

Syllabus (For Gate) :

(a) Number systems.

i) conversions.

ii) signed representation

(b) Boolean algebra.

(c) Logic gates.

(d) K-map

(e) Combinational circuits.

(f) Sequential circuits $\begin{cases} \rightarrow \text{synchronous.} \\ \rightarrow \text{asynchronous.} \end{cases}$

(g) A to D and D to A converters.

Textbooks :

i) Switching theory & logic design — KOHAVI.

ii) Model digital cts \rightarrow R.P. JAIN
(for K-MAP)

iii) (for sequential circuits) \rightarrow V.K. PURI.

iv) Wakerley.

14/9/18

E.K.O.-C.V.V

NUMBER SYSTEMS

$$\begin{array}{c}
 \underbrace{d_n \ d_{n-1} \ \dots \ d_3 \ d_2 \ d_1 \ d_0}_{\text{Integer part}} \cdot \underbrace{d_{-1} \ d_{-2} \ d_{-3} \ \dots \ d_{-m}}_{\text{fraction part}} \\
 \text{radix point.} \\
 r^n \ r^{n-1} \ \dots \ r^2 \ r^1 \ r^0 \cdot r^{-1} \ r^{-2} \ \dots \ r^{-m}
 \end{array}$$

$r^n \rightarrow$ weight of the position.

$d_n r^n \rightarrow$ weighted value of digit in that position.

Base (or) radix :-

The no. of different symbols used to represent a number in that number system is called base (or) radix of the number system.

Every number consists of two parts integer & fraction separated by radix point.

Conversions :- (Base conversions)

(a) Decimal to any base :-

\rightarrow To get integer part take successive divisions of decimal integer part with the required base and accumulate remainders in reverse order.

\rightarrow To get fraction part take successive multiplications of decimal fraction part with the required base & accumulate integers in original order.

Ex:- Convert to base 2, 8, 16, 4 for $(108.6875)_{10}$.

(108)

Sol:-

$$\begin{array}{r}
 2 \overline{) 108} \\
 \underline{2 \ 54 - 0} \\
 2 \overline{) 27} \\
 \underline{2 \ 24 - 0} \\
 2 \overline{) 13} \\
 \underline{2 \ 10 - 1} \\
 2 \overline{) 6} \\
 \underline{2 \ 4 - 1} \\
 2 \overline{) 3} \\
 \underline{2 \ 2 - 0} \\
 1 - 1
 \end{array}$$

$$(108)_{10} \rightarrow (1101100)_2$$

$$0.6875 \times 2 \rightarrow 1.375 \rightarrow 1$$

$$0.375 \times 2 \rightarrow 0.75 \rightarrow 0$$

$$0.75 \times 2 \rightarrow 1.5 \rightarrow 1$$

$$0.5 \times 2 \rightarrow 1.0 \rightarrow 1$$

$$\therefore (108.6875)_{10} \rightarrow (1101100.1011)_2$$

$$\begin{array}{r} 8 \overline{) 108} \\ 8 \overline{) 13-4} \\ 1-5 \end{array} \quad (108)_{10} \rightarrow (154)_8$$

$$0.6875 \times 8 \rightarrow 5.5 \rightarrow 5$$

$$0.5 \times 8 \rightarrow 4 \rightarrow 4$$

$$(0.6875)_{10} \rightarrow (54)_8$$

$$\therefore (108.6875)_{10} \rightarrow (154.54)_8$$

$$\begin{array}{r} 16 \overline{) 108} \\ 16 \overline{) 96-0} \\ 6-0 \end{array} \quad (108)_{10} \quad \begin{array}{r} 16 \overline{) 108} \\ 6-C \end{array}$$

$$(108)_{10} \rightarrow (6C)_{16}$$

$$0.6875 \times 16 \rightarrow 11 \rightarrow B$$

$$(0.6875)_{10} \rightarrow (B)_{16}$$

$$\therefore (108.6875)_{10} \rightarrow (6C.B)_{16}$$

$$\begin{array}{r} 4 \overline{) 108} \\ 4 \overline{) 27-0} \\ 4 \overline{) 6-3} \\ 1-2 \end{array} \quad (108)_{10} \rightarrow (1230)_4$$

$$0.6875 \times 4 \rightarrow 2.75 \rightarrow 2$$

$$0.75 \times 4 \rightarrow 3 \rightarrow 3$$

$$(0.6875)_{10} \rightarrow (.23)_4$$

$$\therefore (108.6875)_{10} \rightarrow (1230.23)_4$$

⑥ Any base to decimal :-

To get decimal equivalent of a number in base 'r' take summation of weighted values of all digits.

$$\text{Ex:- } (10110.011)_2$$

$$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$$

$$\Rightarrow 16 + 4 + 2 + 0.25 + 0.125$$

$$\Rightarrow (22.375)_{10}$$

Ex:- $(627.24)_8$

$$\Rightarrow 6 \times 8^2 + 2 \times 8 + 7 \times 8^0 + 2 \times 8^{-1} + 4 \times 8^{-2}$$

$$\Rightarrow (407.3125)_{10}$$

Ex:- $(3AF.C)_{16}$

$$\Rightarrow 3 \times 16^2 + 10 \times 16 + 15 + 12 \times 16^{-1}$$

$$\Rightarrow (943.75)_{10}$$

Ex:- $(608.74)_9$

Find the eq. number in base 3

$$\Rightarrow 6 \times 9^2 + 8 + 7 \times 9^1 + 4 \times 9^0$$

$$\Rightarrow (494.8271605)_{10}$$

$$\begin{array}{r} 3 \overline{) 494} \\ 3 \overline{) 164-2} \\ 3 \overline{) 54-2} \\ 3 \overline{) 18-0} \\ 3 \overline{) 6-0} \\ 2-0 \end{array}$$

$$(494)_{10} \Rightarrow (200022)_3$$

$$0.8271605 \times 3 \rightarrow 0.827 \times 3 \rightarrow 2.481$$

$$0.481 \times 3 \rightarrow 1.443$$

$$0.443 \times 3 \rightarrow 1.329$$

$$0.329 \times 3 \rightarrow 0.987$$

$$0.987 \times 3 \rightarrow 2.961$$

$$0.961 \times 3 \rightarrow 2.883$$

$$(608.74)_9 \approx (200022.211022)_3$$

Converting base (b^n) to base (b) :-

Each digit in base b^n can be represented using 'n' number of digits in base 'b'.

Ex:- $(608.74)_9 \rightarrow (\quad)_3$

$$9 \rightarrow 3^2 \text{ to } 3.$$

(9)	(3)
0	00
1	01
2	02
3	10
4	11
5	12
6	20
7	21
8	22

$$9 = 3^2$$

$$11 \times 2 = 22$$

$$\therefore (608.74)_9 \rightarrow (200022.2111)$$

Converting hexadecimal to binary

$$\text{base } 16 \rightarrow 2$$

(16)	(2)	(4)
0	0000	00
1	0001	01
2	0010	02
3	0011	03
4	0100	10
5	0101	11
6	0110	20
7	0111	21
8	1000	20
9	1001	21
A	1010	22
B	1011	23
C	1100	30
D	1101	31
E	1110	32
F	1111	33

F

Ex:- $(704.25)_8 \rightarrow (\quad)_2$
 $8 = 2^3 \rightarrow 2$

$\Rightarrow (111000100.010101)_2$

Ex:- $(3AF.C)_{16} \rightarrow (\quad)_2$

$\Rightarrow (001101010111.01100)_2$

$\Rightarrow (032233.30)_4$

(c) Converting base (b) to base (b'):-

Ex:- find base 2 to base 8, 16.

$(1011110.11011)_2$

$\Rightarrow \underline{0010} \underline{11110} . \underline{110110}$

$\Rightarrow (136.66)_8$

$\Rightarrow \overset{8421}{01011110} . \underline{11011000}$

$\Rightarrow (5E.D8)_{16}$

Ex:- $22 + 23 = 100$. Then what is the base value of the numbers.

$(22)_x + (23)_x = (100)_x$

$(2x+2)_{10} + (2x+3)_{10} = (x^2)_{10}$

$2x+2+2x+3 = x^2$

$x^2 - 4x - 5 = 0$

$x^2 - 5x + x - 5 = 0$

$x = 5, -1$

As base value can't be -ve,
base is 5.

Ex:- $\sqrt{41} = 5$. What can be the possible base value.

$\sqrt{41} = 5$

$\Rightarrow \sqrt{(41)_x} = (5)_x$

$\Rightarrow 14x+1 = (25)_x$

$4x \neq 24$

$x = 6$

$\Rightarrow 14x+1 \neq 2x+13$

$\Rightarrow \sqrt{4x+1} = 5x^0 = 5$

$4x+1 = 25$

$4x = 24$

$x = 6$

Ex:- find the base value which satisfies the equation $\frac{55}{5} = 11$.

$\frac{5x+5}{5} = x+1$

$5x+5 = 5x+5$

equation is valid
for any value of x.

$\therefore \frac{55}{5} = 11$ is valid for any base value.

but base ≥ 6 , as 5 is used in the number; so base ≥ 6 .

Ex:- find the base values which satisfies the equation,

$\frac{302}{20} = 12.1$

$\frac{3x^2+2}{2x} = x+2 + \frac{1}{x}$

$\frac{3x^2+2}{2x} = \frac{x^2+2x+1}{x}$

$2 \dots 2 \dots$

$$x^2 - 4x = 0.$$

$$x = 0, 4.$$

∴ Base is 4 as base value cannot be equal to zero.

Ex:- find base value which satisfies the following two equations simultaneously

(i) $2+3=5$ and $2 \times 3=10$

⇒ $2+3=5$

Base value ≥ 6 .

$2 \times 3 = 10$

$2 \times 3 = x$

$x = 6.$

∴ Base is 6.

(ii) $4+5=9$ and $4 \times 2=10$.

$4+5=9$

∴ Base ≥ 6 .

$4 \times 2 = 10$

Base ≥ 5

$4+5=9$

Base ≥ 10

$4 \times 2 = 10$

$8 = x$

These two equations cannot be

satisfied simultaneously with any base.

Ex:- find the base values which satisfy the equation $\sqrt{144} = 12$.

$$\sqrt{x^2 + 4x + 4} = x + 2.$$

$$x^2 + 4x + 4 = (x+2)^2.$$

$$= x^2 + 4x + 4$$

equation is valid for any base value ≥ 5 .

* Find the number of possible values for 'x' which satisfy the equation

$$(43)_8 = (x0)_y.$$

Sol:- $(43)_8 = (x0)_y$

$$4 \times 8 + 3 = xy.$$

$$32 + 3 = xy.$$

$$35 = xy.$$

$$x = \frac{35}{y}.$$

$$y = \frac{35}{x}.$$

x	y	
1	35	✓
35	1	×
5	7	✓
7	5	×

Two possible values of x.

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* Find number of possible solutions

$$(123)_5 = (x8)_y.$$

Sol:- $1 \times 25 + 2 \times 5 + 3 = xy + 8.$

$$25 + 10 + 3 = xy + 8.$$

$$38 = xy + 8.$$

$$xy = 38 - 8 = 30.$$

$$y \geq 9.$$

x	y	
1	30	✓
2	15	✓
3	10	✓
5	6	×
6	5	×
10	3	×
15	2	×
30	1	×

3 possible solutions.

* pgno. 19 Q.1.

Sol:- $(135)_x + (144)_x = (323)_x$

$$\Rightarrow x^2 + 3x + 5 + x^2 + 4x + 4 = 3x^2 + 2x + 3$$

$$\Rightarrow x^2 - 5x - 6 = 0$$

$$x = 6, -1.$$

\therefore Base $x = 6$.

* pgno. 19 Q.4

Sol:- $(657)_8$

$$\Rightarrow (110101111)_2$$

$$\begin{array}{ccccccc} 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ \hline & & & 1 & & & A & & & & F & \end{array}$$

$$\Rightarrow (1AF)_H$$

* pgno. 20 Q.6

Sol:- $\frac{312}{20} = 13.1$

$$\frac{3x^2 + x + 2}{2x} = x + 3 + \frac{1}{x}$$

$$\frac{3x^2 + x + 2}{2x} = \frac{x^2 + 3x + 1}{x}$$

$$3x^2 + x + 2 = 2x^2 + 6x + 2$$

$$x^2 - 5x = 0$$

$$x = 0, 5$$

15/9/14 \therefore Base $x = 5$.

Addition in other base values :-

Ex:-
$$\left(\begin{array}{r} 26578 \\ (+) 35764 \\ \hline 63453 \end{array} \right)_9$$

\rightarrow add every two digits in decimal. If sum is $<$ base, keep the sum as it is.

\rightarrow If sum \geq base, then subtract the sum and base and put carry '1' to the previous digits.

[This procedure only for adding two numbers]

$$\begin{array}{r} 1111 \\ * + \left(\begin{array}{r} 26543 \\ 35264 \\ \hline 65140 \end{array} \right)_7 \end{array}$$

$$\begin{array}{r} 101 \\ * + \left(\begin{array}{r} 46247 \\ 57324 \\ \hline 125573 \end{array} \right)_8 \end{array}$$

Subtraction :-

$$\begin{array}{r} * \left(\begin{array}{r} 64375 \\ - 27846 \\ \hline 35428 \end{array} \right)_9 \end{array}$$

$$\begin{array}{r} * \left(\begin{array}{r} 64025 \\ - 36543 \\ \hline 24152 \end{array} \right)_7 \end{array}$$

Ex:-
$$\left(\begin{array}{r} 62047 \\ - 37645 \\ \hline 22202 \end{array} \right)_8$$

* $(1.2)_4 +$

$(2.3)_4 =$

$(7)_4$

$$\left(\begin{array}{r} 1.2 \\ 2.3 \\ \hline 10.1 \end{array} \right)_4$$

$$\Rightarrow 01.10.$$

$$10.11$$

$$100.01$$

$$\Rightarrow (10.1)_4$$

COMPLEMENTS

i) Radix complements $[r's]$.

ii) Diminished radix complements $[(r-1)'s]$.

$$(N)_r = d_{n-1} d_{n-2} \dots d_1 d_0 . d_{-1} d_{-2} \dots d_{-m}$$

r 's complement :-

Assume a number (N_r) has 'n' number of digits in integer part and 'm' number of digits in fractional part, then

$$\begin{aligned} r's \text{ complement} &= r^n - N \quad (\forall N \neq 0) \\ &= 0 \quad (\forall N = 0). \end{aligned}$$

$(r-1)$'s complement :-

$$(r-1)'s \text{ complement} = r^n - r^{-m} - N.$$

Ex:- find 10's & 9's complement of $(0030700)_{10}$.

$$10's \text{ comp} \rightarrow 10^7 - 0030700$$

$$\Rightarrow \begin{array}{r} 10000000 \\ - 0030700 \\ \hline 9969300 \end{array}$$

$$9's \text{ complements} \rightarrow 10^7 - 10^0 - (0030700).$$

$$\Rightarrow \begin{array}{r} 10000000 \\ - 0030701 \\ \hline 9969299 \end{array}$$

$$\underline{\text{Ex:-}} (0607.89000)_{10}$$

10's complement \rightarrow

$$\begin{array}{r} 10000.00000 \\ - 0607.89000 \\ \hline 9392.11000 \end{array}$$

9's complement \rightarrow

$$\begin{aligned} 10^4 - 10^{-5} - (0607.89000) \\ 10000 - 0.00001 - 0607.8900 \end{aligned}$$

$$\Rightarrow \begin{array}{r} 10000.00000 \\ - 0607.89001 \\ \hline 9392.10999 \end{array}$$

$$\underline{\text{Ex:-}} (00307.20600)_8$$

8's complement $\rightarrow 8^5 = 4096$

$$\begin{array}{r} 4096/06060 \\ 0/307.20600 \end{array}$$

$$\begin{array}{r} 32768/00000 \\ 0/307.20600 \\ \hline 1/260 \end{array}$$

$$(8^5)_8 = 100000.00000.$$

$$\begin{array}{r} 100000.00000 \\ - 00307.20600 \\ \hline 77470.57200 \end{array}$$

$$\begin{aligned} 7's \text{ complement} &\rightarrow (8^5)_8 - (\bar{8}^5) \\ &\quad - (00307.20600) \end{aligned}$$

$$\Rightarrow \begin{array}{r} 100000.00000 \\ - 00307.20601 \\ \hline 77470.57177 \end{array}$$

Ex:- $(0101001.10100)_2$

2's complement \rightarrow

$$\begin{array}{r} 10000000.00000 \\ 0101001.10100 \\ \hline 1010110.01100 \end{array}$$

1's complement \rightarrow

$$\begin{array}{r} 10000000.00000 \\ 0101001.10101 \\ \hline 1010110.01011 \end{array}$$

Ex:- Find 1's

Note:- To get 1's complement of a number in base r , leave least significant zeros as it is, subtract first non-zero least significant digit from r and subtract remaining digits each from $(r-1)$.

To get $(r-1)$'s complement of a number in base r , subtract each digit from $(r-1)$.

Ex:- 01001.10100

2's comp $\rightarrow 10110.01100$

1's comp $\rightarrow 10110.01011$

Signed representation of binary numbers:-

(i) Signed magnitude.

(ii) 1's complement.

(iii) 2's complement.

Signed magnitude representation:-

In signed magnitude representation, MSB is used explicitly for sign which does not have any magnitude and the remaining bits are used for magnitude.

If MSB = 0, number is positive and if MSB = 1, number is negative.

Ex:-

0	0	0	0	$\rightarrow +0$
1	0	0	0	$\rightarrow -0$
0	0	0	1	$\rightarrow +1$
0	1	0	0	$\rightarrow +4$
1	1	0	1	$\rightarrow -5$

The range of 4 bit is

-7 to $+7$

$-(2^{4-1}-1)$ to $(2^{4-1}-1)$

The range of n -bit is

$$\boxed{-(2^{n-1}-1) \text{ to } (2^{n-1}-1)}$$

The range of decimal integers can be represented in n -bits in signed magnitude representation is as above.

In this representation zero has two representations +0 (0000) and -0 (1000).

1's complement representation :-

$$-(2^{n-1}-1) \quad 2^{n-2} \quad 2^{n-3} \quad \dots \quad 2^1 \quad 2^0$$

MSB

$$1000 \rightarrow -7$$

$$1001 \rightarrow -6$$

$$1100 \rightarrow -3$$

$$1111 \rightarrow -0$$

$$0001 \rightarrow +1$$

$$0101 \rightarrow +5$$

$$0111 \rightarrow +7$$

$$0000 \rightarrow +0$$

The range is -7 to +7.

In 1's complement representation, the only bit in MSB has -ve weight and the weight is $-(2^{n-1}-1)$ and the remaining bits have +ve weights.

The range of decimal integers can be represented using 'n' bits in 1's complement representation is

$$-(2^{n-1}-1) \text{ to } (2^{n-1}-1)$$

In this representation, zero has two representations +0 (0000) and -0 (1111)

2's complement representation :-

$$-(2^{n-1}) \quad 2^{n-2} \quad 2^{n-3} \quad \dots \quad 2^1 \quad 2^0$$

$$1000 \rightarrow -8$$

$$1001 \rightarrow -7$$

$$1100 \rightarrow -4$$

$$1111 \rightarrow -1$$

$$0000 \rightarrow 0$$

$$0001 \rightarrow +1$$

$$0101 \rightarrow +5$$

$$0111 \rightarrow +7$$

range is (-8 to +7).

In this representation, the only bit in MSB has -ve weight and the weight is $-(2^{n-1})$ and the remaining bits have positive weights.

The range is

$$-2^{n-1} \text{ to } (2^{n-1}-1)$$

The range of decimal integers can be represented using n-bits in 2's complement representation is as above.

In this representation, zero has only one representation (0000).

Ex:- 1's comp

$$\begin{array}{l} 1's \quad \left\{ \begin{array}{l} 1001 \Rightarrow -6 \\ 0110 \Rightarrow +6 \end{array} \right. \end{array}$$

$$\begin{array}{l} 1's \quad \left\{ \begin{array}{l} 0101 \Rightarrow +5 \\ 1010 \Rightarrow -5 \end{array} \right. \end{array}$$

2's comp

$$\begin{array}{l} 2's \quad \left\{ \begin{array}{l} 1011 \Rightarrow -5 \\ 0101 \Rightarrow +5 \end{array} \right. \end{array}$$

$$\begin{array}{l} 2's \quad \left\{ \begin{array}{l} 0110 \Rightarrow +6 \\ 1010 \Rightarrow -6 \end{array} \right. \end{array}$$

Ex:- $\left. \begin{array}{l} 1000 \rightarrow -8 \\ 2's \rightarrow 1000 \rightarrow -8 \end{array} \right\}$
 not giving complement form becoz of range.

Overflow :-

If the number of bits is not sufficient to represent the result, then we say there is overflow.

1st Ex

1's complement

1010 $\rightarrow -5$
 11010 $\rightarrow -5$
 111010 $\rightarrow -5$
 11111010 $\rightarrow -5$
 0100 $\rightarrow +4$
 00100 $\rightarrow +4$
 00000100 $\rightarrow +4$.

2's complement

1001 $\rightarrow -7$
 111001 $\rightarrow -7$
 111111001 $\rightarrow -7$
 0101 $\rightarrow +5$
 000101 $\rightarrow +5$
 00000101 $\rightarrow +5$.

In 1's comp and 2's comp representation, the extension of sign bit does not change the decimal equivalent value.

Ex:- $\left. \begin{array}{l} 2's \text{ comp} \\ 11000 \rightarrow -8 \\ 2's \rightarrow 01000 \rightarrow +8 \end{array} \right\}$

Ex:- find signed magnitude, 1's comp and 2's comp of -27 using 8 bits.

$\rightarrow (+27) \rightarrow$

16	8	4	2	1
1	1	0	1	1

1st comp of -27 \rightarrow

XXXXXXNN

11100100

2's comp of -27 \rightarrow

11100101

method 2 :-

2 | 27
 2 | 13-1
 2 | 6-1
 2 | 3-0
 1-1

$(27)_{10} \rightarrow (11011)_2$

+27 $\rightarrow 00011011$

-27 $\rightarrow 10011011$ \rightarrow signed mag. representation.

$-31 + 4 = -27$

$= 100100$

$= 11100100$ \rightarrow 1's comp. representation, of -27

$-27 = 100100$

$-32 + 5 = -27$

$= \frac{100101}{-32 \quad +5}$ \rightarrow 2's comp representation of -27.
 $= 11100101$

Method 3 :-

$+27 \rightarrow 00011011$
 $-27 \rightarrow 11100100$

1's comp.

2's comp of +27 \rightarrow 11100101
 $\Rightarrow -27$

Ex:- Find signed magnitude, one's complement and 2's complement representations of +28 using 8 bits.

Sol:- signed mag \rightarrow 00011100.

$2 \mid 28$
 $2 \mid 14-0$
 $2 \mid 7-0$
 $2 \mid 3-1$
 $1-1$

$1^s \text{ comp} \rightarrow 00011100.$
 ~~$\neq 11100011$~~
 $2^s \text{ comp} \rightarrow 00011100.$
 ~~\neq~~

For +ve integers signed magnitude, 1's comp and 2's comp representations are same.

Subtraction with complements :-

is 1's complement.

ii) a 's complement.

2's complement subtraction :-

Add 'A' to the 2's complement of 'B'. Observe the result of obtained in step 1.

② If $A \geq B$, sum produces an end carry. Discard the carry, sum is the desired result.

⑥ If $A < B$, sum ~~for~~ does not produce an end carry. Take 2's complement of sum and produce -ve sign in front to

get the desired result.

ii) 1's complement method :-

→ Add 'A' to the 1's complement of B.

→ Observe the result obtained in step 1.

② If $A > B$, sum produces an end carry, add '1' to the LSB to get the desired result

(b) If $A < B$, sum does not produce an end carry. Take 1's complement of sum and produce -ve sign in front. get the desired result.

Ex:- subtract using 1's comp and 2's comp method.

$$(1001)_2 - (100)_2$$

Sol:- 1st comp:-

$$\begin{array}{r} 1001 \\ 0100 \\ \Rightarrow 1001 \\ 1011 \\ \hline 10100 \\ \xrightarrow{\quad} \\ 01001 \\ \hline 0101 \end{array}$$

2's comp :-

$$\begin{array}{r} 1001 \\ 0100 \\ \hline \Rightarrow 1001 \\ 1100 \\ \hline \textcircled{0}1001 \\ \hline \Rightarrow 0101 \checkmark \end{array}$$

Ex:- $(0101)_2 - (1100)_2$

Sol:-

$$\begin{array}{r} 0101 \\ - 1100 \\ \hline \end{array}$$

1's Comp \Rightarrow

$$\begin{array}{r} 0101 \\ + 0011 \\ \hline 1000 \\ \text{1's Comp} \rightarrow 0111 \\ \Rightarrow -0111 \checkmark \end{array}$$

2's Comp \Rightarrow

$$\begin{array}{r} 0101 \\ + 0100 \\ \hline 1001 \\ \text{2's Comp} \rightarrow 0111 \\ \Rightarrow -0111 \checkmark \end{array}$$

Ex:- Two 2's complement numbers

$X = 01100$ and $Y = 11011$ are added. $X+Y$ in 2's comp format using 6 bits.

- (a) 100111 (b) 000111
(c) 0110001 (d) 111001.

Sol:-

$$\begin{array}{l} X = 01100 \\ Y = 11011 \\ \Rightarrow X = 001100 \\ Y = 111011 \\ X+Y = \cancel{0000111} \\ X+Y = 000111. \end{array}$$

BOOLEAN ALGEBRA

OR (+)	AND (.)	INV (' or -) (NOT)
$0+0=0$	$0 \cdot 0=0$	$\overline{0}=1$
$0+1=1$	$0 \cdot 1=0$	$\overline{1}=0$
$1+0=1$	$1 \cdot 0=0$	
$1+1=1$	$1 \cdot 1=1$	

Dual of a function:-

To get dual of a function replace logical OR with logical AND and viceversa and replace logic 0 with logic 1 and viceversa.

Note:-

If a function is true then dual of that function is also true.

$$\begin{array}{ll} x+0=x & x \cdot 1=x \\ x+1=1 & x \cdot 0=0 \\ x+x=x & x \cdot x=x \\ x+\bar{x}=1 & x \cdot \bar{x}=0 \\ \overline{\bar{x}}=x & \overline{\overline{x}}=x \end{array}$$

$$\begin{array}{ll} x+y=y+x & x \cdot y=y \cdot x \\ x+(y+z)=(x+y)+z & x \cdot (y \cdot z)=(x \cdot y) \cdot z \\ x \cdot (y+z)=x \cdot y+x \cdot z & x+(y \cdot z)=(x+y) \cdot (x+z) \\ x+xy=x & x \cdot (x+y)=x \\ x+\bar{x}y=x+y & x \cdot (\bar{x}+y)=x \cdot y \\ \bar{x}+xy=\bar{x}+y & \bar{x} \cdot (x+y)=\bar{x} \cdot y \\ \overline{(x+y)}=\bar{x} \cdot \bar{y} & \overline{(x \cdot y)}=\bar{x}+\bar{y} \end{array}$$

$$\overline{x+y+z} = \bar{x} \cdot \bar{y} \cdot \bar{z}$$

$$\overline{x \cdot y \cdot z} = \bar{x} + \bar{y} + \bar{z}$$

$$xy + \bar{x}z + yz = xy + \bar{x}z$$

* (Consensus theorem)

$$(x+y) \cdot (\bar{x}+z) \cdot (y+z) =$$

$$(x+y) \cdot (\bar{x}+z)$$

$$(x+y) \cdot (\bar{x}+z)$$

In canonical representation, all the variables must be existed in complemented or uncomplemented form in all terms.

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* pg no. 19 Q.2

Sol:- -127 in 2's comp \longrightarrow

10000001.

-127 in 1's comp \longrightarrow

10000000.

m:n = 2:1.

* Q.3

Sol:- 4-bit 2's comp $\longrightarrow x_3 x_2 x_1 x_0$

8 bit $\longrightarrow x_7 x_6 x_5 x_4 x_3 x_2 x_1 x_0$

sign expansion.

* Q.5

Sol:- $(-539)_{10}$

$\Rightarrow +539 =$

$(01000011011)_2$

$-539 = 2$'s comp of $+539$

$= \underline{010111100101}$

$= (DE5)_{16}$

2		539
2		269-1
2		134-1
2		67-0
2		33-1
2		16-1
2		8-0
2		4-0
2		2-0
		1-0
		8421.

Ex:-

A B C f(A,B,C)

0 0 0 0

1 0 0 1

2 0 1 0

3 0 1 1

4 1 0 0

5 1 0 1

6 1 1 0

7 1 1 1

$$f(A,B,C) = \bar{A} \bar{B} C + \bar{A} B \bar{C} + A \bar{B} C + A B \bar{C}$$

$$= m_1 + m_2 + m_5 + m_6$$

$$= \sum m(1,2,5,6)$$

minterms.

Canonical SOP

$$\bar{f} = \bar{A} \bar{B} C + \bar{A} B C + A \bar{B} \bar{C} + A B C$$

$$f = \bar{\bar{f}} = \overline{\bar{A} \bar{B} C + \bar{A} B C + A \bar{B} \bar{C} + A B C}$$

$$= (A+B+\bar{C}) \cdot (A+\bar{B}+\bar{C}) \cdot (\bar{A}+B+C) \cdot (\bar{A}+\bar{B}+C)$$

$$= M_0 \cdot M_3 \cdot M_4 \cdot M_7$$

$$= \prod M(0,3,4,7)$$

Canonical POS.

maxterms

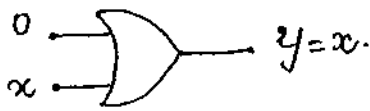
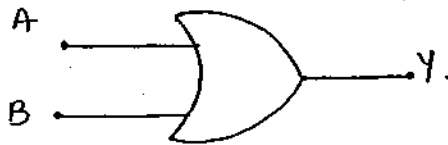
LOGIC GATES

OR GATE :-

A B inputs.

$$OR \Rightarrow A+B=Y.$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1



If one i/p of OR gate is connected to zero, then the o/p depends on other i/p. So zero is called Enabled i/p.



If one i/p is one, then irrespective of other i/p, the o/p is high. So logic '1' is called disabled i/p for other i/p.

Enabled input :-

The i/p which is allowing other i/p's to pass through the gate is called Enabled i/p.

Disabled input :-

The i/p which not allowing other inputs to pass through the gate is disabled input.

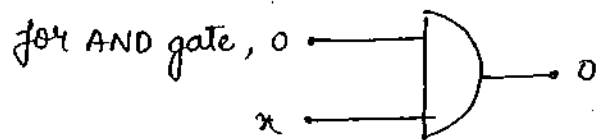
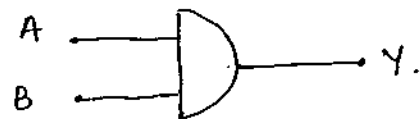
The three i/p OR gate,

A	B	C	Y=A+B+C
0	0	0	→ 0
0	0	1	→ 1
0	1	0	→ 1
0	1	1	→ 1
1	0	0	→ 1
1	0	1	→ 1
1	1	0	→ 1
1	1	1	→ 1

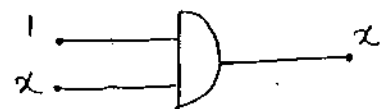
OR gate o/p is zero when all i/p's are zeros.

AND GATE :-

A	B	Y=A.B
0	0	0
0	1	0
1	0	0
1	1	1



∴ '0' is disabled i/p.



'1' is enabled i/p.

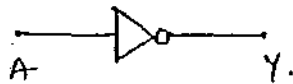
The three i/p AND gate

A	B	C	$Y = ABC$
0	0	0	→ 0
0	0	1	→ 0
0	1	0	→ 0
0	1	1	→ 0
1	0	0	→ 0
1	0	1	→ 0
1	1	0	→ 0
1	1	1	→ 1

AND gate o/p is '1' only when all inputs are ones.

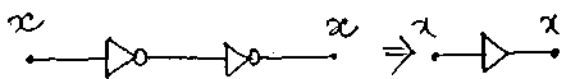
NOT gate :- $A \rightarrow \text{i/p}$.

A	$Y = \bar{A}$
0	1
1	0



Enabled and disabled i/p are not possible for NOT gate.

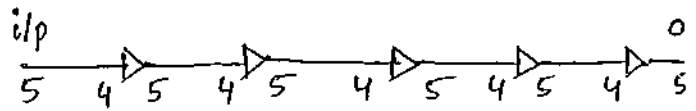
If two not gates are cascaded,



It acts as a buffer.

Buffers helps in transmitting the voltages with less noise, as it introduces noise margin.

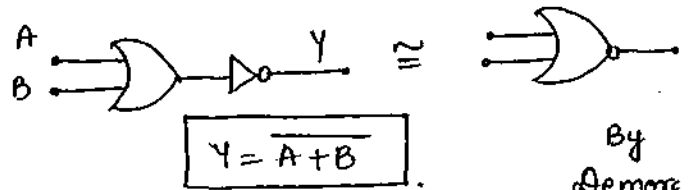
Ex:- If 1cm of wire has 0.5V loss, by introducing buffers in wire, the effect can be reduced.



The AND, OR, NOT are called basic gates as these are sufficient to implement any function.

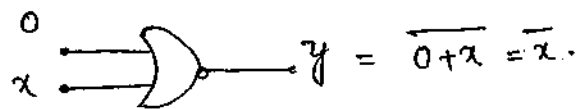
DERIVED GATES :-

NOR gate :-

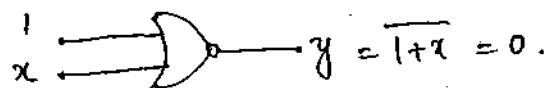


A	B	$Y = \overline{A+B} = \bar{A} \cdot \bar{B}$
0	0	1
0	1	0
1	0	0
1	1	0

By Demorgan Law



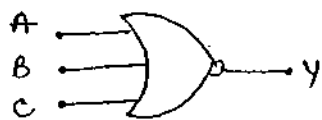
'0' is enabled i/p.



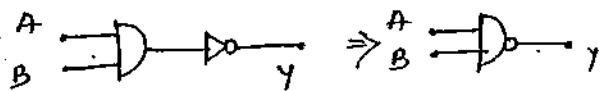
'1' is disabled i/p.

The 3-i/p NOR gate is

A	B	C	$Y = \overline{A+B+C} = \bar{A} \cdot \bar{B} \cdot \bar{C}$
0	0	0	→ 1
0	0	1	→ 0
0	1	0	→ 0
0	1	1	→ 0
1	0	0	→ 0
1	0	1	→ 0
1	1	0	→ 0
1	1	1	→ 0



NAND gate :-



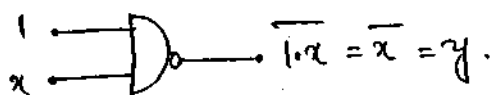
$$A \quad B \quad Y = \overline{A \cdot B} = \overline{A} + \overline{B}$$

$$0 \quad 0 \rightarrow 1$$

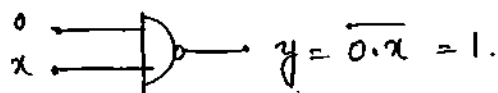
$$0 \quad 1 \rightarrow 1$$

$$1 \quad 0 \rightarrow 1$$

$$1 \quad 1 \rightarrow 0$$



'1' is Enabled i/p.



'0' is disabled i/p.

The 3-i/p NAND gate is

$$A \quad B \quad C \quad Y = \overline{A \cdot B \cdot C} = \overline{A} + \overline{B} + \overline{C}$$

$$0 \quad 0 \quad 0 \rightarrow 1$$

$$0 \quad 0 \quad 1 \rightarrow 1$$

$$0 \quad 1 \quad 0 \rightarrow 1$$

$$0 \quad 1 \quad 1 \rightarrow 1$$

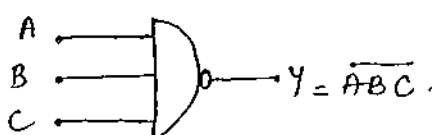
$$1 \quad 0 \quad 0 \rightarrow 1$$

$$1 \quad 0 \quad 1 \rightarrow 1$$

$$1 \quad 1 \quad 0 \rightarrow 1$$

$$1 \quad 1 \quad 1 \rightarrow 0$$

NAND Gate O/p is zero only when all i/p's are 1's.



NAND, NOR and multiplexers are universal gates.

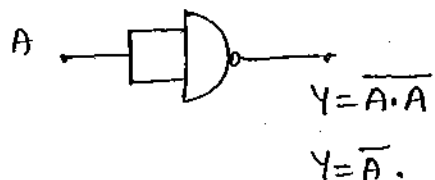
If it is possible to implement any function using a gates, then that gate is universal gate.

So, if basic gates are able to be implemented using universal gate then all the functions can be implemented using universal gate.

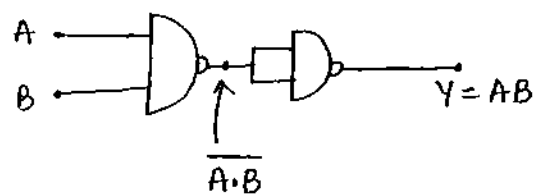
To prove this it must be possible to implement three basic gates (AND, OR, NOT) using only the gates then it is universal gate.

NAND gate as universal gate :-

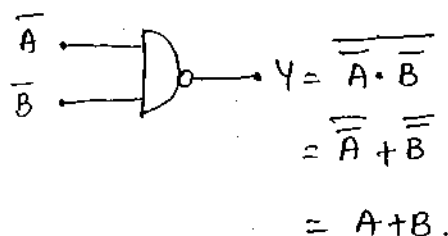
(i) Implementation of NOT gate.

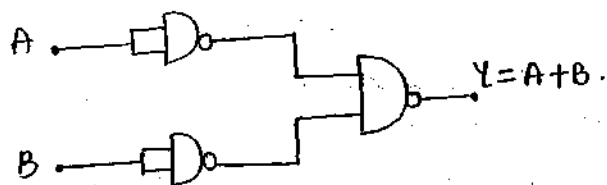


(ii) Implementation of AND gate.



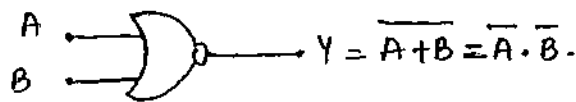
(iii) Implementation of OR gate.



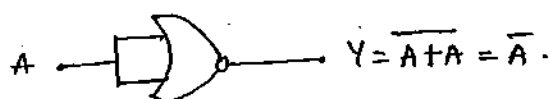


\therefore NAND gate is universal gate.

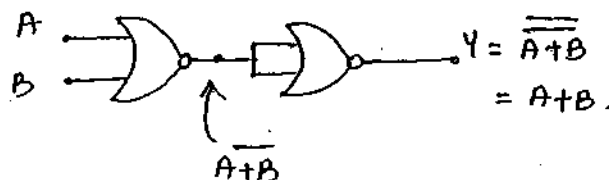
NOR gate as universal gate :-



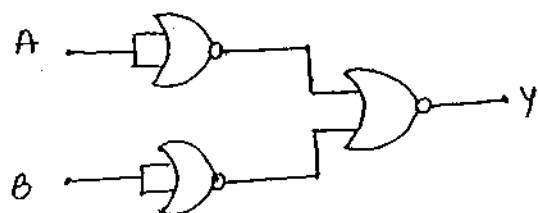
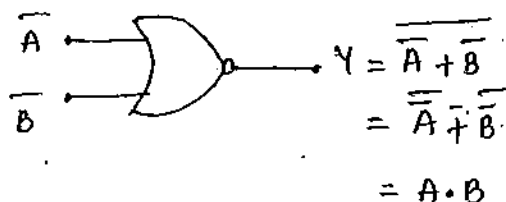
i) NOT gate implementation.



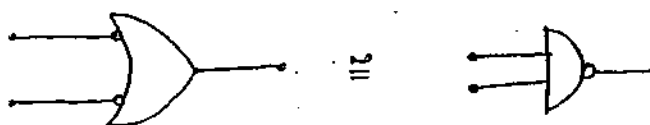
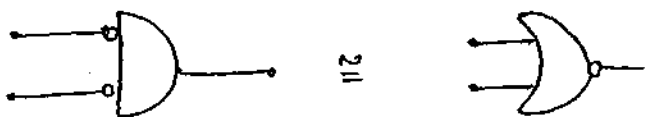
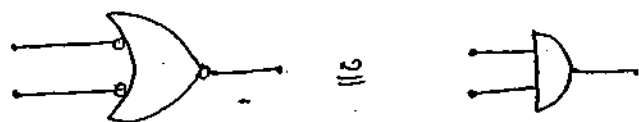
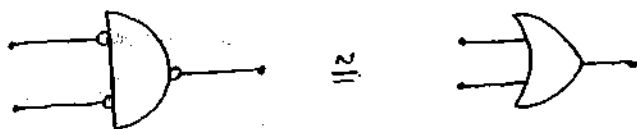
ii) OR gate implementation.



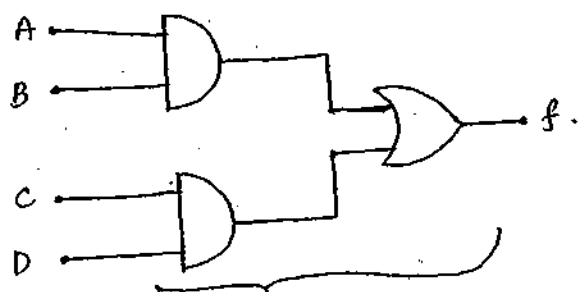
iii) AND gate implementation.



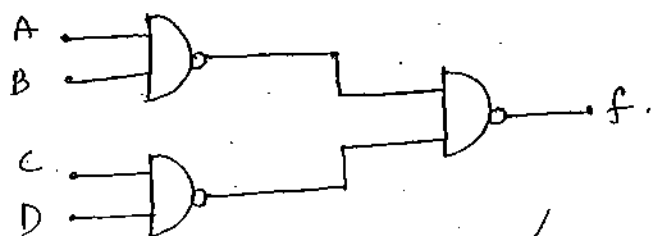
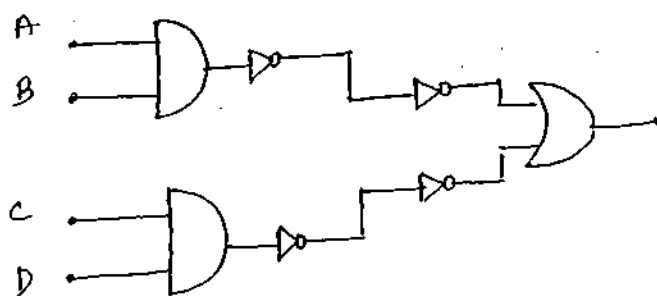
\therefore NOR gate is a universal gate.



Ex:- $AB + CD = f$. Implement f using logic gates.

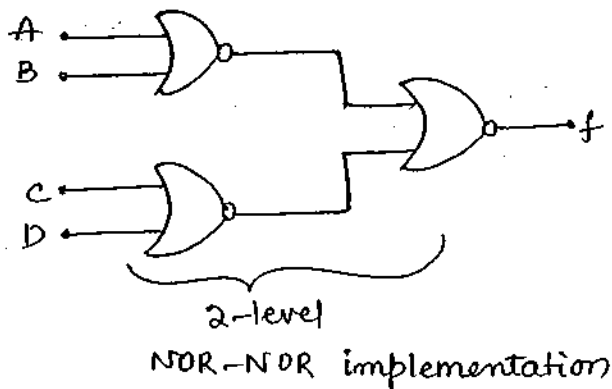
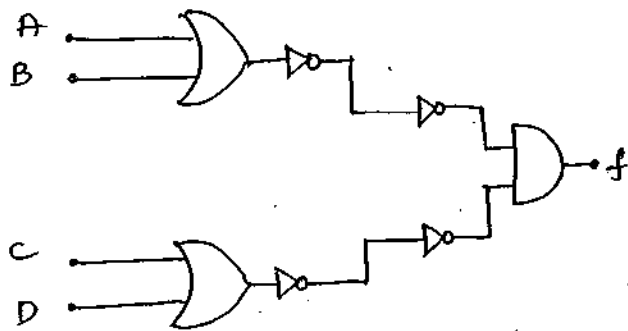
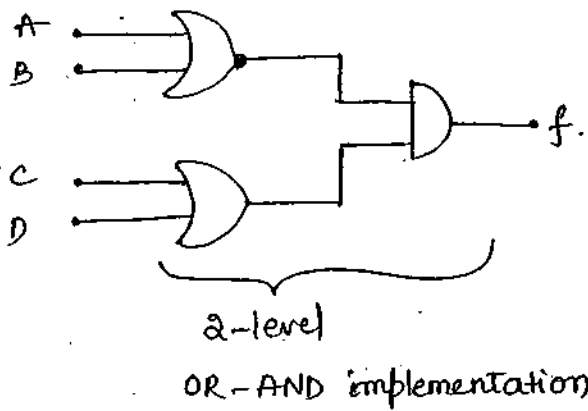


Two-level
AND-OR implementation



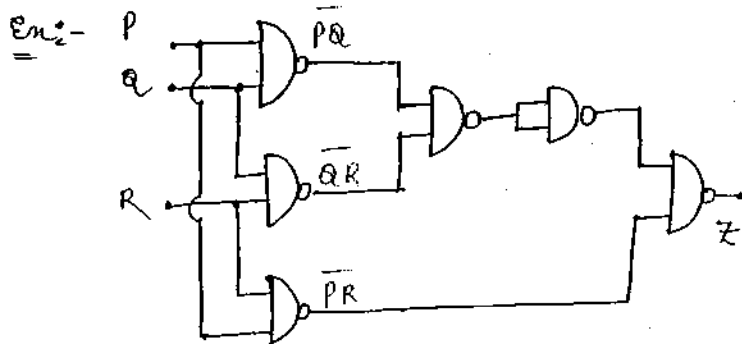
two-level
NAND-NAND implementation.

Ex^o- Implement $f = (A+B)(C+D)$.



∴ SOP form can be implemented with 2-level NAND-NAND.

POS form can be implemented with 2-level NOR-NOR.



The op^t Z will be 1 in the ckt for

(a) two or more of the i/p P, Q, R are zero.

(b) two or more of the i/p's of P, Q, R are one.

(c) odd no. of i/p's of P, Q, R is zero.

(d) odd no. of i/p's of P, Q, R is one.

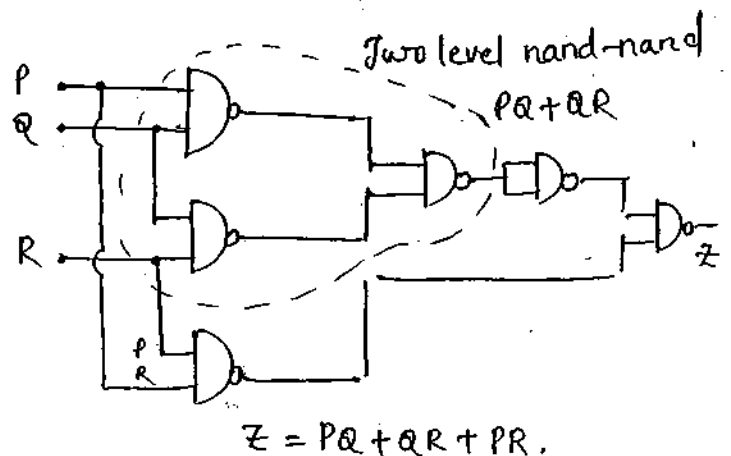
Sol:-

$$F = \overline{PQ \cdot \overline{QR} \cdot \overline{PR}}$$

$$\overline{F} = \overline{PQ \cdot \overline{QR} \cdot \overline{PR}}$$

$$\overline{F} = PQ + \overline{QR} + \overline{PR}$$

$$1 = PQ + \overline{QR} + \overline{PR}$$



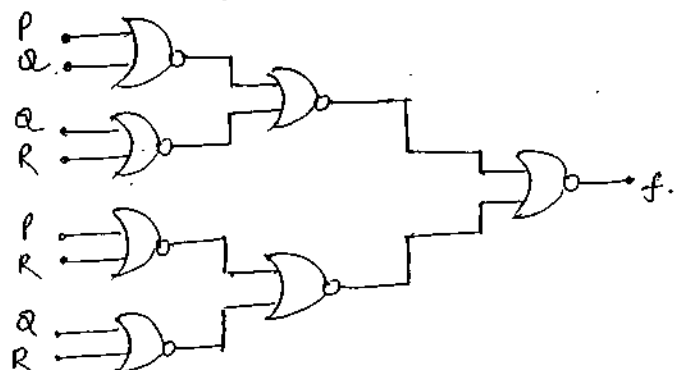
Ex^o- The op^t f' in the ckt

(a) $\overline{P+Q+R}$

(b) $\overline{Q+R}$

(c) $\overline{P+R}$

(d) $\overline{P+Q+R}$



$$f = \overline{(P+Q)(Q+R) + (P+R)(Q+R)}$$

Two-level NOR-NOR given

⇓
OR-AND.

$$f = \overline{(P+Q + Q+R) \cdot (P+R + Q+R)}$$

$$f = \overline{(P+Q \cdot P+R) + Q+R}$$

$$f = \overline{(\overline{P} \cdot \overline{Q} \cdot \overline{P} \cdot \overline{R}) + \overline{Q} \cdot \overline{R}}$$

$$f = \overline{Q \cdot R} [\overline{P} + 1] = \overline{Q \cdot R} = \overline{Q+R}$$

(or)

$$f = \overline{(P+Q)(Q+R) + (P+R)(Q+R)} = \overline{(Q+R)(P+Q+R)}$$

$$= \overline{Q+R}$$

$$\because x(x+y) = x$$

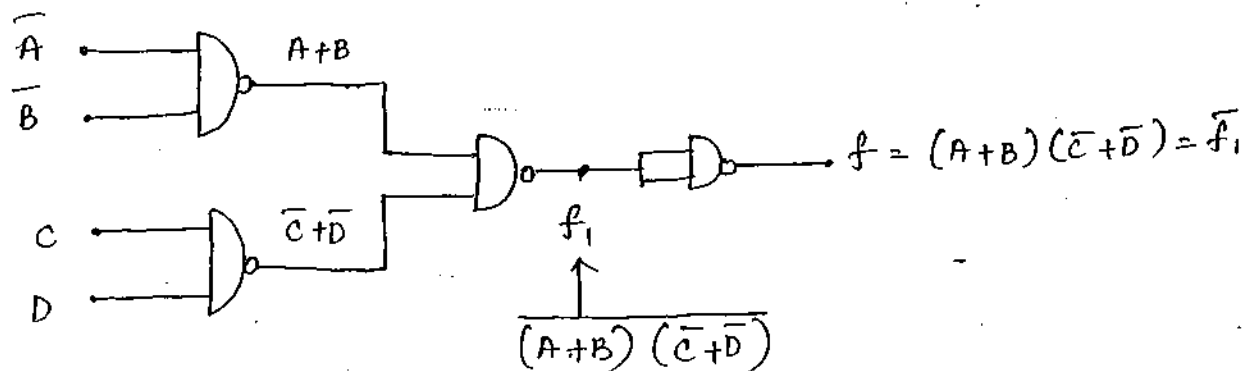
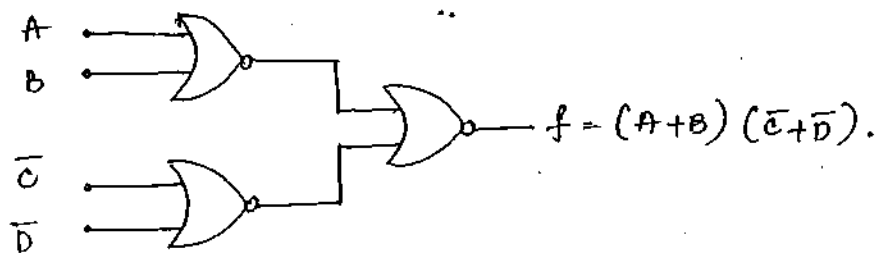
Ex:- Find minimum no. of 2-i/p NAND gates required to implement the function $(A+B)(\overline{C}+\overline{D})$.

[Assume that variables are ^{available} in both complemented and uncomplemented form]. \rightarrow ^{***} literals $(A, \overline{A}, B, \overline{B}, C, \overline{C})$ but

$$f = (A+B)(\overline{C}+\overline{D})$$

variables are
A, B, C
only

\rightarrow two i/p nor-nor implementation

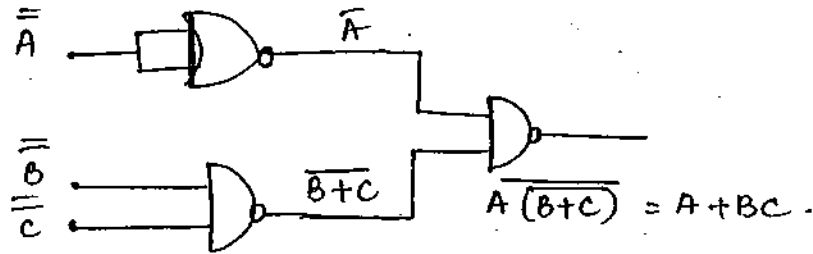


\therefore Minimum 4 nand-gates required.

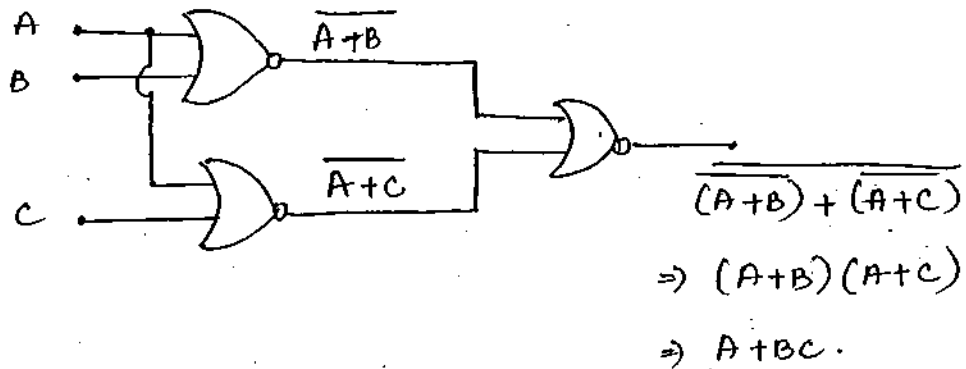
Ex:- min. no. of 2-inp nor gates required to implement $f = A + BC$.

Sol:- $f = A + BC$.

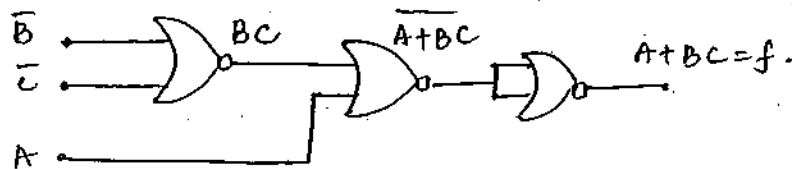
$$f = \overline{\overline{A} \overline{BC}}$$



$$f = A + BC$$



(or)



Ex:- find min no. of NAND gates require to implement $A + AB + A\overline{B}C$.

$$f = A + AB + A\overline{B}C$$

$$f = A(1 + B + \overline{B}C) = A$$

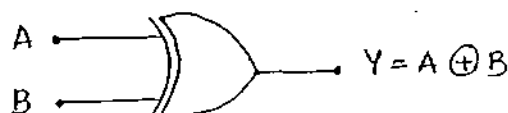
\therefore No NAND gate is required to implement A.

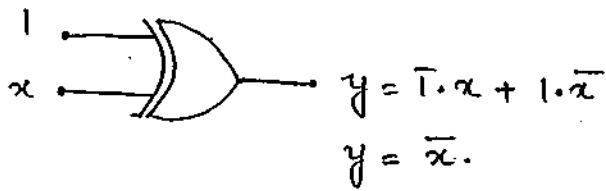
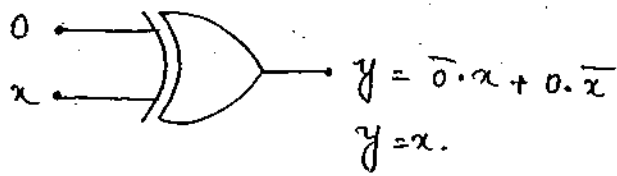
Ex:- 2-4

EX-OR gate :-

$$Y = \overline{A}B + A\overline{B} = (A + B)(\overline{A} + \overline{B})$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

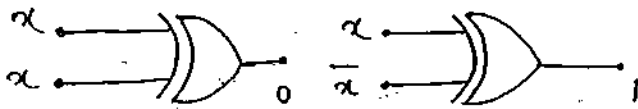




∴ for Ex-OR there is no disable i/p. Both '0' and '1' are enable i/p.

So Ex-OR gate cannot be used for enabling a circuit or gate unlike basic gates AND and OR.

2-i/p Ex-OR gate can be used to check inequality of inputs.



3-i/p Ex-OR gate :-

	A	B	C	$y = A \oplus B \oplus C$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	1

The o/p of gate will be 1, when the i/p is having odd number of ones.

$$y = \sum m(1, 2, 4, 7) = \pi M(0, 3, 5, 6)$$

$$y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

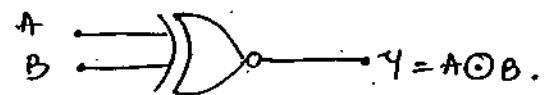
↳ SOP minterms.

$$y = (A+B+C)(A+\bar{B}+\bar{C}) + (\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$$

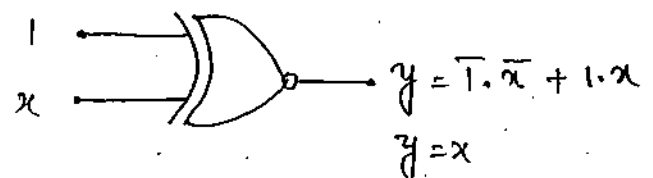
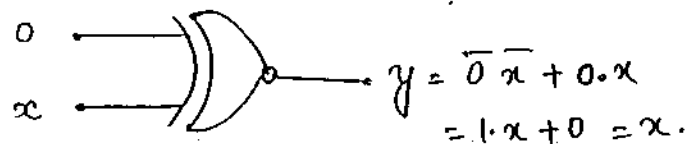
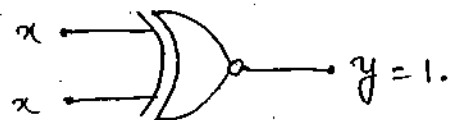


EX-NOR GATE :-

A	B	$y = A \odot B = \bar{A}\bar{B} + AB$
0	0	1
0	1	0
1	0	0
1	1	1



2-i/p EX-NOR gate can be used to check equality of inputs.



Ex-NOR gate cannot be used for enabling and disabling the block.

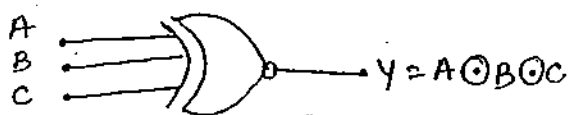
3 i/p Ex-NOR gate :-

	A	B	C	$Y = A \odot B$
0	0	0	0	$\rightarrow 1$
1	0	0	1	$\rightarrow 0$
2	0	1	0	$\rightarrow 0$
3	0	1	1	$\rightarrow 1$
4	1	0	0	$\rightarrow 0$
5	1	0	1	$\rightarrow 1$
6	1	1	0	$\rightarrow 1$
7	1	1	1	$\rightarrow 0$

$$Y = \sum m(0, 3, 5, 6) = \prod M(1, 2, 4, 7)$$

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C}$$

$$Y = (A+B+\bar{C}) \cdot (A+\bar{B}+C) \cdot (\bar{A}+B+C) \cdot (\bar{A}+\bar{B}+\bar{C})$$



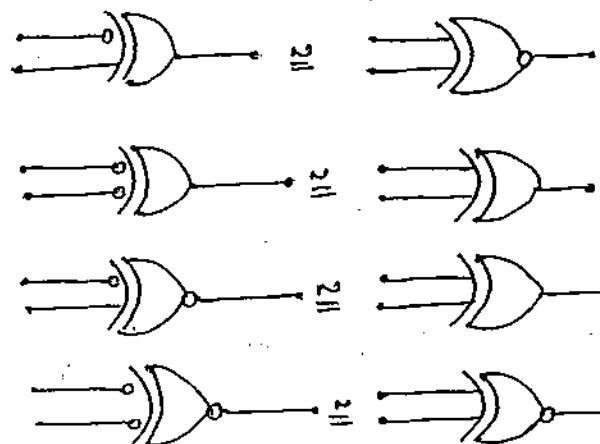
Multi-i/p Ex-NOR gate o/p is 1 when even no. of ones present in i/p.

$$\begin{aligned}
 Y &= \bar{A}B + A\bar{B} \\
 Y &= \bar{x}y + x\bar{y} \\
 Y &= xy + \bar{x}\bar{y} \\
 Y &= x \odot y.
 \end{aligned}$$

$$\begin{aligned}
 Y &= \bar{A}B + A\bar{B} \\
 &= \bar{x}y + x\bar{y} \\
 &= xy + \bar{x}\bar{y} \\
 &= x \oplus y.
 \end{aligned}$$

$$\begin{aligned}
 Y &= \bar{A}\bar{B} + AB \\
 &= \bar{x}\bar{y} + xy \\
 &= xy + \bar{x}\bar{y} \\
 &= x \oplus y.
 \end{aligned}$$

$$\begin{aligned}
 Y &= \bar{A}\bar{B} + AB \\
 &= \bar{x}\bar{y} + xy \\
 &= xy + \bar{x}\bar{y} \\
 &= x \odot y.
 \end{aligned}$$



$$A \oplus B = \bar{A}B + A\bar{B} = \overline{\bar{A}\bar{B} + AB} = \overline{\bar{A}\bar{B} + AB}$$

$$A \odot B = \bar{A}\bar{B} + AB = \overline{A\bar{B} + \bar{A}B} = \overline{A\bar{B} + \bar{A}B}$$

$$A \oplus B \oplus C = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

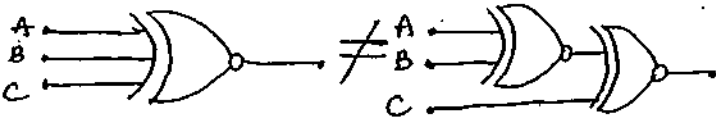
$$A \odot B \odot C = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$$

$$\begin{aligned}
 (A \odot B) \odot C &= (\underbrace{\bar{A}\bar{B} + AB}_P) \odot C \\
 &= P \odot C = \bar{P}\bar{C} + PC \\
 &= \overline{\bar{A}\bar{B} + AB} \bar{C} + (\bar{A}\bar{B} + AB)C \\
 &= (\bar{A}B + A\bar{B})\bar{C} + (\bar{A}\bar{B} + AB)C
 \end{aligned}$$

$$(A \odot B) \odot C = \overline{A} \overline{B} C + \overline{A} B \overline{C} + A \overline{B} \overline{C} + A B C.$$

$$= A \oplus B \oplus C$$

$$\neq A \odot B \odot C.$$



$$(A \oplus B) \oplus C = \underbrace{(\overline{A} B + A \overline{B})}_P \oplus C$$

$$= P \oplus C$$

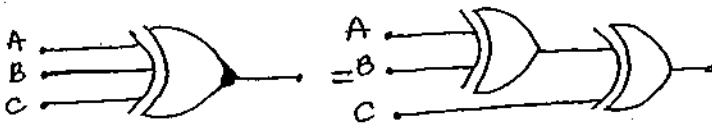
$$= \overline{P} C + P \overline{C}$$

$$= (\overline{A} B + A \overline{B}) C + (\overline{A} B + A \overline{B}) \overline{C}$$

$$= (\overline{A} \overline{B} + A B) C + (\overline{A} \overline{B} + A B) \overline{C}$$

$$= \overline{A} \overline{B} C + A B C + \overline{A} \overline{B} \overline{C} + A B \overline{C}$$

$$= A \oplus B \oplus C$$



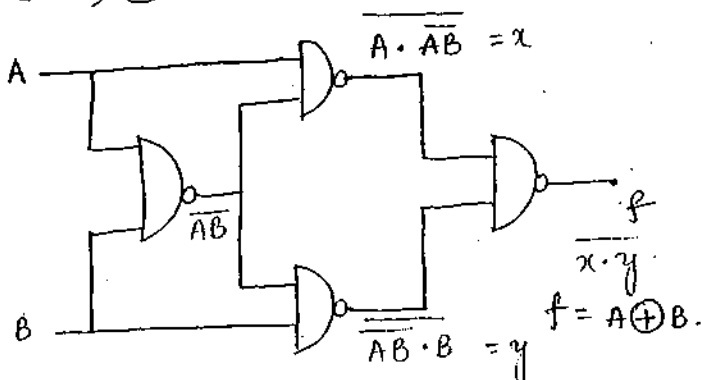
$$(A \oplus B) \oplus A = A \oplus B \oplus A$$

$$= A \oplus A \oplus B$$

$$= 0 \oplus B$$

$$= B$$

$$(A \oplus B) \oplus B = A$$



$$x = \overline{A \cdot \overline{A} B}$$

$$= \overline{A} + A B$$

$$= \overline{A} + B$$

$$y = \overline{A B \cdot B}$$

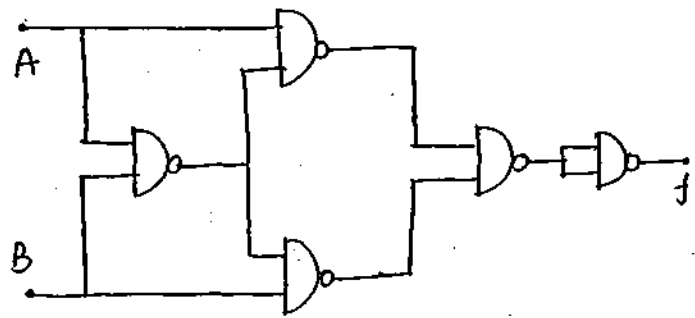
$$= A B + \overline{B}$$

$$= A + \overline{B}$$

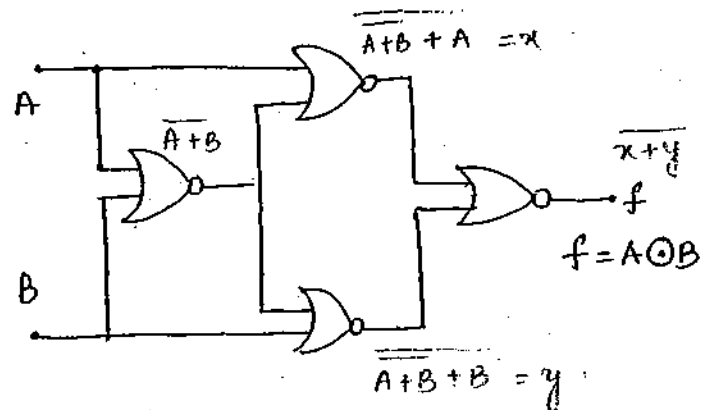
$$f = \overline{x \cdot y} = \overline{(\overline{A} + B)(A + \overline{B})}$$

$$= \overline{(\overline{A} \overline{B} + A B)}$$

$$= \overline{A} B + A \overline{B} = A \oplus B$$



$$f = \overline{A \oplus B} = A \odot B$$



$$x = \overline{A + B + A}$$

$$= \overline{(\overline{A} + B)} \cdot \overline{A}$$

$$= (A + B) \overline{A} = \overline{A} B$$

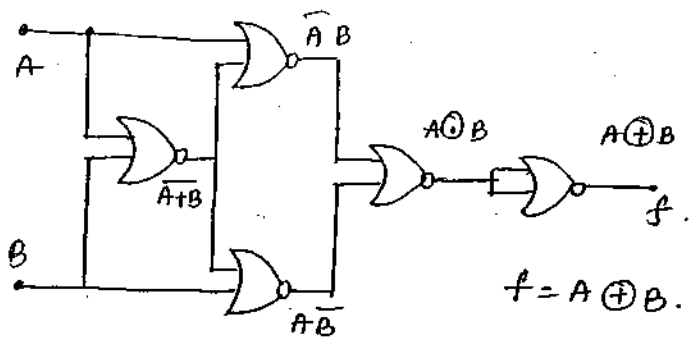
$$y = \overline{A + B + B}$$

$$= (A + B) \overline{B} = A \overline{B}$$

$$f = \overline{x + y} = \overline{\overline{A} B + A \overline{B}}$$

$$= \overline{\overline{A} \overline{B} + A B}$$

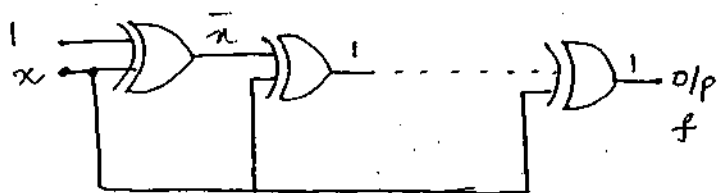
$$= A \odot B$$



Ex:- If 20 EX-OR gates are connected in cascade as shown in figure.

Then o/p 'f' is ____.

- (a) 0 (b) 1 (c) x (d) \bar{x} .



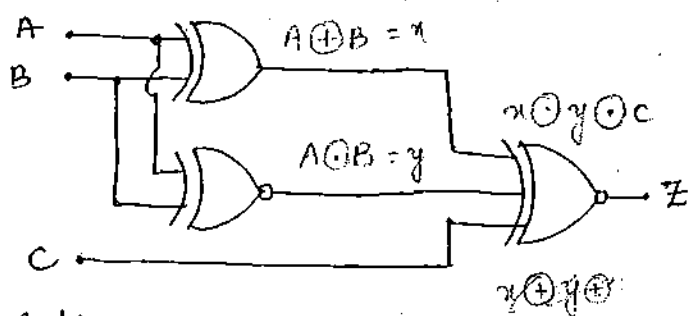
$$1 \oplus x = \bar{1}x + 1\bar{x} = \bar{x}.$$

$$\bar{x} \oplus x = \bar{x}\bar{x} + \bar{x}x = 1.$$

$\therefore f = 1$ as there are even number of EX-OR gates.

Ex:- The i/p combinations A, B, C for which o/p $z = 1$

- (a) 010 (b) 100 (c) 110 (d) 001.



Sol:- $z = x \odot y \odot C$

$$1 = (A \oplus B) \odot (A \odot B) \odot C.$$

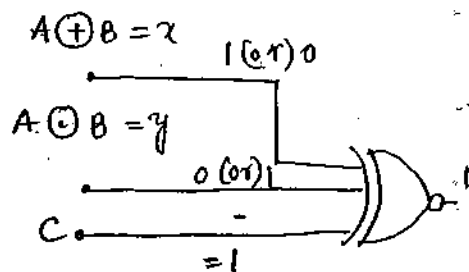
$$x \neq 1.$$

$$1 = [(\bar{A}B + A\bar{B}) \odot (A \odot B) \odot C]$$

For $A \odot B \odot C$.

$$(1 = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC)$$

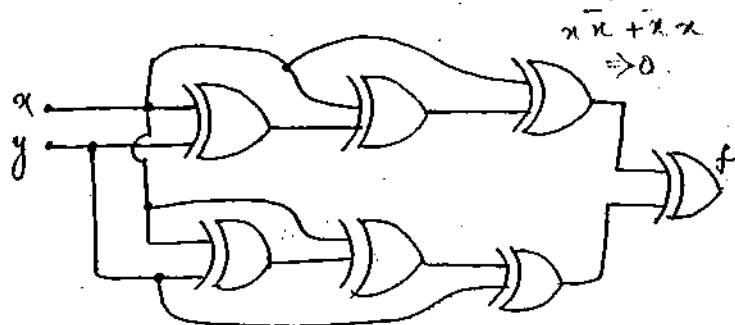
Let



$$\therefore ABC = (101) \text{ or } (011)$$

Ex:- The o/p 'f' in the circuit $0\bar{y} + 1y$.

- (a) 0 (b) 1 (c) $x \oplus y$ (d) $x \odot y$.



Sol:-

$$x \oplus y \oplus x \oplus x \oplus x \oplus y \oplus x \oplus y$$

$$\Rightarrow x \oplus y \oplus 0 \oplus x \oplus y \oplus y \oplus x$$

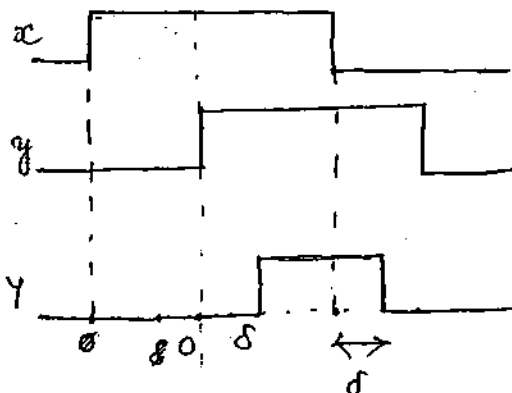
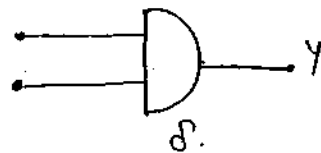
$$\Rightarrow x \oplus y \oplus 0$$

$$\Rightarrow x \oplus y.$$

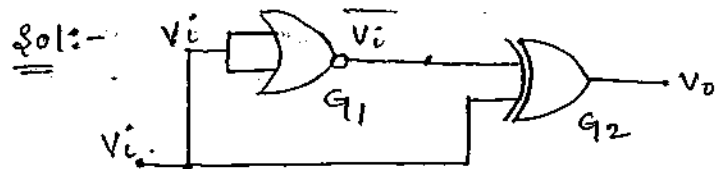
PROPAGATION DELAY OF LOGIC GATES :-

If there is a change in o/p due to changes in the i/p, the change will occur in the o/p after propagation delay time from the point inputs changed.

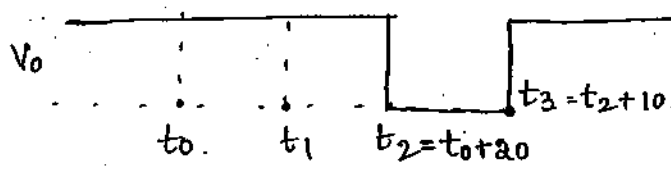
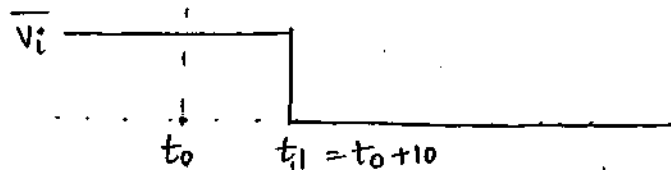
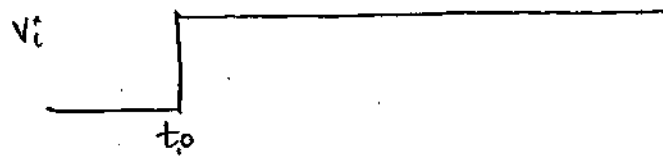
Ex: - x
 y



* Q.37 level-1 pg-35



$G_1 \rightarrow 10 \text{ nsec}$ and $G_2 \rightarrow 20 \text{ nsec}$

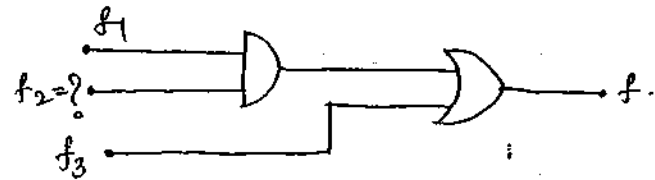


* Q.4 pg no. 35 level 2

Sol: - $f_1 = \sum m(4, 5, 6, 7, 8)$

$f_3 = \sum m(1, 6, 15)$

$f = \sum m(1, 6, 8, 15)$



f_2 contain must 8 and also may contain (1, 6, 15). It should not contain 4.

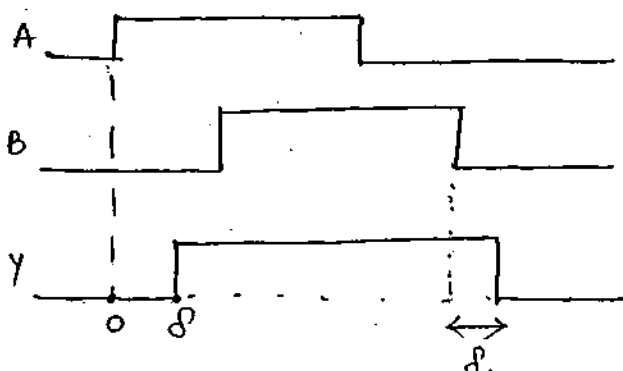
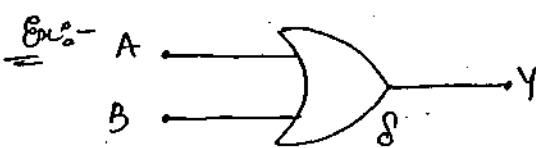
\therefore from options $f_2 = \sum m(6, 8)$

* Q.1 pg no. 35 L-2

Sol: - By assuming the result is also in 2's complement form,

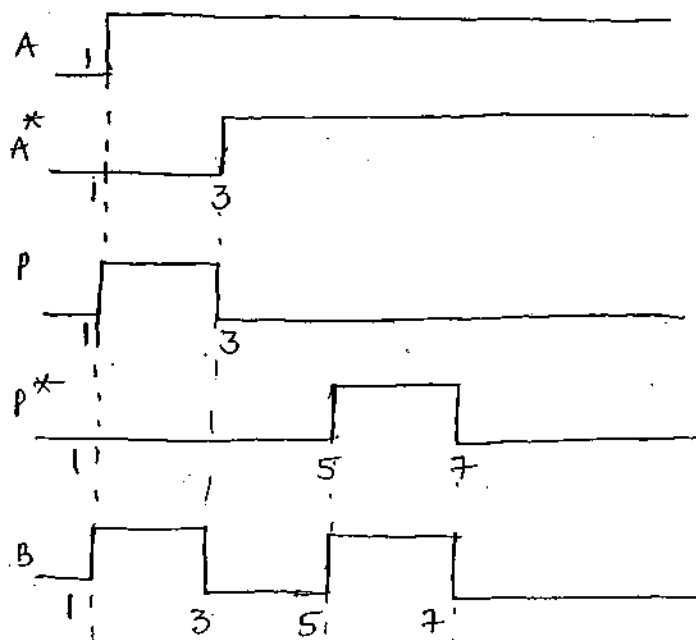
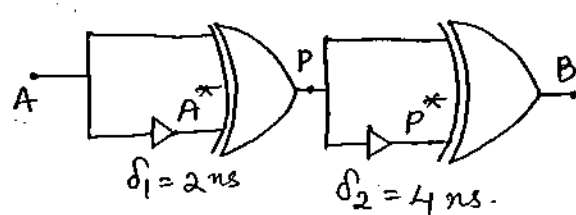
$x, y \rightarrow$ sign bits of inputs.

$z \rightarrow$ sign bit of output.



* Q.5 pg no. 35

Sol: -



Overflow doesn't occur if the two numbers are in the range $(-8 \text{ to } 7)$ during addition (if one is -ve and other is +ve).

If one number is +ve and other number is -ve, then overflow doesn't occur.

If two +ve numbers or two -ve numbers, there is a possibility to occur overflow.

$$\begin{array}{r}
 +4 \\
 +5 \\
 +9 \\
 \hline
 \end{array}
 \begin{array}{r}
 x \textcircled{0} 0 0 \\
 y \textcircled{0} 1 0 1 \\
 \hline
 \textcircled{1} 0 0 1 \\
 \uparrow z \rightarrow \text{overflow}
 \end{array}$$

$$f = xy\bar{z} + \bar{x}\bar{y}z.$$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

$$\begin{array}{r}
 -3 \\
 -5 \\
 -8 \\
 \hline
 \end{array}
 \begin{array}{r}
 x \textcircled{1} 0 1 \\
 y \textcircled{1} 0 1 1 \\
 \hline
 \textcircled{1} 0 0 0 \\
 \Rightarrow \textcircled{1} 0 0 0 \\
 \uparrow z
 \end{array}$$

$$\begin{array}{r}
 -4 \\
 -5 \\
 -9 \\
 \hline
 \end{array}
 \begin{array}{r}
 x \textcircled{1} 1 0 0 \\
 y \textcircled{1} 0 1 1 \\
 \hline
 \textcircled{1} 0 1 1 1 \\
 \downarrow z \rightarrow \text{overflow.}
 \end{array}$$

$$\begin{array}{r}
 +3 \\
 +4 \\
 +7 \\
 \hline
 \end{array}
 \begin{array}{r}
 x \textcircled{0} 0 1 1 \\
 y \textcircled{0} 1 0 0 \\
 \hline
 0 1 1 1 \\
 \uparrow z
 \end{array}$$

K-MAP

→ using K-MAP, the no. of logic gates used decreases and no. of inputs required may also decrease.

Ex:- $\bar{A}\bar{B}C + ABC + A\bar{B}\bar{C} + \bar{A}BC + ABC$

$$\Rightarrow AC(\bar{B}+B) + A\bar{C}(\bar{B}+B) + \bar{A}BC$$

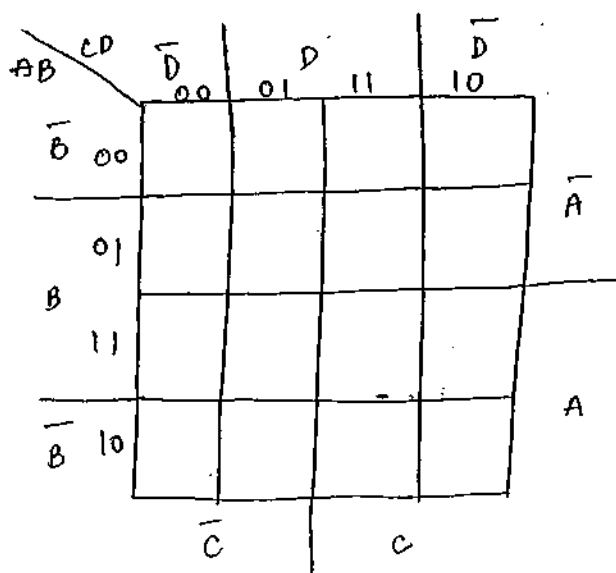
$$\Rightarrow AC + A\bar{C} + \bar{A}BC$$

$$\Rightarrow A(C+\bar{C}) + \bar{A}BC = A + \bar{A}BC = A + BC$$

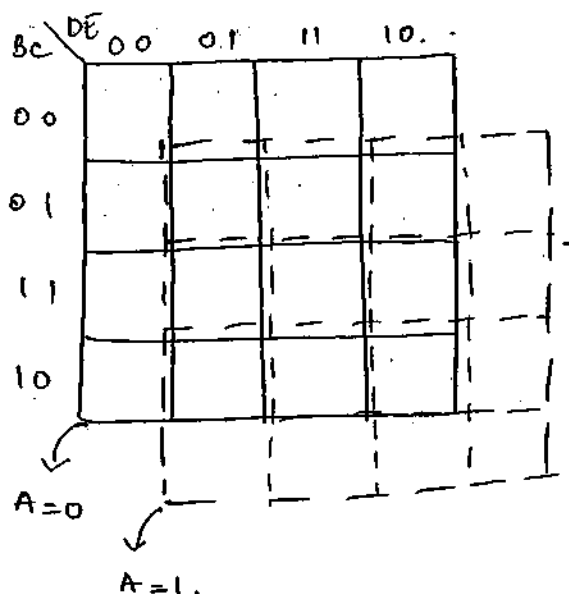
→ In any number of variables, EX-OR and EX-NOR cannot be minimised.

→ In 'n' variable K-map grouping 2^m number of adjacent cubes can eliminate n-variables and the resultant term will be of (n-m) variables.

4-variable K-map :-



5-Variable K-map :-



Prime implicant :-

The smallest possible product term which cannot be reduced further is called prime implicant.

The smallest possible gate count is called minimal expression.

$$\text{gate count} = \frac{\text{Total no. of gates used}}{\text{Total no. of inputs required}}$$

Ex:- $AB + BC + \bar{A}\bar{C}$

\downarrow \downarrow \downarrow
 2 i/p's 2 i/p's 2 i/p's
 AND AND AND

OR gate with 3 i/p's

$$\text{gate count} = 4 + 9 = 13$$

Essential prime implicant :-

If a prime implicant covers at least one minterm which is not covered by other implicants, then that prime implicant is called

$$\bar{B}C + AB + B\bar{C}.$$

④

	BC	00	01	10	11
A	0		1		1
	1		1	1	1

	BC	00	01	11	10
A	0		1	1	
	1		1	1	1

P.I. $\rightarrow 2$.

EPI $\rightarrow 2$.

minimal expression $\rightarrow C + AB$.

⑤

	CD	00	01	11	10
AB	00	1			1
	01	1	1		1
	11	1	1		1
	10	1	1	1	1

P.I. $\rightarrow 3+1=4$.

EPI $\rightarrow 3$

Minimal expression \rightarrow

$$\bar{D} + A\bar{B} + B\bar{C}$$

⑥ $f(A,B,C,D) = \sum m(1,3,5,7,8,9,12,13)$

	CD	00	01	11	10
AB	00		1	1	
	01		1	1	
	11	1	1		
	10	1	1		

P.I. $\rightarrow 3$

EPI $\rightarrow 2$.

Minimal expression \rightarrow

$$A\bar{C} + \bar{A}D$$

The code for gray codes is

	CD	00	01	11	10
AB	00	0	1	3	2
	01	4	5	7	6
	11	12	13	15	14
	10	8	9	11	10

⑦

	CD	00	01	11	10
AB	00	1		1	1
	01	1			
	11				1
	10	1	1		1

P.I. $\rightarrow 8$

EPI $\rightarrow 4$

minimal expression \rightarrow

$$\bar{A}\bar{C}\bar{D} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + ACD$$

⑧

	CD	00	01	11	10
AB	00	1		1	1
	01	1			
	11				1
	10	1	1		1

This quad cannot be a essential prime implicant as there is possibility.

P.I. $\rightarrow 5$

E.P.I. $\rightarrow 4$.

minimal expression \rightarrow

$$\bar{A}\bar{C}\bar{D} + A\bar{B}\bar{C} + ACD + \bar{A}\bar{B}C$$

⑧ $f(A, B, C, D) = \{0, 2, 4, 5, 6, 7, 8, 10, 13, 15\}$.

CD \ AB	00	01	11	10
00	1			1
01	1	1	1	1
11		1	1	
10	1			1

P₁ $\rightarrow 3+1=4$.

EP₁ $\rightarrow 2$.

Minimal expression \rightarrow

$\bar{B}\bar{D} + \bar{A}B + BD$.

⑨ $f(A, B, C, D) = \sum m(1, 3, 4, 5, 10, 11, 12, 13, 14, 15)$.

CD \ AB	00	01	11	10
00		1	1	
01	1	1		
11	1	1	1	1
10			1	1

P₁ $\rightarrow 6$

EP₁ $\rightarrow 2$

Minimal expression \rightarrow

$B\bar{C} + AC + \bar{A}\bar{B}D$.

Minimal expression in POS :-

Ex:- Find minimal expression.

BC \ A	00	01	11	10
0	0	0		
1			0	0

minimal expression in POS \rightarrow

$f = (A+B)(\bar{A}+\bar{B}) = 0 + A\bar{B} + \bar{A}B + 0$

$= A\bar{B} + \bar{A}B$.

Minimal expression in SOP \rightarrow

BC \ A	00	01	11	10
0			1	1
1	1	1		

$f = A\bar{B} + \bar{A}B$.

SOP \equiv POS.

Ex:- Find minimal expression in SOP and POS for the fn

$f(A, B, C, D) = \sum m(1, 4, 5, 6, 12, 13, 14, 15)$.

AB \ CD	00	01	11	10
00		1		
01	1	1		1
11	1	1	1	1
10				

minimal expression in SOP \rightarrow
 $\bar{A}\bar{C}D + B\bar{D} + AB$

AB \ CD	00	01	11	10
00	0		0	0
01			0	
11				
10	0	0	0	0

redundant

minimal expression in POS \rightarrow

$$\bar{B}\bar{D} \neq \bar{A}\bar{C}D + AB$$

$$(B+D)(A+\bar{C}+\bar{D})(\bar{A}+B)$$

$$= (AB + B\bar{C} + B\bar{D} + AD + \bar{C}D)(\bar{A}+B)$$

$$= \bar{A}B\bar{C} + \bar{A}B\bar{D} + \bar{A}\bar{C}D + \underline{AB} + \underline{B\bar{C}} + \underline{B\bar{D}} + \underline{ADB} + \underline{B\bar{C}D}$$

$$= AB + B\bar{C} + B\bar{D} + \bar{A}B\bar{C} + \bar{A}\bar{C}D$$

$$= B(A+\bar{C}) + B\bar{C} + B\bar{D} + \bar{A}\bar{C}D$$

$$= AB + B\bar{C} + B\bar{D} + \bar{A}\bar{C}D$$

redundant

Ex:- find minimal expression for SOP and POS.

$\Sigma M(0, 2, 4, 6, 8, 9, 12, 14)$

AB \ CD	00	01	11	10
00	0			0
01	0			0
11	0			0
10	0	0		

redundant

minimal expression in POS \rightarrow

$$(A+D)(\bar{B}+D)(\bar{A}+B+C)$$

$$\Rightarrow (\bar{A}\bar{B} + AD + \bar{B}D + D)(\bar{A}+B+C)$$

$$\Rightarrow (\bar{A}\bar{B} + D)(\bar{A}+B+C)$$

$$\Rightarrow \bar{A}D + BD + \bar{A}B\bar{C} + \bar{A}CD$$

\Rightarrow

redundant

AB \ CD	00	01	11	10
00		1	1	
01		1	1	
11		1	1	
10			1	1

The minimal expression is

$$(\bar{A}D) + BD + A\bar{B}C.$$

→ redundant.

Ex:- $\pi M(1, 3, 5, 7, 12, 13, 14, 15).$

AB \ CD	00	01	11	10
00	1			1
01	1			1
11				
10	1	1	1	1

$$SOP \rightarrow \bar{A}\bar{D} + A\bar{B}$$

AB \ CD	00	01	11	10
00		0	0	
01		0	0	
11	0	0	0	0
10				

$$POS \rightarrow (\bar{A} + \bar{B})(A + \bar{D}).$$

$$= \bar{A}\bar{D} + A\bar{B} + \bar{B}\bar{D}$$

$$= \bar{A}\bar{D} + A\bar{B} \rightarrow \text{Consensus theorem.}$$

Incompletely specified function (Don't care):-

Function is defined as one for some combinations, defined as zero for some other combinations, for remaining combinations of i/p, o/p is not specified, means we are not expecting the o/p for these i/p combinations. We don't care what system gives the o/p for these combinations which are called don't cares. If don't cares are helping in minimising the expression, take the help of don't cares else don't care about don't cares. We don't need to cover all the don't cares.

NOTE:-

If a grouping is cover in K-MAP is covering extra only

don'tcares compare to other groupings which we are considering is the minimal expression then we don't need to consider this grouping in the minimal expression.

Ex:-

	BC	00	01	11	10
A	0		1	1	
	1			1	

$$f = \bar{A}C + BC$$

function is reduced.

$$f = C$$

	BC	00	01	11	10
A	0		1	1	
	1		X	1	

In this example, by considering the don'tcare, the function is minimised. So consideration of don'tcare is necessary.

Ex:-

	BC	00	01	11	10
A	0	0			0
	1	0			0

$$f = C$$

	BC	00	01	11	10
A	0	0			0
	1	0	X		0

If we consider don'tcare

$$f = C(\bar{A} + B)$$

After considering the don't care, the function expanded. So don'tcare should not be considered in this case.

Ex:- Find the minimal expression in SOP and POS for the function

$$F(A, B, C, D) = \sum m(0, 1, 2, 7, 15) + X(3, 5, 11)$$

Sol:-

	CD	00	01	11	10
AB	00	1	1	X	1
	01		X	1	
	11			1	
	10			X	

This not required as it is not involving extra minterms.

$$F = \bar{A}\bar{B} + CD$$

	CD	00	01	11	10
AB	00			X	
	01	0	X		0
	11	0	0		0
	10	0	0	X	0

By not considering any don'tcares,

$$F = (\bar{A} + C)(\bar{B} + D)(\bar{A} + D)$$

By considering don'tcare,

$$F = (\bar{B} + D)(\bar{A} + C)(\bar{A} + B)$$

Ex:- $F(A, B, C, D) = \sum m(0, 8, 9, 12, 13) + \times(2, 10, 14, 15)$.

Sol:-

CD \ AB	00	01	11	10
00	1			X
01				
11	1	1	X	X
10	1	1		X

$$F = \bar{B}\bar{D} + A\bar{C}$$

CD \ AB	00	01	11	10
00		0	0	X
01	0	0	0	0
11			X	X
10			0	X

$$\begin{aligned} F &= (\bar{C})(A + \bar{B})(A + \bar{D}) \\ &= \bar{C}(A + A\bar{D} + A\bar{B} + \bar{B}\bar{D}) \\ &= \bar{C}(\bar{B}\bar{D} + A) \\ &= A\bar{C} + \bar{B}\bar{C}\bar{D} \end{aligned}$$

Functionality SOP may not be equal to POS in the presence of don't cares. If don't cares all are used in one form and not used in other form, then functionality will be equal.

* Q.3 pgn 0.35

Sol:- $f(v, w, x, y, z) = x + \bar{y}z$.

$$= 2^4 + 2^3$$

$$= 16 + 8 = 24 - 4 = 20.$$

(or) $2^4 + 2^3 - 2^2 = 16 + 8 - 4 = 20.$

wx \ yz	00	01	10	11
00				
01				
11				
10				

$v=0 \rightarrow v=1$

No. of minterms excluding redundant terms is

Ex:- $f(v, w, x, y, z) = w + x + \bar{y}z$

$$= 2^4 + 2^4 + 2^3 - 2^3 - 2^2 - 2^2 + 2^1$$

$$= 16 + 16 + 8 - 8 - 4 - 4 + 2$$

$$= 34 - 8$$

$$= 24.$$

Number of minterms excluding redundant terms is,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A)$$

$$+ n(A \cap B \cap C).$$

Ex:- Find minimal expression in POS for F implemented in the K-map shown

Sol:- $F(A, B, C, D) = ?$

- (a) $A + C$ (b) $A \cdot C$ (c) $\bar{A} + \bar{C}$ (d) $\bar{A} \cdot \bar{C}$

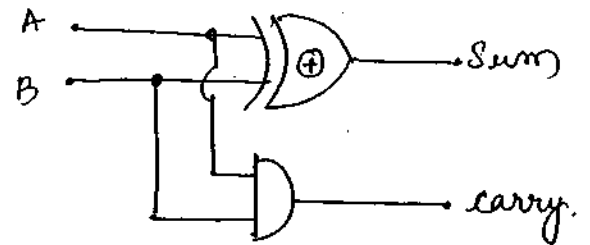
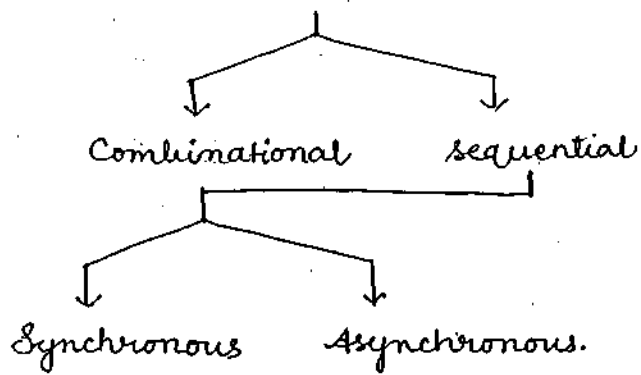
CD \ AB	00	01	11	10
00	0	X	X	0
01	X	0	X	0
11	0	X	X	1
10	0	0	1	X

↓ A

→ C

$$F = A \cdot C$$

DIGITAL CIRCUITS

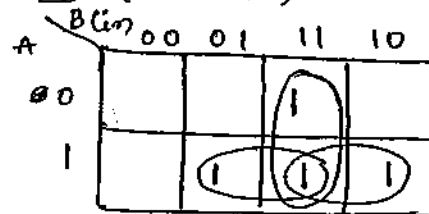


FULL ADDER :-

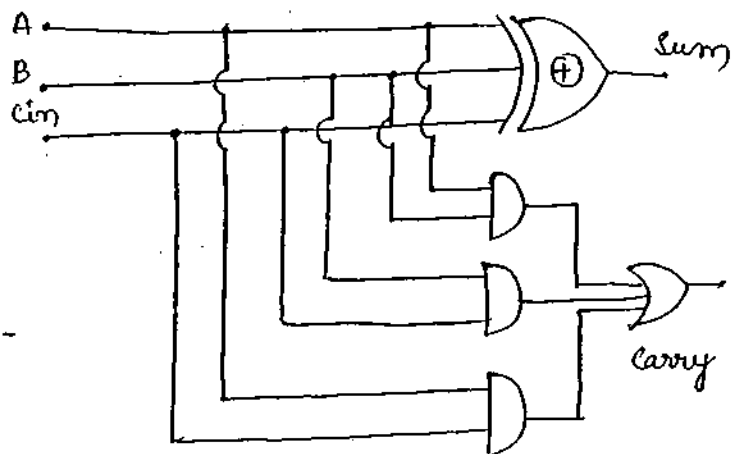
	A	B	C _i	S	C _o
0	0	0	0	0	0
1	0	0	1	1	0
2	0	1	0	1	0
3	0	1	1	0	1
4	1	0	0	1	0
5	1	0	1	0	1
6	1	1	0	0	1
7	1	1	1	1	1

$$S = \sum m(1, 2, 4, 7) = A \oplus B \oplus C_{in}$$

$$C_o = \sum m(3, 5, 6, 7)$$



$$C_o = AC_{in} + AB + BC_{in}$$



The o/p depends on the combination of present inputs is called combinational circuit.

The o/p depends on the sequence of combination of i/p's. For sequence we need a memory element. It is called sequential circuit.

COMBINATIONAL CIRCUITS

It doesnot needs any memory element.

HALF ADDER :-

	A	B	S	C _o
0	0	0	0	0
1	0	1	1	0
2	1	0	1	0
3	1	1	0	1

$$S = \sum m(1, 2)$$

$$C_o = \sum m(3)$$

$$S = \bar{A}B + A\bar{B} = A \oplus B$$

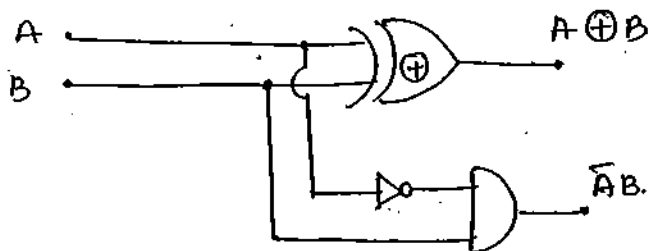
$$C_o = AB$$

HALF SUBTRACTOR :-

	A	B	D	B ₀
0	0	0	0	0
1	0	1	1	1
2	1	0	1	0
3	1	1	0	0

$$D = \sum m(1, 2) = A \oplus B$$

$$B_0 = \sum m(1) = \bar{A}B$$



FULL SUBTRACTOR :-

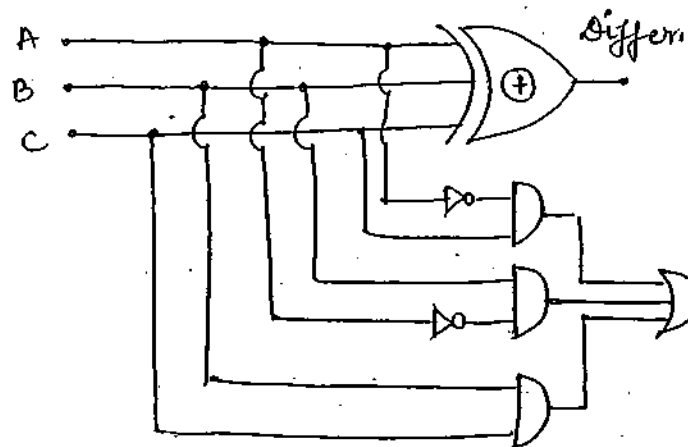
	A	B	C _i	D	B ₀
0	0	0	0	0	0
1	0	0	1	1	1
2	0	1	0	1	1
3	0	1	1	0	1
4	1	0	0	1	0
5	1	0	1	0	0
6	1	1	0	0	0
7	1	1	1	1	1

$$D = \sum m(1, 2, 4, 7) = A \oplus B \oplus C_i$$

$$B_0 = \sum m(1, 2, 3, 7)$$

A \ B C _i	00	01	11	10
0		1	1	1
1			1	

$$B_0 = \bar{A}C_i + \bar{A}B + BC_i$$



Ex:- Design a combinational ckt which is finding 2's complement of 3-bit binary.

	B ₂	B ₁	B ₀	T ₂	T ₁	T ₀
0	0	0	0	0	0	0
1	0	0	1	1	1	1
2	0	1	0	1	1	0
3	0	1	1	1	0	1
4	1	0	0	1	0	0
5	1	0	1	0	1	1
6	1	1	0	0	1	0
7	1	1	1	0	0	1

$$T_2 = \sum m(1, 2, 3, 4)$$

$$T_1 = \sum m(1, 2, 5, 6)$$

$$T_0 = \sum m(1, 3, 5, 7)$$

B ₂ \ B ₁ B ₀	00	01	11	10
0		1	1	1
1	1			

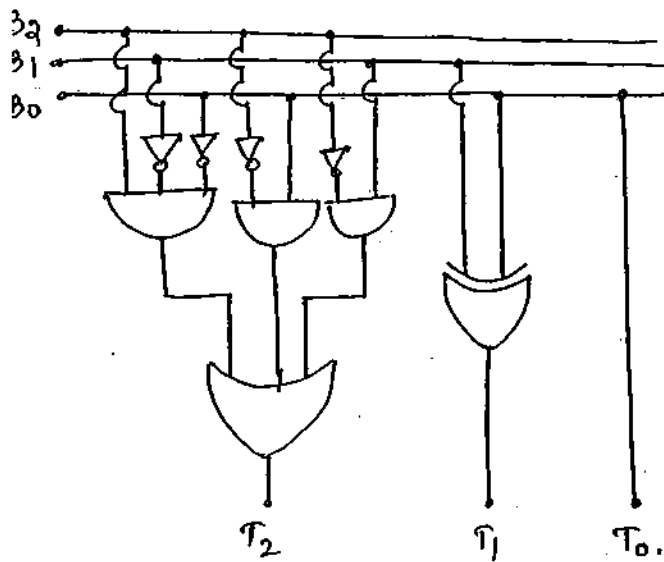
$$T_2 = B_2 \bar{B}_1 \bar{B}_0 + \bar{B}_2 B_0 + \bar{B}_2 B_1$$

		$B_1 B_0$			
		00	01	11	10
T_1	0		1		1
	1		1		1

$$T_1 = \bar{B}_1 B_0 + B_1 \bar{B}_0 = B_1 \oplus B_0$$

		$B_1 B_0$			
		00	01	11	10
T_0	0		1	1	
	1		1	1	

$$T_0 = B_0$$



Ex:- Design a combinational ckt which converts 3-bit binary to gray.

	B_2	B_1	B_0	G_2	G_1	G_0	
0	0	0	0	0	0	0	0
1	0	0	1	0	0	1	1
2	0	1	0	0	1	1	3
3	0	1	1	0	1	0	2
4	1	0	0	1	1	0	6
5	1	0	1	1	1	1	7
6	1	1	0	1	0	1	5
7	1	1	1	1	0	0	4

$$G_2 = \sum m(4, 5, 6, 7)$$

$$G_1 = \sum m(2, 3, 4, 5)$$

$$G_0 = \sum m(1, 2, 5, 6)$$

		$B_1 B_0$			
		00	01	11	10
G_2	0				
	1	1	1	1	1

$$G_2 = B_2$$

		$B_1 B_0$			
		00	01	11	10
G_1	0			1	1
	1	1	1		

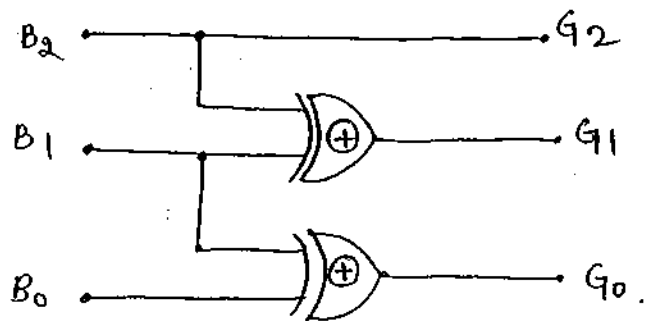
$$G_1 = \bar{B}_2 B_1 + B_2 \bar{B}_1$$

$$G_1 = B_2 \oplus B_1$$

		$B_1 B_0$			
		00	01	11	10
G_0	0		1		1
	1		1		1

$$G_0 = \bar{B}_1 B_0 + B_1 \bar{B}_0$$

$$G_0 = B_1 \oplus B_0$$



Ex:- Conversion of Gray to binary.

	G_2	G_1	G_0	B_2	B_1	B_0
0	0	0	0	0	0	0
1	0	0	1	0	0	1
3	0	1	1	0	1	0
2	0	1	0	0	1	1
6	1	1	0	1	0	0
7	1	1	1	1	0	1
5	1	0	1	1	1	0
4	1	0	0	1	1	1

$$B_2 = \sum m(6, 7, 5, 4).$$

$$B_1 = \sum m(3, 2, 5, 4).$$

$$B_0 = \sum m(1, 2, 4, 7).$$

$$B_0 = A$$

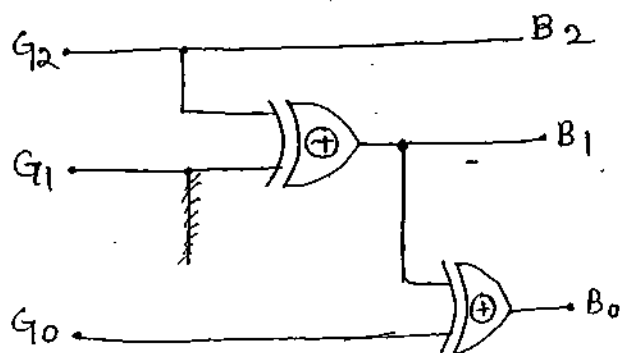
$$B_2 = G_2.$$

$$B_1 = G_2 \oplus G_1.$$

$B_0 = G_1 \oplus G_0$

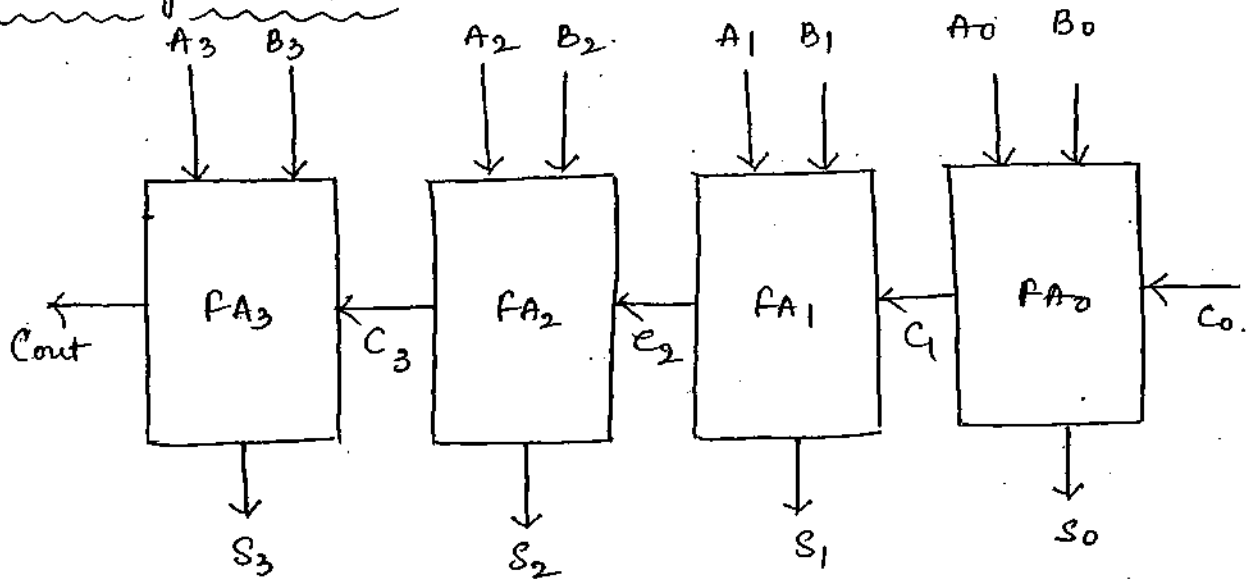
G_2	G_1	G_0	01	11	10
0			1		1
1	1			1	

$$B_0 = G_2 \oplus G_1 \oplus G_0.$$



N-BIT ADDER :-

Ripple carry adder :-



For a 4-bit ripple carry adder, the time taken to give the sum output is $(3t_c + t_s)$ and for carry output is $(4t_c)$.

For a n -bit ripple carry adder, time taken to give o/p of

* sum $\rightarrow (n-1)t_c + t_s$. [for $t_s \geq t_c$]
(expecting only sum) $(n-1)t_c + t_s$ \leftarrow $(n)t_c$ [Expecting sum & final carry]. $\frac{16 \times 12}{192}$
Eg:- 16 bit addition, $t_s = 15 \text{ ns}$ and $t_c = 12 \text{ ns}$.

$$\text{sum} \rightarrow 15 \times 12 \times 10^{-9} + 15 \times 10^{-9} = 180 + 15 = 195 \text{ ns.}$$

$$\text{carry} \rightarrow 16 \times 12 \times 10^{-9} = 192 \text{ ns.}$$

Eg:- 4 bit addition, $t_c = 1.5 \text{ ns}$, $t_s = 1 \text{ ns}$.

$$\text{sum} \rightarrow 3 \times 1.5 + 1 \rightarrow 5.5 \text{ ns.}$$

CARRY LOOK AHEAD ADDER :-

$$C_0 = P_0 + G_0$$

$$C_n = P_n C_{n-1} + G_n$$

$$C_1 = P_0 C_0 + G_0$$

$$= C_0 (A_0 \oplus B_0) + G_0$$

$$= C_0 (A_0 \oplus B_0) + A_0 B_0$$

	A	B	C _i	C _o	S
0	0	0	0	0	0
1	0	0	1	0	1
2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	0	1
5	1	0	1	1	0
6	1	1	0	1	0
7	1	1	1	1	1

$$C_0 = 0$$

$$C_0 = C_i$$

$$C_0 = 1$$

propagation of i/p carry to o/p,

$$P = A \oplus B$$

carry is generated

$$G = AB$$

$$C_2 = P_1 C_1 + G_1$$

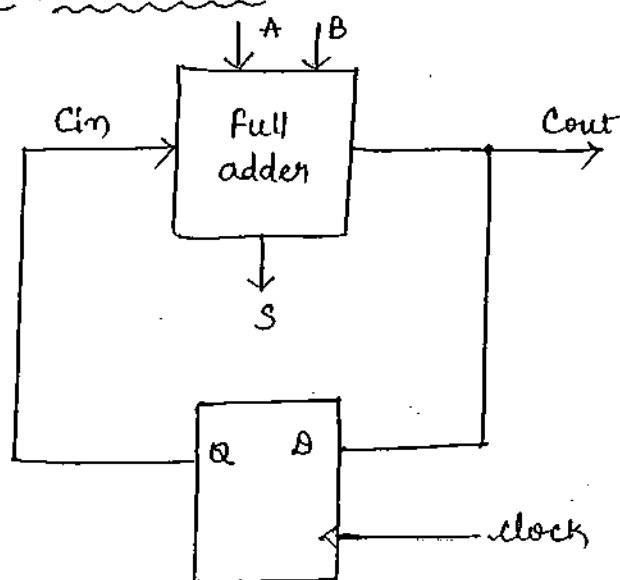
$$= (A_1 \oplus B_1) (C_0 (A_0 \oplus B_0) + A_0 B_0) + A_1 B_1$$

$$C_3 = P_2 C_2 + G_2$$

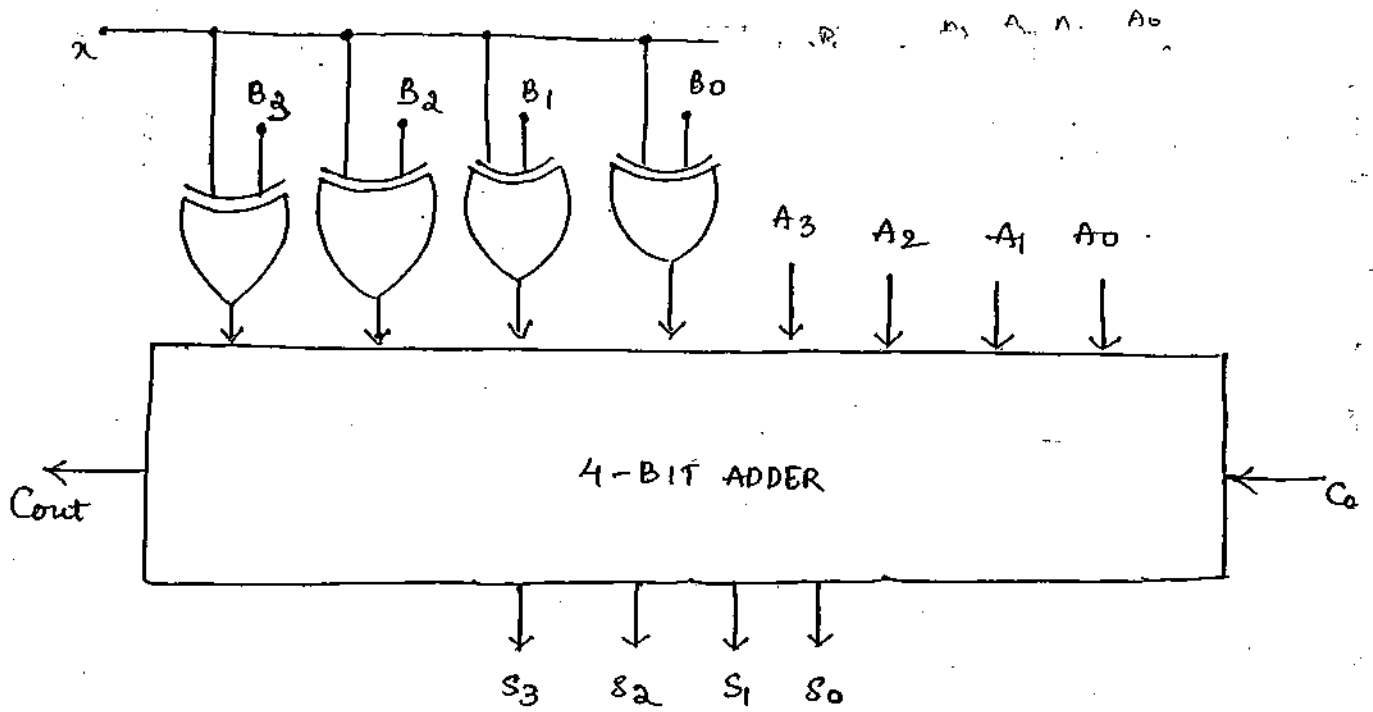
$$= (A_2 \oplus B_2) \{ (A_1 \oplus B_1) [C_0 (A_0 \oplus B_0) + A_0 B_0] + A_1 B_1 \} + A_2 B_2$$

The carry generation itself taking more time. So carry look ahead adder cannot be used for more number of bits.

CARRY SAVE ADDER :-



clock period must be greater than or equal to full adder delay.



for $x=0$ & $C_0=0$

$$A+B$$

$A + 2's \text{ complement of } B \text{ is}$
 $(A-B)$.

for $x=0$ & $C_0=1$

$$A+B+1$$

for $x=1$ & $C_0=0$

$$A+\bar{B}$$

for $x=1$ & $C_0=1$

$$A+\bar{B}+1$$

$$A + 2's \text{ complement of } B$$

$$A-B$$



COMPARATOR :-

A	B	A > B	A < B	A = B
0	0	0	0	1
0	1	0	1	0
1	0	1	0	0
1	1	0	0	1

$$A > B \Rightarrow A\bar{B}$$

$$A < B \Rightarrow \bar{A}B$$

$$A = B \Rightarrow \bar{A}\bar{B} + AB \Rightarrow A \odot B$$

$$f(A > B) = \sum m(4, 8, 9, 12, 13, 14)$$

A ₁ A ₀ \ B ₁ B ₀	00	01	11	10
00				
01				
11	1			1
10	1	1		

$$f(A > B) = A_1\bar{B}_1 + A_0\bar{B}_1\bar{B}_0 + A_1A_0\bar{B}_0$$

$$f(A < B) = \sum m(1, 2, 3, 6, 7, 11)$$

A ₁ A ₀ \ B ₁ B ₀	00	01	11	10
00		1	1	1
01			1	1
11				
10			1	

$$f(A < B) = \bar{A}_1\bar{A}_0B_0 + \bar{A}_0B_1B_0 + \bar{A}_1B_1$$

$$f(A = B) = \sum m(0, 5, 10, 15)$$

A ₁ A ₀ \ B ₁ B ₀	00	01	11	10
00	1			
01		1		
11			1	
10				1

$$f(A = B) = \bar{A}_1\bar{A}_0\bar{B}_1\bar{B}_0 + \bar{A}_1A_0\bar{B}_1B_0 + A_1A_0B_1B_0 + A_1\bar{A}_0B_1\bar{B}_0$$

A ₁ A ₀	B ₁ B ₀	A > B	A < B	A = B
00	00	0	0	1
01	00	0	1	0
10	00	1	0	0
11	00	1	0	0
00	01	0	1	0
01	01	0	0	1
10	01	1	0	0
11	01	1	0	0
00	10	0	1	0
01	10	0	0	1
10	10	1	0	0
11	10	1	0	0
00	11	0	1	0
01	11	0	0	1
10	11	1	0	0
11	11	0	0	1

$$A > B \Rightarrow$$

$$A_1 \bar{B}_1 + (A_1 \odot B_1) A_0 \bar{B}_0$$

$$A < B \Rightarrow$$

$$\bar{A}_1 B_1 + (A_1 \odot B_1) \bar{A}_0 B_0$$

$$A = B \Rightarrow$$

$$(A_1 \odot B_1) (A_0 \odot B_0)$$

comparator o/p for 4 bit inputs;

$$A < B \Rightarrow$$

$$\{ \bar{A}_3 B_3 + (A_3 \odot B_3) \bar{A}_2 B_2 + (A_3 \odot B_3) (A_2 \odot B_2) \bar{A}_1 B_1 + (A_3 \odot B_3) (A_2 \odot B_2) (A_1 \odot B_1) \bar{A}_0 B_0 \}$$

$$A > B \Rightarrow$$

$$\{ A_3 \bar{B}_3 + (A_3 \odot B_3) A_2 \bar{B}_2 + (A_3 \odot B_3) (A_2 \odot B_2) A_1 \bar{B}_1 + (A_3 \odot B_3) (A_2 \odot B_2) (A_1 \odot B_1) A_0 \bar{B}_0 \}$$

$$A = B \Rightarrow$$

$$\{ (A_3 \odot B_3) (A_2 \odot B_2) (A_1 \odot B_1) (A_0 \odot B_0) \}$$

BCD to Excess-3 :-

BCD (B₃B₂B₁B₀) Ex₃ - BCD

0	0000	
1	0001	
2	0010	
3	0011	
4	0100	
5	0101	
6	0110	
7	0111	
8	1000	
9	1001	

representation
of 9025 is

10010000000100101

	B ₃	B ₂	B ₁	B ₀	E ₃	E ₂	E ₁	E ₀
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0

$$E_3 = \sum m(5, 6, 7, 8, 9) + d(10, 11, 12, 13, 14, 15)$$

$$E_2 = \sum m(1, 2, 3, 4, 9) + d(10, 11, 12, 13, 14, 15)$$

$$E_1 = \sum m(0, 3, 4, 7, 8) + d(10, 11, 12, 13, 14, 15)$$

$$E_0 = \sum m(0, 2, 4, 6, 8) + d(10, 11, 12, 13, 14, 15)$$

$B_3 B_2$ \ $B_1 B_0$	00	01	11	10
00				
01		1	1	1
11	x	x	x	x
10	1	1	x	x

$$E_3 = B_3 + B_2 B_0 + B_2 B_1$$

$B_3 B_2$ \ $B_1 B_0$	00	01	11	10
00		1	1	1
01	1			
11	x	x	x	x
10		1	x	x

redundant

$$E_2 = \overline{B_2} B_0 + \overline{B_2} B_1 + B_2 \overline{B_1} \overline{B_0}$$

$B_3 B_2$ \ $B_1 B_0$	00	01	11	10
00	1		1	
01	1		1	
11	x	x	x	x
10	1		x	x

$$E_3 = \overline{B_1} \overline{B_0} + B_0 B_1$$

$B_3 B_2$ \ $B_1 B_0$	00	01	11	10
00	1			1
01	1			1
11	x	x	x	x
10	1		x	x

$$E_0 = \overline{B_0}$$

Excess 3 to BCD :-

	E_3	E_2	E_1	E_0	B_3	B_2	B_1	B_0
3+0	0	0	1	1	0	0	0	0
3+1	0	1	0	0	0	0	0	1
3+2	0	1	0	1	0	0	1	0
3+3	0	1	1	0	0	0	1	1
3+4	0	1	1	1	0	1	0	0
3+5	1	0	0	0	0	1	0	1
3+6	1	0	0	1	0	1	1	0
3+7	1	0	1	0	0	1	1	1
3+8	1	0	1	1	1	0	0	0
3+9	1	1	0	0	1	0	0	1

$$B_3 = \sum m(11, 12) + d(0, 1, 2, 13, 14, 15)$$

$$B_2 = \sum m(7, 8, 9, 10) + d(0, 1, 2, 13, 14, 15)$$

$$B_1 = \sum m(5, 6, 9, 10) + d(0, 1, 2, 13, 14, 15)$$

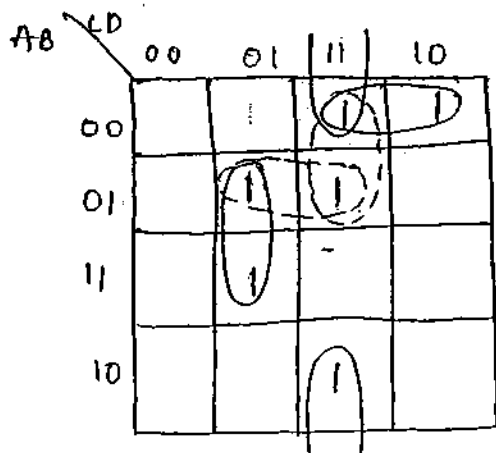
$$B_0 = \sum m(4, 6, 8, 10, 12) + d(0, 1, 2, 13, 14, 15)$$

Eg:- Design a combinational ckt giving o/p,

$$F(A, B, C, D) = 1,$$

when decimal eqv of i/p binary decimal combination is prime number.

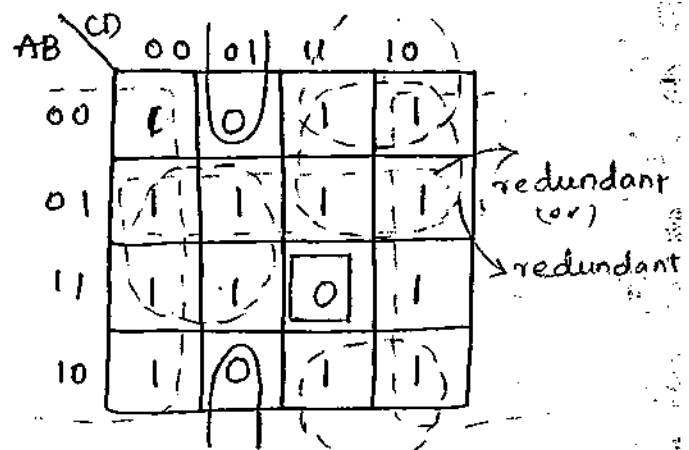
	A	B	C	D	P
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0



$$P = B\bar{C}\bar{D} + \bar{A}\bar{B}C + \bar{B}C\bar{D} + \bar{A}BD$$

$$(or) \bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C + \bar{B}C\bar{D} + \bar{A}CD$$

Eg:- Design a combinational ckt. which is giving o/p one if the decimal equivalent of 4-bit binary is even (or) prime.



$$f(A, B, C, D) = (B + C + \bar{D})(\bar{A} + \bar{B} + \bar{C} + \bar{D})$$

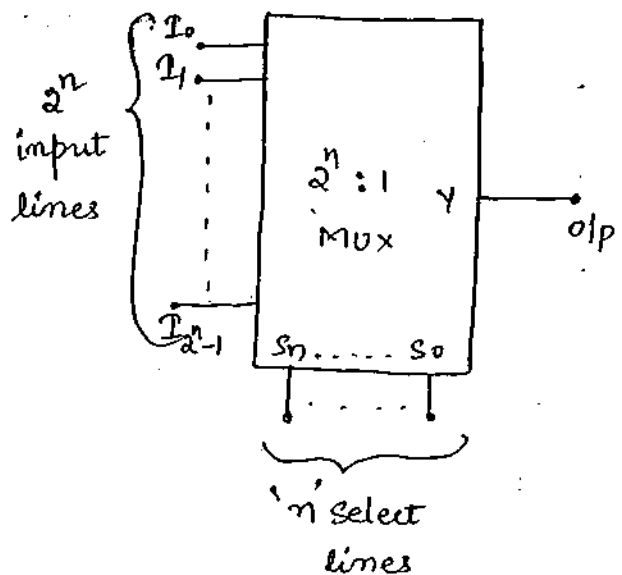
(or)

$$f(A, B, C, D) = \bar{D} + B\bar{C} + \bar{B}C + \bar{A}B$$

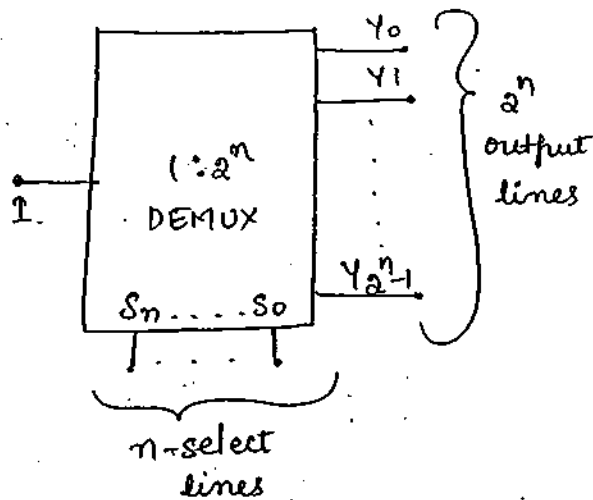
(or)

$$\bar{A}C$$

MULTIPLEXER :-



DEMULTIPLEXER :-

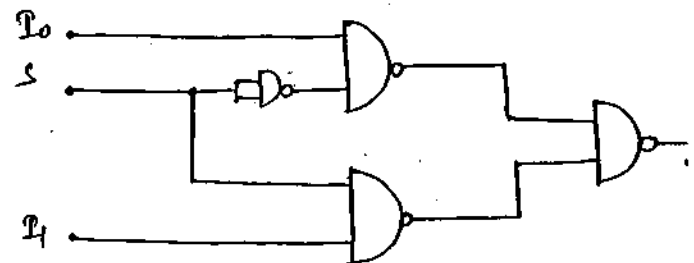
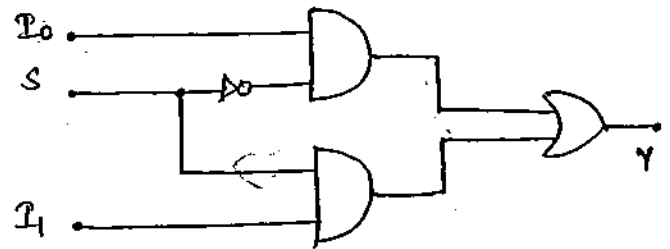


$s \backslash I$	I_0	I_1	I_2	I_3
0	0	1	1	0
1	0	1	1	0

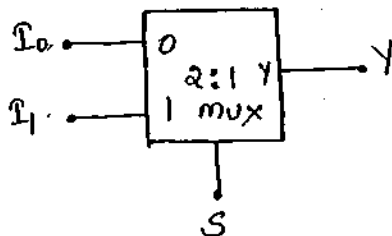
→ reduce

$$Y = \bar{s}I_0 + sI_1$$

Implementation of 2:1 MUX using logic gate :-



2:1 Multiplexer :-



$2:1 \Rightarrow 2^1:1$
1 select line.

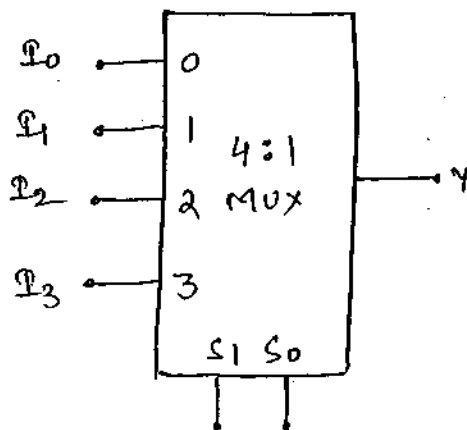
S	Y
0	I_0
1	I_1

$$Y = \bar{s}I_0 + sI_1$$

2:1 Multiplexer :-

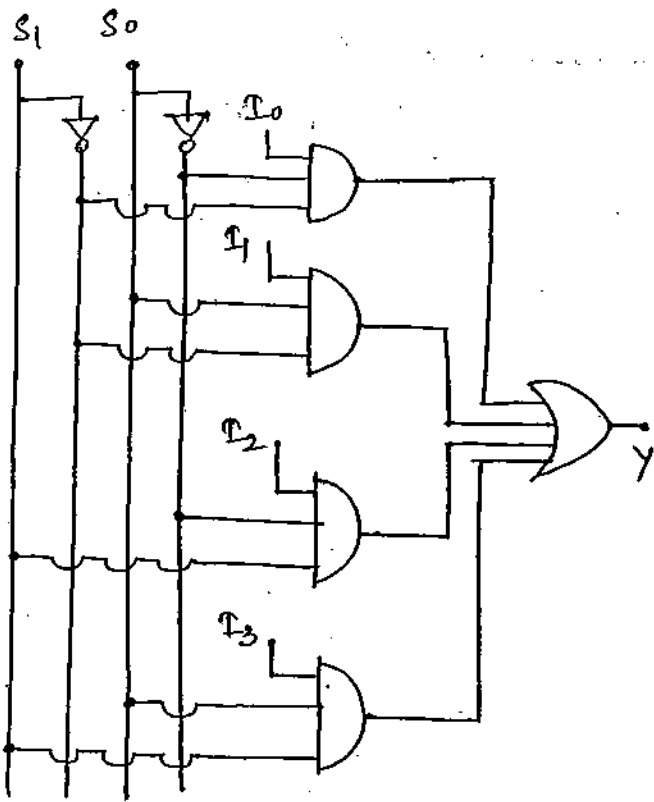
S	I_1	I_0	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

4:1 Multiplexer :-

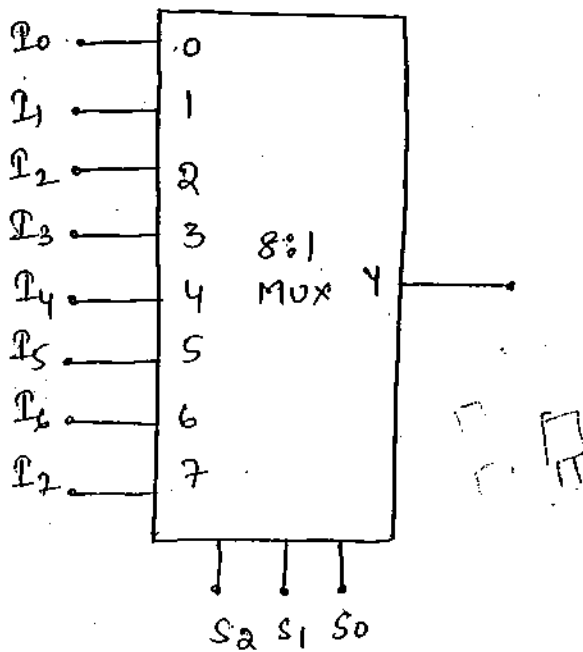


$$Y = \bar{s}_1\bar{s}_0I_0 + \bar{s}_1s_0I_1 + s_1\bar{s}_0I_2 + s_1s_0I_3$$

4:1
2² = 4
2² = 4



8:1 Multiplexer:-



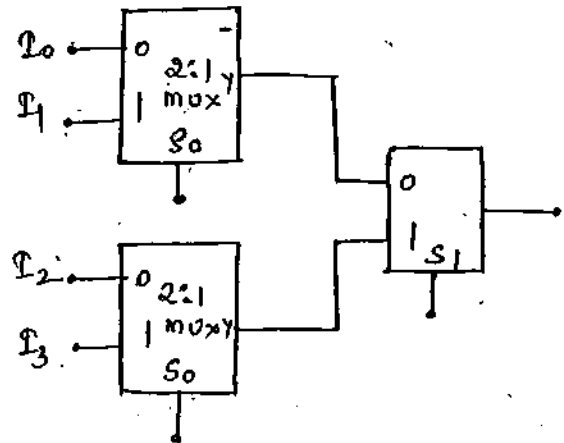
$$Y = \bar{S}_2 \bar{S}_1 \bar{S}_0 I_0 + \bar{S}_2 \bar{S}_1 S_0 I_1 + \bar{S}_2 S_1 \bar{S}_0 I_2 + \bar{S}_2 S_1 S_0 I_3 + S_2 \bar{S}_1 \bar{S}_0 I_4 + S_2 \bar{S}_1 S_0 I_5 + S_2 S_1 \bar{S}_0 I_6 + S_2 S_1 S_0 I_7$$

Eg:- Design a 4:1 Mux using minimum no. of 2:1 multiplexers.

$$4:1$$

$$\frac{4}{2} + 1 = 2 + 1 = 3$$

∴ 3 2:1 mux are required to represent 4:1 mux.



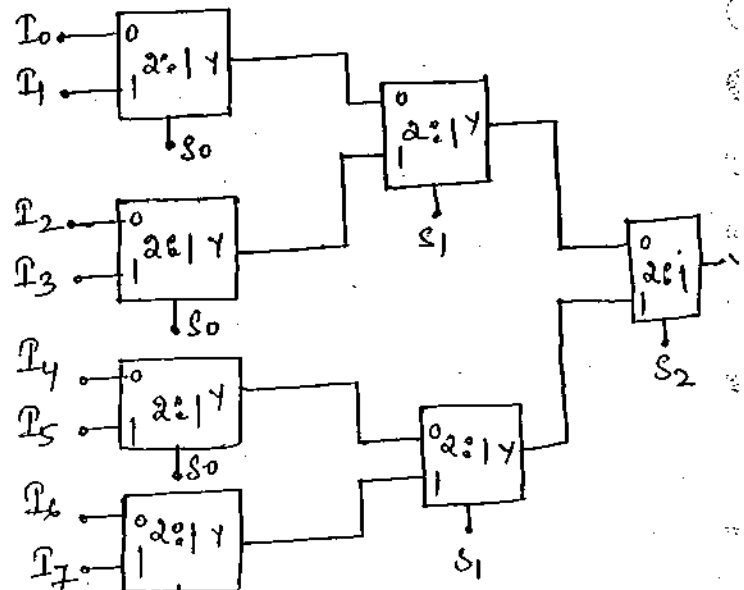
When a higher order mux is represented using a lower order mux, then the LSB select line is represented to the first range mux.

Ex:- Design a 8:1 mux using minimum no. of 2:1 MUX.

$$\frac{8}{2} + \frac{4}{2} + 1$$

$$\Rightarrow 4 + 2 + 1$$

⇒ 7 muxes are required.

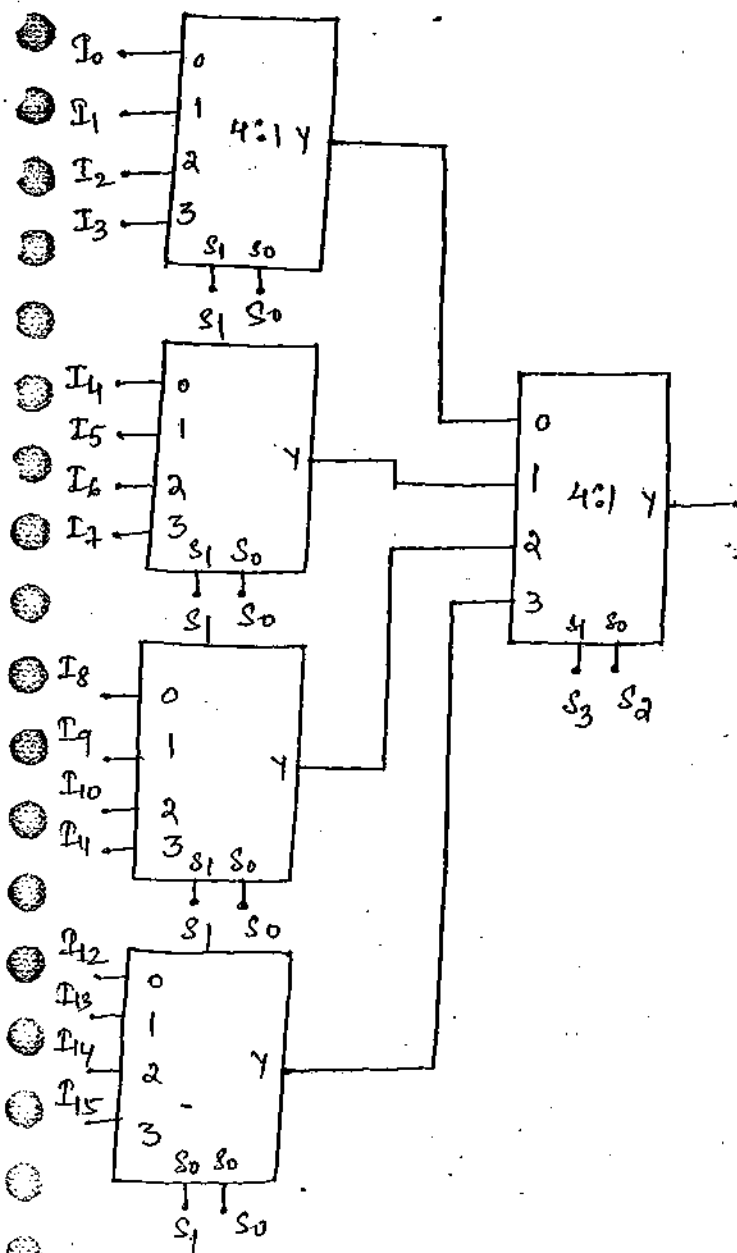


NOTE :-

To construct a $2^n : 1$ Mux, using min no. of $2 : 1$ Mux, then minimum of $(2^n - 1)$ multiplexers are required.

Eg:- Design $16 : 1$ Multiplexer with min no. of $2 : 1$ Mux.

$$\frac{16}{4} + \frac{4}{4} \Rightarrow 4 + 1 = 5.$$



Eg:- find min. no. of $4 : 1$ Mux required to construct $1024 : 1$ Mux.

$$\frac{2^{10}}{4} + \frac{2^8}{4} + \frac{2^6}{4} + \frac{2^4}{4} + \frac{2^2}{4}$$

$$\Rightarrow 256 + 64 + 16 + 4 + 1$$

$$\Rightarrow 341.$$

Eg:- find min no. of $4 : 1$ Mux required to construct $4096 : 1$ and find min no. of $8 : 1$ Mux required to construct $4096 : 1$

Sol:- $4 : 1$ Mux \rightarrow

$$\frac{4096}{4} + \frac{1024}{4} + \frac{256}{4} + \frac{64}{4}$$

$$\frac{16}{4} + \frac{4}{4}$$

$$\Rightarrow 1024 + 256 + 64 + 16 + 4 + 1$$

$$\Rightarrow 1024 + 341$$

$$\Rightarrow 1365.$$

$8 : 1$ Mux \rightarrow

$$\frac{4096}{8} + \frac{512}{8} + \frac{64}{8} + \frac{8}{8}$$

$$\Rightarrow 512 + 64 + 8 + 1$$

$$\Rightarrow 585.$$

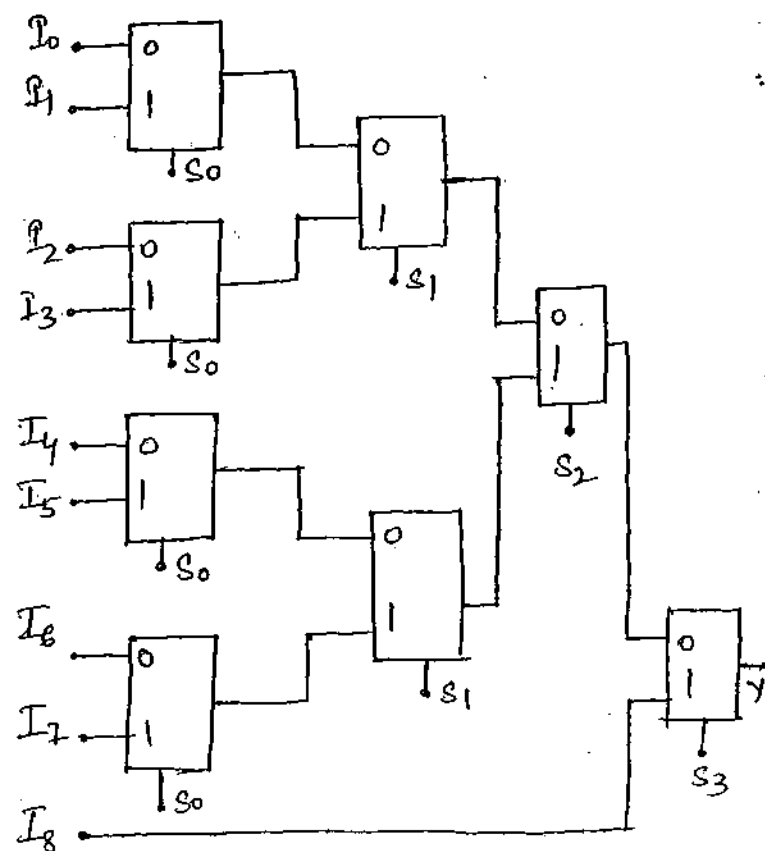
Eg:- find min. no. of $2 : 1$ Mux required to construct $9 : 1$ Mux

Sol:-

S_3	S_2	S_1	S_0	Y
0	0	0	0	I_0
0	0	0	1	I_1
0	0	1	0	I_2
0	0	1	1	I_3
0	1	0	0	I_4
0	1	0	1	I_5
0	1	1	0	I_6
0	1	1	1	I_7
1	0	0	0	I_8

When S_3

When $S_3 = 0$, I_0 to I_7 is transferred to the o/p, When $S_3 = 1$, I_8 is passed to the o/p directly.



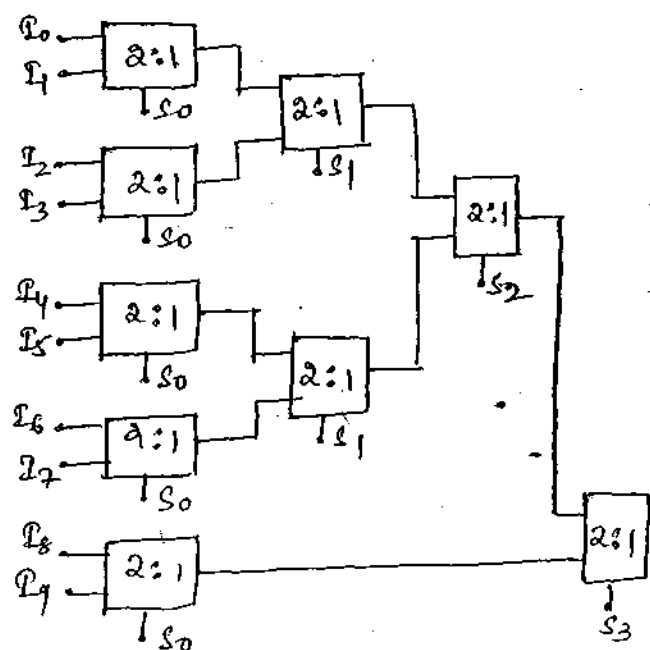
Ex: Design a 10:1 mux using with min no. of 2:1.

S_3	S_2	S_1	S_0	Y
0	0	0	0	I_0
0	0	0	1	I_1
0	0	1	0	I_2
0	0	1	1	I_3
0	1	0	0	I_4
0	1	0	1	I_5
0	1	1	0	I_6
0	1	1	1	I_7
1	0	0	0	I_8
1	0	0	1	I_9

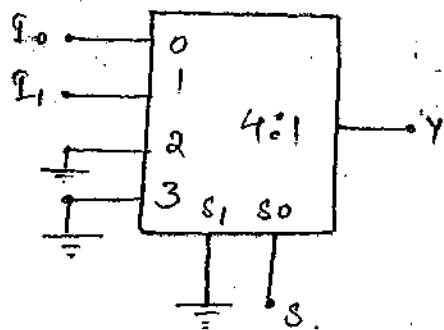
When $S_3 = 0$, I_0 to I_7 is transferred to o/p.

$$\frac{10}{2} + \frac{5}{2} + 1$$

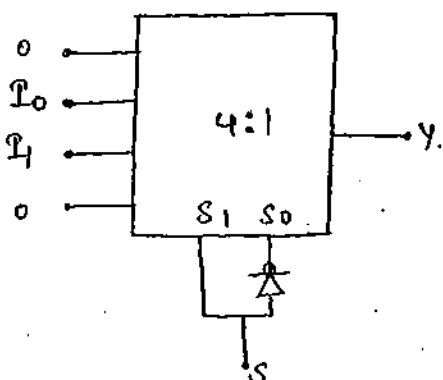
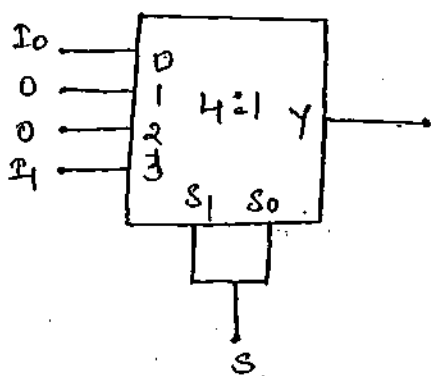
$$\Rightarrow 5 + 2 + 1 + 1 = 9.$$



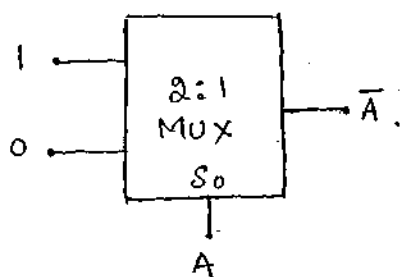
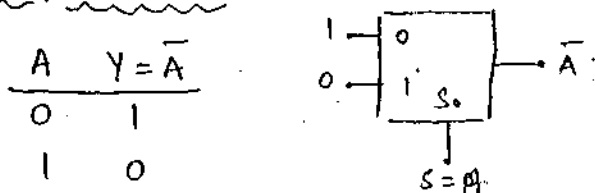
Eg:- Design 2:1 Mux using 4:1 Mux.



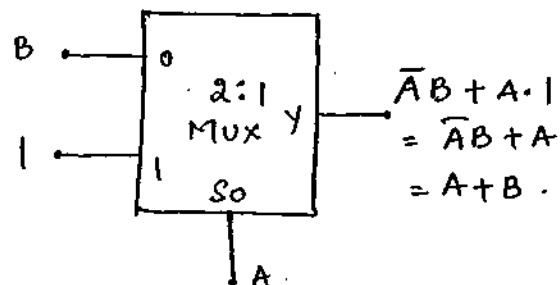
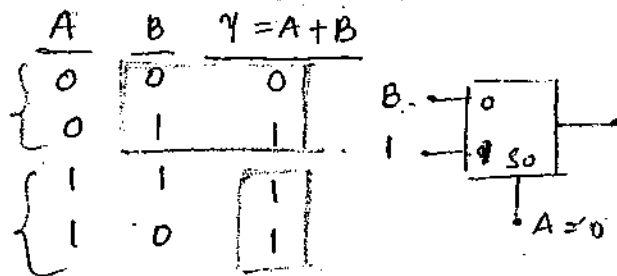
Sol:- There are many possibilities for this implementation.



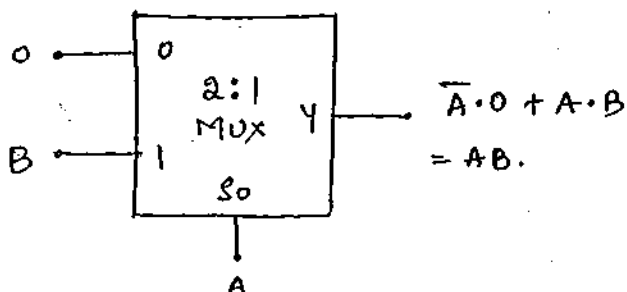
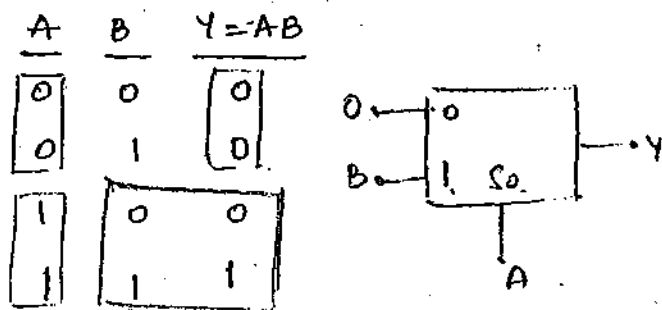
NOT gate implementation using multiplexer:-



OR gate using MUX:-



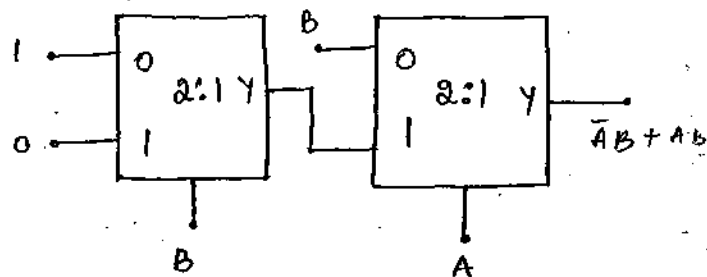
AND gate using MUX:-



As it is possible to implement three basic gates using MUX, so we can say that MUX is an universal gate.

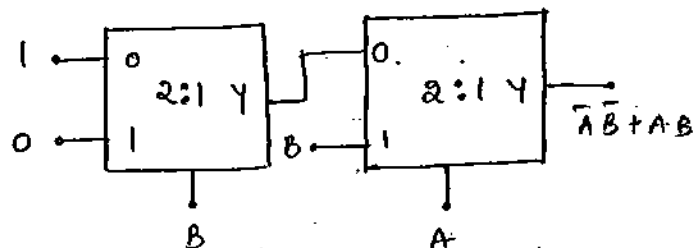
NOR gate using MUX:-

A	B	$Y = \overline{A+B} = \overline{A} \cdot \overline{B}$
0	0	1
0	1	0
1	0	0
1	1	0



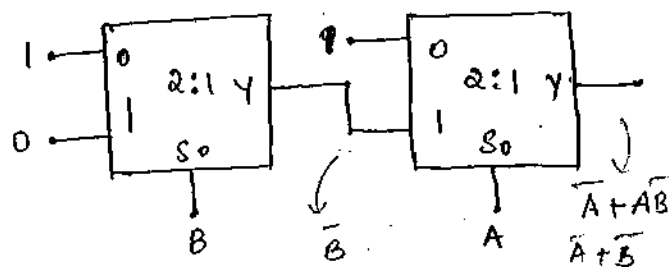
2:1p Ex-NOR Gate using MUX:-

A	B	$Y = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0



NAND gate using MUX:-

A	B	$Y = \overline{AB} = \overline{A} + \overline{B}$
0	0	1
0	1	1
1	0	1
1	1	0



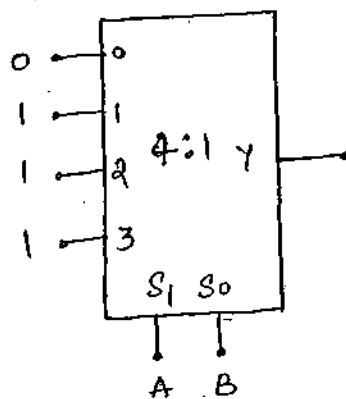
2:1p Ex-OR gate using MUX:-

A	B	$Y = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

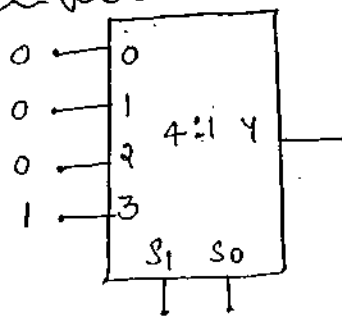
Implementation of logic gates using

4:1 Mux:-

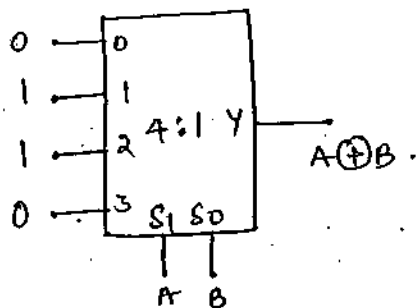
(i) OR gate:-



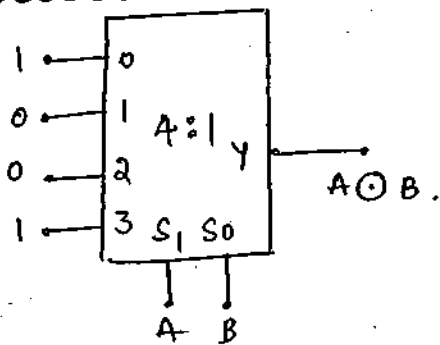
(ii) AND gate:-



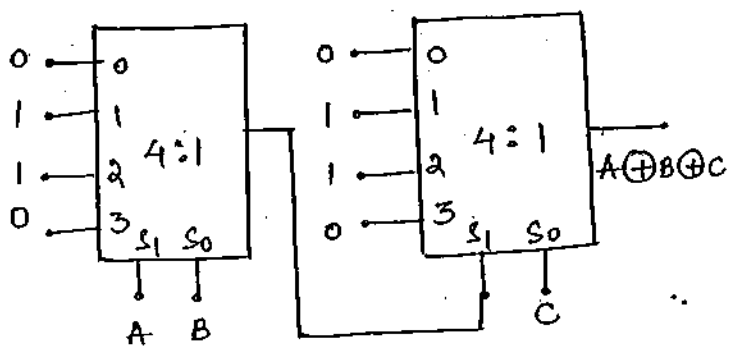
(iii) 2 i/p EXOR :-



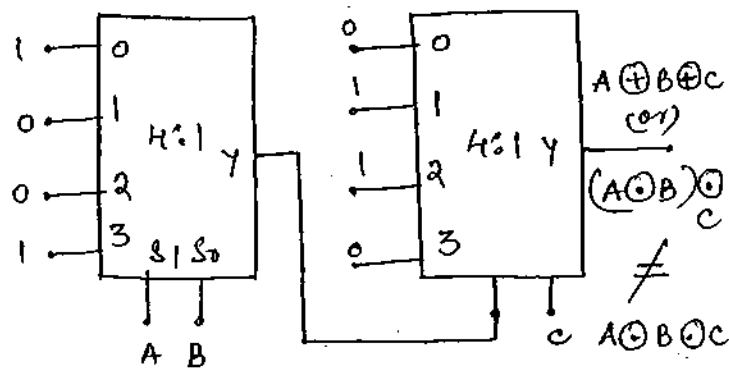
(iv) 2 i/p EX-NOR :-



(v) 3 i/p EX-OR :-



(vi) 3 i/p EX-NOR :-



Eg :- Implement

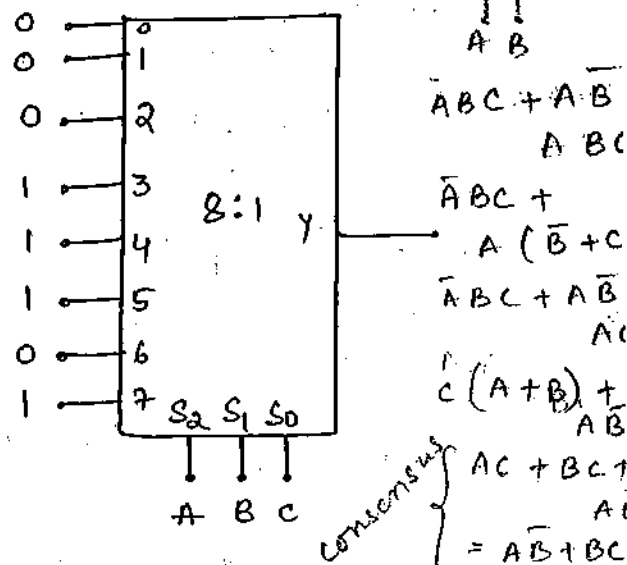
$$f(A, B, C) = \sum m(3, 4, 5, 7) \text{ using}$$

(i) @ 8:1 MUX

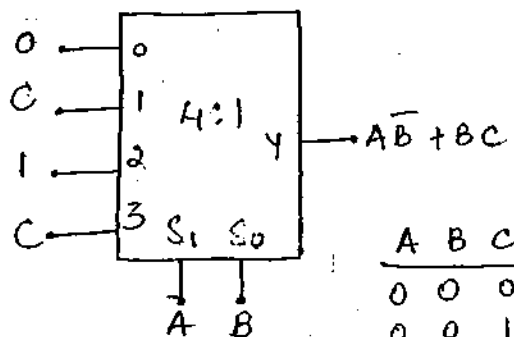
(b) 4:1 MUX and some

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

By using 8:1 MUX ;

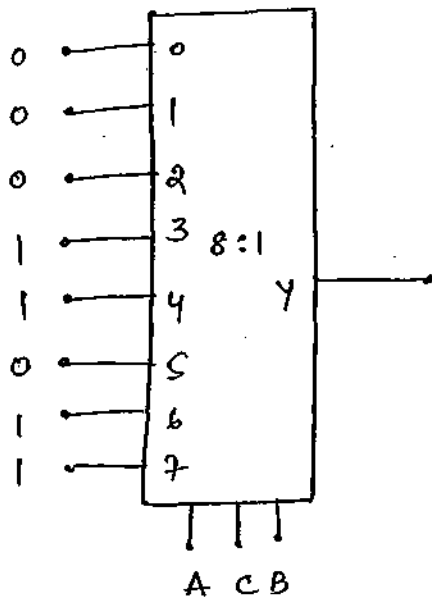


By using 4:1 MUX and some combinational gates,

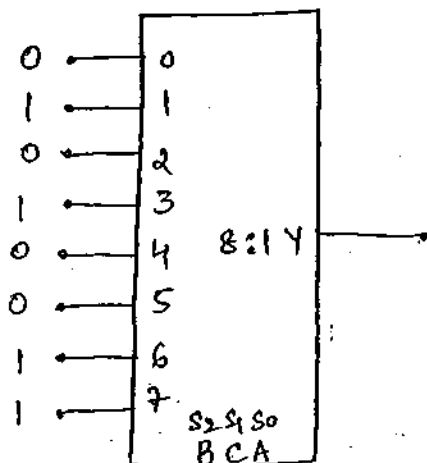


A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Using 8:1 Mux by interchanging
select lines,



	X ₀	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇
ACB	000	001	010	011	100	101	110	111
	0	2	1	(3)	(4)	6	(5)	(7)



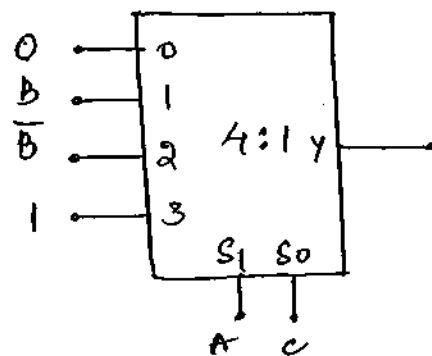
	X ₀	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇
CBA	000	001	010	011	100	101	110	111
	0	(4)	2	6	1	(5)	(3)	(7)

	X ₀	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇
BCA	000	001	010	011	100	101	110	111
	0	(4)	1	(5)	2	6	(3)	(7)

using 4:1 MUX using 'A' and 'C'
as select lines.

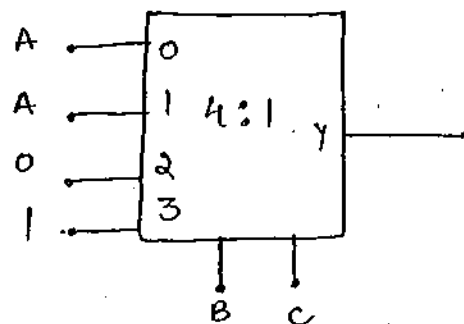
	00	01	10	11
AC	0	1	(4)	(5)
	2	(3)	6	(7)

	A	B	C	f
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1



using 4:1 MUX using 'B' and 'C'
as select lines,

	00	01	10	11
BC	0	1	2	(3)
	(4)	(5)	6	(7)



using 4:1 MUX using 'C' and 'A'
as select lines,

	00	01	10	11
CA	0	(4)	1	(5)
	2	6	(3)	(7)

using \bar{C} and \bar{B} as select lines,

		CB			
A		00	01	10	11
\bar{A}		0	2	1	③
A		④	6	⑤	⑦
		↓	↓	↓	↓
		A	0	A	1

Eg:- Implement the function
 $F(A, B, C, D)$

	A	B	C	D	F
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	1

Eg:- Using only ^{one} 4:1 MUX we can implement

- (a) All 3 variable and 2-variable functions.
- (b) Only 2 variable functions.
- (c) All 2-variable functions and some of 3-variable functions
- (d) none.

Eg:- Implement the function
 $f(A, B, C, D) = \sum m(1, 2, 3, 4, 6, 7, 8, 11, 13, 14)$

- using (a) 8:1 MUX and some combinational gates. Use A, C, D as select lines.
- (b) 4:1 MUX and some combinational gates.

Sol:-

(a) $f(A, B, C, D) = \sum m(1, 2, 3, 4, 6, 7, 8, 11, 13, 14)$

		ACD							
		000	001	010	011	100	101	110	111
B		0	①	②	③	⑧	9	10	⑪
B		④	5	⑥	⑦	12	⑬	⑭	15
		↓	↓	↓	↓	↓	↓	↓	↓
B		\bar{B}	\bar{B}	1	1	\bar{B}	B	B	\bar{B}
		I ₀	I ₁	I ₂	I ₃	I ₄	I ₅	I ₆	I ₇

By using DBA as select lines,

		DBA							
		000	001	010	011	100	101	110	111
D		0	⑧	④	12	①	9	5	⑬
C		②	10	⑥	⑭	③	⑪	⑦	15
		↓	↓	↓	↓	↓	↓	↓	↓
C		\bar{C}	\bar{C}	1	C	1	C	C	\bar{C}
		I ₀	I ₁	I ₂	I ₃	I ₄	I ₅	I ₆	I ₇

$$f(A, B, C, D) = \sum m(1, 2, 3, 4, 6, 7, 8, 11, 13, 14)$$

AB	00	01	10	11
$\bar{C}\bar{D}(00)$	0	(4)	(8)	12
$\bar{C}D(01)$	(1)	5	9	(13)
$C\bar{D}(10)$	(2)	(6)	10	(14)
$CD(11)$	(3)	(7)	(11)	15

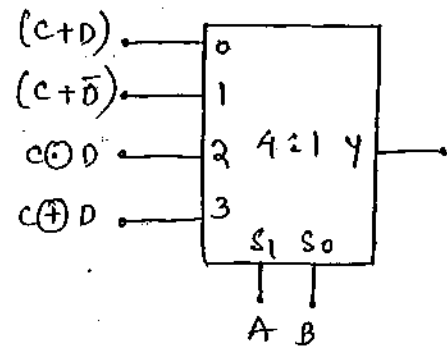
$\bar{C}D + C\bar{D} + CD$ $= C + D$	$\bar{C}\bar{D} + C\bar{D} + CD$ $= C + \bar{D}$	$\bar{C}\bar{D} + CD$ $= \bar{C}D + C\bar{D}$
---	---	--

$$P_0 \rightarrow (C + D)$$

$$P_1 \rightarrow (C + \bar{D})$$

$$P_2 \rightarrow C \oplus D$$

$$P_3 \rightarrow C \oplus \bar{D}$$



By considering B, D as select lines,

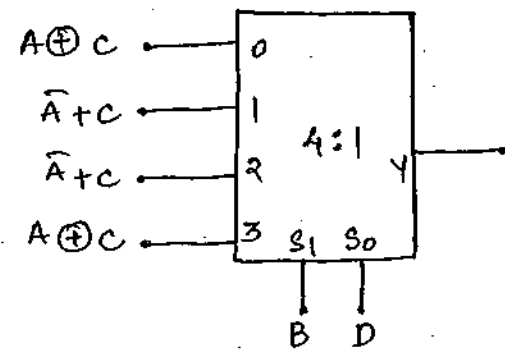
BD	00	01	10	11
$\bar{A}\bar{C}(00)$	0	(1)	(4)	5
$\bar{A}C(01)$	(2)	(3)	(6)	(7)
$A\bar{C}(10)$	(8)	9	12	(13)
$AC(11)$	10	(11)	(14)	15

$$P_0 \rightarrow \bar{A}C + A\bar{C} \rightarrow A \oplus C$$

$$P_1 \rightarrow \bar{A} + C$$

$$P_2 \rightarrow \bar{A} + C$$

$$P_3 \rightarrow \bar{A}C + A\bar{C} \rightarrow A \oplus C$$



Now considering C, A are select lines

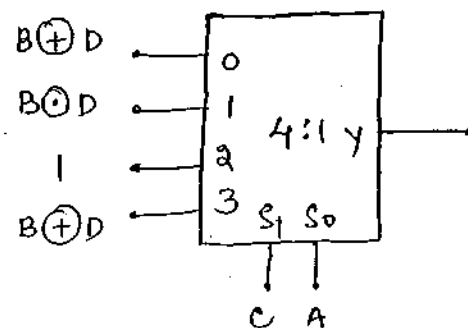
CA	00	01	10	11
$\bar{B}\bar{D}(00)$	0	(8)	(2)	10
$\bar{B}D(01)$	(1)	9	(3)	(11)
$B\bar{D}(10)$	(4)	12	(6)	(14)
$BD(11)$	5	(13)	(7)	15

$$P_0 \rightarrow \bar{B}D + B\bar{D} \rightarrow B \oplus D$$

$$P_1 \rightarrow \bar{B}\bar{D} + BD \rightarrow B \odot D$$

$$P_2 \rightarrow 1$$

$$P_3 \rightarrow \bar{B}D + B\bar{D} \rightarrow B \oplus D$$



By considering D, A as select lines,

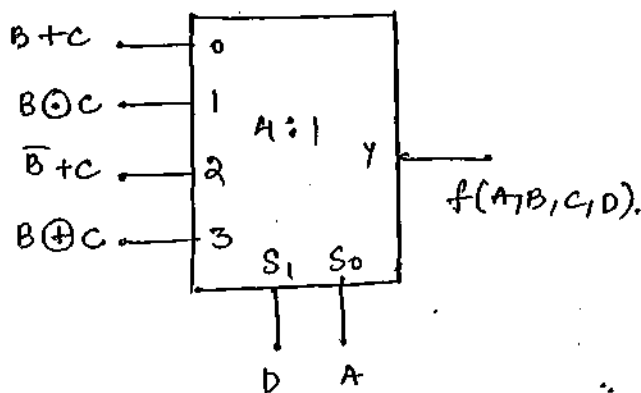
DA	00	01	10	11
$\overline{B}\overline{C}$ (00)	0	⑧	①	9
$\overline{B}C$ (01)	②	10	③	⑪
$B\overline{C}$ (10)	④	12	5	⑬
BC (11)	⑥	⑭	⑦	15

$$I_0 \rightarrow B+C \rightarrow B+C$$

$$I_1 \rightarrow B \odot C$$

$$I_2 \rightarrow \overline{B}+C$$

$$I_3 \rightarrow B \oplus C$$



Implement the sum of full adder using 4:1 MUX :-

$$\text{Sum} = f(A, B, C) = \sum m \{1, 2, 4, 7\}$$

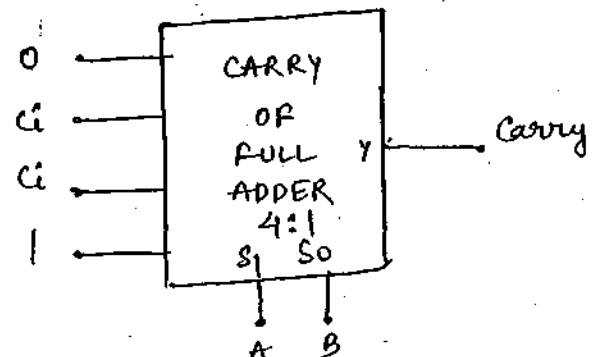
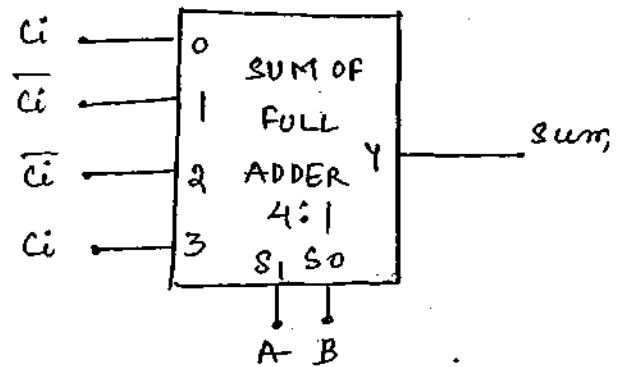
$$= A \oplus B \oplus C_i$$

By considering A, B as select lines

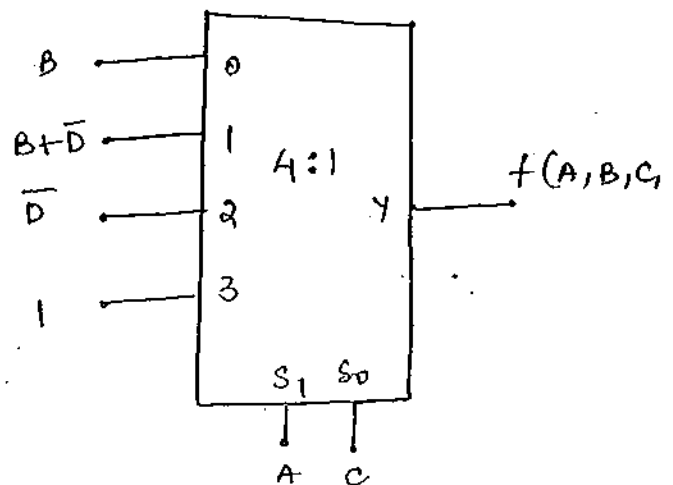
AB	00	01	10	11
\overline{C}_i (0)	0	②	④	6
C_i (1)	①	3	5	⑦
	↓	↓	↓	↓
	C_i	\overline{C}_i	\overline{C}_i	C_i
	I_0	I_1	I_2	I_3

$$\text{Carry} = F(A, B, C_i) = \sum m(3, 5, 6, 7)$$

AB	00	01	10	11
\overline{C}_i (0)	0	2	4	⑥
C_i (1)	1	③	⑤	⑦
	↓	↓	↓	↓
	0	C_i	C_i	1
	I_0	I_1	I_2	I_3



Eg:- Find minimal expression of $f(A, B, C, D)$ implemented by the multiplexer circuit shown in the figure in SOP and POS.



$$f(A, B, C, D) =$$

$$\bar{A}\bar{C}(0) + \bar{A}C(B+\bar{D}) + A\bar{C}(\bar{D}) + AC.$$

$$\Rightarrow \bar{A}B\bar{C} + \bar{A}BC + \bar{A}C\bar{D} + A\bar{C}\bar{D} + AC$$

$$\Rightarrow \bar{A}B + \bar{A}C\bar{D} + A(C + \bar{C}\bar{D})$$

$$\Rightarrow \bar{A}B + \bar{A}C\bar{D} + A(C + \bar{D})$$

$$\Rightarrow \bar{A}B + \bar{A}C\bar{D} + AC + A\bar{D}$$

$$\Rightarrow \bar{A}B + AC + \bar{D}(A + \bar{A}C)$$

$$\Rightarrow \bar{A}B + AC + A\bar{D} + C\bar{D}.$$

AB \ CD	00	01	11	10
00				1
01	1	1	1	1
11	1		1	1
10	1		1	1

$$\text{SOP form} \Rightarrow \bar{A}B + C\bar{D} + AC + A\bar{D}$$

AB \ CD	00	01	11	10
00	0	0	0	
01				
11		0		
10		0	-	

$$\text{POS form} \Rightarrow (A+B+C) \cdot (A+B+\bar{D}) \cdot (\bar{A}+C+\bar{D}).$$

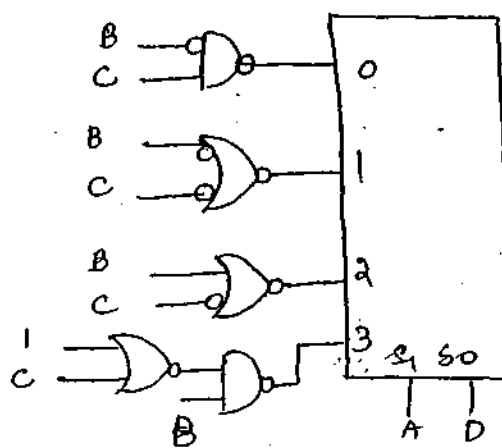
$$f(A, B, C, D) =$$

$$\sum m(2, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15).$$

To know the minterms,

AC	00	01	11	10
$\bar{B}\bar{D}$ (00)	0	2	8	10
$\bar{B}D$ (01)	1	3	9	11
$B\bar{D}$ (10)	4	6	12	14
BD (11)	5	7	13	15

Eg:- Find no. of P.T and E.P.T for $f(A, B, C, D)$ implemented by the circuit shown in the figure.



$$\text{Sol:- } P_0 \rightarrow \overline{B \cdot C} = B + \bar{C}$$

$$P_1 \rightarrow \overline{B + C} = \bar{B} \cdot \bar{C}$$

$$P_2 \rightarrow \overline{B + \bar{C}} = \bar{B} \cdot C$$

$$P_3 \rightarrow \overline{(1 + C) \cdot B} = \bar{0} = 1.$$

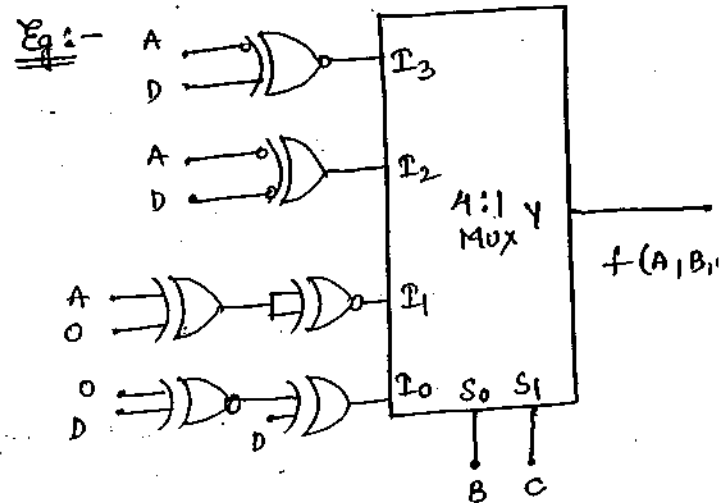
$$f = \sum m(0, 2, 3, 4, 6, 9, 13, 14, 15).$$

AB \ C	00	01	11	10
00	1		1	1
01	1			1
11		1	1	1
10		1		

P.T $\rightarrow 6$

E.P.T $\rightarrow 3$.

$$f(A, B, C, D) = \overline{A} \overline{D} + A \overline{C} D + \overline{A} \overline{B} C + ABC.$$



$$I_0 \rightarrow (\overline{0 \oplus D}) \oplus D$$

$$(\overline{D \oplus D}) \rightarrow 1$$

$$I_1 \rightarrow (A \oplus 0) \oplus (A \oplus 0)$$

$$A \oplus A \rightarrow 1$$

$$I_2 \rightarrow (\overline{A \oplus D})$$

$$I_3 \rightarrow (\overline{A \oplus D}) \rightarrow \overline{A \oplus D}$$

$$\rightarrow A \oplus D$$

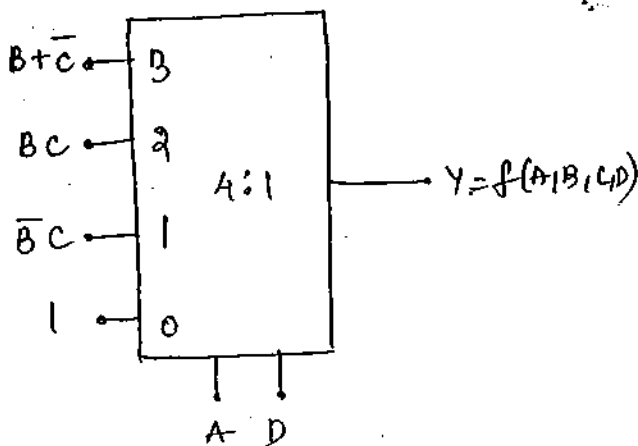
AD	00	01	10	11
$\overline{B} \overline{C}$ (00)	0	1	8	9
$\overline{B} C$ (01)	2	3	10	11
$B \overline{C}$ (10)	4	5	12	13
BC (11)	6	7	14	15

$$f = \sum m(0, 4, 6, 7, 9, 10, 11, 13, 15).$$

AB \ C	00	01	11	10
00	1			
01	1		1	1
11		1	1	
10		1	1	1

P.T $\rightarrow 6$.

E.P.T $\rightarrow 3$



AD	00	01	10	11
$\overline{B} \overline{C}$ (00)	0	1	8	9
$\overline{B} C$ (01)	2	3	10	11
$B \overline{C}$ (10)	4	5	12	13
BC (11)	6	7	14	15

CB	00	01	10	11
$\bar{A}\bar{D}$ (00)	0	4	2	6
$\bar{A}D$ (01)	1	5	3	7
$A\bar{D}$ (10)	8	12	10	14
AD (11)	9	13	11	15

(0) (1) ($\bar{A} \oplus \bar{D}$) ($A \oplus D$)

$$f(A, B, C, D) = \sum m(0, 1, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14)$$

AB \ CD	00	01	11	10
00	1	1	1	
01	1	1	1	
11	1	1		1
10	1	1		1

P.I. $\rightarrow 3$

E.P.I. $\rightarrow 3$

Minimal expression \rightarrow

$$\bar{C} + \bar{A}D + A\bar{D}$$

* Pg. 55 Q. 4

Sol:-

$$I_0 \rightarrow P + \bar{Q}$$

$$I_1 \rightarrow P$$

$$I_2 \rightarrow PQ$$

$$I_3 \rightarrow P$$

RS	00	01	10	11
$\bar{P}\bar{Q}$ (00)	0	1	2	3
$\bar{P}Q$ (01)	4	5	6	7
$P\bar{Q}$ (10)	8	9	10	11
PQ (11)	12	13	14	15

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $P + \bar{Q} \quad P \quad PQ \quad P$

$$f(P, Q, R, S) = \sum m\{0, 8, 9, 11, 12, 13, 14, 15\}$$

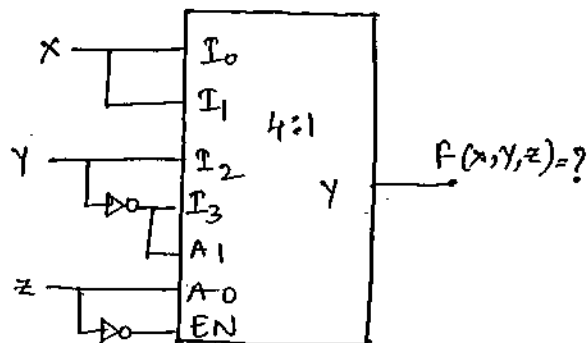
PQ \ RS	00	01	11	10
00	1			
01				
11	1	1	1	1
10	1	1	1	

redundant.

$$f(P, Q, R, S) = PQ + P\bar{R}PS + \bar{Q}\bar{R}\bar{S}$$

* Q. 9 Pg. 56

Sol:-



For enabling MUX, $EN = 1$ must.

$$\therefore Z = 0.$$

$$A_0 = 0.$$

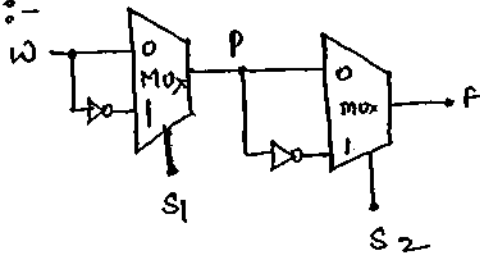
A_1	A_0
0	0
1	0

$$F = (X + Y)$$

$$\begin{aligned}
 F(x, y, z) &= \bar{A}_1 \bar{A}_0 I_0 + \bar{A}_1 A_0 I_1 + \\
 &\quad A_1 \bar{A}_0 I_2 + A_1 A_0 I_3. \\
 &= \bar{A}_1 I_0 + 0 + A_1 I_2 + 0 \\
 &= (\bar{A}_1 I_0 + A_1 I_2) \bar{z} \\
 &= (xy + \bar{y}z) \bar{z} \\
 &= xy \bar{z}.
 \end{aligned}$$

* Q.1 pgno. 57

Sol:-



$$P = \bar{s}_1 w + s_1 \bar{w} = s_1 \oplus w$$

$$\begin{aligned}
 F &= \bar{s}_2 P + s_2 \bar{P} = s_2 \oplus P \\
 &= s_2 \oplus s_1 \oplus w.
 \end{aligned}$$

* Q.2 pgno. 58

Sol:-

$$T_C = 12 \text{ ns.}$$

$$T_S = 15 \text{ ns.}$$

Worst-case delay is

$$(n-1) T_C + T_S.$$

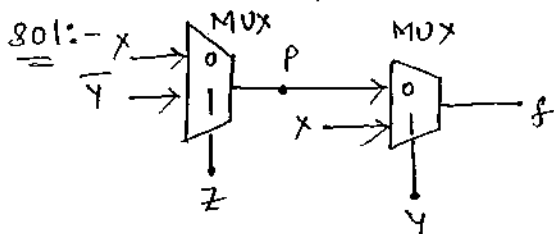
$$\Rightarrow (16-1) T_C + T_S$$

$$\Rightarrow 15 \times 12 + 15$$

$$\Rightarrow 195 \text{ ns.}$$

* Q.2 pgno. 54

Sol:-



$$P = \bar{z}x + z\bar{y}.$$

$$f = \bar{y}P + yx.$$

$$= \bar{y}(\bar{z}x + z\bar{y}) + xy$$

$$= x\bar{y}\bar{z} + z\bar{y} + xy.$$

$$= x(y + \bar{y}\bar{z}) + \bar{y}z.$$

$$= x(y + \bar{z}) + \bar{y}z.$$

$$= xy + x\bar{z} + \bar{y}z.$$

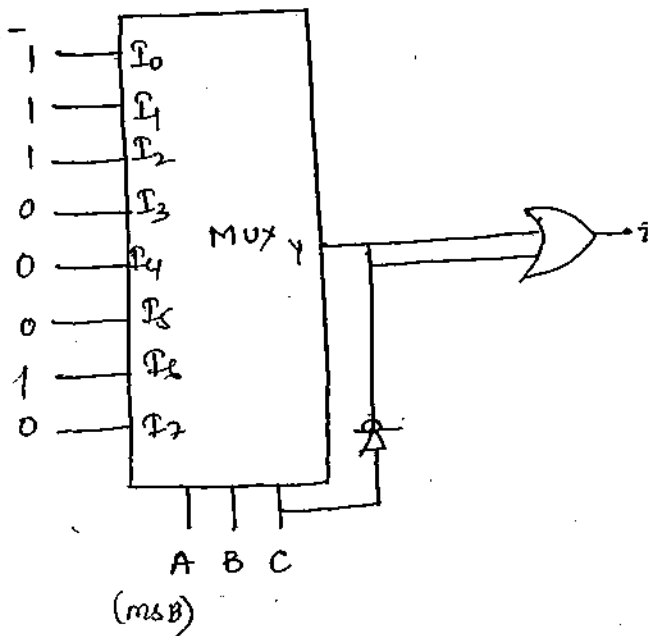
Q.3 pgno. 58

Sol:-

s_1 A_2	s_0 A_1	$A_0 (\bar{E})$	output
0	0	0	I_0
0	0	1	1
0	1	1	1
0	1	0	I_1
1	1	0	I_3
1	1	1	1
1	0	1	1
1	0	0	I_2

Q.5 pgno. 58

Sol:-



$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$$

$$Z = Y + \bar{C}$$

$$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + \bar{C}$$

$$= \bar{A}\bar{B} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + \bar{C}$$

$$= \bar{A}\bar{B} + \bar{A}\bar{B}C + \bar{C}$$

$$= \bar{A}\bar{B} + \bar{C}$$

floating inputs in the TTL

logic family are considered as logic 1.

C B A

1 0 0

1 0 1

1 1 0

1 1 1

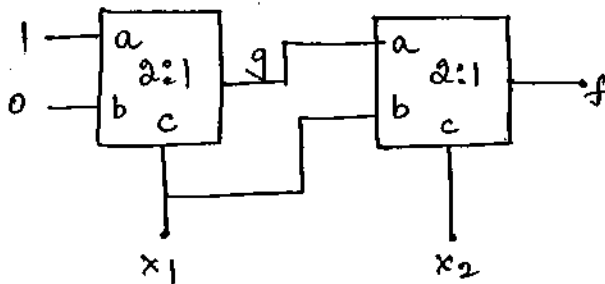
$$\therefore Y = \bar{B}\bar{A}(0) + \bar{B}A(1) + B\bar{A}(1) + BA(0)$$

$$Y = \bar{B}\bar{A} + B\bar{A}$$

$$Y = A \oplus B$$

Q.6 pgno.58

Sol:-



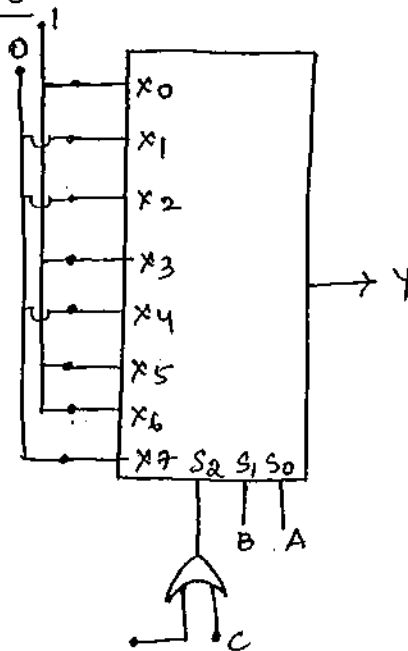
$$f = a\bar{c} + bc$$

$$g = 1\bar{x}_1 + 0x_1 = \bar{x}_1$$

$$f = \bar{x}_2 g + x_2 x_1$$

$$= \bar{x}_2 \bar{x}_1 + x_1 x_2$$

Q.7 pgno.58

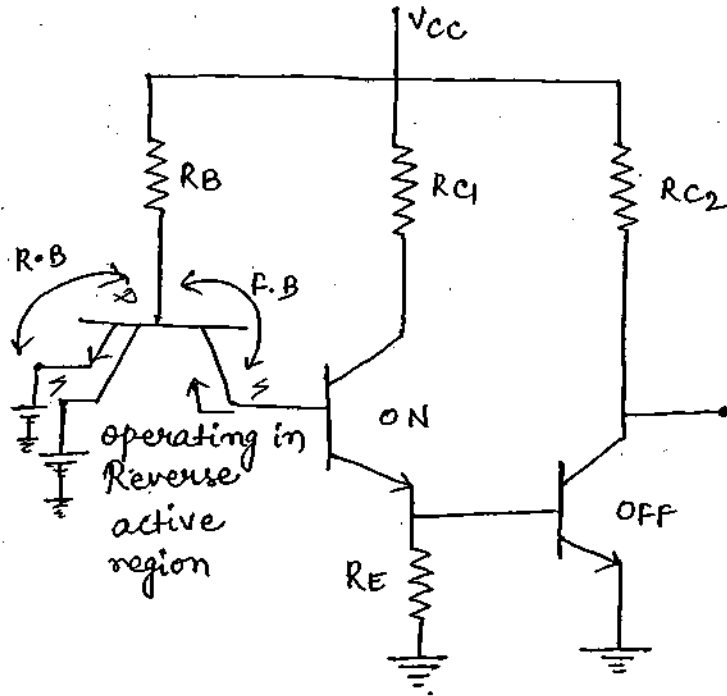


floating logic 1.

$\therefore S_2 = 1$ always.

Basic TTL logic gate :-

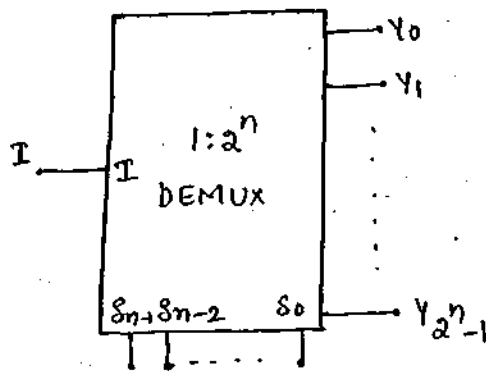
Ex: 6-11



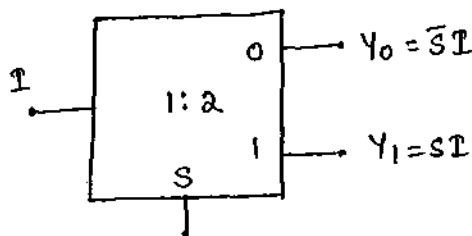
$V = V_{CE(sat)} \rightarrow$ logic '0'.
When both the inputs are at logic '1'.

If one of the i/p's is floating then also logic '0' is o/p. So the floating i/p in TTL gate considered as logic '1'.

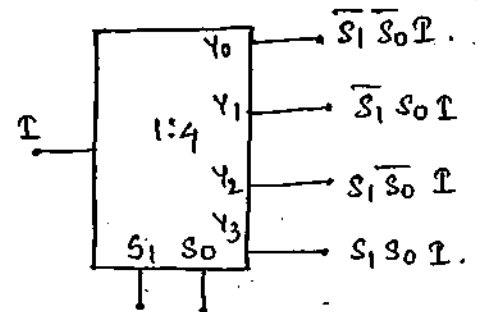
DEMULTIPLEXER :-



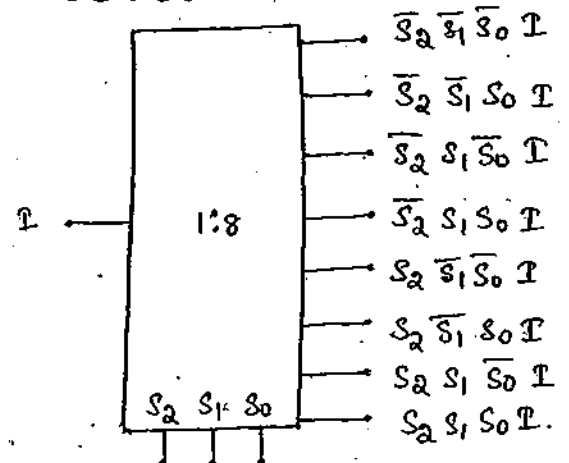
1:2 DEMUX :-



1:4 DEMUX :-

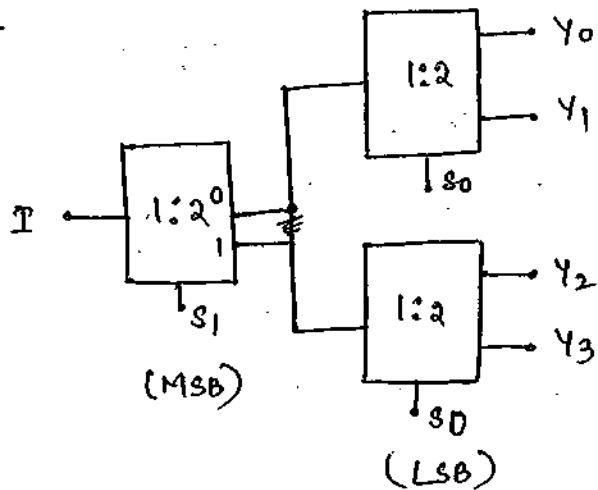


1:8 DEMUX :-



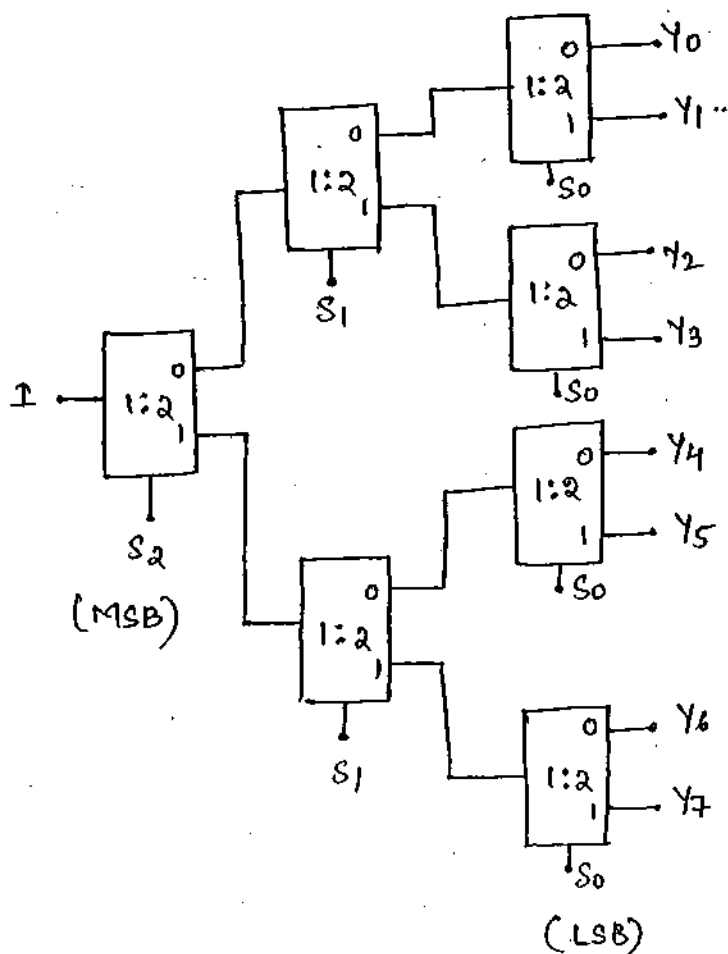
Q:- Implement 1:4 DEMUX using minimum number of 1:2 DEMUX.

Sol:-



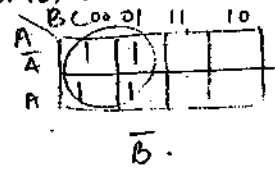
S_1	S_0	
0	0	$Y_0 = I$
0	1	$Y_1 = I$
1	0	$Y_2 = I$
1	1	$Y_3 = I$

Q:- Implement 1:8 DEMUX using minimum number of 1:2 DEMUX.



Q:- Implement the function

$f(A,B,C) = \sum m(0,1,4,5)$ using 1:8 DEMUX and some combinations gates.



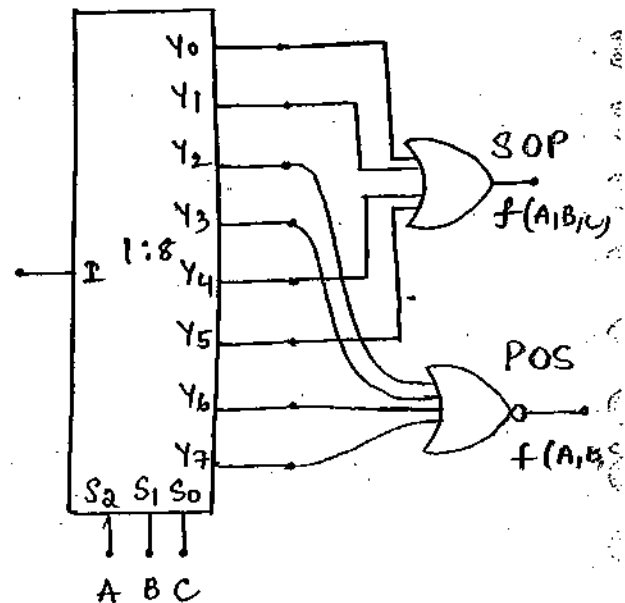
$$f(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC$$

$$= \pi M(2,3,6,7)$$

$$= (A+B+C)(A+B+\bar{C})$$

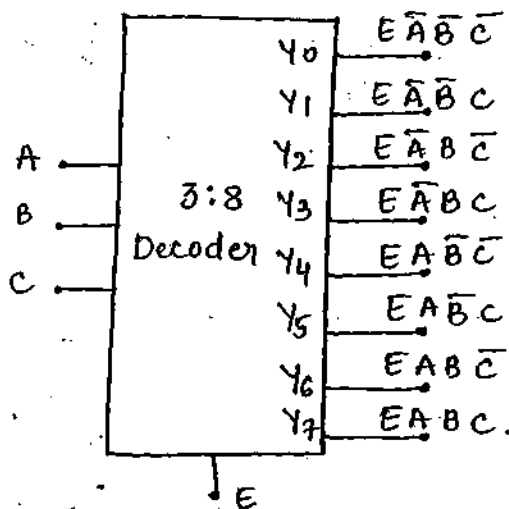
$$(\bar{A}+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})$$

$$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC$$



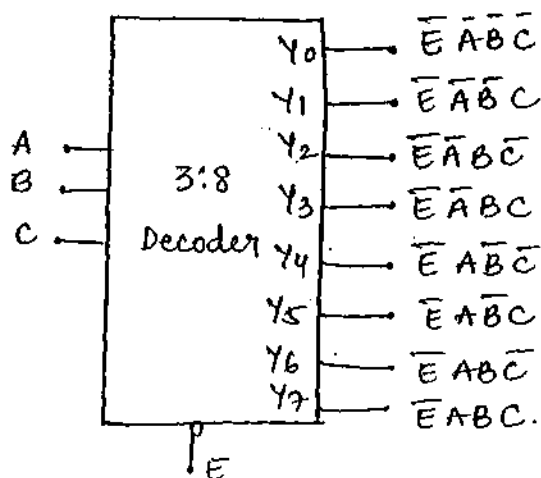
DECODER :- ($n \times 2^n$)

3:8 Decoder :-



E	A	B	C	Y ₀	Y ₁	Y ₂	Y ₃	Y ₄	Y ₅	Y ₆	Y ₇
0	x	x	x	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	0	0	0	0
1	0	0	1	0	1	0	0	0	0	0	0
1	0	1	0	0	0	1	0	0	0	0	0
1	0	1	1	0	0	0	1	0	0	0	0
1	1	0	0	0	0	0	0	1	0	0	0
1	1	0	1	0	0	0	0	0	1	0	0
1	1	1	0	0	0	0	0	0	0	1	0
1	1	1	1	0	0	0	0	0	0	0	1

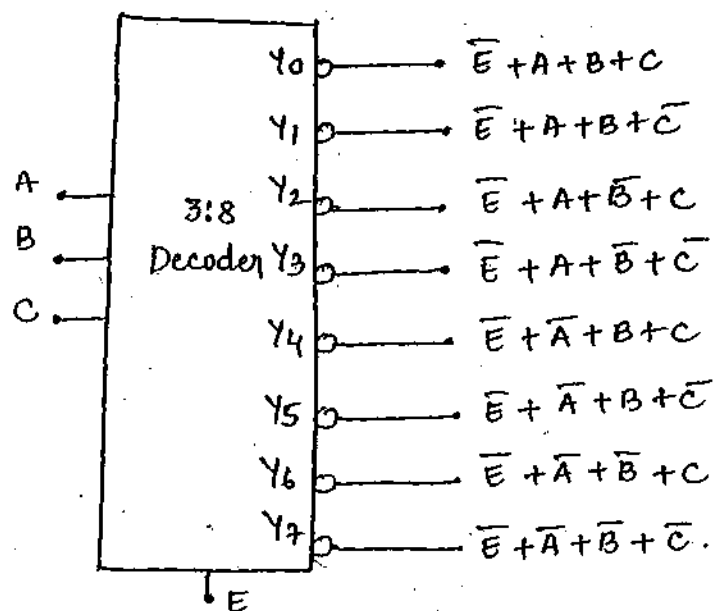
This is for logic high enable decoder block.



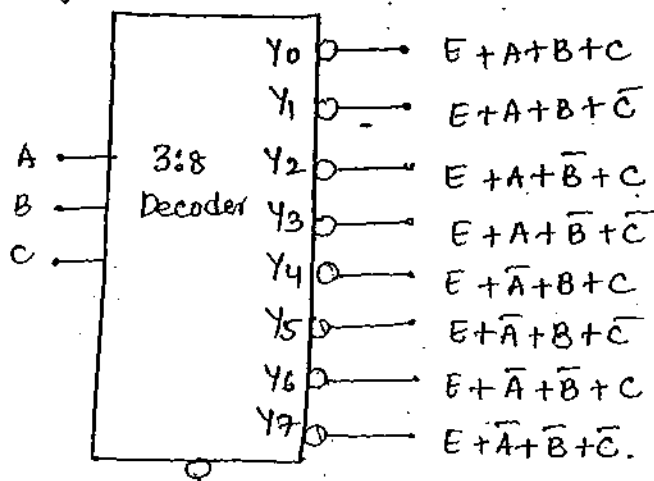
This is logic '0' enable and logic '1' outputs 3x8 decoder.

The truth table for logic '1' enable and logic '0' outputs 3x8 decoder is

E	A	B	C	Y ₀	Y ₁	Y ₂	Y ₃	Y ₄	Y ₅	Y ₆	Y ₇
0	x	x	x	1	1	1	1	1	1	1	1
1	0	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	0	1	1	1	1	1	1
1	0	1	0	1	1	0	1	1	1	1	1
1	0	1	1	1	1	1	0	1	1	1	1
1	1	0	0	1	1	1	1	0	1	1	1
1	1	0	1	1	1	1	1	1	0	1	1
1	1	1	0	1	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1	1	1	0

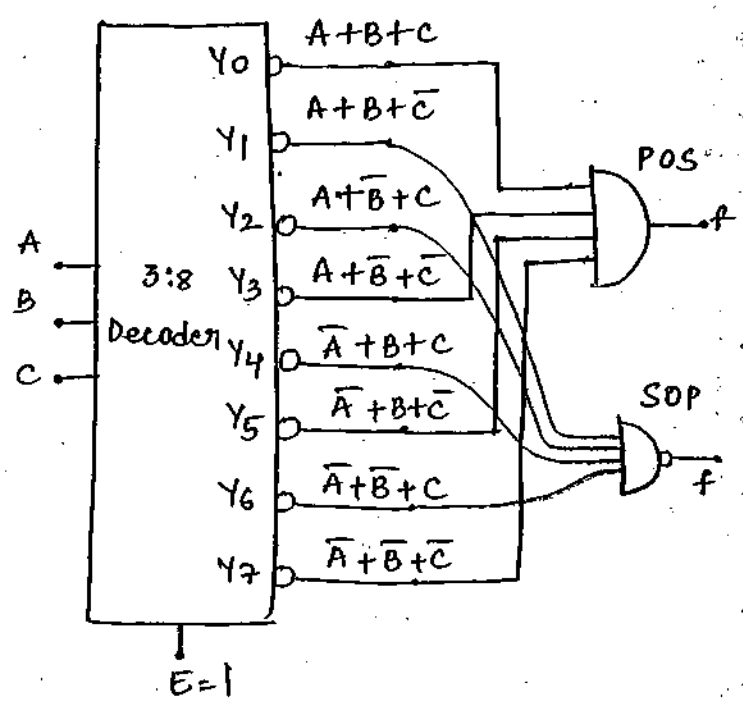


The truth table and pindigram for logic '0' enable and logic '0' o/p 3x8 decoder

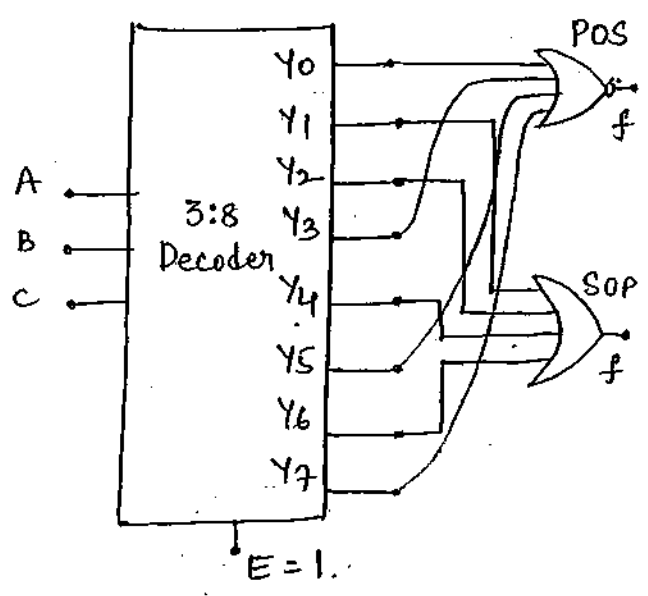


E	A	B	C	Y ₀	Y ₁	Y ₂	Y ₃	Y ₄	Y ₅	Y ₆	Y ₇
1	x	x	x	1	1	1	1	1	1	1	1
2	0	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	0	1	1	1	1	1	1
2	0	1	0	1	1	0	1	1	1	1	1
3	0	1	1	1	1	1	0	1	1	1	1
4	1	0	0	1	1	1	1	0	1	1	1
5	1	0	1	1	1	1	1	1	0	1	1
6	1	1	0	1	1	1	1	1	1	0	1
7	1	1	1	1	1	1	1	1	1	1	0

$$\begin{aligned}
 f(A,B,C) &= (A+B+C)(A+\bar{B}+\bar{C}) \\
 &\quad (\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C) \\
 &= \frac{(A+B+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)}{(\bar{A}+\bar{B}+C)}
 \end{aligned}$$



Ex:- Implement the function $f(A,B,C) = \sum m(1,2,4,6)$ using 3:8 Decoder and some combinational gates.



$$\begin{aligned}
 f(A,B,C) &= \sum m(1,2,4,6) \\
 &= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C \\
 &= \pi M(0,3,5,7) \\
 &= \frac{\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C}{A\bar{B}C}
 \end{aligned}$$

ENCODER ($2^n:n$):-

Mutually exclusive 2^n i/p's to 'n' o/p bits conversion.

I_0	I_1	I_2	I_3	Y_1	Y_0
1	0	0	0	0	0
0	1	0	0	0	1
0	0	1	0	1	0
0	0	0	1	1	1

PRIORITY ENCODER :-

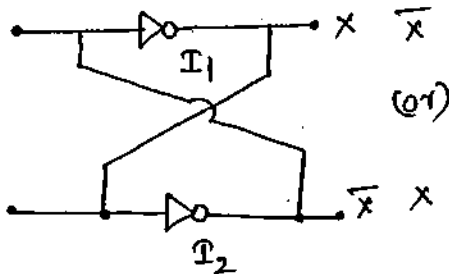
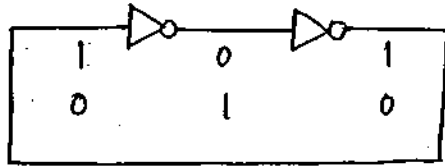
Predefined priority $I_0 > I_1 > I_2 > I_3$

I_0	I_1	I_2	I_3	Y_1	Y_0
1	x	x	x	0	0
0	1	x	x	0	1
0	0	1	x	1	0
0	0	0	1	1	1

For priority $I_0 > I_1 > I_3 > I_2$

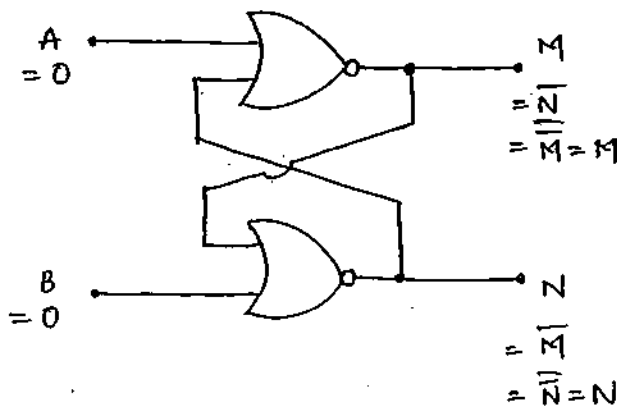
I_0	I_1	I_2	I_3	Y_1	Y_0
1	x	x	x	0	0
0	1	x	x	0	1
0	0	x	1	1	1
0	0	1	0	1	0

SEQUENTIAL CIRCUITS

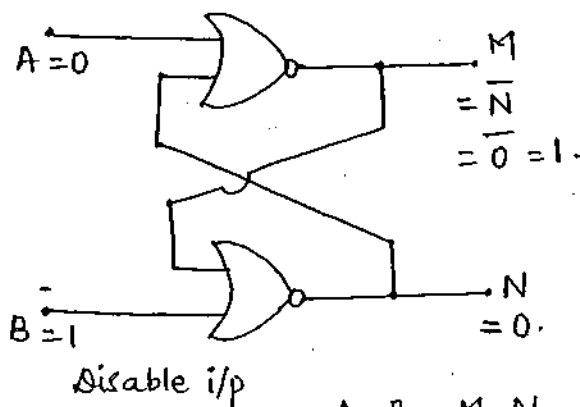


These are basic memory storage elements.

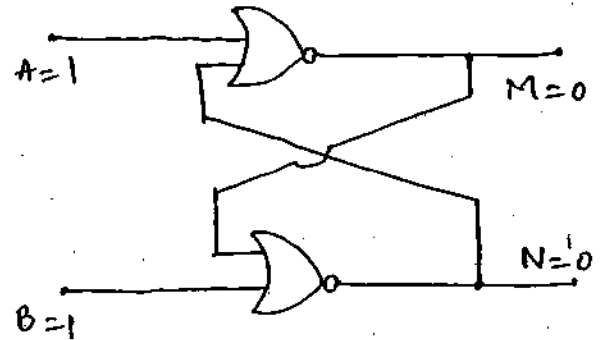
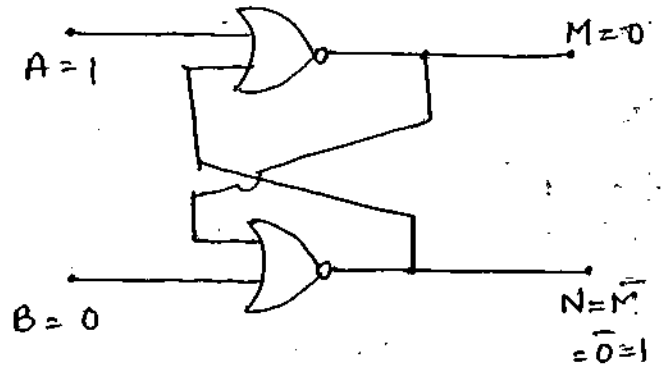
Using NOR gates :-



for i/p's $A=B=0$, the op's remains unchanged i.e., stays in previous state.

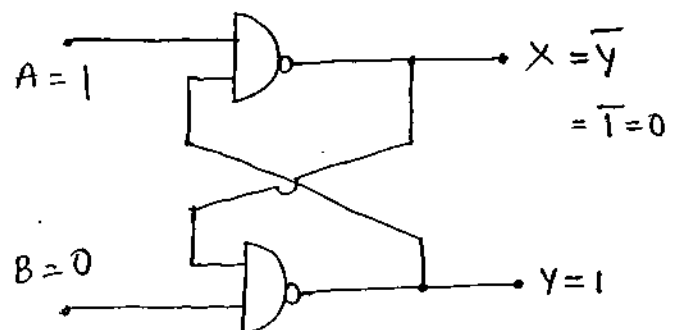
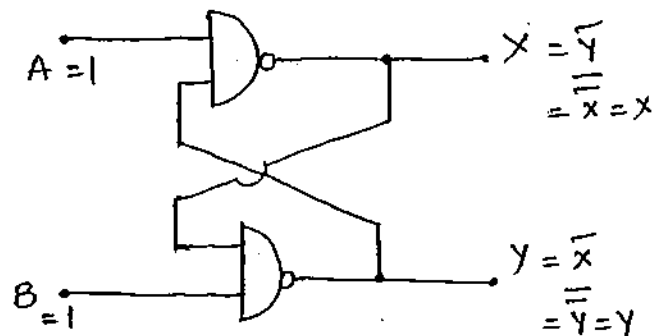


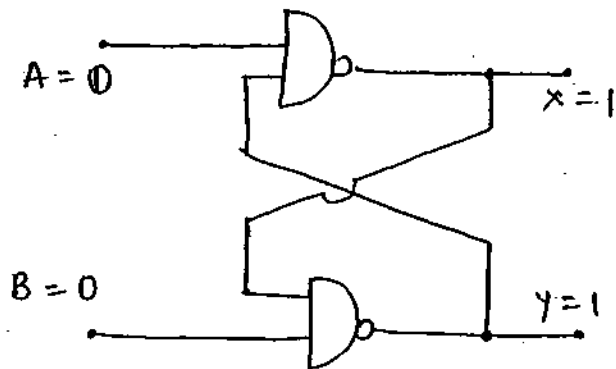
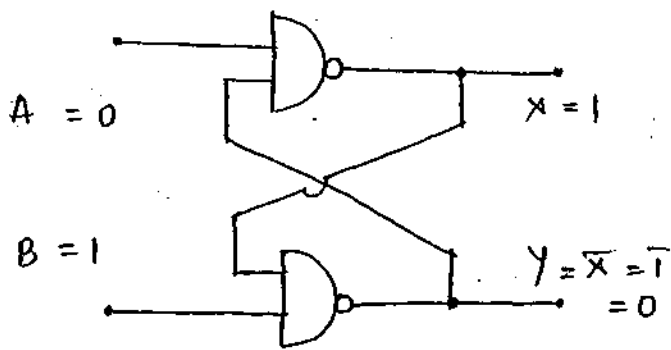
A	B	M	N
0	0	previous	previous
0	1	1	0
1	0	0	1
1	1	0	0



$M=0$ and $N=0$ is an indeterminate states. This indeterminate state will create a problem for analysing the outputs of next inputs applied as the output depends on previous outputs.

Using NAND gates :-

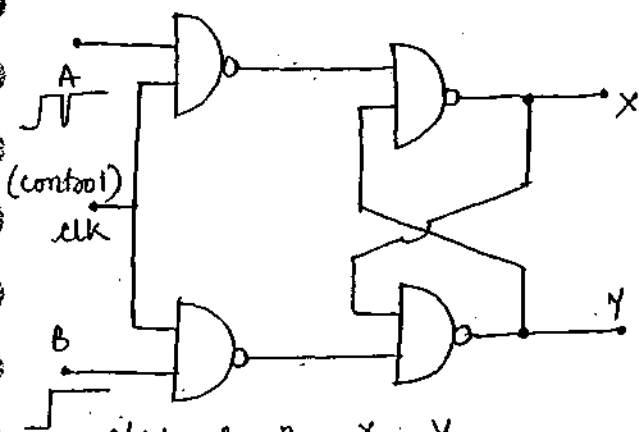




A	B	X	Y
1	1	previous	
0	1	1	0
1	0	0	1
0	0	1	1

Indeterminate state.

To avoid this effect of noise state, two more NAND gates are required.



ctrl	A	B	X	Y
0	x	x	previous states.	
1	0	0	previous states.	
1	0	1	0	1
1	1	0	1	0
1	1	1	1	1

For set, $Q = 1$

Reset, $Q = 0$.

Assume $X = Q$, $Y = \overline{Q}$.

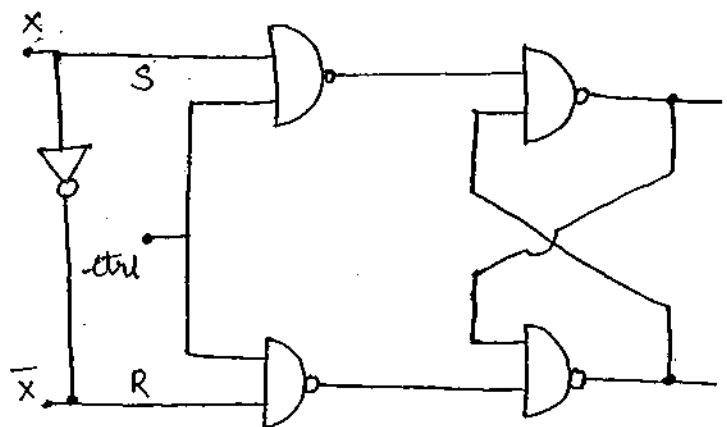
ctrl	S	R	Q	\overline{Q}
0	x	x	previous	
1	0	0	previous	
1	0	1	0	1
1	1	0	1	0
1	1	1	1	1

This is the truth table for S-R latch

$Q(n) \rightarrow$ present state

$Q(n+1) \rightarrow$ next state.

ctrl	S	R	$Q(n+1)$
0	x	x	$Q(n)$
1	0	0	$Q(n)$
1	0	1	0
1	1	0	1
1	1	1	Indeterminate state.



X = D	$Q(n+1)$
0	0
1	1

This function of D-latch or D-flipflop.

<u>ctrl</u>	<u>D</u>	<u>Q(n+1)</u>
0	x	Q(n)
1	0	0
1	1	1

The indeterminate state in S-R flipflop is eliminated in J-K flipflop.

<u>J</u>	<u>K</u>	<u>Q(n+1)</u>
0	0	Q(n)
0	1	0
1	0	1
1	1	$\overline{Q(n)}$

The truth table for Toggle flipflop is

<u>J</u>	<u>Q(n+1)</u>
0	Q(n)
1	$\overline{Q(n)}$

Truth table of S-R flipflop :-

<u>S</u>	<u>R</u>	<u>Q(n)</u>	<u>Q(n+1)</u>
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	x
1	1	1	x

Excitation table of S-R flipflop :-

<u>Q(n)</u>	<u>Q(n+1)</u>	<u>S</u>	<u>R</u>
0	0	0	x
0	1	1	0
1	0	0	1
1	1	x	0

Characteristic table of D-flipflop :-

<u>D</u>	<u>Q(n+1)</u>
0	0
1	1

Truth table of D-flipflop :-

<u>D</u>	<u>Q(n)</u>	<u>Q(n+1)</u>
0	0	0
0	1	0
1	0	1
1	1	1

Excitation table of J-K flipflop :-

<u>Q(n)</u>	<u>Q(n+1)</u>	<u>J</u>
0	0	0
0	1	1
1	0	0
1	1	1

Characteristic table of J-K flipflop :-

<u>J</u>	<u>K</u>	<u>Q(n+1)</u>
0	0	Q(n)
0	1	0
1	0	1
1	1	$\overline{Q(n)}$

Truth table of J-K flipflop :-

<u>J</u>	<u>K</u>	<u>Q(n)</u>	<u>Q(n+1)</u>
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Excitation characteristic table of J-K flipflop :-

<u>Q(n)</u>	<u>Q(n+1)</u>	<u>J</u>	<u>K</u>
0	0	0	x
0	1	1	x
1	0	x	1
1	1	x	0

Characteristic table of T-flipflop :-

<u>T</u>	<u>Q(n+1)</u>
0	Q(n)
1	$\overline{Q(n)}$

Truth table of T-flipflop :-

<u>T</u>	<u>Q(n)</u>	<u>Q(n+1)</u>
0	0	0
0	1	1
1	0	1
1	1	0

Excitation table of T-flipflop :-

<u>Q(n)</u>	<u>Q(n+1)</u>	<u>T</u>
0	0	0
0	1	1
1	0	1
1	1	0

FLIP-FLOP CONVERSIONS :-

- Write the truth table for required flipflop. Table must contain column for given flipflop inputs.
- Flipflop inputs column must be filled with the help of excitation table of given flipflop.
- Find the expression for given flipflop inputs in terms of required flipflop i/p and present state Q.
- Draw the logic diagram.

Conversion of S-R to J-K :-

	<u>J</u>	<u>K</u>	<u>Q(n)</u>	<u>Q(n+1)</u>	<u>S</u>	<u>R</u>
0	0	0	0	0	0	x
1	0	0	1	1	x	0
2	0	1	0	0	0	x
3	0	1	1	0	0	1
4	1	0	0	1	1	0
5	1	0	1	1	x	0
6	1	1	0	1	1	0
7	1	1	1	0	0	1

$$S = \sum m(4, 6) + d(1, 5)$$

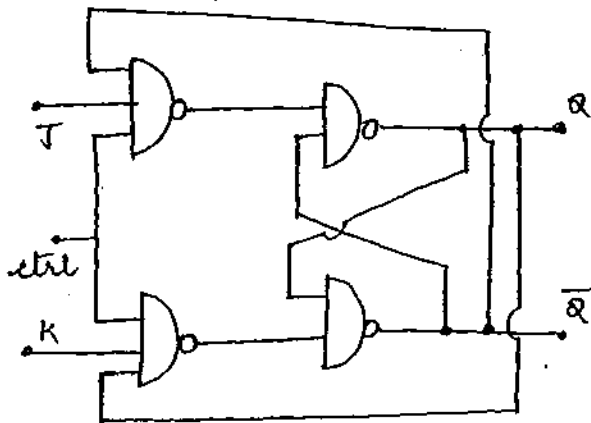
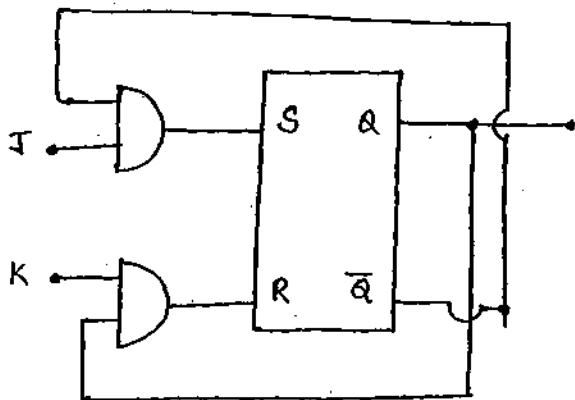
$$R = \sum m(3, 7) + d(0, 2)$$

T \ KQ	00	01	11	10
0		X		
1	1	X		1

$$S = J\bar{Q}$$

T \ KQ	00	01	11	10
0	X		1	X
1			1	

$$R = KQ$$



$$J = \sum m(2) + d(1, 3)$$

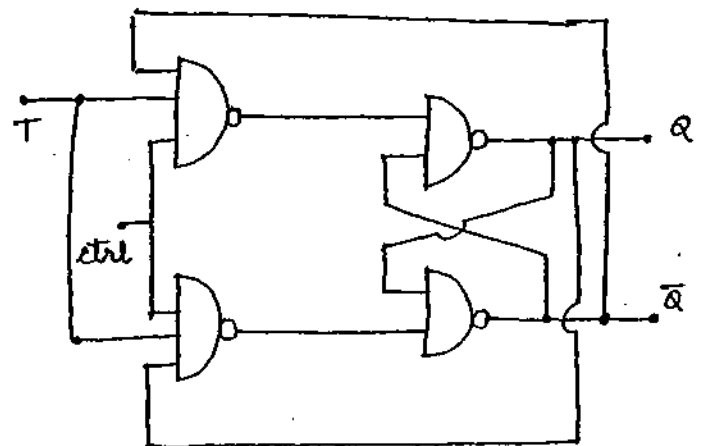
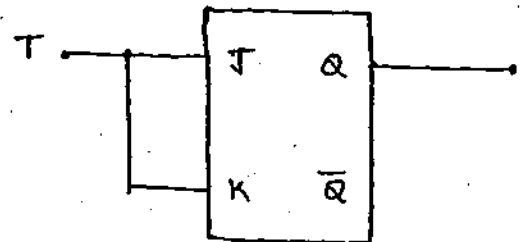
$$K = \sum m(3) + d(0, 2)$$

T \ Q	0	1
0		X
1	1	X

$$J = T$$

T \ Q	0	1
0	X	
1	X	1

$$K = T$$



Delay '0' to J-K flipflop :-

J-K to T-flipflop :-

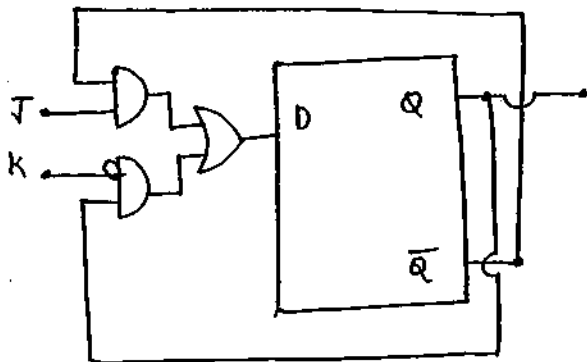
T	Q(n)	Q(n+1)	J	K
0	0	0	0	X
1	0	1	X	0
2	1	0	1	X
3	1	1	X	X

J	K	Q(n)	Q(n+1)	D
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$D = \sum m(1, 4, 5, 6)$$

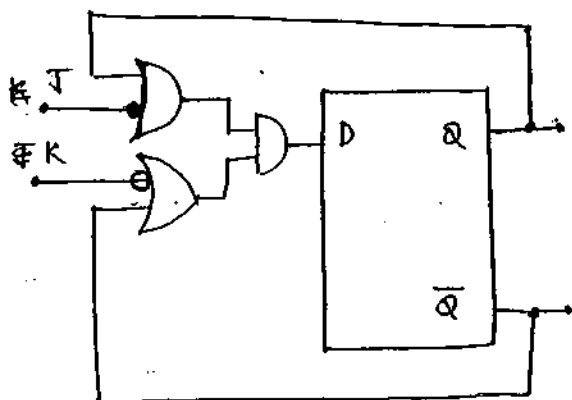
$J \backslash KQ$	00	01	11	10
0		1		
1	1	1		1

$$D = J\bar{Q} + \bar{K}Q \rightarrow \text{SOP form}$$

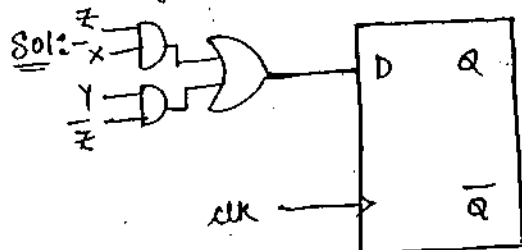


$J \backslash KQ$	00	01	11	10
0	0		0	0
1			0	

$$D = (J+Q)(\bar{K}+\bar{Q}) \rightarrow \text{POS form}$$



* Q.6 Pgno. 78



It is JK-flipflop with
 $Y=J$ and $X=K$, by
 using verification
 method.

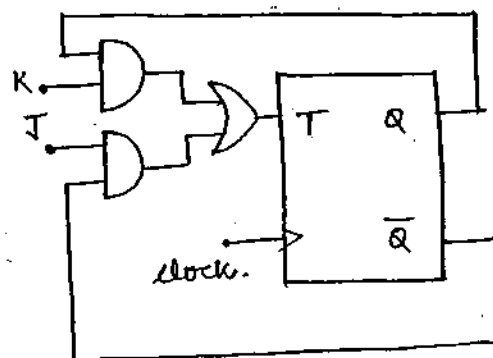
T to J-K flipflop :-

	J	K	$Q(n)$	$Q(n+1)$	T
0	0	0	0	0	0
1	0	0	1	1	0
2	0	1	0	0	0
3	0	1	1	0	1
4	1	0	0	1	1
5	1	0	1	1	0
6	1	1	0	1	1
7	1	1	1	0	1

$$T = \sum m(3, 4, 6, 7)$$

$J \backslash KQ$	00	01	11	10
0			1	
1	1		1	1

$$T = J\bar{Q} + KQ \rightarrow \text{SOP}$$



In POS
 $T = (K+Q)(J+\bar{Q})$

* Q.10 Pgno. 84

$$T = (J+Q)(K+\bar{Q}) \rightarrow \text{POS form}$$

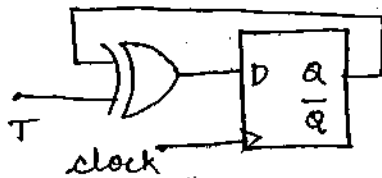
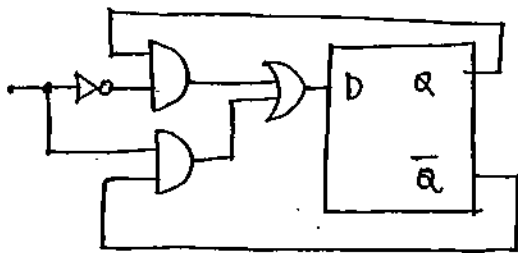
$$\begin{aligned}
 &= JK + J\bar{Q} + KQ + 0 \\
 &= J\bar{Q} + KQ + JK \\
 &= QK + \bar{Q}J + JK \\
 &= QK + \bar{Q}J \\
 &= J\bar{Q} + KQ.
 \end{aligned}$$

Delay to T' flipflop :-

	T	Q(n)	Q(n+1)	D
0	0	0	0	0
1	0	1	1	1
2	1	0	1	1
3	1	1	0	0

$$D = \sum m(1, 2)$$

$$D = \bar{T}Q + \bar{Q}T$$



* Q.5 pgn 0.78
Eg:-

$X = T \rightarrow$ Toggle flipflop.

Eg:- AB flipflop function is defined as shown in table, then construct

D and T flipflops from AB.

A	B	Q(n+1)
0	0	0
0	1	$\overline{Q(n)}$
1	0	$\overline{Q(n)}$
1	1	1

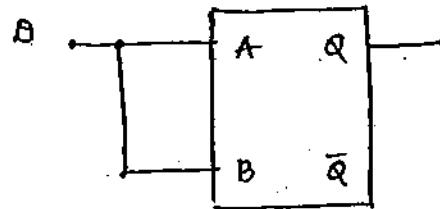
D-flipflop construction,

D	Q(n)	Q(n+1)	A	B
0	0	0		
0	1	0		
1	0	1		
1	1	1		

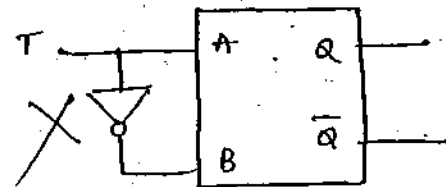
The excitation table cannot be written for AB flipflop.

On observation,

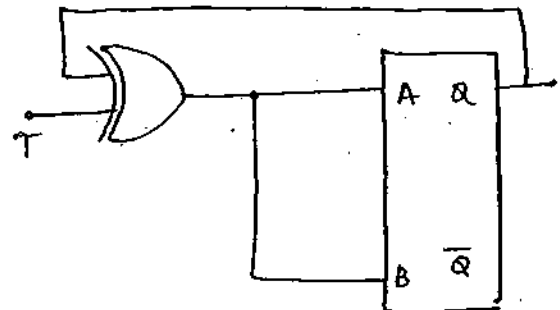
$$D = A = B$$



T-flipflop construction,



from D-flipflop, T-flipflop should be constructed.

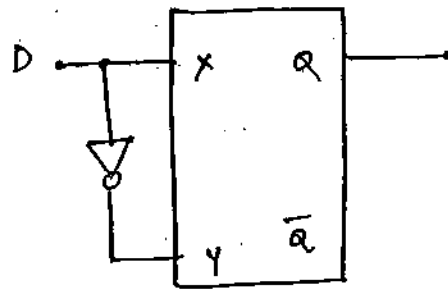


Ex:- The xy flipflop fn is defined as shown in the table. Construct JK from xy.

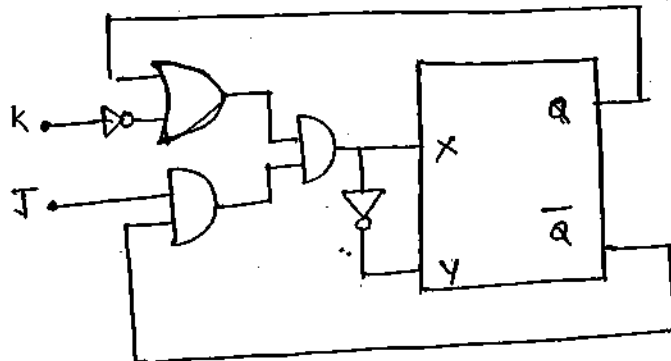
x	y	$Q(n+1)$
0	0	$\overline{Q(n)}$
0	1	0
1	0	1
1	1	$\overline{Q(n)}$

Sol:- Excitation table for xy flipflop cannot be obtained properly.

xy to JK



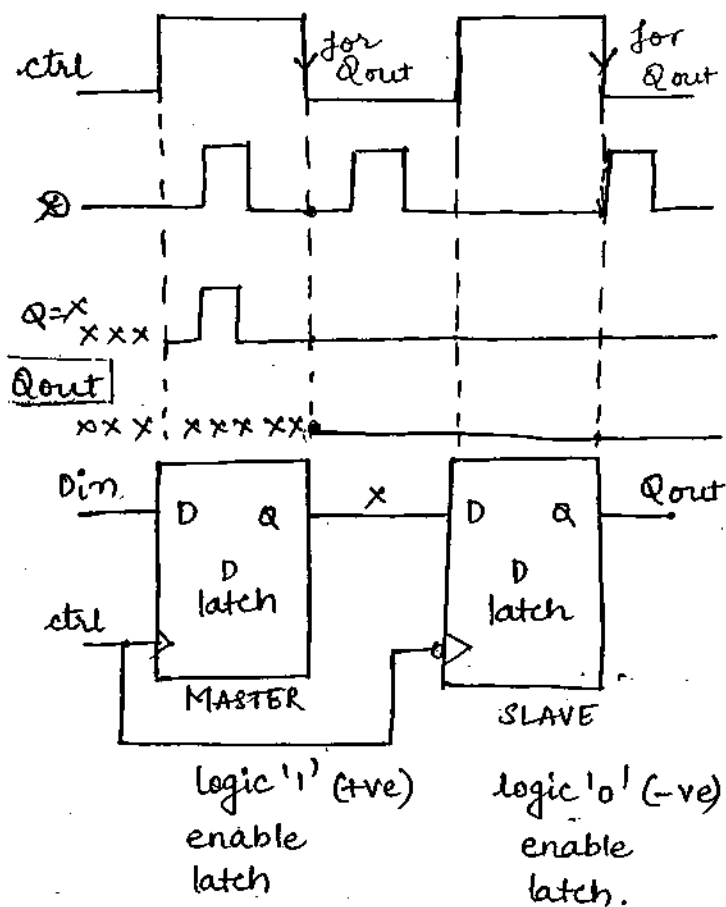
From 'D' flipflop we can construct J-K flipflop.



J	K	Q_n	Q_{n+1}	x	y
0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	1	0	0	1
1	0	0	1	1	0
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	1	1

D-latch :-

for +ve enable latch	$\begin{cases} \text{ctrl} \\ 0 \\ 1 \\ 1 \end{cases}$	$\begin{cases} D \\ x \\ 0 \\ 1 \end{cases}$	$\begin{cases} Q(n+1) \\ Q(n) \\ 0 \\ 1 \end{cases}$
----------------------	--	--	--



for -ve enable latch	$\begin{cases} \text{ctrl} \\ 1 \\ 0 \\ 0 \end{cases}$	$\begin{cases} D \\ x \\ 0 \\ 1 \end{cases}$	$\begin{cases} Q(n+1) \\ Q(n) \\ 0 \\ 1 \end{cases}$
----------------------	--	--	--

When master latch is disabled, the output of master latch depends on the ip at which the master is disabled.

The slave latch is enabled when master is disabled. At this point Qout is changed i.e.,

entire circuit is responding. So the entire o/p depending on the control at which it is changing from high to low.

logic '1' enable latch followed by logic '0' enable latch will give -ve edge triggered flipflops.

logic '0' enable latch followed by logic '1' enable latch will give +ve edge triggered flipflop.

Latch is responding to ip's depending on level of control. So it is called level sensitive (or) level triggered. Flipflop is responding to ip's only once in a clock cycle at clock edge. So it is called edge sensitive (or) edge triggered.

After the change of clk edge, the flipflop takes some time to change its state. It is called clock to Q delay (or) flipflop delay. (t_{clk-Q} (or) t_{FF}) $\Delta = t_{clk-Q}$ (or) t_{FF}

Inputs gets settled before clock edge occurrences for some time. The minimum time is "set-up time" (t_{su}).

Clock to Q delay (or) flipflop delay :-

After clock edge occurrence

flipflop taking some time to change the state is called

flipflop delay (or) clock to Q delay.

Setup time :-

The inputs must get settled atleast setup time before clock edge occurrence to captured by flipflop.

Eg:- Design

SYNCHRONOUS COUNTER :-

2 Bit Binary counter using

D-flipflop :-

(Q_1, Q_0) PS (present state)	NS (Next state)
(Q_1, Q_0)	(Q_1, Q_0)
00	01
01	10
10	11
11	00

← for →
binary
counter

PS is the state before the occurrence of clock edge. NS is the state after the occurrence of clock edge.

For the 'D' flipflop,

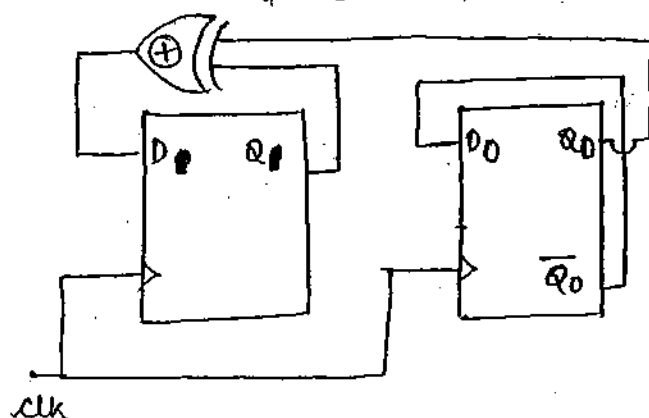
PS	NS	FF i/p's
(Q_1, Q_0)	(Q_1, Q_0)	(D_1, D_0)
0 0 0	0 1	0 1
1 0 1	1 0	1 0
2 1 0	1 1	1 1
3 1 1	0 0	0 0

present state + i/p's → next state

$$\therefore D_1 = \bar{Q}_1 Q_0 + Q_1 \bar{Q}_0 = Q_0 \oplus Q_1$$

$$D_0 = \bar{Q}_1 \bar{Q}_0 + Q_1 Q_0 = Q_1 \oplus Q_0$$

$$(\bar{Q}_1 + Q_1) \bar{Q}_0 = \bar{Q}_0$$

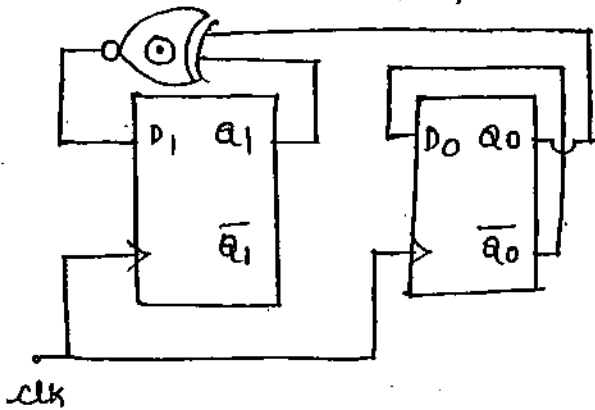


binary
Design 2-bit down-counter :-
 (using D-flipflops).

	<u>PS</u>		<u>NS</u>		<u>FF i/p's</u>
	Q_1	Q_0	Q_1	Q_0	Q_1 Q_0
3	1	1	1	0	1 0
2	1	0	0	1	0 1
1	0	1	0	0	0 0
0	0	0	1	1	1 1

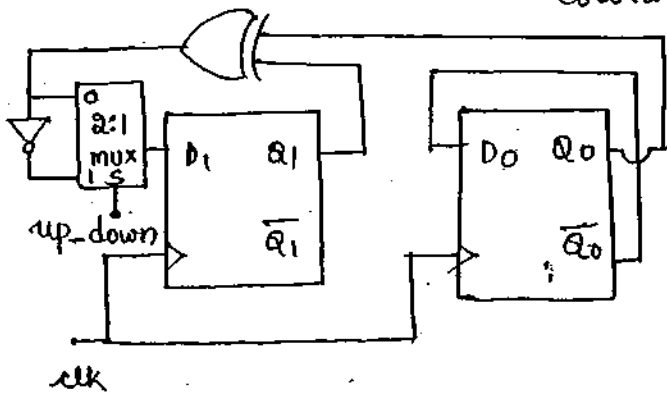
$$D_1 = Q_1 Q_0 + \bar{Q}_1 \bar{Q}_0 = Q_1 \odot Q_0$$

$$D_0 = Q_1 \bar{Q}_0 + \bar{Q}_1 Q_0 = \bar{Q}_0$$

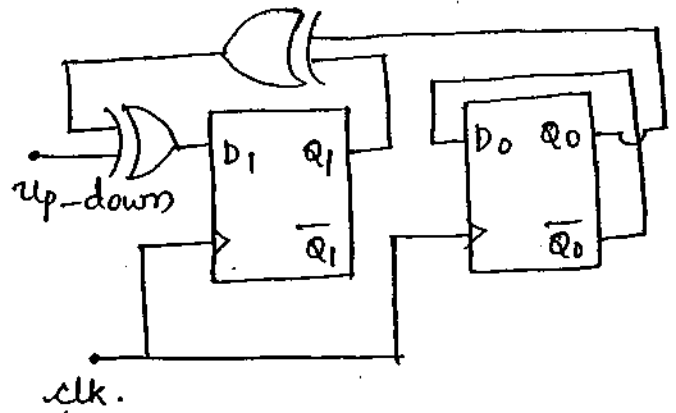


Design 2-bit up/down counter
using D-flipflops :-

up-down = 0 → Up counter
 up-down = 1 → down counter.



Instead of MUX, another X-OR gate also can be used.

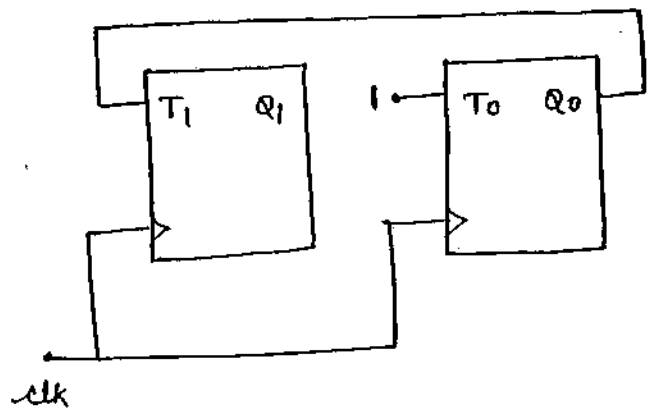


Design 2-bit binary counter using
T-flipflop :-

	<u>PS</u>		<u>NS</u>		<u>FF i/p's</u>
	Q_1	Q_0	Q_1	Q_0	T_1 T_0
0	0	0	0	1	0 1
1	0	1	1	0	1 1
2	1	0	1	1	0 1
3	1	1	0	0	1 1

$$T_1 = \bar{Q}_1 Q_0 + Q_1 Q_0 = Q_0$$

$$T_0 = 1$$



Design 2-bit gray counter using
J-K flipflop :-

	<u>PS</u>	<u>NS</u>	<u>FF i/p's</u>			
	Q_1	Q_0	Q_1	Q_0	T_1 K_1	T_0 K_0
	0	0	0	1	0 X	1 X
	0	1	1	1	1 X	X 0
	1	1	1	0	X 0	X 1
	1	0	0	0	X 1	0 X
	0	0				

$$T_1 = Q_0$$

$Q_1 \backslash Q_0$	0	1
0		1
1	X	X

$$K_1 = \overline{Q_0}$$

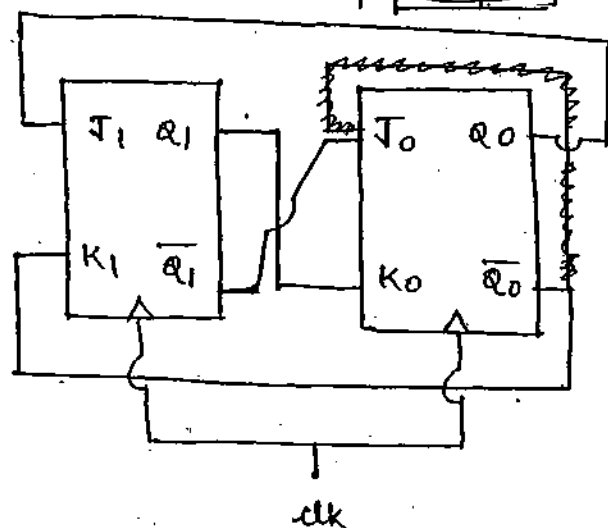
$Q_1 \backslash Q_0$	0	1
0	X	X
1	1	

$$T_0 = \overline{Q_1}$$

$Q_1 \backslash Q_0$	0	1
0	1	X
1	X	1

$$K_0 = Q_1$$

$Q_1 \backslash Q_0$	0	1
0	X	
1	X	1



Design a counter which is going through the states

$00 \rightarrow 10 \rightarrow 11 \rightarrow 01$

using T-flipflops :-

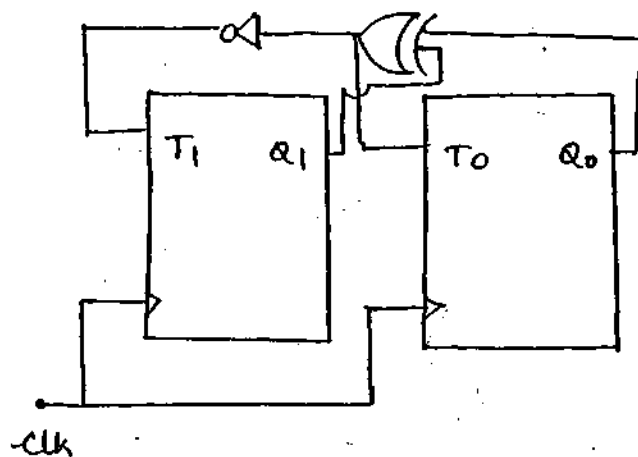
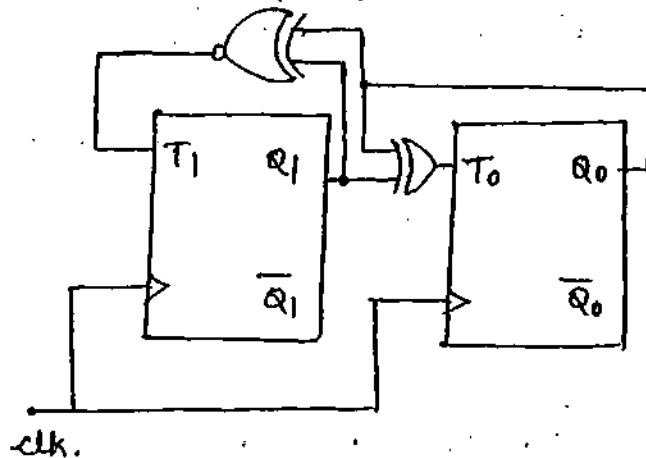
P.S	N.S	FF i/ps
$Q_1 \ Q_0$	$Q_1 \ Q_0$	$T_1 \ T_0$
0 0	1 0	1 0
1 0	1 1	0 1
1 1	0 1	1 0
0 1	0 0	0 1

$$T_1 = \overline{Q_1} \overline{Q_0} + Q_1 Q_0$$

$$= Q_1 \oplus Q_0$$

$$T_0 = Q_1 \overline{Q_0} + \overline{Q_1} Q_0$$

$$= Q_1 \oplus Q_0$$



Design a modulo-6 counter using T-flipflops.

P.S	N.S	FF i/ps
$Q_2 \ Q_1 \ Q_0$	$Q_2 \ Q_1 \ Q_0$	$T_2 \ T_1 \ T_0$
0 0 0	0 0 1	0 0 1
0 0 1	0 1 0	0 1 1
0 1 0	0 1 1	0 0 1
0 1 1	1 0 0	1 1 1
1 0 0	1 0 1	0 0 1
1 0 1	0 0 0	1 0 1

$$T_2 = \overline{Q_2} Q_1 Q_0 + Q_2 \overline{Q_1} Q_0$$

$$+ d(b_1)$$

$$T_1 = \overline{Q_2} \overline{Q_1} Q_0 + \overline{Q_2} Q_1 Q_0$$

$$+ d(b_1)$$

$$= \overline{Q_2} Q_0$$

$$T_0 = \sum m(0,1,2,3,4,5) + d(b_1, b_2)$$

$$T_0 = 1$$

for T_1 ;

$Q_2 \backslash Q_1 Q_0$	00	01	11	10
0		1	1	
1			X	X

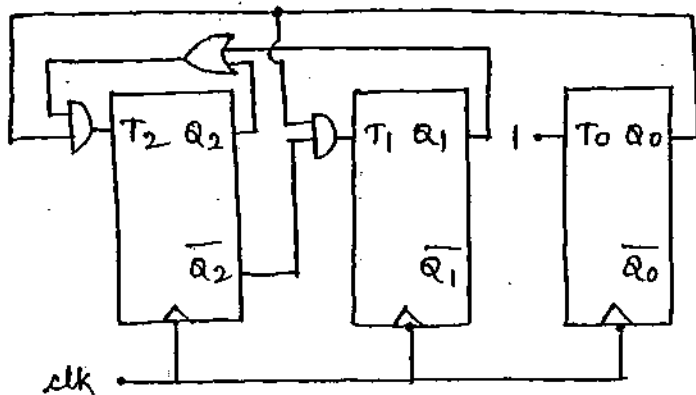
$$T_1 = \bar{Q}_2 Q_0$$

for T_2 ;

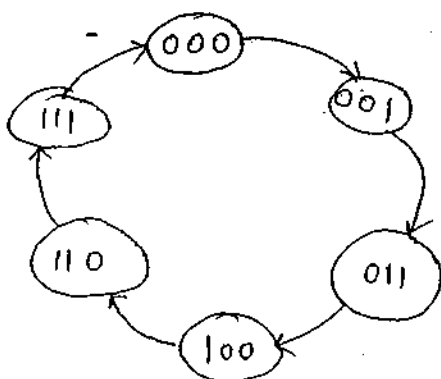
$Q_2 \backslash Q_1 Q_0$	00	01	11	10
0			1	
1		1	X	X

$$T_2 = Q_2 Q_0 + Q_1 Q_0$$

$$T_2 = Q_0 (Q_1 + Q_2)$$



Design a counter which is going through the states specified in the diagram, assuming unused states are initialised to next immediate used state using D-flipflops.



	PS $Q_2 Q_1 Q_0$			NS $Q_2 Q_1 Q_0$			FF i/p's $D_2 D_1 D_0$		
0	0	0	0	0	0	1	0	0	1
1	0	0	1	0	1	1	0	1	1
2	0	1	0	0	1	1	0	1	1
3	0	1	1	1	0	0	1	0	0
4	1	0	0	1	1	0	1	1	0
5	1	0	1	1	1	0	1	1	0
6	1	1	0	1	1	1	1	1	1
7	1	1	1	0	0	0	0	0	0

$D_2 =$	$Q_2 \backslash Q_1 Q_0$	00	01	11	10
0				1	
1		1	1		1

$$D_2 = Q_2 \bar{Q}_1 + Q_2 \bar{Q}_0 + \bar{Q}_2 Q_1 Q_0$$

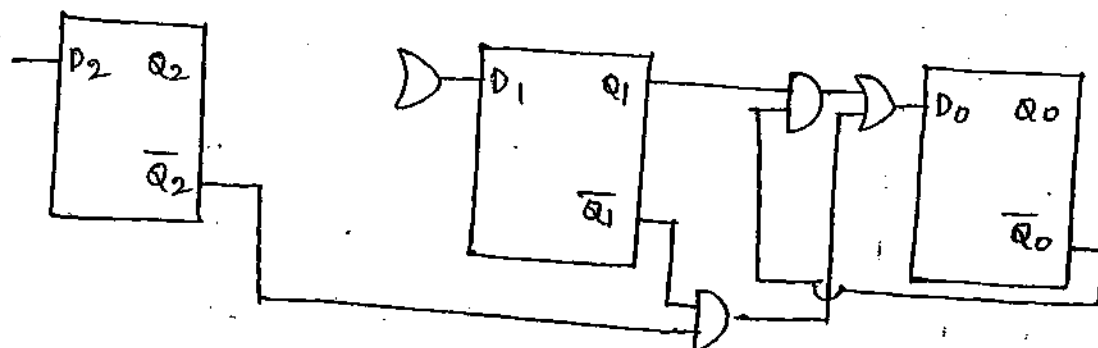
$D_1 =$	$Q_2 \backslash Q_1 Q_0$	00	01	11	10
0			1		1
1		1	1		1

$$D_1 = Q_2 \bar{Q}_1 + \bar{Q}_1 Q_0 + Q_1 \bar{Q}_0$$

$$= Q_2 \bar{Q}_1 + (Q_1 \oplus Q_0)$$

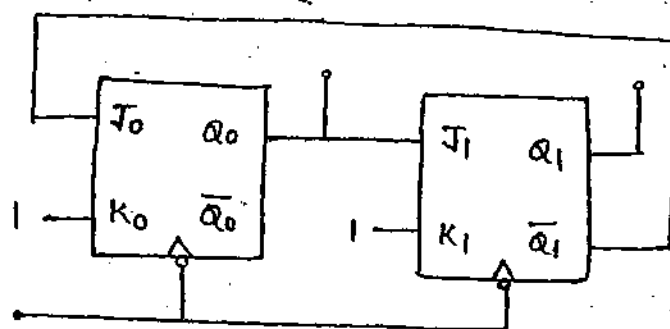
$D_0 =$	$Q_2 \backslash Q_1 Q_0$	00	01	11	10
0		1	1		1
1					1

$$D_0 = \bar{Q}_2 \bar{Q}_1 + Q_1 \bar{Q}_0$$



* Q.8 pgno. 78

Sol:- mod-k counter.



PS PI
Q₁ Q₀ J₁ K₁ J₀ K₀

<u>PS</u>	<u>PI</u>	<u>NS</u>
Q ₁ Q ₀	J ₁ K ₁ J ₀ K ₀	Q ₁ Q ₀
	Q ₀ Q ₁	
0 0	0 1 1	0 1
0 1	0 1 1	1 0
1 0	0 0 1	0 0
0 0	0 1 1	0 1

as we are getting 3 states. It is mod-3 counter.

* Q.4 pgno. 82

Sol:-

<u>PS</u>	<u>PI</u>	<u>NS</u>
Q ₂ Q ₁ Q ₀	Q ₁ (Q ₂ ⊕ Q ₀) D ₂ D ₁ D ₀	Q ₂ Q ₁ Q ₀
0 0 0	0 0 1	0 0 1
0 0 1	0 1 0	0 1 0
0 1 0	1 0 0	1 0 0
1 0 0	0 1 1	0 1 1
0 1 1	1 1 1	1 1 1
1 1 1	1 0 0	1 0 0
1 0 0	0 1 0	0 1 0
0 1 0	1 0 0	1 0 0
1 0 0	0 1 0	0 1 0

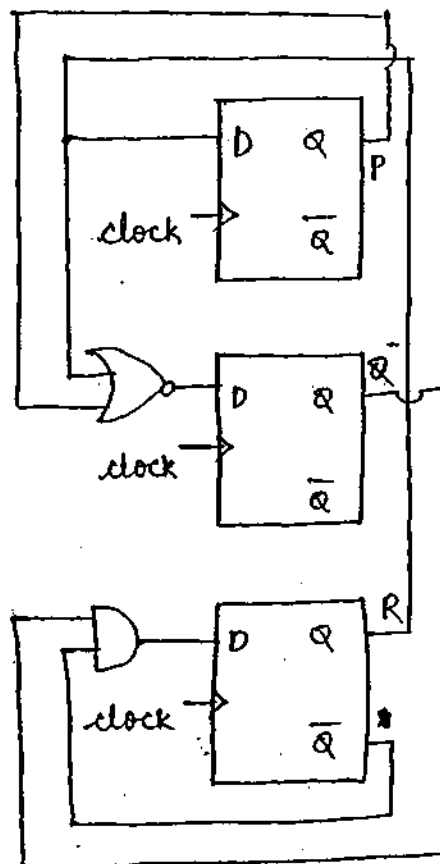
* Q.5 and 6 pgno.82

P.S			P.I			N.S		
P	Q	R	D_P, D_Q, D_R			P	Q	R
0	0	0	$R, \overline{P+R}, \overline{Q\overline{R}}$			0	1	0
0	1	0				0	1	1
0	1	1				1	0	0
1	0	0				0	0	0

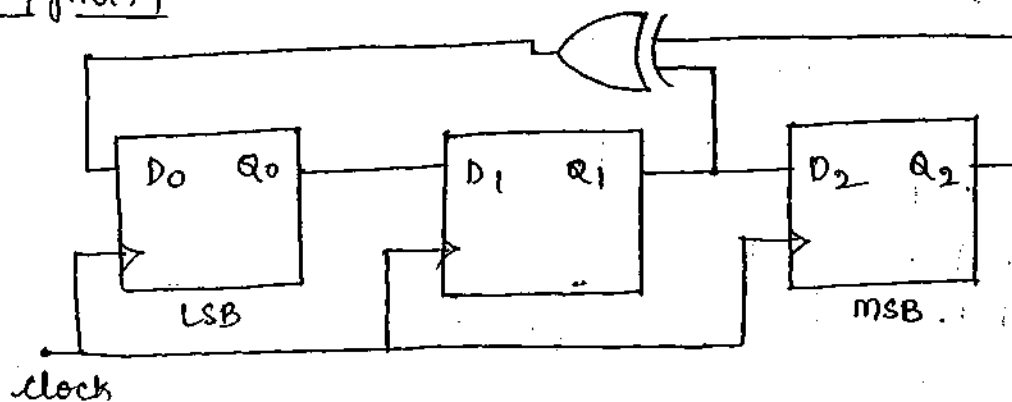
4 states

for PS PAR = 010

NS PAR = 011.



* Q.13 pgno.79



P.S	P.I	N.S	State = $4Q_2 + 2Q_1 + Q_0$
$Q_2 Q_1 Q_0$	$D_2 D_1 D_0$	$Q_2 Q_1 Q_0$	$= 0 + 0 + 1 = 1$
0 0 1	$Q_1 Q_0, Q_1 \oplus Q_2$		
0 0 1	0 1 0	0 1 0	$= 0 + 2 + 0 = 2$
0 1 0	1 0 1	1 0 1	$= 4 + 1 = 5$
1 0 1	0 1 1	0 1 1	$= 2 + 1 = 3$
0 1 1	1 1 1	1 1 1	$= 4 + 2 + 1 = 7$
1 1 1	1 1 0	1 1 0	$= 4 + 2 = 6$
1 1 0	1 0 0	1 0 0	$= 4$

* Q.14 pg no. 79.

Sol:-

PS			PT		
Q_A	Q_B	Q_C	D_A	D_B	D_C
(0	0	0)	$\overline{Q_B} \oplus Q_C$	Q_A	Q_B
0	0	0	$\overline{0} = 1$	0	0
1	0	0	1	1	0
1	1	0	0	1	1
0	1	1	1	0	1
1	0	1	0	1	0
0	1	0	0	0	1
0	0	1	0	0	0

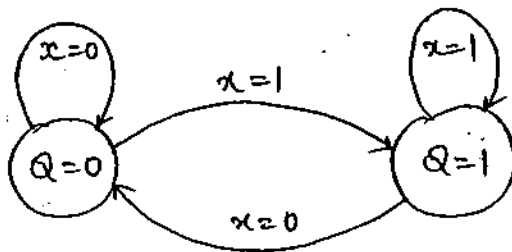
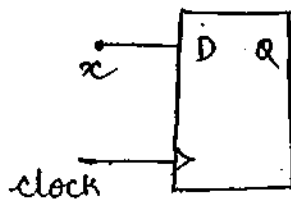
NS:

Q_A	Q_B	Q_C
$\overline{0} = 1$	0	0
1	1	0
0	1	1
1	0	1
0	1	0
0	0	1
0	0	0

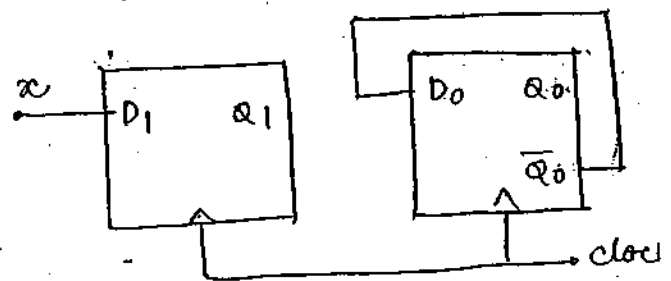
Count sequence at Q_A is 01101000

STATE DIAGRAM:-

State diagram of D-flipflop:-

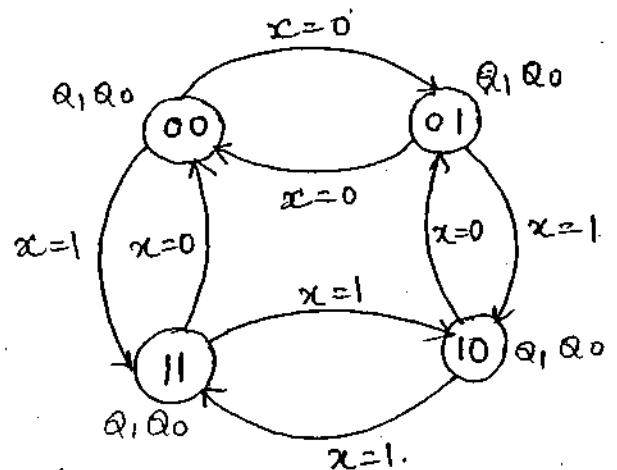
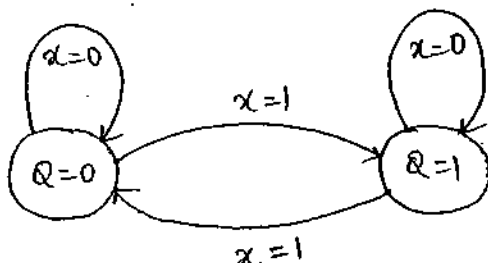
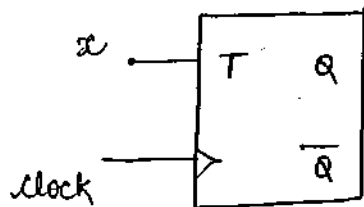


Draw the state diagram w.r.t x for the circuit:-

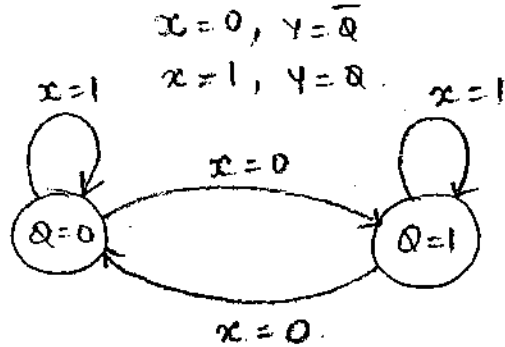
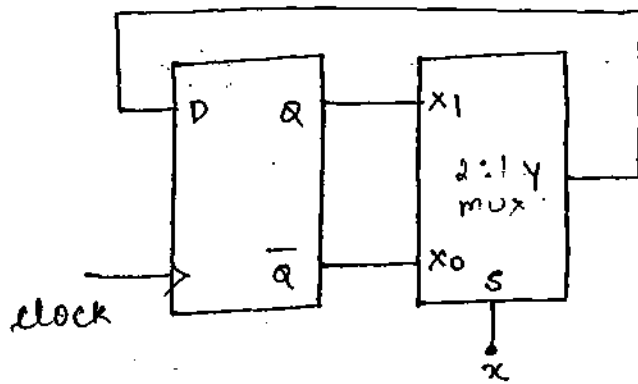


Next state $\rightarrow x \overline{Q_0}$

State diagram of T-flipflop:-



Eg:- Draw the state diagram w.r.t x .



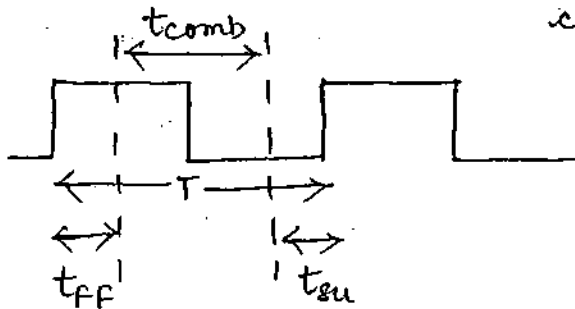
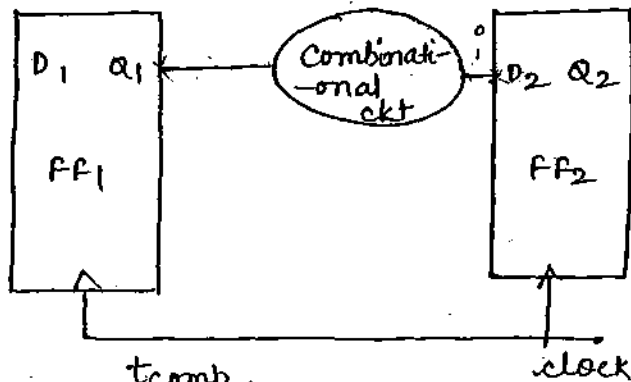
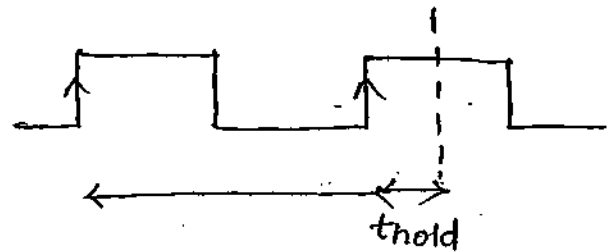
t_{comb} \rightarrow time delay due to combinational circuit.

t_{hold} \rightarrow minimum time required to change the inputs.

$$t_{ck-Q} + t_{comb} \geq t_{hold}$$

(FF1) (FF2)

t_{hold} is to allow the flipflop to respond to the previous input.



$$T_{min} = t_{ff} + t_{comb} + t_{su}$$

\downarrow (FF1) \downarrow (FF2)

$$f_{max} = \frac{1}{T_{min}}$$

t_{ff} \rightarrow time delay for the response

t_{su} \rightarrow setup time for inputs.

Eg:-

$t_{ck-Q} = 1ns$	} for FF1
$t_{su}/t_h = 1ns/1ns$	
$t_{comb} = 2ns$	} for FF2
$t_{ck-Q} = 2ns$	
$t_{su}/t_h = 2ns/2ns$	

find f_{max} .

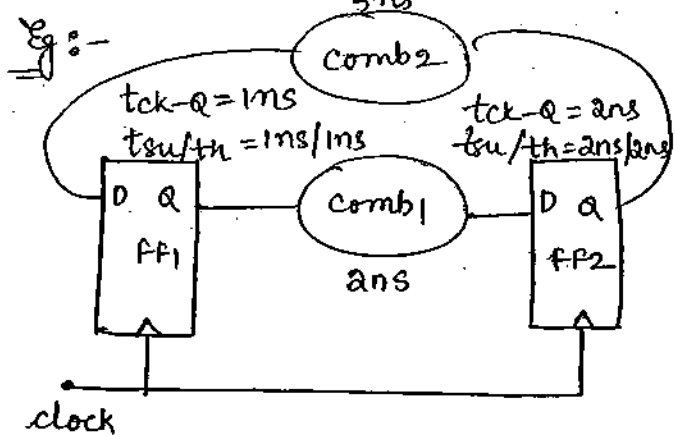
Sol:- $T_{min} = 1ns + 2ns + 2ns$

$= 5ns$.

$$f_m = \frac{1}{T_{min}}$$

$$f_{max} = 0.2 \times 10^9$$

$= 200 MHz$



$$(t_{ck+Q} + t_{comb} \geq t_{hold})$$

find max. freq. at which ckt works properly.

Sol:- FF1 \rightarrow FF2

$$T_{min1} = 1ns + 2ns + 2ns \\ = 5ns$$

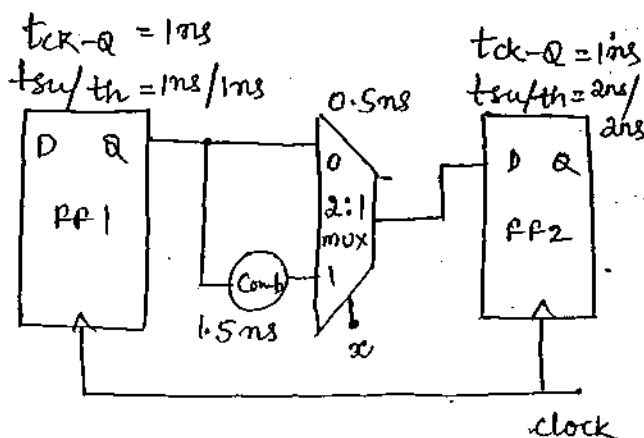
FF2 \rightarrow FF1.

$$T_{min2} = 2ns + 5ns + 1ns \\ = 8ns.$$

$\therefore T_{min} = 8ns$. i.e.,

$$f_{max} = \frac{1}{T_{min}} \quad \text{Max}\{T_{min1}, T_{min2}\} \\ = \frac{1}{8ns} \\ = 125 MHz.$$

Eg:-



find max clock frequency.

for $x=0$,

$$t_{min} = 1ns + 0.5ns + 2ns \\ = 3.5ns.$$

for $x=1$,

$$t_{min} = 1ns + 1.5ns + 0.5ns + 2ns \\ = 5ns.$$

$$\therefore t_{min} = \max\{3.5ns, 5ns\} \\ = 5ns$$

$$f_{max} = \frac{1}{5ns}$$

$$= 0.2 \times 10^9$$

$$= 200 MHz.$$

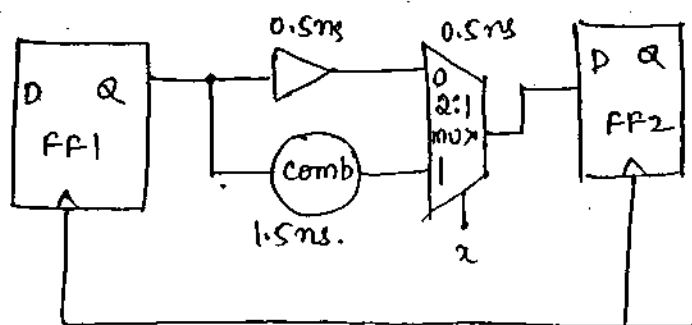
for $x=0$,

$t_{ck+Q} + t_{comb} \geq t_{hold}$ condition is not satisfied.

$$1ns + 0.5ns \neq 2ns$$

In this case we have to add buffer in the path which should have minimum of 0.5ns delay

for $x=1$, the hold condition is satisfied. $1ns + 1.5 + 0.5 > 2ns$.



$$\text{for } x=0, T_{min} = 1ns + 0.5ns + 0.5ns + 2ns$$

for $x \geq 1$,

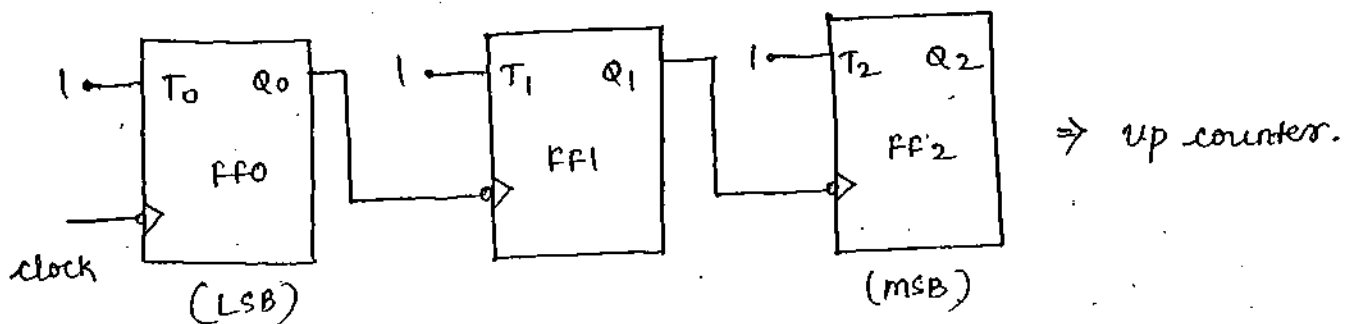
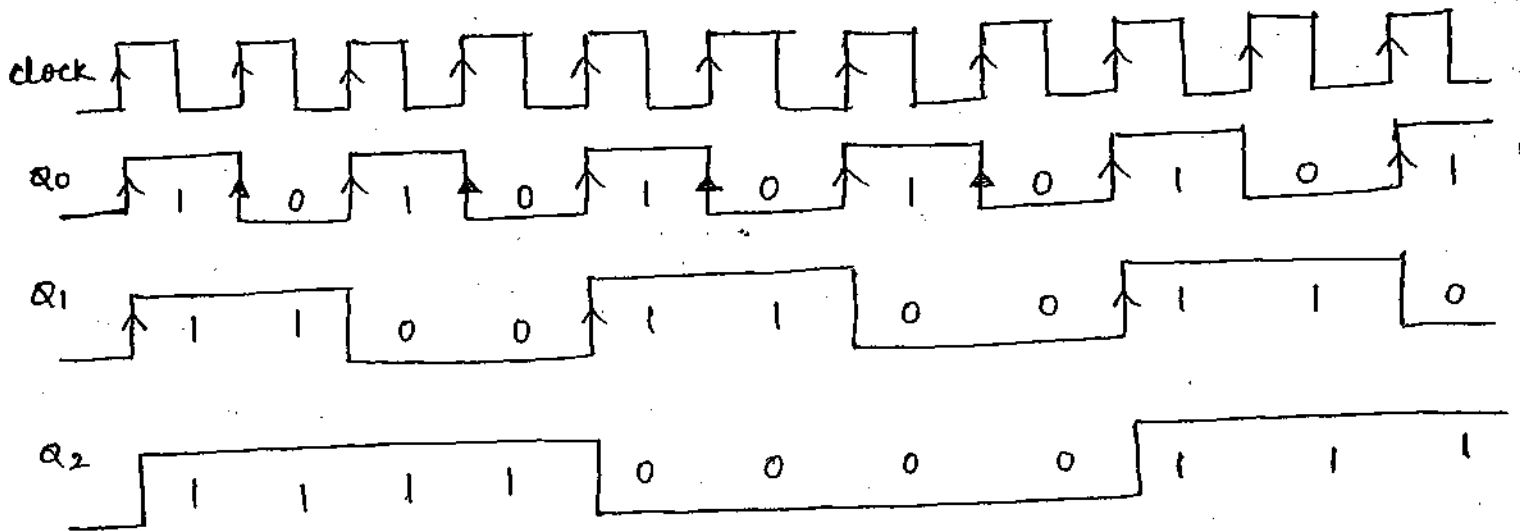
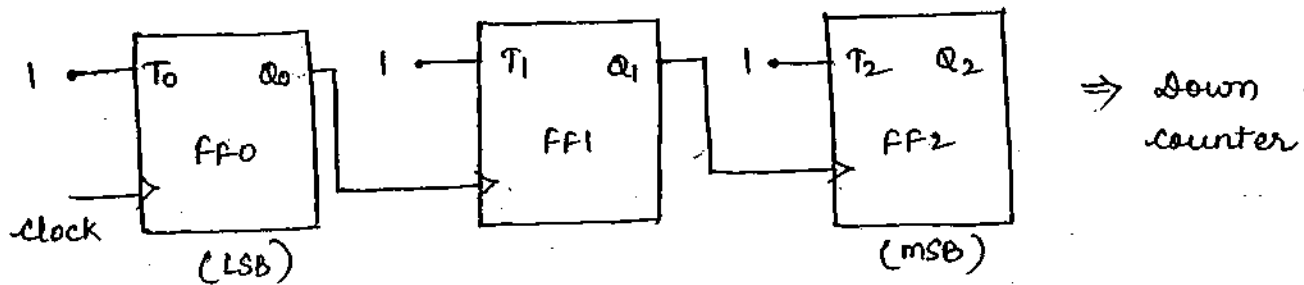
$$t_{min} = 5ns.$$

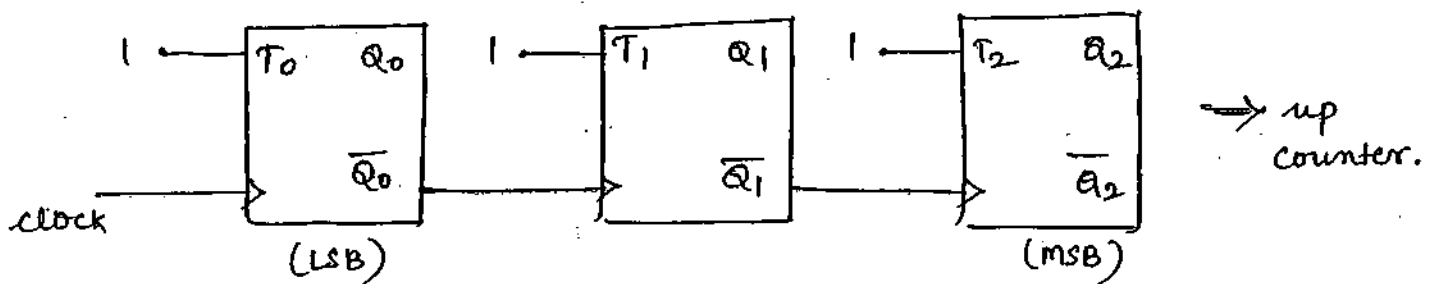
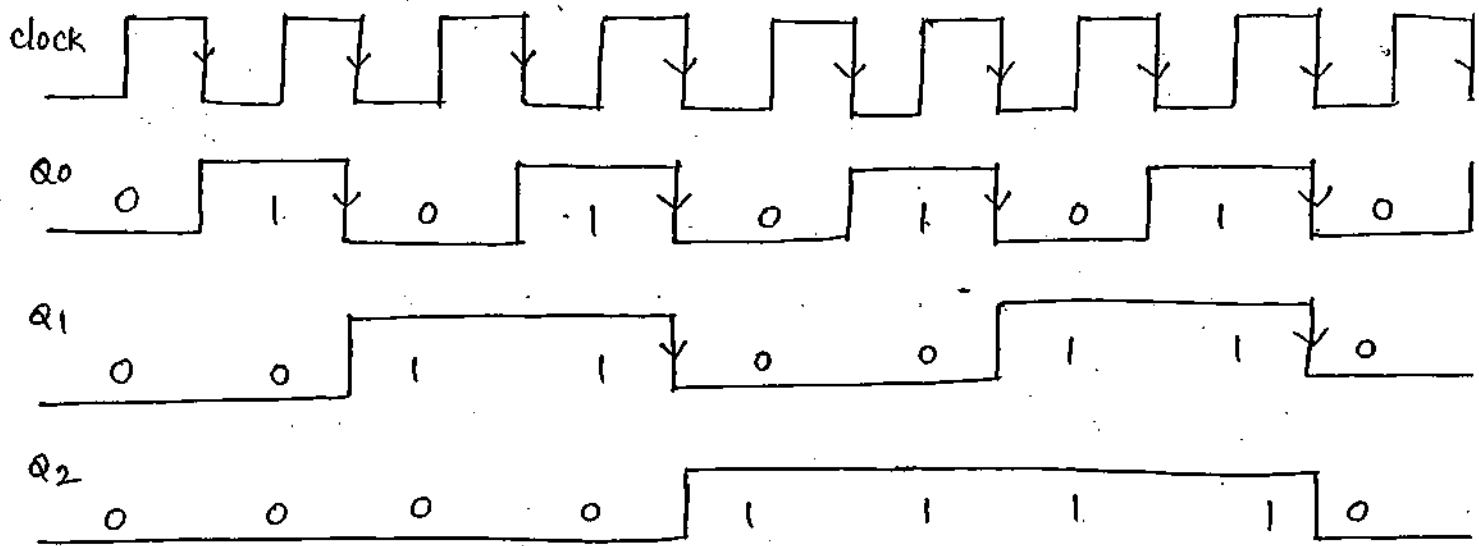
$$\therefore f_{max} = \frac{1}{t_{min}}$$

$$= \frac{1}{5ns}$$

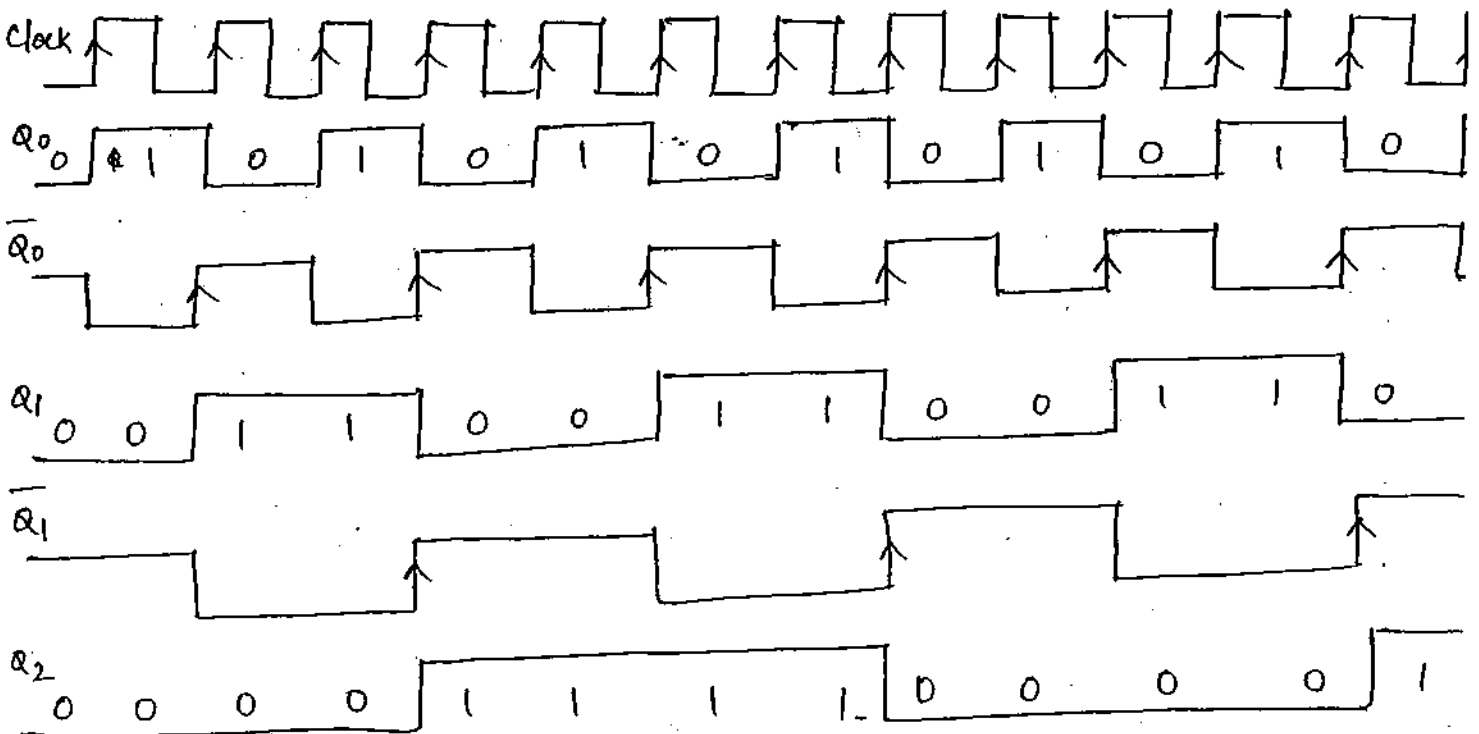
$$= 200 MHz.$$

ASYNCHRONOUS SEQUENTIAL CIRCUIT :-
(RIPPLE COUNTERS).

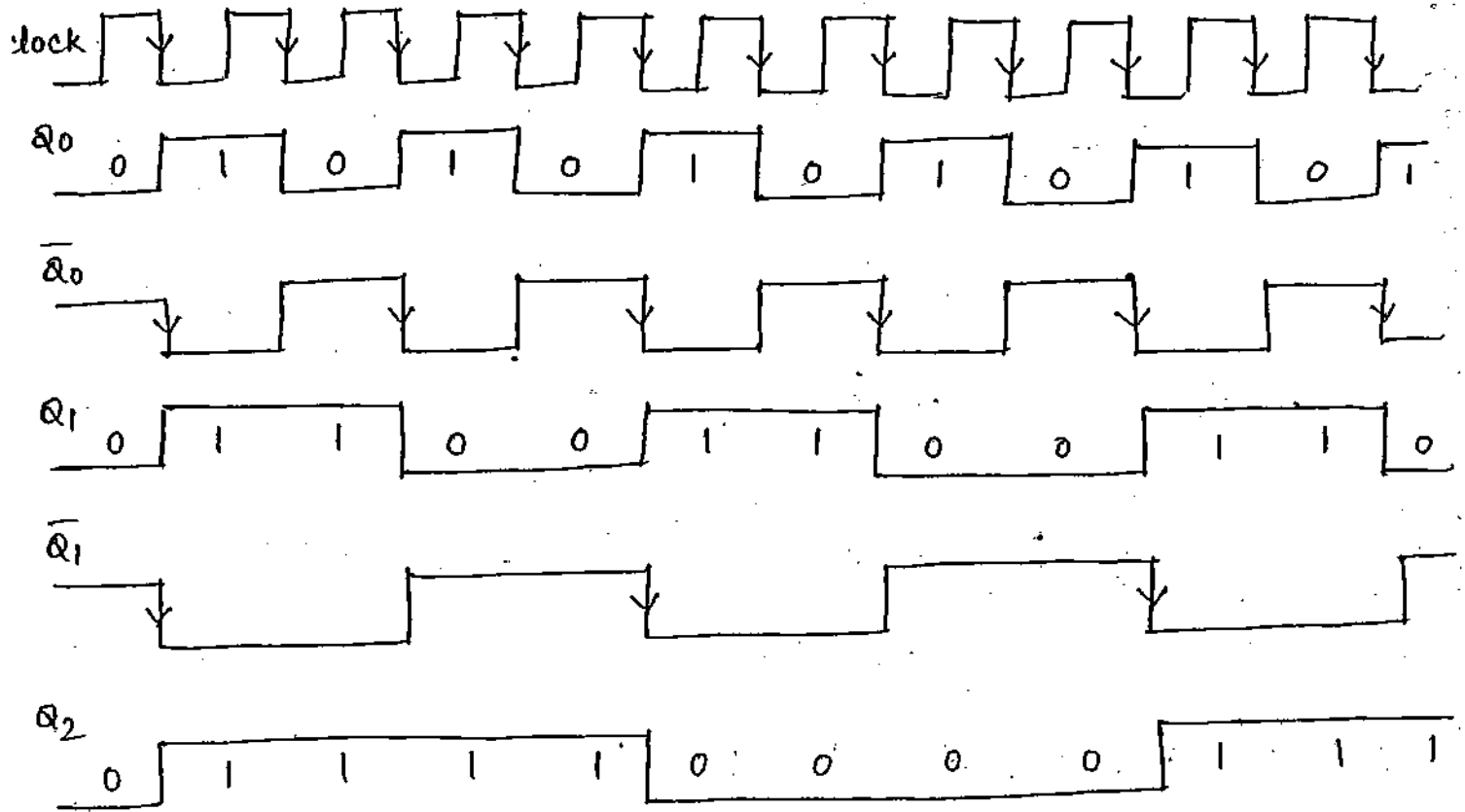
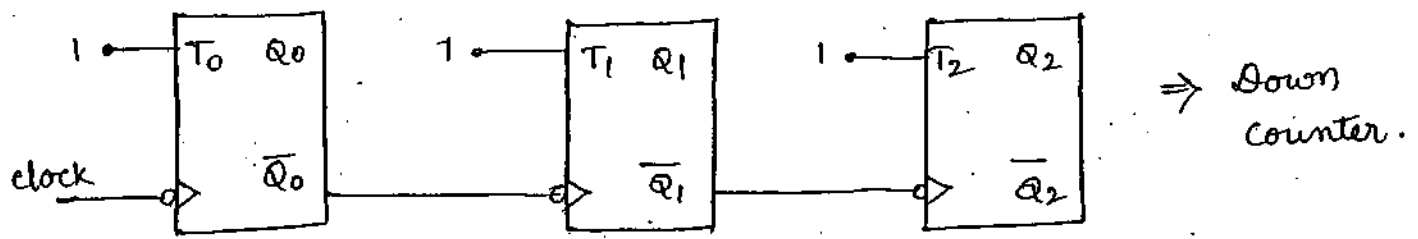




flipflops are +ve edge triggered flipflops.



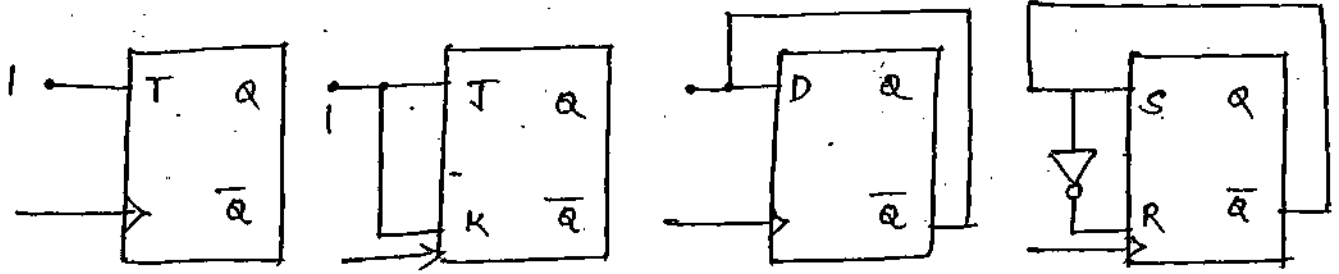
Instead of +ve edge of Q_0 , -ve edge of Q_0 also can be considered



NOTE :-

- (a) positive edge triggered flipflops passing 'Q' as clock to the next stages results down counter.
- (b) -ve edge triggered flipflops passing 'Q' as a clock to the next stages results up counter.
- (c) +ve edge triggered flipflops passing \overline{Q} as clock to the next stages results up counter.
- (d) -ve edge triggered flipflops passing \overline{Q} as a clock to the next stages results down counter.

Instead of T-flipflop, J-K flipflops also can be used in asynchro counters for $J=K=1$. D-flipflop also can be used by $D=\bar{Q}$.

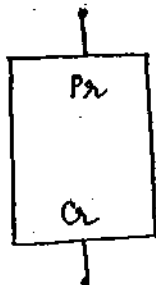


Eg:- Design mod-6 asynchronous up counter. using J-K flipflops.

000
001
010
011
100
101

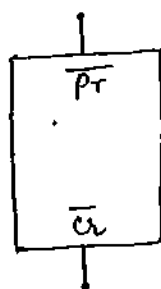
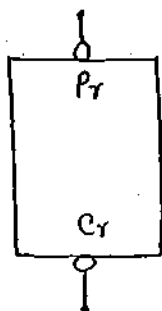
Preset fin $\Rightarrow Q=1$.
clear fin $\Rightarrow Q=0$.
Both should not be applied enabled at same time, it remains in indeterminate state.

Logic '1' enable preset and clear :-



Pr	Cr	function
0	0	normal FF.
0	1	$Q=0$.
1	0	$Q=1$.
1	1	Indeterminate state.

Logic '0' enable preset and clear :-

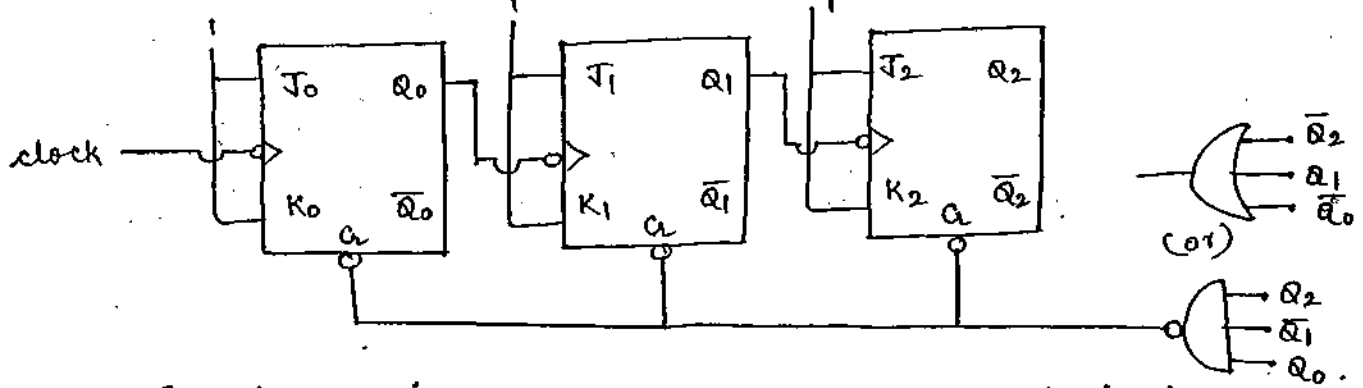


Pr	Cr	function
0	0	Indeterminate state.
0	1	$Q=1$.
1	0	$Q=0$.
1	1	normal FF function

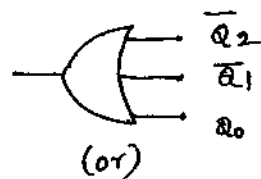
mod-6 counter :

000
001
010
011
100
101

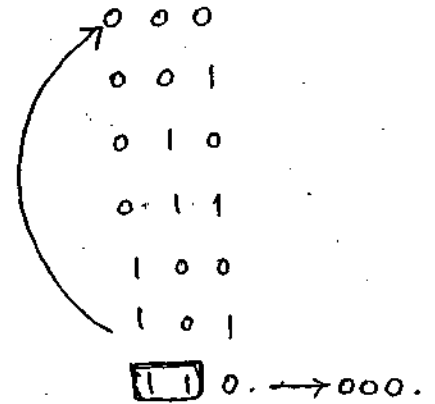
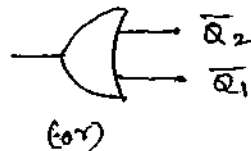
→ Binary up-counter using JK-flipflop :-



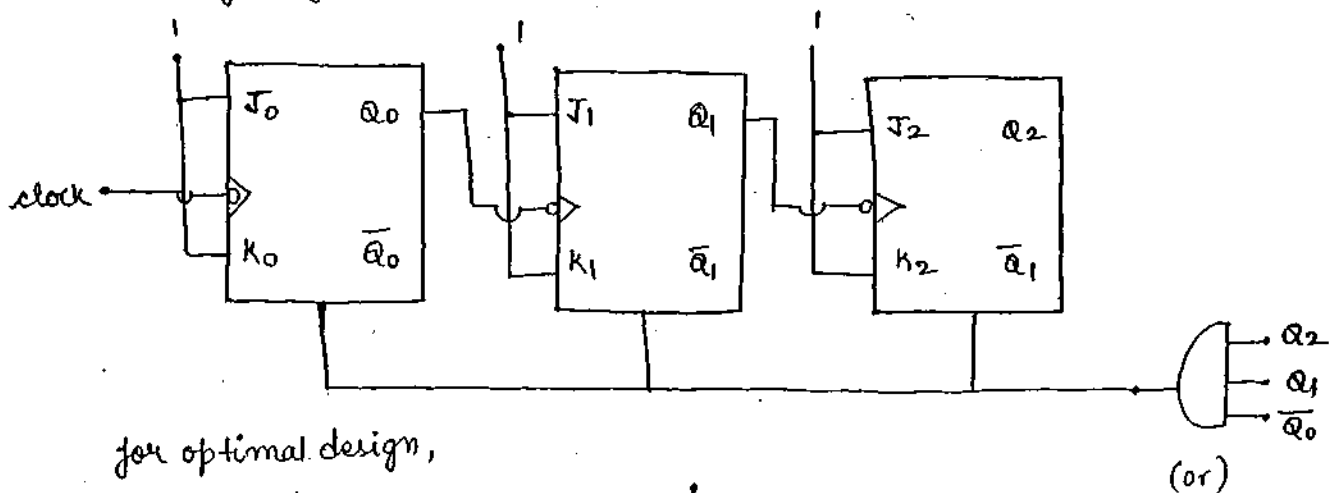
In this 101 is not counted, as soon as 101 obtained, asynchronously the o/p is cleared. So it acts as mod-5 counter. In order to count 101 also, next state 110 is used to clear the o/p, so that it acts as mod-6 counter.



Optimally the gates can be used as,



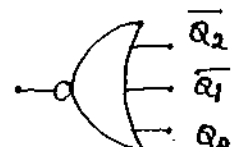
→ By using logic '1' enable pins,



for optimal design,



(or)

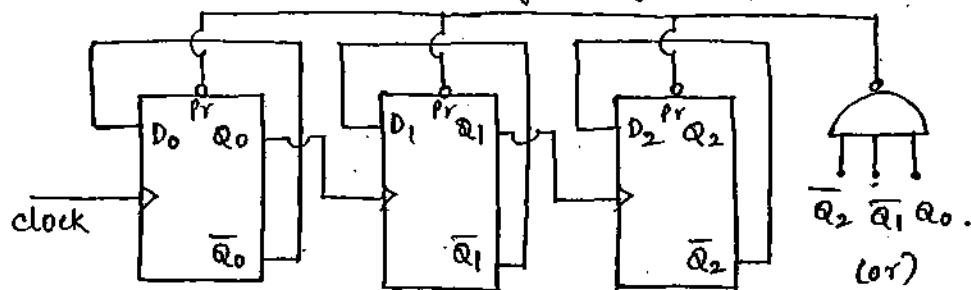


Ex Design mod-6 asynchronous down counter using delay flipflops.

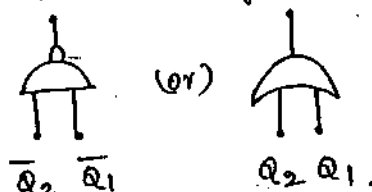
Sol:-

1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0

0 0 1 → 111.



For the optimal design,



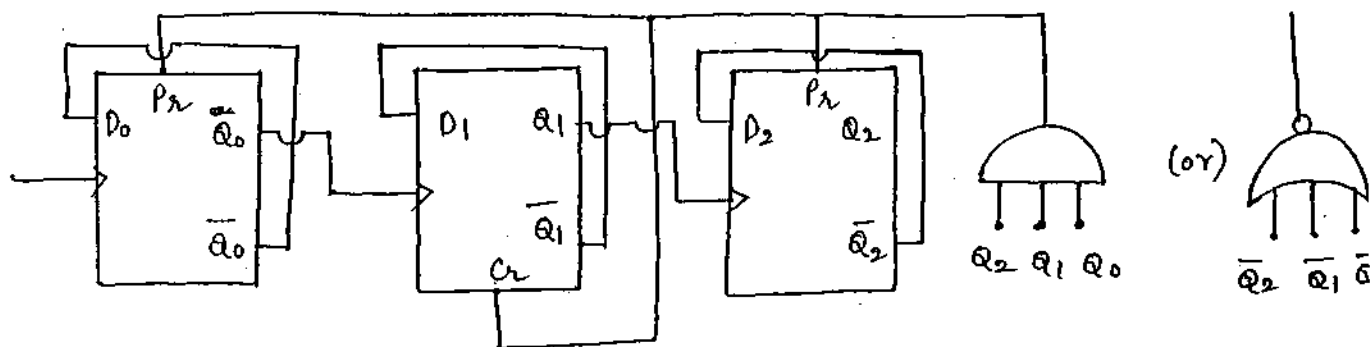
111.

→ By using logic-1 enable preset pins,



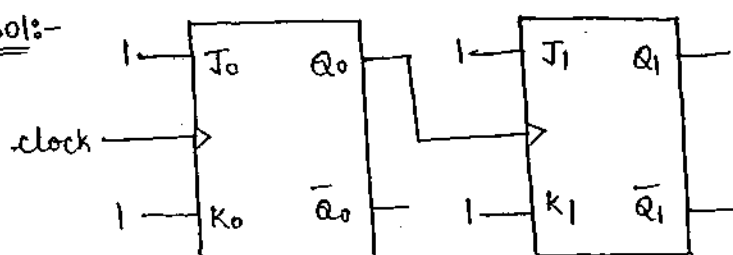
→ To count

1	1	1
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

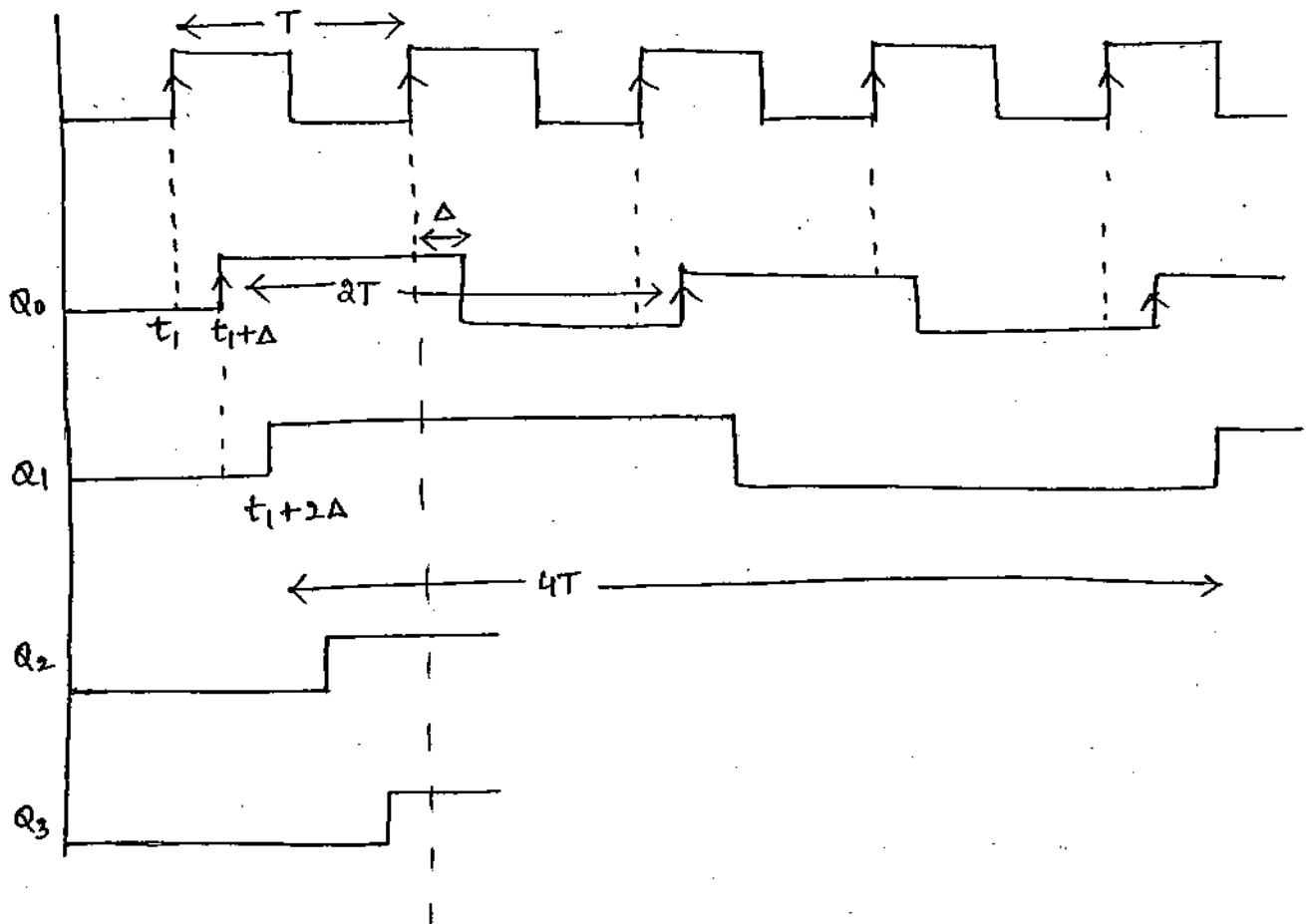


Q.7

Sol:-



propagation delay is ΔT ,



$n\Delta < T \rightarrow$ By using this how many flipflops should be used can be known in asynchronous counter.

$2^n \rightarrow$ Maximum modulus of the counter.

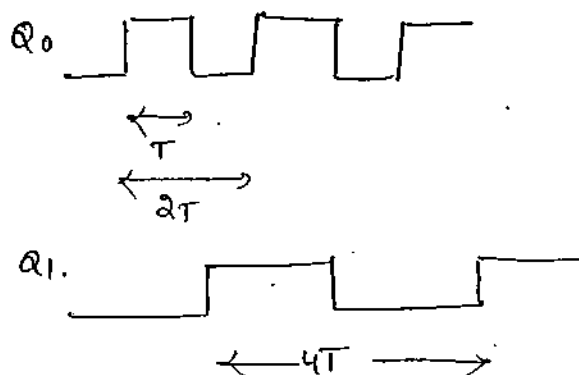
$$T_{Q0} = 2T.$$

$$f_{Q0} = \frac{1}{2T} = \frac{f}{2}.$$

$$T_{Q1} = 4T.$$

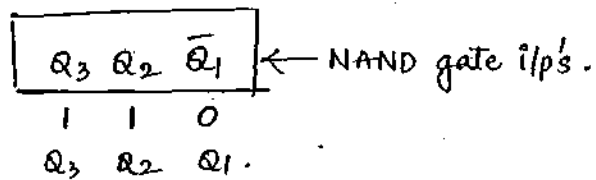
$$f_{Q1} = \frac{1}{4T} = \frac{f}{4}.$$

Q_1	Q_0
0	0
0	1
1	0
1	1



Q.11

Sol:-

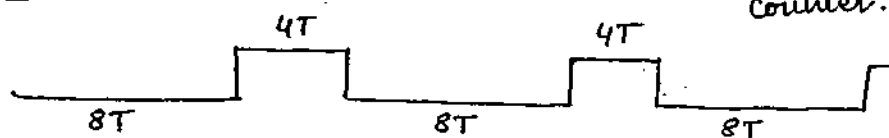


$$f = 10 \text{ KHz.}$$

up counter

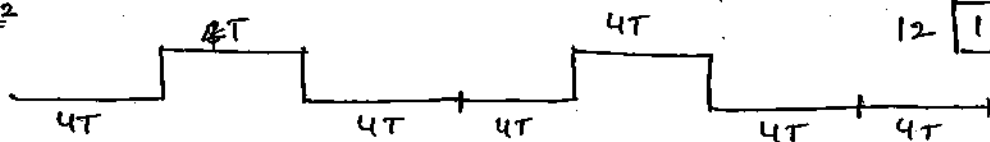
[Q₃ Q₂ Q₁ Q₀]

0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0

mod-12
counter.Q₃

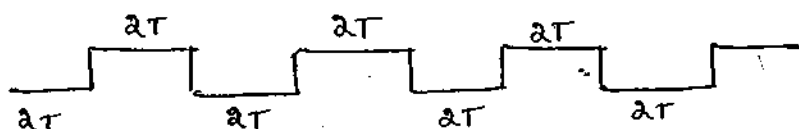
$$T_{Q3} = 4T + 8T = 12T.$$

$$f_{Q3} = f/12 = \frac{10}{12} = 0.833 \text{ KHz.}$$

Q₂

$$T_{Q2} = 8T + 4T = 12T.$$

$$f_{Q2} = f/12 = \frac{10}{12} = 0.833 \text{ KHz.}$$

Q₁

$$T_{Q1} = 2T + 2T = 4T.$$

$$f_{Q1} = f/4 = \frac{10 \text{ KHz}}{4} = 2.5 \text{ KHz.}$$

Q₀

$$T_{Q0} = 2T$$

$$f_{Q0} = f/2 = \frac{10 \text{ KHz}}{2} = 5 \text{ KHz.}$$

Q1

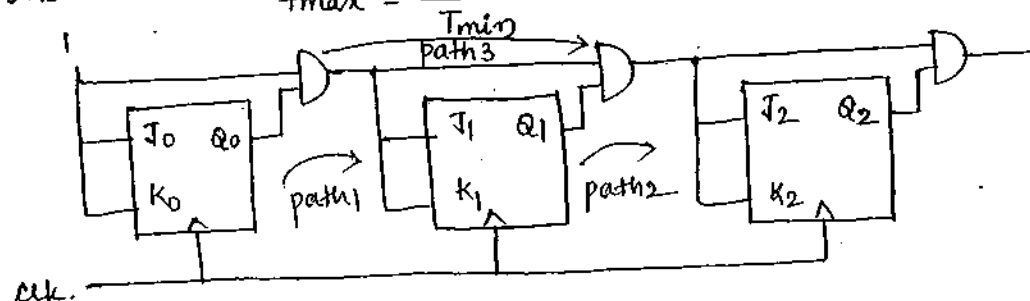
Sol:-

$$T_{FF} = 10 \text{ ns.}$$

$$T_{\text{and}} = 10 \text{ ns.}$$

$$T_{\min} = \max(T_1, T_2).$$

$$f_{\max} = \frac{1}{T_{\min}}$$



$$T_{min1} = 10ns + 10ns + 0$$

$$= 20ns. = T_{min2}$$

$$T_{min3} = 10ns + 10ns + 10ns = 30ns.$$

$$\therefore T_{min} = \max(T_{min1}, T_{min3}).$$

$$= 30ns.$$

$$f_{max} = \frac{1}{T_{min}}$$

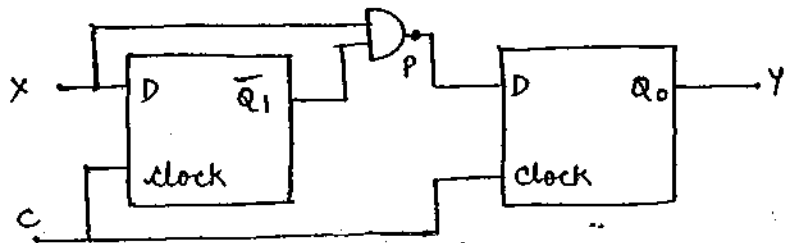
$$= \frac{1}{30ns}$$

$$= 33.3 \text{ MHz}$$

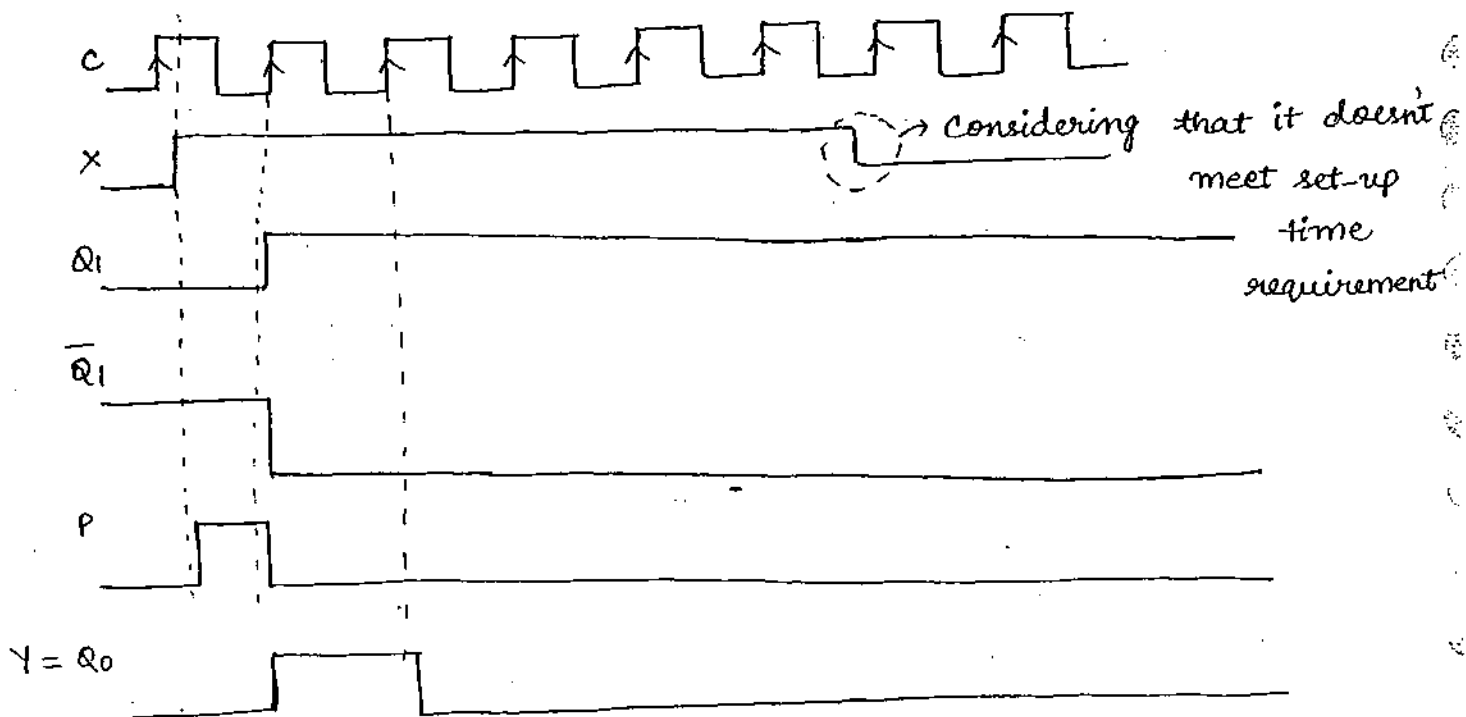
from options, $f_{max} = 25 \text{ kHz}$.

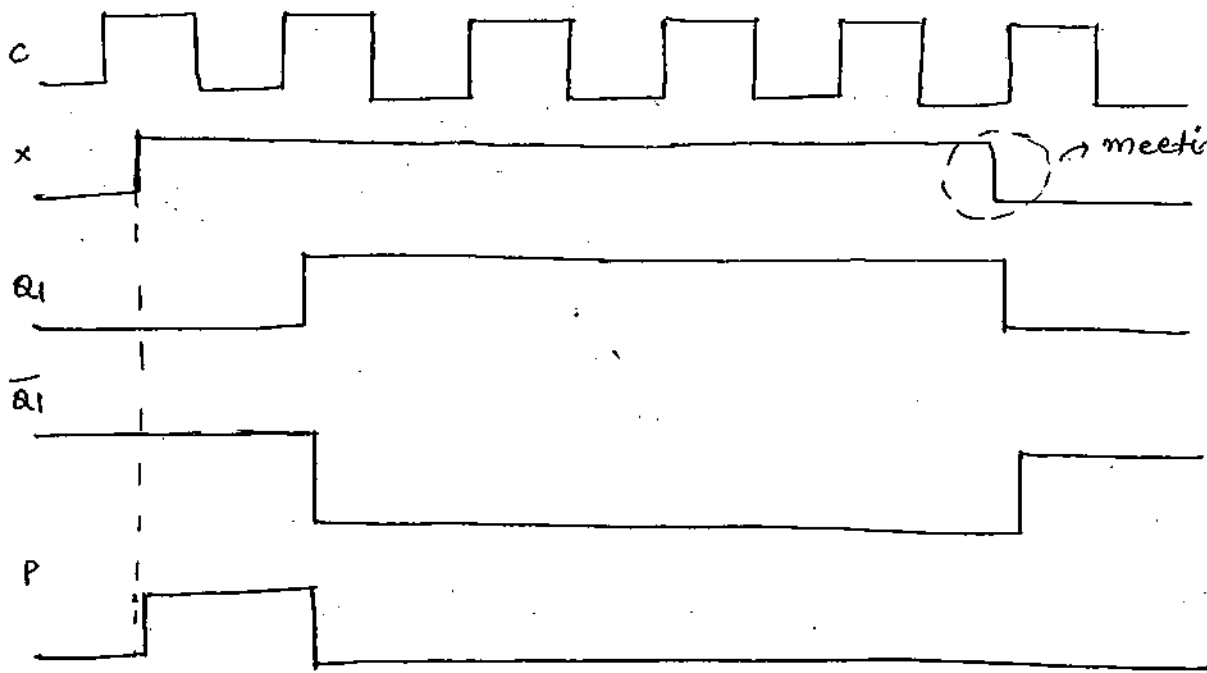
Q.2 Sol:- $Q_0 = Q_1 = 0$

$$T_{su} = 20ns.$$



$$C > 40ns.$$

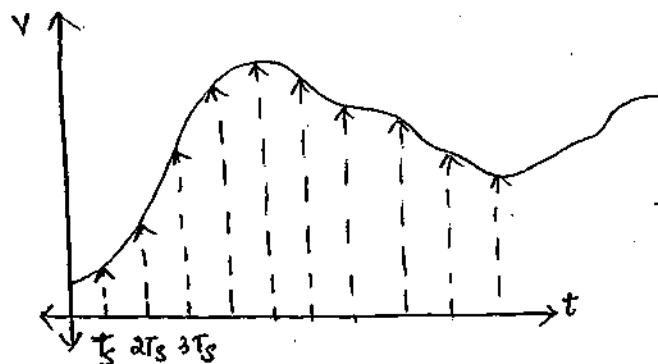
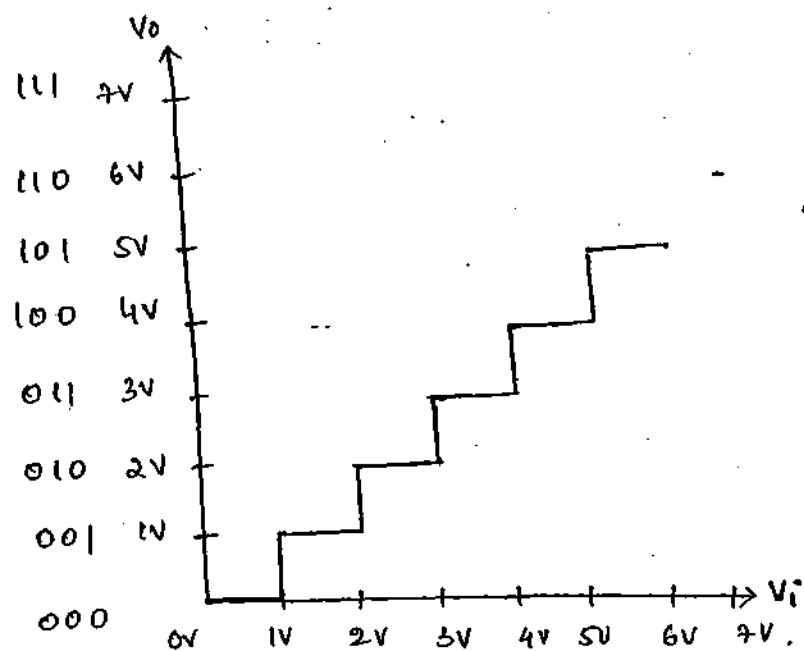




$y = Q_0$

A/D and D/A CONVERTERS

$T_s \rightarrow$ Sampling period.



max possible error = step size.

Analog output, $V_o = K (1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0) \rightarrow 100 \text{ Vp.}$

\downarrow
step size.

$$V_o = K (4 + 0 + 0)$$

$$V_o = 4K$$

$$= 4 \times 1V$$

$$= 4V.$$

DIGITAL TO ANALOG CONVERTERS :-

(i) Weighted resistor D/A.

(ii) R-2R resistor n/w D/A.

Weighted resistor D/A :-

$$I_{n-1} + I_{n-2} + \dots + I_1 + I_0 = I.$$

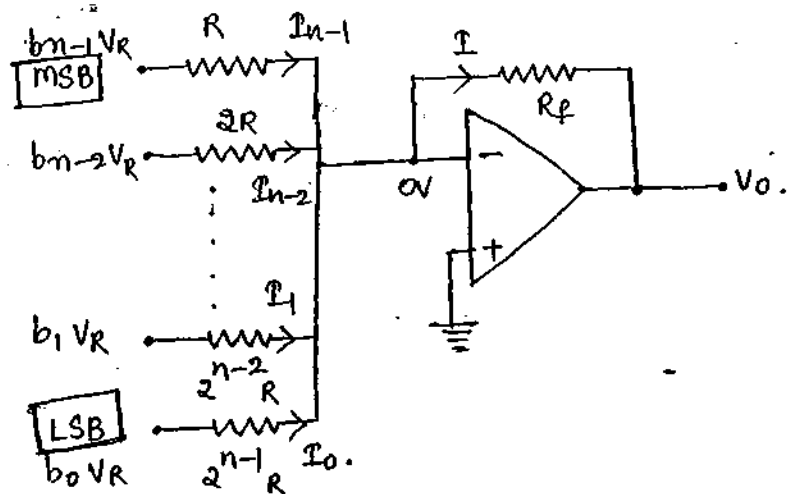
$$\frac{b_{n-1} V_R}{R} + \frac{b_{n-2} V_R}{2R} + \dots + \frac{b_1 V_R}{2^{n-2} R} + \frac{b_0 V_R}{2^{n-1} R} = \frac{-V_o}{R_f}$$

$$\frac{V_R}{R \cdot 2^{n-1}} \left\{ b_{n-1} 2^{n-1} + b_{n-2} 2^{n-2} + \dots + b_1 2 + b_0 \right\} = \frac{-V_o}{R_f}$$

$$V_{out} =$$

$$\frac{-R_f V_R}{R 2^{n-1}} \left\{ b_{n-1} 2^{n-1} + b_{n-2} 2^{n-2} + \dots + b_1 2^1 + b_0 \right\}$$

$$= K \left[\text{Decimal equivalent of Digital Input} \right]$$

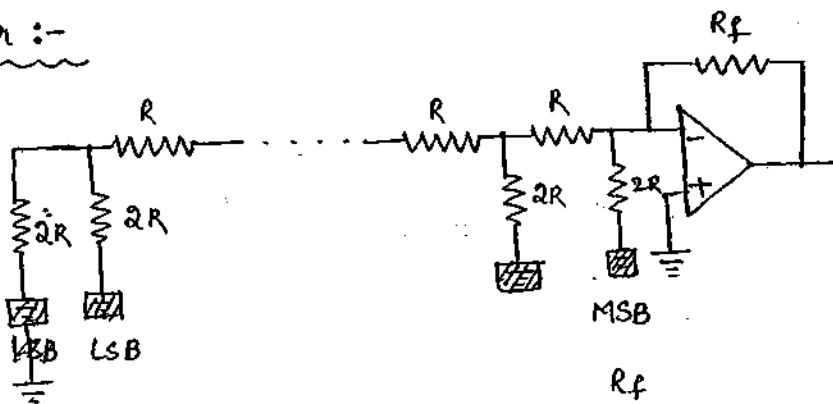


* In weighted resistor D/A converter, for n -bit conversion it requires ' n ' resistances.

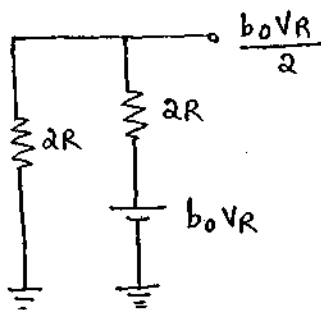
* It requires very large resistance $2^{n-1}R$ for large number of bit conversions.

* As it is using many resistances in the circuit, the linearity is missing.

R-2R resistor network D/A converter :-



3-bit A/D converter :-



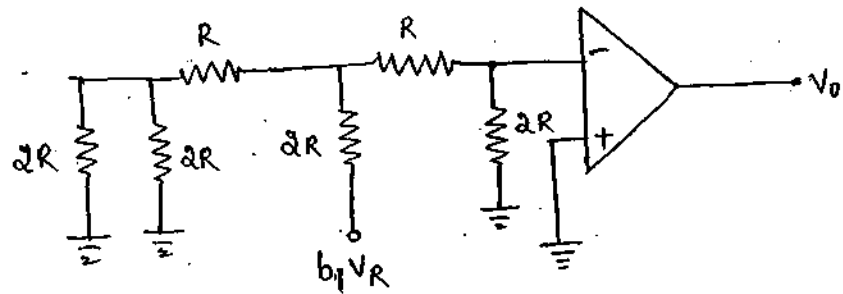
$$R_{th} = 2R \parallel 2R = R$$

$$\therefore R_{th_{eff}} = R$$

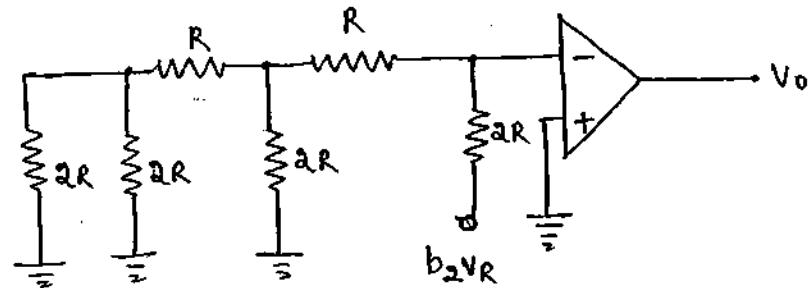
$$V_{o_{eff}} = \frac{b_0 V_R}{8}$$

$$V_{o,0} = -\frac{R_f}{R} \times \frac{b_0 V_R}{8}$$

$$V_{0,1} = -\frac{R_f}{R} \cdot \frac{b_1 V_R}{4}$$



$$V_{0,2} = -\frac{R_f}{R} \cdot \frac{b_2 V_R}{2}$$



$$V_0 = V_{0,0} + V_{0,1} + V_{0,2}$$

$$= -\frac{R_f}{R} \times \frac{V_R}{8} (b_0 + b_1 \cdot 2 + b_2 \cdot 4)$$

$$= -\frac{R_f}{R} \times \frac{V_R}{8} (b_2 \cdot 2^2 + b_1 \cdot 2^1 + b_0 \cdot 2^0)$$

$$= K (\text{Decimal equivalent of Decimal Digital})$$

$$\text{Resolution} = \frac{V_R}{2^n}$$

As only two resistances $R, 2R$ used, the linearity is improved.

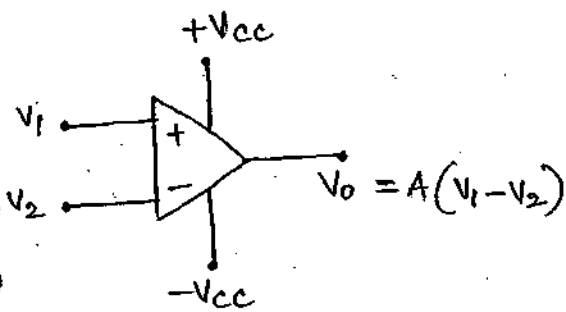
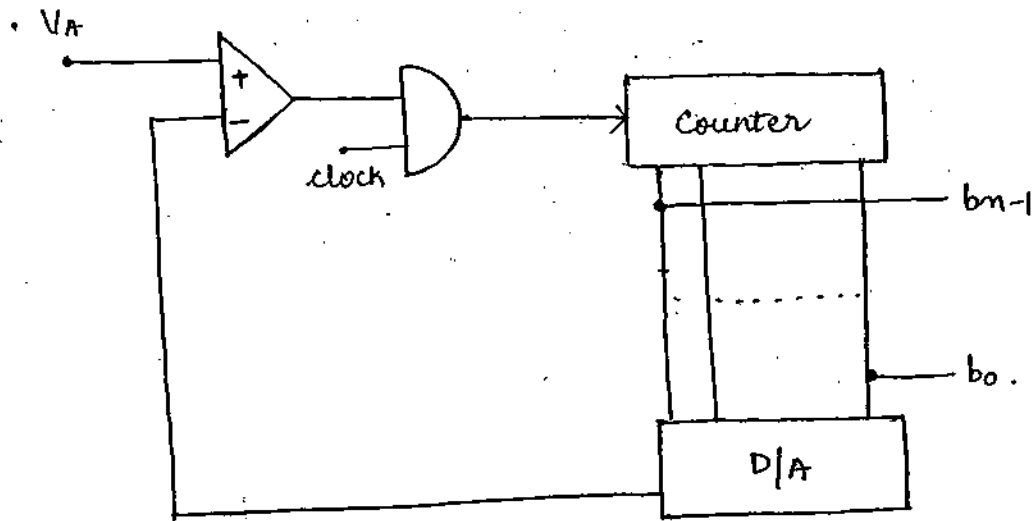
If we consider full scale voltage,

$$\text{Resolution} = \frac{V_{FS}}{2^n - 1}$$

ANALOG TO DIGITAL CONVERTERS :-

- (1) Counter type A/D.
- (2) Flash type A/D.
- (3) Successive approximation type A/D.
- (4) Dual slope type A/D.

Counter type A/D :-



$$V_1 > V_2 \Rightarrow V_0 = +V_{CC} \rightarrow \text{logic '1'}$$

$$V_1 < V_2 \Rightarrow V_0 = -V_{CC} \rightarrow \text{logic '0'}$$

$$V_1 = V_2 \Rightarrow V_0 = 0$$

considering no offset voltages.

conversion time is approximately 2^n clock cycles.

If the analog o/p of D/A converter is less than analog i/p which we want to convert to digital, the comparator o/p is logic '1', AND gate o/p is clock. As the counter is getting the clock cycles, counter is incrementing at each clock edge.

If the analog o/p of D/A converter is exceeding the analog i/p, comparator o/p is logic '0', AND gate o/p is zero. As counter is not getting the clock cycles, counter stops counting.

The count sequence of the counter is considered as approx. digital o/p for analog i/p. The max conversion time is approx. 2^n clock cycles.

Flash type A/D :-

* The fastest A/D.

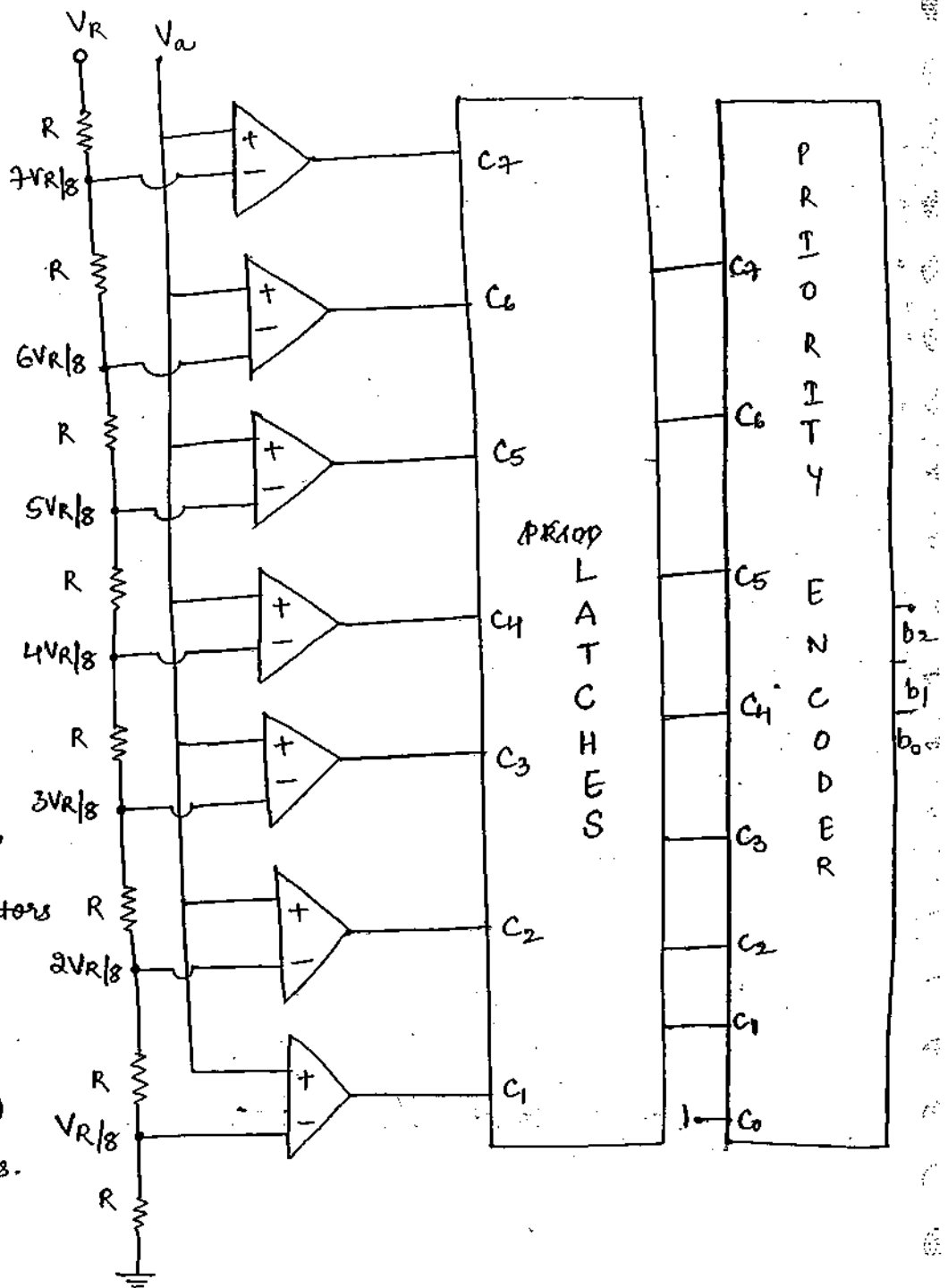
* $2^n - 1$ comparators are required for n-bit conversion.

* Only ideal op-amps are considered such that current entering the op-amp will be zero as the i/p impedance of ideal op-amp is ∞ for an ideal op-amp.

* flash type A/D takes very less conversion time.

* The circuit is very complex as it is using $2^n - 1$ op-amp comparators for n -bit conversion.

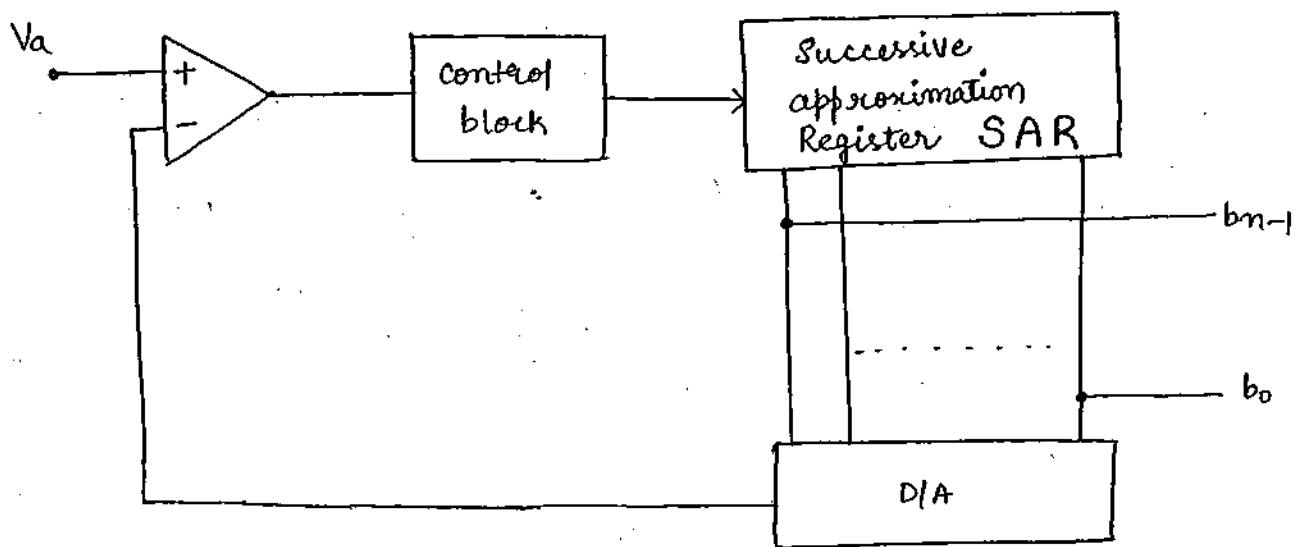
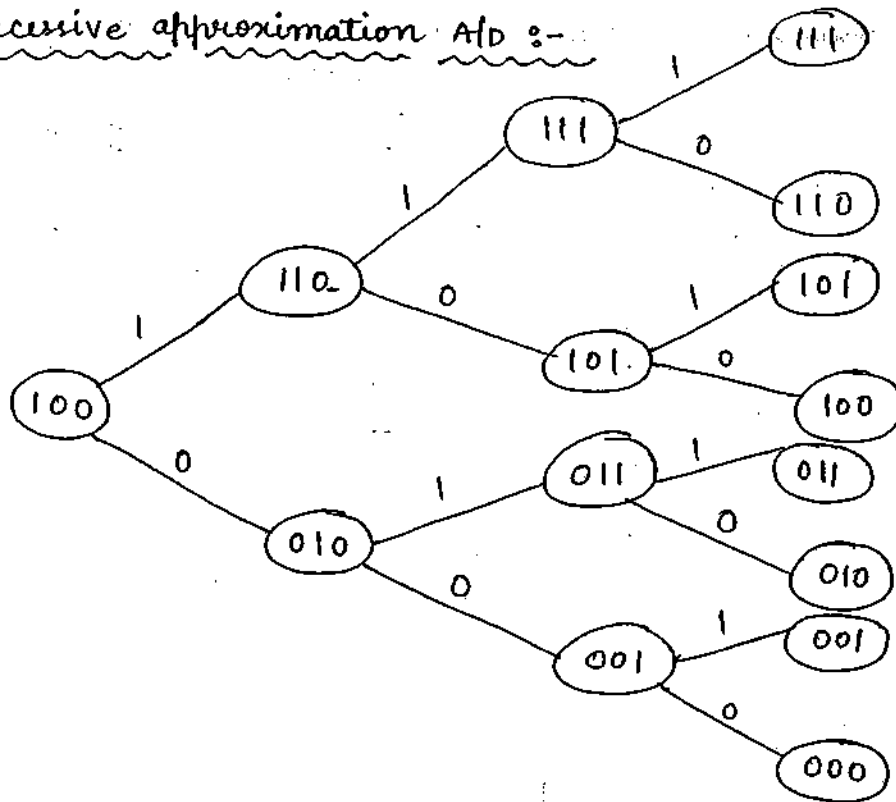
* The approximation time (conversion time) is in μ nanoseconds.



	C_1	C_2	C_3	C_4	C_5	C_6	C_7	ϕ	b_2	b_1	b_0
$V_a < \frac{V_R}{8}$	0	0	0	0	0	0	0		0	0	0
$\frac{V_R}{8} < V_a < \frac{2V_R}{8}$	1	0	0	0	0	0	0		0	0	1
$\frac{2V_R}{8} < V_a < \frac{3V_R}{8}$	1	1	0	0	0	0	0		0	1	0
$\frac{3V_R}{8} < V_a < \frac{4V_R}{8}$	1	1	1	0	0	0	0		0	1	1
$\frac{4V_R}{8} < V_a < \frac{5V_R}{8}$	1	1	1	1	0	0	0		1	0	0
$\frac{5V_R}{8} < V_a < \frac{6V_R}{8}$	1	1	1	1	1	0	0		1	0	1
$\frac{6V_R}{8} < V_a < \frac{7V_R}{8}$	1	1	1	1	1	1	0		1	1	0
$V_a > \frac{7V_R}{8}$	1	1	1	1	1	1	1		1	1	1

* It does not require any clock and counter.

Successive approximation A/D :-



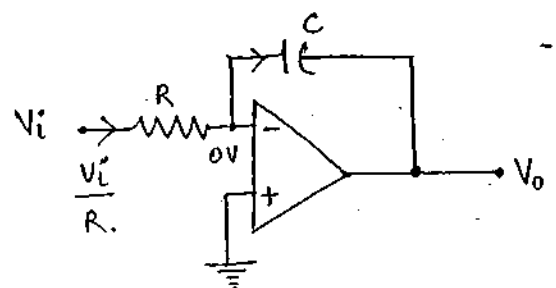
* It is faster than all the A/D converters except flash type. The approximation time is n -clock cycles for n -bit conversion.

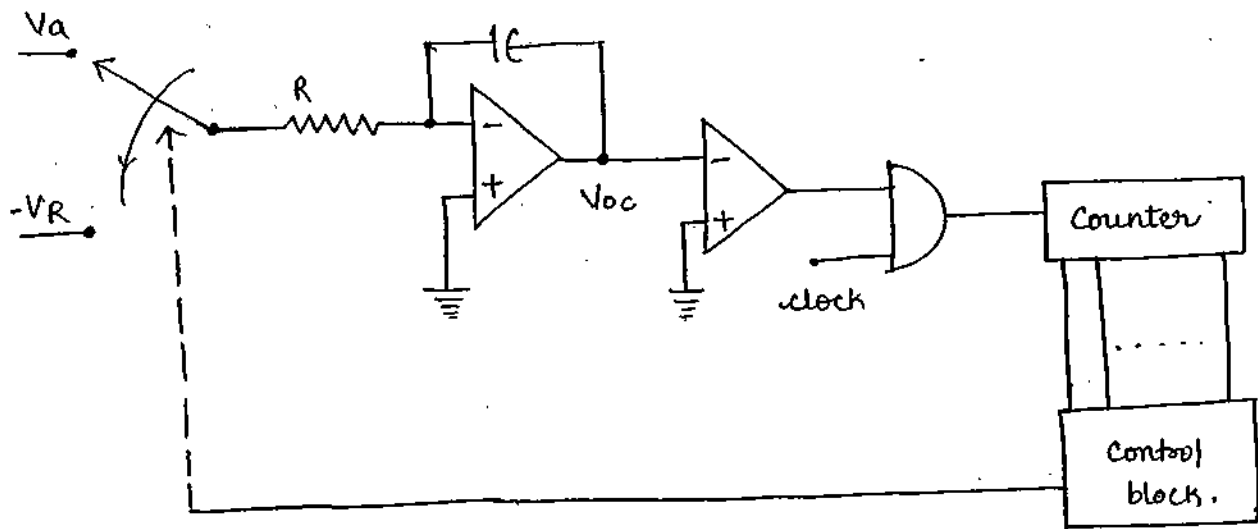
Dual slope A/D :-

$$V_o = -V_c$$

$$V_o = -\frac{1}{C} \int i_c dt$$

$$= -\frac{1}{C} \int \frac{V_i}{R} dt$$





$$V_{0c} = -\frac{1}{\tau} \int v_i dt = -\frac{1}{\tau} \int v_i dt = -\frac{1}{\tau} v_i t \Big|_0^{T_1} = -\frac{1}{\tau} v_i T_1.$$

$$V_0 = -\frac{V_a}{\tau} T_1 + \frac{V_R}{\tau} (t - T_1).$$

at $t = T_2$, $V_0 = 0$.

$$0 = -\frac{V_a}{\tau} T_1 + \frac{V_R}{\tau} (T_2 - T_1).$$

$$\frac{V_a}{\tau} T_1 = \frac{V_R}{\tau} (T_2 - T_1).$$

$$V_a T_1 = V_R (T_2 - T_1)$$

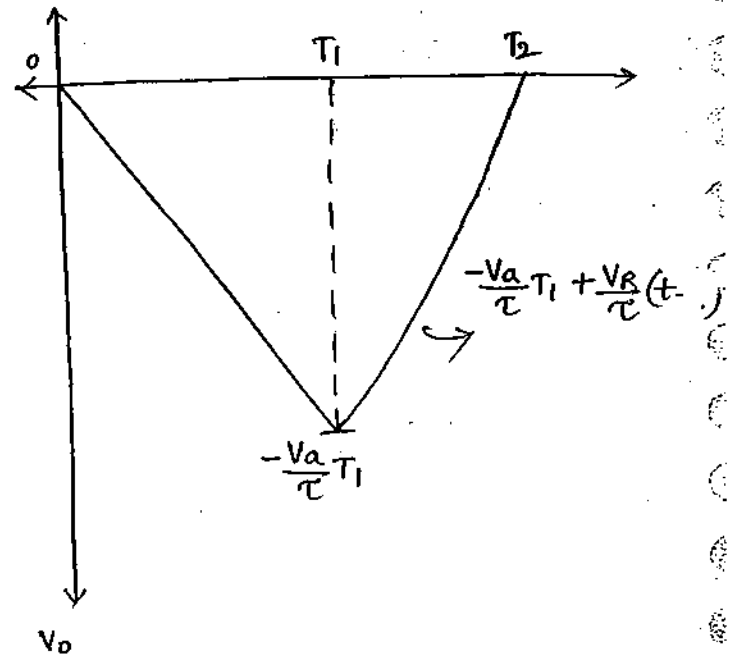
$$V_a (2^n T_{clk}) = V_R (N \cdot T_{clk}).$$

$$V_a = \frac{V_R}{2^n} \cdot N$$

$$V_a \propto N$$

Deintegration time depends on applied analog v_i .

$$V_a t_1 = V_R t_2.$$



* It is the slowest A/D convy 2^{n+1} [i.e., $2^n + 2^n$] clock cycles are required approximately.

Q.1

Sol:-

$$V_0 = \left(1 + \frac{7}{1}\right) \times \frac{1}{2^4} (0 + 1 \times 2^1 + 0 \times 2^2 + 1 \times 2^3)$$

$$= 8 \times \frac{1}{16} (2 + 8) = \frac{8 \times 10}{16} = 5V.$$

[OR]

$$V_{0,1} = \left(1 + \frac{R_f}{R}\right) V_i = \left(1 + \frac{7k}{1k}\right) \times \frac{1}{8} V = 1V.$$

$$V_{0,2} = \left(1 + \frac{R_f}{R}\right) \frac{1}{2} V = \left(1 + \frac{7}{1}\right) \times \frac{1}{2} V = \frac{8}{2} = 4V.$$

$$V_0 = V_{0,1} + V_{0,2} = 1 + 4 = 5V.$$

Q.2

Sol:-

Q_2 D_3	Q_1 D_2	Q_0 D_1	Q_0 D_0	
0	0	0	0	$\rightarrow 0$
0	0	0	1	$\rightarrow 1$
0	0	1	0	$\rightarrow 2$
0	0	1	1	$\rightarrow 3$
1	0	0	0	$\rightarrow 8$
1	0	0	1	$\rightarrow 9$
1	0	1	0	$\rightarrow 10$
1	0	1	1	$\rightarrow 11$

Q.3 & 4

Sol:-

D/A converter.

 $V_R = 10V$ and $R = 10k\Omega$,

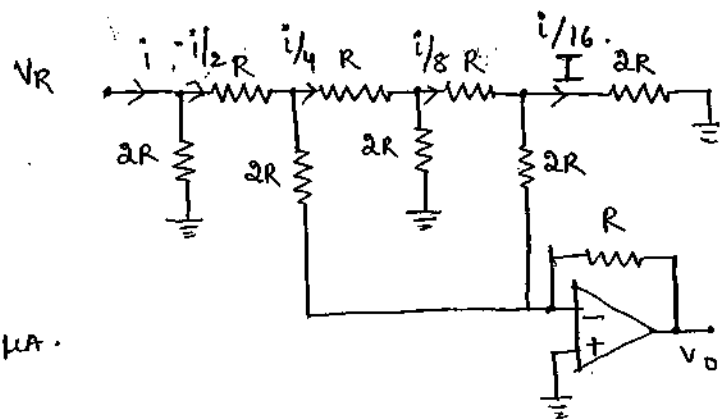
$$i = \frac{V_R}{R} = \frac{10}{10k} = 1mA.$$

$$I = i/16 = \frac{1}{16} mA = 62.5 \mu A.$$

$$V_0 = -I_f R.$$

$$I_f = \frac{i}{4} + \frac{i}{16} = \left(\frac{1}{4} + \frac{1}{16}\right) mA = 0.3125 mA = 312.5 \mu A.$$

$$V_0 = -0.3125 mA \times 10k\Omega = -3.125 V.$$



Q5 & 6

Sol:-

$$V_{DAC} = \sum_{n=0}^3 2^{n-1} b_n \text{ volts.}$$

$$V_{DAC} = 2^{-1} b_0 + 2^0 b_1 + 2^1 b_2 + 2^2 b_3.$$

$$V_{DAC} = 2^{-1}(1) + 0 + 0 + 0 = 0.5V.$$

$$V_{in} = 6.2V.$$

$$\text{no. of clock cycles} = \frac{6.2}{0.5} = 12.4 \text{ cycles.}$$

≈ 13 cycles for stable reading.

$$\begin{aligned} \text{for 13 cycles, } V_{in} &= 13 \times 0.5 \\ &= 6.5V. \end{aligned}$$

$$\text{error} = 6.5 - 6.2 = 0.3V.$$

Q9 Sol:- 8 bit D/A converter,

$$V_{FS} = 20V.$$

$$\text{Resolution} = \frac{20V}{2^8 - 1} = \frac{20}{255} = 0.0784.$$

$$11011011.$$

$$\Rightarrow 2^7 + 2^6 + 2^4 + 2^3 + 0 + 2^1 + 2^0$$

$$\Rightarrow 219. \quad V_0 = \frac{20}{255} \times 219 = 17.146V.$$

$$11011011$$

$$= 219$$

Q11 Sol:- 10 bit A/D converter

$$\frac{5-0}{2^{10}-1} = 4.89 \text{ mV} \approx 5 \text{ mV.}$$

Q12 Sol:- $3\frac{1}{2}$ digit DVM,

$$V_R = 100 \text{ mV.}$$

$$T_1 = 300 \text{ msec.}$$

$$T_2 = 370.2 \text{ ms.}$$

$$V_a \times T_1 = V_R \times T_2.$$

$$V_a = \frac{100 \times 370.2}{300}$$

$$V_a = 123.4 \text{ mV.}$$

Q13 Sol:- $V_{in} = 1.275$, o/p is 11111111.

$$\begin{aligned} \text{quantization error} = \text{Resolution} &= \frac{V_{FS}}{2^n - 1} = \frac{1.275}{2^8 - 1} \\ &= 5 \text{ mV.} \end{aligned}$$

Q.14

Sol:- 8 bit D/A converter

$$V_{ref} = 8V.$$

$$\begin{aligned} \text{to } \overline{00000000} &\rightarrow 0V \\ \underline{11110000} &\rightarrow 240V. \end{aligned}$$

$$V_p = \frac{8}{2^8} \times 240 = 7.5V.$$

$$V_{p-p} = 7.5 - 0 = 7.5V.$$

Q.15

Sol:- 10 bit A/D converter.

10 bit counter.

$$f_{clk} = 1MHz.$$

$T_s \geq$ approximation time of A/D.

$$f_s \leq \frac{1}{\text{approx. time of A/D}}$$

$$T_s \geq 2^{n+1} T_{clk}$$

$$f_s \leq \frac{1}{2^{10+1}} \times 1M$$

$$f_s \leq \frac{10^6}{2^{11}}$$

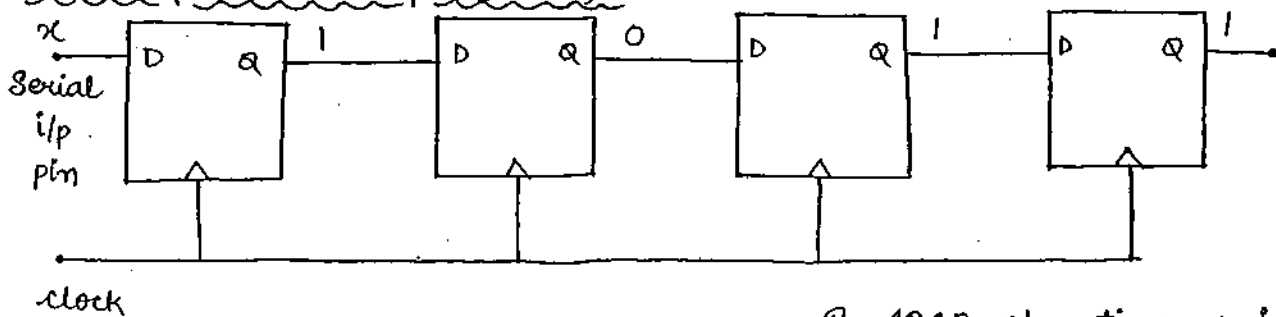
$$f_s \leq 488.281 \text{ Hz.}$$

$$f_s \approx 500 \text{ Hz.}$$

REGISTER :-

To store n -bit information collectively, we use ' n ' number of flipflops collectively is called n -bit register.

Serial i/p & Serial o/p (SISO) :-



In SISO operation using n -bit shift register, it requires $2n-1$ clock cycles to get n -bit information at serial o/p pin which is feeded serially at serial i/p pin.

Ex:- 1001

1st clock : 1 x x x

2nd clock : 0 1 x x

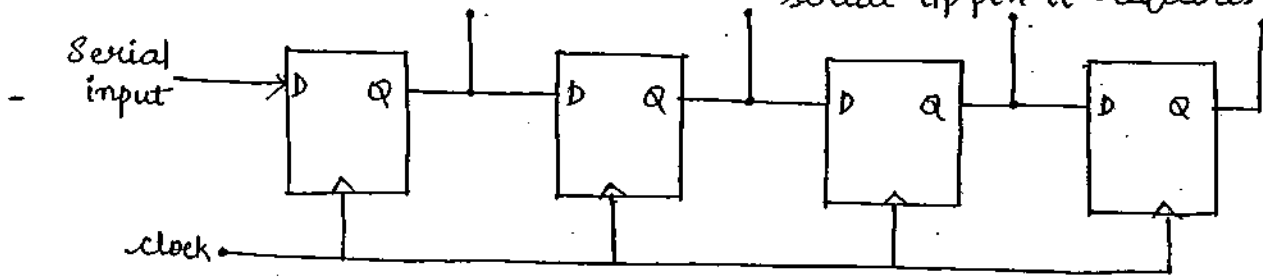
3rd clock : 0 0 1 x

4th clock : 1 0 0 1.

Ex: To get n -bit serial o/p for n -bit serial i/p, it requires $(2n-1)$ clock cycles.

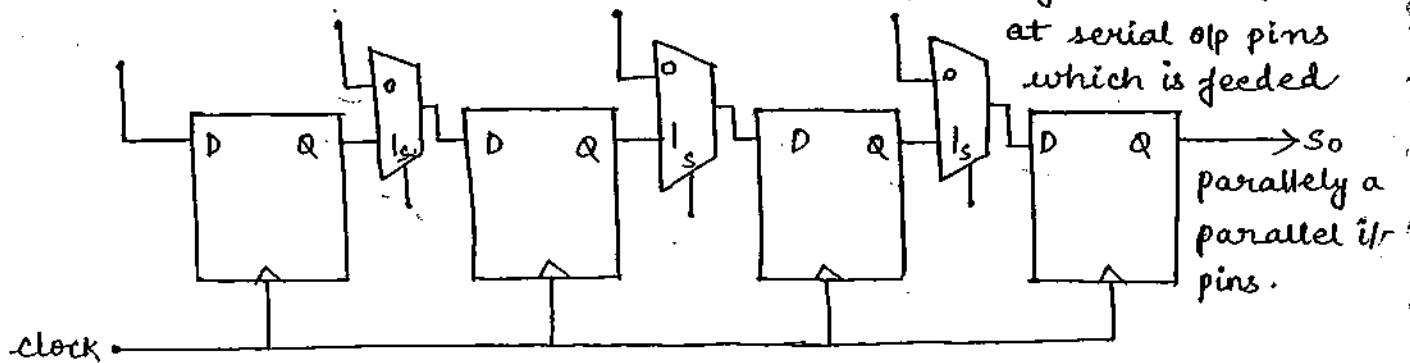
\Downarrow
 $n+(n-1)$

Serial input parallel output :- (SIPO) To get n -bit information on parallel o/p pins which is feeded serially at serial i/p pin it requires ' n ' clock cycles.



* It requires ' n ' clock cycles to get o/p (parallel o/p).

Parallel input & serial output :- (PIPO) In this operation, it requires n -clock cycles to get n -bit information

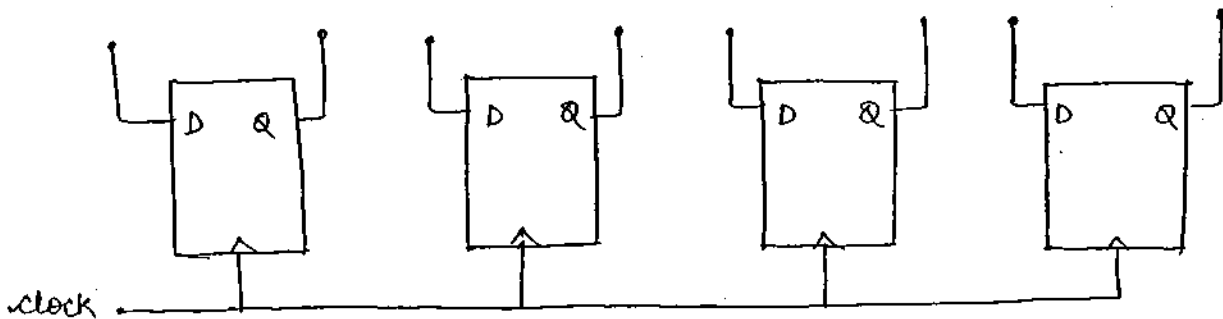


* It requires n clock cycles for required serial n -bit data at o/p.

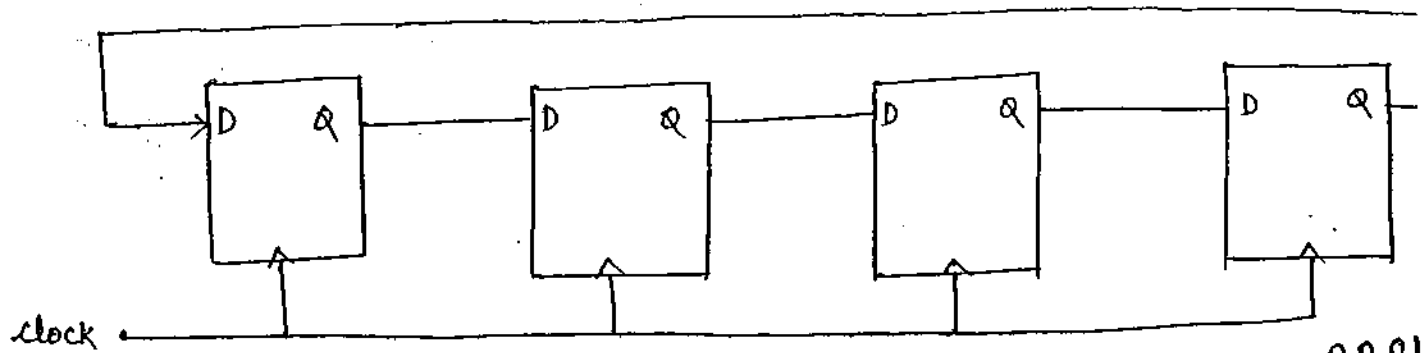
for $s=0$, parallel i/p.

$s=1$, for serial o/p.

Parallel input and parallel output :- (PIPO) It requires 1 clock cycle.



RING COUNTER :-



Op of last flipflop feeded as ip to the first flipflop.

0001
1000
0100
0010

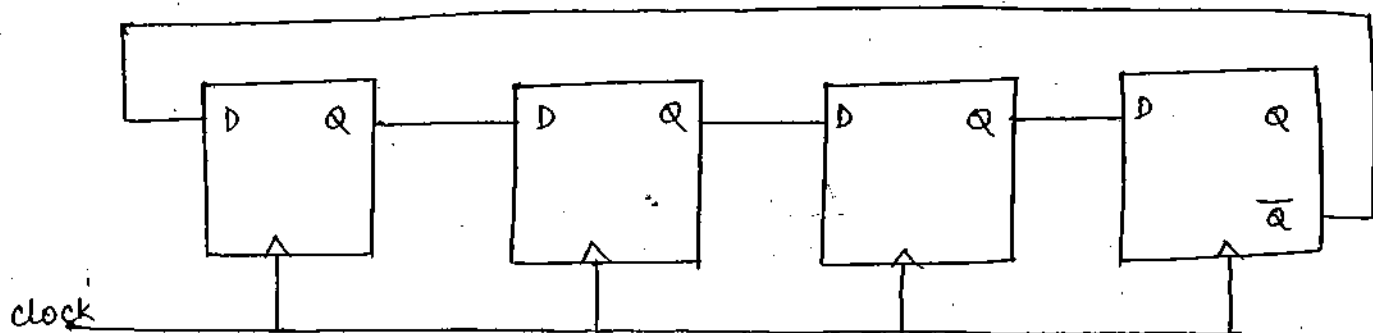
In ring counter using n -bit shift register we can generate maximum n -different vectors Ex:- 0000

→ 0000

⋮

→ 0000.

TWISTED RING COUNTER (JOHNSON COUNTER) :-



In twisted ring counter using n -bit shift register we can generate maximum ' $2n$ ' different vectors.

pg no. 83

Q.8 Sol:-

$$K = 2^n$$

$$H = 2^2$$

$$n = 2, K = 4$$

Q_1	Q_0
0	0
0	1
1	0
1	1

Y_0
1
0
0
0

Y_1
0
1
0
0

Y_2
0
0
1
0

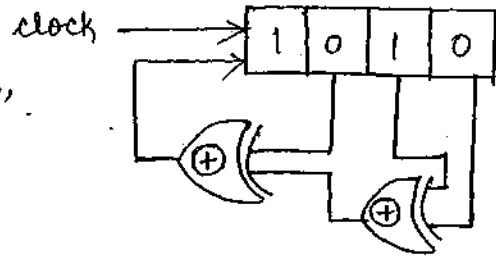
Y_3
0
0
0
1

K-bit
ring
counter

Q.9 Sol:-

1010.

$$\text{as } (A \oplus B) \oplus C = A \oplus B \oplus C,$$



1010
1st clk → 1101
2nd clk → 0110
3rd clk → 0011
4th clk → 0001
5th clk → 1000
6th clk → 0100
7th clk → 1010

Eg:- In the shift register shown in the figure, the contents of the register after occurrence of the first clock cycle is 1100 what must be the state to be initialised.

1001 → Ans
1100

Q.14

Sol:- 8 bit D/A converter

$$\begin{aligned} & \text{to } \begin{matrix} 00000000 \longrightarrow 0V \\ 11110000 \longrightarrow 240V. \end{matrix} \end{aligned}$$

$$V_{ref} = 8V.$$

$$V_p = \frac{8}{2^8} \times 240 = 7.5V.$$

$$V_{p-p} = 7.5 - 0 = 7.5V.$$

Q.15

Sol:- 10 bit A/D converter.

10 bit counter.

$$f_{clk} = 1MHz.$$

 $T_s \geq \text{approximation time of A/D.}$

$$f_s \leq \frac{1}{\text{approx. time of A/D.}}$$

$$T_s \geq 2^{n+1} T_{clk}.$$

$$f_s \leq \frac{1}{2^{10+1}} \times 1M$$

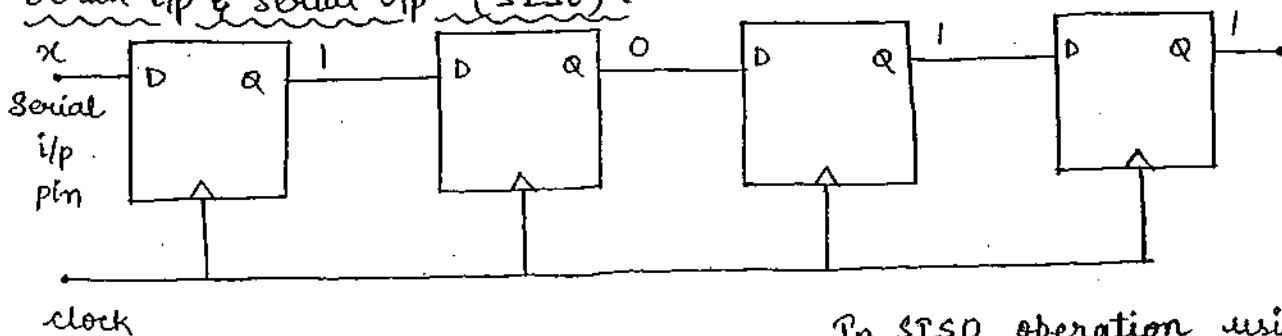
$$f_s \leq \frac{10^6}{2^{11}}$$

$$f_s \leq 488.281 \text{ Hz.}$$

$$f_s \approx 500 \text{ Hz.}$$

REGISTER :-

To store n-bit information collectively, we use 'n' number of flipflops collectively is called n-bit register.

Serial i/p & Serial o/p (SISO) :-

Ex:- 1001

1st clock : 1 x x x

2nd clock : 0 1 x x

3rd clock : 0 0 1 x

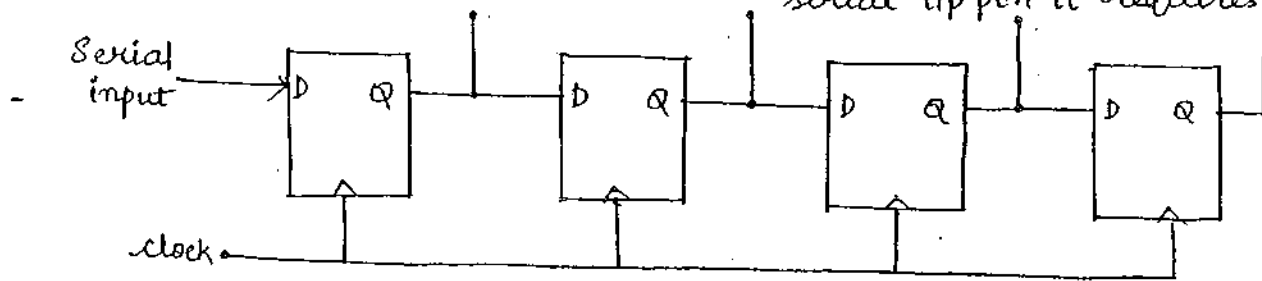
4th clock : 1 0 0 1.

In SISO operation using n-bit shift register, it requires 2n-1 clock cycles to get n-bit information at serial o/p pin which is feeded serially at serial i/p pin.

Ex: To get n -bit serial o/p for n -bit serial i/p, it requires $(2n-1)$ clock cycles.

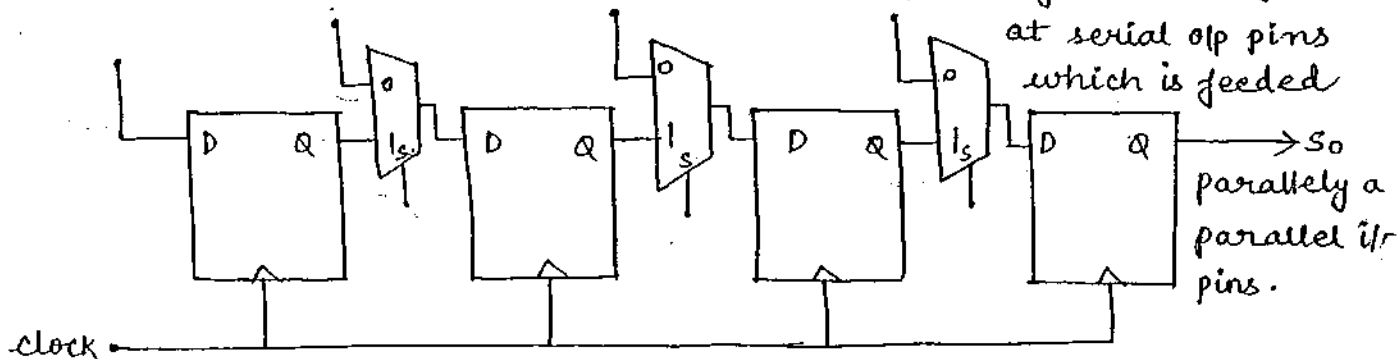
$$\Downarrow \\ n + (n-1)$$

Serial input parallel output :- (SIPO) To get n -bit information on parallel o/p pins which is feeded serially at serial i/p pin it requires ' n ' clock cycles.



* It requires ' n ' clock cycles to get o/p (parallel o/p).

Parallel input & serial output :- (PISO) In this operation, it requires n -clock cycles to get n -bit information at serial o/p pins which is feeded



* It requires n clock cycles for required serial n -bit data at o/p.

for $s=0$, parallel i/p.

$s=1$, for serial o/p.

Parallel input and parallel output :- (PIPO) It requires 1 clock cycle.

