UNIT - 3

BASE BAND PULSE TRANSMISSION

INTRODUCTION:

Generally the digital data transmission takes place in two ways.

i Base Band Pulse Transmission.
ii Pass Band Pulse Transmission.

- In base band pulse transmission, the digital data transmitted directly over the channel without using any modulation technique. This one can be used for shorter distance communication.
- In pass band pulse transmission, the digital data can be transmitted over the channel by using modulation techniques. This one can be used for the longer distance communication.
- In case of base band data transmission, there is a requirement to use low frequency channel whose bandwidth is large enough to pass the input data stream.
- In case the channel is dispersive then the each received pulse is affected somewhat by adjacent pulses. This type of interference is called Inter Symbol Interference (ISI). This is the major source of bit errors. This one can be controlled by maintaining certain pulse shape in the overall system.
- Another source of bit errors is channel noise. Now introducing matched filter for detection of pulse signal effected by the channel noise. Matched filter uses linear time invariant filter.

MATCHED FILTER:

The device uses linear time invariant filter for the detection of pulse transmitted over a channel that is corrupted by channel noise is called marched filter, which is so called because its impulse response is matched to the pulse signal. It is used to improve the S/N ratio.

Let us consider a receiver model shown in fig, with LTI filter of impulse response h(t).

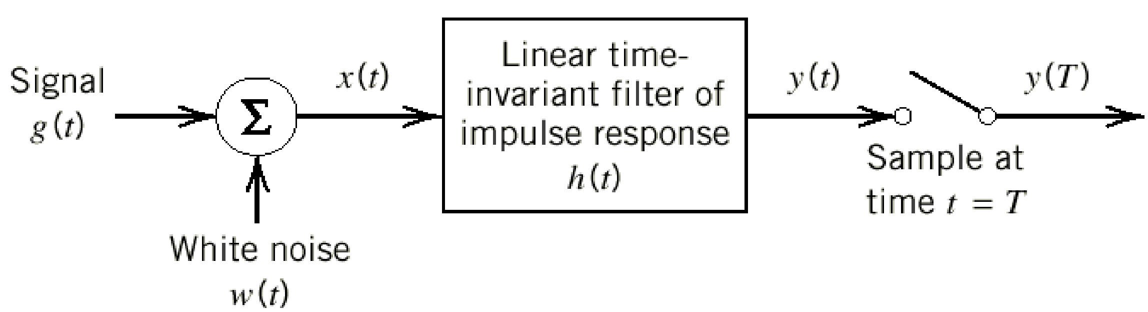

FIG: Linear receiver

The filter input x(t) consist of pulse signal g(t) & noise w(t), it is expressed as

$$x(t) = g(t) + w(t), \qquad 0 \le t \le T \qquad \text{-------------------- (1)}$$

- Where T is observation interval, g(t) is a binary symbol 1 or 0, w(t) is a sample function of white noise, zero mean, psd $N_0/2$

Since the filter is linear, the filter output y(t) is expressed as

$$y(t) = g_o(t) + n(t) \quad \text{-------------------- (2)}$$

Where $g_o(t)$ & n(t) are signal & noise components produced by signal x(t).

Here, the filter has to make at time t=T, the instantaneous power of the output signal as maximum as possible than the average power of the output noise n(t). This is equivalent to maximizing the peak signal to noise ratio, defined as

$$\eta = \frac{|g_o(T)|^2}{E[n^2(t)]} \quad \text{-------------------- (3)}$$

Where $|g_o(T)|^2$ the instantaneous power in the output is signal and $E[n^2(t)]$ is a measure of the average output noise power.

➢ The requirement is to specify the impulse response of the matched filter in order to maximize the output signal to noise ratio in equation 3.

OUTPUT SIGNAL POWER:

Let G(f) denotes the Fourier transform of the known signal g(t) & H(f) denote the frequency response of the filter. Then the F.T of the output signal $g_o(t)$ is equal to $H(f).G(f)$. then

$$g_o(t) = \int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi ft)df \quad \text{-------------------- (4)}$$

When the filter output is sampled at time **t=T**, then take squared magnitude in eq.3.

$$|g_o(T)|^2 = \left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT)df\right|^2 \quad \text{-------------------- (5)}$$

AVERAGE OUTPUT NOISE POWER:

The power spectral density $S_N(f)$ of the output noise $n(t)$ is equal to the power spectral density of the input noise $w(t)$ times the squared magnitude response $|H(f)|^2$, i.e.,

$$S_N(f) = \frac{N_0}{2}|H(f)|^2 \quad \text{-------------------- (6)}$$

Thus the average power of the output noise $n(t)$ is:

$$E[n^2(t)] = \int_{-\infty}^{\infty} S_N(f)df = \frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2\, df \quad \text{-------------------- (7)}$$

Substituting eq.s 5,7 into eq.3, the expression for peak signal to noise ratio is rewritten as,

$$\eta = \frac{\left| \int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi ft)df \right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 \, df} \qquad \text{-------------------- (8)}$$

Apply Schwarz's inequality theorem to the numerator of eq. 8.

- The Schwarz's inequality theorem stated as, if we have two complex functions $\phi_1(x)\phi_2(x)$ in the real variable x, satisfying the conditions

$$\int_{-\infty}^{\infty} |\phi_2(x)|^2 \, dx < \infty \qquad \int_{-\infty}^{\infty} |\phi_1(x)|^2 \, dx < \infty$$

Then we may write,

$$\left| \int_{-\infty}^{\infty} \phi_1(x)\phi_2(x)dx \right|^2 \le \int_{-\infty}^{\infty} |\phi_1(x)|^2 \, dx \int_{-\infty}^{\infty} |\phi_2(x)|^2 \, dx \qquad \text{-------------------- (9)}$$

The equality in eq. 9 holds if, we have $\phi_1(x) = k\phi_2^*(x)$ -------------------- (10)

Where K = arbitrary constant and * = complex conjugation.

From the Schwarz's inequality in eq.9, setting

$\phi_1(x) = H(f)$

$\phi_2(x) = G(f)\exp(j2\pi fT)df$

then the numerator of eq. 8 written as

$$\left| \int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT)df \right|^2 \le \int_{-\infty}^{\infty} |H(f)|^2 \, df \int_{-\infty}^{\infty} |G(f)|^2 df \qquad \text{-------------------- (11)}$$

Substitute eq.11 in eq.8 $\eta \le \frac{2}{N_0}\int_{-\infty}^{\infty} |G(f)|^2 \, df$ -------------------- (12)

The signal to noise ratio does not depends on the frequency response $H(f)$ of the filter but only on the signal energy and noise power spectral density. The max signal to noise ratio of o/p signal is,

$$\eta = \frac{2}{N_0}\int_{-\infty}^{\infty} |G(f)|^2 \, df \qquad \text{-------------------- (13)}$$

Assume that $Hopt(f)$ is the optimum value of $H(f)$ to maximize the signal to noise ratio. From eq.10

$$H_{opt}(f) = kG^*(f)\exp(-j2\pi fT) \qquad \text{-------------------- (14)}$$

Then the optimum value of the filter $hopt(t)$ is

$$h_{opt}(t) = k\int_{-\infty}^{\infty} G^*(f)\exp[-j2\pi f(T-t)]df$$

For real signal g(t), $G^*(f) \ = \ G(-f)$

$$\begin{aligned} h_{opt}(t) &= k\int_{-\infty}^{\infty} G(-f)\exp[-j2\pi f(T-t)]df \\ &= k\int_{-\infty}^{\infty} G(f)\exp[j2\pi f(T-t)]df \\ &= kg(T-t) \end{aligned} \qquad \text{--------------------- (15)}$$

The optimum value of the filter, is except factor k, it is time inversed & delayed version of the input signal g(t). i.e. it is matched to the i/p signal.

PROPERTIES OF MATCHED FILTER:

Matched filter is a optimum device for the detection of received pulse signal effected by noise. By using LTI filter, to improve the output signal to noise ratio.

i The impulse response of the matched filter is, except the scaling factor k, is the time inversed & delayed inversion of the input pulse signal g(t).
   i.e. $\boldsymbol{h_{opt}(t) = k \cdot g(T-t)}$

***Proof*:** From the Schwartz's inequality theorem, from eq.10

$$H_{opt}(f) = kG^*(f)\exp(-j2\pi fT)$$

Then the optimum value of the filter $hopt(t)$ is

$$h_{opt}(t) = k\int_{-\infty}^{\infty} G^*(f)\exp[-j2\pi f(T-t)]df$$

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$$\begin{aligned} h_{opt}(t) &= k\int_{-\infty}^{\infty} G(-f)\exp[-j2\pi f(T-t)]df \\ &= k\int_{-\infty}^{\infty} G(f)\exp[j2\pi f(T-t)]df \\ &= kg(T-t) \end{aligned}$$

ii The max signal to noise ratio of the output signal does not depends on the response of the filter but only depends on signal energy & noise power spectral density.

i.e.

$$\eta_{\max} = \frac{2E}{N_0}$$

Proof: We know that the signal to noise ratio of output signal $\eta = \dfrac{|g_o(T)|^2}{E[n^2(t)]}$ -------- (1)

From the Schwarz's inequality theorem, $H_{opt}(f) = kG^*(f)\exp(-j2\pi fT)$

The Fourier transform of filter o/p $g_o(t)$ may written as

$$\begin{aligned} G_o(f) &= H_{opt}(f)G(f) \\ &= kG^*(f)G(f)\exp(-j2\pi fT) \\ &= k\,|G(f)|^2 \exp(-j2\pi fT) \end{aligned}$$

using the inverse FT

$$\begin{aligned} g_o(T) &= \int_{-\infty}^{\infty} G_0(f)\exp(j2\pi fT)df \\ &= k\int_{-\infty}^{\infty} |G(f)|^2 df \end{aligned}$$ -------------------- (2)

According to Rayleigh's Energy theorem, the integral of the squared magnitude spectrum of a pulse signal with respect to frequency is equal to the signal Energy E.

$$E = \int_{-\infty}^{\infty} g^2(t)dt \quad = \int_{-\infty}^{\infty} |G(f)|^2\, df$$

$$so \quad g_0(T) = k\,E$$ -------------------- (3)

The average output noise power

$$\begin{aligned} E[n^2(t)] &= \int_{-\infty}^{\infty} S_N(f)df \\ &= \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df \end{aligned}$$

$$\begin{aligned} E[n^2(t)] &= \frac{k^2 N_0}{2}\int_{-\infty}^{\infty} |G(f)|^2\, df \\ &= k^2 N_0 E/2 \end{aligned}$$ -------------------- (4)

The peak signal to noise ratio of pulse signal is

$$\eta_{\max} = \frac{(kE)^2}{(k^2 N_0 E/2)} = \frac{2E}{N_0} \qquad \text{-------------------- (5)}$$

The peak signal to noise ratio of the o/p signal does not depends on the response of the filter but only depends on signal energy and noise power spectral density.

ERROR RATE DUE TO NOISE [PROBABILITY ERROR OF MATCHED FILTER]:

Consider a binary PCM system based on polar non return to zero signalling. In this form of signalling, symbol 1 & 0 are represented by positive & negative rectangular pulses with amplitude A for equal duration. The channel noise is modelled as additive white Gaussian noise w(t) of zero mean & power spectral density $N_0/2$

In the signalling interval $0 \le t \le T_b$, the received signal is written as,

$$x(t) = \begin{cases} +A + w(t), & symbol\ 1\ was\ sent \\ \\ -A + w(t), & symbol\ 0\ was\ sent \end{cases} \qquad \text{-------------------- (1)}$$

Where T_b = bit duration, A = transmitted pulse amplitude

Given the noisy signal x(t), the receiver is required to make a decision in each signalling interval as to whether the transmitted symbol is a ‘1’ or a ‘0’.

The structure of the receiver used to perform this decision make process is shown below.

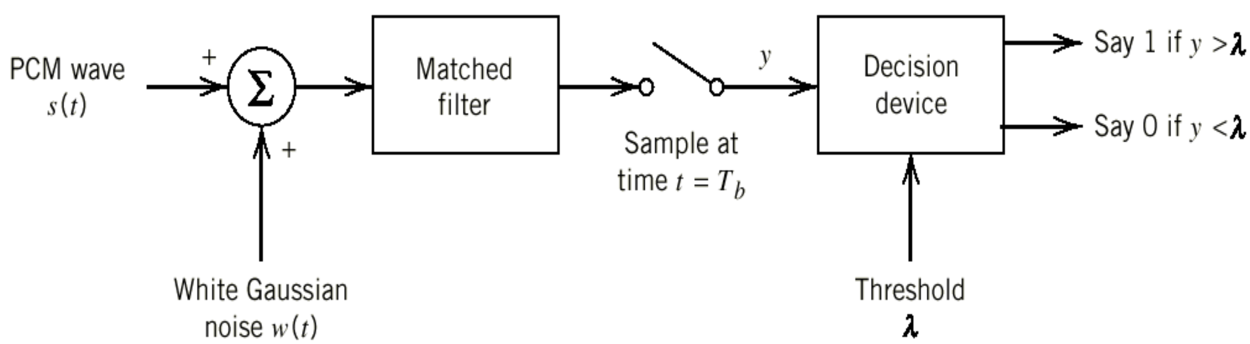


Fig: Receiver for Baseband Transmission

The noisy signal x(t) is passed through the matched filter. Then the o/p of matched filter sampled at time t = T. If this o/p sample value is more than threshold ‘λ’, the receiver make a decision as symbol 0.

If the o/p sample value is exactly equals to threshold value, the receiver may choose it as 1 (or) 0. Here there two possible kinds of errors to be considered

i. Symbol 1 is chosen where a 0 was actually transmitted.
ii. Symbol 0 is chosen where a 1 was actually transmitted.

To determine the average probability of error, we consider these two situations separately.

Symbol '0' was sent

The received signal is expressed as

$$x(t) = -A + w(t), \qquad 0 \le t \le T_b \qquad \text{-------------------- (2)}$$

The output of matched filter is expressed as

$$y = \int_0^{T_b} x(t)\, dt$$
$$= -A + \frac{1}{T_b}\int_0^{T_b} w(t)\, dt \qquad \text{-------------------- (3)}$$

The above equation 3 represents the sample value of random variable. It is characterised as,

- The random variable y is Gaussian distributed with a mean of –A.
- The variance of the random variable y is

$$\sigma_Y^2 = E[(Y + A)^2]$$
$$= \frac{1}{T_b^2} E\left[\int_0^{T_b}\int_0^{T_b} w(t)w(u)\, dt\, du\right]$$
$$= \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} E[w(t)w(u)]\, dt\, du$$
$$= \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} R_W(t, u)\, dt\, du \qquad \text{-------------------- (4)}$$

Where $R_w(t,u)$ is the auto correlation function of the white noise w(t).

$$R_W(t, u) = \frac{N_0}{2}\delta(t - u) \qquad \text{-------------------- (5)}$$

Where $\delta(t - u)$ is the time shifted delta function

$$\sigma_Y^2 = \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} \frac{N_0}{2}\delta(t - u)\, dt\, du$$
$$= \frac{N_0}{2T_b} \qquad \text{-------------------- (6)}$$

Then the conditional probability density function of random variable y where binary '0' is transmitted is

$$f_Y(y|0) = \frac{1}{\sqrt{\pi N_0/T_b}} \exp\left(-\frac{(y+A)^2}{N_0/T_b}\right) \quad \text{-------------------- (7)}$$

This function representation is shown in below fig.(a)

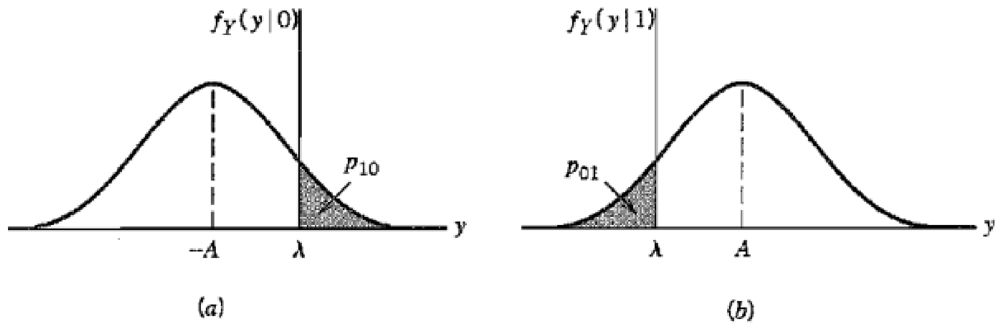


Fig: (a) Pdf of random variable y at matched filter output when '0' is transmitted
Fig: (b) Pdf of random variable y at matched filter output when '1' is transmitted

Let P_{10} denote the conditional probability of error, when symbol 0 was sent. In figure a, the shaded area under the curve $f_y(y/0)$ over the limits λ to ∞ gives the P_{10}

$$\begin{aligned} p_{10} &= P(y > \lambda | \text{symbol 0 was sent}) \\ &= \int_\lambda^\infty f_Y(y|0)\, dy \\ &= \frac{1}{\sqrt{\pi N_0/T_b}} \int_\lambda^\infty \exp\left(-\frac{(y+A)^2}{N_0/T_b}\right) dy \end{aligned} \quad \text{-------------------- (8)}$$

Introduce the complementary error function for simplification of equation 8

$$\text{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty \exp(-z^2)\, dz \quad \text{-------------------- (9)}$$

Simplify the equation 8, let us take

$$z = \frac{y+A}{\sqrt{N_0/T_b}}$$

$$\begin{aligned} p_{10} &= \frac{1}{\sqrt{\pi}} \int_{(A+\lambda)/\sqrt{N_0/T_b}}^\infty \exp(-z^2)\, dz \\ &= \frac{1}{2} \text{erfc}\left(\frac{A+\lambda}{\sqrt{N_0/T_b}}\right) \end{aligned} \quad \text{-------------------- (10)}$$

Symbol 1 was sent

The conditional probability density function of random variable 'y' where binary '1' is transmitted is

$$f_Y(y|1) = \frac{1}{\sqrt{\pi N_0/T_b}} \exp\left(-\frac{(y-A)^2}{N_0/T_b}\right) \quad \text{-------------------- (11)}$$

This function representation is shown in figure b

Let P_{01} denote the conditional probability error, when symbol 1 was transmitted. In figure b, the shaded area under the curve $f_y(y/1)$ over the limits. $-\infty$ to λ gives P_{01}.

$$\begin{aligned} p_{01} &= P(y < \lambda | \text{symbol 1 was sent}) \\ &= \int_{-\infty}^{\lambda} f_Y(y|1)\, dy \\ &= \frac{1}{\sqrt{\pi N_0/T_b}} \int_{-\infty}^{\lambda} \exp\left(-\frac{(y-A)^2}{N_0/T_b}\right) dy \end{aligned} \quad \text{-------------------- (12)}$$

For the simplification of eq. 12, we take

$$z = \frac{A - y}{\sqrt{N_0/T_b}}$$

Interchanging the limits

$$p_{01} = \frac{1}{\sqrt{\pi}} \int_{(A-\lambda)/\sqrt{N_0/T_b}}^{\infty} \exp(-z^2)\, dz \quad \text{-------------------- (13)}$$

From equation 9 & 13 is written as

$$p_{01} = \frac{1}{2} \operatorname{erfc}\left(\frac{A-\lambda}{\sqrt{N_0/T_b}}\right) \quad \text{-------------------- (14)}$$

Therefore the average probability of symbol error, P_e is

$$\begin{aligned} P_e &= p_0 p_{10} + p_1 p_{01} \\ &= \frac{p_0}{2} \operatorname{erfc}\left(\frac{A+\lambda}{\sqrt{N_0/T_b}}\right) + \frac{p_1}{2} \operatorname{erfc}\left(\frac{A-\lambda}{\sqrt{N_0/T_b}}\right) \end{aligned}$$

If symbol 0 & 1 are equiprobable, then $p_1 = p_0 = \frac{1}{2}$

Take the optimum value of threshold is equal to zero

i.e. $$\lambda_{opt} = 0$$

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\frac{A}{\sqrt{N_0/T_b}}\right) \qquad \text{-------------------- (15)}$$

The energy of transmitted signal, E_b is

$$E_b = A^2 T_b$$

$$A = \sqrt{E_b/T_b} \qquad \text{------------------- (16)}$$

Substitute eq. 16 in 15

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \qquad \text{------------------- (17)}$$

The above expression gives the average probability of symbol error. (or) This equation gives the probability error of matched filter.

INTER SYMBOL INTERFERENCE:

This is another source of bit errors in base band data transmission. Suppose the channel is in dispersive nature, the present pulse is affected by the adjacent pulses, this type of interference is called as Inter Symbol Interference.

Let us consider a base band binary data transmission system in order to describe ISI mathematically.

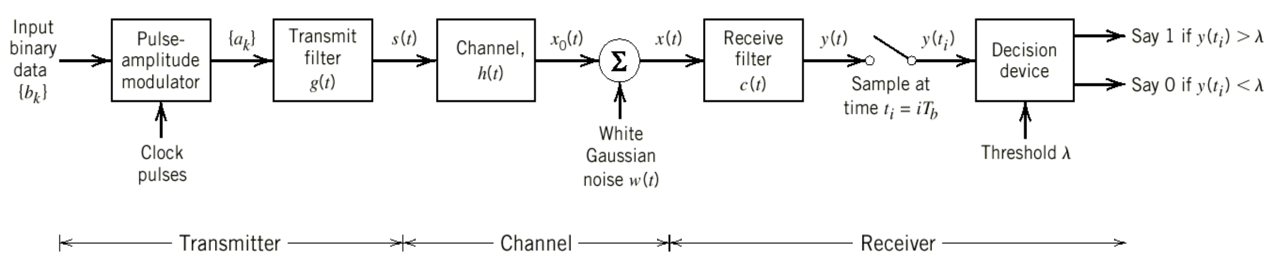


FIG: Base band binary data transmission system

The incoming binary service $\{b_k\}$ consists of symbols 1 & 0, each with T_b duration. It is given to Pulse Amplitude Modulator, it modifies binary sequence into a new sequence of short pulses, whose amplitude a_k is represented in polar form

$$a_k = \begin{cases} +1, & \text{if symbol } b_k \text{ is } 1 \\ -1, & \text{if symbol } b_k \text{ is } 0 \end{cases} \qquad \text{-------------------- (1)}$$

These are given to transmit filter, whose impulse response is g(t), producing the transmitted signal i.e.

$$s(t) = \sum_k a_k g(t - kT_b) \qquad \text{-------------------- (2)}$$

This signal s(t) is modified and transmission over the channel whose impulse response h(t). The channel adds the white noise $\omega(t)$. Then the noisy signal x(t) is then passed through the receiving filter output y(t) sampled & given to decision device. Sampled value is more than the threshold we will say it as binary '1'. If the sampled value is lower than the threshold we will say it as binary '0'.

The receiver filter output is written as

$$y(t) = \mu \sum_k a_k p(t - kT_b) + n(t) \qquad \text{-------------------- (3)}$$

Where μ is the scaling factor & the pulse p(t) is to be defined. For precise value, we have to consider transmission delay to, but in simplification we consider $t_0=0$.

The scaled pulse μ.p(t) is obtained by double convolution of impulse response of transmitter g(t), the impulse response of channel h(t) and impulse response of receiving filter c(t).

$$\mu p(t) = g(t) \bigstar h(t) \bigstar c(t) \qquad \text{-------------------- (4)}$$

p(t) is normalized by setting p(0) = 1

The above eq. gives the use of scaling factor μ to obtain amplitude changes in the signal transmission.

Convolution in time domain transformed into multiplication in frequency domain.

$$\mu P(f) = G(f)H(f)C(f) \qquad \text{-------------------- (5)}$$

The received filter o/p y(t) is sampled at time $t_i=iT_b$

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k p[(i-k)T_b] + n(t_i)$$

$$= \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p[(i-k)T_b] + n(t_i) \qquad \text{-------------------- (6)}$$

In the above eq. first term is contribution i^{th} transmitted bit. The second term indicates residual effect of all other transmitted bits on the decoding of the i^{th} bit, this is because presence of pulses at starting & ending of the sampling instants. This is called ISI. The third term indicates noise at the sample time $t=t_i$.

In the absence of noise & interference eq. 6 written as

$$\boldsymbol{y(t_i)=\mu a_i} \qquad \text{-------------------- (7)}$$

Under these ideal conditions, the i^{th} transmitted bit is decoded correctly. In order to minimize the ISI we have to specify the frequency response of the filters and transmitted signal pulse shape.

NYQUIST'S CRITERION FOR DISTORTION LESS BASE BAND BINARY TRANSMISSION:

In order to eliminate ISI, we need to specify frequency response & pulse shape. First, we need to determine the frequency response of the transmit and receive filter for the better reconstruction of binary sequence. For this, receiver does extracting & decoding. The extraction involves sampling the o/p y(t) at time $t=iT_b$. The decoding done from pulse at k=i, i.e. p(t) is shown as

$$p(iT_b - kT_b) = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases} \qquad \text{-------------------- (1)}$$

Where p(0)=1

If P(t) satisfies the condition in eq. 1 the receiver o/p

$$y(t_i)=\mu a_i \quad \text{for all } i$$

The condition in eq. 1 gives the perfect reception in the absence of noise.

From design point of view, it is used to transform eq. 1 in to the frequency domain.

From the Fourier transform

$$P_\delta(f) = f_b \sum_{n=-\infty}^{\infty} P(f - nR_b) \qquad \text{-------------------- (2)}$$

Where f_b is bit rate in bits per second (b/s)

$P_\delta(f)$ Fourier transform of infinite periodic sequence of delta functions of period T_b.

The time domain representation of pulse signal $P_\delta(t)$ is

$$p_\delta(t) = \sum_{n=-\infty}^{\infty} p(nT_b)\,\delta(t-nT_b) \qquad \text{-------------------- (3)}$$

- **Fourier transform of $p_\delta(t)$ becomes,**

$$P_\delta(f) = \int_{-\infty}^{\infty} p_\delta(t) e^{-j2\pi ft}\,dt$$

$$= \int_{-\infty}^{\infty} \left[\sum_{n=-\infty}^{\infty} p(nT_b)\,\delta(t-nT_b) \right] e^{-j2\pi ft}\,dt$$

Let the integer n = i-k

$$P_\delta(f) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} p\left[(i-k)T_b\right] \delta\left[t-(i-k)T_b\right] e^{-j2\pi ft}\, dt$$

Now let us apply the condition of equation 1 to above equation,

$$P_\delta(f) = \begin{cases} \int_{-\infty}^{\infty} p(0)\,\delta(t)\, e^{-j2\pi ft}\, dt & for\ i=k \\ \int_{-\infty}^{\infty} 0\ \ \delta(t)\, e^{-j2\pi ft}\, dt & for\ i \neq k \end{cases}$$

$$\therefore \quad P_\delta(f) = \int_{-\infty}^{\infty} p(0)\,\delta(t)\, e^{-j2\pi ft}\, dt \quad for\ i=k$$

$$= p(0) \int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi ft}\, dt$$

An integration in above equation is the fourier transform of $\delta(t)$, which is 1. Hence,

$$P_\delta(f) = p(0) \quad for\ i=k \qquad \text{... (4)}$$

$$= 1$$

by normalization of p(0).

Hence equation 2 becomes (with $P_\delta(f) = 1$),

$$1 = f_b \sum_{n=-\infty}^{\infty} P(f-nf_b)$$

$$\text{or} \quad \sum_{n=-\infty}^{\infty} P(f-nf_b) = \frac{1}{f_b}$$

Since $\frac{1}{f_b} = T_b$, $\boxed{\sum_{n=-\infty}^{\infty} P(f-nf_b) = T_b}$ -------------------- (5)

The frequency function p(f) eliminates inter symbol interference for samples taken at intervals T_b provided that it satisfy eq. 5.

CORRELATIVE LEVEL CODING:

By adding inter symbol interference to the transmitted signal in a controlled manner, it is possible to achieve a signalling rate equal to the Nyquist rate of 2w symbols per second in a channel of bandwidth W Hertz. Such schemes are called correlative level coding (or) partial response signalling schemes.

The design of these schemes based on the following assumptions, i.e. ISI introduced into transmitted signal is known value, its effect can be taken at the receiver in a deterministic way.

This is the practical method for receiving theoretical maximum signalling rate of 2w sym/sec in a bandwidth of w Hz.

DUOBINARY SIGNALLING:

- The basic idea of correlative level coding can be implemented by using this signalling scheme. Here Duo implies that doubling of the transmission capacity of a straight binary system.
- This type of correlative coding also called as class 1 practical response.

Consider a binary i/p sequence $\{b_k\}$ consisting of uncorrelated binary symbols 1 and 0, each having duration T_b. First this sequence is applied to a pulse amplitude modulator, it produces two level sequence of short pulses, whose amplitude a_k is defined as,

$$a_k = \begin{cases} +1, & if\ symbol\ b_k\ is\ 1 \\ -1, & if\ symbol\ b_k\ is\ 0 \end{cases} \qquad \text{-------------------- (1)}$$

This sequence is next applied to the duo binary encoder, it is converted into a three level o/p, i.e. -2, 0 & +2. This is produced by the following block diagram.

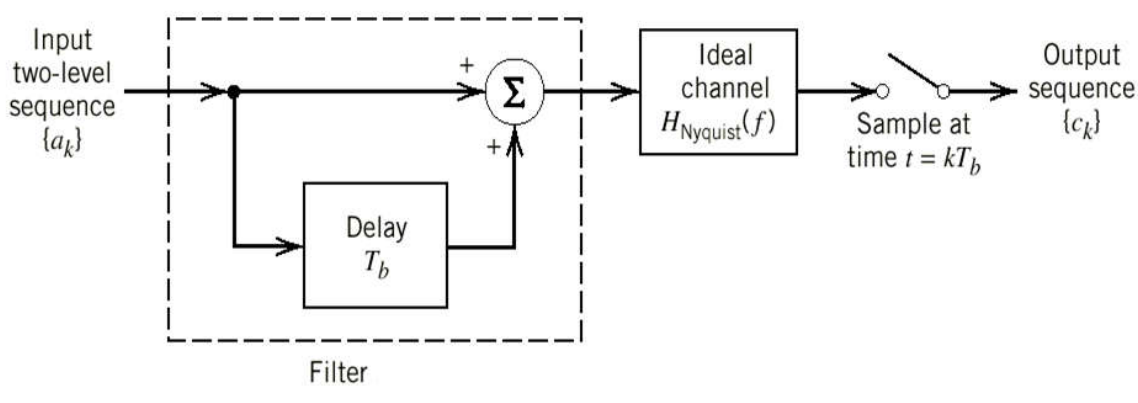


Fig: Duobinary signalling scheme

The two level sequence $\{a_k\}$ is first passed through a simple filter involving a single delay element and summer. For every unit impulse applied to the i/p of this filter, we get two unit impulses spaced T_b seconds apart at the filter o/p.

Therefore the duo binary coder o/p C_k is the sum of present i/p pulse a_k and its previous value a_{k-1}, i.e.

$C_k = a_k + a_{k+1}$ -------------------- (2)

The above eq. describes that, uncorrelated two level sequence converted into three level correlated sequence. This correlation between adjacent pulses is viewed as introducing ISI into a transmitted signal in an artificial manner.

An ideal delay element, producing a delay of T_b seconds, has the frequency response $e^{-i2\pi f T_b}$

The frequency response of simple delay line filter is $1 + e^{-i2\pi f T_b}$. The overall frequency response this filter connected in cascade with ideal Nyquist channel is

$$H_I(f) = H_{Nyquist}(f)[1+\exp(-j2\pi fT_b)]$$
$$= H_{Nyquist}(f)[\exp(j\pi fT_b)+\exp(-j\pi fT_b)]\exp(-j\pi fT_b)$$
$$= 2H_{Nyquist}(f)\cos(\pi fT_b)\exp(-j\pi fT_b) \quad \text{-------------------- (3)}$$

Where $H_1(f)\rightarrow$class 1 partial response.

For ideal Nyquist channel of bandwidth $w=1/2T_b$, we have

$$H_{Nyquist}(f) = \begin{cases} +1, & |f| \le 1/2T_b \\ 0, & otherwise \end{cases} \quad \text{-------------------- (4)}$$

Thus the overall frequency response of the duo binary scheme is half cycle cosine function, i.e.

$$H_I(f) = \begin{cases} 2\cos(\pi fT_b)\exp(-j\pi fT_b), & |f| \le 1/2T_b \\ 0, & otherwise \end{cases} \quad \text{-------------------- (5)}$$

For the above eq. magnitude & phase response spectrums shown below

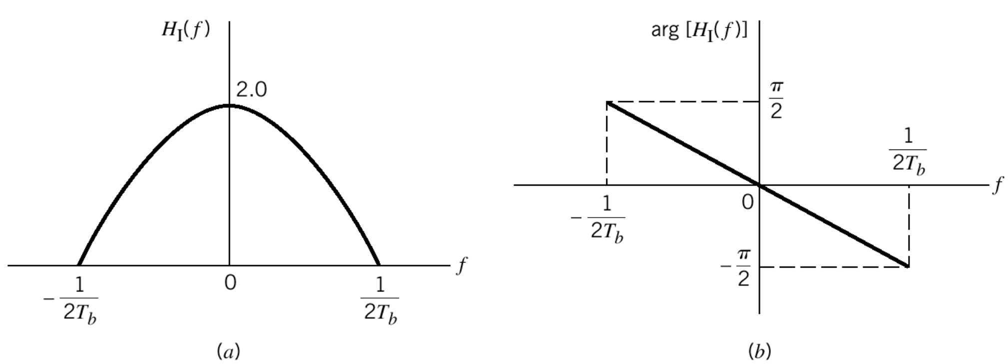


Fig: Frequency response of the duo-binary conversion filter.
(a) Magnitude response. (b) Phase response.

From the frequency response the impulse response of the duo binary signalling scheme is

From eq. 3

$$h_I(t) = \frac{\sin(\pi t / T_b)}{\pi t / T_b} + \frac{\sin[\pi (t - T_b)/T_b]}{\pi (t - T_b)/T_b}$$

$$= \frac{\sin(\pi t / T_b)}{\pi t / T_b} - \frac{\sin(\pi t / T_b)}{\pi (t - T_b)/T_b}$$

$$= \frac{T_b^2 \sin(\pi t / T_b)}{\pi t (T_b - t)} \quad \text{-------------------- (6)}$$

The impulse response spectrum of $h_I(t)$ is shown below

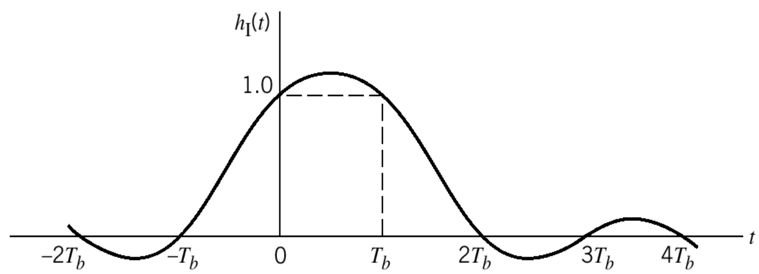


Figure: Impulse response of the duo-binary conversion filter.

In the figure, the response of the input pulse is spread over more than one signalling interval. It is also stated as response in any signalling interval is partial.

The original two level sequence $\{a_k\}$ is detected from the duo binary sequence $\{c_k\}$ by using eq. 2 i.e.

$$\hat{a}_k = c_k - \hat{a}_{k-1} \quad \text{-------------------- (7)}$$

Where $\hat{a}$→Estimate of original pulse at t=kT_b

$\widehat{a_{k-1}}$→Previous Estimate.

The receiver detect correctly if there is no errors in c_k & a_{k-1}.

The technique of using a stored Estimate of the previous symbol, is called "Decision Feedback". It is a just reverse process of delay line filter at transmitter. The major problem in this detection is, either c_k (or) a_{k-1} having error i.e. propagate through the o/p. This error propagation can be eliminated by using proceeding before the duo binary coder this is shown in below fig.

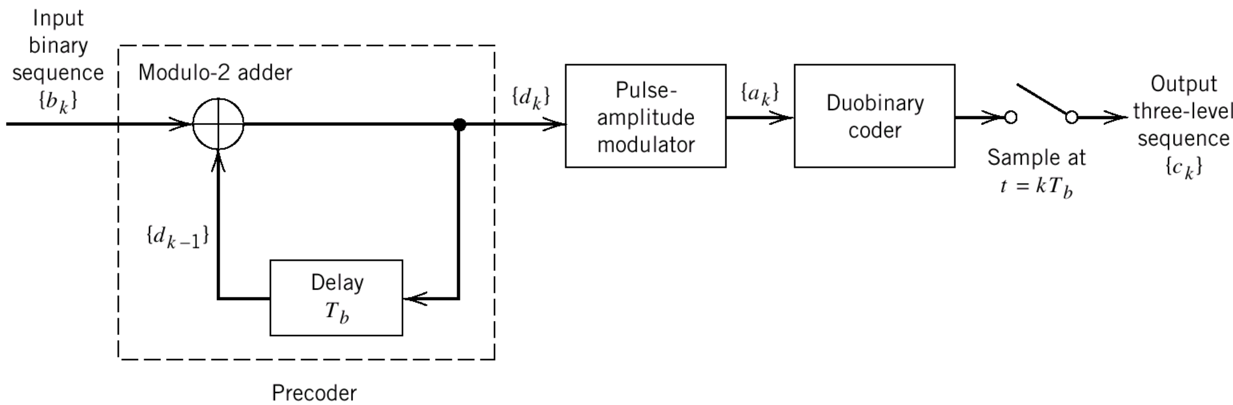


Fig: Precoded Duobinary scheme

From the above diagram, precoder converts i/p binary sequence $\{b_k\}$ into another sequence $\{d_k\}$, i.e.

$$d_k = b_k \oplus d_{k-1} \qquad \text{-------------------- (8)}$$

$$d_k = \begin{cases} symbol\,1, & if\ either\ symbol\ b_k\ or\ symbol\ d_{k-1}\ (but\ not\ both)\ is\,1 \\ symbol\,0, & otherwise \end{cases} \qquad \text{------------ (9)}$$

This precoded sequence is applied to PAM, it generates two level sequence $\{a_k\}$, i.e. $a_k=\pm 1$. After that, this two level sequence given to duo binary coder, it generates three level sequence $\{c_k\}$, i.e. -2, 0, +2

$$c_k = a_k + a_{k-1} \qquad \text{------------------- (10)}$$

$$c_k = \begin{cases} 0, & if\ data\,symbol\ b_k\ is\,1 \\ \pm 2, & if\ symbol\ b_k\ is\ 0 \end{cases}$$

This sequence is applied to the decision device, it is shown in fig.

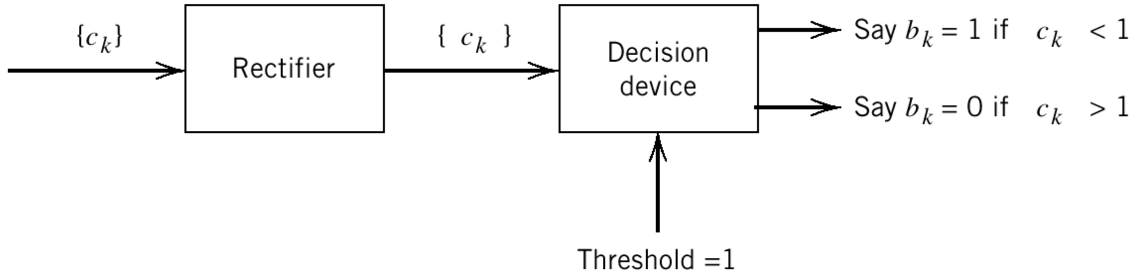


Fig: Detector for recovering original binary sequence from the pre-coded duo-binary coder output.

This device taken the decisions on the duo binary sequence based on following conditions

$$If\ |c_k| < 1,\ \ say\ symbol\ b_k\ is\ 1$$
$$If\ |c_k| > 1,\ \ say\ symbol\ b_k\ is\ 0$$

When $|C_k| = 1$, the receiver simply makes a random guess in favour of symbol 1 or 0.

MODIFIED DUO BINARY SIGNALLING:

In the duo binary signalling technique the frequency response H(f) & the power spectral density of the transmitted pulse is non zero at the origin. This is considered to be an undesirable feature in some applications. This is overcome by using the class IV partial response (or) modified duo binary technique, which involves correlation span of two binary digits (Delay $2T_b$). This is shown in below figure.

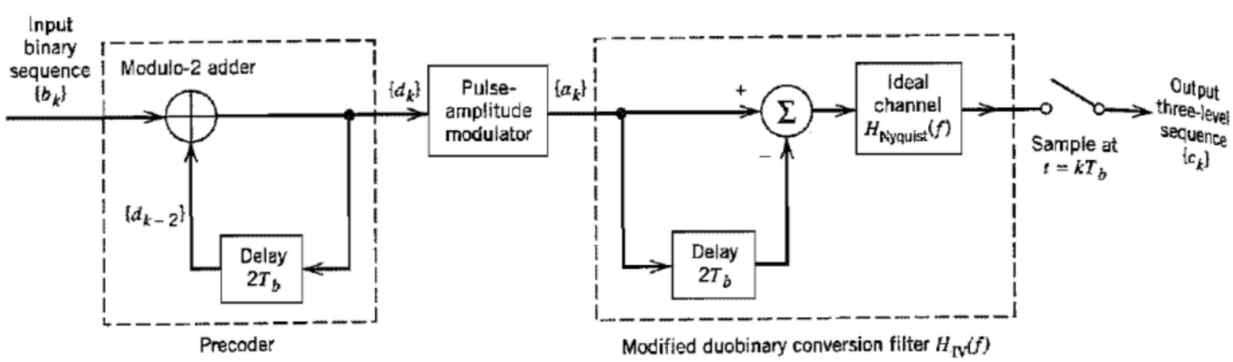


FIGURE 4.16 Modified duobinary signaling scheme.

Modified duo binary encoder involves subtractor & delay $2T_b$. The o/p of the modified duo binary conversion filter is expressed as

$$c_k = a_k - a_{k-2} \qquad \text{-------------------- (1)}$$

Here, the two level sequence ($a_k=\pm1$) converted into three level sequence (-2, 0, +2).

The overall frequency of the delay line filter cascade with the ideal Nyquist channel is

- Spectrum

$$\begin{aligned} H_{IV}(f) &= H_{\text{Nyquist}}(f)[1 - \exp(-j4\pi f T_b)] \\ &= 2jH_{\text{Nyquist}}(f)\sin(2\pi f T_b)\exp(-j2\pi f T_b) \end{aligned}$$

$$H_{IV}(f) = \begin{cases} 2j\sin(2\pi f T_b)\exp(-j2\pi f T_b), & |f| \le 1/2T_b \\ 0, & \text{elsewhere} \end{cases}$$

-------------------- (2)

The corresponding magnitude & phase response is shown in fig.

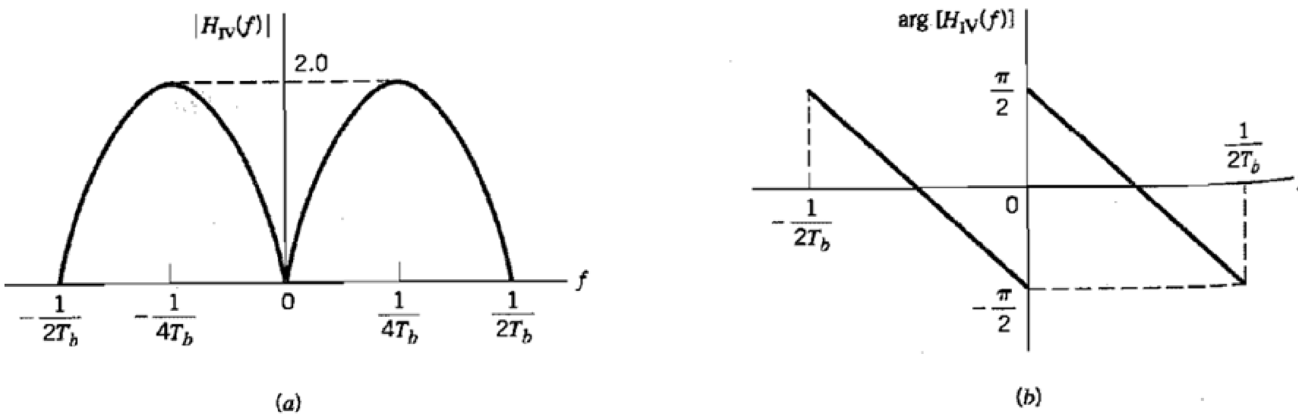


FIGURE Frequency response of the modified duobinary conversion filter. (*a*) Magnitude response. (*b*) Phase response.

From the above spectrum we don't have any frequency response & PSD at zero frequency. It means there is no DC component at o/p.

From the first line of eq. 2, the impulse response of the modified duo binary coder is expressed as below

$$h_{IV}(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin[\pi(t - 2T_b)/T_b)]}{\pi(t - 2T_b)/T_b}$$

$$= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi(t - 2T_b)/T_b}$$

$$= \frac{2T_b^2 \sin(\pi t/T_b)}{\pi t(2T_b - t)} \quad \text{-------------------- (3)}$$

The impulse response is plotted below

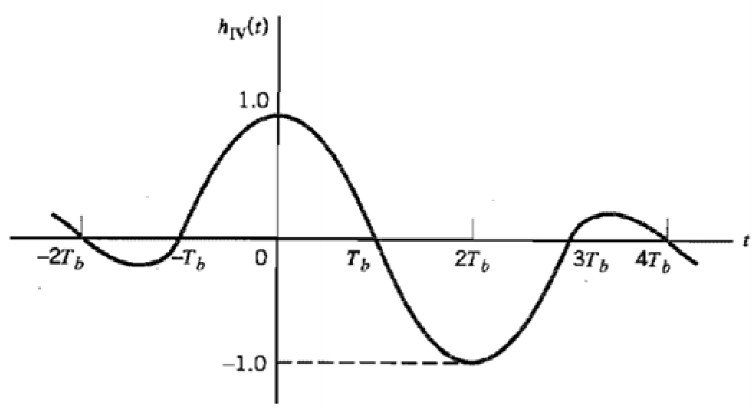


FIGURE Impulse response of the modified duobinary conversion filter.

To eliminate the possibility of error propagation in the detection process of modified duo binary signalling, we use a precoding procedure similar to duo binary scheme. From the fig of modified duo binary, the precoded sequence is expressed as

$$d_k = b_k \oplus d_{k-1} \quad \text{-------------------- (4)}$$

It is given to PAM, it generate two level sequence $\{a_k\}$. This two level sequence a_k is given to modified duo binary coder, it will generate three level sequence based on the following expression

$$c_k = a_k - a_{k-2} \quad \text{-------------------- (5)}$$

For detection, this sequence $\{c_k\}$ given to decision device, it will gives the binary 0 (or) 1 based on the following conditions.

If $|c_k|>1$, say symbol b_k is 1

If $|c_k|<1$, say symbol b_k is 0 -------------------- (6)

If $|c_k|=1$, the receiver makes a random guess in favour of symbol 1 (or) 0.

BASE BAND M-ARY PAM TRANSMISSION:-

In the base band binary PAM transmission, the pulse amplitude modulator produces binary pulses with one of two possible amplitude levels. But in case of base band M-ary PAM transmission, the pulse amplitude modulator produces one of M possible amplitude levels with M>2. This one can be explained with the following example, inarternary system with binary data sequence 0 0 1 0 1 1 0 1 1 1 .

- The above fig a shows the different amplitude levels of gray coded sequence. Fig b shows the electrical representation of given binary data using inarternary system.
- Pulse duration of binary PAM system is denoted by T_b, but in case M-ary PAM transmission the pulse duration $T=2T_b$. We refer 1/T as the signalling rate, which is expressed as sym/sec (or) bauds. In case of M-ary PAM system, 1 baud $=\log_2{}^M$ bits/sec.
- The symbol duration of the M-ary PAM system is related to the symbol duration of binary PAM system is related as

$$T = T_b \log_2 M$$

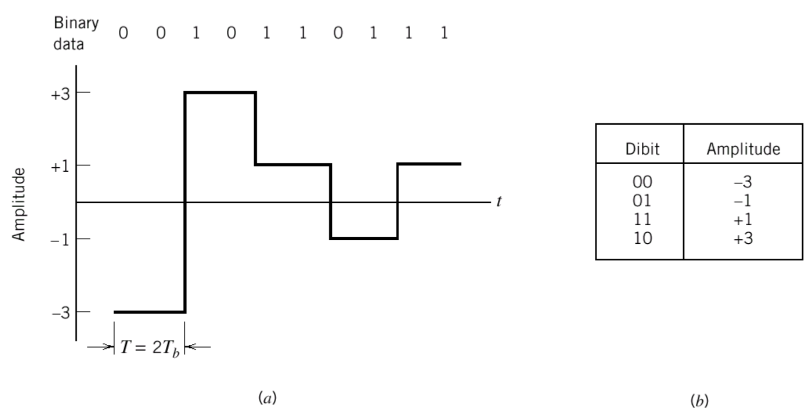


Fig: Output of a quaternary system. (a) Waveform. (b) Representation of the 4 possible dibits, based on Gray encoding.

Therefore, in case of M-ary PAM transmission in a given channel bandwidth, it is possible to transmit $\log_2 M$ times faster than the binary PAM system. The transmitter power must be increased by the factor $\frac{M^2}{\log_2 M}$ compared to a binary PAM system.

- In a base band M-ary PAM transmission system, the input binary sequence converted into a M-level PAM pulse train. The detection process is same as the binary PAM at the receiver section. That is, first this M-level PAM pulses are transmitted over the channel, which is affected by the noise & distortion. The received signal is passed through the receive filter and then sampled at appropriate time intervals. Each sample is compared with the predefined threshold value, and decision is made as to which symbol was transmitted. In order to reduce the bit errors which are introduced due to ISI & noise, we have to design the transmit & receive filter and pulse shape as same as the base band binary PAM system.

QUADRATURE AMPLITUDE MODULATION:

This modulation scheme is also called quadrature carrier multiplexing. This modulation scheme enables two DSB-SC modulated signals to occupy the same transmission band width and therefore it allows for the separation of the two message signals at the receiver output. It is known as band width conversion scheme.

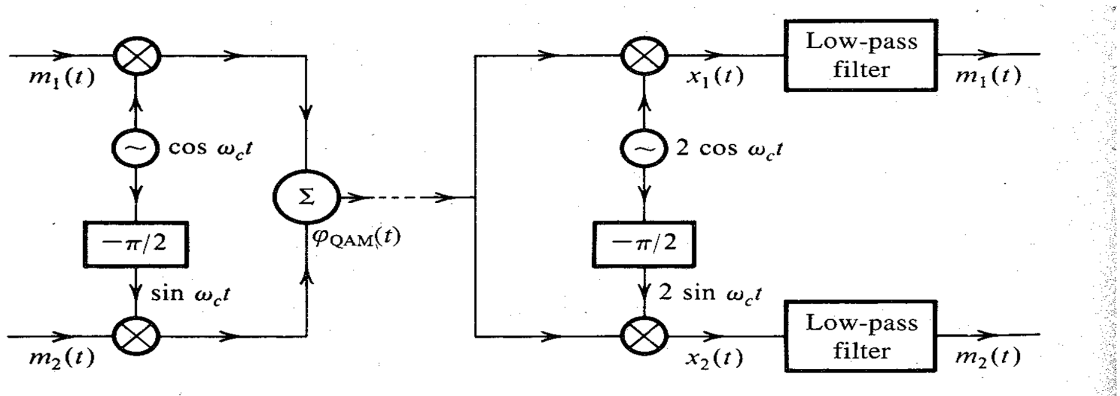


Fig: QAM transmitter and receiver

WORKING OPERATION:

The QAM transmitter consists of two separate balanced modulators (BM) which are supplied with two carrier waves of the same frequency but differing in phase by 90^0.The output of the two balanced modulators are added in the adder and transmitted. The transmitted signal is thus given by,

$$s(t) = Am_1(t)\cos(2\Pi f_c t) + Am_2(t)\sin(2\Pi f_c t)$$

Where $m_1(t)$ and $m_2(t)$ are two different message signals applied to the product modulator. Both $m_1(t)$ and $m_2(t)$ are band limited in the interval $-f_m \le f \le f_m$ then s (t) will occupy a band width of $2f_m$. This band width $2f_m$ is centered at the carrier frequency f_c, where f_m is the band width of message signal $m_1(t)$ and $m_2(t)$.

Hence the multiplexed signal consists of the in-phase component $Am_2(t)$.

The multiplexed signal s (t) from QAM transmitter is applied simultaneously to two separate coherent decoders that are supplied with local carriers of the frequency, but differing in phase by 90 degrees. The output of the detector is $\frac{1}{2}Am_1(t)$ where as the output of the second detector is $\frac{1}{2}Am_2(t)$.

For satisfactory operation of coherent detector, it is essential to maintain coherent phase and frequency relationship between the oscillators used in the QAM transmitter and receiver parts of the system. The QAM finds the application in color television (TV).

Maximum A Posteriori (MAP) and Maximum Likelihood (ML) Decoding:

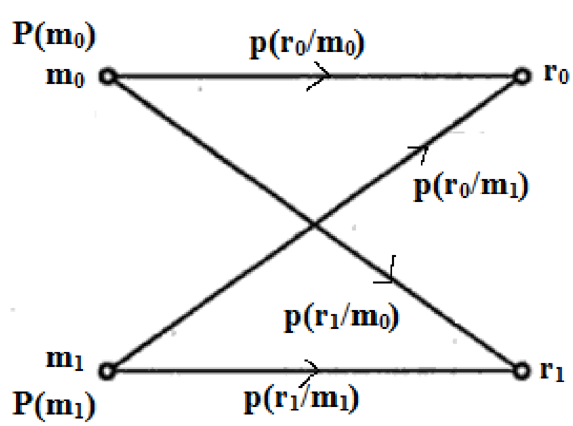
There are two decision based rules on decision threshold

Maximum A Posteriori (MAP) decoding: This rule is used when the a priori probabilities of the transmitted signals are unequal.

Maximum Likelihood (ML) Decoding: This rule is used when the a priori probabilities are equal.

Maximum A Posteriori (MAP) decoding:

In a binary communication system, a 0 or 1 is transmitted. Because of channel noise, a 0 can be received as a 1 and vice-versa. Let m_0 and m_1 denote the events of transmitting 0 and 1, respectively. Let r_0 and r_1 denoted the events of receiving 0 and 1, respectively.



In the absence of noise, r_0 and r_1 is identified for the signals m_0 and m_1 respectively.

In the presence of noise, if m_0 is transmitted, r_1 may be received so there exist an error in making decision.

The transition probabilities

- $P(r_0/m_0)$ – Probability that r_0 is received when m_0 is transmitted.
- $P(r_0/m_1)$ – Probability that r_0 is received when m_1 is transmitted.
- $P(r_1/m_0)$ – Probability that r_1 is received when m_0 is transmitted.
- $P(r_1/m_1)$ – Probability that r_1 is received when m_1 is transmitted.

Maximum A Posteriori probabilities

- $P(m_0/r_0)$ – Probability that m_0 is transmitted when r_0 is received.
- $P(m_1/r_0)$ – Probability that m_1 is transmitted when r_0 is received.
- $P(m_0/r_1)$ – Probability that m_0 is transmitted when r_1 is received.
- $P(m_1/r_1)$ – Probability that m_1 is transmitted when r_1 is received.

$P(m_0)$ and $P(m_1)$ – A Priori Probabilities

If r_0 is received signal

$P(m_0/r_0) > P(m_1/r_0)$ -------->(1)

We will take a decision in favour of message m_0, which is correct decision.

$P(m_0/r_0) < P(m_1/r_0)$ -------->(2)

We will take a decision in favour of message m_1, which is correct decision.

If r_1 is received signal

$P(m_1/r_1) > P(m_0/r_1)$ -------->(3)

We will take a decision in favour of message m_1, which is correct decision.

$P(m_1/r_1) < P(m_0/r_1)$ -------->(4)

We will take a decision in favour of message m_0, which is correct decision.

A receiver which works on the principle MAP that leads to minimum probability of error is defined as the optimum filter.

Multiplying with $P(r_0)$ in both sides of eq. 1 and 2

$P(r_0)\, P(m_0/r_0) > P(r_0)\, P(m_1/r_0)$
$P(m_0)\, P(r_0/m_0) > P(m_1)\, P(r_0/m_0)$ -------->(5)

We will take a decision In favour of message m_0, which is correct decision.

Multiplying with $P(r_1)$ in both sides of eq. 3 and 4

$P(r_1)\, P(m_1/r_1) > P(r_1)\, P(m_0/r_1)$
$P(m_1)\, P(r_1/m_1) > P(m_0)\, P(r_1/m_0)$ -------->(6)

We will take a decision In favour of message m_1, which is correct decision.

To get correct probabilities add eq. 5 and 6,

$P(\,c\,) = P(m_0)\, P(r_0/m_0) + P(m_1)\, P(r_1/m_1)$

Probability of error $Pe = 1 – P\,(\,c\,)$

$Pe = P(m_1)\, P(r_0/m_1) + P(m_0)\, P(r_1/m_0)$

Maximum Likelihood (ML) Decoding:

For Maximum Likelihood Decoding $P(m_0) = P(m_1) = ½$

Note: 1. MAP is an optimum detector and it considers as a general for the maximum likelihood (ML).
2. MAP concept and ML are the same concept for minimizing the error but the difference between them as follows: in ML the events occur are *equiprobable* while in MAP *not equiprobable.*

EYE PATTERN:

It is an experimental tool for observing the combined effect of inter symbol interference and channel noise on the performance of the base band pulse transmission system.

- The eye pattern names comes from human eye because the appearance of binary waves closely equal to human eye.
- The interior portion of the eye pattern is called the eye opening.

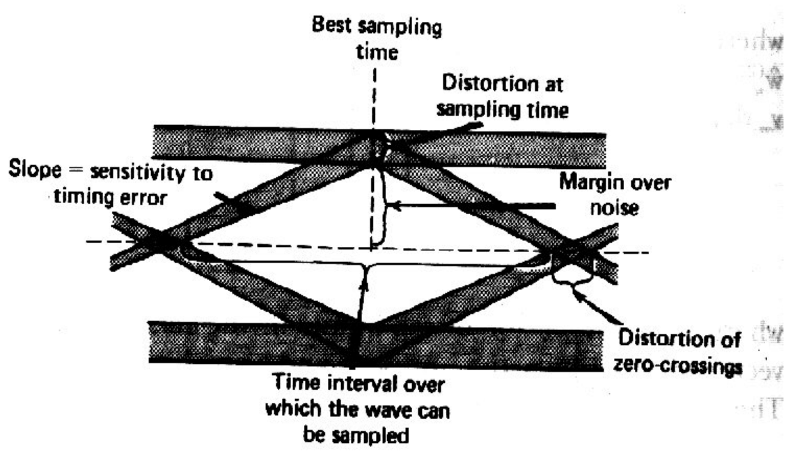


Figure 7.29 Interpretation of the eye pattern.

- The width of the eye opening define the time interval over which the received signal can be sampled without error from ISI.
- The best sampling time is at which the opening eye is widest.
- The height of the eye opening at a specified sampling time, defines the noise margin of the system.
- The sensitivity of the system to timing error is determined by the rate of closure of the eye as the sampling time is varied.