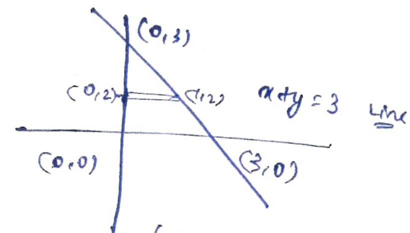
14.) X and Y - 2 Random variables

$$f(x,y) = \begin{cases} 1/8\,(6-x-y), & 0<x<2,\ 2<y<4 \\ 0, & \text{Otherwise} \end{cases}$$

(i) $P(X+Y<3)$



$\left(\begin{array}{l} x \to 2 \text{ to } 3 \\ y \to 0 \text{ to } 3-y \end{array} \right)$

$\therefore P(X+Y<3) = \int_0^1 \int_2^{3-x} f(x,y)\, dx\, dy$

$= \int_0^1 \int_2^{3-x} 1/8\,(6-x-y)\, dx\,dy$

$= 1/8 \int_0^1 \left(6y - xy - y^2/2\right)\Big|_2^{3-x} dx$

$= \frac{1}{8} \int_0^1 \left[\underbrace{6(3-x)}_{6x} - \underbrace{x(3-x)}_{3x} - \frac{(3-x)^2}{2} - 12 + 2x - 2\right] dx$

$= \frac{1}{8} \int_0^1 18 - 6x - 3x + x^2 - \frac{9}{2} + 3x - \frac{x^2}{2} - 10 + 2x$

$= \frac{1}{8} \int_0^1 \left(\frac{x^2}{2} - 4x + 7/2\right) dx$

$= \frac{1}{8} \left(x^3/6 + \left(-\frac{4x^2}{2}\right) + 7/2\,x\right)\Big|_0^1$

$= 1/8\,(1/6 - 2 + 7/2) = 10/48$

$\therefore \boxed{P(X+Y<3) = 5/24}$

$18 - 10 - \frac{9}{2}$ $7/2$
$3\ -9x + 5x$
$4x$
$x^2 - x^2/2$
$\frac{3.5 \cdot 2}{48}$
$10/48$

(ii) $P(X<1 \mid Y<3)$

$$\boxed{P(X<1 \mid Y<3) = \frac{P(X<1 \cap Y<3)}{P(Y<3)}} \quad \text{--- (1)}$$

∴ Find $P(X<1 \cap Y<3)$

$$P(X<1 \cap Y<3) = \int_0^1 \int_2^3 f(x,y)\,dy\,dx$$

$$= 1/8 \int_0^1 \int_2^3 (6-x-y)\,dy\,dx$$

$$= 1/8 \int_0^1 \int_2^3 (6-x-y)\,dy\,dx = 1/8 \int_0^1 (6-x)y - y^2/2 \Big|_2^3 dx$$

$$= 1/8 \int_0^1 \left(\left((6-x)3 - 9/2\right) - \left[2(6-x) - 4/2\right]\right) dx$$

$$= 1/8 \int_0^1 \left(18 - 3x - 9/2\right) - \left(2 - 2x - 2\right) dx$$

$$= 1/8 \int_0^1 \left(\frac{7}{2} - x\right) dx = \frac{1}{8}\left(\frac{7x}{2} - \frac{x^2}{2}\right)\Big|_0^1$$

$$= \frac{1}{8}(7/2 - 1/2) = \boxed{3/8}$$

$$\boxed{P(X<1 \cap Y<3) = 3/8} \quad \text{--- (2)}$$

$$P(Y<3) = \int_0^2 \int_2^3 f(x,y)\,dy\,dx = \int_0^2 \int_2^3 \frac{1}{8}(6-x-y)\,dy\,dx$$

$$= 1/8 \int_0^2 (6-x)y - y^2/2 \Big|_2^3 dx$$

$$= \frac{1}{8} \int_0^2 \left[3(6-x) - 9/2 - 2(6-x) + 4/2\right] dx$$

$= \frac{1}{8}\int_0^2 ((6-x) - 5/2)\,dx \Rightarrow 1/8 \int_0^2 (7/2 - x)\,dx$

$= 1/8 \left(7/2\, x - x^2/2\right|_0^2 \Rightarrow 1/8 \left(7/2(2) - (4/2)\right)$

$= \boxed{5/8}$

$\therefore \boxed{P(Y<3) = 5/8}$ — (3)

By (1), (2) & (3)

$\Rightarrow P(X<1 \mid Y<3) = \frac{3/8}{5/8} = 3/5$

$\therefore \boxed{P(X<1 \mid Y<3) = 3/5}$

(1.) X is Discrete R.V

$P(X=0) = 1 - P(X=1)$ and $E(X) = 3\,Var(X)$.

$P(X=0) = 1 - P(X=1) \Rightarrow X$ takes 0 or 1

Probability Dist of X is : (If $P(X=1) = p$)

X	$P(x)$
0	$1-p$
1	p

$\boxed{E(x) = \sum_{i=1}^{n} x_i p_i}$ $\therefore$ $E(x) = 0(1-p) + 1(p)$

$\boxed{E(x) = p}$ — ①

$Var(x) = E(x^2) - (E(x))^2$

$= 0^2(1-p) + 1^2(p) - p^2$

$\Rightarrow$ $Var(x) = p - p^2$ — ②

Given that $E(x) = 3\,Var(x)$

$\Rightarrow$ $p = 3(p - p^2)$ $\Rightarrow$ $p = 2/3$; (neglect $p=0$)

$= 3p(1-p)$ $\Rightarrow$

$\therefore$ $p = 2/3$ (p is $P(x=1)$)

$\therefore$ $P(x=0) = 1 - P(x=1)$

$= 1 - 2/3$ $\Rightarrow$ $\boxed{P(x=0) = 1/3}$

(3.) 15 interval b/w 7 & 7 30.

X - No of minutes past 7am when passenger arrives,

$f(x) = \frac{1}{30}$; $0 < x < 30$ — ①

(i) $P(\text{wait less than 5 min}) = P(\text{arrive b/w 7:10 and 7:15})$ OR
$+$
$P(\text{arrive b/w 7:25 and 7:30})$

$= P(10 < X < 15) \cup P(25 < X < 30))$

$= P(10 < X < 15) + P(25 < X < 30)$

$= \int_{10}^{15} \frac{1}{30}\, dx + \int_{25}^{30} \frac{1}{30}\, dx = \frac{1}{30}\left(x\Big|_{10}^{15}\right) + \frac{1}{30}\left(x\Big|_{25}^{30}\right)$

$= \frac{1}{30}(15-10) + \frac{1}{30}(30-25) = \frac{10}{30} = \boxed{1/3}$

$\therefore \boxed{P(\text{wait less than 5 min}) = 1/3}$

(ii) $P(\text{wait atleast 12 min}) = P(\text{arrive b/w 7 and 7:03})$ OR
$P(\text{arrive b/w 7:15 and 7:18})$

$= P(0 < x < 3) + P(15 < x < 18)$

$= \int_{0}^{3} 1/30\, dx + \int_{15}^{18} 1/30\, dx = \frac{1}{30} x\Big|_0^3 + 1/30\, x\Big|_{15}^{18}$

$= 1/30\,(3+3) = 6/30 \boxed{1/5}$

$\therefore \boxed{P(\text{wait atleast 12 min}) = 1/5}$

17.) Given Data:

$$\begin{cases} \text{Mean} = \mu \; ; \quad \text{var } \sigma^2 = 1.5 \\ \quad \Rightarrow \sigma = \sqrt{1.5} = 1.2248 \end{cases}$$

$\bar{x} \rightarrow$ Sample Mean.

According to Liapounoff's statement of CLT,
If avg of Random variables follow Normal Distribution,
then $\bar{x}$ follows $N(\mu, \sigma/\sqrt{n})$ by CLT.

$$\therefore \boxed{Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}} \quad \text{—} ①$$

To find n, ~~P(|x̄~~ $P(\mu - 0.5 < \bar{x} < \mu + 0.5) \geq 0.95$

$$P(-0.5 < \bar{x} - \mu < 0.5) \geq 0.95$$

$$P(|\bar{x}-\mu| < 0.5) \geq 0.95$$

$$P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{0.5}{1.2248/\sqrt{n}}\right) \geq 0.95 \quad (\text{Given } \sigma = \sqrt{1.5} = 1.2248)$$

$$\therefore P(|z| < 0.4082\sqrt{n}) \geq 0.95$$

Z is Standard Normal variate.

Least value of n is obtained if

$$P(|z| < 0.4082\sqrt{n}) = 0.95$$

(i.e) $P(-0.4082\sqrt{n} < z < 0.4082\sqrt{n}) = 0.95$

$$2(P(z < 0.4082\sqrt{n})) = 0.95$$

$$P(z < 0.4082\sqrt{n}) = 0.475$$

From std normal Dist Table, $0.475 = P(z < 1.96)$

$$\therefore \quad 0.4082\sqrt{n} = 1.96$$

$$\Rightarrow \boxed{\sqrt{n} = 4.8016}$$

Squaring on both sides $\Rightarrow$ $n = 23.055$

$\therefore$ $\boxed{\text{Size of sample must be ATLEAST 24.}}$

16.) X and Y Independent R.V with

pdf $e^{-x}, x \geq 0$; $e^{-y}, y \geq 0$

Find Density fn of $U = X/X+Y$, $V = X+Y$,

Soln:

Since X & Y are independent,

$\left\{ \begin{array}{l} U = x/v, \quad y = v - x \\ x = uv, \quad y = v - uv \end{array} \right\}$

$$\boxed{f(x,y) = f(x).f(y) = e^{-x}.e^{-y} = e^{-(x+y)}, \quad x,y \geq 0}$$

Density function of U

$$f(u) = \int_{-\infty}^{\infty} f(u,v)\,dv$$

and

Density function of V

$$f(v) = \int_{-\infty}^{\infty} f(u,v)\,du \quad \text{--- (1)}$$

$$f(u,v) = |J| \left(f(x,y)\right) \quad \text{--- (2)}$$

$$\boxed{\begin{array}{ll} u = x/x+y, & v = x+y \\ x = uv, & y = v - uv \end{array}}$$

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} v & u \\ -v & 1-u \end{vmatrix}$$

$\left(\text{Since } \frac{\partial x}{\partial u} = v, \ \frac{\partial y}{\partial u} = -v\right)$

$$\therefore |J| = v - \cancel{uv} + \cancel{uv} = v \quad \Rightarrow \underline{|J| = v} \quad \text{--- (3)}$$

Sub (3) in (2) $\Rightarrow$

$$f(u,v) = v.e^{-v} \quad \text{--- (4)}$$

$\because \left\{ \begin{array}{l} f(x,y) = e^{-(x+y)} \\ \quad = e^{-v} \end{array} \right\}$

$$f(u) = \int_{0}^{\infty} v.e^{-v}\,dv$$

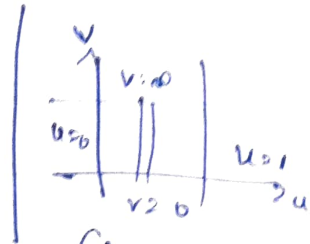
Limits calculation, $x \geq 0$; $uv \geq 0$ $(\because x = uv)$

$$\underline{\underline{v \geq 0}}$$

$y \geq 0$, $-uv + v \geq 0$ $(\because y = v - uv)$

$\therefore$ $v \geq uv \Rightarrow 1 \geq u \Rightarrow \underline{\underline{u \leq 1}}$

$\therefore$ ($v \geq 0$ & $u \leq 1$ are limits)



(Graph for limits)

$\therefore$ $f(u) = \int_0^\infty v e^{-v} \, dv$

$$= \left[v \cdot \frac{e^{-v}}{-1} - 1 \left(\frac{e^{-v}}{(-1)^2} \right) \right] \Big|_0^\infty$$

$$= -v \cdot e^{-v} - e^{-v} \Big|_0^\infty = 0 - 0 - (0 - 1) = \boxed{1}$$

$f(v) = \int_0^1 v e^{-v} \, du = v \cdot e^{-v} \int_0^1 du$

$$= v e^{-v} (u|_0^1 = v e^{-u} (1 - 0) = \underline{\underline{v e^{-v}}}, \; v > 0$$

$\therefore$ $f(u) \cdot f(v) = 1 \cdot v e^{-v}$

$$= v e^{-v}$$

$$= f(u, v) \quad \text{by } (4)$$

$\therefore$ $f(u) \cdot f(v) = f(u, v)$

u & v are independent R.V

(5.)

x\y	-1	1	P(x)
0	1/8	3/8	4/8
1	2/8	2/8	4/8
P(y)	3/8	5/8	1

$E(x) = \sum x_i\, p(x_i) = 0(4/8) + 1(4/8) = 4/8 = 1/2$

$E(x^2) = \sum x_i^2\, p(x_i) = 0^2(4/8) + 1^2(4/8) = 4/8 = 1/2$

$E(y) = \sum y_i\, p(y_i) = (-1)(3/8) + 1(5/8) = 2/8 = 1/4$

$E(y^2) = \sum y_i^2\, p(y_i) = (-1)^2(3/8) + 1^2(5/8) = 8/8 = 1$

$E(xy) = \sum_i \sum_j x_i y_j\, p(x_i, y_j)$

$0(-1)(1/8) + 0(1)(3/8) + 1(-1)(2/8) + (1)(1)(2/8)$

$= -2/8 + 2/8 = \underline{\underline{0}}$

$\sigma_x^2 = E(x^2) - (E(x))^2 = \frac{1}{2} - \left(\frac{1}{2}\right)^2 = 1/4$

$\sigma_y^2 = E(y^2) - (E(y))^2 = 1 - (1/4)^2 = 15/16$

$\boxed{\sigma_x = 1/2 \quad \& \quad \sigma_y = \sqrt{15/16}}$

$$r_{xy} = \frac{Cov(x,y)}{\sigma_x \sigma_y} = \frac{E(xy) - E(x).E(y)}{\sigma_x . \sigma_y}$$

$$= \frac{0 - 1/2 \cdot 1/4}{\frac{1}{2} \cdot \frac{\sqrt{15}}{4}} = \frac{-1/8}{\sqrt{15}/8} = -\frac{1}{\cancel{8}} \times \frac{\cancel{8}}{\sqrt{15}}$$

$$\boxed{r_{xy} = \frac{-1}{\sqrt{15}}} \quad \text{(or)} \quad \boxed{r_{xy} = -0.2582}$$

20.)

Days	Mon	Tues	Wed	Thrus	Fri	Sat
No of accidents	14	18	12	11	15	14

$\boxed{\text{Total} = 84}$

H_0 : Accidents are uniformly distributed over the week

H_1 : Accidents are not uniformly distributed.

$N = 6$

Consider Level of significance $\alpha = 0.05$

Degree of freedom $= n-1 = 6-1 = \boxed{5}$

From table, we see, χ^2 is 11.070 → ①

$\chi^2 = \sum \frac{(O-E)^2}{E}$ is Test statistic ; O is observed freq
E is expected freq

Given that total no of accidents = 84

H_0 is they are uniformly distributed, hence, Expected no of accidents on a day $\}$ = $\frac{84}{6}$ = $\underline{\underline{14}}$

O	E	O − E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
14	14	0	0	0
18	14	4	16	16/14
12	14	−2	4	4/14
11	14	−3	9	9/14
15	14	1	1	1/14
14	14	0	0	0
			Total :	30/14

$$\boxed{\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{30}{14} = 2.143}$$

Table value, by ① is 11.07 ; (2.143 < 11.07)

∴ Calculated χ^2 < table value of χ^2 ; Thus Accept H_0

⇒ Accidents are uniformly Distributed over the week.

21.)

Ho : There is no significant Difference b/w treatments

H1 : There is significant Diff b/w treatments

$\boxed{\text{Treatment } i \text{ is taken as } X_i}$

X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
6	13	7	3	29	36	169	49	9
4	10	9	6	29	16	100	81	36
5	13	11	1	30	25	169	121	1
	12		4	16		144		16
			1	1				1
15	48	27	15	105	77	582	251	63

$N = 15$

$T = 105$; Correction Factor $\frac{T^2}{N} = \frac{11025}{15}$

$$\boxed{T^2/N = 735}$$

TSS :

(Total sum of squares) $= \sum x_1^2 + \sum x_2^2 + \sum x_3^2 + \sum x_4^2 - T^2/N$

$= 77 + 582 + 251 + 63 - 735$ $\therefore$ $\boxed{TSS = 238}$

SST (Sum of squares b/w Treatments)

$$= \frac{(\Sigma x_1)^2}{N_2} + \frac{(\Sigma x_2)^2}{N_2} + \frac{(\Sigma x_3)^2}{N_2} + \frac{(\Sigma x_4)^2}{N_2} - \frac{T^2}{N}$$

(N_2 - No of elements in respective columns)

$$= \frac{(15)^2}{3} + \frac{48^2}{4} + \frac{(27)^2}{3} + \frac{(15)^2}{5} - 735$$

$$= \frac{225}{3} + \frac{2304}{4} + \frac{729}{3} + \frac{225}{5} - 735$$

$$= 75 + 576 + 243 + 45 - 735 = \boxed{204}$$

$$SSE = TSS - SSC = 238 - 204 = \boxed{34}$$

Source of variation	Sum of squares	D.F	Mean squares	variance ratio	table value of 5%
b/w treatments	SST = 204	C-1 = 3	MST = $\frac{SST}{D.F}$ 204/3 = $\boxed{68}$	$F_c = \frac{MST}{MSE}$ = 68 / 3.09 = 22.006	$F_c(3,11)$ = 3.59
Error	SSE = 34	N-C 15-4 = 11	SSE / d.f = 34/11 = 3.09		

$\boxed{F_{calculated} > F_c \text{ from table}}$ 22.006 > 3.59 ∴ Ho is Rejected

$\boxed{\text{There is significant diff b/w treatments}}$

PART-C

23.)

Detergent ↓ / Engine →	1	2	3
A	45	43	51
B	47	46	52
C	48	50	55
D	42	37	49

Two way ANOVA

Subtract each data by 50 for easy calculation.

Detergent	Engine x_1	x_2	x_3	Total	x_1^2	x_2^2	x_3^2
A (y_1)	-5	-7	1	-11	25	49	1
B (y_2)	-3	-4	2	-5	9	16	4
C (y_3)	-2	0	5	3	4	0	25
D (y_4)	-8	-13	-1	-22	64	169	1
Total :	-18	-24	7	-35	102	234	31

H_0 : There is no significant diff b/w column means and Row means

H_1 : There is significant diff b/w column or Row Means

N : 12 - Total No of items

$T = -35$, $\Rightarrow \frac{T^2}{N} = \frac{(-35)^2}{12} = \boxed{102.08}$

Total sum of squares $= \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 - T^2/N$

$= 102 + 234 + 31 - 102.08$

$\boxed{TSS = 264.92}$

Sum of squares b/w columns $= \frac{\Sigma x_1^2}{N_1} + \frac{\Sigma x_2^2}{N_1} + \frac{\Sigma x_3^2}{N_1} + T^2/N_1$

($N_1 \rightarrow$ No of elements in each column)

$= \frac{(-18)^2}{4} + \frac{(-24)^2}{4} + \frac{(7)^2}{4} - 102.08 = 135.17$

$\boxed{SSC = 135.17}$

SSR (Sum of square Rows) $= \frac{\Sigma y_1^2}{N_2} + \frac{\Sigma y_2^2}{N_2} + \dots + \frac{\Sigma y_4^2}{N_2} - T^2/N$.

(N_2 : No of elements in each Row)

$= \frac{(-11)^2}{3} + \frac{(-5)^2}{3} + \frac{(3)^2}{3} + \frac{(-22)^3}{3} - 102.08$

$\therefore \boxed{SSR = 110.91}$

$SSE = TSS - SSC - SSR$

$= 264.92 - 135.17 - 110.91$

$\boxed{SSE = 18.84}$

ANOVA TABLE:

Sources of variation	Sum of squares	D.o.f	Mean squares	variance	Table F.
B/w column	$SSC = 135.17$	$C-1 = \underline{2}$	$MSC = \frac{SSC}{C-1}$ $MSC = \frac{135.17}{2}$ $MSC = 67.585$	$F_c = \frac{MSC}{MSE}$ $= \frac{67.58}{3.14}$ $= 21.52$	$F_{C_{(2,6)}}$ 10.92
B/w Rows	SSR 110.91	$r-1 = 4-1 = \underline{3}$	$MSR = \frac{SSR}{r-1}$ $= 110.91/3$ $= 36.97$	$F_r = \frac{MSR}{MSE}$ $= \frac{36.97}{3.14}$ $= 11.17$	$F_{c\,(3,6)}$ 9.78
Residual	SSE 18.84	$N-C-r+1$ $= 12-3-4+1$ $= \underline{6}$	$MSE = \frac{SSE}{6}$ $= 3.14$		
Total	264.92	11			

Conclusion,

Calculated F_{cal} (for column) = 10.92

$F_c > F_{cal}$ (column)

$\therefore$ Reject H_0

Calculated F_{cal} (for rows) = 9.78

$F_r > F_{cal}$ (row)

$\therefore$ Reject H_0

$\boxed{\text{Thus, conclusion is there is significant difference b/w detergents and engines}}$

Sub code : MA 6351 Reg no : 2019503519

Sub Name : Probability and Statistics Name : Hemanthini.

24.) Defects : 6, 4, 9, 10, 11, 12, 20, 10, 9, 10, 15, 20, 20, 15, 10

Total = 171

Let c denote the no of defects

$\bar{c} = \frac{\Sigma c}{n} = \frac{171}{15} = 11.4$

Let $\bar{c}$ be y → which is the control limit

UCL (Upper control limit) $= \bar{c} + 3\sqrt{\bar{c}}$

$= 11.4 + 3\sqrt{11.4} = 11.4 + 10.12$

$= 21.52$

LCL (Lower control limit) $= \bar{c} - 3\sqrt{c}$

$= 11.4 - 3\sqrt{11.4} = 11.4 - 10.12$

$= 1.28$

$\boxed{\text{Now, UCL} = 21.52,\ \text{CL} = 11.4,\ \text{LCL} = 1.28}$

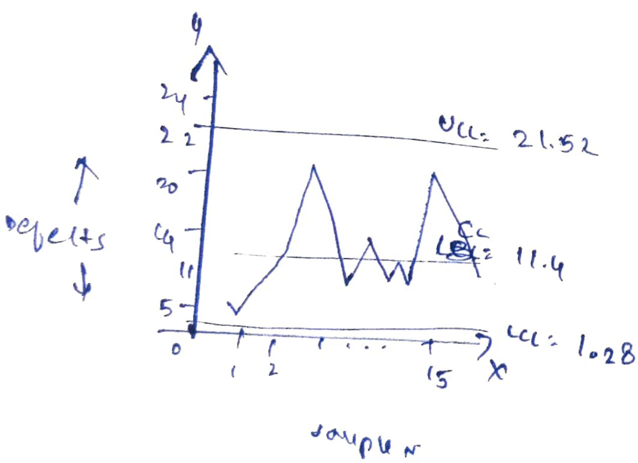
[Graph paper attached next page]

we observe the value of c from the graph.

None of values exceed the upper control limit nor the lower control limit.

∴ we can conclude that, the process is under control since all sample points lie within UCL & LCL

Sample Graph:



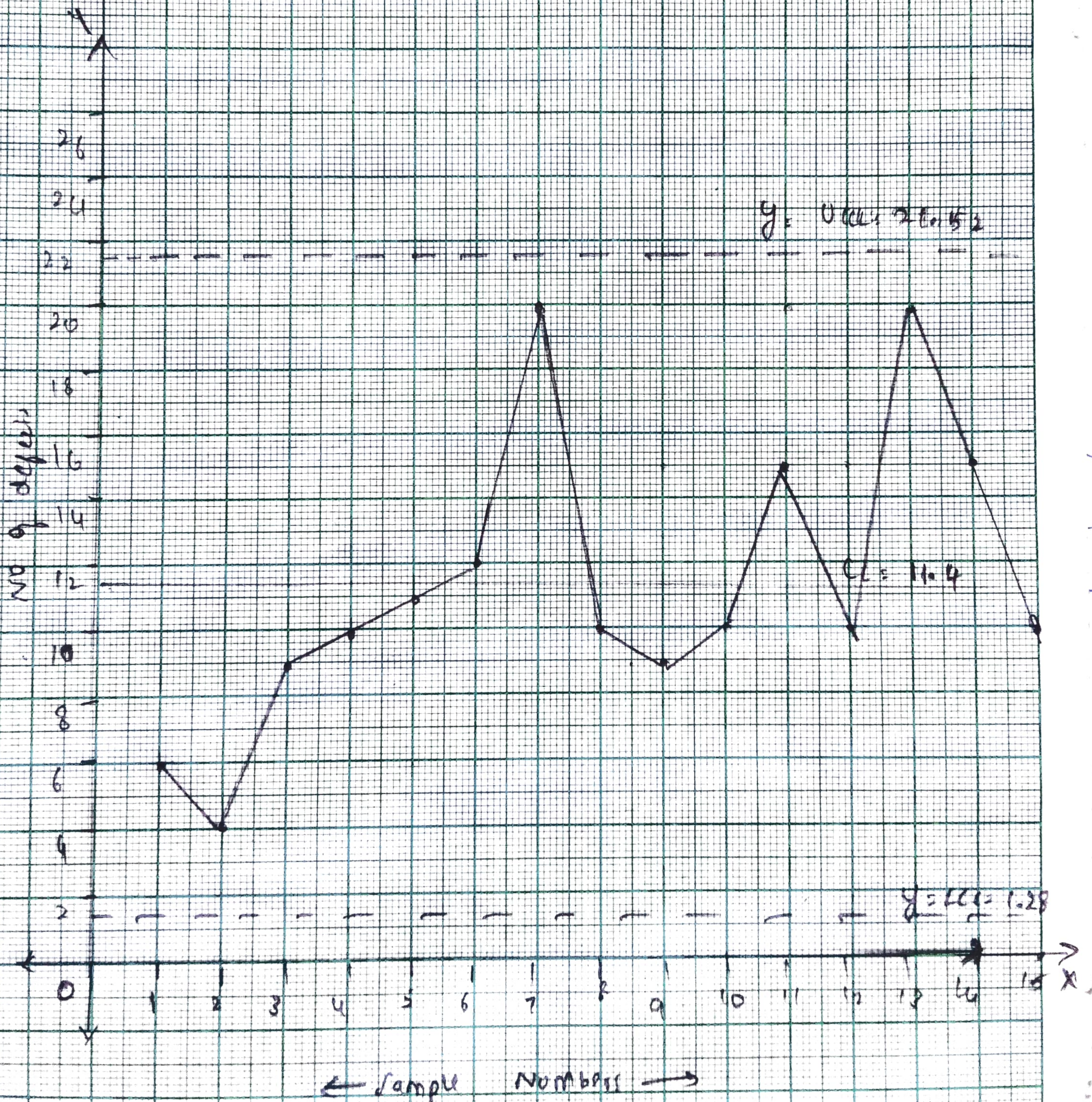
← Sample Graph

[Original graph attached in Prev Page]

Subcode : MA6251
Probability & statistics

Reg no : 2019503519
Name : Hemanth.N.

24.) UCL = 21.52 , CL = 11.4 , LCL = 1.28



Sub Code: MA6[illegible]

Sub Name: Probability and Statistics

Reg No: [illegible]

Name: Hemanth. N

PART A

1.) X outcome when fair die is tossed,

Probability Distribution of X is $P(X=i) = 1/6$, $i = 1,2,3,4,5,6$

$$M(t) = \sum e^{tx} \cdot P_x = \frac{1}{6}(e^t + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t})$$

$$\text{MGF is } = \frac{1}{6}\frac{e^t(e^{6t}-1)}{e^t-1}, \quad t \in R$$

4.) Joint prob dist of (x,y) is given below

x | y	1	2	3
0	3k	6k	9k
1	5k	8k	10k
2	7k	10k	13k
3			

$$\sum_{j=1}^{3}\sum_{i=0}^{2} P(x_i, y_j) = 1$$

Sum of all prob in the table add up to 1

$\therefore$ $72k = 1$ $\Rightarrow$ $\boxed{k = 1/72}$ $\therefore$ $P(x,y) = \frac{1}{72}(2x+3y)$

Date: 30/06/2021

Pg No: 21

Signature: [illegible]

2.) To find $P(X=4)$

Poisson Distribution λ(parameter) = np = $200(0.02)$

$= 4$

$$P_X(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-4}4^4}{4!} = 0.1953$$

$$\boxed{\text{Probability that exactly 4 pins are defective} \Big\} = \cancel{0.1953}\ 0.1953}$$

3.) Given $f(x) = 5(1-x)^4$, $0 \le x \le 1$

Area under the curve $f(x)$ gives Probability $P(X > \text{Capacity}) = 0.01$

Find capacity c , $c < x < 1$; $(1-x = t)$

$$\int_c^1 f(x)\,dx = 0.01 = \int_c^1 5(1-x)^4\,dx = 0.01$$

$(t = 1-x \Rightarrow dt = -dx)$

$$\int_{1-c}^0 t^4\,dt = 0.01 = 5\left(\frac{1}{5}\cdot t^5\right]_{1-c}^0 = 0.01$$

$(1-c)^5 = 0.01 \Rightarrow 1-c = 0.398 \Rightarrow c = 0.602 \times 1000\,l$

$\therefore$ $\boxed{\text{Supply Must be Capacity } 602 \text{ Litres}}$

5.) Lines of regression $\Rightarrow$ $4x + 5y + 30 = 0$ —(1)

$20x + 9y - 107 = 0$ —(2)

(1) $\times 5$

$20x + 25y = -150$

$20x + 9y = 107$

$(-)\quad(-)\quad(-)$

$16y = -257 \Rightarrow$ $\boxed{y = \frac{-257}{16} \; ; \; y = -16.06}$

Substitute $y = 16.06$ in (1)

$\Rightarrow 4x + 5(-16.06) = -30$

$4x = -30 + 80.3$

$x = \frac{+50.3}{4} = 12.576$

$\boxed{x = 12.576}$

Mean values of x & y are 12.576 and -16.06 respectively

6.) Exponential Dist

$f(x) = \lambda e^{-\lambda x}$

$\therefore$ ~~λ~~ $\lambda = 1$ $\Rightarrow$ $f(x) = e^{-x}$, $x > 0$

$Y = \sqrt{x}$, Transformation func is $y = \sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$ $\Rightarrow$ $\frac{dx}{dy} = 2\sqrt{x} = 2y$

PDF of y is $f_Y(y) = \left|\frac{dy}{dx}\right| \cdot f_X(x) = 2ye^{-x}$

$f_Y(y) = 2y \cdot e^{-y^2}$ (as $y = \sqrt{x}$)

$\therefore$ $\boxed{f_Y(y) = 2ye^{-y^2} \quad , y > 0}$

7.) H_0: Sample Drawn from Population with $\mu = 3.25$ cm

H_1: $\mu \neq 3.25$

$\boxed{\text{Two Tailed Test}}$ $\mu = 3.25$, $\sigma = 2.61$ cm

H_0: $z = \frac{3.4 - 3.25}{2.61/\sqrt{900}} = 1.73$ $\boxed{z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$

$|z| < 1.96$. H_0 is accepted.

conclude that the sample is drawn from population of $\mu = 3.25$ cm

9.) 6 treatments , 3 Blocks

$\boxed{k=6, \quad R=3}$

Find Deg of Freedom of error sum of squares

Error = $(k-1)(r-1) = (6-1)(3-1)$

$= 5 \times 2 = \boxed{10}$

$\boxed{\text{D.f of Error sum of squares} = 10}$

10.) $n=100$, no of Defectives noted

$\bar{p} = 0.4$

Find CL, UCL, LCL

P chart,

$CL = \bar{p} = 0.4$

$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.4 + 3\sqrt{\frac{(0.4)(0.6)}{100}} = 0.5469$

$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.4 - 3\sqrt{\frac{(0.4)(0.6)}{100}} = 0.25303$

$\therefore \boxed{CL = 0.4, \quad UCL = 0.5469, \quad LCL = 0.253}$

8.) $n = 400$, $X \to$ No of Defectives $= 30$

p = Proportion of Def. in sample $\Rightarrow \frac{x}{n} = \frac{30}{400} = 0.075$

H_0: Only 5% of Products is Defective

(i.e) $P = 0.05$

H_1: $P > 0.05$ (Right Tailed Test)

H_0:

$$Z_0 = \frac{p - P}{\sqrt{PQ}/\sqrt{n}} = \frac{0.075 - 0.05}{\sqrt{\frac{(0.05)(0.95)}{400}}} = \frac{0.025}{\sqrt{0.0001187}}$$

$$Z_0 = 2.27$$

Expected value $\left|\frac{p-P}{\sqrt{PQ}/\sqrt{n}}\right| \sim N(0,1) \Rightarrow 1.645$ (Right Tailed) from table

Calculated $Z_0 > Z_e$, thus Reject H_0 at 5% level of signi.

Company's claim is wrong.

All answers in this Booklet have been in my own handwriting. Nobody helped me in writing this answers

[signature]
30/06/21