UNIT-5

DIGITAL MODULATION SCHEMES & INFORMATION THEORY

INTRODUCTION:

As baseband transmission is transmission of encoded signal using its own band of frequencies without any shift to higher frequencies which is limited to wired communication and short distances only.

Where pass band transmission is done by shifting baseband frequencies to high frequencies using modulation which can be used for long distances via microwave or satellite links.

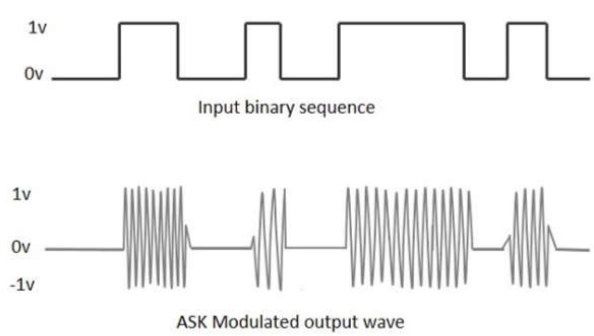
The process of shifting baseband signal to pass band range for transmission is known as modulation which involves switching (keying) the amplitude, frequency or phase of the sinusoidal carrier in accordance with the incoming data. Digital Modulation provides more information capacity, high data security, quicker system availability with great quality communication. Hence, digital modulation techniques have a greater demand, for their capacity to convey larger amounts of data than analog modulation techniques.

Based on that there are three basic signaling schemes:

1. Amplitude shift keying (ASK)
2. Frequency shift keying (FSK)
3. Phase shift keying (PSK)

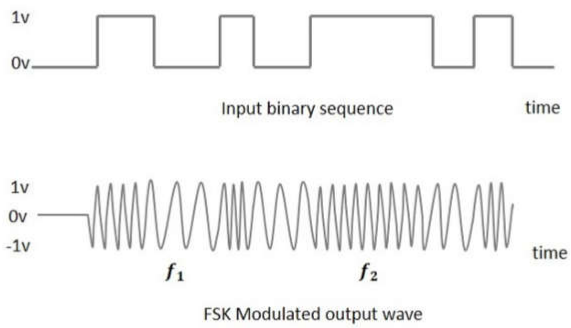
Amplitude shift keying:

Amplitude Shift Keying (ASK) is a type of Amplitude Modulation which represents the binary data in the form of variations in the amplitude of a signal. The binary signal when ASK modulated, gives a zero value for Low input while it gives the carrier output for High input.



Frequency shift keying:

Frequency Shift Keying (FSK) is the digital modulation technique in which the frequency of the carrier signal varies according to the digital signal changes. FSK is a scheme of frequency modulation. The output of a FSK modulated wave is high in frequency for a binary High input and is low in frequency for a binary Low input. The binary 1s and 0s are called Mark and Space frequencies.



Phase shift keying:

Phase Shift Keying (PSK) is the digital modulation technique in which the phase of the carrier signal is changed by varying the sine and cosine inputs at a particular time.

PSK is of two types, depending upon the phases the signal gets shifted. They are:

- Binary phase shift keying (BPSK):

  This is also called as 2-phase PSK or Phase Reversal Keying. In this technique, the sine wave carrier takes two phase reversals such as 0° and 180°.

- Quadrature phase shift keying (QPSK):

  In this phase shift keying technique, in which the sine wave carrier takes four phase reversals such as 0°, 90°, 180°, and 270°.

Except ASK both FSK and PSK have constant envelope. Whenever we are transmitting over pass band transmission channels PSK & FSK are not effected by amplitude non-linearity's so that PSK & FSK are more preferable than ASK for pass band transmission over nonlinear channels.

DIGITAL MODULATION TECHNIQUES:

Depending on whether the receiver is phase recovery circuit or not digital modulation techniques are classified into two types

1. Coherent phase shift keying
2. Non-coherent phase shift keying

In coherent modulation techniques, the receiver will have phase recovery circuit with local oscillator, generates the carrier wave which is synchronized with the carrier wave. Which is actually used to modulate the incoming data. We have two modulation techniques based on this type of detection. They are

1. Coherent phase shift keying
2. Coherent frequency shift keying

In Non-coherent modulation technique, the receiver does not contain phase recovery circuit. In this type receiver will recover the carrier which is same as the carrier used in transmitter. Based on this type of detection. They are

1. Non-coherent binary frequency shift keying
2. Differential phase shift keying

BINARY AMPLITUDE-SHIFT KEYING:

Binary amplitude-shift keying (BASK) is one of the earliest forms of digital modulation used in radio telegraphy at the beginning of the twentieth century. To formally describe BASK, consider a binary data stream which is of the ON–OFF signaling variety. That is, b(t) is defined by

$$b(t) = \begin{cases} \sqrt{E_b}, & \text{for binary symbol 1} \\ 0, & \text{for binary symbol 0} \end{cases} \qquad ----1$$

Then, multiplying b(t) by the sinusoidal carrier wave with the phase $\emptyset_c$ set equal to zero for convenience of presentation, we get the BASK wave

$$S(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c\, t), & for\ symbol\ 1 \\ 0, & for\ symbol\ 0 \end{cases} \qquad ----2$$

The carrier frequency f_c may have an arbitrary value, consistent with transmitting the modulated signal anywhere in the electromagnetic radio spectrum, so long as it satisfies the band-pass assumption.

When a bit duration is occupied by symbol 1, the transmitted signal energy is E_b When the bit duration is occupied by symbol 0, the transmitted signal energy is zero. On this basis, we may express the average transmitted signal energy as

$$E_{av} = \frac{E_b}{2} \qquad ----3$$

For this formula to hold, however, the two binary symbols must be *equiprobable*. In other words, if we are given a long binary data stream, then symbols 1 and 0 occur in essentially equal numbers in that data stream.

Signal space diagram of ASK:

The ASK waveform of symbol '1' is represented by

$$S(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c\, t), & for\ symbol\ 1 \\ 0, & for\ symbol\ 0 \end{cases}$$

Where $\Phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c\, t)$

$$\textbf{i.e. } \boldsymbol{S(t) = \sqrt{E_b}\,\Phi_1(t)}$$

Thus there is only one carrier function $\Phi_1(t)$. The signal space diagram will have two points on $\Phi_1(t)$. One will be zero and other will be at $\sqrt{E_b}$.

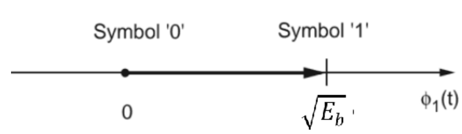


Fig. Signal space diagram of ASK

GENERATION AND DETECTION OF ASK SIGNALS:

From Eqs. 1 and 2, we readily see that a BASK signal is readily generated by using a product modulator with two inputs. One input, the ON–OFF signal of Eq. (1), is the modulating signal. The sinusoidal carrier wave

$$\Phi_1(t) = \sqrt{\frac{2}{T_b}}\, cos(2\pi f_c t) \text{ supplies the other input.}$$

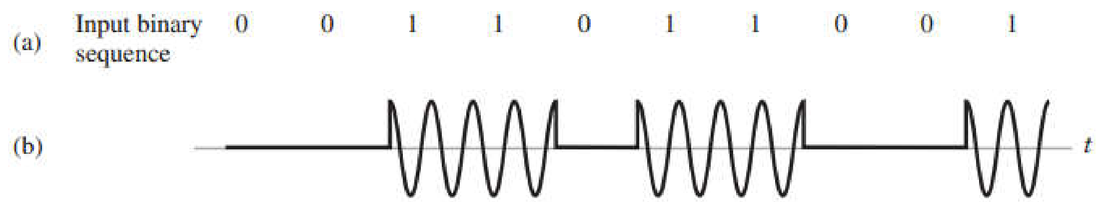


FIGURE 1 The three basic forms of signaling binary information. (*a*) Binary data stream. (*b*) Amplitude-shift keying.

A property of BASK that is immediately apparent from Fig. 1(*b*), which depicts the BASK waveform corresponding to the incoming binary data stream of Fig. 1(*a*), is the *non-constancy of the envelope* of the modulated wave. Accordingly, insofar as detection of the BASK wave is concerned, the simplest way is to use an envelope detector, exploiting the non-constant-envelope property of the BASK signal.

ASK Generator & Detector:

The below figure shows the ASK generator. The input binary sequence is applied to the product modulator. The product modulator amplitude modulates the sinusoidal carrier. It passes the carrier when input bit is '1'. It blocks the carrier (i.e. zero output) when input bit is '0'.

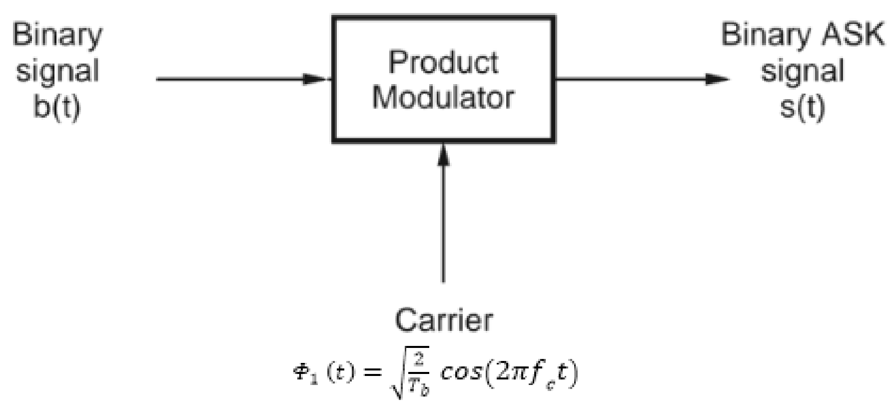


Fig. Block diagram of ASK generator

The below figure shows the block diagram of coherent ASK detector. The ASK signal is applied to the correlator consisting of multiplier and integrator. The locally generated coherent carrier is applied to the multiplier. The output of multiplier is integrated over one bit period. The decision device takes the decision at the end of every bit period. It compares the output of integrator with the threshold. Decision is taken in favor of '1' when threshold is exceeded. Decision is taken as '0' if threshold is not exceeded.

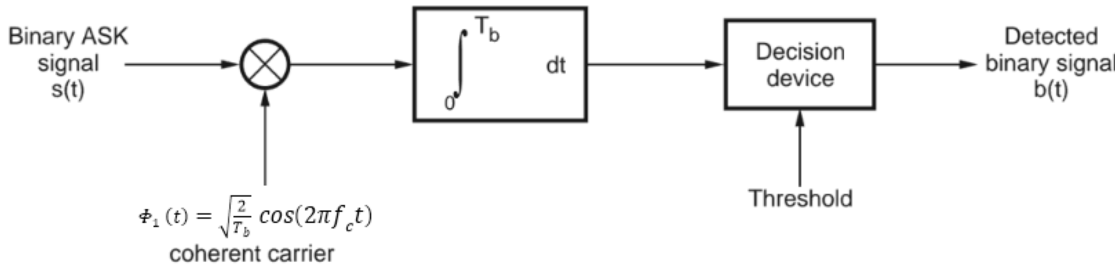


Fig. Block diagram of coherent ASK detector

COHERENT PHASE SHIFT KEYING:

In this section we focus on coherent phase shift keying by considering binary PSK, QPSK.

BINARY PHASE SHIFT KEYING (COHERENT BPSK):

In coherent binary PSK system, the pair of signals $S_1(t)$ and $S_2(t)$ are used to represent binary symbols 1 and o respectively, they are defined as

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t \qquad (1)$$

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}}(\cos 2\pi f_c t + \pi)$$

$$= -\sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t \qquad (2)$$

Where $0 \le t \le T_b$ & E_b is the transmitted signal energy per bit

$$\text{Carrier frequency } f_c = \frac{n_c}{T_b}$$

n_c = integral of no of cycle for carrier wave in one bit duration.

A pair of sinusoidal waves that differ only in a relative phase shift of 180 degrees shown in equations (1) & (2) are referred as Antipodal signals.

From eq's (1) & (2), it is clear that coherent BPSK have only one basis function of unit energy, defined as

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos 2\pi f_c t \qquad 0 \le t \le T_b \qquad (3)$$

$$\left[\because \phi_1(t) = \frac{S_1(t)}{\sqrt{E_b}}\right]$$

Then the transmitted signals $S_1(t)$ and $S_2(t)$ can be written interms of $\phi_1(t)$ as

$$(1) \Rightarrow \quad S_1(t) = \sqrt{E_b}\phi_1(t); \quad 0 \le t \le T_b \qquad (4)$$

$$S_2(t) = -\sqrt{E_b}\phi_1(t); \quad 0 \le t \le T_b \qquad (5)$$

From above eq's coherent BPSK signal space diagram is one (N=1) dimensional, consisting of two message points (N=2). The coordinates of the message points are

$$s_{11} = \int_0^{T_b} s_1(t)\phi_1(t)dt = +\sqrt{E_b} \qquad (6)$$

$$s_{21} = \int_0^{T_b} s_2(t)\phi_1(t)dt = -\sqrt{E_b} \qquad (7)$$

The message point corresponding to $S_1(t)$ is located at $S_{11} = \sqrt{E_b}$ and the message point corresponding to $S_2(t)$ is located at $S_{22} = -\sqrt{E_b}$. It is shown in below figure

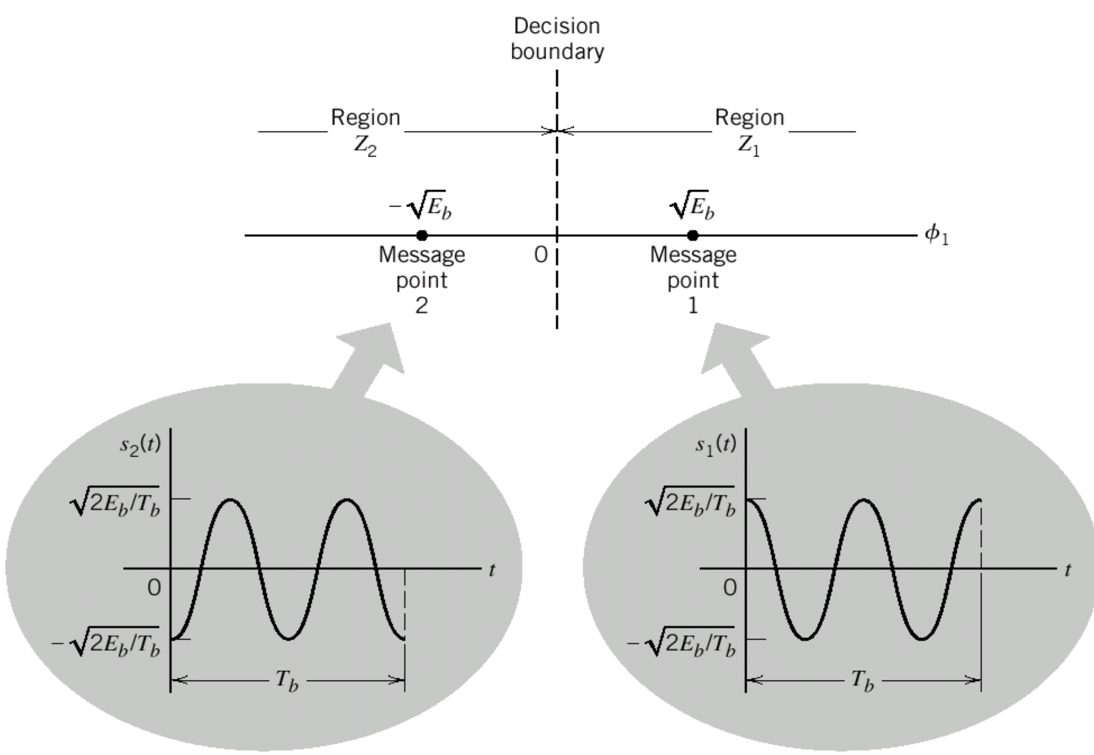


Fig 2: Signal space diagram for coherent BPSK

GENERATION AND DETECTION OF COHERENT BPSK:

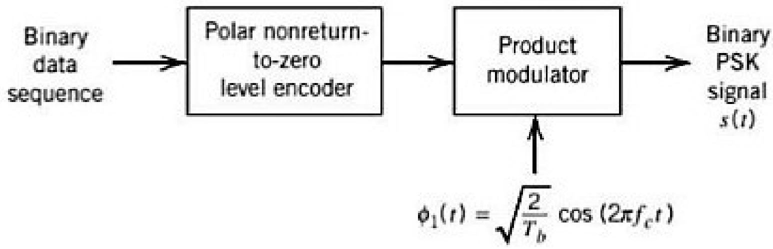


Fig: (a) BPSK Transmitter

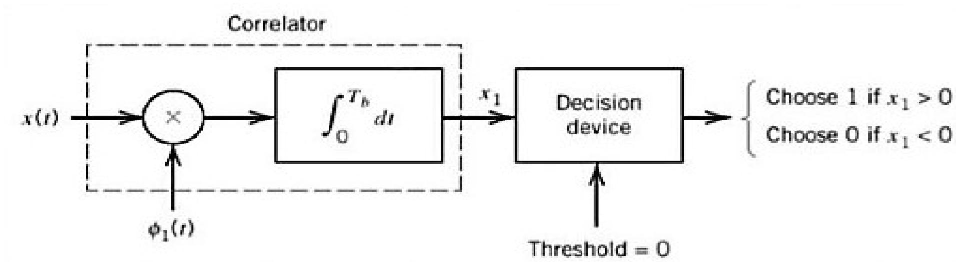


Fig: (b) BPSK Receiver

To generate a binary PSK signal, first the binary data sequence is given to polar non-return to zero level encoder

(i) Non-return-to-zero level encoder: whereby the input binary data sequence is encoded in polar form with symbols 1 and 0 represented by the constant-amplitude of $\sqrt{E_b}$ & $-\sqrt{E_b}$ respectively. The resultant output and sinusoidal carrier is given to product modulator, it will generate BPSK signal.

(ii) Product modulator: which multiplies the level encoded binary wave by the sinusoidal carrier of amplitude to produce the BPSK signal. The timing pulses used to generate the level encoded binary wave and the sinusoidal carrier wave are usually, but not necessarily, extracted from a common master clock.

$$Binary 1 \rightarrow S_1(t) = \sqrt{E_b}\,\phi_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$$

$$Binary 0 \rightarrow S_2(t) = -\sqrt{E_b}\,\phi_1(t) = -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$$

To detect the original binary sequence of 1s and 0s, the BPSK signal at the channel output is applied to a receiver.

(i) Correlator: The noisy PSK signal x(t) is applied which is also supplied with a locally generated reference signal that is a replica of the carrier wave $\phi_1(t)$.The output of the correlator x_1 is given to decision device.

(ii) Decision-making device: Compares the output with the zero threshold. If the threshold is exceeded, the device decides in favor of symbol 1; otherwise, it decides in favor of symbol 0. If x_1 is exactly equals to zero the receiver makes a random guess in favor of symbol 0 (or) 1.

POWER SPECTRA OF BINARY PSK SIGNAL:

The BPSK wave consists of an in phase component only. Depending on the symbol 0 or 1 at the input of modulator during signal interval $0 \le t \le T_b$, the in phase component equals $+g(t)$ or $-g(t)$ respectively, where $g(t)$ is symbol shaping function defined by

$$g(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}}, & 0 \le t \le T_b \\ 0, & otherwise \end{cases} \qquad (1)$$

Suppose the input of the modulator is random wave and equally likely then the psd of $g(t)$ is equal to the energy spectral density of $g(t)$

Energy spectral density of $g(t)$ is equal to the squared magnitude of the signals Fourier transform $\left\| G(t)^2 \right\|$.

$\therefore$ Power spectral density of BPSK signal equals to

$$S_B(f) = \frac{2E_b \sin^2(\pi T_b f)}{(\pi T_b f)^2}$$

$$= 2E_b \operatorname{sinc}^2(T_b f) \qquad (2)$$

The spectrum representation is shown in below figure

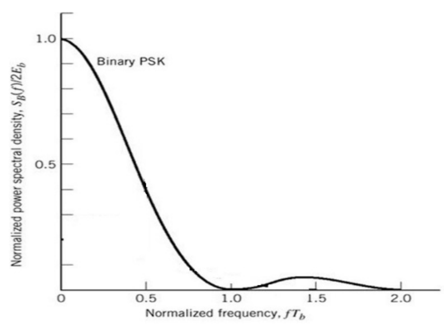


Fig: Power spectra of BPSK.

BANDWIDTH OF BPSK:

The spectra main lobe width gives the bandwidth of BPSK.

$$BW = f_H - f_L$$

$$= \frac{1}{T_b} - \left(-\frac{1}{T_b}\right)$$

$$\therefore \text{Bandwidth } BW = 2f_b$$

QUADRIPHASE SHIFT KEYING (QPSK):

In a digital communication system our goal is to achieve very low probability of error and efficient utilization of channel bandwidth. QPSK is the band conservation modulation scheme which is an example of quadrature carrier multiplication.

Quadrature Phase Shift Keying (QPSK) is a form of Phase Shift Keying in which two bits are modulated at once, selecting one of four possible carrier phase shifts such a $\pi/4, 3\pi/4, 5\pi/4$ and $7\pi/4$. For this set of values the transmitted signal is defined as

$$S_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + (2i-1)\pi/4\right), & 0 \le t \le T \\ 0 & otherwise \end{cases} \quad (1)$$

Where i=1, 2, 3, 4 and E is the transmitted signal energy per symbol.

Each possible value of phase corresponds to unique dibit. For example phase values to represent gray coded set of dibits 10, 00, 01, 11.

Signal space diagram of QPSK:

Using trigonometric functions $S_i(t)$ can be written as

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\pi/4\right] \cos 2\pi f_c t - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\pi/4\right] \sin 2\pi f_c t \quad (2)$$

Where i=1, 2, 3, 4

There are two orthogonal basis functions $\phi_1(t)$ and $\phi_2(t)$ in the above expansion of $S_i(t)$, they are defined as

$$\phi_1(t) = \sqrt{2/T} \cos 2\pi f_c t, \qquad 0 \le t \le T \quad (3)$$

$$\phi_2(t) = \sqrt{2/T} \sin 2\pi f_c t, \qquad 0 \le t \le T \quad (4)$$

$$s_i = \begin{bmatrix} \sqrt{E} \cos[(2i-1)\frac{\pi}{4}] \\ \sqrt{E} \sin[(2i-1)\frac{\pi}{4}] \end{bmatrix}, \quad i = 1,2,3,4 \quad \boxed{(5)}$$

The elements of the signal vector are listed in below table

Gray-encoded Input Dibit	Phase of QPSK Signal (radians)	Coordinates of Message Points	
		s_{i1}	s_{i2}
10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
00	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
11	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$

Table: Signal space characterization of QPSK.

Accordingly, QPSK has two dimensional signal constellation (i.e. N=2) and four message points (i.e., N=4) whose phase angles increase in counter clock wise direction is shown in figure below.

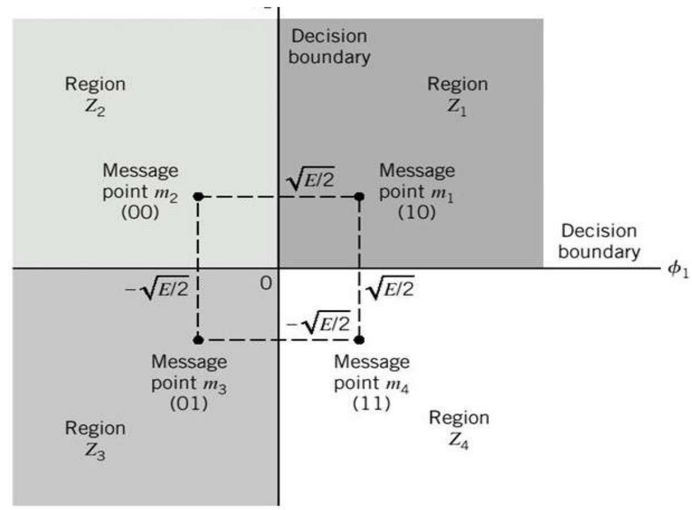


Fig: Signal space diagram of QPSK

GENERATION AND DETECTION OF COHERENT QPSK:

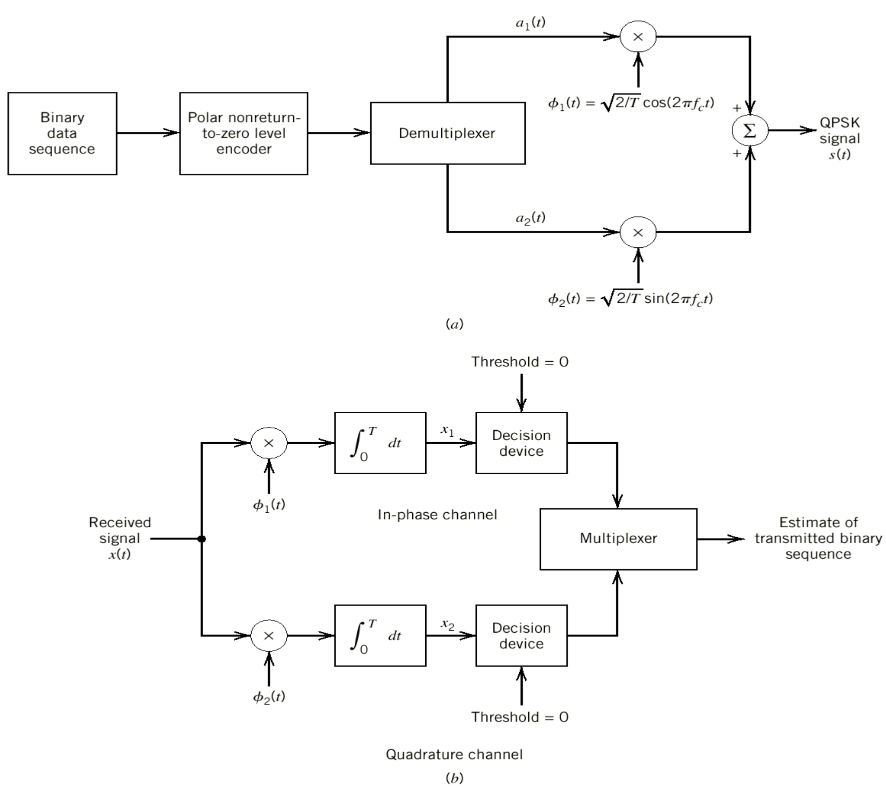


Fig: Block diagrams of (a) QPSK transmitter and (b) coherent QPSK receiver.

The incoming binary data sequence is first transformed into polar form by a non-return to zero level encoder. Thus symbol 1 and 0 are represented by $+\sqrt{E/2}$ and $-\sqrt{E/2}$ respectively. This binary wave divided into two streams by a de-multiplexer (odd and even numbered bits) and represented as $a_1(t)$ and $a_2(t)$. $a_1(t)$ and $a_2(t)$ are modulated with a pair of quadrature carriers $\varphi_1(t)$ and $\varphi_2(t)$ respectively. Thus the two waveforms are added to produce desired QPSK signal.

The QPSK receiver consists of a pair of correlator with a common input and supplied with locally generated carriers $\varphi_1(t)$ and $\varphi_2(t)$. The correlators produce x_1 and x_2 and then these are compared with zero threshold. If the threshold is exceeded, the device decides in favor of symbol 1; otherwise, it decides in favor of symbol 0. Similarly for quadrature phase channel also.

Finally these two binary sequences at the in-phase and quadrature channel outputs are combined using multiplexer to produce the original binary sequence.

POWER SPECTRA OF QPSK SIGNALS:

Assume that the binary wave at the modulator input is random with symbol 1 and 0 being equally likely and with the symbols transmitted during adjacent time slots being statistically independent. QPSK contains in phase and quadrature phase components. We make two observations regarding these two components are

- Depending on dibit sent during the signal interval $-T_b \le t \le T_b$,
  the in-phase component equals +g(t) or – g(t) similar situation exists for the quadrature component. The g (t) denotes the symbol shaping function defined by

$$g(t) = \begin{cases} \sqrt{\dfrac{E}{T}}, & 0 \le t \le T \\ 0, & otherwise \end{cases}$$

The in-phase and quadrature components are statistically independent. The baseband power spectral density of QPSK equals the sum of the individual power spectral densities of the in-phase and quadrature components.

$$\begin{aligned} S_B(f) &= 2E \sin c^2(Tf) \\ &= 4E_b \sin c^2(2T_b f) \end{aligned}$$

The power spectra of QPSK is shown in figure below

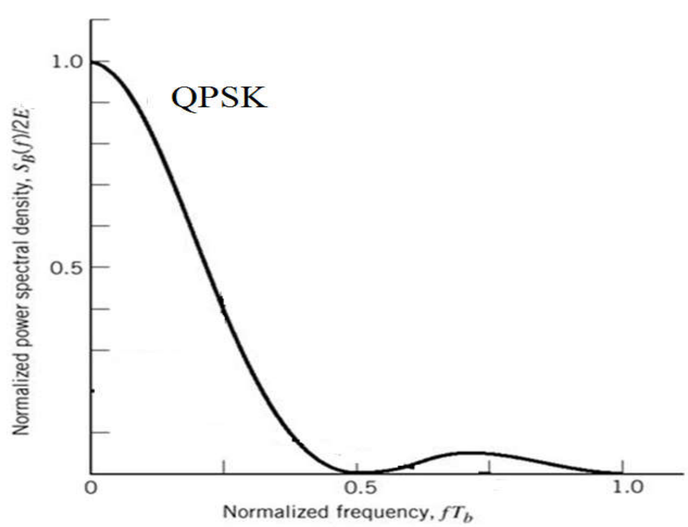


Figure Power spectra of QPSK

BANDWIDTH OF QPSK SIGNALS:

The main lobe width in the power spectra gives the band width of QPSK signals.

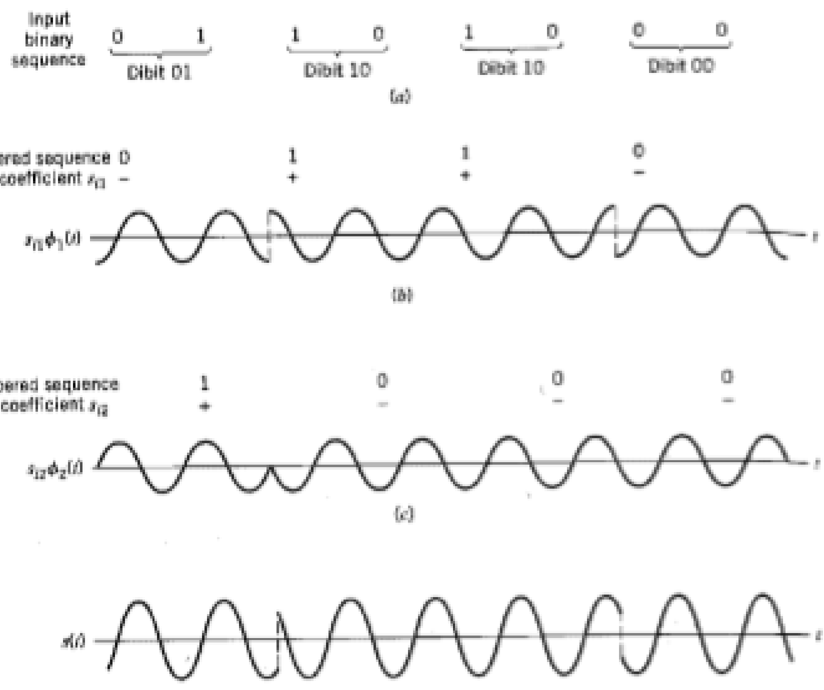
$$BW = f_H - f_L$$

$$= \frac{1}{2T_b} - \left(-\frac{1}{2T_b}\right)$$

$$= 2\left(\frac{1}{2T_b}\right) = f_b$$

QPSK requires half of the bandwidth than BPSK. So QPSK is called as the bandwidth conserving modulation scheme.

Ex: Generate the QPSK signal for given binary sequence 01101000



M-ARY PSK:

In M-ary PSK the carrier will take one of the M possible values, namely $\theta_i = 2(i-1)\pi/M$, where i = 1,2,…M.

The transmitted signal is defined as

$$s_i(t) = \sqrt{\frac{2E}{T}}\cos\left(2\pi f_c t + \frac{2\pi}{M}(i-1)\right), \qquad i = 1,2,.....,M \tag{1}$$

Where E is transmitted signal energy per symbol

f_c is carrier frequency

Using trigonometric function above eq'n can be written as

$$S_i(t)=\sqrt{\frac{2E}{T}}\cos\left[\frac{2\pi}{M}(i-1)\right]\cos 2\pi f_c t-\sqrt{\frac{2E}{T}}\sin\left[\frac{2\pi}{M}(i-1)\pi/4\right]\sin 2\pi f_c t \qquad (2)$$

Where i=1, 2, 3, 4

There are two orthogonal basis functions $\phi_1(t)$ and $\phi_2(t)$ in the above expansion of $S_i(t)$, they are defined as

$$\phi_1(t)=\sqrt{2/T}\cos 2\pi f_c t, \qquad 0\le t\le T \qquad (3)$$

$$\phi_2(t)=\sqrt{2/T}\sin 2\pi f_c t, \qquad 0\le t\le T \qquad (4)$$

From the above equations the signal space diagram of M-ary PSK will be two dimensional with M message points. The M message points are equally spaced on a circle of radius $\sqrt{E_b}$ and centered at the origin.

The Euclidian distance between each two points for M = 8 can be calculated as:

$$d_{12}=d_{18}=2\sqrt{E}\sin(\frac{\pi}{M})$$

For example M=8, the signal space diagram is shown in below figure

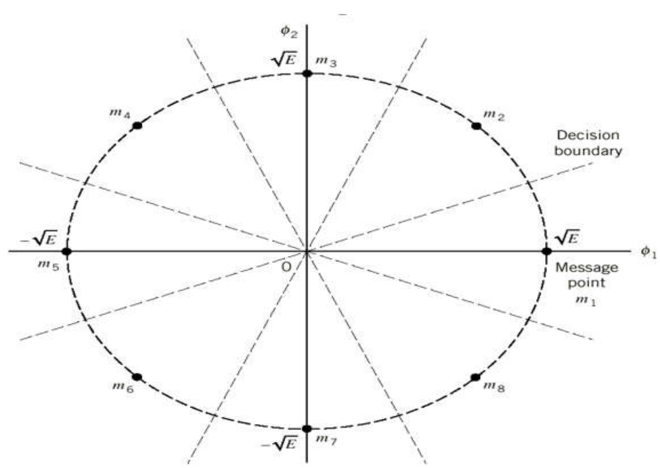


Fig: Signal space diagram of octa phase shift keying (M=8)

POWER SPECTRA OF M-ARY PSK:

The symbol duration of M-ary PSK is

$$T=T_b\log_2 M$$

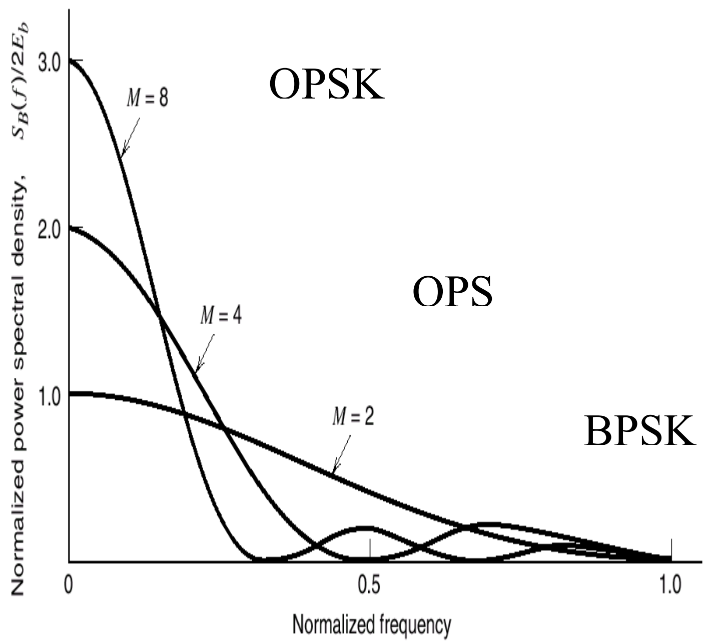


Figure: Power spectra of M-ary PSK signals for M = 2, 4, 8.

BANDWIDTH EFFICIENCY OF M-ARY PSK SIGNAL:

In the power spectra of M-ary PSK main lobe width gives the band width of M-ary PSK signals.

The minimum bandwidth required to pass the M-ary PSK signals over the channel is

$$B = \frac{2}{T}$$

Where T is the symbol duration

$$\because T = T_b \log_2 M$$

$$R_b = \frac{1}{T_b} \, or \quad f_b = \frac{1}{T_b}$$

$$B = \frac{2}{T_b \log_2 M}$$

$$= \frac{2R_b}{\log_2 M} \qquad (1)$$

The band width efficiency is defined as

$$\rho = \frac{R_b}{B}$$

$$= \frac{R_b}{2R_b \log_2 M}$$

$$\therefore \rho = \frac{\log_2 M}{2}$$

As the M value increases, the bandwidth efficiency increases.

M-ARY QUADRATURE AMPLITUDE MODULATION:

M-ary QAM is a two dimensional generalization of M-ary PAM, it involves two orthogonal basis functions, they are

$$\phi_1(t) = \sqrt{2/T}\cos 2\pi f_c t, \qquad 0 \le t \le T \qquad (1)$$

$$\phi_2(t) = \sqrt{2/T}\sin 2\pi f_c t, \qquad 0 \le t \le T \qquad (2)$$

The ith message point in Si in the (Φ1, Φ2) plane be denoted by (a_i dmin/2, b_i dmin/2), where dmin is the minimum distance between any two message points in the constellation, where a_i and b_i are integers.

Let dmin /2 = √E0 where E_0 is the energy of the signal with the lowest amplitude.

The transmitted M-ary QAM signal for symbol k is defined by

$$s_k(t) = \sqrt{\frac{2E_0}{T}}\, a_k \cos(2\pi f_c t) - \sqrt{\frac{2E_0}{T}}\, b_k \sin(2\pi f_c t), \qquad \begin{array}{l} 0 \le t \le T \\ k = 0, \pm 1, \pm 2, \ldots \end{array} \qquad (3)$$

The signal $S_i(t)$ consists of two phase quadrature carriers with each one being modulated by a set of discrete amplitudes, hence the name quadrature amplitude modulation (QAM).

Depending on the number of possible symbols M, we have two different constellation diagrams.

(1) Square constellation: In which number of bits per symbol is even.

(2) Cross constellation: In which the number of bits per symbol is odd.

COHERENT FREQUENCY SHIFT KEYING:

BINARY FSK:

In BFSK, symbol 1 is represented by $S_1(t)$ and symbol 0 is represented by $S_2(t)$, these two transmitted signals differ in frequency by a fixed amount. It is defined as

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_i t), & 0 \le t \le T_b \\ 0, & \text{elsewhere} \end{cases} \qquad (1)$$

Where i=1, 2

E_b is the transmitted signal energy per bit and $f_c = \frac{n_c + i}{2}$

FSK signal described here is known as sunde's FSK. It is a continuous phase signal. It is an example of continuous phase FSK

From the equations, the orthonormal basis function is defined as

$$\phi_i(t) = \begin{cases} \sqrt{\frac{2}{T_b}} \cos 2\pi f_i t & 0 \le t \le T_b \\ 0 & elsewhere \end{cases} \qquad (2)$$

Where i=1, 2

The coefficient S_{ij}, for i=1, 2 is defined as

$$S_{ij} = \int S_i(t) \phi_j(t) dt$$

$$= \int_0^{T_b} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_j t)\, dt$$

$$= \begin{cases} \sqrt{E_b}, & i = j \\ 0, & i \ne j \end{cases} \qquad (3)$$

The two message points are defined as

$$\mathbf{s}_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix} , \qquad \mathbf{s}_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$$

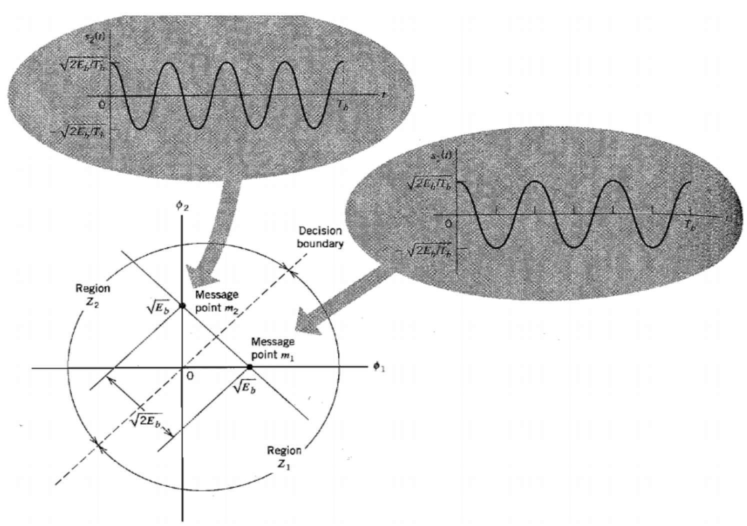


Fig: Signal space diagram for BPSK

GENERATION AND DETECTION OF COHERENT BFSK SIGNALS:

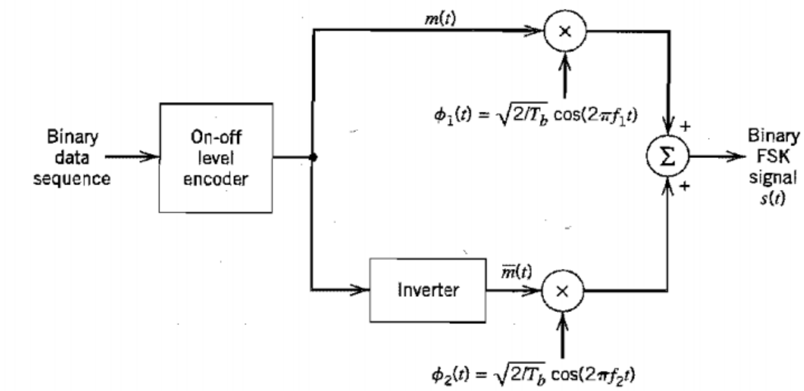


Fig: BFSK Transmitter

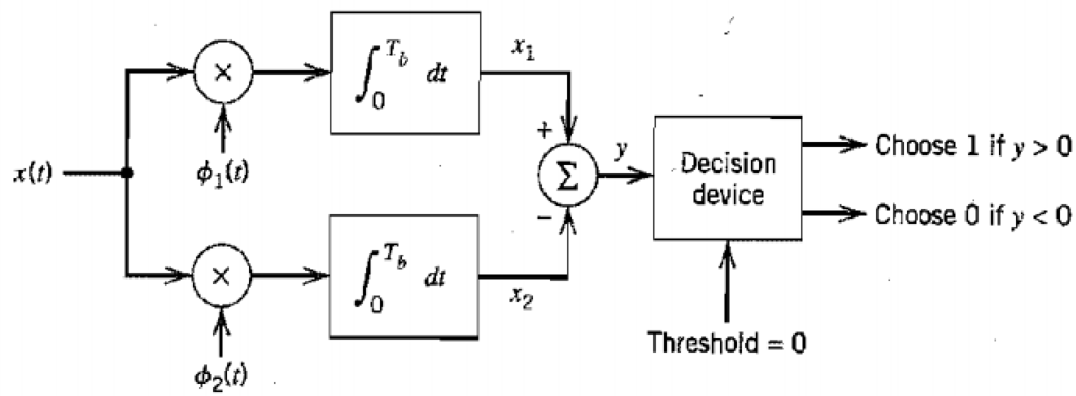


Fig: BFSK Receiver

The generation block diagram is shown in figure above. The incoming binary data sequence is first applied to an on-off level encoder, it generates constant amplitude of $\sqrt{E_b}$ pulse for binary 1, zero for binary 0. When binary 1 is transmitted, it is passed through first channel and it modulates with $\phi_1(t)$ and the second channel is switched off due to inverter. For binary 0, first channel is switched off, due to inverter the output of on-off encoder into pulse with $\sqrt{E_b}$ amplitude and then modulates with $\phi_2(t)$carrier. The output of first channel and second channel are added to generate the BFSK signal.

To detect the original binary sequence from x (t), we use BFSK receiver as shown in above figure. It consists of two correlators with a common input, which are supplied by locally generated carrier $\phi_1(t)$ and $\phi_2(t)$. The output from both the correlators are subtracted from one another and then compared with the zero threshold value.

If y>0, the receiver decides in favor of 1

y<o, the receiver decides in favor of 2

y= 0, the receiver makes a random guess in favor of 1(or) 0.

POWER SPECTRA OF BFSK SIGNALS:

Consider the case of sunde's FSK, carrier frequency f_c is the arithmetic mean of $f1$ and f_2. This special binary FSK signal as follows

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t \pm \frac{\pi t}{T_b}\right), \qquad 0 \le t \le T_b \tag{1}$$

Using trigonometric function the above equation can be rewritten a

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(\pm \frac{\pi t}{T_b}\right) \cos(2\pi f_c t) - \sqrt{\frac{2E_b}{T_b}} \sin\left(\pm \frac{\pi t}{T_b}\right) \sin(2\pi f_c t)$$

$$= \sqrt{\frac{2E_b}{T_b}} \cos\left(\frac{\pi t}{T_b}\right) \cos(2\pi f_c t) \mp \sqrt{\frac{2E_b}{T_b}} \sin\left(\frac{\pi t}{T_b}\right) \sin(2\pi f_c t) \tag{2}$$

In the above equation minus sign corresponds to transmitted symbol 1 and plus sign corresponds to transmitted symbol 0.

Following observations relayed to the in phase and quadrature components of the binary FSK signal with continues phase.

(i) The in phase component is completely independent of the input binary wave. It equals to $\sqrt{\frac{2E_b}{T_b}} \cos\left(\frac{\pi t}{T_b}\right)$ for all values of t. The psd of this component consists of two delta functions, weighted by the factor $\frac{E_b}{2T_b}$ and occurring at $f = \pm\frac{1}{2T_b}$.

(ii) The quadrature component is completely related to the input binary wave. During the signaling interval $0 \le t \le T_b$, it equals $-g(t)$ when we have symbol 1 and $+g(t)$ when we have symbol 0. The symbol shaping function $g(t)$ is defined by

$$g(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \sin\left(\frac{\pi t}{T_b}\right), & 0 \le t \le T_b \\ 0, & \text{elsewhere} \end{cases} \tag{3}$$

The energy spectral density of symbol shaping function equals to

$$\Psi_g(f) = \frac{8E_b T_b \cos^2(\pi T_b f)}{\pi^2(4T_b^2 f^2 - 1)^2} \tag{4}$$

The psd of quadrature components equal to $\frac{\psi_g(f)}{T_b}$

$\therefore$ The psd of coherent BFSK signals is

$$S_B(f) = \frac{E_b}{2T_b}\left[\delta\left(f - \frac{1}{2T_b}\right) + \delta\left(f + \frac{1}{2T_b}\right)\right] + \frac{8E_b \cos^2(\pi T_b f)}{\pi^2(4T_b^2 f^2 - 1)^2}$$

The power spectra is shown in the figure below

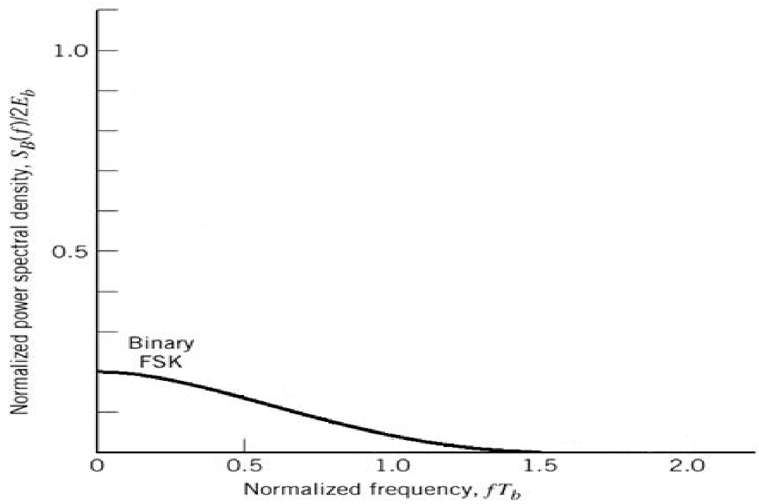


Figure Power spectra of binary FSK signal

BANDWIDTH OF BFSK SIGNAL:

From the spectrum,

$$BW = BW_1 + BW_2$$

$$= \left(\frac{3}{2T_b} + \frac{3}{2T_b} \right) + \left(\frac{1}{2T_b} + \frac{1}{2T_b} \right)$$

$$= 3T_b + T_b = 4T_b$$

$$= 4f_b$$

NON-COHERENT ORTHOGONAL MODULATION:

Non-coherent orthogonal modulation involves two non-coherent receivers. They are BFSK and differential phase shift keying.

Consider a binary signaling scheme that involves the use of two orthogonal signals $S_1(t)$ and $S_2(t)$, which have equal energy, during the interval $0 \le t \le T$, one of these two signals sent over the noisy channels that shifts a carrier phase by a fixed amount. Let $g_1(t)$ and $g_2(t)$ are phase shifted version of $S_1(t)$ and $S_2(t)$. Assume that $g_1(t)$ and $g_2(t)$ are orthogonal we refer such signaling scheme as non-coherent orthogonal modulator.

The received signal x (t) is

$$x(t) = \begin{cases} g_1(t) + w(t), & s_1(t) \text{ sent}, 0 \le t \le T \\ g_2(t) + w(t), & s_2(t) \text{ sent}, 0 \le t \le T \end{cases}$$

For the detection of signals from x(t), The receiver consists of a pair of filters matched to the transmitted signal $S_1(t)$ and $S_2(t)$. Because the phase is unknown the detection depends on amplitude. After that, the matched filter output are envelope detected, sampled and then compared with each other.

If the upper path has an output amplitude l_1 greater than the output amplitude l_2 on lower path, the receiver makes a decision in favor of $S_1(t)$. Else it will take a decision in favor of $S_2(t)$.

This non-coherent matched filter may be viewed as being equivalent to quadrature receiver.

NON-COHERENT BINARY FREQUENCY SHIFT KEYING:

In binary FSK, the transmitted signal is defined by

$$S_i(t)=\begin{cases}\sqrt{\dfrac{2E_b}{T_b}}\cos 2\pi f_i t; & 0\le t\le T_b \\ 0; & elsewhere\end{cases} \tag{1}$$

The transmission of frequency f_1 represents symbol 1, and the transmission of frequency f_2 represents symbol 0.

GENERATION AND DETECTION OF NON COHERENT BFSK SIGNALS:

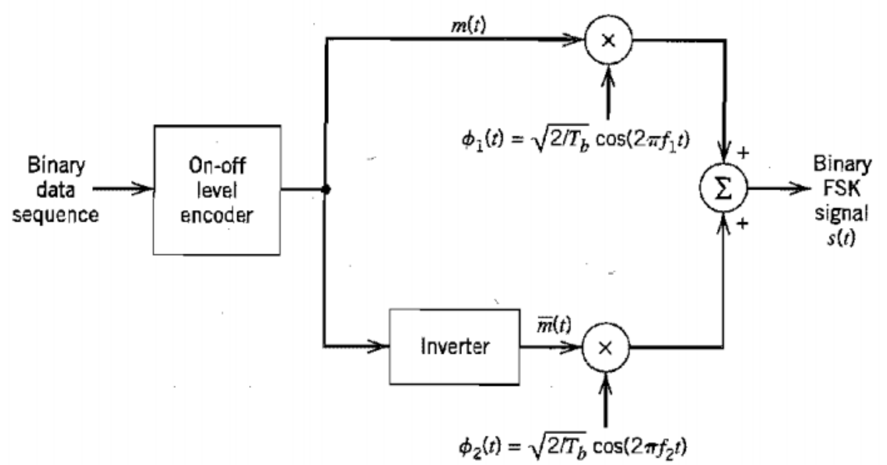


Fig (a): Generation block diagram of BFSK signal.

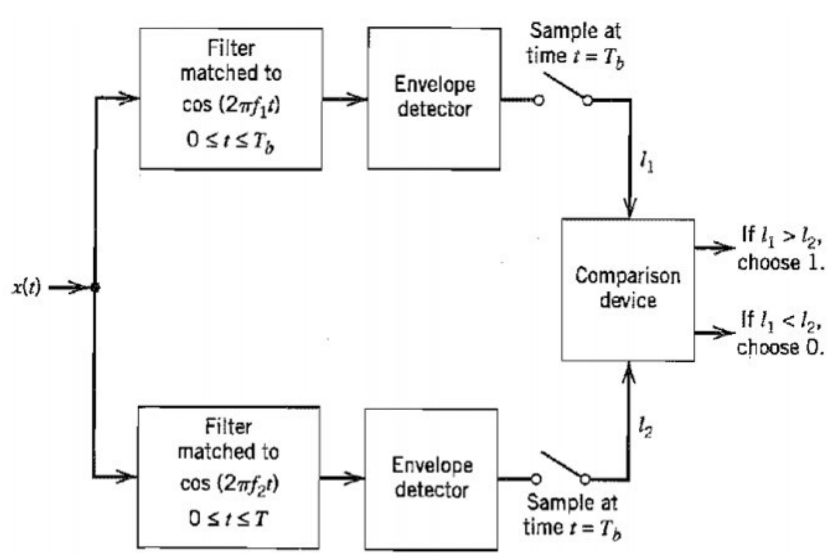


Fig (b): Detection block diagram of BFSK signal.

To generate a binary FSK signal, the incoming binary data sequence is first applied to an on-off level encoder as shown in above figure (a) at the output, symbol 1 is represented by amplitude of $\sqrt{E_b}$ volts and symbol 0 is represented by zero volts.

For binary 1, the second channel is switched off because of inverter with the result frequency f_1 transmitted.

For binary 0, the first channel is switched off, with the result frequency f_2 transmitted.

For non-coherent detection of this frequency modulated wave, the receiver carrier of a pair of matched filter followed by envelope detectors shown in figure (b) are used.

The filter in the upper path of the receiver is matched to $\cos 2\pi f_1 t$ and filter in the lower path is matched to $\cos 2\pi f_2 t$, $0 \le t \le T_b$. The resulting envelope detector outputs are compared at time $t = T_b$. And their values are compared.

The sample values of upper and lower paths are represented by l_1 and l_2.

- If $l_1 > l_2$, the receiver decides in favor of symbol 1
- If $l_1 < l_2$, the receiver decides in favor of symbol 2
- If $l_1 = l_2$, the receiver makes a decision in favor of symbol 1(or)0.

The probability of error (or) bit error rate of Non-coherent BFSK is

$$P_e = \frac{1}{2}\exp\left(\frac{-E_b}{2N_0}\right)$$

DIFFEREENTIAL PHASE SHIFT KEYING:

DPSK is the non-coherent version of PSK. It eliminates the need of phase recovery circuit in the receiver by combining two basic operations at the transmitter.

(i) Differential encoding of the input binary wave.

(ii) Phase shift keying for symbol 0, carrier phase advances by 180^o (or) π. For symbol 1, carrier phase is unchanged.

GENERATION OF DPSK:

In the generation DPSK, first the actual binary sequence converted into differential encoded data based on signal transitions. For example, transition is encoded by binary 0, if no transition is done then it is encoded as binary 1.Generation and detection block diagram is shown in below figure.

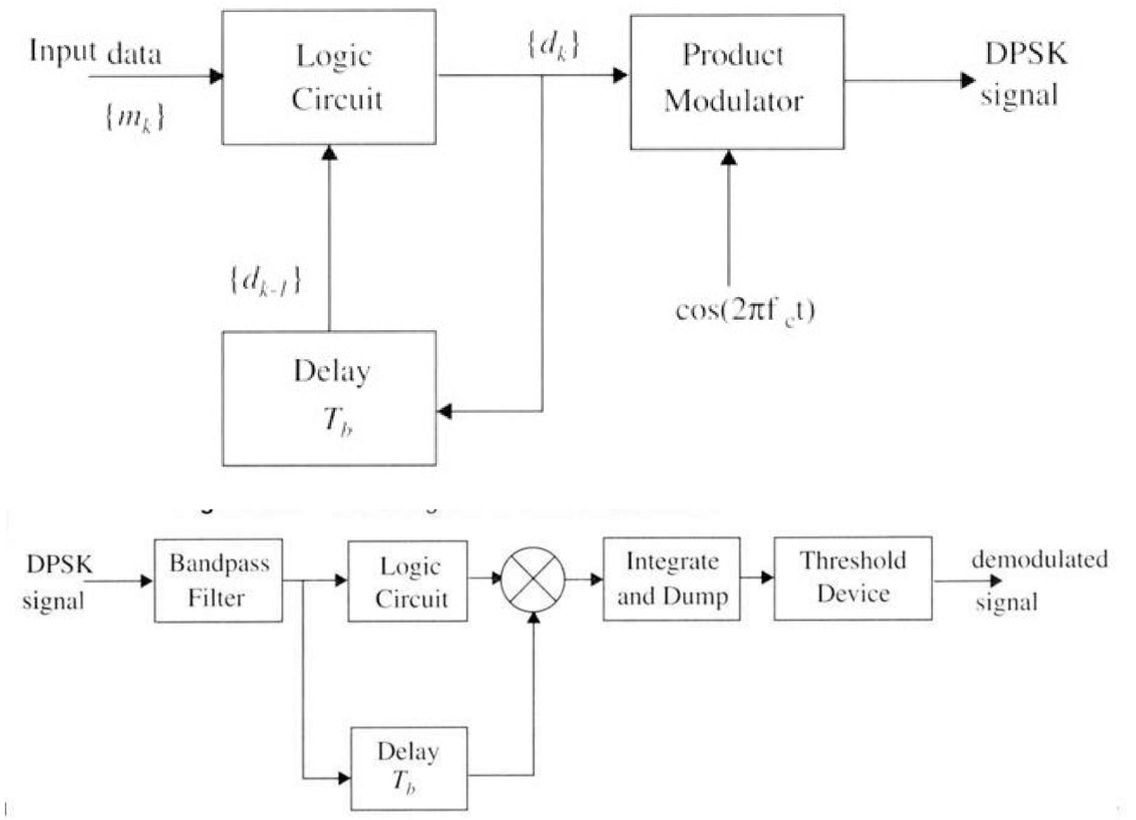


Fig: (a) Generation block diagram of DPSK

(b) Detection block diagram of DPSK

In generation, the binary sequence $\{b(k)\}$ is given to encoder. The output of encoder is applied to one of the input for product modulator and another input for product modulator i.e, carrier modulates with phase of differential encoded data depends on output of the encoder and generates DPSK signal.

In detection DPSK signal is applied as the input for multiplier and other input is delayed version of received DPSK signal by T_b time. If same phase signal are produced the integrator output will be positive and for different phase signals the integrator output will be negative. The output of integrator is represented by l.

- b
- If $l < 0$, choose it as binary 0

<u>BANDWIDTH OF DPSK SIGNAL:</u>

In DPSK, the differential encoded data is generated based on receiver bits.

$\therefore$ Symbol duration $T = 2T_b$

Bandwidth $BW = \frac{2}{T} = \frac{2}{2T_b} = \frac{1}{T_b}$ $\qquad \therefore BW = f_b$

INFORMATION THEORY

Introduction:

The performance of the communication system is measured in terms of its error probability. Zero probability of error represents no error condition.
The performance of system is depends on the properties like available signal power, channel noise and bandwidth. The error less transmission is obtained by defining these properties using Shannon's theory. Information theory is used to mathematical modeling and analysis of communication systems.

Information content of message:

The amount of information transmitted through the message 'm_k' with probability 'p_k' is

$$\boxed{I_k = log_2\left(\frac{1}{p_k}\right)} Bits$$

Properties of Information:

1. If uncertainty is more, then information is more.
2. If receiver knows the message being transmitted then the amount of information is zero
3. If message m_1 carries an information I_1 and information in m_2 is I_2 then the amount of information in combined message is $I_1 + I_2$
4. If there are $M=2^N$ equally likely messages then the amount of information carried by each message will be N bits.

Entropy:

Average information is represented by **Entropy.**

$$Average\ information = \frac{Total\ information}{Number\ of\ messages} = Entropy$$

$$H = \sum_{k=1}^{M} p_k log_2\left(\frac{1}{p_k}\right)\ bits/symbol$$

Where p_k = Probability of k^{th} symbol

M= number of different messages

Entropy of symbols in long independent and dependent sequences:

Consider that there are M different messages. Let these messages be $m_1,m_2,m_3,\ldots..m_M$. They have probabilities as $p_1,p_2,p_3,\ldots\ldots,p_m$. Suppose a L message sequence is transmitted,

p_1L messages of m_1 are transmitted

p_2L messages of m_2 are transmitted

p_3L messages of m_3 are transmitted

.

.

.

P_ML messages of m_M are transmitted

The total information due to message1 will be $I_{1(totaal)} = p_1 L\ log_2\left(\frac{1}{p_1}\right)$

The total information due to message2 will be $I_{2(totaal)} = p_2 L\ log_2\left(\frac{1}{p_2}\right)$

And so on

The total information carried due to sequence of L messages will be

$I_{(total)} = I_{1(total)} + I_{2(total)} + I_{3(total)} \ldots\ldots\ldots + I_{M(total)}$

$= p_1 L\ log_2\left(\frac{1}{p_1}\right) + p_2 L\ log_2\left(\frac{1}{p_2}\right) + p_3 L\ log_2\left(\frac{1}{p_3}\right) + \cdots + p_M L\ log_2\left(\frac{1}{p_M}\right)$

The average information will be,

Avg., Information$=\frac{Total\ information}{Number\ of\ messeges} = \frac{I_{(total)}}{L}$

Therefore the entropy (H) is given by

Entropy (H) $= \frac{I_{(total)}}{L} = p_1 L\ log_2\left(\frac{1}{p_1}\right) + p_2 L\ log_2\left(\frac{1}{p_2}\right) + p_3 L\ log_2\left(\frac{1}{p_3}\right) + \cdots + p_M L\ log_2\left(\frac{1}{p_M}\right)$

Entropy: $H = \sum_{k=1}^{M} p_k log_2\left(\frac{1}{p_k}\right)\ bits/symbol$

Properties of Entropy:

1. Entropy (average information) is zero if the event is sure or impossible.
   H=0 if $P_k = 0$ or 1
2. If symbols are equally likely events then probability is $p_k = 1/M$, then entropy is
   H= $\log_2$ M
3. Upper bound of entropy is given by **$H_{max} = Log_2$ M.**

MUTUAL INFORMATION:

Although conditional entropy can tell us when two variables are completely independent, it is not an adequate measure of dependence. A small value for H(Y| X) may implies that X tells us a great deal about Y or that H(Y) is small to begin with. Thus, we measure dependence using mutual information:

$$I(X,Y) = H(Y) - H(Y|X)$$

Mutual information is a measure of the reduction of randomness of a variable given knowledge of another variable. Using properties of logarithms, we can derive several equivalent definitions

$$I(X,Y) = H(X) - H(X|Y)$$

$$I(X,Y) = H(X) + H(Y) - H(X,Y) = I(Y,X)$$

Properties of Mutual Information:

PROPERTY 1: Symmetry

The mutual information of a channel is symmetric in the sense that

$$I(X,Y) = I(Y,X)$$

PROPERTY 2: Non-negativity

The mutual information is always nonnegative; that is;

$$I(X,Y) \geq 0$$

PROPERTY 3: Expansion of the Mutual Information

The mutual information of a channel is related to the joint entropy of the channel input and channel output by

$$I(X,Y) = H(X) + H(Y) - H(X,Y)$$

where the joint entropy H(X, Y) is defined by $H(X,Y) = \sum_{J=0}^{J-1} \sum_{K=0}^{K-1} P(x_j, y_k) \log_2 \left(\frac{1}{P(x_j, y_k)}\right)$

Shannon – Hartley theorem:

Shannon – Hartley theorem is channel capacity theorem which is applied under Gaussian noise. It is also called information capacity theorem. According to this theorem, the channel capacity of a white band limited Gaussian channel is

$$c = Blog_2\left(1 + \frac{S}{N}\right) \; bits/sec$$

Where B is channel bandwidth, S is transmitted power, N noise power.

Proof:

Le t the noise is AWGN over a channel of bandwidth 'B' having 'N' average power. Then the maximum entropy of the noise is

$$h(N)_{max} = Blog_2(2\pi eN) \; bit/sec$$

let the received signal is 'Y' due to input signal of X over the bandwidth 'B' Hz. Therefore Y will be of the form

Y=X+N; N is the noise in the channel (AWGN assumed)

Let X is having its average power S in the given bandwidth. Since X and N are independent, then the power of Y will be the variance of the received signal.

$$\sigma_y^2 = S + N$$

The maximum differential entropy of Y is given by

$$h(Y)_{max} = Blog_2(2\pi e\sigma_y^2)$$
$$= Blog_2(2\pi e(S+N))$$

The channel capacity is given by

$$C=max\ \{h(Y) - h(N)\}$$

Since Y and N are independent it can be written as

$$C=h(Y)_{max} - h(N)_{max}$$

Substituting the $h(Y)_{max}$ and $h(N)_{max}$ in the above equation

$$C= Blog_2(2\pi e(S+N)) - Blog_2(2\pi eN)$$

On solving it results

$$\boxed{c = Blog_2\left(1+\frac{S}{N}\right)\ bits/sec}$$

<u>Implications of Shannon Hartley Law:</u>

1. It gives upper limit on reliable data transmission rate over Gaussian channels.
2. The channel capacity depends on the SNR and bandwidth.

<u>****Important Note****</u>

Noise less channel has infinite bandwidth, but infinite bandwidth has limited capacity.
Proof: In N=0, then capacity of channel $C=B\ log_2(1+\infty)=\infty$
But when B infinite, then noise power increases. Then the noise power is $N=BN_0$. So SNR will be increased. The upper limit given by $C_\infty = \lim_{B\to\infty} C = 1.44\frac{S}{N_0}$