UNIT-IV

Antenna Arrays

Introduction: In the previous chapter, the radiation characteristics of single-element antennas were discussed and analyzed. Usually the radiation pattern of a single element is relatively wide, and each element provides low values of directivity (gain). In many applications it is necessary to design antennas with very good directional characteristics (very high gains) to meet the demands of long distance communication. This can be accomplished by the antenna arrays.
The antenna array is defined as the group of identical elements that are geometrically arranged and excited properly to get greater directivity in the desired direction.

The antenna array is said to be linear if the antenna elements are arranged along a straight line. The linear antenna array is said to be uniform linear array if all the elements fed with a currents of equal amplitude with progressive phase shift along the line.

The total field of the array is determined by the vector addition of the fields radiated by the individual elements. To provide very good directional patterns, it is necessary that the fields from the elements of the array interfere constructively (add) in the desired directions and interfere destructively (cancel each other) in the remaining space.
In an array of identical elements, there are at least five controls that can be used to shape the overall pattern of the antenna. These are:
1. the geometrical configuration of the overall array (linear, circular, rectangular, spherical, etc.)
2. the relative displacement between the elements
3. the excitation amplitude of the individual elements
4. the excitation phase of the individual elements
5. the relative pattern of the individual elements

The antenna arrays are used for personal, commercial, mobile communication, and military applications. They can provide the capability of a steerable beam (radiation direction change) as in smart antennas.

Array of point sources: The array of point sources is nothing but the array of an isotropic radiator occupying zero volume. This approach is of great value since the pattern of any antenna can be regarded as produced by an array of point sources.
Arrays of Two Isotropic Point Sources:
Let us consider the array of two isotropic point sources, with a distance of separation d between them.

Two Isotropic Point Sources of Same Amplitude and Phase:

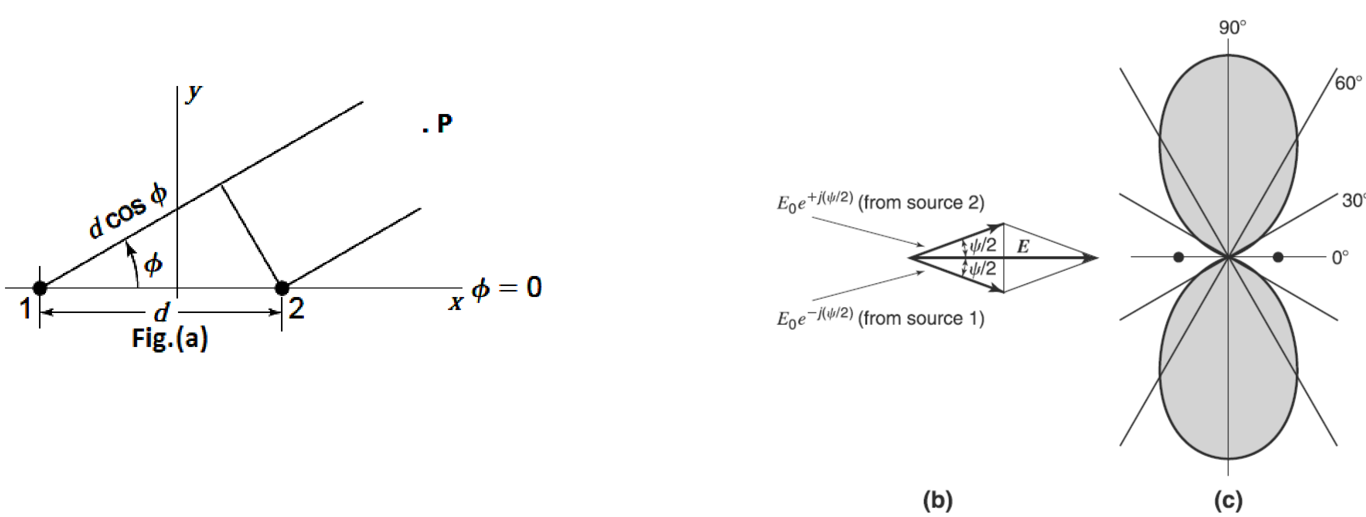


Fig. 1 (a) Two isotropic point sources separated by a distance *d*. (*b*) Vector addition (*c*) Field pattern

Let the two point sources, 1 and 2, be separated by a distance d and located symmetrically with respect to the origin of the coordinates as shown in Fig.1.(a) . The two isotropic point sources having equal amplitudes and exciting in the same phase. The origin of the coordinates is taken as the reference for phase. Then at a distant point in the direction ϕ the field from source 1 is delayed by $\frac{\psi}{2}$ while the field from source 2 is advanced by $\frac{\psi}{2}$.

The total field at a large distance *r* in the direction ϕ is then

$$\mathbf{E = E_0 e^{-j\psi/2} + E_0 e^{+j\psi/2}} \quad \text{-------------(1)}$$

where $\boldsymbol{\psi = \beta d\cos\phi}$ and the amplitude of the field components at the distance r is given by E_0.

The first term in (1) is the component of the field due to source 1 and the second term is the component due to source 2. Equation (1) may be rewritten as

$$\mathbf{E = 2E_0 \frac{e^{-j\psi/2} + e^{+j\psi/2}}{2}} \quad \text{------------(2)}$$

$$\mathbf{E = 2E_0 \cos\frac{\psi}{2} = 2E_0 \cos\left(\frac{\beta d}{2}\cos\phi\right)} \quad \text{--------(3)}$$

The phase of the total field E does not change as a function of ψ.

To normalize (3), that is, make its maximum value unity, set $2E_0 = 1$. If d is $\lambda/2$. Then $\boldsymbol{\beta d = \pi}$

$$\boxed{\mathbf{E = \cos\left(\frac{\pi}{2}\cos\phi\right)}}$$

The field pattern is a bidirectional figure-of-eight with maxima along the *y* axis as shown in fig.

The two sources for this case may be described as a simple “broadside” type of array.

Two Isotropic Point Sources of Same Amplitude but Opposite Phase: This case is identical with the one we have just considered except that the two sources are in opposite phase instead of in the same phase. Let the sources be located as in Fig.1 (*a*) . Then the total field in the direction ϕ at a large distance *r* is given by

$$\mathbf{E = -E_0 e^{-j\psi/2} + E_0 e^{+j\psi/2}} \quad \text{-------------(1)}$$

$$\mathbf{E = 2jE_0 \sin\frac{\psi}{2} = 2jE_0 \sin\left(\frac{\beta d}{2}\cos\phi\right)} \quad \text{--------(2)}$$

The phase reversal of one of the sources in Case 2 results in a 90◦ phase shift of the total field. Thus, putting $2jE_0 = 1$ and considering the special case of $d = \lambda/2$, Equation(2) becomes

$$\boxed{\mathrm{E} = \mathbf{\sin\left(\frac{\pi}{2}\cos\phi\right)}}$$

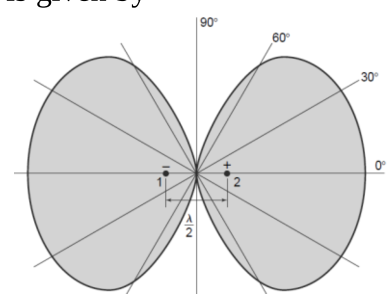


Fig.2. Field pattern of case 2

The pattern is a figure-of-eight with the maximum field in the same direction as the line joining the sources as shown in fig.2. The two sources, in this case, may be described as a simple type of “end-fire” array.

Two Isotropic Point Sources of Equal Amplitude and Any phase Difference:

Let us consider the case of two isotropic point sources of equal amplitude but of any phase difference δ. The total phase difference ψ between the fields from source 2 and source 1 at a distant point in the direction as shown in fig 1.(a) is given by

$$\boldsymbol{\psi = \beta d\cos\phi + \delta} \quad \text{-------- (1)}$$

The origin of the coordinates is taken as the reference for phase. The phase of the field from source 1 at a distant point is given by $-\psi/2$ and that from source 2 by $+\psi/2$. The total field is then

$$\mathbf{E = E_0 e^{-j\psi/2} + E_0 e^{+j\psi/2}} \quad \textbf{------ (2)}$$

$$\mathbf{E = 2E_0 \cos\frac{\psi}{2}} \quad \textbf{-------------- (3)}$$

Normalizing (3), we have the general expression for the field pattern of two isotropic sources of equal amplitude and arbitrary phase,

$$\boxed{\mathbf{E = \cos\frac{\psi}{2}}} \quad \textbf{-------------- (4)}$$

The two cases discussed previously are obtained when $\delta = 0^\circ$ and 180°

Two Isotropic Point Sources of the Same Amplitude and In-Phase Quadrature:

Let the two point sources be located as in Fig.1 (*a*). Taking the origin of the coordinates as the reference for phase, let source 1 be retarded by 45° and source 2 advanced by 45°.

That is $\delta = 90^\circ$

The normalized field pattern is given by

$$\boxed{\mathbf{E = \cos\left(\frac{\pi}{4} + \frac{\pi}{2}\cos\phi\right)}}$$

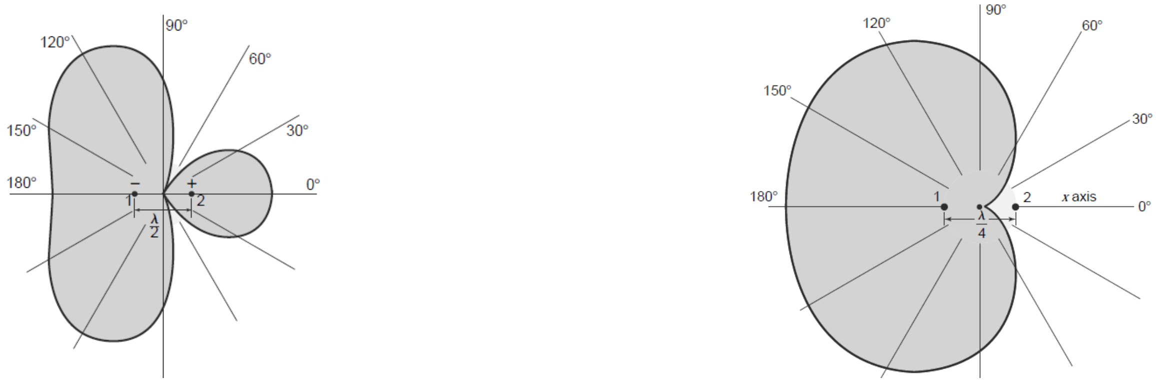


Fig. Field pattern of two isotropic point sources of the same amplitude and in phase quadrature for a spacing of $\lambda/2$ and $\lambda/4$

Two Isotropic Point Sources of Unequal Amplitude and Any Phase Difference:

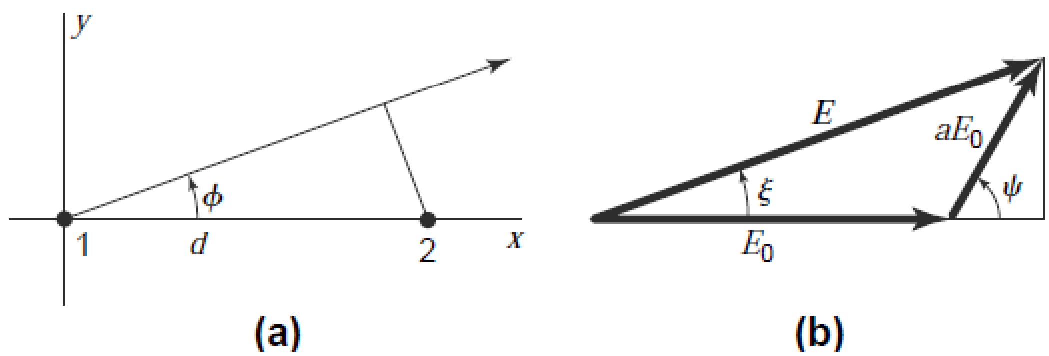


Fig. (*a*) Two isotropic point sources of unequal amplitude and arbitrary phase (*b*) Vector addition of fields

let us consider the case of two isotropic point sources of unequal amplitude but of any phase difference δ shown as in Fig. *a* with source 1 at the origin. Assume that the source 1 has the larger amplitude and that its field at a large distance *r* has amplitude of E_0. Let the field from source 2 be of amplitude aE_0 $(0 \le a \le 1)$ at the distance *r*. Then, referring to Fig., the magnitude and phase angle of the total field *E* is given by

$$\mathbf{E = E_0\sqrt{(1 + a\cos\Psi)^2 + a^2\sin^2\psi}\ \angle\tan^{-1}[a\sin\Psi/(1 + a\cos\Psi)]}$$

Where $\boldsymbol{\psi = \beta d\cos\phi + \delta}$

The Principle of Pattern Multiplication:

The statement of the principle of pattern multiplication as follows:

The total field pattern of an array of nonisotropic but similar sources is the product of the individual source pattern and the pattern of an array of isotropic point sources, while the total phase pattern is the sum of the phase patterns of the individual source and the array of isotropic point sources.

The total field *E* is then

$$E = \underbrace{f(\theta,\phi)F(\theta,\phi)}_{\text{Field pattern}}\ \underline{/\,f_p(\theta,\phi) + F_p(\theta,\phi)}_{\text{Phase pattern}}$$

where

$f(\theta,\phi)$ = field pattern of individual source

$f_p(\theta,\phi)$ = phase pattern of individual source

$F(\theta, \phi)$ = field pattern of array of isotropic sources
$F_p(\theta, \phi)$ = phase pattern of array of isotropic sources

Example (1): Let us consider two vertical short dipoles separated by $d=\lambda/2$ and $\delta=0$ as shown in figure, each source having the pattern given by $E_0 = E_0' \cos\phi$

The total normalized field is given by according to pattern multiplication as

$$\mathbf{E = \cos\phi \cos\frac{\psi}{2}}$$

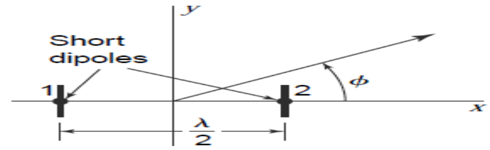


Fig. Array of two nonisotropic sources with respect to the coordinate system.

Where $\psi = \frac{2\pi d}{\lambda}\cos\phi = \pi\cos\phi$

Therefore $\mathbf{E = \cos\phi \cos\left(\frac{\pi}{2}\cos\phi\right)}$

By the principle of pattern multiplication the total normalized field pattern is shown in fig

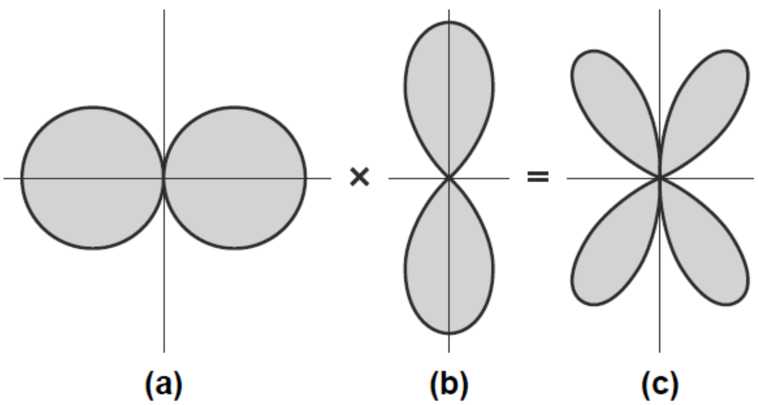


Fig. Total array pattern (*c*) as the product of pattern (*a*) of individual nonisotropic source and (*b*) pattern of array of two isotropic sources.

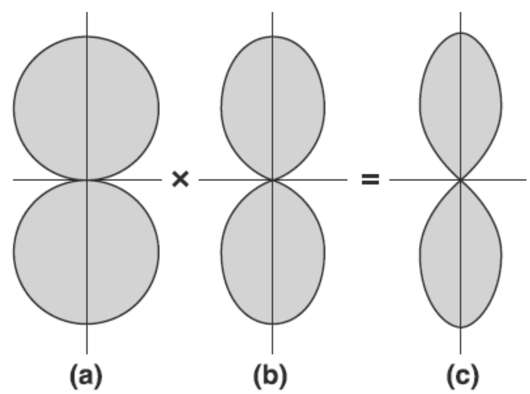
Example (2): Let us consider two horizontal short dipoles separated by $d=\lambda/2$ and $\delta=0$ as shown in figure, each source having the pattern given by

$$\mathbf{E_0 = E_0' \sin\phi}$$

The total normalized field is given by according to pattern multiplication as

$$\mathbf{E = \sin\phi \cos\left(\frac{\pi}{2}\cos\phi\right)}$$

By the principle of pattern multiplication the total normalized field is shown in fig



Uniform Linear Arrays of *n* Isotropic Point Sources of Equal Amplitude and Spacing:

The antenna array is said to be linear if the antenna elements are arranged along a straight line. The linear antenna array is said to be uniform linear array if all the elements fed with a currents of equal amplitude with progressive phase shift along the line.

Let us consider *n* isotropic point sources of equal amplitude and spacing arranged as a linear array, as indicated in Fig. , where *n* is any positive integer and d is spacing. The total field *E* at a large distance in the direction ϕ is given by $\mathbf{E = 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots + e^{j(n-1)\psi}}$ ------------ (1)

Where ψ is $\psi = \beta d\cos\phi + \delta$ $-----$ (2)

where δ is excitation the phase difference between the adjacent sources.

The amplitudes of the fields from the sources are all equal and taken as unity. Source1 is the phase reference.

Thus, at a distant point in the direction ϕ the field from source2 is advanced in phase with respect to source1 by ψ, the field from source 3 is advanced in phase with respect to source 1 by 2ψ, etc.

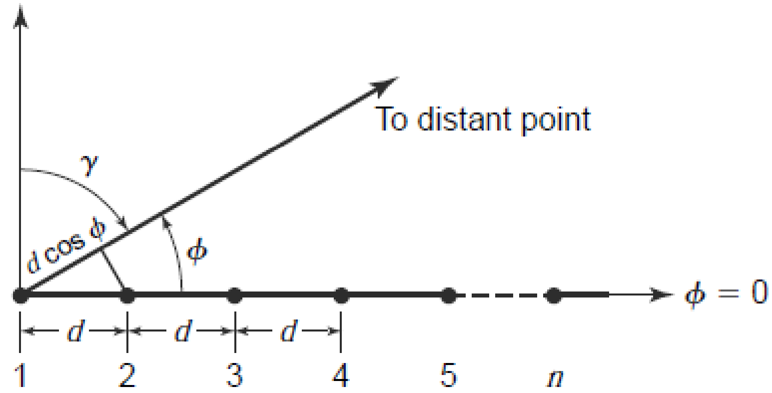


Fig. Arrangement of linear array of *n* isotropic point sources.

Multiply (1) by $e^{j\psi}$, giving

$\mathbf{Ee^{j\psi} = e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots + e^{jn\psi}}$ ------ (2)

Now subtract (2) from (1) and divide by $1 - e^{j\psi}$, yielding

$E = \frac{1-e^{jn\psi}}{1-e^{j\psi}}$ -------- (3)

Equation (3) may be rewritten as

$E = \frac{e^{jn\psi/2}}{e^{j\psi/2}}\left(\frac{e^{jn\psi/2}-e^{-jn\psi/2}}{e^{j\psi/2}-e^{-j\psi/2}}\right)$ -------(4)

$E = e^{j\xi}\frac{\sin n\psi/2}{\sin\psi/2} = \frac{\sin n\psi/2}{\sin\psi/2}\angle\xi$ ------ (5)

where ξ is referred to the field from source 1. The value of ξ is given by

$$\xi = \frac{n-1}{2}\psi$$

If the phase is referred to the center point of the array, (5) becomes

$\boxed{E = \frac{\sin n\psi/2}{\sin\psi/2}}$ ---------- (6)

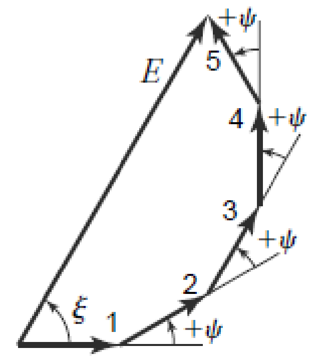


Fig. Vector addition of fields

When $\psi = 0$, equation (6) is indeterminate so that for this case E must be obtained as the limit of (6) as ψ approaches zero. Thus, for $\psi = 0$, The maximum value of E is given by

$E_{max} = n$

Hence, the normalized value of the total field is given by

$\boxed{E_n = \frac{E}{E_{max}} = \frac{1}{n}\frac{\sin n\psi/2}{\sin\psi/2}}$ --------(7)

The field as given by (7) will be referred to as the "array factor." The array will be a maximum in any direction ϕ for which $\psi = 0$.

<u>Broadside Array (Sources in Phase):</u>

Broadside array is the array of antennas in which all the elements are placed parallel to each other along a straight line and the direction of maximum radiation is always perpendicular to the axis of the array. In this array ala the elements are excited with currents of equal amplitude and phase. i.e $\delta=0^\circ$

<u>Properties of broadside array:</u>

<u>Directions of major lobes:</u> The total field is maximum from equation (7), when $\Psi=0$. Therefore

$\Psi = d_r\cos\phi = 0 \Longrightarrow \cos\phi = 0$

Therefore $\phi = 90^\circ$ and 270°

<u>Magnitude of major lobe:</u> The maximum radiation occurs when $\Psi=0$. Hence we can write

$$|\text{Major lobe}| = \left|\frac{E_T}{E_0}\right| = \lim_{\Psi\to 0}\frac{\sin n\psi/2}{\sin\psi/2} = \lim_{\Psi\to 0}\frac{\frac{d\sin n\psi/2}{d\Psi}}{\frac{d\sin\psi/2}{d\Psi}} = n$$

<u>Directions of minor lobes minima:</u>

The directions of minor lobes minima are given when $E = \frac{\sin n\psi/2}{\sin\psi/2} = 0$

i.e $\sin n\psi/2 = 0$ (provided $\sin\frac{\psi}{2} \neq 0$)

$\frac{n\psi}{2} = \pm K\pi$ Where K=1, 2, 3…….

$\Psi = \pm\frac{2K\pi}{n}$

$\beta d\cos\phi = \pm\frac{2K\pi}{n}$

$\frac{2\pi d}{\lambda}\cos\phi = \pm\frac{2K\pi}{n}$

$\cos\phi = \pm\frac{K\lambda}{nd}$

$$\boxed{\phi_{min} = \cos^{-1}\left(\pm\frac{K\lambda}{nd}\right)}$$

Beam width of the major lobe: It is defined as angular width between first nulls or double the angular width between first null and major lobe maximum.

FNBW=2γ

$\phi_{min} = \cos^{-1}\left(\pm\frac{K\lambda}{nd}\right)$

$90 - \gamma = \cos^{-1}\left(\pm\frac{K\lambda}{nd}\right)$

$\cos(90-\gamma) = \pm\frac{K\lambda}{nd}$

$\sin\gamma = \pm\frac{K\lambda}{nd}$ if γ is very small sin γ= γ

$\gamma = \pm\frac{K\lambda}{nd}$

$\gamma = \frac{\lambda}{nd}$ (First null occurs when K=1)

$\text{FNBW} = 2\gamma = \frac{2\lambda}{nd}$ (L=Total length of the array= (n-1)d= n d)

$\text{FNBW} = \frac{2\lambda}{L}$ radians

$\text{FNBW} = 114.6\frac{\lambda}{L}$ degrees (Since 1rad=57.3 degrees)

$\text{HPBW} = \frac{\lambda}{L}$ radiations

$\text{HPBW} = 57.3\frac{\lambda}{L}$ degrees

<u>Directivity of BSA:</u>

$$D = 2\frac{nd}{\lambda} = 2\frac{L}{\lambda}$$

End fire array:

The End array is the array of antennas in which all the elements are placed parallel to each other along a straight line and the direction of maximum radiation is always along the axis of the array. In this array ala the elements are excited with currents of equal amplitude with progressive phase shift. i.e $\delta = \pm\beta d$

Properties of End fire array:

Directions of major lobes: The total field is maximum from equation (7), when Ψ=0. Therefore,

If the maximum radiation is desired towards ϕ=0˚, then $\Psi = |\beta d\cos\phi + \delta|_{\phi=0} = 0$

$\beta d + \delta = 0 \quad \Longrightarrow \quad \delta = -\beta d$

If the maximum radiation is desired towards ϕ=180˚, then $\Psi = |\beta d\cos\phi + \delta|_{\phi=180} = 0$

$-\beta d + \delta = 0 \quad \Longrightarrow \quad \delta = \beta d$

Magnitude of major lobe: The maximum radiation occurs when Ψ=0. Hence we can write

$$|\text{Major lobe}| = \left|\frac{E_T}{E_0}\right| = \lim_{\Psi\to 0}\frac{\sin n\psi/2}{\sin\psi/2} = \lim_{\Psi\to 0}\frac{\frac{d\sin n\psi/2}{d\Psi}}{\frac{d\sin\psi/2}{d\Psi}} = n$$

Directions of minor lobes minima:

The directions of minor lobes minima are given when $E = \frac{\sin n\psi/2}{\sin\psi/2} = 0$

i.e $\sin n\psi/2 = 0$ (provided $\sin\frac{\psi}{2} \neq 0$)

$$\frac{n\psi}{2} = \pm K\pi \quad \text{Where K=1, 2, 3…….}$$

$$\Psi = \pm\frac{2K\pi}{n}$$

$$\beta d\cos\phi + \delta = \pm\frac{2K\pi}{n} \qquad (\text{ substitute } \delta = -d_r \text{ })$$

$$\beta d\cos\phi - d_r = \pm\frac{2K\pi}{n}$$

$$\beta d(\cos\phi - 1) = \pm\frac{2K\pi}{n}$$

$$(\cos\phi - 1) = \pm\frac{2K\pi}{n\beta d}$$

$$-2\sin^2\frac{\phi}{2} = \pm\frac{2K\pi}{n\beta d}$$

$$\sin^2\frac{\phi}{2} = \pm\frac{K\pi}{n\beta d}$$

$$\sin\frac{\phi}{2} = \pm\sqrt{\frac{K\pi}{n\beta d}}$$

$$\frac{\phi}{2} = \sin^{-1}\pm\sqrt{\frac{K\pi}{nd_r}}$$

$$\phi = 2\sin^{-1}\pm\sqrt{\frac{K\pi}{n\beta d}}$$

$$\boxed{\phi_{mim} = 2\sin^{-1}\left[\pm\sqrt{\frac{K\lambda}{2nd}}\right]}$$

Beam width of the major lobe: It is defined as angular width between first nulls or double the angular width between first null and major lobe maximum.

FNBW=2ϕ_{mim}

$$\phi_{mim} = 2\sin^{-1}\left[\pm\sqrt{\frac{K\lambda}{2nd}}\right]$$

$$\sin\frac{\phi_{mim}}{2} = \pm\sqrt{\frac{K\lambda}{2nd}} \qquad \text{If } \phi_{mim} \text{ is small}$$

$$\frac{\phi_{mim}}{2} = \pm\sqrt{\frac{K\lambda}{2nd}}$$

$$\phi_{mim} = \pm 2\sqrt{\frac{K\lambda}{2nd}} = \pm\sqrt{\frac{2K\lambda}{nd}}$$

$$\text{FNBW} = \pm 2\sqrt{\frac{2K\lambda}{nd}} \qquad \text{If length of the array is L=(n-1)d=nd and K=1}$$

$$\text{FNBW} = \pm 2\sqrt{\frac{\lambda}{L}}\ rad$$

$$\text{FNBW} = \pm 2 \times 57.3\sqrt{\frac{\lambda}{L}}\ degrees$$

$$\text{FNBW} = \pm 114.6\sqrt{\frac{\lambda}{L}}\ degrees$$

$$\text{HPBW} = \pm 57.3\sqrt{\frac{\lambda}{L}}\ degrees$$

Directivity of end fire array: $D = 4\left(\frac{L}{D}\right)$

End-Fire Array with Increased Directivity: In ordinary end fire array, for $\delta = -\beta d$, produces a maximum field in the direction $\phi = 0$, but does not give the maximum directivity. It has been shown by Hansen and Woodyard that a larger directivity is obtained by increasing the phase change between sources so that

$\boldsymbol{\delta = -\left(\beta d + \frac{\pi}{n}\right)}$ ---------1. (a)

This condition will be referred to as the condition for "increased directivity." Thus for the phase difference of the fields at a large distance we have

$\boxed{\boldsymbol{\Psi = \beta d(\cos\phi - 1) - \frac{\pi}{n}}}$ ---------- 1. (b)

As an example, the field pattern of an end-fire array of four isotropic point sources for this case is illustrated in fig

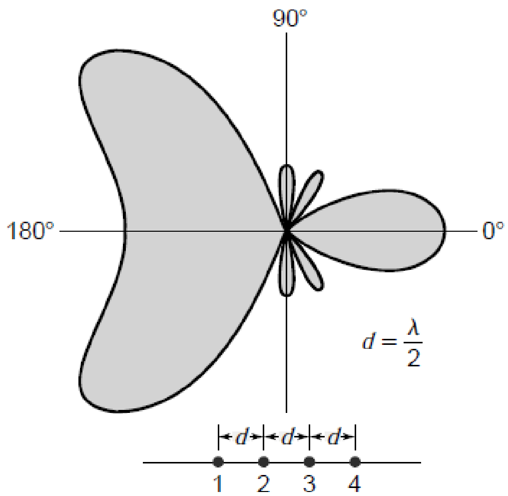


Fig. Field pattern of end-fire array with increased directivity of four isotropic point sources of

equal amplitude spaced $\lambda/2$ apart. $\delta = -(5\pi/4)$

The increased directivity due to Hansen and Woodyard can be realized by using equation (1) along with additional phase difference requires that $|\psi|$ be restricted in its range to a value of $\boldsymbol{\pi/n}$ at $\phi = 0$ and a value in the vicinity of π at $\phi = 180°$.

The spacing between elements **d=λ/4**.

In general, any increased directivity end-fire array, with maximum at $\phi=0$ and $\psi = -\pi/n$, has a normalized field pattern given by

$$\mathbf{E = \sin\left(\frac{\pi}{2n}\right)\frac{\sin n\psi/2}{\sin \psi/2}}$$

Beam width of the major lobe: It is defined as angular width between first nulls or double the angular width between first null and major lobe maximum.

$\text{FNBW} = 2\phi_{mim}$

The directions of minor lobes minima are given when $\mathbf{E} = \frac{\sin n\psi/2}{\sin \psi/2} = \mathbf{0}$

i.e $\sin n\psi/2 = 0$ (provided $\sin\frac{\psi}{2} \neq 0$)

$\frac{n\psi}{2} = \pm K\pi$ Where K=1, 2, 3…….

$$\Psi = \pm\frac{2K\pi}{n}$$

$$\beta d(\cos\phi - 1) - \frac{\pi}{n} = \pm\frac{2K\pi}{n}$$

$$\beta d(\cos\phi - 1) = \pm\frac{2K\pi}{n} + \frac{\pi}{n}$$

$$\beta d(\cos\phi - 1) = \pm\frac{\pi}{n}(2K - 1)$$

$$(\cos\phi - 1) = \pm\frac{\pi}{n\beta d}(2K - 1)$$

$2sin^2\frac{\phi}{2} = \pm\frac{\pi}{n\beta d}(2K-1)$

$sin^2\frac{\phi}{2} = \pm\frac{\pi}{2n\beta d}(2K-1)$

$\sin\frac{\phi}{2} = \pm\sqrt{\frac{\pi}{2n\beta d}(2K-1)}$ Where $\beta = \frac{2\pi}{\lambda}$

$\sin\frac{\phi}{2} = \pm\sqrt{\frac{\lambda}{4nd}(2K-1)}$

If nd is very very large then

$\frac{\phi}{2} = \pm\sqrt{\frac{\lambda}{4nd}(2K-1)}$

$\boldsymbol{\phi_{min}} = \pm\sqrt{\boldsymbol{\frac{\lambda}{nd}(2K-1)}}$

The first nulls either side of the main lobe, occur for k = 1. Thus,

$\boldsymbol{\phi_{min}} = \pm\sqrt{\boldsymbol{\frac{\lambda}{nd}}}$

The total beamwidth of the main lobe between first nulls for a long end-fire array with increased directivity is then

$\textbf{FNBW} = \boldsymbol{2\phi_{min}} = \boldsymbol{\pm 2}\sqrt{\boldsymbol{\frac{\lambda}{nd}}}$

This width is $1/\sqrt{2}$ or 71 percent, of the width of the ordinary end-fire array.

<u>Scanning Array or phased Array:</u> Let us consider the case of an array with a field pattern having a maximum in some arbitrary direction ϕ_1 not equal to $k\pi/2$ where k = 0, 1, 2, or 3. Then

$$\Psi = |\beta d\cos\phi_1 + \delta|_{\phi_1} = 0$$

$$\delta = -\beta d\cos\phi_1 \text{ ------------ (1)}$$

Thus from equation (1) it is clear that the maximum radiation can be obtained in any direction if the progressive phase shift δ between the elements is controlled. This array is called Scanning Array or phased Array.

<u>Nonuniform Amplitude Distributions-General Considerations and Binomial Array:</u>

In case of uniform linear array, to increase the directivity, the array length has to be increased. But when the array length increases, the side lobes appear in the pattern. In some of the special applications, it is desired to have single main lobe with no side lobes. That means the side lobes should be eliminated completely or reduced to minimum level as compared to main lobe. To achieve this nonuniform array is used that is centre element is excited with more current than edge element.

Uniform amplitude arrays produce small half-power beamwidth and possess the largest directivity with side lobes where as nonuniform amplitude linear array produce a pattern with smaller side lobe level and a slightly increased half power beamwidth in comparison to the uniform linear antenna array.

Binomial array is used to reduce the Side-Lobe Level (SLL) of linear broadside arrays, in which all the sources have amplitudes proportional to the coefficients of a binomial series of the form

$$(a+b)^{n-1} = a^{n-1} + (n-1)a^{n-2}b + \frac{(n-1)(n-2)}{2!}a^{(n-3)}b^2 + \cdots$$

Where n is the number of sources. Thus, for arrays of three to six sources the relative amplitudes are given by Table, where the amplitudes are arranged as in Pascal's triangle. Since the coefficients are determined from a binomial series expansion, the array is known as a binomial array.

n	Relative amplitudes (Pascal's triangle)
3	1 2 1
4	1 3 3 1
5	1 4 6 4 1
6	1 5 10 10 5 1

The following fig shows the radiation pattern of the array of five sources spaced $\lambda/2$ apart, in fig (a) the sources have the relative amplitudes 1, 1, 1, 1, 1, and in fig (b) the sources have the relative amplitudes 1, 4, 6, 4, 1.

The uniform distribution has minor lobes where as Binomial Distribution has no minor lobes, but beamwidth is more.
In general, the pattern for the binomial array is given by

$$\boxed{\mathbf{E = \cos^{n-1}\left[\frac{\pi}{2}\cos\theta\right]}}$$

$$\mathbf{HPBW}(d = \lambda/2) = \frac{\mathbf{1.06}}{\sqrt{\mathbf{n-1}}}$$

$$\mathbf{D = 1.77\sqrt{n}}$$

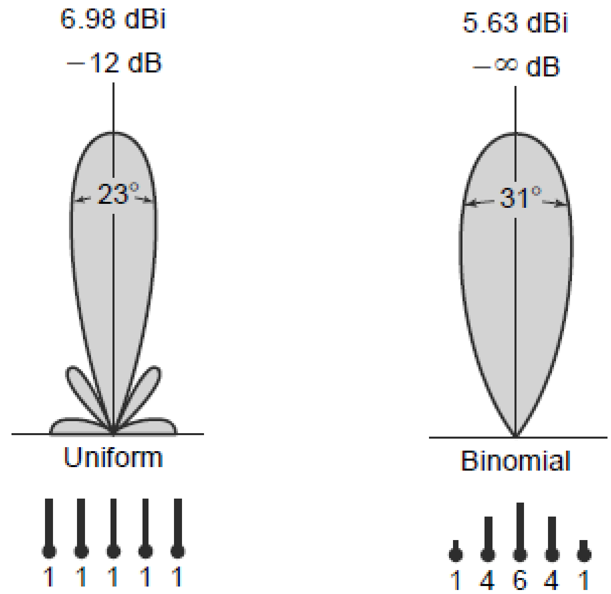


Fig.(a) Uniform (b) Binomial

The increased beamwidth and the large ratio of current amplitudes required in large arrays are disadvantages.

Antenna Measurements

Introduction: Accurate measurements are necessary to establish the actual performance of antennas: their gain, pattern, polarization, bandwidth, efficiency, etc. Antennas having strict specifications are needed in many applications as in mobile and personal communications, satellite communications, remote sensing, and radar. For example, antennas of a point-to-point radio link have to fulfill certain gain, side-lobe level, and cross-polarization requirements set by standards to get type approval.

Basic Concepts: The antenna measurement is used to measure its radiation properties like directional pattern, gain or phase pattern in the far field. The typical configuration of the measurement of radiation properties is shown in Fig. The Antenna under Test (AUT) is located at origin of the coordinate system. The basic procedure is to place *source antenna* at different locations with respect to the *Antenna Under Test* (*AUT*) and get a number of samples of the pattern. The source antenna may be a transmitting or receiving. The different locations are normally achieved by rotating the AUT. To get the sharp sample of the pattern , only one direct signal path should exist between the AUT and the source antenna. This can be achieved in a reflectionless environment like in an anechoic chamber or in free space.

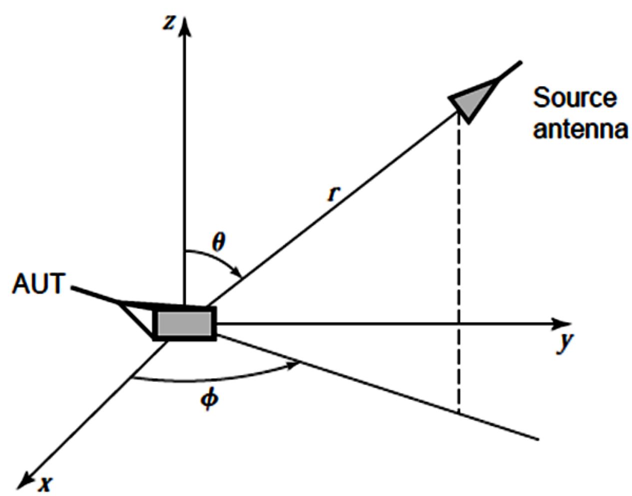


Fig. Typical configuration for the measurement of the radiation properties of an antenna.

Reciprocity in Antenna Measurements: The AUT can act as either a receiving antenna or a transmitting antenna. The two important consequences of the reciprocity principle from the antenna measurement point of view are given:

1. The transmitting and receiving patterns are the same.
2. Power flow is the same in the transmitting and receiving mode.

Thus it is clear that all radiation parameters of the AUT can be measured in either transmission or reception mode. This is especially useful in cases, where, the AUT is an integral part of a larger device acting as either a receiver or a transmitter thus defining the direction of the signal.

Practically while using reciprocity principle the following conditions must be fulfilled.

1. The emfs in the terminals of the transmitting and receiving antennas are of the same frequency.
2. The media are linear, passive and isotropic.
3. The power flow is equal for matched impedances only.

The reciprocity principle is not only applicable to the antenna patterns but also to the other characteristics of the antenna except current distribution.

Near-Field and Far-Field: There are three main regions of the radiated field of the antenna. The region very close to the antenna is called the reactive near-field region (radiansphere), The region next to this is called the radiative near field or Fresnel region. Finally the region located away from the antenna is called the far field or Fraunhofer region.

As we are almost always interested in the radiation properties of the antenna in the far field. So the measurement is done in the far field. There are several advantages of the far-field measurement:

1. The measured field pattern is valid for any distance in the far-field region; only simple transformation of the field strength according to 1/r is required.
2. If a power pattern is required, only power (amplitude) measurement is needed.
3. The result is not very sensitive to the changes in the location of the phase center of the antennas and thus the rotation of the AUT does not cause significant measurement errors.
4. Coupling and multiple reflections between the antennas are not significant.

The main disadvantage of the far-field measurements is the required large distance between the antennas leading to large antenna ranges. .

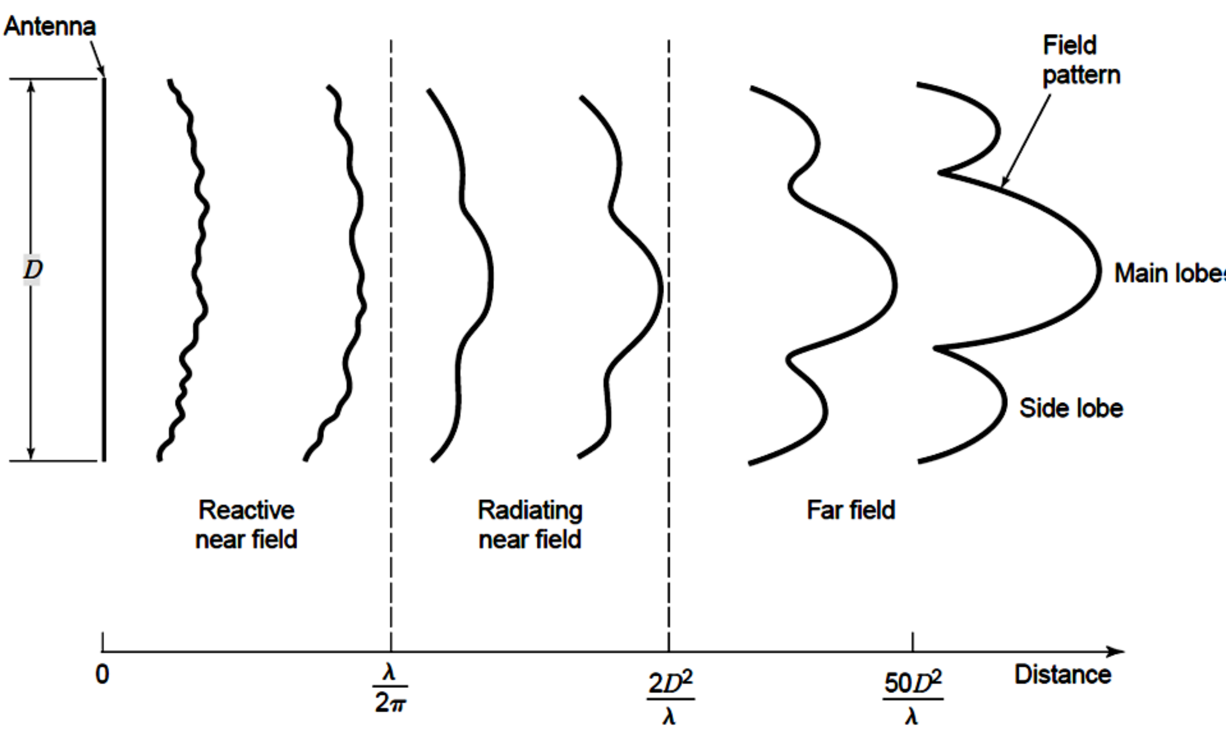


Fig. Radiation patterns in near-field and far-field regions.

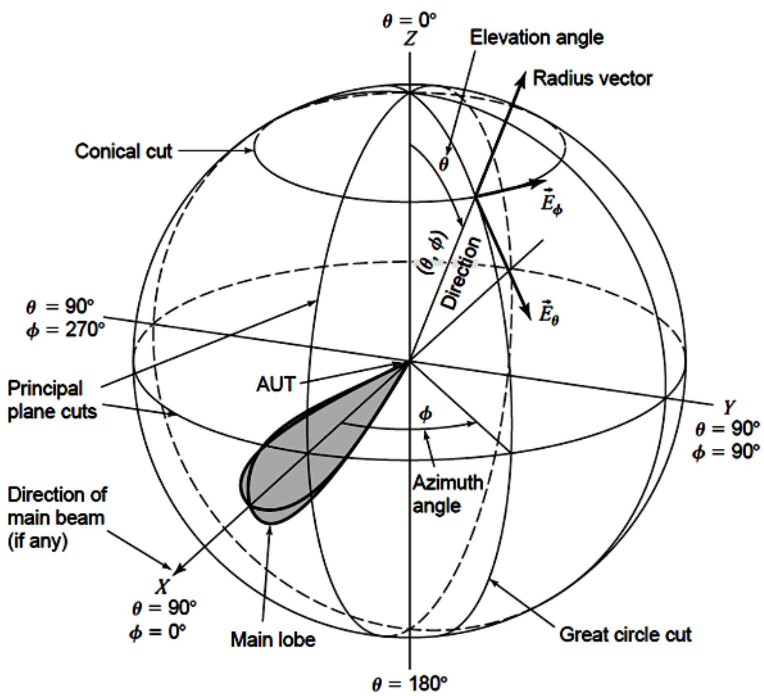
For small elementary dipoles, it is at the distance determined by the radius of the radian sphere, that is, $r = \lambda/2\pi$

The border between the Fresnel and Fraunhofer regions is given by

$$r = \frac{2D^2}{\lambda}$$

Coordinate System for antenna measurement:

The IEEE standard spherical coordinate system is shown in Fig. The AUT is at the origin.



The elevation angle θ is measured from the z-axis (zenith). The azimuth angle ϕ is measured from the projection of the radius vector to the xy- (horizontal) plane with ϕ = 0 at the x-axis increasing counterclockwise.

Normally the coordinate system is defined based on the mechanical structure of the antenna so that the assumed direction of the peak radiation is on the x-axis.

When the source antenna is moving along lines of constant θ results in conical cuts or ϕ cuts. When the source antenna is moving along lines of constant ϕ, results in great-circle cuts or θ cuts . If the cut is taking along the equator with θ = π/2, than such a cut is called θ as well ϕ cut. The two principal-plane cuts are defined as the orthogonal great-circle cuts through the axis of the main lobe of the antenna. With linearly polarized antennas the cuts are selected to coincide with the assumed direction of the E and H fields in the main lobe and then they are called E- and H-plane cuts.

Typical Sources of Error in Antenna Measurements: A pure plane wave (uniform phase and amplitude) is an ideal test field for the measurement of far-field patterns. However, there are inevitably deviations from the plane wave due to

1. insufficient distance between the antennas causes phase curvature and amplitude taper,
2. reflections from surroundings cause amplitude and phase ripple.
3. Phase curvature and amplitude taper can have a significant impact on the main beam, whereas ripples may spoil the accuracy of side-lobe measurements.
4. Errors due to coupling to the reactive near field,
5. Errors due to misalignment of the antenna
6. Errors due to interfering signals
7. Errors due to effects of the atmosphere
8. Errors due to leaking and radiating cables
9. Errors due to manmade interface
10. Errors due to impedance mismatch
11. Errors due to imperfections of instrumentation , etc.,

These are sources of error in antenna measurements.

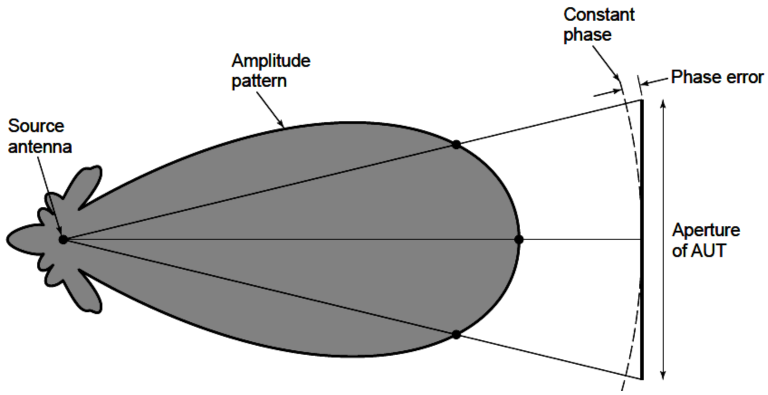
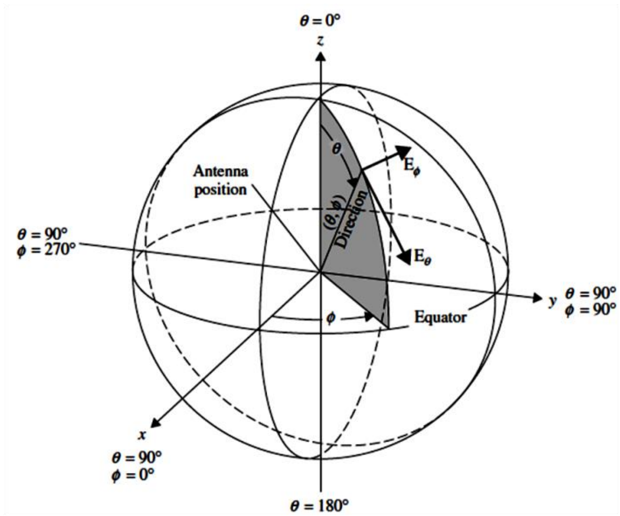


Fig. Phase error and amplitude taper across the aperture of an AUT

Patterns to be Measured: The radiation capabilities of an antenna are characterized by radiation patterns (amplitude and phase), polarization, and gain of an antenna. All these quantities are measured on the surface of the sphere with constant radius. Any point on the sphere is identified using the standard spherical coordinate system of Fig. Since the radial distance is constant, only the two angular coordinates (θ, ϕ) are needed for representing point on the sphere. The graphical representation of the radiation characteristics of the antenna as a function of θ and ϕ for a constant radial distance and frequency is called as the radiation pattern of the antenna.



In general, the pattern of an antenna is three-dimensional. Because it is impractical to measure a three-dimensional pattern, and a number of two-dimensional patterns are used to construct a three-dimensional pattern. The minimum number of two dimensional patterns required to construct three dimensional pattern is two, and they are the orthogonal principal E- and H-plane patterns.

A two-dimensional pattern is also referred to as a pattern cut, and it is obtained by fixing one of the angles (θ or ϕ) while varying the other. For example, pattern cuts can be obtained by fixing $\phi_j (0 < \phi_j < 2\pi)$and varying $\theta (0 < \theta < \pi)$. These are referred to as elevation patterns, and they are also displayed in Figure. Similarly, $\theta_i (0 < \theta_i < \pi)$ can be maintained fixed while $\phi (0 < \phi_j < 2\pi)$ is varied. These are designated as azimuthal patterns. The patterns of an antenna can be measured in the transmitting or receiving mode.

Pattern Measurement Arrangement or Set Up:

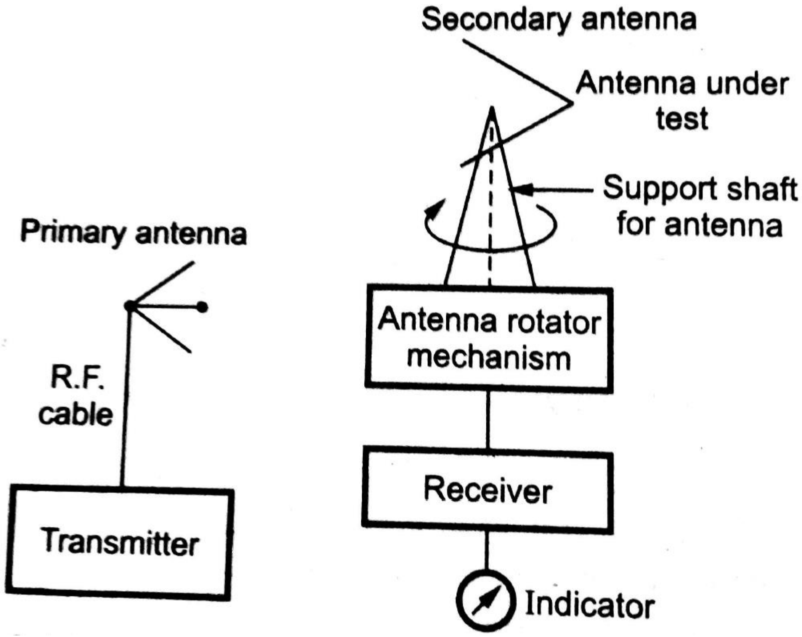


Fig Typical instrumentation for the measurement of the radiation properties of Antenna Under Test (AUT).

Gain Measurements: The most important figure of merit that describes the performance of a radiator is the gain. There are various techniques and antenna ranges that are used to measure the gain. The choice of either depends largely on the frequency of operation.

Basically there are two standard methods used for the measurement of gain of an antenna such as

(a) Gain transfer (or gain comparison) method or direct comparison method
(b) Absolute gain method

Gain-transfer methods must be used in conjunction with standard gain antennas to determine the realized gain of the antenna under test.

The absolute gain method is used to calibrate antennas that can then be used as standards for gain measurements, and it requires no *a priori* knowledge of the gains of the antennas.

Gain transfer (or gain comparison) method or direct comparison method:

In the comparison method (gain-transfer method), the powers received with the AUT and with a known reference antenna are compared as shown in Fig. This measurement can be Performed on either a free-space or a ground-reflection range. The gain of the AUT is

$$\mathbf{G_{AUT}} = \frac{\mathbf{P_{AUT}}}{\mathbf{P_{ref}}}\mathbf{G_{ref}} \qquad -\,-\,-\,-\,-\,(\mathbf{1})$$

Where

P_{AUT} = power received with the AUT

P_{ref} = power received with the reference antenna

G_{ref} = gain of the reference antenna

Half-wave dipoles and horn antennas are commonly used reference antennas because they have a predictable gain and pure polarization. The calibration uncertainty of the reference antenna gain is typically ±0.25 dB. The power ratio can be measured simply with a calibrated attenuator. The attenuation is adjusted to give the same output indication with both antennas, and the power ratio is obtained from the attenuator settings.

It is assumed in Eq. (1) that both the AUT and the reference antenna are perfectly matched to the receiving system and they have the same polarization. Differences in polarization mismatch, impedance mismatch and transmission line losses cause errors.

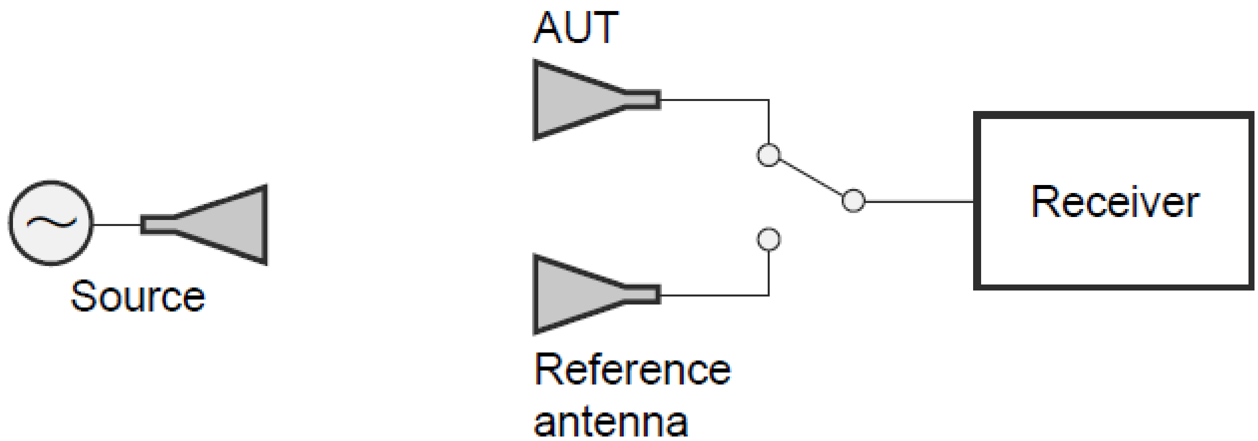


Fig. Gain measurement by comparing the Antenna Under Test (AUT) with a reference antenna.

The gains of circularly or elliptically polarized antennas are usually determined by measuring the partial gains for two orthogonal linear polarizations. First the polarizations of the linearly polarized source and reference antennas are set horizontally and the gain G_H is measured. Then the measurement is repeated for vertically polarized source and reference antennas and the gain G_V is obtained.

The total gain, G_{AUT} , is the sum of the two partial gains

$$\textbf{Total gain} = \mathbf{G_{AUT}} = \mathbf{G_H} + \mathbf{G_V}$$

Where

G_H = gain of the AUT at horizontal polarization

G_V = gain of the AUT at vertical polarization

<u>Absolute gain method:</u> Consider two identical antennas separated by a distance R as shown in fig

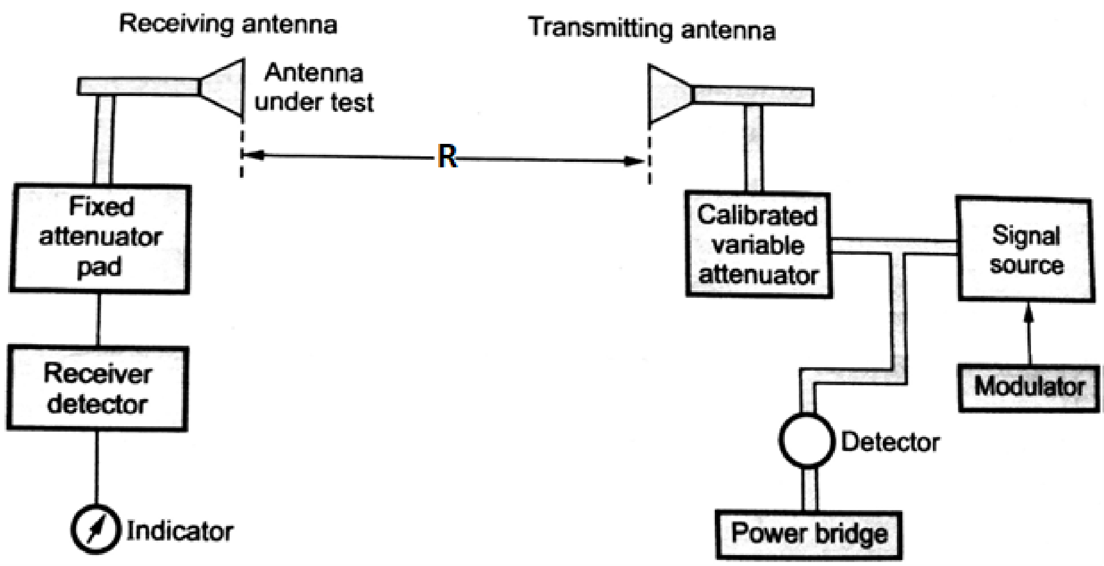


Fig. Transmitting and receiving antennas for absolute gain measurement

The absolute method is based on the Friis transmission formula.

(A) Two-Antenna Method:

$$(G_{0t})_{dB} + (G_{0r})_{dB} = 20\log_{10}\left(\frac{4\pi R}{\lambda}\right) + 10\log_{10}\left(\frac{P_r}{P_t}\right) \quad ------(1)$$

Where
$(G_{0t})_{dB}$ = gain of the transmitting antenna (dB)
$(G_{0r})_{dB}$ = gain of the receiving antenna (dB)
Pr = received power (W)
Pt = transmitted power (W)
R = antenna separation (m)
λ = operating wavelength (m)
If the transmitting and receiving antennas are identical ($G_{0t} = G_{0r}$), equation (1) reduces to

$$(G_{0t})_{dB} = (G_{0r})_{dB} = \frac{1}{2}\left(20\log_{10}\left(\frac{4\pi R}{\lambda}\right) + 10\log_{10}\left(\frac{P_r}{P_t}\right)\right)$$

By measuring *R*, λ, and the ratio of P_r/P_t, the gain of the antenna can be found.

(B). Three-Antenna Method:

If the two antennas in the measuring system are not identical, three antennas (a, b, c) must be employed and three measurements must be made (using all combinations of the three) to determine the gain of each of the three.
Three equations (one for each combination) can be written, and each takes the form of (1). Thus
(a-b Combination)

$$(G_a)_{dB} + (G_b)_{dB} = 20\log_{10}\left(\frac{4\pi R}{\lambda}\right) + 10\log_{10}\left(\frac{P_{rb}}{P_{ta}}\right) \quad ----- (2)$$

(a-c Combination)

$$(G_a)_{dB} + (G_c)_{dB} = 20\log_{10}\left(\frac{4\pi R}{\lambda}\right) + 10\log_{10}\left(\frac{P_{rc}}{P_{ta}}\right) \quad ------(3)$$

(b-c Combination)

$$(G_b)_{dB} + (G_c)_{dB} = 20\log_{10}\left(\frac{4\pi R}{\lambda}\right) + 10\log_{10}\left(\frac{P_{rc}}{P_{tb}}\right) \quad ------(4)$$

From these three equations, the gains $(G_a)_{dB}$, $(G_b)_{dB}$, and $(G_c)_{dB}$ can be determined provided R, λ, and the ratios of P_{rb}/P_{ta}, P_{rc}/P_{ta}, and P_{rc}/P_{tb} are measured.

Directivity measurement:

The directivity of an antenna, *D*, cannot be measured directly.
However, it can be computed from the normalized power pattern $Pn(\theta,\phi)$ as

$$D = \frac{4\pi}{\iint P_n(\theta,\phi)\sin\theta\, d\theta\, d\phi}$$

The total power is divided between two patterns, the co-polar pattern and cross-polar pattern. The power patterns are measured only in the principal planes, e.g., in the *E* and *H* planes of a linearly polarized antenna.