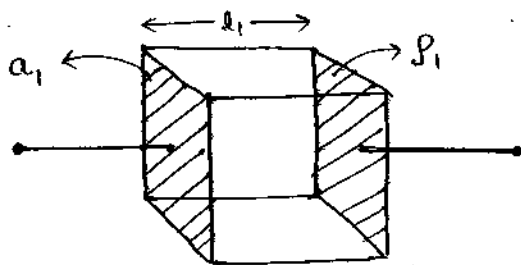
α is (+ve) → Metals (conductors)

α is (-ve) → Semiconductors.

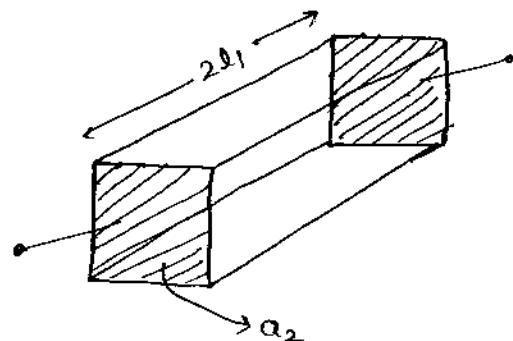
Ex:

- Heater
- Incandescent bulb
- Iron Box
- PCB
- Domestic & Industrial wiring
- Machine winding
- Motor starters & Braking.
- Tr. lines → $\Omega/\text{ph}/\text{km}$

Q the Resistance of a cube shaped material between any of its opposite phases is 3Ω , if this material is stretched in one direction by applying linear force to double its original length then the Resistance b/w its opposite stretched phases is —



$$R_1 = \frac{\rho_1 l_1}{a_1} = 3\Omega$$



$$R_2 = \frac{\rho_2 l_2}{a_2} = \frac{\rho_1 (2l_1)}{a_1/2}$$

$$\begin{aligned} V_1 &= V_2 \\ l_1 a_1 &= l_2 a_2 \\ \therefore a_1 &= 2 a_2 \end{aligned}$$

$$a_2 = \frac{a_1}{2}$$

$$\begin{aligned} &= 4 \cdot \frac{\rho_1 l_1}{a_1} = 4R_1 \\ &= 4(3) \\ &= \underline{\underline{12\Omega}} \end{aligned}$$

$$R = \frac{\rho l}{a}$$

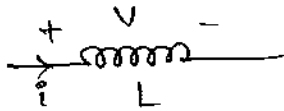
$$= \frac{2\rho l_1}{a_1/2}$$

⇒ if length ↑ by n times then Resistance ↑ by n^2 times

IV Inductance: it is electro magnetic property of matter

Inductor: is a Component to model & design this property

it is Classified based on CORE material.



- Iron
- Ferrite
- SiFe
- air

units: Henry (or) $\frac{\text{Volt-sec}}{\text{amp.}}$

Range: μH , mH , H .

OHM's Law:

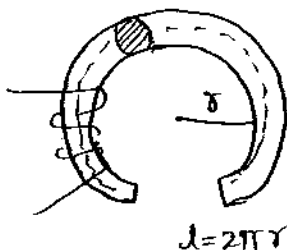
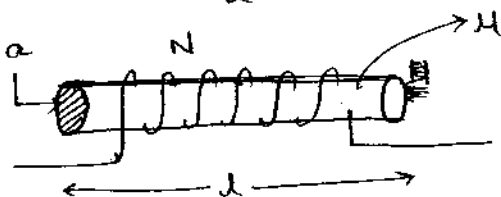
$$V = L \frac{di}{dt} \quad i = \frac{1}{L} \int v dt$$

$$i = \frac{1}{L} \int_{-\infty}^t v dt = \underbrace{\left(\frac{1}{L} \int_{-\infty}^0 v dt \right)}_{\text{I.C. (Initial Condition)}} + \frac{1}{L} \int_0^t v dt$$

$$i(t) = I(0) + \frac{1}{L} \int_0^t v dt$$

Basic formulae:

$$L = \frac{\mu N^2 a}{l}$$



$\mu = \mu_0 \mu_r$] permeability of CORE

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m.}$$

$$\mu_r = 1 \text{ (air)}$$

$$\mu_r > 1000 \text{ (iron)}$$

$N \rightarrow$ no. of Turns of coil

$a \rightarrow$ Cross-Sectional area of CORE (m^2)

$l \rightarrow$ effective length of magnetic flux path (m)

14

Ex:

\rightarrow choke coil

\rightarrow C.L.R (Current Limiting Resistor)

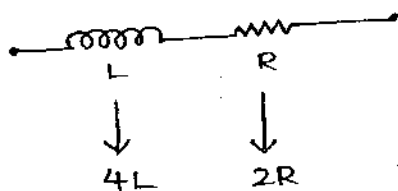
\rightarrow Transformer

\rightarrow Electromagnets

\rightarrow A/C machine wdg

\rightarrow Tr lines \rightarrow mH/ph/km

A practical coil has a time constant of $\tau = \frac{L}{R}$ sec. if no. of turns in the coil are doubled then its new time constant is —

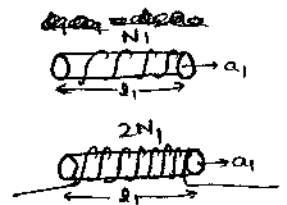


$$L_1 = \frac{\mu N_1^2 a_1}{l_1}$$

$$L_2 = \frac{\mu (2N_1)^2 a_1}{l}$$

$$L_2 = 4L_1$$

$$N_2 = 2N_1$$

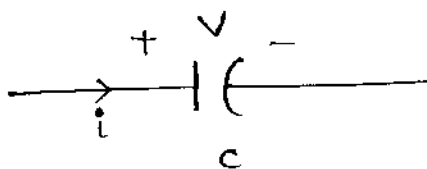


$$\tau_{\text{new}} = \frac{4L}{2R} = 2 \frac{L}{R} \text{ sec}$$

V. Capacitance: it is electro-static property of matter.

Capacitor: is a Component to model & design this property.

it is classified based on Dielectric material.



units: farad (or) $\frac{\text{Amp-sec}}{\text{Volt}}$

Range: pF, nF, μ F, mF

- polyester
- mica
- distilled water
- oil
- paper
- glass.

OHM's Law: $i = C \frac{dv}{dt}$ $V = \frac{1}{C} \int i dt$

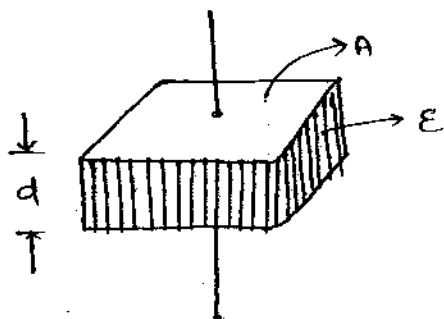
$$V = \frac{1}{C} \int_{-\infty}^t i dt = \left(\frac{1}{C} \int_{-\infty}^0 i dt \right) + \frac{1}{C} \int_0^t i dt$$

Initial Condition

$$V(t) = V(0) + \frac{1}{C} \int_0^t i(t) dt$$

Basic formulae:

$$C = \frac{\epsilon A}{d} \text{ F.}$$



$\epsilon = \epsilon_0 \epsilon_r \rightarrow$ permittivity of dielectric

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$\epsilon_r = 1 \text{ (air)}$$

$$\epsilon_r = 6 \text{ (polyester)}, \epsilon_r = 200 \text{ (glass)}$$

$A \rightarrow$ Common cross sectional area b/w electrodes (m^2)

$d \rightarrow$ distance b/w electrodes (m)

Ex:

- \rightarrow passive R.C filter
- \rightarrow phase shifting
- \rightarrow pulse, digital, wave shaping
- \rightarrow P.F & VAR Correction
- \rightarrow Touch - Screens of Cell phone.
- \rightarrow Tr. lines \rightarrow $\mu\text{F/ph/km}$

* TouchScreen of Cellphone is Capacitor

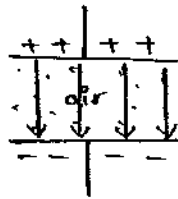
Screen is one plate (electric field)

glass is dielectric

our finger is another plate
acting

Q a parallel plate capacitor is dipped in ethyl alcohol with rel. permittivity $\epsilon_r = 25$ upto half the distance b/w plates. determine the ratio of Capacitance before & after dipping in ethyl alcohol.

$$C_{\text{Before}} = \frac{\epsilon_0 (1) \cdot A}{d}$$



as $\epsilon_r = 1$
very less
charge distribution
across plates
less uniform Elec. fld Inten.

$$C = \frac{\epsilon A}{d}$$

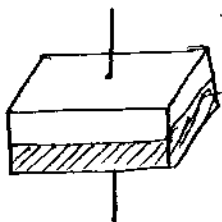
$$\epsilon_r = \epsilon_0 \epsilon_r$$

$$\epsilon_r = \epsilon_0 \cdot 25$$

$$C = \frac{\epsilon_0 \cdot 25 \times A}{4 \times 2}$$

(15)

after dipping



Series Combination
of Capacitor

$$C_{\text{After}} = \frac{\epsilon_0 (1) A}{d/2} + \frac{\epsilon_0 (25) A}{d/2}$$

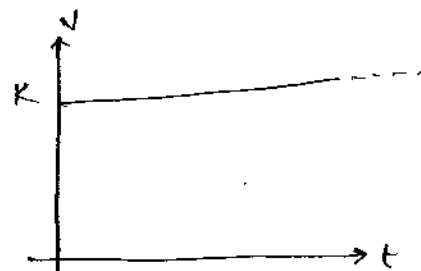
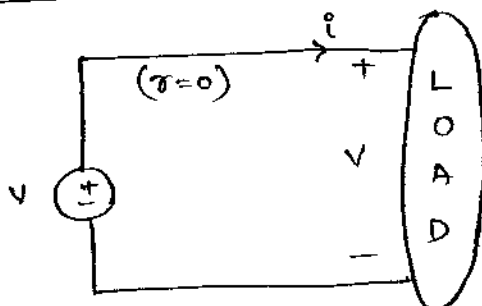
$$\frac{\epsilon_0 A}{d/2} [1 + 25]$$

$$C_{\text{After}} = \frac{50}{26} \left[\frac{\epsilon_0 A}{d} \right]$$

→ C_{Before}

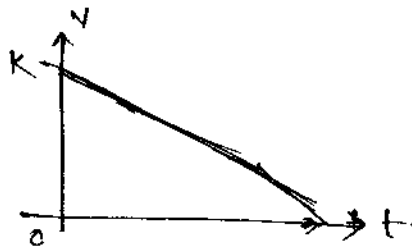
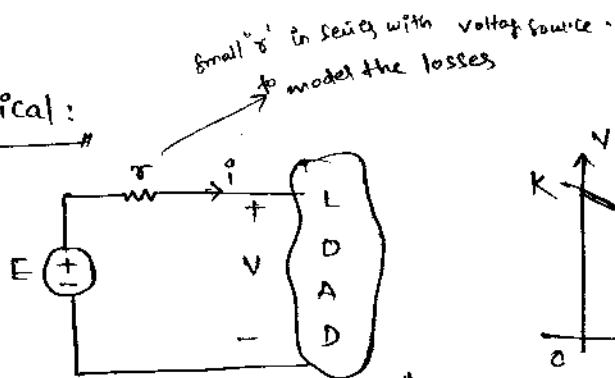
$$\frac{C_{\text{Before}}}{C_{\text{After}}} = \frac{26}{50} = \frac{13}{25}$$

Ideal voltage Source:



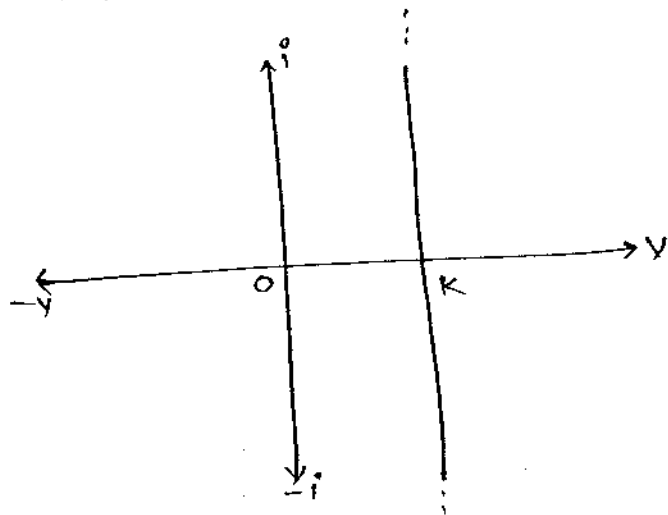
⇒ no volt. Reg
⇒ 100% eff.

practical:



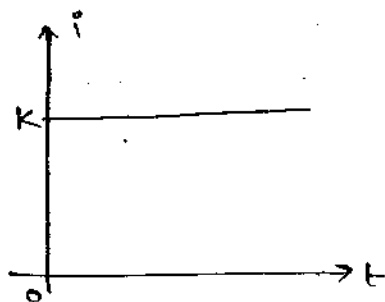
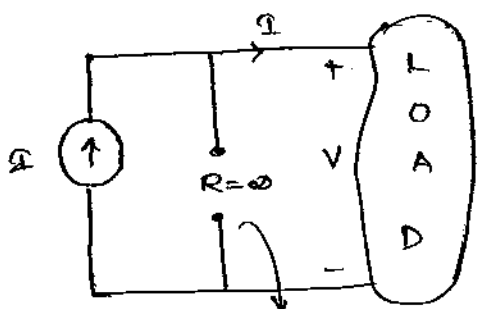
$V = E - ir$ as load $\uparrow \Rightarrow V < E$

Based on static $V-i$:



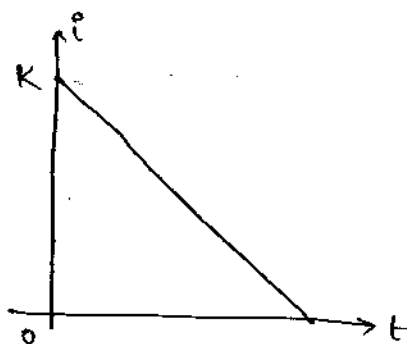
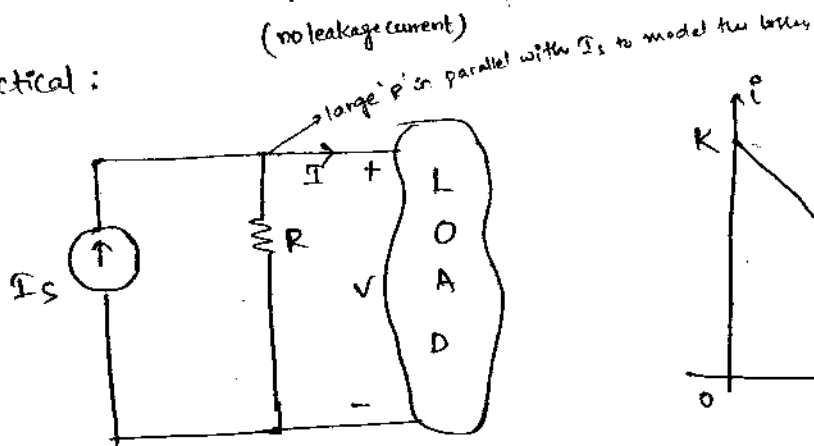
voltage should be stiff for any current.

Ideal Current Source.



(no leakage current)

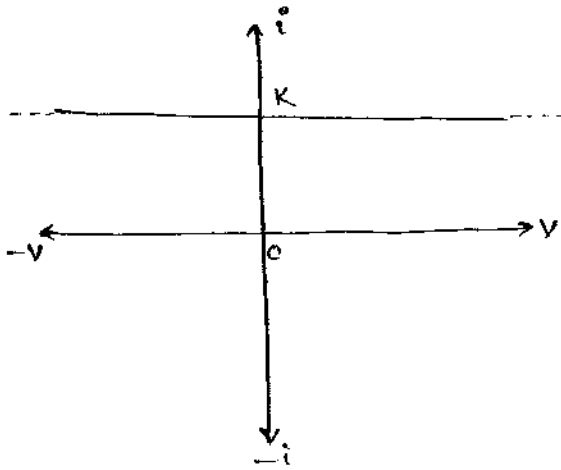
practical:



$I = I_s - \frac{V}{R}$

Based on static $V-i$:

16



Independent Sources:

⇒ if the properties & characteristics are Independent to any other parameter within or outside the ckt.

⇒ We assume ideal Sources to be Independent.

⇒ They are Represented by "○" → circle symbol.

dependent Sources:

⇒ if the properties & characteristics depends upon any other parameter within or outside the ckt.

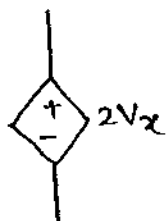
Example: 1. a BJT acts as Current Controlled Current Source

2. a Solar cell is the light dependent. Voltage Source

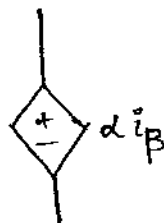
3. power electronic Converters are dependent Sources as their characteristics depends upon type of load (R, R_L, R_{LE} etc)

⇒ They are represented by "◇" → diamond symbol.

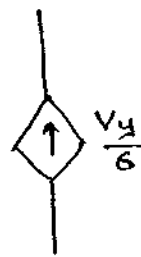
⇒ They are standard 4-types.



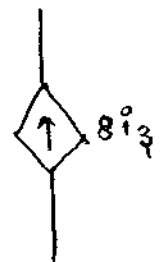
VDVS
(v)
VCVS



CDVS
(v)
CCVS



VDCS
(a)
VCCS



CCCS
(a)
CDCS

Note: unlike independent source these dependent sources will have Internal Subcircuit Resistance which must be considered during analysis of ckt.

power: it is Rate of change in Elec. Energy.

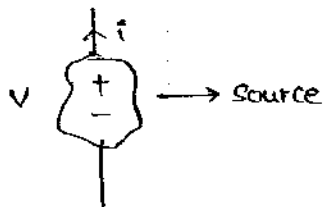
$$P = \frac{dE}{dt} = \frac{dw}{dt}$$

$$P = \frac{E}{t}$$

units: Watts (or) $\frac{\text{Joule}}{\text{sec.}}$

1 hp = 746 Watts.

Range: mw, w, kw, mw, GW.



$$\begin{aligned} P_{\text{delivered}} &= +V \cdot i \text{ W} \\ &= \frac{dw}{dq} \cdot \frac{dq}{dt} \\ &= \frac{dw}{dt} \end{aligned}$$

R

$$P_R(t) = V_R(t) \cdot I_R(t) = [I_R(t)]^2 \cdot R = \frac{[V_R(t)]^2}{R} \text{ W.}$$

→ if Excitation is T.T [S.S]

$$P_R = V_R I_R = I_R^2 R = \frac{V_R^2}{R}$$

~~AC~~
for AC → RMS value

L

→ $P_{\text{avg}} = 0$ → for one cycle supply waveform
→ Instantaneous Value

$$P_L = V_L \cdot i_L = L \cdot i \frac{di}{dt} \text{ W}$$

C

→ $P_{avg} = 0$ → for one cycle of supply waveform
 → Instantaneous Value.

17

$$P_C = V_C \cdot i_C = C \cdot V \cdot \frac{dV}{dt} \rightarrow W. \quad \text{for AC} \rightarrow \text{RMS Values.}$$

Energy: it is the capacity to do work. (or to drive system)

$$E = \int_{-\infty}^t P dt = \underbrace{\int_{-\infty}^0 P dt}_{\text{Initial stored energy}} + \int_0^t P dt$$

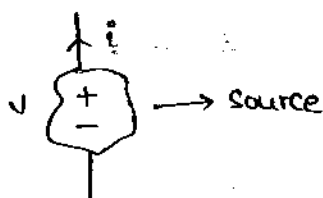
$$E(t) = E(0) + \int_0^t P(t) dt \quad J$$

$$\boxed{E = P * t.}$$

→ units: J (or) Watt-sec.

→ 1 unit of Elec. Energy = 1 KWH.

$$\begin{aligned} 1 \text{ KWH} &= 1000 \text{ W} * 1 \text{ hr} \\ &= 500 \text{ W} * 2 \text{ hr} \\ &= 100 \text{ W} * 10 \text{ hr} \\ &= 2000 \text{ W} * \frac{1}{2} \text{ hr}. \end{aligned}$$



$$E_{\text{delivered}} = + V \cdot i \cdot t \quad \begin{matrix} \text{r(sec)} \\ J \end{matrix}$$

$$1 \text{ KWH} = 36 * 10^5 \text{ J}.$$

R

$$E_R(t) = \int P_R(t) dt = \int V_R(t) \cdot I_R(t) \cdot dt = \int [I_R(t)]^2 \cdot R dt \quad J$$

⇒ in S.S

$$E_R = V_R \cdot I_R t = I_R^2 \cdot R \cdot t = \frac{V_R^2}{R} \cdot t \quad J.$$

□

$$E_L(t) = \int P_L(t) \cdot dt = \int L i(t) \cdot \frac{di(t)}{dt} \cdot dt \quad J.$$

→ in S.S [if Excitation is T.I]

in L
i → state variable

$$E_L = \int L i \cdot \frac{di}{dt} \cdot dt = \frac{1}{2} L i^2 \quad J.$$

$$\boxed{\psi = Li}$$

$$E_L = \frac{1}{2} L i^2 = \frac{1}{2} \psi \cdot i = \frac{\psi^2}{2L} \quad J.$$

□

$$E_C(t) = \int P_C(t) dt = \int C \cdot V(t) \cdot \frac{dV(t)}{dt} \cdot dt$$

in C
V → state variable.

→ in S.S [if Excitation is T.I]

$$E_C = \int C \cdot V \cdot \frac{dV}{dt} \cdot dt = \frac{1}{2} C V^2 \quad J.$$

$$\therefore \boxed{q = CV}$$

$$E_C = \frac{1}{2} C V^2 = \frac{1}{2} q V = \frac{q^2}{2C}$$

In Batteries Energy storage Capacity is given by Ampere-Hour Rating
(assuming Voltage is constant)

ex: AA pencil cell → 1.5V, 500 mAh

cell phone → 3.7V, 5400 mAh

CAR → 12V, 40 Ah

$$40 \text{ Ah} = 40 \text{ A} \cdot 1 \text{ hr}$$

$$= 20 \text{ A} \cdot 2 \text{ hr}$$

$$= 10 \text{ A} \cdot 4 \text{ hr}$$

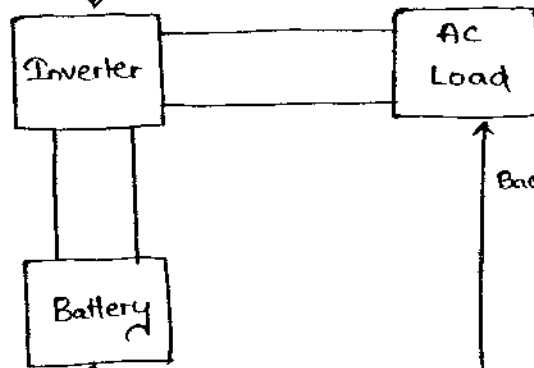
$$= 80 \text{ A} \cdot \frac{1}{2} \text{ hr}$$

$$= 1 \text{ A} \cdot 40 \text{ hr}$$

*

Inverter Rating decide the Load capacity.

* PF of P.E device is low



Backup time.

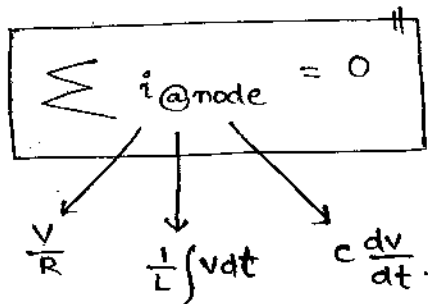
Battery AHR Rating decide

18

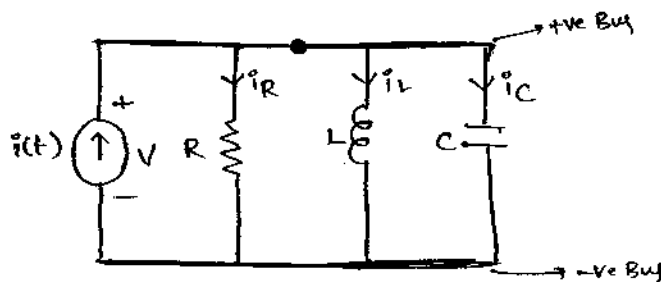
* Kirchhoff's Laws :

I. KCL (or) K "Node" L

→ Based on Law of Conservation of charge.



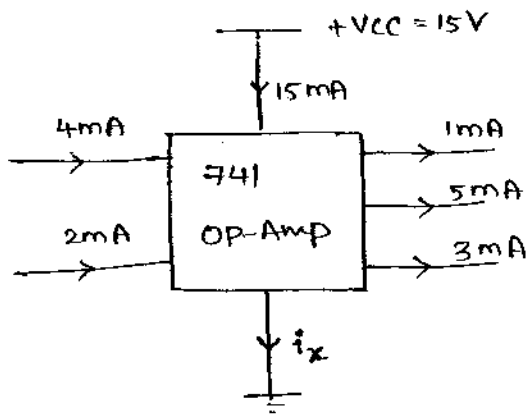
→ Notation :



KCL: $-i(t) + i_R + i_L + i_C = 0$

$$i(t) = \frac{V}{R} + \frac{1}{L} \int v dt + C \frac{dv}{dt}$$

Q.



$$i_x =$$

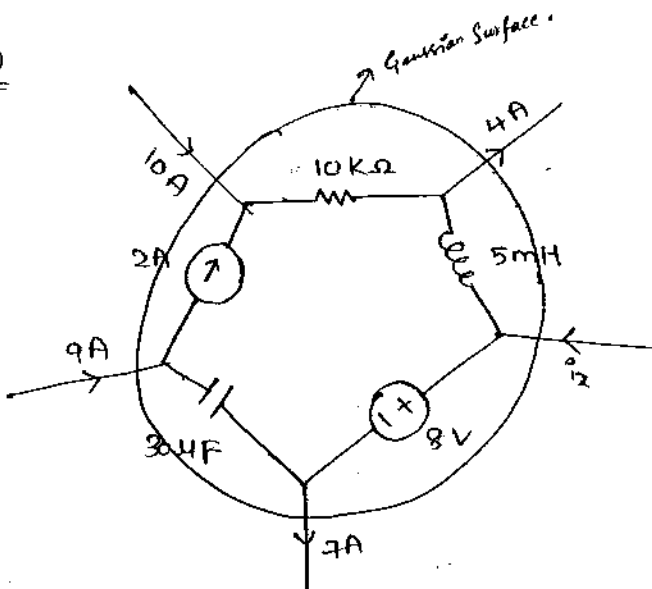
$$15 + 4 + 2 = i_x + 5 + 1 + 3$$

$$i_x = 21 - 9 = \underline{\underline{+12 \text{ mA}}}$$

21mA
-9mA
12mA
21mA
12mA

Analog IC
741-Op-amp
↓
Int.ckt
circuit
↓
At DC 130 MHz/fn
Texas Inst
Min 60 Transistors

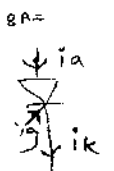
Q



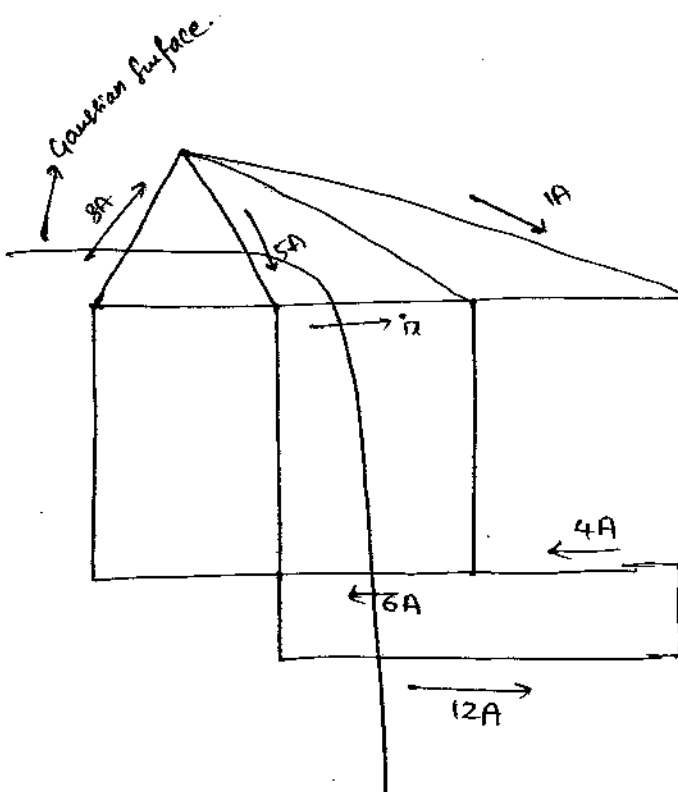
$$i_x =$$

$$10 + i_x + 9 = 4 + 7$$

$$i_x = \underline{\underline{+8 \text{ A}}}$$



Q



$$8 + i_x + 12 = 5 + 6$$

$$i_x = \underline{\underline{+9 \text{ A}}}$$

Gaussian surface
divide the n/w
into two parts.

II). KVL \longrightarrow K^u Mesh^u L

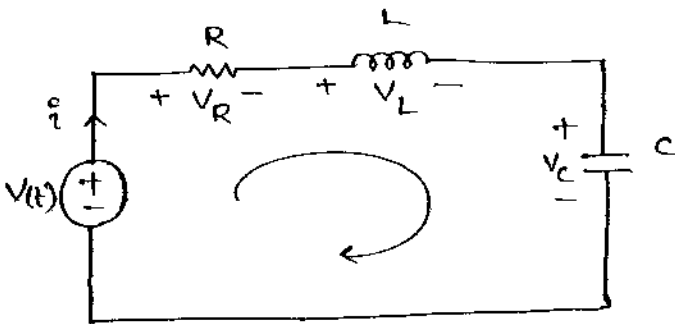
→ Based on Law of Conservation of Energy.

$\sum V_{@mesh} = 0$

↓

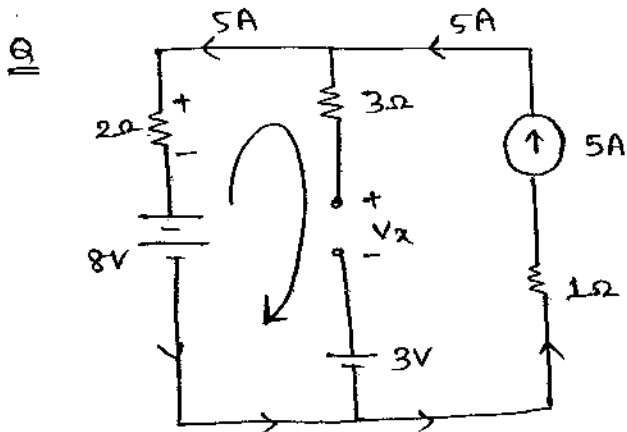
iR $L \frac{di}{dt}$ $\frac{1}{c} \int i dt$

→ Notation :



$$\text{KVL} \rightarrow -V(t) + V_R + V_L + V_C = 0$$

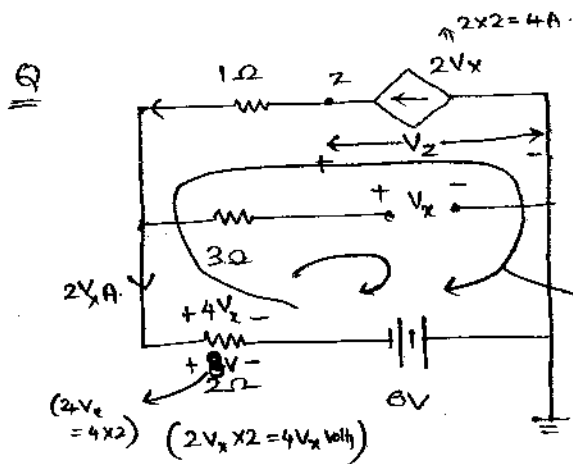
$$V(t) = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt$$



$$V_X =$$

$$-8 - 10 + V_2 + 3 = 0$$

$$V_2 = +15V.$$



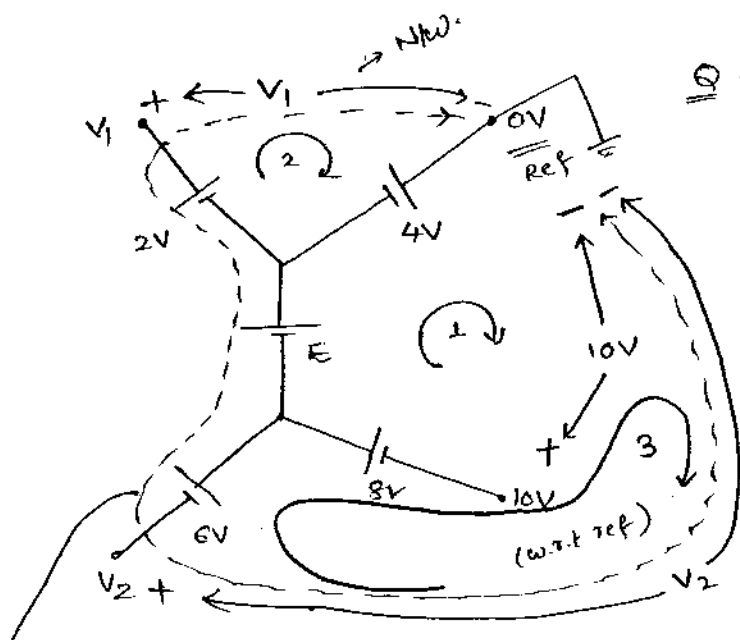
$$V_2 =$$

$$6 - 4V_2 + V_2 = 0$$

$$V_2 = 2V$$

$$+6 - 8 - 4 + V_2 = 0$$

$$V_Z = +6 \text{ Volts.}$$



Q find $E = \underline{\hspace{1cm}}$, $V_1 = \underline{\hspace{1cm}}$, $V_2 = \underline{\hspace{1cm}}$

if 0V or
if not given
take any ref.
outside the N/W

$$\text{KVL1: } -10 - 8 - E - 4 = 0 \Rightarrow E = -22\text{V}$$

$$\text{KVL2: } 4 - 2 + V_1 = 0 \Rightarrow V_1 = -2\text{V}$$

$$\text{KVL3: } -V_2 - 6 + 8 + 10 = 0 \Rightarrow V_2 = +12\text{V}$$

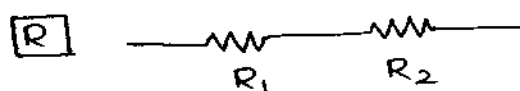
Check:

$$-V_1 + 2 + E + 6 + V_2$$

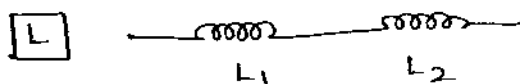
$$+2 + 2 - 22 + 6 + 12 = 0 \checkmark$$

Series Connection of elements:

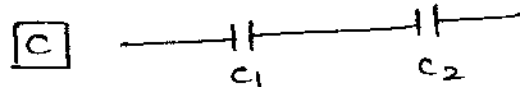
→ if current through them is Equal.



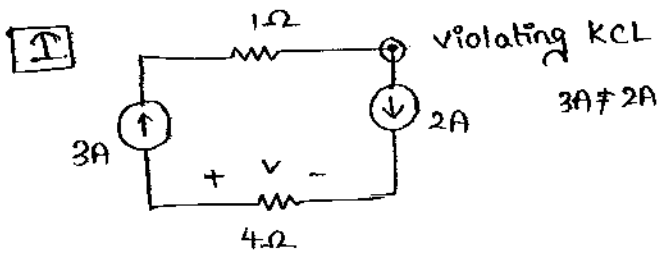
$$R_s = R_1 + R_2$$



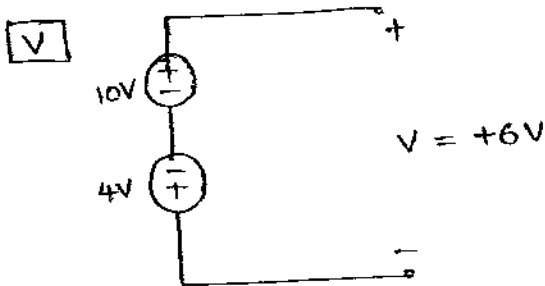
$$L_s = L_1 + L_2$$



$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

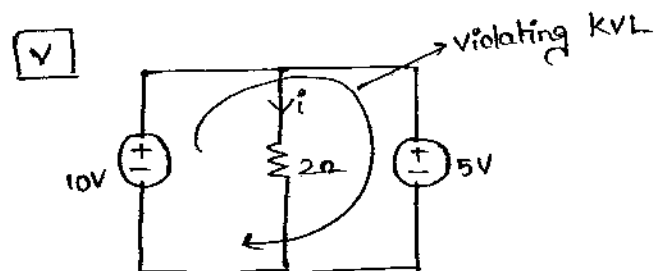
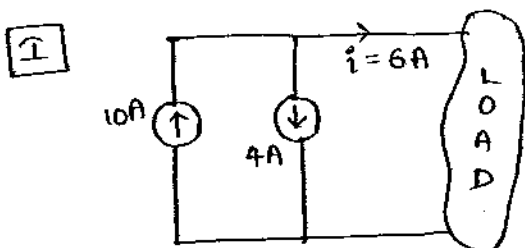
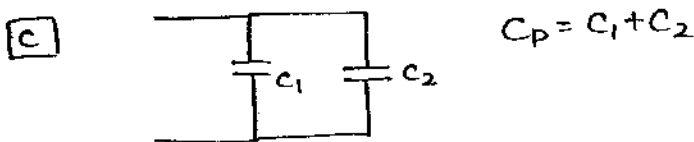
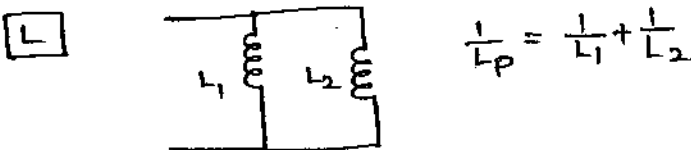
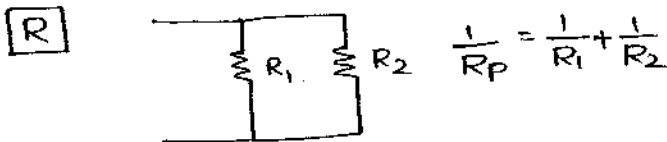


20



parallel Connection of elements:

→ Voltages across them is Equal (in magnitude & direction).



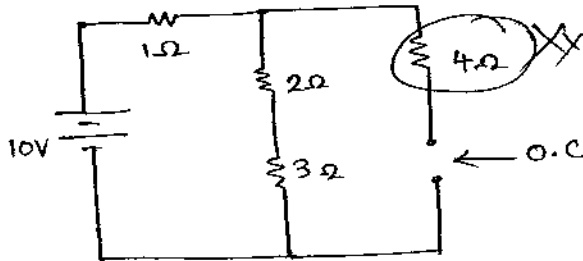
HVDC single pole = 800 kV.
 Thyristor Blocking Voltage = 10 kV
 Blocking current = 5 kA

Open circuit : (O.C)

⇒ In an O.C, $i = 0$ for any voltage

$$R_{oc} = \frac{V}{0} = \infty \Omega$$

⇒ Any passive element completely in series to O.C can be neglected.

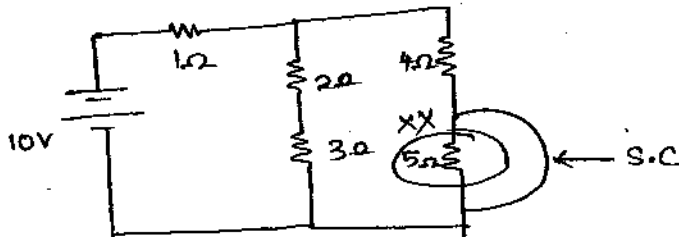


Short circuit (S.C):

⇒ In a S.C, $V = 0$, for any current

$$R_{sc} = \frac{0}{i} = 0 \Omega$$

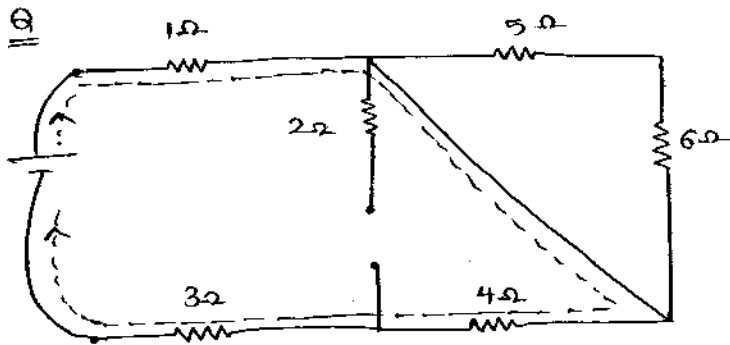
⇒ any passive element completely in parallel to S.C, can be neglected.



⇒ open ckt live wire faults in High voltage Engineering are more dangerous than S.C due to insulation material limitations. However S.C can always be protected both at L.V & H.V level by using fuses & C.B.

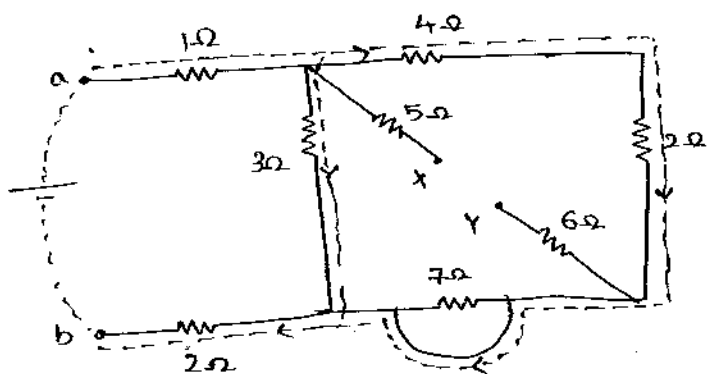
⇒ Resistance is offered by the path where current can flow, as seen by target terminals

⊙
 C.B. ON → UP → Becz going against gravity is more
 OFF → DOWN
 S.C are protectable at H.V, L.V side by providing fuses & C.B

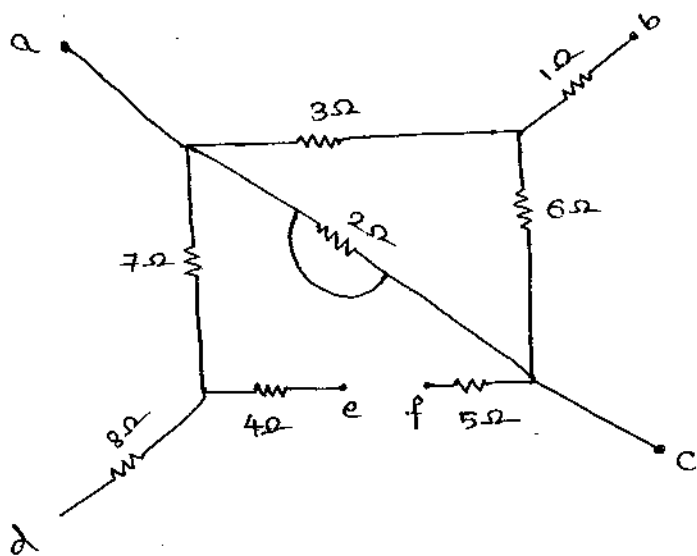
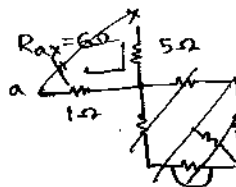
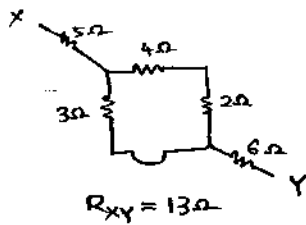
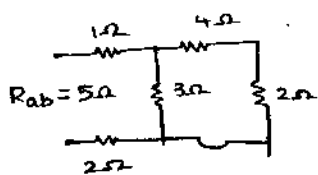


$$R = 1 + 4 + 3 = 8\Omega$$

21



$$R_{ab} = \frac{5\Omega}{}, R_{xy} = \frac{13\Omega}{}, R_{ax} = \frac{6\Omega}{}, R_{by} = \frac{8\Omega}{}$$



$$R_{ab} = 3\Omega$$

$$R_{ac} = 0\Omega$$

$$R_{ad} = 15\Omega$$

$$R_{bd} = 18\Omega$$

$$R_{ef} = 16\Omega$$

$$R_{fb} = 8\Omega$$

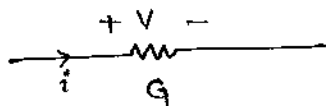
Conductance: it is the ability to conduct electrically.

\Rightarrow it is used to further classify metals (conductors)

$$G = \frac{1}{R}$$

units: (S) mho

Siemens



OHM's Law:

$$V = \frac{I}{G}$$

$$I = V \cdot G$$

Basic formulae:

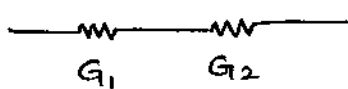
$$G = \frac{a}{\rho l} = \frac{\sigma \cdot a}{l}$$

$\sigma \rightarrow$ conductivity

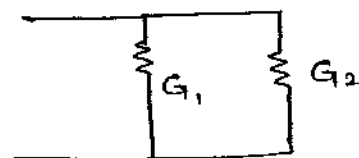
units of $\sigma \Rightarrow [\Omega \cdot m]^{-1}$

S/m (or) S/m

$\rho \rightarrow \Omega \cdot m$



$$\frac{1}{G_s} = \frac{1}{G_1} + \frac{1}{G_2}$$



$$G_p = G_1 + G_2$$

\Rightarrow power

$$P_G = V_G \cdot i_G = \frac{i_G^2}{G} = V_G^2 \cdot G$$

Based on Conductivity

Rank I → Silver

-II → Copper

-III → Gold

-IV → Aluminium

it is used in High current Density (compact)
used in - Domestic, Industrial, machines

22

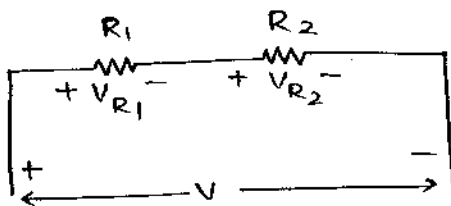
light weight
cheap

used in External 'T & D' lines
⇒ A.C.S.R.

* Voltage division Rule :

→ applicable for Series Connected elements only.

[R]



$$V_{R1} = V \left[\frac{R_1}{R_1 + R_2} \right]$$

$$V_{R2} = V \left[\frac{R_2}{R_1 + R_2} \right]$$

[L]

$$V_{L1} = V \left[\frac{L_1}{L_1 + L_2} \right]$$

$$V_{L2} = V \left[\frac{L_2}{L_1 + L_2} \right]$$

[C]

$$V_{C1} = V \left[\frac{C_2}{C_1 + C_2} \right]$$

$$V_{C2} = V \left[\frac{C_1}{C_1 + C_2} \right]$$

[G]

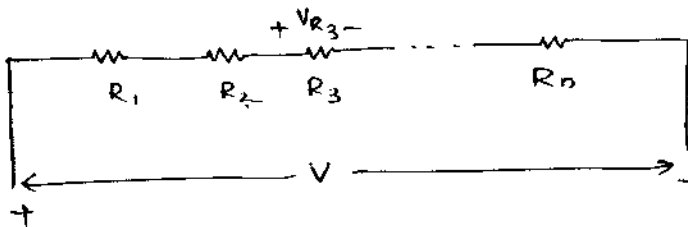
$$V_{G1} = V \left[\frac{G_2}{G_1 + G_2} \right]$$

$$V_{G2} = V \left[\frac{G_1}{G_1 + G_2} \right]$$

Note:

$$\left. \begin{array}{l} 'R' \Omega \longleftrightarrow 'L' H \longleftrightarrow 'X' \Omega \longleftrightarrow 'Z' \Omega \\ 'G' \Omega \longleftrightarrow 'C' F \longleftrightarrow 'B' \Omega \longleftrightarrow 'Y' \Omega \end{array} \right\} \begin{array}{l} \text{for voltage division \&} \\ \text{Current division} \end{array}$$

*



$$V_{R_3} = V \left[\frac{R_3}{\sum_{i=1}^n R_i} \right]$$

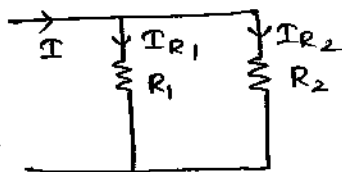
* for capacitors in series,

$$V_{C_3} = V \left[\frac{\frac{1}{C_3}}{\sum_{i=1}^n \frac{1}{C_i}} \right]$$

Current division Rule:

→ applicable for parallel Connected elements only.

[R]



$$I_{R_1} = I \left[\frac{R_2}{R_1 + R_2} \right]$$

$$I_{R_2} = I \left[\frac{R_1}{R_1 + R_2} \right]$$

[L]

$$I_{L_1} = I \left[\frac{L_2}{L_1 + L_2} \right]$$

$$I_{L_2} = I \left[\frac{L_1}{L_1 + L_2} \right]$$

[C]

$$I_{C_1} = I \left[\frac{C_1}{C_1 + C_2} \right]$$

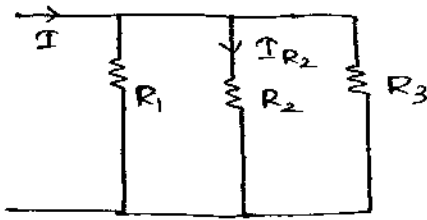
$$I_{C_2} = I \left[\frac{C_2}{C_1 + C_2} \right]$$

[G]

$$I_{G_1} = I \left[\frac{G_1}{G_1 + G_2} \right]$$

$$I_{G_2} = I \left[\frac{G_2}{G_1 + G_2} \right]$$

⇒

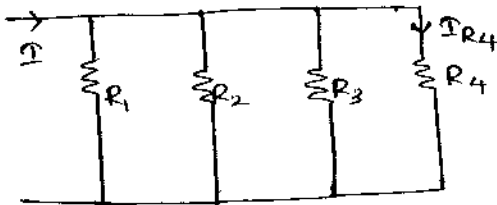


$$I_{R_2} = I \left[\frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} \right]$$

⇒ for 3 capacitors in parallel 23

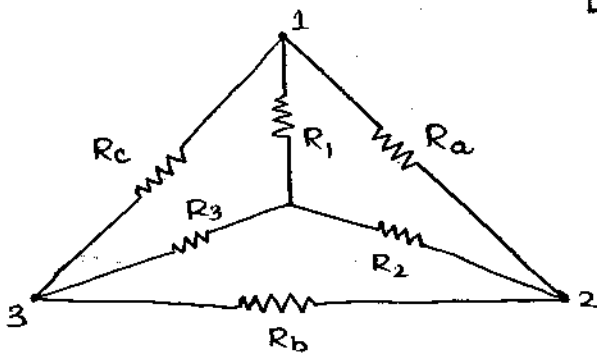
$$I_{C_2} = I \left[\frac{\frac{1}{C_1} \cdot \frac{1}{C_2}}{\frac{1}{C_1} \cdot \frac{1}{C_2} + \frac{1}{C_2} \cdot \frac{1}{C_3} + \frac{1}{C_3} \cdot \frac{1}{C_1}} \right]$$

⇒



$$I_{R_4} = I \left[\frac{R_1 R_2 R_3}{R_1 R_2 R_3 + R_2 R_3 R_4 + R_3 R_4 R_1 + R_4 R_1 R_2} \right]$$

Δ - Y Transformation : (Invent 1 node)



[R]

$$R_1 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_b R_c}{R_a + R_b + R_c}$$

[L]

$$L_1 = \frac{L_a L_c}{L_a + L_b + L_c}$$

$$L_2 = \frac{L_a L_b}{L_a + L_b + L_c}$$

$$L_3 = \frac{L_b L_c}{L_a + L_b + L_c}$$

[C]

$$\frac{1}{C_1} = \frac{\frac{1}{C_a} \cdot \frac{1}{C_c}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

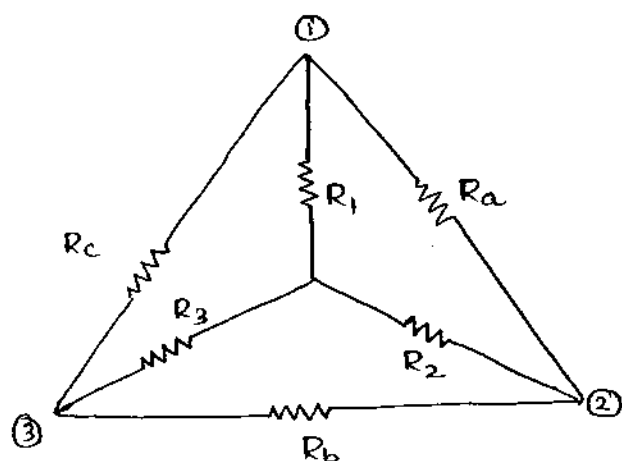
$$\frac{1}{C_2} = \frac{\frac{1}{C_a} \cdot \frac{1}{C_b}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

$$\frac{1}{C_3} = \frac{\frac{1}{C_b} \cdot \frac{1}{C_c}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

[G]

$$\frac{1}{G_1} = \frac{\frac{1}{G_a} \cdot \frac{1}{G_c}}{\frac{1}{G_a} + \frac{1}{G_b} + \frac{1}{G_c}}$$

Y- Δ Transformation: (reduces 1 node)



[R]

$$R_a = \frac{R_1 R_3 + R_2 R_3 + R_1 R_2}{R_3}$$

$$R_b = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

$$R_c = \frac{R_2 R_3 + R_1 R_2 + R_3 R_1}{R_2}$$

[L]

$$L_a = \frac{L_1 L_3 + L_2 L_3 + L_1 L_2}{L_3}$$

$$L_b = -$$

$$L_c = -$$

[C]

$$\frac{1}{C_a} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{\frac{1}{C_1} \cdot \frac{1}{C_2}}{\frac{1}{C_3}}$$

$$\frac{1}{C_b} =$$

$$\frac{1}{C_c} =$$

[G]

$$\frac{1}{G_a} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{\frac{1}{G_1} \cdot \frac{1}{G_2}}{\frac{1}{G_3}}$$

$$\frac{1}{G_b} =$$

$$\frac{1}{G_c} =$$

Rating's (or) Specifications:

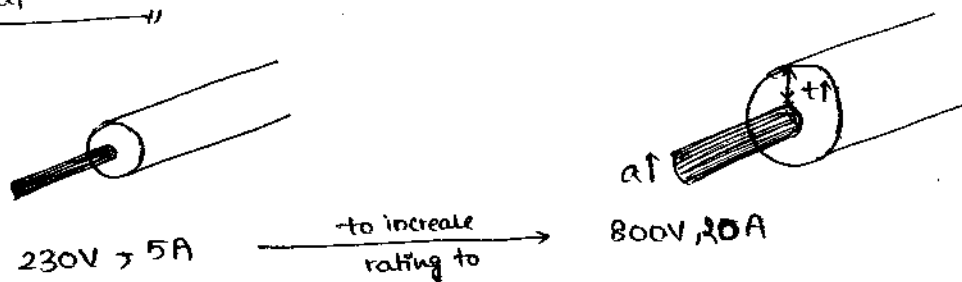
⇒ They represent the maximum permissible safe values for continuous operation of an electrical device or application.

⇒ We generally consider Voltage, Current, power, frequency, Insulation class rating with Temperature limitations.

⇒ operating any device or application above the rated values leads to deterioration & damage

⇒ but operating below the rated values leads to under utilisation i.e., degeneracy.

practical wire:



24

to $\uparrow \Rightarrow$ Conductor cross-sectional Area \uparrow

$\uparrow \Rightarrow$ Insulation level \uparrow

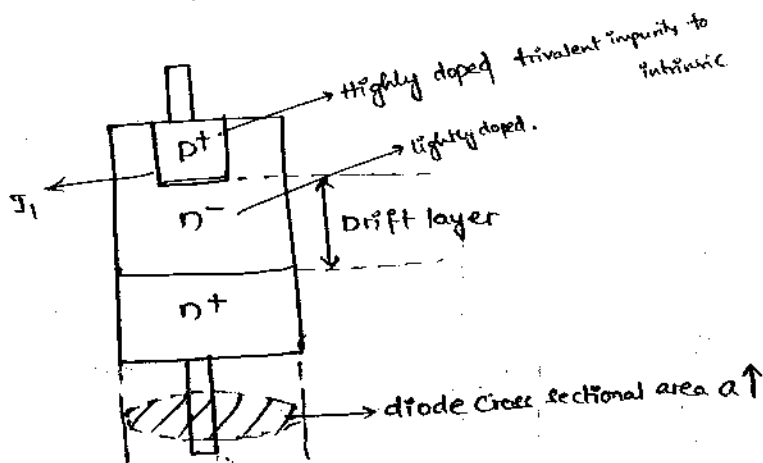
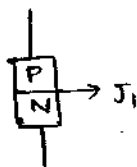
* Signal Diode vs power diode.

1N4007

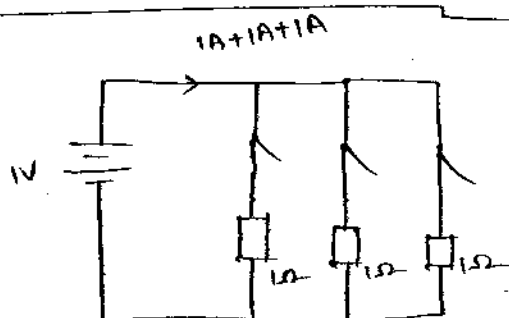
- 60V, 800mA.

DI04

→ 1000V, 40A



$$\left[\text{Breakdown Voltage} \right] \propto \frac{1}{\text{Doping density}}$$



Note: most of our applications are designed to work for constant Rated voltage only.

\Rightarrow But Current through them depends upon Loading level.

Low Wattage

V
 $I \downarrow$
 $a \downarrow$
 (thin wire)
 $R_{High} \uparrow$

High Wattage

V
 $I \uparrow$
 $a \uparrow$
 (thick wire)
 $R_{Low} \downarrow$

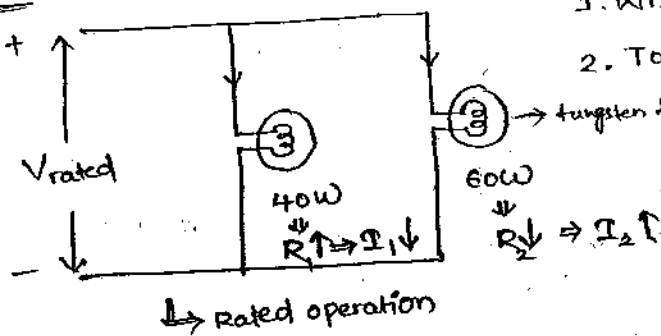
⇒ DC Motors

220V, $R_a = 1 \Omega$
 $\xrightarrow{\quad} R \text{ is High} \Rightarrow a \downarrow \Rightarrow I \downarrow \Rightarrow P (= V \cdot I) \text{ is less} \Rightarrow \text{Low H.P.}$

220V, $R_a = 0.2 \Omega$
 $\xrightarrow{\quad} R \text{ is low} \Rightarrow a \uparrow \Rightarrow I \uparrow \Rightarrow P (= V \cdot I) \text{ is more} \Rightarrow \text{more H.P.}$

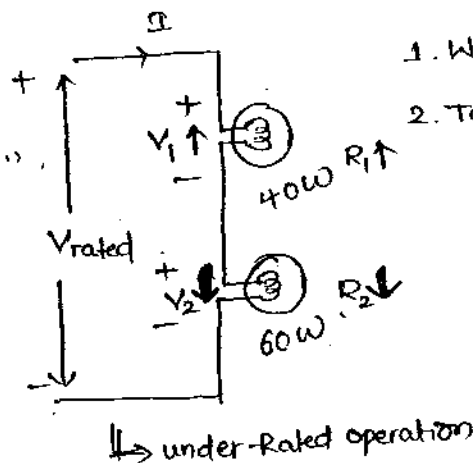
CFL - II Gen. lamp
 \hookrightarrow Inverter, 400Hz
 High freq \Rightarrow less weight

IESQ



1. Which bulb glows brighter 60W
2. Total power absorbed $60 + 40 = 100W$ 60W 40W
 tungsten filament lamps, pure Resistive, consumes more energy for less lumens.
 (unity P.F.)

Gate:



1. Which bulb glows brighter 40W.
2. Total power absorbed 100W

$$= \frac{P_1 P_2}{P_1 + P_2}$$

$$= \frac{40 \times 60}{40 + 60}$$

$$= 24 \text{ watts only.}$$

lowest thermal cost \Rightarrow Tungsten
 lowest thermal time const

Heater - Nichrome.

$$V = V_1 + V_2$$

$$V = V_1 + V_2$$

$$60 + 40 = 100W$$

$$R_{\text{designed}} = \frac{[V_{\text{rated}}]^2}{P_{\text{rated}}} \Omega \rightarrow \text{for under rated.}$$

25

$$\therefore P_T = \frac{V_T^2}{R_1 + R_2} = \frac{V_T^2}{\frac{V_T^2}{P_1} + \frac{V_T^2}{P_2}}$$

$$P_T = \frac{V_T^2}{V_T^2 \left[\frac{1}{P_1} + \frac{1}{P_2} \right]} = \frac{P_1 P_2}{P_1 + P_2}$$

Ex! INDIA.
Incandescent Bulb.
220V, 40W.

OFF

$$R_{\text{cold}} = 20 \Omega \text{ [Multimeter]}$$

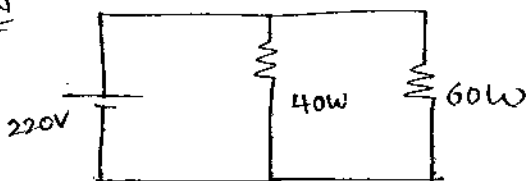
$$I = \frac{V}{R_{\text{cold}}} = \frac{220}{20} = 11 \text{ A} \times \text{Wrong}$$

ON

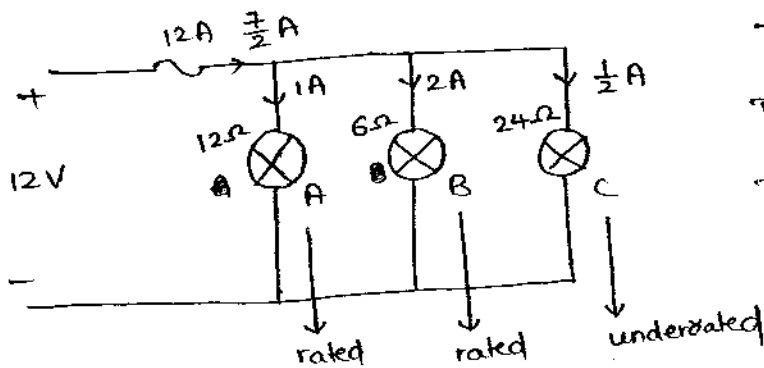
$$R_{\text{hot}} = \frac{(220)^2}{40} = 1210 \Omega$$

$$I = \frac{V}{R_{\text{hot}}} = \frac{220}{1210} \approx 180 \text{ mA} \checkmark \text{ Correct}$$

4W Same bulb problem with 'R'



Q How many Extra type 'c' bulbs can be Connected to the Supply without Blowing off the fuse.



Type (A) Bulb] 12V, 12W

Type (B) Bulb] 12V, 24W

Type (C) Bulb] 24V, 24W.

$$\begin{aligned} 12 + 24 &= 36 \\ 24 + \frac{24 \times 12}{12} &= 48 \\ 12, 6, 24 & \\ \frac{6 \times 24}{12 \times 6 + 6 \times 24 + 24 \times 12} & \end{aligned}$$

$$R_A = \frac{(12)^2}{12} = 12\Omega$$

$$i_A = \frac{12V}{12\Omega} = 1A$$

$$R_B = \frac{(12)^2}{24} = 6\Omega$$

$$i_B = \frac{12V}{6\Omega} = 2A$$

$$R_C = \frac{(24)^2}{24} = 24\Omega$$

$$i_C = \frac{12V}{24\Omega} = \frac{1}{2}A$$

$$\begin{aligned} \text{Extracurrent allowed by fuse} &< \left[12 - \frac{7}{2} \right] \\ &< \frac{17}{2}A \end{aligned}$$

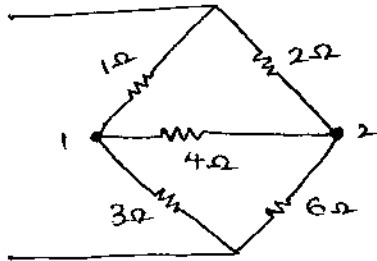
$$\text{But each bulb (C)} \rightarrow \frac{1}{2}A$$

$$[N_C] \cdot \left[\frac{1}{2} \right] < \frac{17}{2}$$

$$N_C < 17$$

\rightarrow 16 Bulbs only.

Resistor Reduction Techniques;

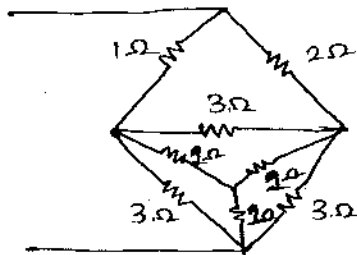


$$(1)(6) = 2(3) \Rightarrow \text{Balance}$$

P.D. doesn't exist b/w 1 & 2 nodes

\therefore no current flows through $4\Omega \Rightarrow \text{O.C}$

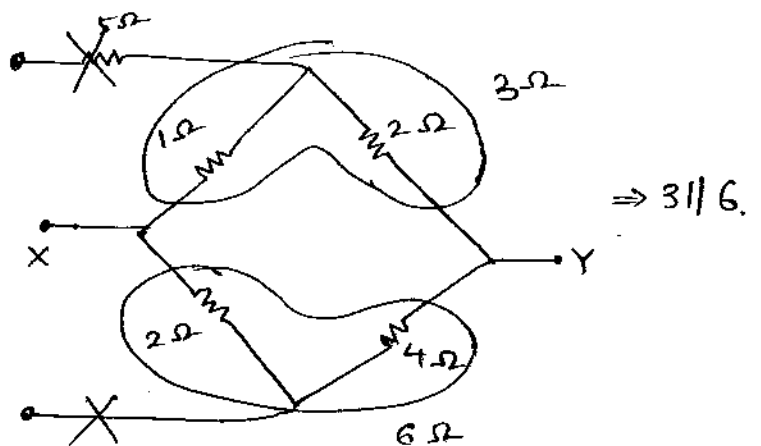
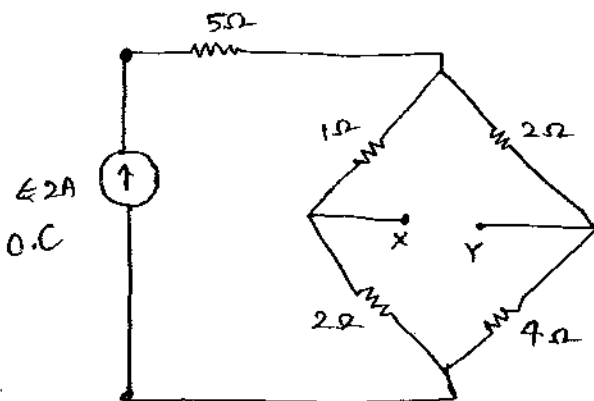
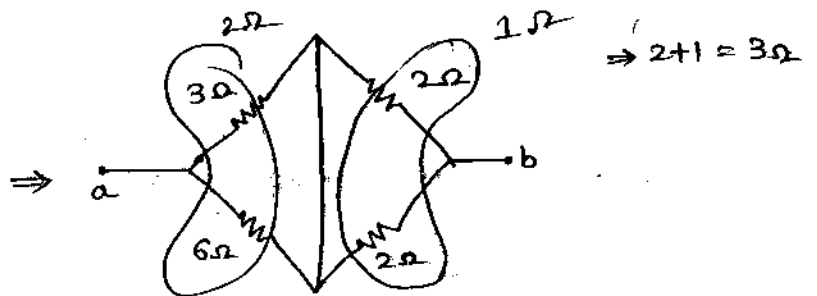
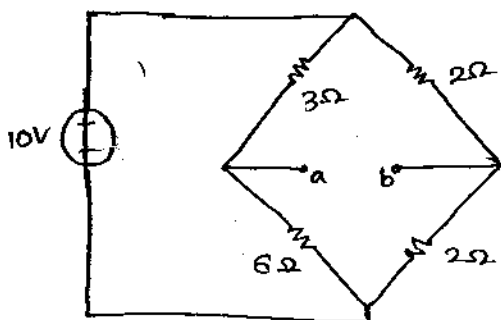
$$\therefore 4 \parallel 8 = \frac{4 \times 8}{12} = \frac{8}{3} \Omega$$

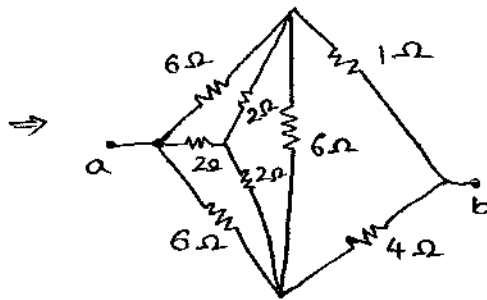
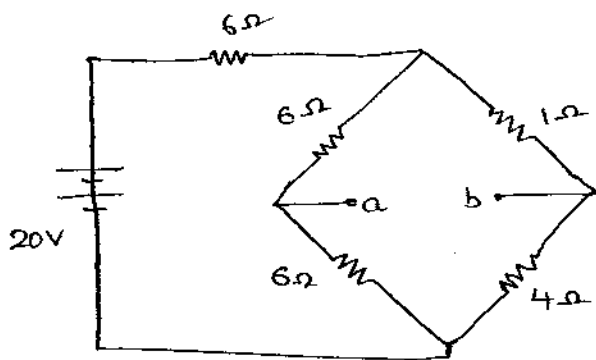


$$1 + [2 \parallel 3] = 1 + \frac{6}{5} = \frac{11}{5} \Omega$$

$$\frac{3 \times 3}{2}$$

$$10 \times 11$$



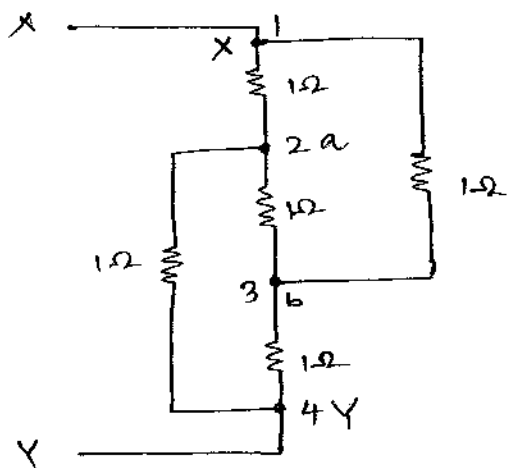


$$\Rightarrow 2\Omega + \left\{ (2+1) \parallel (2+4) \right\}$$

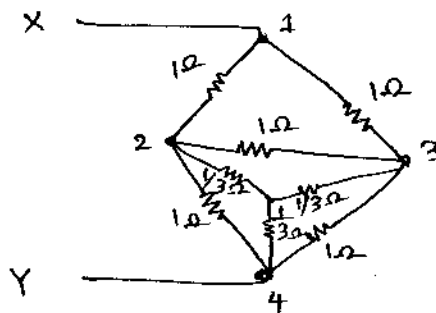
$$\Rightarrow 2\Omega + \{ 3 \parallel 6 \}$$

$$\Rightarrow 2+2$$

$$\Rightarrow 4\Omega$$

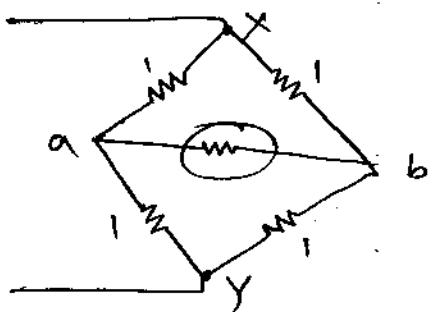


$$R_{XY} = \frac{2}{3}\Omega$$



$$\frac{3}{\frac{1}{2} + \frac{1}{2}} = \frac{3}{1} = 3\Omega$$

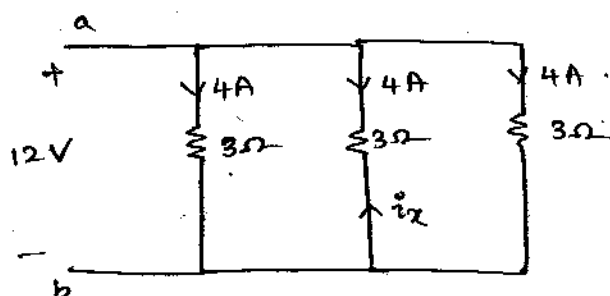
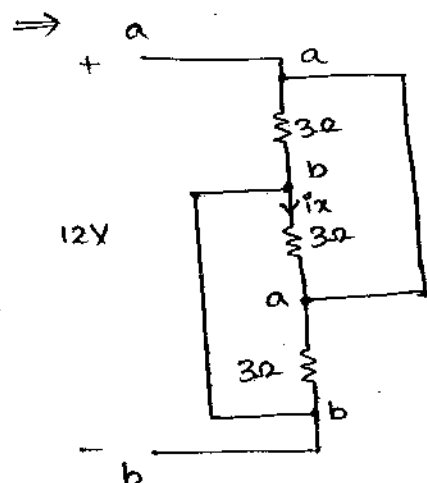
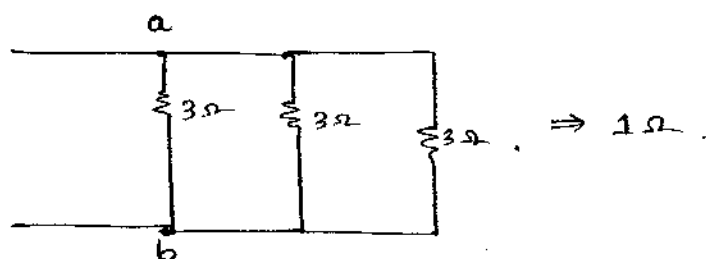
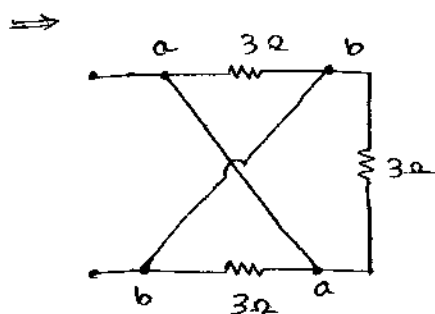
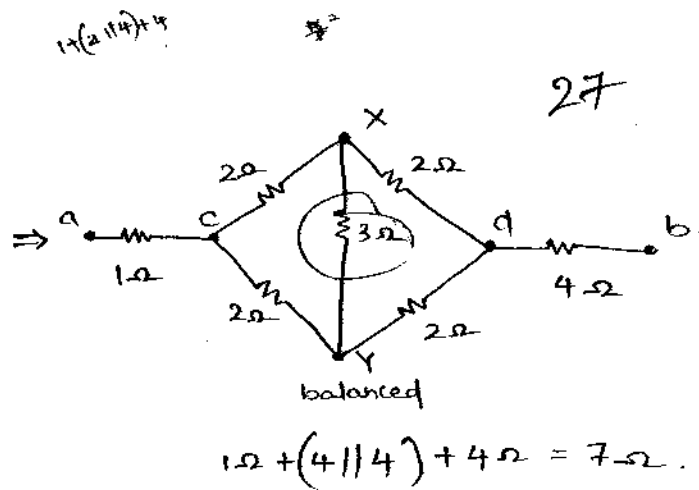
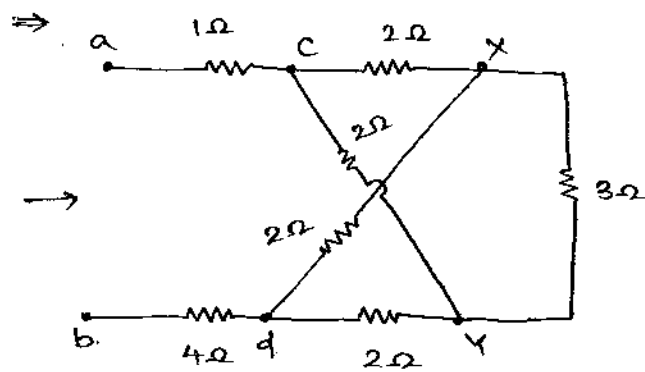
⇓ node shifting.



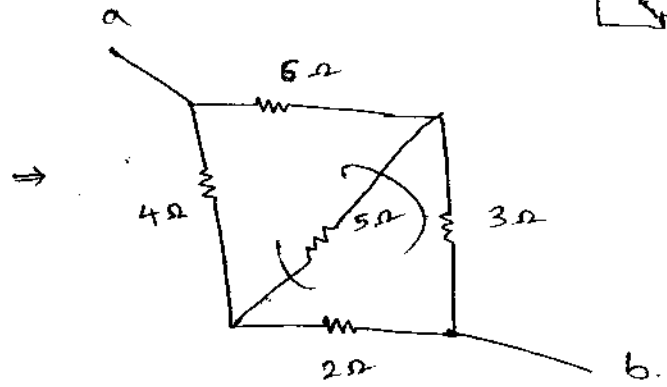
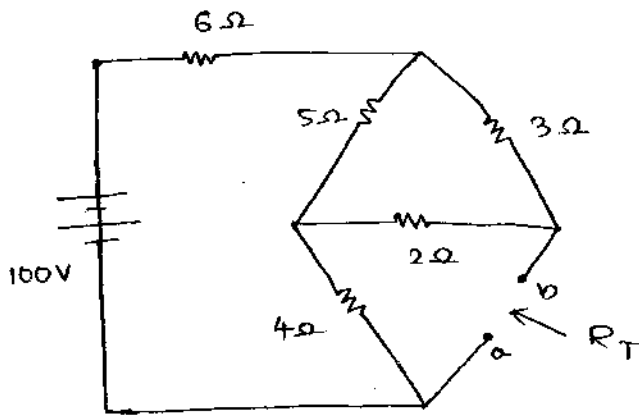
⇒ bridge balanced

$$1 \times 1 = 1 \times 1$$

$$\therefore 2 \parallel 2 = 1\Omega$$

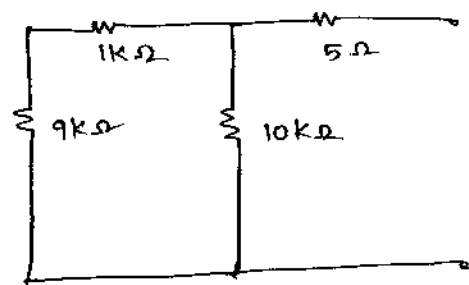
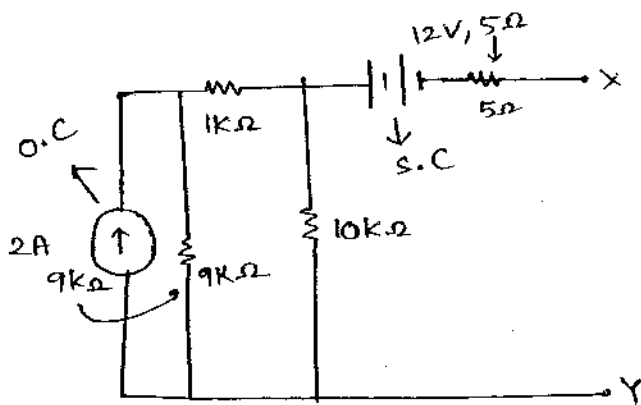


$$i_x = -4A$$

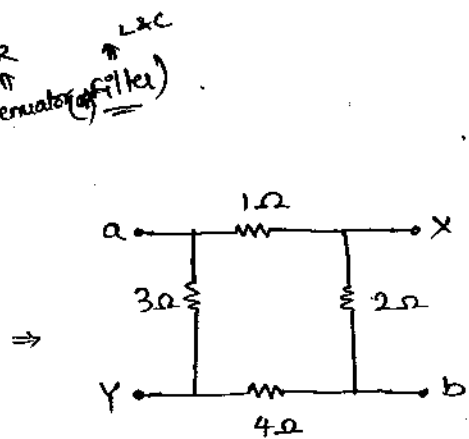
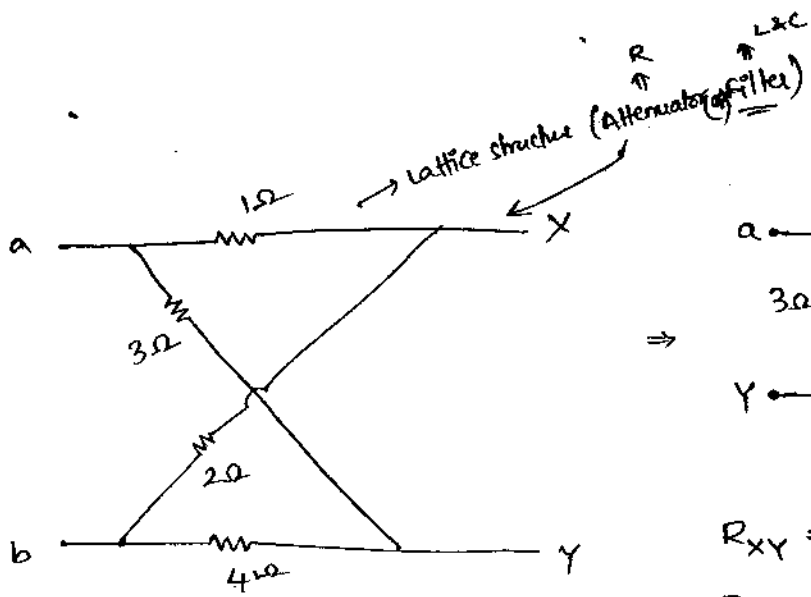


Bridge is Balanced $\Rightarrow 6 \times 2 = 3 \times 4$

$$\therefore 9\Omega \parallel 6\Omega = \frac{18}{5}\Omega$$



$$= 5005\Omega$$

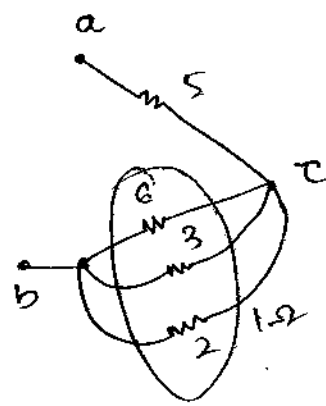
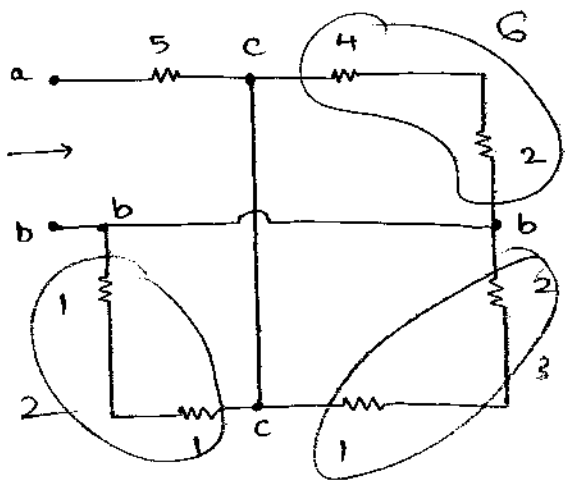


$$R_{XY} = 4\Omega \parallel 6\Omega = 2.4\Omega$$

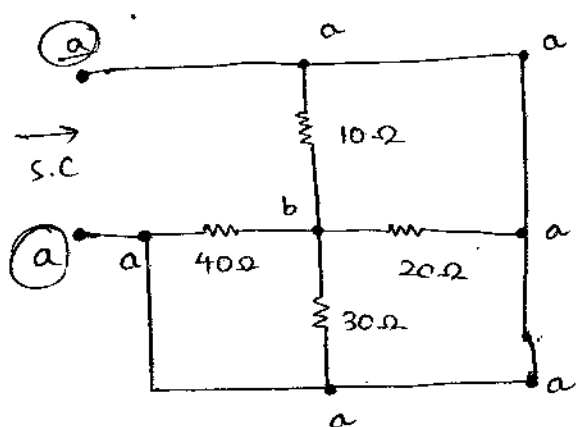
$$R_{ab} = 7\Omega \parallel 3\Omega = 2.1\Omega$$

$$R_{aY} = 3\Omega \parallel 7\Omega = 2.1\Omega$$

$$R_{bY} = 4\Omega \parallel 6\Omega = 2.4\Omega$$



$\Rightarrow 5 + 1 = 6\Omega$



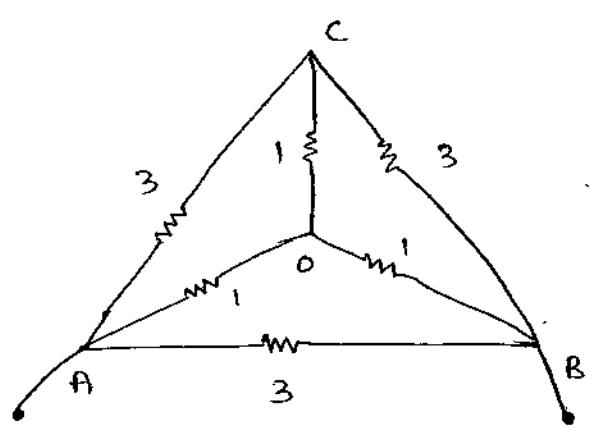
\Rightarrow dead S.C.

$\Rightarrow 0$

10/20

10/20/30

10



Bridge Balanced.

\Rightarrow

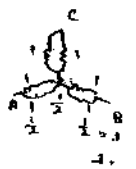
$R_{AB} = 1$

$R_{CO} = 2 // 1 = \frac{2}{3}$

$\frac{R_{AB}}{R_{CO}}$

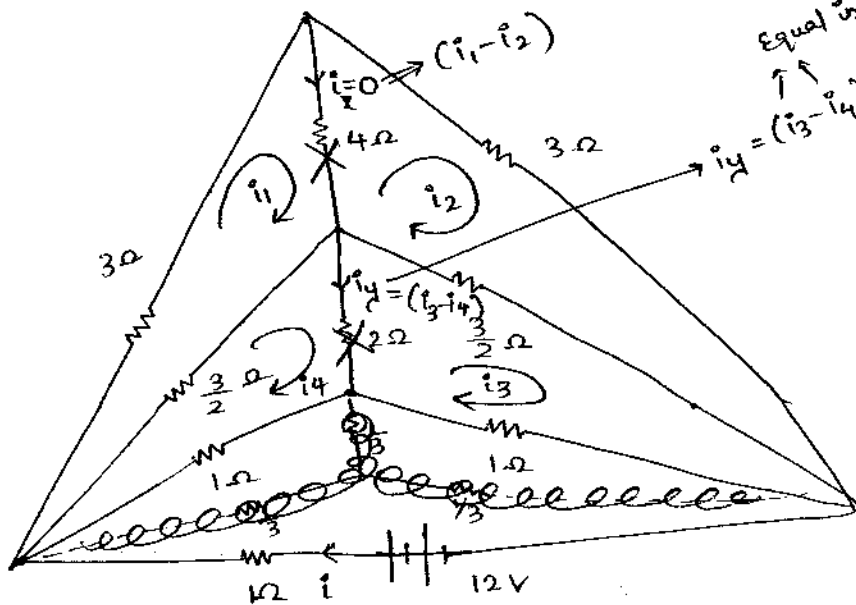
$\frac{1}{2/3} = \frac{3}{2}$

$\frac{R_{AB}}{R_{CO}} = \frac{1}{2/3} = \frac{3}{2}$



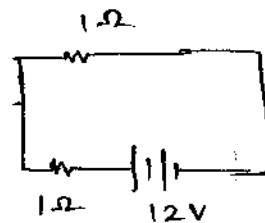
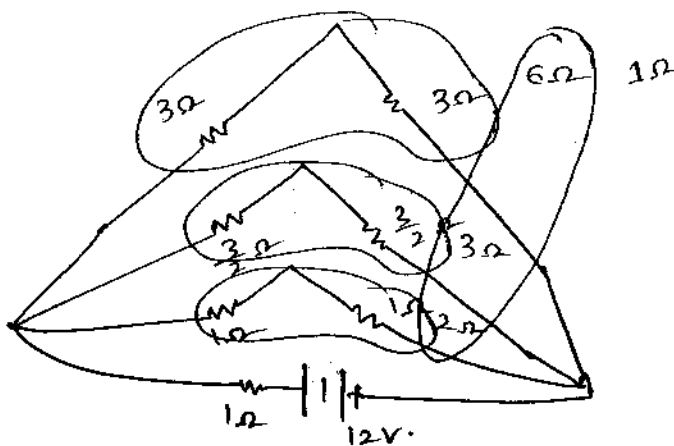
$$i_2 =$$

Equal in magnitude (\because components are same on both sides)



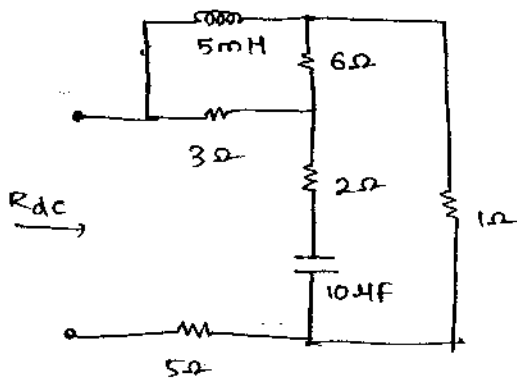
$$i_2 = \frac{12}{1+1} = 6A$$

$$\frac{1}{3} + 2 = \frac{7}{3}$$

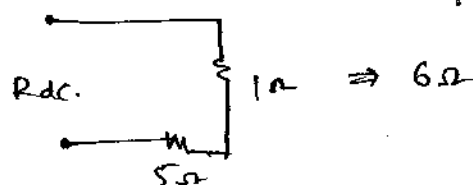
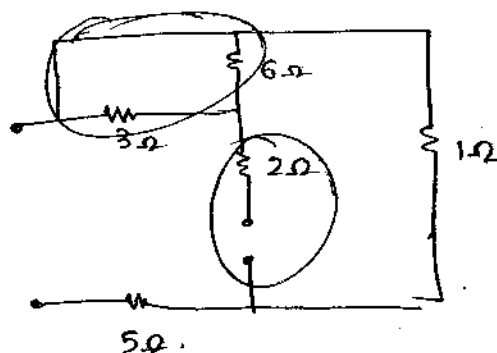


$$\Rightarrow i = \frac{12}{1+1} = 6A$$

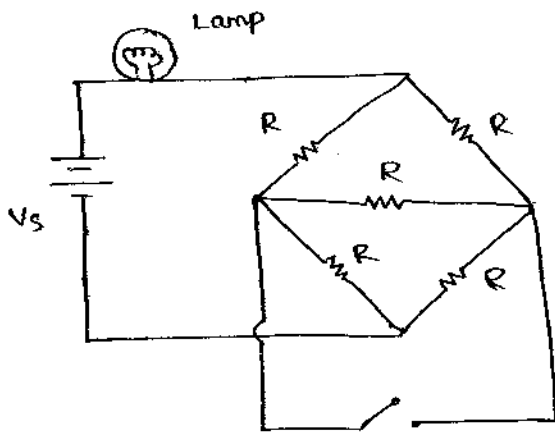
$$\frac{2}{\frac{6 \times 2}{2+1}}$$



$$R_{dc} = (1+5) = 6\Omega$$

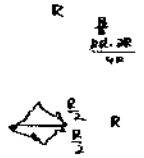


If Switch 'S' is closed then intensity of Lamp is

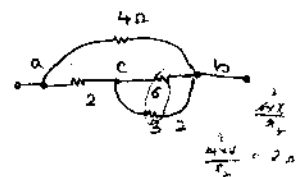
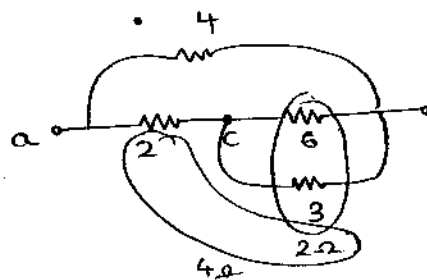
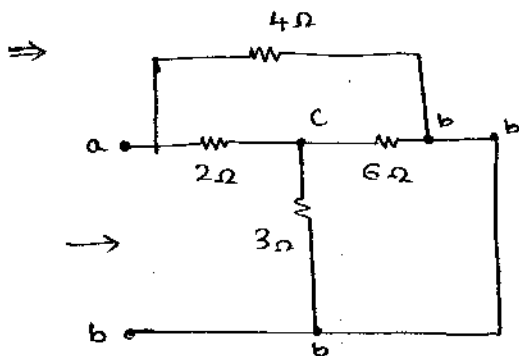


- (a) Increases
(b) Decreases
(c) Remaining Same
(d) Blow off

29

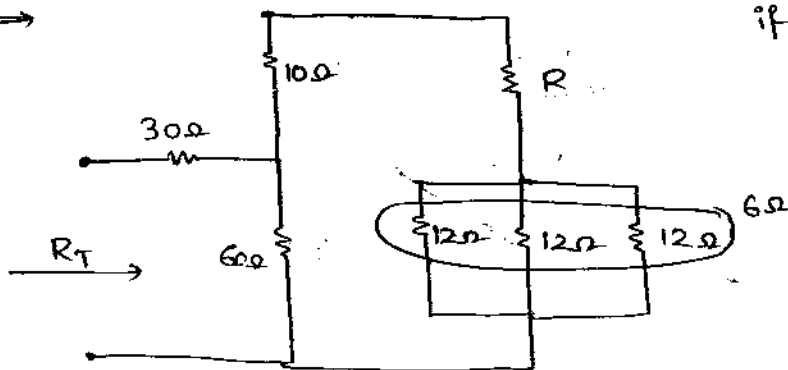


⇒ Eff. Resistance seen by ckt before & After closing switch is Same.



$$4 \parallel 4 = 2\Omega$$

if $R_T = 50\Omega$ then $R = ?$



$$50 = 30 + \left[60 \parallel (14 + R) \right]$$

$$20 = \frac{60 \times (14 + R)}{74 + R}$$

$$74R = 42 + 3R$$

$$R = 16\Omega$$

$$\frac{12}{3} = 4$$

$$(14 + R \parallel 6)$$

$$50 = 30 + (14 + R) \parallel 60$$

$$20 = \frac{(14 + R) \times 60}{74 + R}$$

$$20 \times 74 + 20R = 14 \times 60 + 60R$$

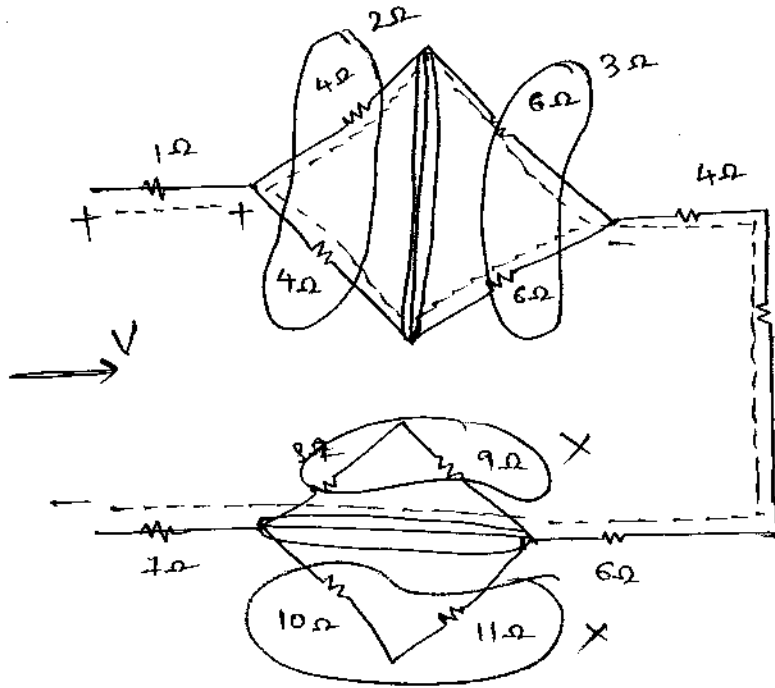
$$1480 + 20R = 840 + 60R$$

$$60R - 20R = 1480 - 840$$

$$40R = 640$$

$$R = \frac{640}{40} = 16$$

$\sim 13+5+5+6$
 $+$
 $(19/21)$
 $+7$



5Ω

$$\Rightarrow \Sigma (1,7) = 28\Omega$$

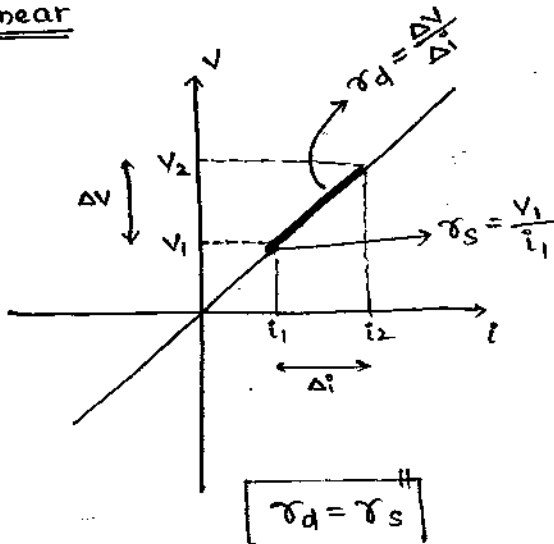
$$= 1\Omega + 2\Omega + 3\Omega + 4\Omega +$$

$$5\Omega + 6\Omega + 7\Omega$$

$$= 28\Omega$$

Q The voltage-current relation in a non-Linear Component is $V=i^2$ then the relation b/w static resistance (r_s) & dynamic Resistance (r_d) is.

Linear



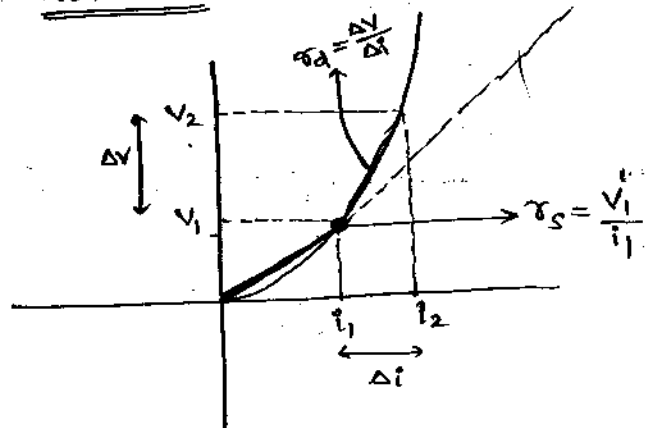
~~(a) $r_d = 2r_s$~~

(b) $r_d = \frac{r_s}{2}$

(c) $r_d = r_s^2$

(d) \therefore

Non Linear ($V=i^2$)



$$V=i^2$$

$$\frac{dV}{di} = 2i \frac{di}{di}$$

$$V=i^2$$

$$\frac{V}{i} = i$$

$$\rightarrow r_s = i$$

$$\frac{dV}{di} = r_d = 2i$$

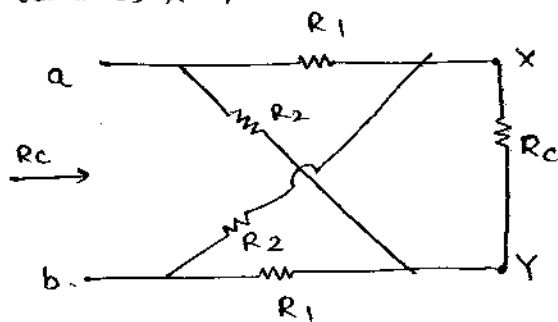
→ steady state op. point

where we initially calculate ' r_s '

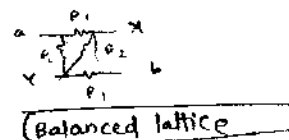
$$r_d = 2i$$

$$r_d = 2r_s$$

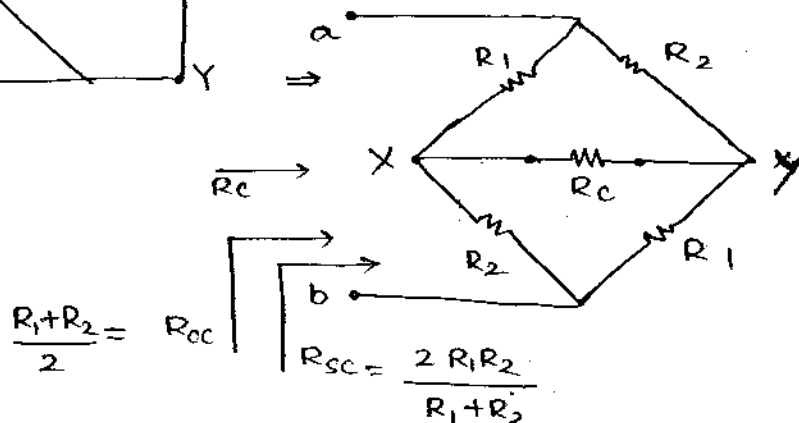
find characteristic resistance of N/w at terminals a-b, if Load is Connected b/w terminals X-Y



Here
 \Rightarrow this N/w is
 balanced & Symmetrical lattice.

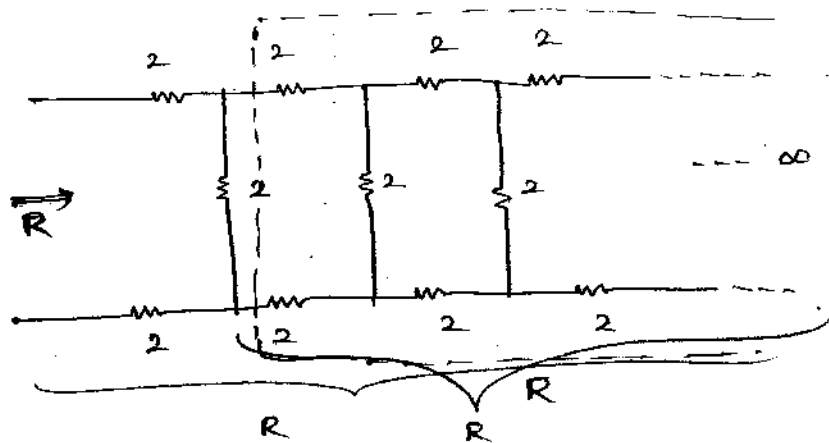


30



$$R_c = \sqrt{R_{oc} * R_{sc}}$$

$$= \sqrt{\frac{R_1 + R_2}{2} * \frac{2 * R_1 R_2}{R_1 + R_2}} \Rightarrow R_c = \sqrt{R_1 R_2}$$



\Rightarrow all 'R' are linear
 passive.
 Bilateral NW

$$R = I R_{III}$$

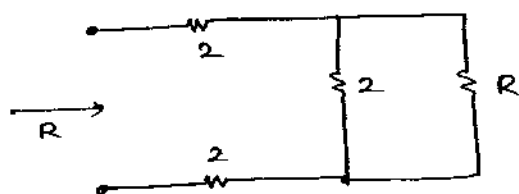
\Downarrow Q.

$$R = +ve \text{ slope}$$

$$\Downarrow$$

 $R = +ve$

$$R = (2 + 2\sqrt{3}) R$$



$$R = 4 + [2 // R]$$

$$R = 4 + \frac{2R}{2 + R}$$

$$R^2 + 2R = 8 + 4R + 2R$$

$$R^2 - 4R - 8 = 0$$

$$R = \frac{4 \pm \sqrt{16 - 4(-8)}}{2(1)} = \frac{4 \pm 4\sqrt{3}}{2} = (2 \pm 2\sqrt{3}) R$$

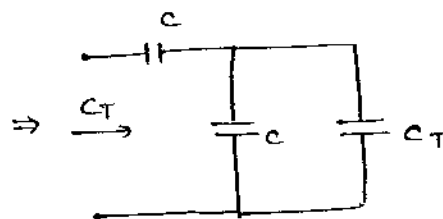
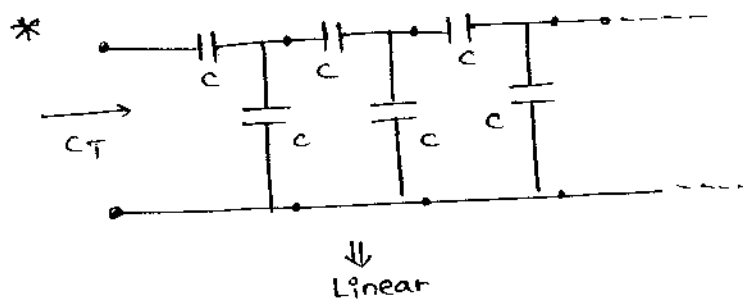
Characteristic Resistance:

it is that Resistance with which if we terminate the n/w the total Reflected ^{Resistance} ~~At~~ including the n/w is same

it is possible to construct char. Resistance only if ^{the} n/w is balanced and Symmetrical.

Condition for Char. Resistance

$$R_c = \sqrt{R_{oc} \cdot R_{sc}}$$



$$C_T = C \oplus [C_T + C]$$

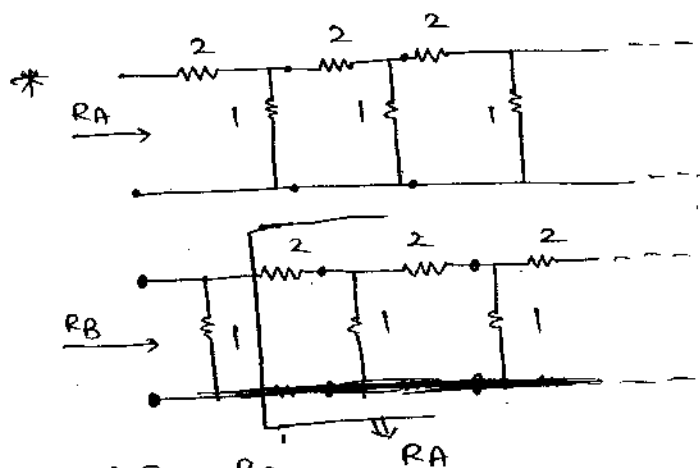
$$C_T = \frac{C \cdot C_T + C^2}{C_T + 2C}$$

$$C_T^2 + 2CC_T = C \cdot C_T + C^2$$

$$C_T^2 + C \cdot C_T - C^2 = 0$$

$$C_T = \frac{-C \pm \sqrt{C^2 - 4(-C^2)}}{2(1)}$$

$$C_T = \frac{-C \pm \sqrt{5}C}{2} \neq$$



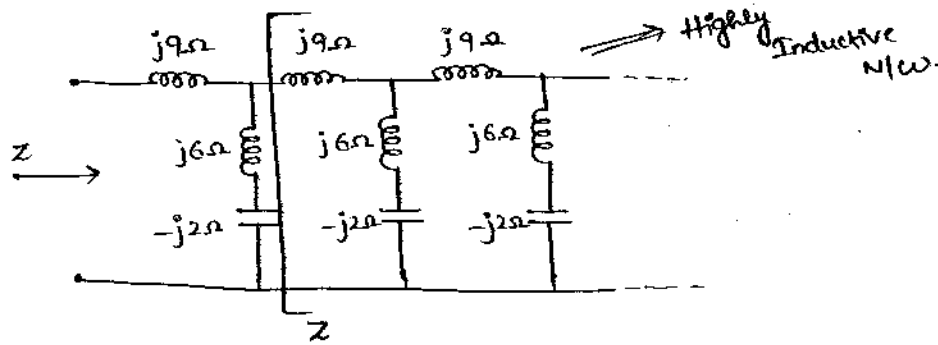
$$R_B = \frac{R_A \cdot 1}{1 + R_A}$$

$$R_B = \frac{R_A}{1 + R_A}$$

(a) $R_A = R_B$

(b) $R_B = R_A = 0$

(c) $R_A = \frac{R_B}{1 + R_B}$ (d) $R_B = \frac{R_A}{1 + R_A}$



31

$j \Rightarrow$ max operator

reactance = +ve \Rightarrow Inductive

= -ve \Rightarrow Capacitive

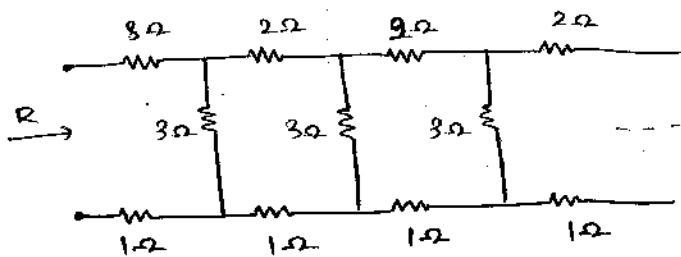
$$Z = j9 + [Z \parallel (j6 - j2)] = j9 + \frac{j4Z}{Z + j4}$$

$$Z^2 + Zj4 = j9Z - 36 + j4Z$$

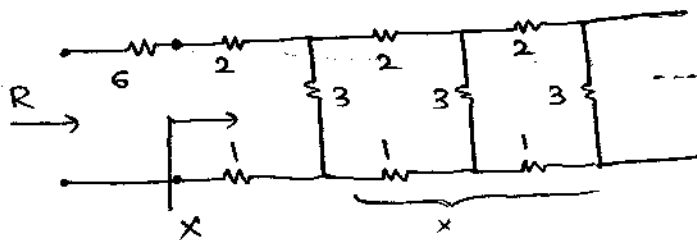
$$Z^2 - j9Z + 36 = 0$$

$$Z = \frac{j9 \pm \sqrt{-81 - 4(36)}}{2(1)} = \frac{j9 \pm \sqrt{-225}}{2}$$

$$Z = \frac{j9 \pm j15}{2} \left\{ \begin{array}{l} \rightarrow +j12\Omega \quad \checkmark \text{ Inductive} \\ \rightarrow -j3\Omega \quad \times \end{array} \right.$$



accurate value of R.



$$X = 2 + (X \parallel 3) + 1$$

$$X = 2 + \frac{3X}{3+X} + 1$$

$$(3+X)X = 3(3+X) + 3X$$

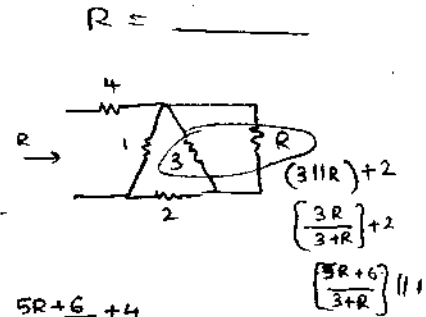
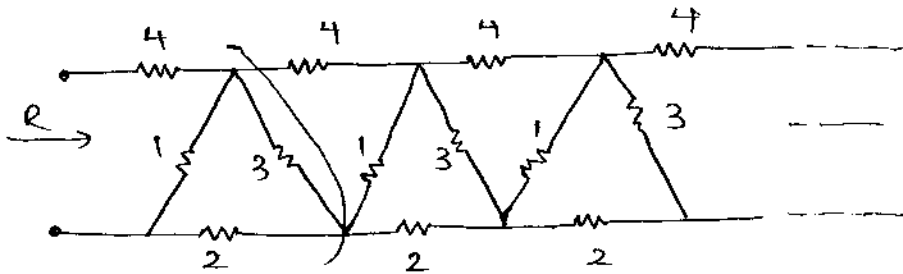
$$3X + X^2 = 9 + 3X + 3X$$

$$X^2 - 3X - 9 = 0$$

$$X = \frac{3 \pm \sqrt{9 - 4(1)(-9)}}{2(1)}$$

$$X = \frac{3 \pm \sqrt{45}}{2}$$

$$R = X + 6$$



$$R = \frac{5R+6}{6R+9} + 4$$

$$6R^2 + 9R = 5R + 6 + 24R + 36$$

$$6R^2 - 20R - 30 = 0$$

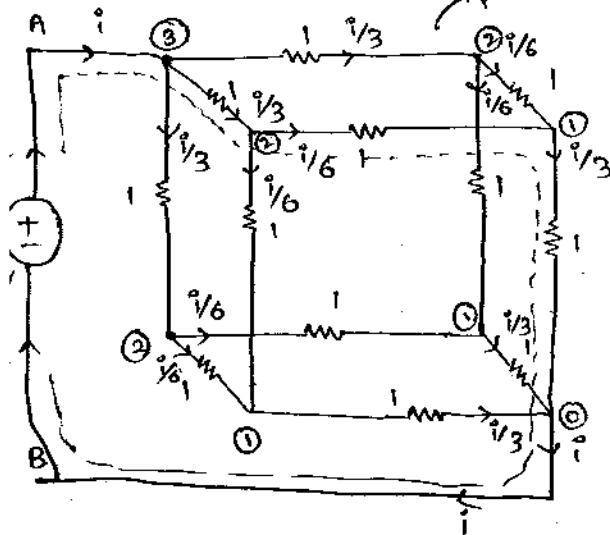
$$3R^2 - 10R - 15 = 0$$

$$R = \frac{10 \pm \sqrt{100 - 4 \times 3 \times (-15)}}{6}$$

$$R = \frac{10 + \sqrt{250}}{6}$$

$$\frac{5R+6}{5R+6+2R} \times \frac{5R+6}{6R+9}$$

* find $R_{AB} = \frac{V}{I} = \frac{5}{6} \Omega$



⇒ Symmetrical

if (i) All resistors are equal [not mandatory]

(ii) The path resistance b/w A & B in all the paths is Equal.

→ must condition.

KVL: $-V + \frac{i}{3}(1) + \frac{i}{6}(1) + \frac{i}{3}(1) = 0$

$$V = i \left[\frac{1}{3} + \frac{1}{6} + \frac{1}{3} \right]$$

$$V = i \frac{5}{6}$$

$$R_{AB} = \frac{V}{i} = \frac{5}{6} \Omega$$

for capacitors; [Time-variance]

$$-V(t) + \frac{1}{C} \int \frac{i(t)}{3} dt + \frac{1}{C} \int \frac{i(t)}{6} dt + \frac{1}{C} \int \frac{i(t)}{3} dt = 0$$

$$V(t) = \frac{1}{C} \left[\frac{1}{3} + \frac{1}{6} + \frac{1}{3} \right] \int i(t) dt$$

$$V(t) = \frac{5}{6C} \int i(t) dt$$

$$V(t) = \frac{1}{(6C/5)} \int i(t) dt$$

for $\tau = X \Omega = 2 \Omega$

$$R_{AB} = \frac{5}{6} \tau \Omega$$

$$\frac{1}{C} \quad L_{AB} = \frac{5}{6} L$$

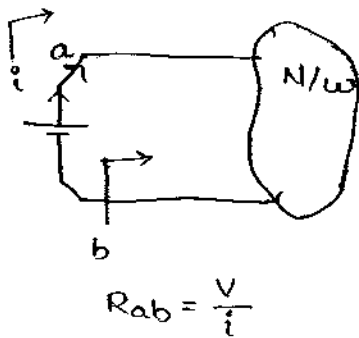
$$\frac{C}{C} \quad C_{AB} = \frac{6C}{5} F$$

$$g = B \tau = Y \tau$$

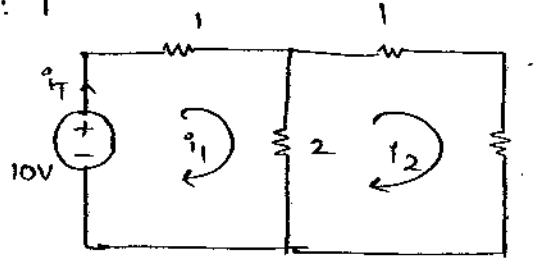
$$G_{AB} = \frac{6g}{5}$$

OHM'S Law

32



EX: 1

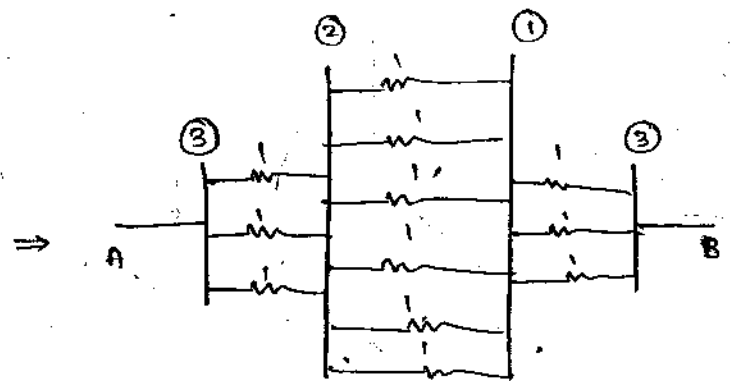
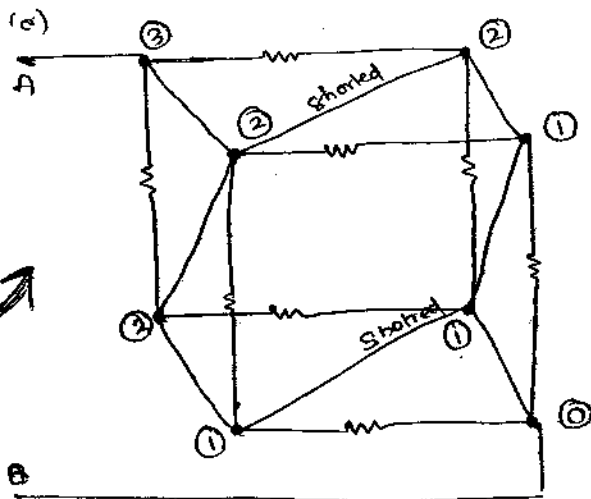


$$\begin{cases} -10 + i_1 + 2[i_1 - i_2] = 0 \rightarrow 3i_1 - 2i_2 = 10 \\ 2[i_2 - i_1] + 2i_2 = 0 \rightarrow -i_1 + 2i_2 = 0 \end{cases} \quad \underline{i_T = 5A}$$

$$R = \frac{V}{i_T} = \frac{10}{5} = 2\Omega$$

Note: 1:

if we can identify such nodes in any big n/w that are at same potential we can virtually join them to reduce faster



$$R_{AB} = \frac{1}{3} + \frac{1}{6} + \frac{1}{3} = \frac{5}{6}$$

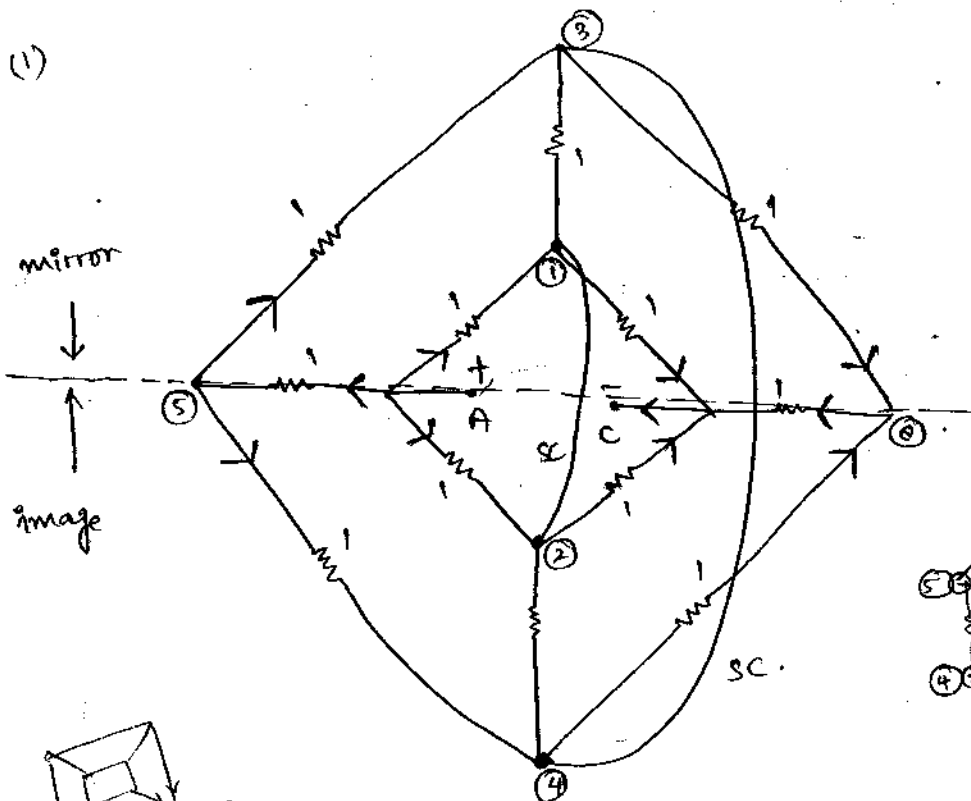
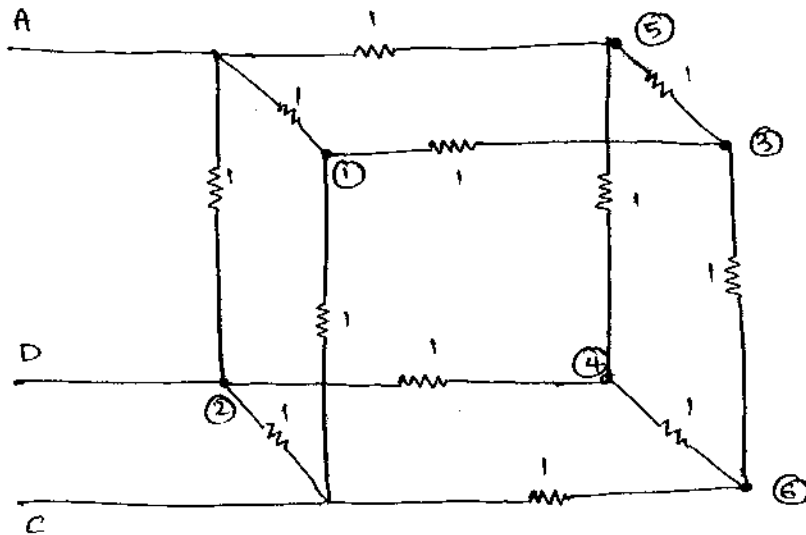
Note: 2:

another Important property of Symmetrical N/ws is that they can be Expressed as mirror images w.r.t Common axis drawn including the target terminals.

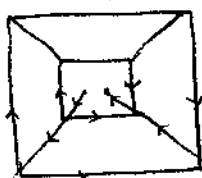
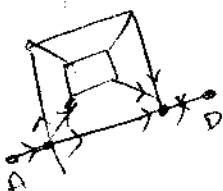
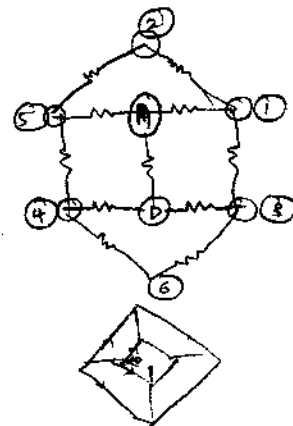
Q use the above two ideas given in notes 1 & notes 2 to find.

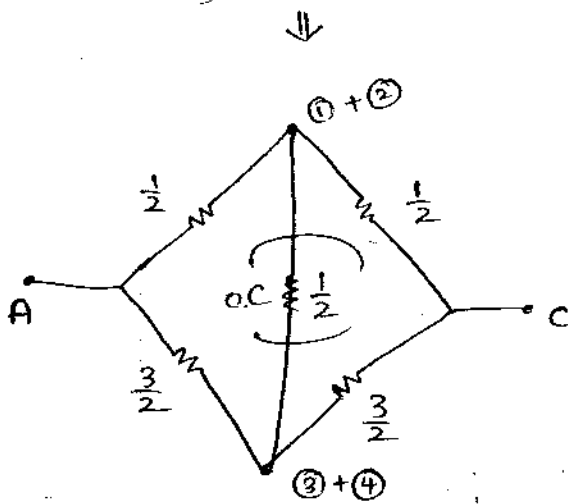
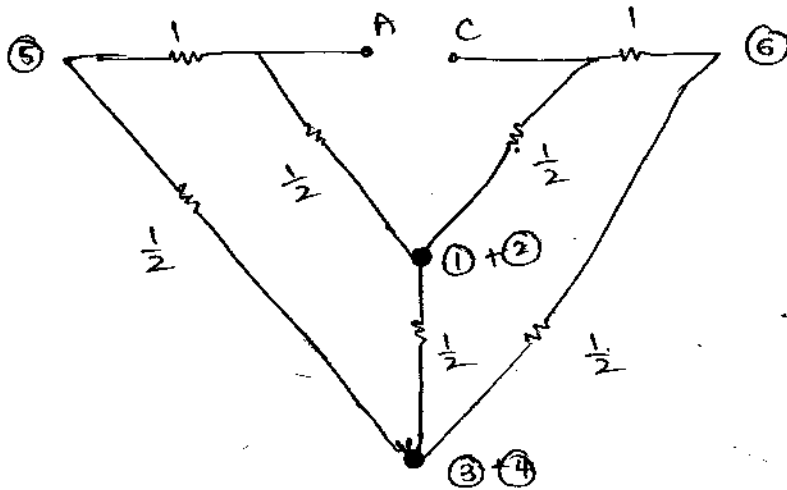
(1) $R_{AC} = \underline{\hspace{2cm}}$ (2) $R_{AD} = \underline{\frac{7}{12} \Omega}$
How

\Rightarrow we can't apply
 Current division rule
 \because path Resistances of
 all paths from A to C are
 not Equal.



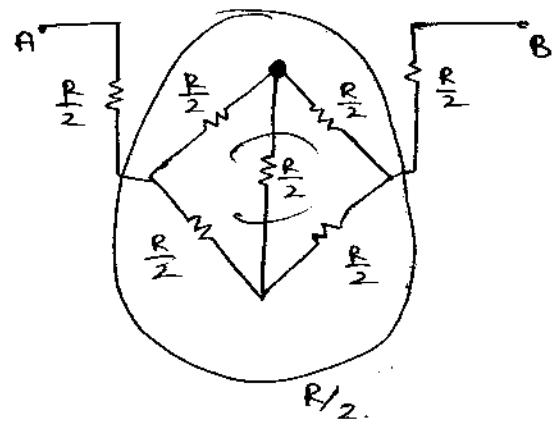
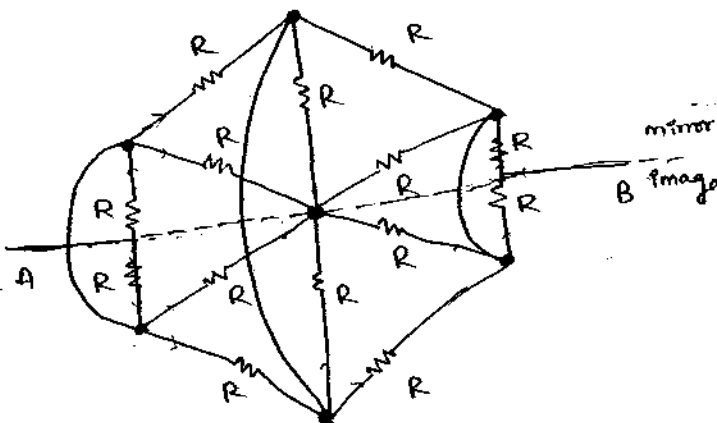
$V_1 = V_2 \rightarrow$ same potential
 $V_3 = V_4 \rightarrow$ (S.C.)



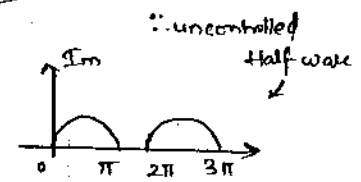
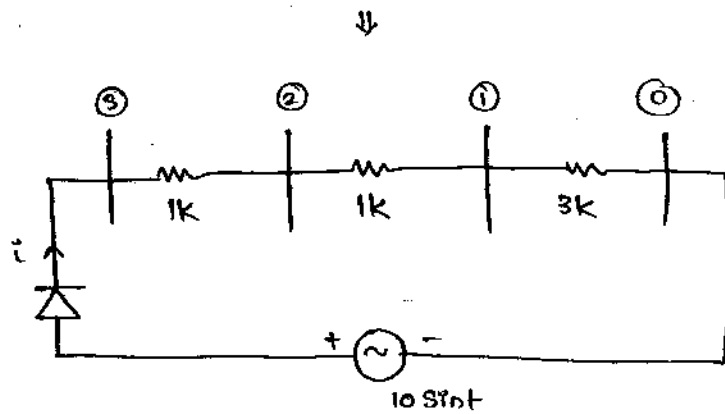
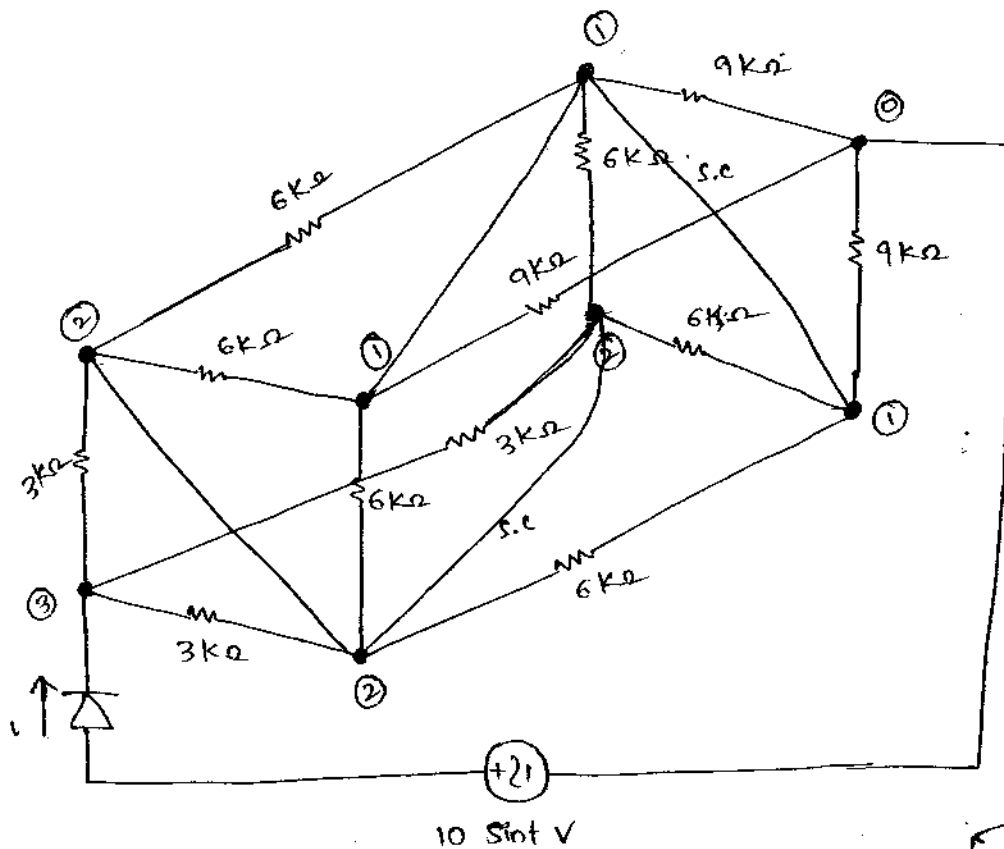


$$R_{AC} = 1 // 3 = \underline{\underline{\frac{3}{4}}}$$

IES (Q) $R_{AB} = \underline{\underline{\frac{3}{2} R \Omega}}$



$$R_{AB} = \underline{\underline{\frac{3}{2} R \Omega}}$$



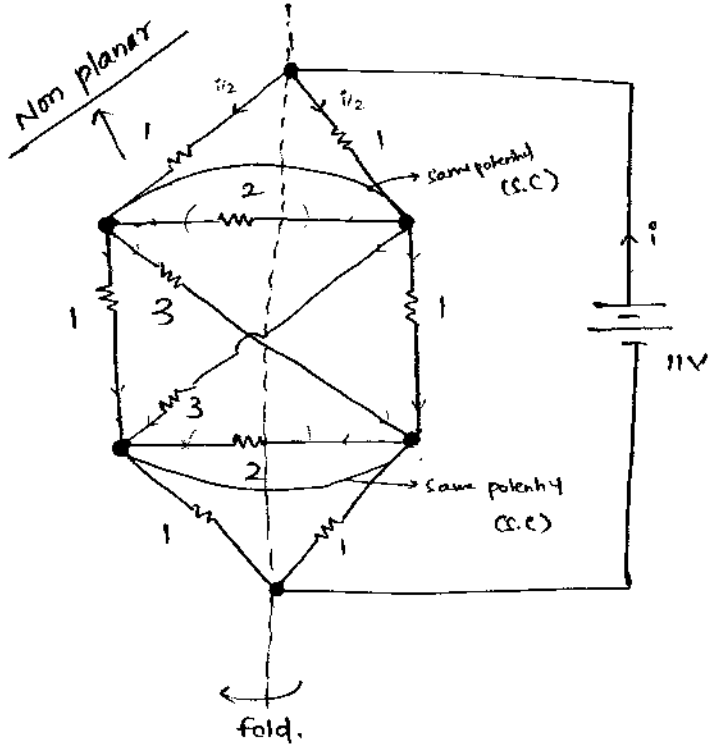
$$I_{rms} = \sqrt{\frac{1}{2\pi} \int_0^\pi I_m^2 \sin^2 t dt}$$

→ Without 'D'

$$i = \frac{V}{R_T} = \frac{10 \sin t}{5k} = 2 \text{ mill. } \sin t \text{ A.}$$

→ with 'D'

$$I_{rms} = \frac{I_m}{2} = \frac{2m}{2} = \underline{\underline{1mA}}$$



$$i =$$

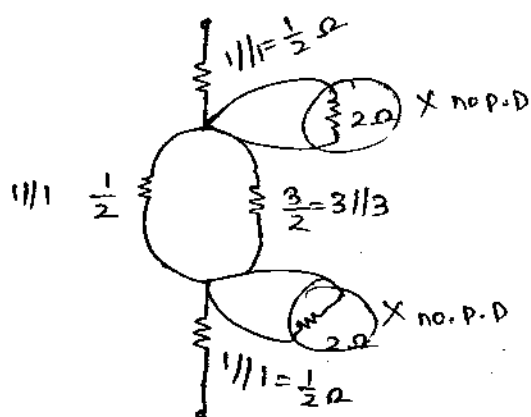
$$34$$

$$V = \frac{P}{i}$$

$$i_2 \propto P$$

$$= 11 \times \frac{5}{2}$$

$$= 27.5$$

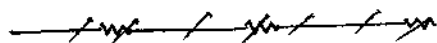


$$R_T = 1 + \left[\frac{1}{2} \parallel \frac{3}{2} \right] = 1 + \left[\frac{\frac{3}{4}}{2} \right]$$

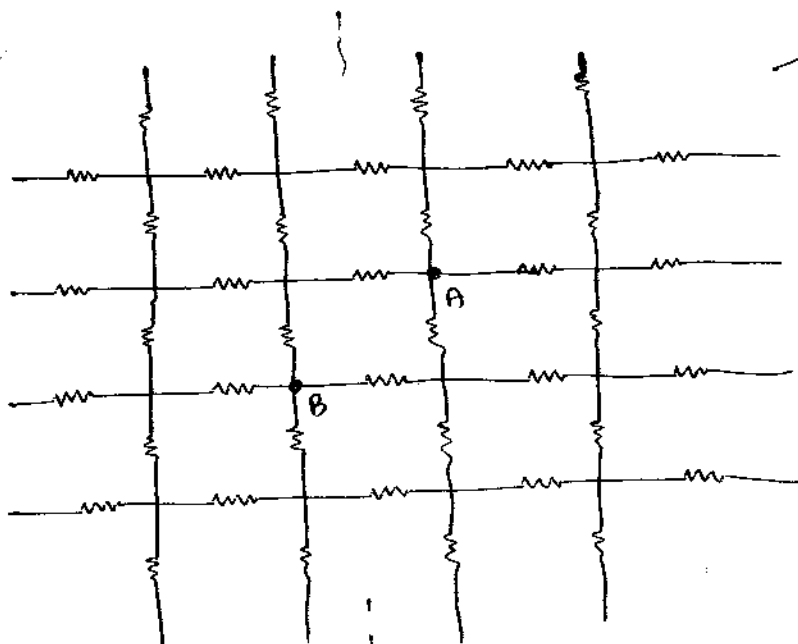
$$R_T = 1 + \frac{3}{8} = \frac{11}{8} \Omega$$

$$i = \frac{V}{R_T} = \frac{11}{11/8} = 8 A$$

IES(c)



16810)



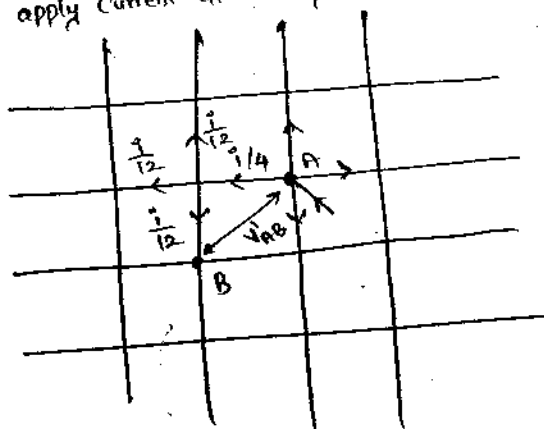
→ Infinite grid.

→ parallel natured network
(current division)
result is lower than least
($\therefore R_{AB} < 1\Omega$)

If each Branch Resistance is 1Ω then find R_{AB} .

Apply Superposition theorem:

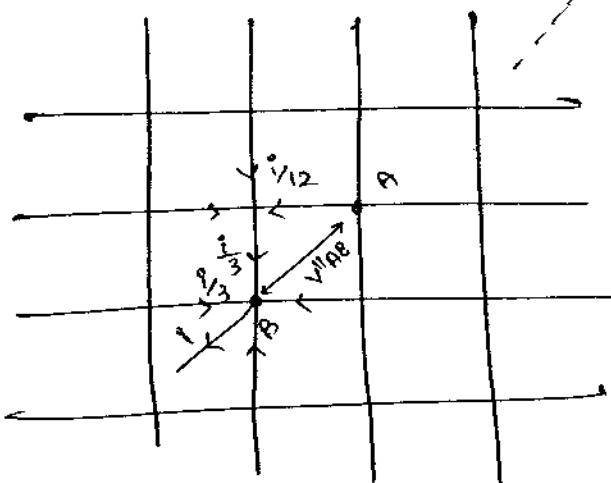
Step-I: apply current division by inject current at A & collect current at ∞ of path.
 \Rightarrow ~~path of~~ Resistance is Equal
 from A to ∞



$$V_{AB}' = \frac{i}{4}(1) + \frac{i}{12}(1) = \frac{i}{3}$$



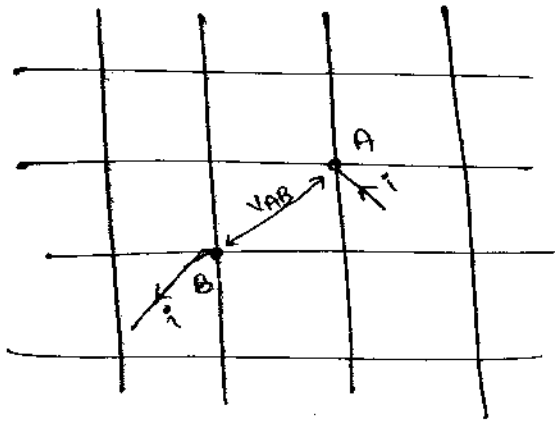
Step-II:



$$V_{AB}'' = \frac{i}{12}(1) + \frac{i}{4}(1) = \frac{i}{3}$$

Superposition theorem: S-I + S-II

35

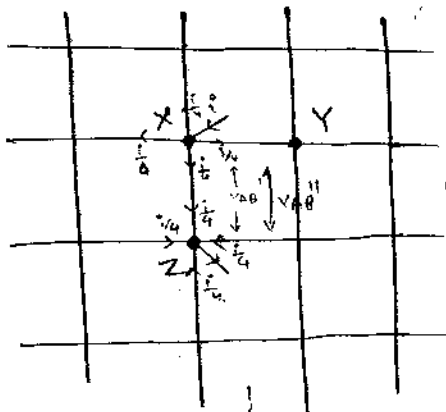


$$V_{AB} = V_{AB}^I + V_{AB}^{II}$$

$$V_{AB} = \frac{i}{3} + \frac{i}{3} = \frac{2i}{3}$$

$$R_{AB} = \frac{V_{AB}}{i} = \frac{2}{3} \Omega$$

HW-1



(i) if each Branch Resistance is $\frac{8}{3} \Omega$

find $R_{XZ} =$

(ii) if current of 1 Amp injected at X & collected at Y, then determine the current in Branch XY.

$$V_{XZ}^I = \frac{i}{4} (8)$$

$$V_{XZ}^{II} = \frac{i}{4} (0) =$$

$$V_{XZ} = \frac{2i}{4} \Omega$$

$$R_{XZ} = \frac{V_{XZ}}{i} = \frac{8}{2}$$

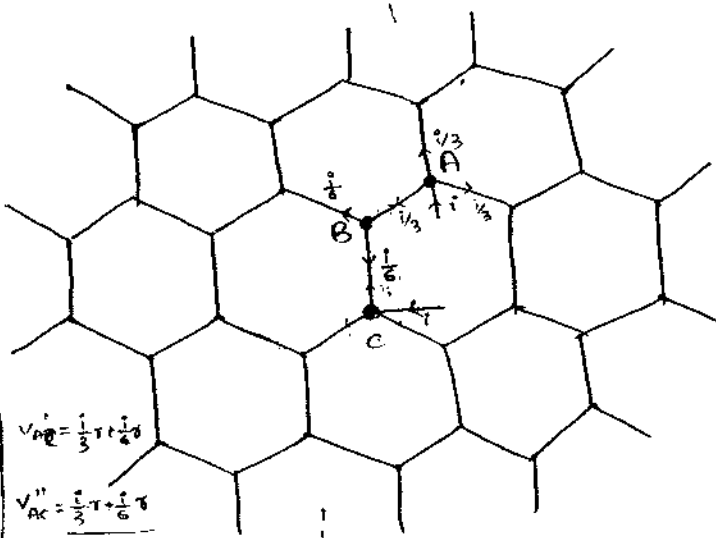
$$i_{XZ} = \frac{V_{XZ}}{R_{XZ}} = \frac{\frac{2i}{2}}{\frac{8}{2}} = \frac{i}{4} = \frac{1}{4}$$

HW-2

if each Branch resistance is $\frac{1}{3} \Omega$

find R_{AB}

R_{AC}



$$V_{AB} = \frac{1}{3} \cdot 1$$

$$V_{AB} = \frac{1}{3} \cdot 1 + \frac{1}{6} \cdot 1$$

$$V_{AB} = \frac{1}{3} \cdot 1$$

$$V_{AC} = \frac{1}{3} \cdot 1 + \frac{1}{6} \cdot 1$$

$$V_{AB} = \frac{2}{3} \cdot 1$$

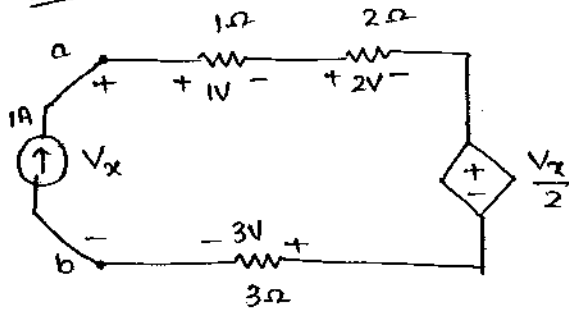
$$\frac{2}{3} \cdot 1 + \frac{2}{3} \cdot 1 =$$

$$R_{AB} = \frac{2}{3}$$

$$V_{AC} = \frac{2}{3} \cdot 1$$

$$\frac{V_{AC}}{1} = R_{AC} = \frac{2}{3}$$

IES (0)



$R_{ab} = \underline{\hspace{2cm}}$

pure dependent ckt are not working.

KVL:

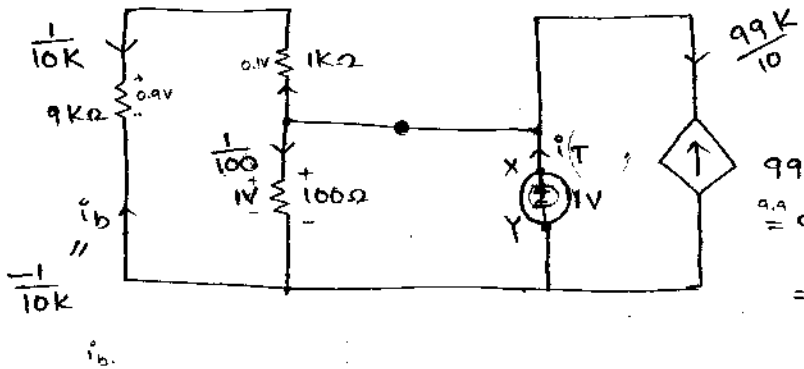
$$-V_x + 1 + 2 + \frac{V_x}{2} + 3 = 0$$

36

$$\frac{V_x}{2} = 6 \Rightarrow V_x = 12V$$

$$R_{ab} = \frac{V_x}{1A} = 12\Omega$$

* Gate 2012/11T'D'



$R_{xy} = ?$

$99i_b + 10$

$V = 99$
 $0.9 = 99i$
 $i = \frac{0.9}{99} = 0.009 \text{ mA}$
 $V = 10K$
 $\frac{1}{0.1}$

$99i_b$
 $= 99 \times \frac{-1}{10K}$
 $= \frac{-99}{10K}$

KCL at node

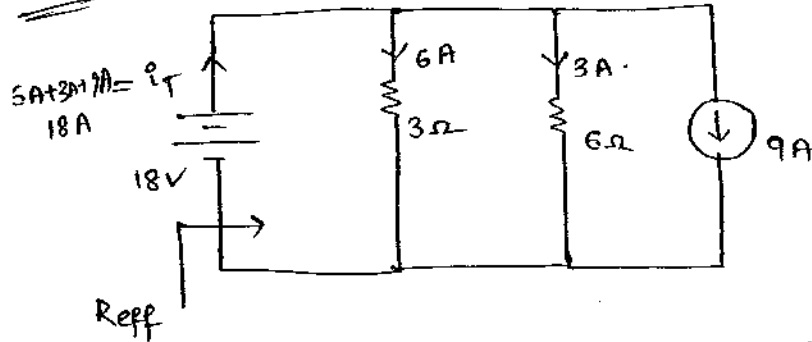
$$i_T = \frac{1}{100} + \frac{1}{10K} + \frac{99}{10K}$$

$$i_T = \frac{1}{100} + \frac{1}{10K} (1 + 99)$$

$$i_T = \frac{1}{100} + \frac{1}{100} = \frac{1}{50}$$

$$R_{xy} = \frac{1}{i_T} = \frac{1}{1/50} = 50\Omega$$

IES(0)



Determine the eff Resistance
Seen by the Voltage source

⇒ Effective Resistance is Resistance offered by the N/w to the source
under working condition

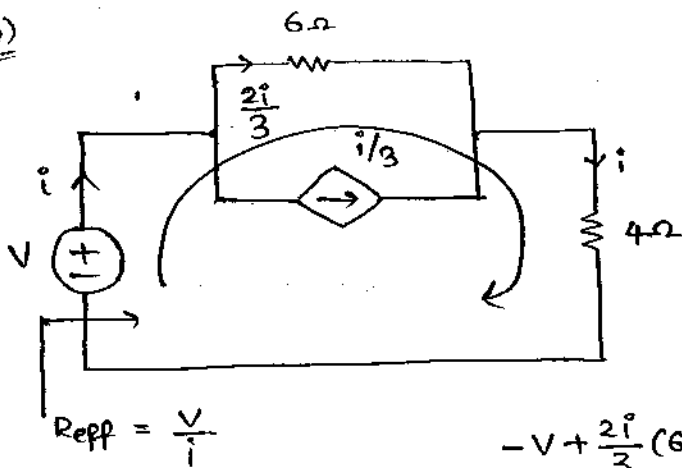
⇒ if the N/w is purely passive then eff. Resistance = Thevenins Resistance
 $R_{eff} = R_{Th}$

⇒ But if the N/w has an active element then Both are different

$$R_{eff} = \frac{18V}{i_T} = \frac{18}{18} = 1\Omega$$

$$[R_{Th} = R_{oc} = 3 \parallel 6 = 2\Omega]$$

IES(0)



What is the eff. Resistance
Seen by voltage source

$$V = iR$$

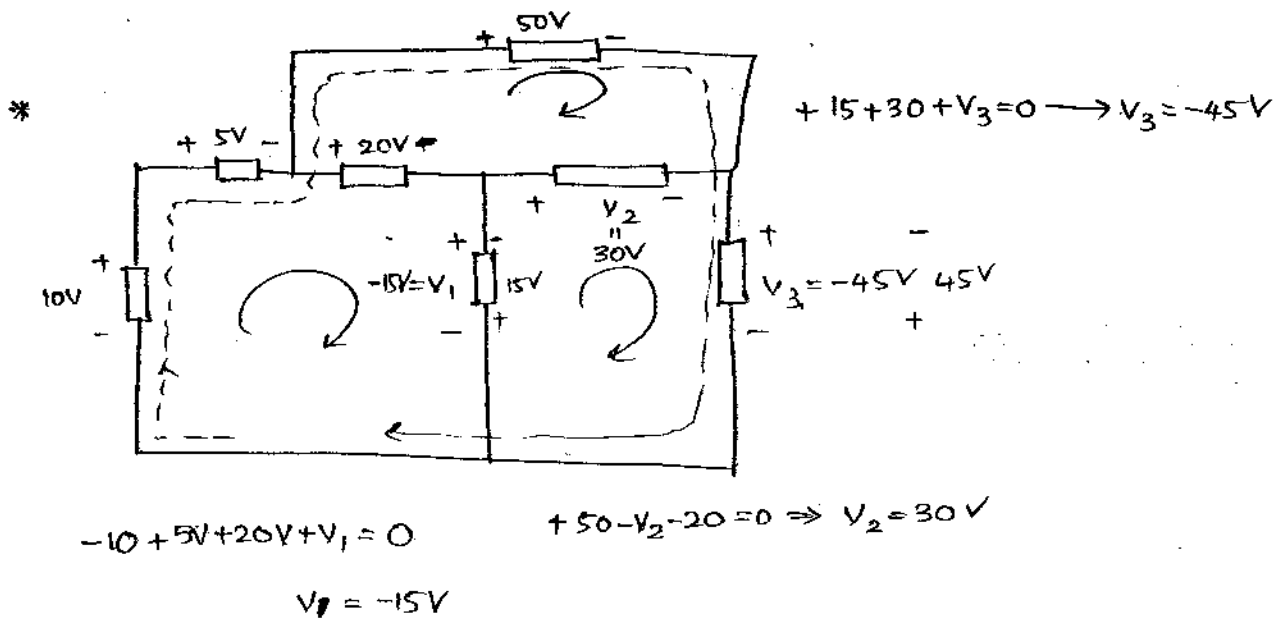
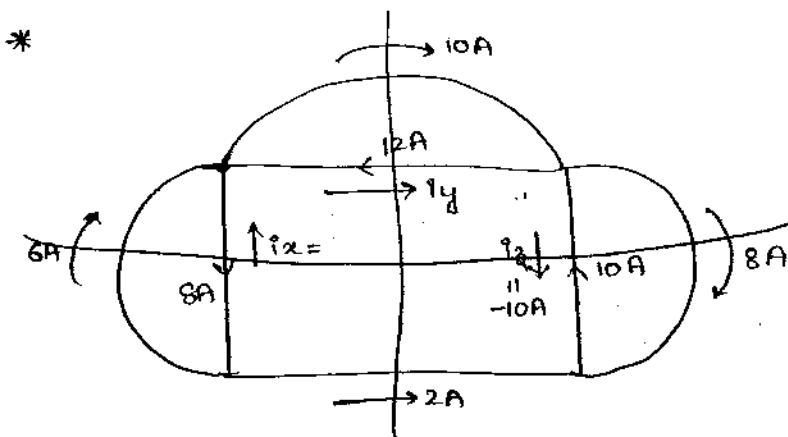
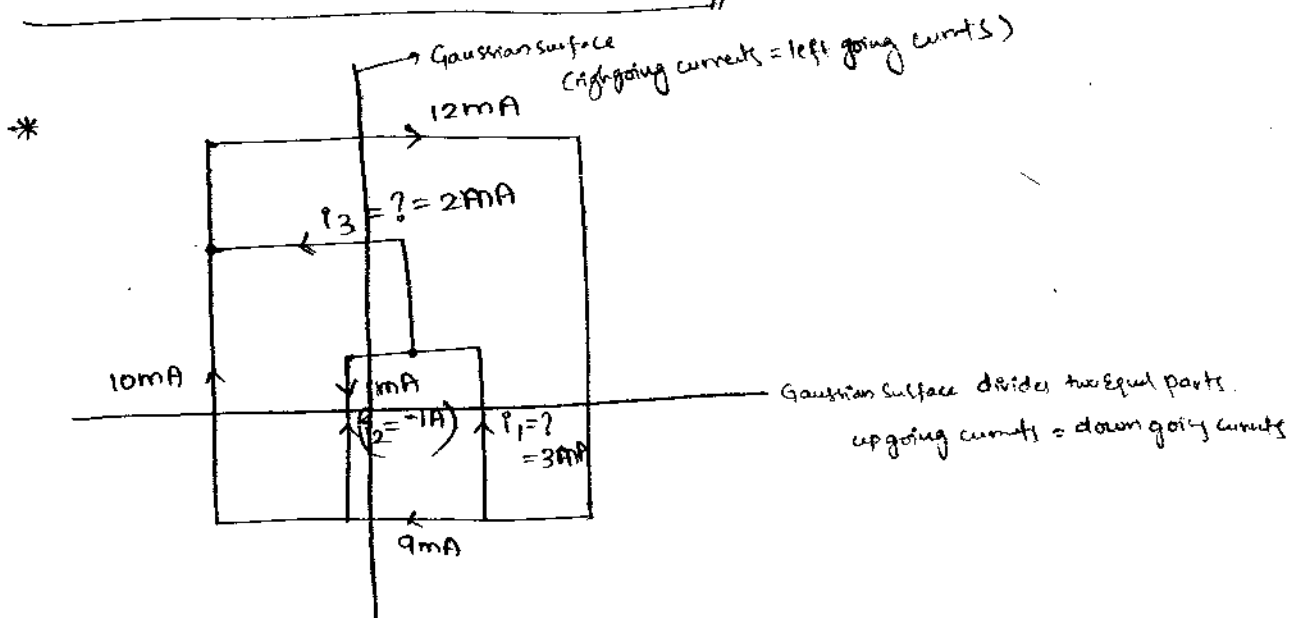
$$-V + \frac{2i}{3}(6) + 4i = 0$$

$$V = 8i$$

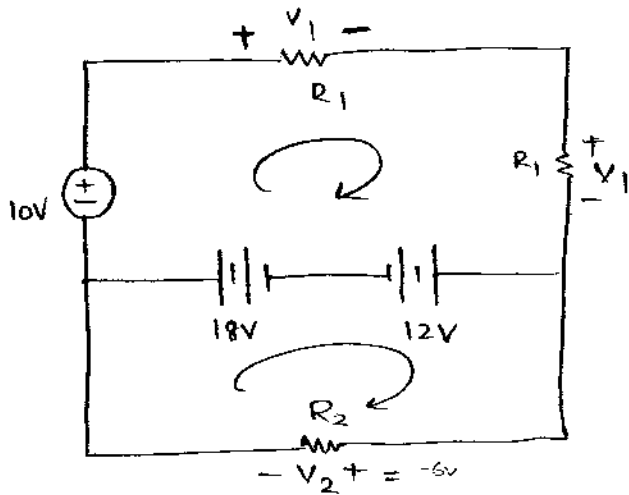
$$R_{eff} = \frac{V}{i} = 8\Omega$$

Basic DC Circuit Analysis:

37



*



$$-10 + 2V_1 + 12 - 18 = 0$$

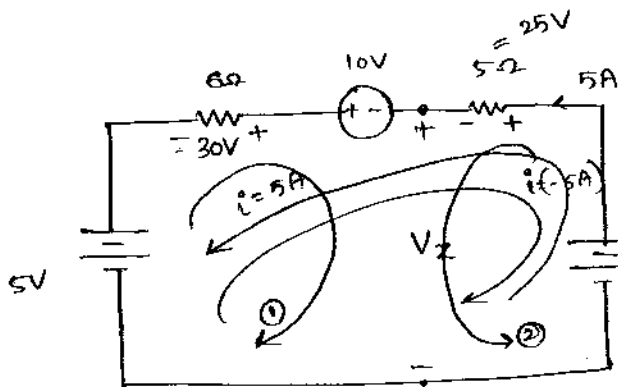
$$2V_1 = 16$$

$$V_1 = 8V$$

$$+18 - 12 + V_2 = 0$$

$$V_2 = -6V$$

*



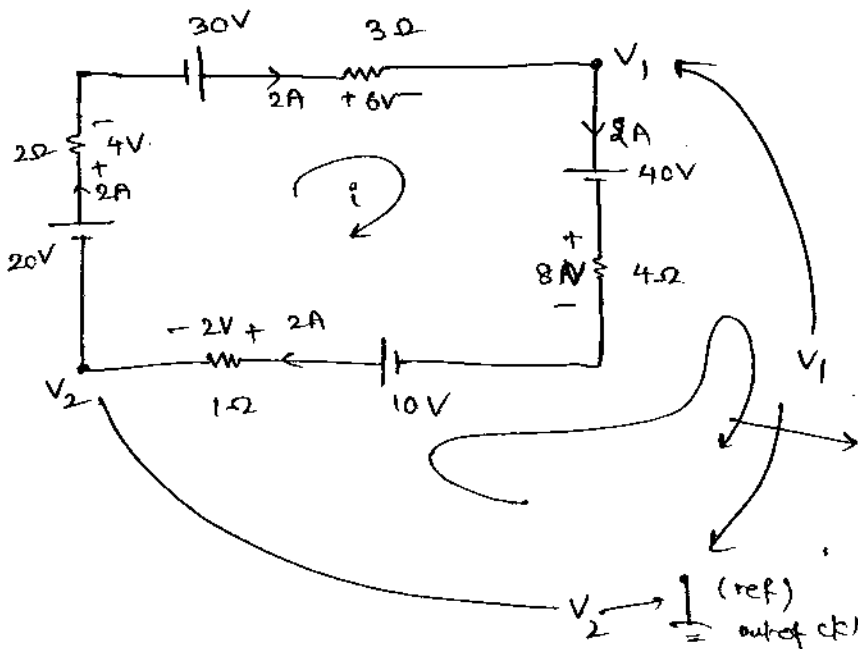
$$-5 + 6i + 10 + 5i + 50 = 0$$

$$i = -5A$$

$$\textcircled{1} -5 - 30 + 10 + V_2 = 0 \Rightarrow V_2 = +25V$$

$$\textcircled{2} -50 + 25 + V_2 = 0 \Rightarrow V_2 = +25V$$

*



$$V_2 - V_1$$

$$\frac{V_2 - 20V - 30V - V_1}{5} + \frac{V_2 + 10V - 40V - V_1}{5}$$

$$2V_2 - 2V_1 = 80 - 10 = 70V$$

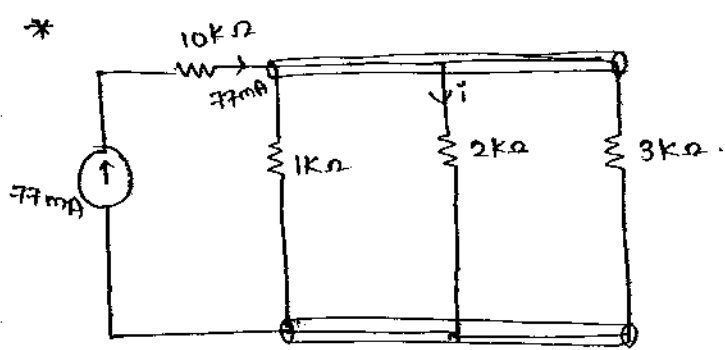
$$-20 + 10i - 30 + 40 - 10 = 0$$

$$i = +2A$$

$$-V_2 - 2 + 10 - 8 - 40 + V_1 = 0$$

$$(V_1 - V_2) = 40V$$

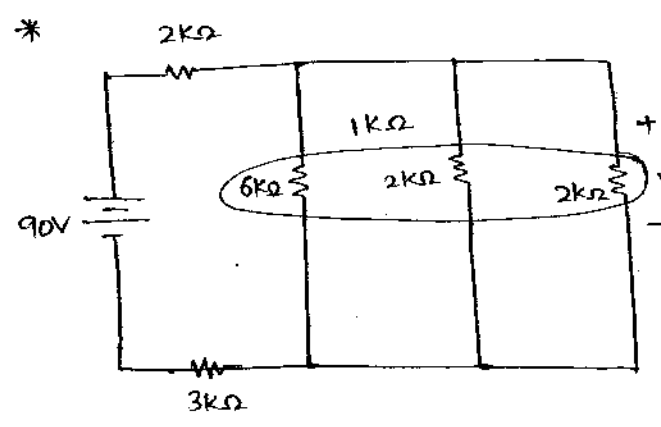
38



$$i = 77m \left[\frac{1k \times 3k}{1k \times 2k + 2k \times 3k + 3k \times 1k} \right]$$

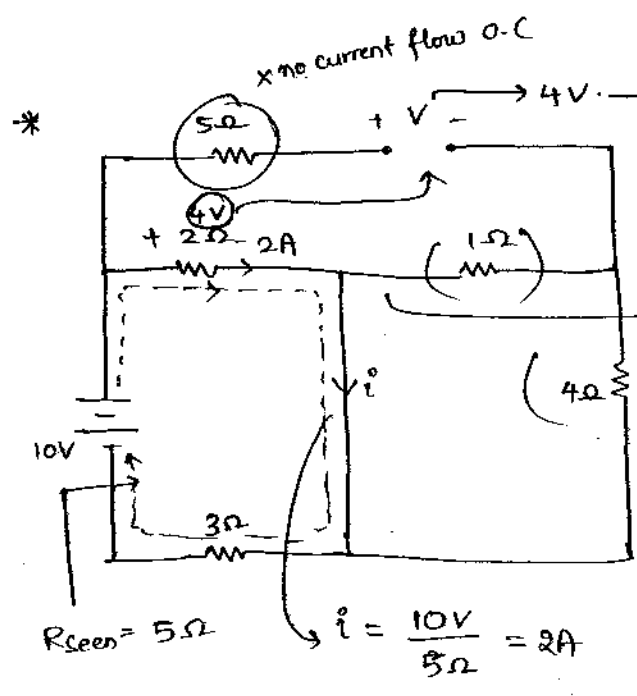
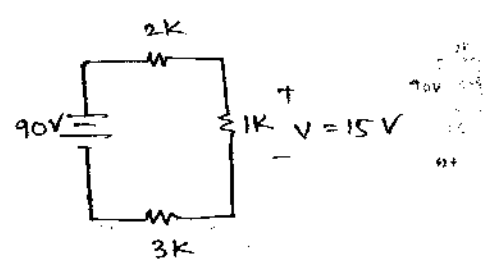
$$= 77m \left[\frac{3M}{2m + 6m + 3m} \right]$$

$$i = 21mA$$



$$V = 90 \left[\frac{1k}{6k} \right]$$

$$= 15V$$



x no current flow o.c

total voltage across "2Ω" is appears across o.c

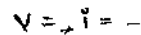
no volt. appears across 5Ω

∴ no current flowing through 5Ω

no current flows ∴ S.C

$$R_{seen} = 5\Omega$$

$$i = \frac{10V}{5\Omega} = 2A$$


$$V = 32 \text{ V}$$

3A

24

34

41.

$$\frac{1}{2}x^2$$

22

210

2. ~~3.~~ ~~4.~~ ~~5.~~ ~~6.~~ ~~7.~~ ~~8.~~ ~~9.~~ ~~10.~~ ~~11.~~ ~~12.~~ ~~13.~~ ~~14.~~ ~~15.~~ ~~16.~~ ~~17.~~ ~~18.~~ ~~19.~~ ~~20.~~ ~~21.~~ ~~22.~~ ~~23.~~ ~~24.~~ ~~25.~~ ~~26.~~ ~~27.~~ ~~28.~~ ~~29.~~ ~~30.~~ ~~31.~~ ~~32.~~ ~~33.~~ ~~34.~~ ~~35.~~ ~~36.~~ ~~37.~~ ~~38.~~ ~~39.~~ ~~40.~~ ~~41.~~ ~~42.~~ ~~43.~~ ~~44.~~ ~~45.~~ ~~46.~~ ~~47.~~ ~~48.~~ ~~49.~~ ~~50.~~ ~~51.~~ ~~52.~~ ~~53.~~ ~~54.~~ ~~55.~~ ~~56.~~ ~~57.~~ ~~58.~~ ~~59.~~ ~~60.~~ ~~61.~~ ~~62.~~ ~~63.~~ ~~64.~~ ~~65.~~ ~~66.~~ ~~67.~~ ~~68.~~ ~~69.~~ ~~70.~~ ~~71.~~ ~~72.~~ ~~73.~~ ~~74.~~ ~~75.~~ ~~76.~~ ~~77.~~ ~~78.~~ ~~79.~~ ~~80.~~ ~~81.~~ ~~82.~~ ~~83.~~ ~~84.~~ ~~85.~~ ~~86.~~ ~~87.~~ ~~88.~~ ~~89.~~ ~~90.~~ ~~91.~~ ~~92.~~ ~~93.~~ ~~94.~~ ~~95.~~ ~~96.~~ ~~97.~~ ~~98.~~ ~~99.~~ ~~100.~~ ~~101.~~ ~~102.~~ ~~103.~~ ~~104.~~ ~~105.~~ ~~106.~~ ~~107.~~ ~~108.~~ ~~109.~~ ~~110.~~ ~~111.~~ ~~112.~~ ~~113.~~ ~~114.~~ ~~115.~~ ~~116.~~ ~~117.~~ ~~118.~~ ~~119.~~ ~~120.~~ ~~121.~~ ~~122.~~ ~~123.~~ ~~124.~~ ~~125.~~ ~~126.~~ ~~127.~~ ~~128.~~ ~~129.~~ ~~130.~~ ~~131.~~ ~~132.~~ ~~133.~~ ~~134.~~ ~~135.~~ ~~136.~~ ~~137.~~ ~~138.~~ ~~139.~~ ~~140.~~ ~~141.~~ ~~142.~~ ~~143.~~ ~~144.~~ ~~145.~~ ~~146.~~ ~~147.~~ ~~148.~~ ~~149.~~ ~~150.~~ ~~151.~~ ~~152.~~ ~~153.~~ ~~154.~~ ~~155.~~ ~~156.~~ ~~157.~~ ~~158.~~ ~~159.~~ ~~160.~~ ~~161.~~ ~~162.~~ ~~163.~~ ~~164.~~ ~~165.~~ ~~166.~~ ~~167.~~ ~~168.~~ ~~169.~~ ~~170.~~ ~~171.~~ ~~172.~~ ~~173.~~ ~~174.~~ ~~175.~~ ~~176.~~ ~~177.~~ ~~178.~~ ~~179.~~ ~~180.~~ ~~181.~~ ~~182.~~ ~~183.~~ ~~184.~~ ~~185.~~ ~~186.~~ ~~187.~~ ~~188.~~ ~~189.~~ ~~190.~~ ~~191.~~ ~~192.~~ ~~193.~~ ~~194.~~ ~~195.~~ ~~196.~~ ~~197.~~ ~~198.~~ ~~199.~~ ~~200.~~ ~~201.~~ ~~202.~~ ~~203.~~ ~~204.~~ ~~205.~~ ~~206.~~ ~~207.~~ ~~208.~~ ~~209.~~ ~~210.~~ ~~211.~~ ~~212.~~ ~~213.~~ ~~214.~~ ~~215.~~ ~~216.~~ ~~217.~~ ~~218.~~ ~~219.~~ ~~220.~~ ~~221.~~ ~~222.~~ ~~223.~~ ~~224.~~ ~~225.~~ ~~226.~~ ~~227.~~ ~~228.~~ ~~229.~~ ~~230.~~ ~~231.~~ ~~232.~~ ~~233.~~ ~~234.~~ ~~235.~~ ~~236.~~ ~~237.~~ ~~238.~~ ~~239.~~ ~~240.~~ ~~241.~~ ~~242.~~ ~~243.~~ ~~244.~~ ~~245.~~ ~~246.~~ ~~247.~~ ~~248.~~ ~~249.~~ ~~250.~~ ~~251.~~ ~~252.~~ ~~253.~~ ~~254.~~ ~~255.~~ ~~256.~~ ~~257.~~ ~~258.~~ ~~259.~~ ~~260.~~ ~~261.~~ ~~262.~~ ~~263.~~ ~~264.~~ ~~265.~~ ~~266.~~ ~~267.~~ ~~268.~~ ~~269.~~ ~~270.~~ ~~271.~~ ~~272.~~ ~~273.~~ ~~274.~~ ~~275.~~ ~~276.~~ ~~277.~~ ~~278.~~ ~~279.~~ ~~280.~~ ~~281.~~ ~~282.~~ ~~283.~~ ~~284.~~ ~~285.~~ ~~286.~~ ~~287.~~ ~~288.~~ ~~289.~~ ~~290.~~ ~~291.~~ ~~292.~~ ~~293.~~ ~~294.~~ ~~295.~~ ~~296.~~ ~~297.~~ ~~298.~~ ~~299.~~ ~~300.~~ ~~301.~~ ~~302.~~ ~~303.~~ ~~304.~~ ~~305.~~ ~~306.~~ ~~307.~~ ~~308.~~ ~~309.~~ ~~310.~~ ~~311.~~ ~~312.~~ ~~313.~~ ~~314.~~ ~~315.~~ ~~316.~~ ~~317.~~ ~~318.~~ ~~319.~~ ~~320.~~ ~~321.~~ ~~322.~~ ~~323.~~ ~~324.~~ ~~325.~~ ~~326.~~ ~~327.~~ ~~328.~~ ~~329.~~ ~~330.~~ ~~331.~~ ~~332.~~ ~~333.~~ ~~334.~~ ~~335.~~ ~~336.~~ ~~337.~~ ~~338.~~ ~~339.~~ ~~340.~~ ~~341.~~ ~~342.~~ ~~343.~~ ~~344.~~ ~~345.~~ ~~346.~~ ~~347.~~ ~~348.~~ ~~349.~~ ~~350.~~ ~~351.~~ ~~352.~~ ~~353.~~ ~~354.~~ ~~355.~~ ~~356.~~ ~~357.~~ ~~358.~~ ~~359.~~ ~~360.~~ ~~361.~~ ~~362.~~ ~~363.~~ ~~364.~~ ~~365.~~ ~~366.~~ ~~367.~~ ~~368.~~ ~~369.~~ ~~370.~~ ~~371.~~ ~~372.~~ ~~373.~~ ~~374.~~ ~~375.~~ ~~376.~~ ~~377.~~ ~~378.~~ ~~379.~~ ~~380.~~ ~~381.~~ ~~382.~~ ~~383.~~ ~~384.~~ ~~385.~~ ~~386.~~ ~~387.~~ ~~388.~~ ~~389.~~ ~~390.~~ ~~391.~~ ~~392.~~ ~~393.~~ ~~394.~~ ~~395.~~ ~~396.~~ ~~397.~~ ~~398.~~ ~~399.~~ ~~400.~~ ~~401.~~ ~~402.~~ ~~403.~~ ~~404.~~ ~~405.~~ ~~406.~~ ~~407.~~ ~~408.~~ ~~409.~~ ~~410.~~ ~~411.~~ ~~412.~~ ~~413.~~ ~~414.~~ ~~415.~~ ~~416.~~ ~~417.~~ ~~418.~~ ~~419.~~ ~~420.~~ ~~421.~~ ~~422.~~ ~~423.~~ ~~424.~~ ~~425.~~ ~~426.~~ ~~427.~~ ~~428.~~ ~~429.~~ ~~430.~~ ~~431.~~ ~~432.~~ ~~433.~~ ~~434.~~ ~~435.~~ ~~436.~~ ~~437.~~ ~~438.~~ ~~439.~~ ~~440.~~ ~~441.~~ ~~442.~~ ~~443.~~ ~~444.~~ ~~445.~~ ~~446.~~ ~~447.~~ ~~448.~~ ~~449.~~ ~~450.~~ ~~451.~~ ~~452.~~ ~~453.~~ ~~454.~~ ~~455.~~ ~~456.~~ ~~457.~~ ~~458.~~ ~~459.~~ ~~460.~~ ~~461.~~ ~~462.~~ ~~463.~~ ~~464.~~ ~~465.~~ ~~466.~~ ~~467.~~ ~~468.~~

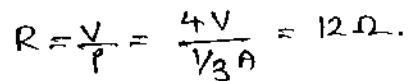
33

✱

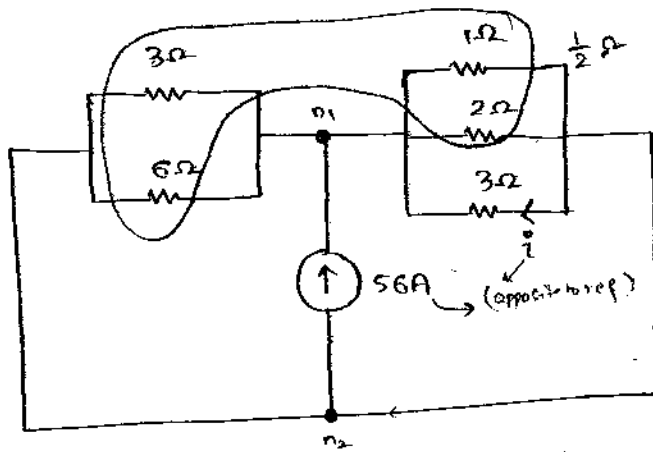
 t_{4v}

416-v

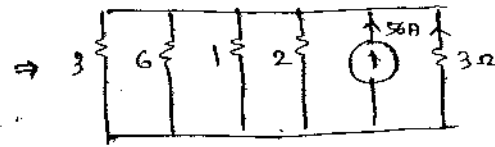
4v



*

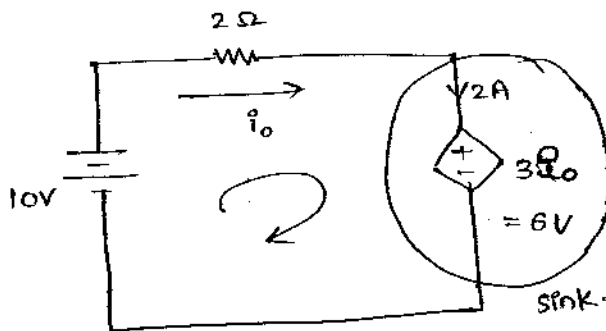


n_1 degree = 6 (6 branches)
 n_2 degree = 6



$$i = -56 \left[\frac{\frac{1}{2}}{\frac{7}{2}} \right] = -8A$$

* power delivered by dependent source



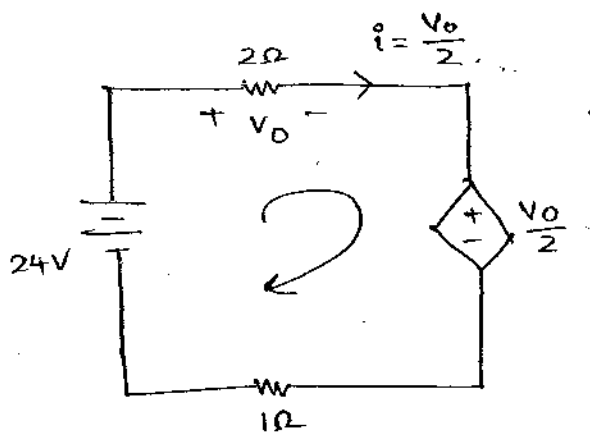
$$-10 + 2i_o + 3i_o = 0$$

$$i_o = 2A$$

$$P_{\text{deliv}} = -6(2) = -12W$$

$$-10V + 2i_o + 3i_o$$

* $V_o =$



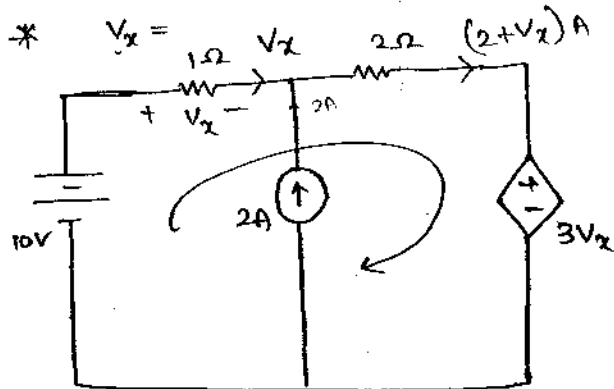
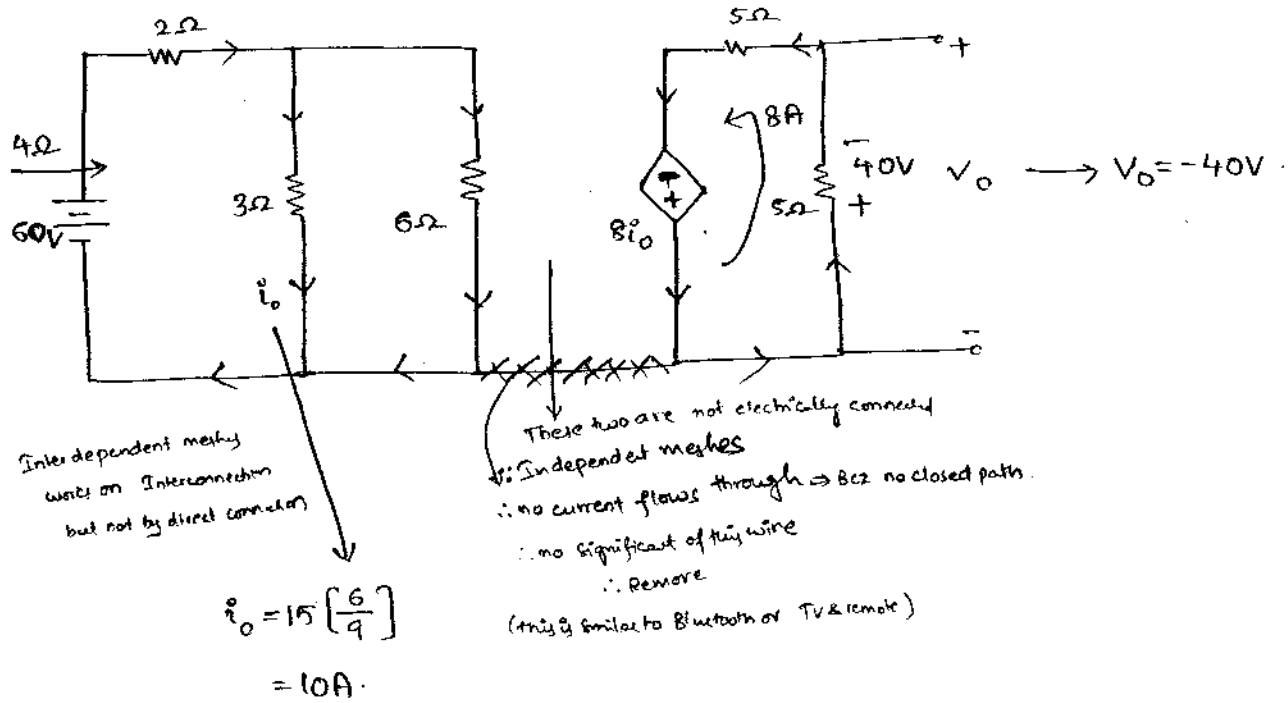
$$-24 + V_o + \frac{V_o}{2} + \frac{V_o}{2}(1) = 0$$

$$V_o = 12V$$

$$24 + V_o + \frac{V_o}{2} + V_o$$

VDCS \rightarrow TV & Remote

* $V_0 =$

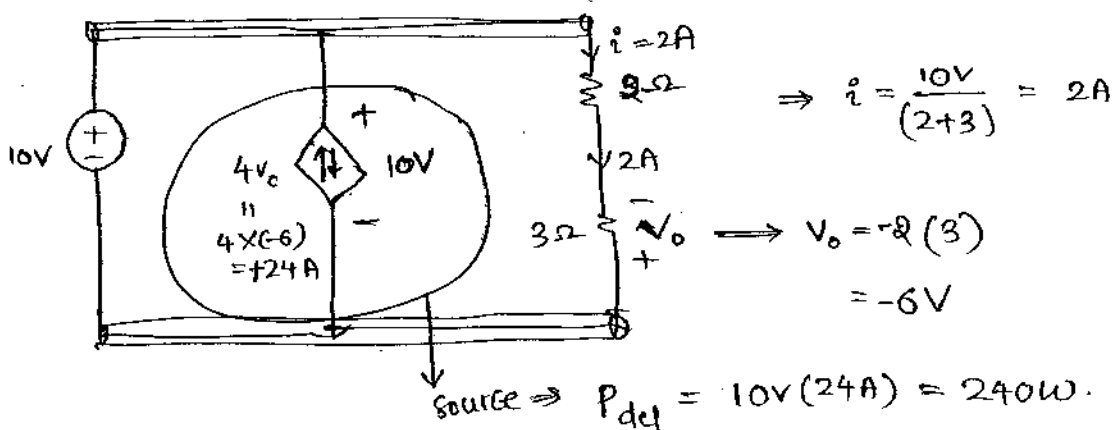


~~$10 + V_x(1)$~~

$-10 + V_x + (2+V_x) \cdot 2 + 3V_x = 0$

$6V_x = 6 \rightarrow V_x = 1V$

* power delivered by dependent source



$4V_0 = 10$

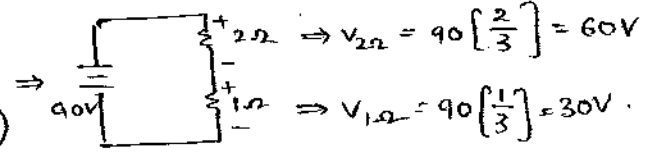
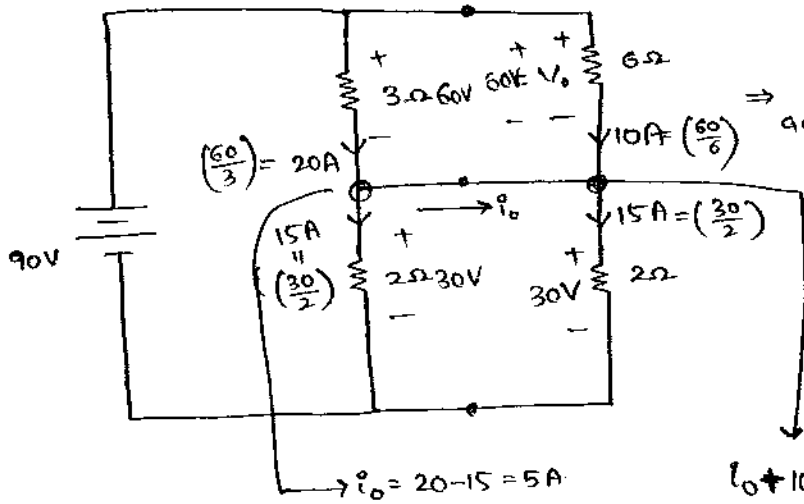
$V_0 = \frac{10}{4} = 2.5$

$2 \times \frac{10}{2} = 10$

$3 \times \frac{10}{3} = 10$

Gate.
*

$$V_o = 60V \quad i_o = 5A$$

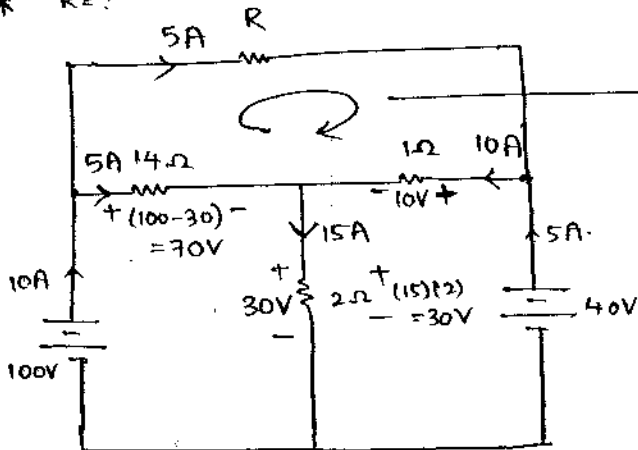


40

$$i_o + 10 = 15$$

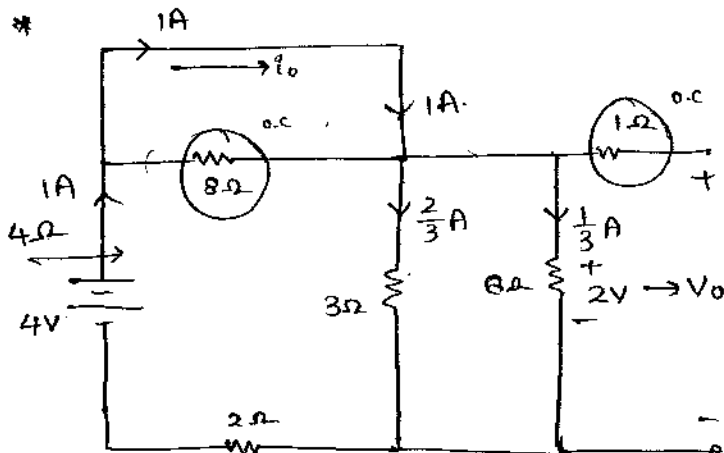
$$i_o = 5A$$

* R = ?



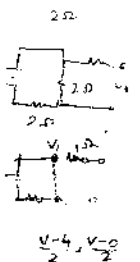
$$5R + 10 - 70 = 0 \rightarrow R = 12\Omega$$

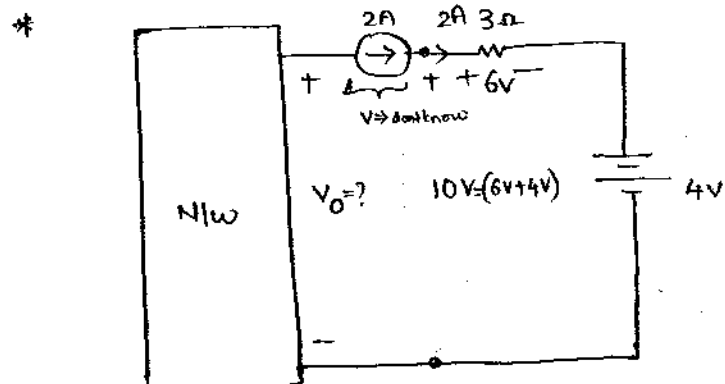
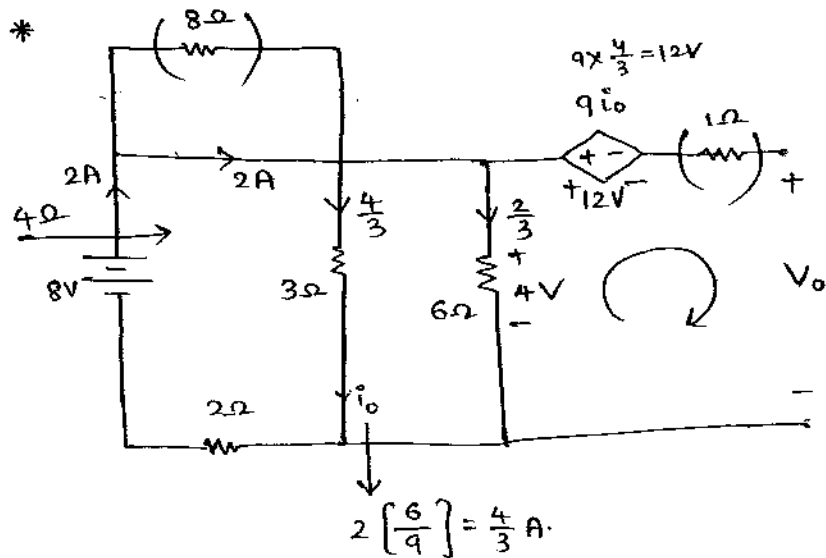
*



$$i_o = 1A$$

$$V_o = 2V$$

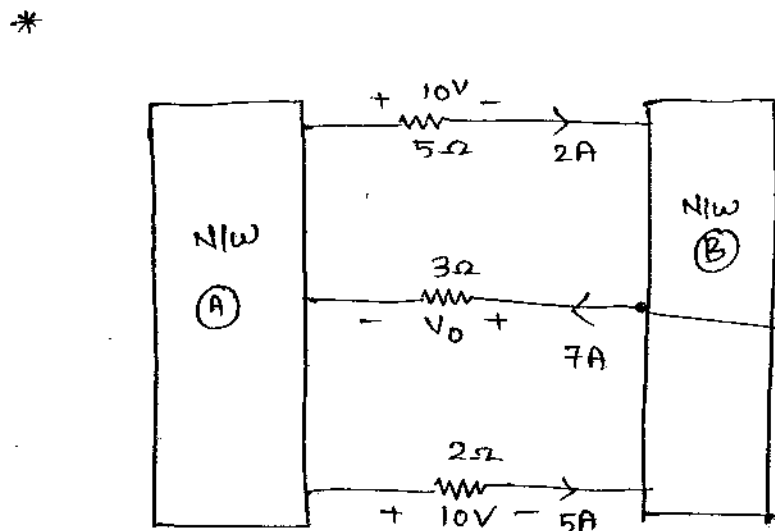




(a) 4V (b) 10V (c) 12V (d) None.

∴ data is insufficient

∴ we don't know Volt. across $2A$ source.



(a) 10V (b) -10V (c) +21V (d) -21V

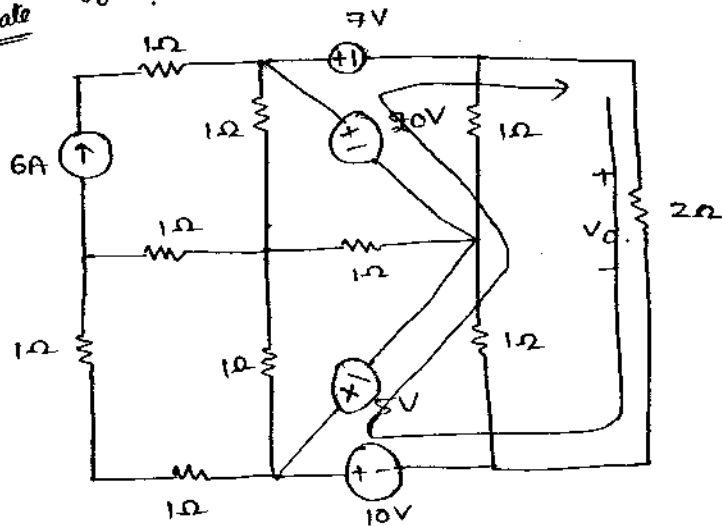
By KCL $2A + 5A = 7A$

$$= 7 \times 3\Omega$$

$$= 21V$$

Gate

$$V_0 = ?$$



Write KVL in which voltages are visible.

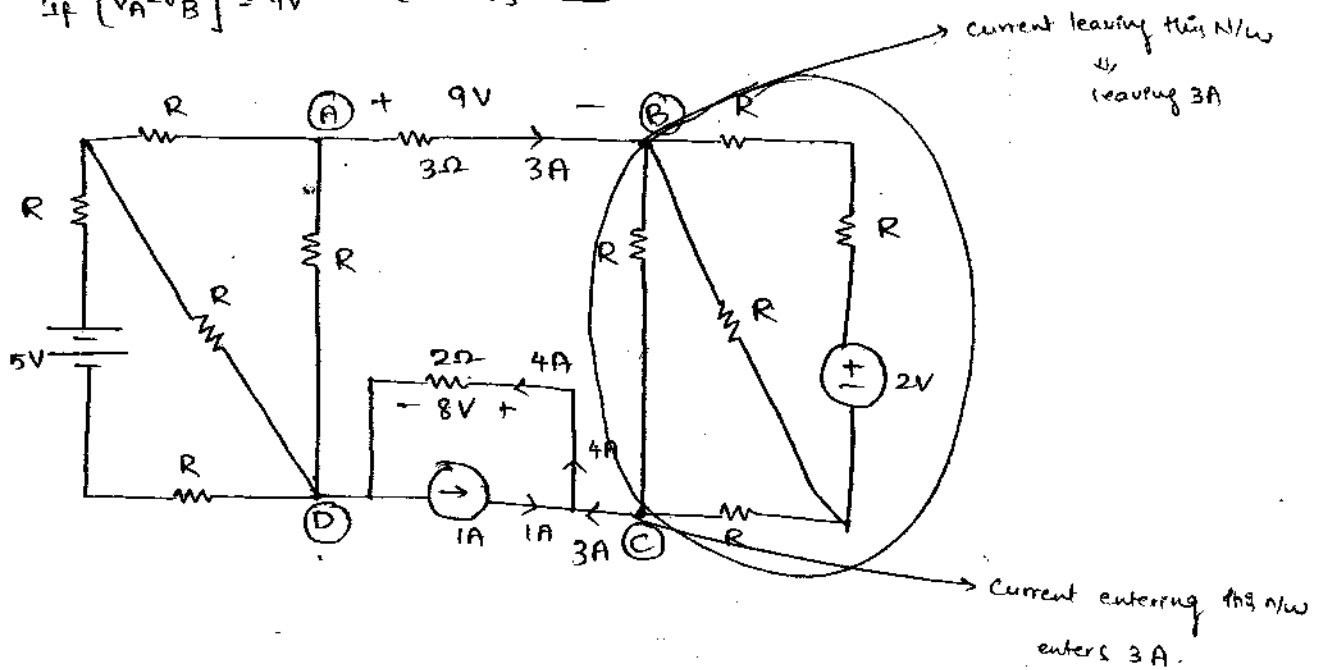
$$-10 + 5 - 20 + 7 + V_0 = 0$$

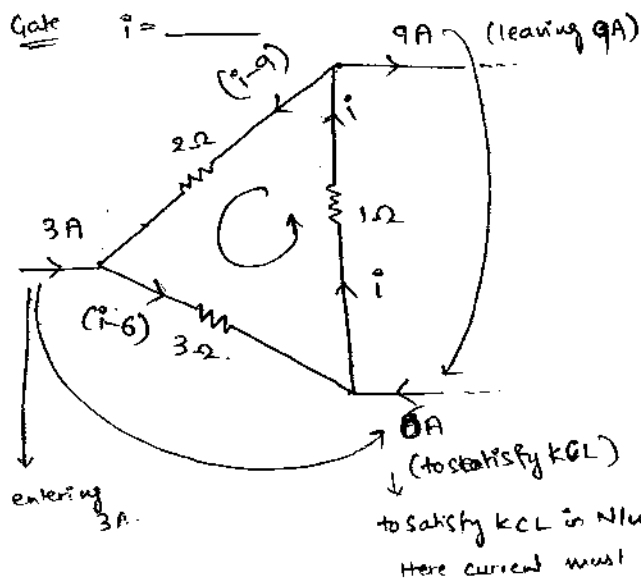
$$V_0 = +18V.$$

41

Gate 115 D

$$\text{If } [V_A - V_B] = 9V \text{ then } [V_C - V_D] = +18V$$

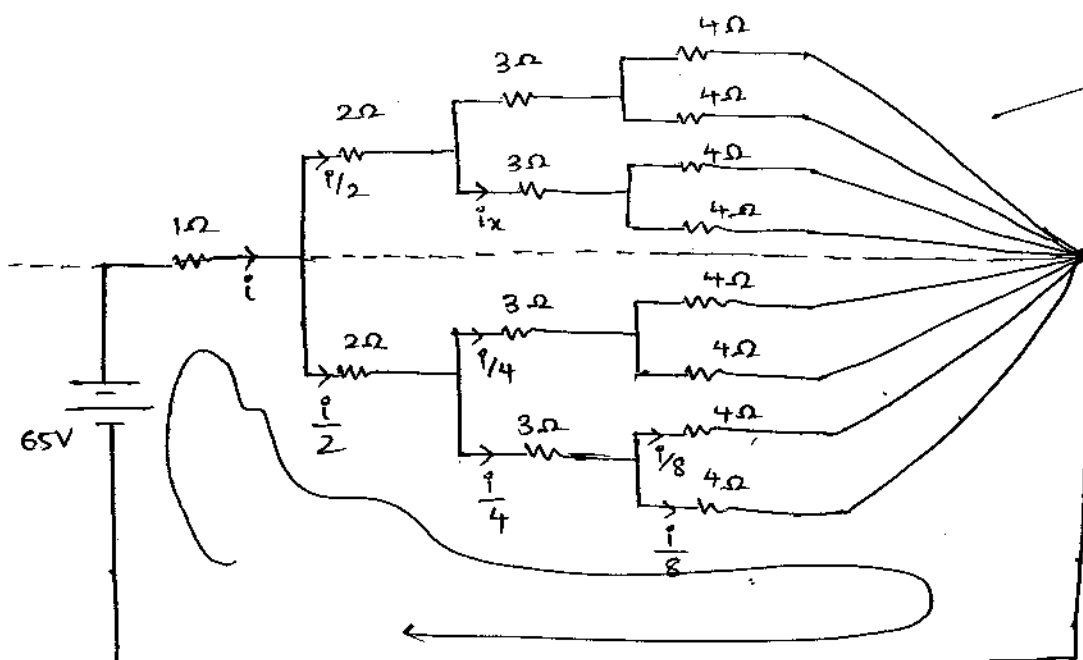




$$3 \times \frac{2}{5} = \frac{6}{5}$$

$$3 \times \frac{3}{5} = \frac{9}{5}$$

gate $i_x =$ _____



path R is all paths
 is equal
 → symmetry
 → draw mirror image

$$-65 + i \left[1 + \frac{1}{2} \times 1 + 3 \times \frac{1}{4} + 4 \times \frac{1}{8} \right] = 0$$

$$i \left[\frac{13}{4} \right] = 65$$

$$i = 20A$$

$$\text{But } i_x = \frac{i}{4} = \frac{20}{4} = \underline{\underline{5A}}$$

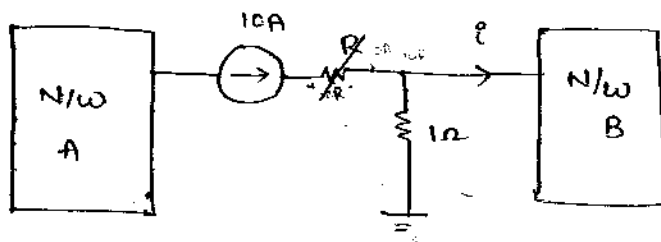
IES If $R = 2\Omega$, then $i = 5A$

Now if $R = 5 \Omega$ then $i = \underline{5A}$

$$R = 2\ \Omega \longrightarrow i = 5\text{ A}$$

$$R = 5\Omega \longrightarrow i = 5A$$

42



$\therefore 'R'$ dissipates \leftarrow

no loss in current value

\therefore for any value of R $\rightarrow 100 + \frac{0.15}{1}$

Current will be same

Current Source to
any element i

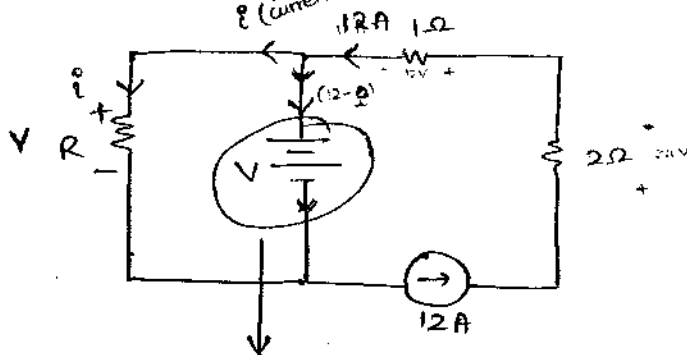
Redundant.

gate

If Voltage Source absorb power then Current $i =$ _____
(must be less than 12A) \rightarrow from options

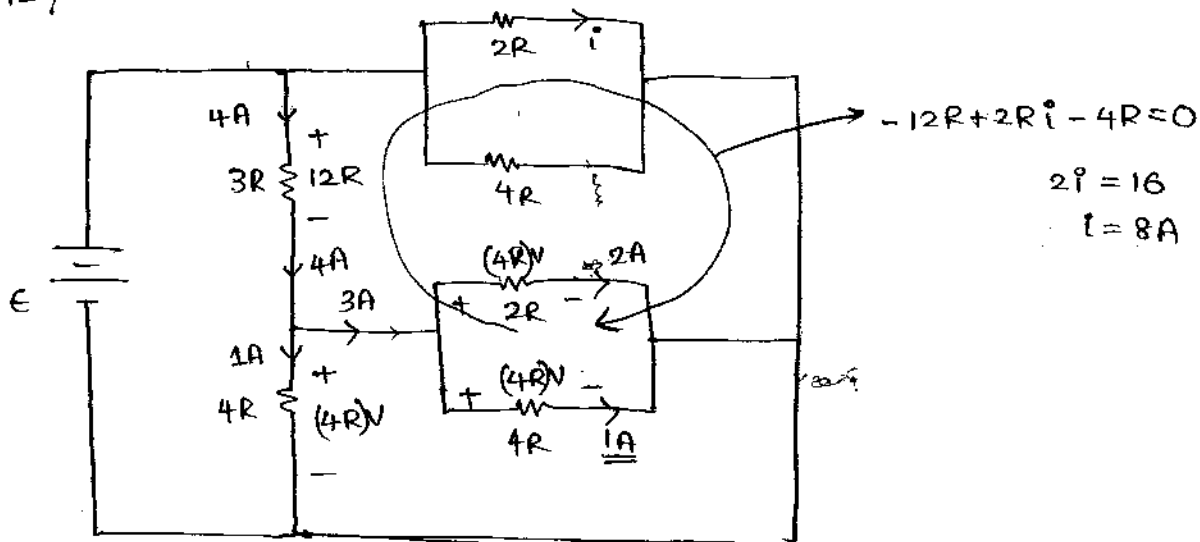
(i) current must be less than 12A) \rightarrow from options
12A 12 $\cancel{12}$ (a)

c) ~~(a)~~ 10 A (b) 12 A (c) 13 A (d) 14 A.



Sink \therefore source absorbing power

942



$$-12R + 2Ri - 4R = 0$$

$$2^i = 16$$

$$I = 8 \text{ A}$$

2×4

$$\frac{1}{2} + \frac{1}{4}$$

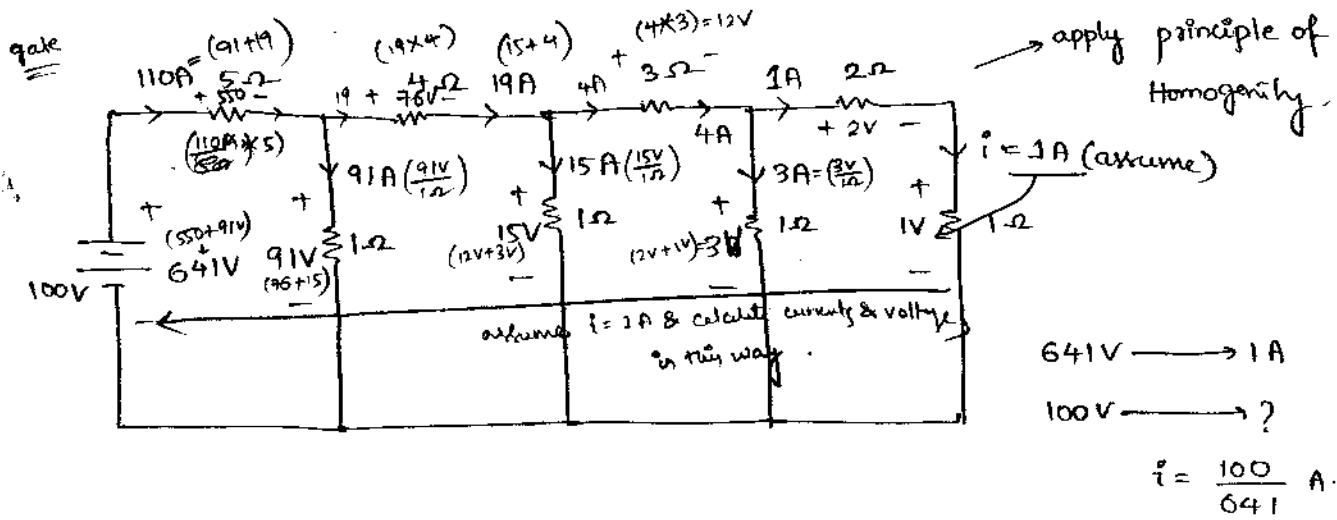
f.

6.

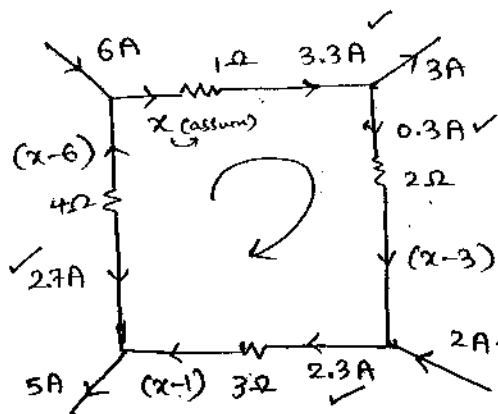
\mathbf{R}_i

343

2004



165(c)
* Total power lost in NIW



$$x + 2[x - 3] + 3[x - 1] + 4[x - 6] = 0$$

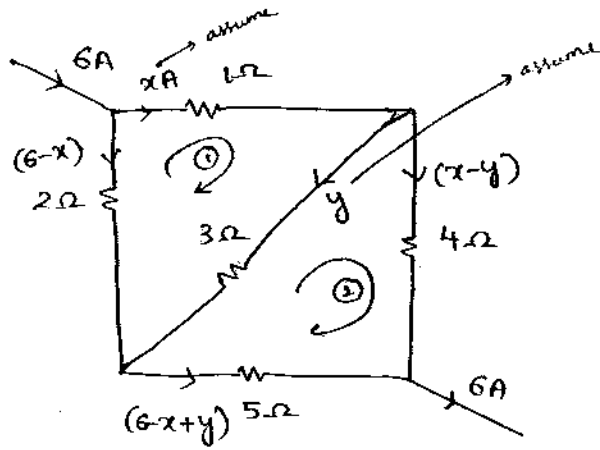
$$10x = 33$$

$$x = 3.3 A$$

$$\text{Total power loss} = (3.3)^2(1) + (0.3)^2(2) + (2.3)^2(3) + (2.9)^2(4)$$

1ES(C)

Determine all Branch currents



$$(1) x + 3(y) + 2(6-x) = 0 \rightarrow (1)$$

$$4(x-y) - 5(6-x+y) - 3y = 0 \rightarrow (2)$$

solve (1) & (2)

43

$$x = \underline{\hspace{2cm}}$$

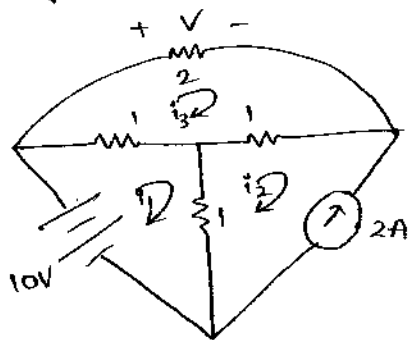
$$y = \underline{\hspace{2cm}}$$

Methods of Analysis

1. Mesh Analysis \rightarrow KVL + OHM's Law (More Current Sources than Voltage sources)
Mesh is faster.
 \rightarrow PLANAR N/w is Required.

2. Nodal Analysis \rightarrow KCL + OHM's Law (More voltage sources than Current sources)
Nodal is faster.

determine voltage 'V' by mesh & nodal analysis



Mesh:

$$-10 + 1[i_1 - i_3] + 1[i_1 - i_2] = 0$$

$$2i_1 - i_2 - i_3 = 10 \rightarrow (1)$$

$$i_2 = -2 \quad (\text{Comparison Eqn}) \rightarrow (2)$$

Current source in mesh analysis is Comparison Eqn.

$$2i_3 + 1[i_3 - i_2] + 1[i_3 - i_1] = 0$$

$$-i_1 - i_2 + 4i_3 = 0 \rightarrow (3)$$

$i_2 = -2$

$$(1) \rightarrow 2i_1 - i_3 = 10$$

$$(3) \times 2 \rightarrow -2i_1 + 8i_3 = -4$$

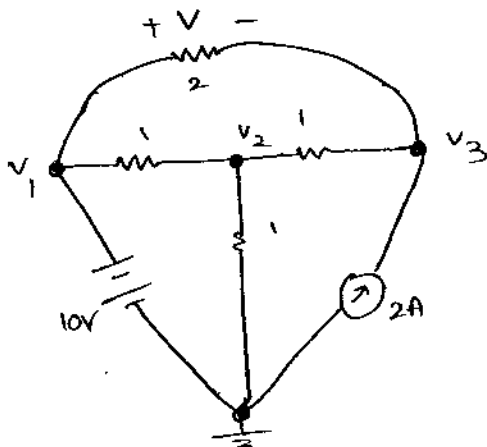
$$7i_3 = 4$$

$$i_3 = \frac{4}{7} \text{ A}$$

But $V = 2i_3$

$$V = 2 \left[\frac{4}{7} \right]$$

$$V = \frac{8}{7} \text{ volts}$$



Nodal:

$$V_1 = 10 \text{ V} \rightarrow (1) \quad (\text{Comparison Eqn})$$

$$\frac{V_2 - V_1}{1} + \frac{V_2 - 0}{1} + \frac{(V_2 - V_3)}{1} = 0$$

$$3V_2 - V_3 = 10 \rightarrow (2)$$

$$-2 + \frac{(V_3 - V_2)}{1} + \frac{(V_3 - 10)}{2} = 0$$

$$-2V_2 + 3V_3 = 14 \rightarrow (3)$$

$$(2) \times 2 \rightarrow 6V_2 - 2V_3 = 20$$

$$(3) \times 3 \rightarrow -6V_2 + 9V_3 = 42$$

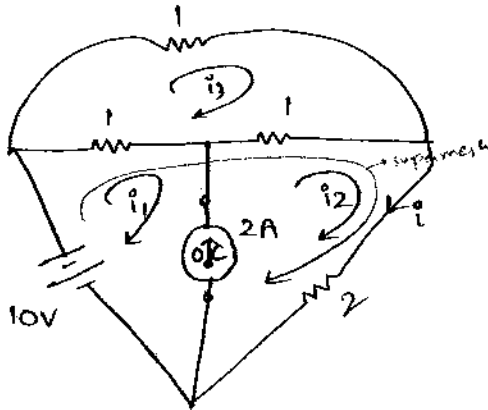
$$7V_3 = 62$$

$$V = V_1 - V_3$$

$$= 10 - \frac{62}{7}$$

$$V = \frac{8}{7} \text{ V}$$

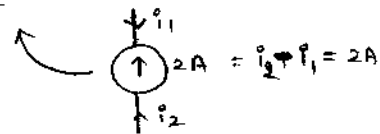
Q determine Current (i) in ~~circuit~~^{ckt} shown below by using mesh & nodal analysis



$$-10 + 1[i_1 - i_3] + 1[i_2 - i_3] + 2i_3 = 0$$

$$i_1 + 3i_2 - 2i_3 = 10 \rightarrow (1)$$

$$-i_1 + i_2 = 2A \rightarrow (2)$$



Current Sources
b/w two meshes
treat as O.C

ie, KVL for
Both meshes
↓
Super mesh

(44)

$$i_3 + 1[i_3 - i_2] + 1[i_3 - i_1] = 0$$

$$-i_1 - i_2 + 3i_3 = 0 \rightarrow (3)$$

$$4i_2 - 2i_3 = 12$$

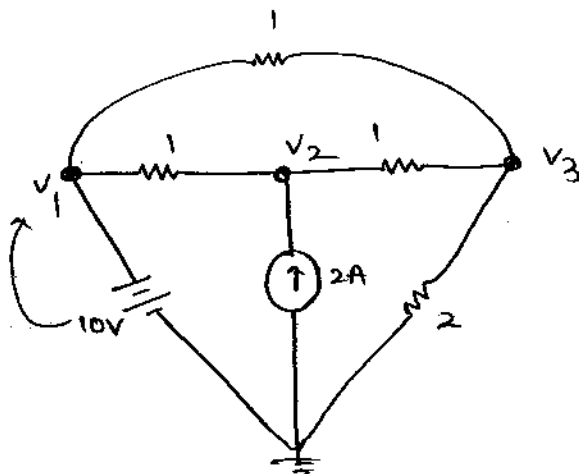
$$2i_2 + i_3 = 10$$

$$8i_2 = 32$$

$$i_2 = 4A$$

But $i_2 = i$

$$\Rightarrow i = 4A$$



Nodal
KCL

$$V_1 = 10 \rightarrow (1)$$

$$\frac{V_2 - 10}{1} - 2 + \frac{(V_2 - V_3)}{1} = 0$$

$$2V_2 - V_3 = 12 \rightarrow (2)$$

$$\frac{V_3}{2} + \frac{(V_3 - V_2)}{1} + \frac{(V_3 - 10)}{1} = 0$$

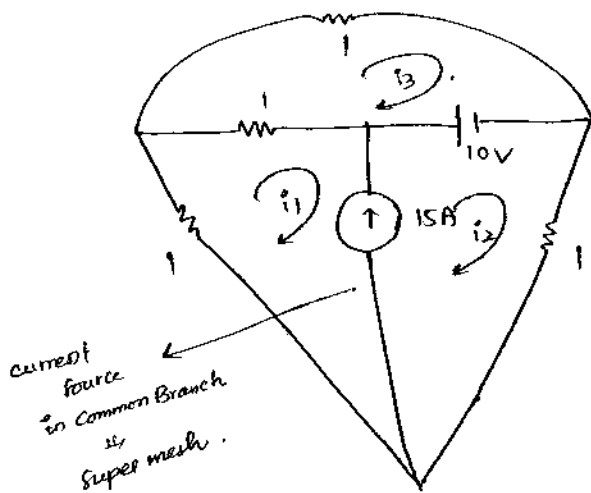
$$-2V_2 + 5V_3 = 20 \rightarrow (3)$$

$$(2) + (3)$$

$$4V_3 = 32 \rightarrow V_3 = 8V$$

$$\text{But } i = \frac{V_3}{2} = \frac{8}{2} = \underline{\underline{4A}}$$

Q determine the power delivered by the voltage source using mesh & Nodal Analysis.



Mech

$$i_1 + i_1[i_1 - i_3] + 10 + i_2 = 0$$

$$2i_1 + i_2 - i_3 = -10 \rightarrow (1)$$

$$-i_1 + i_2 = 15 \rightarrow (2)$$

$$i_3 - 10 + i_1[i_3 - i_1] = 0$$

$$-i_1 + 2i_3 = 10 \rightarrow (3)$$

$$3i_1 - i_3 = -25$$

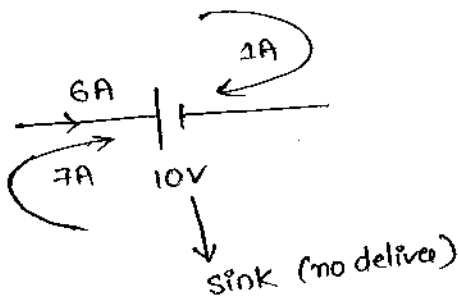
$$-3i_1 + 6i_3 = 30$$

$$5i_3 = 5$$

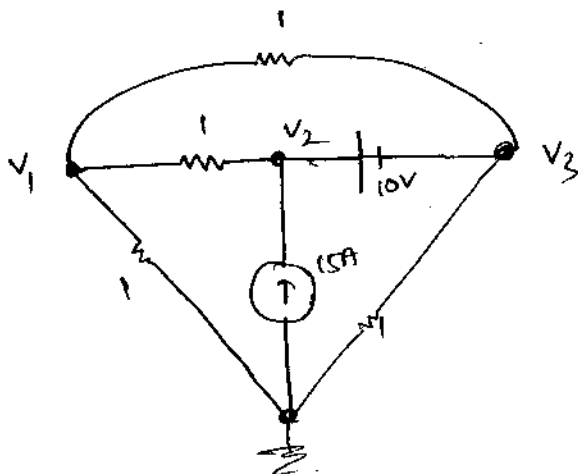
$$i_3 = 1A$$

$$i_1 = -8A$$

$$i_2 = 7A$$



$$P_{\text{deliver}} = -10(6) = -60W$$



Nodal

$$\frac{V_1}{1} + \frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{1} = 0$$

$$3V_1 - V_2 - V_3 = 0 \rightarrow (1)$$

$$\underbrace{\frac{V_2 - V_1}{1}}_{\text{at } V_2} - 15 + \underbrace{\frac{V_3}{1} + \frac{V_3 - V_1}{1}}_{\text{at } V_3} = 0$$

$$-2V_1 + V_2 + 2V_3 = 15 \rightarrow (2)$$

$$V_2 - V_3 = 10 \rightarrow (3) \text{ (comp. eqn)}$$

ideal volt source
b/w two principle nodes
treat it like
short circuit ($R_{VS} = 0$)
ie super mesh.
& write KCL combining
for two nodes
which u.s.c.

$$\textcircled{1} \times 2 \quad 6V_1 - 2V_2 - 2V_3 = 0$$

$$\textcircled{2} \times 3 \quad -6V_1 + 3V_2 + 6V_3 = 45$$

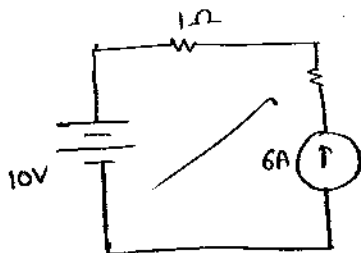
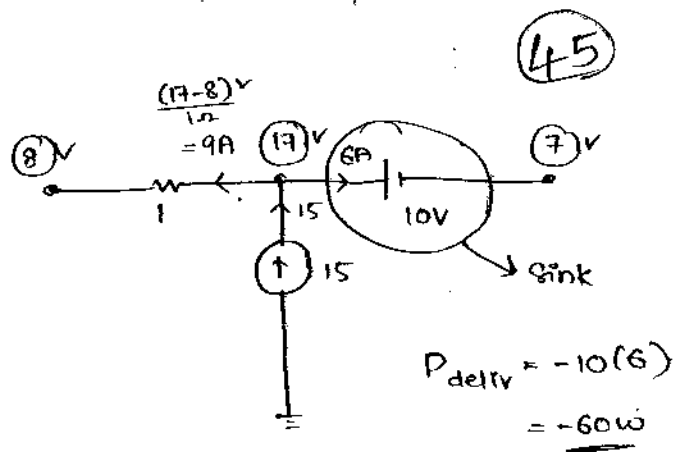
$$V_2 + 4V_3 = 45$$

$$5V_3 = 35$$

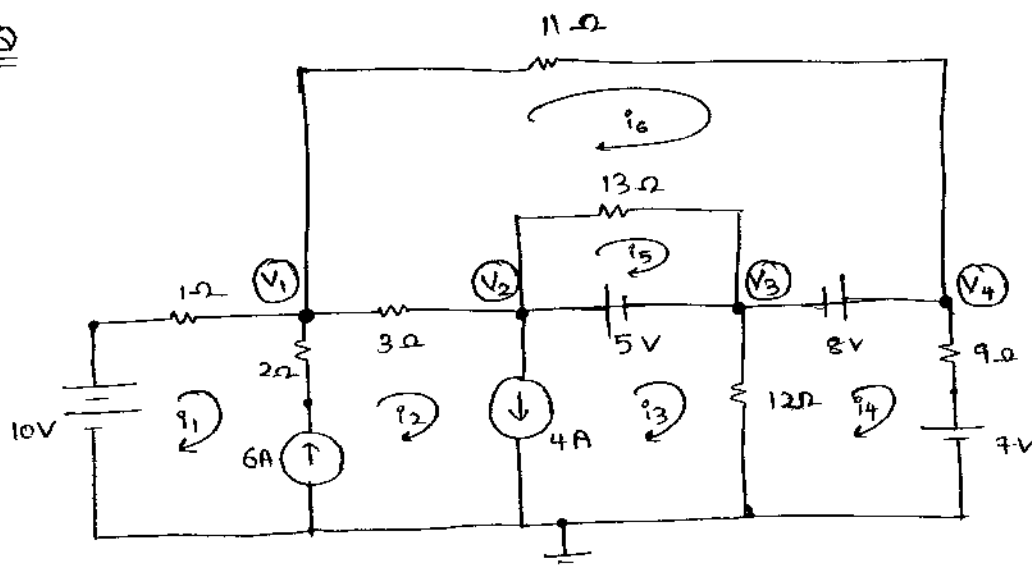
$$V_3 = 7V$$

$$V_2 = 17V$$

$$V_1 = 8V$$



Q



$$\begin{aligned} n &= 8 \\ b &= 13 \\ m &= 13 - 8 + 1 \\ &= 6. \end{aligned}$$

Mesh

$$-10 + i_1(1) + 2(i_1 - i_2) + 3(i_2 - i_6) + 5V + 12(i_3 - i_4) = 0$$

$$-i_1 + i_2 = 6$$

$$i_2 - i_3 = 4$$

$$12(i_4 - i_3) - 8V + 9(i_4) + 7V = 0$$

$$-5V + 13(i_5 - i_6) = 0$$

$$3(i_6 - i_2) + 11(i_6) + 8V + 13(i_6 - i_5) = 0$$

Nodal

$$\frac{V_1 - 10}{1\Omega} + 6A + \frac{V_1 - V_2}{3} + \frac{V_1 - V_4}{11} = 0$$

$$\frac{V_2 - V_1}{3} + 4A + \frac{V_2 - V_3}{13} + \frac{V_3 - 0}{12} + \frac{V_3 - V_2}{13} +$$

$$\frac{V_4 - V_1}{11} + \frac{V_4 - 7}{9} = 0$$

$$V_2 - V_3 = 5V$$

$$V_4 - V_3 = 8V$$

Mesh

$$-10 + i_1(1) + 2(i_1 - i_2) + 2(i_2 - i_1) + 3(i_2 - i_6) + 5 + 12(i_3 - i_4) = 0 \rightarrow (1)$$

$$-i_1 + i_2 = 6 \rightarrow (2)$$

$$i_2 - i_3 = 4 \rightarrow (3)$$

$$12(i_4 - i_3) - 8 + 9i_4 + 7 = 0 \rightarrow (4)$$

$$13(i_5 - i_6) - 5 = 0 \rightarrow (5)$$

$$11i_6 + 8 + 13(i_6 - i_5) + 3(i_6 - i_2) = 0 \rightarrow (6)$$

Nodal

$$\frac{V_1 - 10}{1} + 8V + \frac{(V_1 - V_2)}{3} + \frac{(V_1 - V_4)}{11} = 0 \rightarrow (1)$$

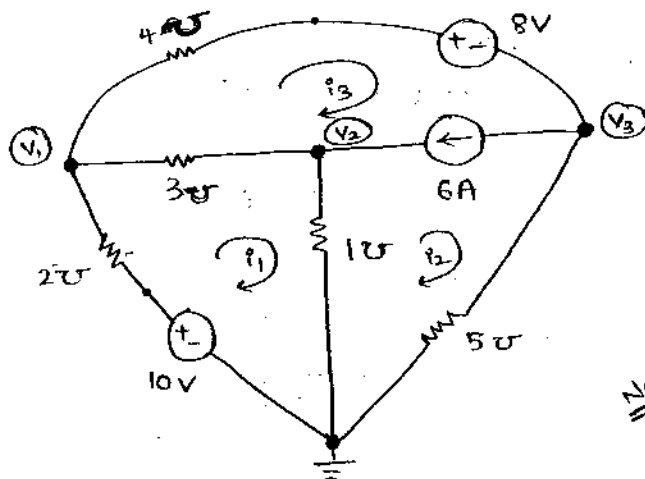
46

$$\frac{V_2 - V_1}{3} + 4 + \frac{V_2 - V_3}{3} + \frac{V_3 - V_2}{3} + \frac{V_3}{12} + \frac{(V_4 - 7)}{9} + \frac{(V_4 - V_1)}{11} = 0 \rightarrow (2)$$

$$V_2 - V_3 = 5 \rightarrow (3)$$

$$-V_3 + V_4 = 8V \rightarrow (4)$$

Q Determine power absorbed by (3 Siemens) Conductance by using Mesh & nodal.



$$n=6, b=8, m=8-6+1=3.$$

Mesh

$$-10 + \frac{i_1}{2} + \frac{(i_1 - i_3)}{3} + \frac{(i_1 - i_2)}{1} = 0$$

$$\frac{(i_2 - i_1)}{1} + \frac{(i_2)}{5} + \frac{(i_3 - i_1)}{3} + \frac{(i_3)}{4} + 8V = 0$$

$$i_3 - i_2 = 6A$$

Nodal

$$(V_1 - 10)2 + (V_1 - V_2)(3) + (V_1 - 8 - V_3)(4) = 0$$

$$(V_2 - 0)(1) + (V_2 - V_1)(3) + (-6)A = 0$$

$$(V_3 - 0)(5) + (V_3 + 8V - V_1)(4) + (6)A = 0$$

Nodal

$$2[V_1 - 10] + 3[V_1 - V_2] + 4[V_1 - V_3 - 8] = 0 \rightarrow (1)$$

$$3[V_2 - V_1] + 1(V_2) + (-6)A = 0 \rightarrow (2)$$

$$5V_3 + 4(V_3 + 8V - V_1) + (6)A = 0 \rightarrow (3)$$

$$V_1 = \text{---} V$$

$$V_2 = \text{---} V$$

$$P_{3\Omega} = |V_1 - V_2|^2 (3) = \text{---} W$$

Mesh

$$-10 + \frac{i_1}{2} + \frac{(i_1 - i_3)}{3} + \frac{(i_1 - i_2)}{1} = 0 \rightarrow (1)$$

$$\frac{(i_2 - i_1)}{1} + \frac{(i_3 - i_1)}{3} + \frac{i_3}{4} + 8 + \frac{i_2}{5} = 0 \rightarrow (2)$$

$$-i_2 + i_3 = 6 \rightarrow (3)$$

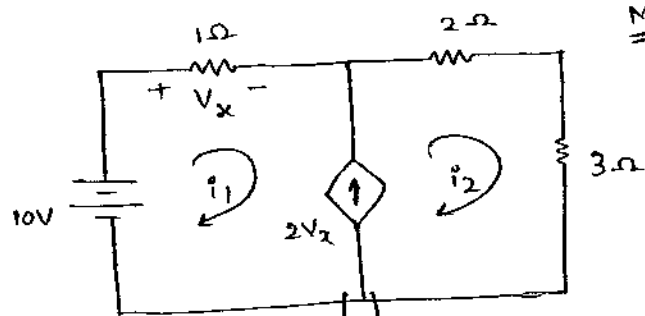
Solve $i_1 = \text{---} A$

$$i_3 = \text{---} A$$

then

$$P_{3\Omega} = \frac{|i_1 - i_3|^2}{3} = \text{---} W$$

Q Determine Voltage V_x by using mesh & Nodal Analysis.



Current sources common
super mesh
even dependent source
treat as normal source.

Mesh

$$-10 + i_1(1) + 2(i_2) + 3(i_2) = 0$$

$$-10 + i_1 + 5i_2 = 0 \rightarrow (1)$$

$$-i_1 + i_2 = 2V_x \rightarrow (2)$$

$$V_x = 1 \cdot i_1 \text{ [link Eqn]}$$

link & link
b/w mesh current &
dependent parameter

* if dependent Sources involved

no. of Eqns required

$$= \text{no. of mesh Eqns} + \text{no. of dependent source link Eqns}$$

$$= \text{no. of nodal Eqns} + \text{no. of dependent source link Eqns}$$

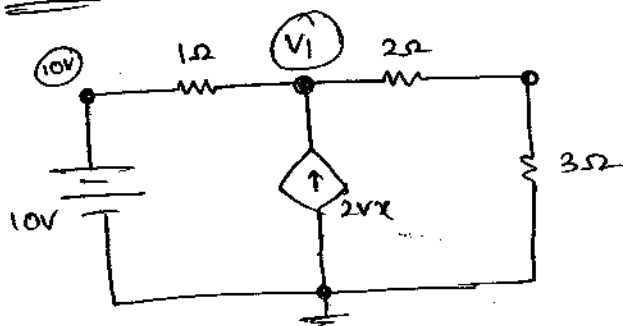
$$3i_1 - i_2 = 0$$

$$15i_1 - 5i_2 = 0$$

$$16i_1 = 10 \rightarrow i_1 = \frac{5}{8} \text{ A}$$

$$\text{But } V_x = i_1(1) = \frac{5}{8} \text{ V.}$$

Nodal



$$\frac{V_1 - 10}{1} - 2V_x + \frac{V_1}{5} = 0$$

$$6V_1 - 10V_x = 50$$

$$3V_1 - 5V_x = 25 \rightarrow (1)$$

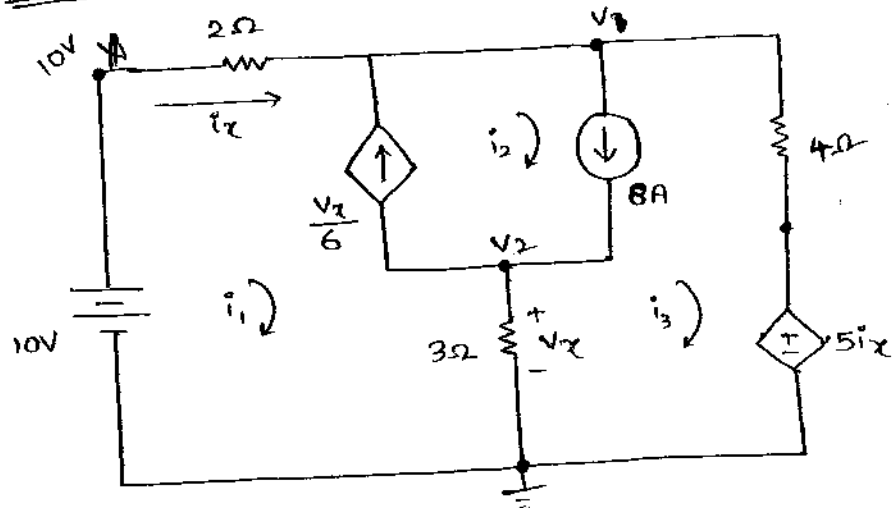
$$V_x = 10 - V_1 \rightarrow (2)$$

$$3[10 - V_x] - 5V_x = 25$$

$$8V_x = 5$$

$$V_x = \frac{5}{8} \text{ V}$$

IES (C)



Write mesh & nodal analysis.

$$n = 5 \quad b = 7 \quad m = 3$$

Mesh = 3 mesh eqns + 2 link eqns
(∵ 2 dependent source)

Nodal = 2 nodal eqns + 2 link eqns.

47

Mesh

$$-10 + 2i_1 + 4i_3 + 5i_x = 0 \rightarrow (1)$$

$$-i_1 + i_2 = \frac{V_x}{6} \rightarrow (2)$$

$$i_2 - i_3 = 8 \rightarrow (3)$$

$$i_x = i_1 \rightarrow (4) \quad \left. \begin{array}{l} (2) \\ (3) \end{array} \right\} \text{link}$$

$$V_x = 3[i_1 - i_3] \rightarrow (5)$$

Nodal

$$\frac{(V_1 - 10)}{2} - \frac{V_x}{6} + 8 + \frac{(V_1 - 5i_x)}{4} = 0 \rightarrow (1)$$

$$+ \frac{V_x}{6} - 8 + \frac{V_2}{3} = 0 \rightarrow (2)$$

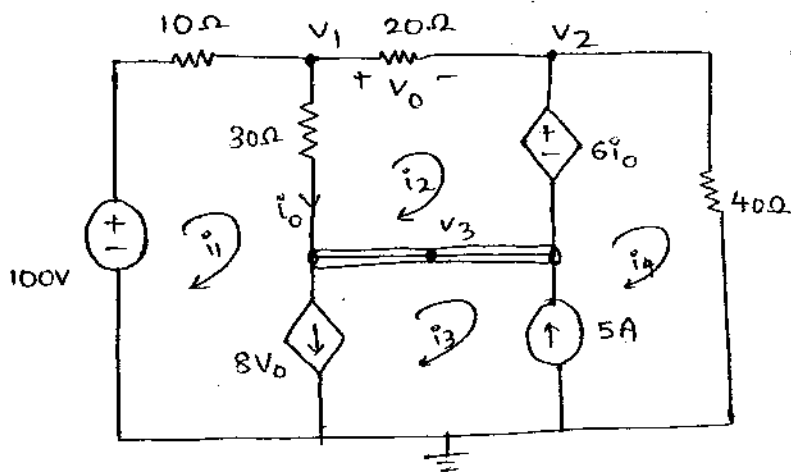
$$i_x = \frac{10 - V_1}{2} \rightarrow (3) \quad \left. \begin{array}{l} (1) \\ (2) \end{array} \right\} \text{link}$$

$$V_x = V_2 \rightarrow (4)$$

$$n = 5, b = 8, m = 4$$

$$\text{mesh} = 4 + 2 = 6$$

$$\text{Nodal} = 3 + 2 = 5$$



Mesh

$$-100 + 10i_1 + 30[i_1 - i_2] - 6i_o + 40i_4 = 0 \rightarrow (1)$$

$$30[i_2 - i_1] + 20i_2 + 6i_o = 0 \rightarrow (2)$$

$$(i_1 - i_3) = 8V_o \rightarrow (3)$$

$$(-i_3 + i_4) = 5 \rightarrow (4)$$

$$i_o = (i_1 - i_2) \rightarrow (5)$$

$$V_o = 20i_2 \rightarrow (6)$$

Nodal

$$\frac{(v_1 - 100)}{10} + \frac{(v_1 - v_2)}{20} + \frac{(v_1 - v_3)}{30} = 0 \rightarrow (1)$$

$$\frac{(v_2 - v_1)}{20} + \frac{v_2}{40} - 5 + 8V_o + \frac{(v_3 - v_1)}{30} = 0 \rightarrow (2)$$

$$(v_2 - v_3) = 6i_o \rightarrow (3)$$

$$i_o = \frac{(v_1 - v_3)}{30} \rightarrow (4)$$

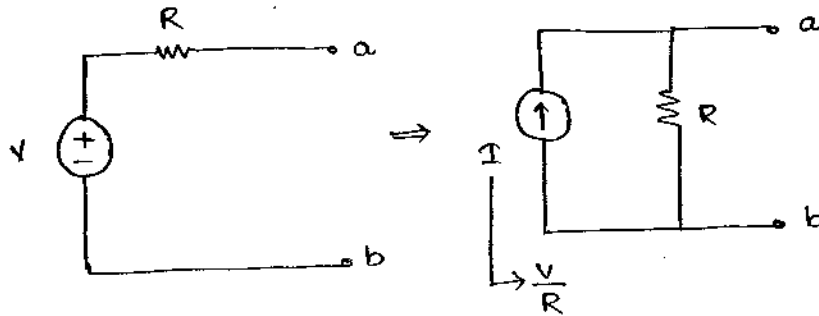
$$V_o = (v_1 - v_2) \rightarrow (5)$$

DC Network Theorems

48

Theorem 1: Source Transformation Technique (S.T.T)

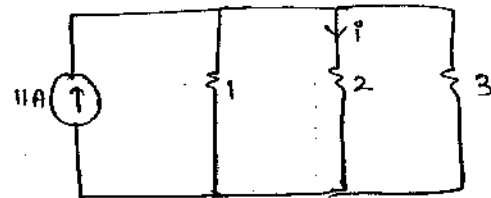
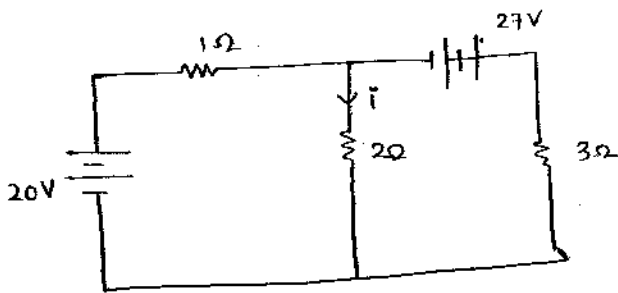
an ideal Voltage Source in Series with Resistance can be converted into ideal Current Source in parallel to the same Resistance across the same two terminals and vice versa.



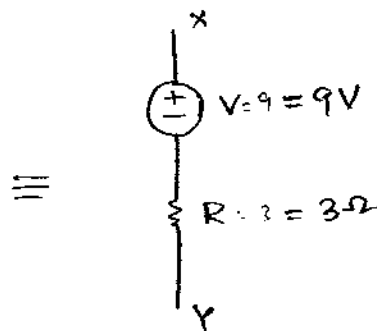
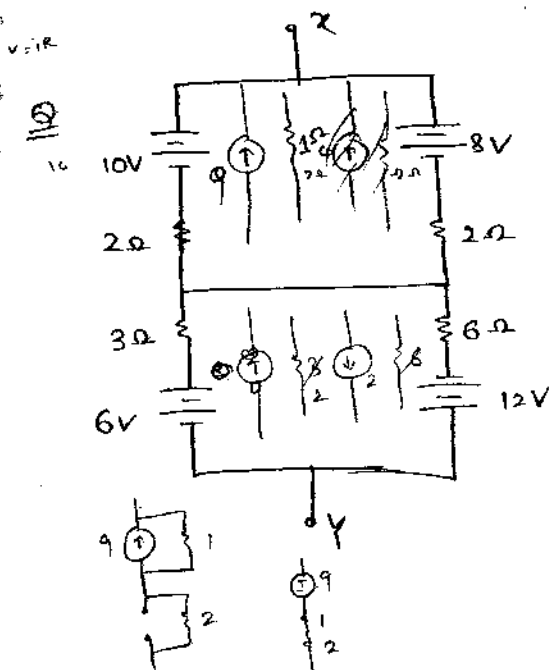
Volt. Reg \rightarrow MOSFET
Cur. Reg \rightarrow BJT circuit
 \downarrow
Const. Current

S.T.T based on ohm's law
based Reduction
Technique.

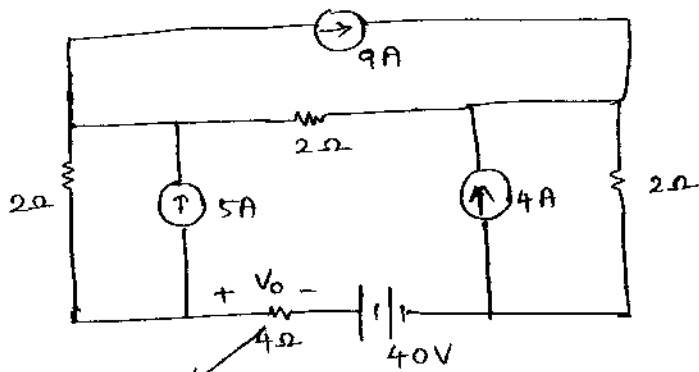
Q Determine the Current i in the ckt shown by S.T.T.



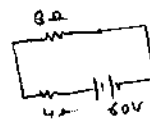
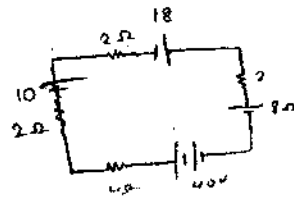
$$i = 11 \left[\frac{3 \times 1}{2 + 6 + 3} \right] = 3A.$$



IES(0) $V_0 = \underline{\hspace{2cm}}$



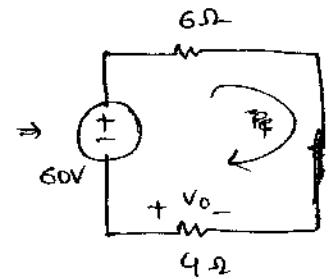
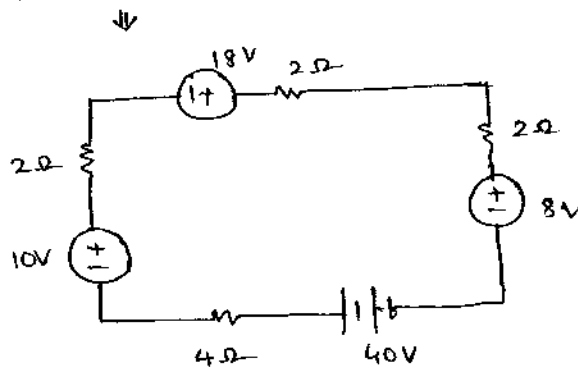
Shouldn't distribute Target
w/ Target rest of w/w reduce.



28.

$\frac{60}{1.2} = 24$

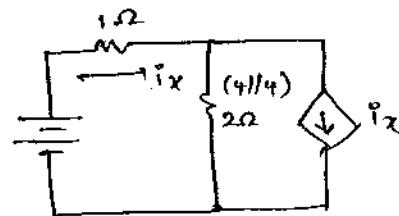
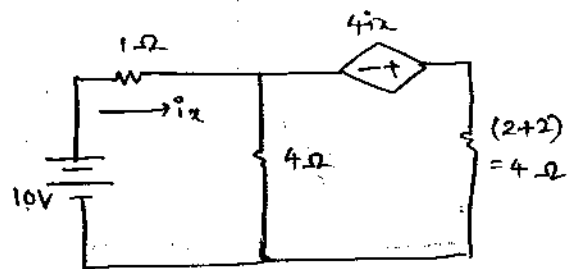
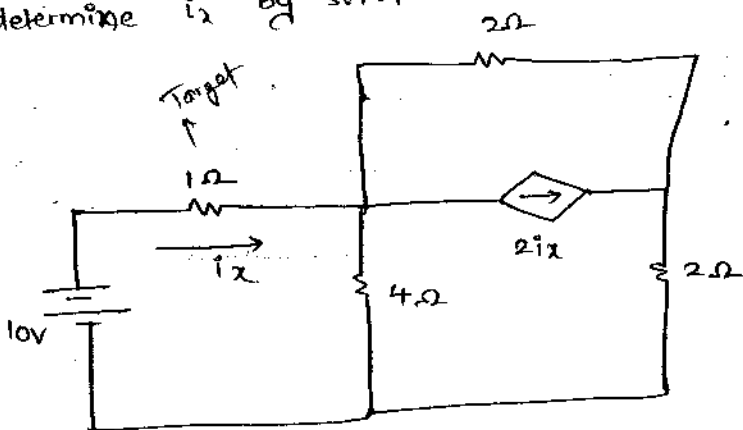
$\frac{60}{1.2} = 24$



$V_0 = -60 \left[\frac{4}{10} \right]$

$= -24V$

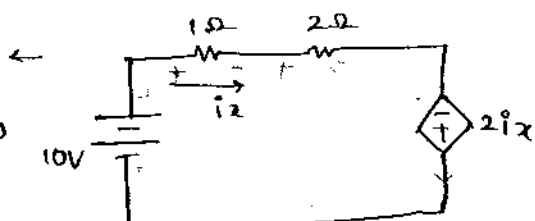
Q determine i_2 by S.T.T

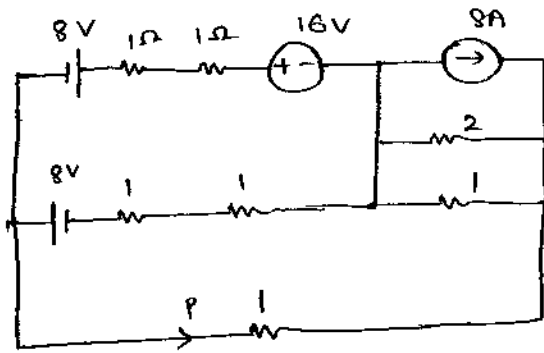


KVL

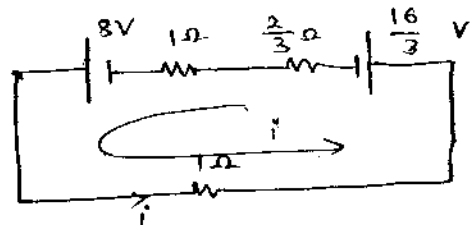
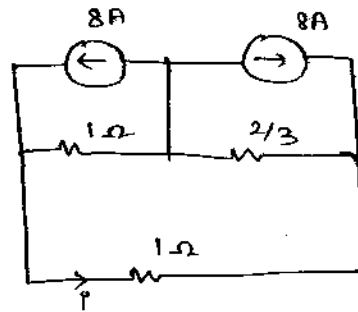
$-10 + i_2(1) + i_2(2) - 2i_2 = 0$

$i_2 = 10A$





\Rightarrow



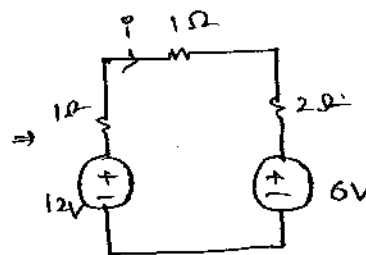
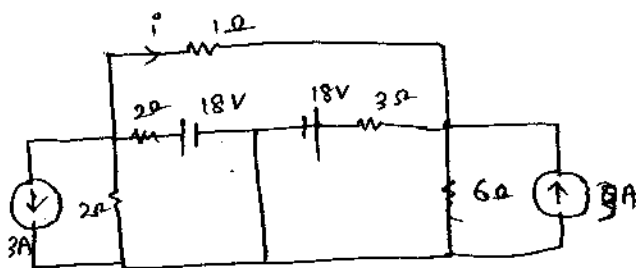
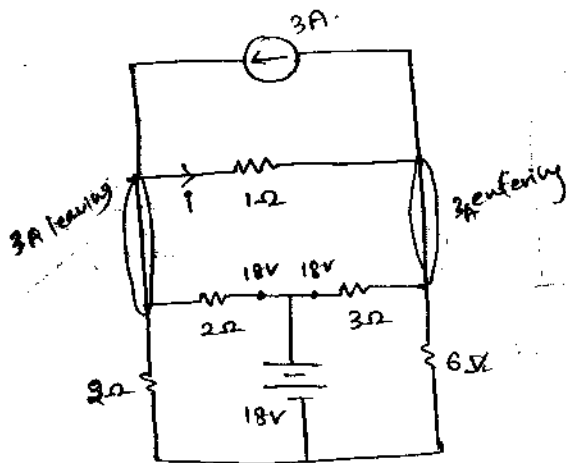
$$+\frac{16}{3} + \frac{2}{3}i + i - 8 + i = 0$$

$$i\left[\frac{2}{3} + 2\right] = 8 - \frac{16}{3}$$

$$i \frac{8}{3} = \frac{8}{3}$$

$$i = 1A$$

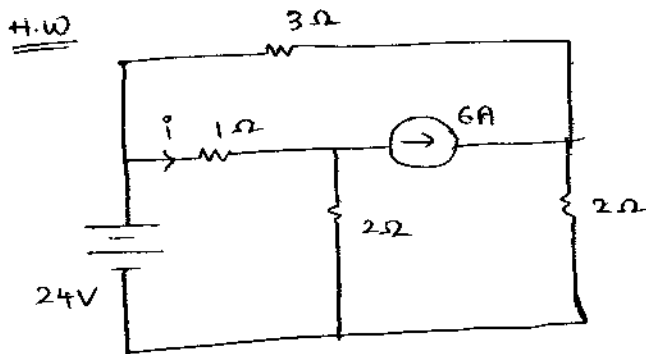
Determine Current 'i' by using S.T.T



$$-12 + 4i + 6 = 0$$

$$4i = 6$$

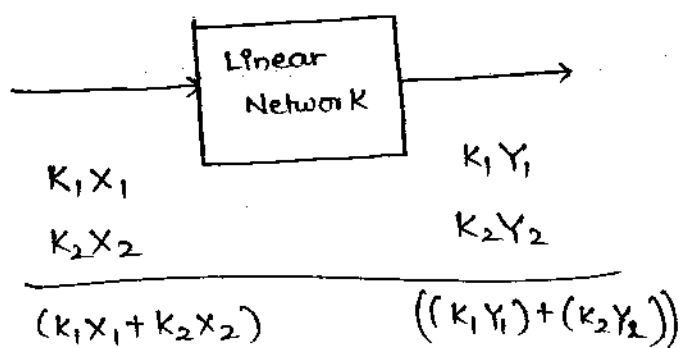
$$i = 1.5A$$



50

Theorem 2: Superposition theorem (S.P.T)

In any Linear active Bilateral N/w Consisting of n no. of energy sources, Resistances etc. the effect produced in any element when all ~~element~~ sources act at a time is Equal to sum of effects produced in the same elements when each source is considered individually.



SPT: Combination of

- (1) Linearity (ohms law) $X_1 \rightarrow Y_1$
- (2) Homogeneity $X_1 \rightarrow Y_1$ (proportionality)
 $KX_1 \rightarrow KY_1$
- (3) Additive.

Q The no. of sub cks to be solved while applying SPT is No. of Independent Sources only.
(dependent sources can't be considered)

Q Which of the following elec. parameter can't be directly evaluated by using SPT ?

(a) V

(b) I

(c) P $\rightarrow \because$ power is non linear (additive property not satisfies)

(d) Q

electrical parameter

additive property is applicable for linear & Homogeneity

Note: While applying SPT each subcircuit has only one Independent Source.

The other Independent Sources are deactivated

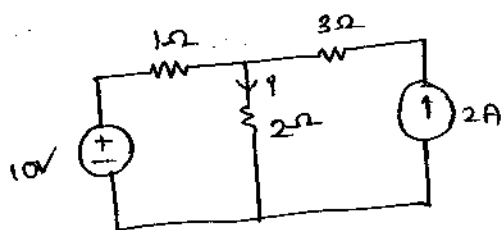
V \rightarrow short circuitd

I \rightarrow open circuitd

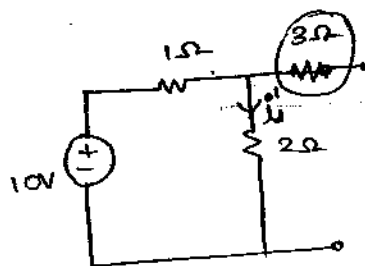
However dependent sources can't be suppressed

* Voltages have specific polarities & Currents have unique directions. they must be Respected while applying this theorem

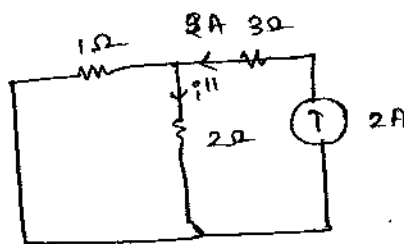
Q determine Current 'i' in the ckt shown below by using SPT.



S-I



$$i^1 = \frac{10}{3} \text{ A}$$

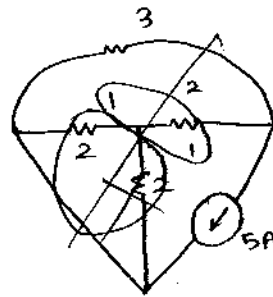
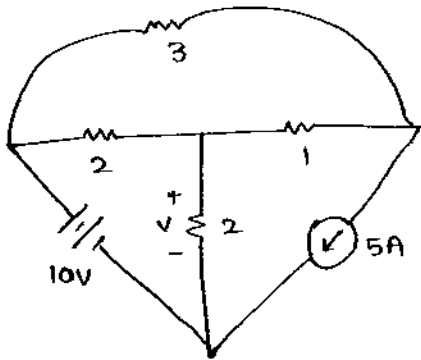


$$i^{11} = 2 \left[\frac{1}{3} \right] = \frac{2}{3} \text{ A}$$

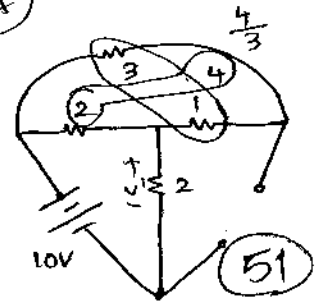
$$i = i^1 + i^{11}$$

$$i = \frac{10}{3} + \frac{2}{3} = 4 \text{ A}$$

determine voltage 'V' by using SPT



(S-I)

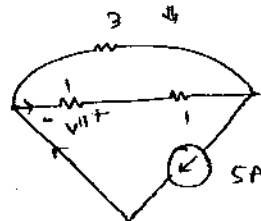
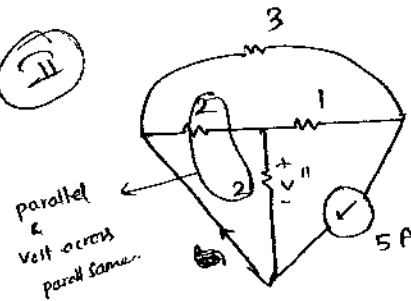


$$V' = 10 \left[\frac{2}{2 + \frac{4}{3}} \right]$$

$$= 6V$$

no. of subckts = 2 sources = 2

(S-II)



$$i'' = 5 \cdot \left(\frac{3}{3+2} \right)$$

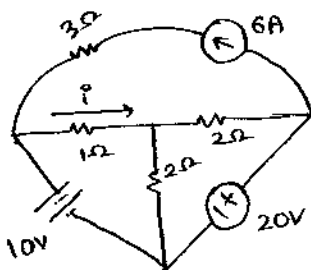
$$V'' = -1 \left[5 \left(\frac{3}{3+2} \right) \right]$$

$$V'' = -3$$

$$V = V' + V'' = (6 - 3) = 3V$$

Determine Current through 1Ω Resistance using SPT

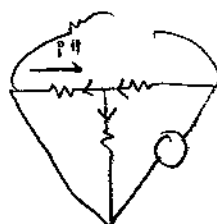
let i be the current through 1Ω.



S-I 10V only

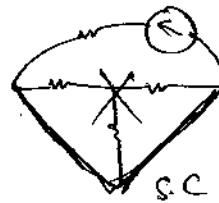
$$i' = \frac{10}{1 + \left[\frac{2 \parallel 2}{2} \right]} = \frac{10}{2} = 5A$$

S-II : 20V only

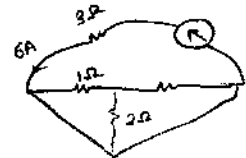


$$i'' = - \left[\frac{20}{2 + \frac{2}{3}} \right] \cdot \frac{2}{3} = - \frac{20}{4} = -5A$$

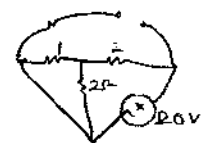
S-III :



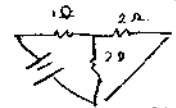
$$i''' = 0$$



$$2 \times \frac{2}{3} = 4A$$



$$20 \times \frac{2}{2 + \frac{2}{3}} = 10A$$

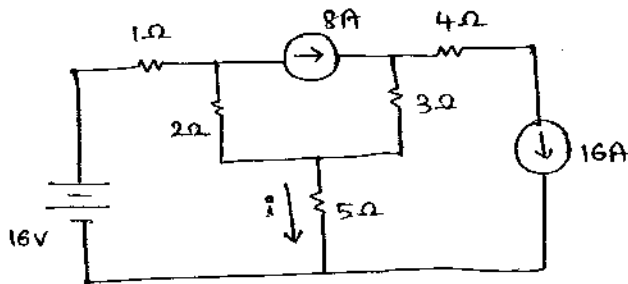


$$\frac{20}{3} + 10 + 4$$

$$= \frac{20 + 30 + 12}{3} = \frac{62}{3}$$

$$\begin{aligned}\text{By SPT } i &= i' + i'' + i''' \\ &= +5 - 5 + 0 \\ i &= 0 \text{ A}\end{aligned}$$

Determine power abs. by 5Ω using SPT.



16V only

$$\begin{aligned}i' &= \frac{16}{1+2+5} \\ &= \underline{\underline{2 \text{ A}}}\end{aligned}$$

8A only

$$\begin{aligned}i'' &= 8 \left[\frac{2}{2+6} \right] \\ &= 2 \text{ A}\end{aligned}$$

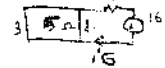
let i be current (assume)

16A only

$$\begin{aligned}i''' &= -16 \left[\frac{3}{3+5} \right] \\ &= -6 \text{ A}\end{aligned}$$

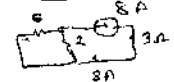
$$V = \frac{16 \times 1}{8} =$$

$$i = 2 \text{ A}$$



$$-\left[\frac{16 \times 3}{8} \right]$$

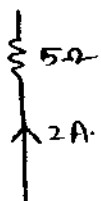
$$= -6 \text{ A}$$



$$8 \times \frac{2}{8} = 2$$

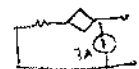
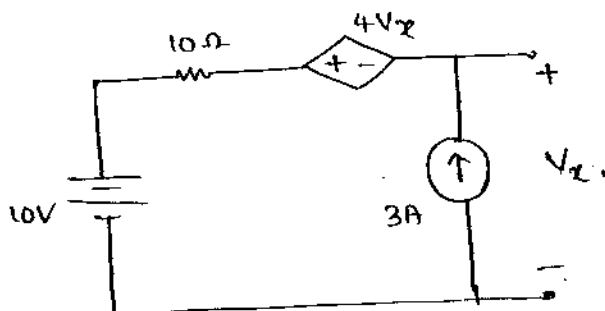
$$\underline{\underline{-2 \text{ A}}}$$

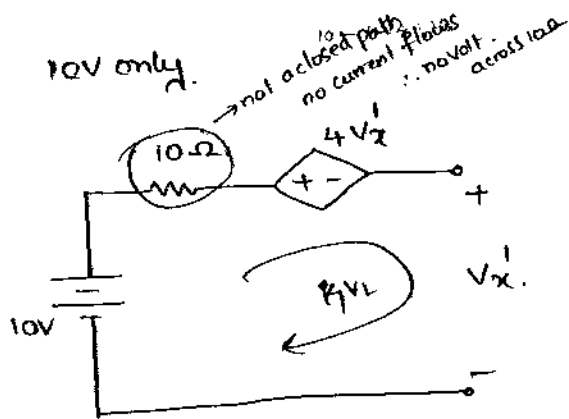
$$\begin{aligned}\text{By SPT } i &= i' + i'' + i''' \\ &= 2 + 2 - 6 \\ i &= \underline{\underline{-2 \text{ A}}}\end{aligned}$$



$$\begin{aligned}P_{\text{abs}} &= [I_{\text{net}}]^2 R \\ &= (2)^2 \cdot (5) \\ &= \underline{\underline{20 \text{ W}}}\end{aligned}$$

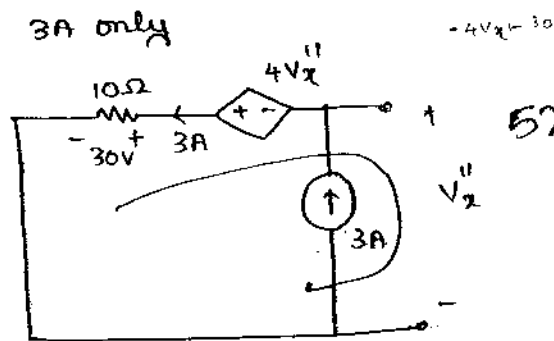
Determine Voltage V_x by using S.P.T.





$$\text{KVL: } -10 + 4V_x' + V_x' = 0$$

$$V_x' = 2V$$



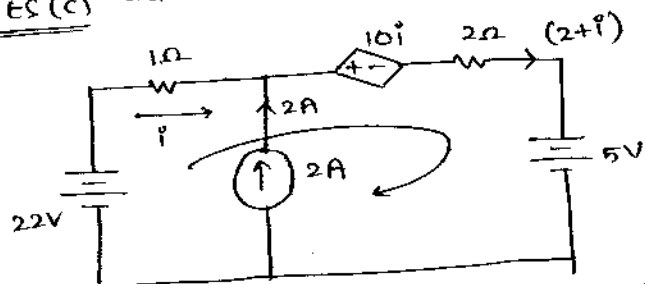
$$\text{KVL: } -30 + 4V_x'' + V_x'' = 0$$

$$V_x'' = 6V$$

By SPT = $V_x = V_x' + V_x''$

$$= 2V + 6V = \underline{8V}$$

IES (c) determine 'i' by SPT

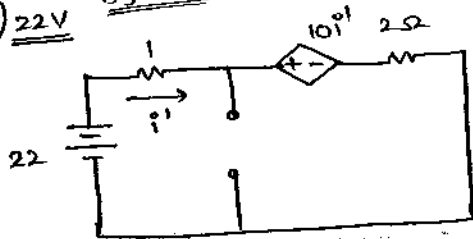


Direct method.

$$-22 + i + 10i + 2(2+i) + 5 = 0$$

$$13i = 13 \rightarrow i = 1A$$

I] 22V By SPT



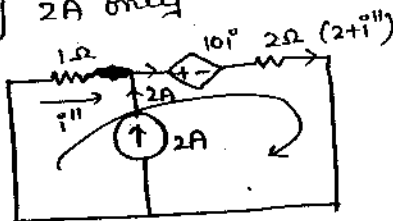
$$i' \neq \frac{22V}{1\Omega} \neq 22A$$

$$\text{KVL: } -22 + i' + 10i' + 2(2+i') = 0$$

$$13i' = 22$$

$$i' = \frac{22}{13}$$

II] 2A only



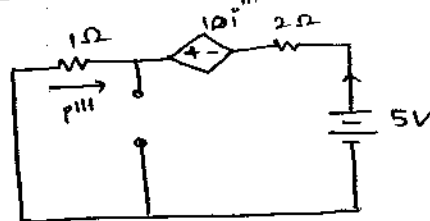
$$i'' = 2A$$

KCL + KVL

$$i'' + 10i'' + 2(2+i'') = 0$$

$$i'' = -\frac{4}{13} A$$

III] 5V only



$$i''' \neq$$

$$-5V + 2i''' + (10i''') - 2 = 0$$

$$i''' + 10i''' + 2i''' + 5 = 0$$

$$13i''' + 5 = 0$$

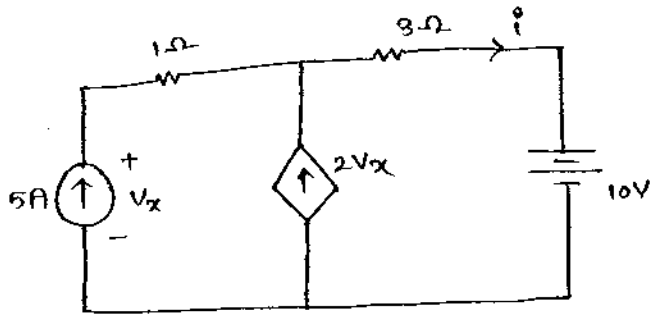
$$i''' = -\frac{5}{13}$$

By SPT

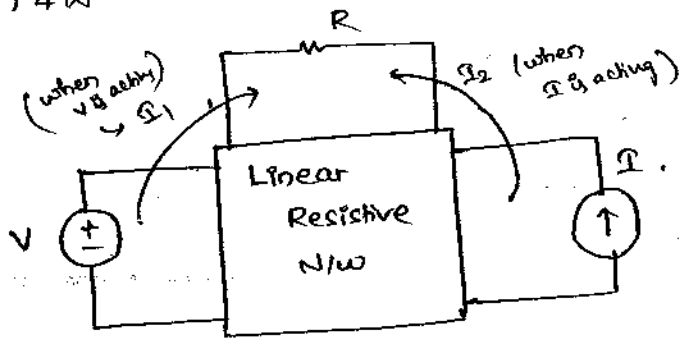
$$i = i' + i'' + i'''$$

$$= \frac{22}{13} - \frac{4}{13} - \frac{5}{13} = \frac{13}{13} = 1A$$

Ans
Q Determine current 'i' in ckt shown below by using SPT.



Q The power lost in Resistor 'R' when Voltage source alone acts is 25W. and Current source alone acts is 9W. determine the total power lost in Resistor 'R' when both sources act simultaneously
(a) 4W (b) 16W (c) 34W (d) 64W.



power is non linear
∴ powers can't be added
(25 ± 9) = 16 or 34

V alone

$$P_1 = 25W = I_1^2 R$$

$$|I_1| = \frac{5}{\sqrt{R}}$$

I alone

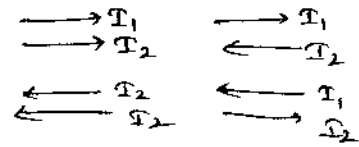
$$P_2 = 9W = I_2^2 R$$

$$|I_2| = \frac{3}{\sqrt{R}}$$

by SPT

$$I_T = [\pm I_1 \pm I_2]$$

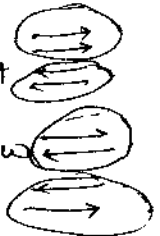
∴ we don't know the inside of N/w
∴ we don't know the direction of I_1 & I_2



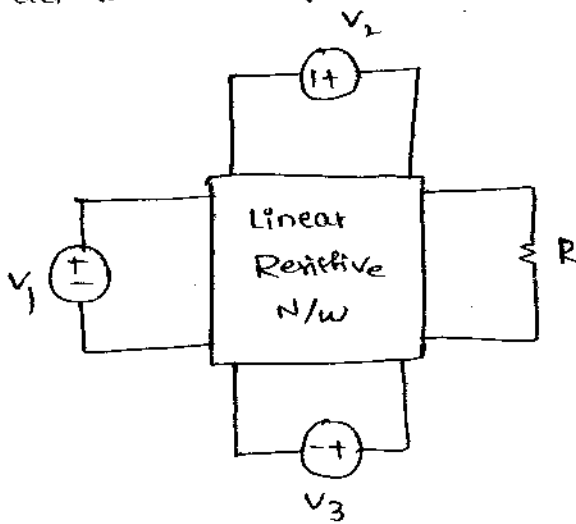
$$P_T = [I_T^2] R$$

$$= [\pm I_1 \pm I_2]^2 R = \left[\pm \frac{5}{\sqrt{R}} \pm \frac{3}{\sqrt{R}} \right]^2 R$$

$$P_T = [\pm 5 \pm 3]^2 W$$



Determine the values of max & min power lost in Resistance 'R' when all sources act simultaneously



If,

\$V_1\$ alone \$\rightarrow 18W\$

\$V_2\$ alone \$\rightarrow 50W\$

\$V_3\$ alone \$\rightarrow 98W\$

$$P_i = I_i^2 R$$

$$18 = I_1^2 R$$

$$I_1 = \frac{18}{R}$$

$$I_2 = \frac{50}{R}$$

$$I_3 = \frac{98}{R}$$

$$I_T = \left(\frac{18}{R} + \frac{50}{R} + \frac{98}{R} \right)$$

$$18 + 50 + 98$$

$$\frac{166}{R}$$

$$\frac{98}{R}$$

$$90W$$

(53)

$$P_T = \left[\pm \sqrt{P_1} \pm \sqrt{P_2} \pm \sqrt{P_3} \right]^2$$

$$P_T = \left[\pm \sqrt{18} \pm \sqrt{50} \pm \sqrt{98} \right]^2$$

$$P_T = 2 \left[\pm 3 \pm 5 \pm 7 \right]^2$$

$$\rightarrow \text{for max } P_T = 2 \left[3 + 5 + 7 \right]^2 = 450W \quad \text{max}$$

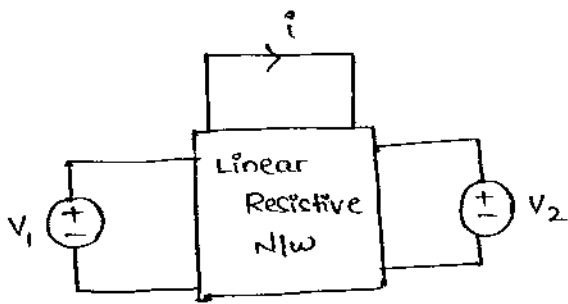
$$\rightarrow \text{for min } P_T = 2 \left[3 + 5 - 7 \right]^2 = 2W \quad \text{min}$$

if two sources acting simultaneously

then $P_T = \left[\pm \sqrt{P_1} \pm \sqrt{P_2} \right]^2$

if three sources acting simultaneously on 'R'

$$P_T = \left[\pm \sqrt{P_1} \pm \sqrt{P_2} \pm \sqrt{P_3} \right]^2$$



if $V_1 = V_2 = 15V$ then $i =$ _____

If

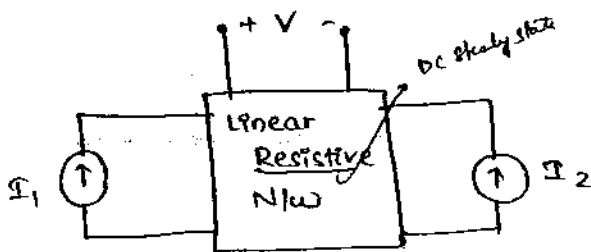
V_1	V_2	i
10V	0V	5A
0V	-5V	1A

$$\begin{aligned} i &= \frac{10}{2} + \frac{-5}{10} \\ &= 5 - \frac{1}{2} \\ &= 4.5 \end{aligned}$$

V_1 alone } $10V \longrightarrow 5A$
 $V_2 = 0$ } $15V \longrightarrow 7.5A$

V_2 alone } $-5V \longrightarrow 1A$
 $V_1 = 0$ } $+5V \longrightarrow -1A$
 $15V \longrightarrow -3A$

When both $V_1 = V_2 = 15V$ acting $\rightarrow i = (7.5 - 3)A = \underline{\underline{4.5A}}$



If $I_1 = I_2 = 15A$ then $V =$ _____

I_1	I_2	V
10A	5A	20V
5A	-10A	5V

15

$$V = K_1 I_1 + K_2 I_2$$

$$20 = K_1 10 + K_2 5 \longrightarrow (1)$$

$$5 = K_1 5 + K_2 (-10) \longrightarrow (2)$$

$$K_1 = 9/5$$

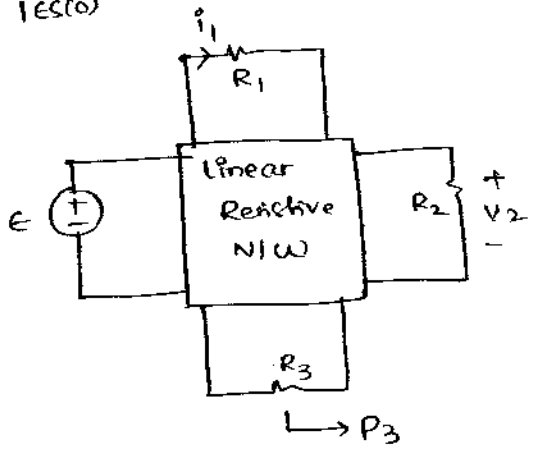
$$K_2 = 2/5$$

$$V = \frac{9}{5} I_1 + \frac{2}{5} I_2$$

$$V = \frac{9}{5} (15) + \frac{2}{5} (15)$$

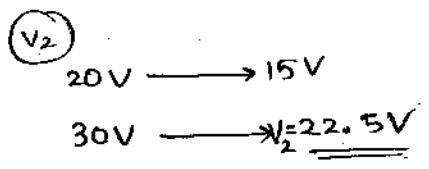
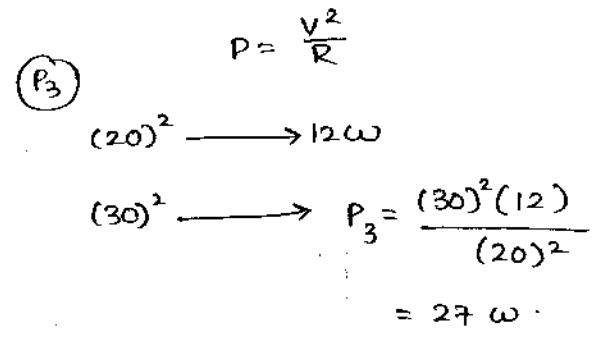
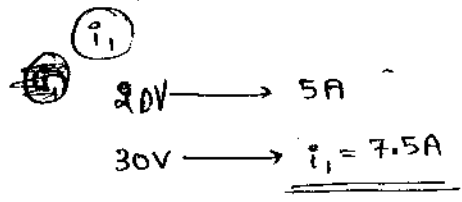
$$V = 33 \text{ Volts}$$

IES (0)



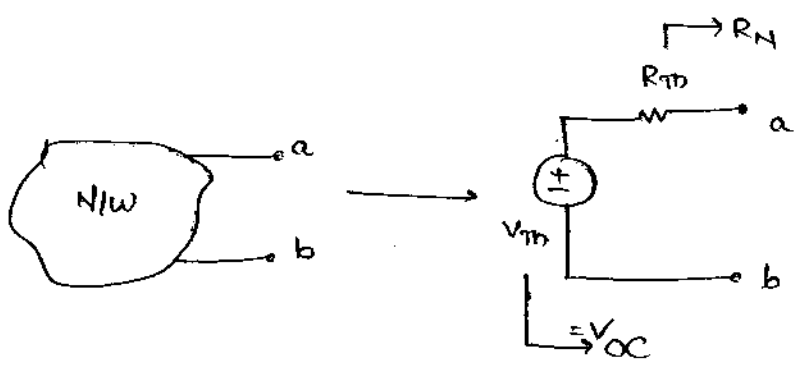
if $E = 20V$ then $i_1 = 5A$, $V_2 = 15V$, $P_3 = 12W$

if $E = 30V$ then $i_1 = \underline{\hspace{1cm}}$, $V_2 = \underline{\hspace{1cm}}$, $P_3 = \underline{\hspace{1cm}}$



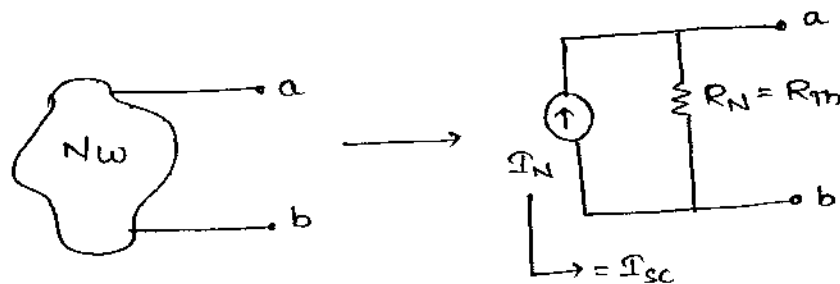
Theorem-3 Thévenin's Theorem: \rightarrow (Active Reduction) ^{Reduction including active elements.}

In any Linear active Bilateral N/w Consisting of no. of Energy Sources, Resistances, etc., with open output target terminals defined can be Converted into Simple network Consisting of Voltage source in Series with Resistance



Theorem 4: Norton's Theorem:

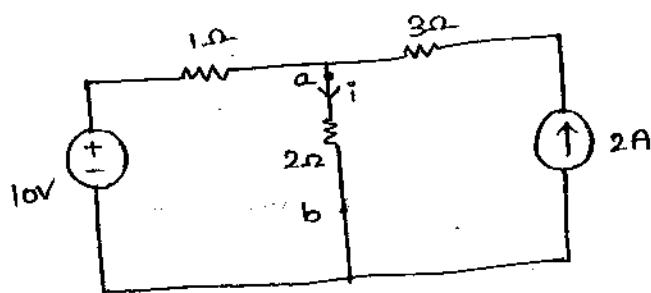
in any linear active Bilateral N/w consisting of Energy Sources, Resistances etc. with open o/p target terminals defined can be converted into a simple N/w consisting of Current Source in parallel with Resistance.



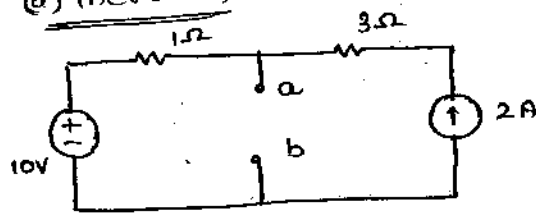
Note: Thevenin's & Norton's Equivalents are dual of Each other i.e., they are Source Transformable.

Category-I problems : problems with only Independent Sources.

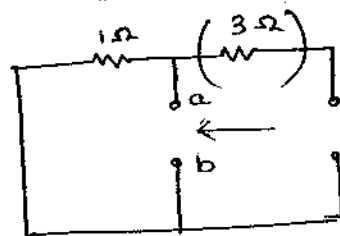
a determine Current i by using (a) Thevenin (b) Norton's theorem.



(a) Thevenin

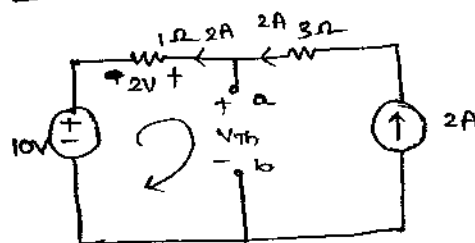


Step-I: R_{Th}



$$R_{Th} = 1\Omega$$

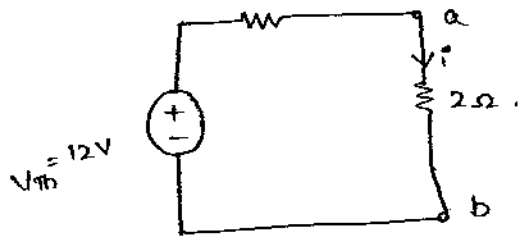
Step-II: V_{Th}



$$-10 - 2 + V_{Th} = 0$$

$$V_{Th} = 12V$$

Thevenin's Eq: $R_{Th} = 1\Omega$



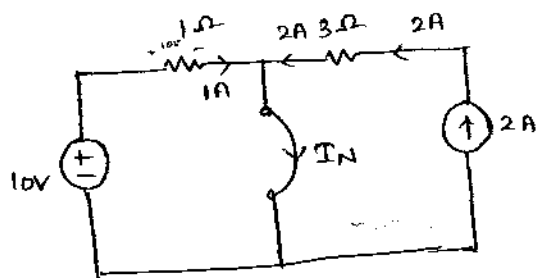
$$i = \frac{12}{1+2} = 4A$$

55

(b) Norton's:

Step-I: $R_N = R_{Th} = 1\Omega$

Step-II: I_N

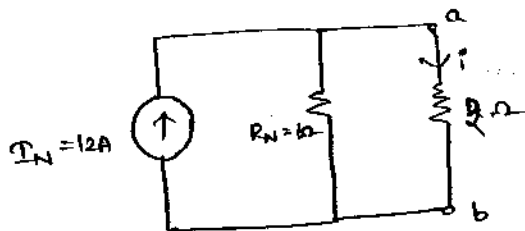


KCL

$$10 + 2 = I_N$$

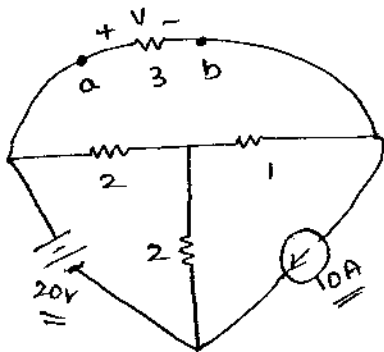
$$I_N = 12A$$

Norton's Eq:



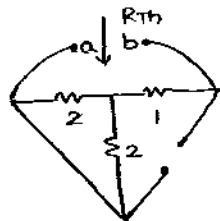
$$i = 12 \left[\frac{1}{3} \right] = 4A$$

Q determine Voltage 'V' by (i) Thevenin (ii) Norton.



(a) Thevenin.

[I] R_{Th}

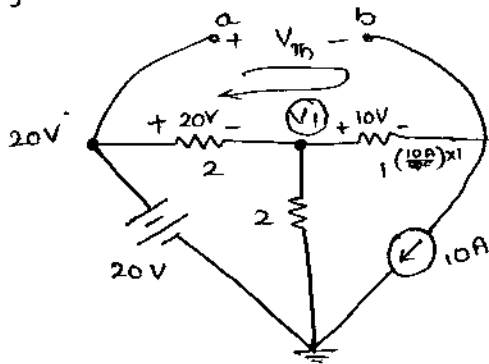


[II] V_{Th}

$$R_{Th} = (2 \parallel 2) + 1$$

$$= 2\Omega$$

[I] V_{Th} .



Nodal:

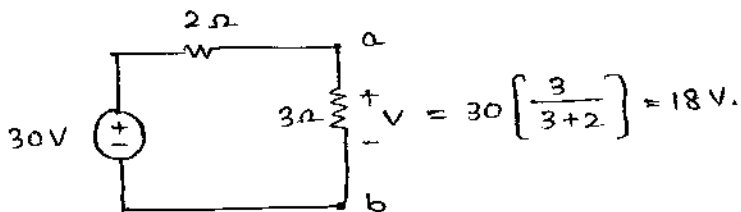
$$\frac{V-20}{2} + \frac{V}{2} + 10 = 0$$

$$\underline{V = 0}$$

KCL

$$+V_{Th} - 10 - 20 = 0 \Rightarrow V_{Th} = 30V$$

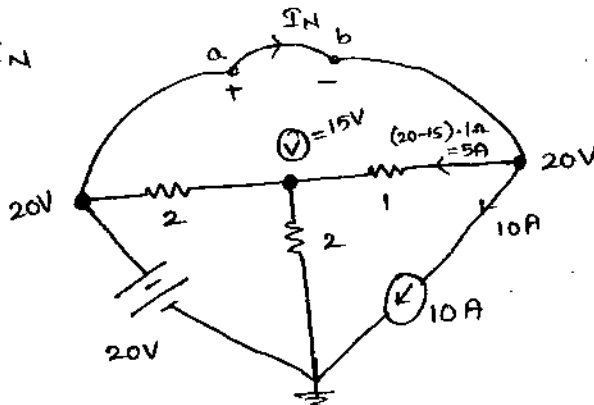
Thevenins N/w:



(b). Nortons:

[S-I]: $R_N = R_{Th} = 2\Omega$

[S-II]: I_N



Nodal:

$$\frac{V-20}{2} + \frac{V-20}{1} + \frac{V-0}{2} = 0$$

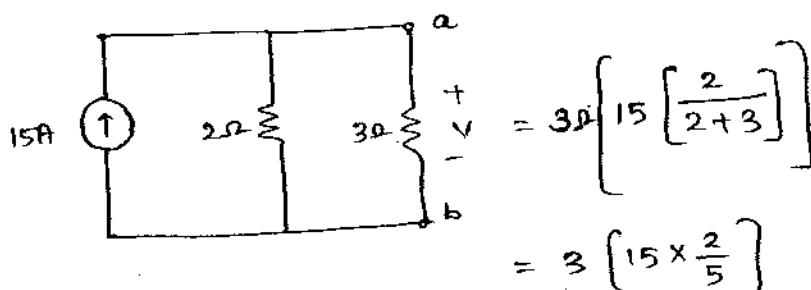
$$4V = 60$$

$$V = 15V$$

KCL.

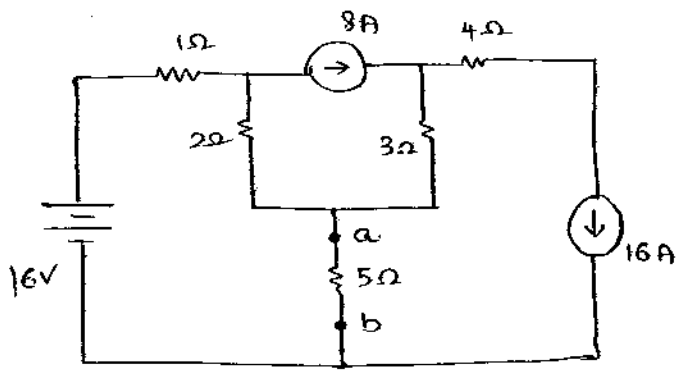
$$I_N = 10 + 5 = 15A$$

Nortons. N/w:



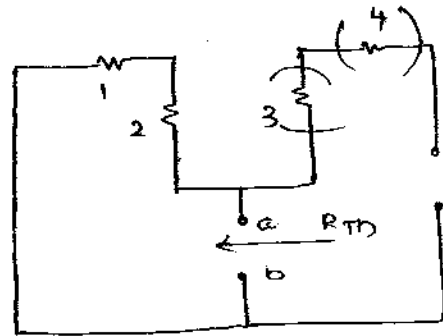
$$\underline{= 18V}$$

Q Determine the power absorbed by 5Ω Resistance using Thevening & Nortons theorem.



Thevenins:

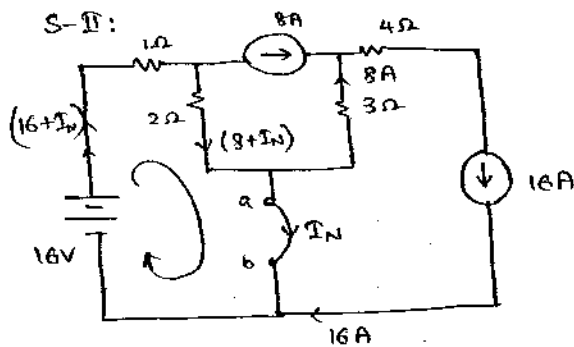
Step-I: R_{Th}



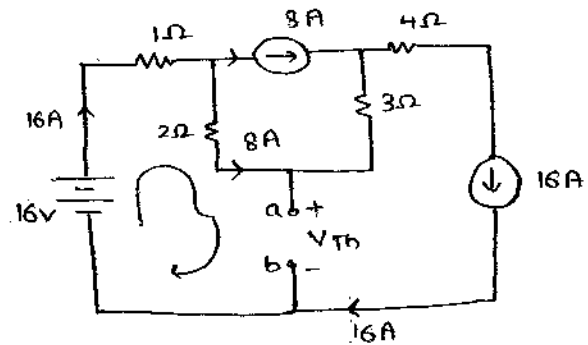
$$R_{Th} = 3\Omega$$

Nortons:

S-I: $R_N = R_{Th} = 3\Omega$



Step-II: V_{Th}

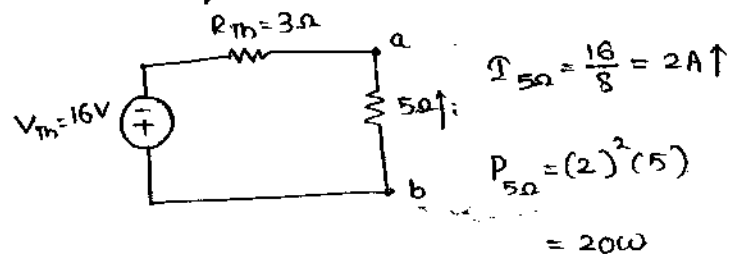


KVL:

$$-16 + 16(1) + 2(8) + V_{Th} = 0$$

$$V_{Th} = -16V$$

Thevening Eq.

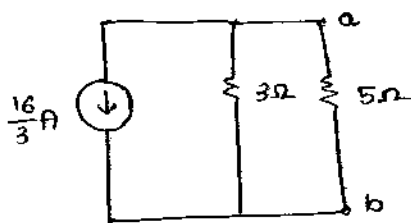


KCL+KVL:

$$-16 + 1[16 + I_N] + 2[8 + I_N] = 0$$

$$I_N = -\frac{16}{3}A$$

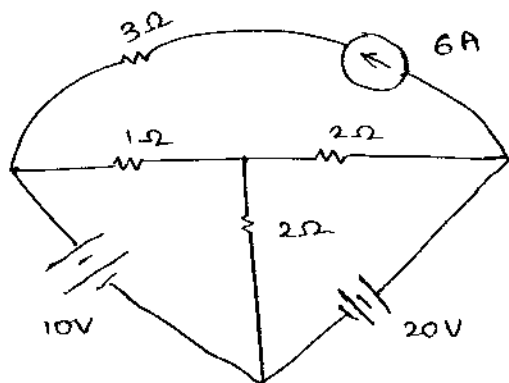
N.E:



$$I_{5\Omega} = \frac{16}{3} \left[\frac{3}{8} \right] = 2A \uparrow$$

$$P_{5\Omega} = (2)^2(5) = 20W$$

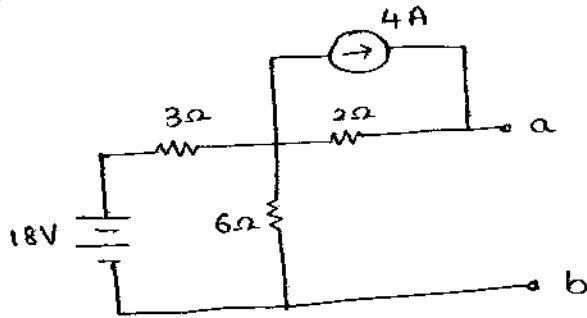
H.W



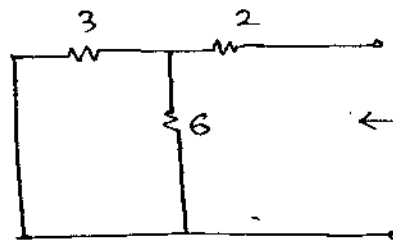
Determine Current through 1Ω Resistance
using Thevenin's & Norton's theorem

941

Q Determine the Thevening & Norton Equivalent across terminals a, b

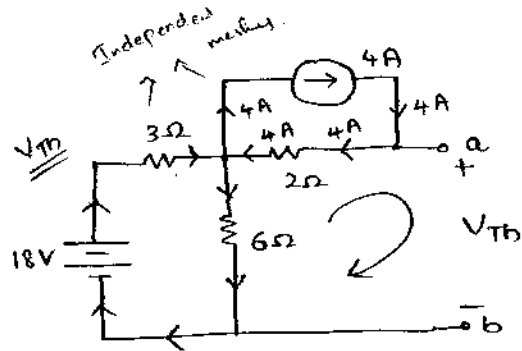


Thevening
 R_{Th}



57

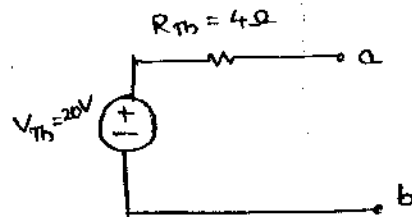
$$R_{Th} = (3 \parallel 6) + 2 = 4\Omega$$



KVL

$$- [2(6)] - [4(2)] + V_{Th} = 0$$

$$V_{Th} = 12 + 8 = 20V$$

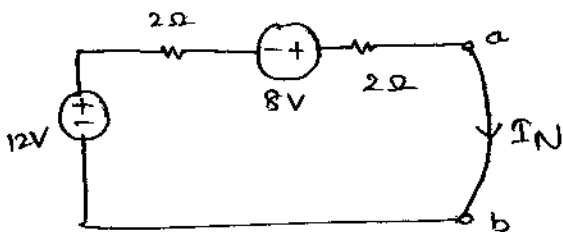
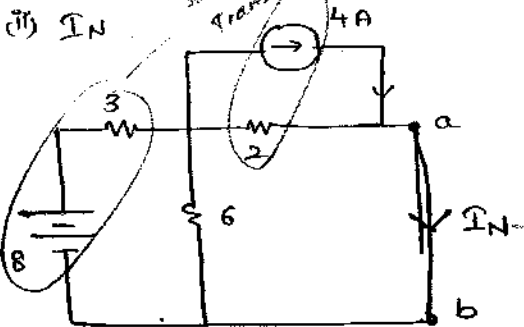


Norton:

(i) $R_{Th} = R_N = 4\Omega$

(ii) I_N

Source Transformation

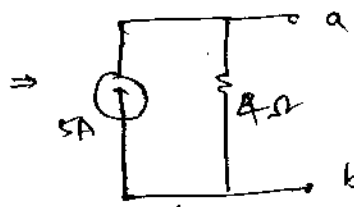


$I_N \Rightarrow$ STT + KCL

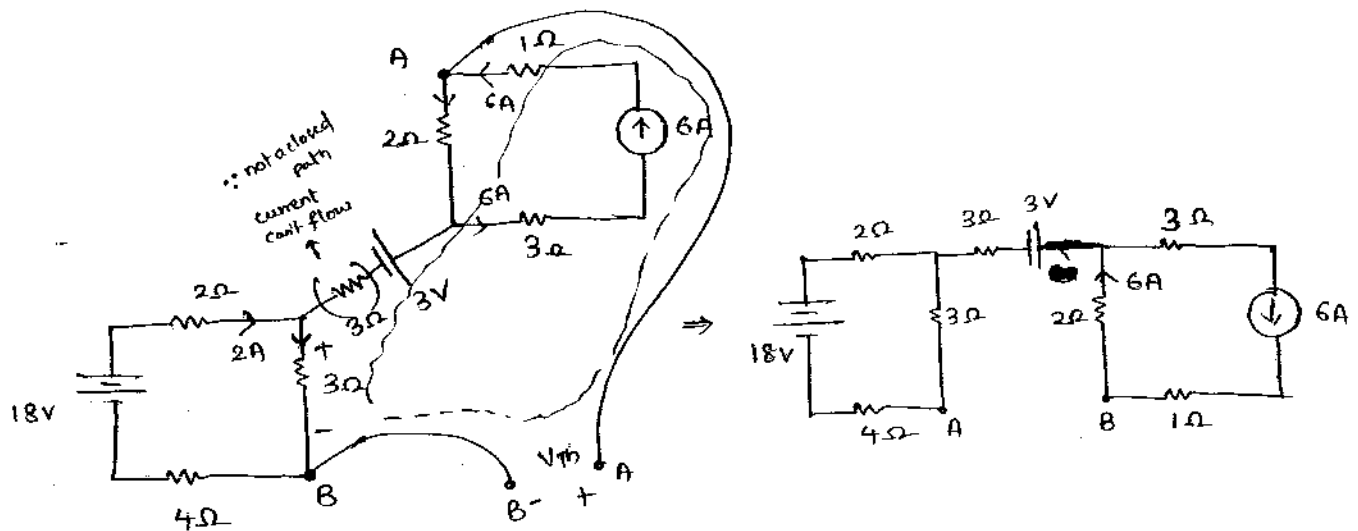
$$-12 + 2I_N - 8 + 2I_N = 0$$

$$4I_N = 20$$

$$I_N = 5A$$



Determine the Thevenin's Equivalent across terminals AB.



$$R_{Th} = 7\Omega$$

$$R_{Th} = 2 + 3 + [3 \parallel 6]$$

$$= 2 + 3 + 2$$

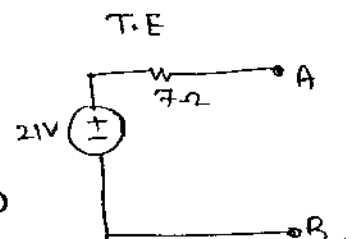
$$= 7\Omega$$

$$V_{Th}$$

L → KVL

$$- [2(3)] + 0 - 3 - [6(2)] + V_{Th} = 0$$

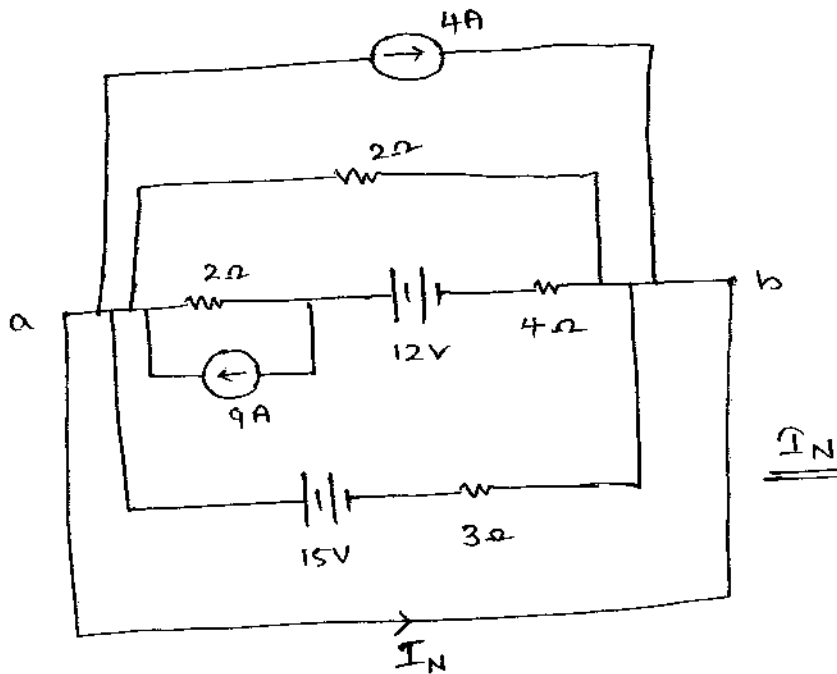
$$V_{Th} = 21V$$



for the above problem, find the Norton's Eq. across AB.

Q determine the Norton's Equivalent across terminals AB.

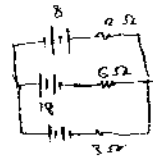
58



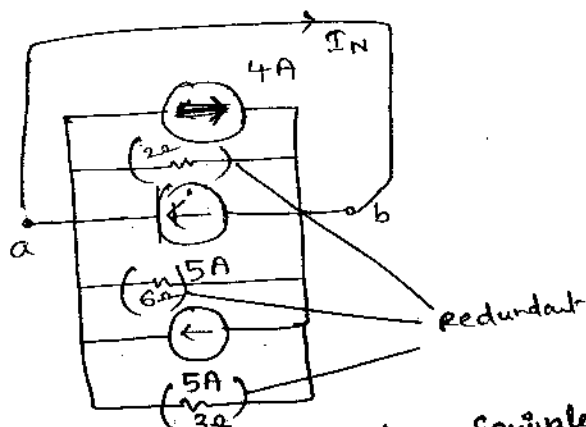
Norton's

$$R_N = 2 \parallel 6 \parallel 3$$

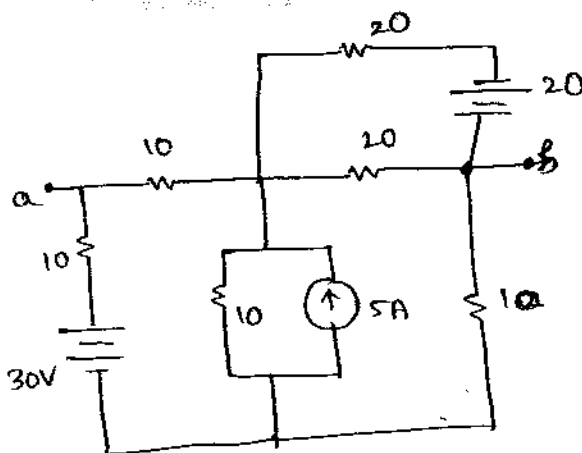
$$= 1$$



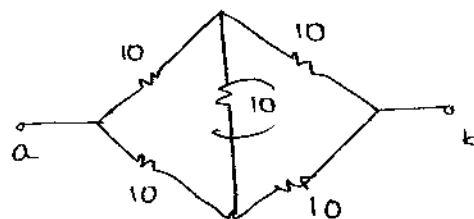
S.c to passive elements
 is Redundant \Rightarrow
 no p.d \Rightarrow no current flows



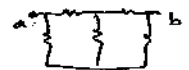
Q determine Thevenin's & Norton's Equivalent across AB.



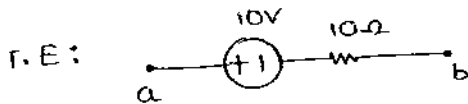
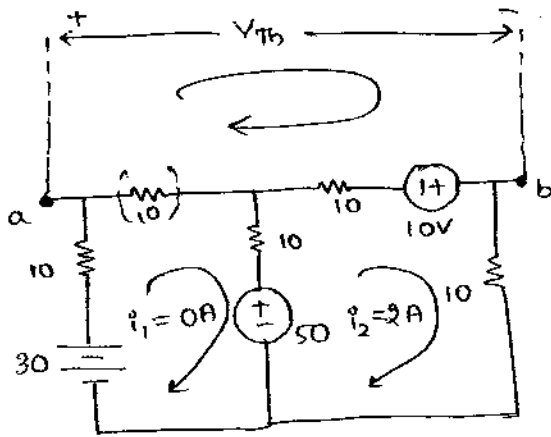
$[I]$ $R_{th} = R_N$



$$= 20 \parallel 20 = 10 \Omega$$



S-II] V_{Th} .



Mesh

$$-30 + 20i_1 + 10[i_1 - i_2] + 50 = 0$$

$$3i_1 - i_2 = -2 \rightarrow (1)$$

$$-50 + 10[i_2 - i_1] + 20i_2 - 10 = 0$$

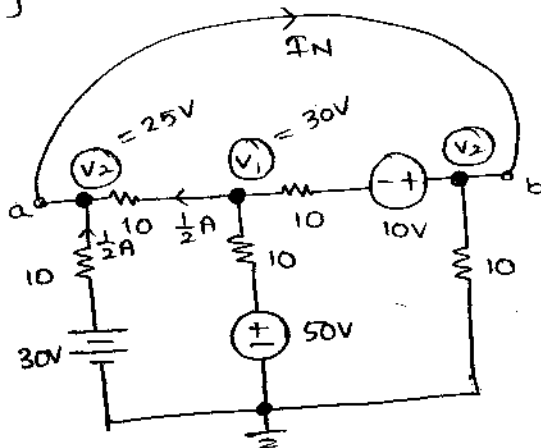
$$-i_1 + 3i_2 = 6 \rightarrow (2)$$

solving
① & ②

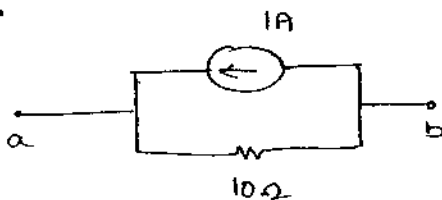
$$i_2 = 2A$$

$$i_1 = 0A$$

S-III] I_N



N.E:



Nodal -1

$$\frac{(V_1 - V_2)}{10} + \frac{(V_1 - 50)}{10} + \frac{(V_1 - V_2 + 10)}{10} = 0$$

$$3V_1 - 2V_2 = 40 \rightarrow (1)$$

$$\frac{(V_2 - V_1)}{10} + \frac{(V_2 - 30)}{10} + \frac{V_2}{10} + \frac{(V_2 - V_1 - 10)}{10} = 0$$

$$-2V_1 + 4V_2 = 40$$

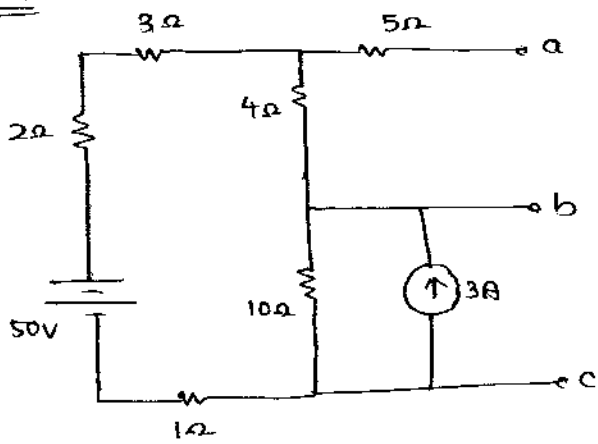
$$-V_1 + 2V_2 = 20 \rightarrow (2)$$

KCL

$$\frac{1}{2} + \frac{1}{2} = I_N$$

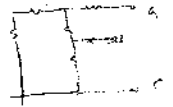
$$I_N = 1A$$

H.W



Determine

- (1) Th & N. across terminals ac
- (2) Th & N Eq. across terminals bc

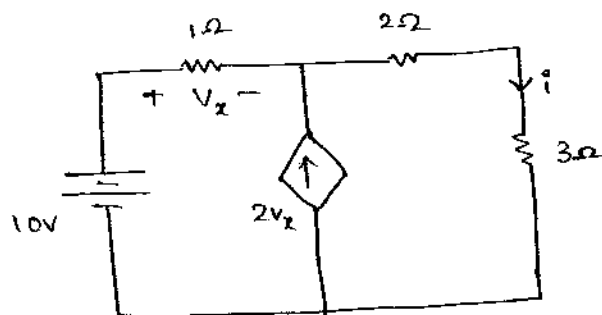


59

Category-II problems with Both Independent & dependent Sources.

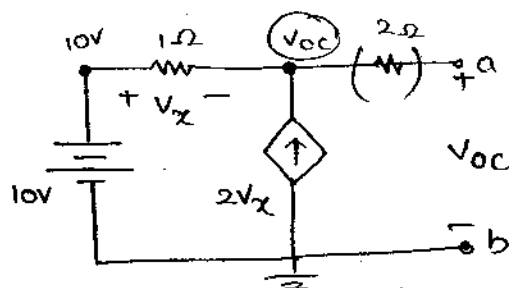
In Such Networks determining R_{Th} or R_N is not possible directly due to the presence of dependent Sources. Moreover the N/w is already active & working due to the presence of Independent Sources in it. In such N/w we use OHM's law to indirectly find Resistance where $R_{Th} = R_N = \frac{V_{oc}}{I_{sc}}$ at target terminals.

Q determine Current 'i' in the ckt shown by using Thevenins & Nortons Theorem.



S-I $V_{oc} = V_{Th}$

Nodal:



$$\frac{V_{oc} - 10}{1} - 2V_x = 0 \rightarrow (1)$$

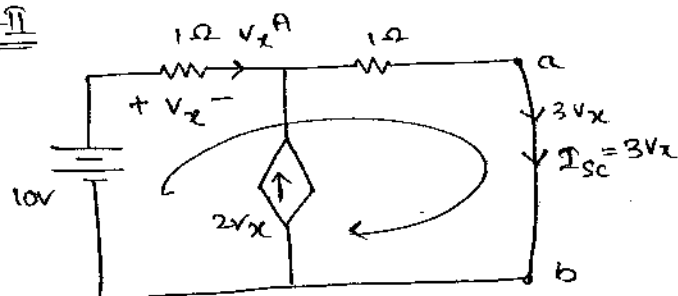
$$V_x = 10 - V_{oc} \rightarrow (2)$$

$$V_{oc} - 10 - 20 + 2V_{oc} = 0$$

$$3V_{oc} = 30$$

$$V_{oc} = 10V = V_{Th}$$

S-II



$$-10 + V_x + 2(3V_x) = 0$$

$$V_x = \frac{10}{7}$$

$$\text{But } I_{sc} = 3V_x$$

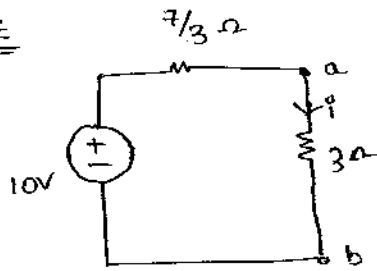
$$= 3\left(\frac{10}{7}\right)$$

$$I_{sc} = \frac{30}{7} = I_N$$

$$R_{Th} = R_N = \frac{V_{oc}}{I_{sc}} = \frac{10}{30/7} = \frac{7}{3} \Omega$$

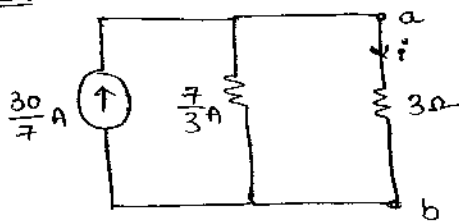
60

T.E



$$i = \frac{10}{3 + \frac{7}{3}} = \frac{30}{18} = \frac{15}{8} \text{ A}$$

N.E



$$i = \frac{30}{7} \left[\frac{\frac{7}{3}}{3 + \frac{7}{3}} \right] = \frac{15}{8} \text{ A}$$

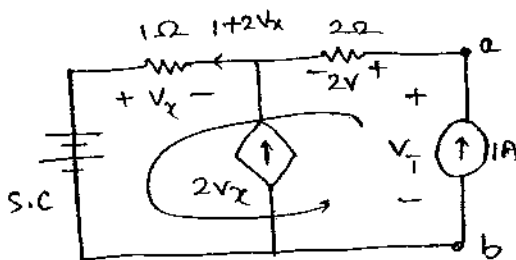
Note: In this Category Sometimes if it is Required only to find Resistance but not Equivalent then deactivate N/w & apply ohm's law directly.

Ex: In determining i/p Impedence, o/p impedance of electronic devices & ckt,

In Maximum Power Transfer Theorem, Time Constant problems.

In determining two port N/w parameters etc

Q find only R_{Th} (or) R_N :



KVL

$$-V_T + 2 + 1[1 + 2V_x] = 0$$

$$V_T = 3 + 2V_x \rightarrow (1)$$

$$V_x = -1[1 + 2V_x]$$

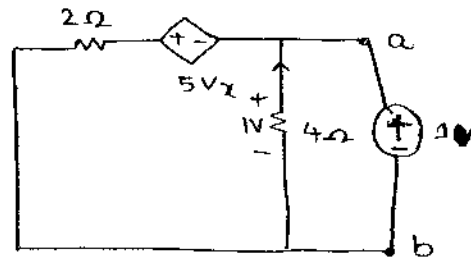
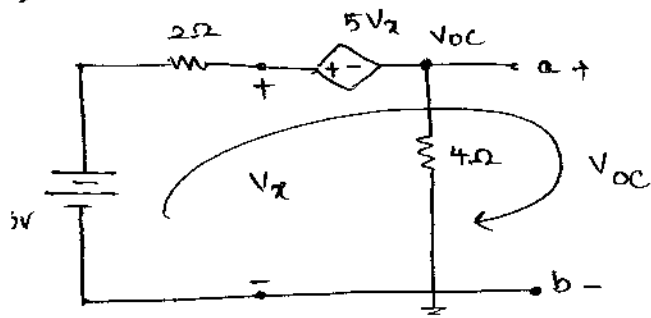
$$3V_x = -1 \rightarrow (2)$$

$$V_T = 3 + 2\left[\frac{-1}{3}\right] = \frac{7}{3} \text{ V}$$

$$R_{Th} = R_N = \frac{V_T}{1 \text{ A}} = \frac{7}{3} \Omega$$

fat

Determine the Norton Eq. across terminals ab



V_OC

Nodal:

$$\frac{(V_{OC} - 6 + 5V_x)}{2} + \frac{V_{OC}}{4} = 0$$

$$3V_{OC} + 10V_x = 12 \rightarrow (1)$$

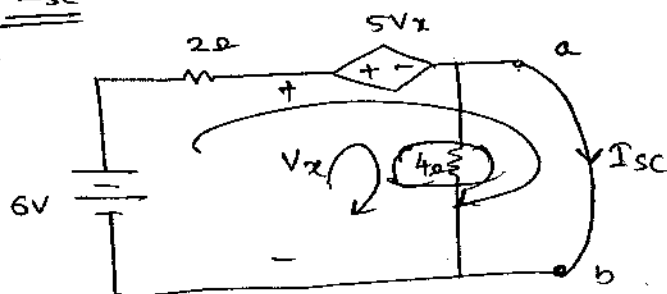
KCL

$$-V_x + 5V_x + V_{OC} = 0 \rightarrow V_{OC} = -4V_x \rightarrow (2)$$

$$3V_{OC} + 10\left[\frac{-V_{OC}}{4}\right] = 12$$

$$V_{OC} = 24V$$

I_SC



Mesh

$$-6 + 2I_{SC} + 5V_x = 0 \rightarrow (1)$$

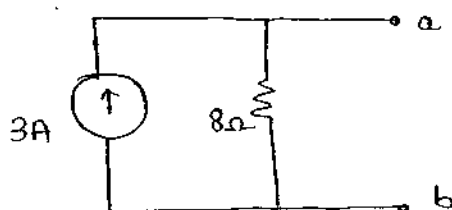
$$-V_x + 5V_x = 0$$

$$V_x = 0 \rightarrow (2)$$

$$2I_{SC} = 6 \rightarrow I_{SC} = 3A$$

$$R_N = \frac{V_{OC}}{I_{SC}} = \frac{24}{3} = 8\Omega$$

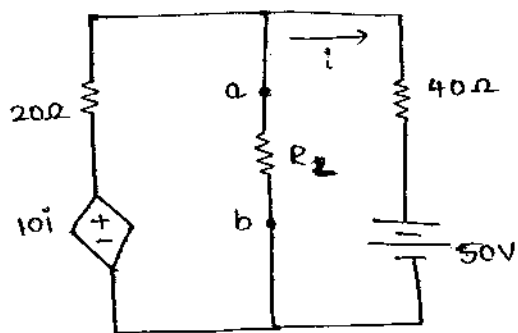
N.E



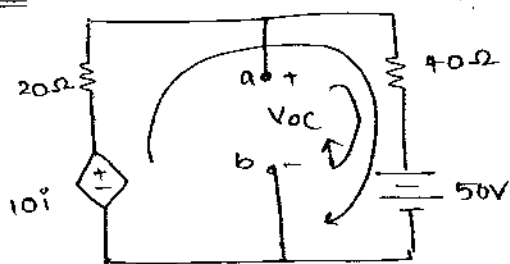
Gate

Determine the Thevenius Eq. across the load.

61



Voc



Mesh

$$-10i + 20i + 40i + 50 = 0$$

$$i = -1 \rightarrow (1)$$

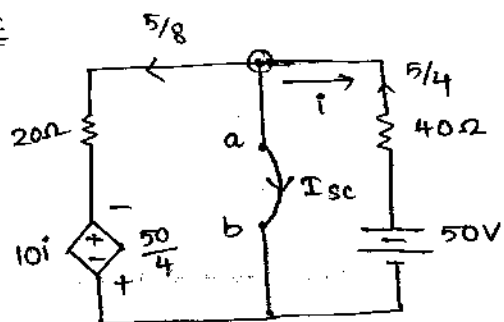
KVL

$$-V_{oc} + 40i + 50 = 0$$

$$V_{oc} = 40i + 50 \rightarrow (2)$$

$$V_{oc} = 40(-1) + 50 \\ = 10V$$

Isc



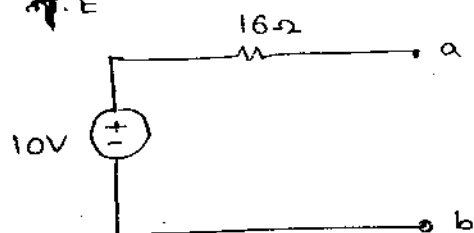
KCL

$$\frac{5}{4} = I_{sc} + \frac{5}{8}$$

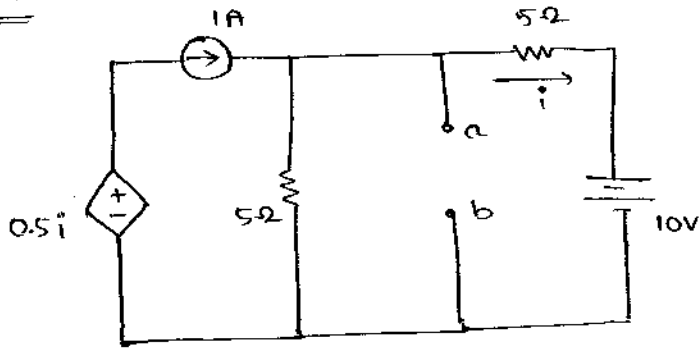
$$I_{sc} = \frac{5}{8} A$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{10}{5/8} = 16\Omega$$

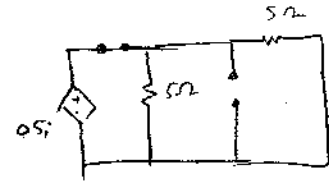
T.E



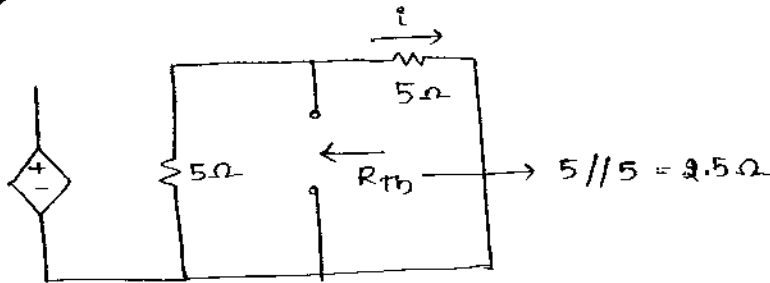
gate:



The Thevenin Resistance across terminals a, b .

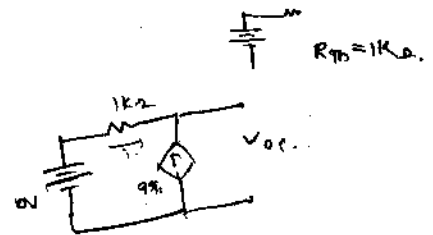
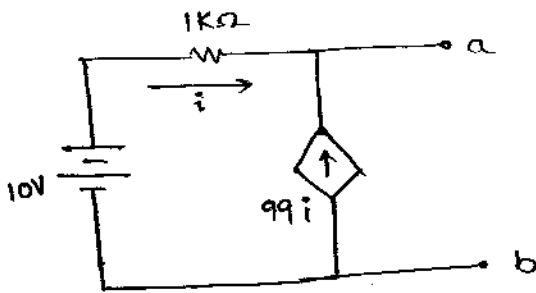


- (a) 0Ω (b) 2.5Ω (c) 5Ω (d) 7.5Ω



Q T.E b/w a, b

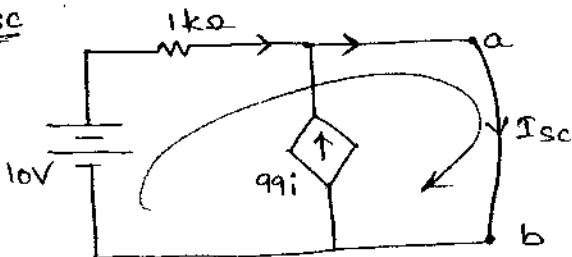
- (a) $1V, 1\Omega$ (b) $10V, 1\Omega$
(c) $1V, 10\Omega$ (d) $10V, 10\Omega$



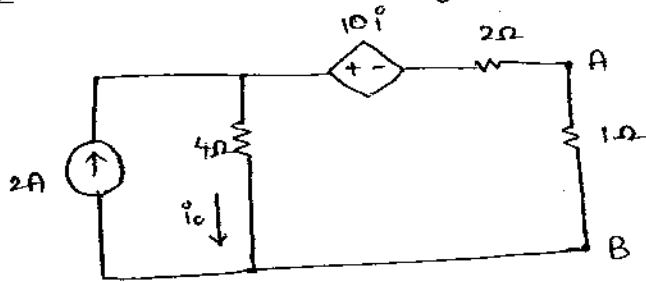
$-10V + KI$

$99i = -250$

I_{sc}

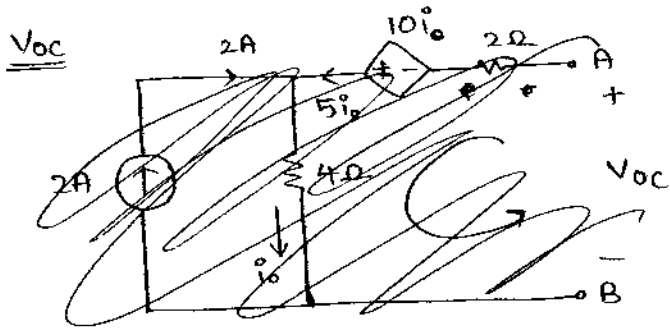


Q determine Current through 1Ω Resistance Using Norton's theorem.

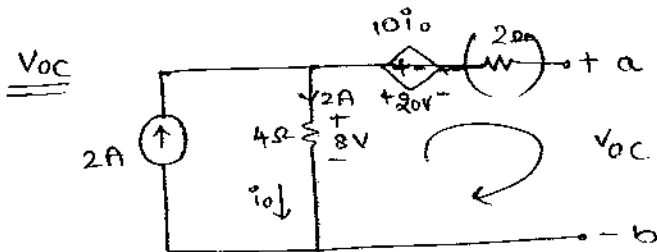


actively generative p/w

62

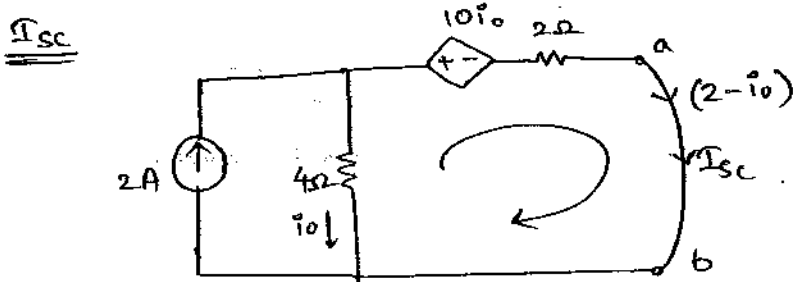


$$-V_{oc} + 2\Omega(5i_o) - 10i_o + 4\Omega(5i_o + 2) = 0$$



$$\Rightarrow -8 + 20 + V_{oc} = 0$$

$$V_{oc} = -12V$$



$$-4i_o + 10i_o + 2(2 - i_o) = 0$$

$$i_o = -1A$$

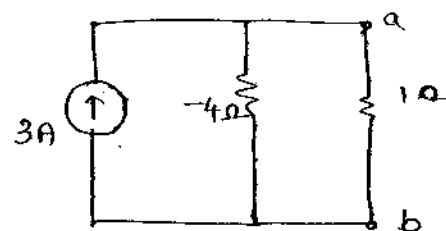
$$I_{sc} = 2 - i_o$$

$$= 2 - (-1)$$

$$= +3A$$

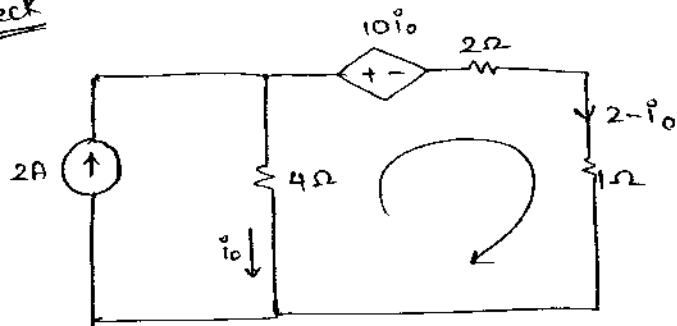
$$R_N = \frac{V_{oc}}{I_{sc}} = \frac{-12}{+3} = -4\Omega$$

N.E



$$I_{1\Omega} = 3 \left[\frac{-4}{-4+1} \right] = +4A$$

check



$$-4i_o + 10i_o + 3(2-i_o) = 0$$

$$i_o = -2A$$

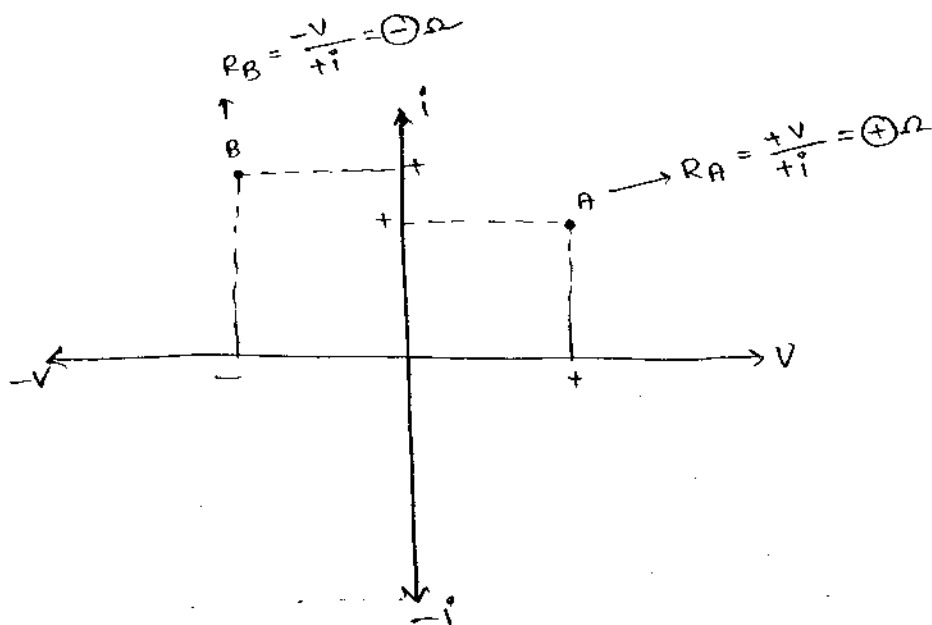
But

$$I_{1\Omega} = 2 - i_o$$

$$= 2 - (-2)$$

$$= +4A$$

*



+ve feedback
Ex: solar cell, oscillator

Note: In the above problem R_{th} or R_N is -ve, -ve Resistance is characteristic where to model active Regenerative N/w is electrical Engg. whose operating V-i characteristic appears in Q-II or Q-IV

Ex: High gain amplifiers, photo Transistors, solar cells, +ve feedback N/w like oscillators etc.

⇒ However, -ve Resistance is not a physically Reliable Quantity

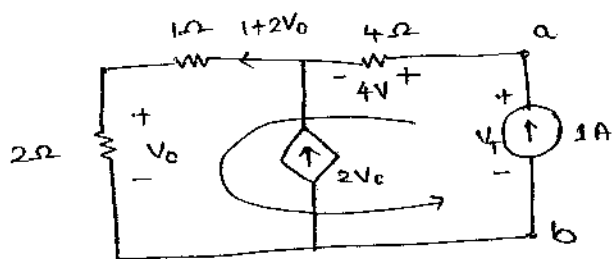
Category III: problems with only dependent sources.

Such N/ws do not work on their own as there is no Independent active element to drive them. In such N/w $V_{Th} = 0V$, $I_N = 0A$. However they have Resistance. This Resistance can be Indirectly determined by ohm's law but by Externally exciting them where $R_{Th} = R_N = \frac{1V}{i_T}$ (or) $= \frac{V_T}{1A}$.

63

Gate:

Determine Thevenin & Norton Equivalent across terminals ab.



$$-V_T + 4 + 3(1 + 2V_0) = 0$$

$$V_T = 7 + 6V_0 \rightarrow (1)$$

$$V_0 = 2(1 + 2V_0)$$

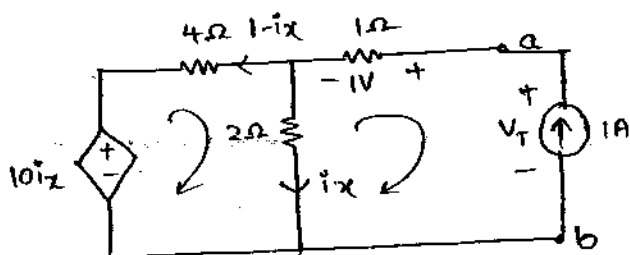
$$3V_0 = -2 \rightarrow (2)$$

$$V_T = 7 + 6\left[\frac{-2}{3}\right] = 3V$$

$$R_{Th} = R_N = \frac{V_T}{1} = 3\Omega$$

ISRO:

T.E across a-b



$$(a) 1V, 1\Omega \quad (b) 1V, 1\Omega$$

$$(c) 0V, 1\Omega \quad (d) 0V, -1\Omega$$

$$-10i_x - 4(1 - i_x) + 2i_x = 0$$

$$i_x = -1$$

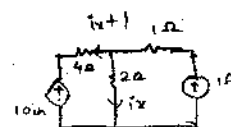
$$-2i_x - 1 + V_T = 0$$

$$V_T = 1 + 2i_x$$

$$= 1 - 2$$

$$= -1$$

$$R_{Th} = \frac{V_T}{1A} = \frac{-1}{1A} = -1\Omega$$



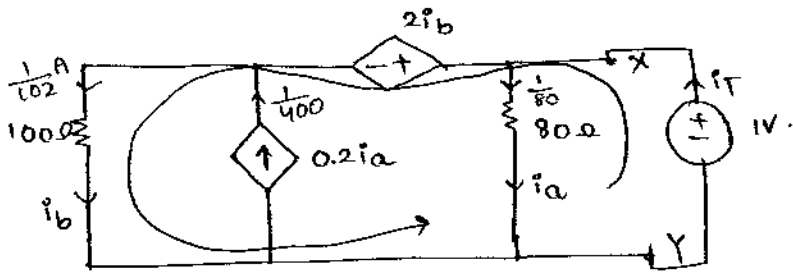
$$-10i_x - 4(i_x + 1) + 2i_x = 0$$

$$-14i_x - 4 = 0$$

$$i_x = -\frac{2}{7}$$

$$V_T = -1$$

Gate/2m. T.E & N.E across X-Y



KCL

$$-1 + 2i_b + 100i_b + 0$$

$$i_b = \frac{1}{102}$$

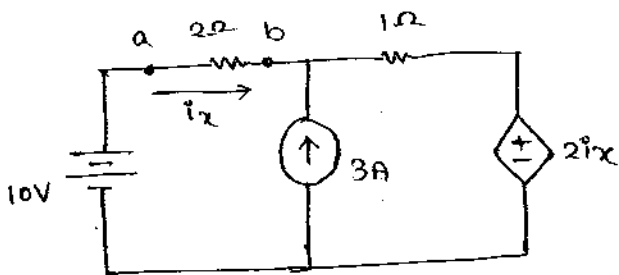
KCL

$$\frac{1}{102} + \frac{1}{80} = \frac{1}{400} + i_T$$

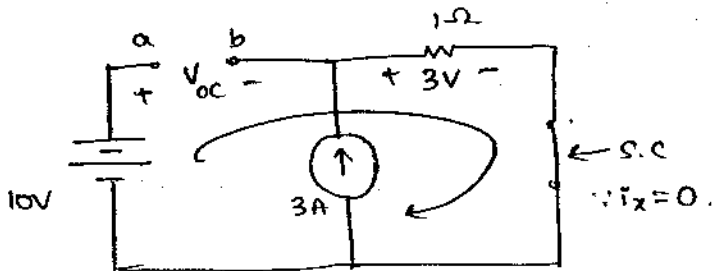
$$i_T = 0.0198 \text{ A}$$

$$R_{Th} = R_N = \frac{1V}{i_T} = 50.5 \Omega$$

* Special model.

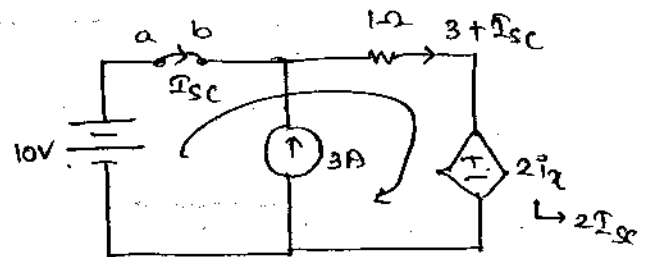


Determine i_x by using Norton's Theorem.



$$-10 + V_{oc} + 3 = 0$$

$$V_{oc} = 7V$$

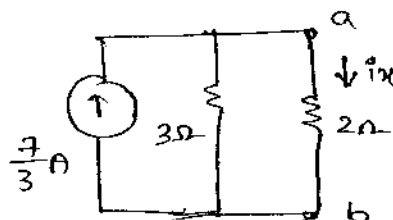


$$-10 + 1[3 + I_{sc}] + 2I_{sc} = 0$$

$$I_{sc} = \frac{7}{3} \text{ A}$$

$$R_N = \frac{V_{oc}}{I_{sc}} = \frac{7}{7/3} = 3 \Omega$$

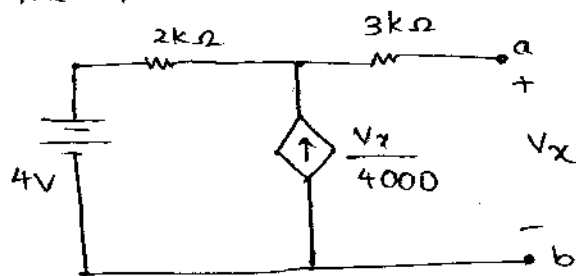
N.E



$$i_x = \frac{7}{3} \left[\frac{3}{5} \right] = \frac{7}{5} \text{ A}$$

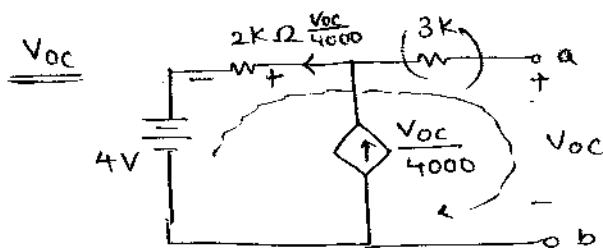
Q.4

T.E b/w a-b



1

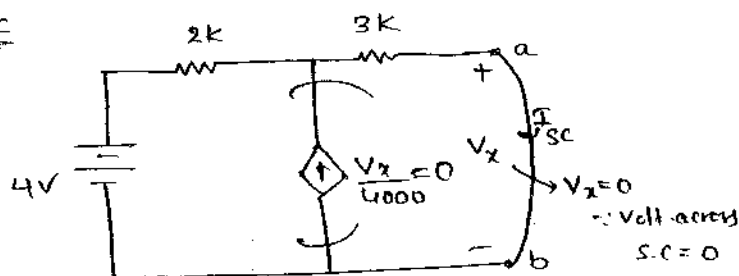
64



$$-4 - \left[\frac{V_{oc}}{4000} \right] \times 2000 + V_{oc} = 0$$

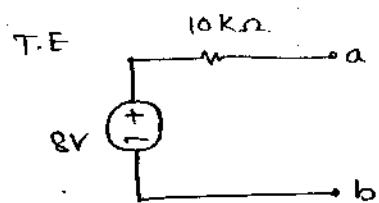
$$V_{oc} = 8V$$

I_{sc}



$$I_{sc} = \frac{4}{5K}$$

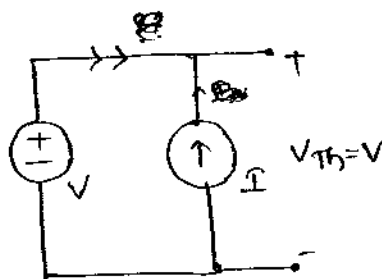
$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{8}{4/5K} = 10K\Omega$$



an Ideal 'V' source is in parallel to Ideal 'I' source across two terminals then

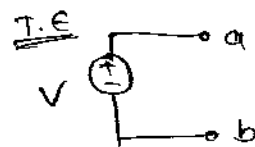
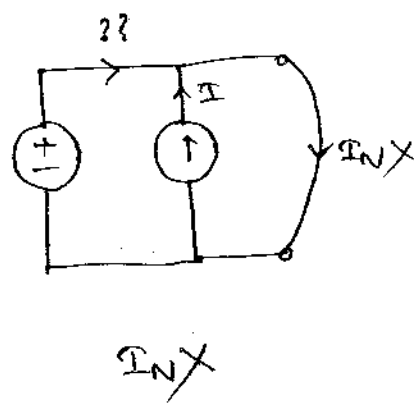
This combination has

- (a) Thevenin Eq. only
- (b) Norton Eq. only
- (c) Both
- (d) none



$$R_{th} = R_N = 0\Omega$$

$$V_{th} = V$$



* an Ideal \mathcal{I} -source ~~can't~~ is in series with Ideal \mathcal{V} -source this combination has Norton's Equivalent only.

IES(0)



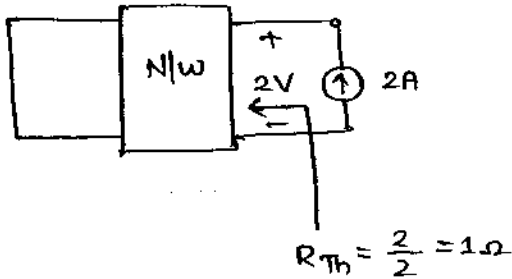
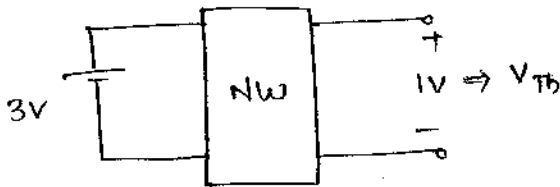
If

	E	V	I
1.	3V	1V	0A
2.	0V	2V	2A

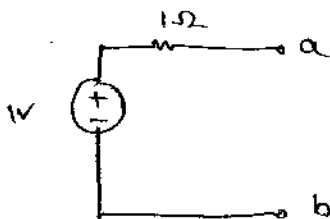
Now if $E = 30V$, & I is replaced by $2A$

then $V = \underline{\hspace{2cm}}$

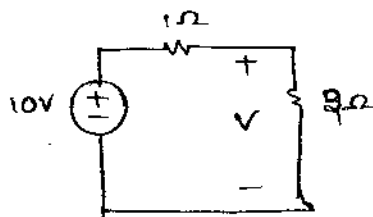
~~30V~~
2V
2A



T.E for $E = 3V$



T.E = $E = 30V$



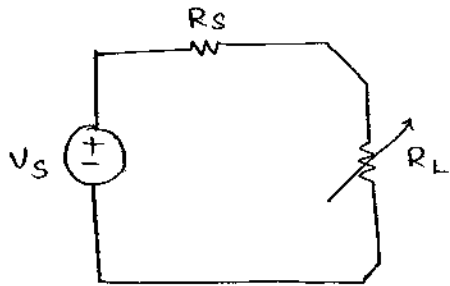
$$V = 10 \left[\frac{2}{3} \right]$$

$$= \underline{\underline{\frac{20}{3} V}}}$$

Maximum power Transfer Theorem in D.C N/w:

65

In any linear^{active} Bilateral N/w consisting of No. of Energy sources with their Internal Resistances is driving an external load then Max. power is transferred to the load When Load Resistance is Equal to Source Resistance i.e. the Thevenins Equivalent Resistance Seen by the load.



$$I_L = \frac{V_S}{R_S + R_L}$$

$$P_L = I_L^2 R_L$$

$$= \left(\frac{V_S^2}{(R_S + R_L)^2} \right) \cdot R_L$$

$$\text{At } P_{L\max}, \frac{dP_L}{dR_L} = 0$$

$$\Rightarrow V_S^2 \left[\frac{(R_S + R_L)^2 (1) - (R_L)(2)(R_S + R_L)}{(R_S + R_L)^4} \right] = 0$$

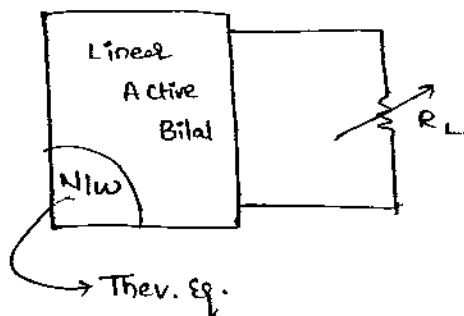
$$2R_L(R_S + R_L) = (R_S + R_L)^2$$

$$2R_L = R_S + R_L$$

$$R_L = R_S$$

$$P_{L\max} = \frac{V_S^2}{4R_S} \quad W$$

In General:

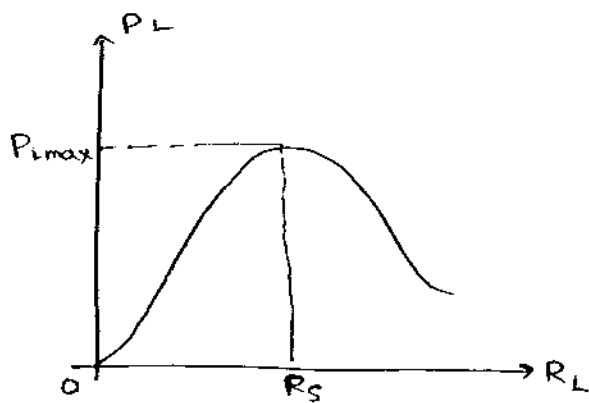


Then for

$$P_{\max} \Rightarrow R_L = R_{Th}$$

$$P_{\max} = \frac{[V_{Th}]^2}{4R_{Th}} \quad W$$

IES



IES(0) During P_{max} transfer to the Load, output efficiency is 50%

$100W$ V_s $50W$ R_s R_{L50W} $\% \eta = \frac{50}{100} \times 100\% = 50\%$

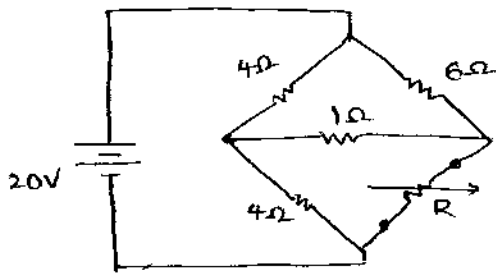
IES(0) During P_{max} transfer to the Load, 50% % of Supply voltage will appear across the load.

Applications:

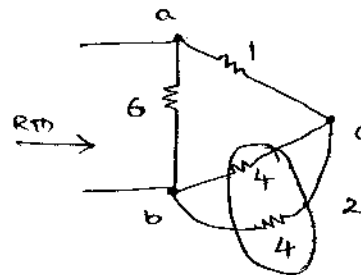
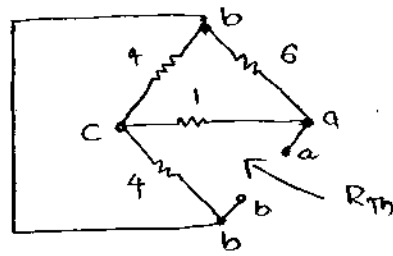
- (1) Audio Speaker Design
- (2) Automobile lighting
- (3) Cascaded Connection of multistage amplifier with Impedance matching.

Q for what value of Resistance 'R' max. power is transferred to it

66



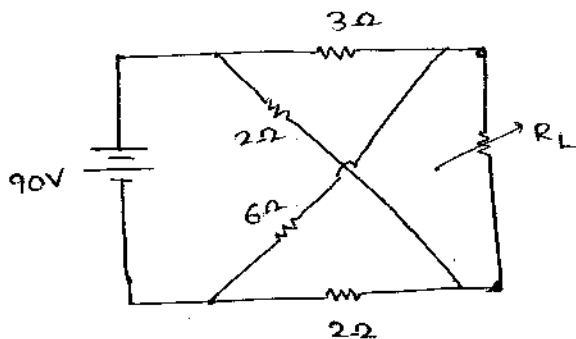
⇒



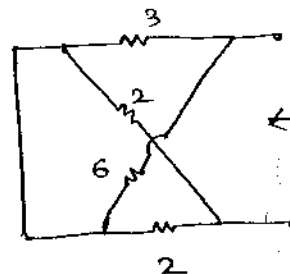
$$R_{Th} = R = 3 \parallel 6 = 2 \Omega$$

gate 2007

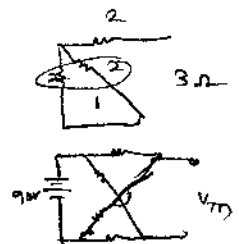
Q



What is the max power Transferred to the load

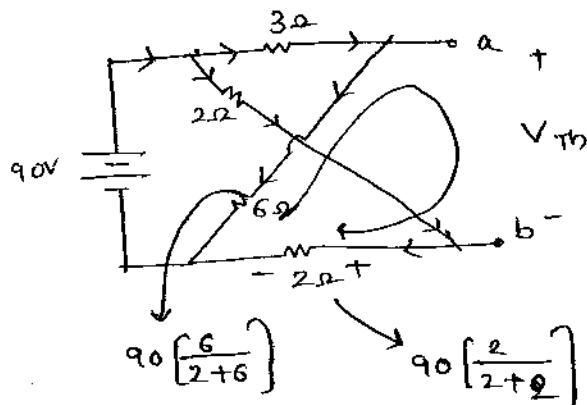


$$R_{Th} = 3 \Omega$$



$$\rightarrow R_L = R_{Th} = [3 \parallel 6] + [2 \parallel 2] = 3 \Omega$$

$$\rightarrow \frac{V_{Th}}{L} \rightarrow \text{Voll. Div. Rule + KVL}$$



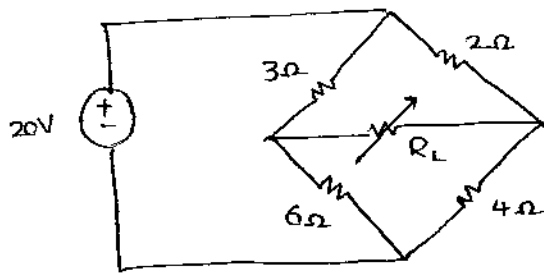
$$+V_{Th} + \left[90 \left(\frac{2}{2+2} \right) - 90 \left(\frac{6}{6+3} \right) \right] = 0$$

$$V_{Th} = 60 - 45 = 15V$$

$$P_{max} = \frac{[V_{Th}]^2}{4R_{Th}}$$

$$= \frac{(15)(15)}{4 \times (3)} = 18.75 W$$

IES(0) What is the max power Transferred to the load.



∴ Bridge is Balanced

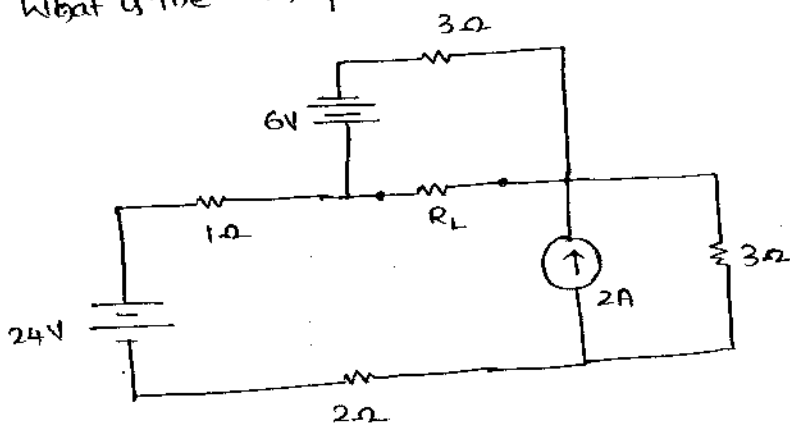
Current can never flow through R_L for any value.

$$\therefore P_{\max} = 0W.$$

In a Balance Bridge V_{Th} across cross arm element is 0V.

Gate 1M

What is the max power Transferred to the load.



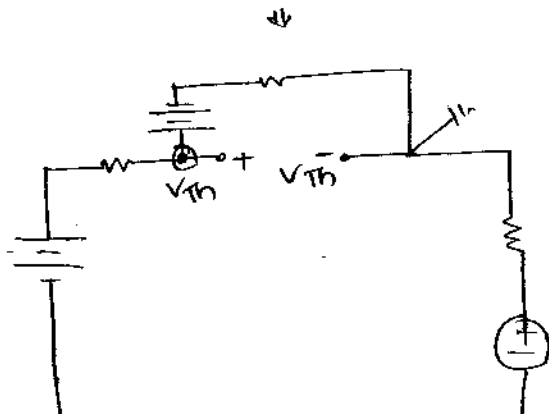
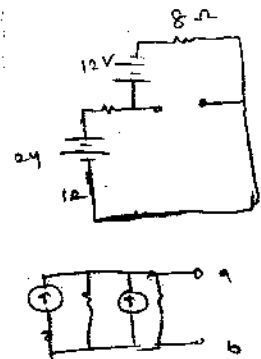
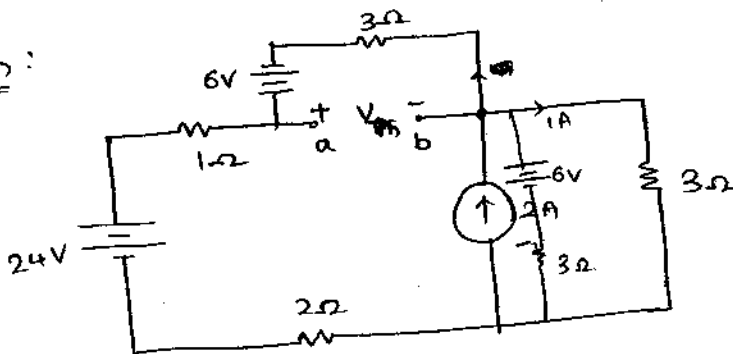
$$R_{Th} = (1+2+3) \parallel 3\Omega$$

$$= 6\Omega \parallel 3\Omega$$

$$= 2\Omega$$

6/11

V_{Th} :

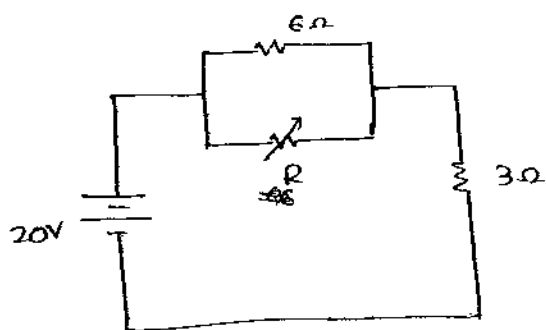


$$\frac{V_{Th}}{L} \rightarrow S.T.T + Nodal$$

$$\frac{(V_{Th}-24+6)}{6} + \frac{(V_{Th}-6)}{3}$$

$$P_{\max} = \frac{[V_{Th}]^2}{4R_{Th}} = \frac{(10)^2}{4(2)} = 12.5W.$$

20
Q for what value of Resistance 'R' max power is Transferred to 3Ω Resistance.



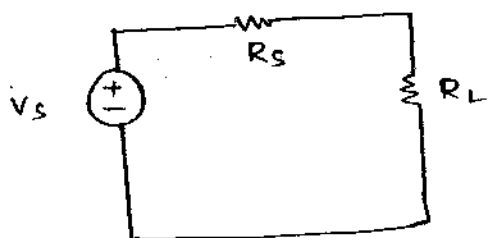
- ~~(a) 2Ω~~
 (b) 3Ω
 (c) 6Ω ✗
~~(d) infinity.~~

- (a) 2Ω
 (b) 3Ω
 (c) 6Ω
(d) zero
- 67

The Question has no Relation with original statement of P_{max} theorem as here Source Side Resistance is being Varied.

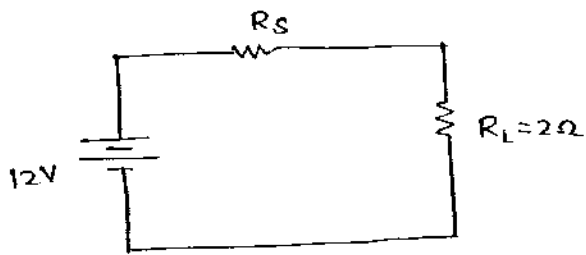
When $R = 6\Omega \Rightarrow$ current through $3\Omega \Rightarrow \frac{20}{6} \Rightarrow P_{max} = \left(\frac{20}{6}\right)^2 \times 3\Omega$
 $R = 3\Omega \Rightarrow$ " " $3\Omega \Rightarrow \frac{20}{5} = \left(\frac{20}{5}\right)^2 \times 3\Omega$
 $R = 2\Omega \Rightarrow$ " " $3\Omega \Rightarrow \frac{20}{4.5} = \left(\frac{20}{4.5}\right)^2 \times 3\Omega \checkmark_{max}$
 When $R = 0\Omega \Rightarrow$ " " $3\Omega \Rightarrow \frac{20}{3} \Rightarrow P_{max} = \left(\frac{20}{3}\right)^2 \times 3\Omega$

H.W
Q Derive mathematically the Condition Req. to maximise power in the load by Varying Source Side Resistance.



$\frac{dP_L}{dR_S}$
 $R_S = 0$

IES (A) What is the max power Transferred to the load.



$$I_p (1 \leq R_S \leq 10) \Omega$$

$$\left(\frac{12}{1+2}\right)^2 \cdot 2$$

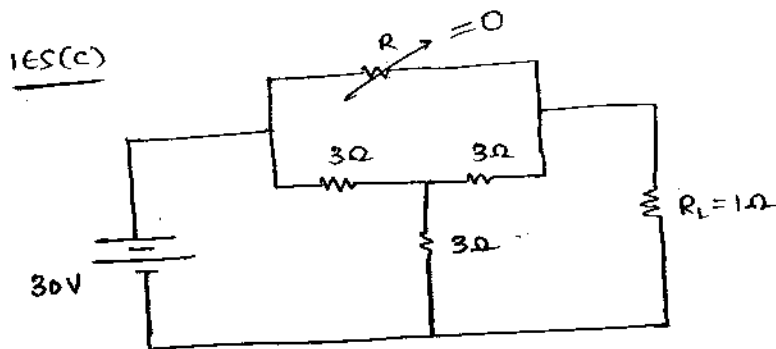
$16 \times 2 = 32 \text{ W}$

for P_{\max} in R_L

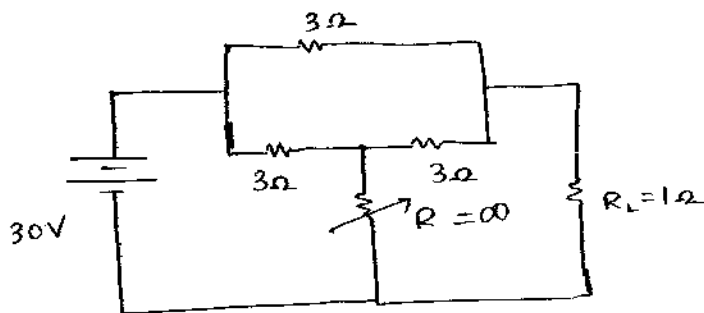
R_S should be min (i.e. we are Varying R_S)
i.e. $R_S = 1\Omega$

$$\begin{aligned} P_{L\max} &= (I_L)^2 R_L \\ &= \left(\frac{12}{1+2}\right)^2 \cdot (2) \\ &= 16 \times 2 \\ &= 32 \text{ W} \end{aligned}$$

for the following ckt's shown determine the value of Resistance 'R' for which max Power Transferred to the load.



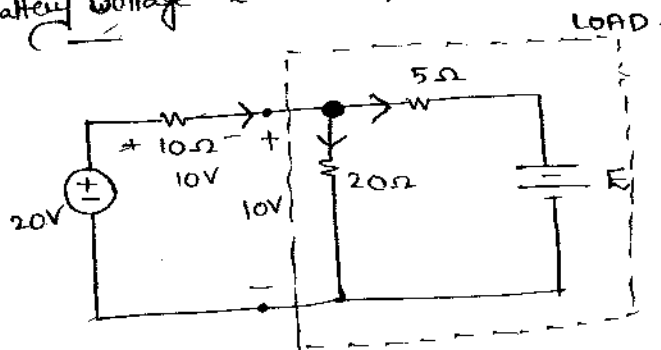
$R=0$ will make High power.



$R = \infty$ will make Highest power.

68

Q the ckt represent an approx. DC Equivalent of a Battery charger for what value of Battery voltage 'E' max power is Transferred to the entire Load.



P_{max} occurs in load when R_s is Equal to Resistance of total load

Including the effect of 'E' Voltage ~~charge~~ ^{Battery} ~~charge~~.

$$R_s = (R_{LOAD})_{\text{including Battery voltage 'E'}}$$

then if $R_s = R_{load}$ then ^{total} Voltage appears. half across ~~each~~ R_s & R_{load} .

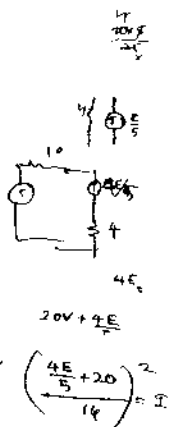
KCL

$$\frac{20-10}{10} = \frac{10}{20} + \frac{(10-E)}{5}$$

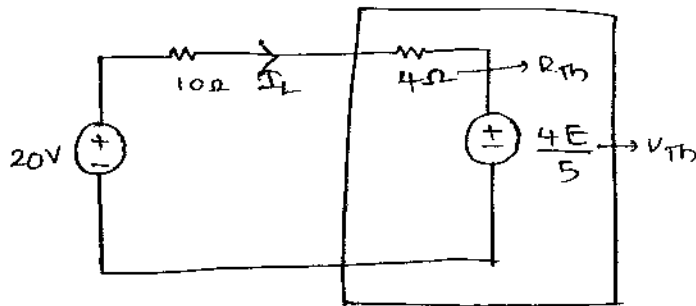
$$1 = \frac{1}{2} + \frac{10-E}{5}$$

$$\frac{10-E}{5} = \frac{1}{2}$$

$$E = 10 - \frac{5}{2} = 7.5 \checkmark$$



Exact method:



$$I_L = \frac{20 - \frac{4E}{5}}{14} = \frac{100 - 4E}{70} = \left[\frac{50 - 2E}{35} \right]$$

$$P_L = I_L^2 (4) + I_L \left(\frac{4E}{5} \right)$$

$$P_L = \left[\frac{50 - 2E}{35} \right]^2 4 + \left[\frac{50 - 2E}{35} \right] \left[\frac{4E}{5} \right]$$

$$P_L = \frac{4}{35} [50 - 2E] [50 + 3E]$$

$$P_L = \frac{40}{(35)^2} (25 - E)(10 + E)$$

$$P_L = \frac{40}{(35)^2} [-E^2 + 15E + 250]$$

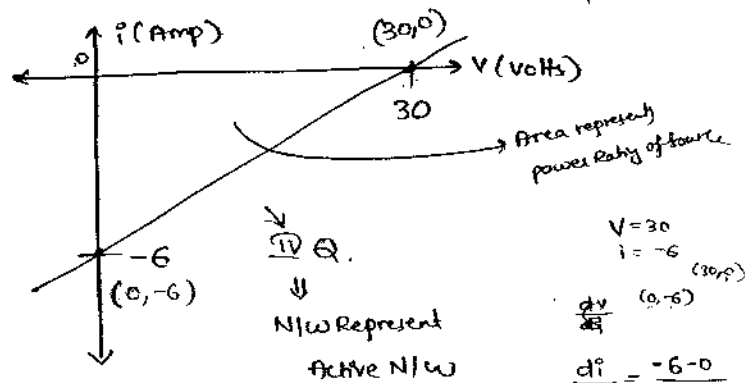
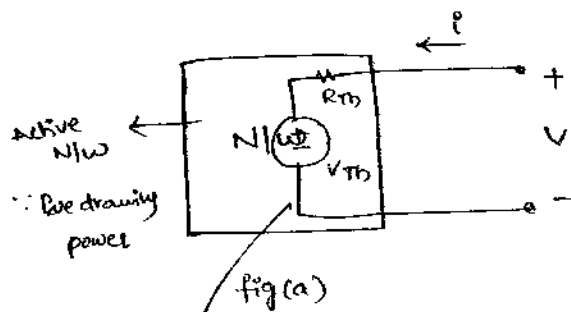
for P_{Lmax} $\frac{dP_L}{dE} = 0$

$$\frac{40}{(35)^2} [-2E + 15] = 0$$

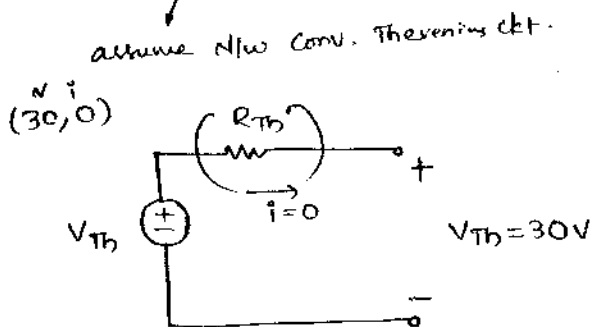
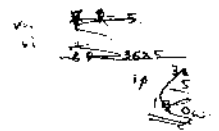
$$2E = 15 \longrightarrow E = \underline{\underline{7.5V}}$$

Q The static V-I characteristics of N/w shown in figure (a) are plotted in fig (b).
What is max power that can be drawn from the N/w.

69

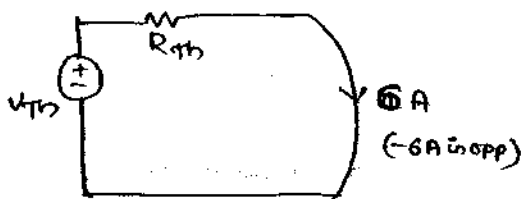


$$\begin{aligned} V &= 30 \\ i &= -6 \quad (30, 0) \\ \frac{dV}{di} &= \frac{0 - 30}{-6 - 0} \\ \frac{dV}{di} &= \frac{1}{5} \end{aligned}$$



⇒ When $i = 0$
 $R_{Th} = 0 \cdot C$
∴ Voltage across terminals $= V_{Th} = 30V$. — (1)

(0, -6)
when $V = 0$, $i = -6A$.



$$V_{Th} = 6R_{Th}$$

$$R_{Th} = \frac{V_{Th}}{6} = \frac{30}{6} = 5\Omega$$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{30 \times 30}{4 \times 5} = 45W$$

(or)

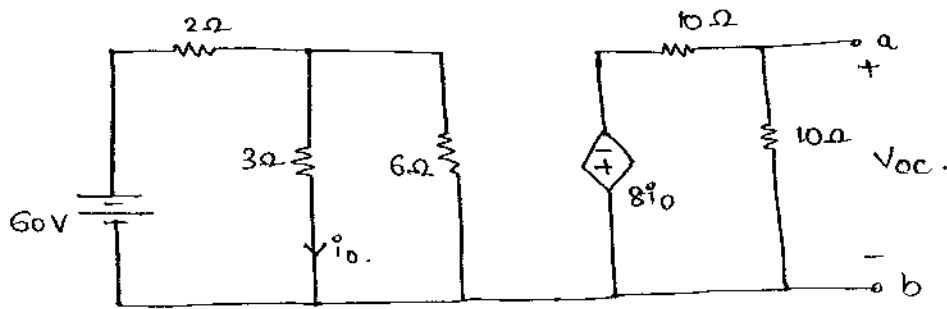
$$P_S = \frac{1}{2} (6) (30) = 90W \quad (\text{100\%})$$

free

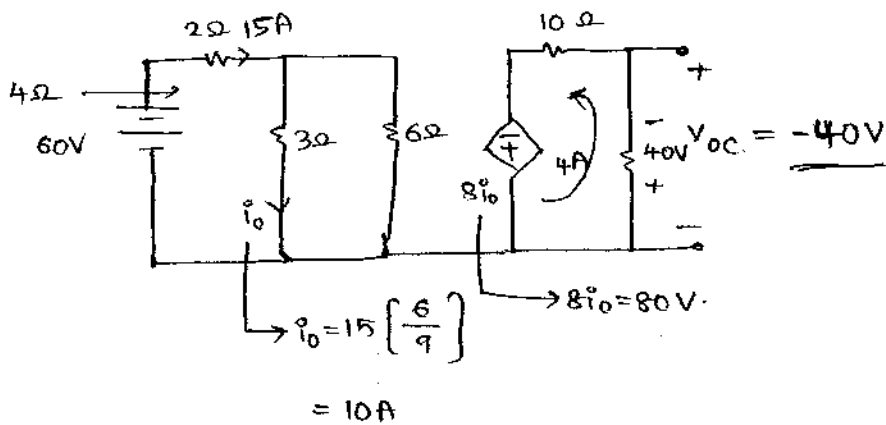
at P_{max}

$$\% \eta = 50\% \Rightarrow P_{Smax} = 45W \quad (\text{50\%})$$

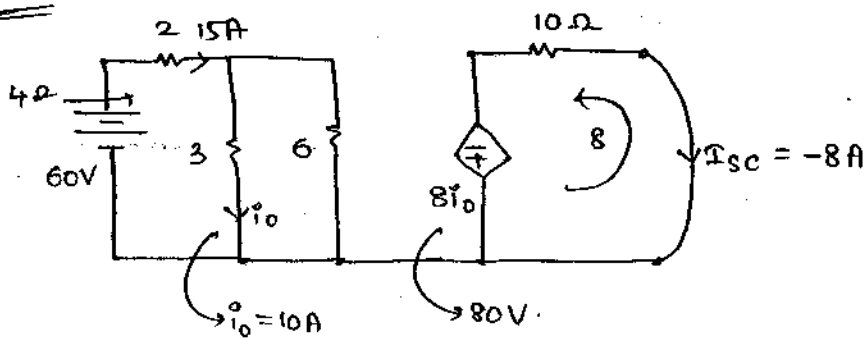
What Resistance Connected b/w terminals AB will draw max power from the N/w & also find P_{max} .



V_{oc}



I_{sc}



$$R_{ab} = R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{-40}{-8} = +5\Omega \rightarrow \text{sink}$$

$$P_{max} = \frac{[V_{th}]^2}{4R_{th}} = \frac{40 \times 40}{4 \times 5} = 80W.$$

VI Tellegan's Theorem:

This theorem is about Clarification of law of Conservation of Energy. However in any linear Time Invariant N/w or System we can also Verify this theorem by measuring power at an Instant. 70

$$\therefore E = \int P dt \cdot J$$

So, Here in this theorem we need to mathematically verify

$$\sum_{k=1}^b V_k \cdot I_k = 0$$

b = no. of branches in the N/w.

$$\begin{array}{ccc} \text{i.e., } \sum V \cdot I \Big|_{\text{Sources}} & = & \sum V \cdot I \Big|_{\text{Sinks.}} \\ \downarrow & & \downarrow \\ P_{\text{delivered by Sources}} & = & P_{\text{absorbed by sinks.}} \end{array}$$

Note: (1) Tellegan's theorem is the only theorem in elec. ckt analysis which is also valid for any nonlinear element or Network.

(2) While applying this theorem the topological arrangement of N/w shouldn't be altered. However we can still verify Tellegan's theorem independently for the ~~with~~ SubCircuits involved either before reduction or after reduction in N/w.

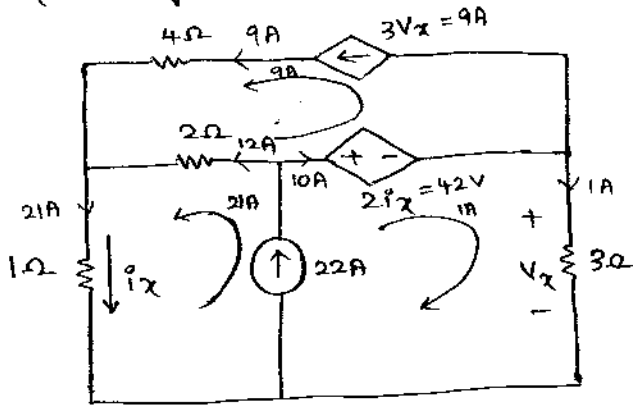
(3) Tellegan's theorem can also be verified for any LTI N/w but for Time Variance in the input Excitations where we can mathematically prove

$$\sum_{k=1}^b V_k(t_1) I_k(t_2) = \sum_{k=1}^b V_k(t_2) I_k(t_1)$$

t_1 & t_2 are two different Instances in time for measuring voltages & currents in N/w

$b \rightarrow$ no. of branches in N/w.

Q. Verify Tellegans theorem for the ckt shown below.



$$1(i_1) + (i_1 - i_3)2 + 2i_3 + 3i_2 = 0$$

mesh

$$i_1 + 2[i_1 - i_3] + 2i_3 + 3i_2 = 0 \rightarrow (1)$$

$$-i_1 + i_2 = 22 \rightarrow (2)$$

$$i_3 = -3V_x \rightarrow (3)$$

$$i_x = -i_1 \rightarrow (4)$$

$$V_x = 3i_2 \rightarrow (5)$$

Solve

$$\begin{cases} i_1 + 3i_2 - 2i_3 = 0 \\ -i_1 + i_2 = 22 \\ 9i_2 + i_3 = 0 \end{cases}$$

$$\begin{aligned} 4i_2 - 2i_3 &= 22 \\ 18i_2 + 2i_3 &= 0 \end{aligned}$$

x2

$$i_2 = 1A$$

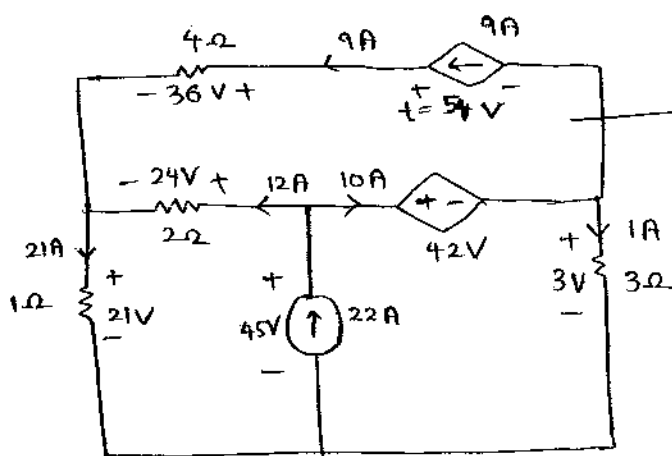
$$V_x = 3i_2$$

$$V_x = 3V$$

$$i_3 = -3V_x = -9A$$

$$i_1 = i_2 - 22 = -21A$$

$$i_x = +21A$$



let t be volt. across 9A dep. source.

$$\text{KVL} \rightarrow -42 + 24 - 36 + t = 0$$

$$t = 54V$$

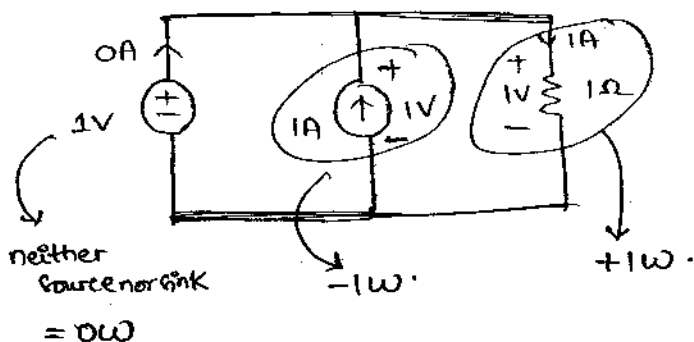
71

$$\sum P_{\text{del}} = 45(22) + (54)(9) = \underline{1476W} \checkmark$$

$$\sum P_{\text{abs}} = (21)(21) + (24)(12) + (3)(1) + (42)(10) + (36)(9) = \underline{1476W} \checkmark$$

165(6)

The power absorbed by each element in order is _____



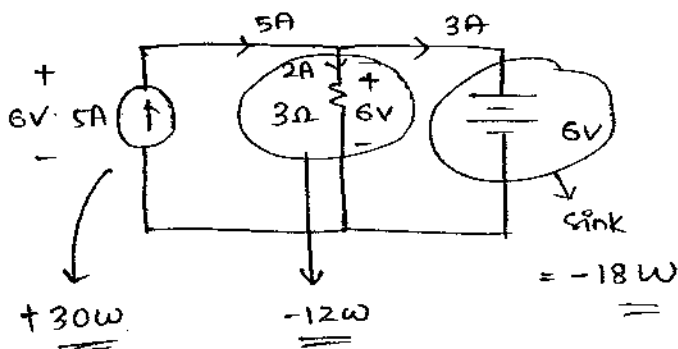
(a) $1W, 0W, -1W$

(b) $-1W, 0W, 1W$

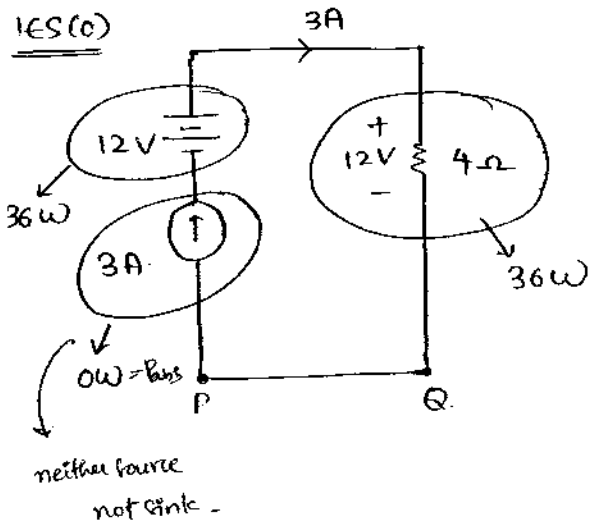
(c) $0W, -1W, 1W$

(d) $0W, 1W, -1W$

The power delivered by each element in order is _____



$$\underline{\underline{\sum P = 0}}$$



The power absorbed by each element

Travelling from CW from P to Q in order is
(virtual travelling)

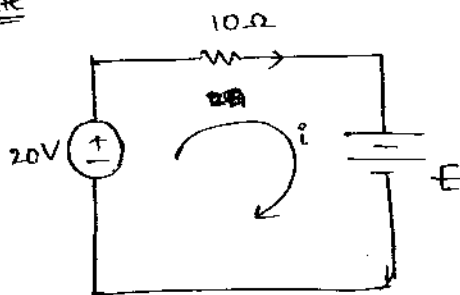
$$0W, -36W, +36W$$

(I) (V) (R)
source. source. sink

$$\underline{\underline{\sum P = 0}}$$

element source to any element
is Redundant

gate



If the power absorbed ^{by the} battery is 10W
then determine its Voltage E.

$$E \cdot i = 10W \text{ (given)}$$

By Tellegen's theorem

$$P_{deli} = P_{abs.}$$

~~20V source~~

$$20(i) = i^2(10) + E \cdot i$$

$$20i = 10i^2 + 10 \rightarrow \text{gets satisfied}$$

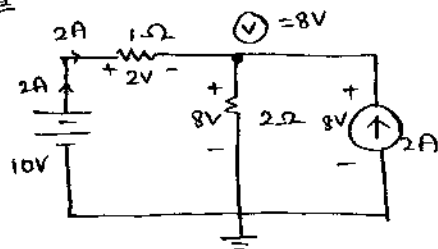
if $i = 1$

$$\therefore i = 1$$

$$E(1A) = 10W$$

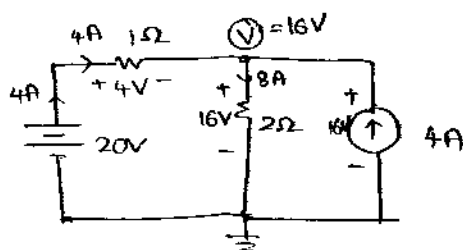
$$\underline{\underline{E = 10V}}$$

N/w at time $-t_1$



$$b=4$$

N/w at time $-t_2$



Verify Tellegans theorem for LTI N/w shown but for Time Variance in the I/p Excitations referred at two different instances in time as shown

72

$$1 \frac{V-10}{1} + \frac{V-0}{2} - 2 = 0$$

$$V = 8V$$

We need to prove

$$\sum_{k=1}^4 V_k(t_1) I_k(t_2) = \sum_{k=1}^4 V_k(t_2) I_k(t_1)$$

$$\begin{aligned} \text{L.H.S.} & \xrightarrow{\quad} (10)(4) + (2)(4) + (8)(8) + (8)(4) = 144W \\ \text{R.H.S.} & \xrightarrow{\quad} (20)(2) + (4)(2) + (16)(4) + (16)(2) = 144W \end{aligned}$$

Source branch
1Ω
2Ω
C. Source

**

VII Reciprocity Theorem:

In any linear passive Bilateral N/w the Ratio of Response to Excitation Remains Constant even if the positions of Source & load are Interchanged

fig (a)

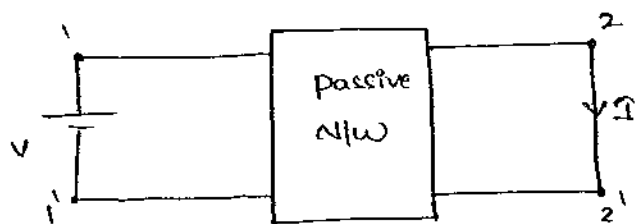
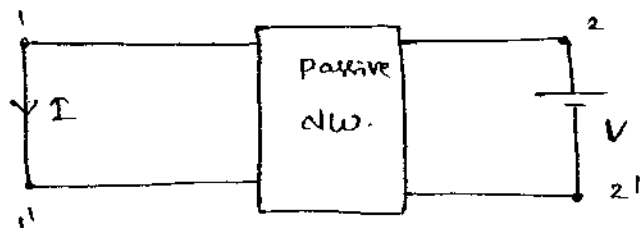


fig (b)



If N/w is Reciprocal, then

$$\frac{I}{V} = \text{Constant} \xrightarrow[\text{Homogeneity}]{\text{also by}} \frac{I_1}{V_1} = \frac{I_2}{V_2}$$

Note: 1. This theorem is valid for N/ws excited with only a single source

2. This theorem is not valid for N/ws with dependent sources.

3. While writing the Reciprocal N/w of a given N/w Ideal voltage source must be connected in series to the Target branch & Ideal current source must be connected parallel to the Target branch.

4. any linear passive bilateral N/w is by default Reciprocal. Since, Reciprocity means passivity.

Reciprocity \iff passivity.

5. The principle of Reciprocity can be verified for any passive N/w as a Ratio b/w Response to Excitation being Constant by considering any parameter as i/p or o/p.

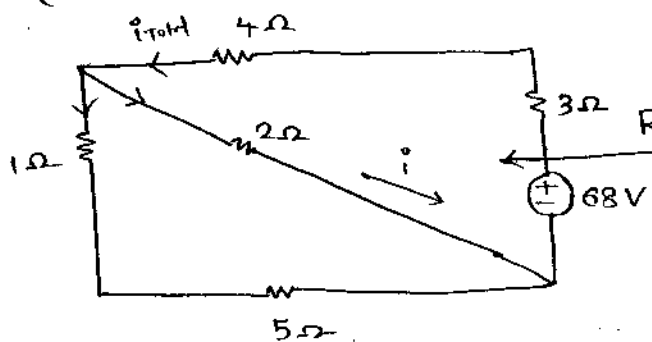
applications:

Ideally our P.S Tr. line N/w & Telephone line in Communication system are Reciprocal where in terms of Transmission line parameters the following Condition exists.

$$AD - BC = 1$$

Q. verify Reciprocity for ckt shown below by determining Current 'i'

V source parallel to element is Redundant.



$$i_{Total} = \frac{68}{7 + [2||6]}$$

$$i_{2\Omega} = i = i_{Total} \times \left[\frac{6}{2+6} \right]$$

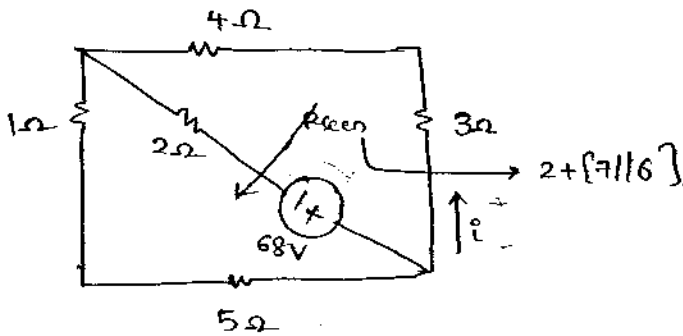
$$i = \frac{68}{7 + [2||6]} \times \frac{6}{8}$$

$$= \frac{68}{7 + \frac{12}{8}} \times \frac{6}{8} = \frac{68}{7 + 1.5} \times \frac{6}{8}$$

$$i = 6A$$

$$\frac{6 \times 7}{6 + 7} = \frac{42}{13}$$

Reciprocal N/w.



$$i_{\text{Total}} = \frac{68}{2 + [7//6]} \quad 73$$

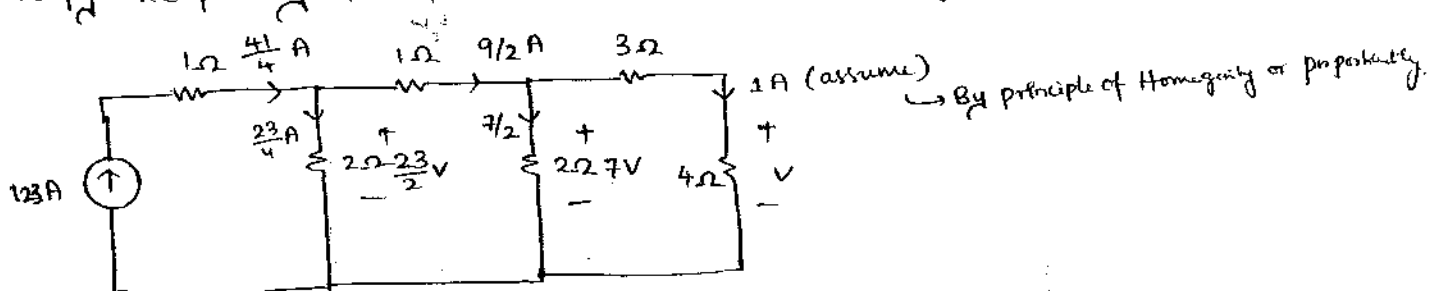
$$i_{3\Omega} = i = \frac{68}{2 + 4.2} \quad i_{\text{Total}} \times \frac{6}{13}$$

$$= \left[\frac{68}{2 + \frac{42}{13}} \right] \times \frac{6}{13}$$

$$= \frac{68}{13} \times \frac{6}{13}$$

$$i = 6A$$

Verify Reciprocity principle for ckt shown by determining voltage V.



$$\frac{41}{4} A \longrightarrow 1A$$

$$123A \longrightarrow ? \quad i = \frac{123}{\frac{41}{4}} = 12A$$

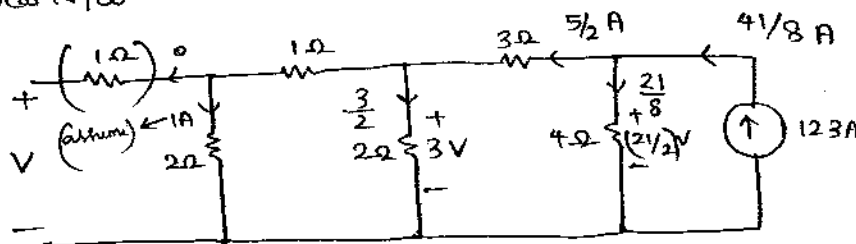
But

$$V = i[4]$$

$$= 12(4)$$

$$V = 48V$$

Reciprocal N/w



$$\frac{41}{8} A \longrightarrow 1A$$

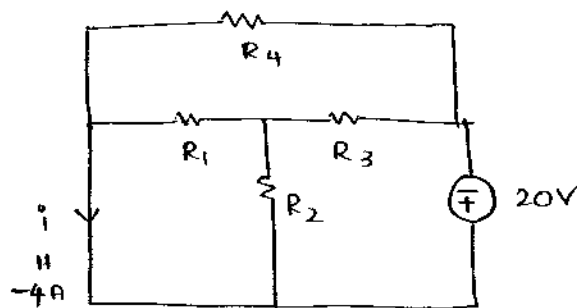
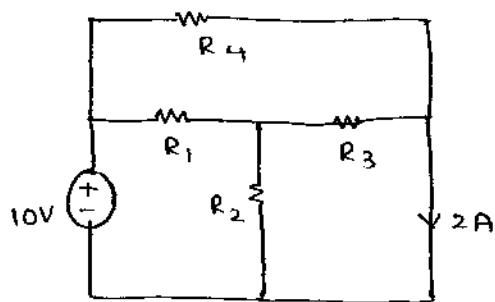
$$123A \longrightarrow i = \frac{123}{\frac{41}{8}} = 24A$$

$$V = i[2]$$

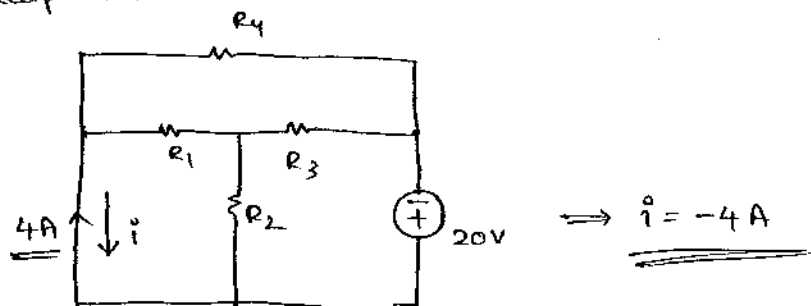
$$= 24[2]$$

$$= 48 \text{ volts.}$$

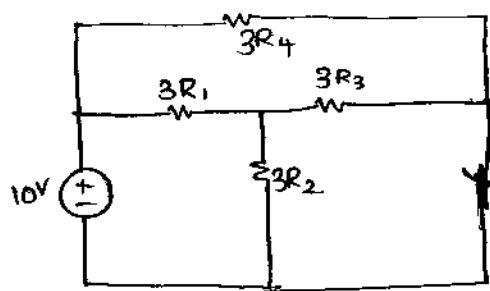
Q Use the data given in fig (a) to find Current i in fig (b).



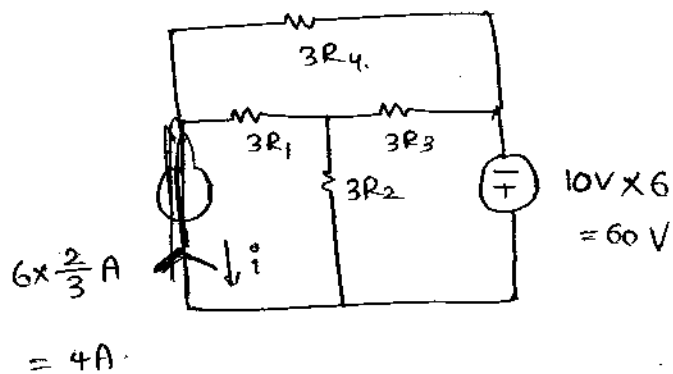
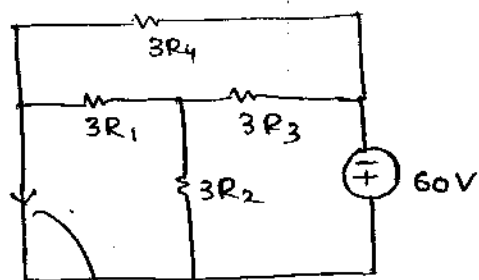
Reciprocal N/w



Q Use the data given in fig (a) to find current i in fig (b)



By
Ther.
eq.
if RT by 3 times
 i ↓ by 3 times

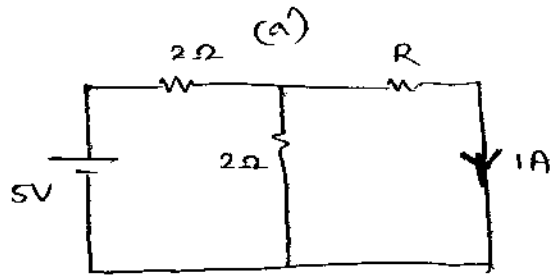


$i = -4A$

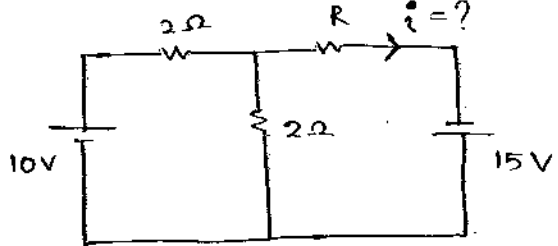
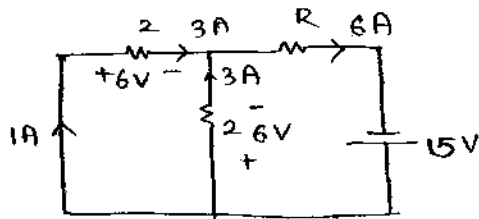
Q use data given in fig (a) to find 'i' in fig (b)

(b)

74

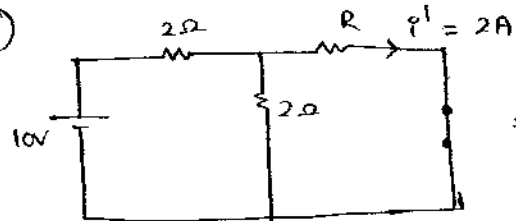


↓



↓
apply S.P.T

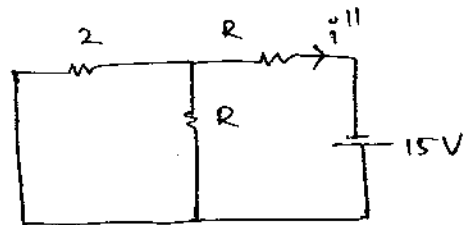
(I)



($\because 5V \rightarrow 1A$
 $10V \rightarrow 2A$)

$\Rightarrow i = 2A$

(II)



$i'' = 6A$

By applying (1) SPT
(2) R.T.

(3) H.R

(4) KCL

$$i = i' + i''$$

$$= 2 + 6 = 8A$$

Q find 'i' in fig (b) by using fig (a) data & verify Result by using Tellegen's Theorem

fig (a) $\rightarrow t_1$

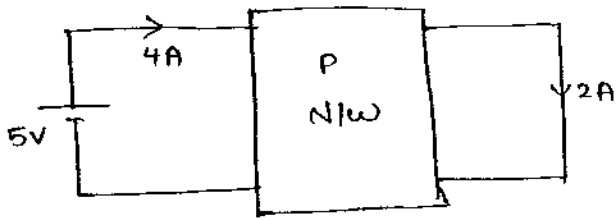
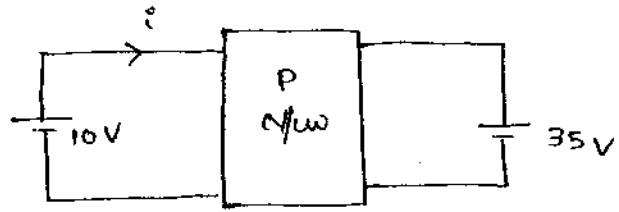
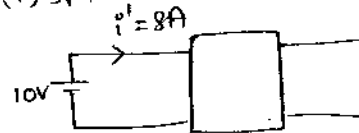


fig (b) $\rightarrow t_2$

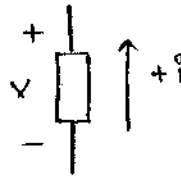


(1) SPT



$$\begin{array}{c} \xrightarrow{8A} \\ \xleftarrow{14A} \end{array} \left. \vphantom{\begin{array}{c} \xrightarrow{8A} \\ \xleftarrow{14A} \end{array}} \right\} \xleftarrow{6A} \Rightarrow i = -6A$$

assume
+ve Notation



By Tellegen's

$$\sum_{k=1}^2 V_k(t_1) I_k(t_2) = \sum_{k=1}^2 V_k(t_2) I_k(t_1)$$

$$(+5)(+i) + (0)(\underset{\substack{\uparrow \\ \text{unknown}}}{X}) = (+10)(+4) + (35)(-2)$$

$$5i = 40 - 70$$

$$5i = -30$$

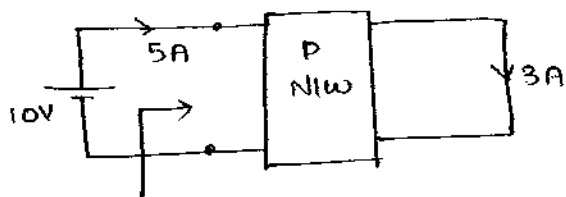
$$\underline{\underline{i = -6A}}$$

Q Use the data given in fig(a) to find current 'i' in fig(b)

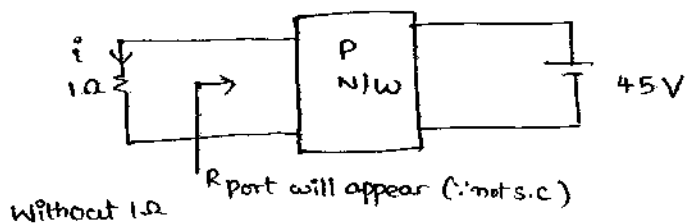
75

fig(a) $\rightarrow t_1$

fig(b) $\rightarrow t_2$

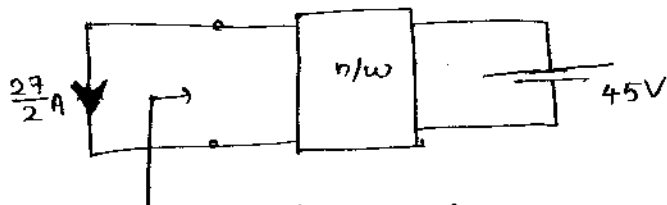


$$R_{port} = \frac{10}{5} = 2 \Omega$$



Without 1Ω

R_{port} will appear (\therefore not s.c)

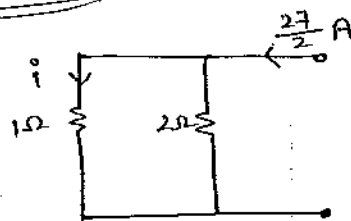


R_{port} will not appear

\therefore s.c across n/w is Redundant.

s.c Bypass the port n/w & draws full current.

With 1Ω



$$i = \frac{27}{2} \left(\frac{2}{3} \right) = 9A$$

(or) By Tellegans.

$$\sum_{k=1}^2 V_k(t_1) I_k(t_2) = \sum_{k=1}^2 V_k(t_2) I_k(t_1)$$

$$(10)(-i) + (0) \cdot (\text{X}) = (i) (+5A) + (45)(-3)$$

(unknown)

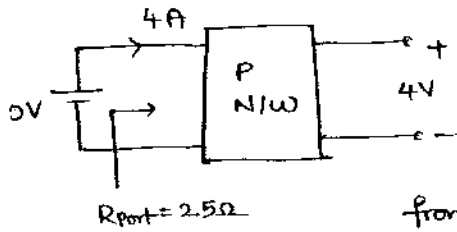
$$-10i = 5i - 135$$

$$15i = 135$$

$$i = 9A$$

IES (d)

fig (a)



from Note (5)

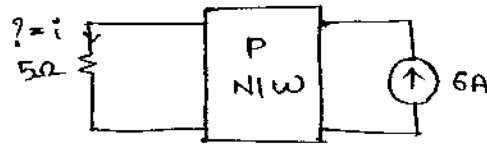
Ratio. $\frac{\text{Excitation}}{\text{Response}} = \text{same.}$

$$10V \longrightarrow 4V$$

$$6A \longrightarrow ? \quad i = \frac{6 \times 4}{10}$$

$$i = 2.4A$$

fig (b)

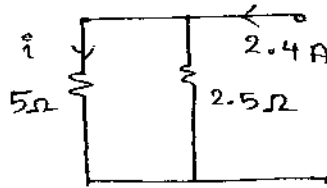


Without 5Ω

$$10V \longrightarrow 4V$$

$$6A \longrightarrow i = \frac{6 \times 4}{10}$$

$$i = 2.4A$$



$$i = 2.4 \left[\frac{2.5}{7.5} \right] = \underline{\underline{0.8A}}$$

By Tellegen's.

$$\sum_{k=1}^2 V_k(t_1) I_k(t_2) = \sum_{k=1}^2 V_k(t_2) I_k(t_1)$$

$$(10)(-i) + (4)(6) = (5i)(4) + (\text{X})(0)$$

↑
unknown.

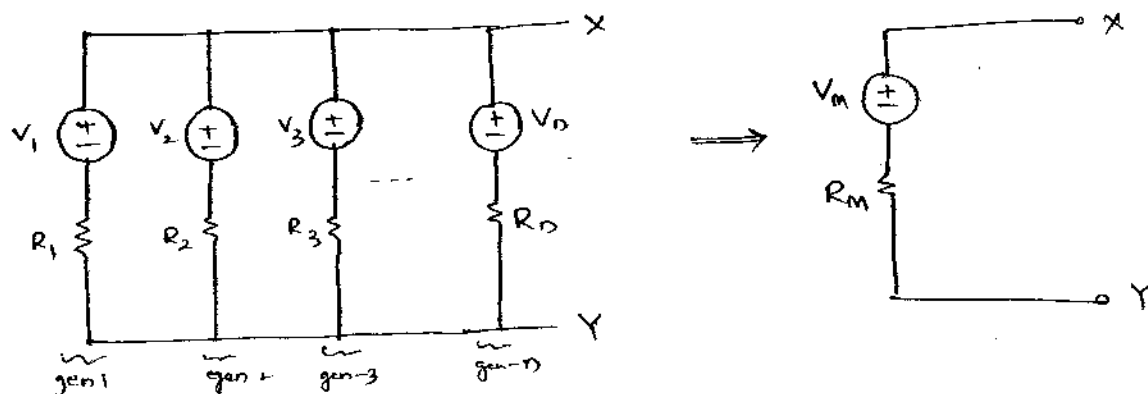
$$-10i + 24 = 20i$$

$$30i = 24$$

$$i = 0.8A$$

VIII. Milliman's Theorem: (or) parallel Generator Theorem.

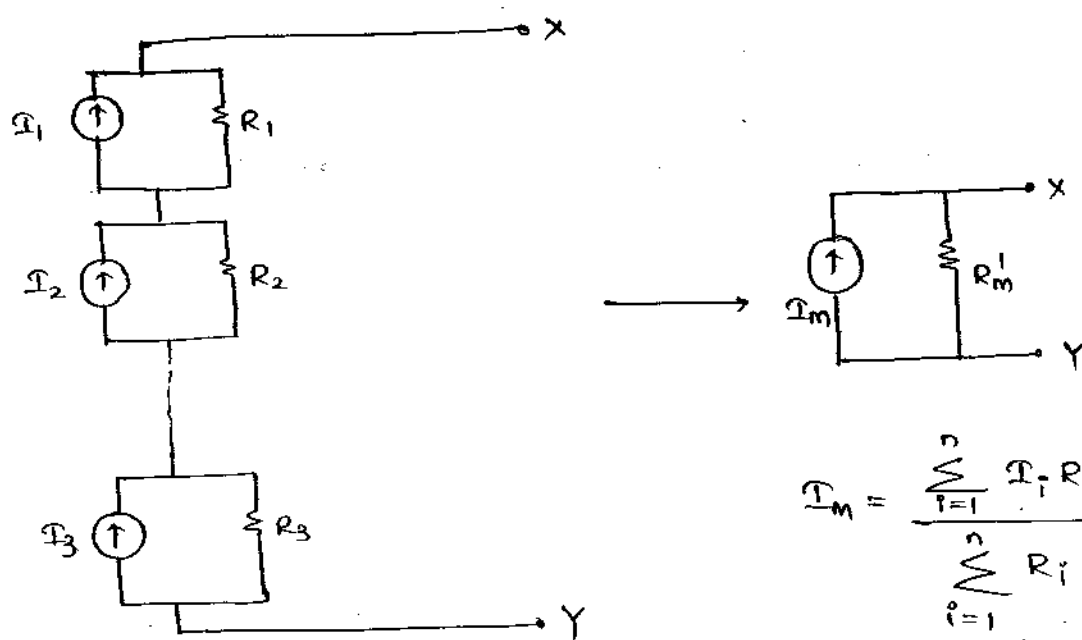
76



$$V_M = \frac{\sum_{i=1}^n \frac{V_i}{R_i}}{\sum_{i=1}^n \frac{1}{R_i}} = \frac{\sum_{i=1}^n V_i G_i}{\sum_{i=1}^n G_i}$$

$$R_M = \frac{1}{\sum_{i=1}^n \frac{1}{R_i}} = \frac{1}{\sum_{i=1}^n G_i}$$

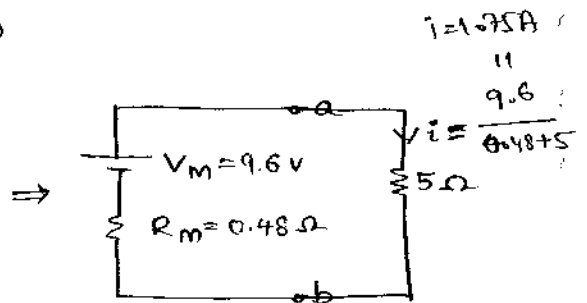
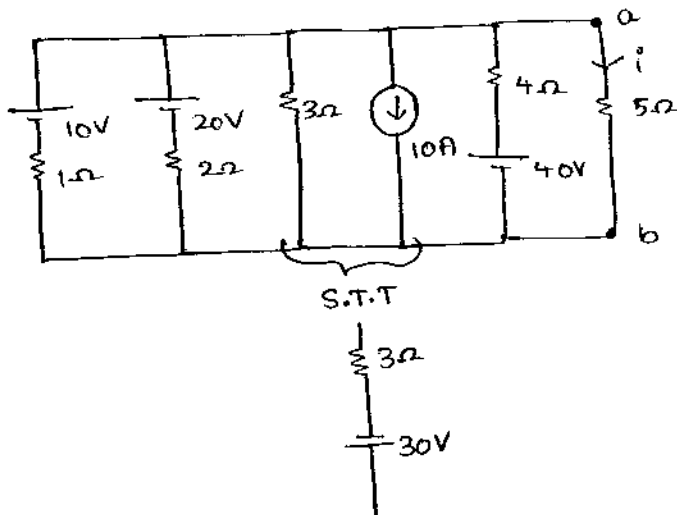
Dual of Millimans.



$$I_M = \frac{\sum_{i=1}^n I_i R_i}{\sum_{i=1}^n R_i}$$

$$R'_M = \sum_{i=1}^n R_i$$

Q determine Current 'i' by using milliman's Theorem

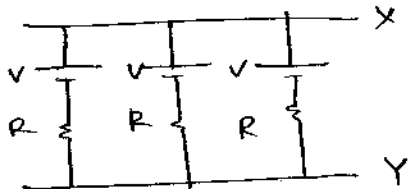


$$V_M = \frac{\frac{+10}{1} + \frac{+20}{2} - \frac{30}{3} + \frac{40}{4}}{\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}} = 9.6V$$

$$R_M = \frac{1}{\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}} = 0.48\Omega$$

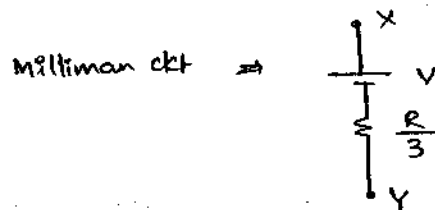
$$i = \frac{9.6}{5 + 0.48} = 1.75A$$

1ES(10)

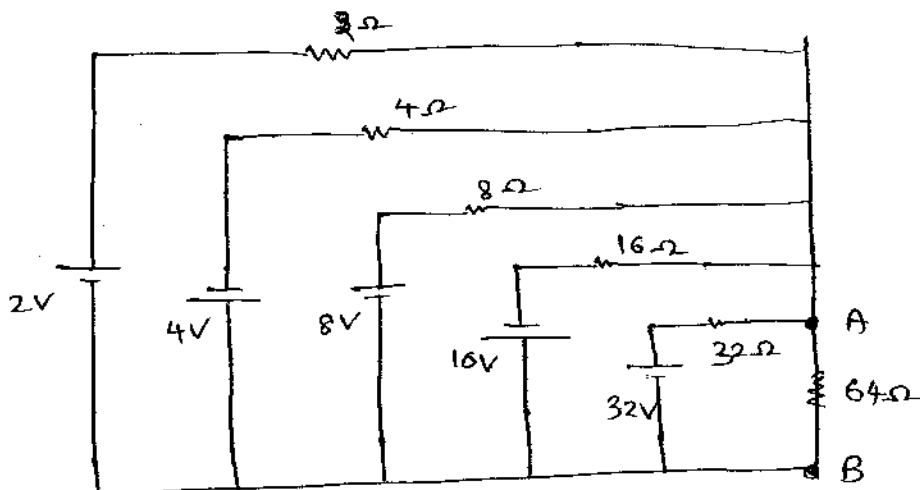


$$V_M = \frac{\frac{V}{R} + \frac{V}{R} + \frac{V}{R}}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R}} = \frac{\frac{3V}{R}}{\frac{3}{R}} = V$$

$$R_M = \frac{1}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R}} = \frac{1}{\frac{3}{R}} = \frac{R}{3}$$



1ES(10) Find $V_{AB} =$ _____

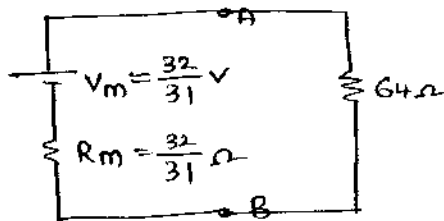


$$V_m = \frac{\frac{2}{2} - \frac{4}{4} + \frac{8}{8} - \frac{16}{16} + \frac{32}{32}}{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}} = \frac{32}{31} V$$

77

$$R_m = \frac{1}{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}} = \frac{32}{31} \Omega$$

Milliman Eq.



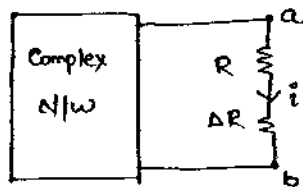
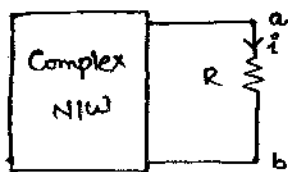
$$V_{AB} = \frac{32}{31} \left[\frac{64}{64 + \frac{32}{31}} \right]$$

$$V_{AB} = \frac{64}{63} V$$

IX Compensation Theorem:

This Theorem allows to determine the correct value of current in any Branch of the N/w directly in one step when N/w is subjected to any parametric change.

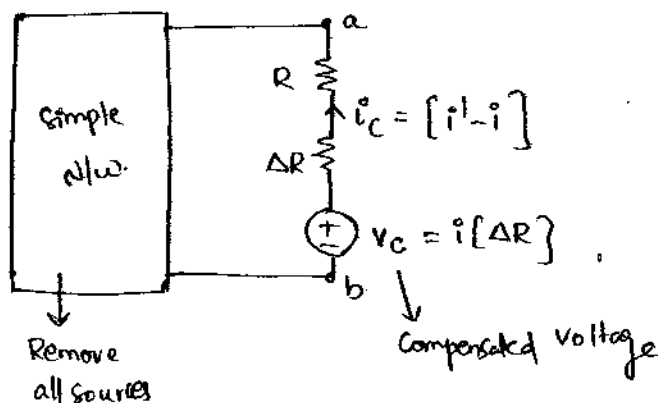
This Theorem is Exclusively used in electrical measurements to determine the steady state error introduced by the meters by writing the Compensated N/w of the Given N/w.



as ΔR is extra then i' is very low compared to i

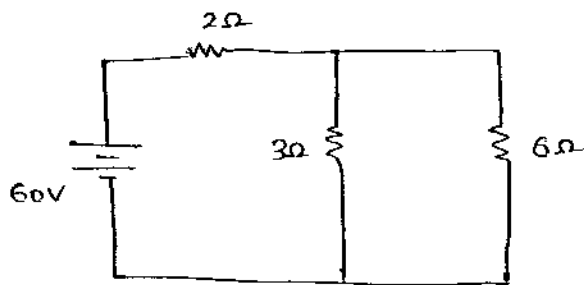
$$\therefore i_c = [i' - i] = -ve$$

Compensated N/w:



$\therefore i_c$ direction is reversed

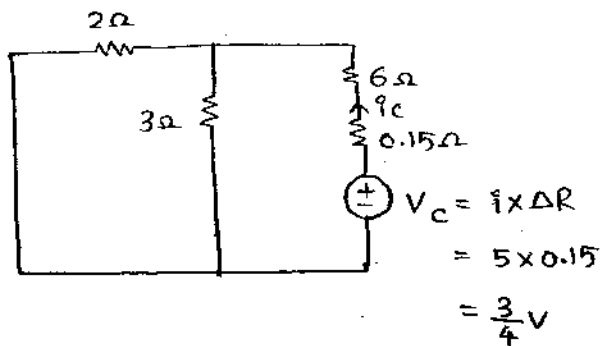
determine the ^{compensation} change in Current in 6Ω Resistance Branch when an ammeter with Internal Resistance of 0.15Ω is used to measure Current through it. & also determine the steady state error Introduced by the meter.



original Current in 6Ω :

$$i = \frac{60}{2 + [3//6]} * \frac{3}{9} = \frac{20}{4} = 5A$$

Compensated N/w:



$$i_c = \frac{3/4}{6.15 + [2//3]} = \underline{\underline{0.102A}}$$

By Connecting a practical Ammeter With small Internal Resistance in Series to 6Ω Branch the Current in that Branch is reduced by $\underline{\underline{0.102A}}$

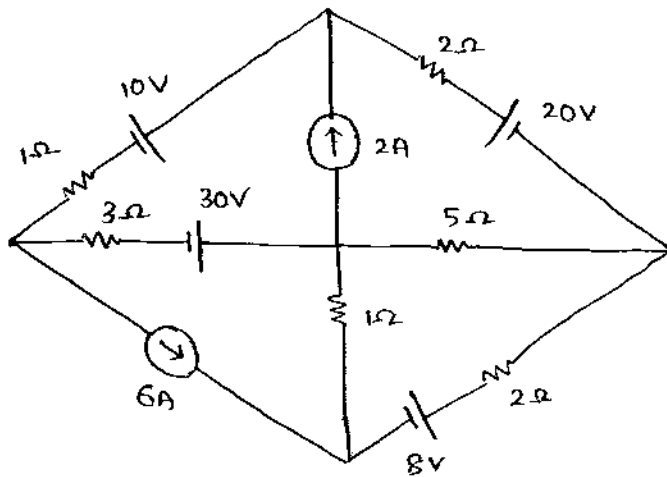
$$\% \text{ Error} = \frac{i_c}{i} \times 100\%$$

$$= \frac{0.102}{5} \times 100\%$$

$$= 2.04\%$$

→ acceptable error.

H.W

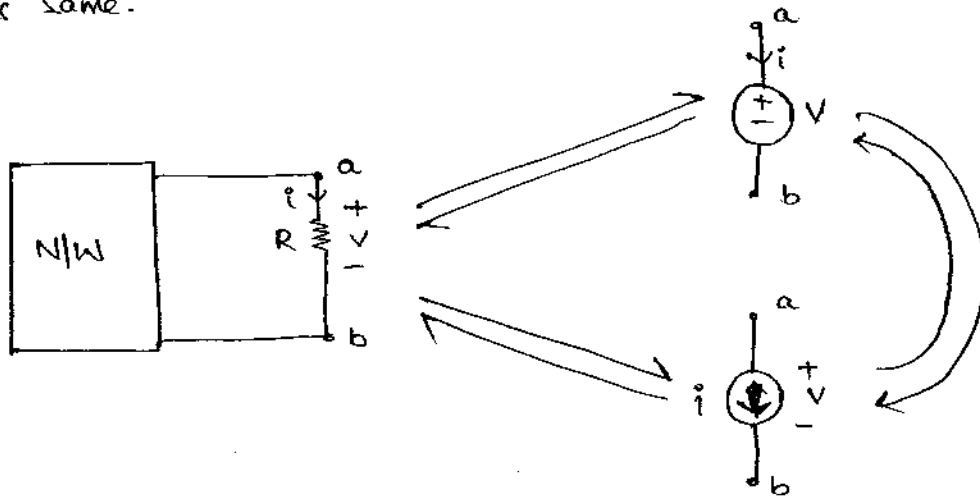


Determine the change in Current in 5Ω Resistance Branch when an additional 1Ω Resistance is Connected in Series to it.

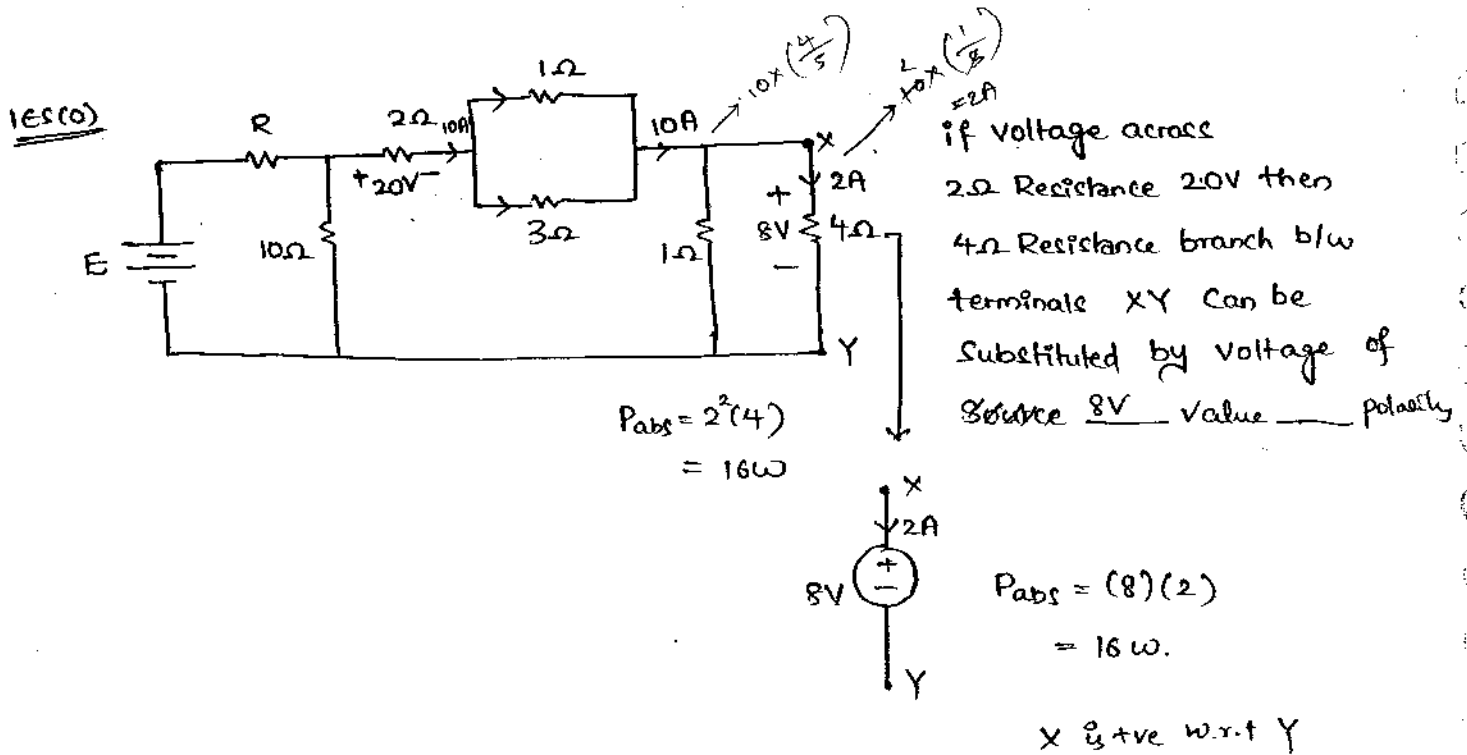
78

Substitution:

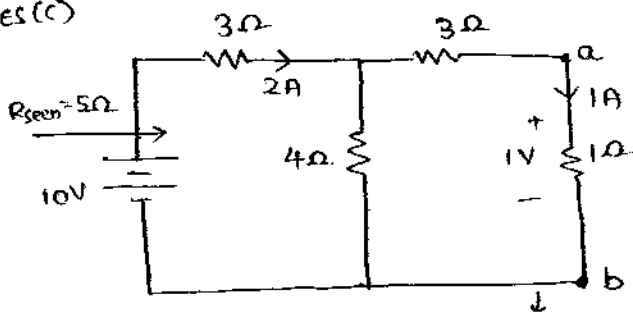
In any linear active Bilateral N/w consisting of ^{no. of} Energy Sources & Resistances etc. any passive element can be substituted in terms of its Equivalent Voltage or Current & Vice Versa. for further analysis & Reduction in the N/w without disturbing the Rest of the N/w provided the power absorbed by the passive element & its Equivalent substituted voltage or current Remains Same.



$$P_{abs} = i^2 R = \frac{V^2}{R} = V \cdot i \text{ W}$$



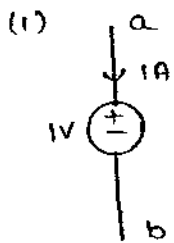
ies (c)



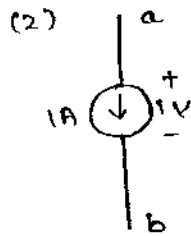
$$P_{abs} = I^2(1) = 1W$$

use substitution theorem to represent
1Ω Resistance branch b/w a b atleast
in 5 different ways.

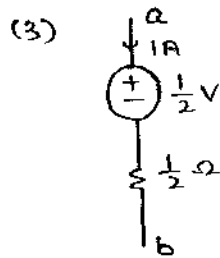
79



$$P_{abs} = 1W$$

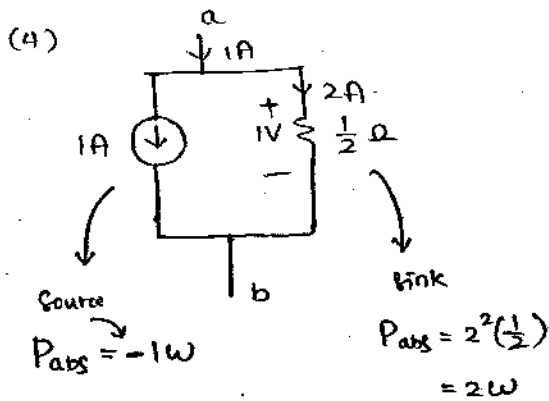


$$P_{abs} = 1W$$

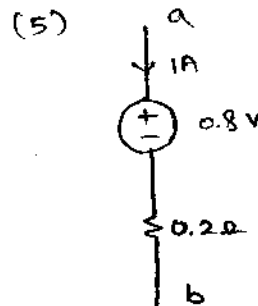


$$P_{abs} = \frac{1}{2}(1) + (1)^2 \frac{1}{2} = 1W$$

(4) - 1



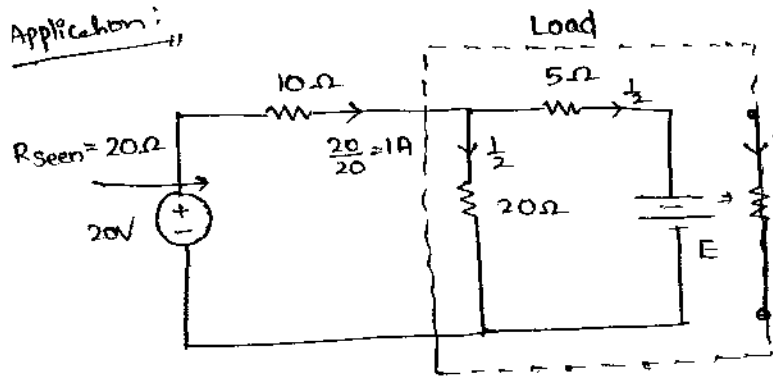
$$P_{Total abs} = (-1 + 2) = 1W$$



$$P_{abs} = 0.8(1) + 1^2(0.2) = 1W$$

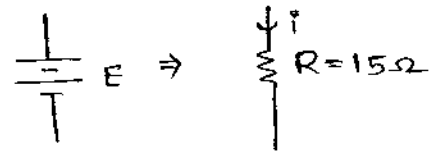
... ∞ ways.

Application:



use Substitution theorem to determine the value of Battery voltage E for which max power is Transferred to entire load

Let $E = iR$ → sink



for P_{max}

Load $R =$ Source R

$[20 || (5+R)] = 10$

$\frac{20(5+R)}{25+R} = 10$

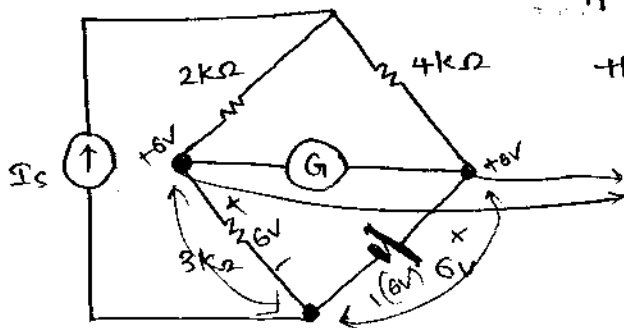
$R = 15\Omega$

for P_{max}

$E = iR$

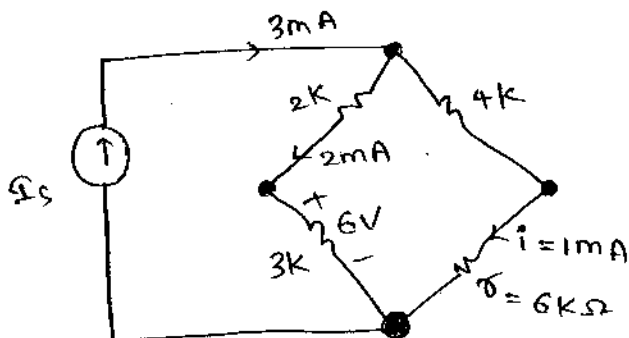
$E = \frac{1}{2}(15) = 7.5 \text{ Volts.}$

1ES(6)



if Reading of Galvanometer is Zero. find the value of Source Current I_s .

when Bridge is Balance
P.D. across terminals is Zero
∴ Voltage across these terminals w.r.t any of the ref. terminals is same.



let $V = iR$

∴ Bridge Balanced

$(2k)(7) = (3k)(4k)$

$7 = 6k\Omega$

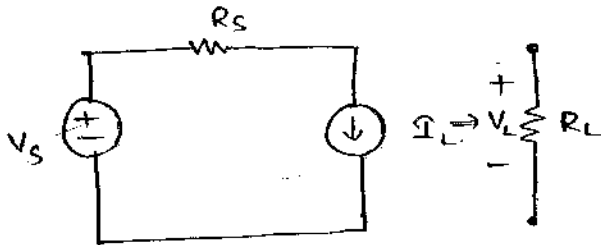
$i = \frac{6}{6k} = 1mA$

If V_s is Const. Source Voltage & I_L is Const load current then P_{max} occurs in the load

If I_L is _____

(a) $\frac{V_s}{R}$ (b) $\frac{V_s}{2R}$ (c) $\frac{V_s}{4R}$ (d) $\frac{2V_s}{R}$

$R_L =$
Example of const current load.
Const. Torque DC motor
draws const. current



let $I_L = \frac{V_L}{R_L}$

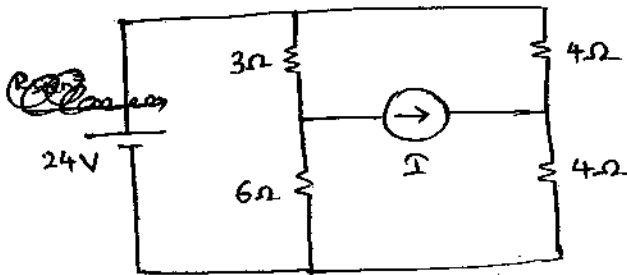
at P_{max}

$R_L = R_s$

$V_L = \frac{V_s}{2}$

$I_L = \frac{V_L}{R_L} = \frac{V_s}{2R_s}$

Q. for what value of current i max power absorbed



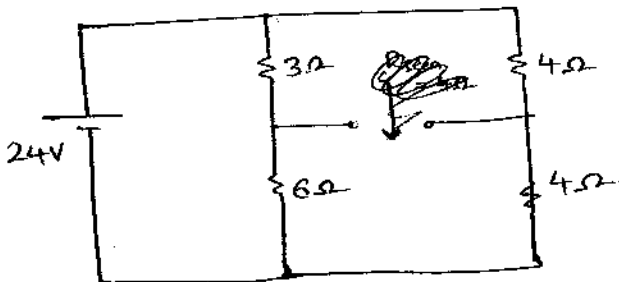
$R_L = [3 \parallel 6] + [4 \parallel 4] = 4\Omega$

$V_L = 12V$

$I = \frac{V_L}{R_L} = \frac{12}{4} = 3A$

at P_{max} 50% appears

$V_L = 12V$



Basis of Time Variance.

Properties of Inductors & Capacitors.

Properties of Inductors :

$$\Rightarrow \therefore V = L \frac{di}{dt}$$

\Rightarrow for Ideal dc Excitation $\frac{di}{dt} = 0 \Rightarrow$ Voltage across Inductor = 0
ie, Inductor acts as S.C for Ideal DC in Steady state

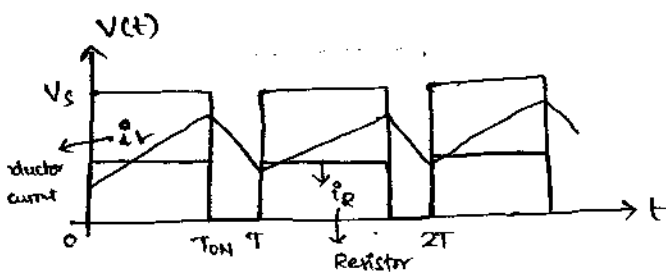
\Rightarrow Inductor will never allow Sudden change in Current through it (ie, in Zero time)

\Rightarrow But if we do allow Sudden change in Current through Inductor in a short Interval of time it develops large Impulse Voltage. Ex: Fluorescent lamp (tube light)

\Rightarrow Ideal Inductors are Coiled Wires with Zero Internal Resistance. So power loss is zero.

\Rightarrow Practical Inductors do have Some Internal Wdg Resistance & they are Represented as Coils as shown.

\Rightarrow Inductor is a Versatile freq. dependent Component & its analysis is different in different applications based on the type of Input & mode of operation.



$$V = L \frac{di}{dt}$$

$$\int di = \frac{V}{L} \int dt$$

$$i = \frac{V}{L} t$$

$$0 < t < T_{on}$$

$$V \rightarrow V_s \text{ (pulse)}$$

$$i = \left[\frac{V_s}{L} \right] t$$

$$y = mx$$

Current is Ramp fn.

choke in tube light is impulse voltage generator to ignite gas

High freq \rightarrow compact in size
High power density
power density handling is high
Inductor doesn't allow sudden change in current
ie Inductor is large

Inertia mag domain element for current

$$\infty \uparrow V = L \frac{di}{dt} \rightarrow 0$$

It is violation of law of Conservation of Energy

Properties of Capacitors:

81

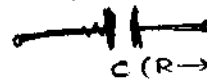
$$\Rightarrow i = C \frac{dv}{dt}$$

\Rightarrow for ideal DC Excitation $\frac{dv}{dt} = 0 \Rightarrow$ Current through cap = 0
 \Rightarrow Cap. acts as O.C for Ideal DC in steady state.

\Rightarrow Cap. will never allow sudden change in Voltage ~~time~~ across it in zero time

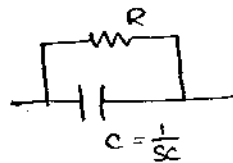
\Rightarrow if we do allow sudden change in Voltage across cap in short interval of time they drive large Impulse Currents.

\Rightarrow Ideal Capacitors are Considered to have Infinite dielectric Resistance so, no leakage Currents so, P_{losses} are zero.

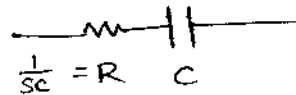


$$C (R \rightarrow \infty)$$

\Rightarrow Practical Capacitors do have large dielectric Resistance which (incurs) undergo losses & they are modelled as



Low freq Model
 $(f \rightarrow 0)$



High freq Model
 $(f \rightarrow \infty)$

used in modelling of the losses at high freq of cap

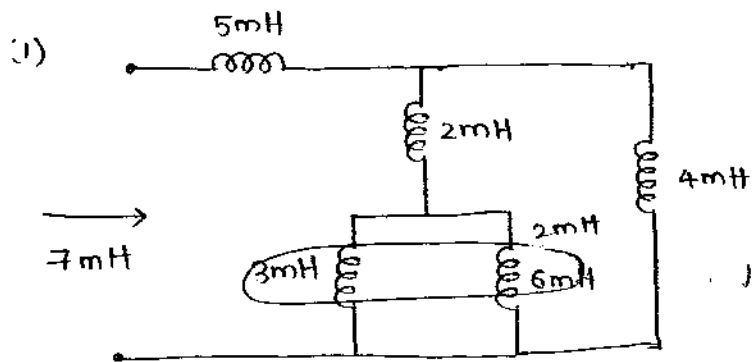
\Rightarrow Capacitor is a Versatile freq dependent Component & its analysis is different in different applications based on type of i/p & mode of operation

\Rightarrow $i = C \frac{dv}{dt} = 0$ \rightarrow violation of law of Conservation of charge.

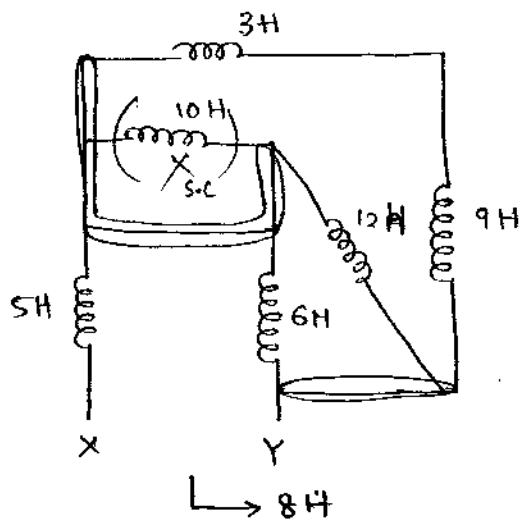
capacitors
 Inertia element in
 electrostatic domain

camera flash
 is impulse current

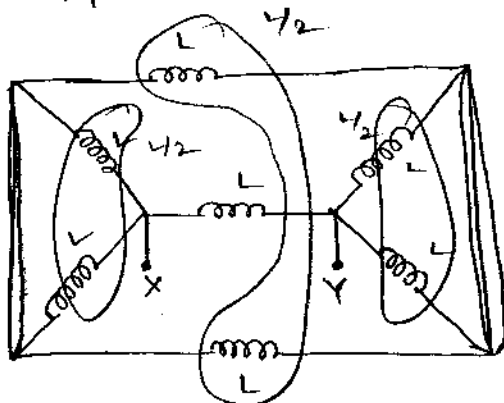
Reduction Techniques:



* $L_{xy} = \underline{\hspace{2cm}}$

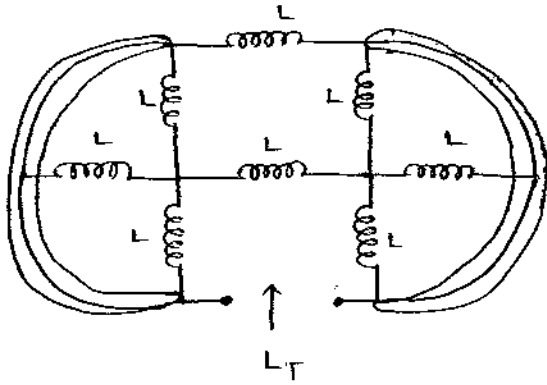


*
IES $L_{xy} = \underline{\hspace{2cm}}$



$$L_{xy} = L \parallel \frac{3L}{2} = \frac{3}{5} L \text{ H.}$$

IES



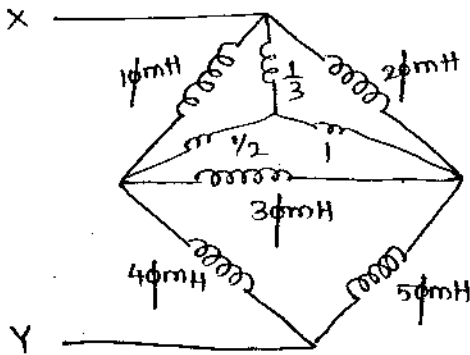
$$L_T = \left(\frac{5L}{3} \right) \parallel L$$

$$= \frac{5L}{8} \text{ H.}$$

82

Gate

$$L_{XY} = \underline{\hspace{2cm}}$$



magnitude scaling is divide by $\frac{1}{10}$ 10mH

$\Delta - Y$ Conversion.

$$L_{XY} = \left[\frac{1}{3} + \left(6 \parallel \frac{9}{2} \right) \right] \times 10 \text{ mH}$$

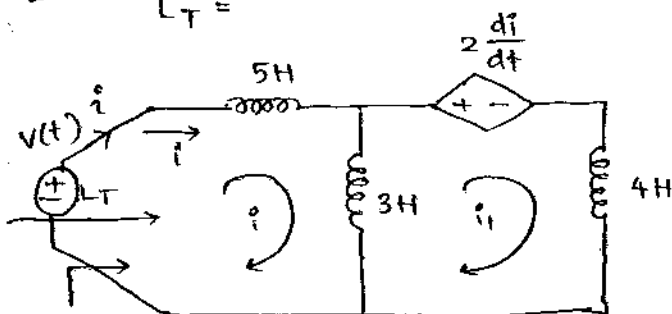
$$= 29.04 \text{ mH}$$

or 2 mag. scaling.

Ther. cat. III

IES (C)

$$L_T =$$



$$V(t) = L_T \frac{di}{dt}$$

$$-V(t) + 5 \frac{di}{dt} + 3 \left[\frac{di}{dt} - \frac{dii}{dt} \right] = 0$$

$$8 \frac{di}{dt} - 3 \frac{dii}{dt} = V(t) \rightarrow (1)$$

$$3 \left[\frac{dii}{dt} - \frac{di}{dt} \right] + 2 \frac{di}{dt} + 4 \frac{dii}{dt} = 0$$

$$\frac{di}{dt} = 7 \frac{dii}{dt} \rightarrow (2)$$

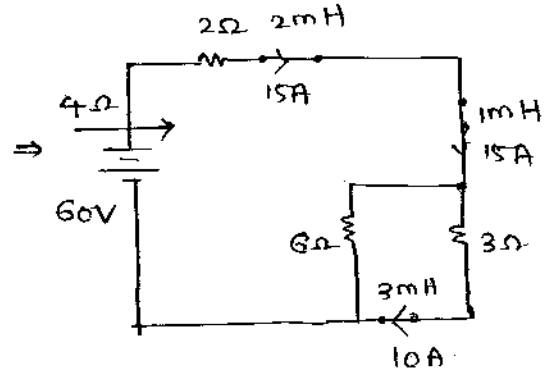
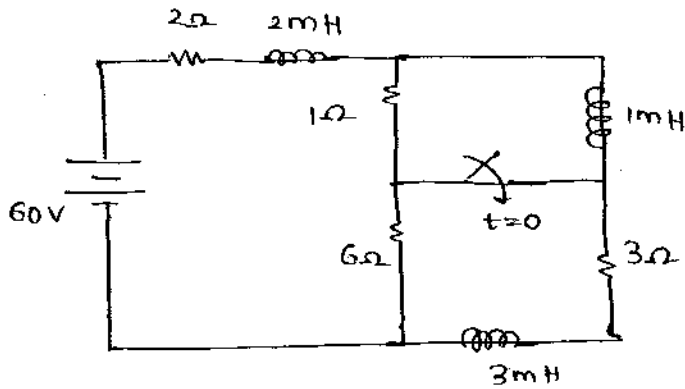
$$V(t) = \frac{di}{dt} \left[8 - 3 \left(\frac{1}{7} \right) \right]$$

$$V(t) = \left[\frac{53}{7} \right] \frac{di}{dt}$$

$$\rightarrow L_T = \underline{\underline{\frac{53}{7} \text{ H}}}$$

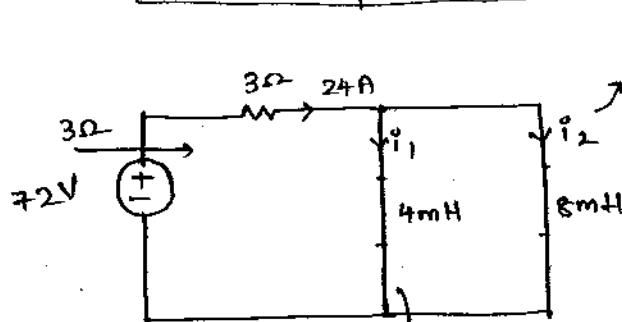
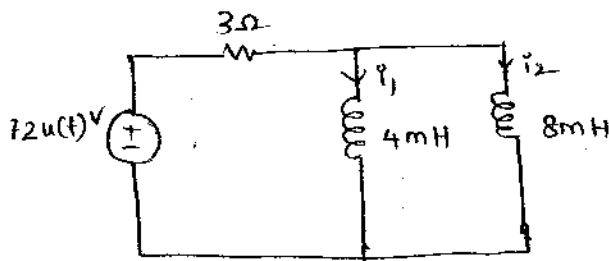
Q Find steady state currents in each inductor & energy stored in each.

↳ for dc $L \rightarrow S.C$



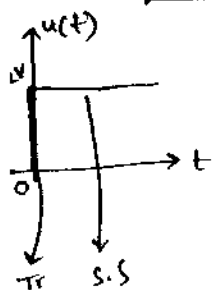
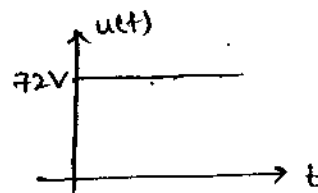
$$\left. \begin{aligned} E_{2m} &= \frac{1}{2} (2m) (15)^2 = 225 \text{ mJ} \\ E_{1m} &= \frac{1}{2} (1m) (15)^2 = 112.5 \text{ mJ} \\ E_{3m} &= \frac{1}{2} (3m) (10)^2 = 150 \text{ mJ} \end{aligned} \right\} \begin{array}{l} \text{Electromag form} \\ \text{E-m Form} \\ \downarrow \\ \text{DC flux} \end{array}$$

IESCO Find currents i_1 & i_2



$$24 \left(\frac{4}{8+4} \right) = 8A$$

$$24 \left(\frac{4}{8+4} \right) = 8A$$



$$i_1 = 24 \left(\frac{8m}{8+4m} \right) = 16A$$

$t=0^+ \rightarrow$ Transient state

$$e_1 = e_2 \quad [\because \text{parallel}]$$

$$\psi_1 = \psi_2 \quad [\because \psi = Li]$$

$$L_1 i_1 = L_2 i_2$$

$$4m(i_1) = 8m(i_2)$$

$$i_1 = 2i_2 \rightarrow \textcircled{1}$$

$$\text{as } t \rightarrow \infty$$

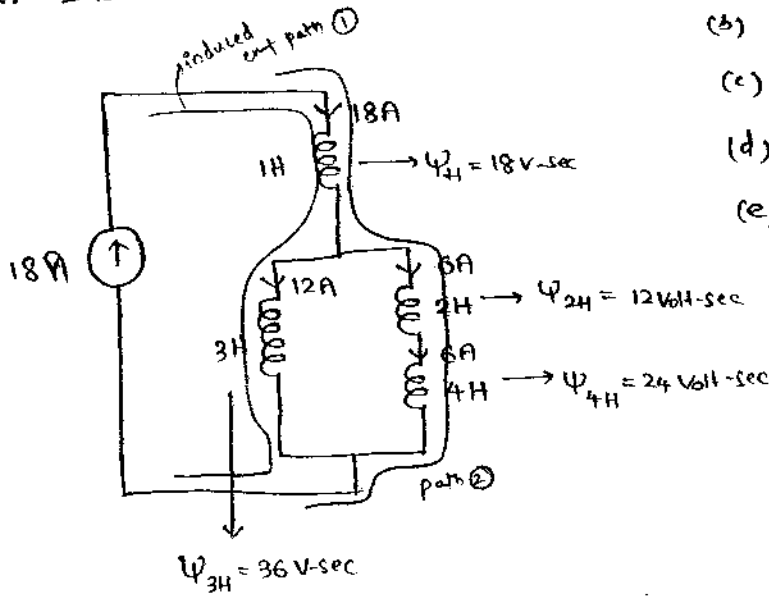
$$i_1 + i_2 = 24 \rightarrow \textcircled{2}$$

By law of Conserv. charge

$$\textcircled{1} \text{ \& \& } \textcircled{2} \text{ solving } i_1 = 16A, i_2 = 8A$$

for Inductive ckt shown determine

- c.s currents in each Inductor
- flux linkage in each Inductor
- Energy stored in Inductor
- Verify law of Cons.v. of Energy
- Verify law of Cons. of flux



83

$$\psi = \Phi \cdot N \quad \text{v.s.c}$$

$$\begin{aligned} \Phi &= L \cdot i \\ &= \frac{V \cdot dt}{di} \times i \\ &= \frac{V \cdot \text{sec}}{\text{amp}} \times \text{amp} \\ \psi &= V \cdot \text{sec} \end{aligned}$$

$$\begin{aligned} \textcircled{c} \quad E_{1H} &= \frac{1}{2} L i^2 = \frac{1}{2} (1) (18)^2 = 162 \text{ J} \\ E_{2H} &= \frac{1}{2} \times 2 \times 6^2 = 36 \text{ J} \\ E_{3H} &= \frac{1}{2} (3) (12)^2 = 216 \text{ J} \\ E_{4H} &= \frac{1}{2} (4) (6)^2 = 72 \text{ J} \end{aligned}$$

E-M form
↓
DC flux

When Inductors are parallel
current will divide &
flux will be same.
When Ind. in series
currents same
flux divid..

$$(d) \quad E_{\text{deli}} = E_{\text{stored}} \quad (\text{in 'L' } E_{\text{stored}} = E_{\text{absor}})$$

$$\begin{aligned} \text{LHS} \quad E_{\text{deli}} &= \frac{1}{2} L_T i_T^2 = \frac{1}{2} [3] \cdot [18]^2 = 486 \text{ J} \\ \text{RHS} \quad E_{\text{stored}} &= [162 + 36 + 216 + 72] = 486 \text{ J} \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{LHS} \\ \text{RHS} \end{aligned}} \right\} \text{law of Cons.v. Energy.}$$

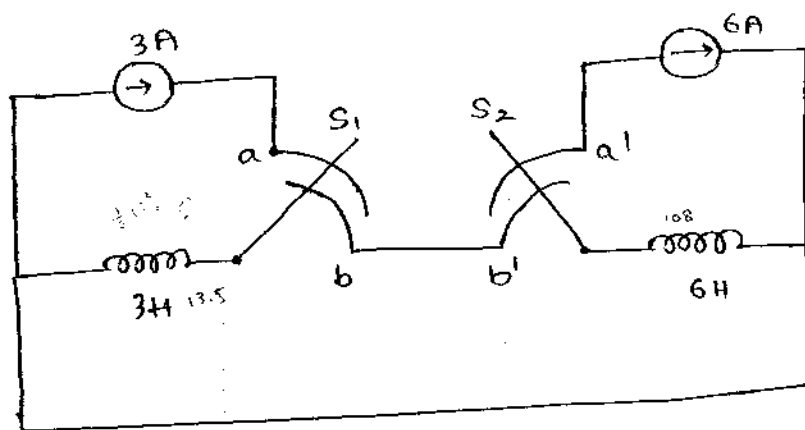
(e) L.C. of flux

$$\psi_{\text{established}} = \psi_{\text{retained}}$$

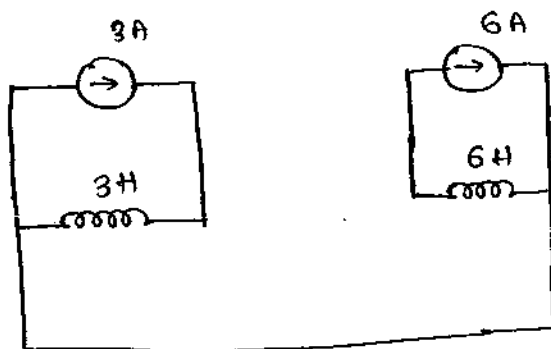
$$\text{LHS} \quad \psi_{\text{est}} = L_T \cdot i_T = [3] [18] = 54 \text{ volt-sec}$$

$$\begin{aligned} \text{RHS} \quad \psi_{\text{retained}} &= \text{path 1 (KVL)} \\ &= \psi_{1H} + \psi_{3H} = 18 + 36 = 54 \text{ Volt-sec} \\ &= \text{path 2 (KVL)} \\ &= \psi_{1H} + \psi_{2H} + \psi_{4H} = [18 + 12 + 24] = 54 \text{ volt-sec} \end{aligned}$$

Initially switches are at position A & A' respectively then determine the steady state current, flux linkages & Energy stored in each Inductor. Now if the switches are simultaneously moved to positions B & B' respectively & Remained there forever then determine their s.s current, flux linkages, & Energy stored in each Inductor. Verify L.C. of flux, L.C. of Energy before & after the change. assume the switches are Ideal & there is no loss in Energy during the Transient as they are made before break Contacts.



Before



$$i_1 = 3A$$

$$\psi = 9 \text{ volt-sec}$$

$$E_1 = \frac{1}{2} (3) (3)^2 = 13.5 \text{ J}$$

$$i_2 = 6A$$

$$\psi_2 = 36 \text{ volt-sec}$$

$$E_2 = \frac{1}{2} (6) (6)^2 = 108 \text{ J}$$

$$\psi_T = \psi_1 + \psi_2 \text{ (independent loops)}$$

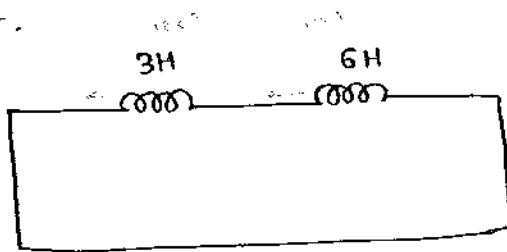
$$= (9 + 36)$$

$$= 45 \text{ volt-sec} \rightarrow \textcircled{1}$$

$$E_T = E_1 + E_2 = (13.5 + 108) \text{ J}$$

$$= 121.5 \text{ J} \rightarrow \textcircled{2}$$

After.



84

\therefore in series $i_1 = i_2$ [$\psi = Li$]

$$\frac{\psi_1}{L_1} = \frac{\psi_2}{L_2} \Rightarrow \frac{\psi_1}{3} = \frac{\psi_2}{6}$$

$$\psi_2 = 2\psi_1 \longrightarrow \textcircled{3}$$

Respecting L.C. of flux only

$$\psi_T(\text{Before}) = \psi_T(\text{after})$$

So solving ① & ③

$$\psi + \psi_2 = 45$$

$$3\psi_1 = 45$$

$$\psi_1 = 15 \text{ volt-sec}$$

$$\psi_2 = 30 \text{ volt-sec.}$$

Then $i_1 = i_2 \Rightarrow \frac{15}{3} = \frac{30}{6} = 5A \longrightarrow i_1 = i_2$

$$E_1 = \frac{1}{2}(3)(5)^2 = 37.5 \text{ J}$$

$$E_2 = \frac{1}{2}(6)(5)^2 = 75 \text{ J.}$$

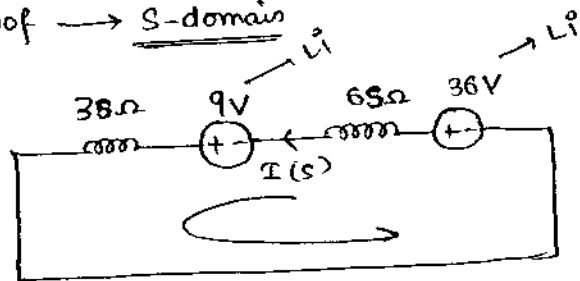
Now $\psi_T = [15 + 30] = 45 \text{ volt-sec} \longrightarrow \textcircled{4}$

$$E_T = [37.5 + 75] \text{ J} = 112.5 \text{ J} \longrightarrow \textcircled{5}$$

Here L.C. of flux satisfied

But L.C. of Energy has to be justified.

proof \rightarrow S-domain



KCL

$$-36 + I(s)6S - 9 + I(s)38 = 0$$

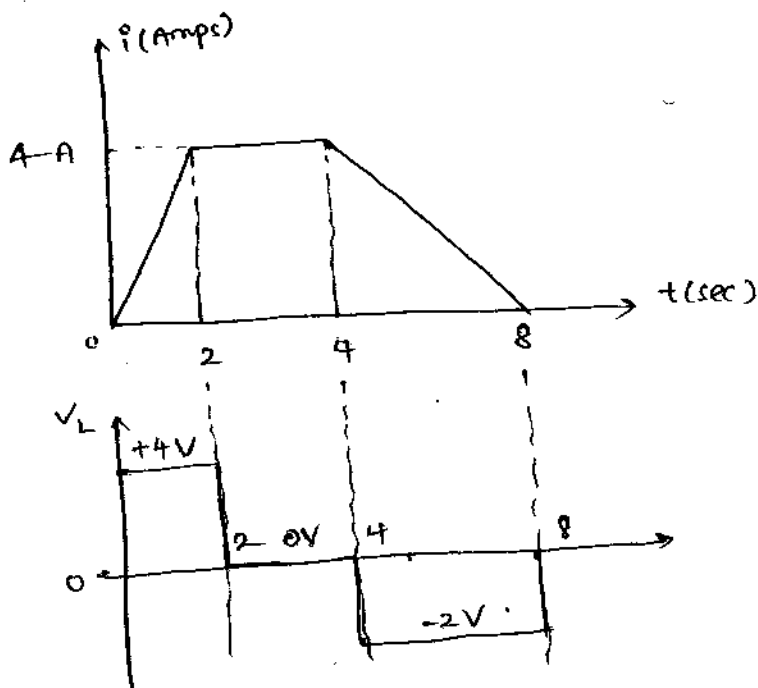
$$I(s)[9S] = 45$$

$$I(s) = \frac{45}{9S} = \frac{5}{S}$$

$$i(t) = L^{-1}[I(s)] = 5A \quad \rightarrow i_1 = i_2$$

Here flux is ^{only} Considered as matter but Energy can be in any form. Sum of the Energy from previous state has been utilized to redistribute the flux among Inductors to maintain Constant Equal Currents.

Q Plot the Voltage across 2H Inductor if Current through it as shown below also plot Power & Energy fr. in the Inductor & verify law. of Cons. of Energy. Hence determine (a) pow. in Inductor at $t=5\text{sec}$
(b) energy stored in Inductor upto $t=5\text{sec}$.



$$V = L \frac{di}{dt}$$

$$0 < t < 2$$

$$i(t) = 2t \rightarrow V = L \frac{di}{dt}$$

$$V = 2 \cdot \frac{d}{dt}(2t)$$

$$V = +4V \text{ (pulse)}$$

$$2 < t < 4$$

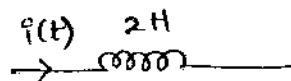
$$i(t) = 4 \rightarrow V = 2 \cdot \frac{d}{dt}(4) = 0V$$

$$4 < t < 8$$

$$4 - 0 = \frac{4 - 0}{4 - 8} (t - 8)$$

$$i(t) = (-t + 8)$$

$$V = 2 \frac{d}{dt}(-t + 8) = -2V \text{ (pulse)}$$



85

$$\frac{di}{dt} = \oplus \rightarrow \begin{matrix} 4A \\ \text{---} \\ 0A \end{matrix} + 4V - \left. \vphantom{\frac{di}{dt}} \right\} E_{\text{stored}}$$

$$\frac{di}{dt} = 0 \quad \begin{matrix} 4A \\ \text{---} \\ 0A \end{matrix} + 0V - \left. \vphantom{\frac{di}{dt}} \right\} E_{\text{retained}} = \frac{1}{2}(2)(4)^2 = 16J$$

$$\frac{di}{dt} = \ominus \quad \begin{matrix} 1A \\ \text{---} \\ 0A \end{matrix} - 2V + \left. \vphantom{\frac{di}{dt}} \right\} E_{\text{released}} \quad (+(-2)-)$$

$$P_L(t) = V_L(t) \cdot i_L(t) \text{ W}$$

$$0 < t < 2$$

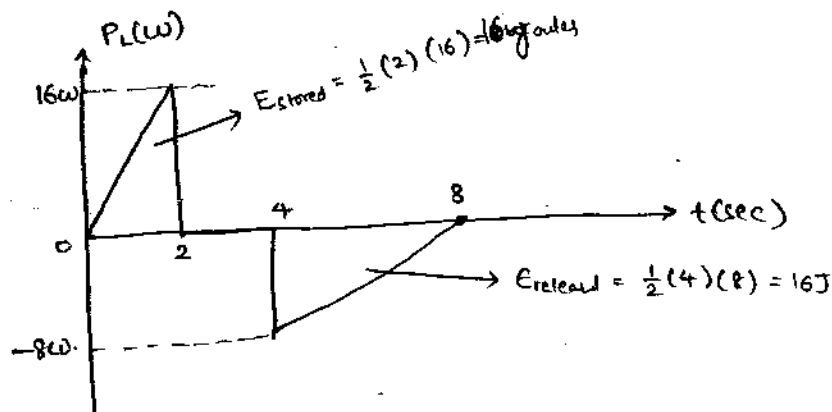
$$P_L(t) = 8t \text{ W}$$

$$2 < t < 4$$

$$P_L(t) = 0 \text{ W}$$

$$4 < t < 8$$

$$P_L(t) = (+2t - 16) \text{ W}$$



$$E_L(t) = \int_{-\infty}^t P_L(t) dt = \int_{-\infty}^0 P_L(t) dt + \int_0^t P_L(t) dt \rightarrow \left[E_L(t) = E(0) + \int_0^t P_L(t) dt \right] J$$

$$0 < t < 2$$

$$0 < t < 2$$

$$E_L(t) = E(0) + \int_0^t 8t \, dt$$

$$= 0 + 4t^2 \Big|_0^2$$

$$t=0 \longrightarrow E_L = 0 \text{ J}$$

$$t=2 \longrightarrow E_L = 16 \text{ J}$$

parabolic

$$2 < t < 4$$

$$E_L(t) = E(0) + \int_2^t 0 \, dt$$

$$= 16 + 0$$

$$= 16 \text{ J}$$

E remain at 16 J

$$4 < t < 8$$

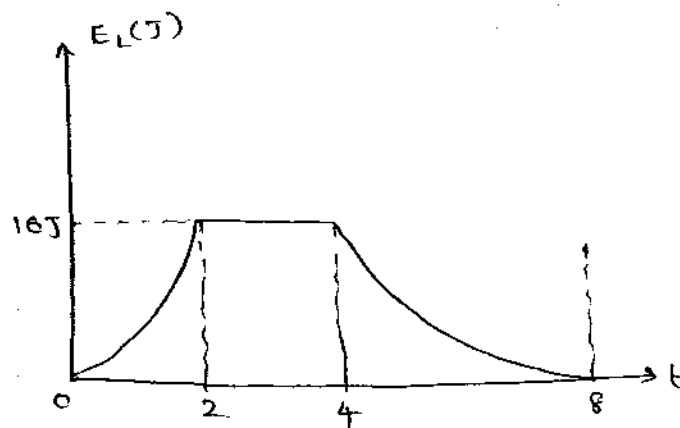
$$E_L(t) = E(0) + \int_4^t (2t-16) \, dt$$

$$= 16 + t^2 \Big|_4^8 - 16t \Big|_4^8$$

$$t=4 \longrightarrow E_L = 16 \text{ J}$$

$$t=8 \longrightarrow E_L = 0 \text{ J}$$

parabolic



power is instantaneous
we use at that instant

$$(a) P_L(t=5) = 2(5) = 10$$

$$= -6 \text{ W}$$

-ve power indicates at that instant of 6th second Energy is released by the Inductor Back to the Supply.

$$(b) E_L(4 \text{ to } 5 \text{ sec})$$

$$E_2(t) = E(0) + \int_4^5 (2t-16) \, dt$$

$$= 16 + t^2 \Big|_4^5 - 16t \Big|_4^5$$

$$= 16 + [25 - 16] - 16$$

$$= 9 \text{ J}$$

(c) \therefore Energy stored upto 5 sec =
Energy at 5 sec.

$$4 < t < 8:$$

$$i(t) = (-t+8)$$

$$i(t=5) = (-5+8) = 3 \text{ A}$$

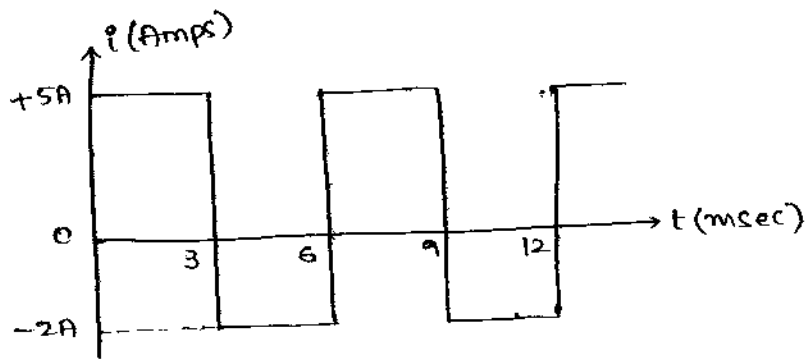
$$E_L = \frac{1}{2} \times (2) (3)^2$$

$$= 9 \text{ J}$$

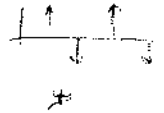
Repeat the above power to plot V , Power, Energy for the same given current function in the case of 2Ω Resistor



Q.10
plot the Voltage across 5mH Inductor if Current through it is as shown below.



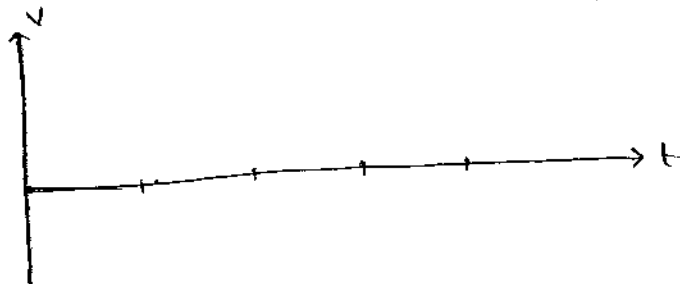
87



$$0 < t < 3 \Rightarrow i(t) = 5A$$

$$V = L \frac{di}{dt}$$

$$= 5 \frac{d(5)}{dt} = 0$$



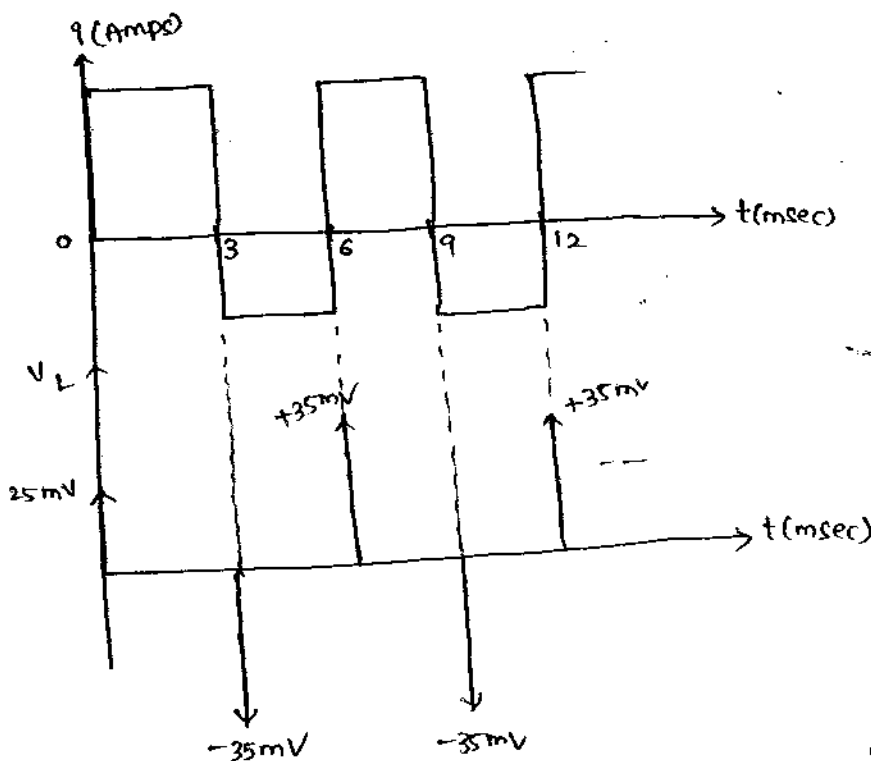
$$V = L \frac{di}{dt} \rightarrow \text{if } \frac{di}{dt} = 0 \text{ then } V = 0$$

$$\rightarrow \text{if } dt \rightarrow 0 \text{ then } V = \text{Impulse}$$

$$V = L \left[\frac{i_{\text{final}} - i_{\text{initial}}}{\Delta t} \right]$$

↓
impulse

↑ it controls the magnitude of impulse.



$$t=0 \Rightarrow$$

$$V = 5m \left[\frac{5-0}{\Delta t} \right] = +25mV \text{ (impulse)}$$

$$t=3m$$

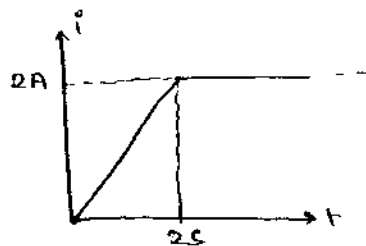
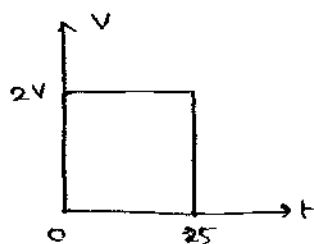
$$V = 5m \left[\frac{-2-(+5)}{\Delta t} \right] = -35V \text{ (impulse)}$$

$$t=6m$$

$$V = 5m \left[\frac{5-(-2)}{\Delta t} \right] = +35mV \text{ (impulse)}$$

IES (0)

If voltage-current plot is a passive Component is shown below, then Component is



(a) $L = 1H$

(b) $L = 2H$

(c) $C = \frac{1}{2}F$

(d) $C = 2F$

$V = iR$ ✗

↓ ↓

V & i wff should be same

$V = \frac{1}{C} \int i dt$ ✗

↓
parabolic

↓
Ramp

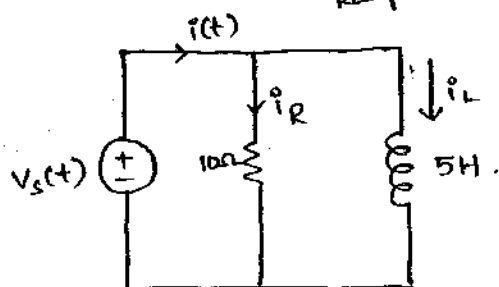
$V = L \frac{di}{dt} \rightarrow \text{Ramp} \checkmark$
↓
step.

$V = L \cdot \left[\frac{i_2 - i_1}{t_2 - t_1} \right]$

$2 = L \cdot \left[\frac{2 - 0}{2 - 0} \right]$

$L = 2H$

Gate If $V_s(t) = 40t$ V, $i_L(0) = 5A$, then find Current ' i ' at $t = 2$ sec.



$V \rightarrow$ Excitation is Ramp

for $R \rightarrow i \Rightarrow \text{Ramp}$

for $L \rightarrow i \Rightarrow \text{parabolic}$

$i(t) = i_R + i_L$

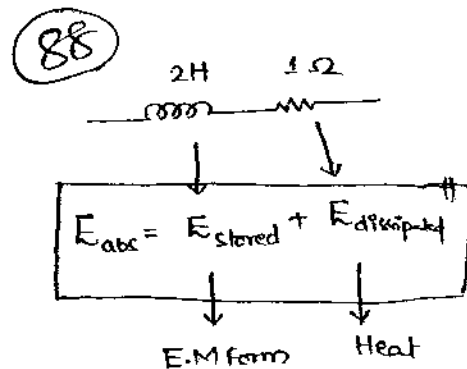
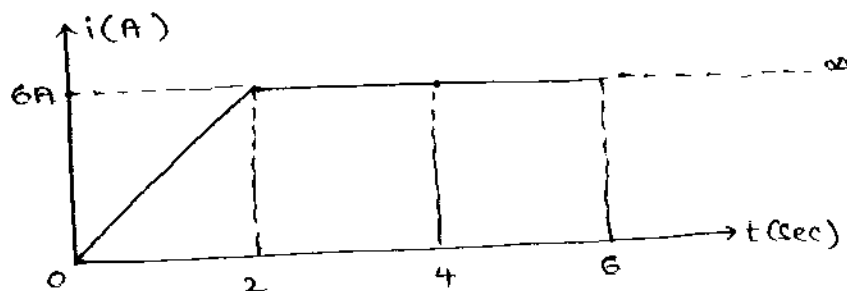
$= \frac{V_s(t)}{R} + \left\{ i_L(0) + \frac{1}{L} \int_0^t V_s(t) dt \right\}$

$i(t) = \frac{40t}{10} + 5 + \frac{1}{5} \int_0^t 40t dt$

$i(t) = (4t + 5 + 4t^2)$

$i(t=2) = 8 + 5 + 16 = \underline{29A}$

gate a practical coil of $2H$ Inductance & 1Ω Resistance is excited by Time Varying Current shown. determine the total Energy absorbed by the coil upto first 4 sec.



~~Res~~ E_{diss} $0 < t < 2 : i(t) = 3t$
 $2 < t < 4 : i(t) = 6.$

$$E_{diss} = \int P_R dt = \int_0^4 [i(t)]^2 \cdot R dt = \int_0^2 \underbrace{(3t)^2 (1)}_{24J} dt + \int_2^4 \underbrace{(6)^2 (1)}_{42J} dt = 96J$$

$$E_{stored} = \int P_L dt = \int_0^4 L i(t) \cdot \frac{di}{dt} dt = \int_0^2 \underbrace{2(3t) \cdot \frac{d}{dt}(3t)}_{36J} dt + \int_2^4 \underbrace{2(6) \cdot \frac{d(6)}{dt}}_{0J} dt = 36J$$

total $E_{abs} = 96 + 36 = 132J$

Note: In the above problem Resistor will Convert & dissipate Energy in the form of Heat External to the ckt Where ckt Cannot Recover it Back. So Resistor is Converting & lossy Component of Energy.

However Inductor is a state element (memory ele) & inductor current is its state Variable as this Component Response when there is change in current through it which actually happened from 0 to 2sec as its stored Energy. later it will Retain this Energy as a memory element as long as the Excitation is maintained Constant. Hence we can say Energy stored in inductor upto 4 secs is a Energy at 4th sec provided we know state variable i.e., current at 4 sec. which is $E_L(t=4) = \frac{1}{2}(2)(\underline{6})^2 = 36J$ at $t=4$ sec.

⇒ Similarly for Capacitor its Voltage is its state variable
 → Energy stored in Inductor for one sine cycle is 0J

Q in the above problem determine

- (1) Energy dissipated by the coil 4 sec to ∞
- (2) Energy stored by coil upto $t = \infty$
- (3) total Energy absorbed by the coil 4th sec to ∞
- (4) Change in stored Energy in coil from 4th sec to ∞

$$(1) E_{\text{diss.}} (t=4 \text{ to } \infty) = \int_4^{\infty} (6)^2 (1) dt = \infty \text{ J}$$

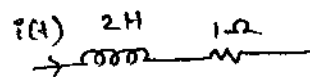
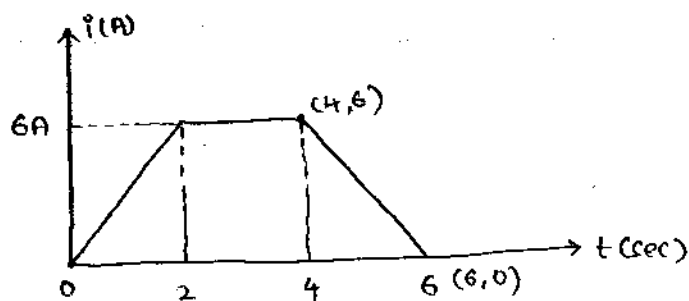
$$(2) E_{\text{stored}} = \frac{1}{2} (2)(6)^2 = 36 \text{ J}$$

↳ i, at $t = \infty$

$$(3) (\infty + 36) \text{ J} = \infty \text{ J}$$

$$(4) \underline{\underline{0 \text{ J}}}$$

Q In the above problem determine total Energy absorbed by coil if current function changed to



$$0 < t < 2 : i(t) = 3t$$

$$2 < t < 4 : i(t) = 6$$

$$4 < t < 6 : i(t) = ?$$

$$y - 0 = \frac{6 - 0}{4 - 6} (x - 6)$$

$$i(t) = (-3t + 18)$$

89

$$E_{\text{diss. (R)}} = \int_0^6 P_R dt = \int_0^6 (i(t))^2 \cdot R dt$$

$$= \underbrace{\int_0^2 (3t)^2 (1) dt}_{24 \text{ J}} + \underbrace{\int_2^4 (6)^2 (1) dt}_{72 \text{ J}} + \underbrace{\int_4^6 (-3t+18)^2 \cdot (1) dt}_{24 \text{ J}}$$

$$= \underline{120 \text{ J}}$$

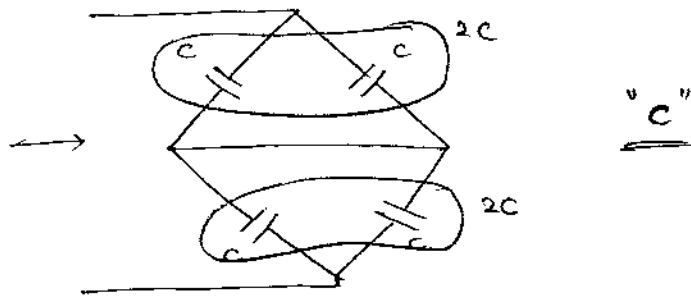
$$E_{\text{stored (L)}} = \int_0^6 P_L dt = \int_0^6 L i(t) \frac{di(t)}{dt} dt = \underbrace{\int_0^2 2(3t) \cdot \frac{d(3t)}{dt} dt}_{36 \text{ J}} + \underbrace{\int_2^4 2(6) \cdot \frac{d(0)}{dt} dt}_{0 \text{ J}} + \underbrace{\int_4^6 (2)(-3t+18) \cdot \frac{d(-3t+18)}{dt} dt}_{-36 \text{ J}}$$

$$= \underline{0 \text{ J}}$$

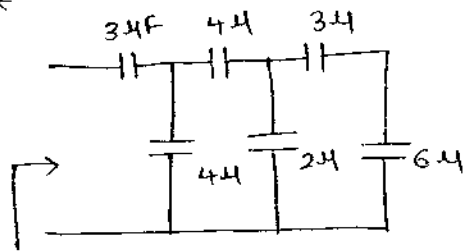
$$\text{Total } E_{\text{abs}} = (120 + 0) \text{ J}$$

$$= \underline{120 \text{ J}}$$

ies(0)



*

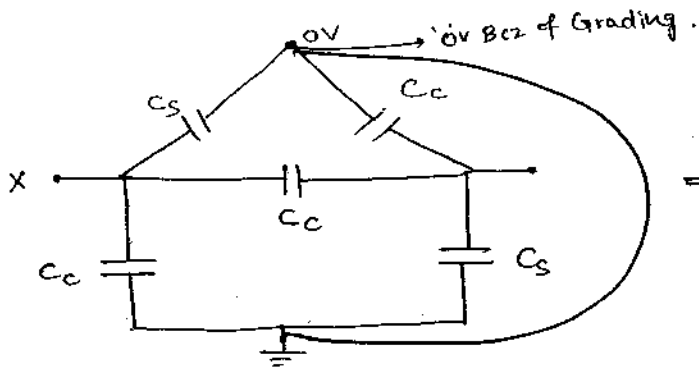


$C_T = 2\mu F$

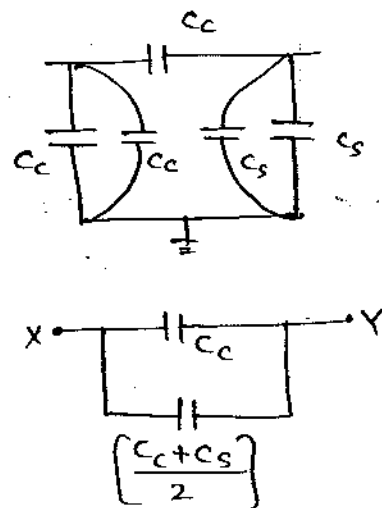
*
gate

$C_{XY} = \text{---}$

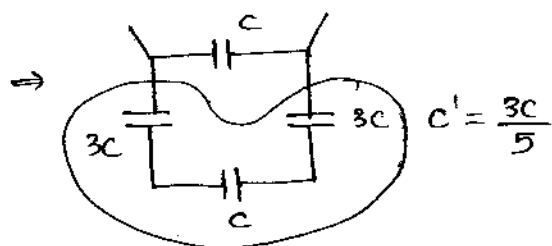
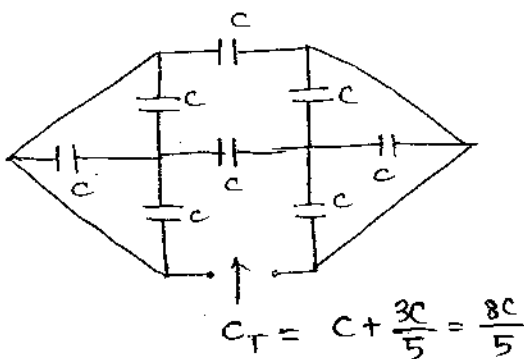
→ p.s application (cable grading)



$C_{XY} = \left[\frac{3C_c + C_s}{2} \right] F$



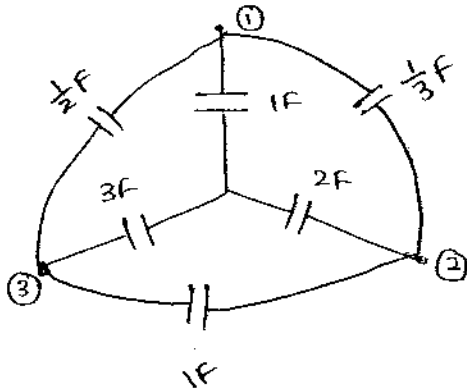
ies



$\frac{1}{C'} = \frac{1}{3C} + \frac{1}{C} + \frac{1}{3C} = \frac{5}{3C}$

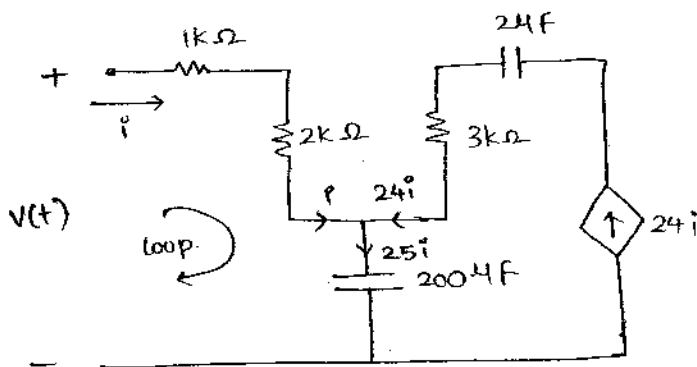
$C_T = C + C' = C + \frac{3C}{5} = \frac{8C}{5}$

IES Convert into Δ



90

Gate Determine Input loop capacitance



$$-V(t) + i[3k] + \frac{1}{200\mu} \int 25i dt = 0$$

$$V(t) = i(3000) + \frac{1}{84} \int i dt$$

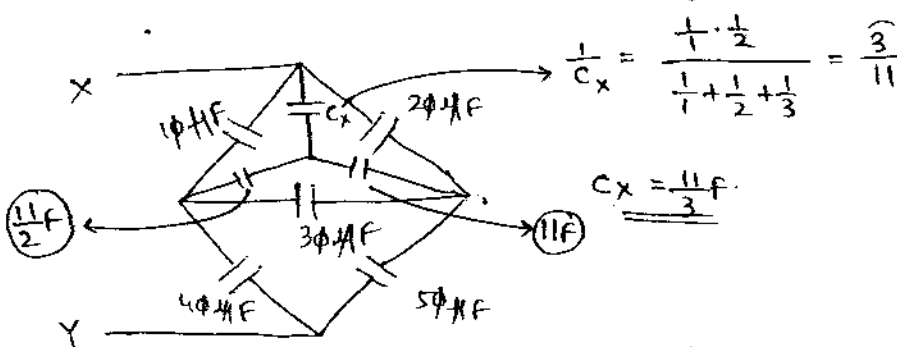
iR $\frac{1}{C} \int i dt$

$C_{loop} = 84F.$

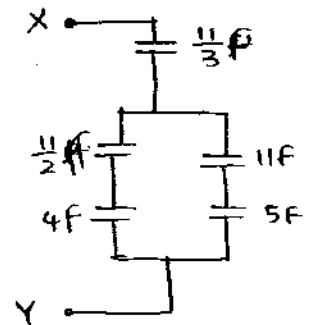
Gate

Gate $C_{xy} =$ _____

Magnitude scaling $\rightarrow 10\mu F$.



$$C_{xy} = 22.39 \mu F$$



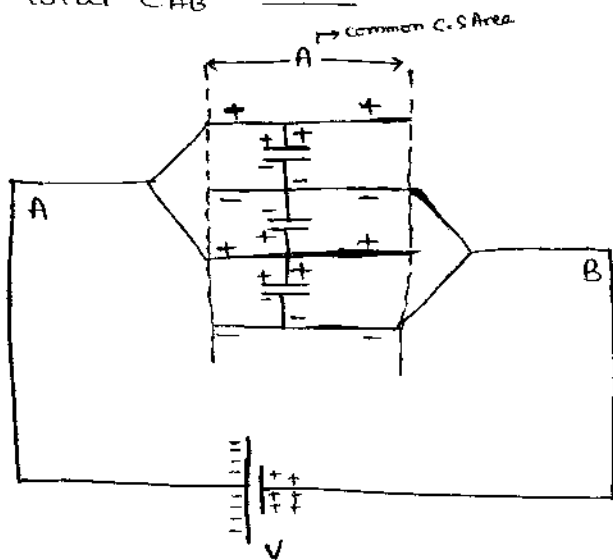
$\frac{55}{12}$

$\frac{11}{2} \times 4$

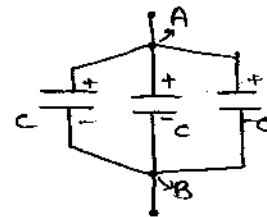
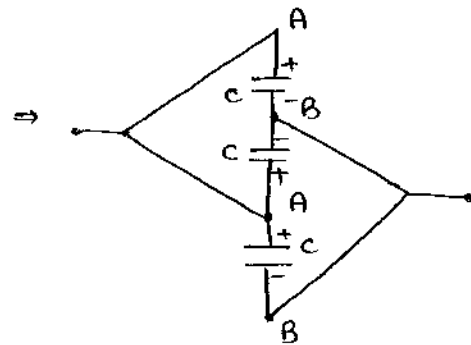
$\frac{44}{19} + \frac{55}{16}$

IES if Equivalent Capacitance b/w the opposite electrodes is 'c' farads then

total $C_{AB} =$ _____

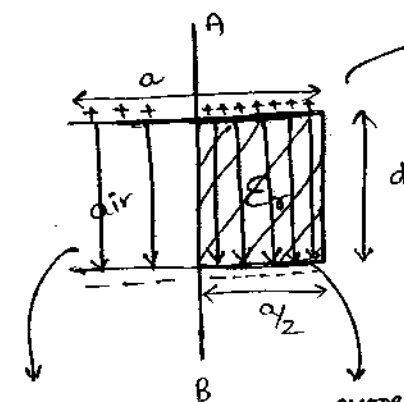


N-S closed
 + - not closed



$$C_{AB} = 3C \text{ farads.}$$

Gate $C_{AB} =$ _____



Working voltages are Same but currents different \Rightarrow parallel

$$C_{AB} = \frac{\epsilon_0(1) a}{2d} + \frac{\epsilon_0 \epsilon_r a}{2d}$$

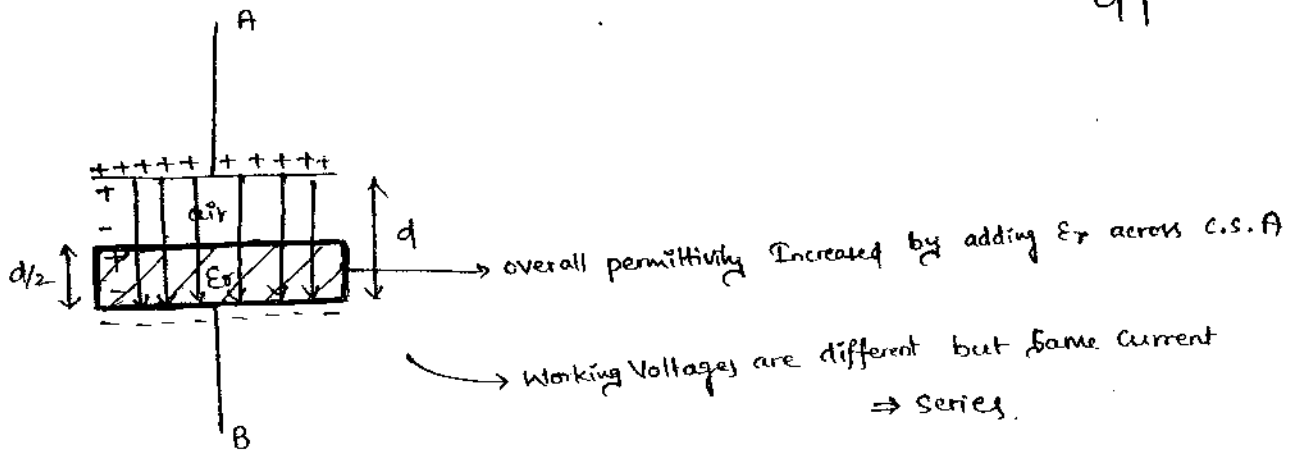
$$C_{AB} = \frac{\epsilon_0 a}{2d} (1 + \epsilon_r)$$

less current flows
 due to less
 accumulation
 of charge

more
 accumulation of charge \Rightarrow Bcz of more polarisation
 more current flows

Gate $C_{AB} = \underline{\hspace{2cm}}$

91

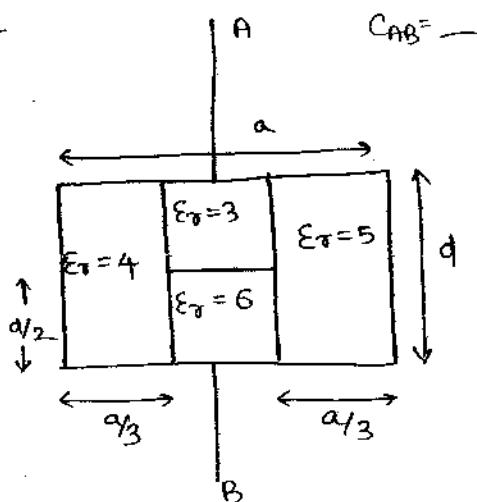


$$C_{AB} = \frac{\epsilon_0 \epsilon_r(a)}{d/2} * \frac{\epsilon_0 \epsilon_r \cdot a}{d/2}$$

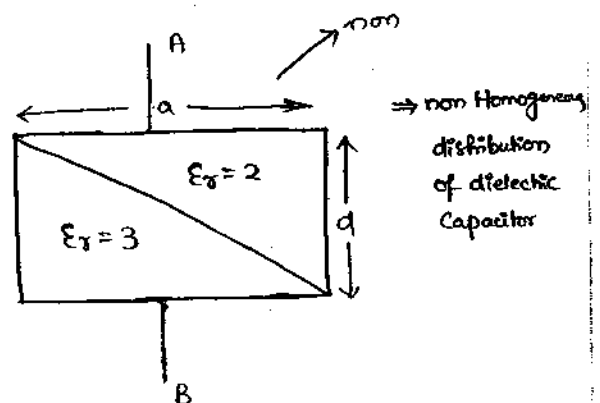
$$\frac{\epsilon_0 a}{d/2} (1 + \epsilon_r)$$

$$C_{AB} = \frac{2\epsilon_0 \epsilon_r a}{d(1 + \epsilon_r)}$$

HW-1



$C_{AB} = \underline{\hspace{2cm}}$



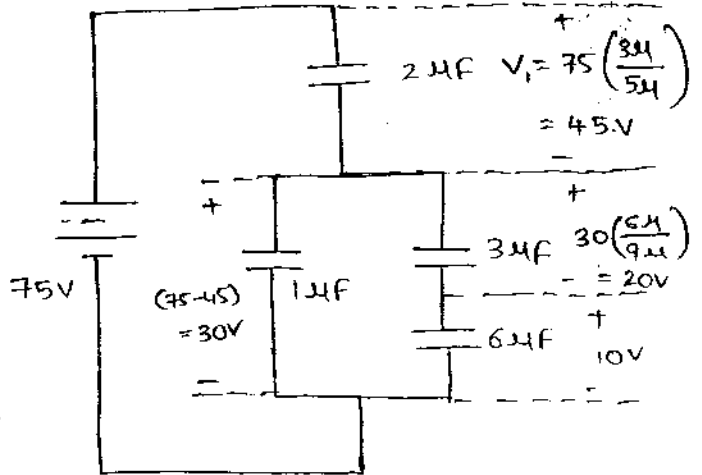
for the Capacitive N/w shown determine (i) Steady State Voltages across each ~~plate~~ capacitor

(2) Charge accumulated by each cap.

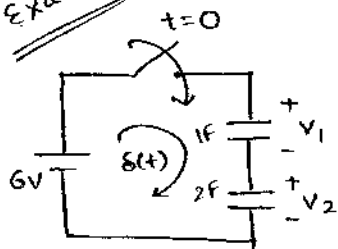
(3) Energy stored in each cap.

(4) Verify law of Consrv. Energy

(5) Verify law of Consrv. charge



Example



In Series

$$q_1 = q_2$$

$$C_1 V_1 = C_2 V_2$$

$$1(V_1) = 2(V_2) \rightarrow (1)$$

$$V_1 + V_2 = 6V \rightarrow (2)$$

L.C. of Energy

Solve eqns (1) & (2)

$$3V_2 = 6V \rightarrow V_2 = 2V$$

$$V_1 = 4V$$

(or) By Voltage division Rule.

$$V_1 = 6 \left[\frac{2}{3} \right] = 4V$$

$$V_2 = 6 \left[\frac{1}{3} \right] = 2V.$$

(1)

$$V_{2\mu} = 75 \left(\frac{3\mu}{2\mu + 3\mu} \right) = 45V \quad ; \quad V_{1\mu} = 30V$$

$$V_{3\mu} = 30 \left(\frac{6\mu}{3\mu + 6\mu} \right) = 20V \quad ; \quad V_{6\mu} = 10V \quad (30 - 20)$$

(2) $q = CV$

$q_{2\mu} = 90\mu C$

$q_{1\mu} = 30\mu C$

$q_{3\mu} = 60\mu C$

$q_{6\mu} = 60\mu C$

92

(3)
$$\left. \begin{aligned} E_{2\mu} &= \frac{1}{2} (2\mu) (45)^2 = 2025 \mu J \\ E_{1\mu} &= \frac{1}{2} (1\mu) (30)^2 = 450 \mu J \\ E_{3\mu} &= \frac{1}{2} (3\mu) (20)^2 = 600 \mu J \\ E_{6\mu} &= \frac{1}{2} (6\mu) (10)^2 = 300 \mu J \end{aligned} \right\} \begin{array}{l} \text{Electrostatic form} \\ \downarrow \\ \text{charge} \end{array}$$

(4) L.C. Energy

$E_{deli} = E_{stored}$

LHS $E_{deli} = \frac{1}{2} C_T V_T^2 = \frac{1}{2} \left[\frac{6}{5} \mu \right] [75]^2 = 3375 \mu J$

RHS $E_{stored} = [2025 + 450 + 600 + 300] \mu J = 3375 \mu J$

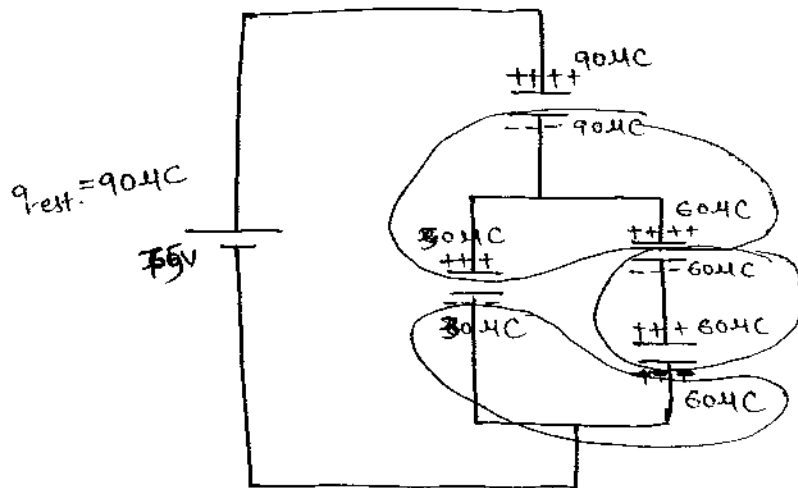
(5) L.C. charge

$q_{reestablished} = q_{accumlated}$

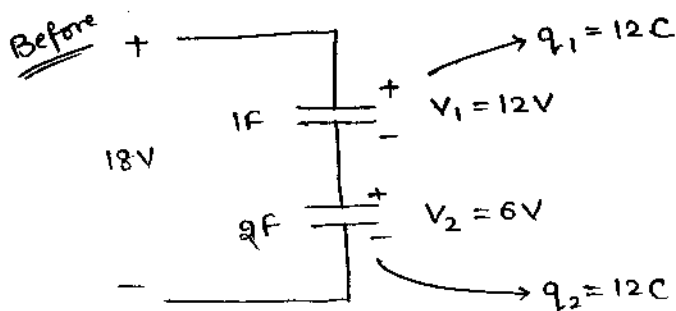
LHS $q_{est.} = C_T \cdot V_T = \left[\frac{6}{5} \mu \right] [75] = 90\mu C$

RHS $q_{acc} = 90\mu C$

Here the total charge accumulated by the ckt is the charge stored on one series capacitor to the source which $2\mu F$ capacitor which is equal to $90\mu F$ only Hence law of Conservation of charge verified



Q. Two capacitors of $1F$ & $2F$ are connected in series across $18V$ DC source. determine their steady state voltages & charge & Energy stored in it. Now if these two capacitors are disconnected from supply & connected with like polarities together then determine their steady state voltage & charge & Energy stored in each. Verify L.C of charge & L.C of Energy before and after the change.

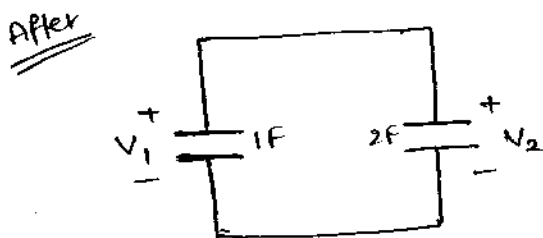


$$E_1 = \frac{1}{2}(1)(12)^2 = 72 \text{ J}$$

$$E_2 = \frac{1}{2}(2)(6)^2 = 36 \text{ J}$$

$$q_T = C_T \cdot V_T = \frac{2}{3}[18] = 12 \text{ C} \rightarrow (1)$$

$$E_T = (72 + 36) = 108 \text{ J} \rightarrow (2)$$



\therefore parallel

$$V_1 = V_2$$

$$q = CV \Rightarrow V = \frac{q}{C}$$

$$\frac{q_1}{C_1} = \frac{q_2}{C_2}$$

$$q_1 \times 2F = q_2 \times 1F$$

$$q_2 = 2q_1 \rightarrow (3)$$

$$\begin{aligned} q_1 &= q_2 \\ \frac{q_1}{C_1} &= \frac{q_2}{C_2} \\ \frac{q_1}{1} &= \frac{q_2}{2} \\ q_1 &= \frac{q_2}{2} \\ q_1 + q_2 &= 12 \end{aligned}$$

$$q_1 + q_2 = 24 \rightarrow (4) \quad (q_1 + q_2 \neq 12C \text{ bec})$$

solving (3) & (4)

$$3q_1 = 24$$

$$q_1 = 8C$$

$$q_2 = 16C$$

$$V_1 = V_2$$

$$\frac{8}{1} = \frac{16}{2} = 8V$$

But the moment when we separate them from source they become Isolated Bodies

\therefore charge on each cap. = 12C

then ^{total} charge

$$q_T = q_1 + q_2 = 12C + 12C$$

$$q_1 + q_2 = 24C \rightarrow (4)$$

(93)

$$E_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} (1) (8)^2 = 32J$$

$$E_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2} \times 2 \times (8)^2 = 64J$$

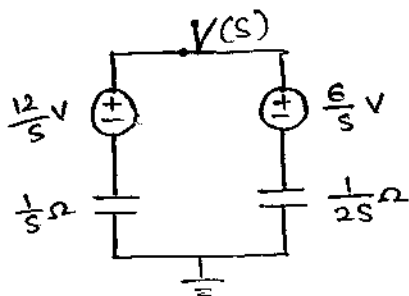
$$q_T = q_1 + q_2 = 8 + 16 = 24C \rightarrow (5)$$

$$E_T = E_1 + E_2 = 32 + 64 = 96J \rightarrow (6) \quad \left. \begin{array}{l} 12J \text{ of energy lost (or) utilised} \\ \text{to redistribute the charge to maintain same} \\ \text{across each cap.} \end{array} \right\}$$

\Rightarrow Here, L.C of Charge satisfied

\Rightarrow L.C. of Energy has to be justified,

proof (S-domain):



Nodal

$$\frac{\left[V(s) - \frac{12}{s} \right]}{\frac{1}{s}} + \frac{\left[V(s) - \frac{6}{s} \right]}{\frac{1}{2s}} = 0$$

$$V(s) [s + 2s] = 24$$

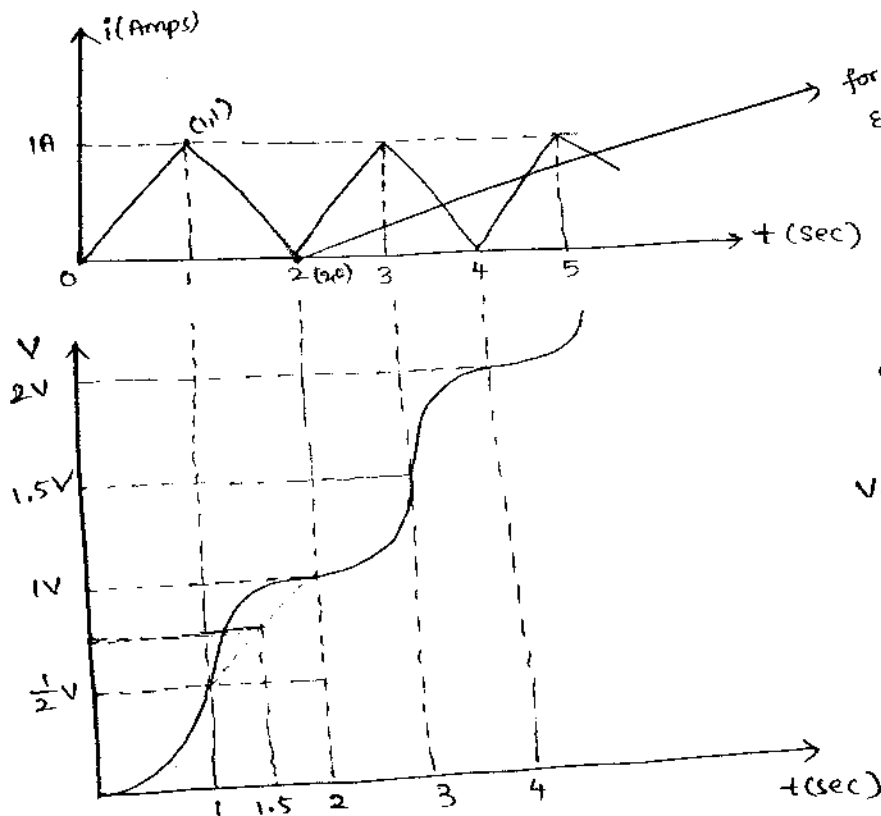
$$V(s) = \frac{24}{3s} = \frac{8}{s}$$

$$V(t) = \mathcal{L}^{-1} [V(s)] = 8 \text{ volts}$$

$$\rightarrow V_1 = V_2$$

Here, charge is considered as matter but Energies can be in any form. So, law of Conservation of charge is satisfied but some of the Energy from previous state has been utilised to redistribute the charge among capacitors to maintain constant Equal Voltages.

Q if $V(0) = 0V$ plot the Voltage across 1F capacitor if current through it as shown below.



for $i=0$
Energy stored in cap $\frac{1}{2} C V^2$
 $\frac{1}{2} \times 1 \times 1^2$

even $i=0$, the Energy stored is not zero

∴ Cap are better storing elements & 'L'

But in 'L' to store energy current has to flow

$$V = \frac{1}{C} \int_{-\infty}^t i dt \quad \text{in 'L', if } i=0 \text{ energy stored is zero}$$

$$V(t) = V(0) + \frac{1}{C} \int_0^t i(t) dt$$

$$0 < t < 1:$$

$$i(t) = t$$

$$1 < t < 2:$$

$$i(t) = (-t+2)$$

$$0 < t < 1$$

$$V(t) = V(0) + \frac{1}{1} \int_0^t t dt$$

$$= 0 + \frac{t^2}{2} \Big|_0^1$$

$$V(t) = \frac{t^2}{2} \Rightarrow \text{parabolic}$$

$$t=0 \longrightarrow V=0$$

$$t=1 \longrightarrow V = \frac{1}{2} V$$

$$1 < t < 2$$

$$V(t) = V(0) + \frac{1}{1} \int_0^2 (-t+2) dt$$

$$= \frac{1}{2} + \left(\frac{-t^2}{2} \right) \Big|_1^2 + 2t \Big|_1^2$$

$$V(t) = \frac{1}{2} - \frac{t^2}{2} + 2t \Rightarrow \text{parabolic}$$

$$t=1 \longrightarrow V = \frac{1}{2} V$$

$$t=2 \longrightarrow V = 1V$$

check

Value of 'V' at $t = 1.5 \text{ Sec}$

94

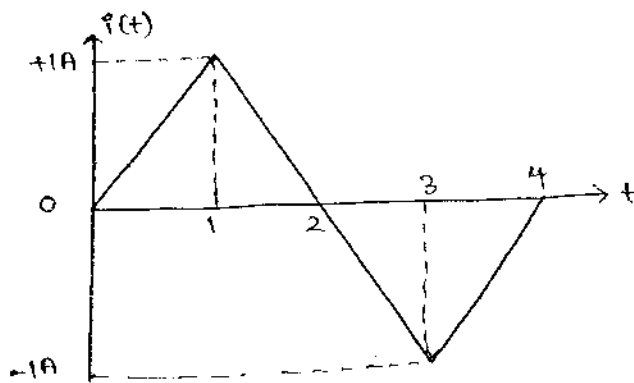
$$= \frac{1}{2} - \frac{t^2}{2} \Big|_1^{1.5} + 2t \Big|_1^{1.5}$$

$$= \frac{1}{2} - \frac{1}{2} [2.25 - 1] + 2 [1.5 - 1]$$

$$= 1.5 - 0.625$$

$$= 0.875 \text{ volts.}$$

Q In the above problem plot V, P, Energy in 1 farad Capacitor if the current w/f is changed to $i(t)$ [$V(0) = 0V$]



0

0

0

0

0

0

0

0

0

0

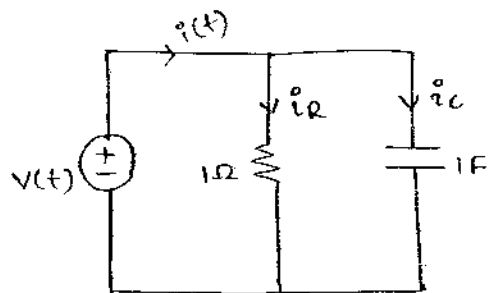
0

0

0

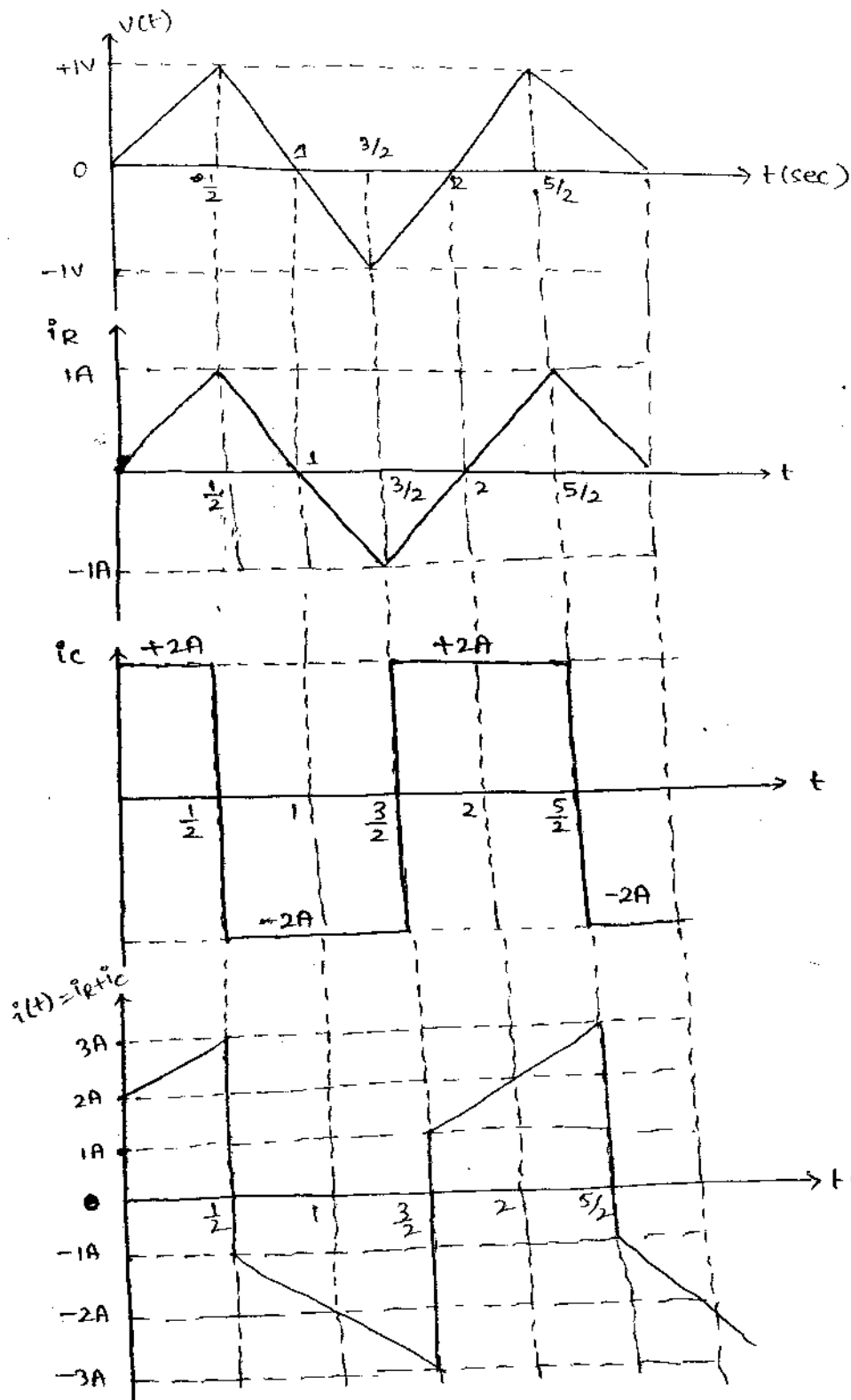
Q plot $i(t)$ if voltage $V(t)$ is given.

95



KCL

$$i(t) = i_R + i_C$$



$$V = i_R$$

$$|i| = \frac{V}{R} = \frac{1}{1} = 1$$

\therefore in Resistor
 i follows V

$$i_C = C \frac{dV}{dt}$$

$$0 < t < \frac{1}{2}$$

$$i_C = 1 \cdot \frac{d(2t)}{dt} = +2A \text{ (pulse)}$$

$$\frac{1}{2} < t < \frac{3}{2}$$

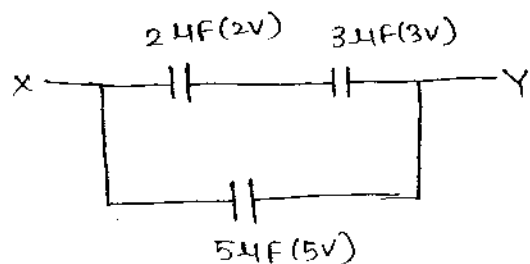
$$i_C = 1 \cdot \frac{d(-2t + C_1)}{dt} = -2A \text{ (pulse)}$$

$$\frac{3}{2} < t < \frac{5}{2}$$

$$i_C = 1 \cdot \frac{d(2t - C_2)}{dt}$$

$$i_C = +2A \text{ (pulse)}$$

gate/ES



5 μF

$$V_{\max} = 5V \longrightarrow (1)$$

3 μF

$$V_{\max} * \left[\frac{2 \mu}{5 \mu} \right] = 3V \longrightarrow V_{\max} = \frac{15}{2} V = 7.5 \text{ volts} \longrightarrow (2)$$

2 μF

$$V_{\max} * \left[\frac{3 \mu}{5 \mu} \right] = 2V \longrightarrow V_{\max} = \frac{10}{3} V = 3.33 \text{ volts} \longrightarrow (2)$$

Overall

$$\text{Safe, } V_{\max} = \frac{10}{3} V = 3.33 V.$$

$$q_{\max} = C_T \cdot V_{\max}$$

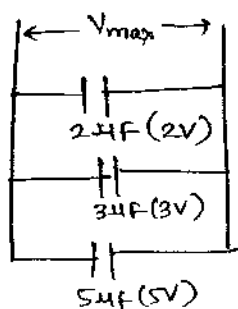
$$= \left[\frac{6}{5} + 5 \right] \mu \left[\frac{10}{3} \right]$$

$$q_{\max} = \frac{62}{3} \mu C.$$

Ex:

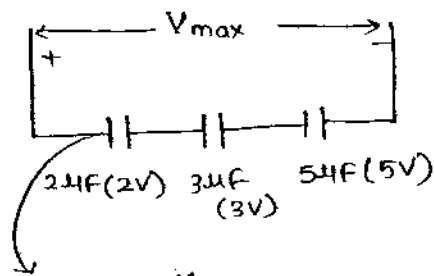
Suppose

Sol



$$\text{Safe, } V_{\max} = \underline{2V}$$

*



∴ this has the total pressure of V_{max} on $2\mu F$ (small) rated cap

∴ if we provide safety to this capacitor then overall safety can be provided.

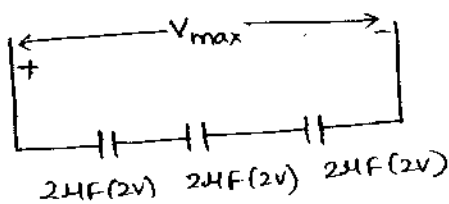
$$V_{max} \left[\frac{\frac{1}{2\mu}}{\frac{1}{2\mu} + \frac{1}{3\mu} + \frac{1}{5\mu}} \right] = 2 \text{ Volts. } 96$$

$$V_{max} \left[\frac{\frac{1}{2\mu}}{\frac{31}{30\mu}} \right] = 2 \text{ Volts.}$$

$$V_{max} = \frac{62}{15} = 4.133 \text{ Volt}$$

if blocks 4.13V.

* Suppose all cap. are symmetrically rate.



Safe, $V_{max} = 6V$.

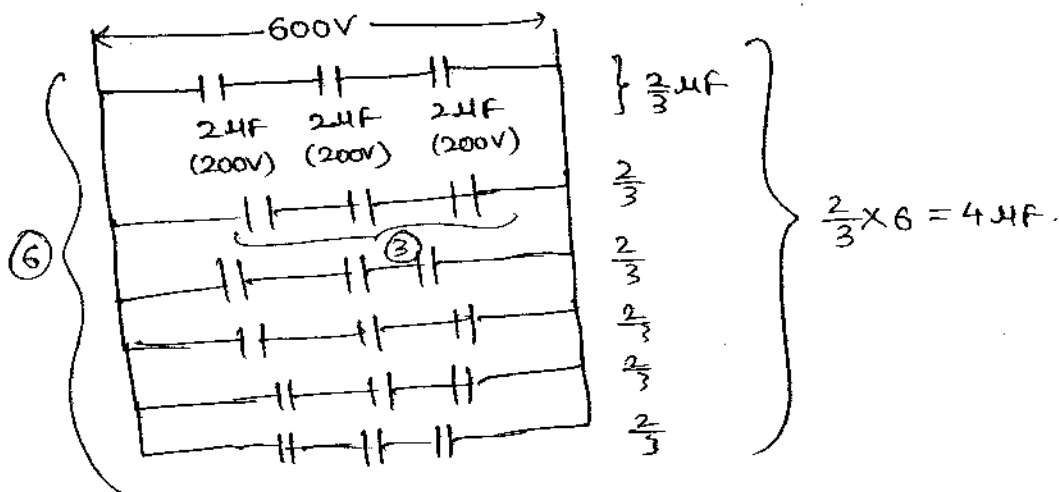
⇒ Symmetrical Rated all much more better performance if blocks 6V

④ How many Capacitors of each rated 200V, $2\mu F$ are required to be connected to work at an application level of 600V, $4\mu F$.

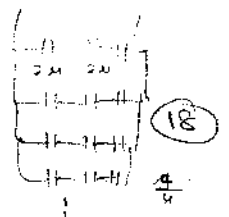
to Increase Voltage Rating Insulation level has to be Increased

∴ In cap. dielectric is the Insulation

∴ Connect 3, 200V cap. in Series



$$\Rightarrow 6 \times 3 = 18 \text{ capacitors}$$

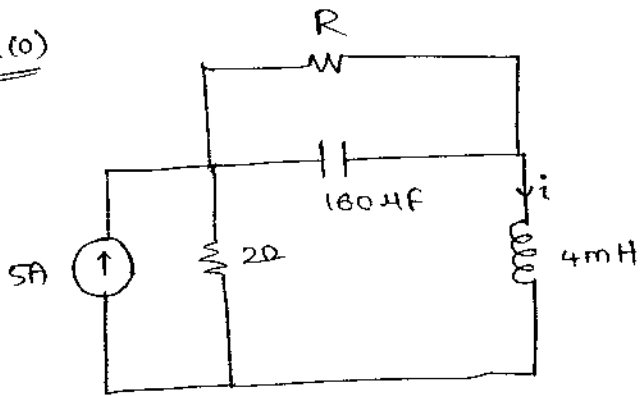


$$\frac{2}{3} \times \frac{18}{1} = 12$$

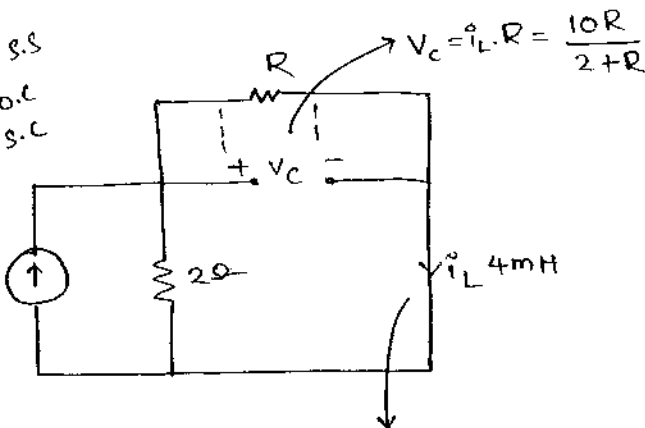
$$\frac{2}{3} \times 6 = 4$$

$$\frac{2}{3} \times 6 = 4$$

IES(0)



for D.C S.S
Cap - O.C
Ind - S.C



$$v_c = i_L R = \frac{10R}{2+R}$$

$$i_L = 5 \left[\frac{2}{2+R} \right] = \frac{10}{2+R}$$

for what value of 'R'
Energy stored in Inductor &
Capacitor are Equal in S.S

$$\frac{1}{2} C V_c^2 = \frac{1}{2} L i_L^2$$

$$180 \times 10^{-6} V_c^2 = \frac{1}{2} \times 4 \times 10^{-3} i_L^2$$

$$\frac{4}{100} V_c^2 = i_L^2$$

$$\frac{V_c^2}{100} = i_L^2$$

$$\frac{\left(\frac{10R}{2+R} \right)^2}{100} = \left(\frac{10}{2+R} \right)^2$$

$$\frac{\left(\frac{10R}{2+R} \right)^2}{\left(\frac{10}{2+R} \right)^2} = \frac{100}{4}$$

$$\frac{100R^2}{100} = \frac{100}{4}$$

$$R^2 = 25$$

$$R = 5\Omega$$

Steady State AC Circuit Analysis

S³A → Steady state Sinusoidal Analysis.

$$\begin{cases} V = V_m \sin \omega t \text{ [IES]} \\ V = V_m \cos \omega t \end{cases}$$

97

The Best waveform in electrical power Engg. where our entire generation, Transmission, distribution & utilisation of Elec. Energy where our major Components such as alternator, Transformers, Tr. lines & I. Motors gives Highest Efficiency & power factor to only Sinusoid.

⇒ T.F → % $\eta = 90\%$.

Alternator → 92%.

Tr. lines → > 70%.

> 70% of Elec. Energy generated

↳ used to drive Mech. loads

} gives Higher η for Sinusoid.

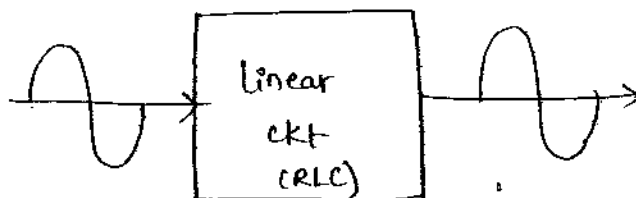
⇒ King of loads of pow. sys → Ind. Motors

↳ Best performance only for Sinusoid.

Side effects:

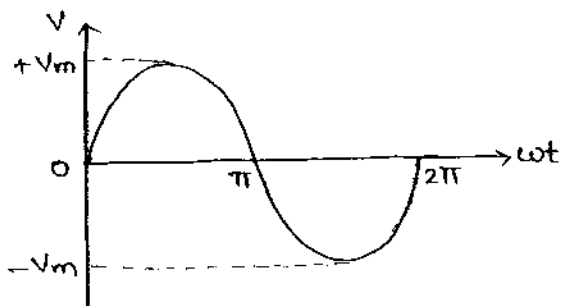
if we Excite our Components in power Engg. with other than Sinusoidal w/f then we have Torque ripples, Harmonics, electromag. Interference, additional losses, low p.f & so low efficiency

—x—
The only w/f if we give as Input to any linear circuit will also the Same o/p is Sinusoid.



(a) Radian

$$V = V_m \sin \omega t$$



$V_m \rightarrow$ Amplitude [volts]

$\omega \rightarrow$ angular Velocity [rad/sec]

$\omega t \rightarrow$ Argument

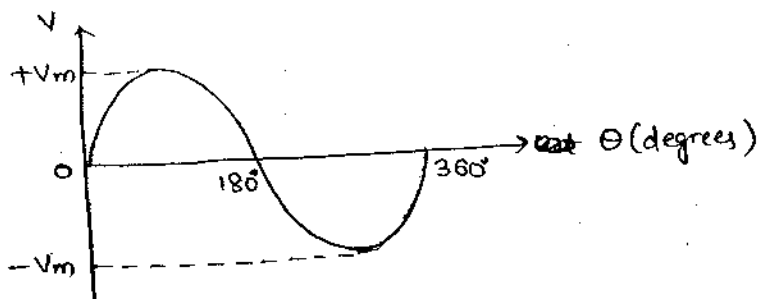
$$\omega = 2\pi f$$

$$\therefore f = \frac{1}{T}$$

$$\omega = \frac{2\pi}{T}$$

(b) degree

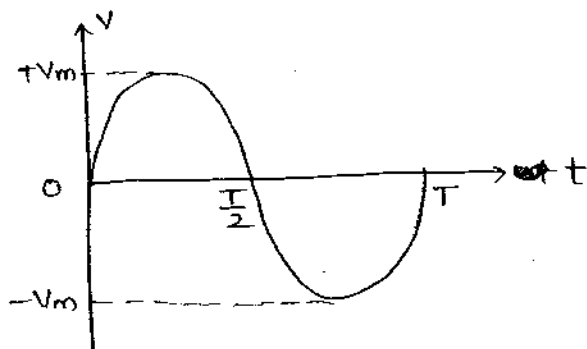
$$V = V_m \sin \theta$$



(c) Time

$$V = V_m \sin \left(\frac{2\pi}{T} \cdot t \right)$$

$$f = \frac{1}{T}$$



INDIA //

$$f = 50 \text{ Hz}$$

$$1T = \frac{1}{50} = 20 \text{ mSec}$$

$$\begin{aligned} \omega &= 2\pi f = 2\pi(50) \\ &= 100\pi \\ &\approx 314 \end{aligned} \left. \vphantom{\begin{aligned} \omega &= 2\pi f = 2\pi(50) \\ &= 100\pi \\ &\approx 314 \end{aligned}} \right\} \text{ rad/sec.}$$

INDIA @ 50Hz

$$1T = \underset{\substack{\downarrow \\ \text{Radian}}}{2\pi} = \underset{\substack{\downarrow \\ \text{degrees}}}{360^\circ} = \underset{\substack{\downarrow \\ \text{Time}}}{20 \text{ mSec}}$$

98

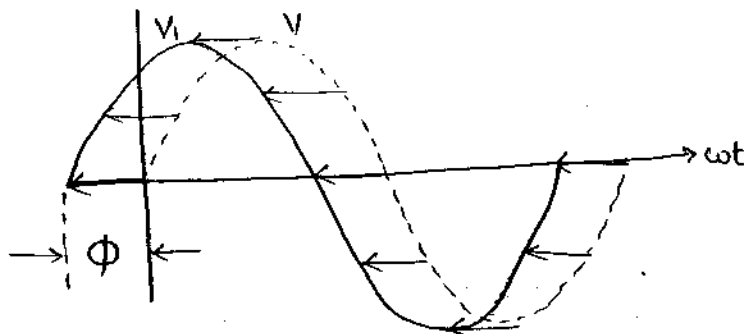
Standard Sinusoid:

$$V_x = V_m \sin(\omega t \pm \phi)$$

$\phi \rightarrow$ phase shift [deg.]

\hookrightarrow Time shift.

Example: 1: $V_1 = V_m \sin(\omega t + \phi)$

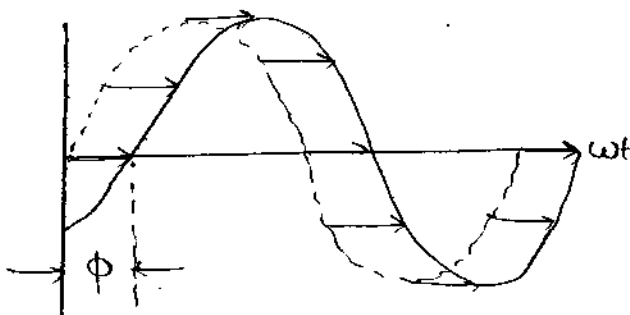


V_1 comes early than V by ϕ

V_1 leads V by ϕ

$+\phi \rightarrow$ leading.

Example: 2: $V_2 = V_m \sin(\omega t - \phi)$



V_2 comes late than V by ϕ

V_2 lags V by ϕ

$-\phi \rightarrow$ lagging.

INDIA, $f = 50\text{Hz}$,

let $\phi = 60^\circ$

$360^\circ \longrightarrow 20\text{msec}$

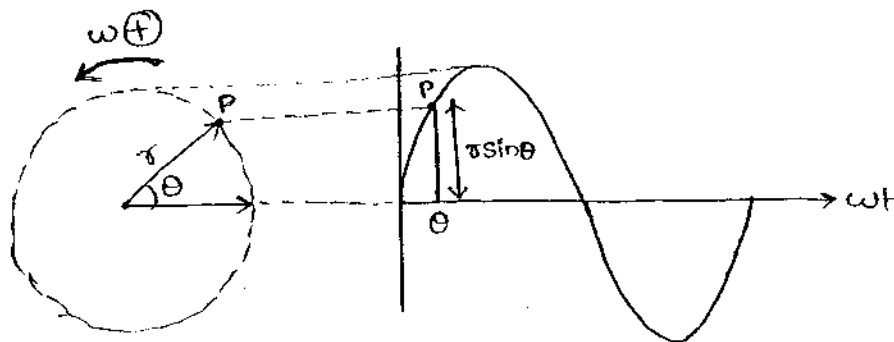
$60^\circ \longrightarrow t_{\text{shift}}$

$$t_{\text{shift}} = \frac{60^\circ}{360^\circ} * 20\text{msec}$$

$$t_{\text{shift}} = 3.33\text{msec}$$

Convolute
↓
operate

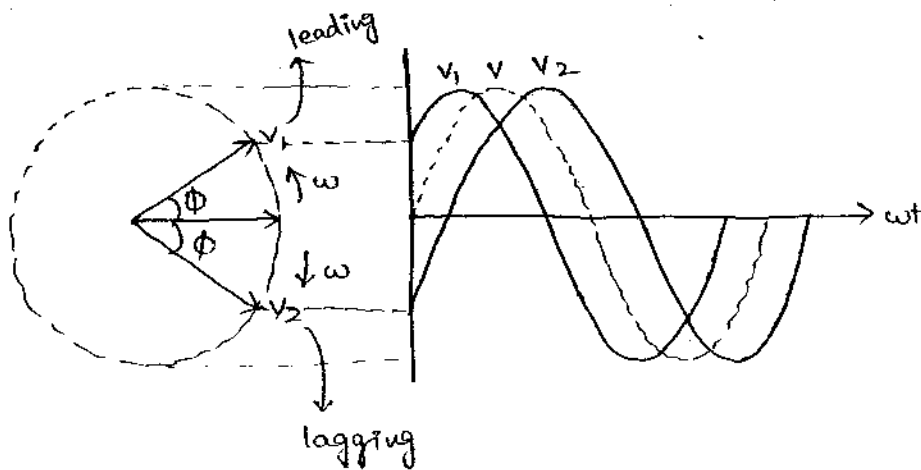
phasor \longrightarrow Represent Sinusoids only



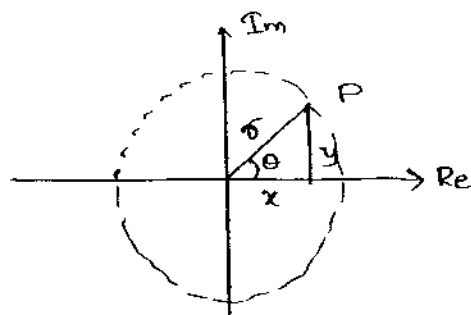
+ new

$$P = r \angle \theta$$

$$\overline{V} = |V_m| \angle 0^\circ, \overline{V}_1 = |V_m| \angle +\Phi^\circ, \overline{V}_2 = |V_m| \angle -\Phi^\circ$$



s-plane



99

Electrical Quantity:



P

Rectangular form $\rightarrow x + jy$

Polar form $\rightarrow r \angle \theta$

Euler form $\rightarrow r e^{j\theta}$

RECT \rightarrow POL

$0 + j0 \rightarrow 0 \angle 0^\circ$

$1 + j0 \rightarrow 1 \angle 0^\circ$

$0 + j1 \rightarrow 1 \angle 90^\circ$ \rightarrow maths operator is e, e

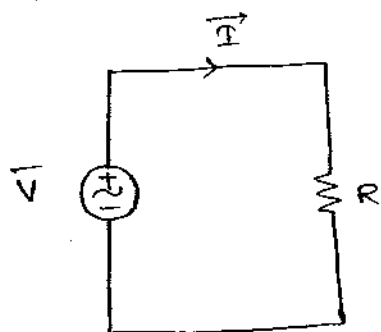
$1 + j1 \rightarrow \sqrt{2} \angle 45^\circ$

$0 - j1 \rightarrow 1 \angle -90^\circ$

j operator shifts the phase by 90° & multiplies mag by "1"

phasor Relationship b/w voltages & currents in passive elements:

(a) Resistor.



$$\text{let } \bar{V} = V_m \sin \omega t \rightarrow (1)$$

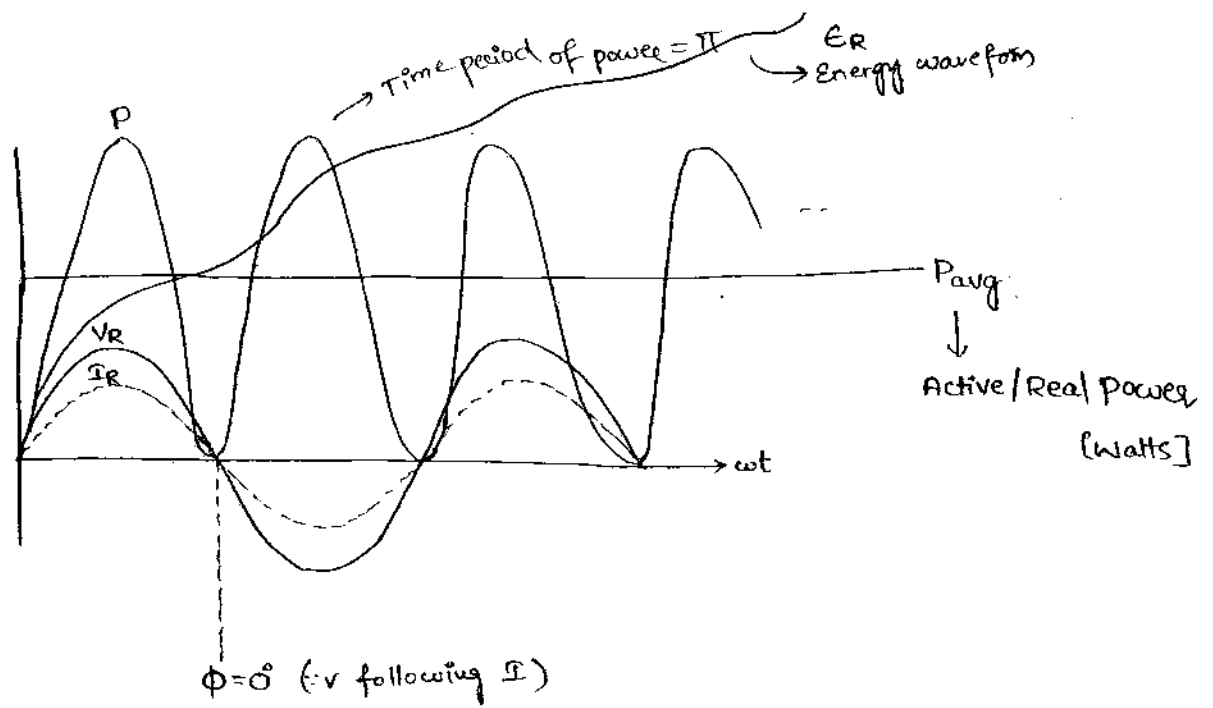
$$\bar{I} = \frac{V}{R}$$

$$\bar{I} = \frac{V_m \sin \omega t}{R} = \left[\frac{V_m}{R} \right] \sin \omega t$$

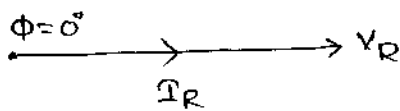
$$\bar{I} = I_m \sin \omega t \rightarrow (2)$$

$$\boxed{\bar{V} = \bar{I} \cdot R} \rightarrow \text{7th form of Ohm's law.}$$

\rightarrow Resistor is P_R phase element (V & I are in phase)



phasor diagram:



power factor:

$$\cos \phi = \cos 0^\circ = 1 \text{ [UPF]}$$

$$P_{avg} = \frac{1}{T} \int_0^T P(t) dt$$

Time period of P is π

$$P_{avg} = \frac{1}{\pi} \int_0^\pi V_m I_m \sin^2 \omega t d\omega t$$

$$= \frac{V_m I_m}{2} \left[\frac{1}{\pi} \int_0^\pi (1 - \cos 2\omega t) dt \right]$$

$$P_{avg} = \frac{V_m I_m}{2} \left[\underbrace{\frac{1}{\pi} (\pi - 0)}_1 - \underbrace{\frac{1}{2} (0 - 0)}_0 \right]$$

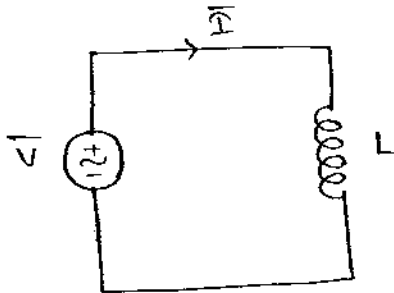
$$P_{avg} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$P_{avg} = V_{rms} \cdot I_{rms}$$

* Instantaneous freq. of ^{Power of} ~~Indian~~ Heater in Indian System = 100 Hz
Resistor

(b) Inductor:

100



$$\bar{I} = I_m \sin \omega t \rightarrow (1)$$

$$V = L \frac{dI}{dt}$$

$$V = \omega L I_m \cos \omega t$$

$$V = \omega L I_m \sin(90^\circ + \omega t)$$

$$V = \omega L I_m \sin(\omega t + 90^\circ) \rightarrow (2)$$

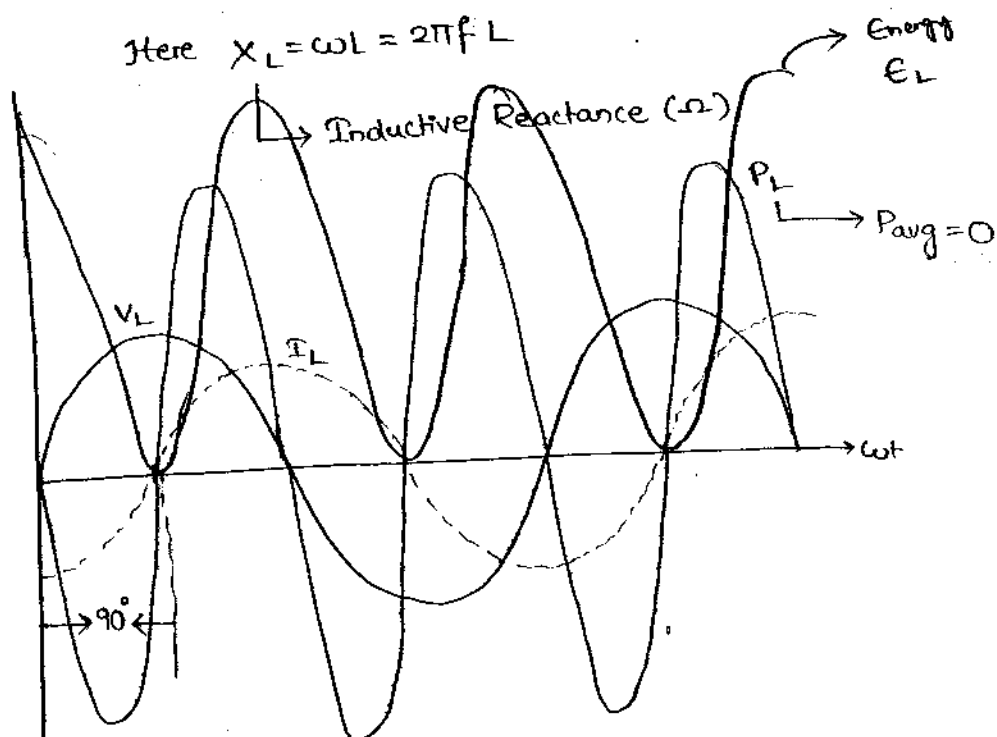
$$V = \omega L I_m \sin \omega t [j] \rightarrow \because j \text{ shifts by } 90^\circ \text{ phase}$$

$$\bar{V} = j \omega L \bar{I}$$

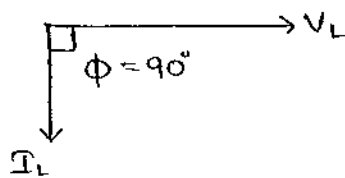
$$\bar{V} = +j X_L \bar{I} \rightarrow \text{8th form of Ohm's law.}$$

\Rightarrow 'L' is lagging element (w.r.t Voltage Current is lagging)

$$\text{Here } X_L = \omega L = 2\pi f L$$



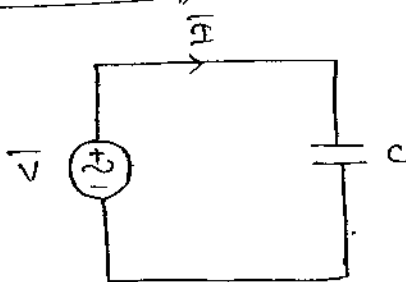
phasor diagram:



power factor:

$$\cos \phi = \cos 90^\circ = 0$$

(c) Capacitor:



$$\text{Let } \bar{V} = V_m \sin \omega t$$

$$I = C \frac{dV}{dt}$$

$$I = \omega C V_m \cos \omega t$$

$$I = \omega C V_m \sin(90^\circ + \omega t)$$

$$I = \omega C V_m \sin(\omega t + 90^\circ) \rightarrow (2)$$

$$I = \omega C \underbrace{V_m \sin \omega t}_{\downarrow} [j]$$

$$\bar{I} = j\omega C \bar{V}$$

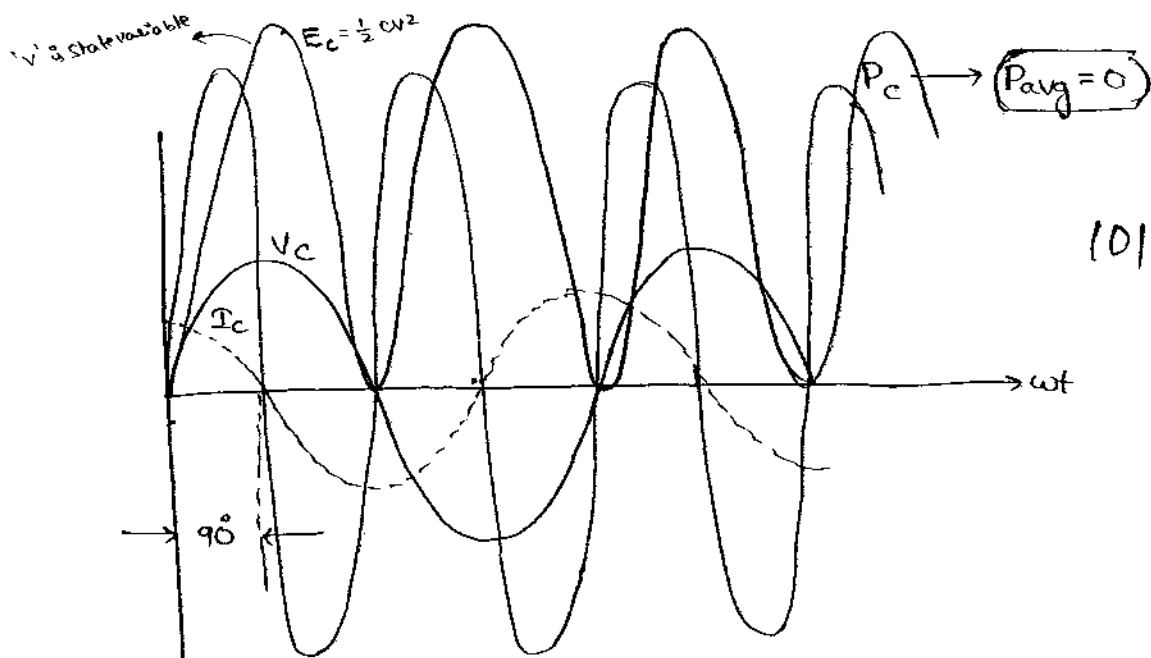
$$\bar{V} = \frac{\bar{I}}{j\omega C} = \frac{-j}{\omega C} \cdot \bar{I}$$

$$\boxed{\bar{V} = -j X_C \bar{I}} \rightarrow \text{9th form of ohm's law.}$$

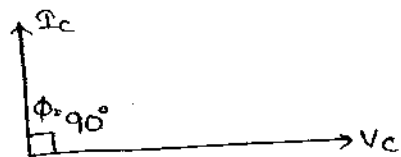
\Rightarrow capacitor is leading element (current leading voltage)

$$\text{Here } X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

\rightarrow capacitive Reactance ($-\Omega$)
(AC Resistance)



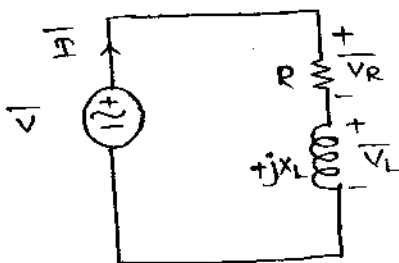
phasor diagram:



power factor:

$$\cos \phi = \cos 90^\circ = 0$$

(d) Series RL //



KVL //

$$\bar{V} = \bar{V}_R + \bar{V}_L$$

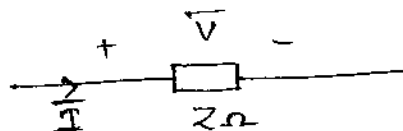
$$\bar{V} = \bar{I} \cdot R + jX_L \cdot \bar{I}$$

$$\bar{V} = \bar{I} [R + jX_L]$$

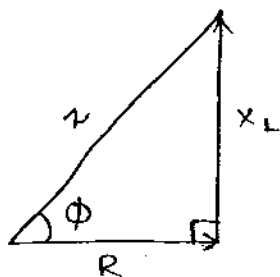
$$\boxed{\bar{V} = \bar{I} Z}$$

$$Z = R + jX_L$$

\rightarrow Impedance (Ω)



→ Impedance ΔI_e



$$|Z| = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{R^2 + (\omega L)^2}$$

$$\phi = \tan^{-1} \left[\frac{X_L}{R} \right] = \tan^{-1} \left[\frac{\omega L}{R} \right]$$

↓
Impedance angle

→ power factor

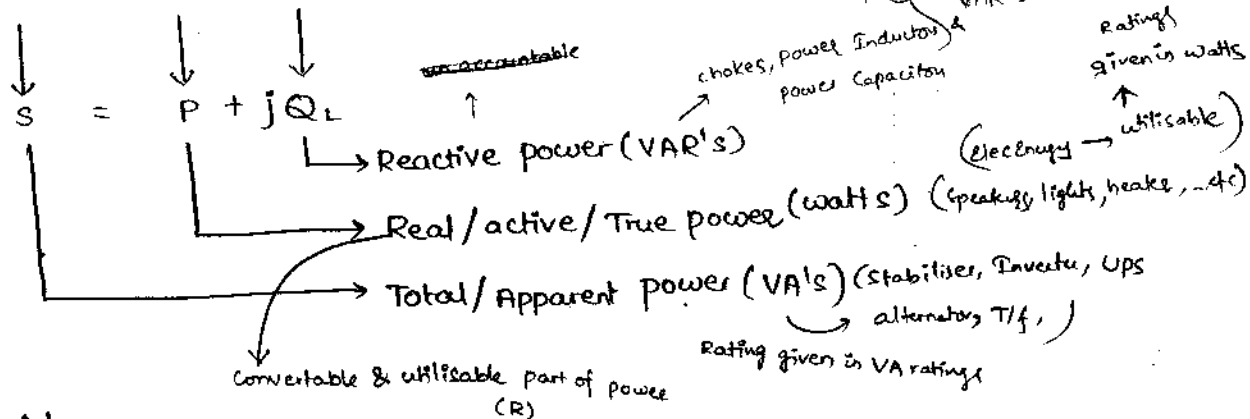
$$\cos \phi = \frac{R}{|Z|}$$

$$\text{also, } \sin \phi = \frac{X_L}{|Z|}$$

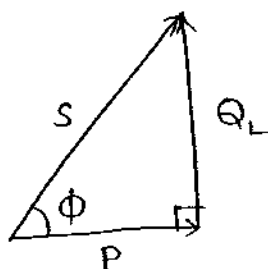
→ scalar addition
vector addition at
instant

$$I^2 Z = I^2 R + j X_L I^2$$

$$V \cdot I = I^2 R + j I^2 X_L$$



⇒ power ΔI_e



$$|S| = \sqrt{P^2 + Q_L^2}$$

$$\phi = \tan^{-1} \left[\frac{Q_L}{P} \right]$$

power factor:

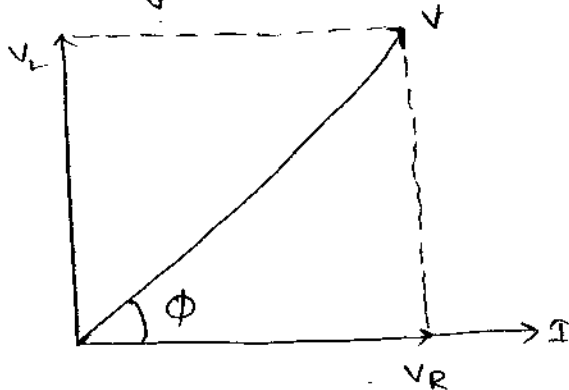
$$\cos\phi = \frac{P}{S} \rightarrow P = S \cos\phi = V \cdot I \cdot \cos\phi \text{ W}$$

$$\text{also } \sin\phi = \frac{Q_L}{S} \rightarrow Q_L = S \sin\phi = V \cdot I \cdot \sin\phi \text{ VARS.}$$

102

Here V, I are Rms values

phasor diagram:



\bar{I} lags \bar{V} by $\phi < 90^\circ$ (lagging element)

$$\bar{I} = \frac{\bar{V}}{Z} = \frac{V_m \sin\omega t}{R + jX_L} = \frac{V_m \sin\omega t}{|Z| \angle +\phi}$$

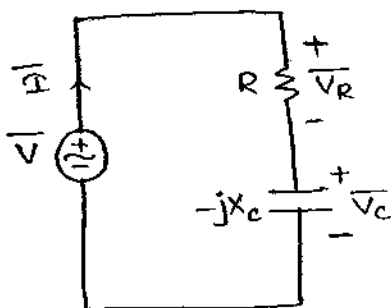
$$\bar{I} = \frac{V_m}{|Z|} \sin(\omega t - \phi) \quad \text{Amp}$$

Here V, I

Note: In Linear ckt analysis Impedance angle, phasor angle, power factor angle all are same.

A source Referred
with Apparent (m)
Total Power
(VA)

(e) Series RC:



KVL

$$\bar{V} = \bar{V}_R + \bar{V}_C$$

$$\bar{V} = \bar{I}R + (jX_C)\bar{I}$$

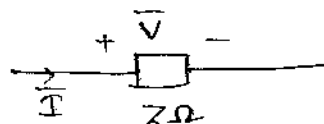
$$\bar{V} = \bar{I}R - jX_C\bar{I}$$

$$\bar{V} = \bar{I}[R - jX_C]$$

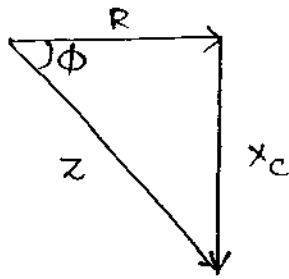
$$\bar{V} = \bar{I} \cdot Z$$

$$Z = R - jX_C$$

\rightarrow Impedance (Ω)



Impedance Δ le.



for series
for easy
series - I ref. take
parallel - V ref. take

$$|Z| = \sqrt{R^2 + X_c^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_c}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

↓
Impedance angle.

⇒ power factor

$$\cos \phi = \frac{R}{|Z|}$$

$$\text{also } \sin \phi = \frac{X_c}{|Z|}$$

$$I^2 Z = I^2 R - j X_c I^2$$

$$V \cdot I = I^2 R - j I^2 X_c$$

$$\downarrow \quad \downarrow \quad \downarrow$$

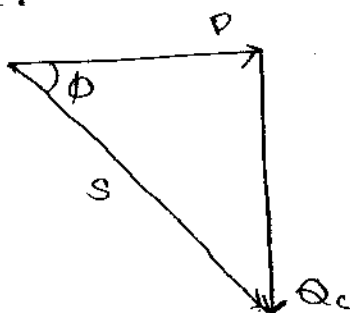
$$S = P - j Q_c$$

→ Reactive power (VAR's)

→ Real/Active/True power (watts)

→ Total/Apparent power (VA's).

power Δ le:



$$|S| = \sqrt{P^2 + Q_c^2}$$

$$\phi = \tan^{-1}\left[\frac{Q_c}{P}\right]$$

power factor:

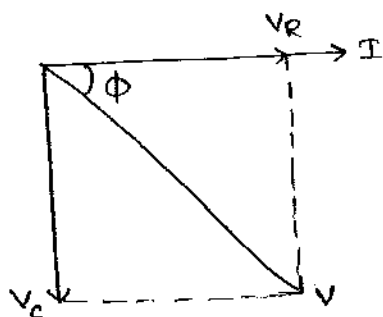
$$\cos \phi = \frac{P}{S} \rightarrow P = S \cos \phi = V \cdot I \cdot \cos \phi \quad \underline{\omega}$$

$$\text{also } \sin \phi = \frac{Q_c}{S} \rightarrow Q_c = S \sin \phi = V \cdot I \sin \phi \quad \underline{\text{VARs}}$$

103

Here
V, I are RMS values.

phasor diagram:

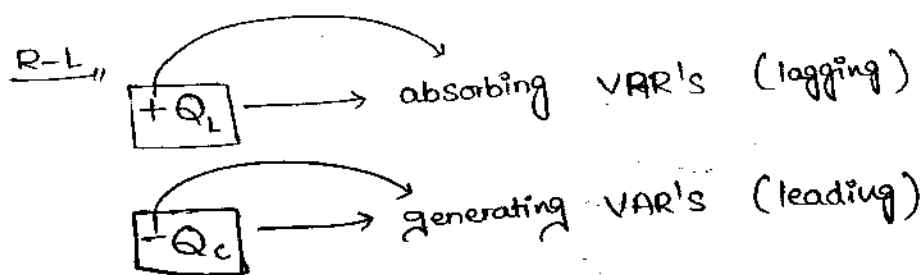


\bar{I} leads \bar{V} by $\phi < 90^\circ$ (leading element)

$$\bar{I} = \frac{\bar{V}}{Z} = \frac{V_m \sin \omega t}{R - jX_c} = \frac{V_m \sin \omega t}{|Z| \angle -\phi}$$

$$= \frac{V_m}{|Z|} \sin(\omega t + \phi) \text{ Ampe.}$$

as L, C are ~~active~~ passive



in passive sink

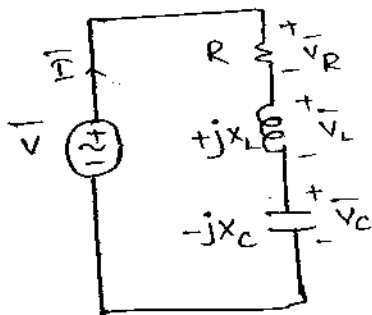
+P → absorb energy

-P → generate energy

$$\downarrow \bar{Q} = \frac{V_2 \downarrow}{X} [V_1 \cos \delta - V_2]$$

Compensation.

(f) Series R-L-C



KVL

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$$

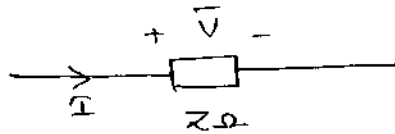
$$\bar{V} = \bar{I}R + jX_L \bar{I} - jX_C \bar{I}$$

$$\bar{V} = \bar{I} [R + j(X_L - X_C)]$$

$$\bar{V} = \bar{I} \cdot Z$$

$$Z = R + j \underbrace{(X_L - X_C)}_{X_{net}}$$

Impedance (Ω)



$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

↳ net Impedance angle

power factor:

$$\cos \phi = \frac{R}{|Z|}$$

$$\text{also } \sin \phi = \frac{X_{net}}{|Z|} = \frac{[+X_L - X_C]}{|Z|}$$

$$\bar{I}^2 \bar{Z} = \bar{I}^2 R + j [\bar{I}^2 X_L - \bar{I}^2 X_C]$$

$$V \cdot \bar{I} = \bar{I}^2 R + j [\bar{I}^2 X_L - \bar{I}^2 X_C]$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$S = P + j \underbrace{[Q_L - Q_C]}_{Q_{net}}$$

$$|S| = \sqrt{P^2 + (Q_L - Q_C)^2}$$

$$\phi = \tan^{-1} \left[\frac{Q_L - Q_C}{P} \right]$$

power factor:

$$\cos \phi = \frac{P}{S} \Rightarrow P = S \cos \phi = V \cdot I \cdot \cos \phi$$

104

$$\text{also } \sin \phi = \frac{Q_{\text{net}}}{S} = \frac{[Q_L - Q_C]}{S}$$

$$Q_{\text{net}} = [+Q_L - Q_C] = S \sin \phi = V \cdot I \cdot \sin \phi \text{ VAR's.}$$

Case (i):

$$\text{If } |X_L| > |X_C|$$

$$Z = R + jX_{\text{net}}$$

→ R-L

'I' lags 'V' by $\phi < 90^\circ$

(lagging P.F)

General Nature of
Electrical power System

Case (ii):

$$\text{If } |X_L| < |X_C|$$

$$Z = R - jX_{\text{net}}$$

→ R-C

'I' leads 'V' by $\phi < 90^\circ$

(leading P.F)

Case (iii):

$$\text{If } |X_L| = |X_C|$$

$$Z = R$$

→ purely Resistive

'I' in phase with 'V'

$$\phi = 0^\circ [\text{UPF}]$$

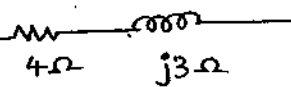
→ Electrical Resonance

$$Z = R + jX$$

$$\text{Ex: } Z = (4 + j3) \Omega$$

$$Z = R + jX_L$$

+ve Reactance
is Inductive



$$Z = R - jX_C$$

-ve Reactance
is Capacitive

$$Y = \frac{1}{Z}$$

↓
admittance

Units: mho (Ω)
Siemens.

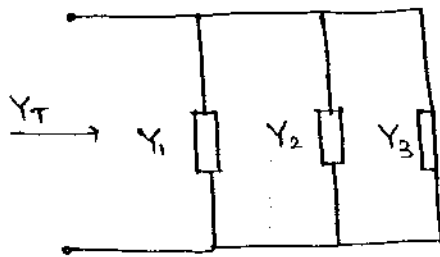
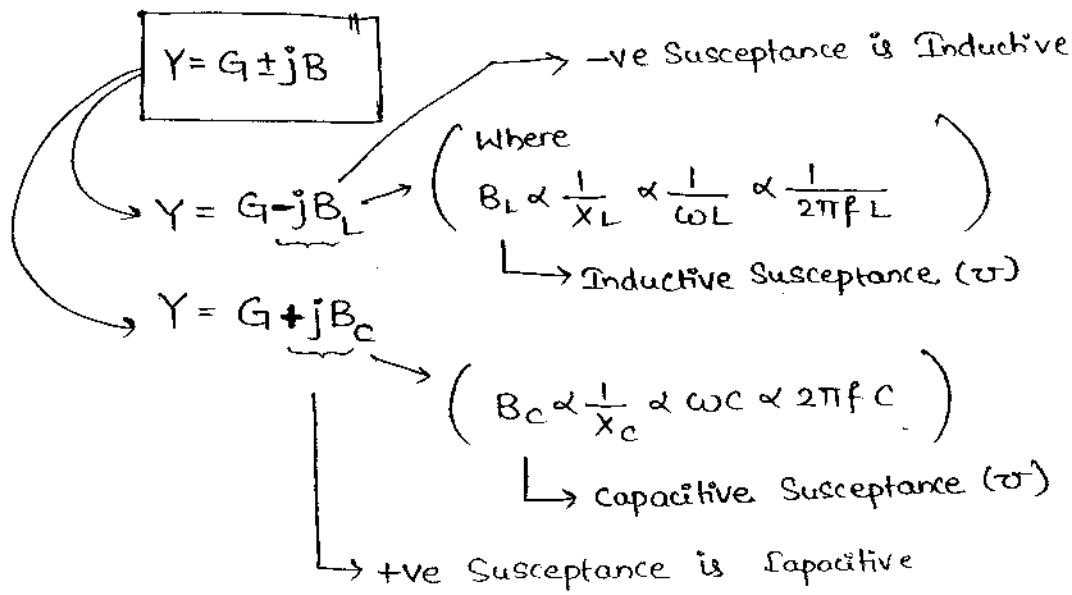
$$Y = \frac{1}{4 + j3} \times \frac{4 - j3}{4 - j3}$$

$$Y = \frac{4 - j3}{25} = \frac{4}{25} - j\frac{3}{25}$$

$$Y = 0.16 - j0.12 \text{ } \Omega$$

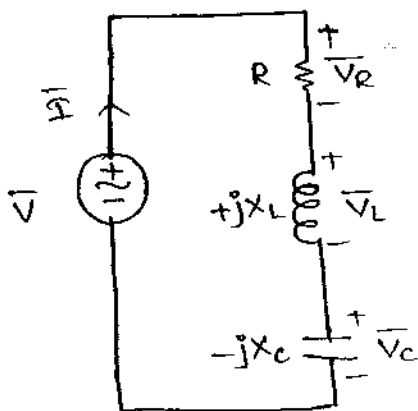
→ -ve Susceptance

is Inductive.



$$Y_T = Y_1 + Y_2 + Y_3$$

Q Draw the phasor diagram of Series RLC ckt if $|X_L| > |X_C|$



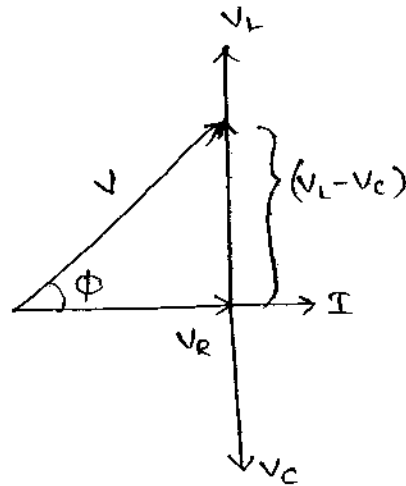
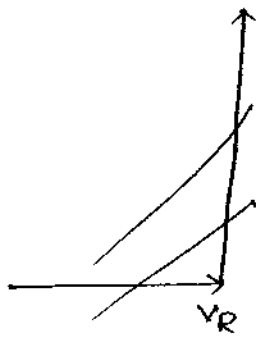
$$\bar{V}_R = I \cdot R \angle 0^\circ$$

$$V_L = +jX_L I = I \cdot X_L \angle +90^\circ$$

$$V_C = -jX_C I = I \cdot X_C \angle -90^\circ$$

$$\therefore |X_L| > |X_C|$$

$$|V_L| > |V_C|$$



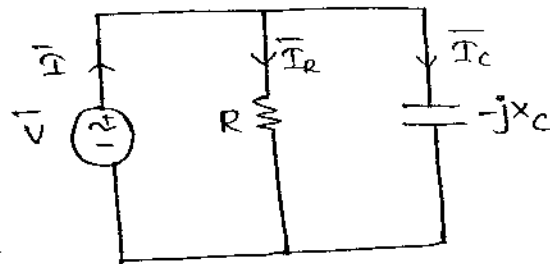
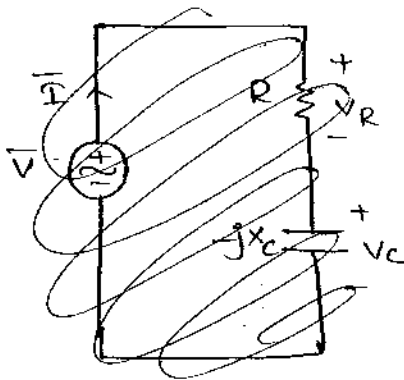
$$|V| = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\phi = \tan^{-1} \left[\frac{V_L - V_C}{V_R} \right]$$

$$\text{P.F.}, \cos \phi = \frac{V_R}{V} [\text{lag}]$$

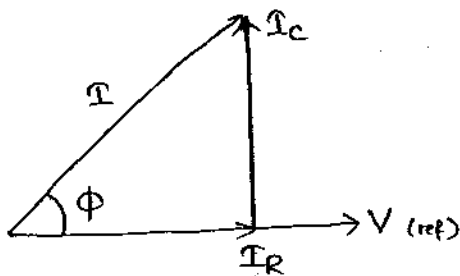
105

draw the phasor diagram of a parallel R-C kkt.



$$\bar{I}_R = \frac{V}{R} \angle 0^\circ$$

$$\bar{I}_C = \frac{V}{-jX_C} = \frac{V}{X_C} \angle +90^\circ$$

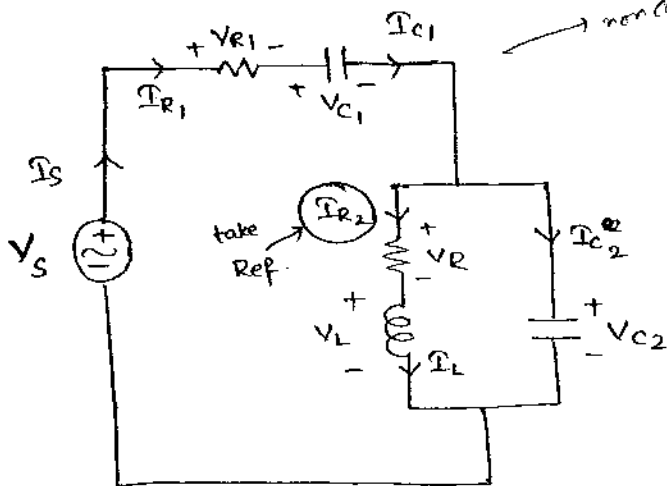


$$|I| = \sqrt{I_R^2 + I_C^2}$$

$$\phi = \tan^{-1} \left[\frac{I_C}{I_R} \right]$$

$$\text{P.F.} : \cos \phi = \frac{I_R}{I} (\text{lead})$$

Draw the phasor diagram Relating all the elemental Voltages & Currents in the ckt shown.

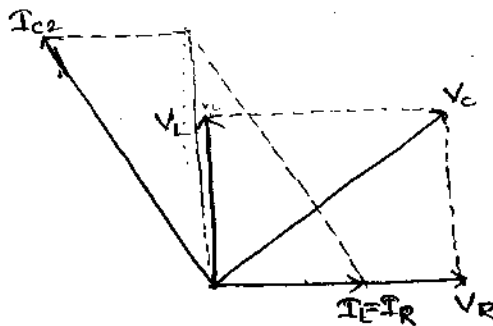


→ non canonical form
neither V , nor I is same
for each element

if V, I is same for all
elements → canonical form

↓

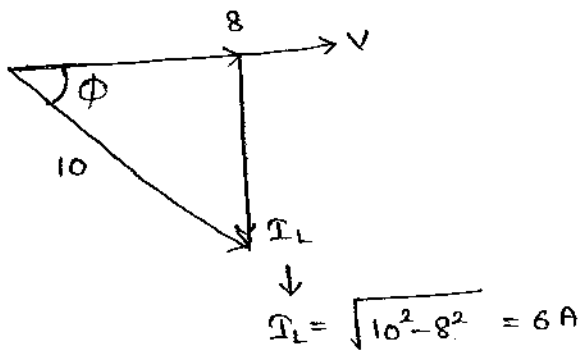
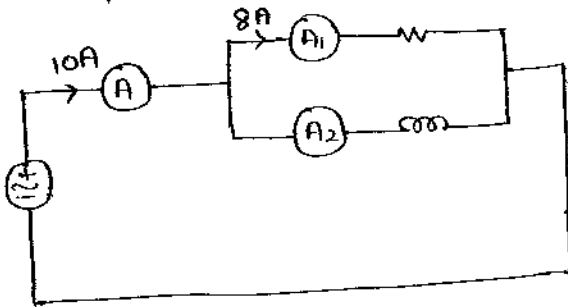
(parallel → V same)
(series → I same)



Gate/119.

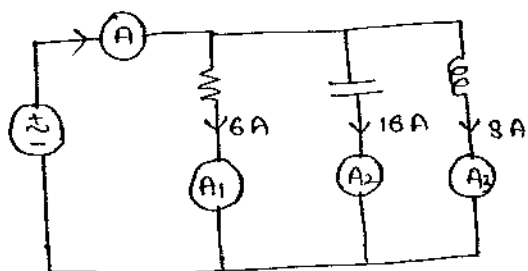
If $(A) = 10A$, $(A_1) = 8A$ then $(A_2) = \underline{\hspace{2cm}}$
ckt power factor is

106

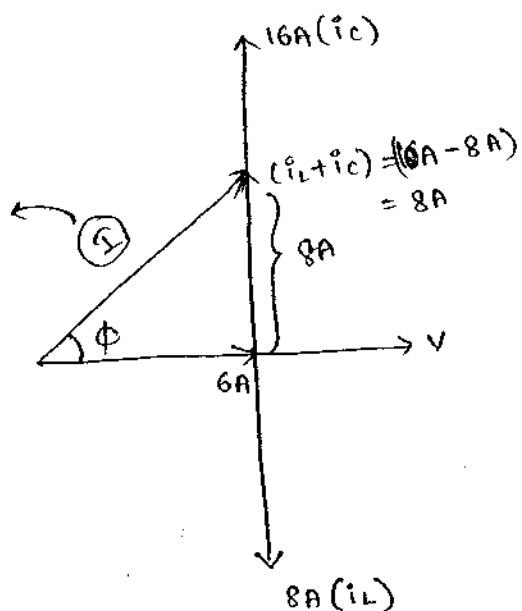


$$P.F = \cos \phi = \frac{8}{10} = 0.8 \text{ (lag)} \\ (\because \text{current lagging})$$

165(0) If $A_1 = 6A$, $A_2 = 16A$, $A_3 = 8A$ then $A = 10A$
 total supply P.F is 0.6 lead



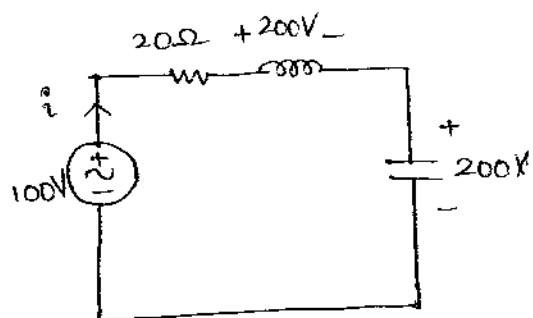
$$I = \sqrt{6^2 + 8^2} = 10A$$

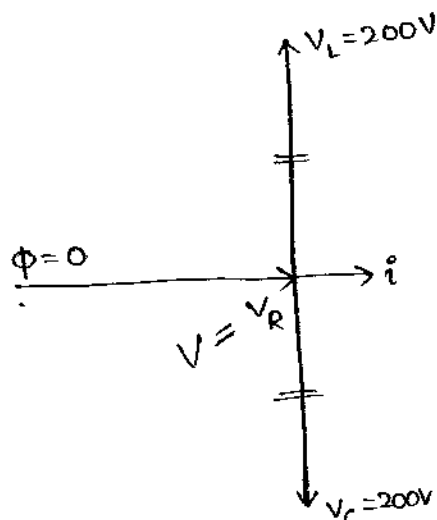


$$\cos \phi = \frac{6}{10} = 0.6 (\text{lead})$$

Find current $i =$ _____

Total Input P.F = _____





$$I_R = \frac{V_R}{R} = \frac{V}{R} = \frac{100}{20} = 5A = I$$

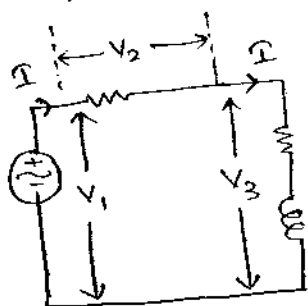
$$\begin{aligned} \text{P.F.} &= \cos \phi \\ &= \cos 0 \\ &= 1 \text{ (UPF)} \end{aligned}$$

107

If $V_1 = 220V$, $V_2 = 122V$, $V_3 = 136V$, then load P.F. = _____

also if load resistance is 5Ω , then average power absorbed by load is _____

Active (or) Real power in watts
(\because for P_{avg} for pure L & $C = 0$)
avg. pow. only exist of Resistor.

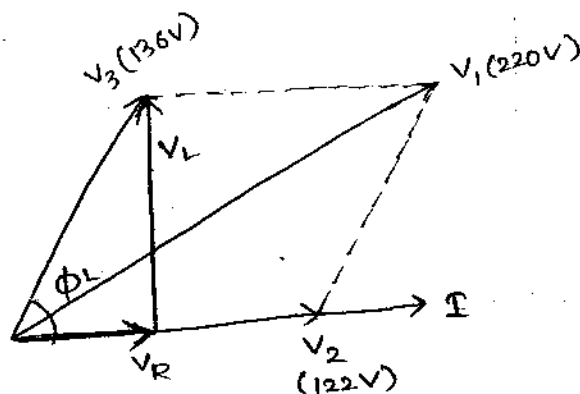


$$R_T = \sqrt{R_1^2 + R_2^2 + 2R_1R_2 \cos \theta}$$

$$V_1 = \sqrt{V_2^2 + V_3^2 + 2V_2V_3 \cos \phi}$$

$$220 = \sqrt{122^2 + 136^2 + 2(122)(136) \cos \phi_L}$$

$$\cos \phi_L = 0.45 \text{ (lag)}$$

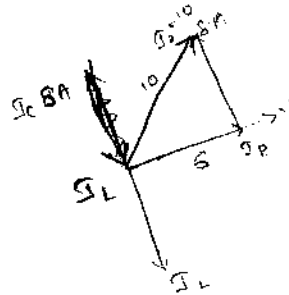
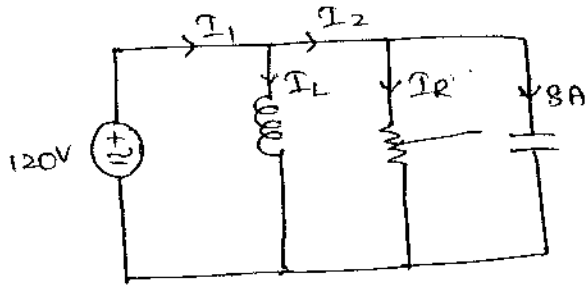


$$\cos \phi_L = \frac{V_R}{V_3} \rightarrow V_R = V_3 \cos \phi_L = 136(0.45) = 61.2 \text{ Volts}$$

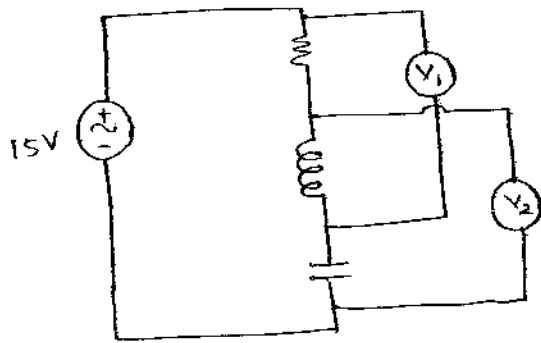
$$P_{avg} = P = \frac{[V_R]^2}{R} = \frac{(61.2)^2}{5} = 750W$$

H.W, If $|I_1| = |I_2| = 10A$ then find $I_P = \underline{6A}$, $I_L = \underline{16A}$

Total Current P.F =



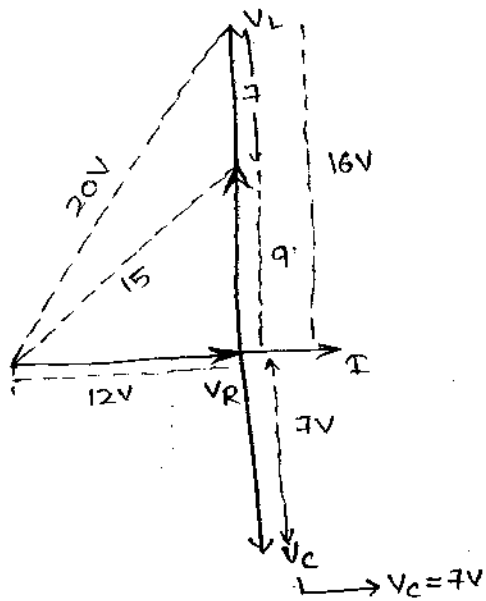
7. Gen If $V_1 = 20V$, $V_2 = 9V$ then voltage across capacitor alone is



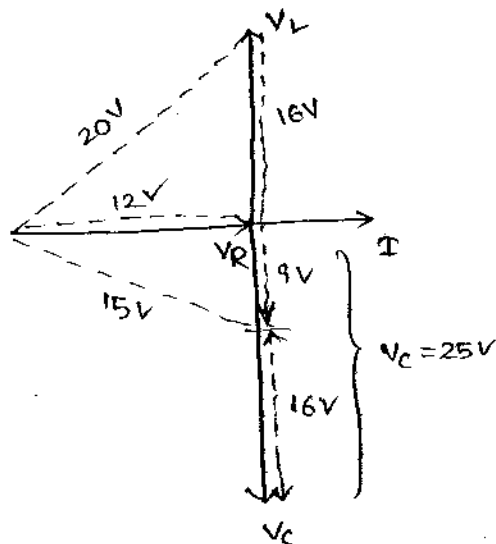
- (a) 12V
- (b) 16V
- ~~(c) 25V~~
- (d) none.

108

lag P.F.
 $(X_L > X_C)$
 $(V_L > V_C)$



lead P.F.
 $(X_L < X_C)$
 $(V_L < V_C)$



I. RMS / EFFECTIVE / TRUE VALUE of a Time Varying Signal:

It is that steady Equivalent value of Time Varying waveform which could also develop the same amount of heat as given by the Original waveform for a definite period of Time in the circuit.

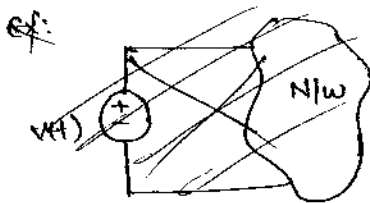
$$V_{RMS} = \sqrt{\frac{1}{T} \int_0^T [v(t)]^2 dt} \text{ Volts.}$$

RMS: V, I, ϕ

↳ exists for single elec. parameter
↳ does not exist for "P"
↓

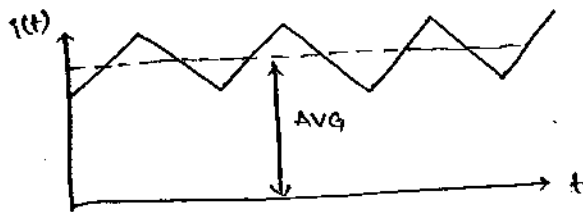
∴ Power is product of V, I
↓

∴ two elec. parameters

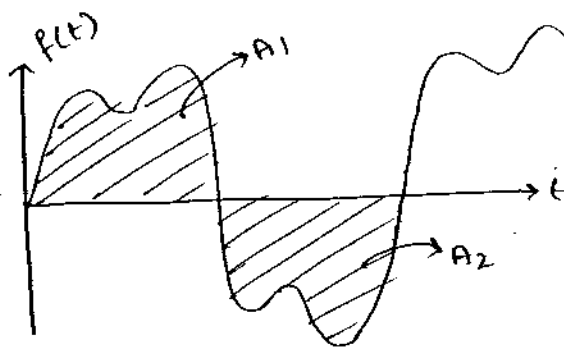


II. MEAN / AVERAGE VALUE

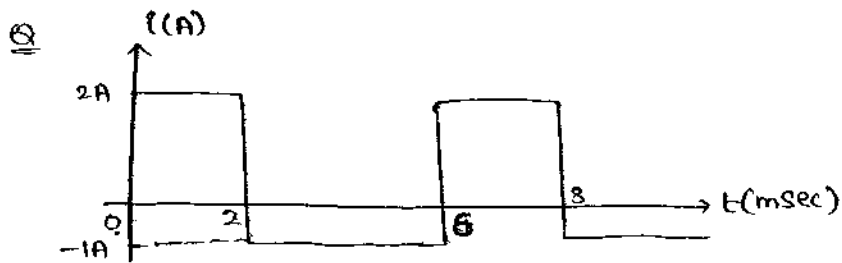
It is that steady Equivalent value of Time Varying waveform which could also Transfer the same amount of charge as given by original waveform for a definite period of Time in the ckt.



Concept of Symmetry in waveform:



if $|A_1| = |A_2| \rightarrow$ Symmetrical
 $|A_1| \neq |A_2| \rightarrow$ Asymmetrical



109

$$\left. \begin{aligned} \text{+ve Area} &= |2 \times 2m| = 4mC \\ \text{-ve Area} &= |-1 \times 4m| = 4mC \end{aligned} \right\} \text{Symmetrical.}$$

Note:

The avg value of any Symmetrical waveform for one full cycle is always zero.

(a) For Symmetrical waveform:

$$V_{AVG} \begin{cases} \rightarrow \text{Full cycle} \\ \rightarrow \frac{1}{T/2} \int_0^{T/2} V(t).dt \rightarrow \text{Half cycle.} \end{cases}$$

(b) Asymmetrical waveform:

$$V_{AVG} = \frac{1}{T} \int_0^T V(t).dt$$

III] peak/crest factor = $\frac{V_{max}}{V_{RMS}}$

IV] form/shape factor = $\frac{V_{RMS}}{V_{AVG}}$

V] peak-to-peak = $|V_{max} - V_{min}|$

Notes

1. AC Analog meter \rightarrow M.I Type \rightarrow RMS Values
2. DC Analog meter \rightarrow PMMC Type \rightarrow Average Values
3. Rectifier Type meter.

$$[\text{Actual Value}] = [\text{Average Value}] * [F.F.]$$

\downarrow
if calibrated for sinusoid \rightarrow (1.11)

Note:

In most of our Elec. Engg applications directly or Indirectly lot of Heat is generated during InterConversion and Transfer of elec. Energy. & Hence we prefer Calculating RMS Values in general.

ex: In India for domestic Supply System

1 ϕ , 230 Volts, 50Hz

\downarrow
RMS Voltage [~~V~~ V_{L-N} (phase Voltage)]

$$V(t) = 230\sqrt{2} \sin(2\pi 50t)$$

$$V(t) = 325 \sin(314t) \text{ Volts.}$$

Ex: INDIA, in power System,

3- ϕ , 66kV, 50Hz (Balanced)

\downarrow
RMS value [V_{L-L} (line Voltage)]

$$V_{RY} = 66000\sqrt{2} \sin(314t)$$

$$V_{YB} = 66000\sqrt{2} \sin(314t - 120)$$

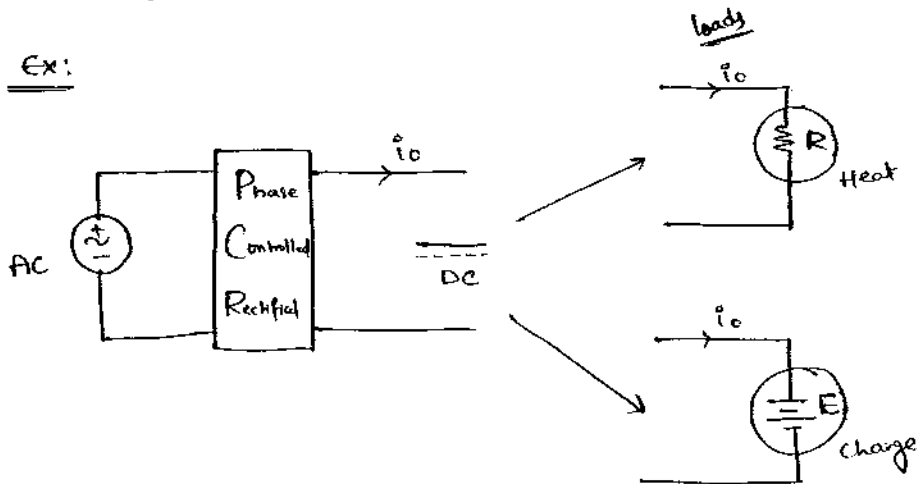
$$V_{BR} = 66000\sqrt{2} \sin(314t - 240)$$

However, in Certain specific appl. like Battery charging, electroplating, electrometallurgical Refineries, electrolysis,

Speed Control of DC motor with Back emf etc., We prefer Calculating Average Values.

110

Ex:



$$P_o = i_o^2 R \text{ W}$$

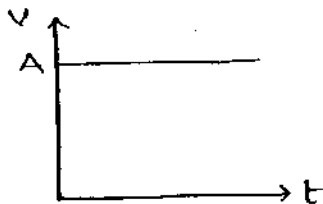
\uparrow
RMS

$$P_o = E i_o \text{ W}$$

\uparrow
avg

Standard Waveforms:

(1) Ideal DC.



$$V_{RMS} = A$$

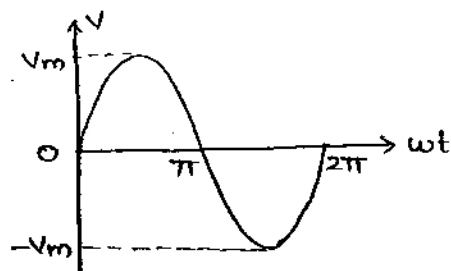
$$V_{AVG} = A$$

$$\text{Peak Factor} = 1$$

$$F.F = 1$$

$$V_{p-p} = A$$

(2) Ideal AC



$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$$V_{AVG} \rightarrow 0 \rightarrow \text{full cycle}$$

$$V_{AVG} \rightarrow \frac{2V_m}{\pi} \rightarrow \text{Half cycle.}$$

$$P.F = \sqrt{2}$$

$$F.F = \frac{\pi}{2\sqrt{2}} = 1.11$$

$$V_{pp} = 2V_m$$

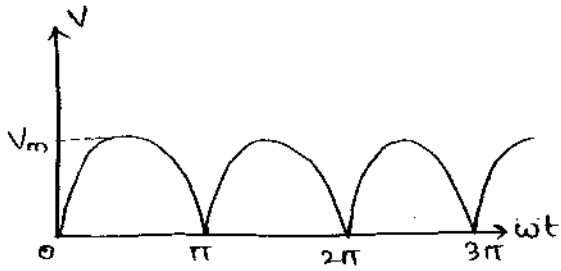
$$V_{AVG} = \frac{1}{2\pi} \int_0^{2\pi} V_m \sin \omega t d\omega t$$

(Full cycle)

$$= \frac{1}{\pi} \int_0^{\pi} V_m \sin \omega t d\omega t$$

(Half cycle)

uncontrolled
(3) Full Wave Rectifier



$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

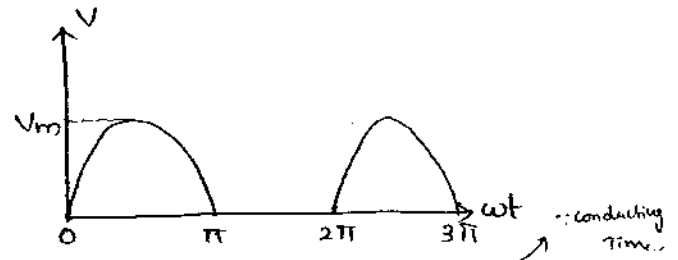
$$V_{AVG} = \frac{2V_m}{\pi}$$

$$P.F. = \sqrt{2}$$

$$F.F. = 1.11$$

$$V_{p-p} = V_m$$

Uncontrolled
(4) Half wave Rectifier



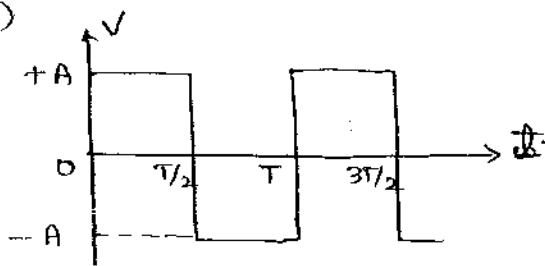
$$V_{RMS} = \frac{V_m}{2} \sqrt{\frac{1}{2\pi} \int_0^\pi V_m^2 \sin^2 \omega t d\omega}$$

$$V_{AVG} = \frac{V_m}{\pi}$$

$$P.F. = 2, F.F. = \frac{\pi}{2} = 1.57$$

$$V_{p-p} = V_m$$

(5)



$$V_{RMS} = A$$

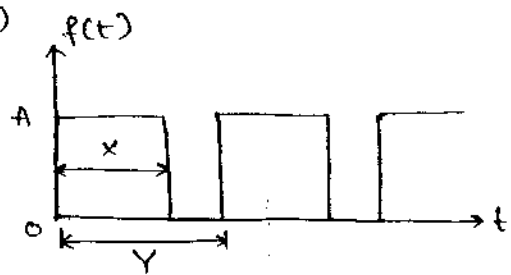
$$V_{AVG} = \begin{cases} 0 & \rightarrow \text{Full cycle} \\ A & \rightarrow \text{Half cycle} \end{cases}$$

$$P.F. = 1, F.F. = 1$$

$$V_{p-p} = 2A$$

chopped DC

(6)

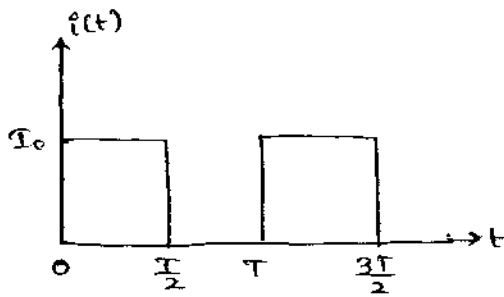


$$f_{RMS} = A \sqrt{\frac{X}{Y}} \rightarrow \text{on period} \rightarrow \text{Total period}$$

$$f_{AVG} = A \left[\frac{X}{Y} \right]$$

$$RMS = \text{Amp} \times \sqrt{\frac{\text{ON period}}{\text{Total period}}}$$

(7)



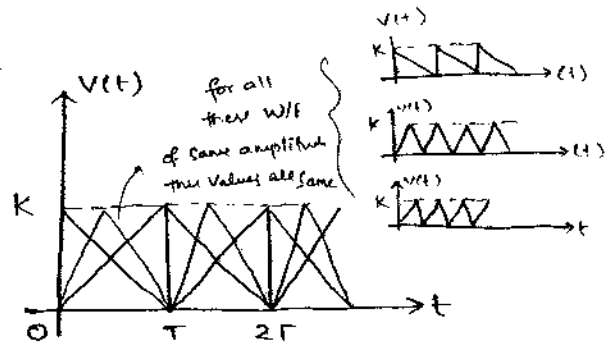
$$I_{RMS} = \frac{I_0}{\sqrt{2}}$$

$$I_{AVG} = \frac{I_0}{2}$$

$$P.F = \sqrt{2}, F.F = \sqrt{2}$$

$$I_{pp} = I_0$$

(8)



$$V_{RMS} = \frac{K}{\sqrt{3}} = \sqrt{\frac{1}{T} \int_0^T \left(\frac{K}{T} \cdot t\right)^2 dt}$$

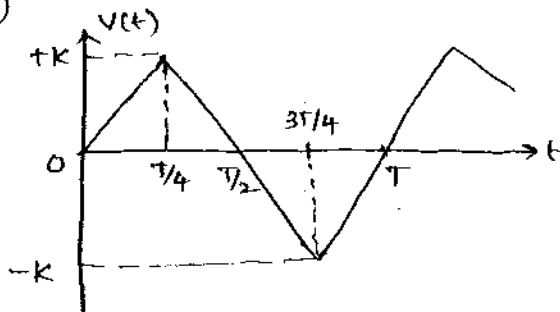
$$V_{AVG} = \frac{K}{2}$$

$$P.F = \sqrt{3}, F.F = \frac{2}{\sqrt{3}}$$

$$V_{p-p} = K$$

(111)

(9)



$$V_{RMS} = \frac{K}{\sqrt{3}}$$

$$V_{AVG} \begin{cases} \rightarrow 0 \rightarrow \text{Full cycle} \\ \rightarrow \frac{K}{2} \rightarrow \text{Half cycle} \end{cases}$$

$$P.F = \sqrt{3}, F.F = \frac{2}{\sqrt{3}}$$

$$V_{p-p} = 2K$$

Note: In practical applications to Signals & Systems at Signal level & power electronics are at power level the w/fs are neither ideal nor standard but mostly periodic. Hence we do Fourier Series Expansion to Express any periodic non sinusoidal w/f in terms of Sine & Cosine of higher order frequencies.

Conclusion I]

$$V(t) = V_0 + V_1 \sin \omega_1 t + V_2 \sin \omega_2 t + V_3 \sin \omega_3 t + \dots$$

$$\downarrow \text{DC}$$

$$\rightarrow V_{\text{AVG}} = V_0$$

$$\rightarrow V_{\text{RMS}} = \sqrt{V_0^2 + \left(\frac{V_1}{\sqrt{2}}\right)^2 + \left(\frac{V_2}{\sqrt{2}}\right)^2 + \left(\frac{V_3}{\sqrt{2}}\right)^2 + \dots}$$

multiple frequencies } Fourier Series

II]

$$i(t) = I_0 + I_1 \cos(\omega_1 t - \phi_1) + I_2 \cos(\omega_2 t - \phi_2) + I_3 \cos(\omega_3 t - \phi_3) + \dots$$

phase shift has no impact on RMS

$$\rightarrow I_{\text{AVG}} = I_0$$

$$\rightarrow I_{\text{RMS}} = \sqrt{I_0^2 + \left(\frac{I_1}{\sqrt{2}}\right)^2 + \left(\frac{I_2}{\sqrt{2}}\right)^2 + \left(\frac{I_3}{\sqrt{2}}\right)^2 + \dots}$$

III]

$$V(t) = V_0 + V_1 \sin(\underbrace{n\omega t + \phi_1}_{\text{same frequencies}}) + V_2 \sin(\underbrace{n\omega t - \phi_2}_{\text{same frequencies}})$$

Same frequencies. \Rightarrow not Fourier Series.

$$\rightarrow V_{\text{avg}} = V_0$$

$$\rightarrow V(t) = V_0 + \underbrace{[V_1 \sin(\phi_1)] + [V_2 \sin(-\phi_2)]}_{\text{add phasor}}$$

$$\text{let } V(t) = V_0 + [V_x \sin(\phi_x)]$$

$$V(t) = V_0 + V_x \sin(n\omega t + \phi_x)$$

$$V_{\text{RMS}} = \sqrt{V_0^2 + \left(\frac{V_x}{\sqrt{2}}\right)^2} = \underline{V}$$

Case [I], Sinusoidal Supply to linear load. \Rightarrow Elec. ckt Analysis / Network Theory.

Let $V(t) = V \sin(\omega t + \theta) \underline{V}$

$i(t) = I \sin(\omega t + \phi) \underline{A}$

112

Then $P = \frac{V}{\sqrt{2}} \cdot \frac{I}{\sqrt{2}} \cdot \cos(\theta - \phi) \underline{W}$

$Q = \frac{V}{\sqrt{2}} \cdot \frac{I}{\sqrt{2}} \sin(\theta - \phi) \underline{\text{VARs}}$

$S = \frac{V}{\sqrt{2}} \cdot \frac{I}{\sqrt{2}} \underline{\text{VA's}}$

$P.F. = \frac{P}{S} = \cos(\theta - \phi)$

Case [II], Non Sinusoidal Supply to linear load. \Rightarrow Signals & Systems.

Let $V(t) = V_0 + V_1 \sin(\omega_1 t + \theta_1) + V_2 \sin(\omega_2 t + \theta_2) + V_3 \sin(\omega_3 t + \theta_3) + \dots$

$i(t) = I_0 + I_1 \sin(\omega_1 t + \phi_1) + I_2 \sin(\omega_2 t + \phi_2) + I_3 \sin(\omega_3 t + \phi_3) + \dots$

$P = V_0 I_0 + \frac{V_1}{\sqrt{2}} \cdot \frac{I_1}{\sqrt{2}} \cos(\theta_1 - \phi_1) + \frac{V_2}{\sqrt{2}} \cdot \frac{I_2}{\sqrt{2}} \cos(\theta_2 - \phi_2) + \frac{V_3}{\sqrt{2}} \cdot \frac{I_3}{\sqrt{2}} \cos(\theta_3 - \phi_3) + \dots \xrightarrow{\text{①}}$

$Q = \frac{V_1}{\sqrt{2}} \cdot \frac{I_1}{\sqrt{2}} \sin(\theta_1 - \phi_1) + \frac{V_2}{\sqrt{2}} \cdot \frac{I_2}{\sqrt{2}} \sin(\theta_2 - \phi_2) + \frac{V_3}{\sqrt{2}} \cdot \frac{I_3}{\sqrt{2}} \sin(\theta_3 - \phi_3) + \dots \xrightarrow{\text{②}}$

for DC
Reactive pow = 0
 $\therefore V_0 I_0 = 0$

$S = [V_{\text{TRMS}}] [I_{\text{TRMS}}]$

$= \left[\sqrt{V_0^2 + \left(\frac{V_1}{\sqrt{2}}\right)^2 + \left(\frac{V_2}{\sqrt{2}}\right)^2 + \dots} \right] \left[\sqrt{I_0^2 + \left(\frac{I_1}{\sqrt{2}}\right)^2 + \left(\frac{I_2}{\sqrt{2}}\right)^2 + \dots} \right] \underline{\text{VA's}} \xrightarrow{\text{②}}$

$P.F. = \frac{P}{S} = \frac{\text{①}}{\text{②}}$

Case [III]: Sinusoidal Supply to Non Linear Load. \Rightarrow pow. electronics or Analog electronics

Let $v(t) = V_1 \sin(\omega_1 t + \theta_1)$ V

$i(t) = I_0 + I_1 \sin(\omega_1 t + \phi_1) + I_2 \sin(\omega_2 t + \phi_2) + I_3 \sin(\omega_3 t + \phi_3) + \dots$ A

$P = \frac{V_1}{\sqrt{2}} \cdot \frac{I_1}{\sqrt{2}} \cos(\theta_1 - \phi_1)$ W

$Q = \frac{V_1}{\sqrt{2}} \cdot \frac{I_1}{\sqrt{2}} \sin(\theta_1 - \phi_1)$ VAR's

$S = [V_{\text{RMS}}] [I_{\text{RMS}}]$ VA's

$S = \left[\frac{V_1}{\sqrt{2}} \right] \left[\sqrt{I_0^2 + \left(\frac{I_1}{\sqrt{2}} \right)^2 + \left(\frac{I_2}{\sqrt{2}} \right)^2 + \dots} \right]$ VA's

$$\text{P.F.} = \frac{P}{S} = \frac{\left[\frac{V_1}{\sqrt{2}} \right] \cdot \left[\frac{I_1}{\sqrt{2}} \right] \cdot \cos(\theta_1 - \phi_1)}{\left[\frac{V_1}{\sqrt{2}} \right] \left[\sqrt{I_0^2 + \left(\frac{I_1}{\sqrt{2}} \right)^2 + \left(\frac{I_2}{\sqrt{2}} \right)^2 + \dots} \right]}$$

$$\text{P.F.} = \left[\frac{I_{1\text{RMS}}}{I_{\text{RMS}}} \right] \cdot \cos(\theta_1 - \phi_1)$$

\downarrow

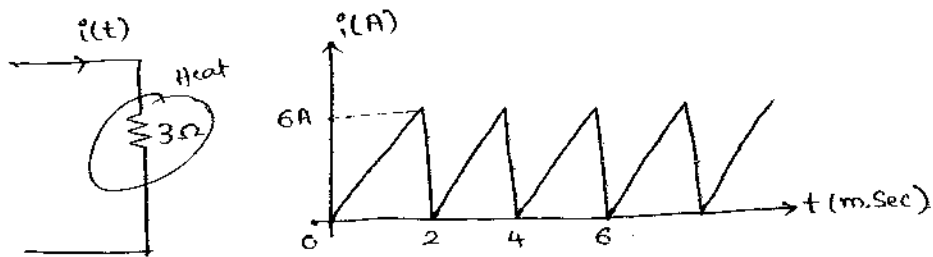
(< 1)

(< 1)

$$\left[\text{Total power factor} \right] = \left[\text{Distortion factor} \right] \left[\text{fundamental power factor} \right]$$

IES(0) power absorbed =

113

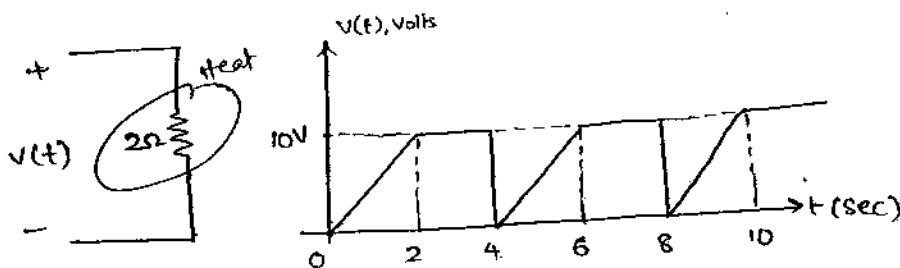


$$\begin{aligned}
 P_{abs} &= I_{RMS}^2 \cdot R \\
 &= \left[\frac{6}{\sqrt{3}} \right]^2 \cdot 3 \\
 &= 36 \text{ W}
 \end{aligned}$$

for saw tooth

$$I_{rms} = \frac{A}{\sqrt{3}}$$

B. power absorbed =



$$0 < t < 2: v(t) = 5t$$

$$2 < t < 4: v(t) = 10$$

$$V_{rms} = \sqrt{\frac{1}{4} \left[\int_0^2 (5t)^2 dt + \int_2^4 (10)^2 dt \right]}$$

$$= \frac{1}{2} \sqrt{\frac{25}{3} (8) + [100 \times 2]}$$

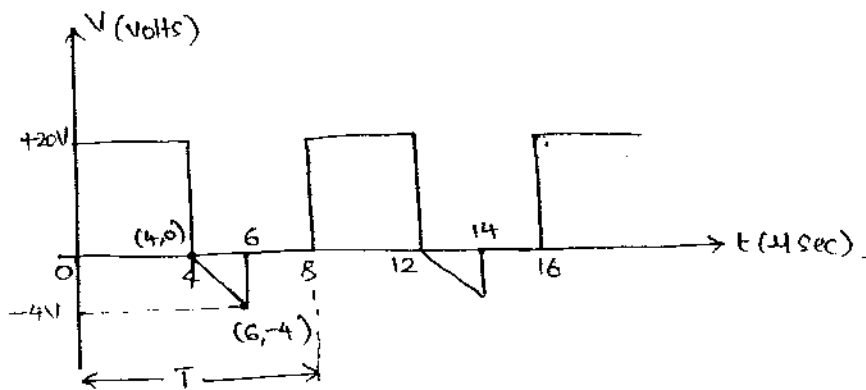
$$= \frac{5}{2} \sqrt{\frac{4}{3} + 2} = \frac{5\sqrt{6}}{\sqrt{3}}$$

$$P_{abs} = \frac{(V_{rms})^2}{R}$$

$$= \frac{(8.16)^2}{2}$$

$$= 33.33 \text{ W}$$

Q Find V_{RMS} , V_{AVG} , Peak factor, F.F, V_{PP}



$$0 < t < 4\mu: V(t) = 20$$

$$4\mu < t < 6\mu: V(t) = (-2t + 8) \left[\because (4-0) = \frac{-4-0}{6-4}(x-4) \right]$$

$$6\mu < t < 8\mu: V(t) = 0$$

$$V_{RMS} = \sqrt{\frac{1}{8} \left[\int_0^4 (20)^2 dt + \int_4^6 (-2t+8)^2 dt + \int_6^8 (0)^2 dt \right]} = 14.18 \text{ W}$$

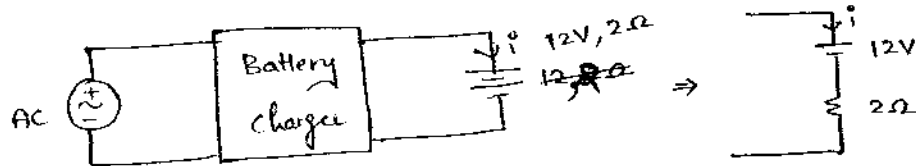
$$V_{AVG} = \frac{1}{8} \left[\int_0^4 (20) dt + \int_4^6 (-2t+8) dt + \int_6^8 (0) dt \right] = 9.5 \text{ W}$$

$$P.F = \frac{+20}{14.18} = 1.41$$

$$F.F = \frac{V_{RMS}}{V_{AVG}} = \frac{14.18}{9.5} = 1.49$$

$$V_{P-P} = | +20 - (-4) | = 24 \text{ Volts}$$

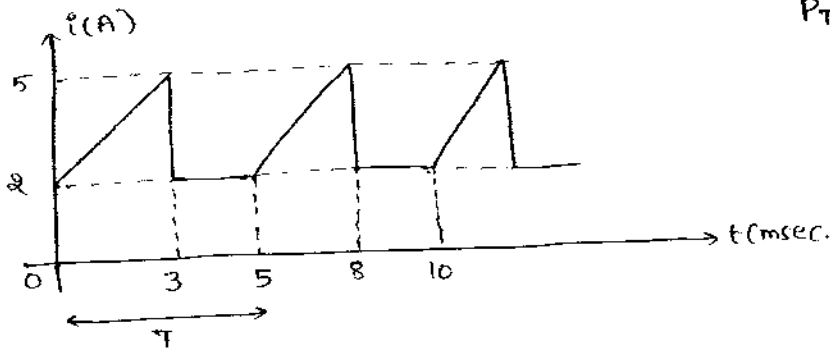
1ES(6) Total power absorbed by the Battery



avg \rightarrow charging (Battery)
RMS \rightarrow sink (R)

114

$$P_T = 12 \underset{\substack{\downarrow \\ \text{avg}}}{[i]} + \underset{\substack{\downarrow \\ \text{RMS}}}{[i]^2} \cdot R$$



$$\begin{aligned} \therefore P_T &= 12[2.9] + [3.06]^2 \cdot 2 \\ &= 34.8 + 18.72 \\ &= 53.52 \text{ W} \end{aligned}$$

$$0 < t < 3\text{m}$$

$$i(t) = (t+2)$$

$$3\text{m} < t < 5\text{m}$$

$$i(t) = 2$$

$$I_{\text{RMS}} = \sqrt{\frac{1}{5} \left[\int_0^3 (t+2)^2 dt + \int_3^5 (2)^2 dt \right]} = 3.06 \text{ A}$$

$$\frac{1}{5} \left(\frac{9}{2} + 2.3 + 2(2) \right) = 4.5 + 6 + 4 = 10.5$$

$$I_{\text{avg}} = \frac{1}{5} \left[\int_0^3 (t+2) dt + \int_3^5 (2) dt \right] = 2.9 \text{ A}$$

Gate If $v(t) = 8 + 20 \sin(\omega_1 t + 25) + 10 \cos(\omega_2 t - 25)$

find

(a) V_{AVG}

(b) V_{RMS} if $\omega_2 = 2\omega_1$

(c) V_{RMS} if $\omega_2 = \omega_1$

$$(a) V_{\text{AVG}} = 8 \text{ V (DC only)}$$

$$(b) v(t) = 8 + 20 \sin(\omega_1 t + 25) + 10 \cos(\underline{2\omega_1 t - 25}) \Rightarrow \text{FS}$$

$$V_{\text{RMS}} = \sqrt{8^2 + \left[\frac{20}{\sqrt{2}} \right]^2 + \left[\frac{10}{\sqrt{2}} \right]^2} = 17.7 \text{ V}$$

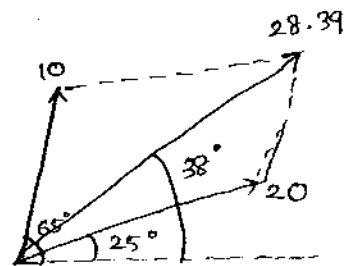
(c) $V(t) = 8 + 20 \sin(\omega_1 t + 25^\circ) + 10 \cos(\omega_1 t - 25^\circ) \Rightarrow \text{not F.S.}$

$$\downarrow$$

$$10 \sin(90 + \omega_1 t - 25^\circ)$$

$$10 \sin(\omega_1 t + 65^\circ)$$

$$V(t) = 8 + \underbrace{(20 \angle 25^\circ + 10 \angle 65^\circ)}_{\text{add phasors.}}$$



$$V(t) = 8 + [8.12 + j8.45] + [4.22 + j9.03]$$

$$= 8 + [22.34 + j17.5]$$

$$= 8 + [28.39 \angle 38^\circ]$$

$$V(t) = 8 + 28.39 \sin(\omega_1 t + 38^\circ)$$

$$V_{\text{RMS}} = \sqrt{8^2 + \left(\frac{28.39}{\sqrt{2}}\right)^2}$$

$$= 21.61 \text{ V}$$

Q14 If a current of $i(t) = 5 \cos(1000t + 100^\circ) \text{ A}$ is flowing through a load impedance of $(4 - j3) \Omega$ then the avg power absorbed by the load is _____

Q14

$$i(t) = 5 \cos(1000t + 100^\circ) \text{ A}$$

$$P_{\text{avg}} = \text{active power} = I_{\text{RMS}}^2 \cdot R$$

$$= \left[\frac{5}{\sqrt{2}}\right]^2 \cdot (4)$$

$$= \underline{\underline{50 \text{ W}}}$$

$$Q_{\text{net}} = ?$$

$$Q_{\text{net}} = I_{\text{RMS}}^2 X_{\text{net}} = \left[\frac{5}{\sqrt{2}}\right]^2 (3) = \underline{\underline{37.5 \text{ VAR}}} \text{ (generated } \therefore \text{ capacitor)}$$

$$S = I_{\text{RMS}}^2 \cdot Z = \left[\frac{5}{\sqrt{2}}\right]^2 (5) = \underline{\underline{62.5 \text{ VA's}}}$$

check pow. Δte

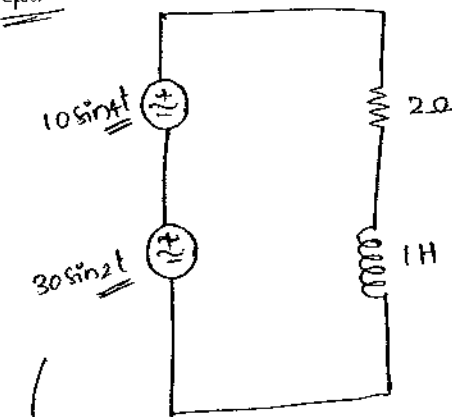
$$\text{P.F.} = \frac{R}{|Z|} = \frac{4}{5} = 0.8 \text{ (lead)}$$

$$\stackrel{(\infty)}{=} \frac{P}{S} = \frac{50}{62.5} = \underline{\underline{0.8}}$$

$$|S| = \sqrt{P^2 + Q_{\text{net}}^2}$$

$$= \underline{\underline{62.5 \text{ VA}}}$$

Gate



two sinusoidal functions of different freq \Rightarrow non sinusoidal supply \Rightarrow Fourier Series \Rightarrow Case (ii)

find (a) P_{avg}

(b) Total Supply power factor.

115

\Rightarrow two sources \Rightarrow apply S.P.T (Superposition Theorem)

Step-I: When $30 \sin 2t$ alone acting

$$I_1 = \frac{V_1}{Z_1} = \frac{30 \sin 2t}{2 + j(2)(1)} = \frac{30 \sin 2t}{2 + j2}$$

$$= \frac{30 \sin 2t}{2\sqrt{2} \angle 45^\circ}$$

$$I_1 = 10.6 \sin(2t - 45^\circ) \text{ A}$$

Step-II: when $10 \sin 4t$ alone acting

$$I_2 = \frac{V_2}{Z_2} = \frac{10 \sin 4t}{2 + j(4)(1)} = \frac{10 \sin 4t}{2 + j4}$$

$$= \frac{10 \sin 4t}{\sqrt{20} \angle 63.43^\circ}$$

$$= 2.235 \sin(4t - 63.43^\circ) \text{ A}$$

$$(a) P_{avg} = \frac{30}{\sqrt{2}} \times \frac{10.6}{\sqrt{2}} \cos(45^\circ) + \frac{10}{\sqrt{2}} \times \frac{2.23}{\sqrt{2}} \cos(63.43^\circ)$$

$$= 117.41$$

$$(b) S = \sqrt{\left(\frac{30}{\sqrt{2}}\right)^2 + \left(\frac{10}{\sqrt{2}}\right)^2} \cdot \sqrt{\left(\frac{10.6}{\sqrt{2}}\right)^2 + \left(\frac{2.23}{\sqrt{2}}\right)^2}$$

$$= 171.26$$

Then

$$P.F = \frac{P}{S} = 0.68 \text{ (lagging)}$$

165(0) If $V(t) = 230\sqrt{2} \sin(100\pi t)$

$$i(t) = 8 + 15\sqrt{2} \sin(100\pi t - 40^\circ) + 12\sqrt{2} \sin(300\pi t - 60^\circ) + 9\sqrt{2} \sin(500\pi t - 80^\circ)$$

$\underbrace{\hspace{10em}}_{\text{lag}}$

(a) $P = \left[\frac{230\sqrt{2}}{\sqrt{2}} \right] \left[\frac{15\sqrt{2}}{\sqrt{2}} \right] \cos(40^\circ)$

$$= 2642 \text{ W}$$

(b) $Q = \left[\frac{230\sqrt{2}}{\sqrt{2}} \right] \left[\frac{15\sqrt{2}}{\sqrt{2}} \right] \sin(40^\circ)$

$$= 2217 \text{ VAR}$$

(c) $S = [230] \sqrt{8^2 + 15^2 + 12^2 + 9^2}$

$$= 5212 \text{ VA}$$

(d) $\text{F.P.F} = \cos 40^\circ = 0.76$

(e) $\text{D.F} = \frac{15}{\sqrt{8^2 + 15^2 + 12^2 + 9^2}} = 0.66$

(f) $\text{T.P.F} = \frac{P}{S} = (\text{D.F})(\text{F.P.F}) = 0.5 \text{ (lag)} \Rightarrow \text{fundamental P.F} = \text{lag}$

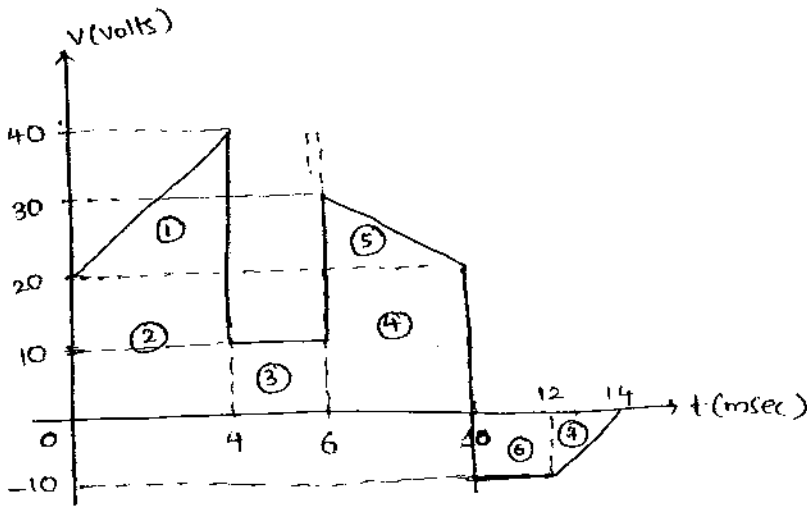
Here, power triangle is not valid

\therefore ~~Extra P~~ \because utilisation is fundamental only
but ^{some} power is utilized for Harmonics

$$|S| = \sqrt{P^2 + Q^2 + (\text{Harmonic factor})^2}$$

Gate

116



periodic Voltage w/f shown is given to a PMMC voltmeter then what is its Reading.
 ↓
 reads avg value.

$$V_{Avg} = \frac{1}{T} \int_0^T v(t) dt \Rightarrow \text{Area under curve.}$$

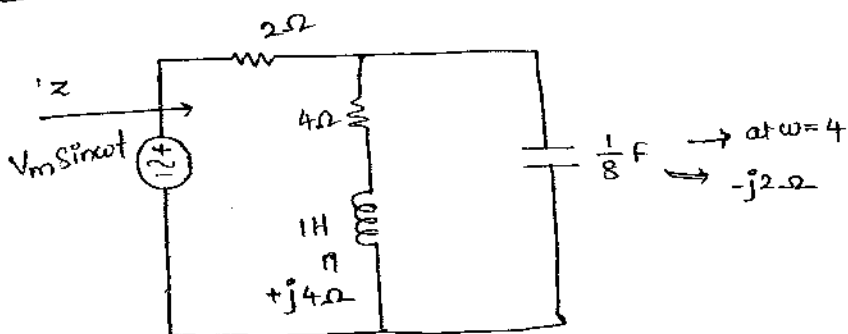
$$(+)\text{Area} = \frac{1}{2} (4) (20) + (20)(4) + (2)(10) + \frac{1}{2} (4)(20) + \frac{1}{2} \times 10 \times 4 = 240$$

$$(-)\text{Area} = \frac{1}{2} 10 \times 2 + \frac{1}{2} \times (2) \cdot (10) = 30$$

$$V_{Avg} = \frac{\text{Area}}{T} = \frac{(240-30)}{14} = \frac{210}{14} = \underline{\underline{15V}}$$

$$\frac{(4+j4) \times (j2)}{4-j2}$$

Gate At $\omega = 4$ rad/sec. find Total ckt P.F.



$$Z = 2 + [(4+j4) \parallel (-j2)]$$

$$= (2.8 \angle 2.4)$$

overall it behaves like

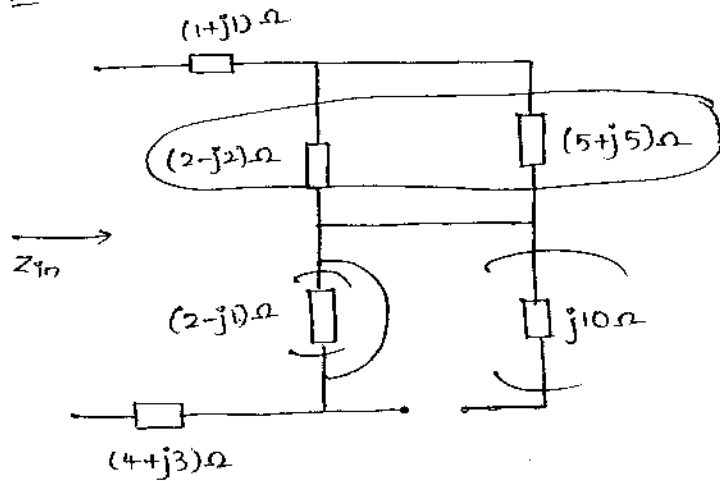
RC Network

\Rightarrow leading

$$P.F. = \frac{R}{|Z|}$$

$$= \frac{2.8}{\sqrt{(2.8)^2 + (2.4)^2}} = 0.75 (\text{lead})$$

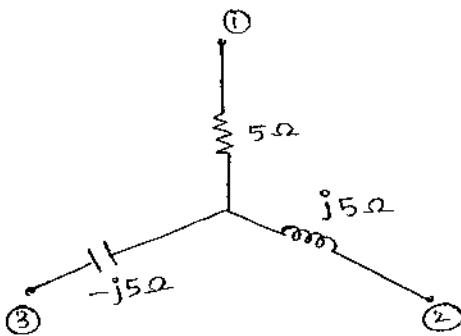
Q $Z_{in} =$



$$Z_{in} = (1+j1) + [(2-j2) \parallel (5+j5)] + (4+j3)$$

$$= 7.4 + j2.96$$

ES(0) Convert into Δ .

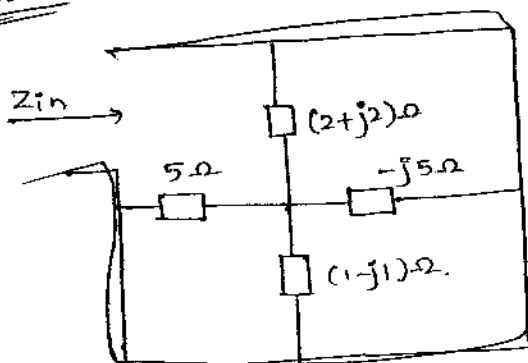


$$R_{12} = 5 + (j5) + \frac{(5)(j5)}{-j5} = j5 \Omega$$

$$R_{23} = j5 - j5 + \frac{(j5)(-j5)}{5} = +5 \Omega$$

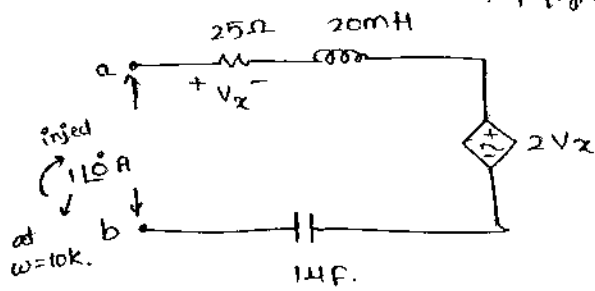
$$R_{31} = -j5 + 5 + \frac{(j5)(5)}{j5} = \underline{\underline{-j5 \Omega}}$$

IES(0)

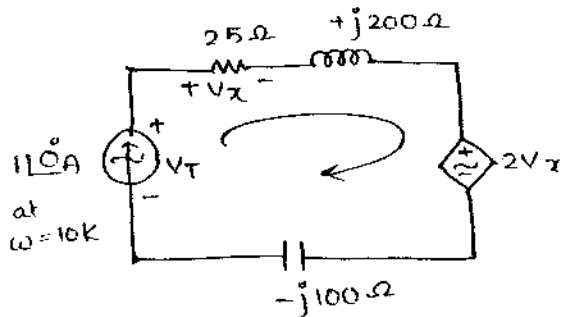


$\Rightarrow 0$ (dead S.C)

GATE At $\omega = 10 \text{ krad/sec}$, find total input Impedance & n/w power factor
 → if freq. given in rad/sec $\Rightarrow \sin$. but if given in Hz it may \sin or \cos ... etc.



117



KVL

$$-V_T + 110\angle 0^\circ [25 + j200 - j100] + 50\angle 0^\circ = 0$$

$$V_T = (75 + j100) \text{ V}$$

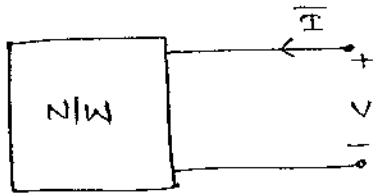
$$Z_{in} = \frac{V_T}{110\angle 0^\circ} = (75 + j100)\Omega$$

$$\text{P.F.} = \frac{R}{|Z|}$$

$$= \frac{75}{\sqrt{75^2 + 100^2}} = 0.6 (\text{lag})$$

($\because X_L > X_C$)

ES(10) The minimal realisation of N/W has _____ components.



If $\vec{V} = 30 \sin(1000t - 20^\circ) V = 30 \angle -20^\circ V$

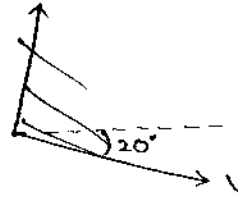
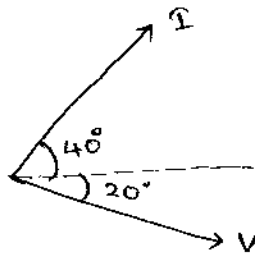
$\vec{I} = 5 \cos(1000t - 50^\circ) A = 5 \sin(1000t - 50^\circ + 90^\circ) = 5 \sin(1000t + 40^\circ)$
 $= 5 \angle 40^\circ A$

(a) R only

~~(b) R & C~~

(c) R & L

(d) L & C



$\Rightarrow \vec{I}$ leading Voltage by 60°
 \Rightarrow acts R-C circuit

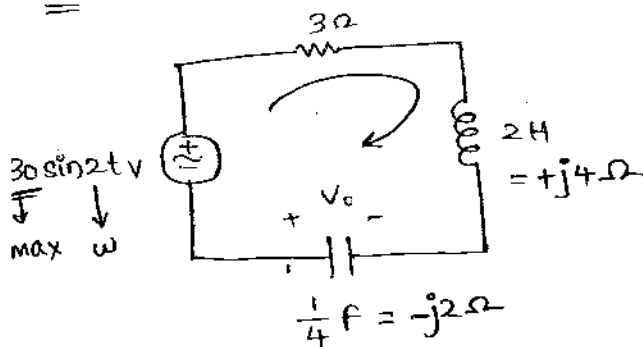
$Z = \frac{V}{I} = \frac{30 \angle -20^\circ}{5 \angle 40^\circ} = 6 \angle -60^\circ \Omega$

$Z = [3 - j5.19] \Omega$
 \downarrow
 $R = 3 \Omega$ $X_C = 5.19 \Omega$

But $X_C = \frac{1}{\omega C} \rightarrow C = \frac{1}{\omega X_C}$

$C = \frac{1}{(1000)(5.19)}$
 $= 192.6 \mu F$

Q find $V_o =$ _____



without mentioning anything if given in polar representation it is RMS value
 if functions \Rightarrow avg value

Q / Find $V_o = - [30 \angle 0^\circ] \left[\frac{-j2}{3+j4-j2} \right] = \frac{60 \angle 90^\circ}{(3+j2)}$

118

(-ve) due
in cap. (sink)
current entering
-ve side

$$= \frac{60 \angle 90^\circ}{3.6 \angle 33.6^\circ}$$

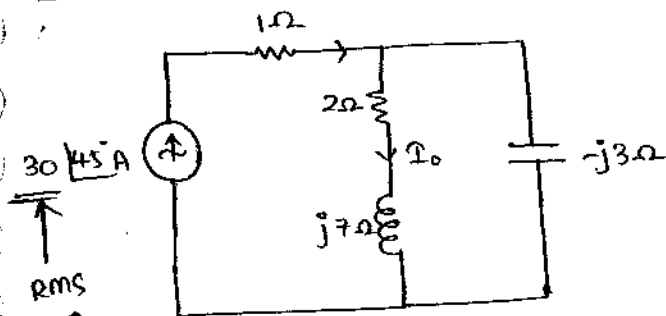
$$= 16.66 \angle 56.4^\circ$$

$$V_o = \underbrace{16.66}_{\text{max value}} \angle 56.4^\circ \text{ V} \longrightarrow \text{Phase Value}$$

$$V_o = 16.66 \sin(2t + 56.4^\circ) \longrightarrow \text{function}$$

$$V_o = \underbrace{\frac{16.66}{\sqrt{2}}}_{\text{RMS}} \angle 56.4^\circ = 11.78 \angle 56.4^\circ \longrightarrow \begin{array}{l} \text{RMS value (or)} \\ \text{Meter value (or)} \\ \text{Working value} \end{array}$$

Q find $I_o =$



$$I_o (\text{RMS}) = 30 \angle 45^\circ \left[\frac{-j3}{2+j7-j3} \right]$$

$$= \frac{90 \angle -45^\circ}{(2+j4)}$$

$$I_o (\text{RMS}) = \frac{90 \angle -45^\circ}{4.47 \angle 63.43^\circ}$$

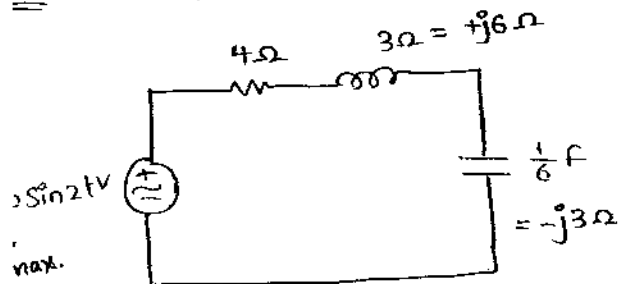
$$= 20.13 \angle -108.43^\circ$$



\Rightarrow phasor represents max value.

* The length of phasor Represents maximum value of the Electrical Quantity.

Q Calc. problems



Z
Y
I
 V_R, V_L, V_C
Verify KVL

P, Q, S
P.F [cos ϕ]
Phasor diagram

(a) $Z = (4 + j6 - j3) = (4 + j3) \Omega$

$|Z| = 5 \Omega$

(b) $Y = \frac{1}{Z} = \frac{1}{4 + j3} \cdot \frac{(4 - j3)}{(4 - j3)} = (0.16 - j0.12) \text{ S}$

$|Y| = \frac{1}{|Z|} = \frac{1}{5} = 0.2 \text{ S}$

(c) $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{\frac{50}{\sqrt{2}} \angle 0^\circ}{(4 + j3)} = 7.07 \angle -36.86^\circ \text{ A}$
 \downarrow
 ϕ (lagging)
 $\rightarrow I_S = I_R = I_L = I_C$ [series]

(d) $V_R = I_R \cdot R = [7.07 \angle -36.86^\circ] (4) = 28.28 \angle -36.86^\circ \text{ V}$

$V_L = +jX_L I_L = [7.07 \angle -36.86^\circ] [j6] = 42.42 \angle 52.14^\circ \text{ V}$

$V_C = -jX_C I_C = [7.07 \angle -36.86^\circ] [-j3] = 21.21 \angle -126.86^\circ \text{ V}$

(e) Verify KVL

$\vec{V}_S = \vec{V}_R + \vec{V}_C + \vec{V}_L$

LHS
 V_S
 (rms) $= \frac{50}{\sqrt{2}} \angle 0^\circ = 35.35 \angle 0^\circ \text{ V}$

$$\underline{RHS} = [28.28 \angle -36.86^\circ] + [42.42 \angle 53.14^\circ] + [21.21 \angle -126.86^\circ]$$

119

$$\approx 35.35 \angle 0.009$$

KVL \longrightarrow verified.

(f) for power calculations Consider RMS magnitudes only.

$$P = V_R \cdot I_R(1) = I_R^2 \cdot R = \frac{V_R^2}{R}$$

$$= (28.28)(7.07) = (7.07)^2(4) = \frac{(28.28)^2}{4} = 200 \text{ W}$$

$$Q_{net} = [+Q_L - Q_C]$$

$$Q_L = V_L I_L(1) = I_L^2 X_L = \frac{V_L^2}{X_L}$$

$$= (42.42)(7.07) = (7.07)^2(6) = \frac{(42.42)^2}{6} = 300 \text{ VARs}$$

$$Q_C = V_C I_C(1) = I_C^2 X_C = \frac{V_C^2}{X_C}$$

$$= (21.21)(7.07) = (7.07)^2(3) = \frac{(21.21)^2}{3} = 150 \text{ VARs}$$

$$Q_{net} = [+300 - 150] = +150 \text{ VAR}$$

$$S = V_S I_S = I_S^2 Z = \frac{V_S^2}{Z}$$

$$S = [35.35](7.07) = (7.07)^2(5) = \frac{(35.35)^2}{5}$$

$$S = 250 \text{ VARs}$$

$$(g) \text{ P.F} = \cos \phi = \frac{R}{|Z|} = \frac{P}{S}$$

$$\cos(36.86^\circ) = \frac{4}{5} = \frac{200}{250}$$

$$= 0.8 \text{ (lagging)}$$

also check

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1} \left[\frac{X_L - X_C}{R} \right]$$

$$|S| = \sqrt{P^2 + (Q_L - Q_C)^2}$$

$$\phi = \tan^{-1} \left[\frac{Q_L - Q_C}{P} \right]$$

$$P = S \cos \phi = V_S I_S \cos \phi$$

$$Q_{net} = \{+Q_L - Q_C\} = S \sin \phi = V_S I_S \sin \phi \text{ VARs.}$$

(b) phasor diagram:

→ Max. values.

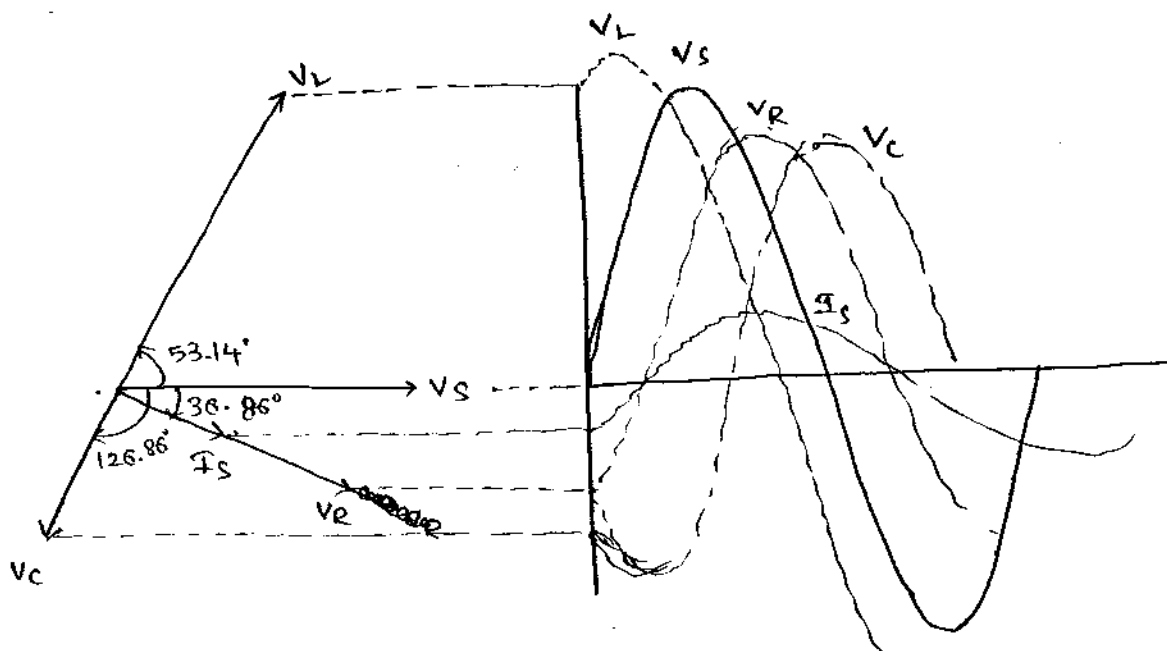
$$V_S = 50 \angle 0^\circ \text{ [Ref]}$$

$$I_S = I_R = I_L = I_C = 10 \angle -36.86^\circ \text{ A.}$$

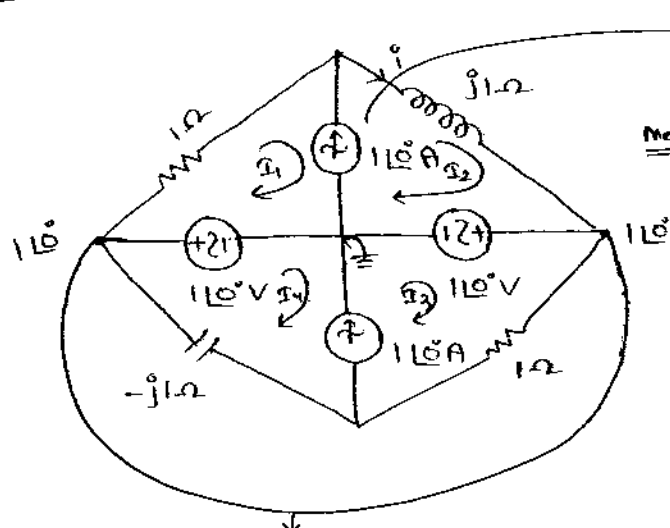
$$V_R = 40 \angle -36.86^\circ \text{ V}$$

$$V_L = 60 \angle +53.14^\circ \text{ V}$$

$$V_C = 30 \angle -126.86^\circ \text{ V}$$



Q determine Current ' i ' in the circuit shown.



→ Current source in parallel

120

\therefore Super mesh

Mesh $I_1(1) + j4[I_2] + [1\cancel{0}] - [1\cancel{0}] = 0$

$$\mathbf{I}_1 + j\mathbf{I}_2 = 0$$

$$-I_1 + I_2 = 110^\circ \text{ (link eqn)}$$

$$(1+j)I_2 = 1 \angle 0^\circ$$

$$T_2 = \frac{110^\circ}{1+j} = \frac{110^\circ}{\sqrt{2} \angle 45^\circ}$$

$$i = I_2 = 0.707 \text{ [45] n}$$

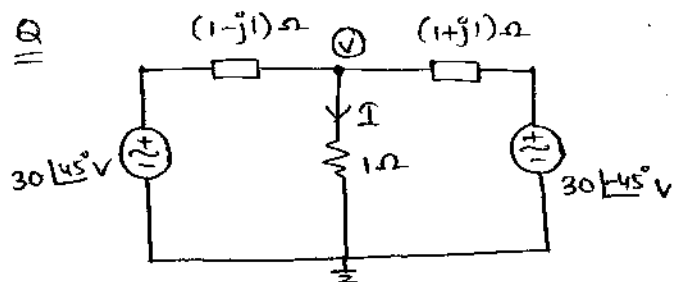
Nodal

∴ at same potential
Short & fold

\therefore By current division (1Ω & $j1\Omega$ are in parallel)

$$i = 1 \angle 0^\circ \text{ A} \left[\frac{1 \Omega}{1 + j1} \right]$$

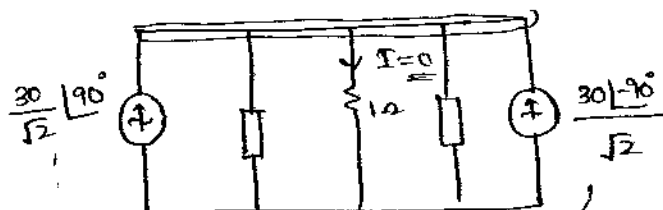
$$i = 0.707 \angle -45^\circ \text{ A}$$



Determine current 'I' by using Nodal analysis & verify result by applying any theorem in the easiest possible way

SPY

Source Tr. Tech.



Equal & opposite

1. T. O.

ନୋଡ଼ା

$$\frac{V - [30 \angle 45^\circ]}{(1-j)} + \frac{V}{1} + \frac{[V - [30 \angle 45^\circ]]}{1+j} = 0$$

$$\frac{(V - (30 \angle 45^\circ))(1+j)}{2} + \frac{V}{1} + \frac{(V - (30 \angle 45^\circ))(1-j)}{2} = 0$$

$$4V = (30 \angle 45^\circ)(1+j) + (30 \angle -45^\circ)(1-j)$$

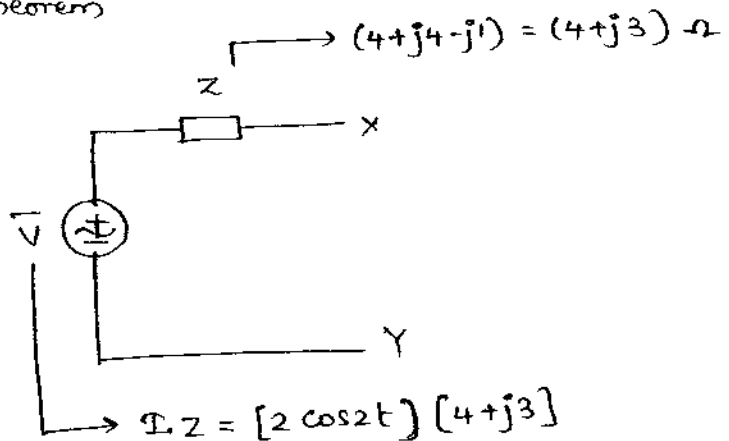
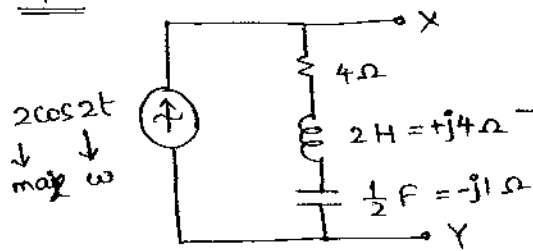
$$4V = 30\sqrt{2} \angle 90^\circ + 30\sqrt{2} \angle 90^\circ$$

$$V = 0 \implies \underline{I} \in \partial A$$

AC Network Theorems

Theorem I: Source Transformation Theorem

Gate

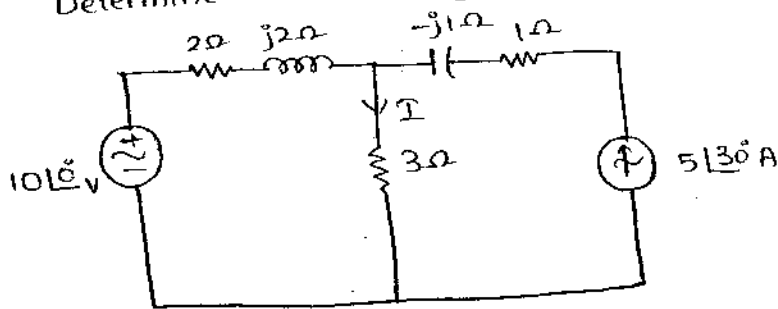


$$= [2 \cos 2t] [5 \angle 36.86^\circ]$$

$$\bar{V} = 10 \cos(2t + 36.86^\circ)$$

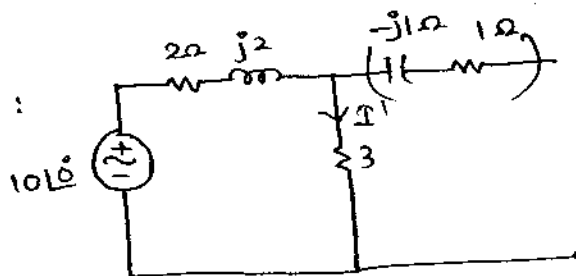
Theorem - II, III, IV

Determine Current "I" by using (a) SPT (b) Thevenins (c) Nortons.



(a) SPT

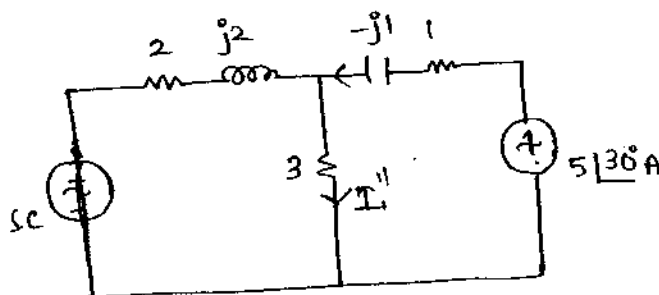
Step I:



$$\mathcal{I}' = \frac{10 \angle 0^\circ}{(5 + j2)}$$

$$= 1.85 \angle -21.8^\circ$$

Step II:



$$\mathcal{I}'' = 5 \angle 30^\circ \left[\frac{2 + j2}{2 + j2 + 3} \right]$$

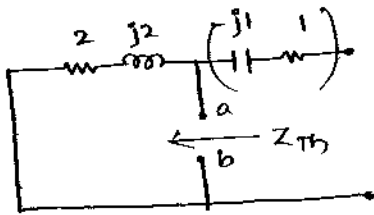
$$= 2.62 \angle 53.19^\circ \text{ A}$$

$$\mathcal{I} = \mathcal{I}' + \mathcal{I}'' = 3.58 \angle 23.2^\circ \text{ A}$$

(b) Thevenins.

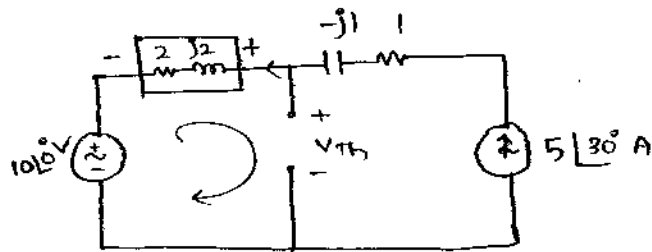
121

S-I Z_{Th}



$$Z_{Th} = (2 + j2)$$

S-II V_{Th}

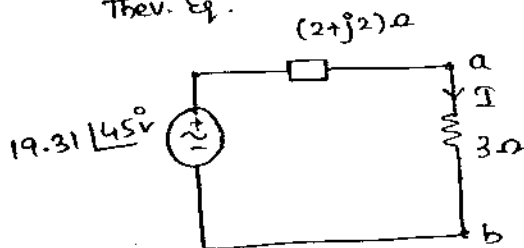


KVL

$$-10\angle 0^\circ - [5\angle 30^\circ][2 + j2] + V_{Th} = 0$$

$$V_{Th} = 19.31\angle 45^\circ$$

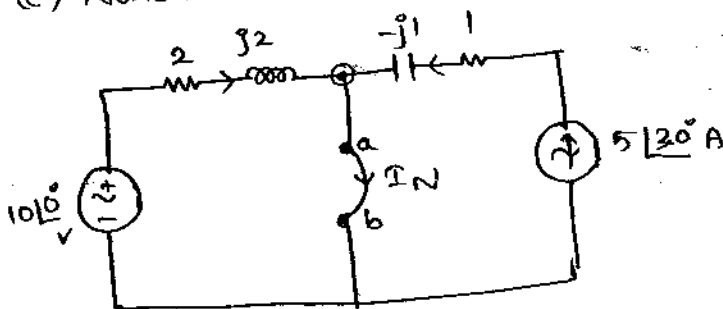
Thev. Eq.



$$I = \frac{19.31\angle 45^\circ}{(5 + j2)}$$

$$= 3.58\angle 23.2^\circ \text{ A}$$

(c) Nortons.

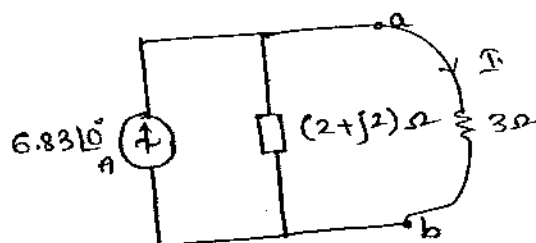


KCL

$$\frac{10\angle 0^\circ}{(2 + j2)} + [5\angle 30^\circ] = I_N$$

$$I_N = 6.83\angle 0^\circ$$

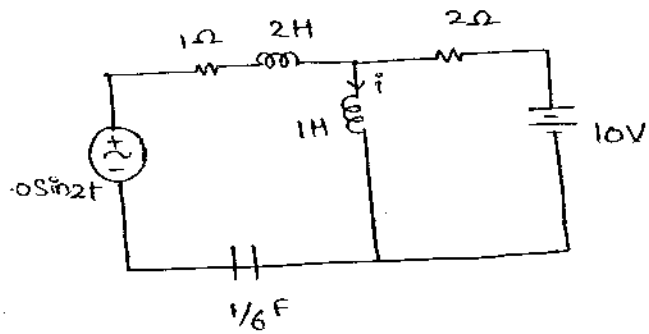
Nortons Eq.



$$I = 6.83\angle 0^\circ \left[\frac{2 + j2}{5 + j2} \right]$$

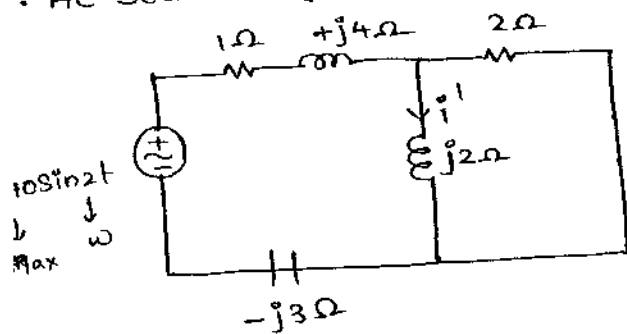
$$I = 3.58\angle 23.2^\circ \text{ A}$$

Q determine 'i' in the ckt shown.



⇒ It is a multifreq. (AC: $\omega=2$, DC: $\omega=0$)
Excited N/w & Solution in time domain be easily determined by SPT.

I. AC Source only



total cur^t ($\frac{V}{Z}$) Current division ↓

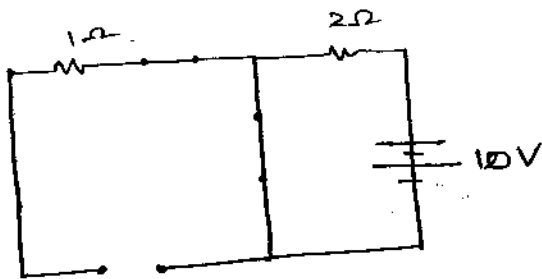
$$i' = \left[\frac{40 \sin 2t}{(1+j) + [2 \parallel j2]} \right] * \frac{2}{(2+j2)}$$

$$(1+j4-j3) + (2 \parallel j2)$$

$$= (40 \sin 2t) / (4 \angle 90^\circ) \text{ A}$$

$$i' = 10 \sin(2t - 90^\circ) \text{ A}$$

II DC Source only

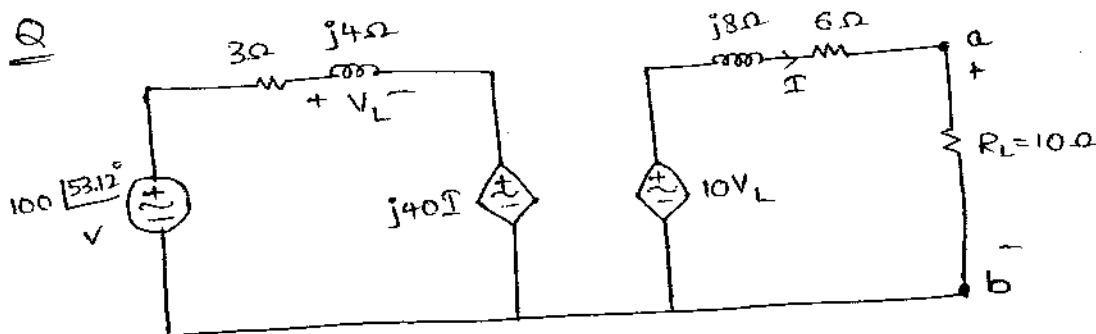


$$i'' = \frac{10}{2} = 5 \text{ A}$$

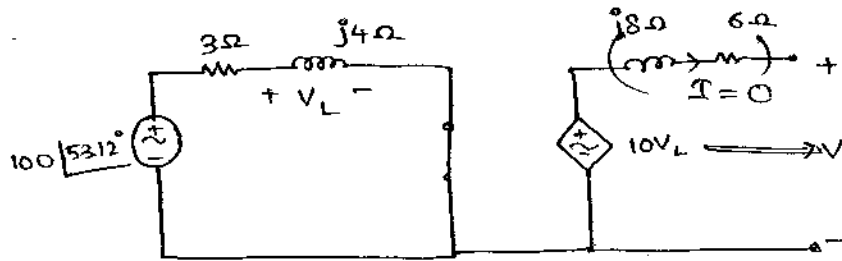
⇒ SPT
 $i = i' + i''$

$$i = 10 \sin(2t - 90^\circ) + 5 \text{ A}$$

Q



determine
Thevenins Voltage
across load



122

$$V_{Th} = 10V_L = 10[80\angle 90^\circ]$$

$$V_{Th} = 800\angle 90^\circ$$

$$= j800 \text{ V.}$$

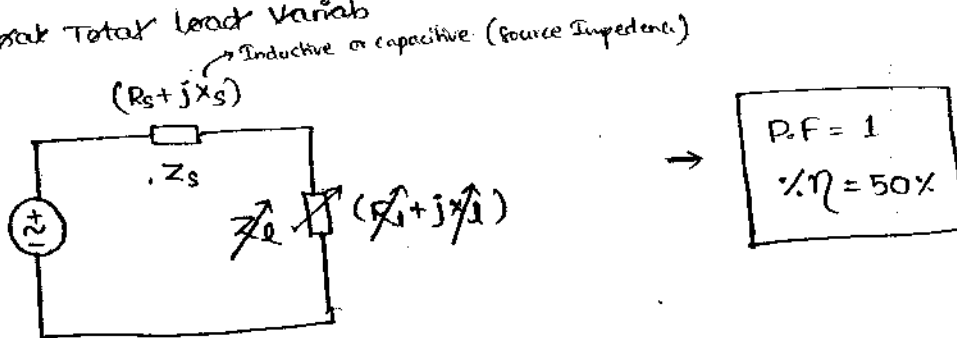
$$V_L = 100\angle 53.12^\circ \left[\frac{j4}{3+j4} \right] = 80\angle 90^\circ \text{ V}$$

Theorem - II

maximum power Transfer Theorem in AC N/w's:

Though There are 3 types of physical powers in AC N/w i.e. P, Q, S but the power that is Convertible or utilisable in any form is active power or Real power "P" Watts. So, maximum power Transfer theorem in AC Networks is Confined to Real power "P" & that to in the Resistive part of load N/w.

Show that Total load Variab



$$P.F = 1$$

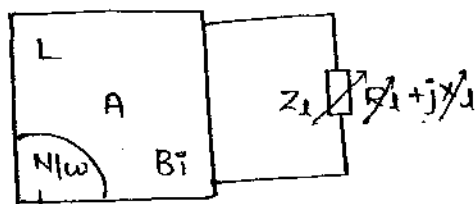
$$\% \eta = 50\%$$

Here both load Resistance & Reactance can be Independently Controlled. So, we can Compensate the Net Reactance to zero & ckt operates at UPF (i.e. only R). So, Here Both phase Balancing & Magnitude Balancing of Impedences can be achieved. So, P_{max} occurs in the load if load Impedence is Complex Conjugate of Source Impedence.

$$Z_L = Z_S^*$$

$$P_{max} = \frac{|V_S|^2}{4R_S} \text{ Watts}$$

In General



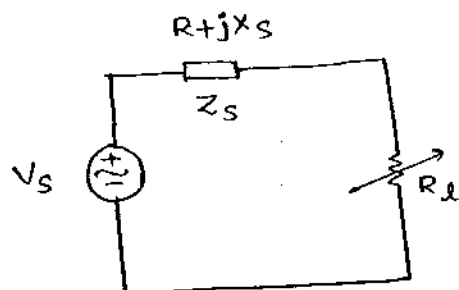
First Convert to Thevenin Equivalent.

Then for P_{max}

$$Z_L = Z_{Th}^*$$

$$P_{max} = \frac{[V_{Th}]^2}{4 R_{Th}} \omega.$$

Special case (a)



$$P.F. < 1.$$

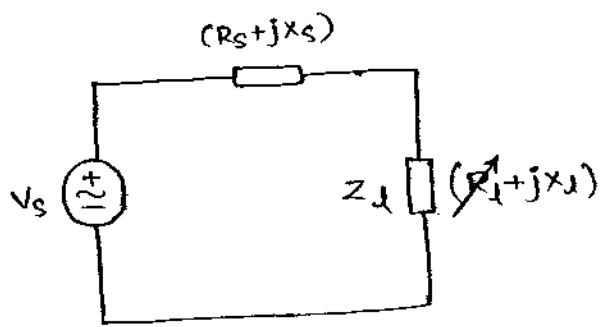
\therefore Here load is purely Resistive but Source has Impedance, ^{therefore} phase Balancing of Impedance is not possible. So, atleast match the Impedance

So, P_{max} occurs in load if

$$R_L = |Z_s| = \sqrt{R_s^2 + X_s^2}$$

but to find the P_{max} , Resubstitute this R_L Back into ckt & find RMS Current through it.

Special Case (b)



So P_{max} occurs in load if,

$$R_L = |R_s + jX_s + \cancel{R_s} + jX_L|$$

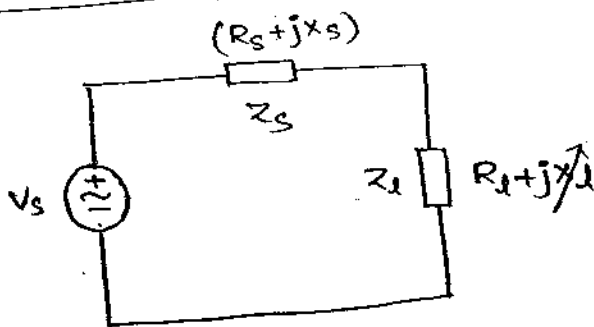
$$R_L = |R_s + j(X_s + X_L)|$$

$$R_L = \sqrt{R_s^2 + (X_s + X_L)^2}$$

Here, Though Both Source & Load have Impedances, only Load Resistance is adjustable, So we Can't Compensate the net Reactance to Zero. Here also Phase Balancing of Impedance is not possible. So, at least match the magnitude.

P.f < 1

Special Case (c)



So, P_{max} Occurs in the load,

if, $X_L = -X_s$

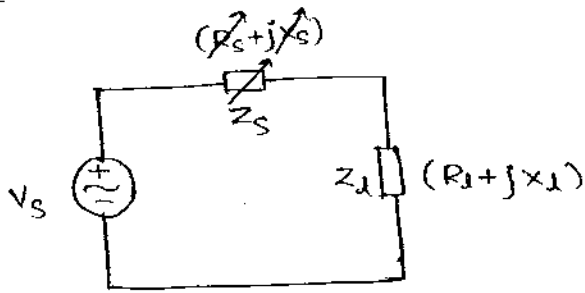
$X_L + X_s = 0$

Here, Since only Load Reactance is adjustable we Can Compensate the net Reactance to Zero. So, Here Phase Balancing of Impedance Can be achieved but magnitudes are fixed bcz R_L is Constant. So,

P.f < 1

$0 < \eta < 100$

Q for what value of Source Impedence Z_s , P_{max} Occure in the load .



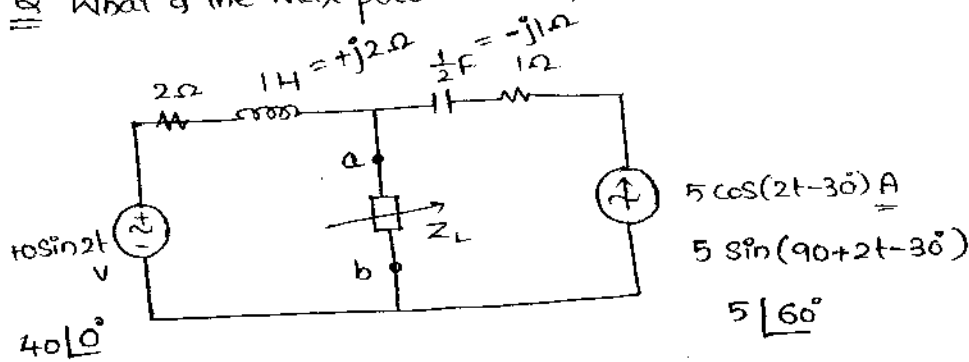
$$X_s \neq X_L$$

$X_s = -X_L \rightarrow$ Reactive power Compensation

$R_s = 0 \rightarrow$ Minimization of losses

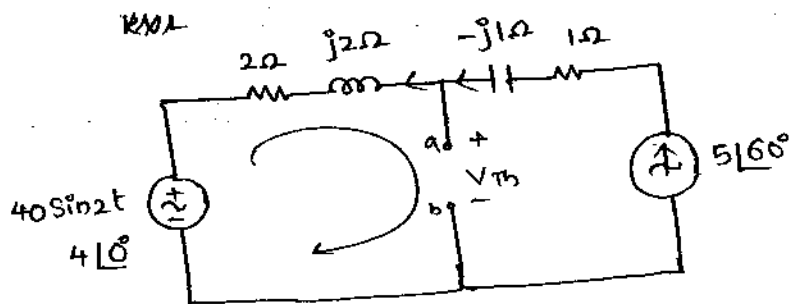
$$\underline{\underline{Z_s = -jX_L}}$$

Q What is the max power Transferred to load .



$$Z_{Th} = (2 + j2) \Omega$$

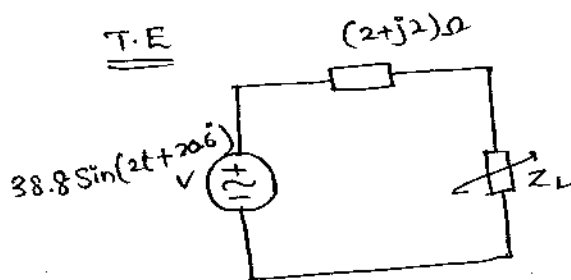
V_{Th} :



$$\underline{\text{KVL}} \quad -[40\angle 0^\circ] - [5\angle 60^\circ][2 + j2] + V_{Th} = 0$$

$$V_{Th} = 38.8 \angle 20.6^\circ \text{ V.}$$

→ phasor (max)



for P_{max}

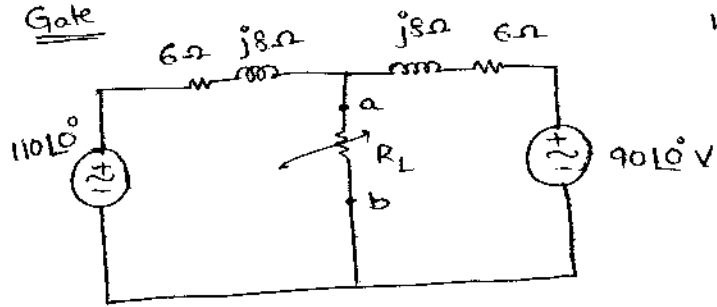
$$Z_L = Z_s^*$$

$$Z_L = 2 - j2$$

$$P_{max} = \frac{(V_{Th})^2}{4 R_{Th}} = \frac{\left[\frac{38.8}{\sqrt{2}} \right]^2}{4 [2]}$$

$$= 94.1 \text{ W}$$

Gate

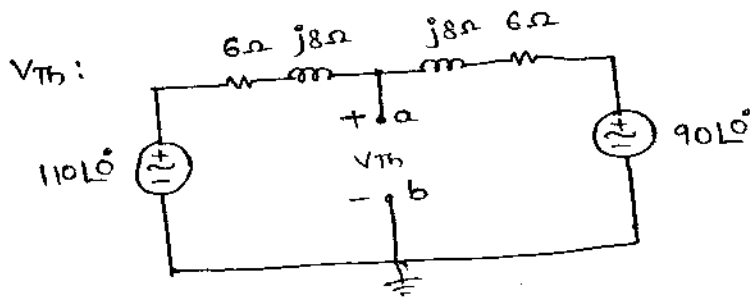


What is the max power Transferred to the load

- (a) 500 W (b) 1000 W
(c) 625 W (d) 1250 W

124

$$Z_{Th} = (6 + j8) \parallel (6 + j8) = 3 + j4$$



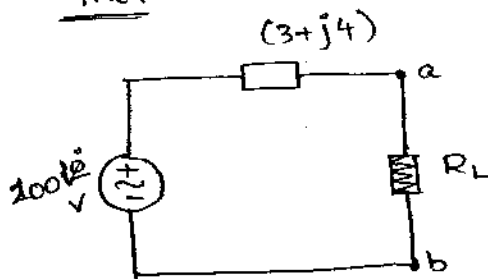
$$\frac{V_{Th} - 110\angle 0^\circ}{6 + j8} + \frac{V_{Th} - 90\angle 0^\circ}{6 + j8} = 0$$

$$V_{Th} - 110\angle 0^\circ - 90\angle 0^\circ = 0$$

$$2 V_{Th} = 200$$

$$V_{Th} = 100 \text{ V}$$

T.E:



at P_{max}

$$R_L = |Z_s|$$

$$= \sqrt{9 + 16}$$

$$R_L = 5$$

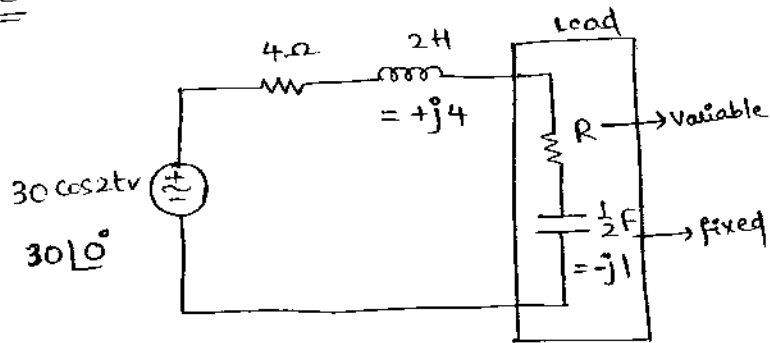
11.88

$$I_{L(rms)} = \frac{100\angle 0^\circ}{(3 + j4 + 5)} = 11.88 \angle -26.56^\circ$$

$$P_{max} = |I_L|^2 \cdot R_L = (11.88)^2 (5)$$

$$= 624.9 \text{ W}$$

14e
 Q. What is the max power Transferred to the load.



$$= 30 \angle 0^\circ \left(\frac{R - j1}{R + 4 + j3} \right)$$

∴ special case (b)

$$R_L = \sqrt{R_S^2 + (X_S + X_L)^2}$$

$$= \sqrt{4^2 + (4 - 1)^2}$$

$$R = R_L = 5$$

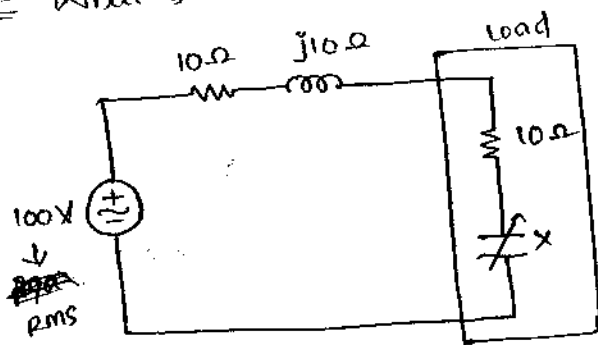
for P_{max}

$$R_L = |4 + j4 - j1| = 5 \Omega$$

$$I_L (Rms) = \frac{\left[\frac{30}{\sqrt{2}} \angle 0^\circ \right]}{(4 + j4 + 5 - j1)} = 2.296 \angle -18.14^\circ$$

$$P_{max} = I_L^2 \cdot R = 25 \text{ W.}$$

15e What is the max power Transferred to the load.



→ special case (c)

→ For P_{max} $X = -j10 \Omega$

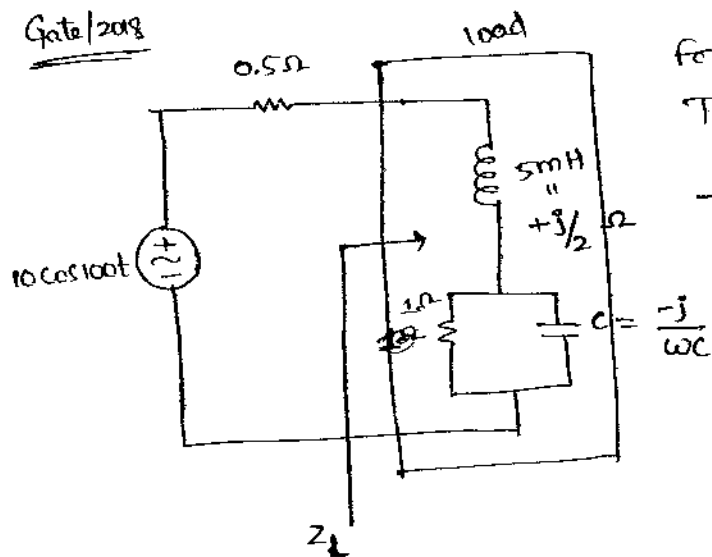
$$I_L (Rms) = \frac{100}{10 + j10 + 10 - j10} = 5 \text{ A}$$

$$P_{max} = I_L^2 R_L$$

$$= (5)^2 (10)$$

$$= 250 \text{ W}$$

GATE/2018



for what value of capacitor 'C' max power Transferred to entire load.

125

→ case (c)

'C' should be such that net load Reactance is zero

$$\begin{aligned}
 Z_L &= \frac{j}{2} + \left[1 \parallel \frac{-j}{\omega C} \right] \\
 &= \frac{j}{2} + \left[\frac{-j/\omega C}{1 - \frac{j}{\omega C}} \right] \\
 &= \frac{j}{2} + \left[\frac{-j/\omega C}{\omega C - j/\omega C} \right] \\
 &= \frac{j}{2} + \left[\frac{-j (\omega C + j)}{(\omega C - j)(\omega C + j)} \right] \\
 &= \frac{j}{2} + \frac{(1 - j\omega C)}{\omega^2 C^2 + 1} \\
 &= \left[\frac{1}{\omega^2 C^2 + 1} \right] + j \underbrace{\left[\frac{1}{2} - \frac{\omega C}{\omega^2 C^2 + 1} \right]}_{X_{\text{net}}}
 \end{aligned}$$

for P_{max} ,

$$X_{\text{net}} = 0$$

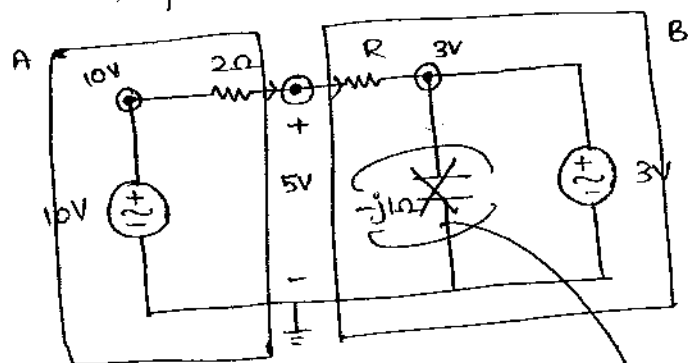
$$\frac{1}{2} - \frac{\omega C}{\omega^2 C^2 + 1} = 0 \Rightarrow \omega^2 C^2 + 1 = 2\omega C$$

$$\omega^2 C^2 - 2\omega C + 1 = 0$$

$$(\omega C - 1)^2 = 0 \Rightarrow \omega C = 1 \Rightarrow C = \frac{1}{\omega}$$

$$C = \frac{1}{100} = \underline{\underline{10 \text{ mF}}}$$

Gate Assuming Both the Sources are in phase for what Value of "R"
max. power Transferred from ckt A to ckt B.



$$\frac{10-5}{2} = \frac{5-3}{R}$$

$$R = \frac{4}{5} = 0.8\Omega$$

Redundant = parallel to Voltage Source.

$\therefore -j1\Omega$ is Redundant

\therefore Voltage drop is 50% across each side

parallel to
Ideal voltage
sources are
Redundant

$$2V (R=1)$$

$$\frac{2V}{-j1}$$

$$\frac{2j}{1 \angle 0^\circ}$$

$$2j$$

do above problem in Exact Method

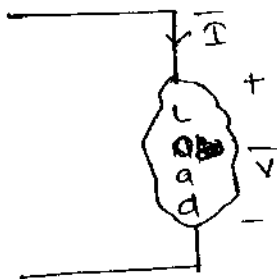
Concept of Complex power ($S^* = V I^*$)

Complex power is a mathematical Concept which will allow us to evaluate at three ^{physical} powers P, Q, S in any element or N/w or system directly by absorbing the Voltage & Current phasors but without declaring the N/w & its components.

126

Here I^* is Complex Conjugate of total Current " I " & units of Complex power is also Volt amperes.

Assuming Lagging P.F. load.



$$\underline{V} = |V| \angle \alpha$$

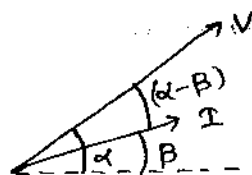
$$\underline{I} = |I| \angle \beta$$

$$S = \underline{V} \cdot \underline{I} = |V| |I| \angle \alpha + \beta \rightarrow \text{Wrong phase angle}$$

$$S^* = \underline{V} \cdot \underline{I}^* = \cancel{|V| |I| \angle \alpha + \beta}$$

$$= |V| \angle \alpha \cdot |I| \angle -\beta$$

$$= |V| |I| \angle \alpha - \beta \rightarrow \text{Correct phase angle}$$



Note: In Verifying Tellegans Theorem in AC Network & to derive the power flow Eqn in power N/w we should consider the Complex power S^* only. Where we need Mathematically prove

$$\sum_{k=1}^b \underline{V}_k \cdot \underline{I}_k^* = 0$$

where b = no. of Branches in N/w.

continued

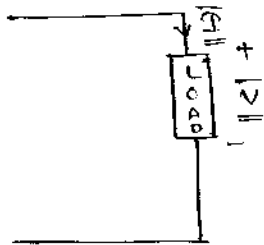
$$\text{i.e., } \sum \underline{V} \cdot \underline{I}^* \Big|_{\substack{\text{Sources} \\ \downarrow \\ (V, I)}} = \sum \underline{V} \cdot \underline{I}^* \Big|_{\substack{\text{Sinks} \\ \downarrow \\ (R, L, C)}}$$

gate

$$\text{If } \underline{V} = (7+j2) \text{ V}$$

$$\underline{I} = (1+j5) \text{ A}$$

Determine $\underline{P}, \underline{Q}, \underline{S}$ & total load N/w P.F



$$\underline{S}^* = \underline{V} \cdot \underline{I}^*$$

$$= [7+j2][1+j5]^*$$

$$= (7+j2)(1-j5)$$

$$\underline{S}^* = 17 - j33$$

$$\downarrow$$

$$P = 17 \text{ W}$$

$Q_c = 33 \text{ VARs}$
(generally)
 \downarrow
Capacitor

$$|\underline{S}^*| = \sqrt{P^2 + Q_c^2}$$

$$= \sqrt{17^2 + 33^2}$$

$$= 37.1 \text{ VA}$$

$$\text{P.F.} = \frac{P}{S} = \frac{17}{37.1} = 0.45 \text{ (lead)}$$

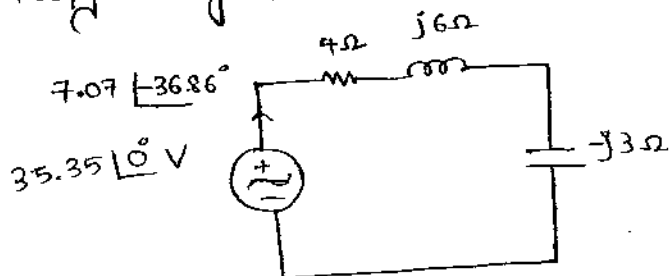
(a)

$$\underline{V} = 7.28 \angle 15.9^\circ$$

$$\underline{I} = 5.09 \angle 78.9^\circ$$

current leading
V by 62°
 \downarrow
RC N/w.

Verify Tellegans Theorem in previous Calculation problem (before 8 pm)



Tellegans:

$$\underline{V}_S \underline{I}_S^* = \underline{V}_R \underline{I}_R^* + \underline{V}_L \underline{I}_L^* + \underline{V}_C \underline{I}_C^*$$

L.H.S:

$$\Rightarrow (35.35 \angle 0^\circ)(7.07 \angle 36.86^\circ)$$

$$= 250 \angle 36.86^\circ \text{ VAs}$$

RHS

127

$$\Rightarrow (28.28 \angle -36.86^\circ)(7.07 \angle +36.86^\circ) + (42.42 \angle 53.14^\circ)(7.07 \angle +36.86^\circ) +$$

$$(21.21 \angle -126.96^\circ)(7.07 \angle +36.86^\circ)$$

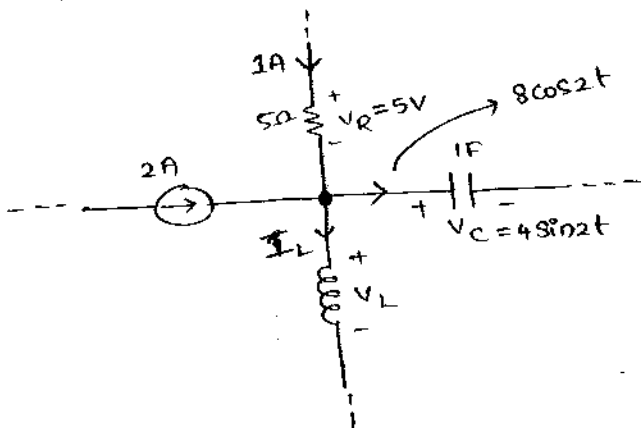
$$= 250 \angle 36.86^\circ$$

$$= (200 + j150) \text{ VA}$$

LHS = RHS \rightarrow verified.

Q.10

If $V_R = 5\text{V}$, $V_C = 4\sin 2t\text{V}$ then $V_L =$ _____



$$i = C \frac{dv}{dt}$$

$$= 1\text{F} \times \frac{d}{dt}(4\sin 2t)$$

$$= 1\text{F} \cdot 4\cos 2t \cdot 2$$

$$I_C = i = 8\cos 2t$$

$$I_C = \frac{V}{-jX_C} = 4\sin 2t$$

KCL

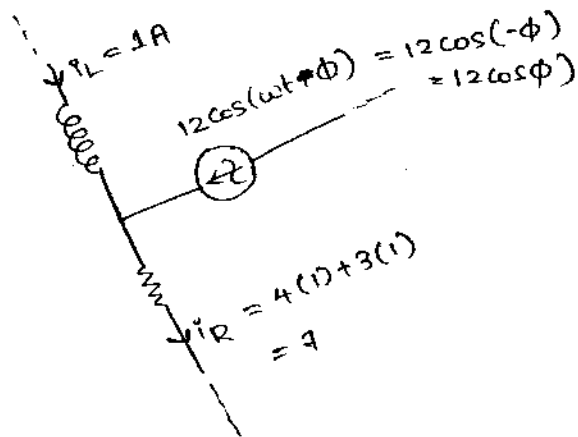
$$2 + i = 8\cos 2t + I_L$$

$$I_L = 2 - 8\cos 2t$$

$$V_L = L \frac{dI_L}{dt} = 2 [0 + 8 \cdot 2 \cdot \sin 2t]$$

$$V_L = 32\sin 2t$$

Gate If $i_L(0) = 1A$, $i_R(t) = 4e^{-3t} + 3e^{-4t}$, $t > 0$ find $\phi =$ _____



KCL at $t=0$

~~$i_L + i_R = 0$~~

$$1 + 12 \cos \phi = 7$$

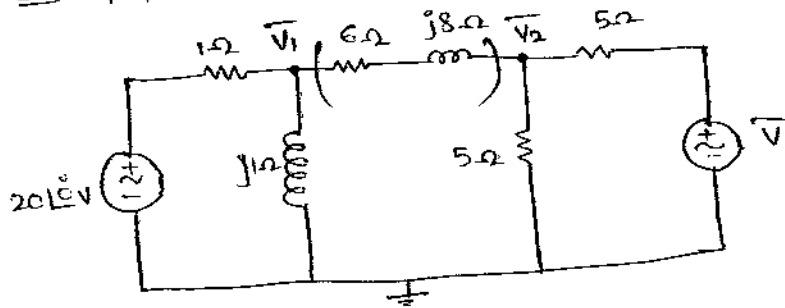
$$12 \cos \phi = 6$$

$$\cos \phi = \frac{1}{2}$$

$$\phi = 60^\circ$$

IES if power lost in 6Ω is Zero, then Voltage $\bar{V} = \underline{\hspace{2cm}}$

128



power lost = 0

when current doesn't flow

i.e., Voltage diff. is same

i.e., in AC same volt. mag, phase & freq.

$$\bar{V}_1 = \bar{V}_2$$

$$\therefore V_1 = 20 \angle 0^\circ \left[\frac{j1}{1+j} \right]$$

$$V_2 = V \left[\frac{5}{5+5} \right]$$

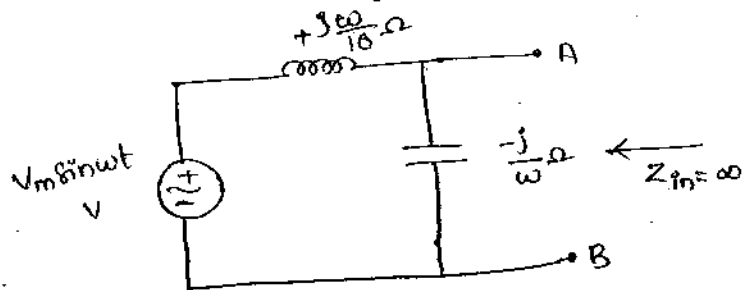
$$V_1 = 2V_2$$

$$V_1 = 10\sqrt{2} \angle 45^\circ$$

$$V = 2(10\sqrt{2} \angle 45^\circ)$$

$$V = 20\sqrt{2} \angle 45^\circ$$

Gate for what Supply freq. ω the N/w b/w 'A' 'B' acts as Ideal Current Source.
 $Z_{internal} = \infty$

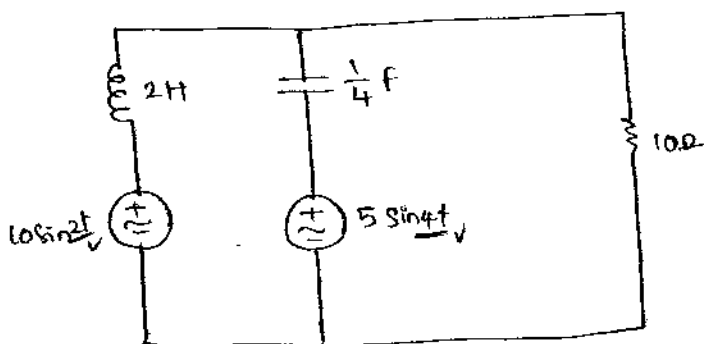


$$Z_{in} = \left[\frac{j\omega}{16} \parallel \frac{-j}{\omega} \right]$$

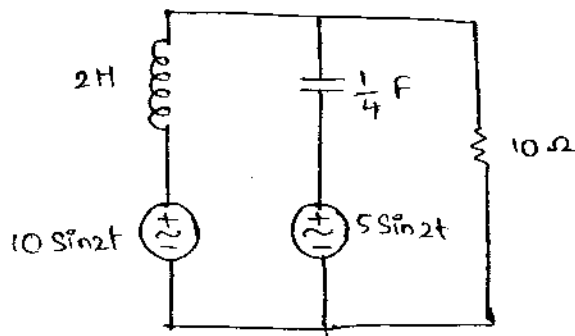
$$Z_{in} = \frac{\frac{1}{16}}{j \left[\frac{\omega}{16} - \frac{1}{\omega} \right]} = \infty$$

$$\frac{\omega}{16} = \frac{1}{\omega} \Rightarrow \omega^2 = 16$$

$$\omega = 4 \text{ rad/sec}$$



which is the Best theorem to evaluate Response in 10Ω Resistance.
 two multiple Sources with multiple freq.
 \hookrightarrow Superposition Theorem



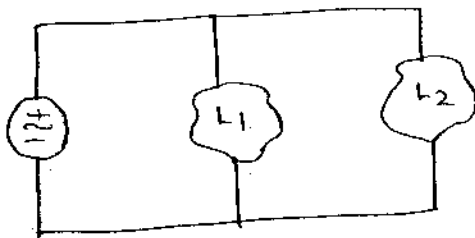
which is best theorem to evaluate response in 10Ω .

⇒ Millman's.

$$\bar{V}_M = \frac{\sum_{i=1}^n \frac{V_i}{Z_i}}{\sum_{i=1}^n \frac{1}{Z_i}} = \frac{\sum_{i=1}^n V_i Y_i}{\sum_{i=1}^n Y_i}$$

$$Z_M = \frac{1}{\sum_{i=1}^n \frac{1}{Z_i}} = \frac{1}{\sum_{i=1}^n Y_i}$$

Q determine the total Supply power factor :



Load 1 : 5 KVA , 0.6 (lag)

Load 2 : 1 KW , 0.8 (lead)

Load - I

$$S_1 = 5000 \text{ VA}$$

$$\cos \phi_1 = 0.6 (\text{lag})$$

$$P_1 = S_1 \cos \phi_1 = 3000 \text{ W}$$

$$Q_1 = S_1 \sin \phi_1 = 4000 \text{ VARs}$$

$$S_1 = P_1 + jQ_1$$

Load - II

$$P_2 = 1000 \text{ W}$$

$$\cos \phi_2 = 0.8 (\text{lead})$$

$$S_2 = \frac{P_2}{\cos \phi_2} = 1250 \text{ VA}$$

$$Q_2 = S_2 \sin \phi_2 = 750 \text{ VARs}$$

$$S_2 = P_2 - jQ_2$$

Tellegan : $S_T = S_1 + S_2$ ↑ lag → lead

$$S_T = [P_1 + P_2] + j[Q_1 - Q_2]$$

$$S_T = [3000 + 1000] + j[4000 - 750]$$

$$S_T = 4000 + j3250$$

overall lag

$$\text{P.F.} = \frac{P_T}{|S_T|} = \frac{4000}{\sqrt{4000^2 + 3250^2}} = 0.77 (\text{lag})$$

power factor & its Correction:

129

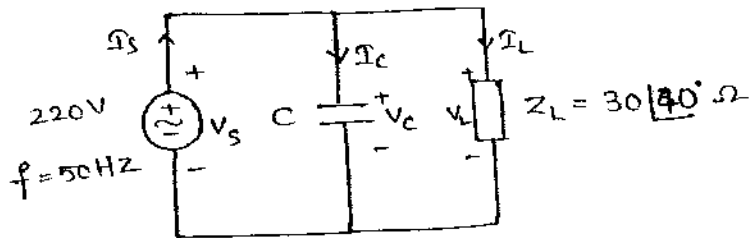
I. Correction at UPF level:

Q determine the Value of Capacitor 'C' required to make Supply power factor to Unity.

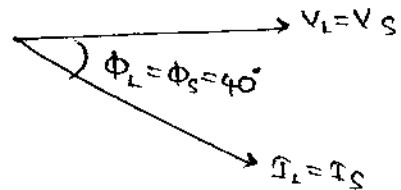
Correct the

Supply power factor to

Without 'C'

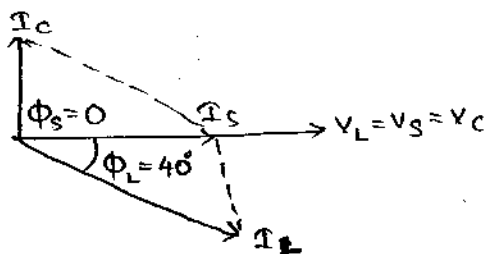


$$\text{KCL: } \bar{I}_S = \bar{I}_L + \bar{I}_C$$



$$\left[\begin{matrix} \text{load} \\ \text{P.F} \end{matrix} \right] = \left[\begin{matrix} \text{Supply} \\ \text{P.F} \end{matrix} \right] = \cos 40^\circ = 0.766 (\text{lag})$$

With Capacitor:



$$\Rightarrow \text{Load P.F} = \cos 40^\circ = 0.766 (\text{lag})$$

$$\text{Supply P.F} = \cos 0^\circ = 1 [\text{UPF}]$$

→ E.C.A (Elec. ckt analysis)

(a) Series: $Z = R \pm jX_{\text{net}}$

at UPF, $Z = R$, so $X_{\text{net}} = 0$

(b) parallel: $Y = G \pm jB_{\text{net}}$

at UPF, $Y = G$, so $B_{\text{net}} = 0$

sol parallel.

$$Y_T = Y_1 + Y_2$$

$$Y_T = \frac{1}{-jX_C} + \frac{1}{Z_L} = \frac{1}{-j/\omega C} + \frac{1}{30 \angle 40^\circ}$$

$$Y_T = j2\pi fC + \frac{1}{30 \angle 40^\circ}$$

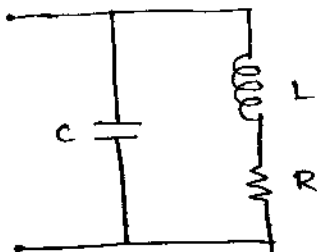
$$Y_T = j100\pi C + [0.0255 - j0.02142]$$

$$Y_T = 0.0255 + j[100\pi C - 0.02142]$$

at UPF, $B_{net} = 0$

$$100\pi C = 0.02142$$

$$C = \frac{0.02142}{100\pi} = \underline{\underline{0.8 \mu F}}$$



$$Y_T = Y_1 + Y_2 = \frac{1}{-jX_C} + \frac{1}{R + jX_L}$$

$$Y_T = \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$Y_T = \left[\frac{R}{R^2 + X_L^2} \right] + j \left[\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2} \right]$$

at UPF, $\rightarrow B_{net} = 0$

$$\frac{1}{X_C} = \frac{X_L}{R^2 + X_L^2}$$

$$\cancel{\omega C} = \frac{\omega L}{R^2 + (\omega L)^2}$$

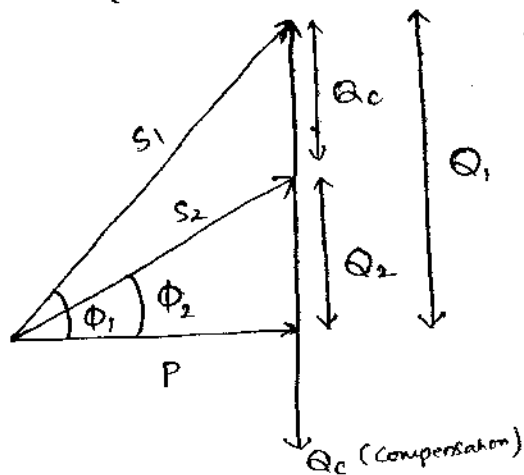
$$C = \frac{L}{R^2 + \omega^2 L^2} \quad f$$

II. power factor Correction at desired level:

Q In the above ckt determine the "C" value. to improve supply P.F to 0.85 lagging

130

power Triangle [lagging P.F]



Before Correction $\rightarrow \cos \phi_1$

After Correction $\rightarrow \cos \phi_2$

$$\cos \phi_2 > \cos \phi_1$$

$$\phi_2 < \phi_1$$

$$\tan \phi_1 = \frac{Q_1}{P} \rightarrow Q_1 = P \tan \phi_1$$

$$\tan \phi_2 = \frac{Q_2}{P} \rightarrow Q_2 = P \tan \phi_2$$

$$Q_c = [Q_1 - Q_2]$$

$$Q_c = P [\tan \phi_1 - \tan \phi_2]$$

$$Q_c = \frac{V^2}{X_c} = \frac{V^2}{1/\omega C} = \omega C V^2$$

$$C = \frac{Q_c}{\omega V^2} \text{ F}$$

Sol Before Correction $\rightarrow \phi_1 = 40^\circ$

After Correction, $\cos \phi_2 = 0.85 \rightarrow \phi_2 = \cos^{-1}(0.85)$
 $\phi_2 = 31.78^\circ$

$$P = V_L I_L \cos \phi_1$$

$$I_L = \frac{V_L}{Z_L} = \frac{220 \angle 0^\circ}{30 \angle 40^\circ} = 7.33 \angle -40^\circ$$

$$P = (220)(7.31) \cos 40^\circ$$

$$P = 1235 \text{ Watts}$$

$$Q_c = 1235 [\tan 40^\circ - \tan 31.78^\circ]$$

$$= 271 \text{ VARs}$$

$$C = \frac{Q_c}{\omega V^2} = \frac{(271)}{2\pi 50 (220)^2}$$

$$= 17.8 \mu\text{f}$$

Duals & Duality

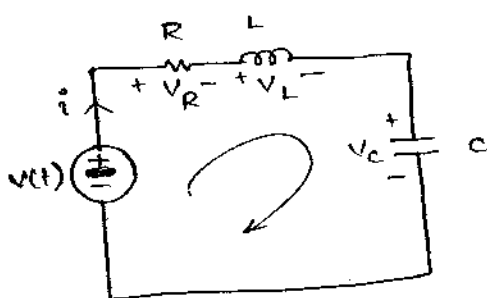
131

Duals: Two ckt's are said to be duals of each other if Mesh Eqns that characterised one of them has the same Mathematical form as the nodal Eqns that characterise the other.

principle of Duality:

Identical Behaviour patterns observed b/w Voltages & Currents of two Independent Ntws demonstrate the principle of duality.

ex: Series R-L-C

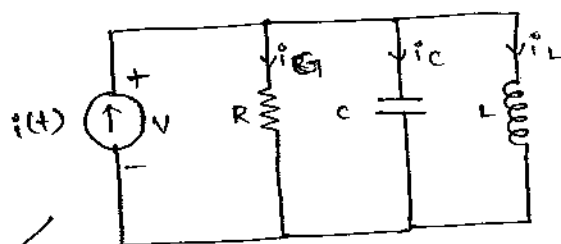


Mesh \rightarrow KVL

$$-V(t) + V_R + V_L + V_C = 0$$

$$V(t) = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

parallel R-L-C



Nodal \rightarrow KCL

$$-i(t) + i_G + i_C + i_L = 0$$

$$i(t) = v.G + C \frac{dv}{dt} + \frac{1}{L} \int v dt$$

Mathematically identical

List of Dual Elements:

$$V(t) \longleftrightarrow i(t)$$

$$V \longleftrightarrow I$$

$$V_m \sin \omega t \longleftrightarrow I_m \sin \omega t$$

$$R \longleftrightarrow G$$

$$C \longleftrightarrow L$$

$$KVL \longleftrightarrow KCL$$

$$\text{Series} \longleftrightarrow \text{parallel}$$

$$\text{Mesh} \longleftrightarrow \text{Node}$$

$$\int v dt \longleftrightarrow \int i dt$$

$$\frac{di}{dt} \longleftrightarrow \frac{dv}{dt}$$

$$Z \longleftrightarrow Y$$

$$X \longleftrightarrow B$$

$$O.C \longleftrightarrow S.C$$

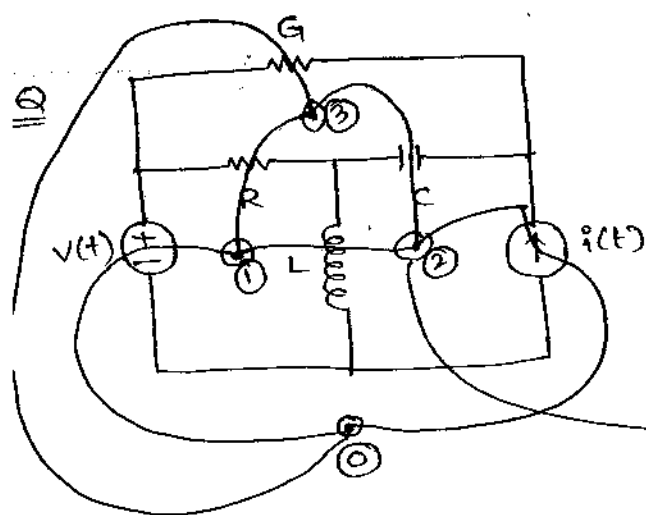
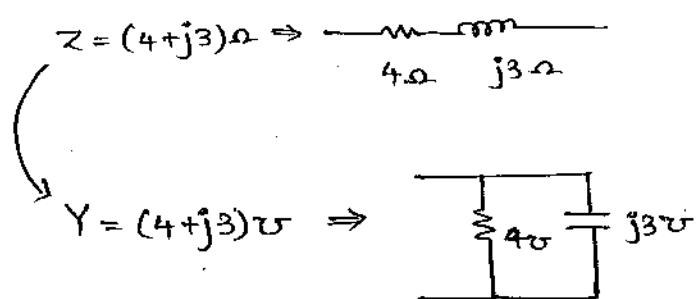
$$\text{Thevenin} \longleftrightarrow \text{Norton}$$

$$Y \longleftrightarrow \Delta$$

$\pi \longleftrightarrow \tau$	$V_C(0) \longleftrightarrow i_L(0)$
$q \longleftrightarrow \phi$	$E_C \longleftrightarrow E_L$
$\sigma \longleftrightarrow \mu$	Time const of voltage $T_V \longleftrightarrow T_I$ (Time const of current)
Tree \longleftrightarrow Co-Tree	Switch in Series (getting closed) \longleftrightarrow Switch in parallel (getting opened)
Twigs \longleftrightarrow Links/chords	$\nabla \uparrow 2V_x \longleftrightarrow \nabla \downarrow 2i_x$
Cut-set \longleftrightarrow Tie-set	$V(\text{polarity}) \longleftrightarrow i(\text{direction})$
$[Q] \longleftrightarrow [B]$	

Note: Dual of 5Ω is $5V$ (or) element only.

Q What is the dual of $(4+j3)\Omega$

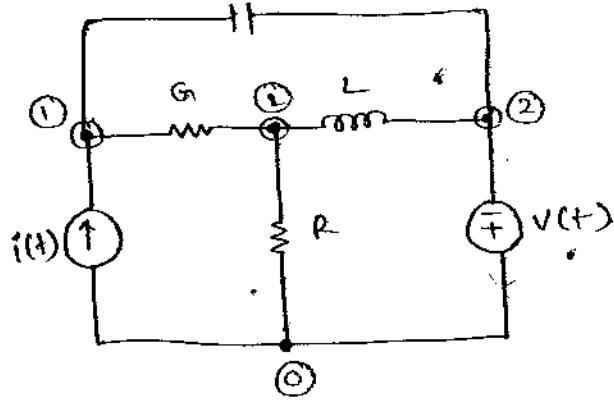
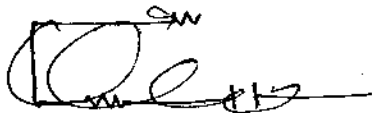


Construct the dual of the following ckt & verify by writing Mesh & Nodal eqns.

meshes becomes nodes.

① ② ③ ④ Meshes becomes nodes ① ② ③ ④

132



e^{-t}
 $1-A$

Mesh

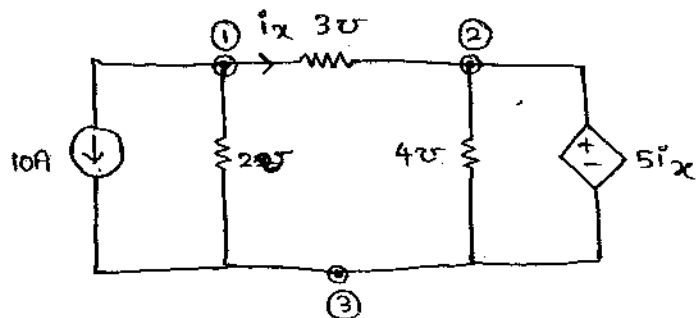
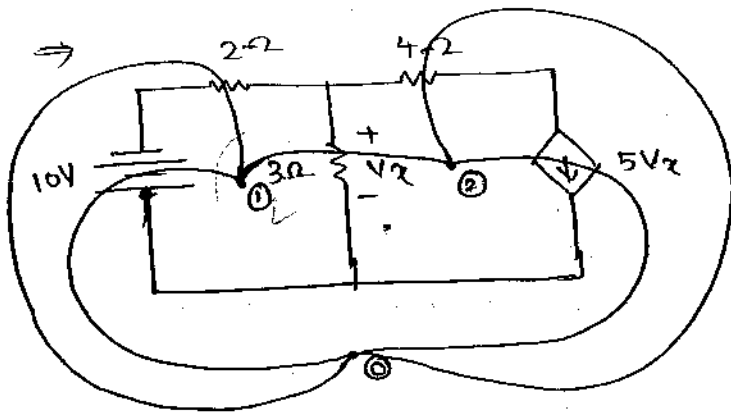
$$-V(t) + R[i_1 - i_3] + L \left[\frac{di_1}{dt} - \frac{di_2}{dt} \right] = 0 \quad \text{--- ①}$$

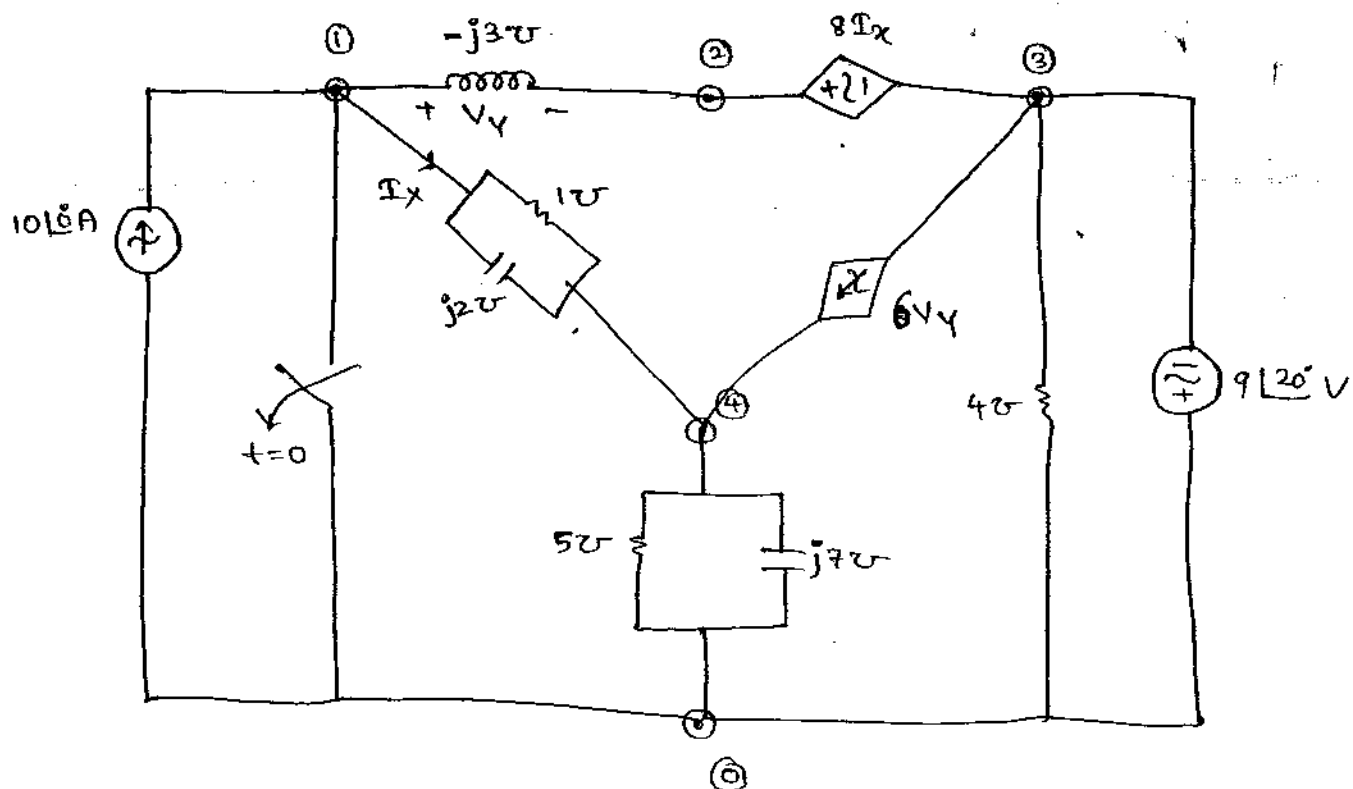
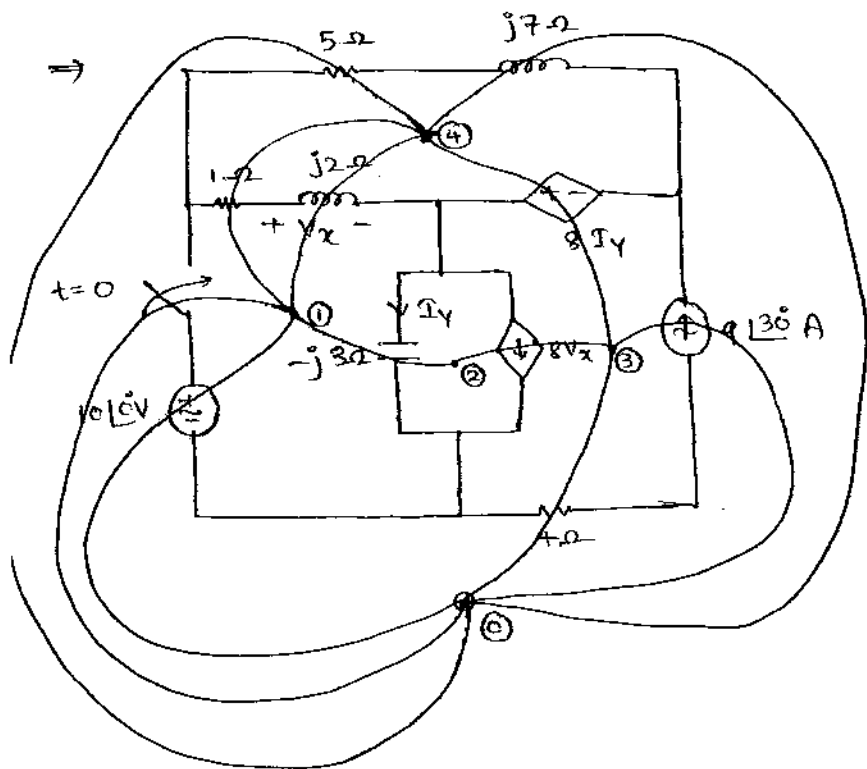
Nodal

$$-i(t) + G[v_1 - v_3] + C \left[\frac{dv_1}{dt} - \frac{dv_2}{dt} \right] = 0 \quad \text{--- ①}$$

$$i_2 = -i(t) \quad \text{--- ②} \quad \xleftrightarrow{\text{dual}} \quad v_2 = -V(t) \quad \text{--- ②}$$

$$\frac{i_3}{G} + \frac{1}{C} \int (i_3 - i_2) dt + R[i_3 - i_1] = 0 \quad \text{--- ③} \quad \xleftrightarrow{\text{dual}} \quad \frac{v_3}{R} + \frac{1}{L} \int (v_3 - v_2) dt + G[v_3 - v_1] = 0 \quad \text{--- ③}$$





Locus diagram

133

These are Variable ~~Phasor~~ diagrams when one of the electrical ckt parameter is varied.

The parameters that can be varied are values of R, L, C , load current, Supply freq. ... etc.

Locus diagrams allows us to predetermine all the possible operating points of a network application from which we can observe the optimum operating point for the best performance of the N/w application.

Ex: (1) Circle diagram of a Ind. Motor is a locus diagram from which we can determine the load at which highest running power factor of Motor can be achieved.

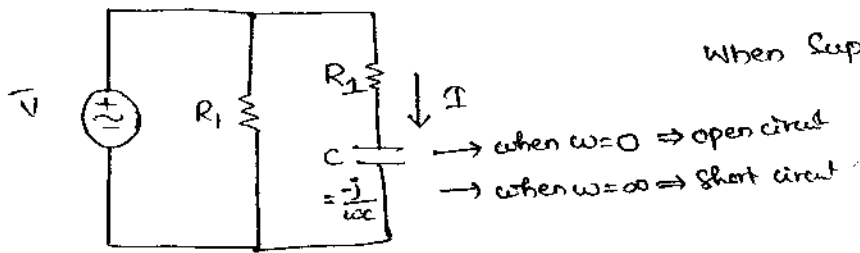
(2) Resonance plots are locus diagrams from which we can observe the performance of N/w & its Natural frequency.

(3) Relay characteristics ^{in p.s} for its Zonal protection of Tr. line such as mho Relay, Reactance Relay ... etc are also locus diagrams.

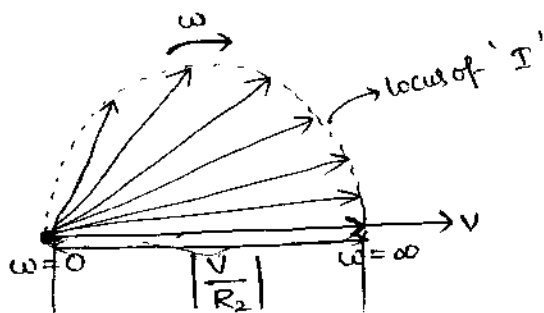
Since most of our Network applications are designed to work for Rated Supply Voltage, we observe their Current locus diagrams.

Since most of our Network applications are Connected in parallel we prefer their admittance locus diagrams.

Q plot the locus of Current 'i' w.r.t Voltage 'V' as Supply freq ω is varied from 0 to ∞
 (or) What is the Radius of Current locus inckt shom.
 When Supply freq ω varied from 0 to ∞



$$I = \frac{V}{Z} = \frac{V}{R_2 - jX_C}$$



Radius of 'I' $\rightarrow \frac{V}{2R_2}$

$$0 < \omega < \infty$$

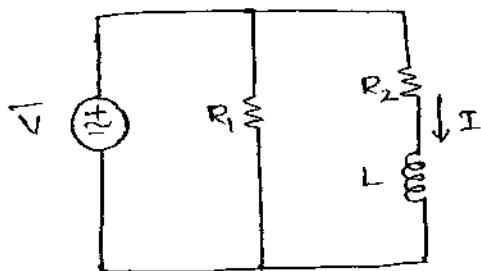
\rightarrow R-C Branch

$$R_2^2 + \left(\frac{1}{\omega C}\right)^2 = \left[\frac{V}{I}\right]^2 \quad \left. \vphantom{\frac{V}{I}} \right\} \begin{matrix} X^2 + Y^2 = K^2 \\ \text{circle} \end{matrix}$$

\downarrow
 only ' ω ' is varied

\rightarrow Semi circle shape in ω Variation

Q plot the locus 'i' ^{w.r.t} voltage 'V' if R_2 is Varied from 0 to ∞ also
 determine the radius of Current locus



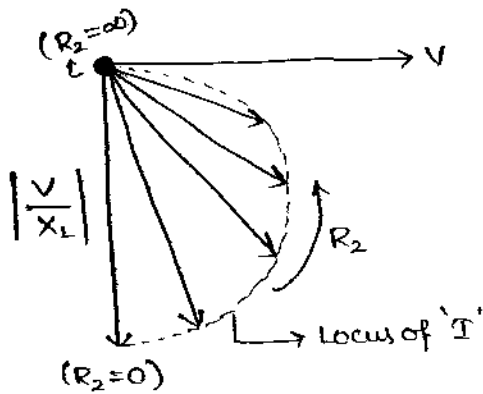
$$I = \frac{V}{Z} = \frac{V}{R_2 + jX_L}$$

$$\left. \begin{aligned} |I| &= \frac{|V|}{\sqrt{R_2^2 + X_L^2}} \\ \phi &= \tan^{-1} \left(\frac{X_L}{R_2} \right) \end{aligned} \right\}$$

$$R_2^2 + X_L^2 = \left[\frac{V}{I}\right]^2$$

\downarrow
 only R_2 is varied

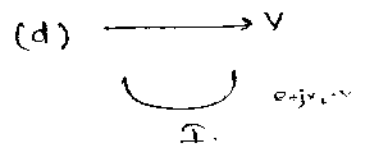
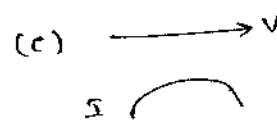
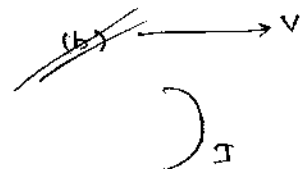
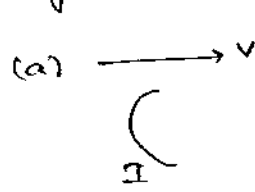
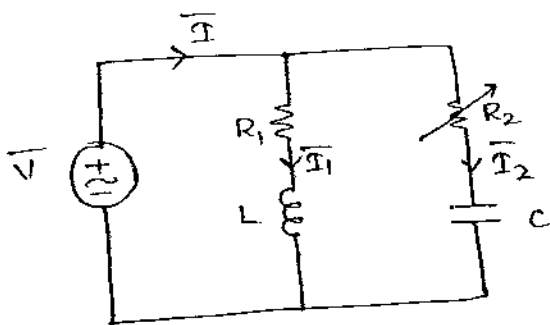
\rightarrow semi circle
 shape in variation



$$\text{Radius of 'I'} = \frac{V}{2X_L} = \frac{V}{2\omega L} \text{ A.}$$

134

Q plot the locus of Current I w.r.t Voltage V as R_2 is varied from 0 to ∞



→ Based on options [N/w is lagging]
 V, I in Q4 ⇒ ∴ Current lagging.

$$P = \frac{V}{R_2 + jX_L - jX_C} = \frac{V}{R_2 + j(X_L - X_C)}$$

KCL

$$I = I_1 + I_2 \quad (|I_1| > |I_2|)$$

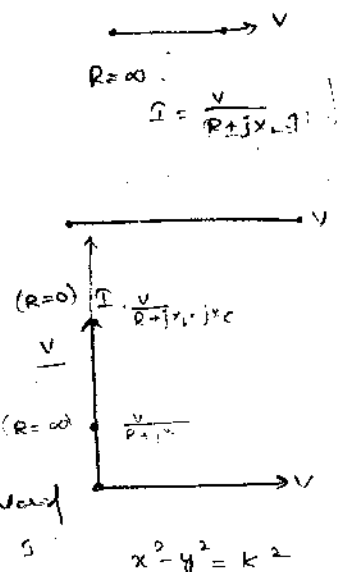
 ↓ ↓ ↓
 Varied Const. Varied
 ∵ R_2 is Varied

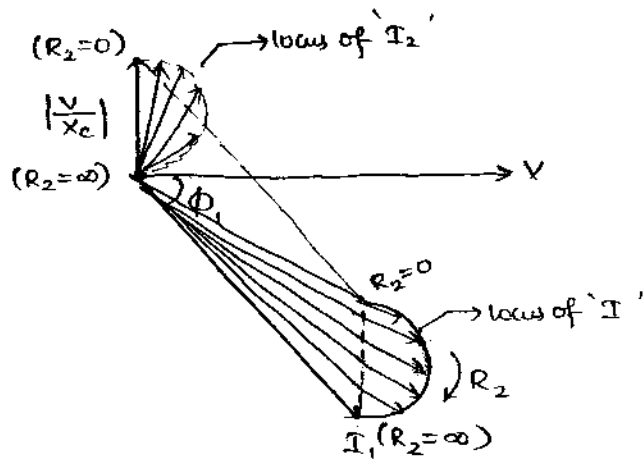
$$I_1 = \frac{V}{Z_1} = \frac{V}{R_1 + jX_L}$$

$$I_2 = \frac{V}{Z_2} = \frac{V}{R_2 - jX_C}$$

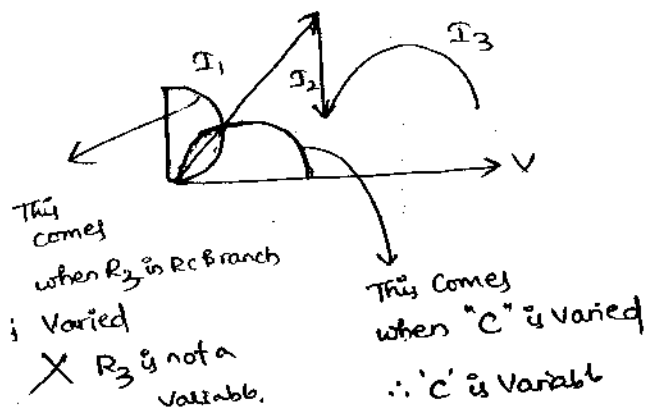
$$\left. \begin{aligned} |I_1| &= \frac{|V|}{\sqrt{R_1^2 + X_L^2}} \\ \phi_1 &= -\tan^{-1} \left[\frac{X_L}{R_1} \right] \end{aligned} \right\} \text{Const.}$$

$$\left. \begin{aligned} |I_2| &= \frac{|V|}{\sqrt{R_2^2 + X_C^2}} \\ \phi_2 &= +\tan^{-1} \left[\frac{X_C}{R_2} \right] \end{aligned} \right\} \text{Varied}$$





Q draw the Relevant ckt & Identify the variable passive parameter in the ckt whose Current locus diagram is as shown

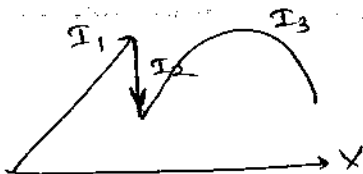
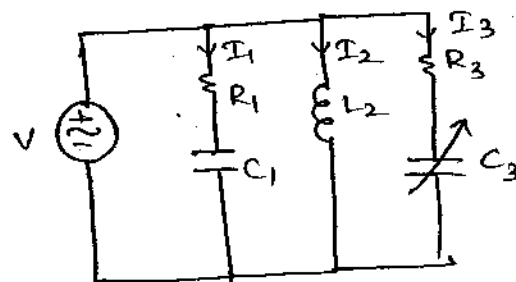


\Rightarrow parallel ckt \because V is Constant
 & currents are dividing.

$\Rightarrow I_1 \Rightarrow$ R-C Branch \because Current leading voltage. ($\phi < 90^\circ$)

$I_2 \Rightarrow$ pure Inductor \because Current lagging V by 90°

$I_3 \Rightarrow$ R-C Branch \because Current leading
 & it is variable




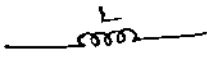
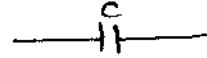
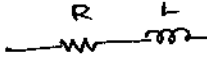
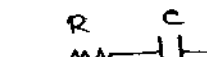
- ⇒ Resonance is the freq Response of a ckt on N/w when it is operating at its Natural freq. called Resonance freq.
- ⇒ Under Resonance, the total Supply Voltage & Supply Current are in phase
So, phase angle $\phi = 0^\circ$ & Ckt Exhibits Unity power factor (UPF)
- ⇒ Under Resonance, the Nature of the N/w is purely Resistive & maximum power is Transferred to the entire Network by the Source.
- ⇒ Resonance Can happen in Such electrical N/w s When we have two similar but opposite Natured Energy storage Components L & C.
- ⇒ These Energy storage Components Used in Such Tuned circuits must have Very High Quality which is defined as Quality factor or figure of Merit, given by

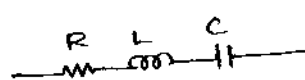
$$Q_{\text{factor}} = 2\pi * \left[\frac{\text{Max. stored Energy per cycle of supply}}{\text{Energy dissipated per cycle of supply}} \right]$$

$$Q_{\text{factor}} \approx \frac{Q_{\text{VAR's}}}{P_{\text{watts}}}$$

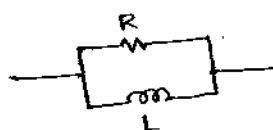
In practical applications of Resonance to passive filter design at power levels analog Communication Based Receivers & Antennae, Q should be ≥ 5

$$Q \geq 5 \text{ [index for design]}$$

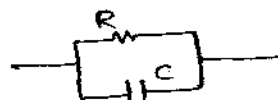
Element	Q-factor
	0
	∞
	∞
	$\frac{\omega L}{R}$
	$\frac{1}{\omega RC}$



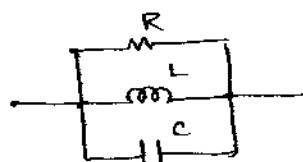
$$Q_0 = \frac{1}{R} \sqrt{\frac{L}{C}}$$



$$\frac{R}{\omega L}$$



$$\omega RC$$



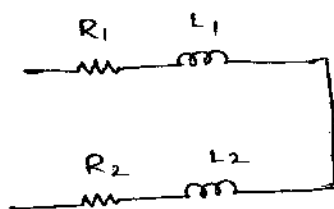
$$Q_0 = R \sqrt{\frac{C}{L}}$$

Note: The Combined Quality factor of a Series Combination of one Inductor & one Capacitor including their losses modelled either in Series or shunt Resistance is given by

$$Q_T = \frac{1}{\frac{1}{Q_L} + \frac{1}{Q_C}} = \frac{Q_L \cdot Q_C}{Q_L + Q_C}$$

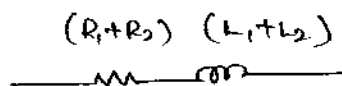
Q two practical Coils with Internal Resistances of R_1, R_2 have Quality factors of Q_1, Q_2 Respectively. if Both these coils are Connected in Series then the Combined Quality factor is _____

$$Q_T = \frac{Q_1 Q_2}{Q_1 + Q_2} \quad \text{or} \quad \frac{Q_1 Q_2}{R_1 + R_2} \left[\frac{Q_1 Q_2}{Q_1 + Q_2} \right]$$



$$Q_1 = \frac{\omega L_1}{R_1}$$

$$Q_2 = \frac{\omega L_2}{R_2}$$

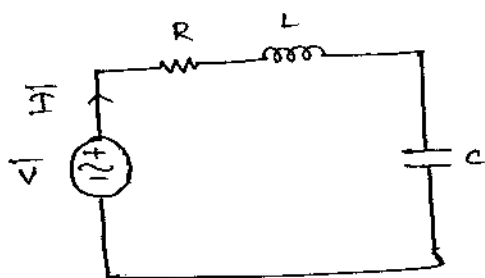


$$Q_T = \frac{\omega(L_1 + L_2)}{R_1 + R_2}$$

$$Q_T = \frac{\left(\frac{\omega L_1}{R_1}\right) * R_1 + \left(\frac{\omega L_2}{R_2}\right) * R_2}{R_1 + R_2}$$

$$Q_T = \frac{Q_1 R_1 + Q_2 R_2}{R_1 + R_2}$$

I]. Series Resonance,



at Resonance →

\bar{V} & $\bar{I} \rightarrow$ inphase

$$\phi = 0^\circ \text{ [UPF]}$$

$$Z = R$$

$$Z = R + j[X_L - X_C]$$

136

at Resonance (ω_0)

$$X_{net} = 0$$

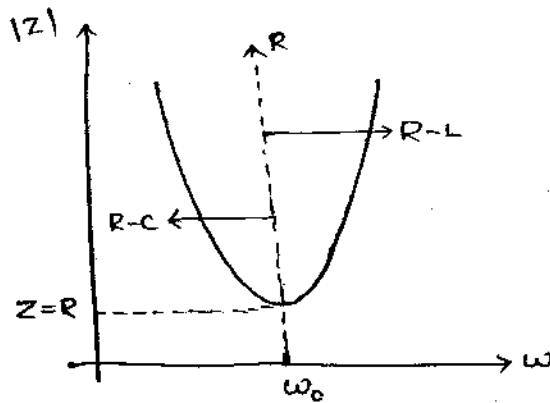
$$X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

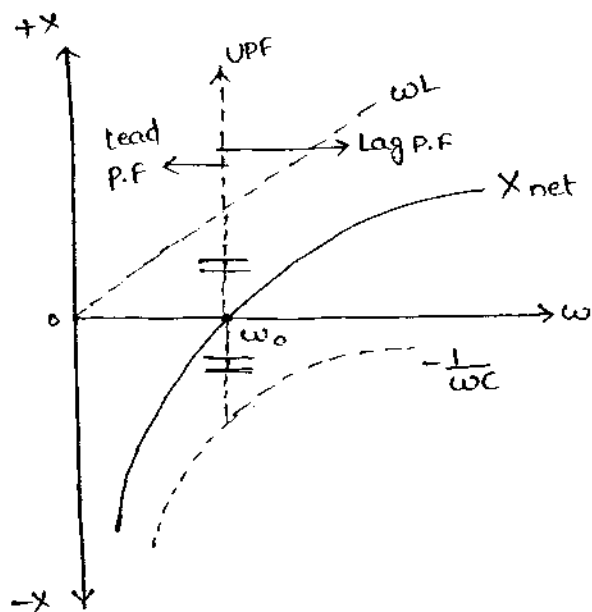
Graph-2 || $|Z|$ vs ' ω '

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$



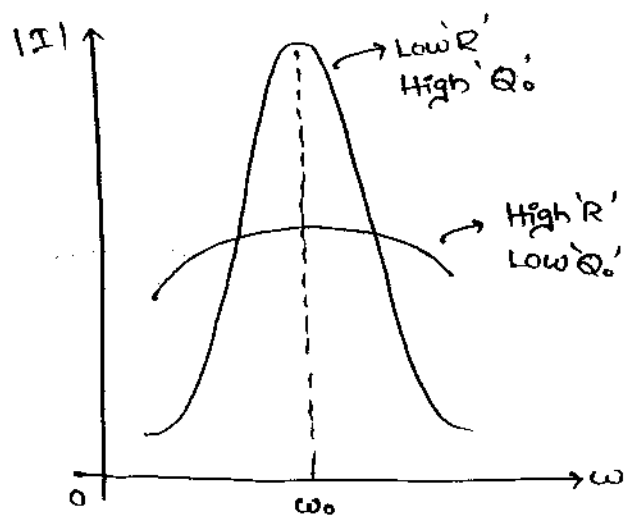
Graph-II

$$X_{\text{net}} \text{ vs } \omega \left[Z = R + j \underbrace{\left(\omega L - \frac{1}{\omega C} \right)}_{X_{\text{net}}} \right]$$



Graph-III

$|I|$ vs. ω



phasor diagram:

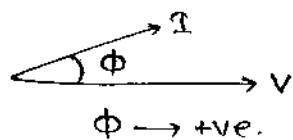
137

case (i): If $\omega < \omega_0$

$$Z = R - jX_{\text{net}}$$

$\rightarrow R-C$

'I' leads 'V' by $\phi < 90^\circ$
(leading P.F.)

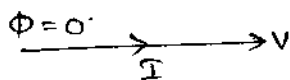


case (ii): $\omega = \omega_0$

$$Z = R$$

\rightarrow purely Resistive

'I' Inphase with 'V'
 $\phi = 0^\circ$ [UPF]

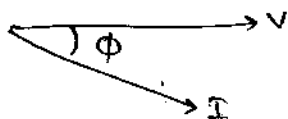


Case (iii): $\omega > \omega_0$

$$Z = R + jX_{\text{net}}$$

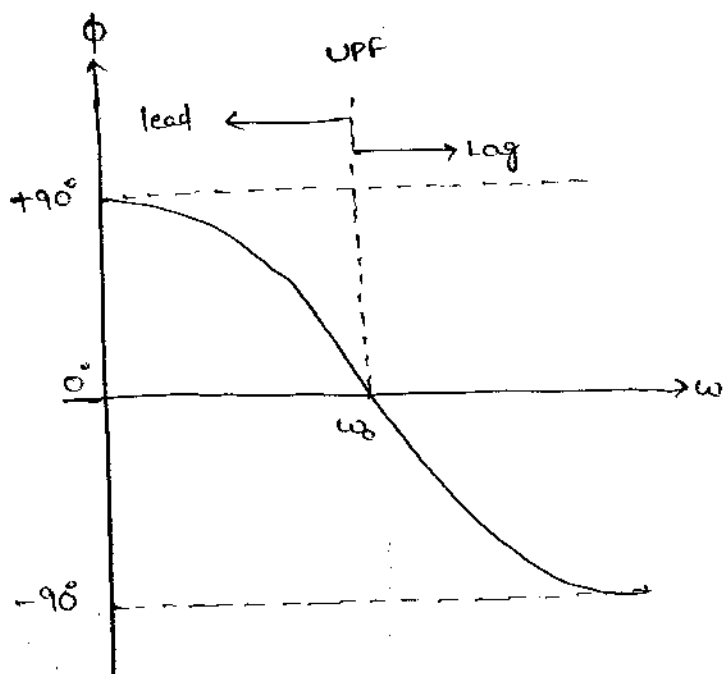
$\rightarrow R-L$

'I' lags 'V' by $\phi < 90^\circ$
(lagging P.F.)

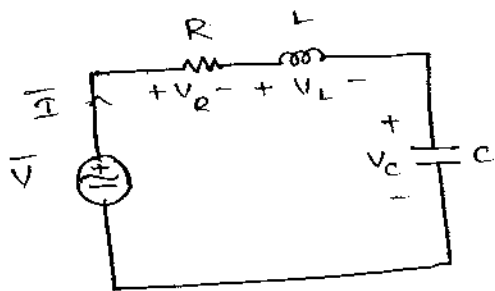


$\phi \rightarrow -ve$

Graph IV ϕ vs ω



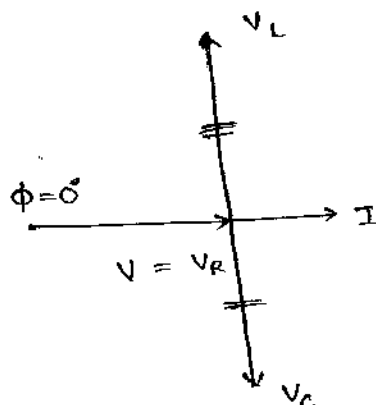
Complete phasor diagram Under Resonance .



at Resonance

$$|X_L| = |X_C|$$

$$|V_L| = |V_C|$$



$$V = |V_R|$$

$$\phi = 0^\circ$$

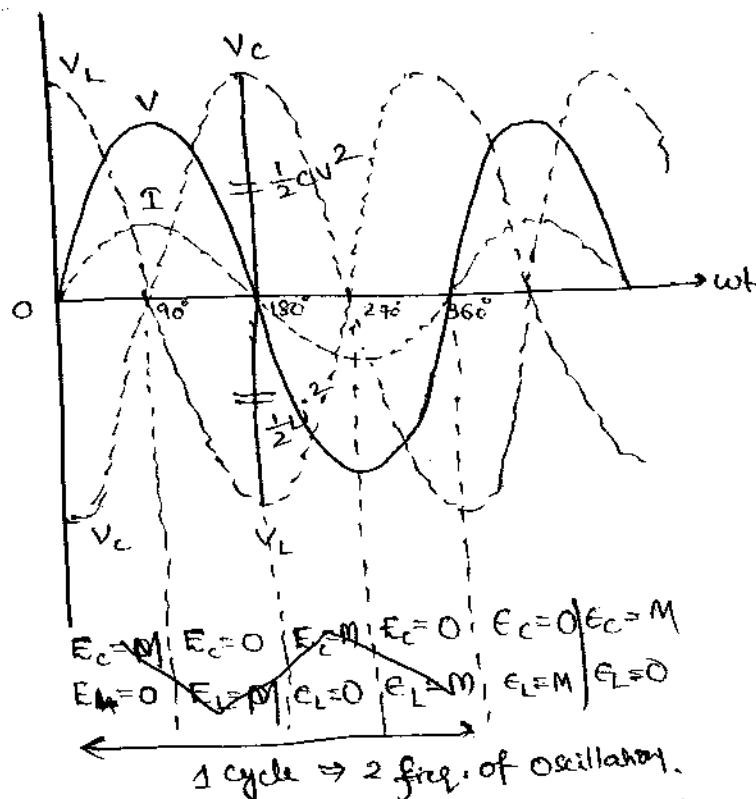
$$P.F = 1 \text{ (UPF)}$$

Note: Under Series Resonance Condition the pure Series LC Segment of Ckt acts like a short circuit & Source Supply Energy only to Resistor. However L & C Components Work Together by Exchanging the stored mag. & static (E_C) Energy b/w them as an Oscillatory phenomena.

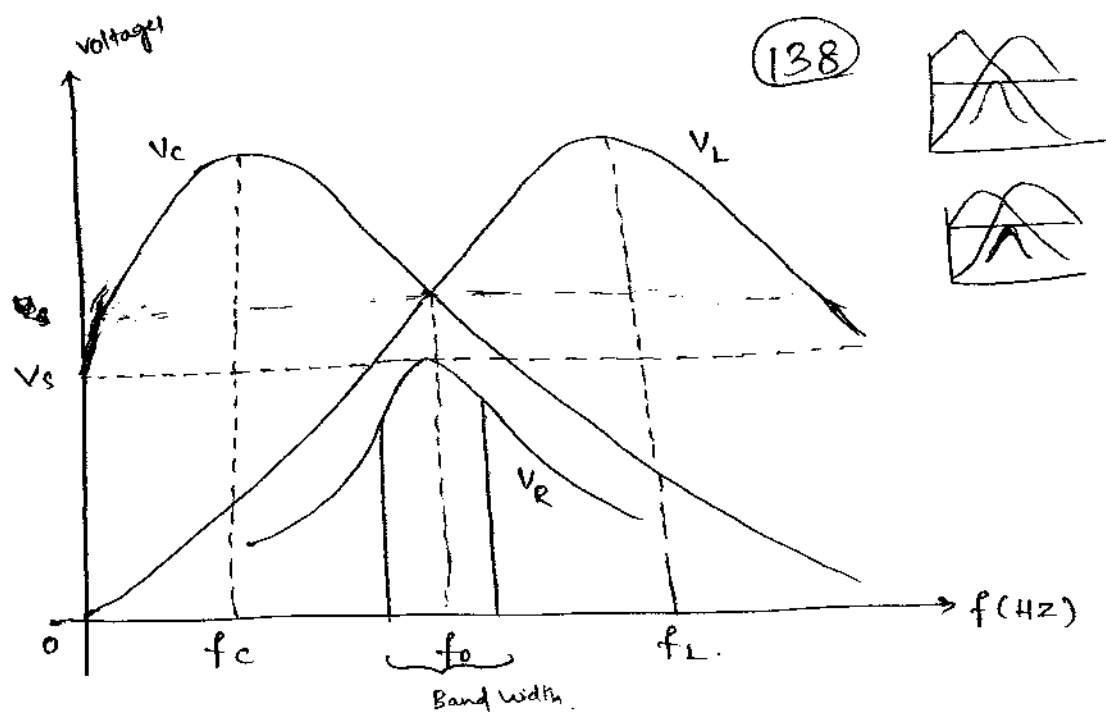
Q the freq. of Energy Oscillations b/w L & C Components under Resonance is —

$$= 2f_0$$

$E_C \rightarrow$ static Energy stored



Variation of voltages across passive elements with change in supply frequency



Q the freq. at which max Voltage appears across Capacitor is

Derive, $\frac{dV_c}{d\omega} = 0$

$$f_c = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} \text{ Hz}$$

$$= \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{R^2C}{2L}}$$

$$f_c = f_0 \sqrt{1 - \frac{R^2C}{2L}} \text{ Hz.}$$

The freq. at which max Voltage appears across Inductance

derive, $\frac{dV_L}{d\omega} = 0$

$$f_L = \frac{1}{2\pi \sqrt{LC - \frac{R^2C^2}{2}}}$$

$$f_L = \frac{1}{2\pi\sqrt{LC} \sqrt{1 - \frac{R^2C}{2L}}}$$

$$f_L = \frac{f_0}{\sqrt{1 - \frac{R^2C}{2L}}} \text{ Hz.}$$

from ckt

$$|I| = \frac{|V|}{|Z|} = \frac{|V|}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

at $\omega = \omega_0 \rightarrow X_{net} = 0$

$$|I| = \frac{|V|}{\sqrt{R^2 + 0^2}} \Rightarrow |I| = \frac{|V|}{R}$$

└─ maximum

\Rightarrow So, power Transferred to entire circuit is also maximum, $P_0 = |I_0|^2 \cdot R = \frac{V^2}{R} \omega$

at $\omega = \omega_0$

R $V_R = I_R \cdot R = I_0 \cdot R = \frac{|V|}{R} \cdot R \rightarrow V_R = V$

L $V_L = +jX_L I_L = +j\omega_0 L \cdot I_0 = +j \left[\frac{\omega_0 L}{R} \right] \cdot V \rightarrow V_L = +jQ_0 V$

C $V_C = -jX_C I_C = \frac{-j}{\omega_0 C} \cdot I_0 = -j \left[\frac{1}{\omega_0 RC} \right] \cdot V \rightarrow V_C = -jQ_0 V$

voltage amplification

Note: Under Series Resonance Condition, the Net Impedance is minimum.

So Current is Maximum, ~~the~~ hence it is an acceptor ckt.

at Series Resonance freq. the Voltages across L & C are Quality factor times the Supply Voltage. Hence this ckt is Considered as Voltage amplification ckt

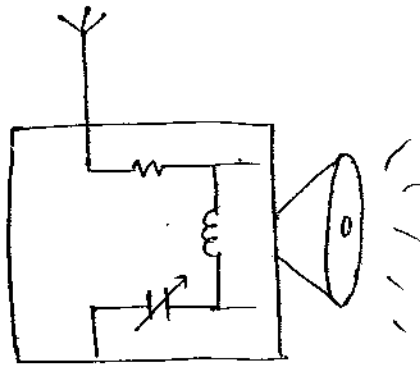
This Series Resonance phenomena Can be used in design of Analog Communication based Receivers Using passive elements
Ex: Old Radios before Transistor Technology.

Application of Series Resonance phenomena to Communication:

139

BandWidth (BW):

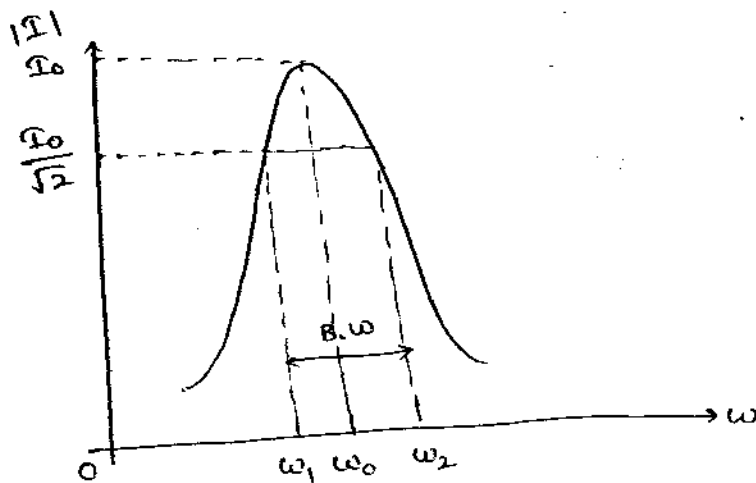
B.W represents the Range of frequencies where the power level in the signal is at least half of max power that ckt can handle.



for Tuning Variable Cap is used \because if Inductor is used it Bulky to handle & lower

at cut off frequencies:

$$P = \frac{P_0}{2} = \frac{I_0^2 R}{2} = \left[\frac{I_0}{\sqrt{2}} \right]^2 R = [0.707 I_0]^2 R$$



$\omega_1 \rightarrow$ lower cut off frequency

$\omega_2 \rightarrow$ upper cut off frequency

At cut-off frequencies

$$|I| = \frac{I_0}{\sqrt{2}}$$

$$\frac{|V|}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{|V|}{R\sqrt{2}}$$

$$(\omega L - \frac{1}{\omega C})^2 = R^2$$

$$\omega L - \frac{1}{\omega C} = \pm R$$

Radio Ravi India

88 - 110 MHz

$$\text{at } \omega = \omega_2 \quad \left[\omega_2 L - \frac{1}{\omega_2 C} \right] = +R \rightarrow \textcircled{A}$$

$$\text{at } \omega = \omega_1 \quad \left[\omega_1 L - \frac{1}{\omega_1 C} \right] = -R \rightarrow \textcircled{B}$$

From Graph II concept
X_{net} vs ω graph

$\textcircled{A} + \textcircled{B}$

$$L(\omega_1 + \omega_2) - \frac{1}{C} \left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2} \right) = 0$$

$$L = \frac{1}{C \omega_1 \omega_2}$$

$$\omega_1 \omega_2 = \frac{1}{LC} = \omega_0^2$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$f_0 = \sqrt{f_1 f_2}$$

→ Geometric progression

Resonance freq is geometric mean of cut off frequencies

$$\textcircled{A} - \textcircled{B} \quad L(\omega_2 - \omega_1) + \frac{1}{C} \left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2} \right) = 2R$$

$$(\omega_2 - \omega_1) \left[L + \frac{1}{C \omega_1 \omega_2} \right] = 2R$$

$$(\omega_2 - \omega_1) \left[L + \frac{1}{\cancel{Q} \cdot \frac{1}{LQ}} \right] = 2R$$

140

$$[\omega_2 - \omega_1] \cancel{L} = \cancel{L} R$$

$$\boxed{\begin{aligned} \text{B.W} &= \omega_2 - \omega_1 = \frac{R}{L} \text{ rad/sec.} \\ (f_2 - f_1) &= \frac{R}{2\pi L} \text{ Hz.} \end{aligned}}$$

$$\underline{\underline{\text{B.W} \propto R}}$$

But f_0 is Independent to 'R'

Most accurate values of ω_1 & ω_2

$$(\omega_2 - \omega_1) = \frac{R}{L} \rightarrow \textcircled{1}$$

$$\omega_1 \omega_2 = \frac{1}{\cancel{L} \cancel{C}} \rightarrow \textcircled{2} \Rightarrow \omega_1 \omega_2 = \frac{1}{LC} \rightarrow \textcircled{2}$$

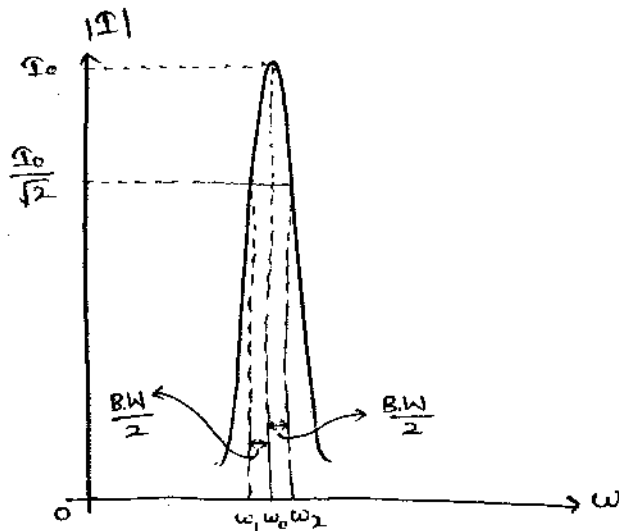
$$(\omega_2 + \omega_1) = \sqrt{(\omega_2 - \omega_1)^2 + 4\omega_1 \omega_2} \rightarrow \textcircled{3}$$

Solve $\textcircled{1}$ & $\textcircled{2}$, We get

$$\left. \begin{aligned} \omega_1 &= \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{\sqrt{LC}}\right)^2} \\ \omega_2 &= \frac{+R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{\sqrt{LC}}\right)^2} \end{aligned} \right\} \text{rad/sec}$$

* if $Q_0 \geq 10$ then the Current vs frequency graph becomes Very narrow & steep. then the following approximate mathematical Relations hold good.

if $Q_0 \geq 10$



$$[\omega_0 - \omega_1] = \frac{B.W.}{2} \rightarrow \omega_1 = \omega_0 - \frac{B.W.}{2}$$

$$\omega_1 = \omega_0 - \frac{R}{2L}$$

$$f_1 = f_0 - \frac{R}{4\pi L}$$

$$\omega_0 = \frac{\omega_1 + \omega_2}{2} \Rightarrow \text{Arithmetic mean}$$

$$[\omega_2 - \omega_0] = \frac{B.W.}{2} \rightarrow \omega_2 = \omega_0 + \frac{B.W.}{2}$$

$$\omega_2 = \omega_0 + \frac{R}{2L}$$

$$f_2 = f_0 + \frac{R}{4\pi L}$$

Q determine the Power factor of Series RLC ckt at lower & Upper Cutoff frequencies.

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\text{at } \omega = \omega_1 \Rightarrow Z = R - jR$$

$$\therefore \omega_1 L - \frac{1}{\omega_1 C} = -R$$

$$P.F = \frac{R}{|Z|} = \frac{R}{\sqrt{R^2 + R^2}} = \frac{1}{\sqrt{2}} = 0.707 \text{ (lead)}$$

$$\text{at } \omega = \omega_2 \Rightarrow Z = R + jR$$

$$\therefore \omega_2 L - \frac{1}{\omega_2 C} = +R$$

$$P.F = \frac{R}{|Z|} = \frac{R}{\sqrt{R^2 + R^2}} = \frac{1}{\sqrt{2}} = 0.707 \text{ (lag)}$$

IES determine the Relation b/w Quality factor & damping Ratio in a series RLC ckt

Series R-L-C: $Q_0 = \frac{1}{R} \sqrt{\frac{L}{C}}$

141

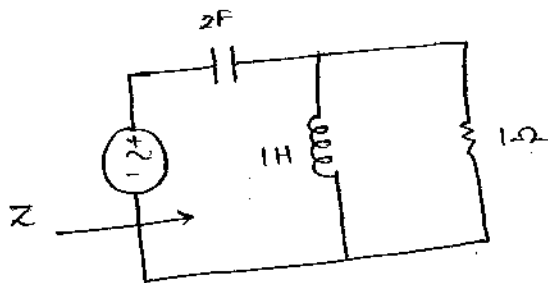
$$\xi = \frac{R}{2} \sqrt{\frac{C}{L}}$$

$$\Rightarrow \underline{\underline{\xi = \frac{1}{2Q_0} *}}$$

$$\underline{\underline{Q_0 = \frac{1}{2\xi}}}$$

Gate

determine the Resonance freq. of ckt shown.



$$Z = \frac{-j}{2\omega} + [1 \parallel j\omega]$$

$$= \frac{-j}{2\omega} + \frac{j\omega}{1+j\omega} \times \frac{(1-j\omega)}{(1-j\omega)}$$

$$= \frac{-j}{2\omega} + \frac{\omega^2 + j\omega}{1+\omega^2}$$

$$Z = \frac{\omega^2}{1+\omega^2} + j \left[\frac{\omega}{1+\omega^2} - \frac{1}{2\omega} \right]$$

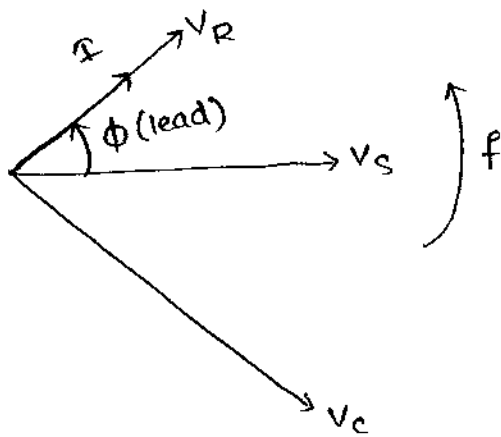
at $\omega = \omega_0 \rightarrow X_{net} = 0$

$$\frac{\omega_0}{1+\omega_0^2} = \frac{1}{2\omega_0}$$

$$2\omega_0^2 = 1 + \omega_0^2$$

$$\omega_0^2 = 1 \Rightarrow \omega_0 = 1 \text{ rad/sec.}$$

Q The partial phasor diagram of a series RLC ckt is shown as below.
then the operating freq. of a ckt is.



(a) $f = f_0$

(b) $f < f_0$

(c) $f > f_0$

(d) $f = 0$

1. ...
P.T. 6/10

In RLC Series Current Same

in R Current is in phase.

$\therefore V_R$ & I are $\therefore I$ leading V_S

\Rightarrow major RC A/W

in RC $f < f_0$

Selectivity (S):

Communication

Selectivity is the ability of 'N/w' to discriminate or distinguish b/w desired & undesired frequencies. mathematically

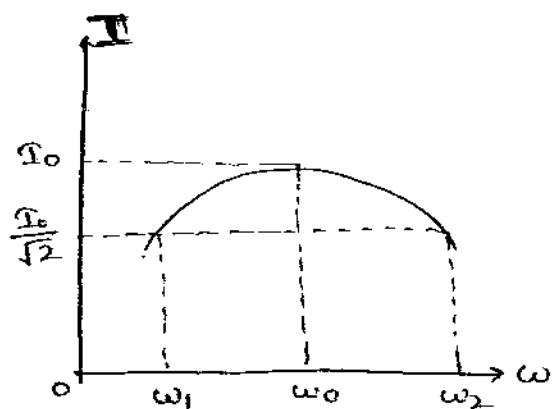
Selectivity is given by

$$S = \frac{f_0}{\text{B.W.}} = \frac{f_0}{|f_2 - f_1|} = \frac{\frac{1}{2\pi \sqrt{LC}}}{\frac{R}{2\pi L}} = \frac{1}{R} \sqrt{\frac{L}{C}} \quad \text{--- } Q_0$$

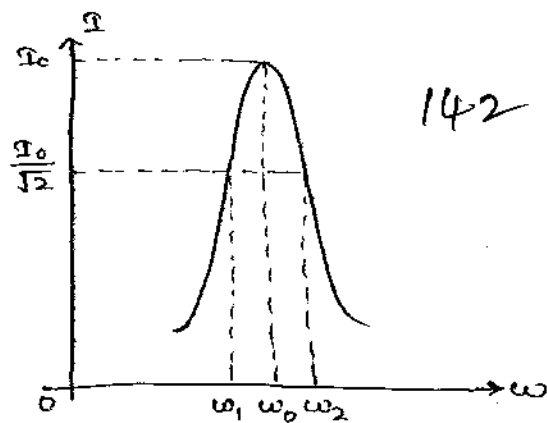
$$\boxed{S = Q_0}$$

Selectivity is Quality factor under Resonance.

$$S \propto \frac{1}{\text{B.W.}}$$



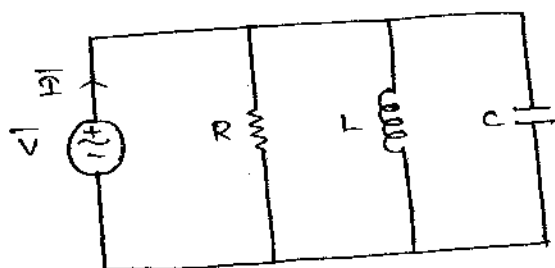
B.W ↑, S ↓, Q₀ ↓



B.W ↓, S ↑, Q₀ ↑

II] Parallel Resonance:

(a) General Circuit.



At Resonance:

\bar{V} and $\bar{I} \rightarrow$ in phase

$$\phi = 0^\circ \text{ [UPF]}$$

$$Y = G$$

$$Y_T = Y_R + Y_L + Y_C$$

$$= \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$Y_T = \frac{1}{R} + j \left[\frac{1}{X_C} - \frac{1}{X_L} \right]$$

$$\text{at } \omega = \omega_0 \rightarrow B_{\text{net}} = 0$$

$$\frac{1}{X_C} = \frac{1}{X_L} \Rightarrow \omega_0 L = \frac{1}{\omega_0 C}$$

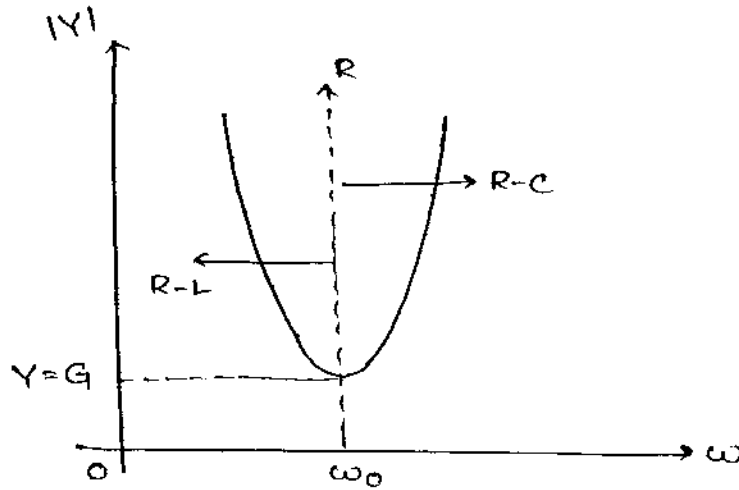
$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

Graph-I

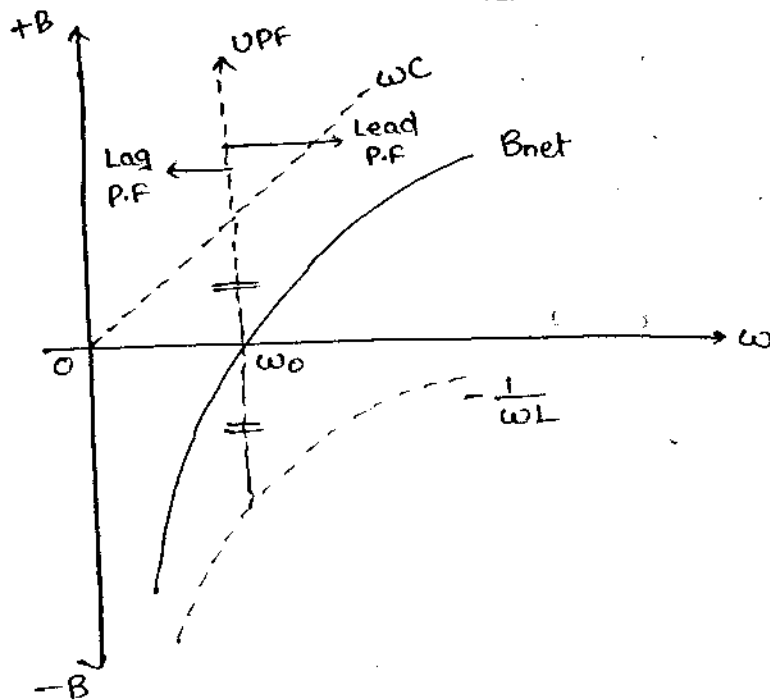
$|Y|$ vs ω

$$|Y| = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$



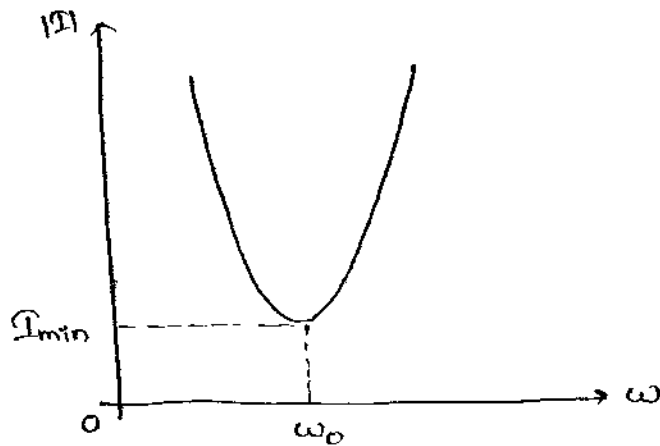
Graph-II

B_{net} vs ω $\left[Y = \frac{1}{R} + j \underbrace{\left(\omega C - \frac{1}{\omega L}\right)}_{B_{net}} \right]$



Graph III

$|I|$ vs ω



143

Phasor diagram:

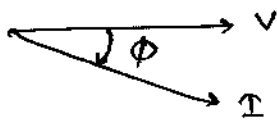
Case (i) $\omega < \omega_0$

$$Y = G - jB_{\text{net}}$$

$\rightarrow R-L$

'I' lags 'V' by $\phi < 90^\circ$

(lagging P.F.)



$\phi \rightarrow (-ve)$

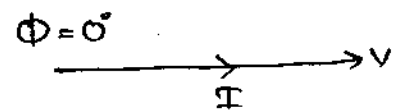
Case (ii) $\omega = \omega_0$

$$Y = G$$

\rightarrow purely Resistive.

'I' in phase with 'V'

$$\phi = 0^\circ \text{ [UPF]}$$



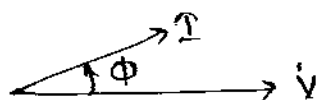
Case (iii) $\omega > \omega_0$

$$Y = G + jB_{\text{net}}$$

$\rightarrow R-C$

'I' leads 'V' by $\phi < 90^\circ$

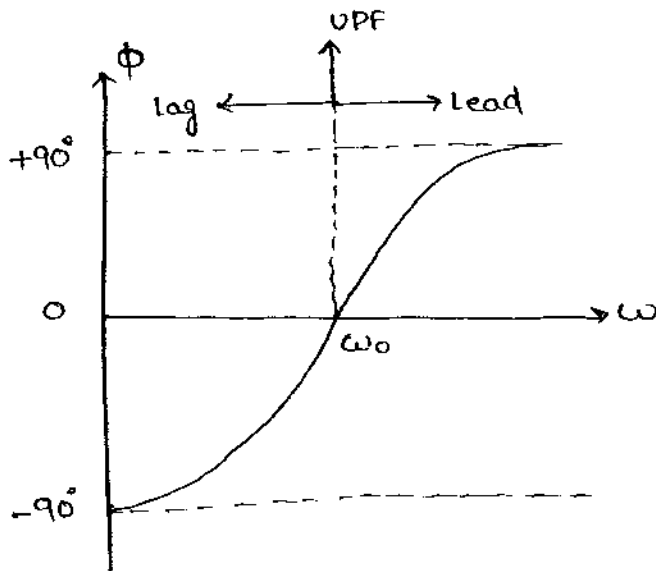
(leading P.F.)



$\phi \rightarrow (+ve)$

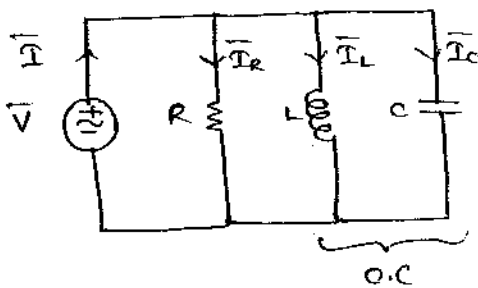
Graph-IV

ϕ vs ω



Complete phasor diagram Under Resonance:

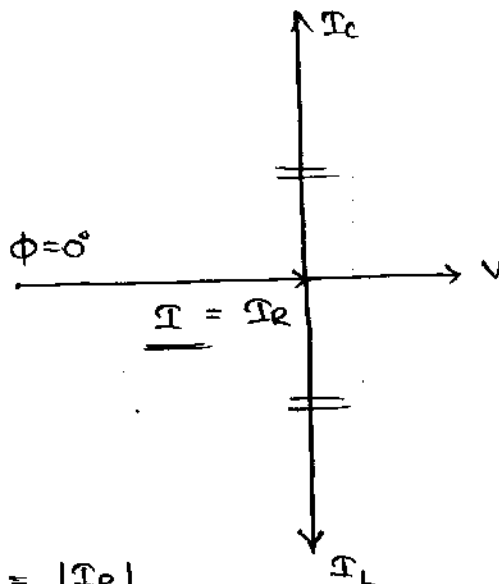
$\phi = 180^\circ \rightarrow C-L$



At Resonance:

$$|B_L| = |B_C|$$

$$|I_L| = |I_C|$$



$$I = |I_R|$$

$$\phi = 0^\circ$$

$$P.F = 1 \text{ (UPF)}$$

under Resonance: LC acts as O.C $\therefore B_{net} = 0$

$$\text{Reactance} = X_{net} = \frac{\infty}{(-\frac{1}{L})}$$

at parallel Resonance Condition, the pure parallel LC ^{segment} of the ckt acts as Open circuit. and Source Supplying Energy only to Resistor. However, L & C Components Work together by Exchanging the store magnetic & static Energies Between them as an Oscillatory phenomena.

⇒ The freq. of these Energy Oscillations B/w L & C is $2\omega_0$ (or) $2f_0$

144

from ckt

$$|V| = \frac{|I|}{|Y|} = \frac{|I|}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

at $\omega = \omega_0 \rightarrow \underline{B_{net} = 0}$

$$|V| = \frac{|I|}{\sqrt{\frac{1}{R^2} + 0^2}}$$

$$|V| = |I| \cdot R \quad \xrightarrow{\text{minimum}}$$

at $\omega = \omega_0$

$$\boxed{R} \quad I_R = \frac{V_R}{R} = \frac{V}{R} = \frac{|I| \cdot R}{R} \rightarrow \boxed{I_R = I}$$

$$\boxed{L} \quad I_L = \frac{V_L}{+jX_L} = \frac{V}{+j\omega_0 L} = -j \left[\frac{R}{\omega_0 L} \right] \cdot I \rightarrow \boxed{I_L = -jQ_0 I}$$

$$\boxed{C} \quad I_C = \frac{V_C}{-jX_C} = \frac{V}{-j/\omega_0 C} = +j [\omega_0 R C] \cdot I \rightarrow \boxed{I_C = +jQ_0 I}$$

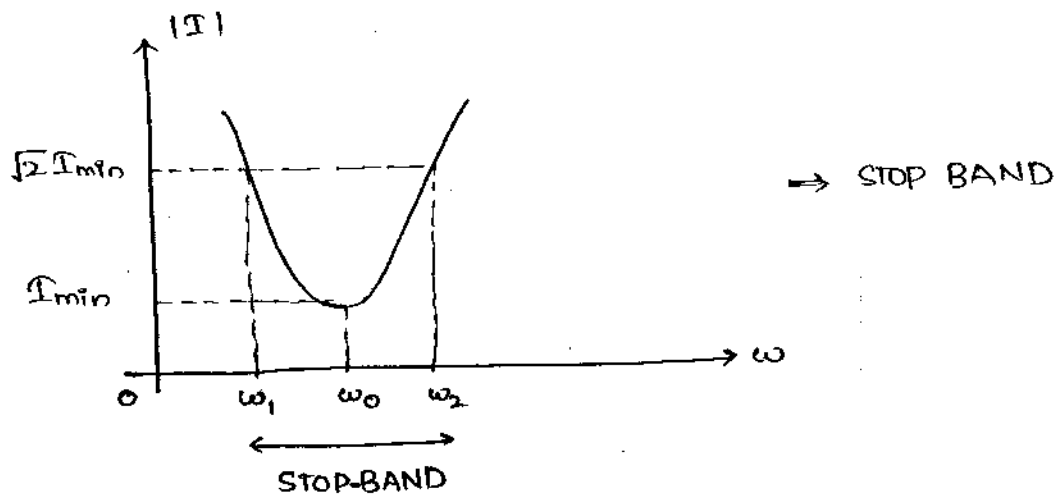
Current Amplification

Note:

1. At parallel Resonance Condition the Net admittance is minimum.
So, Current also minimum. Hence it is a Rejector circuit
2. At parallel Resonance freq. the Currents through L & C Components are Quality factor times the Supply Current. Hence this circuit is Considered as Current Amplification circuit.

Note

This parallel Resonance phenomena Can be used in design of Band stop filter or Band Rejection filter.



$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \left(\frac{1}{\sqrt{LC}}\right)^2}$$

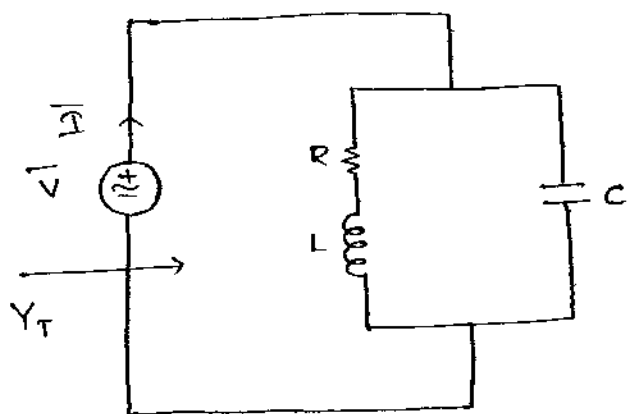
$$\omega_2 = +\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \left(\frac{1}{\sqrt{LC}}\right)^2}$$

STOP-BAND:

$$(\omega_2 - \omega_1) = \frac{1}{RC} \text{ rad/sec}$$

(b) practical parallel Resonance circuit [TANK Circuit]

145



$$Y_T = Y_1 + Y_2$$

$$Y_T = \frac{1}{R + jX_L} + \frac{1}{-jX_C}$$

$$Y_T = \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$Y_T = \left[\frac{R}{R^2 + X_L^2} \right] + j \left[\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2} \right]$$

at $\omega = \omega_0 \rightarrow B_{net} = 0$

$$\frac{1}{X_C} = \frac{X_L}{R^2 + X_L^2}$$

$$R^2 + X_L^2 = X_C X_L = \omega_0 \cdot L \cdot \frac{1}{\omega_0 C}$$

$$\boxed{R^2 + X_L^2 = \frac{L}{C}}$$

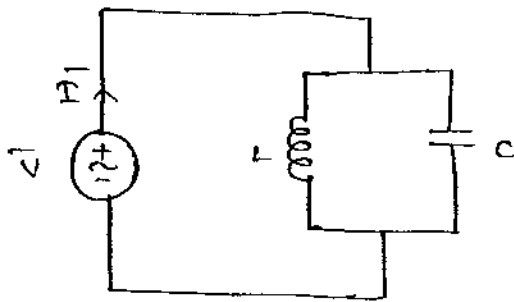
$$R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

$$\omega_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\boxed{\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \text{ rad/sec}}$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \text{ Hz}$$

(c) Ideal Tank [$R=0$, $Q_0=\infty$]



$$Y_T = Y_1 + Y_2$$

$$Y_T = \frac{1}{+jX_L} + \frac{1}{-jX_C}$$

$$Y_T = +j \left[\frac{1}{X_C} - \frac{1}{X_L} \right]$$

at $\omega = \omega_0 \longrightarrow B_{net} = 0$

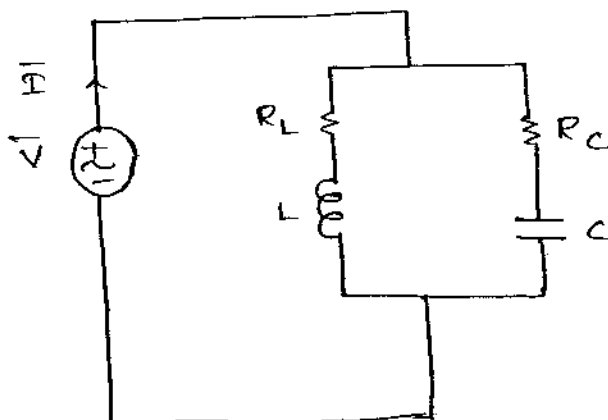
$$\frac{1}{X_C} = \frac{1}{X_L}$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz.}$$

(d) most Accurate parallel Resonance circuit Including losses.



$$f_0 = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{\left(\frac{L}{C} - R_L^2\right)^2}{\left(\frac{L}{C} - R_C^2\right)^2}} \text{ Hz}$$

Gate
For what circuit condition shown above^s (d) the circuit Resonates all frequencies?

146

for circuit in (d)

$$Y_T = Y_1 + Y_2 = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$= \left[\frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} \right] + j \left[\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2} \right]$$

→ if $R_L = R_C = \sqrt{\frac{L}{C}}$ then the net Susceptance part of total Admittance by default becomes zero

$$B_{\text{net}} = 0$$

Hence, circuit is purely Resistive $\Rightarrow Y_T = G_{net}$

So, we can say circuit Resonates at all frequency

Dynamic Impedance ($Z_{dynamic}$):

⇒ it is Impedance offered by the Network to Source under Resonance Condition

→ (a) Series R-L-C : $Z_{dyn} = R - j$

(b) parallel R-L-C: $Z_{dyn} = R \parallel \omega$

$$[\because Y_T = \frac{1}{R} \Rightarrow Z_{dyn} = R]$$

(c) Tank circuit: $Z_{\text{dyn}} = \frac{L}{RC}$ $\left[I_0 = \frac{V}{Z_{\text{dyn}}} = \frac{V}{L/RC} = \frac{VRC}{L} \text{ A} \right]$

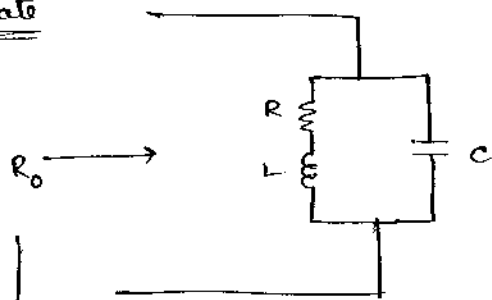
$$Y_T = \left[\frac{R}{R^2 + X_L^2} \right] + j \left[\underbrace{\frac{1}{X_L} \times \frac{X_L}{R^2 + X_L^2}}_{=0} \right]$$

$$\downarrow$$

$$Z_{dyn} = \frac{R^2 + X_L^2}{R} = \frac{L/c}{R} = \frac{L}{RC} \Omega \quad \therefore R^2 + X_L^2 = \frac{L}{C}$$

(d) Ideal Tank: $Z_{dyn} = \frac{\omega \cdot \Omega}{\rightarrow o.c}$

Gate



at Resonance

(a) $R_o = R$

(b) $R_o > R$

(c) $R_o < R$

(d) $R_o = 0$

→ Here $R_o = Z_{dyn}$

$$R_o = \frac{L}{RC}$$

we know, $R^2 + X_L^2 = \frac{L}{C}$

$$R^2 \left[1 + \frac{X_L^2}{R^2} \right] = \frac{L}{C}$$

$$\boxed{R^2 \left[1 + \frac{\omega_0^2 L^2}{R^2} \right] = \frac{L}{C}}$$

$$R^2 [1 + Q_o^2] = \frac{L}{C}$$

$$\frac{L}{RC} = R [1 + Q_o^2]$$

↓

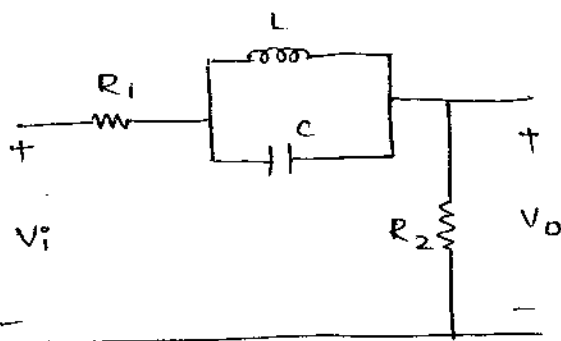
$$Z_{dyn} = R [1 + Q_o^2]$$

$$R_o = R [1 + Q_o^2]$$

$$\therefore \underline{\underline{R_o > R}}$$

* Q. factor of RL = $\frac{\omega_0 L}{R}$

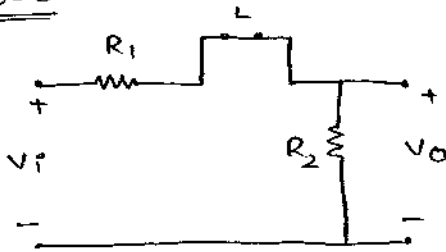
2. The N/w acts as low pass filter.



L or $C \Rightarrow$ first order filter

L & $C \Rightarrow$ second order filter

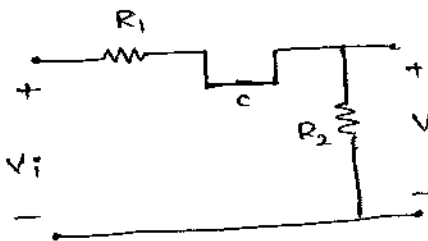
$\omega = 0$



$$V_o = V_i \left[\frac{R_2}{R_1 + R_2} \right] \Rightarrow \text{Gain} = |H| = \left| \frac{V_o}{V_i} \right| = \frac{R_2}{R_1 + R_2}$$

147

$\omega = \infty$

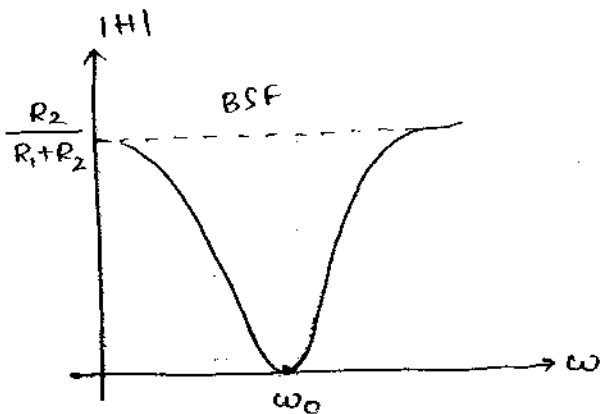


$$V_o = V_i \left[\frac{R_2}{R_1 + R_2} \right] \Rightarrow |H| = \left| \frac{V_o}{V_i} \right| = \frac{R_2}{R_1 + R_2}$$

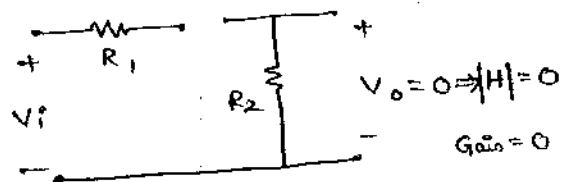
at $\omega = \omega_0$ (Resonance) $\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

for Tank ckt

$Z_{dyn} = \infty \Rightarrow \text{o.c}$



Band stop filter



Magnetic Circuits

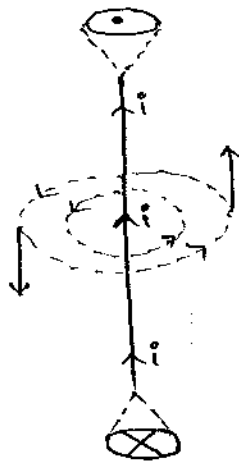
⇒ charge at rest produce only electric fld (electrostatics)

⇒ charge in motion ie, Current electric fld, produce both elec. & mag flds.

(a) steady Currents DC produce Time Invariant mag. flds

(b) Time Varying Currents AC produce Time Varying mag. flds.

Ampere Right Hand Thumb Rule:

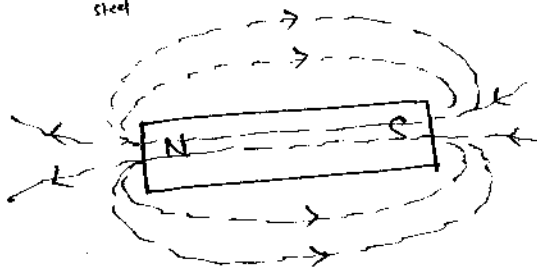


permanent magnet:

↳ High Retentivity

↳ Hard magnetic Materials

ex: Alnico's, Se_2Co_3 (Selenium cobalt)



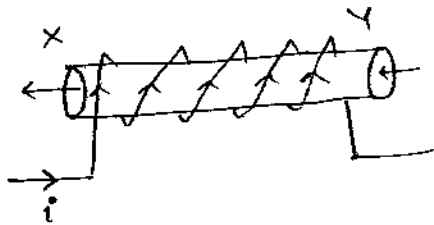
Electro magnet :

↳ Low Retentivity

↳ Soft magnetic Materials

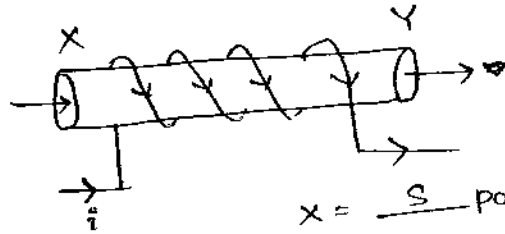
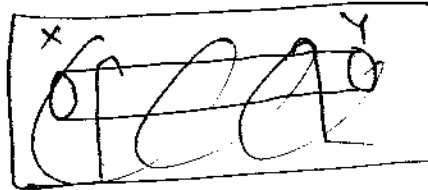
ex: Soft Iron, SiFe, ferrites.

148



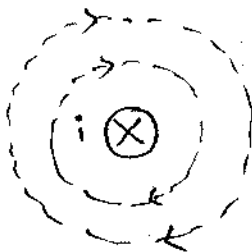
X = N pole

Y = S pole

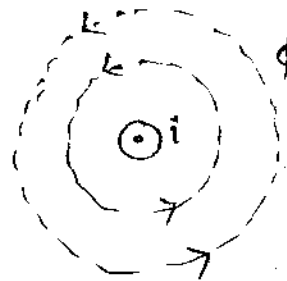


X = S pole

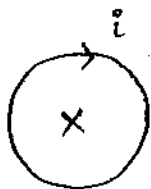
Y = N pole



ϕ = Clock oriented



ϕ = anticlock oriented

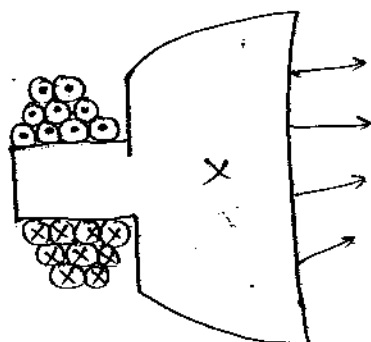


X = South pole



Y = North pole.

Ex: pole Core \rightarrow DC machine



X = North pole

Analogy between electric & magnetic circuit:

Elec. ckt

Mag. ckt

(1) Voltage / EMF



$V \rightarrow (\text{volts})$

(1) $\text{MMF} = N \cdot i$



$\text{MMF} \rightarrow (\text{AT})$

(2) Current



$I \rightarrow (\text{Amps})$

(2) Flux



$\Phi \rightarrow (\text{wb})$

(3) Resistance $= \frac{V}{I}$



$R \rightarrow (\text{ohms})$

(3) Reluctance $= \frac{\text{MMF}}{\Phi} = \frac{N^2}{\Phi}$



$\mathcal{S} \rightarrow (\text{AT/wb})$

(4) $R = \frac{\rho l}{a} = \frac{l}{\sigma a}$

(4) $\mathcal{S} = \frac{l}{\mu a}$

(5) $E = \frac{V}{d}$



(volt/m)

(5) $H = \frac{\text{MMF}}{l} = \frac{N i}{l}$



(AT/m)

(6) $J = \frac{I}{a}$



Amp/m^2

(6) $B = \frac{\Phi}{a}$



$(\text{wb/m}^2) \rightarrow \text{Tesla}$

(7) $J = \sigma E$

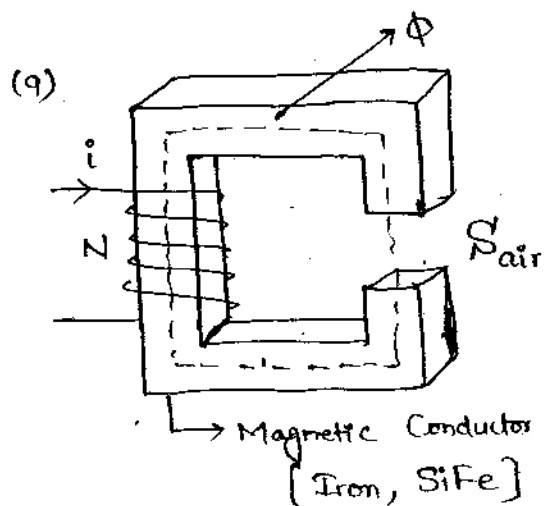
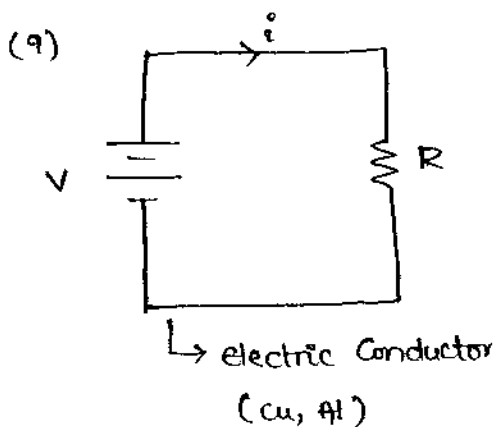
(7) $B = \mu H$

(8) $R_s = R_1 + R_2$

$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$

(8) $\mathcal{S}_s = \mathcal{S}_1 + \mathcal{S}_2$

$\frac{1}{\mathcal{S}_p} = \frac{1}{\mathcal{S}_1} + \frac{1}{\mathcal{S}_2}$



Faraday's Law of Electromagnetic Induction:

The mathematical form of Faraday's Law is

149

$$e = - \frac{d\psi}{dt}$$

$$\psi = N\Phi$$

\rightarrow Flux linkages (wb-T)

$$e = -N \frac{d\Phi}{dt}$$

\rightarrow Rate of change in flux (wb/sec)

\rightarrow No. of Turns of Coil

\rightarrow -ve sign [Lenz's law]

\rightarrow Induced emf in Coil (Volts)

$$e = -N \left[\frac{\Phi_{\text{final}} - \Phi_{\text{initial}}}{\Delta t} \right] \text{ volts.}$$

Note:

(1) Flux is a proportional function of Current

$$\text{MMF} = \Phi \cdot S$$

$$N \cdot i = \Phi \cdot S$$

\downarrow Fixed \downarrow Fixed

$$\frac{\Phi}{i} = \left(\frac{N}{S} \right)$$

\rightarrow fixed

$$\Rightarrow \frac{\Phi}{i} = k$$

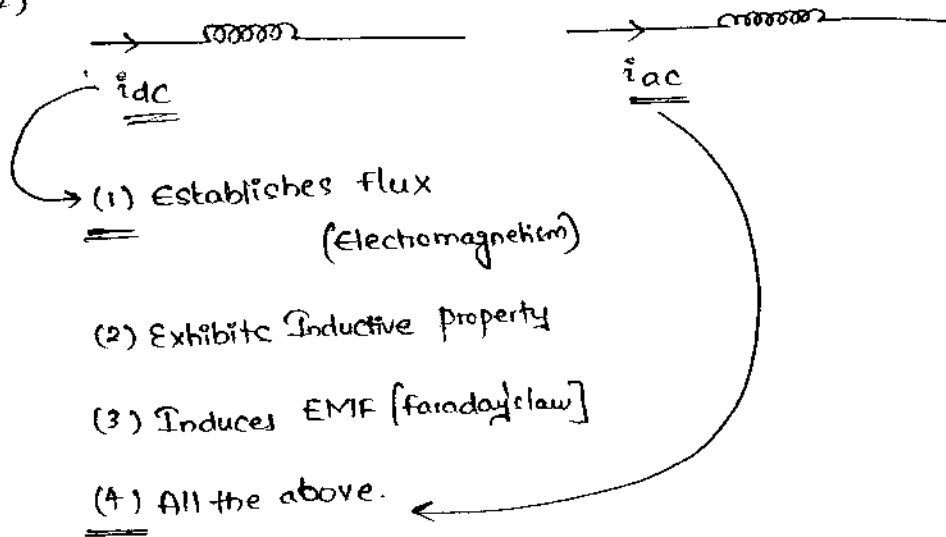
$$\Phi = k \cdot i$$

$$\boxed{\Phi \propto i}$$

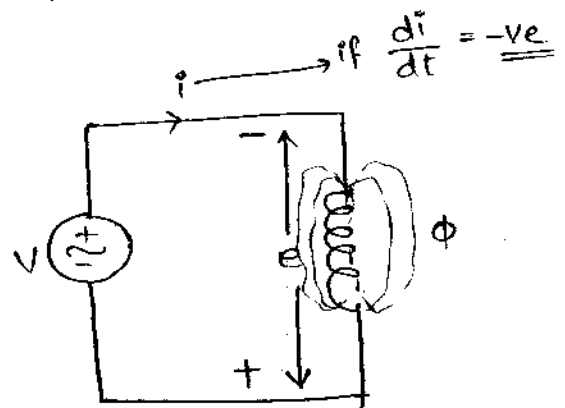
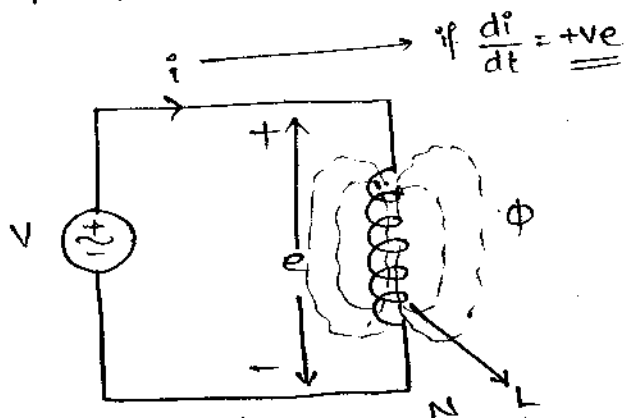
\Rightarrow neglecting saturation

Note:

(2)



Concept of Self Induced EMF & Self Inductance:



$$\Phi = \Phi * \frac{i}{i}$$

$$\Phi = \left[\frac{\Phi}{i} \right] * i$$

$$\frac{d\Phi}{dt} = \left[\frac{\Phi}{i} \right] \frac{di}{dt} \quad \text{--- ①}$$

$$e = -N \frac{d\Phi}{dt} \quad \text{--- ②}$$

~~$$e = -N \left[\frac{N\Phi}{i} \right] \frac{di}{dt}$$~~

150

$$e = -[L] \frac{di}{dt}$$

Self Induced emf (volts)

$$L = \frac{N\Phi}{i} = \frac{\mathcal{P}}{i}$$

self Inductance (H)

$$\mathcal{P} = L.i \longrightarrow \text{O.L (3rd form)}$$

Ohm's law

$$e = -L \left[\frac{i_{\text{final}} - i_{\text{initial}}}{\Delta t} \right] \text{ Volt.}$$

$$\text{MMF} = \Phi \cdot \mathcal{S}$$

$$N.i = \Phi \cdot \mathcal{S}$$

$$N \left[\frac{N\Phi}{L} \right] = \Phi \cdot \mathcal{S}$$

$$L = \frac{N^2}{\mathcal{S}}$$

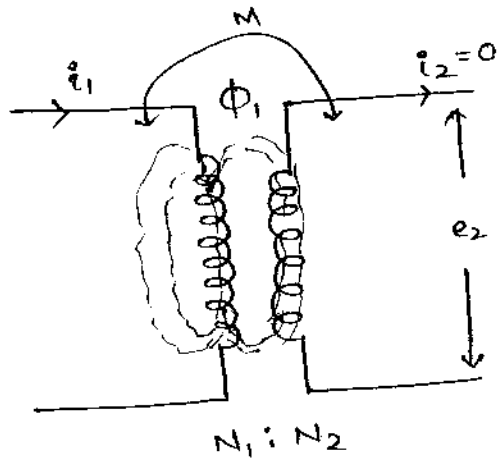
$$\underline{\underline{L \propto N^2}}$$

$$\mathcal{S} = \frac{l}{\mu a}$$

$$L = \frac{N^2}{l/\mu a}$$

$$L = \frac{\mu N^2 a}{l} \underline{\underline{\text{H.}}}$$

Concept of Mutually Induced EMF & Mutual Inductance:



$$\Phi_1 = \Phi_{11} + \Phi_{12}$$

Total Flux \leftarrow Φ_{11} \leftarrow Leakage flux \leftarrow Φ_{12} \leftarrow Common flux (or)

$$\Phi_{12} = k \Phi_1$$

$$0 \leq k \leq 1$$

$$\Phi_{12} = \Phi_{12} * \frac{i_1}{i_1}$$

$$\Phi_{12} = \left[\frac{\Phi_{12}}{i_1} \right] * i_1$$

$$\frac{d\Phi_{12}}{dt} = \left[\frac{\Phi_{12}}{i_1} \right] \frac{di_1}{dt} \longrightarrow (1)$$

$$e_2 = -N_2 \frac{d\Phi_{12}}{dt} \longrightarrow (2)$$

$$e_2 = - \left[\frac{N_2 \Phi_{12}}{i_1} \right] \frac{di_1}{dt}$$

$$e_2 = - [M_{12}] \frac{di_1}{dt} \text{ volts}$$



Mutually Induced emf

$$M_{12} = \frac{N_2 \Phi_{12}}{i_1} = \frac{k \cdot \Phi_1 N_2}{i_1}$$

↓
Mutual Inductance.

151

also vice-versa

$$\Phi_{21} = k \Phi_2 \quad (0 \leq k \leq 1)$$

so,
$$e_1 = - \left[\frac{N_1 \Phi_{12}}{i_2} \right] \frac{di_2}{dt}$$

$$e_1 = - [M_{21}] \cdot \frac{di_2}{dt} \quad \text{Volts.}$$

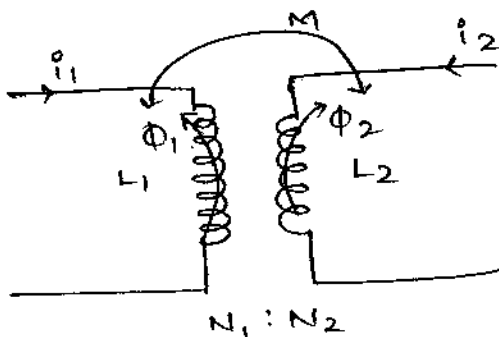
$$M_{21} = \frac{N_1 \Phi_{21}}{i_2} = \frac{k \Phi_2 N_1}{i_2} \quad \text{H.}$$

Note:

If distance b/w coils & permeability of Medium is constant

then,

$$M_{12} = M_{21} = M$$



$$L_1 = \frac{N_1 \Phi_1}{i_1} \quad \left. \vphantom{\frac{N_1 \Phi_1}{i_1}} \right\} \text{ self Inductance of Coil-1}$$

$$L_2 = \frac{N_2 \Phi_2}{i_2} \quad \left. \vphantom{\frac{N_2 \Phi_2}{i_2}} \right\} \text{ self Inductance of Coil-2}$$

$$M = \frac{k \Phi_1 N_2}{i_1} = \frac{k \Phi_2 N_1}{i_2} \quad \left. \vphantom{\frac{k \Phi_2 N_1}{i_2}} \right\} \text{ Mutual Inductance b/w coils}$$

Relationship B/w Self & Mutual Inductances:

$$[M][M] = \left[\frac{K\Phi_1 N_2}{i_1} \right] \left[\frac{K\Phi_2 N_1}{i_2} \right]$$

$$M^2 = K^2 \left[\frac{N_1 \Phi_1}{i_1} \right] \left[\frac{N_2 \Phi_2}{i_2} \right]$$

$$M^2 = K^2 [L_1][L_2]$$

$$M = K \sqrt{L_1 L_2}$$

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

↑
coefficient of coupling

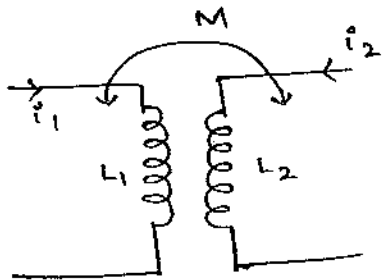
$$(0 \leq K \leq 1)$$

$$M \leq \sqrt{L_1 L_2}$$

Note: In Ideal Transformer $K=1$.

then $M = \sqrt{L_1 L_2}$

Energy stored in a System of two mutually Coupled Coils:



$$E_T = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \pm M i_1 i_2$$

$\oplus \rightarrow$ Mutually adding fluxes

$\ominus \rightarrow$ Mutually opposing fluxes

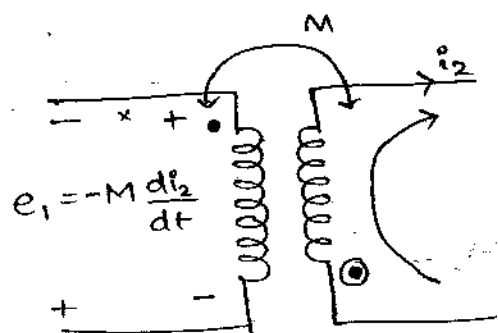
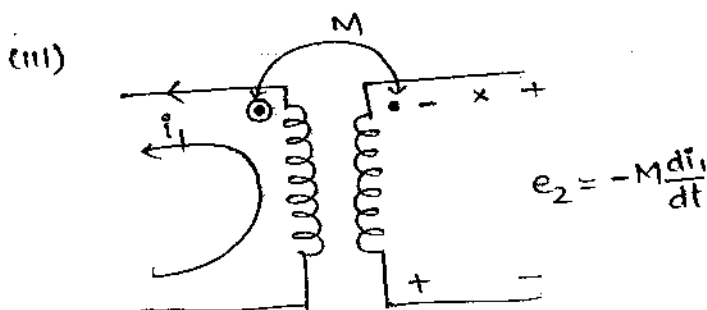
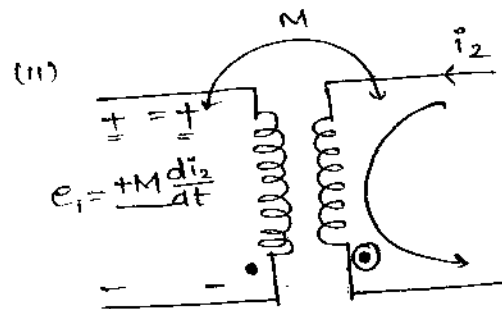
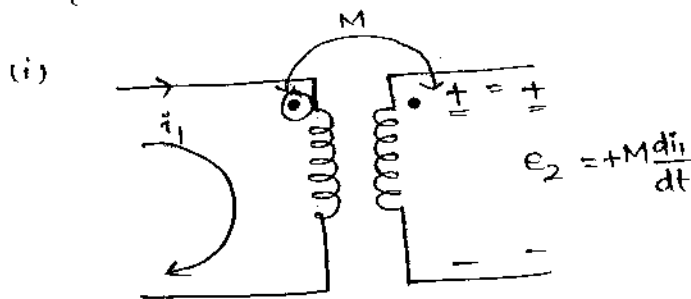
Note: Mutual Inductance is always a positive Quantity but mutually Induced emf can be +ve or -ve. determining the Correct polarity of Mutual Voltage is not possible directly & Hence we used "dot Convention" or "dot Notation".

(152)

St-I: If Current enters the dotted terminale of first coil then the polarity of Mutual ~~Vol~~ Voltage will be +ve at dotted terminal of second coil.

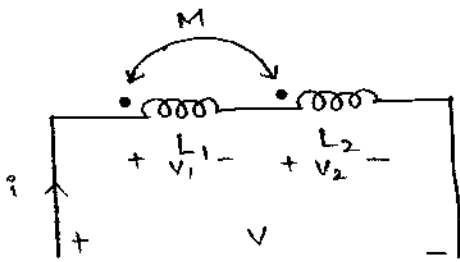
St-II: if Current leaves the dotted terminal of first coil then the polarity of Mutual Voltage is -ve at the dotted terminal of second coil.

Example: determine the Correct polarity & magnitude of mutual voltage w.r.t the given Reference Voltage for System of coils shown below using dot Convention.



I] Coils in Series:

(a) Mutually adding:



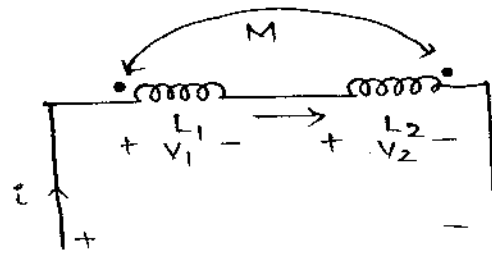
$$V_1 = L_1 \frac{di}{dt} + M \frac{di}{dt}$$

$$V_2 = L_2 \frac{di}{dt} + M \frac{di}{dt}$$

$$L_T = L_1 + L_2 + 2M$$

$$V = V_1 + V_2 \Rightarrow V = (L_1 + L_2 + 2M) \frac{di}{dt}$$

(b) Mutually opposing:



$$L_T = L_1 + L_2 - 2M$$

$$V_1 = L_1 \frac{di}{dt} - M \frac{di}{dt}$$

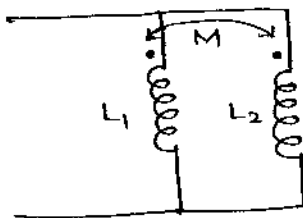
$$V_2 = L_2 \frac{di}{dt} - M \frac{di}{dt}$$

$$V = V_1 + V_2$$

$$V = (L_1 + L_2 - 2M) \frac{di}{dt}$$

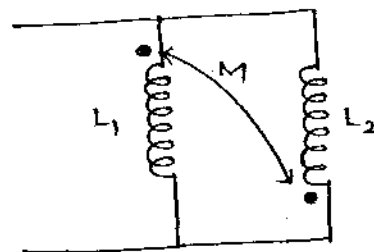
II] Coils in parallel:

(a) Mutually adding:



$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

(b) Mutually opposing:

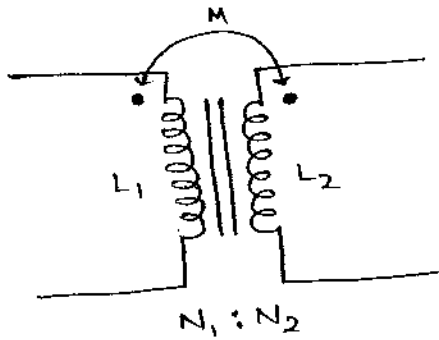


$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

Ideal Transformer circuit in electrical circuits:

Ideal Transformer in Electrical ckt;

153



$$K = 1 \Rightarrow \text{Coeff. of Coupling}$$

$$\text{Ideal Core : } \mu_r = \infty$$

- \$\Rightarrow L_1 \rightarrow \infty H \quad \Rightarrow\$ No losses
- \$\Rightarrow L_2 \rightarrow \infty H \quad \Rightarrow\$ No saturation
- \$\Rightarrow M \rightarrow \infty H \quad \Rightarrow \therefore\$ Linear Transformer

$$\Rightarrow L \propto N^2$$

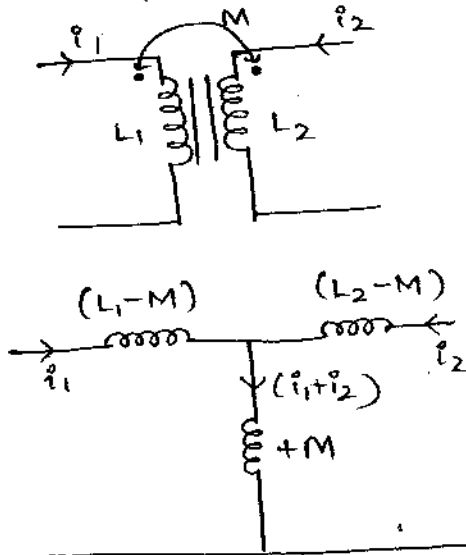
$$L_1 : L_2 : M = N_1^2 : N_2^2 : N_1 N_2$$

\$\Rightarrow\$ Turns Ratio

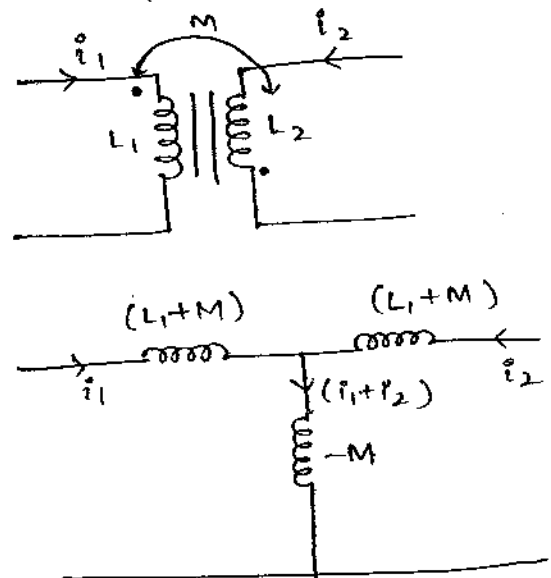
$$a = \frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}} = \frac{L_1}{M} = \frac{M}{L_2}$$

T-Equivalent representation of Ideal T/F: (circuit)

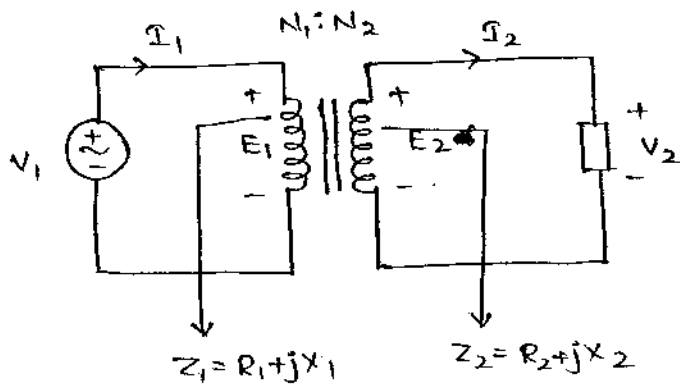
(a) Mutually aiding.



(b) Mutually opposing



Transformer [Machines]



$$K = \frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

→ Voltage Transformation Ratio (Transformer Ratio)

$$R_2' = \frac{R_2}{K^2} \quad (\because K = \frac{N_2}{N_1})$$

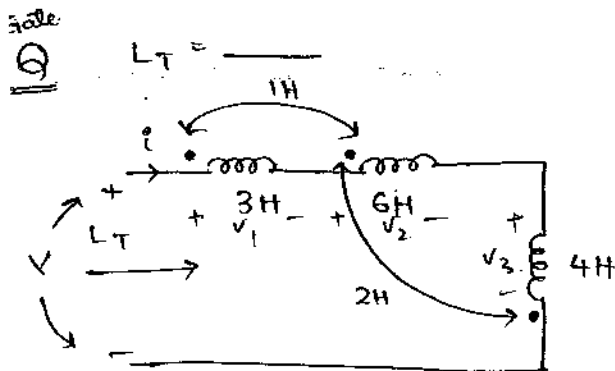
$$R_1' = K^2 R_1$$

$$X_2' = \frac{X_2}{K^2}$$

$$X_1' = K^2 X_1$$

$$Z_1' = K^2 Z_1$$

$$Z_2' = \frac{Z_2}{K^2}$$



current entering in both

$$V_1 = 3 + 1 = 4$$

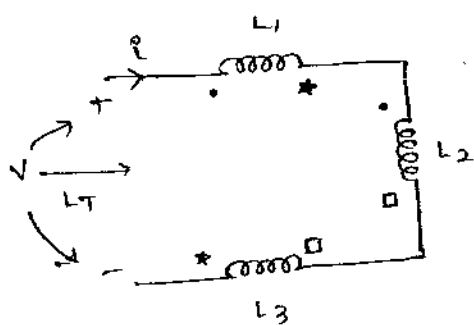
$$V_2 = 6 + 1 - 2 = 5$$

$$V_3 = 4 - 2 = 2$$

$$L_T = 4 + 5 + 2 = 11 H$$

current entering at $6H$
current leaving at $2H$ } mutual will be opposite

185 $L_T =$ _____



$\bullet \rightarrow M_{12}$

$\square \rightarrow M_{23}$

$\star \rightarrow M_{31}$

154

$V_1 = L_1 + M_{12} + M_{31}$

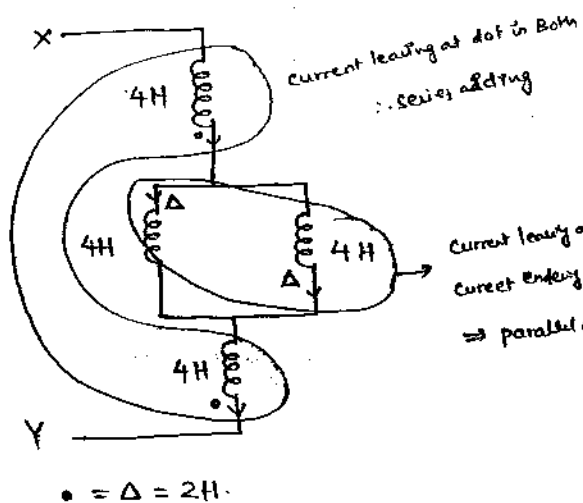
lead.

$V_2 = L_2 + M_{12} - M_{23}$

$V_3 = L_3 + M_{31} - M_{23}$

$L_T = L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} + 2M_{31}$

186 $L_{XY} =$ _____



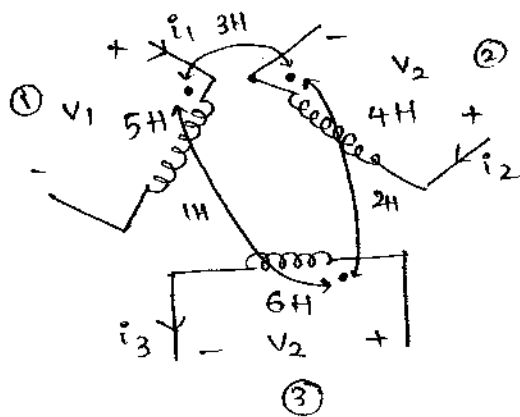
$\Rightarrow 4 + 4 + 2(2) = 12H$

Current leaving at one dot
Current entering at one dot
 \Rightarrow parallel op.

$\Rightarrow \frac{(4)(4) - 2^2}{4 + 4 + 2(2)} = 1H$

$L_{XY} = (12 + 1)H = 13H$

ES write Complete Inductance Matrix for mutually Coupled Coils Below.



open delta connection it is not electrically connected
 but Inductively Connected
 magnetically connected

$$V_1 = 5 \frac{di_1}{dt} - 3 \frac{di_2}{dt} + 1 \frac{di_3}{dt}$$

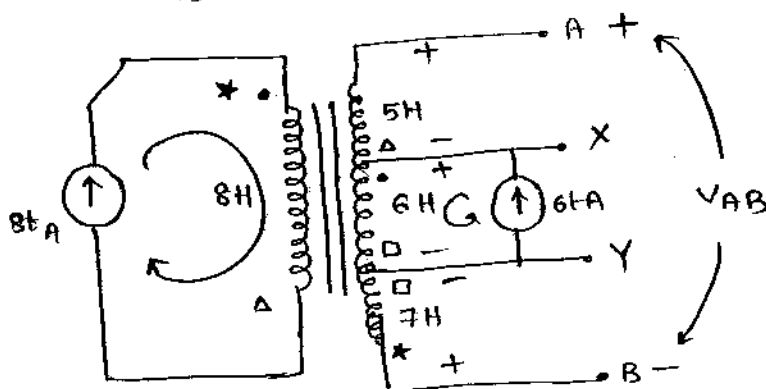
$$V_2 = -3 \frac{di_1}{dt} + 4 \frac{di_2}{dt} - 2 \frac{di_3}{dt}$$

$$V_3 = +1 \frac{di_1}{dt} - 2 \frac{di_2}{dt} + 6 \frac{di_3}{dt}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 5 & -3 & 1 \\ -3 & 4 & -2 \\ 1 & -2 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

Gate

Find $V_{AB} =$ _____



$$V_{AB} = V_{AX} + V_{XY} + V_{YB}$$

- $\rightarrow 4H$
- ★ $\rightarrow 3H$
- △ $\rightarrow 2H$
- $\rightarrow 1H$

$$V_{AX} = 0V_{\text{self}} + V_{M(\Delta)} = +2 \cdot \frac{d(8t)}{dt} = +16 \text{ volts}$$

Self volt = 0
 ∴ no current circulating

$$V_{XY} = V_S + V_{M(\bullet)} + 0(V_{M(\square)}) = 6 \frac{d(6t)}{dt} + 4 \frac{d(8t)}{dt} = 68 \text{ volts}$$

Mutual Inductance = 0
 ∴ Not excited to induce flux in the other coil

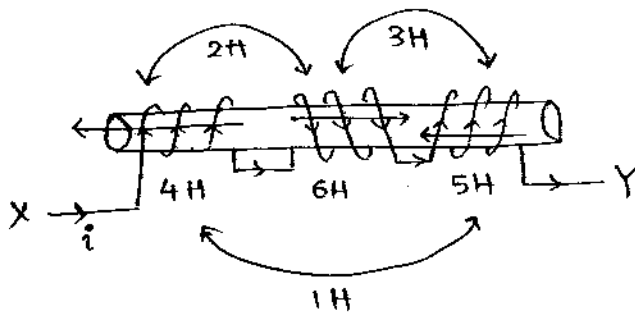
$$V_{YB} = 0V_{\text{self}} + V_{M(\star)} + V_{M(\square)} = -3 \frac{d(8t)}{dt} - 1 \cdot \frac{d(6t)}{dt} = -30 \text{ volts}$$

no current circulating

$$V_{AB} = V_{AX} + V_{XY} + V_{YB} = 16 + 68 - 30 = 54 \text{ volts}$$

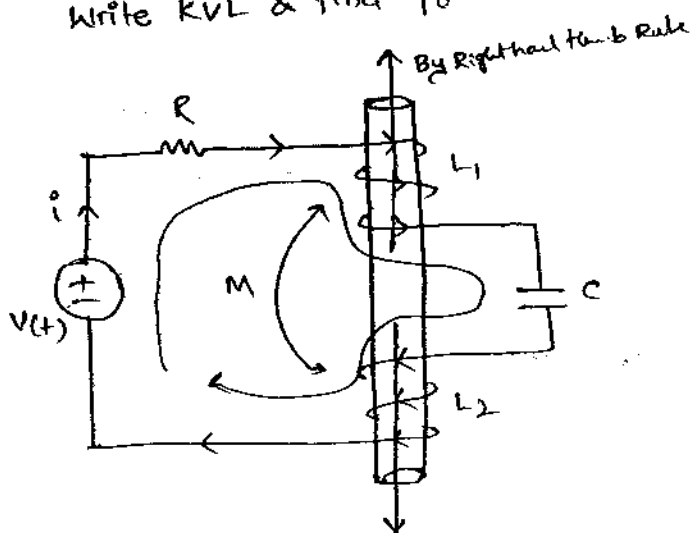
$$L_{xy} = \underline{\hspace{2cm}}$$

155



$$\left. \begin{aligned} V_1 &= 4 - 2 + 1 = 3H \\ V_2 &= 6 - 2 - 3 = 1H \\ V_3 &= 5 - 3 + 1 = 3H \end{aligned} \right\} 7H.$$

Write KVL & find 'fo'

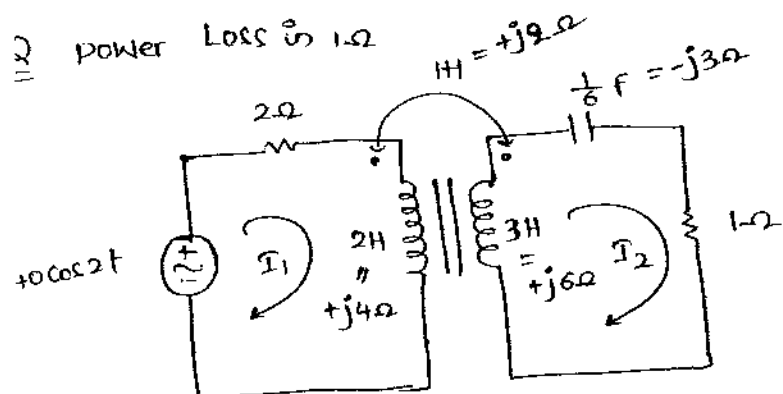


KCL

$$-V(t) + iR + (L_1 + L_2 - 2M) \frac{di}{dt} + \frac{1}{C} \int i dt = 0$$

$$f_0 = \frac{1}{2\pi \sqrt{(L_1 + L_2 - 2M) \cdot C}} \text{ Hz.}$$

ii) power loss in 1Ω



$$\Rightarrow - \left[\frac{40}{\sqrt{2}} \angle 0^\circ \right] + I_1 [2 + j4] - j2 [I_2] = 0 \longrightarrow (1)$$

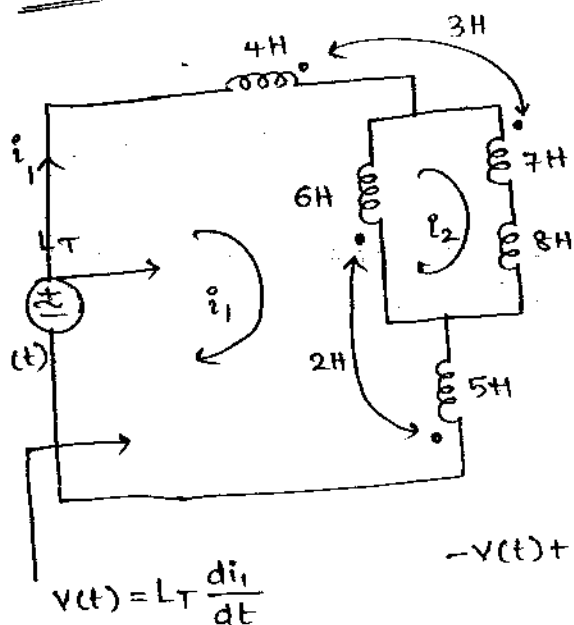
\downarrow
rms

$$I_2 [j6 - j3 + 1] - j2 [I_1] = 0 \longrightarrow (2)$$

Solving (1) & (2) $I_2 = \underline{\hspace{2cm}}$

$$P_{1\Omega} = [I_2]^2 (1) = \underline{\hspace{2cm}} \text{ watts}$$

IES(C) What is $L_{\text{Total}} = \underline{\hspace{2cm}}$



$$-v(t) + 4 \frac{di_1}{dt} + 6 \left[\frac{di_1}{dt} - \frac{di_2}{dt} \right] + 5 \frac{di_1}{dt} -$$

$$-v(t) + 4 \frac{di_1}{dt} + 6 \left[\frac{di_1}{dt} - \frac{di_2}{dt} \right] + 5 \frac{di_1}{dt} - 3 \frac{di_1}{dt} + 2 \frac{di_1}{dt} + 2 \left[\frac{di_1}{dt} - \frac{di_2}{dt} \right] = 0$$

$$19 \frac{di_1}{dt} - 11 \frac{di_2}{dt} = v(t) \longrightarrow (1)$$

$$6 \left[\frac{di_2}{dt} - \frac{di_1}{dt} \right] + 7 \frac{di_2}{dt} + 8 \frac{di_2}{dt} - 2 \frac{di_1}{dt} - 3 \frac{di_1}{dt} = 0$$

$$-11 \frac{di_1}{dt} + 21 \frac{di_2}{dt} = 0 \Rightarrow 21 \frac{di_2}{dt} = 11 \frac{di_1}{dt} \longrightarrow (2)$$

$$V(t) = \frac{di_1}{dt} \left[19 - 11 \left(\frac{11}{22} \right) \right]$$

$$V(t) = [13.23] \cdot \frac{di_1}{dt}$$



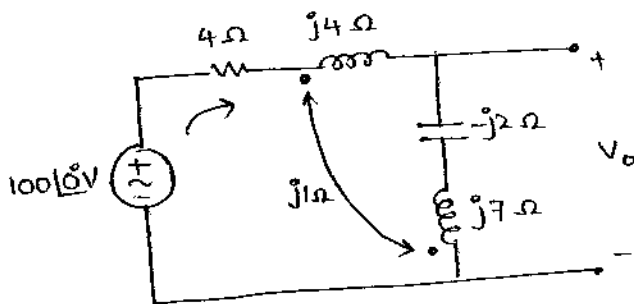
$$L_T = 13.23.$$

Rules to write mesh voltage.

1. Which mesh
2. Which two dots
3. aiding \oplus or subtracting \ominus
4. Resultant current?

156

gate $V_o =$ _____

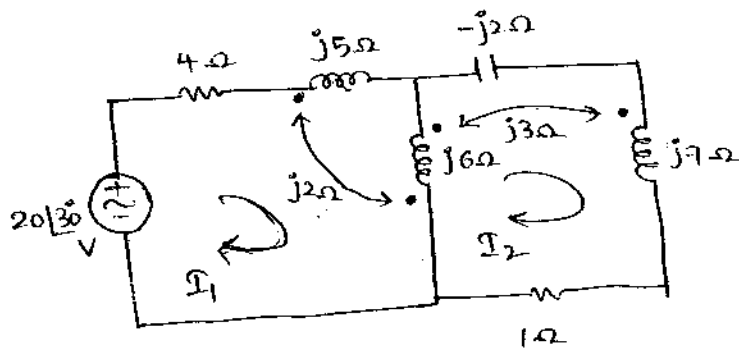


$$V_o = 100 \angle 0^\circ \left[\frac{-j2 + j7 - j1}{4 + j4 - j2 + j7 - \underbrace{2(j1)}_{2j1}} \right]$$

$$V_o = \frac{400 \angle 90^\circ}{(4 + j7)}$$

=

IES Write Mesh Equation



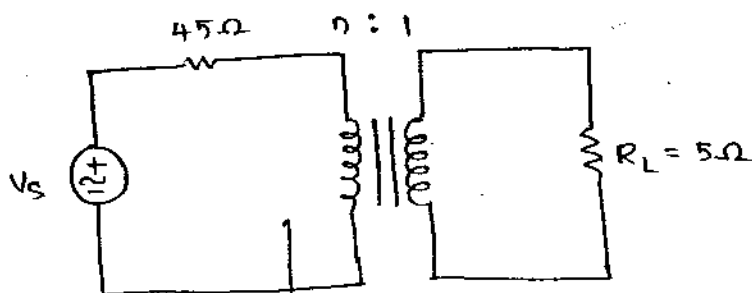
$$- [20\angle 30^\circ] + I_1 [4 + j5] + j6 [I_1 - I_2] - j2 [I_1 - I_2] - j2 [I_1] + j3 [I_2] = 0$$

$$(4 + j7) I_1 - j1 [I_2] = 20\angle 30^\circ \rightarrow (1)$$

$$j6 [I_2 - I_1] + I_2 [-j2 + j7 + 1] + j2 [I_1] - j3 [I_2] - j3 [I_2 - I_1] = 0$$

$$-j1 [I_1] + [1 + j5] I_2 = 0 \rightarrow (2)$$

Gate For what value of 'n', P_{max} occurs in load.



P_{max} Source R = Load R in Conductively Connected ckt
but not in Inductively Connected ckt

\therefore make this ckt into Conductively Connected ckt
by using Reflection

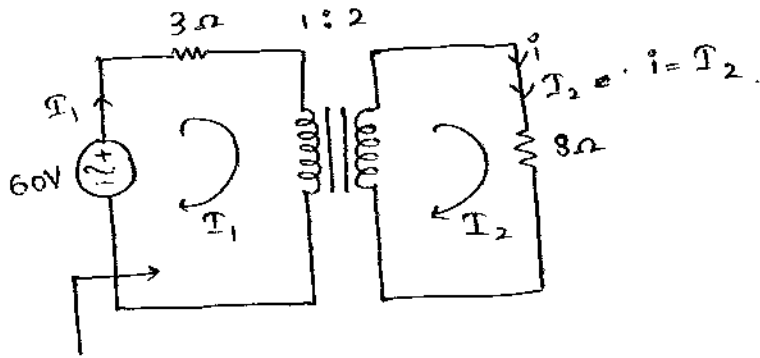
$$R_2' = \frac{R_2}{k^2} = \frac{5}{(1/n)^2} = n^2 5$$

$$\text{for } P_{max} \Rightarrow n^2 5 = 45$$

$$n^2 = 9$$

$$\underline{n = 3}$$

IES find $i =$ _____



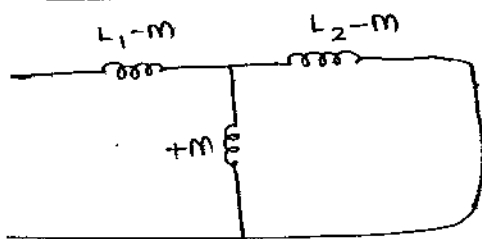
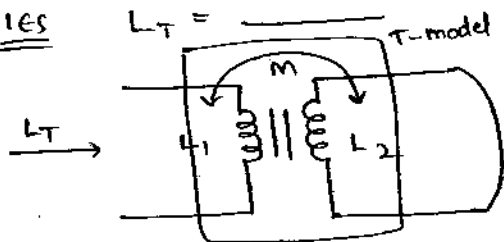
157

$$I_1 = \frac{V_s}{R_T} = \frac{V_s}{R_1 + R_2'} = \frac{60}{3 + \frac{8}{(2/1)^2}} = \frac{60}{3+2} = \frac{60}{5} = 12A$$

$$K = \frac{2}{1} = \frac{I_1}{I_2}$$

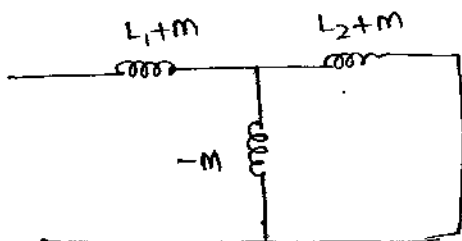
$$\Rightarrow \frac{2}{1} = \frac{12}{I_2} \Rightarrow I_2 = 6A \Rightarrow I_2 = i = 6A$$

IES $L_T =$ _____ \Rightarrow to find L_T convert it into T model (\because it is in ckt model)



$$L_1 - M + \left[\frac{M L_2}{L_2} - \frac{M^2}{L_2} \right]$$

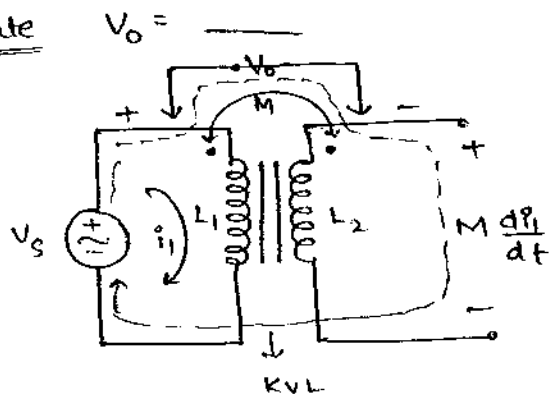
$$L_T = L_1 - \frac{M^2}{L_2}$$



$$L_1 + M + \left[\frac{-M L_2}{L_2} - \frac{M^2}{L_2} \right]$$

$$L_T = L_1 - \frac{M^2}{L_2}$$

Gate



$$-V_s + V_0 + M \cdot \frac{di_1}{dt} = 0$$

$$V_0 = V_s - M \frac{di_1}{dt}$$

(ohm's law
in 1st Mesh)

$$V_s = L_1 \frac{di_1}{dt}$$

$$V_0 = V_s - M \cdot \left[\frac{V_s}{L_1} \right]$$

$$V_0 = V_s \left[1 - \frac{M}{L_1} \right]$$

$$(a) V_s \left[1 - \frac{M}{L_1} \right]$$

$$(b) V_s \left[1 - \frac{L_1}{M} \right]$$

$$(c) V_s \left[1 - \frac{L_2}{M} \right]$$

$$(d) V_s \left[1 - \frac{M}{L_2} \right]$$

(in 1st mesh)

Self Inductance Exist

Mutual Ind = 0.

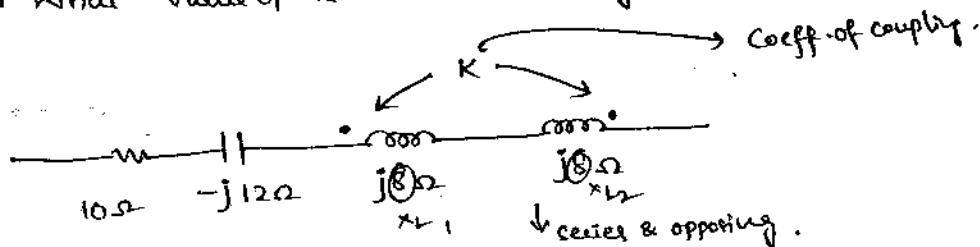
(in 2nd mesh)

Self = 0 (∵ no current)

Mutual Exist (due to current flowing in L_1)

Gate

For What value of 'k' branch undergoes Resonance.



$$X_L = \omega L_T = \omega [L_1 + L_2 - 2M]$$

$$= \omega L_1 + \omega L_2 - 2\omega k \sqrt{L_1 L_2}$$

$$= \omega L_1 + \omega L_2 - 2k \sqrt{(\omega L_1)(\omega L_2)}$$

$$X_L = X_1 + X_2 - 2k \sqrt{X_1 X_2}$$

$$= 8 + 8 - 2k \sqrt{8 \times 8}$$

$$X_L = (16 - 16k)$$

at resonance

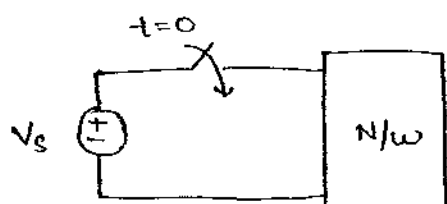
$$|X_L| = |X_C|$$

$$(16 - 16k) = 12$$

$$16k = 4$$

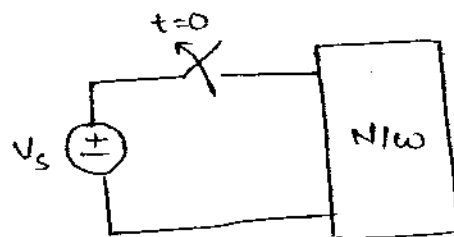
$$k = \frac{1}{4} = 0.25$$

⇒ Transients are regarded as Sudden change in the state of N/w or ckt, Indicated by Switch operation or Special Input function.



⇒ Step Response

Ex: Turning-ON a motor (Excitation)



⇒ Source-free Response

Ex: Turning-OFF a motor (deenergise)

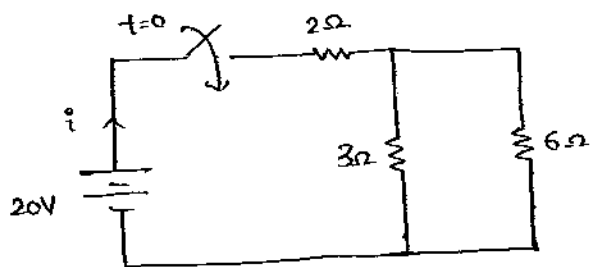
⇒ Transients Occur in nature usual in electric ckts as Networks & Systems are not adaptable for Sudden changes in their energy states.

⇒ allowing Sudden change in Energy in Zero time is not possible which is Violation of law of Conservation Energy.

⇒ Note: In electric circuits Capacitor Cannot allow Sudden change in voltage across it & Inductor Can Never allow Sudden change in Current through it. However Resistor has the ability to Convert & dissipate Energy in the form of Heat can allow Sudden changes.

IES (CO) \rightarrow Current just after the sudden change

What is $i(0^+) = \underline{\hspace{2cm}}$



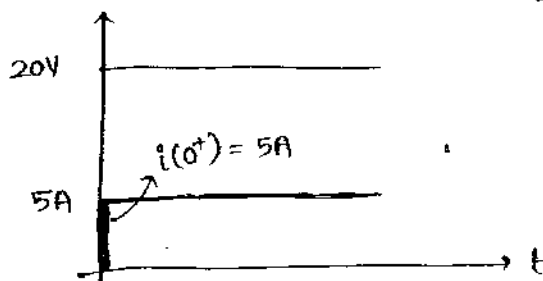
$i(0^-) =$ previous steady state (ie. Sw is open)

$$\therefore i(0^-) = 0A$$

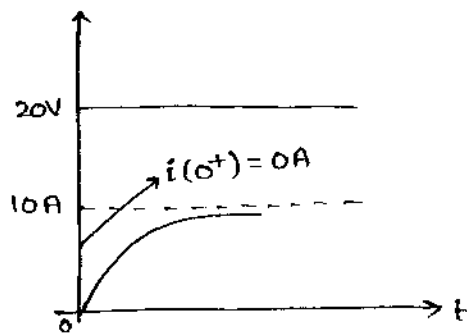
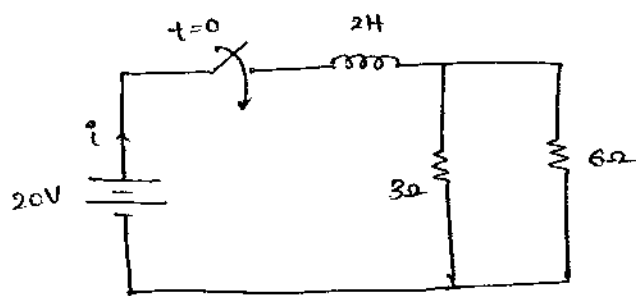
$$i(0^+) = \frac{20}{2 + [3 \parallel 6]} = 5A$$

$$i(\infty) = 5A$$

↳ DC steady state



Q $i(0^+) = \underline{\hspace{2cm}}$



$$i(0^-) = 0A$$

$$i(0^+) = 0A \quad \left[\because \text{Inductor is Inertia element in magnetic domain} \right. \\ \left. \text{Which never allow Sudden change in Current} \therefore i(0^+) = 0A \right]$$

$$i(\infty) = \frac{20}{3 \parallel 6} = 10A$$

⇒ Transient is non linear mode to linear ckt

⇒ Transients are regarded as argument b/w input Excitation & output Response as Network reaches its Next Steady state from previous Steady state.

⇒ Though Transients Occur for Very short duration in time but their analysis is Very Critical as this state Can determine the overall Steady state Stability.

⇒ Customers of electrical N/w applications look into its steady state performance but designer is more Interested in its Transient state as this state Could determine the overall Critical Design parameter Value

State Variables :

These are the Critical parameters that should be selected in any ckt. which Can determine its Correct state at any Instant in time.

(a) In a ckt Involving Capacitor, $\therefore i = C \frac{dV}{dt}$

↓
Voltage across Capacitor is Correct State Variable

$KCL \longrightarrow \text{Nodal.}$

(RC)

(b) In a ckt Involving Inductor $\therefore V = L \frac{di}{dt}$

159

↓
Current through Inductor is Correct state variable

$\therefore \text{KVL} \longrightarrow \text{mesh}^{\text{th}}$
(for RL ckt)

\Rightarrow any ckt or N/w can have two types of Responses

I. The Response of a ckt or N/w with Source present in it, is called as Forced Response that leads to steady state Response. This Response is Independent to Nature of passive elements but purely depends Upon the Type of Input. This Response can be different for different types of Inputs.

\Rightarrow Ex: Dc & AC steady state Responses.

\Rightarrow The solution to this part of diff. Eqns can be obtained by solving the particular Integral part.

II. The Response of a ckt or N/w without any Source in it is called as Natural Response that leads to Transients. This Response is Independent to the type of Input but purely depends upon the Nature of passive elements. This Response is always Unique which can be determined from the characteristic Equation Governing the N/w.

\Rightarrow Ex: Source free Response of ckt.

\Rightarrow The solution to this part of Eqn Diff. Eqn. can be obtained by Solving Complementary Function part.

Note: Ckts & N/w can Response even without Source provided they have some Initial stored Energy

(a) in Capacitor : $q_0 = CV_0 \longrightarrow \text{initial Voltage}$

↓
Initial stored Electrostatic Energy \nearrow will give rise to

(b) In a Inductor,

$$\psi_0 = L i_0 \xrightarrow{\text{initial current}}$$

\downarrow
 internal stored
 Electro magnetic Energy

\nearrow will give rise to.

So,

$$[\text{Total Response}] = [\text{forced Response}] + [\text{Natural Response}]$$

$$\downarrow \qquad \qquad \qquad \downarrow$$

$$= [\text{zero State Response}] + [\text{zero input Response}]$$

$$\downarrow \qquad \qquad \qquad \downarrow$$

$$= [\text{leads to steady state}] + [\text{leads to Transients}]$$

$$[\text{Total Solution to D.E}] = \downarrow \qquad \qquad \qquad \nwarrow$$

$$= [P.I] + [C.F]$$

Ex: $V(t) = 10 - 5 \cdot e^{-2t}$

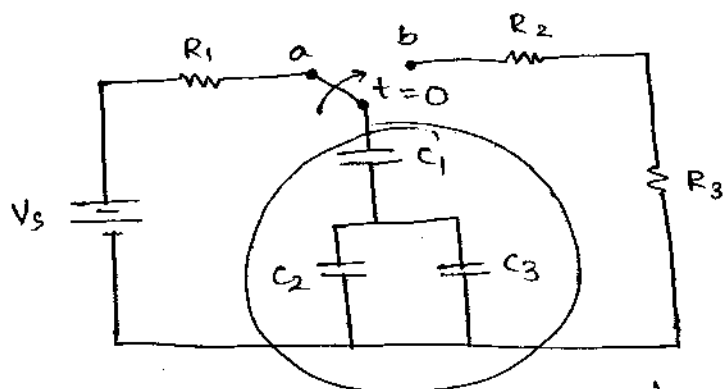
$$i(t) = 2 \cos 5t + \frac{5}{3} \cdot e^{3t} - \frac{2}{3} \cdot e^{-t}$$

Order of a Circuit (or) Network:

The Number of Energy storage elements available in distributed form but Interacting Represents its "order"

$R-L, R-C \longrightarrow 1^{\text{st}} \text{ order}$

$R-L-C, L-R-L, C-R-C, L-C \longrightarrow 2^{\text{nd}} \text{ order}$



\hookrightarrow can be lumped
 $\therefore 1^{\text{st}} \text{ order only}$

Initial Conditions:

These are the Critical Values of Voltages across Capacitors & Current through Inductors that must be Considered from prev. steady state so as to determine the solution to its next steady state, which are specifically Indicated in time as, $t = 0^- \rightarrow$ ^{instant} Just before switch separation (s.s before switch operation)

$t = 0$, Exact instant of switch operation

$t = 0^+$, instant just after switch operation [Transient state]

$t \rightarrow \infty$, steady state after switch operation.

160

Note: $V_C(0^-) = V_C(0) = V_C(0^+)$ [\because cap. doesn't allow sudden change in Voltage]
 $i_L(0^-) = i_L(0) = i_L(0^+)$ [\because Inductor doesn't allow sudden change in current]

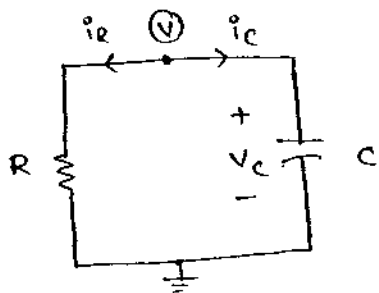
Behaviour of passive elements in Transient state:

\Rightarrow if analysing ckts for sudden changes in time i.e. $t \rightarrow 0^+$ is considered as Transients. then analysing the same ckt as $s \rightarrow \infty$ which is steady state frequency solution will also include the Transient part in it. Hence laplace Transforms are powerfull tools to analyse ckts during Transients.

Element	D.C S.S $s = 0$	A.C S.S $s = j\omega$ \downarrow const	TRANSIENT state $t \rightarrow 0^+ \quad s \rightarrow \infty$	$s = j\omega$ \downarrow Complex Eq
R	R	R	R	$Z_R = R \Omega$
L	S.C	I_L lags V_L by $\phi = 90^\circ$	S.C	$Z_L = j\omega L = sL \Omega$
C	O.C	I_C leads V_C by $\phi = 90^\circ$	S.C	$Z_C = \frac{1}{j\omega C} = \frac{1}{sC} \Omega$

Category-I: Source free first order Circuits.

(a) R-C Circuit.



Let $V(0) = V_0$

Nodal

$$i_C + i_R = 0$$

$$C \frac{dv}{dt} + \frac{V}{R} = 0$$

$$C \frac{dv}{dt} = -\frac{V}{R}$$

$$\int \frac{dv}{V} = -\int \frac{dt}{RC}$$

$$\ln[V] = -\frac{t}{RC} + \ln[A]$$

$$\ln\left[\frac{V}{A}\right] = -\frac{t}{RC}$$

$$V = A \cdot e^{-t/RC}$$

at $t=0 \rightarrow V = V_0$

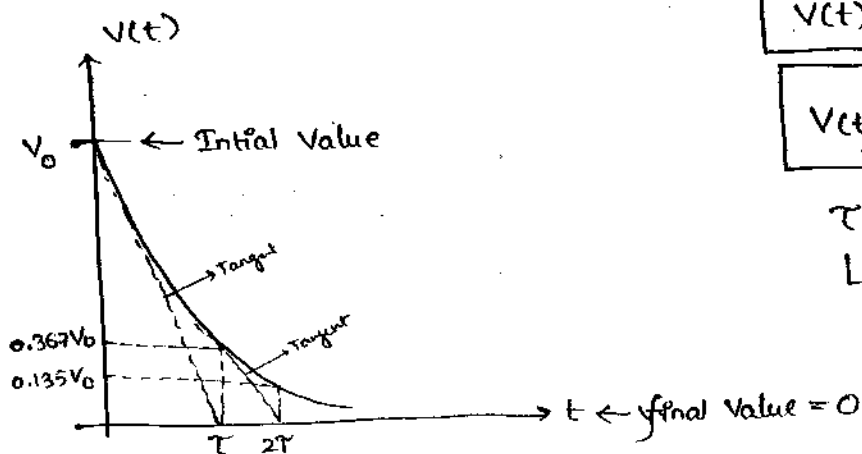
So, $A = V_0$

$$V(t) = V_0 \cdot e^{-t/RC} \quad t > 0$$

$$V(t) = V_0 \cdot e^{-t/\tau} \quad , t > 0$$

$$\tau = RC$$

\rightarrow Time Constant.



$$V(t=0) \rightarrow V_0$$

$$V(t=\tau) = e^{-1} V_0 = 0.367 V_0$$

$$V(t=2\tau) = e^{-2} V_0 = 0.135 V_0$$

$$V(t=3\tau) = e^{-3} V_0 = 0.049 V_0$$

$$V(t=4\tau) = e^{-4} V_0 = 0.018 V_0$$

$$V(t=5\tau) = e^{-5} V_0 = 0.006 V_0$$

$$0 \leq t \leq 4\tau \Rightarrow \text{TRANSIENT}$$

$$t \geq 5\tau \Rightarrow \text{S.S}$$

$$i_c(t) = C \frac{d}{dt} \{ V_0 \cdot e^{-t/\tau} \} = C \cdot V_0 \cdot e^{-t/\tau} * \frac{-1}{\tau} \longrightarrow i_c(t) = \frac{-V_0}{R} e^{-t/\tau} \underline{\underline{A.}}, t > 0$$

$$P_R(t) = \frac{[V(t)]^2}{R} = \frac{V_0^2}{R} \cdot e^{-t/(\tau/2)} \underline{\underline{W}}$$

161

$$E_c(t) = \frac{1}{2} C [V(t)]^2 = \frac{1}{2} C V_0^2 \cdot e^{-t/(\tau/2)} \underline{\underline{J}}$$

$$q(t) = C V(t) = C V_0 \cdot e^{-t/\tau} \underline{\underline{C.}}$$

Time Constant:

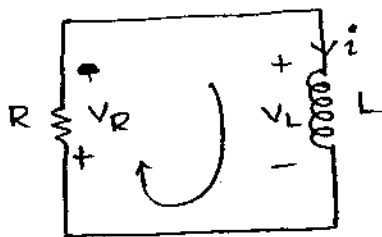
it is the Time taken by Response to reach 36.7% of its Initial value

While decaying

(or)

** it is the Time taken by Response to Reach 63.2% of its final value

(b) R-L Circuit:



Let $i(0) = I_0$

KVL

$$V_L + V_R = 0$$

$$L \frac{di}{dt} + iR = 0$$

$$L \frac{di}{dt} = -iR$$

$$\int \frac{di}{i} = \frac{-R}{L} \int dt$$

$$\ln[i] = \frac{-R}{L} \cdot t + \ln[A]$$

$$\ln\left[\frac{i}{A}\right] = \frac{-R}{L} \cdot t$$

$$i = A \cdot e^{\frac{-R}{L} t}$$

$$\text{at } t=0 \longrightarrow i = I_0$$

$$\text{So, } A = I_0$$

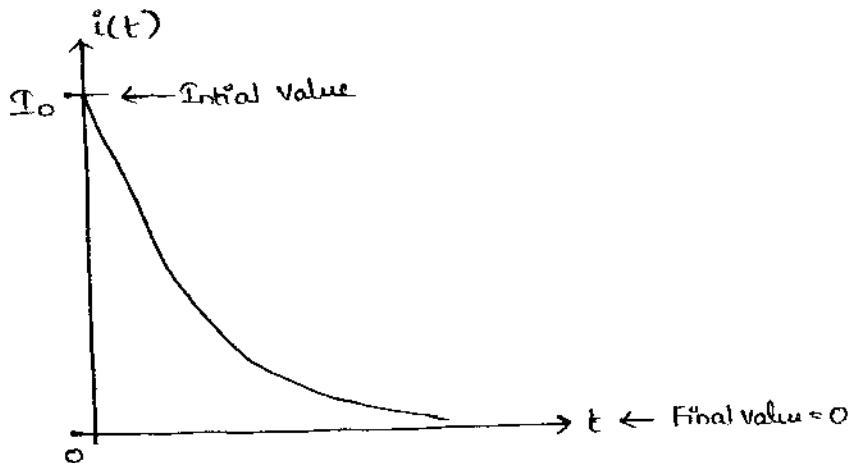
$$i(t) = I_0 e^{-t/(L/R)}, t > 0$$

$$i(t) = I_0 \cdot e^{-t/(L/R)}, t > 0$$

$$i(t) = I_0 \cdot e^{-t/\tau}, t > 0$$

$$\tau = \frac{L}{R}$$

→ Time Constant.



$$V_L(t) = L \frac{d}{dt} \{ I_0 \cdot e^{-t/\tau} \} = L I_0 \cdot e^{-t/\tau} * \frac{1}{\tau}$$

$$V_L(t) = -I_0 \cdot R \cdot e^{-t/\tau} \quad \underline{\text{Volts.}}$$

$$P_R(t) = [i(t)]^2 \cdot R = I_0^2 \cdot R \cdot e^{-t/(\tau/2)} \quad \underline{\underline{W}}$$

$$E_L(t) = \frac{1}{2} L [i(t)]^2 = \frac{1}{2} L \cdot I_0^2 \cdot e^{-t/(\tau/2)} \quad \underline{\underline{J}}$$

$$\psi(t) = L \cdot i(t) = L \cdot I_0 \cdot e^{-t/\tau} \quad \underline{\underline{\text{Volt-sec.}}}$$

⇒ The units of ' τ ' is seconds

⇒ In terms of units only

$$\tau_v = \tau_i$$

$$RC = \frac{L}{R}$$

⇒ The units of $\frac{L}{RC}$ is $\underline{\Omega}$

⇒ The units of $\frac{L}{R^2}$ is \underline{F}

⇒ The units of R^2C is \underline{H}

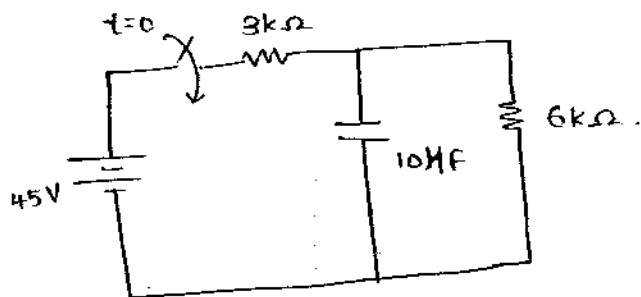
⇒ The units of $\frac{RC}{L}$ is \underline{V}

⇒ The units of $\sqrt{\frac{L}{C}}$ is $\underline{\Omega}$

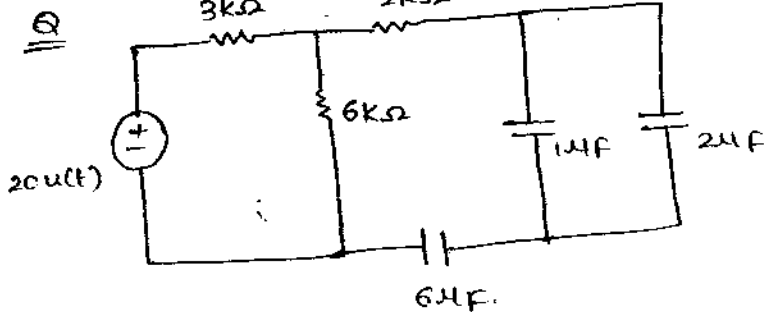
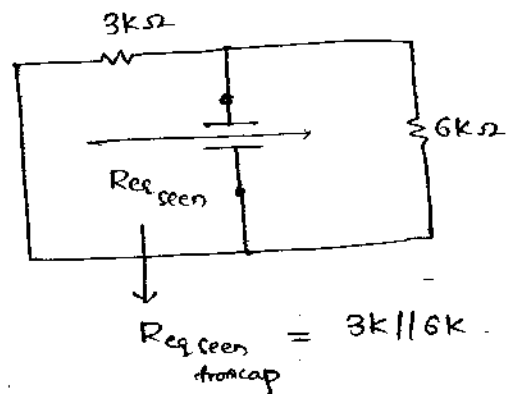
⇒ The units of $\frac{L}{R^2C}$ is $\underline{1}$ (unitless)

162

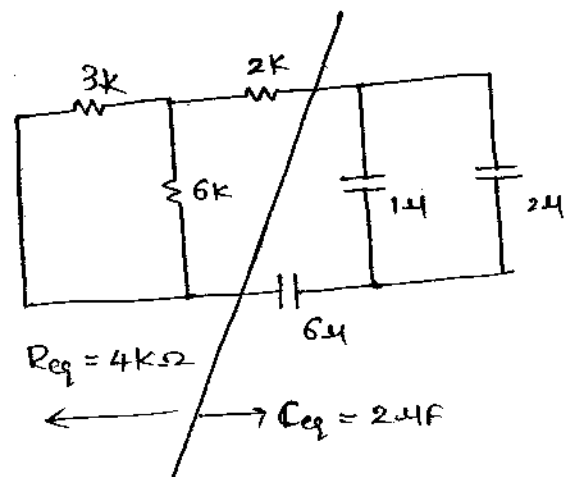
Q Determine ' τ ' of circuit

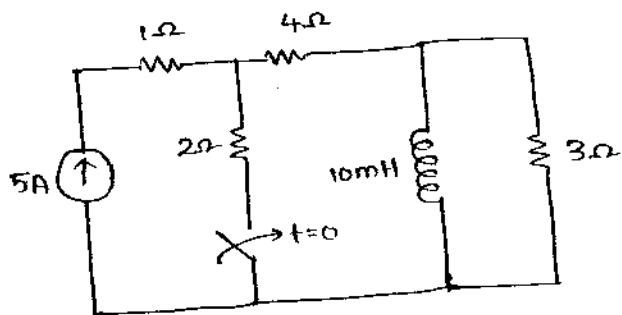


$$\begin{aligned}\tau &= R_{eq} \cdot C_{eq} \\ &= [3k \parallel 6k] 10\mu \\ &= 20\text{msec.}\end{aligned}$$

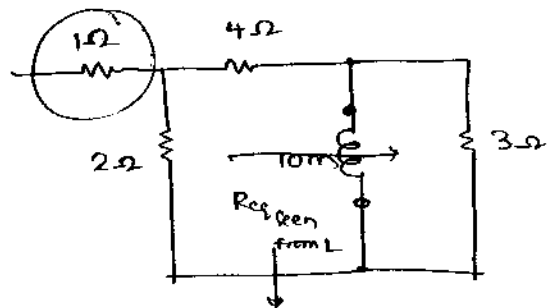


$$\begin{aligned}\tau &= R_{eq} C_{eq} \\ \tau &= (4k)(2\mu F) \\ &= 8\text{msec.}\end{aligned}$$

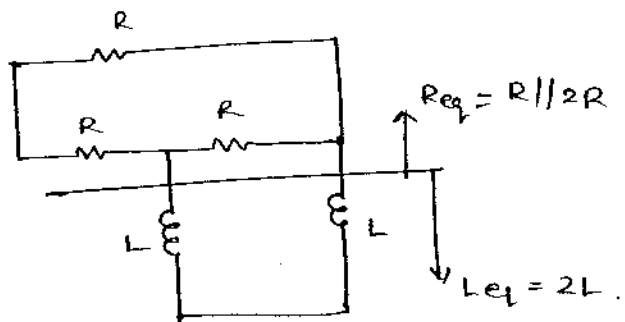
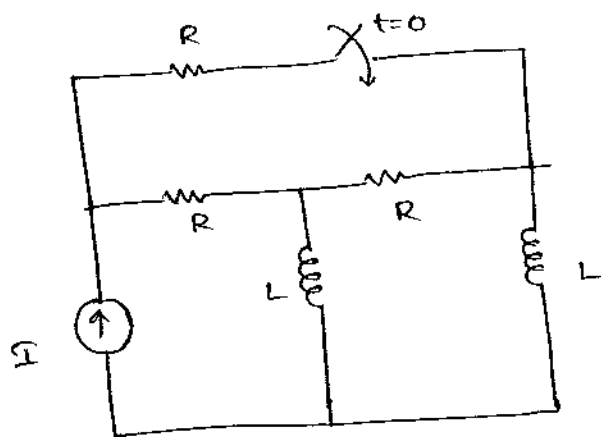




$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{10m}{(6//3)} = \frac{10m}{2} = 5msec.$$

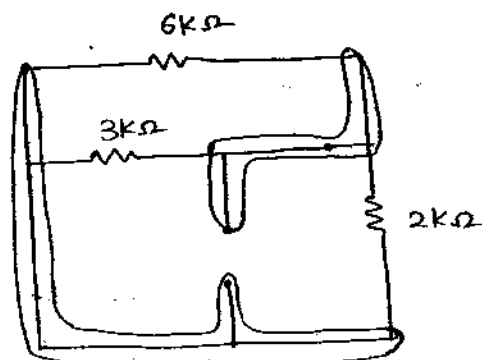
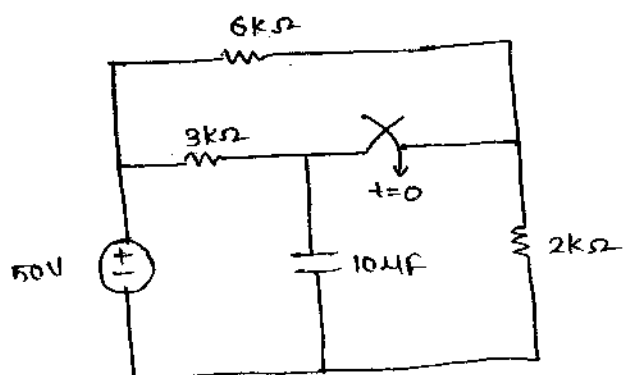


$$R_{eq \text{ seen from } L} = (6\Omega // 3\Omega)$$



$$\tau = \frac{2L}{\frac{2}{3}R} = \frac{3L}{R} \text{ sec.}$$

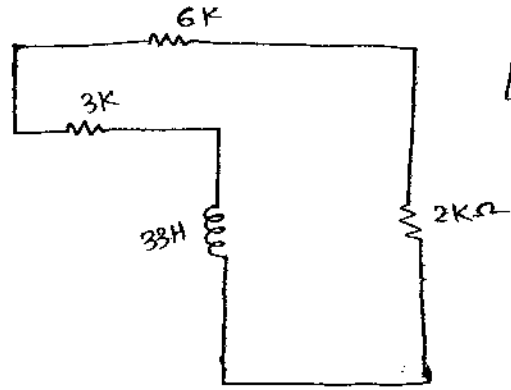
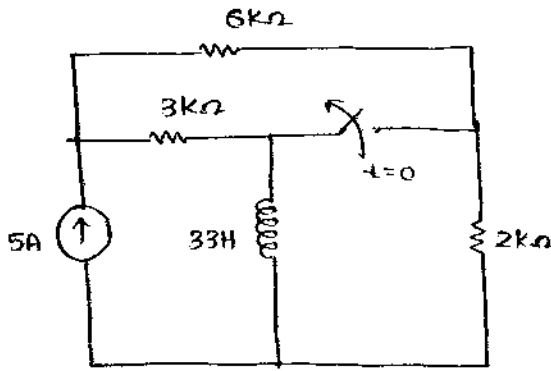
Gate find τ .



$$\tau = R_{eq} \cdot C = [6k // 3k // 2k] \times 10\mu$$

$$= 10 \text{ mSec.}$$

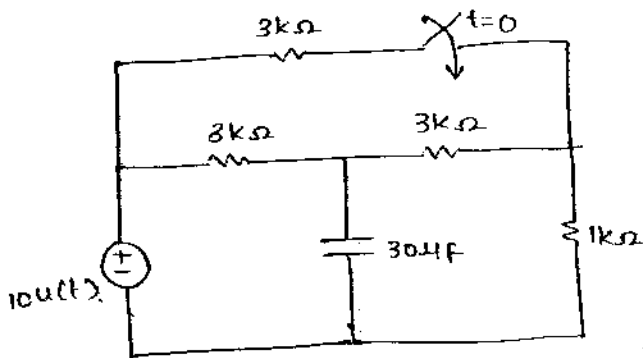
10



163

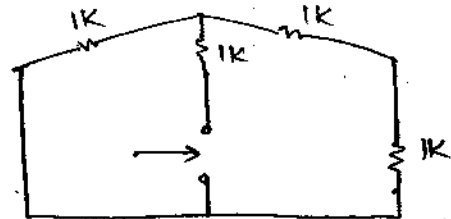
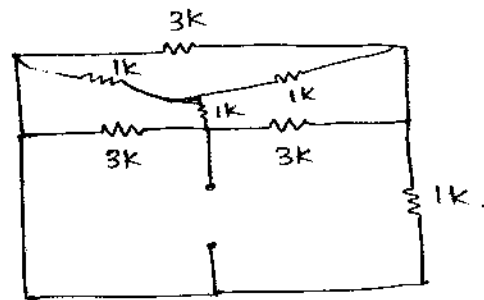
$$\tau = \frac{33}{11K} = 3\text{msec}$$

10 $\tau =$

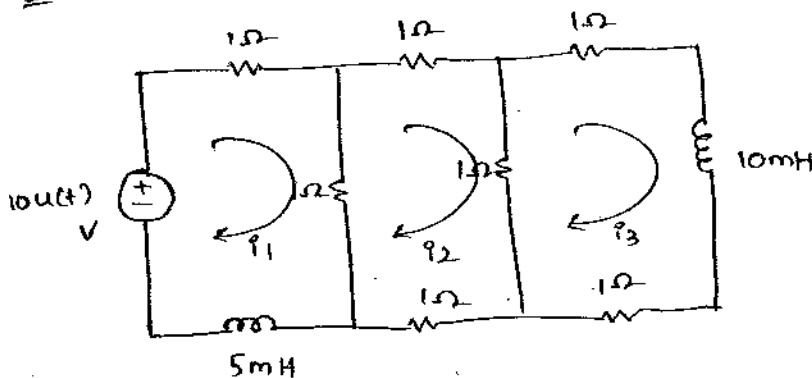


$$R_{eq} = 1K [1K // 2K] = \left[1 + \frac{2}{3}\right]K = \frac{5}{3}K$$

$$\tau = R_2 C = \frac{5}{3}K \cdot 30\mu \Rightarrow \tau = 50\text{msec}$$



10 $\tau =$

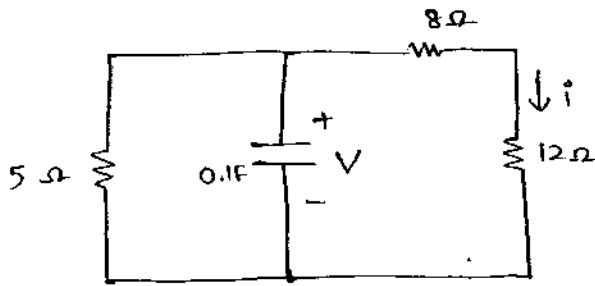


Here the two Inductors Cannot be lumped Together. So, it is a Second order circuit that to Non Canonical form.

Here, the two State Variable Inductor Currents will have different Time Constants in different Segment of ckt.

& the solution to these diff Eqns Can be easily determined by Laplace Tlf method

if $V(0) = 120V$. determine the Complete Expression for Current $i(t)$ for all $t > 0$



⇒ it is Source free Response, 1st order

⇒ State Variable (S.V) = 'V'

$$V(t) = V_0 e^{-t/\tau}$$

$$V_0 = 120V$$

$$\tau = R_2 C = [5 || 20] [0.1]$$

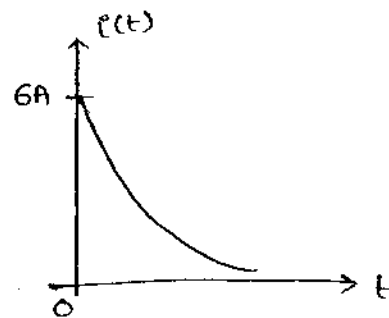
$$= 0.4 \text{ sec.}$$

$$V(t) = 120 \cdot e^{-t/0.4}$$

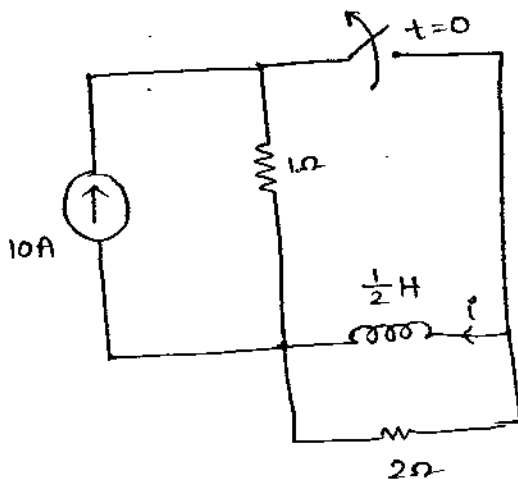
$$V(t) = 120 \cdot e^{-2.5t}, t > 0$$

But $i(t) = \frac{V(t)}{8+12} = \frac{120 \cdot e^{-2.5t}}{20}$

$$i(t) = 60 \cdot e^{-2.5t}, t > 0$$



Gate determine the Complete Expression for Current $i(t)$ for all $t > 0$.

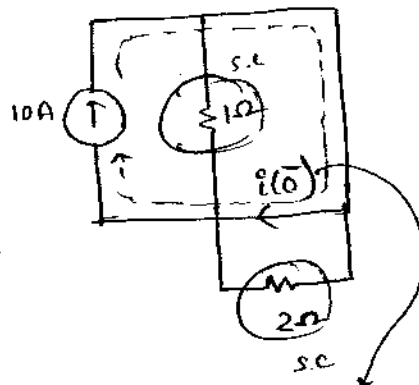


⇒ Source free, 1st order

⇒ S.V ⇒ 'i'

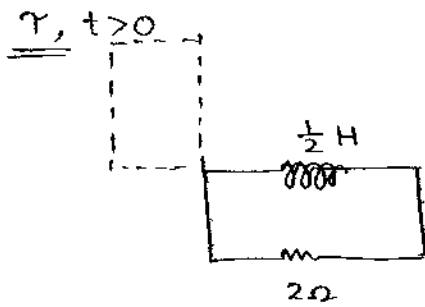
Initial Condition not given

∴ find Initial Condition for D.C.S.S



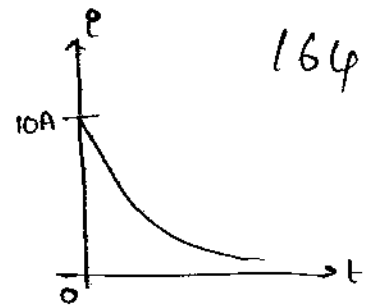
$$i(0^-) \Rightarrow I_0 = 10A$$

$$i(t) = I_0 \cdot e^{-t/\tau}$$

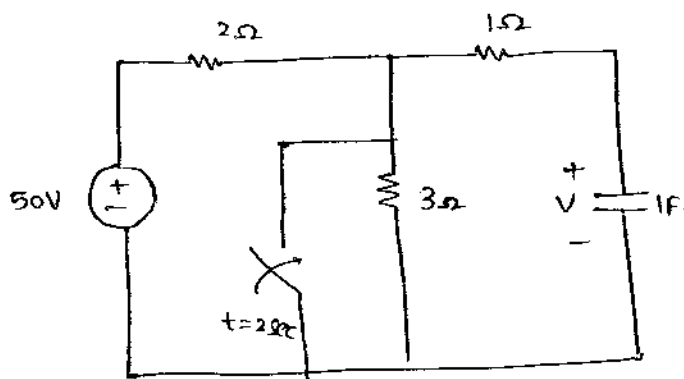


$$\tau = \frac{\frac{1}{2}}{2} = \frac{1}{4} \text{ sec}$$

$$i(t) = 10 e^{-4t}, t > 0$$



Q Determine the Energy stored by Cap. upto $t = 2.5 \text{ Sec}$

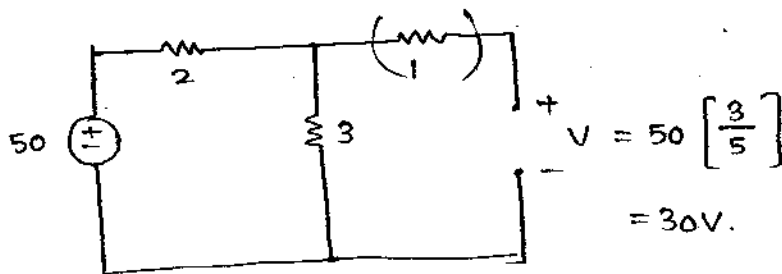


⇒ When switch is closed, S.C
 ∴ Source is Isolating from State Variable
 (when switch is open ⇒ it is in ^{prev} Steady state)
 i.e. from $0 < t < 2$

↪ Source free 1st order, R-C

↪ S.V ⇒ 'V'

P-I $0 < t < 2 \Rightarrow \text{DC. S.S}$



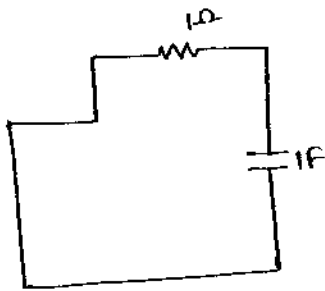
P-II $t > 2 \Rightarrow \text{TRANSIENT}$

$$V(t) = V_0 \cdot e^{-t/\tau}$$

$V_0 \Rightarrow V(2^-)$, from prev. S.S.

$$V(2^-) = 30V.$$

$T, t > 2$

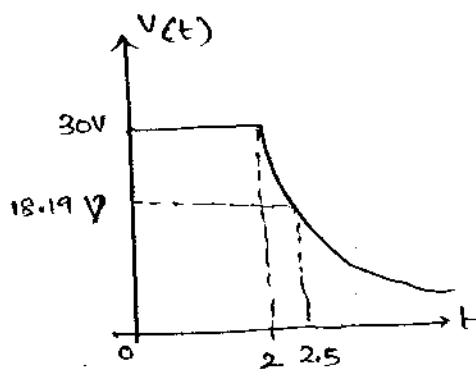


$$\tau = 1 \text{ sec.}$$

$$V(t) = 30 \cdot e^{-\frac{(t-2)}{1}}, t > 2$$

Total Response is

$$V(t) = \begin{cases} 30V, & 0 < t < 2 \text{ sec} \\ 30 \cdot e^{-(t-2)} V, & t > 2 \text{ sec.} \end{cases}$$

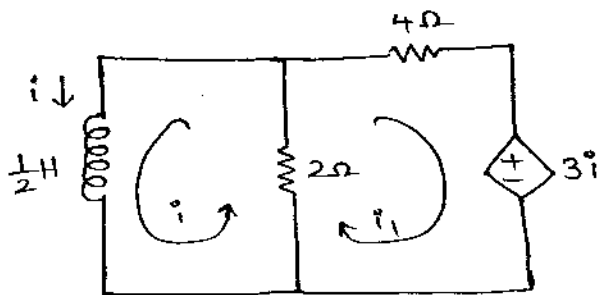


$$E_c (\text{upto } t = 2.5 \text{ sec}) = E_c (\text{at } t = 2.5 \text{ sec})$$

$$V_c(t=2.5) = 30 \cdot e^{-1/2} = 18.19 \text{ V}$$

$$E_c = \frac{1}{2} (1) (18.19)^2 = 165.5 \text{ J}$$

Q. if $i(0) = 12 \text{ A}$. determine the Complete Expression for all $t > 0$ for current $i(t)$



\Rightarrow Source free, 1st order

$$\Rightarrow s.v = i$$

Method-I, Diff Eqn.

$$\frac{1}{2} \frac{di}{dt} + 2[i + i_1] = 0 \quad \text{--- (1)}$$

$$4i_1 + 3i + 2[i + i_1] = 0$$

$$5i = -6i_1 \quad \text{--- (2)}$$

$$\frac{1}{2} \frac{di}{dt} + 2i + 2 \left[\frac{-5}{6} \right] i = 0$$

165

$$\frac{1}{2} \frac{di}{dt} + \frac{i}{3} = 0$$

$$\frac{di}{dt} = -\frac{2}{3} i$$

$$\int \frac{di}{i} = -\frac{2}{3} \int dt$$

$$\ln[i] = -\frac{2}{3} t + \ln[A]$$

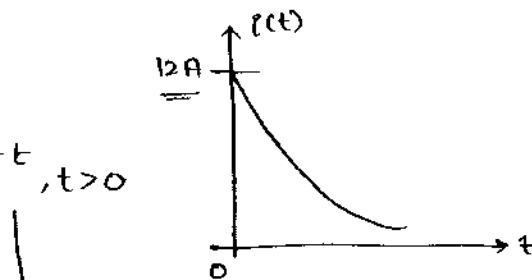
$$\ln\left[\frac{i}{A}\right] = -\frac{2}{3} t$$

$$i = A \cdot e^{-\frac{2}{3} t}$$

at $t=0 \rightarrow i=12$

So, $A=12$

$$i(t) = 12 \cdot e^{-\frac{2}{3} t}, t > 0$$



L.T. $I(s) = \frac{12}{(s+3)}$

Method - II

$$i(t) = I_0 \cdot e^{-t/\tau}$$

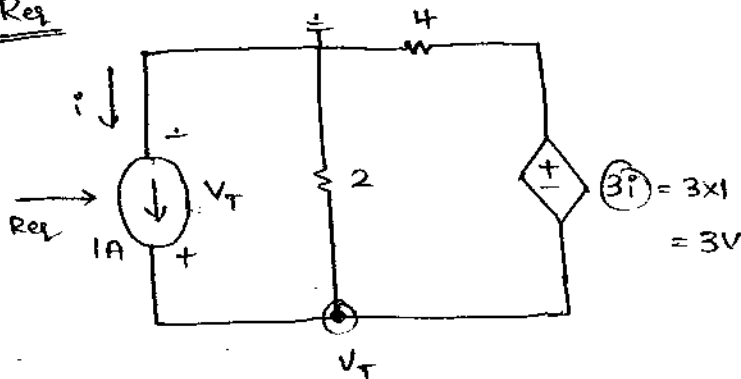
$$I_0 = 12A$$

$$\tau = \frac{L}{R_{eq}}$$

LHS $\frac{-2}{3}$

\therefore Stable

Req



Nodal

$$-1 + \frac{V_T}{2} + \frac{(V_T+3)}{4} = 0$$

$$-4 + 2V_T + V_T + 3 = 0$$

$$3V_T = 1$$

$$V_T = \frac{1}{3} V$$

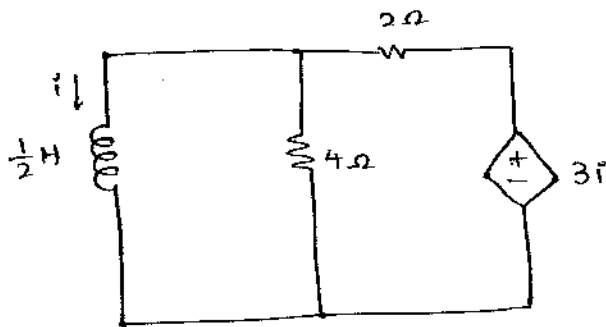
$$R_{eq} = \frac{V_T}{1A} = \frac{\frac{1}{3} V}{1A} = \frac{1}{3} \Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{1/2}{1/3} = \frac{3}{2} \text{ sec.}$$

$$\therefore i(t) = 12 \cdot e^{-2/3 t}, t > 0$$

Q In the above problem, determine the Complete Expression for Current i , if the Values of Resistors are Interchanged.

Reg. +ve feedback



Method-2

$$i(t) = I_0 \cdot e^{-t/\tau}$$

$$I_0 = 12 \text{ A}$$

$$\tau = \frac{L}{R_{eq}}$$

Nodal:

$$-1 + \frac{V_T}{4} + \frac{(V_T + 3)}{2} = 0$$

$$-4 + V_T + 2V_T + 6 = 0$$

$$3V_T = -2$$

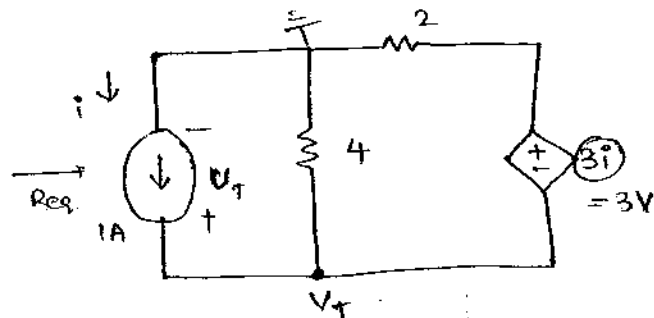
$$V_T = -\frac{2}{3} \text{ V}$$

$$R_{eq} = \frac{V_T}{1} = -\frac{2}{3} \text{ V}$$

-ve Resistance

Reg +ve feedback N/w

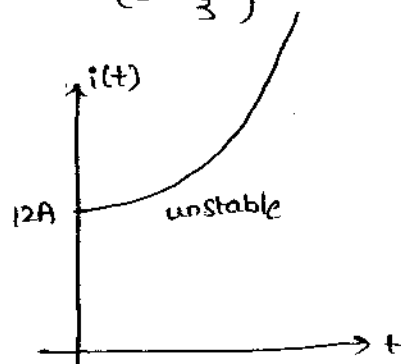
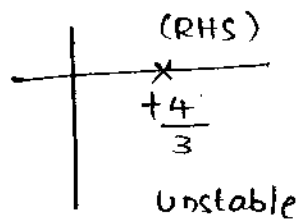
makes system Unstable



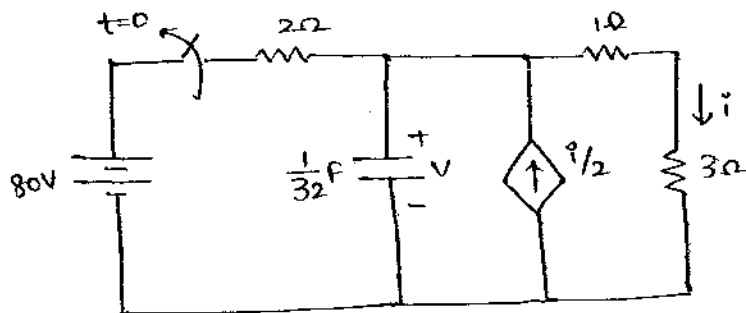
$$\therefore \tau = \frac{L}{R_{eq}} = \frac{1/2}{-2/3} = -\frac{3}{4}$$

$$i(t) = 12 \cdot e^{+\frac{4}{3}t}, t > 0$$

$$I(s) = \frac{12}{(s - \frac{4}{3})}$$



IES Determine the Complete Expression for Current $i(t)$, for all $t > 0$.



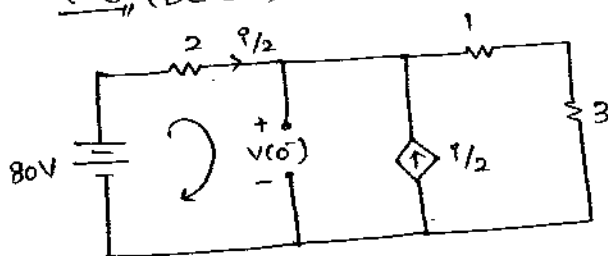
\Rightarrow Source Free, 1st order
 \Rightarrow S.V $\rightarrow v_c$

166

Direct Method:

$$v(t) = V_0 \cdot e^{-t/\tau}$$

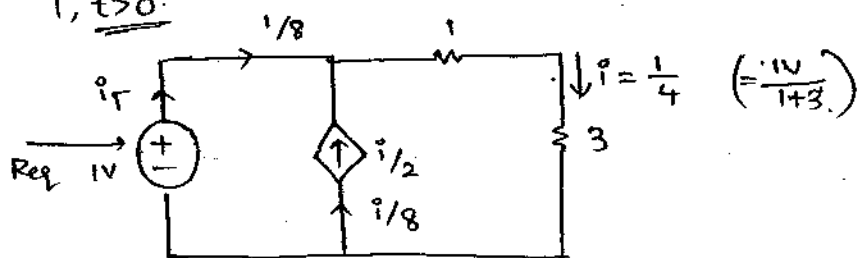
$t = 0^-$ (DC S.S)



$$-80V + \frac{i}{2}(2) + 4i = 0 \Rightarrow i = 16A$$

$$-80 + \frac{i}{2}(2) + v(0^-) = 0 \Rightarrow v(0^-) = 80 - i = 80 - 16 = 64V$$

$\tau, t > 0$



$$i_T = \frac{1}{8}$$

$$R_2 = \frac{1V}{i_T} = \frac{1}{1/8} = 8\Omega$$

$$\left. \begin{array}{l} i_T = \frac{1}{8} \\ R_2 = 8\Omega \end{array} \right\} \tau = R_{eq} \cdot C = 8 \times \left[\frac{1}{32} \right] = \frac{1}{4} \text{ Sec.}$$

$$v(t) = 64 \cdot e^{-4t}, t > 0$$

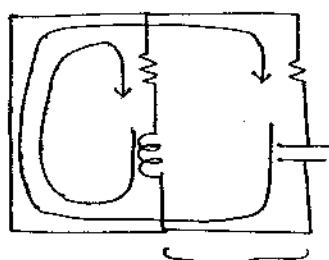
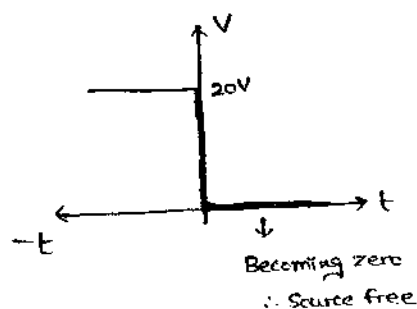
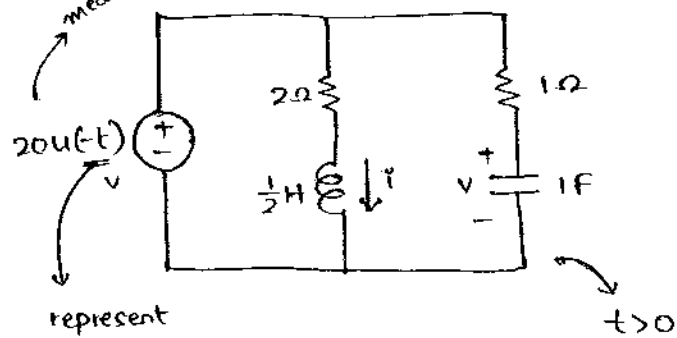
$$i(t) = \frac{v(t)}{R} = \frac{v(t)}{1+3} = \frac{64 \cdot e^{-4t}}{4}$$

$$i(t) = 16 \cdot e^{-4t}, t > 0$$

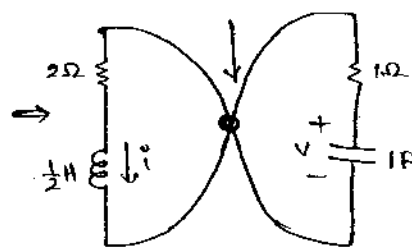
Q find (a) Ratio τ_v to τ_i ($\frac{\tau_v}{\tau_i}$)

(b) Complete Expression for $v(t), i(t)$ for all $t > 0$.

mean 20 becoming 0 at $t=0$



here short circuit not allowing interaction b/w L & C
L & C are not interacting with each other
∴ it is not a 2nd order
∴ it become two Independent first orders.



$$\tau_i = \frac{1/2}{2} = \frac{1}{4} \text{ Sec (very less)}$$

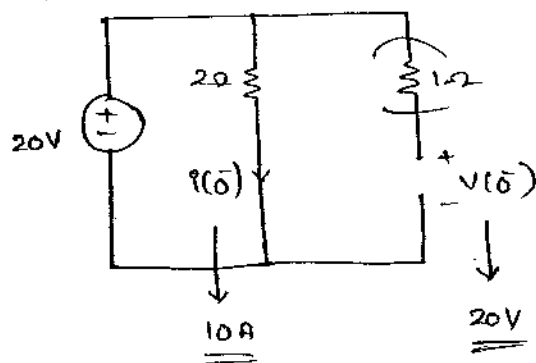
$$\tau_v = 1(1) = 1 \text{ Sec. (more)}$$

$$(a) \frac{\tau_v}{\tau_i} = \frac{1}{1/4} = 4$$

$$(b) v(t) = v_0 \cdot e^{-t/\tau_v}$$

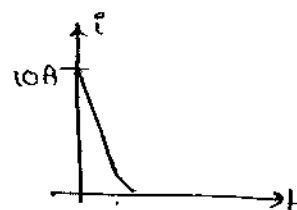
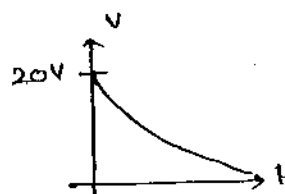
$$i(t) = I_0 \cdot e^{-t/\tau_i}$$

$t = 0^-$

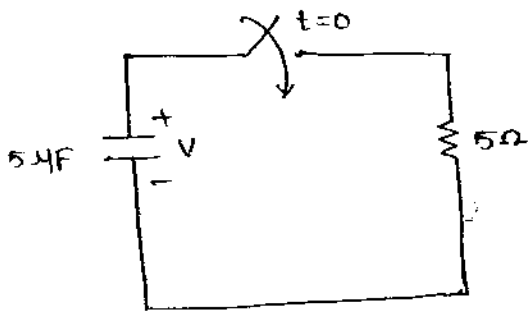


$$v(t) = 20 \cdot e^{-t}, t > 0$$

$$i(t) = 10 \cdot e^{-4t}, t > 0$$



Gate. Q if $V(0) = 4V$, determine the charge Transferred by Capacitor from $25\mu\text{sec}$ to $100\mu\text{sec}$.



⇒ Source free, first order R-C

167

⇒ S.V → 'V'

$$\tau = RC$$

$$\tau = 5\mu \times 5$$

$$= 25\mu$$

$$V(t) = V_0 \cdot e^{-t/RC}$$

$$V(t) = 4 \cdot e^{-t/25\mu}$$

$$q(t) = CV(t)$$

$$q(t) = 20 \cdot e^{-t/25\mu} \mu\text{C}$$

$$\begin{cases} q(t=25\mu) = 20 \cdot e^{-1} \mu\text{C} = 7.35 \mu\text{C} \\ q(t=100\mu) = 20 \cdot e^{-4} \mu\text{C} = 0.35 \mu\text{C} \end{cases}$$

$$q_{\text{transferred}} = (7.35 - 0.35) \mu\text{C}$$

$$= 7 \mu\text{C}$$

Category - II.

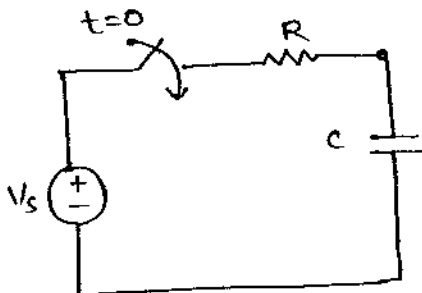
Step Response of first order.

(a) R-C ckt:

Where $V(0)$ = Voltage across Cap. before Switch operation in prev. S.S

$V(\infty)$ = Voltage across Cap after Switch operation in Next S.S

$\tau = RC$ Time Const. of ckt.



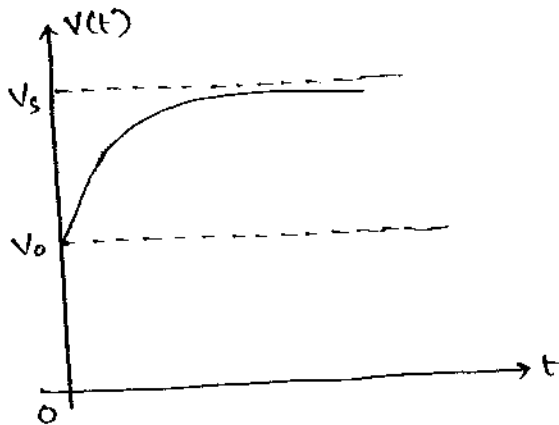
$$V(t) = V_{ss}(t) + V_{tr}(t)$$

$$V(t) = V(\infty) + [V(0) - V(\infty)] e^{-t/\tau}$$

Case (i) With Initial Condition.

$$V(0) = V_0$$

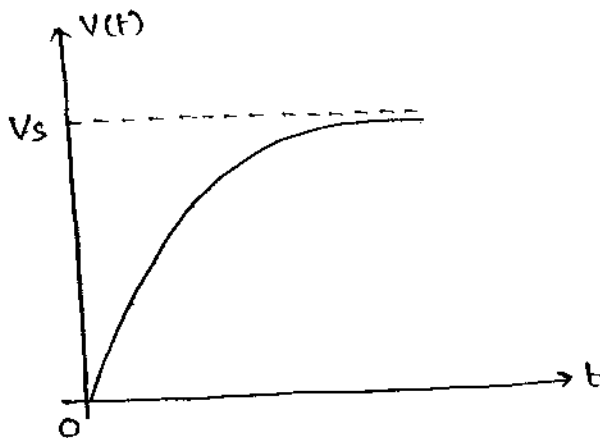
$$V(t) = V_s + [V_0 - V_s] e^{-t/\tau}$$



Case (ii) Without Initial Condition.

$$V(0) = 0$$

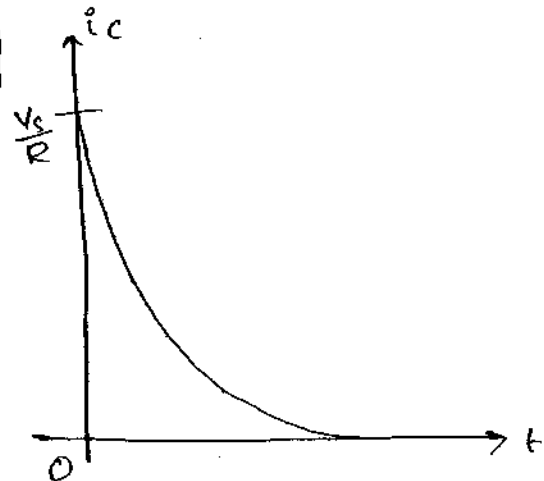
$$V(t) = V_s [1 - e^{-t/\tau}]$$



$$i_c(t) = C \frac{d}{dt} [V_s (1 - e^{-t/\tau})]$$

$$= C V_s \cdot [0 - e^{-t/\tau} \times -\frac{1}{\tau}]$$

$$i_c(t) = \frac{V_s}{R} e^{-t/\tau}$$



$$P_c(t) = V_c(t) \cdot i_c(t)$$

$$P_c(t) = \frac{V_s^2}{R} \left[e^{-t/\tau} - e^{-2t/\tau} \right] \quad \underline{\omega}$$

168

$$E_c(t) = \int_0^{\infty} P_c(t) dt$$

$$= \frac{1}{2} C V_s^2 \quad \underline{\underline{J}}$$

$$i_R(t) = \frac{V_s}{R} \cdot e^{-t/\tau} \quad \underline{\underline{A}}$$

$$V_R(t) = V_s \cdot e^{-t/\tau} \quad \underline{\underline{V}}$$

$$P_R(t) = \frac{V_s^2}{R} e^{-2t/\tau} \quad \underline{\omega}$$

$$E_R(t) = \int_0^{\infty} P_R(t) dt = \frac{1}{2} C V_s^2 \quad \underline{\underline{J}}$$

$$\% \eta_{RC} = \frac{E_o}{E_{in}} \times 100\% = \frac{E_o}{E_o + \text{losses}} \times 100\% \quad \rightarrow \text{energy loss in 'R'}$$

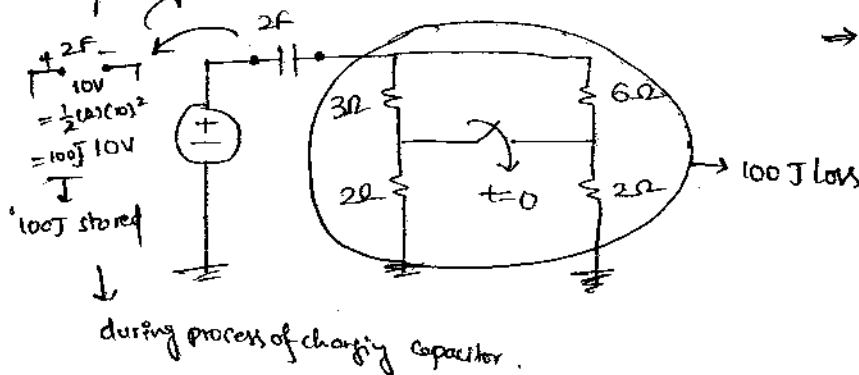
$$\% \eta_{RC} = \frac{\frac{1}{2} C V_s^2}{\frac{1}{2} C V_s^2 + \frac{1}{2} C V_s^2} \times 100\%$$

$$\% \eta_{RC} = \underline{\underline{50\%}}$$

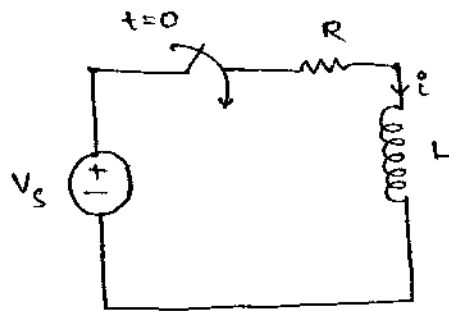
Note: In the above ckt total Energy Supplied by the Source in charging the Capacitor Completely is $\frac{1}{2} C V_s^2 + \frac{1}{2} C V_s^2 = \underline{\underline{C V_s^2}}$

Q The total Energy Supplied by the Source in charging the Capacitor

Completely is _____



(b) R-L Circuit.



$$i(t) = I_{ss}(t) + I_{tr}(t)$$

$$i(t) = I(\infty) + [I(0) - I(\infty)] \cdot e^{-t/\tau}$$

Where,

$I(0)$ = Current through Inductor before switch operation in prev. S.S

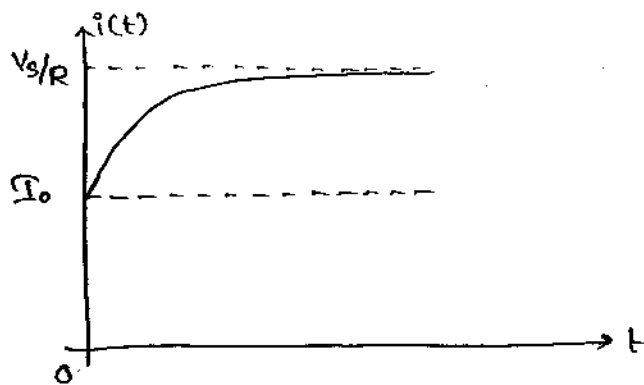
$I(\infty)$ = Current through Inductor after switch operation in next S.S

$$\tau = \text{time const. of ckt} = \frac{L}{R}$$

Case(i): With Initial Condition:

$$\text{let } I(0) = I_0$$

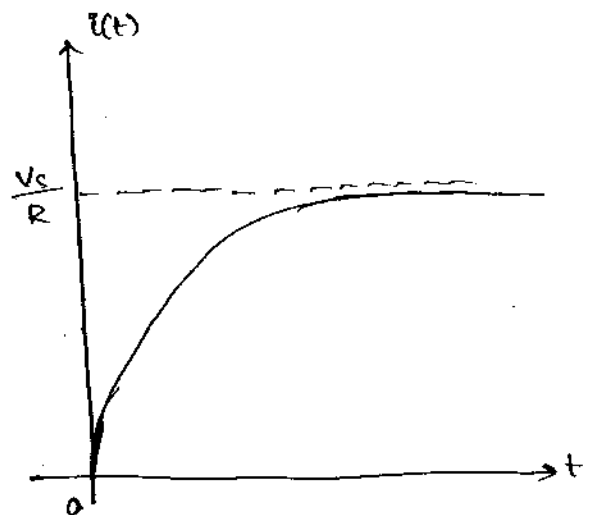
$$i(t) = \frac{V_s}{R} + \left[I_0 - \frac{V_s}{R} \right] e^{-t/\tau} \text{ A}$$



Case(ii): Without Initial Condition:

$$I(0) = 0$$

$$i(t) = \frac{V_s}{R} [1 - e^{-t/\tau}] \text{ A}$$

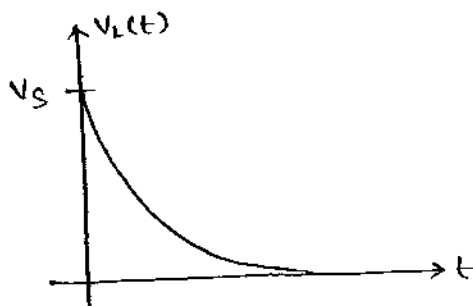


$$V_L(t) = L \frac{d}{dt} \left\{ \frac{V_S}{R} (1 - e^{-t/\tau}) \right\}$$

$$= L \cdot \frac{V_S}{R} \left\{ 0 - e^{-t/\tau} * \frac{-1}{\tau} \right\}$$

169

$$V_L(t) = V_S \cdot e^{-t/\tau}, t > 0.$$



$$P_L(t) = V_L(t) \cdot i_L(t)$$

$$P_L(t) = \frac{V_S^2}{R} [e^{-t/\tau} - e^{-2t/\tau}] \quad \underline{\underline{W}}$$

$$E_L(t) = \int_0^{\infty} P_L(t) dt$$

$$= \frac{1}{2} L \left[\frac{V_S}{R} \right]^2 \quad \underline{\underline{J}}$$

$$i_R(t) = \frac{V_S}{R} [1 - e^{-t/\tau}] \quad \underline{\underline{A}}$$

$$V_R(t) = V_S [1 - e^{-t/\tau}] \quad \underline{\underline{V}}$$

$$P_R(t) = \frac{V_S^2}{R} [1 - e^{-t/\tau}]^2 \quad \underline{\underline{W}}$$

$$E_R(t) = \int_0^{\infty} P_R(t) dt = \infty \quad \underline{\underline{J}}$$

$$\% \eta_{RL} = \frac{E_o}{E_{in}} \times 100 = \frac{E_o}{E_o + \text{losses}} \times 100\%$$

$$\% \eta_{RL} = \frac{\frac{1}{2} L \left[\frac{V_S}{R} \right]^2}{\frac{1}{2} L \left[\frac{V_S}{R} \right]^2 + \infty} \times 100\%$$

$$\% \eta_{RL} = 0\%$$

current has to flow in Inductor upto ∞ time

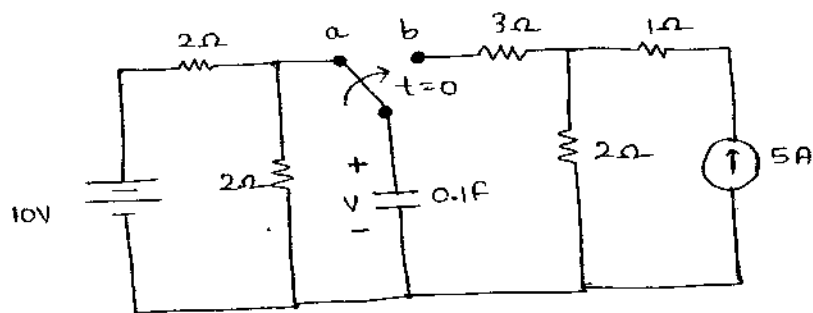
the R produces ∞ losses

'L' is most inefficient passive element.

\therefore R-L η is very less.

\therefore most of switching ckt's use R-only.

determine Complete Expression for Voltage $V(t)$ for all $t > 0$.

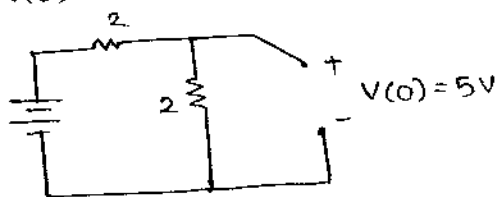


\Rightarrow Step Response, First order

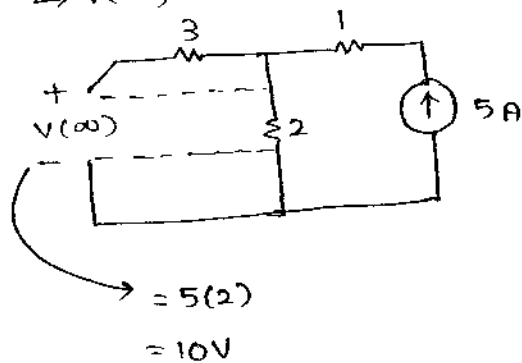
$\Rightarrow SV \rightarrow 'V'$

$$V(t) = V(\infty) + [V(0) - V(\infty)] \cdot e^{-t/\tau}$$

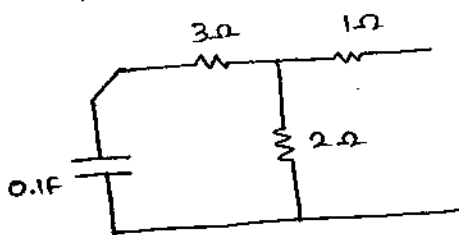
$\Rightarrow V(0)$



$\Rightarrow V(\infty)$



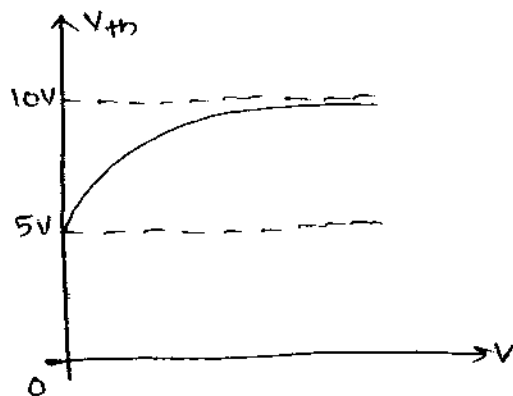
$\tau, t > 0$



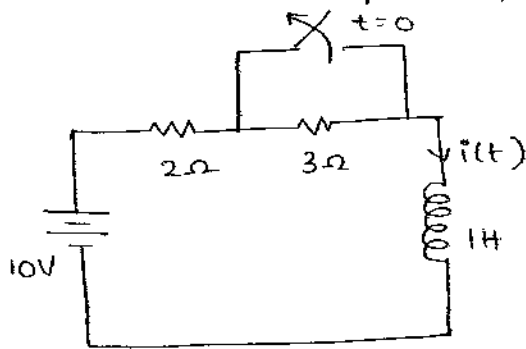
$$\tau = R_{eq} \cdot C = 5(0.1) = \frac{1}{2} \text{ Sec.}$$

$$V(t) = 10 + [5 - 10] e^{-t/1/2}$$

$$V(t) = 10 - 5e^{-2t}, t > 0$$



determine the Complete Expression for current 'i' for all $t > 0$.



\Rightarrow Step Resp, 1^{st} order

\Rightarrow S.V \Rightarrow 'i'

$$s(0) = 0$$

$$s(0) =$$

$$V(0) =$$

170

$$\frac{1}{s} =$$

$$5 - (5) \cdot e^{-5t}$$

$$33(1 - e^{-5t})$$

$$i(t) = I(\infty) + [I(0) - I(\infty)] \cdot e^{-t/\tau}$$

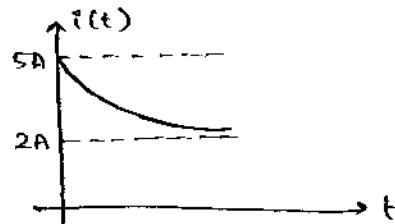
$$I(0) = \frac{10}{2} = 5A$$

$$I(\infty) = \frac{10}{5} = 2A$$

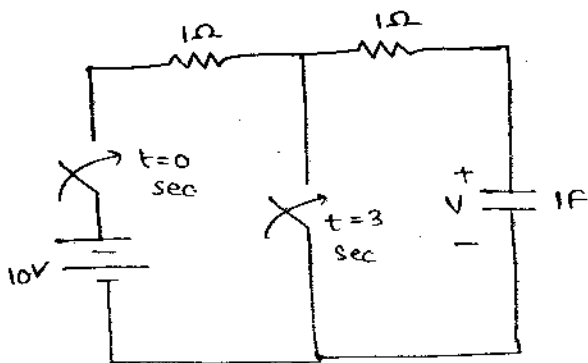
$$\tau = \frac{L}{R} = \frac{1}{5} \text{ sec.}$$

$$i(t) = 2 + [5 - 2] \cdot e^{-t/0.2}$$

$$i(t) = 2 + 3 \cdot e^{-5t}; t > 0$$



Q plot Voltage $V(t)$ for all $t > 0$.

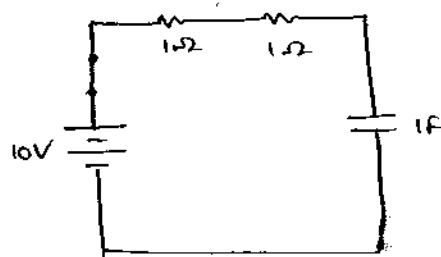


part I:

$0 < t \leq 3 \text{ sec.}$

\Rightarrow Step-Resp, 1^{st} order

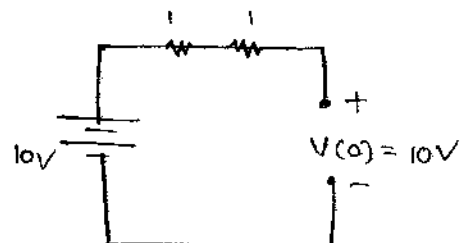
\Rightarrow S.V $\rightarrow V$



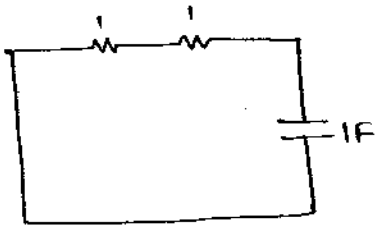
$$V(t) = V(\infty) + [V(0) - V(\infty)] e^{-t/\tau}$$

$$V(0) = 0$$

$$V(\infty) = 10V$$



$\tau, t > 0$



$$\tau = 2 \text{ Sec.}$$

$$V(t) = 10 \left[1 - e^{-t/2} \right] \quad 0 < t \leq 3 \text{ sec.}$$

P-II $t > 3$

Source free 1st order

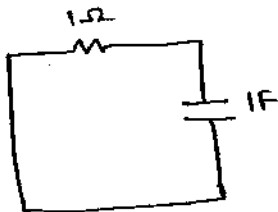
$$S.V \longrightarrow V$$

$$V(t) = V_0 \cdot e^{-t/\tau}$$

$V_0 \longrightarrow V(3^-)$ from previous state

$$V(3^-) = 10 \left[1 - e^{-3/2} \right] \\ = 7.76 \text{ V}$$

$\tau, t > 3$

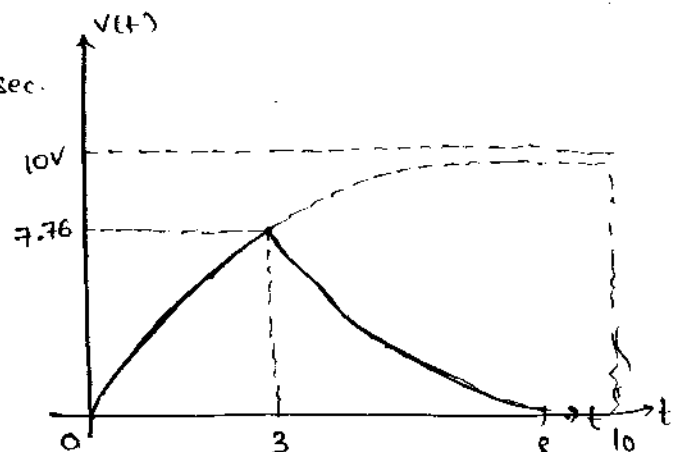


$$\tau = 1 \text{ sec}$$

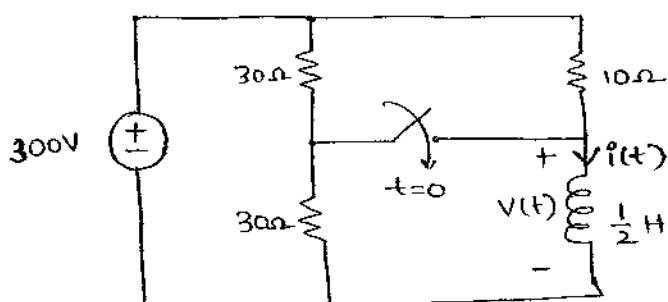
$$V(t) = 7.76 \cdot e^{-\frac{(t-3)}{1}}, t > 3.$$

Total Response \ddot{u}

$$V(t) = \begin{cases} 10(1 - e^{-t/2}) \text{ V}, & 0 < t \leq 3 \text{ sec.} \\ 7.76 e^{-(t-3)} \text{ V}, & t > 3 \text{ sec.} \end{cases}$$



Determine the Voltage Exp. for all $t > 0$.

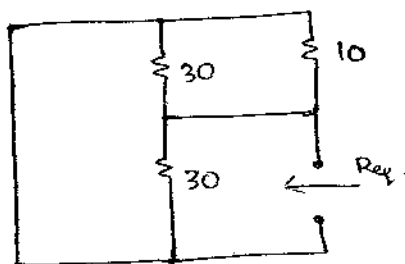


$$i(t) = I(\infty) + [I(0) - I(\infty)]e^{-t/\tau}$$

$$I(0) = \frac{300}{10} = 30A \quad |71$$

$$I(\infty) = \frac{300}{(30 \parallel 10)} = 40A$$

$$\tau = \frac{L}{R_{eq}} = \frac{1/2}{(30 \parallel 30 \parallel 10)} = \frac{1}{12} \text{ Sec.}$$

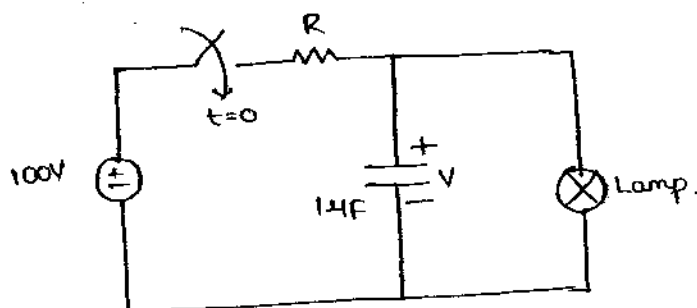


$$i(t) = 40 - 10e^{-12t}$$

$$V(t) = L \frac{di(t)}{dt} = \frac{1}{2} [0 - 10e^{-12t}(-12)]$$

$$V(t) = 60e^{-12t}, t > 0$$

Q a Neon Lamp Ionises at 75 volts, is Connected across 1μF capacitor, determine the Value of Resistance 'R' that should be Connected in Series to this Combination In order to Trigger the lamp Exactly after 20 sec from an Instant DC 100V is applied to the entire circuit.



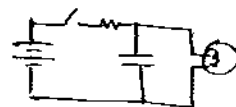
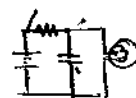
$$V(t) = 100 + [0 - 100]e^{-t/R\mu}$$

$$V(t) = 100[1 - e^{-t/R\mu}]$$

$$75V = 100[1 - e^{-20/R\mu}]$$

$$1 - e^{-20/R\mu} = \frac{3}{4}$$

$$e^{-20/R\mu} = \frac{1}{4}$$



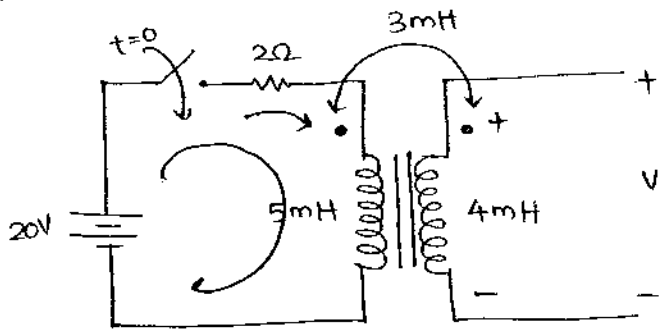
$$\frac{-20}{RM} = \ln\left[\frac{1}{4}\right]$$

$$-\frac{20}{RM} = -1.38$$

$$R = \frac{20}{1.384}$$

$$\underline{R = 14.4 \text{ M}\Omega}$$

Gate Determine the max. Value of Voltage 'V'



$$i(t) = I(\infty) + [I(0) - I(\infty)] \cdot e^{-t/\tau}$$

$$I(0) = 0 \text{ A}$$

$$I(\infty) = \frac{20}{2} = 10 \text{ A}$$

$$\tau = \frac{L}{R} = \frac{5\text{m}}{2}$$

$$i(t) = 10[1 - e^{-400t}], t > 0$$

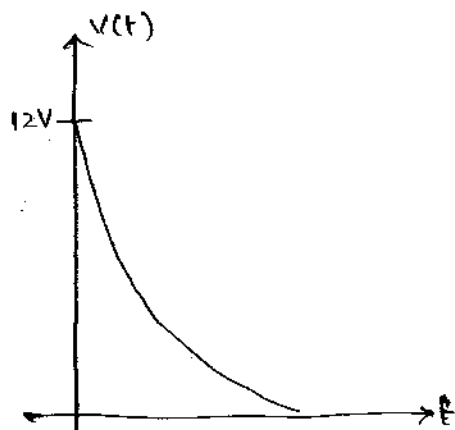
$$e_2 = +M \frac{di}{dt}$$

$$V(t) = +3\text{m} \cdot \frac{d}{dt} \{10(1 - e^{-400t})\}$$

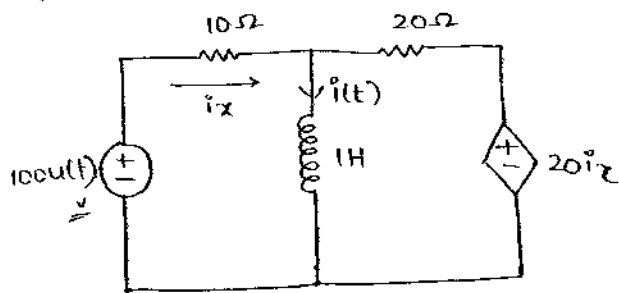
$$= 30\text{m} \cdot [0 - e^{-400t}(-400)]$$

$$V(t) = 12e^{-400t}$$

$$\therefore V_{\max} = 12\text{V}$$



gate



Determine the Complete Exp for $i(t)$ for all $t > 0$

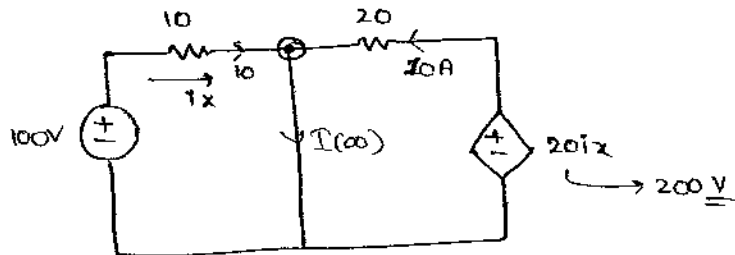
$$i(t) = I(\infty) + [I(0) - I(\infty)] e^{-t/\tau}$$

$$I(0) = 0A$$

$$I(\infty)$$

172

DC. S.S

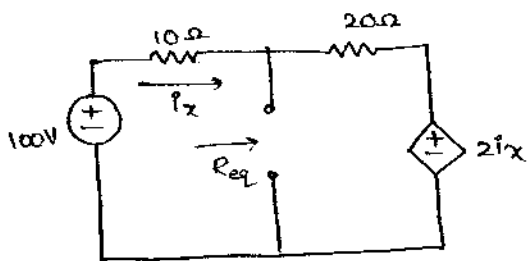


KCL

$$10 + 10 = I(\infty)$$

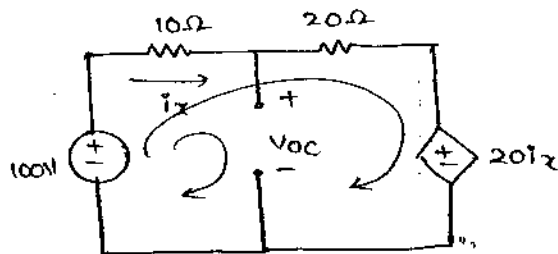
$$I(\infty) = 20A$$

$$\Rightarrow \tau = \frac{L}{R_2}$$



$$R_{eq} = \frac{V_{oc}}{I_{sc}}$$

s.I] V_{oc}



$$-100 + 30i_x + 20i_x = 0 \Rightarrow i_x = 2A$$

KVL

$$-100 + 10i_x + V_{oc} = 0 \Rightarrow V_{oc} = 100 - 20 = 80V$$

s-II] I_{sc}

↳ But Here

$$I_{sc} = I(\infty)$$

$$I_{sc} = 20A$$

$$R_{eq} = \frac{V_{oc}}{I_{sc}} = \frac{80}{20} = 4\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{1}{4} \text{ Sec}$$

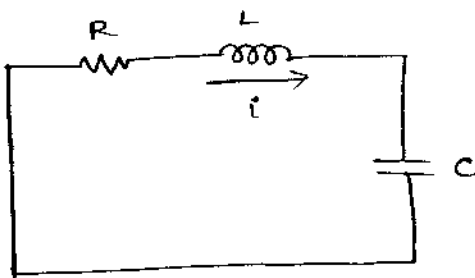
$$i(t) = 20 + [0 - 20] \cdot e^{-t/1/4}$$

$$i(t) = 20 [1 - e^{-4t}], t > 0.$$

Category - III

Source-free Second order circuits [Canonical forms]

(a) Series R-L-C:



preferable s.v. $\rightarrow i$

Mesh

$$iR + L \frac{di}{dt} + \frac{1}{C} \int i dt = 0$$

$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} = 0$$

Use L.T [Homogeneous]

$$L \cdot s^2 I(s) + R \cdot s I(s) + \frac{I(s)}{C} = 0$$

$$I(s) \left[s^2 + \frac{R}{L} s + \frac{1}{LC} \right] = 0$$

$$\therefore I(s) \neq 0$$

$$s^2 + \frac{R}{L} s + \frac{1}{LC} = 0$$

The two roots,

$$s_1, s_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)^2}$$

Let $\alpha = \frac{R}{2L} \Rightarrow$ Damping factor
 (or)
 Damping Coefficient

173

$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow$ Undamped Natural freq.

The two roots are

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Case (i): If $\alpha > \omega_0 \Rightarrow$ over damped (Sluggish & slow)

$$\frac{R}{2L} > \frac{1}{\sqrt{LC}}$$

Then two roots s_1, s_2 are -ve Real & Unequal. (or)

Then

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, t > 0$$

$$\tau = \frac{-1}{\text{Dominant pole}} \text{ sec}$$

A_1 & A_2 are Constants that can be determined from initial condition.

Case (ii): If $\alpha = \omega_0 \Rightarrow$ Critically damped.

$$\frac{R}{2L} = \frac{1}{\sqrt{LC}}$$

Then two roots s_1, s_2 are -ve Real & equal.

Then

$$i(t) = e^{-\alpha t} [A_1 + A_2 t], t > 0$$

$$\tau = \frac{1}{\alpha} = \frac{2L}{R} \text{ sec}$$

Case (iii): If $\alpha < \omega_0 \Rightarrow$ Underdamped.

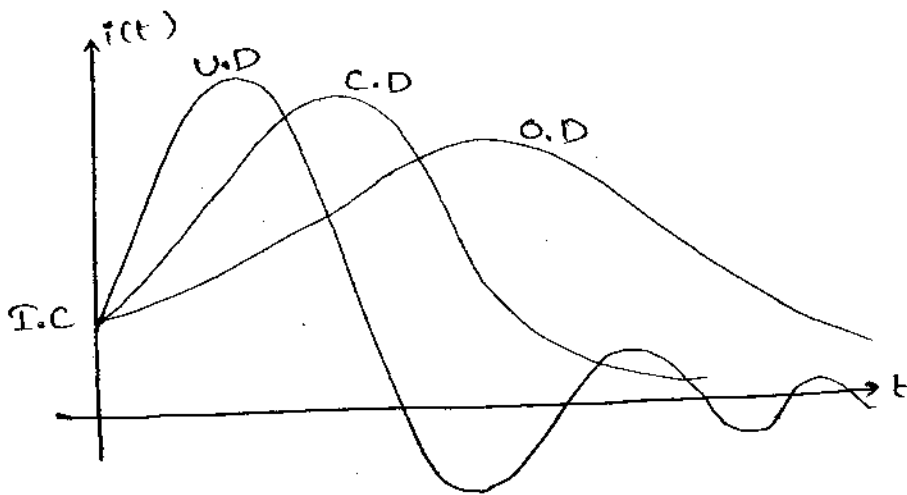
$$\frac{R}{2L} < \frac{1}{\sqrt{LC}}$$

Then the two roots s_1, s_2 are Complex Conjugate with (-ve) Real part.

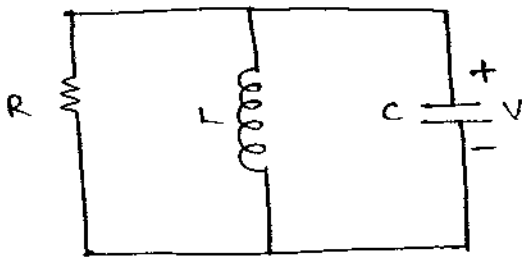
Then $i(t) = e^{-\alpha t} [A_1 \cos \omega_d t + A_2 \sin \omega_d t], t > 0$ $\tau = \frac{1}{\alpha} = \frac{2L}{R} \text{ sec.}$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

→ damped / forced / Ringing free.



(b) parallel R-L-C:



pref. s.v $\rightarrow 'v'$

KCL

$$\frac{v}{R} + \frac{1}{L} \int v dt + C \frac{dv}{dt} = 0$$

$$C \frac{d^2 v}{dt^2} + \frac{1}{R} \frac{dv}{dt} + \frac{v}{L} = 0$$

U.S.E L.T [Homogeneous]

$$C \cdot s^2 V(s) + \frac{S}{R} V(s) + \frac{V(s)}{L} = 0$$

174

$$V(s) \left[s^2 + \frac{S}{RC} + \frac{1}{LC} \right] = 0$$

$$\therefore V(s) \neq 0$$

$$\boxed{s^2 + \frac{S}{RC} + \frac{1}{LC} = 0}$$

The two roots are,

$$s_1, s_2 = \frac{-1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

Let $\alpha = \frac{1}{2RC} \Rightarrow$ damping factor
(r)
Damping Coefficient.

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \text{Undamped Natural Freq.}$$

The two roots,

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

case(1): If $\alpha > \omega_0 \Rightarrow$ overdamped

$$\frac{1}{2RC} > \frac{1}{\sqrt{LC}}$$

Then Two roots s_1, s_2 are -ve Real & Unequal.

Then,

$$\boxed{V(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, t > 0}$$

$$\tau = \frac{-1}{\text{Dominant pole}}$$

A_1, A_2 are Constants that Can be determined from I.C.

Case (ii): If $\alpha = \omega_0$ \Rightarrow Critically damped

$$\frac{1}{2RC} = \frac{1}{\sqrt{LC}}$$

The two roots are $S_1, S_2 \rightarrow$ -ve, Real & equal.

Then

$V(t) = e^{-\alpha t} (A_1 + A_2 t), t > 0.$

$$\tau = \frac{1}{\alpha} = 2RC \text{ sec.}$$

Case (iii): If $\alpha < \omega_0$ \Rightarrow Underdamped.

$$\frac{1}{2RC} < \frac{1}{\sqrt{LC}}$$

The two roots S_1, S_2 are Complex Conjugate with (-ve) Real part.

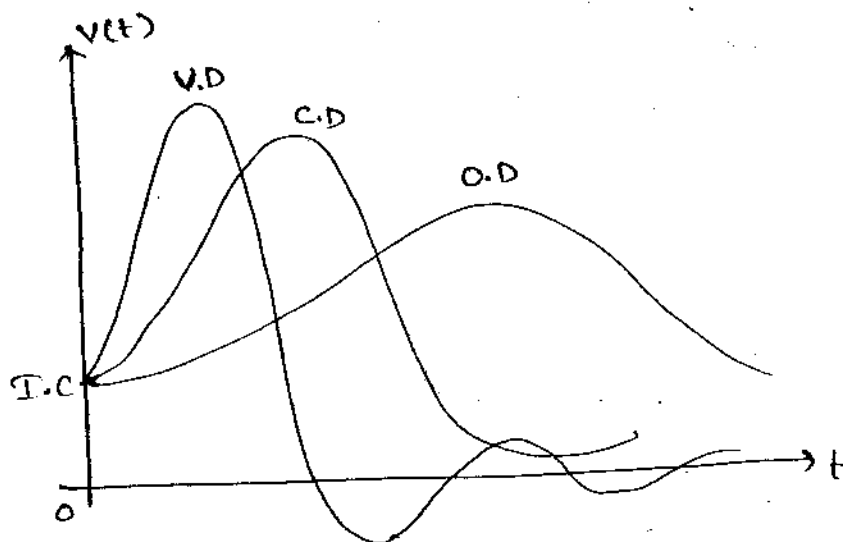
Then

$V(t) = e^{-\alpha t} [A_1 \cos \omega_d t + A_2 \sin \omega_d t], t > 0$

$$\tau = \frac{1}{\alpha} = 2RC \text{ sec}$$

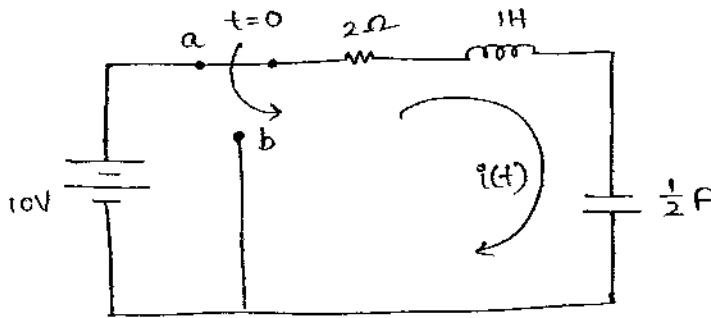
$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

\hookrightarrow damped / forced / Ringing freq.



Q The Response of Current $i(t)$ is of the form.

175



- (a) O.D
- (b) C.D
- ☒ (c) U.D
- (d) sinusoidal

$$\alpha = \frac{R}{2L} = \frac{2}{2(1)} = 1$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \cdot \frac{1}{2}}} = \sqrt{2}$$

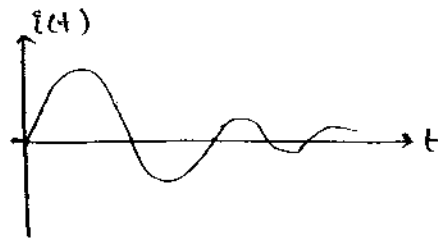
$$\left. \begin{matrix} \alpha < \omega_0 \\ \text{U.D} \end{matrix} \right\}$$

$i(0^-) = \text{Current in prev. s.s} = \underline{0A}$

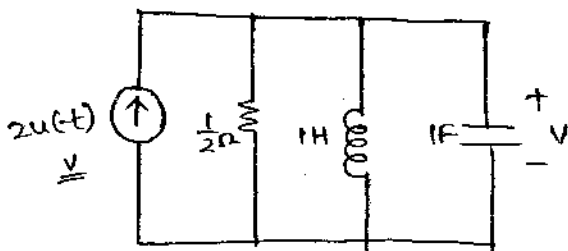
$E(0^-) = \cancel{K}, \textcircled{C}, \cancel{B \text{ or } A}$

→ energy stored in prev. s.s
($\because i=0 \Rightarrow \frac{1}{2}Li^2=0$)

$i(\infty) = 0A$



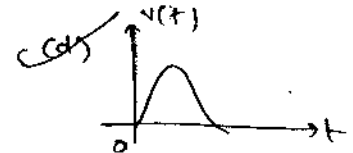
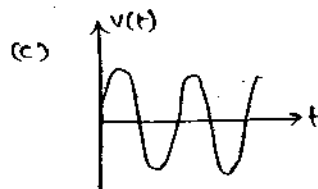
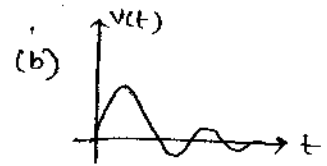
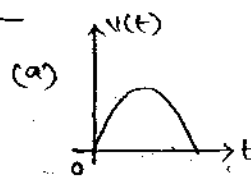
Q The Response for Voltage $V(t)$ —



$$\alpha = \frac{1}{2 \cdot \frac{1}{2} \cdot 1} = 1$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 1$$

$$\left. \begin{matrix} \alpha = \omega_0 \\ \text{C.D.} \end{matrix} \right\}$$

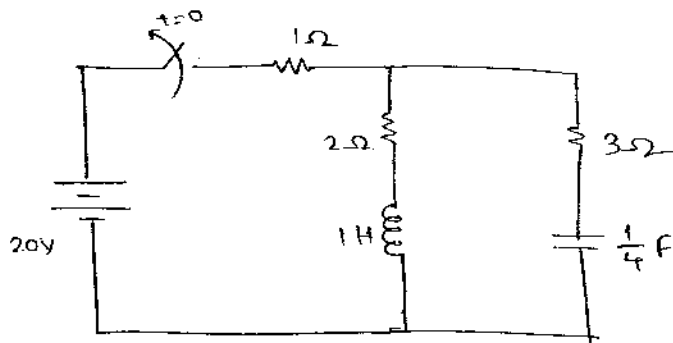


$V(0^-) = 0V$

$E(0^-) \rightarrow \textcircled{L} \cancel{K}, \cancel{B \text{ or } A}$
($\frac{1}{2}Cv^2=0$)

$V(\infty) = \underline{0V}$

The time Constant of ckt given below.



$$\alpha = \frac{2+3}{2(1)} = 2.5$$

$$\omega_0 = \frac{1}{\sqrt{1 \cdot \frac{1}{4}}} = 2$$

$$\alpha > \omega_0 \Rightarrow \text{O.D.}$$

$$s_1 s_2 = -2.5 \pm \sqrt{(2.5)^2 - 2^2}$$

$$= -1, -4$$

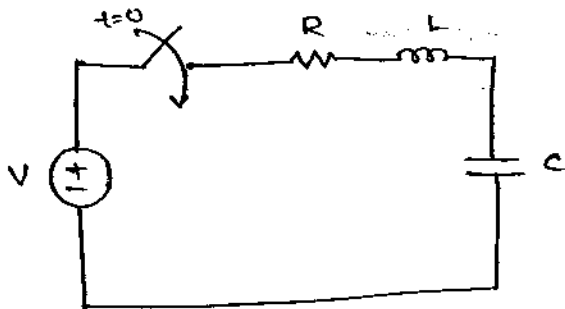
↑
dom. pole

$$T = \frac{-1}{\text{D.P.}} = \frac{-1}{(-1)} = \underline{\underline{1 \text{ sec}}}$$

Category - IV

Step Response of Second order ckt's [canonical forms]

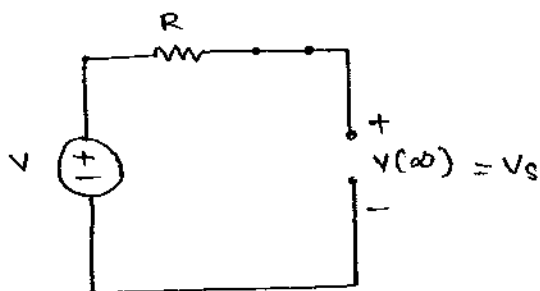
(a) Series R-L-C



Dominat s.v \rightarrow 'v' only

$$v(t) = V_{ss}(t) + V_{tr}(t)$$

$V_{ss}(t)$ at $t \rightarrow \infty$



But $V_{tr}(t)$ depends Upon R, L, C

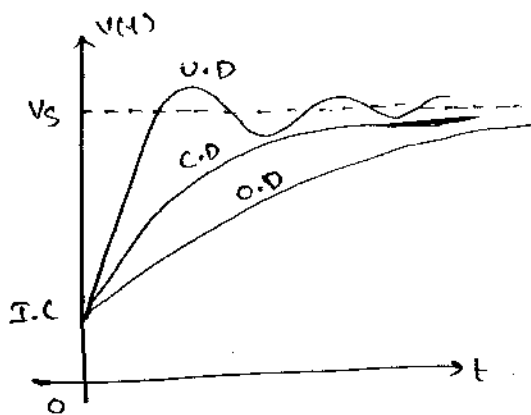
$$\alpha = \frac{R}{2L}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

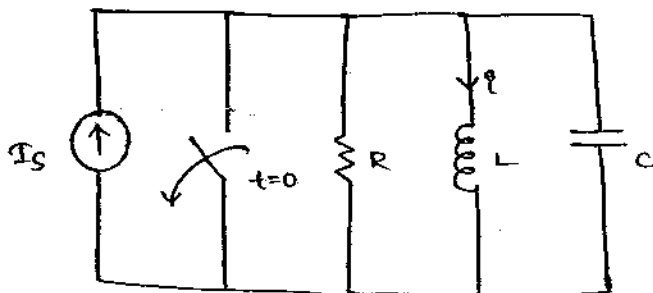
Total Response

176

$$V(t) = \begin{cases} V_S + A_1 e^{s_1 t} + A_2 e^{s_2 t}, t > 0 & \text{if } \alpha > \omega_0 \Rightarrow \text{overdamped} \\ V_S + e^{-\alpha t} [A_1 + A_2 t], t > 0 & \text{if } \alpha = \omega_0 \Rightarrow \text{critically damped} \\ V_S + e^{-\alpha t} [A_1 \cos \omega_d t + A_2 \sin \omega_d t], t > 0 & \text{if } \alpha < \omega_0 \Rightarrow \text{underdamped} \end{cases}$$



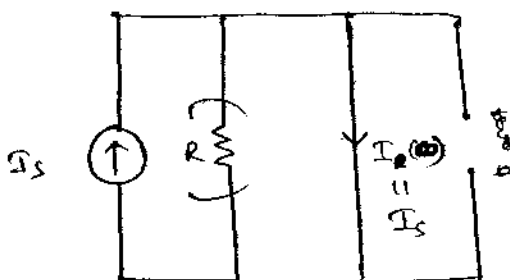
(b) parallel R-L-C



Dominant s.v. \rightarrow 'i' only.

$$i(t) = I_{ss}(t) + I_{tr}(t).$$

$I_{ss}(t)$, at $t \rightarrow \infty$



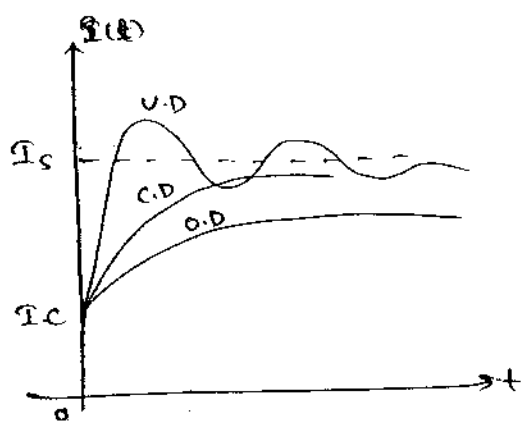
But $I_{tr}(t)$ depends upon R, L, C

$$\alpha = \frac{1}{2RC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

total Response,

$$i(t) = \begin{cases} I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}, t > 0 & \text{if } \alpha > \omega_0 \rightarrow \text{over damped} \\ \text{(or)} \\ I_s + e^{-\alpha t} [A_1 + A_2 t], t > 0 & \text{if } \alpha = \omega_0 \rightarrow \text{critically damped} \\ \text{(or)} \\ I_s + e^{-\alpha t} [A_1 \cos \omega_d t + A_2 \sin \omega_d t], t > 0 & \text{if } \alpha < \omega_0 \rightarrow \text{underdamped} \end{cases}$$



Note: 1. Never we should represent sudden change in voltage across capacitor

(\because Results in high $\frac{di}{dt}$)

2. Never we should represent a sudden change in current through inductor

(\because Results in high $\frac{dv}{dt}$)

3. Never we should a switch across Ideal voltage source & switch is

Series to Ideal Current source.

Category V

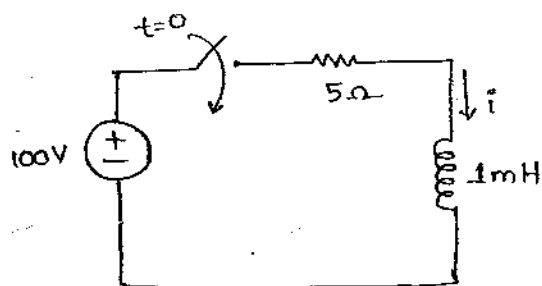
Initial Condition problems (a) Transient state problem i.e., problems at $t=0^+$.

Equivalent ckt representation of passive elements in Transient state:

177

Element	ckt at $t=0^+$

Q find $i(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{d^2i(0^+)}{dt^2}$



at $t=0^-$, switch was open,

$$i(0^-) = 0A = i(0^+)$$

↓
∴ Inductor never allow sudden change in current

KVL, $t > 0$

$$-100 + 5i + 1m \frac{di}{dt} = 0 \quad \text{--- (1)}$$

└→ $t=0^+$

$$-100 + 5i(0^+) + 1m \cdot \frac{di(0^+)}{dt} = 0$$

$$\frac{di(0^+)}{dt} = \frac{+100}{1m} = \underline{\underline{100 \text{ kA/sec}}}$$

diff. Eq ① again

$$5 \cdot \frac{di}{dt} + 1m \cdot \frac{d^2i}{dt^2} = 0$$

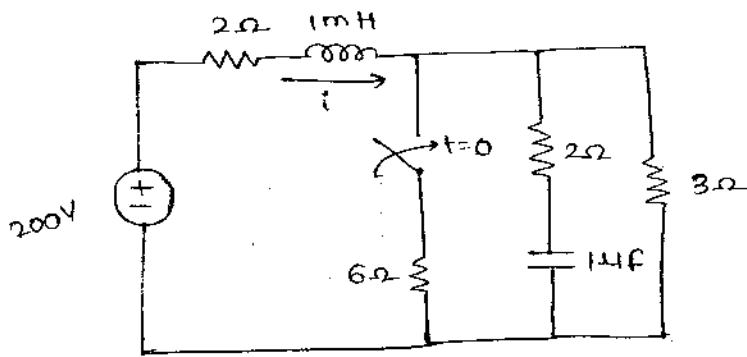
$$L \rightarrow i = 0^+$$

$$5 \cdot \frac{di(0^+)}{dt} + 1m \cdot \frac{d^2i(0^+)}{dt^2} = 0$$

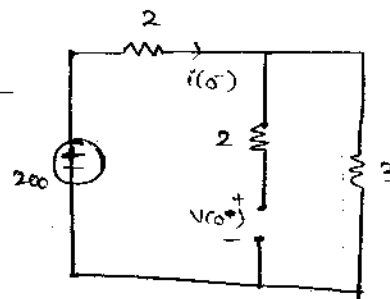
$$\frac{d^2i(0^+)}{dt^2} = \frac{-500k}{1m} = \underline{\underline{-500 \text{ MA/sec}^2}}$$

to determine stability

Q find $i(0^+)$, $V(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{dV(0^+)}{dt}$, $i(\infty)$, $V(\infty)$



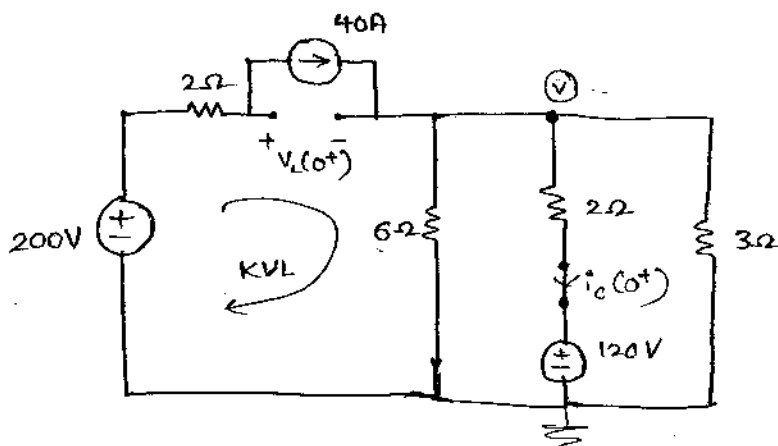
part-I $[t=0^-]$
 $i(0^-) = \frac{200}{5}$



$$i(0^-) = \frac{200}{5} = 40A = i(0^+)$$

$$V(0^-) = 40(3) = 120V = V(0^+)$$

P-II] Transient state $[t=0^+]$



$$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L}$$

$$\frac{dV(0^+)}{dt} = \frac{i_c(0^+)}{C}$$

Nodal, $[t=0^+]$

178

$$-40 + \frac{V}{6} + \frac{(V-120)}{2} + \frac{V}{3} = 0$$

$$-240 + V + 3V - 360 + 2V = 0$$

$$6V = 600$$

$$V = 100V$$

$$-200 + 40(2) + V_L(0^+) + 100 = 0$$

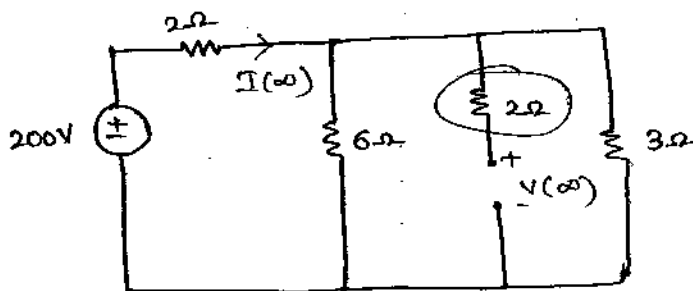
$$V_L(0^+) = 20$$

$$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L} = \frac{20}{1m} = 20 \text{ kA/sec.}$$

$$i_C(0^+) = \frac{V-120}{2} = \frac{100-120}{2} = -10 \text{ A.}$$

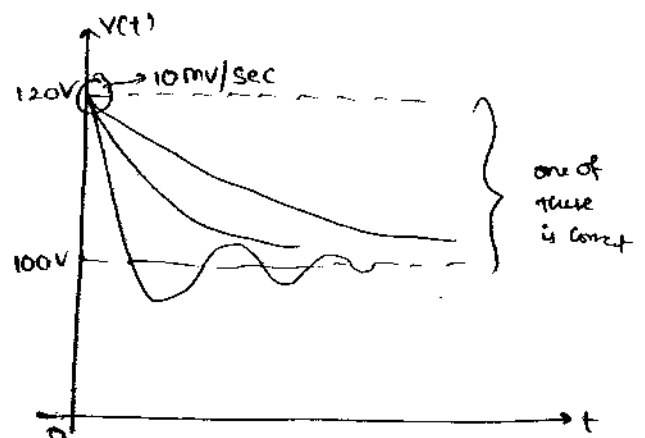
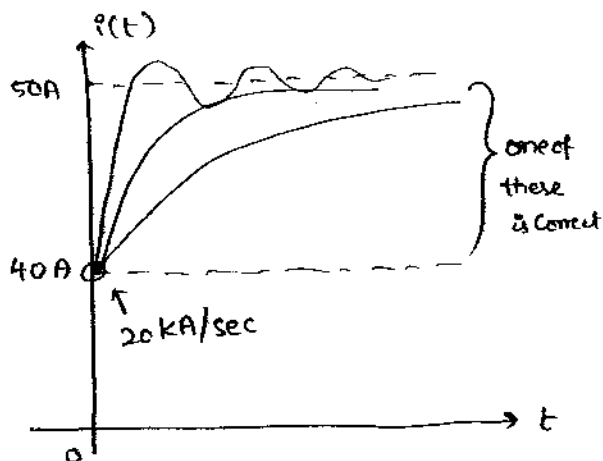
$$\frac{dV(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{-10}{1\mu} = -10 \text{ MV/sec.}$$

P-III] future $[t \rightarrow \infty]$



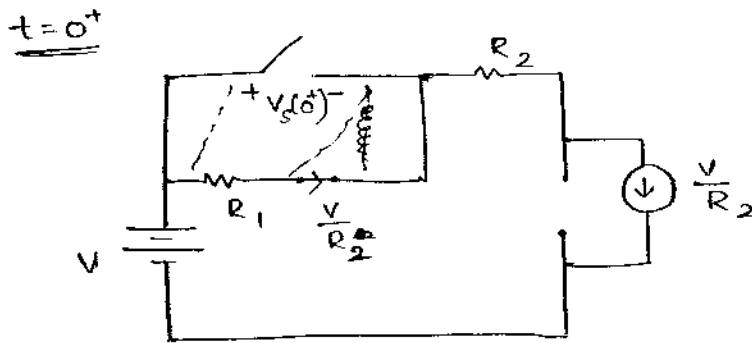
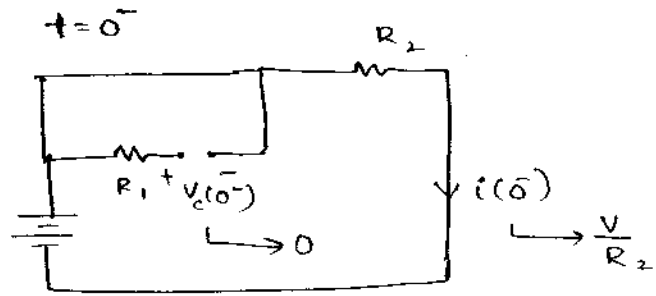
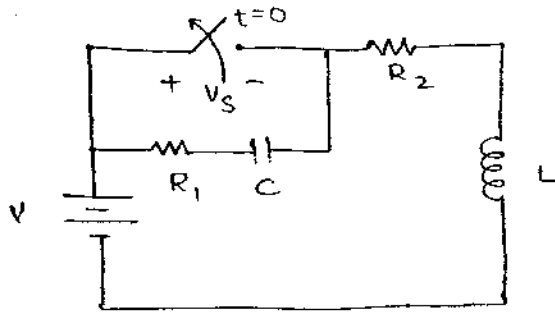
$$I(\infty) = \frac{200}{2 + [6||3]} = 50 \text{ A}$$

$$V(\infty) = 200 \left[\frac{2}{2+2} \right] = 100 \text{ V}$$



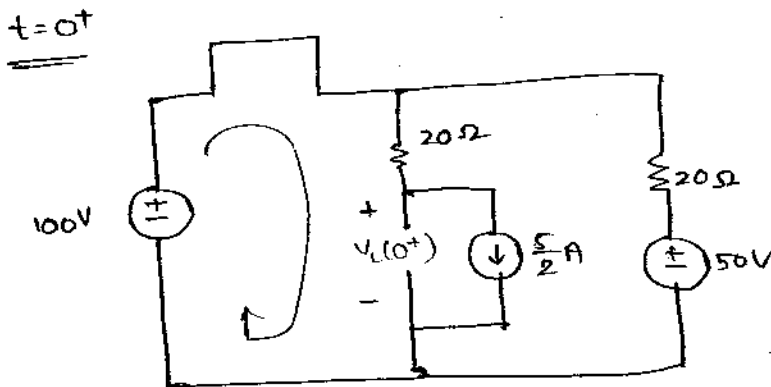
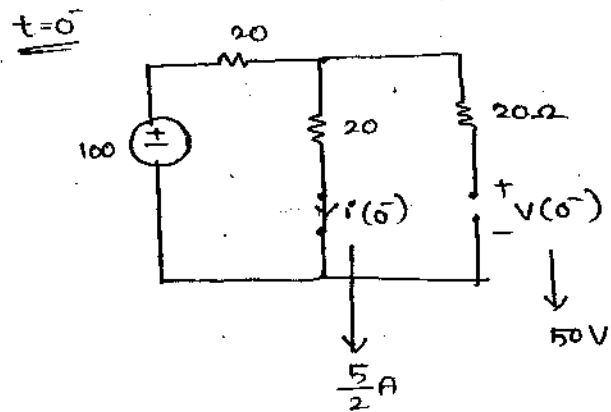
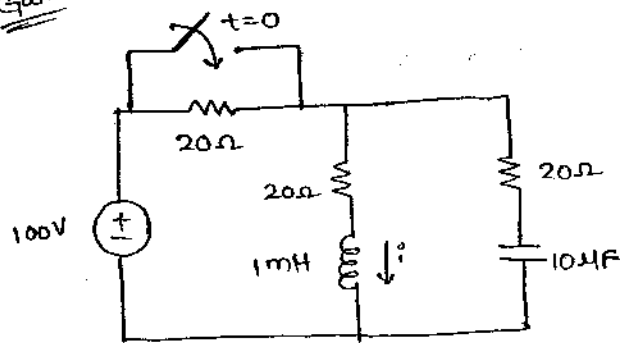
Q $V_S(0^+) = \underline{\hspace{2cm}}$

(a) $V \left[\frac{R_1}{R_2} \right]$ (b) $V \left[\frac{R_2}{R_1} \right]$ (c) $V \left[\frac{R_1}{R_1 + R_2} \right]$ (d) $V \left[\frac{R_2}{R_1 + R_2} \right]$



$V_S(0^+) = \frac{V}{R_2} [R_1]$

Q2 $\frac{di(0^+)}{dt} = \underline{\hspace{2cm}}$



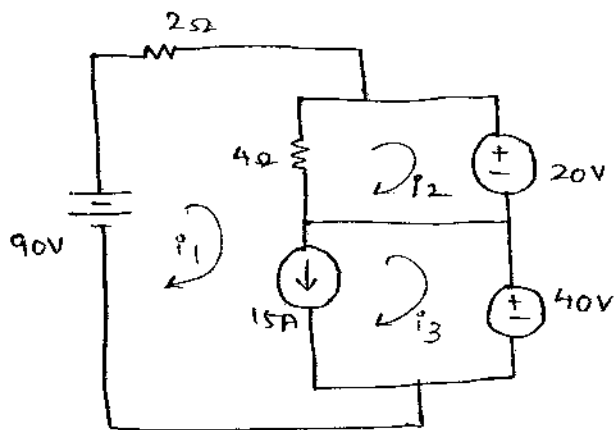
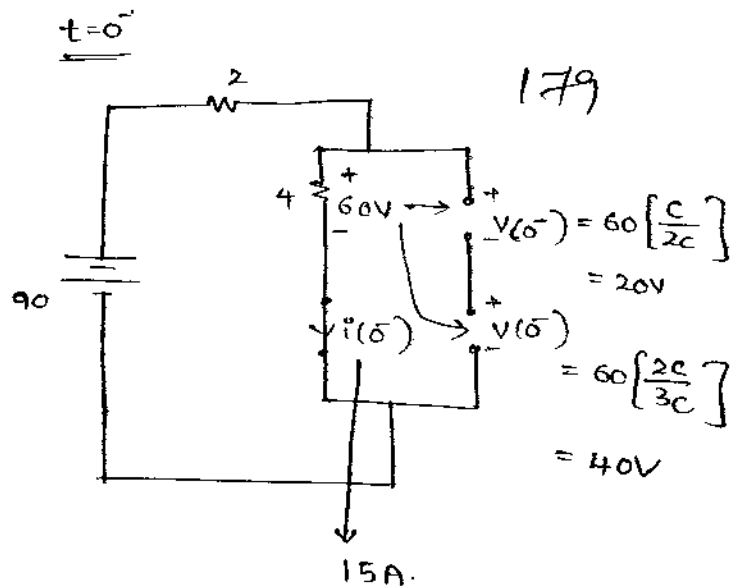
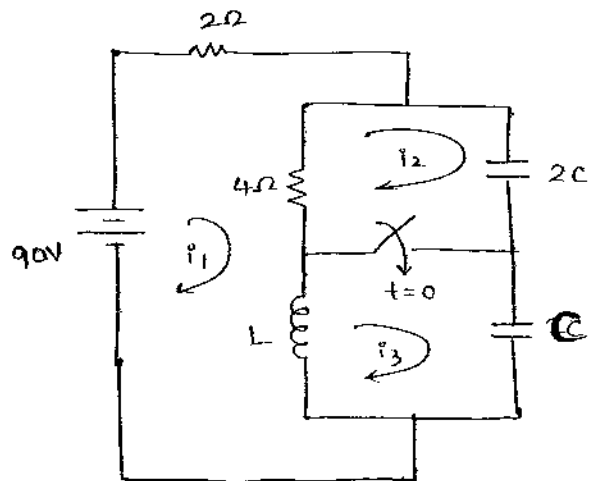
KVL
 $-100 + \frac{5}{2}(20) + V_L(0^+) = 0 \rightarrow$

$V_L(0^+) = 50$

$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L} = \frac{50}{1m}$

$= 50 \text{ KA/sec}$

Q find $i_1(0^+)$, $i_2(0^+)$, $i_3(0^+)$



mesh

$$-90 + 2i_1 + 4[i_1 - i_2] + 40 = 0 \rightarrow (1)$$

$$4[i_2 - i_1] + 20 = 0 \rightarrow (2)$$

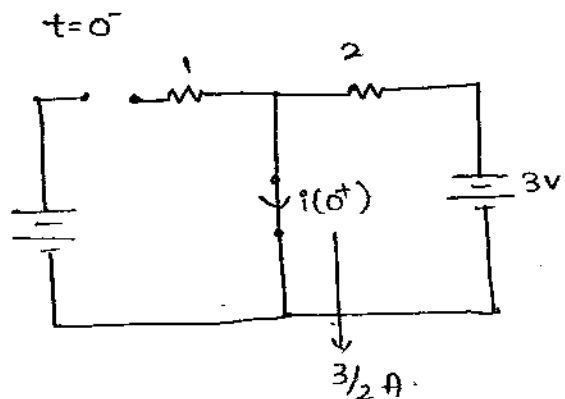
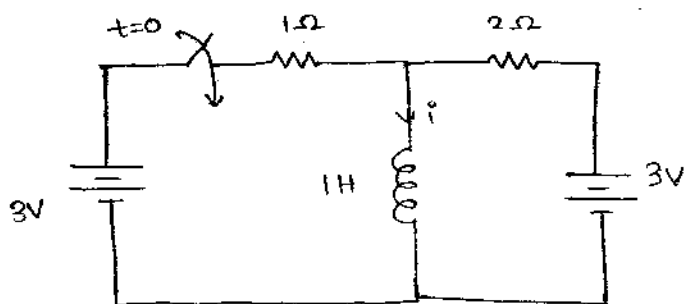
$$(i_1 - i_3) = 15 \rightarrow (3)$$

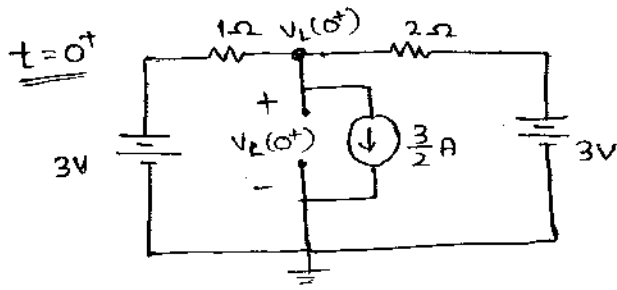
$$i_1(0^+) = 15A$$

$$i_2(0^+) = 10A$$

$$i_3(0^+) = 0A$$

Q2 $\frac{di(0^+)}{dt} = \underline{\hspace{2cm}}$





Nodal

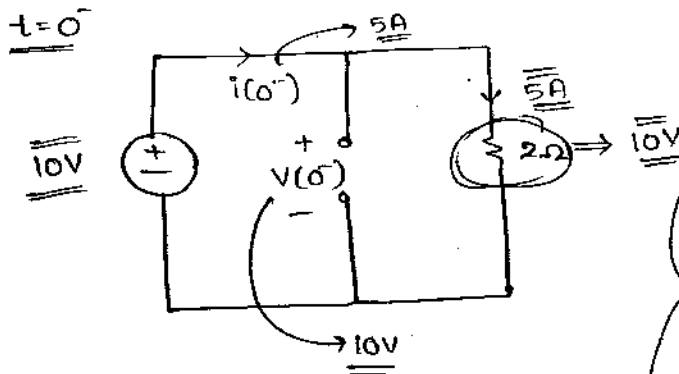
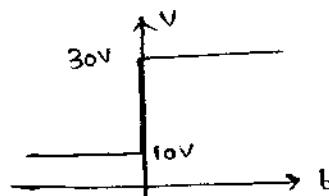
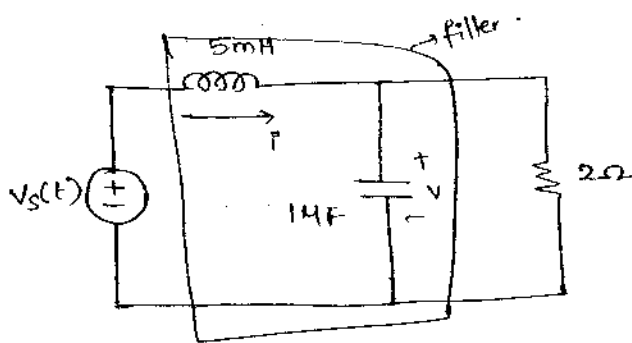
$$\frac{[V_L(0^+) - 3]}{1} + \frac{3}{2} + \frac{[V_L(0^+) - 3]}{2} = 0$$

$$3V_L(0^+) = 6 \rightarrow V_L(0^+) = 2$$

$$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L} = 2 \text{ A/sec.}$$

Ex If $V_s(t) = 10 + 20u(t)$

find $\frac{di(0^+)}{dt} = \underline{\hspace{2cm}}$, $\frac{dv(0^+)}{dt} = \underline{\hspace{2cm}}$



even the Supply suddenly changed
but load side
 V, I are same
it's appl. of shock absorber, filter.

KVL

$$-30 + V_L(0^+) + 10 = 0 \rightarrow V_L(0^+) = 20V$$

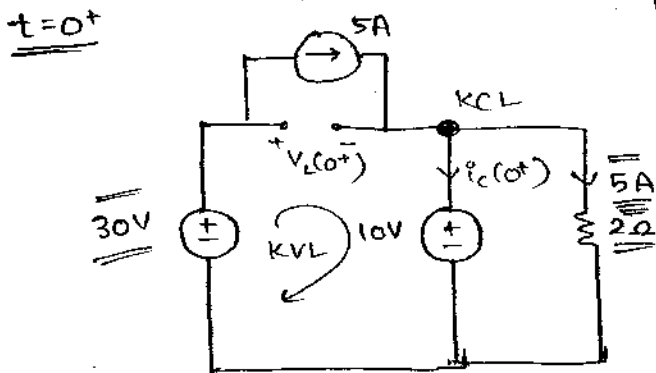
$$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L} = \frac{20}{5m} = 4 \text{ kA/sec}$$

KCL

$$5 = i_c(0^+) + 5$$

$$\therefore i_c(0^+) = 0 \text{ A}$$

$$\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C} = 0 \text{ V/sec}$$



Complete Solution to Second order ckt:

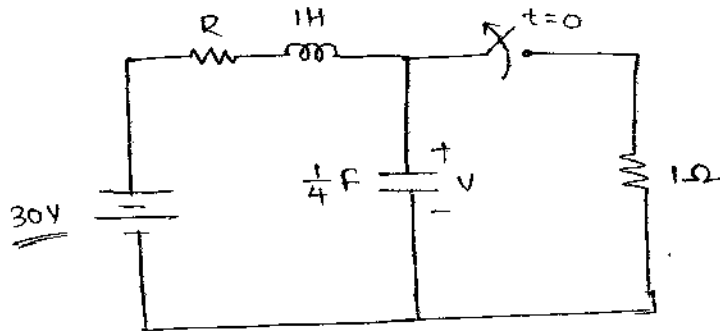
180

Q Determine the Complete Exp. for voltage $V(t)$ for all $t > 0$ if

(a) $R = 5 \Omega$

(b) $R = 4 \Omega$

(c) $R = 1 \Omega$

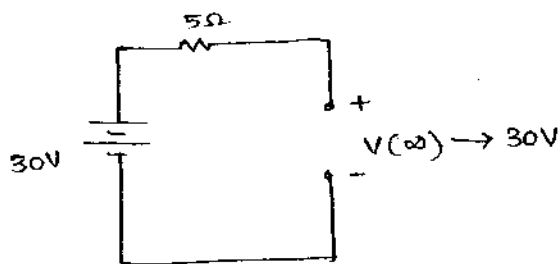


\Rightarrow Step Resp. 2nd order

\Rightarrow Series R-L-C

$V(t) = V_{ss}(t) + V_{tr}(t)$

$V_{ss}(t)$, at $t \rightarrow \infty$



\Rightarrow But $V_{tr}(t)$ depends Upon R, L, C

$\alpha = \frac{5}{2(1)} = 2.5$

$\omega_0 = \frac{1}{\sqrt{1(\frac{1}{4})}} = 2$

$\alpha > \omega_0 \rightarrow$ O.D

$V_{tr}(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$s_1, s_2 = -2.5 \pm \sqrt{(2.5)^2 - 2^2}$

$= -4, -1$

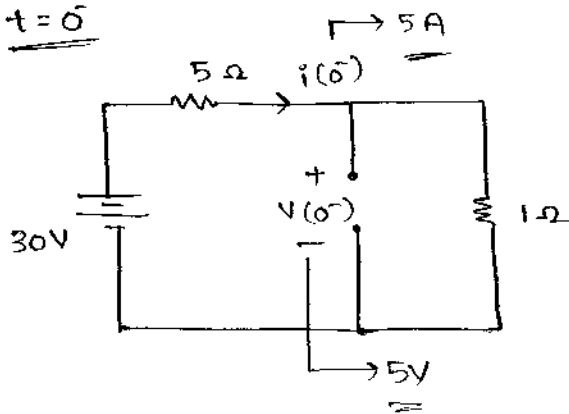
Total Response is

$V(t) = 30 + A_1 e^{-4t} + A_2 e^{-t}, t > 0$

1. $V(0) = \underline{\hspace{2cm}}$ } cat - V

2. $\frac{dV(0)}{dt} = \underline{\hspace{2cm}}$

$t = 0^-$

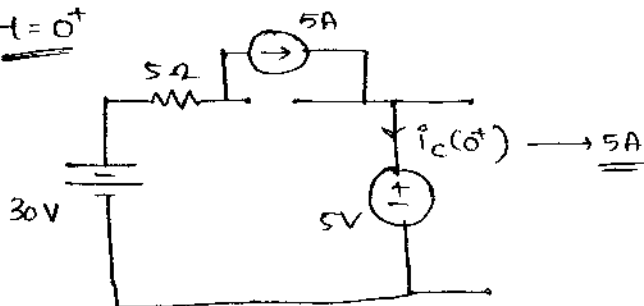


$$V(0^-) = V(0) = 5V$$

$$5 = 30 + A_1 + A_2$$

$$A_1 + A_2 = 25 \rightarrow \textcircled{1}$$

$t = 0^+$



$$\frac{dV(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{5}{1/4} = 20 \text{ V/sec.} \quad \textcircled{2}$$

$$\frac{dV(t)}{dt} = -4A_1 e^{-4t} - A_2 e^{-t}$$

$$\downarrow t=0$$

$$20 = -4A_1 - A_2 \rightarrow \textcircled{2}$$

① & ②

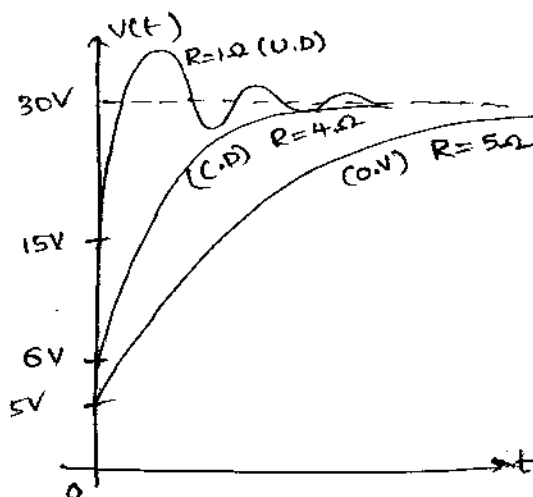
$$A_1 = \frac{5}{3}$$

$$A_2 = -\frac{80}{3}$$

Total Response is

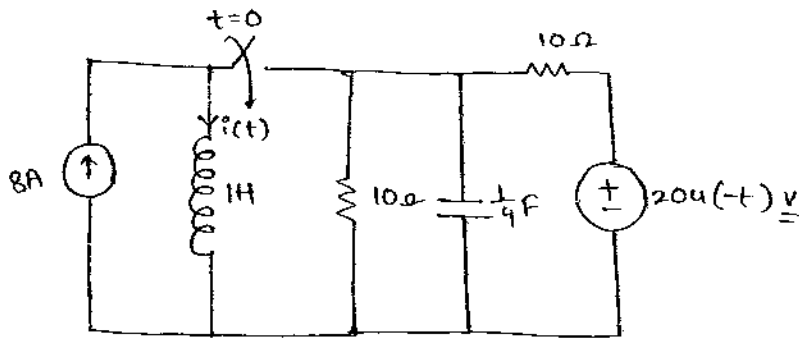
$$V(t) = 30 + \frac{5}{3} e^{-4t} - \frac{80}{3} e^{-t}, t > 0$$

Resistor \rightarrow dampers



Q determine the Complete Exp for Current $i(t)$ for all $t > 0$

181

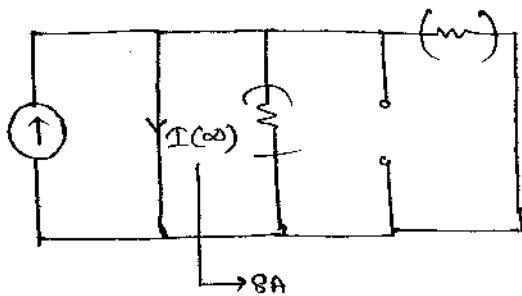


⇒ step Response, II order

⇒ parallel R-L-C

$$i(t) = I_{ss}(t) + I_{tr}(t)$$

I_{ss} , at $t \rightarrow \infty$



But $I_{tr}(t)$ depends upon, R, L, C

$$\alpha = \frac{1}{2 \left[10 // 10 \right] \frac{1}{4}} = \frac{2}{5}$$

$$\omega_0 = \frac{1}{\sqrt{1 \left(\frac{1}{4} \right)}} = 2$$

$$\left. \begin{array}{l} \alpha < \omega_0 \\ \downarrow \\ \text{U.D} \end{array} \right\}$$

$$I_{tr}(t) = e^{-\alpha t} \left[A_1 \cos \omega_d t + A_2 \sin \omega_d t \right]$$

$$\omega_d = \sqrt{\left(4 - \frac{4}{25} \right)} = \sqrt{\frac{96}{25}} = 1.96 \text{ rad/sec}$$

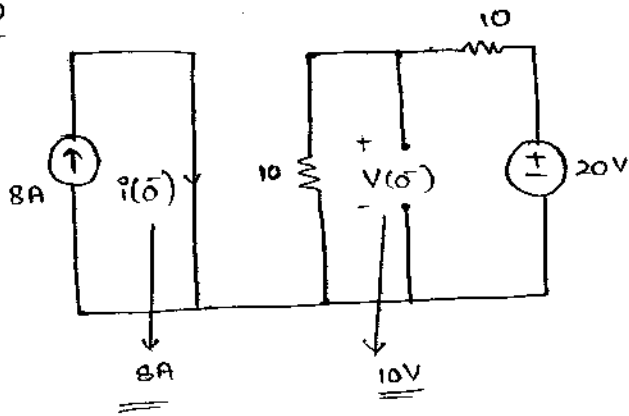
⇒ Total Response \vec{i}

$$i(t) = 8 + e^{-\frac{2t}{5}} \left[A_1 \cos 1.96t + A_2 \sin 1.96t \right], t > 0$$

Idea, (i) $i(0) = \underline{\hspace{2cm}}$

(ii) $\frac{di(0)}{dt} = \underline{\hspace{2cm}}$

$t=0^-$

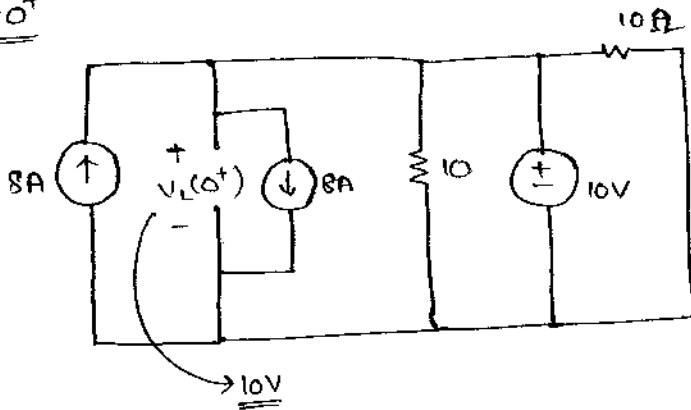


$$i(0^-) = i(0) = 8A \text{ [use]}$$

$$8 = 8 + A_1$$

$$A_1 = 0 \rightarrow \textcircled{1}$$

$t=0^+$



$$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L} = \frac{10}{1} = 10A/sec \text{ (use)}$$

$$\frac{di(t)}{dt} = e^{-\frac{2t}{5}} \left[-1.96 A_1 \sin 1.96t + 1.96 A_2 \cos 1.96t \right] + \left[A_1 \cos 1.96t + A_2 \sin 1.96t \right] \left[-\frac{2}{5} e^{-\frac{2t}{5}} \right]$$

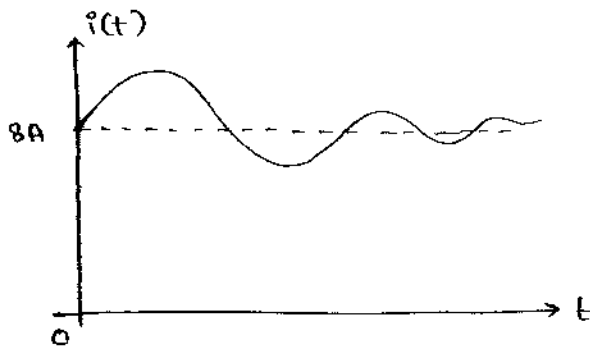
$\rightarrow t=0$

$$10 = 1.96 A_2 - \frac{2 A_1}{5} \rightarrow \textcircled{2}$$

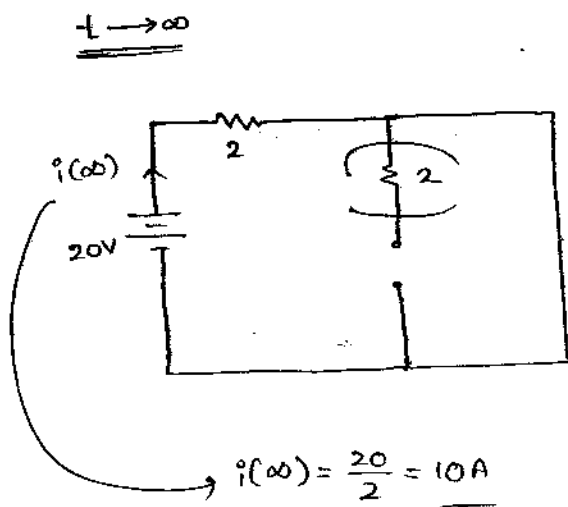
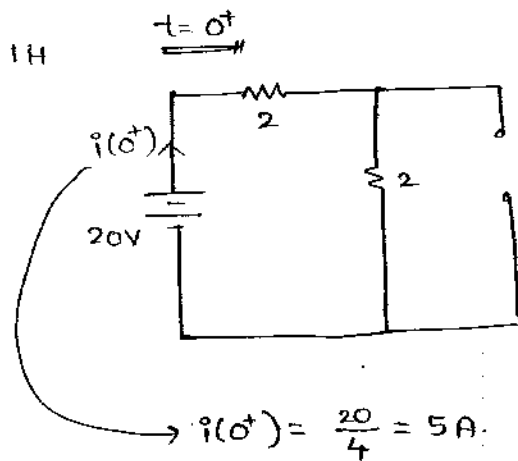
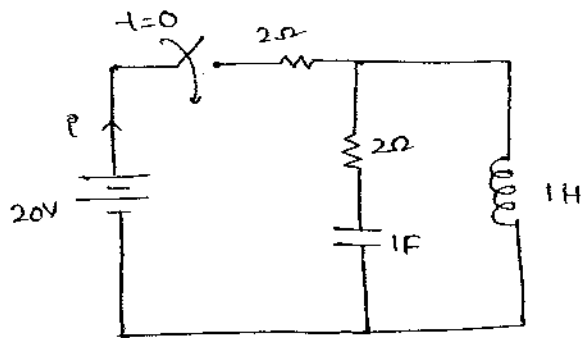
$$\text{So, } A_2 = \frac{10}{1.96} = 5.1$$

total Response \hat{i}

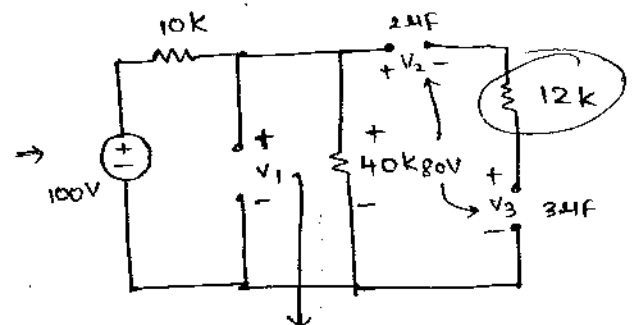
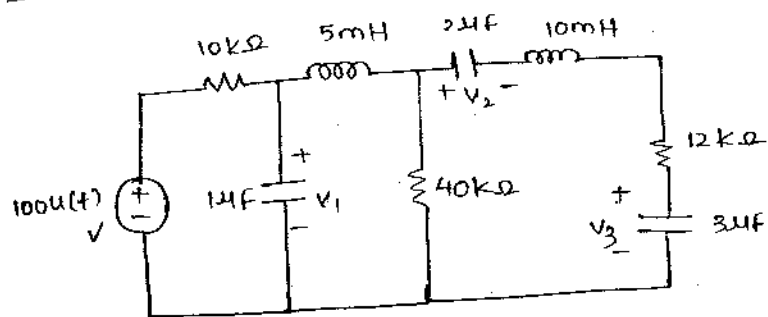
$$i(t) = 8 + 5.1 e^{-\frac{2t}{5}} \sin 1.96t, t > 0.$$



Q determine Current through Battery at $t=0^+$ & $t \rightarrow \infty$.



⇒ determine S-S Voltages across Capacitor



$$V_1 = 100 \left[\frac{40}{40+10} \right]$$

$$V_1 = 80V,$$

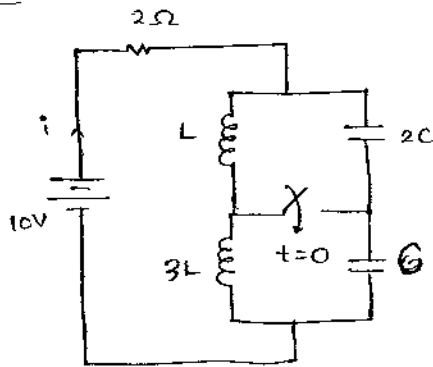
$$V_2 = 80 \left[\frac{3\mu}{5\mu} \right] = 48V$$

$$V_3 = 80 \left[\frac{2\mu}{5\mu} \right] = 32V.$$

H.W-1

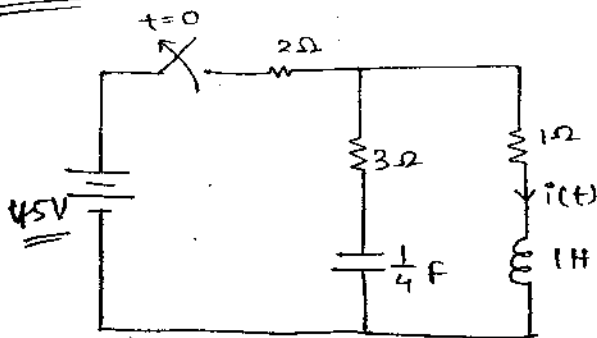
Determine Current through Battery at $t=0^+$ & $t \rightarrow \infty$

183



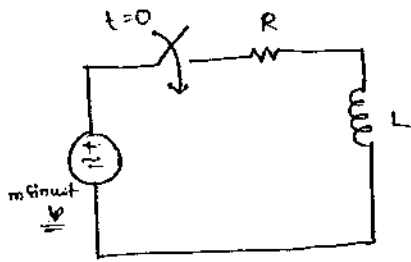
H.W-2

Determine the Complete solution for Current $i(t)$ for all $t > 0$



Category : VI : AC Transients.

(a) R-L ckt:



$$i(t) = I_{ss}(t) + I_{tr}(t)$$

The solution is of the form

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t - \phi) + A \cdot e^{-t/\tau}$$

When

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

$$\phi = \tan^{-1} \left[\frac{\omega L}{R} \right]$$

$$\tau = \frac{L}{R}$$

at $t = 0 \longrightarrow i = 0$

$$0 = \frac{V_m}{|Z|} \sin(-\phi) + A$$

$$A = + \frac{V_m}{|Z|} \sin \phi$$

Total Response is;

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t - \phi) + \overbrace{\frac{V_m}{|Z|} \sin \phi}^{\text{dc offset value}} \cdot e^{-t/\tau}, t > 0.$$

Note: 1: If Input Supply is phase shifted to $V(t) = V_m \sin(\omega t + \theta)$ then output Response is also phase shifted.

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \theta - \phi) + A \cdot e^{-t/\tau}, t > 0.$$

at $t = 0 \longrightarrow i = 0$

$$0 = \frac{V_m}{|Z|} \sin(\theta - \phi) + A$$

So, total Response is,

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \theta - \phi) - \frac{V_m}{|Z|} \sin(\theta - \phi) e^{-t/\tau}, t > 0.$$

In the above Current Expression for what circuit Condition the current is free from Transient.

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \theta - \phi) - \underbrace{\frac{V_m}{|Z|} \sin(\theta - \phi) e^{-t/\tau}}_{\text{"0"}}$$

184

$$\sin(\theta - \phi) = 0 \Rightarrow \sin(\theta - \phi) = \sin 0$$

$$\theta - \phi = 0 \Rightarrow \underline{\underline{\theta = \phi}}$$

In the above Current Expression for what circuit Condition the dc offset value is +ve maximum

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \theta - \phi) - \underbrace{\frac{V_m}{|Z|} \sin(\theta - \phi) e^{-t/\tau}}_{\text{positive max}}$$

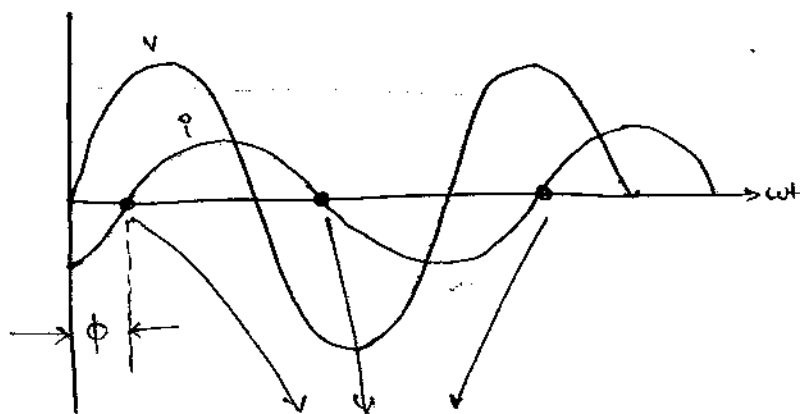
$$\sin(\theta - \phi) = -1 = \sin(-90^\circ)$$

$$\theta - \phi = -90^\circ$$

$$\theta = -90^\circ + \phi$$

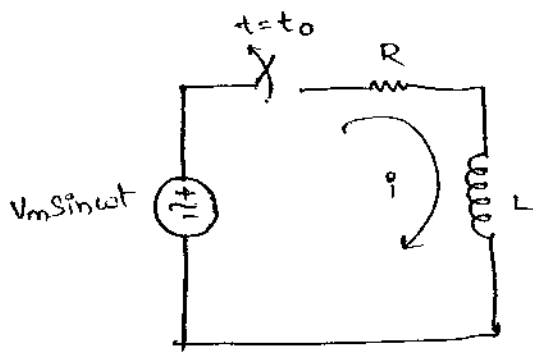
Note 2: If input Excitation is in Cosine terms then also Express the Response in Cosine terms.

Note 3: In general A.C S.S R-L circuit



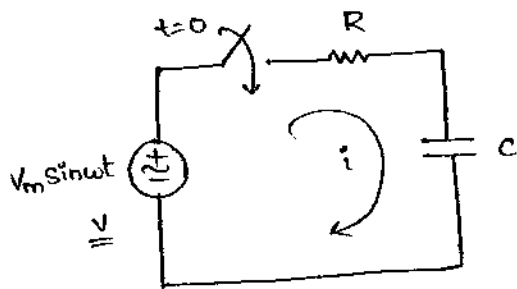
Natural Zero Current Instances in time.

\Rightarrow say if we can interrupt the circuit Exactly at these instances when Current is zero, we can completely avoid Transients



at $\omega t_0 = \phi$ → open the switch to avoid TRANSIENT current
 ↓
 calculate t_0

(b) R-C Circuit:



This solution of the form

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \phi) + A \cdot e^{-t/\tau}$$

When $|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

$$\phi = \tan^{-1} \left[\frac{1}{\omega R C} \right]$$

$$\tau = RC$$

$$i(t) = I_{ss}(t) + I_{tr}(t)$$

at $t=0 \rightarrow i=0$

$$0 = \frac{V_m}{|Z|} \sin(\phi) + A$$

$$A = -\frac{V_m}{|Z|} \sin \phi$$

Total Response is,

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \phi) - \frac{V_m}{|Z|} \sin \phi \cdot e^{-t/\tau}$$

Note 1: If Input Supply is phase shifted to $V(t) = V_m \sin(\omega t + \theta)$, then Output Response is also phase shifted

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \theta + \phi) + A \cdot e^{-t/\tau}, \quad t > 0$$

$$\text{at } t=0 \longrightarrow i=0$$

$$0 = \frac{V_m}{|Z|} \sin(\theta + \phi) + A$$

185

So, Total Response is,

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \theta + \phi) - \frac{V_m}{|Z|} \sin(\theta + \phi) \cdot e^{-t/\tau}$$

Q In the above Current Expression for what circuit Condition the Current in the ckt is free from Transient.

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \theta + \phi) - \underbrace{\frac{V_m}{|Z|} \sin(\theta + \phi)}_{=0} \cdot e^{-t/\tau}$$

$$\sin(\theta + \phi) = 0 = \sin 0^\circ \Rightarrow \theta + \phi = 0^\circ$$

$$\underline{\underline{\theta = -\phi}}$$

Q In the above Current Expression for what circuit Condition the Current dc offset value of Current is +ve maximum.

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \theta + \phi) - \frac{V_m}{|Z|} \sin(\theta + \phi) \cdot e^{-t/\tau}$$

$$\sin(\theta + \phi) = -1$$

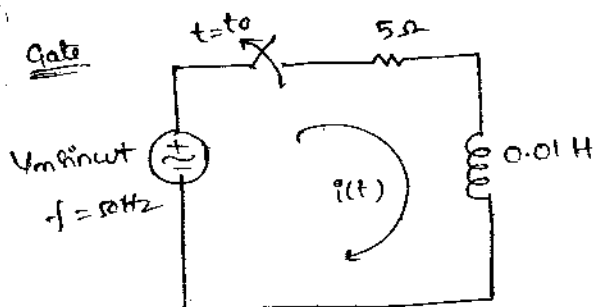
$$= \sin(-90^\circ)$$

$$\theta + \phi = -90^\circ$$

$$\boxed{\theta = -90^\circ - \phi} \quad \phi = \tan^{-1} \left[\frac{1}{\omega R C} \right]$$

Note: 2: If Input Excitation is in Cosine terms then also Express the

Response in Cosine terms



at what Switching Instant $t = t_0$ the Current in the circuit is free from Transients?

$$\omega t_0 = \phi$$

$$2\pi f t_0 = \tan^{-1} \left[\frac{2\pi f L}{R} \right]$$

$$100\pi t_0 = \tan^{-1} \left[\frac{100\pi (0.01)}{5} \right]$$

Radian

$$100\pi t_0 = 0.56$$

$$t_0 = \frac{0.56}{100\pi} = 1.78$$

We can completely avoid Transient

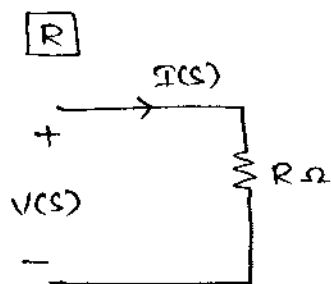
if we open the switch ^{can} Exactly after 1.78 msec, from instant of Zero crossing in Voltage

Category - VII Laplace Transforms & its applications to elec. ckt analysis and the
 title is Circuit Analysis in S-domain (or) Advance Circuit Analysis.

Equivalent ckt Representation of passive elements in

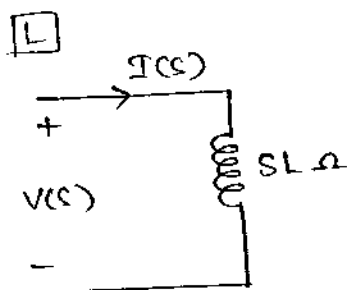
Laplace domain for any type of Input excitation:

Higher order
 non constant
 special function } s-domain
 time domain X



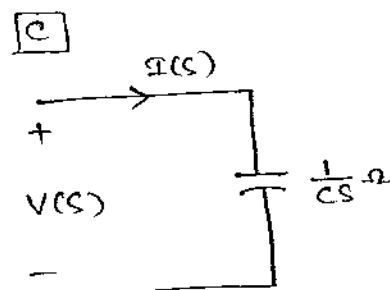
$$V(s) = I(s) \cdot R$$

$$I(s) = \frac{V(s)}{R}$$



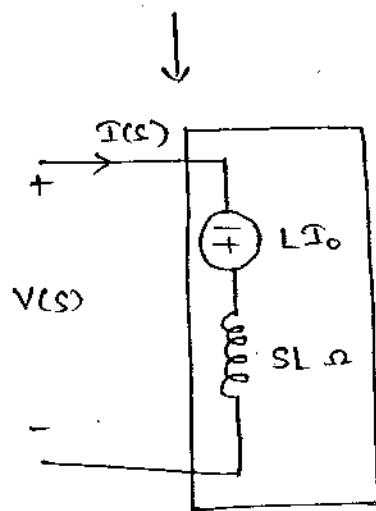
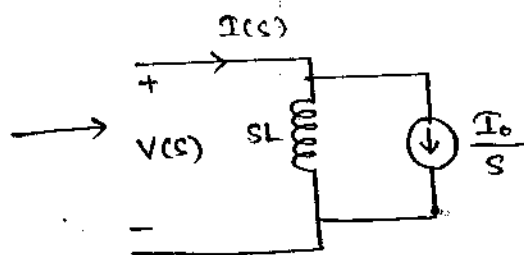
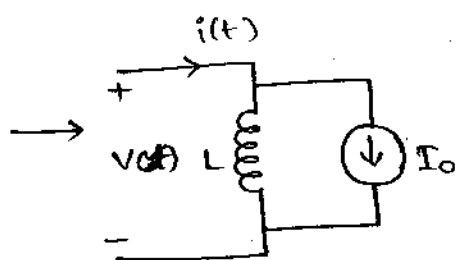
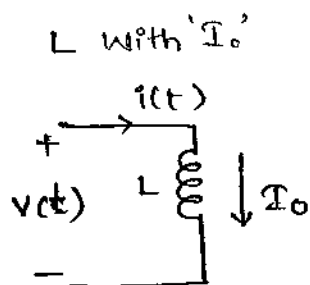
$$V(s) = s \cdot L \cdot I(s)$$

$$I(s) = \frac{V(s)}{s \cdot L}$$



$$V(s) = \frac{I(s)}{s \cdot C}$$

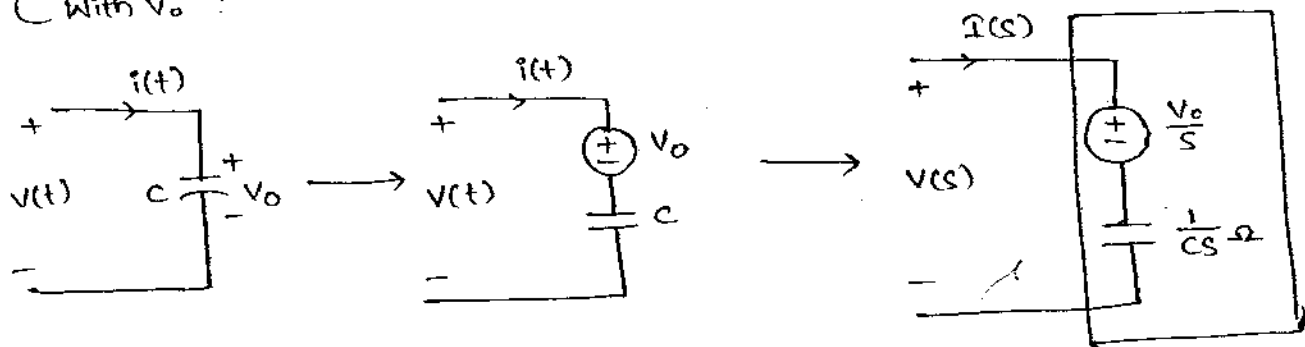
$$I(s) = s \cdot C \cdot V(s)$$



$$V(s) = s \cdot L \cdot I(s) - L I_0$$

$$I(s) = \frac{V(s)}{s \cdot L} + \frac{I_0}{s}$$

C with V_0 :

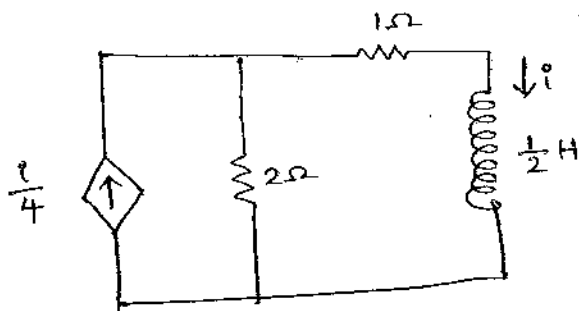


$$V(s) = \frac{I(s)}{Cs} + \frac{V_0}{s}$$

186

$$I(s) = C.s.V(s) - C.V_0$$

Q If $i(0) = 10A$, determine the Complete Expression for current $i(t)$, for all $t > 0$. (a) in time domain ^{analysis} (b) Laplace ^{Using Transform} Analysis

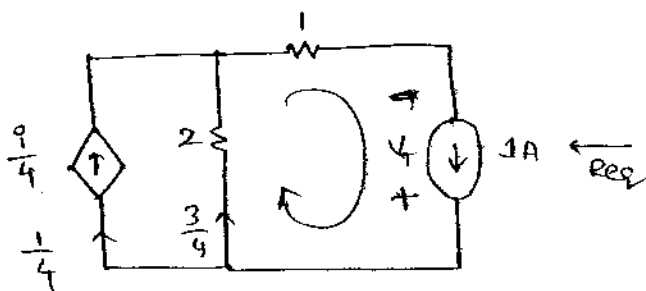


(a) time domain

→ Source free, 1st order

→ S.V → 'i'

$$i(t) = I_0 \cdot e^{-t/\tau} \quad \tau = \frac{L}{R_{eq}} \quad I_0 = 10A$$



$$KVL: \frac{3}{2} + 1 - V_T = 0$$

$$V_T = \frac{5}{2} V$$

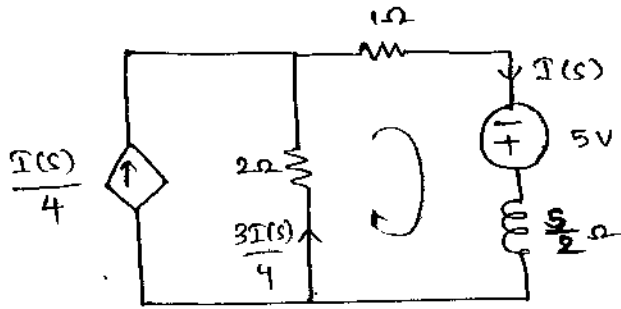
$$R_{eq} = \frac{V_T}{1A} = \frac{5/2}{1} = \frac{5}{2} \Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{1/2}{5/2} = \frac{1}{5} \text{ sec.}$$

$$i(t) = 10 \cdot e^{-5t}, t > 0$$

(b) S-domain

KCL + KVL

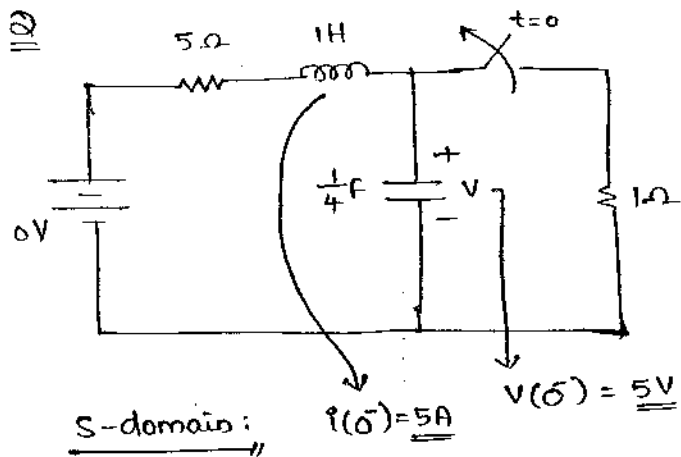


$$\frac{3}{2} I(s) + I(s) - 5 + I(s) \cdot \frac{s}{2} = 0$$

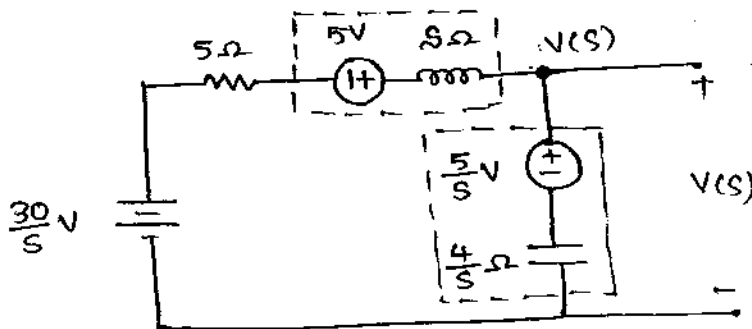
$$I(s) \left\{ \frac{s}{2} + \frac{5}{2} \right\} = 5$$

$$I(s) = \frac{10}{s+5}$$

$$i(t) = \mathcal{L}^{-1}[I(s)] = \underline{\underline{10 \cdot e^{-5t}, t > 0}}$$



Determine the Complete Expression for Voltage $V(t)$ for all $t > 0$ using Laplace Transform method.



Nodal

$$\frac{V(s) - \frac{30}{s} - 5}{(s+5)} + \frac{V(s) - \frac{5}{s}}{\frac{4}{s}} = 0$$

$$V(s) \left[\frac{1}{s+5} + \frac{s}{4} \right] = \frac{30}{s(s+5)} + \frac{5}{(s+5)} + \frac{5}{4}$$

$$V(s) \left[\frac{4 + s^2 + 5s}{4(s+5)} \right] = \frac{120 + 20s + 5s^2 + 25s}{4s(s+5)}$$

$$V(s) = \frac{5s^2 + 45s + 120}{s(s+4)(s+1)} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+1}$$

Partial fraction

$$A = V(s) \cdot s \Big|_{s=0} = 30$$

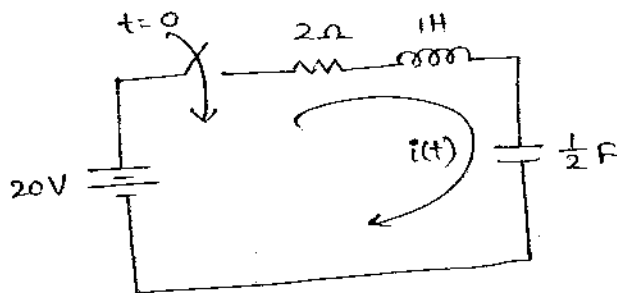
$$B = V(s)(s+4) \Big|_{s=-4} = \frac{20}{12} = \frac{5}{3}$$

$$C = V(s)(s+1) \Big|_{s=-1} = \frac{80}{(-1)(3)} = -\frac{80}{3}$$

$$V(s) = \frac{30}{s} + \frac{5}{3(s+4)} - \frac{80}{3(s+1)}$$

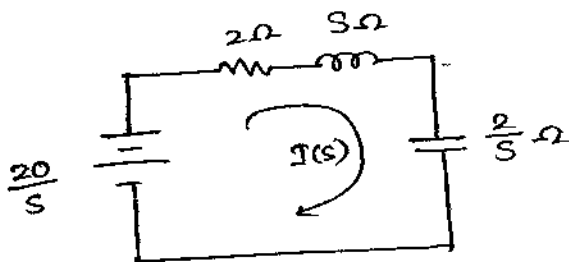
$$V(t) = \mathcal{L}^{-1}(V(s)) = 30 + \frac{5}{3}e^{-4t} - \frac{80}{3}e^{-t}, t > 0.$$

Gate



Determine the Complete Expression for Current $i(t)$ for all $t > 0$.

↳ Initially Relaxed i.e. no Initial Condition



KVL

$$-\frac{20}{s} + I(s) \left[2 + s + \frac{2}{s} \right] = 0$$

$$I(s) \left[\frac{s^2 + 2s + 2}{s} \right] = \frac{20}{s}$$

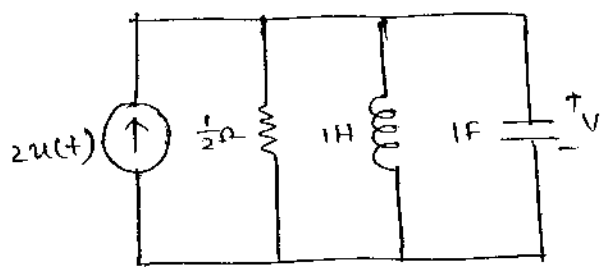
$$I(s) = \frac{20}{s^2 + 2s + 2}$$

$$I(s) = \frac{20}{(s+1)^2 + (1)^2}$$

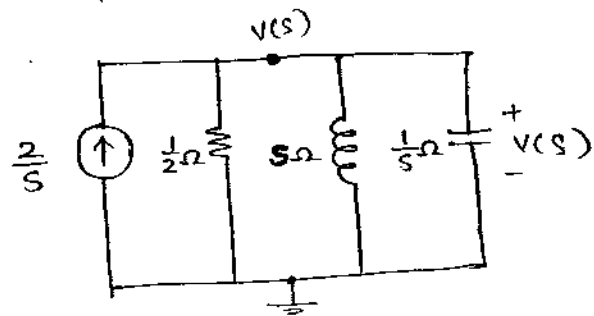
$$i(t) = \mathcal{L}^{-1}(I(s))$$

$$= 20e^{-t} \sin t, t > 0.$$

determine the Complete Expression for Voltage $V(t)$ for all $t > 0$.



→ Initially relaxed i.e. no Int. Cond.



$$-\frac{2}{s} + \frac{V(s)}{1/2} + \frac{V(s)}{s} + \frac{V(s)}{1/s} = 0$$

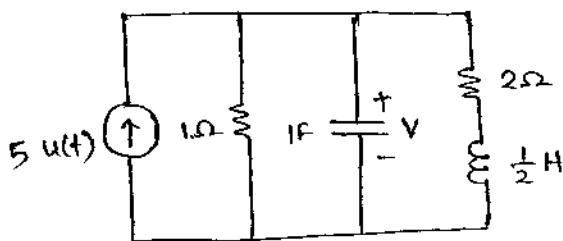
$$V(s) \left[s + 2 + \frac{1}{s} \right] = \frac{2}{s}$$

$$V(s) \left[\frac{s^2 + 2s + 1}{s} \right] = \frac{2}{s}$$

$$V(s) = \frac{2}{(s+1)^2}$$

$$V(t) = \mathcal{L}^{-1}[V(s)] = 2t \cdot e^{-t}, t > 0$$

determine the Complete Expression for voltage $V(t)$ for all $t > 0$.



→ second order, Non Canonical

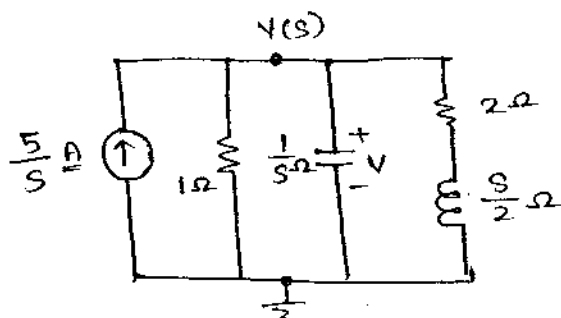
→ Initially relaxed

Nodal

$$-\frac{5}{s} + \frac{V(s)}{1} + \frac{V(s)}{1/s} + \frac{V(s)}{(2 + \frac{s}{2})} = 0$$

$$V(s) \left[1 + s + \frac{2}{s+4} \right] = \frac{5}{s}$$

$$V(s) \left[\frac{s+4+s^2+4s+2}{s+4} \right] = \frac{5}{s}$$



$$V(s) = \frac{5(s+4)}{s(s+3)(s+2)} \xrightarrow{\text{overdamped}} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+2}$$

188

$$A = V(s) \cdot s \Big|_{s=0} = \frac{10}{3}$$

$$B = V(s) \cdot (s+3) \Big|_{s=-3} = \frac{5}{3}$$

$$C = V(s) \cdot (s+2) \Big|_{s=-2} = -5$$

$$V(s) = \frac{10}{3s} + \frac{5}{3(s+3)} - \frac{5}{(s+2)}$$

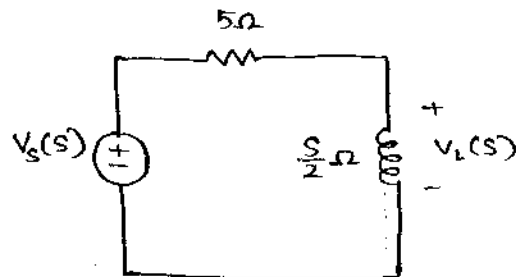
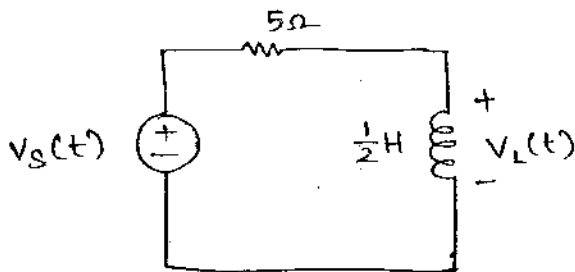
$$V(t) = L^{-1}[V(s)]$$

$$= \frac{10}{3} + \frac{5}{3} \cdot e^{-3t} - 5 \cdot e^{-2t}, t > 0.$$

Q find $V_L(t), t > 0$

if (a) $V_S(t) = 10 \cdot e^{-t} u(t)$

(b) $V_S(t) = 3\delta(t)$



$$V_L(s) = V_S(s) \left[\frac{s/2}{5 + s/2} \right]$$

$$V_L(s) = V_S(s) \left[\frac{s}{s+10} \right]$$

(a) $V_S(t) = 10 \cdot e^{-t} u(t)$

$$V_S(s) = \frac{10}{s+1}$$

$$V_L(s) = \frac{10s}{(s+1)(s+10)} = \frac{A}{s+10} + \frac{B}{s+1}$$

$$A = \frac{100}{9}, B = \frac{-10}{9}$$

$$V_L(s) = \frac{100}{9(s+10)} - \frac{10}{9(s+1)}$$

$$V_L(t) = L^{-1}[V_L(s)] = \frac{100}{9} e^{-10t} - \frac{10}{9} e^{-t}, t > 0$$

$$(b). \quad V_S(t) \longrightarrow 3\delta(t)$$

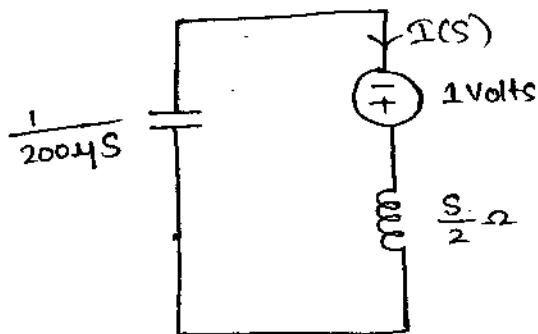
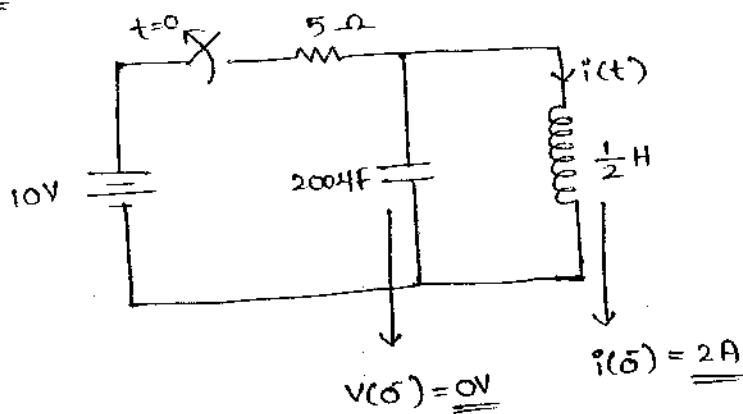
$$V_S(s) \longrightarrow 3$$

$$V_L(s) = \frac{3s}{s+10} = 3 \left[\frac{s+10-10}{s+10} \right]$$

$$V_L(s) = 3 \left[1 - \frac{10}{s+10} \right]$$

$$V_L(t) = \mathcal{L}^{-1}[V_L(s)] = 3 \left[\delta(t) - 10 \cdot e^{-10t} \right], \underline{t > 0}$$

Q determine the Complete Expressions for current $i(t)$ for all $t > 0$.



$$\text{KVL} \quad I(s) \left[\frac{S}{2} + \frac{1}{2004S} \right] = 1$$

$$I(s) \left[9 + \frac{10^6}{100S} \right] = 2$$

$$I(s) \left[\frac{S^2 + 10^4}{S} \right] = 2$$

$$I(s) = \frac{2S}{S^2 + 10^4}$$

$$I(s) = \frac{2S}{S^2 + (100)^2}$$

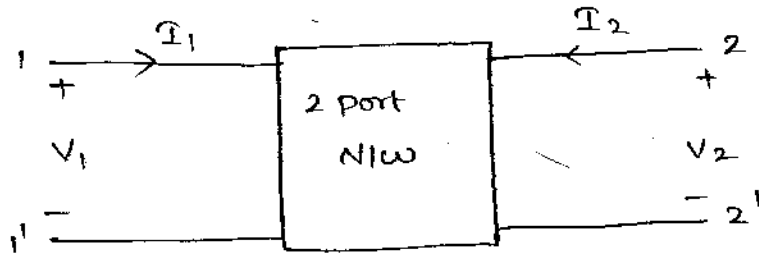
$$i(t) = \mathcal{L}^{-1}[I(s)]$$

$$= 2 \cos 100t, \underline{t > 0}$$

TWO PORT NETWORKS

189

⇒ It consists of two pairs of terminals describing the N/w where the +ve reference Current Voltage polarities & Current directions are shown as below.



\Rightarrow Here, out of 4 parameters, V_1, V_2, I_1 & I_2 two of them are considered as dependent & other two as Independent, so we have 4C_2 i.e. = 6 types of parameters with which we can model this two port Nlws.

port N/Ws.
they are $[Z]$, $[Y]$, $[h]$, $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$, $[g]$, $\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$
 \downarrow \downarrow \downarrow \downarrow \downarrow \times
ZBUS YBUS Transistor Power system

- ⇒ The Nlws inside the ports ~~is~~ Considered a Block Box where it should consist of linear, passive, Bilateral elements.
- ⇒ The Nlw inside ^{also} may have Energy storage elements L & C but their Initial Conditions must be zero.
- ⇒ The Nlw inside ^{may} also have dependent sources but never an Independent Source in it.

Concept of Symmetry in 2-port N/w:

if the ratio of Excitation to Response at Both the ports are Independently Equal w.r.t the given N/w Conditions such as open circuit (or) Short circuit etc. is said to be Symmetrical.

fig (a)

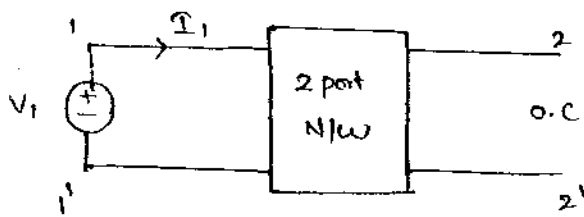
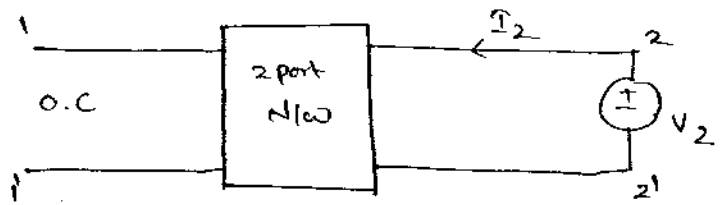


fig (b)



If N/w is Symmetrical, then

$$\frac{V_1}{I_1} = \frac{V_2}{I_2}$$

Note: In the case of Small Independent passive N/w's Symmetry can be Identified as mirror Image property w.r.t ports.

Concept of Reciprocity in 2-port N/w's:

If the 2-port N/w Obey's Reciprocity theorem, However any Linear, passive, Bilateral N/w is always Reciprocal. Since, Reciprocity means passivity.

Z-parameters (Open Circuit Impedance parameters):

Here $V_1, V_2 \rightarrow$ dependent
 $I_1, I_2 \rightarrow$ Independent

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Condition for Symmetry: $Z_{11} = Z_{22}$

Reciprocity: $Z_{12} = Z_{21}$

II. $[Y] \rightarrow$ short circuit admittance parameters.

$I_1, I_2 \rightarrow$ dependent

$V_1, V_2 \rightarrow$ Independent.

190

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Symmetry : $Y_{11} = Y_{22}$

Reciprocity : $Y_{12} = Y_{21}$

III. $[h] \xrightarrow{\text{circuit}} \text{hybrid parameters.}$

$V_1, I_2 \rightarrow$ dependent

$I_1, V_2 \rightarrow$ Independent.

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

Symmetry : $(h_{11} h_{22} - h_{12} h_{21}) = 1$

Reciprocity : $h_{12} = -h_{21}$

Note: $[g] \rightarrow$ Inverse Hybrid circuit parameters.

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$

Symmetry : $(g_{11} g_{22} - g_{12} g_{21}) = 1$.

Reciprocity : $g_{12} = -g_{21}$

Q. iv. $\begin{bmatrix} A & B \\ C & D \end{bmatrix} / [T] \rightarrow$ Transmission circuit parameters.

$V_1, I_1 \rightarrow$ dependent

$V_2, I_2 \rightarrow$ Independent.

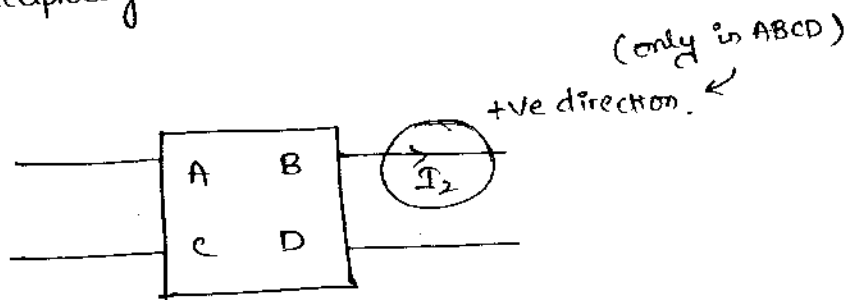
$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$

Symmetry : $A = D$

Reciprocity : $(AD - BC) = 1$

Note :



Note: $\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \rightarrow$ Inverse Transmission circuit parameters.

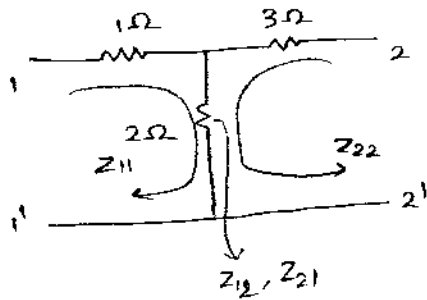
$$V_2 = A'V_1 + B'I_1$$

$$I_2 = C'V_1 + D'I_1$$

Symmetry : $A' = D'$

Reciprocity : $(A'D' - B'C') = 1$.

Q Determine the Z-parameters



$$[Z] = \begin{bmatrix} 1+2 & 2 \\ 2 & 2+3 \end{bmatrix} \\ = \begin{bmatrix} 3 & 2 \\ 2 & 5 \end{bmatrix} \underline{\underline{\Omega}}$$

191

(or) procedure

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

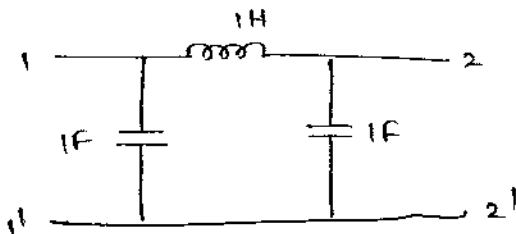
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$Z_{11} \& Z_{21} \longrightarrow I_2 = 0 = \text{O.C}$$

$$V_1 = 3I_1 \longrightarrow Z_{11} = 3\Omega$$

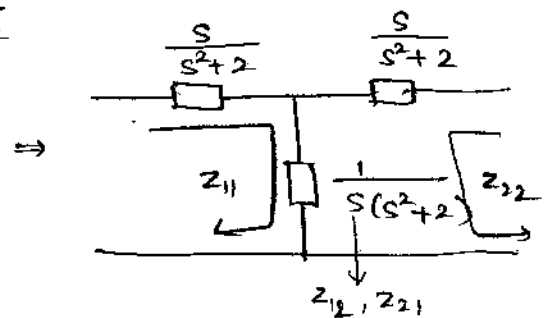
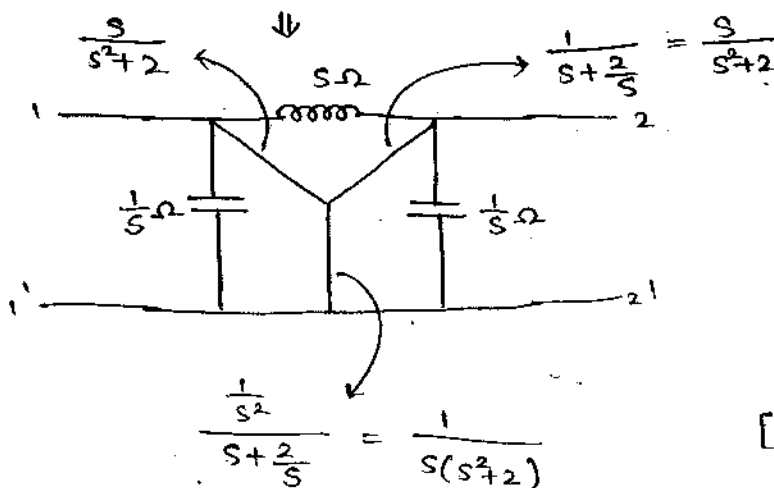
$$V_2 = 2I_1 \longrightarrow Z_{21} = 2\Omega$$

1ES [Z] =



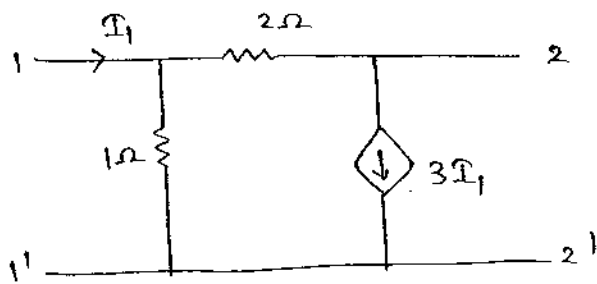
⇒ Convert to s-domain

∴ We want to Convert into parameters of any freq.

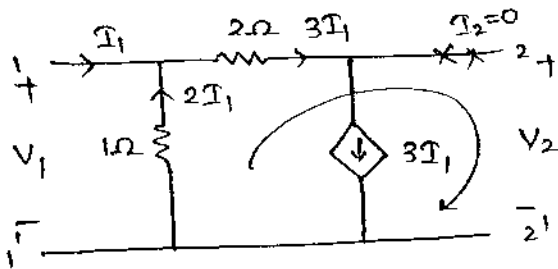


$$[Z] = \begin{bmatrix} \frac{s^2+1}{s(s^2+2)} & \frac{1}{s(s^2+2)} \\ \frac{1}{s(s^2+2)} & \frac{s^2+1}{s(s^2+2)} \end{bmatrix}$$

Gate. $[Z] =$ _____



Z_{11} & $Z_{21} \longrightarrow I_2 = 0$



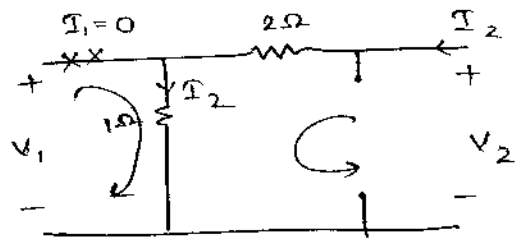
$$V_1 = -2I_1 \longrightarrow Z_{11} = -2\Omega$$

$$2I_1 + 6I_1 + V_2 = 0 \longrightarrow V_2 = -8I_1$$

$$Z_{21} = -8\Omega$$

$$[Z] = \begin{bmatrix} -2 & 1 \\ -8 & 3 \end{bmatrix} \Omega$$

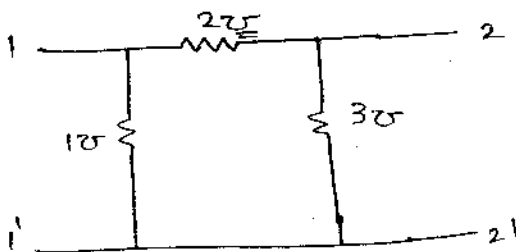
Z_{22} & $Z_{12} \longrightarrow I_1 = 0$



$$V_2 = 3I_2 \longrightarrow Z_{22} = 3\Omega$$

$$V_1 = I_2 \longrightarrow Z_{12} = 1\Omega$$

* $[Y] =$ _____



Y is admittance & Π network, then apply short-circuit

$$[Y] = \begin{bmatrix} 3 & -2 \\ -2 & 5 \end{bmatrix}$$

procedure -

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$Y_{11} \text{ \& } Y_{21} \longrightarrow V_2 = 0$$

$$V_1 = \frac{I_1 + I_2}{1} \longrightarrow \textcircled{1}$$

$$\frac{I_2}{2} + \frac{I_1 + I_2}{1} = 0$$

$$I_1 = -\frac{3}{2} I_2 \longrightarrow \textcircled{2}$$

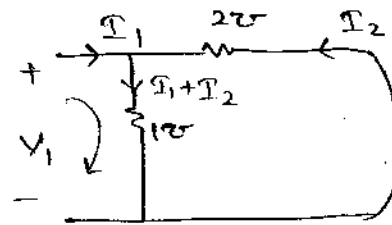
$$\underline{\underline{Y_{11}}} \quad V_1 = I_1 + \left[\frac{-2}{3} \right] I_1$$

$$V_1 = \frac{I_1}{3} \longrightarrow \underline{\underline{Y_{11} = 3\Omega}}$$

$$\underline{\underline{Y_{21}}} \quad V_1 = \frac{-3}{2} I_2 + I_2$$

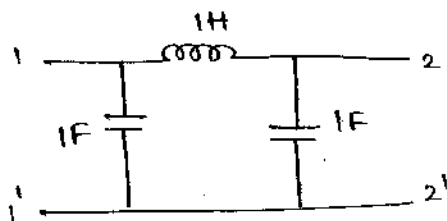
$$V_1 = \frac{-I_2}{2}$$

$$\underline{\underline{Y_{21} = -2\Omega}}$$



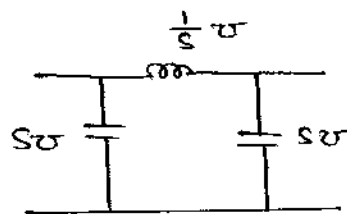
192

Q



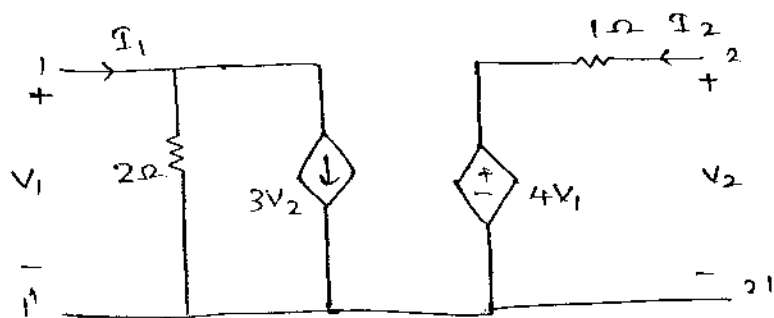
$[Y] =$

→ Convert to s-domain admittances

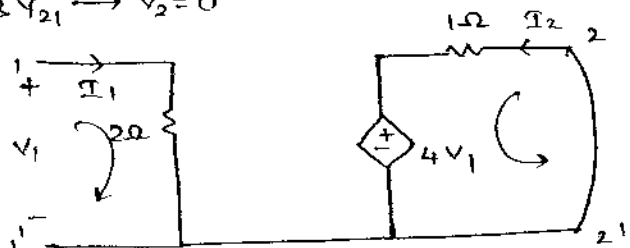


$$\begin{bmatrix} s + \frac{1}{s} & -\frac{1}{s} \\ \frac{1}{s} & s + \frac{1}{s} \end{bmatrix} = \begin{bmatrix} \frac{s^2+1}{s} & -\frac{1}{s} \\ \frac{1}{s} & \frac{s^2+1}{s} \end{bmatrix} \underline{\underline{Y}}$$

gate $[Y] = \underline{\hspace{2cm}}$



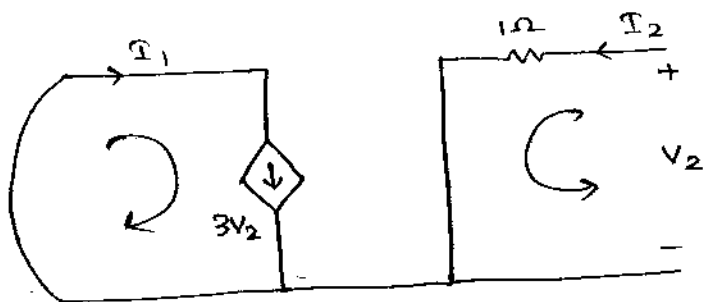
$Y_{11} \text{ \& } Y_{21} \rightarrow V_2 = 0$



$$V_1 = 2I_1 \rightarrow Y_{11} = \frac{1}{2} \text{ S}$$

$$I_2 = -4V_1 \rightarrow Y_{21} = -4 \text{ S}$$

$Y_{22} \text{ \& } Y_{12} \rightarrow V_1 = 0$

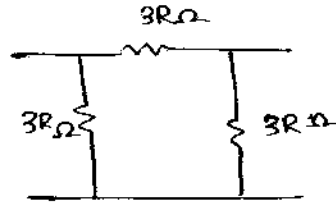
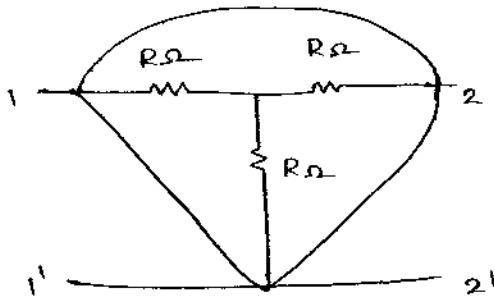


$$V_2 = I_2 \rightarrow Y_{22} = 1 \text{ S}$$

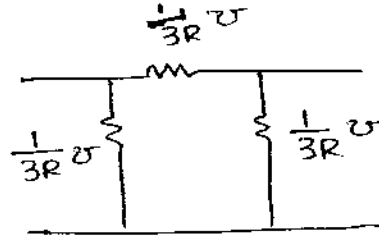
$$I_1 = 3V_2 \rightarrow Y_{12} = 3 \text{ S}$$

$$[Y] = \begin{bmatrix} 1/2 & 3 \\ -4 & 1 \end{bmatrix} \text{ S}$$

Gate $Y_{21} =$ _____



↓



193

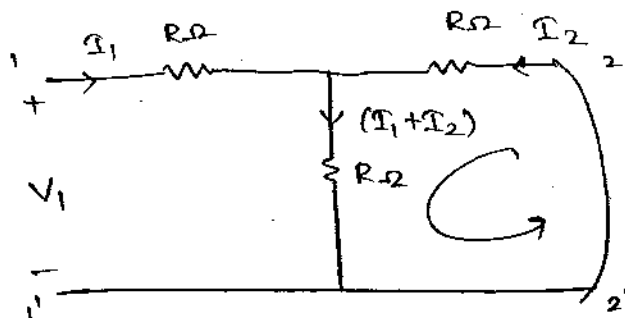
$$\Rightarrow Y = \left[\begin{array}{cc} \frac{1}{3R} + \frac{1}{3R} & -\frac{1}{3R} \\ -\frac{1}{3R} & \frac{1}{3R} + \frac{1}{3R} \end{array} \right]$$

↓

$$Y_{21} = -\frac{1}{3R}$$

for the above N/w find h_{21}

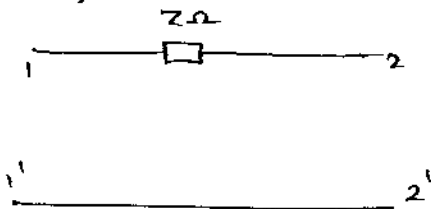
$$h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0}$$



$$R I_2 + R (I_1 + I_2) = 0$$

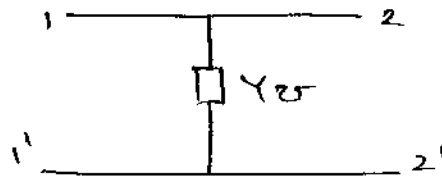
$$I_1 = -2I_2 \rightarrow h_{21} = \frac{I_2}{I_1} = -\frac{1}{2}$$

$[T] =$ _____



$$[T] = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

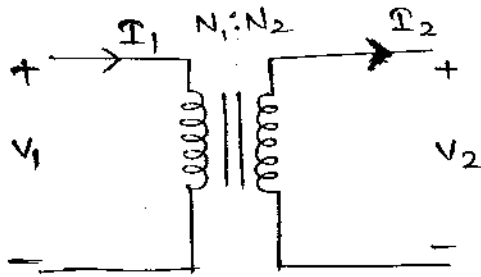
$[T] =$ _____



$$[T] = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

Gate $[T] =$ _____

Ideal T/F is Reciprocal



$$K = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

$$V_1 = \left[\frac{N_1}{N_2} \right] V_2 + [0] I_2$$

$$I_1 = [0] V_2 + \left[\frac{N_2}{N_1} \right] I_2$$

$$[T] = \begin{bmatrix} (N_1/N_2) & 0 \\ 0 & (N_2/N_1) \end{bmatrix}$$

↓

$$AD - BC = 1$$

↓

Ideal Transformer is Reciprocal

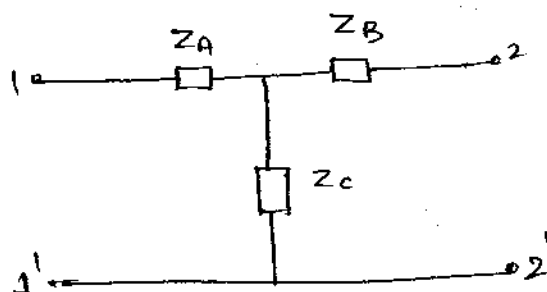
Note: two port N/w allows us to model our Complex practical N/w & systems into their Simple mathematical Subcircuit models. Such as T, TT, Lattice, T-T, Bridged-T, Ladder N/ws... etc.

Using such Simplified mathematical Equivalents of Original N/w we can still determine the performance of original Network in terms of these two port N/w parameters.

Ex: Modelling our medium distance Tr. line N/w as T (or) TT model, using A-BCD parameters from which we can determine the Regulation, power factor, Efficiency of original Tr. line N/w

⇒ 1. T Eq. Representation of a Two port Reciprocal N/w in terms of

Z-parameters:



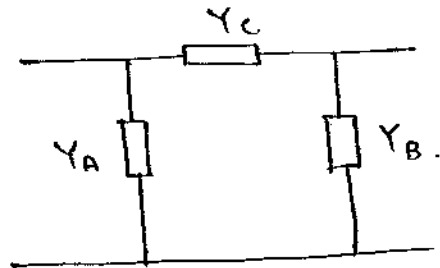
$$Z_A = (Z_{11} - Z_{12})$$

194

$$Z_B = (Z_{22} - Z_{21})$$

$$Z_C = Z_{12} = Z_{21}$$

2. Π Eq. Representation of a two port Reciprocal N/w in terms of Y-parameters

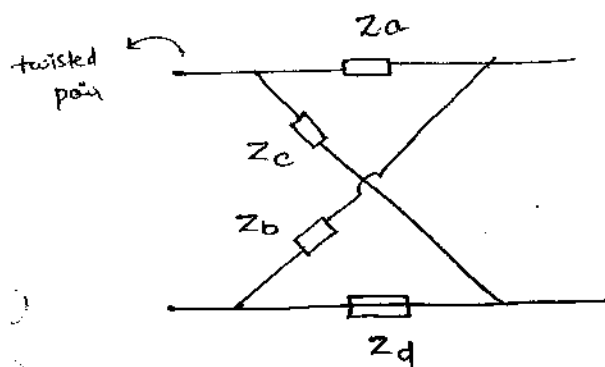


$$Y_A = Y_{11} + Y_{12}$$

$$Y_B = Y_{22} + Y_{21}$$

$$Y_C = -Y_{12} = -Y_{21}$$

3. Lattice N/w:



if $(Z_a = Z_d) \text{ \& } (Z_b = Z_c)$ } Symmetrical & Balanced Lattices

if $(Z_a \neq Z_d) \text{ or } (Z_b \neq Z_c)$ } Asymmetrical & unbalanced lattice

Representation of Lattice N/w in terms of Z-parameters provided it is

Symmetrical & Balanced:

$$Z_{11} = Z_{22} = \frac{Z_b + Z_a}{2}$$

$$Z_{12} = Z_{21} = \frac{Z_b - Z_a}{2}$$

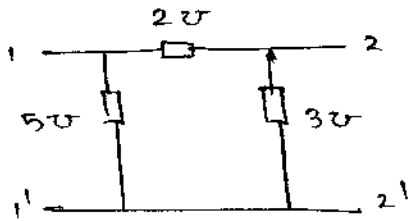
$$\left. \begin{array}{l} Z_{11} + Z_{12} = Z_b \\ Z_{11} - Z_{12} = Z_a \end{array} \right\}$$

Gate The port currents of a 2 port N/w are given by $I_1 = 7V_1 - 2V_2$
 $I_2 = -2V_1 + 5V_2$

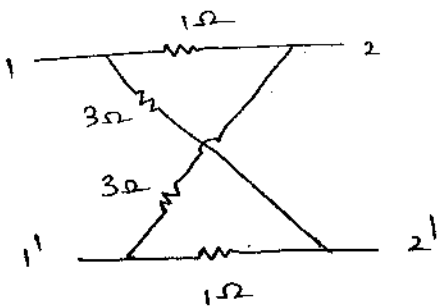
then its Eq. π model is _____

$$[Y] = \begin{bmatrix} 7 & -2 \\ -2 & 5 \end{bmatrix} \text{ S}$$

$\therefore Y_{12} = Y_{21} \Rightarrow$ N/w is Reciprocal
 so, π model is possible



IES $[Z] =$ _____



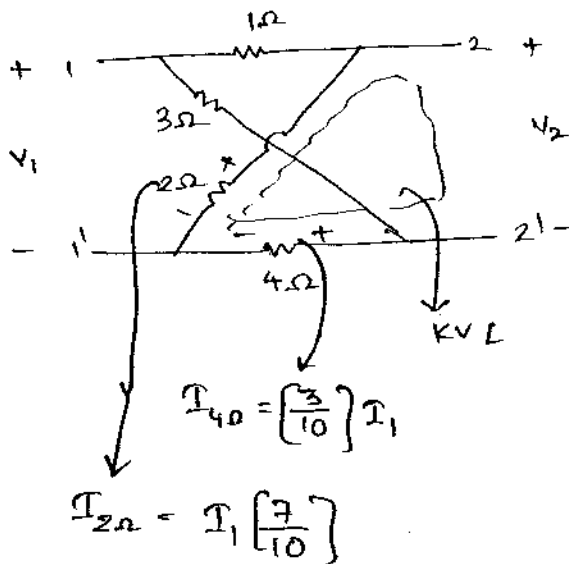
\therefore it is symmetrical & balanced lattice. so we can determine $[Z]$ parameters directly

$$Z_{11} = Z_{22} = \frac{3+1}{2} = 2 \Omega$$

$$Z_{12} = Z_{21} = \frac{3-1}{2} = 1 \Omega$$

$$[Z] = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \Omega$$

Gate find $[Z] =$ _____



$$Z_{11} = 3 \parallel 7 = 2.1 \Omega$$

$$Z_{22} = 6 \parallel 4 = 2.4 \Omega$$

$$Z_{11} = Z_{21} \Rightarrow [\because \text{passive N/w is Reciprocal}]$$

find $Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}$

KVL + C. Div. Rule

$$+V_2 + 4 \cdot \frac{6}{10} I_1 - \frac{7}{10} I_1 \cdot 2 = 0$$

$$V_2 = \frac{1}{5} I_1$$

$$Z_{21} = 0.2 \Omega = Z_{12}$$

$$[Z] = \begin{bmatrix} 2.1 & 0.2 \\ 0.2 & 2.4 \end{bmatrix} \Omega$$

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}$$

$$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

195

Note:

$$[Z] = [Y]^{-1} \text{ \& } [Y] = [Z]^{-1}$$

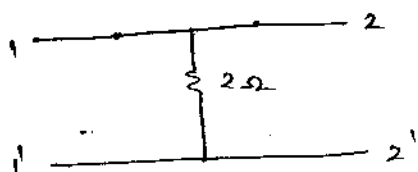
$$[h] = [g]^{-1} \text{ \& } [g] = [h]^{-1}$$

$$[T] = [T']^{-1} \text{ \& } [T'] = [T]^{-1}$$

→ Invertible only if N/w is Reciprocal.

Gate

find $[Y] =$



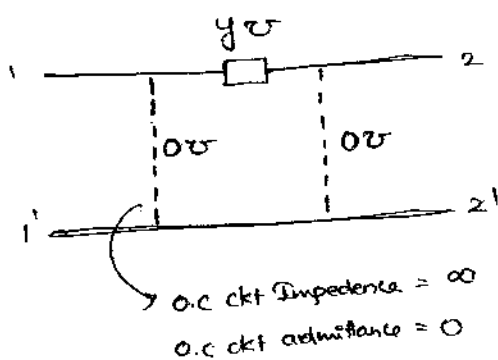
$$[Z] = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$\det [Z] = 0 \rightarrow$ singular Matrix

so, $[Y]$ does not exist.

IES

find $[Y]$ & then $[Z]$



$$[Y] = \begin{bmatrix} y & -y \\ -y & y \end{bmatrix}$$

$$\det [Y] = 0$$

→ singular matrix

so, $[Z]$ does not exist.

InterConversion b/w two port N/w parameters:

Ex: determine h-parameters in terms of Y-parameters.

Y

$$\begin{cases} I_1 = Y_{11}V_1 + Y_{12}V_2 \\ I_2 = Y_{21}V_1 + Y_{22}V_2 \end{cases}$$

$$\rightarrow \underline{V_2 = 0} \begin{cases} I_1 = Y_{11}V_1 \\ I_2 = Y_{21}V_1 \end{cases}$$

h

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$h_{11} \text{ \& } h_{21} \rightarrow \underline{V_2 = 0}$$

$$h_{11} = \frac{V_1}{I_1} = \frac{1}{Y_{11}}$$

$$h_{21} = \frac{I_2}{I_1} = \frac{Y_{21}}{Y_{11}}$$

$$h_{22} \text{ \& } h_{12} \rightarrow \underline{I_1 = 0}$$

$$h_{22} = \frac{I_2}{V_2} = \frac{\Delta Y}{Y_{11}}$$

$$I_2 = Y_{21} \left[\frac{-Y_{12}}{Y_{11}} \right] V_2 + Y_{22}V_2$$

$$I_2 = V_2 \left[\frac{Y_{11}Y_{22} - Y_{21}Y_{12}}{Y_{11}} \right]$$

$$I_2 = \left[\frac{\Delta Y}{Y_{11}} \right] \cdot V_2$$

$$h_{12} = \frac{V_1}{V_2} = \frac{-Y_{12}}{Y_{11}}$$

$$[h] = \begin{bmatrix} \frac{1}{Y_{11}} & \frac{-Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{bmatrix}$$

196
ies find Z_{12} & Z_{21} in terms of $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ In ABCD I_2 direction is opposite to the other parameters

\therefore take +ve direction of I_2 by -ve sign

$$Z_{12} = \frac{V_1}{I_2} \Bigg|_{\underline{I_1=0}}$$

$$Z_{21} = \frac{V_2}{I_1} \Bigg|_{\underline{I_2=0}}$$

$$\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}$$

$$\textcircled{Z_{12}} \Rightarrow \underline{I_1=0} \quad CV_2 = DI_2$$

$$\textcircled{Z_{21}} \Rightarrow \underline{I_2=0} \quad I_1 = CV_2$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{1}{C}$$

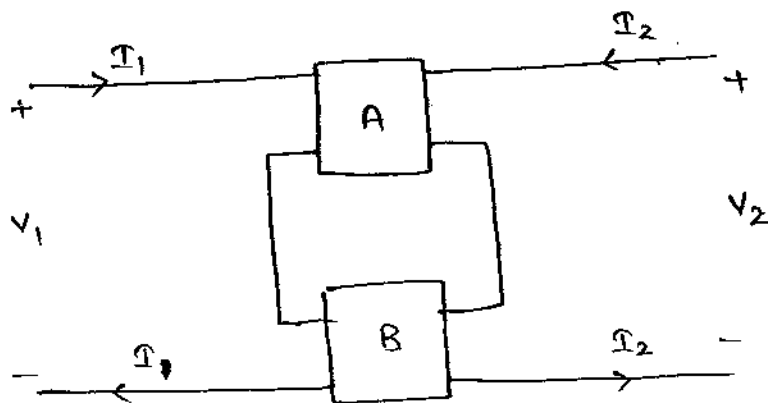
$$V_1 = A \left[\frac{D}{C} \right] I_2 - BI_2$$

$$V_1 = \left[\frac{\Delta T}{C} \right] I_2$$

$$Z_{12} = \frac{\Delta T}{C}$$

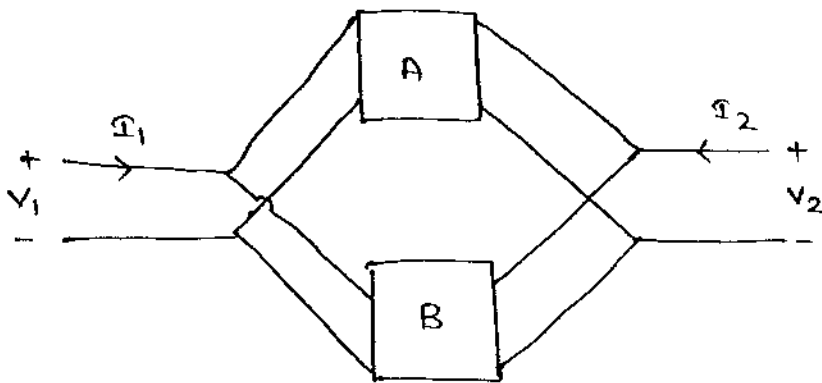
Inter Connection b/w two two-port Nlws :

I. Series - Series



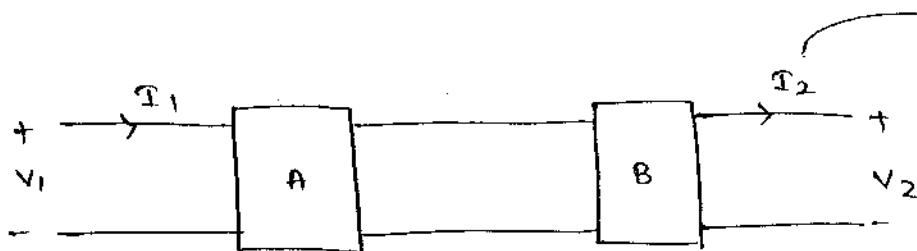
$$[Z_T] = [Z_A] + [Z_B]$$

II. parallel-parallel.



$$[Y_T] = [Y_A] + [Y_B]$$

III. Cascaded/chain/Tandem:

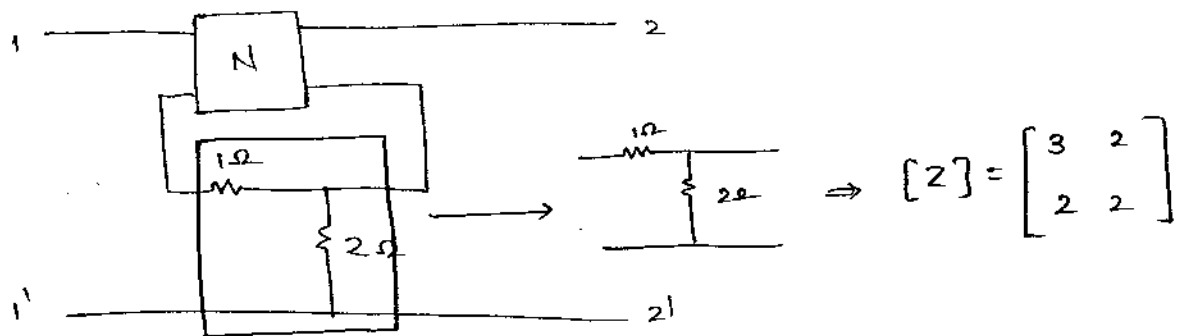


∵ it is a P.S N/w
P.S N/w is a cascaded sys.
Distribution sys is a parallel sys.

$$[T_T] = [T_A] \cdot [T_B]$$

↑
multiply.

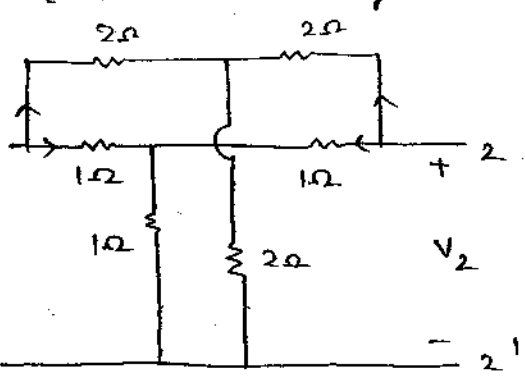
Gate If $[Z_n] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is total $[Z]$ of entire N/w is

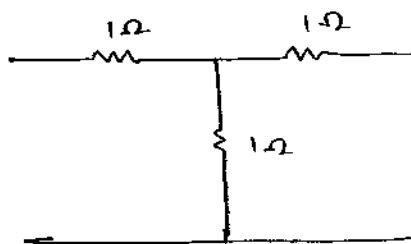


$$[Z]_{\text{Total}} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$$

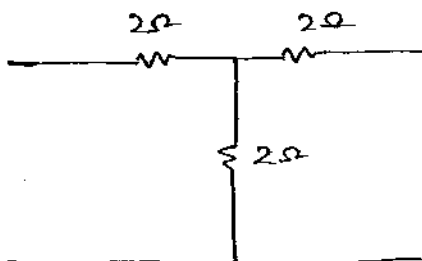
197

$$= \begin{bmatrix} 4 & 4 \\ 5 & 6 \end{bmatrix} \quad \text{Ans}$$

165(6) $[Y] =$  *two T networks connected in parallel.*



$$\rightarrow [Z_A] = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \rightarrow [Y_A] = [Z_A]^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} \quad \text{Ans}$$

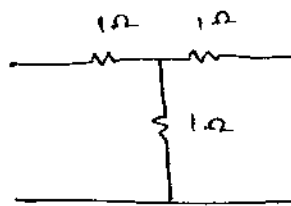
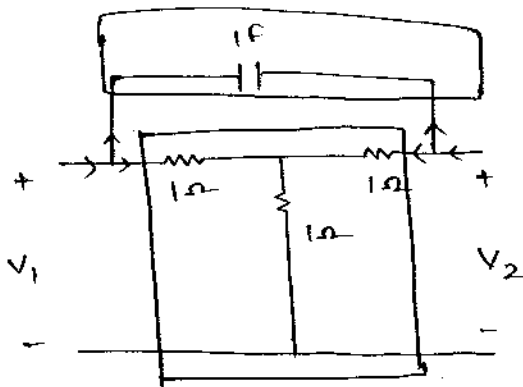


$$\rightarrow [Z_B] = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} = [Y_B] = [Z_B]^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix} \quad \text{Ans}$$

$$[Y_T] = [Y_A] + [Y_B] = \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix} \Omega$$

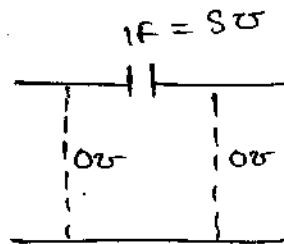
Gate

$$[Y] = \underline{\hspace{2cm}}$$



$$\downarrow$$

$$[Y_A] = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \Omega$$

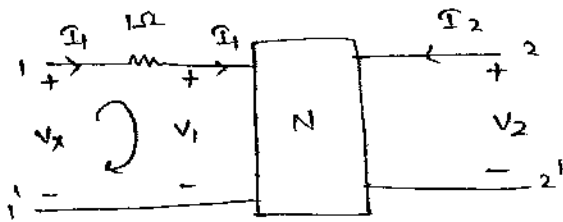


$$\Rightarrow [Y_B] = \begin{bmatrix} s & -s \\ -s & s \end{bmatrix} \Omega$$

$$[Y]_{\text{total}} = [Y_A] + [Y_B] = \begin{bmatrix} s + \frac{2}{3} & -(s + \frac{1}{3}) \\ -(s + \frac{1}{3}) & s + \frac{2}{3} \end{bmatrix}$$

Gate If $[Z_N] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$, then total $[Z]$ of entire N/w is

198



$$V_1 = Z_{11}I_1 + Z_{12}I_2 \rightarrow (1)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \rightarrow (2)$$

KVL

$$-V_x + I_1(1) + V_1 = 0$$

$$V_x = V_1 + I_1$$

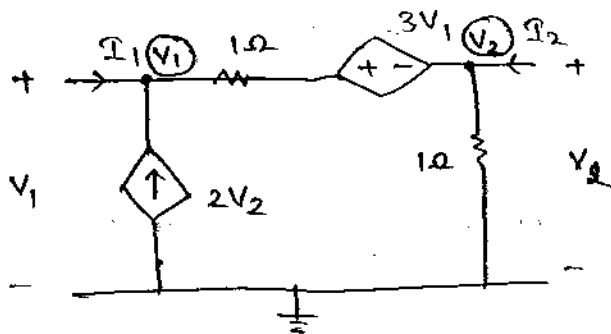
$$V_x = Z_{11}I_1 + Z_{12}I_2 + I_1$$

$$V_x = (Z_{11} + 1)I_1 + Z_{12}I_2 \rightarrow (3)$$

from (3) & (2)

$$[Z] = \begin{bmatrix} (Z_{11} + 1) & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

Gate $[Y] =$ _____



KCL \rightarrow Nodal

$$-I_1 - 2V_2 + \frac{(V_1 - V_2 - 3V_1)}{1} = 0$$

$$I_1 = -2V_1 - 3V_2 \rightarrow (1)$$

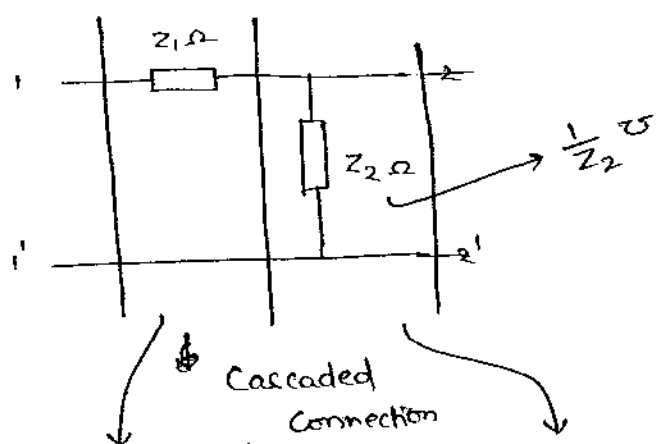
$$-I_2 + \frac{V_2}{1} + \frac{(V_2 - V_1 + 3V_1)}{1} = 0$$

$$I_2 = 2V_1 + 2V_2 \rightarrow (2)$$

from (1) & (2)

$$[Y] = \begin{bmatrix} -2 & -3 \\ 2 & 2 \end{bmatrix} \text{ S}$$

Gate $[T] = \underline{\hspace{2cm}}$

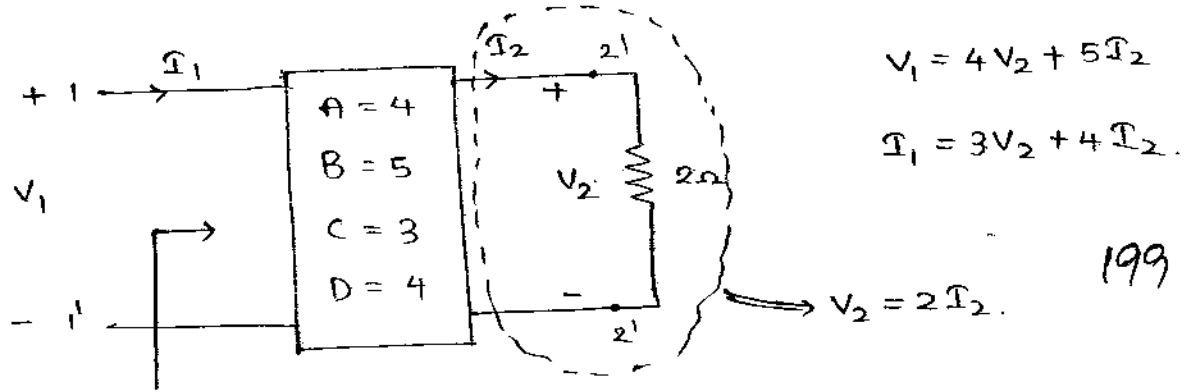


$$\begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix}$$

$$[T]_{\text{Total}} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix} = \begin{bmatrix} 1 + \frac{Z_1}{Z_2} & Z_1 \\ \frac{1}{Z_2} & 1 \end{bmatrix}$$

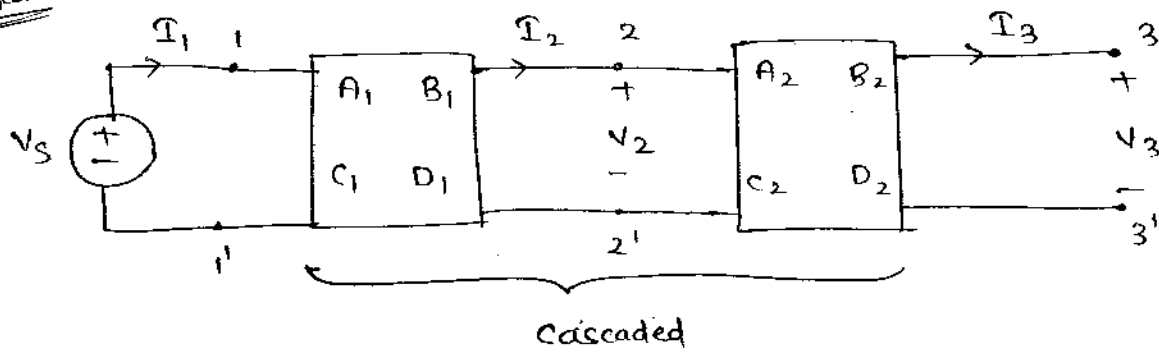
Gate find total input Impedance of A/w.



$$Z_{in} = \frac{V_1}{I_1} = \frac{4V_2 + 5I_2}{3V_2 + 4I_2} = \frac{4(2I_2) + 5I_2}{3(2I_2) + 4I_2} = \frac{13I_2}{10I_2}$$

$$= \underline{\underline{\frac{13}{10} \Omega}}$$

Gate Determine the Thevenin's Eq. of N/w at port 3



$$[T_1][T_2] = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

$$[T]_{\text{total}} = \begin{bmatrix} A_1 A_2 + B_1 C_2 & A_1 B_2 + B_1 D_2 \\ C_1 A_2 + D_1 C_2 & C_1 B_2 + D_1 D_2 \end{bmatrix}$$

$$\therefore V_S = (A_1 A_2 + B_1 C_2) V_3 + (A_1 B_2 + B_1 D_2) I_3 \rightarrow (1)$$

$$I_1 = (C_1 A_2 + D_1 C_2) V_3 + (C_1 B_2 + D_1 D_2) I_3 \rightarrow (2)$$

S-I V_{th} of port 3

$$\rightarrow I_3 = 0 \quad \text{in Eq (1)}$$

& let $V_3 = V_{th}$

So.

$$V_{th} = \left[\frac{V_S}{A_1 A_2 + B_1 C_2} \right] \underline{V}$$

S-II $Z_{th} \Rightarrow$ (by ohm's law)

$$\rightarrow V_S = 0 \text{ (o.c voltage source)}$$

$$Z_{th} = \frac{V_S}{I_3} \Rightarrow \text{But } I_3 = -ve \quad \therefore \text{if we give excitation at port 3 the current enters according to passive notation but we are considering as leaving} \therefore \text{apply -ve sign.}$$

put $V_s = 0$ & $I_3 = (-ve)$ in Eq (1)

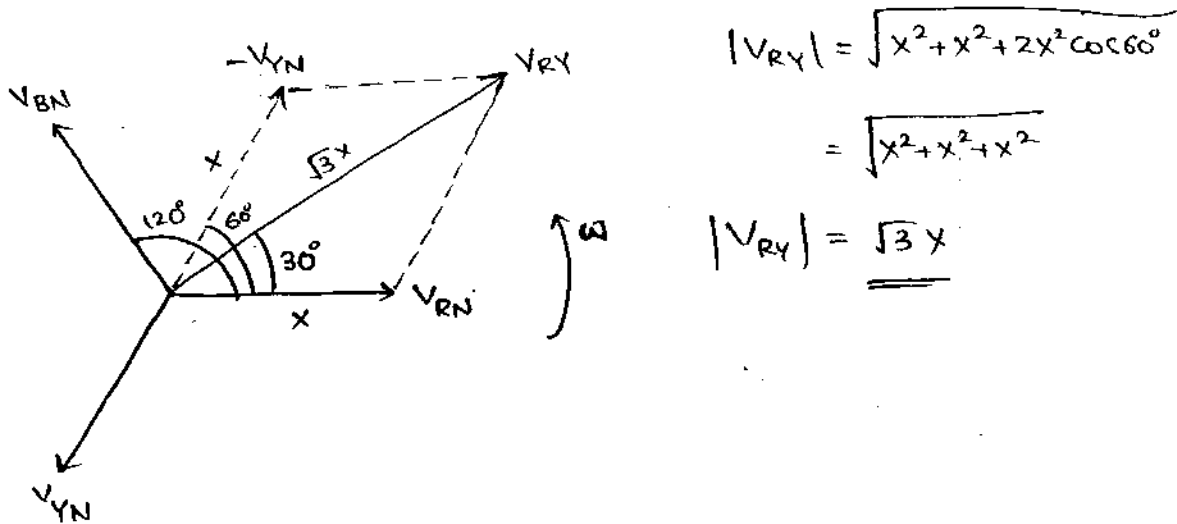
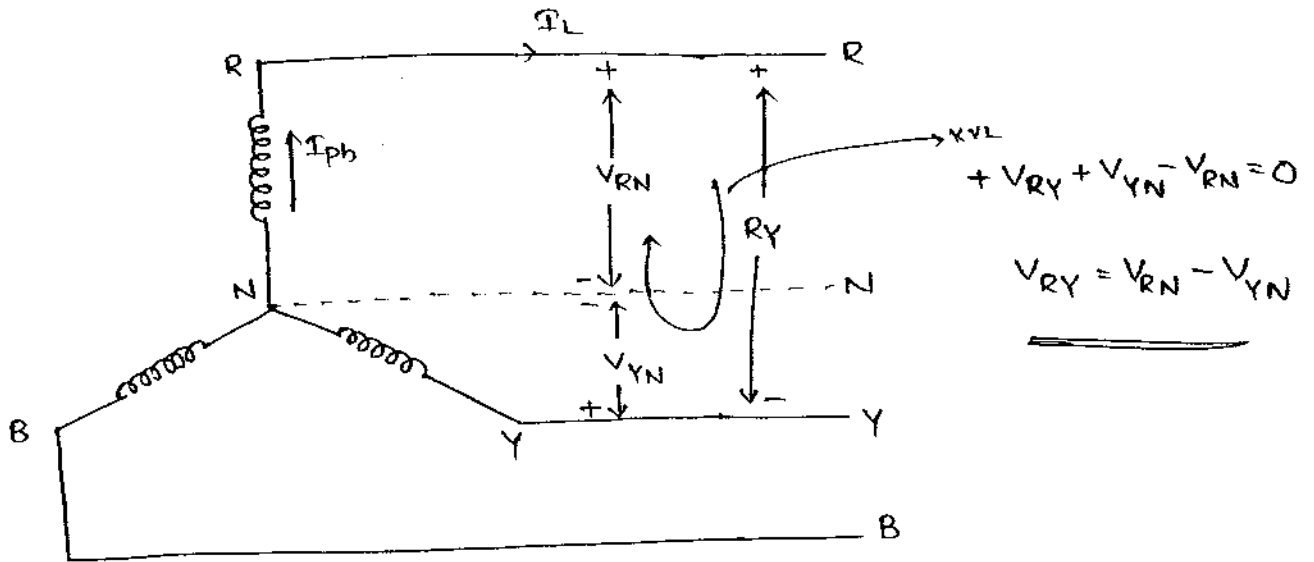
$$(A_1 B_2 + B_1 D_2) I_3 = (A_1 A_2 + B_1 C_2) V_3$$

250

$$Z_{th} = \frac{V_3}{I_3} = \left[\frac{A_1 B_2 + B_1 D_2}{A_1 A_2 + B_1 C_2} \right] \Omega$$

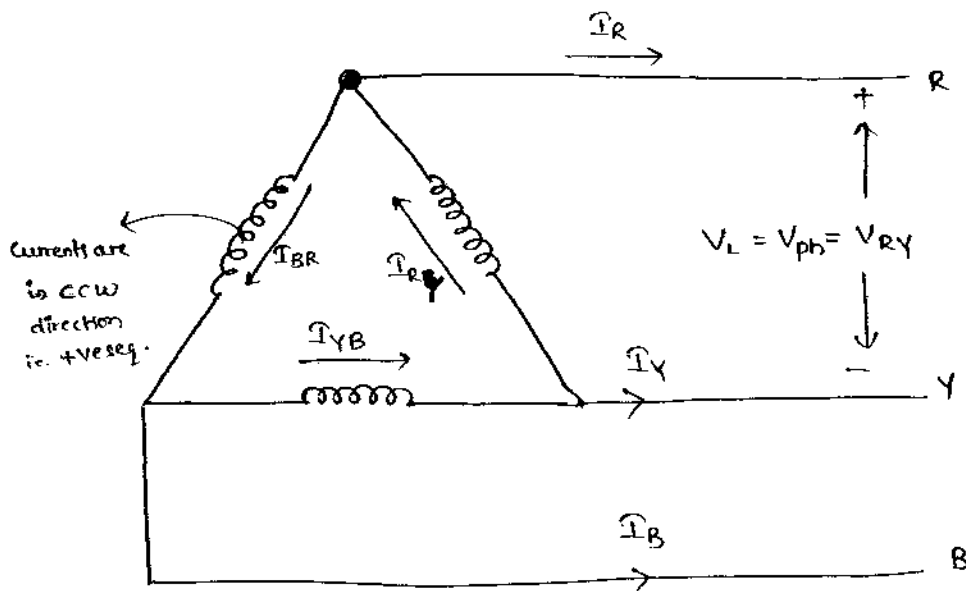
3- ϕ Circuits

(a) star Connection.



In a Balanced 3ϕ Star Connected Supply line Voltage is $\sqrt{3}$ times phase Voltage in Magnitude & line Voltage leads phase Voltage by 30° in a positive R-Y-B phase Sequence system

(b) Delta Connection:

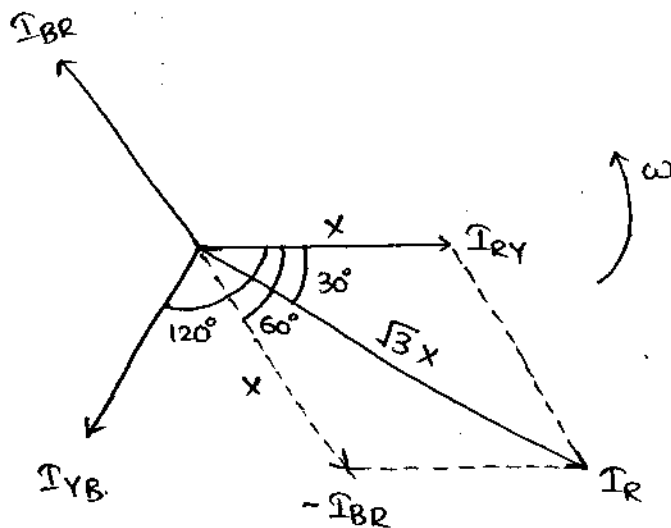


201

KCL

$$\bar{I}_{RY} = \bar{I}_R + \bar{I}_{BR}$$

$$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR}$$



$$|I_R| = \sqrt{X^2 + X^2 + 2X^2 \cos 60^\circ}$$

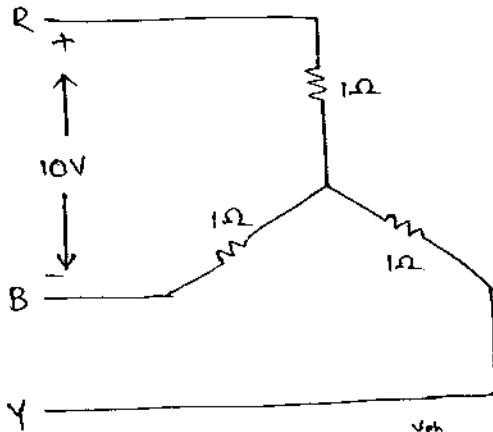
$$= \sqrt{X^2 + X^2 + X^2}$$

$$|I_R| = \sqrt{3} X$$

In a balanced 3 ϕ delta Connected Supply, line Current is $\sqrt{3}$ times phase Current in Magnitude. and line Current lags phase Current by 30° in a positive R-Y-B Sequence.

Q1 Calculate the Total power absorbed in the 3 ϕ N/w shown

(1)

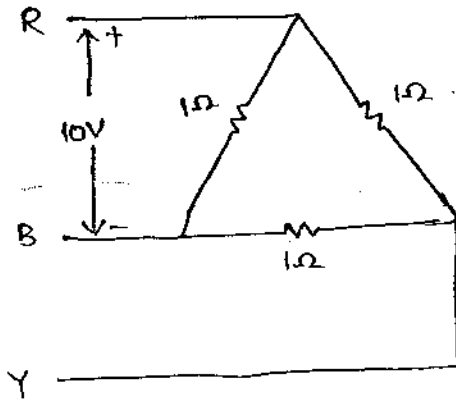


$$P_T = 3 \cdot V_{ph} \cdot I_{ph} \cos \phi$$

$$= 3 \left[\frac{10}{\sqrt{3}} \right] \cdot \left[\frac{10}{\sqrt{3} \times 1} \right] \times 1$$

$$= \underline{\underline{100 \text{ W}}}$$

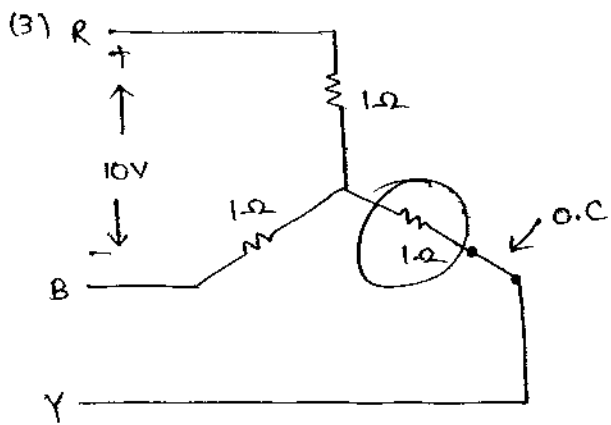
(2)



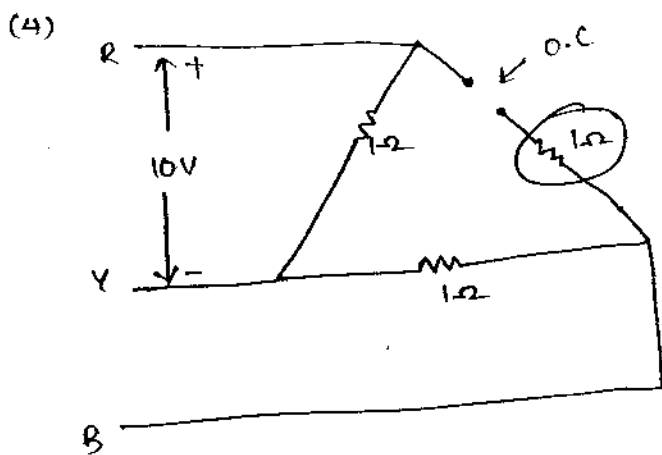
$$P_T = 3 [10] \left[\frac{10}{1} \right] \cdot (1)$$

$$= \underline{\underline{300 \text{ W}}}$$

202



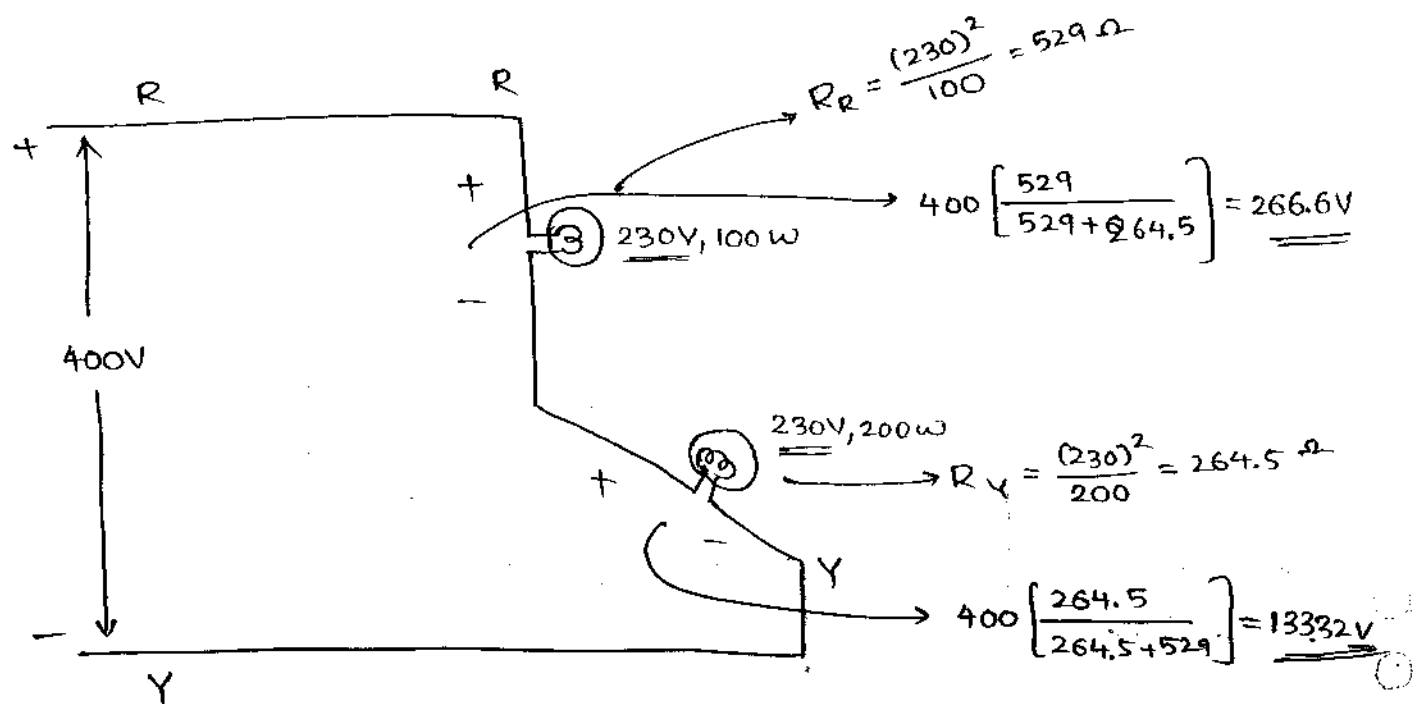
$$P_T = \frac{V^2}{R} = \frac{10^2}{1+1} = 50W$$



$$P_T = \frac{10^2}{1} + \frac{10^2}{1} = 200W$$

Q.10 two Incandescent bulbs of rating 230V, 100W ; 230V, 200W are connected to a balanced 3 ϕ 400V supply in R-N-Y phases respectively. If Neutral wire Breaks Suddenly

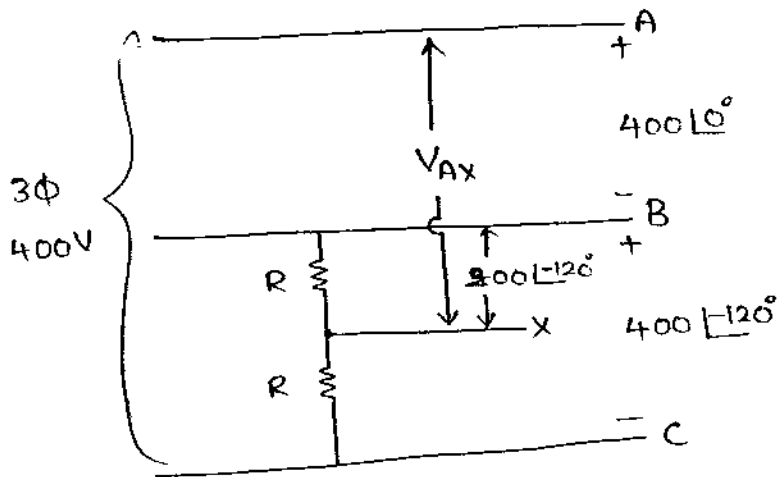
- (a) Bulb in R-ph will fuse off
 (b) Bulb in Y-ph will fuse off
 (c) Both Bulbs will fuse off
 (d) Both Bulbs are safe.



once Neutral Wire Breaks over Voltage appears in R-ph & under Voltage appears in Y-ph. therefore R-ph Bulb will Blow off.

In a balanced 3 ϕ 400V supply lines 2 Equal Resistors are Connected in Series between Line-B & Line-C. determine the magnitude of Voltage B/w Line-A & Junction of these two Resistors.

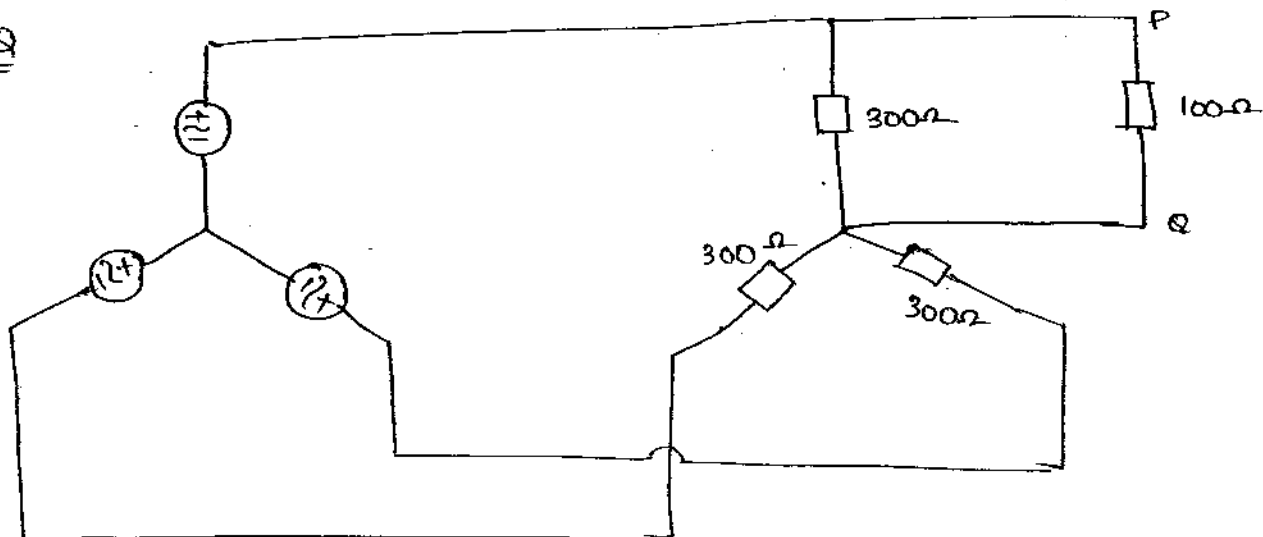
203



$$|V_{Ax}| = 400\angle 0^\circ + 200\angle -120^\circ$$

$$|V_{Ax}| = \frac{200\sqrt{3}}{1} \text{ V} = 346.4 \text{ V}$$

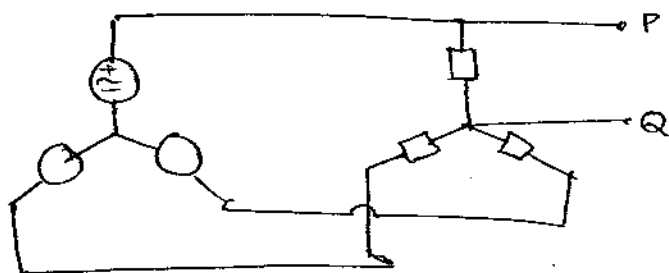
Determine the Voltage across 100 Ω Resistor by using Thevenin's theorem



3 ϕ balanced 400V supply

$\hookrightarrow \therefore$ No Neutral Current i.e. $I_N = 0$

\therefore Voltage across PQ $\Rightarrow V_{Th} = V_{Ph}$

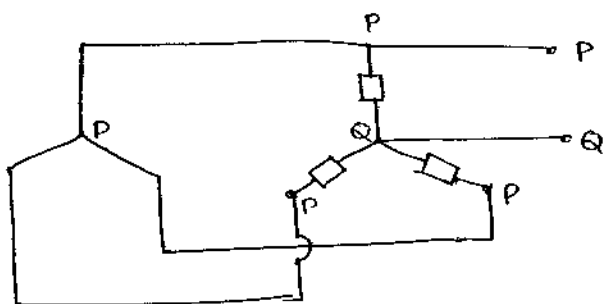


Step-I

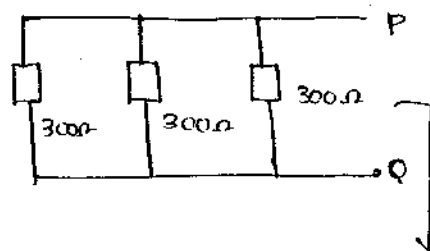
$$V_{Th} = V_{ph}$$

$$V_{Th} = \frac{400}{\sqrt{3}} \angle 0^\circ$$

Step-II

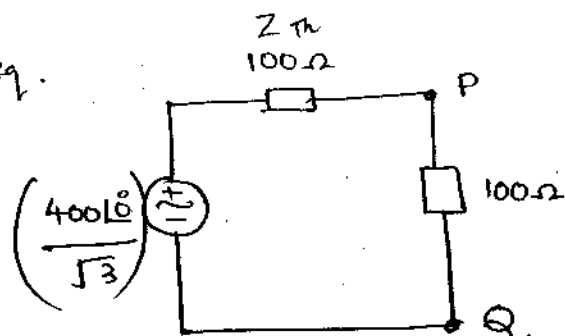


\Rightarrow



$$Z_{Th} = \underline{\underline{100 \Omega}}$$

Ther. eq.



$$V_{PQ} = \frac{400 \angle 0^\circ}{\sqrt{3}} \left[\frac{100}{200} \right]$$

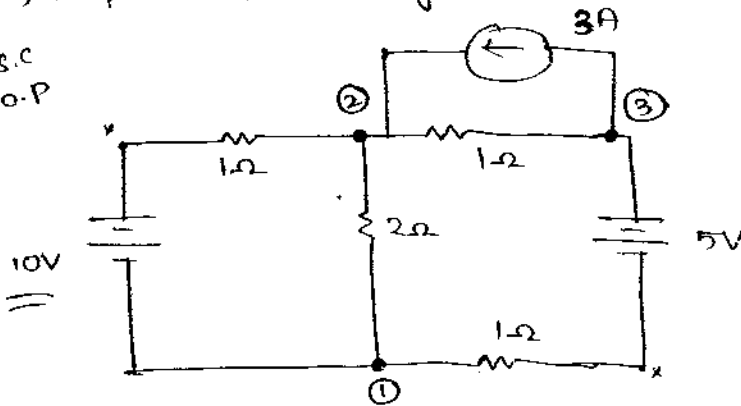
$$= \frac{200}{\sqrt{3}} \angle 0^\circ = \underline{\underline{\quad \quad \quad}} \underline{\underline{V}}$$

Topology

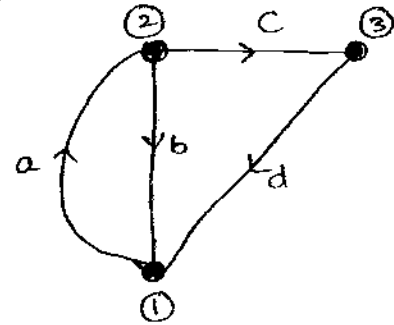
204

determine all branch currents for Nlw shown by writing Nlw eqns in standard KVL form using Tieset matrix

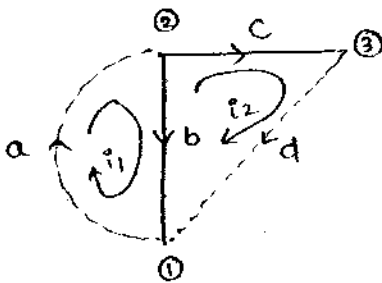
V-S.C
I-O.P



Graph



Tree



$$[B] = \begin{bmatrix} i_1 & a & b & c & d \\ i_2 & 0 & -1 & +1 & +1 \end{bmatrix} \quad 2 \times 4$$

b, c twigs
a, d links

one link &
rem. twigs
forms
fund. loop

— → Tree

--- → CoTree

dir. of current
in direction of
link

$$[Z_b] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad 4 \times 4$$

$$[I_{link}] = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad 2 \times 1 \quad [V_s] = \begin{bmatrix} +10 \\ 0 \\ 0 \\ -5 \end{bmatrix} \quad [I_s] = \begin{bmatrix} 0 \\ 0 \\ +3 \\ 0 \end{bmatrix} \quad 4 \times 1$$

$$[B][Z_b][B]^T[I_{link}] = [B][V_s] - [B][Z_b][I_s]$$

LHS

$$[B][z_b]$$

$$= \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & -2 & 1 & 1 \end{bmatrix}_{2 \times 4} \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}_{4 \times 2} = \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$
$$= \begin{matrix} \text{LHS} \\ \begin{bmatrix} 3i_1 - 2i_2 \\ -2i_1 + 4i_2 \end{bmatrix} \end{matrix} = \begin{matrix} \text{RHS} \\ \begin{bmatrix} +10 \\ -8 \end{bmatrix} \end{matrix}$$

RHS

$$[B][V_s] - [B][z_b][I_r]$$

$$\begin{bmatrix} +10 \\ -5 \end{bmatrix}_{2 \times 1} - \begin{bmatrix} 0 \\ 3 \end{bmatrix}_{2 \times 1}$$