REVISION SESSION

GATE-2025

ELECTROMAGNETIC THEORY

SYLLABUS:

- Vector calculas & co-ordinate systems
- Maxwells equations
- Em – wave propagation in unbound medium
- Wave guides
- Transmission lines
- Antenna theory

VECTOR CALCULAS

&

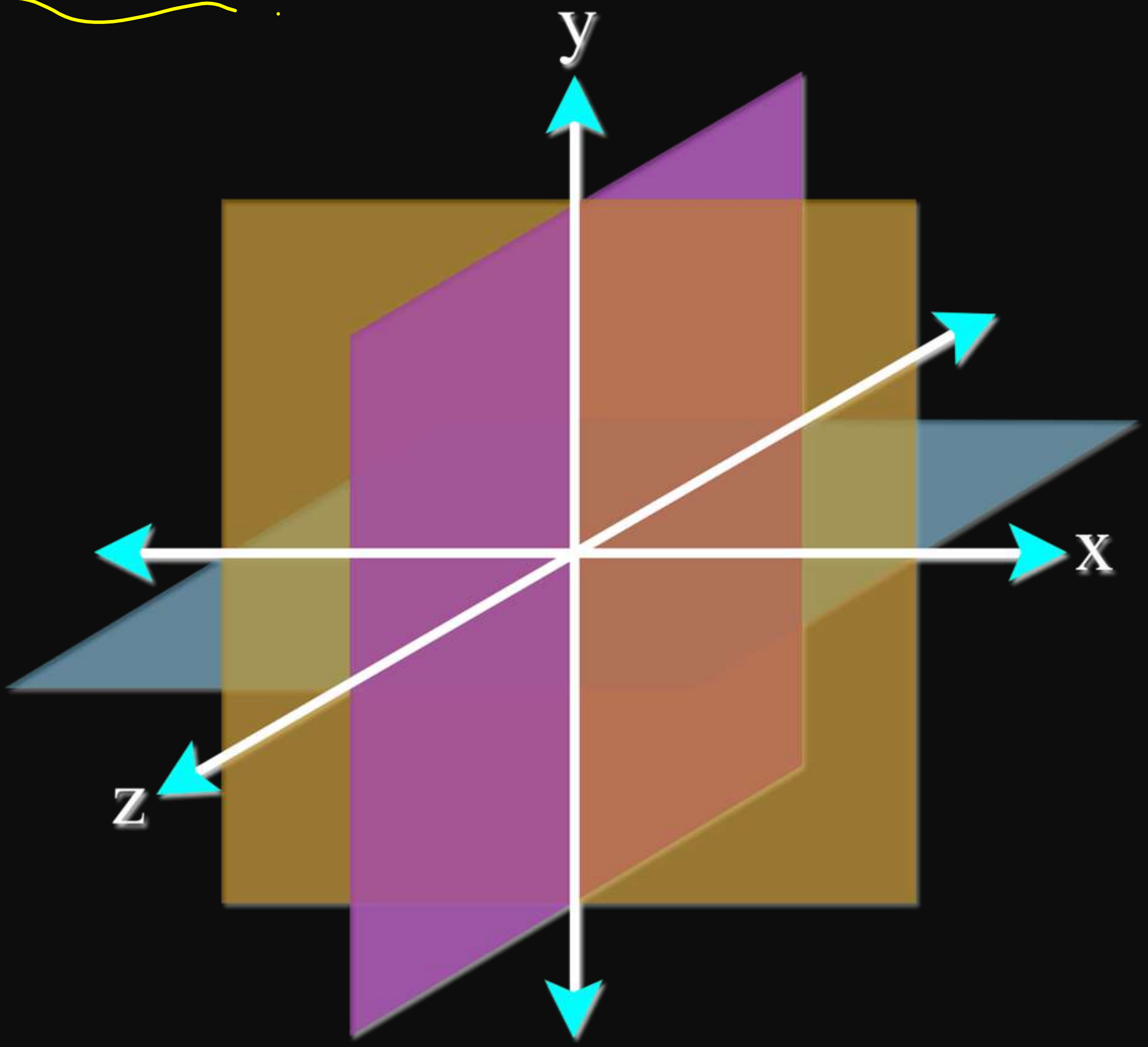
COORDINATE SYSTEM

CO-ORDINATE SYSTEMS

(i) Cartesian co-ordinate system (x, y, z)

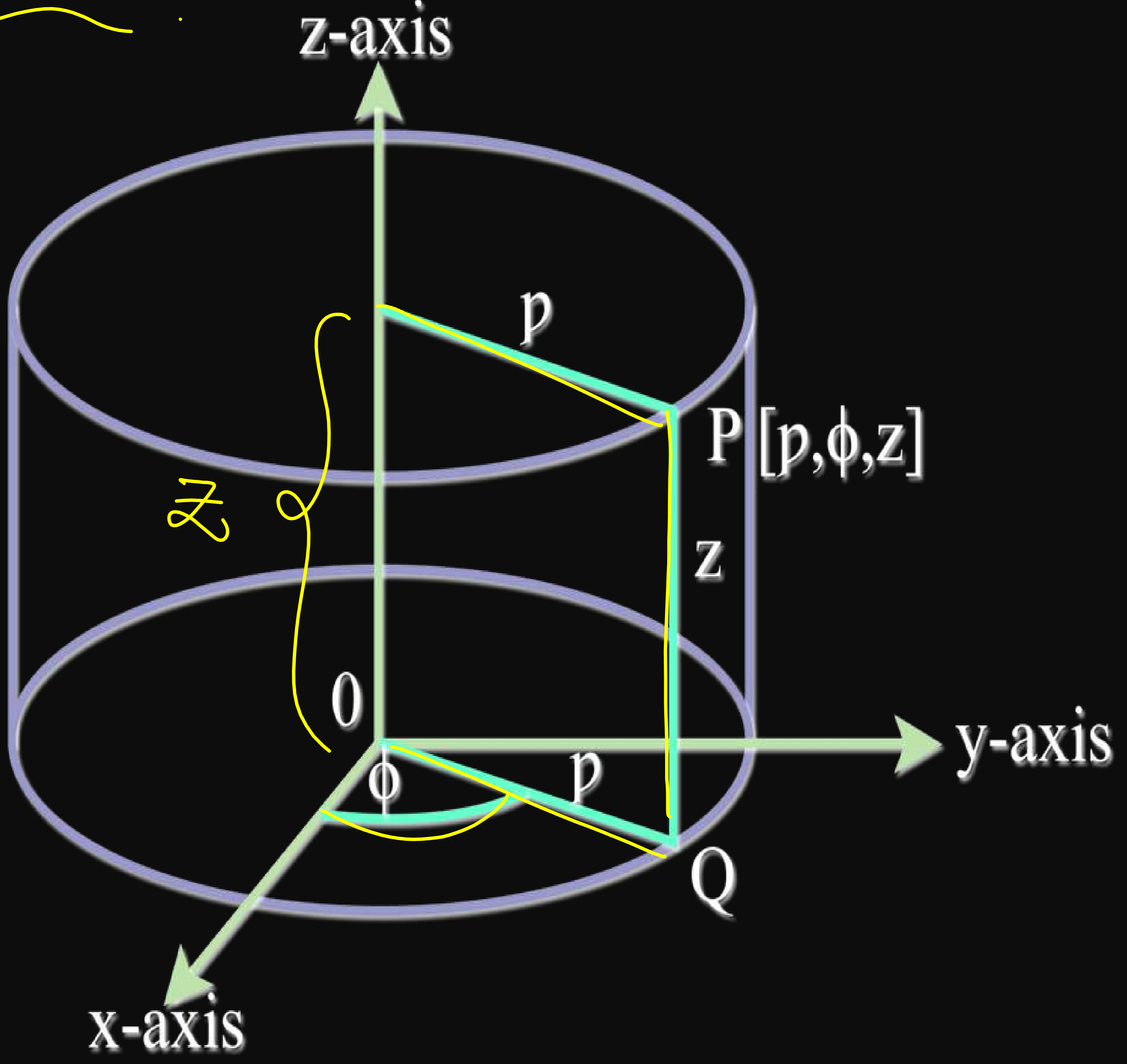
$$\left\{\begin{aligned} -\infty &< x < +\infty \\ -\infty &< y < +\infty \\ -\infty &< z < +\infty \end{aligned}\right\}$$

$$\hat{x} \rightarrow \hat{y} \rightarrow \hat{z} \rightarrow \hat{x}$$



(II) CYLINDRICAL CO-ORDINATE (ρ, ϕ, z)

$$\begin{cases} 0 \le \rho < \infty \\ 0 \le \phi \le 2\pi \\ -\infty < z < +\infty \end{cases}$$



CONVERSIONS

$(\rho, \phi, z) \rightarrow (x, y, z)$

$$x = \rho \cos\phi$$

$$y = \rho \sin\phi$$

$$z = z$$

$(x, y, z) \rightarrow (\rho, \phi, z)$

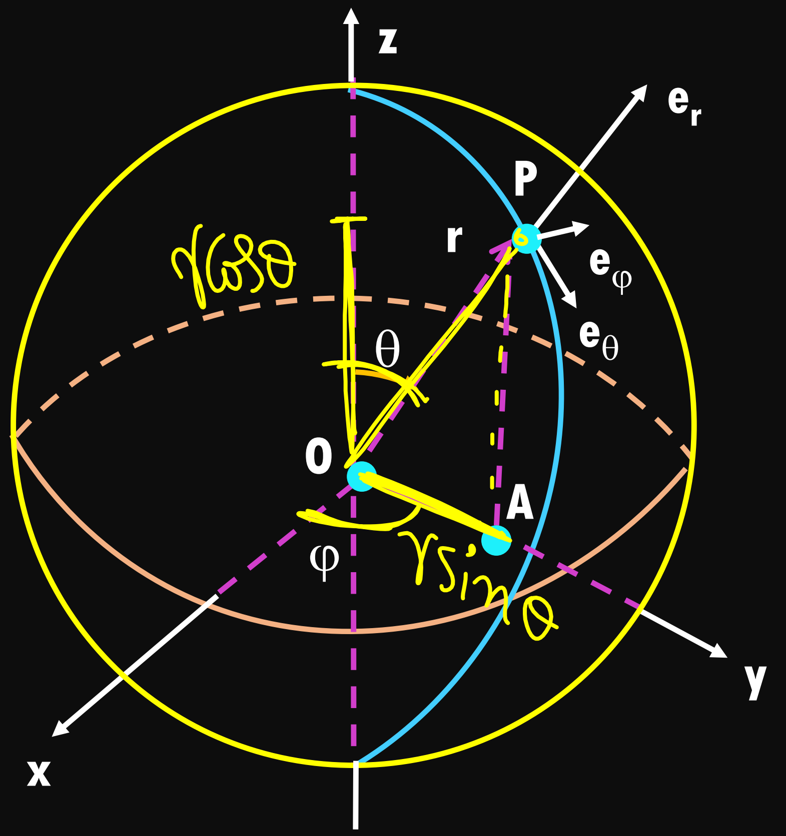
$$\rho = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}(y/x)$$

$$z = z$$

(III) SPHERICAL CO-ORDINATE (r,θ,ϕ)

$$* \left\{ \begin{array}{l} 0 \le r < \infty \\ 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{array} \right\}$$



CONVERSIONS

$(r, \theta, \phi) \rightarrow (x, y, z)$

$$z = r\cos\theta$$

$$x = (r\sin\theta)\cos\phi$$

$$y = (r\sin\theta)\sin\phi$$

$(x, y, z) \rightarrow (r, \theta, \phi)$

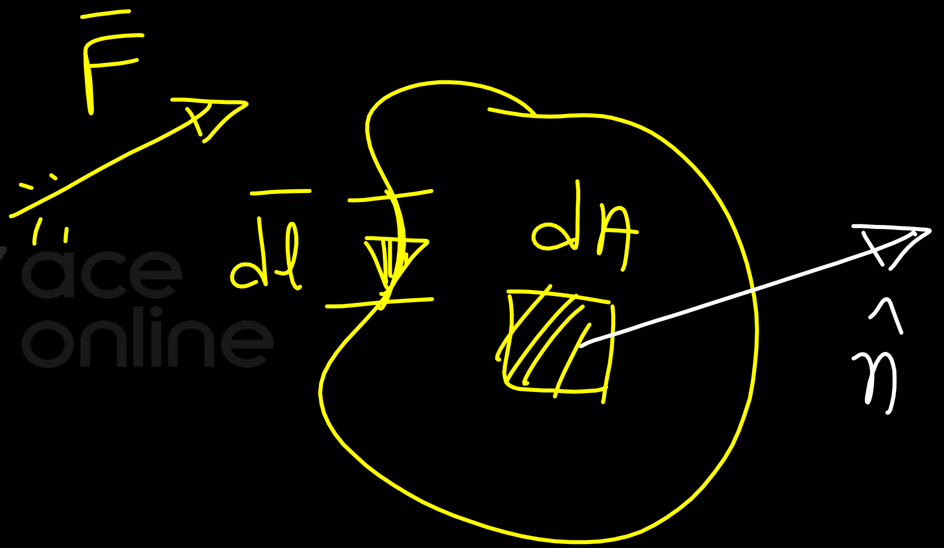
$$r = \sqrt{x^2+y^2+z^2}$$

$$\theta = \cos^{-1}\left(\frac{z}{r}\right)$$

$$\phi = \tan^{-1}(y/x)$$

STOKE'S THEOREM

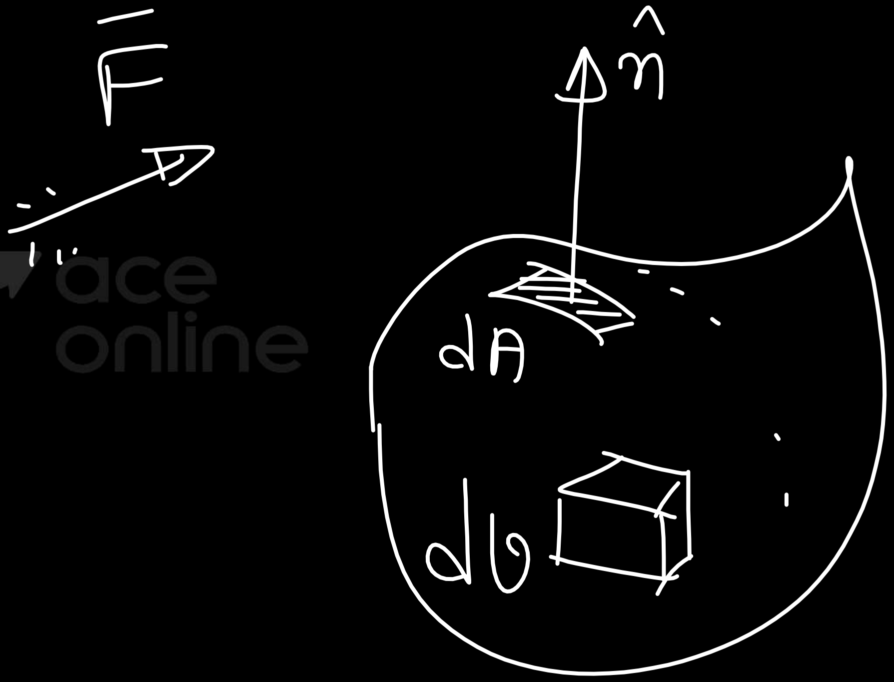
Closed Line Integral ↔ Open Surface Integral



$$\boxed{\oint \bar{F} \cdot \bar{dl} = \iint \nabla \times \bar{F} \cdot \bar{dA}}$$

DIVERGENCE THEOREM

Closed Surface Integral ↔ Volume Integral

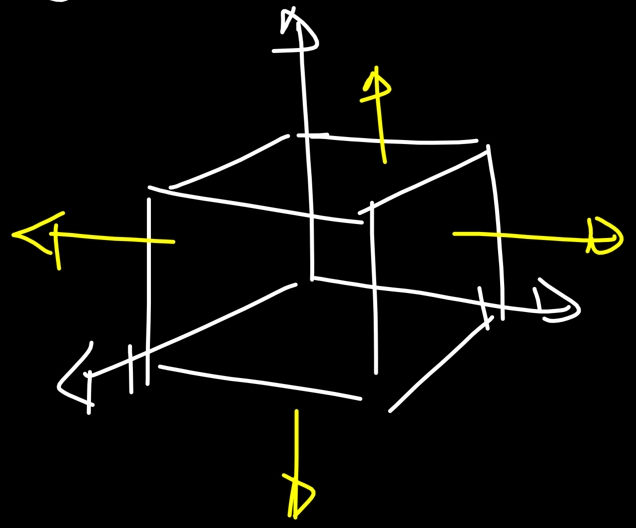


$$\oiint \bar{F} \cdot \bar{dA} = \iiint \nabla \cdot \bar{F} \, d\upsilon$$

Q. Given $\bar{V} = x\cos^2 y\hat{i} + x^2 e^z\hat{j} + z\sin^2 y\hat{k}$ and S. The surface of a unit cube with one corner at the origin and edges parallel to co-ordinate axes. The value of interval $\oiint \bar{V}.\hat{n}ds$ Is __________

Soln

$$I = \oiint \bar{V}.\bar{ds} = \iiint \nabla.\bar{V}\,dv$$



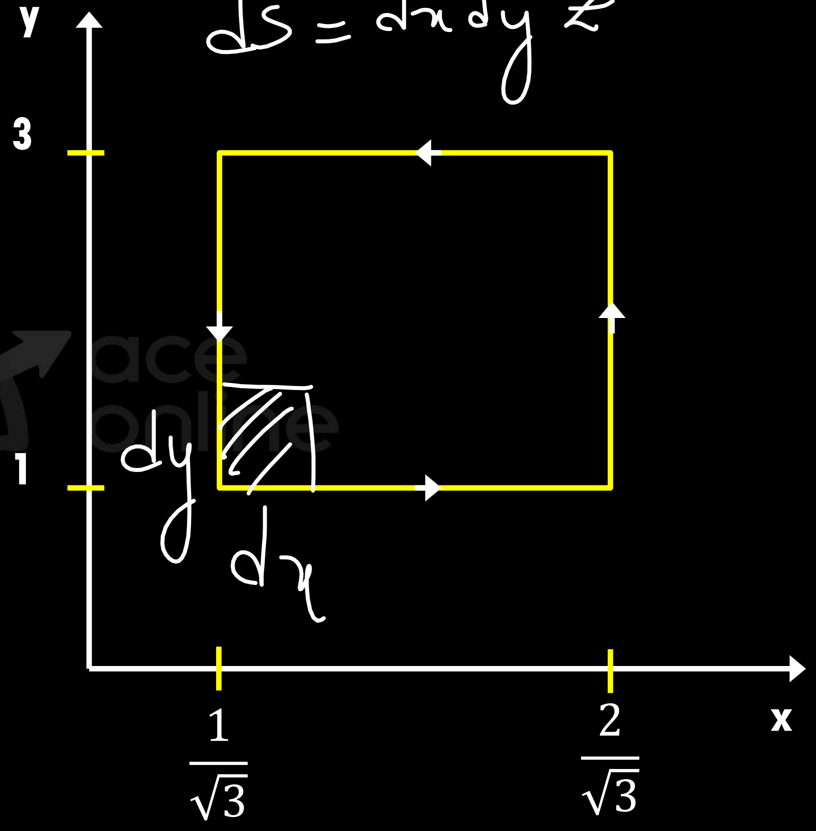
$$\nabla.\bar{V} = \cos^2 y + 0 + \sin^2 y = 1$$

$$I = \iiint \nabla.\bar{V}\,dv = \iiint (1)\,dv$$

$$= \underbrace{\iiint dv}_{\text{Volume}} = 1 \times 1 \times 1 = \underline{\underline{1}}$$

Q. If $\bar{A} = xy\hat{x} + x^2\hat{y}$, then $\oint \bar{A}.\overline{d\ell}$ over the path shown in figure is **(GATE-10)**

$\overline{ds} = dx\,dy\,\hat{z}$



$$I = \oint \bar{A} \cdot \overline{dl} = \iint \nabla \times \bar{A} \cdot \overline{ds}$$

$$\nabla \times \bar{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & x^2 & 0 \end{vmatrix}$$

$$\nabla \times \bar{A} = \hat{x}(0-0) - \hat{y}(0-0) + \hat{z}(2x - x)$$

$$\nabla \times \bar{A} = x\hat{z}$$

$$I = \iint x\hat{z} \cdot dx\,dy\,\hat{z} = \iint x\,dx\,dy = \int x\,dx \int dy.$$

$$= \left[\frac{x^2}{2}\right]_{1/\sqrt{3}}^{2/\sqrt{3}} \left[y\right]_1^3 = \frac{1}{2}\left[\frac{4}{3} - \frac{1}{3}\right]\left[3-1\right]$$

$$I = \underline{\underline{1}}.$$

STANDARD FORMS

1. $d\bar{l}$
2. $d\bar{A}$
3. dv
4. $\nabla \cdot (\ )$
5. $\nabla \times (\ )$
6. $\nabla (\ )$
7. $\nabla^2 (\ )$

$$\left\{ \begin{array}{l} (x, y, z) \\ (\rho, \phi, z) \\ (r, \theta, \phi) \end{array} \right\}$$

CO-ORDINATES **SCALING FACTORS**

u	v	w	h_1	h_2	h_3
x	y	z	1	1	1
ρ	ϕ	z	1	ρ	1
r	θ	ϕ	1	r	$r\sin\theta$

DIFFERENTIAL LENGTH ($\overline{d\ell}$)

$$\boxed{\overline{d\ell} = h_1\, du\, \hat{u} + h_2\, dv\, \hat{v} + h_3\, dw\, \hat{w}}$$

DIFFERENTIAL SURFACE ($\overline{ds}$ / $\overline{dA}$)

$$\boxed{\overline{dA} = h_1 h_2\, du\, dv\, \hat{w} + h_2 h_3\, dv\, dw\, \hat{u} + h_3 h_1\, dw\, du\, \hat{v}}$$

DIFFERENTIAL VOLUME (dv)

$$\boxed{dV = h_1 h_2 h_3 \, du \, dv \, dw}$$

GRADIENT:

SCALAR FIELD : f (u, v, w) $\longrightarrow f$

$$\boxed{\nabla f = \frac{1}{h_1}\frac{\partial f}{\partial u}\hat{u} + \frac{1}{h_2}\frac{\partial f}{\partial v}\hat{v} + \frac{1}{h_3}\frac{\partial f}{\partial w}\hat{w}}$$

DIVERGENE AND CURL

Vector Field : $\bar{F} = F_u\hat{u} + F_v\hat{v} + F_w\hat{w}$

$$\nabla \cdot \bar{F} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u}(h_2h_3F_u) + \frac{\partial}{\partial v}(h_1h_3F_v) + \frac{\partial}{\partial w}(h_1h_2F_w)\right]$$

$$\nabla \times \bar{F} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{u} & h_2\hat{v} & h_3\hat{w} \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1F_u & h_2F_v & h_3F_w \end{vmatrix}$$

LAPLACIAN OPERATOR (∇^2)

Scalar Field : $f(u, v, w)$ $\Longrightarrow f$.

$$\nabla^2 f = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2 h_3}{h_1}\frac{\partial f}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_1 h_3}{h_2}\frac{\partial f}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{h_1 h_2}{h_3}\frac{\partial f}{\partial w}\right)\right].$$

Q. **The scalar field in certain region is defined as $r^2\sin\theta\cos\phi$. The magnitude of gradient at point $(1, \pi/4, \pi/4)$ is**

Soln.

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla f = 2r\sin\theta\cos\phi\,\hat{r} + \frac{1}{r}r^2\cos\theta\cos\phi\,\hat{\theta} + \frac{1}{r\sin\theta}\left(-r^2\sin\theta\sin\phi\right)\hat{\phi}$$

$$\nabla f = 2r\sin\theta\cos\phi\,\hat{r} + r\cos\theta\cos\phi\,\hat{\theta} - r\sin\phi\,\hat{\phi}$$

$$\nabla f = 2\times 1\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\hat{r} + 1\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\hat{\theta} - 1\times\frac{1}{\sqrt{2}}\hat{\phi}$$

$$\nabla f = \hat{r} + \frac{1}{2}\hat{\theta} - \frac{1}{\sqrt{2}}\hat{\phi}$$

$$|\nabla f| = \sqrt{1^2 + \left(\frac{1}{2}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2} = \sqrt{1 + \frac{1}{4} + \frac{2}{4}} = \sqrt{\frac{7}{4}}$$

$$= \frac{\sqrt{7}}{2} = \underline{1.32}.$$

Q. If $\overline{F}(\rho,\phi,z)=\rho\hat{\rho}+\rho sin^2\phi\hat{\phi}-z\hat{z}$, which one of the following is TRUE?

(a) $\dfrac{\nabla.\overline{F}}{at\ \phi=0^\circ} < \dfrac{\nabla.\overline{F}}{at\ \phi={}^{\pi}/_{2}}$

(b) $\dfrac{\nabla.\overline{F}}{at\ \phi={}^{\pi}/_{4}} = \dfrac{\nabla.\overline{F}}{at\ \phi=0^\circ}$

(c) $\dfrac{\nabla.\overline{F}}{at\ \phi=0^\circ} > \dfrac{\nabla.\overline{F}}{at\ \phi={}^{\pi}/_{2}}$

(d) $\dfrac{\nabla.\overline{F}}{at\ \phi={}^{\pi}/_{4}} = \dfrac{2\nabla.\overline{F}}{at\ \phi=0^\circ}$

Soln. $\nabla\cdot\bar{F} = \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho F_\rho) + \frac{\partial}{\partial\phi}(F_\phi) + \frac{\partial}{\partial z}(\rho F_z)\right]$

$$= \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho\,\rho) + \frac{\partial}{\partial\phi}(\rho\sin^2\phi) + \frac{\partial}{\partial z}[\rho(-z)]\right]$$

$$= \frac{1}{\rho}\left[2\rho + \rho\, 2\sin\phi\cos\phi - \rho\right]$$

$$\nabla\cdot\bar{F} = 1 + \sin 2\phi$$

AT $\phi = 0^\circ$

$\nabla\cdot\bar{F} = 1 + 0 = 1$

AT $\phi = \pi/4$

$\nabla\cdot\bar{F} = 1 + 1 = 2$

AT $\phi = \pi/2$

$\nabla\cdot\bar{F} = 1 + 0 = 1.$

Q. the magnitude of CURL of vector field $\overline{F}(\rho, \phi, z) = \frac{\hat{\phi}}{\rho}$ is

Soln

$$\nabla \times \overline{F} = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ F_\rho & \rho F_\phi & F_z \end{vmatrix} = 0$$

$F_\rho = 0$, $F_\phi = \left(\frac{1}{\rho}\right)$, $F_z = 0$

$0 \quad 1 \quad 0$

MAXWELL'S EQUATIONS

MAXWELL'S EQUATIONS

DIFFERENTIAL FORM		INTEGRAL FORM
① $\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$	COUPLED EQUATIONS	① $\oint \bar{H} \cdot \bar{dl} = \iint \left(\bar{J} + \frac{\partial \bar{D}}{\partial t} \right) \cdot \bar{dA}$
② $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$		② $\oint \bar{E} \cdot \bar{dl} = -\iint \frac{\partial \bar{B}}{\partial t} \cdot \bar{dA}$
③ $\nabla \cdot \bar{D} = \rho_v$	SOURCE EQUATIONS	③ $\oiint \bar{B} \cdot \bar{dA} = \iiint \rho_v \, dv$
④ $\nabla \cdot \bar{B} = 0$		④ $\oiint \bar{B} \cdot \bar{dA} = 0$

FOR STATIC FIELDS

$\left(\frac{\partial}{\partial t} = 0\right)$

DIFFERENTIAL FORM	INTEGRAL FORM
① $\nabla \times \bar{H} = \bar{J}$	① $\oint \bar{H} \cdot \bar{dl} = \iint \bar{J} \cdot \bar{dA}$
② $\nabla \times \bar{E} = 0$	② $\oint \bar{E} \cdot \bar{dl} = 0$
③ $\nabla \cdot \bar{D} = \rho_v$	③ $\oiint \bar{D} \cdot \bar{dA} = \iiint \rho_v \, dv$
④ $\nabla \cdot \bar{B} = 0$	④ $\oiint \bar{B} \cdot \bar{dA} = 0$

Q. In an electrostatic field, the electric displacement density vector, $\vec{D}$, is given by

$$\vec{D}(x,y,z) = \left(x^3\,\vec{i} + y^3\,\vec{j} + xy^2\,\vec{k}\right)\ C/m^2$$

Consider a cubical region R centered at the origin with each side of length 1m, and vertices at (±0.5m, ±0.5m, ±0.5m). The electric charge enclosed within R is __________ C (rounded off to two decimal places).

Soln.

$$Q_{ENC} = (\Psi_e)_{NET} = \oint\!\!\oint \bar{D}\cdot\bar{dA} = \iiint \nabla\cdot\bar{D}\,dv = \iiint \rho_v\,dv$$

$$\nabla\cdot\bar{D} = 3x^2 + 3y^2 + 0$$

$$\nabla\cdot\bar{D} = 3(x^2+y^2) = \rho_v$$

$$Q_{ENC} = \iiint 3(x^2+y^2)\,dx\,dy\,dz$$

$$= 3\iiint x^2\,dx\,dy\,dz + 3\iiint y^2\,dx\,dy\,dz$$

$$= 3\int x^2 dx\int dy\int dz + 3\int dx\int y^2 dy\int dz$$

$$= 3\left[\frac{x^3}{3}\right]_{-1/2}^{+1/2}\left[y\right]_{-1/2}^{+1/2}\left[z\right]_{-1/2}^{+1/2} + 3\left[x\right]_{-1/2}^{+1/2}\left[\frac{y^3}{3}\right]_{-1/2}^{+1/2}\left[z\right]_{-1/2}^{+1/2}$$

$$= \left(\frac{2}{8}\times 1\times 1\right) + \left(1\times\frac{2}{8}\times 1\right)$$

$$= \frac{4}{8} = \underline{\underline{0.5\,C}}$$

Q. The given equation represents a magnetic field strength $\bar{H}(r, \theta, \varnothing)$ in the spherical coordinate system, in free space. The value of P in the equation should be ____________ (rounded off to the nearest integer)

$$\bar{H}(r, \theta, \varphi) = \frac{1}{r^3}(\hat{r} P \cos\theta + \hat{\theta}\sin\theta)$$

Soln

$\nabla \cdot \bar{B} = 0$, $\bar{B} = \mu \bar{H}$

$\bar{B} = \mu_0 \bar{H}$

$\nabla \cdot \mu_0 \bar{H} = 0$

$\mu_0 \nabla \cdot \bar{H} = 0$

$\Rightarrow \nabla \cdot \bar{H} = 0.$

$$\nabla \cdot \bar{H} = \frac{1}{r^2 \sin\theta}\left[\frac{\partial}{\partial r}\left(r^2 \sin\theta \frac{p\cos\theta}{r^3}\right) + \frac{\partial}{\partial \theta}\left(r\sin\theta \frac{\sin\theta}{r^3}\right) + \frac{\partial}{\partial \phi}(r\ 0)\right] = 0$$

$$= \frac{1}{r^2 \sin\theta}\left[\frac{-p\sin\theta\cos\theta}{r^2} + \frac{2\sin\theta\cos\theta}{r^2}\right] = 0.$$

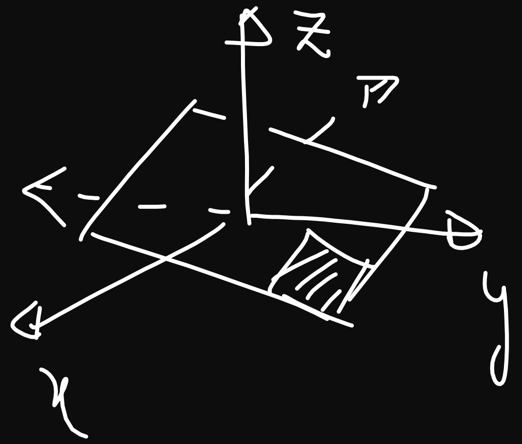
$$\Rightarrow \frac{p\sin\theta\cos\theta}{r^2} = \frac{2\sin\theta\cos\theta}{r^2}$$

$$\Rightarrow p = 2$$

Q. **Let** $\mathrm{H} = -\mathrm{y}(\mathrm{x}^2 + \mathrm{y}^2)\hat{\mathrm{x}} + \mathrm{x}(\mathrm{x}^2 + \mathrm{y}^2)\hat{\mathrm{y}}$ **A/m, the amount of current passing through loop placed in z = 0 plane in z-direction defined by $-1\mathrm{m} \le x \le 1\mathrm{m}$, $-2\mathrm{m} \le y \le 2\mathrm{m}$ is**

Soln

$$I_{ENC} = MMF_L = \oint \bar{H} \cdot \bar{dl} = \iint \nabla \times \bar{H} \cdot \bar{dA} = \iint \bar{J} \cdot \bar{dA}$$



$\bar{dA} = dx\,dy\,\hat{z}$

$$\nabla \times \overline{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -yx^2-y^3 & x^3+xy^2 & 0 \end{vmatrix}$$

$$= \hat{x}(0-0) - \hat{y}(0-0) + \hat{z}\left\{3x^2+y^2-(-x^2-3y^2)\right\}$$

$$\nabla \times \overline{H} = \overline{J} = 4(x^2+y^2)\hat{z} \ \mathrm{A/m^2}$$

$$I_{ENC} = \iint 4(x^2+y^2)\hat{z} \cdot dx\,dy\,\hat{z}$$

$$= \iint 4(x^2+y^2)\,dx\,dy$$

$$= 4\iint x^2\,dx\,dy + 4\iint y^2\,dx\,dy.$$

$$= 4\int x^2\,dx \int dy + 4\int dx \int y^2\,dy$$

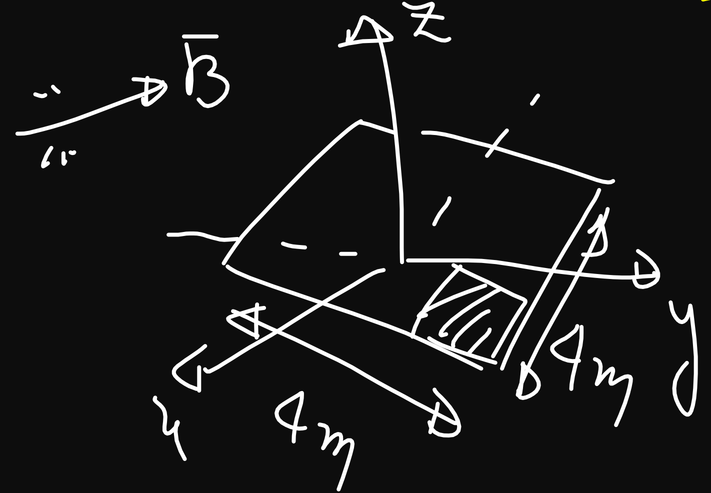
$$= 4\left[\frac{x^3}{3}\right]_{-1}^{+1}\left[y\right]_{-2}^{+2} + 4\left[x\right]_{-1}^{+1}\left[\frac{y^3}{3}\right]_{-2}^{+2}$$

$$= \frac{4}{3}\left\{(2\times 4) + (2\times 16)\right\} = \frac{4}{3}\times 40$$

$$= \frac{160}{3} = 53.33\,\mathrm{A}.$$

Q. A square loop of 4m side is placed in xy-plane with it's center at the origin and sides along the co-ordinate axes. If the magnetic flux density in the region is given by $\bar{B} = (0.2\hat{x} - 0.3\hat{y} +$

Soln



$\overline{dA} = dx\,dy\,\hat{z}$

$EMF_h = V = \oint \bar{E}\cdot\overline{dl} = -\iint \frac{\partial \bar{B}}{\partial t}\cdot\overline{dA}$

$$= -\iint \frac{\partial}{\partial t}\left\{(0.2\hat{x} - 0.3\hat{y} + 0.4\hat{z})\, e^{-0.1t}\right\} \cdot dx\,dy\,\hat{z}$$

$$= -\frac{\partial}{\partial t} e^{-0.1t} \iint (0.2\hat{x} - 0.3\hat{y} + 0.4\hat{z}^{*}) \cdot dx\,dy\,\hat{z}^{*}$$

$$= -(-0.1)\, e^{-0.1t} \iint 0.4\, dx\,dy$$

$$= 0.1 \times 0.4\, e^{-0.1t} \underbrace{\iint dx\,dy}_{AREA} = 0.1 \times 0.4 \times 4 \times 4\, e^{-0.1t} = 0.64\, e^{-0.1t}$$

<u>AT $t = 10\,\text{sec}$</u> $\therefore$ $V = 0.64\, e^{-0.1 \times 10} = 0.64\, e^{-1} = \frac{0.64}{e} \simeq 0.2356$

$$V \simeq 0.23 \text{ Volt}$$

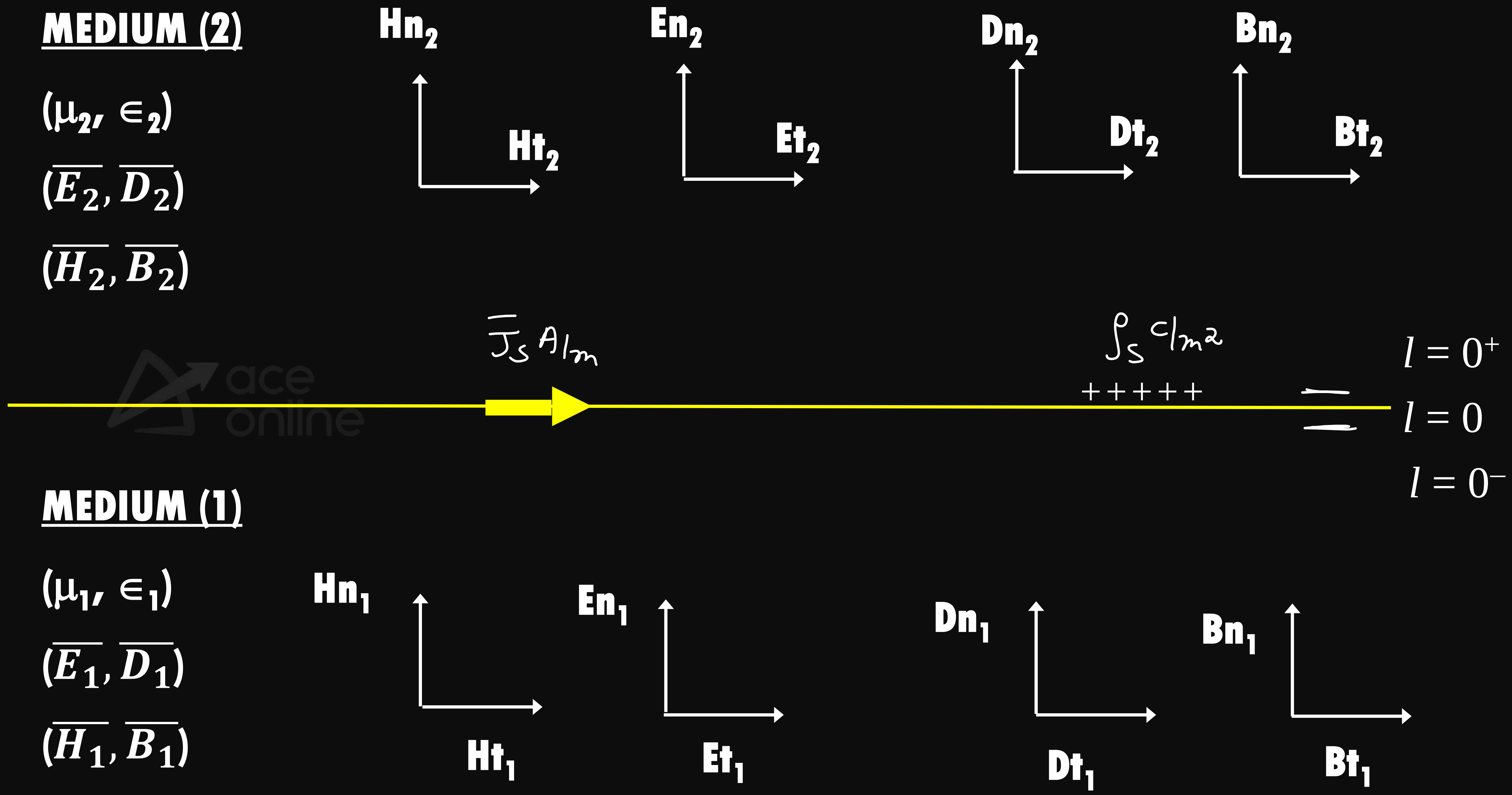
BOUNDARY CONDITIONS

MEDIUM (2)

(μ_2, ϵ_2)

$(\overline{E_2}, \overline{D_2})$

$(\overline{H_2}, \overline{B_2})$



MEDIUM (1)

(μ_1, ϵ_1)

$(\overline{E_1}, \overline{D_1})$

$(\overline{H_1}, \overline{B_1})$

DIELECTRIC - DIELECTRIC	DIELECTRIC - CONDUCTOR
① $H_{t_1} - H_{t_2} = J_s$	① $H_t = J_s$
② $E_{t_2} = E_{t_1}$	② $E_t = 0$
③ $D_{n_2} - D_{n_1} = \rho_s$	③ $D_n = \rho_s$
④ $B_{n_2} = B_{n_1}$	④ $B_n = 0$

Q. The displacement flux density at a point on the surface of a perfect conductor is $\bar{D} = 2(\hat{a}_x - \sqrt{3}\hat{a}_z)$ C/m² and is pointing away from the surface. The surface charge density at that point c/m² will be

(a) 2

(b) – 2

(c) 4

(d) –4

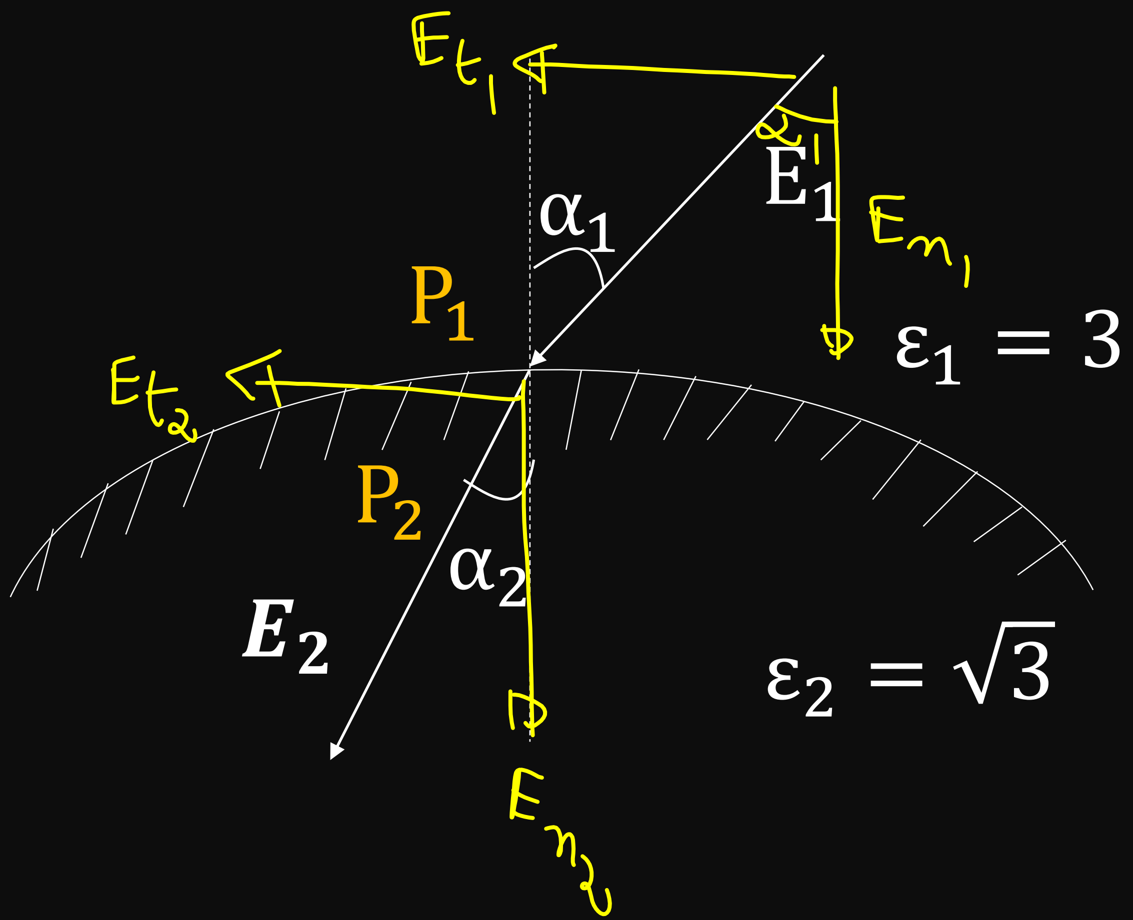
Soln $\rho_s = D_n = |\bar{D}_n|$

$E_t = 0, \quad \epsilon E_t = 0, \quad D_t = 0$

$\rho_s = \sqrt{(2)^2 + (-2\sqrt{3})^2} = \sqrt{4 \times 4} \rightarrow +4, \ -4$

$\rho_s = +4$ C/m² (AWAY)

Q. Two die-electric media with permittivity's 3 and $\sqrt{3}$ are separated by a charge-free boundary as shown in figure. The electric filed intensity in medium 1 at point P_1 has magnitude E_1 and makes an angle $\alpha_1 = 60^o$ with the normal. The direction of the electric field intensity at point P_2, α_2 is



(a) $sin^{-1}\left(\frac{\sqrt{3}E_1}{2}\right)$ (b) 45°

(c) $cos^{-1}\left(\frac{\sqrt{3}E_1}{2}\right)$ (d) 30°

$$E_{n_1} = E_1 \cos\alpha_1$$

$$E_{t_1} = E_1 \sin\alpha_1$$

$$E_{n_2} = E_2 \cos\alpha_2$$

$$E_{t_2} = E_2 \sin\alpha_2.$$

$$E_{t_2} = E_{t_1}$$

$$E_2 \sin\alpha_2 = E_1 \sin\alpha_1 \rightarrow 1$$

$$D_{n_2} - D_{n_1} = \cancelto{0}{\rho_s}$$

$$D_{n_2} = D_{n_1}$$

$$\epsilon_2 E_{n_2} = \epsilon_1 E_{n_1}$$

$$\epsilon_2 E_2 \cos\alpha_2 = \epsilon_1 E_1 \cos\alpha_1 \rightarrow 2$$

$$\frac{1}{2} \quad \frac{\tan\alpha_2}{\epsilon_2} = \frac{\tan\alpha_1}{\epsilon_1}$$

$$\tan\alpha_2 = \frac{\epsilon_2}{\epsilon_1} \tan\alpha_1$$

$$\tan\alpha_2 = \frac{\sqrt{3}\epsilon_0}{3\epsilon_0} \tan 60^\circ$$

$$= \frac{1}{\sqrt{3}} \cdot \sqrt{3} = 1$$

$$\tan\alpha_2 = 1$$

$$\alpha_2 = 45^\circ$$

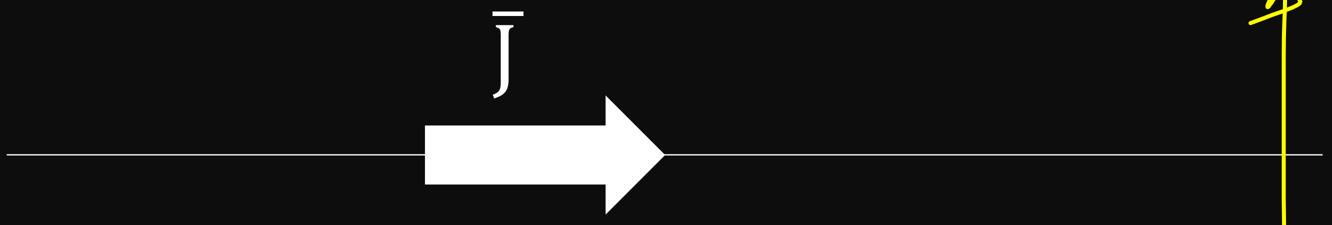
Q. A current sheet $\bar{J} = 10\hat{y}$ A/m lies on the di-electric interface $x = 0$ between two di-electric media with $\varepsilon_{r1} = 5, \mu_{r1} = 1$ in region 1 $(x < 0)$ and $\varepsilon_{r2} = 2, \mu_{r1} = 2$ in region 2 $(x > 0)$. If the magnetic field in region 1 at $x = 0^-$ is $\bar{H}_1 = 3\hat{V}_x + 30\hat{V}_y$ A/m. The magnetic field in region 2 at $x = 0^+$ is

$\bar{H}_{t1}$ $\quad$ $\bar{H}_{n1}$

$\bar{H}_{n1}$ $\quad$ $\bar{H}_{t1}$

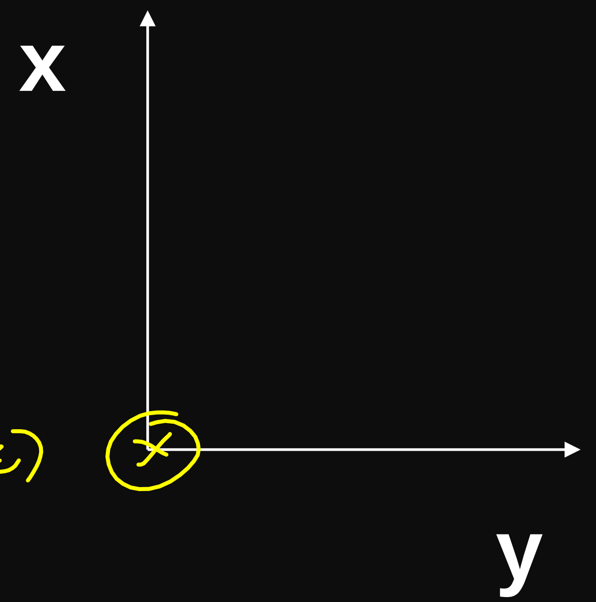
$x > 0$ (region 2): $\varepsilon_{r2} = 2, \mu_{r2} = 2$

$\hat{n}_{12} = +\hat{x}$



$x = 0$

(yz-plane)



$x < 0$ (region 1): $\varepsilon_{r1} = 5, \mu_{r1} = 1$

(a) $\bar{H}_2 = 1.5\hat{V}_x + 30\hat{V}_y - 10\hat{V}_z$ A/m

(b) $\bar{H}_2 = 3\hat{V}_x + 30\hat{V}_y - 10\hat{V}_z$ A/m

(c) $\bar{H}_2 = 1.5\hat{V}_x + 40\hat{V}_y$ A/m

(d) $\bar{H}_2 = 3\hat{V}_x + 30\hat{V}_y + 10\hat{V}_z$ A/m

Soln

$$B_{n_2} = B_{n_1}$$

$$\mu_2 H_{n_2} = \mu_1 H_{n_1}$$

$$\bar{H}_{n_2} = \frac{\mu_1}{\mu_2}\,\bar{H}_{n_1}$$

$$= \frac{1}{2} \times 3\hat{x}$$

$$\bar{H}_{n_2} = 1.5\,\hat{x}$$

NOTE

$$(\bar{H}_{t_1} - \bar{H}_{t_2}) \rightarrow \hat{n}_{12} \rightarrow \bar{J}_s$$

$$\bar{H}_{t_1} - \bar{H}_{t_2} = \hat{n}_{12} \times \bar{J}_s$$

$$\bar{H}_{t_2} = \bar{H}_{t_1} - \hat{n}_{12} \times \bar{J}_s$$

$$= 30\hat{y} - \hat{x} \times 10\hat{y}$$

$$= 30\hat{y} - 10\hat{z}$$

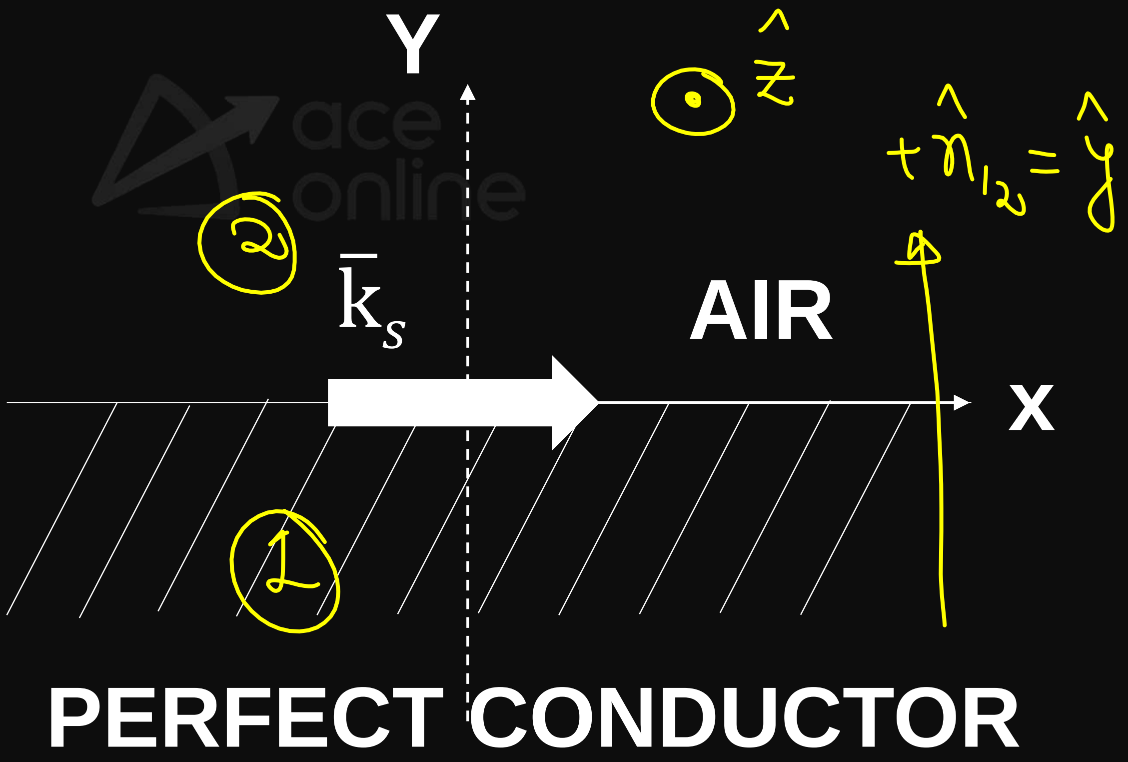
$$\bar{H}_{t_2} = 30\hat{y} - 10\hat{z}$$

$$\bar{H}_2 = \bar{H}_{t_2} + \bar{H}_{n_2}$$

$$\bar{H}_2 = 30\hat{y} - 10\hat{z} + 1.5\hat{x}$$

$$\bar{H}_2 = 1.5\hat{x} + 30\hat{y} - 10\hat{z}$$

Q. The region show below contains a perfect conducting half-space and air. The surface current $\bar{k}_s$ on the surface of the perfect conductor is $\bar{k}_s = 2\hat{x}$ A/m. The tangential $\bar{H}$ filed in the air just above the perfect conductor is



(a) $(\hat{x}+\hat{z})2\ A/m$

(b) $2\hat{x}$ A/m

(c) $-2\hat{z}$ A/m

(d) $2\hat{z}$ A/m

Soln

NOTE

$$\begin{tikzcd}
& (\bar{H}_{t_1} - \bar{H}_{t_2}) \arrow[dr, bend left] & \\
\bar{J}_s \arrow[ur, bend left] & & \hat{n}_{12} \arrow[ll]
\end{tikzcd}$$

$$\underbrace{\bar{H}_{t_1}}_{0} - \bar{H}_{t_2} = \hat{n}_{12} \times \bar{J}_s$$

$$-\bar{H}_{t_2} = \hat{n}_{12} \times \bar{J}_s$$

$$\bar{H}_{t_2} = \bar{J}_s \times \hat{n}_{12}$$

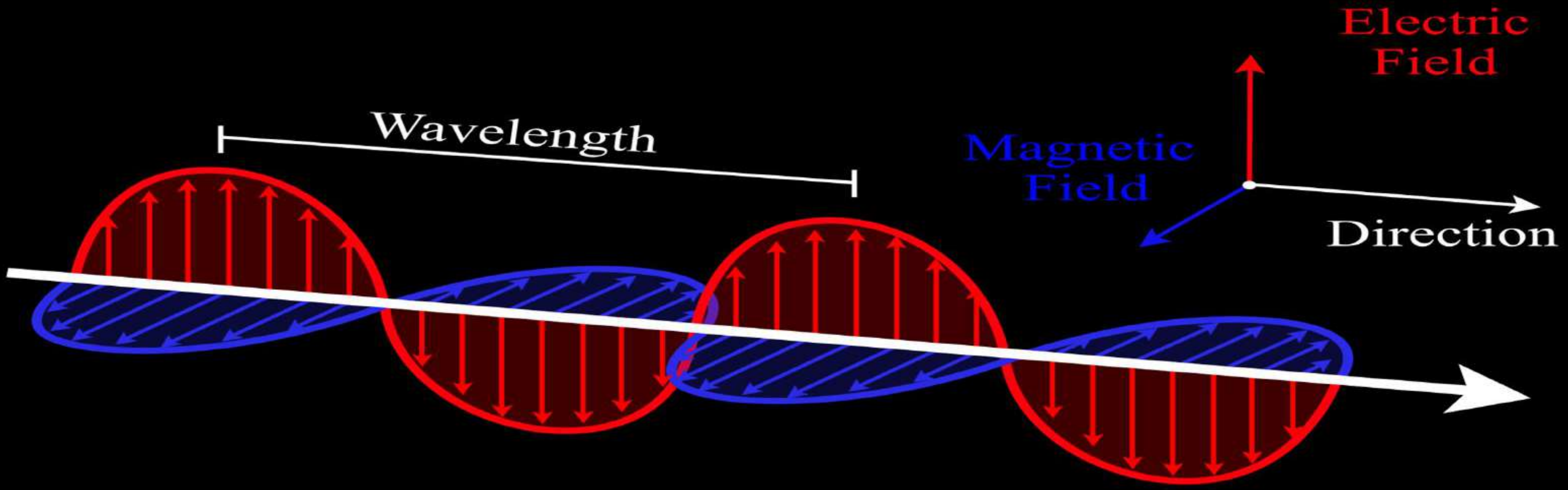
$$\bar{H}_{t_2} = 2\,\hat{x} \times \hat{y}$$

$$\bar{H}_{t_2} = 2\,\hat{z} \quad A/m$$

EM-WAVE PROPAGATION

IN

UN-BOUND MEDIUM



Em-waves

(1) $\bar{E} \quad \bar{H} : \bar{E} \perp \bar{H} \perp \bar{V}$

(2) WAVE EQUATIONS

$$\frac{\partial^2 \bar{E}}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 \bar{E}}{\partial t^2} = 0$$

$$\frac{\partial^2 \bar{H}}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 \bar{H}}{\partial t^2} = 0$$

$$\bar{E}(z,t) \;\Big|\; \bar{H}(z,t)$$

(3) General solutions

$$E(z,t) = f(z \mp vt)$$

$$H(z,t) = \tilde{f}(z \mp vt)$$

(4) Wave variable

$$(z \mp vt)$$

$-$: $+$ve z

$+$: $-$ve z

(5) $\bar{E} \perp \bar{H} \perp \bar{p}$

(6)

$$\hat{E} \to \hat{H} \to \hat{p} \to \hat{E}$$

$$\hat{E} \times \hat{H} = \hat{p} \qquad \hat{E} \cdot \hat{H} = 0$$

$$\hat{H} \times \hat{p} = \hat{E} \qquad \hat{H} \cdot \hat{p} = 0$$

Q. Consider the following wave equation, $\frac{\partial^2 f(x,t)}{\partial t^2} = 10000 \frac{\partial^2 f(x,t)}{\partial x^2}$

Which of the given options is / are solution(s) to the given wave equation?

(GATE-22)

(a) $f(x,t) = e^{-(x-100t)^2} + e^{-(x+100t)^2}$

(b) $f(x,t) = e^{-(x-100t)} + 0.5e^{-(x+1000t)}$

(c) $f(x,t) = e^{-(x-100t)} + \sin(x+100t)$

(d) $f(x,t) = e^{j100\pi(-100x+t)} + e^{j100\pi(100x+t)}$

Soln

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{10^4}\frac{\partial^2 f}{\partial t^2}$$

$$\frac{\partial^2 f}{\partial x^2} - \frac{1}{(100)^2}\frac{\partial^2 f}{\partial t^2} = 0$$

$$\boxed{\begin{gathered}\frac{\partial^2 E}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2} = 0 \\ E = f'(z \mp vt)\end{gathered}}$$

$$f(x,t) = F(x \mp 100t)$$

GENERAL WAVE EQUATIONS

$$\nabla^2 \bar{E} = \mu \sigma \frac{\partial \bar{E}}{\partial t} + \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\nabla^2 \bar{H} = \mu \sigma \frac{\partial \bar{H}}{\partial t} + \mu \epsilon \frac{\partial^2 \bar{H}}{\partial t^2}$$

Example:

For X- directed wave travelling in free space

$\bar{E}(x,t)$ $\quad (\alpha = 0,\ \mu = \mu_0,\ \epsilon = \epsilon_0)$

$\bar{H}(x,t)$

$$\nabla^2 \phi = \frac{\partial^2}{\partial x^2} + \cancel{\frac{\partial^2}{\partial y^2}} + \cancel{\frac{\partial^2}{\partial z^2}} = \frac{\partial^2}{\partial x^2}$$

$$\frac{\partial^2}{\partial x^2}\bar{E} = \mu_0 \epsilon_0 \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\frac{\partial^2}{\partial x^2}\bar{H} = \mu_0 \epsilon_0 \frac{\partial^2 \bar{H}}{\partial t^2}$$

Example:

For Y- directed EM-Wave travelling in material medium

$\bar{E}(y,t) / \bar{H}(y,t)$ (σ, μ, ϵ).

$$\nabla^2 = \frac{\partial^2}{\partial y^2}$$

$$\frac{\partial^2}{\partial y^2}\bar{E} = \mu\sigma\frac{\partial\bar{E}}{\partial t} + \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2}$$

$$\frac{\partial^2}{\partial y^2}\bar{H} = \mu\sigma\frac{\partial\bar{H}}{\partial t} + \mu\epsilon\frac{\partial^2\bar{H}}{\partial t^2}.$$

UPW (+ve z)

1. TEM $(E_z \equiv 0, H_z \equiv 0)$

2. WAVE FORMAT

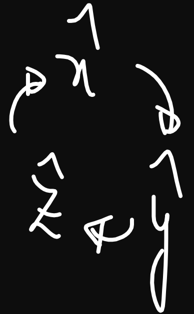
$$\boxed{\bar{E}(z,t) = \bar{E}_0 e^{-\alpha z} e^{j(\omega t - \beta z)}}$$

$\rightarrow \cos(\omega t - \beta z)$

$\rightarrow \sin(\omega t - \beta z)$

$$\boxed{\bar{E}_0 \cdot \hat{z} = 0}$$

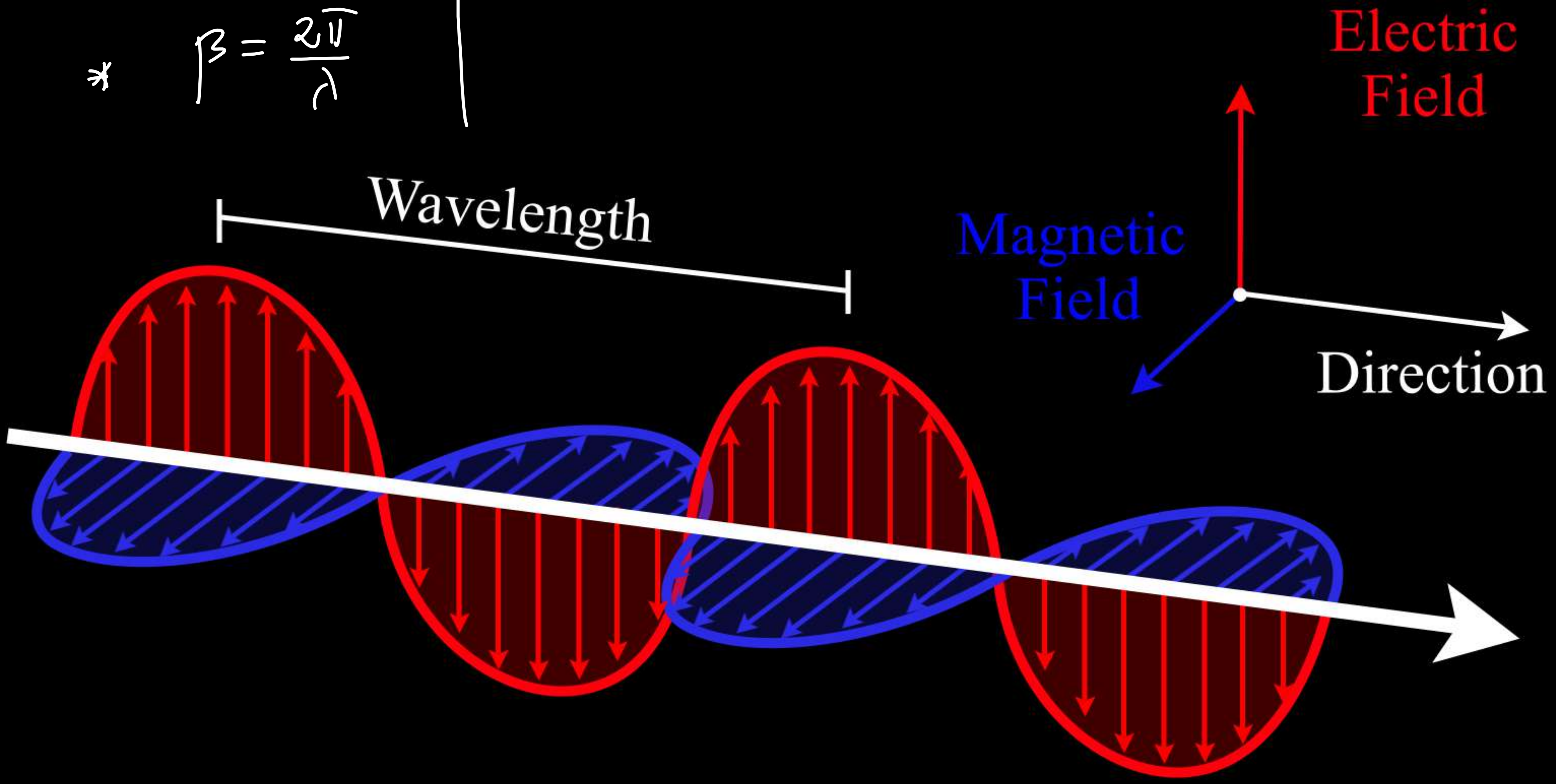
3. INTRINSIC IMPEDANCE



$$\frac{E_x}{H_y} = \frac{-E_y}{H_x} = \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

* $\gamma = \alpha + j\beta$

* $\beta = \frac{2\pi}{\lambda}$



Q. Which one of the following statements is (are) correct for a plane wave with $\bar{H} = 0.5e^{-0.1x}\cos(10^6 t - 2x)\,\hat{a}_z$ A/m

(a) The wave frequency is 10^6 r.p.s

(b) The wavelength is 3.14 m

(c) The wave travels along + x - direction

(d) wave attenuates as it travels $(\alpha = 0.1)$

$(\omega t - \beta x) = (10^6 t - 2x)$

* $\omega = 10^6$ r.p.s

* $\beta = 2 = \frac{2\pi}{\lambda} \Rightarrow \lambda = \pi = 3.14$ m

PHASE VELOCITY (V_P):

$$\boxed{v_p = \frac{\omega}{\beta}}$$

$$v_p = f\lambda$$

Q. An EM-wave in un-known medium has

$$\overline{E(y,t)} = 25\sin\left(10^8 t - y\right)\hat{z}\ V/m$$

The medium is

(a) ~~Free space~~

(b) ~~Lossy-Di-electric~~

(c) Loss-less Di-electric

(d) ~~Good conductor~~

$$v_p = \frac{\omega}{\beta} = \frac{10^8}{1} = 10^8 \neq c = 3\times10^8\ m/s$$

CHARACTERIZATION OF MEDIUM

$$\boxed{\tan\delta = \frac{\sigma}{\omega\epsilon} = \frac{f_q}{f}}$$

WHERE

$$f_q = \frac{\sigma}{2\pi\epsilon}$$

$$\tan\delta = \frac{|J_c|}{|J_D|}$$

* IF $\frac{\sigma}{\omega\epsilon} = 0$. LOSS-LESS
* IF $\frac{\sigma}{\omega\epsilon} << 1$. LOW-LOSS $(f >> f_q)$
* IF $\frac{\sigma}{\omega\epsilon} >> 1$ HIGH-LOSS $(f << f_q)$

	NO-LOSS $\left(\frac{\sigma}{\omega\epsilon}=0\right)$	LOW LOSS $\left(\frac{\sigma}{\omega\epsilon}\ll 1\right)$	HIGH LOSS $\left(\frac{\sigma}{\omega\epsilon}\gg 1\right)$
α	0	$\simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\simeq \sqrt{\frac{\omega\mu\sigma}{2}}$
β	$\omega\sqrt{\mu\epsilon}$	$\simeq \omega\sqrt{\mu\epsilon}$	$\simeq \sqrt{\frac{\omega\mu\sigma}{2}}$
V_p	$\frac{1}{\sqrt{\mu\epsilon}}$	$\simeq \frac{1}{\sqrt{\mu\epsilon}}$	$\simeq \sqrt{\frac{2\omega}{\mu\sigma}}$
η	$\sqrt{\frac{\mu}{\epsilon}}$	$\simeq \sqrt{\frac{\mu}{\epsilon}}$	$\simeq \sqrt{\frac{\omega\mu}{\sigma}}\, e^{j\pi/4}$ $\simeq \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}} = R_s + jX_s$

Q. A material has conductivity of 10^{-2} mho/m and a relative permittivity of 4. The frequency at which the conduction current in the medium is equal to the displacement current is

(GATE - 01)

(a) 45 MHz

(b) 90 MHz

(c) 450 MHz

(d) 900 MHz

Soln

$$f_q = \frac{\sigma}{2\pi\epsilon} = \frac{10^{-2}}{2\pi \times 4 \times \frac{1}{36\pi \times 10^{9}}} = \frac{36}{8} \times 10^{11} = 45\,\text{MHz}$$

Q. Consider the two fields

$\vec{E} = 120\pi\cos(10^6\pi t - \beta x)\hat{a}_y$ V/m

$\vec{H} = A\cos(10^6\pi t - \beta x)\hat{a}_z$ A/m

The values of A and β which will satisfy the Maxwell's equation in a linear isotropic, homogeneous, loss-less medium with $\varepsilon_r = 8$ and $\mu_r = 2$ will be

A (in A/m)	**β (in rad/m)**
(a) 1	0.0105
(b) 1	0.042
(c) 2	0.0105
(d) 2	0.042

Soln

$\beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}$

$= \frac{10^6\pi}{3\times10^8}\sqrt{8\times2} = \frac{10^6\pi\times4}{3\times10^8} = 0.042$

$\beta = 0.042$ rad/m

$$\frac{E_y}{H_Z} = \eta = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}}$$

$$\frac{120\pi}{A} = \sqrt{\frac{\mu_0 2}{\epsilon_0 8}}$$

$$\frac{120\pi}{A} = \frac{120\pi}{2}$$

$$\Rightarrow A = 2$$

SKIN DEPTH (δ m):

- The effective distance travelled by EM-wave inside the lossy medium is described by the concept of skin depth (δ m).

$$\boxed{\delta = \frac{1}{\alpha} = \frac{1}{\beta} = \frac{1}{\alpha R_s} = \frac{1}{\alpha x_s} = \sqrt{\frac{2}{\omega\mu\sigma}}}$$

LOSSY (HIGH-LOSS)

Definition: It is the distance travelled by the EM-Wave inside the lossy medium over which its magnitude falls to 1/e times (37%) of its initial value.

Q. EM-Waves travelling in free space strikes a block of brass ($\sigma = 1.6 \times 10^7$ ℧/m) and with $\eta = 0.05 \angle 45^o$, the effective distance travelled by EM - waves in conducting medium is

Soln

$$\delta = \frac{1}{\sigma R_s} = \frac{1}{\sigma x_s}$$

$$\eta = 0.05\, e^{j\pi/4}$$

$$= 0.05\left[\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right]$$

$$= R_s + j x_s$$

$$\delta = \frac{1}{1.6 \times 10^7 \times \frac{0.05}{\sqrt{2}}} = 1.7 \times 10^{-6}$$

$$\delta = 1.7\ \mu m$$

Q. Skin depth at 2GHz for a good conductor with $\sigma = 4.55\times10^7$ S/m is 1.5 μm. Skin depth (in μm) at 8GHz and 18GHz is

(a) 3.00, 4.50

(b) 0.75, 0.50

(c) 0.375, 0.055

(d) 1.50, 1.00

Soln.

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \propto \frac{1}{\sqrt{f}}$$

$\delta_1 = 1.5\,\mu m$ — $f_1 = 2\,GHz$

$\delta_2 = ?$ — $f_2 = 8\,GHz$

$\delta_3 = ?$ — $f_3 = 18\,GHz$

$$\delta_2 = \delta_1\sqrt{\frac{f_1}{f_2}} = 1.5\,\mu m\sqrt{\frac{2G}{8G}}$$

$$\delta_2 = 0.75\,\mu m$$

$$\frac{\delta_3}{\delta_1} = \sqrt{\frac{f_1}{f_2}} \Rightarrow \delta_3 = 1.5\,\mu\sqrt{\frac{2G}{18G}}$$

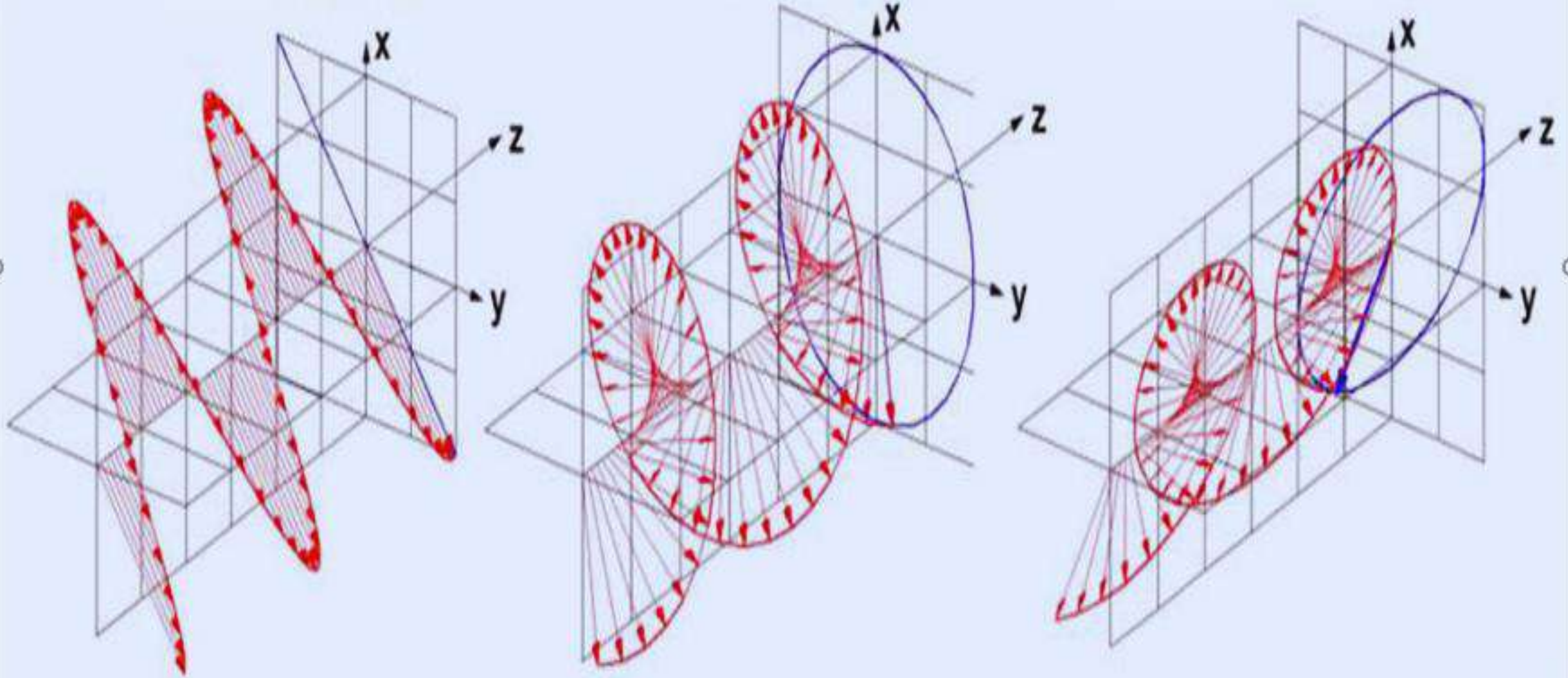
$$\delta_3 = 0.5\,\mu m$$

POLARIZATION

- It is the study of time behavior of EM-waves
- Polarization is always defined with respect to electric field and polarization of magnetic field is obtained by intrinsic impedance relation.

Definition: It is the locus drawn by tip of electric field vector with respect to time.

Linear Polarization | Circular Polarization | Elliptical Polarization



$\delta = \phi_y - \phi_x$ $+ve\ z$	$\delta = 0^o$	$\delta = +\pi/2$	$\delta = -\pi/2$
$E_x\ (E_y = 0)$	LINEAR (X-ORI)		
$E_y\ (E_x = 0)$	LINEAR (Y-ORI)		
$\lvert E_x \rvert = \lvert E_y \rvert$	LINEAR ($\theta = 45^0$)	LH-CP	RH-CP
$\lvert E_x \rvert \neq E_y \rvert$	LINEAR ($\theta = \tan^{-1}\left[\frac{\lvert E_y \rvert}{\lvert E_x \rvert}\right]$)	LH-EP	RH-EP

STEPS TO IDENTITY TYPE OF POLARIZATION

1. Number of components
2. Phase difference and magnitudes
3. Direction of propagation

5. $\bar{E} = 40\sin(wt - \cancel{\beta y})\,\hat{x} + 50\cos(wt - \cancel{\beta y})\,\hat{z}$ v/m Ep

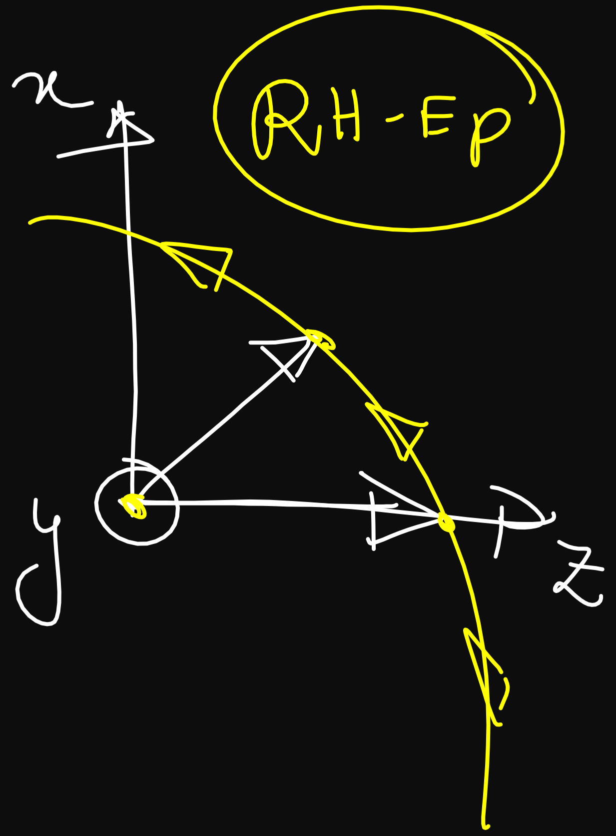
$\bar{E} = 40\sin\omega t\,\hat{x} + 50\cos\omega t\,\hat{z}$

AT $t = 0,\ \omega t = 0$

$\bar{E} = \cancel{40}\ 0\,\hat{x} + 50\,\hat{z}$

AT $t = T/8,\ \omega t = \frac{2\pi}{T}t = \frac{\pi}{4}$

$\bar{E} = \frac{40}{\sqrt{2}}\hat{x} + \frac{50}{\sqrt{2}}\hat{z}$



10. $\bar{E} = 20\sin(wt - \beta y)\hat{x} + 30\sin(wt - \beta y + 45^o)\hat{z}$ Ep

~~AT t=0, wt=0~~

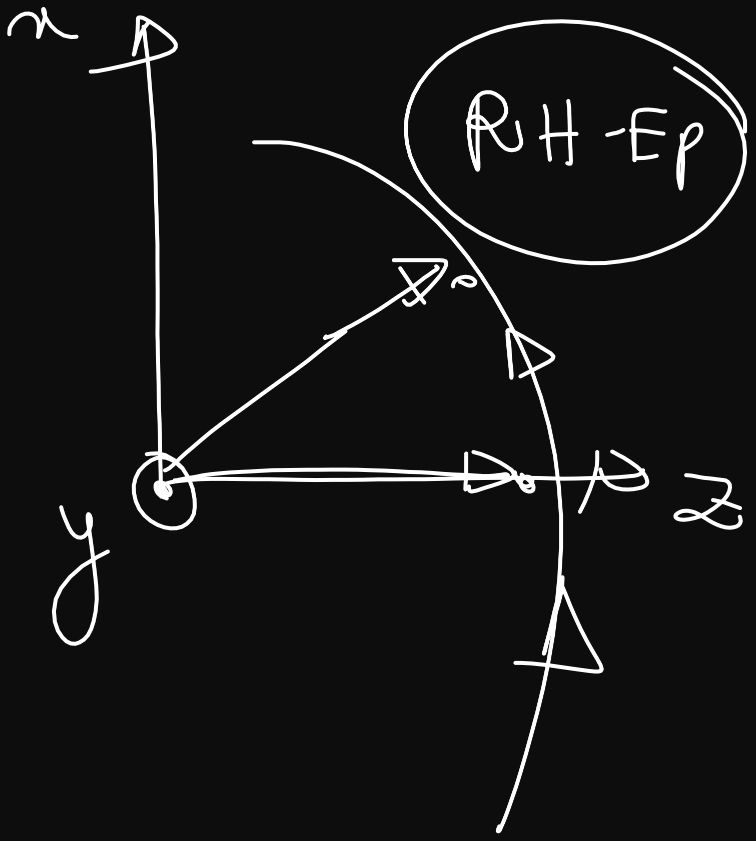
$\bar{E} = 20\sin wt\,\hat{x} + 30\sin(wt + 45^\circ)\hat{z}$

AT $t=0,\ wt=0$

$\bar{E} = 0\hat{x} + \frac{30}{\sqrt{2}}\hat{z}$

AT $t = T/8,\ wt = \pi/4$

$\bar{E} = \frac{20}{\sqrt{2}}\hat{x} + 30\hat{z}$



12. $\bar{E} = 20\cos(wt + \beta z)\hat{x} + 30\sin(wt + \beta z)\hat{y}\ v/m$ CP

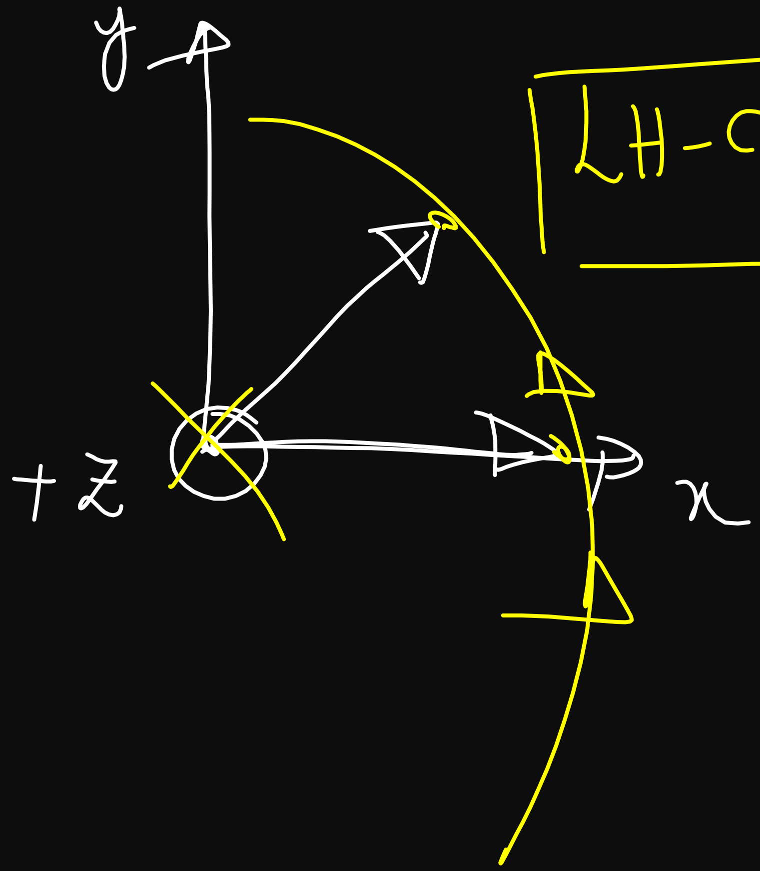
$\bar{E} = 30\cos wt\,\hat{x} + 30\sin wt\,\hat{y}$

AT $t=0$, $wt=0$

$\bar{E} = 30\hat{x} + 0\,\hat{y}$

AT $t = T/8$, $wt = \pi/4$

$\bar{E} = \frac{30}{\sqrt{2}}\hat{x} + \frac{30}{\sqrt{2}}\hat{y}$



LH-CP

POWER FLOW IN EM-FIELDS / EM-WAVES

① $\vec{P}_{avg} = \frac{1}{2}\mathrm{Re}\{\bar{E}\times\bar{H}^{*}\}$ Watts/m^2

② $W_{avg} = \oint \vec{P}_{avg}\cdot\bar{dA}$ Watts

$= \frac{V_m}{\sqrt{2}}\cdot\frac{I_m}{\sqrt{2}}$

$P_{avg} = \frac{V_m I_m}{\sqrt{2}}$ watts

③ UPW (+ve z)

$$\vec{P}_{avg} = \frac{|\bar{E}|^2}{2\eta}\hat{z} = \eta\frac{|\bar{H}|^2}{2}\hat{z} \text{ Watts/}m^2.$$

Q. In free space $\bar{E}(x,t) = 60\cos(\omega t - 2x)\hat{a}_y$ V/m.

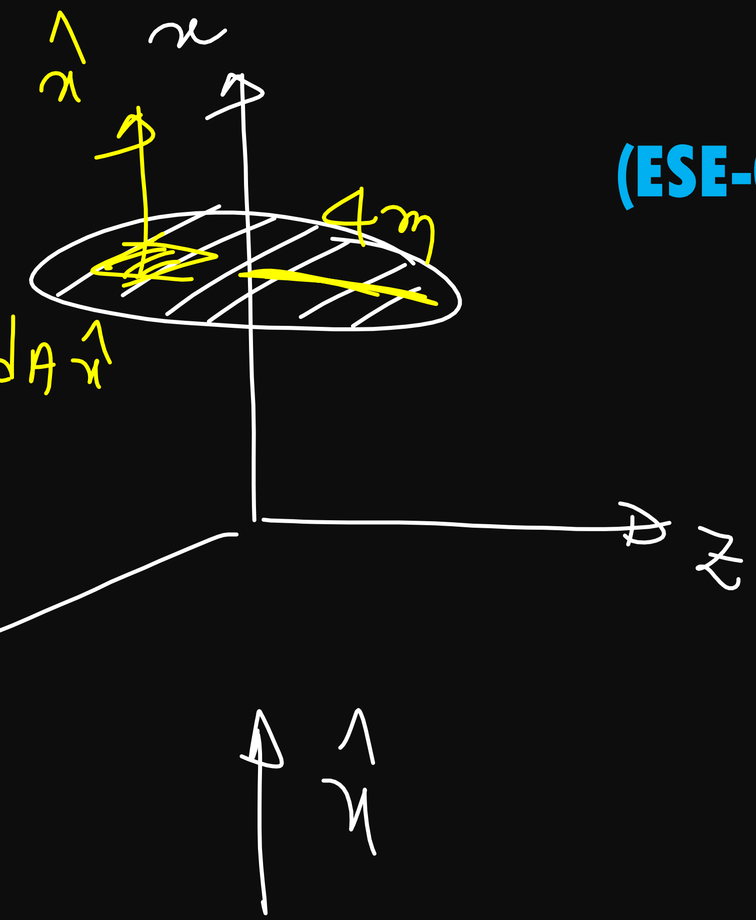
What is the average power crossing a circular area of radius 4 m in the plane x = constant ?

(ESE-04)

(a) 480 W
(b) 240 W
(c) 120 W
(d) 60 W

Soln

$\bar{dA} = dA\,\hat{x}$



$$\vec{P}_{avg} = \frac{|E|^2}{2\eta_0}\hat{n} = \frac{60\times60}{2\times120\pi}\hat{n}$$

$$|W|_{avg} = \oiint \vec{P}_{avg}\cdot\overline{dA}$$

$$= \iint \frac{60\times60}{2\times120\pi}\hat{n}\cdot dA\,\hat{n}$$

$$= \frac{60\times60}{2\times120\pi}\underbrace{\iint dA}_{\pi\times4^2}$$

$$|W|_{avg} = \frac{60\times60}{2\times120\pi}\times\pi\times4^2 = \frac{\overset{30\;\;15}{\cancel{60}\times\cancel{60}}\times16}{\cancel{2}\times\underset{2}{\cancel{120}}}$$

$$|W|_{avg} = 15\times16 = 240\ \text{Watts}$$

Q. If $\bar{E} = (\hat{a}_x + j\hat{a}_y)e^{jkz-j\omega t}$ and $\bar{H} = \left(\frac{k}{\omega\mu}\right)(\hat{a}_y + j\hat{a}_x)e^{jkz-j\omega t}$

the time averaged poynting vector is (GATE - 2004)

(a) Null vector

(b) $\left(\frac{k}{\omega\mu}\right)\hat{a}_z$

(c) $\left(\frac{2k}{\omega\mu}\right)\hat{a}_z$

(d) $\left(\frac{k}{2\omega\mu}\right)\hat{a}_z$

$* \; \vec{P}_{avg} = \frac{1}{2}\mathrm{Re}\{\bar{E}\times\bar{H}^*\}$ W/m²

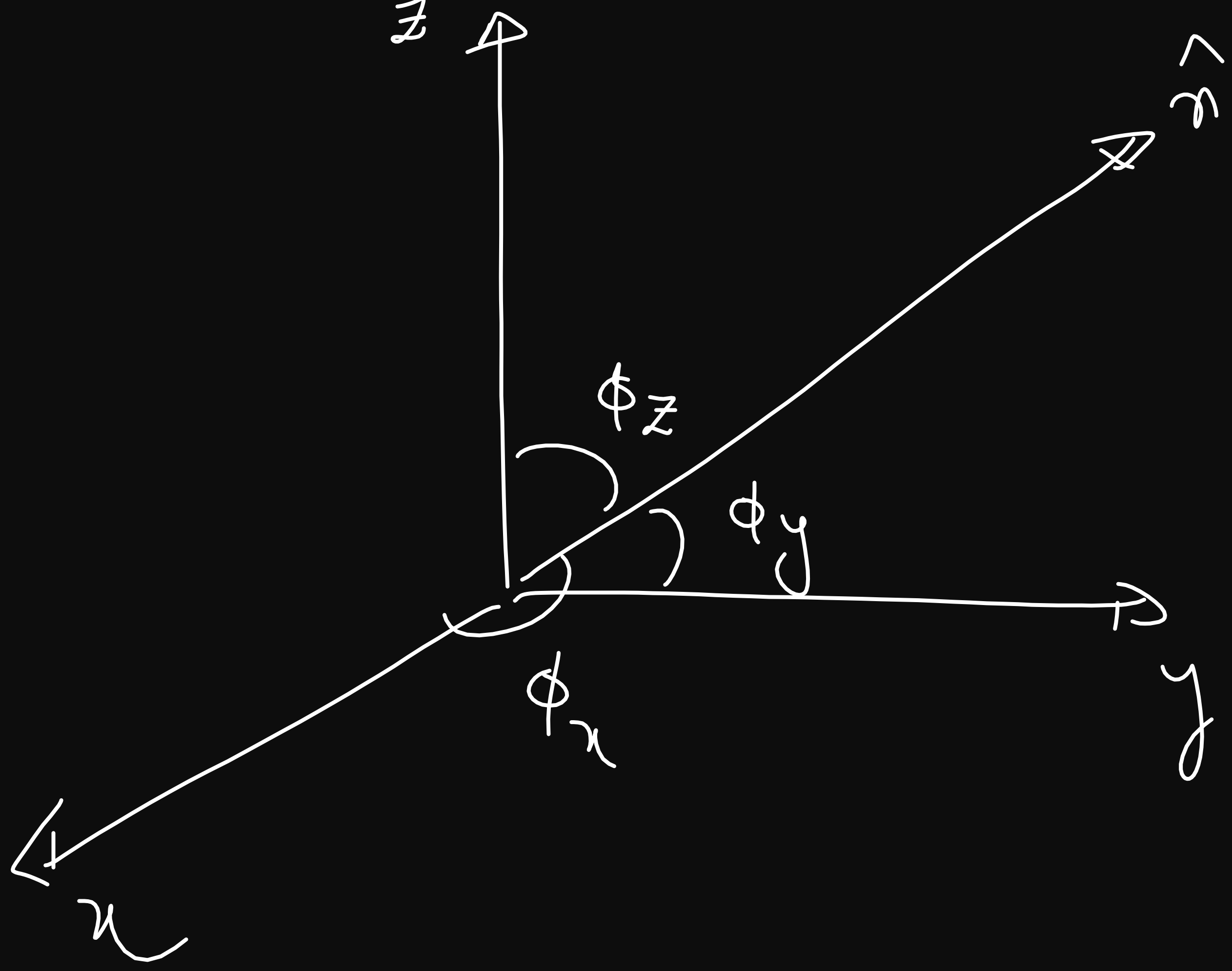
$\bar{E} = (\hat{x} + j\hat{y})\, e^{jkz-j\omega t}$

$\bar{H}^* = \left(\frac{k}{\omega\mu}\right)^*(\hat{y} - j\hat{x})\, e^{-jkz+j\omega t}$

$\bar{E}\times\bar{H}^* = \left(\frac{k}{\omega\mu}\right)^*\left\{\hat{z} + j\hat{y}\times(-j\hat{x})\right\} = \left(\frac{k}{\omega\mu}\right)^*\left\{\hat{z} - \hat{z}\right\} = 0.$

$\vec{P}_{avg} = 0$

UPW IN ARBITRARY DIRECTION



(+ve z)

$$\bar{E} = \bar{E}_0 e^{j(\omega t - \beta \cancel{z})}$$

CONCLUSIONS

(1) PHASE CONSTANT FORM:

$$\bar{E} = \bar{E}_0 e^{j(\omega t - \beta_x x - \beta_y y - \beta_z z)}, \quad \boxed{\bar{E}_0 \cdot \hat{n} = 0}$$

$$\beta_x = \beta \cos\phi_x, \quad \beta_y = \beta \cos\phi_y, \quad \beta_z = \beta \cos\phi_z$$

(2) WAVE VECTOR FORM:

$$\bar{E} = \bar{E}_0 e^{j(\omega t - \bar{K} \cdot \bar{R})}, \quad \boxed{\bar{E}_0 \cdot \hat{K} = 0}$$

$$\bar{K} = \beta_x \hat{x} + \beta_y \hat{y} + \beta_z \hat{z}$$

$$\bar{R} = x\hat{x} + y\hat{y} + z\hat{z}$$

(3) WAVE VECTOR:

$$\bar{K} = \beta_x \hat{x} + \beta_y \hat{y} + \beta_z \hat{z}$$

(4) MAGNITUDE:

$$|\bar{K}| = \beta \text{ rad/m}$$

(5) DIRECTION.

$$\hat{K} = \hat{n} = \cos\phi_x \hat{x} + \cos\phi_y \hat{y} + \cos\phi_z \hat{z}$$

(6) DIRECTION OF PROPAGATION

$$\hat{n} = \cos\phi_x \hat{x} + \cos\phi_y \hat{y} + \cos\phi_z \hat{z}$$

(7) MAGNETIC FIELD

$$\bar{H} = \frac{1}{\omega\mu} \bar{K} \times \bar{E}$$

Q.A plane wave of wavelength λ is travelling in a direction making an angle 30° with +ve X-axis and 90° with +ve Y-axis. The $\bar{E}$ field of the plane wave can be represented as (E_0 is a constant)

(GATE - 2007)

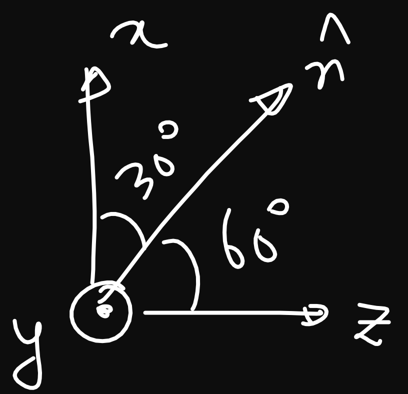
(a) $\bar{E} = \hat{y}E_0 e^{j\left(\omega t - \frac{\sqrt{3}\pi}{\lambda}x - \frac{\pi}{\lambda}z\right)}$

(c) $\bar{E} = \hat{y}E_0 e^{j\left(\omega t + \frac{\sqrt{3}\pi}{\lambda}x + \frac{\pi}{\lambda}z\right)}$

(d) $\bar{E} = \hat{y}E_0 e^{j\left(\omega t - \frac{\pi}{\lambda}x + \frac{\sqrt{3}\pi}{\lambda}z\right)}$

(b) $\bar{E} = \hat{y}E_0 e^{j\left(\omega t - \frac{\pi}{\lambda}x - \frac{\sqrt{3}\pi}{\lambda}z\right)}$

Soln



$$\phi_x = 30^\circ, \ \phi_y = 90^\circ, \ \phi_z = 60^\circ$$

$$\beta_x = \beta \cos\phi_x = \frac{2\pi}{\lambda}\,\frac{\sqrt{3}}{2} = \frac{\sqrt{3}\pi}{\lambda}$$

$$\beta_y = \beta\cos\phi_y = 0$$

$$\beta_z = \beta\cos\phi_z = \frac{2\pi}{\lambda}\,\frac{1}{2} = \frac{\pi}{\lambda}$$

$$\bar{E} = \bar{E}_0\, e^{j\left\{\omega t - \frac{\sqrt{3}\pi}{\lambda}x - 0y - \frac{\pi}{\lambda}z\right\}}$$

$$\bar{E} = \bar{E}_0\, e^{j\left(\omega t - \frac{\sqrt{3}\pi}{\lambda}x - \frac{\pi}{\lambda}z\right)}$$

Q. The expression for an electric field in free space is $E = E_0(\hat{x} + \hat{y} + j2\hat{z})e^{-j(\omega t - 4x + 4y)}$, where x, y, z represent the spatial coordinates, t represents time, and ω, k are constants. This electric field

Soln

$$\bar{E} = E_0(\hat{x} + \hat{y} + j2\hat{z})e^{-j\{\omega t - (4\hat{x} - 4\hat{y})\cdot(x\hat{x} + y\hat{y})\}}$$

$$\bar{k} = 4\hat{x} - 4\hat{y}$$

$$\hat{k} = \hat{n} = \left(\frac{\hat{x} - \hat{y}}{\sqrt{2}}\right)$$

$$\bar{E}_0 \cdot \hat{n} = 0$$

$$E_0\left(\hat{x} + \hat{y} + 2j\hat{z}\right) \cdot \left(\frac{\hat{x} - \hat{y}}{\sqrt{2}}\right)$$

$$= E_0\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) = \underline{\underline{0}}$$

$$\bar{E} = E_0\left(\hat{x} + \hat{y} + 2j\hat{z}\right)e^{-j(\omega t - 4x + 4y)}$$

$\parallel^{le}$ $\quad$ $\perp^{le}$

$\sqrt{1+1}$ $\quad$ $|2j|$

$= \sqrt{2}$ $\quad$ 2

$\pm 90^\circ$

$\Rightarrow$ Ep

~~(a)~~ does not represent a plane wave.

(b) represents a circularly polarized plane wave propagating normal to the z-axis.

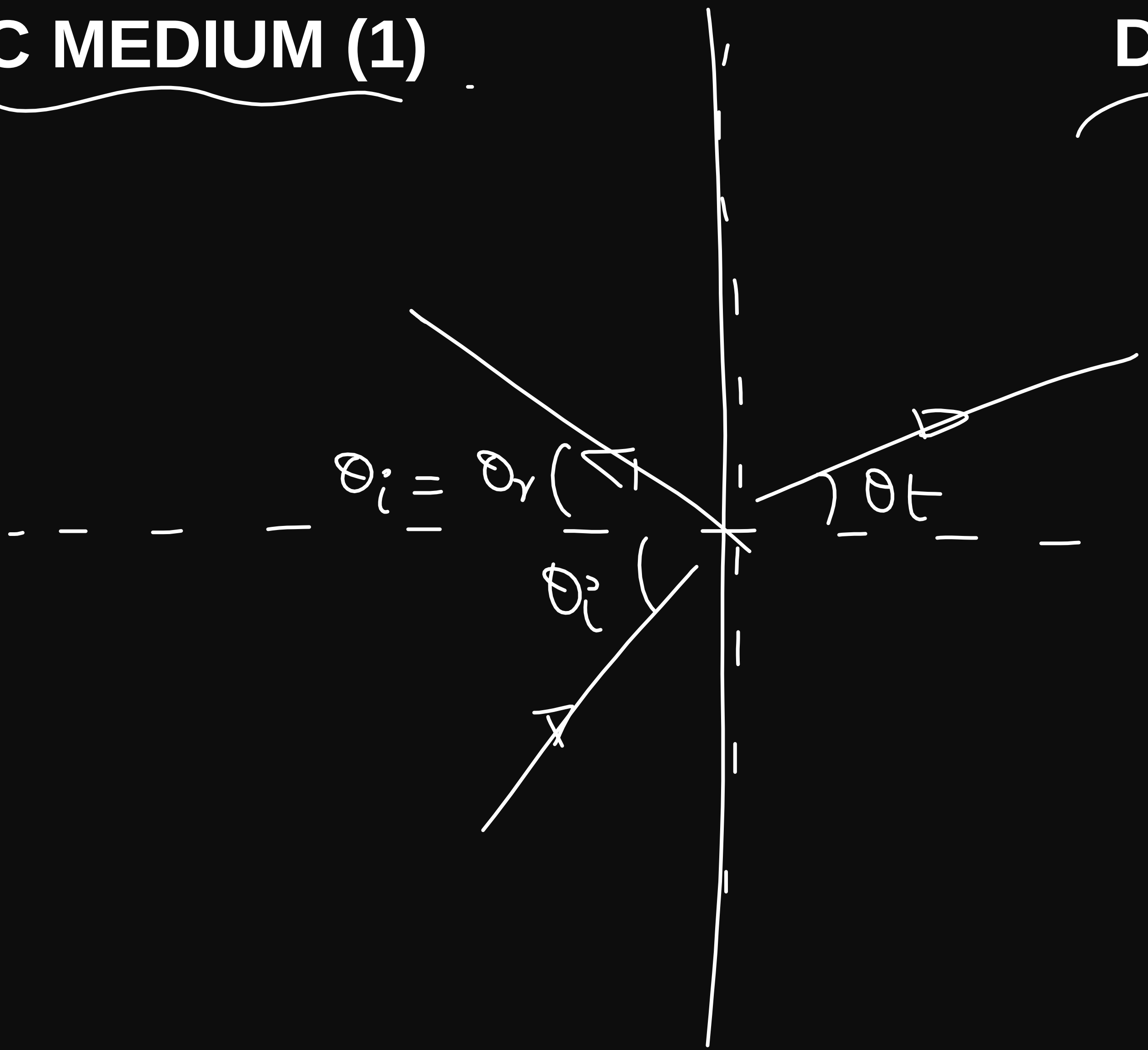
(c) represents an elliptically polarized plane wave propagating along the x-y plane.

(d) represents a linearly polarized plane wave.

UPW AT DIELECTRIC MEDIA INTERFACE

DIELECTRIC MEDIUM (1)

DIELECTRIC MEDIUM (2)



$$\sqrt{\epsilon_{r_1}}\,\sin\theta_i = \sqrt{\epsilon_{r_2}}\,\sin\theta_t$$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

Q. Assume that a plane wave in air with an electric field $\bar{E} = 10\cos(\omega t - 3x - \sqrt{3}z)\hat{a}_y$ is incident on a non-magnetic dielectric slab of relative permittivity 3 which covers the region z > 0. The angle of transmission in the dielectric slab is ______ degrees.

Soln $\bar{E} = 10\cos\{\omega t - (3\hat{x} + \sqrt{3}\hat{z})\cdot(x\hat{x} + z\hat{z})\}\hat{y}$ **(GATE - 14)(Set3)**

$\bar{K} = 3\hat{x} + \sqrt{3}\hat{z}$

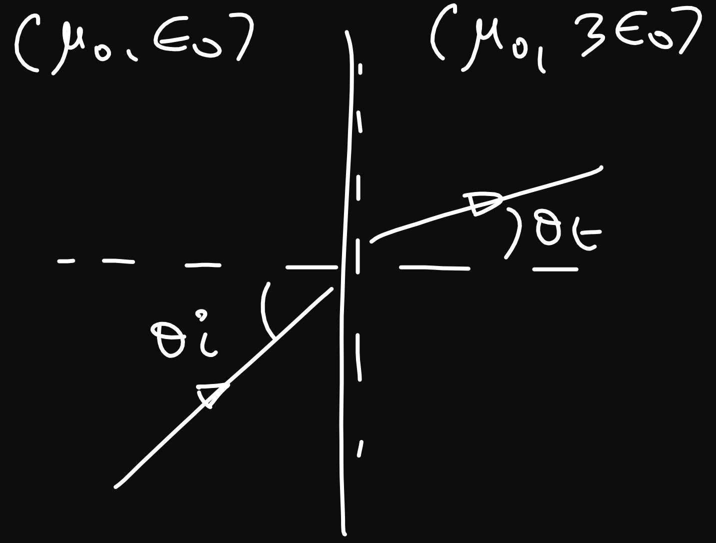
$\hat{n}_i = \hat{K}_i = \dfrac{3\hat{x} + \sqrt{3}\hat{z}}{\sqrt{9+3}} = \dfrac{\sqrt{3}}{2}\hat{x} + \dfrac{1}{2}\hat{z}$

FOR θ_i:

$\hat{n}_i \cdot \hat{n}_n = \cos\theta_i$

$\left(\dfrac{\sqrt{3}}{2}\hat{x} + \dfrac{1}{2}\hat{z}\right)\cdot\hat{z} = \cos\theta_i$

$\cos\theta_i = \dfrac{1}{2} \Rightarrow \theta_i = 60^\circ$



$$\sqrt{\epsilon_{r_1}}\,\sin\theta_i = \sqrt{\epsilon_{r_2}}\,\sin\theta_t$$

$$\sqrt{1}\,\sin 60^\circ = \sqrt{3}\,\sin\theta_t$$

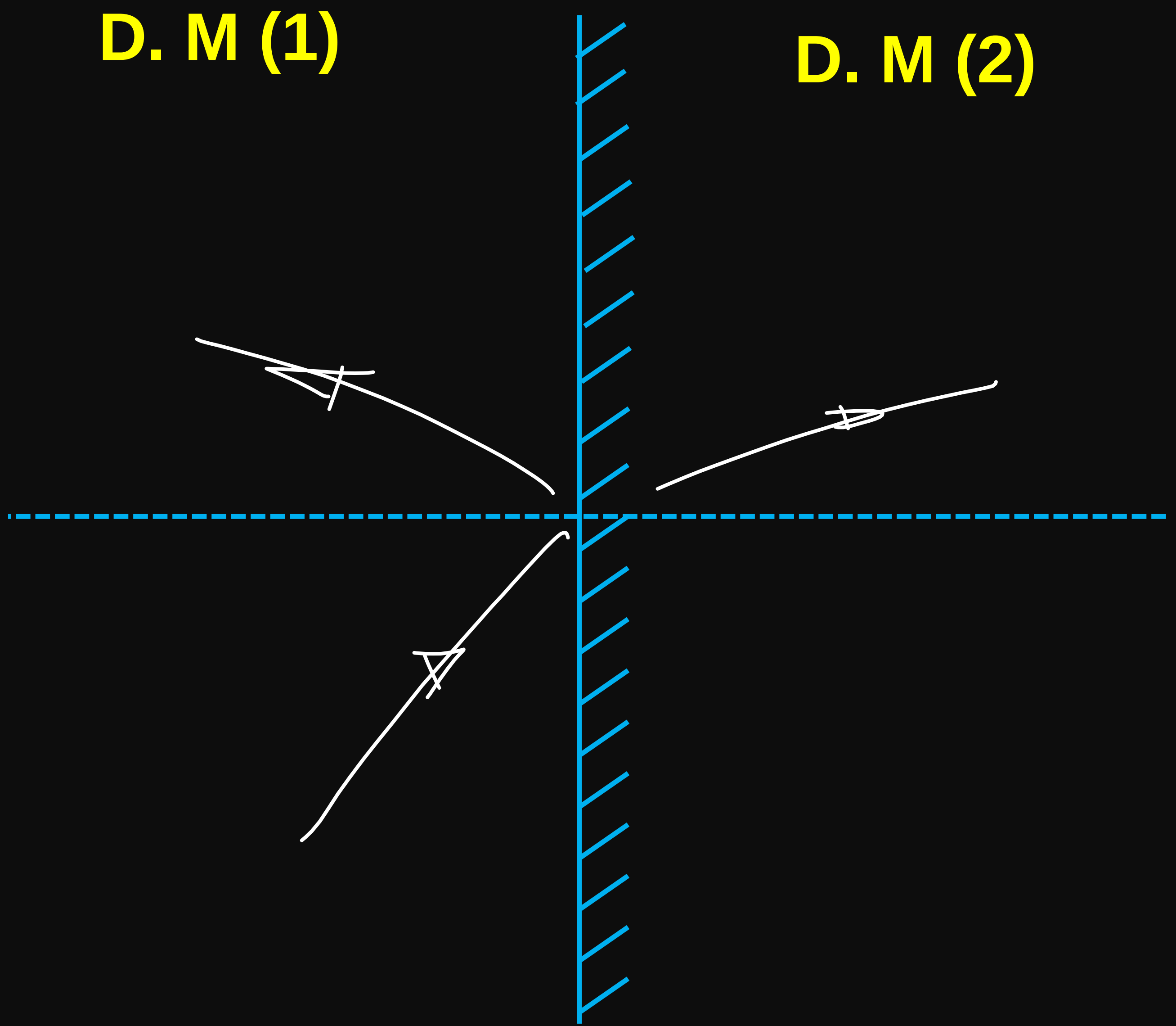
$$\frac{\sqrt{3}}{2} = \sqrt{3}\,\sin\theta_t$$

$$\sin\theta_t = \frac{1}{2}$$

$$\cancel{\theta_t = 60^\circ}$$

$$\underline{\theta_t = 30^\circ}$$

TRANSMISSION AND REFLECTION CO-EFFICIENTS



REFLECTION CO-EFFICIENT

$$\rho = \frac{E_r}{E_i}$$

TRANSMISSION CO-EFFICIENT

$$\tau = \frac{E_t}{E_i}$$

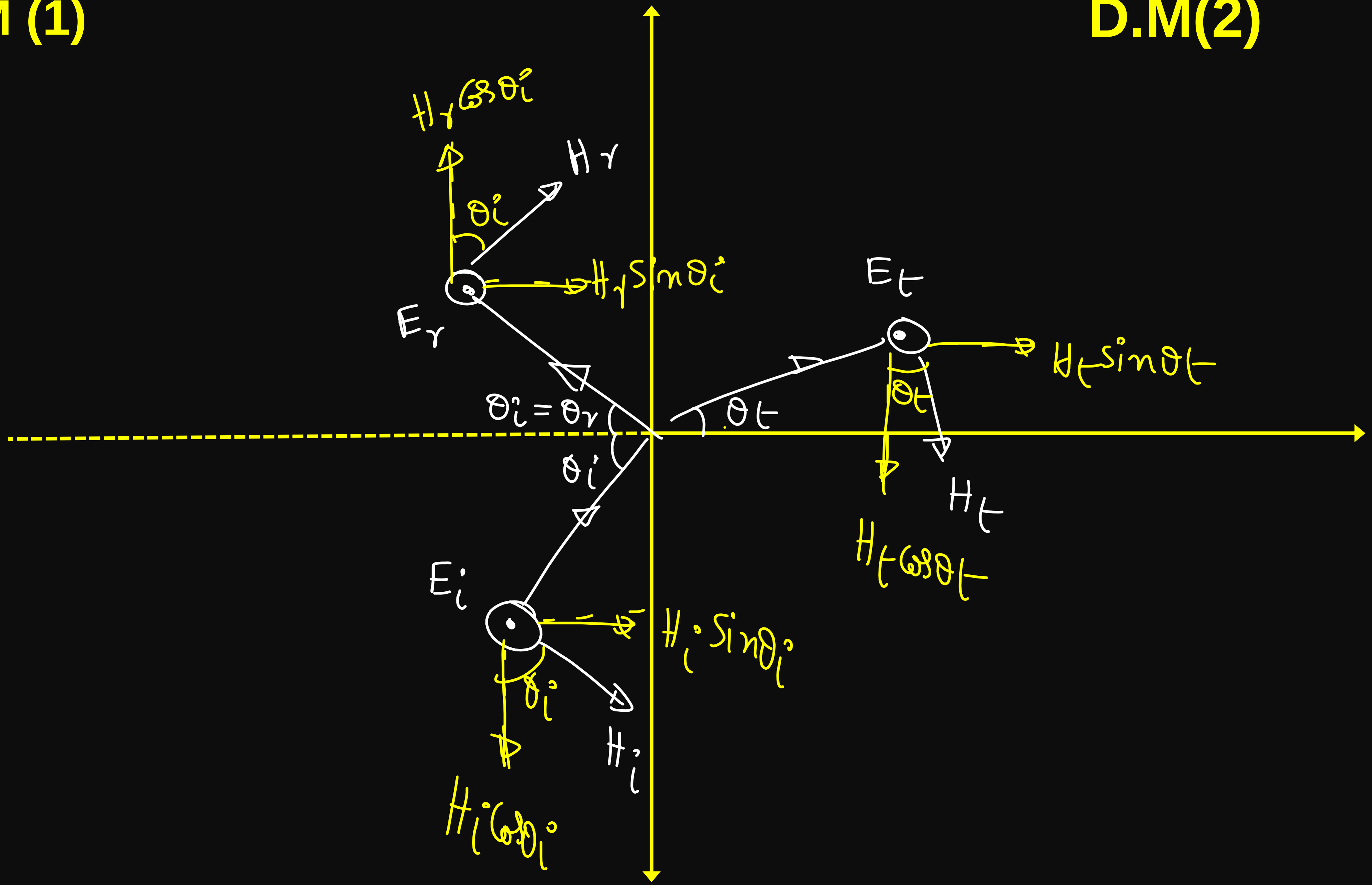
CASE (I) :

REFLECTION AND TRANSMISSI CO-EFFICIENTS

FOR HORIZONTAL (OR) PERPENDICULARLY ($\perp^{le}$)

POLARIZED EM-WAVE AT OBLIQUE INCIDENCE.

D.M (1) **D.M(2)**



$H_r \cos\theta_i$
H_r
θ_i
$H_r \sin\theta_i$
E_r
E_t
$H_t \sin\theta_t$
θ_t
$\theta_i = \theta_r$
θ_t
θ_i
H_t
$H_t \cos\theta_t$
E_i
$H_i \sin\theta_i$
θ_i
H_i
$H_i \cos\theta_i$

REFLECTION CO-EFFICIENT

$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

TRANSMISSION CO-EFFICIENT

$$\tau_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

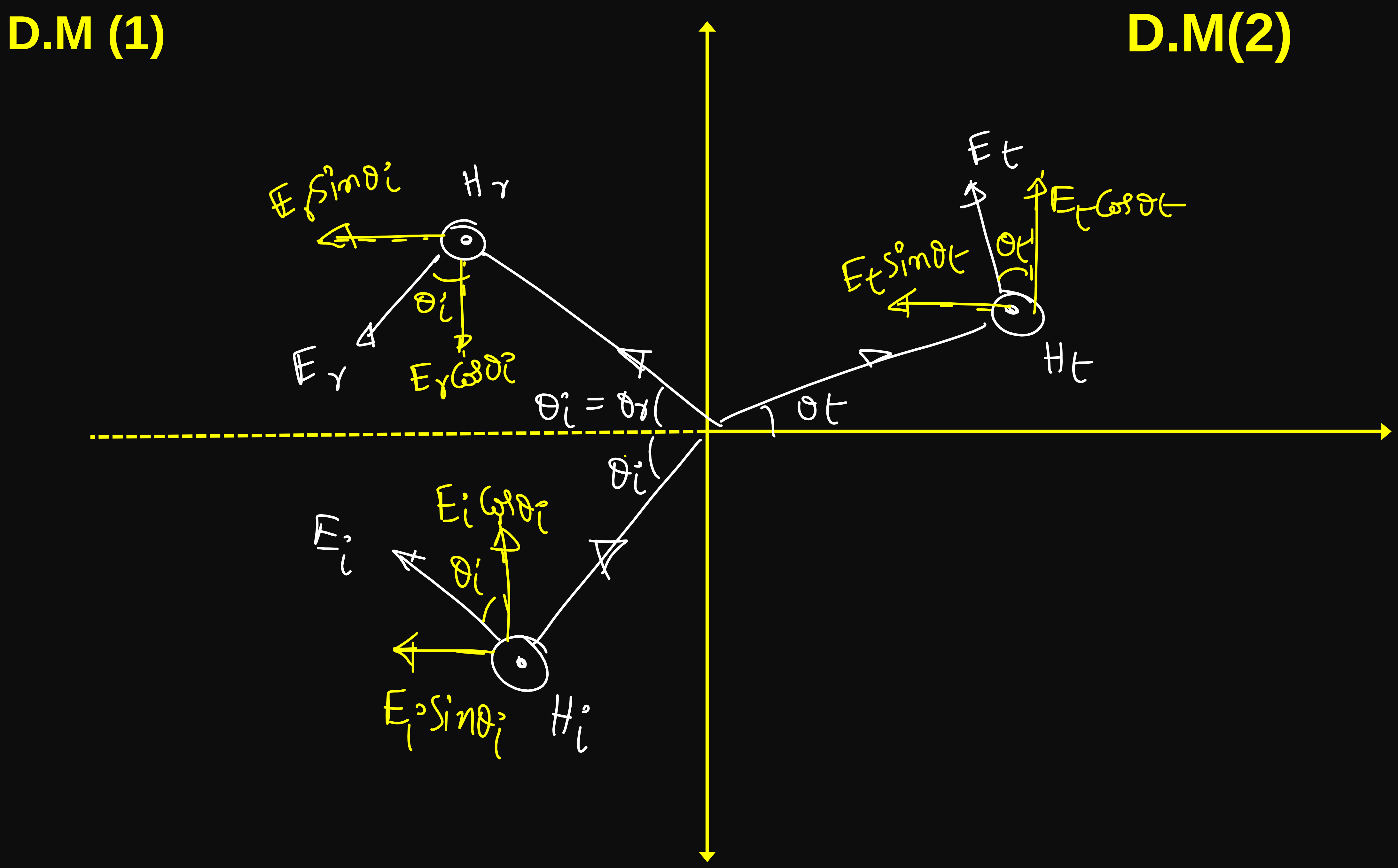
CASE (II) :

REFLECTION AND TRANSMISSION CO-EFFICIENTS FOR VERTICALL (or) PARALLEL ($\parallel^{le}$) POLARIZED EM-WAVE AT OBLIQUE INCIDENCE.

NOTE : IF ELECTRIC FIELD IS PARALLEL POLARIZED ( )

THEN MAGNETIC FIELD WILL BE OF PERPENDICULAR

POLARIZATION ( ).


D.M (1)
D.M(2)
E sinθi
Hr
Er
Er cosθi
θi
θi = θr
θi
Ei cosθi
Ei
θi
Ei sinθi
Hi
Et
Et cosθt
Et sinθt
θt
Ht
θt

REFLECTION CO-EFFICIENT

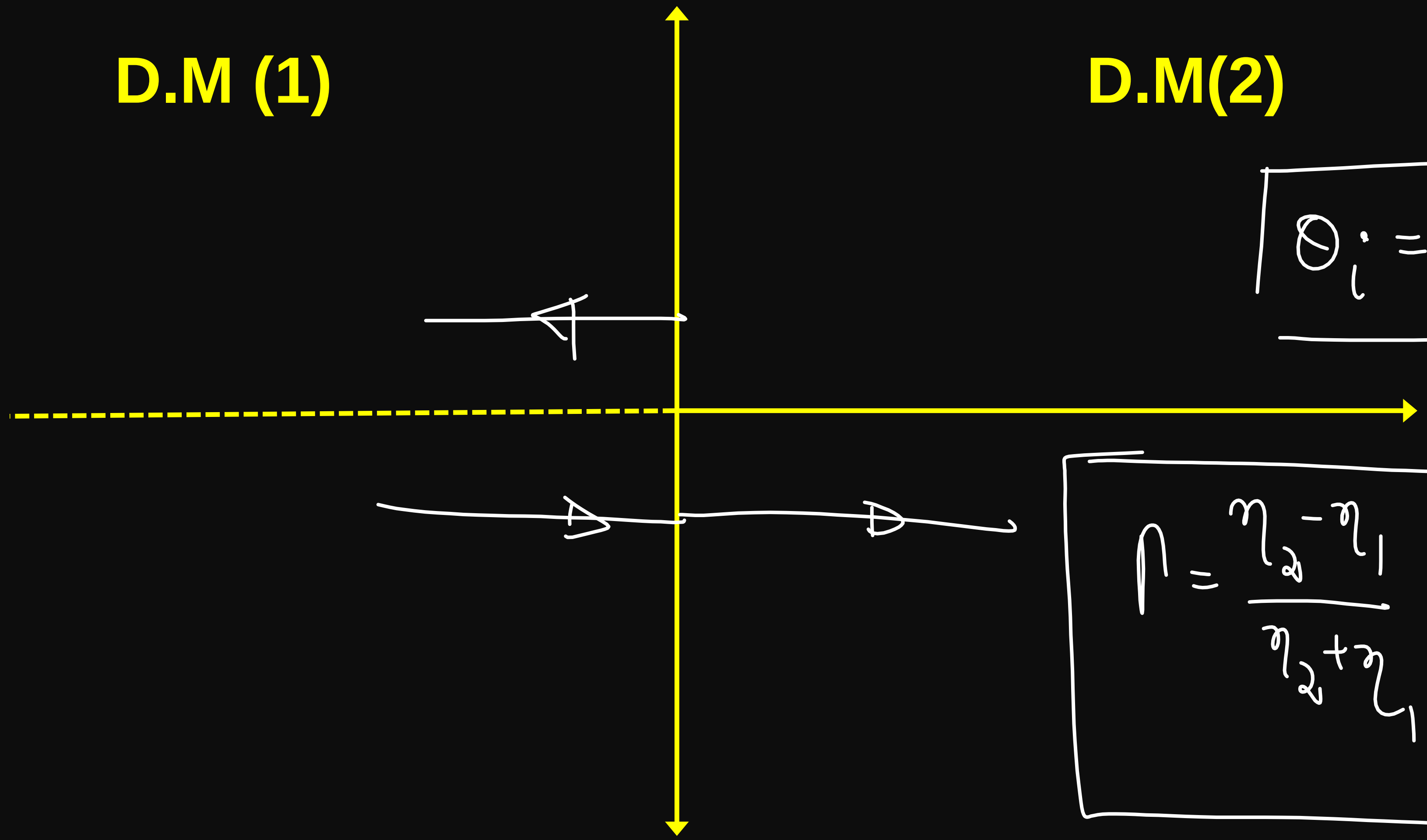
$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

TRANSMISSION CO-EFFICIENT

$$\tau_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

CASE (III) : REFLECTION AND TRANSMISSION CO-EFFICIENTS

FOR NORMAL INCIDENCE



$$\boxed{\theta_i = \theta_r = \theta_t = 0^\circ}$$

$$\boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \quad \tau = \frac{2\eta_2}{\eta_2 + \eta_1}}$$

Q.7 A plane wave having the electric field component

$$\vec{E}_i = 24\cos(3 \times 10^8 t - \beta y)\hat{a}_z \text{ V/m}$$

and traveling in free space is incident normally on a lossless medium with $\mu = \mu_0$ and $\varepsilon = 9\varepsilon_0$ which occupies the region $y \geq 0$. The reflected magnetic field component is given by

Soln

(μ_0, ϵ_0) | $(\mu_0, 9\epsilon_0)$

$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \eta_0 \qquad \eta_2 = \sqrt{\frac{\mu_0}{9\epsilon_0}} = \frac{\eta_0}{3}$$

$$\beta_1 = \omega\sqrt{\mu_0\epsilon_0} = 3\times10^8 \times \frac{1}{3\times10^8}$$

$$\beta_1 = 2$$

$$\Gamma = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_0/3 - \eta_0}{\eta_0/3 + \eta_0} = -\frac{1}{2}$$

$$\bar{E}_r = -\frac{1}{2}\bar{E}_i = -\frac{1}{2} \times 24\cos(3\times10^8 t + \beta y)\hat{z}$$

$$\bar{E}_r = -12\cos(3\times10^8 t + y)\hat{z}$$

(a) $\frac{1}{10\pi}\cos(3\times10^8 t + y)\hat{a}_x$ A/m

(b) $\frac{1}{20\pi}\cos(3\times10^8 t + y)\hat{a}_x$ A/m

(c) $-\frac{1}{20\pi}\cos(3\times10^8 t + y)\hat{a}_x$ A/m

(d) $-\frac{1}{10\pi}\cos(3\times10^8 t + y)\hat{a}_x$ A/m

Upward (-ve y)

$\hat{x} \to \hat{y} \to \hat{z}$

$$-\frac{E_z}{H_x} = \eta, = 120\pi$$

$$H_x = \frac{-E_z}{120\pi}$$

$$H_x = \frac{-1}{120\pi}\{-12\cos(3\times10^8 t + y)\}$$

$$H_x = \frac{1}{10\pi}\cos(3\times10^8 t + y)$$

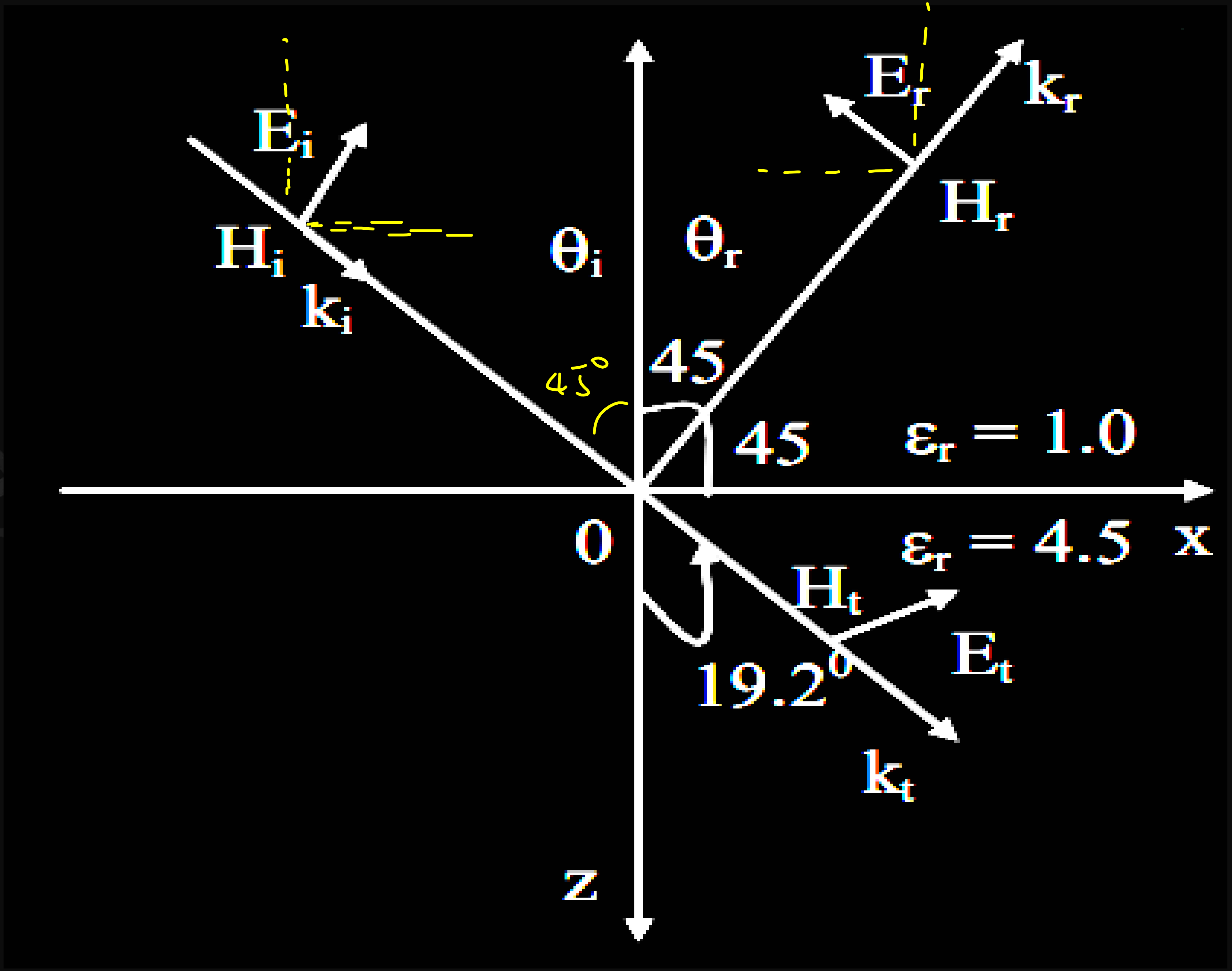
$$\bar{H} = \frac{1}{10\pi}\cos(3\times10^8 t + y)\hat{x} \text{ A/m}.$$

Statement for Linked Answer Questions 1 & 2

A monochromatic plane wave of wavelength $\lambda = 600\mu m$ is propagating in the direction as shown in the figure below.

$\vec{E}_i, \vec{E}_r$ and $\vec{E}_t$ denote incident, reflected, and transmitted electric field vectors associated with the wave.

(GATE-13)



E_i
H_i
k_i
θ_i
θ_r
E_r
k_r
H_r
45°
45
45
ε_r = 1.0
0
ε_r = 4.5
x
H_t
E_t
19.2°
k_t
z

1. The angle of incidence θ_i and the expression for $\vec{E}_i$ are

(a) 60^o and $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times10^4(x+z)}{3\sqrt{2}}}$ V/m

(b) 45^o and $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times10^4 z}{3}}$ V/m

(c) 45^o and $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times10^4(x+z)}{3\sqrt{2}}}$ V/m

(d) 60^o and $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times10^4 z}{3}}$ V/m

2. The expression for $\vec{E}_r$ is

(a) $-0.23\frac{E_0}{\sqrt{2}}(\hat{a}_x+\hat{a}_z)e^{-j\frac{\pi\times10^4(x-z)}{3}}$ V/m

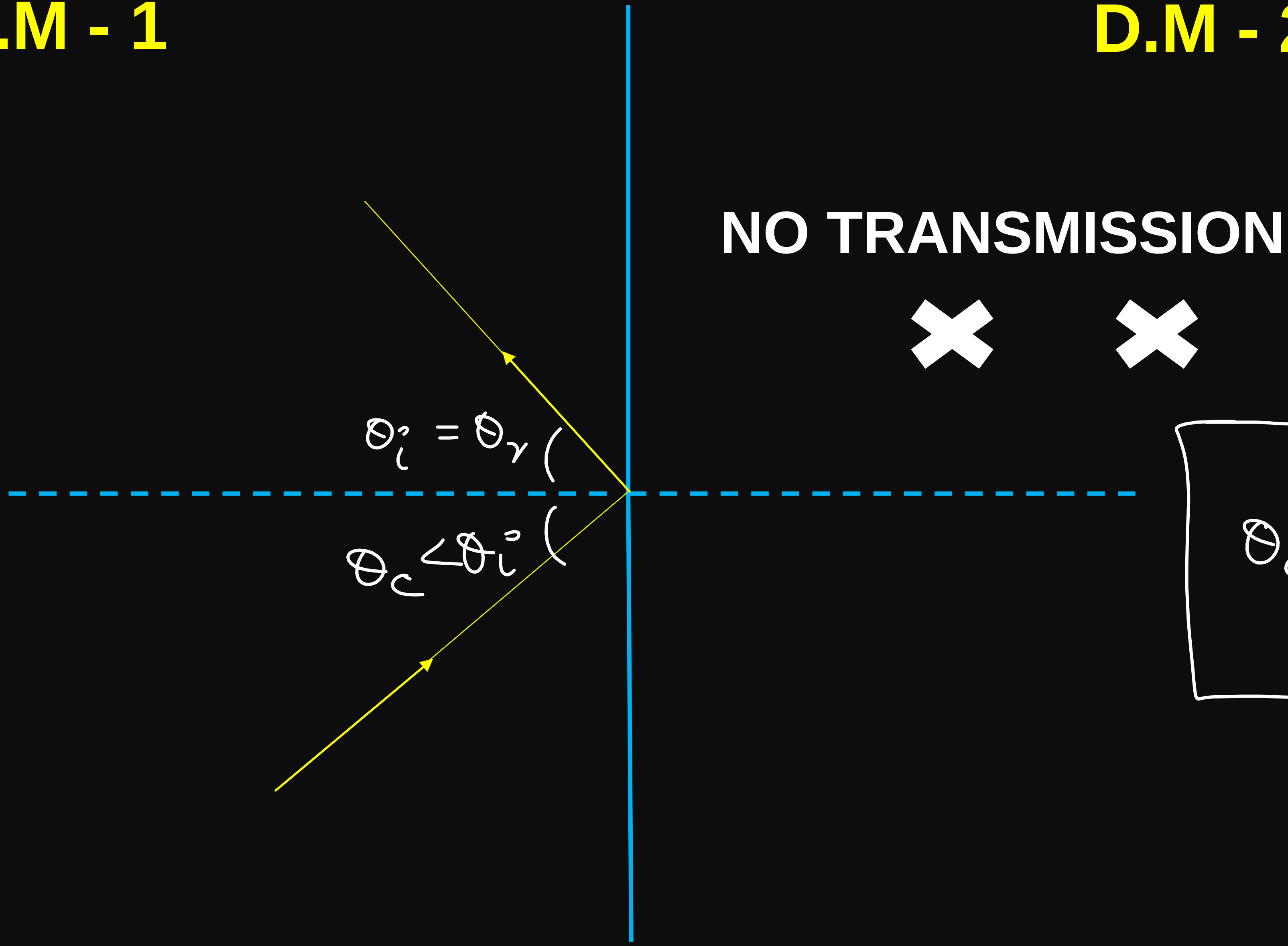
(b) $-\frac{E_0}{\sqrt{2}}(\hat{a}_x-\hat{a}_z)e^{-j\frac{\pi\times10^4 z}{3}}$ V/m

(c) $0.44\frac{E_0}{\sqrt{2}}(\hat{a}_x-\hat{a}_z)e^{-j\frac{\pi\times10^4(x-z)}{3\sqrt{2}}}$ V/m

(d) $\frac{E_0}{\sqrt{2}}(\hat{a}_x-\hat{a}_z)e^{-j\frac{\pi\times10^4(x+z)}{3\sqrt{2}}}$ V/m

TOTAL INTERNAL REFLECTION (T.I.R)

D.M - 1 **D.M - 2**



$$\boxed{\theta_c = \sin^{-1}\left[\sqrt{\frac{\epsilon_{r_2}}{\epsilon_{r_1}}}\right] = \sin^{-1}\left[\frac{n_2}{n_1}\right]}$$

NOTE:

- **FOR T.I.R THE REFLECTION CO-EFFICIENTS ARE COMPLEX.**

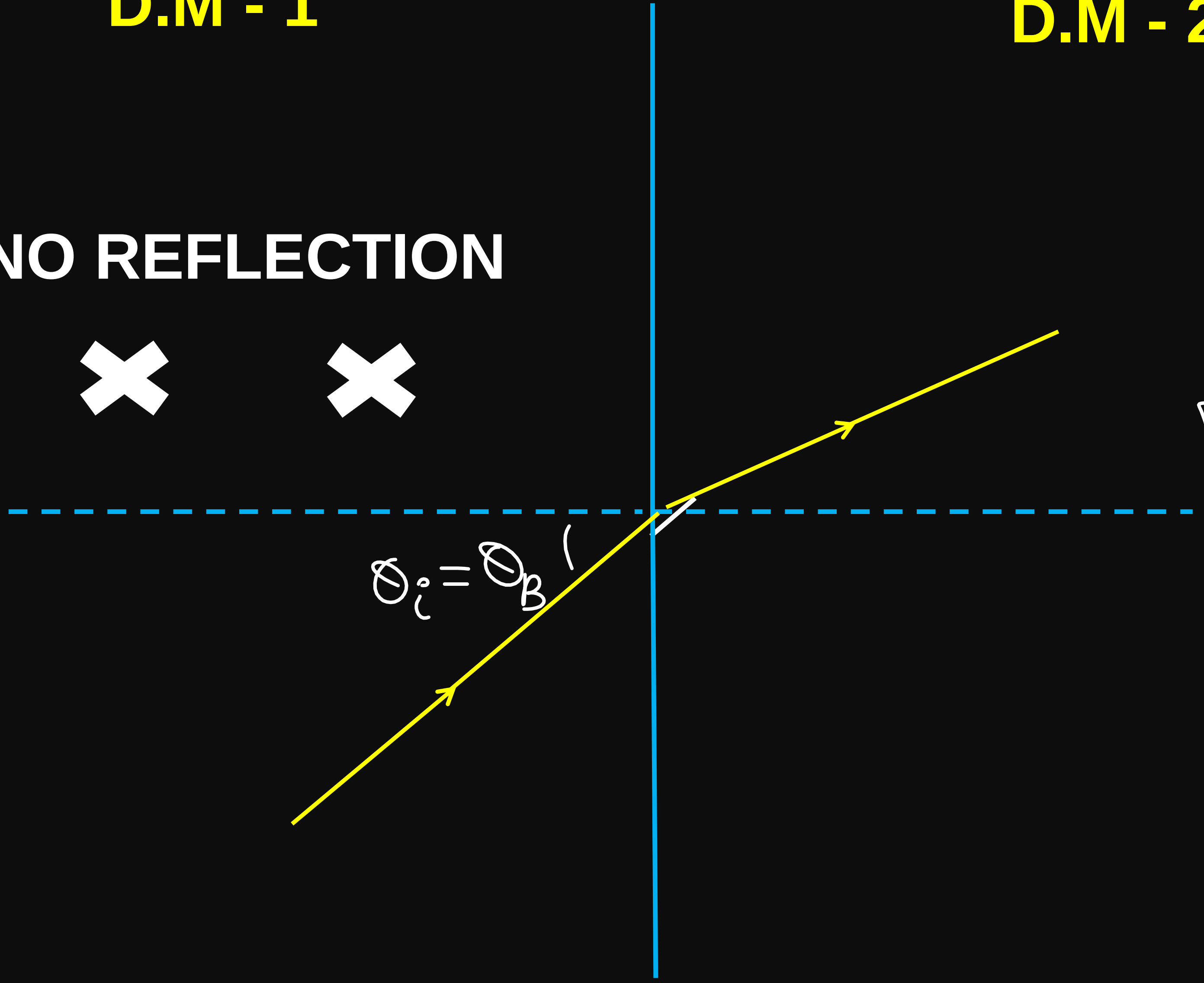
$$\Gamma_{\perp} = |\Gamma_{\perp}| \angle \Gamma_{\perp} \quad , \quad \Gamma_{\parallel} = |\Gamma_{\parallel}| \angle \Gamma_{\parallel} \qquad |\Gamma_{\perp}| = |\Gamma_{\parallel}| = 1$$

- **THE T.I.R PHENOMENON IS APPLICABLE TO BOTH PARALLEL ($\parallel^{le}$) AND PERPENDICULAR ($\perp^{le}$) POLARIZED EM-WAVES BUT PHASE SHIFT UNDERGONE BY REFLECTED WAVE IS DIFFERENCE FOR DIFFERENT POLARIZATIONS.**

TOTAL TRANSMISSION:

D.M - 1 **D.M - 2**

NO REFLECTION



$$\boxed{\theta_B = \theta_{B_{\parallel}}}$$

$$\boxed{\theta_B = \tan^{-1}\left[\sqrt{\frac{\epsilon_{r_2}}{\epsilon_{r_1}}}\right] = \tan^{-1}\left[\frac{n_2}{n_1}\right]}$$

- The concept of total transmission is not valid for perpendicularly ($\perp^{r}$) polarized wave at (pure) dielectric media interface.
- By default Brewster angle refers to parallelly ($\parallel^{le}$) polarized EM-Wave.

$$\theta_B = \theta_{B_{\parallel}}$$

Q. For incidence from dielectric medium1(ε_1) on to dielectric medium 2(ε_2),the Brewster angle θ_B and the corresponding angle of transmission θ_t for $\frac{\varepsilon_1}{\varepsilon_2} = 3$ will be respectively

(a) 30° and 30°

(b) 30° and 60°

(c) 60° and 30°

(d) 60° and 60°

① ②

$\tan\theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \frac{1}{\sqrt{3}} \Rightarrow \theta_B = 30^\circ$

IF $\theta_i = \theta_B = 30^\circ$, $\theta_t = ?$

$\sqrt{\epsilon_{r_1}}\sin\theta_i = \sqrt{\epsilon_{r_2}}\sin\theta_t$

$\sin\theta_t = \sqrt{\frac{\epsilon_{r_1}}{\epsilon_{r_2}}}\sin\theta_i = \sqrt{3}\cdot\frac{1}{2} = \frac{\sqrt{3}}{2}$

$\theta_t = 60^\circ$

Q.A uniform plane wave travelling in free space and having the electric field

$$\bar{E} = \left(\sqrt{2}\hat{a}_x - \hat{a}_z\right)\cos\left[6\sqrt{3}\pi \times 10^8 t - 2\pi\left(x + \sqrt{2}z\right)\right] V/m$$

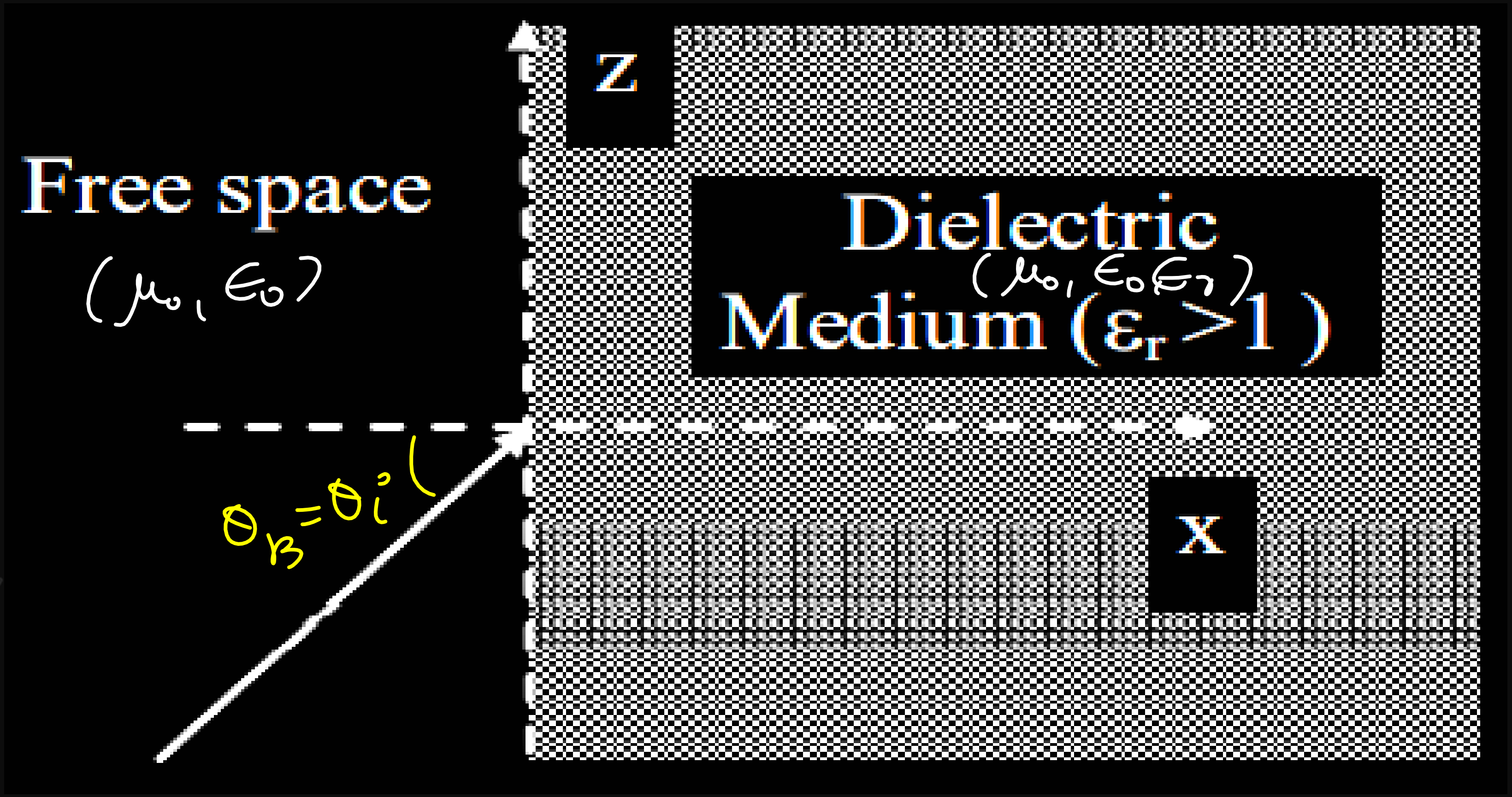
is incident on a dielectric medium (relative permittivity >1, relative permeability = 1) as shown in the figure and there is no reflected wave

Soln $\theta_i = \theta_B = \tan^{-1}\left[\sqrt{\frac{\epsilon_{r_2}}{\epsilon_{r_1}}}\right]$

$\theta_i = \theta_B = \tan^{-1}\left[\sqrt{\frac{\epsilon_r}{1}}\right]$

$\theta_i = \theta_B = \tan^{-1}\left[\sqrt{\epsilon_r}\right]$

$\tan\theta_i = \tan\theta_B = \sqrt{\epsilon_r}$.



The relative permittivity (correct to two decimal places) of the dielectric medium is ____________. **(GATE-18)**

$$\bar{K} = 2\pi\hat{x} + 2\pi\sqrt{2}\,\hat{z}$$

$$\hat{K}_i = \hat{n}_i = \frac{2\pi\hat{x} + 2\pi\sqrt{2}\,\hat{z}}{\sqrt{(2\pi)^2 + (2\pi\sqrt{2})^2}}$$

$$= \frac{1}{\sqrt{3}}\hat{x} + \frac{\sqrt{2}}{\sqrt{3}}\hat{z}$$

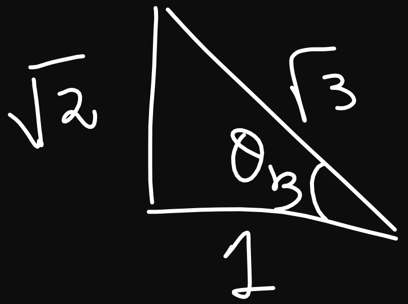
$$\hat{K}_i = \hat{n}_i = \frac{1}{\sqrt{3}}\hat{x} + \frac{\sqrt{2}}{\sqrt{3}}\hat{z}$$

$$\hat{n}_i \cdot \hat{n}_n = \cos\theta_i$$

$$\left(\frac{1}{\sqrt{3}}\hat{x} + \frac{\sqrt{2}}{\sqrt{3}}\hat{z}\right)\cdot\hat{x} = \cos\theta_i$$

$$\cos\theta_i = \frac{1}{\sqrt{3}} = \cos\theta_B$$

$$\tan\theta_i = \tan\theta_B = \sqrt{\epsilon_r}.$$



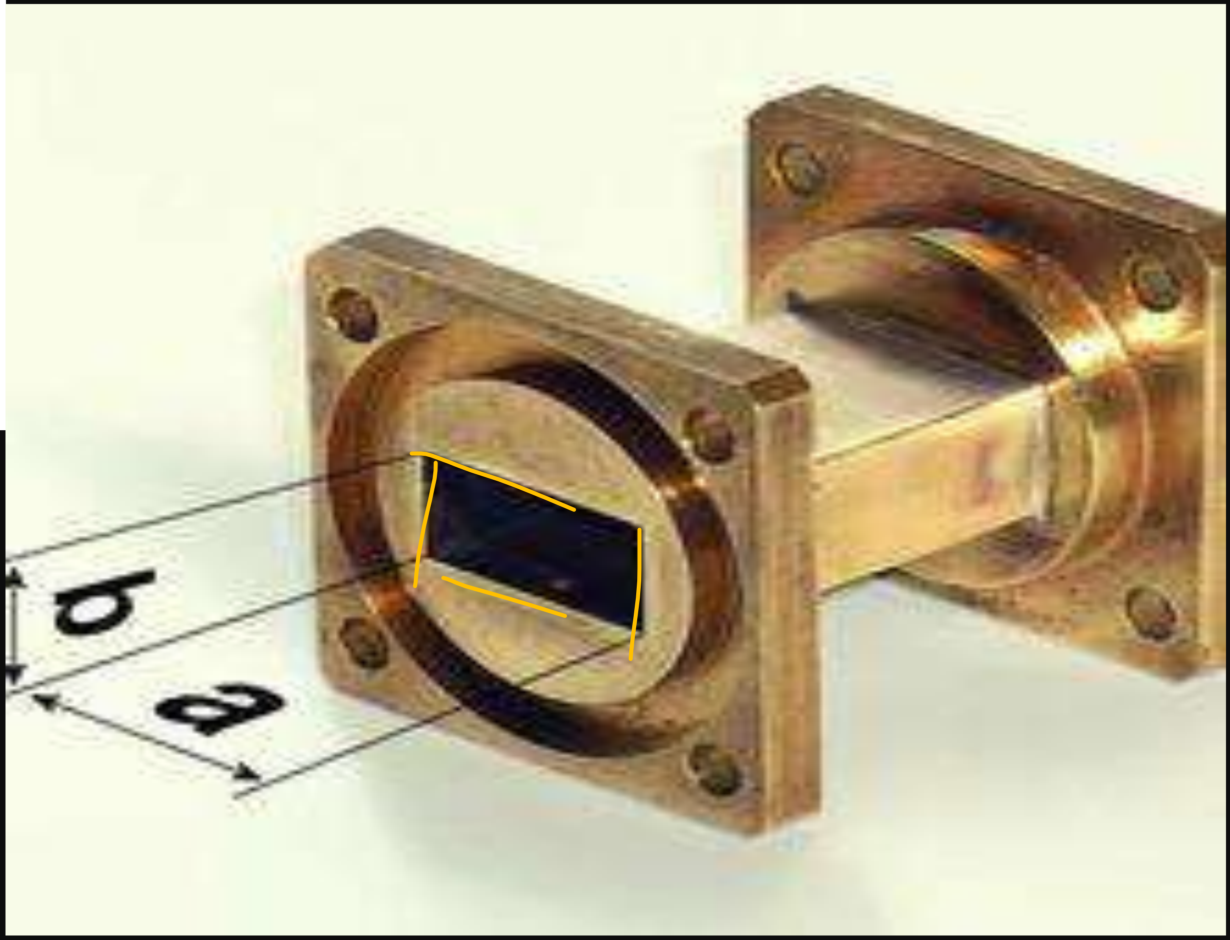
$$\tan\theta_B = \sqrt{2} = \sqrt{\epsilon_r}$$

$$\underline{\epsilon_r = 2}$$

WAVE-GUIDES (W/G)

(3GHz – 3000 GHz)

* Wave-Guide is hollow metallic pipe, which guides or propagates EM-Waves along its structure from one point to other point.
* TYPES OF WAVE-GUIDES
  1. Rectangular wave-guide
  2. Circular wave-guide
  3. Elliptical wave-guide

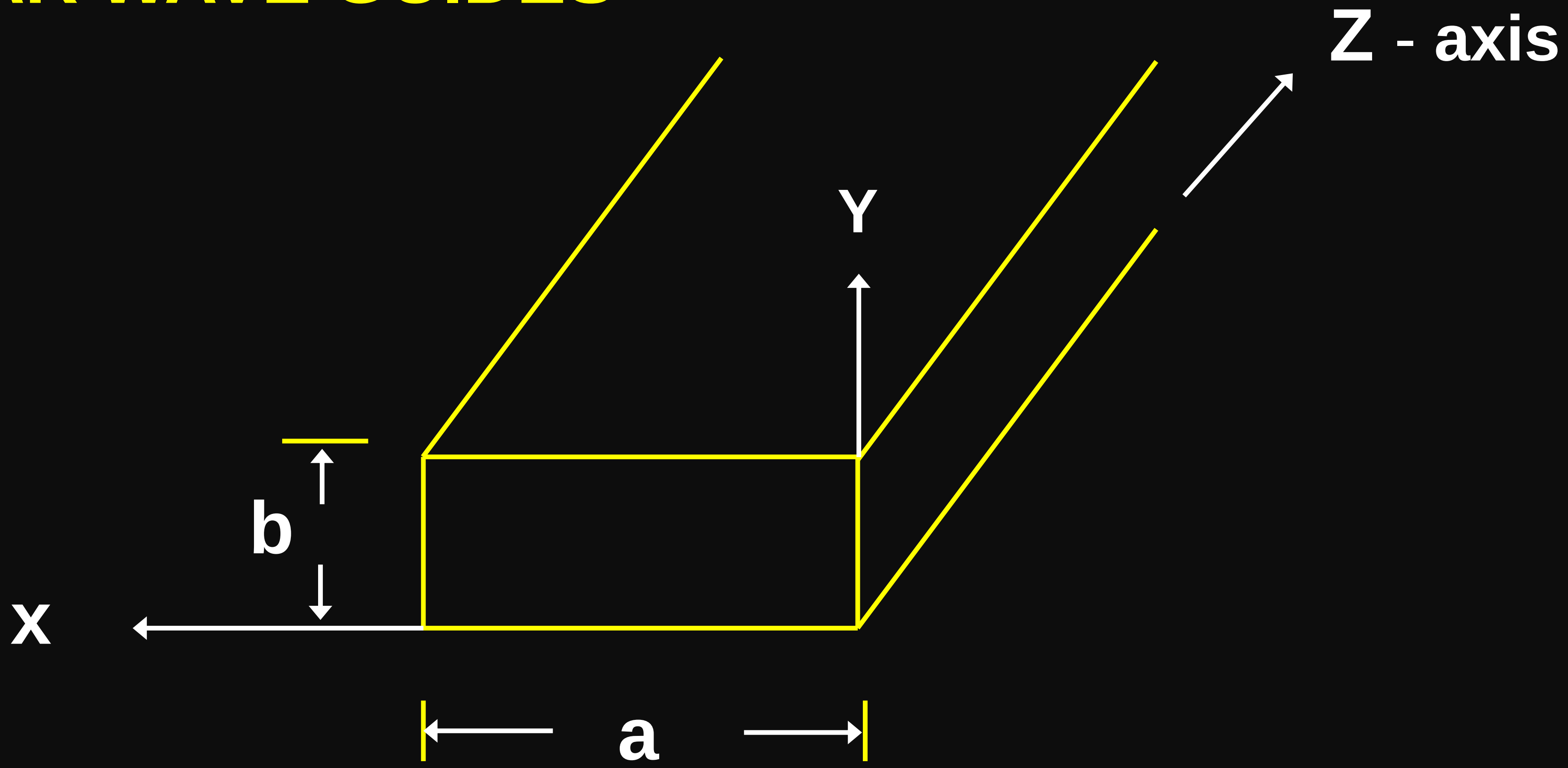


b
a


RECTANGULAR WAVE GUIDES

$a \times b$

$a > b$



a : Broader Dimension (or) Width

b: Narrow Dimension (or) Height

(1) TEM ($E_z \equiv 0$, $H_z \equiv 0$) ✗

(2) TE ($E_z \equiv 0$, $H_z \neq 0$) ✓

(3) TM ($H_z \equiv 0$, $E_z \neq 0$) ✓

Conclusion (I): By inspecting above set of equations for transverse fields, which are formulated in terms of longitudinal fields by modifying Maxwell's Equations, indicates impossibility of TEM ($E_z \equiv 0$, $H_z \equiv 0$) inside the rectangular Wave-Guide.

i.e. E_z, H_z can't be zero simultaneously.

i.e. Wave-Guide supports TE ($E_z \equiv 0, H_z \neq 0$) with the help of extra magnetic field.

(or)

It supports TM ($H_z \equiv 0, E_z \neq 0$) with the help of extra electric field.

CONCLUSION – II

- By inspecting above set of expressions for longitudinal and transverse fields of TM-Wave, which are obtained by solving the wave equations, indicates dependency of all the fields on integers m, n. Hence, different propagating modes in TM-Wave are designated as TM_{mn}.

TM_{mn} :-

1. TM_{00} ✗
2. TM_{m0} (OR) TM_{0n} ✗
3. TM_{11} ← LOWEST
4. TM_{12}, TM_{21} - - - -

CONCLUSION – III

By inspecting above set of expressions for longitudinal and transverse fields for TE-Wave, which are obtained by solving the wave equation, indicates dependency of all the fields on the integers m,n. Hence different propagating modes in TE-Wave are designated as TE_{mn}.

TE_{mn} :-

1. TE_{00} ✗
2. TE_{m0} OR TE_{0n} ✓
3. TE_{10} OR TE_{01} ← LOWEST
4. TE_{11} ~~Φ~~, TE_{12}, TE_{21} . . . HIGHER ORDER.

Conclusion: IV

$$\begin{tikzcd}[column sep=large]
TE(H_z) \arrow[r] & \cos\left(\frac{m\pi}{a}x\right) \arrow[dr, "1,\,4"] & \cos\left(\frac{n\pi}{b}y\right) \\
TM(E_z) \arrow[r] & \sin\left(\frac{m\pi}{a}x\right) \arrow[ur, "2,\,3"'] & \sin\left(\frac{n\pi}{b}y\right)
\end{tikzcd}$$

1 E_x 2 E_y 3 H_x 4 H_y

Q. The $\bar{E}$ - field in a rectangular wave guide of inner dimension a × b is given by $\bar{E} = \frac{w\mu}{h^2}\left(\frac{\pi}{a}\right) H_0 sin\left(\frac{2\pi}{a}x\right) sin(wt - \beta z)\hat{y}$ where H_0 is a constant and a and b are the dimensions along the x-axis and the y-axis respectively. The mode of propagation in the wave-guide is

$E_y = E_{0y} \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right)$

(GATE: 2007)

(a) TE_{20}

$\boxed{m=2 \quad n=0} \rightarrow TE_{20}$

(b) TM_{11}

(c) ~~TM_{20}~~

(d) TE_{10}

Q. The rectangular wave-guide of inner dimension a × b has

$\mathbf{E_z} = E_0 sin\left(\frac{4\pi}{a}x\right) sin\left(\frac{2\pi}{b}y\right) \cos(wt - \beta z)$ The mode of propagation is

(a) TE_{42}

(b) TM_{42}

(c) Doesn't exist

(d) None

$$E_z = E_{0z} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right)$$

$$\boxed{\begin{matrix} m=4 \\ n=2 \end{matrix}} \rightarrow TM_{42}$$

CUT-OFF FREQUENCY

$$\boxed{f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

TM$_{11}$ $\longrightarrow$ $f_c\big|_{TM_{11}} = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\frac{a^2+b^2}{ab}}$

TE$_{01}$ $\longrightarrow$ $f_c\big|_{TE_{01}} = \frac{1}{2\sqrt{\mu\epsilon}}\left(\frac{1}{b}\right)$

TE$_{10}$ $\longrightarrow$ $f_c\big|_{TE_{10}} = \frac{1}{2\sqrt{\mu\epsilon}}\left(\frac{1}{a}\right)$

$a > b$

$f_c\big|_{TE_{10}} < f_c\big|_{TE_{01}} < f_c\big|_{TM_{11}}$

$\downarrow$

DOMINANT MODE

- The mode with the lowest cut-off frequency is called dominant mode and it’s corresponding wave is called dominant wave.

- TE_{10} is the dominant mode in rectangular Wave-Guide.

Note: In general practical Wave-Guides are operated in dominant mode.

Q. An air filled rectangular Wave-Guide has inner wall dimensions of 4 cm × 3 cm. Find

(a) Cut-off frequency of dominant mode.

(b) Cut-off frequency of lowest mode in TM-Wave

Soln

(a) $$f_c\big|_{TE_{10}} = \frac{1}{2\sqrt{\mu_0\epsilon_0}}\left(\frac{1}{a}\right) = \frac{3\times10^8}{2}\times\frac{1}{4\times10^{-2}} = 0.375\times10^{10} = 3.75\,\text{GHz}.$$

(b) $$f_c\big|_{TM_{11}} = \frac{1}{2\sqrt{\mu_0\epsilon_0}}\sqrt{\frac{a^2+b^2}{ab}} = \frac{3\times10^8}{2}\sqrt{\frac{(4\times10^{-2})^2+(3\times10^{-2})^2}{4\times3\times10^{-2}\times10^{-2}}} = \underline{6.25\,\text{GHz}}.$$

SQUARE WAVE-GUIDE

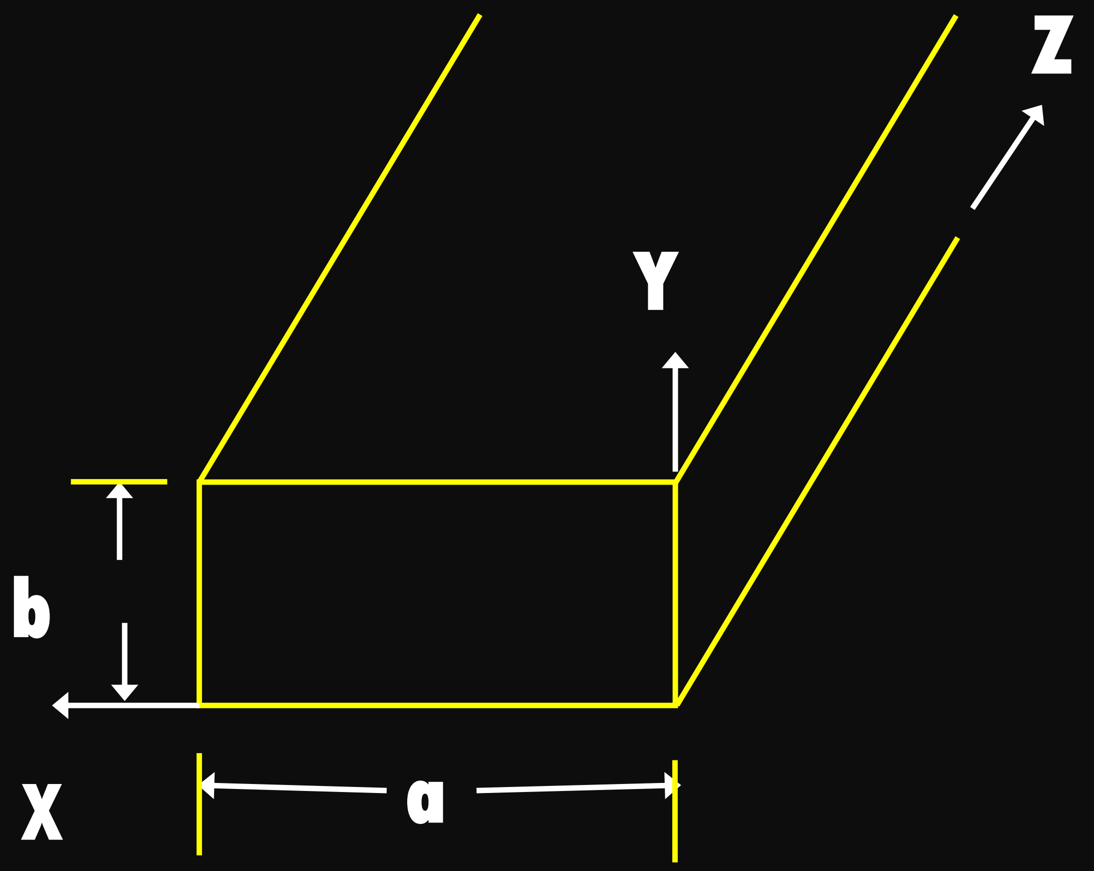
For square Wave-Guide

$(a = b)$

$$f_c|_{TE_{10}} = \frac{1}{2\sqrt{\mu\epsilon}}\left(\frac{1}{a}\right)$$

$$f_c|_{TE_{01}} = \frac{1}{2\sqrt{\mu\epsilon}}\left(\frac{1}{b}\right)$$

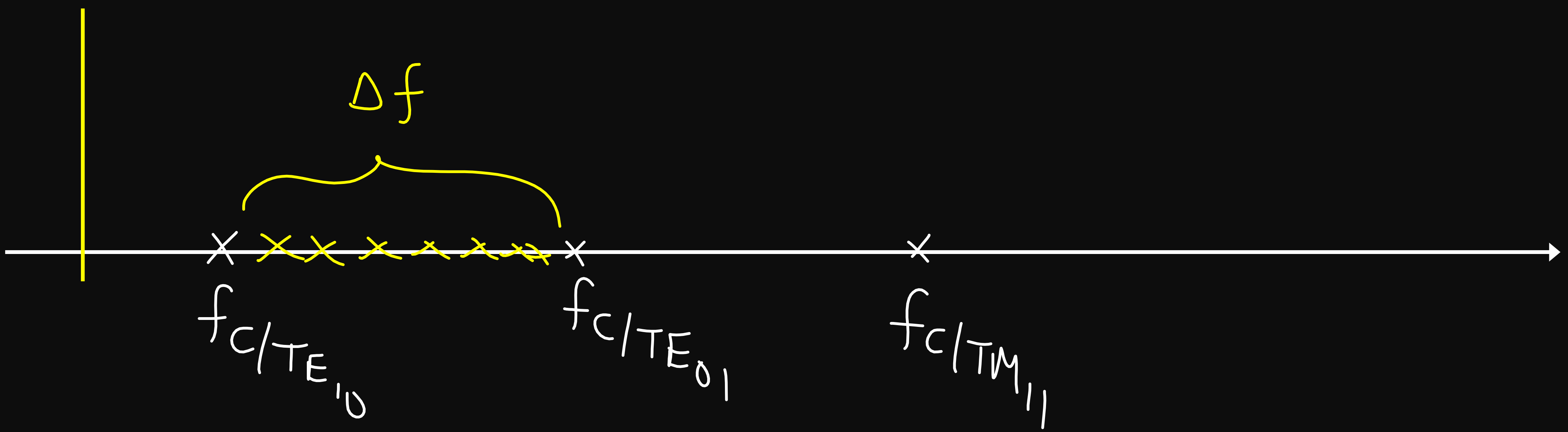
$$f_c|_{TE_{10}} = f_c|_{TE_{01}}$$



NOTE: For square Wave-Guide ($a = b$) both TE_{10} and TE_{01} are the dominant modes.

DOMINANT REGION

For rectangular Wave-Guide (a > b)



DE-GENERATIVE MODES

The different, possible modes with same cut-off frequency are called de-generative modes.

Ex:

(1) TE_{11} , TM_{11} ✓

(2) TE_{20} , TM_{20} ✗

(3) TM_{31} , TE_{13} ✗

(4) TM_{42} , TE_{42} ✓

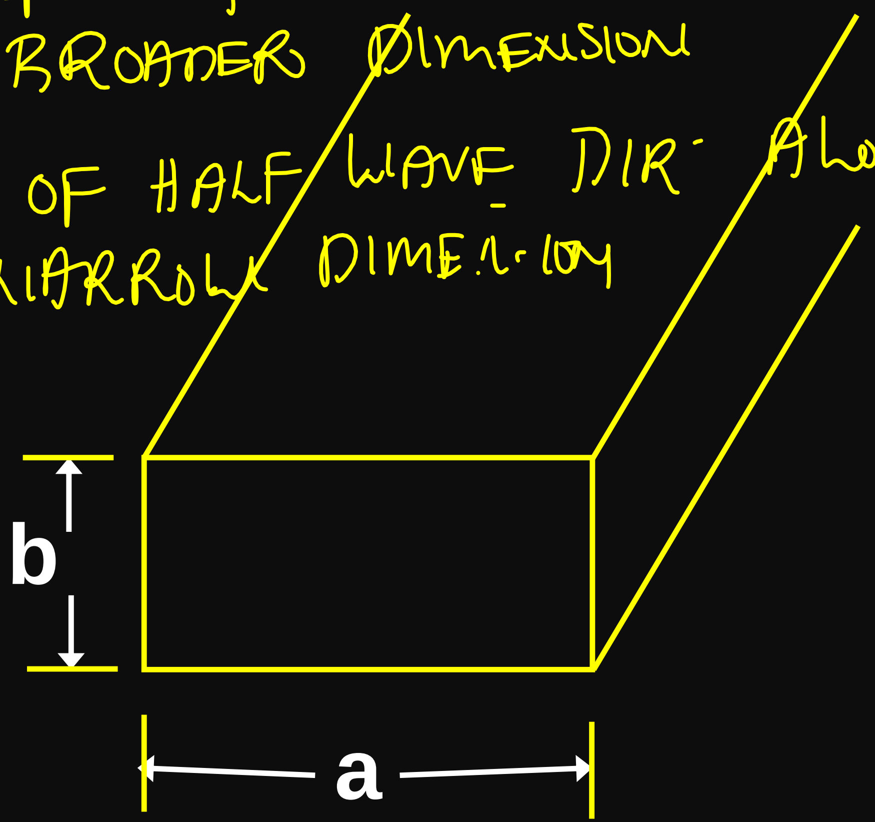
PHYSICAL SIGNIFICANCE OF INTEGERS m, n.

Coupling Techniques

- Probe Coupling ($\bar{E}$- Field)
- Loop Coupling ($\bar{H}$- Field)

m · # OF HALF WAVE DIR^n ALONG BROADER DIMENSION

n # OF HALF WAVE DIR· ALONG NARROW DIMENSION



NOTE (1): INSIDE RECTANGULAR WAVE-GUIDE

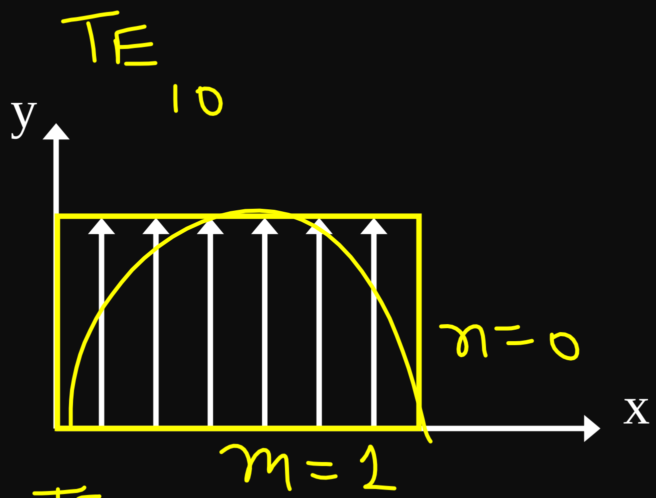
- Electric field lines exists as lines.
- Magnetic field lines exists as loops.

NOTE (2): IN CROSS-SECTION OF RECTANGULAR WAVE-GUIDE

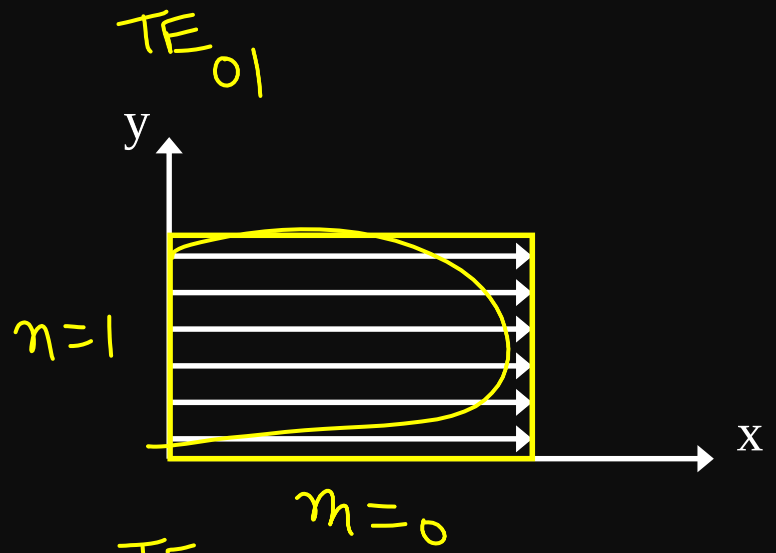
- If lines are observed then it is TE-wave.
- If loops are observed then it is TM-wave.

Q. Which one of the following does represent the electric field lines for the TE_{02} mode in the cross-section of a hollow rectangular metallic waveguide? **(GATE - 05)**

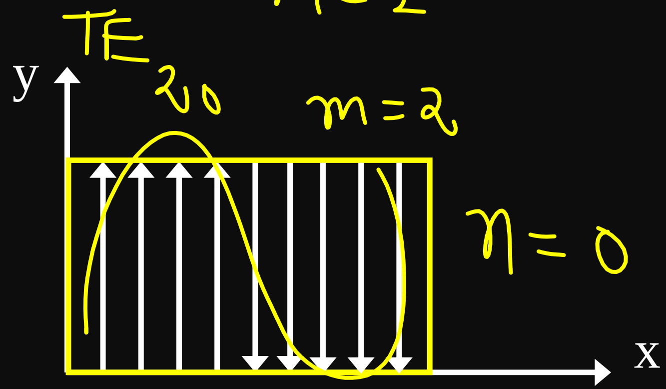
(a)



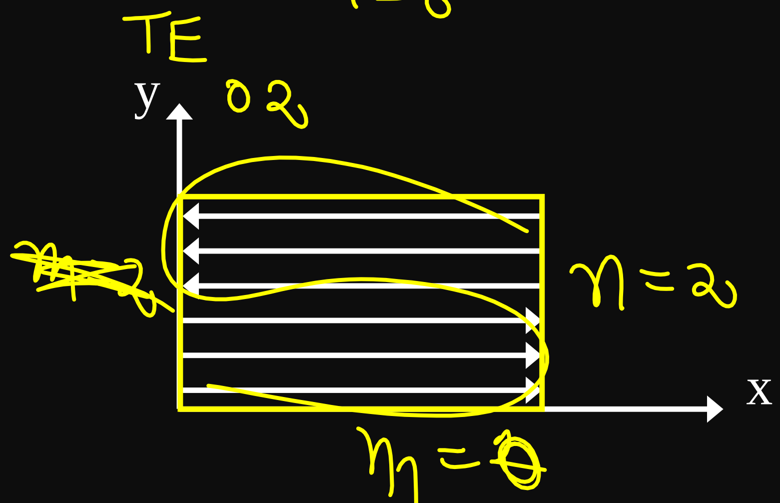
(b)



(c)



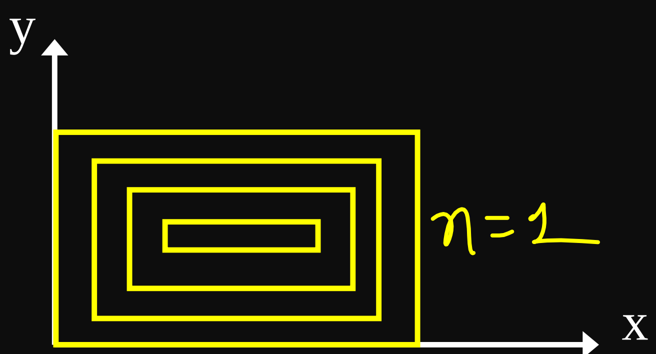
(d)



Q. Which one of the following does represent the magnetic field lines for TM_{22} mode in the cross-section of hollow rectangular Wave-Guide?

(a)

TM_{11}

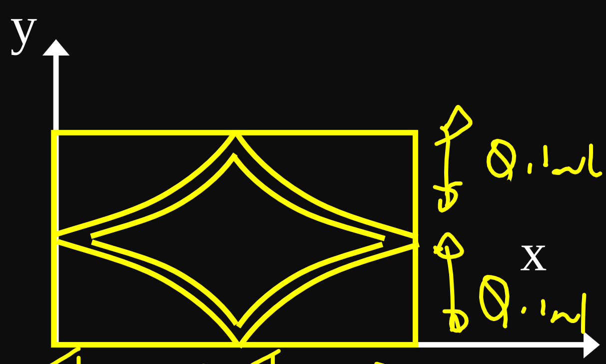


$n=1$

$m=1$

(b)

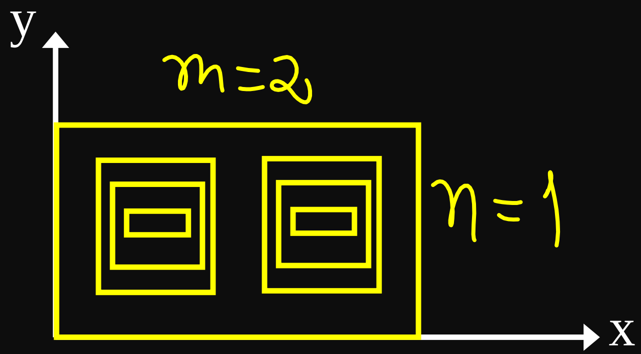
TE_{11}



(c)

TM_{21}

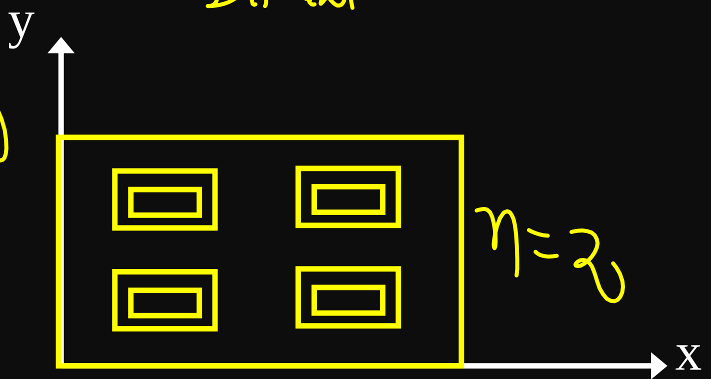
$m=2$



$n=1$

(d)

TM_{22}

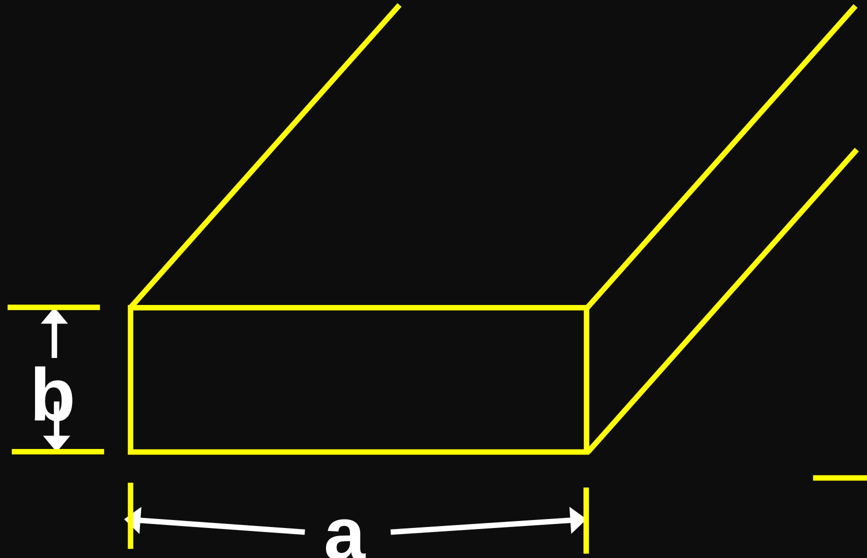


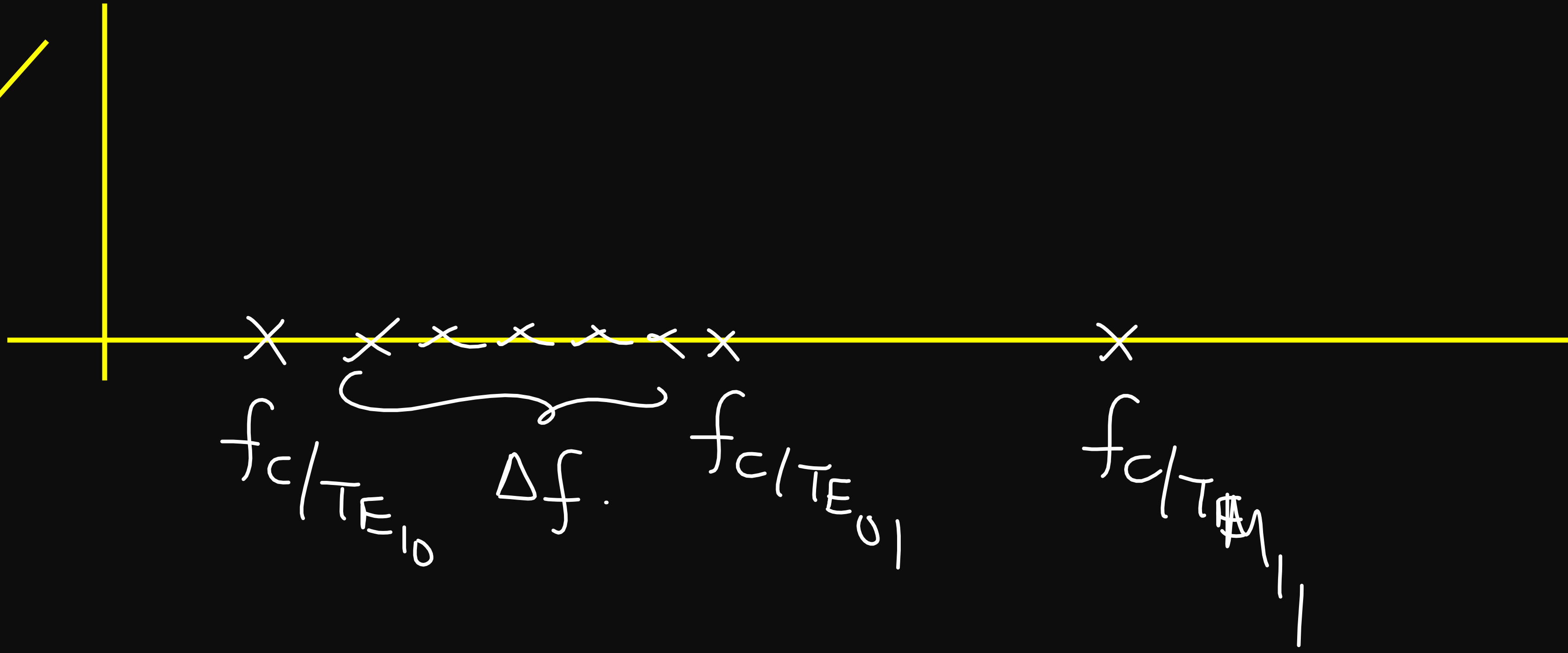
$n=2$

$m=2$

EFFECT OF ASPECT RATIO ON DOMINANT REGION

a > b





Aspect Ratio
(a : b)

NOTE:

For the Wave-Guide to support high dominant region (Δf), the Wave-Guide dimensions are chosen as $b \leq \frac{a}{2}$.

$$\left[\Delta f_{b \leq a/2}\right] > \left[\Delta f_{b > a/2}\right].$$

Condition for single mode (or) dominant region of operation.

IF $b \leq a/2$ $\quad f_c|_{TE_{10}} < f < f_c|_{TE_{20}}$

IF $b > a/2$. $\quad f_c|_{TE_{10}} < f < f_c|_{TE_{01}}$

Q: An air filled rectangular waveguide has dimension of a = 2 cm and b = 1 cm. Determine the range of frequencies over which the guide will operate single mode (dominant mode)

(a) 7.5 GHz < f < 15 GHz

(b) 15 GHz < f < 30 GHz

(c) 0 GHz < f < 7.5 GHz

(d) 0 GHz < f < 30 GHz

Soln

$$\left.\begin{array}{l} a = 2\,cm \\ b = 1\,cm \end{array}\right\} b = a/2$$

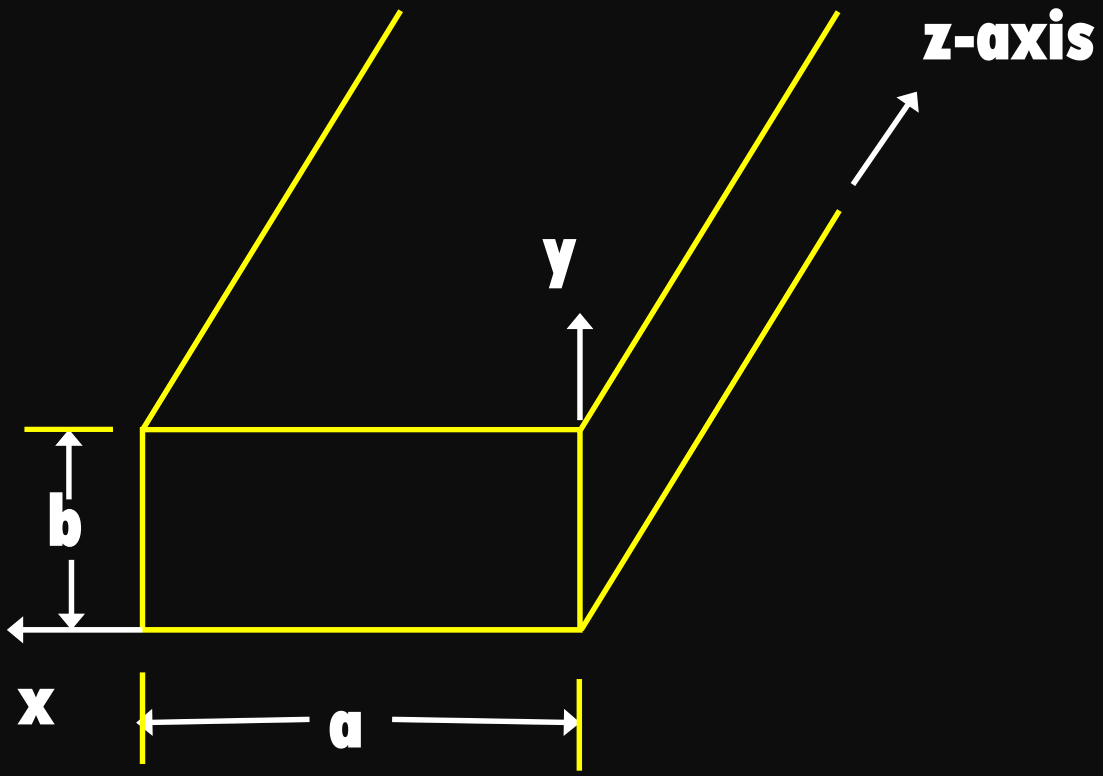
$$f_c|_{TE_{10}} < f < f_c|_{TE_{20}}$$

$$f_c|_{TE_{10}} = \frac{1}{2\sqrt{\mu_0\epsilon_0}}\left(\frac{1}{a}\right) = \frac{3\times10^8}{2} \times \frac{1}{2\times10^{-2}} = 7.5\,GHz.$$

$$f_c|_{TE_{20}} = \frac{1}{2\sqrt{\mu_0\epsilon_0}}\left(\frac{2}{a}\right) = \frac{3\times10^8}{2} \times \frac{2}{2\times10^{-2}} = 15\,GHz$$

$$7.5\,GHz < f < 15\,GHz.$$

POWER FLOW INSIDE WAVE-GUIDE



$$\boxed{W = \frac{|E_0|^2}{4\eta_{TE_{10}}} ab}$$ Watts

PROPAGATING CHARACTERISTICS INSIDE WAVE-GUIDE

(I) INTRINSIC IMPEDANCE

(II) WAVE LENGTH

(III) VELOCITY

I. INTRINSIC IMPEDANCE:

* $\eta_{TM} < \eta_{TEM} < \eta_{TE}$

$$\text{HPF} = \sqrt{1-\left(\frac{f_c}{f}\right)^2} < 1$$

* $\eta_{TM} = \eta_{TEM}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

* $\eta_{TE} = \dfrac{\eta_{TEM}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

* $\eta_{TEM} = \sqrt{\eta_{TE}\,\eta_{TM}}$

Q. An air-filled rectangular waveguide has inner dimensions of 3 cm × 2 cm. The wave impedance of the TE_{20} mode of propagation in the waveguide at a frequency of 30 GHz is (free space impedance $\eta_0 = 377\ \Omega$)

(GATE - 07)

(a) 308 Ω

(b) 355 Ω

(c) 400 Ω

(d) 461 Ω

Soln.

$$\eta_{TE_{20}} = \frac{\eta_{TEM}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{f_c}{30G}\right)^2}}$$

$$f_c/_{TE_{20}} = \frac{1}{2\sqrt{\mu_0\epsilon_0}}\left(\frac{2}{a}\right) = \frac{3\times10^8}{2}\times\frac{2}{3\times10^{-2}} = 10\,GHz$$

$$\eta_{TE_{20}} = \frac{377}{\sqrt{1-\left[\frac{10G}{30G}\right]^2}} \approx 400\,\Omega$$

(II) WAVE LENGTH:

λ : Intrinsic Wave Length

(Inside: Apparent)

$\bar{\lambda}$: Guided Wave Length

(Un-Bound medium)

λ_c : Cut-off Wave Length

* $\lambda < \bar{\lambda}$ AND λ_c

* $$\bar{\lambda} = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$$

* $$\frac{1}{\lambda^2} = \frac{1}{\bar{\lambda}^2} + \frac{1}{\lambda_c^2}.$$

(III) VELOCITY:

(1) Phase Velocity ($\bar{V}_p / V_p$)

(Inside: Apparent)

(2) Intrinsic Velocity (V_o)

(Un-Bound medium)

(3) Group Velocity (V_g)

(Inside: Actual)

* $V_g < V_o < V_p$

* $V_g = V_o \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$

* $V_p = \dfrac{V_o}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$

* $V_o = \sqrt{V_p V_g}$

Q. The dispersion equation of a waveguide, which relates the wavenumber k to the frequency ω, is $k(\omega) = (1/c)\sqrt{\omega^2 - \omega_0^2}$ where the speed of light $c = 3\times10^8$ m/s, and ω_0 is a constant. If the group velocity is 2×10^8 m/s, then the phase velocity is

(a) 2×10^8 m/s

(b) 4.5×10^8 m/s,

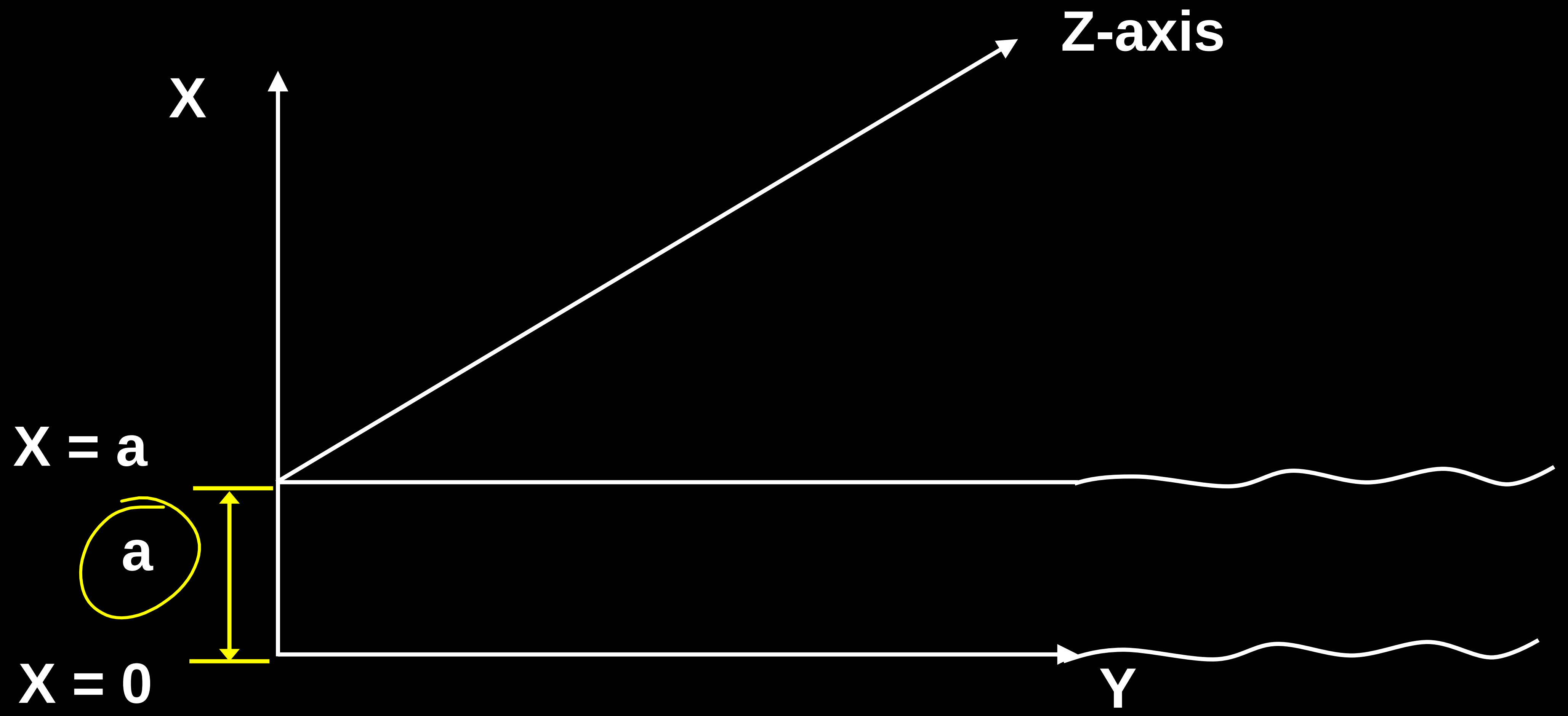
(C) 3×10^8 m/s

(d) 1.5×10^8 m/s,

$$v_p = \frac{v_0^2}{v_g} = \frac{(3\times10^8)^2}{2\times10^8}$$

$$v_p = 4.5\times10^8 \text{ m/s}$$

PARALLEL PLATE WAVE-GUIDE:



1. TEM ($E_z \equiv 0$, $H_z \equiv 0$)

2. TE ($E_z \equiv 0$, $H_z \neq 0$)

3. TM ($H_z \equiv 0$, $E_z \neq 0$)

TM_m:

1. Tm_0 (TEM) $\longrightarrow$ SUPPORT

2. Tm_1 - - - .

TE_m:

1. TE_0 ×

2. TE_1 ✓ → LOWEST

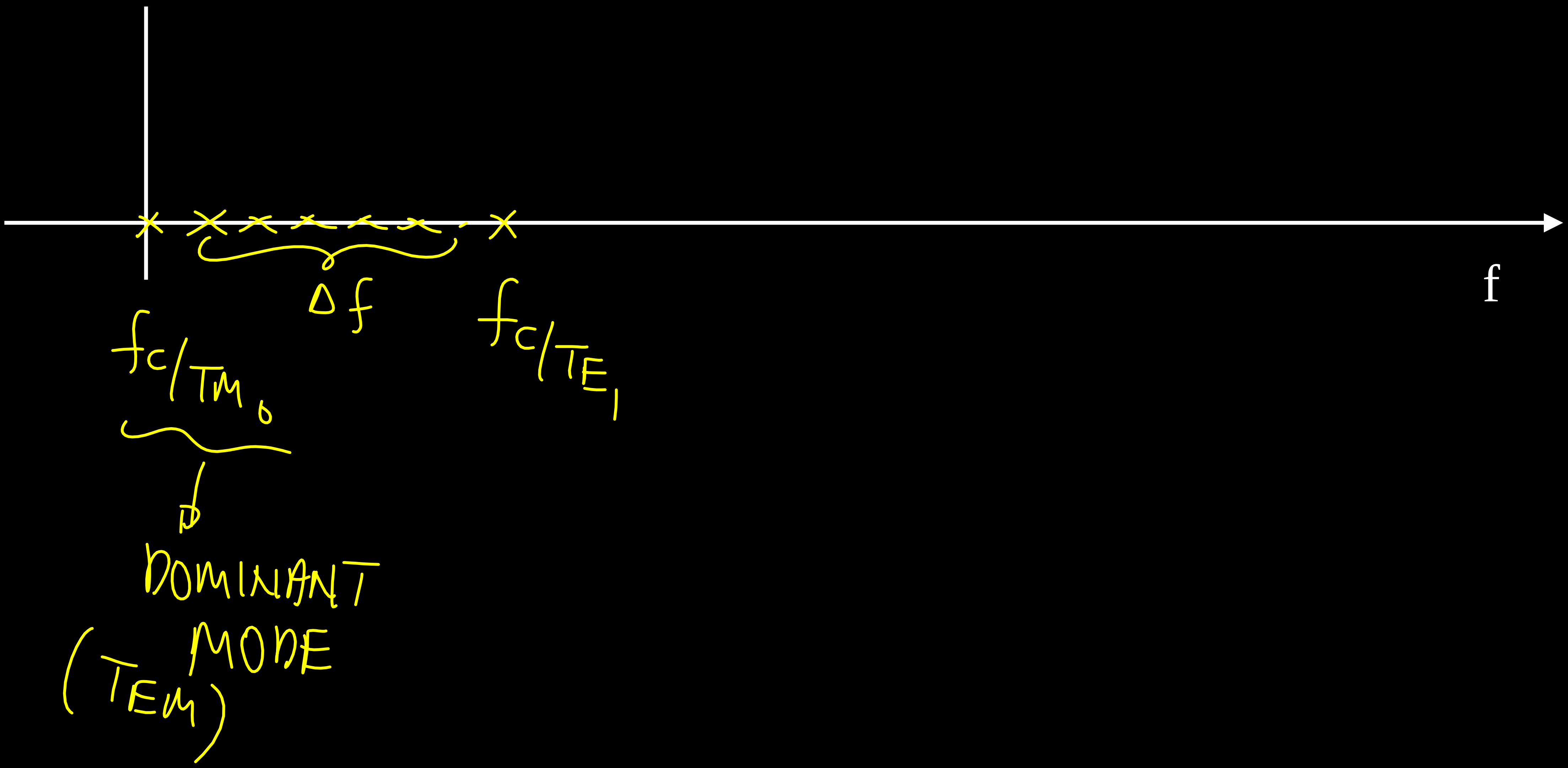
CUT-OFF FREQUENCY (f_C)

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}}\left(\frac{m}{a}\right)$$

$TM_0\ (TEM) \longrightarrow f_c/_{TM_0} = 0$

$TE_1 \longrightarrow f_c/_{TE_1} = \frac{1}{2\sqrt{\mu\epsilon}}\left(\frac{1}{a}\right)$

DOMINANT REGION



Q. A waveguide consists of two infinite parallel plates (perfect conductors) at a separation of 10^{-4} cm, with air as the dielectric. Assume the speed of light in air to be 3×10^8 m/s. The frequency/frequencies of TM waves which can propagate in this waveguide is/are ________. (GATE - 22)

(a) 6×10^{15} Hz

(b) 0.5×10^{12} Hz

(c) 8×10^{14} Hz

(d) 1×10^{13} Hz

$f > f_c|_{TM_0} = 0$

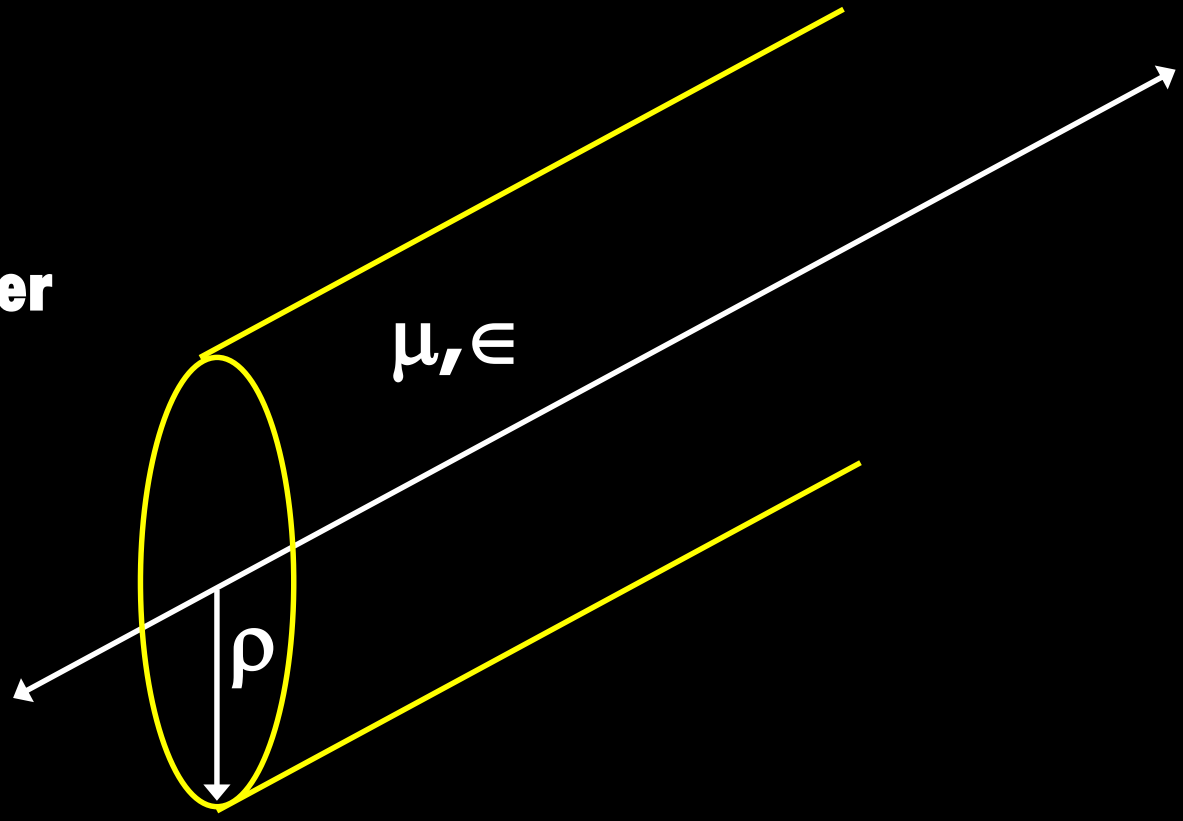
$f > 0$

CIRCULAR WAVE-GUIDE

(W/G)

ρ = r

r = Radius of cylinder



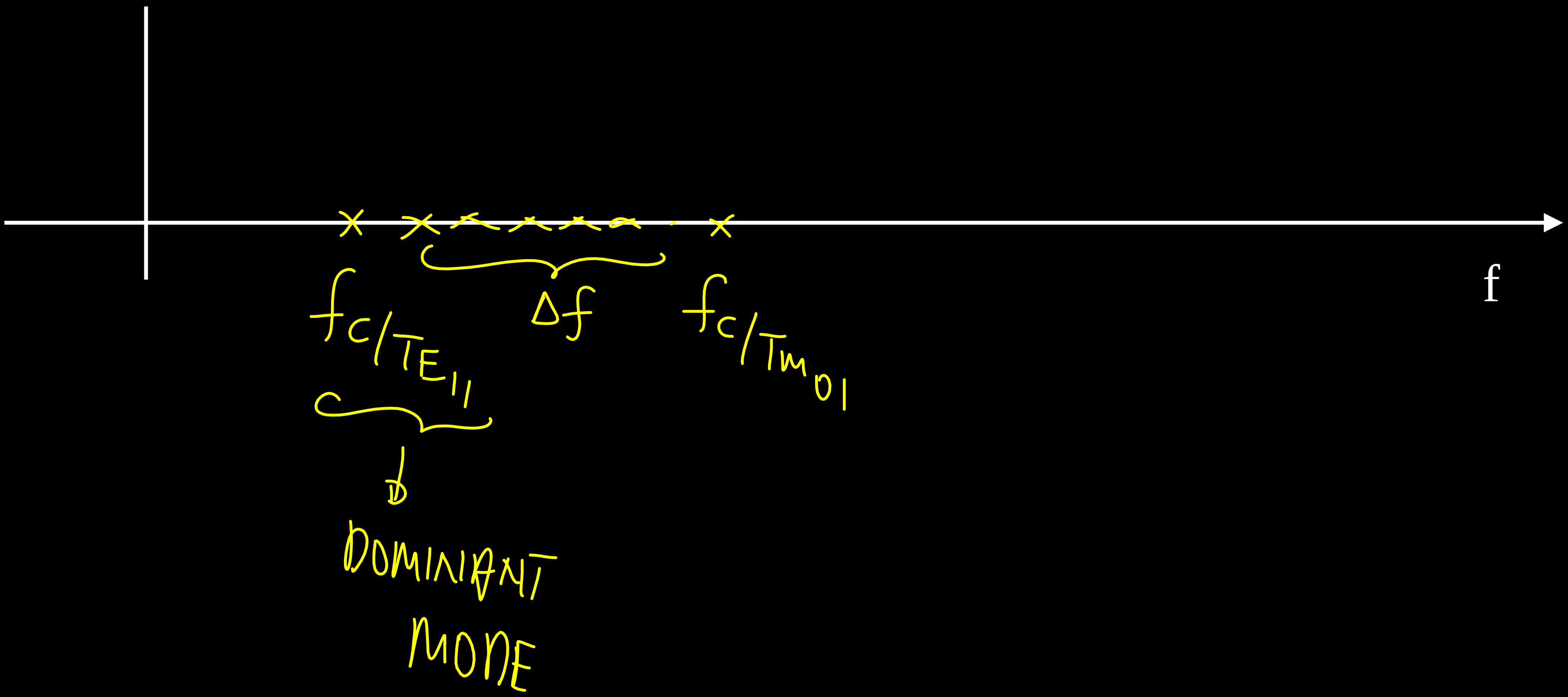
z-Axis ($e^{-\bar{\gamma} z}$)
Direction of propagation

CUT-OFF FREQUENCY (f_C)

$$f_c/_{TE_{11}} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\left(\frac{1.841}{r}\right)$$

$$f_c/_{TM_{01}} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\left(\frac{2.405}{r}\right)$$

DOMINANT REGION



Q. An air filled circular Wave-Guide has 1cm radius

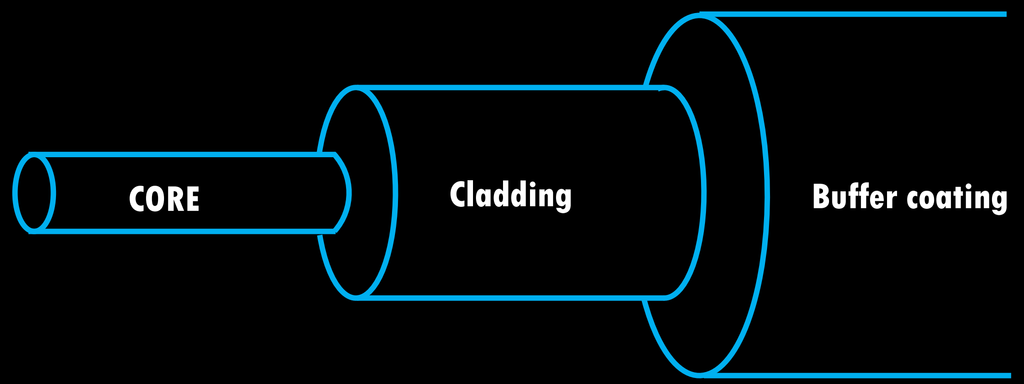
(a) Find the cut off frequency of dominant mode

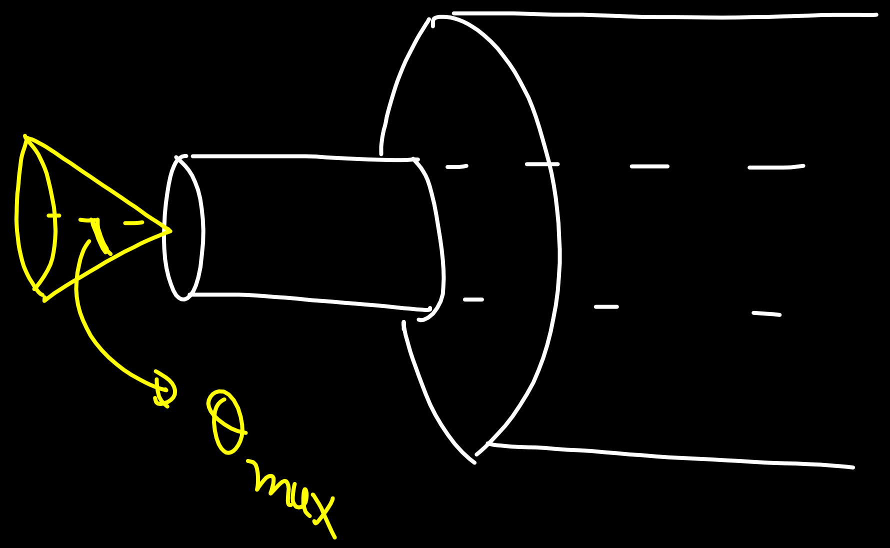
(b) Find the cut off frequency of lowest mode in TM

(a) $$f_c\Big|_{TE_{11}} = \frac{1}{2\pi\sqrt{\mu_0\epsilon_0}}\left(\frac{1.841}{r}\right) = \frac{3\times10^8}{2\pi}\times\frac{1.841}{1\times10^{-2}} = \underline{8.7\,\text{GHz}}$$

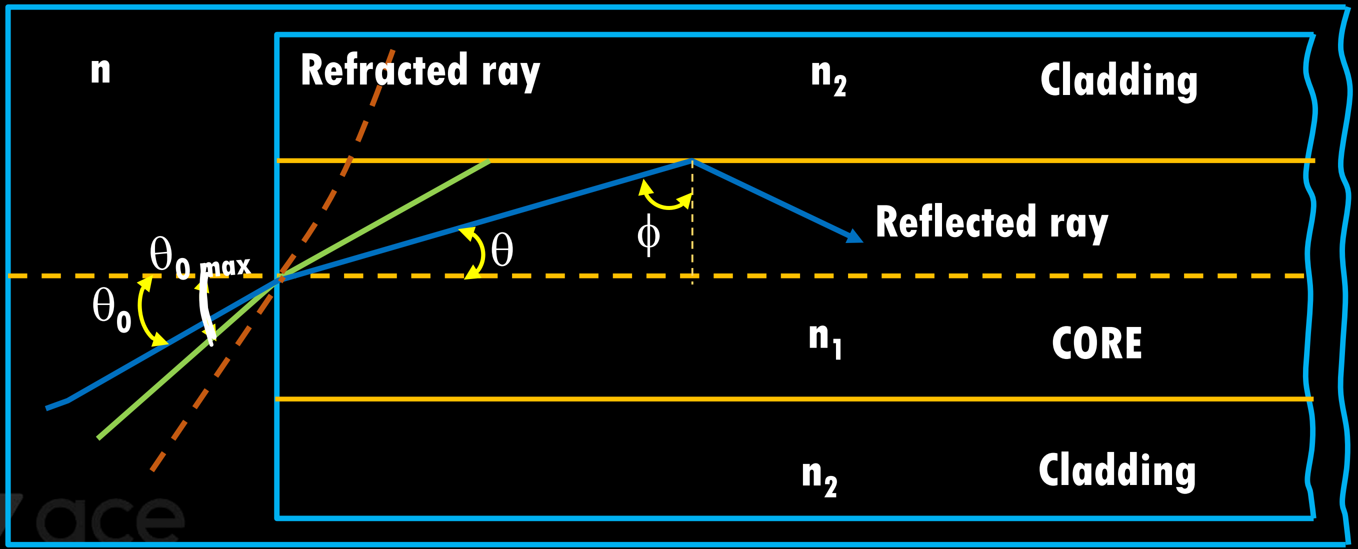
(b) $$f_c\Big|_{TM_{01}} = \frac{1}{2\pi\sqrt{\mu_0\epsilon_0}}\left(\frac{2.405}{r}\right) = \frac{3\times10^8}{2\pi}\times\frac{2.405}{1\times10^{-2}} = \underline{11.4\,\text{GHz}}$$

Single Fiber structure





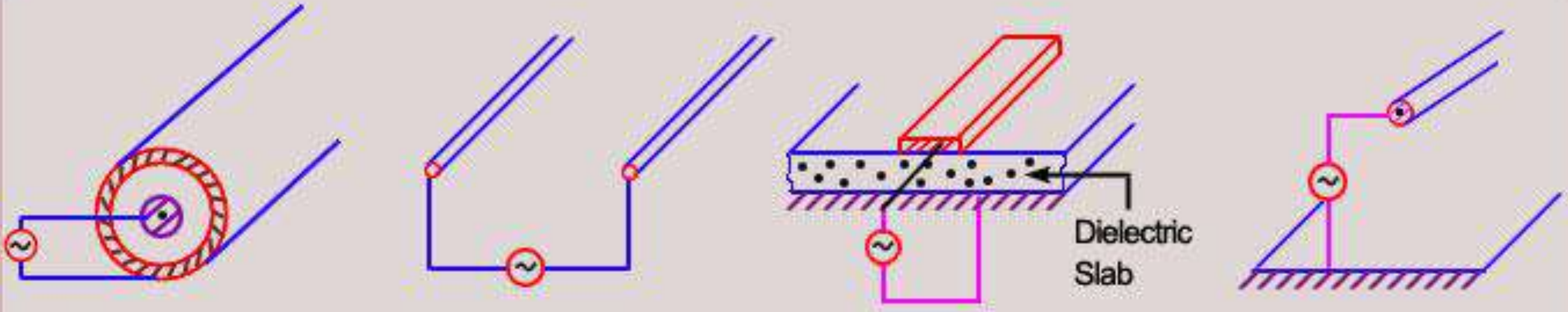
Meridional ray representation



$$\boxed{\sin\theta_{max} = \sqrt{n_1^2 - n_2^2} = NA}$$

$$\boxed{n_1 > n_2}$$

TRANSMISSION LINES



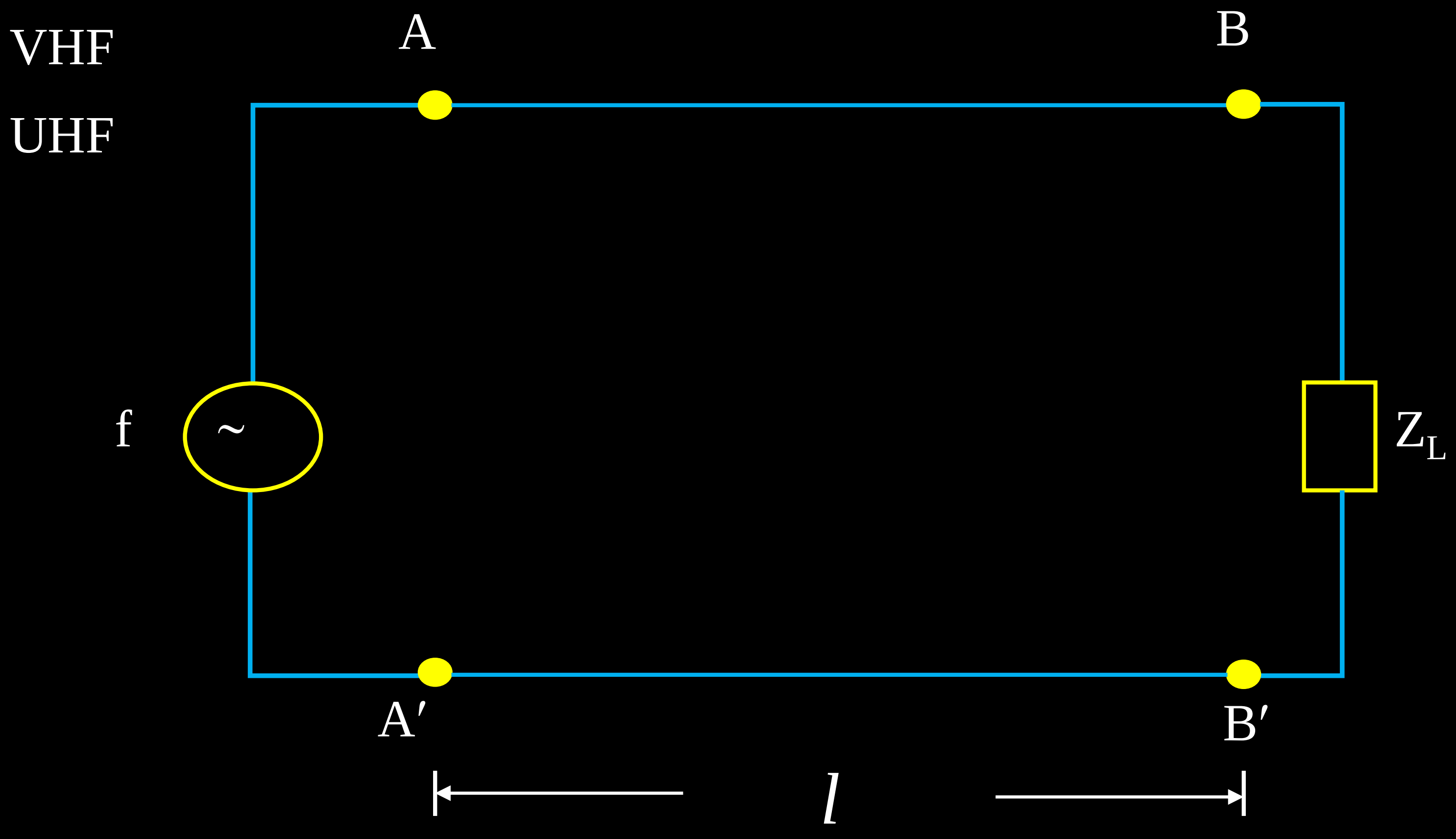
(a) Co - axial cable

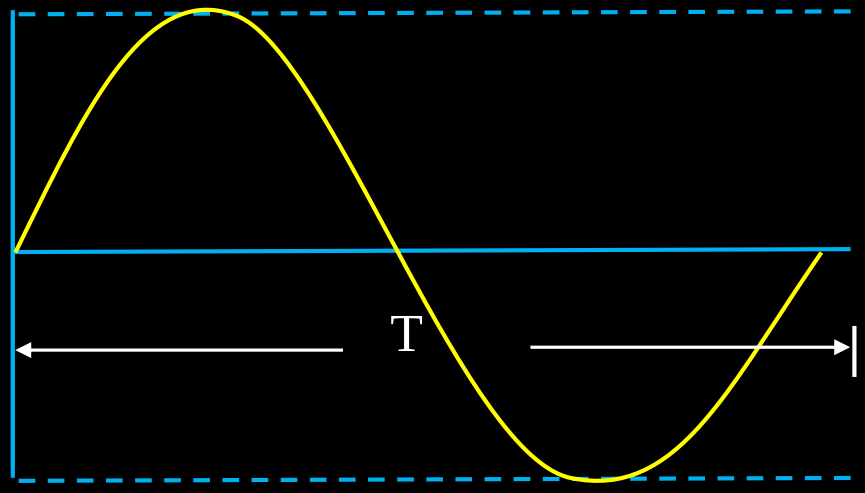
(b) Parallel wire Transmission line

(c) Microstrip line

(d) Unbalanced line

TWO CONDUCTOR SYSTEM:





Transit Time (t_r) $t_r = \dfrac{\ell}{v}$

EM-Theory	**Tx-Line**
$(\sigma, \mu, \in)$	(R, L, C, G)
γ, η	γ, Z_o
E / H – waves	V / I – waves
	The EM-energy propagation on Tx-line is analysed in terms of voltage, current waves and with the help of primary (R, L, C, G) and secondary (γ, z_o) constants of Tx-line.

Q. The propagation constant of a lossy transmission line is $(2 + j5)\ m^{-1}$ and its characteristic impedance is $(50 + j0)\ \Omega$ at $\omega = 10^6$ rad S^{-1}. The values of the line constants L,C,R,G are, respectively,

(GATE-16)(SET-1)

(a) L = 200 μH/m, C = 0.1 μF/m, R = 50 Ω/m, G = 0.02 S/m

(b) L = 250 μH/m, C = 0.1 μF/m, R = 100 Ω/m, G = 0.04 S/m

(c) L = 200 μH/m, C = 0.2 μF/m, R = 100 Ω/m, G = 0.02 S/m

(d) L = 250 μH/m, C = 0.2 μF/m, R = 50 Ω/m, G = 0.04 S/m

Soln $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \alpha + j\beta = 2+j5$

$Z_0 = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}} = 50$

$(R+j\omega L) = 50(2+j5) = 100 + j250$

$R = 100,\quad \omega L = 250,\quad L = \dfrac{250}{\omega} = 250\times 10^{-6}$

$L = 250\ \mu H/m.$

Reflection coefficient

$$\Gamma(l) = \Gamma_L e^{-j2\beta l}$$

$$\Gamma_L = \left[\frac{Z_L - Z_0}{Z_L + Z_0}\right]$$

$$\Gamma_L = |\Gamma_L| e^{j\phi_L}$$

$$\left[\begin{array}{l} 0 \le |\Gamma_L| \le 1 \\ 0^\circ \le \phi_L \le 180^\circ \end{array}\right]$$

Q. In a transmission line the reflection coefficient at the load end is given by $0.3.\ e^{-j30°}$. What is the reflection coefficient at a distance of 0.1 wavelengths toward source?

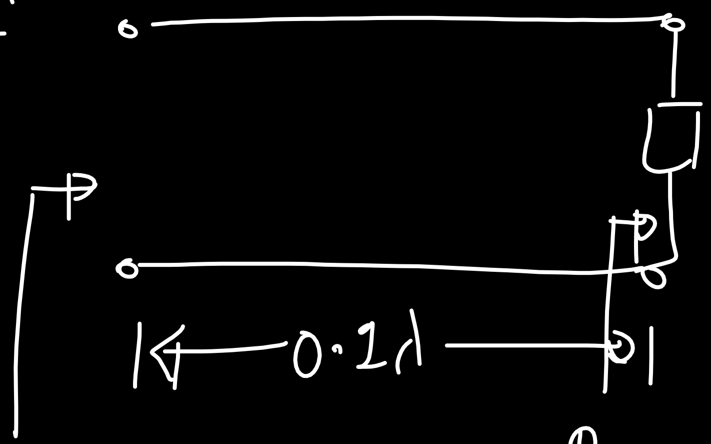
(a) $0.3e^{+j30^o}$

(b) $0.3\ e^{-j102^o}$

(c) $0.3\ e^{+j25^o}$

(d) $0.3\ e^{-j66^o}$

Sol$_n$



$\Gamma_L = 0.3 e^{-j30°}$

$\Gamma(l) = ?$

$\Gamma(l) = \Gamma_L e^{-j2\beta l}$

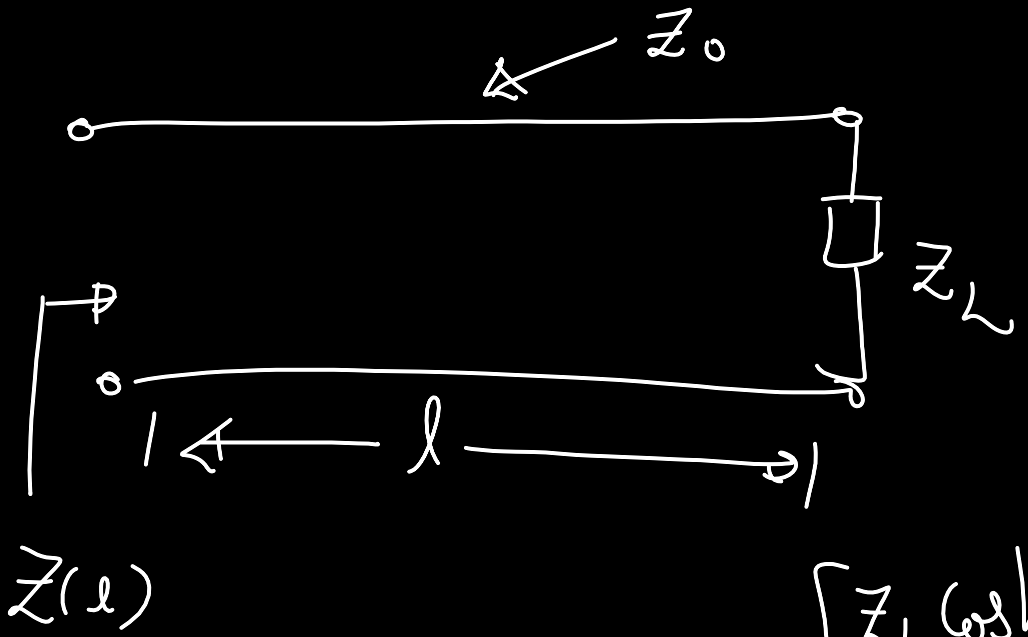
$2\beta l = 2 \times \frac{2\pi}{\lambda} \times \frac{\lambda}{10} = \frac{4}{10} \times 180°$

$2\beta l = 72°$

$\Gamma(l) = 0.3 e^{-j30°} \cdot e^{-j72°}$

$\Gamma(l) = 0.3 e^{-j102°}$

IMPEDANCE TRANSFORMATION RELATION



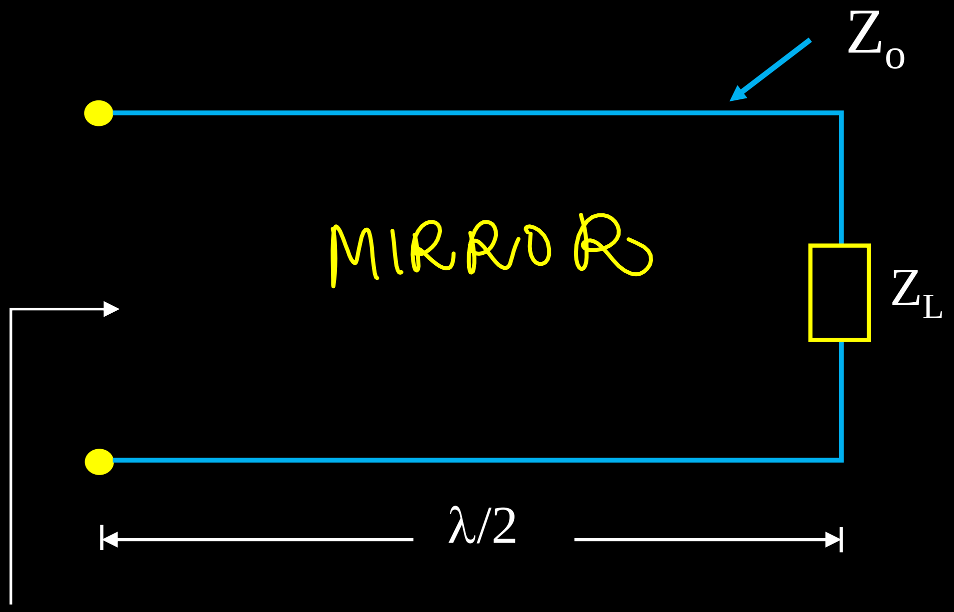
$$Z(l) = Z_0\left[\frac{Z_L \cosh\gamma l + Z_0 \sinh\gamma l}{Z_0 \cosh\gamma l + Z_L \sinh\gamma l}\right], \quad \gamma = \alpha + j\beta.$$

IMPEDANCE TRANSFORMATION ON LOSS-LESS TX-LINE

$$\gamma = j\beta$$

$$Z(l) = Z_0 \left[\frac{Z_L \cos\beta l + j Z_0 \sin\beta l}{Z_0 \cos\beta l + j Z_L \sin\beta l} \right]$$

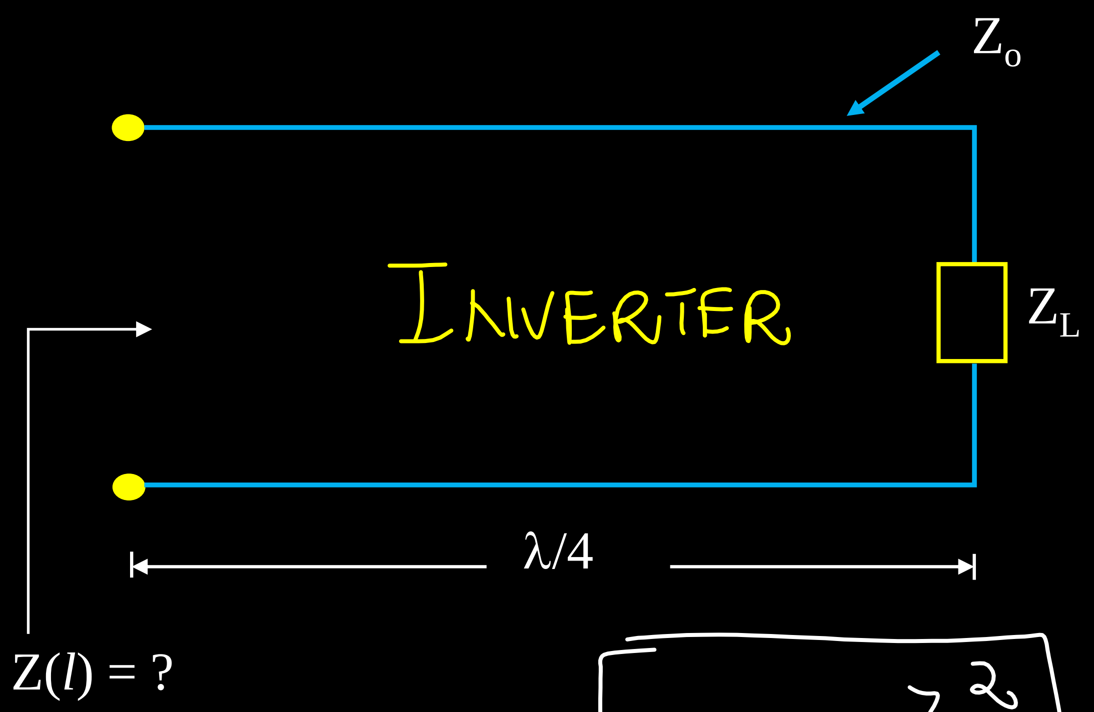
Case 1 $\frac{\lambda}{2}$: Tx – Line



$Z(l) = ?$

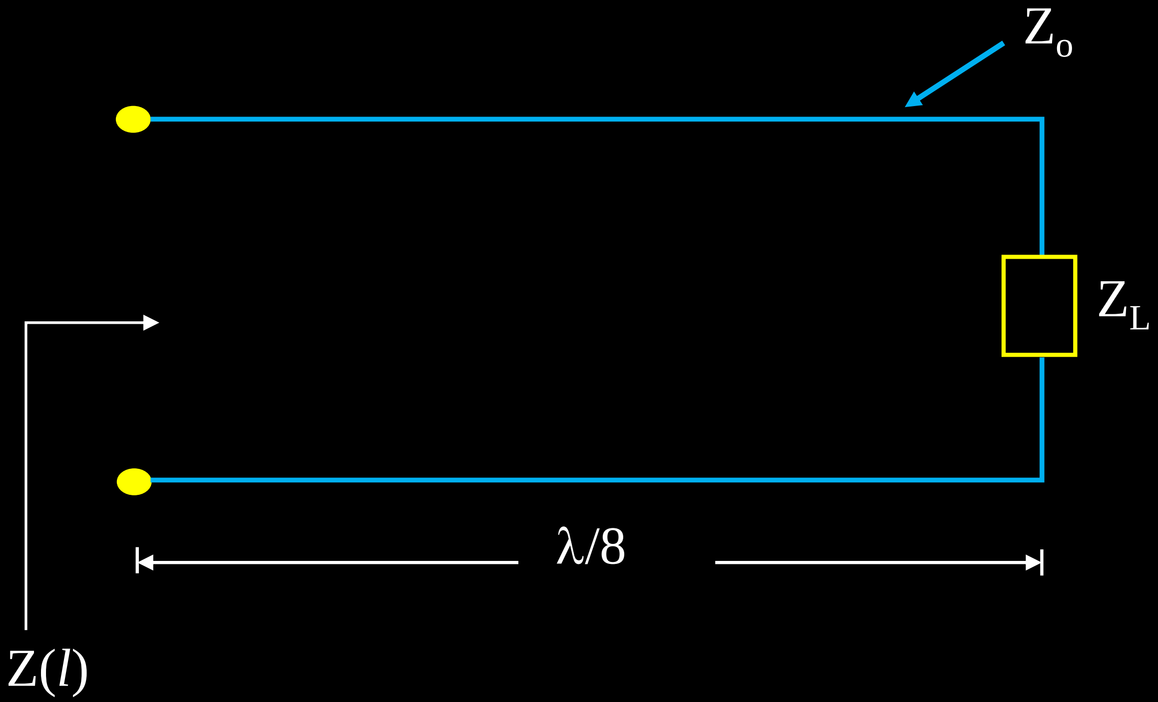
$$\boxed{Z(l) = Z_L}$$

Case 2 $\frac{\lambda}{4} : Tx - Line$



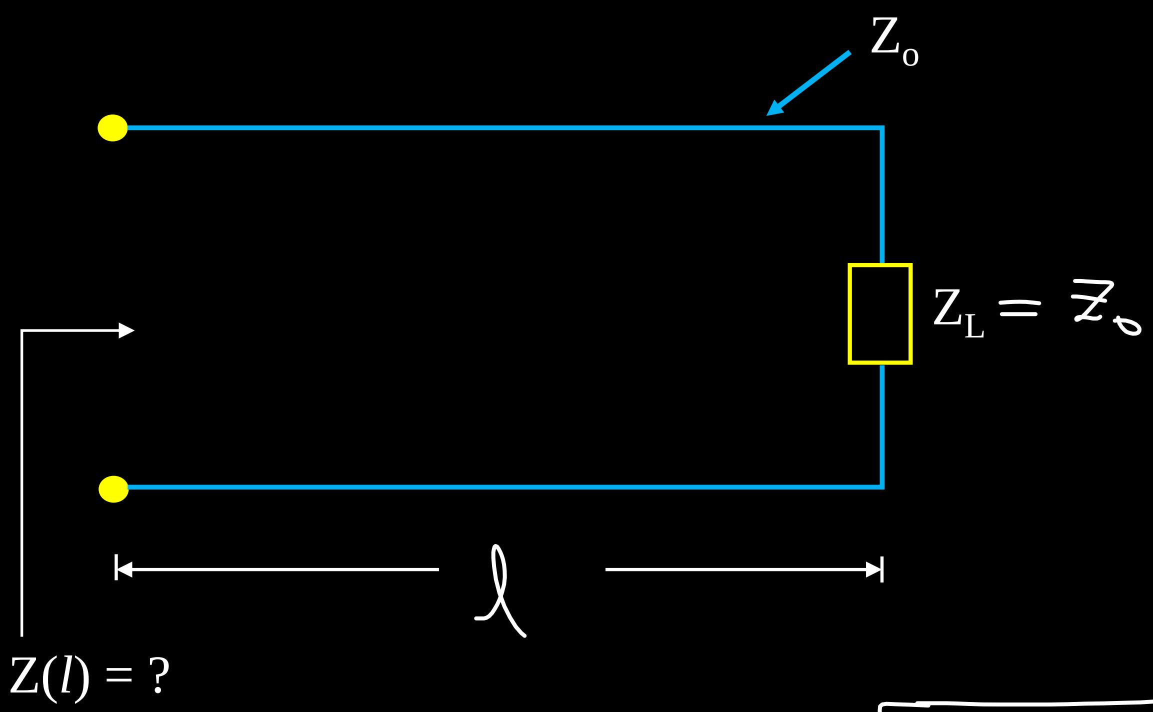
$$\boxed{Z(l) = \frac{Z_0^2}{Z_L}}$$

Case 3 $\frac{\lambda}{8}$: Tx – Line



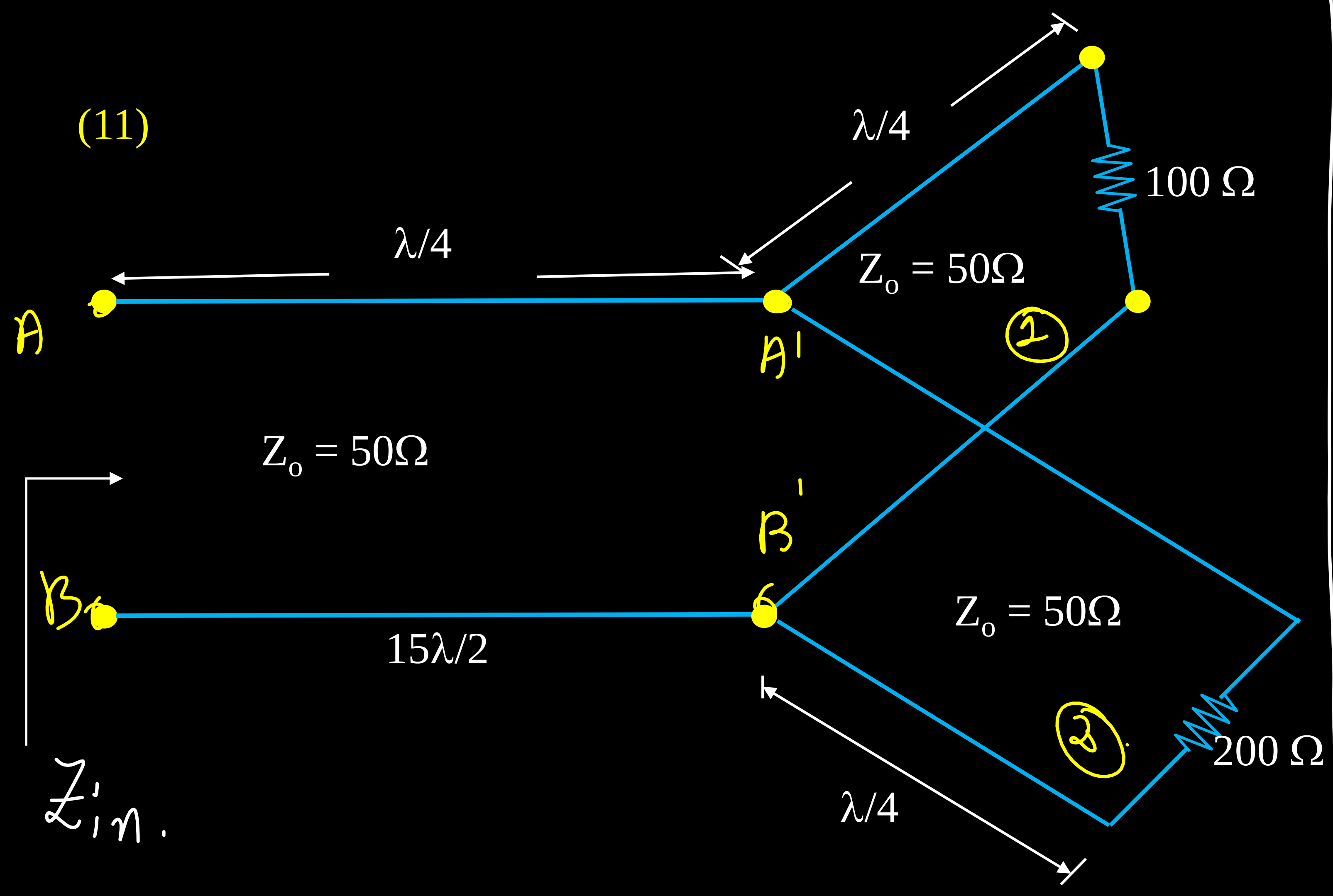
$$\boxed{Z(l) = Z_o \left[\frac{Z_L + j Z_o}{Z_o + j Z_L} \right]}$$

Case 4: Matched Line

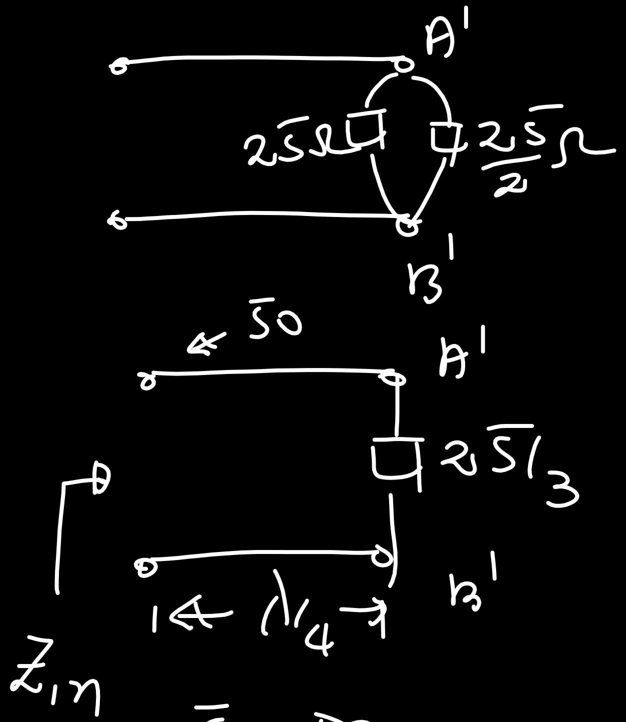


$$\boxed{Z(l) = Z_o}$$

(11)



$$Z_1 = \frac{50\times 50}{100} = 25,\quad Z_2 = \frac{50\times 50}{200} = \frac{25}{2}$$



$$Z_{in} = \frac{50\times 50}{(25/3)} = 2\times 50\times 3$$

$$= \underline{300\,\Omega}$$

CHARACTERIZATION OF TX-LINE:

1. No-Loss

$R = 0, \; G = 0$

$\alpha = 0, \; \beta = \omega\sqrt{LC}$

$v_p = \frac{1}{\sqrt{LC}}, \; Z_0 = \sqrt{\frac{L}{C}}$

2. Low-Loss

$R \ll \omega L, \; G \ll \omega C$

$\alpha \simeq \frac{1}{2}\left[R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right]$

$\beta \simeq \omega\sqrt{LC}$

$v_p \simeq \frac{1}{\sqrt{LC}}$

$Z_0 \simeq \sqrt{\frac{L}{C}}$

3. High-Loss

$R \gg \omega L, \; G \gg \omega C$

4. Distortion-Less

***

$\boxed{LG = RC}$

$\alpha = \sqrt{RG}, \; Z_0 = \sqrt{\frac{R}{G}}$

$\beta = \omega L\sqrt{\frac{G}{R}} = \omega C\sqrt{\frac{R}{G}}$

Q. A lossy transmission line has resistance per unit length R = 0.05Ω/m. The line is distortionless and has characteristic impedance of 50Ω. The attenuation constant (in Np/m, correct to three decimal places) of the line is ______________.

(GATE - 18)

Soln

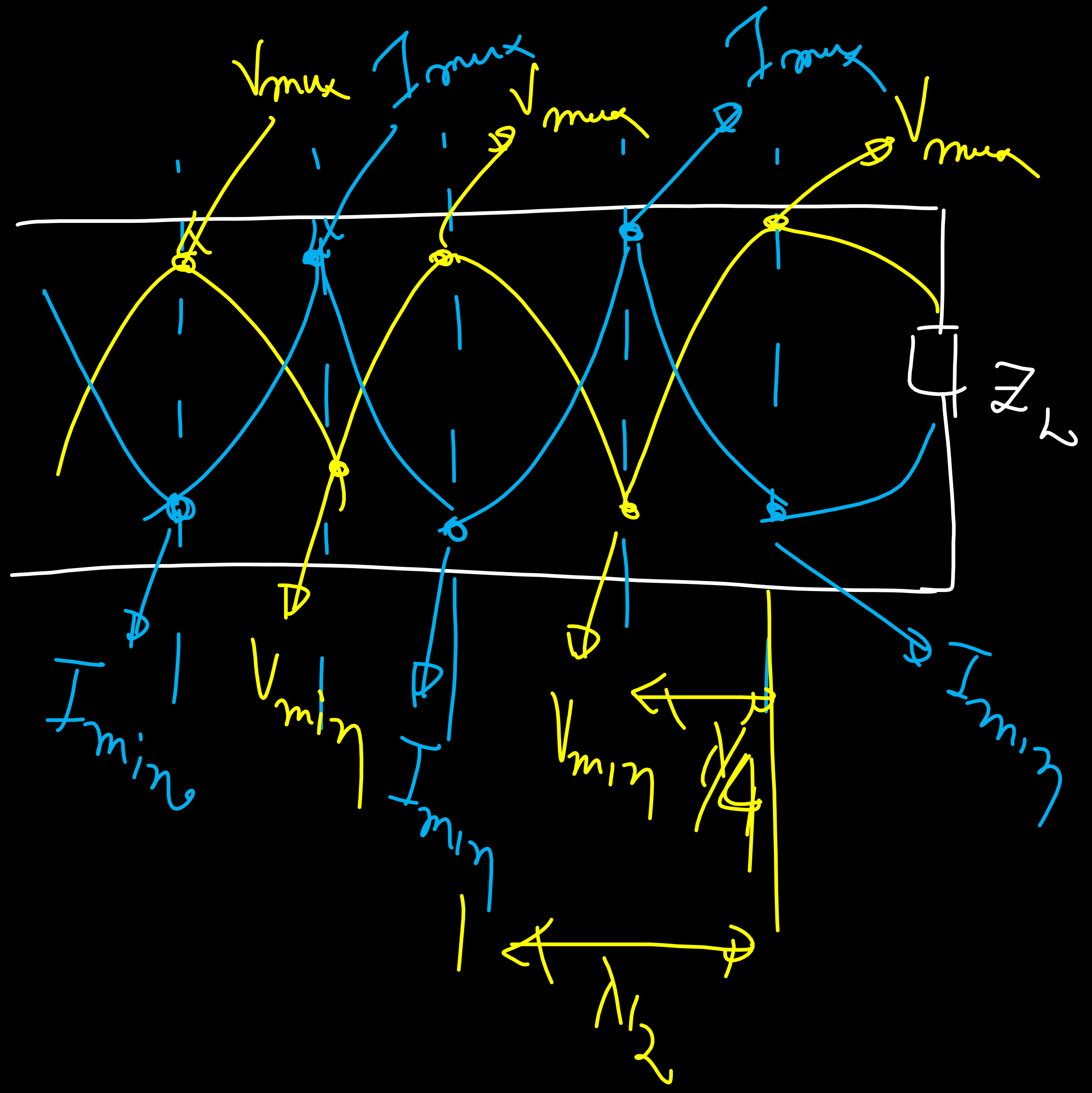
$$\alpha = \sqrt{RG}$$

$$Z_0 = \sqrt{\frac{R}{G}}$$

$$\alpha Z_0 = R.$$

$$\alpha = \frac{R}{Z_0} = \frac{0.05}{50} = \frac{5}{100 \times 50} = 0.001 \text{ Np/m}$$

Voltage and currents on loss-less Tx-lines



Maxima : (V_{max} , I_{min})

$$(\phi_L - 2\beta l) = 2n\pi$$

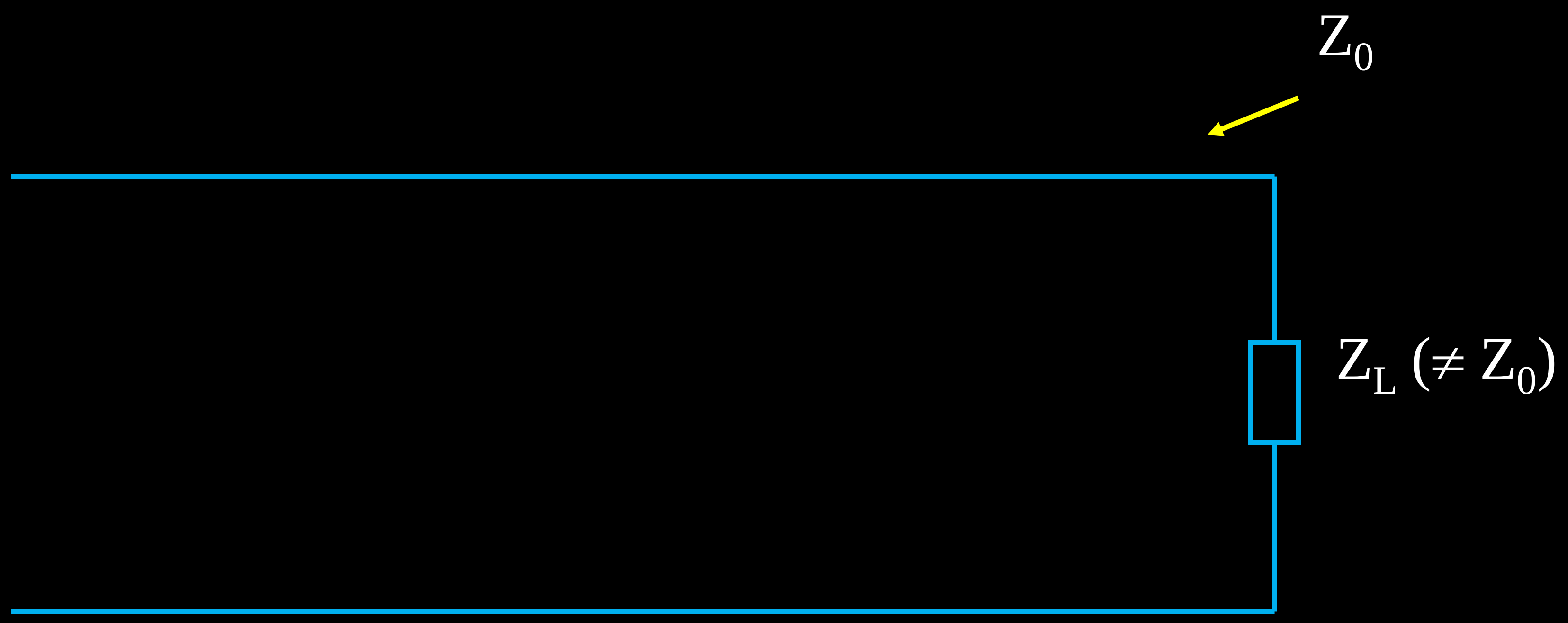
$$R_{max} = Z_0 \rho$$

Minima: (V_{min} , I_{max})

$$(\phi_L - 2\beta l) = (2m+1)\pi$$

$$R_{min} = \frac{Z_0}{\rho}$$

EX :



VOLTAGE STANDING WAVE RATIO (ρ)

$$\rho = \frac{|V_{max}|}{|V_{min}|}$$

$$\boxed{\rho = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}}$$

$$\boxed{|\Gamma_L| = \frac{\rho - 1}{\rho + 1}}$$

$$\boxed{1 \leq \rho \leq \infty}$$

* A two wire transmission line terminates in a television set. The VSWR measured on the line is 5.8. The percentage of power that is reflected from the television set is ______

(GATE - 17) (Set 2)

Soln. $\rho = 5.8$

$$|\Gamma_L| = \frac{\rho - 1}{\rho + 1} = \frac{5.8 - 1}{5.8 + 1} = \frac{4.8}{6.8}$$

$$|\Gamma_L| = \frac{48}{68}$$

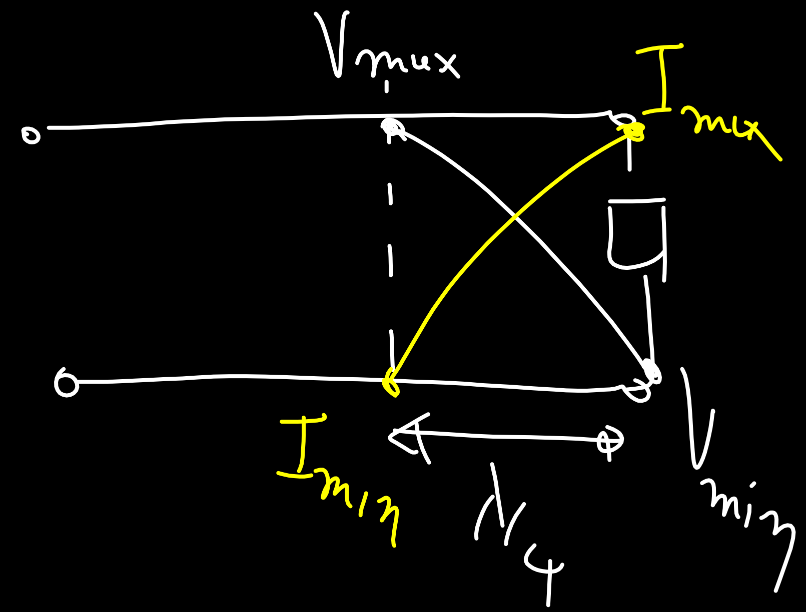
$$\frac{P_r}{P_i} = |\Gamma_L|^2 = \left(\frac{48}{68}\right)^2$$

$$\% \frac{P_r}{P_i} = 49.8\%$$

Q. A Tx-Line of characteristic impedance 50 Ω is terminated in a load impedance Z_L. The VSWR of the line is measured as 5 and the first of the maxima on the line is observed at the distance of $\lambda/4$ from the load. The value of Z_L is ______

GATE-11

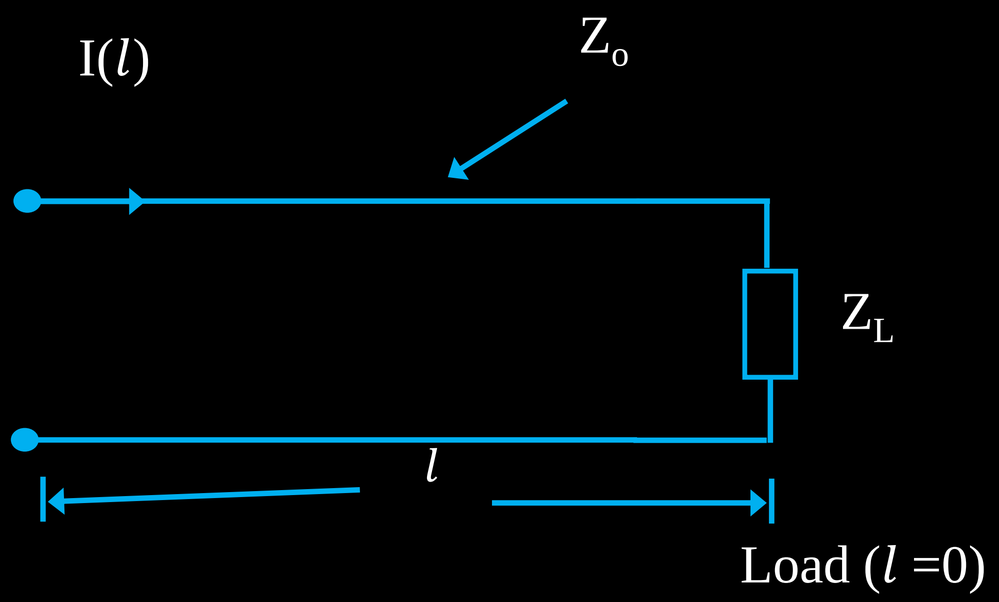
Soln.



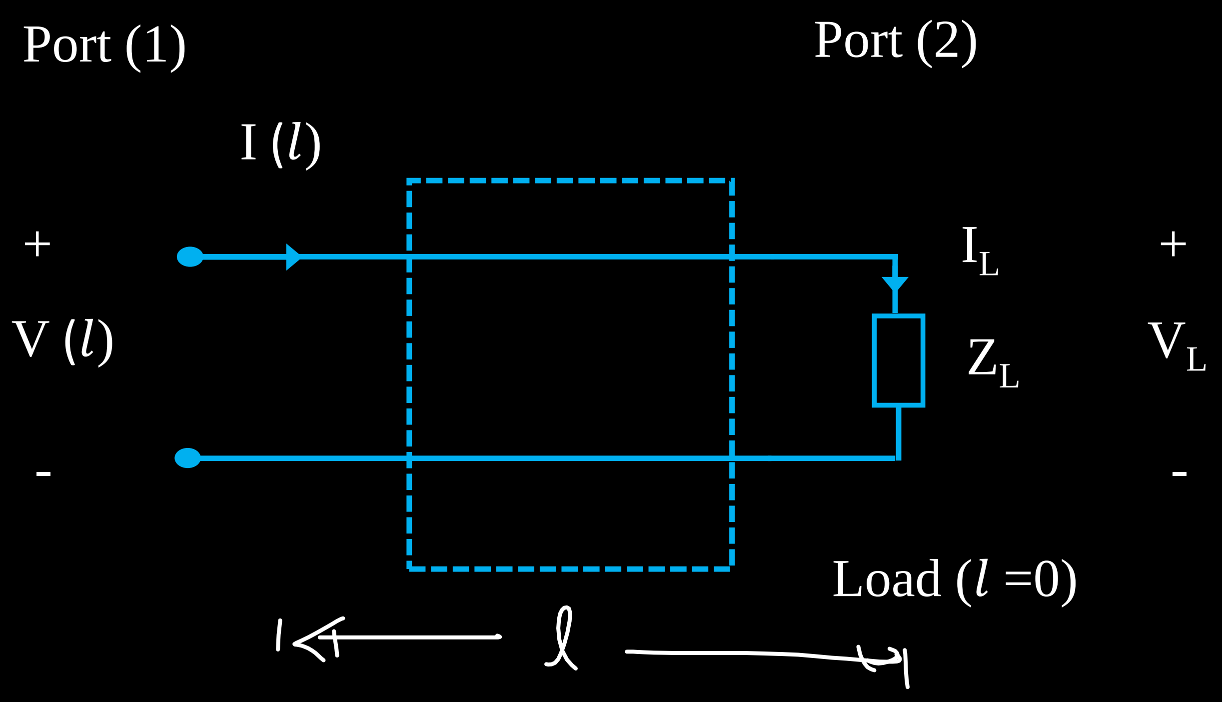
$$Z_L = \frac{V_{min}}{I_{max}} = Z_0/\rho = 50/5$$

$$Z_L = \cancel{250\,\Omega} \quad 10\,\Omega$$

Evaluation of Arbitrary constants V^+ , V^-



Consider



$$V(l) = V_L cos\beta l + I_L jZ_o sin\beta l$$

$$I(l) = V_L \frac{jsin\beta l}{Z_o} + I_L cos\beta l$$

$$\begin{bmatrix} V(l) \\ I(l) \end{bmatrix} = \begin{bmatrix} \cos\beta l & jZ_0 \sin\beta l \\ \frac{j\sin\beta l}{Z_0} & \cos\beta l \end{bmatrix} \begin{bmatrix} V_L \\ I_L \end{bmatrix}$$

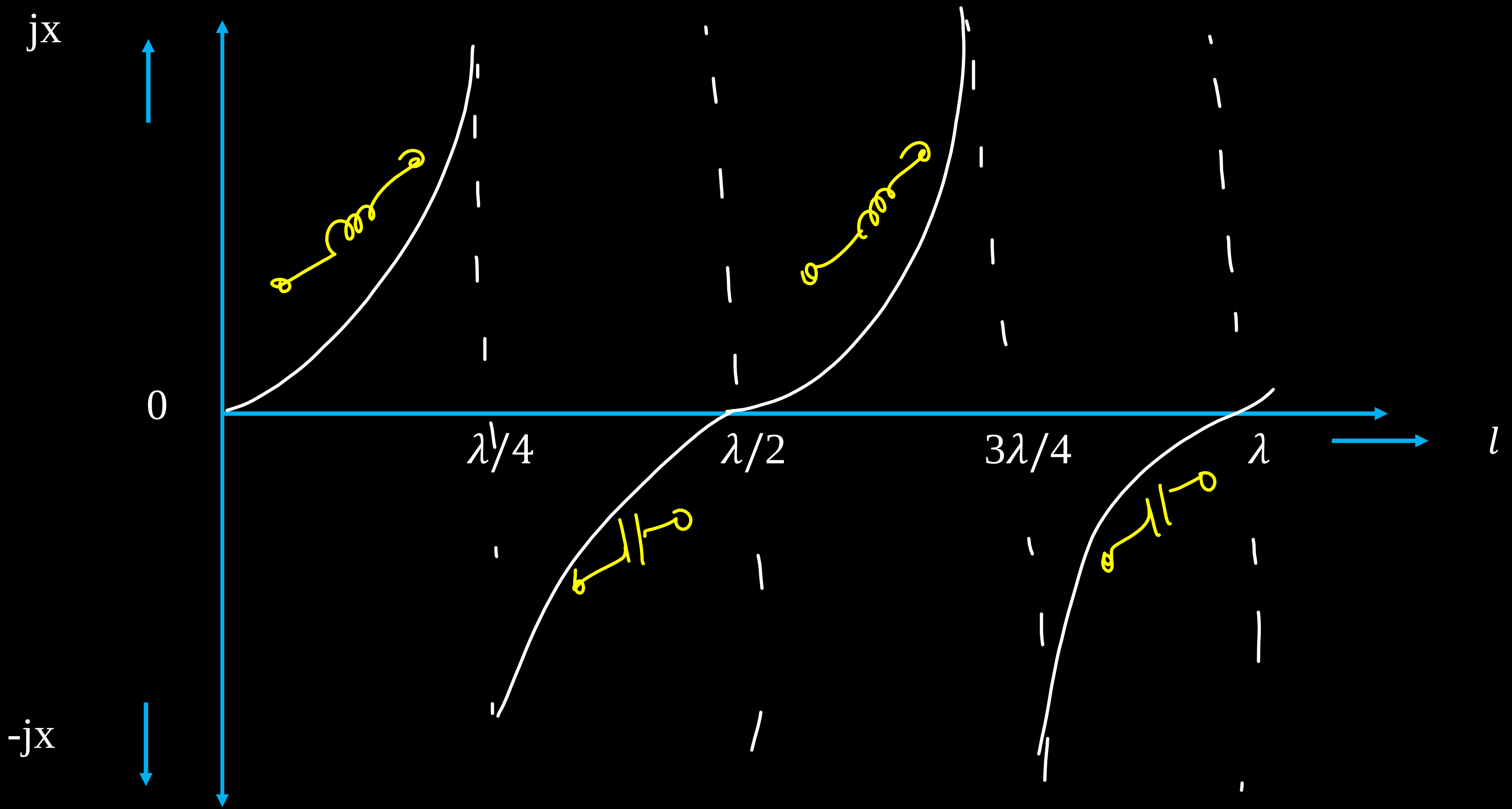
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta l & jZ_0 \sin\beta l \\ \frac{j\sin\beta l}{Z_0} & \cos\beta l \end{bmatrix}$$

Applications of Tx-Line

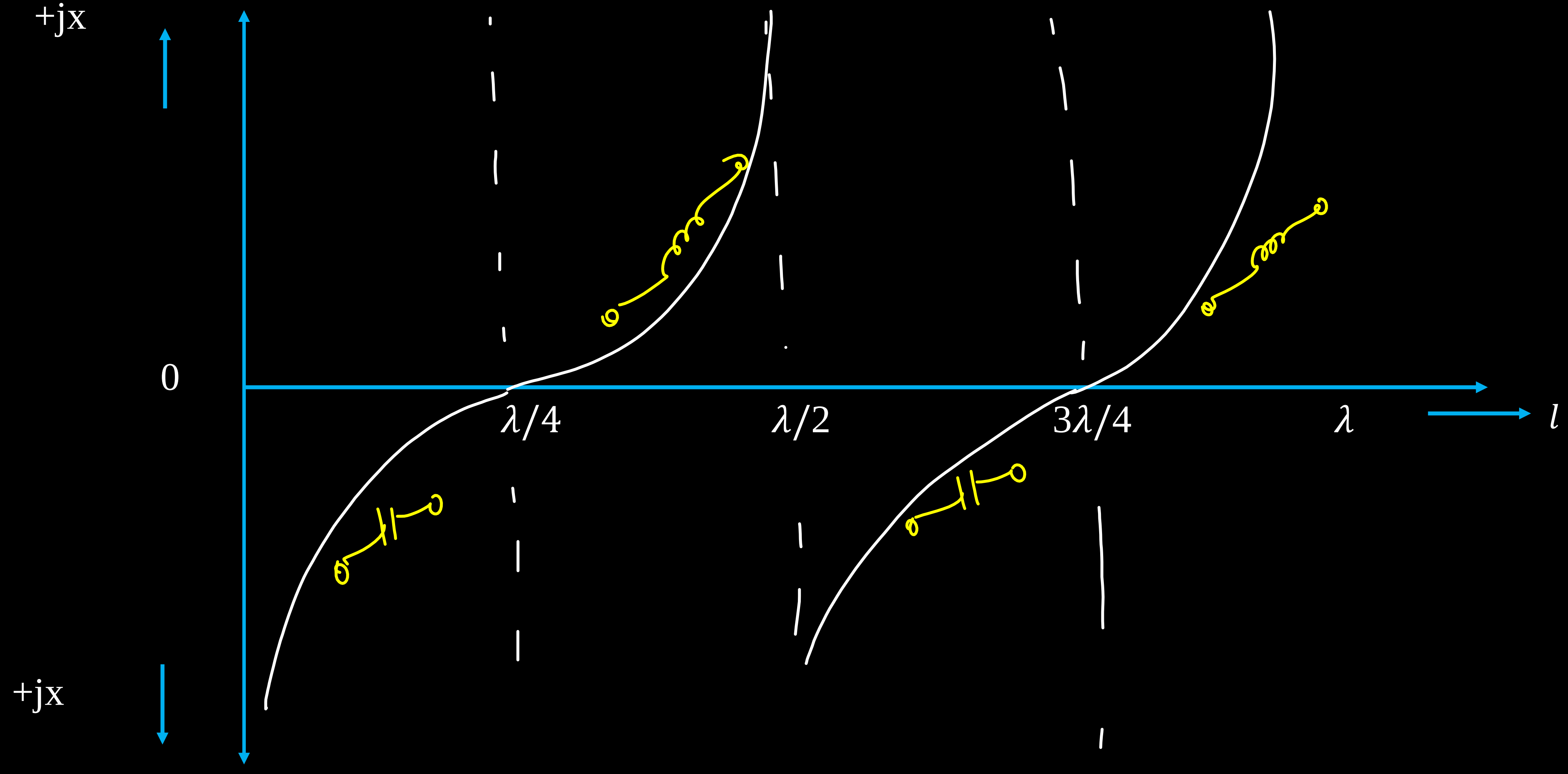
(I) Tx-Line as circuit element

$$\boxed{Z_0 = \sqrt{Z_{oc} Z_{sc}}}$$

$$\boxed{Z_{S.C} = j\,Z_o \tan\beta l}$$



$$\boxed{Z_{O.C} = -j\, Z_o \cot\beta l}$$



Q. One end of a loss-less Tx-line having the characteristic impedance of 75Ω and length of 1cm is short-circuited. At 3GHz, the input impedance at the other end of the Tx-line is.

G-2008

(a) 0

(b) Resistive

(c) Capacitive

(d) Inductive

soln

$Z_{s.c} = jZ_0 \tan\beta l$

$\lambda = \frac{3\times10^8}{3\times10^9} = 0.1m = \frac{1}{10}$

$l = 1\times10^{-2} = \frac{1}{100}$

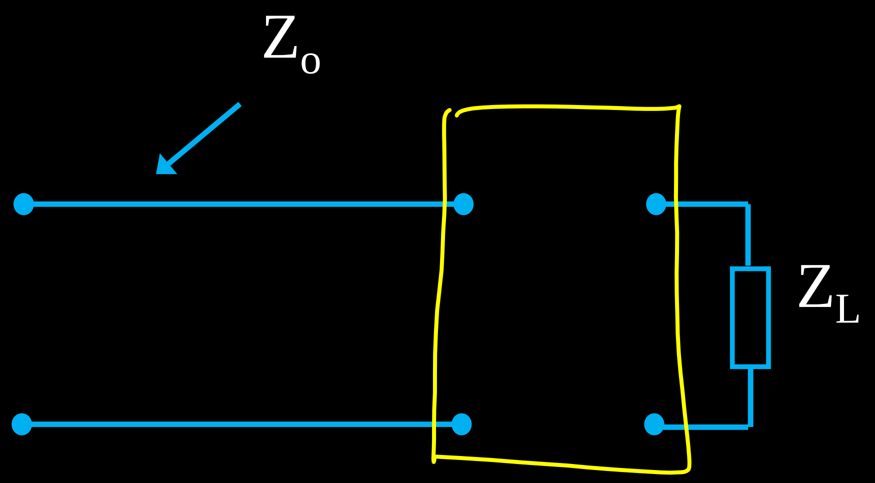
$= j\times75 \tan\left\{\frac{2\pi}{\left(\frac{1}{10}\right)} \times \left(\frac{1}{100}\right)\right\}$

$= j75\tan\left(\frac{2\pi}{10}\right)$ (+ve)

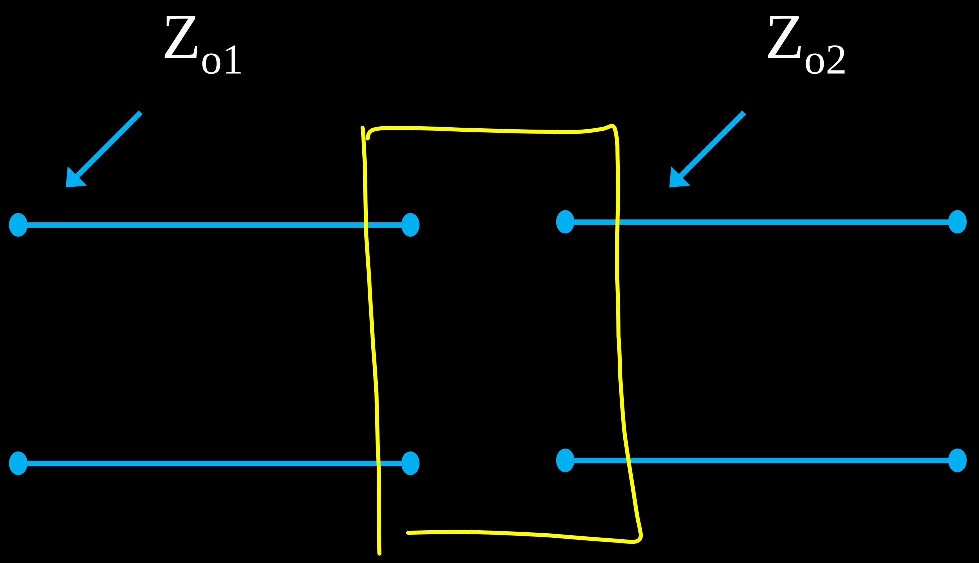
$= j54$

(II) Impedance Matching

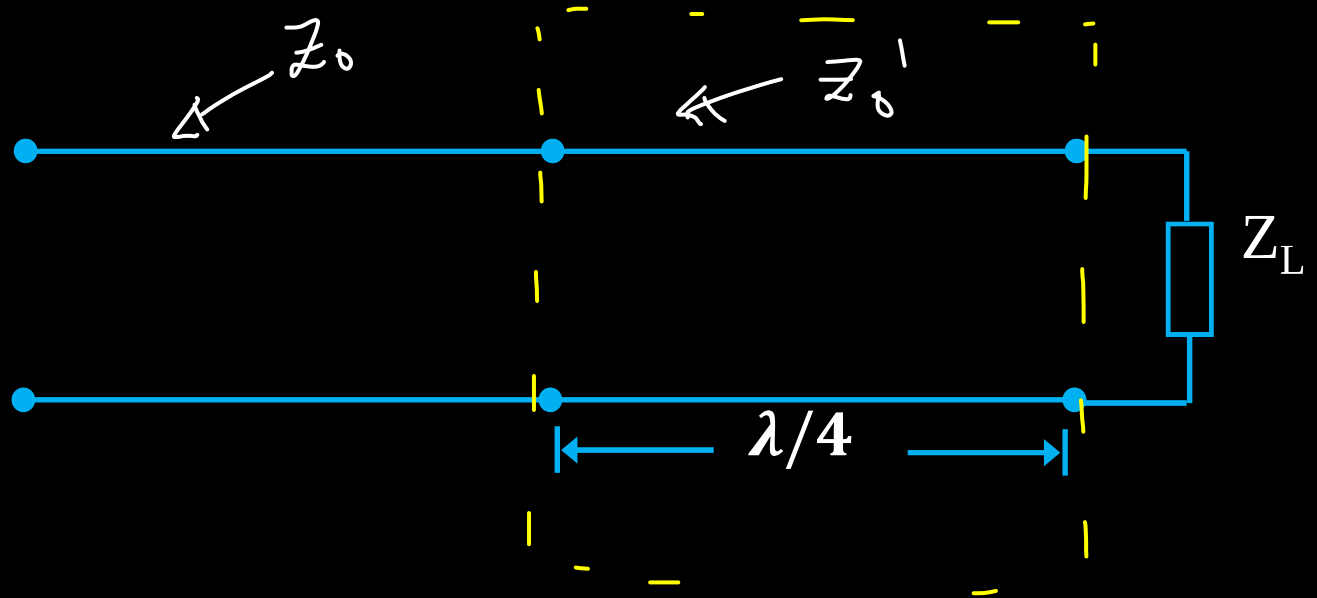
(1)



(2)



$\left(\frac{\lambda}{4}\right)$: Impedance Matching Transformer



$$\boxed{Z_0' = \sqrt{Z_0 Z_L}}$$

Q. Two very long loss-less cables of characteristic impedance 50Ω and 100Ω respectively are to be joined for reflection-less transmission. Find characteristic impedance of matching transformer.

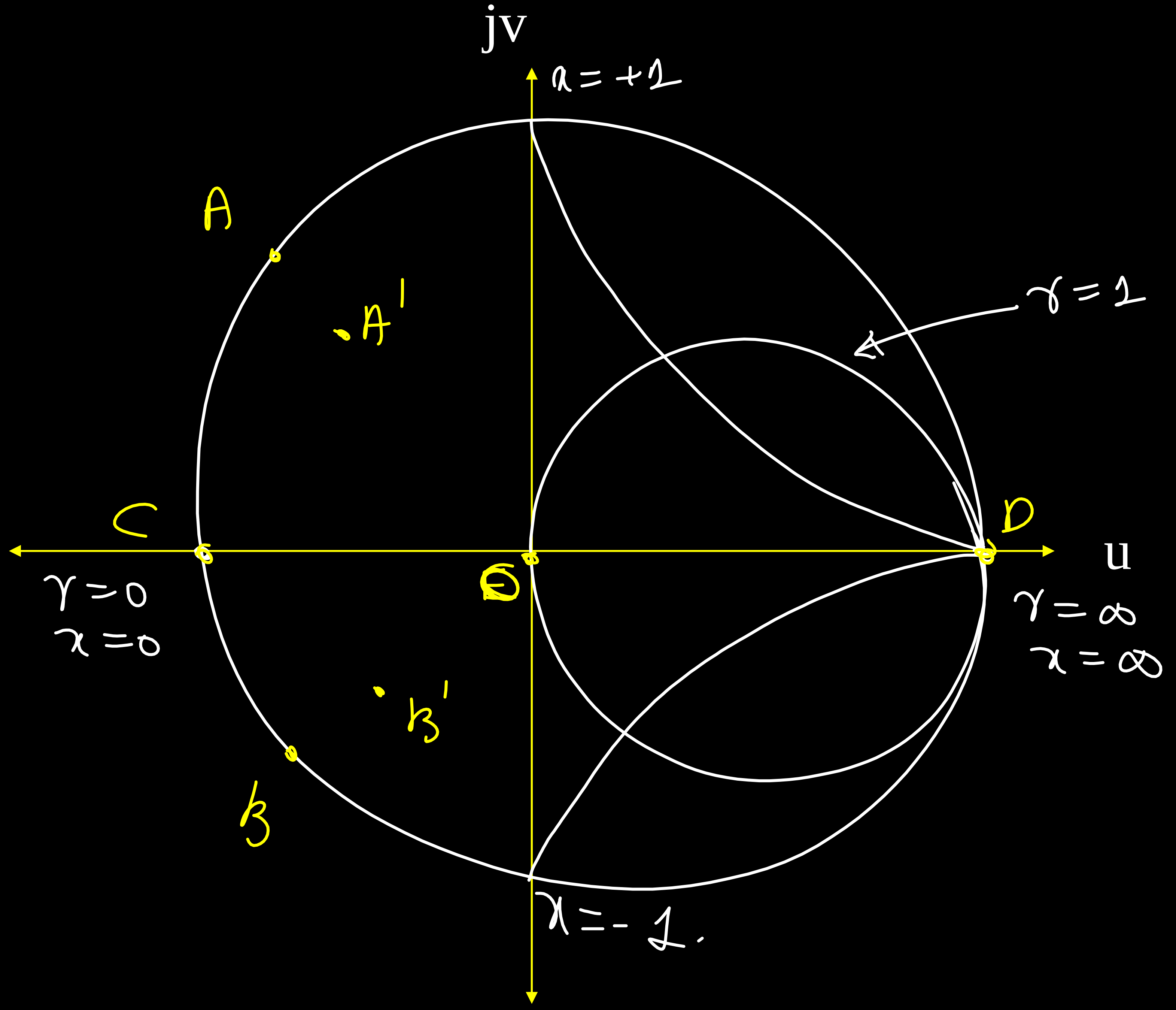
$$Z_0' = \sqrt{50 \times 100}$$

$$Z_0' = \underline{70.71\,\Omega}$$

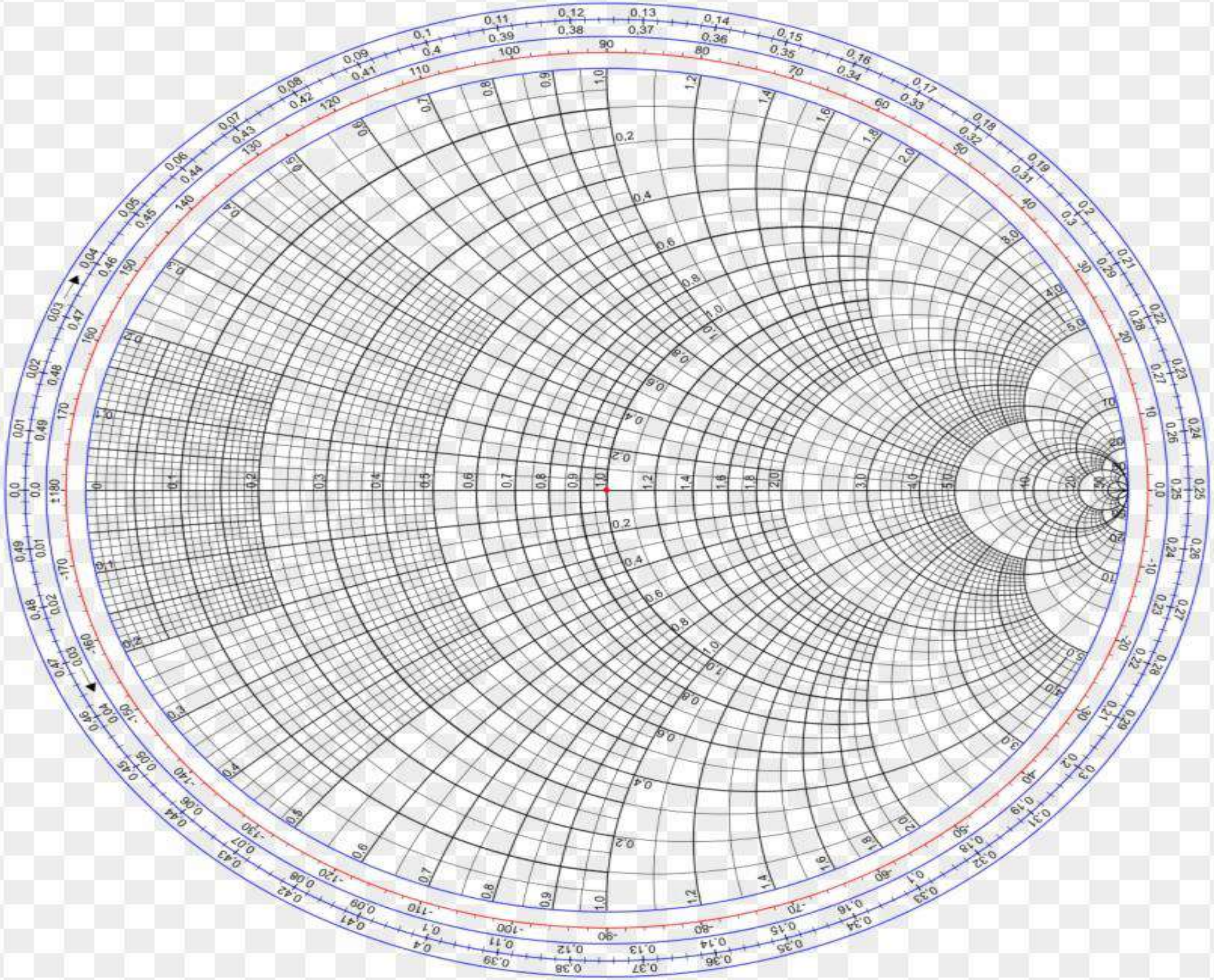
Graphical Approach

(Smith Chart)

- Smith chart is the graphical tool in which complex impedances are plotted on to the complex reflection co-efficient plane.



jv
x = +1
A
A'
r = 1
C
D
u
r = 0
x = 0
O
r = ∞
x = ∞
B'
B
x = -1.



A: jx (Pure inductive)

A¹: r + jx (Inductive)

B : - jx (Pure Capacitive)

B¹: r - jx (Capacitive)

C : Short Circuit

D : Open Circuit

Centre (0):

$\bar{Z} = 1 + j\,0$

$$\frac{Z}{Z_0} = 1$$

$Z = Z_0$

Matche load

Clock Wise Rotation: Moving towards the generator.

Anti Clock Wise Rotation: Moving towards the load.

2π Radians: $\frac{\lambda}{2}$ Length movement on Tx – line.

π Radians: $\frac{\lambda}{4}$ Length movement on Tx – line.

Constant VSWR Circle:

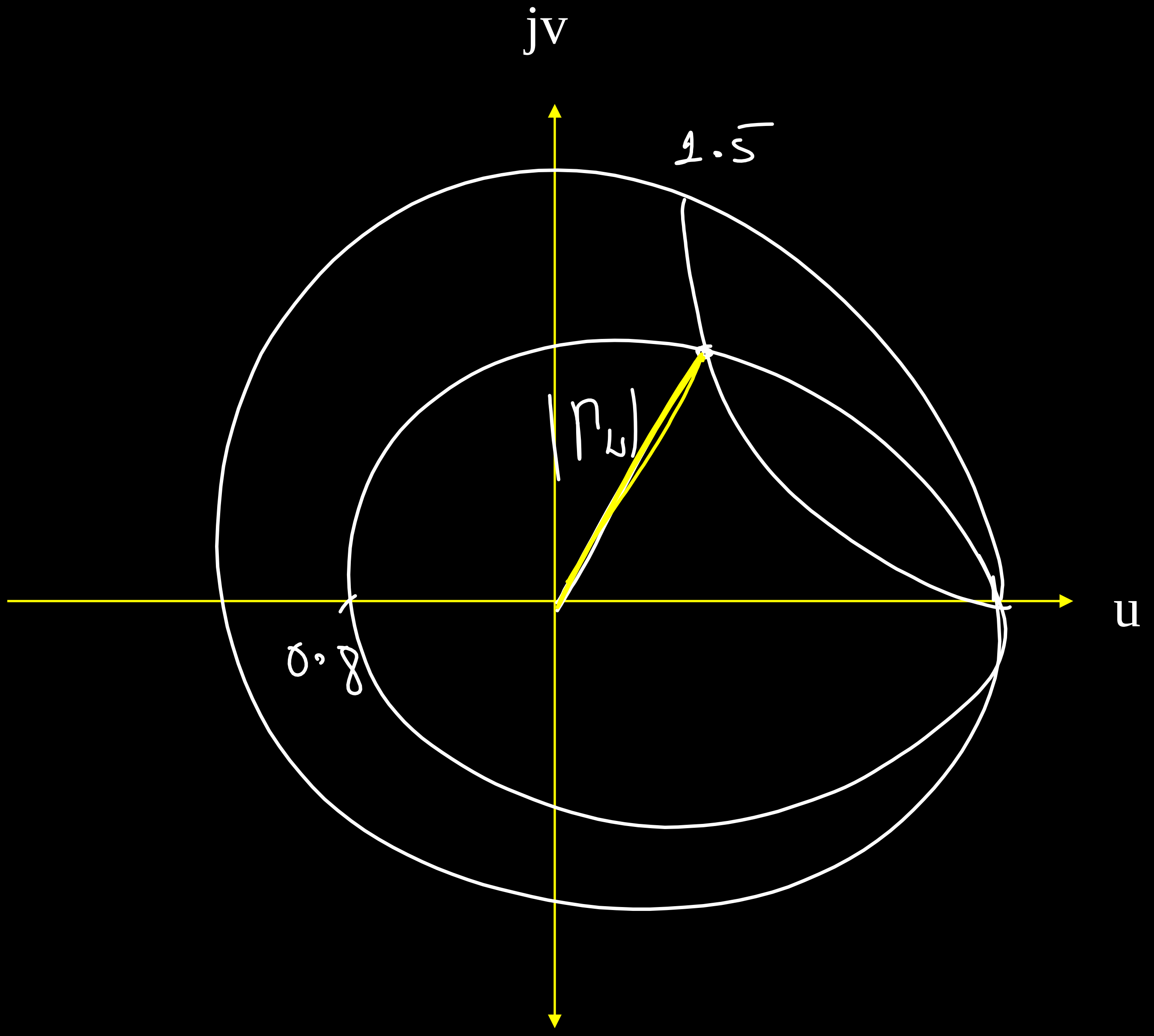
Ex: $Z = 80 + j\,150\ \Omega$

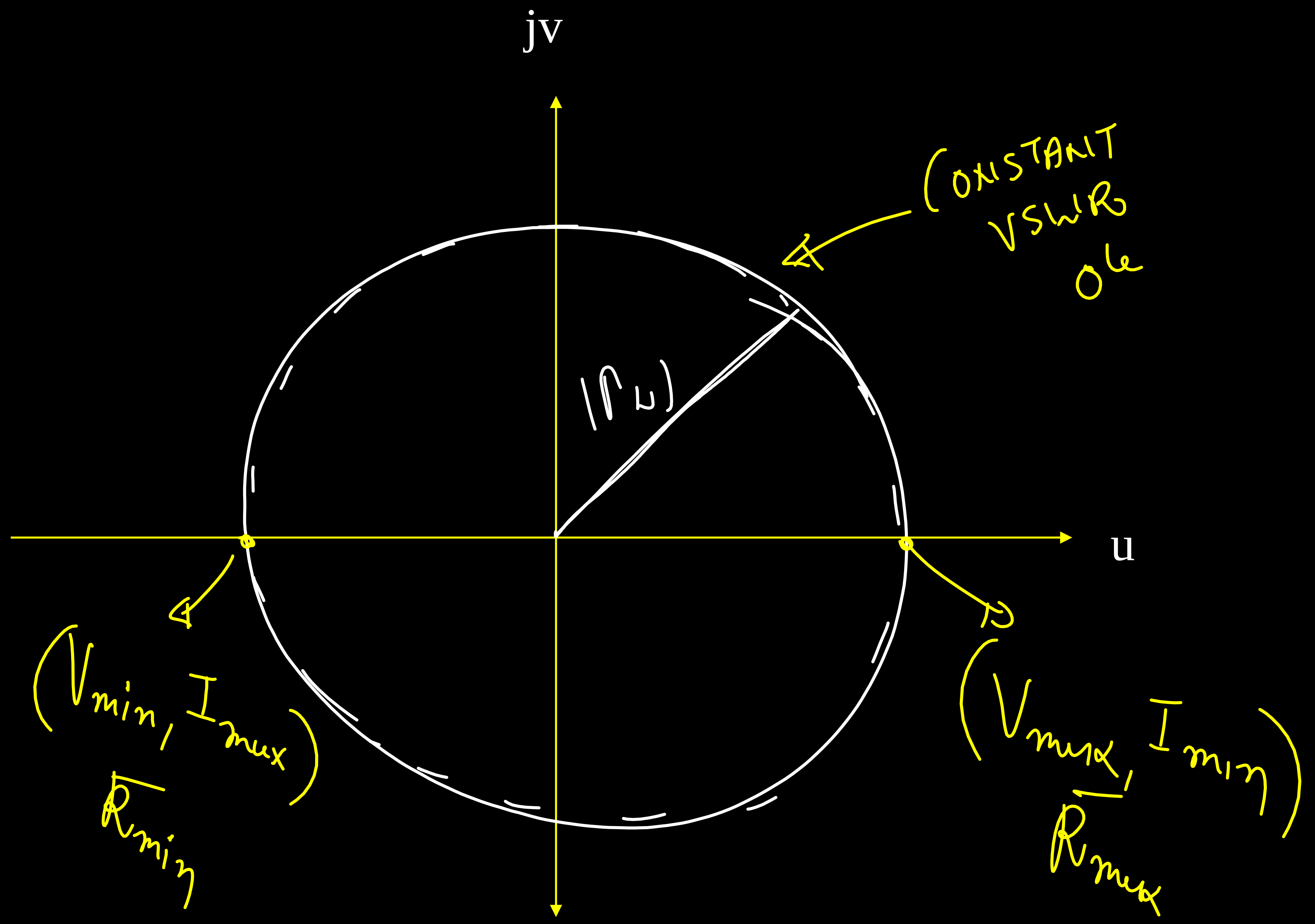
$Z_0 = 100\ \Omega$

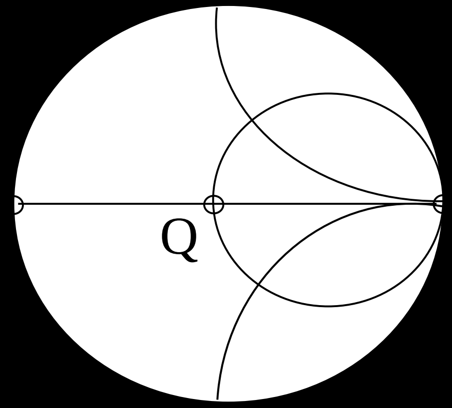
$$\overline{Z} = \frac{Z}{Z_0} = \frac{80 + j\,150}{100}$$

$\overline{Z} = 0.8 + j\,1.5$

Normalized Impedance

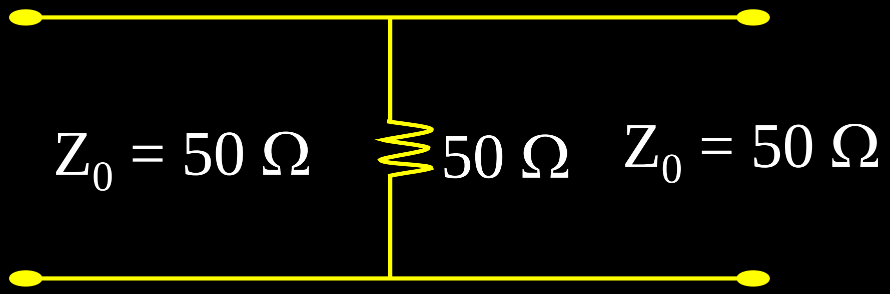



jv
u
CONSTANT VSWR CC
$|\Gamma_L|$
(V_{min}, I_{max})
R_{min}
(V_{max}, I_{min})
R_{max}


Q

Scattering Parameters

(S – Parameters)



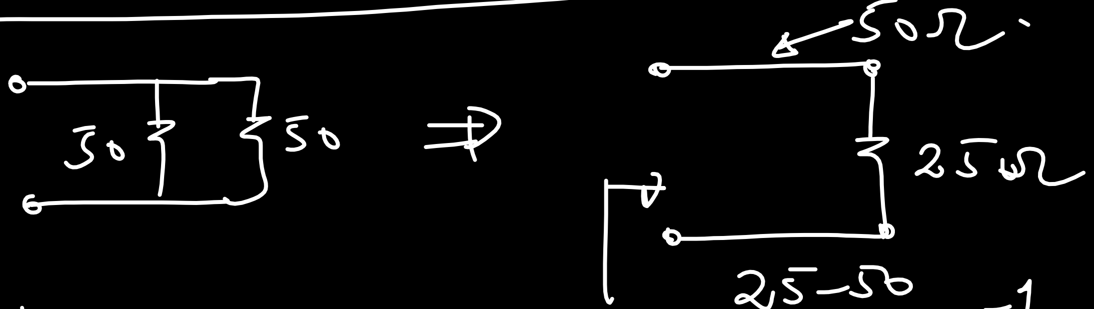
(a) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

(d) $\begin{bmatrix} \frac{1}{4} & \frac{-3}{4} \\ \frac{-3}{4} & \frac{1}{4} \end{bmatrix}$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$



$$S_{11} = \frac{25-50}{25+50} = \frac{-1}{3}.$$

$$|\Gamma| + \tau = 1$$

$$|S_{11}| + S_{21} = 1$$

$$S_{21} = 1 - \left|-1/3\right| = 2/3$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{bmatrix}$$

Symmetry Of S – Matrix:

For reciprocal microwave junction S – matrix is symmetrical.

i.e.,

$$S_{ij} = S_{ji} \Rightarrow [S] = [S]^T$$

S – Matric For Loss-less Junction:

$$\boxed{[s]\,[s^*]^T = [U]}$$

$R_1 R_1^* \rightarrow 1$ (UNIT PROPERTY)

+

$R_1 R_2^* \rightarrow 0$ (ZERO PROPERTY)

$\left.\right\}$ UNITARY PROPERTY.

⋮

Q. A two-port network characterized by the S-parameter matrix

$$S = \begin{bmatrix} 0.3\angle 0^o & 0.9\angle 90^o \\ 0.9\angle 90^o & 0.2\angle 0^o \end{bmatrix}$$

(a) both reciprocal and lossless

(b) reciprocal, but not lossless

(c) lossless, but not reciprocal

(d) neither reciprocal nor lossless

$\boxed{R_1 R_1^* \Rightarrow 1}$

$R_2 R_2^* \Rightarrow 1$

$R_1 R_2^* \Rightarrow 0$

$R_2 R_1^* \Rightarrow 0.$

$(0.3 e^{j0^\circ})(0.3 e^{j0^\circ})^* + (0.9 e^{j90^\circ})(0.9 e^{j90^\circ})^*$

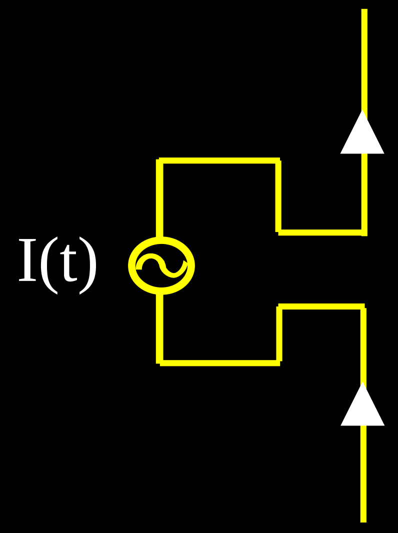
$(0.3)^2 + (0.9)^2$

$= 0.09 + 0.81 = 0.9 \neq 1$

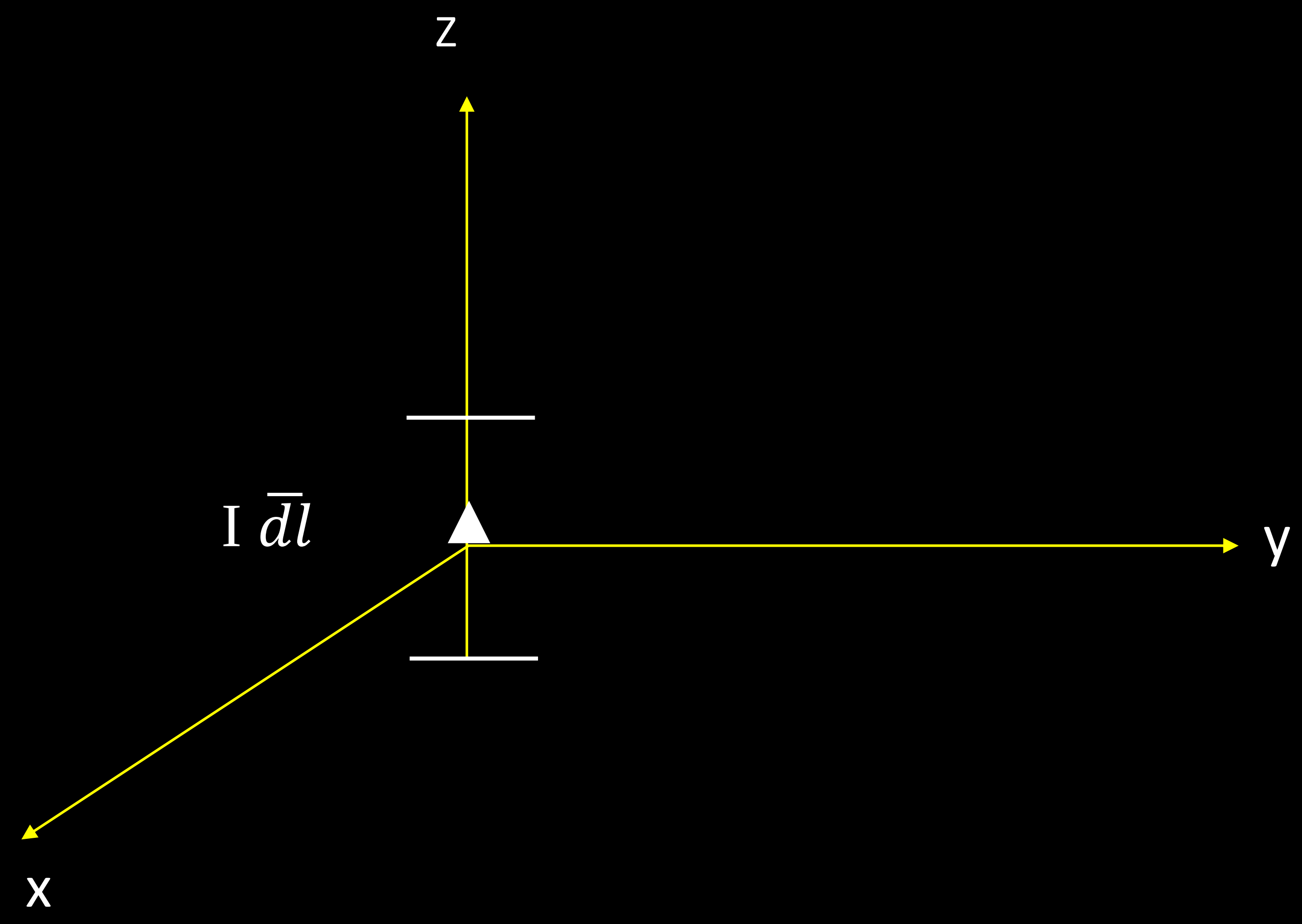
$\Rightarrow$ Lossy.

ANTENNA THEORY

Analysis Of Small Current Element Hertz Dipole ($dl \ll \lambda$):



$I\ (t) = I_0\ e^{jwt}$



Current moment

$I\ \bar{dl} = I_0\ e^{jwt}\ \bar{dl}$

Types Of Fields:

1. Radiation fields $\propto \frac{1}{r}$

2. Induction field $\propto \frac{1}{r^2}$

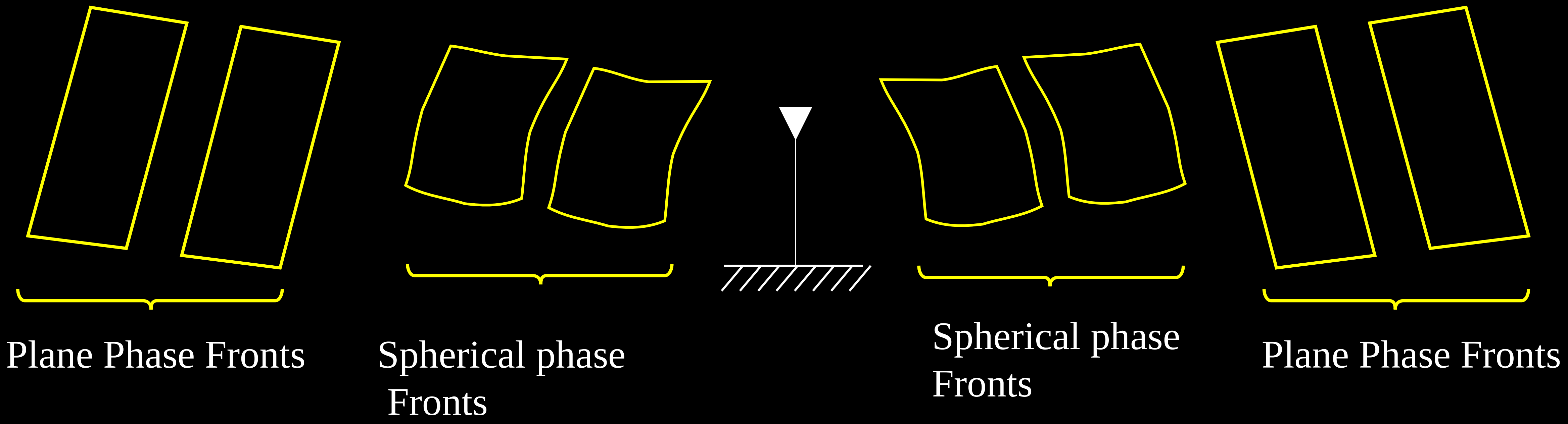
3. Electric field $\propto \frac{1}{r^3}$

Ex: r = 0.1 m

$$\frac{1}{r} = 10, \ \frac{1}{r^2} = 100, \ \frac{1}{r^3} = 1000$$

Ex: r = 10 m

$$\frac{1}{r} = 0.1, \ \frac{1}{r^2} = 0.01, \ \frac{1}{r^3} = 0.001$$


Plane Phase Fronts
Spherical phase Fronts
Spherical phase Fronts
Plane Phase Fronts

* Hertz dipole generates (radiates) linearly polarized transvers spherical electromagnetic wave.

Note:

* $$E_\theta = \frac{I_0\, dl\, sin\theta}{4\pi\epsilon}\left[\frac{\beta^2}{wr}\right]$$

* $$H_\phi = \frac{E_\theta}{\eta_0}$$

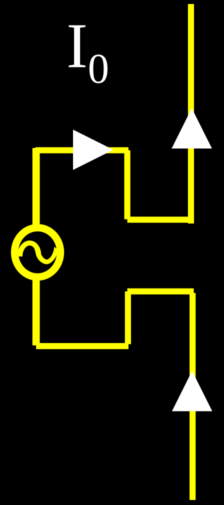
Power Radiated By Hertz Dipole:

$$\overrightarrow{P_{avg}} = \frac{1}{2} \mathrm{Re}\left\{\bar{E} \times \bar{H}^{*}\right\}$$

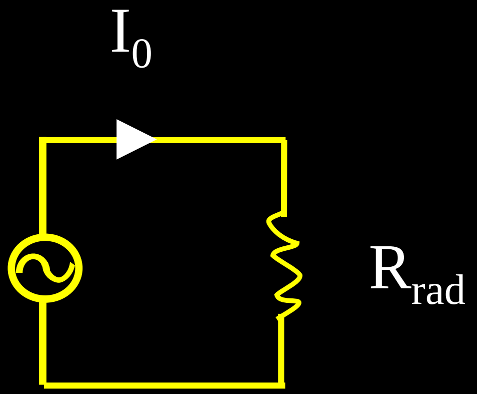
$$W_{avg} = \oiint \overrightarrow{P_{avg}} \cdot \overline{dA}$$

$$\boxed{W = 40\pi^2 I_0^2 \left[\frac{dl}{\lambda}\right]^2} \text{ Watts.}$$

Hertz Dipole From Circuit Point Of View:



$$\boxed{R_{rad} = 80\pi^2\left[\frac{dl}{\lambda}\right]^2}$$



$$W = 40\,\pi^2 I_0^2\left[\frac{dl}{\lambda}\right]^2$$

$$W = \left[\frac{I_0}{\sqrt{2}}\right]^2 R_{\,rad}$$

$R_{\,rad}$: Radiation resistance

Radiation pattern (3D):

- Directional dependency of fields radiated on fields radiated.

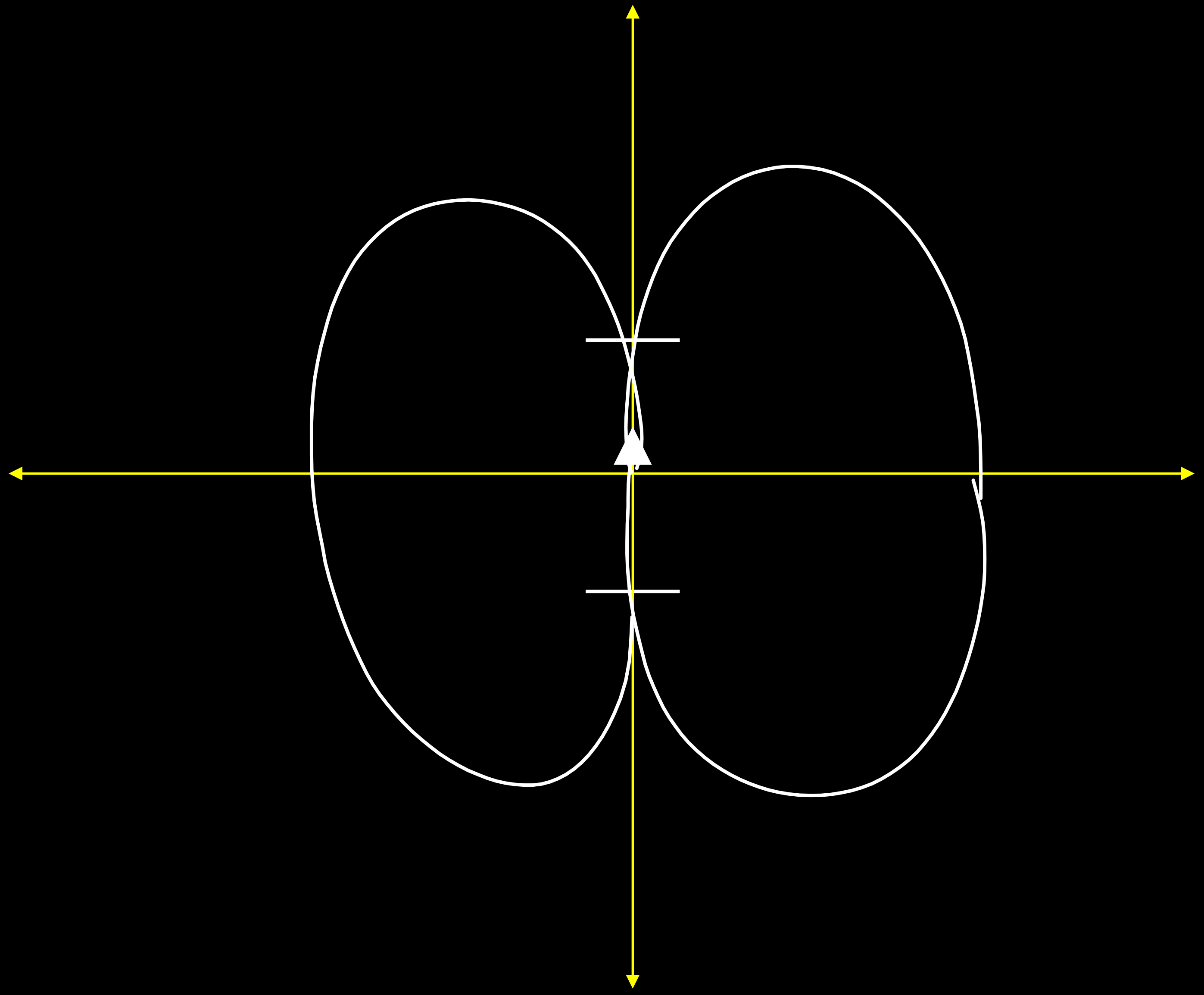
- Normalized radiation pattern ($F(\theta,\phi)$)

$$0 \leq |F(\theta,\phi)| \leq 1$$

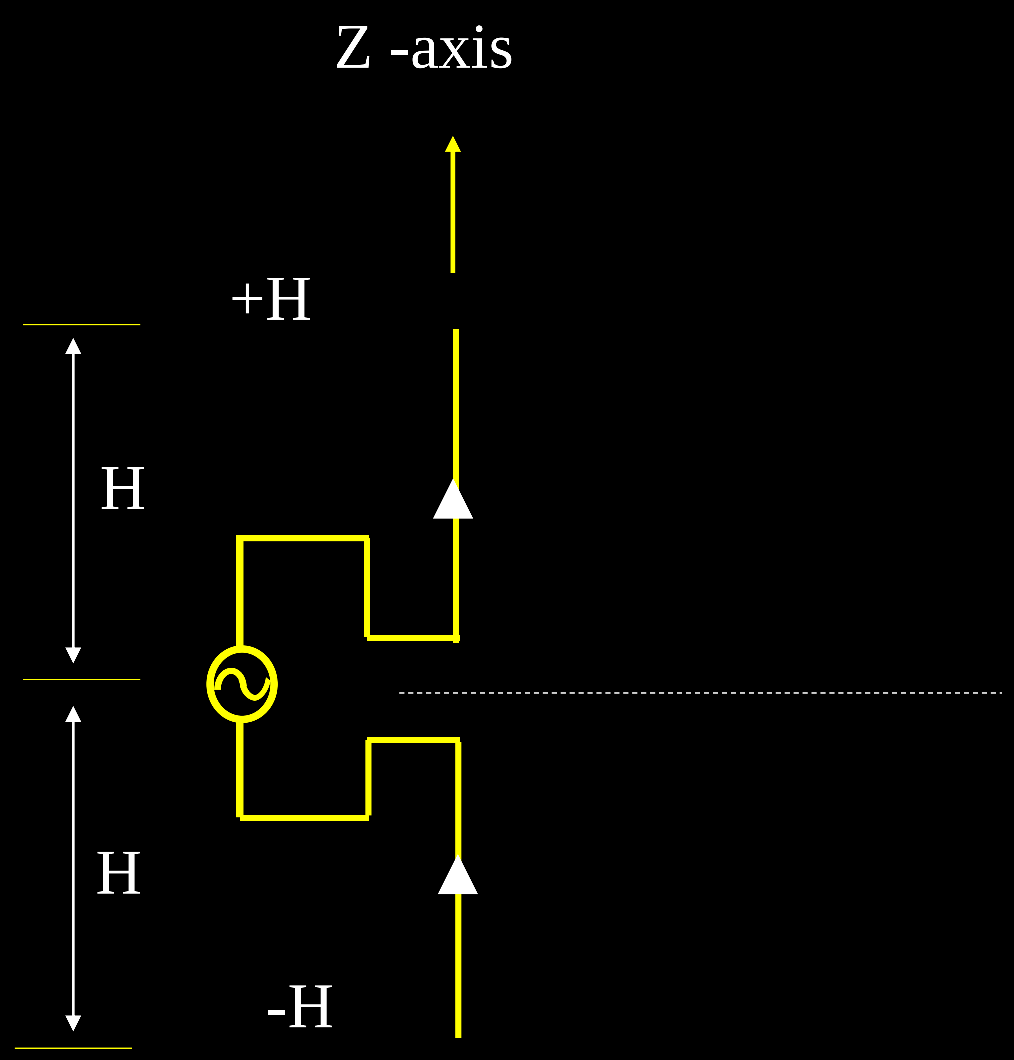
$E_\theta \sim k_1 \sin\theta$

$H_\phi \sim k_2 \sin\theta$

$\boxed{F(\theta) = \sin\theta}$



Dipole Antenna (2H):



Current distribution on dipole antenna (2H):

$$I(Z) = I_m \text{ Sin } [\beta(H-|Z|)]$$

If $Z > 0$

$$I(Z) = I_m \sin [\beta(H - Z)]$$

If $Z < 0$

$$I(Z) = I_m \sin [\beta(H+Z)]$$

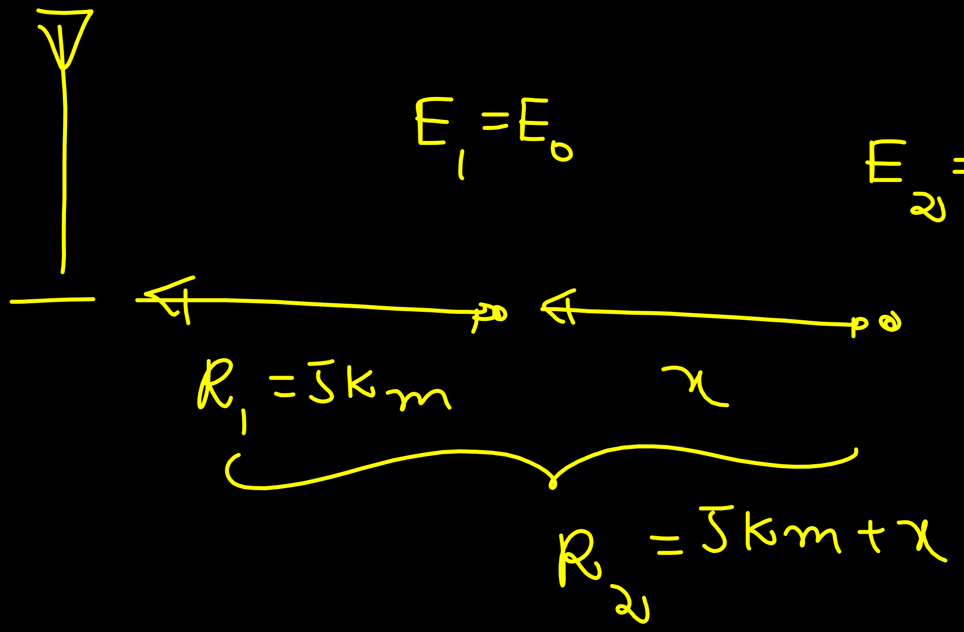
Q. A person with a receiver is 5 km away from the transmitter. What is the distance that this person must move further to detect a 3-dB decrease in signal strength? **(GATE - 02)**

(a) 942 m

(b) 2070 m

(c) 4978 m

(d) 5320 m



$$E \propto \frac{1}{R}$$

$$\frac{E_2}{E_1} = \frac{R_1}{R_2}$$

$$\frac{\left(\frac{E_0}{\sqrt{2}}\right)}{E_0} = \left(\frac{5 \times 10^3}{5 \times 10^3 + x}\right)$$

$$\underline{x = 2070\,m}$$

Q. Radiation resistance of a small dipole current element of length l at a frequency of 3GHz is $3\ \Omega$. If the length is changed by 1%, then the percentage change in the radiation resistance, rounded off to two decimal places, is ______ %

$$R_{rad} = 80\pi^2\left(\frac{dl}{\lambda}\right)^2$$

$$R_{rad} \propto l^2$$

$$dR_{rad} \propto 2l\,dl$$

$$\frac{dR_{rad}}{R_{rad}} = \frac{2l\,dl}{l^2} = 2\left(\frac{dl}{l}\right)$$

$$\frac{dR_{rad}}{R_{rad}} = 2\times 1\%$$

$$= 2\%$$

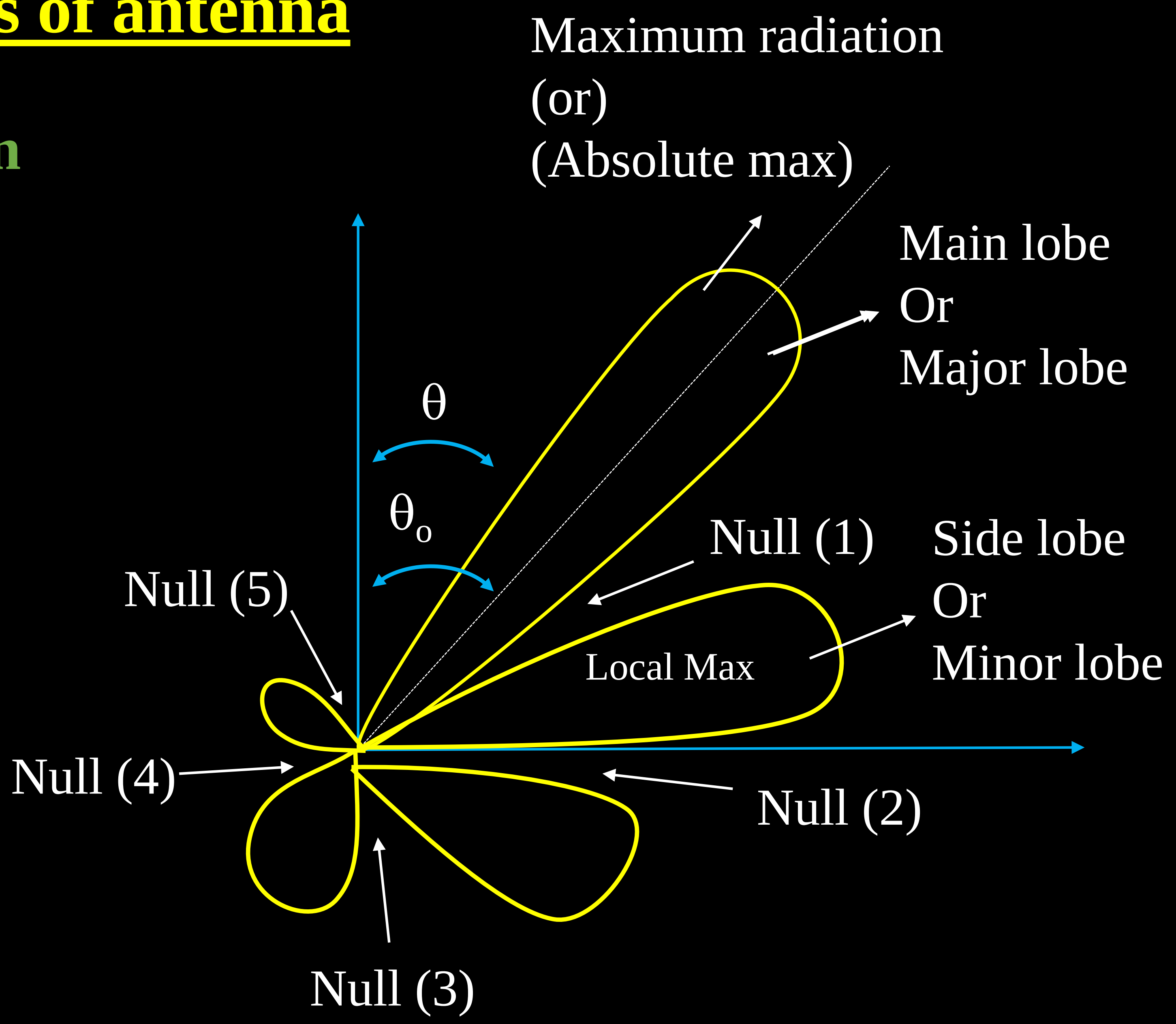
Antenna Parameters:

- Transmitting Antenna
- Receiving Antenna

Radiation characteristics of antenna

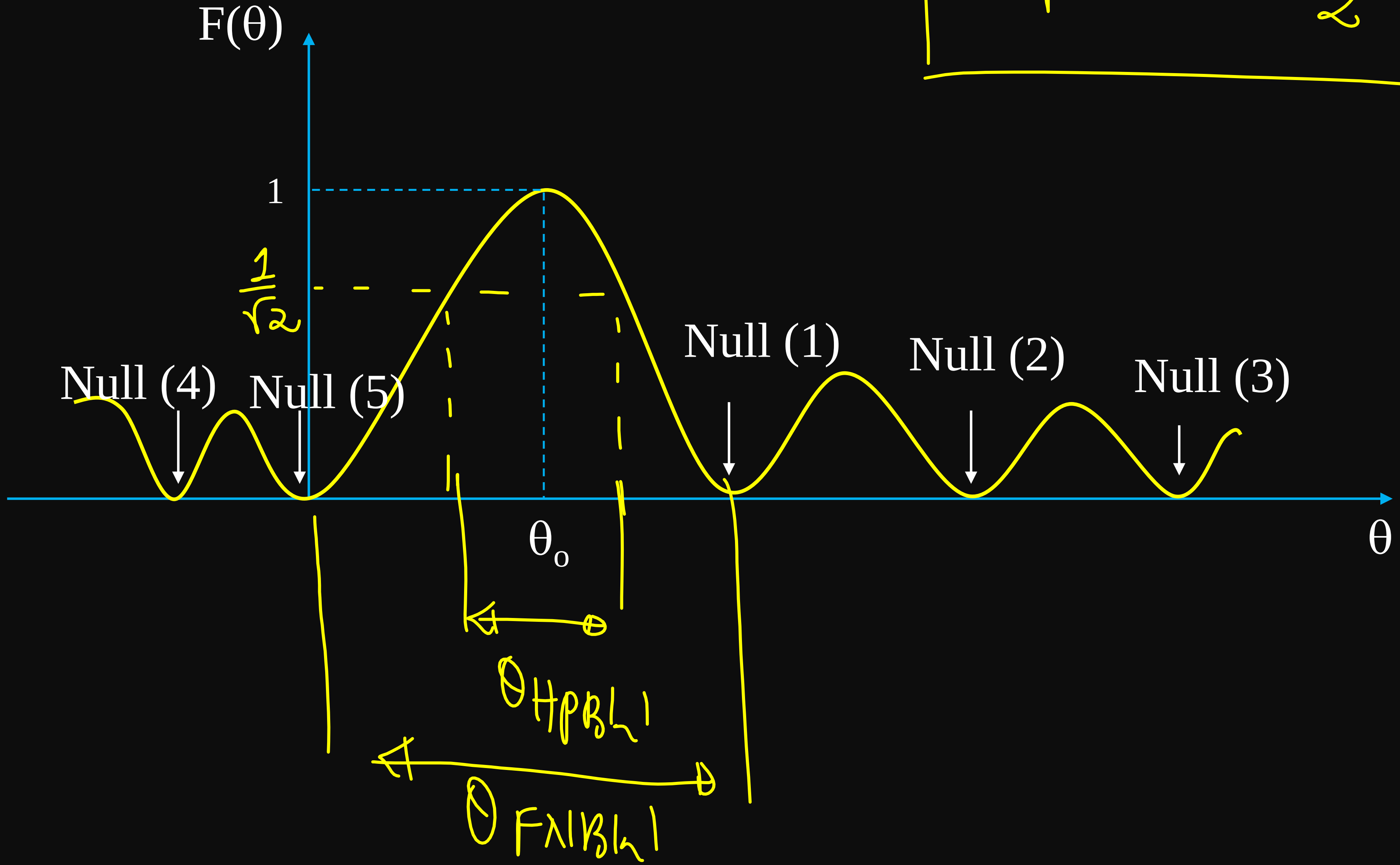
General Radiation pattern

Polar Plot



Cartesian Plot:

$$\boxed{\theta_{HPBW} \simeq \frac{\theta_{FNBW}}{2}}$$



1. Direction of maximum radiation (θ_o, ϕ_o)

For E – Plane : θ_o

For H – Plane : ϕ_o

2. Half power beam width (HPBW)

For E – Plane : θ_{HPBW}

For H – Plane : ϕ_{HPBW}

Half power region

(or)

3dB - Region

It is the effective angular region over which effective radiation is possible.

3. Beam width between first nulls (Or) First null beam width

For E – Plane : θ_{BWFN}

For H – Plane : ϕ_{BWFN}

- This is angular region without any side lobes (Leakage)

Directive gain / Directive function (G_D)

$$G_D = \left[\frac{\text{Radiation intensity in given direction}}{\text{Average radiation intensity}} \right]$$

$$G_D = \frac{U(\theta,\phi)}{U_{avg}(\theta,\phi)} = \frac{U(\theta,\phi)}{\left(\frac{W_r}{4\pi}\right)} = \frac{4\pi U(\theta,\phi)}{W_r}$$

The focusing ability of antenna compared to isotropic antenna.

Power Gain / Gain function (G_P)

$$G_D = \left[\frac{\textit{Radiation intensity in given direction}}{\textit{Average radiation intensity of input}}\right]$$

$$G_D = \frac{U(\theta, \phi)}{\left(\frac{W_i}{4\pi}\right)} = \frac{U(\theta, \phi)}{W_i} = \left[\frac{\text{Output Intensity}}{\text{Input average Intensity}}\right]$$

Efficiency (η)

$$\eta = \frac{W_r}{W_i} = \frac{I^2 R_{rad}}{I^2 R_{rad} + I^2 R_L}$$

$$\boxed{\eta = \frac{R_{rad}}{R_{rad} + R_L}}$$

R_{rad}: Radiation resistance

R_L: Resistance due to all losses on antenna

Directivity (D)

$$D = \left[\frac{Radiation\ intensity\ in\ maximum\ direction}{Average\ radiation\ intensity}\right]$$

D = maximum (G_D)

$$D = \frac{U_{max}(\theta,\emptyset)}{U_{avg}(\theta,\emptyset)} = \frac{U_{max}(\theta,\emptyset)}{\left(\frac{W_r}{4\pi}\right)} = \frac{4\pi U_{max}(\theta,\emptyset)}{W_r}$$

For directivity (D) formula :

Consider

$$D = \frac{4\pi U_{max}(\theta, \emptyset)}{W_r}$$

We know

$$W_r = \iint u(\theta, \emptyset) d\Omega$$

$$d\Omega = sin\theta d\theta d\emptyset$$

$$W_r = \iint u(\theta, \emptyset) sin\theta d\theta d\emptyset$$

We have

$$u(\theta, \emptyset) = s(\theta, \emptyset) r^2$$

$$u(\theta, \emptyset) = \frac{|E(\theta, \emptyset)|^2}{2\eta_0} r^2$$

$$U_{max}(\theta, \phi) = \frac{|E_{max}(\theta, \phi)|^2}{2\eta_0} r^2$$

$$D \approx \frac{4\pi}{(\theta_{HPBW}\phi_{HPBW})_{rad}} = \frac{41253}{(\theta_{HPBW}\phi_{HPBW})_{deg}}$$

$$\boxed{D = \frac{4\pi}{\int_{\theta}\int_{\phi} [F(\theta,\phi)]^2 \sin\theta \, d\theta \, d\phi}}$$

Q. The half power beam width (HPBW) of an antenna in the two orthogonal planes are 100° and 60° respectively the directivity of the antenna is approximately equal to

G-2000

(a) 2dB (b) 5dB (c) 8dB (d) 12dB

Soln

$$D \simeq \frac{41253}{100 \times 60} = 6.875$$

$$D(dB) = 10 \log(6.875) = 8.3 dB$$

Characteristics of receiving antennas

Reciprocity Theorem :

- The properties of transmitting and receiving antennas are related through the reciprocity theorem.

- The summary of theorem is what ever properties the antenna has while it was transmitting, the same properties it would have in receiving mode also.

Example :

1. If transmitting antenna has high power in one direction, when same antenna is used as receiving antenna it will receive more power in that direction.

2. The antenna will respond to that polarization which is capable of transmitting. [The antenna has state of polarization to which antenna respond maximally to input]

Polarization Efficiency Factor (P.E.F)

The power received by the antenna is determined by the polarization of incident EM-wave with respect to polarization of the antenna which is given by polarization efficiency factor (P.E.F)

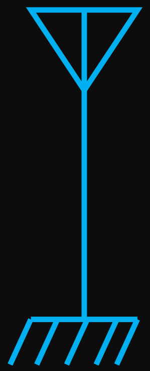
$$\boxed{\text{P.E.F} = |\,\hat{a}_w \cdot \hat{a}_a^{*}\,|^2}$$

$\hat{a}_w$: Unit vector associated with polarization of the incident wave.

$\hat{a}_a$: Unit vector associated with polarization of the antenna.

EX : (1)

$$\bar{E}_w = 5\hat{\theta}e^{j(wt+\beta r)}$$

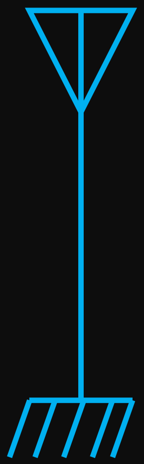


$$\bar{E}_a = 4\hat{\theta}e^{j(wt-\beta r)}$$

$$PLF = \left|\hat{\theta}\cdot(\hat{\theta})^*\right|^2$$

$$= \left|\hat{\theta}\cdot\hat{\theta}\right|^2 = 1$$

EX : (2)



$$\bar{E}_w = 2\hat{y}e^{j(wt+\beta z)}$$

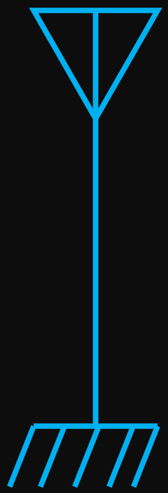
$$P.L.F = \left| \hat{y} \cdot \left[\frac{2\hat{x} + 3j\hat{y}}{\sqrt{4+9}} \right]^* \right|^2$$

$$= \left| \frac{3}{\sqrt{13}} \right|^2 = \frac{9}{13}$$

$$\bar{E}_a = (2\hat{x} + 3j\hat{y})e^{j(wt-\beta z)}$$

EX : (3)

$$\bar{E}_w = 2\hat{y}e^{j(wt+\beta z)}$$

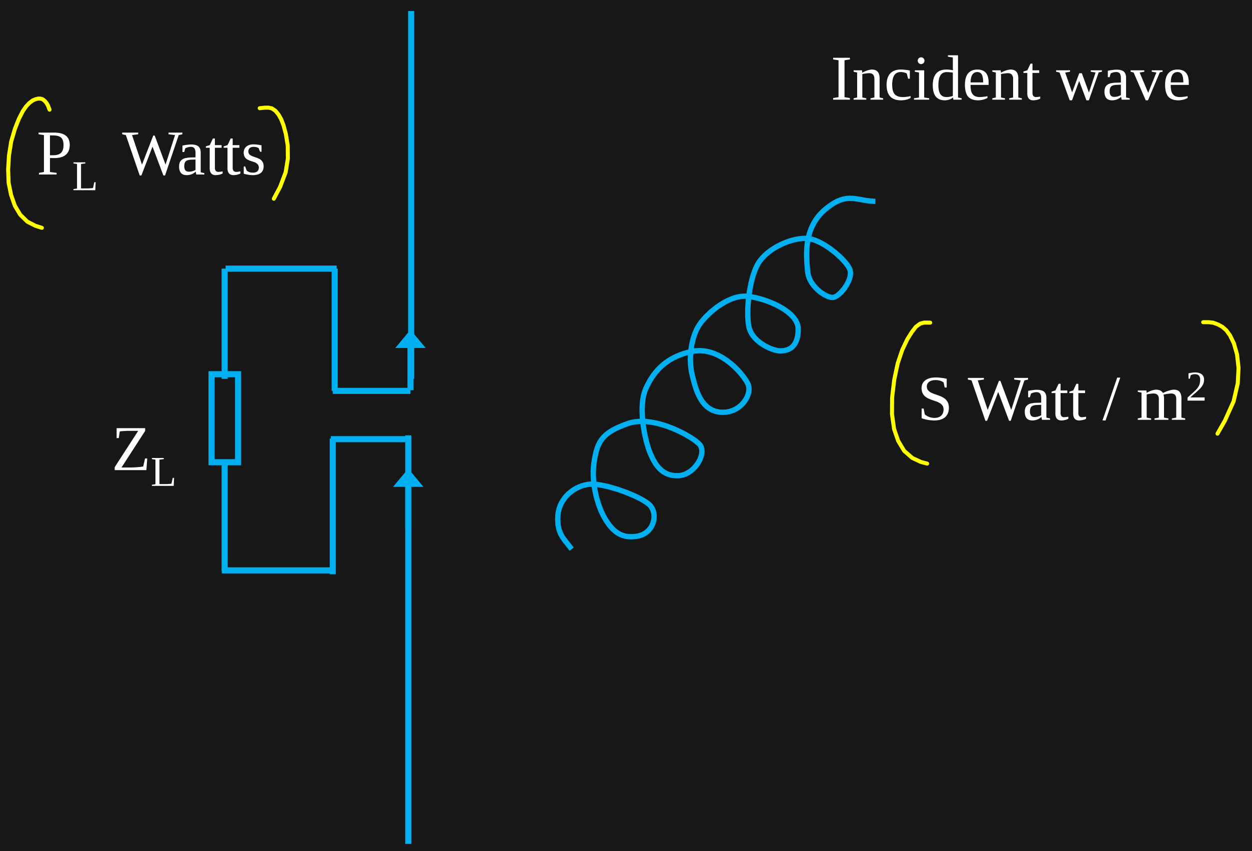


$$P.E.F = |\hat{y}\cdot(\hat{x})^*|^2$$

$$= |\hat{y}\cdot\hat{x}|^2 = 0$$

$$\bar{E}_a = 4\hat{x}e^{j(wt-\beta z)}$$

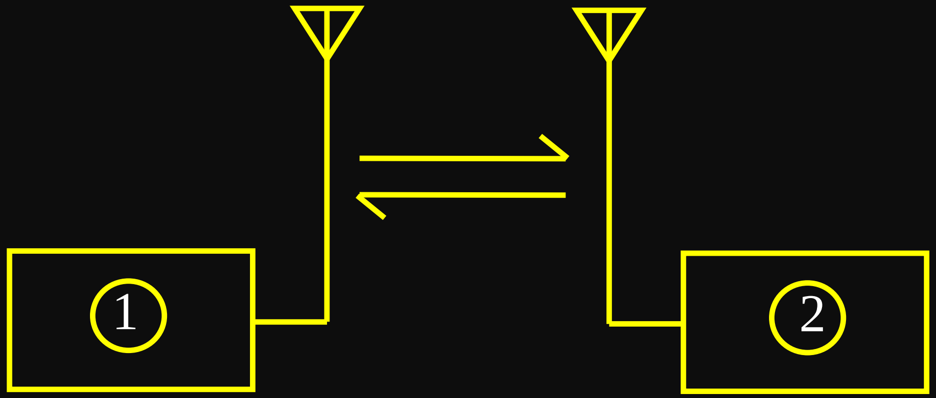
Effective Aperture :



$\cancel{A_{em} = P_L / S \;\; m^2}$

$$\boxed{A_e = \frac{P_L}{S}}\; m^2$$

Relation between A_{em} m^2 and D



$D_{1_1}, A_{e_1} m^2$ $\qquad$ $D_{2_1}, A_{e_2} m^2$

$$\frac{D_1}{A_{e_1}} = \frac{D_2}{A_{e_1}} = constant$$

$$\frac{D}{A_e} = \frac{4\pi}{\lambda^2}$$

$$\boxed{D = \frac{4\pi}{\lambda^2} A_e}$$

D: Unique property of Txing antenna

A_{em} : Unique property of Rxing antenna

Q. An antenna in free space receives 2 μW of power when the incident electric field is 20 mV/m rms. The effective aperture of the antenna is

(a) 0.005 m^2

(b) 0.05 m^2

(c) 1.885 m^2

(d) 3.77 m^2

$$A_e = \frac{P_L}{S}, \quad S = \frac{|E_0|^2}{2\eta_0} = \frac{(E_0/\sqrt{2})^2}{\eta_0} = \frac{(E_{rms})^2}{\eta_0}$$

$$S = \frac{(20\times10^{-3})^2}{120\pi}$$

$$A_e = \frac{2\times10^{-6}}{\left[\frac{(20\times10^{-3})^2}{120\pi}\right]} = 1.885\ m^2$$

Q. The radiation intensity of a certain antenna is

$U(\theta, \phi) = 2\sin\theta\sin^3\phi; \qquad 0 \le \theta \le \pi,\ 0 \le \phi \le \pi$

$\qquad\qquad = 0; \qquad \text{elsewhere}$

The directivity (in dB) of the antenna is ________

$U(\theta,\phi) \propto |E|^2$

$|E|^2 \propto 2\sin\theta\sin^3\phi$

$|E| \propto \left(\sqrt{2\sin\theta\sin^3\phi}\right)$

$\propto \sqrt{2}\sqrt{\sin\theta\sin^3\phi}$

$F(\theta,\phi) = \sqrt{\sin\theta\sin^3\phi}$

$D = \dfrac{4\pi}{\iint [F(\theta,\phi)]^2 \sin\theta\, d\theta\, d\phi}$

$= \dfrac{4\pi}{\iint \sin\theta\sin^3\phi \sin\theta\, d\theta\, d\phi}$

$$D = \frac{4\pi}{\underbrace{\int \sin^2\theta \, d\theta}_{I_1} \underbrace{\int \sin^3\phi \, d\phi}_{I_2}}$$

$$I_1 = \int \sin^2\theta \, d\theta = \int \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$I_1 = \frac{\pi}{2}$$

$$I_2 = \int \sin^3\phi \, d\phi = \int (1-\cos^2\phi)\sin\phi \, d\phi$$

$$= \int (\sin\phi - \cos^2\phi \sin\phi) \, d\phi$$

$$= \left[-\cos\phi + \frac{\cos^3\phi}{3}\right]_0^{\pi}$$

$$= \left\{\left[-(-1) - \frac{1}{3}\right] - \left[-1 + \frac{1}{3}\right]\right\}$$

$$= 1 + 1 - 1/3 - 1/3 = 2 - 2/3$$

$$I_2 = 4/3$$

$$D = \frac{4\pi}{\frac{\pi}{2} \times \frac{4}{3}} = 6$$

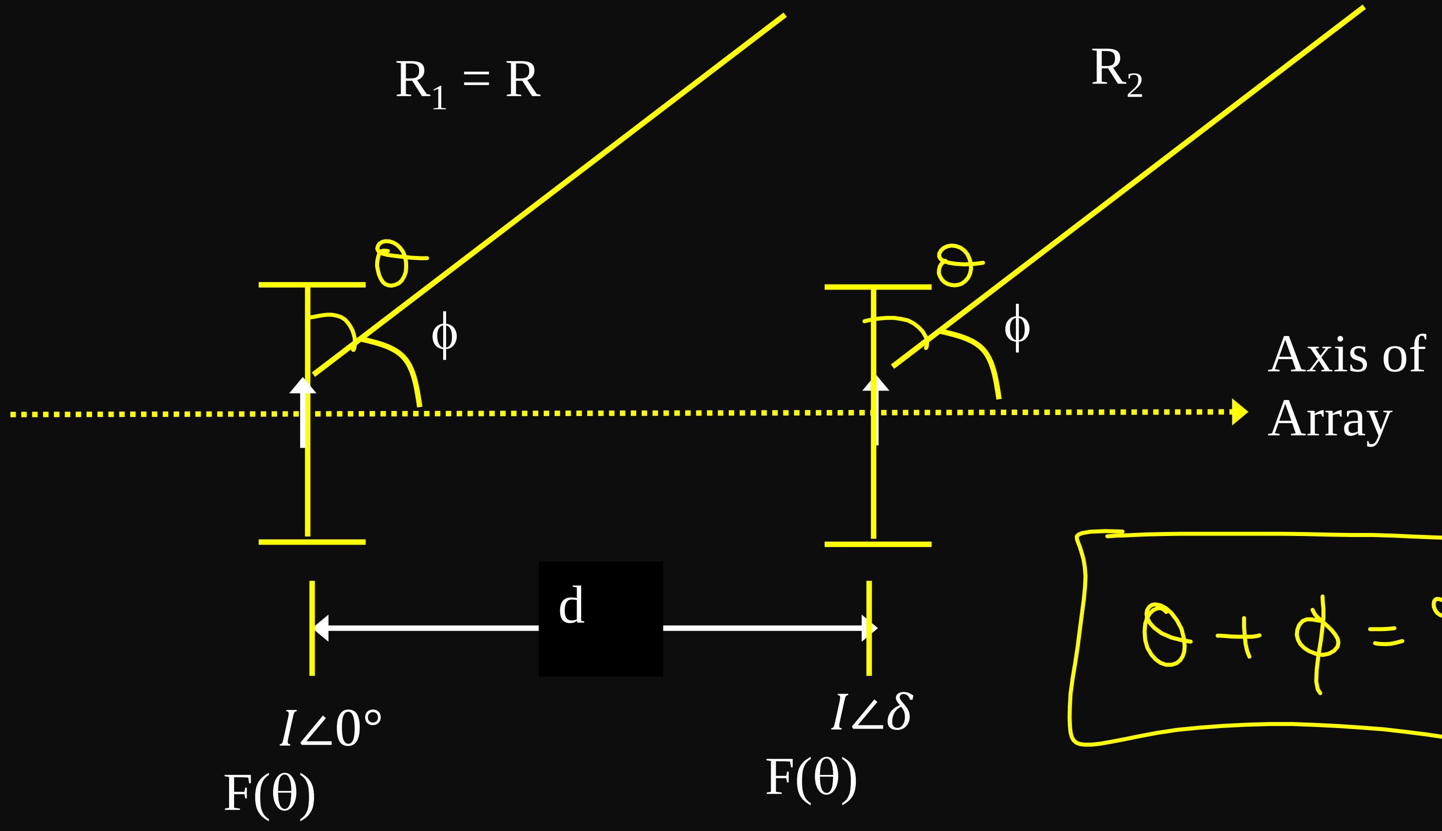
$$D(dB) = 10 \log 6 = 7.7 \, dB.$$

ANTENNA ARRAYS

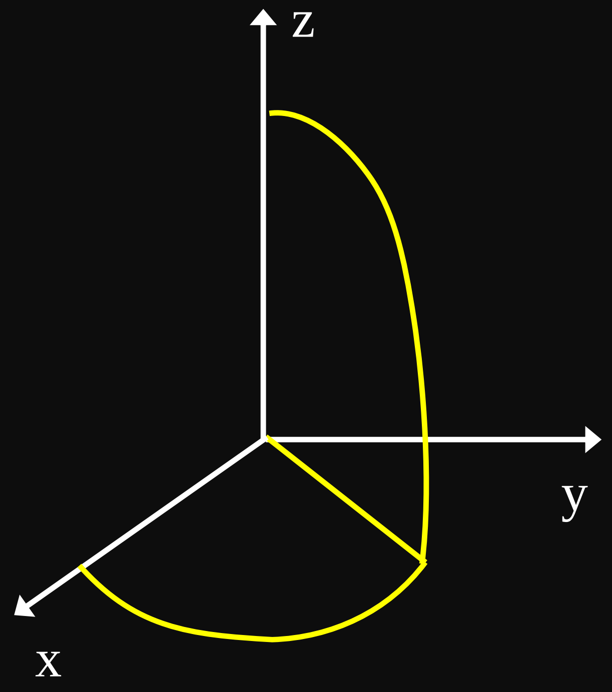
- Antenna arrays provides flexibility in obtaining required radiation pattern with effecting the terminal characteristics (or) input impedance.

Uniform Linear Array of the Element:

Model: (I)



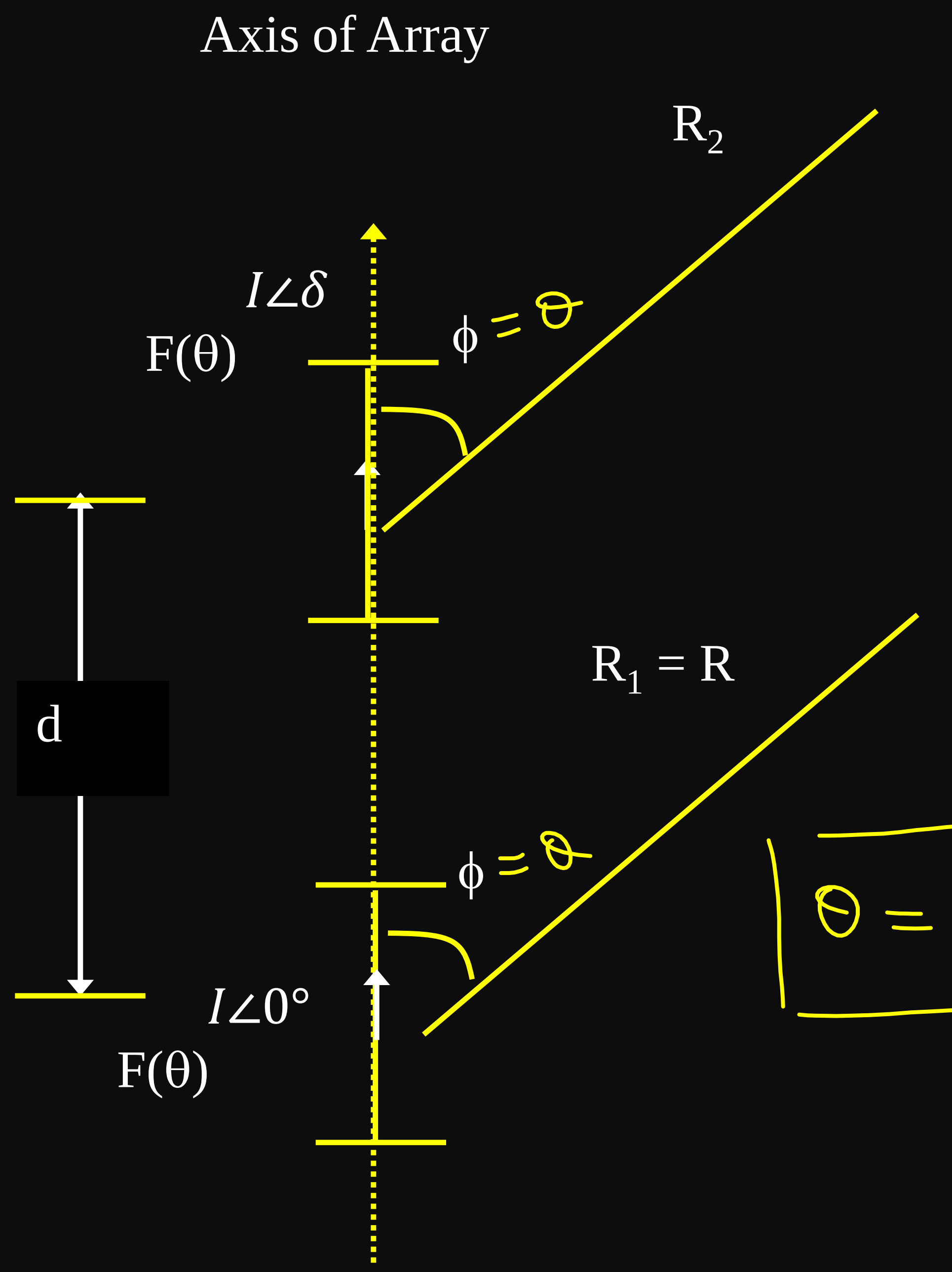
$$\boxed{\theta + \phi = 90°}$$



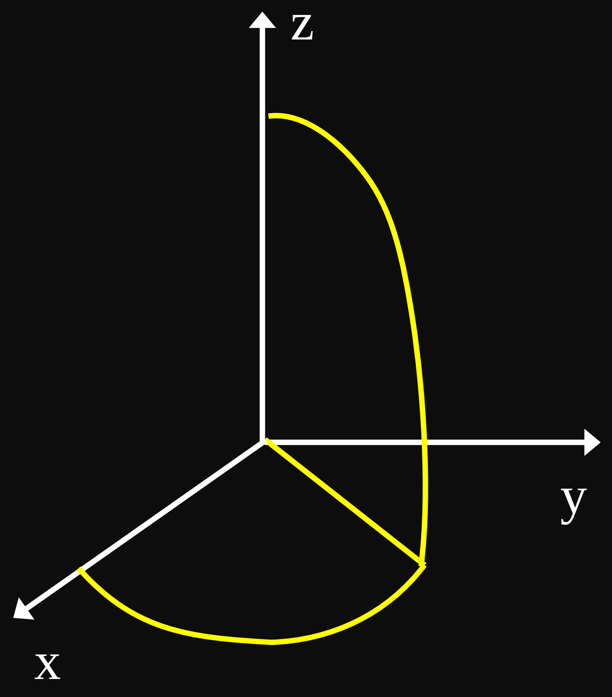
- Displaced in $\bar{H}$ - plane
- Parallelly placed

$$\boxed{0 \le \phi \le \pi}$$

Module: (II)



$\boxed{\theta = \phi}$



- Displaced in E - plane
- Axially placed

Note:

1. Normalized Array Factor

$$(A.F)_n = \cos(\psi/2) = f_{Gp}$$

2. Direction of Maximum Radiation (ϕ_{max})

$$\frac{\psi}{2} = \pm m\pi, \;\; m = 0, 1, 2, \ldots\ldots$$

3. Direction of Null (ϕ_n)

$$\frac{\psi}{2} = \pm(2m+1)\frac{\pi}{2}, \;\; m = 0, 1, 2, \ldots\ldots$$

4. $f_T = f_{Up} \times f_{Gp}$

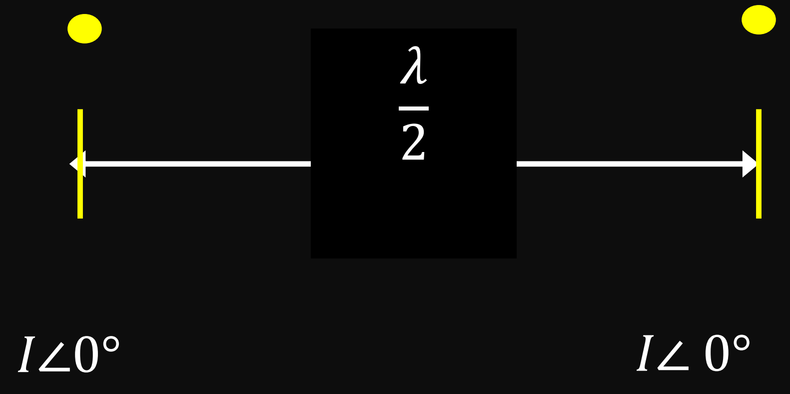
Where

$$\psi = \delta + \beta d \cos\phi$$

Q. Consider an array of two isotropic radiators with spacing between elements as $\frac{\lambda}{2}$

(a) Determine the direction of maximum radiation

(b) If the maximum radiation to exist along array axis what is the excitation phase required



$f_T = f_{up} * f_{ap} = (1) * (AF)_n$

$f_T = \cos(\psi/2)$, $\psi = \delta + \beta d \cos\phi$

$\psi = 0° + \frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\phi$

$\psi = \pi\cos\phi$

FOR MAX

$\frac{\psi}{2} = \pm m\pi$

$$\psi = \pm 2m\pi$$

$$\pi \cos\phi_{max} = \pm 2m\pi$$

$$\cos\phi_{max} = \pm 2m$$

$$\underline{\underline{m = 0}}$$

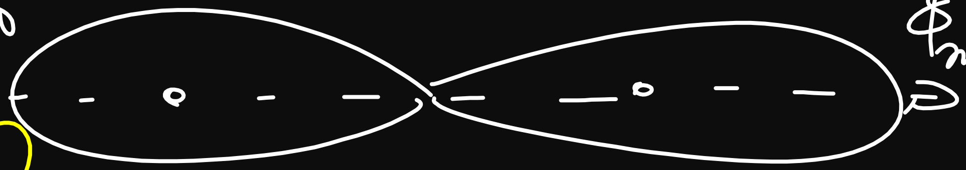
$$\cos\phi_{max} = 0$$

$$\phi_{max} = 90^\circ$$

---

(2)

$\phi_{max} = 180^\circ$ $\underline{\underline{(\delta = +\pi)}}$



$\phi_{max} = 0^\circ$ $\underline{\underline{(\delta = -\pi)}}$

$I\angle 0^\circ$ $\qquad$ $I\angle\delta$

$$\psi = \delta + \beta d\cos\phi = \delta + \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}\cos\phi = \delta + \pi\cos\phi$$

FOR $\frac{\psi}{2} = \pm m\pi$

$$\psi = \pm 2m\pi$$

$$\delta + \pi\cos\phi_{max} = \pm 2m\pi$$

$$\delta = \pm 2m\pi - \pi\cos\phi_{max}$$

$\underline{\underline{m = 0}}$

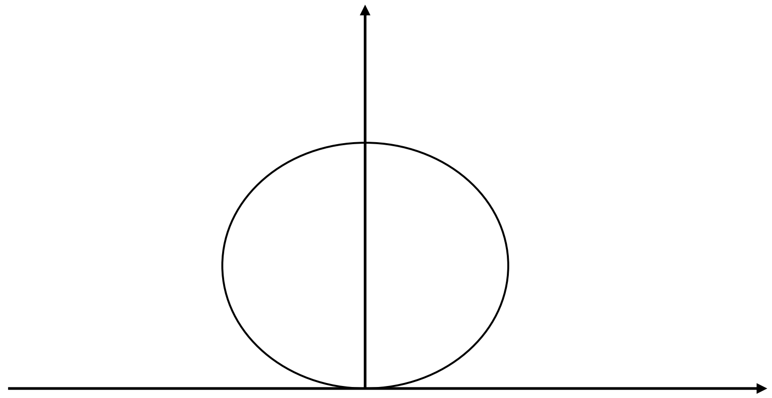
$$\delta = -\pi\cos\phi_{max}$$

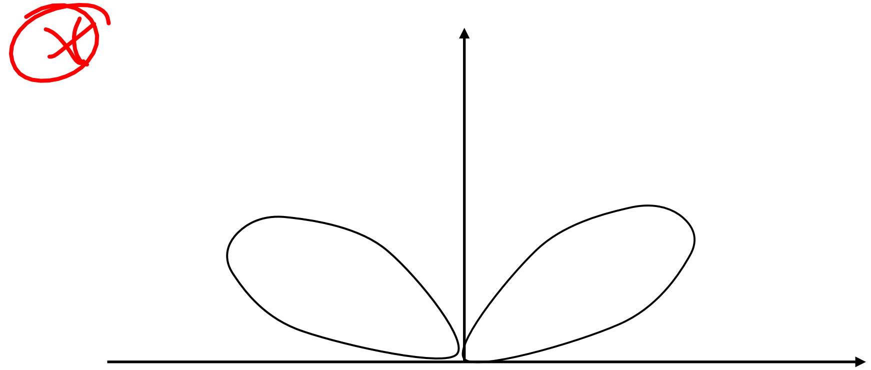
IF $\phi_{max} = 0^\circ \Rightarrow \delta = -\pi$

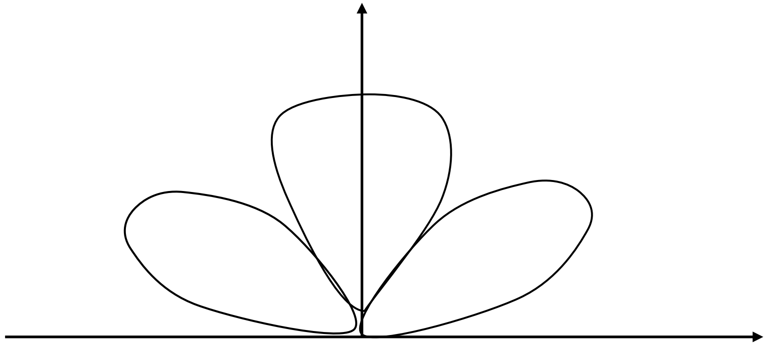
IF $\phi_{max} = \pi \Rightarrow \delta = +\pi$

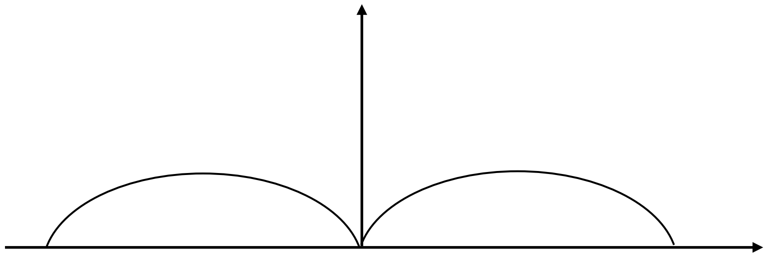
Q. A $\lambda/2$ dipole is kept horizontally at a height of $\lambda_0/2$ above a perfectly conducting infinite ground plane. The radiation pattern in the plane of the dipole ($\vec{E}$ plane) looks approximately as

(GATE – 07)

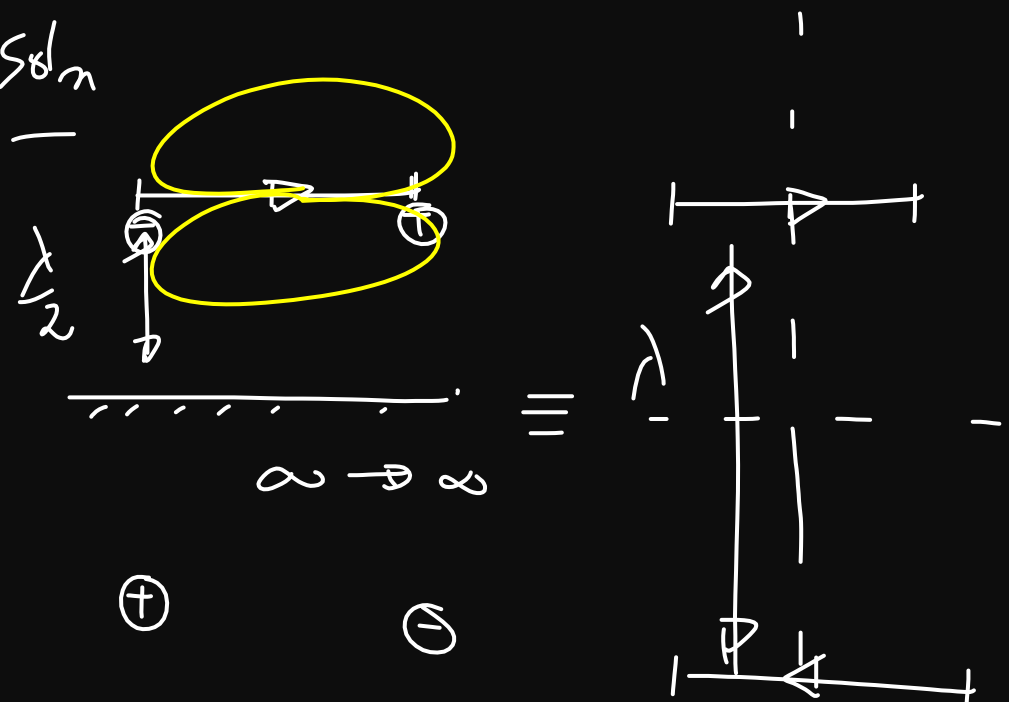








Soln



$$f_T = f_{up} * f_{Gp}$$

$$f_{Gp} = (AF)_n = \cos(\psi/2)$$

$$\psi = \delta + \beta d \cos\phi$$

$$\psi = \pi + \frac{2\pi}{\lambda}\lambda \cos\phi$$

$$\psi = \pi + 2\pi \cos\phi$$

FOR NULL (ϕ_n)

$$\frac{\psi}{2} = \pm(2m+1)\frac{\pi}{2}$$

$$\psi = \pm(2m+1)\pi$$

$$\pi + 2\pi\cos\phi_n = \pm(2m+1)\pi$$

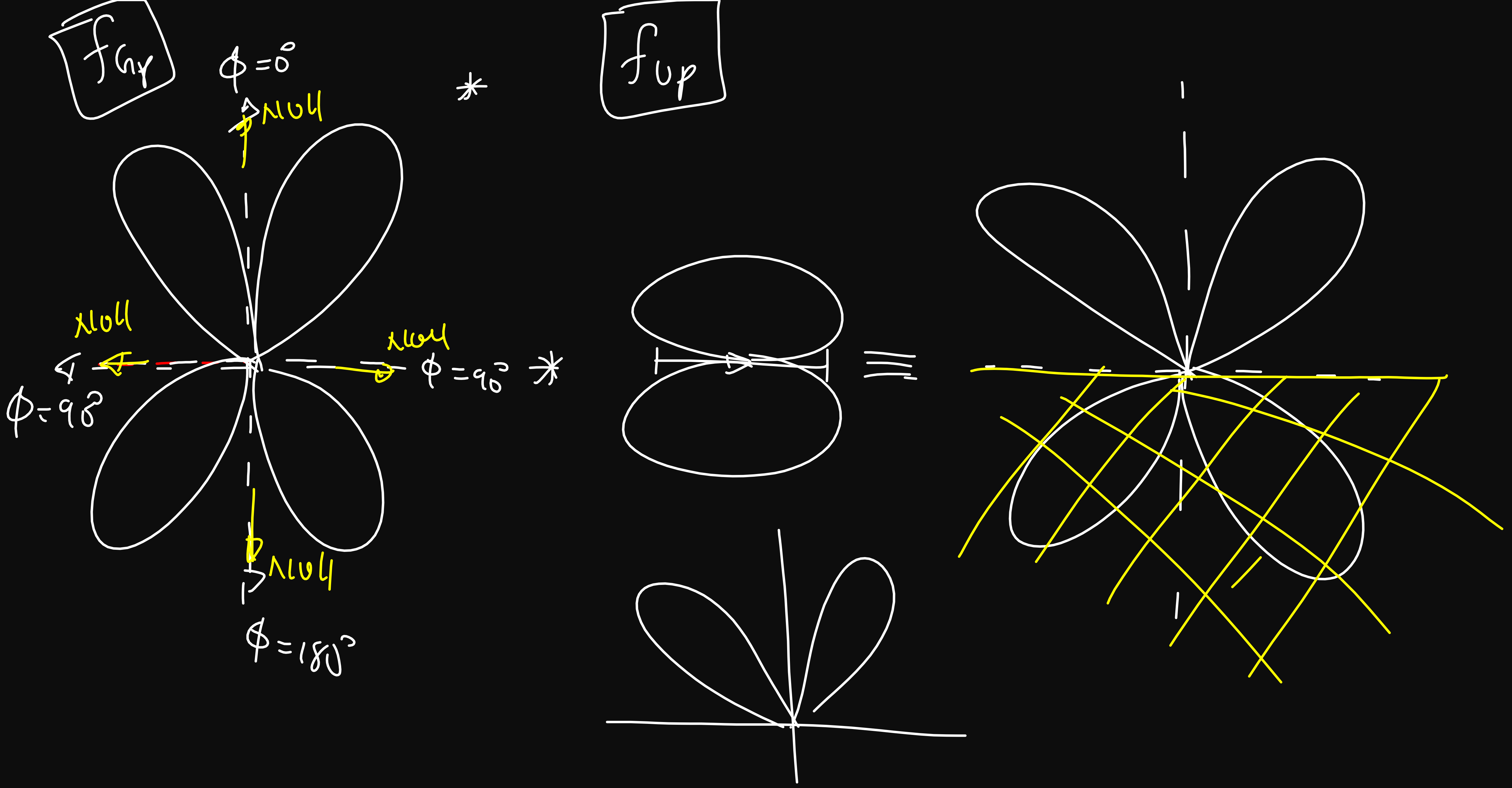
$$\cos\phi_n = \frac{\pm(2m+1) - 1}{2}$$

$m = 0$

$$\cos\phi_n = \frac{\pm 1 - 1}{2} \rightarrow \phi_n = 90^\circ,\quad \phi_n = 180^\circ$$

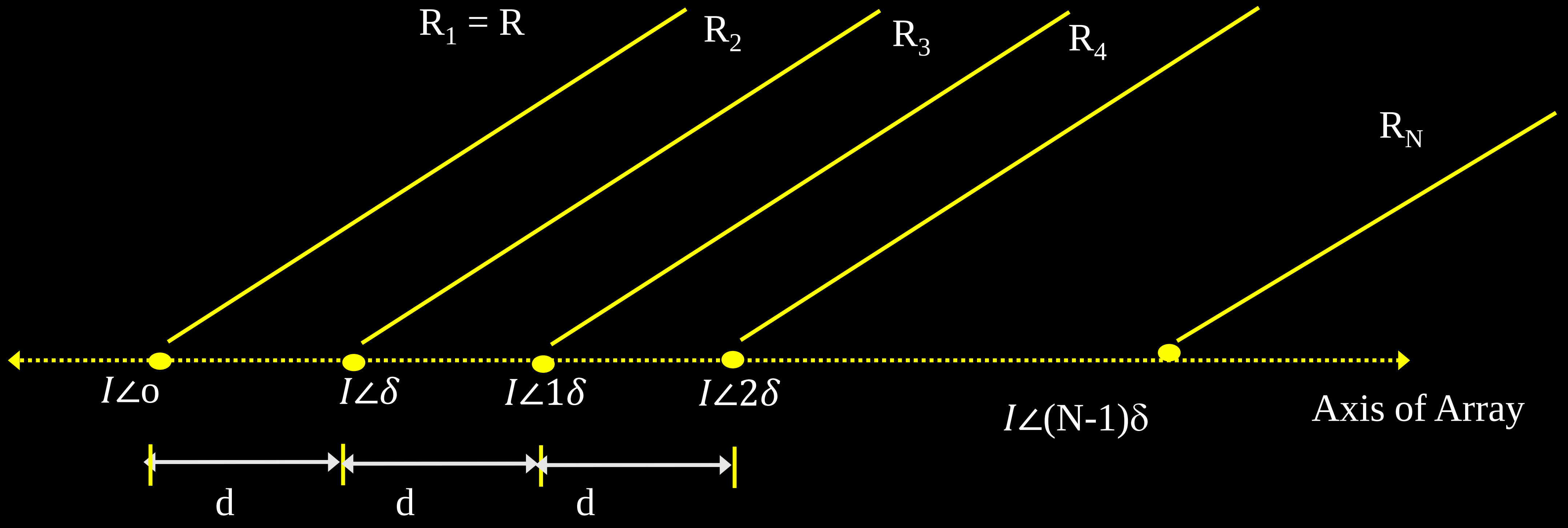
$m = 1$

$$\cos\phi_n = \frac{\pm 3 - 1}{2} \rightarrow \phi_n = 0^\circ$$



f_{xy}
$\phi = 0°$
$\phi = 90°$
$\phi = 90°$
$\phi = 180°$
f_{up}

Uniform Linear Array (N-Element)



Condition For Direction of Maximum Radiation (ϕ_{max})

** $\boxed{\cos\phi_{max} = -\frac{\delta}{\beta d}}$

$$\boxed{\cos\phi_{max} = -\frac{\delta}{\beta d}}$$

Case (1)

If $\delta = \mp\beta d$

$$\cos\phi_{max} = \pm 1$$

$$\phi_{max} = 0^\circ, 180^\circ$$

$\boxed{\text{END SIDE}}$

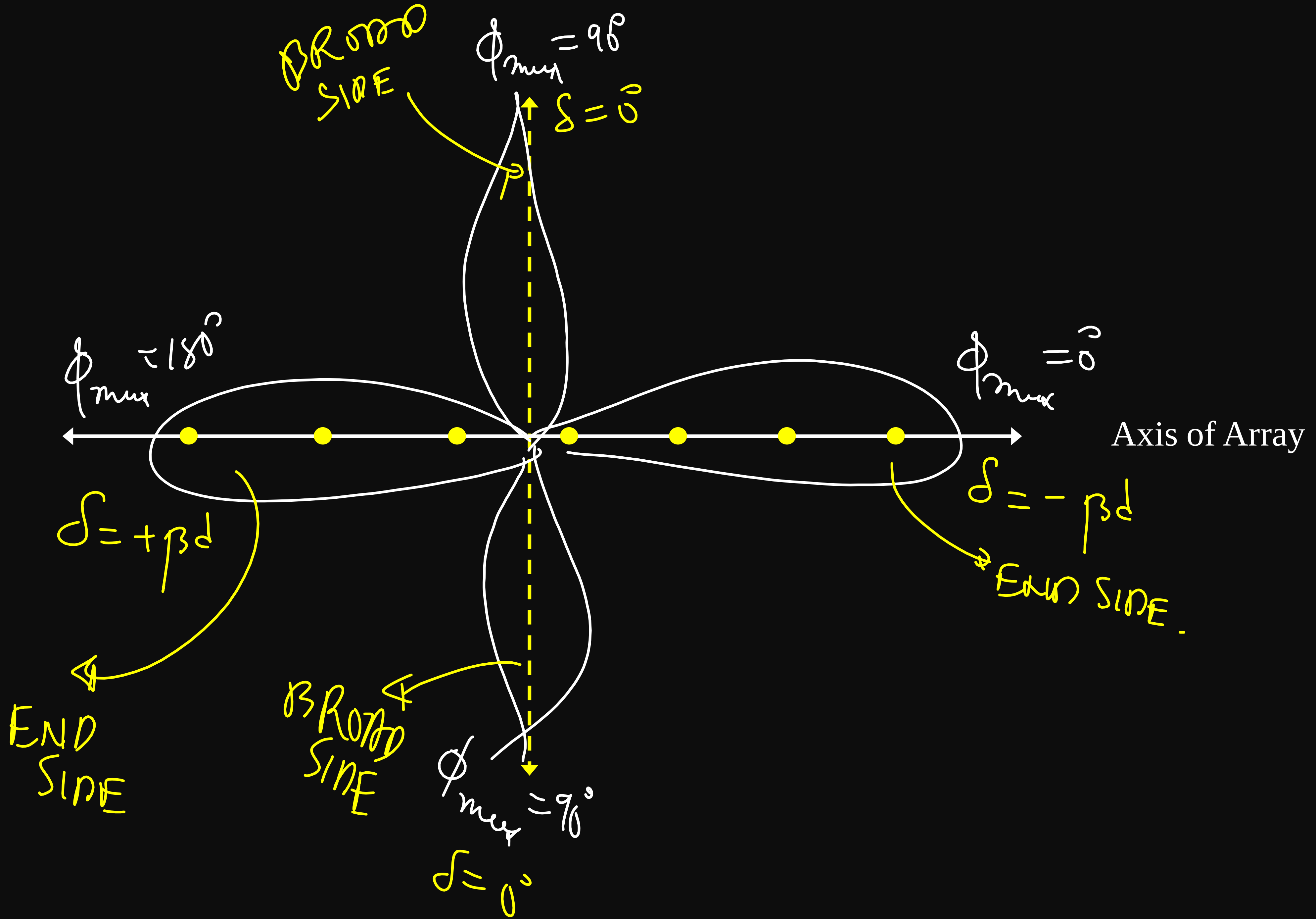
Case (2)

If $\delta = 0^\circ$

$$\cos\phi_{max} = 0$$

$$\phi_{max} = 90^\circ$$

$\boxed{\text{BROAD SIDE}}$


BROAD SIDE
$\phi_{max} = 90°$
$\delta = 0°$
$\phi_{max} = 180°$
$\phi_{max} = 0°$
Axis of Array
$\delta = +\beta d$
$\delta = -\beta d$
END SIDE
END SIDE
BROAD SIDE
$\phi_{max} = 90°$
$\delta = 0°$

End Fire Array	Broad Side Array
1. $\phi_{max} = 0°, 180°$	1. $\phi_{max} = 90°$
2. $\delta = \pm \beta d$	2. $\delta = 0°$
3. $\phi_{HPBW_E} \cong \sqrt{\frac{2\lambda}{Nd}}$	3. $\phi_{HPBW_B} \cong \frac{\lambda}{Nd}$
4. $D_E \cong \frac{8Nd}{\lambda}$	4. $D_B \cong \frac{2Nd}{\lambda}$
• $\phi_{HPBW} \cong \frac{\phi_{FNBW}}{2}$	• $Length\ of\ array = (N-1)d$

Q. The BWFN of uniform linear array of N equally spaced (element spacing = d) equally excited antennas is determined by

(GATE : 92)

(a) N alone and not By d

(b) D alone and not by N

(c) The ratio $\left(\frac{N}{d}\right)$

(d) The product (Nd)

Q. In a uniform linear array, four isotropic radiating elements are spaced $\frac{\lambda}{4}$ apart. The progressive phase shift between the elements required for forming the main beam at 30° from the broad side is

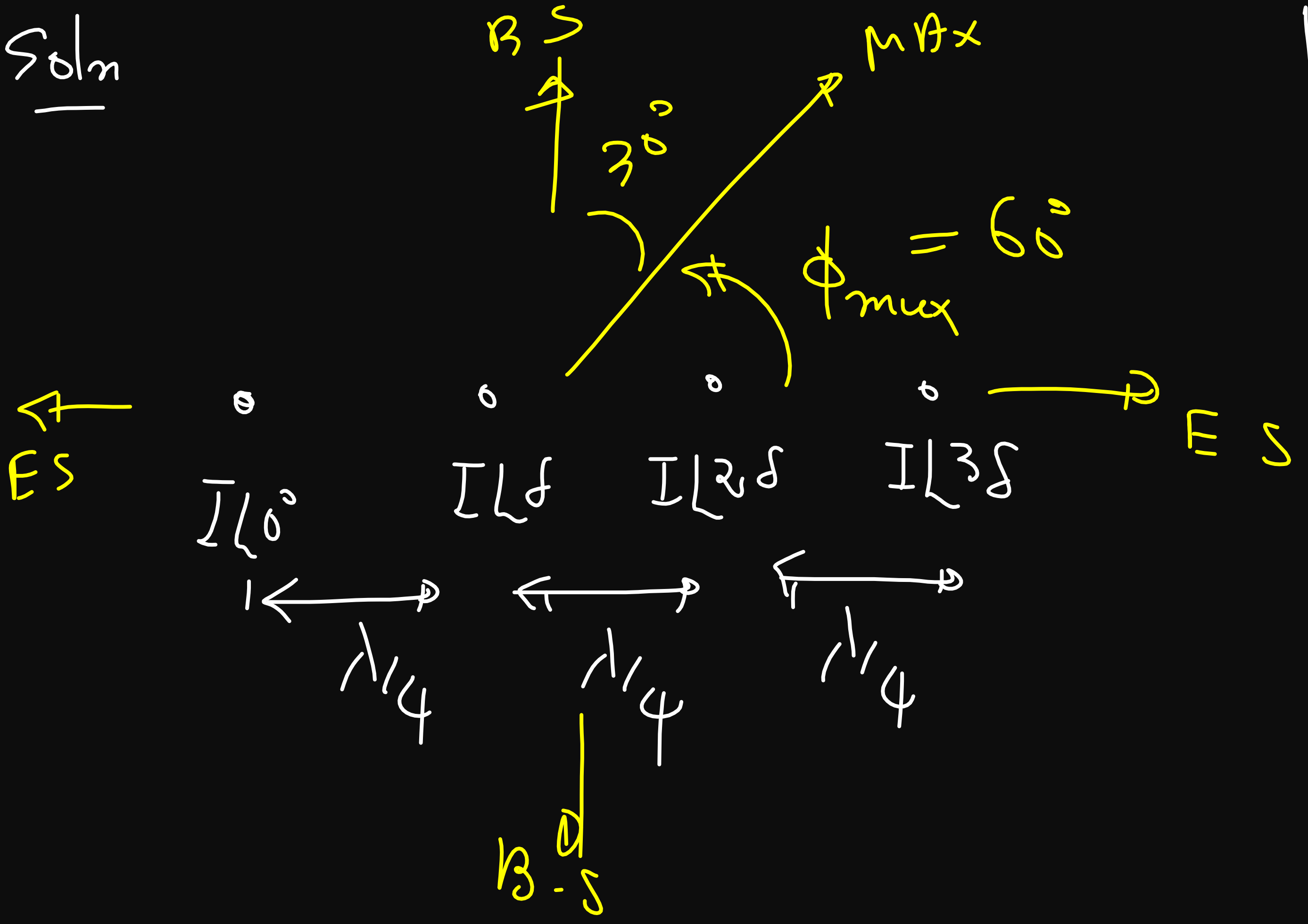
(a) $-\pi^c$

(b) $-\frac{\pi^c}{2}$

(c) $-\frac{\pi^c}{4}$

(d) $-\frac{\pi^c}{8}$

Soln



$$\cos\phi_{max} = -\frac{\delta}{\beta d}$$

$$\delta = -\beta d \cos\phi_{max}$$

$$= -\frac{2\pi}{\lambda}\,\frac{\lambda}{4}\cos 60°$$

$$= -\frac{\pi}{2}\,\frac{1}{2} = \frac{-\pi}{4}$$

$$\delta = -\frac{\pi}{4}$$