Code: 20A54402 **R20**

B.Tech II Year I Semester (R20) Supplementary Examinations August/September 2023

NUMERICAL METHODS & PROBABILITY THEORY

(Food Technology)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) How Regula-falsi method is different from Secant method. 2M

(b) How many iterations is needed to obtain an approximation with accuracy 10^{-5} to the solution $f(x) = x^3 + 4x^2 - 10 = 0$ lying in the interval [1, 2]? 2M

(c) Construct Lagrange's interpolating polynomial using the following data: 2M
$f(0) = 1, f(-1) = 2, f(1) = 3$

(d) If $f(x) = x^2$, then find the second order divided difference for the points x_0, x_1, x_2. 2M

(e) Use Trapezoidal rule to find the value of $\int_0^2 f(x)dx$ from the following data; 2M

x :	0	1	2
f(x):	4	3	12.

(f) If $f(0) = 1$ *and* $f(1) = 2.72$, then what is the approximate value of $\int_0^1 f(x)dx$ by Simpson's 1/3 rule? 2M

(g) What is the probability of getting an odd numbers if a fair dice is thrown once? 2M

(h) In class, 30% of students study Hindi, 45% study Maths, and 15% study both Hindi and Maths. If a student is randomly selected, what is the probability that he/she studies Hindi or maths? 2M

(i) Poisson distribution is of discrete or continuous type? Justify your answer. 2M

(j) Draw the normal distribution curve and write its two characteristics. 2M

PART – B

(Answer all the questions: 05 X 10 = 50 Marks)

2 (a) Solve the following system by using Gauss-Seidal method 5M

$20x + y - 2z = 17;$
$3x + 20y - z = -18;$
$2x - 3y + 20z = 25.$

(b) Find a root of $x\,logx - 1.2 = 0$ by using Newton Raphson's method correct up to 3 decimal places. 5M

OR

3 (a) Solve the following system by Gauss-Jordan method. 5M

$2x + 3y + z = 9;$
$x + 2y + 3z = 6;$
$3x + y + 2z = 8.$

(b) Find a positive root of the equation $cosx - 3x + 1 = 0$ by using method of fixed point iteration. 5M

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4 (a) Using Lagrange's formula find the polynomial for the following data: 5M

X:	0	1	2	4
F(x):	2	3	12	147.

(b) Using Newton's Forward Difference formula compute $f(x) = e^x \; for \; x = 0.02$ from the table below: 5M

x :	0	0.1	0.2	0.3	0.4
e^x :	1.0000	1.1052	1.2214	1.3499	1.4918.

OR

5 (a) If f (0) = 1, f (1) = 2, f (2) = 33 and f (3) = 244 then find a cubic spline approximation, assuming M(0) = M(3) = 0. Also, find f (2.5). 5M

(b) The value of x and e^x are given in the following table: 5M

x :	0.61	0.62	0.63	0.64	0.65
e^x :	1.840	1.858	1.877	1.896	1.934.

Find the approximate value of $e^x \; at \; x = 0.644$ by using Stirling formula (up to second differences).

6 (a) Use Taylor series method to find y(0.1) and y(0.2), given that $\frac{dy}{dx} = 3e^x + 2y, \;\; y(0) = 0,$ correct up to 4 decimal accuracy. 5M

(b) Find the value of $\log 2^{1/3}$ from $\int_0^1 \frac{x^2}{1+x^3}$by using Simpson's 1/3 rule with h = 0.25. 5M

OR

7 (a) Use Runge-Kutta method of 4th order to find y(0.2), given $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$, y(0) =1 taking h = 0.2. 5M

(b) Find the approximate value of $\int_0^1 \frac{dx}{1+x}$ by using Trapezoidal rule and estimate the error. 5M

8 (a) If F(x) is the distribution function of x is given by $F(x) = \begin{cases} o, & x \le 1 \\ k(x-1)^4, & 1 < x \le 3 \\ 1, & x > 3 \end{cases}$ then determine the density function f(x) and the value of k. 5M

(b) For any two events, prove that $p\left(B/_A\right) \ge 1 - \frac{p(\bar{B})}{p(A)}$ in general. 5M

OR

9 (a) If A and B are independent events, then prove that A^c and B^c are independent. 5M

(b) From the numbers 1,2, ... 2n+1, three are chosen at random. Prove the probability of these are in A.P is $\frac{3n}{4n^2-1}$. 5M

10 (a) Find the mean and variance of the Binomial distribution. 5M

(b) Let X be normal with mean 105 and variance 25, Find $P(X \leqq 112.5), P(X > 100), P(110.5 < X < 111.25)$. 5M

OR

11 (a) An insurance company has discovered that only about 0.1 percent of the population is involved in a certain type of accident each year. If its 10,000 policy holders were randomly selected from the population, what is the probability that not more than 5 of its clients are involved in such an accident next year? ($e^{-10} = 0.000045$). 5M

(b) If a ticket office can serve at most 4 customers per minute and the average number of customers is 120 per hour, what is the probability that during a given minute the customers will have to wait? 5M

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Code: 20A54402 **R20**

B.Tech II Year I Semester (R20) Supplementary Examinations April/May 2024

NUMERICAL METHODS & PROBABILITY THEORY

(Food Technology)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Explain Iterative method for finding a root of a transcendental equation. 2M

(b) Isolate the roots of the equation $x^3 - 6x + 2 = 0$. 2M

(c) Define the Backward Differences. 2M

(d) Write the Gauss forward Interpolation formula. 2M

(e) Write the Trapezoidal Rule. 2M

(f) Using Euler's method, find $y(0.2)$ given that $\frac{dy}{dx} = y + e^x, y(0) = 0$. 2M

(g) What is the probability that a number selected from the numbers 1, 2, ..., 20 is an even number, when each of the given numbers is equally likely to be selected? 2M

(h) Let X be the random variable that denotes the life in hours of a certain electronic device. The probability density function is; 2M

$$f(x) = \begin{cases} \frac{20,000}{x^3}, & x > 100, \\ 0 & , \text{elsewhere} \end{cases}$$

Find the expected life of this type of device.

(i) The number of emergency admissions each day to a hospital is found to have Poisson distribution with mean 4. Find the probability that on a particular day there will be no emergency admissions. 2M

(j) The continuous random variable X is uniformly distributed with mean 1.5 and variance 27/4. Find P{X > 0). 2M

PART – B

(Answer all the questions: 05 X 10 = 50 Marks)

2 (a) Obtain an approximate root using bisection method, for the equation: $x^4 - 4x - 9 = 0$. 5M

(b) Solve the equations by Jacobi's iteration method: 5M

$20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$.

OR

3 (a) Find a positive root of $x^4 - x = 10$ using Newton-Raphson's method. 5M

(b) Solve the system of equations; 5M

$5x_1 + x_2 - 2x_3 = 2, 3x_1 + 4x_2 - x_3 = -2$, $2x_1 - 3x_2 + 5x_3 = 10$ by using the Gauss-Seidel iteration method.

4 (a) Evaluate the following, the interval of differencing being unity. 5M

(i) $\Delta\left(\tan^{-1} ax\right)$; (ii) $\Delta\left(e^{2x} \log 3x\right)$.

(b) In a certain experiment, the values of x and y were found as follows: 5M

x	0	1	2	3	4	5	6
y	0	1	16	81	256	625	1296

Find the value of y when $x = 2.5$, using Newton's forward interpolation formula.

OR

Contd. in page 2

5 (a) Using Stirling's formula, compute $f(1.22)$ from the following data: 5M

x	1.0	1.1	1.2	1.3	1.4
$f(x)$	0.841	0.891	0.932	0.963	0.985

(b) Using Lagrange's interpolation formula, find the value of y when $x=10$, form the following table: 5M

x	5	6	9	11
y	12	13	14	16

6 (a) Evaluate correct to 4 decimal places, by Simpson's $\frac{3}{8}^{th}$ rule $\int_0^9 \frac{dx}{1+x^3}$. 5M

(b) Using Taylor series method, find $y(0.1)$ correct to 3-decimal places given that $\frac{dy}{dx}=e^x-y^2$, $y(0)=1$. 5M

OR

7 (a) Calculate the value of $\int_0^{\pi/2} \sqrt{\sin x}\, dx$ by Simpson's $\frac{1}{3}^{rd}$ rule, using 7 ordinates. 5M

(b) Apply Runge-Kutta method of 4th order, to find an approximate value of y when $x=0.2$ given that $\frac{dy}{dx}=x+y,\ \ y(0)=1$. 5M

Take $h=0.1$.

8 (a) From a pack of well shuffled cards, one card is drawn. Find the probability that this card is either a king or an ace. 5M

(b) Two cards are drawn one after the other from a well-shuffled deck of 52 cards. Find the probability that both are spade cards, if the first card is (i) replaced, (ii) not replaced. 5M

OR

9 (a) A company has 4 machines A, B, C and D manufacturing bulbs. The machines A, B, C, D produce 50%, 25%, 15% and 10% bulbs respectively. The percentages of defective bulbs produced by the machines A, B, C and D are 2%, 1%, 1%, and 0.5% respectively. Out of the output, one bulb is chosen at random. What is the probability that it is defective? 5M

(b) A random variable X has the following probability distribution: 5M

Values of x	0	1	2	3	4	5	6	7	8
$P(x)$	a	3a	5a	7a	9a	11a	13a	15a	17a

(i) Determine the value of a.

(ii) Find $P(X<3)$, $P(X\geq 3)$, $P(0<X<5)$.

(iii) What is the smallest value of x for which $P(X\leq x)>0.5$?

(iv) Find out the distribution function of X.

10 (a) Out of 800 families with 5 children each, how many would you expect to have (i) 3 boys, (ii) 5 girls, (iii) either 2 or 3 boys? Assume equal probabilities for boys and girls. 5M

(b) The average number of phone calls/minute coming into a switch board between 2 and 4 PM is 2.5. Determine the probability that during one particular minute there will be (i) 0, (ii) 1, (iii) 2, (iv) 3, (v) 4 or fewer, (vi) more than 6, (vii) at most 5 (viii) at least 20 calls. 5M

OR

11 (a) Find the probabilities that a random variable having the standard normal distribution will take on a value: 5M

(i) between 0.87 and 1.28;

(ii) between − 0.34 and 0.62;

(iii) greater than 0.85;

(iv) greater than − 0.65.

(b) In a distribution which is exactly normal, 12% of the items are under 30 and 85% are under 60. Find the mean and standard deviation of the distribution. 5M

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Code: 20A54402 **R20**

B.Tech II Year II Semester (R20) Regular & Supplementary Examinations August/September 2023

NUMERICAL METHODS & PROBABILITY THEORY

(Common to EEE and ME)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

*****

1 Answer the following: (10 X 02 = 20 Marks)

(a) Solve the equation $e^x - x^2 = 0$ using the Regula Falsi method. Use an initial bracket [a, b] = [1, 2]. 2M

(b) Solve the system of algebraic equations using the Gauss-Seidel method: $4x + y = 5$, $3x + 7y = 10$.

(c) State Newton's backward interpolation formula. 2M

(d) State Lagrange's interpolation formula. 2M

(e) Apply Simpson's 1/3 Rule to approximate the value of the definite integral $\int_0^2 (3x^3 - 2x^2 + 5x)\,dx$. 2M

(f) Solve the initial value problem (IVP) using the Modified Euler's method: $\frac{dy}{dx} = x^2 + y$, y(0) = 1, over the interval [0, 0.2] with a step size of h = 0.1. 2M

(g) State the three probability axioms that form the foundation of probability theory. 2M

(h) Explain the addition law of probability and when it is applicable. 2M

(i) Define the uniform distribution and explain its key properties. 2M

(j) Explain the exponential distribution and its relevance in modeling certain types of real-world phenomena. 2M

PART – B

(Answer all the questions: 05 X 10 = 50 Marks)

2 Using Regula falsi position method find the positive root of $xe^x = 2$. 10M

OR

3 Consider the function f(x) = $x^3 - 2x - 5$. Use the bisection method to find a root of the equation f(x) = 0 in the interval [1, 2] correct to five decimal places. 10M

4 Use Newton's backward interpolation formula to estimate the value of f(2.5) based on the following data points: 10M

x:	2.0	2.2	2.4	2.6	2.8
f(x):	2.5	3.0	3.5	4.0	4.5

OR

5 (a) A second degree polynomial passes the points (1, -1), (2, -1), (3, 1), (4, 5). Find the Polynomial f(x). Also find (1.2).

(b) The value of x and e^x are give in the table below:

x	0.61	0.62	0.63	0.64	0.65
e^x	1.840	1.858	1.877	1.896	1.934

Find the approximate value of e^x at x = 0.644 by using Bessel's formula (upto 4rd differences). Choose x_0 = 0.63.

6 Use the Modified Euler's method to approximate the solution of the initial value problem (IVP): $\frac{dy}{dx} = x^2 + y$, y(0) = 1, over the interval [0, 0.4] with a step size of h = 0.2. 10M

OR

7 Solve the initial value problem (IVP) using the fourth-order Runge-Kutta method: $\frac{dy}{dx} = x^2 + y$, y(0) = 1, over the interval [0, 0.6] with a step size of h = 0.2. 10M

Contd. in Page 2

8 (a) If X and Y are independent variables prove that E(X+Y) = E(X) +E(Y) and E (XY) = E(X) E (Y). 5M

(b) For a discrete random variable X with the following probability distribution: 5M

x	0	1	2
P(x)	0.2	0.3	0.5

(i) Calculate the expected value of X (ii) Calculate the variance of X.

OR

9 For a continuous random variable Y with the probability density function (PDF): $f(y) = 3y^2$, $0 \leq y \leq 1$. 10M

(i) Show that the given function is a valid PDF (ii) Calculate the cumulative distribution function (CDF) of Y (iii) Calculate the probability that Y takes a value between 0.2 and 0.6.

10 Consider a binomial distribution with parameters n = 8 and p = 0.6. 10M

(i) Calculate the probability mass function (PMF) for each possible value of X (ii) Find the mean and variance of the distribution.

OR

11 Suppose a random variable X follows a normal distribution with a mean of 70 and a standard deviation of 5. 10M

(i) Calculate the probability that X is between 65 and 75 (ii) Find the value of X that corresponds to the 90th percentile.

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Code: 20A54402

R20

B.Tech II Year II Semester (R20) Regular & Supplementary Examinations April/May 2024

NUMERICAL METHODS & PROBABILITY THEORY

(Common to EEE and ME)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

*****

1 Answer the following: (10 X 02 = 20 Marks)

(a) Write the difference between Bisection, Regula Falsi and Newton Rapson method. 2M

(b) Explain the rate of convergence. 2M

(c) Show that $\left((1+\Delta)(1-\nabla)\equiv 1\right)$. 2M

(d) Evaluate the $f(x)$ $at\ x = 4$ by using Lagrange's interpolation formula: 2M

x :	3	5
$y = f(x)$:	6	24

(e) Evaluate the value of y (1) by Taylor's series method for the differential equation 2M

$\frac{dy}{dx} = -x\,y^2, y(0) = 1.$

(f) Discuss Trapezoidal rule for integration. 2M

(g) The distribution function of a random variable X is given by 2M

$$F(X) = \begin{cases} 1-(1+x)\,e^{-x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

Determine the density function of random variable X.

(h) A die is thrown. Find the probability of getting a composite number. 2M

(i) A manufacturer knows that the razor blades he makes contain on an average 0.5% of defectives. He packs them in packets of 5. What is the probability that a packet picked at random will contain 3 or more faulty blades? 2M

(j) Suppose that X has a Poisson distribution. If $P(X=2) = \frac{2}{3}P(X=1)$. Find $P(X=0)$. 2M

PART – B

(Answer all the questions: 05 X 10 = 50 Marks)

2 Solve the following equations by Gauss-Jorden method: 10M

$2x - 3y + z = -1;\quad x + 4y + 5z = 25$ and $3x - 4y + z = 2$.

OR

3 Calculate the root of the equation $x\,log_{10}x = 1.2$ using the Newton Rapson method correct to four decimal places. 10M

4 Find the value of e^x when $x = 0.644$ by using Stirling's formula: 10M

x :	0.61	0.62	0.63	0.64	0.65	0.66	0.67
$y = e^x$:	1.840431	1.858928	1.877610	1.896481	1.915541	1.934792	1.954237

OR

Contd. in Page 2

5 Evaluate f(1.235) using backward interpolation formula from the following table. 10M

x :	1.00	1.05	1.10	1.15	1.20	1.25
f(x) :	0.682689	0.706282	0.728668	0.749856	0.769861	0.788700

6 Apply Modified Euler's method to find an approximate value of *y* when *x* = 0.3, given that $\frac{dy}{dx} = y\,x$ and *y* = 2 when *x* = 1. Taking *h* = 0.2. 10M

OR

7 Use Runge-kutta method of fourth order to approximate y when x = 1.4, given that $\frac{dy}{dx} = y + x$ and *y* = 1 when *x* = 0. 10M

8 A businessman goes to hotels X, Y, Z, 20%, 50%, 30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in X, Y, Z hotels have faulty plumbings. What is the probability that business man's room having faulty plumbing is assigned to hotel Z? 10M

OR

9 A can hit a target 3 times in 5 shots, B hits target 2 times in 5 shots, C hits target 3 times in 4 shots. Find the probability of the target being hit when all of them try. 10M

10 Determine the mean and variance of Poisson Distribution. 10M

OR

11 In a normal distribution 31% of items are under 45 and 8% are over 64. Obtain the mean and standard deviation of the distribution. [Area 0.19 is Z = 0.496 and Area 0.42 is Z = 1.405]. 10M

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