UNIT-IV
BRIDGES

Syllabus Contents:

Review of DC Bridges: Wheatstone bridge, Wein Bridge, errors and precautions in using bridges, AC bridges: Measurement of inductance-Maxwell‟s bridge, Anderson Bridge. Measurement of capacitance - Schearing Bridge. Kelvin Bridge, Q-meter, EMI and EMC, Interference and noise reduction techniques

INTRODUCTIPON:

A bridge circuit in its simplest form consists of a network of four resistance arms forming a closed circuit, with a de source of current applied to two opposite junctions and a current detector connected to the other two junctions, as shown in Fig.1

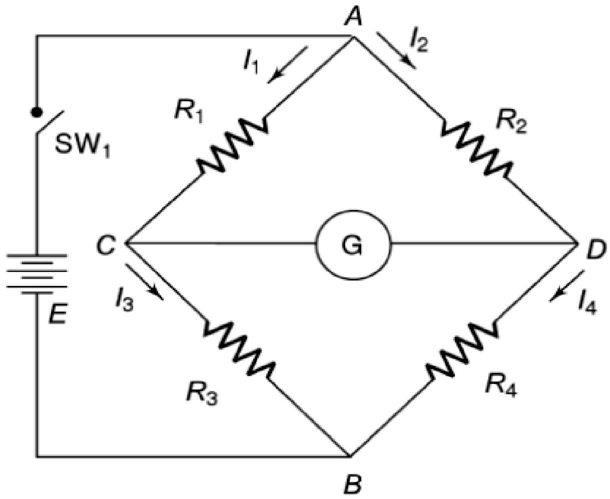


Figure 1: Wheatstone's Bridge

Bridge circuits are extensively used for measuring component values such as *R*, Land C. Since the bridge circuit merely compares the value of an unknown component with that of an accurately known component, its measurement accuracy can be very high. This is because the readout of this comparison is based on the null indication at bridge balance, and is essentially independent of the characteristics of the null detector.

The measurement accuracy is therefore directly related to the accuracy of the bridge component and not to that of the null indicator used. The basic dc bridge is used for accurate measurement of resistance and is called Wheatstone's bridge.

WHEATSTONE'S BRIDGE (MEASUREMENT OF RESISTANCE):

Wheatstone's bridge is the most accurate method available for measuring resistances and is popular for laboratory use. The circuit diagram of a typical Wheatstone bridge is given in Figure 1. The source of emf and switch is connected to points *A* and *B*, while a sensitive current indicating meter, the galvanometer, is connected to points C and *D*.

The galvanometer is a sensitive microammeter, with a zero center scale. When there is no current through the meter, the galvanometer pointer rests at 0, i.e. mid scale. Current in one direction causes the pointer to deflect on one side and current in the opposite direction to the other side.

When SW1 is closed, current flows and divides into the two arms at point *A*, i.e. I_1 and 1_2. The bridge is balanced when there is no current through the galvanometer, or when the potential difference at points C and *D* is equal, i.e. the potential across the galvanometer is zero.

To obtain the bridge balance equation, we have from the Figure 1.

$$I_1R_1 = I_2R_2 \qquad (1)$$

For the galvanometer current to be zero, the following conditions should be satisfied.

$$I_1 = I_3 = \frac{E}{R_1 + R_3} \qquad (2)$$

$$I_2 = I_4 = \frac{E}{R_2 + R_4} \qquad (3)$$

Substituting in Eq. (1)

$$\frac{E \times R_1}{R_1 + R_3} = \frac{E \times R_2}{R_2 + R_4}$$

$$R_1 \times (R_2 + R_4) = (R_1 + R_3) \times R_2$$

$$R_1\, R_2 + R_1\, R_4 = R_1\, R_2 + R_3\, R_2$$

$$R_4 = \frac{R_2\, R_3}{R_1}$$

This is the equation for the bridge to be balanced.

In a practical Wheatstone's bridge, at least one of the resistance is made adjustable, to permit balancing. When the bridge is balanced, the unknown resistance (normally connected at *R4)* may be determined from the setting of the adjustable resistor, which is called a standard resistor because it is a precision device having very small tolerance. Hence

$$R_x = \frac{R_2\, R_3}{R_1} \qquad (4)$$

Problem: Figure 1 consists of the fallowing parameters. R_1 = 10 k, R_2 = 15 k and R_3 = 40 k. Find the unknown resistance R_x.

Solution: From the equation for bridge balance we have

$$R_1\, R_4 = R_2\, R_3, \text{ i.e. } R_1\, R_x = R_2\, R_3$$

Therefore

$$R_x = \frac{R_2\, R_3}{R_1} = \frac{15\text{ k} \times 40\text{ k}}{10\text{ k}} = 60\text{ k}\Omega$$

Sensitivity of a Wheatstone Bridge:

When the bridge is in an unbalanced condition, current flows through the galvanometer, causing a deflection of its pointer. The amount of deflection is a function of the sensitivity of the galvanometer. Sensitivity can be thought of as deflection per unit current.

A more sensitive galvanometer deflects by a greater amount for the same current. Deflection may be expressed in linear or angular units of measure, and sensitivity can be expressed in units of S = mm/µA or degree/µA or radians/µA.

Therefore it follows that the total deflection D is D = S x I, where S is defined above and *I* is the current in microamperes.

Application of Wheatstone's Bridge:

A Wheatstone bridge may be used to measure the dc resistance of various types of wire, either for the purpose of quality control of the wire itself, or of some assembly in which it is used. For example, the resistance of motor windings, transformers, solenoids, and relay coils can be measured.

Wheatstone's bridge is also used extensively by telephone companies and others to locate cable faults. The fault may be two lines shorted together, or a single line shorted to ground.

Limitations of Wheatstone's Bridge:

For low resistance measurement, the resistance of the leads and contacts becomes significant and introduces an error. This can be eliminated by Kelvin's Double bridge.

For high resistance measurements, the resistance presented by the bridge becomes so large that the galvanometer is insensitive to imbalance. Therefore, a power supply has to replace the battery and a dc VTVM replaces the galvanometer.

In the case of high resistance measurements in mega ohms, the Wheatstone's bridge cannot be used.

Another difficulty in Wheatstone's bridge is the change in resistance of the bridge arms due to the heating effect of current through the resistance. The rise in temperature causes a change in the value of the resistance, and excessive current may cause a permanent change in value.

KELVIN'S BRIDGE:

When the resistance to be measured is of the order of magnitude of bridge contact and lead resistance, a modified form of Wheatstone's bridge, the Kelvin bridge is employed.

Kelvin's bridge is a modification of Wheatstone 's bridge and is used to measure values of resistance below 1Ω. In low resistance measurement, the resistance of the leads connecting the unknown resistance to the terminal of the bridge circuit may affect the measurement.

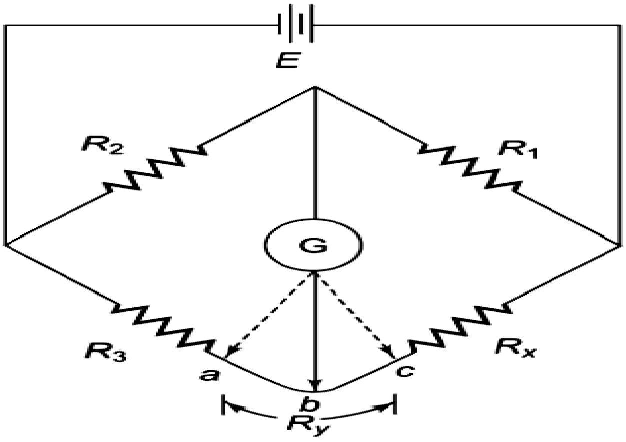


Figure 1: Kelvin's Bridge

Consider the circuit in Figure 1, where R_y represents the resistance of the connecting leads from R_3 to R_x (unknown resistance). The galvanometer can be connected either to point c or to point *a*. When it is connected to point *a*, the resistance R_y the connecting lead is added to the unknown resistance R_x, resulting in too high indication for R_x.

When the connection is made to point c, R_y is added to the bridge arm R_3 and resulting measurement of R_x is lower than the actual value, because now the actual value of R_3 is higher than its nominal value by the resistance R_y.

If the galvanometer is connected to point *b*, in between points c and *a*, in such a way that the ratio of the resistance from c to *b* and that from *a* to *b* equals the ratio of resistances R_1 and R_2.

Then

$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2} \qquad (1)$$

and the usual balance equations for the bridge give the relationship

$$(R_x + R_{cb}) = \frac{R_1}{R_2}(R_3 + R_{ab}) \qquad (2)$$

but $$R_{ab} + R_{cb} = R_y \text{ and } \frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$$

$$\frac{R_{cb}}{R_{ab}} + 1 = \frac{R_1}{R_2} + 1$$

$$\frac{R_{cb} + R_{ab}}{R_{ab}} = \frac{R_1 + R_2}{R_2}$$

$$\text{i.e.} \quad \frac{R_y}{R_{ab}} = \frac{R_1 + R_2}{R_2}$$

$$\text{Therefore} \quad R_{ab} = \frac{R_2 \, R_y}{R_1 + R_2} \quad \text{and as} \quad R_{ab} + R_{cb} = R_y$$

$$\therefore \quad R_{cb} = R_y - R_{ab} = R_y - \frac{R_2 \, R_y}{R_1 + R_2}$$

$$\therefore \quad R_{cb} = \frac{R_1 \, R_y + R_2 \, R_y - R_2 \, R_y}{R_1 + R_2} = \frac{R_1 \, R_y}{R_1 + R_2}$$

Substituting for R_{ab} and R_{cb} in **Eq. (2)**

$$R_x + \frac{R_1 \, R_y}{R_1 + R_2} = \frac{R_1}{R_2}\left(R_3 + \frac{R_2 \, R_y}{R_1 + R_2} \right)$$

$$R_x + \frac{R_1 \, R_y}{R_1 + R_2} = \frac{R_1 \, R_3}{R_2} + \frac{R_1 \, R_2 \, R_y}{R_2 \, (R_1 + R_2)}$$

$$\text{Hence} \quad R_x = \frac{R_1 \, R_3}{R_2} \qquad (3)$$

Equation (3) is the usual Wheatstone's balance equation and it indicates that the effect of the resistance of the connecting leads from point *a* to point c has been eliminated by connecting the galvanometer to an intermediate position, *b*.

The above principle forms the basis of the construction of Kelvin's Double Bridge, popularly known as Kelvin's Bridge. It is a Double bridge because it incorporates a second set of ratio arms. Figure 2 shows a schematic diagram of Kelvin's double bridge.

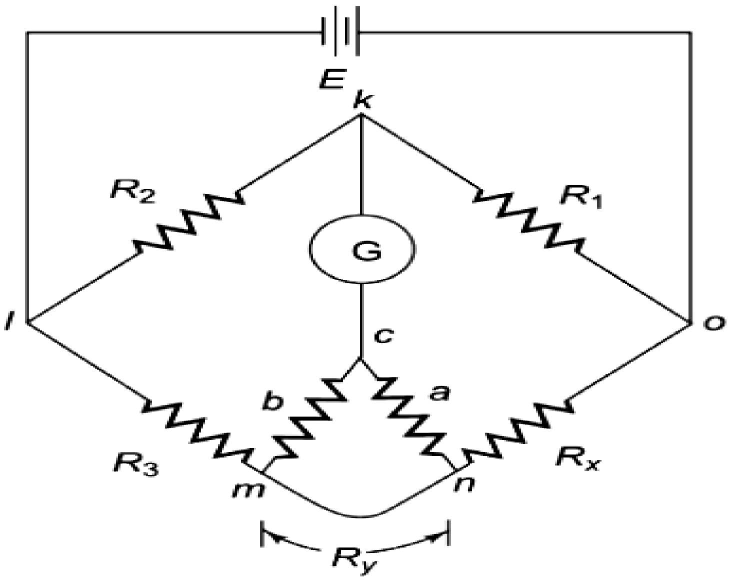


Figure 2: Kelvin's Double bridge

The second set of arms, a and b, connects the galvanometer to a point c at the appropriate potential between m and n connection, i.e. R_y. The ratio of the resistances of arms a and b is the same as the ratio of R_1 and R_2. The galvanometer indication is zero when the potentials at *k* and c are equal.

$$\therefore \quad E_{lk} = E_{lmc}$$

$$\text{But} \quad E_{lk} = \frac{R_2}{R_1 + R_2} \times E \qquad (4)$$

$$\text{and} \quad E = I\left(R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right)$$

Substituting for E in Eq. (4),

$$\text{we get} \quad E_{lk} = \frac{R_2}{R_1 + R_2} \times I\left(R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right) \qquad (5)$$

$$\text{Similarly,} \quad E_{lmc} = I\left(R_3 + \frac{b}{a+b}\left[\frac{(a+b)R_y}{a+b+R_y}\right]\right) \qquad (6)$$

$$\text{But} \quad E_{lk} = E_{lmc}$$

$$\text{i.e.} \quad \frac{I R_2}{R_1 + R_2}\left(R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right) = I\left[R_3 + \frac{b}{a+b}\left\{\frac{(a+b)R_y}{a+b+R_y}\right\}\right]$$

$$\therefore \quad R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \frac{R_1 + R_2}{R_2}\left(R_3 + \frac{b R_y}{a+b+R_y}\right)$$

$$\therefore \quad R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \left(\frac{R_1}{R_2} + 1\right)\left(R_3 + \frac{b R_y}{a+b+R_y}\right)$$

$$R_x + \frac{(a+b)R_y}{a+b+R_y} + R_3 = \frac{R_1 R_3}{R_2} + R_3 + \frac{b R_1 R_y}{R_2(a+b+R_y)} + \frac{b R_y}{a+b+R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_1 R_y}{R_2(a+b+R_y)} + \frac{b R_y}{a+b+R_y} - \frac{(a+b)R_y}{a+b+R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_1 R_y}{R_2(a+b+R_y)} + \frac{b R_y - aR_y - bR_y}{a+b+R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_1 R_y}{R_2(a+b+R_y)} - \frac{aR_y}{a+b+R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_y}{(a+b+R_y)}\left(\frac{R_1}{R_2} - \frac{a}{b}\right)$$

$$\text{But} \quad \frac{R_1}{R_2} = \frac{a}{b}$$

$$\text{Therefore,} \quad R_x = \frac{R_1 R_3}{R_2}$$

This is the usual equation for Kelvin's bridge. It indicates that the resistance of the connecting lead R_y has no effect on the measurement, provided that the ratios of the resistances of the two sets of ratio arms are equal. In a typical Kelvin's bridge the range of a resistance covered is 1 - 0.00001 Ω (10 µohm) with an accuracy of ± 0.05% to ± 0.2%.

Problem: *If in Figure 1 the ratio of R_a to R_b is 1000 Ω, R_1 is 5Ω and $R_1 = 0.5\ R_2$. What is the value of R_x.*

Solution: Resistance R_x can be calculated as follows.

$$\frac{R_x}{R_2} = \frac{R_b}{R_a}$$

Therefore,

$$\frac{R_x}{R_2} = \frac{R_b}{R_a} = \frac{1}{1000}$$

Since

$$R_1 = 0.5\ R_2,\ R_2 = 5/0.5 = 10\ \Omega.$$

Therefore

$$R_x/10 = 1/1000 = 10 \times 1/1000 = 1/100 = 0.01\ \Omega.$$

Now, $R_x = 0.1\Omega$

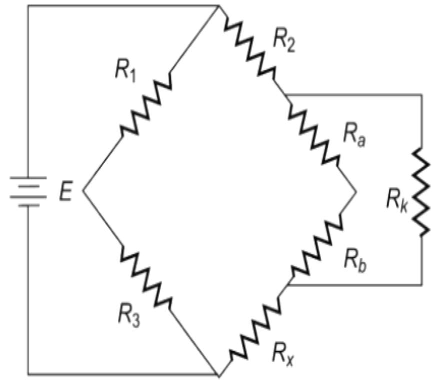


Fig.1 Kelvin's bridge

PRACTICAL KELVIN'S DOUBLE BRIDGE:

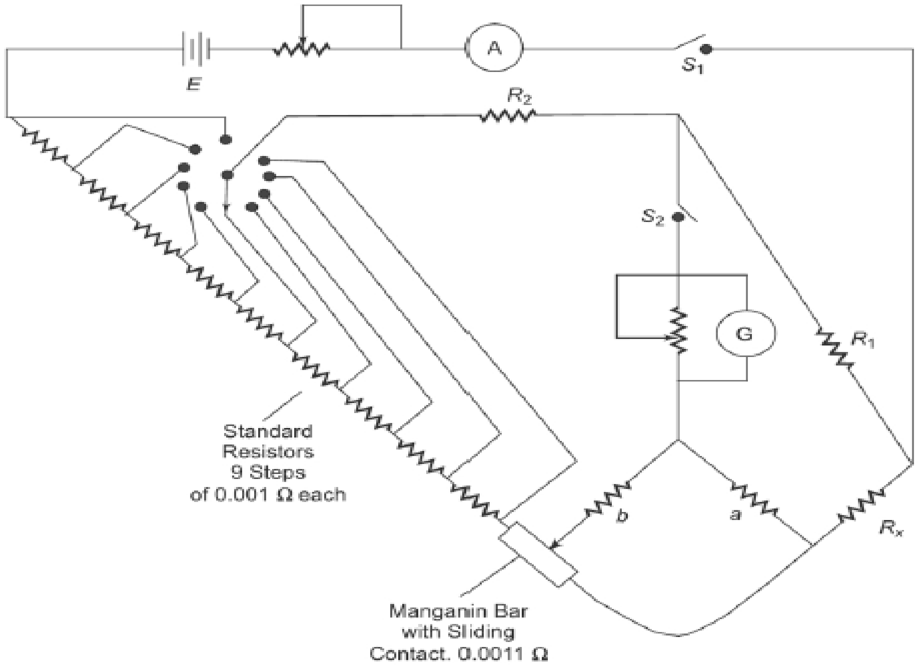


Fig. Practical kelvin's bridge

Figure shows a commercial Kelvin's bridge capable of measuring resistances from 10 - 0.00001 Ω.

Contact potential drops in the circuit may cause large errors. This effect is reduced by varying a standard resistance consisting of nine steps of 0.001 Ω each, plus a calibrated manganin bar of 0.0011 Ω with a sliding contact. When both contacts are switched to select the suitable value of standard resistance, the voltage drop between the ratio arm connection points is changed, but the total resistance around the battery circuit is unchanged.

This arrangement places any contact resistance in series with the relatively high resistance value of the ratio arms, rendering the contact resistance effect negligible. The ratio *R/R2* in Figure is selected such that a relatively large part of the standard resistance is used and hence R_x is determined to the largest possible number of significant figures. Therefore, measurement accuracy Improves.

AC BRIDGES:

Impedances at AF or RF are commonly determined by means of an ac Wheatstone bridge. The diagram of an ac bridge is given in Figure1. This bridge is similar to a dc bridge, except that the bridge arms are impedances.

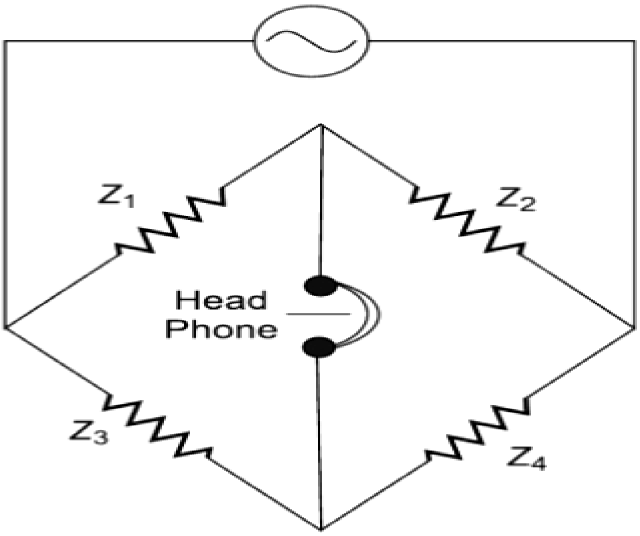


Fig. 1. ac Wheatstone's bridge

The bridge is excited by an ac source rather than dc and the galvanometer is replaced by a detector, such as a pair of headphones, for detecting ac. When the bridge is balanced,

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$$

where Z_1, Z_2, Z_3 and Z_4 are the impedances of the arms, and are vector complex quantities that possess phase angles. It is thus necessary to adjust both the magnitude and phase angles of the impedance arms to achieve balance, i.e. the bridge must be balanced for both the reactance and the resistive component.

WIEN'S BRIDGE (MEASSUREMENT OF FREQUENCY):

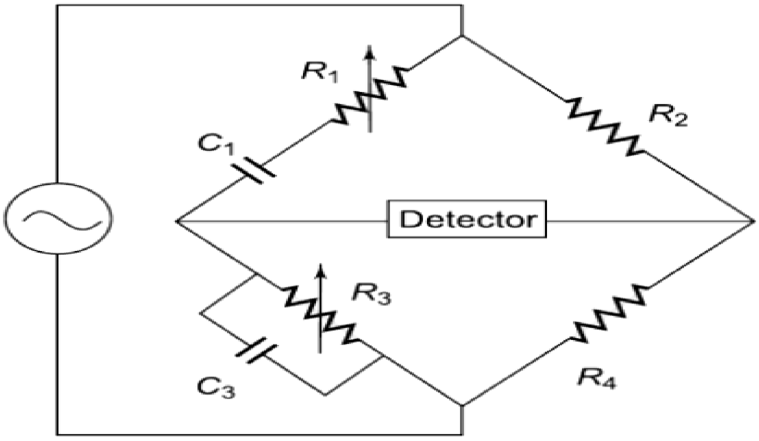


Fig.1. Wein's bridge

The Wien bridge shown in Fig.1 has a series *RC* combination in one arm and a parallel combination in the adjoining arm. Wien's bridge in its basic form is designed to measure frequency. It can also be used for the measurement of an unknown capacitor with great accuracy.

The impedance of one arm is

$$Z_1 = R_1 - j/\omega C_1.$$

The admittance of the parallel arm is

$$Y_3 = 1/R_3 + j\ \omega\ C_3.$$

Using the bridge balance equation,
we have $Z_1 Z_4 = Z_2 Z_3$
Therefore, $Z_1 Z_4 = Z_2/Y_3$ i.e. $Z_2 = Z_1 Z_4 Y_3$

$$\therefore \quad R_2 = R_4\left(R_1 - \frac{j}{\omega C_1}\right)\left(\frac{1}{R_3} + j\omega C_3\right)$$

$$R_2 = \frac{R_1 R_4}{R_3} - \frac{j R_4}{\omega C_1 R_3} + j\omega C_3 R_1 R_4 + \frac{C_3 R_4}{C_1}$$

$$R_2 = \left(\frac{R_1 R_4}{R_3} + \frac{C_3 R_4}{C_1}\right) - j\left(\frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4\right)$$

Equating the real and imaginary terms we have

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{C_3 R_4}{C_1} \quad \text{and} \quad \frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4 = 0$$

$$\text{Therefore} \quad \frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \qquad (1)$$

$$\text{and} \quad \frac{1}{\omega C_1 R_3} = \omega C_3 R_1 \qquad (2)$$

$$\therefore \quad \omega^2 = \frac{1}{C_1 R_1 R_3 C_3}$$

$$\omega = \frac{1}{\sqrt{C_1 R_1 C_3 R_3}}$$

$$\text{as} \quad \omega = 2\pi f$$

$$\therefore \quad f = \frac{1}{2\pi\sqrt{C_1 R_1 C_3 R_3}} \qquad (3)$$

The two conditions for bridge balance, Eq. (1) and Eq. (2), result in an expression determining the required resistance ratio R_2/R_4 and another expression determining the frequency of the applied voltage. If we satisfy Eq. (1) and also excite the bridge with the frequency of Eq. (3), the bridge will be balanced.

In most Wien bridge circuits, the components are chosen such that $R_1 = R_3 = R$ *and* $C_1 = C_3 = C$. Equation (1) therefore reduces to $R_2/R_4 = 2$ and Eq. (3) *to* $f = 1/2\pi RC$, which is the general equation for the frequency of the bridge circuit.

The bridge is used for measuring frequency in the audio range. The audio range is normally divided into 20 - 200 - 2 k - 20 kHz ranges. In this case, the resistances can be used for range changing and capacitors C_1 and C_3 for fine frequency control within the range. An accuracy of 0.5% - 1% can be readily obtained using this bridge.

Problem 1: A Wien bridge circuit consists of the following:
R_1=4.7kΩ, C_1 =5nf, R_2 = 20 kΩ, C_3 = 10 nf, R3= 10kΩ, R_4 = 100 kΩ.
Determine the frequency of the circuit.
Solution: The frequency is given by the equation

$$f = \frac{1}{2\pi\sqrt{C_1 \; R_1 \; R_3 \; C_3}}$$

$$f = \frac{1}{2\pi\sqrt{5\times10^{-9}\times4.7\times10^{3}\times10\times10^{-9}\times10\times10^{3}}}$$

$$f = \frac{1}{2\pi\sqrt{5\times10^{-10}\times4.7}}$$

$$f = \frac{10^5}{2\pi\sqrt{5\times4.7}} = 3.283\ \text{kHz}$$

Problem 2: Find the equivalent parallel resistance and capacitance that causes a Wien bridge to null with the following component values.

R_1 = 3.1 kΩ, C_1= 5.2 pF, R_2 = 25 kΩ, f = 2.5 kHz, R_4 = 100kΩ

Solution:

Given $\omega = 2\,\pi f = 2 \times 3.14 \times 2500 = 15.71$ k rad/s.

Substituting the value of C_3 from Eq. (2) in Eq. (1) we get,

$$R_3 = \frac{R_4}{R_2}\left(R_1 + \frac{1}{\omega^2 R_1 C_1^2}\right)$$

$$= \frac{100\text{ k}}{25\text{ k}}\left(3.1\text{ k} + \frac{1}{(15.71\text{ k})^2 \times 3.1\text{ k} \times (5.2\times10^{-6})^2}\right)$$

$$= 12.4\text{ k}\Omega$$

$$C_3 = \frac{R_2}{R_4}\left(\frac{C_1}{1+\omega^2 R_1^2 C_1^2}\right)$$

$$= \frac{25\text{ k}}{100\text{ k}}\left(\frac{5.2\times10^{-6}}{1+(15.71\text{ k})^2\times(3.1\text{ k})^2\times(5.2\times10^{-6})^2}\right)$$

$$= 1.3\times10^{-6}\left(\frac{1}{1+64133.07}\right)$$

$$= 20.3\text{ pf}$$

Problem 3: An ac bridge with terminals ABCD has in Arm AB a resistance of 800 Ω in parallel with a capacitor of 0.5 µF, Arm BC - a resistance of 400 Ω in series with a capacitor of 1 µF, Arm CD - a resistance of 1000 Ω, Arm DA - a pure resistance R.

(a) Determine the value of frequency for which the bridge is balanced

(b) Calculate the value of R required to produce balance.

Solution:

The bridge configuration is of Wien Bridge.

Given : C_1= 0.5 µF, $R_1 = 800\ \Omega$
C_2= 1.0 µF, $R_2 = 400\ \Omega$
R_4= 1000 Ω, $R_3 = R = ?$

Step 1 : Frequency calculated by

$$f = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}}$$

$$= \frac{1}{2\pi\sqrt{800\times0.5\,\mu\text{F}\times400\times1\,\mu\text{F}}}$$

$$= \frac{1}{2\pi\sqrt{800\times400\times0.5\times10^{-12}}}$$

$$= \frac{10^6}{2\pi\sqrt{800\times200}}$$

$$= \frac{10^6}{2\pi\times400} = \frac{1000\text{ kHz}}{2\times3.14\times400} = \frac{1000}{314\times8} = 0.398\text{ kHz}$$

Step 2 : Also given,

$$\frac{R_2}{R_1} + \frac{C_1}{C_2} = \frac{R_4}{R_3}$$

$$\therefore \quad \frac{400}{800} + \frac{0.5\ \mu F}{1\ \mu F} = \frac{1000}{R}$$

$$\therefore \quad 0.5 + 0.5 = \frac{1000}{R}$$

$$\therefore \quad R = 1000\ \Omega$$

MAXWELL'S BRIDGE (MEARUMENT OF INDUCTANCE):

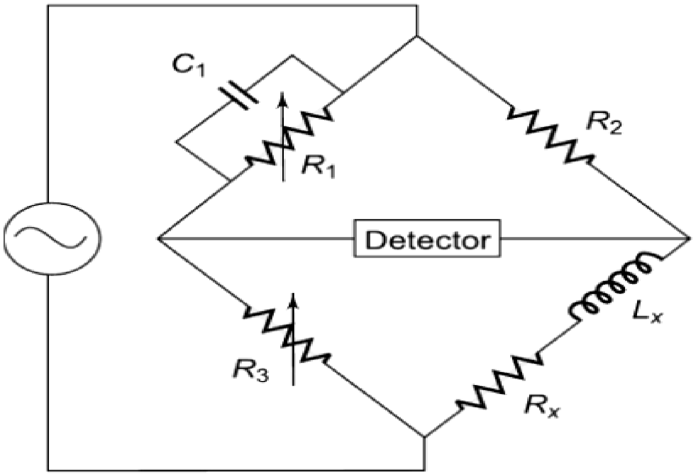


Fig.1 Maxwell's bridge

Maxwell's bridge, shown in Fig.1, measures an unknown inductance in terms of a known capacitor. The use of standard arm offers the advantage of compactness and easy shielding. The capacitor is almost a loss-less component.

One arm has a resistance R_1 in parallel with C_1, and hence it is easier to write the balance equation using the admittance of arm 1 instead of the impedance.

The general equation for bridge balance is

$$Z_1 Z_x = Z_2 Z_3$$

$$\text{i.e.} \quad Z_x = \frac{Z_2 Z_3}{Z_1} = Z_2 Z_3 Y_1 \qquad (1)$$

Where $\quad Z_1 = R_1$ in parallel with C_1 i.e. $Y_1 = \dfrac{1}{Z_1}$

$$Y_1 = \frac{1}{R_1} + j\omega C_1$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$Z_x = R_x$ in series with $L_x = R_x + j\omega L_x$

From Eq. (1), we have

$$R_x + j\omega L_x = R_2 R_3 \left(\frac{1}{R_1} + j\omega C_1\right)$$

$$R_x + j\omega L_x = \frac{R_2 R_3}{R_1} + j\omega C_1 R_2 R_3$$

Equating real terms and imaginary terms we have

$$R_x = \frac{R_2 R_3}{R_1} \text{ and } L_x = C_1 R_2 R_3 \qquad (2)$$

$$\text{Also} \quad Q = \frac{\omega L_x}{R_x} = \frac{\omega C_1 R_2 R_3 \times R_1}{R_2 R_3} = \omega C_1 R_1$$

Maxwell's bridge is limited to the measurement of low Q values (1 - 10). The measurement is independent of the excitation frequency. The scale of the resistance can be calibrated to read inductance directly**.**

The Maxwell bridge using a fixed capacitor has the disadvantage that there is an interaction between the resistance and reactance balances. This can be avoided by varying the capacitances, instead of R_2 and R_3, to obtain a reactance balance. However, the bridge can be made to read directly in Q.

The bridge is particularly suited for inductances measurements, since comparison with a capacitor is more ideal than with another inductance. Commercial bridges measure from 1-1000 H, with ± 2% error.

Problem 1: A Maxwell bridge is used to measure an inductive impedance. The bridge constants at balance are C_1 = 0.01 μF, R_1 = 470 kΩ, R_2 = 5.1 kΩ, and R_3 = 100 kΩ. Find the series equivalent of the unknown impedance.

Solution: We need to find R_x and L_x.

$$R_x = \frac{R_2 R_3}{R_1} = \frac{100 \text{ k} \times 5.1 \text{ k}}{470 \text{ k}} = 1.09 \text{ k}\Omega$$

$$\begin{aligned} L_x &= R_2 R_3 C_1 \\ &= 5.1 \text{ k} \times 100 \text{ k} \times 0.01 \ \mu\text{f} \\ &= 5.1 \text{ H} \end{aligned}$$

The equivalent series circuit is shown in below Fig.

Problem 2: The arms of an ac Maxwell's bridge are arranged as follows:

AB and BC are non-reactive resistors of 100 Ω each, DA a standard variable reactor $L_1$of resistance 32.7 Ω and CD consists of a standard variable resistor R in series with a coil of unknown impedance Z, balance was found with L_1 = 50 mH and Z = 1.36R. Find the Rand L of coil.

Solution: Given: R_1 = 32.7 Ω, L_1= 50 mH

R_2 = 1.36Ω, R_3= 100 Ω, R_4 = 100 Ω

Step 1: To find *'r'* and *L2* where *r* is the resistance of the coil

Given that $R_4R_1 = R_3(R_2 + r)$

32.7 x 100 = 100 (l.36 + r)

100(32.7 - 1.36) = 100 r

r = 32.7 - 1.36

r = 31.34Ω

Step 2: To find L_2,

$$L_2 = L_1 \times \frac{R_4}{R_3} = 50 \text{ mH} \times \frac{100}{100}$$

$$L_2 = 50 \text{ mH}$$

HAY'S BRIDGE:

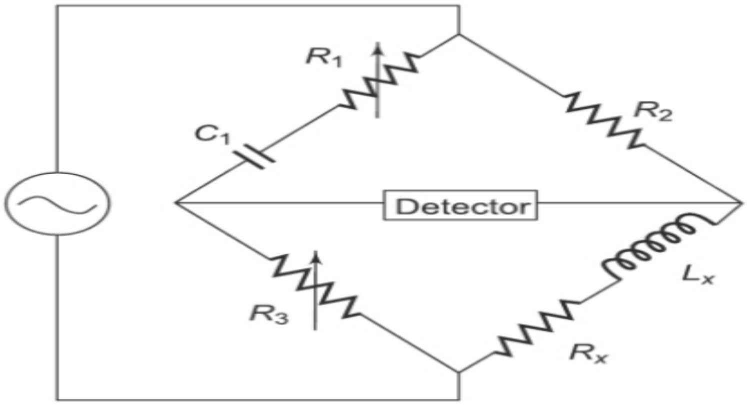


Fig. 1 Hay's Bridge

The Hay bridge, shown in Fig. 1, differs from Maxwell's bridge by having a resistance R_1 in series with a standard capacitor C_1 instead of a parallel. For large phase angles, R_1 needs to be low; therefore, this bridge is more convenient for measuring high-Q coils. For Q = 10, the error is ± 1%, and for Q = 30, the error is ± 0.1%. Hence Hay's bridge is preferred for coils with a high Q, and Maxwell's bridge for coils with a low Q.

At balance,

$$Z_1 Z_X = Z_2 Z_3$$

Where

$$Z_1 = R_1 - j/\omega C_1$$
$$Z_2 = R_2$$
$$Z_3 = R_3$$
$$Z_x = R_x + j\omega L_x$$

Substituting these values in the balance equation we get

$$\left(R_1 - \frac{j}{\omega c_1}\right)(R_x + j\omega L_x) = R_2 R_3$$

$$R_1 R_x + \frac{L_x}{C_1} - \frac{j R_x}{\omega C_1} + j\omega L_x R_1 = R_2 R_3$$

Equating the real and imaginary terms we have

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \qquad (1)$$

$$\text{and} \qquad \frac{R_x}{\omega C_1} = \omega L_x R_1 \qquad (2)$$

Solving for L_x and R_x we have, $R_x = \omega^2 L_x C_1 R_1$.

Substituting for R_x in Eq. (1)

$$R_1 (\omega^2 R_1 C_1 L_x) + \frac{L_x}{C_1} = R_2 R_3$$

$$\omega^2 R_1^2 C_1 L_x + \frac{L_x}{C_1} = R_2 R_3$$

Multiplying both sides by C_1 we get

$$\omega^2 R_1^2 C_1^2 L_x + L_x = R_2 R_3 C_1$$

$$\text{Therefore,} \qquad L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 R_1^2 C_1^2} \qquad (3)$$

Substituting for L_x in Eq. (2)

$$R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1 + \omega^2 R_1^2 C_1^2} \qquad (4)$$

The term ω appears in the expression for both L_x and R_x. This indicates that the bridge is frequency sensitive.

An inconvenient feature of this bridge is that the equation giving the balance condition for inductance, contains the multiplier $1/(1 + 1/Q^2)$. The inductance balance thus depends on its Q and frequency.

$$\text{Therefore,} \qquad L_x = \frac{R_2 R_3 C_1}{1 + (1/Q)^2}$$

For a value of Q greater than 10, the term $1/Q^2$ will be smaller than $1/100$ and can be therefore neglected. Therefore $L_x = R_2 R_3 C_1$, which is the same as Maxwell's equation**.**

A commercial bridge measure from 1 µH - 100 H with ± 2% error.

Problem 1: Find the series equivalent inductance and resistance of the network that causes an opposite angle (Hay bridge) to null with the following bridges arms. (See Fig. 1)
ω= 3000 rad/s, R2 = 10 kΩ, R1= 2 kΩ, C1 = 1 µF, R3 = 1kΩ.

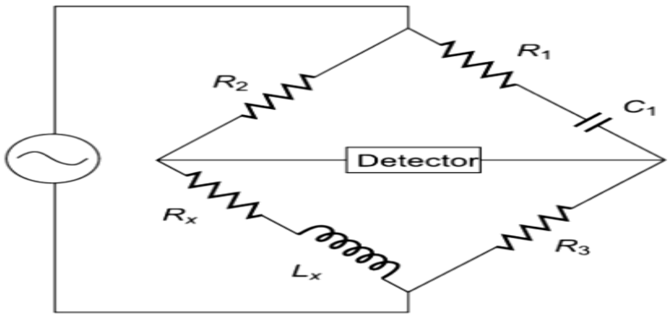


Fig. 1

Solution: We need to find R_x and L_x.

We know that

$$R_x = \frac{\omega^2 R_1 R_2 R_3 C_1^2}{1 + \omega^2 R_1^2 C_1^2}$$

$$= \frac{(3000)^2 \times 10\,\text{k} \times 2\,\text{k} \times 1\,\text{k} \times (1 \times 10^{-6})^2}{1 + (3000)^2 \times (2\,\text{k})^2 \times (1 \times 10^{-6})^2}$$

$$= \frac{180 \times 10^3}{1 + 36} = \frac{180}{37} \times 10^3$$

$$= 4.86\ \text{k}\Omega$$

We know that

$$L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 R_1^2 C_1^2}$$

$$= \frac{10\,\text{k} \times 1\,\text{k} \times (1 \times 10^{-6})}{1 + (3000)^2 \times (2\,\text{k})^2 \times (1 \times 10^{-6})^2}$$

$$= \frac{10}{1 + 36} = \frac{10}{37} = 0.27 = 270\ \text{mH}$$

Therefore R_x = 4.86 kΩ and L_x = 270 mH

ANDERSON BRIDGE:

The Anderson Bridge is a very important and useful modification of the Maxwell-Wien Bridge as shown in Fig. 1.

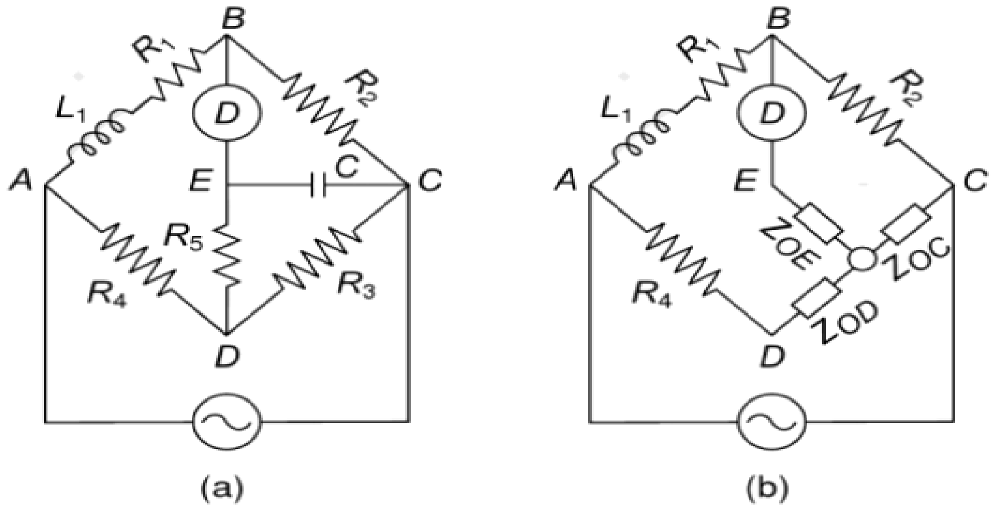


Fig. 1 Anderson's bridge

The balance condition for this bridge can be easily obtained by converting the mesh impedances C, R_3, R_5 to an equivalent star with the star point O as shown in Fig. 1(b) by using star/delta transformation.

As per delta to star transformation

$$Z_{OD} = \frac{R_3\, R_5}{(R_3 + R_5 + 1/j\omega C)} \qquad Z_{OC} = \frac{R_3\, /\, j\omega C}{(R_3 + R_5 + 1/j\omega C)} = Z_3$$

Hence with reference to Fig. 1(b) it can be seen that

$$Z_1 = (R_1 + j\omega L_1),\ Z_2 = R_2,\ Z_3 = Z_{OC} = \frac{R_3 /\, j\omega C}{(R_3 + R_5 + 1/\, j\omega C)} \text{ and } Z_4 = R_4 + Z_{OD}$$

For balance condition,

$$Z_1 Z_3 = Z_2\, Z_4$$

$$\text{Therefore, } (R_1 + j\omega L_1) \times Z_{OC} = Z_2 \times (Z_4 + Z_{OD})$$

$$(R_1 + j\omega L_1) \times \left(\frac{R_3 / j\omega C}{(R_3 + R_5 + 1/j\omega C} \right) = R_2 \left(R_4 + \frac{R_3 R_5}{(R_3 + R_5 + 1\, j\omega C} \right)$$

Simplifying,

$$(R_1 + j\omega L_1) \times \frac{R_3 / j\omega C}{(R_3 + \cancel{R_5 + 1} / j\omega C} = R_2 \left(R_4\, (R_3 + R_5 + 1/j\omega C) + \frac{R_3 R_5}{(R_3 + \cancel{R_5 + 1} / j\omega C} \right)$$

$$(R_1 + j\omega L_1) \times \frac{R_3}{j\omega C} = R_2\, R_4\, (R_3 + R_5 + 1/j\omega C) + R_2 R_3 R_5$$

$$\frac{R_1\ R_3}{j\omega C} + \frac{j\omega L_1 R_3}{j\omega C} = R_2 R_3 R_4 + R_2 R_4 R_5 + \frac{R_2 R_4}{j\omega C} + R_2 R_3 R_5$$

$$\frac{-j\, R_1\ R_3}{\omega C} + \frac{L_1 R_3}{C} = R_2 R_3 R_4 + R_2 R_4 R_5 - \frac{j\, R_2 R_4}{\omega C} + R_2 R_3 R_5$$

Equating the real terms and imaginary terms

$$\frac{L_1 R_3}{C} = R_2\, R_3 R_4 + R_2\, R_4\, R_5 + R_2\, R_3 R_5$$

$$L_1 = \frac{C}{R_3}\left(R_2R_3R_4 + R_2R_4R_5 + R_2R_3R_5\right)$$

$$L_1 = CR_2\left[R_4 + \frac{R_4R_5}{R_3} + R_5\right]; \quad L_1 = CR_2\left[R_4 + R_5 + \frac{R_4R_5}{R_3}\right]$$

$$\frac{-j\,R_1\,R_3}{\omega C} = \frac{-jR_2\,R_4}{\omega C}; \quad R_1R_3 = R_2R_4, \text{ therefore, } R_1 = \frac{R_2\,R_4}{R_3}$$

This method is capable of precise measurement of inductances and a wide range of values from a few **µH** to several Henries**.**

Problem 1: An inductive coil was tested by an Anderson bridge. The following were the values on balance. Arm AB having unknown impedance of resistance R_1 and inductance L_1. Arm BC, CD, DA are resistors having 1000 Ω, 1000 Ω and 2000 Ω respectively. A capacitor of 10 µF and resistance 400Ω are connected between CE and ED respectively, Source between A and C, r = 496. Determine $L_1$and R_1.

Solution:

Given : $R_2 = 200\ \Omega$, $R_3 = 1000\ \Omega$, $R_4 = 1000\ \Omega$, $C = 10\ \mu\text{F}$, $r = 496$

Step 1: To calculate

$$R_1 = \frac{R_2\,R_3}{R_4} = \frac{200 \times 1000}{1000} = 200\ \Omega$$

Step 2: Similarly to calculate

$$L_1 = \frac{CR_3}{R_4}\left(rR_4 + R_2R_4 + rR_2\right)$$

$$= \frac{10 \times 10^{-6} \times 1000}{1000} \times (496 \times 10^3 + 200 \times 10^3 + 496 \times 200)$$

$$= 10^{-5} \times 10^3 \times (496 + 200 + 0.496 \times 200)$$

$$= 10^{-2}\,(496 + 200 + 99.2)$$

$$= 795.2 \times 10^{-2}$$

$$= 7.952\ \text{H}$$

SCHERING'S BRIDGE (MEASUREMENT OF CAPACITANCE):

A very important bridge used for the precision measurement of capacitors and their insulating properties is the Schering bridge. Its basic circuit arrangement is given in Fig.1.

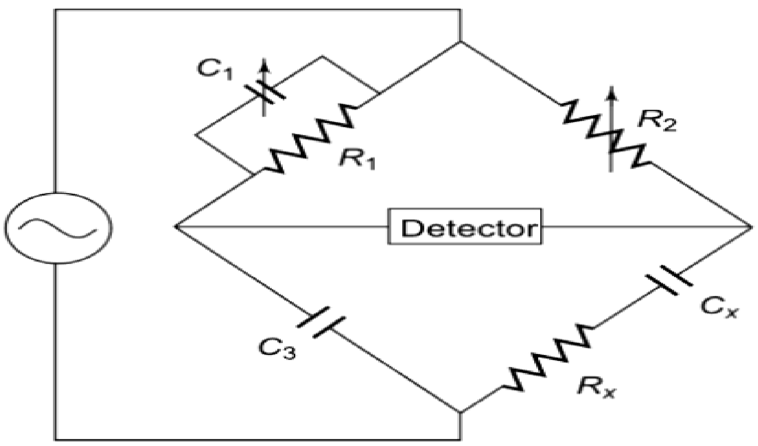


Fig.1. Schering's bridge

The standard capacitor C_3 is a high quality mica capacitor for general measurements, or an air capacitor for insulation measurement.

For balance, the general equation is

$$Z_1 Z_x = Z_2 Z_3$$

$$\therefore \quad Z_x = \frac{Z_2 Z_3}{Z_1}, \; Z_x = Z_2 Z_3 Y_1$$

where
$$Z_x = R_x - j/\omega C_x$$
$$Z_2 = R_2$$
$$Z_3 = -j/\omega C_3$$
$$Y_1 = 1/R_1 + j\,\omega C_1$$

as
$$Z_x = Z_2 Z_3 Y_1$$

$$\therefore \quad \left(R_x - \frac{j}{\omega C_x}\right) = R_2 \left(\frac{-j}{\omega C_3}\right) \times \left(\frac{1}{R_1} + j\omega C_1\right)$$

$$\left(R_x - \frac{j}{\omega C_x}\right) = \frac{R_2(-j)}{R_1(\omega C_3)} + \frac{R_2 C_1}{C_3}$$

Equating the real and imaginary terms, we get

$$R_x = \frac{R_2 C_1}{C_3} \qquad (1)$$

and
$$C_x = \frac{R_1}{R_2} C_3 \qquad (2)$$

The dial of capacitor C_1 can be calibrated directly to give the dissipation factor at a particular frequency.

The dissipation factor D of a series RC circuit is defined as the cotangent of the phase angle.

$$D = \frac{R_x}{X_x} = \omega C_x R_x$$

Also, D is the reciprocal of the quality factor Q, i.e. D =1/Q. D indicates the quality of the capacitor.

Commercial units measure from 100 pf - 1µf, with ± 2% accuracy. The dial of C_3 is graduated in terms of direct readings for C_x, if the resistance ratio is maintained at a fixed value.

This bridge is widely used for testing small capacitors at low voltages with very high precision.

Problem 1: An ac bridge A has the following constants (refer Fig. 1).
Arm AB - capacitor of 0.5 µF in parallel with 1kΩ resistance
Arm AD - resistance of 2 kΩ
Arm BC- capacitor of 0.5 µF
Arm CD - unknown capacitor Cx and Rx in series
Frequency - 1kHz. Determine the unknown capacitance and dissipation factor.

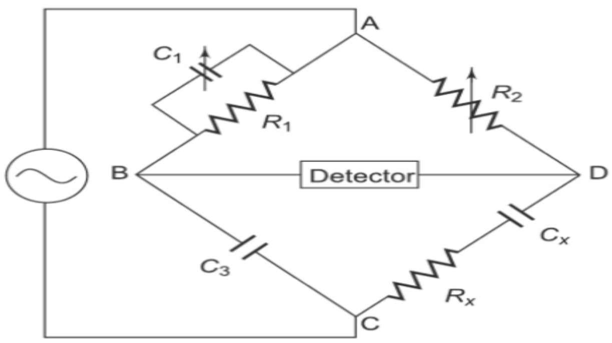


Fig. 1

Solution: From Eqs (1)and (2), we have

$$R_x = \frac{C_1}{C_3} R_2 = \frac{0.5\ \mu F}{0.5\ \mu F} \times 2\ k = 2\ k\Omega$$

$$C_x = \frac{R_1}{R_2} \times C_3 = \frac{1\ k}{2\ k} \times 0.5\ \mu F = 0.25\ \mu F$$

The dissipation factor is given by

$$\begin{aligned} D &= \omega C_x R_x \\ &= 2 \times 3.142 \times 1000 \times 2\ k \times 0.25\ \mu F \\ &= 4 \times 3.142 \times 0.25 \\ &= 3.1416 \end{aligned}$$

ERRORS IN BRIDGES:

Practically some factors like stray couplings between bridge arms etc, modify the balance conditions making the balance impracticable or wrong balance condition. The following are the factors causing errors in bridges:

1. Stray conductance effects, due to imperfect insulation.

2. Mutual inductance effects due to magnetic coupling between various components of the bridge.

3. Stray capacitance effects, due to electrostatic fields between conductors at different potentials.

4. Residuals in components for example the presence of small magnitudes of series inductance or shunt capacitance in non-reactive resistors.

PRECAUTIONS TO BE TAKEN WHEN USING A BRIDGE:

Assuming that a suitable method of measurement has been selected and that the source and detector are given, there are some precautions which must be observed to obtain accurate readings.

1. The leads should be carefully laid out in such a way that no loops or long lengths enclosing magnetic flux are produced, with consequent stray inductance errors.

2. With a large *L*, the self-capacitance of the leads is more important than their inductance, so they should be spaced relatively far apart.

3. In measuring a capacitor, it is important to keep the lead capacitance as low as possible. For this reason the leads should not be too close together and should be made of fine wire.

4. In very precise inductive and capacitances measurements, leads are encased in metal tubes to shield them from mutual electromagnetic action, and are used or designed to completely shield the bridge.

Q METER:

The overall efficiency of coils and capacitors intended for RF applications is best evaluated using the Q value. The Q meter is an instrument designed to measure some electrical properties of coils and capacitors.

The principle of the Q meter is based on series resonance; the voltage drop across the coil or capacitor is Q times the applied voltage (where Q is the ratio of reactance to resistance, X_L/R).
If a fixed voltage is applied to the circuit, a voltmeter across the capacitor can be calibrated to read Q directly.

At resonance $X_L = X_C$ and $E_L = I X_L$, $E_C = I X_C$, $E = I R$

where E — applied voltage, E_C — capacitor voltage
E_L — inductive voltage, X_L — inductive reactance
X_C — capacitive reactance, R — coil resistance
I — circuit current

Therefore $$Q = \frac{X_L}{R} = \frac{X_C}{R} = \frac{E_C}{E}$$

From the above equation, if E is kept constant the voltage across the capacitor can be measured by a voltmeter calibrated to read directly in terms of Q.

A practical Q meter circuit is shown in Fig. 1.

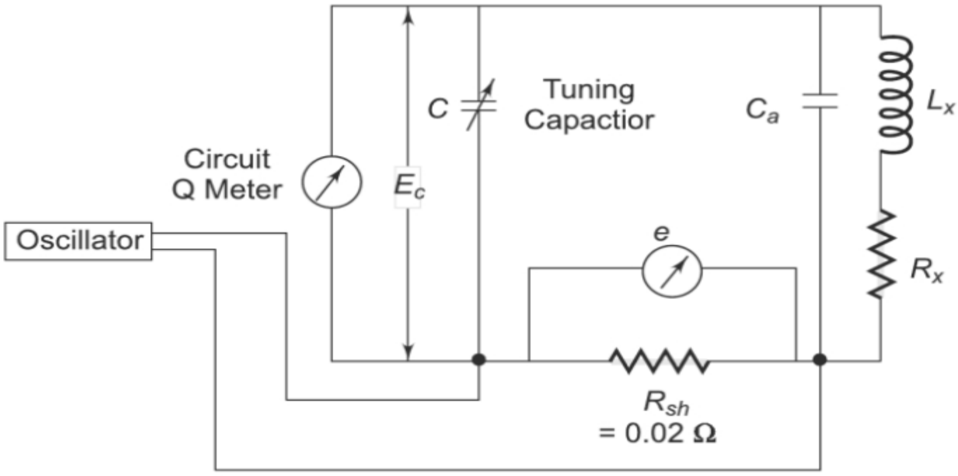


Fig.1 Circuit diagram of a Q-meter

The wide range oscillator, with frequency range from 50 kHz to 50 MHz, delivers current to a resistance R_{sh} having a value of 0.02 Ω. This shunt resistance introduces almost no resistance into the tank circuit and therefore represents a voltage source of a magnitude e with a small internal resistance.

The voltage across the shunt is measured with a thermocouple meter. The voltage across the capacitor is measured by an electronic voltmeter corresponding to E_C and calibrated directly to read Q.

The oscillator energy is coupled to the tank circuit. The circuit is tuned to resonance by varying C until the electronic voltmeter reads the maximum value.

The resonance output voltage E, corresponding to E_C, is $E = Q \times e$, that is, Q= E/e. Since e is known, the electronic voltmeter can be calibrated to read Q directly.

The inductance of the coil can be determined by connecting it to the test terminals of the instrument. The circuit is tuned to resonance by varying either the capacitance or the oscillator frequency.

If the capacitance is varied, the oscillator frequency is adjusted to a given frequency and resonance is obtained. If the capacitance is pre-setted to a desired value, the oscillator frequency is varied until resonance occurs.

The Q reading on the output meter must be multiplied by the index setting or the "Multiply Q by" switch to obtain the actual Q value. The inductance of the coil can be calculated from known values of the coil frequency and resonating capacitor (C).

$$X_L = X_C,\ f = \frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad L = \frac{1}{(2\pi f)^2\, C}$$

The Q indicated is not the actual Q, because the losses of the resonating capacitor, voltmeter and inserted resistance are all included in the measuring circuit.

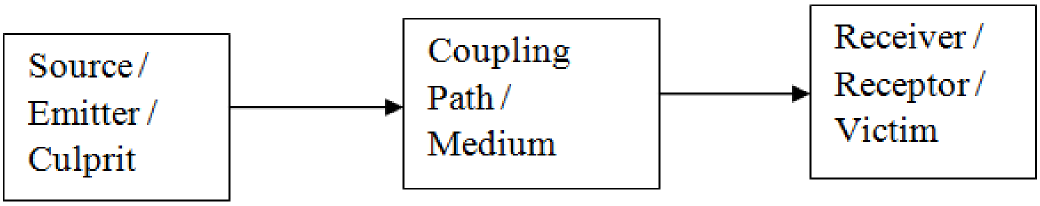
The actual Q of the measured coil is somewhat greater than the indicated Q. This difference is negligible except where the resistance of the coil is relatively small compared to the inserted resistance R_{sh}.

EMI and EMC:

An electromagnetic disturbance which may degrade the performance of an equipment (device, system or sub-system) or causes malfunction of the equipment is called electromagnetic interference (EMI).

Electromagnetic compatibility (EMC) is a near perfect state in which a receptor (device , system or subsystem) functions satisfactorily in common electromagnetic environment, without introducing intolerable electromagnetic disturbance to any other devices / equipments / system in that environment.

Basic elements of EMI situations:



- Interference occurs if the received energy causes the receptor to function in unwanted manner.
- Whether the receiver is functioning in wanted or unwanted manner, depends on the coupling path as well as the source and victim.
- The coupling path is to be made as inefficient as possible.

Causes of EMI:

Sources

- Refrigerator, washing machine, electric motors.
- Arc welding machine.
- Electric shavers, AC, computers.
- Fast switching digital devices, ICs.
- Power cords of computers, UPS etc.
- Air craft navigation and military equipments

Victims

- Communication receivers.
- Microprocessors, computers.
- Industrial controls.
- Medical devices.
- House hold appliances.
- Living beings.

Purpose and Methodology for EMC System:

A system is said to be electro magnetically compatible if :-

- It doesn't cause interference with other system .
- It is not susceptible to emissions from other systems.
- It doesn't cause interference with itself.

The methodologies to prevent EMI are

- Suppress the emissions at source point, best method to control EMI .
- Make the coupling path as inefficient as possible.
- Make the receiver less susceptible to emission.

Interference and Noise Reduction Techniques:

- Noise and Interference:- Noise can be characterized as any disturbance that tends to obscure a desired signal. Noise can be generated within a circuit or picked up from external natural or artificial sources.
- Interference is noise that tends to obscure the useful signal. It is usually caused by electrical sources but can be induced from other physical sources such as mechanical vibration, acoustical feedback, or electrochemical sources.
- As surface biopotential recording involves the measurement of extremely small potential differences, noise is likely to play an important role.

Noise Reduction: The type of connecting wires used between electrical devices can have a significant impact on the noise level of signal. Low-level signals of <100mV are particularly susceptible to errors induced by noise.
Three simple rules will help to keep noise levels low:
(1) keep the connecting wires as short as possible;
(2) keep signal wires away from noise sources; and
(3) use a wire shield and proper ground.