Code: 20A54304 **R20**

B.Tech II Year I Semester (R20) Supplementary Examinations August/September 2023

DISCRETE MATHEMATICS & GRAPH THEORY

(Common to CSE(CS),IT,CSE,CSE(AI),CSE(AI&ML),AI&DS,CSE(IoT),CSE(DS),(AI&ML) and CS&D)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Define existential quantifier with example. 2M

(b) Symbolize the statement “everyone in the final year class has a cellular phone”. 2M

(c) Give an example of a relation which is neither symmetric nor antisymmetric on any set A. 2M

(d) Determine how many integers below 100 are not divisible by 5 and 7. 2M

(e) Find the coefficient of x^{10} in the expansion of $\frac{1}{(1-2x)}$. 2M

(f) How many 3-digit numbers can be formed using the 6 digits 2, 3, 4, 5, 6 and 8, if the number is to be odd and repetitions are not allowed? 2M

(g) Find the generating function of $a_r = 5.2^r$. 2M

(h) Write the recursive definition of the sequence: $1, 3, 5, 7, 9, 11, \ldots$ 2M

(i) Define Bipartite graph and give an example. 2M

(j) Find chromatic number(s) of cycle graph C_n. 2M

PART – B

(Answer all the questions: 05 X 10 = 50 Marks)

2 Check the validity of the following argument. 10M

If I go to school, then I attend all the classes. If I attend all classes, then I get A grade. I do not get grade A and I do not feel happy. Therefore, if I do not go to school then, I do not feel happy.

OR

3 (a) Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a Tautology. 5M

(b) What do you mean by equivalent statements? Show that $P \to (Q \to R) \equiv (P \wedge Q) \to R$. 5M

4 In a class of 50 students, there are 2 choices for optional subjects. If it is found that 18 students have physics as an optional subject but not chemistry and 25 students have chemistry as optional subject not physics, then How many students 10M

(i) have both optional subjects?

(ii) have chemistry as an optional subject?

(iii) have physics as an optional subject?

OR

5 (a) Let R be a relation defined on a set of positive integers such that for all $x, y \in Z^+$, xRy if and only if $x+y$ is an even number. Prove that R is an equivalence relation. 8M

(b) Define transitive closure of a relation. Find the transitive closure of the relation $R=\{(1,2),(2,3),(3,3)\}$ on the set $A=\{1,2,3\}$. 2M

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6	(a)	Use a combinatorial argument to prove $C(n+r+1,r)=\sum_{j=0}^{r}C(n+j,j)$.	5M
	(b)	In how many ways can 7 persons sit around a round table? In how many ways can 7 gentlemen and 7 ladies sit around a round table, no 2 ladies being sit together?	5M
		OR	
7	(a)	How many strings of length 3 or less can be generated using the letters $\{a,b,c\}$ if repetition is allowed?	5M
	(b)	There are some cans of Coke, Pepsi and Sprite in a refrigerator. In how many ways can 5 cans be selected if repetition is allowed?	5M
8	(a)	Solve the recurrence relation $a_n=6a_{n-1}-12a_{n-2}+8a_{n-3}+n^2$ using the characteristic equation.	6M
	(b)	Find the solution of recurrence relation $a_n=-3a_{n-1}-3a_{n-2}-a_{n-3}$ with initial conditions $a_0=1, a_1=-2, a_2=-1$.	4M
		OR	
9	(a)	Use generating function to solve the recurrence relation $a_k=3a_{k-1}$ for $k=1,2,3,..$ and the initial condition is $a_0=2$.	5M
	(b)	Find the solution of the recurrence relation $a_n=6a_{n-1}-9a_{n-2}+3^n$.	5M
10	(a)	Prove that a connected multi graph possesses an Eulerian trial if and only if there are exactly two vertices of odd degree.	5M
	(b)	If G is a bipartite graph, then prove that that there is no cycle of odd length.	5M
		OR	
11	(a)	Prove that a connected graph is a tree if there is a unique path in between every pair of vertices.	5M
	(b)	If G is a connected planner graph with e edges and v vertices, where v is greater than 3 then show that $e \le 3v-6$.	5M

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Code: 20A54304 **R20**

B.Tech II Year I Semester (R20) Supplementary Examinations April/May 2024

DISCRETE MATHEMATICS & GRAPH THEORY

(Common to CS&IT, IT, AI&DS, CSE (AI), CSE (DS), CSE (AI&ML), CSE (IOT), CSE (CS), CSE, CS&D and AI&ML)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Find the negation of $p \to q$. 2M

(b) Show that the following implication: $(p \wedge q) \Rightarrow (p \to q)$. 2M

(c) Define the Lattices and write two properties. 2M

(d) Define the subgroup and Homomorphism. 2M

(e) How many “words” of three distinct letters can be formed from the letters of the word MAST? 2M

(f) In how many ways can a six-card hand be dealt from a deck of 52 cards? 2M

(g) Suppose that f is defined recursively by $f(0) = 3,\ f(n+1) = 2f(n) + 3$. Find $f(1),\ f(2),\ f(3)$, and $f(4)$. 2M

(h) Solve the recurrence relation $a_n = a_{n-1} + n^2$, where $a_0 = 7$. 2M

(i) Can a simple graph exist with 15 vertices each of degree five? 2M

(j) Give an example of a graph that has an Eulerian circuit and a Hamiltonian circuit, which are distinct. 2M

PART – B

(Answer all the questions: 05 X 10 = 50 Marks)

2 (a) Construct the truth table of the compound proposition $(p \vee \neg q) \to (p \wedge q)$. 5M

(b) How can this English sentence be translated into a logical expression? 5M
“You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.”

OR

3 (a) Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent. 5M

(b) Show that the premises “It is not sunny this afternoon and it is colder than yesterday,” “We will go swimming only if it is sunny,” “If we do not go swimming, then we will take a canoe trip,” and “If we take a canoe trip, then we will be home by sunset” lead to the conclusion “We will be home by sunset.” 5M

4 (a) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ for all sets A, B and C. 5M

(b) Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of f and g? What is the composition of g and f? 5M

OR

5 (a) There are 345 students at a college who have taken a course in calculus, 212 who have taken a course in discrete mathematics, and 188 who have taken courses in both calculus and discrete mathematics. How many students have taken a course in either calculus or discrete mathematics? 5M

(b) Let $(S_1, *_1), (S_2, *_2)$, and $(S_3, *_3)$ be semi groups and $f : S_1 \to S_2$ and $g : S_2 \to S_3$ be homeomorphisms. Prove that $g \circ f$ is a homomorphism from S_1 to S_3. 5M

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6 (a) How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest? 5M

(b) Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have the same number of men and women? 5M

OR

7 (a) What is the coefficient of x^3y^7 in $(x+y)^{10}$ in $(2x-9y)^{10}$? 5M

(b) Use the multinomial theorem to expand $(x_1+x_2+x_3+x_4)^4$. 5M

8 (a) Solve the recurrence relation: 5M

$a_n - 4a_{n-1} + 4a_{n-2} = 0$ *for* $n \geq 2$ *and* $a_0 = 5/2$, $a_1 = 8$ by using the characteristic roots.

(b) Solve $a_n - 9a_{n-1} + 20a_{n-2} = 0$ for $n \geq 2$ and $a_0 = -3$, $a_1 = -10$ by generating functions. 5M

OR

9 (a) Solve the following recurrence relation using generating function; 5M

$a_n - 4a_{n-2} = 0$ for $n \geq 2$, $a_0 = 0$, $a_1 = 1$.

(b) Solve the recurrence relation $a_n - 6a_{n-1} + 8a_{n-2} = 3^n$ where $a_0 = 3$ and $a_1 = 7$. 5M

10 (a) Prove that a connected graph with n vertices and $(n-1)$ edges is a tree. 5M

(b) Define Euler's graph and Hamiltonian graph with suitable examples. 5M

OR

11 (a) Find the in-degree and out-degree of each vertex in the graph *G* with directed edges shown in Figure 1. 5M

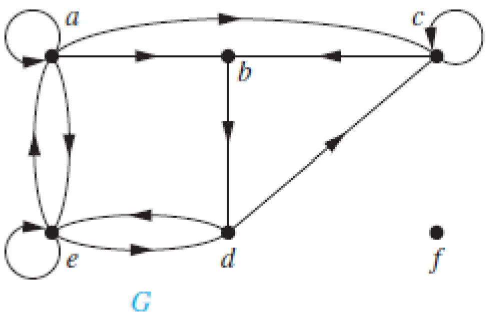

Figure 1. The Directed Graph *G*.

(b) Determine whether the given pair of graphs in Figure 2, is isomorphic. 5M

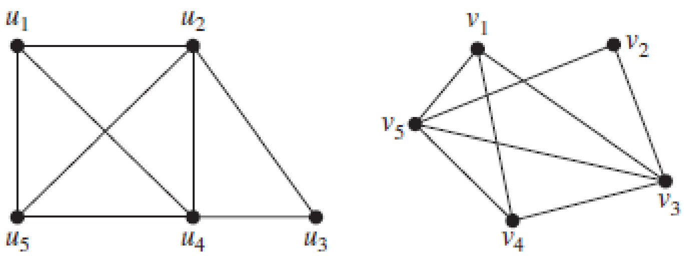

Figure 2. The Pair of Graphs.

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