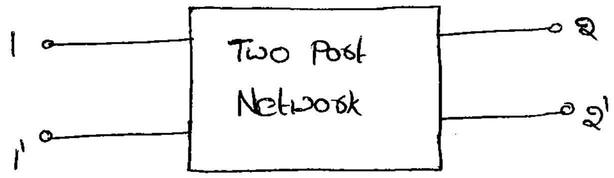
Two Port Networks

Port :-

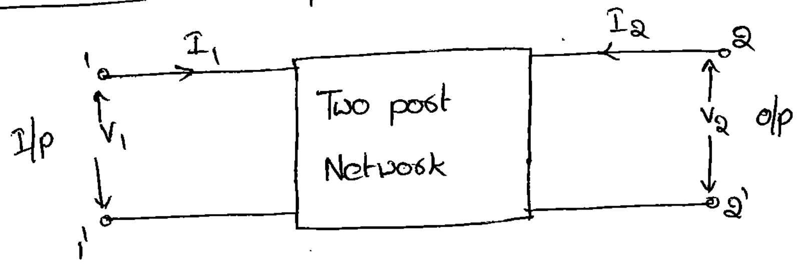
A pair of terminals at which electrical signal may enter or leave the network is called "Port."

One Port Network :- A Network having only one pair of terminals is called "One Port Network."

Two Port Network :- A Network having two pairs of terminals is called "Two Port Network."



Z- Parameters (Impedence (or) Open Circuit Parameters) :-



Here V_1, V_2 are dependent variables and I_1, I_2 are independent variables.

$\therefore$ Z- Parameter equations are

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{——} ①$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{——} ②$$

Z_{11}, Z_{12}, Z_{21}, Z_{22} are called Impedence parameters.

Matrix Representation:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[V] = [Z] \; [I]$$

To obtain Z_{11} and Z_{21}, Open Circuit Port '2' i.e, $I_2 = 0$

$\therefore$ From eqn ①, $V_1 = Z_{11} I_1$

$$\Rightarrow \boxed{Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}}$$

$\therefore$ Z_{11} is called "Open Circuit Driving Point input impedence."

From eqn ②, $V_2 = Z_{21} I_1$

$$\Rightarrow \boxed{Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}}$$

$\therefore$ Z_{21} is called "Open Ckt Forward Transfer impedence."

To obtain Z_{12} and Z_{22} parameters, Open ckt port '1' i.e, $I_1 = 0$

$\therefore$ From eqn ①,

$$\boxed{Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}}$$

$\therefore$ Z_{12} is called "Open Ckt Reverse Transfer impedence."

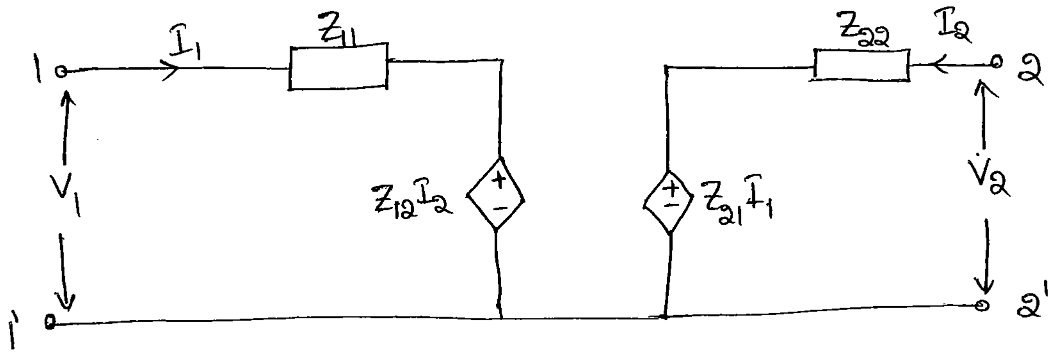
From eqn ②,

$$\boxed{Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}}$$

Z_{22} is called "Open Ckt Driving Point output impedence."

The Z-parameters are obtained only when the current in One port is zero ie, one of the port is open circuited. Hence the Z-parameters are also called "Open Circuit parameters" (or) "Impedence parameters"

Equivalent Circuit :-



NOTE:-

→ If the voltages and currents corresponding to the same ports, then the parameters are called "Driving Point parameters."

→ If the voltages or currents corresponding to the different ports, then the parameters are called "Transfer parameters."

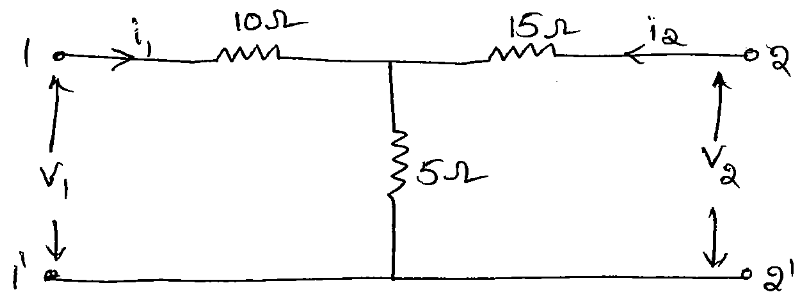
Reciprocal Network :- (Reciprocity condition)

When $Z_{12} = Z_{21}$ then network is called "Reciprocal Network."

Symmetrical Network :-

When $Z_{11} = Z_{22}$ then network is called "Symmetrical Network."

→ Find the z-parameters for the given circuit. (T- Network)



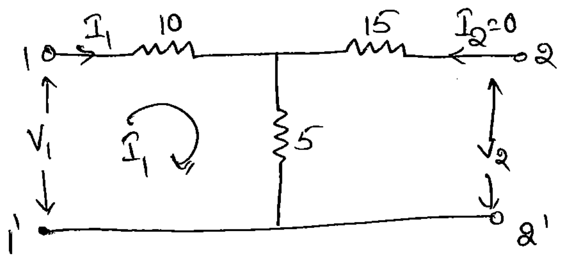
Sol:

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

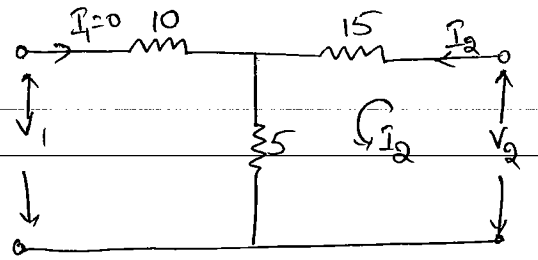
To find parameters Z_{11}, Z_{21} open circuit port 2 i.e $I_2 = 0$



$$V_1 = 10 I_1 + 5 I_1$$
$$= 15 I_1$$
$$\Rightarrow \frac{V_1}{I_1} = 15\Omega = Z_{11}$$

$$V_2 = 5 I_1 \Rightarrow \frac{V_2}{I_1} = 5\Omega = Z_{21}$$

To find parameters Z_{12}, Z_{22} open circuit port 1 i.e $I_1 = 0$

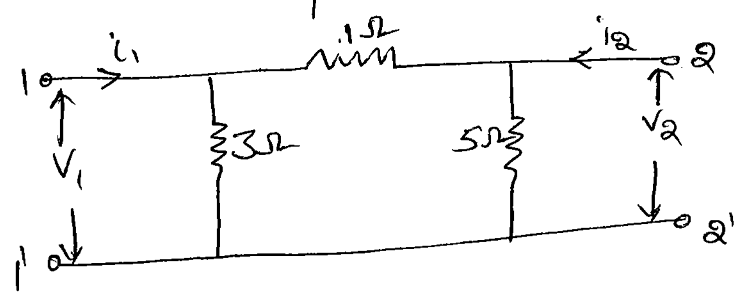


$$V_2 = 15 I_2 + 5 I_2$$
$$= 20 I_2$$
$$\Rightarrow \frac{V_2}{I_2} = 20\Omega = Z_{22}$$

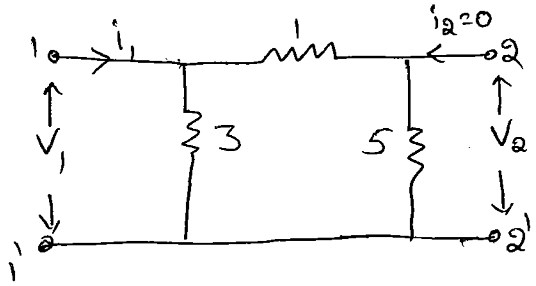
$$V_1 = 5 I_2 \Rightarrow \frac{V_1}{I_2} = 5\Omega = Z_{12}$$

$$\therefore \text{Z-parameters are } \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ 5 & 20 \end{bmatrix}$$

→ Determine the z-parameters for the given circuit (π-Network)



Sol:- To find parameters z_{11}, z_{21} open ckt Port 2 i.e $I_2 = 0$



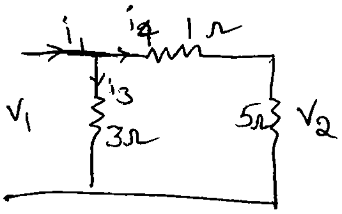
$$V_1 = z_{eq} i_1$$

$$\Rightarrow z_{eq} = \frac{V_1}{i_1}$$

$$z_{eq} = R_{eq} = (1+5) \parallel 3 = \frac{3 \times 6}{9} = 2\Omega$$

$$\therefore z_{11} = \frac{V_1}{i_1} = 2\Omega$$

$$z_{21} = \left.\frac{V_2}{i_1}\right|_{i_2=0}$$

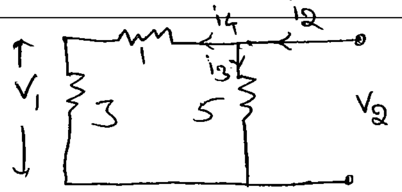


$$V_2 = 5 i_4$$

$$= 5 \times \frac{3}{3+5+1} i_1 = \frac{15}{9} i_1 = \frac{5}{3} i_1$$

$$z_{21} = \frac{V_2}{i_1} = \frac{5}{3}\,\Omega$$

To find parameters z_{12}, z_{22} Open ckt port 1 i.e, $I_1 = 0$



$$z_{eq} = (1+3) \parallel 5 = \frac{5 \times 4}{9} = \frac{20}{9}\,\Omega$$

$$V_2 = z_{eq} i_2$$

$$\Rightarrow z_{22} = z_{eq} = \frac{V_2}{i_2} = \frac{20}{9}\,\Omega$$

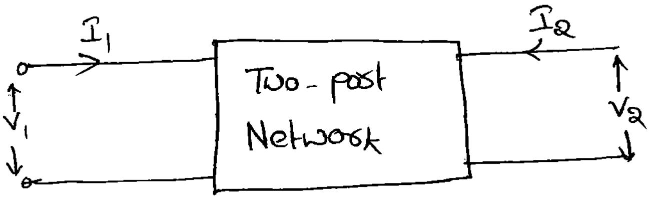
$$V_1 = 3 \times i_4 = 3 \times \frac{5\, i_2}{3+5+1} = \frac{15}{9}\, i_2$$

$$\Rightarrow \quad Z_{12} = \frac{V_1}{i_2} = \frac{5}{3}\,\Omega$$

$\therefore$ Z-parameters are
$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 2 & 5/3 \\ 5/3 & \frac{20}{9} \end{bmatrix}$$

$Z_{12} = Z_{21} \Rightarrow$ The given n/w is Reciprocal Network

Y-Parameters (Admittance Parameters):-



Here I_1, I_2 are dependent variables and V_1, V_2 are independent variables.

$$\therefore \quad I_1 = f(V_1, V_2)$$
$$I_2 = f(V_1, V_2)$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{——} \ (1)$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{——} \ (2)$$

Here $Y_{11}, Y_{12}, Y_{21}, Y_{22}$ are the admittance parameters

$$\therefore Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

To obtain Y_{11}, Y_{21} parameters short ckt port '2' i.e $V_2 = 0$

$$\therefore (1) \Rightarrow I_1 = Y_{11} V_1 \Rightarrow \boxed{Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \ \mho}$$

Here Y_{11} is called Short ckt Driving point i/p admittance

② $\Rightarrow$ $I_2 = Y_{21} V_1$

$$\boxed{Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \mho}$$

Here Y_{21} is called "Short ckt forward Transfer admittance".

To obtain Y_{12}, Y_{22} parameters short ckt port-1 i.e $V_1 = 0$

$\therefore$ ① $\Rightarrow$ $I_1 = Y_{12} V_2$ $\Rightarrow$ $\boxed{Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \mho}$

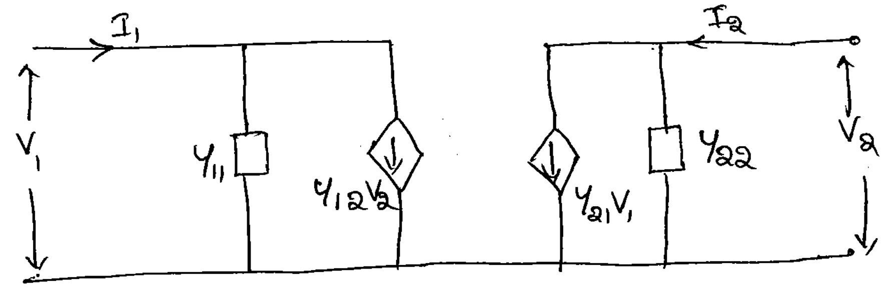
Here Y_{12} is called "Short ckt reverse transfer admittance".

② $\Rightarrow$ $I_2 = Y_{22} V_2$ $\Rightarrow$ $\boxed{Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \mho}$

Here Y_{22} is called "Short ckt Driving point o/p admittance."

$\therefore$ Inorder to determine Y-parameters, the voltage in any of the port must become zero i.e short circuited. Hence Y-parameters are also called "Short ckt parameters".

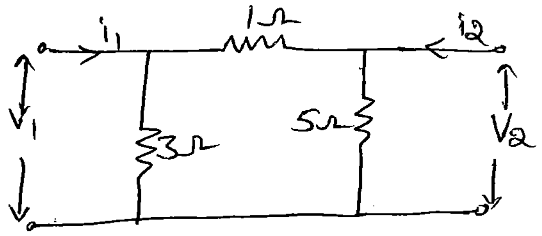
The equivalent ckt representing the Y-parameters is shown below:



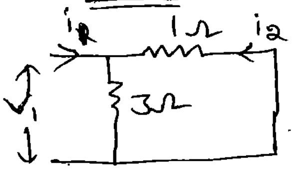
Reciprocal condition is $Y_{12} = Y_{21}$

Symmetrical condition is $Y_{11} = Y_{22}$

→ Determine Y- parameters for the given circuit.



Sol:- Port '2' is short circuited



$$Z_{eq} = 3 \| 1 = \frac{3}{4}\,\Omega$$

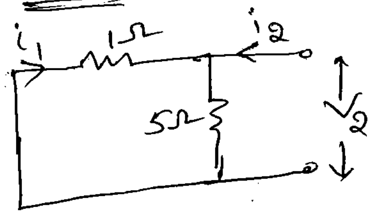
$$Y_{eq} = \frac{4}{3}\,\mho$$

$$I_1 = Y_{11} V_1 \Rightarrow Y_{11} = \frac{I_1}{V_1} = \frac{4}{3}\,\mho$$

$$I_2 = -\frac{3}{4} i_1 \;, \quad V_1 = \frac{3}{4} i_1$$

$$\therefore Y_{21} = \frac{I_2}{V_1} = \frac{-3/4\, i_1}{3/4\, i_1} = -1\,\mho$$

Port '1' is short circuited



$$Z_{eq} = 1 \| 5 = \frac{5}{6}\,\Omega$$

$$I_2 = Y_{22} V_2 \Rightarrow Y_{22} = \frac{I_2}{V_2} = \frac{6}{5}\,\mho$$

$$i_1 = -\frac{5}{6} i_2 \;, \quad V_2 = \frac{5}{6} i_2$$

$$\therefore \frac{i_1}{V_2} = -1 = Y_{12}$$

$$\therefore \text{Y-parameters} \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 4/3 & -1 \\ -1 & 6/5 \end{bmatrix}$$

Hence this is Reciprocal Network

Alternate Method:-

$$Y = \frac{1}{Z} = Z^{-1}$$

$$Y = \begin{bmatrix} 2 & 5/3 \\ 5/3 & 20/9 \end{bmatrix}^{-1} \qquad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{\frac{40}{9} - \frac{25}{9}} \begin{bmatrix} 20/9 & -5/3 \\ -5/3 & 2 \end{bmatrix}$$

$$= \frac{9}{15} \begin{bmatrix} 20/9 & -5/3 \\ -5/3 & 2 \end{bmatrix} = \begin{bmatrix} 4/3 & -1 \\ -1 & 6/5 \end{bmatrix}$$

ABCD (or) Transmission Parameters :-

These are generally used in the analysis of transmission in which input port is referred as Sending end and output port is referred as Receiving End.

Characteristic eqn's :- $V_1 = AV_2 - BI_2$ —— ①

$I_1 = CV_2 - DI_2$ —— ②

Where A, B, C, D are called Chain (or) Transmission Parameters.

i. To obtain parameters A, C open ckt port '2' i.e $I_2 = 0$

$\therefore$ ① $\Rightarrow$ $\boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0}}$

A is called "Open ckt reverse Voltage gain".

② $\Rightarrow$ $\boxed{C = \left.\frac{I_1}{V_2}\right|_{I_2=0}}$

'C' is called "Open circuit transfer admittance."

ii. To obtain the parameters B, D Short ckt port '2' i.e. $V_2 = 0$

① $\Rightarrow$ $\boxed{B = \left.\frac{-V_1}{I_2}\right|_{V_2=0}}$

'B' is called "Short Circuit transfer impedence."

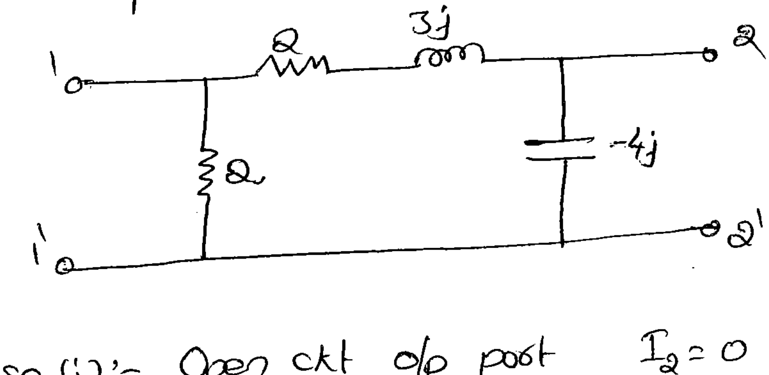
② $\Rightarrow$ $\boxed{D = \left.-\frac{I_1}{I_2}\right|_{V_2=0}}$

'D' is called "Short ckt Reverse current gain."

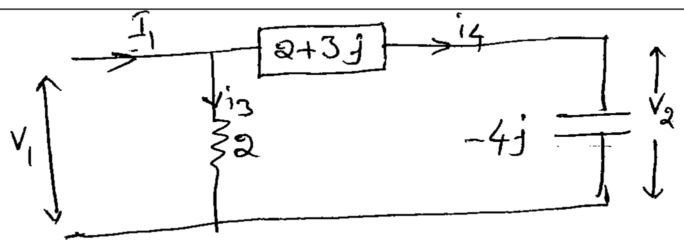
For symmetrical Network $\boxed{A = D}$

For Reciprocal Network $\boxed{AD - BC = 1}$

→ Verify whether the given n/w is reciprocal or not using ABCD parameters.



Sol:- Case (i):- Open ckt o/p port $I_2 = 0$



$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ $\quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$

$$V_2 = i_4(-4j) = \frac{2}{2+2+3j-4j} \times I_1(-4j) = \frac{-8j\, I_1}{4-j}$$

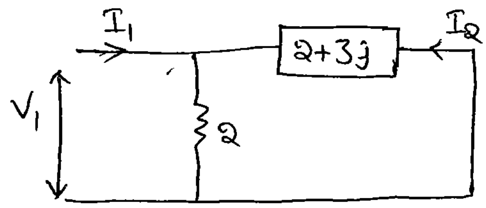
$$Z_{eq} = (2+3j-4j) \parallel 2 = \frac{(2-j)2}{2-j+2} = \frac{4-2j}{4-j} = 1.05-0.23j$$

$$V_1 = I_1 Z_{eq} = (1.05-0.23j) I_1$$

$$A = \frac{V_1}{V_2} = \frac{(1.05-0.23j)I_1}{-8jI_1} \times (4-j) = \underline{0.24+0.525j}$$

$$C = \frac{I_1}{V_2} = \frac{4-j}{-8j} = \underline{0.125+0.5j}$$

Case(ii) :- Short ckt o/p port , $V_2 = 0$



$$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0} \qquad D = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$$

$$Z_{eq} = \frac{2\times(2+3j)}{2+2+3j} = \underline{1.36+0.48j}$$

$$V_1 = Z_{eq} I_1 = (1.36+0.48j) I_1$$

$$I_2 = -\frac{2}{2+3j+2} I_1 = \frac{-2}{4+3j} I_1$$

$$\therefore B = \frac{-V_1}{I_2} = -\frac{(1.36+0.48j)I_1}{-2I_1} \times (4+3j) = \underline{2.72+2.46j}$$

$$D = \frac{-I_1}{I_2} = \frac{4+3j}{2} = \underline{2+1.5j}$$

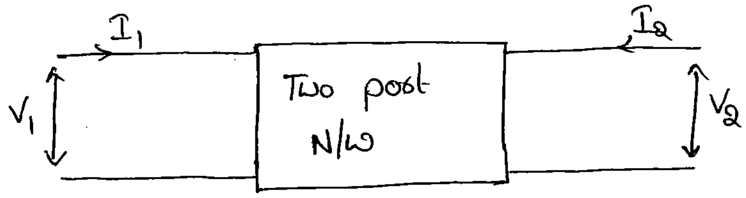
$$AD-BC = (0.24+0.525j)(2+1.5j) - (2.72+2.46j)(2+1.5j)$$

$$= \underline{-2.057-7.59j} \neq 1$$

$\therefore$ The given network is not reciprocal network

Hybrid Parameters :- (H-Parameters)

H-parameter representation is used in modelling of electronic components & circuits, particularly transistors.



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{——} ①$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{——} ②$$

Case(i) :- To obtain the parameters h_{11}, h_{21} short ckt port ② i.e, $V_2 = 0$.

$$① \Rightarrow \boxed{h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} \Omega}$$

h_{11} is called "Short ckt driving point i/p impedence."

$$② \Rightarrow \boxed{h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0}}$$

h_{21} is called "Short ckt forward current gain"

Case(ii) :- To obtain the parameters h_{12}, h_{22} open ckt port ① i.e, $I_1 = 0$

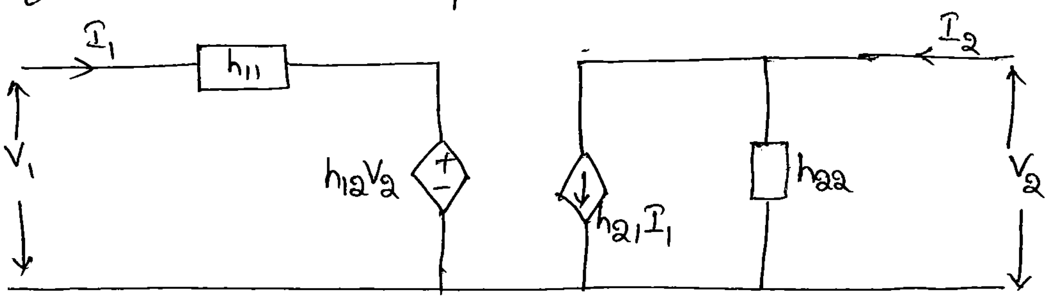
$$① \Rightarrow \boxed{h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0}}$$

h_{12} is called "Open ckt reverse voltage gain".

$$② \Rightarrow \boxed{h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} (\mho)}$$

h_{22} is called "Open ckt driving point o/p admittance"

The equivalent ckt of h-parameters is shown below:

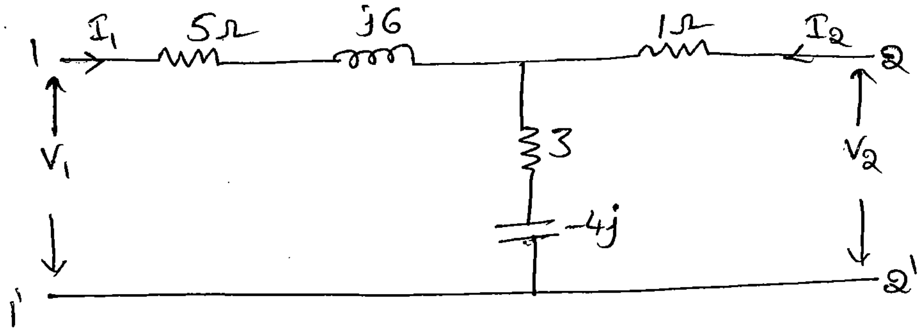


Reciprocal condition:- $\boxed{h_{12} = -h_{21}}$

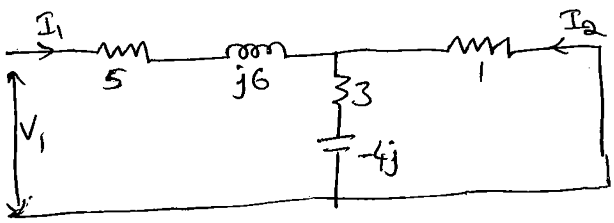
Symmetrical condition:- $\Delta h = 1$

$$\boxed{h_{11}h_{22} - h_{12}h_{21} = 1}$$

→ Obtain hybrid parameters for the given network :-



Sol:- case(i):- Short ckt o/p port $V_2 = 0$



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$Z_{eq} = \left[1 \parallel (3-4j)\right] + 5 + j6 = \frac{3-4j}{4-4j} + 5 + 6j$$

$$= 5.875 + 5.875j$$

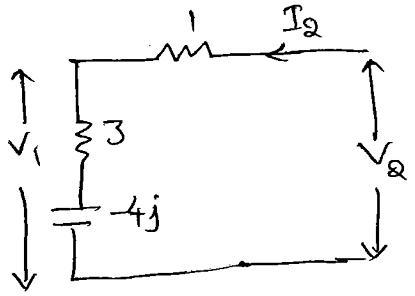
$$V_1 = Z_{eq} I_1 = (5.875 + 5.875j) I_1 \quad \text{--- ①}$$

$$I_2 = -\frac{(3-4j)}{3-4j+1} I_1 = (0.875 - 0.125j) I_1 \quad \text{--- ②}$$

$$① \Rightarrow h_{11} = \frac{V_1}{I_1} = \underline{5.875 + 5.875j}$$

(2) $\Rightarrow$ $h_{21} = \dfrac{I_2}{I_1} = 0.875 - 0.125j$

Case (ii):- Open ckt the input port, $I_1 = 0$



$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$$Z_{eq} = 1+3-4j = 4-4j$$

$$V_2 = I_2 Z_{eq} = (4-4j)I_2 \quad \text{--- (3)}$$

$$V_1 = \frac{3-4j}{3-4j+1} V_2 = \frac{3-4j}{4-4j} V_2 \quad \text{--- (4)}$$

(3) $\Rightarrow$ $h_{22} = \dfrac{I_2}{V_2} = \dfrac{1}{4-4j} = 0.125 + 0.125j$

(4) $\Rightarrow$ $h_{12} = \dfrac{V_1}{V_2} = \dfrac{3-4j}{4-4j} = 0.875 - 0.125j$

Relationship between parameters :-

1. Z-parameters interms of Y-parameters :-

$$Z = [Y]^{-1} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$$= \frac{1}{\underbrace{Y_{11}Y_{22} - Y_{21}Y_{12}}_{\Delta Y}} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

$$\therefore \boxed{\begin{array}{ll} Z_{11} = \dfrac{Y_{22}}{\Delta Y} & Z_{12} = -\dfrac{Y_{12}}{\Delta Y} \\[2ex] Z_{21} = \dfrac{-Y_{21}}{\Delta Y} & Z_{22} = \dfrac{Y_{11}}{\Delta Y} \end{array}}$$

2. Z in terms of ABCD parameters :-

We know, $V_1 = Z_{11} I_1 + Z_{12} I_2$ —— ① (V_1, I_1, I_2)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ —— ② (V_2, I_1, I_2)

For ABCD parameters,

$V_1 = A V_2 - B I_2$ —— ③ (V_1, V_2, I_2)

$I_1 = C V_2 - D I_2$ —— ④ (I_1, V_2, I_2)

From eqn ④ $\Rightarrow$ $V_2 = \dfrac{I_1 + D I_2}{C} = \dfrac{I_1}{C} + \dfrac{D}{C} I_2$ —— ⑤

Compare ⑤ with ②

$$\Rightarrow \boxed{Z_{21} = \frac{1}{C}, \quad Z_{22} = \frac{D}{C}}$$

Substitute ⑤ in ③,

$$V_1 = A\left[\frac{I_1}{C} + \frac{D}{C} I_2\right] - B I_2$$

$$= \frac{A}{C} I_1 + \left(\frac{AD}{C} - B\right) I_2 \quad \text{—— ⑥}$$

Compare ⑥ with ①

$$\Rightarrow \boxed{Z_{11} = \frac{A}{C} \qquad Z_{12} = \frac{AD - BC}{C}}$$

3. Z in terms of 'H' parameters :-

We know, $V_1 = Z_{11} I_1 + Z_{12} I_2$ —— ① (V_1, I_1, I_2)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ —— ② (V_2, I_1, I_2)

For h-parameters,

$V_1 = h_{11} I_1 + h_{12} V_2$ —— ③ (V_1, I_1, V_2)

$I_2 = h_{21} I_1 + h_{22} V_2$ —— ④ (I_2, I_1, V_2)

From eqn ④ $\Rightarrow$ $h_{22} V_2 = I_2 - h_{21} I_1$

$$V_2 = \frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1$$

$$= -\left(\frac{h_{21}}{h_{22}}\right) I_1 + \frac{1}{h_{22}} I_2 \quad \text{——⑤}$$

Compare ⑤ with ②

$$\Rightarrow \quad z_{21} = \frac{-h_{21}}{h_{22}} , \quad z_{22} = \frac{1}{h_{22}}$$

Substitute ⑤ in ③

$$\Rightarrow \quad V_1 = h_{11} I_1 + h_{12} \left[\frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \right]$$

$$= \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2 \quad \text{——⑥}$$

From eqns ⑤ & ⑥

$$\boxed{\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}$$

Y-Parameters in terms of other parameters:-

1. Y in terms of Z-parameters:-

The equations representing Y-parameters are

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{——①} \quad (I_1, V_1, V_2)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{——②} \quad (I_2, V_1, V_2)$$

For Z-parameters, $V_1 = Z_{11} I_1 + Z_{12} I_2$ ——③ (V_1, I_1, I_2)

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{——④} \quad (V_2, I_1, I_2)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}$$

where $\Delta = Z_{11}Z_{22} - Z_{12}Z_{21}$

2. Y in terms of ABCD parameters :-

For Y-parameters, $I_1 = Y_{11}V_1 + Y_{12}V_2$ —① (I_1, V_1, V_2)

$I_2 = Y_{21}V_1 + Y_{22}V_2$ —② (I_2, V_1, V_2)

For ABCD parameters, $V_1 = AV_2 - BI_2$ —③ (V_1, V_2, I_2)

$I_1 = CV_2 - DI_2$ —④ (I_1, V_2, I_2)

From eqn ③ $\Rightarrow$ $I_2 = \frac{A}{B}V_2 - \frac{1}{B}V_1$

$= \frac{-1}{B}V_1 + \frac{A}{B}V_2$ —⑤

Compare ② & ⑤ $\Rightarrow$ $Y_{21} = \frac{-1}{B}$ $\quad Y_{22} = \frac{A}{B}$

Substitute eqn ⑤ in ④

$\Rightarrow I_1 = CV_2 - D\left[\frac{-1}{B}V_1 + \frac{A}{B}\right]V_2$

$= \frac{D}{B}V_1 + \frac{BC-AD}{B}V_2$ —⑥

Compare ⑥ & ① $\Rightarrow$ $Y_{11} = \frac{D}{B}$, $\quad Y_{12} = \frac{BC-AD}{B}$

$$\therefore \boxed{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{D}{B} & \frac{BC-AD}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}$$

3. Y in terms of h-parameters :-

For y-parameters, $I_1 = y_{11} V_1 + y_{12} V_2$ — ① (I_1, V_1, V_2)

$I_2 = y_{21} V_1 + y_{22} V_2$ — ② (I_2, V_1, V_2)

For h-parameters, $V_1 = h_{11} I_1 + h_{12} V_2$ — ③ (V_1, I_1, V_2)

$I_2 = h_{21} I_1 + h_{22} V_2$ — ④ (I_2, I_1, V_2)

From eqn ③ $\Rightarrow$ $I_1 = \frac{1}{h_{11}} V_1 + \left(-\frac{h_{12}}{h_{11}}\right) V_2$ — ⑤

Substitute eqn ⑤ in eqn ④

$$\Rightarrow I_2 = h_{21}\left[\frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2\right] + h_{22} V_2$$

$$= \frac{h_{21}}{h_{11}} V_1 + \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}} V_2 \quad \text{— ⑥}$$

From eqns ⑤ & ⑥

$$\Rightarrow \boxed{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}$$

ABCD parameters in terms of other parameters :-

1. ABCD in terms of Z-parameters :-

For ABCD parameters, $V_1 = AV_2 - BI_2$ — ① (V_1, V_2, I_2)

$I_1 = CV_2 - DI_2$ — ② (I_1, V_2, I_2)

For Z parameters, $V_1 = Z_{11} I_1 + Z_{12} I_2$ — ③ (V_1, I_1, I_2)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ — ④ (V_2, I_1, I_2)

From eqn ④, $Z_{21} I_1 = V_2 - Z_{22} I_2$

$$\Rightarrow I_1 = \frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2 \quad \text{—— ⑤}$$

Substitute eqn. ⑤ in equation ③

$$\Rightarrow V_1 = Z_{11}\left[\frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2\right] + Z_{12} I_2$$

$$= \frac{Z_{11}}{Z_{21}} V_2 - \frac{Z_{22}Z_{11} - Z_{12}Z_{21}}{Z_{21}} I_2 \quad \text{—— ⑥}$$

From eqn's ⑤ & ⑥

$$\boxed{\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{Z_{22}Z_{11} - Z_{12}Z_{21}}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}}$$

2. ABCD in terms of Y-parameters:-

For ABCD parameters,
$V_1 = AV_2 - BI_2$ —— ① (V_1, V_2, I_2)
$I_1 = CV_2 - DI_2$ —— ② (I_1, V_2, I_2)

For Y-parameters,
$I_1 = Y_{11} V_1 + Y_{12} V_2$ —— ③ (I_1, V_1, V_2)
$I_2 = Y_{21} V_1 + Y_{22} V_2$ —— ④ (I_2, V_1, V_2)

From eqn ④, $Y_{21} V_1 = -Y_{22} V_2 + I_2$

$$\Rightarrow V_1 = \frac{-Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2 \quad \text{—— ⑤}$$

Substitute eqn. ⑤ in ③

$$\Rightarrow I_1 = Y_{11}\left[\frac{-Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2\right] + Y_{12} V_2$$

$$= -\left(\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{21}}\right) V_2 - \left(-\frac{Y_{11}}{Y_{21}}\right) I_2 \quad \text{—— ⑥}$$

From equations 5 & 6

$$\Rightarrow \boxed{\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\left(\frac{Y_{11}Y_{22}-Y_{12}Y_{21}}{Y_{21}}\right) & -\frac{Y_{11}}{Y_{21}} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}}$$

3. ABCD in terms of h-parameters :-

For ABCD parameters, $V_1 = AV_2 - BI_2$ —— 1 (V_1, V_2, I_2)

$I_1 = CV_2 - DI_2$ —— 2 (I_1, V_2, I_2)

For h-parameters, $V_1 = h_{11}I_1 + h_{12}V_2$ —— 3 (V_1, I_1, V_2)

$I_2 = h_{21}I_1 + h_{22}V_2$ —— 4 (I_2, I_1, V_2)

From eqn 4 $\Rightarrow h_{21}I_1 = -h_{22}V_2 + I_2$

$$I_1 = \frac{-h_{22}}{h_{21}}V_2 - \left(\frac{-1}{h_{21}}\right)I_2 \quad \text{—— 5}$$

Substitute eqn 5 in 3 $\Rightarrow V_1 = h_{11}\left[\frac{-h_{22}}{h_{21}}V_2 + \frac{1}{h_{21}}I_2\right] + h_{12}V_2$

$$= \frac{-h_{11}h_{22}+h_{21}h_{12}}{h_{21}}V_2 - \left(\frac{-h_{11}}{h_{21}}\right)I_2 \quad \text{—— 6}$$

From eqn's 5 & 6 $\Rightarrow$

$$\boxed{\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{-h_{11}h_{22}+h_{12}h_{21}}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}}$$

H-parameters in terms of other parameters:-

1. H in terms of z-parameters:-

For h-parameters, $V_1 = h_{11} I_1 + h_{12} V_2$ — ① (V_1, I_1, V_2)

$I_2 = h_{21} I_1 + h_{22} V_2$ — ② (I_2, I_1, V_2)

For z-parameters, $V_1 = Z_{11} I_1 + Z_{12} I_2$ — ③ (V_1, I_1, I_2)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ — ④ (V_2, I_1, I_2)

From eqn ④ $\Rightarrow Z_{22} I_2 = -Z_{21} I_1 + V_2$

$$\Rightarrow I_2 = \frac{-Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2 \quad \text{— ⑤}$$

From substitute eqn ⑤ in eqn ③

$$\Rightarrow V_1 = Z_{11} I_1 + Z_{12}\left[\frac{-Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2\right]$$

$$= \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}} I_1 + \frac{Z_{12}}{Z_{22}} V_2 \quad \text{— ⑥}$$

From eqn's ⑤ & ⑥

$$\boxed{\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}}$$

2. H in terms of Y-parameters:-

For h-parameters, $V_1 = h_{11} I_1 + h_{12} V_2$ — ① (V_1, I_1, V_2)

$I_2 = h_{21} I_1 + h_{22} V_2$ — ② (I_2, I_1, V_2)

For Y-parameters, $I_1 = Y_{11} V_1 + Y_{12} V_2$ — ③ (I_1, V_1, V_2)

$I_2 = Y_{21} V_1 + Y_{22} V_2$ — ④ (I_2, V_1, V_2)

$\therefore$ From eqn ③ $\Rightarrow$ $y_{11}V_1 = I_1 - y_{12}V_2$

$$V_1 = \frac{1}{y_{11}} I_1 - \frac{y_{12}}{y_{11}} V_2 \quad \text{---} ⑤$$

Substitute eqn ⑤ in eqn ④

$$\Rightarrow \quad I_2 = y_{21}\left[\frac{1}{y_{11}} I_1 - \frac{y_{12}}{y_{11}} V_2\right] + y_{22}V_2$$

$$= \frac{y_{21}}{y_{11}} I_1 + \frac{y_{11}y_{22} - y_{21}y_{12}}{y_{11}} V_2 \quad \text{---} ⑥$$

From eqn's ⑤ & ⑥ $\Rightarrow$

$$\boxed{\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{y_{11}} & -\frac{y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{y_{11}y_{22} - y_{21}y_{12}}{y_{11}} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}}$$

3. h in terms of ABCD parameters :-

For h-parameters, $V_1 = h_{11}I_1 + h_{12}V_2$ — ① (V_1, I_1, V_2)

$I_2 = h_{21}I_1 + h_{22}V_2$ — ② (I_2, I_1, V_2)

For ABCD parameters; $V_1 = AV_2 - BI_2$ — ③ (V_1, V_2, I_2)

$I_1 = CV_2 - DI_2$ — ④ (I_1, V_2, I_2)

From eqn ④ $\Rightarrow$ $DI_2 = -I_1 + CV_2$

$$I_2 = \frac{-1}{D} I_1 + \frac{C}{D} V_2 \quad \text{---} ⑤$$

Substitute eqn. ⑤ in eqn ③ $\Rightarrow$ $V_1 = AV_2 - B\left[\frac{-1}{D}I_1 + \frac{C}{D}V_2\right]$

$$\Rightarrow \quad V_1 = \frac{B}{D} I_1 + \frac{AD-BC}{D} V_2 \quad \text{---} ⑥$$

From ⑤&⑥ $\Rightarrow$

$$\boxed{\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{B}{D} & \frac{AD-BC}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}}$$

S.No.	Parameter	Dependent Variables	Independent Variables	Equations	Condition for Symmetry	Condition for Reciprocity
1.	Z-parameters (Impedence (or) Open ckt parameters)	V_1, V_2	I_1, I_2	$V_1 = Z_{11} I_1 + Z_{12} I_2$ $V_2 = Z_{21} I_1 + Z_{22} I_2$	$Z_{11} = Z_{22}$	$Z_{12} = Z_{21}$
2.	Y-parameters (Admittance (or) Short ckt parameters)	I_1, I_2	V_1, V_2	$I_1 = Y_{11} V_1 + Y_{12} V_2$ $I_2 = Y_{21} V_1 + Y_{22} V_2$	$Y_{11} = Y_{22}$	$Y_{12} = Y_{21}$
3.	ABCD parameters (Chain (or) Transmission parameters)	V_1, I_1	V_2, I_2	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$	$A = D$	$AD - BC = 1$
4.	H-parameters (Hybrid parameters)	V_1, I_2	V_2, I_1	$V_1 = h_{11} I_1 + h_{12} V_2$ $I_2 = h_{21} I_1 + h_{22} V_2$	$h_{11}h_{22} - h_{12}h_{22} = 1$	$h_{12} = -h_{21}$

Table for Interrelation between various parameters :-

	$[Z]$	$[Y]$	$[h]$	$[T]$
$[Z]$	$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$
$[Y]$	$\begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$	$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & -\frac{\Delta T}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$
$[h]$	$\begin{bmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{21}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{Y_{11}} & \frac{-Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{AD-BC}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{bmatrix}$
$[T]$	$\begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-Y_{22}}{Y_{21}} & \frac{-1}{Y_{21}} \\ \frac{-\Delta Y}{Y_{21}} & \frac{-Y_{11}}{Y_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

Where,

$$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$$

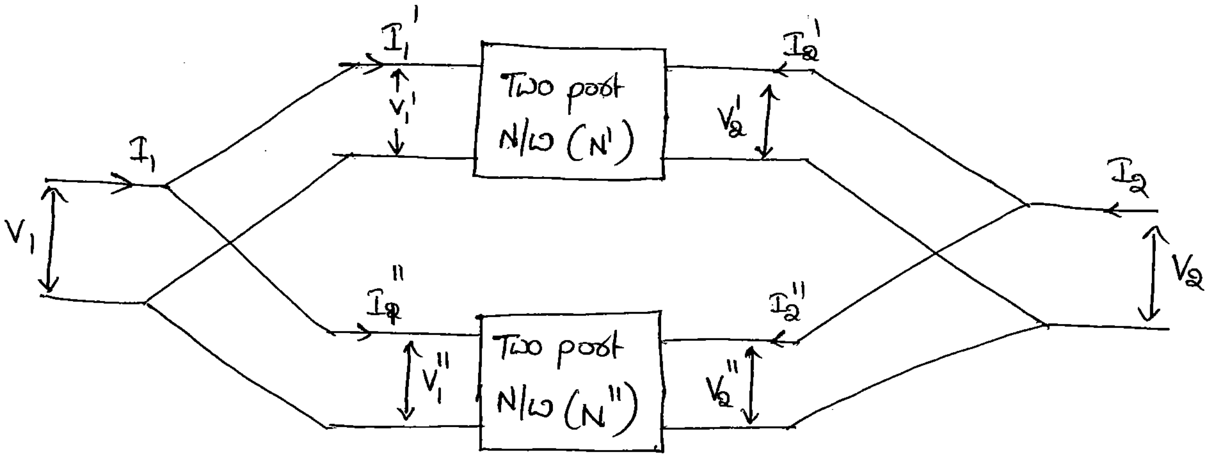
$$\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$$

$$\Delta h = h_{11} h_{22} - h_{12} h_{21}$$

$$\Delta T = AD - BC$$

Parallel Connection of 2-port Networks :-

Consider two networks N' and N'' are connected in parallel as shown in fig. below. When 2-port Networks are connected in parallel we can add their "Y-parameters" to get overall Y-parameters of the parallel connection.



Let the Y-parameters for the Network N' are Y_{11}', Y_{12}', Y_{21}', Y_{22}'. Let the Y-parameter for the Network N'' are Y_{11}'', Y_{12}'', Y_{21}'', Y_{22}''. Let the overall Y-parameters for parallel connection are Y_{11}, Y_{12}, Y_{21}, Y_{22}.

For a parallel connected N/W, $V_1 = V_1' = V_1''$

$$V_2 = V_2' = V_2''$$

$$I_1 = I_1' + I_1''$$

$$I_2 = I_2' + I_2''$$

For the N/W N' the eqn's representing Y-parameters are,

$$I_1' = Y_{11}' V_1' + Y_{12}' V_2' \quad \text{—①}$$

$$I_2' = Y_{21}' V_1' + Y_{22}' V_2' \quad \text{—②}$$

For the N/w N'' the equations representing y-parameters are;

$$I_1'' = y_{11}'' V_1'' + y_{12}'' V_2'' \quad \text{--- } ③$$

$$I_2'' = y_{21}'' V_1'' + y_{22}'' V_2'' \quad \text{--- } ④$$

But we know, $I_1 = I_1' + I_1''$

$$= [y_{11}' V_1' + y_{12}' V_2'] + [y_{11}'' V_1'' + y_{12}'' V_2'']$$

$$= [y_{11}' + y_{11}''] V_1 + [y_{12}' + y_{12}''] V_2 \quad \text{--- } ⑤ \quad \left(\because V_1 = V_1' = V_1'' ,\ V_2 = V_2' = V_2''\right)$$

Similarly, $I_2 = I_2' + I_2''$

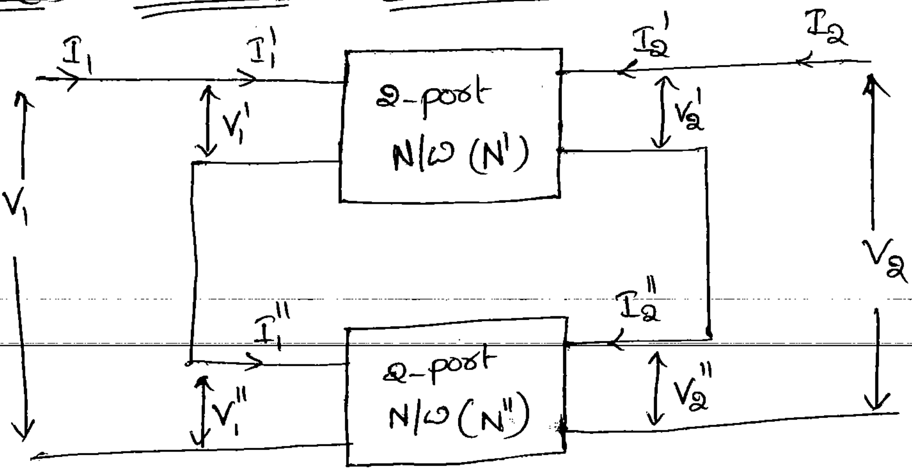
$$= [y_{21}' V_1' + y_{22}' V_2'] + [y_{21}'' V_1'' + y_{22}'' V_2'']$$

$$= [y_{21}' + y_{21}''] V_1 + [y_{22}' + y_{22}''] V_2 \quad \text{--- } ⑥$$

From equations ⑤ & ⑥, The overall y-parameters of parallel Connected N/w's,

$$\boxed{\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} y_{11}' + y_{11}'' & y_{12}' + y_{12}'' \\ y_{21}' + y_{21}'' & y_{22}' + y_{22}'' \end{bmatrix}}$$

Series connection of 2-port N/w's :-



When two ports are connected in series we can add their "Z-parameters" to get the overall Z-parameters.

Here for series connection, $V_1 = V_1' + V_1''$

$$V_2 = V_2' + V_2''$$

$$I_1 = I_1' + I_1''$$

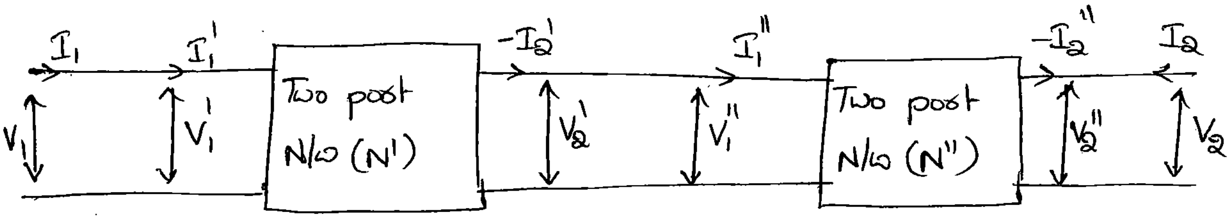
$$I_2 = I_2' = I_2''$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11}' + Z_{11}'' & Z_{12}' + Z_{12}'' \\ Z_{21}' + Z_{21}'' & Z_{22}' + Z_{22}'' \end{bmatrix}$$

Cascade Connection of two port Networks:-

The cascade connection is also called

Connection." Consider two Networks N' and N'' are connected in cascade as shown in fig. below: When two ports are connected in cascade, we can multiply their individual transmission parameters to get overall transmission parameters of cascaded connection.



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix}$$