UNIT- 4

DIGITAL PASSBAND TRANSMISSION

PASS BAND TRANSMISSION MODEL:

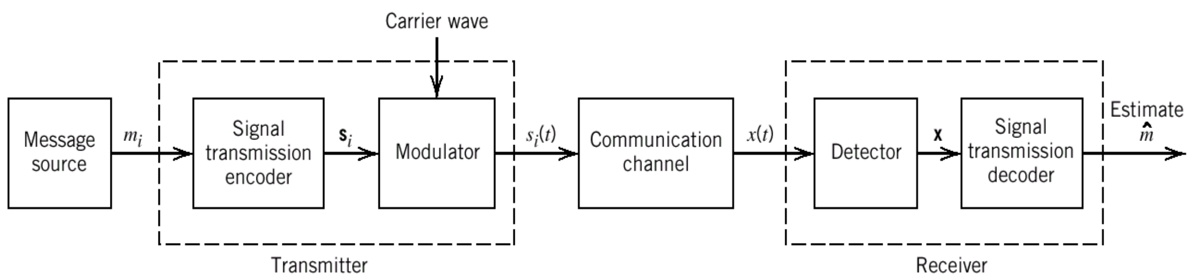


Fig: Functional model of pass band transmission model

First, the message source emits M symbols on symbol for every τ seconds, denoted by $m_1, m_2, \ldots m_N$. These symbols are transmitted with equal probability i.e.,

$$p_i = p(m_i)$$

$$= \frac{1}{M} \quad for\ all \quad i$$

These symbols are given to signal transmission decoder, it produces a vector S_i made up of N real elements$[N \le M]$. With Vector S_i as the input, modulator then constructs a distinct signal $S_i(t)$ of duration τ seconds. The signal $S_i(t)$ is necessarily an energy signal given as

$$E_i = \int_0^t S_i^2(t)dt, \quad i = 1,2,\ldots..M$$

S_i modulates with sinusoidal carrier. The energy signal $S_i(t)$ transmits over the channel. The channel has following characteristics.

- Channel is linear, with a sufficient bandwidth to transmit $S_i(t)$ without distortion.
- The noise added to the channel is AWGN with a process of zero mean and power spectral density $\frac{N_0}{2}$

The receiver which consist of a detector followed by a signal transmission decoder, performs two functions.

- It reverses the operations performed in the transmitter

It minimizes the effect of channel noise on the estimate of $\hat{m}$ from the transmitted symbol m_i.

GEOMETRIC REPRESENTATION OF SIGNALS:

The main idea of geometric representation of signals is to represent any set of M signals $\{s_i(t)\}$ as a linear combination of N Orthonormal basis functions where $N \leq M$, then the real valued energy signals $s_1(t)$, $s_2(t), \ldots s_M(t)$ each of duration T seconds , written as

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \qquad \begin{cases} 0 \leq t \leq T \\ i = 1, 2, \ldots, M \end{cases} \qquad \text{------1}$$

Where the coefficients of the expression are expressed as

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)\, dt, \qquad \begin{cases} i = 1, 2, \ldots, M \\ j = 1, 2, \ldots, N \end{cases} \qquad \text{-----2}$$

The real valued basis functions are Orthonormal it means,

$$\int_0^T \phi_i(t)\phi_j(t)\, dt = \delta_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$$

In the above eqn first condition state that the basis functions have unit energy. The second condition states that the basis functions are ortho normal to each other.

The set of coefficients $\{S_{ij}\}_{j=1}^{N}$ is naturally viewed as N dimensional vector, denoted by s_i. It has one to one relationship with the transmitted signal $s_i(t)$.

Where ,

$$\mathbf{s}_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix}, \qquad i = 1, 2, \ldots, M$$

Given the N elements of the vector S_i operating as input, we use fig(a) generating the signal $S_i(t)$. It is derived from eqn(1). It consists of N bank multipliers, with each multiplier have one basis function followed by summer, this scheme is called as synthesizer.

Given the signal $S_i(t)$, i=1, 2, …..M operating as input, we use fig(b) generating signal vector. it is derived from eqn(2) It consists of N product integrators (or) correlators, with each multiplier have own basis function. This scheme is called analyzer.

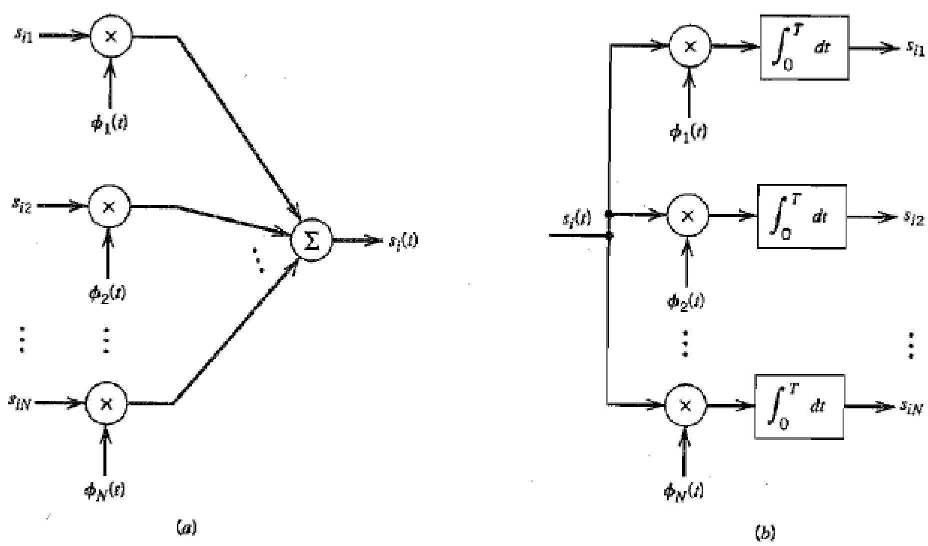


FIGURE (*a*) Synthesizer for generating the signal $s_i(t)$. (*b*) Analyzer for generating the set of signal vectors $\{s_i\}$.

Consider an N dimensional Euclidian space, having M points corresponds to signal vector, N mutually perpendicular axes labeled $\phi_1, \phi_2, \ldots, \phi_N$. This N dimensional Euclidian space is called Signal Space.

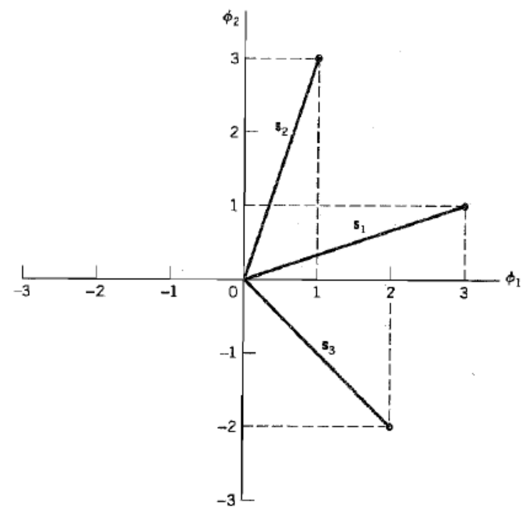


Fig : Geo metric representation of signals for N= 2 & M=3

In N dimensional Euclidian space, we may define length of the vectors, Distance between vectors and angles between vector.

The squared length of any signal vector is defined as the inner product or dot product of S_i itself

$$\begin{aligned} \| \mathbf{s}_i \|^2 &= \mathbf{s}_i^T \mathbf{s}_i \\ &= \sum_{j=1}^{N} s_{ij}^2, \qquad i = 1, 2, \ldots, M \end{aligned}$$

The relation between energy of the signal & representation of vector is derived as

$$E_i = \int_0^T s_i^2(t)\, dt$$

$$E_i = \int_0^T \left[\sum_{j=1}^{N} s_{ij}\phi_j(t) \right] \left[\sum_{k=1}^{N} s_{ik}\phi_k(t) \right] dt$$

$$E_i = \sum_{j=1}^{N} \sum_{k=1}^{N} s_{ij} s_{ik} \int_0^T \phi_j(t)\phi_k(t) dt$$

For j=k,

$$E_i = \sum_{j=1}^{N} s_{ij}^2$$
$$= \| \mathbf{s}_i \|^2$$

In case of pair of signal $s_i(t)$ and $s_k(t)$, we have

$$\int_0^T s_i(t) s_k(t)\, dt = \mathbf{s}_i^T \mathbf{s}_k$$

The squared distance between the points represented by the signals $s_i(t)$ & $s_k(t)$ is denoted as

$$\| \mathbf{s}_i - \mathbf{s}_k \|^2 = \sum_{j=1}^{N} (s_{ij} - s_{kj})^2$$
$$= \int_0^T (s_i(t) - s_k(t))^2 dt$$

The angle between two signal vectors s_i & s_k is expressed as

$$\cos \theta_{ik} = \frac{\mathbf{s}_i^T \mathbf{s}_k}{\| \mathbf{s}_i \| \, \| \mathbf{s}_k \|}$$

The cosine of the angle θ_{ik} is equal to inner product of these two vectors divided by the product of their individual norms.

GRAM SCHMIDT ORTHOGONALIZATION PROCEDURE:

In case of geometric representation of signals, the M energy signals are linear combination of N orthonormal basis functions, this one can be verified mathematically by using Gram-Schmidt orthogonalization.

Let us consider M energy signals $S_1(t), S_2(t) \ldots\ldots S_M(t)$. By using $S_1(t)$, the first basis function is defined as,

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} \quad \text{-------1}$$

$$s_1(t) = \sqrt{E_1}\phi_1(t)$$
$$= s_{11}\phi_1(t)$$

By using $S_2(t)$, the coefficient S_{21} is defined as,

$$s_{21} = \int_0^T s_2(t)\phi_1(t)dt \qquad \text{-----3}$$

We may thus introduce new a new intermediate function,

$$g_2(t) = s_2(t) - s_{21}\phi_1(t) \qquad \text{------4}$$

Which is orthogonal to $\phi_1(t)$,over the interval $0 \le t \le T$, $\phi_2(t)$ is defined as

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)dt}} \qquad \text{--------5}$$

Substituting eqn (4) in eqn (5), We get

$$\phi_2(t) = \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{E_2 - s_{21}^2}} \qquad \text{-------6}$$

It is clear that from eqn 5,

$$\int_0^T \phi_2^2(t)dt = 1 \qquad \text{------7}$$

From eqn 6,

$$\int_0^T \phi_1(t)\phi_2(t)dt = 0 \qquad \text{------8}$$

The set of basis functions defined as,

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)dt}}, \qquad i = 1, 2, \ldots, N$$

RESPONSE OF BANK OF CORRELATORS IN NOISE:

Suppose that the input to the bank of N product integrators or correlators is not the transmitted signal $S_i(t)$ but actually it is a received signal x(t).From the equivalent model of channel it is,

$$x(t) = s_i(t) + w(t), \qquad \begin{cases} 0 \le t \le T \\ i = 1, 2, \ldots, M \end{cases} \text{------1}$$

The output of the correlator j, is the sample function of RV x_j is,

$$\begin{aligned} x_j &= \int_0^T x(t)\phi_j(t)dt \\ &= s_{ij} + w_j, \qquad j = 1, 2, \ldots, N \text{---------2} \end{aligned}$$

Consider a new random variable X'(t) whose sample function x'(t) is related to the received signal x(t) as follows,

$$x'(t) = x(t) - \sum_{j=1}^{N} x_j\phi_j(t) \text{------ 3}$$

Substitute 1 and 2 eqns in 3, then we get

$$\begin{aligned} x'(t) &= s_i(t) + w(t) - \sum_{j=1}^{N} (s_{ij} + w_j)\phi_j(t) \\ &= w(t) - \sum_{j=1}^{N} w_j\phi_j(t) \\ &= w'(t) \quad \text{----------4} \end{aligned}$$

The sample function x'(t) therefore depends only on the channel noise w(t). From 3&4 eqns,

$$x(t) = \sum_{j=1}^{N} x_j \phi_j(t) + x'(t)$$
$$= \sum_{j=1}^{N} x_j \phi_j(t) + w'(t) \text{-------} 5$$

From the above eqn, w'(t) must be included in the right to preserve the equality in eqn 5.

Note:

- ✓ The signal $s_i(t)$, i=1, 2, …M is applied to a bank of correlators, with a common input and supplied with a suitable set of N orthogonal basis functions, N. The resulting output defines the signal vector s_i.
- ✓ We represent each signal $s_i(t)$ as a point in the Euclidian space, $N \leq M$ (referred to as transmitted signal point or message point).The set of message points corresponding to the set of transmitted signals $s_i(t)$ {i = 1 to M} is called ***signal constellation***.
- ✓ The received signal x(t) is applied to a bank of N correlators and the correlator outputs define the observation vector x.
- ✓ On the receiving side the representation of the received signal x(t) is complicated by the additive noise w(t).
- ✓ The vector x differs from the vector s_i by the noise vector w.
- ✓ However only the portion of it which interferes with the detection process is of importance to us, and this is fully described by w(t).
- ✓ Based on the observation vector 'x' we may represent the received signal x(t) by a point in the same Euclidian space used to represent the transmitted signal.

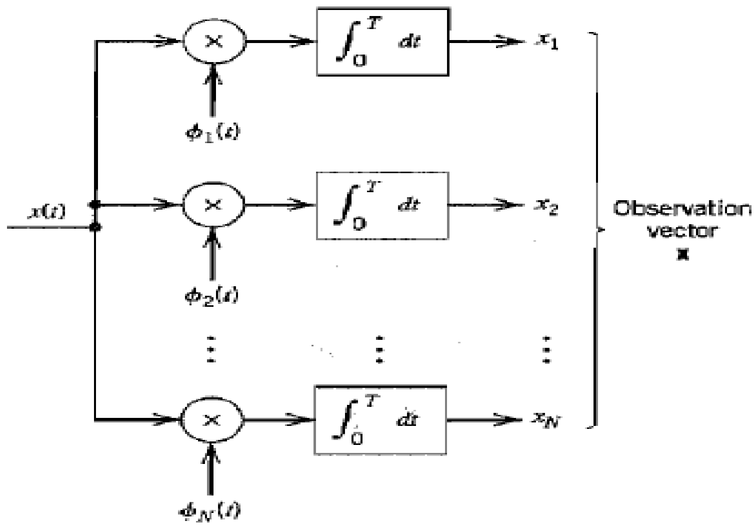
CORRELATION RECEIVER:

The optimum receiver consists of two sub systems

i) detector or demodulator

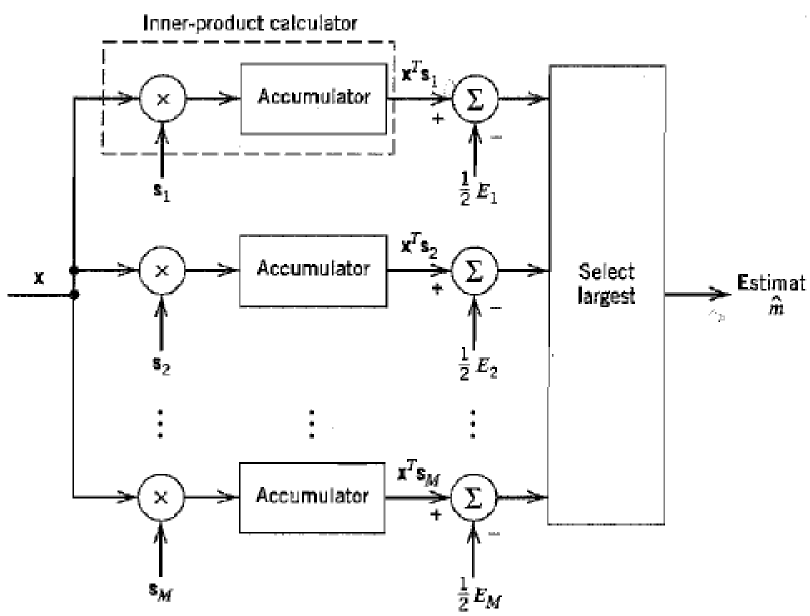
ii) signal transmission decoder

Detector or demodulator is shown in fig a, in consists of N-bank of correlators, each one having one orthogonal basis function that are locally generated. This bank of correlators operates on received signal x(t) and produce the observation vector x.



(a)

The second part of the receiver is signal transmission decoder, it is shown in fig b. It is implemented in the form of maximum likelihood decoder that operates on the observation vector x to produce an estimate $\hat{m}$ of the transmitted symbol m_i in the way to minimize the avg probability error.



(b)

First the observation vector x are first multiplied by the individual signal vectors S_1, S_2-----S_M and the resulting products are successively summed by accumulators to produce corresponding set of inner products. Next these inner products are subtracted by transmitted signal energies. Finally the largest in the resulting set of numbers is selected and takes the appropriate decision on the transmitted message.

The optimum receiver in eqn 1 & 2 is commonly referred to as correlation receiver.

PROBABILITY OF ERROR:

The average probability of symbol error, P_e is

$$\begin{aligned} P_e &= \sum_{i=1}^{M} p_i \, P(\mathbf{x} \text{ does not lie in } Z_i | m_i \text{ sent}) \\ &= \frac{1}{M} \sum_{i=1}^{M} P(\mathbf{x} \text{ does not lie in } Z_i | m_i \text{ sent}) \\ &= 1 - \frac{1}{M} \sum_{i=1}^{M} P(\mathbf{x} \text{ lies in } Z_i | m_i \text{ sent}) \end{aligned}$$

The above eqn is rewritten interms of likelihood function as follows

$$P_e = 1 - \frac{1}{M} \sum_{i=1}^{M} \int_{Z_i} f_{\mathbf{X}}(\mathbf{x} | m_i) \, d\mathbf{x}$$

The above equation represents avg probability of error interms of likelihood functions for an N dimensional vector.

DETECTION OF SIGNALS WITH UNKNOWN PHASE:

Consider a digital communication system in which the transmitted signal equals

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_i t) \qquad 0 \le t \le T \qquad ----1$$

Where E is the signal energy, T is the duration of the signaling interval, and the f_i *is* an integral multiple of 1/2T. When no provision is made to phase synchronize the receiver with transmitter, the received signal will, for AWGN channel, be of the form

$$x(t) = \sqrt{\frac{2E}{T}}\cos(2\pi f_i\, t + \theta) \; + w(t) \quad 0 \le t \le T \qquad ----2$$

Where *w(t)* is the sample function of a white Gaussian noise process of zero mean and power spectral density $N_0/2$. The phase θ is unknown, and it is usually considered to be the sample value of a random variable uniformly distributed between 0 and 2π radians. This implies a complete lack of knowledge of the phase. A digital communication system characterized in this way is said to be non-coherent.

From eq. 2, the received signal, the output of the associated correlators in the receiver will be a function of unknown phase θ. From trigonometric identity, we may rewrite equation 2 in the expanded form

$$x(t) = \sqrt{\frac{2E}{T}}\cos\theta\cos(2\pi f_i t) - \sqrt{\frac{2E}{T}}\sin\theta\sin(2\pi f_i t) \; + w(t)$$

$$0 \le t \le T \qquad ----3$$

Suppose that the received signal x(t) is applied to a pair of correlators, we assume that one correlators is supplied with the reference signal $\sqrt{\frac{2}{T}}\cos(2\pi f_i t)$ and other is supplied with the reference signal $\sqrt{\frac{2}{T}}\sin(2\pi f_i t)$

For the both correlators, the observation interval is $0 \le t \le T$. Then in the absence of noise we find that the first correlator output equals $\sqrt{E}\ \cos\theta$ and second correlator output equals $-\sqrt{E}\ \sin\theta$. The dependence on the unknown phase θ may be removed by summing the squares of the two correlator outputs, and then taking the square root of the sum. Thus when the noise w(t) is zero, the result of these operations is simply $\sqrt{E}$, which is independent of phase θ. So, for the detection of a sinusoidal signal of arbitrary phase, and which is corrupted by an additive white Gaussian noise (which is in eq.2), we can use "Quadrature receiver ", this receiver is optimum in the sense that it realizes detection of signals with the minimum probability of error.

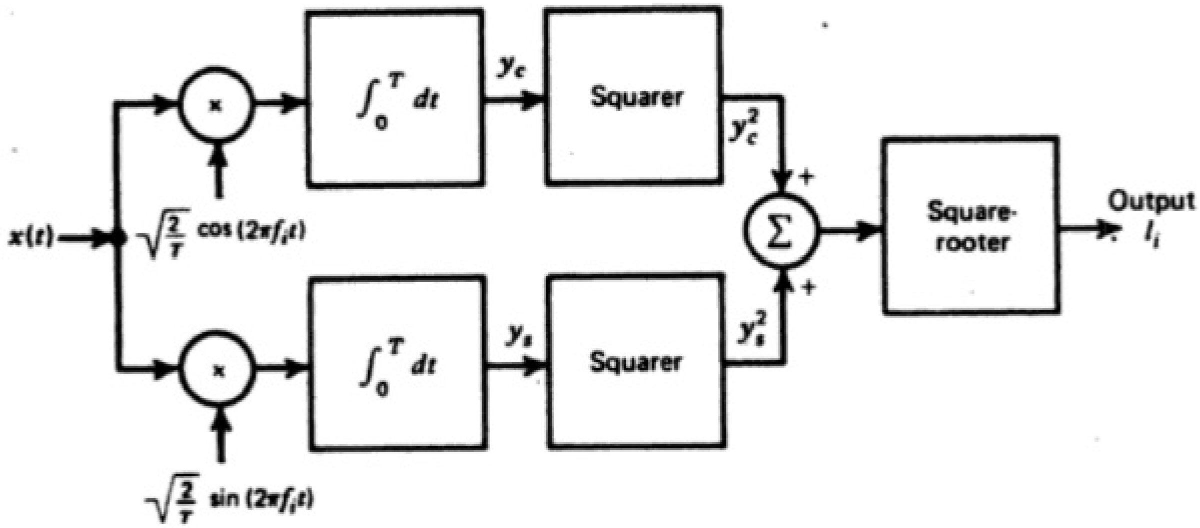


Fig: Quadrature receiver