UNIT 2

SHEAR FORCE & BENDING MOMENT DIAGRAMS

Course Objectives:

- To plot the variation of shear force and bending moments over the beams under different types of loads.

Course Outcomes:

- Draw the shear force and bending moment diagrams for the beam subjected to different loading conditions.

UNITII

SHEAR FORCE AND BENDING MOMENT DIAGRAMS

Shear force

The algebraic sum of the vertical forces at any section of a beam to the right or left of the section is known as shear force

Bending moment

The algebraic sum of the moments of all the forces acting to the right or left of the section is known as beading moment

Shear force and bending moment diagrams

A shear force diagram is one which shows the variation of the shear force along the length of the, beam. And a bending moment diagram is one which shows the variation of the bending moment along the length of the beam.

Important points for Shear force and bending moment

1.Shear Force (V) ≡ equal in magnitude but opposite in direction to the algebraic sum (resultant) of the components in the direction perpendicular to the axis of the beam of all external loads and support reactions acting on either side of the section being considered.

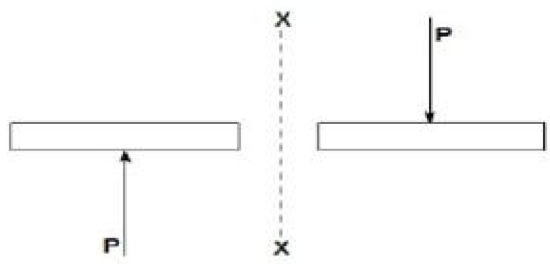
2. Bending Moment (M) equal in magnitude but opposite in direction to the algebraic sum of the moments about (the centroid of the cross section of the beam) the section of all external loads and support reactions acting on either side of the section being considered.

Notation and sign convention

1. **Shear force (V)**

   Positive Shear Force

A shearing force having a downward direction to the right hand side of a section or upwards to the left hand of the section will be taken as 'positive'. It is the usual sign conventions to be followed for the shear force. In some book followed totally opposite sign convention.

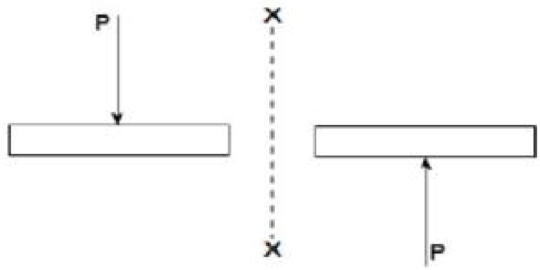


The **upward direction** shearing force which is on the left hand of the section XX is positive shear force

The **downward direction** shearing force which is on the right hand of the section XX is positive shear force.

Negative Shear Force

A shearing force having an upward direction to the right hand side of a section or downwards to the left hand of the section will be taken as 'negative'.



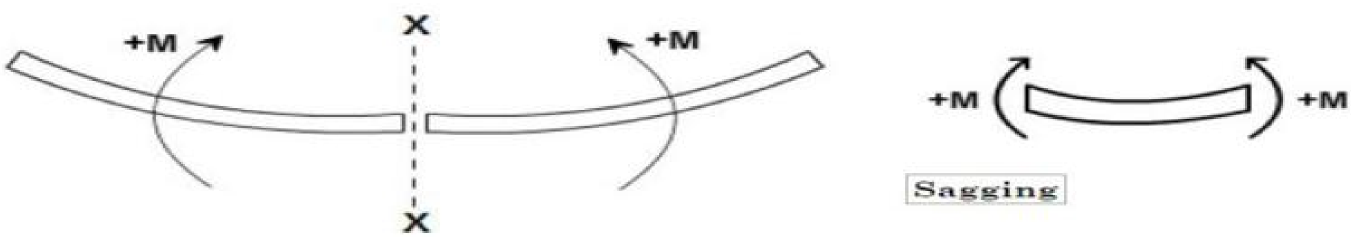
The downward direction shearing force which is on the left hand of the section XX is negative shear force.

The upward direction shearing force which is on the right hand of the section XX is negative shear force.

Bending Moment (M)

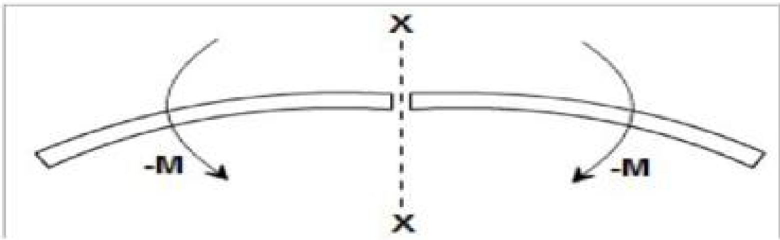
Positive Bending Moment

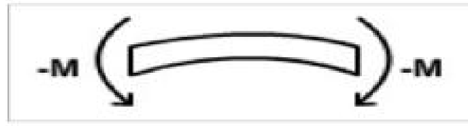
A bending moment causing concavity upwards will be taken as 'positive' and called as sagging bending moment.



- If the bending moment of the left hand of the section XX is clockwise then it is a positive bending moment.
- If the bending moment of the right hand of the section XX is anti-clockwise then it is a positive bending moment.
- A bending moment causing concavity upwards will be taken as 'positive' and called as sagging bending moment

Negative Bending Moment





Hogging

- If the bending moment of the left hand of the section XX is anti-clockwise then it is a negative bending moment.
- If the bending moment of the right hand of the section XX is clockwise then it is a negative bending moment.
- **Hogging**
  A bending moment causing convexity upwards will be taken as 'negative' and called as hogging bending moment.

Relation between S.F (Vx), B.M. (Mx) & Load (w)

$$\frac{dV_x}{dx} = -w \text{ (load)}$$

The value of the distributed load at any point in the beam is equal to the slope of the shear force curve. (Note that the sign of this rule may change depending on the sign convention used for the external distributed load).

$$\frac{dM_x}{dx} = V_x$$

The value of the shear force at any point in the beam is equal to the slope of the bending moment curve.

Procedure for drawing shear force and bending moment diagram

Construction of shear force diagram

- From the loading diagram of the beam constructed shear force diagram.
- First determine the reactions.
- Then the vertical components of forces and reactions are successively summed from the left end of the beam to preserve the mathematical sign conventions adopted. The shear at a section is simply equal to the sum of all the vertical forces to the left of the section.
- The shear force curve is continuous unless there is a point force on the beam. The curve then "jumps" by the magnitude of the point force (+ for upward force).
- When the successive summation process is used, the shear force diagram should end up with the previously calculated shear (reaction at right end of the beam). No shear force acts through the beam just beyond the last vertical force or reaction. If the shear

force diagram closes in this fashion, then it gives an important check on mathematical calculations. i.e. The shear force will be zero at each end of the beam unless a point force is applied at the end.

Construction of bending moment diagram

- The bending moment diagram is obtained by proceeding continuously along the length of beam from the left hand end and summing up the areas of shear force diagrams using proper sign convention.
- The process of obtaining the moment diagram from the shear force diagram by summation is exactly the same as that for drawing shear force diagram from load diagram.
- The bending moment curve is continuous unless there is a point moment on the beam. The curve then "jumps" by the magnitude of the point moment (+ for CW moment).
- We know that a constant shear force produces a uniform change in the bending moment, resulting in straight line in the moment diagram. If no shear force exists along a certain portion of a beam, then it indicates that there is no change in moment takes place. We also know that dM/dx= Vx therefore, from the fundamental theorem of calculus the maximum or minimum moment occurs where the shear is zero.
- The bending moment will be zero at each free or pinned end of the beam. If the end is built in, the moment computed by the summation must be equal to the one calculated initially for the reaction.

A Cantilever beam with a concentrated load 'P' at its free end

Shear force:

At a section a distance x from free end consider the forces to the left, then

$(V_x) = - P$ (for all values of x) negative in sign

i.e. the shear force to the left of the x-section are in downward direction and therefore negative.

Bending Moment:

Bending Moment

Taking moments about the section gives (obviously to the left of the section)

$$Mx = -P.x$$

(negative sign means that the moment on the left hand side of the portion is in the anticlockwise direction and is therefore taken as negative according to the sign convention)

so that the maximum bending moment occurs at the fixed end i.e.

$$Mmax = - PL(at\ x = L)$$

A Cantilever beam with uniformly distributed load over the whole length

When a cantilever beam is subjected to a uniformly distributed load whose intensity is given w /unit length.

Shear force:

Consider any cross-section XX which is at a distance of x from the free end. If we just take the resultant of all the forces on the left of the X-section, then

$$Vx = -w.x \quad \text{for all values of 'x'.}$$

$$\text{At } x = 0,\quad Vx = 0$$

$$\text{At } x = L,\quad Vx = -wL \text{ (i.e. Maximum at fixed end)}$$

Plotting the equation Vx = -w.x, we get a straight line because it is a equation of a straight line y

$$(Vx) = m(- w) .x$$

Bending Moment:

Bending Moment at XX is obtained by treating the load to the left of XX as a concentrated load of the same value (w.x) acting through the centre of gravity at x/2.

Therefore, the bending moment at any cross-section XX is

$$M_x = (-w.x).\frac{x}{2} = -\frac{w.x^2}{2}$$

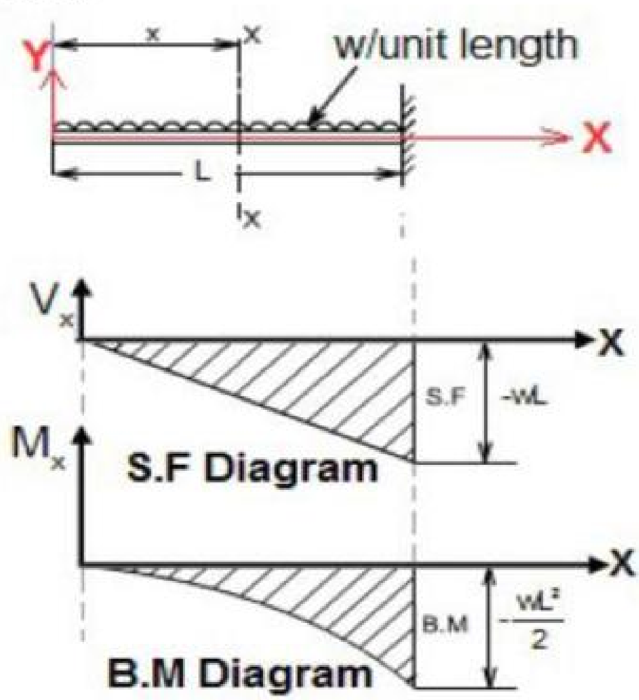
Therefore *the variation of bending moment is according toparabolic law.*

The extreme values of B.M would be

at $x = 0$, $M_x = 0$

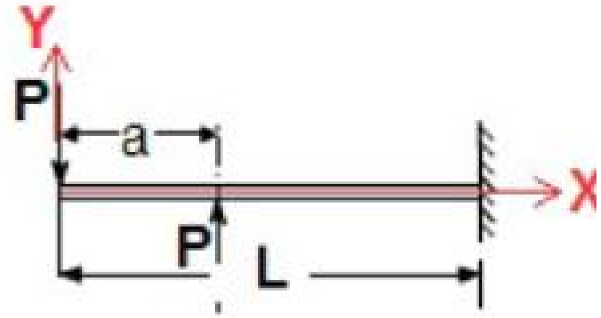
and $x = L$, $M_x = -\frac{wL^2}{2}$

Maximum bending moment, $M_{max} = \frac{wL^2}{2}$ at fixed end



S.F and B.M diagram

A Cantilever beam loaded as shown below draw its S.F and B.M diagram



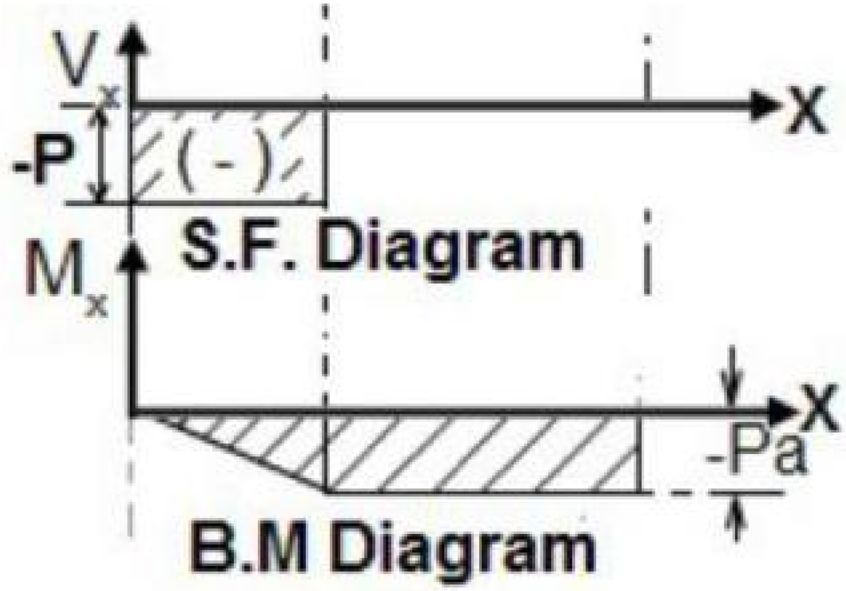
In the region $0 < x < a$

Following the same rule as followed previously, we get

$V_x = -P$; and $M_x = -P.x$

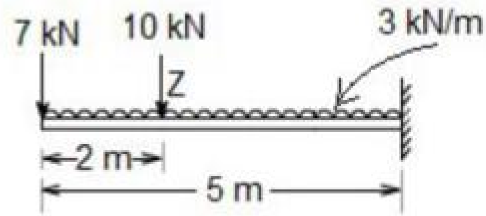
In the region $a < x < L$

$V_x = -P + P = 0$; and $M_x = -P.x + P(x-a) = P.a$



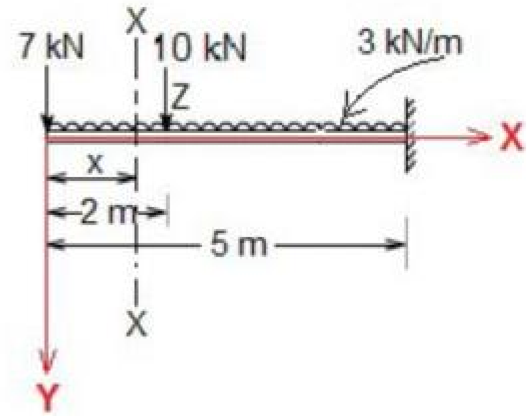
S.F and B.M diagram

Example 1: A cantilever bean of 5 m length. It carries a uniformly distributed load 3 KN/m and a concentrated load of 7 kN at the free end and 10 kN at 3 meters from the fixed end.



Draw SF and BM diagram.

Answer:In the region 0 < x < 2 m



Consider any cross section XX at a distance x from free end.

Shear force (V_x) = -7- 3x

So, the variation of shear force is linear.

at x = 0, V_x = -7 kN

at x = 2 m , $V_x = -7 - 3\times2 = -13$ kN

at point Z $V_x = -7 -3\times2-10 = -23$ Kn

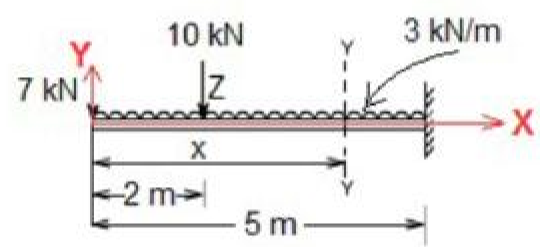
Bending moment $(M_x) = -7x - (3x).\ \dfrac{x}{2} = -\dfrac{3x^2}{2} - 7x$

So, the variation of bending force is parabolic.

at x = 0, $M_x = 0$

at x = 2 m. $M_x = -7\times2 - (3\times2)\times\dfrac{2}{2} = -20$ kNm

In the region 2 m < x < 5 m



Consider any cross section YY at a distance x from free end

Shear force $(V_x) = -7 - 3x - 10 = -17- 3x$

So, the variation of shear force is linear.

at x = 2 m, V_x = - 23 kN

at x = 5 m, V_x = - 32 kN

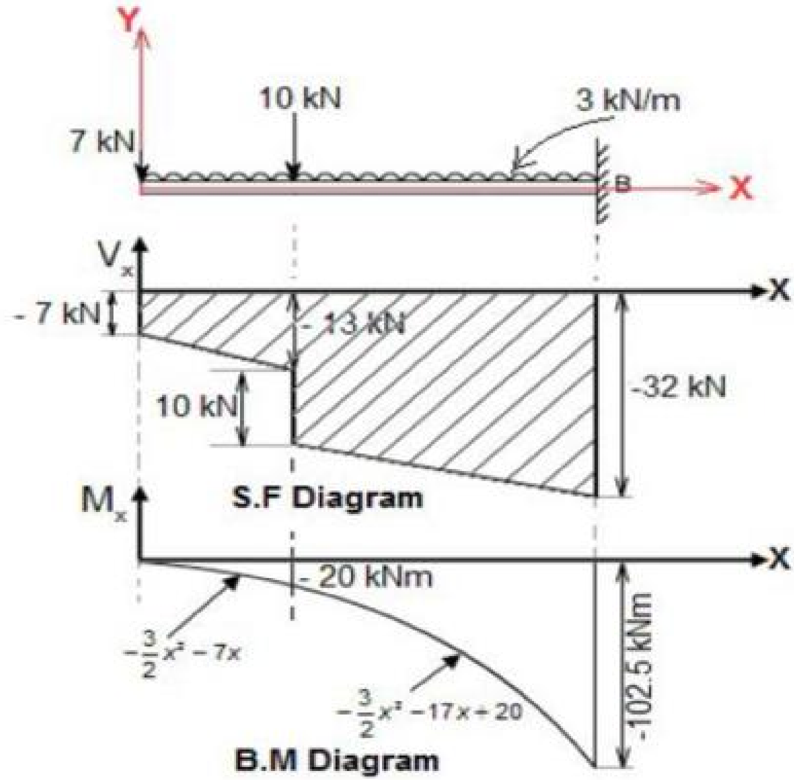
Bending moment $(M_x) = -7x - (3x)\times\left(\dfrac{x}{2}\right) - 10(x-2)$

$$= -\frac{3}{2}x^2 - 17x + 20$$

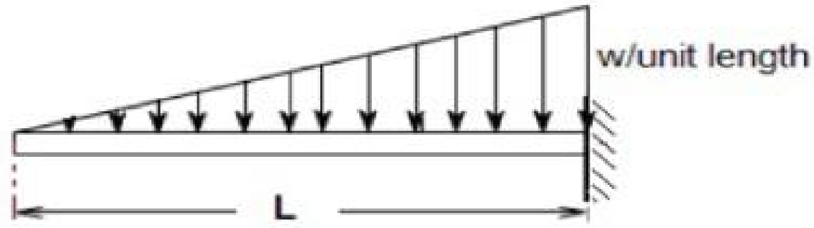
So, the variation of bending force is parabolic.

at $x = 2$ m, $M_x = -\frac{3}{2} \times 2^2 - 17 \times 2 + 20 = -20$ kNm

at $x = 5$ m, $M_x = -102.5$ kNm



A Cantilever beam carrying uniformly varying load from zero at free end and w/unit length at the fixed end



Consider any cross-section XX which is at a distance of x from the free end.

At this point load $(w_x) = \frac{w}{L}.x$

Therefore total load (W) $= \int_0^L w_x dx = \int_0^L \frac{w}{L}.xdx = \frac{wL}{2}$

Shear force $(\mathbf{V_x})$ = area of ABC (load triangle)

$$= -\frac{1}{2}.\left(\frac{w}{L}x\right).x = -\frac{wx^2}{2L}$$

$\therefore$ The shear force variation is parabolic.

at $x = 0$, $V_x = 0$

at $x = L$, $V_x = -\frac{WL}{2}$ i.e. Maximum Shear force $(V_{max}) = \frac{-WL}{2}$ at fixed end

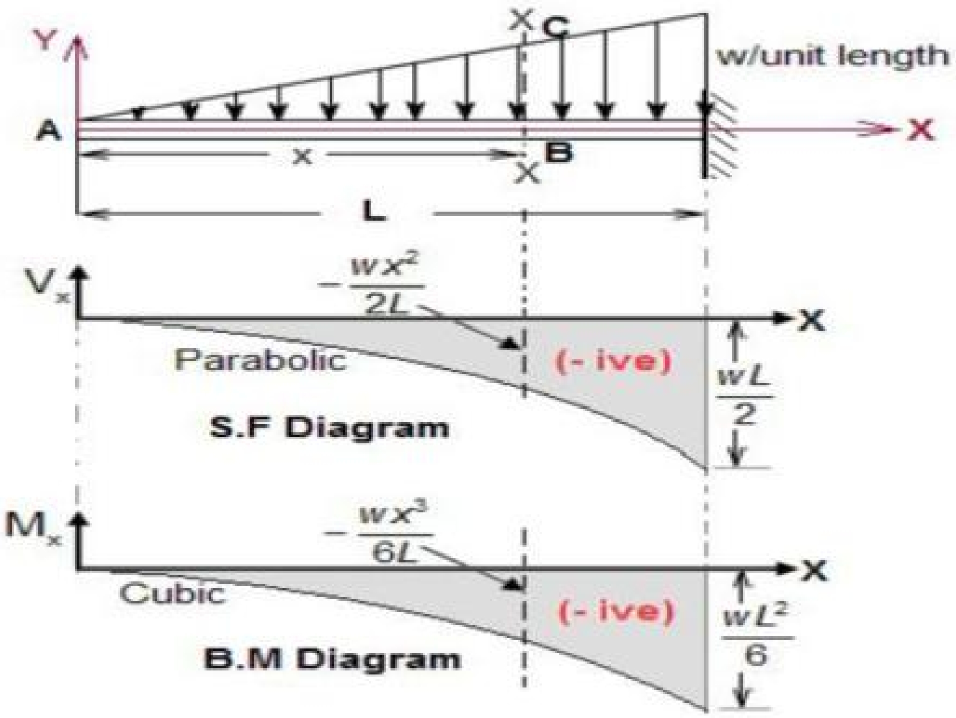
Bending moment $(\mathbf{M_x})$ = load $\times$ distance from centroid of triangle ABC

$$= -\frac{wx^2}{2L}.\left(\frac{x}{3}\right) = -\frac{wx^3}{6L}$$

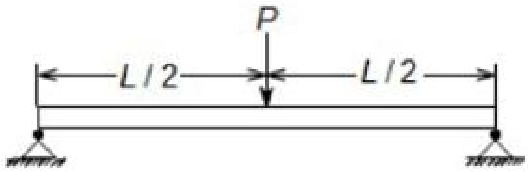
$\therefore$ The bending moment variation is cubic.

at $x= 0$, $M_x = 0$

at $x = L$, $M_x = -\frac{wL^2}{6}$ i.e. Maximum Bending moment $(M_{max}) = \frac{wL}{6}$ at fixed en



A Simply supported beam with a concentrated load 'P' at its mid span



Considering equilibrium we get, $R_A = R_B = \dfrac{P}{2}$

Now consider any cross-section XX which is at a distance of x from left end A and section YY at a distance from left end A, as shown in figure below.

Shear force:In the region 0 < x < L/2

$V_x = R_A = + P/2$ (it is constant)

In the region L/2 < x < L

$V_x = R_A - P = \dfrac{P}{2} - P = - P/2$ (it is constant)

Bending moment: In the region 0 < x < L/2

$M_x = \dfrac{P}{2}.x$ (its variation is linear)

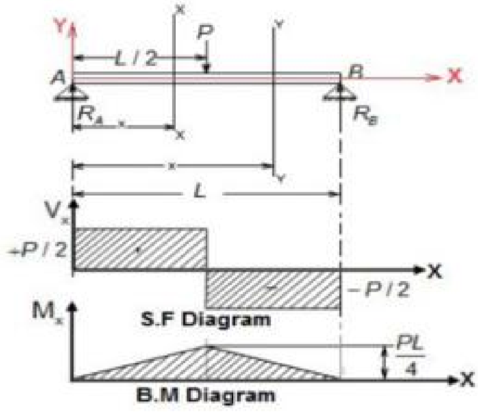
at $x = 0$, $M_x = 0$ and at $x = L/2$ $M_x = \dfrac{PL}{4}$ i.e. maximum

Maximum bending moment, $M_{max} = \dfrac{PL}{4}$ at $x = L/2$ (at mid-point)

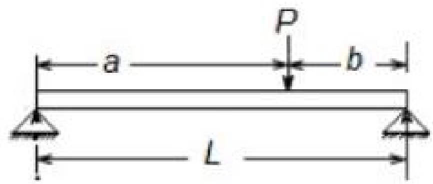
In the region L/2 < x < L

$M_x = \dfrac{P}{2}.x - P(x - L/2) = \dfrac{PL}{2} - \dfrac{P}{2}.x$ (its variation is linear)

at $x = L/2$, $M_x = \dfrac{PL}{4}$ and at $x = L$, $M_x = 0$



A Simply supported beam with a concentrated load 'P' is not at its mid span



Considering equilibrium we get, $R_A = \frac{Pb}{L}$ and $R_B = \frac{Pa}{L}$

Now consider any cross-section XX which is at a distance x from left end A and another section YY at a distance x from end A as shown in figure below.

Shear force: In the range $0 < x < a$

$$V_x = R_A = +\frac{Pb}{L} \qquad \text{(it is constant)}$$

In the range $a < x < L$

$$V_x = R_A - P = -\frac{Pa}{L} \qquad \text{(it is constant)}$$

Bending moment: In the range $0 < x < a$

$$M_x = +R_A.x = \frac{Pb}{L}.x \qquad \text{(it is variation is linear)}$$

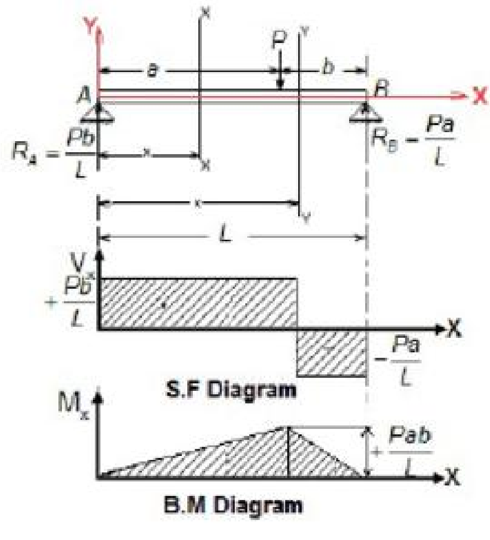
at $x = 0$, $M_x = 0$ and at $x = a$, $M_x = \frac{Pab}{L}$ (i.e. maximum)

In the range $a < x < L$

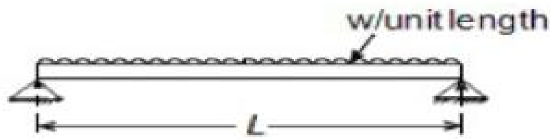
$$M_x = R_A.x - P(x-a) = \frac{Pb}{L}.x - P.x + Pa \quad \text{(Put } b = L - a\text{)}$$

$$= Pa\ (1 - Pa\left(1-\frac{x}{L}\right))$$

at $x = a$, $M_x = \frac{Pab}{L}$ and at $x = L$, $M_x = 0$



A Simply supported beam with a uniformly distributed load (UDL) through out its length



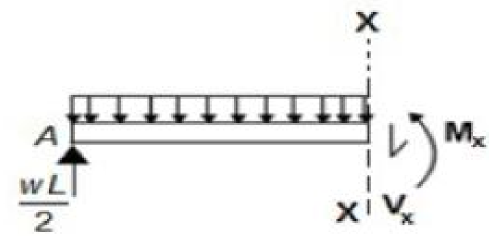
We will solve this problem by following two alternative ways.

(a) By Method of Section

Considering equilibrium we get $R_A = R_B = \frac{wL}{2}$

Now Consider any cross-section XX which is at a distance x from left end A.

Then the section view



Shear force: $V_x = \frac{wL}{2} - wx$

(i.e. S.F. variation is linear)

at $x = 0$, $V_x = \frac{wL}{2}$

at $x = L/2$, $V_x = 0$

at $x = L$, $V_x = -\frac{wL}{2}$

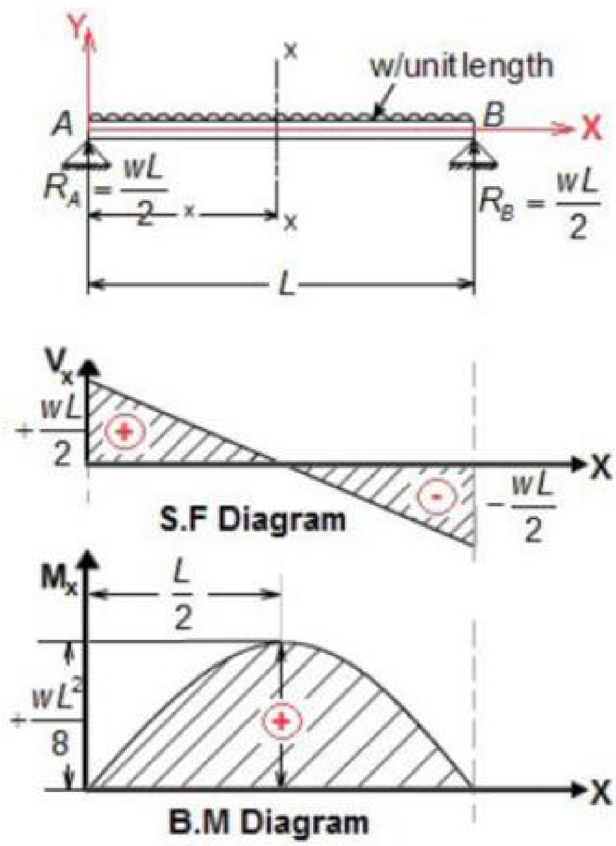
Bending moment: $M_x = \frac{wL}{2}.x - \frac{wx^2}{2}$

(i.e. B.M. variation is parabolic)

at $x = 0$, $M_x = 0$

at $x = L$, $M_x = 0$

Now we have to determine maximum bending moment and its position.



For maximum B.M: $\frac{d(M_x)}{dx} = 0 \;\; i.e. \; V_x = 0 \quad \left[\because \frac{d(M_x)}{dx} = V_x\right]$

$$\text{or } \frac{wL}{2} - wx = 0 \quad or \quad x = \frac{L}{2}$$

Therefore, maximum bending moment, $M_{max} = \dfrac{wL^2}{8}$ at x = L/2

(a) By Method of Integration

Shear force:

We know that, $\dfrac{d(V_x)}{dx} = -w$

or $\quad d(V_x) = -wdx$

Integrating both side we get (at x =0, $V_x = \dfrac{wL}{2}$)

$$\int_{\frac{wL}{2}}^{V_x} d(V_x) = -\int_0^x wdx$$

$$or\ V_x - \frac{wL}{2} = -wx$$

$$or\ V_x = \frac{wL}{2} - wx$$

Bending moment:

We know that, $\dfrac{d(M_x)}{dx} = V_x$

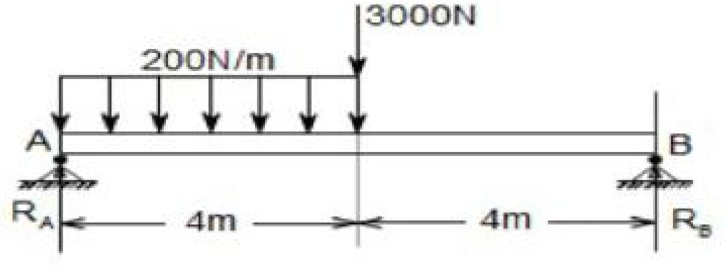
$$or \qquad d(M_x) = V_x dx = \left(\frac{wL}{2} - wx\right)dx$$

Integrating both side we get (at x =0, V_x =0)

$$\int_0^{M_x} d(M_x) = \int_0^x \left(\frac{wL}{2} - wx\right)dx$$

$$or\ M_x = \frac{wL}{2}.x - \frac{wx^2}{2}$$

Example 2 :A loaded beam as shown below. Draw its S.F and B.M diagram



Considering equilibrium we get

$$\sum M_A = 0 \text{ gives}$$

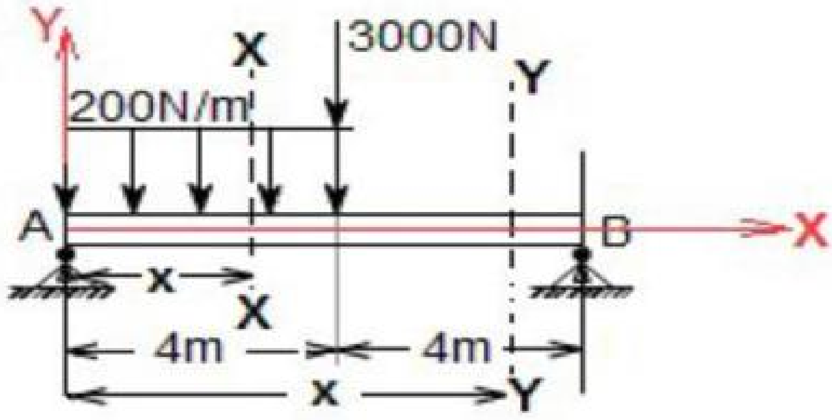
$$-(200 \times 4) \times 2 - 3000 \times 4 + R_B \times 8 = 0$$

$$or \qquad R_B = 1700\,N$$

$$And \quad R_A + R_B = 200 \times 4 + 3000$$

$$or \qquad R_A = 2100\,N$$

Now consider any cross-section *XX* which is at a distance 'x' from left end A and as shown in figure



In the region $0 < x < 4m$

Shear force $(V_x) = R_A - 200x = 2100 - 200\,x$

Bending moment $(M_x) = R_A.x - 200\,x\,.\left(\frac{x}{2}\right) = 2100\,x - 100\,x^2$

at $x = 0$, $V_x = 2100$ N, $M_x = 0$

at $x = 4m$, $V_x = 1300$ N, $M_x = 6800$ N.m

In the region $4\,m < x < 8\,m$

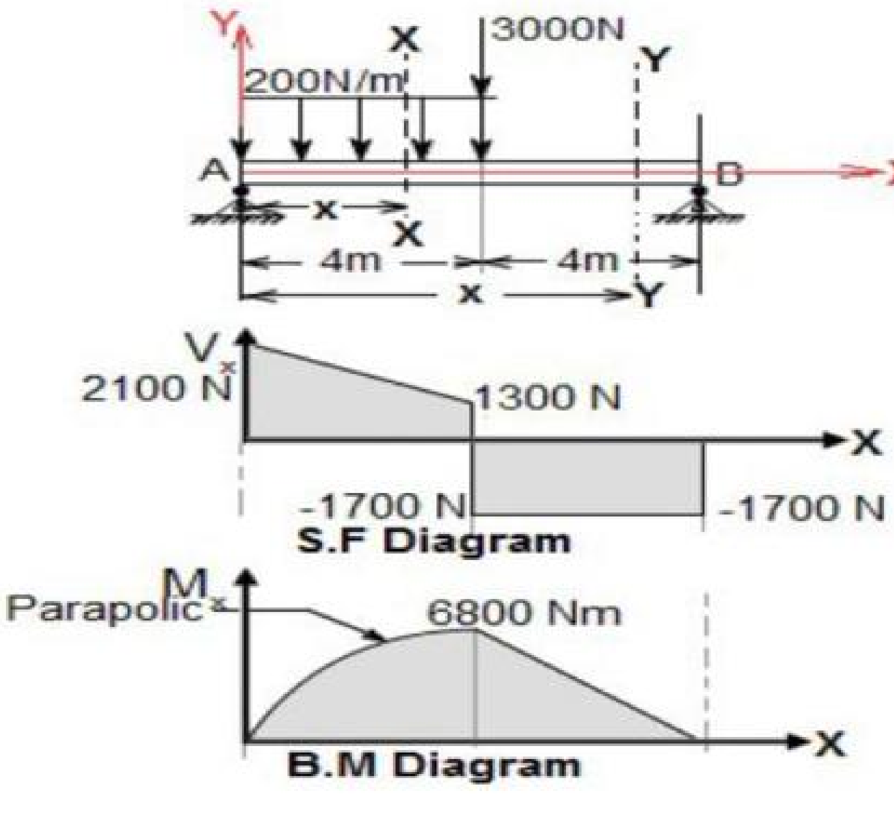
Shear force $(V_x) = R_A - 200 \times 4 - 3000 = -1700$

Bending moment $(M_x) = R_A.\,x - 200 \times 4\,(x-2) - 3000\,(x-4)$

$$= 2100\,x - 800\,x + 1600 - 3000x + 12000 = 13600 - 1700\,x$$

at $x = 4$ m, $V_x = -1700$ N, $M_x = 6800$ Nm

at $x = 8$ m, $V_x = -1700$ N, $M_x = 0$



Shear force and bending moment diagrams for over-hanging beams

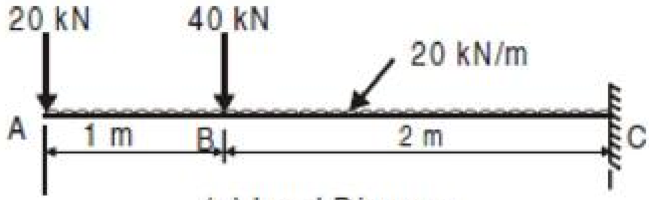
If the end portion of a beam is extended beyond the support, such beam is known as overhanging beam. In case of overhanging beams, the B.M. is positive between the two supports, whereas the S.M. is negative for the over-hanging portion. Hence at some point, the B.M. is zero after changing its sign from positive to negative or vice-versa. That point is known as the point of Contraflexure or point of inflexion

Point of Contraflexure:

It is the point where the B.M. is zero after changing its sign from positive to negative or vice-versa.

Overhanging Beam Subjected to a Concentrated Load at Free End

Draw shear force and bending moment diagram for the cantilever beam shown in Fig.



(a) Load Diagram

Solution: Portion *AB*:

At distance x, from A,

$$F = -20 - 20\,x, \text{ linear variation.}$$

At $x = 0$, $F_A = -20$ kN

At $x = 1$, $F_B = -20 - 20 \times 1 = -40$ kN.

$$M = -20x - 20x \cdot \frac{x}{2}, \text{ parabolic variation}$$

At $x = 0$, $M_A = 0$

At $x = 1$ m, $M_B = -20 - 20 \times 1 \times \frac{1}{2} = -30$ kN-m.

Portion *BC*:

Measuring x from A,

$$F = -20 - 40 - 20x, \text{ linear variation.}$$

At $x = 1$ m, $F_B = -80$ kN

At $x = 3$ m, $F_C = -120$ kN.

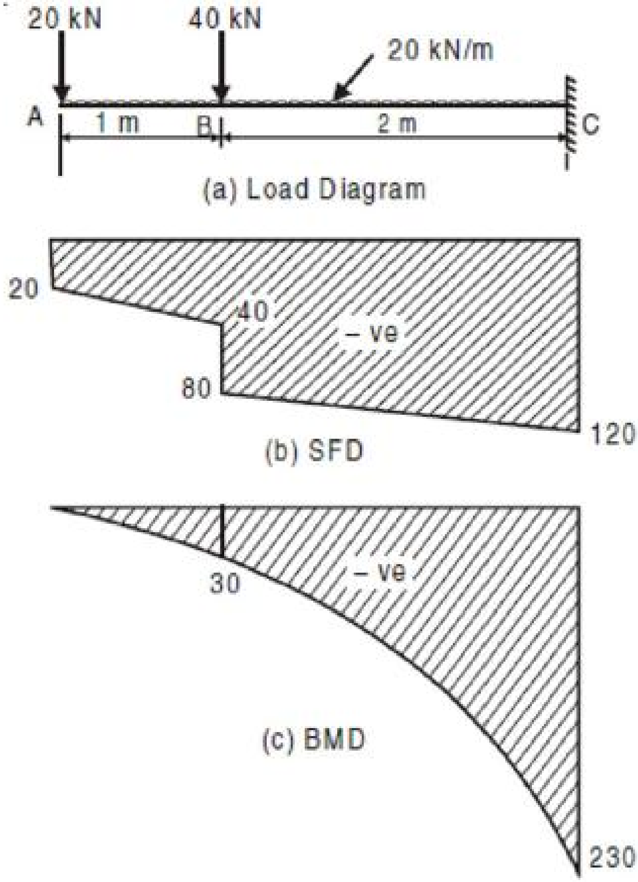
$$M = -20x - 40(x - 1) - 20x \,.\, \frac{x}{2}, \text{ parabolic variation;}$$

At $x = 1$ m, $M = -30$ kN-m

At $x = 3$ m, $M = -60 - 40 \times 2 - 20 \times 3 \times \frac{3}{2}$

$= -230$ kN-m

Hence *SFD* and *BMD* are shown in Fig. 9.37(*b*) and 9.37(*c*) respectively.



Statically determinate & Statically Indeterminate beams

Beams for which reaction forces and internal forces cannot be found out from static equilibrium equations alone are called statically indeterminate beam. This type of beam requires deformation equation in addition to static equilibrium equations to solve for unknown forces.

Statically determinate - Equilibrium conditions sufficient to compute reactions.

Statically indeterminate - Deflections (Compatibility conditions) along with equilibrium equations should be used to find out reactions.