Unit - 4

MAGNETIC POTENTIAL

→ Scalar Magnetic Potential :

In case of electric field, the Term potential & EFI can be expressed as

$$E = -\nabla V \longrightarrow (1)$$

In The above expression, The Term V is called scalar Electric potential since it is a scalar quantity.

similarly in magnetic field, the magnetic field Intensity $\bar{H}$ can be expressed in terms of potential as,

$$\bar{H} = -\nabla V_m \longrightarrow (2)$$

where V_m is known as Scalar magnetic potential (or) MMF.

we have $\nabla \times \bar{H} = \bar{J} \longrightarrow (3)$

Taking curl on both sides of eq (2)

$$\nabla \times \bar{H} = -\nabla \times (\nabla V_m)$$

since curl of gradient of scalar quantity is zero, $\nabla \times \nabla V_m = 0$

$$\therefore \nabla \times \bar{H} = 0.$$

∴ From the above eq,

$$\nabla \times \bar{H} = \bar{J} = 0 \longrightarrow (4)$$

Thus Scalar Magnetic potential V_m can be defined for source free region where $\bar{J}$ is zero.

$$\therefore \bar{H} = -\nabla V_m \quad \text{only for } \bar{J} = 0 \longrightarrow (5)$$

Magnetic scalar potential can be expressed interms of $\bar{H}$ as

$$V_{m\,a,b} = -\int_b^a \bar{H}\cdot d\bar{L} \text{ ....Specified path.}$$

→ Laplace's Equation for Scalar Magnetic Potential :-

It is known that as monopole of magnetic field is non existing.

$$\oint \bar{B}\cdot d\bar{s} = 0$$

using Divergence theorem,

$$\oint \bar{B}\cdot d\bar{s} = \int_{vol} \nabla\cdot\bar{B}\; dv = 0$$

$$\nabla\cdot\bar{B} = 0$$

$$\nabla\cdot(\mu_0\bar{H}) = 0 \qquad \text{but } \mu_0 \neq 0$$

$$\nabla\cdot\bar{H} = 0$$

$$\nabla\cdot(-\nabla V_m) = 0 \quad , \quad \text{since } \bar{H} = -\nabla V_m$$

$$\boxed{\nabla^2 V_m = 0 \qquad \text{for } \bar{J} = 0}$$

This is Laplace's equation for scalar magnetic potential.

→ Limitations of Scalar Magnetic potential (or) Properties of scalar magnetic potential :-

1) Scalar Magnetic potential V_m is defined for the regions of zero current density ($\bar{J}=0$), where as there is no such restrictions for scalar electric potential.

2) The scalar magnetic potential satisfies the Laplace Equation's only. ($\nabla^2 V_m = 0$). where as Scalar Electric potential satisfies both poisson's and Laplace Eq's.

3) The scalar magnetic potential V_m is obtained through the Solution of laplace Eq's and is multi valued but scalar electric potential is single valued.

4) $\oint \bar{H} \cdot d\bar{l} = I_{enclosed}$ according to Ampere circuit law.

But in case of Electric field $\oint \bar{E} \cdot d\bar{l} = 0$.

Hence scalar magnetic potential field is non conservative field but scalar electric potential is conservative.

→ Vector Magnetic Potential

The vector magnetic potential is denoted as $(\bar{A})$ and measured in wb/m.

It is defined as that its curl is equal to the magnetic field

$$\bar{B} = \nabla \times \bar{A} \longrightarrow (1)$$

~~proof:~~ ~~for~~ vector magnetic potential has to satisfy the following relation, where $\bar{A}$ = vector mag. potential.

$$\nabla \cdot (\nabla \times \bar{A}) = 0 \longrightarrow (2)$$

$$\nabla \cdot \bar{B} = 0 \longrightarrow (3)$$

$$\therefore \bar{B} = \nabla \times \bar{A}$$

Now $\nabla \times \bar{H} = \bar{J}$

$$\nabla \times \frac{\bar{B}}{\mu_0} = \bar{J} \quad (\text{since } \bar{B} = \mu_0 \bar{H})$$

$$\nabla \times \bar{B} = \mu_0 \bar{J} \quad \text{since } \bar{B} = \nabla \times \bar{A}$$

$$\nabla \times \nabla \times \bar{A} = \mu_0 \bar{J}$$

using vector quantity to express left hand side we can write

$$\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu_0 \bar{J}$$

$$\therefore \bar{J} = \frac{1}{\mu_0}[\nabla \times \nabla \times \bar{A}] = \frac{1}{\mu_0}[\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}].$$

Thus if Vector magnetic field is known then current density $\bar{J}$ can be obtained. For ~~A~~ defining $\bar{A}$, the $\bar{J}$ need not be zero.

↳ Poisson's Equation for Magnetic field :-

For vector magnetic potential $\bar{A}$, its curl is defined as

$$\nabla \times \bar{A} = \bar{B} \quad \text{which is known.}$$

But to complete define $\bar{A}$, its divergence must be known.

Assume $\nabla \cdot \bar{A} = 0$.

$$\text{Then } \bar{J} = \frac{1}{\mu_0}\left[\nabla(\nabla\cdot\bar{A}) - \nabla^2\bar{A}\right]$$

$$= \frac{1}{\mu_0}\left[\nabla(0) - \nabla^2\bar{A}\right]$$

$$\bar{J} = \frac{1}{\mu_0}\left[-\nabla^2\bar{A}\right]$$

$$\therefore \boxed{\nabla^2\bar{A} = -\mu_0\bar{J}}$$

This is the poisson's Equation for magneto static field.

Properties of vector magnetic potential:

1) Uniqueness: Although Vector magnetic potential is not uniquely determined, The magnetic field $\bar{B}$ derived from it is unique. This property allows for different choices of $\bar{A}$ to yield the same physical field.

2) Gauge invariance: The physical mag field $\bar{B}$ derived from $\bar{A}$ is guage invariant means it remains unchanged under different ~~condit~~ choices of guage.

3) Relation to magnetic field: $\overline{B}$ can be expressed as the curl of ~~vector~~ vector magnetic potential $\overline{A}$.

$$\overline{B} = \nabla \times \overline{A}$$

4) ~~This~~ Convenient for solving problems: In certain situations, such as when dealing with static or quasi static electromagnetic fields in linear, isotropic materials, using the vector mag. potential can simplify calculations and solutions of Maxwell's equation.

5) Boundary conditions: The vector magnetic potential can help in imposing appropriate boundary conditions particularly in problems involving conductors and magnetic materials.

6) Energy considerations: The vector magnetic potential is related to magnetic vector potential energy, which is useful in studying energy stored in magnetic fields and their interactions with other fields and materials.

Vector magnetic potential due to Simple configurations :

(Or)

$\bar{A}$ due to Differential Current element :-

Consider the differential element $d\bar{L}$ carrying current I. According to Biot savart law, the vector magnetic potential $\bar{A}$ at a distance R from the differential current element is given by,

$$\bar{A} = \oint \frac{\mu_0 I \, d\bar{L}}{4\pi R} \quad wb/m.$$

For the distributed current sources, $I d\bar{L}$ can be replaced by $\bar{K} dS$ where $\bar{K}$ is surface current density.

$$\bar{A} = \oint_S \frac{\mu_0 \bar{K} \, d\bar{S}}{4\pi R} \quad wb/m.$$

The line integral becomes a surface integral.

If the volume current density $\bar{J}$ is given in A/m^2 then $I d\bar{L}$ can be replaced by $\bar{J} \, dv$, where dv is differential volume element

$$\bar{A} = \oint_{vol} \frac{\mu_0 \bar{J} \, dv}{4\pi R} \quad wb/m.$$

It can be noted that,

1) The zero reference for $\bar{A}$ is at infinity
2) No finite current can produce the contributions as $R \to \infty$.

Inductance (L):

It is the property of the material by virtue of which it opposes the changes in current. It is defined as the number of flux linkages per ampere.

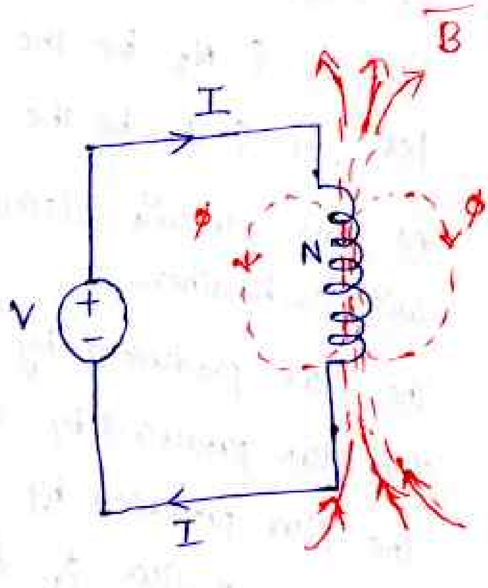
$$L = \frac{\lambda}{I}.$$

A part of the circuit which has the property of Inductance is called Inductor.

- A wire or conductor of certain length when twisted into coil becomes a basic inductor.

Self Inductance:

when a closed conducting path or a circuit carries current I, a magnetic field $\bar{B}$ is produced. This causes a magnetic flux ϕ which is given by



$$\phi = \int_s \bar{B} \cdot d\bar{S}. \longrightarrow (1)$$

If circuit consists of N turns, the flux produced links with each turn of circuit, then flux linkage λ is

$$\lambda = N\phi \quad , \quad wb\cdot t \rightarrow (2)$$

The Self Inductance: The ratio of total flux linkage to the current flowing through the circuit is called Inductance (or) self Inductance

$$\boxed{L = \frac{N\phi}{I} = \frac{\lambda}{I}} \text{ Henry}$$

Mutual Inductance :

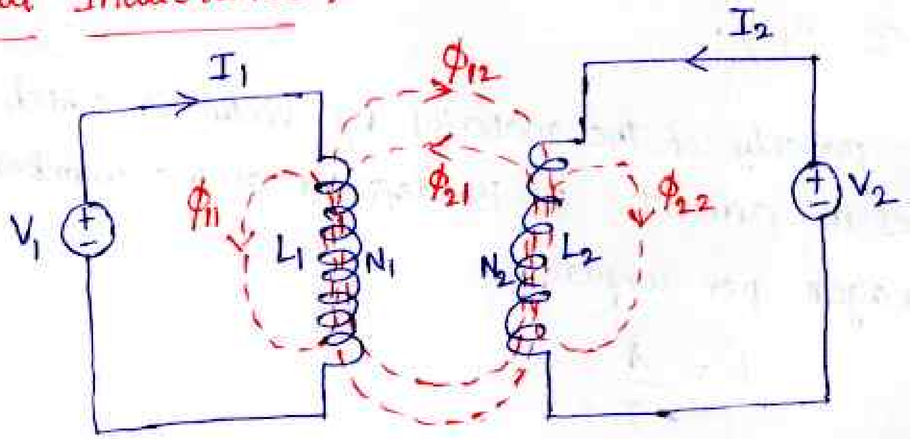


Fig (2): Linking of flux b/w two circuits

→ Consider that Two different circuits with self Inductances, L_1 & L_2 are kept close to each other as shown in fig. Let N_1 & N_2 be the no. of Turns for two circuits and let I_1 & I_2 be the currents through two circuits.

→ As two circuits placed very close, they interact magnetically with each other.

The flux produced by circuit 1, due to current I_1 is ϕ_{11}.
The flux produced by circuit 2 due to current I_2 is ϕ_{22}.
The flux produced by each circuit links with other circuit also.
The flux produced by circuit 2 is denoted as ϕ_{12}.
The part of flux ϕ_{11} links with circuit 2 is ϕ_{21}.
Similarly, the part of flux ϕ_{22} links with circut 1 is ϕ_{12}... b/w two circuits

→ The mutual Inductance : The mutual Inductance b/w two circuits is defined as the flux linkage of one circuit to the current in other circuit. M_{12} is given by

$$M_{12} = \frac{\text{flux linkage of circuit 1}}{\text{current in circuit 2}} = \frac{N_1 \phi_{21}}{I_2}.$$

Similarly $$M_{21} = \frac{\text{flux linkage of circuit 2}}{\text{current in circuit 1}} = \frac{N_2 \phi_{12}}{I_1}$$

for linear medium around two circuits, we can write

$$M_{12} = M_{21} = M.$$

Mutual Inductance is also measured in henry (H). (or) $\frac{wb\text{-}turn}{ampere}$.

Co-efficient of coupling between two circuits :-

→ consider the example of magnetically coupled circuits as shown in fig

The self Inductances of circuit 1 & circuit 2 are given as

$$L_1 = \frac{N_1 \phi_{11}}{I_1} = \frac{\lambda_{11}}{I_1} \rightarrow (1)$$

$$L_2 = \frac{N_2 \phi_{22}}{I_2} = \frac{\lambda_{22}}{I_2} \rightarrow (2)$$

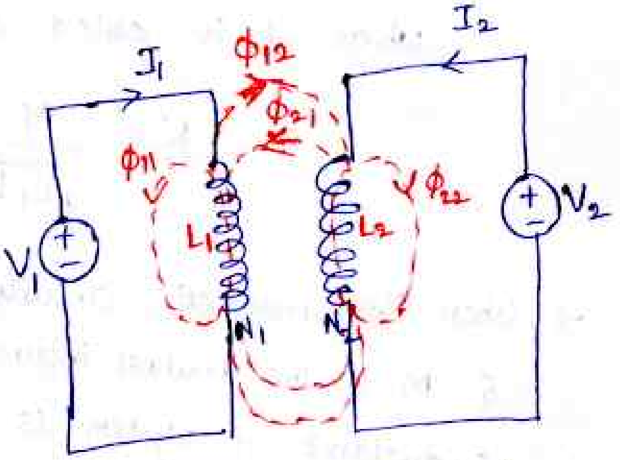


Fig: flux linking b/w Two circuits.

The flux linking with circuit 2, due to current I_1 is ϕ_{12}.

$$\phi_{12} = K_1 \phi_{11}. \longrightarrow (3)$$

Similarly, for flux ϕ_{21}, which is the part of flux ϕ_{22}, can be written as

$$\phi_{21} = K_2 \phi_{22}. \longrightarrow (4)$$

Rewriting the expressions for mutual inductances,

$$M_{12} = \frac{N_1 \phi_{21}}{I_2} = \frac{N_1 (K_2 \phi_{22})}{I_2} \longrightarrow (5)$$

$$\& \quad M_{21} = \frac{N_2 \phi_{12}}{I_1} = \frac{N_2 (K_1 \phi_{11})}{I_1} \longrightarrow (6)$$

Assuming linear medium surrounding two circuits,

$$M_{12} = M_{21} = M$$

$$M^2 = M_{12} \cdot M_{21} = \frac{N_1(K_2 \phi_{22})}{I_1} \cdot \frac{N_2(K_1 \phi_{11})}{I_1}$$

$$= (K_1 K_2)\left(\frac{N_1 \phi_{11}}{I_1}\right)\left(\frac{N_2 \phi_{22}}{I_2}\right)$$

$$M^2 = (K_1 K_2)(L_1)(L_2)$$

$$M = \sqrt{K_1 K_2}\sqrt{L_1 L_2}$$

Let $K = \sqrt{K_1 K_2}$

$$\therefore \boxed{M = K\sqrt{L_1 L_2}}$$

where K is called co-efficient of coupling b/w two coils.

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

→ When two magnetic circuits are coupled together in "series aiding" & M is the mutual inductance between them, then the effective inductance of system is

$$L_{eq} = (L_1 + L_2 + 2M), H$$

→ when two magnetic circuits with inductances L_1 & L_2 are magnetically coupled in "series opposing", then effective inductance of system is

$$L_{eq} = (L_1 + L_2 - 2M), H$$

→ when Two magnetic circuits with inductances L_1 & L_2, are coupled magnetically in parallel aiding, then Effective inductance of system is

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

when Two magnetic circuits with inductances L_1 & L_2 are coupled magnetically in parallel opposing, then equivalent inductance of system is,

$$\therefore \quad L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

→ Neuman's Formula :

Consider two closed loops C_1 & C_2 of any random shape as shown in Fig. Both closed loops are stationary loops placed in a linear medium.

Let I_1 & I_2 be the currents flowing through closed paths.

Let 'r' be the distance of separation between C_1 & C_2.

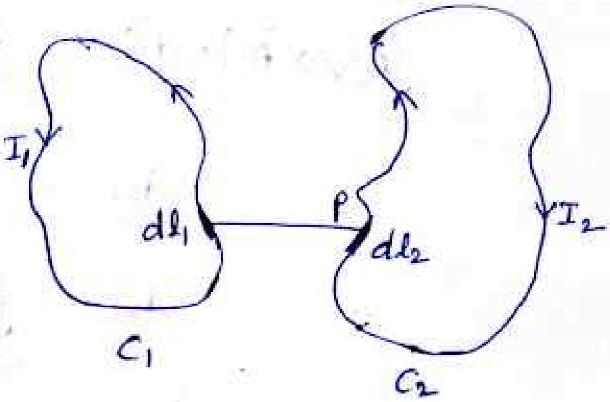


Fig: Two Closed loops Carrying current I_1 & I_2.

Let S_1 & S_2 be the surfaces of loop 1 & loop 2 respectively.

Consider a point P located along the surface of loop 2. The vector magnetic potential at point P due to loop 1 is given by,

$$A_1 = \frac{\mu}{4\pi} \oint_{C_1} \frac{I_1 \, d\bar{L}_1}{r} = \frac{\mu I_1}{4\pi} \oint \frac{d\bar{L}_1}{r} \longrightarrow (1)$$

The magnetic flux density $\bar{B}_1$ can be expressed interms of vector magnetic potential $\bar{A}_1$ as given below. $\longrightarrow$ (2)

$$\bar{B}_1 = \nabla \times \bar{A}_1$$

Let ϕ_{12} be the flux linking coil 2 due to current in coil 1.

$$\lambda_{12} = N_1 \phi_{12} = \bar{B}_1 \, ds_2 . \longrightarrow (3)$$

Assuming both the circuits of single turn,

i.e., $N_1 = N_2 = 1$.

Then $\lambda_{12} = \phi_{12} = \int_{S_2} \bar{B}_1 \cdot d\bar{S}_2$. $\longrightarrow$ (4)

$$\lambda_{12} = \phi_{12} = \int_{S_2} (\nabla \times \bar{A}) \cdot d\bar{S}_2 . \longrightarrow (5)$$

By stokes theorem, converting surface integral to the line integral, we get

$$\int_{S_2} (\nabla \times \bar{A}) \cdot d\bar{S}_2 = \oint_{C_2} \bar{A}_1 \cdot d\bar{L}_2 \longrightarrow (6)$$

$$\lambda_{21} = \oint_{C_2} \bar{A}_1 \cdot d\bar{L}_2$$

$$\lambda_{21} = \oint_{C_2} \left[\frac{\mu}{4\pi} \oint_{C_1} \frac{I_1 \, d\bar{L}_1}{r} \cdot d\bar{L}_2 \right]$$

$$\lambda_{21} = \frac{\mu I_1}{4\pi} \oint_{C_2} \oint_{C_1} \frac{d\bar{L}_1 \cdot d\bar{L}_2}{r} \longrightarrow (7)$$

But mutual inductance can be defined as

$$M = \frac{\lambda_{21}}{I_1} = \frac{\lambda_{12}}{I_2} \longrightarrow (8)$$

$$\boxed{M_{21} = \frac{\mu}{4\pi} \oint_{C_2} \oint_{C_1} \frac{d\bar{L}_1 \cdot d\bar{L}_2}{r}} \longrightarrow (9)$$

Similarly we can write

$$\boxed{M_{12} = \frac{\mu}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\bar{L}_1 \cdot d\bar{L}_2}{r}} \longrightarrow (10)$$

Eqn's (9) & (10) are called Neuman's Integrals (or) Neuman's Formulae.

→ Inductance of a Solenoid:

consider a solenoid of N turns as shown in fig (1).

Let I be the current flowing through the solenoid.

let l be the length of solenoid. and A be the cross sectional area.

The Field Intensity inside solenoid is given by,

$$H = \frac{NI}{l}, \; A/m$$

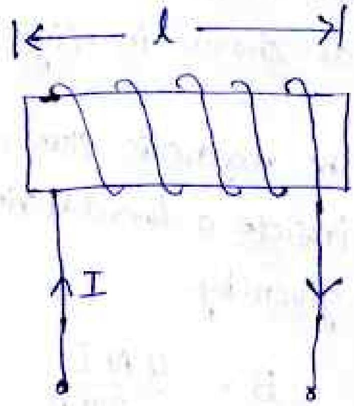


Fig: Solenoid with N turns.

$$\text{The total flux linkage} = N\phi = N(BA)$$
$$= N(\mu H)A$$

$$\text{Total flux linkage} = \mu NHA$$
$$= \mu N\left[\frac{NI}{l}\right]A$$

$$\text{Total flux linkage} = \frac{\mu N^2 I A}{l}$$

Thus the Inductance of a solenoid is given by,

$$L = \frac{\text{Total flux linkage}}{\text{Total current}}$$

$$L = \frac{\frac{\mu N^2 I A}{l}}{I}$$

$$\boxed{L = \frac{\mu N^2 A}{l} \quad \text{Henry}}$$

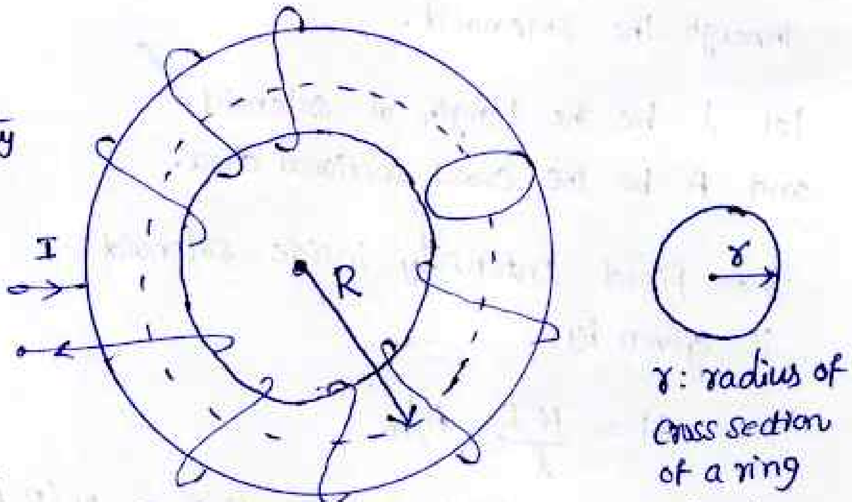
→ Inductance of a Toroid :-

Consider a toroidal ring with N turns and carrying current I.

→ Let radius of Toroid be R, as shown in fig.

→ The magnetic flux density inside a toroidal ring is given by

$$B = \frac{\mu N I}{2\pi R}$$



r: radius of cross section of a ring

fig(a): Toroidal ring

fig(b)

Fig(1): Toroidal ring & its cross sectional views.

→ The total flux linkage of toroidal ring having N turns is given by,

Total flux linkage $= N\phi$

But $\phi = (B)(A)$, where A = Area of cross section of toroidal ring.

∴ Total flux linkage $= N(B)(A)$

$$= N\left(\frac{\mu N I}{2\pi R}\right) A$$

$$= \frac{\mu N^2 I A}{2\pi R}$$

The Inductance of a Toroid is given by,

$$L = \frac{\text{Total flux linkage}}{\text{Total current}} = \frac{\frac{\mu N^2 I A}{2\pi R}}{I}$$

$$\boxed{L = \frac{\mu N^2 A}{2\pi R}, \ H}$$

where A = Area of cross section of toroidal ring

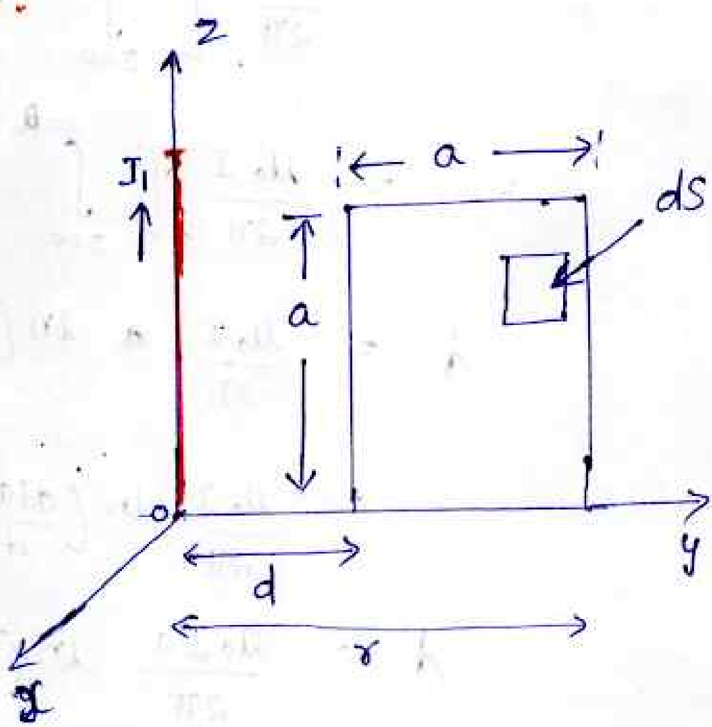
$= \pi r^2$, m^2.

For a toroid with N No. of turns, h as the height of Toroid with r_1 as inner radius and r_2 as outer radius, the inductance is given by

$$L = \frac{\mu N^2 h}{2\pi} \ln\left(\frac{r_2}{r_1}\right), \ H.$$

→ Mutual Inductance between a long, straight wire and square Loop lying in same plane :

→ Consider a square loop with sides a and b as shown in fig. A straight long conductor is kept parallel to one side of loop along z axis.



→ Consider long straight wire is circuit 1 and square loop is circuit 2.

→ The ~~magnetic field intensity~~ at a distance ~~'ρ' from long conductor~~

Mutual Inductance is given by the flux linkages with the loop per unit current in the wire

$$M = \frac{\lambda}{I}$$

$$\lambda = \int_S \bar{B} \cdot d\bar{S} = \int_{z=0}^{a} \int_{r=d}^{r=d+a} \left(\frac{\mu_0 I}{2\pi r}\, \bar{a}_\phi\right) \cdot \left(dr\, dz\, \bar{a}_\phi\right)$$

$$= \int_{z=0}^{a} \int_{r=d}^{r=d+a} \frac{\mu_0 I}{2\pi r}\, dr\, dz$$

$$= \frac{\mu_0 I}{2\pi} \int_{z=0}^{z=a} \left[\int_{r=d}^{r=d+a} \frac{dr}{r} \right] dz$$

$$= \frac{\mu_0 I}{2\pi} \left[\int_{z=0}^{z=a} \ln \left(r \right)_{d}^{d+a} dz \right]$$

$$= \frac{\mu_0 I}{2\pi} \left[\int_{z=0}^{z=a} \ln(d+a) - \ln d \ \ dz \right]$$

$$= \frac{\mu_0 I}{2\pi} \left[\int_{z=0}^{z=a} \ln \left(\frac{d+a}{d} \right) dz \right.$$

$$\lambda = \frac{\mu_0 I}{2\pi} \ln \left(\frac{d+a}{d} \right) \int_{z=0}^{z=a} dz$$

$$= \frac{\mu_0 I}{2\pi} \ln \left(\frac{d+a}{d} \right) (z)_{0}^{a}$$

$$\lambda = \frac{\mu_0 I a}{2\pi} \ln \left(\frac{d+a}{d} \right)$$

Mutual Inductance $M = \frac{\lambda}{I}$

$$M = \frac{\frac{\mu_0 I a \ln\left(\frac{d+a}{d}\right)}{2\pi}}{I}$$

$$\Rightarrow \boxed{M = \frac{\mu_0 a}{2\pi} \ln \left(\frac{d+a}{d} \right) \quad \text{— Henry}}$$

→ Energy stored in a Magnetic field:

The inductor stores energy in the form of magnetic field. The energy stored by an inductor is given by

$$W_m = \frac{1}{2} L I^2$$

consider a differential volume in a magnetic field $\bar{B}$ as shown in fig.

consider that at the top & bottom surfaces of a differential volume conducting sheets with current ΔI are present.

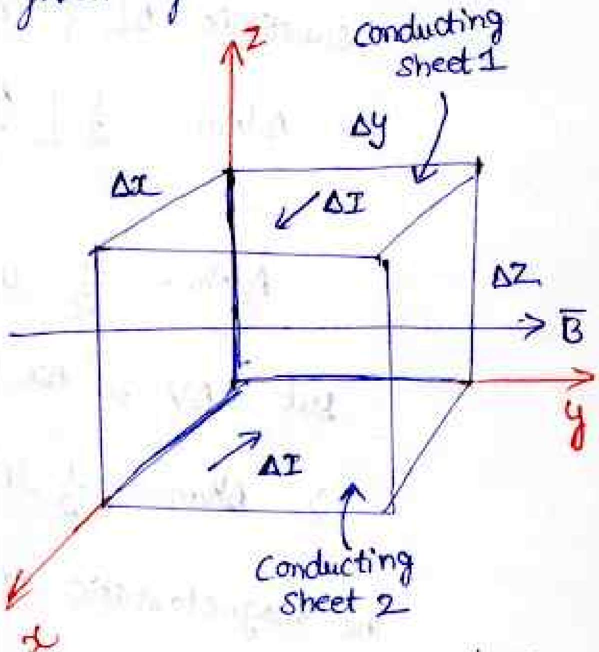


Fig: Differential volume in magnetic field $\bar{B}$.

From the definition of Inductance, ΔL can be written as

$$\Delta L = \frac{\Delta \phi}{\Delta I} = \frac{B\,\Delta S}{\Delta I} \qquad \left(\because \phi = \int_S \bar{B} \cdot d\bar{S}\right). \longrightarrow (1)$$

where ΔS = differential surface = $\Delta x\, \Delta z$.

$$\Delta L = \frac{B(\Delta x\, \Delta z)}{\Delta I} \longrightarrow (2)$$

But $B = \mu H$

$$\Delta L = \frac{\mu H(\Delta x\, \Delta z)}{\Delta I}. \longrightarrow (3)$$

→ Now the differential current ΔI can be expressed in terms of H. The current flowing through conducting sheets at top & bottom is in y-direction.

$$\Delta I = (H)\, \Delta y. \longrightarrow (4)$$

Energy stored in inductance of differential volume is

$$\Delta W_m = \frac{1}{2}\Delta L\,\Delta I^2$$

substitute ΔL & ΔI values from Eq (3) & (4)

$$\Delta W_m = \frac{1}{2}\left[\frac{\mu H\,\Delta x\,\Delta z}{H\,\Delta y}\right][H\,\Delta y]^2$$

$$\Delta W_m = \frac{1}{2}\mu H^2(\Delta x\,\Delta y\,\Delta z)$$

But $\Delta V = \Delta x\,\Delta y\,\Delta z$

$$\Rightarrow \Delta W_m = \frac{1}{2}\mu H^2\,\Delta V \longrightarrow (5)$$

The magnetostatic energy density function is defined as

$$w_m = \lim_{\Delta V \to 0}\frac{\Delta W_m}{\Delta V} = \frac{\frac{1}{2}\mu H^2\,\Delta V}{\Delta V}$$

$$\boxed{w_m = \frac{1}{2}\mu H^2} \longrightarrow (6)$$

Energy density is expressed in Joule/m^3.

The magnetostatic Energy density can be Expressed in different forms as

$$w_m = \frac{1}{2}(\mu H)(H) = \frac{1}{2}BH \longrightarrow (7)$$

and $$w_m = \frac{1}{2}B\left(\frac{B}{\mu}\right) = \frac{B^2}{2\mu} \longrightarrow (8)$$

In linear medium, the Energy in magnetostatic field is given by

$$W_m = \int w_m\,dV = \frac{1}{2}\int \mu H^2\,dV = \frac{1}{2}\int\frac{B^2}{\mu}\,dV$$

$$\therefore W_m = \frac{1}{2}\int \bar{B}\cdot\bar{H}\,dV \longrightarrow (9)$$

1) If a coil of 800 μH is magnetically coupled to another coil of 200 μH. The co-efficient of coupling between two coils is 0.05. calculate inductance if two coils are connected in (i) series aiding (ii) series opposing (iii) parallel aiding (iv) Parallel opposing.

Sol:

Given $L_1 = 800\ \mu H$

$L_2 = 200\ \mu H$

Co-efficient of coupling $K = 0.05$

mutual inductance between two coils $M = K\sqrt{L_1 L_2}$

$$M = (0.05)\sqrt{(800\times10^{-6})(200\times10^{-6})}$$

$$= 20\ \mu H$$

(i) series aiding : $L_{eq} = L_1 + L_2 + 2M$

$$= (800\times10^{-6}) + (200\times10^{-6}) + (2\times20\times10^{-6})$$

$$= 1040\ \mu H$$

(ii) series opposing : $L_{eq} = L_1 + L_2 - 2M$

$$= (800\times10^{-6}) + (200\times10^{-6}) - (2\times20\times10^{-6})$$

$$= 960\ \mu H$$

(iii) Parallel aiding : $L_{eq} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$

$$L_{eq} = \frac{(800\times10^{-6}\times200\times10^{-6}) - (20\times10^{-6})^2}{(800\times10^{-6}) + (200\times10^{-6}) - 2(20\times10^{-6})} = 166.25\ \mu H$$

(iv) Parallel opposing : $L_{eq} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$

$$L_{eq} = \frac{800\times10^{-6}\times200\times10^{-6} - (20\times10^{-6})^2}{800\times10^{-6} + 200\times10^{-6} + 2(20\times10^{-6})} = 153.46\ \mu H$$

2) Calculate the inductance of a solenoid of 200 turns wound tightly on a cylindrical tube of 6cm diameter. The length of tube is 60 cm and the solenoid is in air.

Sol: Given No. of Turns $N = 200$ turns

diameter $= d = 6cm = 6\times10^{-2}$ m

radius $r = \frac{d}{2} = 3\times10^{-2}$ m.

length of tube $l = 60$ cm $= 60\times10^{-2}$ m

$\mu = \mu_0 \mu_r$

$\mu_r = 1$ for air

Inductance of a solenoid is $L = \frac{\mu N^2 A}{l}$

$A =$ Area of solenoid $= \pi r^2$

$$L = \frac{\mu_0 N^2 (\pi r^2)}{l} = \frac{4\pi\times10^{-7}\times(200)^2\times \pi\times(3\times10^{-2})^2}{60\times10^{-2}}$$

$L = 2.3687\times10^{-4}$ H.

3) A coil of 500 turns is wound on a closed iron ring of mean radius 10cm and cross section area of $3cm^2$. Find the self inductance of the winding if the relative permeability of iron is 800.

Given, No. of turns $N = 500$.

Area $A = 3\,cm^2 = 3\times(10^{-2})^2 = 3\times10^{-4}$ m

radius $R = 10$ cm $= 10\times10^{-2}$ m

relative permeability $\mu_r = 800$.

Inductance of iron Toroid is $L = \frac{\mu N^2 A}{2\pi R} = \frac{\mu_0 \mu_r N^2 A}{2\pi R}$

$$L = \frac{4\pi\times10^{-7}\times800\times(500)^2(3\times10^{-4})}{2\pi(10\times10^{-2})} = 0.12\text{ H}$$

4) Calculate the inductance of a toroid formed by surfaces $\rho = 3$ cm & $\rho = 5$ cm, $z = 0$ & $z = 1.5$ cm wrapped with 5000 turns of wire and filled with a magnetic material with $\mu_r = 6$.

Sol: For a toroid,

outer radius $r_2 = 5$ cm $= 5 \times 10^{-2}$ m

inner radius $r_1 = 3$ cm $= 3 \times 10^{-2}$ m

Height, $h = 1.5$ cm $= 1.5 \times 10^{-2}$ m

No. of Turns $N = 5000$

$\mu_r = 6$

$\mu = \mu_0 \mu_r$

The Inductance of Toroid $L = \frac{\mu N^2 h}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$

$$L = \frac{(4\pi \times 10^{-7})(6)(5000)^2(1.5 \times 10^{-2})}{2\pi} \ln\left(\frac{5 \times 10^{-2}}{3 \times 10^{-2}}\right)$$

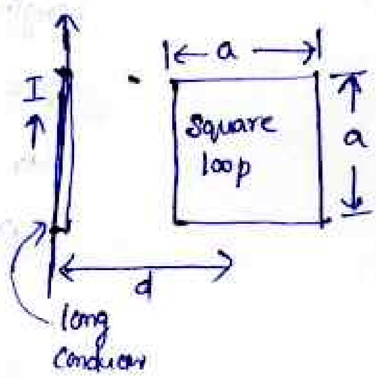
$L = 0.2298$ H

5) A straight long wire is situated parallel to one side of a square coil. Each side of coil has a length of 10 cm. The distance between straight wire and centre of coil is 20 cm. find mutual inductance of the system.

Sol. Given Side of coil $a = 10$ cm $= 10 \times 10^{-2}$ m

distance between straight wire and centre of coil

$d = 20$ cm $= 20 \times 10^{-2}$ m



Mutual Inductance b/w long straight wire & square coil is

$$M = \frac{\mu_0 a}{2\pi} \ln\left[1 + \frac{a}{d}\right]$$

$$M = \frac{4\pi \times 10^{-7} \times 10 \times 10^{-2}}{2\pi} \ln\left(1 + \frac{10 \times 10^{-2}}{20 \times 10^{-2}}\right)$$

$$M = 8.1093 \ nH$$

6) Compute energy density in free space on account of field $H = 1000$ A/m.

Sol. Energy density in free space in a magnetic field

$$\omega_m = \frac{1}{2} \mu H^2$$

$$= \frac{1}{2} \mu_0 \mu_r H^2$$

for free space $\mu_r = 1$

$$\omega_m = \frac{1}{2} \times 4\pi \times 10^{-7} \times 1 \times (1000)^2$$

$$= 0.6283 \ J/m^3.$$

7) A solenoid of the 500 turns has a length of 50 cm and the radius of 10 cm. A steel rod of circular cross section is fitted in the solenoid coaxially. Relative permeability of steel is 3000. A dc current of 10A is passed through solenoid. Compute inductance of system, energy stored in the system and mean flux density inside the solenoid

Sol: Given No. of turns $N = 500$

length of solenoid $l = 50$ cm $= 50 \times 10^{-2}$ m

r = radius = 10 cm = 10×10^{-2} m = 0.1 m

μ_r = relative permeability = 3000.

Current $I = 10$ A.

magnetic field inside solenoid is

$$H = \frac{NI}{l} \quad A/m$$

magnetic flux density is $B = \mu H$

$$B = \frac{\mu_0 \mu_r NI}{l}$$

Hence Total flux $\phi = B \times A = \frac{\mu_0 \mu_r NI}{l} A$

$$\phi = \frac{\mu_0 \mu_r NI}{l} (\pi r^2)$$

(i) Inductance of system is

$$L = \frac{N\phi}{I} = \frac{N}{I}\left[\frac{\mu_0 \mu_r NI}{l} (\pi r^2)\right]$$

$$= \frac{\mu_0 \mu_r N^2 (\pi r^2)}{l}$$

$$= \frac{4\pi \times 10^{-7} \times 3000 \times (500)^2 (\pi \times (0.1)^2)}{0.5}$$

$$L = 59.2176 \text{ H}$$

(ii) Energy stored in the system, $W_H = \frac{1}{2} L I^2$

$$W = \frac{1}{2}(59.2176)(10)^2$$

$$= 2.96 \text{ KJ}$$

(iii) Mean flux density inside solenoid is $B = \frac{\mu NI}{l}$

$$B = \frac{\mu_0 \mu_r NI}{l} = \frac{4\pi \times 10^{-7} \times 3000 \times 500 \times 10}{0.5}$$

$$= 37.69 \text{ Wb/m}^2.$$

8) A Toroidal coil of 500 turns is wound on a steel ring of 0.5 m mean diameter and $2 \times 10^{-3}\ m^2$ cross sectional area. An excitation of $4000\ Am^{-1}$ produces a flux density of 1T. Find the inductance of coil.

Sol. Given No. of Turns $N = 500$.

diameter of steel ring $= D = 0.5\ m$

radius of steel ring $= R = \frac{D}{2} = \frac{0.5}{2} = 0.25\ m$.

Area of cross section $A = 2 \times 10^{-3}\ m^2$.

magnetic field Intensity, $H = 4000\ A m^{-1}$.

flux density $B = 1T$. (1 Tesla)

Inductance of Toroid. $\Rightarrow L = \frac{\mu N^2 A}{2\pi R}$

$$\mu = \frac{B}{H} = \frac{1}{4000} = 0.25 \times 10^{-3}$$

$$\therefore L = \frac{0.25 \times 10^{-3} \times (500)^2 \times 2 \times 10^{-3}}{2\pi \times 0.25}$$

$\Rightarrow L = 80\ mH$