L.A. Assignment 5

i) WKT $y = A + Bx + Cx^2$

at (1,1)

$1 = A + B + C \rightarrow ①$

at (2,-1)

$-1 = A + 2B + 4C \rightarrow ②$

at (3,1)

$1 = A + 3B + 9C \rightarrow ③$

~~unknowns = 3~~.

$Ax = b$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$\approx \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 2 & 8 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

by backward subtitution

$2C = 4 \Rightarrow \boxed{C = 2}$

$B + 3C = -2 \Rightarrow \boxed{B = -2 - 6 = -8}$

$A + B + C = 1$

$A - 8 + 2 = 1$

$\boxed{A = 7}$

$y = 7 + (-8)x + 2x^2$

(ii) LU decomposition of a matrix

$$A = LU$$

$$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix}$$

Ans

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - (-5) R_1$
$R_4 \to R_3 - 5 R_1$

$$\simeq \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & -4 & 5 & 13 \\ 0 & -4 & 11 & 19 \end{bmatrix}$$

$R_3 \to R_3 - (-2) R_2$
$R_4 \to R_4 - (-2) R_2$

$$\simeq \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 9 & 11 \end{bmatrix}$$

$$R_4 \to R_4 - 3R_3 \simeq \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix} = U \qquad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ -5 & -2 & 3 & 1 \end{bmatrix}$$

3.) $T(x,y,z) =$ ~~$(x+2y-z)$~~ $(x+2y-z,\ y+z,\ x+y-2z)$

(i) find T wrt standard basis of R_3

basis for $R^3 = \{(1,0,0), (0,1,0), (0,0,1)\}$

$T(1,0,0) = (1,0,1)$
$T(0,1,0) = (2,1,1)$
$T(0,0,1) = (-1,1,-2)$

therefore columnwise transforma gives us

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$R_3 \to R_3 - R_1 \simeq \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 3 & 1 \end{bmatrix}$$

}

tranforma

$$\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \Rightarrow$$

$$R_3 \to R_3 + R_2 \quad \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

column 1,2 in T produce pivots. hence they are linear indipendent

$$C(A) = \{(1,0,1), (2,1,1)\}$$

$$\boxed{\dim(C(A)) = 2}$$

row space

$$C(A^T) = \{(1,2,-1), (0,1,1)\}$$

$$\boxed{\dim(C(A^T)) = 2}$$

finding $N(A)$ & $N(A^T)$

Convert T to row-reduced form

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow_{R_1 \to R_1 - 2R_2} \begin{array}{ccc} x & y & z \\ \hline 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

z is free variable

~~x-3z~~ $x - 3z = 0$ $\quad x = 3z$

$y + z = 0$ $\quad y = -z$

$0x + 0y + 0z = 0$ $\quad$ let $z = 1$

$$N(A) = z \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

$$N(A) = \{(3,-1,1)\}$$

$$\dim(N(A)) = 1 //$$

finding $N(A^T)$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 1 & 1 & 2 & b_3 \end{array}\right] \quad R_3 \to R_3 - R_1 \quad \left[\begin{array}{ccc|c} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 0 & -1 & -1 & b_3 - b_1 \end{array}\right]$$

$$R_3 \to R_3 + R_2 \quad \left[\begin{array}{ccc|c} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 0 & 0 & 0 & b_3 - b_1 + b_2 \end{array}\right]$$

for consisteny

$$(-b_1 + b_2 + b_3 = 0)$$

therefore $N(A^T) = \{(-1,1,1)\}$

$$\boxed{\dim(N(A^T)) = 1 //}$$

(iii) eigen value & eigen vectors

$$\begin{bmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{bmatrix} : (A-\lambda I)$$

$$|A-\lambda I| = 0$$

$$(1-\lambda)([1-\lambda][-2-\lambda]-1) + 1(2+(1-\lambda)) = 0$$

$$(1-\lambda)^2(2+\lambda) = 2$$

$$(1-\lambda)(1-\lambda)(2+\lambda)-2=0$$

$$\lambda^3 - 0(\lambda^2) + (-3-1+1)\lambda = 0$$

$$\lambda^3 = 3\lambda \qquad \boxed{\lambda = \sqrt{3}, -\sqrt{3}, 0}$$

eigen vector for $\lambda = \sqrt{3}$

$$\begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 1 & 1 & -2-\sqrt{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_3 \rightarrow \frac{R_3}{1} - \frac{1}{1-\sqrt{3}} R_1$$

$$\approx \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 1-\frac{2}{1-\sqrt{3}} & -(2+\sqrt{3})+\frac{1}{1-\sqrt{3}} \end{bmatrix} = \begin{bmatrix} -0.732 & 2 & -1 \\ 0 & -0.732 & 1 \\ 0 & 3.732 & -5.098 \end{bmatrix}$$

R

$$\approx \begin{bmatrix} -0.732 & 2 & -1 \\ 0 & -0.732 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Ux = 0$$

$$-0.732x + 2y - Z = 0$$

$$-0.732y + Z = 0$$

$$y = \frac{-Z}{-0.732} = 1.366Z //$$

$$-0.732x = -1.7322Z$$

$$\boxed{x = 2.3664Z} //$$

eigen vector : $z\begin{pmatrix} 2.3664 \\ 1.366 \\ 1 \end{pmatrix}$

for $\lambda = \sqrt{3}$

$$\begin{pmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 1 & 1 & -2+\sqrt{3} \end{pmatrix} = \begin{pmatrix} 2.732 & 2 & -1 \\ 0 & 2.732 & 1 \\ 1 & 1 & -0.2679 \end{pmatrix}$$

$$R_3 \rightarrow R_3 - \frac{1}{2.732} R_1 = \begin{pmatrix} 2.732 & 2 & -1 \\ 0 & 2.732 & 1 \\ 0 & 0.2679 & 0.0981 \end{pmatrix}$$

$$= \begin{pmatrix} 2.732 & 2 & -1 \\ 0 & 2.732 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Let $z = 1$//

$2.732y + z = 0$

$2.732y = -z$

$y = -0.3660$

$2.732x + 2y = z$

$x = z - 2y / 2.732$

$x = 0.63396$//

$x_2 = \begin{pmatrix} 0.63396 \\ -0.3660 \\ 1 \end{pmatrix}$

if $\lambda = 0$

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_3 = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$ from N(A) solution obtained previously

(iv) factorize $T = QR$

$$T = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix}$$

$$q_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \qquad q_2 = \frac{l_2}{\|l_2\|}$$

$$l_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \cdot - \left(\frac{2}{\sqrt{2}} + 0 + \frac{1}{\sqrt{2}} \right) \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \frac{1}{\sqrt{2}}$$

$$= \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 3/2 \\ 0 \\ 3/2 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0 \\ -0.5 \end{pmatrix}$$

$$q_2 = \frac{1}{\sqrt{1.5}} \begin{pmatrix} 0.5 \\ 0 \\ -0.5 \end{pmatrix}$$

$$q_3 = \frac{l_3}{\|l_3\|} \qquad l_3 = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix} - \frac{1}{1.5} \begin{pmatrix} 0.5 \\ 1 \\ -0.5 \end{pmatrix} \left(-\frac{1}{2} + 1 + 1 \right) + \frac{3}{2} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$q_3 = \begin{pmatrix} -2/7 \\ 3/7 \\ -6/7 \end{pmatrix}$$

$$Q = \begin{bmatrix} 1/\sqrt{2} & 0.5/\sqrt{1.5} & -2/7 \\ 0 & 1/\sqrt{1.5} & 3/7 \\ 1/\sqrt{2} & -0.5/\sqrt{1.5} & -6/7 \end{bmatrix}$$

$A = QR$

$Q^T A = R$

$$R = \begin{bmatrix} 1.4142 & 2.1213 & -2.121 \\ 0 & 1.2247 & 1.2247 \\ 0 & 0 & 0 \end{bmatrix}$$

4) find a best fit line.

$y = mx + c$

let $y = \begin{bmatrix} \\ \\ \end{bmatrix}_{n \times 1}$ $\qquad A = \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ \vdots & x_n \end{bmatrix}_{n \times 2}$

$$\begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} c \\ m \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

A x B

WKT

$$\hat{x} = (A^T A)^{-1} \cdot A^T \cdot B$$

$$\hat{x} = \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}^T \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \cdot A^T \cdot B$$

$$A^T A = \begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix}$$

$$A^T B = \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}^T \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$= \underset{2\times4}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix}} \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix} = \begin{bmatrix} 28 \\ 28+6 \end{bmatrix}_{2\times1} = \begin{bmatrix} 28 \\ 34 \end{bmatrix}$$

$$(A^T A)^{-1} = \begin{bmatrix} 15/58 & -1/58 \\ -1/58 & 1/29 \end{bmatrix}$$

$$\hat{x} = \begin{pmatrix} 193/29 \\ 20/29 \end{pmatrix}$$

$m = 20/29$
$x = 193/29$

5.) consider the eq of plane

let $V = [\underset{\substack{\downarrow \\ \text{pivot}}}{1} \quad 1 \quad 3 \quad 0 \quad 4]$

$x_1 = -x_2 + (-3)x_3 - 4x_5$ //

let

~~x~~ $x \quad x_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = k_1 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + k_3 \begin{bmatrix} -4 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$$\therefore A = \begin{bmatrix} -1 & -3 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{5\times 3}$$

WKT $P = A(A^TA)^{-1}.A^T$

using calculati

$$P = \begin{bmatrix} 26/27 & -1/27 & -1/9 & 0 & -4/27 \\ -1/27 & 26/27 & -1/9 & 0 & -4/27 \\ -1/9 & -1/9 & 2/3 & 0 & -4/9 \\ 0 & 0 & 0 & 0 & 0 \\ -4/27 & -4/27 & -4/9 & 0 & 11/27 \end{bmatrix}$$

WKT $P + Q = I$

so $Q = I - P = \begin{bmatrix} 1/27 & 1/27 & 1/9 & 0 & 4/27 \\ 1/27 & 1/27 & 1/9 & 0 & 4/27 \\ 1/9 & 1/9 & 1/3 & 0 & 4/9 \\ 0 & 0 & 0 & 1 & 0 \\ 4/27 & 4/27 & 4/9 & 0 & 16/27 \end{bmatrix}$

6} for what values of a is the matrix +ve definite

Ans WKT determine of principle sub-matrices ≥ 0

$$A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix}$$

$|a| > 0$ so $a > 0$

$\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix} > 0$ so $a^2 - 4 > 0$

$a^2 > 4$

$a > \pm 2$

$(a-2)(a+2) > 0$

$(a-2) > 0$

$a > 2$ $a+2 > 0$

$a > -2$

so $a \neq 0, -1, 1, -2, 2$

$a \notin [-2, 2]$

$|A| > 0$

$a(a^2-4) - 2(2a-4) + 2(4-2a) > 0$

$a(a-2)(a+2) - 4(a-2) + 4(2-a) > 0$

$a(a-2)(a+2) + 4(2-a) + 4(2-a) > 0$

$a(a-2)(a+2) + 8(2-a) > 0$

$(2-a)$

∴

$f = x^TAx$

$f = 2x$

a

$\therefore a_{31} =$

$A =$

7} SVD of A =

find A^TA

λ_1, λ_2

A

$|A - \lambda I|$

$(2-a)\,(a\,(a+2)\,(-1)+8) > 0$

$a^3 - 12a + 16 > 0$

$2-a > 0$ 4 $\Rightarrow$ $\underline{\underline{2 > a}}$

$a > 2$

$a > -4$

$a > 1$

$$\boxed{\therefore a \in (2, \infty)}$$

$f = x^T A x$

$$f = 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 + (-2)(x_2x_3)$$

$$\begin{array}{ccccccc} \downarrow & \downarrow & \downarrow & \overset{-1}{\swarrow} & \overset{-1}{\downarrow} & \overset{-1}{\swarrow} & \overset{-1}{\searrow} \\ a_{11} & a_{22} & a_{33} & a_{12} & a_{21} & a_{23} & a_{321} \end{array}$$

$\therefore a_{31} = a_{13} = 0$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

7.} SVD of $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$

find A^TA

$$A^TA_{(2\times3\times3\times2)} = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$$

λ_1, λ_2 of A^TA ? $\begin{bmatrix} 81-\lambda & -27 \\ -27 & 9-\lambda \end{bmatrix} = 0$

$A - \lambda I$

$|A - \lambda I| = ?$

$(81-\lambda)(9-\lambda) - 27^2 = 0$

$729 - 81\lambda - 9\lambda + \lambda^2 - 27^2 = 0$

$\lambda_1 = 90$ $\sqrt{\lambda_1} = 3\sqrt{10}$

$\lambda_2 = 0$

$$\Sigma = \begin{bmatrix} 3\sqrt{10} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

λ_1 vector x_1

$\underline{x_1} =$

$$\begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$+9x + 27y = 0$$

$$27x + 81y = 0$$

$$y \begin{bmatrix} 27/9 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix} // \qquad y \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

if $\lambda = 0$

$$x_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} //$$

for $v = \lambda_1 = 90$

$\lambda_2 = 0$

$$v_1 = \frac{x_1}{\|x_1\|} = \frac{1}{\sqrt{10}} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$v_2 = \frac{x_2}{\|x_2\|} = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$$

$$V = \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix} = V^T$$

finding U

eigen values are 90,0,0

$$U_i = \frac{A \cdot V_i}{\sigma_i}$$

$$U_1 = \frac{A \cdot V_1}{\sigma_1} = \frac{1}{3\sqrt{10}} A \cdot V = \begin{bmatrix} -0.266 \\ 0.533 \\ 0.533 \end{bmatrix}$$

$U_2 = ??$ $\sigma_2 = 0$ here

we do $(AA^T - 0 \cdot I)x = 0$

$$\overset{80}{AA^T} \cdot x = 0$$

$$\begin{bmatrix} 10 & -20 & -20 \\ -20 & 40 & 40 \\ -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

finding null space

$R_2 \to R_2 + 2R_1$ $\begin{bmatrix} 10 & -20 & -20 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$10x - 20y - 20z = 0$

$x - 2y - 2z = 0$

$x = 2y + 2z$

$= y\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

for $\lambda = 0$ we get 2 vectors eigen

choosing $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

$u = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

* $U = \begin{bmatrix} -0.266 & 0.8944 & a \\ 0.533 & 0.44721 & b \\ 0.533 & 0 & c \end{bmatrix}$

u is orthogonal so $u_3 \perp u_2$ & $u_3 \perp u_1$

$-0.266a + 0.533b + 0.533c = 0$

$0.8944a + 0.44721b + 0c = 0$

Let $c = 1$

$-0.266a + 0.533b + 0.533C = 0$ — ①

$0.8944a + 0.44721 = 0$ — (ii)

eq ① × 3.3624

$-0.8944a + 1.792162b = 1.79216$

$+0.8944a + 0.44721b = 0$

$2.239b = 1.79216$

$\boxed{b = 0.8}$

$a = 0.4$

$c = 1$

$V_3 = \frac{1}{\sqrt{1.8}}\begin{bmatrix} 0.4 \\ 0.8 \\ 1 \end{bmatrix}$

$$A = \underset{u}{\begin{bmatrix} \cancel{1/\sqrt{5}}\ 1/3 & 2/\sqrt{5} & 0.4/\sqrt{1.8} \\ -2/3 & 1/\sqrt{5} & 0.8/\sqrt{1.8} \\ -2/3 & 0 & 1/\sqrt{1.8} \end{bmatrix}} \underset{\Sigma}{\begin{bmatrix} \sqrt{90} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}} \underset{V^T}{\begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}}$$