END Semester Re Examination

Registration no: 2019503553 | Subject code: MA6351

Name: G. Rubakpreyan | Title: Probability & Statistics

## Part-C

23. Two way ANOVA

| | 1 | 2 | 3 |
|---|---|---|---|
| A | 45 | 43 | 51 |
| B | 47 | 46 | 52 |
| C | 48 | 50 | 55 |
| D | 42 | 37 | 49 |

In order to simplify the data we need to subtract all the data with 50

| Detergent | Engine $x_1$ | $x_2$ | $x_3$ | Total | $x_1^2$ | $x_2^2$ | $x_3^2$ |
|---|---|---|---|---|---|---|---|
| A $(y_1)$ | -5 | -7 | 1 | -11 | 25 | 49 | 1 |
| B $(y_2)$ | -3 | -4 | 2 | -5 | 9 | 16 | 4 |
| C $(y_3)$ | -2 | 0 | 5 | 3 | 4 | 0 | 25 |
| D $(y_4)$ | -8 | -13 | -1 | -22 | 64 | 169 | 1 |
| Total | -18 | -24 | 7 | -35 | 102 | 234 | 31 |

(i) Defining $H_0$: There is no significance difference between column means as well as row means

(ii) $H_1$: There is significance difference between column means or row means

Here $N=12$ ( Total number of items)

$T = -35$

so $\frac{T^2}{N} = \frac{(-35)^2}{12} = 102.08$

$\Rightarrow$ Now TSS (Total sum of square)

$= \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^3 - T^2/N$

$= 102 + 284 + 31 - 102.08 = 264.92$

$\Rightarrow$ SSC (Sum of Square between column)

$= \frac{\Sigma x_1^2}{N_1} + \frac{\Sigma x_2^2}{N_1} + \frac{\Sigma x_3^2}{N_1} - T^2/N_0$ ($N_1 \to$ No. of element in each column)

$= \frac{(-18)^2}{4} + \frac{(-24)^2}{4} + \frac{(7)^2}{4} - 102.08$

$= 81 + 144 + 12.25 - 102.08 = 135.17$

$\Rightarrow$ SSR = (sum of square rows)

$= \frac{\Sigma y_1^2}{N_2} + \frac{\Sigma y_2^2}{N_2} + \frac{\Sigma y_3^2}{N_2} + \frac{\Sigma y_4^2}{N_2} - \frac{T^2}{N}$

($N_2 \to$ Num of elements in each row)

$= \frac{(-11)^2}{3} + \frac{(-5)^2}{3} + \frac{(3)^2}{3} + \frac{(-22)^2}{3} - 102.08$

$= 40.33 + 8.33 + 3 + 161.33 - 102.08$

$= 110.91$

SSE = TSS − SSC − SSR

$= 264.92 - 135.17 - 110.91 = 18.84$

Registration no: 2019503553 Subject code: MA6351

Name: G. Rubak preyan Title Probability & statistics

Anova Table:

| Success of Variation | Sum of Square | d.f | Mean Squares | Variance | Table 1.1 ($\alpha$ 0.01) |
|---|---|---|---|---|---|
| Between column | SSC $= 135.17$ | $C-1$ $=3-1$ 2 | $MSC = \frac{SSC}{C-1}$ $= \frac{135.17}{2}$ $= 67.585$ | $F_c = \frac{MSC}{MSE}$ $= \frac{67.58}{3.14}$ $= 21.52$ | $F_{cal}(2,6)$ $= 10.92$ |
| Between rows | SSR $= 110.91$ | $r-1$ $= 4-1$ | $MSR = \frac{SSR}{r-1}$ $= \frac{110.91}{3}$ $= 36.97$ | $F_r = \frac{MSR}{MSE}$ $= \frac{36.97}{3.14}$ $= 11.77$ | $F_{cal}(3,6)$ $= 9.78$ |
| Residual | SSE $= 18.84$ | $N-(C+r-1)$ $= 12-3-4+1$ $= 6$ | $MSE = \frac{SSE}{6}$ $= \frac{18.84}{6}$ $= 3.14$ | | |
| Total | 264.92 | 11 | | | |

Final:

* Here the calculate $F_{cal}$ (for column) = 10.92
  This is $F_c > F_{cal}$ (column)
  So we reject $H_0$
* Here the calculated $F_{cal}$ (for row) = 9.78
  This $F_r > F_{cal}$ (row)
  So we reject $H_0$

Date: 30.6.2021 Sign: G. Rubak preyan Page: 3

Control chart

24. Defects : 6, 4, 9, 10, 11, 12, 20, 10, 9, 10, 15, 10, 20, 15, 16

Let c denote the number of defects:

$$\bar{c} = \frac{\Sigma c}{n} = \frac{171}{15} = 11.4 \ (y)$$

Here $\bar{c}$ represent the 'y' which is the CL

UCL (upper control limit) $= \bar{c} + 3\sqrt{\bar{c}}$

$= 11.4 + 3\sqrt{11.4}$

$= 11.4 + 10.12$

$= 21.52$

LCL (lower control limit)

$= \bar{c} - 3\sqrt{\bar{c}}$

$= 11.4 - 3\sqrt{11.4}$

$= 11.4 - 10.12$

$= 1.28$

Now: UCL = 21.52, CL = 11.4, LCL = 1.28

From the graph we can inspect the value of c we see that no values ~~excess~~ exceed the upper control limit and lower control limits.
Since all the samples points lie with UCL & LCL lines the process is under control.

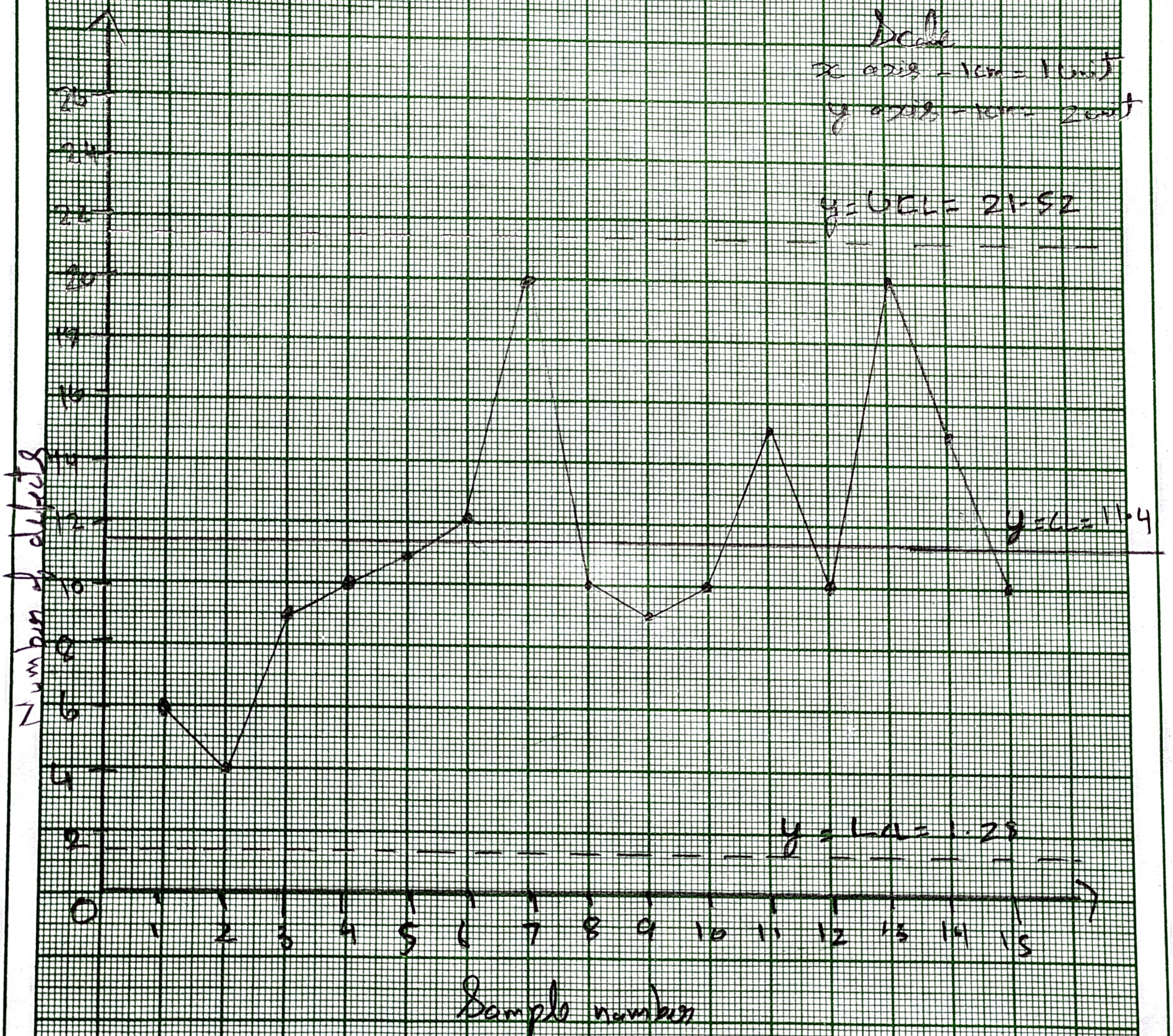
Name: G. Rubakpreyan

Reg no: 2019503553

Title: Probability & Statistics

Subject code: MA6351

24)



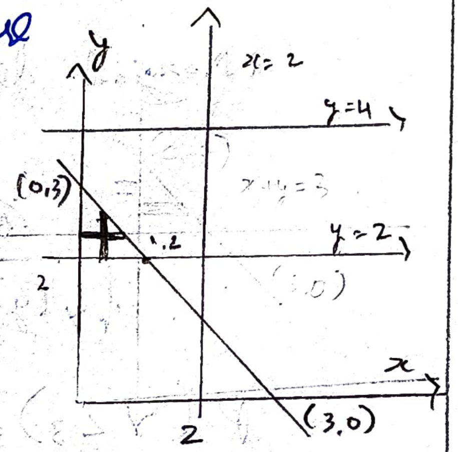

Date: 30.6.2021

Sign: G. Rubakpreyan

Part -B

14. X and Y are Two random Variable

$$f(x,y) = \begin{cases} \frac{1}{8}(6-x-y), & 0 \le x < 2,\ 2 < y < 4 \\ 0 & \text{otherwise} \end{cases}$$




(i) $P(x+y<3)$

$$\cancel{\int_0^1 \int_2^{3-x} f(x,y)\,dx\,dy}$$

y varies from 2 to 3

x varies from 0 to 3-y

$$\int_2^3 \int_0^{3-y} f(x,y)\,dx\,dy$$

$$= \int_2^3 \int_0^{3-y} \frac{1}{8}(6-x-y)\,dx\,dy$$

$$= \frac{1}{8}\int_2^3 \left[6x - \frac{x^2}{2} - xy\right]_0^{3-y} dy$$

$$= \frac{1}{8}\int_2^3 6(3-y) - \frac{(3-y)^2}{2} - y(3-y)\Big]$$

$$= \frac{1}{8}\int_2^3 18 - 9y + y^2 - \frac{(3-y)^2}{2}\Big].dy$$

$$= \frac{1}{8}\Big[18y - \frac{9y^2}{2} + \frac{y^3}{3} + \frac{(3-y)^3}{6}\Big]_2^3$$

$$= \frac{1}{8}\left[18 - \frac{9}{2}\times 5 + 9 - \frac{8}{3} - \frac{1}{6}\right]$$

$$= \frac{1}{8}\left[\frac{162-16-1-135}{6}\right] = \frac{5}{24}$$

$$P(x+y<3) = \frac{5}{24}$$

(ii) $P(x<1 / y<3) = \dfrac{P(x<1 \cap Y<3)}{P(Y<3)}$

Marginal density of Y $\quad f_Y(y) = \int_0^2 \frac{1}{8}(6-x-y) \cdot dx$

$$= \frac{1}{8}\left[6x - \frac{x^2}{2} - xy\right]_0^2 = \frac{1}{8}\left[12-2-2y\right]$$

$$f_Y(y) = \frac{1}{4}(5-y)$$

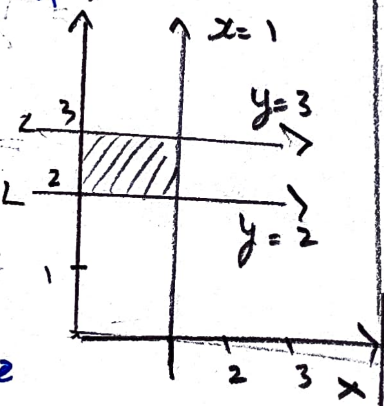
$$P(Y<3) = \int_2^3 f(y) \cdot dy = \int_2^3 \frac{1}{4}(5-y) \cdot dy$$

$$= \int_2^3 \frac{5}{4} - \frac{y}{4} \cdot dy$$

$$= \frac{1}{4}\left[5y - \frac{y^2}{2}\right]_2^3$$

$$= \frac{1}{4}\left[7 - \frac{9}{2}\right] = \frac{5}{8}$$

$$P(X<1 \cap Y<3) = \int_0^1 \int_2^3 \frac{1}{8}(6-x-y) \cdot dy \cdot dx$$




$$= \frac{1}{8} \int_0^1 \left[6y - xy - \frac{y^2}{2}\right]_2^3 \cdot dx$$

$$= \frac{1}{8} \int_0^1 \left[18 - 3x - \frac{9}{2}\right] - \left[12 - 2x - \frac{4}{2}\right] \cdot dx$$

$$= \frac{1}{8} \int_0^1 6 - \frac{5}{2} - x.$$

$$= \frac{1}{8}\int_0^1 \left(\frac{7}{2} - x\right) \cdot dx$$

$$= \frac{1}{8}\left[\frac{7}{2}x - \frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{8}\left[\frac{7}{2} - \frac{1}{2}\right] = \frac{3}{8}$$

$$P(x<1 / y<3) = \frac{P(x<1 \cap y<3)}{P(y<3)} = \frac{3/8}{5/8} = \frac{3}{5}$$

$$P(x<1 / y<3) = \frac{3}{5}$$

13. Let $x$ denote number of minutes from 7 am when the passenger arrives at stop

$$f(x) = \frac{1}{30} \quad 0 < x < 30$$

(i) P( less than 5 minutes) =
P( He arrives between 7.10 and 7.15 or 7.25 and 7.30)

$$= P[(10 < x < 15) \cup (25 < x < 30)]$$

$$= \int_{10}^{15} \frac{1}{30} \cdot dx + \int_{25}^{30} \frac{1}{30} \cdot dx$$

$$= \frac{1}{30}\left[[x]_{10}^{15} + [x]_{25}^{30}\right]$$

$$= \frac{1}{30}[5+5] = \frac{1}{3}$$

$$P(\text{less than 5 minutes}) = \frac{1}{3}$$

(ii) P(waits atleast 12 minutes) = P( He arrives between 7 and 7.03 or between 7.15 and 7.18)

$= P(0 < x < 3) + P(15 < x < 18)$

$= \int_0^3 \frac{1}{30} dx + \int_{15}^{18} \frac{1}{30} dx$

$= \frac{1}{30}\left[[x]_0^3 + [x]_{15}^{18}\right]$

$= \frac{1}{30}[3+3] = \frac{6}{30} = \frac{1}{5}$

P( atleast 12 minutes) = $\frac{1}{5}$

15. Correlation Coefficient

| x \ y | -1 | 1 | $P(x) = x_i$ |
|---|---|---|---|
| 0 | 1/8 | 3/8 | 4/8 |
| 1 | 2/8 | 2/8 | 4/8 |
| $P_j = P(y)$ | 3/8 | 5/8 | 1 |

$E(x) = \sum x_i P(x)_i = 0 \times 4/8 + 1 \times 4/8 = 1/2$

$E(x^2) = \sum x_i^2 P(x)_i = 0 \times 4/8 + 1 \times 4/8 = 1/2$

$E(y) = \sum y_i P(y)_i = -1 \times 3/8 + 1 \times 5/8 = 1/4$

$E(y^2) = \sum y_i^2 P(y)_i = 1 \times 3/8 + 1 \times 5/8 = 1$

$$E(xy) = \sum_i \sum_j x_i y_j \, P(x_i y_j)$$

$$= 0 \times -1 \times 1/8 + 0 \times 1 \times 3/8 + 1 \times -1 \times 2/8 + 1 \times 1 \times 2/8$$

$$= 0 + 0 - 2/8 + 2/8 = 0$$

$$\sigma_x^2 = E(x^2) - (E(x))^2 = 1/2 - (1/2)^2 = 1/4$$

$$\sigma_y^2 = E(y^2) - (E(y))^2 = 1 - (1/4)^2 = 1 - 1/10 = 15/16$$

$$Cov(xy) = E(xy) - E(x)E(y)$$

$$= 0 - 1/4 \times 1/2$$

$$= -1/8$$

Correlation coefficient

$$r_{xy} = \frac{Cov(x,y)}{\sigma_x \cdot \sigma_y} = \frac{-1/8}{\sqrt{1/4}\sqrt{15/16}} = -1/\sqrt{15}$$

$$r_{xy} = -1/\sqrt{15} = -0.25819$$

16. Since X & Y are independent

$$f(x,y) = e^{-x} \cdot e^{-y} = e^{-(x+y)} \text{ where } x,y \geq 0$$

We know: $U = \frac{X}{X+Y}$ $V = X+Y$

So: $U = \frac{X}{V} \Rightarrow x = UV$

$\cancel{V} = x + y \Rightarrow \cancel{y = U}$

$V = UV + y$

$y = V - UV$

so: $x = UV, \quad y = V - UV$

$$|J| = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

Here $f(x,y) = e^{-(x+y)}$

$g(u,v) = |J| \cdot f(x,y)$

$$= \begin{vmatrix} \frac{\partial(uv)}{\partial u} & \frac{\partial(uv)}{\partial v} \\ \frac{\partial(v-uv)}{\partial u} & \frac{\partial(v-uv)}{\partial v} \end{vmatrix} = \begin{vmatrix} v & u \\ -v & 1-u \end{vmatrix}$$

$= (1-u)v - u(-v)$

$= v$

$\Rightarrow e^{-(v)} \cdot v$

$g(u,v) = v \cdot e^{-v}$

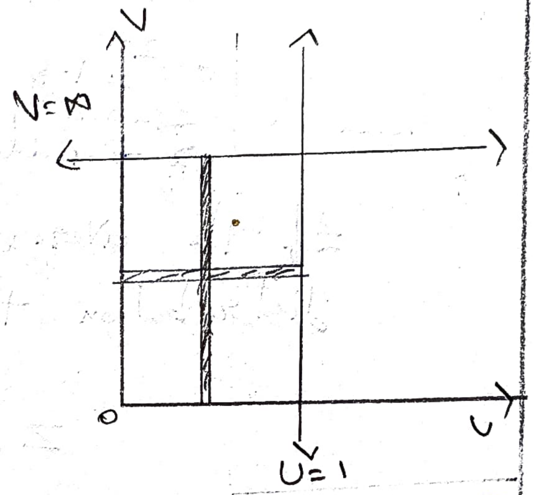
Limits:

$x \geq 0$ $y \geq 0$

$uv \geq 0$ $v - uv \geq 0$

$v \geq 0$ $v \geq uv$

$y \leq 1$




Density function of u ffu

$$f(u) = \int_{-\infty}^{\infty} f(u,v) \cdot dv$$

$$= \int_0^{\infty} v \cdot e^{-v} \cdot dv = \left[ v\left(\frac{e^{-v}}{-1}\right) - 1\left(\frac{e^{-v}}{(-1)^2}\right)\right]_0^{\infty}$$

$$= \left[-ve^{-v} - e^{-v}\right]_0^{\infty}$$

$$= [-0-0-0+1] = 1$$

$$f(u) = 1$$

$$f(v) = \int_{-\infty}^{\infty} f(u,v) \cdot du$$

$$= \int_0^1 v \cdot e^{-v} \cdot du = ve^{-v} \int_0^1 du$$

$$= ve^{-v} [u]_0^1$$

$$= ve^{-v}$$

To check independent $f(u)$ $f(u) f(v) = 1 \times ve^{-v}$

$$= ve^{-v}$$

$$= f(u,v)$$

u and v are independendent variable

17. Given Mean $= \mu$

$\sigma^2 = 1.5 \quad \sigma = \sqrt{1.5} = 1.224$

$\bar{x}$ will be Sample mean

If the average of random variable follows Normal distribution then $\bar{x}$ follow

$N(\mu, \sigma/\sqrt{n})$ Using CLT

$$Z = \frac{\bar{x} - \mu}{(\sigma/\sqrt{n})}$$

To find 'n', $P[\mu - 0.5 < \bar{x} < \mu + 0.5] \geq 0.95$

$P[-0.5 < \bar{x} - \mu < 0.5] \geq 0.95$

$P[|\bar{x} - \mu| < 0.5] \geq 0.95$

$$P\left[\frac{|\bar{x}-\mu|}{(\sigma/\sqrt{n})} < \frac{0.5}{\left(\frac{1.2248}{\sqrt{n}}\right)}\right] \geq 0.95$$

$P[|Z| < 0.482\sqrt{n}] \geq 0.95$

~~here~~ here Z is standard normal Variable

So Least value of 'n' is obtained when,

$P[|Z| < 0.4082\sqrt{n}] = 0.95$

$P[-0.4082\sqrt{n} < Z < 0.4882\sqrt{n}] = 0.95$

$2[P[Z < 0.4082\sqrt{n}] = 0.475$

From Table, we know

$P[Z < 1.96] = 0.475$

$0.4082\sqrt{n} = 1.96$

$\sqrt{n} = 1.96 / 0.4082 = 4.8016$

So approximately the size of sample should be atleast "24"

21.

| | | | | | |
|---|---|---|---|---|---|
| Treatment 1 | 6 | 4 | 5 | | |
| Treatment 2 | 13 | 10 | 13 | 12 | |
| Treatment 3 | 7 | 9 | 11 | | |
| Treatment 4 | 3 | 6 | 1 | 4 | 1 |

| Treatment $x_1$ 1 | $x_2$ 2 | $x_3$ 3 | $x_4$ 4 | Total | $x_1^2$ | $x_2^2$ | $x_3^2$ | $x_4^2$ |
|---|---|---|---|---|---|---|---|---|
| 6 | 13 | 7 | 3 | 29 | 36 | 169 | 49 | 9 |
| 4 | 10 | 9 | 6 | 29 ~~34~~ | 16 | 100 | 81 | 36 |
| 5 | 13 | 11 | 1 | 30 | 25 | 169 | 121 | 1 |
| – | 12 | – | 4 | 16 ~~18~~ | – | 144 | – | 16 |
| – | – | – | 1 | 1 | – | – | – | 1 |
| 15 | 48 | 27 | 15 | 105 | 77 | 582 | 251 | 63 |

$H_0$ : There is no significance between 4 treatment

$H_1$ : There is a significance between 4 treatment

* N = 15

Total = 105

$$\text{Factor} = \frac{T^2}{N} = \frac{11025}{15} = \cancel{918.75}\ 735$$

Now calculating Tss

$$(\text{Total sum of square}) = \sum x_1^2 + \sum x_2^2 + \sum x_3^2 + \sum x_4^2 - T^2/N$$

$$= 77 + 582 + 251 + 63 - 733$$

$$= 238$$

$$\text{Sum of square between column (SSC)} = \frac{(\sum x_1)^2}{N_1} + \frac{(\sum x_2)^2}{N_2} + \frac{(\sum x_3)^2}{N_3} + \frac{(\sum x_4)^2}{N_4} - T^2/N$$

$$= \frac{15^2}{3} + \frac{48^2}{4} + \frac{(27)^2}{3} + \frac{(15)^2}{5} - 735$$

$$= \frac{225}{3} + \frac{2304}{4} + \frac{729}{3} + \frac{225}{5} - 735$$

$$= 75 + 576 + 243 + 45 - 735$$

$$= 204$$

$$SSE = Tss - SSC$$

$$= 238 - 204 = 34$$

| Source of Variation | Sum of Square | d.f | Mean Square | Variance (f) | Table Calculated(f) |
|---|---|---|---|---|---|
| Between Treatment | SSC = 204 | c-1<br>4-1<br>= 3 | MSC = $\frac{SSC}{d.f}$<br>= 204/3<br>= 68 | $F_c = \frac{68}{3.09}$<br>= $\frac{MSC}{MSE}$<br>=<br>22.006 | $F_{cal}$ = (3, 11)<br>= 3.59 |
| Error | SSE = 34 | N-C<br>15-4<br>= 11 | MSE = $\frac{SSE}{d.f}$<br>= 34/11<br>= 3.09 | | |

From the above $F_c > F_{cal}$ (for column)

So here Ho will be rejected

11. Consider X is a discrete random Variable

given $P(X=0) = 1 - P(X=1)$

& $E(x) = 3 \, Var(X)$

So $P(X=0) + P(X=1) = 1$

We know that $\Sigma p(x) = 1$

Which Tells that X can Take either 0 or 1

Registration no: 2019503553 Subject Code: MA6351

Name: G. Rubakpreyan Title Probability & statistics

Let the probability of $P(x=1) = p$

| X | 0 | 1 |
|---|---|---|
| P(x) | 1-p | p |

So $E(x) = \sum_{i=1}^{n} x_i p_i = 0 \times (1-p) + 1 \times p$
Mean

$E(x) = p$

$Var(x) = E(x^2) - (E(x))^2$

$= \sum_{i=1}^{n} x_i^2 p_i - (p)^2$

$= 0^2(1-p) + 1^2(p) - p^2$

$= p - p^2$

Given $E(x) = 3\,Var(x)$

$p = 3(p - p^2)$

$\Rightarrow p = 3p(1-p)$

$p = 1 - 1/3 \Rightarrow p = 2/3$

Here $p = 2/3$

For $P(x=0) = 1 - p = 1 - 2/3 = 1/3$

So $P(x=0) = 1/3$

20.

| Days | Mon | Tue | Wed | Thu | Fri | Sat |
|---|---|---|---|---|---|---|
| No. of Accident | 14 | 18 | 12 | 11 | 15 | 14 |

Total = 84

$H_0$ : Aircraft Accidents are uniformly distributed over the week

$H_1$ : Accidents are not uniformly distributed

N = 6 , Level of significance = 0.05

D.f (Degree of freedom) = n-1 = 5

From Table $\chi^2$ = 11.070

Given that Total no. of Accidents = 84

Expected no. of Accidents = $\frac{84}{6}$ = 14

| O | E | O - E | $(O-E)^2/E$ |
|---|---|---|---|
| 14 | 14 | 0 | 0 |
| 18 | 14 | 4 | 16/14 |
| 12 | 14 | -2 | 4/14 |
| 11 | 14 | -3 | 9/14 |
| 15 | 14 | 1 | 1/14 |
| 14 | 14 | 0 | 0 |

$$\text{Total} = \frac{\Sigma(O-E)^2}{E}$$

$$= \frac{16}{14} + \frac{4}{14} + \frac{9}{14} + \frac{1}{14}$$

$$= \frac{30}{14} = 2.143$$

$$= 2.143 < 11.07$$

$X^2_{cal} = 2.143$ , $X^2_{Table} = 11.07$

From the value $X^2_{cal} < X^2_{Table}$

Thus $H_0$ Accepted so which tells that Accidents are uniformly distributied over the week.

## Part -A

1. outcome for a Fair die, Moment generating Function of X

let x be the outcome of a fair dice roll

Thus, we have $P(x=i) = 1/6$, $i = 1, 2, 3, 4, 5, 6$

Moment generating Function of x

$$M_x(t) = E[e^{tx}] = \sum_{i=1}^{6} p(x=x) e^{tx}$$

$$= 1/6 \sum_{x=1}^{6} e^{tx}$$

$$= 1/6 \, \frac{e^t(e^{6t}-1)}{e^t-1} \qquad t \in R$$

2. Probability = ~~0.02~~ 2.1.

Total (n) = 200

$P = 2\cdot 1 \% = \frac{4}{200} = \frac{1}{50}$

$\mu = np$ (mean for poission distribution)

$= 200 \times \frac{1}{50} = 4$

$P(x=4) = \frac{e^{-4} \cdot 4^4}{4!}$

$= \underline{0.1953}$

3. Supplied → once a day

$f(x) = 5(1-x)^4, \quad 0 \le x \le 1$

supply is exhausted in day ⇒ Sales ≥ capacity

Area under the curve $f(x)$ ⇒ total probability

$P(x > \text{capacity})$

$f(x) = 5$ at $x = 0$

$f(x) = 0$ at $x = 1$

As $x > 0$ & $x < 1$ limits will be from $c < x < 1$

Let $1 - x = t$, then $-dx = dt$

$\Rightarrow \int_c^1 f(x)\,dx = 0.01$

$= \int_c^1 5(1-x)^4\,dx$

$= 5 \int_{1-c}^{0} -t^4\,dt$

$$= 5 \times \left[ \frac{1}{5} \times t^5 \right]$$

$$\Rightarrow (1-c)^5 = 0.01$$

$$1 - c = 0.398$$

$$c = 0.602 \times 1000$$

$$= 602 \text{ litres}$$

4. Joint probability Mass Function

$$p(x, y) = k(2x+3y) \quad x = 0, 1, 2, \ y = 1, 2, 3$$

| x \ y | 1 | 2 | 3 | |
|---|---|---|---|---|
| 0 | 3k | 6k | 9k | 18k |
| 1 | 5k | 8k | 11k | 24k |
| 2 | 7k | 10k | 13k | 30k |
| | 15k | 24k | 33k | 72k |

Since $\sum_j \sum_i = P_{ij} = 1 \quad k = 1/72$

$$k = 1/72$$

5. Given

Equation : $4x + 5y + 30 = 0$ , $20x + 9y = 107$

$$4x + 5y = -30 \rightarrow (1)$$

$$20x + 9y = 107 \rightarrow (2)$$

① × 5

$$20x + 25y = -150$$
$$20x + 9y = 107$$

$$16y = -257$$

$$y = \frac{-257}{16}$$

$$y = -16.06$$

Substitute y in ①

$$4x + 5(-16.06) = -30$$

$$4x = -30 + 80.3$$

$$x = \frac{50.3}{4}$$

$$x = 12.575$$

7. $H_0 : \mu = 3.25$ $H_1 : \mu \neq 3.25$

$\alpha = 5\%$.

$\mu = 3.25\text{cm}$ $\sigma = 2.61$

So $H_0 = Z = \dfrac{3.40 - 3.25}{2.61/\sqrt{900}} = 1.73$

Since $|Z| < 1.96$ we can say that the data does not provide any evidence against null hypothesis $H_0$ which may therefore be accepted at 5% level of significance

10. Average fraction defective $= \bar{p} = 0.4$

For p-chart:

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.4 + 3\sqrt{\frac{0.4(0.6)}{100}}$$

$$= 0.4 + 3\sqrt{0.024}$$

$$= 0.4 + 3 \times 0.154$$

$$= 0.184$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(p-\bar{p})}{n}} = 0.4 - 0.464$$

$$= -0.062$$

Since LCL is -ve value of LCL will be 0

$$CL = \bar{p} = 0.4$$

So: $CL = 0.4$, $LCL = 0$, $UCL = 0.184$

8. $n = 400$, $X$ = No. of defects in sample = 30

$$p = \frac{X}{n} = \frac{30}{400} = 0.075$$

$H_0: P = 0.05$

$H_1: p > 0.05$ ( Right tail)

$\alpha = 5\%$

$$Z_0 = \left| \frac{p - P}{\sqrt{\frac{PQ}{n}}} \right|$$

$$= \left| \frac{0.075 - 0.050}{\sqrt{\frac{0.05 \times 0.95}{400}}} \right|$$

$$= \frac{0.025}{\sqrt{0.0001187}} = 2.27$$

$Z_c$ for $\sim N(0,1) = 1.645$ $(\alpha = 0.05)$

since $Z_0 > Z_c$ we reject $H_0$. So the company's claim is not tenable

6. Exponential dist

$f(x) = \lambda e^{-\lambda x}$ $\lambda = 1 \Rightarrow f(x) = e^{-x}$

$y = \sqrt{x}$ ; Transformed to $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \quad \frac{dx}{dy} = 2\sqrt{x} = 2y$$

P.D.F of $y = f_y(y) = \left| \frac{dx}{dy} \right| \cdot f_x(x) = 2y e^{-x}$

$f_y(y) = 2y e^{-y^2}$ as $(y = \sqrt{x})$

$f_y(y) = \underline{2y . e^{-y^2}}$ $y \geq 0$

9) 6 - Treatments , 3 - blocks

$k=6, \quad r=3$

$$\text{Error} = (k-1)(r-1)$$
$$= (6-1)(3-1)$$
$$= 5 \times 2 = 10$$

Df of Error Sum of Squares = 10

10. $n = 100$ , $\bar{p} = 0.4$

P chart

$CL = \bar{p} = 0.4$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{100}} = 0.4 + 3\sqrt{\frac{0.4 \times 0.6}{100}}$$
$$= 0.5469$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{100}} = 0.4 - 3\sqrt{\frac{0.4 \times 0.6}{100}}$$
$$= 0.253$$

$\Rightarrow$ $CL = 0.4$ , $UCL = 0.546$ , $LCL = 0.253$

All the answers in this Answer book have been in my own handwriting . Nobody has ~~been~~ helped me in writing the answers

- G. Rubak preyan
30.6.2021