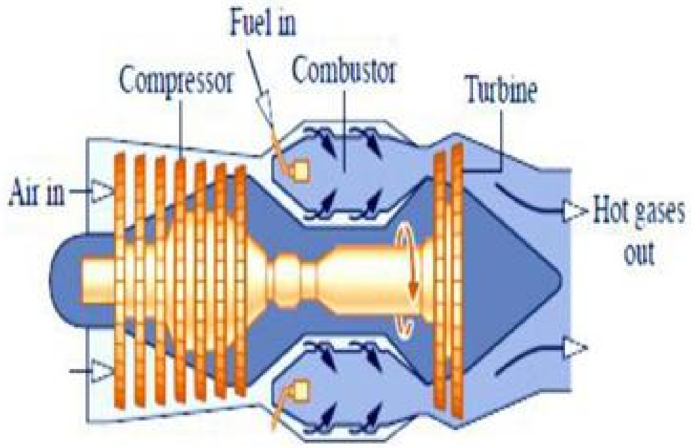
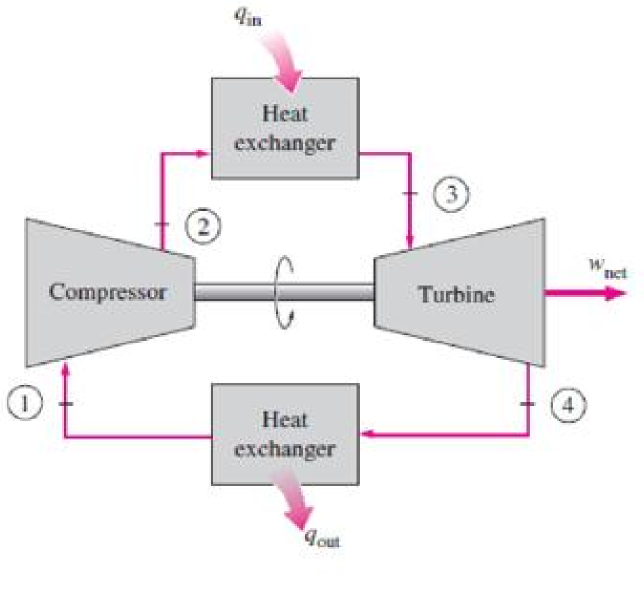
Unit - V

Air Standard Cycles





Course Contents

7.1 Terminology Used in Gas Power Cycles

a) Cycle: "A Cycle is defined as a repeated series of operations occurring in a certain order."

b) Air standard cycle: "The thermodynamics cycle with air as the working fluid is called an air standard cycle."

c) Compression ratio (r):

$$r = \frac{Total\ cylinder\ volume}{Clearance\ volume}$$

$$r = \frac{V_C + V_S}{V_C} - - - - - - - - - (7.1)$$

– Higher the compression ratio better will be the performance of an engine.

d) Piston Speed: "The distance travelled by the piston in one minute is called piston speed."

$$Piston\ Speed = \frac{2LN}{60}\frac{m}{sec} - - - - - - - - - (7.2)$$

e) Mechanical Efficiency: It is defined as the ratio of the brake power and the indicated power. Mechanical efficiency is indicator of losses due to friction.

$$\eta_{mech} = \frac{B.P.}{I.P.} - - - - - - - - - (7.3)$$

f) Thermal Efficiency: "It is the ratio of work done to heat supplied by fuel."

$$\eta_{th} = \frac{\text{Work output}}{\text{Heat input}} = \frac{Q_1 - Q_2}{Q_1} - - - - - - - - - (7.4)$$

Where,

Q_1 = Heat addition

Q_2 = Heat rejection

[Assuming no friction & heat losses, so $W = Q_1 - Q_2$]

i. **Indicated thermal efficiency** = Indicated Power/ Heat supplied by fuel

$$\eta_{ith} = \frac{I.P.}{m_f \times CV} - - - - - - - - - (7.5)$$

Where, m_f = mass of fuel supplied, Kg/sec and CV = calorific value of fuel, J/kg

ii. **Brake thermal efficiency** = Brake Power/ Heat supplied by fuel

$$\eta_{bth} = \frac{B.P.}{m_f \times CV} - - - - - - - - - (7.6)$$

Also

$$\eta_{mech} = \frac{\eta_{bth}}{\eta_{ith}} ---------(7.7)$$

g) Air standard efficiency: The efficiency of engine using air as the working medium is known as an "Air standard efficiency" or "Ideal efficiency".

- The actual efficiency of a cycle is always less than the air standard efficiency of that cycle under ideal conditions.
- This is taken into account by introducing a new term "Relative efficiency".

$$\eta_{\text{relative}} = \frac{\text{Actual thermal efficiency}}{\text{Air standard efficiency}} ---------(7.8)$$

- The analysis of all air standard cycles is based upon the following assumptions.

<u>Assumptions:</u>

1. The gas in the engine cylinder is a perfect gas i.e. it obeys the gas laws and has constant specific heat.
2. The compression and expansion processes are adiabatic and they take place without internal friction i.e. these processes are Isentropic.
3. No chemical reaction takes place in the cylinder. Heat is supplied or rejected by bringing a hot body or a cold body in contact with cylinder at appropriate points during the process.
4. The engine operates in a closed cycle. The cylinder is filled with constant amount of working medium and the same fluid is used repeatedly.

The approach and concept of ideal air cycle helps to.......

1. Indicate the ultimate performance i.e. to determine the maximum ideal efficiency of a specific thermodynamics cycle.
2. Study qualitatively the influence of different variables on the performance of an actual engine.
3. Evaluate one engine relative to another.

7.2 Mean Effective Pressure

- The pressure variation versus volume inside the cylinder of a reciprocating engine is plotted with the help of an engine indicator. The resulting contour is closed one and is referred to as indicator diagram as shown in Fig. 7.1.

- The area enclosed by the contour is a measure of the work output per cycle from the engine.
- **Mean effective pressure** is defined as the average pressure acting on the piston which will produce the same output as is done by the varying pressure during a cycle.
- Therefore

  Area of indicator loop = Area of rectangle abcd

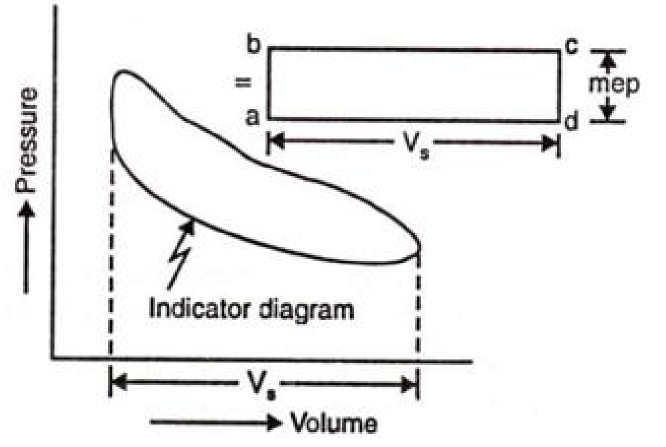


Fig. 7.1 Engine Indicator Diagram

- The height of the rectangle than represents the mean effective pressure.

$$mep = \frac{work\ done\ per\ cycle}{swept\ volume}$$

$$= \frac{Area\ of\ indicator\ loop}{length\ of\ loop} \quad ---------(7.9)$$

<u>**Unit:**</u> bar or KN/m^2

- Mean effective pressure is used as a parameter to compare the performance of reciprocating engines of equal size.
- An engine that has a large volume of mep will deliver more net work and will thus perform better.

7.3 The Carnot Gas Power Cycle

- A Carnot cycle is a hypothetical cycle consisting four different processes: two reversible isothermal processes and two reversible adiabatic (isentropic) processes.
- According to Carnot theorem ***"No cycle can be more efficient than a reversible cycle operating between the same temperature limits."***

  Assumptions made in the working of the Carnot cycle

a) Working fluid is a perfect gas.

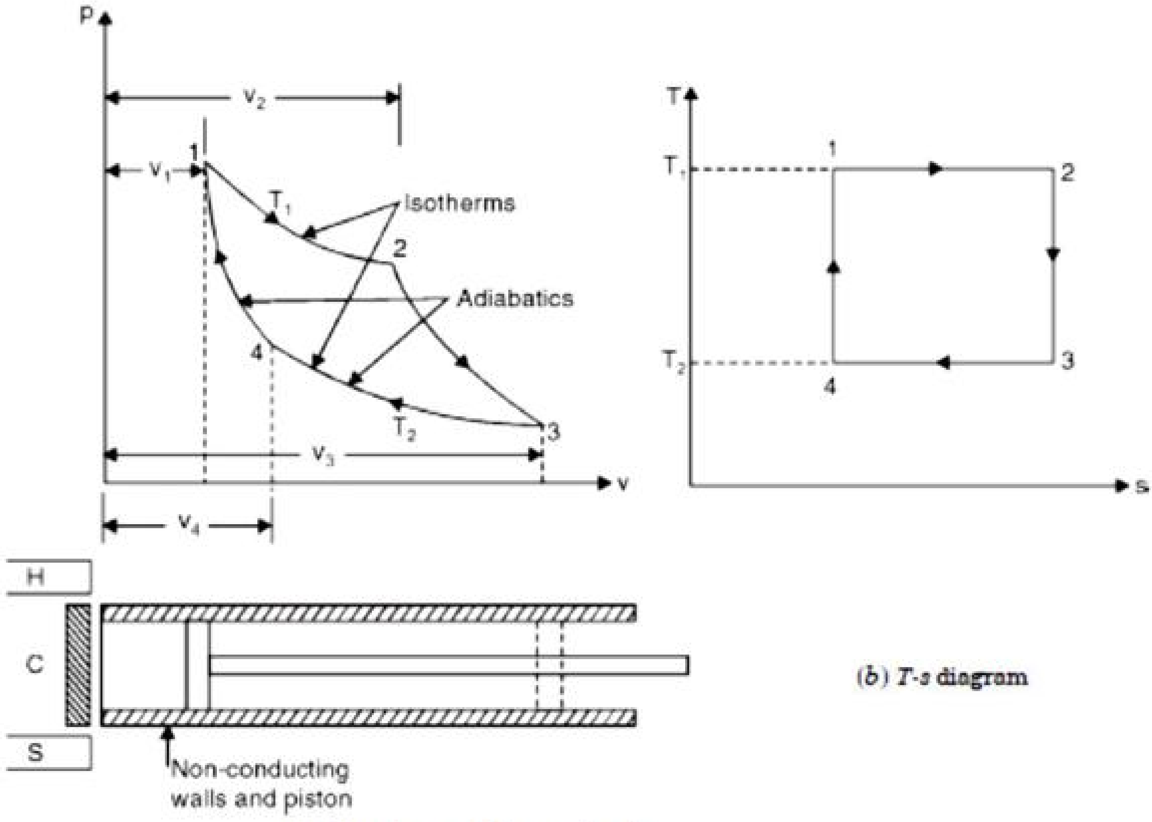
b) Piston cylinder arrangement is weightless and does not produce friction during motion.

c) The walls of cylinder and piston are considered as perfectly insulated.

d) Compression and expansion are reversible.

e) The transfer of heat does not change the temperature of sources or sink.

– Fig. 7.2 shows essential elements for a Carnot cycle, P-v and T-S diagrams.



(a) Four stages of the carnot cycle

(b) T-s diagram

Fig. 7.2 P-v, T-S and schematic diagram of Carnot gas power cycle

– This cycle has the highest possible efficiency and it consists four simple operations as below:

a) Isothermal Expansion (1 – 2)
b) Isentropic Expansion (2 – 3)
c) Isothermal Compression (3 – 4)
d) Isentropic Compression (4 – 1)

a) Isothermal expansion (1 – 2) :-

The source of heat (H) is applied to the end of the cylinder and isothermal reversible expansion occurs at temperature T_1. During this process q_1 heat is supplied to the system.

b) Adiabatic expansion (2 – 3) :-

Non conducting cover (C) is applied to the end of the cylinder and the cylinder becomes perfect. Adiabatic cover is brought in contact with the cylinder head. Hence no heat transfer takes place. The fluid expands adiabatically and reversibly. The temperature falls from T_1 to T_2.

c) Isothermal compression (3 – 4) :-

Adiabatic cover is removed and sink (S) is applied to the end of the cylinder. The heat, q_2 is transferred reversibly and isothermally at temperature T_2 from the system to the sink (S).

d) Adiabatic compression (4 – 1) :-

Adiabatic cover is brought in contact with cylinder head. This completes the cycle and system is returned to its original state at 1. During the process, the temperature of system is raised from T_2 to T_1.

Efficiency of Carnot Gas Cycle:

- Consider 1 kg of working substance (air) is enclosed in the cylinder.
- **Heat supplied** during isothermal process (1 – 2):

$$q_1 = p_1 V_1 \ln \frac{V_2}{V_1}$$

$$\therefore q_1 = RT_1 \ln \frac{V_2}{V_1}$$

- **Heat rejected** during isothermal compression (3 – 4):

$$q_2 = p_3 V_3 \ln \frac{V_4}{V_3}$$

$$\therefore q_2 = RT_2 \ln \frac{V_4}{V_3}$$

- During adiabatic expansion (2 – 3) and adiabatic compression (4 – 1), the heat transfer from or to the system is zero.
- **Work done**,

$$W = q_1 - q_2$$

$$\therefore W = RT_1 \ln \frac{V_2}{V_1} - RT_2 \ln \frac{V_4}{V_3} - - - - - - - - -(7.10)$$

- Let r = ratio of expansion for process (1 – 2) $= \frac{V_2}{V_1}$

  = ratio of compression for process (3 – 4) $= \frac{V_4}{V_3}$

- by substituting the value of r in equation 7.10, we get,

$$W = RT_1 \ln r - RT_2 \ln r - - - - - - - - -(7.11)$$

- **Thermal efficiency,**

$$\eta = \frac{Work\ done}{Heat\ supplied}$$

$$\therefore \eta = \frac{RT_1 \ln r - RT_2 \ln r}{RT_1 \ln \frac{V_2}{V_1}} = \frac{RT_1 \ln r - RT_2 \ln r}{RT_1 \ln r}$$

$$\therefore \eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} - - - - - - - - -(7.12)$$

Where,

T_1 = Maximum temperature of the cycle (K)

T_2 = Minimum temperature of cycle (K)

- In equation 7.12, if temperature T_2 decreases, efficiency increases and it becomes 100% if temperature T_2 becomes absolute zero; which is ***impossible*** to attain.

Limitations of Carnot Gas Cycle:

✓ The Carnot cycle is hypothetical.

✓ The thermal efficiency of Carnot cycle depends upon absolute temperature of heat source T1 and heat sink T2 only, and independent of the working substance.

✓ Practically it is not possible to neglect friction between piston and cylinder. It can be minimized but cannot be eliminated.

✓ It is impossible to construct cylinder walls which are perfect insulator. Some amount of heat will always be transferred. Hence perfect adiabatic process cannot be achieved.

✓ The isothermal and adiabatic processes take place during the same stroke. Therefore the piston has to move very slowly for isothermal process and it has to move very fast during remaining stoke for adiabatic process which is practically not possible.

✓ The output obtained per cycle is very small. This work may not be able to overcome the friction of the reciprocating parts.

7.4 The Otto Cycle *OR* Constant Volume Cycle (Isochoric)

- The cycle was successfully applied by a German scientist Nicolous A. Otto to produce a successful 4 – stroke cycle engine in 1876.

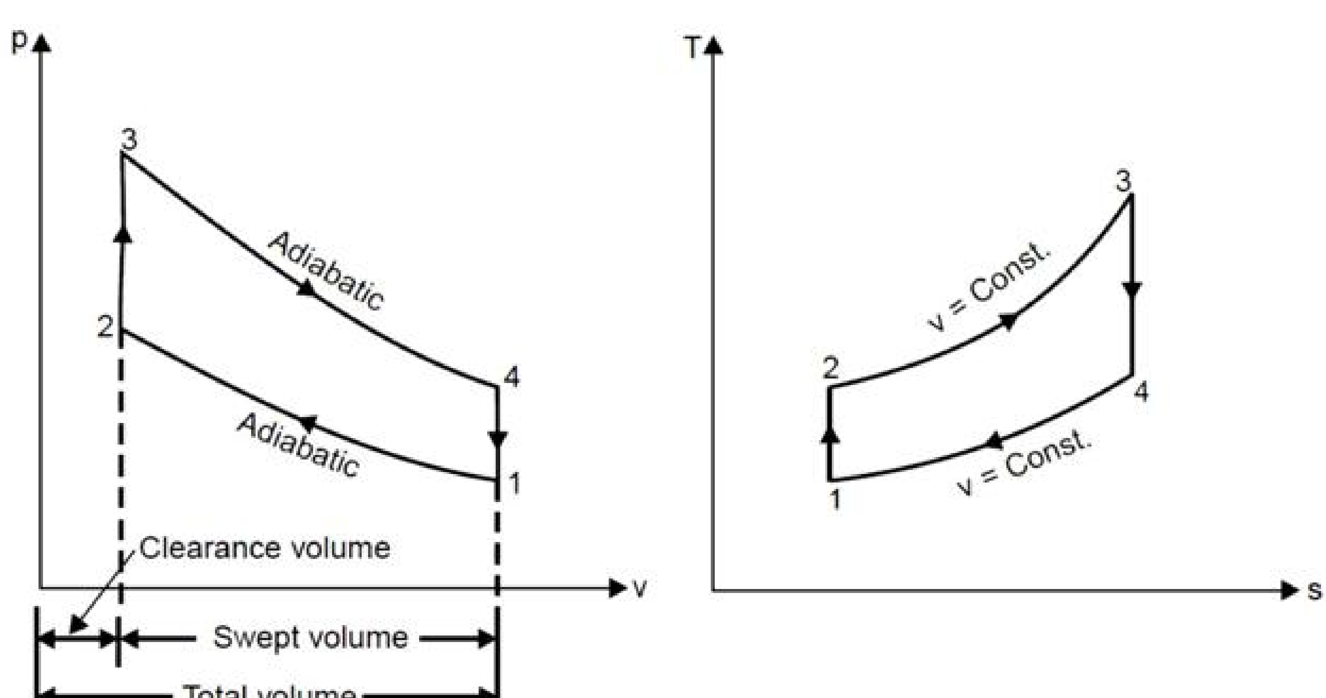


Fig. 7.3 p-V and T-s diagrams of Otto cycle

- The thermodynamic cycle is operated with isochoric (constant volume) heat addition and consists of two adiabatic processes and two constant volume changes.
- Fig. 7.3 shows the Otto cycle plotted on p – V and T – s diagram.

Adiabatic Compression Process (1 – 2):

- At pt. 1 cylinder is full of air with volume V_1, pressure P_1 and temp. T_1.
- Piston moves from BDC to TDC and an ideal gas (air) is compressed isentropically to state point 2 through compression ratio,

$$r = \frac{V_1}{V_2}$$

Constant Volume Heat Addition Process (2 – 3):

- Heat is added at constant volume from an external heat source.
- The pressure rises and the ratio $r_p\ or\ \alpha = \frac{P_3}{P_2}$ is called expansion ratio or pressure ratio.

Adiabatic Expansion Process (3 – 4):

- The increased high pressure exerts a greater amount of force on the piston and pushes it towards the BDC.
- Expansion of working fluid takes place isentropically and work done by the system.
- The volume ratio $\frac{V_4}{V_3}$ is called isentropic expansion ratio.

Constant Volume Heat Rejection Process (4 – 1):

- Heat is rejected to the external sink at constant volume. This process is so controlled that ultimately the working fluid comes to its initial state 1 and the cycle is repeated.
- Many petrol and gas engines work on a cycle which is a slight modification of the Otto cycle.
- This cycle is called constant volume cycle because the heat is supplied to air at constant volume.

Thermal Efficiency of an Otto Cycle:

- Consider a unit mass of air undergoing a cyclic change.
- **Heat supplied** during the process 2 – 3,

$$q_1 = C_V(T_3 - T_2)$$

- **Heat rejected** during process 4 – 1 ,

$$q_2 = C_V(T_4 - T_1)$$

- **Work done,**

$$\therefore W = q_1 - q_2$$

$$\therefore W = C_V (T_3 - T_2) - C_V (T_4 - T_1)$$

- **Thermal efficiency,**

$$\eta = \frac{Work\ done}{Heat\ supplied} = \frac{W}{q_1}$$

$$= \frac{C_V (T_3 - T_2) - C_V (T_4 - T_1)}{C_V (T_3 - T_2)}$$

$$= 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} - - - - - - - - - (7.13)$$

- For Adiabatic compression process (1 – 2),

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1}$$

$$\therefore \mathrm{T_2} = \mathrm{T_1 r^{\gamma-1}} - - - - - - - - - (7.14)$$

- For Isentropic expansion process (3 – 4),

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$\therefore T_3 = T_4 \left(\frac{V_4}{V_3}\right)^{\gamma-1}$$

$$\therefore T_3 = \mathrm{T_4} \left(\frac{\mathrm{V_1}}{\mathrm{V_2}}\right)^{\gamma-1} (\because \mathrm{V_1} = \mathrm{V_4}, \mathrm{V_2} = \mathrm{V_3})$$

$$\therefore T_3 = T_4 (r)^{\gamma-1} - - - - - - - - - (7.15)$$

- From equation 7.13, 7.14 & 7.15, we get,

$$\eta_{otto} = 1 - \frac{(T_4 - T_1)}{T_4\, r^{\gamma-1} - T_1\, r^{\gamma-1}}$$

$$\therefore \eta_{otto} = 1 - \frac{(T_4 - T_1)}{r^{\gamma-1}(T_4 - T_1)}$$

$$\therefore \boldsymbol{\eta_{otto} = 1 - \frac{1}{r^{\gamma-1}} - - - - - - - - - (7.16)}$$

Expression 7.16 is known as the air standard efficiency of the Otto cycle.

- It is clear from the above expression that efficiency increases with the increase in the value of r (as is constant).
- We can have maximum efficiency by increasing r to a considerable extent, but due to practical difficulties its value is limited to 8.

- In actual engines working on Otto cycle, the compression ratio varies from 5 to 8 depending upon the quality of fuel.
- At compression ratios higher than this, the temperature after combustion becomes high and that may lead to spontaneous and uncontrolled combustion of fuel in the cylinder.
- The phenomenon of uncontrolled combustion in petrol engine is called detonation and it leads to poor engine efficiency and in structural damage of engine parts.
- Fig. 7.4 shows the variation of air standard efficiency of Otto cycle with compression ratio.

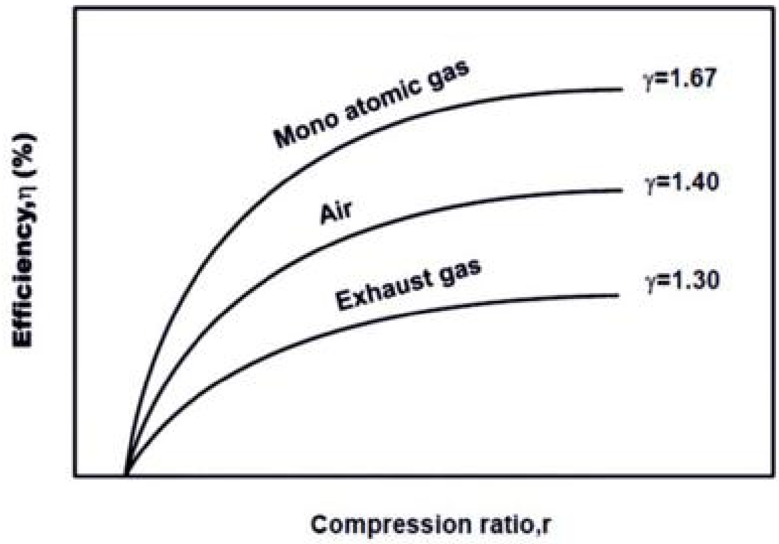


Fig. 7.4 Variation of Otto cycle efficiency with compression ratio

Mean Effective Pressure:

- **Net work done** per unit mass of air,

$$W_{net} = C_V\,(T_3 - T_2) - C_V(T_4 - T_1) - - - - - - - - - (7.17)$$

- **Swept volume,**

$$Swept\ volume = V_1 - V_2 = V_1\left(1 - \frac{V_2}{V_1}\right) = \frac{RT_1}{P_1}\left(1 - \frac{1}{r}\right)$$

$$= \frac{RT_1}{P_1 r}(r - 1) - - - - - - - - - (7.18)$$

- **Mean effective pressure,**

$$mep = \frac{Work\ done\ per\ cycle}{swept\ volume}$$

$$= \frac{C_V\,(T_3 - T_2) - C_V(T_4 - T_1)}{\frac{R\,T_1}{P_1 r}\,(r - 1)}$$

$$= \frac{C_V}{R}\,\frac{P_1 r}{(r - 1)}\left[\frac{(T_3 - T_2) - (T_4 - T_1)}{T_1}\right] - - - - - - - - - (7.19)$$

- For process 1 – 2,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\boldsymbol{T_2 = T_1\, r^{\gamma-1}}$$

- Process 2 – 3,

$$\frac{T_3}{T_2} = \frac{P_3}{P_2} (\because\ V_2 = V_3)$$

$$\therefore\ T_3 = T_2\, \alpha \qquad (\alpha = explosion\ pressure\ ratio)$$

$$\therefore\ \boldsymbol{T_3 = T_1\, \alpha\, r^{\gamma-1}}$$

- Process 3 – 4,

$$T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$\therefore T_4 = T_1\ \alpha\ r^{\gamma-1} \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\therefore T_4 = T_1\ \alpha\ r^{\gamma-1} \times \frac{1}{r^{\gamma-1}}$$

$$\therefore\ \boldsymbol{T_4 = T_1 \cdot \alpha}$$

Substituting all these temperature values in equation 7.19, We get,

$$mep = \frac{C_V}{R}\frac{P_1\, r}{(r-1)}\left[\frac{(T_1\, \alpha\, r^{\gamma-1} - T_1\, r^{\gamma-1}) - (T_1\, \alpha - T_1)}{T_1}\right]$$

$$\therefore mep = \frac{C_V}{R}\frac{P_1\, r}{(r-1)}\left[\frac{T_1\, r^{\gamma-1}(\alpha-1) - T_1\,(\alpha-1)}{T_1}\right]$$

$$\therefore mep = \frac{C_V}{R}\frac{P_1\, r}{(r-1)}[(r^{\gamma-1}-1)(\alpha-1)]$$

$$\therefore mep = \frac{P_1\, r}{(r-1)(\gamma-1)}[(r^{\gamma-1}-1)(\alpha-1)] ---------(7.20)$$

$$\left(\because \frac{C_V}{R} = \frac{1}{\gamma-1}\right)$$

$$\begin{bmatrix} \dfrac{C_P}{C_V} = \gamma, \\ C_P - C_V = R, \\ C_V\left(\dfrac{C_P}{C_V} - 1\right) = R, \\ \dfrac{C_V}{R} = \dfrac{1}{\gamma-1} \end{bmatrix}$$

Condition for Maximum Work:

- For unit mass of air,

$$W = q_1 - q_2$$

$$\therefore W = C_V\ (T_3 - T_2) - C_V\ (T_4 - T_1)$$

$$\therefore \frac{W}{C_V} = T_3 - T_2 - T_4 + T_1 - - - - - - - - - (7.21)$$

- We know that,

$$T_2 = T_1 r^{\gamma-1}$$

$$T_4 = T_3 \frac{1}{r^{\gamma-1}} = T_3 \frac{T_1}{T_2} \qquad \left(\because r^{\gamma-1} = \frac{T_2}{T_1}\right)$$

So

$$\frac{W}{C_V} = T_3 - T_2 - \frac{T_3 T_1}{T_2} + T_1 - - - - - - - - - (7.22)$$

- The intermediate temperature T_2 for maximum work output can be obtained by differentiating the above equation with respect to T_2 & setting the derivatives equal to zero.

$$\therefore \frac{1}{C_V}\frac{dW}{dT_2} = -1 + \frac{T_1 T_3}{T_2^2} = 0 \quad (for\ max\ work)$$

$$\therefore T_2^2 = T_1 T_3$$

$$\therefore \boldsymbol{T_2 = \sqrt{T_1 T_3} - - - - - - - - - (7.23)}$$

- Similarly for temperature T_4

$$\frac{W}{C_V} = T_3 - \frac{T_1 \cdot T_3}{T_4} - T_4 + T_1$$

$$\therefore \frac{1}{C_V}\frac{dW}{dT_4} = \frac{T_1 T_3}{T_4^2} - 1 = 0 \qquad (for\ max\ work)$$

$$\therefore \boldsymbol{T_4 = \sqrt{T_1 T_3} - - - - - - - - - (7.24)}$$

- Thus for maximum work,

$$\boldsymbol{T_2 = T_4 = \sqrt{T_1 T_3} - - - - - - - - - (7.25)}$$

i.e. the intermediate temperature T_2 & T_4 must be equal for maximum work.

Maximum work,

$$W_{max} = C_V(T_3 - T_2 - T_4 + T_1)$$

$$\therefore W_{max} = C_V\left(T_3 - \sqrt{T_1 T_3} - \sqrt{T_1 T_3} + T_1\right)$$

$$\therefore \boldsymbol{W_{max} = C_V\left(T_3 + T_1 - 2\sqrt{T_1 T_3}\right) - - - - - - - - - (7.26)}$$

7.5 The Diesel Cycle *__OR__* Constant Pressure Cycle (Isobaric)

- This cycle was discovered by a German engineer Dr. Rudolph Diesel. Diesel cycle is also known as ***constant pressure heat addition cycle.***

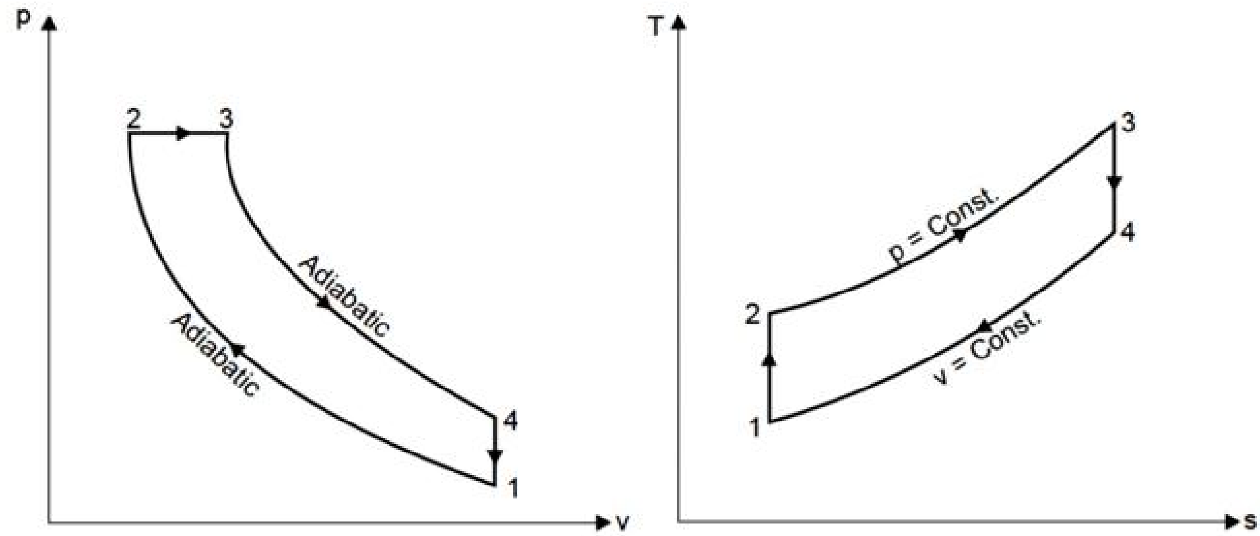


Fig. 7.5 p-V and T-s diagrams of Diesel cycle

Adiabatic Compression Process (1 – 2):

- Isentropic (Reversible adiabatic) compression with $= \frac{V_1}{V_2}$.

Constant Pressure Heat Addition Process (2 – 3):

- The heat supply is stopped at point 3 which is called the cut – off point and the volume ratio $\rho = \frac{V_3}{V_2}$ is called **cut off ratio** or Isobaric expansion ratio.

Adiabatic Expansion Process (3 – 4):

- Isentropic expansion of air $\frac{V_4}{V_3}$ = isentropic expansion ratio.

Constant Volume Heat Rejection Process (4 – 1):

- In this process heat is rejected at constant volume.

This thermodynamics cycle is called constant pressure cycle because heat is supplied to the air at constant pressure.

Thermal Efficiency for Diesel Cycle:

- Consider unit mass of air.
- **Heat supplied** during process 2 – 3,

$$q_1 = C_P(T_3 - T_2)$$

- **Heat rejected** during process 4 – 1,

$$q_2 = C_V(T_4 - T_1)$$

Work done,

$$W = q_1 - q_2$$

$$W = C_P(T_3 - T_2) - C_V(T_4 - T_1)$$

- **Thermal efficiency,**

$$\eta = \frac{Work\ done}{Heat\ supplied}$$

$$\therefore \eta = \frac{C_P(T_3 - T_2) - C_V(T_4 - T_1)}{C_P(T_3 - T_2)}$$

$$\therefore \eta = 1 - \frac{C_V}{C_P}\frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$\therefore \eta = 1 - \frac{1}{\gamma}\frac{(T_4 - T_1)}{(T_3 - T_2)} - - - - - - - - - (7.27)$$

- For adiabatic compression process (1 – 2),

$$r = \frac{V_1}{V_2} - - - - - - - - - (a)$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$P_2 = P_1 r^{\gamma} - - - - - - - - - (b)$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1\, r^{\gamma-1} - - - - - - - - - (c)$$

- For constant pressure heat addition process (2 – 3)

$$P_3 = P_2 = P_1 r^{\gamma} - - - - - - - - - (d)$$

$$\rho = \frac{V_3}{V_2}\ (Cutoff\ ratio) - - - - - - - - - (e)$$

$$T_3 = T_2\,\frac{V_3}{V_2}$$

$$= T_2\,\rho$$

$$\therefore T_3 = T_1\, r^{\gamma-1}\rho - - - - - - - - - (f)$$

- For adiabatic expansion process (3 – 4),

$$P_4 = P_3(V_3/V_4)^{\gamma} = P_3(V_3/V_1)^{\gamma}$$

$$\therefore P_4 = P_3\left(\frac{V_3/V_2}{V_1/V_2}\right)^{\gamma} = P_3(\rho/r)^{\gamma} - - - - - - - - - (g)$$

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} = T_3\left(\frac{\rho}{r}\right)^{\gamma-1}$$

$$\therefore T_4 = \frac{T_1 r^{\gamma-1} \rho \rho^{\gamma-1}}{r^{\gamma-1}}$$

$$\therefore T_4 = T_1 \rho^{\gamma} \text{ --------- } (h)$$

From equation 7.27,

$$\eta = 1 - \frac{1}{\gamma}\frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$\therefore \eta = 1 - \frac{1}{\gamma}\frac{(T_1 \rho^{\gamma} - T_1)}{(T_1 r^{\gamma-1}\rho - T_1 r^{\gamma-1})}$$

$$\therefore \eta = 1 - \frac{1}{\gamma}\frac{(\rho^{\gamma} - 1)}{(r^{\gamma-1}\rho - r^{\gamma-1})}$$

$$\therefore \boldsymbol{\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{(\rho^{\gamma} - 1)}{\gamma(\rho - 1)}\right]} \text{ --------- } \mathbf{(7.28)}$$

- Apparently the efficiency of diesel cycle depends upon the compression ratio (r) and cutoff ratio (ρ) and hence upon the quantity of heat supplied.
- Fig. 7.6 shows the air standard efficiency of diesel cycle for various cut off ratio.

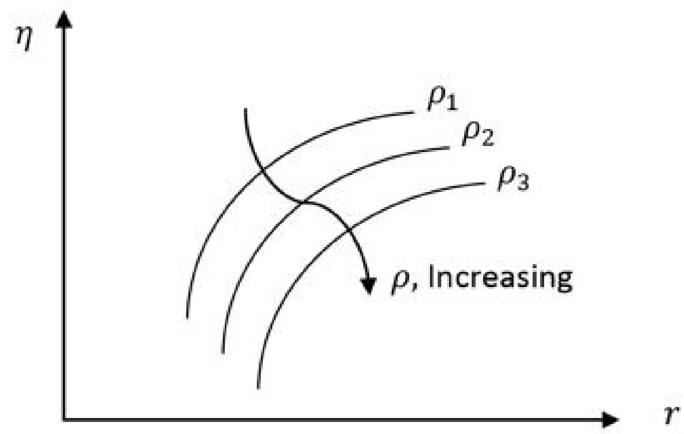


Fig. 7.6 Efficiency of Diesel cycle for various cut-off ratio

- Further,

$$K = \frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)}$$

reveals that with an increase in the cut – off ratio (ρ) the value of factor K increases.

That implies that for a diesel engine at constant compression ratio, the efficiency would increase with decrease in ρ and in the limit $\rho \rightarrow 1$, the efficiency would become

$$1 - \frac{1}{r^{\gamma-1}}$$

- Since the factor $K = \frac{\rho^{\gamma}-1}{\gamma(\rho-1)}$ is always greater than unity, the Diesel cycle is always less efficient than a corresponding Otto cycle having the same compression ratio.

- However Diesel engine operates on much higher compression ratio (14 to 18) compared to those for S.I. Engines operating on Otto cycle.
- High compression ratios for Diesel engines are must not only for high efficiency but also to prevent diesel knock; a phenomenon which leads to uncontrolled and rapid combustion in diesel engines.

Mean Effective Pressure:

- **Net work done** per unit mass of air,

$$W_{net} = C_p (T_3 - T_2) - C_V(T_4 - T_1) - - - - - - - - - (7.29)$$

- **Swept volume,**

$$Swept\ volume = V_1 - V_2 = V_1\left(1 - \frac{V_2}{V_1}\right) = \frac{RT_1}{P_1}\left(1 - \frac{1}{r}\right)$$

$$= \frac{RT_1}{P_1 r}(r - 1) - - - - - - - - - (7.30)$$

- **Mean effective pressure,**

$$mep = \frac{Work\ done\ per\ cycle}{swept\ volume}$$

$$\therefore mep = \frac{C_P (T_3 - T_2) - C_V(T_4 - T_1)}{\frac{R\,T_1}{P_1 r}\,(r - 1)}$$

$$\therefore mep = \frac{C_V}{R}\frac{P_1 r}{(r-1)}\left[\frac{\gamma(T_3 - T_2) - (T_4 - T_1)}{T_1}\right] - - - - - - - - - (7.31)$$

- From equation (c), (f) and (h),

$T_2 = T_1\, r^{\gamma-1}$

$T_3 = T_1\, r^{\gamma-1}\rho$

$T_4 = T_1\, \rho^{\gamma}$

$$\therefore mep = \frac{C_V}{R}\frac{P_1 r}{(r-1)}\left[\frac{\gamma(T_1\, r^{\gamma-1}\rho - T_1\, r^{\gamma-1}) - (T_1\, \rho^{\gamma} - T_1)}{T_1}\right]$$

$$\boldsymbol{\therefore mep = \frac{P_1 r}{(\gamma-1)(r-1)}[\gamma r^{\gamma-1}(\rho - 1) - (\rho^{\gamma} - 1)] - - - - - - - - - (7.32)}$$

7.6 The Dual Combustion Cycle <u>OR</u> The Limited Pressure Cycle

- This is a cycle in which the addition of heat is partly at constant volume and partly at constant pressure.

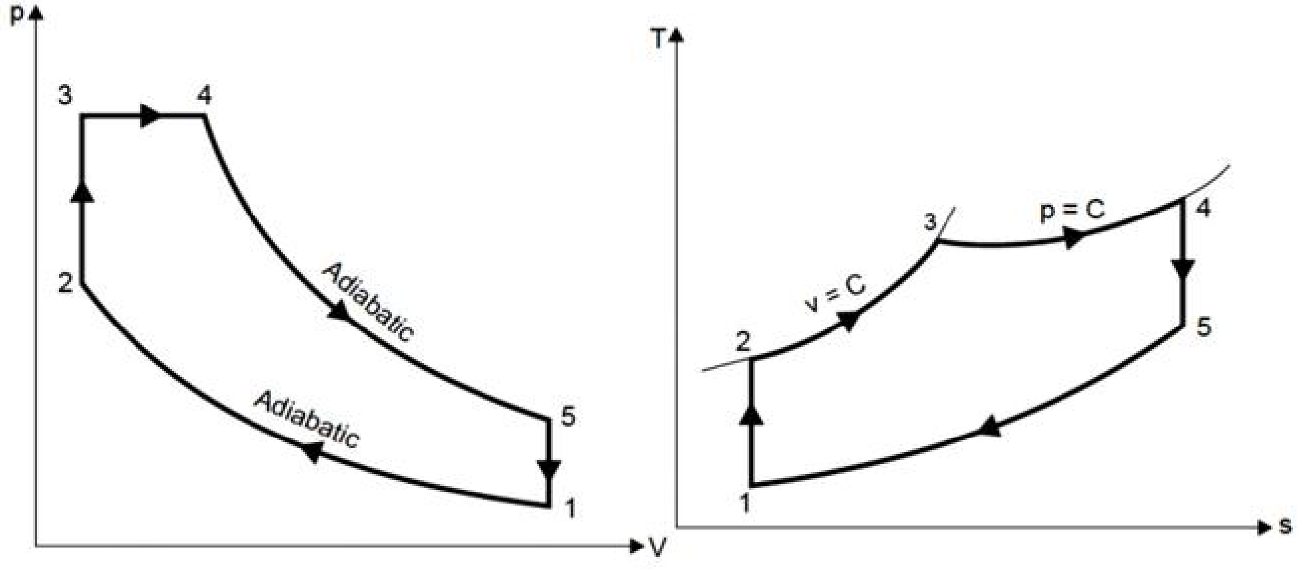


Fig. 7.7 p-V and T-s diagrams of Diesel cycle

Adiabatic Compression Process (1 – 2):

- Isentropic (Reversible adiabatic) compression with $= \frac{V_1}{V_2}$.

Constant Volume Heat Addition Process (2 – 3):

- The heat is supplied at constant volume with explosion ratio or pressure ratio $\alpha = \frac{P_3}{P_2}$.

Constant Pressure Heat Addition Process (3 – 4):

- The heat supply is stopped at point 4 which is called the cut – off point and the volume ratio $\rho = \frac{V_4}{V_3}$ is called **cut off ratio**.

Adiabatic Expansion Process (4 – 5):

- Isentropic expansion of air with $\frac{V_5}{V_4}$ = isentropic expansion ratio.

Constant Volume Heat Rejection Process (5 – 1):

- In this process heat is rejected at constant volume.

The high speed Diesel engines work on a cycle which is slight modification of the Dual cycle.

Thermal Efficiency for Dual Cycle:

- Consider unit mass of air undergoing the cyclic change.
- **Heat supplied**,

$$q_1 = q_{2-3} + q_{3-4}$$
$$q_1 = C_V(T_3 - T_2) + C_P(T_4 - T_3)$$

- **Heat rejected** during process 5 – 1,

$$q_2 = C_V(T_5 - T_1)$$

- **Work done,**

$$W = q_1 - q_2$$

$$W = C_V(T_3 - T_2) + C_P(T_4 - T_3) - \mathrm{C_V(T_5 - T_1)}$$

- **Thermal efficiency,**

$$\eta = \frac{Work\ done}{Heat\ supplied}$$

$$\therefore \eta = \frac{C_V(T_3 - T_2) + C_P(T_4 - T_3) - \mathrm{C_V(T_5 - T_1)}}{C_V(T_3 - T_2) + C_P(T_4 - T_3)}$$

$$\therefore \eta = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)} ---------(7.33)$$

- For adiabatic compression process (1 – 2),

$$r = \frac{V_1}{V_2} ---------(a)$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$P_2 = P_1 r^{\gamma} ---------(b)$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1\, r^{\gamma-1} ---------(c)$$

- For constant volume heat addition process (2 – 3)

$$V_3 = V_2 = \frac{V_1}{r}$$

$$\alpha = \frac{P_3}{P_2}\ (Pressure\ ratio) ---------(d)$$

$$\therefore P_3 = P_2\alpha = P_1 r^{\gamma}\alpha$$

$$T_3 = T_2\,\frac{P_3}{P_2}$$

$$= T_2\,\alpha$$

$$\therefore T_3 = T_1\, r^{\gamma-1}\alpha ---------(e)$$

- For constant pressure heat addition process (3 – 4)

$$P_3 = P_4 = P_1 r^{\gamma}\alpha ---------(f)$$

$$\rho = \frac{V_4}{V_3}\ (Cutoff\ ratio) ---------(g)$$

$$T_4 = T_3\,\frac{V_4}{V_3}$$

$$\therefore T_4 = T_3\,\rho$$

$$\therefore T_4 = T_1\,r^{\gamma-1}\rho\alpha \text{ --------- }(h)$$

- For adiabatic expansion process (4 – 5),

$$P_4 V_4^{\gamma} = P_5 V_5^{\gamma}$$

$$P_5 = P_4(V_4/V_5)^{\gamma} = P_3(V_4/V_1)^{\gamma} \qquad (\because V_1 = V_5 \;\&\; P_3 = P_4)$$

$$P_5 = P_3\left(\frac{V_4}{V_1}\frac{V_3}{V_3}\right)^{\gamma} = P_3\left(\frac{V_4}{V_1}\frac{V_2}{V_3}\right)^{\gamma} \qquad (\because V_3 = V_2)$$

$$\therefore P_5 = P_3\left(\frac{V_4/V_3}{V_1/V_2}\right)^{\gamma} = P_3(\rho/r)^{\gamma} \text{ --------- }(i)$$

And

$$T_5 = T_4\left(\frac{V_4}{V_5}\right)^{\gamma-1}$$

$$\therefore T_5 = T_4\left(\frac{\rho}{r}\right)^{\gamma-1}$$

$$\therefore T_5 = \frac{T_1\,r^{\gamma-1}\,\rho\alpha\,\rho^{\gamma-1}}{r^{\gamma-1}}$$

$$\therefore T_5 = T_1\,\alpha\,\rho^{\gamma} \text{ --------- }(j)$$

From equation 7.33,

$$\eta = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

$$\therefore \eta = 1 - \frac{(T_1\,\alpha\,\rho^{\gamma} - T_1)}{(T_1\,r^{\gamma-1}\alpha - T_1\,r^{\gamma-1}) + \gamma(T_1\,r^{\gamma-1}\alpha\rho - T_1\,r^{\gamma-1}\alpha)}$$

$$\therefore \eta = 1 - \frac{(\rho^{\gamma}\alpha - 1)}{[r^{\gamma-1}\{(\alpha - 1\alpha) + \gamma\alpha(\rho - 1)\}]}$$

$$\therefore \boldsymbol{\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{(\alpha\rho^{\gamma} - 1)}{(\alpha - 1) + \gamma\alpha(\rho - 1)}\right]} \text{ --------- }\mathbf{(7.34)}$$

- It can be seen from the equation 7.34 that the thermal efficiency of a Dual cycle can be increased by supplying a greater portion of heat at constant volume (high value of α) and smaller portion at constant pressure (low value of ρ).
- In the actual high speed Diesel engines operating on this cycle, it is achieved by early fuel injection and an early cut-off.
- It is to be noted that Otto and Diesel cycles are special cases of the Dual cycle.
- If $\rho = 1$ $(V_3 = V_4)$

  Hence, there is no addition of heat at constant pressure. Consequently the entire heat is supplied at constant volume and the cycle becomes the Otto cycle.

By substituting $\rho = 1$ in equation 7.34, we get,

$$\eta = 1 - \frac{1}{r^{(\gamma-1)}} = Efficiency\ of\ Otto\ cycle$$

- Similarly if $\alpha = 1$, the heat addition is only at constant pressure and cycle becomes Diesel cycle.

By substituting $\alpha = 1$ in equation 7.34, we get,

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{(\rho^{\gamma} - 1)}{\gamma(\rho - 1)}\right] = Efficiency\ of\ Diesel\ cycle$$

Mean Effective Pressure:

- **Net work done** per unit mass of air,

$$W_{net} = C_V(T_3 - T_2) + C_p\,(T_4 - T_3) - C_V(T_5 - T_1) - - - - - - - - - (7.35)$$

- **Swept volume,**

$$Swept\ volume = V_1 - V_2 = V_1\left(1 - \frac{V_2}{V_1}\right) = \frac{RT_1}{P_1}\left(1 - \frac{1}{r}\right)$$

$$= \frac{RT_1}{P_1 r}(r - 1) - - - - - - - - - (7.36)$$

- **Mean effective pressure,**

$$mep = \frac{Work\ done\ per\ cycle}{swept\ volume}$$

$$\therefore mep = \frac{C_V(T_3 - T_2) + C_p\,(T_4 - T_3) - C_V(T_5 - T_1)}{\frac{R\,T_1}{P_1 r}\,(r - 1)}$$

$$\therefore mep = \frac{C_V}{R}\,\frac{P_1 r}{(r-1)}\left[\frac{(T_3 - T_2) + \gamma(T_4 - T_3) - (T_5 - T_1)}{T_1}\right]$$

- From equation (c), (e), (h) and (j),

$T_2 = T_1\,r^{\gamma-1}$

$T_3 = T_1\,r^{\gamma-1}\alpha$

$T_4 = T_1\,r^{\gamma-1}\alpha\rho$

$T_5 = T_1\,\alpha\,\rho^{\gamma}$

$\therefore mep$

$$= \frac{C_V}{R}\,\frac{P_1 r}{(r-1)}\left[\frac{\gamma(T_1\,r^{\gamma-1}\alpha - T_1\,r^{\gamma-1}) + \gamma(T_1\,r^{\gamma-1}\alpha\rho - T_1\,r^{\gamma-1}\alpha) - (T_1\,\alpha\rho^{\gamma} - T_1)}{T_1}\right]$$

$$\boldsymbol{\therefore mep = \frac{P_1 r}{(\gamma-1)(r-1)}[(\alpha - 1)r^{\gamma-1} + \gamma\alpha r^{\gamma-1}(\rho - 1) - (\alpha\rho^{\gamma} - 1)]}$$

$$- - - - - - - - - \mathbf{(7.37)}$$

7.7 Comparison of Otto, Diesel and Dual Cycles

- Following are the important variable factors which are used as a basis for comparison of the cycles:
  - Compression ratio
  - Maximum pressure
  - Heat supplied
  - Heat rejected
  - Net work.

A. For the Same Compression Ratio and the Same Heat Input

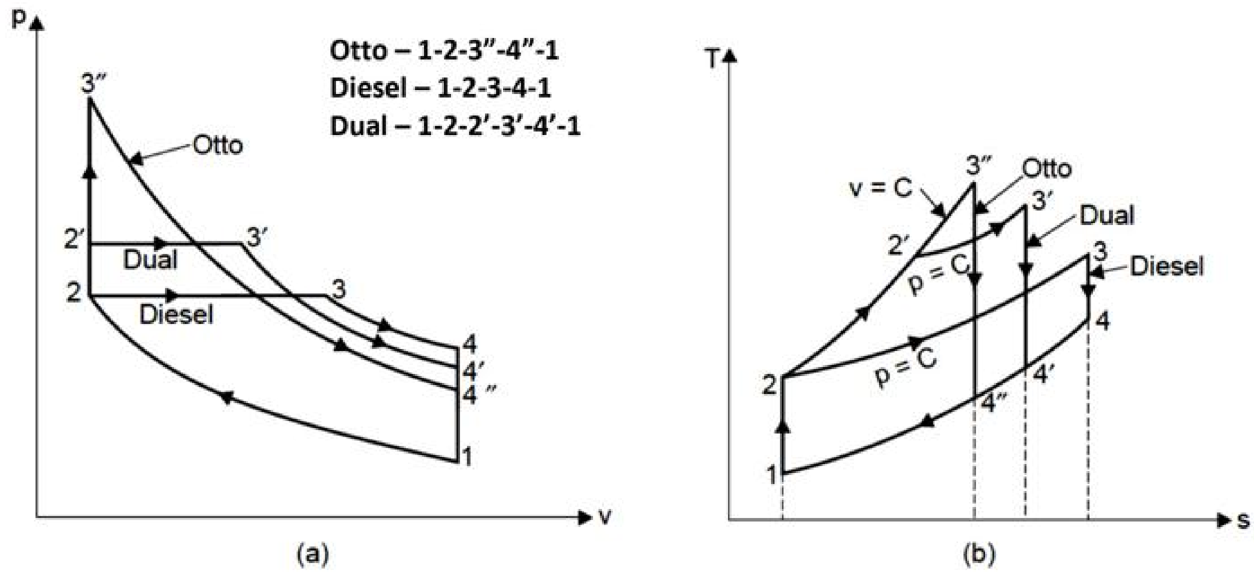


Fig. 7.8 (a) P-V diagram and (b) T-S diagram

- We know that,

$$\eta = 1 - \frac{Heat\ Rejected}{Heat\ Supplied} = 1 - \frac{q_2}{q_1} - - - - - - - - - (7.38)$$

- The quantity of heat rejected from each cycle is represented by the appropriate area under the line 4 – 1 on the T – S diagram.
- From equation 7.38; it is clear that the cycle which has the least heat rejected will have the highest efficiency.

$$\therefore \eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$$

B. Same Maximum Pressure and Temperature

- When pressure is the limiting factor in engine design, it becomes necessary to compare the air standard cycles on the basis of same maximum pressure & temperature.

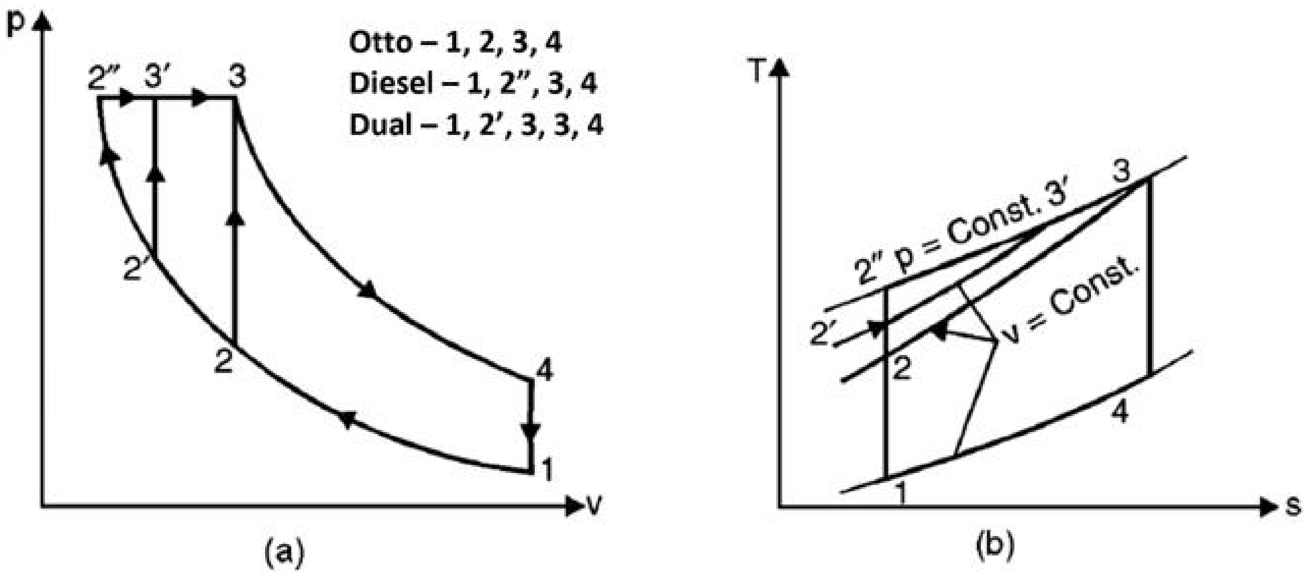


Fig. 7.9 (a) P-V diagram and (b) T-S diagram

- Here the Otto cycle must be limited to low compression ratio to fulfill the condition that point 3 (same maximum pressure & temperature) is to be a common state for all the three cycles.
- From Fig. 7.9 it is clear that the heat rejected is same for all the three cycles. Hence with the same heat rejected, the cycle with greater heat addition is more efficient.
- We know that,

$$\eta = 1 - \frac{Heat\ Rejected}{Heat\ Supplied} = 1 - \frac{q_2}{q_1} - - - - - - - - - (7.39)$$

- From Fig. 7.9,

$$\therefore \boldsymbol{\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}}$$

C. For Constant Maximum Pressure and Heat Input

- Fig. 7.10 shows the Otto and Diesel cycles on P-V and T-S diagrams for constant maximum pressure and heat input respectively.

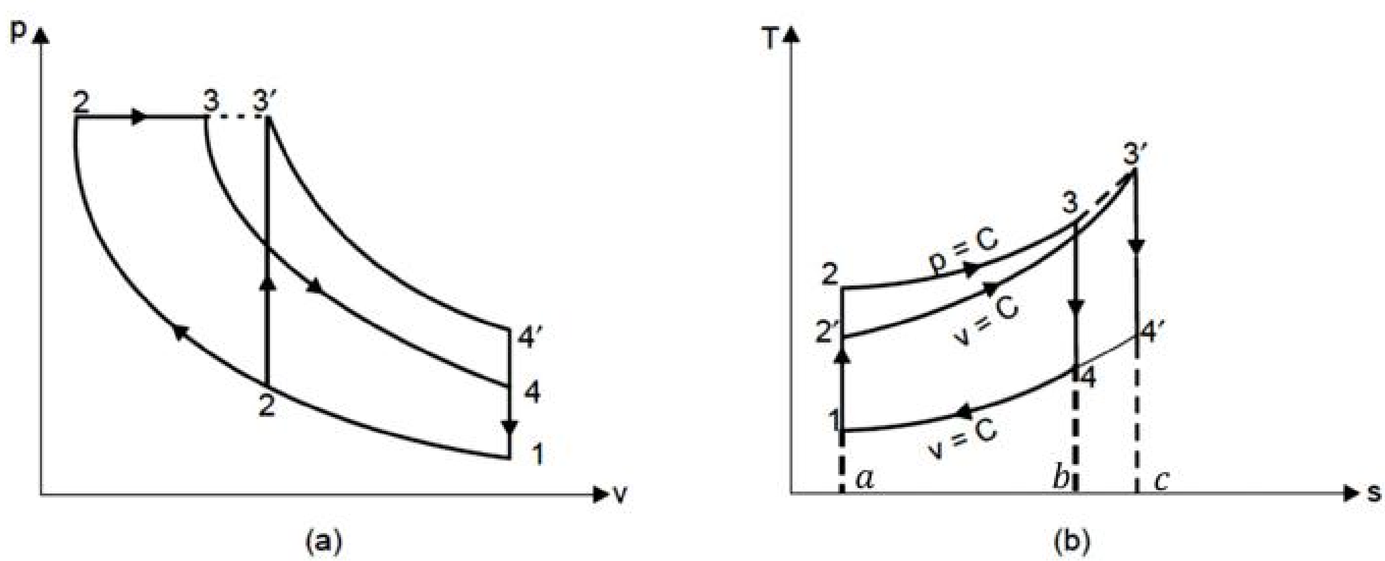


Fig. 7.10 (a) P-V diagram and (b) T-S diagram

- For the constant maximum pressure, points 3 and 3′ must lie on the constant pressure line.
- Also for the same heat input the areas $a - 2 - 3 - b$ and $a - 2' - 3' - c$ on the T-S plot must be equal.
- Now,

$$\eta = 1 - \frac{Heat\ Rejected}{Heat\ Supplied} = 1 - \frac{q_2}{q_1} - - - - - - - - -(7.40)$$

- Hence for the same amount of heat supplied the cycle with less heat rejected has a higher value of thermal efficiency.
- From Fig. 7.10,

$$\therefore \boldsymbol{\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}}$$

7.8 The Brayton Cycle OR The Joule Cycle

- The Brayton cycle is a constant pressure cycle for a perfect gas. It is also called Joule cycle.
- It is a theoretical cycle on which constant pressure gas turbine works.

The various operations are as follows:

- **Isentropic Compression (1 – 2):**

The air is compressed isentropically from the lower pressure p_1 to the upper pressure p_2, the temperature rising from T_1 to T_2. No heat flow occurs.

- **Constant Pressure Heat Addition (2 – 3):**

The compressed air is passed through a heat exchanger, where heat is externally supplied to it at constant pressure. Heat flows into the system increasing the volume from V_2 to V_3 and temperature from T_2 to T_3 whilst the pressure remains constant at p_2.

- **Isentropic Expansion (3 – 4):**

Isentropic expansion of high pressure & high temperature air takes place in the turbine during which the work is done by the system. The air is expanded isentropically from p_2 to p_1, the temperature falling from T_3 to T_4. No heat flow occurs.

- **Constant Pressure Heat Rejection (4 – 1):**

The air at state point 4 is passed through a heat exchanger and heat is rejected at constant pressure. The volume decreases from V_4 to V_1 and the temperature from T_4 to T_1 whilst the pressure remains constant at p_1.

- The closed Brayton cycle is shown in the Fig. 7.11 (a) and it is represented on p-v and T-s diagrams as shown in Figs. 7.11 (b) and (c) respectively.

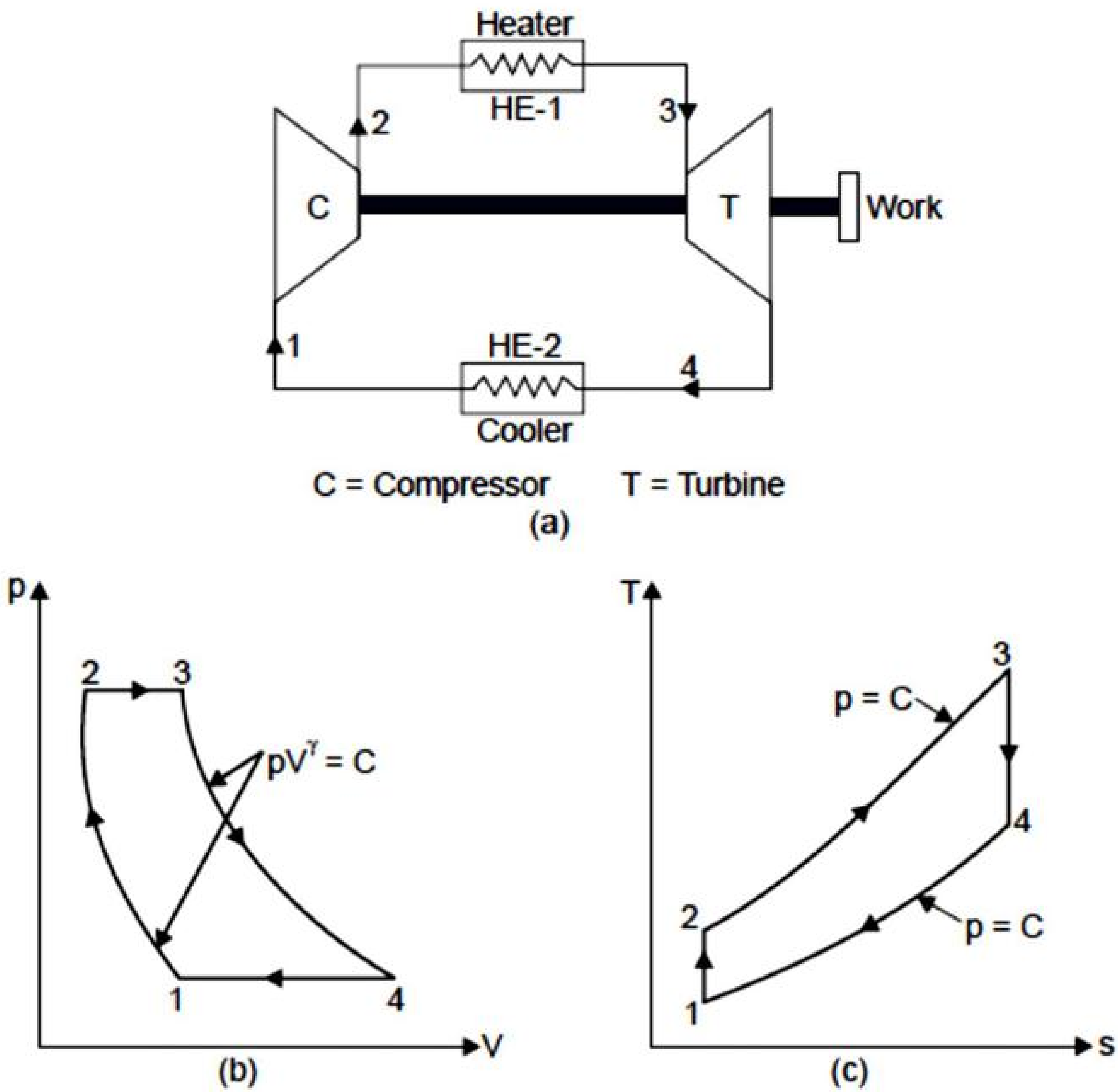


Fig. 7.11 The P-v, T-s and Schematic diagram of Brayton cycle

Thermal Efficiency for Closed Brayton Cycle:

- For unit mass of air,
- **Heat supplied** during process 2 – 3,

$$q_1 = C_P(T_3 - T_2)$$

- **Heat rejected** during process 4 – 1,

$$q_2 = C_P(T_4 - T_1)$$

- **Work done,**

$$W = q_1 - q_2$$

$$\therefore W = C_P(T_3 - T_2) - C_P(T_4 - T_1)$$

- **Thermal efficiency,**

$$\eta = \frac{Work\ done}{Heat\ supplied}$$

$$\therefore \eta = \frac{C_P(T_3 - T_2) - C_P(T_4 - T_1)}{C_P(T_3 - T_2)}$$

$$\therefore \eta = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} - - - - - - - - - (7.41)$$

- Take **pressure ratio,**

$$r_p = \frac{P_2}{P_1} = \frac{P_3}{P_4} - - - - - - - - - (7.41a)$$

- For isentropic compression process (1 – 2),

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}} - - - - - - - - - (7.41b)$$

- For isentropic expansion process (3 – 4),

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}} - - - - - - - - - (7.41c)$$

- Thus from equation (7.41b) and (7.41c),

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$\therefore \frac{T_4}{T_1} = \frac{T_3}{T_2} - - - - - - - - - (7.41d)$$

- From equation 7.41,

$$\eta = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$\therefore \eta = 1 - \frac{T_1\left(\frac{T_4}{T_1} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1\right)} = 1 - \frac{T_1\left(\frac{T_3}{T_2} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1\right)} \qquad (\because equation\ (7.41d))$$

$$\therefore \eta = 1 - \frac{T_1}{T_2}$$

$$\therefore \eta = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \boldsymbol{\eta = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}} - - - - - - - - - \mathbf{(7.42)}$$

- Thermal efficiency of Brayton cycle is function of pressure ratio. Efficiency increases with pressure ratio as shown in Fig. 7.12.

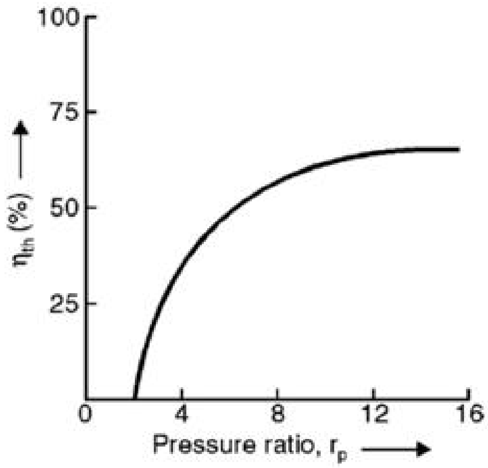


Fig. 7.12 Effect of pressure ratio on the efficiency of Brayton cycle

- The curve tends to become flat at higher pressure ratios, which implies that though the efficiency is increasing, the rate of increase starts diminishing at higher pressures.

7.9 The Open Cycle Gas Turbine *OR* Actual Brayton Cycle:

- The fundamental gas turbine unit is one operating on the open cycle. In Open cycle gas turbine, the products of combustion coming out from the turbine are exhausted to the atmosphere as they cannot be used any more. The working fluids (air and fuel) must be replaced continuously as they are exhausted into the atmosphere.
- In practice, it is not possible to achieve either isentropic compression or isentropic expansion because of internal friction, turbulence and leakage.
- If pressure drop is neglected in combustion chamber, the actual Brayton cycle on T-S diagram is shown by process 1-2'-3-4' in Fig. 7.13.

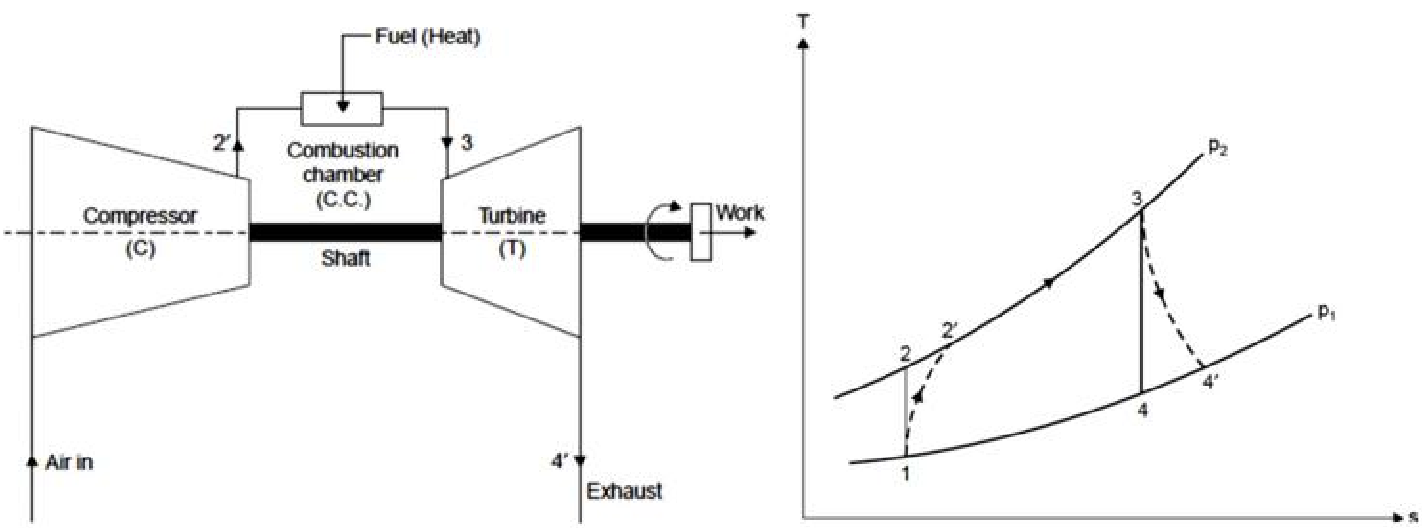


Fig. 7.13 Schematic and T-S diagram of an actual Brayton cycle

- In actual cycle, the temperatures at the end of compression and at the end of expansion are higher than in an ideal case for the same pressure ratio.
- **Efficiency of compressor,**

$$\eta_c = \frac{Isentropic\ temerature\ rise}{Actual\ temperature\ rise}$$

$$\therefore \boldsymbol{\eta_t = \frac{T_2 - T_1}{T_2' - T_1} - - - - - - - - - (7.43)}$$

- **Efficiency of turbine,**

$$\eta_t = \frac{Actual\ decrease\ in\ temerature}{Isentropic\ decrease\ in\ temperature}$$

$$\therefore \boldsymbol{\eta_t = \frac{T_3 - T_4'}{T_3 - T_4} - - - - - - - - - (7.44)}$$

- The performance of Brayton cycle can be improved by using multi stage compression with inter-cooling, multi stage expansion with reheating and regeneration.

Pressure Ratio for Maximum Net Work

- From equation 7.41a to 7.41c (Refer Page No. 7.25),

Pressure ratio,

$$r_p = \frac{P_2}{P_1} = \frac{P_3}{P_4} - - - - - - - - - (7.41a)$$

For isentropic compression process (1 – 2),

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}} - - - - - - - - - (7.41b)$$

For isentropic expansion process (3 – 4),

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}} - - - - - - - - - (7.41c)$$

Thus from equation (7.41b) and (7.41c),

$$\boldsymbol{\frac{T_2}{T_1} = \frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} = x}$$

- Actual compression work,

$$Wc_a = h_2' - h_1 = \frac{h_2 - h_1}{\eta_c} = \frac{C_p(T_2 - T_1)}{\eta_c}$$

- Actual turbine work,

$$Wt_a = h_3 - h_4' = \eta_t(h_3 - h_4) = C_p(T_3 - T_4)\eta_t$$

- Actual net work,

$$W_{net_a} = Wt_a - Wc_a$$

$$\therefore W_{net_a} = C_p(T_3 - T_4)\eta_t - \frac{C_p(T_2 - T_1)}{\eta_c}$$

$$\therefore W_{net_a} = C_p\eta_t T_3\left(1 - \frac{T_4}{T_3}\right) - \frac{C_pT_1}{\eta_c}\left(\frac{T_2}{T_1} - 1\right)$$

$$\therefore W_{net_a} = C_p\eta_t T_3\left(1 - \frac{1}{x}\right) - \frac{C_pT_1}{\eta_c}(x - 1)$$

- For maximum work, differentiate above equation w.r.t. x while keeping T_1 & T_3 constants.

$$\therefore \frac{dW_{net_a}}{dx} = 0$$

$$\therefore C_p\eta_t T_3\left(\frac{1}{x^2}\right) - \frac{C_pT_1}{\eta_c} = 0$$

$$\therefore x = \sqrt{\eta_t\eta_c\frac{T_3}{T_1}}$$

$$\therefore (r_p)^{\frac{\gamma-1}{\gamma}} = \left(\eta_t\eta_c\frac{T_3}{T_1}\right)^{1/2}$$

$$\therefore r_{p_{opt}} = \left(\eta_t\eta_c\frac{T_3}{T_1}\right)^{\gamma/2(\gamma-1)}$$

$$\therefore \boldsymbol{r_{p_{opt}}} = \left(\boldsymbol{\eta_t\eta_c\frac{T_{max}}{T_{min}}}\right)^{\boldsymbol{\gamma/2(\gamma-1)}}$$

- For Ideal cycle,

$$\boldsymbol{r_{p_{opt}}} = \left(\boldsymbol{\frac{T_{max}}{T_{min}}}\right)^{\boldsymbol{\gamma/2(\gamma-1)}}$$

- For maximum work, the temperature T_1 at compressor entry should be as low as possible and the temperature T_3 at entry to the turbine should be as high as possible.

- The compressor inlet temperature is normally at atmospheric temperature (say 288K at sea level), while the turbine inlet temperature is decided by metallurgical considerations (the maximum value of about 1000K that the metal can withstand).

- The performance of an actual gas turbine plant depends upon both the pressure ratio and the temperature ratio.

- The maximum temperature to which the air could be heated in the heat exchanger is ideally that of exhaust gases, but less than this is obtained in practice.
- The effectiveness of the heat exchanger is given by,

$$\varepsilon = \frac{Increase\ in\ enthalpy\ per\ kg\ of\ air}{Available\ increase\ in\ enthalpy\ per\ kg\ of\ air}$$

$$\therefore \varepsilon = \frac{T_3 - T_2'}{T_5' - T_2'} - - - - - - - - - (7.49)$$

Thermal Efficiency for Open Brayton Cycle with Regeneration:

- **Turbine work,**

$$W_t = (h_4 - h_5') = C_p(T_4 - T_5')$$

- **Compressor work,**

$$W_c = (h_2' - h_1) = C_p(T_2' - T_1)$$

- **Heat Supplied,**

$$Q_s = (h_4 - h_3) = C_p(T_4 - T_3)$$

- **Thermal efficiency,**

$$\eta_{th} = \frac{Net\ Work}{Heat\ Supplied} = \frac{W_t - W_c}{Q_s}$$

$$\therefore \eta_{th} = \frac{C_p(T_4 - T_5') - C_p(T_2' - T_1)}{C_p(T_4 - T_3)}$$

$$\therefore \boldsymbol{\eta_{th}} = \frac{\boldsymbol{(T_4 - T_5') - (T_2' - T_1)}}{\boldsymbol{(T_4 - T_3)}} - - - - - - - - - \boldsymbol{(7.50)}$$

- Under ideal conditions, $T_3 = T_5'$ for $\varepsilon = 1$, then

$$\therefore \boldsymbol{\eta_{th} = 1 - \frac{(T_2' - T_1)}{(T_4 - T_3)}}$$

7.11 Solved Numerical

Ex 9.1. ***[GTU; Jun-2010; 3 Marks]***

An engine uses 6.5 Kg of oil per hour of calorific value of 30,000 kJ/Kg. If the Brake power of engine is 22 kW and mechanical efficiency is 85% calculate (a) indicate thermal efficiency (b) Brake thermal efficiency (c) Specific fuel consumption in Kg/B.P/hr.

Solution:

Given Data:

$\dot{m}_f = 6.5\, kg/hr$

$C.V. = 30000\ kJ/kg$

$B.P. = 22kW$

$\eta_m = 85\%$

To be Calculated:

a) $\eta_{ith} = ?$

b) $\eta_{bth} = ?$

c) $BSFC = ?$

⇨ Indicated Power,

$$\eta_m = \frac{B.P.}{I.P.}$$

$$\therefore I.P. = \frac{22}{0.85} = 25.8824\ kW$$

⇨ **Indicated Thermal Efficiency:**

$$\eta_{ith} = \frac{I.P.}{\dot{m}_f \times C.V.}$$

$$\therefore \eta_{ith} = \frac{25.8824}{\left(\frac{6.5}{3600}\right) \times 30000}$$

$$\therefore \boldsymbol{\eta_{ith} = 0.4778 = 47.78\%}$$

⇨ **Break Thermal Efficiency:**

$$\eta_{bth} = \frac{B.P.}{\dot{m}_f \times C.V.}$$

$$\therefore \eta_{bth} = \frac{22}{\left(\frac{6.5}{3600}\right) \times 30000}$$

$$\therefore \boldsymbol{\eta_{bth} = 0.4062 = 40.62\%}$$

⇨ **Break Specific Fuel Consumption:**

$$BSFC = \frac{\dot{m}_f(kg/hr)}{B.P.(kW)}$$

$$\therefore BSFC = \frac{6.5}{22}$$

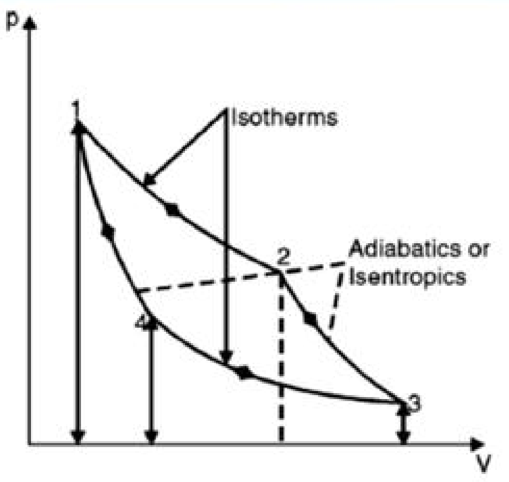
$$\therefore \boldsymbol{BSFC = 0.2955\ kg/kWh}$$

Ex 9.2. ***[Ex 13.3; P. K. Nag]***

In a Carnot cycle, the maximum pressure and temperature are limited to 18 bar and 410°C. The ratio of isentropic compression is 6 and isothermal expansion is 1.5. Assuming the volume of the air at the beginning of isothermal expansion as 0.18 m³, determine :

(a) The temperature and pressures at main points in the cycle.
(b) Change in entropy during isothermal expansion.
(c) Mean thermal efficiency of the cycle.
(d) Mean effective pressure of the cycle.
(e) The theoretical power if there are 210 working cycles per minute.

Solution:



Given Data:

$p_1 = 18\ bar$

$T_H = T_1 = T_2 = 410°C$

$\frac{V_4}{V_1} = 6$

$\frac{V_2}{V_1} = 1.5$

$V_1 = 0.18\ m^3$

No. of cycles = 210 /min

Assume

$\gamma = 1.4$ *for air*

To be Calculated:

a) $T_L, p_2, p_3, p_4 = ?$

b) $\Delta S = ?$

c) $\eta_{th} = ?$

d) $p_m = ?$

e) $P = ?$

⇨ **Temperatures & Pressures at the main point of the cycle:**

For Process 4-1 (Isentropic Compression),

$$\frac{T_L}{T_H} = \left(\frac{V_1}{V_4}\right)^{\gamma-1}$$

$$\therefore T_L = 683 \times \left(\frac{1}{6}\right)^{1.4-1}$$

$$\therefore \boldsymbol{T_L = T_3 = T_4 = 333.5494\ K}$$

Also,

$$p_1 V_1^{\gamma} = p_4 V_4^{\gamma}$$

$$\therefore p_4 = p_1 \times \left(\frac{V_1}{V_4}\right)^{\gamma}$$

$$\therefore p_4 = 18 \times \left(\frac{1}{6}\right)^{1.4}$$

$$\therefore \boldsymbol{p_4 = 1.465\ bar}$$

For Process 1-2 (Isothermal Expansion),

$$p_1 V_1 = p_2 V_2$$

$$\therefore p_2 = p_1 \times \left(\frac{V_1}{V_2}\right)$$

$$\therefore p_2 = 18 \times \left(\frac{1}{1.5}\right)$$

$$\therefore \boldsymbol{p_2 = 12\ bar}$$

For Process 2-3 (Isentropic Expansion),

$$p_2 V_2^{\gamma} = p_3 V_3^{\gamma}$$

$$\therefore p_3 = p_2 \times \left(\frac{V_2}{V_3}\right)^{\gamma}$$

$$\therefore p_3 = p_2 \times \left(\frac{V_1}{V_4}\right)^{\gamma} \qquad \left(\because \frac{V_4}{V_1} = \frac{V_3}{V_2}\right)$$

$$\therefore p_3 = 12 \times \left(\frac{1}{6}\right)^{1.4}$$

$$\therefore \boldsymbol{p_3 = 0.9767\ bar}$$

➪ **Change in Entropy:**

From T-S diagram,

$$Q_s = T_H \times (S_2 - S_1)$$

$$\therefore \Delta S = S_2 - S_1 = \frac{Q_s}{T_H} = \frac{p_1 V_1 \ln \frac{V_2}{V_1}}{T_H}$$

$$\therefore \Delta S = \frac{18 \times 10^5 \times 0.18 \times \ln 1.5}{683}$$

$$\therefore \boldsymbol{\Delta S = 192.3436\ J/K}$$

➪ **Thermal Efficiency of the Cycle:**

Heat Supplied,

$$Q_s = p_1 V_1 \ln \frac{V_2}{V_1}$$

$$\therefore Q_s = T_H \times (S_2 - S_1)$$

$$\therefore Q_s = 683 \times 192.3436$$

$$\therefore Q_s = 131370.6788\, J = 131.370\, kJ$$

Heat Rejected,

$$Q_r = p_4 V_4 \ln \frac{V_3}{V_4}$$

$$\therefore Q_r = T_L \times (S_3 - S_4) = T_L \times (S_2 - S_1)$$

$$\therefore Q_r = 333.5494 \times 192.3436$$

$$\therefore Q_r = 64156.0923\, J = 64.1561\, kJ$$

Efficiency,

$$\eta = \frac{Q_s - Q_r}{Q_s} = \frac{131.370 - 64.1561}{131.370}$$

$$\therefore \boldsymbol{\eta = 0.5116 = 51.16\%}$$

⇨ **Mean Effective Pressure of the Cycle:**

$$\frac{V_4}{V_1} = 6 \;\; \& \;\; \frac{V_2}{V_1} = 1.5$$

Also,

$$\frac{V_4}{V_1} = \frac{V_3}{V_2}$$

$$\therefore \frac{V_3}{V_2} \times \frac{V_2}{V_1} = 6 \times 1.5$$

$$\therefore \frac{V_3}{V_1} = 9$$

Swept Volume,

$$V_s = V_3 - V_1 = 9V_1 - V_1 = 8V_1$$

$$\therefore V_s = 8 \times 0.18$$

$$V_s = 1.44\, m^3$$

Mean Effective Pressure,

$$p_m = \frac{Net\ Work}{Swept\ Volume} = \frac{Q_s - Q_r}{V_s}$$

$$\therefore p_m = \frac{131370.6788 - 64156.0923}{1.44}$$

$$\therefore \boldsymbol{p_m = 46676.7961\, Pa}$$

⇨ **Power of the Engine:**

$$P = \frac{Work\ Done}{Cycle} \times \frac{No.of\ Cycle}{Sec}$$

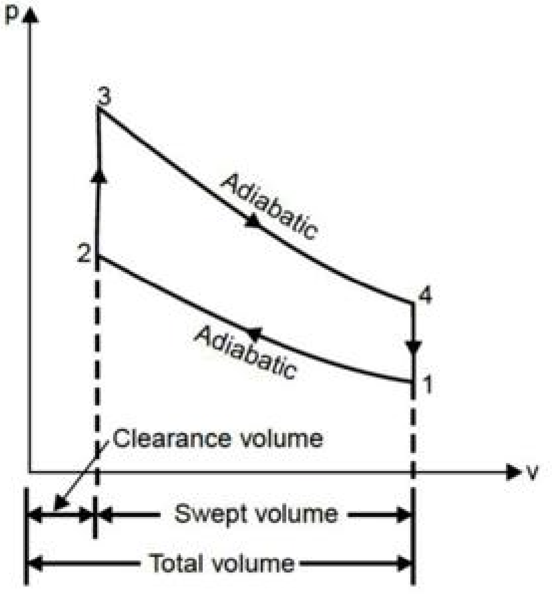
$$\therefore P = (131370.6788 - 64156.0923) \times \frac{210}{60}$$

$$\therefore \boldsymbol{P = 235251.0528\, W = 235.251\, kW}$$

Ex 9.3. ***[GTU; Jan-2015; 7 Marks]***

In an I C Engine working with the Otto cycle, the cylinder diameter is 250mm and a stroke is 375mm. If the clearance volume is 0.00263m^3, and the initial pressure and temperature are 1bar and 50°C, calculate (a) The air standard efficiency and (b) Mean effective pressure of the cycle. The maximum cycle pressure is limited to 25bar.

Solution:



Given Data:

$D = 0.250\ m$

$L = 0.375\ m$

$V_c = 0.00263\ m^3$

$p_1 = 1\ bar$

$T_1 = 323\ K$

$p_3 = 25\ bar$

To be Calculated:

a) $\eta = ?$

b) $p_m = ?$

⇨ Swept Volume,

$$V_s = \frac{\pi}{4} D^2 L$$

$$\therefore V_s = \frac{\pi}{4} 0.250^2 \times 0.375$$

$$\therefore V_s = 0.0184\ m^3$$

⇨ Total Volume,

$$V_1 = V_s + V_c$$

$$\therefore V_1 = 0.0184 + 0.00263$$

$$\therefore V_1 = 0.02103\ m^3$$

⇨ Compression Ratio,

$$r = \frac{V_1}{V_2} = \frac{0.02103}{0.00263}$$

$$\therefore r = 7.9961$$

⇨ **Air Standard Efficiency:**

$$\eta = 1 - \frac{1}{r^{\gamma - 1}}$$

$$\therefore \eta = 1 - \frac{1}{(7.9961)^{1.4-1}}$$

$$\therefore \boldsymbol{\eta = 0.5646 = 56.46\%}$$

⇨ For Process 1-2 (Isentropic Compression),

$$\frac{T_2}{T_1} = (r)^{\gamma - 1}$$

$$\therefore T_2 = 323 \times (7.9961)^{1.4-1}$$

$$\therefore T_2 = 741.9144\ K$$

And,

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

$$\therefore p_2 = p_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$\therefore p_2 = 1 \times (7.9961)^{1.4}$$

$$\therefore p_2 = 18.3666\ bar$$

For Process 2-3 (Constant Volume Heat Addition)

$$\frac{T_3}{T_2} = \frac{P_3}{P_2} \qquad (\because V_2 = V_3)$$

$$\therefore T_3 = 741.9144 \times \frac{25}{18.3666}$$

$$\therefore T_3 = 1009.869\ K$$

⇨ Mass of Air,

$$p_1V_1 = mRT_1$$

$$\therefore m = \frac{1 \times 10^5 \times 0.02103}{0.287 \times 10^3 \times 323}$$

$$\therefore m = 0.02268\ kg$$

⇨ Heat Supplied,

$$Q_s = mC_v(T_3 - T_2)$$

$$\therefore Q_s = 0.02268 \times 0.718 \times 10^3 \times (1009.869 - 741.9144)$$

$$\therefore Q_s = 4363.437\ J = 4.3634\ kJ$$

⇨ Net Work,

$$\eta = \frac{W_{net}}{Q_s}$$

$$\therefore W_{net} = 0.5646 \times 4363.437$$

$$\therefore W_{net} = 2463.5965\ J$$

⇨ **Mean Effective Pressure:**

$$p_m = \frac{W_{net}}{V_s}$$

$$\therefore p_m = \frac{2463.5965}{0.0184}$$

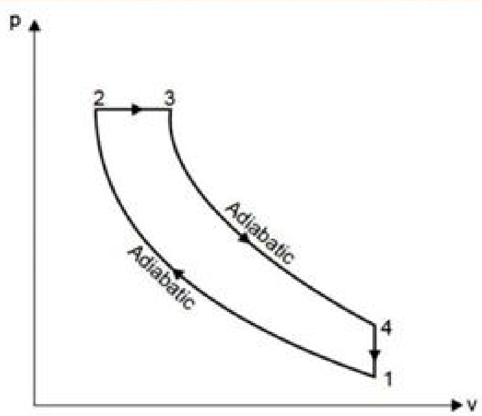
$$\therefore p_m = 133891.1141\ Pa$$

$$\therefore \boldsymbol{p_m = 1.3389\ bar}$$

Ex 9.4. ***[GTU; Nov-2011; 7 Marks]***

In an air standard diesel cycle the compression ratio is 16. At the beginning of isentropic compression the temperature is 15 °C and pressure is 0.1 MPa. Heat is added until the temperature at the end of constant pressure process is 1480°C. Calculate: (a) cut off ratio, (b) cycle efficiency and (c) M. E. P.

Solution:



Given Data:

$r = 16$

$p_1 = 0.1\ MPa = 1 bar$

$T_1 = 288\ K$

$T_3 = 1753\ K$

To be Calculated:

a) $\rho = ?$

b) $\eta = ?$

c) $p_m = ?$

⇨ **Cut off Ratio:**

For Process 1-2 (Isentropic Compression),

$$\frac{T_2}{T_1} = (r)^{\gamma-1}$$

$$\therefore T_2 = 288 \times (16)^{1.4-1}$$

$$\therefore T_2 = 873.0527\ K$$

For Process 2-3 (Constant Pressure Heat Addition)

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \qquad (\because\ p_2 = p_3)$$

$$\therefore \frac{V_3}{V_2} = \frac{1753}{873.0527}$$

$$\therefore \boldsymbol{\rho = \frac{V_3}{V_2} = 2.007}$$

⇨ **Cycle Efficiency:**

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{(\rho^{\gamma} - 1)}{\gamma(\rho - 1)}\right]$$

$$\therefore \eta = 1 - \frac{1}{16^{0.4}}\left[\frac{(2.007^{1.4} - 1)}{1.4(2.007 - 1)}\right]$$

$$\therefore \boldsymbol{\eta = 0.6134 = 61.34\%}$$

⇨ **Mean Effective Pressure:**

Heat Supplied (per unit mass),

$$q_s = C_P(T_3 - T_2)$$

$$\therefore q_s = 1.005(1753 - 873.0527)$$

$$\therefore q_s = 884.347\ \frac{kJ}{kg}$$

Net Work,

$$\eta = \frac{w_{net}}{q_s}$$

$$\therefore w_{net} = 0.6134 \times 884.347$$

$$\therefore w_{net} = 542.4584\ \frac{kJ}{kg}$$

Swept Volume,

$$V_s = V_1 - V_2 = V_1\left(1 - \frac{V_2}{V_1}\right)$$

$$\therefore v_s = \frac{RT_1}{p_1}\left(1 - \frac{1}{r}\right)$$

$$\therefore v_s = \frac{287 \times 288}{1 \times 10^5}\left(1 - \frac{1}{16}\right)$$

$$\therefore v_s = 0.7749\ \frac{m^3}{kg}$$

Mean Effective Pressure,

$$p_m = \frac{W_{net}}{V_s}$$

$$\therefore p_m = \frac{542.4584 \times 10^3}{0.7749}$$

$$\therefore p_m = 700036.6499\ Pa$$

$$\therefore \boldsymbol{p_m = 7.0003\ bar}$$

7.12 References

[1] Engineering Thermodynamics by P.K. Nag, McGraw-Hill Publication.

[2] Thermal Engineering by Mahesh M Rathore, McGraw-Hill Publication.

[3] Thermodynamics – An Engineering Approach by Yunus Cengel & Boles, McGraw-Hill Publication.

[4] Fundamentals of Engineering Thermodynamics by Moran & Shapiro, 6th Edition, Wiley India (P) Ltd.

[5] Thermal Science and Engineering by D. S. Kumar, S K Kataria and Sons Publications.

[6] Engineering Thermodynamics by R. K. Rajput, Laxmi Publications.

[7] National Programme on Technology Enhanced Learning (NPTEL), A Joint Initiate by IIT's and IISc. (Web: http://nptel.ac.in/)