Using technology

The use of technology is an integral part of DP mathematics courses. Developing an appreciation of how developments in technology and mathematics have influenced each other is one of the aims of the courses and using technology accurately, appropriately and efficiently both to explore new ideas and to solve problems is one of the assessment objectives. Learning how to use different forms of technology is an important skill in mathematics and time has been allowed in each topic of the syllabus and through the "toolkit" in order to do this.

Technology is a powerful tool in mathematics and in recent years increased student and teacher access to this technology has supported and advanced the teaching and learning of mathematics. Discerning use of technology can make more mathematics accessible and motivating to a greater number of students.

Teachers can use technology to support and enhance student understanding in many ways including:

- to bring out teaching points
- to address misconceptions
- to aid visualisation
- to enhance understanding of concepts that would otherwise be restricted by lengthy numerical calculations or algebraic manipulation
- to support students in making conjectures and checking generalizations
- to explicitly make the links between different mathematical representations or approaches.

Students can also use technology to engage with the learning process in many ways, including the following:

- to develop and enhance their own personal conceptual understanding
- to search for patterns
- to test conjectures or generalizations
- to justify interpretations
- to collaborate on project-based work
- to help organize and analyse data.

In the classroom teachers and students can use technology working individually or collaboratively to explore mathematical concepts. Key to successful learning of mathematics with technology is the fine balance between the teacher and student use of technology, with carefully chosen use of technology to support the understanding and the communication of the mathematics itself.

Many topics within the DP mathematics courses lend themselves to the use of technology. Graphical calculators, dynamic graphing software, spreadsheets, simulations, apps, dynamic geometry software and interactive whiteboard software are just a few of the many kinds of technology available to support the teaching and learning of mathematics.

Within the guide the term "technology" is used for any form of calculator, hardware or software that may be available in the classroom. The terms "analysis" and "analytic approach" are generally used in the guide to indicate an algebraic approach that may not require the use of technology. It is important to note there will be restrictions on which technology may be used in examinations, which will be detailed in relevant documents.

Finance packages on the graphical display calculator

Graphical calculators have functionality that allows finance calculations to be carried out with ease. The following unit plans illustrate two different scenarios which might arise in the course.

Financial applications of geometric sequences and series

Amortization and annuities

Monte Carlo simulations

Modern technology allows us to answer many questions to a high degree of accuracy, even if we cannot find the exact solution. One extremely powerful modern method of doing this is called Monte Carlo simulation, where we use random numbers to generate many possible sets of data and use this to investigate the question.

Although computers are an important part of the simulation process there are still three key skills which students need to develop. In many situations, these are far more important mathematical skills than the analytic methods traditionally emphasized in mathematics teaching.

- Describing the situation mathematically
- Converting this description into a computer simulation
- Interpreting the results of the simulation, including realising when a simulation fails

The following two classic examples of simulation should give students a valuable insight into modern mathematics and allow them to develop these skills. Sample datasets are available here as well, which can be adapted or used as a model for creating original datasets.

Shooting arrows at a target

Shooting arrows at a target–dataset

Overloading lifts

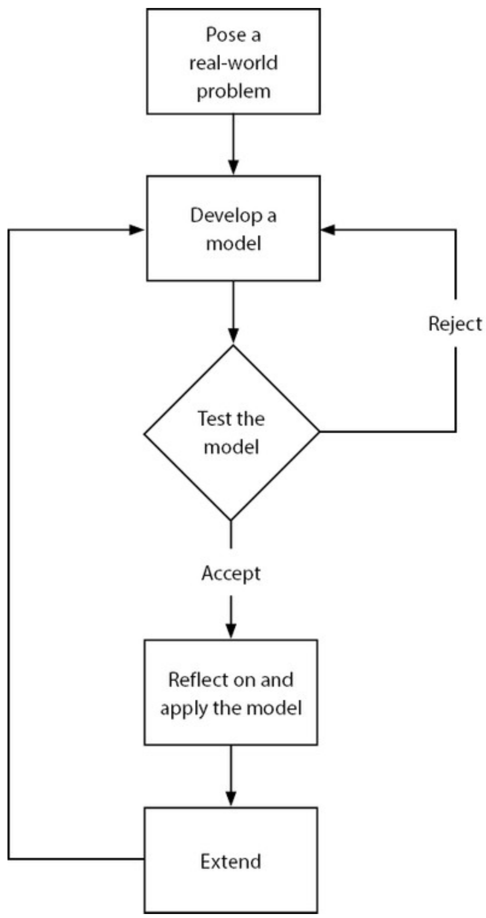
Overloading lifts–dataset

Mathematical modelling

Modelling is an important skill in mathematics and is becoming increasingly important as technology expands the bounds of what can and cannot be modelled. The guide gives an introduction and overview of the modelling process. Here we give one example of a modelling activity which has been designed to bring out the most important stages of modelling and act as an example of what the modelling cycle below looks like when this type of activity is carried out in the classroom.

The guide defines a range of different functions to be used for modelling and guidance is given as to the different contexts where these functions could arise. There are many resources with data available online, however it is often more engaging for students to gather their own data if the context allows this.

The cycle of mathematical modelling is illustrated below.



The following activity seeks to bring out the important elements of mathematical modelling.

Poachers on the game reserve–student activity

Poachers on the game reserve–teacher notes

Voronoi diagrams

Voronoi diagrams are being used increasingly widely and are a very authentic use of a mathematics application. They have many practical uses in ecology, epidemiology, urban planning, deliveries, service areas, control of robots, rovers and driverless cars, and in graphic design.

Voronoi diagrams partition space and can be used for questions such as these.

- How can we accurately map the territories of animals to prevent overcrowding?
- Where's the best place to open a new restaurant to steal competition from another restaurant?
- If a city has several hospitals with a helicopter, what's the service area of each helicopter?
- If I know how much it has rained in several locations, how can I estimate rainfall in other nearby locations?

Voronoi diagrams are a rich application of mathematics and are becoming of more and more of interest to mathematicians as technology has developed to support their creation. They are a fine example of how developments in technology and mathematics influence each other (aim 7). This resource aims to introduce teachers and students to the concept of the diagram, how they are constructed and two important techniques connected to the Voronoi diagram–the incremental algorithm and the nearest neighbour interpolation. There are also six learning activities which teachers may use directly or adapt for their own uses.

What's a Voronoi diagram?

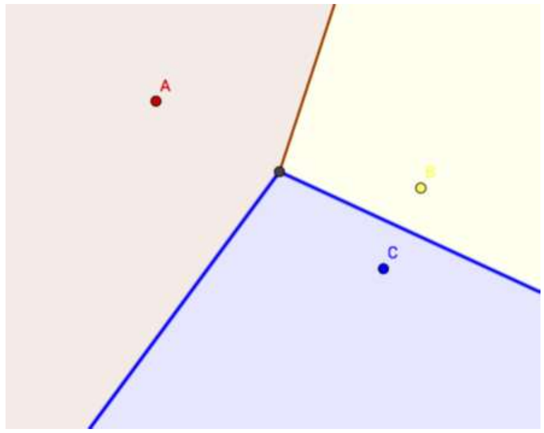
Suppose we are given a set of **sites** (points) in a bounded or unbounded plane (Fig. 1). A **Voronoi diagram** answers this question: which points are closest to point A? To point B? To point C? The Voronoi diagram for this set of sites divides the plane into Voronoi **cells or regions** (polygons). Each cell contains all the points in the plane that are closer to that site than any other (Fig 2). The line segments dividing cells are **edges or boundaries** and intersection points of edges are **vertices**.

Figure 1

Sites A, B, C

Figure 2

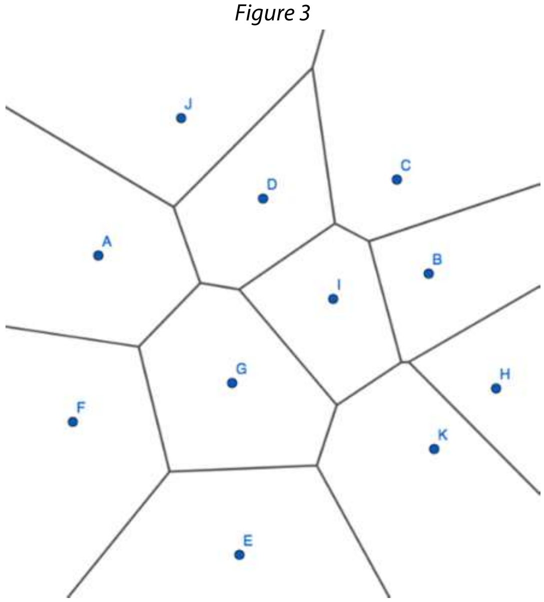
Cells for sites A, B, C



Now, given any point in the plane, we can determine which site it is closest to. Any point in the blue region, for example, is closer to site C than to sites A or B.

A more complex example is shown in Figure 3:

Figure 3



Constructing a Voronoi diagram

There are many algorithms for constructing Voronoi diagrams. The one demonstrated here is an **incremental algorithm** that builds the diagram recursively, adding one site at a time.

To understand it, we first look at the structure of a Voronoi diagram. Because each edge of a Voronoi diagram is equidistant to two sites, it must lie on the perpendicular bisector of the segment joining those sites (Fig 4). This implies that each vertex of the Voronoi diagram is a circumcentre of a triangle formed by a set of three sites (Fig 5).

Figure 4

Edges lie on perpendicular bisectors of AB, BC, AC

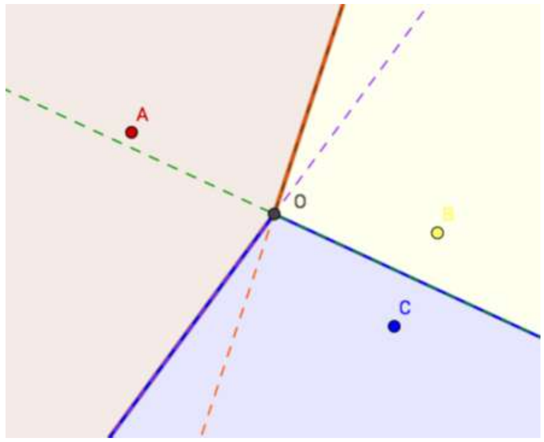
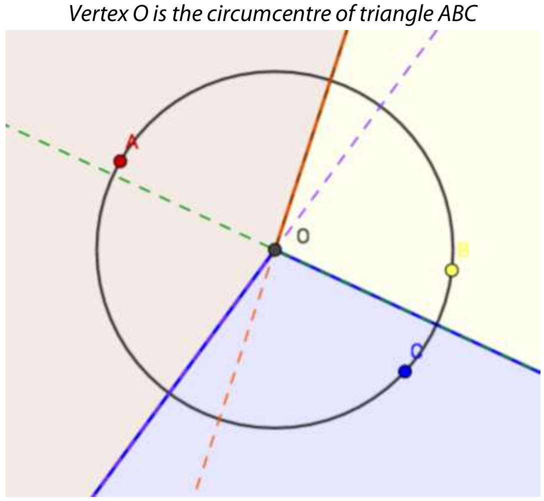


Figure 5

Vertex O is the circumcentre of triangle ABC



The incremental algorithm utilizes these relationships to build the diagram recursively, one site at a time. Given the diagram above, suppose we wish to add a fourth site, D. We draw the perpendicular bisectors with all nearby sites, $\underline{AD}$, $\underline{BD}$, and $\underline{CD}$, as shown in Figure 6.