UNIT-3

TRACKING RADARS

➢ INTRODUCTION

- Tracking of the targets with the help of tracking radar.
- Tracking means follow the path of the target using the radar.
- There are following parameters of the targets are important:

  i. Range of the target
  ii. The velocity of the target
  iii. The azimuth angle of the target
  iv. Elevation angle of the target

- There are different types of techniques of tracking the targets such as

  i. Sequential Lobbing,
  ii. Conical Scanning,
  iii. Mono Pulse Tracking

- Determine the target path and to predict its future position. Different co-ordinates such as

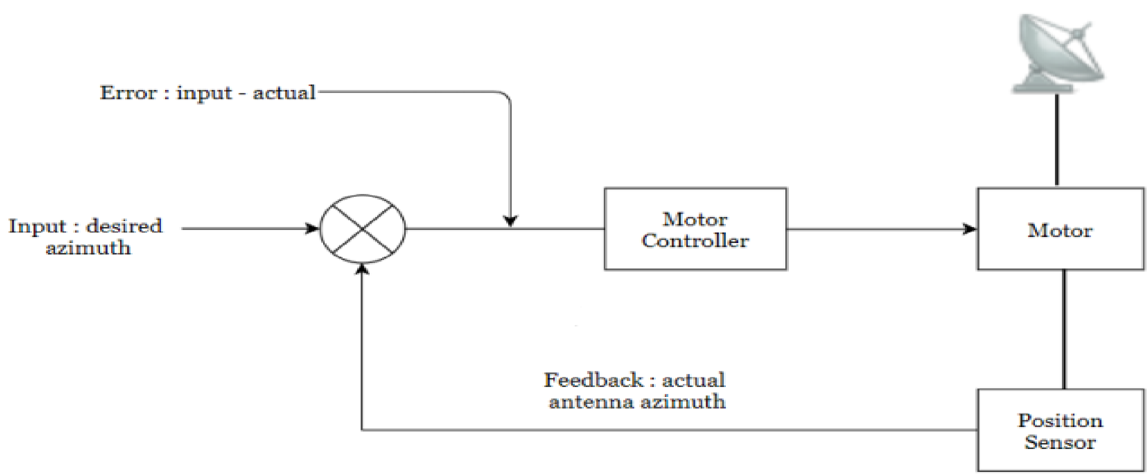
  i. Range
  ii. Elevation Angle
  iii. Azimuth Angle
  iv. Doppler Shift Frequency

It is the method by which **ANGLE TRACKING** accomplished may be called the tracking radar.

- One of the most useful features of radar is the ability of a radar set to continuously predict the next location of the target from the information received from the target and to align itself to continuously point at the predicted location. When this is occurring, the radar is said to be **Tracking Radar.**

Servo Mechanism of Tracking Radar

- One of the most basic tracking systems is the servo tracking system, as shown in the figure below



Servo Mechanism

- Here, the radar antenna is initially trained on the target after which it automatically remains pointed at the target as it follows its motion.
- Furthermore, the system provides continuous position information to the operator and possibly to a fire control system. T
- The antenna is rotated by a motor which provides a negative position feedback signal to a controller. This system is known as a **servo mechanism**.

- Sometimes single antenna is not favorable for both search and tracking the target at the same time.
- So in this condition, a separate search radar is employed to provide the information related to the position of the target to the tracking system.
- When a separate search radar is employed for giving the information of the target to the tracking radar system is called **acquisition radar**.

Types of Tracking Radar

There are many types of tracking radar, which is being used to track the targets. They are as follows:

1. Single Target Tracker (STT)
2. Track While Scan (TWS)
3. Automatic Detection And Track (ADT)
4. Phased Array Tracking

1. **Single Target Tracker (STT)**

- A single target tracker is used where continuous tracking of a single target with a higher data rate is required.
- Generally, this type of tracking is used in controlling the missile movements as it is carried out by a missile guidance radar.
- In this type of tracker, a closed-loop servomechanism is used to keep the angle coordinates error very small.
- If the errors are less then there will be more accurate.

2. **Track While Scan (TWS)**

- TWS radar scans their beam over relatively large areas. It rapidly scans in the angular sector to keep track of the targets, maybe move one target at a time.
- The radar computer still measures returned power as a function of beam location to provide tracking but the large scanning area enables the radar to still see the target even if the track has beam broken or lost.
- However, this large scan makes the TWS highly vulnerable to ECM jamming.

3. **Automatic Detection and Track**

- This type of technique is used in air traffic control radar.
- This technique is employed with the air surveillance radar, which has 7.5 to 15 RPM.
- It does provide updating data according to the rotation of the antenna.
- It can track an enormous number of targets simultaneously may be few hundred.
- This antenna position is not controlled by closed-loop, it works in the open-loop system.

4. **Phased Array Radar Tracking**

- In this technique, multiple targets are tracked on a time-sharing basis by the computer.
- Large numbers of tracks can be scanned rapidly in this technique because of electronically steered phased array method is used here.
- The antenna beam is scanned electronically make to switch in from one angular direction to another in a small-time such as a few microseconds.

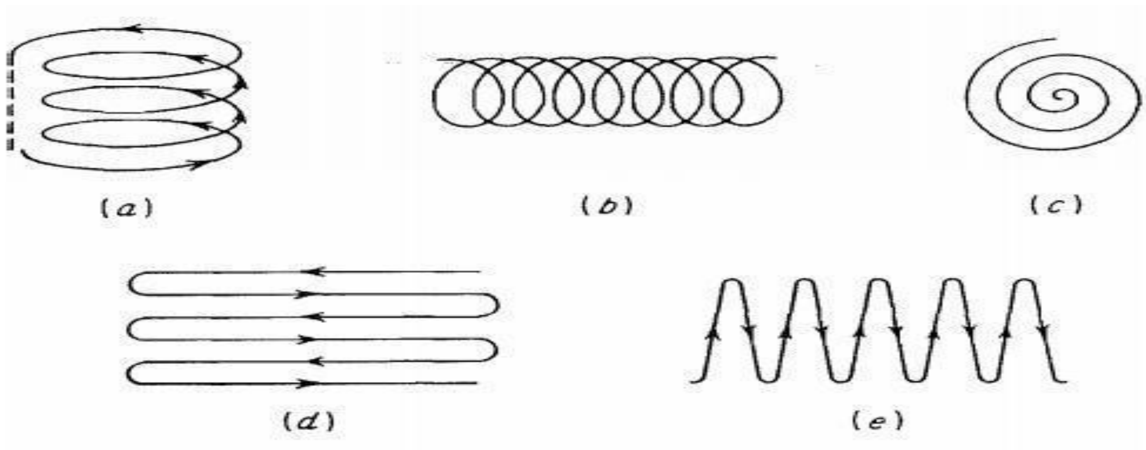
➢ **'Acquisition and Scanning Patterns'::**

- A radar which detects a target determines its location and trajectory in the future is called tracking radar.
- Tracking is the process of continuously maintaining the antenna beam on the target and also the echo signal within the range gate (present Range value).
- The antenna beam in the continuous tracking radar is positioned in angle by a servomechanism actuated by an error signal.
- The tracking radar must first find and acquire its target before it can track. This is explained in detail subsequently under the heading ***'Acquisition and Scanning Patterns'***

a) Acquisition of radar :: A radar set capable of locking onto a received signal and tracking the object emitting the signal; the radar may be airborne or on the ground.

b) **Scanning Patterns :: The process or action of directing a radar beam through a space search pattern for the purpose of locating a target**.

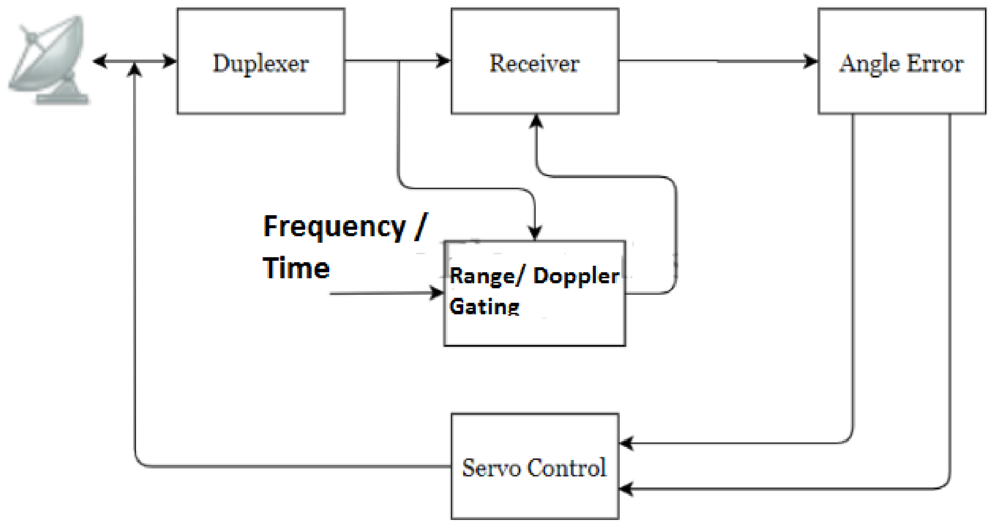
- A tracking radar must first find and acquire its target before it can operate as a tracker.
- Therefore it is usually necessary for the radar to scan an angular sector in which the presence of the target is suspected.
- Most tracking radars employ a narrow pencil-beam antenna.
- Examples of the common types of scanning patterns employed with pencil-beam antennas are illustrated in Fig.



a) In the ***helical scan***, the antenna is continuously rotated in azimuth while it is simultaneously raised or lowered in elevation. It traces a helix in space.
b) The ***Palmer scan*** consists of a rapid circular scan (conical scan) about the axis of the antenna, combined with a linear movement of the axis of rotation.
   When the axis of rotation is held stationary the Palmer scan reduces to the ***conical*** scan.
c) The ***spiral scan*** covers an angular search volume with circular symmetry.
   Both the spiral scan and the Palmer scan suffer from the disadvantage that all parts of the scan volume do not receive the same energy unless the scanning speed is varied during the scan cycle.
   As a consequence, the number of hits returned from a target when searching with a constant scanning rate depends upon the position of the target within the search area.
d) The ***raster or TV, scan***, unlike the Palmer or the spiral scan, scans the search area in a uniform manner.
e) The raster scan is a simple and convenient means for searching a limited sector, rectangular in shape.
f) Similar to the raster scan is the ***nodding scan*** produced by oscillating the antenna

beam rapidly in elevation and slowly in azimuth. Although it may be employed to cover a limited sector-as does the raster scan-nodding scan may also be used to obtain hemispherical coverage, that is, elevation angle extending to $\mathbf{90^0}$and the azimuth scan angle to $\mathbf{360^0}$

BLOCK DIAGRAM OF TRACKING RADAR::



Block Diagram of Tracking Radar

- Most tracking radars use angular information as the basis for tracking operations.
- For better accuracy, it is important that radar concentrates on one target at a time.
- Range gating/Doppler filtering can be used for that purpose.
- Time and frequency control for range and doppler gating is done in range and doppler trackers respectively. **<u>The angular error signal</u>** for the desired target to be tracked is developed in the error demodulator block which is also controlled by range/doppler gate generation block and then fed back to the steerable antenna in a closed-loop for tracking.

REAL TIME EXAMPLE

- Here we are tracking the missile which was launched

ANGLE TRACKING

- Angle tracking employs two (or four) different beams at different angles.
- The angle between them is called the squint angle relative to ‘boresight’.
- Boresight is the angle at which the two beams’ polar shapes meet.
- These two beams need to be positioned such that the boresight is always in the direction of the target.
- This is done by the amplitudes of the echo signals A and B, they help in finding the distance of the target from boresight.

➢ **Let’s look at the two cases of angle tracking.**

Case 1:

- Two beams A & B for one angle co-ordinate.
- The beams that we use here can be simultaneous or switched. Which means that they are time-shared – One beam switches OFF for the other to light up and vice versa.
- This is an older technique. It is used in sequential lobing and conical trackers.

Case 2:

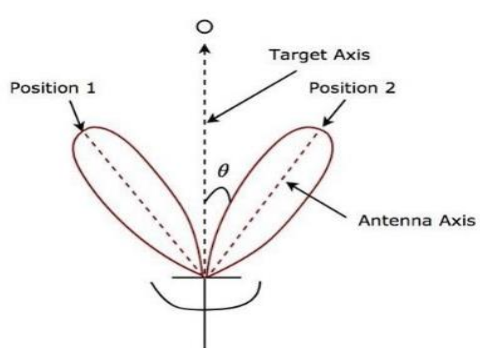
- Four beams are used to get two angle co-ordinates
- This is a modern technique. And it has higher precision than that of using just two beams.
- Simultaneous lobing trackers. Example: Monopulse tracking.

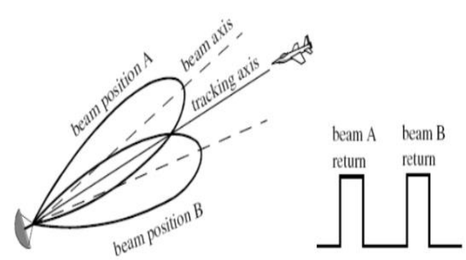
- The various methods for generating the error signal are classified as

1. **Sequential Lobing,**
2. **Conical Scan,** And
3. **Simultaneous lobing** Or **Monopoles**

1. <u>Sequential Lobing</u>

- Sequential lobing is also known as lobe switching or sequential switching.
- As we understood above, there is generally a difference between the actual target location and the reference direction of the tracking radar. This difference is known as the angular error.
- In sequential lobing, the position of the antenna beam is switched between two positions.
- This gives us the direction and magnitude of the angular error.
- The method of switching a single beam between two squinted angular positions to obtain an angle of measurement is called sequential lobing.
- Though it is possible to use a time-shared antenna beam to obtain an angle measurements simpler. However, it's **<u>not very accurate.</u>**
- The polar representation and the rectangular representation of the antenna beams in the two switched positions is shown below





Sequential lobing – polar and rectangular representations

- The error obtained from the target being NOT on the boresight is also shown.

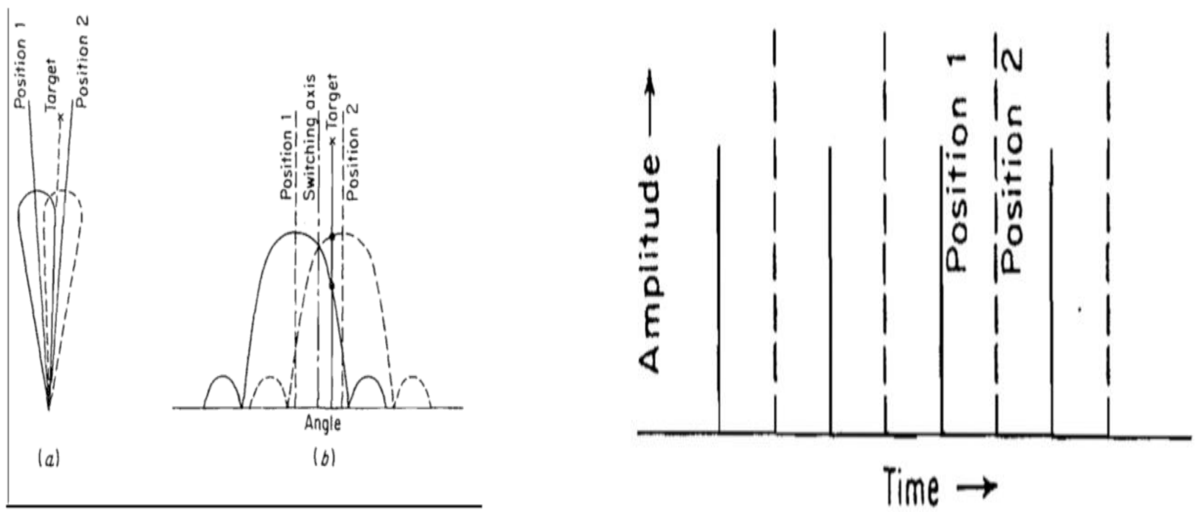


Fig::Sequential lobing – error signals

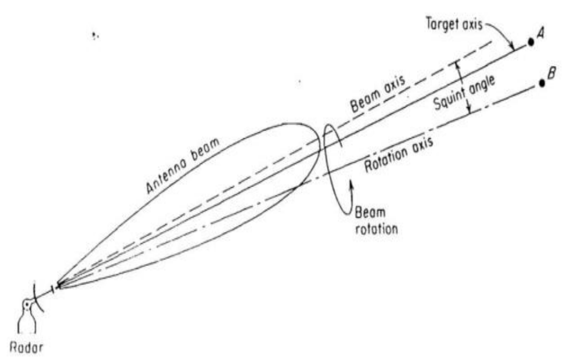
- The difference in the amplitude between the voltages that we obtain from the two positions gives the angular displacement of the target from the boresight.
- The direction in which to move the antenna comes from the beam that has the larger signal.

2. **Conical Scan**

- A conical beam is used in a conical scan
- The conical scan is another method to find the angular error in angle tracking. This error is then used to control the servo motors to find and predict the accurate location, movement and direction of a moving object.
- Angular error is the error signal that we get when we compare the amplitudes of the voltage signals that we receive from the two switching beams that we use to track an object.

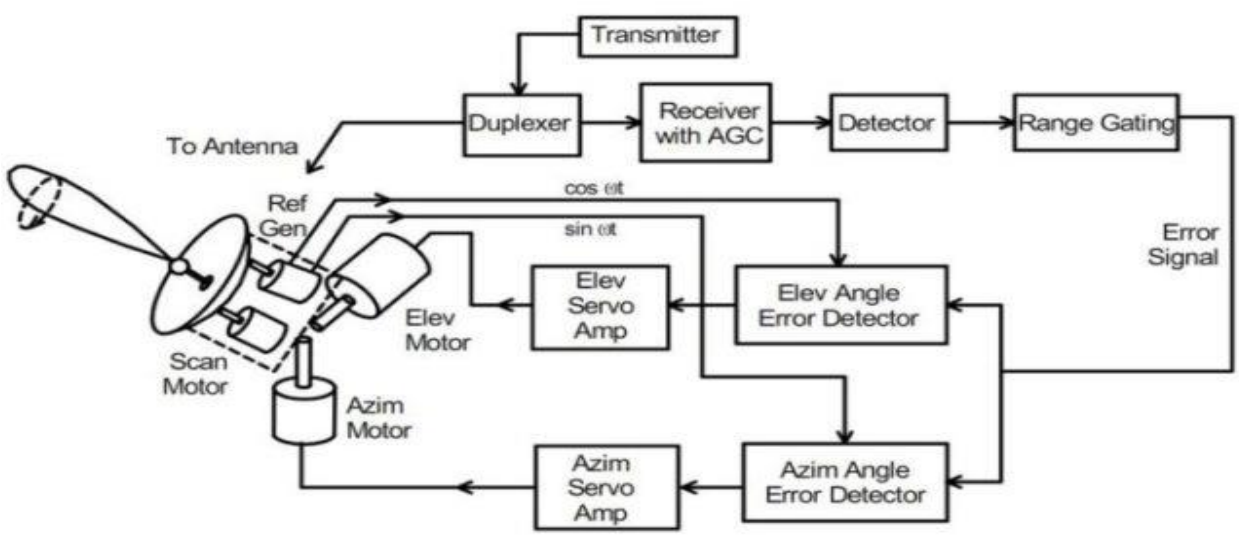
Working of the conical scan method

- The rotating beam of a conical scan traces out the shape of a cone. Hence, it is named as a conical scan.
- This beam can be achieved by using a rotating feed that is driven by a motor.
- This motor is kept in a housing or enclosure at the end of the dish.
- Let's take a look at the different elements of the beam that we use in a conical scan.



- If you are standing at point A and look at the lobe.
- Assuming that the lobe is visible.
- Then you would see it tracing the shape of a circle.
- If you are the centre of this circle constantly then the amplitude of the reflected signal will be constant.
- However, if you start moving a bit towards the left or right, your position relative to the axis changes.
- Correspondingly, this causes a change in the amplitude of the reflected signal too.

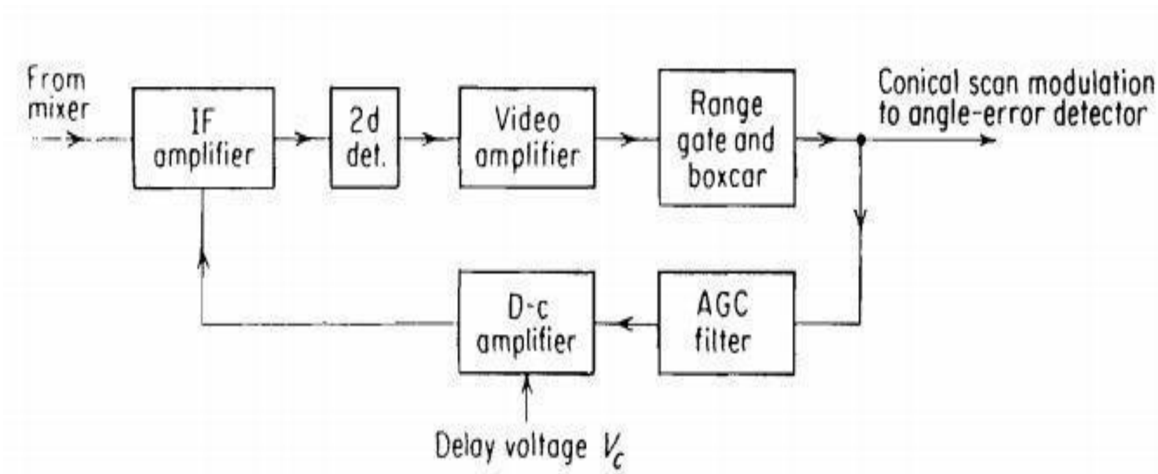
- This data from the echo signal is sent to an angle-error detector circuit.
- The data from this circuit drives the servo motors that control the antenna.
- A nutating feed design is given preference over a rear feed design in the conical scan.
- This is because we need to maintain the plane of polarization.
- A rear feed design rotates it.
- This can cause amplitude shifts and hence, is undesirable
- Nutating means to spin without affecting the polarization.
- However, the nutating feed is more complex than the rear feed.
- Let’s take a look at the block diagram of the conical scan to understand its working.



Block diagram of a conical scan radar

- The reference generator has two outputs that extract elevation and azimuth errors. The echo signal that is received is fed to the receiver from the antenna via two rotary joints (not shown in the block diagram).
- One rotary joint controls the azimuth movement. And the other controls the elevation movement.
- The receiver is a superhetrodyne receiver.
- The error signal is extracted after the second detector.
- Range gate is used to search and lock the target and continuously track the target.

Automatic Gain Control



- The function of the automatic gain control (AGC) is to maintain the d-c level of the receiver output constant and to smooth or eliminate as much of the noise like amplitude fluctuations as possible without disturbing the extraction of the desired error signal at the conical-scan frequency.
- One of the purposes of AGC in any receiver is to prevent saturation by large signals. The scanning modulation and the error signal would he lost if the receiver were to saturate.

- In the conical-scan tracking radar an AGC that maintains the d-c level constant results in an error signal that is a true indication of the angular pointing error. The d-c level of the receiver must be maintained constant if the angular error is to be linearly related to the angle-error signal voltage.
- An example of the AGC portion of a tracking-radar receiver is shown in Fig.
- A portion of the video-amplifier output is passed through a low-pass or smoothing filter and fed back to control the gain of the IF amplifier.
- The larger the video output, the larger will be the feedback signal and the greater will be the gain reduction.
- The filter in the AGC loop should pass all frequencies from direct current to just below the conical-scan-modulation frequency.
- The loop gain of the AGC filter measured at the conical-scan frequency should be low so that the error signal will not be affected by AGC action.
- The output of the feedback loop will be zero unless the feedback voltage exceeds a pre-specified minimum value V_c.
- In the block diagram the feedback voltage and the voltage V_c are compared in the d-c amplifier.
- If the feedback voltage exceeds V_c, the AGC is operative, while if it is less, there is no AGC action. The voltage V_c is called the delay voltage.
- The purpose of the delay voltage is to provide a reference for the constant output signal and permit receiver gain for weak signals.

Disadvantages of The conical-scan and sequential-lobing tracking radars:

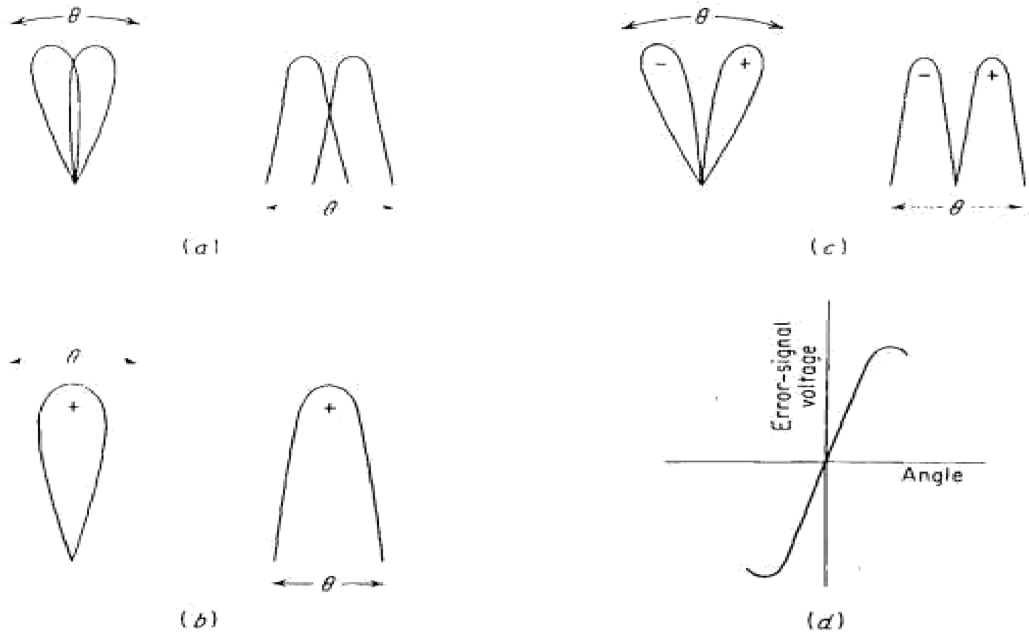
- They require a minimum number of pulses to extract the angle-error signal.
- The train of echo pulses must not contain amplitude-modulation components other than the modulation produced by scanning.

- If the echo pulse-train contains additional modulation components, caused, for example, by a fluctuating **target cross section**, the tracking accuracy will be degraded.
- The fluctuating echo can severely limit the accuracy of those tracking radars which require many pulses to be processed to extract the error signal.

These disadvantages are over come in the Monopulse Tracking Radar

2. Monopulse tracking radar

- The monopulse radar system is mainly used for target angle measurement and tracking. The information on the target angular position is determined by comparison of signals received in two or more simultaneous beams.
- The term "monopulse" comes from the ability of this system to extract the angular position from only one pulse
- The main advantage of a monopulse system in comparison to standard angle measurement methods is that it is not affected by amplitude fluctuations of the target echo
- In this technique, the RF signals received from two offset antenna beams are combined so that both the **sum and the difference** signals are obtained simultaneously.
- The sum and difference signals are multiplied in a **phase-sensitive detector** to obtain both the **magnitude** and **the direction** of the error signal



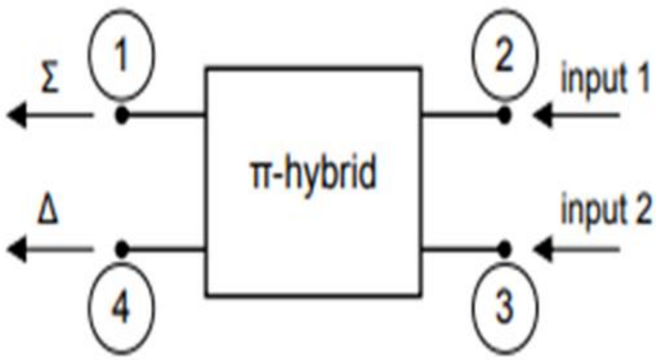


Fig : Symbol of Magic T

- The amplitude-comparison monopulse employs two overlapping antenna patterns (Fig 7 (a)) to obtain the angular error in one coordinate.
- The two overlapping antenna beams may be generated with a single reflector or with a lens antenna illuminated by two adjacent feeds. (A cluster of four feeds may be used if both elevation- and azimuth-error signals are wanted.)
- The sum of the two antenna patterns of Fig (a) is shown in Fig (b), and the difference in Fig (c). The sum patterns are used for transmission, while both the sum pattern and the difference pattern are used on reception.
- The signal received with the difference pattern provides the magnitude of the angle error.

- The sum signal provides the range measurement and is also used as a reference to extract the sign of the error signal.
- Signals received from the sum and the difference patterns are amplified separately and combined in a phase-sensitive detector to produce the error-signal characteristic shown in Fig (d).

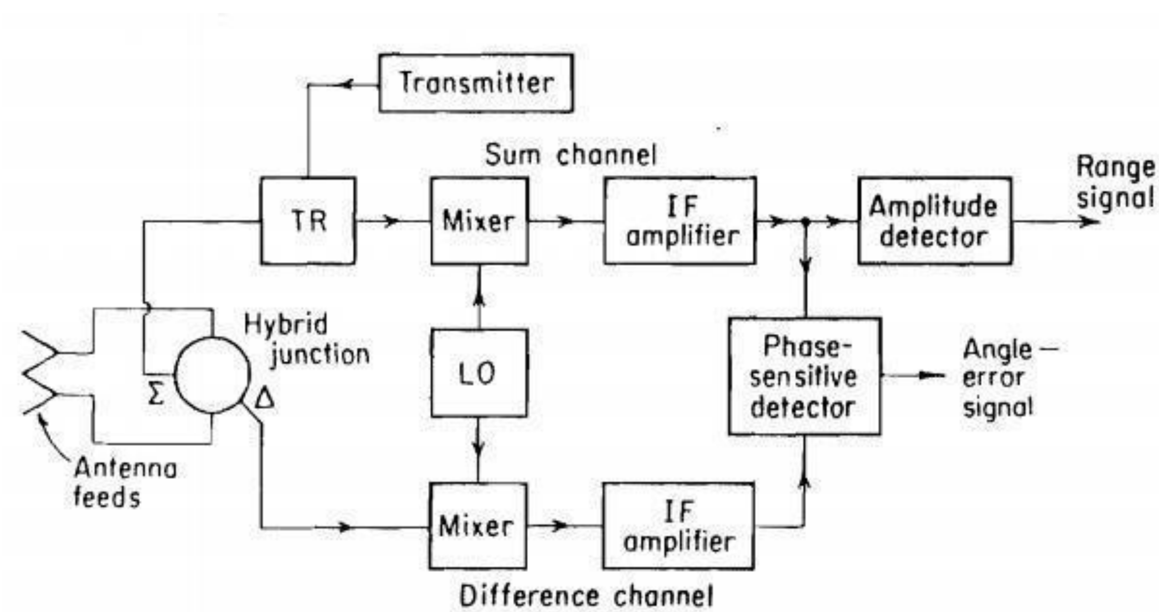
Monopulse Techniques

1. Amplitude comparison monopulse radar
2. Phase comparison monopulse radar

Amplitude comparison monopulse radar

1. One Angular coordinate
2. Two angular coordinate (azimuth and elevation)

<u>Block Diagram of Amplitude-Comparison Monopulse Radar for Single Angular Coordinate</u>



- A block diagram of the amplitude-comparison-monopulse tracking radar for a single angular coordinate is shown in Fig.

- The two adjacent antenna feeds are connected to the two arms of a hybrid junction such as a magic T, a rat race, or a short-slot coupler.
- The sum and difference signals appear at the two other arms of the hybrid.
- On reception, the outputs of the sum arm and the difference arm are each heterodyned to an intermediate frequency and amplified as in any superhetrodyne receiver.
- The transmitter is connected to the sum arm. Range information is also extracted from the sum channel.
- A duplexer is included in the sum arm for the protection of the receiver.
- The output of the phase-sensitive detector is an error signal whose magnitude is proportional to the angular error and whose sign is proportional to the direction.
- The output of the monopulse radar is used to perform automatic tracking.
- The angular error signal actuates a servo-control system to position the antenna, and the range output from the sum channel feeds into an automatic-range-tracking unit.
- The sign of the difference signal (and the direction of the angular error) is determined by comparing the phase of the difference signal with the phase of the sum signal.
- If the sum signal in the IF portion of the receiver were $A_s \cos(\omega_{IF} t)$, the difference signal would be either $A_d \cos(\omega_{IF} t)$ or $-A_d \cos(\omega_{IF} t)$ ($A_s > 0$, $A_d > 0$), depending on which side of center is the target.
- Since $-A_d \cos(\omega_{IF} t) = A_d \cos[\omega_{IF}(t + \pi)]$, the sign of the difference signal may be measured by determining whether the difference signal is in phase with the sum or 180^o out of phase.

Block Diagram of Amplitude-Comparison Monopulse Radar for Two Angular Coordinate (extracting error signals in both elevation and azimuth)

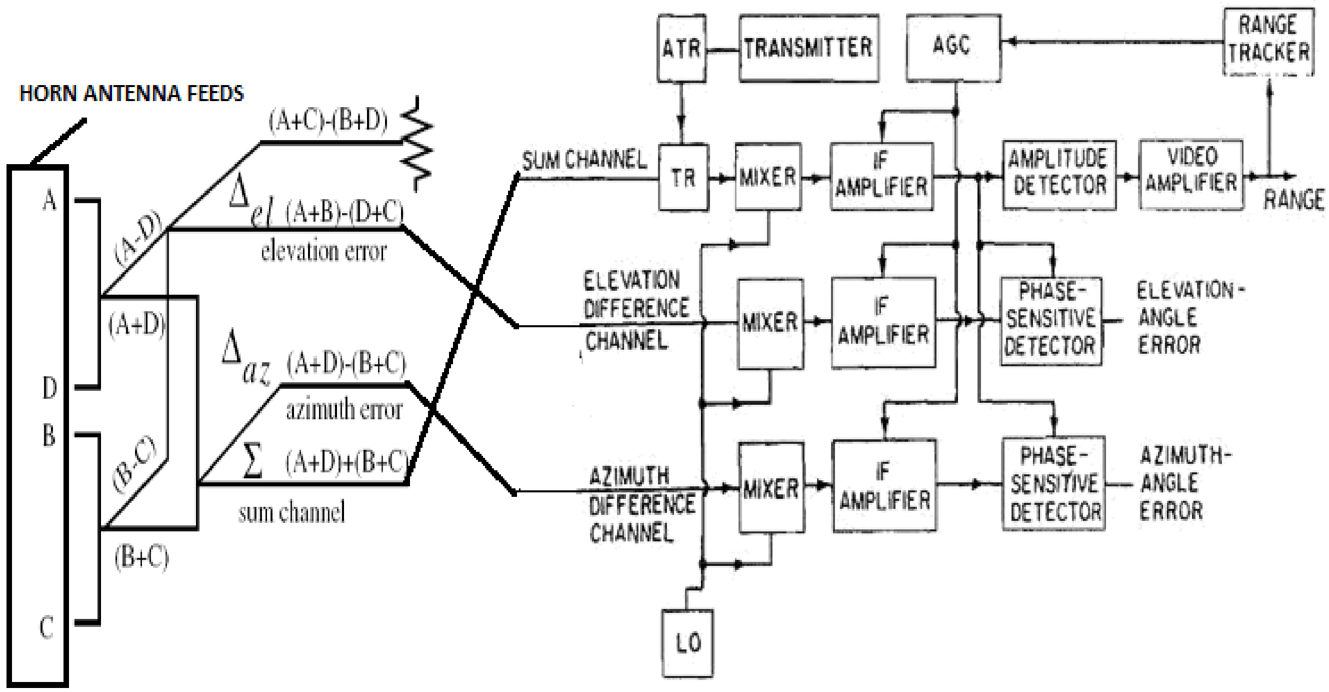


Fig::Microwave comparator circuitry used with a four-horn monopulse feed

i. **Hybrid junction-1 (A,D)**
 SUM :: A+D
 DIFFERENCE :: A-D

ii. **Hybrid junction-1 (B,C)**
 SUM :: B+C
 DIFFERENCE :: B-C

iii. **Hybrid junction-3 (A+D, B+C)**
 SUM :: A+D+ B+C
 DIFFERENCE :: (A+D)-(B+C)
 =

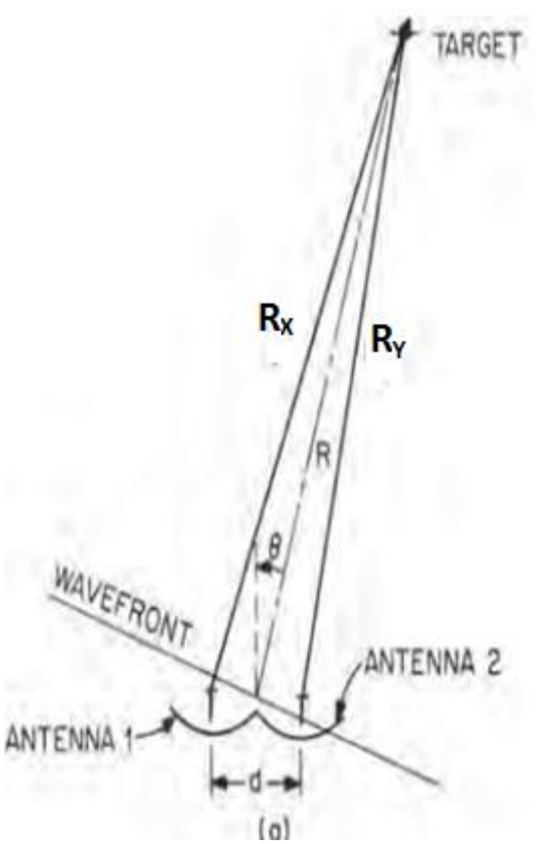
iv. **Hybrid junction-4 (A-D , B-C)**
 SUM :: A-D + B-C=A+B-(D+C)
 DIFFERENCE :: A-D –(B-C)
 =A-D-B+C
 =A+C –(D+B)

- A block diagram of monopulse radar with provision for extracting error signals in both elevation and azimuth is shown in Fig.
- The cluster of four feeds generates four partially overlapping antenna beams.
- The feeds might be used with a parabolic reflector, Cassegrain antenna, or a lens.
- All four feeds generate the sum pattern.
- The difference pattern in one plane is formed by taking the sum of two adjacent feeds and subtracting this from the sum of the other two adjacent feeds.

- The difference pattern in the orthogonal plane is obtained by adding the differences of the orthogonal adjacent pairs.
- A total of four hybrid junctions generate the sum channel, the azimuth difference channel, and the elevation difference channel.
- Three separate mixers and IF amplifiers are shown, one for each channel.
- All three mixers operate from a single local oscillator in order to maintain the phase relationships between the three channels.
- Two phase-sensitive detectors extract the angle-error information, one for azimuth, the other for elevation.
- Range information is extracted from the output of the sum channel after amplitude detection.
- Since a phase comparison is made between the output of the sum channel and each of the difference channels, it is important that the phase shifts introduced by each of the channels be almost identical.
- The phase difference between channels must be maintained to within 25° or better for reasonably proper performance.
- The gains of the channels also must not differ by more than specified amounts.

PHASE-COMPARISON MONOPULSE TRACKING RADAR TECHNIQUE

- The measurement of angle of arrival by comparison of the phase relationships in the signals from the separated antennas of a radio interferometer has been widely used by the radio astronomers for precise measurements of the positions of radio stars.
- The interferometer as used by the radio astronomer is a passive instrument, the source of energy being radiated by the target itself.
- A Tracking radar which operates with phase information is similar to an active interferometer and might be called an **interfero meter radar.** It has also been called **Simultaneous phase comparison radar**, **or phase-comparison monopulse**.



Two antennas separated by a distance 'd' and the target is at a distance of 'R' from the antenna.

The distance of target from the antenna 'x' is given by

$$R_x = R + \frac{d}{2}\sin\theta \text{ ---------------- } 1$$

The distance of target from the antenna 'y' is given by

$$R_y = R - \frac{d}{2}\sin\theta \text{ ---------------- } 2$$

Therefore, the phase difference between the two echo signals is approximately

$$\Delta\varphi = \frac{2\pi}{\lambda}(R_x - R_y)$$

$$\Delta\varphi = \frac{2\pi}{\lambda}\left(R + \frac{d}{2}\sin\theta - \left(R - \frac{d}{2}\sin\theta\right)\right)$$

$$\Delta\varphi = \frac{2\pi}{\lambda}\left(R + \frac{d}{2}\sin\theta - R + \frac{d}{2}\sin\theta\right)$$

$$\Delta\varphi = \frac{2\pi}{\lambda}\left(\frac{2d}{2}\sin\theta\right)$$

$$\Delta\varphi = \frac{2\pi}{\lambda}\left(d\sin\theta\right)$$

Fig : Wavefront phase relationships in phase- comparison monopulse radar

In the early versions of the phase-comparison monopulse radar, the angular error was determined by measuring the phase difference between the outputs of receivers connected to each antenna.

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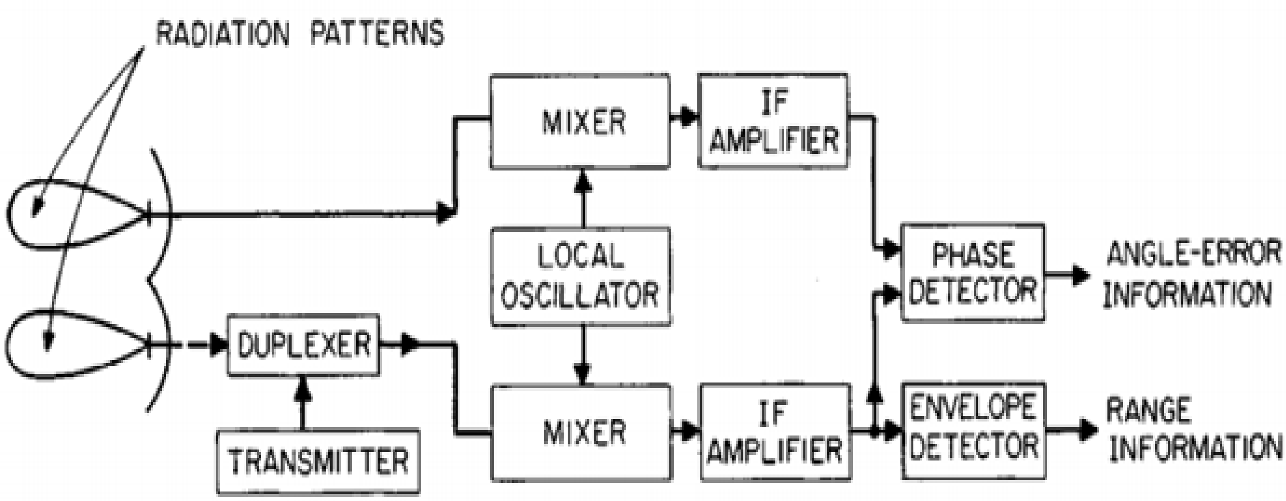


Fig : Block diagram of a phase comparison monopulse radar

- Figure shows the antenna and receiver for one angular-coordinate tracking by phase-comparison monopulse.
- Any phase shifts occurring in the mixer and IF amplifier stages cause a shift in the boresight of the system.
- The disadvantages of phase-comparison monopulse compared with amplitude-comparison monopulse are the relative difficulty in maintaining a highly stable boresight and the difficulty in providing the desired antenna illumination taper for both sum and difference signals.
- The longer paths from the antenna outputs to the comparator circuitry make the phase-comparison system more susceptible to boresight change due to mechanical loading or sag, differential heating, etc

TRACKING IN RANGE IS ACHIEVED USING SPLIT RANGE GATES:

- The tracking radar tracks the target both in azimuth and elevation . This is done manually but isn't efficient and satisfactory.
- To overcome above problem it is replaced by automatic tracking.
- It consist of two gates:

  1-Early gate

  2- Late gate
- The early gate produces positive voltage output but the late gate produces negative voltage output.
- Subtracting & integration, Output is: zero, negative or positive

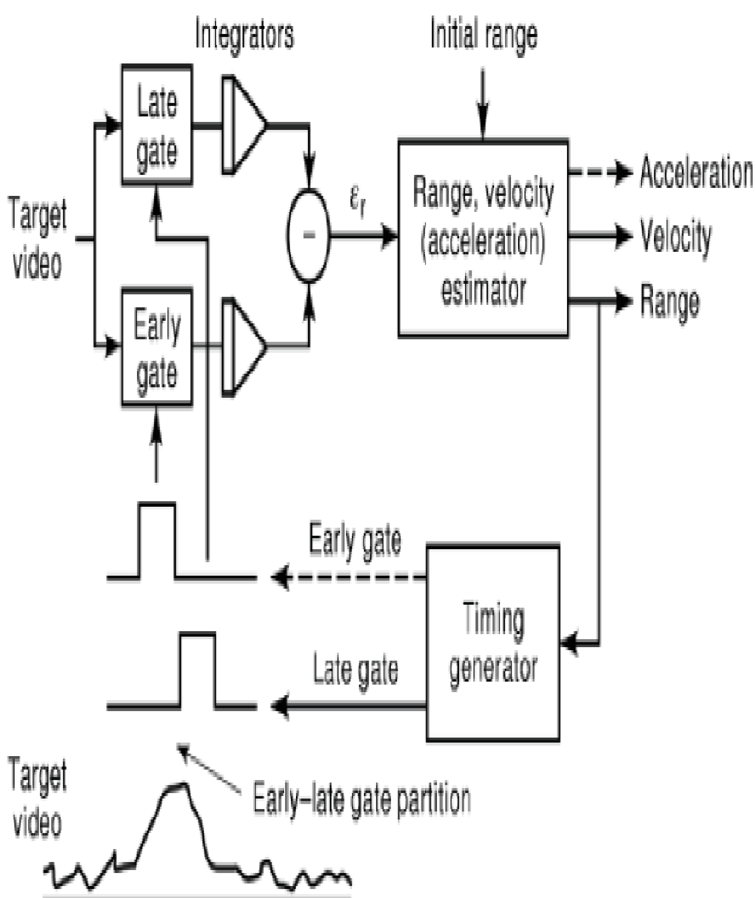
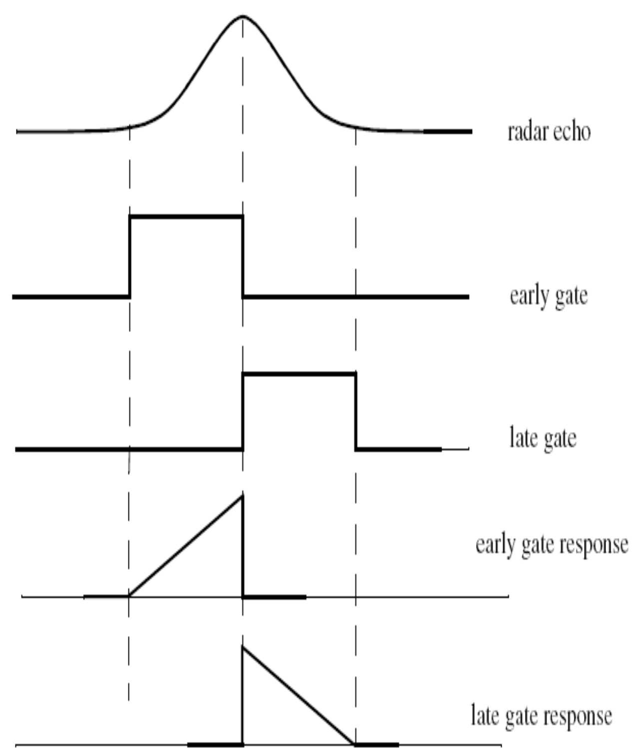
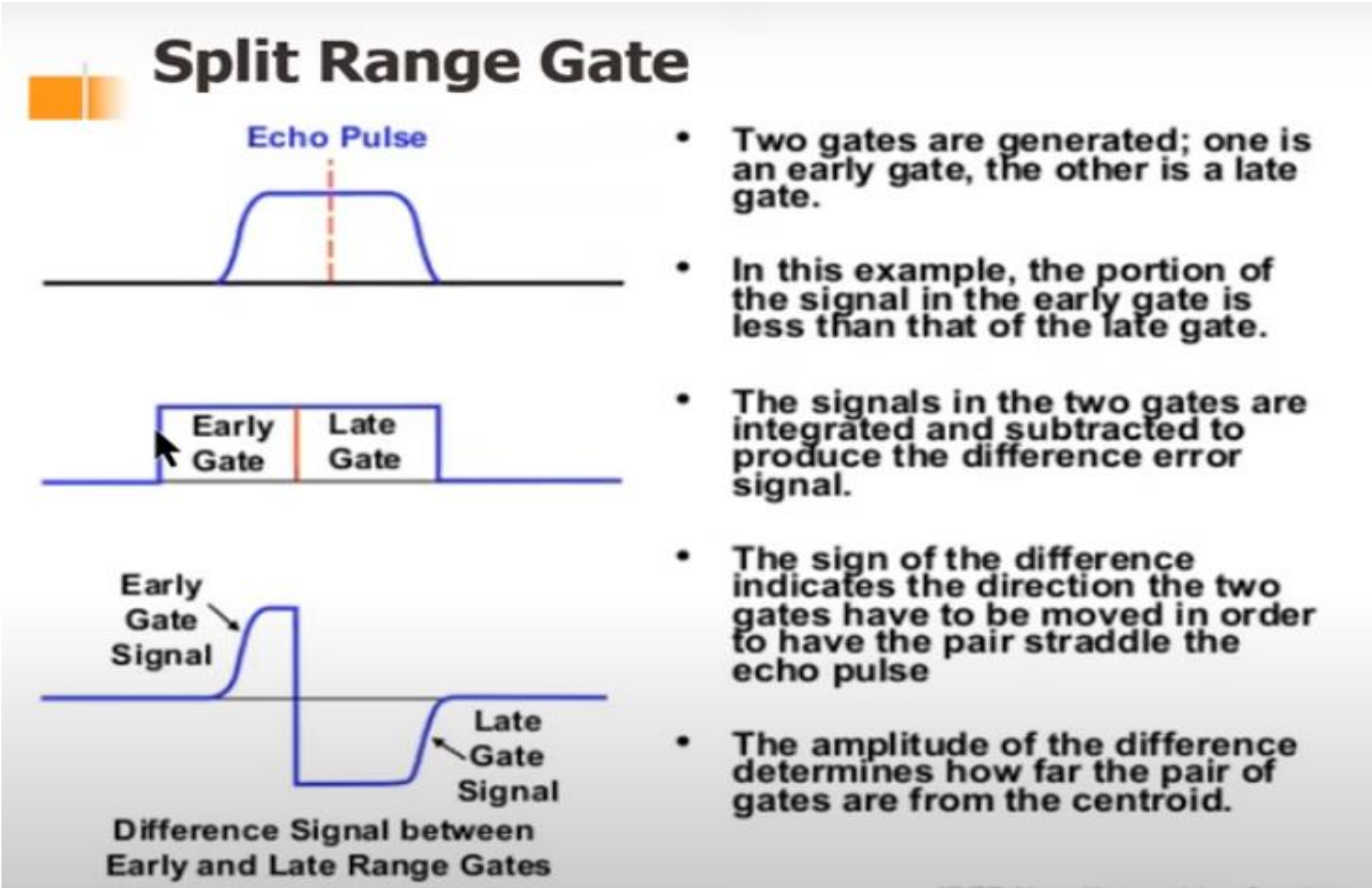


Fig:: Split gate Tracker



Split range gate - Its predict the target movement



MATCHED FILTER

- Matched Filter Is Used Detection Of Radar Signals In Noise.
- The two basic operations performed by the radar are
  1. Detection of the presence of reflecting objects
  2. Extraction of information from the received waveform to obtain target data such as Position ,Velocity & Size.
- A network whose frequency response function maximizes the output peak signal to mean noise power ratio is called a matched filter.

- If a filter produces an output in such a way that it maximizes the ratio of output peak power to mean noise power in its frequency response, then that filter is called Matched filter.
- This is an important criterion, which is considered while designing any Radar receiver but ultimately Noise limits the capability of any receiver.
- In this chapter, let us discuss the frequency response function of Matched filter and impulse response of Matched filter.

Frequency Response Function of Matched Filter

The frequency response of the Matched filter will be proportional to the complex conjugate of the input signal's spectrum. Mathematically, we can write the expression for **frequency response function**, H(f) of the Matched filter as

$$H(f) = G_a S^*(f) e^{-j2\pi f t_1} \longrightarrow 1$$

Where

G_a is the maximum gain of the Matched filter

S(f) is the Fourier transform of the input signal, s(t)

$S^*(f)$ is the complex conjugate of S(f)

t_1 is the time instant at which the signal observed to be maximum

In general, the value of G_a is considered as one. We will get the following equation by substituting G_a =1 in Eq 1.

$$H(f) = S^*(f) e^{-j2\pi f t_1} \longrightarrow 2$$

The frequency spectrum of the received signal may be written as an amplitude spectrum & phase spectrum

$$\mathbf{S(f)= \ |S(f)|}\text{ and }\mathbf{e^{-j\text{Ø}_S(f).}}$$

$$\mathbf{S^*(f)= \ |S(f)|}\text{ and }\mathbf{e^{j\text{Ø}_S(f).}}$$

The matched filter frequency response function may also be similarly written in terms of its amplitude and phase spectrarim

$$H(f)= |\mathbf{H(f)}|\text{and }\mathbf{e^{-j\text{Ø}_m(f).}}$$

Then Eq. **(1)** for the matched filter may be written as

$$\mathbf{H (f) = S^*(f)\, e^{(-j2\pi ft_1)}}$$

$$| H(f) |\, \mathbf{e^{-j\text{Ø}_m(f).}} = | S(f) |\, \mathbf{e^{j\text{Ø}_s(f).}} * \mathbf{e^{-j2\pi ft_1}}$$

$$\mathbf{=}| S(f) |\, \mathbf{e^{-j\text{Ø}_s(f)}* e}$$

By comparing amplitudes and phase in both side

$$\boxed{\begin{array}{l} \mathbf{|H(f)| = |S(f)|} \\ \mathbf{\text{Ø}_m(f) = -\text{Ø}_S(f) + 2\pi ft_1} \end{array}}$$

Thus, the amplitude spectrum of the matched filter is the same as the amplitude spectrum of the signal, but the phase spectrum of the matched filter is the negative of the phase spectrum of the signal plus a phase shift proportional to frequency**.**

The matched filter may also be specified by its impulse response h(t), which is the inverse Fourier transform of the frequency-response function.

2. Impulse Response of Matched Filter

In **time domain**, we will get the o/p, h(t) of Matched filter receiver

By applying the inverse Fourier Transform of the frequency response function, H(f).

$$h(t) = \int_{-\infty}^{\infty} \{G_a S^*(f) e^{-j2\pi f t_1}\} e^{j2\pi f t} df \longrightarrow 3$$

Substitute, Eq - 1 in Eq -3.

$$h(t) = \int_{-\infty}^{\infty} H(f) e^{-j2\pi f t_1} df$$

$$\Rightarrow h(t) = \int_{-\infty}^{\infty} G_a S^*(f) e^{-j2\pi f (t_1 - t)} df \longrightarrow 4$$

We know the following relation.

$$S^*(f) = S(-f) \longrightarrow 5$$

Substitute, Eq- 5 in Eq- 4.

$$h(t) = \int_{-\infty}^{\infty} G_a S(-f) e^{-j2\pi f (t_1 - t)} df$$

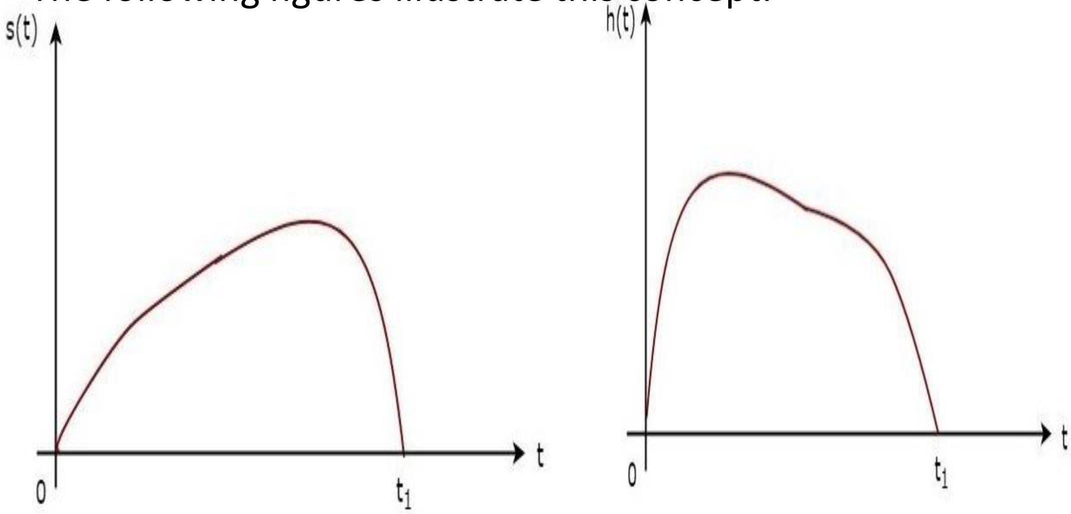
$$\Rightarrow h(t) = \int_{-\infty}^{\infty} G_a S^{(f)} e^{j2\pi f (t_1 - t)} df$$

$$\Rightarrow h(t) = G_a s(t_1 - t) \longrightarrow 6$$

In general, the value of Ga is considered as one. We will get the following equation by substituting Ga=1 in Eq-6.

$$h(t) = s(t_1 - t)$$

- The above equation proves that the **impulse response of Matched filter** is the mirror image of the received signal about a time instant t_1. The following figures illustrate this concept.



- The above equation indicates the impulse respose of a matched filter i.e **time inverse** of the received signal.
- The impulse response of a matched filter is not defined t<0.We must always have $t < t_1$

Derivation of the Matched filter characterstic::

The frequency response function of the matched filter can be derived using either the

1. **Calculus Of Variation::** (Functions that maximize or minimize functional may be found using the Euler–Lagrange equation of the calculus of variations.)
2. **Schwartz Inequality::** The Cauchy–Schwarz inequality is used **to prove that the inner product is a continuous function with respect to the topology induced by the inner product itself**.

$$\left|\int_{\mathbb{R}^n} f(x)\overline{g(x)}\,dx\right|^2 \leq \int_{\mathbb{R}^n} |f(x)|^2\,dx \int_{\mathbb{R}^n} |g(x)|^2\,dx.$$

The frequency response function of linear time invariant

$$H(f) = G_a S^*(f) e^{-j2\pi f t_1}$$

When the input noise is stationary and white noise(which is consider uniform spectral density).

The ratio to be maximized is

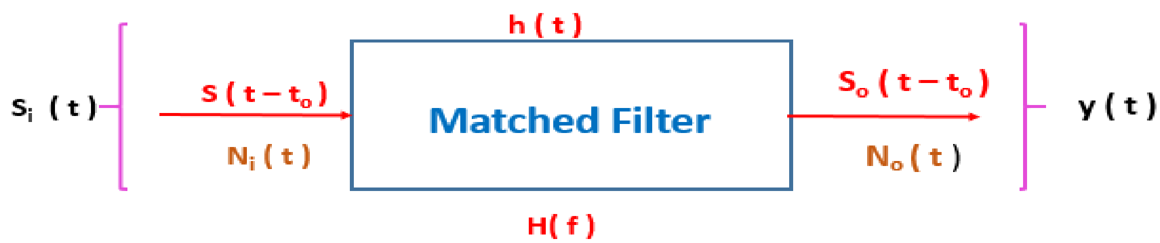
$$R_f = \frac{|s_0(t)|^2_{max}}{N} \quad \text{----------- } 1$$

Where

$|s_o(t)|_{max}$ = Maximum value of output signal voltage

N = Mean noise power at receiver output

The ratio R_f is not quite the same as the SNR which is been considered previously in the Radar Eq.



Note that the **peak power as used here is actually the peak instantaneous power**

The output voltage of a filter with frequency response for H(f) is

$$|s_o(t)| = \left|\int_{-\infty}^{\infty} s(f) * H(f) * e^{j2\pi f t_1}\right| \quad \text{---------- } 2$$

Where s(f) is the Fourier transform of the received signal

The mean output noise power is

$$N=\frac{N_o}{2}\int_{-\infty}^{\infty}|H(f)|^2 \qquad \text{-----------3}$$

Where N_o-Input noise power/ unit bandwidth

The factor $\frac{1}{2}$ appears before the integral because the limit extends from -∞ to ∞, whereas N_o is defined as the noise power per cycle of bandwidth over positive values only.

Substituting Eq's 2 &3 in 1

Assuming that the **Maximum value of $|s_o(t)|^2$ occurs at time t=t_1**,

The ratio R_f becomes

$$R_f=\frac{\left|\int_{-\infty}^{\infty}|s(f)|*H(f)*e^{j2\pi f t_1}\right|^2}{\frac{N_o}{2}\int_{-\infty}^{\infty}|H(f)|^2 df} \qquad \text{----------- 4}$$

Schwartz's. inequality states that if f & g are two complex functions then

$$\left|\int_{\mathbb{R}^n} f(x)\overline{g(x)}\,dx\right|^2 \leq \int_{\mathbb{R}^n}|f(x)|^2\,dx\int_{\mathbb{R}^n}|g(x)|^2\,dx.$$

$$(e^{j2\pi})^2=(\cos 2\pi+j\sin 2\pi)^2=(1+0)^2=1$$

$$(e^{j2\pi})^2=1$$

$$R_f=\frac{\left|\int_{-\infty}^{\infty}|s(f)|\right|^2 df\left|\int_{-\infty}^{\infty}H(f)\right|^2 df}{\frac{N_o}{2}\int_{-\infty}^{\infty}|H(f)|^2\,df}$$

$$R_f=\frac{\left|\int_{-\infty}^{\infty}|s(f)|\right|^2 df}{\frac{N_o}{2}}$$

From the parseval's theorem

$$\left|\int_{-\infty}^{\infty}|s(f)|\right|^2 df = \left|\int_{-\infty}^{\infty}|s(t)|\right|^2 dt = \text{Signal Energy(E)}$$

Therefore we have

$$\text{SNR}=R_f \leq \frac{2E}{N_O}$$

The maximum ratio of the peak signal power to the mean noise power is simply twice the energy 'E' contained in the signal divided by the noise power per hertz of band width N_O .

Where, $N_O = kT_O B_n$

$N_O = kT_O F$

K = Boltzmann's constant

T_o = Standard temperature

B_n = Noise Bandwidth

F= Noise figure

Matched filter with nonwhite noise

- In the derivation of frequency response function of a matched filter the noise is assumed as white noise.it is independent of frequency.
- If this assumption were not true, the filter which maximizes the output SNR would not be the same as the matched filter.
- The input power spectrum of the interfering noise is given by $[N_i(f)]^2$.
- The frequency-response function of the filter which maximizes the output SNR is

$$H(f) = \frac{G_a * S^*(f) * e^{-j2\pi f t_1}}{\frac{N_o}{2}} = \frac{G_a * S^*(f) * e^{-j2\pi f t_1}}{|N_i(f)|^2}$$

When the noise is nonwhite, the filter which maximizes the output SNR is called the NWN (nonwhite noise) matched filter.

Equation above can be written as

$$\mathbf{H\ (\ f\ )} = \frac{1}{\mathbf{N_i}(\mathrm{f})} * \frac{G_a * S^*(f) * e^{-\mathbf{j\,2\,\pi\,ft_1}}}{\mathbf{N_i}(\mathrm{f})}$$

This indicates that the NWN matched filter can be considered as the cascade of two filters.

The first filter, with frequency-response function l/Ni (f), acts to make the noise spectrum uniform, or white. It is sometimes called the whitening filter.

The second is the matched filter, when the input is white noise and a signal whose spectrum is **S (f)/$\mathbf{N_i}$ (f).**

THE MATCHED FILTER AND THE CORRELATION FUNCTION

The convolution function between two signals $x_1(\ t\ )$ and $x_2(\ t\ )$ is defined as

$$\mathrm{y}\ (\ \mathrm{t}\ ) = \int_{-\infty}^{+\infty} x_1(\ \tau\ ) x_1(\ t - \ \tau\ )\, \mathrm{d}\tau$$

Thecross - correlation function between two signals $x_1(\ t\ )$ and $x_2(\ t\ )$ is defined as

$$\mathrm{y}\ (\ \mathrm{t}\ ) = \int_{-\infty}^{+\infty} x_1(\ \tau\ ) x_1(\ \tau - \ t\ )\, \mathrm{d}\tau$$

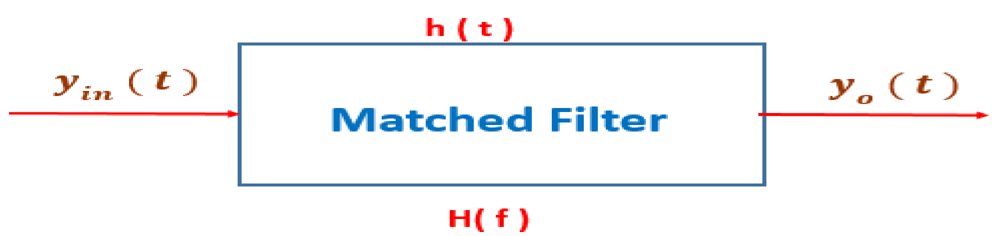


Fig 2: Block diagram of matched filter

From linear filter theory the output $y_o\ (t\ )$of a filter is the convolution of the input

$y_{in}(\ t\ ) = \mathrm{s}\ (\ \mathrm{t}\ ) + n\ (\ t\ )$ and

the filter's impulse response h (t). It may be written as

$$y_o\ (\ t\ ) = \int_{-\infty}^{+\infty} y_{in}(\ \tau\ )\ h\ (\ t - \ \tau\ )\ \mathrm{d}\tau \qquad \ldots.\ (7)$$

Already we know that h **(t) = $\mathbf{G_a}$ s ($\mathbf{t_0}$ - t) ….** For convenience $G_a = 1$ and $t_0 = 0$

So h (t) = s (-t) Then h (t - τ) = s (- t + τ) = s (τ – t)

Sub above value in equation 7 becomes

$$\boxed{\boldsymbol{y_o\ (\ t\ ) = \int_{-\infty}^{+\infty} y_{in}(\ \tau\ )\mathrm{s}\ (\tau - \ \mathrm{t}\ )\ \mathrm{d}\tau}}$$

The output of a matched filter is the cross correlation between the received echo signal $y_{in}(\ t\ )$ and time reversal of transmitted signal s (t).

- When the signal-to-noise ratio large $y_{in}(\ t\ )\approx$ s (t), and the output signal from the matched filter is approximately by the autocorrelation function of the transmitted signal.

CROSS- CORRELATION RECEVIER::

- The output of the matched filter as the cross correlation between the input signal and a delayed replica of the transmitted signal.
- The matched filter and cross-correlation receiver have same mathematical operation
- This implies that the matched-filter receiver can be replaced by a cross-correlation receiver

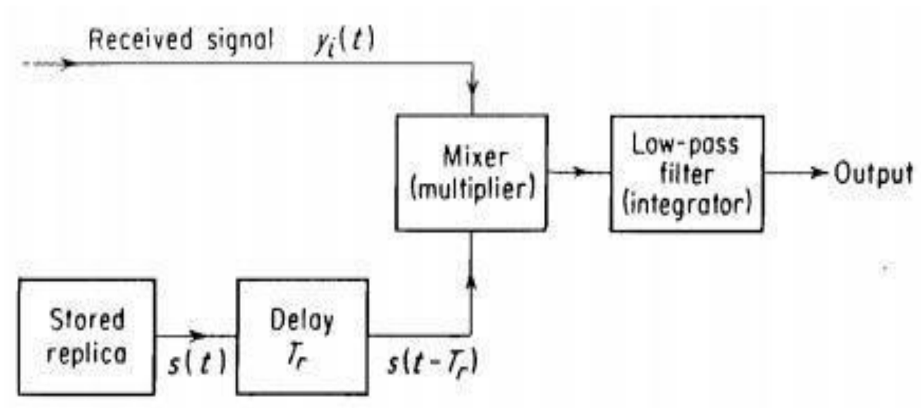


Fig 3:.Block diagram of cross correlation receiver

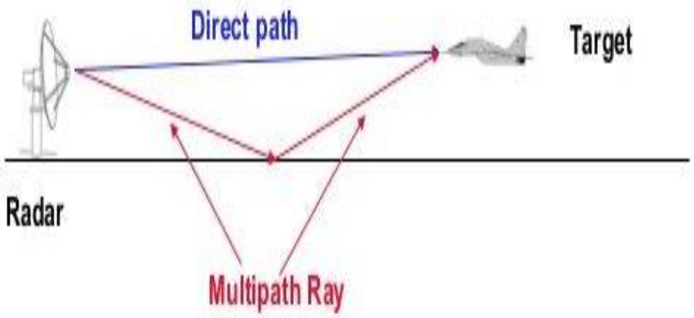
- The input signal $y_i(t)$is multiplied by a delayed replica of the transmitted signal $s(t-T_r)$and the product is passed through a low-pass filter to perform the integration.
- The cross-correlation receiver tests for the presence of a target at only a single time delay T_r.
- Targets at other time delays, or ranges, might be found by varying T_r.
- However, this requires a longer search time. The search time can be reduced by adding parallel channels.

- Since the cross-correlation receiver and the matched-filter receiver are equivalent mathematically, the choice as to which one to use in a particular radar application is determined by which is more practical to implement.
-

EFFICIENCY OF NON-MATCHED FILTERS:

- In practice the matched filter cannot always be obtained exactly. it is appropriate, therefore, to examine the efficiency of non-matched filters compared with the ideal matched filter.
- The measure of efficiency is taken as the peak signal-to-noise ratio from the non-matched filter divided by the peak signal-to-noise ratio (2E/No) from the matched filter.
- Table lists the values of B_τ which maximize the signal-to-noise ratio (SNR) for various combinations of filters and pulse shapes.

WHAT IS LOW ANGLE TRACKING?

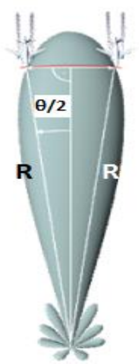


- A radar that tracks low elevation angles illuminates the target via two paths.
- One is the direct path from radar to target and the other is the reflection from the earth's surface.
- This is the equivalent to the radar illuminating two targets, one above the surface, the other below the surface.

- The effect of glint is quite prevalent here, with errors in the measured elevation angle of the target.
- At small angles, (assuming) over a perfectly reflecting surface, the reflection coefficient from the surface is approximately 1.
- Since they are almost in line, the phase is equal to 180 degrees and the magnitude is equal to 1.
- Thus the amplitudes of the incident signal are equal to the amplitude of the reflected signal.

ANGULAR RESOLUTION

Radar angular resolution is the minimum distance between two equally large targets at the same range which radar is able to distinguish and separate to each other.



$$\boldsymbol{S_A = 2RSin\left(\frac{\theta}{2}\right)}$$

S_A = Angular Resolution As A Distance Between Two Targets

θ = Antenna beamwidth (Theta)

R = Slant range aim - antenna [m]

- The angular resolution characteristics of radar are determined by the antenna beamwidth represented by the −3 dB angle θ which is defined by the half-power (−3 dB) points.
- The half-power points of the antenna radiation pattern (i.e., the −3 dB beamwidth) are normally specified as the limits of the antenna beamwidth for the purpose of defining angular resolution; two identical targets at

the same distance are, therefore, resolved in angle if they are separated by more than the antenna −3 dB beamwidth.

- The angular resolution as a distance between two targets is given by following formula:

$$\boxed{S_A = 2RSin\left(\frac{\theta}{2}\right)}$$

- The angular resolution of targets on an analog PPI-scope,
- In practical terms, is dependent on the operator being able to distinguish the two targets involved.
- Systems having Target-Recognition features can improve their angular resolution.
- Cause such systems are able to compare individual Target-Pulse-Amplitudes.

RANGE AMBIGUITY RESOLUTION

It is a technique used with medium Pulse repetition frequency (PRF) radar to obtain range information for distances that exceed the distance between transmit pulses.

This signal processing technique is required with pulse-Doppler radar.

The raw return signal from a reflection will appear to be arriving from a distance less than the true range of the reflection when the wavelength of the pulse repetition frequency (PRF) is less than the range of the reflection.

This causes reflected signals to be **folded**, so that the apparent range is a **modulo function** of true range.

Range aliasing occurs when reflections arrive from distances that exceed the distance between transmit pulses at a specific pulse repetition frequency (PRF).

Range ambiguity resolution is required to obtain the true range when the measurements are made using a system where the following inequality is true.

$$\text{Distance} > \left(\frac{c}{2 \times \text{PRF}}\right)$$

Here c is the signal speed, which for radar is the speed of light.

The range measurements made in this way produces a modulo function of the true range.

$$\text{Apparent Range} = (\text{True Range}) \mod \left(\frac{c}{2 \times \text{PRF}}\right)$$

To find the true range, the radar must measure the apparent range using two or more different PRF.

Suppose a two PRF combination is chosen where the distance between transmit pulses (pulse spacing) is different by the pulse width of the transmitter.

$$\textit{Two PRF Combination} \begin{cases} \textit{Pulse Spacing (Ambiguous Range)} = \frac{1}{\text{PRF}} \\ \frac{1}{\text{PRF}_A} - \frac{1}{\text{PRF}_B} = \pm \textit{Transmit Pulse Width} \end{cases}$$

Each transmit pulse is separated in distance the ambiguous range interval. Multiple samples are taken between transmit pulses.

If the receive signal falls in the same sample number for both PRF, then the object is in the first ambiguous range interval.

If the receive signal falls into sample numbers that are different by one, then the object is in the second ambiguous range interval.

If the receive signal falls into sample numbers that are different by two, then the object is in the third ambiguous range interval.

The general constraints for range performance are as follows.

- Each sample is processed to determine if there is a reflected signal (detection). This is called signal detection.
- The detection made using both PRF can be compared to identify the true range. This comparison depends upon the transmitter duty cycle (the ratio between on and off).
- The duty cycle is the ratio of the width of the transmit pulse width (T) and the period between pulses(1/PRF)
- $$\begin{matrix} Transmitter \\ Characteristics \end{matrix} \begin{cases} Duty\ Cycle = \mathrm{PRF} \times Transmit\ Pulse\ Width \\ Pulse\ Spacing = \left(\frac{c}{\mathrm{PRF}}\right) \end{cases}$$
- Pulse-Doppler can reliably resolve true range at all distances less than the **Instrumented Range**.
- The optimum pair of PRF used for a pulse-Doppler detection scheme must be different by a minimum of **PRF*Duty cycle**.
- This makes the range of each PRF different by the width of the sample period.

Example::

The difference between the sample numbers where reflection signal is found for these two PRF will be about the same as the number of the ambiguous range intervals between the radar and the reflector (i.e.: if the reflection falls in sample 3 for PRF 1 and in sample 5 for PRF 2, then the reflector is in ambiguous range interval 2=5-3).

$$\begin{matrix} Instrumented \\ Range \end{matrix} \begin{cases} Minimum\ Sample\ Width = \left(\frac{Duty\ Cycle}{\mathrm{PRF}}\right) \\ Maximum\ Distance = \left(\frac{Pulse\ Spacing}{Sample\ Width}\right) = \left(\frac{c}{Sample\ Width \times 2 \times \mathrm{PRF}^2}\right) \\ Samples\ Per\ Transmit\ Pulse = \left(\frac{1}{Minimum\ Sample\ Width} - 1\right) \end{cases}$$

THIS TECHNIQUE HAS TWO LIMITATIONS.

1. **Blind Zones**
2. **Multiple Targets**

- The process described above is slightly more complex in real systems because more than one detection signal can occur within the radar beam.
- The pulse rate must alternate rapidly between at least 4 different PRF to handle these complexities.

1. Blind Zones

- Each individual PRF has blind ranges, where the transmitter pulse occurs at the same time as the target reflection signal arrives back at the radar.
- Each individual PRF has blind velocities where the velocity of the aircraft will appear stationary.
- This causes scalloping, where the radar can be blind for some combinations of speed and distance.
- A four PRF scheme is generally used with two pair of PRF for the detection process so that blind zones are eliminated.
- The antenna must dwell in the same position for at least three different PRF.
- This imposes a minimum time limit for the volume to be scanned.

2. Multiple Targets

- Multiple aircraft within the radar beam that are separated by over 500 meters introduces additional degrees of freedom that requires additional information and additional processing.

- This is mathematically equivalent to multiple unknown quantities that require multiple equations.
- Algorithms that handle multiple targets often employ some type of clustering to determine how many targets are present.
- Doppler frequency shift induced by changing transmit frequency reduces unknown degrees of freedom.
- Sorting detections in order of amplitude reduces unknown degrees of freedom.
- Ambiguity resolution relies on processing detections with similar size or speed together as a group.