




gwsnr: A python package for efficient signal-to-noise calculation of gravitational-waves


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Summary

Gravitational waves (GWs), ripples in spacetime predicted by Einstein's theory of General Relativity, have revolutionized astrophysics since their first detection in 2015. Emitted by cataclysmic events such as mergers of binary black holes (BBHs), binary neutron stars (BNSs), and black hole-neutron star pairs (BH-NSs), these waves provide a unique window into the cosmos.

A central quantity in GW analysis is the Signal-to-Noise Ratio (SNR), which measures the strength of a GW signal relative to the background noise in detectors such as LIGO (The LIGO Scientific Collaboration et al. (2015), B. P. Abbott et al. (2020), Buikema et al. (2020)), Virgo (F. Acernese et al. (2014), F. Acernese et al. (2019)), and KAGRA (Akutsu et al. (2020), Aso et al. (2013)). While real detections are established using a False-Alarm Rate (FAR) threshold, under stationary Gaussian noise assumptions the condition that the SNR exceeds a chosen threshold can serve as a practical proxy (Essick (2023), Essick and Fishbach (2024)), especially in simulations of detectable events and in studies aimed at extracting astrophysical information (Abbott, B.P. et al. (2016)).

Applications such as population simulations for rate estimation (B. P. Abbott et al. (2016)) and hierarchical Bayesian inference with selection effects (Thrane and Talbot (2019), Essick and Fishbach (2024)) require repeated and efficient computation of the Probability of Detection (P_{det}), which is generally derived from SNR. However, traditional approaches that rely on noise-weighted inner products for SNR evaluation are computationally demanding and often impractical for such large-scale analyses (Taylor and Gerosa (2018), Gerosa et al. (2020)).

Statement of Need

The *gwsnr* Python package addresses this challenge by providing efficient and flexible tools for computing the optimal SNR (ρ_{opt}). This quantity depends on the intrinsic and extrinsic source parameters, the detector antenna response ($F_{+, \times}$), and the noise power spectral density (PSD) (Allen et al. (2012)). The primary use case of ρ_{opt} in *gwsnr* is the estimation of P_{det} , which is evaluated against a detection statistics threshold.

The package provides a flexible and user-friendly interface for combining detector noise models, waveform families, detector configurations, and signal parameters. It accelerates ρ_{opt} evaluation using a **partial-scaling interpolation** method for non-precessing binaries and a multiprocessing **inner-product** routine for frequency-domain waveforms implemented in *lalsuite* (LIGO Scientific Collaboration, Virgo Collaboration, and KAGRA Collaboration (2018)), including those with spin precession and subdominant modes. For rapid P_{det} estimation, *gwsnr* also supports ANN-based models and a Hybrid SNR recalculation scheme. Finally, using an optimal-SNR threshold $\rho_{\text{opt,thr}}$, the package computes the horizon distance (D_{hor}), a standard measure of detector sensitivity, via both analytical (Allen et al. (2012)) and numerical methods.

High performance is achieved through *NumPy* vectorization (NumPy Community (2022)) and Just-in-Time (JIT) compilation with *Numba* (Lam, Pitrou, and Seibert (2022)), with optional

GPU acceleration available via *JAX* (James Bradbury and others (2018)) and *MLX* (Hannun et al. (n.d.)). These JIT compilers translate Python code into optimized machine code at runtime, while built-in parallelization strategies such as `numba.prange`, `jax.vmap`, and `mlx.vmap` maximize efficiency on both CPUs and GPUs.

This combination of efficiency and usability makes *gwsnr* a valuable tool for GW data analysis. It enables large-scale simulations of compact binary mergers, facilitates the estimation of detectable lensed and unlensed event rates (as demonstrated in the *ler* package; Phurailatpam et al. (2024), Ng et al. (2024), More and Phurailatpam (2025), Janquart et al. (2023), R. Abbott et al. (2021), Collaboration et al. (2023), Wierda et al. (2021), Wempe et al. (2022)), and supports the treatment of selection effects through P_{det} in hierarchical Bayesian frameworks (Thrane and Talbot (2019), Essick (2023)).

Mathematical Formulation and Methods Overview

Following are the key mathematical formulations and methods implemented in *gwsnr* for SNR calculation, P_{det} estimation, and D_{hor} computation.

Noise-Weighted Inner Product

The standard frequency-domain inner product (Allen et al. (2012)) between two signals $\tilde{a}(f)$ and $\tilde{b}(f)$ is

$$\langle a|b \rangle = 4\Re \int_{f_{\min}}^{f_{\max}} \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)} df,$$

where $S_n(f)$ is the detector PSD. The optimal SNR is $\rho = \sqrt{\langle h|h \rangle}$, and for polarizations h_+, h_\times :

$$\rho = \sqrt{F_+^2 \langle \tilde{h}_+|\tilde{h}_+ \rangle + F_\times^2 \langle \tilde{h}_\times|\tilde{h}_\times \rangle}.$$

Although waveform generation is costly, *gwsnr* accelerates it using multiprocessing, `numba.njit`, and optional `jax` backends (with `ripple`; Edwards et al. (2024)).

Partial Scaling Interpolation

For aligned-spin or non-spinning binaries, *gwsnr* adapts FINDCHIRP (Allen et al. (2012)) to precompute a partial-scaled SNR,

$$\rho_{1/2} = \frac{D_{\text{eff}}}{\mathcal{M}^{5/6}} \rho_{\text{opt}},$$

where \mathcal{M} is the chirp mass and D_{eff} the effective distance. $\rho_{1/2}$ is stored on a parameter grid (2D for non-spinning, 4D for aligned spins). New SNRs are recovered by spline interpolation and rescaling:

$$\rho = \rho_{1/2} \frac{\mathcal{M}^{5/6}}{D_{\text{eff}}}.$$

This replaces costly integrations with interpolation, enabling major speed-ups.

ANN-based P_{det} Estimation

gwsnr includes an ANN built with `tensorflow` (Abadi et al. (2015)) and `scikit-learn` (Pedregosa et al. (2011)), trained to approximate ρ_{opt} for BBH systems with the IMRPhenomXPHM waveform, which includes spin precession and subdominant modes. While the ANN is poor at estimating ρ_{opt} directly, its outputs are effective for P_{det} , since detectability depends on threshold crossing rather than precise values.

Trained on large *ler* datasets, the model uses partial-scaled SNRs to reduce input dimensionality (15 to 5) and accelerate detectability estimates under stationary Gaussian noise. Users can also retrain the ANN for different detectors or astrophysical settings. Related work includes (Chapman-Bird et al. (2023), Gerosa et al. (2020), Callister et al. (2024)).

Hybrid SNR Recalculation for P_{det} Estimation

The Partial Scaling method is efficient for aligned-spin systems but unreliable for precessing binaries, and the ANN-based approach is less accurate. To address this, *gwsnr* uses a hybrid strategy: it first estimates SNRs with Partial Scaling or ANN, identifies signals near the threshold ρ_{th} , and then recalculates them with the Noise-Weighted Inner Product.

This approach retains the speed of approximations while ensuring accuracy for systems close to the detection limit, producing more reliable P_{det} estimates.

Statistical Models for P_{det}

In *gwsnr*, estimation of P_{det} is based on a detection threshold for the observed (matched-filter) SNR, $\rho_{\text{obs,thr}}$. The observed SNR, ρ_{obs} , is modeled either as a Gaussian random variate centered at ρ_{opt} (or $\rho_{\text{opt,net}}$ for a detector network) with unit variance (Fishbach, Farr, and Holz (2020), B. P. Abbott et al. (2019)), or as a non-central χ distribution (`scipy.stats.ncx2`; Virtanen et al. (2020)) with non-centrality parameter $\lambda = \rho_{\text{opt}}$ (or $\rho_{\text{opt,net}}$) and two degrees of freedom for a single detector, extended to $2N$ for a network of N detectors (Essick (2023)).

gwsnr uses precomputed $\rho_{\text{obs,thr}}$ values derived from semianalytic sensitivity estimates of GW transient injection catalogues (following Essick (2023)). The package also supports custom threshold computation from user-provided catalogue data, including parameter-dependent thresholds that vary with intrinsic properties such as total observed mass ($m_{\text{tot,obs}}$).

Horizon Distance Calculation

D_{hor} is a standard measure of detector sensitivity, defined as the maximum distance at which an optimally oriented source can be detected with a given threshold $\rho_{\text{opt,thr}}$ (Allen et al. (2012)). *gwsnr* computes D_{hor} using two methods.

The **analytical method** rescales a known D_{eff} by

$$D_{\text{hor}} = \frac{\rho_{\text{opt}}}{\rho_{\text{th}}} D_{\text{eff}}.$$

The **numerical method** maximizes SNR over sky location, then solves for the luminosity distance (d_L) where

$$\rho(d_L) - \rho_{\text{opt,thr}} = 0.$$

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