Code: 20A54302

R20

B.Tech II Year I Semester (R20) Supplementary Examinations April/May 2024

COMPLEX VARIABLES & TRANSFORMS

(Common to EEE & ECE)

Time: 3 hours | Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Show that f(z) = z^3 is analytic for all z. 2M

(b) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. 2M

(c) State (i) Cauchy integral theorem, (ii) Liouvillies theorem. 2M

(d) Determine the poles of the function $\frac{z}{cosz}$. 2M

(e) Find: $L\{t^{3/2}\}$. 2M

(f) If f(t) is a periodic function then find $L\{f(t)\}$. 2M

(g) State Dirichlet's conditions. 2M

(h) Define Odd and Even function with an example each. 2M

(i) State Fourier integral theorem. 2M

(j) If Z[f(n)] = F(z) then find z[a^{-n} f(n)]. 2M

PART – B

(Answer all the questions: 05 X 10 = 50 Marks)

2 Find the analytic function whose imaginary part is $e^x(xsiny + ycosy)$. 10M

OR

3 Find the bilinear transformation which maps the points z = 1, i, -1 onto the points w = i, 0, -i. Also find the image of $|z|$<1. 10M

4 (a) Using Cauchy's integral formula, evaluate $\int_c \frac{z}{(z-1)(z-2)^2} dz$. 5M

(b) Find the residue of $\frac{ze^{zt}}{(z-3)^2}$ at its poles. 5M

OR

5 Using the method of contour integration, Prove that $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{a+b}$ $a > 0, b > 0, a \neq b$. 10M

6 (a) Find $L\{tsinat\}$. 5M

(b) Find $L^{-1}\left\{\frac{s^2}{(s+1)(s+2+(s+3)}\right\}$. 5M

OR

7 (a) Using convolution theorem find $L^{-1}\{\frac{1}{(s+1)(s^2+4)}\}$. 5M

(b) Using Laplace transform, solve (D^2 + 4D + 5) y = 5, given that y(0) = 0, y''(0) = 0. 5M

8 Find a Fourier expansion for $f(x) = x + x^2, -\pi \leq x \leq \pi$ hence find $\sum_1^\infty \frac{1}{n^2}$. 10M

OR

9 Find a Fourier sine series expansion of $f(x) = x(\pi - x), 0 < x < \pi$. Hence Find $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots$.... 10M

10 (a) Find the Fourier transform of $\frac{1}{x}$. 5M

(b) Find the Fourier Sine transform of e^{-ax}. 5M

OR

11 (a) Find Z(2.3^n + 5n) and deduce Z[2.3^{n+3} + 5(n+3)] using shifting theorem. 5M

(b) Find the inverse Z-transform of $\frac{Z}{(Z-1)(Z^2+1)}$. 5M

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Code: 20A54302 **R20**

B.Tech II Year I Semester (R20) Supplementary Examinations August/September 2023

COMPLEX VARIABLES & TRANSFORMS

(Common to EEE & ECE)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

*****

1		Answer the following: (10 X 02 = 20 Marks)	
	(a)	Describe the harmonic function with suitable example.	2M
	(b)	Discuss the continuity of a complex function.	2M
	(c)	State the Taylor's series for a function $f(z)$.	2M
	(d)	Obtain the residue of $f(z)=\frac{z-3}{z(z^2+1)}$ at a simple pole $z=0$.	2M
	(e)	Compute the Laplace transform of $f(t)=e^{3t}+\sin 5t$.	2M
	(f)	Obtain the inverse Laplace transform of $F(s)=\frac{1}{s^2-a^2}$.	2M
	(g)	Write Dirichlet Conditions for the existence of Fourier series.	2M
	(h)	Write Fourier series for Even and Odd Numbers.	2M
	(i)	Define Z-transform and discuss the linear property.	2M
	(j)	State Fourier integral theorem of f(x).	2M

PART – B

(Answer all the questions: 05 X 10 = 50 Marks)

2	Suppose $w=\phi+i\psi$ represents the complex potential function for an electric field and $\psi=x^2-y^2+\frac{x}{x^2-y^2}$. Determine the function ϕ.	10M
	OR	
3	If $f(z)$ is an analytic function with constant modulus then show that $f(z)$ is constant.	10M
4	Evaluate $\frac{z-3}{z^2+2z+5}dz$ where 'c' is $\lvert z+1-i\rvert=2$ using Cauchy's integral formula.	10M
	OR	
5	Obtain the Taylors expansion of; $(i)\ f(z)=\frac{1}{(z+1)^2}$ about the point $z=-i$. $(ii)\ f(z)=\frac{2z^3+1}{z^2+z}$ about the point $z=i$.	10M

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6 Find the Laplace transform of $f(t) = \frac{\cos at - \cos bt}{t} + t \sin at.$ 10M

OR

7 Solve by the method of transforms, the equation; 10M

$y''' + 2y'' - y' - 2y = 0, \quad y(0) = y'(0) = 0 \text{ and } y''(0) = 6.$

8 Find the Fourier series for the function: 10M

$f(x) = -\pi, \quad -\pi < x < 0;$
$\quad = x, \quad 0 < x < \pi.$

OR

9 Find the Fourier series expansion of $f(x) = -x^{2}$, $-\pi \le x \le \pi$. Deduce the series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}.$ 10M

10 Find the Fourier transform *f(x)* given by 10M

$f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ Hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx.$

OR

11 Determine the inverse Z-transform of $\frac{2z}{(z-1)(z^2+1)}$. 10M

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Code: 20A54303 **R20**

B.Tech II Year I Semester (R20) Supplementary Examinations August/September 2023

COMPLEX VARIABLES, TRANSFORMS AND APPLICATION OF PDE

(Mechanical Engineering)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Describe the harmonic function with suitable example. 2M

(b) Discuss the limit of a complex function. 2M

(c) State the Laurent's series for the function $f(z)$. 2M

(d) Obtain the residue of $f(z)=\frac{\sin z}{z\cos z}$ at a simple pole $z=0$. 2M

(e) Compute the Laplace transform of $f(t)=\sin 2t$. 2M

(f) Obtain the inverse Laplace transform of $F(s)=\frac{2as}{s^2+a^2}$. 2M

(g) Describe the Fourier series of $f(x)$. 2M

(h) Explain periodic function with give suitable example. 2M

(i) Form the partial differential equation by eliminating the arbitrary constants from $z=x^2+y^2+ax+by$. 2M

(j) find the solution of the partial differential equation $xp+yq=z$. 2M

PART – B

(Answer all the questions: 05 X 10 = 50 Marks)

2 Obtain the stream function of an electrostatic field in the xy- plane is given by the potential function $\phi=3x^2y-y^3$. 10M

OR

3 If $f(z)$ is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left|f(z)\right|^2=4\left|f'(z)\right|^2$. 10M

4 Evaluate $\int_0^{2+i}\left(\bar{z}\right)^2 dz$, along (i) the line $y=\frac{x}{2}$, (ii) the real axis to 2 and then vertically to $2+i$. 10M

OR

5 Compute the closed integral C for the function $f(z)=\frac{e^z}{\left(z^2+\pi^2\right)^2}$, where C is $|z|=4$. 10M

6 Find the Laplace transform of $f(t)=\frac{\cos at-\cos bt}{t}+t\sin at$. 10M

OR

Contd. in Page 2

7 Solve by the method of transforms, the equation; 10M

$y''' + 2y'' - y' - 2y = 0, \quad y(0) = y'(0) = 0 \text{ and } y''(0) = 6.$

8 Expand $f(x) = \sqrt{(1 - cosx)}, 0 < x < 2\pi$ in a Fourier series. Hence evaluate $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + ...$ 10M

OR

9 Obtain Fourier sine series expansion for *f(x)* given by; 10M

$$f(x) = \begin{cases} \frac{1}{4} - x, & 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \frac{1}{2} < x < 1. \end{cases}$$

10 Solve the equations $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(x,0) = 3\sin \pi x$, $u(0,t) = 0$ and $u(1,t) = 0$, where $0 < x < 1$, $t > 0$. 10M

OR

11 Solve by the Method of Separation of variables $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, give $u(x,0) = 6e^{-3x}$. 10M

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R20

B.Tech II Year I Semester (R20) Supplementary Examinations April/May 2024

COMPLEX VARIABLES, TRANSFORMS AND APPLICATION OF PDE

(Mechanical Engineering)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) An annular domain in the complex plane is defined by 0 < *amp* (z) < $\frac{\pi}{4}$. Find the mapping which maps this region onto the left half plane. 2M

(b) If $z = re^{i\theta}$, then find the image of θ = *constant* under the mapping $w(z) = Re^{i\varphi} = iz^3$. 2M

(c) Find the value of $\int_C Z^4 e^{\frac{1}{z}}$, where C is $|z| = 1$. 2M

(d) Find the singular points of $\frac{\cos \pi z}{(z-1)(z-2)}$. 2M

(e) Find the value of $L(sin\ t\ cos\ t)$. 2M

(f) For the periodic function 2π, find the value of $\int_{a+2\pi}^{b+2\pi} f(x)dx$. 2M

(g) Write the formulae for evaluation of Fourier coefficients for a given set of points $(x_i,\ y_i)$: $i = 0, 1, 1, \ldots ,\ n$. 2M

(h) Calculate the value of b_n in the Fourier series $f(x) = |x|$ in $(-\pi,\ \pi)$. 2M

(i) If the ends $x = 0$ and $x = l$ are insulated in one dimensional heat flow problem, then what are the expected boundary conditions? 2M

(j) Write the three possible solutions of the Laplace equations $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$. 2M

PART – B

(Answer all the questions: 05 X 10 = 50 Marks)

2 (a) Check for the analyticity of the function $f(z) = \sqrt{|xy|}$ at the origin. 5M

(b) Show that $w = \frac{i-z}{i+z}$ maps the real axis of z-plane into the circle $|w|$ = 1 and the half plane *y* > 0 into the interior of the unit circle $|w|$ = 1 in the *w*-plane. 5M

OR

3 If $w = \varphi + i\psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2+y^2}$, determine the function φ. 10M

4 Verify the Cauchy's theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the point $1 + i, -1 + i$ and $-1 - i$. 10M

OR

5 By integrating around a unit circle, evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\ cos\theta} d\theta$. 10M

6 (a) Find the inverse transform of $\frac{1}{s(s+a)^3}$. 5M

(b) Solve by the method of transforms, the equation $y''' + 2y'' - y' - 2y = 0$ given that $y'(0) = 0$ and $y''(0) = 6$. 5M

OR

Contd. in Page 2

7	(a)	Find the inverse transform of $\frac{s}{s^4+4a^4}$.	5M
	(b)	Solve $(D^3 - 3D^2 + 3D - 1)y = t^2e^t$ given that $y(0) = 1$, $y^{I}(0) = 0$ and $y^{II}(0) = -2$.	5M
8		Find the Fourier series expansion of $f(x) = 2x - x^2$ in (0, 3) and hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots - \infty = \frac{\pi}{12}$.	10M
		OR	
9	(a)	Express $f(x) = x/2$ as Fourier series in the interval $-\pi < x < \pi$.	4M
	(b)	Expand $f(x) = \frac{1}{4} - x,\ if\ 0 < x < \frac{1}{2}$, $= x - \frac{3}{4},\ if\ \frac{1}{2} < x < 1$, as Fourier series of sine terms.	6M
10	(a)	Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$.	3M
	(b)	A tightly stretched string with end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If each of its points is given a velocity $\lambda x(l - x)$, find the displacement of the string at any distance *x* from one end at any time *t*.	7M
		OR	
11		An infinitely long plate is bounded by two parallel edges and an end at right angles to them. The breadth is π; this end is maintained at a temperature u_0 at all points and the other edges are at zero temperature. Determine the temperature at any point of the plate in the steady state.	10M

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