4. Testing of Hypothesis

A statistical Hypothesis is a statement about a Parameter in a Population using data measured in a sample. This process is called Testing of hypothesis. That is the process for deciding to accept / reject a lot by using this test. Testing of hypothesis are of two types:

1. Null Hypothesis
2. Alternative Hypothesis

Null Hypothesis:

A Hypothesis of no difference is called Null Hypothesis. It is denoted as H_0. $\boxed{H_0 : \mu = \mu_0}$

Alternative Hypothesis:

A Hypothesis which is contract to the Null Hypothesis is called Alternative Hypothesis. It is denoted as H_1.

$$\boxed{H_1 : \mu \neq \mu_0}$$

Four steps to hypothesis Testing:

Step 1: state the hypothesis that is (Null and Alternative Hypothesis)

Step 2: set the criteria for a decision. i.e (Level of significance - α %)

Step 3: compute the test statistic. i.e (calculated value).

Step 4: Make a Decision. i.e., (compare calculated value with Table value to make a decision).

Errors of sampling:

There are two types of errors:

Type-1 Error:

Reject H_0 when it is true i.e., (Rejected a correct hypothesis) It is denoted as 'α'.

Type-2 Error:

Accept Ho when it is false and it is denoted by 'β'.

The only Way to reduce both of errors is to increase the sample size if Possible.

Critical Region:

A Region corresponding to a statistic in the sample space 's' which leads to Reject Ho is called critical Region

Acceptance Region:

The Region which leads to accept Ho is called Acceptance Region.

Critical Value:

The Value of Test statistic which seperates rejection region and Acceptance Region is called critical Value (or) significant Value.

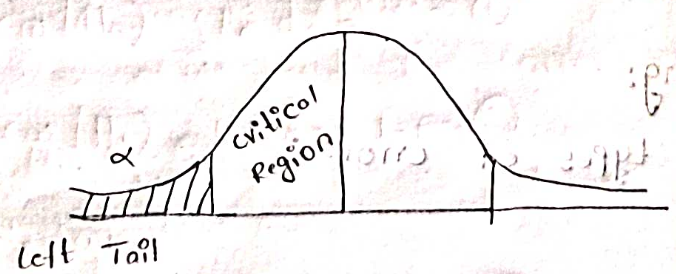
Level of significance:

The criterion of Judgement which a decision is made Regarding the Value stated at α% is known as "Level of significance".

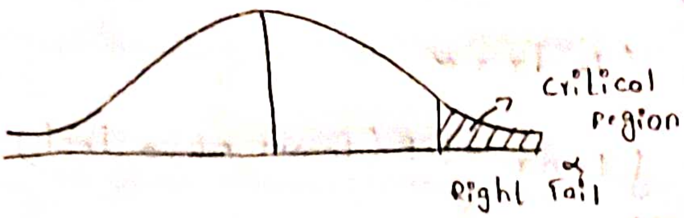
One-Tail and Two-Tail tests:

For one-Tail Tests critical Region is represented at only one side.

Left tail:

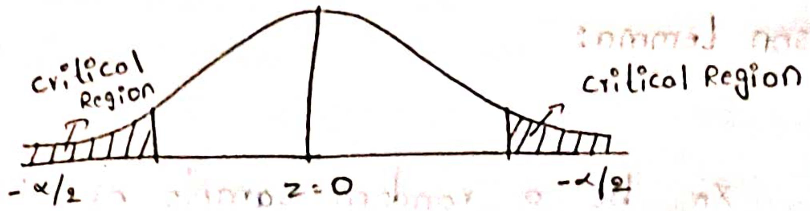


Right Tail:



Two-Tailed Test:

The two tail critical region represents two sides.



Power of a Test:

The Power of a Test is also known as Purchasing Power of consumer i.e., (Bad lot is rejected). It is denoted as $(1-\beta)$.

$$\text{Power} = P(\text{reject } H_0 \mid H_1 \text{ is true})$$
$$= 1 - P(\text{type II error})$$
$$= 1-\beta$$

Here $1-\beta$ is known as a "Power function".

Definition:

The Power of a hypothesis test is the Probability of rejecting a false null hypothesis.

$$1-\beta = \text{Probability of } (x \in w \mid H_1)$$
$$= \int_w L_1\, dx$$

Where L_1 is Likelihood function under H_1.

Most Powerful Test:

The critical region (W) is the most Powerful region of size 'α' and the corresponding test of most Powerful test for level α for testing $H_0: \theta = \theta_0$,
$H_1: \theta = \theta_1$

Under these conditions W is called most powerful critical region of size α.

$$P(x \in \omega \mid H_0) = \int_W L_0 \, dx = \alpha$$

$$P(x \in \omega \mid H_1) = \int_\omega L_1 \, dx = 1-\beta \text{ (Power of a test)}$$

$$1-\beta \geq 1-\beta_1$$

where $\omega \geq W_1$

Neymann Pearson Lemma:

statement:

Let $X_1, X_2 \ldots X_n$ be a random sample of size n drawn from a population with density function $(f(x), \theta)$. Let $k > 0$ be a constant and ω is the most powerful test critical Region of size 'α' for testing a simple null Hypothesis $H_0 : \theta = \theta_0$ against an alternative hypothesis $H_1 : \theta = \theta_1$. such that

$$\omega = \{x \in S ; \frac{L_1}{L_0} > k\} \text{ i.e., } \omega = \{x \in S ; \frac{f(x, \theta)}{f(x_0, \theta_0)} > k\}.$$

$$\overline{W} = \{x \in S ; \frac{L_1}{L_0} \leq k\}$$

Where L_0, L_1 are likelihood functions under H_0 and H_1 respectively.

Proof:

for W, we take as

$$P(x \in \omega \mid H_0) = \int_\omega L_0 \, dx = \alpha \rightarrow (1)$$

$$P(x \in \omega \mid H_1) = \int_\omega L_1 dx = 1-\beta \rightarrow (2)$$

for W_1 we take as

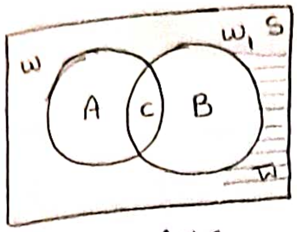
$$P(x \in \omega_1 \mid H_0) = \int_{\omega_1} L_0 \, dx = \alpha_1 \rightarrow (3)$$

$$P(x \in \omega_1 \mid H_1) = \int_{\omega_1} L_1 \, dx = 1-\beta_1 \rightarrow (4)$$

Now considering the sizes of ω and ω_1

$\omega_1 \le \omega$

that is $\alpha_1 \le \alpha$



Here $\omega = A \cup C$

$\omega_1 = B \cup C$

if $\alpha_1 \le \alpha$ which means

$$\int_{\omega_1} L_0 \, dx \le \int_{\omega} L_0 \, dx$$

$$\int_{B \cup C} L_0 dx \le \int_{A \cup C} L_0 dx$$

By cancelling c on b.s

$$\int_{B} L_0 dx \le \int_{A} L_0 dx$$

$$\int_{A} L_0 dx \ge \int_{B} L_0 dx \rightarrow (5)$$

From statement I we get

$$x \in \omega, \ \frac{L_1}{L_0} > k$$

$$\Rightarrow L_1 > L_0 k$$

$$\Rightarrow \int_{\omega_1} L_1 dx > k \int_{\omega} L_0 dx$$

By diagram $A C \omega$

$$\int_{A} L_1 dx > k \int_{A} L_0 dx \rightarrow (6)$$

eq (5) is multiplied by k

$$k \int_{A} L_0 \, dx \ge k \int_{B} L_0 dx \rightarrow (7)$$

By comparing (6) & (7)

$$\int_{A} L_1 dx \ge k \int_{B} L_0 dx \rightarrow (8)$$

From statement II

for $x \in \bar{\omega}$, $\frac{L_1}{L_0} \le k' \Rightarrow L_1 \le k \cdot L_0$

$$\int_{\bar{\omega}_1} L_1 dx \le k \int_{\bar{\omega}} L_0 dx$$

$B \subset \bar{\omega}$

$$\int_B L_1 dx \le k \int_B L_0 dx$$

$$k \cdot \int_B L_0 dx \ge \int_B L_1 dx \rightarrow (9)$$

from (8) & (9)

$$\int_A L_1 dx \ge \int_B L_1 dx$$

adding $\int_C L_1 dx$ to above eq

$$\int_A L_1 dx + \int_C L_1 dx \ge \int_B L_1 dx + \int_C L_1 dx$$

$$\int_{A \cup C} L_1 dx \ge \int_{B \cup C} L_1 dx$$

from eq (2) & eq (3)

$$1 - \beta \ge 1 - \beta_1$$

By this

$$\alpha \ge \alpha_1$$

$\therefore$ The Power of critical Region is always greater.

$\therefore \omega \ge \omega_1$

Hence the theorem is Proved.

Uniformly most Powerful Test:

The Region ω is called uniformly most Powerful critical size of α and the corresponding test $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$. This is known as uniformly most Powerful Test.

Likelihood Ratio[LR] Test:

The Likelihood Ratio test is a test of hypothesis in which two different maximum likelihood estimates of parameters are compared in order to decide whether to reject (or) not to reject a restriction on parameter. The likelihood Ratio test, the Null hypothesis is rejected if $LR_N > Z$

where z is three pre-specified critical value

The size of the test is like α = Probability of $LR_N > Z$ $(\alpha = P(LR_N > Z)) = 1 - P(LR_N < Z)$.

In statistics the likelihood ratio test access the goodness of fit of two competing statistical methods. General Method of a test construction is called likelihood Ratio test which was introduced by Neymann & Pearson for testing of hypothesis is simple (or) composite. This test is related to maximum likelihood estimates.

-um likelihood ...

* LR-test Properties:

1. Nested models: The LR test Compares 2 nested models, typically a Simpler model and a more Complex model. The null model is a restricted version of the alternative model, achieved by imposing constraints

2. Test Statistic: The LR test statistic follows a Chi-square distribution under the null hypothesis that the simpler model is correct. It is calculated as the difference in the log likelihoods of 2 models, scaled by the degrees of freedom.

3 Hypothesis testing: The L.R. test is used to determined whether the additional parameters in the alternative model significantly improve the model fit compared to the null model. Thus, it tests the overall fit of the alternative model.

4. Degrees of freedom: The degrees for freedom for the L.R test statistic are equal to the difference in the number of Parameters estimated in the two models.

5. Interpretation :- A significant LR test statistic (i.e., a p-value below a choosen significance level, often 0.05) suggests that the alternative model provides a significiantly better fit to the data than the null model. In this case, you would reject the null hypothesis in favor of the alternative hypothesis

6. Applications :- LR tests are commonly used in SEM and surv models for model comparison, assessing the adequacy of model specifications and testing specific hypothesis about the relationships among variables.