UNIT-3
ANALYSIS OF PIPE FLOW

Boundary Layer Characteristics

The concept of boundary layer was first introduced by a German scientist, Ludwig Prandtl, in the year 1904. Although, the complete descriptions of motion of a viscous fluid were known through Navier-Stokes equations, the mathematical difficulties in solving these equations prohibited the theoretical analysis of viscous flow. Prandtl suggested that the viscous flows can be analyzed by dividing the flow into two regions; one close to the solid boundaries and other covering the rest of the flow. Boundary layer is the regions close to the solid boundary where the effects of viscosity are experienced by the flow. In the regions outside the boundary layer, the effect of viscosity is negligible and the fluid is treated as inviscid. So, the boundary layer is a buffer region between the wall below and the inviscid free-stream above. This approach allows the complete solution of viscous fluid flows which would have been impossible through Navier-Stokes equation. The qualitative picture of the boundary-layer growth over a flat plate is shown in Fig. 1.

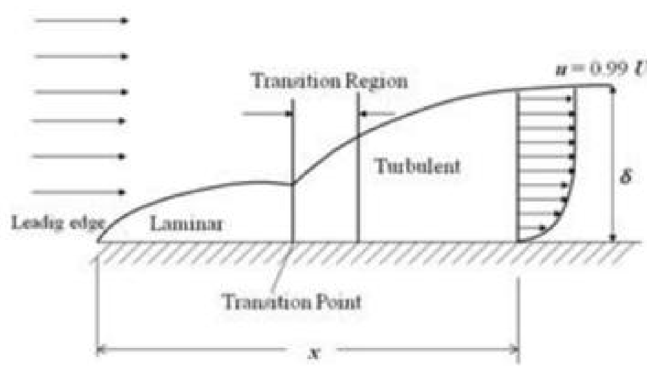


Fig. 1: Representation of boundary layer on a flat plate.

A laminar boundary layer is initiated at the leading edge of the plate for a short distance and extends to downstream. The transition occurs over a region, after certain length in the downstream followed by fully turbulent boundary layers. For common calculation purposes, the transition is usually considered to occur at a distance where the Reynolds number is about 500,000. With air at standard conditions, moving at a velocity of 30m/s, the transition is expected to occur at a distance of about 250mm. A typical boundary layer flow is characterized by certain parameters as given below;

Boundary layer thickness: It is known that no-slip conditions have to be satisfied at the solid surface: the fluid must attain the zero velocity at the wall. Subsequently, above the wall, the effect of viscosity tends to reduce and the fluid within this layer will try to approach the free stream velocity. Thus, there is a velocity gradient that develops within the fluid layers inside the small regions near to solid surface. The *boundary layer thickness* is defined as the distance from the surface to a point where the velocity is reaches 99% of the free stream velocity. Thus, the velocit y profile merges smoothly and asymptotically into the free stream as shown in Fig. 2.

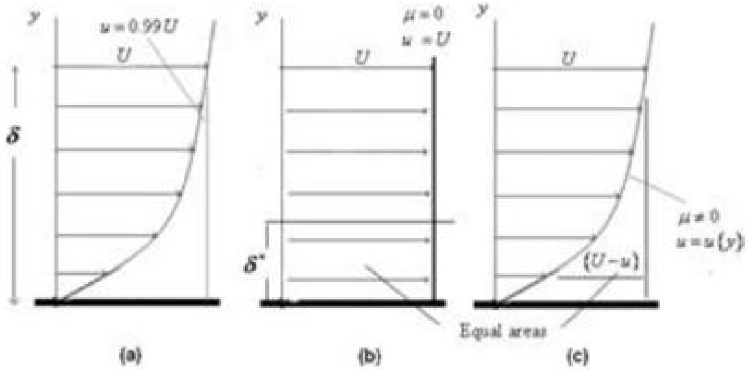


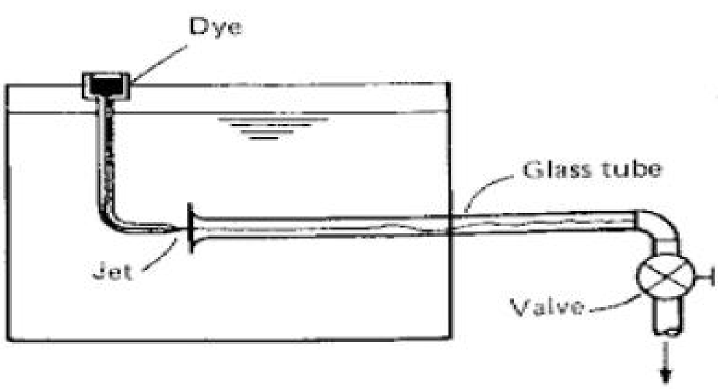
Fig. 5.7.3: (a) Boundary layer thickness; (b) Free stream flow (no viscosity);
(c) Concepts of displacement thickness.

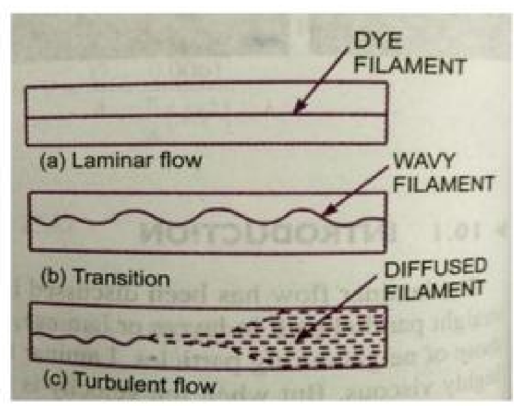
Fig. 2: (a) Boundary layer thickness; (b) Free stream flow (no viscosity);

(c) Concepts of displacement thickness.

CLOSED CONDUIT FLOW:

REYNOLDS EXPERIMENT: It consists of a constant head tank filled with water, a small tank containing dye, a horizontal glass tube provided with a bell-mouthed entrance a and regulating valve. The water was made to flow from the tank through the glass tube in to the atmosphere and the velocity if flow was varied by adjusting the regulating valve. The liquid dye having the same specific weight as that of water was introduced in to the flow at the bell – mouth through a small tube





From the experiments it was disclosed that when the velocity of flow was low, the dye remained in the form of a straight line and stable filament passing through the glass tube so steady that it scarcely seemed to be in motion with increase in the velocity of flow a critical state was reached at which the filament of dye showed irregularities and began of waver. Further increase in the velocity of flow the fluctuations in the filament of dye became more intense and ultimately the dye diffused over the entire cross-section of the tube, due to intermingling of the particles of the flowing fluid

Reynolds"s deduced from his experiments that at low velocities the intermingling of the fluid particles was absent and the fluid particles moved in parallel layers or lamina, sliding past the adjacent lamina but not mixing with them. This is the laminar flow. At higher velocities the dye filament diffused through the tube it was apparent that the intermingling of fluid particles was occurring in other words the flow was turbulent. The velocity at which the flow changes from the laminar to turbulent for the case of a given fluid at a given temperature and in a given pipe is known as Critical Velocity. The state of flow in between these types of flow is known as transitional state or flow in transition.

Reynolds discovered that the occurrence of laminar and turbulent flow was governed by the relative magnitudes of the inertia and the viscous forces. At low velocities the viscous forces become predominant and flow is viscous. At higher velocities of flow the inertial forces predominance over viscous forces. Reynolds related the inertia to viscous forces and arrived at a dimension less parameter.

$$□_□ \; □□ \; □ = \frac{\text{inertia force}}{\text{viscous force}} = \frac{□_□}{□_□}$$

According to Newton"s 2nd law of motion, the inertia force F_i is given by

F_i = mass × acceleration

= ρ × volume × acceleration ρ = mass density

$= \rho \times L^3 \times \frac{L}{T^2} = \rho L^2 V^2$ ---------- (1) L = Linear dimension

Similarly viscous force F_V is given by Newton's 2nd law of velocity as

$F_V = \tau \times$ area τ = shear stress

$= \mu \frac{dv}{dy} \times L^2 = \mu VL$ ---------------- (2) V = Average Velocity of flow

μ = Viscosity of fluid

$$R_e \text{ or } N_R = \frac{\rho L^2 V^2}{\mu VL} = \frac{\rho VL}{\mu}$$

In case of pipes L = D

In case of flow through pipes $= \frac{\rho DV}{\mu}$ or $\frac{□□}{□}$ □

$□_□$

Where μ/ρ = kinematic viscosity of the flowing liquid ν

The Reynolds number is a very useful parameter in predicting whether the flow is laminar or turbulent.

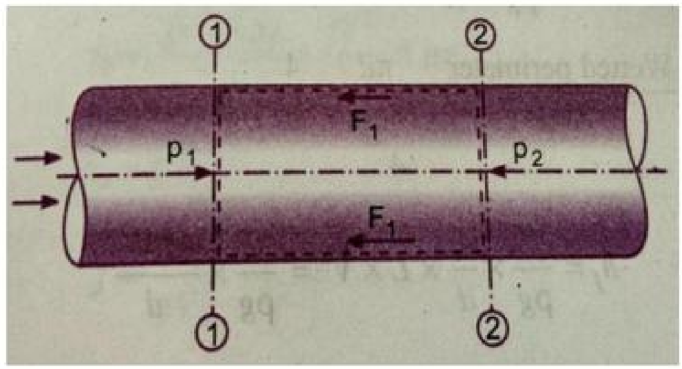
$R_e < 2000$ viscous / laminar flow

$R_e \rightarrow 2000$ to 4000 transient flow

$R_e > 4000$ Turbulent flow

FRICTIONAL LOSS IN PIPE FLOW - DARCY WEISBACK EQUATION

When a liquid is flowing through a pipe, the velocity of the liquid layer adjacent to the pipe wall is zero. The velocity of liquid goes on increasing from the wall and thus velocity gradient and hence shear stresses are produced in the whole liquid due to viscosity. This viscous action causes loss of energy, which is known as frictional loss.



Consider a uniform horizontal pipe having steady flow. Let 1-1, 2-2 are two sections of pipe.

Let P_1 = Pressure intensity at section 1-1

V_1 = Velocity of flow at section 1-1

L = Length of pipe between section 1-1 and 2-2

d = Diameter of pipe

f' = Fractional resistance for unit wetted area per a unit velocity h_f = Loss of head due to friction

And P_2, V_2 = are values of pressure intensity and velocity at section 2-2

Applying Bernoulli's equation between sections 1-1 and 2-2

Total head at 1-1 = total head at 2-2 + loss of head due to friction between 1-1 and 2-2

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

$Z_1 = Z_2$ as pipe is horizontal

$V_1 = V_2$ as dia. of pipe is same at 1-1 and 2-2

$$\frac{p_1}{\rho g} = \frac{p_2}{\rho g} + h_f \quad \text{Or}$$

$$\square_{\square} = \frac{\square_1}{\square\square} - \frac{\square_2}{\square\square} \qquad (1)$$

But h_f is head is lost due to friction and hence the intensity of pressure will be reduced in the direction flow by frictional resistance.

Now, Frictional Resistance = Frictional resistance per unit wetted area per unit velocity × Wetted Area × $(\text{velocity})^2$

$\square_1 = \square' \times \square\square\square \times \square^2$ [∵ Wetted area $= \square\square \times \square$, Velocity $= V = V_1 = V_2$]

$\square_1 = \square' \times \square\square\square^2$ ____________(2) [∵ $\square d$ = perimeter = p]

The forces acting on the fluid between section 1-1 and 2-2 are
Pressure force at section 1-1 = $P_1 \times A$ where A = area of pipe
Pressure force at section 2-2 = $P_2 \times A$
Frictional force = F_1
Resolving all forces in the horizontal direction, we have

$$P_1 A - P_2 A - F_1 = 0$$

$$(P_1 - P_2)A = F_1 = \square' \times \square \times \square \times \square^2 \quad \text{from equation} - (2)$$

$$P_1 - P_2 = \frac{\square' \times \square \times \square \times \square^2}{A}$$ But from equation (1) $P_1 - P_2 = \rho g h_f$

Equating the value of $P_1 - P_2$, we get

$$\rho g h_f = \frac{\square' \times \square \times \square \times \square^2}{A}$$

$$\frac{\square}{\square} = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(3)}$$

In the equation (3) $\frac{P}{A} = \frac{\text{Wetted Permiter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{\pi}{d}$

$$\frac{\square}{\square} = \frac{\square'}{\square\square} \times \frac{4}{\square} \times \square \times \square^2 = \frac{\square'}{\square\square} \times \frac{4\square\square^2}{\square}$$

Putting $\frac{\square'}{\square} = \frac{\square}{2}$ Where f is known as co-efficient of friction.

Equation (4) becomes as

$$h_f = \frac{4f}{2g} \times \frac{LV^2}{d}$$

$$h_f = \frac{4fLV^2}{2gd}$$

This Equation is known as Darcy – Weisbach equation, commonly used for finding loss of head due to friction in pipes

Then f is known as a friction factor or co-efficient of friction which is a dimensionless quantity. f is not a constant but, its value depends upon the roughness condition of pipe surface and the Reynolds number of the flow.

MINOR LOSSES IN PIPES:

The loss of energy due to friction is classified as a major loss, because in case of long pipe lines it is much more than the loss of energy incurred by other causes.

The minor losses of energy are caused on account of the change in the velocity of flowing fluids (either in magnitude or direction). In case of long pipes these loses are quite small as compared with the loss of energy due to friction and hence these are termed as „"minor losses „"

Which may even be neglected without serious error However in short pipes these losses may sometimes outweigh the friction loss. Some of the losses of energy which may be caused due to the change of velocity are:

1 Loss of energy due to sudden enlargement $h_e = \frac{(V_1 - V_2)^2}{2g}$

2 Loss of energy due to sudden contraction $h_c = 0.5 \frac{V_2^2}{2g}$

3 Loss of energy at the entrance to a pipe $h_i = 0.5 \frac{V^2}{2g}$

4 Loss of energy at the exit from a pipe $h_o = \frac{V^2}{2g}$

5 Loss of energy due to gradual contraction or enlargement $h_L = \frac{k(V_1 - V_2)^2}{2g}$

6 Loss of energy in the bends $h_b = \frac{kV^2}{2g}$

7 Loss of energy in various pipe fittings h

$$= \frac{\square\square^2}{2\square}$$

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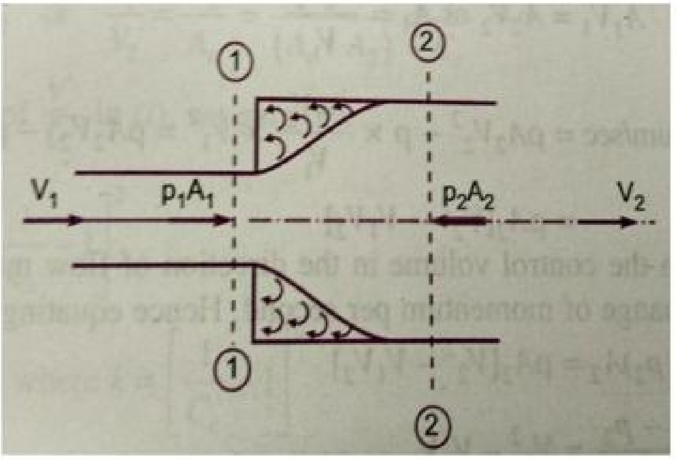
1. LOSS OF HEAD DUE TO SUDDEN ENLARGEMENT

Consider a liquid flowing through a pipe which has sudden enlargement. Consider two sections 1-1 and 2-2 before and after enlargement. Due to sudden change of diameter of the pipe from D_1 to D_2. , The liquid flowing from the smaller pipe is not able to fallow the abrupt change of the boundary. Thus the flow separates from the boundary and turbulent eddies are formed. The loss of head takes place due to the formation of these eddies.



Let p' = Pressure intensity of the liquid eddies on the area $(A_2 - A_1)$

h_e = loss of head due to the sudden enlargement.

Applying Bernoulli"s equation at section 1-1 and 2-2

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{Loss of head due to sudden enlargement}$$

But $z_1 = z_2$ as pipe ishorizontal

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

Or

$$h_e = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} \qquad \text{——————(1)}$$

The force acting on the liquid in the control volume in the direction of flow

$$F_x = p_1 A_1 + p'(A_2 - A_1) - p_2 A_2$$

But experimentally it is found that $p' = p_1$

$$F_x = p_1 A_1 + p_1(A_2 - A_1) - p_2 A_2$$

$$= p_1 A_2 - p_2 A_2$$

$$= (p_1 - p_2) A_2 \qquad \text{——————(2)}$$

Momentum of liquid/ second at section 1-1 = mass × velocity

$$= \rho A_1 V_1 \times V_1$$

$$= \rho A_1 V_1^2$$

Momentum of liquid/ second at section 2-2 $= \rho A_2 V_2 \times V_2 = \rho A_2 V_2^2$

Change of momentum/second = $\square\square_2\square_2^2 - \square\square_1\square_1^2$ —————— (3)

But from continuity equation, we have

$$A_1V_1 = A_2V_2$$

Or

$$V_1 = \frac{A_2V_2}{A_1}$$

$\therefore$ Change of momentum/sec $= \rho A_2 V_2^2 - \rho A_1 \times \frac{A_2V_2}{A_1} \times V_1$

$$= \rho A_2 V_2^2 - \rho A_2 V_1 V_2$$

$$= \rho A_2 \left[V_2^2 - V_1V_2\right] \quad \text{(4)}$$

Now the net force acting on the control volume in the direction of flow must be equal to rate of change of momentum per second. Hence equating equation (2) and equation (4)

$$(p_1 - p_2)A_2 = \rho A_2 \left[V_2^2 - V_1V_2\right]$$

$$\frac{p_1 - p_2}{\rho} = V_2^2 - V_1V_2$$

Dividing both sides by „g" we have $\frac{p_1 - p_2}{\rho g} = \frac{V_2^2 - V_1V_2}{g}$

Or $$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{V_2^2 - V_1V_2}{g}$$

Substituting in equation (1)

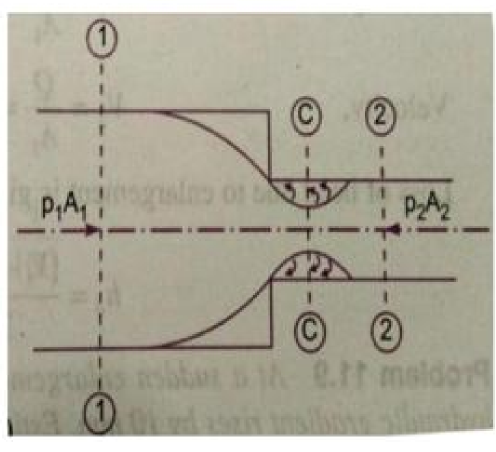
$$h_e = \frac{V_2^2 - V_1V_2}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g}$$

$$= \frac{V_1^2 + V_2^2 - 2V_1V_2}{2g}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

2. LOSS OF HEAD DUE TO SUDDEN CONTRACTION

Consider a liquid flowing in a pipe, which has a sudden contraction in area. Consider two sections 1-1 and 2-2 before and after contraction. As the liquid flows from larger pipe to smaller pipe, the area of flow goes on decreasing and becomes minimum at section C - C. This section is called Vena-contracta. After section C-C, a sudden enlargement of area takes place. The loss of head due to sudden contraction is actually due to sudden enlargement of area from vena-contracta to smaller pipe.



Let A_c = Area of flow at section C - C

V_c = Velocity of flow at section C – C

A_2 = Area of flow at section 2 - 2

V_2 = Velocity of flow at section 2 - 2

h_c = Loss of head due to sudden contraction

Now, h_c = Actual loss of head due to sudden enlargement from section C - C to section 2-2 is

$$h_c = \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left(\frac{V_c}{V_2} - 1\right)^2 \qquad \text{——————} \quad (1)$$

From continuity equation, we have

$$A_c V_c = A_2 V_2 \qquad \text{Or} \qquad \frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{\frac{A_c}{A_2}} = \frac{1}{C_c} \qquad \because C_c = \frac{A_c}{A_2}$$

Substituting the value of $\frac{V_c}{V_2}$ in equation (1)

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2 = \frac{k V_2^2}{2g}, \qquad \text{Where } k = \left(\frac{1}{C_c} - 1\right)^2$$

If the value of C_c is assumed to be equal to 0.62, then

$$k = \left(\frac{1}{0.62} - 1\right)^2 = 0.375$$

Then h_c becomes as

$$h_c = \frac{k V_2^2}{2g} = 0.375\,\frac{V_2^2}{2g}$$

If the value of C_c is not given, then the head loss due to contraction is taken as

$$= 0.5\,\frac{V_2^2}{2g}$$

3. LOSS OF HEAD AT THE ENTRANCE OF A PIPE

This is the loss of energy which occurs when a liquid enters a pipe which is connected to a large tank or reservoir. This loss is similar to the loss of head due to sudden contraction. This loss depends on the form of entrance. For a sharp edge entrance, this loss is slightly more than a rounded or bell mouthed entrance. In practice the value of loss of head at the entrance. In practice the value of loss of head at the entrance (inlet) of a pipe with sharp cornered entrance is taken as 0.5 $v^2/2g$ where v = velocity of liquid in pipe. This loss is denoted by h_i

$$h_i = 0.5\,\frac{V^2}{2g}$$

4. LOSS OF HEAD AT THE EXIT OF A PIPE

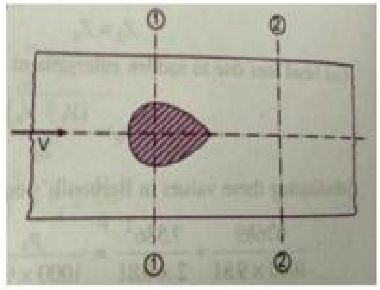
This loss of head (or energy) due to the velocity of the liquid at the out let of the pipe, which is dissipated either in the form of a free jet (if the out let of the pipe is free) or it is lost in the tank or reservoir. This loss is equal to $\frac{V^2}{2g}$, where V is the velocity of liquid at the out let of the pipe. This loss is denoted by h_o.

$$h_o = \frac{V^2}{2g} \qquad V = \text{velocity at outlet of the pipe}$$

5. LOSS OF HEAD DUE TO OBSTRUCTION IN A PIPE

Whenever there is an obstruction in a pipe, the loss of energy take place due to reduction of the area of the cross section of the pipe at the place where obstruction is present. There is a sudden enlargement of the area of flow beyond the obstruction due to which loss of head takes place.

Consider a pipe of area of cross- section A having an obstruction



Let a = max. Area of obstruction

A = Area of pipe

V = velocity of liquid in pipe

Then (A - a) = Area of flow of liquid at section 1-1

As the liquid flows and passes through section 1-1, a vena - contracta is formed beyond section 1-1- after which the stream of liquid widens again and velocity of flow at section on 2-2 become uniform and equal to velocity, v in the pipe. This situation is similar to the flow of liquid through sudden enlargement.

Let V_c = velocity of liquid at vena –contracta

Then loss of head due to obstruction = loss of head due to enlargement from vena – contracta to section 2-2

$$= \frac{(V_c - V)^2}{2g} \text{ -------- (1)}$$

From continuity equation we have $a_c \times V_c = A \times V$ ______________(2)

Where a_c = Area of cross section at vena-contracta

If C_c = co-efficient of contraction

Then $$C_c = \frac{\text{Area at vena contracta}}{A-a} = \frac{a_c}{A-a}$$

$$\therefore a_c = C_c (A-a)$$

Substituting this value in equation (2) $C_c (A-a) V_c = A V$

$$\therefore V_c = \frac{A \times V}{C_c (A-a)}$$

Substituting this value of V_C in equation (1)

$$\text{Head loss due to obstruction} = \frac{(V_c - V)^2}{2g} = \frac{\left(\frac{A \times V}{C_c(A-a)} - V\right)^2}{2g} = \frac{V^2}{2g}\left(\frac{A}{C_c(A-a)} - 1\right)^2$$

6. LOSS OF HEAD DUE TO BEND IN PIPE:

When there is a bend in a pipe, the velocity flow changes, due to which separation of the flow from the boundary and also formation of eddies takes place, thus the energy is lost.

Loss of head in pipe due to bend is expressed as

$$h_b = \frac{kV^2}{2g}$$

Where, = Loss of head due to bend, V = Velocity of flow, k = Co-efficient of bend.

The value of k depends on

1. Angle of bend,
2. Radius of curvature of bend,
3. Diameter of pipe.

7. LOSS OF HEAD IN VARIOUS PIPE FITTINGS:

The loss of head in various pipe fittings such as valves, couplings etc. is expressed as

$$= \frac{\square\square^2}{2\square}.$$

Where V= Velocity of flow, k= co-efficient of pipe fitting.

LOSS OF ENERGY DUE TO GRADUAL CONTRACTION OR ENLARGEMENT:

The loss of energy can be considerably reduced if in place of a sudden contraction or sudden enlargement a gradual contraction or gradual enlargement is provided. This is because in gradual contraction or enlargement the velocity of flow is gradually increased or reduced, the formation of eddies responsible for dissipation of energy are eliminated.

The loss of head in gradual contraction or gradual enlargement is expressed as

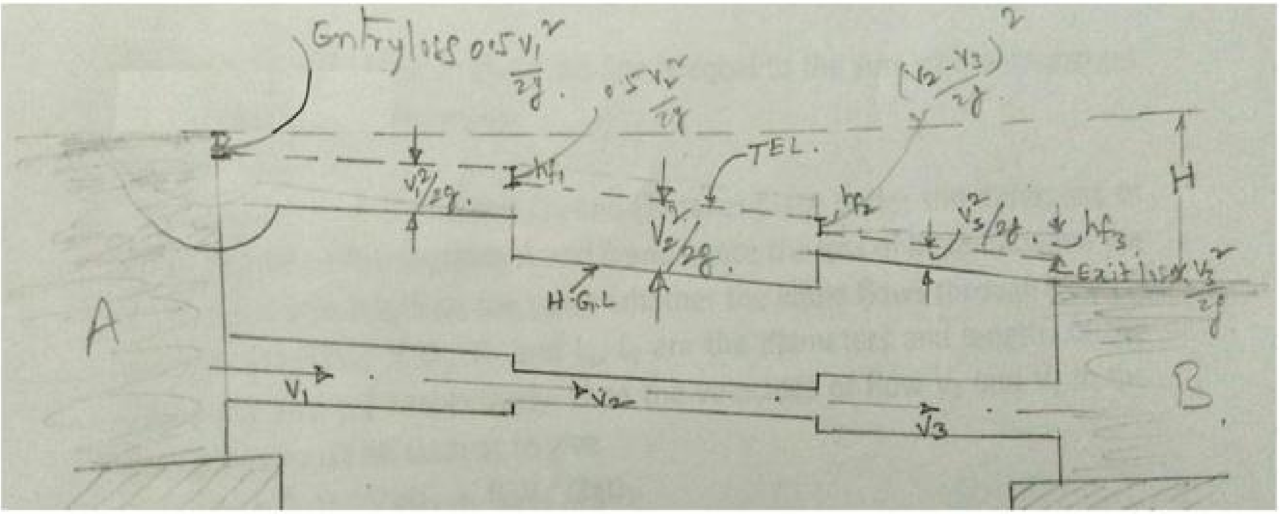
$$\square_{\square} = \frac{\square\;\square_1 - \square_2{}^2}{2\square}$$

Where k is a co-efficient and V_1 and V_2 are the mean velocities at the inlet and the out let the value of K depends on the angle of convergence or divergence and on the ratio of the upstream and the downstream cross-sectional areas. For gradual contraction the value of K is very small even for larger values of the angle of convergence. For gradual contraction without sharp corners the loss of energy caused is so small that it may be neglected.

For gradual enlargement the value of K depends on the angle of divergence.

The value of K increases as the angle of divergence increases for a given ratio of the cross-sectional areas at the inlet and at the outlet. In the case of gradual enlargement, except for very small angles of divergence, the flow of fluid is always subjected to separation from the boundaries and consequent formation of the eddies resulting in loss of energy. Therefore in the case of gradual enlargement the loss of energy can't be completely eliminated.

PIPES IN SERIES:



If a pipe line connecting two reservoirs is made up of several pipes of different diameters d_1, d_2, d_3, etc. and lengths L_1, L_2, L_3 etc. all connected in series (i.e. end to end), then the difference in the liquid surface levels is equal to the sum of the head losses in all the sections. Further the discharge through each pipe will be same.

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2g\,d_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2g\,d_2} + \frac{0.5V_3^2}{2g} + \frac{4f_3L_3V_3^2}{2g\,d_3}$$

Also $$Q = \frac{\pi \times d_1^2}{4} \times V_1 = \frac{\pi \times d_2^2}{4} \times V_2 = \frac{\pi \times d_3^2}{4} \times V_3$$

However if the minor losses are neglected as compared with the loss of head due to friction in each pipe, then

$$H = \frac{4f_1L_1V_1^2}{2g\,d_1} + \frac{4f_2L_2V_2^2}{2g\,d_2} + \frac{4f_3L_3V_3^2}{2g\,d_3}$$

The above equation may be used to solve the problems of pipe lines in series. There are two types of problems which may arise for the pipe lines in series. Viz.

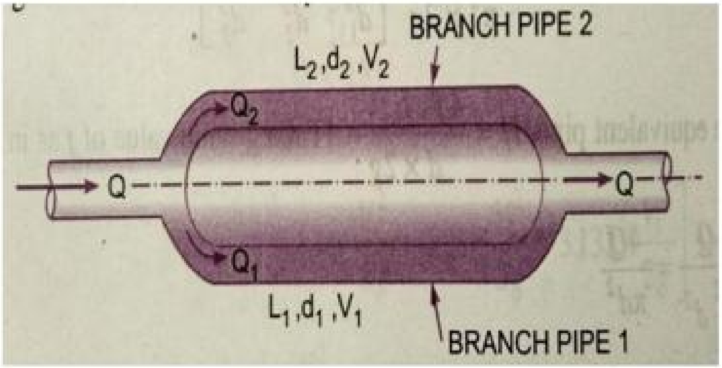
a) Given a discharge Q to determine the head H and
b) Given H to determine discharge Q.

If the co-efficient of friction is same for all the pipes i.e. $f_1 = f_2 = f_3$, then

$$H = \frac{4f_1L_1V_1^2}{d_3^2\,2g\,d_3} + \frac{L_2V_2^2}{d_1} + \frac{L_3}{d_2}$$

PIPES IN PARALLEL:

When a main pipeline divides in to two or more parallel pipes, which may again join together downstream and continue as main line, the pipes are said to be in parallel. The pipes are connected in parallel in order to increase the discharge passing through the main. It is analogous to parallel electric current in which the drop in potential and flow of electric current can be comp ared to head loss and rate of discharge in a fluid flow respectively.



The rate of discharge in the main line is equal to the sum of the discharges in each of the parallel pipes.

Thus $Q = Q_1 + Q_2$

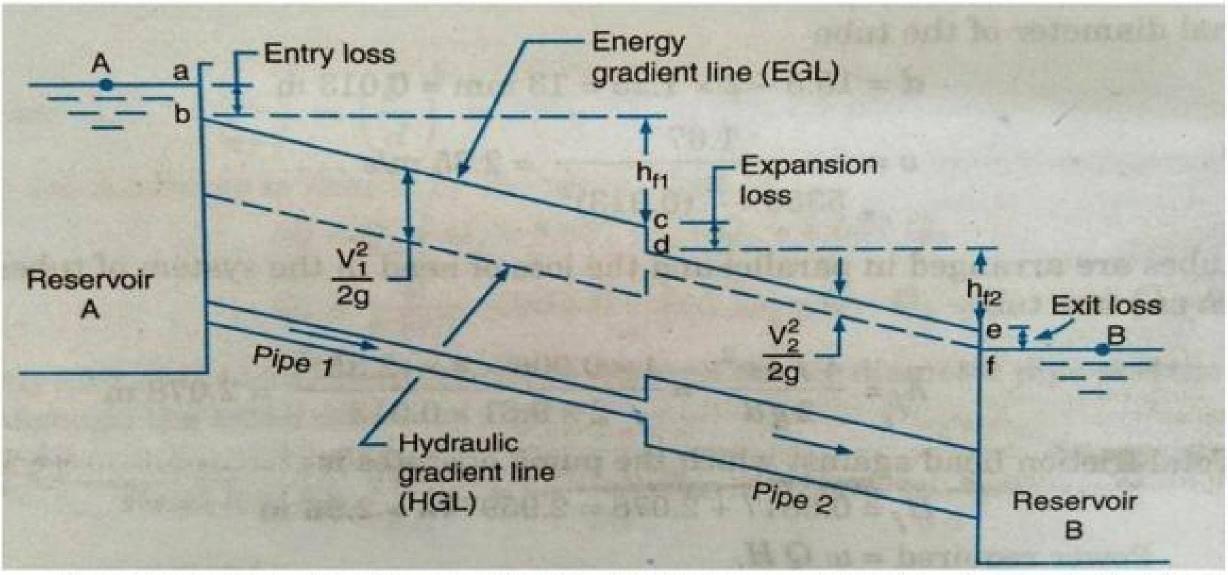
The flow of liquid in pipes (1) and (2) takes place under the difference of head between the sections A and B and hence the loss of head between the sections A and B will be the same whether the liquid flows through pipe (1) or pipe (2). Thus if D_1, D_2 and L_1, L_2 are the diameters and lengths of the pipes (1) and (2) respectively, then the velocities of flow V_1 and V_2 in the two pipes must be such as to give

$$h_f = \frac{f L_1 V_1^2}{2 g D_1} = \frac{f L_2 V_2^2}{2 g D_2}$$

Assuming same value of f for each parallel pipe

$$\frac{L_1 V_1^2}{2 g D_1} = \frac{L_2 V_2^2}{2 g D_2}$$

HYDRAULIC GRADIENT LINE AND TOTAL ENERGY LINE:



Consider a long pipe line carrying liquid from a reservoir A to reservoir B. At several points along the pipeline let piezo meters be installed. The liquid will rise in the piezometers to certain heights corresponding to the pressure intensity at each section. The height of the liquid surface above the axis of the pipe in the piezometer at any section will be equal to the pressure head (p/w) at that section. On account of loss of energy due to friction, the pressure head will decrease gradually from section to section of pipe in the direction of flow. If the pressure heads at the different sections of the pipe are plotted to scale as vertical ordinates above the axis of the pipe and all these points are joined by a straight line , a sloping line is obtained, which is known as Hydraulic Gradient Line (H.G.L).

Since at any section of pipe the vertical distance between the pipe axis and Hydraulic gradient line is equal to the pressure head at that section, it is also known as pressure line. Moreover if Z is the height of the pipe axis at any section above an arbitrary datum, then the vertical height of the Hydraulic gradient line above the datum at that section of pipe represents the piezometric head equal to (p/w + z). Sometimes the Hydraulic gradient line is also known as piezometric head line.

At the entrance section of the pipe for some distance the Hydraulic gradient line is not very well defined. This is because as liquid from the reservoir enters the pipe, a sudden drop in pressure head takes place in this portion of pipe. Further the exit section of pipe being submerged, the pressure head at this section is equal to the height of the liquid surface in the reservoir B and hence the hydraulic gradient line at the exit section of pipe will meet the liquid surface in the reservoir B.

If at different sections of pipe the total energy (in terms of head) is plotted to scale as vertical ordinate above the assumed datum and all these points are joined, then a straight sloping line will be obtained and is known as energy grade line or Total energy line (T.E.L). Since total energy at any section is the sum of the pressure head (p/w), datum head z and velocity head $\frac{V^2}{2g}$ and the vertical distance between the datum and hydraulic grade line is equal to the piezometric head (p/w + z), the energy grade line will be parallel to the hydraulic grade line, with a vertical distance between them equal to —

2□

. at the entrance section of the

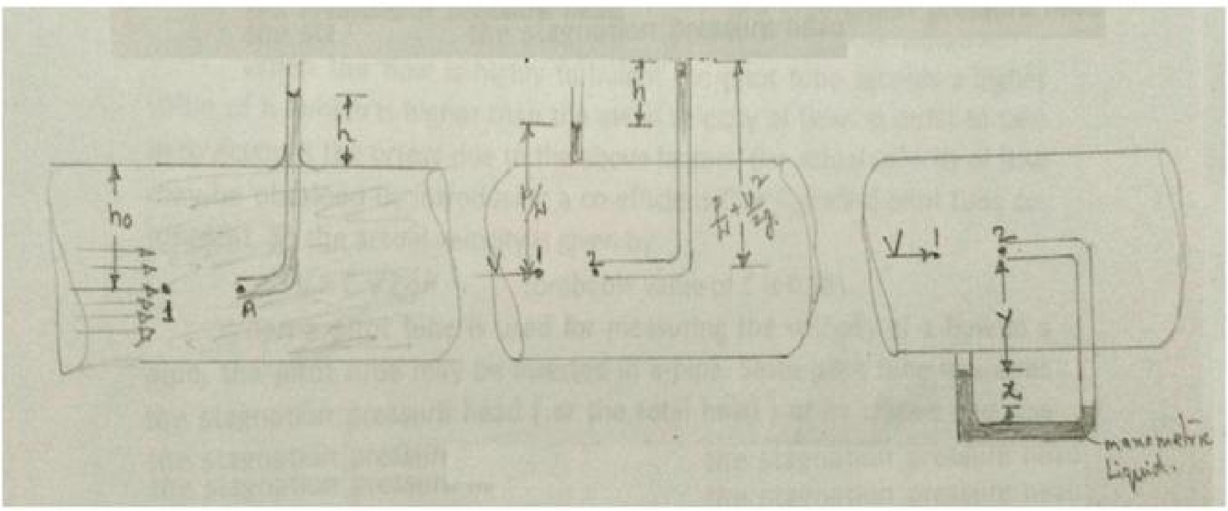
pipe there occurs some loss of energy called "Entrance loss" equal to $h_L = 0.5\frac{V^2}{2g}$ and hence the energy grade line at this section will lie at a vertical depth equal to $0.5\frac{V^2}{2g}$ below the liquid surface in the reservoir A. Similarly at the exit section of pipe, since there occurs an exit loss equal to $h_L = \frac{V^2}{2g}$. The energy gradiant line at this section will lie at a vertical distance equal to $\frac{V^2}{2g}$ above the liquid surface in the reservoir B. Since at any section of pipe the vertical distance between the energy grade line and the horizontal line drawn through the liquid surface in reservoir A will represents the total loss of energy incurred up to that section.

If the pipe line connecting the two reservoirs is horizontal, then the datum may be assumed to be along the pipe axis only. The piezometric head and the pressure head will then become the same.

If a pipe line carrying liquid from reservoir A discharges freely in to the atmosphere at its exit end, the hydraulic grade line at the exit end of the pipe will pass through the centre line of the pipe, since the pressure head at the exit end of the pipe will be zero (being atmospheric). The energy grade line will again be parallel to the hydraulic grade line and it will be at a vertical distance of $\frac{V^2}{2g}$ above the Hydraulic grade line

PITOT – TUBE

A Pitot tube is a simple device used for measuring the velocity of flow. The basic principle used in this is that if the velocity of flow at a particular point is reduced to zero, which is known as stagnation point, the pressure there is increased due to conversion of the kinetic energy in to pressure energy and by measuring the increase in pressure energy at this point , the velocity of flow may be determined.



Simplest form of a pitot tube consists of a glass tube, large enough for capillary effects to be negligible and bent at right angles. A single tube of this type is used for measuring the velocity of flow in an open channel. The tube is dipped vertically in the flowing stream of fluid with its open end A directed to face the flow and other open end projecting above the fluid surface in the stream. The fluid enters the tube and the level of the fluid in the tube exceeds that of the fluid surface in the surrounding stream. This is so because the end A of the tube is a stagnation point, where the fluid is at rest, and the fluid approaching end A divides at this point and passes around tube. Since at stagnation point the kinetic energy is converted in to pressure energy, the fluid in the tube rises above the surrounding fluid surface by a height, which corresponds to the velocity of flow of fluid approaching end A of the tube. The pressure at the stagnation point is known as stagnation pressure.

Consider a point 1 slightly upstream of end A and lying along the same horizontal plane in the flowing stream of velocity V. Now if the point 1 and A are at a vertical depth of h_o from the free surface of fluid and h is the height of the fluid raised in the pitot tube above the free surface of the liquid. Then by applying Bernoulli‟s equation between the point 1 and A, neglecting loss of energy, we get

$$h_o + \frac{V^2}{2g} = (h_o + h)$$

$(h_o + h)$ is the stagnation pressure head at a point A, which consists of static pressure head h_o and dynamic pressure head h. Simplifying the expression,

$$\frac{V^2}{2g} = h \qquad \text{Or} \qquad V = \sqrt{2gh} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (1)}$$

This equation indicates that the dynamic pressure head h is proportional to the square of the velocity of flow close to end A.

Thus the velocity of flow at any point in the flowing stream may be determined by dipping the Pitot tube to the required point and measuring the height „h‟ of the fluid raised in the tube above the free surface. The velocity of flow given by the above equation (1) is more than actual velocity of flow as no loss of energy is considered in deriving the above equation.

When the flow is highly turbulent the Pitot tube records a higher value of h, which is higher than the mean velocity of flow. In order to take in to account the errors due to the above factors, the actual velocity of flow may be obtained by introducing a co-efficient C or C_V called Pitot tube co-efficient. So the actual velocity is given by

$$V = C\sqrt{2gh} \quad \text{(Probable value of C is 0.98)}$$

When a pitot tube is used for measuring the velocity of a flow in a pipe, the Pitot tube may be inserted in a pipe. Since pitot tube measures the stagnation pressure head (or the total head) at its dipped end, the static pressure head is also required to be measured at the same section, where tip of pitot tube is held, in order to determine the dynamic pressure head „h‟. For measuring the static pressure head a pressure tap is provided at this section to which a piezo meter may be connected. Alternatively a dynamic pressure head may also be determined directly by connecting a suitable differential manometer between the pitot tube and pressure tap.

Consider point 1 slightly up stream of the stagnation point 2.

Appling Bernoulli‟s equation between the points 1 and 2, we get

$$\frac{p_1}{w} + \frac{V^2}{2g} = \frac{p_2}{w} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (2)}$$

Where P_1 and P_2 are the pressure intensities at points 1 and 2, V is velocity of flow at point 1 and □ is the specific weight of the fluid flowing through the pipe. P_1 is the static pressure and P_2 is the stagnation pressure. The equation for the pressure through the manometer in meters of water may be written as,

$$\frac{□_1}{□}□□ + □□ + \mp □ + □ □ \frac{□}{2} □ \qquad (3)$$

Where s and s_m are the specific gravities of the fluid flowing in the pipe and the manometric liquid respectively. By simplifying

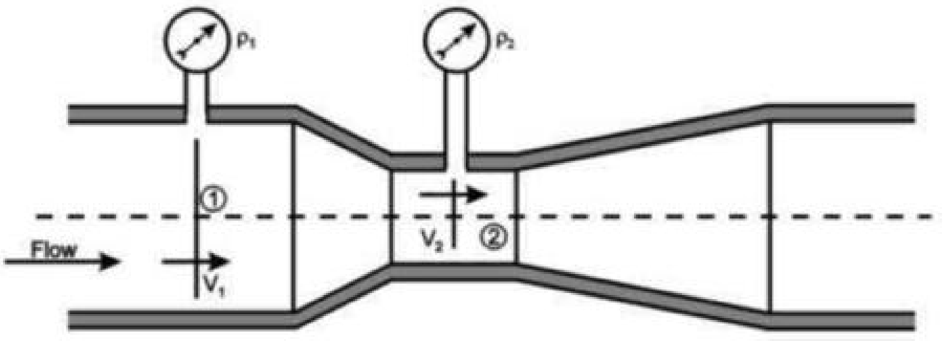
$$\frac{□_2}{□} - \frac{□_1}{□} = □ \frac{□_□}{□} - 1$$

After substituting for $\left(\frac{2}{□} - \frac{□_1}{□} =\right)$ in the equation (2) and solving for V

$$= □ \overline{2□□} \frac{□_□}{□} - 1$$

VENTURI METER

A venture meter is a device used for measuring the rate of flow of fluid through a pipe. The basic principle on which venture meter works is that by reducing the cross-sectional area of the flow passage, a pressure difference is created and the measurement of the pressure difference enables the determination of the discharge through the pipe.



A venture meter consists of (1) an inlet section, followed by a converging cone (2) a cylindrical throat and (3) a gradually divergent cone. The inlet section of venture meter is the same diameter as that of the pipe which is followed by a convergent cone. The convergent cone is a short pipe, which tapers from the original size of the pipe to that of the throat of the venture meter. The throat of the venture meter is a short parallel – sided tube having its cross-sectional area smaller than that of the pipe. The divergent cone of the venture meter is a gradually diverging pipe with its cross-sectional area increasing from that of the throat to the original size of the pipe. At the inlet section and the throat i.e sections 1 and 2 of the venture meter pressure gauges are provided.

The convergent cone of the venture meter has a total included angle of 21° + 1° and its length parallel to the axis is approx. equal to 2.7(D-d), where D is the dia. of pipe at inlet section and d is the dia. of the throat. The length of the throat is equal to d. The divergent cone has a total included angle 5° to 15°, preferably about 6°. This results in the convergent cone of the venture meter to be of smaller length than its divergent cone. In the convergent cone the fluid is being accelerated from the inlet section 1 to the throat section 2, but in the divergent cone the fluid is retarded from the throat section 2 to the end section 3 of the venture meter. The acceleration of the flowing fluid may be allowed to take place rapidly in a relatively small length without resulting in loss of energy. However if the retardation of the flow is allowed to take place rapidly in small length, then the flowing fluid will not remain in contact with the boundary of the diverging flow passage or the flow separates from the walls and eddies are formed and consequent energy loss. Therefore to avoid flow separation and consequent energy loss, the divergent cone is made longer with a gradual divergence. Since separation may occur in the divergent cone this portion is not used for discharge measurement.

Since the cross-sectional area of the throat is smaller than the cross-sectional area of the inlet section, the velocity of flow at the throat will become greater than that at inlet section, according to continuity equation. The increase in the velocity of flow at the throat results in the decrease in the pressure. As such a pressure difference is developed between the inlet section and the throat section of the venture meter. The pressure difference between these sections can be determined either by connecting differential manometer or pressure gauges. The measurement of the pressure difference between these sections enables the rate of flow of fluid to be calculated. For greater accuracy the cross-sectional area of the throat is reduced so that the pressure at the throat is very much reduced. But if the cross-sectional area of the throat is reduced so much that pressure drops below the vapour pressure of the flowing liquid. The formation of vapour and air pockets results in cavitation, which is not desirable. Therefore in order to avoid cavitation to occur, they diameter of the throat can be reduced to 1/3 to 3/4 of pipe diameter, more commonly the diameter of the throat is 1/2 of pipe diameter.

Let a_1 and a_2 be the cross-section l areas at inlet and throat sections, at which P_1 and P_2 the pressures and velocities V_1 and V_2 respectively. Assuming the flowing fluid is incompressible and there is no loss of energy between section 1 and 2 and applying Bernoulli"s equation between sections 1 and 2, we get,

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2$$

Where w is the specific weight of flowing fluid.

If the venturi meter is connected in a horizontal pipe, then $Z_1 = Z_2$, then

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g}$$

$$\frac{P_1}{w} - \frac{P_2}{w} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

In the above expression $\frac{P_1}{w} - \frac{P_2}{w}$ is the difference between the pressure heads at section 1 and 2 , is known as venture head and is denoted by h

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$Q_{th} = a_1 V_1 = a_2 V_2 ,$$

$$V_1 = \frac{Q_{th}}{a_1}, \quad V_2 = \frac{Q_{th}}{a_2}$$

$$h = \frac{Q_{th}^2}{2g}\left(\frac{1}{a_2^2} - \frac{1}{a_1^2}\right)$$

$$Q_{th} = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$Q = C_d Q_{th}$$

$$= \frac{C_d a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

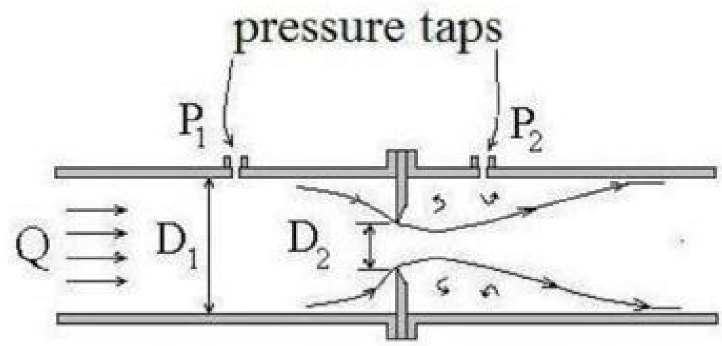
$$= C_d K \sqrt{h} \qquad \because K = \frac{a_1 a_2 \sqrt{2g}}{\sqrt{a_1^2 - a_2^2}}$$

$$Q_{actual} = C_d K \sqrt{h}$$

C_d = Co-efficient of discharge < 1

ORIFICE METER

An orifice meter is a simple device for measuring the discharge through pipes. Orifice meter also works on the same principle as that of venture meter i.e by reducing cross-sectional area of the flow passage, a pressure difference between the two sections is developed and the measurement of the pressure difference enables the determination of the discharge through the pipe. Orifice meter is a cheaper arrangement and requires smaller length and can be used where space is limited.



An orifice meter consists of a flat circular plate with a circular hole called orifice, which is concentric with the pipe axis. The thickness of the plate t is less than or equal to 0.05 times the diameter of pipe. From the upstream face of the plate the edge of the orifice is made flat for a thickness 0.02 times the diameter of pipe and the remaining thickness the plate is beveled with the bevel angle 45°. The diameter of the orifice is kept at 0.5 times the diameter of pipe. Two pressure taps are provided one at section 1 on upstream side of the orifice plate and other at section 2 on the downstream side of the orifice plate. The upstream tap is located at a distance of 0.9 to 1.1 times the pipe diameter from the orifice plate. The position of downstream pressure tap, however depends on the ratio of the orifice diameter and the pipe diameter. Since the orifice diameter is less than pipe diameter as the fluid flows through the orifice, the flowing stream converges, which results in the acceleration of the flowing fluid in accordance with the consideration of continuity. The effect of convergence of flowing stream extends up to a certain distance upstream from the orifice plate and therefore the pressure tap on the upstream side is provided away from the orifice plate at a section where this effect is non-existent. However on the downstream side the pressure tap is provided quite close to the orifice plate at a section where the converging jet of fluid has the smallest cross-sectional area (which is known as veena- contracta) resulting in the max.velocity of flow and consequently the min. pressure at this section. Therefore a max. Pressure difference exists between the section 1 and 2, which is measured by connecting a differential manometer between the pressure taps at these sections or connecting separate pressure gauges. The jet of fluid coming out of the orifice gradually expands from the veena-contracta to again fill the pipe. In case of an orifice meter an abrupt change in the cross-sectional area of the flow passage is provided and there being no gradual change in the cross-sectional area of flow passage as in the case of venture meter, there is a greater loss of energy in an orifice meter.

Let p_1,p_2 and v_1,v_2 be the pressures and velocities at sections 1 and 2 respectively. Then for an incompressible fluid, applying Bernoulli"s equation between section 1 and 2 and neglecting losses, we have

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

Or

$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \qquad (1)$$

Where h is the difference between piezo metric heads at sections 1 and 2. However if the orifice meter is connected in a horizontal pipe, then $z_1 = z_2$, in which case h will represent the pressure head difference between sections 1 and 2. From equation (1) we have

$$V_2 = \sqrt{2gh + V_1^2} \quad \text{———————— (2)}$$

In deriving the above expression losses have not been considered, this expression gives the theoretical velocity of flow at section 2. To obtain actual velocity, it must be multiplied by a factor C_V, called co-efficient of velocity, which is defined as the ratio between the actual velocity and theoretical velocity. Thus actual velocity of flow at section 2 is obtained as

$$V_2 = C_v\sqrt{2gh + V_1^2} \quad \text{———————————— (3)}$$

Further if a_1 and a_2 are the cross-sectional areas of pipe at section 1 and 2,
By continuity equation $Q = a_1 v_1 = a_2 v_2$ ———————————— (4)
The area of jet a_2 at section 2 (i.e. at veena- contracta) may be related to the area of orifice a_o by the fallowing expression

$a_2 = C_C a_o$ where C_C is known as co-efficient of contraction, which is defined as the ratio between the area of the jet at veena - contracta and the area of orifice. Thus introducing the value of a_2 in equation (4), we get

$V_1 = \frac{C_c a_0}{a_1} V_2$ By substituting this value of V_1 in equation (3), we get

$$V_2 = C_v\sqrt{2gh + \frac{C_c^2 a_0^2}{a_1^2} V_2^2}$$

Solving for V_2, we get
$$V_2 = \frac{C_v\sqrt{2gh}}{\sqrt{1 - C_v^2 C_c^2 \frac{a_0^2}{a_1^2}}}$$

Now $Q = a_2 V_2 = C_c a_0 V_2$ and $C_c C_v = C_d$

$$Q = \frac{C_d a_0\sqrt{2gh}}{\sqrt{1 - C_v^2 C_c^2 \frac{a_0^2}{a_1^2}}} = \frac{C_d a_0\sqrt{2gh}}{\sqrt{1 - C_c^2 \frac{a_0^2}{a_1^2}}}$$

Where C_d is the co-efficient of discharge of the orifice.
Simplifying the above expression for the discharge through the orifice meter by using a co-efficient C expressed as

$$C = \frac{C_d\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1 - C_c^2\left(\frac{a_0}{a_1}\right)^2}}$$

So that
$$Q = \frac{C a_0\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} = C a_0 a_1 \frac{\sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

$$Q = C a_0 a_1 \frac{\sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

This gives the discharge through an orifice meter and is similar to the discharge through venture meter. The co-efficient C may be considered as the co-efficient of discharge of an orifice meter. The co-efficient of discharge for an orifice meter is smaller than that for a venture meter. This is because there are no gradual converging and diverging flow passages as in the case of venture meter, which results in a greater loss of energy and consequent reduction of the co-efficient of discharge for an orifice meter.

PROBLEMS ON FLOW THROUGH PIPES

1. At a sudden enlargement of a water main from 240mm to 480mm diameter, the hydraulic gradient rises by 10mm. Estimate the rate of flow.

Given: Dia. of smaller pipe $D_1 = 240\text{mm} = 0.24\text{m}$

Area $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.24)^2$

Dia. of larger pipe $D_2 = 480\text{mm} = 0.48\text{m}$

Area $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.48)^2$

Rise of hydraulic gradient i.e. $\left(Z_2 + \frac{P_2}{\rho g}\right) - \left(Z_1 + \frac{P_1}{\rho g}\right) = 10\text{mm} = \frac{10}{1000}\text{m} = \frac{1}{100}\text{m}$

Let the rate of flow = Q

Applying Bernoulli's equation to both sections i.e smaller and larger sections

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \text{Head loss due to enlargement} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

But head loss due to enlargement, $h_e = \frac{(V_1 - V_2)^2}{2g}$ ______(2)

From continuity equation, we have $A_1 V_1 = A_2 V_2$ $\quad V_1 = \frac{A_2 V_2}{A_1}$

$$V_1 = \frac{\frac{\pi}{4} D_2^2 V_2}{\frac{\pi}{4} D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{0.48}{0.24}\right)^2 V_2 = 2^2 V_2 = 4V_2$$

Substituting this value in equation (2), we get

$$h_e = \frac{(4V_2 - V_2)^2}{2g} = \frac{(3V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

Now substituting the value of h_e and V_1 in equation (1)

$$\frac{P_1}{\rho g} + \frac{(4V_2)^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \frac{9V_2^2}{2g}$$

$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \left(\frac{P_2}{\rho g} + Z_2\right) - \left(\frac{P_1}{\rho g} + Z_1\right)$$

But Hydraulic gradient rise $= \left(\frac{P_2}{\rho g} + Z_2\right) - \left(\frac{P_1}{\rho g} + Z_1\right) = \frac{1}{100}\text{ m}$

$$\frac{6V_2^2}{2g} = \frac{1}{100}\text{ m} \qquad V_2 = \sqrt{\frac{2 \times 9.81}{6 \times 100}} = 0.1808 = 0.181\ m/sec$$

Discharge $\quad \mathbf{Q} = A_2 V_2 = \frac{\pi}{4} D_2^2 V_2$

$$= \frac{\pi}{4}(0.48)^2 \times 0.181 = 0.03275\text{m}^3/\text{sec}$$

= **32.75Lts/sec**

2. A 150mm dia. pipe reduces in dia. abruptly to 100mm dia. If the pipe carries water at 30lts/sec, calculate the pressure loss across the contraction. Take co-efficient of contraction as 0.6

Given: Dia. of larger pipe $D_1 = 150mm = 0.15m$

Area of larger pipe $A_1 = \frac{\pi}{4}(0.15)^2 = 0.01767m^2$

Dia. of smaller pipe $D_2 = 100mm = 0.10m$

Area of smaller pipe $A_2 = \frac{\pi}{4}(0.10)^2 = 0.007854m^2$

Discharge $Q = 30$ lts/sec $= 0.03m^3/sec$

Co-efficient of contraction $C_C = 0.6$

From continuity equation, we have $Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{Q}{A_1} = \frac{0.03}{0.01767} = 1.697\ m/sec$$

$$V_2 = \frac{Q}{A_2} = \frac{0.03}{0.007854} = 3.82\ m/sec$$

Applying Bernoulli"s equation before and after contraction

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_c \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

But $Z_1 = Z_2$ and h_c the head loss due to contraction is given by the equation

$$= \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2 = \frac{3.82^2}{2\times 9.81}\left(\frac{1}{0.6} - 1\right)^2 = 0.33$$

Substituting these values in equation (1), we get

$$\frac{P_1}{\rho g} + \frac{1.697^2}{2\times 9.81} = \frac{P_2}{\rho g} + \frac{3.82^2}{2\times 9.81} + 0.33$$

$$\frac{P_1}{\rho g} + 0.1467 = \frac{P_2}{\rho g} + 0.7438 + 0.33$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = 0.7438 + 0.33 - 0.1467 = 0.9271\ m\ of\ water$$

$$P_1 - P_2 = \rho g \times 0.9271 = 1000 \times 9.81 \times 0.9271 = 0.909 \times 10^4\ N/m^2$$

= 0.909 N/cm²

Pressure loss across contraction = $P_1 - P_2$ = 0.909N/cm²

3. Water is flowing through a horizontal pipe of diameter 200mm at a velocity of 3m/sec. A circular solid plate of diameter150mm is placed in the pipe to obstruct the flow. Find the loss of head due to obstruction in the pipe, if C_C =0.62.

Given: Diameter of pipe D = 200mm = 0.2m

Velocity V = 3m/sec

Area of pipe $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.2)^2 = 0.03141m^2$

Diameter of obstruction d = 150mm =0.15m

Area of obstruction $a = \frac{\pi}{4}(0.15)^2 = 0.01767m$

$C_C = 0.62$

The head loss due to obstruction $= \frac{V^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]^2$

$$= \frac{3\times3}{2\times9.81}\left[\frac{0.03141}{0.62\times 0.03141 - 0.01767} - 1\right]^2$$

$$= \frac{9}{19.62}\,[3.687 - 1]^2$$

$= \underline{\mathbf{3.311m}}$

Problems on Pitot tube

1. A pitot tube is placed in the centre of a 300mm pipe line has one end pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe, if the pressure difference between the two orifices is 60mm of water. Co-efficient of Pitot tube C_V=0.98

Given: Diameter of pipe = 300mm =0.3m

Difference of pressure head h = 60mm of water = 0.06m of water

Mean velocity $\bar{V}$ = 0.80 × central velocity

Central velocity $= C_v\sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 0.06} = 1.063\ m/sec$

Mean velocity =0.8 × 1.063 = 0.8504m/sec

Discharge Q = Area of pipe × Mean velocity = $A \times \bar{V}$

$= \frac{\pi}{4}(0.3)^2 \times 0.8504$

$= 0.06m^3/sec$

2. Find the velocity of flow of an oil through a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tappings of the pitot tube is 100mm. Co-efficient of pitot tube C = 0.98 and sp.gr.of oil =0.8.

Given: Difference of mercury level x = 100mm = 0.1m

Sp.gr. of oil = 0.8, C_V=0.98

$$\text{Difference of pressure head } h = x\left(\frac{S_g}{S_o} - 1\right) = 0.1\left(\frac{13.6}{0.8} - 1\right)$$

= 1.6 m of oil

$$\text{Velocity of flow} = C_V\sqrt{2 \times g \times h} = 0.98\sqrt{2 \times 9.81 \times 1.6} = 5.49\text{m/sec}$$

= **5.49m/sec**

3. A pitot tube is used to measure the velocity of water in a pipe. The stag nature pressure head is 6m and static pressure head is 5m. Calculate the velocity of flow. Co-efficient of pitot tube is 0.98.

Given: Stag nature pressure head h_s =6m

Static pressure head h_t =5m

$h = h_s - h_t = 6 - 5 = 1\text{m}$

$$\text{Velocity of flow } V = C_V\sqrt{2 \times g \times h}$$

$$= 0.98\sqrt{2 \times 9.81 \times 1}$$

= **4.34 m/sec**

4. A submarine moves horizontally in a sea and has its axis 15m below the surface of the water. A pitot tube is properly placed just in front of the submarine and along its axis is connected to the two limbs of a U- tube containing mercury. The difference of mercury level is found to be 170mm. Find the speed of the sub-marine. Specific gravity of mercury is 13.6 and sea water is 1.026 with respect to fresh water.

Given: Difference of mercury level = 170mm = 0.17m

Specific gravity of mercury S_m=13.6,

Specific gravity of sea water S_o = 1.026

$$h = x\left(\frac{S_m}{S_o} - 1\right) = 0.17\left(\frac{13.6}{1.026} - 1\right) = 3.0834$$

$$V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 2.0834}$$

= 6.393 m/sec

$$= \frac{6.393 \times 60 \times 60}{1000}$$

$$= 23.01 \text{ km / hr}$$

5. A pitot tube is inserted in a pipe of 300mm diameter. The static pressure in the pipe is 100mm of mercury (Vacuum). The stagnation pressure at the centre of the pipe is recorded by Pitot tube is $0.981 N/cm^2$. Calculate the rate of flow of water through the pipe. The mean velocity of flow is 0.85 times the central velocity $C_V = 0.98$

Given: Diameter of pipe d = 0.3m

Area of pipe a $= \frac{\square}{4}(0.3)^2 = 0.07068 m^2$

Static pressure head = 100mm of mercury $= \frac{100}{1000} \times 13.6 = 1.36$m of water

Stag nature pressure head $= \frac{0.981 \times 10^4}{1000 \times 9.81} = 1$

□

h= Stagnation pressure head –

static pressure head = 1 – (-1.36) = 2.36m

Velocity at centre = $C_V \sqrt{2 \square \square} = 0.98 \sqrt{2 \times 9.81 \times 2.36} = 6.668$m/sec

Mean velocity □ = 0.85× 6.668 = 5.6678m/sec

Rate of flow of water = □ × Area of pipe

= 5.6678 × 0.07068

= $0.4006 m^3/sec$