

GATE | PSUs

ELECTRONICS & COMMUNICATION ENGINEERING

Network Theory

(Text Book: Theory with worked out Examples

and Practice Questions)



Chapter 1

Basic Concepts

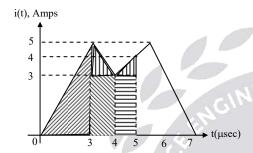
(Solutions for Text Book Practice Questions)

01. Ans: (c)

Sol: We know that;

$$i(t) = \frac{dq(t)}{dt}$$

$$dq(t) = i(t).dt$$

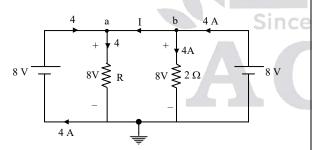


$$q = \int_{0}^{5\mu sec} i(t)dt = Area under i(t) upto 5 \mu sec$$

$$\begin{split} q &= q_1 | + q_2 | + q_3 \mid \\ &= \left(\frac{1}{2} \times 3 \times 5\right) + \left(\frac{1}{2} \times 1 \times 2 + (1 \times 3)\right) + \left(\frac{1}{2} \times 1 \times 1 + (1 \times 3)\right) \\ q &= 15 \mu C \end{split}$$

02. Ans: (a)

Sol:



Applying KCL at node 'b'

$$I + 4 = 4$$

$$\Rightarrow$$
 I = 0A

And
$$\frac{8}{R} = 4$$

$$\Rightarrow$$
 R = 2 Ω

03. Ans: (a)

Sol: The energy stored by the inductor $(1\Omega, 2H)$ upto first 6 sec:

$$\begin{split} E_{\text{stored upto 6 sec}} &= \int_{0}^{6} P_{L} dt = \int_{0}^{6} v_{L}(t) i_{L}(t) dt \\ &= \int_{0}^{2} \left(L \frac{di(t)}{dt} . i(t) \right) dt \\ &= \int_{0}^{2} \left(2 \left[\frac{d}{dt} (3t) \right] \times 3t \right) dt + \int_{2}^{4} \left(2 \left[\frac{d}{dt} (6) \right] \times 6 \right) dt \\ &+ \int_{4}^{6} \left(2 \left[\frac{d}{dt} (-3t + 18) \right] \times (-3t + 18) \right) dt \\ &= \int_{0}^{2} 18t \ dt + \int_{2}^{4} 0 \ dt + \int_{4}^{6} \left(-6 \left[-3t + 18 \right] \right) dt \\ &= 36 + 0 - 36 = 0 \ J \\ &\text{(or)} \\ E_{\text{stored upto 6 sec}} &= E_{L}|_{t = 6 \text{sec}} \\ &= \frac{1}{2} L \left(i(t) |_{t = 6} \right)^{2} \\ &= \frac{1}{2} \times 2 \times 0^{2} = 0 \ J \end{split}$$

04. Ans: (d)

Sol: The energy absorbed by the inductor $(1\Omega, 2H)$ upto first 6sec:

$$E_{absorbed} = E_{dissipated} + E_{stored}$$

Energy is dissipated in the resistor

$$E_{dissipated} = \int P_R dt = \int (i(t))^2 R dt$$

$$= \int_{0}^{2} (3t)^{2} \times 1 dt + \int_{2}^{4} (6)^{2} \times 1 dt + \int_{4}^{6} (-3t + 18)^{2} \times 1 dt$$

$$= \int_{0}^{2} 9t^{2} dt + \int_{2}^{4} 36 dt + \int_{4}^{6} (9t^{2} + 324 - 108t) dt$$



$$= 24 + 72 + 24$$

= 120J

$$\therefore$$
 E_{dissipated} = 120 J

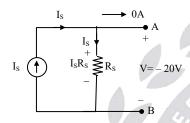
And
$$E_{\text{stored upto 6 sec}} = 0 J$$

$$\therefore E_{absorbed} = E_{dissipated} + E_{stored}$$

$$\Rightarrow$$
 E_{absorbed} = 120J + 0J = 120J

05. Ans: (a)

Sol: Point $(-20, 0) \Rightarrow V = -20V$ and I = 0A

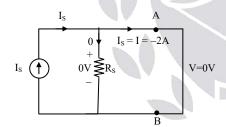


By KVL
$$\Rightarrow$$
 I_S R_S – V = 0

$$\Rightarrow$$
 I_SR_S + 20 = 0

$$\Rightarrow$$
 I_SR_S = -20V(1)

Point:
$$(0, -2) \Rightarrow V = 0V$$
 and $I = -2A$

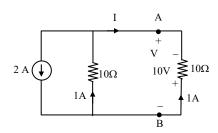


$$I_s = I$$

$$\Rightarrow I_s = -2A$$

Substituting I_s in eq. (1)

$$R_S = 10\Omega$$

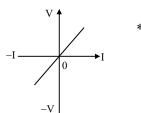


From the diagram;

$$I = -1A$$
 and $V = -10V$

Ans: (a) **06.**

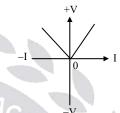
Sol:



- * linear
 - * Passive
 - * bilateral

07. Ans: (b)

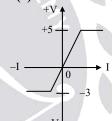
Sol:



- * Non linear
- * Active
- * Unilateral

Ans: (e) 08.

Sol:

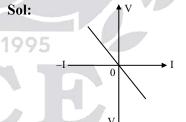


- * Non linear
- * Passive
- * Unilateral

09. Ans: (c)

Sol:

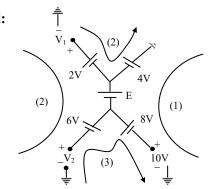
Since



- * Linear
- * Active
- * Bilateral

10.

Sol:







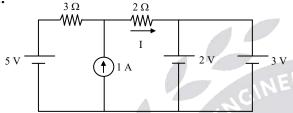
(1) By KVL
$$\Rightarrow$$
 + 10 + 8 + E + 4 = 0
E = -22V

(2) By KVL
$$\Rightarrow$$
 + V₁ - 2 + 4 = 0
V₁ = -2V

(3) By KVL
$$\Rightarrow$$
 + V₂ + 6 - 8 - 10 = 0
V₂ = 12V

11. Ans: (d)

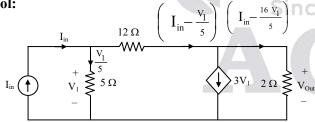
Sol:



Here the 2V voltage source and 3V voltage source are in parallel which violates the KVL. Hence such circuit does not exist. (But practical voltage sources will have some internal resistance so that when two unequal voltage sources are connected in parallel current can flow and such a circuit may exist).

12. Ans: (d)

Sol:



Applying KVL,

$$-V_{1} + 12\left(I_{in} - \frac{V_{1}}{5}\right) + 2\left(I_{in} - \frac{16V_{1}}{5}\right) = 0$$

$$-V_{1} + 12I_{in} - \frac{12V_{1}}{5} + 2I_{in} - \frac{32V_{1}}{5} = 0$$

$$14I_{in} = \frac{49}{5}V_{1}$$

$$\Rightarrow V_1 = \frac{70}{49} I_{in} \dots (1)$$

$$\therefore V_{\text{out}} = 2 \left(I_{\text{in}} - \frac{16V_1}{5} \right) \dots (2)$$

Substitute equation (1) in equation (2)

$$V_{out} = 2\left(I_{in} - \frac{16}{5} \times \frac{70}{49}I_{in}\right)$$
$$= 2\left(\frac{-25}{7}\right)I_{in}$$
$$= \frac{-50}{7}I_{in}$$

$$\therefore V_{out} = -7.143 I_{in}$$

13. Ans: (c)

Sol: 1Ω V=12V 4A 12A 12A 12V 12V

By nodal
$$\Rightarrow$$

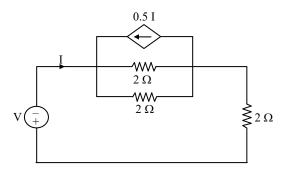
$$V - 20 + V - 4 = 0$$

$$V = 12 \text{ volts}$$

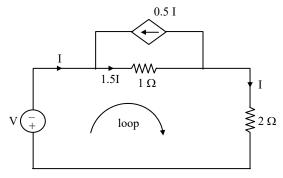
Power delivered by the dependent source is $P_{del} = (12 \times 4) = 48$ watts

14. Ans: (d)

Sol:







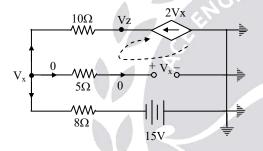
Applying KVL,

$$\Rightarrow$$
 V + 1.5I +2I = 0

$$\Rightarrow$$
 V = $-3.5 I$

15. Ans: (c)

Sol:



By using Nodal Analysis

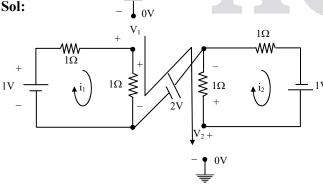
$$\frac{V_x + 15}{8} - 2V_x = 0 \implies V_x = 1 \text{ V}$$

By using nodal Analysis at V_z node

$$\frac{V_z + 15}{18} - 2 = 0 \implies V_z = +21V$$

16.





By KVL
$$\Rightarrow 1 - i_1 - i_1 = 0$$

 $i_1 = 0.5A$

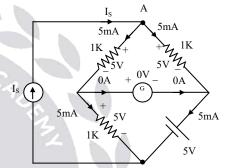
By KVL
$$\Rightarrow -i_2 - i_2 + 1 = 0$$

 $i_2 = 0.5A$
By KVL $\Rightarrow V_1 - 0.5 + 2 + 0.5 - V_2 = 0$
 $V_2 = V_1 + 2 V$

17.

Sol: As the bridge is balanced; voltage across (G) is

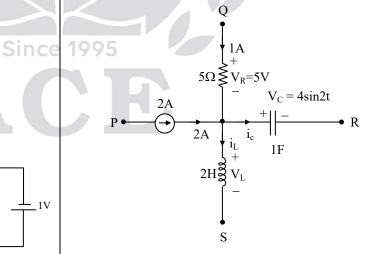
By KCL at node "A" \Rightarrow - I_s + 5mA + 5mA = 0 $I_S = 10 \text{mA}$



18.

Sol: Given data:

$$V_R = 5V$$
 and $V_C = 4\sin 2t$ then $V_L = ?$



$$i_c = \frac{CdV_c}{dt} = \frac{d}{dt}(4\sin 2t) = 8\cos 2t$$

By KCL;
$$-1 - 2 + i_L + i_c = 0$$

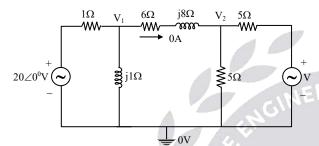
$$i_L = 3 - 8\cos 2t$$

We know that;

$$V_L = L \frac{di_L}{dt} = 2 \frac{d}{dt} (3 - 8\cos 2t)$$
$$= 2(-8)(-2)\sin 2t$$
$$V_L = 32\sin 2t \text{ volt}$$

19.

Sol: V = ? If power dissipated in 6Ω resistor is zero.



 $P_{6\Omega} = 0 \text{ W (Given)}$

$$\Rightarrow i_{60}^2.6 = 0$$

$$\Rightarrow i_{6\Omega} = 0 \ (V_{6\Omega} = 0)$$

$$\frac{V_1 - V_2}{6 + i8} = 0; V_1 = V_2$$

By Nodal ⇒

$$\frac{V_1 - 20 \angle 0^0}{1} + \frac{V_1}{i1} + 0 = 0$$

$$V_1 = 10\sqrt{2} \angle 45^0 = V_2$$

By Nodal ⇒

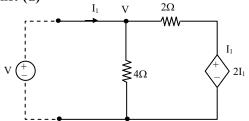
$$0 + \frac{V_2}{5} + \frac{V_2 - V}{5} = 0$$

$$V = 2V_2 = 2(10\sqrt{2} \angle 45^0)$$

$$\therefore V = 20\sqrt{2} \angle 45^{0}$$

20. Ans: (d)

Sol:



Note: Since no independent source in the network, the network is said to be unenergised, so called a DEAD network".

The behavior of this network is a load resistor behavior.

By Nodal \Rightarrow

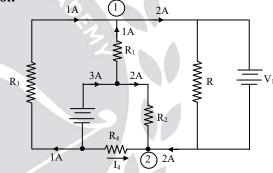
$$-I_1 + \frac{V}{4} + \frac{V - 2I_1}{2} = 0$$

$$3V = 8I_1$$

$$R_{eq} = \frac{V}{I_1} = \frac{8}{3}\Omega$$

21. Ans: (a)

Sol:



Apply KCL at Node − 1,

$$I = I_{R1} + I_{R3} = 1 + 1 = 2A$$

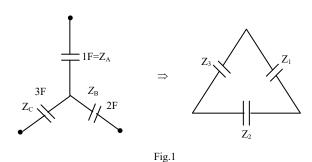
Apply KCL at Node-2,

$$I_4 = -I_2 - I = -2 - 2 = -4A$$

22.

Since

Sol:



7

$$Z_1 = Z_A + Z_B + \left(\frac{Z_A Z_B}{Z_C}\right)$$
$$= \frac{1}{s} + \frac{1}{2s} + \frac{\left(\frac{1}{s}\right)\left(\frac{1}{2s}\right)}{\left(\frac{1}{3s}\right)}$$

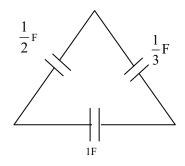
$$Z_1 = \frac{1}{s(\frac{1}{3})}$$
; $C = \frac{1}{3} F$

$$Z_2 = Z_B + Z_C + \frac{Z_B Z_C}{Z_A} = \frac{1}{2s} + \frac{1}{3s} + \frac{\left(\frac{1}{2s}\right)\left(\frac{1}{3s}\right)}{\left(\frac{1}{s}\right)}$$

$$Z_2 = \frac{1}{S(1)}$$
; $C = 1F$

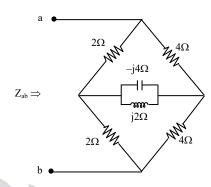
$$Z_3 = Z_A + Z_C + \frac{Z_A Z_C}{Z_B}$$
$$= \frac{1}{s} + \frac{1}{3s} + \frac{\left(\frac{1}{s}\right)\left(\frac{1}{3s}\right)}{\left(\frac{1}{2s}\right)}$$

$$Z_3 = \frac{1}{s(\frac{1}{2})}$$
; $C = \frac{1}{2}F$

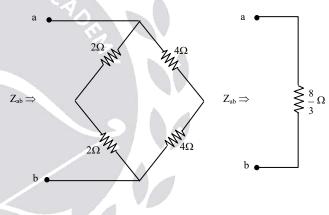




Sol:
$$Z_{ab} = ?$$



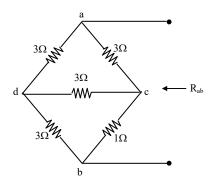
Since $2 \times 4 = 4 \times 2$; the given bridge is balanced one, therefore the current through the middle branch is zero. The bridge acts as below:



$$Z_{ab} = \frac{4 \times 8}{4 + 8} = \frac{8}{3}\Omega$$

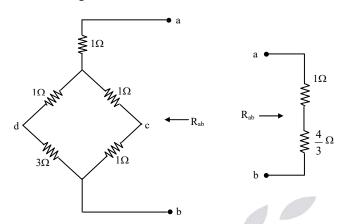
24.

Sol: Redraw the circuit diagram as shown below:





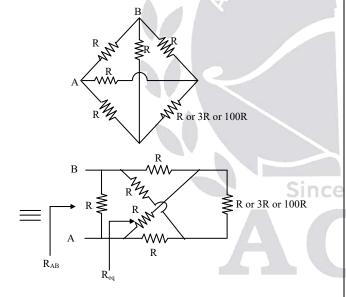
Using Δ to star transformation:



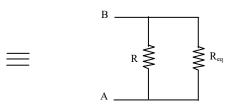
25.

Sol: On redrawing the circuit diagram

 $\therefore \ R_{ab}=1+\frac{4}{3}=\frac{7}{3}\,\Omega$

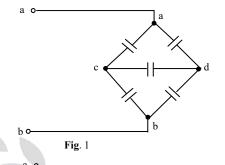


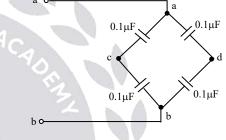
As bridge is balanced, $R_{eq} = R$ So $R_{AB} = R || R_{eq} = R || R = R/2$

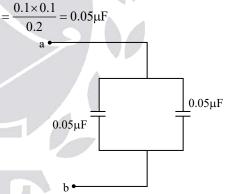


26. Ans: (b)

Sol: The equivalent capacitance across a, b is calculated by simplifying the bridge circuit as shown in Fig. .1 [: $C = 0.1\mu F$]







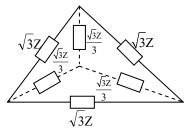
 $C_{ab} = 0.1 \mu F$

Note: The bridge is balanced and the answer is easy to get.

27. Ans: (a)

1995

Sol: Consider a Δ connected network



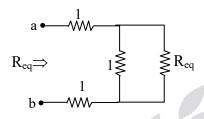


Then each branch of the equivalent Y-connected

impedance is
$$\frac{\sqrt{3}Z}{3} = \frac{Z}{\sqrt{3}}$$

28. Ans: (a)

Sol: Network is redrawn as



$$R_{eq} = 1 + 1 + \frac{R_{eq}}{1 + R_{eq}}$$

$$= 2 + \frac{R_{eq}}{1 + R_{eq}} = \frac{2 + 2R_{eq} + R_{eq}}{1 + R_{eq}}$$

$$R_{eq} + R_{eq}^2 = 2 + 3 R_{eq}$$

$$R_{eq}^2 - 2R_{eq} - 2 = 0$$

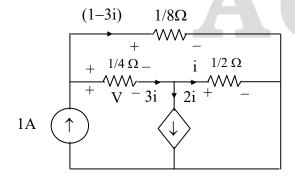
$$R_{eq} = (1 + \sqrt{3})\Omega$$

29. Ans: (c)

Sol: Applying KCL

$$I_{0.250} = 2i + i = 3i$$

$$I_{0.125\Omega} = (1 - 3 i) A$$



Applying KVL in upper loop.

$$-\frac{(1-3i)}{8} + \frac{i}{2} + \frac{3i}{4} = 0$$

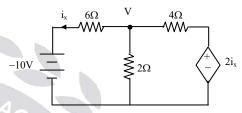
$$\frac{5i}{4} = \frac{1 - 3i}{8} \Rightarrow 10i = 1 - 3i$$

$$\therefore i = \frac{1}{13}A$$

$$V = \frac{3i}{4} = \frac{3}{4} \times \frac{1}{13} = \frac{3}{52}V$$

30. Ans: (a)

Sol:



Applying KCL at Node V

$$\frac{V}{2} + \frac{V - 2i_x}{4} + i_x = 0 \dots (1)$$

$$i_x = \frac{V+10}{6} \Rightarrow V = 6i_x - 10$$

Put in equation (1), we get

$$3i_x - 5 + i_x - 2.5 + i_x = 0$$

$$5i_x = 7.5$$

$$i_{x} = 1.5A$$

$$V = -1V$$

$$I_{\text{dependent souce}} = \frac{V - 2i_x}{4} = \frac{-1 - 3}{4} = -1A$$

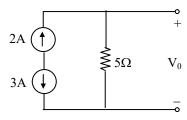
$$\therefore$$
 Power absorbed = $(I_{dependent source}) (2i_x)$

$$=(-1)(3)=-3W$$

31. Ans: (d)

Sol:
$$V_0 = ?$$

Since



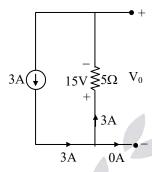
By KCL
$$\Rightarrow$$
 $+2+3=0$
 $+5 \neq 0$



Since the violation of KCL in the circuit; physical connection is not possible and the circuit does not exist.

32. **Ans: (b)**

Sol: Redraw the given circuit as shown below:



By KVL
$$\Rightarrow$$

$$-15 - V_0 = 0$$

$$V_0 = -15V$$

33. Ans: (d)

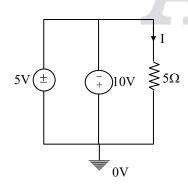
Sol: Redraw the circuit diagram as shown below:

Across any element two different voltages at a time is impossible and hence the circuit does not exist.

Another method:

By KVL
$$\Rightarrow$$

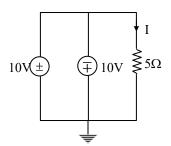
 $5 + 10 = 0$
 $15 \neq 0$



Since the violation of KVL in the circuit, the physical connection is not possible.

34. Ans: (d)

Sol: Redraw the given circuit as shown below:



By KVL
$$\Rightarrow$$

-10 -10 = 0

$$-20 \neq 0$$

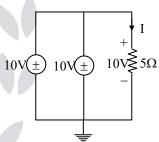
Since the violation of KVL in the circuit, the physical connection is not possible.

Ans: (b) 35.

Sol: Redraw the given circuit as shown below:

By KVL
$$\Rightarrow$$

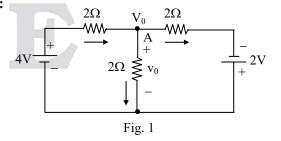
 $10-10=0$
 $0=0$
KVL is satisfied
 $I_{5\Omega} = \frac{10}{5} = 2A$
 $I_{5\Omega} = 2A$



36. Ans: (d)

Sol:

Since



The diode is forward biased. Assuming that the diode is ideal, the Network is redrawn with node A marked as in Fig. 1.

Apply KCL at node A

$$\frac{4 - v_0}{2} = \frac{v_0}{2} + \frac{v_0 + 2}{2}$$



$$\frac{3\,\mathrm{v_0}}{2} = 1$$

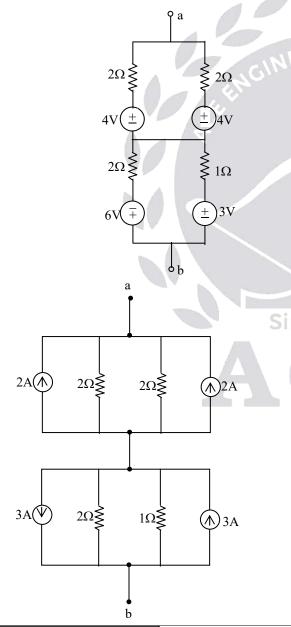
$$\mathbf{v}_0 = \frac{2}{3}\mathbf{V}$$

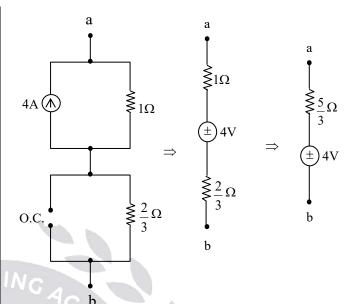
(Here polarity is different what we assume so

$$V_0 = \frac{-2}{3}V$$

37.

Sol: The actual circuit is





38. Ans: (b)

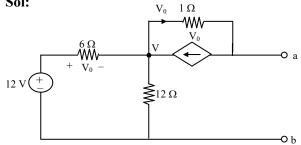
10Ω -Sol: -20V + $5\Omega \ge 10V$ 10V _ 5V

Voltage across 2A = 10 + 20 + 10 - 5= 35 V

> ∴ Power supplied = VI $= 35 \times 2 = 70 \text{ W}$

39. Ans: (d)

Sol:







Applying KCL at node V

$$\frac{V - 12}{6} + \frac{V}{12} - V_0 + V_0 = 0$$

$$\Rightarrow \frac{V}{6} + \frac{V}{12} = 2 \Rightarrow V = 8V$$

$$\therefore V_0 = 4V$$

Applying KVL in outer loop

$$\Rightarrow$$
 -V+1(V₀) +V_{ab} = 0

$$\Rightarrow$$
 $V_{ab} = V - V_0 = 8 - 4 = 4V$

40.

Sol: By KVL

$$\Rightarrow$$
 V_i $-6 - 10 = 0$

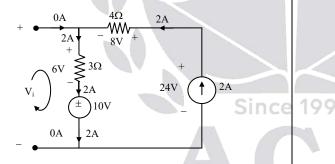
$$V_i = 16V$$

$$P_{4\Omega} = (8 * 2) = 16$$
watts – absorbed

$$P_{2A} = (24 * 2) = 48$$
 watts delivered

$$P_{3\Omega} = (6*2) = 12 \text{ watts} - \text{absorbed}$$

$$P_{10V} = (10 * 2) = 20 \text{ watts} - \text{absorbed}$$



$$48 = 16 + 12 + 20$$

$$48 = 48 \text{ W}$$

Since; $P_{del} = P_{abs} = 48$ watts. Tellegen's Theorem is satisfied.

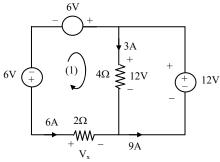
41.

Sol: By KVL in first mesh

$$\Rightarrow$$
V_v - 6 + 6 - 12 = 0

$$V_{\rm v} = 12V$$

$$P_{12v} = (12 \times 9) = 108$$
 watts delivered



$$P_{4\Omega} = (12 \times 3) = 36 \text{ watts} - \text{absorbed}$$

$$P_{6V} = (6 \times 6) = 36 \text{ watts} - \text{absorbed}$$

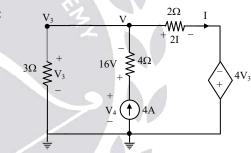
$$P_{6V} = (6 \times 6) = 36 \text{ watts } - \text{delivered}$$

$$P_{2\Omega} = (12 \times 6) = 72 \text{ watts} - \text{absorbed}$$

Since $P_{del} = P_{abs}$; Tellegen's theorem is satisfied.

42.

Sol:



By Nodal ⇒

$$\frac{V}{3} - 4 + \frac{V + 4V_3}{20} = 0$$

$$\frac{5V}{6} = 4 - 2V_3 \dots (1)$$

By KVL ⇒

$$V_3 - 2I + 4V_3 = 0$$

$$5V_3 - 2I = 0 \dots (2)$$

By KVL \Rightarrow

$$V = V_3$$
(3)

Substitute (3) in (1), we get

$$V_3 = \frac{24}{17}$$
; $V_4 = V + 16 = \frac{24}{16} + 16 = \frac{296}{17} V$

$$V_3 = \frac{24}{17}$$
 Volt and $I = \frac{60}{17}$ A





 $P_{3\Omega} = 0.663W$ absorbed

 $P_{4\Omega} = 64W$ absorbed

 $P_{4A} = 69.64W$ delivered

 $P_{2\Omega} = 24.91W$ absorbed

 $P_{4V3} = 19.92$ Wdelivered

Since $P_{del} = P_{abs} = 89.57W$; Tellegen's Theorem is satisfied.

43. Ans: (a, d)

Sol: \rightarrow For practical voltage source V_S is connected in series with its internal resistance R_S as low as possible. For ideal $V.S \Rightarrow R_S = 0 \ \Omega$

ightarrow For practical current I_S its internal resistance R_S connected in parallel as maximum as possible. For ideal $C.S \Rightarrow R_S = \infty \ \Omega$

Any element connected with an ideal current source is not effect.

Any element connected in parallel with an ideal voltage source is not effect.

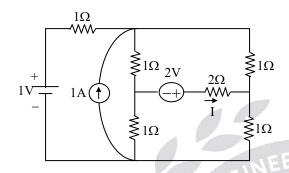


Chapter 2

Circuit Theorems

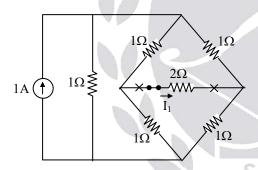
01.

Sol: The current "I" = ?



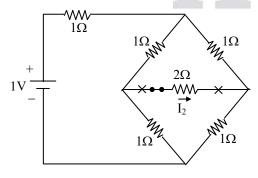
By superposition theorem, treating one independent source at a time.

(a) When 1A current source is acting alone.



Since the bridge is balanced; $I_1 = 0A$

(b) When 1V voltage source is acting alone



 $I_2 = 0A$

Since the bridge is balanced.

(c) When 2V voltage source is acting alone and apply Reciprocity theorem, interchange source 2 volt and 1 Ω positions.

$$\frac{\text{Excitation}}{\text{Re sponse}} = \text{same}$$

$$I_3 \geqslant 2\Omega \qquad \qquad \geqslant 1\Omega \qquad \qquad \geqslant 1\Omega$$

$$2V^{\pm} \qquad \qquad 1\Omega \geqslant 1\Omega$$

$$I_3 = \frac{2}{3} = 0.66A$$

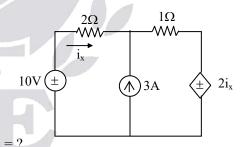
By superposition theorem; $I = I_1 + I_2 + I_3$

$$I = 0 + 0 + 0.66A$$

$$I = 0.66A$$

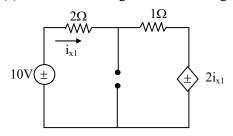
02.

Sol:



By super position theorem; treating only one independent source at a time

(a) When 10V voltage source is acting alone

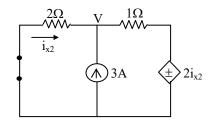


By KVL \Rightarrow



$$10 - 2i_{x1} - i_{x1} - 2i_{x1} = 0$$
$$i_{x1} = 2A$$

(b) When 3A current source is acting alone



By Nodal ⇒

$$\frac{V}{2} - 3 + \frac{(V - 2i_{x2})}{1} = 0$$

$$3V - 4i_{x2} = 6 \dots (1)$$

And

$$i_{x2} = \frac{0 - V}{2} \Rightarrow V = -2i_{x2} \dots (2)$$

Put (2) in (1), we get

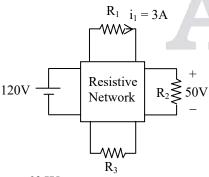
$$i_{x2} = -\frac{3}{5}A$$

By SPT;

$$i_x = i_{x1} + i_{x2} = 2 - \frac{3}{5} = \frac{7}{5}$$

$$\therefore i_x = 1.4A$$

03. Sol:



$$P_{R_3} = 60 \text{ W}$$

For 120 V
$$\rightarrow$$
 $i_1 = 3$ A

For 105 V
$$\rightarrow i_1 = \frac{105}{120} \times 3 = 2.625 A$$

For 120 V
$$\rightarrow$$
 V₂ = 50 V

For 105 V
$$\rightarrow$$
 V₂ = $\frac{105}{120} \times 50 = 43.75$ V

$$V_2 = 120 \text{ V} \Rightarrow I^2 R_3 = 60 \text{ W} \Rightarrow I = \sqrt{\frac{60}{R_3}}$$

For
$$V_S = 105 \text{ V}$$

$$P_3 = \left(\frac{105}{120}\sqrt{\frac{60}{R_3}}\right)^2 \times R_3 = 45.9 \text{ W}$$

04. Ans: (b)

Sol: It is a liner network

 $\therefore V_x$ can be assumed as function of i_{s1} and i_{s2}

$$V_x = Ai_{s_1} + Bi_{s_2}$$

$$80 = 8A + 12 B \rightarrow (1)$$

$$0 = -8A + 4B \qquad \rightarrow (2)$$

From equation 1 & 2

$$A = 2.5, B = 5$$

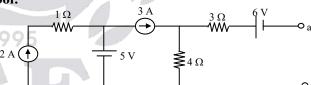
Now,
$$V_X = (2.5)(20) + (5)(20)$$

$$V_x = 150V$$

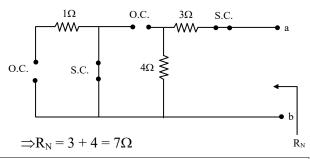
05. Ans: (c)

Sol:

Since



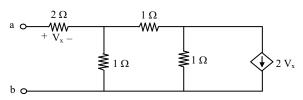
For finding Norton's equivalent resistance independent voltage sources to be short circuited and independent current sources to be open circuited, then the above circuit becomes



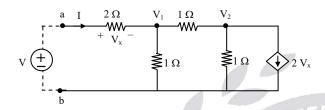


06. Ans: (b)

Sol:



Excite with a voltage source 'V'



Apply KCL at node V₁

$$-I + \frac{V_1}{1} + \frac{V_1 - V_2}{1}$$

$$\Rightarrow 2V_1 - V_2 - I = 0 \dots (1)$$

Apply KCL at node V₂

$$\frac{V_2 - V_1}{1} + \frac{V_2}{1} + 2V_x = 0$$

$$2V_2 - V_1 + 2V_x = 0 \dots (2)$$

But from the circuit,

$$V_x = 2I \dots (3)$$

Substitute (3) in (2)

$$\Rightarrow 2V_2 - V_1 + 4I = 0$$

$$4V_2 - 2V_1 + 8I = 0$$

From (1),

$$2 V_1 = V_2 + I$$

$$\therefore 4 V_2 - (V_2 + I) + 8I = 0$$

$$\Rightarrow$$
 3V₂ +7I = 0

$$\Rightarrow$$
 V₂ = $-\frac{7 \text{ I}}{3}$

Substitute (V₂) in (1)

$$2V_1 - \left(-\frac{7I}{3}\right) - I = 0$$

$$2V_{1} + \frac{7}{3}I - I = 0 \Rightarrow 2V_{1} = \frac{-4I}{3}$$

$$\Rightarrow V_{1} = \frac{-2I}{3}$$

$$\therefore V = V_{x} + V_{1} = 2I + \left(-\frac{2I}{3}\right)$$

$$= \frac{4I}{3}$$

$$\Rightarrow V = \frac{4I}{3}$$

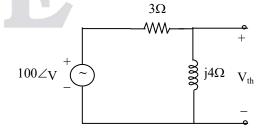
$$\Rightarrow \frac{V}{I} = \frac{4}{3}\Omega$$

07.

Sol: $\begin{array}{c|c} 3\Omega \\ \hline \\ 100 \angle 0 \\ \hline \end{array} \begin{array}{c} -j1\Omega \\ \hline \\ \end{array} \begin{array}{c} j1\Omega \\ \hline \end{array} \begin{array}{c} j4\Omega \\ V \end{array}$

199 Here $j1\Omega$ and $-j1\Omega$ combination will act as open circuit.

The circuit becomes

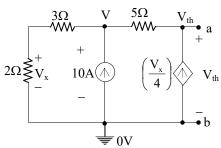


$$\Rightarrow V_{th} = \frac{100\angle 0^{\circ} \times j4}{3 + j4}$$
$$= 80\angle 36.86^{\circ} \text{ V}$$



08.

Sol: Thevenin's and Norton's equivalents across a, b.



By Nodal ⇒

$$\frac{V_{th}}{5} - \frac{V}{5} - \frac{V_x}{4} = 0 \qquad \dots (2)$$

$$V_{x} = \left(\frac{2V}{5}\right)$$
(3)
$$\frac{2V}{5} = \left(10 + \frac{V_{th}}{5}\right)$$
(4)

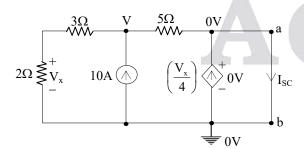
$$\frac{2V}{5} = \left(10 + \frac{V_{th}}{5}\right)$$
(4)

$$\frac{V_{th}}{5} = \left(\frac{V}{10} + \frac{V}{5}\right)$$

VDR:
$$V_x = V \times \frac{2}{2+3}$$

Solve eq (1) and (2) & (3)

$$V_{th} = 150V, V = 100 V$$



$$\frac{\mathbf{V}}{5} - 10 + \frac{\mathbf{V}}{5} = 0$$

$$\frac{2V}{5} = 10$$

$$V = 25V$$

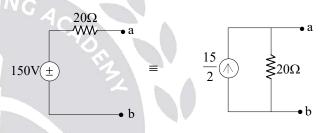
$$V_x = \frac{2V}{5} = \frac{2 \times 25}{5}$$

$$V_x = 10V, \ \frac{0-V}{5} - \frac{V_x}{4} + I_{Sc} = 0$$

$$I_{SC} = \left(\frac{10}{4} + 5\right) = \frac{15}{2}A$$

$$I_{SC} = \frac{15}{2} A$$

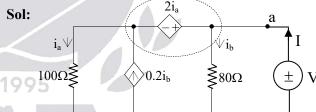
$$R_{th} = \frac{V_{th}}{I_{SC}} = \frac{150}{\frac{15}{2}} = 20\Omega$$



09.

Since

17



Super nodal equation

$$\Rightarrow$$
 i_a -0.2 i_b + i_b -I = 0

$$I = i_a + 0.8i_b$$

$$V = 80i_b$$
; $i_b = \frac{V}{80}$

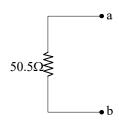
- Inside the supernode, always the KVL is written.

By KVL
$$\Rightarrow$$

$$100i_a + 2i_a - 80i_b = 0$$



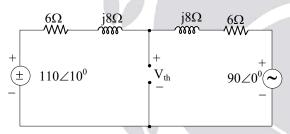




$$\begin{split} I &= \frac{V}{102} + \frac{0.8 \times V}{80} \\ \frac{V}{I} &= R_L = \frac{1}{\frac{1}{102} + \frac{1}{100}} \\ &= 50.5\Omega. \\ R_L &= 50.5\Omega \end{split}$$

10.

Sol: V_{th}:

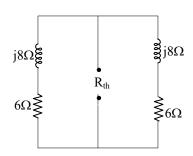


By Nodal ⇒

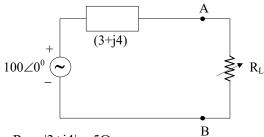
$$\frac{V_{th}}{(6+j8)} - \frac{110\angle 0^0}{(6+j8)} + \frac{V_{th}}{(6+j8)} - \frac{90\angle 0^0}{(6+j8)} = 0$$

$$2V_{th} = 200 \angle 0^0 \implies V_{th} = 100 \angle 0^0$$
.

R_{th}:



$$R_{th} = (6 + i8) || (6+i8) \equiv (3+i4)\Omega$$



$$R_L = |3 + j4| = 5\Omega$$

$$I = \frac{100 \angle 0^0}{(8 + i4)}$$

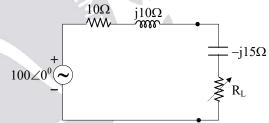
$$P = |I|^2 \times R_L$$

$$P_{\text{max}} = 125 \times 5 = 625 \text{ W}$$

$$\therefore P_{\text{max}} = 625 \text{ watts}$$

11.

Sol:



The maximum power delivered to "R_L" is

$$R_{L} = \sqrt{R_{S}^{2} + (X_{S} + X_{L})^{2}}$$

Here
$$R_S = 10\Omega$$
; $X_S = 10\Omega \& X_L = -15\Omega$

$$R_L = \sqrt{10^2 + (10 - 15)^2}$$

$$R_L = 5\sqrt{5} \Omega$$
.

$$I = \frac{100 \angle 0^0}{(10 + j10 - j15 + 5\sqrt{5})}$$

$$P_{\text{max}} = |I|^2 . 5\sqrt{5} = 236W$$

12. Sol:



The maximum power delivered to 10Ω load resistor is:

$$Z_L = 10 - jX_C = 10 + j(-X_C)$$

$$X_L = -X_C$$

So for MPT;
$$(X_S + X_L) = 0$$

$$10 - X_C = 0;$$

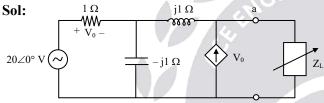
$$X_{\rm C} = 10$$

$$I = \frac{100 \angle 0^0}{(10 + j10 - j10 + 10)} = 5 \angle 0^0$$

$$P_{\text{max}} = |I|^2 R_L = 5^2 (10) = 250 W$$

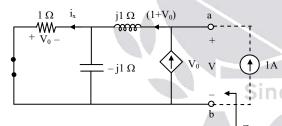
$$P_{max} = 250 \text{ Watts}$$

13. Ans: (b)



For maximum power delivered to Z_{L}

$$Z_{\rm L} = Z_{\rm th}^*$$



$$i_x = (1 + V_0) \times \frac{-j1}{1 - j1} = (1 + V_0) (0.5 - j0.5)$$

But

$$V_0 = -i_x$$
= - (1+V₀) (0.5 - j0.5)
(-1-j) V₀ = 1 + V₀

$$\Rightarrow V_0 (-1 - j - 1) = 1$$

$$V_0 = \frac{1}{-2 - i} = -0.4 + j0.2$$

Applying KVL

$$+ V_0 - i1(1 + V_0) + V = 0$$

$$\Rightarrow V = -V_0 + j1(1+V_0)$$

= 0.4 - j0.2+ j1(0.6+j0.2)

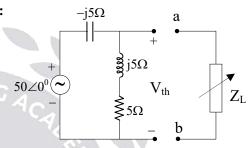
$$V = (0.2 + j 0.4)V$$

$$\therefore Z_{th} = \frac{V}{1} = V = (0.2 + j0.4)\Omega$$

$$\therefore Z_{L} = Z_{th}^{*} = (0.2 - j \ 0.4) \ \Omega$$

14.

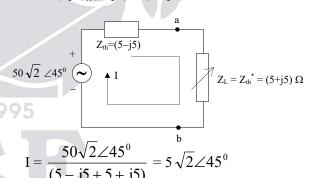
Sol:



The maximum true power delivered to " Z_L " is :

$$V_{th} = \left(\frac{50 \angle 0^0}{-j5 + j5 + 5}\right)(j5 + 5) = 50\sqrt{2} \angle 45^0$$

$$Z_{th} = (-j5)||(5+j5) = (5-j5)\Omega$$

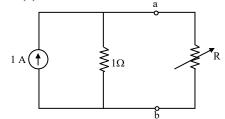


$$P = |I|^2 5 = |5\sqrt{2}|^2 .5 = 250 \text{ Watts}$$

$$\therefore P_{\text{max}} = 250 \text{ watts}$$

15. Ans: (c)

Sol:

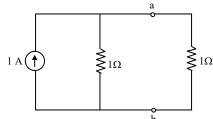






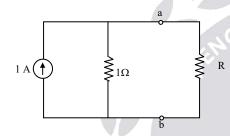
Maximum power will occurs when $R = R_s$

$$\Rightarrow$$
 R = 1 Ω



$$\therefore P_{\text{max}} = \left(\frac{1}{2}\right)^2 \times 1 = \frac{1}{4} W$$

25% of
$$P_{max} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16} W$$



current passing through 'R'

$$I = 1 \times \frac{1}{1+R} = \frac{1}{1+R}$$

$$\therefore P = I^2 R = \left(\frac{1}{1+R}\right)^2 R = \frac{1}{16}$$

$$\Rightarrow (R+1)^2 = 16R$$

$$\Rightarrow$$
R² +2R+1 = 16R

$$\Rightarrow R^2 - 14R + 1 = 0$$

 $R = 13.9282\Omega \text{ or } 0.072\Omega$

From the given options $72m\Omega$ is correct.

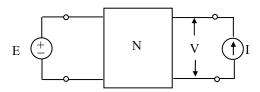
16. The network 'N' shown in figure contains only resistances.

E = 10V and 0V

I = 0A and 2A

V = 3V and 2V respectively.

If E = 100V and I is replaced by R = 2Ω , then determine V.



Sol: For, E = 10V, I = 0A then V = 3V

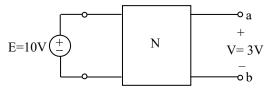


Fig.(b)

 $V_{oc} = 3V$ (with respect to terminals a and b)

For, E = 0V, I = 2A then V = 2V

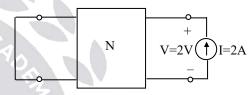
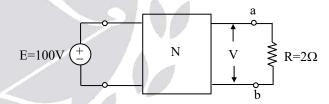


Fig.(c)

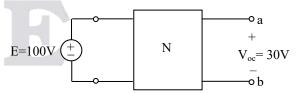
Now when E=100V, and I is replaced by $R=2\Omega$ then V=?



When E = 100V,

Since

From Fig.(b) using homogeneity principle



For finding Thevenin's resistance across ab independent voltage sources to be short circuited & independent current sources to be open circuited.

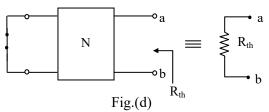
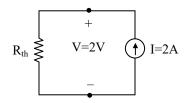




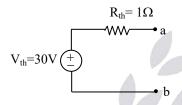


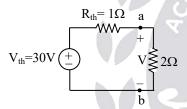
Fig.(c) is the energized version of Fig. (d)



$$\Rightarrow R_{th} = \frac{2}{2} = 1\Omega$$

:. With respect to terminals a and b the Thevenin's equivalent becomes.





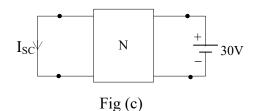
$$V = 30 \times \frac{2}{2+1} = 20V$$

$$\therefore V = 20V$$

17.

Sol: Superposition theorem cannot be applied to fig (b)

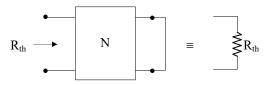
Since there is only voltage source given:



By homogeneity and Reciprocity principles to fig (a);

$$I_{SC} = 6A$$

For R_{th}:



Statement: Fig (a) is the energized version of figure (d)

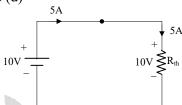
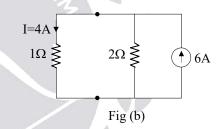


Fig (a)

$$10 = R_{th}. 5 \Big|_{by \, ohm's \, law}$$

$$R_{th} = 2\Omega$$
.



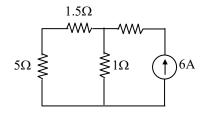
$$I = \frac{6 \times 2}{(2+1)} = 4A$$

Since

Sol:
$$\begin{bmatrix} 10 \\ 4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$10 = Z_{11}(4) + Z_{12}(0)$$

$$4 = Z_{21}(4) + Z_{22}(0)$$



$$Z_{11} = \frac{10}{4} = 2.5$$



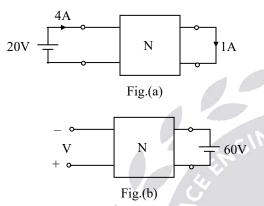


$$Z_{21} = \frac{4}{4} = 1$$

 $I_{5\Omega} = \frac{6 \times 1}{6.5 + 1} = \frac{6}{7.5} = 0.8 \text{ A}$

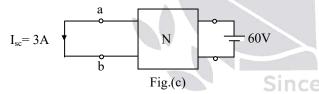
19. Ans: (b)

Sol:



Using reciprocity theorem, for Fig.(a)





Norton's resistance between a and b is

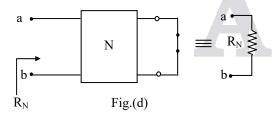
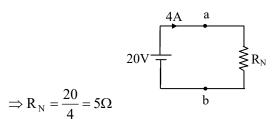
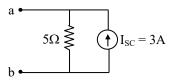


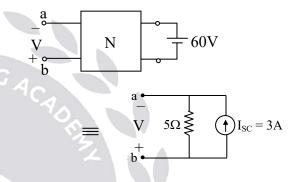
Fig.(a) is the energized version of Fig.(d)



With respect to terminals a and b the Norton's equivalent of Fig.(b) is

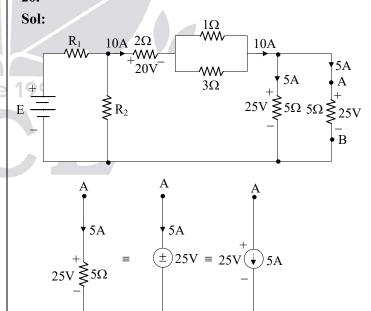


∴ From Fig.(b)



$$\Rightarrow$$
V = -15 V

20.



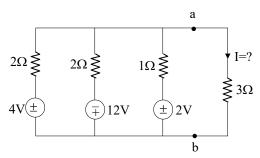
$$P_{AB} = P_{5\Omega} = P_{25V} = P_{5A} = 5*25 = 125$$
 watts (ABSORBED)





21.

Sol:



By Mill Man's theorem;

$$V' = \frac{V_1G_1 + V_2G_2 + V_3G_3}{G_1 + G_2 + G_3}$$

$$\equiv \frac{\frac{4}{2} - \frac{12}{2} + \frac{2}{1}}{\left(\frac{1}{2} + \frac{1}{2} + 1\right)} = \frac{4 - 12 + 4}{2 \cdot 2} \equiv -1V$$

$$\frac{1}{2} = \frac{1}{2} = -1V$$

$$\frac{1}{2} = \frac{1}{2} = -1V$$

$$\therefore V' = -1V$$

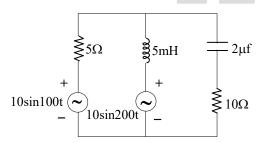
$$\frac{1}{R^{1}} = \frac{1}{R_{1}} + \frac{1}{R_{2}} + \frac{1}{R_{3}} = \frac{1}{2} + \frac{1}{2} + 1 = 2$$

$$\therefore R^1 = \frac{1}{2}\Omega$$

$$I = \frac{-1}{\left(\frac{1}{2} + 3\right)} \Rightarrow I = \frac{-2}{7} A$$

22. Ans: (d)

Sol:



Since the two different frequencies are operating on the network simultaneously; always the super position theorem is used to evaluate the responses since the reactive elements are frequency sensitive

i.e.,
$$Z_L = j\omega L$$
 and $Z_C = \frac{1}{j\omega c} \Omega$.

23.

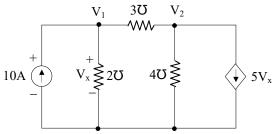
Sol: In the above case if both the source are 100 rad/sec, each then Millman's theorem is more conveniently used.

Sol: $\begin{array}{c|c} 2\Omega & 4\Omega \\ \hline & & \\ \hline & &$

Nodal equations

Since

$$\begin{split} i &= GV \\ i_x &= i_1 \\ 10 &= 2i_1 + 3(i_1 - i_2) \dots (1) \\ 0 &= 4i_2 + 2i_x + 3(i_2 - i_1) \dots (2) \\ V_x &= V_1 \\ 10 &= 2V_1 - 3(V_1 - V_2) \dots (3) \\ 0 &= 4V_2 + 2V_x + 3(V_2 - V_1) \dots (4) \end{split}$$



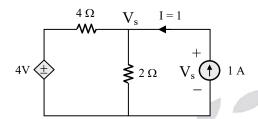


26. (b, c)

Sol: Tellegen's Theorem is applicable to any nonlinear Network.

27. Ans: (c, d)

Sol:



$$I = 1 \Rightarrow 4I = 4(1) = 4 \text{ V}$$

$$R_{th} = \frac{V_S}{I}$$

$$\frac{V_{S}-4}{4} + \frac{V_{S}}{2} - 1 = 0$$

$$\frac{3V_S}{4} = 2 \Rightarrow V_S = \frac{8}{3}V$$

$$R_{\text{th}} = \frac{V_{\text{S}}}{I} = \frac{8}{3}\,\Omega$$

 \therefore There is no independent source, $V_{th} = 0$

 \therefore (c, d) are correct.





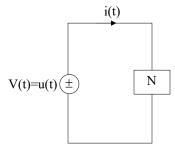
Chapter

3

Transient Circuit Analysis

01.

Sol:



$$i(t) = e^{-3t}A$$
 for $t > 0$ (given)

Determine the elements & their connection

Response Laplace transform = System Excitation Laplace transform

transfer function

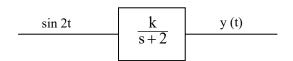
i.e.,
$$\frac{I(s)}{V(s)} = H(s) = \frac{\frac{1}{(s+3)}}{\frac{1}{s}}$$
$$= \frac{s}{(s+3)} = y(s) = \frac{1}{Z(s)}$$
$$\therefore Z(s) = \left(\frac{s+3}{s}\right)$$
$$= 1 + \frac{1}{s\left(\frac{1}{3}\right)} = R + \frac{1}{SC}$$

$$\therefore$$
 R = 1 Ω and C = $\frac{1}{3}$ F are in series

02. Ans: (c)

Sol: The impulse response of first order system is Ke^{-2t} .

So T/F = L(I.R) =
$$\frac{K}{s+2}$$



$$G(s) = \frac{K}{s+2}$$

$$|G(j\omega)| = \frac{K}{\sqrt{\omega^2 + 2^2}} = \frac{K}{2\sqrt{2}}$$

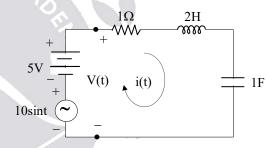
$$\angle G(j\omega) = -\tan^{-1}\frac{\omega}{2} = -\tan^{-1}1 = -\frac{\pi}{4}$$

So steady state response will be

$$y(t) = \frac{K}{2\sqrt{2}} \sin\left(2t - \frac{\pi}{4}\right)$$

03.

Sol:



By KVL \Rightarrow v(t) = (5 + 10sint)volt

Evaluating the system transfer function H(s).

 $\frac{\text{Desired response L.T}}{\text{Excitation response L.T}} = \text{System transfer function}$

$$\frac{I(s)}{V(s)} = H(s) = Y(s) = \frac{1}{Z(s)} = \frac{1}{\left(R + SL + \frac{1}{SC}\right)}$$

$$H(s) = \frac{S}{\left(2s^2 + s + 1\right)}$$

$$H(j\omega) = \frac{1}{\left(1 + \frac{1}{j\omega} + 2j\omega\right)}$$

II. Evaluating at corresponding $\boldsymbol{\omega}_s$ of the input

$$H(j\omega)|_{\omega=0}=0$$

$$H(j\omega)|_{\omega=1} = \frac{1}{\sqrt{2}} \angle -45^{\circ}$$



III.
$$\frac{I(s)}{V(s)} = H(s)$$

$$I(s) = H(s)V(s)$$

$$i(t) = 0 \times 5 + \frac{1}{\sqrt{2}} \times 10\sin(t - 45^{\circ})$$

$$i(t) = 7.07\sin(t - 45^{\circ})A$$

OBS: DC is blocked by capacitor in steady state.

04.

Sol:
$$\frac{V(s)}{I(s)} = H(s) = Z(s) = \frac{1}{Y(s)} = \frac{1}{\left(\frac{1}{R} + \frac{1}{sL} + sC\right)}$$

$$H(s) = \frac{1}{\left(1 + \frac{1}{s} + s\right)}$$

$$H(j\omega)\Big|_{\omega=1} = \frac{1}{\left(1+\frac{1}{j}+j\right)} = 1$$

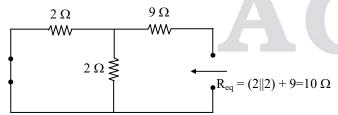
$$V(s) = I(s) H(s) = \sin t$$

$$v(t) = \sin t \text{ volts}$$

05.

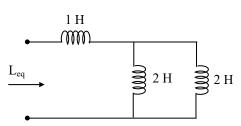
Sol:
$$\tau = \frac{L_{eq}}{R_{eq}}$$

 R_{eq} :



$$R_{eq} = (2 \parallel 2) + 9 = 10 \Omega$$

 L_{eq} :

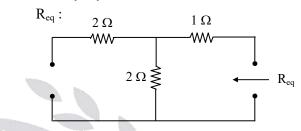


$$L_{eq} = (2 \parallel 2) + 1 = 2 H$$

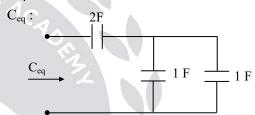
$$\therefore \ \tau = \frac{L_{eq}}{R_{eq}} = \frac{2}{10} = 0.2 \text{ sec}$$

06.

Sol:
$$\tau = R_{eq} C_{eq}$$



$$R_{eq} = 3 \Omega$$

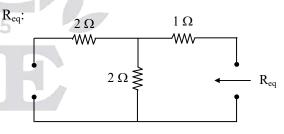


$$C_{eq} = 1 F$$

$$\therefore \tau = 3 \times 1 = 3 \text{ sec}$$

07.

Sol:
$$\tau = R_{eq} C$$



$$R_{eq} = 3 \Omega$$

$$\therefore \tau = 3 \times 1 = 3 \text{ sec}$$

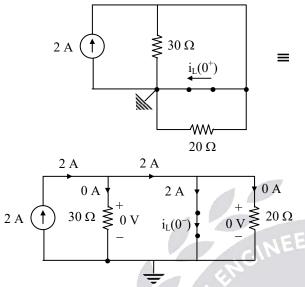
08.

Sol: Let us assume that switch is closed at $t = -\infty$, now we are at $t = 0^-$ instant, still the switch is closed i.e., an infinite amount of time, the independent dc source is connected to the network and hence it is said to be in steady state.





In steady state, the inductor acts as short circuit and nature of the circuit is resistive.

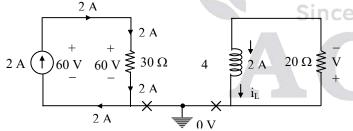


At $t = 0^-$: Steady state: A resistive circuit

Note: The number of initial conditions to be evaluated at just before the switching action is equal to the number of memory elements present in the network.

(i)
$$t = 0^{-}$$

 $i_{L}(0^{-}) = 2 = i_{L}(0^{+})$
 $E_{L}(0^{-}) = \frac{1}{2} L i_{L}^{2}(0^{-})$
 $= \frac{1}{2} \times 4 \times 2^{2} = 8J = E_{L}(0^{+})$

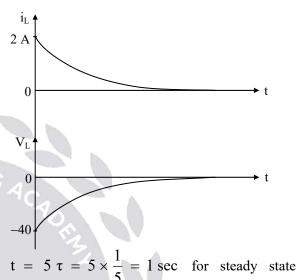


For $t \ge 0$

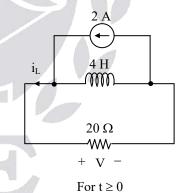
i_L 4 H 2 A 20 Ω WW

For $t \ge 0$: Source free circuit

$$\begin{split} I_0 &= 2 \; A \; ; \; \tau \; = \; \frac{L}{R} \; = \; \frac{4}{20} \; = \; \frac{1}{5} \; sec \\ i_L &= \; 2 \; e^{-5t} \; \; for \; \; 0 \leq t \leq \infty \\ V_L &= \; L \; \frac{d \, i_L}{d \, t} \; = \; -40 \; e^{-5t} \; \; V \; \; for \; \; 0 \leq t \leq \infty \end{split}$$



practically i.e., with in 1 sec the total 8 J stored in the inductor will be delivered to the resistor.



 $\begin{array}{c|c}
2 A \\
\downarrow \\
I_L(0^+) & V_L(0^+) \\
\hline
 & - & + \\
20 \Omega & WW \\
 & + & V(0^+) & \end{array}$

At $t = 0^+$: Resistive circuit: Network is in transient state



1995



By KCL:

$$-2 + i_L(0^+) = 0$$

$$i_{\rm I}(0^+) = 2 {\rm A}$$

$$V(0^{+}) = R i_L(0^{+}) |_{By Ohm's law}$$

$$V(0^+) = 20 (2) = 40 V$$

By KVL:

$$V_L(0^+) + V(0^+) = 0$$

$$V_L(0^+) = -V(0^+) = -40 \text{ V} = V_L(t)|_{t=0^+}$$

Observations:

$$t = 0^{-}$$

$$i_L(0^-) = 2 A$$

$$i_L(0^+)=2 A$$

$$i_{20\Omega}(0^-)=0 A$$

$$i_{20\Omega}(0^+) = 2 A$$

$$V_{20\Omega}(0^-) = 0 \text{ V}$$

$$V_{20\Omega}(0^+) = 40 \text{ V}$$

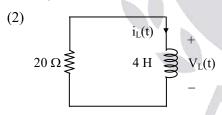
 $V_L(0^+) = -40 \text{ V}$

$$V_{L}(0^{-}) = 0 \text{ V}$$

$$V_{L}(0^{+}) = -40 \text{ V}$$

Conclusion:

To keep the same energy as $t = 0^-$ and to protect the KCL and KVL in the circuit (i.e., to ensure the stability of the network), the inductor voltage, the resistor current and its voltage can change instantaneously i.e., within zero time at $t = 0^{+}$.



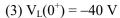
For $t \ge 0$

$$i_L(t) = 2 e^{-5t} A \text{ for } 0 \le t \le \infty$$

 $V_L(t) = -40 e^{-5t} V \text{ for } 0 \le t \le \infty$

Conclusion:

For all the source free circuits, $V_L(t) = -ve$ for $t \ge 0$, since the inductor while acting as a temporary source (upto 5τ), it discharges from positive terminal i.e., the current will flow from negative to positive terminals. (This is the must condition required for delivery, by Tellegan's theorem)



$$V_{L}(t)|_{t=0^{+}} = -40 \text{ V}$$

$$L \left. \frac{d i_L(t)}{d t} \right|_{t=0^+} = -40$$

$$\frac{di_L(t)}{dt}\Big|_{t=0^+} = -\frac{40}{L} = -\frac{40}{4} = -10 \text{ A/sec}$$

Check:

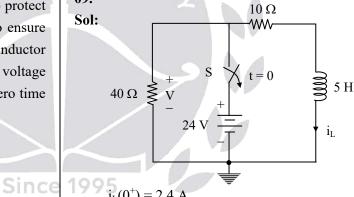
$$i_L(t) = 2 e^{-5t} A$$
 for $0 \le t \le \infty$

$$\frac{di_L(t)}{dt} = -10 e^{-5t} \text{ A/sec for } 0 \le t \le \infty$$

$$\frac{di_L(t)}{dt}$$
 = -10 A/sec

09.

Sol:



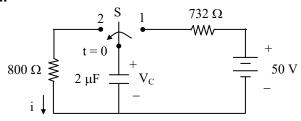
$$i_L(0^+) = 2.4 A$$

$$V(0^+) = -96 \text{ V}$$

$$i_L(t) = 2.4 e^{-10 t} A \text{ for } 0 \le t \le \infty$$

10.

Sol:



$$V_C(0^+) = 50 \text{ V} ; i(0^+) = 62.5 \text{ mA}$$

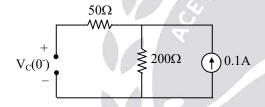




$$\begin{split} V_C(t) &= 50 \ e^{-\frac{t}{1.6\times 10^{-3}}} \ V \ \text{ for } \ t \geq 0 \\ i_C &= C \left. \frac{d\,V_C}{d\,t} \, \right|_{By\,Ohm's\,law} \\ &= 2\times 10^{-6} 50 \ e^{-\frac{t}{1.6\times 10^{-3}}} \times \frac{-1}{1.6\times 10^{-3}} \\ &= \frac{100\times 10^{-6}}{1.6\times 10^{-3}} \\ &= \frac{1}{16} \end{split}$$

11.

Sol: Case (i): t < 0

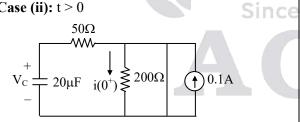


$$V_C(0^-) = 20V \& i(0^-) = 0.1A$$

: Capacitor never allows sudden changes in voltages

$$V_C(0^-) = V_C(0) = V_C(0^+) = 20V$$

Case (ii): t > 0



To find the time constant $\tau = R_{eq}C$

After switch closed

$$R_{eq} = 50\Omega$$
 $C = 20\mu F$

$$i(0^+) = 0A$$

$$\tau = 50 \times 20 \mu$$

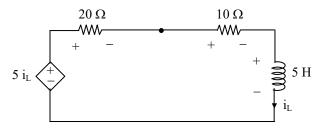
 $\tau = 1$ msec

$$V_C(t) = V_0 e^{-t/\tau} = 20 e^{-t/1m}$$

$$V_C(t) = 20e^{-t/1m}V; \quad 0 \le t \le \infty$$

12.

Sol: After performing source transformation;



By KVL;

$$5 i_L - 30 i_L - 5 \frac{di_L}{dt} = 0$$

$$\frac{di_L}{dt} + 5i_L = 0$$

$$(D + 5) i_L = 0$$

$$i_L(t) = K e^{-5t} A \text{ for } 0 \le t \le \infty$$

$$\tau = \frac{1}{5} \sec$$

13.

Sol:
$$i_{L_1}(0) = 10 \text{ A}$$
; $i_{L_2}(0) = 2 \text{ A}$

$$i_{L_1}(t) = I_0 e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R} = \frac{1}{1} = 1 \text{ sec}$$

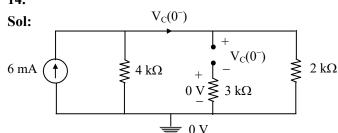
$$i_{L_1}(t) = 10 e^{-t} A$$

Similarly, $i_{L_2}(t) = I_0 e^{-\frac{t}{\tau}}$

$$\tau = \frac{L}{R} = 2 \text{ sec}$$

$$i_{L_2}(t) = 20 e^{-\frac{t}{2}} A$$

14.

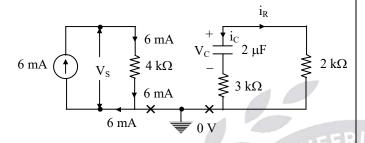






At $t = 0^-$: Steady state: A resistive circuit By Nodal:

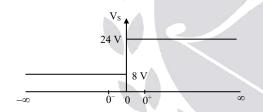
$$-6 \text{ mA} + \frac{V_{c}(0^{-})}{4 \text{ K}} + \frac{V_{c}(0^{-})}{2 \text{ K}} = 0$$
$$V_{c}(0^{-}) = 8 \text{ V} = V_{c}(0^{+})$$



For $t \ge 0$: A source free circuit

$$V_s = 6 \text{ m} \times 4 \text{ K} = 24 \text{ V}$$

$$\tau = R_{eq} C = (5 \text{ K}) 2 \mu = 10 \text{ m sec}$$



By KCL:

$$i_C + i_R = 0$$

$$i_R = - i_C = 1.6 \ e^{-100 \ t} \ mA \quad for \ 0 \leq t \leq \infty$$

Observation:

In all the source free circuit, $i_C(t) = -ve$ for $t \ge 0$ because the capacitor while acting as a temporary source it discharges from the +ve terminal i.e., current will flow from -ve to +ve terminals.

15.

$$i(t) = i_{R}(t) + i_{L}(t)$$

$$= \frac{V_{R}(t)}{R} + \frac{1}{L} \int_{-\infty}^{t} V_{L}(t) dt$$

$$= \frac{V_{S}(t)}{10} + i_{L}(0) + \frac{1}{L} \int_{0}^{t} V_{S}(t) dt$$

$$i(t) = 4.4 + 5 + 4.4^{2}$$

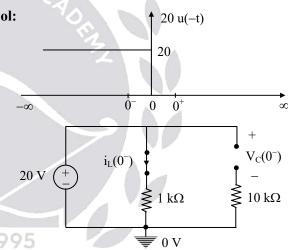
$$i(t) = 4 t + 5 + 4 t^2$$

$$i(t)|_{t=2 \text{ sec}} = 8 + 16 + 5 = 29 \text{ A} = 29000 \text{ mA}$$

Ans: (c) 16.

17.

Sol:

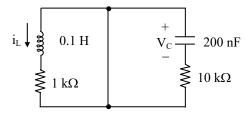


At $t = 0^-$: steady state: A resistive circuit.

(i)
$$t = 0^{-}$$

$$V_C(0^-) = 20 \text{ V} = V_C(0^+)$$

$$i_L(0^-) = \frac{20}{1K} = 20 \text{ mA} = i_L(0^+)$$



For $t \ge 0$: A source free RL & RC circuit





$$\tau = \frac{0.1}{1 \, \text{K}} = 100 \, \mu \text{sec}$$

$$\tau_C = 200 \times 10^{-9} \times 10 \times 10^3 = 2 \text{ m sec}$$

$$\frac{\tau_{_{C}}}{\tau_{_{L}}}{=}20~;~\tau_{_{C}}=20~\tau_{_{L}}$$

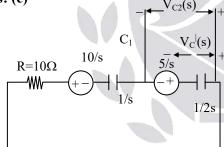
Observation:

 $\tau_L < \tau_C$; therefore the inductive part of the circuit will achieve steady state quickly i.e., 20 times faster.

$$\begin{split} &V_{_{C}} \,=\, 20\; e^{-\frac{t}{\tau_{_{C}}}}\;\; V \quad for \quad 0 \leq t \leq \infty \\ &i_{_{L}} \,=\, 20\; e^{-\frac{t}{\tau_{_{L}}}}\;\; mA \quad for \quad 0 \leq t \leq \infty \\ &V_{_{L}} \,=\, L\; \frac{di_{_{L}}}{d\,t}\; \bigg|_{By\;Ohm's\;law} \\ &i_{_{C}} \,=\, C\; \frac{d\,V_{_{C}}}{d\,t}\; \bigg|_{By\;Ohm's\;law} \end{split}$$

18. Ans: (c)

Sol:



$$V_{c}(s) = \frac{\frac{5}{s} (\frac{1}{2s})}{R + \frac{1}{s} + \frac{1}{2s}}$$

$$= \frac{\frac{5}{2s^{2}}}{\frac{2Rs + 2 + 1}{2s}} = \frac{5}{s(2Rs + 3)}$$

$$V_{c_2}(\infty) - V_{c_1}(s) - \frac{5}{s} = 0$$

$$V_{c}(\infty) = V_{c}^{\dagger}(s) + \frac{5}{s}$$

$$V_{c}(\infty) = \text{Lt s.} \left[\frac{5}{s(2Rs+3)} + \frac{5}{s} \right] = \frac{5}{3} + 5 = \frac{20}{3}$$

19. Ans: (d)

Sol: at t = 0

$$L\frac{\mathrm{di}(0)}{\mathrm{dt}} = V_{L}(0)$$

$$V_L = 2 \times 3 = 6$$

$$V_L = 6V$$

$$E_2 + 6 - 8R = 0$$

$$E_2 = 8R - 6$$

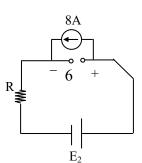


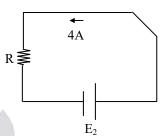
$$E_2 = 4R$$

$$8R - 6 = 4R$$

$$4R = 6$$

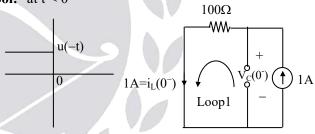
$$R = 1.5\Omega$$





20. Ans: (d)

Sol: at t < 0



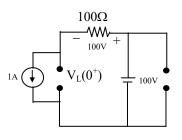
Apply KVL in loop1
$$\Rightarrow$$
 V_C(0⁻)-100 = 0
 \Rightarrow V_C(0⁻) = 100V

At
$$t = 0^+$$

$$V_L(0^+) = 0$$

$$L\frac{di(0^+)}{dt} = 0$$





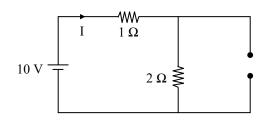
21.

Sol: Case -1 at
$$t = 0^+$$

By redrawing the circuit







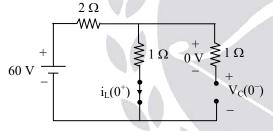
Current through the battery at $t = 0^+$ is

$$\frac{10}{3}$$
 Amp

Case -2 at $t = \infty$ $I \quad I \quad \Omega$ $2 \quad \Omega$

Current through the battery at $t = \infty$ is 10 A

22. Sol:

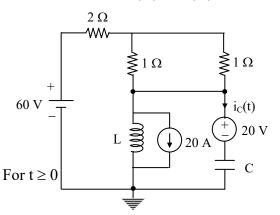


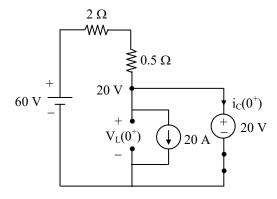
At $t = 0^-$: Steady state: A resistive circuit

(i)
$$t = 0^-$$
:

$$i_L(0^-) = \frac{60}{3} = 20 \text{ A} = i_L(0^+)$$

$$V_{1\Omega} = 20 \text{ V} = V_{C}(0^{-}) = V_{C}(0^{+})$$





At $t = 0^+$: A resistive circuit: Network is in transient state

$$V_{L}(0^{+}) = 20 \text{ V}$$

Nodal:

$$\frac{20-60}{2.5} + 20 + i_{\rm C}(0^+) = 0$$

$$i_C(0^+) = -4 A$$

23.

Sol: Repeat the above problem procedure :

$$\frac{\mathrm{d}i_{\mathrm{L}}(t)}{\mathrm{d}t} = \frac{V_{\mathrm{L}}(0^{+})}{\mathrm{L}} = 0 \text{ A/sec}$$

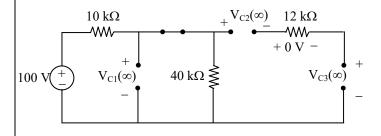
$$\left. \frac{dV_{C}(t)}{dt} \right|_{t=0^{+}} = \frac{i_{C}(0^{+})}{C} = -10^{6} \text{ V/sec}$$

24.

1995

Since

Sol: Observation: So, the steady state will occur either at $t = 0^-$ or at $t = \infty$, that depends where we started i.e., connected the source to the network.



At $t = \infty$: Steady state: A Resistive circuit

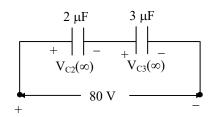


India's Best Online Coaching Platform for GATE, ESE, PSUs, SSC-JE, RRB-JE, SSC, Banks, Groups & PSC Exams

Since



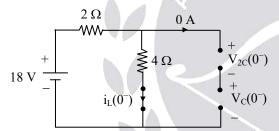
$$V_{C_1}(\infty) = \frac{100}{50 \, \text{K}} \times 40 \, \text{K} = 80 \, \text{V}$$



$$V_{C_2}(\infty) = \frac{80 \times 3 \,\mu\,F}{(2+3)\,\mu\,F} = 48 \text{ V}$$

$$V_{C_3}(\infty) = \frac{80 \times 2 \mu F}{5 \mu F} = 32 \text{ V}$$

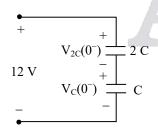
25. Sol:



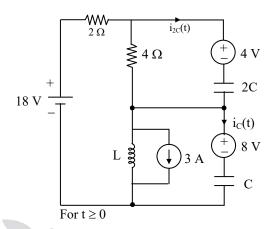
At $t = 0^-$: Circuit is in Steady state: Resistive circuit

$$i_L(0^-) = 3 A = i_L(0^+)$$

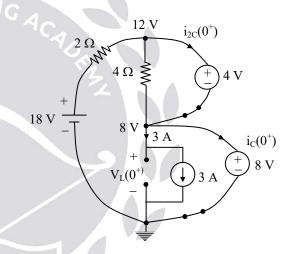
 $V_{4O} = 4 \times 3 = 12 V$



$$V_{2c}(0^{-}) = \frac{12 \times C}{2C + C}$$
$$= 4 V = V_{2c}(0^{+})$$
$$V_{C}(0^{-}) = 8 V = V_{C}(0^{+})$$



and redrawing the circuit



By Nodal; $\frac{12-18}{2} + \frac{12-8}{4} + i_{2C}(0^{+}) = 0$ $\frac{-6}{2} + \frac{4}{4} + i_{2C}(0^{+}) = 0$ $i_{2C}(0^{+}) = 2 \text{ A} = i_{2C}(0^{-})$ $\frac{8-12}{4} - i_{2C}(0^{+}) + 3 + i_{C}(0^{+}) = 0$ $i_{C}(0^{+}) = 0 \text{ A} = i_{C}(0^{-})$

26. Sol:
$$t = 0^ t = 0^+$$
 $t = 0^+$ $i_L(0^-) = 5 \text{ A}$ $i_L(0^+) = 5 \text{ A}$





$$\frac{di_{L}(0^{+})}{dt} = \frac{V_{L}(0^{+})}{I_{L}} = 40$$

$$i_R(0^-) = -5 A$$

$$i_{R}(0^{+}) = -1A$$

$$\frac{\mathrm{di}_{\mathrm{R}}(0^{+})}{\mathrm{dt}} = -40 \; \mathrm{A/sec}$$

$$i_{\rm C}(0^-) = 0 \text{ A}$$

$$i_{\rm C}(0^+) = 4A$$

$$\frac{di_{\rm C}(0^+)}{dt} = -40 \text{ A/sec}$$

$$V_{L}(0^{-}) = 0 \text{ V}$$

$$V_L(0^+) = 120 \text{ V}$$

$$\frac{dV_L(0^+)}{dt} = 1098 \text{ V/sec}$$

$$V_R(0^-) = -150 \text{ V}$$

$$V_R(0^+) = -30 \text{ V}$$

$$\frac{dV_R(0^+)}{dt} = -1200 \text{ V/sec}$$

$$V_{\rm C}(0^-) = 150 \text{ V}$$

$$V_L(0^+) = 150 \text{ V}$$

$$\frac{dV_{\rm C}(0^+)}{dt} = 108 \text{ V/sec}$$

(i).
$$t = 0^-$$

By KCL
$$\Rightarrow$$
 $i_L(t) + i_R(t) = 0$

$$t = 0^- \implies i_1(0^-) + i_R(0^-) = 0$$

$$i_{\rm P}(0^-) = -5$$
 A

$$V_R(t) = R i_R(t) |_{By Ohm's law}$$

$$V_R(0^-) = R i_R(0^-) = 30(-5) = -150 V$$

By KVL
$$\Rightarrow$$
 V_L(t) - V_R(t) - V_C(t) = 0

$$V_C(0^-) = V_L(0^-) - V_R(0^-) = 150 \text{ V}$$

(ii). At
$$t = 0^+$$

By KCL at
$$1^{st}$$
 node \Rightarrow

$$-4 + i_{\rm L}(t) + i_{\rm R}(t) = 0$$

$$-4 + i_1(0^+) + i_2(0^+) = 0$$

$$i_R(0^+) = -i_L(0^+) + 4$$

$$i_R(0^+) = -5 + 4 = -1 A$$

$$V_R(t) = R i_R(t) |_{By Ohm's law}$$

$$V_R(0^+) = R i_R(0^+)$$

$$V_{R}(0^{+}) = -30 \text{ V}$$

By KVL
$$\Rightarrow$$
 V_L(t) - V_R(t) - V_C(t) = 0

$$V_L(0^+) = V_R(0^+) + V_C(0^+)$$

$$= 150 - 30 = 120 \text{ V}$$

By KCL at 2nd node;

$$-5 + i_C(t) - i_R(t) = 0$$

$$i_{\rm C}(0^+) = 4 {\rm A}$$

(iii).
$$t = 0^+$$

By KCL at
$$1^{st}$$
 node \Rightarrow

$$-4 + i_{\rm L}(t) + i_{\rm R}(t) = 0$$

$$0 + \frac{di_{L}(t)}{dt} + \frac{d}{dt}i_{R}(t) = 0$$

$$V_R(t) = R i_R(t) |_{By Ohm's law}$$

$$\frac{d}{dt} V_R(t) = R \frac{d}{dt} i_R(t)$$

$$V_{L}(t) - V_{R}(t) - V_{C}(t) = 0$$

$$\frac{d V_L(t)}{dt} - \frac{d V_R(t)}{dt} - \frac{d V_C(t)}{dt} = 0$$

By KCL at node 2:

$$-5 + i_C(t) - i_R(t) = 0$$

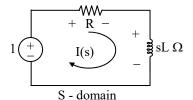
$$0 + \frac{\mathrm{d}}{\mathrm{d}t} i_{\mathrm{C}}(t) - \frac{\mathrm{d}}{\mathrm{d}t} i_{\mathrm{R}}(t) = 0$$

$$\frac{d}{dt} i_C(0^+) = -(-40) = 40 \text{ A/sec}$$

27.

Since 1995

Sol: Transform the network into Laplace domain



$$V(s) = Z(s) I(s)$$

By KVL in S-domain ⇒





$$1 - R I(s) - s L I(s) = 0$$

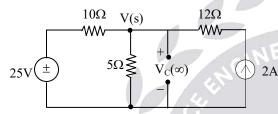
$$I(s) = \frac{1}{L} \frac{1}{\left(s + \frac{R}{L}\right)}$$

$$i(t) = \frac{1}{L} e^{-\frac{R}{L}t} A \text{ for } t \ge 0$$

28.

Sol: By Time domain approach;

$$V_C(0^-) = 5 \times 2 = 10 \text{ V} = V_C(0^+)$$



At $t = \infty$: Steady state: A resistive circuit

Nodal
$$\Rightarrow \frac{V_C(\infty) - 25}{10} + \frac{V_C(\infty)}{5} - 2 = 0$$

$$V_C(\infty) = 15 \text{ V}$$

$$\tau = R_{eq} C = (5 \parallel 10) . 1 = (10/3) sec$$

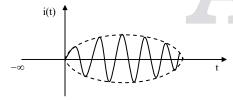
$$V_{\rm C} = 15 + (10 - 15) e^{-\frac{\tau}{(10/3)}}$$

$$V_C = 15 - 5 e^{-3t/10} V \text{ for } t \ge 0$$

$$i_{C} = C \frac{dV_{C}}{dt} = 1.5 e^{-3t/10} \text{ A for } t \ge 0$$

29.

Sol:



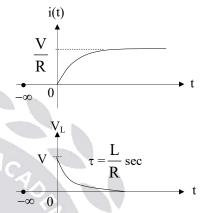
That is the response is oscillatory in nature

30.

Sol:
$$i(0^-) = 0$$
 A = $i(0^+)$

$$i(\infty) = \frac{V}{R} A$$

$$\begin{split} \tau &= \frac{L}{R} sec \\ i(t) &= \frac{V}{R} + \left(0 - \frac{V}{R}\right) e^{-t/\tau} = \frac{V}{R} \left(1 - e^{-t/\tau}\right) \\ V_L &= \frac{L di(t)}{dt} = V \ e^{-Rt/L} \ for \ t \! \ge \! 0 \end{split}$$



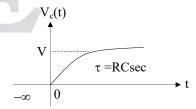
Exponentially Increasing Response

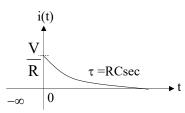
31.

$$\begin{split} V_C(\infty) &= 15 \ V \\ \tau &= R_{eq} \ C = (5 \parallel 10) \ . \ 1 = (10/3) \ sec \\ V_C &= 15 + (10 - 15) \ e^{-\frac{t}{(10/3)}} \\ V_C &= 15 - 5 \ e^{-3t/10} \ V \ \text{ for } \ t \geq 0 \end{split}$$

$$\begin{aligned} &\text{Sol: } \ V_C(0^{\circ}) = 0 = V_C(0^{+}) \\ V_C(\infty) &= V \\ \tau &= RC \\ V_C &= V + (0 - V) e^{-t/\tau} \\ &= V(1 - e^{-t/RC}) \ \text{ for } \ t \geq 0 \end{aligned}$$

$$\begin{aligned} &\text{i.c.} \ &= C \frac{d V_C}{dt} = 1.5 \ e^{-3t/10} \ A \ \text{ for } \ t \geq 0 \end{aligned}$$





Exponentially Decreasing Response

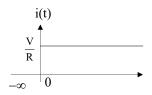




32.

Sol: It's an RL circuit with $L = 0 \Rightarrow \tau = 0$ sec

$$i(t) = \frac{V}{R} \,, \; \forall t \geq 0 \text{ So, } 5\tau = 0 \text{ sec}$$



i.e., the response is constant

33.

Sol:
$$i_1 = \frac{100u(t) - V_L}{10}$$

$$i1 = \left(10u(t) - \frac{1}{100} \frac{di_L}{dt}\right) A$$

 $Nodal \Rightarrow$

$$-i_1 + i_L + \frac{V_L - 20i_1}{20} = 0$$

$$-2i_1+i_L+\frac{1}{200}\frac{di_L}{dt}=0$$

Substitute i₁;

$$\frac{\mathrm{di}_{\mathrm{L}}}{\mathrm{dt}} + 40\mathrm{i}_{\mathrm{L}} = 800\mathrm{u}(\mathrm{t})$$

$$SI_L(s) - i_L(0+) + 40I_L(s) = \frac{800}{s}$$

$$i_L(0^-) = 0A = i_L(0^+)$$

$$I_L(s) = \frac{800}{s(s+40)} = \frac{20}{s} - \frac{20}{s+40}$$

$$I_L t$$
) = 20u(t) – 20e^{-40t} u(t)

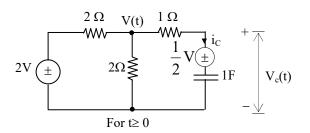
$$I_L(t) = 20(1-e^{-40t}) u(t)$$

$$i_l = 10u(t) - \frac{1}{100} d \frac{i_L}{dt}$$

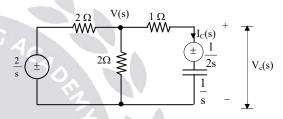
$$i_1 = (10-8e^{-40t}) u(t)$$

34.

Sol: By Laplace transform approach:



Transform the above network into the Laplace domain



For t≥ 0

Nodal ⇒

Since

$$\frac{V(s) - \frac{2}{s}}{2} + \frac{V(s)}{2} + \frac{V(s) - \frac{1}{2s}}{1 + \frac{1}{s}} = 0$$

$$I_c(s) = \left(\frac{V(s) - \frac{1}{2s}}{1 + \frac{1}{s}} \right)$$

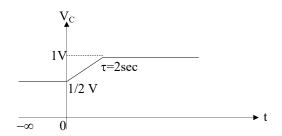
$$\Rightarrow i_c(t) = \frac{1}{4} e^{-\frac{t}{2}} A \text{ for } t \ge 0$$

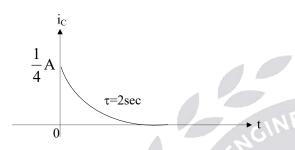
$$V_{c}(s) - \frac{1}{2s} - \frac{1}{s} I_{c}(s) = 0$$

$$V_{c}(s) = \frac{1}{2s} + \frac{1}{s} I_{c}(s)$$

$$v_{c}(t) = 1 - \frac{1}{2} e^{-\frac{t}{2}} V \text{ for } t \ge 0$$



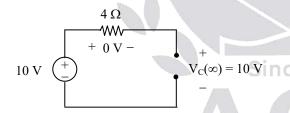




Sol: By Time domain approach;

$$V_C(0) = 6 V \text{ (given)}$$

$$V_{\rm C}(\infty) = 10 \text{ V}$$



At $t = \infty$: Steady state: Resistive circuit

$$\tau = R C = 8 \text{ sec}$$

$$V_C = 10 + (6 - 10) e^{-t/8}$$

$$V_C = 10 - 4 e^{-t/8}$$

$$V_C(0) = 6 V$$

$$i_C = C \frac{dV_C}{dt} = e^{-t/8} = i(t)$$

$$E_{4\Omega} = \int_0^{\infty} (e^{-t/8})^2 4 dt = 16 J$$

36.

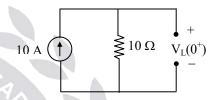
Sol: 10 A $10 \text$

At $t = 0^-$: Network is not in steady state i.e., unenergised

$$t = 0^-$$
:

$$i_L(0^-) = 0 A = i_L(0^+)$$

$$V_{I}(0^{+}) = 10 \times 10 = 100 \text{ V}$$



At $t = 0^+$: Network is in transient state : A resistive circuit

 $i_L(\infty) = 10 \text{ A (since inductor becomes short)}$

$$\tau = \frac{L}{R} = \frac{5}{10} = 0.5 \text{ sec}$$

$$i_L(t) = 10 + (0 - 10) e^{-t/\tau}$$

$$= 10 (1 - e^{-t/0.5}) A \text{ for } 0 \le t \le \infty$$

$$V_L(t) = L \frac{d}{dt} i_L(t) = 100 e^{-2t} V \text{ for } 0 \le t \le \infty$$

$$E_L \mid_{t=5\tau \text{ or } t=\infty} = \frac{1}{2} Li^2 = \frac{1}{2} \times 5 \times 10^2 = 250 J$$

37. Ans: (b)

Sol: $V_{C1}(0^{-})$ $+ V_{C1}(0^{-})$ $V_{C1}(0^{-})$ $V_{C1}(0^{-})$ $V_{C1}(0^{-})$ $V_{C2}(0^{-})$

At $t = 0^-$: Steady state: A resistive circuit By KVL \Rightarrow



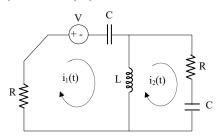


$$V - V_{c1}(0) = 0$$

$$V_{C1}(0-) = V = V_{C1}(0^+)$$

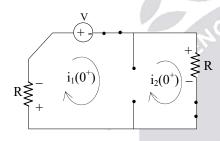
$$V_{C2}(0^-) = 0V = V_{C2}(0^+)$$

$$i_L(0^-) = 0A = i_L(0^+)$$



For $t \ge 0$

Fig (a)



At $t = 0^+$: A resistive circuit: Network is in transient state.

$$i_1(0^+) = i_2(0^+)$$

By KVL \Rightarrow

$$-Ri_1(0^+)-V-Ri_1(0^+)=0$$

$$i_1(0^+) = \frac{-V}{2R} = i_2(0^+)$$

OBS:
$$i_1(t) = i_1(t) \sim i_2(t)$$

At
$$t = 0^+ \Rightarrow$$

$$i_L(0+) = i_1(0+) \sim i_2(0+)$$

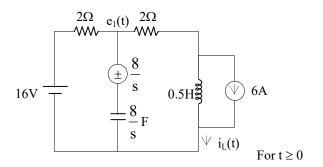
 $= 0A \Rightarrow$ Inductor: open circuit



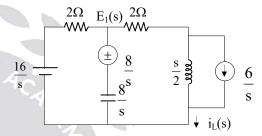
Sol: Evaluation of $i_L(t)$ and $e_1(t)$ for $t \ge 0$ by Laplace transform approach.

$$i_L(0^+) = 6A; i_L(\infty) = 4A$$

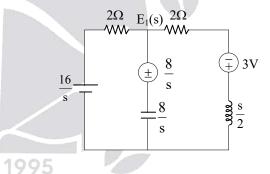
$$e_1(0^+) = 8V; e_1(\infty) = 8V$$



Transform the above network into Laplace domain.



S-domain:



Nodal in S-domain

Since

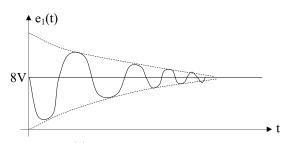
$$\frac{E_1(s) - 16/s}{2} + \frac{E_1(s) - \frac{8}{s}}{\frac{8}{s}} + \frac{E_1(s) + 3}{2 + \frac{s}{2}} = 0$$

$$E_1(s) = \frac{8}{s} \left(\frac{s^2 + 6s + 32}{s^2 + 8s + 32} \right)$$

$$E_1(s) = \frac{8}{s} \left(1 - \frac{2s}{(s+4)^2 + 4^2} \right)$$

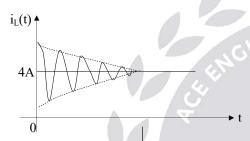
$$e_1(t) = 8 - 4e^{-4t} \sin 4t \text{ V for } t \ge 0$$





$$I_L(s) = \frac{E_1(s) + 3}{2 + \frac{s}{2}}$$

 $i_L(t) = 4+2e^{-4t} \cos 4t A$ for $t \ge 0$ $\omega_n = 4$ rad/sec



OBS:
$$\tau = \frac{1}{4} \sec = \frac{1}{\xi \omega_n} |_{\omega_n} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{2} \times \frac{1}{8}}} = 4$$

$$\frac{1}{4} \times \omega_{n} = \frac{1}{\xi}$$

$$\xi = \frac{4}{\omega} = \frac{4}{4} = 1$$

 $\xi = 1$ (A critically damped system)

39.

Sol:
$$\omega t \Big|_{t=t_0} = \tan^{-1} \left(\frac{\omega L}{R} \right)$$

 $\omega t_o = \tan^{-1} \left(\frac{\omega L}{R} \right)$
 $2\pi (50) t_o = \tan^{-1} \left(\frac{2\pi (50)(0.01)}{5} \right)$
 $t_o = 32.14 \times \frac{\pi}{180^\circ}$
 $t_o = 1.78$ msec.

So, by switching exactly at 1.78msec from the instant voltage becomes zero, the current is free from Transient.

40.

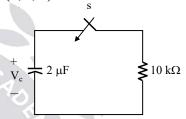
Sol:
$$\omega t_o + \phi = \tan^{-1}(\omega CR) + \frac{\pi}{2}$$

 $2t_o + \frac{\pi}{4} = \tan^{-1}(\omega CR) + \frac{\pi}{2}$
 $2t_o + \frac{\pi}{4} = \tan^{-1}\left(2\left(\frac{1}{2}\right)(1)\right) + \frac{\pi}{2} = \frac{\pi}{4} + \frac{\pi}{2}$
 $2t_o = \frac{\pi}{2} \Rightarrow t_o = 0.785 \sec$

41. Ans: (b, c, d)

Sol:

Since



(b)
$$V_c(t) = V_c(0)e^{-t/\tau}$$

$$V_c = 20e^{-t/\tau} = 20.e^{-t/(1/50)}$$

$$= 20e^{-50t} V$$

$$V_c(t)$$

(c)
$$i_C(t) = C \frac{dV_C(t)}{dt}$$

= $2 \times 10^{-6} \times 20 e^{-50t} \times (-50)$
 $i_C(t) = -2e^{-50t} mA$

(d)
$$\tau = RC = (10k) (2\mu)$$

= 20 ms
= $\frac{20}{1000} = 1/50 \text{ sec}$

42. Ans: (a, c)

Sol: At
$$t = 0^+$$
; $i_1(0) = 2$ A $\neq i_2(0) = 1$ A So, 2 A $\neq 1$ A

The given network violates KCL at $t = 0^+$ Constant current applied to inductor the voltage across inductor is impulse.

Chapter 4

AC Circuit Analysis

01.

Sol:
$$I_{avg} = I_{dc} = \frac{1}{T} \int_{0}^{T} i(t) dt$$

 $= 3 + 0 + 0 = 3A$
 $I_{rms} = \sqrt{\frac{1}{T} \int_{0}^{T} i^{2}(t) dt}$
 $= \sqrt{3^{2} + \left(\frac{4\sqrt{2}}{\sqrt{2}}\right)^{2} + \left(\frac{5\sqrt{2}}{\sqrt{2}}\right)^{2} + 0 + 0 + 0}$
 $= 5\sqrt{2}A$

02.

Sol:
$$V_{dc} = V_{avg} = \frac{1}{T} \int_0^T V(t) dt = 2V$$

Here the frequencies are same, by doing simplification

$$v(t) = 2 - 3\sqrt{2} \left(\cos 10t \times \frac{1}{\sqrt{2}} - \sin 10t \times \frac{1}{\sqrt{2}}\right) + 3\cos 10t$$

$$= 2 + 3\sin 10t \text{ V}$$

So
$$V_{rms} = \sqrt{(2)^2 + (\frac{3}{\sqrt{2}})^2} = \sqrt{8.5} \text{ V}$$

03.

Sol:
$$X_{avg} = X_{dc} = \frac{1}{T} \int_0^T x(t) dt = 0$$

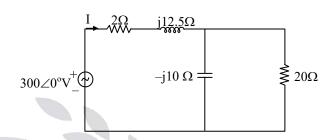
 $X_{rms} = \sqrt{\frac{1}{T} \int_0^T x^2(t) dt} = \frac{A}{\sqrt{3}}$

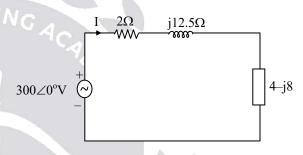
04. Ans: (a)

Sol: For a symmetrical wave (i.e., area of positive half cycle = area of negative half cycle.) The RMS value of full cycle is same as the RMS value of half cycle.

05.

Sol: Complex power, $S = VI^*$





$$\Rightarrow I = \frac{300 \angle 0^{\circ}}{2 + j12.5 + 4 - j8}$$

$$\Rightarrow$$
 I = 40 \angle -36.86°

∴ Complex power,
$$S = VI^*$$

$$= 300 \angle 0^{\circ} \times 40 \angle 36.86^{\circ}$$

= $9600 + i7200$

:. Reactive power delivered by the source

$$Q = 72000 \text{ VAR}$$
$$= 7.2 \text{ KVAR}$$

06.

Sol:
$$Z = j1 + (1-j1)||(1+j2) = 1.4 + j 0.8$$

$$I = \frac{E_1}{Z}\Big|_{By \text{ ohm's law}} = \frac{10\angle 20}{1.4 + j8}$$

$$= 6.2017\angle -9.744^{\circ} \text{ A}$$

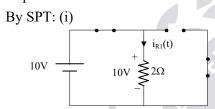
$$I_1 = \frac{I(1+j2)}{1-j1+1+j2}$$

$$= 6.2017\angle 27.125^{\circ} \text{ A}$$



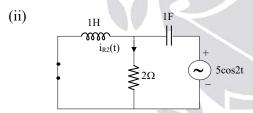
$$\begin{split} I_2 &= \frac{I(1-j1)}{1-j1+1+j2} \\ &= 3.922 \angle -81.31^{\circ} \, A \\ E_2 &= (1-j1)I_1 = 8.7705 \angle -17.875^{\circ} \, V \\ E_0 &= 0.5I_2 = 1.961 \angle -81.31^{\circ} \, V \end{split}$$

Sol: Since two different frequencies are operating on the network simultaneously always the super position theorem is used to evaluate the response.



Network is in steady state, therefore the network

is resistive.
$$I_{R1}(t) = \frac{10}{2} = 5A$$



Network is in steady state

As impedances of L and C are present because of $\omega = 2$. They are physically present.

$$Z_L = j\omega L; \ Z_c = \frac{1}{j\omega C}\Big|_{\omega=2}$$

$$\downarrow j2\Omega \qquad V \qquad \downarrow i_{R2}(t) \qquad \downarrow j2\Omega \qquad V \qquad \downarrow j_{R2}(t) \qquad \downarrow j_{$$

Network is in phasor domain

Nodal ⇒

$$\frac{V}{j2} + \frac{V}{2} + \frac{V - 5 \angle 0^0}{-j0.5} = 0$$

$$V = 6.32 \angle 18.44^{0}$$

$$I_{R2} = \frac{V}{2} = 3.16 \angle 18.44^{0} = 3.16 e^{j18.14^{0}}$$

$$i_{R2}(t) = R.P[I_{R2}e^{j2t}]A$$

$$= 3.16\cos(2t + 18.44^{0})$$

By super position theorem,

$$i_R(t) = i_{R1}(t) + i_{R2}(t)$$

= 5 + 3.16cos (2t + 18.44°)A

Sol:
$$\frac{1}{s^2 + 1} - I(s) \left(2 + 2s + \frac{1}{s} \right) = 0$$
$$I(s) \left(\frac{2s + 2s^2 + 1}{s} \right) = \frac{1}{s^2 + 1}$$
$$I(s) + 2s^2 I(s) + 2sI(s) = \frac{s}{s^2 + 1}$$
$$i(t) + \frac{2d^2 i}{dt^2} + 2\frac{di}{dt} = \cos t$$

$$2\frac{d^2i}{dt^2} + 2\frac{di}{dt} + i(t) = \cos t$$

09.

Sol:
$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

 $V = V_R = I.R$
 $100 = I.20$; $I = 5A$
Power factor $= \cos\phi = \frac{V_R}{V} = \frac{V_R}{V_R} = 1$

So, unity power factor.

10.

Sol: By KCL in phasor - domain
$$\Rightarrow -I_1 -I_2 -I_3 = 0$$

$$I_3 = -(I_1 + I_2)$$

$$i_1(t) = \cos(\omega t + 90^0)$$

$$I_1 = 1 \angle 90^0 = i1$$





$$I_{2} = 1 \angle 0^{0} = (1 + j0)$$

$$I_{3} = \sqrt{2} \angle \pi + 45^{0} = \sqrt{2} e^{j(\pi + 45)}$$

$$i_{3}(t) = \text{Real part}[I_{3}.e^{j\omega t}]\text{mA}$$

$$= -\sqrt{2} \cos(\omega t + 45^{0} + \pi)\text{mA}$$

$$i_{3}(t) = -\sqrt{2} \cos(\omega t + 45^{0})\text{mA}$$

Sol:
$$I = \frac{V}{R} + \frac{V}{Z_L} + \frac{V}{Z_C} = 8 - j12 + j18$$

 $I = 8 + 6j$
 $|I| = \sqrt{100} = 10A$

12.

Sol: By KCL
$$\Rightarrow$$

$$-I + I_L + I_C = 0$$

$$I = I_L + I_C$$

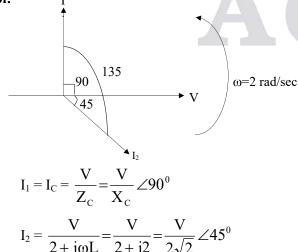
$$I_L = \frac{V}{Z_L} = \frac{V}{j\omega L} = \frac{3\angle 0^{\circ}}{j(3) \cdot \left(\frac{1}{3}\right)}$$

$$I_L = \frac{3\angle 0^{\circ}}{j} = \frac{3\angle 0^{\circ}}{\angle 90^{\circ}} = 3\angle -90^{\circ}$$

$$I = 3\angle -90^{\circ} + 4\angle 90^{\circ} = -j3 + j4 = j1 = 1\angle 90^{\circ}$$

13. Ans: (d)

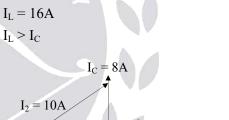
Sol:



Therefore, the phasor I_1 leads I_2 by an angle of 135°.

14.

$$\begin{aligned} &\textbf{Sol:} \quad I_2 = \sqrt{I_R^2 + I_C^2} & \implies 10 = \sqrt{I_R^2 + 8^2} \\ &I_R = 6A \\ &I_1 = I = \sqrt{I_R^2 + \left(I_L - I_C\right)^2} \\ &10 = \sqrt{6^2 + \left(I_L - I_C\right)^2} \\ &I_L - I_C = \pm 8A \\ &I_L - 8 = \pm 8 \\ &I_L - 8 = -8 (\text{Not acceptable}) \\ &\text{Since } I_L = \frac{V}{Z_L} \neq 0. \\ &I_L - 8 = 8 \end{aligned}$$



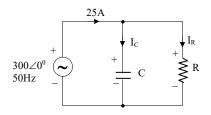
$$I_2$$
 leads $120 \angle 0^0$ by $tan^{-1} \! \left(\frac{8}{6} \right)$

$$I_1 \text{ lags } 120 \angle 0^0 \text{ by } \tan^{-1} \left(\frac{8}{6}\right)$$

Power factor
$$\cos\phi = \frac{I_R}{I} = \frac{I_R}{I}$$
$$= \frac{6}{10} = 0.6 \text{ (lag)}$$



15. Sol:



Network is in steady state.

$$|I_{\text{C}}| = \left| \frac{V}{Z_{\text{C}}} \right| = \left| \frac{300 \angle 0^{0}}{\left(1/j\omega c \right)} \right| = v\omega c$$

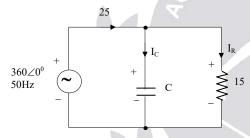
$$=300 \times 2\pi \times 50 \times 159.23 \times 10^{-6}$$

$$I_C = 15A$$

$$I = \sqrt{I_R^2 + I_C^2}$$

$$25 = \sqrt{I_R^2 + 15^2}$$

$$I_R = 20A$$



 $V_R = RI_R|By \text{ ohm's law}$

$$300 = R.20$$

$$R = 15\Omega$$

Network is in steady state

$$I_R = \frac{360}{15} = 24A$$

So the required $I_C = \sqrt{25^2 - 24^2}$

$$v\omega c = 7$$

$$360 \times 2\pi \times f \times 159.23 \times 10^{-6} = 7$$

$$f = 19.4Hz$$

OBS:
$$I_C = \frac{V}{Z_C}$$

$$Z_{\rm C} = \frac{1}{\mathrm{j}\omega c}\Omega$$

As
$$f \downarrow \Rightarrow Z_C \uparrow \Rightarrow I_C \downarrow$$

16.

Sol:
$$P_{5\Omega} = 10$$
Watts (Given)
= $P_{avg} = I_{rms}^2 R$

$$10 = I_{rms}^{2}.5$$

$$I_{rms} = \sqrt{2} A$$

Power delivered = Power observed

(By Tellegen's Theorem)

$$P_T = I_{rms}^2 (5 + 10)$$

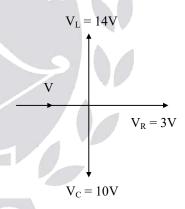
$$V_{\rm rms} I_{\rm rms} \cos \phi = \left(\sqrt{2}\right)^2 (15)$$

$$\frac{50}{\sqrt{2}} \times \sqrt{2} \cos \phi = 2 \times 15$$

$$\cos\phi = 0.6 \text{ (lag)}$$

17. Ans: (d)

Sol:



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$= \sqrt{(3)^2 + (14 - 10)^2}$$
$$V = 5 V$$

18.

1995

Since

Sol:
$$Y = Y_1 + Y_c = \frac{1}{Z_L} + \frac{1}{Z_C}$$
$$= \frac{1}{30 \angle 40^0} + \frac{1}{\left(\frac{1}{\text{i}\omega c}\right)}$$





$$= j\omega c + \frac{1}{30} \angle -40^{0}$$
$$= j\omega c + \frac{1}{30} (\cos 40^{0} - j\sin 40^{0})$$

Unit power factor \Rightarrow j-term = 0

$$\omega c = \frac{\sin 40^{0}}{30}$$

$$C = \frac{\sin 40^{0}}{2\pi \times 50 \times 30} = 68.1 \mu F$$

$$C = 68.1 \mu F$$

19. Ans: (b)

Sol: To increase power factor shunt capacitor is to be placed.

VAR supplied by capacitor

= P
$$(\tan\phi_1 - \tan\phi_2)$$

= $2 \times 10^3 [\tan(\cos^{-1} 0.65) - \tan(\cos^{-1} 0.95)]$
= 1680 VAR

VAR supplied =
$$\frac{V^2}{X_C} = V^2 \omega C = 1680$$

$$\therefore C = \frac{1680}{(115)^2 \times 2\pi \times 60} = 337 \,\mu\text{F}$$

20.

Sol:
$$Z = \frac{V}{I} = \frac{160 \angle 10^{\circ} - 90^{\circ}}{5 \angle -20^{\circ} - 90^{\circ}} = 32 \angle 30^{\circ}$$

$$\phi = 30^{\circ}$$
 (Inductive)

$$V_{rms} = \frac{160}{\sqrt{2}} Vj, I_{rms} = \frac{5}{\sqrt{2}}$$

Real power (P) =
$$\frac{160}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times \cos 30^{\circ}$$

= $200\sqrt{3}$ W

Reactive power (Q) =
$$\frac{160}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times \frac{1}{2}$$

= 200 VAR

Complex power = P + jQ =
$$200(\sqrt{3} + j1)$$
 VA

Sol:
$$V = 4 \angle 10^{\circ}$$
 and $I = 2 \angle -20^{\circ}$

Note: When directly phasors are given the magnitudes are taken as rms values since they are measured using rms meters.

$$V_{rms}$$
 = 4V and I_{rms} = 2A
 $Z = \frac{V}{I} = 2 \angle 30^{\circ}$; $\phi = 30^{\circ}$ (Inductive)
 $P = 10 \sqrt{3}$ W, $Q = 10 VAR$

22. Ans: (a)

Sol:
$$S = VI^*$$

= $(10 \angle 15^\circ) (2 \angle 45^\circ)$
= $10 + j17.32$
 $S = P + jQ$
 $P = 10 \text{ W} O = 17.32 \text{ VAR}$

 $S = 10(\sqrt{3} + j1) VA$

Sol:
$$P_R = (I_{rms})^2 \times R$$

$$I_{rms} = \frac{10}{\sqrt{2}}$$

$$P_R = \left(\frac{10}{\sqrt{2}}\right)^2 \times 100$$

24

Since

Sol:
$$P_{avg} = \frac{V_{rms}^2}{R} = \frac{\left(\frac{240}{\sqrt{2}}\right)^2}{60} = 480 \text{ Watts}$$

$$V = 240 \angle 0^0$$

$$I_R = \frac{V}{R} = \frac{240}{60} = 4A$$

$$I_L = \frac{V}{Z_L} = \frac{V}{X_L} = \frac{240}{40} = 6A$$

$$I_C = \frac{V}{Z_C} = \frac{V}{X_C} = \frac{240}{80} = 3A$$





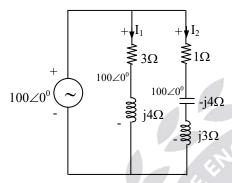
 $I_L > I_C$: Inductive nature of the circuit.

$$I = \sqrt{I_R^2 + (I_L - I_C)^2} = \sqrt{4^2 + 3^2} = 5A$$

Power factor = $\frac{I_R}{I} = \frac{4}{5} = 0.8$ (lagging)

25. Ans: (a)

Sol:



NW is in Steady state.

$$V = 100 \angle 0^0 \Rightarrow V_{rms} = 100V$$

$$I_1 = \frac{100\angle 0^0}{(3+i4)\Omega} \implies |I_1| = 20 = I_{1rms}$$

$$I_2 = \frac{100\angle 0^0}{(1-i1)\Omega} \implies |I_2| = \frac{100}{\sqrt{2}} A = I_{2rms}$$

$$P = P_1 + P_2$$

$$= (I_{1rms})^2 . 3 + (I_{2rms})^2 . 1$$

$$= 20^2 . 3 + \left(\frac{100}{\sqrt{2}}\right)^2 . 1$$

$$P = 6200 \text{ W}$$

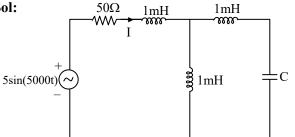
$$Q = Q_1 + Q_2$$

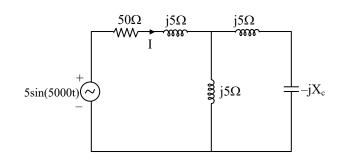
= $(I_{1rms})^2 \cdot 4 + (I_{2rms})^2 \cdot (1)$
= 3400VAR

So,
$$S = P + jQ = (6200 + j3400) \text{ VA}$$

26.

Sol:





when I = 0,

⇒ impedance seen by the source should be infinite

$$\Rightarrow$$
 Z = ∞

$$\therefore Z = (50+j5) + (j5) \parallel j(5-X_c)$$

$$= 50 + j5 + \frac{j5 \times j(5-X_c)}{j5 + j(5-X_c)} = \infty$$

$$\Rightarrow j (10-X_c) = 0$$

$$\Rightarrow X_c = 10 \Rightarrow \frac{1}{\omega c} = 10$$

$$\Rightarrow C = \frac{1}{5000 \times 10} = 20 \ \mu\text{F}$$

27. Ans: (c)

Sol:
$$I_{\text{rms}} = \sqrt{3^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2}$$

= $\sqrt{25} = 5 \,\text{A}$

Power dissipation =
$$I_{rms}^2 R$$

= $5^2 \times 10$
= 250 W

28.

Sol:
$$X_C = X_L$$

 $\Rightarrow \omega = \omega_0$, the circuit is at resonance $V_C = QV_S \angle -90^0$
 $Q = \frac{\omega_0 L}{R} = \frac{X_L}{R} = 2$





$$= \frac{1}{\omega_0 cR} = \frac{X_C}{R} = 2$$

$$\Rightarrow V_C = 200 \angle -90^0$$

$$= -j200V$$

Sol: Series RLC circuit

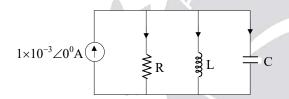
$$f = f_L PF = \cos \phi = 0.707 (lead)$$

$$f = f_H \cdot PF = \cos \phi = 0.707(lag)$$

$$f = f_0$$
, $PF = \cos \phi = 1$

30. Ans: (b)

Sol: Network is in steady state (since no switch is given)



Let
$$I = 1mA$$

$$\omega = \omega_0(Given)$$

$$\Rightarrow I_R = I$$

$$I_{L} = OI \angle -90^{0} = -iOI$$

$$I_C = QI \angle 90^0 = iQI$$

$$I_L + I_C = 0$$

$$|I_R + I_L| = |I - jQI|$$

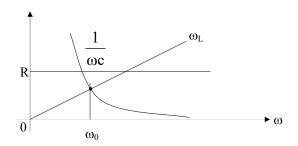
$$= I\sqrt{1 + Q^2} > I$$

$$|I_R + I_C| = |I + jQI|$$

$$= I\sqrt{1 + Q^2} > I$$

31. Ans: (c)

Sol: Since; "I" leads voltage, therefore capacitive effect and hence the operating frequency $(f < f_0)$



32.

Sol:
$$Y = \frac{1}{R_L + j\omega L} + \frac{1}{R_C - \frac{j}{\omega C}}$$

$$= \frac{R_{L} - j\omega L}{R_{L}^{2} + (\omega L)^{2}} + \frac{R_{C} + j/\omega c}{R_{C}^{2} + (1/\omega C)^{2}}$$

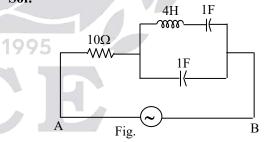
$$j - term \Rightarrow 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}} \text{ rad/sec}$$

33.

Sol:

Since



The given circuit is shown in Fig.

$$Z_{AB} = 10 + Z_1$$

where,
$$Z_1 = \left(\frac{-j}{\omega}\right) \| \left(j4\omega - \frac{j}{\omega}\right) \|$$

$$= \frac{\left(\frac{-j}{\omega}\right) \left(j4\omega - \frac{j}{\omega}\right)}{\frac{-j}{\omega} + j4\omega - \frac{j}{\omega}}$$





$$=\frac{4-\frac{1}{\omega^2}}{\mathrm{j}4\omega-\frac{\mathrm{j}2}{\omega}}$$

For circuit to be resonant i.e., $\omega^2 = \frac{1}{4}$

$$\omega = \frac{1}{2} = 0.5 \text{ rad/sec}$$

 $\therefore \omega_{\text{resonance}} = 0.5 \text{ rad/sec}$

34.

Sol: (i) $\frac{L}{C} = R^2$ \Rightarrow circuit will resonate for all the

frequencies, out of infinite number of frequencies we are selecting one frequency.

i.e.,
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{2} \text{ rad/sec}$$

then $Z = R = 2\Omega$.

$$I = \frac{V}{Z} = \frac{10\angle 0^0}{2} = 5\angle 0^0$$

$$i(t) = 5\cos\frac{t}{2}A$$

$$Z_L = j\omega_0 L = j2\Omega$$
; $Z_C = \frac{1}{j\omega_0 c} = -j2\Omega$.

$$I_L = \frac{I(2-j2)}{2+j2+2-j2} = \frac{I}{\sqrt{2}} \angle -45^0$$
 Since 1995

$$i_L = \frac{5}{\sqrt{2}} \cos \left(\frac{t}{2} - 45^0\right) A$$

$$i_c = \frac{I(2+j2)}{2+i2+2-i2} = \frac{I}{\sqrt{2}} \angle 45^0$$

$$i_{c} = \frac{5}{\sqrt{2}} \cos \left(\frac{t}{2} + 45^{\circ}\right) A$$

$$P_{avg} = I_{L(rms)}^{2}.R + I_{c(rms)}^{2}.R$$

$$= \left(\frac{5/\sqrt{2}}{\sqrt{2}}\right)^2 \cdot 2 + \left(\frac{5/\sqrt{2}}{\sqrt{2}}\right)^2 \cdot 2$$

= 25 watts

(ii) $\frac{L}{C} \neq R^2$ circuit will resonate at only one frequency.

i.e., at
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{4} \text{ rad/sec}$$

Then
$$Y = \frac{2R}{R^2 + \frac{L}{C}}$$
 mho

$$Y = \frac{2(2)}{2^2 + \frac{4}{4}} = \frac{4}{5} \text{ mho}$$

$$Z = \frac{5}{4}\Omega$$

$$I = \frac{V}{Z} = \frac{10 \angle 0^0}{\frac{5}{4}} = 8 \angle 0^0$$

$$i(t) = 8\cos\frac{t}{4}A$$

$$Z_L = j\omega_0 L = j1\Omega$$

$$Z_{c} = \frac{1}{j\omega_{0}C} = -j1\Omega$$

$$I_L = \frac{I(2-j1)}{2+j1+2-j1} = \frac{\sqrt{5}}{4} \text{ I.} \angle \tan^{-1} \left(\frac{1}{2}\right)$$

$$i_{L} = \frac{8\sqrt{5}}{4} \cos\left(\frac{t}{4} - \tan^{-1}\left(\frac{1}{2}\right)\right)$$

$$I_c = \frac{I(2+jl)}{2+il+2-il} = \frac{\sqrt{5}}{4} I \angle \tan^{-1} \left(\frac{1}{2}\right)$$

$$i_c = \frac{8\sqrt{5}}{4} \cos\left(\frac{t}{4} + \tan^{-1}\left(\frac{1}{2}\right)\right)$$

$$\begin{aligned} P_{avg} &= I_{Lrms}^2.R + I_{Crms}^2R \\ &= \left(\frac{2\sqrt{5}}{\sqrt{2}}\right)^2.2 + \left(\frac{2\sqrt{5}}{\sqrt{2}}\right)^2.2 \\ &= 40 \text{ watts} \end{aligned}$$





Sol: (i)
$$Z_{ab} = 2 + (Z_L \parallel Z_C \parallel 2)$$

$$= 2 + jX_L \parallel - jX_C \parallel 2$$

$$= \frac{2 + 2X_L X_C (X_L X_C - j2(X_L - X_C))}{(X_L X_C)^2 + 4(X_L - X_C)^2}$$

$$j$$
-term = 0
 $\Rightarrow -2(X_L - X_C) = 0$

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4.4}} = \frac{1}{4} \, \text{rad/sec}$$

At resonance entire current flows through 2Ω only.

(ii)
$$Z_{ab}\big|_{\omega=\omega_0}=2+2=4\Omega$$

$$X_L=X_C$$

(iii)
$$V_i(t) = V_m \sin\left(\frac{t}{4}\right)V$$

$$Z = 4\Omega$$

$$i(t) = \frac{V_i(t)}{Z} = \frac{V_m}{4} \sin\left(\frac{t}{4}\right) = \dot{i}_R$$

$$V = 2i_{R} = \frac{V_{m}}{2} \sin\left(\frac{t}{4}\right) V = V_{C} = V_{L}$$

$$i_C = C \frac{dV_C}{dt} = \frac{V_m}{2} \cos\left(\frac{t}{4}\right)$$

$$i_c = \frac{V_m}{2} \sin \left(\frac{t}{4} + 90^0\right) A$$

$$i_{L} = \frac{1}{L} \int V_{L}.dt = \frac{-V_{m}}{2} \cos\left(\frac{t}{4}\right)$$

$$i_{L} = \frac{V_{m}}{2} \sin \left(\frac{t}{4} - 90^{0}\right) A$$

OBS: Here $i_L + i_C = 0$

⇒ LC Combination is like an open circuit.

36. Ans: (d)

Sol:

$$Q = \frac{\omega L}{R}$$

$$V \xrightarrow{R}$$

$$Q = \frac{2\omega L}{R} = 2 \times \text{orginal} \rightarrow Q - \text{doubled}$$

$$S = V.I = V.\frac{V}{R + i\omega L} \times \frac{R - j\omega L}{R - i\omega L}$$

$$S = \frac{V^2}{R^2 + (\omega L)^2} - \frac{V^2 \cdot j\omega L}{R^2 + (\omega L)^2}$$

$$S = P + iQ$$

Active power (P) =
$$\frac{V^2}{R^2 + (\omega L)^2}$$

$$P = \frac{V^2}{R^2(1+Q^2)}$$

$$P \approx \frac{V^2}{R^2 Q^2}$$

As Q is doubled, P decreases by four times.

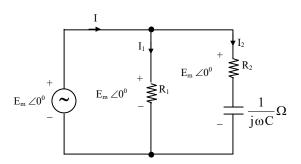
37.

Sol:
$$Z_C = \frac{1}{i\omega C}$$

$$\omega = 0; Z_C = \infty \implies C : \text{open circuit} \implies i_2 = 0$$

$$\omega = \infty; Z_C = 0 \Rightarrow C : \text{Short Circuit} \Rightarrow i_2 = \frac{E_m}{R_2} \angle 0^\circ$$

Transform the given network into phasor domain.





Network is in phasor domain.

By KCL in P-d \Rightarrow I = I₁ + I₂

$$I_1 = \frac{E_m \angle 0^o}{R_1}$$

$$I_{2} = \frac{E_{m} \angle 0^{\circ}}{R_{2} + \frac{1}{j\omega C}} = \frac{E_{m} \angle 0^{\circ}}{R_{2} - \frac{j}{\omega C}}$$

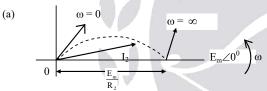
$$I_{2} = \frac{E_{m} \angle \tan^{-1} \left(\frac{1}{\omega CR_{2}}\right)}{\sqrt{R^{2} + \left(\frac{1}{\omega C}\right)}}$$

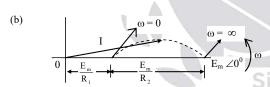
$$\omega = \infty \Rightarrow I_2 = \frac{E_m \angle 0^o}{R_2}$$

$$\omega = 0 \Longrightarrow I_2 = 0A$$

 $\omega:(0 \text{ and } \infty)$ j the current phasor I_2 will always

lead the voltage $E_m \angle 0^o$.

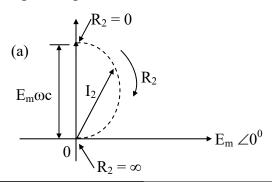


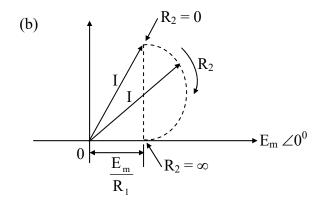


38.

Sol:
$$R_2 = 0 \Rightarrow I_2 = \frac{E_m \angle 0^\circ}{0 + \frac{1}{j\omega C}} = E_m \omega C \angle 90^\circ$$

$$R_2 = \infty \Longrightarrow I_2 = 0 A$$

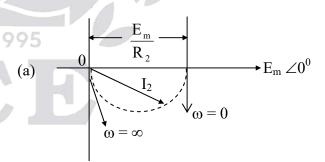


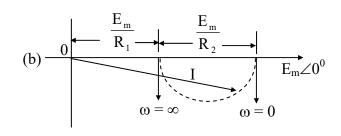


39.

Sol:
$$I = I_1 + I_2$$
; $I_1 = \frac{E_m \angle 0^o}{R_1}$
 $I_2 = \frac{E_m \angle 0^o}{R_2 + j\omega L}$
 $= \frac{E_m}{\sqrt{R_2^2 + (WL)^2}} \angle - tan^{-1} \left(\frac{\omega L}{R_2}\right)$

(i) If "ω" Varied

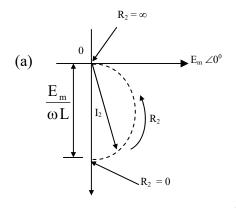


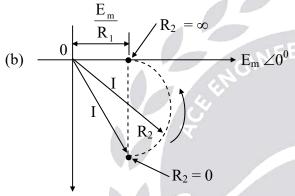






ii. If "R2" is varied





40. Ans: (a)

Sol: The given circuit is a bridge.

 $I_R=0$ is the bridge is balanced. i.e., $Z_1Z_4=R_2\ R_3$ Where $Z_1=R_1+j\omega L_1,$

$$Z_4 = R_4 - \frac{j}{\omega C_4}$$

As R_2 R_3 is real, imaginary part of Z_1 $Z_4 = 0$

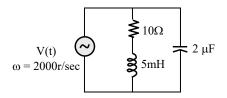
$$\omega L_1 R_4 - \frac{R_1}{\omega C_4} = 0$$
 or $\frac{\omega L_1}{R_1} = \frac{1}{\omega C_4 R_4}$

or
$$Q_1 = Q_4$$

Where Q is the Quality factor.

41. Ans: (b, c)

Sol:



$$\begin{split} Y_T &= \frac{1}{10 + j\omega(5m)} + j\omega(2\mu) \\ &= \frac{10 - j\omega(5m)}{100 + \omega^2(25\mu)} + j\omega(2\mu) \\ &= \frac{j\omega(5m)}{100 + \omega^2(25\mu)} + j\omega(2\mu) \end{split}$$

$$2500 = 100 + \omega^2 (25\mu)$$

$$2400 = \omega^2 (25) \mu$$

$$24 \times 4 \text{ M} = \omega^2$$

 $\omega = 9.8 \text{ rad/sec}$

(Q) pf of coil =
$$\frac{R}{Z}$$
 = $\frac{10}{\sqrt{10^2 + (5m \times 2000)^2}}$
= $\frac{1}{\sqrt{2}}$ = 0.707 lag
(R) Q-factor = $\frac{\omega L}{R}$ = $\frac{(2000)(5m)}{10}$ = 1

∴ (b, c) are correct

42. Ans: (a, d)

Since

Sol:
$$R = 30 \Omega$$
, $X_L = 60 \Omega$, $X_C = 20 \Omega$

 $V(t) = 100\sin 10\omega t$

(a)
$$\phi = \tan^{-1} \left(\frac{X_{L} - X_{C}}{R} \right)$$

= $\tan^{-1} \left(\frac{40}{30} \right) = 53.13^{\circ} \log 10^{\circ}$

(b) p.f =
$$\cos 53.13^{\circ} = 0.6 \log$$

(c) current is lagging by 53.13°

(d) p.f =
$$\cos 53.13^{\circ} = 0.6 \log$$

∴ a, d are correct

Chapter 5

Magnetic Circuits

01.

Sol: $X_C = 12$ (Given)

 $X_{eq} = 12$ (must for series resonance)

So the dot in the second coil at point "Q"

$$L_{eq} = L_1 + L_2 - 2M$$

$$L_{eq} = L_1 + L_2 - 2K\sqrt{L_1L_2}$$

$$\omega L_{eq} = \omega L_1 + \omega L_2 - 2K \sqrt{L_1 L_2 \omega \omega}$$

$$12 = 8 + 8 - 2K\sqrt{8.8}$$

$$\Rightarrow$$
 K = 0.25

02.

Sol: $X_C = 14$ (Given)

 $X_{Leq} = 14$ (must for series resonance)

So the dot in the 2nd coil at "P"

$$L_{eq} = L_1 + L_2 + 2M$$

$$L_{eq} = L_1 + L_2 + K \sqrt{L_1 L_2}$$

$$\omega L_{eq} = \omega L_1 + \omega L_2 + 2K \sqrt{\omega L_1 L_2 \omega}$$

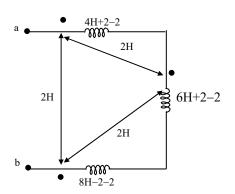
$$14 = 2 + 8 + 2K\sqrt{2(8)}$$

$$\Rightarrow$$
 K = 0.5

03.

Sol:
$$L_{ab} = 4H + 2 - 2 + 6H + 2 - 2 + 8H - 2 - 2$$

 $L_{ab} = 14H$



04. Ans: (c)

Sol: Impedance seen by the source

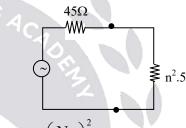
$$Z_{s} = \frac{Z_{L}}{16} + (4 - j2)$$

$$= \frac{10 \angle 30^{\circ}}{16} + (4 - j2)$$

$$= 4.54 - j1.69$$

05.

Sol:



$$Z_{\rm in} = \left(\frac{N_1}{N_2}\right)^2 . Z$$

$$R'_{in} = n^2.5$$

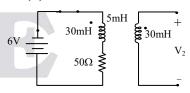
For maximum power transfer; $R_L = R_s$

$$n^2 5 = 45 \Rightarrow n = 3$$

06. Ans: (b)

Sol:

Since



Apply KVL at input loop

$$-6-30\times10^{3}\frac{di_{1}}{dt}+5\times10^{3}\frac{di_{2}}{dt}-50i_{1}=0...(1)$$

Take Laplace transform

$$-\frac{6}{s} + [-30 \times 10^{-3} (s) - 50] I_{1}(s) + 5 \times 10^{-3} s I_{2}(s) = 0 \dots (2)$$

Apply KVL at output loop

$$V_2(s) - 30 \times 10^{-3} \frac{di_2}{dt} + 5 \times 10^{-3} \frac{di_1}{dt} = 0$$



Take Laplace transform

$$V_2(s) - 30 \times 10^{-3} s I_2(s) + 5 \times 10^{-3} s I_1(s) = 0$$

Substitute $I_2(s) = 0$ in above equation

$$V_2 + 5 \times 10^{-3} \text{ sI}_1(\text{s}) = 0 \dots (3)$$

From equation (2)

$$-\frac{6}{s} + (-30 \times 10^{-3}(s) + 50)I_1(s) = 0$$

$$I_{1}(s) = \frac{-6}{s (30 \times 10^{-3} (s) + 50)}$$
(4)

Substitute eqn (4) in eqn (3)

$$V_2(s) = \frac{-5 \times 10^{-3} (s) (-6)}{s (30 \times 10^{-3} (s) + 50)}$$

Apply Initial value theorem

Lt s
$$\frac{-5 \times 10^{-3} (s)(-6)}{s (30 \times 10^{-3} (s) + 50)}$$

$$v_2(t) = \frac{-5 \times 10^{-3} \times (-6)}{30 \times 10^{-3}} = +1$$

07.

Sol:
$$R_{in}' = \frac{8}{2^2} = 2\Omega$$

$$R_{in} = 3 + R_{in}' = 3 + 2 = 5\Omega$$

$$I_1 = \frac{10\angle 20}{5} = 2\angle 20^0$$

$$\frac{I_1}{I_2} = n = 2 \implies I_2 = 1 \angle 20^{\circ} A$$

08.

Sol: By the definition of KVL in phasor domain

$$V_{S} - V_{0} - V_{2} = 0$$

$$V_0 = V_S - V_2 = V_S \left(1 - \frac{V_2}{V_S} \right)$$

$$V = ZI$$

$$V_S = j\omega L_1.I_1 + j\omega M (0)$$

$$V_2 = j\omega L_2(0) + j\omega MI_1$$

$$V_0 = V_s \left(1 - \frac{M}{L_1} \right)$$



Chapter 6

Two Port Networks

01.

Sol: The defining equations for open circuit impedance parameters are:

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$\begin{bmatrix} Z \end{bmatrix} = \begin{bmatrix} \frac{10}{s} & \frac{4s+10}{s} \\ \frac{10}{s} & \frac{3s+10}{s} \end{bmatrix} \Omega$$

02. Ans: (b)

Sol: The matrix given is $\begin{bmatrix} 0 & \frac{-1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$

since $y_{11} \neq y_{22}$

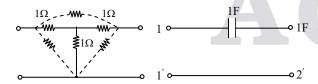
 \Rightarrow Asymmetrical, and

$$y_{12} \neq y_{21}$$

⇒ Non reciprocal network

03.

Sol: Convert Y to Δ :



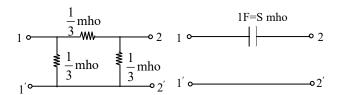


Fig: A

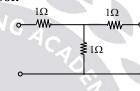
Fig: B

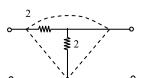
$$Y_{A} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \qquad Y_{B} = \begin{bmatrix} S & -S \\ -S & S \end{bmatrix}$$

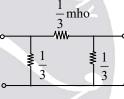
$$Y = \begin{bmatrix} S + \frac{2}{3} & -S - \frac{1}{3} \\ -S - \frac{1}{3} & S + \frac{2}{3} \end{bmatrix} \text{ mho}$$

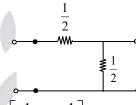
04.

Sol:









$$Y_{A} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \quad Y_{B} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

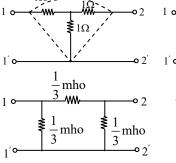
$$Y = \begin{bmatrix} \frac{7}{6} & -\frac{5}{6} \\ -\frac{5}{6} & \frac{5}{3} \end{bmatrix}$$

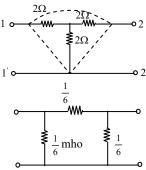
05.

Since

Sol: Convert Y to Δ :

Convert Y to Δ :







$$Y_{A} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \text{mho} \quad Y_{B} = \begin{bmatrix} \frac{2}{6} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{2}{6} \end{bmatrix} \text{mho}$$

$$Y = \begin{bmatrix} \frac{6}{6} & -\frac{3}{6} \\ -\frac{3}{6} & \frac{6}{6} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

Sol:
$$T_1 = T_2 = \begin{bmatrix} 1 + \frac{1}{-j1} & 1 \\ \frac{1}{-j1} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+j & 1 \\ j & 1 \end{bmatrix}$$

$$T_3 \Rightarrow Z_1 = 1\Omega; Z_2 = \infty$$

$$T_3 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$T = (T_1)(T_2)(T_3)$$

$$T = \begin{bmatrix} j3 & 2+j4 \\ -1+j2 & j3 \end{bmatrix}$$

07.

Sol:
$$T_1: Z = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

 $T_1 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$
 $T_2: Z_1 = 0$; $Z_2 = 2 \Omega$

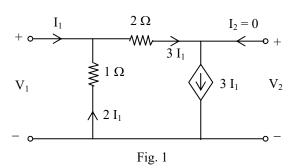
$$T_2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$$

$$T = [T_1] [T_2]$$

$$T = \begin{bmatrix} 3.5 & 3 \\ 2 & 2 \end{bmatrix}$$

08. Ans: (a)

Sol: For $I_2 = 0$ (O/P open), the Network is shown in Fig.1



$$V_1 = -2 I_1 \dots (1)$$

$$Z_{11} = \frac{V_1}{I_1} = -2$$

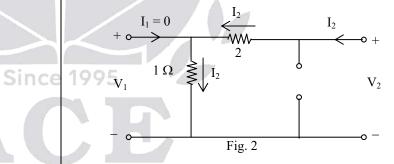
$$V_2 = -6 I_1 + V_1 \dots (2)$$

From (1) and (2)

$$V_2 = -6 I_1 - 2 I_1$$

or $V_2 = -8 I_1$
 $Z_{21} = \frac{V_2}{I_1} = -8$

For $I_1 = 0$ (I/P open), the network is shown in Fig.2



Note: that the dependent current source with current 3 I_1 is open circuited.

$$V_1 = 1 I_2$$
, $Z_{12} = \frac{V_1}{I_2} = 1$
 $V_2 = 3 I_2$, $Z_{22} = \frac{V_2}{I_2} = 3$

$$\begin{bmatrix} Z \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -8 & 3 \end{bmatrix}$$





Sol: By Nodal

$$-I_{1} + V_{1} - 3V_{2} + V_{1} + 2V_{1} - V_{2} = 0$$

$$-I_{2} + V_{2} + V_{2} - 2V_{1} = 0$$

$$Y = \begin{bmatrix} 4 & -4 \\ -3 & 2 \end{bmatrix} \nabla$$

$$[Z] = Y^{-1}$$

We can also obtain [g], [h], [T] and [T]⁻¹ by rewriting the equations.

10.

Sol: The defining equations for open-circuit impedance parameters are:

$$V_1\!\!=\!\!Z_{11}I_1\!\!+\!\!Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

In this case, the individual Z-parameter matrices get added.

$$(Z) = (Z_a) + (Z_b)$$

$$\begin{bmatrix} Z \end{bmatrix} = \begin{bmatrix} 10 & 2 \\ 2 & 7 \end{bmatrix} \Omega$$

11.

Sol: For this case the individual y-parameter matrices get added to give the y-parameter matrix of the overall network.

$$Y = Y_a + Y_b$$

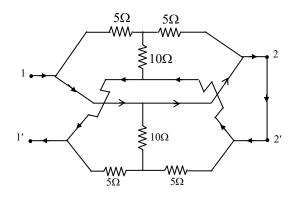
The individual y-parameters also get added

$$Y_{11} = Y_{11a} + Y_{11b}$$
 etc

$$[Y] = \begin{bmatrix} 1.4 & -0.4 \\ -0.4 & 1.4 \end{bmatrix}$$
mho

12. Ans: (c)

Sol:
$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$



$$\mathbf{Y}_{11} = \frac{\mathbf{I}_1}{\mathbf{0}} = \infty$$

13.

Sol: (i).
$$[T_a] = \begin{bmatrix} 1 + \frac{Z_1}{Z_2} & Z_1 \\ \frac{1}{Z_2} & 1 \end{bmatrix}$$

(ii).
$$[T_a] = \begin{bmatrix} 1 & Z_1 \\ \frac{1}{Z_2} & 1 + \frac{Z_1}{Z_2} \end{bmatrix}$$

 $\begin{bmatrix} T_a \end{bmatrix}$ and $\begin{bmatrix} T_b \end{bmatrix}$ are obtained by defining equations for transmission parameters.

14.

Sol: In this case, the individual T-matrices get multiplied

$$(T) = (T_1) \times (T_{N1})$$

$$(T) = (T_1)(T_{N1}) = \begin{pmatrix} 1+s/4 & s/2 \\ 1/2 & 1 \end{pmatrix} \begin{pmatrix} 8 & 4 \\ 2 & 5 \end{pmatrix}$$

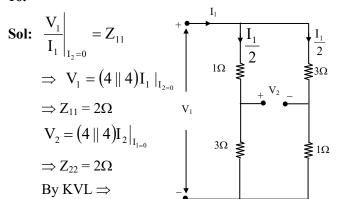
$$= \begin{pmatrix} 3s+8 & 3.5s+4 \\ 6 & 7 \end{pmatrix}$$

15.

Sol:
$$Z_{in} = R_{in} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2} = \frac{V_2 - 2I_2}{V_2 - 3I_2}$$
, $V_2 = 10(-I_2)$ $Z_{in} = R_{in} = \frac{12}{13}\Omega$







$$\frac{3I_1}{2} - V_2 - \frac{I_1}{2} = 0$$

$$V_2 = I_1$$

$$\Rightarrow Z_{21} = 1\Omega = Z_{12}$$

$$Z = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \Omega$$

 $Y = Z^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix} \mathbf{U}$

Now [T] parameters;

$$V_1 = 2I_1 + I_2 \dots (1)$$

$$V_2 = I_1 + 2I_2 \dots (2)$$

$$\Rightarrow I_1 = V_2 - 2I_2 \dots (3)$$

Substituting (3) in (1):

$$V_1 = 2(V_2 - 2I_2) + I_2 = 2V_2 - 3I_2 \dots (4)$$

$$T = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\mathbf{T}^1 = \mathbf{T}^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

Now h parameters

$$2I_2 = -I_1 + V_2$$

$$I_2 = \frac{-I_1}{I_2} + \frac{V_2}{2}$$
(5)

Substitute (5) in (1)

$$V_{_{1}}=2I_{_{1}}\frac{-\,I_{_{1}}}{2}+\frac{V_{_{2}}}{2}$$

$$V_1 = \frac{3}{2}I_1 + \frac{1}{2}V_2$$
(6)

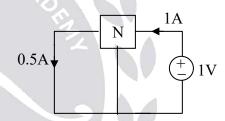
$$h = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{1}{2} \end{bmatrix}$$

$$g = [h]^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

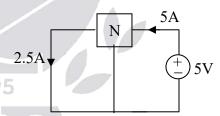
17. Ans: (a)

Sol:
$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

Just use reciprocity of fig (a)



Now use Homogeneity



So,
$$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} = \frac{5}{5} = 1 \text{ mho}$$

This has noting to do with fig (b) since fig (b) also valid for some specific resistance of 2 Ω at port-1, but Y_{22} , V_1 = 0. So S.C port-1

18.

Sol:
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n = \frac{-I_1}{I_2}$$

 $\frac{V_2}{V_1} = n$





$$\Rightarrow V_{1} = \frac{1}{n} V_{2} - (0) I_{2}$$

$$\Rightarrow T = \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

$$T^{1} = T^{-1} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$

$$T^{1} = T^{-1} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$

Now h-parameters

$$V_{1} = (0)I_{1} + \frac{1}{n}V_{2}$$

$$I_{2} = \frac{-I_{1}}{n} + (0)V_{2}$$

$$g = \begin{bmatrix} 0 & \frac{1}{n} \\ -\frac{1}{n} & 0 \end{bmatrix}$$

$$I_{2} = \begin{bmatrix} 0 & -\frac{1}{n} \\ -\frac{1}{n} & 0 \end{bmatrix}$$

Note: In an ideal transformer, it is impossible to express V_1 and V_2 in terms of I_2 and I_2 , hence the 'Z' parameters do not exist. Similarly, the y-parameters.

19. Ans: (c)
Sol:
$$Z_{22} = \frac{V_2}{I_2^1}\Big|_{V_1=0}$$

$$\frac{V_1}{V_2} = \frac{1}{n} = \frac{I_2}{I_1}$$

$$V_1 = \frac{1}{n}V_2$$

$$\frac{V_2 - V_1}{R} = I_1$$

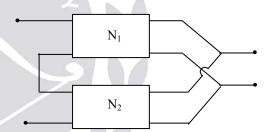
$$I_2^1 = I_2 + I_1$$

$$\frac{1}{n} = \frac{I_2}{I_1} = \frac{I_2^1 - I_1}{I_1} = \frac{I_2^1}{I_2} - 1$$

$$\begin{split} &\frac{I_{2}^{1}}{I_{1}} = \frac{1}{n} + 1 = \frac{1+n}{n} \\ &I_{2}^{1} = \left(\frac{1+n}{n}\right) I_{1} \\ &I_{2}^{1} = \left(\frac{1+n}{n}\right) \left(\frac{V_{2} - V_{1}}{R}\right) \\ &I_{2}^{1} = \left(\frac{1+n}{n}\right) \left(\frac{V_{2} - \frac{1}{n}V_{2}}{R}\right) \\ &\frac{I_{2}^{1}}{V_{2}} = \left(\frac{1+n}{n}\right) \left(\frac{n-1}{nR}\right) \\ &\frac{V_{2}}{I_{2}^{1}} = \frac{n^{2}R}{n^{2} - 1} \end{split}$$

20. Sol:

Since



For series parallel connection individual h-parameters can be added.

$$\therefore \text{ For network } 1, h_1 = g_1^{-1}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

For network 2, $h_2 = g_2^{-1}$

$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$
$$\therefore h = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

.. overall g-parameters.

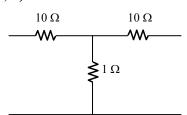
$$g = h^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}^{-1} = \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$
$$g = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$





21. Ans: (a, b)

Sol:



$$[Z] = \begin{bmatrix} 11 & 1 \\ 1 & 11 \end{bmatrix}$$

$$Z_{22} = 11, Z_{12} = 1$$

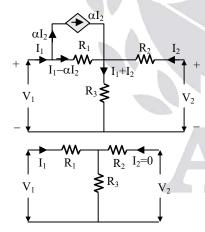
$$[y] = [Z]^{-1} = \frac{1}{121 - 1} \begin{bmatrix} 11 & -1 \\ -1 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{11}{120} & \frac{-1}{120} \\ \frac{-1}{120} & \frac{11}{120} \end{bmatrix}$$

$$Y_{11} = \frac{11}{120} \text{ U}, Y_{12} = \frac{-1}{120} \text{ U}$$

22. Ans: (a, c)

Sol: $V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$



$$A = \frac{V_1}{V_2} \Big|_{I_2=0}$$

$$V_2 = V_1 \frac{R_3}{R_1 + R_3}$$

$$\Rightarrow A = \frac{V_1}{V_2} = \frac{R_1 + R_3}{R_3}$$

$$\Rightarrow C = \frac{I_1}{V_2} \Big|_{I_2=0} = \frac{\frac{V_1}{R_1 + R_3}}{V_1 \frac{R_3}{R_1 + R_3}} = \frac{1}{R_3}$$

$$B = -\frac{\mathbf{V}_1}{\mathbf{I}_2}\Big|_{\mathbf{V}_2 = 0}$$

$$\alpha \mathbf{I}_2$$

$$\mathbf{R}_1$$

$$\mathbf{R}_2$$

$$\mathbf{R}_1$$

$$\mathbf{I}_1 + \mathbf{I}_2$$

$$\mathbf{R}_3$$

$$\mathbf{V}_2 = 0$$

$$V_1 = (I_1 - \alpha I_2)R_1 - I_2R_2$$
(1)

$$V_1 = (I_1 - \alpha I_2)R_1 + (I_1 + I_2)R_3 \dots (2)$$

$$I_2R_2 + (I_1 + I_2)R_3 = 0$$
(3)

From eq. (3)

Since

$$I_2R_2 + I_1R_3 + I_2R_3 = 0$$

$$D = \frac{I_1}{-I_2} = \frac{R_2 + R_3}{R_3}$$

$$B = \frac{V_1}{-I_2} = \left[\frac{(R_2 + R_3)(R_1 + R_3)}{R_3} - R_3 + \alpha R_1 \right]$$

Chapter 7

Graph Theory

01. Ans: (c)

Sol:
$$n > \frac{b}{2} + 1$$

Note: Mesh analysis simple when the nodes are more than the meshes.

02. Ans: (c)

Sol: Loops =
$$b - (n-1) \Rightarrow loops = 5$$

 $n = 7$ $\therefore b = 11$

03. Ans: (a)

04.

Sol: Nodal equations required = f-cut sets
$$= (n-1) = (10-1) = 9$$

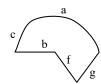
$$= b - n + 1 = 17 - 10 + 1 = 8$$

So, the number of equations required

= Minimum (Nodal, mesh) =
$$Min(9.8) = 8$$

05. Ans: (c)

Sol: Not a tree (Because trees are not in closed path)



06. Ans: (a)

07.

Sol: For a complete graph;

$$b = n_{C_2} \Rightarrow \frac{n(n-1)}{2} = 66$$

$$n = 12$$

f-cut sets =
$$(n-1) = 11$$

$$f$$
-loops = $(b-n+1) = 55$

f-loop = f-cutset matrices =
$$n^{(n-2)}$$

= $12^{12-2} = 12^{10}$

08. Ans: (a)

Nodes=1, Branches =
$$0$$
; f-loops = 0

Let N=2



Nodes =
$$2$$
; Branches = 1 ; f-loop = 0



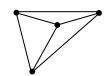
Nodes =
$$3$$
; Branches = 3 ; f-loop = 1

$$\Rightarrow$$
 Links = 1

Let
$$N = 4$$



Still
$$N = 4$$



Branches =
$$6$$
; f -loops = Links = 3

Let
$$N = 5$$



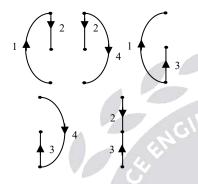


Nodes = 5; Branches = 8; f –loops = Links = 4 etc

Therefore, the graph of this network can have at least "N" branches with one or more closed paths to exist.

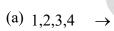
09. Ans: (b)

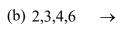
Sol:

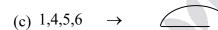


10. Ans: (d)

Sol:







(d)
$$1,3,4,5 \rightarrow$$

11. Ans: (b)

Sol:
$$m = b - n + 1 = 8 - 5 + 1 = 4$$

12. Ans: (d)

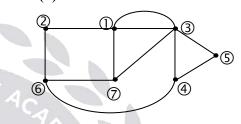
13. Ans: (d)

Sol: The valid cut –set is (1,3,4,6)



14. Ans: (b)

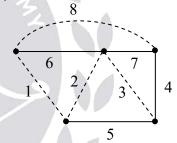
Sol:



15. Ans: (d)

Sol:

Since



Fundamental loop should consist only one link, therefore option (d) is correct.



Passive Filters

01.

Sol:

$$\omega = 0 \Rightarrow V_0 = V_i$$

 $\omega = \infty \Rightarrow V_0 = 0$ \Rightarrow Low pass filter

02.

Sol:
$$\omega = 0 \Rightarrow V_0 = \frac{V_i R_2}{R_1 + R_2}$$

" V_0 " is attenuated $\Rightarrow V_0 = 0$

$$\omega = \infty \Longrightarrow V_0 = V_i$$

It represents a high pass filter characteristics.

03.

Sol:
$$H(s) = \frac{V_i(s)}{I(s)} = \frac{S^2LC + SRC + 1}{SC}$$

Put
$$s = j\omega i = -\frac{\omega^2 LC + j\omega RC + 1}{j\omega C}$$

$$\omega = 0 \Rightarrow H(s) = 0$$

$$\omega = \infty \Rightarrow H(s) = 0$$

It represents band pass filter characteristics

04.

Sol:
$$\omega = 0 \Rightarrow V_0 = 0$$

$$\omega = \infty \Rightarrow V_0 = 0$$

It represents Band pass filter characteristics

05.

Sol:
$$\omega = 0 \Rightarrow V_0 = 0$$

$$\omega = \infty \Longrightarrow V_0 = V_i$$

It represents High Pass filter characteristics.

06.

Sol:
$$H(s) = \frac{1}{s^2 + s + 1}$$

$$\omega = 0 : S = 0 \Rightarrow H(s) = 1$$

$$\omega = \infty : S = \infty \Rightarrow H(s) = 0$$

It represents a Low pass filter characteristics

07.

Sol:
$$H(s) = \frac{s^2}{s^2 + s + 1}$$

$$\omega = 0 : S = 0 \Rightarrow H(s) = 0$$

$$\omega = \infty : S = \infty \Rightarrow H(s) = 1$$

It represents a High pass filter characteristics

08.

Sol:
$$\omega = 0$$
; $V_0 = V_1$

$$\omega = \infty; V_0 = 0$$

It represents a low pass filter characteristics.

09.

Sol:
$$\omega = 0 \Rightarrow V_0 = V_{ir}$$

$$\omega = \infty \Rightarrow V_0 = V_{in}$$

It represents a Band stop filter or notch filter.

110.95

Sol:
$$H(s) = \frac{S}{s^2 + s + 1}$$

$$\omega = 0 : S = 0 \Rightarrow H(s) = 0$$

$$\omega = \infty : S = \infty \Rightarrow H(s) = 0$$

It represents a Band pass filter characteristics

11.

Sol:
$$H(s) = \frac{S^2 + 1}{s^2 + s + 1}$$

$$\omega = 0 \Rightarrow S = 0 \Rightarrow H(s) = 1$$

$$\omega = \infty \Rightarrow S = \infty \Rightarrow H(s) = 1$$

It represents a Band stop filter

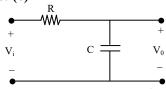


Sol:
$$H(s) = \frac{1-s}{1+s}$$

 $\omega = 0 \Rightarrow S = 0 \Rightarrow H(s) = 1$
 $\omega = \infty \Rightarrow S = \infty \Rightarrow H(s) = -1 = 1 \angle 180^{0}$
It represents an All pass filter

13. Ans: (c)

Sol.



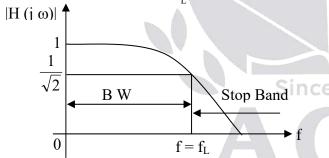
$$\begin{split} \omega &= 0 \Longrightarrow V_0 = V_i \\ \omega &= \infty \Longrightarrow V_0 = 0 \end{split}$$

$$\omega = \infty \Longrightarrow V_0 = 0$$

$$V_0(s) = \left(\frac{V_i(s)}{R + \frac{1}{sc}}\right) \left(\frac{1}{sc}\right)$$

$$\frac{V_0(s)}{V_i(s)} = H(s) = \frac{1}{SscR + 1}$$

$$H(j\omega) = \frac{1}{1 + j\omega cR} = \frac{1}{1 + j\frac{f}{f_1}}$$



Where
$$f_L = \frac{1}{2\pi RC}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_L}\right)^2}}$$

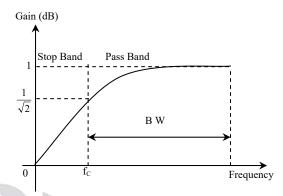
$$\angle H(j\omega) = -\tan^{-1}\left(\frac{f}{f_L}\right)$$

$$f = 0 \Rightarrow \phi = 0^0 = \phi_{min}$$

$$f = f_L \Longrightarrow \phi = -45^0 = \phi_{max}$$

14. Ans: (b)

Sol:



First order high pass filter =
$$\frac{s}{1+sT}$$

Phase shift = $90 - \tan \omega T$

Max. phase shift is at corner frequency

$$\omega = \frac{1}{T}$$

Max. phase shift =
$$90 - \tan^{-1}\omega T$$

= $90 - \tan^{-1}\left(\frac{1}{T} \times T\right)$
= $90 - 45$
= 45°

15. Ans: (d)

1995

16. Ans: (a)

Sol: Half power of series RC circuit is at t = T(Time constant)

$$T = RC$$

Frequency =
$$\frac{1}{RC}$$

17. Ans: (c)

Sol: Magnitude of voltage gain 0.707 is at half power frequency

$$\omega = \frac{1}{RC}$$

