① Form the partial differential equation by eliminating the orbitary constants 'a' and 'b' from. $\log(az-1) = x+ay+b$.

Ⓐ Given: $\log(az-1) = x+ay+b$ —①.

D.w.r to 'x' to eq① on both si'

$$\frac{1}{az-1}\frac{d}{dx}(az-1) = 1+0+0$$

$$\frac{1}{az-1}a\cdot\frac{\partial z}{\partial x} = 1$$

$$\frac{1}{az-1}\cdot ap = 1$$

$$ap = az-1 \quad —②$$

eq① D.w.r to 'y'.

$$\frac{1}{az-1}\cdot\frac{\partial}{\partial y}(az-1) = 0+a+0.$$

$$\frac{1}{az-1}\cdot a\cdot\frac{\partial z}{\partial y} = a$$

$$\frac{1}{az-1}\cdot aq = a$$

$$aq = a(az-1) \quad —③$$

Solving eq ② & ③

$$ap = az-1$$
$$aq = a(az-1)$$

$$\frac{p}{q} = \frac{1}{a}$$

$$pa = q$$

$$\boxed{ap=q}$$

② Form the P.D.E by eliminating the orbitary functions $f(x)$ and $g(y)$ from $z = y f(x) + xg(y)$.

Ⓐ Given:- $z = y.f(x) + xg(y)$ —①

D.w.r to 'x'.

$$\frac{\partial z}{\partial x} = y.f'(x) + g(y)$$

$$p = y.f'(x) + g(y) \quad —②$$

eq① D.w.r to 'y'.

$$\frac{\partial z}{\partial y} = f(x) + x \cdot g'(y)$$

$$q = f(x) + x \cdot g'(y) \quad \text{—— (3)}.$$

Eq(2) and eq(3) are not enought to remove $f(x), g(y), f'(x), g'(y)$. So, we take second order derivatives.

eq(2) D.w.r to 'x':

$$\frac{\partial p}{\partial x} = y \cdot f''(x) + 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = y f''(x)$$

$$\frac{\partial^2 z}{\partial x^2} = y f''(x). \quad \text{—— (4)}$$

eq(2) D.w.r to 'y'.

$$\frac{\partial p}{\partial y} = f'(x) + g'(y).$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = f'(x) + g'(y)$$

$$\frac{\partial^2 z}{\partial x \partial y} = f'(x) + g'(y) \quad \text{—— (5)}$$

eq(3) D.w.r to 'y'.

$$\frac{\partial q}{\partial y} = 0 + x \cdot g''(y)$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = x\, g''(y)$$

$$\frac{\partial^2 z}{\partial y^2} = x g''(y) \quad \text{—— (6)}$$

From eq(2)

$$p = y f'(x) + g(y)$$

$$p - g(y) = y f'(x)$$

$$f'(x) = \frac{1}{y}\left[p - g(y)\right].$$

From eq(3)

$$q = f(x) + x \cdot g'(y)$$

$$q - f(x) = x \cdot g'(y).$$

$$g'(y) = \frac{1}{x} \cdot \left[q - f(x)\right]$$

$f'(x)$ and $g'(y)$ values sub in eq(5).

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{y} \cdot \left[p - g(y)\right] + \frac{1}{x}\left(q - f(x)\right].$$

$$s = \frac{p}{y} - \frac{g(y)}{y} + \frac{q}{x} - \frac{f(x)}{x}.$$

Now multiply with 'xy'. on both sides.

$$xys = xp - xg(y) + yq - yf(x).$$

$$xys = xp + yq - [yf(x) + xg(y)].$$

$$xys = xp + yq - z \quad \text{is required P.D.E.}$$

③ Form the P.D.E by eliminating the orbitary function ϕ from $\phi(x^2+y^2+z^2; z^2-2xy)$.

Ⓐ Given $\phi(x^2+y^2+z^2; z^2-2xy) = 0$

it is in the form of $\phi(u,v) = 0$.

Here $u = x^2+y^2+z^2$, $v = z^2-2xy$.

$\phi(u,v) = 0$

$$\begin{vmatrix} \frac{\partial u}{\partial x} + p\frac{\partial u}{\partial z} & \frac{\partial v}{\partial x} + p\cdot\frac{\partial v}{\partial z} \\ \frac{\partial u}{\partial y} + q\cdot\frac{\partial u}{\partial z} & \frac{\partial v}{\partial y} + q\cdot\frac{\partial v}{\partial z} \end{vmatrix} = 0$$

we find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial z}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial z}, \frac{\partial v}{\partial y}$.

$$\frac{\partial u}{\partial x} = 2x \qquad \frac{\partial v}{\partial x} = -2y$$

$$\frac{\partial u}{\partial z} = 2z \qquad \frac{\partial v}{\partial z} = 2z$$

$$\frac{\partial u}{\partial y} = 2y \qquad \frac{\partial v}{\partial y} = -2x.$$

$$\begin{vmatrix} 2x + p(2z) & -2y + p(2z) \\ 2y + q(2z) & -2x + q(2z) \end{vmatrix} = 0$$

$$\cancel{[2x + p(2z)][-2x + q(2z)] - [(2y + 2zq)(-2y + 2zp)] = 0}$$

$$\cancel{-2x^2 + 4xz - 4xzp + 4pqz^2}$$

$$2\begin{vmatrix} x + pz \\ y + qz \end{vmatrix}$$

$\bar{e} = \frac{1}{3}(2\bar{i} - \bar{j} - 2\bar{k})$.

Now :- $D.D = \nabla\phi_{at\ P(1,-2,-1)} \cdot \bar{e}$.

$= (8\bar{i} - \bar{j} - 10\bar{k}) \cdot \frac{1}{3}(2\bar{i} - \bar{j} - 2\bar{k})$

$= \frac{1}{3}\left[8\times2(\bar{i}.\bar{i}) + (\bar{j}.\bar{j}) + (10\times2)(\bar{k}.\bar{k})\right]$

$= \frac{1}{3}\left[16(1) + 1 + 20(1)\right]$

$= \frac{37}{3}$ units //

⑥ Find curl $\bar{f}$. where $\bar{f} = \text{grad}(x^2+y^2+z^2-3xyz)$

Ⓐ Given $\bar{f} = \text{grad}(x^2+y^2+z^2-3xyz)$

$\bar{f} = \text{grad}(\phi)$.

$\bar{f} = \bar{i}.\frac{\partial\phi}{\partial x} + \bar{j}.\frac{\partial\phi}{\partial y} + \bar{k}.\frac{\partial\phi}{\partial z}$.

$\bar{f} = (2x - 3yz)\bar{i} + (2y - 3xz)\bar{j} + (2z - 3xy).\bar{k}$.

$\bar{f} = f_1\bar{i} + f_2\bar{j} + f_3\bar{k}$.

Now curl of $\bar{f}$:- $\nabla\times\bar{f} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$

$$= \begin{vmatrix} \overset{+}{\bar{i}} & \overset{-}{\bar{j}} & \overset{+}{\bar{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (2x-3yz) & (2y-3xz) & (2z-3xy) \end{vmatrix}$$

Explanation:-

$= \bar{i}\left[-3x - (-3x)\right] - \bar{j}\left[-3y - [-3y]\right] + \bar{k}\left[-3z - [-3z]\right]$.

$= \bar{i}[0] - \bar{j}[0] + \bar{k}[0]$.

$\boxed{\text{Curl } \bar{f} = \bar{0}}$ //

⑦ Find the angle b/w two surfaces $x^2+y^2+z^2=9$. and $z = x^2+y^2-3$ at the point $P(2,-1,2)$.

Ⓐ Given $\left.\begin{array}{l} \phi_1 = x^2+y^2+z^2-9 \\ \phi_2 = x^2+y^2-z-3 \end{array}\right\}$ at point $(2,-1,2)$.

The angle b/w the two surfaces are.

case ② from ③ & ④

$$\frac{dy-dz}{(y-z)+(x+y+z)} = \frac{dz-dx}{(z-x)+(x+y+z)}$$

$$\frac{dy-dz}{y-z} = \frac{dz-dx}{z-x}$$

I.O.B.S.

$$\int \frac{dy-dz}{y-z} = \int \frac{dz-dx}{z-x}$$

$$\log(y-z) = \log(z-x) + \log c_2$$

$$\log\left(\frac{y-z}{z-x}\right) = \log c_2$$

$$\frac{y-z}{z-x} = c_2 \quad \text{——⑥}$$

∴ The G.S. $\phi(c_1, c_2) = 0$.

$$\phi\left(\frac{x-y}{y-z}, \frac{y-z}{z-x}\right) = 0 \;//$$

⑤ Find the D.D of the function $\phi = x^2yz + 4xz^2$ at $P(1,-2,-1)$ along the direction of the vector $2\bar{i} - \bar{j} - 2\bar{k}$.

Ⓐ Given $\phi = x^2yz + 4xz^2$.

the D.D of 'ϕ' at point $P(x,y,z)$ along the direction of unit normal vector $\bar{e}$ is:

$$DD = \nabla\phi_{\text{at } P(x,y,z)} \cdot \bar{e}.$$

$$\nabla\phi = \bar{i}\frac{\partial\phi}{\partial x} + \bar{j}\cdot\frac{\partial\phi}{\partial y} + \bar{k}\cdot\frac{\partial\phi}{\partial z}$$

$$= \bar{i}\left[2xyz + 4z^2\right] + \bar{j}\left[x^2z + \cancel{0}\right] + \bar{k}\cdot\left[x^2y + 8xz\right]$$

$$= (2xyz + 4z^2)\bar{i} + x^2z\,\bar{j} + (x^2y + 8xz)\bar{k}.$$

$$\nabla\phi_{\text{at } P(1,-2,-1)} = \left(2(1)(-2)(-1) + 4(-1)^2\right)\bar{i} + (1)^2(-1)\,\bar{j} + \left[(1)^2(-2) + 8(1)(-1)\right]\bar{k}$$

$$= (4+4)\bar{i} - \bar{j} - 10\bar{k}$$

$$\nabla\phi = 8\bar{i} - \bar{j} - 10\bar{k}.$$

The directional vector: $\bar{a} = 2\bar{i} - \bar{j} - 2\bar{k}$.

Now unit vector: $\bar{e} = \frac{\bar{a}}{|\bar{a}|} = \frac{2\bar{i} - \bar{j} - 2\bar{k}}{\sqrt{(2)^2 + (-1)^2 + (-2)^2}}$

$$= \frac{2\bar{i} - \bar{j} - 2\bar{k}}{\sqrt{4+1+4}} \Rightarrow \frac{2\bar{i} - \bar{j} - 2\bar{k}}{\sqrt{9}}$$

(4) Solve: $(x^2-yz)p+(y^2-zx)q=z^2-xy$.

(A) Given:- $(x^2-yz)p+(y^2-zx)q=z^2-xy$ —(1)

It is in the form of $Pp+Qq=R$.

Here $P=x^2-y^2$, $Q=y^2-zx$, $R=z^2-xy$.

The A.E:- $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$

$$\frac{dx}{x^2-yz}=\frac{dy}{y^2-zx}=\frac{dz}{z^2-xy}.$$

Grouping

$$\frac{dx}{x^2-yz}=\frac{dy}{y^2-zx}$$

$$\frac{dx}{x^2-yz}=\frac{dy}{y^2-zx}$$

$$\frac{dx-dy}{x^2-y^2+z(x-y)}$$

$$\frac{dx-dy}{(x+y)(x-y)+z(x-y)}$$

$$\frac{dx-dy}{(x-y)(x+y+z)} \quad \text{—(2)}$$

$$\frac{dy}{y^2-zx}=\frac{dz}{z^2-xy}.$$

$$\frac{dy-dz}{y^2-z^2+x(y-z)}$$

$$\frac{dy-dz}{(y+z)(y-z)+x(y-z)}$$

$$\frac{dy-dz}{(y-z)+(x+y+z)} \quad \text{—(3)}$$

$$\frac{dz}{z^2-xy}=\frac{dx}{x^2-yz}$$

$$\frac{dz-dx}{z^2-x^2+y(z-x)}$$

$$\frac{dz-dx}{(z+x)(z-x)+y(z-x)}$$

$$\frac{dz-dx}{(z-x)(x+y+z)} \quad \text{—(4)}$$

Case (1) From (2) & (3)

$$\frac{dx-dy}{(x-y)\cancel{(x+y+z)}}=\frac{dy-dz}{(y-z)\cancel{(x+y+z)}}$$

$$\frac{dx-dy}{x-y}=\frac{dy-dz}{y-z}$$

Integrating,

$$\int\frac{dx-dy}{x-y}=\int\frac{dy-dz}{y-z}$$

$$\log(x-y)=\log(y-z)+\log c$$

$$\log(x-y)-\log(y-z)=\log c$$

$$\log\left(\frac{x-y}{y-z}\right)=\log c$$

$$\frac{x-y}{y-z}=c_1 \quad \text{—(5)}$$

$$\cos\theta = \frac{\bar{n}_1 \cdot \bar{n}_2}{|\bar{n}_1| \cdot |\bar{n}_2|}$$

now $\bar{n}_1 \Rightarrow \nabla\phi_1$

$$= \bar{i} \cdot \frac{\partial\phi_1}{\partial x} + \bar{j} \cdot \frac{\partial\phi_2}{\partial y} + \bar{k} \cdot \frac{\partial\phi_3}{\partial z}.$$

$$= 2x\,\bar{i} + 2y\,\bar{j} + 2z\,\bar{k}.$$

$$\nabla\phi_1 \text{ at } P(2,-1,2) = 2(2)\,\bar{i} + 2(-1)\,\bar{j} + 2(2)\,\bar{k}.$$

$$= 4\bar{i} - 2\bar{j} + 4\bar{k} \Rightarrow \bar{n}_1$$

now $\bar{n}_2 = \nabla\phi_2$

$$= \bar{i} \cdot \frac{\partial\phi_2}{\partial x} + \bar{j} \cdot \frac{\partial\phi_2}{\partial y} + \bar{k} \cdot \frac{\partial\phi_3}{\partial z}.$$

$$= \bar{i}\,(2x) + \bar{j}\,(2y) + \bar{k} \cdot (-1)$$

$$= 2x\,\bar{i} + 2y\,\bar{j} - \bar{k}.$$

$$\nabla\phi_2 \text{ at } P(2,-1,2) = 2(2)\,\bar{i} + 2(-1)\,\bar{j} - \bar{k}$$

$$= 4\bar{i} - 2\bar{j} - \bar{k} = \bar{n}_2.$$

Now

$$\cos\theta = \frac{\bar{n}_1 \cdot \bar{n}_2}{|\bar{n}_1| \cdot |\bar{n}_2|}$$

$$= \frac{(4\bar{i} - 2\bar{j} + 4\bar{k}) \cdot (4\bar{i} - 2\bar{j} - \bar{k})}{\sqrt{16+4+16} \cdot \sqrt{16+4+1}}$$

$$= \frac{16(\bar{i}\cdot\bar{i}) + 4(\bar{j}\cdot\bar{j}) - 4(\bar{k}\cdot\bar{k})}{\sqrt{36} \cdot \sqrt{21}}$$

$$= \frac{16+4-4}{6\sqrt{21}}$$

$$= \frac{16}{6\sqrt{21}}$$

$$\cos\theta = \frac{8}{3\sqrt{21}}$$

$$\theta = \cos^{-1}\left(\frac{8}{3\sqrt{21}}\right) //$$

(8) Find work done in moving particle force field $\bar{F} = 3x^2\bar{i} + (2xz-y)\bar{j} + z\bar{k}$ along the straight line from origin $(0,0,0)$ to $(2,1,3)$.

(A) Given $\bar{F} = 3x^2\bar{i} + (2xz-y)\bar{j} + z\bar{k}$.

The straight line from origin $\underset{x_1, y_1, z_1}{(0,0,0)}$ to $\underset{x_2, y_2, z_2}{(2,1,3)}$

Equation of OA $\Rightarrow \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} = t$ (say)

$$\frac{x-0}{2-0} = \frac{y-0}{1-0} = \frac{z-0}{3-0} = t.$$

$$\frac{x}{2} = \frac{y}{1} = \frac{z}{3} = t$$

$\therefore x = 2t,\ y = t,\ z = 3t.$ | $dx = 2dt$, $dy = dt$, $dz = 3dt$.

Now work done is :-

$$\int_C \bar{F}\cdot d\bar{r} = \int_{t=0}^{1} \left(3x^2\bar{i} + (2xz-y)\bar{j} + z\bar{k}\right)\left(dx\,\bar{i} + dy\,\bar{j} + dz\,\bar{k}\right).$$

$$= \int_{t=0}^{1} \left(3(2t)^2\cdot\bar{i} + (2(2t)(3t) - t)\bar{j} + 3t\,\bar{k}\right)\left(2dt\,\bar{i} + dt\,\bar{j} + 3dt\,\bar{k}\right)$$

$$= \int_0^1 \left(12t^2\bar{i} + (12t^2-t)\bar{j} + 3t\,\bar{k}\right)\left(2\,dt\,\bar{i} + dt\,\bar{j} + 3dt\,\bar{k}\right)$$

$$= \int_0^1 24t^2\,dt\,(\bar{i}\cdot\bar{i}) + (12t^2-t)\,dt\,(\bar{j}\cdot\bar{j}) + 9t\,dt\,(\bar{k}\cdot\bar{k})$$

$$= \int_0^1 24t^2\,dt + (12t^2-t)\,dt + 9t\,dt$$

$$= \int_0^1 36t^2\,dt + 8t\,dt.$$

$$= 36\int_0^1 t^2\,dt + 8\int_0^1 t\,dt.$$

$$= 36\left[\frac{t^3}{3}\right]_0^1 + 8\cdot\left(\frac{t^2}{2}\right)_0^1 + c$$

$$= 36\left(\frac{1}{3}\right) + 8\left(\frac{1}{2}\right) + c$$

$$= 12 + 4 + c$$

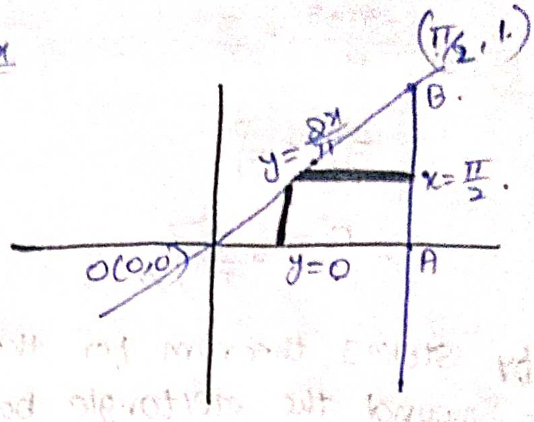
$$= 16 + c$$

9) verify green's theorem in a plane for $\oint (y-\sin x)dx + \cos x\, dy$ where c: the triangle enclosed by the lines $y=0$, $x=\frac{\pi}{2}$, $y=\frac{2x}{\pi}$

(A) Given $\int_c (y-\sin x)\,dx + \cos x\, dy$.

limits

Given $y=0$, $x=\frac{\pi}{2}$, $y=\frac{2x}{\pi}$



x-limits:- $\left(\frac{\pi y}{2}, \frac{\pi}{2}\right)$

y-limits :- $(0,1)$.

Apply green's theorem:-

$$\int_c M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy.$$

$$= \int_{y=0}^{1} \int_{x=\frac{\pi y}{2}}^{\pi/2} \left[(-\sin x - 1)\,dx\right] dy.$$

$$= \int_{y=0}^{1} \left[-(-\cos x) - x\right]_{\frac{\pi y}{2}}^{\pi/2} dy.$$

$$= \int_{y=0}^{1} (\cos x - x)_{\frac{\pi y}{2}}^{\pi/2}\, dy.$$

$$= \int_{y=0}^{1} \left[\cos\frac{\pi}{2} - \frac{\pi}{2} - \left(\cos\left(\frac{\pi y}{2}\right) - \frac{\pi y}{2}\right)\right] dy$$

$$= \int_{y=0}^{1} \left(0 - \frac{\pi}{2} - \cos\left(\frac{\pi y}{2}\right) + \frac{\pi y}{2}\right) dy.$$

$$= \int_{0}^{1} \left(-\frac{\pi}{2} - \cos\left(\frac{\pi y}{2}\right) + \frac{\pi y}{2}\right) dy.$$

$$= \left[-\frac{\pi}{2} y - \frac{\sin \frac{\pi y}{2}}{\pi/2} + \frac{\pi}{2}\,\frac{y^2}{2}\right]_0^1$$

$$= -\frac{\pi}{2} - \frac{\sin \pi/2}{\pi/2} + \frac{\pi}{2}\cdot\frac{1}{2} - [0]$$

RHS :- $\iint_S (\nabla \times \bar{F}) \hat{n} \cdot ds$.

$$\nabla \times \bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x^2+y^2) & -2xy & 0 \end{vmatrix}$$

$$= \bar{i}[0] - \bar{j}[0] + \bar{k}[-2y-2y]$$

$$= \bar{k}[-4y].$$

$$= -4y\bar{k}$$

$\bar{n} = \bar{k}$

$ds = dx\,dy$.

$$\iint -4y\,\hat{k}\cdot\hat{k}\,dx\,dy.$$

$$\int_0^b \int_{-a}^{a} -4y\,dx\,dy.$$

$$\int_0^b -4y[x]_{-a}^{a}\,dy$$

$$\int_0^b -4y(a+a)\,dy$$

$$\int_0^b -8ay\,dy$$

$$-8a\int_0^b y\,dy$$

$$-\cancel{8}^{4}a\left[\frac{y^2}{\cancel{2}}\right]_0^b$$

$$-4ab^2$$

$\therefore$ LHS = RHS

NOW

$$\int_{BC} (x^2+y^2)\, dx.$$

$y=b$

$dy=0$

$$= \int_{a}^{-a} (x^2+y^2)\, dx.$$

$$= \left[\frac{x^3}{3}\right]_a^{-a} + y^2 [x]_a^{-a}.$$

$$= \frac{1}{3}(-a^3-a^3) + y^2(-a-a)$$

$$= \frac{-2a^3}{3} - 2ay^2 \qquad \backslash y=b$$

$$= \frac{-2a^3}{3} - 2ab^2.$$

NOW

$$\int_{CD} -2xy \cdot dy.$$

$x=-a$

$dx=0$

$$= -2x\int_b^0 y\, dy$$

$$= -2x\left[\frac{y^2}{2}\right]_b^0$$

$$= +a(0-b^2)$$

$$= -ab^2$$

NOW

$$\int_{DA} (x^2+y^2)\, dx.$$

$y=0$

$dy=0$

$$= \int_{-a}^{a} (x^2+y^2)\, dx.$$

$$= \left[\frac{x^3}{3}\right]_{-a}^{a} \Rightarrow \frac{a^3}{3}+\frac{a^3}{3} = \frac{2a^3}{3}.$$

NOW

LHS:- $\displaystyle\int_C = -ab^2 - \cancel{\frac{2a^3}{3}} - 2ab^2 + \cancel{\frac{2a^3}{3}} - ab^2$

$$\int_C = -4ab^2.$$

$$\left[\frac{-\pi}{2} - \frac{1}{\pi/2} + \frac{\pi}{4}\right]$$

$$\left[\frac{-\pi}{2} - \frac{2}{\pi} + \frac{\pi}{4}\right]$$

$$\frac{-\pi}{2} + \frac{\pi}{4} - \frac{2}{\pi}$$

$$\frac{-2\pi+\pi}{4} - \frac{2}{\pi}$$

$$\frac{-\pi}{4} - \frac{2}{\pi}$$

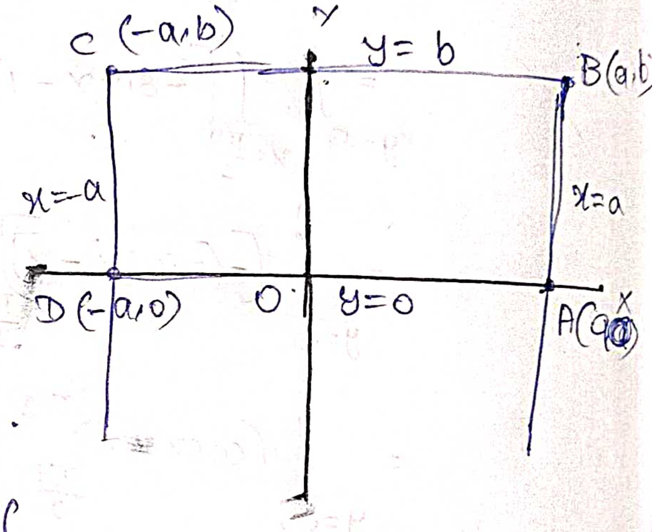
$$-\left[\frac{\pi}{4} + \frac{2}{\pi}\right]$$

(10) verify stokes theorem for the vector $\bar{F} = (x^2+y^2)\bar{i} - 2xy\bar{j}$. take round the rectangle bounded by the lines $x = \pm a$, $y=0$, $y=b$.

(A) Given $\bar{F} = (x^2+y^2)\bar{i} - 2xy\bar{j}$.

Given $x = \pm a$, $y=0$, $y=b$.

$$\int_C \vec{F}\cdot d\vec{r} = \iint_S (\nabla\times\bar{F})\cdot\hat{n}\, ds.$$



LHS :-

$$d\bar{r} = dx\,\bar{i} + dy\,\bar{j} + dz\,\bar{k}.$$

$$\vec{F}\cdot d\vec{r} = (x^2+y^2)\,dx - 2xy\,dy.$$

$$\int_C = \int_{AB} + \int_{BC} + \int_{CD} + \int_{DA}.$$

NOW

$$\int_{AB} 0 - 2xy\,dy.$$

$x=a$
$dx=0$

$$-2\int_0^b xy\,dy.$$

$$-2x\left(\frac{y^2}{2}\right)_0^b$$

$$= -ab^2$$