Code: 20A54302 **R20**

B.Tech II Year I Semester (R20) Supplementary Examinations April/May 2024

COMPLEX VARIABLES & TRANSFORMS

(Common to EEE & ECE)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Show that f(z) = z^3 is analytic for all z. 2M

(b) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. 2M

(c) State (i) Cauchy integral theorem, (ii) Liouvillies theorem. 2M

(d) Determine the poles of the function $\frac{z}{cosz}$. 2M

(e) Find: $L\{t^{3}/_{2}\}$. 2M

(f) If f(t) is a periodic function then find $L\{f(t)\}$. 2M

(g) State Dirichlet's conditions. 2M

(h) Define Odd and Even function with an example each. 2M

(i) State Fourier integral theorem. 2M

(j) If Z[f(n)] = F(z) then find z[a^{-n} f(n)]. 2M

PART – B

(Answer all the questions: 05 X 10 = 50 Marks)

2 Find the analytic function whose imaginary part is $e^x(xsiny + ycosy)$. 10M

OR

3 Find the bilinear transformation which maps the points z = 1, i, -1 onto the points w = i, 0, -i. Also find the image of $|z|<1$. 10M

4 (a) Using Cauchy's integral formula, evaluate $\int_c \frac{z}{(z-1)(z-2)^2} dz$. 5M

(b) Find the residue of $\frac{ze^{zt}}{(z-3)^2}$ at its poles. 5M

OR

5 Using the method of contour integration, Prove that $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{a+b}$ $a > 0, b > 0, a \neq b$. 10M

6 (a) Find $L\{tsinat\}$. 5M

(b) Find $L^{-1}\left\{\frac{s^2}{(s+1)(s+2+(s+3)}\right\}$. 5M

OR

7 (a) Using convolution theorem find $L^{-1}\{\frac{1}{(s+1)(s^2+4)}\}$. 5M

(b) Using Laplace transform, solve (D^2 + 4D + 5) y = 5, given that y(0) = 0, y''(0) = 0. 5M

8 Find a Fourier expansion for $f(x) = x + x^2, -\pi \leq x \leq \pi$ hence find $\sum_1^{\infty} \frac{1}{n^2}$. 10M

OR

9 Find a Fourier sine series expansion of $f(x) = x(\pi - x), 0 < x < \pi$. Hence Find $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots$.... 10M

10 (a) Find the Fourier transform of $\frac{1}{x}$. 5M

(b) Find the Fourier Sine transform of e^{-ax}. 5M

OR

11 (a) Find Z(2.3^n + 5n) and deduce Z[2.3^{n+3} + 5(n+3)] using shifting theorem. 5M

(b) Find the inverse Z-transform of $\frac{Z}{(Z-1)(Z^2+1)}$. 5M

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Code: 20A54302

R20

B.Tech II Year I Semester (R20) Supplementary Examinations August/September 2023

COMPLEX VARIABLES & TRANSFORMS

(Common to EEE & ECE)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Describe the harmonic function with suitable example. 2M

(b) Discuss the continuity of a complex function. 2M

(c) State the Taylor's series for a function $f(z)$. 2M

(d) Obtain the residue of $f(z)=\frac{z-3}{z(z^2+1)}$ at a simple pole $z=0$. 2M

(e) Compute the Laplace transform of $f(t)=e^{3t}+\sin 5t$. 2M

(f) Obtain the inverse Laplace transform of $F(s)=\frac{1}{s^2-a^2}$. 2M

(g) Write Dirichlet Conditions for the existence of Fourier series. 2M

(h) Write Fourier series for Even and Odd Numbers. 2M

(i) Define Z-transform and discuss the linear property. 2M

(j) State Fourier integral theorem of f(x). 2M

PART – B

(Answer all the questions: 05 X 10 = 50 Marks)

2 Suppose $w=\phi+i\psi$ represents the complex potential function for an electric field and $\psi=x^2-y^2+\frac{x}{x^2-y^2}$. Determine the function ϕ. 10M

OR

3 If $f(z)$ is an analytic function with constant modulus then show that $f(z)$ is constant. 10M

4 Evaluate $\frac{z-3}{z^2+2z+5}dz$ where 'c' is $|z+1-i|=2$ using Cauchy's integral formula. 10M

OR

5 Obtain the Taylors expansion of; 10M

$(i)\ f(z)=\frac{1}{(z+1)^2}$ about the point $z=-i$.

$(ii)\ f(z)=\frac{2z^3+1}{z^2+z}$ about the point $z=i$.

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6 Find the Laplace transform of $f(t)=\frac{\cos at-\cos bt}{t}+t\sin at.$ 10M

OR

7 Solve by the method of transforms, the equation; 10M

$y'''+2y''-y'-2y=0,\ \ y(0)=y'(0)=0$ and $y''(0)=6.$

8 Find the Fourier series for the function: 10M

$f(x)$ = - π , - π < x < 0;
= x, 0 < x < π.

OR

9 Find the Fourier series expansion of $f(x)$ = - $x^{2,}$ - π ≤ x ≤ π. Deduce the series 10M
$1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots=\ \frac{\pi^2}{12}$.

10 Find the Fourier transform *f(x)* given by 10M

$f(x)=\begin{cases}1-x^2, & |x|\le 1\\ 0, & |x|>1\end{cases}$ Hence evaluate $\int_0^\infty \frac{x\cos x-\sin x}{x^3}\cos\frac{x}{2}dx.$

OR

11 Determine the inverse Z-transform of $\frac{2z}{(z-1)(z^2+1)}$. 10M

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