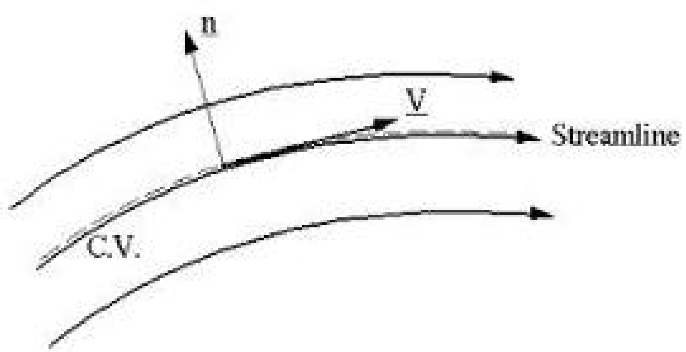
UNIT-2

FLUID KINEMATICS

Kinematics is defined as a branch of science which deals with motion of particles without considering the forces causing the motion. The velocity at any point in a flow field at any time is studied in this. Once the velocity is known, then the pressure distribution and hence the forces acting on the fluid can be determined.

Stream line: A stream line is an imaginary line drawn in a flow field such that the tangent drawn at any point on this line represents the direction of velocity vector. From the definition it is clear that there can be no flow across stream line. Considering a particle moving along a stream line for a very short distance 'ds' having its components dx , dy and dz, along three mutually perpendicular co-ordinate axes. Let the components of velocity vector Vs along x, y and z directions be u, v and w respectively. The time taken by the fluid particle to move a distance 'ds' along the stream line with a velocity Vs is:

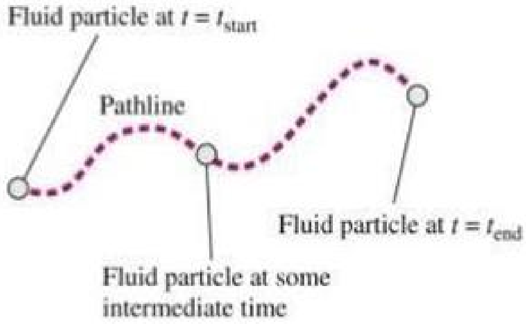
$$=\frac{ds}{Vs} \quad \text{Which is same as } \square = \frac{dx}{u} = \frac{dy}{v} = \frac{ds}{w}$$



Hence the differential equation of the steam line may be written as:

$$\frac{dx}{u}=\frac{dy}{v}=\frac{ds}{w}$$

Path line: A path line is locus of a fluid particle as it moves along. In other words a path line is a curve traced by a single fluid particle during its motion. A stream line at time t_1 indicating the velocity vectors for particles A and B. At times $t_2$and t_3 the particle A occupies the successive positions. The line containing these various positions of A represents its **Path line**



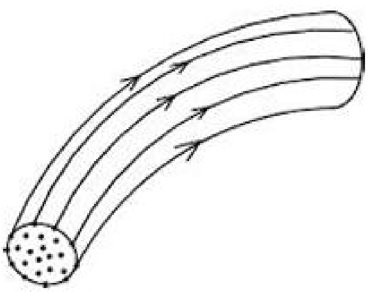
Streak line: When a dye is injected in a liquid or smoke in a gas, so as to trace the subsequent motion of fluid particles passing a fixed point, the path fallowed by dye or smoke is called the **streak line**. Thus the streak line connects all particles passing through a given point.

In steady flow, the stream line remains fixed with respect to co-ordinate axes. Stream lines in steady flow also represent the path lines and streak lines. In unsteady flow, a fluid particle will not, in general, remain on the same stream line (except for unsteady uniform flow). Hence the stream lines and path lines do not coincide in unsteady non-uniform flow.

Instantaneous stream line: in a fluid motion which is independent of time, the position of stream line is fixed in space and a fluid particle fallowing a stream line will continue to do so. In case of time dependent flow, a fluid particle fallows a stream line for only a short interval of time, before changing over to another stream line. The stream lines in such cases are not fixed in space, but change with time. The position of a stream line at a given instant of time is known as **Instantaneous stream line.** For different instants of time, we shall have different Instantaneous stream lines in the same space. The Stream line, Path line and the streak line are one and the same, if the flow is steady.

Stream tube: If stream lines are drawn through a closed curve, they form a boundary surface across which fluid cannot penetrate. Such a surface bounded by stream lines is known as **Stream tube**.

From the definition of stream tube, it is evident that no fluid can cross the bounding surface of the stream tube. This implies that the quantity of fluid entering the stream tube at one end must be the same as the quantity leaving at the other end. The Stream tube is assumed to be a small cross-sectional area, so that the velocity over it could be considered uniform.



CLASSIFICATION OF FLOWS

The fluid flow is classified as:

i) Steady and unsteady flows.
ii) Uniform and Non-uniform flows.
iii) Laminar and Turbulent flows.
iv) Compressible and incompressible flows.
v) Rotational and Ir-rotational flows.
vi) One, two and three dimensional flows.

i) Steady and Un-steady flows: Steady flow is defined as the flow in which the fluid characteristics like velocity, pressure, density etc. at a point do not change with time. Thus for a steady flow, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x,y,z} = 0, \qquad \left(\frac{\partial p}{\partial t}\right)_{x,y,z} = 0, \qquad \left(\frac{\partial \rho}{\partial t}\right)_{x,y,z} = 0$$

Un-Steady flow is the flow in which the velocity, pressure, density at a point changes with respect to time. Thus for un-steady flow, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x,y,z} \neq 0, \qquad \left(\frac{\partial p}{\partial t}\right)_{x,y,z} \neq 0, \qquad \left(\frac{\partial \rho}{\partial t}\right)_{x,y,z} \neq 0$$

ii) Uniform and Non-uniform flows:

Uniform flow is defined as the flow in which the velocity at any given time does not change with respect to space. (i.e. the length of direction of flow)

For uniform flow $\left(\frac{\partial V}{\partial s}\right)_{t=const} = 0$

Where ∂V = Change of velocity

∂s = Length of flow in the direction of – S

Non-uniform is the flow in which the velocity at any given time changes with respect to space.

For Non-uniform flow $\left(\frac{\partial V}{\partial s}\right)_{t=const} \neq 0$

iii) Laminar and turbulent flow:

Laminar flow is defined as the flow in which the fluid particles move along well-defined paths or stream line and all the stream lines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called streamline flow or viscous flow.

Turbulent flow is the flow in which the fluid particles move in a zigzag way. Due to the movement of fluid particles in a zigzag way, the eddies formation takes place, which are responsible for high energy loss. For a pipe flow, the type of flow is determined by a non-Dimensional number $\frac{VD}{\nu}$ called the Reynolds number.

Where D = Diameter of pipe.

V = Mean velocity of flow in pipe.

ν = Kinematic viscosity of fluid.

If the Reynolds number is lesthan2000, the flow is called Laminar flow.

If the Reynolds number is more than 4000, it is called Turbulent flow.

If the Reynolds number is between 2000 and 4000 the flow may be Laminar or Turbulent flow.

iv) Compressible and Incompressible flows:

Compressible flow is the flow in which the density of fluid changes from point to point or in other words the density is not constant for the fluid.

For compressible flow $\rho \neq$ Constant.

In compressible flow is the flow in which the density is constant for the fluid flow. Liquids are generally incompressible, while the gases are compressible.

For incompressible flow ρ = Constant.

v) Rotational and Irrotational flows:

Rotational flow is a type of flow in which the fluid particles while flowing along stream lines also rotate about their own axis. And if the fluid particles, while flowing along stream lines, do not rotate about their own axis, the flow is called Ir-rotational flow.

vi) One, Two and Three - dimensional flows:

One dimensional flow is a type of flow in which flow parameter such as velocity is a function of time and one space co-ordinate only, say '*x*'. For a steady one- dimensional flow, the velocity is a function of one space co-ordinate only. The variation of velocities in other two mutually perpendicular directions is assumed negligible.

Hence for one dimensional flow $\mathbf{u = f(x), v = 0}$ **and** $\mathbf{w = 0}$

Where u, v and w are velocity components in x, y and z directions respectively.

Two – dimensional flow is the type of flow in which the velocity is a function of time and two space co-ordinates, say x and y. For a steady two-dimensional flow the velocity is a function of two space co-ordinates only. The variation of velocity in the third direction is negligible.

Thus for two dimensional flow $\mathbf{u = f_1 (x, y), v = f_2 (x, y)}$ **and** $\mathbf{w = 0}$**.**

Three – dimensional flow is the type of flow in which the velocity is a function of time and three mutually perpendicular directions. But for a steady three-dimensional flow, the fluid parameters are functions of three space co-ordinates (x, y, and z) only.

Thus for ***three- dimensional flow*** $u = f_1 (x, y, z), v = f_2 (x, y, z), z = f_3 (x, y, z)$.

Rate of flow or Discharge (Q)

It is defined as the quantity of a fluid flowing per second through a section of pipe or channel. For an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of the liquid flowing cross the section per second. or compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.

Thus i) For liquids the unit of Q is m^3/sec or Litres/sec. Ii)

For gases the unit of Q is Kg f/sec or Newton/sec.

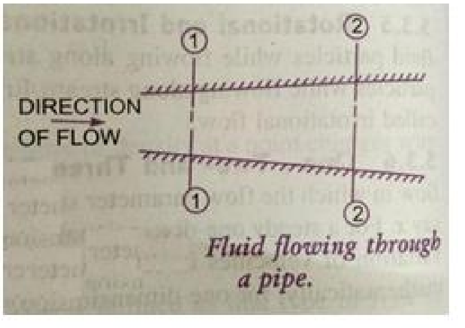
The discharge $Q = A \times V$

Where, A = Area of cross-section of pipe.

V= Average velocity of fluid across the section.

CONTINUITY EQUATION

The equation based on the principle of conservation of mass is called Continuity equation. Thus for a fluid flowing through the pipe at all cross- sections, the quantity of fluid per second is constant. Consider two cross- sections of a pipe.



Fluid flowing through a pipe.

Let V_1 = Average velocity at cross- section 1-1

ρ_1 = Density of fluid at section 1-1

A_1 = Area of pipe at section 1-1

And V_2, ρ_2, A_2 are the corresponding values at section 2—2

Then the rate flow at section 1-1 $= \rho_1 A_1 V_1$

Rate of flow at section 2—2 = $\rho_2 A_2 V_2$

According to law of conservation of mass

Rate of flow at section 1---1 = Rate of flow at section 2---2

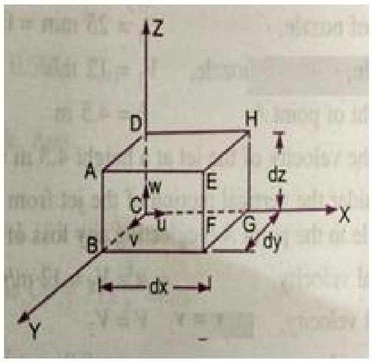
$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

This equation is applicable to the compressible as well as incompressible fluids and is called **"Continuity equation"**. If the fluid is incompressible, then $\rho_1 = \rho_2$ and the continuity equation reduces to

$$A_1 V_1 = A_2 V_2$$

CONTINUITY EQUATION IN THREE DIMENSIONAL FLOW

Consider a fluid element of lengths dx, dy and dz in the direction of x, y and z. Let u, v and w are the inlet velocity components in x, y and z directions respectively.



Mass of fluid entering the face ABCD per second

$= \rho \times$ velocity in x – direction $\times$ Area of ABCD

$= \rho \times u \times (dy \times dz)$

Then the mass of fluid leaving the face EFGH per second

$= \rho \times u \times (dy \times dz) + \frac{\partial}{\partial x} \rho u \, dy \, dz \, dx$

Gain of mass in x- direction

= Mass through ABCD – Mass through EFGH per second.

$= \rho u \, dy \, dz - \rho u \, dy \, dz - \frac{\partial}{\partial x} \rho u \, dy \, dz \, dx$

$= -\frac{\partial}{\partial x} \rho u \, dy \, dz \, dx$

$= -\frac{\partial}{\partial x} \rho u \; dxdydz$ ______________ (1)

Similarly the net gain of mass in y- direction.

$= -\frac{\partial}{\partial y} (\rho v) \, dx dy dz$ ____________ (2)

In z – direction $= -\frac{\partial}{\partial z}(\rho w) dx \, dy \, dz$______________ (3)

Net gain of mass $= -\left[\frac{\partial}{\partial x} \rho u + \frac{\partial}{\partial y} \rho v + \frac{\partial}{\partial z} \rho w\right] dxdydz$ ____________________ (4)

Since mass is neither created nor destroyed in the fluid element , the net increase of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element. But the mass of fluid in the element is ρ dx dy dz and its rate of increase with time is $\frac{\partial}{\partial t}(\rho.\ dx.\ dy.\ dz.)$ or $\frac{\partial \rho}{\partial t}$. dx. dy. dz. ______________ (5)

Equating the two expressions (4) & (5)

$-\left(\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w\right) dx \, dydz = \frac{\partial \rho}{\partial t}.\ dx.\ dy\ .dz.$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = \mathbf{0} \quad \text{\_\_\_\_\_\_\_\_\_\_ (6)}$$

This equation is applicable to

i) Steady and unsteady flow
ii) Uniform and non- uniform flow , and
iii) Compressible and incompressible flow.

For steady flow $\frac{\partial \rho}{\partial t} = 0$ and hence equation (6) becomes

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = \mathbf{0} \quad \text{\_\_\_\_\_\_ (7)}$$

If the fluid is incompressible, then ρ is constant and the above equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \_\_\_\_\_ (8)$$

This is the continuity equation in three - dimensional flow.

FLUID DYNAMICS

A fluid in motion is subjected to several forces, which results in the variation of the acceleration and the energies involved in the flow of the fluid. The study of the forces and energies that are involved in the fluid flow is known as Dynamics of fluid flow.

The various forces acting on a fluid mass may be classified as:

1. Body or volume forces
2. Surface forces
3. Line forces.

Body forces: The body forces are the forces which are proportional to the volume of the body.

Examples: Weight, Centrifugal force, magnetic force, Electromotive force etc.

Surface forces: The surface forces are the forces which are proportional to the surface area which may include pressure force, shear or tangential force, force of compressibility and force due to turbulence etc.

Line forces: The line forces are the forces which are proportional to the length.

Example is surface tension.

The dynamics of fluid flow is governed by Newton"s second law of motion which states that the resultant force on any fluid element must be equal to the product of the mass and acceleration of the element and the acceleration vector has the direction of the resultant vector. The fluid is assumed to be in compressible and non- viscous.

$$\sum F = M.\ a$$

Where $\sum F$ represents the resultant external force acting on the fluid element of mass **M** and **a** is total acceleration. Both the acceleration and the resultant external force must be along same line of action. The force and acceleration vectors can be resolved along the three reference directions x, y and z and the corresponding equations may be expressed as ;

$$\sum F_x = M.\ a_x$$

$$\sum F_y = M.\ a_y$$

$$\sum F_z = M.\ a_z$$

Where $\sum F_x$, $\sum F_y$ and $\sum F_z$ are the components of the resultant force in the x, y and z directions respectively, and a_x , a_y and a_z are the components of the total acceleration in x, y and z directions respectively.

FORCES ACTING ON FLUID IN MOTION:

The various forces that influence the motion of fluid are due to gravity, pressure, viscosity, turbulence and compressibility.

The gravity force Fg is due to the weight of the fluid and is equal to Mg . The gravity force per unit volume is equal to "ρ g".

The pressure force Fp is exerted on the fluid mass, if there exists a pressure gradiant between the two points in the direction of the flow.

The viscous force Fv is due to the viscosity of the flowing fluid and thus exists in case of all real fluids.

The turbulent flow Ft is due to the turbulence of the fluid flow.

The compressibility force Fc is due to the elastic property of the fluid and it is important only for compressible fluids.

If a certain mass of fluid in motion is influenced by all the above forces, then according to Newton"s second law of motion

The net force $F_x = M.\ a_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$

i) if the net force due to compressibility(F_c) is negligible, the resulting net force

$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$ and the equation of motions are called

Reynolds's equations of motion.

ii) For flow where (Ft) is negligible, the resulting equations of motion are known as

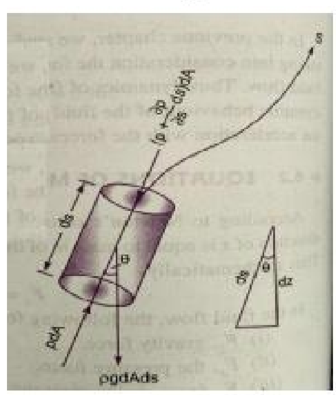
Navier – Stokes equation.

iii) If the flow is assumed to be ideal, viscous force (F_v) is zero and the equations of motion are known as **Euler's equation of motion.**

EULER'S EQUATION OF MOTION

In this equation of motion the forces due to gravity and pressure are taken in to consideration. This is derived by considering the motion of the fluid element along a stream-line as:

Consider a stream-line in which flow is taking place in s- direction. Consider a cylindrical element of cross-section dA and length ds.



The forces acting on the cylindrical element are:

1. Pressure force pdA in the direction of flow.
2. Pressure force □ + $\frac{\partial p}{\partial s}$ds □□
3. Weight of element □ g dA.ds

Let □ is the angle between the direction of flow and the line of action of the weight of the element.

The resultant force on the fluid element in the direction of S must be equal to the mass of fluid element× acceleration in the direction of s.

$$□□□ - □ + \frac{\partial p}{\partial s}□□\ □□ - □□□□□□ \cos\theta$$

$$= □□□□□ \times □_{□} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (1)$$

Whereas is the acceleration in the direction of s.

Now $a_s = \frac{dv}{dt}$ where „v" is a function of s and t.

$$= \frac{□□}{□□}\frac{□□}{□□} + \frac{□□}{□□} = \frac{□□□}{□□} + \frac{□□}{□□}$$

If the flow is steady, then $\frac{□□}{□□} = 0$

$$a_s = \frac{□□□}{□□}$$

Substituting the value of a_s in equation (1) and simplifying, we get

$$-\frac{\partial p}{\partial s} dsdA - \rho gdAds \cos\theta = \rho dAds \times \frac{\partial v}{\partial s}$$

Dividing by □ dA.ds $\quad -\times\frac{1}{\rho}\frac{\partial p}{\partial s} - g\cos\theta = \frac{v\,\partial v}{\partial s}$

,

$$\frac{1}{\rho} \times \frac{\partial p}{\partial s} + g\cos\theta + \frac{v\,\partial v}{\partial s} = 0$$

But we have $\cos\theta = \frac{dz}{ds}$

$$\frac{1}{\rho} \times \frac{\partial p}{\partial s} + g\frac{dz}{ds} + \frac{v\,\partial v}{\partial s} = 0$$

$$\frac{□□}{□} + □□□ + □□□ = □$$

∴ This equation is known as Euler"s equation of motion.

BERNOULLI'S EQUATION FROM EULER'S EQUATION

Bernoulli"s equation is obtained by integrating the Euler"s equation of motion as

$$\frac{□□}{□} + □□□ + □□□ = □□□□□□□□□$$

If the flow is incompressible, □ is constant and

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

The above equation is Bernoulli"s equation in which

$\frac{p}{\rho g}$ = Pressure energy per unit weight of fluid or pressure head.

$\frac{v^2}{2g}$ = Kinetic energy per unit weight of fluid or Kinetic head.

z = Potential energy per unit weight of fluid or Potential head.

The following are the assumptions made in the derivation of Bernoulli"s equation.

i. The fluid is ideal. i.e. Viscosity is zero.

ii. The flow is steady.

iii. The flow is incompressible.

iv. The flow is irrotational.

MOMENTUM EQUATION

It is based on the law of conservation of momentum or on the momentum principle, which states that the net force acting on a fluid mass equal to the change in the momentum of the flow per unit time in that direction. The force acting on a fluid mass ,, m ,, is given by Newton's second law of motion.

$$F = m \times a$$

Where 'a' is the acceleration acting in the same direction as force

F. But $\square = \frac{\square\square}{\square\square}$

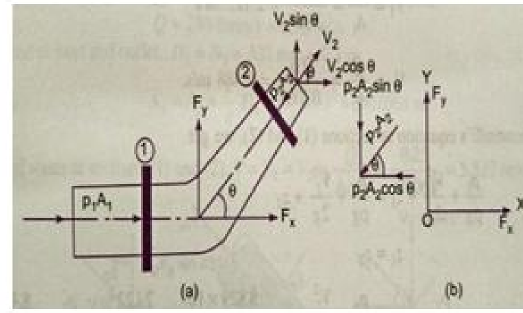
$$\square = \square \frac{\square}{\square\square} = \frac{\square\,\square\square}{\square}$$ (Since m is a constant and can be taken inside differential)

$$\square = \frac{\square\,\square\square}{\square\square}$$ is known as the momentum principle.

$F.\,dt = d(mv)$ ___________Is known as the impulse momentum equation.
It states that the impulse of a force F acting on a fluid mass m in a short interval of time dt is equal to the change of momentum d(m v) in the direction of force.

Force exerted by a flowing fluid on a pipe-bend:

The impulse momentum equation is used to determine the resultant force exerted by a flowing fluid on a pipe bend.



Consider two sections (1) and (2) as above

Let v_1 = Velocity of flow at section (1)

P_1= Pressure intensity at section (1)

A_1 = Area of cross-section of pipe at section (1)

And V_2, P_2, A_2 are corresponding values of Velocity, Pressure, Area at section (2)

Let F_x and F_y be the components of the forces exerted by the flowing fluid on the bend in x and y directions respectively. Then the force exerted by the bend on the fluid in the directions of x and y will be equal to F_X and F_Y but in the opposite directions. Hence the component of the force exerted by the bend on the fluid in the x – direction = - F_x and in the direction of y = - F_y. The other external forces acting on the fluid are $p_1 A_1$ and $p_2 A_2$ on the sections (1) and (2) respectively. Then the momentum equation in x-direction is given by

Net force acting on the fluid in the direction of x = Rate of change of momentum in x –direction

$p_1 A_1 - p_2 A_2 \cos\theta - F_x$ = (Mass per second) (Change of velocity)

= ρ Q (Final velocity in x-direction – Initial velocity in x-direction)

$= \rho Q (V_2 \cos\theta - V_1)$

$F_x = \rho Q (V_1 - V_2 \cos\theta) + p_1 A_1 - p_2 A_2 \cos\theta$ _______________(1)

Similarly the momentum equation in y-direction gives

$0 - p_2 A_2 \sin\theta - F_y = \rho Q (V_2 \sin\theta - 0)$

$F_y = \rho Q (-V_2 \sin\theta) - p_2 A_2 \sin\theta$ ______________(2)

Now the resultant force (F_R) acting on the bend

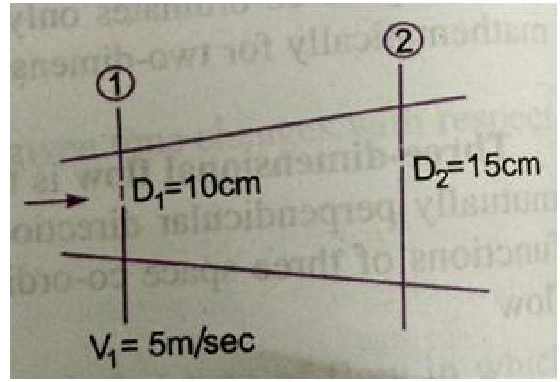
$$F_R = \sqrt{F_x^2 + F_y^2}$$

And the angle made by the resultant force with the horizontal direction is given by

$$\tan\theta = \frac{F_y}{F_x}$$

PROBLEMS

1. The diameter of a pipe at sections 1 and 2 are 10 cm and 15cms respectively. Find the discharge through pipe, if the velocity of water flowing through the pipe at section 1 is 5m/sec. determine the velocity at section 2.



Given:

At section 1,

$$D_1 = 10\text{cms} = 0.1\text{m}$$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} 0.1^2 = 0.007254\ m^2$$

$$V_1 = 5\text{m/sec}$$

At section 2, $D_2 = 15\text{cms} = 0.15\text{m}$

$$A_2 = \frac{\pi}{4} 0.15^2 = 0.01767\ m^2$$

Discharge through pipe $Q = A_1 \times V_1$

$$= 0.007854 \times 5 = 0.03927\ m^3/\text{sec}$$

We have $A_1 V_1 = A_2 V_2$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.007854}{0.01767} \times 5.0 = 2.22\ m/s$$

2. Water is flowing through a pipe of 5cm dia. Under a pressure of 29.43N/cm^2 and with mean velocity of 2 m/sec. find the total head or total energy per unit weight of water at a cross-section, which is 5m above datum line.

Given: dia. Of pipe $= 5\text{cm} = 0.05\text{m}$

Pressure P $= 29.43\text{N/cm}^2 = 29.43 \times 10^4\text{N/m}^2$
Velocity V $= 2$ m/sec
Datum head Z $= 5$m

Total head = Pressure head + Kinetic head + Datum head

$$\text{Pressure head} = \frac{p}{\rho g} = \frac{29.43\times10^4}{1000\times9.81} = 30m$$

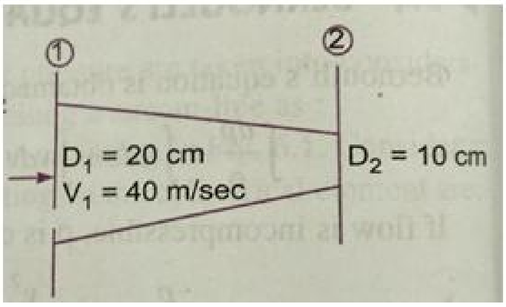
$$\text{Kinetic head} = \frac{V^2}{2g} = \frac{2\times2}{\ldots} = 0.204m$$

Datum head = Z = 5m

$$\frac{p}{\rho g} + \frac{V^2}{2g} + Z = 30 + 0.204 + 5 = 35.204m$$

Total head = 35.204m

3. A pipe through which water is flowing is having diameters 20cms and 10cms at cross-sections 1 and 2 respectively. The velocity of water at section 1 is 4 m/sec. Find the velocity head at section 1 and 2 and also rate of discharge?



Given: D_1 = 20cms = 0.2m

$A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314m^2$

V_1 = 4 m/sec

D_2 = 10 cm = 0.1 m

$A_2 = \frac{\pi}{4} \times 0.1^2 = 0.007854m^2$

i) Velocity head at section 1 $\frac{V_1^2}{2g} = \frac{4 \times 4}{2 \times 9.81} = 0.815m$

ii) Velocity head at section 2 $\frac{V_2^2}{2g}$

To find V_2, apply continuity equation $A_1 V_1 = A_2 V_2$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.0314 \times 4}{0.00785} = 16m/sec$$

Velocity head at section 2

$$\frac{V_2^2}{2g} = \frac{16 \times 16}{2 \times 9.81} = 13.047m$$

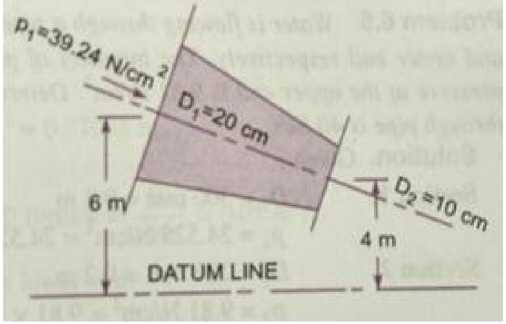
iii) Rate of discharge

$Q = A_1 V_1 = A_2 V_2$

$= 0.0314 \times 4 = 0.1256 \, m^3/sec$

Q = 125.6 Liters/sec

4. Water is flowing through a pipe having diameters 20cms and 10cms at sections 1 and 2 respectively. The rate of flow through pipe is 35 liters/sec. The section 1 is 6m above the datum and section 2 is 4m above the datum. If the pressure at section 1 is 39.24n/cm². Find the intensity of pressure at section 2?



Given: At section 1 D_1 = 20cm =0.2m

$A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314m^2$

P_1 = 39.24N/cm² =39.24 x 10^4N/m²

Z_1 = 6m

At section 2 D_2 =10cm = 0.1m

$A_2 = \frac{\pi}{4} \times 0.1^2 = 0.0007854m^2$

Z_2 = 4m, P_2 = ?

Rate of flow Q = 35 lt/sec = (35/1000) m³/sec = 0.035 m³/sec

$Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{Q}{A_1} = \frac{0.035}{0.0314} = 1.114m/sec$$

$$V_2 = \frac{Q}{A_2} = \frac{0.035}{0.007854} = 4.456m/sec$$

Applying Bernoulli"s equation at sections 1 and 2 $\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{1.114^2}{2 \times 9.81} + 6 = \frac{p_2}{1000 \times 9.81} + \frac{4.456^2}{2 \times 9.81} + 4$$

$$40 + 0.063 + 6 = \frac{p_2}{9810} + 1.102 + 4$$

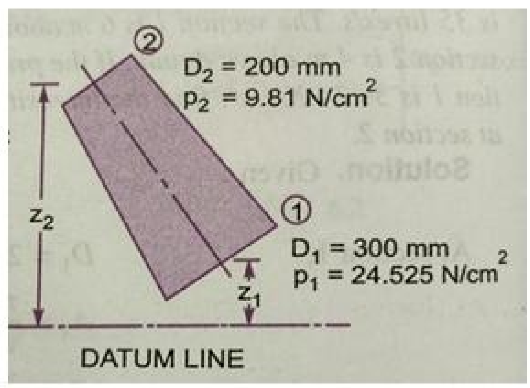
$$46.063 = \frac{p_2}{9810} + 5.102$$

$$\frac{p_2}{9810} = 46.063 - 5.102 = 41.051$$

$$P_2 = 41.051 \times 9810 = 402710 \text{N/m}^2$$

$$\mathbf{P_2 = 40.271 N/cm^2}$$

5) Water is flowing through a pipe having diameter 300mm and 200mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525N/cm^2 and the pressure at the upper end is 9.81N/cm^2. Determine the difference in datum head if the rate of flow through is 40lit/sec?



Given:

section 1 D_1= 300mm =0.3m

$$A_1 = \frac{\pi}{4} \times 0.3^2 = 0.07065 \text{ m}^2$$

P_1= 24.525N/cm^2 = 24.525 × 10^{4}N/m^2

Section 2 D_2 = 200mm0.2m

$$A_2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314 \text{ m}^2$$

P_2 = 9.81N/cm^2 = 9.81 × 10^4 N/m^2

Rate of flow Q = 40 lit/Sec =40/1000 = 0.04 m^3/sec

$$Q = A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{Q}{A_1} = \frac{0.04}{0.07065} = 0.566 \text{m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.04}{00.0314} = 1.274 \text{m/sec}$$

Applying Bernoulli"s equation at sections 1 and 2

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

24.525 × 10^4 0.566 ^{2} 9.81 × 10^4 1.274 ^{2}

$$\frac{}{1000 \times 9.81} + \frac{}{2 \times 9.81} + \square_1 = \frac{}{1000 \times 9.81} + \frac{}{2 \times 9.81} + \square_2$$

$$25 + 0.32 + Z_1 = 10 + 1.623 + Z_2$$

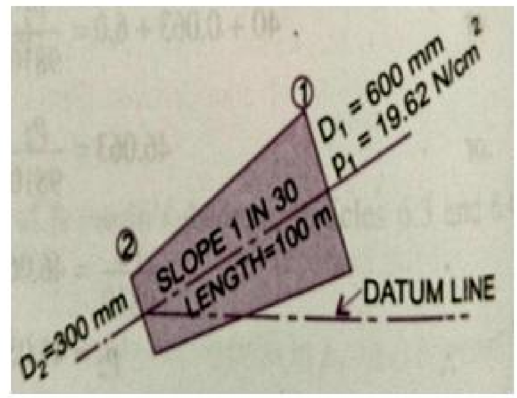
$$Z_2 - Z_1 = 25.32 - 11.623 = 13.697 \text{ or say } 13.70\text{m}$$

The difference in datum head = $Z_2 - Z_1$ = 13.70m

6) The water is flowing through a taper pipe of length 100m having diameters 600mm at the upper end and 300mm at the lower end, at the rate of 50lts/sec. the pipe has a slope of 1 in 30. Find the pressure at the lower end, if the pressure at the higher level is 19.62N/cm^2?

Given: Length of pipe L = 100m

Dia. At the upper end D_1 = 600mm = 0.6m



$$\square_1 = \frac{\pi}{4} \times 0.6^2 = 0.2827\text{m}^2$$

$$P_1 = 19.62\text{N/cm}^2 = 19.62 \times 10^4 \text{ N/m}^2$$

Dia. at the lower end D_2 = 300mm = 0.3m

$$\square_2 = \frac{\pi}{4} \times 0.3^2 = 0.07065\text{m}^2$$

Rate of flow Q = 50 Lts/sec = $\frac{50}{1000}$ = 0.05 m^3/sec

Let the datum line is passing through the centre of the lower end. Then $Z_2 = o$

As slope is 1 in 30 means $\square_1 = \frac{1}{30} \times 100 = \frac{10}{3}$ m

We also know that $Q = A_1 V_1 = A_2 V_2$

$$\square_1 = \frac{Q}{A_1} = \frac{0.05}{0.2827} = 0.177\text{m/sec}$$

$$\square_2 = \frac{Q}{A_2} = \frac{0.05}{0.07065} = 0.707\text{m/sec}$$

Applying Bernoulli"s equation at sections 1 and 2

$$\frac{\square_1}{\square\square} + \frac{\square_1^2}{2\square} + \square_1 = \frac{\square_2}{\square\square} + \frac{\square_2^2}{2\square} + \square_2$$

$$\frac{19.62\times10^4}{1000\times9.81} + \frac{0.177^2}{2\times9.81} + \frac{10}{3} = \frac{\square_2}{1000\times9.81} + \frac{0.707^2}{2\times9.81} + 0$$

$$20 + 0.001596 + 3.334 = \frac{P_2}{9810} + 0.0254$$

$$23.335 = \frac{P_2}{9810} + 0.0254$$

$$\frac{P_2}{9810} = 23.335 - 0.0254 = 23.31$$

$$P_2 = 23.31 \times 9810 = 228573 N/m^2$$

$$\mathbf{P_2 = 22.857 N/cm^2}$$

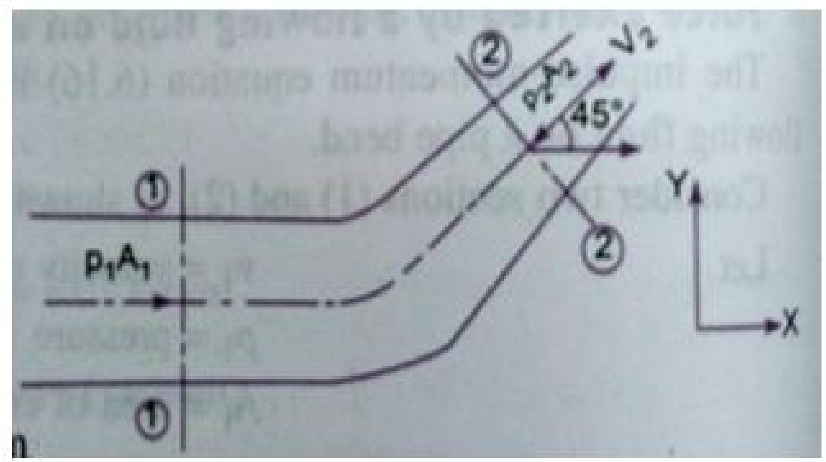
7) A 45° reducing bend is connected to a pipe line, the diameters at inlet and out let of the bend being 600mm and 300mm respectively. Find the force exerted by the water on the bend, if the intensity of pressure at the inlet to the bend is 8.829N/cm² and rate of flow of water is 600 Lts/sec.

Given: Angle of bend $\theta = 45°$

Dia. at inlet $D_1 = 600mm = 0.6m$ $\qquad A_1 = \frac{\pi}{4} \times (0.6)^2 = 0.2827\ m^2$

Dia. at out let $D_2 = 300mm = 0.3m$ $\qquad A_2 = \frac{\pi}{4} \times (0.3)^2 = 0.07065\ m^2$

Pressure at inlet $P_1 = 8.829 N/cm^2 = 8.829 \times 10^4 N/m^2$



$$Q = 600 Lts/sec = 0.6\ m^3/sec$$

$$V_1 = \frac{Q}{A_1} = \frac{0.6}{0.2827} = 2.122 m/sec$$

$$V_2 = \frac{Q}{A_2} = \frac{0.6}{0.07068} = 8.488 m/sec$$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

But $Z_1 = Z_2$, then

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{8.829 \times 10^4}{1000 \times 9.81} + \frac{2.122^2}{2 \times 9.81} = \frac{P_2}{1000 \times 9.81} = \frac{8.488^2}{2 \times 9.81}$$

$$9 + 0.2295 = \frac{P_2}{9810} + 3.672 \qquad \frac{P_2}{9810} = 9.2295 - 3.672 = 5.5575 \text{ m of water}$$

$P_2 = 5.5575 \times 9810 = 5.45 \times 10^4\ N/m^2$

Force exerted on the bend in X and Y – directions

$F_x = \square\ Q\ (V_1 - V_2 \, Cos\ \square) + P_1 A_1 - P_2 A_2 \, Cos\ \square$

$= 1000 \times 0.6\ (2.122 - 8.488\ Cos45^{\circ}) + 8.829 \times 10^4 \times 0.2827 - 5.45 \times 10^4 \times 0.07065 \times Cos45^{\circ}$

$= -\ 2327.9 + 24959.6 - 2720.3 \quad = 24959.6 - 5048.2 = 19911.4N$

$$\mathbf{F_x = 19911.4\ N}$$

$F_y = \square\ Q\ (-V_2 \, Sin\ \square) - P_2 A_2 \, Sin\ \square$

$= 1000 \times 0.6\ (-\ 8.488\ Sin45^{\circ}) - 5.45 \times 10^4 \times 0.07068\ Sin45^{\circ}$

$= -\ 3601.1 - 2721.1 = -\ 6322.2N$

(- ve sign means Fy is acting in the down ward direction)

$$\mathbf{F_y = -\ 6322.2N}$$

Therefore the Resultant Force $F_R = \square_{\square}{}^2 + \square_{\square}{}^2 = (19911.4)^2 + (-\ 6322.2)^2 = 20890.9N$

$$\mathbf{F_R = 20890.9\ N}$$

The angle made by resultant force with X – axis is Tan $\square = \frac{F_x}{F_y}$

$= (6322.2/19911.4) = 0.3175$

$$\mathbf{\square = tan^{-1} 0.3175 = \square\square^{\square}\square\square'}$$