UNIT-1

INTRODUCTION TO FLUID STATICS

Fluid Mechanics is basically a study of:

1) Physical behavior of fluids and fluid systems and laws governing their behavior.

2) Action of forces on fluids and the resulting flow pattern.

Fluid is further sub-divided in to liquid and gas. The liquids and gases exhibit different characteristics on account of their different molecular structure. Spacing and latitude of the motion of molecules is large in a gas and weak in liquids and very strong in a solid. It is due to these aspects that solid is very compact and rigid in form, liquid accommodates itself to the shape of the container, and gas fill up the whole of the vessel containing it.

Fluid mechanics cover many areas like:

1. Design of wide range of hydraulic structures (dams, canals, weirs etc) and machinery (Pumps, Turbines etc).
2. Design of complex network of pumping and pipe lines for transporting liquids. Flow of water through pipes and its distribution to service lines.
3. Fluid control devices both pneumatic and hydraulic.
4. Design and analysis of gas turbines and rocket engines and air–craft.
5. Power generation from hydraulic, stream and Gas turbines.
6. Methods and devices for measurement of pressure and velocity of a fluid in motion.

UNITS AND DIMENSIONS:

A dimension is a name which describes the measurable characteristics of an object such as mass, length and temperature etc. a unit is accepted standard for measuring the dimension. The dimensions used are expressed in four fundamental dimensions namely Mass, Length, Time and Temperature.

Mass (M) – Kg

Length (L) – m

Time (T) – S

Temperature (t) – ^{0}C or K (Kelvin)

1. **Density:** Mass per unit volume=kg/m^3
2. **Newton:** Unit of force expressed in terms of mass and acceleration, according to Newton's 2nd law motion. Newton is that force which when applied to a mass of 1 kg gives an acceleration 1m/Sec^2. F=Mass x Acceleration = kg – m/sec^2 = N.
3. **Pascal:** A Pascal is the pressure produced by a force of Newton uniformly applied over an area of 1 m^2. Pressure = Force per unit area = N/ m^2 = Pascal or P_a.
4. **Joule:** A joule is the work done when the point of application of force of 1 Newton is displaced Work = Force per unit area = N m = J or Joule.

5. **Watt:** A Watt represents a work equivalent of a Joule done per second.
   Power = Work done per unit time = J/ Sec = W or Watt.

Density or Mass Density: The density or mass density of a fluid is defined as the ratio of the mass of the fluid to its volume. Thus the mass per unit volume of the fluid is called density. It is denoted by ℓ

The unit of mass density is Kg/m^3

$$\square = \frac{\text{Mass of fluid Volume}}{\text{of fluid}}$$

The value of density of water is $1000 Kg/m^3$.

Specific weight or Specific density: It is the ratio between the weights of the fluid to its volume. The weight per unit volume of the fluid is called weight density and it is denoted by **w**.

$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{\text{Mass of fluid} \times \text{Acceleration due to gravity}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}} = \rho \times g$$

Specific volume: It is defined as the volume of the fluid occupied by a unit mass or volume per unit mass of fluid is called Specific volume.

$$\text{Specific volume} = \frac{\text{Volume of the fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume of the fluid}}} = \frac{1}{\rho}$$

Thus the Specific volume is the reciprocal of Mass density. It is expressed as m^3/kg and is commonly applied to gases.

Specific Gravity: It is defined as the ratio of the Weight density (or density) of a fluid to the Weight density (or density) of a standard fluid. For liquids the standard fluid taken is water and for gases the standard liquid taken is air. The Specific gravity is also called relative density. It is a dimension less quantity and it is denoted by **s**.

$$\mathbf{S} \text{ (for liquids)} = \frac{\text{weight density of liquid}}{\text{weight density of water}}$$

$$\mathbf{S} \text{ (for gases)} = \frac{\text{weight density of gas}}{\text{weight density of air}}$$

Weight density of liquid=S × weight density of water = S×1000 ×9.81 N/m^3
Density of liquid= S × density of water = S × 1000 Kg/m^3
If the specific gravity of fluid is known, then the density of fluid will be equal to specific gravity of the fluid multiplied by the density of water

Example: The specific gravity of mercury is 13.6
Hence density of mercury = 13.6 ×1000 = 13600 Kg/m^3

VISCOSITY: It is defined as the property of a fluid which offers resistance to the movement of one layer of the fluid over another adjacent layer of the fluid. When the two layers of a fluid, at a distance 'dy' apart, move one over the other at different velocities, say u and u+du. The viscosity together with relative velocities causes a shear stress acting between the fluid layers.

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.

This shear stress is proportional to the rate of change of velocity with respect to y. it is denoted by symbol τ (tau)

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

Where μ is the constant of proportionality and is known as the co-efficient of dynamic viscosity or only viscosity. $\frac{du}{dy}$ represents the rate of shear strain or rate of shear deformation or velocity gradient.

From the above equation, we have $\mu = \dfrac{\tau}{\frac{du}{dy}}$

Thus, viscosity is also defined as the shear stress required producing unit rate of shear strain.

The unit of viscosity in CGS is called poise and is equal to dyne-see/ cm^2

KINEMATIC VISCOSITY: It is defined as the ratio between dynamic viscosity and density of fluid. It is denoted by symbol □ (nu)

$$\square = \frac{\text{viscosity}}{\text{density}} = \frac{\mu}{\rho}$$

The unit of kinematic viscosity is m^2 /sec

Thus one stoke = cm^2/sec = $\left(\frac{1}{100}\right)^2$ m^2/sec= 10^{-4} m^2/sec

NEWTONS LAW OF VISCOSITY: It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co- efficient of viscosity .It is expressed as:

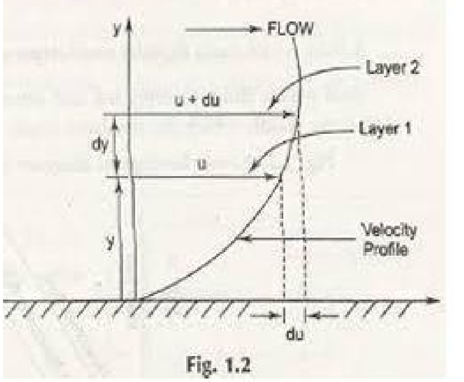


Fig. 1.2

$$\tau = \mu \frac{du}{dy}$$

Fluids which obey above relation are known as NEWTONIAN fluids and fluids which do not obey the above relation are called NON-NEWTONIAN fluids.

UNITS OF VISCOSITY

The units of viscosity is obtained by putting the dimensions of the quantities in equation

$$\mu = \frac{\tau}{\frac{du}{dy}}$$

$$\mu = \frac{\text{Shear stress}}{\frac{\text{Change of velocity}}{\text{change of distance}}}$$

$$= \frac{\text{Force/Area}}{\left(\frac{\text{Length}}{\text{Time}}\right) \times \frac{1}{\text{Length}}} = \frac{\text{Force/Length}^2}{\frac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{\text{Length}^2}$$

In MKS System Force is represented by (Kg f) and Length by meters (m)
In CGS System Force is represented by dyne and length by cm and
In SI System Force is represented by Newton (N) and Length by meter (m)

MKS unit of Viscosity = $\frac{\text{Kg f} - \text{Sec}}{\text{m}^2}$

CGS Unit of Viscosity = $\frac{\text{dyne} - \text{sec}}{\text{cm}^2}$

S I Unit of Viscosity = $\frac{\text{Newton} - \text{sec}}{\text{m}^2} = \frac{\text{Ns}}{\text{m}^2}$

The unit of Viscosity in CGS is called **Poise,** which is equal to $\frac{(\text{dyne} - \text{sec})}{\text{cm}^2}$.

The numerical conversion of the unit of viscosity from MKS units to CGS unit is as follows:

$$\frac{\text{One Kg f} - \text{sec}}{\text{m}^2} = \frac{9.81\text{N} - \text{sec}}{\text{m}^2} \qquad (1\text{Kgf} = 9.81 \text{ Newton})$$

But one Newton = One Kg (mass) $\times$ one $\frac{\text{m}}{\text{sec}^2}$ (Acceleration)

$$= \frac{(1000 \text{ gms} \times 100\text{cm})}{\text{sec}^2}$$

$$= 1000 \times 100 \frac{\text{gm} - \text{cm}}{\text{sec}^2}$$

$$= 1000 \times 100 \text{ dyne} \qquad \left(\text{dyne} = \text{gm} \times \frac{\text{cm}}{\text{sec}^2}\right)$$

$$\frac{\text{One Kg f} - \text{sec}}{\text{m}^2} = 9.81 \times 100000 \frac{\text{dyne} - \text{sec}}{\text{cm}^2}.$$

$$= 9.81 \times 100000 \frac{\text{dyne} - \text{sec}}{\text{cm}^2} = 9.81 \times 100000 \frac{\text{dyne} - \text{sec}}{100 \times 100 \times \text{cm}^2}$$

$$= 98.1 \frac{\text{dyne} - \text{sec}}{\text{cm}^2}$$

$$= 98.1 \text{ Poise} \qquad \frac{(\text{dyne} - \text{sec})}{\text{cm}^2} = \text{Poise}$$

$$\frac{\text{One Ns}}{\text{m}^2} = \frac{9.81}{9.81} \text{ Poise} = 10 \text{ Poise}$$

Or 1 Poise $= \dfrac{1}{10} \dfrac{Ns}{m^2}$

VARIATION OF VISCOSITY WITH TEMPERATURE:

Temperature affects the viscosity. The viscosity of liquids decreases with the increase of temperature, while the viscosity of gases increases with the increase of temperature. The viscous forces in a fluid are due to cohesive forces and molecular momentum transfer. In liquids cohesive forces predominates the molecular momentum transfer, due to closely packed molecules and with the increase in temperature, the cohesive forces decreases resulting in decreasing of viscosity. But, in case of gases the cohesive forces are small and molecular momentum transfer predominates with the increase in temperature, molecular momentum transfer increases and hence viscosity increases. The relation between viscosity and temperature for liquids and gases are:

i) For **liquids** $\mu = \mu_0 \frac{1}{1+\alpha t+\beta t^2}$

Where, μ = Viscosity of liquid at t c in Poise.

μ_0 = Viscosity of liquid at o c

α and β are constants for the Liquid.

For Water, $\mu_0 = 1.79 \times 10^{-3}$ poise, $\alpha = 0.03368$ and $\beta = 0.000221$

The above equation shows that the increase in temp. The Viscosity decreases.

ii) **For gases** $\mu = \mu_0 + \alpha t - \beta t^2$

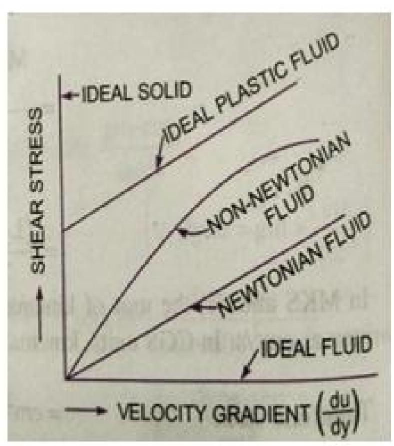
For air $\mu_0 = 0.000017$, $\alpha = 0.056 \times 10^{-6}$, $\beta = 0.118 \times 10^{-9}$

The above equation shows that with increase of temp. The Viscosity increases.

TYPES of FLUIDS: The fluids may be classified in to the following five types.

1. Ideal fluid 2. Real fluid 3.Newtonian fluid 4. Non-Newtonian fluid 5. Ideal plastic fluid

1. **Ideal fluid**: A fluid which is compressible and is having no viscosity is known as ideal fluid. It is only an imaginary fluid as all fluids have some viscosity.



2. **Real fluid**: A fluid possessing a viscosity is known as real fluid. All fluids in actual practice are real fluids.

3. **Newtonian fluid**: A real fluid, in which the stress is directly proportional to the rate of shear strain, is known as Newtonian fluid.

4. **Non-Newtonian fluid**: A real fluid in which shear stress is not Proportional to the rate of shear strain is known as Non-Newtonian fluid.

5. **Ideal plastic fluid:** A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain is known as ideal plastic fluid.

SURFACE TENSION:

Surface tension is defined as the tensile force acting on the surface of a liquid is contact with a gas or on the surface behaves like a membrane under tension. The magnitude of this force per unit length of free surface will have the same value as the surface energy per unit area. It is denoted by □ (sigma). In MKS units it is expressed as Kg f/m while in SI units as N/m

Surface Tension on Liquid Droplet:

Consider a small spherical droplet of a liquid of radius 'r' on the entire surface of the droplet, the tensile force due to surface tension will be acting

Let σ = surface tension of the liquid

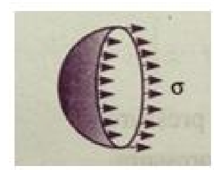
p= pressure intensity inside the droplet (In excess of outside pressure intensity)

d= Diameter of droplet

Let, the droplet is cut in to two halves. The forces acting on one half (say left half) will be

i) Tensile force due to surface tension acting around the circumference of the cut portion

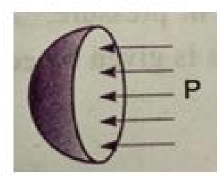
$= \sigma \times$circumference $= \sigma \times$ □d



ii) Pressure force on the area $\frac{□}{4}d^2 = p \times \frac{□}{4}d^2$

These two forces will be equal to and opposite under equilibrium conditions i.e.

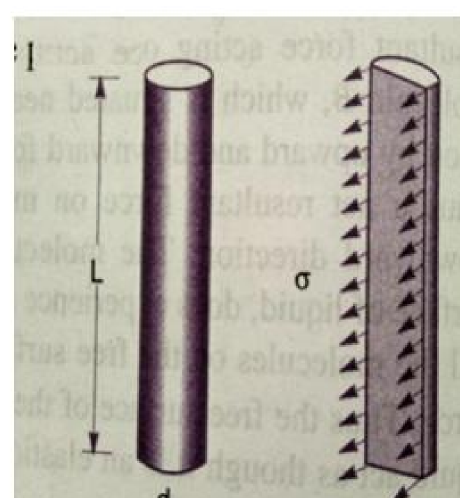
$□ \times \frac{-}{4} d^2 = \sigma$ □d, $\quad p = \frac{\varsigma □d}{\frac{□}{4}d^2} \quad p = \frac{□□}{□}$



Surface Tension on a Hallow Bubble: A hallows bubble like soap in air has two surfaces in contact with air, one inside and other outside. Thus, two surfaces are subjected to surface tension.

$□ \times \frac{-}{4}d^2 = 2(\sigma □□)$ $\mathbf{p} = \frac{□□}{□}$

SURFACE TENSION ON A LIQUID JET:



Consider a liquid jet of diameter 'd' length 'L'

Let, p = pressure intensity inside the liquid jet above the outside pressure

σ = surface tension of the liquid

Consider the equilibrium of the semi-jet

Force due to pressure = p × area of the semi-jet = p × L × d

Force due to surface tension = $\sigma \times 2L$

$p \times L \times d = \sigma \times 2L$,

$$\mathbf{p} = \frac{□□}{□}$$

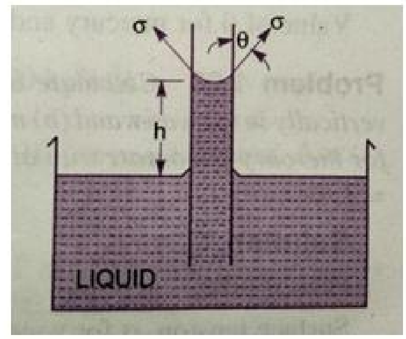
CAPILLARITY

Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise, while the fall of the liquid surface is known as capillary depression. It is expressed in terms of 'cm' or 'mm' of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

EXPRESSION FOR CAPILLARY RISE:

Consider a glass tube of small diameter 'd' opened at both ends and is inserted in a liquid; the liquid will rise in the tube above the level of the liquid outside the tube.

Let 'h' be the height of the liquid in the tube. Under a state of equilibrium, the weight of the liquid of height 'h' is balanced by the force at the surface of the liquid in the tube. But, the force at the surface of the liquid in the tube is due to surface tension.



Let σ = surface tension of liquid

ϴ = Angle of contact between the liquid and glass tube

The weight of the liquid of height 'h' in the tube

$= (\text{area of the tube} \times h) \times \rho \times g = \frac{\pi}{4} d^2 \times h \times \rho \times g$

Where 'ρ' is the density of the liquid.

The vertical component of the surface tensile force = (σ× circumference) × cosϴ = σ × πd × cosϴ

For equilibrium, $\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \pi d \cos\theta$, $\quad h = \frac{\sigma \pi d \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma \cos\theta}{\rho \times g \times d}$.

The value of ϴ is equal to '0' between water and clean glass tube, then cosϴ = 1, $\quad h = \frac{4\sigma}{\rho \times g \times d}$

EXPRESSION FOR CAPILLARY FALL:

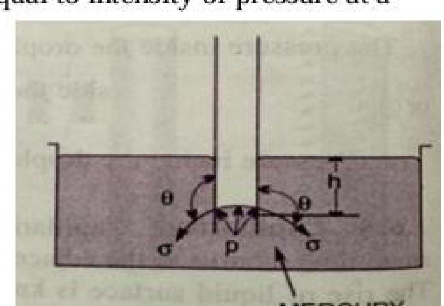
If the glass tube is dipped in mercury, the Level of mercury in the tube will be lower than the general level of the outside liquid.

Let, h = height of the depression in the tube. Then, in equilibrium, two forces are acting on the mercury inside the tube. First one is due to the surface tension acting in the downward direction = $\sigma \times \pi d \times \cos\theta$

The second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth 'h' × area = $p \times \frac{\pi}{4} d^2 = \rho g h \frac{\pi}{4} d^2 \; (p = \rho g h)$,

$$\sigma \pi d \cos\theta = \rho g h \frac{\pi d^2}{4}$$

(The value of ϴ for glass and mercury 128^0)



$$h = \frac{\sigma \pi d \cos\theta}{\rho g h \frac{\pi d^2}{4}}$$

$$\mathbf{h} = \frac{4\sigma\cos\theta}{\rho g d}$$

VAPOUR PRESSURE AND CAVITATION

A change from the liquid state to the gaseous state is known as Vaporizations. The vaporization (which depends upon the prevailing pressure and temperature condition) occurs because of continuous escaping of the molecules through the free liquid surface.

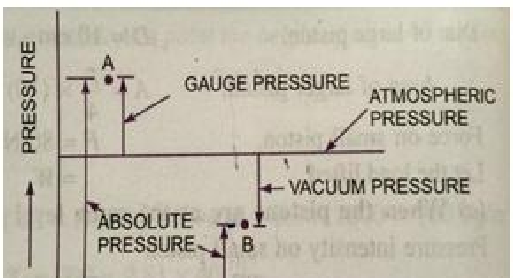
Consider a liquid at a temp. of 20 ℃ and pressure is atmospheric is confined in a closed vessel. This liquid will vaporize at 100 ℃, the molecules escape from the free surface of the liquid and get accumulated in the space between the free liquid surface and top of the vessel. These accumulated vapours exert a pressure on the liquid surface. This pressure is known as vapour pressure of the liquid or pressure at which the liquid is converted in to vapours.

Consider the same liquid at 20 $^{\circ}$c at atmospheric pressure in the closed vessel and the pressure above the liquid surface is reduced by some means; the boiling temperature will also reduce. If the pressure is reduced to such an extent that it becomes equal to or less than the vapour pressure, the boiling of the liquid will start, though the temperature of the liquid is 20 ℃. Thus, the liquid may boil at the ordinary temperature, if the pressure above the liquid surface is reduced so as to be equal or less than the vapour pressure of the liquid at that temperature.

Now, consider a flowing system, if the pressure at any point in this flowing liquid becomes equal to or less than the vapour pressure, the vapourisation of the liquid starts. The bubbles of these vapours are carried by the flowing liquid in to the region of high pressure where they collapse, giving rise to impact pressure. The pressure developed by the collapsing bubbles is so high that the material from the adjoining boundaries gets eroded and cavities are formed on them. This phenomenon is known as **CAVITATION.**

Hence the cavitations is the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below the vapour pressure and sudden collapsing of these vapour bubbles in a region of high pressure,. When the vapour bubbles collapse, a very high pressure is created. The metallic surface, above which the liquid is flowing, is subjected to these high pressures, which cause pitting actions on the surface. Thus cavities are formed on the metallic surface and hence the name is **cavitation.**

ABSOLUTE, GAUGE, ATMOSPHERIC and VACCUM PRESSURES



The pressure on a fluid is measured in two different systems. In one system, it is measured above the absolute zero or complete vacuum and it is called the Absolute pressure and in other system, pressure is measured above the atmospheric pressure and is called Gauge pressure.

1. ABSOLUTE PRESSURE: It is defined as the pressure which is measured with reference to absolute vacuum pressure

2. GAUGE PRESSURE: It is defined as the pressure, which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric on the scale is marked as zero.

3. VACUUM PRESSURE: It is defined as the pressure below the atmospheric pressure

i) Absolute pressure = Atmospheric pressure+ gauge pressure

$p_{ab} = p_{atm} + p_{guag}$

ii) Vacuum pressure = Atmospheric pressure - Absolute pressure

The atmospheric pressure at sea level at 15^0C is $10.13N/cm^2$ or $101.3KN/m^2$ in S I Unitsand 1.033 Kg f/cm^2 in M K S System.

The atmospheric pressure head is 760mm of mercury or 10.33m of water.

<u>MEASUREMENT OF PRESSURE</u>

The pressure of a fluid is measured by the fallowing devices.

1. Manometers 2.Mechanical gauges.

1. Manometers: Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of fluid. They are classified as:

a) Simple Manometers b) Differential Manometers.

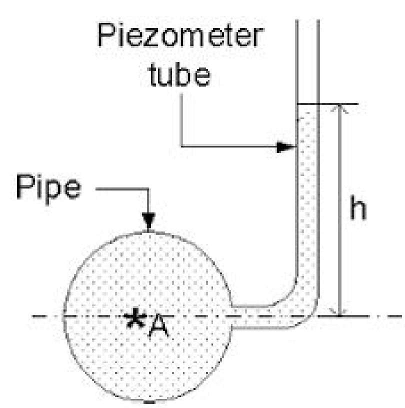
2. Mechanical Gauges: are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used Mechanical pressure gauges are:

a) Diaphragm pressure gauge b) Bourdon tube pressure gauge

c) Dead – Weight pressure gauge d) Bellows pressure gauge.

Simple Manometers: A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and the other end remains open to the atmosphere. The common types of simple manometers are:

1. Piezo meter.
2. U-tube manometer.
3. Single column manometer.



1. Piezometer: It is a simplest form of manometer used for measuring gauge pressure. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere. The rise of liquid in the Piezometer gives pressure head at that point A.

The height of liquid say water is ‘h’ in piezometer tube, then

Pressure at A = □ g h $\frac{□}{□□}$

2. U- tube Manometer:

It consists of a glass tube bent in u-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

a) For Gauge Pressure: Let B is the point at which pressure is to be measured, whose value is p. The datum line A – A

Let h_1 = height of light liquid above datum line

h_2 =height of heavy liquid above datum line

S_1 = sp. gravity of light liquid
ρ_1 = density of light liquid = 1000 S_1
S_2 = sp. gravity of heavy liquid
ρ_2 = density of heavy liquid = 1000 S_2

As the pressure is the same for the horizontal surface. Hence the pressure above the horizontal datum line A – A in the left column and the right column of U – tube manometer should be same.

Pressure above A—A ion the left column = $p + \rho_1$

gh_1 Pressure above A – A in the left column = $\rho_2 gh_2$

Hence equating the two pressures $p + \rho_1 gh_1 = \rho_2 gh_2$

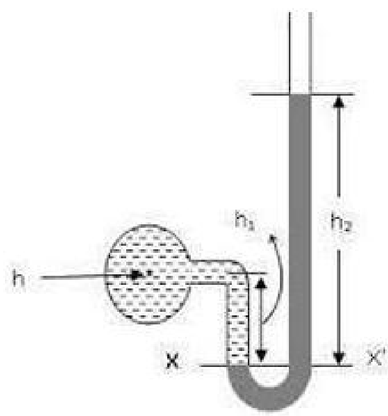
$$\mathbf{p = \rho_2 gh_2 - \rho_1 gh_1}$$

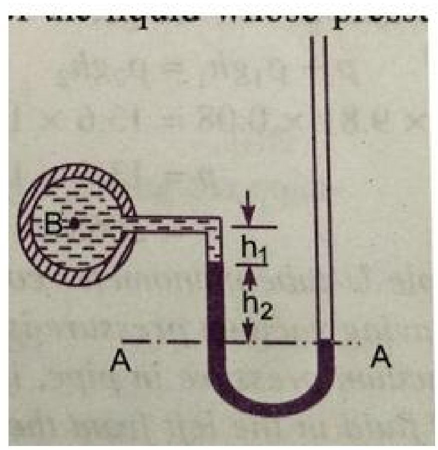
(b) For Vacuum Pressure:

For measuring vacuum pressure, the level of heavy fluid in the manometer will be as shown in fig.
Pressure above A A in the left column = $\rho_2 g\, h_2 + \rho_1 g\, h_1 + P$
Pressure head in the right column above A A = O

$\rho_2 g\, h_2 + \rho_1 g\, h_1 + P = O$

$$\mathbf{p = -(\rho_2 g\, h_2 + \rho_1 g\, h_1)}$$





(a) For Gauge Pressure (b) For Vacuum Pressure

<u>SINGLE COLUMN MANOMETER:</u>

Single column manometer is a modified form of a U- tube manometer in which a reservoir, having a large cross sectional area (about. 100 times) as compared to the area of tube is connected to one of the limbs (say left limb) of the manometer. Due to large cross sectional area of the reservoir for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of the liquid in the other limb. The other limb may be vertical or inclined. Thus, there are two types of single column manometer

1. Vertical single column manometer. 2. Inclined single column manometer.

VERTICAL SINGLE COLUMN MANOMETER:

Let X – X be the datum line in the reservoir and in the right limb of the manometer, when it is connected to the pipe, when the Manometer is connected to the pipe, due to high pressure at A The heavy in the reservoir will be pushed downwards and will rise in the right limb.

Let, Δ h= fall of heavy liquid in the reservoir

h_2= rise of heavy liquid in the right limb

h_1= height of the centre of the pipe above X – X

p_A= Pressure at A, which is to be measured.

A = Cross- sectional area of the reservoir

a = cross sectional area of the right limb

S_1= Specific. Gravity of liquid in pipe

S_2= sp. Gravity of heavy liquid in the reservoir and right limb

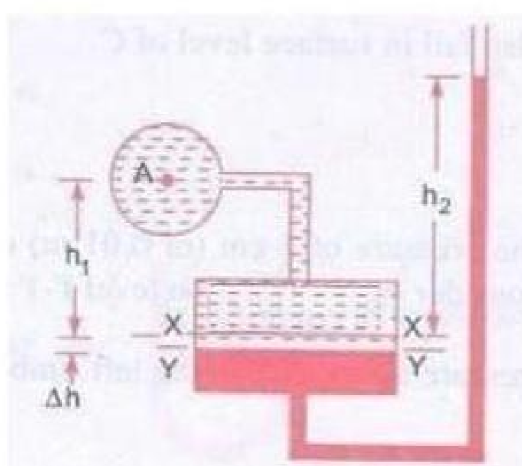
ρ_1 = density of liquid in pipe

ρ_2= density of liquid in reservoir

Fall of heavy liquid reservoir will cause a rise of heavy liquid level in the right limb

$$A \times \Delta h = a \times h_2$$

$$\Delta h = \frac{a \times h_2}{A} \qquad \text{-------- (1)}$$



Now consider the datum line Y –Y

The pressure in the right limb above Y – Y

$$= \rho_2 \times g \times (\Delta h + h_2)$$

Pressure in the left limb above Y—Y

$$= \rho_1 \times g \times (\Delta h + h_1) + P_A$$

Equating the pressures, we have

$$\square_2 g \times (\Delta h+h_2) = \square_1 \times g \times (\Delta h+h_1) + p_A$$

$$p_A = \square_2 \times g \times (\Delta h+h_2) - \square_1 \times g \times(\Delta h+h_1)$$

$$= \Delta h\,(\square_2 g\ \square_1 g) + h_2 \square_2 g - h_1 \square_1 g$$

But, from eq (1) $\Delta h = \frac{\square \times h^2}{\square}$

$$p_A = \frac{\quad}{\square} (\square_2 g - \square_1 g) + h_2 \square_2 g - h_1 \square_1 g$$

As the area A is very large as compared to a, hence the ratio $\frac{a}{A}$ becomes very small and can be neglected Then,

$$\mathbf{p_A = h_2 \square_2 g - h_1 \square_1 g \text{------- (2)}}$$

INCLINED SINGLE COLUMN MANOMETER:

The manometer is more sensitive. Due to inclination the distance moved by heavy liquid in the right limb will be more.

Let L= length of heavy liquid moved in the rite limb

Θ = inclination of right. Limb with horizontal.

H_2 = vertical rise of heavy liquid in the right limb above X – X

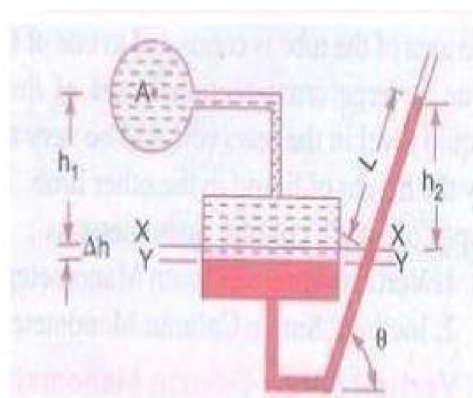
$= L \sin\Theta$

From above eq (2), the pressure at A is

$$p_A = h_2 \square_2 g - h_1 \square_1 g$$

Substituting the value of h_2

$$p_A = L \sin \Theta\ \square_2 g - h_1 \square_1 g$$



DIFFERENTIAL MANOMETERS:

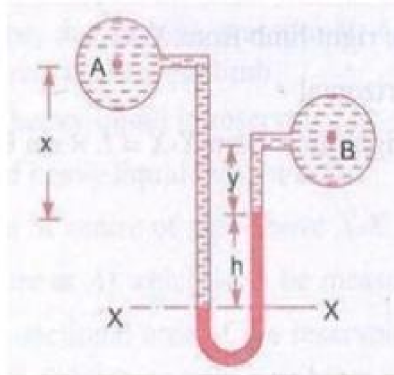
Differential manometers are the devices used for measuring the difference of pressure between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. The common types of U- tube differential manometers are:

1. U- Tube differential manometer 2. Inverted U- tube differential manometer.

1. U- Tube differential manometer:

a) Let the two points A and B are at different levels and also contains liquids of different sp.gr.

b) These points are connected to the U – Tube differential manometer.
Let the pressure at A and B are p_A andp_B.



Let h = Difference of mercury levels in the u – tube

y = Distance of centre of B from the mercury level in the right limb

x = Distance of centre of A from the mercury level in the left limb

$\square_1$ = Density of liquid A

$\square_2$ = Density of liquid B

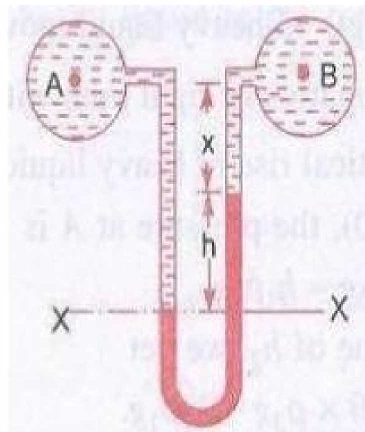
$\square_\square$ = Density of heavy liquid or mercury Taking datum line at X – X

Pressure above X – X in the left limb $=\square_1 g(h + x) + p_A$ (where p_A = Pressure at A)

Pressure above X – X in the right limb $= \square_\square\, g\, h + \square_2 g\, y + P_B$ (where p_B = Pressure at B)

Equating the above two pressures, we have

$$\square_1 g(h + x) + p_A = \square_\square\, g\, h + \square_2\, g\, y + p_B$$



$$p_A - p_B = \square_\square\, g\, h + \square_2 g\, y - \square_1 g\,(h + x)$$

$$= h\, g\,(\square_\square - \square_1) + \square_2 g\, y - \square_1 g\, x$$

∴ Difference of Pressures at A and B = $h\, g\,(\square_\square - \ell_1) + \square_2\, g\, y - \square_1 g\, x$

Let the two points A and B arte at the same level and contains the same liquid of density ℓ_1

Then pressure above X – X in the right limb $= \square_\square\, g\, h + \square_1 g\, x + P_B$

Pressure above X—X in the left limb $= \square_1 g\,(h + x) + P_A$

Equating the two pressures

$$\square_\square\, g\, h + \square_1 g\, x + p_B = \square_1 g\,(h + x) + p_A$$

$$P_A - P_B = \square_\square\, g\, h + \square_1 g\, x - \square_1 g(h + x)$$

$$= g\, h\,(\square_\square - \square_1)$$

Difference of pressure at A and B = $g\, h\,(\square_\square - \square_\square)$

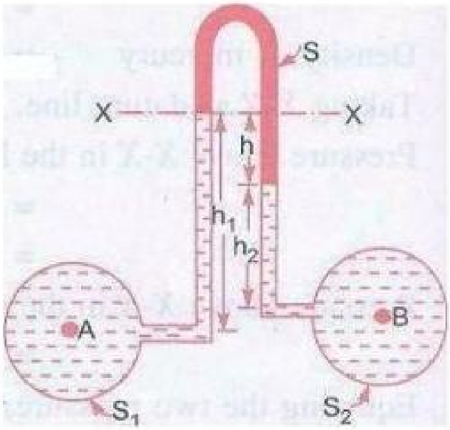
<u>Inverted U – Tube differential manometer</u>

It consists of a inverted U – tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Let an inverted U – tube differential manometer connected to the two points A and B. Let pressure at A is more than pressure at B.

Let h_1 = Height of the liquid in the left limb below the datum line X-X

h_2 = Height of the liquid in the right limb.

h = Difference of height of liquid



$□_1$ = Density of liquid A

$□_2$ = Density of liquid B

□□ = Density of light liquid

p_A = Pressure at A

p_B= Pressure at B

Taking x – x as datum line

The pressure in the left limb below $x - x = p_A$ - $□_1$ g h_1

Pressure in the right limb below $x - x = p_B$ - $□_2$ g h_2 - □□ g h

Equating the above two pressures

p_A - $□_1$ g h_1 = p_B - $□_2$ g h_2 - □□ g

h $p_A - p_B$ = $□_1$ g h_1 - $□_2$ g h_2 - □□

g h

Difference of pressure at A and B = □□ g h_1 - □□ g h_2 -□□ g h

PROBLEMS

1. Calculate the density, specific weight and weight of one liter of petrol of specific gravity= 0.7

Sol: i) Density of a liquid = S × Density of water = S x 1000 kg/ m^3

□= 0.7 × 1000

□ = 700Kg/m^3

ii) Specific weight w = □ × g = 700 × 9.81 = 6867 N/ m^3

iii) Weight (w) Volume = 1 liter = 1 × 1000 cm^3 = $\frac{1000 m^3}{10^6}$ = **0.001m^3**

We know that, specific weight w = $\frac{\text{weight of fluid}}{\text{volume of the fluid}}$

Weight of petrol = w× volume of petrol

= w × 0.001

= 6867 × 0.001 = **6.867 N**

2. Two horizontal plates are placed 1.25 cm apart, the space between them being filled with oil of viscosity 14 poises. Calculate shear stress in oil, if the upper plate is moved velocity of m/sec.

Sol: Given distance between the plates dy= 1.25 cm = 0.0125m

Viscosity μ = 14 poise = $\frac{14}{10}$ N s/m^2

Velocity of upper plate u = 2.5 m/sec

Shear stress $\tau = \mu \frac{du}{dy}$

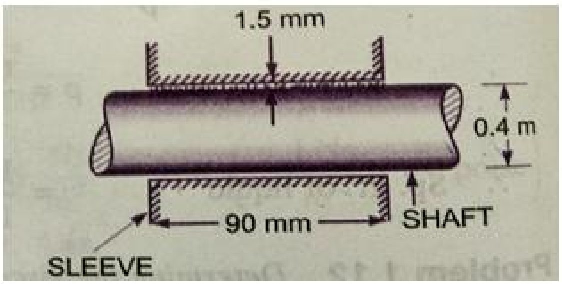
Where du = change of velocity between plates = u – 0 = u = 2.5 m/sec

$$\tau = \frac{14}{10} \times \frac{2.5}{0.0125}$$

<u>**Shear stress τ = 280 N/m^2**</u>

3. The dynamic viscosity of oil used for lubrication between a shaft and sleeve is 6 poise. The shaft dia. is 0.4m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 90mm. The thickness foil film is 1.5mm

Sol: Given, Viscosity μ = 6 poise = $\frac{6}{10} \frac{N\ s}{m^2}$ = 0.6 $\frac{N\ s}{m^2}$



Dia. of shaft D = 0.4M

Speed of shaft N = 190 rpm

Sleeve length L = 90mm = 90 × 10^{-3} m

Thickness of a film t= 1.5mm = 1.5mm = 1.5 × 10^{-3} m

Tangential velocity of shaft = u = $\frac{\pi DN}{60} = \frac{\pi \times 0.4 \times 190}{60}$ = 3.98 m/sec

Using the relation $\tau = \mu \frac{du}{dy}$

Where du = change of velocity = u – 0 = u = 3.98 m/sec

dy = change of distance = t = 1.5 × 10^{-3}m

$$\tau = 0.6 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592 \text{ N/m}^2$$

This is the shear stress on the shaft

Shear force on the shaft F = shear stress × area =1592 × π DL = 1592 × π × 0.4 x 90 × 10^{-3} = 180.05 N

Torque on the shaft $T = \text{Force} \times \frac{D}{2} = 180.05 \times \frac{0.4}{2} = 36.01$ Nm

Power lost $= \frac{2\pi NT}{60} = 36.01 \times \frac{2\pi \times 190}{60} \times 36.01$

Power lost = 716.48 W

4. A cylinder 0.12m radius rotates concentrically inside a fixed cylinder of 0.13 m radius. Both cylinders are 0.3m long. Determine the viscosity of liquid which fills the space between the cylinders, if a torque of 0.88 Nm is required to maintain an angular velocity of 2 π rad / sec.

Sol : Diameter of inner cylinder = 0.24m

Diameter of outer cylinder = 0.26 m

Length of cylinder $L = 0.3$ m

Torque $T = 0.88$ NM

$w = 2\pi N/60 = 2\pi$

N = speed = 60rpm

Let the viscosity = μ

Tangential velocity of cylinder $u = \frac{\pi DN}{60} = \frac{\pi \times 0.24 \times 60}{60} = 0.7536$ m/sec

Surface area of cylinder $A = \pi DL = \pi \times 0.24 \times 0.3 = 0.226\ m^2$

Now using the relation $\tau = \mu \frac{du}{dy}$

Where $du = u - 0 = 0.7536$ m/sec

$$dy = \frac{0.26-0.24}{2} = 0.02 = 0.01 \text{ m}$$

$$\tau = \mu \times \frac{0.7536}{0.01}$$

Shear force, F = shear stress x area

$$= \mu \times 75.36 \times 0.226 = 17.03\ \mu$$

Torque $T = F \times D/2 = 17.03\ \mu \times \frac{0.24}{2}$

$$0.88 = \mu \times 2.0436$$

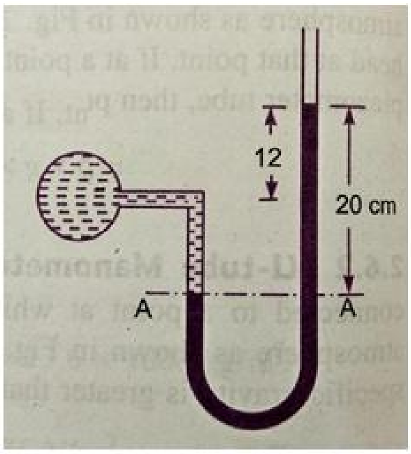
$$\mu = \frac{0.88}{2.0436}$$

$$= 0.4306 \text{ Ns/ m}^2$$

$$= 0.4306 \times 10 \text{ poise}$$

Viscosity of liquid = 4.306 poise

5. The right limb of a simple U – tube manometer containing mercury is open to the atmosphere, while the left limb is connected to a pipe in which a fluid of sp.gr.0.9 is flowing. The centre of pipe is 12cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe, if the difference of mercury level in the two limbs is 20 cm.



Given, Sp.gr. of liquid $S_1 = 0.9$

Density of fluid $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900$ kg/ m^3

Sp.gr. of mercury $S_2 = 13.6$

Density of mercury $\rho_2 = 13.6 \times 1000 = 13600$ kg/m^3

Difference of mercury level $h_2 = 20\text{cm} = 0.2\text{m}$

Height of the fluid from A – A $h_1 = 20 - 12 = 8\text{cm} = 0.08$ m

Let 'P' be the pressure of fluid in pipe

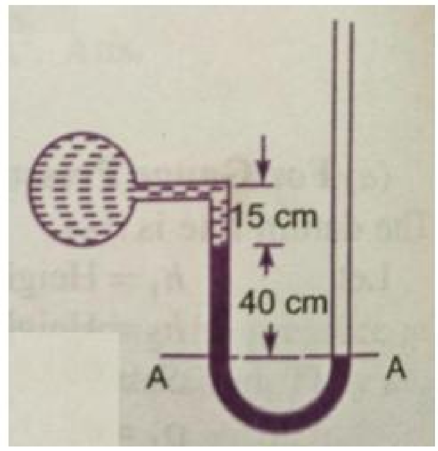
Equating pressure at A – A, we get

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times 0.2$
$p = 13.6 \times 1000 \times 9.81 \times 0.2 - 900 \times 9.81 \times 0.08$
$p = 26683 - 706$
$p = 25977 \text{ N/m}^2$
$p = 2.597 \text{ N/cm}^2$

Pressure of fluid = 2.597 N/ cm²

6. A simple U – tube manometer containing mercury is connected to a pipe in which a fluid of sp.gr. And having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40cm. and the height of the fluid in the left tube from the centre of pipe is 15cm below.



Given,
Sp.gr of fluid $S_1 = 0.8$
Sp.gr. of mercury $S_2 = 13.6$
Density of the fluid $= S_1 \times 1000 = 0.8 \times 1000 = 800$
Density of mercury = 13.6×1000
Difference of mercury level $h_2 = 40\text{cm} = 0.4\text{m}$
Height of the liquid in the left limb = 15cm =0.15m
Let the pressure in the pipe = p
Equating pressures above datum line A-- A

$$\square_2 g h_2 + \square_1 g h_1 + P = 0$$

$$P = - [\square_2 g h_2 + \square_1 g h_1]$$

$$= - [13.6 \times 1000 \times 9.81 \times 0.4 + 800 \times 9.81 \times 0.15]$$

$$= 53366.4 + 1177.2$$

$$= -54543.6 \text{ N/m}^2$$

$P = - 5.454 \text{ N/cm}^2$

7. What are the gauge pressure and absolute pressure at a point 3m below the surface of a liquid having a density of 1.53×10^3 kg/m³? If the atmospheric pressure is equivalent to 750mm of mercury. The specific gravity of mercury is 13.6 and density of water 1000 kg/m³
Given:

Depth of the liquid, $z_1 = 3\text{m}$
Density of liquid $\square_1 = 1.53 \times 10^3 \text{ kg/ m}^3$
Atmospheric pressure head $z_0 = 750$ mm of mercury $= \frac{750}{1000} = 0.75\text{m of Hg}$
Atmospheric pressure $p_{atm} = \square_0 \times g \times z_0$

Where $\square_0$ = density of Hg = sp.gr. of mercury x density of water
$= 13.6 \times 1000 \text{ kg/ m}^3$

And z_0 = pressure head in terms of mercury = 0.75m of Hg
$P_{atm} = (13.6 \times 1000) \times 9.81 \times 0.75 \text{ N/m}^2$
$= 100062 \text{ N/ m}^2$

Pressure at a point, which is at a depth of 3m from the free surface of the liquid is
$P = \square_1 \times g \times z_1 = 1.53 \times 10^3 \times 9.81 \times 3$
Gauge pressure $P = 45028 \text{ N/ m}^2$

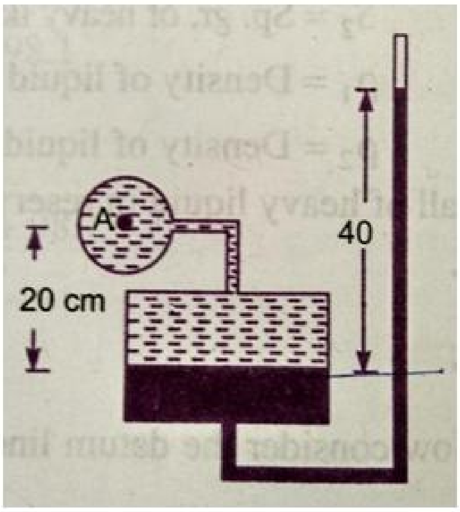
Absolute Pressure = Gauge pressure + Atmospheric pressure

$= 45028 + 100062$

Absolute Pressure = 145090 N/m^2

8. A single column manometer is connected to the pipe containing liquid of sp.gr.0.9. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube of manometer. sp.gr. of mercury is 13.6. Height of the liquid from the centre of pipe is 20cm and difference in level of mercury is 40cm.

Given,



Sp.gr. of liquid in pipe $S_1 = 0.9$

Density $\rho_1 = 900$ kg/m^3

Sp.gr. of heavy liquid $S_2 = 13.6$

Density $\rho_2 = 13600$

$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$

Height of the liquid $h_1 = 20\text{cm} = 0.2\text{m}$

Rise of mercury in the right limb $h_2 = 40\text{cm} = 0.4\text{m}$

Pressure in pipe A

$$p_A = \frac{A}{a} \times h_2 [\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

$$= \frac{1}{100} \times 0.4\, [13600 \times 9.81 - 900 \times 9.81] + 0.4 \times 13600 \times 9.81 - 0.2 \times 900 \times 9.81$$

$$= \frac{0.4}{100} [133416 - 8829] + 53366.4 - 1765.8$$

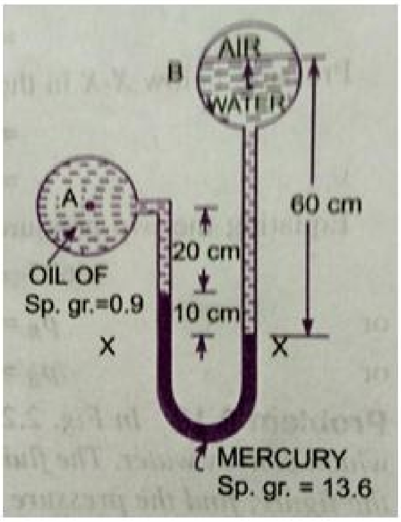
$$= 533.664 + 53366.4 - 1765.8$$

$$= 52134 \text{ N/m}^2$$

Pressure in pipe A= 5.21 N/ cm^2

9. A pipe contains an oil of sp.gr.0.9. A differential manometer is connected at the two points A and B shows a difference in mercury level at 15cm. find the difference of pressure at the two points.

Given: Sp.gr. of oil $S_1 = 0.9$: density $\rho_1 = 0.9 \times 1000 = 900$ kg/ m^3 Difference of level in the mercury $h = 15\text{cm} = 0.15$ m



Sp.gr. of mercury = 13.6, Density = $13.6 \times 1000 = 13600$ kg/m^3

The difference of pressure $p_A - p_B = g \times h \times (\rho_2 - \rho_1)$

$= 9.81 \times 0.15\,(13600 - 900)$

$p_A - p_B$ = 18688 N/ m^2

10. A differential manometer is connected at two points A and B .At B air pressure is 9.81 N/cm^2. Find absolute pressure at A.

Density of air = $0.9 \times 1000 = 900$ kg/m^3

Density of mercury = 13.6×10^3 kg/ m^3

Let pressure at A is p_A

Taking datum as X – X

Pressure above X – X in the right limb

$$= 1000 \times 9.81 \times 0.6 + p_B = 5886 + 98100 = 103986$$

Pressure above X – X in the left limb

$$= 13.6 \times 10^3 \times 9.81 \times 0.1 + 0900 \times 9.81 \times 0.2 + p_A$$

$$= 13341.6 + 1765.8 + p_A$$

Equating the two pressures heads

$$103986 = 13341.6 + 1765.8 + p_A$$

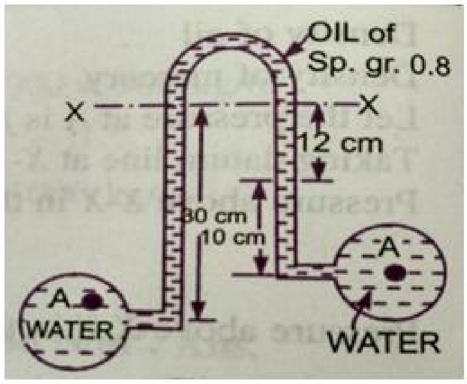
$$= 15107.4 + p_A$$

$$p_A = 103986 - 15107.4$$

$$= 88878.6 \text{ N/m}^2$$

<u>**p_A = 8.887 N/cm^2**</u>

11. Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp.gr. 0.8 is connected. The pressure head in the pipe A is 2m of water. Find the pressure in the pipe B for the manometer readings shown in fig.



Given: Pressure head at A = $\dfrac{p_A}{\rho g}$ = 2m of water

$$p_A = \rho \times g \times 2 = 1000 \times 9.81 \times 2 = 19620 \text{ N/m}^2$$

Pressure below X – X in the left limb

$$= p_A - \square_1 g h_1$$

$$= 19620 - 1000 \times 9.81 \times 0.3 = 16677 \text{ N/m}^2$$

Pressure below X – X in the right limb

$$= p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$$

$$= p_B - 981 - 941.76 = p_B - 1922.76$$

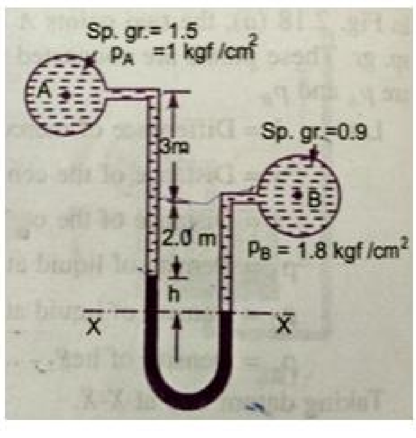
Equating the two pressures, we get,

$$16677 = p_B - 1922.76$$

$$p_B = 16677 + 1922.76$$

<u>**p_B = 18599.76 N/m^2**</u>

12. A different manometer is connected at two points A and B of two pipes. The pipe A contains liquid of sp.gr. = 1.5 while pipe B contains liquid of sp.gr. = 0.9. The pressures at A and B are 1 kgf/cm^2 and 1.80 Kg f/cm^2 respectively. Find the difference in mercury level in the differential manometer.



Sp.gr. of liquid at A $S_1 = 1.5$

Sp.gr. of liquid at B $S_2 = 0.9$

Pressure at A $p_A = 1\ kgf/cm^2 = 1 \times 10^4 \times kg/m^2 = 1 \times 10^4 \times 9.81 N/m^2$

Pressure at B $p_B = 1.8\ kgf/cm^2 = 1.8 \times 10^4 \times 9.81\ N/m^2$ [1kgf = 9.81 N]

Density of mercury = $13.6 \times 1000\ kg/m^3$

Taking X – X as datum line

Pressure above X – X in left limb

$$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81(2+3) + (9.81 \times 10^4)$$

Pressure above X – X in the right limb = $900 \times 9.81(h + 2) + 1.8 \times 9.81 \times 10^4$

Equating the two pressures, we get

$$13.6 \times 1000 \times 9.81h + 1500 \times 9.81 \times 5 + 9.81 \times 10^4 = 900 \times 9.81(h + 2) + 1.8 \times 9.81 \times 10^4$$

Dividing both sides by 1000×9.81

$$13.6\ h + 7.5 + 10 = 0.9(h+2) + 18$$

$$(13.6 - 0.9)h = 1.8 + 18 - 17.5 = 19.8 - 17.5 = 2.3$$

$$h = \frac{2.3}{12.7} = 0.181m$$

h = 18.1 cm