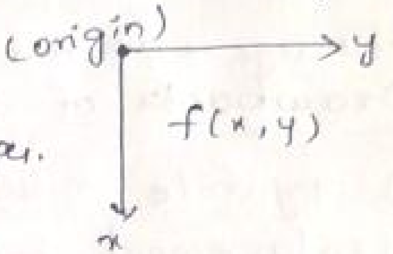
Unit - I

Introduction to Image procesing:-

what is an image?

An image is defined as a two dimensional function f(x,y) where x and y are spatial coordinates in a plane. The amplitude of f at any pair of coordinates (x,y) is called as the intensity or gray level of the image at that point.



An Image is categoriced into two types.

Analogy image:-

An image that can be mathematically represented as a continuous range of values representing position and intensity. Analog image is characterized by a physical magnitude varying continuously in space.

Digital image:-

An image that can be mathematically represented as a discrete range of values representing position and itensity

* Digital image is defined as the discipline in which both input and output of a process are image.
* Digital image processing is the process of Extracting attributes from images (upto the level till we are able to recognise the individual object). A digital image is composed of picture Elements as PIXEL.
* pixels are the smallest sample of an image. pixels represented the brightness at one point to convert an image into digital image involves two operations sampling and Quantization.

Advantages of Digital Image:-

1) processing of image is fast.
2) cost Effective
3) Effective storage
4) Efficient transfer
5) Image copy is easy
6) Image Quality is standard even when transfered multiple.
7) Reproduction of image is fast and Cheap.
8) Image captured can be viewed immediately to see the image

Drawbacks of digital Images:-

1) Copywhile misuse is easy.
2) Enlargement beyond a Certain size cannot be done.
3) Violating leads to loose of quality.
4) Memory required to store the good image is high.

* The process of processing an image by using a pcn; a processor based System is called as digital image processing. where the complete task is do to digitally.

A general digital image processing system Consists of

Input → Sense from vision → Imaging system → Samples and quantities → Digital Storage

↓

Digital computer → Buffers → Display output

Buffers → Recorder

How digital image is formed:-

A digital image is formed by multiple steps like the above image is Captured with the help of a sensor and the data is first stored in analog form by the process of sampling and Quantization. Then this digital data is stored.

Representation of a digital Image:-

As a digital image is formed by the process of sampling and quantization, so it can be represented as a matrix.

* If we consider an image as a function $f(x,y)$ which when sampled and quantised resulting a digital image of M rows and N columns, then

(a) The coordinates at the origin are $(0,0)$

(b) The coordinates at the first row will be

$(0,0), (0,1), (0,2), \ldots\ldots (0, N-1)$

where the value of M and N are always positive and cannot be negative.

* The amplitude at the coordinates (x,y) as the function $f(x,y)$ is called as gray level which can also be digitilized as 0 to $L-1$ a total of L levels, which is dynamic range of an image.

* when we want to store it digitally we have to calculate the gray level in bits if no. of bits are k then gray levels will be $L = 2^k$

$$A = \begin{bmatrix} a_{00} & a_{01} & a_{02} & \cdots & a_{0N} \end{bmatrix}$$

if gray level is 256 it means

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(1,N-1) \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{bmatrix}$$

$L = 256$

$L = 2^8$

$\boxed{k = 8}$

so it requires 8 bits to store the each pixel information.

Fundamentals steps in Digital Image processing:-

As digital image processing is a very wide area to study the technique clearly the whole process is divided into some fundamental steps as

(i) Image Acquisition

(ii) Image Enhancement

(iii) Image Restoration

(iv) Color Image processing

(v) wavelets and multiresolution processing.

(vi) Compression

(vii) Morphological processing

(viii) segmentation.

(ix) Representation and description

(x) object Recognition

(xi) knowledge data base

Image Acquisition:-

It is the first step in digital image processing where an image is acquired in the digital form. Here we have a image sensor. which when subjected to object produces the electrical equivalent signal. later it is digitized using ADC. Hence the image is sensed by the ilumination of light from the source and reflection (or) observation from sensor. This steps are also involves the preprocessing tasks such as scaling.

Image Enhancement:-

It is the second step in digital image processing where when an image is acquired by the reflection of iluminated light. The clarity may not be good when acquired. due to excess (or) low amount of luminuous. so the acquired image need to be Enhanced by highlighting the certain features as image using some of the mathematical tools like transforms etc...,. It is mainly required impose the quality of the captured image.

Image Restoration:-

In this step it is very well known that the previous step has Enhanced the Capture image features for the better quality so now we need to replace the captured with the Enhanced image data.

This is a objective process and it is based on the mathematical model and probabilistic models of image degradation.

Here we will go with a question "why the image is degraded. After getting the reason we will remove that problem by some methods.

Color Image processing:-

Here we deal with the modelling and processing of color images. It is mostly used in the present era by using multiple advanced algorithms.

wavelets and multiresolution processing:-

wavelets are the foundation for representing images in multiple resolutions (or) various degree of resolutions. It is used for the image data compression.

Compression:-

It is a technique that is mainly used for the storage of image at a concentrated memory here we can compress the size of pixel as well as the bandwidth for transmission of an image. But the process is done without changing the actual quality of the image.

Morphological processing:-

It is a process that deals with the tools which are required for extracting the useful image components in the representation and description by shape we Extract the image components we get the output of this stage as image attributes.

* All the steps that are seen here are called as the preprocessing steps. These steps are used for improving the image quality. Here we Enhance the contrast, remove noise Elements, setting the isolate regions,

brightness adjustment Etc.., The output of these preprocessing steps is also an image.

Segmentation:-

It is the process of converting a image into small images (or) segments so that we can Extract more accurate information from the segments. If the two segments are properly autonomous in nature then the two segments of an image should not have any identical information then the representation and description of image will be accurate and vice versa.

Normally the output of the segmentation stage is a raw pixel data which describes the pixels as Either boundary (or) region. For this initially it has to be decided that whether the data should be represented boundary (or) a complete region. Boundary representation is prefered when the External shape characteristics of the image due to corners (or) inflections are interested. Region representation is preferred when internal properties of the image such as texture (or) skeletal shape are interested.

Description:-

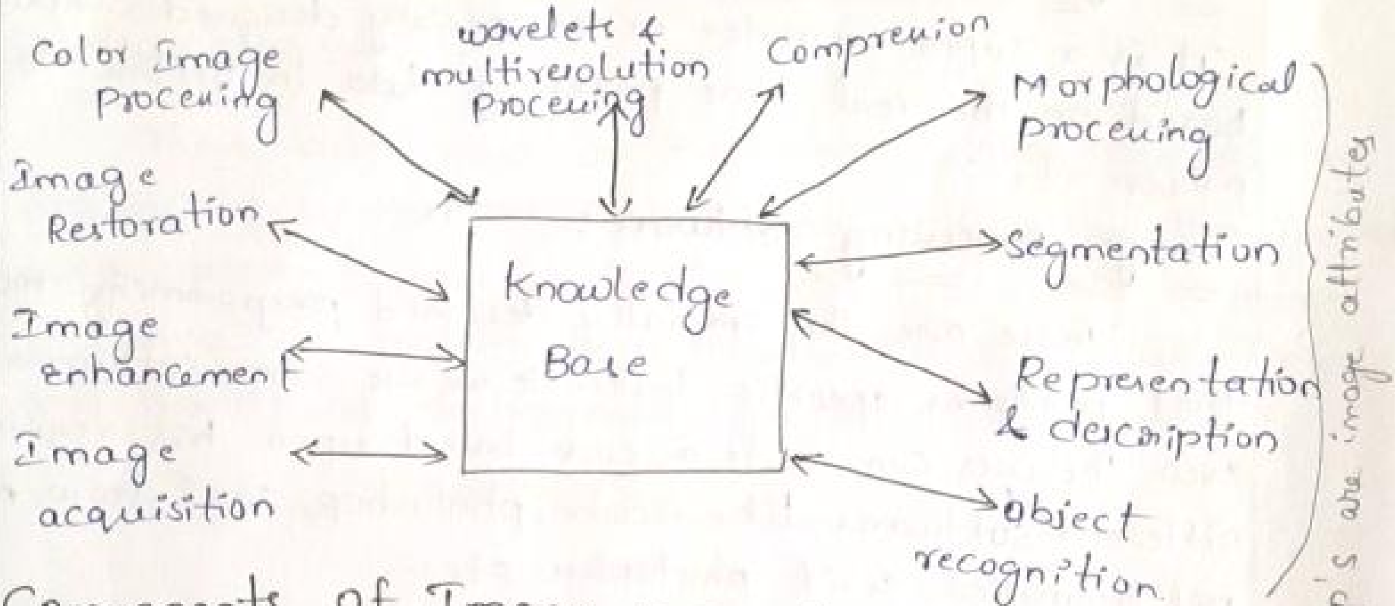
It is also called as feature selection. It deals with Extracting the attributes that results in some quantities information of interest. It is also used for differentiating one object from another.

Recognition:-

It is a process that is used to assign a lable to an object based on the description.

Knowledge Base:-

It is defined as the database or a software that helps the user for the proper image enhancement, restoration or compression techniques. It controls all the steps of the digital image processing. It is a base which supports every step in digital image processing by doing the required action over the data.



Components of Image processing System:-

Actually the components of digital image processing system various with different types of images and cannot be a single or fixed type. But in general there are some basic components which are used for the digital image processing using general purpose s/m. The system has to perform some of the operations like acquistion, storage, processing, Communication and display. The components are.

Image sensor/Acquistion:-

Sensor is a physical device which senses the Energy radiated by the object we wish to image.

* Digitizer is used for converting the output of the physical sensing device into digital form.

Specialized Image procesing Hardware:-

Image procesing hardware is used just before the intelligent device because it will perform the specialized proces on the image but with high speed.

It is a hardware which can work at high speed for real time procesing with high data throughputs which Cannot be procesed by a normal computer.

Intelligent procesing Machine:-

It is a super computer or a specially designed Computer based on the real time procesing data in offline to proces it.

Image procesing software:-

There are the specially designed programming modules that performs specific tasks. He we use some softwares where even the user can write a code based upon his requirement. Different softwares like Adobe, photoshop, corel draw, matlab pci Geomatica, serif photoplus etc.,

Mass storage/storage devices:-

These devices are used to store an image for different purposes as we do some type of proces in each step of digital image procesing. The changes and the old data most also be preserved until and unless we request to overwrite (or) save it else the actual raw information data will be lost. So here we have the importance of using mass storage.

Here storing devices can be of 3 types:-

① short-term storage for use during proces.

② on-line storage for relatively fast recall.

③ storage for frequent use.

Display devices:-

There devices mostly include Color TV, monitor preferably a flat screen monitor are driven by the output of image and graphics display cards which are placed in a Computer System.

Hard copy:-

There devices are used for recording images over a hard materials. There include lazer printers, film Cameras, heat sensitive devices, inkjet units, etc...,

Networking:-

These days most of the applications demand for networking so that one system can also process the s/m at another place. Because of large amount of data in image processing we require high bandwidth so optical fibers and broadband technologies are better options.

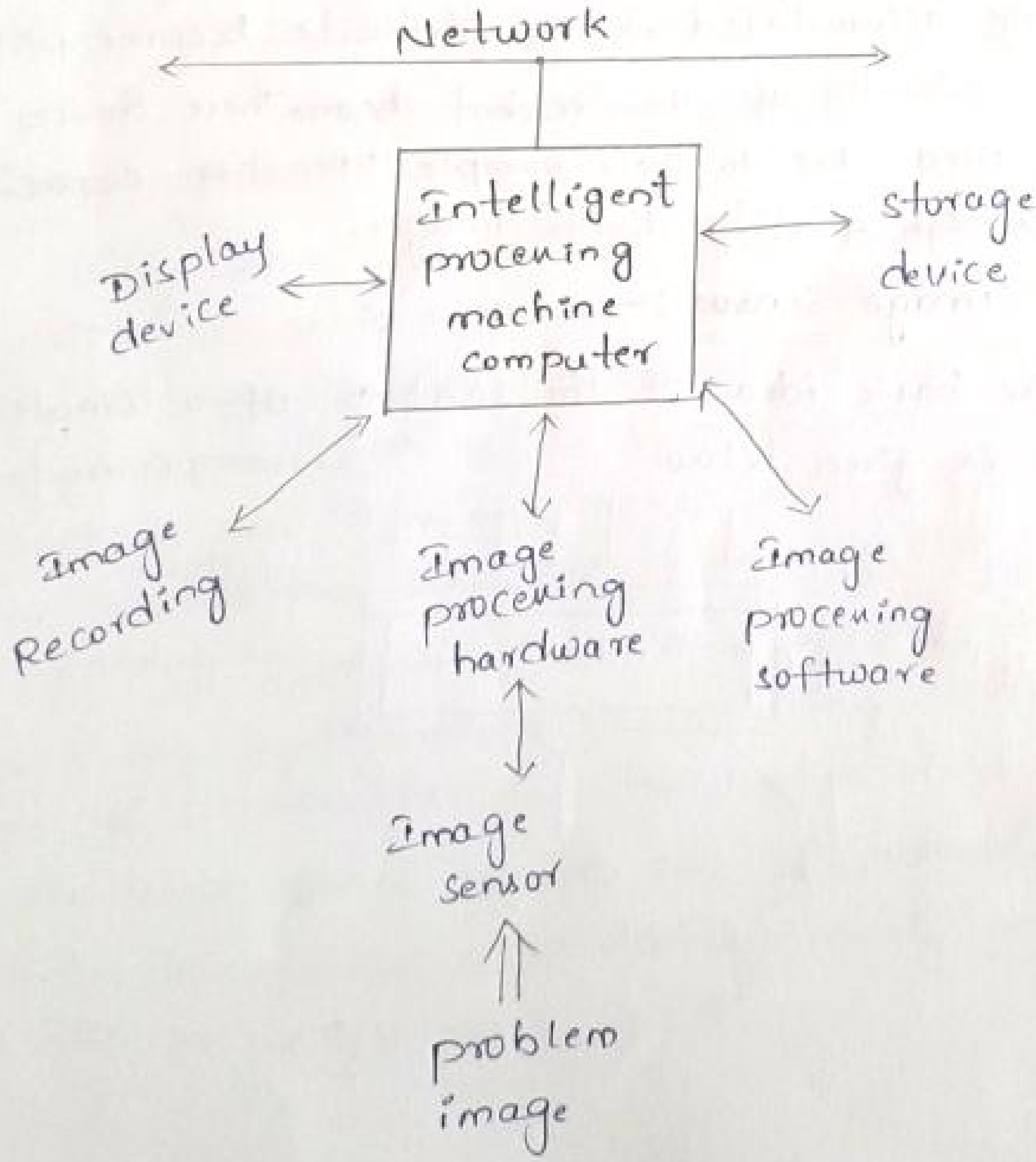


Image Sensing And Acquisition:-

A image is normally formed by the generation of an illumination source and the reflection (or) absorption of Energy from the souce by the Elements of scene bring imaged.

where the illumination can be obtained from any type of source.

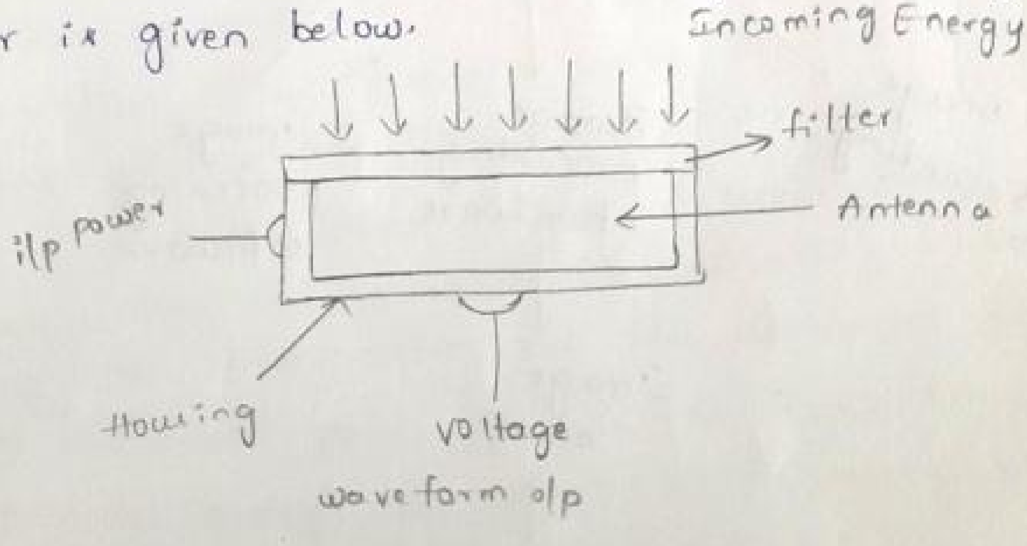
so, to sense the image we use sensor according to the nature of illumination. And the process of image sense is called as image acquistion.

Most of the places we use change coupled diode sensor where CCD is a collection of very tiny light sensitive diode which converts light into Electrical signals. These acts as photosites.

At every photosite, a beam of light strikes and the change accumulated at the photosite becomes proportional to the intensity of the incident beam. These devices are mostly used due to their sample operation capacity to produce high quality digital images.

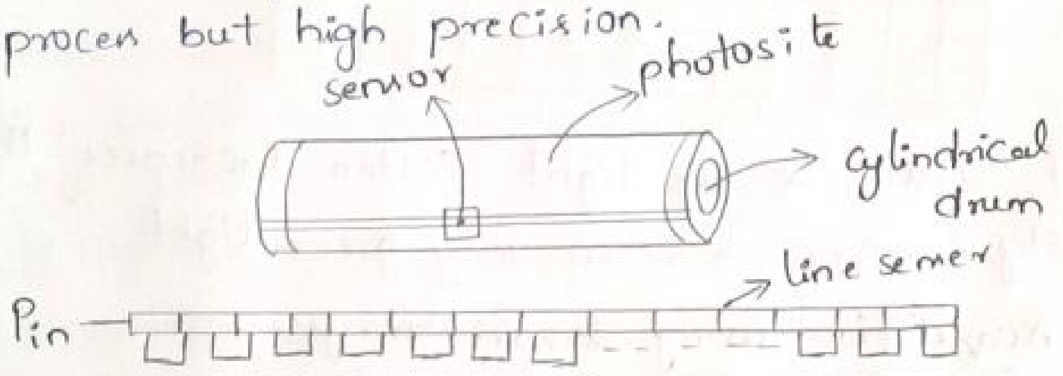
Single Image sensor:-

The basic idea of the working of a single image sensor is given below.



Here the photosite/photodiode is also a single sensor that generates output waveforms according to the illumination comming on the photodiode.

To generate a 2D Image the sensor must have a relative displacement in both x and y direction b/w the sensor and the area to be imaged. Here a negative film is mounted on drum. The mechanical movement of the drum provides the displacement in 1-direction and the single sensor is mounted on a lead screw that leads the motion in perpendicular direction. This method is a slow process but high precision.



voltage waveform outputs.

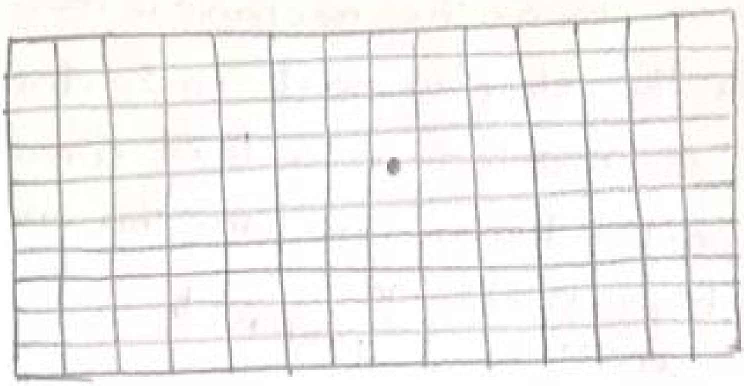
Image sensor strip:-

Image sensor strip is nothing but the method of arranging multiple sensors in a single line is called as Image sensor strip. This strip provides imaging element in one direction and the motion perpendicular to the strip provides in another direction. This type of image sensing is used in air bone applications like xerox, scanning etc..,

In another method. A sensor strip is mounted over a ring configuration. The rotating x-ray source provides illumination and the portion of sensor opposite to the sensor source collects the solar energy that provides through the object. It is mostly used in applications like CAT, ~~Com~~ MRI, PLT

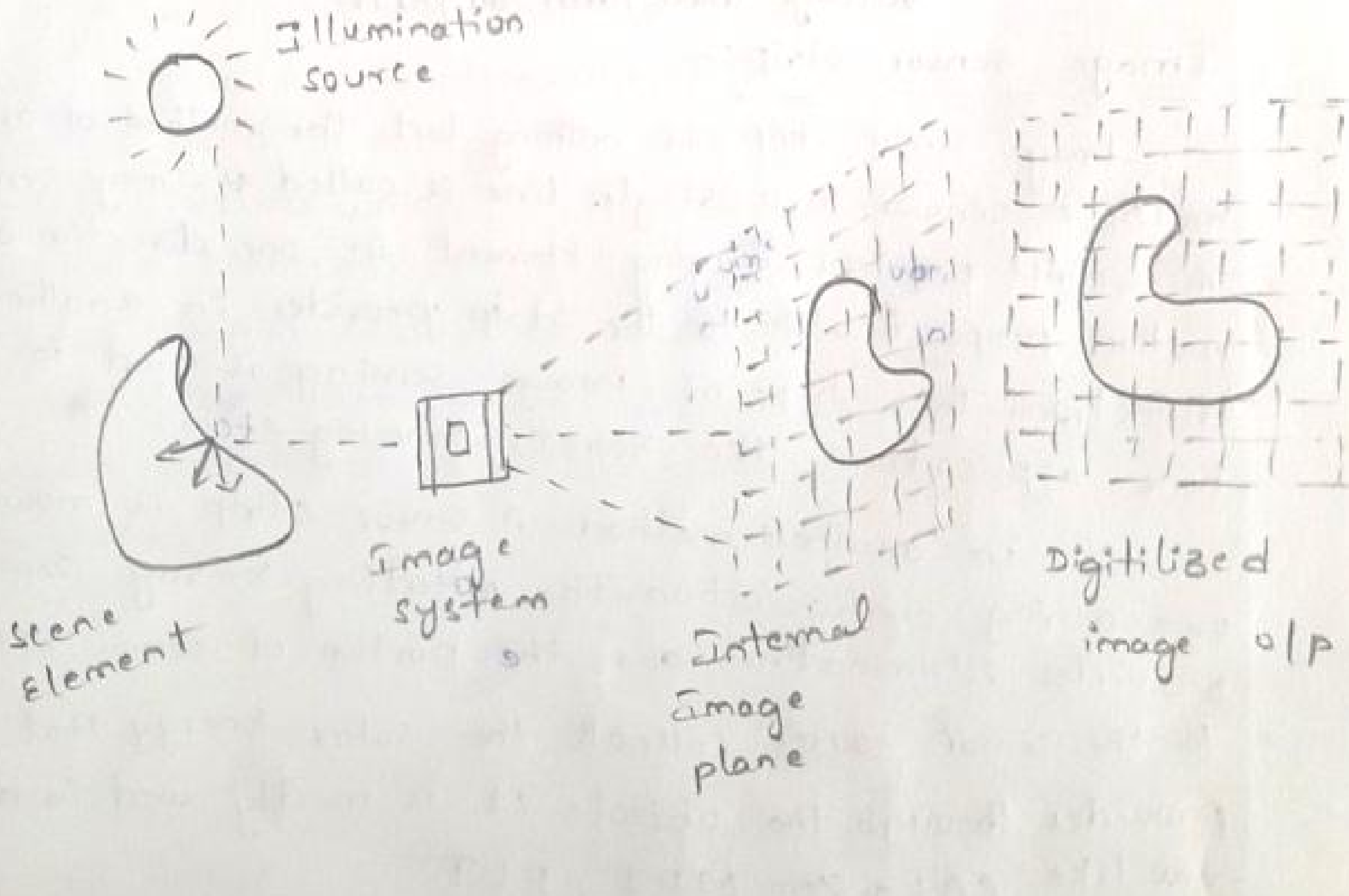
Image Sensor Array:-

This is nothing but the process of arranging the photosites | photodiodes in a two dimensional Array pattern. This is mostly used in digital camera. Normally we have 4000 X 4000 Elements in this type of sensor Array. This is used in wide applications including astronomical studies,



o/p of sensor $\propto$ i/p light filter increases the selectivity $\rightarrow$ GPF $\rightarrow$ allow only green light.

Image Acquistion using Sensor Arrays:-



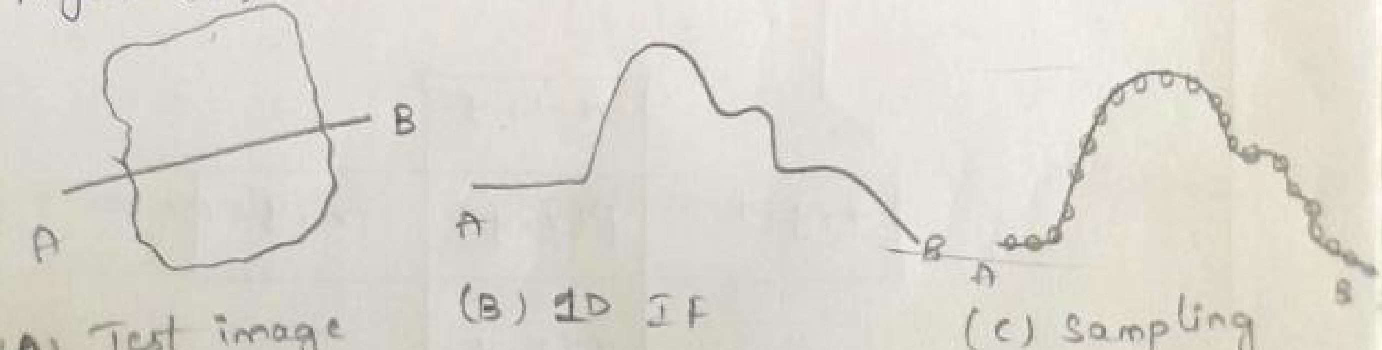
when Even light falls on an object some amount of light is refracted and some is reflected. The light which is reflected and refracted is absorbed by the imaging sensor or the photosites (or) photodiodes and stored in the form of wavelets. These wavelets are converted into digital data and stored as the digitilized image in the Pixel.

Image sampling and Quantization:-

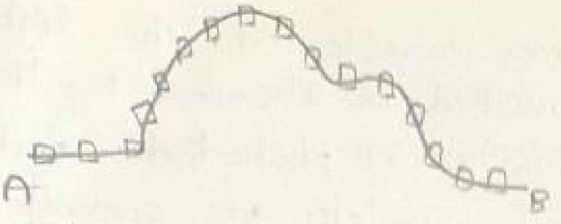
whenever we acquire image by different sensor methods. Image is acquired as an analog (or) Continuous form. How to create a digital image we had to convert the continuous form into digital form. This involves two steps.

* Digitilizing the coordinate values is called sampling.
* Digitilizing the Amplitude values is called Quantization-

when we consider a image as shown in the figure (a) need to be digitilized then we need to first convert the image into a 1D-function. Then it will be as figure (b). Now we need to apply the concept of sampling to the coordinate values which leads to figure (C). Filtering the unwanted elements we get figure (d). which contains filtered samples. Now these samples have to be quantized in amplitude then we get figure (e) which is the complete quantized digitilized data of the image in figure (a).



(A) Test image (B) 1D IF (C) sampling O/P



(D)

(c) Quantization

Some Basic Relationship b/w pixels:-

Neighbours of a pixel:-

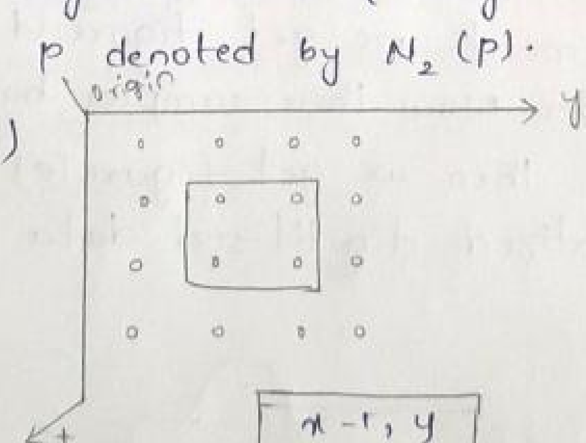
* A pixel p at co-ordinates (x, y) has 4 horizontal & vertical neighbours whose co-ordinates are given by
$(x+1, y)$, $(x-1, y)$, $(x, y+1)$, $(x, y-1) = N_4(P)$

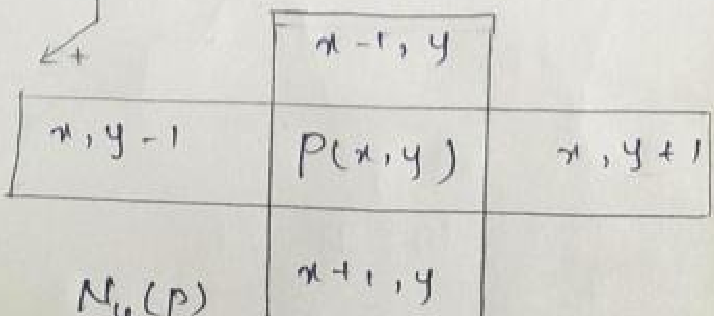
This set of pixels called the 4-neighbours of p is denoted by $N_4(P)$.

* Each pixel is a unit distance from (x, y). Some of the neighbours of p lie outside the digital image.
* If (x, y) is on the border of the image. The 4 diagonal neighbours of p hence co-ordinates.
$(x+1, y+1)$ $(x+1, y-1)$ $(x-1, y+1)$ $(x-1, y-1) = N_D(P)$

These points together with 4 neighbour are called 8 neighbours of p denoted by $N_8(P)$.

Image $f(x, y)$





$N_4(P)$

$x-1, y-1$	$(x-1, y)$	$x-1, y+1$
$x, y-1$	$p(x,y)$	$x, y+1$
$(x+1, y-1)$	$x+1, y$	$x+1, y+1$

Connectivity/Adjacency:-

Two pixels p and q are said to be connected as adjacent if p and q belongs to two different sets S_1 & S_2 such that the neighbours of p and q are distant. In this case adjacency / connectivity can be defined as.

i) **4-connectivity / Adjacency:-**

Two pixels p and q with values of V are 4-adjacent if q is in the set $N_4(P)$.

ii) **8-connectivity / Adjacency:-**

Two pixels p and q with values of V are 8-adjacent if q is in the set $N_8(P)$.

iii) **m-connectivity / Adjacency:-**

Two pixels p and q with values of V are m-adjacent if (i) q is in $N_4(P)$ (or) q is in $N_D(P)$

(ii) Set has no. of pixels whose values are from V.

m-Adjacency is called as mixed Adjacency which is mainly used to remove the ambiguities that often arise when 8-Adjacency is used.

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Arrangement of pixels (a) — 8-Adjacency (b) — m-Adjacency (c)

aR_c. ... tive if aR_b a

Path:-

* A digital path or a curve from pixel p with coordinates (x, y) to pixel q with coordinates (s, t) is a sequence of distance pixels with coordinates.

$(x_0, y_0), (x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$

where $(x, y) = (x_0, y_0)$ and $(s, t) = (x_n, y_n)$ here if $(x_0, y_0) = (x_n, y_n)$ then the path is closed path.

* Here whenever (x_i, y_i) is taken as a pixel then this will be adjacent to $(x_i - 1, y_i - 1)$ for $1 \leq i \leq n$. where n is called the length of the path.
* If S represents a subset of pixels of an image and if p and q are two pixels in the subset S. The S is said to be connected if the path between p and q connects all the pixels of subset S.
* If any pixel p in S is taken and if the set of pixels S and connected to p then that pixel p is called as the connected component of S.

p_{p_i} & $q_{p_i} \in S$

Relation :-

It tells us about the nearness or fartherness of neighbouring elements. If a pair (A, B) is in relation then we represent it as ARB as A is related to B.

For Example

P_1 P_2

P_3 P_4

If we have from pixels as shown $A = \{P_1, P_2, P_3, P_4\}$ and arranged as shown then the relation of 4 connected is

$R = \{(P_1, P_2) (P_2, P_1), (P_2, P_3), (P_3, P_2)\}$

so, when ever we come across a relation it has three properties.

(i) Reflective :- A Relation R is said to be reflective if each a in A is aR_a.

(ii) Symmetric :- A Relation R is said to be symmetric if for each a and b in A is $aR_b \Leftrightarrow bR_a$

(iii) Transitive :- A relation R is transitive if aR_b and bR_c implies aR_c.

which ever relation that satisfies these three properties is called an Equivalence relation.

Distance Measures :-

Let us consider 3 pixels p, q, z with coordinates as (x,y), (s,t) and (u,v) respectively. Then D is a distance function or a metric if

① $D(p,q) \geq 0$ i.e., if $D(p,q) = 0$ if $p = q$

② $D(p,q) = D(q,p)$

③ $D(p,z) \leq D(p,q) + D(q,z)$

Euclidean distance :-

If p and q are two pixels of coordinates (x,y) and (s,t) then the distance b/w them is

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

D_4 distance (or) City Block distance :-

If p and q are two pixels of coordinates (x,y) and (s,t) then the distance between them is

$$D_4 = |(x-s)| + |(y-t)|$$

If p and q are two pixels of coordinates (x,y) and (s,t) then the distance b/w them is

$$D_8 = \text{Max}(|x-s|, |y-t|)$$

Introduction to Mathematical tools using Digital Image processing:-

There are the tools which are used in for processing the digital images. They are.

(i) Arrays V_s matrix
(ii) Linear V_s non-linear
(iii) Arithmetic operations
(iv) set and logical operations
(v) spatial operations
(vi) vector and matrix operations
(vii) Image transforms.
(viii) probabilistic methods

(i) Array V_s matrix:-

Array multiplication × matrix multiplication

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \qquad \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix} \qquad \begin{bmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \end{bmatrix}$$

Most of the operations that we come across the digital image processing are array operations until and unless it is specified as matrix operation.

(ii) Linear V_s non-linear:-

One of the important methodology in image processing is Linear V_s non-linear.

* If we consider an input image as $f(x,y)$ and H is an operator applied on input image $f(x,y)$ which yields to the o/p image $g(x,y)$.

i.e., $g(x,y) = H\{f(x,y)\}$

where here in this operation we tell it as a linear operator ~~applied on input image~~ $f(x,y)$ if it satisfies the superposition principle.

i.e., Homogeneity and additivity

$f(ax) = af(x) \forall a$ $f(x+y) = f(x)+f(y)$

Let us consider a_i, a_j as the arbitary constants then H is said to be linear operator if

$$H[a_i f_i(x,y) + a_j (f_j(x,y)] = a_i H[f_i(x,y) + a_j H[f_j(x,y)]$$

$$= a_i g_i(x,y) + a_j g_j(x,y)$$

Then it is said to be linear.

P) Let us consider H is an operator which gives the max value of the pixel in the image. The operator can be checked as linear (or) non-linear as follows.

$$f_1 = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} \quad f_2 = \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix} \quad \text{if } a_1 = 1 \text{ and } a_2 = -1$$

Sol

$$H[a_1 f_1(x,y) + a_2 f_2(x,y)] = \max\left[\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -6 & -5 \\ -4 & -7 \end{bmatrix}\right]$$

$$= \max\begin{bmatrix} -6 & -3 \\ -2 & -4 \end{bmatrix} = -2$$

$$a_1 H[f_1(x,y)] + a_2 H[f_2(x,y)] = \max\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + (-1)\max\begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}$$

$$= 1(3) + (-1)(7)$$

$$= 3 - 7$$

$$= -4$$

where LHS ≠ RHS. So the operator H is a non linear operator.

(iii) Arithmetic operations:-

There are different types of arithmetic operators and operations that is used for the operation b/w corresponding pixel pairs:

Addition :- If we consider two images $f(x,y)$ and $g(x,y)$ the addition of two images $f(x,y)$ and $g(x,y)$ produces the resultant image $s(x,y)$ where

$$s(x,y) = f(x,y) + g(x,y)$$

This operation is mainly used for the reduction of noise, to change brightness, it is also possible to add a constant value to the image to increase the brightness

$$s(x,y) = f(x,y) + k$$

Applications :-

(i) superimpose of an image on another image is called as double Exposure.

(ii) Increases the brightness of the image by adding constant value.

Subtraction :-

If we consider two images $f(x,y)$ and $g(x,y)$ then the subtraction is given as

$s(x,y) = |f(x,y) - g(x,y)|$ here we apply modulus operator such as to avoid the negative values. It is also possible to subtraction of a constant value from the image.

$$s(x,y) = |f(x,y) - k|$$

Applications :-

(i) Background Elimination (ii) Brightness reduction

(iii) Dissimilarity b/w images (iv) change detection.

multiplication :- If we consider two images $f(x,y)$ and $g(x,y)$ then multiplication is given by

$$s(x,y) = f(x,y) \times g(x,y)$$

Images can be also multiplied by a constant for adjusting contrast

$$s(x,y) = f(x,y) \times k$$

if $k > 1$ contrast increases

$k < 1$ contrast decreases

Pratical Applications:-

(i) Adjustment of contrast
(ii) Designing of filter masks
(iii) Designing (or) creating mask to highlight the area of interest.

Division:-

If we Consider two images $f(x,y)$ and $g(x,y)$ then division is given as $S(x,y) = \frac{f(x,y)}{g(x,y)}$ division can also be done by single constant $S(x,y) = \frac{f(x,y)}{k}$.

Applications:-

1) Contrast reduction
2) Change detection
3) Separation of luminance component from reflection.

Problem of Arithmetic operations:-

If a pixel is 8 bit it can hold 0-255 range of intensity values. If the values exceeds the 255 range due to any operations it is set as 255 and if value falls below 0 then it is set as 0.

Let $f_1 = \begin{bmatrix} 1 & 3 & 7 \\ 5 & 15 & 75 \\ 200 & 50 & 150 \end{bmatrix}$ $f_2 = \begin{bmatrix} 50 & 150 & 125 \\ 45 & 55 & 155 \\ 200 & 50 & 75 \end{bmatrix}$

i) $g = f_1 + f_2 = \begin{bmatrix} 50+1 & 3+150 & 7+125 \\ 5+45 & 15+55 & 75+155 \\ 200+200 & 50+50 & 150+75 \end{bmatrix} = \begin{bmatrix} 51 & 153 & 132 \\ 50 & 70 & 230 \\ 400 & 100 & 225 \end{bmatrix}$

$$= \begin{bmatrix} 51 & 153 & 132 \\ 50 & 70 & 230 \\ 255 & 100 & 225 \end{bmatrix}$$

$$g = f_1 - f_2 \Rightarrow \begin{bmatrix} 1-50 & 3-150 & 7-125 \\ 5-45 & 15-55 & 75-155 \\ 200-200 & 50-50 & 150-75 \end{bmatrix} = \begin{bmatrix} -49 & -147 & -118 \\ -40 & -40 & -80 \\ 0 & 0 & 75 \end{bmatrix}$$

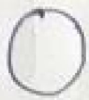
$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 75 \end{bmatrix}$$

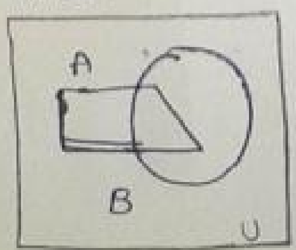
$$g = f_1 \times f_2 \Rightarrow \begin{bmatrix} 1\times 50 & 3\times 150 & 7\times 125 \\ 5\times 45 & 15\times 55 & 75\times 155 \\ 200\times 200 & 50\times 50 & 150\times 75 \end{bmatrix} = \begin{bmatrix} 50 & 450 & 875 \\ 225 & 825 & 11625 \\ 40000 & 2500 & 11250 \end{bmatrix}$$

$$= \begin{bmatrix} 50 & 255 & 255 \\ 255 & 255 & 255 \\ 255 & 255 & 255 \end{bmatrix}$$

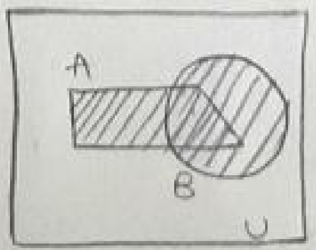
$$g = f_1 / f_2 = \begin{bmatrix} 1/50 & 3/150 & 7/125 \\ 5/45 & 15/55 & 75/155 \\ 200/200 & 50/50 & 150/75 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

SET And logical operations:-

Set and logical operations and applied on images. If we consider two objects   as A and B then

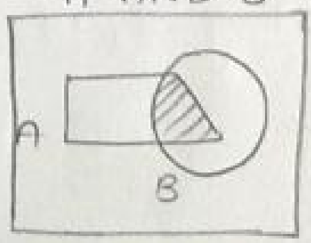


A OR B



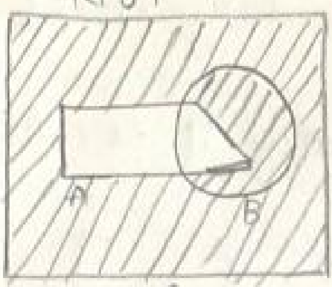
A ∪ B

A AND B



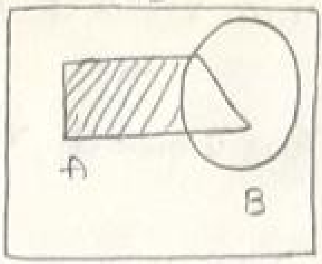
A ∩ B

NOT A



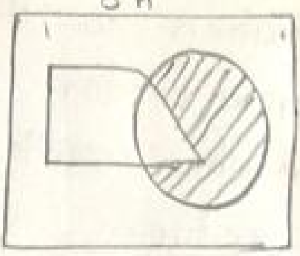
A^C

$A\bar{B}$



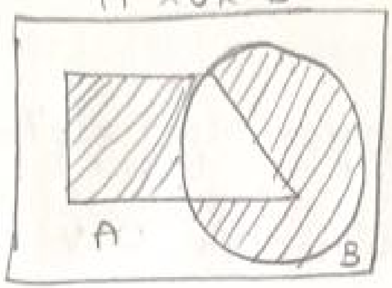
$A - B$

$B\bar{A}$



$B - A$

A XOR B



Most of the logical operations come with set operations. so, Mathematically we represent sets and practically we use logical operations.

Logical operations also consists of the set of comparative conditions that we use in comparision of two images. Like > then < than , <= >= $b^{>}$ $b^{<}$ $b^{=}$.

Image Transforms:-

Image transforms are the direct transforms which act on the pixels in spatial domain. Sometimes we transform the domain using some specific transform to get the required data, and applying inverse transform will return back to spatial domain.

Vector Vs Matrix:-

The Algebraic operations based on vectors and matrices are very much useful in image manipulation. In an image, represented as a rectangular matrix, a row (or) a column may be considered as a row (or) column vector respectively.

$$V = (v_0, v_1, v_2, v_3, v_4 \cdots v_{n-1})$$

* when written (or viewed horizontally it is called row vector.
* when written (or viewed vertically it is called column vector.

$$V = (v_0, v_1, v_2, v_3 - - - v_{n-1})^T = \begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ \vdots \\ v_{n-1} \end{bmatrix}$$

$$\|v\| = (v_0^2 + v_1^2 + - - - + v_{n-1}^2)^{1/2}$$

if we consider two vector u, v whose

$$u = (u_0, u_1, - - - u_{n-1})$$
$$v = (v_0, v_1, - - - v_{n-1})$$

$$u \pm v = (u_0 \pm v_0 \quad u_1 \pm v_1 \quad u_2 \pm v_2 - - - u_{n-1} \pm v_{n-1})$$

$$kv = (kv_0, kv_1, kv_2 - - - , kv_{n-1})$$

if $v = u$ ⇒ $u \cdot v = v \cdot u = \|u\|^2 = \|v\|^2$

* vector addition, multiplication operations are commutative, associative and distributive

(i) $u + v = v + u$, $v \cdot u = u \cdot v$

(ii) $u + (v + w) = (u + v) + w$

(iii) $u \cdot (v + w) = u \cdot v + u \cdot w$

The most common form of representation of an image is a rectangular matrix consisting intensity values as its elements. The matrix operations are very much useful in image processing applications.

$$F = f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & - - - & f(0,N-1) \\ f(1,0) & f(1,1) & - - - & f(1,N-1) \\ f(2,0) & f(2,1) & - - - & f(2,N-1) \\ \vdots & & & \\ f(m-1,0) & f(m-1,1) & - - - & f(m-1,N-1) \end{bmatrix}$$

Here $f(i,j)$ represents the intensity at a specific cell in the image.

If $f(i,j) = 0$ for all i,j then it is Null matrix.

If $N = M$ then the matrix is called as square matrix.

If $f(i,j) = k$ $i=j$ else 0, then it is called diagonal matrix

If $f(i,j) = 1$ $i=j$ else 0, then it is called Identity matrix, I

Some of the algebraic operations can be performed like addition, subtraction Etc..., if both the matrices are of same size.

$H = F+G$ where $H(i,j) = F(i,j) + G(i,j)$ for all i,j

$$\left.\begin{aligned} F+G &= G+F \\ F+(G+H) &= (F+G)+H \end{aligned}\right\} \text{Matrix addition is Commutative and distributive.}$$

Matrix multiplication is non-commutative but distributive

$$FG \neq GF$$

$$FGH = (FG)H$$

$$F(G+H) = FG + FH$$

An important 2D transform $T(u,v)$ can be Expressed as

$$T(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, r(x,y,u,v)$$

where $f(x,y)$ is input image

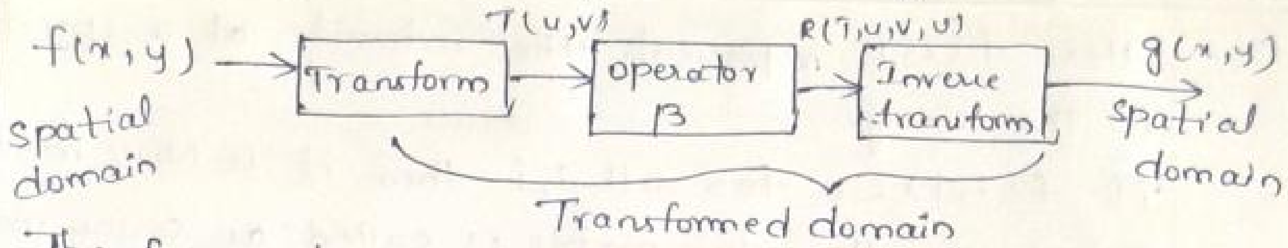
$r(x,y,u,v)$ is a forward transform kernel

similarly to Inverse transformation of $T(u,v)$

$$f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} T(u,v)\, s(x,y,u,v)$$

where $s(x,y,u,v)$ is called Inverse transform.

characteristics of an image like Average intensity,



The forward Transform is said to be separable if

$$r(x,y,u,v) = r_1(x,u)\, r_2(y,v)$$

In addition the kernel is said to be symmetric

$$r(x,y,u,v) = r_1(x,u)\, r_1(y,v)$$

if $r(x,y,u,v) = e^{-j2\pi\left(\frac{ux}{m} + \frac{vy}{N}\right)}$ then

$$s(x,y,u,v) = \frac{1}{MN} e^{j\pi\left(\frac{ux}{m} + \frac{vy}{N}\right)}$$

$$T(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{m} + \frac{vy}{N}\right)}$$

$$f(x,y) = \frac{1}{MN} \sum_{m=0}^{m-1} \sum_{n=0}^{N-1} T(u,v)\, e^{j2\pi\left(\frac{ux}{m} + \frac{vy}{N}\right)}$$

$$\boxed{T = AFA}$$

Probablistic Methods :-

These methods use the concepts of probability in image processing. Here we treat the intensity values as random values of variables.

Ex :- Let z_i denotes all possible intensities of $M \times N$ digital image where $i = 0, 1, 2, 3, \ldots L-1$

Then the probability of occurance of an intensity z_i in the given image is given by.

$$\boxed{p(z_k) = \frac{n_k}{MN}}$$

where n_k is the no. of times z_k occurs in the image.

MN is the total no. of pixels in image.

where $\sum_{k=0}^{L-1} p(z_k) = 1$

In these process if $p(z_k)$ is known then the Important characteristics of an image like Average intensity, variance of intensities can be Easily calculated.

$$m = \sum_{k=0}^{L-1} z_k \, p(z_k)$$

$$\sigma^2 = \sum_{k=0}^{L-1} (z_k - m)^2 p(z_k)$$

The n^{th} order central moment is given by

$$\mu_n(z) = \sum_{k=0}^{L-1} (z_k - m)^n p(z_k)$$

where $\mu_0(z) = 0$, $\mu_1(z) = 0$, $\mu_2(z) = \sigma^2$

Spatial operations:-

when Ever we perform any operation directly over a pixel then we called it as spatial operation. They are categorized as

(i) single pixel operation

(ii) Neighbourhood operation.

(iii) Geometric spatial operations

(i) singles pixel operations:-

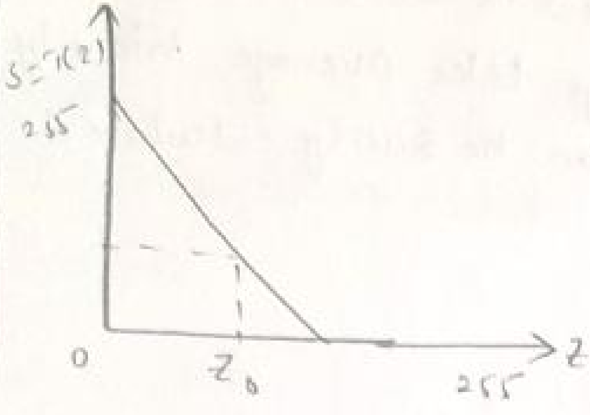
The intensity value of single pixel is changed in this single pixel operation. This is the simple Pixel operation that we can perform on a pixel z

$$S = T(z)$$

$S \rightarrow$ processed image

z = original image

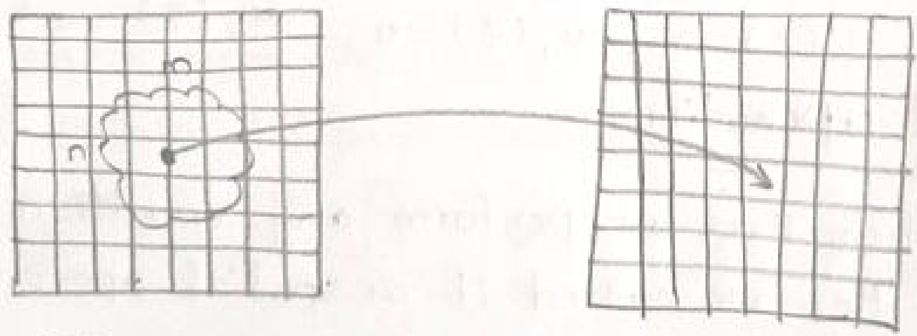
$T \rightarrow$ Transform operator for change it value we use this to obtain the negative of an 8 bit input



Here the intensity value of the (z) input is mapped with the O/P using transfer function $s=T(z)$

Neighbourhood operation:-

In this the operation of a task over a pixel depends upon the neighbourhood pixels and its output is projected at the same pixel.



If we consider an image and select a region say with centre as (x,y) then S_{xy}, is a set of pixels which act on the particular point (x,y) and is set as a resultant at (x,y) location only.

$$g(x,y) = \frac{1}{MN} \sum_{S_{xy}} f(xy)$$

Geometric Transformation:-

These are the transforms which are used mainly to transform. The spatial relationship b/w pixels in an image. It is similar to printing an image on the screen (or) sheet (or) rubber and stretching it.

Different translations can be made over an image.
This generally consists of two steps.

(i) A spatial transformation of coordinates.

(ii) Intensity, Interpolation, Cassigning intensity values to the spatially transformed pixels.

If $g(u,v) = T(f(x,y))$

$(u,v) = T(x,y) = (x/2, y/2)$

This transforms shrints the image to by affine transform.

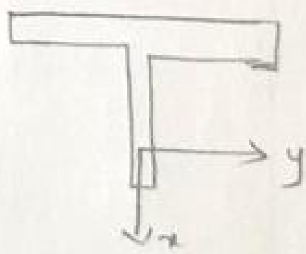
$[x \ y \ 1] = [u \cdot v \cdot 1]$ is the condition.

$$[u, v, 1] = [x \ y \ 1] T$$

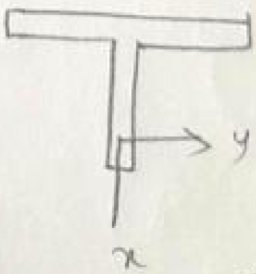
$$= [x \ y \ 1] \begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & \phi \end{bmatrix}$$

Here Based upon the values of the matrix Elements we can obtain different transforms.

Identity $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $u = x$, $v = y$



scaling. $\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $u = c_x x$, $v = c_y y$



Rotation $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $u = x\cos\theta - y\sin\theta$, $v = x\sin\theta + y\cos\theta$

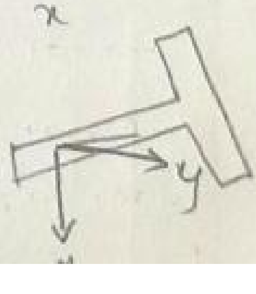


Image Transforms :-

Image transforms are mathematical tools that helps us to Convert images from spatial domain to frequency domain.

DFT $T(u,v)$ $R[T(u,v)]$

$f(x,y)$ → Transform → operations (R) → Inverse fourier transform

spatial domain

← Transform Domain →

Need for Image transforms :-

1. TO do image procesing task like Image filtering, Image compresion, feature Extraction etc...,
2. TO Convert the image from one domain to another
3. TO Extract information.

* The Image transforms are
(i) DFT
(ii) DCT
(iii) DST
(iv) DWT
(v) DHT
(vi) Haar transform
(vii) slant transform
(viii) KL transform

DFT in Digital Image procesing :-

Properties of DFT

1. superability
2. Translation
3. periodicity
4. conjugate
5. Rotation
6. Distributive
7. scaling
8. convolution
9. Co-relation

Rectangular image (M x N):-

$$F(k,l) = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f(m,n)\, e^{-\frac{j2\pi}{N} mk} \cdot e^{-\frac{j2\pi}{N} nl}$$

Square image:-

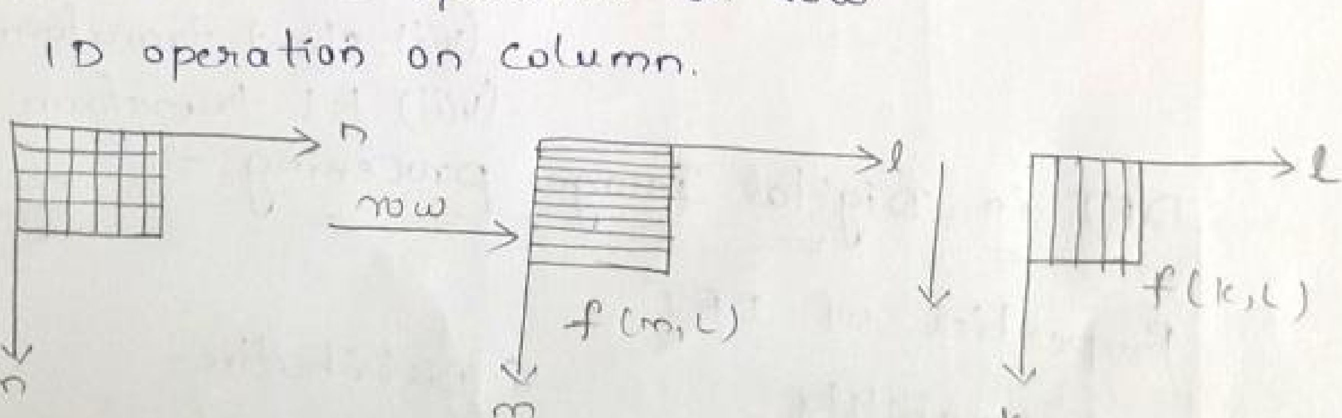
$$F(k,l) = \sum_{m=0}^{N-1} \sum_{N=0}^{N-1} f(m,n)\, e^{-\frac{2\pi}{N} mk} \cdot e^{-\frac{j2\pi}{N} nl}$$

Separability:- when 2D image is being transformed to two 1-D image can be computed one along with Row & column.

Eg:- 2D image $f(m,n) \xrightarrow[\text{1-D row transform}]{} f(m,l) \xrightarrow[\text{1-D Column transform}]{} f(k,l)$

It is done in 2 steps.

1. performing 1D transform on each row of an image to get $f(x,v)$.
2. Perform 1D transform on
   a) Succesive 1D operation on row
   1D operation on column.



$$f(m,n) \longrightarrow f(k,l)$$

Mathematically $\longrightarrow$ consider an image of N X N

$$f(k,l) = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f(m,n)\, \cancel{\sum}\, e^{-\frac{j2\pi}{N} mk}\, e^{-\frac{j2\pi}{N} nl}$$

$$= \sum_{m=0}^{N-1} \left(\sum_{n=0}^{n-1} f(m,n) e^{-\frac{j2\pi}{N} nl} \right) e^{-\frac{j2\pi}{N} mk}$$

→ solve using repeatable property convert n to L

$$= \sum_{m=0}^{N-1} f(m,L) e^{-\frac{j2\pi}{N} mk}$$

→ Converting m to k

$$f(k,l) = f(k,l)$$

Translation/ spatial shift property - creates additional phase.

* It tells the image is being shifted from m to m

consider a 2D image

$$f(x,y) \xrightarrow{x_0, y_0} f(x-x_0, y-y_0)$$

2D DFT $F(u,v) = \sum_{x,y=0}^{N-1} \sum f(x-x_0, y-y_0) e^{-\frac{j2\pi}{N} u(x-x_0)+v(y-y_0)}$

$$= \underbrace{\sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x-x_0, y-y_0) e^{-\frac{j2\pi}{N}(ux+vy)}}_{\text{magnitude}} \underbrace{e^{-\frac{j2\pi}{N}(ux_0+vy_0)}}_{\text{Additional phase}}$$

$$F(u,v) = F(u,v) e^{-\frac{j2\pi}{N}(ux_0+vy_0)}$$

Periodicity:- periodic with period of N

$$F(u,v) = F(u+N, v) = F(u, v+N) = F(u+N, v+N)$$

$$F(u,v) = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) e^{-\frac{j2\pi}{N}(ux+vy)}$$

$$F(u+N, v+N) = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) e^{-\frac{j2\pi}{N}(ux+vy+Nx+Ny)}$$

$$= \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) e^{-\frac{j2\pi}{N}(ux+vy)} e^{-\frac{j2\pi}{N}(Nx+Ny)}$$

$$= \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-\frac{j2\pi}{N}(ux+vy)}\, \underbrace{e^{-\frac{j2\pi}{\alpha} M(x+y)}}_{1}$$

$$= \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-\frac{j2\pi}{N}(ux+vy)}$$

$$= F(u,v)$$

Conjugate :- To visualise fourier spectrum. If $f(x,y)$ is real value of the function.

$F(u,v) = F(-u,-v) \rightarrow$ Complex

$|F(u,v)| = |F(-u,-v)|$

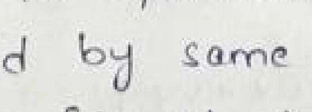
* The complex conjugate of a complex number, z is its mirror image with respect to the horizontal axis.
* Modulus of a complex number gives the distance of the complex number from the origin.
* Conjugate of a complex number gives the reflection of the complex number about the real axis.

Rotation :- Rotation property tells that if the image in the spatial domain is rotated by some amount then its spectrum which is obtained by DFT is also rotated by same amount.

E.g :- $f(x,y) \Rightarrow$ Image

$DFT\left[f(r\cos\theta, r\sin\theta)\right] = F(R\cos\theta, R\sin\theta)$

To rotate $f(r, \theta+\theta_0) = F\left[R\cos\theta+\theta_0, R\sin(\theta+\theta_0)\right]$



Distributive property:-

Applicable on addition not for multiplication.

scaling $\longrightarrow a f(x,y) \Leftrightarrow a F(u,v)$ a - scaling quantity

$$f\left(\frac{x}{a}, \frac{y}{b}\right) \Leftrightarrow \frac{1}{(ab)} F\left(\frac{u}{a}, \frac{v}{b}\right)$$

Convolution :-

$$f(x) * g(x) \Leftrightarrow F(u) \cdot G(u)$$

Convolution in time domain — Multiplication in frequency domain

In DFT:-

1. one - dimension DFT

$$F(k) = \sum_{x=0}^{N-1} f(x) e^{-\frac{j2\pi}{N} kx} \quad \text{where } k = 0, 1, 2 \ldots N-1$$

* Inverse DFT

$$f(x) = \frac{1}{N} \sum_{k=0}^{N-1} F(k) e^{\frac{j2\pi kx}{N}} \quad \text{where } x = 0, 1, 2, \ldots N-1$$

Compute DFT of the sequence $f(x) : \{ \underset{\uparrow}{1}, 0, 0, 1 \}$

A) $$F(k) = \sum_{x=0}^{N-1} f(x) e^{-\frac{j2\pi}{N} kx} \quad \text{where } k = 0, 1, \ldots N-1$$

Here $N = 4$

$$F(k) = \sum_{x=0}^{3} f(x) e^{-\frac{j2\pi}{4} kx}$$

$$= f(0) e^{0} + f(1) e^{-\frac{j2\pi}{4} k \cdot 1} + f(2) e^{-\frac{j2\pi}{4} k \cdot 2} + f(x) e^{-\frac{j2\pi}{4} kx}$$

$$= 1 + 0 + 0 + 1 \cdot e^{-\frac{j6\pi k}{4}}$$

$$= 1 + 0 + 0 + e^{-j3\pi/2 k}$$

when $k=0 \Rightarrow F[0] = 1+e^0 = 1+1 = 2$

$k=1 \Rightarrow F[1] = 1+e^{\frac{-j3\pi \cdot 1}{2}} = 1+j$

$k=2 \Rightarrow F[2] = 1+e^{\frac{-j3\pi \cdot 2}{2}} = 1-1 = 0$

$k=3 \Rightarrow F[3] = 1+e^{\frac{-j3\pi}{2} \cdot 3} = 1-j$

$F[k] = \{2, 1+j, 0, 1-j\}$

Important formula

$e^{j\pi} = -1$

$e^{-j\pi} = -1$

$e^{+j\pi/2} = j$

$e^{-j\pi/2} = -j$

$e^{-j2\pi} = 1$

$e^{-j3\pi} = -1$

$e^{\frac{-j3\pi}{2}} = j$

$e^{\frac{-j9\pi}{2}} = -j$

$e^{+j\theta} = \cos\theta + j\sin\theta$

$e^{-j\theta} = \cos\theta - j\sin\theta$

Kernel for 4-point DFT :-

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

1D DFT $F[k] = \text{kernel} * f(x)$

2D DFT $F[k,l] = \text{kernel} * f(x,y) * \text{kernel}^T$

P) Compute 4 point DFT for the sequence $\{0, 1, 2, 3\}$ using matrix method.

A) The 4-point DFT in one-dimensional

= kernel × Input sequencing

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+2+3 \\ 0-j-2+3j \\ 0-1+2-3 \\ 0+j-2-3j \end{bmatrix} = \begin{bmatrix} 0 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

P) Compute the 2D DFT of the gray scale image is given by $f(m,n) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

A) $F(k,l) = \text{kernel} * f(x,y) * \text{kernel}^T$

kernel of a 4-point DFT $= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$

$$= \begin{bmatrix} 4 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Discrete cosine transform - DCT:

* DCT is similar to DFT. It transforms a signal (or) image from the spatial.

Spatial domain $\xrightarrow{\text{DCT}}$ frequency domain

* The difference b/w the two is the type of basis function used in each transform, The DFT uses a set of harmonically related complex Exponential

function which the DCT uses only real-valued cosine function.

$$F[u,v] = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-\frac{j2\pi}{N}(ux+vy)} \longrightarrow \text{DFT-2D}$$

$$C[u,v] = a(u), a(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \cos\left[\frac{(2x+1)u\pi}{2N}\right] \cos\left[\frac{(2y+1)vN}{2N}\right]$$

what is DCT in Image processing.

A) * DCT represents an image as a sum of sinusoids of varying magnitude & frequencies.

* It helps separate the image into parts of differing importance (with respect to image's visual quality)

* DCT is used in image compression application.

* used in JPEG algorithm, biometric, cryptography, video compression, face recognition.

1-D DCT square of length N can be written as

$$F(u) = \sqrt{2}\, \frac{Q(u)}{\sqrt{N}} \sum_{x=0}^{N-1} f(x) \cos\left(\frac{2x+1}{2N}\right) u\pi \quad \text{for } u=0,1,\ldots N-1$$

$$u = 0,1,2 \ldots N-1$$

$$\text{where } w(u) = \begin{cases} 1/\sqrt{2} & \text{if } u=0 \\ 1 & \text{if } u \neq 0 \end{cases}$$

Inverse DCT:-

$$f(x) = \sqrt{\frac{2}{N}} \sum_{u=0}^{N-1} a(u) * F(u) * \cos\left(\frac{2x+1}{2N}\right) u\pi$$

$$a(u) = \begin{cases} 1/\sqrt{2} & \text{if } u=0 \\ 1 & \text{if } u \neq 0 \end{cases}$$

2D IDCT :-

$$f(x,y) = \frac{2}{\sqrt{MN}} \sum_{x=0}^{m-1} \sum_{y=0}^{N-1} a(u).a(v)F(u,v) \cos\left(\frac{2x+1}{2m}\right)u\pi$$

$$\cos\left(\frac{2x+1}{2N}\right)v\pi$$

$u = 0, 1, 2, \ldots N-1$

$v = 0, 1, 2, \ldots N-1$

$x = 0, 1, 2 \ldots N-1$

$y = 0, 1, 2, \ldots N-1$

$$a(u), a(v) = \begin{cases} 1/\sqrt{2} & \text{if } u = v = 0 \\ 1 & \text{otherwise} \end{cases}$$

Eg:- Let us consider $N=4$ Apply DCT

$N=4, \quad u = 0, 1, 2, 3$

1-D DCT

$$F(u) = \sqrt{2}\, \frac{a(u)}{\sqrt{N}} \sum_{x=0}^{N-1} f(x) \cos \frac{(2x+1)}{2N} u\pi$$

$$u=0 \quad F(0) = \frac{\sqrt{2}}{4} \times 1/\sqrt{2} \sum_{x=0}^{N-1} f(x) \cos \frac{(2x+1)}{2N} 0 \cdot \pi$$

$$= 1/2 \left[\cos 0, \cos 0, \cos 0, \cos 0\right]$$

$$= \left[0.5 \quad 0.5 \quad 0.5 \quad 0.5\right] \text{— } 1^{st} \text{ row}$$

$$u=1. \quad F(1) \Rightarrow \frac{\sqrt{2}}{\sqrt{4}} \cdot 1 \sum_{x=0}^{N-1} f(x) \cos \frac{(2x+1)}{2 \cdot 4} 1\pi$$

$$= 1/\sqrt{2} \left[\cos\frac{\pi}{8} \quad \cos\frac{3\pi}{8} \quad \cos\frac{5\pi}{8} \quad \cos\frac{7\pi}{8}\right]$$

$$= \left[0.65 \quad 0.27 \quad -0.27 \quad 0.65\right] \; 2^{nd} \text{ row}$$

$u=2 \Rightarrow F(2) = 1/\sqrt{2} \cdot 1 \sum_{x=0}^{N-1} f(x) \cos \frac{(2x+1) 2\pi}{8}$

$= 1/\sqrt{2} \left[\cos \pi/4, \cos 3\pi/4, \cos \frac{5\pi}{4}, \cos 7\pi/4 \right]$

$= [0.5 \quad -0.5 \quad -0.5 \quad 0.5] \rightarrow$ 3rd row.

$F(3) \Rightarrow 1/\sqrt{2} \sum_{x=0}^{3} f(x) \cdot \cos \left(\frac{2x+1}{8}\right) 3\pi$

$= 1/\sqrt{2} \left[\cos \frac{3\pi}{8} \quad \cos \frac{9\pi}{8} \quad \cos \frac{15\pi}{8} \quad \cos \frac{21\pi}{8} \right]$

$= [0.27, -0.65, -0.61, 0.27] \rightarrow$ 4th row

$$\Rightarrow \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.65 & 0.27 & -0.27 & -0.65 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ 0.27 & -0.65 & +0.65 & -0.27 \end{bmatrix}$$ $\Rightarrow$ kernel for 1-D DCT of $N=4$

IDCT kernel is given by $$\begin{bmatrix} 0.5 & 0.65 & 0.5 & 0.27 \\ 0.5 & 0.27 & -0.5 & -0.65 \\ 0.5 & -0.27 & -0.5 & 0.65 \\ 0.5 & -0.65 & 0.5 & -0.27 \end{bmatrix}$$

1) Find the DCT of $f(x) = (1, 2, 4, 7)$

A) $F(X) = \text{kernel} \times f(x)$

$$= \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.65 & 0.27 & -0.27 & -0.65 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ 0.27 & -0.65 & +0.65 & -0.27 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 7 \\ -4.459 \\ 1 \\ -0.370 \end{bmatrix}$$

2) DCT of a grey scale image

$$f(x,y) \quad \begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 2 & 1 \end{bmatrix}$$

A) $F(u,v) = \text{kernel} * f(x,y) * \text{kernel}^T$

$$= \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.65 & 0.27 & -0.27 & -0.65 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ 0.27 & -0.65 & 0.65 & -0.27 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0.65 & 0.5 & 0.27 \\ 0.5 & 0.27 & -0.5 & -0.65 \\ 0.5 & -0.27 & -0.5 & 0.65 \\ 0.5 & -0.65 & 0.5 & -0.27 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0.3025 & -1 & -0.9235 \\ 0 & -0.1463 & -0.3825 & -0.3532 \\ 0 & 0 & 0 & 0 \\ 0 & -0.3532 & -0.923 & -0.852 \end{bmatrix}$$

Walsh-Hadmard transform :-

* It is a non-sinusoidal & orthogonal transform
* It decomposes a signal into a set of basic functions
* walsh transform function co-efficients are real & they take only two values (+1 / -1)

Applications :-

* used in power spectrum analysis
* Filtering
* speech processing
* Medical signal
* Multiplexing & coding in Communication
* Logic design & Analysis
* solving Non-linear differential Equation etc...,

1-D walsh transform

$$w(u) = \frac{1}{N} \sum_{x=0}^{N-1} f(x) \prod_{i=0}^{n-1} (-1)^{b_i(x) \cdot b_{n-1-i}(u)}$$

Inverse 1-D walsh transform

$$f(x) = \sum_{x=0}^{N-1} w(u) \prod_{i=0}^{n-1} (-1)^{b_i(x)\, b_{n-1-i}(u)}$$

2-D walsh transform

$$w(u,v) = \frac{1}{N} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \prod_{i=0}^{n-1} (-1)^{b_i(x)\, b_{n-1-i}(u) + b_i(y) \cdot b_{n-1-i}(v)}$$

Inverse 2D walsh transform

$$f(x,y) = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} w(u,v) \prod_{i=0}^{n-1} (-1)^{b_i(x)\, b_{n-1-i}(u) + b_i(y) \cdot b_{n-1-i}(v)}$$

Find the 1-D walsh basis for 4^{th} order system

1 D $$g(n,k) = \frac{1}{N} \prod_{i=0}^{m-1} (-1)^{b_i(n)\, b_{m-1-i}(k)}$$

n - time index
k - frequency index } varies from 0 to N-1

m = no. of bits represent the number $m = \log_2 N$

N - order

sol

$N = 4$

$m = \log_2 4$, $\log_2 2^2 = 2 \log_2 2 = 2$

n, k varies from 0 to $N-1 = 0, 1, 2, 3$

i varies from 0 to $n-1 = 0$ to 1

then $k/N = 0$ the basis value will be $1/N = 1/4$

$$g(n,k) = 1/4, \prod_{i=0}^{2-1} (-1)^{b_i(n)\, b_{m-1-i}(k)}$$

$$\begin{array}{c|cccc} k \backslash n & 0 & 1 & 2 & 3 \\ \hline 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 1 & 1/4 & 1/4 & -1/4 & -1/4 \\ 2 & 1/4 & -1/4 & 1/4 & -1/4 \\ 3 & 1/4 & -1/4 & -1/4 & 1/4 \end{array}$$

$$g(1,1) = \frac{1}{4} \prod_{i=0}^{1} (-1)$$

$n\ k$

$$= \frac{1}{4}\left[(-1)^{b_0(1) \cdot b_1(1)} \cdot (-1)^{b_1(1) \cdot b_0(1)}\right]$$

$$= \frac{1}{4}\left[(-1)^{1 \cdot 0} \cdot (-1)^{0 \cdot 1}\right]$$

$$= \frac{1}{4}\left[1 \cdot 1\right] = 1/4$$

$$g(2,1) = \frac{1}{4}\left[(-1)^{b_0(2)\, b_1(1)} \cdot (-1)^{b_1(2)\, b_0(1)}\right]$$

$n\ k$

$$= 1/4\left[(-1)^{0 \cdot 0} \cdot (-1)^{1 \cdot 1}\right]$$

$$= 1/4\left[1 \cdot -1\right] = -1/4$$

$$g(3,1) = \frac{1}{4}\left[(-1)^{b_0(3)\, b_1(1)} \cdot (-1)^{b_1(1)}\right]$$

$$= 1/4\left[(-1)^{1 \cdot 0} \times (-1)^{1 \times 1}\right]$$

$$= 1/4\left[1 \times (-1)^{1}\right]$$

$$= -1/4$$

Easy method to calculate walsh kernel.

1. write the binary representation of n.
2. write the binary representation of k in reverse order.

3. check for the no. of overlap of '1' b/n n & k

a. If the no. of overlap is '0' sign +ve

Even sign +ve

odd sign -ve

E.g:- $g(1,1)$ = n - 01 01 no overlap ∴ +ve = 1/4

n k k - 01 10

n = 2, 3 n = 1 0 odd no. of overlap ∴ -ve = -1/4

k = 1 1

P) Find the walsh transform for the $f(x) = \{1, 2, 0, 3\}$

A)
$$F = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & +1 \\ 1 & +1 & +1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+0+3 \\ 1+2-0-3 \\ 1-2-0+3 \\ 1+2+0-3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ -2 \\ 0 \end{bmatrix}$$

Hadamard Transform :-

* It is also known as walsh - hadamard transform. and also called as Hadamard - rademacher - walsh transform.
* It is an Example of generalized class of fourier transform.
* It performs on orthogonal, Symmetric operations.
* It transforms 2^m real numbers x_n into 2^m real number x_k.
* used in signal processing, data compression, data Encryption.

* In many scientific methods such as NMR, mass spectroscopy & crystallography Etc...,
* Basic function of hadamard is also contain -1 & +1
* The kernel of 1D hadamard transform is

$$g(x,u) = \frac{1}{N}(-1)^{\sum_{i=0}^{n-1} b_i(x)\, b_i(u)}$$

1-D hadamard transform $H(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x)(-1)^{\sum_{i=0}^{N-1} b_i(x).b_i(u)}$

$N = 2^n$

1-D inverse hadamard transform : $f(x) = \sum_{x=0}^{N-1} H(u)(-1)^{\sum_{i=0}^{N-1} b_i(x)\, b_i(u)}$

2-D Hadamard transform $H(u,v) = \frac{1}{N}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)(-1)^{\sum_{i=0}^{N-1}[b_i(x)b_i(u) + b_i(y)\, b_i(v)]}$

2D-Inverse transform $f(x,y) = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} H(u,v)(-1)^{\sum_{i=0}^{N-1}[b_i(x)b_i(u) + b_i(y)\, b_i(v)]}$

* Hadamard kernel → Separable, → Symmetric

* The Hadamard transform of an image f is denoted as $g = H * f * H$
* The matrix H is related to hadamard matrix as $H = \frac{1}{\sqrt{N}} H_{q,q}$ $N \rightarrow$ dimension of an image
* Hadamard 2x2 matrix is , 4x4 matrix is

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & +1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

8 × 8 matrix is

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ \hline 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{array}\right]$$

<u>E.g</u>:- $f(x) = \{1, 2, 0, 3\}$ apply the hadamard transform.

It is 1D

$$F = H \cdot f \quad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 0 \\ 2 \end{bmatrix}$$

* 2D $f(x, y) = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 2 \end{bmatrix}$

$$F = H f H^T$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 8 & 12 & 8 \\ 2 & 0 & 0 & 0 \\ 0 & -2 & -2 & -2 \\ 0 & -2 & -2 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 32 & 4 & -6 & -6 \\ 2 & 2 & 2 & 2 \\ -6 & -2 & 2 & 2 \\ -6 & 2 & 2 & 2 \end{bmatrix}$$

Haar transform :-

* Haar proposes the haar transform in 1910.
* It has low computing requirement so mainly used in image processing & pattern recognition.
* oldest & simplest of wavelet transforms.
* It is real & orthogonal
* It is separable & symmetric
* It is non sinusoidal - non square function.
* It has poor Energy compaction for images
* the Elements of basic function Consists of +1, -1, 0.

Procedure to generate the kernel of the Haar transform :-

1. Find the order N

N represents size of the kernel.

2. Determine the total no. of bits required based upon the $n = \log N$.

3. Determine P & q

$P \in [0, N-1]$ i.e.. $0 \le P \le N-1$

if $P = 0$ then $q = 0/1$

$P \neq 0$ then $q \in [1, 2^P]$ $1 \le q \le 2^P$

4. Determine $k \rightarrow$ total no. of rows in the kernel

$k = 2^P + q - 1$

5. Determine the value of z $z \in \left[0, \frac{1}{N}, \frac{2}{N}, \ldots \frac{N-1}{N}\right]$ columns.

6. If $k=0$ $H(z) = \frac{1}{\sqrt{N}}$ $\forall z$

$k \neq 0$

Design a kernel of Haar transform of $N=2$

(1) $N=2$

(2) $n = \log_2 2$

(3) p & q = ?

$p \in [0 \quad n-1]$

$p \in [0 \quad 1-1]$

$p = 0$

If $p=0$ $q = 0/1$

4) $k = 2^p + q - 1$

$p=0$ $q=0$ $k = 2^0 + 0 - 1 = 0$

$p=0$ $q=1$ $k = 2^0 + 1 - 1 = 1$

5) $z \in \left[0, 1/2\right]$

6) $k=0$ then $H(z) = 1/\sqrt{N}$ $\forall z$

$k=0$ $H(0) = 1/\sqrt{2}$

$H(1/2) = 1/\sqrt{2}$

7) when $k=1$ $p=0, q=1$

$$H(z) = \frac{1}{\sqrt{2}} \begin{bmatrix} 2^{0/2}, & \frac{0-0}{2^0} \leq z < \frac{1-1/2}{2^0} \\ -2^{0/2}, & \frac{1-1/2}{2^0} \leq z < 1/2^0 \\ 0, & \text{otherwise} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \leq z < 1/2 \\ -1 & 1/2 \leq z < 1 \\ 0 & \text{otherwise} \end{bmatrix}$$

8) kernel

	z = 0	z = 1/2
k = 0	$1/\sqrt{2}$	$1/\sqrt{2}$
1	$1/\sqrt{2}$	$-1/\sqrt{2}$

$$1/\sqrt{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Design a kernel of Haar transform for N = 4

1) $N = 4$
2) $n = \log_2 4$
3) $p, q = ?$

$p \in [0 \quad n-1]$

$[0 \quad 2-1]$

$[0 \ , 1]$

$p = 0 \quad q = 0/1$

$p = 1 \quad q = [1 \quad 2^p]$

$= [1, 2^1]$

$= [1, 2]$

4) $k = 2^p + q - 1$

p	q	k
0	0	0
0	1	1
1	1	2
1	2	3

k

$2^0 + 0 - 1 = 1 + 0 - 1 = 0$

$2^0 + 1 - 1 = 1 + 1 - 1 = 1$

$2^1 + 1 - 1 = 2$

$2^1 + 2 - 1 = 3$

5) $\frac{N-1}{N} = \frac{4-1}{4} = 3/4$

$z \in [0, 1/4, 2/4, 3/4$

$z \in [0, 1/4, 1/2, 3/4]$

6) $k=0 \quad H(z) = 1/\sqrt{4} \; \forall (z)$

$H(0) = H(1/4) = H(1/2) = H(3/4) = 1/\sqrt{4}$

7) $k=1 \quad p=0 \quad q=1$

$$H(z) = 1/\sqrt{4} \left[\begin{array}{ll} 2^{p/2} & \frac{q-1}{2^p} \leq z < \frac{q-1/2}{2^p} \\ 2^{p/2} & \frac{q-1/2}{2^p} \leq z < q/2^p \\ 0 & \text{otherwise} \end{array} \right]$$

$$= 1/\sqrt{4} \left[\begin{array}{ll} 2^{0/2} & \frac{1-1}{2^0} \leq z < \frac{1-1/2}{2^0} \\ -2^{p/2} & \frac{1-1/2}{2^0} \leq z < 1/2^0 \\ 0 & \text{otherwise} \end{array} \right]$$

$$= 1/\sqrt{4} \left[\begin{array}{ll} 1 & 0 \leq z < 1/2 \\ -1 & 1/2 \leq z < 1 \\ 0 & \text{otherwise} \end{array} \right]$$

$H(0) = 1/\sqrt{4}$, $H(1/2) = -1/\sqrt{4}$, $H(1/4) = 1/\sqrt{4}$, $H(3/4) = -1/\sqrt{4}$

$k=2 \quad p=1 \quad q=1$

$$H(z) = 1/\sqrt{4} \left[\begin{array}{ll} 2^{1/2} & \frac{1-1}{2^1} \leq z < \frac{1-1/2}{2^1} \\ -2^{1/2} & \frac{1-1/2}{2^1} \leq z < 1/2^1 \\ 0 & \text{otherwise} \end{array} \right]$$

$$= 1/\sqrt{4} \left[\begin{array}{ll} \sqrt{2} & 0 \leq z < 1/4 \\ -\sqrt{2} & 1/4 \leq z < 1/2 \\ 0 & \text{otherwise} \end{array} \right]$$

$H(0) = \sqrt{2}/\sqrt{4}$, $H(1/2) = \sqrt{2} \cdot 0 = 0$

$H(1/4) = -\sqrt{2}/\sqrt{4}$, $H(3/4) = 0$

$k = 3, \; p = 1, \; q = 2$

$$H(z) = \frac{1}{\sqrt{4}} \left[\begin{array}{ll} 2^{1/2}, & \frac{2-1}{2^1} \le z < \frac{2 - 1/2}{2^1} \\ -2^{1/2}, & \frac{2-1/2}{2^1} \le z < \frac{2}{2^1} \\ 0, & \text{otherwise} \end{array} \right]$$

$$= 1/\sqrt{4} \left[\begin{array}{ll} \sqrt{2}, & 1/2 \le z < 3/4 \\ \sqrt{2}, & 3/4 \le z < 1 \\ 0 & \text{otherwise} \end{array} \right]$$

$H(0) = 0 \qquad H(1/2) = \sqrt{2}/\sqrt{4}$

$H(1/4) = 0 \qquad H(3/4) = -\sqrt{2}/\sqrt{4}$

$z=0$		$1/4$	$1/2$	$3/4$
$k=0$	$1/\sqrt{4}$	$1/\sqrt{4}$	$1/\sqrt{4}$	$1/\sqrt{4}$
$k=1$	$1/\sqrt{4}$	$1/\sqrt{4}$	$-1/\sqrt{4}$	$-1/\sqrt{4}$
$k=2$	$\sqrt{2}/\sqrt{4}$	$-\sqrt{2}/\sqrt{4}$	0	0
$k=3$	0	0	$\sqrt{2}/\sqrt{4}$	$-\sqrt{2}/\sqrt{4}$

Kernel

$$= 1/\sqrt{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ \sqrt{2} & -\sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & -\sqrt{2} \end{bmatrix}$$

Slant transform:-

* Slant transform was introduced by Enomoto & Shibata.
* It is an orthogonal transform containing discrete sawtooth wave-form or slant basis vector.
* orthogonality makes the basis function to be independent & makes it able to recover the original sampled rate.
* specially designed for image coding.
* It is used in image coding systems for monochrome & color images.
* The kernel can be generated recursively in hadamard transform.
* The slant transform of order 2×2 is designed as

$$S_1 = 1/\sqrt{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad S_n = 1/\sqrt{n}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

In general

$n > 1$

$$S_N = \begin{array}{c} \left[\begin{array}{cccc} \begin{pmatrix} 1 & 0 \\ a_N & b_N \end{pmatrix} & 1 & \begin{pmatrix} 1 & 0 \\ -a_N & b_N \end{pmatrix} & 0 \\ 0 & I_{N/2-2} & 0 & I_{N/2-2} \\ \begin{pmatrix} 1 & 0 \\ -b_N & a_N \end{pmatrix} & 0 & \begin{pmatrix} 0 & -1 \\ b_N & a_N \end{pmatrix} & 0 \\ 0 & I_{N/2-2} & 0 & -I_{(N/2)-2} \end{array}\right] \begin{array}{c} N/2 \\ \\ N/2 \end{array} \\ \quad N/2 \qquad\qquad\qquad N/2 \end{array}$$

$$\begin{bmatrix} S_{N/2} & 0 \\ \end{bmatrix}$$

Let S_N denotes an $N \times N$ slant matrix with $N = 2^n$ then $S_2 = 1/\sqrt{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ for order of 2.

order = 4 $\quad S_4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ a & b & -a & b \\ 0 & 1 & 0 & -1 \\ -b & a & b & a \end{bmatrix} \begin{bmatrix} S_2 & 0 \\ 0 & S_2 \end{bmatrix}$

If $a = 2b$ & $b = 1/\sqrt{5}$ the slant matrix

$$S_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3/\sqrt{5} & 1/\sqrt{5} & -1/\sqrt{5} & -3/\sqrt{5} \\ 1 & -1 & -1 & 1 \\ 1/\sqrt{5} & -3/\sqrt{5} & 3/\sqrt{5} & -1/\sqrt{5} \end{bmatrix} \begin{array}{l} \rightarrow \text{zero sign change} \\ \rightarrow 1 \text{ sign change} \\ \rightarrow 2 \text{ sign change} \\ \rightarrow 3 \text{ sign change} \end{array}$$

* The sequence of the matrix of order 4 is given by the slant transform reproduces linear variations of brightness very well.