

Cryptography.

Principles of Security:

- * Confidentiality
- * Authenticity
- * Integrity

To overcome CIA there should be no,

- * Interception
- * Modification
- * Fabrication
- * Availability
- * Access control
- * Attacks

→ Criminal

→ Publicity

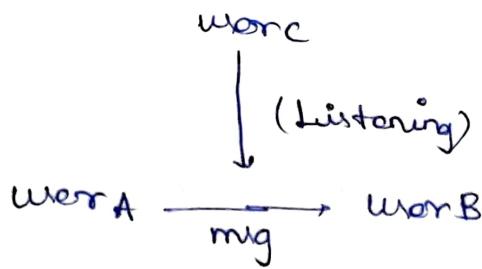
→ Legal

Attacks :

→ Passive

→ Active

eg: DOS



(Passive attacks)

29/08/22

Vulnerability - Some problem in program exploiting.

Fuzzing - try different combination to find whether Vulnerable or not

Registers :

* General purpose registers

- EAX Accumulator
- EBX Base / offset
- ECX Counter
- EDX Data

* Segment

- CS Counter Segment
- DS Data Segment
- SS Stack Segment.

* Control register

- EIP Next Instruction

Every program will have,

- * Text → read only
- * Data ↴ Global / static variables.
- * BSS
- * Heap ↪ uninitialized.
 FIFO.
- * Stack ↪ LIFO

Dynamic Register allocation → we use heap or stack.

To define word in Assembly : dw.

e.g.: int number; number dw 0;

code

...

number1;

mov eax, number;

inc eax;

move number, eax;

Int number;

number dw 0;

If (number < 0) {

mov eax, number;

...

dm, eax, eax;

lge label

< No code >

g

label

< Yes code >

Debugging :

- b Main → Stops after main
- b → Stops in current line
- b N → Stops after N lines.

r → run till the first occurrence of
break points.

s → same as next but if it is a block it moves
next → move to next line to next block.

bt

info b → list all break points.

Buffer :

* Temporary Storage

* fast processing

Cryptology :

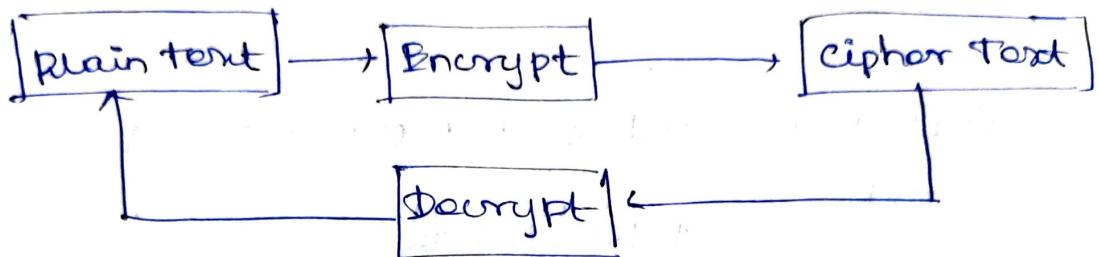
* Study of Cryptography.

* Plain text : human readable

* Cipher text : unreadable form

Encryption : Technique to convert plain text to cipher text

Decryption : to convert cipher text to plain text.



Substitution

Cesar Cipher :

A B C D E F G H I J K L M N O
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

P Q R S T U V W X Y Z

15 16 17 18 19 20 21 22 23 24 25

$$C = (P, K)$$

Cipher = $(P + K) \bmod 26$,
text

$$P = (C, K)$$

Plain
text = $(C - K) \bmod 26$

eg:

P = Meet Me,

Cipher = PHHHLW DH,

C = Nihar is a friend of Antony

Cipher = QLVKDQW LV Ⓛ ZULJQJ RP
text Ⓛ QWRQB

Cipher = GBXXEQQEXSRG&
text Ⓛ BZBBLBU

MonoAlphabetic cipher!

* Each character is mapped to some random alphabet.

Eg: P.T = CIPHER

Cipher = LVZSJT

* Total combinations = $26!$

* Can able to decipher it in $26!$

ways of combination.

Playfair cipher:

Key : APPLE

Rules:

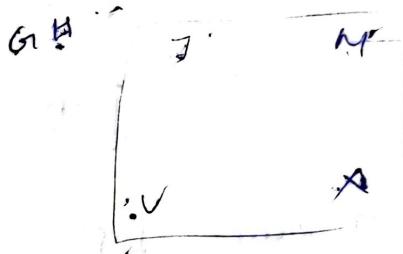
A	P	L	E	B
C	D	F	G	H
I	K	M	N	O
Q	R	S	T	U
V	W	X	Y	Z

- * Same row →
- * Same column ↓
- * Rectangle / Square Swap

P.T

(A T)	T A	C K	E
E Q	Q E	I	D I

C.T



P.T =

E	N	C	R	Y	P	T	I	O
---	---	---	---	---	---	---	---	---

C.T =

G	T	D	Q	W	P	Q	I	O
---	---	---	---	---	---	---	---	---

Hill cipher: (1929).

* Using mathematical operation

$$C.T = P.T \times K \pmod{26}$$

$$P.T = C \times K^{-1} \pmod{26}$$

$$\text{Key} \quad \begin{bmatrix} 3 & 3 \\ 2 & 5 \end{bmatrix} \quad PT = \text{HELP.}$$

Taking HE :

$$= [7 \ 4] \begin{bmatrix} 3 & 3 \\ 2 & 5 \end{bmatrix}^{-1} \mod 26$$

$$= [29 \ 41] \mod 26$$

$$= [3 \ 15]$$

$$[D \ P]$$

Taking LP :

$$= [11 \ 15] \begin{bmatrix} 3 & 3 \\ 2 & 5 \end{bmatrix}^{-1} \mod 26$$

$$= [63 \ 108] \mod 26$$

$$= [11 \ 4] \mod 26$$

$$= [L \ E]$$

$$\therefore C.T = \text{DPLE}$$

Decryption:

Find K^{-1} .

$$\therefore K^{-1} = \frac{1}{9} \begin{bmatrix} 5 & -3 \\ -2 & 3 \end{bmatrix}$$

$$9x \equiv 1 \pmod{26}$$

$$C.T. = DPL E$$

Taking $D P$:

$$= 3 \begin{bmatrix} 3 & 15 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 5 & -3 \\ -2 & 3 \end{bmatrix} \pmod{26}.$$

$$= 3 \begin{bmatrix} -15 & 66 \end{bmatrix} \pmod{26}$$

$$= \begin{bmatrix} -75 & 198 \end{bmatrix} \pmod{26}.$$

$$= \begin{bmatrix} -49 & 198 \end{bmatrix} \pmod{26} \quad (+26 \text{ to make +ve})$$

Polyalphabetic cipher:

Vigenere

$$c_i = (p_i + k_i) \bmod 26.$$

Eg:

Key = Deceptive

P.T = Pay once more for the class.

If P.T is longer than key size,

repeat key until it becomes equal to P.T

Encryption:

$$\begin{array}{l} \text{P.T: } \text{Pay once more for the class.} \\ \text{Key: } \text{Deceptive} \\ \hline \end{array} \quad \begin{aligned} &= (3+15) \bmod 26 = 18 = \text{S} \\ &= (4+0) \bmod 26 = 4 = \text{E} \\ &= (2+24) \bmod 26 = 0 = \text{A} \end{aligned}$$

⋮

Instead of repeating key characters

we can also repeat the characters of

P.T ⋮

Vernam cipher:

$$c_i = (p_i \oplus k_i)$$

One Time Pad:

Same as polyalphabetic cipher but here we use a key only once.

Transposition:

Rail Fence cipher:

Depth \Rightarrow 2.

P.T = Attack at once.

A	T	C	A	O	I	C
T	A	K	P	A	T	N
E						

C.T = read characters now wise.

$$\therefore C.T = ATCAOCTAKTNB$$

Depth = 3

A		C	O	I	C
T	A	K	T	N	B
E	F	A	T	N	E

$$C.T = ACOTAKTNBETAC.$$

Row Column Transposition:

- * Writing in row wise
- * reading column wise using priority
- * remaining cells can be filled with any combination of alphabet.

PT = ATTACK ONCE ^{AT}

4 3 1 2
A T T A

C K A T
/ O N C E

CT = STARGATE TKNACO

Decryption:

key = MONARCHY

M	O	N	A	R
C	H	Y	B	D
E	F	G	I/J	K
L	P	Q	T	Z
V	U	W	X	Z

R S/S R/D E

ATTACK

ii) ON(TS|ML)

MOSQUE.

Key = Caesar.

C	A	E	S	R
B	D	F	G	H
Z	K	L	M	N
O	P	Q	T	U
V	W	X	Y	Z

ii). NTmalkdqaf

MULTIPLE

ii) RLACWTHOMFU

BNCRYPTION.

10/7/22

Sample Space - \mathcal{S} :

* Set of all possible events.

eg: coin toss $\mathcal{S} = \{\text{H}, \text{T}\}$.

Event - E :

* Occurrence of an action.

* Types :

→ Favorable event

→ non-Favorable event.

m - Number of times the favourable event occurs,

$\frac{m}{n} \rightarrow$ event

$n \rightarrow$ Sample Space.

Addition:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

↳ for dependent events

$$P(A \cup B) = P(A) + P(B)$$

↳ for independent events.

Binomial distribution = $\binom{n}{k} P^k (1-P)^{n-k}$.

DES: Algorithm: (Data Encryption Standards)

* based on symmetric cipher.

* Stream cipher:

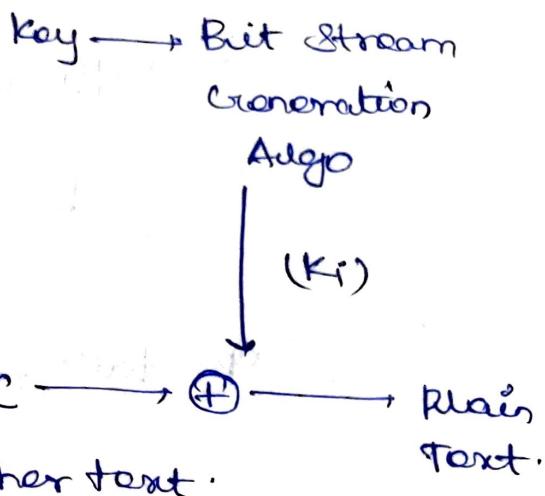
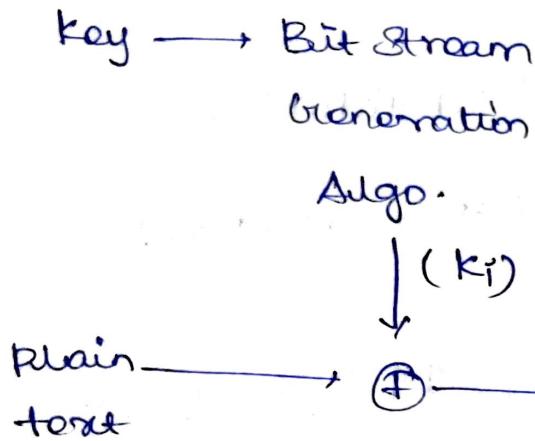
→ change PT to CT character by character

* Block cipher!

→ change PT to CT block by block

→ Minimum 64 bit / 128 bit.

* DES → first block structure



* mapping :

- reversible (eg. Caesar cipher)
- irreversible.

Eg for irreversible :

P.T	C.T
00	10
01	11
10	00
11	11

here 01 and 11 both are mapped

to 11. So it requires 2^n combination

↳ identify correct P.T.

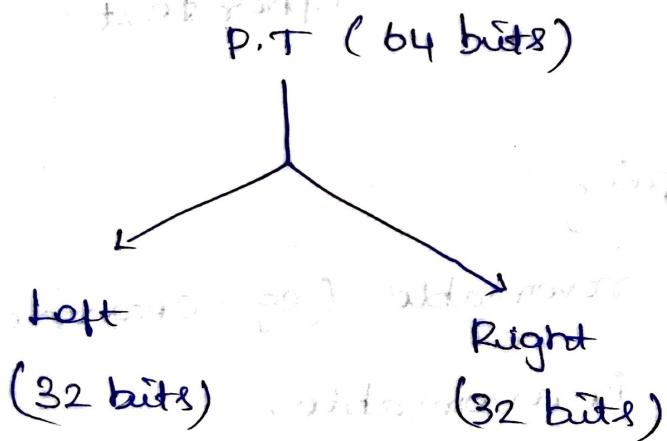
* First block structure \rightarrow idle block structure.

* Concepts used in first:

→ Diffusion. (Substitution)

→ confusion. (Transposition)

* Steps followed in DES encryption



→ every 8th bit is removed from key

→ Initially key is 64 bit after removing parity it becomes 56 bit.

e.g:

1 2 3 4 5, 6 7 8
.....
16

24

(remove 8, 16, 24, ...)

64 bit P.T



Initial permutation

↓ 64
Round 1. ← 48 key ...

↓ 64.

Round 2. ← 48 key ...

↓

↓

Round 16

↓

32 bit Swap.

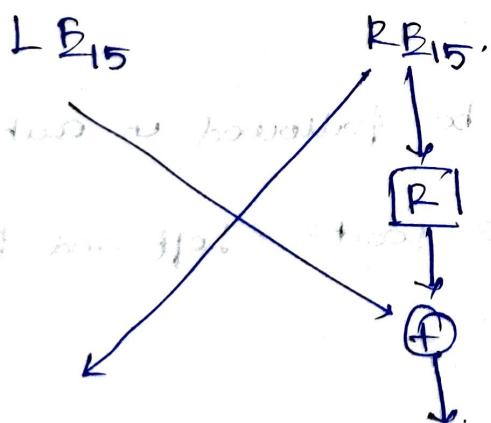
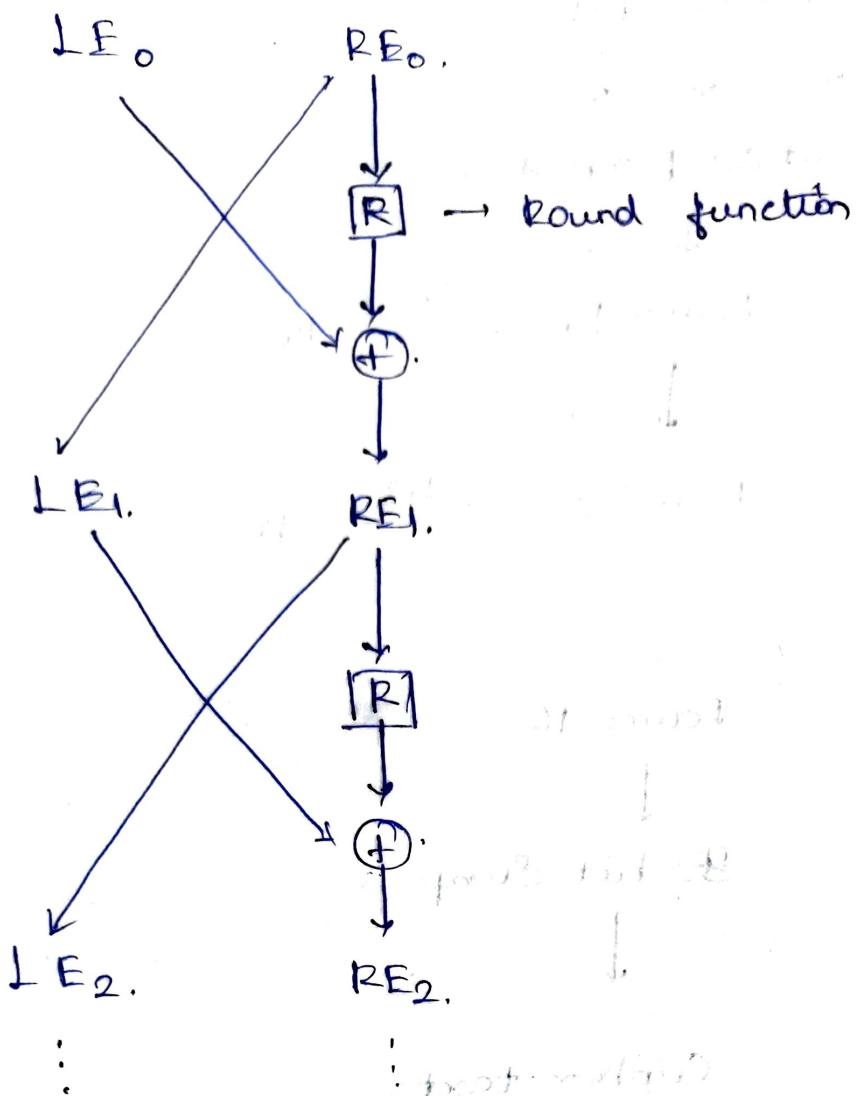
↓

Cipher text.

Round (Operation to be followed in each Round)

- * Divide into 2 parts Left and Right
- * bit Shuffling
- * Substitution (any Substitution can be applied)

- * XOR key with input



After completing 15 rounds swap left and right part and apply inverse Permutation.

Hw! See problem, (any 1).

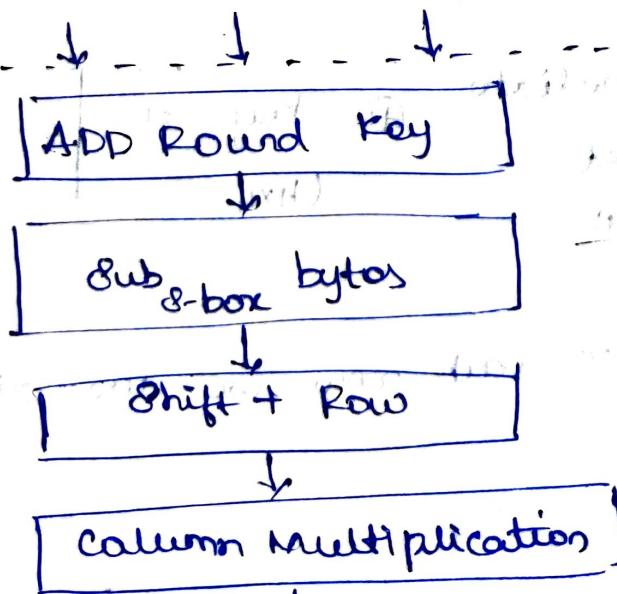
- * Number of round is important
- * operation performed in round function is also very important
- + Initially DES is ~~was~~ impossible to crack but later it become possible so we move to AES.

12/09/22

AES: (Advanced Encryption Standards)

- * Initially proposed by National Institute of Standards.

Plain Text. (128 bits)



ADD Round Key

* Repeat the same process for 9 rounds.

* For 10th round (last) alone column multiplication is not used.

State array : \rightarrow Intermediate array

at odd step and at 10th step

$$\text{if odd step} \quad S = \begin{bmatrix} S_{0,0} & S_{0,1} & \dots \\ S_{1,0} & S_{1,1} & \dots \\ S_{2,0} & S_{2,1} & \dots \\ S_{3,0} & S_{3,1} & \dots & S_{3,B} \end{bmatrix}$$

(Output = intermediate + result) \leftarrow CEA

$$\begin{bmatrix} 0_1 \\ 0_2 \\ \vdots \\ 0_3 \\ \vdots \\ 0_{15} \end{bmatrix}$$

for 10th round $S = \begin{bmatrix} 0_1 \\ 0_2 \\ \vdots \\ 0_3 \\ \vdots \\ 0_{15} \end{bmatrix} \leftarrow$ output of previous step

Add round key (3x4), last round

$$\text{Intermediate} \oplus \text{key} = \begin{bmatrix} \text{key} \\ \text{key} \\ \text{key} \end{bmatrix} \leftarrow \text{round.}$$

text (4×4)

For each row we use different key.

Final output

Root word:

X₁ 69 6E 67 \$2

S-Box:

Y₁ F9 9F 85 40

Rand const:

[R₁ → R₁₀]

[01] [02] ... [36] ↗ g(m₃) ↗
[00] [00] ... [00] ↗ f_q ↗ 01
[00] [00] ... [00] ↗ qf ↗ 00
[00] [00] ... [00] ↗ 85 ↗ 00
[00] [00] ... [00] ↗ 40 ↗ 00

Block cipher Mode:

* Electronic Code book

Plain Text

Cipher Text

Key

Encrypt

Key

Decrypt

$$C_1 = E(K, P_1)$$

Cipher Text

$$P_1 = D(K, C_1)$$

Plain Text

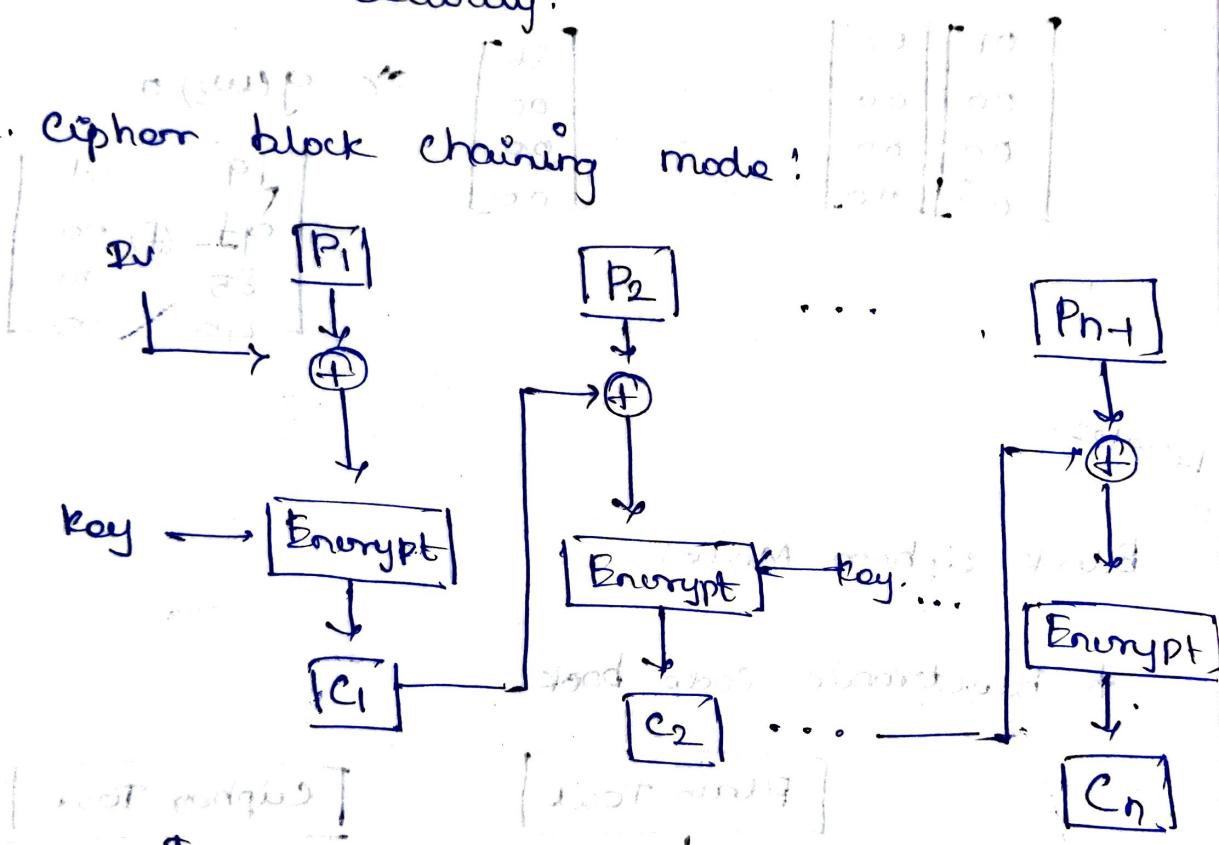
→ Used for small size text.

→ If there is repetition of words it is easier to crack.

* Other constraints to be maintained

1. Overhead
2. Error Recovery.
3. Error Propagation.
4. Diffusion.
5. Security.

2. Cipher block chaining mode:



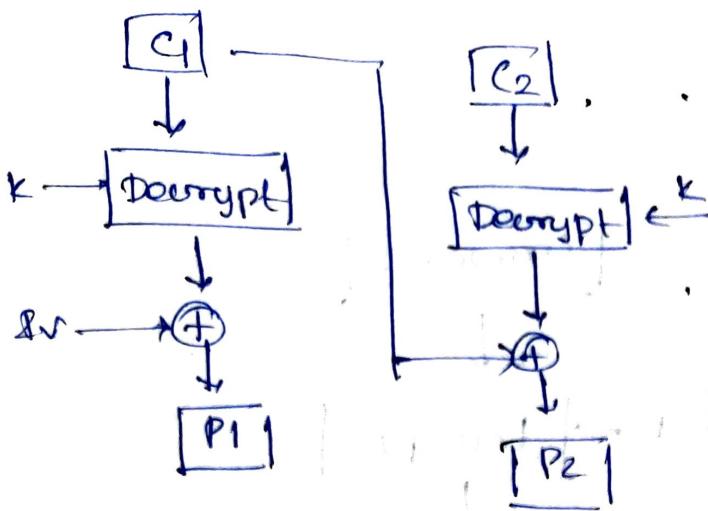
IV - Initialization Vector. can be

time stamp or random number generated.

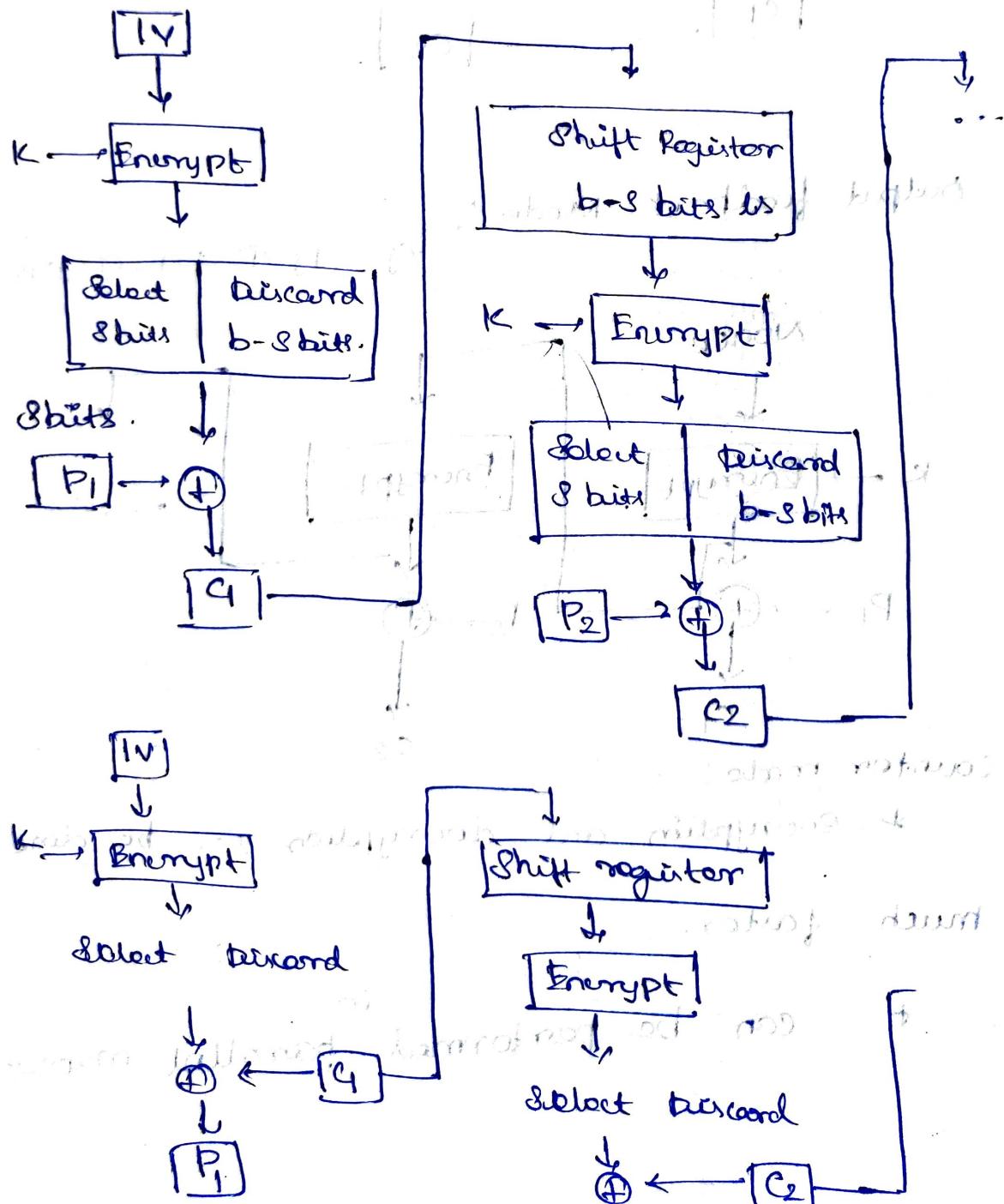
$$C_j = E[k, (c_{j-1} \oplus p_j)]$$

$$D = D[k, E(k_1, c_j \oplus p_j)]$$

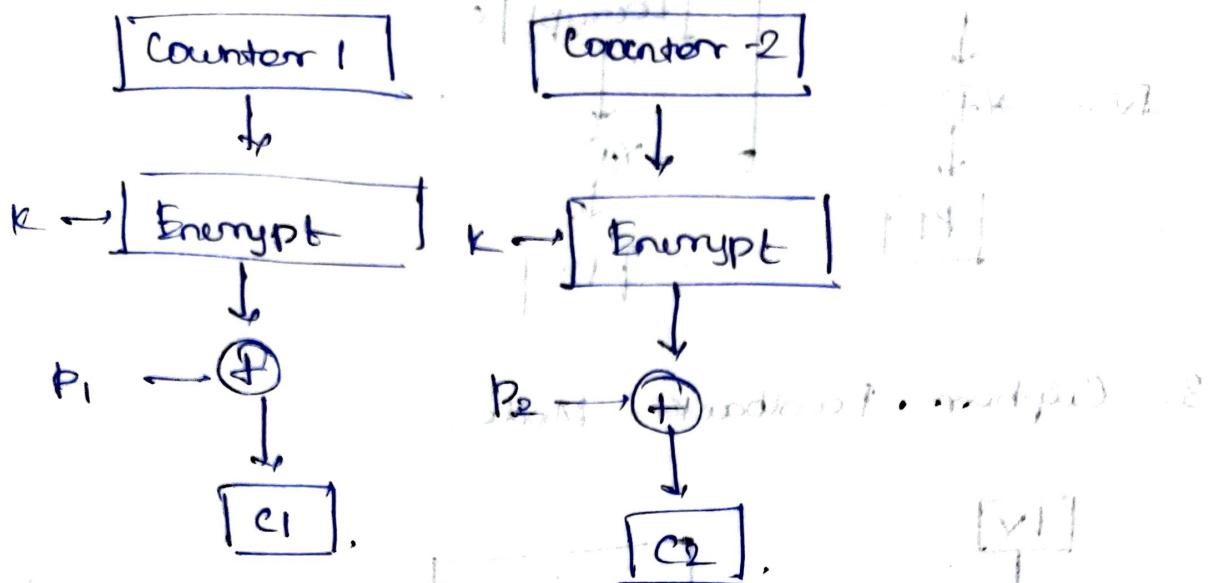
If above is padded in length



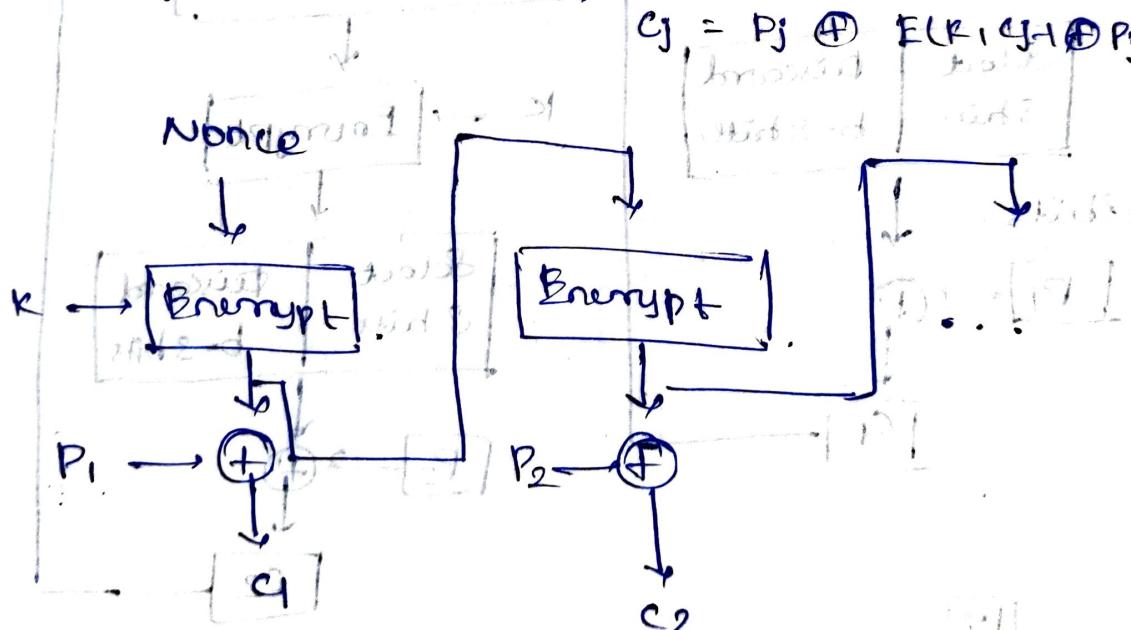
3. Cipher Feedback Mode:



Counter Mode:



Output feedback Mode:



Counter mode:

- * encryption and decryption can be done much faster.
- * can be performed in parallel manner.

Padding:

Simple - 10^*

Multirate - $10^* \dots 1$.

20/09/2022.

Divisibility:

- * A nonzero b divides a if $a = mb$ for some m , where a, b and m are integers.
- * b divides a if there is no remainder on division.
- * notation $b|a$ is commonly used to mean b divides a .

Prime numbers:

Numbers that are divisible by only 1 and by the number itself.

Relative prime numbers:

a and b are relative prime numbers if $\gcd(a, b) = 1$.

e.g.: 3 and 4.

congruent mod n :

$$a \bmod n \equiv b \bmod n.$$

Eg: $73 \bmod 23 = 4 \bmod 23$

Properties of divisibility:

- * If $a \mid 1$ then $a = \pm 1$
- * If $a \mid b$ and $b \mid a$ then $a = \pm b$.
- * Any $b \neq 0$ divides 0.
- * If $a \mid b$ and $b \mid c$ then $a \mid c$.
- * If $b \mid g$ and $b \mid h$ then $b \mid (mg + nh)$ for arbitrary integers m and n .

Division Algorithm:

Given any positive integer n and any nonnegative integer a , if we divide a by n

then there exist unique integers q and r such that $a = nq + r$ and $0 \leq r < n$.
This is called the division algorithm.

GCD of a and b is the largest integer that divides both a and b .

$$\gcd(a, b) = \gcd(|a|, |b|)$$

$$ii) \gcd(72345, 43215).$$

$$i) \gcd(3486, 10295).$$

$$72345 = 1 \times 43215 + 29130.$$

$$43215 = 1 \times 29130 + 14085$$

$$29130 = 2 \times 14085 - 960$$

$$14085 = 14 \times 960 + 645$$

$$960 = 1 \times 645 + 315$$

$$645 = 2 \times 315 + 15$$

$$315 = 21 \times 15$$

$$\gcd(72345, 43215) = 15$$

$$iii) \gcd(45, 32)$$

$$iv) \gcd(117, 218).$$

$$45 = 2 \times 32 + 11$$

$$218 = 1 \times 117 + 101.$$

$$32 = 2 \times 11 + 10$$

$$117 = 1 \times 101 + 16$$

$$11 = 1 \times 10 + 1.$$

$$101 = 6 \times 16 + 5$$

$$10 = 10 \times 1.$$

$$5 = 5 \times 1.$$

$$\gcd(45, 32) = 1$$

$$\gcd(117, 218) = 1.$$

Modular Arithmetic:

* Congruent modulo n:

two integers a and b are said to be

Congruent modulo n if $(a \bmod n) = (b \bmod n)$

* Modulus:

If a is an integer and n is a positive integer we define $a \bmod n$ to be the remainder when a is divided by n .

The integer n is called modulus.

Properties:

1. $a \equiv b \pmod{n}$ if $n|(a-b)$

2. $a \equiv b \pmod{n}$ implies $b \equiv a \pmod{n}$

3. $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$

implies $a \equiv c \pmod{n}$.

Let $a = 23$, $b = 8$, $n = 5$.

$$23 \equiv 8 \pmod{5}$$

$$23 - 8 = 15 = 5 \times 3.$$

$$\Rightarrow -11 \equiv 5 \pmod{8}$$

$$-11 = -16 = -2 \times 8,$$

$$[(a \bmod n) + (b \bmod n)] \bmod n = (a+b) \bmod n.$$

$$[(a \bmod n) - (b \bmod n)] \bmod n = (a-b) \bmod n$$

$$[(a \bmod n) * (b \bmod n)] \bmod n = (a * b) \bmod n.$$

Commutative laws

Associative laws

Distributive law

Identities

Additive inverse law

Format's Theorem:

If p is prime and a is a positive integer not divisible by p then,

$$a^{p-1} \equiv 1 \pmod{p}.$$

Alternative form is:

If p is a prime and a is a positive integer then

$$a^p \equiv a \pmod{p}.$$

$$5^{19-1} \equiv 1 \pmod{19} = 1$$

$$5^{19} \equiv 5 \pmod{19} \equiv 5$$

$$9^{79} \mod 73$$

$$= 9^{(79 \times 10 + 64)} \mod 73$$

$$= [(9^{73})^{10} \cdot 9^{64}] \mod 73.$$

$$= [(9)^{10} \cdot 9^{64}] \mod 73.$$

$$= 9^{74} \mod 73 = (9^{73} \mod 73)(9 \mod 73)$$

$$= 9 \mod 73 \times 9 \mod 73$$

$$= 9 \times 9 \mod 73 = 81 \mod 73 = 8.$$

$$x^{86} = 6 \mod 29.$$

$$x^{72} \cdot x^{14} = 6 \mod 29.$$

$$(1 \mod 29) x^{14} = 6 \mod 29.$$

$$x^{14} = 6 \mod 29.$$

$$x^{12} \cdot x^2 = 6 \mod 29$$

$$(1 \mod 14) x^2 = 6 \mod 29,$$

$$x^2 \mod = 6$$

$$29$$

$$x^2 \mod 29 = 6$$

6, 35, 64, 93

$$\therefore x = 8.$$

$$(iii) 4^{225} \pmod{13}.$$

$$= ((4^{\frac{17}{13}})^4 \cdot 4^4) \pmod{13}$$

$$= (4^{\frac{17}{13}} \pmod{13} \cdot 4^4) \pmod{13}.$$

$$= ((4^{\frac{13+4}{13}} \pmod{13}) \cdot 16) \pmod{13}.$$

$$= (4^{\frac{13}{13}} \cdot 4^4 \pmod{13} \cdot 16) \pmod{13}.$$

$$= (4 \cdot (3)(16)) \pmod{13}.$$

$$= 102$$

24/09/2022

Overflow Buffer:

array [30]; NOP - no operation;

func () {

getchar()

printf()

g

main () {

func()

g.

26/09/2022

Shell code (User) :

- * getting input
- * accessing binary files
- * for direct kernel access.

```
int main() {
```

```
    exit(0);
```

g.

For Sys call : 0x80,

Call the function. It will be thrown into
accumulator along with parameters and
then execute sys call using 0x80.

```
int $0x80
```

```
mov 0x1
```

```
int $0x80.
```

store 0 into ebx

store 1 into eax

execute 0x80 syscall (not group exit),

Section .text

global = start

start

1. mov ebx storage bb 00 00 00 00
2. mov eax storage b8 01 00 00 00
3. int 0x80 storage cd 80 - - -

char shellcode [] = { 'Xbb'\x00\00\x00\x00\x00
 \xb8'\x01\00\00\00 };

1. xor ebx ebx.

2. mov bd, 1.

3. int 0x80

* fork() + execve() → copy the program.

* execve() → replace with another program.

* execve() three arguments: function, argb
and argb endp.

* 4 registers used: 1 → Sys call 3 → argument

* relative addressing used.

* placeholders.

→ Call by name trap live

→ Sys call is loaded into ebx

→ placing arguments into register.

→ CPU is changed to kernel mode.

→ Sys call function is executed.

→ group exit is not needed. (optimization)

• Write desired code in high level language.

Format strings:

→ modifying output.

Used in:

* Output statements.

Different Format Strings

%d

Integer value.

%s - String

%u

unsigned value

%x

hexadecimal

%n

number of bytes

```
#include <stdio.h>
```

```
int main() {
```

```
    A = 5, B = 10, count_a, count_b;
```

```
    pf ("No. of bytes x %d till now and no.  
        of bytes x %d till pt", count_a,  
        count_b);
```

```
    pf ("%d", count_a);
```

```
    pf ("%d", count_b);
```

```
    pf ("Adds %d x %d for %d, %d is value of  
        B, &A, A, B);
```

3

Format String Vulnerability:

X86 - 8 bytes in pack.

Reading from arbitrary memory address:

Pars virtual ~~and~~ 4 bytes

Direct parameter packing:

```
pf ("f: %f fd, h: %4$05d", 10, 20, 30, 40, 50  
    60, 70, 80, 90)...
```

Output:

f: 40, h: 00040.

BL!

Dotours using divisors

Short writer

table

Overwriting offset modes.

node search.

12/10/2022

Euler's theorem: $a^{\phi(n)} \pmod{n} = 1$.

$\phi(n) = n-1$, if n is a prime number,

$\phi(n) = (p-1)(q-1)$ if $n = pq$, where p and q are prime numbers.

$\phi(n) = n(1 - \frac{1}{p_1})$ if $n = ab$ where a and b or either one of them is composite

$$\phi(7000) :$$

$$7000 = 7 \times 1000$$

$$= 7 \times 2^3 \times 5^3$$

$$\therefore \phi(7000) = 7000 \left(1 - \frac{1}{7}\right) \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{5}\right)$$

$$= 7 \times 1000 \left(1 - \frac{1}{7}\right) \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{5}\right)$$

Using Euler theorem solve:

$$4^{99} \pmod{35}$$

$$\phi(35) = 24.$$

$$= 4^{96} \cdot 4^3 \pmod{35}$$

$$= 35 \left(1 - \frac{1}{5}\right) \left(1 - \frac{1}{7}\right)$$

$$= 35 \left(\frac{4}{5}\right) \left(\frac{6}{7}\right)$$

$$= 24.$$

$$= (1 \pmod{35})^4 \cdot 4^3 \pmod{35}$$

$$= 1 \cdot 64 \pmod{35}$$

$$= 29.$$

$$3^{202} \pmod{13}.$$

$$\phi(13) = 12,$$

$$= (3^{12})^{16} \cdot 3^1 \pmod{13}$$

$$= (1 \pmod{13})^{16} \cdot 3^1 \pmod{13}$$

$$= 3^1 \pmod{13}.$$

$$= 3^3 \cdot 3^3 \cdot 3^3 \cdot 3^1 \pmod{13}.$$

$$= 27 \pmod{13} \quad 9 \times 9 \times 9 \times 3$$

$$= 3.$$

Chinese remainder theorem:

Step 1: Identify or calculate m_1, m_2, m_3

Step 2: $M = m_1 \times m_2 \times m_3 \times \dots \times m_i$

Step 3: $M_i = \frac{M}{m_i}$

Step 4: Calculate a_1, a_2, \dots, a_i

Step 5: Calculate M_i^{-1}

Step 6: $x = \sum_i a_i m_i M_i^{-1}$

$$x = a_i \bmod m_i$$

$$x = 2 \bmod 3$$

$$x = 3 \bmod 5 \quad \text{Using CRT.}$$

$$x = 2 \bmod 7.$$

It is used when $m_i > n$ ($a_i^n \bmod m_i$)

Step 1: $m_1 = 3 \quad m_2 = 5 \quad m_3 = 7$

Step 2: $M = 3 \times 5 \times 7 = 105$

Step 3: $M_1 = \frac{105}{3} = 35 \quad \underline{\underline{M_2}} = \frac{105}{5} = 21$

$$\underline{\underline{M_3}} = \frac{105}{7} = 15$$

Step 4: $a_1 = 2, a_2 = 3, a_3 = 2$

Step 5: M_i^{-1} :

$$M_i M_i^{-1} \equiv 1 \pmod{m_i}$$

$$35 \times 1 \pmod{3} \equiv 2$$

$$M_1^{-1} \equiv 2$$

$$35 \times 2 \pmod{3} \equiv 1$$

$$M_2^{-1} \equiv 1$$

$$21 \times 1 \pmod{5} \equiv 1$$

$$M_3^{-1} \equiv 1.$$

$$15 \times 1 \pmod{7} \equiv 1$$

Step 6:

$$X = 2 \times 35 \times 2 + 3 \times 21 \times 1 + 2 \times 15 \times 1.$$

$$= 12 + 15 + 14.$$

$$X \equiv 233 \pmod{115} \equiv 23.$$

HW:

1) $4x \equiv 5 \pmod{9}$

$2x \equiv 6 \pmod{20}$

2) $x \equiv 5 \pmod{3}$

$x \equiv 2 \pmod{5}$

$x \equiv 1 \pmod{11}$

Miller-Rabin Primality Test:

$$n-1 = 2^k \cdot m$$

$$a \quad 1 \leq a \leq (n-1).$$

$$b_0 = a^m \pmod{n}.$$

$$b_1 = b_0^2 \pmod{n}$$

$$b_2 = b_1^2 \pmod{n}.$$

i) 561 a P.number

i) $561 = (n-1) \Rightarrow 561-1 = 560 = 2^4 \times 35$.

ii) $a \equiv 2$

iii) $b_0 = 2^{35} \pmod{561}$,

$$= 263 \neq \pm 1$$

$$b_1 = (263)^2 \pmod{561},$$

$$= 166 \neq \pm 1$$

$$b_2 = (166)^2 \pmod{561},$$

$$= 67 \neq \pm 1,$$

$$b_3 = (67)^2 \pmod{561},$$

$$= 1.$$

Composite number,

2) Is 53 a prime number?

i) $53 \Rightarrow (n-1) = 52 = 2^2 \times 13$.

Additive inverse of 53 is -1.

∴ It is prime number.

G.F. Galois Field of Prime number:

- * Order of finite field is given as p^n .
- * used in polynomial prime
- * used in AES.

G.F.(2) :

$$(0,1) = Z_2[x] + g$$

*	0	1
0	0	1
1	1	0

*	0	1
0	0	0
1	0	0

a	0	1
a	0	1
a ²	-	1

G.F.(5) :

*	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	4	0	1	2
3	3	4	0	1	2
4	4	0	1	2	3

*	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	2	3	1

a	$-a$	a^{-1}
0	0	0
1	4	5
2	3	3
3	2	2
4	1	4.

GF(7), GF(8).

$+_{\#}$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

$*_{\#}$	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4.
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1.

a	$-a$	a^{-1}
0	0	-
1	6	1
2	5	4
3	4	5
4	3	2
5	2	3
6	1	6.

$+_8$	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4.
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6.

$*_8$	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	0	2	4	6
3	0	3	6	1	4	7	2	5
4	0	4	0	4	0	4	0	4.
5	0	5	2	7	4	1	6	3
6	0	6	4	2	0	6	4	2
7	0	7	6	5	4	3	2	1

a	$-a$	a^7
0	0	1
1	1	1
2	6	1
3	5	3
4	4	1
5	3	5
6	2	1
7	1	7

26/10/22

Not every element have inverse so

we move to polynomial \Rightarrow carrying \Rightarrow

maximum degree = n+1. (eg: 2^3 degree = 3-
= 2)

Polynomial addition \Rightarrow Xor.

$$\begin{array}{r}
 0 - 000 \xrightarrow{\text{Xor.}} 000 \\
 1 - 001 \xrightarrow{\text{Xor.}} 110 \\
 2 - 010 \xrightarrow{\text{Xor.}} 101 \\
 \vdots \\
 7 - 111 \xrightarrow{\text{Xor.}} 000
 \end{array}
 \quad
 \begin{aligned}
 & x^2 + x + 1 \\
 & \underline{x^2 + 1} \\
 & \hline
 & 0 + x + 0 \\
 & \hline
 & = x \quad (2)
 \end{aligned}$$

Polynomial multiplication \Rightarrow multiply and then
Xor.

eg:

$$\begin{array}{r}
 x^2 + x + 1 \\
 \times x^2 + 1 \\
 \hline
 x^4 + x^2 + x^3 + x + x^2 + 1
 \end{array}$$

$$m(x) = x^3 + x + 1.$$

$$\begin{array}{r} x+1 \\ \hline x^4 + x^3 + x + 1. \\ x^4 + x^2 + x \\ \hline x^3 + x^2 + 1 \\ x^3 + x + 1 \\ \hline x^2 + x \end{array} = 6.$$

$$m(x) = x^8 + x^4 + x^3 + x + 1 \quad \text{for } \text{GF}(2^8).$$

(2)

$$\begin{array}{r} x+1 \\ \hline x^2 + x \\ x^3 + x^2 + x^2 + x \\ - \\ x^3 + x \end{array}$$

$$\begin{array}{r} x^3 + x + 1 \\ \hline x^3 + x \\ x^3 + x + 1 \\ \hline 11 \end{array}$$

$$f(x) = x^6 + x^4 + x^2 + x + 1$$

$$g(x) = x^7 + x + 1$$

$$m(x) = x^8 + x^4 + x^3 + x + 1.$$

$$f(x) + g(x) = x^7 + x^6 + x^4 + x^2$$

multiplication

$$= x^{13} + \underline{x^7} + x^6 + x^{11} + x^5 + x^4 + \\ x^9 + x^3 + \underline{x^2} + \underline{x^8} + x^2 + x + \\ \underline{x^7} + x + 1.$$

$$= x^{13} + x^{11} + x^9 + x^b + x^5 + x^4 + x^3 \\ + 1.$$

$$\underline{x^5 + x^3 + 1}$$

$$x^8 + x^4 + x^3 + x + 1,$$

$$x^{13} + x^{11} + x^9 + x^b + x^5 + x^4 + x^3 + 1$$

$$x^{13} + x^9 + x^8 + x^b + x^5$$

$$= x^{11} + x^8 + x^4 + x^3 + 1.$$

$$\underline{x^{11} + x^7 + x^6 + x^4 + x^3}$$

$$x^8 + x^7 + x^6 + 1.$$

$$x^8 + x^4 + x^3 + x + 1$$

$$\underline{x^7 + x^6 + x^4 + x^3 + x}$$

Assignment :

Take one application apply any hashing function to get output. (eg: SHA-256)

$$f(x) = x^2 + x + 1$$

$$g(x) = x^2.$$

Addition: $x+1 \quad (3)$

multiplication: $x^4 + x^3 + x^2$

$$\begin{array}{r} x+1 \\ \hline x^4 + x^3 + x^2 \\ x^4 + x^2 + x \\ \hline x^3 + x \\ x^3 + x+1 \\ \hline \boxed{11} \end{array}$$

24. $f(x) = x^7 + x^1$

$$g(x) = x^6 + x^5 + x^3 + 1.$$

$$m(x) = x^8 + x^4 + x^3 + x + 1$$

Addition: $x^7 + x^5 + x^3 + 1.$

multiplication: $x^{13} + \underline{x^{12}} + x^{10} + x^7 + \underline{x^{12}} + x^{11} + x^9 + x^6$
 $x^5 + x^3 + x^2 + 1$

$$\begin{array}{r} x^{13} + x^{14} + x^{11} + x^9 + x^7 + x^6 \\ x^{13} + \underline{x^9} + x^8 + x^6 + x^5 \\ \hline x^{11} + x^{10} + x^8 + x^7 + x^5 \\ x^{11} + x^7 + x^6 + x^4 + x^3 \\ \hline x^{10} + x^8 + x^6 + x^5 + x^4 + x^3 \\ x^{10} + x^6 + x^5 + x^3 + x^2 \\ \hline \end{array}$$

9/11/2022

Affine Cipher:

key : (α, β)

$$f(x) = \alpha x + \beta \\ \hookrightarrow P.T$$

constraints:

(i) $\text{GCD}(\alpha, 26) = 1$

(ii) $1 \leq \alpha \leq 25$

(iii) $0 \leq \beta \leq 25$

Problem:

1) $\alpha = 7 \quad \beta = 2$

P.T = 41.

$$\therefore f(x) = (7(6) + 2)$$

$$\therefore x = 44, (44 \gamma 26) \Rightarrow 44 \text{ mod } 26$$

$$f(x) = 18$$

$$\therefore C.T = 8$$

2). P.T = Hello.

for H :

$$f(x) = \#(7) + 2$$

$$f(x) = 51 \bmod 26 = 25$$

$$\therefore C.T = Z$$

for E :

$$f(x) = \#(4) + 2$$

$$= 30 \bmod 26 = 4$$

$$C.T = E$$

for L :

$$f(x) = \#(11) + 2$$

$$= 49 \bmod 26 = 1$$

$$C.T = B$$

for L :

$$C.T = B$$

For O :

$$f(x) = \#(14) + 2$$

$$= 98 + 2$$

$$= 100 \bmod 26 = 22$$

$$C.T = W$$

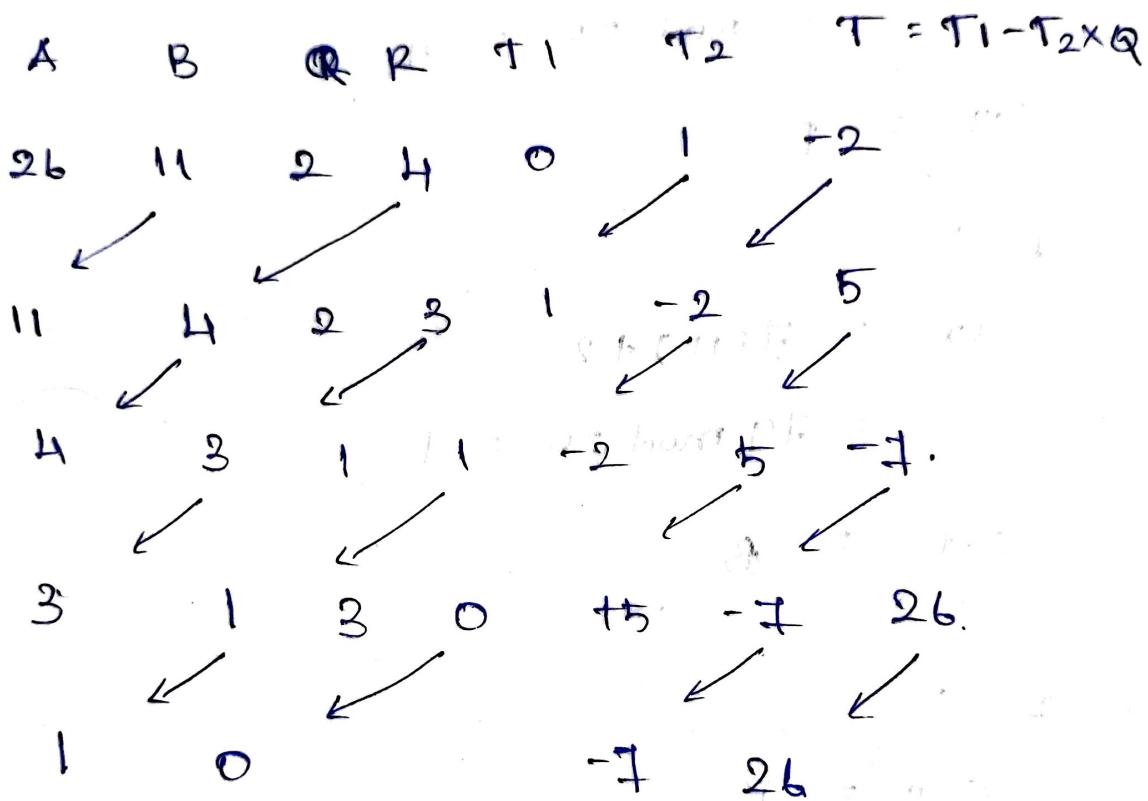
$\therefore C.T = ZEBBW.$

Decryption :

$$x = \alpha^{-1} (y - B) \quad [d \cdot d^{-1} \bmod 26 = 1]$$

Pending inverse:

$$A \bmod B \quad 11 \bmod 26$$



$$A^T = -7 \bmod 26$$

$$A^T = 19$$

$\# \text{ mod } 26$

R	A	B	R	T ₁	T ₂	T ₃
3	26	#	5	0	1	-3
1	#	5	2	1	-3	4
2	5	2	1	-3	4	-11
2	2	1	0	4	-11	26
-	1	0	-	-11	26	-

$$\therefore x^{-1} = -11 \text{ mod } 26$$

$$x^{-1} = 15$$

C.T = ZEBBW.

For Z:

$$x = 15(25 - 2)$$

$$= 15 \times 23.$$

$$= 345 \text{ mod } 26$$

$$= # \quad (\text{H})$$

For B:

$$x = 15(1 - 2)$$

$$= -15 \text{ mod } 26$$

$$= 11 \quad (\text{L})$$

For W:

For E:

$$x = 15(4 - 2)$$

$$= 30 \text{ mod } 26$$

$$= 4. \quad (\text{E})$$

$$x = 15(22 - 20)$$

$$= 300 \text{ mod } 26$$

$$= 14. \quad (\text{O})$$

$\therefore \text{P.T} = \text{HELLO.}$

Public key cryptography:

- * Consist of 2 keys (public and private).
- * Encryption done using public key.
- * Decryption done using private key.

$$E \rightarrow (P_{\text{Pub}} \times X) \quad P_{\text{Pub}} = \text{Public}$$

$$D \rightarrow (P_{\text{Priv}} \times E) \quad P_{\text{Priv}} = \text{Private.}$$

Terminologies used:

- * Encryption Algorithm (symmetric)
- * Decryption Algorithm

RSA!

* Not very secure

* Fairly prone to attack

3 Steps:

* Key generation

* Encryption

* Decryption

key generation:

* # Step process

(i) Select prime numbers P and q

(ii) Calculate $n = P \times q$

(iii) Find your $\phi(n) = (P-1)(q-1)$
↑
Euler function

(iv) choose this value ' e ' such that,

$$1. 1 < e < \phi(n)$$

$$2. \gcd(e, \phi(n)) = 1.$$

(v) calculate ' d ' = $e^{-1} \pmod{\phi(n)}$.

can be rewritten, $ed \pmod{\phi(n)} = 1$.

(vi) Public key $P_{\text{pub}} = \{e, n\}$

private key $P_{\text{priv}} = \{d, n\}$

Enc $\Rightarrow C \cdot t = M^e \pmod{n}$, $M \leq n$
 \downarrow length of the
message.

$$\text{Dec} \Rightarrow P \cdot t = C^d \pmod{n}$$

Problem: (Commonly asked problem $\ddot{\wedge}$)

$$p = 3, q = 11, n = 33.$$

(I) $p = 3, q = 11$

(II) $n = pq = 3 \times 11 = 33$

(III) $\phi(n) = (p-1)(q-1) = (2)(10) = 20.$

(IV) $e = ? \quad \because \gcd(\# \text{, } 20) = 1$
 $1 < e < 20$

(V) $d = ?^{-1} \pmod{20} \quad e \cdot d \pmod{20} = 1$

$$e = 3$$

$$3 \times 3 \pmod{20} = 1$$

$$1 \cdot 3^{-1} = 3$$

(VI) $P_{\text{pub}} = \{e, n\}$

$$P_{\text{pub}} = \{3, 33\}.$$

(VII) $P \cdot T = M \dots (M \leq 31)$

$$\text{Enc} \Rightarrow C \cdot T = 31^3 \pmod{33}$$

$$C \cdot T = ?$$

$$\text{Dec} \Rightarrow P \cdot T = (4)^3 \pmod{33}$$

$$P \cdot T = 31.$$

- * Brute Force (trying with all possible combinations)
- * Mathematical
- * Timing
- * chosen cipher text

Mathematical:

- * Using factors of public key, we can find private key.

Timing:

- * Add constant time
- * Add random delay

Chosen cipher text:

- * Based on properties

$$P = 17 \quad q = 19.$$

$$(i) \quad P = 17, \quad q = 11$$

$$(ii) \quad n = 17 \times 11 = 187 \quad d = 107$$

$$(iii) \quad \phi(n) = 16 \times 10 = 160$$

$$(iv) \quad e = 3.$$

$$(v) \quad d = e^{-1} \bmod 160$$

$$(vi) \quad \text{Pub} = \{3, 187\}$$

$$\text{Priv} = \{107, 187\}$$

$$M = 2$$

$$C \cdot T = M^e \pmod{n}$$

$$(2)^3 \pmod{187}$$

$$C \cdot T = 8$$

$$P \cdot T = 8 \cdot C^d \pmod{n}$$

$$= 8^{107} \pmod{187}$$

$$P \cdot T = 2$$

28/11/2022

Diffie-Hellman

* Private key $x_A \rightarrow$ Value or number, less than q (prime number),

* α - primitive root q .

$a \pmod{q}, \alpha^2 \pmod{q}, \alpha^3 \pmod{q}, \dots, \alpha^{p-1} \pmod{q}$.

↓

1, 2, 3, ..., $p-1$.

* Only public key is exchanged,

$$Y_A = \alpha^{x_A} \pmod{q}$$

* Key Generation :

$$\text{i)} \quad K = (Y_B)^{X_A} \bmod q.$$

$$\text{ii)} \quad K = (Y_A)^{X_B} \bmod q.$$

$$\Rightarrow K = (Y_A)^{X_B} \bmod q$$

$$= (\alpha^{X_A} \bmod q)^{X_B} \bmod q$$

$$= (\alpha^{X_A X_B}) \bmod q.$$

$$= (\alpha^{X_B})^{X_A} \bmod q.$$

$$= (\alpha^{X_B} \bmod q)^{X_A} \bmod q$$

$$= (Y_B)^{X_A} \bmod q.$$

Problem :

$$q = 11 \quad \alpha = 2 \quad X_A = 6 \quad X_B = 8.$$

$$Y_A = 2^6 \bmod 11.$$

$$= 9.$$

$$Y_B = 2^8 \bmod 11$$

$$= 3.$$

$$\therefore K = (3)^6 \bmod 11 = 3$$

$$K = (9)^8 \bmod 11 = 3.$$

$\frac{23}{256}$

Module - 3

* ELF

* SQL Injections.

3/12/2022

* Fuzzing

* Port Scanning.

* ARP.

7/12/2022

Authentication :

* To determine authenticity.

* Traffic analysis:

* Masquerade:

* Content modification:

* Sequence modification:

* Timing (delay).

* Source reputation.

Message authentication function:

* 2 levels of auth.

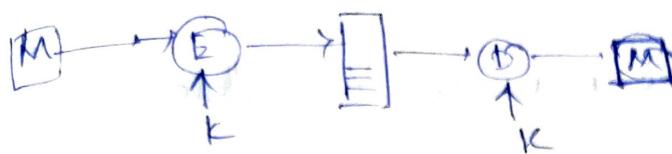
* Low level → Authentication

* High / Message integrity.

frame check sequence.

* cryptographic check sum.

$$\text{HMAC} = C(K, M)$$



over:

* Network broadcasting



Disadv:

* can decrypt using brute force attack

HMAC:

12/12/2022.

PKI. (Public Key Infrastructure).

Certificate authority:

* collect all public keys.

* should be trustable

* secure communication

* provide authorized access.

Example:

Universal PKI:

* Highly impossible to exist.

Everyone trust this PKI which is "the center to all, only person to hold all PKI".

- * To remotely access resources through virtual location.
- * Person outside network can also access resources.

Electronic Banking:

- * Financial transaction
- * Person making transaction should be authorized
- * Need PKI to secure and accept authorized transaction.
- * Need multiple level of certificate authorities
- * Individual CA for each bank.

Postman Services:

- * every process is automated.
- * need PKI to prevent hacker from modifying the information from sender.
- * CA to validate info from sender.

Credit Card Validation:

- * authorization of transaction.
- * CA to validate authorized transaction.

Multilevel certification :

- * Split CA into multiple levels.
eg: Bank transaction.
- * PKI uses clock through which a particular digital signature will get expired. This improves security.

PKI reality :

- * Name
- * Authority
- * Trust

Indirect authorisation :

- * access control list.
- * Which is a drawback.

Direct authorisation :

- * For every key, authorization is provided,

Credential system :

- * Contains additional information.
- * who can access along with time to access.
- * micromanaging the entire system.
- * Very complex.

Revocation:

- * Very difficult task.
- * Online certification protocol. for verification of certificate
- *
 - Speed : max. time to revoke
 - reliability : effective or not
 - connectivity : verify certificate valid or not
 -
- * Expiration can be used to overcome difficulties in revocation

Life of a key:

- * Create a key
- * obtain a certificate.
- * Broadcast certificate details using PKI
- * Active use / Partue use
- * Expiration (must be stored in CT)

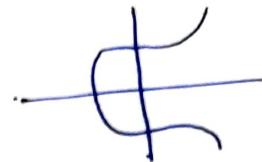
Certification format:

- * required + optional details
- * Permission
- * Root key,

Module - 9,

Elliptic curve cryptography:

$$y^2 = x^3 + ax + b.$$



Characteristics:

- * Faster
- * Efficient

Disadv:

- * Difficult to implement
- * Secure but slow.

Algorithm for Ecc : ECDSA

Digital signature operation:

- * key generation
- * signing alg.
- * Signature verification

Digital certificates: