UNIT-I

Antenna Basics & Dipole antennas

Introduction: Antenna is an important device which has become an integral part of our day to day life . we find antenna everywhere ,at home ,work places ,on cars,vehicles, aircraft,ships,and not only that we carry antenna along with us in our mobiles. There are number of types of antennas but all operate with the basic principles of electromagnetics. First radio antenna was assembled in 1886 by Heinrich Hertz. He developed a circuit resembling a radio system with end loaded dipoles as a transmitting antenna while resonant square loop antenna as a receiving antenna operating at one meter wavelength. The laboratory work done by Hertz was further completed by Guglielmo Marconi. He demonstrated world communication of signal over long distances in 1901. Now a days ,antennas are the most essential communication link for aircraft and ships. Antennas are our electronic eyes and ears on the world. They are our links with space. The radiation is produced by accelerated or decelerated charges.

Basic Definitions of Antenna:

1. An antenna is a metallic device in the form of either wire or rod used for radiating or receiving radio waves .
2. According to The *IEEE Standard Definitions of Terms for Antennas* defines the antenna or aerial as "a means for radiating or receiving radio waves."
3. An *antenna* may be defined as the structure associated with the region of transition between a guided wave and a free-space wave, or vice versa.
4. An antenna can also be defined as transducer which converts electrical current into EM waves.
5. An antenna can also be defined as an impedance matching device between transmission line and free space and vice-versa.

Antenna as a transition device:

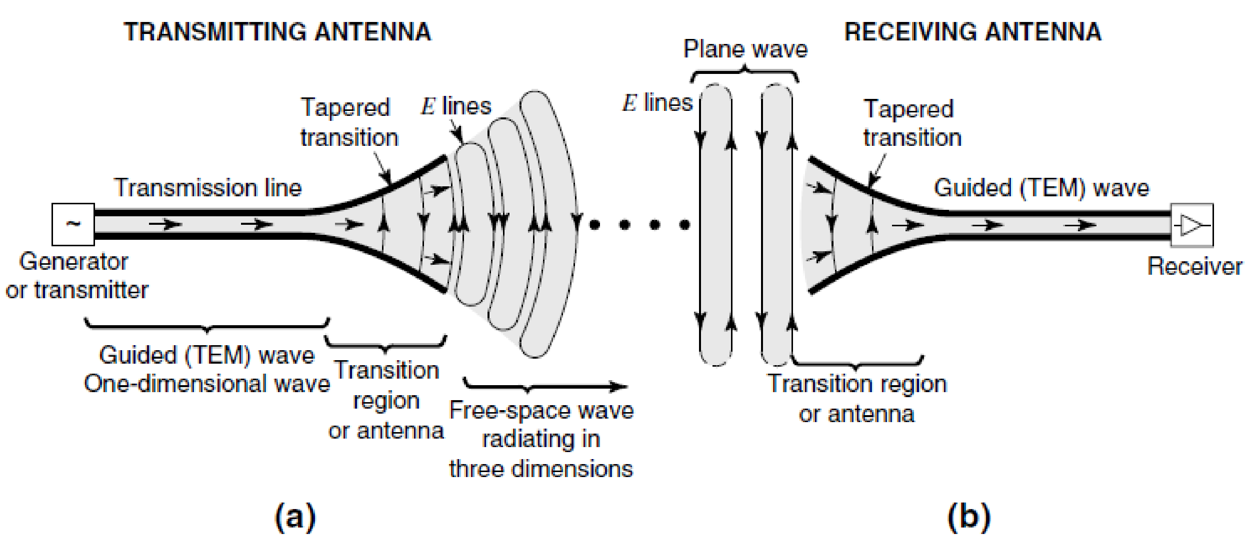


Fig.(a) transmitting antenna and (*b*) receiving antenna.

The two wire transmission line in fig is connected to a transmitter or generator. Along the uniform part of the line , the energy is guided as a plane TEM wave with little loss. The spacing between wires is assumed to be a small fraction of a wavelength. Further on the transmission line opens out in tapered region. As separation approaches the order of the wave length or more ,the wave tends to be radiated so that open out transmission line acts like an antenna which launches a free space wave. The currents on the transmission line flow out on the antenna and end there but the fields associated with them keep on going.

The transmitting antenna in Fig. 1*a* is a region of transition from a guided wave on a transmission line to a free-space wave. The receiving antenna (Fig. 1*b*) is a region of transition from a space wave to a guided wave on a transmission line. Thus, ***an antenna is a transition device, or transducer, between a guided wave and a free-space wave, or vice-versa.*** The antenna is a device which interfaces a circuit and space.

RADIATION MECHANISM:

Regardless of antenna type, all involve the same basic principle that radiation is produced by accelerated (or decelerated) charge. The *basic equation of radiation* may be expressed simply as

$\boxed{\dot{I}L = Q\dot{v}}$ Basic radiation equation

where

$\dot{I}$ = time-changing current, A s^{-1}

L = length of current element, m

Q = charge, C

$\dot{v}$= time change of velocity which equals the acceleration of the charge, m s^{-2}

L = length of current element, m

It simply states that *to create radiation, there must be a time-varying current or an acceleration (or deceleration) of charge*. The phenomenon of transmitting energy into the free space in the form of electromagnetic fields is called as radiation.

1. If a charge is not moving, current is not created and there is no radiation.
2. If charge is moving with a uniform velocity: (a). There is no radiation if the wire is straight, and infinite in extent.(b.) There is radiation if the wire is curved, bent, discontinuous, terminated, or truncated, as shown in Figure
3. If charge is oscillating in a time hormonic-motion, it radiates even if the wire is straight.

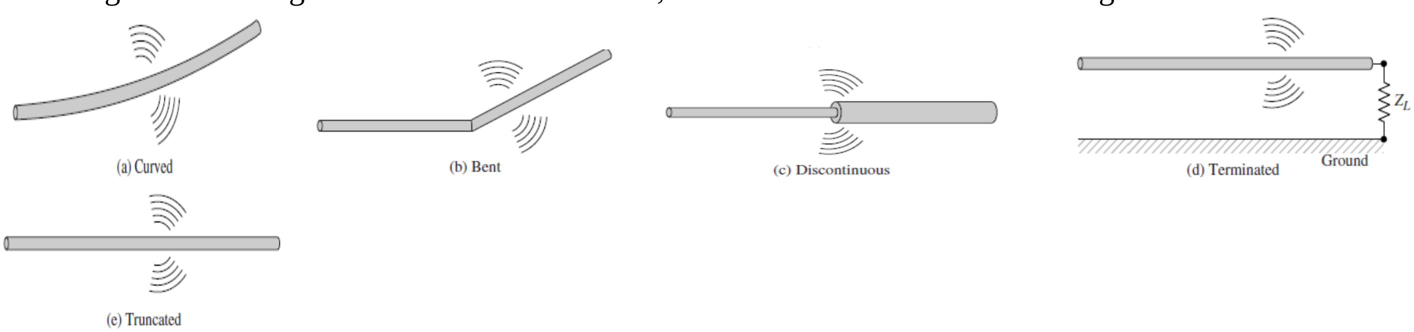


Fig Wire configurations for radiation.

<u>Types of antennas:</u>

1. Wire antennas(Dipole,loop and helical antennas)
2. Aperture antennas(Horn antenna)
3. Microstrip Antennas
4. Array antennas(Yagi-Uda array)
5. Lens antennas
6. Reflector antennas**(**parabolic reflector)

<u>Isotropic Radiator</u>: An *isotropic* radiator is defined as "a hypothetical lossless antenna having equal radiation in all directions". It is also called isotropic source. As it radiates uniformly in all directions, it is also called omnidirectional radiator or unipole. Basically isotropic radiator is a lossless ideal radiator or antenna. Generally all the practical antennas are compared with the characteristics of the isotropic radiator. The isotropic antenna or radiator is used as reference antenna. Practically all antennas show directional properties i.e. directivity property. That means none of the antennas radiate energy uniformly in all directions. Hence practically isotropic radiator cannot exist.

Consider that an isotropic radiator is placed at the centre of sphere of radius r. Then all the power radiated by the isotropic radiator passes over the surface area of the sphere given by $4\pi r^2$. The average power density Pavg at any point the surface of the sphere is defined as "power radiated per unit area in any direction". The total power radiated by the isotropic radiator is given by $P_{rad} = \iint P_{avg} \,.\, dS = P_{avg}(4\pi r^2)$

$P_{avg} = \frac{P_r}{4\pi r^2}$

Where P_{rad}= total power radiated in watts

P_{avg}=Average power density

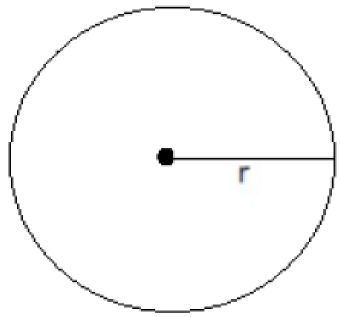


Fig. Isotropic Radiator

Note: 1. A *directional* antenna is one "having the property of radiating or receiving electromagnetic waves more effectively in some directions than in others.

2. Omnidirectional antenna is defined as the antenna having an essentially nondirectional pattern in a given plane (e.g., in azimuth) and a directional pattern in any orthogonal plane (in elevation). An *omnidirectional* pattern is then a special type of a *directional* pattern

Basic Antenna parameters: To describe the performance of an antenna, definitions of various parameters are necessary. They are given by the following

1. Radiation pattern (a).Field radiation pattern (2). Power pattern
2. Radiation Intensity
3. Directivity and gain
4. Antenna beam width
5. Antenna bandwidth
6. Antenna beam area
7. Resolution
8. Antenna impedance
9. Effective height
10. Effective length
11. Antenna apertures
12. Antenna temperature
13. Antenna polarizations

Radiation pattern:

The radiation patterns are the graphical representation of the three dimensional variations of field or power as a function of the spherical coordinates **θ** and ϕ. The pattern has its main lobe (maximum radiation) in the z direction (θ = 0) with minor lobes (side and back) in other directions.

To completely specify the radiation pattern with respect to field intensity and polarization requires three patterns:

1. The θ component of the electric field as a function of the angles θ and ϕ or $E_\theta(\theta,\phi)$ (V m−1)
2. The φ component of the electric field as a function of the angles θ and ϕ or $E_\phi(\theta,\phi)$ (V m−1).
3. The phases of these fields as a function of the angles θ and ϕ or δθ (θ, φ) and δφ(θ, φ) (rad or deg).

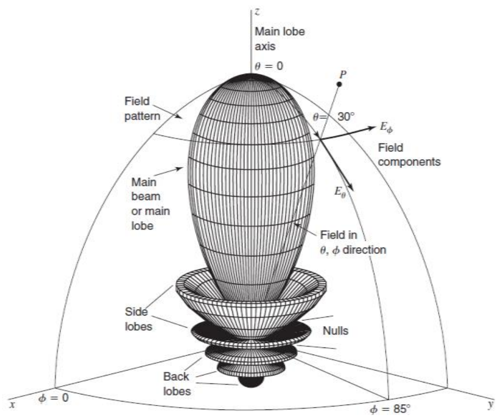


Fig Three-dimensional field pattern of a directional antenna with maximum radiation in *z*-direction at $\theta = 0^\circ$.

To represent the radiation pattern in two dimensionally two plane cuts are required. Two such cuts at right angles, called the *principal plane patterns* (as in the *xz* and xy planes) may be required but if the pattern is symmetrical around the *z* axis, one cut is sufficient.

Note: For a linearly polarized antenna, performance is often described in terms of its principal E and H plane patterns.

1. The E-plane is defined as "the plane containing the electric-field vector and the direction of maximum radiation," . the x-z plane (elevation plane; $\phi = 0$) is the principal E-plane
2. The H-plane is defined as "the plane containing the magnetic-field vector and the direction of maximum radiation." the x-y plane (azimuthal plane; $\theta = \pi/2$) is the principal H-plane.

The power pattern can be represented two dimensionally as shown below.

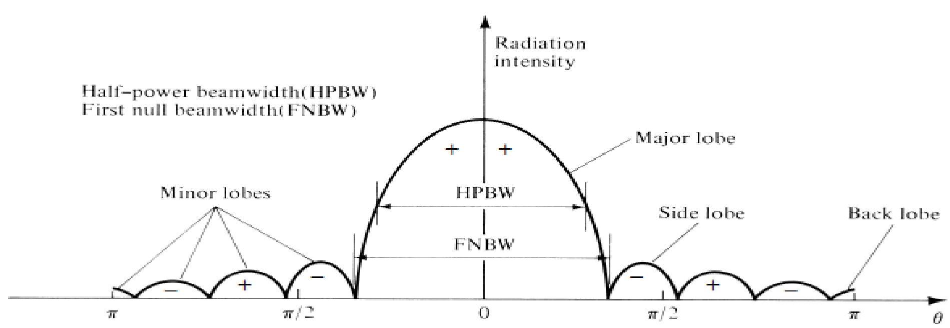


Fig . Linear plot of power pattern and its associated lobes and beamwidths.

Field pattern : The field patterns are the graphical representation of the three dimensional variations of fields as a function of the spherical coordinates **θ** and ϕ **.**

Normalized field pattern: The normalized field pattern or relative field pattern is defined as the ratio of the field components to its maximum value. There are no units for the normalized field pattern.

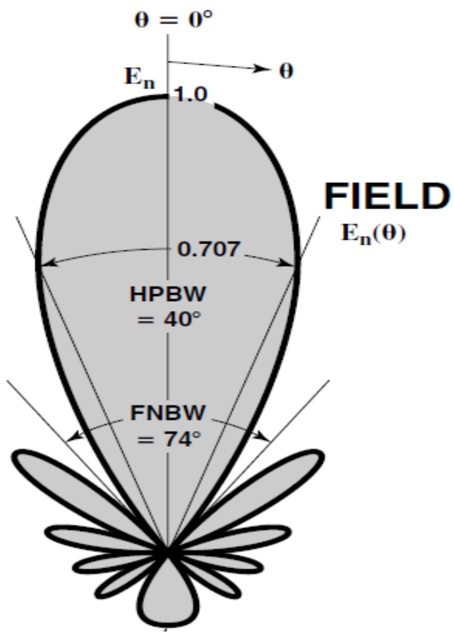
$$\boxed{\textit{Normalized field pattern} = E_\theta(\theta,\phi)_n = \frac{E_\theta(\theta,\phi)}{E_\theta(\theta,\phi)_{\max}} \qquad \text{(dimensionless)}}$$

The half-power level occurs at those angles θ and φ for which $E\theta\ (\theta,\ \varphi)n = 1/\sqrt{2}=0.707$

Power pattern: The radiation patterns are the graphical representation of the three dimensional variations of power as a function of the spherical coordinates **θ** and **φ.**
Normalized power pattern: The normalized power pattern or relative power pattern is defined as the ratio of the power per unit area to its maximum value. There are no units for the normalized power pattern.

$$\boxed{\textbf{Normalized power pattern} = \mathbf{P}_{\boldsymbol{n}}(\boldsymbol{\theta},\emptyset) = \frac{\mathbf{P_d}(\boldsymbol{\theta},\emptyset)}{\mathbf{P}_{\boldsymbol{d}}(\boldsymbol{\theta},\emptyset)_{\boldsymbol{max}}} \quad \text{(dimensionalless)}}$$

The decibel level is given by $dB = 10log_{10}(P_n(\theta,\phi)$



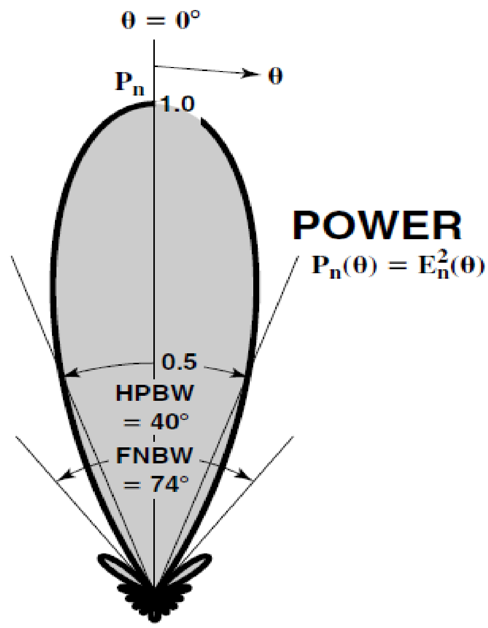


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Fig (a) Normalized Field pattern (b).Normalized power pattern

Note:1. Various parts of a radiation pattern are referred to as *lobes*, which may be subclassified into *major* or *main, minor, side,* and *back* lobes.

2. A *major lobe* (also called main beam) is defined as "the radiation lobe containing the direction of maximum radiation."

3.A *minor lobe* is any lobe except a major lobe and A *side lobe* is "a radiation lobe in any direction other than the intended lobe.

4.A *back lobe* is "a radiation lobe whose axis makes an angle of approximately 180◦ with respect to the beam of an antenna."

<u>Antenna Beam width:</u> The beam width is defined as the angular width in degrees between the two points on the major lobe of the radiation pattern. It is the measure of directivity of the antenna

<u>Half power beam width (HPBW):</u> The HPBW is defined as the angular width in degrees between the two half power points on the major lobe of the radiation pattern.

<u>First null beam width (FNBW):</u> The FNBW is defined as the angular width in degrees between the two null points on the major lobe of the radiation pattern.

<u>Front to Back ratio(FBR)</u>:

$$\boldsymbol{FBR} = \frac{\textbf{Power radiated in the desired direction}}{\textbf{power radiated in opposite direction}}$$

<u>Radian and Steradian</u> : The radian is the measure of the plane angle while the Steradian is the measure of a solid angle.

Radian: One *radian* is defined as the plane angle with its vertex at the center of a circle of radius *r* that is subtended by an arc whose length is *r*. Since the circumference of a circle of radius *r* is C = 2πr, there are 2π rad (2πr/r) in a full circle.

<u>Steradian:</u> One *steradian* is defined as the solid angle with its vertex at the center of a sphere of radius *r* that is subtended by a spherical surface area equal to that of a square with each side of length *r*. Since the area of a sphere of radius *r* is S = 4πr2, there are 4π sr (4πr2/r2) in a closed sphere.

The infinitesimal area *dA* on the surface of a sphere of radius *r*, shown in Figure 2.1, is given by

$dS = r^2 \sin\theta d\theta d\emptyset$ (m^2)

Therefore, the element of solid angle $d\Omega$ of a sphere can be written as

$d\Omega = {}^{\mathrm{dS}}/_{\mathrm{r}^2} = \sin\theta\, d\theta d\phi$ (sr)

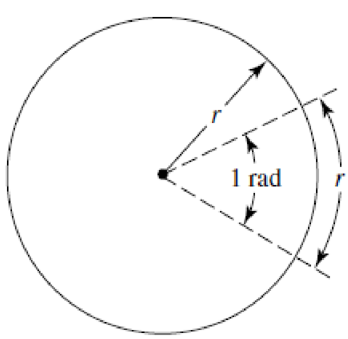
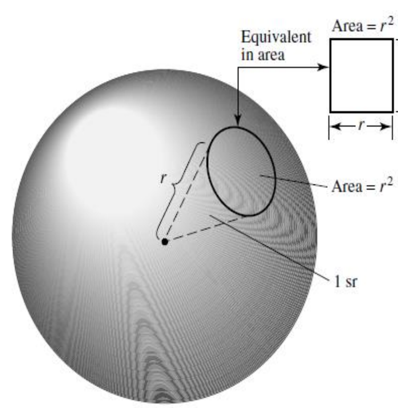


Fig .(a) Radian



(b)Steradian

<u>Beam Area or beam solid angle</u> (Ω_A): The beam area or *beam solid angle* or $\Omega_A$of an antenna is given by the integral of the normalized power pattern over a sphere (4π sr)

$$\Omega_A = \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi} P_n(\theta, \phi) \sin\theta \, d\theta \, d\phi$$

$$\boxed{\Omega_A = \iint_{4\pi} P_n(\theta, \phi)\, d\Omega \qquad \text{(sr)} \qquad \textbf{\textit{Beam area}}}$$

Where $d\Omega = \sin\theta \, d\theta d\phi$

The beam area of an antenna can often be described approximately in terms of the angles subtended by the half-power points of the main lobe in the two principal planes. Thus,

$$\boxed{\textit{Beam area} \cong \Omega_A \cong \theta_{HP}\phi_{HP} \qquad \text{(sr)}}$$

where θ_{HP} and ϕ_{HP} are the half-power beamwidths (HPBW) in the two principal planes, minor lobes being neglected

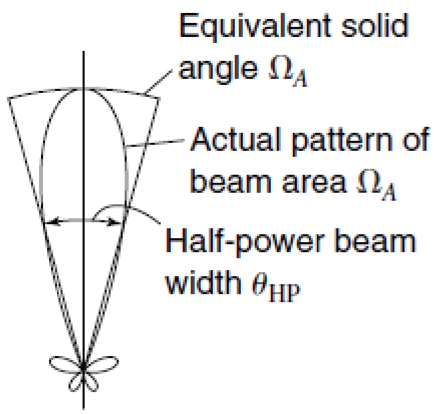


Fig . Antenna power pattern and its equivalent beam solid angle or beam area $d\Omega_A$

<u>BEAM EFFICIENCY:</u> The (total) *beam area* Ω_A (or *beam solid angle*) consists of the main beam area (or solid
angle) Ω_M plus the minor-lobe area (or solid angle) Ω_m. Thus
$\Omega_A = \Omega_M + \Omega_m$
The ratio of the main beam area to the total beam area is called the *beam efficiency* ε_M. Thus

$$Beam\ efficiency = \varepsilon_M = \frac{\Omega_M}{\Omega_A} \quad (dimensionless)$$

The ratio of the minor-lobe areaΩ_m to the total beam area *is* called the *stray factor*.
Thus,,$\boldsymbol{\varepsilon} = \frac{\boldsymbol{\Omega_m}}{\boldsymbol{\Omega_A}} = \boldsymbol{Stray\ factor}$ **. It** follo ws that $\varepsilon_M + \varepsilon_m = 1$

<u>RADIATION INTENSITY:</u> *Radiation intensity* in a given direction is defined as "the power radiated from an antenna per unit solid angle."

In mathematical form it is expressed as $U(\theta,\phi) = r^2 P_d(\theta,\emptyset)$
where
$U(\theta,\phi)$ = radiation intensity (Watts/Steradian)
$P_d(\theta,\emptyset)$= radiation power density (W/m^2)

The total power radiated is obtained by integrating the radiation intensity over the entire solid angle of 4π, as given by

$$P_r = \oiint U(\theta,\phi)\, d\Omega = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi) \sin\theta \, d\theta d\phi$$

Where $d\Omega = \sin\theta \, d\theta d\phi$

For an isotropic source U will be independent of the angles θ and ϕ, therefore

$$P_{rad} = \oiint U_o \, d\Omega = U_o \oiint \mathrm{d}\Omega = 4\pi U_o$$

Therefore the radiation intensity of an isotropic source is given by

$$U_o = \frac{P_{rad}}{4\pi}$$

<u>Directivity (D):</u> the *directivity of an antenna* defined as "the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions".

$$D = \frac{U}{U_{avg}}$$

The average radiation intensity is equal to the total power radiated by the antenna divided by 4π. Therefore

$$D = \frac{4\pi U}{P_{rad}}$$

If direction is not given , the *directivity of an antenna* defined as "the ratio of the maximum radiation intensity to it's average radiation "

$$D = \frac{U_{max}}{U_{avg}} = \frac{4\pi U_{max}}{P_{rad}}$$

Or

The *directivity* of an antenna is defined as the ratio of the maximum power density to it's its average value over a sphere as observed in the far field of an antenna.

$$D = \frac{P_d(\theta, \emptyset)_{max}}{P_d(\theta, \emptyset)_{avg}} \quad Directivity\ from\ pattern$$

The average power density over a sphere is given by

$$P_d(\theta, \emptyset)_{avg} = \frac{1}{4\pi} \int_{\emptyset=0}^{\emptyset=2\pi} \int_{\theta=0}^{\theta=\pi} P_d(\theta, \emptyset) \sin\theta \, \mathrm{d\theta d\emptyset}$$

Therefore, the directivity

$$D = \frac{P_d(\theta, \emptyset)_{max}}{\frac{1}{4\pi} \int_{\emptyset=0}^{\emptyset=2\pi} \int_{\theta=0}^{\theta=\pi} P_d(\theta, \emptyset) \sin\theta \, \mathrm{d\theta d\emptyset}} = \frac{1}{\frac{1}{4\pi} \left(\int_{\emptyset=0}^{\emptyset=2\pi} \int_{\theta=0}^{\theta=\pi} \frac{P_d(\theta, \emptyset)}{P_d(\theta, \emptyset)_{max}} \mathrm{d}\Omega \right)}$$

$$\boxed{\text{Normalized power pattern} = P_n = \frac{P_d(\theta, \emptyset)}{P_d(\theta, \emptyset)_{max}}}$$

$$\boxed{D = \frac{4\pi}{\iint_{4\pi} P_n(\theta,\phi)\, d\Omega} = \frac{4\pi}{\Omega_A} \qquad \textit{Directivity from beam area } \Omega_A}$$

where

$$\Omega_A = \iint_{4\pi} P_n(\theta,\phi)\, d\Omega$$

$$\boxed{\textit{Beam area} \cong \Omega_A \cong \theta_{\mathrm{HP}}\phi_{\mathrm{HP}} \qquad (\mathrm{sr})}$$

$$\boxed{D = \frac{4\pi}{\theta_{HP}\phi_{HP}}}$$

$$4\pi steradians = 4\pi(180|\pi)^2 = 4\pi \times 3282.8064 = 41{,}253 \text{ square degrees} = 41{,}253$$

Therefore

$$\boxed{D = \frac{41{,}253}{\theta_{HP}\phi_{HP}} \cong \frac{40{,}000}{\theta_{HP}\phi_{HP}}}$$

$$\boxed{\textit{Directivity in dB} \qquad D = 10\log_{10} D}$$

GAIN: The gain of the antenna is defined as the ratio of the radiation intensity to the average total input power.

$$\boxed{\mathrm{G} = \frac{\text{radiation intensity}}{\text{average total input power}} = 4\pi\frac{U}{P_{in}}}$$

The average total input power is the ratio of total input power to 4π

Antenna Efficiency: The ratio of the gain to the directivity is the *antenna efficiency factor*. Thus, $G = kD$

where k = efficiency factor $(0 \le k \le 1)$, dimensionless.

In practice, G is always less than D.

$$\boxed{K = \frac{G}{D} = \frac{P_{rad}}{P_{in}} = \frac{\mathrm{I}_{rms}^2 R_r}{\mathrm{I}_{rms}^2 R_r + \mathrm{I}_{rms}^2 R_l} = \frac{R_r}{R_r + R_l}}$$

Resolution: The resolution of an antenna may be defined as equal to half the beamwidth between first nulls (FNBW)/2.

For example, an antenna whose pattern FNBW = 2° has a resolution of 1° and, accordingly, should be able to distinguish between transmitters on two adjacent satellites.

Half the beamwidth between first nulls is approximately equal to the half-power beamwidth (HPBW)

$$\boxed{\mathrm{HPBW} = \mathrm{FNBW}/2}$$

Thus antenna beam area is given by $\boxed{\Omega_A = \theta_{HP}\phi_{\mathrm{HP}} = \left(\frac{\mathrm{FNBW}}{2}\right)_\theta \left(\frac{\mathrm{FNBW}}{2}\right)_\phi}$

The N number of point sources of radiation distributed uniformly over the sky which the antenna can resolve is given approximately by $\boxed{N = \frac{4\pi}{\Omega_A}}$

But the directivity of the antenna is given by $\boxed{D = \frac{4\pi}{\Omega_A}}$

Hence we can write $\boxed{N = D}$

Therefore the ***directivity is equal to the number of point sources in the sky that the antenna can resolve***

EFFECTIVE APERTURE: Aperture of an Antenna is the area through which the power is radiated or received.The concept of aperture is most simply introduced by considering a receiving antenna. The effective aperture is ability of the antenna to extract power from the travelling EM waves.
The effective aperture is defined as the ratio of power received at the antenna terminal to the poynting vector of the the incident wave. Thus

$$\boxed{\text{effective aperture or area} = \frac{\text{power received}}{\text{poynting vector of incident wave}}}$$

$$\boxed{A_e = \frac{P}{S} \quad (m^2)}$$

Where P = power received at the antenna terminals
The effective aperture is always less than physical aperture.
Aperture efficiency: It is defined as the ratio of the effective aperture to physical aperture.

$\boxed{\varepsilon_{ap} = \frac{A_e}{A_p}}$ (Dimensionless)

For horn and parabolic reflector antenna, aperture efficiencies are commonly in the range of 50 to 80%

Effective Aperture and Beam area: Consider now an antenna with an *effective aperture Ae*, which radiates all of its power in a conical pattern of beam area Ω_A, as shown in fig. Assuming a uniform field *Ea* over the aperture, the power radiated is

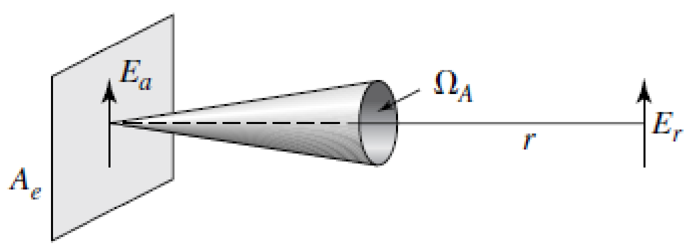


Fig . Radiation over beam area Ω_A from aperture A_e

$P = \frac{E_a^2}{\eta_o} A_e$ ------(1)

Assuming a uniform field *Er* in the far field at a distance *r* , the power radiated is also given by

$P = \frac{E_r^2}{\eta_o} r^2 \Omega_A$ ----(2)

Equating (1) and (2) and noting that *Er* = *Ea Ae/rλ* yields the aperture–beam-area relation

$\lambda^2 = A_e \Omega_A$ $\quad$ ***Aperture–beam – area relation*** ---(3)

We know that Directivity of the antenna as

$D = \frac{4\pi}{\Omega_A}$ ------(4)

From equations (3) and (4) $\boxed{D = \frac{4\pi}{\lambda^2} A_e \quad \textit{Directivity from aperture}}$

All antennas have an effective aperture which can be calculated or measured. Even the hypothetical, idealized isotropic antenna, for which $D = 1$, has an effective aperture

$$\boxed{A_e = D\frac{\lambda^2}{4\pi} = \frac{\lambda^2}{4\pi} = 0.0796\lambda^2}$$

EFFECTIVE HEIGHT: the effective height may be defined as the ratio of the induced voltage to the incident field

$$h_e = \frac{V}{E} \qquad (m)$$

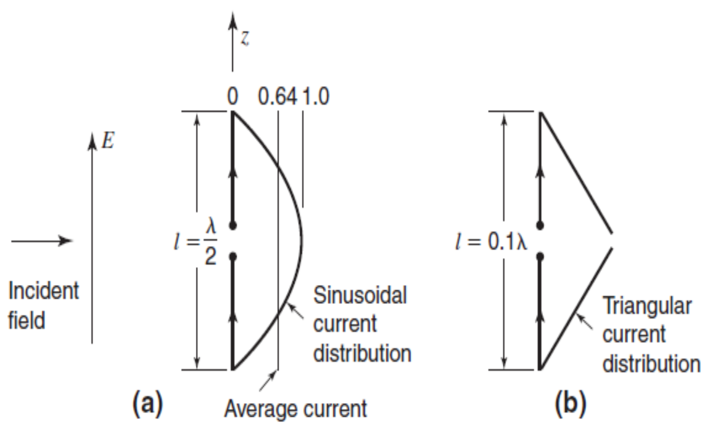


Fig (*a*) Dipole of length $l = \lambda/2$ with sinusoidal current distribution. (*b*) Dipole of length $l = 0.1\lambda$ with triangular current distribution.

Consider, for example, a vertical dipole of length $l = \lambda/2$ immersed in an incident field E, as in Fig. (*a*). If the current distribution of the dipole were uniform, its effective height would be l. The actual current distribution, however, is nearly sinusoidal with an *average value* $2/\pi = 0.64$ (of the maximum) so that its effective height $h = 0.64\, l$. If the length of thedipole is 0.1λng so that current distribution is triangular as in Fig. (*b*). The average current is 1/2 of the maximum so that the effective height is $0.5l$

<u>The relation between effective height and effective aperture</u> : For an antenna of radiation resistance Rr matched to its load, the power delivered to the load is equal to

$P = \frac{V^2}{4R_r} = \frac{h^2E^2}{4R_r}$ ----(1)

In terms of effective aperture the same power is given by

$P = SA_e = \frac{E^2A_e}{\eta_o}$ ----(2)

From above equations

$\boxed{h_e = 2\sqrt{\frac{R_r A_e}{\eta_o}}}$ $\boxed{A_e = \frac{h_e^2\eta_o}{4R_r}}$ $\boxed{D = \frac{4\pi}{\lambda^2}A_e = \frac{\pi h_e^2 \eta_o}{\lambda^2 R_r}}$

Radiation resistance: The radiation resistance is defined as the fictitious resistance which when connected in series with an antenna will consume the same power as it actually radiated by the antenna and is denoted by R_r

Power radiated $P_{rad} = I_{rms}^2 R_r$

Antenna Bandwidth: The antenna bandwidth is defined as the range of frequency over which an antenna maintains its certain characteristics such as gain ,radiation resistance ,polarization , FBR ,vswr and impedance to specified value.

Fields from oscillating dipole: Although a charge moving with uniform velocity along a straight conductor does not radiate, a charge moving back and forth in simple harmonic motion along the conductor is subject to acceleration (and deceleration) and radiates.

To illustrate radiation from a dipole antenna, let us consider a the dipole with two equal and opposite charges oscillating up and down in harmonic motion with instantaneous separation l (maximum separation $l0$) while focusing attention on the electric field. For clarity only a single electric field line is shown.

1. At time t = 0 the charges are at maximum separation and undergo maximum acceleration $\dot{v}$ as they reverse direction (Fig. *a*). At this instant the current *I* is zero.
2. At t=1/8 the charges are moving toward each other (Fig.)
3. At time t=1/4 they pass at the midpoint (Fig. *c*). As this happens, the field lines detach and new ones of opposite sign are formed. At this time the equivalent current *I* is a maximum and the charge acceleration is zero.
4. As time progresses to a 1/2, the fields continue to move out as in Fig. *d* and *e*.

In this way, electric field lines are detached or radiated into free space by an oscillating electric dipole .

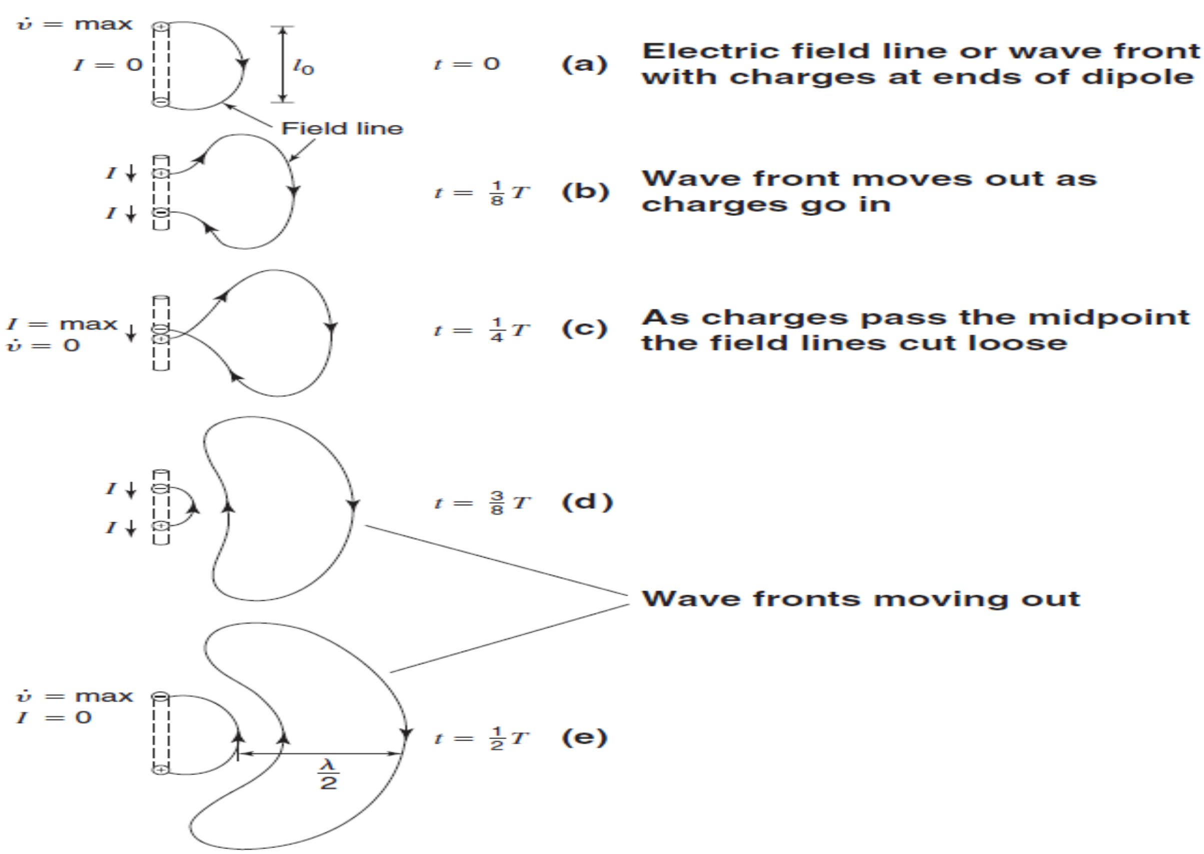


Fig Oscillating electric dipole consisting of two electric charges in simple harmonic motion, showing propagation of an electric field line and its detachment (radiation) from the dipole.

ANTENNA FIELD ZONES: The fields around an antenna may be divided into two principal regions

1. one near the antenna called the *near field* or *Fresnel zone*
2. other at a large distance called the *far field* or *Fraunhofer zone*

The boundary between the two may be arbitrarily taken to be at a radius $R = \frac{2L^2}{\lambda} \qquad (m)$

Where L = maximum dimension of the antenna, m

λ = wavelength, m

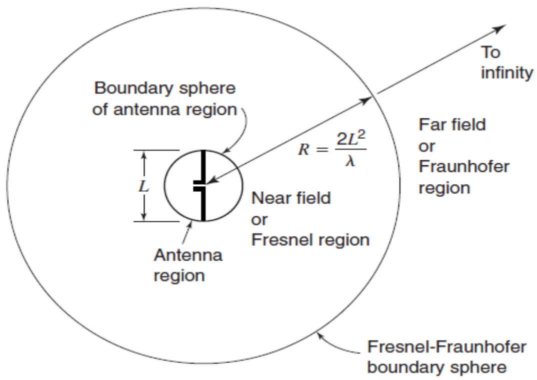


Fig . Antenna region, Fresnel region and Fraunhofer region.

In the far or Fraunhofer region, the measurable field components are transverse to the radial direction from the antenna and all power flowis directed radially outward. In the far field the shape of the field pattern is independent of the distance. In the near or Fresnel region, the longitudinal component of the electric field may be significant and power flowis not entirely radial. In the near field, the shape of the field pattern depends, in general, on the distance.

SHAPE-IMPEDANCE CONSIDERATIONS: In most of the cases , the quantitative behaviour of an antenna cab be guessed from its shape. This may be illustrated with the aid of Fig.

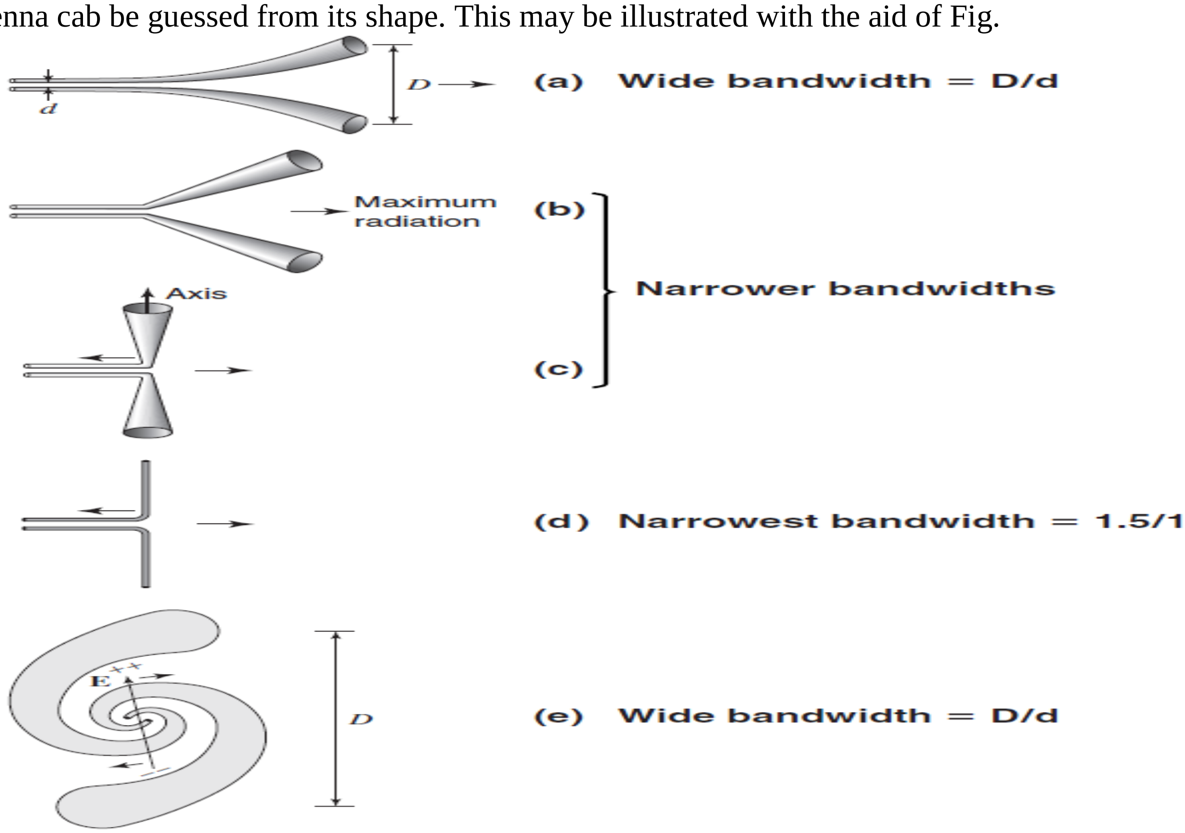


Fig . Evolution of a thin cylindrical antenna (*d*) from an opened-out twin line (*a*).Curving the conductors as in (*e*) results in the spiral antenna.

Starting with the opened-out two conductor transmission line of Fig. *a*, we find that, if extended far enough, a nearly constant impedance will be provided at the input (left) end for $d \ll \lambda$ and $D \geq \lambda$.

In Fig.*b* the curved conductors are straightened into regular cones and in Fig. c the cones are aligned collinearly, forming a biconical antenna.
In Fig. *d* the cones degenerate into straight wires.
In going from Fig.*a* to *d*, the bandwidth of relatively constant impedance tends to decrease.
Another difference is that the antennas of Fig. 2–19*a* and *b* are unidirectional with beams to the right, while the antennas of Fig. 2–19*c* and *d* are omnidirectional in the horizontal plane (perpendicular to the wire or cone axes).

A different modification is shown in Fig. 2*e*. Here the two conductors are curved more sharply and in opposite directions, resulting in a spiral antenna with maximum radiation broadside (perpendicular to the page) and with polarization which rotates clockwise. This antenna, just like the antenna in Fig. *a*, exhibits very broadband characteristics .
The dipole antennas of Fig. are balanced, i.e., they are fed by two-conductor , balanced transmission lines.

Antennas with large and abrupt discontinuities have large reflections and act as reflectionless transducers only over narrow frequency bands where the reflections cancel.

Antennas with discontinuities that are small and gradual have small reflections and are, in general, relatively reflectionless transducers over wide frequency bands.

<u>Polarization:</u> *Polarization of an antenna* in a given direction is defined as "the polarization of the wave transmitted or radiated by the antenna.
The polarization *of a radiated wave* is defined as the figure traced by the end point of the time-varying electric field vector at a fixed observation point in space.
Polarization may be classified as linear, circular, or elliptical.

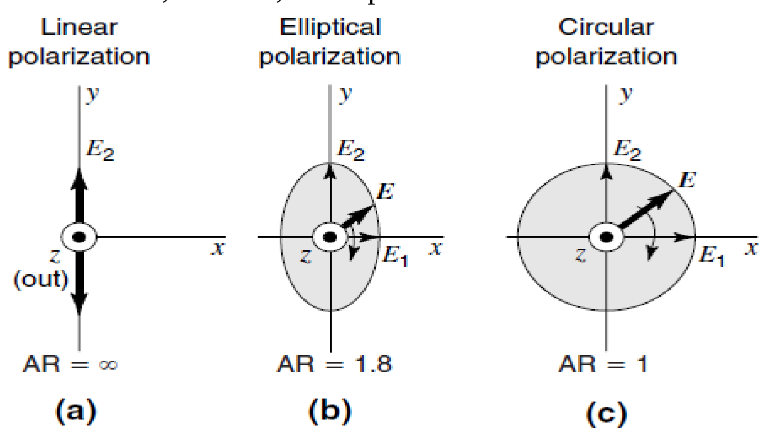


Fig. (*a*) Linear, (*b*) elliptical, and (*c*) circular polarization for left-circularly polarized wave

Consider a plane wave travelling along positive *z* direction as shown in figure.
As a function of time and position, the electric field is given by

$$E = E_1 \sin(\omega t - \beta Z)\, \bar{a}_x + E_2 \sin(\omega t - \beta Z + \delta)\, \overline{a_y}$$

When electric field vector at any point in the free space is the function of time and if it is directed always along the line , then the field is said to linearly polarized.

When the electric field vector $\bar{E}$ lies in the vertical plane , the wave is said to be vertically polarized wave ($E1 = 0$). Similarly when the electric field vector $\bar{E}$ lies in horizontal plane , the wave is said to be horizontally polarized wave($E2 = 0$). Note the orientation of the antenna and the polarizations are always same. That means

the vertical antenna produces vertically polarized wave and horizontal antenna produces horizontally polarized wave.
If $\delta = 0$ and $E1 = E2$, the wave is also linearly polarized but in a plane at an angle of 45◦ with respect to the x axis .

When the figure traced by the instantaneous electric field vector is circle, the field is said to be circularly polarized field. If $E1 = E2$ and $\delta = \pm 90^\circ$, the wave is circularly polarized. When $\delta = +90^\circ$, the wave is *left circularly polarized,* and when $\delta = -90^\circ$, the wave is *right circularly polarized*.
When the figure traced by the instantaneous electric field vector is ellipse , the field is said to be elliptically polarized field ($E_1 \neq E_2$ and for any δ)
The ratio of the major to minor axes of the polarization ellipse is called the ***Axial Ratio*** (AR). AR = $E2/E1$.
($1 \leq AR \leq \infty$)
Linear and circular polarizations are special cases of elliptical and they can be obtained when the ellipse becomes a straight line or a circle, respectively. *Clockwise* rotation of the electric-field vector is also designated as left-*hand polarization* and *counterclockwise* as right-*hand polarization*.

Antenna theorems:

Reciprocity Theorem: The reciprocity theorem is most powerful theorem in circuit and field tehories. The reciprocity theorem for antennas is stated as follows .
Statement: " If a current I_1 at the terminals of an antenna no.1 induces an emf E_{21} at the open terminals of an antenna no.2 and a current I_2 at terminals of an antenna no.2 induces an emf E_{12} at the open terminals of antenna no.1 , then $E_{12}=E_{21}$ provided $I_1=I_2$ "

The reciprocity is used to derive the following very important properties of transmitting and receiving antennas.

1. Antenna has identical impedance when it is used as transmitting or receiving purposes. This property is called equality of impedances.
2. Antenna has identical directional characteristics/patterns when it is used as transmitting or receiving purposes. This property is called equality of directional patterns.
3. Antenna has same effective length when it is used as transmitting or receiving purposes. This property is called equality of effective length.

Basic Maxwell's Equations:

Maxwell's equations can be written in differential and integral forms.

$$\nabla \times H = J + \partial D/\partial t \text{ (in general)},$$

$$\nabla \times H = \partial D/\partial t \text{ (if J = 0) and } \nabla \times H = J \text{ (for dc field)}$$

$$\nabla \times E = -\partial B/\partial t \text{ (in general) and } \nabla \times E = 0 \text{ (for static field)}$$

$$\nabla \cdot D = \rho \text{ (in general), and } \nabla \cdot D = 0 \text{ (for charge-free region, i.e., } \rho = 0)$$

$$\nabla \cdot B = 0$$

Retarded potential : Maxwell's equations can be written in differential and integral forms.

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \text{-------------------(1)}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{-------------------(2)}$$

$$\nabla \cdot \mathbf{D} = \rho \quad \text{--------------------(3)}$$

$$\nabla \cdot \mathbf{B} = 0 \quad \text{--------------------(4)}$$

From equation(4) it is clear that the divergence of **B** is zero. But from the vector identity "the divergence of a curl of a vector is zero"

This clearly indicates that to satisfy equation (4) **B** must be expressed as a curl of some vector. So defining vector potential **A** as $\mathbf{B} = \mu\mathbf{H} = \nabla \times \mathbf{A}$ **-------(5)**

Substituting in equation (2) we get

$\nabla \times \mathbf{E} = -\frac{\partial(\nabla\times\mathbf{A})}{\partial t}$

Interchanging the operators on R.H.S of the above equation, we get

$$\nabla \times \mathbf{E} = -\left(\nabla \times \frac{\partial \mathbf{A}}{\partial t}\right)$$

$$\nabla \times \mathbf{E} + \nabla \times \frac{\partial \mathbf{A}}{\partial t} = \mathbf{0}$$

$\nabla \times \left(\mathrm{E} + \frac{\partial \mathrm{A}}{\partial t}\right) = 0$ --------------(6)

According to vector identity "curl of a gradient of a scalar is always zero". So the equation (6) will satisfied only if the term $(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t})$ is defined as a gradient of a scalar. let us introduce a scalar V such that $\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla \cdot \mathbf{V}$

$\mathbf{E} = -\nabla\mathbf{V} - \frac{\partial \mathbf{A}}{\partial t}$ -----------(7)

Hence from equations (5) and (7),it is clear that the electric and magnetic fields E and H can be expressed interms of a scalar potential V and vector potential A.

Substituting the values of E and H from equations (5) and (7) respectively in equation (1) we get

$$\nabla \times \left(\frac{\mathbf{1}}{\mu}\nabla \times \mathbf{A}\right) = \mathbf{J} + \in \frac{\partial\left(-\nabla\mathbf{v} - \frac{\partial \mathbf{A}}{\partial t}\right)}{\partial t}$$

Interchanging the operators

$$\frac{\mathbf{1}}{\mu}(\nabla \times \nabla \times \mathbf{A}) = \mathbf{J} + \in \left(-\nabla\frac{\partial \mathbf{V}}{\partial t} - \frac{\partial^2 \mathbf{A}}{\partial t^2}\right)$$

$$\nabla \times \nabla \times \mathbf{A} = \mu\mathbf{J} - \mu \in \nabla\frac{\partial \mathbf{A}}{\partial t} - \mu \in \frac{\partial^2 \mathbf{A}}{\partial t^2}$$

From the vector identity

$$\nabla \times \nabla \times \mathrm{A} = \nabla(\nabla \cdot \mathrm{A}) - \nabla^2\mathrm{A}$$

$$\nabla(\nabla \cdot \mathrm{A}) - \nabla^2\mathrm{A} = \mu\mathbf{J} - \mu \in \nabla\frac{\partial \mathbf{A}}{\partial t} - \mu \in \frac{\partial^2 \mathbf{A}}{\partial t^2}$$

$\nabla^2\mathrm{A} - \nabla(\nabla \cdot \mathrm{A}) = \mu\mathbf{J} - \mu \in \nabla\frac{\partial \mathbf{A}}{\partial t} - \mu \in \frac{\partial^2 \mathbf{A}}{\partial t^2}$ **-----------(8)**

Substituting value of E from equation (7) in (3),we get

$$\nabla \cdot \left(-\nabla\mathbf{V} - \frac{\partial \mathbf{A}}{\partial t}\right) = \frac{\rho}{\in}$$

$\nabla^2\mathrm{V} + \nabla \cdot \frac{\partial \mathbf{A}}{\partial t} = -\frac{\rho}{\in}$ **----------(9)**

Equations (8) and (9) are the differential equations in which both unknowns A and V appear. Hence there equations are called coupled equations .

According to Helmholtz Theorem , *"A vector field is completely defined only when both its curl and divergence are known*".

According to Lorentz gauge condition and Coulomb's gauge condition

$$\nabla \cdot \mathbf{A} = -\mu\epsilon\frac{\partial \mathbf{V}}{\partial t}$$

$$\nabla \cdot \mathbf{V} = \mathbf{0}$$

Using the Lorentz gauge condition, (8) and (9) can be rewritten as

$\nabla^2 V = -\boldsymbol{\rho}/\varepsilon - \partial\,(\mu\varepsilon\partial V/\partial t)/\partial t = -\rho_{\mathrm{V}}/\varepsilon - \mu\varepsilon\,(\partial^2 V/\partial t^2)$ -------------(10)

$$\nabla^2 A = -\mu J + \mu\varepsilon\,(\partial^2 A/\partial t^2) \quad \text{----------------------------(10)}$$

The solutions of the above equations are given by

$$\mathbf{V(r,t)} = \frac{\mathbf{1}}{\mathbf{4\pi\epsilon}}\int_{\mathbf{v}} \frac{\boldsymbol{\rho}(\mathbf{r}',\mathbf{t}-\mathbf{R}/\boldsymbol{\upsilon})}{\mathbf{R}}\,\mathbf{dv} \quad \text{------------}\mathbf{(11)}$$

$$A(r,t) = \frac{\mu}{4\pi}\int_V \frac{J(r',t-R/\upsilon)}{R}\,dv \quad \text{------------(12)}$$

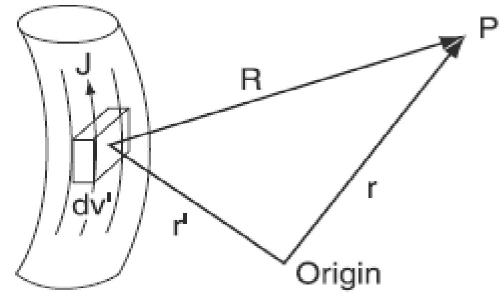


Fig . Retarded potential

In the above equations a time delay of R/υ seconds has been introduced , so that now the potentials have been delayed or retarded by this amount . For this reason they are retarded potentials. Similarly, advanced potential expression can be obtained by replacing t− R/υ by t+R/υ in equations (11) and (12).
Equation (12), the starting point for the study of radiation process, is rewritten in the following alternating form on replacing R by r

$$\mathbf{A(r)} = \frac{\boldsymbol{\mu}}{\mathbf{4\pi}}\int_{\mathbf{v}} \frac{\mathbf{J(t-r/\upsilon)}}{\mathbf{r}}\mathbf{dv}.$$

For sinusoidal time variation characterized by $e^{j\omega t}$

$$\nabla^2 V = -\rho_v/\varepsilon + \omega^2\mu\varepsilon V$$

$$\nabla^2 A = -\mu J + \omega^2\mu\varepsilon J$$

The corresponding retarded potentials are given by

$$V(r) = \frac{1}{4\pi\epsilon}\int \boldsymbol{\rho}(r)\frac{e^{-j\omega r}}{r}\,dv$$

$$A(r) = \frac{\mu}{4\pi}\int J(r)\frac{e^{-j\omega r}}{r}\,dv$$

<u>Radiation from Small Electric Dipole (or) infinitesimal dipole (or) current element (or) short electric dipole (or)Hertizian dipole:</u>

An infinitesimal linear dipole ($l \ll \lambda$ and $a \ll \lambda$) is positioned symmetrically at the origin of the coordinate system and oriented along the *z* axis, as shown in Figure

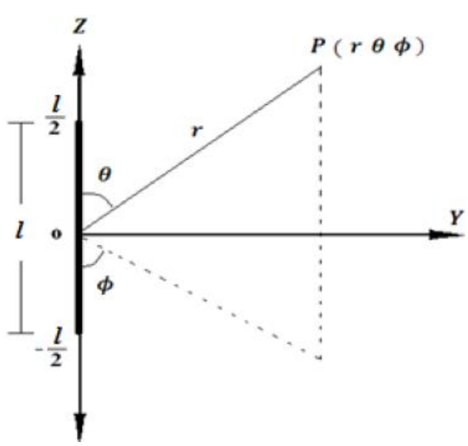


Fig (a) An infinitesimal dipole

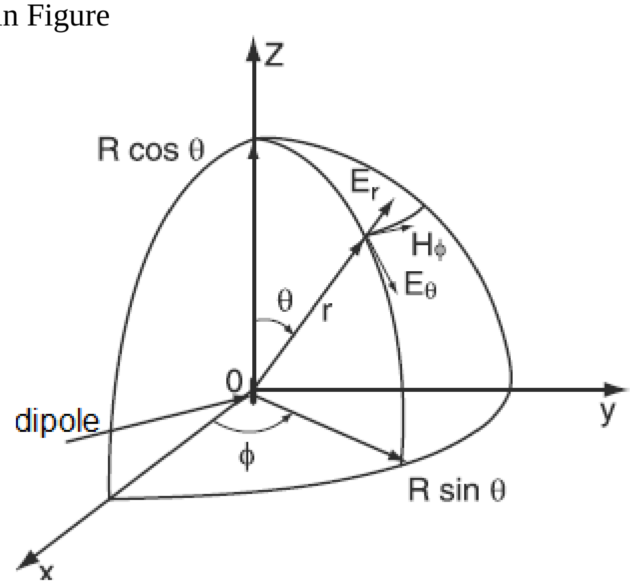


Fig (b) Electric-field components

As the dipole length is very small,The spatial variation of the current is assumed to be constant and given by

$\mathbf{I(x,y,z) = I_o \bar{a}_z}$

where **I0** = constant

Radiated Fields:

To find the fields radiated by the current element, the two-step procedure is used.

The vector magnetic potential *A* is given by

$$\boxed{\mathbf{A(x,y,z)} = \frac{\mu}{4\pi}\int \mathbf{I(x,y,z)}\frac{e^{-j\omega r}}{r}\mathbf{dl}} \quad \text{--------(1)}$$

As current element is along Z-axis $\mathrm{I(x,y,z)} = \mathbf{I_o \bar{a}_z}$ **and** dl=dz

Therefore $\boxed{\mathbf{A}(x,y,z) = \frac{\mu I_o}{4\pi r} e^{-j\beta r}\int_{l/2}^{1/2} dz\,\bar{a}_z = \frac{\mu I l_o}{4\pi r}e^{-j\beta r}\bar{a}_z}$ **------(2)**

The vector magnetic potential has only Z-component and remaining components are zeros

Taht is $A_x = 0 \; and \; A_y = 0$

A in spherical coordinate system is given by

$$A_r = A_z\cos\theta = \frac{\mu I l_o e^{-j\beta r}}{r}\cos\theta \quad \text{--------(3)}$$

$$A_\theta = -A_z\sin\theta = -\frac{\mu I l_o e^{-j\beta r}}{r}\sin\theta \quad \text{-----(4)}$$

$$A_\phi = 0 \quad \text{---------(5)}$$

We know that $B = \nabla \times A$**,** the components of$\nabla \times A$ are obtained in spherical as below.

$$(\nabla \times A)_r = \frac{1}{\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\phi - \frac{\partial A_\theta}{\partial\phi})\right] = B_r = 0$$

$$(\nabla \times A)_\theta = \left[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{1}{r}\frac{\partial(rA_\phi)}{\partial r}\right] = B_\theta = 0$$

$$(\nabla \times A)_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right] = B_\phi = \mu H_\phi$$

$$\mathbf{H_r = 0 \;\; \mathit{and}\;\; H_\theta = 0}$$

Substituting equations (3) to (5) in above equations, they reduce to

$$\boxed{\mathbf{H_r = H_\theta = 0}} \quad \text{-------}\mathbf{(6)}$$

$$\boxed{\mathbf{H_\phi = j\frac{\beta I_o l\sin\theta}{4\pi r}\left[1 + \frac{1}{j\beta r}\right]e^{-j\beta r}}} \quad \text{-------}\mathbf{(7)}$$

The electric field **E** can now be found by using the following equation

$$\mathbf{E = \frac{\nabla\times H}{j\omega\epsilon}} \quad \text{----------}\mathbf{(8)}$$

Substituting equations (6) and (7) into equation(8) ,it reduces to

$$\boxed{E_r = \eta\frac{I_o l\cos\theta}{2\pi r^2}\left[1 + \frac{1}{j\beta r}\right]e^{-j\beta r}} \quad \text{---------(9)}$$

$$\boxed{E_\theta = j\eta\frac{\beta I_o l\sin\theta}{4\pi r}\left[1 + \frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right]e^{-j\beta r}} \text{----(10)}$$

$$\boxed{E_\phi = 0} \quad \text{---------------------------------(11)}$$

The expressions of $E_\theta, E_r \; and \; H_\phi$ given above are involved three types of terms, which represent three different types of fields. These are noted below:

1. The terms which are inversely proportional to r represent radiation (distant or far) field and are involved in the expressions of E_θ and H_ϕ.

2. The terms inversely proportional to r^2 represent induction or near field. Such terms are involved in all the field components, i.e., in $E_\theta, E_r \; and \; H_\phi$.

3. The terms inversely proportional to r^3 represent electrostatic field. Such terms are involved in the expressions of E_θ *and* E_r

Note: The distance at which near and far field equal is given $\beta r \cong 1$ or $r = \frac{\lambda}{6}$. If $r < \frac{\lambda}{6}$, the field is near field and if $r > \frac{\lambda}{6}$, field is far field. Where $\beta = \frac{2\pi}{\lambda}$

The far field ($\beta r \gg 1$) components are given by

$$\boxed{E_\theta \cong j\eta \frac{\beta I_o l e^{-j\beta r}}{4\pi r} \sin\theta} \qquad \boxed{E_r = E_\phi = H_r = H_\theta = 0} \qquad \boxed{H_\phi \cong j \frac{\beta I_o l e^{-j\beta r}}{4\pi r} \sin\theta}$$

Power Density and Radiation Resistance:

The Poynting vector is formed in terms of the **E**- and **H**-fields radiated by the antenna. By integrating the Poynting vector over a closed surface (usually a sphere of constant radius), the total power radiated by the source is found.

the average power density is given by

$$P_{avg} = \frac{1}{2} Re(E \times H^*) = \frac{1}{2\eta} |E_\theta|^2 \overline{a_r} = \frac{\eta}{2} \left|\frac{\beta l I_o}{4\pi}\right|^2 \frac{\sin^2\theta}{r^2} \overline{a_r}$$

Power radiated is given by

$$\mathbf{P}_{rad} = \iint P_{avg}\, \boldsymbol{ds}$$

$$\mathbf{P}_{rad} = \int_0^{2\pi} \int_0^{\pi} \frac{\eta}{2} \left|\frac{\beta l I_o}{4\pi}\right|^2 \frac{\sin^2\theta}{r^2}\, r^2 \sin\theta\, \mathbf{d\theta d\phi}$$

$$\mathbf{P}_{rad} = \int_0^{2\pi} \int_0^{\pi} \frac{\eta}{2} \left|\frac{\beta l I_o}{4\pi}\right|^2 \sin^2\theta\, \mathbf{d\theta d\phi}$$

The power radiated is given by

$$\boxed{P_{rad} = \eta \left(\frac{\pi}{3}\right) \left|\frac{I_o l}{\lambda}\right|^2}$$

We know that power radiated in terms of radiation resistance is given by $P_{rad} = \frac{1}{2} |I_o|^2 R_r$

Therefore $\quad P_{rad} = \eta \left(\frac{\pi}{3}\right) \left|\frac{I_o l}{\lambda}\right|^2 = \frac{1}{2} |I_o|^2 R_r$

The radiation resistance is given by

$$\boxed{R_r = \eta \left(\frac{2\pi}{3}\right) \left(\frac{l}{\lambda}\right)^2 = 80\pi^2 \left(\frac{l}{\lambda}\right)^2}$$

Field patterns of the Hertzian or infinitesimal dipole are shown below fig

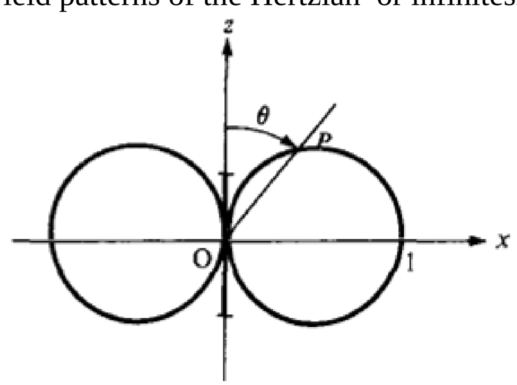


Fig (a) E-plane pattern

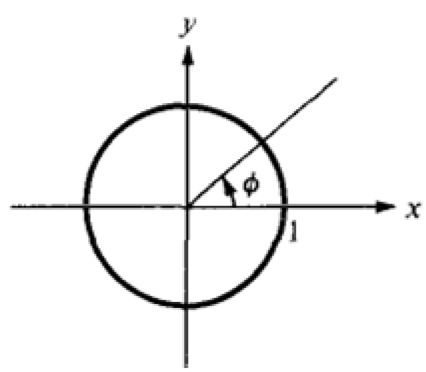


Fig (b) H-plane pattern

Directivity and effective aperture:

The average power density of An infinitesimal dipole is given by

$$\boxed{\mathrm{P}_{avg} = \frac{1}{2} Re(\mathrm{E} \times \mathrm{H}^*) = \frac{1}{2\eta} |\mathrm{E}_\theta|^2 \overline{\mathrm{a}_r} = \frac{\eta}{2} \left|\frac{\beta l \mathrm{I_o}}{4\pi}\right|^2 \frac{\sin^2\theta}{\mathrm{r}^2} \overline{\mathrm{a}_r}}$$

The radiation intensity U which is given by

$$\mathrm{U} = \mathrm{r}^2 \mathrm{P}_{avg} = \frac{\eta}{2} \left|\frac{\beta l \mathrm{I_o}}{4\pi}\right|^2$$

The maximum value occurs at $\theta = \pi/2$ and it is equal to

$$\mathbf{U}_{max} = \frac{\eta}{2} \left|\frac{\beta l \mathrm{I_o}}{4\pi}\right|^2$$

the directivity is given by $\boxed{\mathrm{D}_o = 4\pi \frac{\mathrm{U}_{max}}{\mathrm{P}_{rad}} = \frac{3}{2} = 1.5 \; or \; 1.76dB}$

The maximum effective aperture is given by $\boxed{\mathrm{A}_e = \left(\frac{\lambda^2}{4\pi}\right) \mathrm{D}_o = \frac{3\lambda^2}{8\pi}}$

The Half-wave Dipole and Quarter-wave monopole:

One of the most commonly used antennas is the half-wave dipole with length one half of the free space wavelength of the radiated wave i.e (L= $\lambda/2$). It gives a current distribution which is approximately sinusoidal with maximum at centre and zero at ends. It is a vertical antenna fed in the centre. It produces maximum radiation in the plane normal to the axis .

The vertical antenna of height $\mathrm{H} = \frac{\mathrm{L}}{2} = \frac{\lambda}{4}$ fed against an infinitely perfectly conducting plane has the same radiation characteristics above the plane as does the half wave dipole antenna in free space. This vertical antenna is reffered to Quarter-wave monopole.

Radiation from the Half-wave Dipole and Quarter-wave monopole: The sinusoidal current distribution is given by **I = Im sin β(H − z) for z > 0 and I = Im sin β(H + z) for z < 0 ----------(1)**

The field is to be obtained at a point which is so distantly located that the distances r and R (shown in the Fig. 4–7) can be considered to bear the following relation.

R=r for the estimation of amplitude and $\mathrm{R} = \mathrm{r} - \mathrm{z}\cos\theta$ for the estimation of phase.-------- (2)

Since the current in the dipole is in the z-direction, the z component of the differential vector magnetic potential is

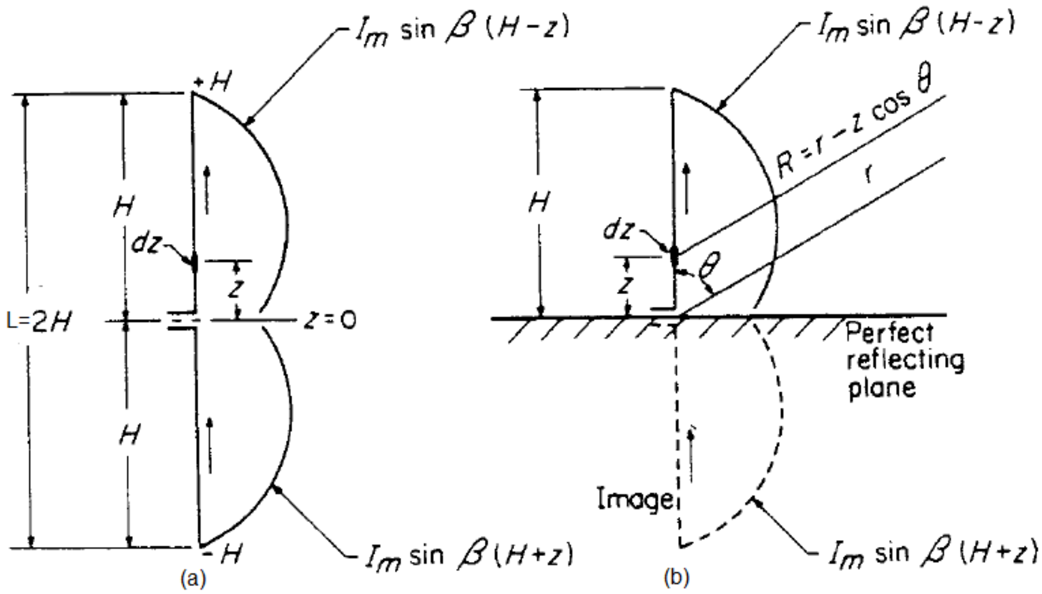


Fig. Half-wave dipole or quarter-wave monopole with assumed sinusoidal current distribution.

$$dA_z = \frac{\mu I dz}{4\pi R} e^{-j\beta R} \tag{3}$$

$$A_z = \frac{\mu}{4\pi} \int_{-H}^{0} \frac{I_m \sin\beta(H+z)}{R} e^{-j\beta R} dz + \frac{\mu}{4\pi} \int_{0}^{H} \frac{I_m \sin\beta(H-z)}{R} e^{-j\beta R} dz$$

$$= \frac{\mu I_m}{4\pi r} e^{-j\beta r} \left[\int_{-H}^{0} \sin\beta(H+z) e^{j\beta z\cos\theta} dz + \int_{0}^{H} \sin\beta(H-z) e^{j\beta z\cos\theta} dz \right] \tag{4}$$

For $H = \lambda/4$, $\beta H = \pi/2$, $\sin\beta(H+z) = \sin\beta(H-z)$ and $\sin(\pi/2 + \beta z) = \sin(\pi/2 - \beta z) = \cos\beta z$

$$A_z = \frac{\mu I_m}{4\pi r} e^{-j\beta r} \int_{0}^{H} \cos\beta z (e^{j\beta z\cos\theta} + e^{-j\beta z\cos\theta}) dz$$

$$= \frac{\mu I_m}{4\pi r} e^{-j\beta r} 2 \int_{0}^{H} \cos\beta z \cos(\beta z\cos\theta) dz$$

$$= \frac{\mu I_m}{4\pi r} e^{-j\beta r} 2 \int_{0}^{H} [\cos\beta z(1+\cos\theta) + \cos\beta z(1-\cos\theta)] dz$$

$$= \frac{\mu I_m}{4\pi r} e^{-j\beta r} 2 \left[\frac{\sin\beta z(1+\cos\theta)}{\beta(1+\cos\theta)} + \frac{\sin\beta z(1-\cos\theta)}{\beta(1-\cos\theta)} \right]_0^{\lambda/4}$$

$$= \frac{\mu I_m}{4\pi\beta r} e^{-j\beta r} \left[\left\{ (1-\cos\theta)\cos\left(\frac{\pi}{2}\cos\theta\right) + (1+\cos\theta)\cos\left(\frac{\pi}{2}\cos\theta\right) \right\} \frac{1}{\sin^2\theta} \right]$$

$$= \frac{\mu I_m}{2\pi\beta r} e^{-j\beta r} \left[\frac{\cos\{(\pi/2)\cos\theta\}}{\sin^2\theta} \right] \tag{5}$$

If $J = J_z$ and $B = \nabla \times A = B_\phi a_\phi$ only, thus $\mu H_\phi = -\frac{\partial A_z}{\partial r}\sin\theta$

$$H_\phi = \frac{j I_m e^{-j\beta r}}{2\pi r} \left[\frac{\cos\{(\pi/2)\cos\theta\}}{\sin\theta} \right] \tag{6}$$

$$E_\theta = \eta H_\phi = 120\pi H_\phi = \frac{j60 I_m e^{-j\beta r}}{r} \left[\frac{\cos\{(\pi/2)\cos\theta\}}{\sin\theta} \right] \tag{7}$$

The magnitudes of E_θ and H_ϕ are

$$|E_\theta| = \frac{60 I_m}{r} \left[\frac{\cos\{(\pi/2)\cos\theta\}}{\sin\theta} \right] \tag{8}$$

$$|H_\phi| = \frac{I_m}{2\pi r} \left[\frac{\cos\{(\pi/2)\cos\theta\}}{\sin\theta} \right] \tag{9}$$

$$P_{av} = |E_\theta||H_\phi| = \frac{\eta I_m^2}{8\pi^2 r^2} \left[\frac{\cos^2\{(\pi/2)\cos\theta\}}{\sin^2\theta} \right] \tag{10}$$

Note; Average poynting vector is half of the product of the peak values of E_θ *and* H_ϕ as in equ(10)
The total power radiated through a hemispherical surface of radius is r is given by

$$P = \frac{\eta I_m^2}{4\pi} \int_{0}^{\pi/2} \left[\frac{\cos^2\{(\pi/2)\cos\theta\}}{\sin^2\theta} \right] d\theta$$

The evaluation of above equation gives as below

$$P = \frac{0.609\eta I_m^2}{4\pi} = \frac{0.609\eta I_{eff}^2}{2\pi} = 36.5 I_{eff}^2 = I_{eff}^2 R_{rad}$$

where $R_{rad} = 36.5\Omega$ is the radiation resistance of a quarter-wave monopole.
For the half wave dipole antenna in free space ,the power would be radiated through a complete spherical surface.
Therefore ,for the same current the power radiated would be twice that of the monople.
Therefore The radiation resistance of a half-wave dipole is twice that of the monopole, i.e. $R_{rad} = 73\Omega$

Directivity of half wave dipole: Directivity is given by $D_o = 4\pi \frac{U_{max}}{P_{rad}}$

the time-average power density and radiation intensity can be written, respectively, as

$$P_{avg} = \frac{1}{2} Re(E \times H^*) = \frac{1}{2\eta} |E_\theta|^2$$

$$P_{avg} = \eta \frac{|I_m|^2}{8\pi^2 r^2} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right]^2 \simeq \eta \frac{|I_m|^2}{8\pi^2 r^2} \sin^3\theta$$

$$U = r^2 P_{av} \simeq \eta \frac{|I_m|^2}{8\pi^2} \sin^3\theta$$

$$U_{max} \simeq \eta \frac{|I_m|^2}{8\pi^2}$$

The power radiated by half wave dipole is twice taht of monopole and is given by

$$P_{rad} = \frac{0.609 \eta {I_m}^2}{2\pi}$$

Directivity is given by $\boxed{D_o = 4\pi \frac{U_{max}}{P_{rad}} \simeq 1.643}$

The corresponding maximum effective area is equal to

$$\boxed{A_e = \left(\frac{\lambda^2}{4\pi}\right) D_o = \left(\frac{\lambda^2}{4\pi}\right)(1.643) \simeq 0.13\lambda^2}$$

The far-field patterns of half wave diole are shown in below fig

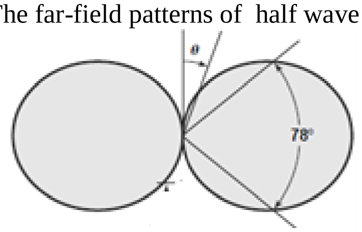


Fig (a) E-plane pattern

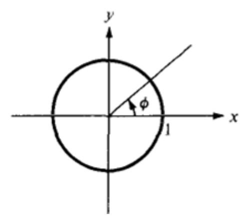


Fig (b) H-plane pattern

Natural current distributions, far fields and patterns of Thin Linear Centre-fed Antennas of different lengths:

When the antenna diameter is less than $\frac{\lambda}{100}$, the antenna is called as thin linear antenna.The antennas are symmetrically fed at the center by a balanced two-wire transmission line. The antennas may be of any length, but it is assumed that the current distribution is sinusoidal.

Examples of the approximate natural-current distributions on a number of thin, linear center-fed antennas of different length are illustrated in Fig

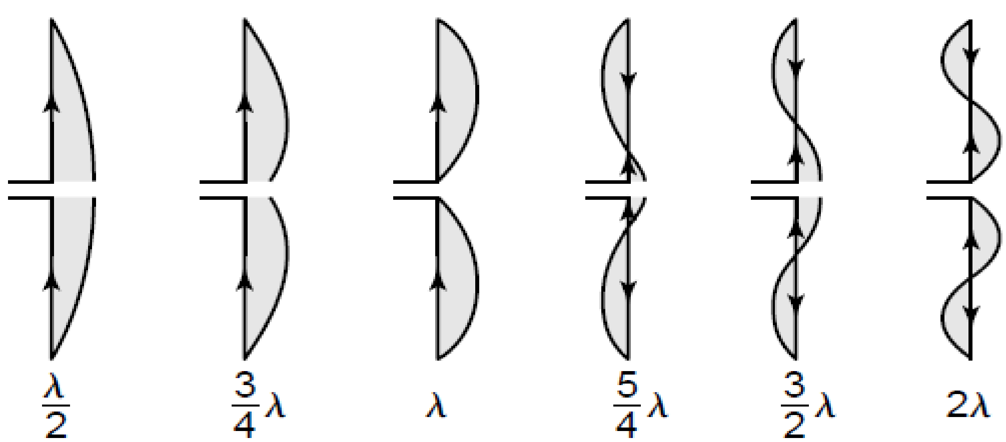


Fig . Approximate natural-current distribution for thin, linear, center-fed antennas of various lengths.

The retarded value of the current at any point z on the antenna referred to a point at a distance s is

$$[I] = I_0 \sin\left[\frac{2\pi}{\lambda}\left(\frac{L}{2} \pm z\right)\right] e^{j\omega[t-(r/c)]}$$

The far-fields of thin linear center-fed antennas of any length are given by

Far fields of center-fed dipole

$$H_\phi = \frac{j[I_0]}{2\pi r}\left[\frac{\cos[(\beta L\cos\theta)/2] - \cos(\beta L/2)}{\sin\theta}\right]$$

$$E_\theta = \frac{j60[I_0]}{r}\left[\frac{\cos[(\beta L\cos\theta)/2] - \cos(\beta L/2)}{\sin\theta}\right]$$

The far-field patterns of thin linear center-fed three different antennas are shown in below figures

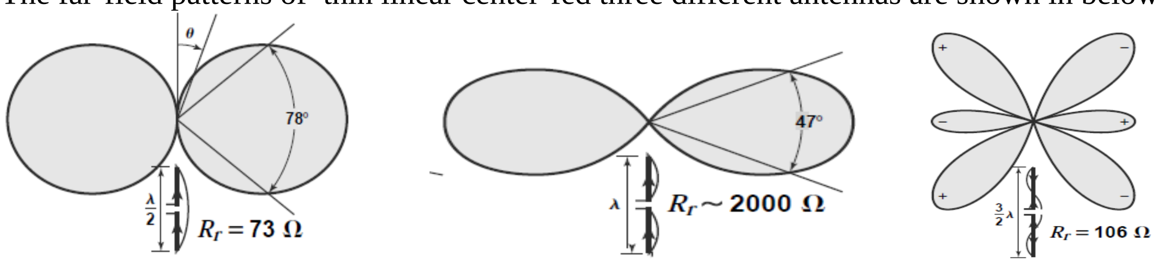


Fig .$\frac{\lambda}{2}$ Antenna Fig .λ Antenna Fig $3\lambda/2$ Antenna

Fig . polar plots of the far field patterns of $\lambda/2$, λ, and $3\lambda/2$ antennas.

Antenna(equivalent noise) temperature:

It is the fictitious temperature at the input of the antenna, which would account for noise ΔN at the out . ΔN is the additional noise introduced by the antenna itself.

$$F = 1 + \frac{T_e}{T_o}$$

T_o =290k

Antenna input impedance: or Draw Thevenin's equivalent circuit of an antenna in transmitting mode and power delivered to the anteena for radiation in terms of circuit parameters:

Antenna Input impedance is defined as "the impedance presented by an antenna at its terminals or the ratio of the voltage to current at a pair of terminals"

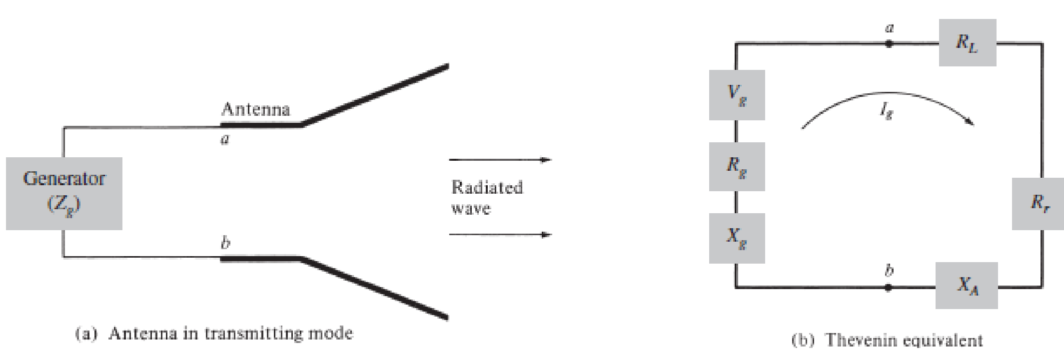
ZA = RA + jXA where RA = Rr + RL

Rr = radiation resistance of the antenna

RL = loss resistance of the antenna

If we assume that the antenna is attached to a generator with internal impedance

Zg = Rg + jXg



The current developed within the loop which is given by

$$I_g = \frac{V_g}{Z_t} = \frac{V_g}{Z_A + Z_g} = \frac{V_g}{(R_r + R_L + R_g) + j(X_A + X_g)}$$

and its magnitude by

$$|I_g| = \frac{|V_g|}{[(R_r + R_L + R_g)^2 + (X_A + X_g)^2]^{1/2}}$$

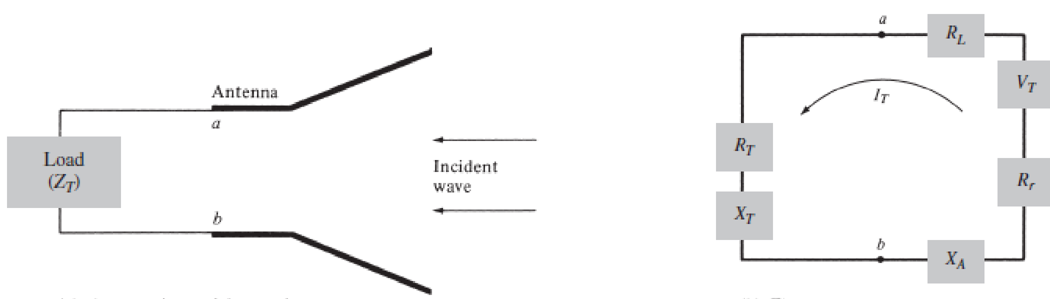
The power delivered to the antenna for radiation is given by

$$P_r = \frac{1}{2}|I_g|^2 R_r = \frac{|V_g|^2}{2}\left[\frac{R_r}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2}\right]$$

The maximum power delivered to the antenna occurs when we have conjugate matching; that is when $R_r + R_L = R_g$ and $X_A = -X_g$

$$P_r = \frac{|V_g|^2}{2}\left[\frac{R_r}{4(R_r + R_L)^2}\right] = \frac{|V_g|^2}{8}\left[\frac{R_r}{(R_r + R_L)^2}\right]$$

Antenna and its equivalent circuits in the receiving mode.



(a) Antenna in receiving mode (b) Thevenin equivalent

Half power beamwidth of short :

Normalized power pattern of short or infinitesimal dipole is $P_n = [E_n]^2 = sin^2\theta$

At HPBW P_n=1/2=0.5 therefore θ=45 degrees

HPBW=2θ=90 degrees

Half power beamwidth of half wave dipole antenna:

Normalized power pattern of half wave dipole is $P_n = [E_n]^2 = \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2 \cong sin^3\theta$

At HPBW P_n=1/2=0.5 therefore θ=39 degrees

HPBW=2θ=78 degrees