RADAR ENGINEERING

UNIT-2

CW AND FREQUENCY MODULATED RADAR

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DOPPLER EFFECT

A radar detects the presence of objects and locates their position in space by transmitting electromagnetic energy and observing the returned echo.

It is well known in the fields of optics and acoustics that if either the source of oscillation or the observer of the oscillation is in motion, an apparent shift in frequency will result. This is the ***Dopplereffect***and is the basis of CW radar.

When wave energy like sound or radio waves travels between two objects, the frequency can seem to be changed if one or both of them are moving. This is called the **Doppler effect.**

If R is the distance from the radar to target, the total number of wavelengths λcontained in the two-way path between the radar and the target is $2R/\lambda$. The distance R and the wavelength λ are assumed to be measured in the same units.

Since one wavelength corresponds to an phase angle excursion of $\mathbf{2\pi}$radians, the total phase angle excursion **Ø** made by the electromagnetic wave during its transit to and from the target is $\mathbf{2\pi * 2R/\lambda = 4\pi R/\lambda}$radians.

If the target is in motion, **R** and the phase **Ø** are continually changing. A change in **Ø** with respect to time is equal to frequency. This is the Doppler angular frequency $\boldsymbol{\omega_d}$and is given by:

$$Ø = (4\pi R/\lambda) \qquad \text{--Apply differentiate on both sides}$$

$$dØ/dt = d(4\pi R/\lambda)/dt \Rightarrow \omega_d = (4\pi/\lambda) . dR/dt \qquad \text{Where-- } dR/dt = Vr$$

$$2\pi f_d = (4\pi/\lambda). Vr = 4\pi.Vr / \lambda$$

$$\boxed{f_d = \frac{2\,Vr}{\lambda} = \frac{2\,Vr\,f_0}{c}} \qquad \text{Hence} \qquad \boxed{\mathbf{V_r = f_d\lambda / 2}}$$

Where $\mathbf{f_d}$ is the Doppler frequency shift in Hz, and

Vr= relative velocity of the target with respect to the Radar.

$\mathbf{f_o}$= transmitted frequency,

c = velocity of propagation of the electromagnetic waves (same as that of light) = $\mathbf{3 \times 10^8}$ m/s.

If $\mathbf{f_d}$ is **in** hertz. **Vr** in **knots**, and **λ** in meters then the Doppler frequency $\mathbf{f_d}$ is given by

$$\mathbf{f_d = 1.03\ Vr / \lambda}$$

The relative velocity may be written as $\mathbf{V_r = V .\cos\theta}$ where **V** is the target speed and **θ** is angle made by the target trajectory and the line joining radar and target. When **θ=0** the Doppler frequency is maximum. The Doppler is zero when the trajectory is perpendicular to the radar line of sight $\mathbf{(\theta = 90^o)}$.

CW RADAR

Consider the simple CW radar as illustrated by the block diagram of Figure below. **T**he transmitter generates a continuous (unmodulated) oscillation of frequency **fo**, which is radiated by the antenna. A portion of the radiated energy is intercepted by the target and is scattered, some of it in the direction of the radar, where it is collected by the receiving antenna.

If the target is in motion with a velocity **Vr** relative to the radar, the received signal will be shifted in frequency from the transmitted frequency **fo** by an amount $+/-f_d$ as given by the equation : $f_d = 2Vr / \lambda = 2\ Vrfo / c$ **.** The plus sign associated with the Doppler frequency applies if the distance between target and radar is decreasing (approaching target) that is, when the received signal frequency is greater than the transmitted signal frequency. The minus sign applies if the distance is increasing (receding target).

The received echo signal at a frequency $fo +/- f_d$ enters the radar via the antenna and is heterodyned in the detector (mixer) with a portion of the transmitter signal **fo** to produce a Doppler beat note of frequency f_d. The sign of f_d is lost in this process.

The purpose of the Doppler amplifier is to eliminate echoes from stationary targets and to amplify the Doppler echo signal to a level where it can operate an indicating device. Its frequency response characteristic is shown in the figure (b) below. The low-frequency cutoff must be high enough to reject the d-c component caused by stationary targets, but yet it must be low enough to pass the smallest Doppler frequency expected. The upper cutoff frequency is selected to pass the highest Doppler frequency expected.

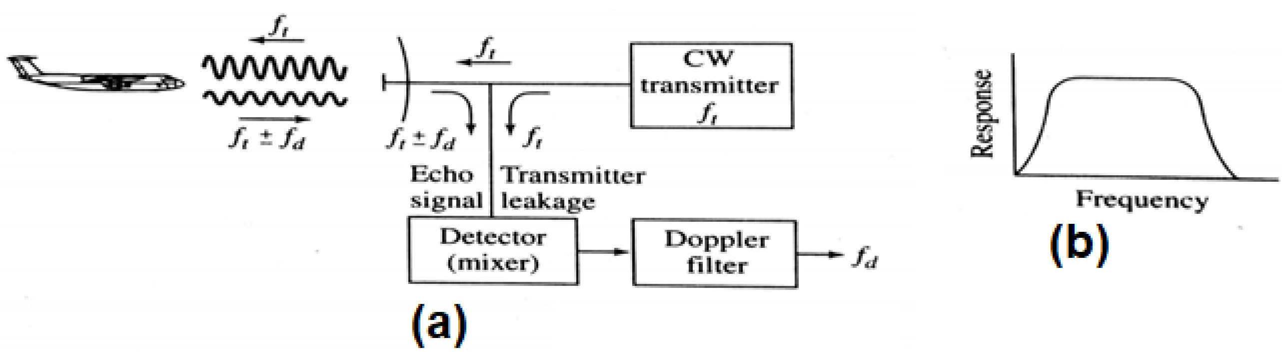


Figure 1: Simple CW radar block diagram (b) the frequency response of the Doppler filter

Isolation between Transmitter and Receiver

A single antenna serves the purpose of transmission and reception in the simple CW radar described above. In principle, a single antenna may be employed since the necessary isolation between the transmitted and the received signals is achieved via separation in frequency as a result of the doppler effect.

In practice, it is not possible to eliminate completely the transmitter leakage. However, transmitter leakage is not always undesirable. A moderate amount of leakage entering the receiver along with the echo signal supplies the reference necessary for the detection of the doppler frequency shift. If a leakage signal of sufficient magnitude were not present, a sample of the transmitted signal would have to be deliberately introduced into the receiver to provide the necessary reference frequency.

There are two practical effects which limit the amount of transmitter leakage power which can be tolerated at the receiver. These are

(1) The maximum amount of power the receiver input circuitry can withstand before it is physically damaged or its sensitivity reduced (burnout)

(2) The amount of transmitter noise due to hum, microphones, stray pick-up, and instability which enters the receiver from the transmitter.

The amount of isolation required depends on the transmitter power and the accompanying transmitter noise as well as the ruggedness and the sensitivity of the receiver. Isolation between transmitter and receiver might be obtained with a single antenna by using a hybrid junction, circulator, turnstile junction, or with separate polarizations.

Non-zero IF Receiver or Intermediate-frequency receiver:

Limitation of Zero IF receiver: The receiver of the simple CW radar of above Fig is in some respects analogous to a superheterodyne receiver. Receivers of this type are called homodyne receivers, or superheterodyne receivers with zero IF.

This simpler receiver is not very sensitive because of increased noise at the lower intermediate frequencies caused by flicker effect. Flicker-effect noise occurs in semiconductor devices such as diode detectors and cathodes of vacuum tubes. The noise power produced by the flicker effect varies as $\mathbf{1/f^{\alpha}}$ where $\boldsymbol{\alpha}$ is approximately unity. This is in contrast to shot noise or thermal noise, which is independent of frequency

Thus, at the lower range of frequencies (audio or video region), where the Doppler frequencies usually are found, and the detector of the CW receiver can introduce a considerable amount of flicker noise, resulting in reduced receiver sensitivity.

Non-zero IF Receiver: The effects of flicker noise are overcome in the normal superheterodyne receiver by using an intermediate frequency high enough to render the flicker noise small compared with the normal receiver noise. This results from the inverse frequency dependence of flicker noise. Figure 2 shows a block diagram of the CW radar whose receiver operates with a nonzero IF**.**

Separate antennas are used for transmission and reception. Instead of the usual local oscillator found in the conventional susuperheterodyne receiver, the local oscillator (or reference signal) is derived in this receiver from a portion of the transmitted signal mixed with a locally generated signal of frequency equal to that of the receiver IF.

Since the output of the mixer consists of two sidebands on either side of the carrier plus higher harmonics, a narrowband filter selects one of the sidebands as the reference signal. The improvement in receiver sensitivity with an intermediate-frequency super heterodyne might be as much as 30 dB over the simple zero IF receiver discussed earlier.

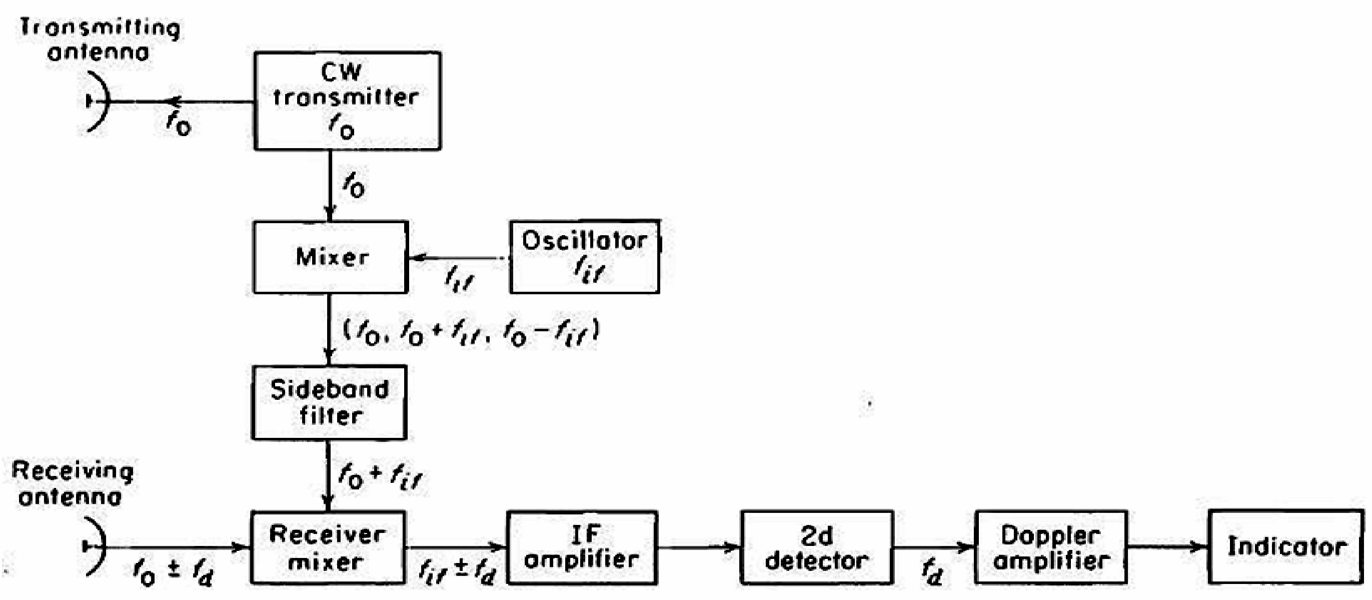


Figure 2: Block diagram of a CW Doppler radar with nonzero IF receiver, also called sideband super heterodyne Receiver.

Receiver bandwidth

One of the requirements of the doppler-frequency amplifier in the simple CW radar or the IF amplifier of the sideband superheterodyne (Fig. 2) is that it must be wide enough to pass the expected range of doppler frequencies.

In most cases of practical interest the expected range of doppler frequencies will be much wider than the frequency spectrum occupied by the signal energy. Consequently, the use of a wideband amplifier covering the expected doppler range will result in an increase in noise and a lowering of the receiver sensitivity.

If the frequency of the doppler-shifted echo signal were known beforehand, a narrowband filter-one just wide enough to reduce the excess noise without eliminating a significant amount of signal energy-might be used.

If the received waveform were a sine wave of infinite duration, its frequency spectrum would be a delta function (Fig.3.a) and the receiver bandwidth would be infinitesimal. But a sine wave of infinite duration and an infinitesimal bandwidth cannot occur in nature.

The more normal situation is an echo signal which is a sine wave of finite rather than infinite duration. The frequency spectrum of a finite-duration sine wave has a shape of the form$[\sin\pi(f-f_o)\delta]/\ \pi(f-f_o)$ where f_o and δ are the frequency and duration of the sine wave, respectively, and f is the frequency variable over which the spectrum is plotted . Practical receivers can only approximate this characteristic.

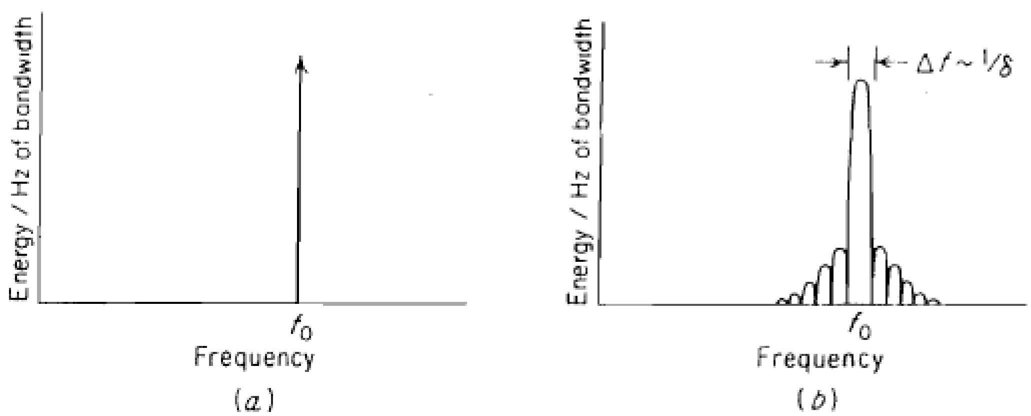


Fig 3 Frequency spectrum of CW oscillation of (a) infinite duration and (b) finite duration.

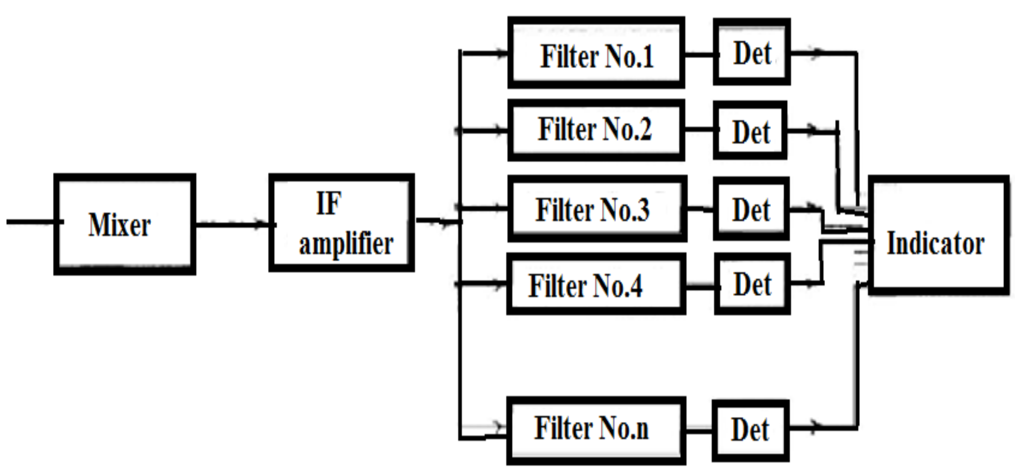
In many instances, the echo is not a pure sine wave of finite duration but is perturbed by fluctuations in cross section, target accelerations, scanning fluctuations, etc., which tend to broaden the bandwidth still further. In addition to the spread of the received signal spectrum caused by the finite time on target, the spectrum may be further widened if the target cross section fluctuates.

If the target's relative velocity is not constant, a further widening of the received signal spectrum can occur. If a_r is the acceleration of the target with respect to the radar, the signal will occupy a bandwidth

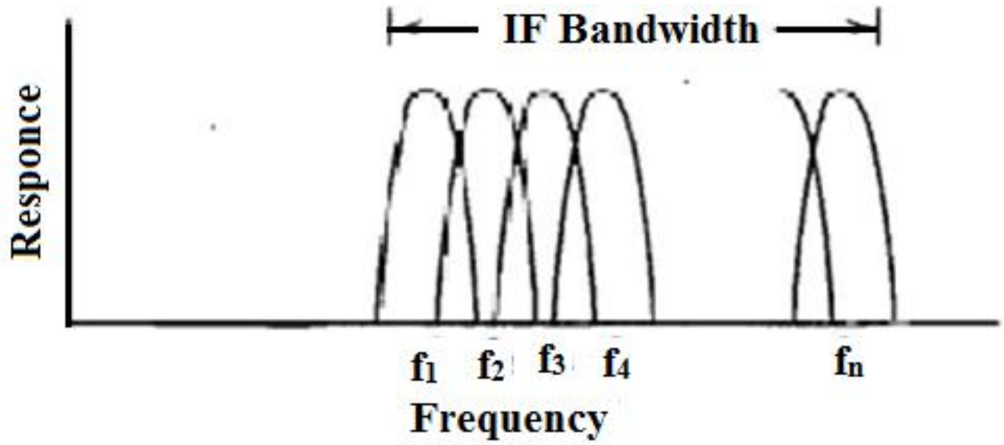
$$\boxed{\Delta f_d = \left(\frac{2a_r}{\lambda}\right)^{1/2}}$$

When the Doppler-shifted echo signal is known to lie somewhere within a relatively wideband of frequencies, a bank of narrowband filters as shown below spaced throughout the frequency range permits a measurement of frequency and improves the signal-to-noise ratio.

The bandwidth of each individual filter should be wide enough to accept the signal energy, but not so wide as to introduce more noise. The center frequencies of the filters are staggered to cover the entire range of Doppler frequencies



(a)



(b)

Figure 4: (a) Block diagram of IF Doppler filter bank (b) frequency-response characteristic of Doppler filter bank.

Sign of the Radial Velocity

In some applications of CW radar it is of interest to know whether the target is approaching or receding. This might be determined with separate filters located on either side of the intermediate frequency. If the echo-signal frequency lies below the carrier, the target is receding; if the echo frequency is greater than the carrier, the target is approaching.

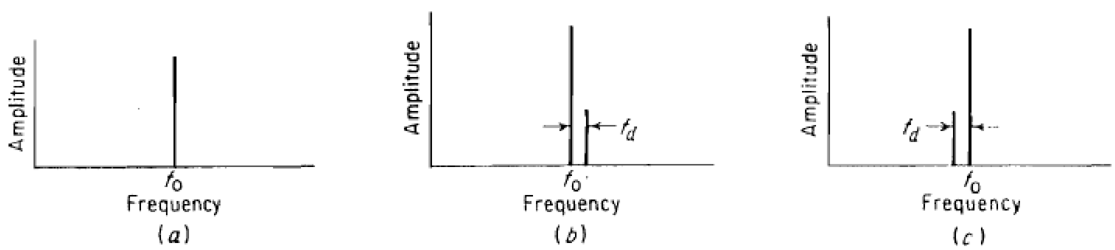


Fig 4: Spectrum of received signals (a) No doppler shift, no relative target motion; (b) approachingTarget, (c) Receding target.

It is possible to determine its sign from a technique borrowed from single-sideband communications. If the transmitter signal is given by

$$E_t = E_0 \cos \omega_0 t$$

the echo signal from a moving target will be

$$E_r = k_1 E_0 \cos \left[(\omega_0 \pm \omega_d)t + \phi\right]$$

The sign of the doppler frequency, and therefore the direction of target motion, may be found by splitting the received signal into two channels as shown in Fig.5

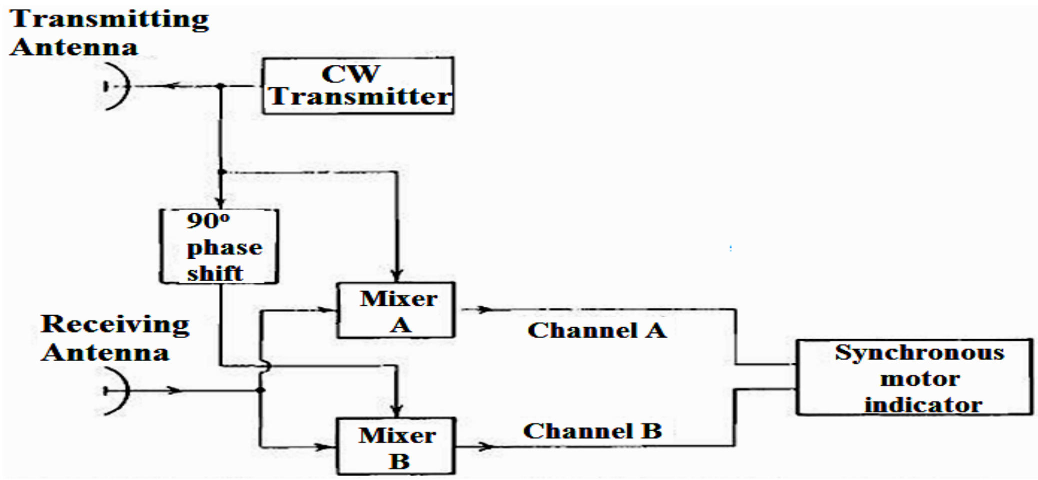


Fig 5: Measurement of doppler direction by two phase synchronus two phase moto

.In channel A, the signal is processed as in the simple CW radar. The received signal and a portion of the transmitter heterodyne in the detector (mixer) to yield a difference signal

$$E_A = k_2 E_0 \cos(\pm\omega_d t + \phi)$$

The other channel is similar, except for a 90° phase delay introduced in the reference signal. The output of the channel B mixer is

$$E_B = k_2 E_0 \cos\left(\pm\omega_d t + \phi + \frac{\pi}{2}\right)$$

If the **target is approaching** (positive doppler). the outputs from the two channels are

$$E_A(+) = k_2 E_0 \cos(\omega_d t + \phi) \qquad E_B(+) = k_2 E_0 \cos\left(\omega_d t + \phi + \frac{\pi}{2}\right)$$

The sign of ω_d and the direction of the target's motion may be determined according to whether the output of channel B leads or lags the output of channel A.

I.e. **B** output **lags** with **A** output from above equation, then motor rotates in **clockwise direction**. This means **target is approaching**

On the other hand, if the **targets are receding** (negative doppler),

$$E_A(-) = k_2 E_0 \cos(\omega_d t - \phi) \qquad E_B(-) = k_2 E_0 \cos\left(\omega_d t - \phi - \frac{\pi}{2}\right)$$

I.e. **B** output **leads** with **A** output, then motor rotates in **anti - clockwise direction**. This means **targets are receding**

APPLICATIONS OF CW RADAR

- The chief use of the simple, unmodulated CW radar is for the measurement of the relative velocity of a moving target, as in the police speed monitor.
- Rate-of-climb meter for vertical-take-off aircraft.
- CW radar has been suggested for the control of traffic lights,
- regulation of tollbooths
- vehicle counting
- For railways, CW radar can be used as a speedometer to replace the conventional axle-driven tachometer.
- It has also used for the measurement of the velocity of missiles, baseballs.
- In industry this has been applied to the measurement of turbine-blade vibration,
- Measurement of the speed of grinding wheels.
- Monitoring of vibrations in the cables of suspension bridges.
- High-power CW radars for the detection of aircraft and other targets have been developed and have been used in such systems as the Hawk missile systems.

Advantages and disadvantages (Limitations) of CW Radars:

- The principal advantage of CW Doppler radar over the other (non radar) methods of measuring speed is that there need not be any physical contact with the object whose speed is being measured

- Most of the above applications can be satisfied with a simple, solid-state CW source with powers in tens of milli watts

- The difficulty of eliminating the leakage of the transmitter signals into the receiver

- Transmitter noise reduces noise sensitivity

- Major disadvantage of the simple CW radar is its inability to obtain a **measurement of range**. This limitation can be overcome by modulating the CW carrier, as in the **frequency-modulated radar**.

Frequency Modulated CW Radar (FMCW)

Introduction: The inability of the simple CW radar to measure range is mainly due to the lack of a Timing mark. The timing mark permits the time of transmission and the time of return to be recognized but it increases the spectrum of the transmitted waveform.

This follows from the properties of the Fourier transform. Therefore a finite spectrum of necessity must be transmitted if transit time or range is to be measured. This means if you take more spectrum, automatically it gives accurate measurement of transient time. The spectrum of a CW transmission can be broadened by the application of a modulation – amplitude modulation, frequency modulation, or phase modulation.

The greater the transmitter frequency deviation in a given time interval, the more accurate is the measurement of the transit time but the transmitted spectrum also becomes larger.

A block diagram illustrating the principle of the FM-CW radar is shown in the figure below. A portion of the transmitter signal acts as the reference signal required to produce the beat frequency. It is introduced directly into the receiver via a cable or other direct connection.

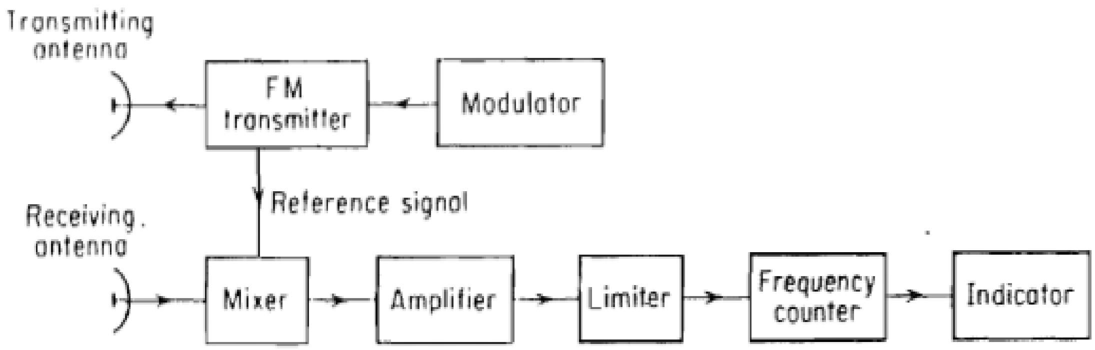


Figure 7: Block diagram of FM-CW RADAR

- Ideally the isolation between transmitting and receiving antennas is made sufficiently large so as to reduce to a negligible level the transmitter leakage signal which arrives at the receiver via the coupling between antennas.
- The beat frequency is amplified and limited to remove any amplitude fluctuations. The frequency of the amplitude-limited beat note is measured with a cycle-counting frequency meter calibrated in distance
- In the above, the target was assumed to be stationary. If this assumption is not applicable, a doppler frequency shift will be superimposed on the FM range beat note and an erroneous range measurement results, **if target is moving.**

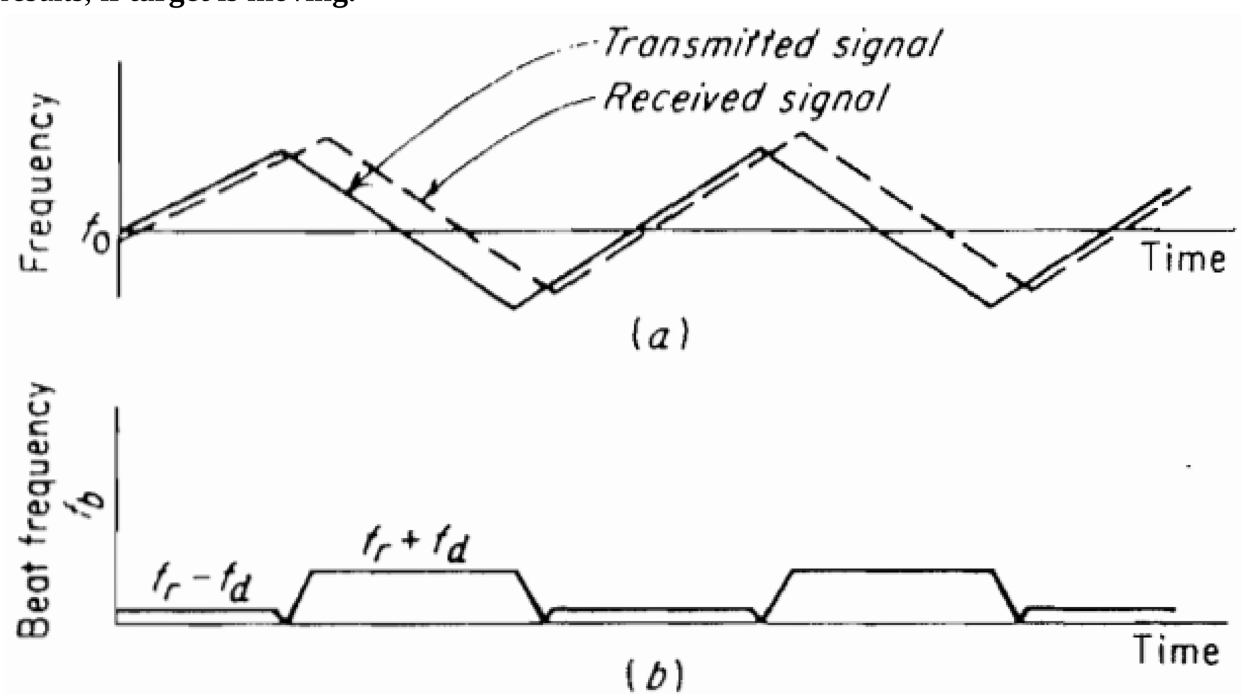


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Fig 8: Frequency-time relation-ships in FM-CW radar when the fr + fd received signal is shifted in frequency by the doppler effect (a) Transmitted (solid curve) and echo (dashed curve); (b) beat frequency

 The doppler frequency shift causes thefrequency-time plot of the echo signal to be shifted up or down (Fig.#). On one portion of the frequency-modulation cycle the heat frequency (Fig.#) is increased by the doppler shift, while on the other portion it is decreased.

Figure 8: Frequency-time relationships in FM-CW radar when the received signal is shifted in frequency by the Doppler effect (a) Transmitted (solid curve) and echo (dashed curve) (b) beat frequency

If for example, the target is approaching the radar, the beat frequency f_b(up) produced during the increasing, or up, portion of the FM cycle will be the difference between thebeat frequency due to the range fr, and the doppler frequency shift $\mathbf{f_d}$. Similarly, on the decreasing portion, the beat frequency, f_b(down) is the sum of the two.

Case 1 if $f_r > f_d$ (closing target)

$$f_b(up) = f_r - f_d \quad \text{and} \quad f_b(down) = f_r + f_d$$

The range frequency fr, may be extracted by measuring the average beat frequency; that is,

$$\boxed{\mathbf{fr = [fb(up) + fb(down)]/2 \quad f_d = [fb(up) - fb(down)]/2}}$$

- If f_b(up) and f_b(down) are measured separately, for example, by switching a frequency counter every half modulation cycle, one-half the difference between the frequencies will yield the doppler frequency.
- If, on the other hand, $f_r < f_d$ such as might occur with a high-speed target at short range, the roles of the averaging and the difference-frequency measurements are reversed; the averaging meter will measure Doppler velocity, and the difference meter, range.

Case 1 if $f_r < f_d$ (Receding target)

$$f_b\,(up) = f_r + f_d \quad \text{and} \quad f_b\,(down) = f_r - f_d$$

$$\boxed{\mathbf{fr = [fb(up) - fb(down)]/2 \quad and \quad f_d = [fb(up) + fb(down)]/2}}$$

With the help of f_r and f_d, we can easily measure range and velocity of the target

Range and Doppler Measurement:-

- The frequency-modulated CW radar (abbreviated as FM-CW), the transmitter frequency is changed as a function of time in a known manner. Assume that the transmitter frequency increases linearly with time, as shown by the solid line in Fig.6

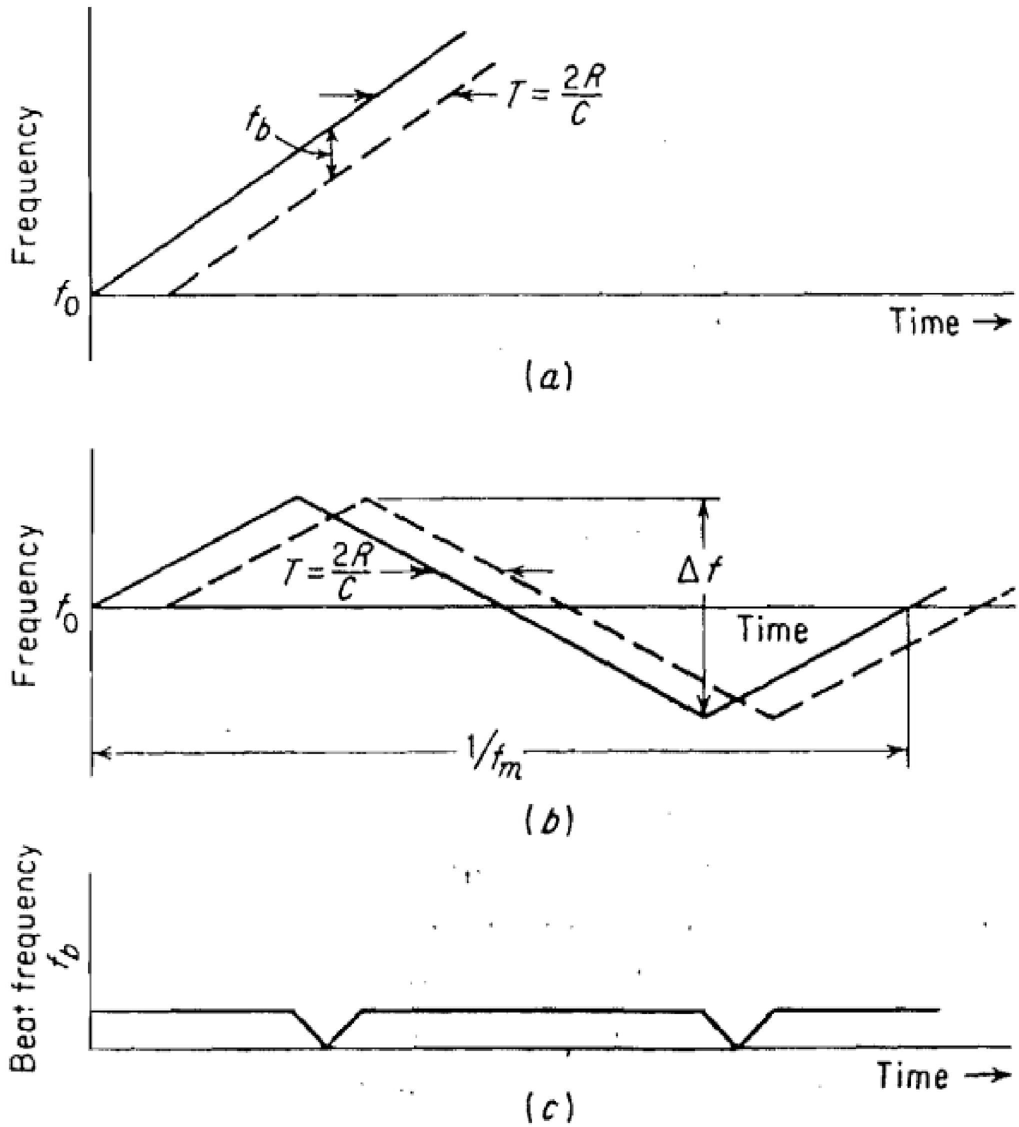

Fig 6:Frequency-time relationships in FM-CW radar. Solid curve represents transmitted signal, dashed curve represents echo. (a) Linear frequency modulation; (b) triangular frequency modulation; (c) beat note of (b)

If there is a reflecting object at a distance **R**, the echo signal will return after a time $\mathbf{t_d = 2R/c}$. The dashed line in the figure represents the echo signal. When the echo signal is heterodyned with a portion of the transmitter signal in a nonlinear element such as a diode, a beat note ***fb*** will be produced

If target is stationary, there is no doppler frequency shift, the beat note (difference frequency) is a measure of the target's range and ***fb* = *f*r** where ***fr*** is the beat frequency due only to the target's range

The beat frequency depending upon transient time ($\mathbf{t_d = 2R/c}$) and frequency deviation (i.e. the rate of change of carrier frequency w.r.t time **fo(dot)**) is given by

$$f_b = f_r = f^{\wedge}_{o} \boldsymbol{t_d} = \frac{\boldsymbol{\Delta f}}{\boldsymbol{\Delta T}} \frac{\boldsymbol{2R}}{\boldsymbol{c}} \qquad -- \Delta \boldsymbol{T} = \frac{\boldsymbol{T_m}}{\mathbf{2}} = \frac{\mathbf{1}}{\boldsymbol{2f_m}}$$

$$f_r = \frac{\Delta f}{\frac{1}{2f_m}} \frac{2R}{c} \Rightarrow \quad f_r = \frac{4 f_m R \Delta f}{c}$$

$$\boxed{R = \frac{f_r\, c}{4 f_m \Delta f}}$$

Thus the measurement of the beat frequency determines the range R.

Where f_r is beat frequency

f_m is modulated signal frequency

Δf is the rate change in carrier frequency (higher frequency - lower frequency

c= velocity of propagation of the electromagnetic waves (same as that of light) = **3 X 10^8** m/s.

FMCW Altimeter:

- The FM-CW radar principle is used in the aircraft radio altimeter to measure height above the surface of the earth. The large backscatter cross section and the relatively short ranges required of altimeters permit low transmitter power and low antenna gain.
- Since the relative motion between the aircraft and ground is small, the effect of the Doppler frequency shift may usually be neglected.
- The band from 4.2 to 4.4 G Hz is reserved for radio altimeters, although they have in the past operated at UHF.
- The transmitter power is relatively low and can be obtained from a CW magnetron, a backward-wave oscillator, or a reflex klystron, but these have been replaced by the solid state transmitter.
- The altimeter can employ a simple homodyne receiver, but for better sensitivity and stability the superheterodyne is to be preferred whenever its more complex construction can be tolerated.

A block diagram of the FM-CW radar with a sideband superheterodyne receiver shown in Fig. 9

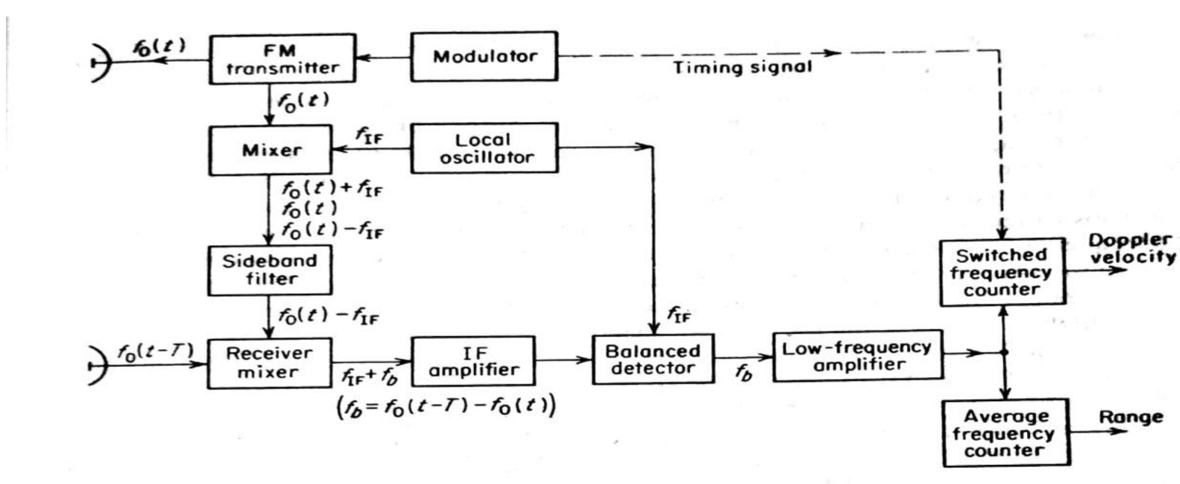

Fig 9:Block diagram of FM-CW radar using sideband superheterodyne receiver

- A portion of the frequency-modulated transmitted signal is applied to a mixer along with the oscillator signal.

The local-oscillator frequency f_{IF} should be the same as the intermediate frequency used in the receiver, whereas in the conventional superheterodyne the LO frequency is of the same order of magnitude as the RF signal.

- The output of the mixer consists of the varying transmitter frequency $f_o(t)$ plus two sideband frequencies, one on either side of $f_o(t)$ and separated from $f_o(t)$ by the local-oscillator frequency f_{IF}.
- The filter selects the lower sideband $f_o(t)$ - f_{IF} and rejects the carrier and the upper sideband. The sideband that is passed by the filter is modulated in the same fashion as the transmitted signal.
- The sideband filter must have sufficient bandwidth to pass the modulation, but not the carrier or other sideband.
- The filtered sideband serves the function of the local oscillator. When an echo signal is present, the output of the receiver mixer is an IF signal of frequency f_{IF}+ f_b where $\mathbf{f_b}$ is composed of the range frequency fr and the doppler velocity frequency f_d.
- The IF signals is amplified and applied to the balanced detector along with the local oscillator signal f_{IF}
- The output of the detector contains the beat frequency (range frequency and the Doppler velocity frequency), which is amplified to a level where it can actuate the frequency measuring circuits.
- In Fig. the output of the low-frequency amplifier is divided into two channels. one feeds an average-frequency counter to determine range, the other feeds a switched frequency counter to determine the doppler velocity (assuming f_r>f_d) Only the averaging frequency counter need be used in an altimeter application, since the rate of change of altitude is usually small.

EFFECT OF NOISE SIGNALS ON FM ALTIMETER:

The different noise signals occurring in a typical FM altimeter are:

- Due to the mismatch in impedance a part transmitted signal gets reflected from the space causing error in the altimeter.
- The mismatch between the sideband filter and receiving gives rise to standing wave pattern.
- The leakage signal due to the transmitting and receiving antennas reach the receiver and cause error.
- The double bounce signal.

Multiple Frequency CW Radar (MFCW)

The CW radar does not measure range, it is possible under some circumstances to do so by measuring the phase of the echo signal relative to the phase of the transmitted signal.

Consider a CW radar radiating a single-frequency sine wave of the form sin$(2\pi f_o t)$ (The amplitude of the signal is taken to be unity since it does not influence the result) the signal travels to the target at a range **R** and returns to the radar after a time **T = 2R/c** where **c** is the velocity of propagation.

The echo signal received at the radar is $\sin[2\pi f_o(t - T)]$. If the transmitted and received signals are compared in a phase detector, the output **is** proportional to the phase difference between the two and is given by :

$\Delta\varnothing = 2\pi f_0 T = 4\pi f_o R / c$.   *Derivation w.r.t to time domain*

Or $\Delta\varnothing = 4\pi R / \lambda = 4\pi f_o R / c$ *Derivation w.r.t to frequency domain, but both are same.*

The phase difference may therefore be used as a measure of the range, or

$$\boxed{R = \frac{\Delta\text{Ø}\ \mathbf{c}}{\mathbf{4\pi fo}} = \frac{\Delta\text{Ø}\ \lambda}{\mathbf{4\pi}}} \qquad \text{………… [Eq. 2]}$$

- The variation of phase with freq. is the fundamental basis of radar measurement of time delay or range measurement.
- However, the measurement of the phase difference ΔØ is unambiguous only if ΔØ does not exceed 2π radians. Substituting ΔØ= ***2π*** into the above equation gives the maximum unambiguous range as **λ/*2.*** At radar frequencies this unambiguous range is much too small to be of any practical interest

.

Phase measurement of range detection has two problems

1. The separation of Tx and Rx signal w.r.t phase is very difficult (the calculation of phase difference is difficult) if you transmit single frequency continously
2. Ambiguity of phase measured beyond 2π

- Unambiguous range may be extended considerably by utilizing two separate CW signals differing slightly in frequency. The principal used in multiple freq. CW radar is the measurement of range by computing the phase difference.

The transmitted waveform is assumed to consist of two continuous sine waves of frequency ***f_1**and* ***f_2***separated by an amount **Δf.** For convenience, the amplitudes of all signals are set equal to unity. The voltage waveforms of the two components of the transmitted signal ***V_{1T} and V_{2T}*** may be written as

$V_{1T} = \sin(2\pi f_1 t + Ø_1)$ $\qquad$ $V_{2T} = \sin(2\pi f_2 t + Ø_2)$

where **$Ø_1$ and $Ø_2$** are arbitrary (constant) phase angles. The echo signal is shifted in frequency by the Doppler Effect. The form of the Doppler-shifted signals corresponding to the two frequencies ***f_1** and* ***f_2*** are:

$$v_{1R} = \sin\left[2\pi(f_1 \pm f_{d1})t - \frac{4\pi f_1 R_0}{c} + \phi_1\right]$$

$$v_{2R} = \sin\left[2\pi(f_2 \pm f_{d2})t - \frac{4\pi f_2 R_0}{c} + \phi_2\right]$$

Where **Ro** = Range to target at a particular time ***t*** = ***to*** (range that would be measured if target were not moving)

f_{d1} = Doppler frequency shift associated with frequency **f_1**

f_{d2}= Doppler frequency shift associated with frequency **f_2**

- Since the two RF frequencies **f_1**and **f_2**are approximately the same (that is **f_2=f_1+ Δf,** where **Δf<<f $_1$)**the Doppler frequency shifts **fd1**and **fd2** can be assumed to be equal to each other. Therefore we may write **f_{d1}**= **f_{d2}= f_d**
- The receiver separates the two components of the echo signal and heterodynes each received signal component with the corresponding transmitted waveform and extracts the two Doppler-frequency components given below

$$v_{1D} = \sin\left(\pm 2\pi f_d t - \frac{4\pi f_1 R_0}{c}\right)$$

$$v_{2D} = \sin\left(\pm 2\pi f_d t - \frac{4\pi f_2 R_0}{c}\right)$$

These two signals give to the phase detector. It will give the phase difference between two signals is given by

$$\Delta\emptyset = \frac{4\pi\,(f_2 - f_1)R_0}{c} = \frac{4\pi\,\Delta f R_0}{c}$$

Hence $$\boxed{R_0 = \frac{c\ \Delta\emptyset}{4\pi\,\Delta f}}$$

Which is same as that of **Eq..2,** with **Δf** substituted in place of **fo**. We conclude that the phase difference determination is very easy when if you transmit two frequency signals. It is difficult when if you transmit single frequency

Sub $\Delta\emptyset = 2\pi$ in above equation

$$R_{unamb} = \frac{c}{2\ \Delta f}$$

The above equation describes **the maximum unambiguous range** increases when we use double frequency radar than CW radar

- A large difference in frequency between the two transmitted signals improves the accuracy. The two-frequency CW radar is essentially a single-target radar since only one phase difference can be measured at a time.

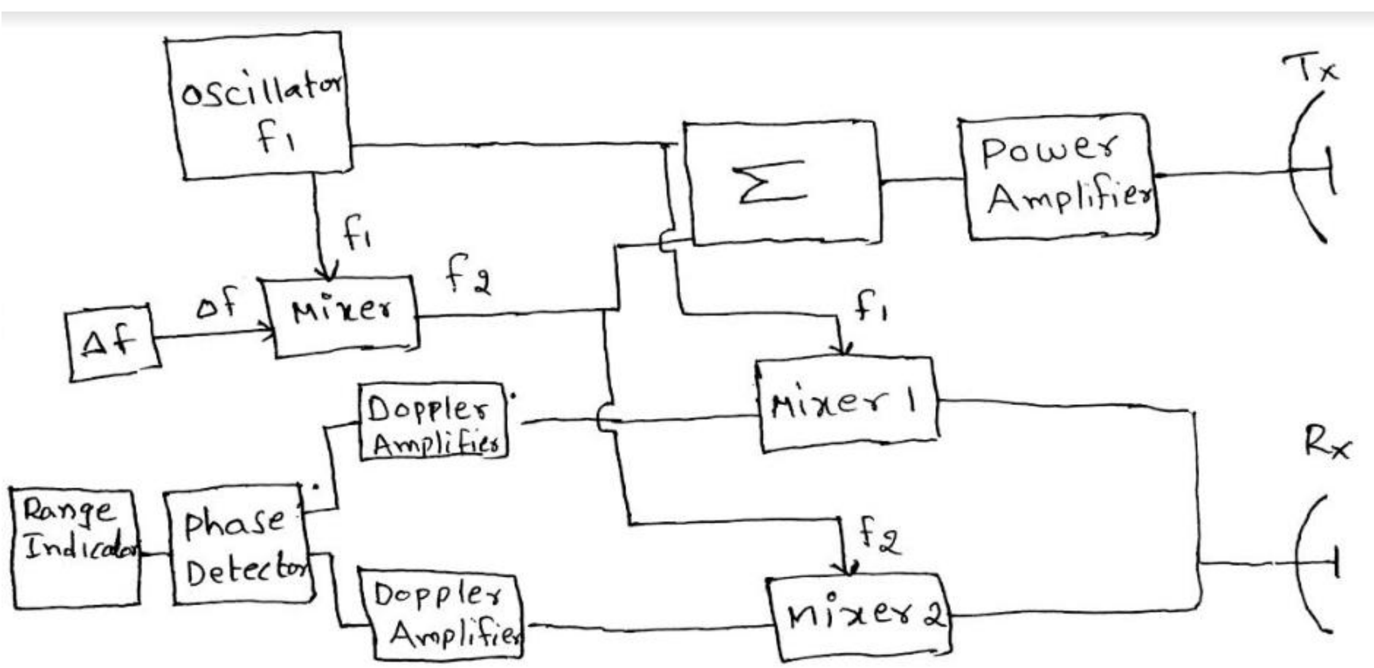

Fig: Double Frequency CW Radar Block Diagram

Important Formulae:

- Relation between Relative velocity Vr and Doppler frequency f_d : $\mathbf{f_d = 2Vr / \lambda = 2Vrf_0 / c}$
- Change in Doppler frequency due to target's acceleration:

$$\Delta f_d = \left(\frac{2a_r}{\lambda}\right)^{1/2}$$

- In a FM CW Radar:
  - Target's beat frequency due to rangef_r is given by (Assuming there is no Doppler shift):

$$f_r = \frac{4f_m R\Delta f}{c} \quad R = \frac{f_r c}{4f_m \Delta f}$$

Where f_m = modulating frequency and Δf = frequency swing

- Target's beat frequency **fr** and Doppler frequency $\mathbf{f_d}$ are given by (with Doppler shift for Approaching target):

fr = [fb(up) + fb(down)]/2 $\quad$ $\mathbf{f_d}$ **= [fb(up) - fb(down)]/2**

- In Double frequency radar the target range $\quad R_0 = \frac{c\ \Delta\text{Ø}}{4\pi\ \Delta f}$

Example1: Determine the Range and Doppler velocity of an approaching target using a triangular modulation FMCW Radar. Given : Beat frequency fb(up) = 15KHz and fb (down) = 25KHz , modulating frequency : 1MHz, Δf : 1KHz and Operating frequency : 3Ghz

Solution: We know**fr**= ½[**fb(up)**+ **fb (down)**] = ½(15+25) = 20 Khz

$\mathbf{f_d}$= ½[**fb (down) - fb(up)**] = ½(25-15) = 5 Khz

The Range **R** in terms of **fr ,fm and Δf**is given by **:**

R = c fr / 4fm.Δf = $(3\text{x}10^8)20\text{x}10^3 / 4(1\text{x}10^6\text{x}1\text{x}10^3)$ mtrs = 1500 mtrs = **1.5 Kms**

$$V_r = \frac{Cf_d}{2f_o}$$

Example2: What is the Doppler shift when tracking a car moving away from the RADAR at 50m/s and operating frequency 5 GHz?

Solution: $\quad f_d = \frac{2v_r}{\lambda}$

v_r= 50 m/s

f = 5 GHz

$\lambda = \frac{c}{f} = \frac{3\times10^8}{5\times10^9} = 0.06$ m

f_d=1667 Hz

Example 3: Calculate the Doppler of a stationary CW radar transmitting at 6MHz frequency when a moving target approaches the radar with a radial velocity of 100Km/Hour.

Solution: $f_d = \frac{2v_r}{\lambda}$

v_r= 100 x 10^3/360 = 1666.6 m/s

f = 6 MHz

$\lambda = \frac{c}{f} = \frac{3\times10^8}{6\times10^6} = 50$ m

f_d=1.1 Hz