Unit-3 Magneto statics

Introduction of Magnetic field :

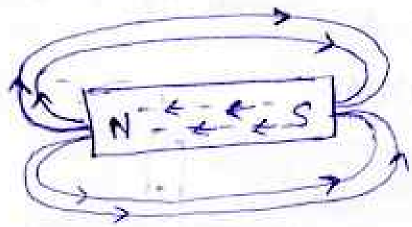
A magnet having two poles North and South. The region around a magnet with which the influence of magnet can be experienced is called magnetic field.

The field is represented by imaginary lines around the magnet are called magnetic lines of force.

These are introduced by scientist Michael faraday.

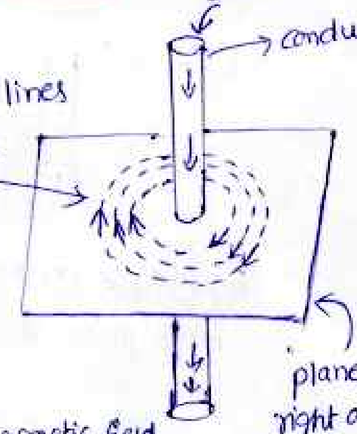
The lines direction is always from N pole to S pole. These lines of force is also called magnetic lines of flux (or) magnetic flux lines.

Fig(1): Permanant magnet and magnetic lines of force.



Magnetic field due to current carrying conductor :-

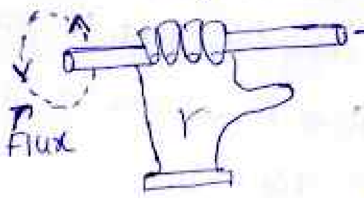
→ When a straight conductor carries a current, it produces a magnetic field around it, all along its length. The lines of force are in the form of concentric circles in the planes at right angles to the conductor. This is shown in fig(2).



Fig(2): magnetic field due to current carrying conductor.

→ A right hand Thumb rule is used to determine the direction of magnetic field around a conductor carrying a current.

→ It states that hold the current carrying conductor in a right hand, such that the Thumb pointing in the direction of current,



Fig(3): Right Hand Thumb Rule

and parallel to the conductor, then curled fingers point in the direction of magnetic lines of flux around it. This is shown in Fig (3).

→ The current carrying conductor, practically is represented by a small circle. i.e, top view of straight conductor and the direction of current is represented by cross or dot.

→ The cross indicates that the current direction is going in to the plane of plane, away from observer.

→ The dot indicates that the current direction is coming out of the plane of plane, towards the observer.

→ Using Right hand thumb rule, the direction of magnetic flux around such a conductor is either clockwise or anticlockwise as shown in fig(4).

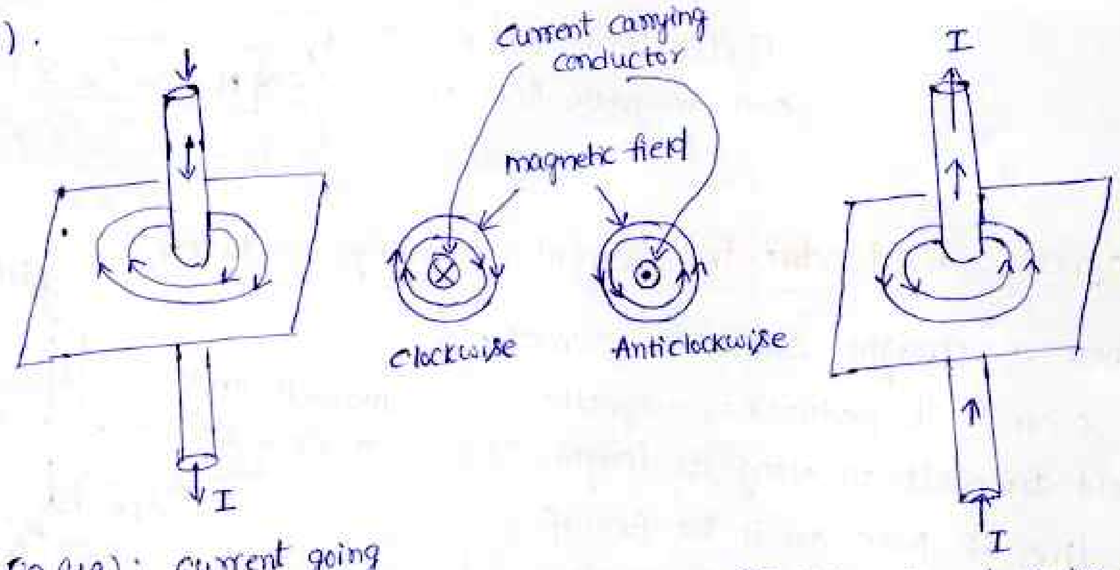


fig (4a): current going in to plane of paper

Fig (4b): Current coming out of the plane of paper.

→ Another method of identifying the direction of magnetic flux around a conductor is Right handed Screw rule. It states that imagine a right handed screw to be along the conductor carrying current with its axis parallel to the conductor and tip pointing in the direction of current flow. The direction of magnetic field is given by the direction in which screw must be turned so as to advance in the direction of current flow. This is explained in fig (5).

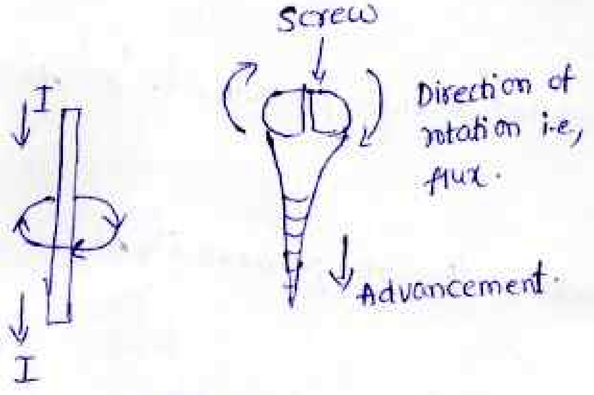


Fig (5a)

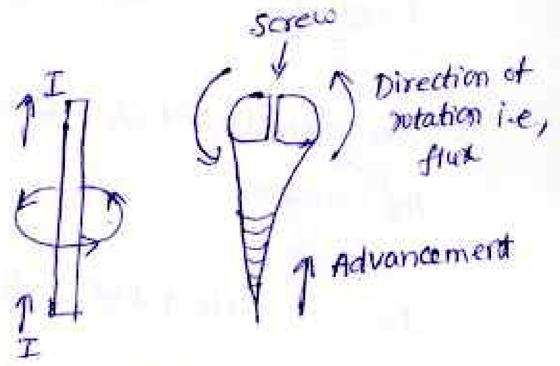


Fig (5b).

Fig (5): Right handed screw Rule.

→ **Magnetic field Intensity ($\bar{H}$) (or) Magnetic Field Strength :-**

The quantitative measure of strongness or weakness of magnetic field is given by magnetic field intensity.

The magnetic field Intensity at a point in the magnetic field is defined as the force experienced by a unit north pole of one weber strength, when placed at that point.

→ It is denoted as $\bar{H}$. and measured in newtons/weber (N/wb) or ampere per metre (A/m) or ampere-turns/metre (AT/m).

→ $\bar{H}$ is vector quantity. This is similar to Electric field Intensity $\bar{E}$ in electrostatics.

→ **Magnetic flux Density ($\bar{B}$) :**

The total magnetic flux or lines of force crossing a unit area in a plane at right angles to the direction of force (or) flux is called magnetic flux density.

It is denoted as $\bar{B}$ and measured in weber per square metre (wb/m^2). also called as Tesla (T).

→ $\bar{B}$ is a vector quantity. This is similar to Electric flux density $\bar{D}$ in Electrostatics.

Relation between $\bar{B}$ and $\bar{H}$:-

In Electrostatics, $\bar{E}$ & $\bar{D}$ are related through permittivity 'ε' of the region.

In magnetostatics $\bar{B}$ & $\bar{H}$ are related through permeability 'μ'.

$\bar{B}$ & $\bar{H}$ are related as,

$$\boxed{\bar{B} = \mu \bar{H} = \mu_0 \mu_r \bar{H}}$$

for free space $\mu_r = 1$, $\bar{B} = \mu_0 \bar{H}$

For all non magnetic media $\mu_r = 1$ &

For all magnetic materials, μ_r is greater than unity.

Oersted's Experiment :-

→ In 1820, christian Oersted, a professor of science at university of Copenhagen, Denmark conducted an experiment to find the relation between electricity and magnetism.

→ Oersted conducted an experiment in which a current carrying conductor was taken. A compass needle was kept under this conductor as shown in fig.

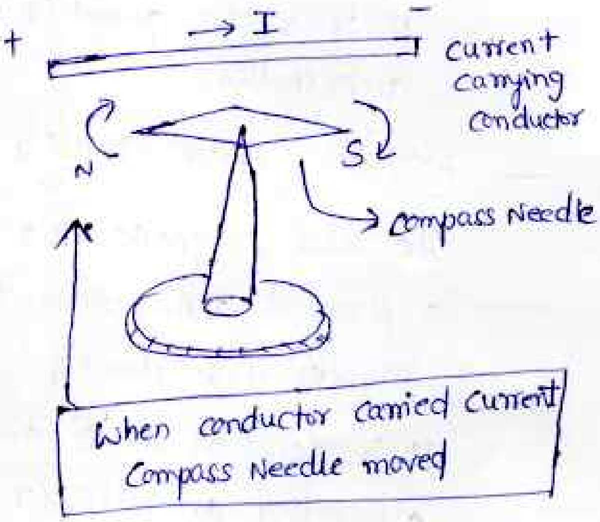


Fig: Oersted's Experiment.

→ When there was no current through the conductor, then needle was pointing along North and south of Earth.

→ If conductor carries current, then the needle was neither attracted to conductor and nor repelled from it, but it moved and tended to stand at right angles to conductor.

→ From this Experiment, oersted showed that an Electric current produces a Magnetic field.

→ Biot Savart Law :-

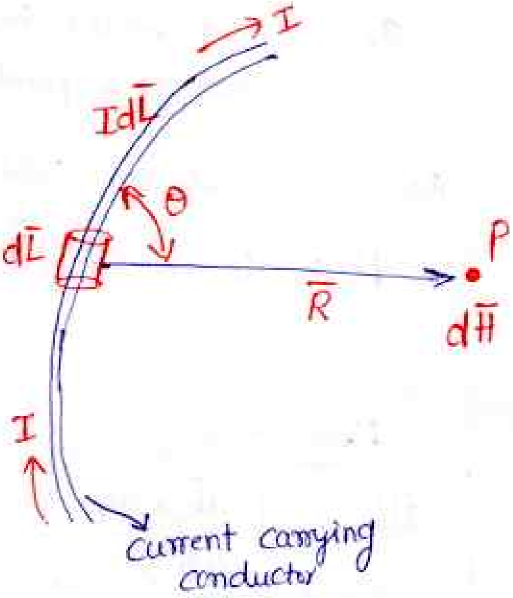
Consider a conductor carrying a direct current I and a steady magnetic field produced around it.

→ The Biot-savart law allows us to obtain the differential magnetic field Intensity $d\bar{H}$, produced at a point P, due to differential current element IdL.

→ Consider a differential length dL hence the differential current element is IdL.

→ The point P is at a distance 'R' from IdL.

→ θ is the angle between the differential current element & the line Joining point P.



Fig(1):

→ Biot Savart Law states that, The magnetic field intensity $d\bar{H}$ produced at a point P due to a differential current element IdL is :

(1) proportional to the product of current I and differential length dL.

(2) Proportional to the sine of the angle between the element & the line Joining point P to the Element.

(3) And inversely proportional to the square of the distance R between point P and the Element.

Mathematically, The Biot Savart law can be stated as

$$d\bar{H} \propto \frac{IdL \sin\theta}{R^2} \longrightarrow (1)$$

$$d\bar{H} = \frac{K\, IdL \sin\theta}{R^2} \longrightarrow (2)$$

where K = constant of proportionality $= \frac{1}{4\pi}$

$$d\bar{H} = \frac{I\, dL \sin\theta}{4\pi R^2} \longrightarrow (3)$$

dH Expression in Vector form :

Let dL = magnitude of vector length $d\bar{L}$

$\bar{a}_R$ = unit vector in the direction from differential current element to point P.

Then from rule of cross product,

$$d\bar{L} \times \bar{a}_R = dL\,|\bar{a}_R|\sin\theta$$
$$= dL \sin\theta \qquad (\because |\bar{a}_R| = 1)$$

From Eq (3),

$$\boxed{d\bar{H} = \frac{I\,d\bar{L} \times \bar{a}_R}{4\pi R^2}} \text{ Ampere/metre} \longrightarrow (4)$$

But $\bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{\bar{R}}{R}$

Hence $$\boxed{d\bar{H} = \frac{I\,d\bar{L} \times \bar{R}}{4\pi R^3}} \text{ A/m} \longrightarrow (5)$$

Eq (4) & Eq (5) is the mathematical form of Biot savart Law.

$\bar{H}$ in Integral form :-

$$\bar{H} = \oint \frac{I\,d\bar{L} \times \bar{a}_R}{4\pi R^2} \longrightarrow (6)$$

where $\oint$ is closed line Integral.

If the current element is considered at Point 1 & point P at point 2; as shown in fig(2), then

$$d\bar{H}_2 = \frac{I_1\,d\bar{L}_1 \times \bar{a}_{R12}}{4\pi (R_{12})^2}, \text{ A/m} \longrightarrow (7)$$

$$\bar{a}_{R12} = \frac{\bar{R}_{12}}{|\bar{R}_{12}|} = \frac{\bar{R}_{12}}{R_{12}}$$

$$\bar{H}_2 = \oint \frac{I_1\,d\bar{L}_1 \times \bar{a}_{R12}}{4\pi (R_{12})^2}, \text{ A/m} \longrightarrow (8)$$

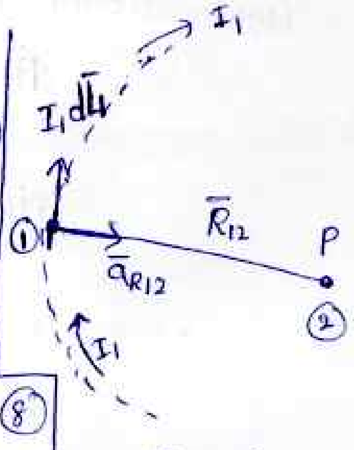
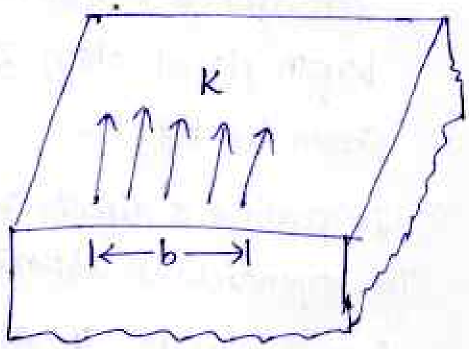


Fig (2).

This is called Integral form of Biot savart law.

Biot Savart Law interms of Distributed Sources:

→ Consider a surface carrying a uniform current over its surface as shown in fig(3). The surface current density is denoted as $\bar{K}$, measured in Ampere/metre (A/m).



Fig(3): Surface current density.

→ For uniform current density, the current I in any width b is given by

$$I = K b$$

where width b is perpendicular to direction of current flow.

→ If differential surface area ds is considered having current density $\bar{K}$ then,

$$I\, d\bar{L} = \bar{K}\, dS. \longrightarrow (9)$$

→ If current density in a volume of given conductor is J, then

$$I\, d\bar{L} = \bar{J}\, dv. \longrightarrow (10)$$

Hence the Biot savart law can be expressed for surface current considering $\bar{K}$ ds while for volume current considering $\bar{J}$ dv.

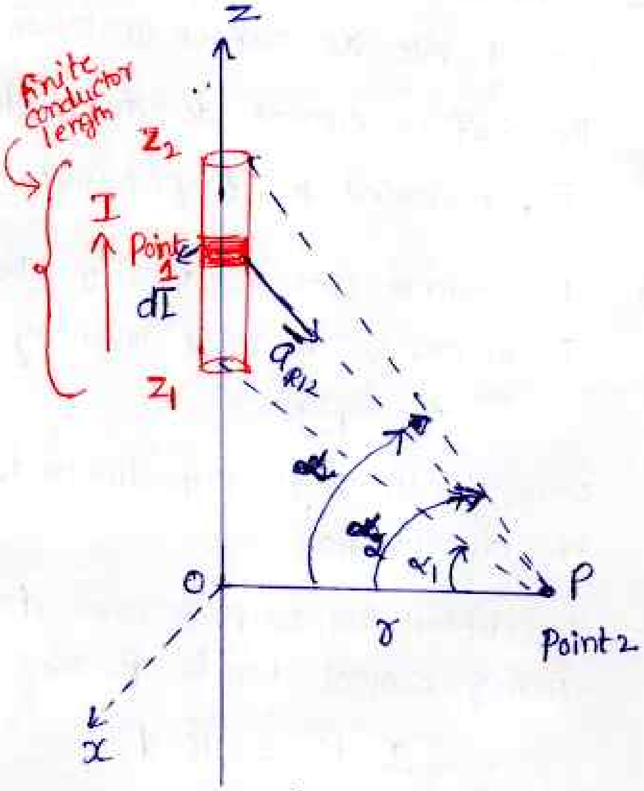
$$\bar{H} = \int_S \frac{\bar{K} \times \bar{a}_R}{4\pi R^2} \quad A/m \longrightarrow (11)$$

and

$$\bar{H} = \int_{vol} \frac{\bar{J} \times \bar{a}_R}{4\pi R^2} \quad A/m \longrightarrow (12)$$

The Biot savart law is also called "Ampere's Law for current Element."

→ Magnetic field Intensity ($\bar{H}$) due to straight conductor of finite length:-



fig(1):

→ consider a conductor of finite length placed along z axis as shown in fig.(1).

→ It carries a direct current I.
The perpendicular distance of point P from z axis is r. as shown in fig.

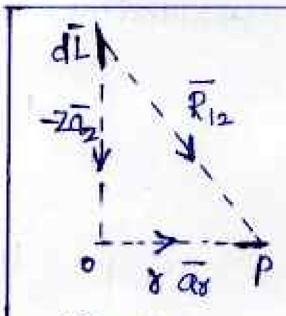
→ The conductor is placed such that its one end is at $z = z_1$ and other end is at $z = z_2$.

consider a differential length $d\bar{L}$ along z axis at a distance z from origin.

$$d\bar{L} = dz\, \bar{a}_z \longrightarrow (1).$$

The unit vector in the direction joining differential element to point P is $\bar{a}_{R12}$.

$$\bar{a}_{R12} = \frac{\bar{R}_{12}}{|\bar{R}_{12}|} = \frac{-z\,\bar{a}_z + r\,\bar{a}_r}{\sqrt{(-z)^2 + (r^2)}}$$



Fig(2)

$$\bar{a}_{R12} = \frac{r\,\bar{a}_r - z\,\bar{a}_z}{\sqrt{r^2+z^2}} \longrightarrow (2).$$

For cross Product, Neglect $|\bar{R}_{12}|$ term in above Eq (2). i.e., neglect $\sqrt{r^2+z^2}$ for convenience and must be considered for further calculations.

$$\therefore\ d\bar{L} \times \bar{a}_{R12} = \begin{vmatrix} \bar{a}_r & \bar{a}_\phi & \bar{a}_z \\ 0 & 0 & dz \\ r & 0 & -z \end{vmatrix}$$

$$= r\,dz\,\bar{a}_\phi \longrightarrow (3)$$

$\therefore\ I\, dL \times \bar{a}_{R12} = \dfrac{I\, r dz\, \bar{a}_\phi}{\sqrt{r^2+z^2}} \longrightarrow (4)$

According to Biot savart law, $d\bar{H}$ at point P is,

$$d\bar{H} = \frac{I\, d\bar{L} \times \bar{a}_{R12}}{4\pi R_{12}^2}$$

$$= \frac{I\, r\, dz\, \bar{a}_\phi}{4\pi\left(\sqrt{r^2+z^2}\right)^2\left(\sqrt{r^2+z^2}\right)}$$

$$d\bar{H} = \frac{I r\, dz\, \bar{a}_\phi}{4\pi\,(r^2+z^2)^{3/2}} \longrightarrow (5)$$

The total $\bar{H}$ at P due to conductor of finite length can be obtained by integrating $d\bar{H}$ over $Z = Z_1$ to $Z = Z_2$.

$$\bar{H} = \int_{Z_1}^{Z_2} d\bar{H} = \int_{Z_1}^{Z_2} \frac{I r\, dz\, \bar{a}_\phi}{4\pi\,(r^2+z^2)^{3/2}} \longrightarrow (6)$$

use $z = r\tan\alpha$, $\quad z^2 = r^2\tan^2\alpha$.

$dz = r\sec^2\alpha\, d\alpha$.

for $z = Z_1$; $\left.\begin{array}{l} Z_1 = r\tan\alpha_1 \\ Z_2 = r\tan\alpha_2 \end{array}\right\}$ from fig (1).

$$\left.\begin{array}{l} \alpha_1 = \tan^{-1}\left(\frac{Z_1}{r}\right) \\ \alpha_2 = \tan^{-1}\left(\frac{Z_2}{r}\right) \end{array}\right\} \longrightarrow (7)$$

$$\bar{H} = \int_{\alpha_1}^{\alpha_2} \frac{I r\; r\sec^2\alpha\, d\alpha\; \bar{a}_\phi}{4\pi\left[r^2 + r^2\tan^2\alpha\right]^{3/2}} = \int_{\alpha_1}^{\alpha_2} \frac{I\, \cancel{r^2}\sec^2\alpha\, d\alpha\, \bar{a}_\phi}{4\pi\, \cancel{r^3}\left[1+\tan^2\alpha\right]^{3/2}}$$

$$= \int_{\alpha_1}^{\alpha_2} \frac{I\sec^2\alpha\, d\alpha\, \bar{a}_\phi}{4\pi r\,(\sec^2\alpha)^{3/2}} = \int_{\alpha_1}^{\alpha_2} \frac{I\sec^2\alpha\, d\alpha\, \bar{a}_\phi}{4\pi r\sec^3\alpha}$$

$$\bar{H} = \int_{\alpha_1}^{\alpha_2} \frac{I \, d\alpha \, \bar{a}_\phi}{4\pi r \sec\alpha}$$

$$= \frac{I}{4\pi r} \int_{\alpha_1}^{\alpha_2} \cos\alpha \, d\alpha \;\; \bar{a}_\phi$$

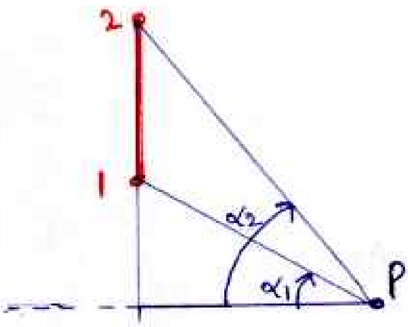
$$= \frac{I}{4\pi r} \left[\sin\alpha\right]_{\alpha_1}^{\alpha_2} \bar{a}_\phi$$

$$\bar{H} = \frac{I}{4\pi r}\left[\sin\alpha_2 - \sin\alpha_1\right] \bar{a}_\phi \;\;, \;\; A/m \rightarrow (8)$$

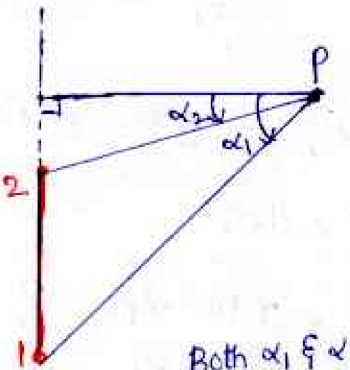
Magnetic flux density $\bar{B} = \mu \bar{H}$

$$\bar{B} = \frac{\mu I}{4\pi r}\left[\sin\alpha_2 - \sin\alpha_1\right] \bar{a}_\phi \;\;, \;\; wb/m^2 \rightarrow (9)$$

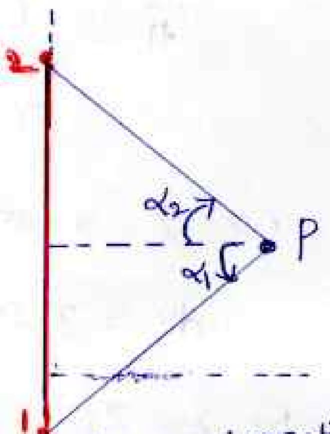
→ Sign convention for α_1 and α_2 :-



Fig(a): Both α_1, α_2 positive



Fig(b): Both α_1 & α_2 negative.

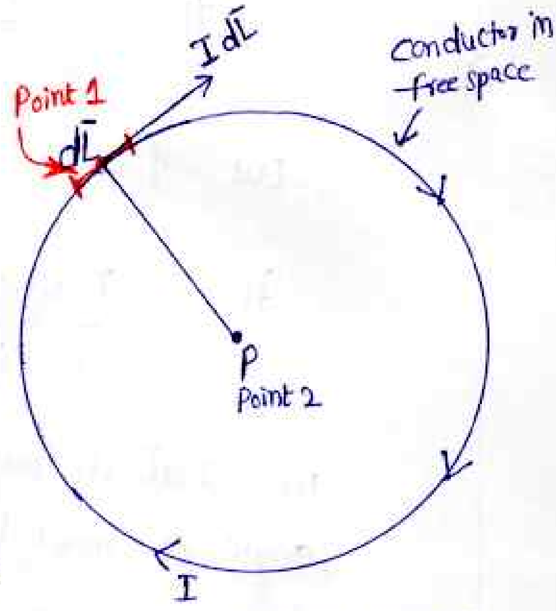


Fig(c): α_1 negative, α_2 positive

If both ends of conductor are above point P, then α_1 & α_2 are positive. If both ends of conductor are below point P, then α_1 & α_2 are negative.

If one end of conductor is above 'P' and other is below 'P' then α_1 is negative & α_2 is positive, as shown in fig above.

→ Magnetic field Intensity $\bar{H}$ at the centre of circular conductor:-



Fig(1): circular conductor

Consider the current carrying conductor arranged in a circular form as shown in fig.

The $\bar{H}$ at the centre of circular loop is to be obtained. The conductor carries the direct current.

consider differential length $d\bar{L}$ at point 1

The Tangential of $d\bar{L}$ at a point 1 is tangential to the circular conductor at point 1.

Let θ = angle between $I\,d\bar{L}$ and $\bar{a}_{R12}$

$\bar{a}_{R12}$ = Unit vector in the direction of $\bar{R}_{12}$

$\bar{R}_{12}$ = Distance Vector Joining differential current element at point 1 to point P at point 2, which is centre of circle.

using the definition of cross product,

$$\therefore I\,d\bar{L} \times \bar{a}_{R12} = I|d\bar{L}|\,|\bar{a}_{R12}| \sin\theta\,\bar{a}_N$$

$$= I\,dL \sin\theta\,\bar{a}_N \rightarrow ① \quad (\because |\bar{a}_{R12}| = 1)$$

Here, $\bar{a}_N$ = unit vector normal to plane containing $d\bar{L}$ and $\bar{a}_{R12}$ i.e., normal to the plane in which circular conductor is lying.

According to Biot savart law,

$$d\bar{H} = \frac{I\,d\bar{L} \times \bar{a}_{R12}}{4\pi R_{12}^2} = \frac{I\,d\bar{L}\sin\theta\,\bar{a}_N}{4\pi R^2} \quad (\because R = R_{12} = \text{radius}) \rightarrow ②$$

Hence Total magnetic field intensity $\bar{H}$ at point P can be obtained by integrating $d\bar{H}$ around circular closed path.

$$\bar{H} = \oint d\bar{H} = \oint \frac{I\,d\bar{L}\sin\theta\,\bar{a}_N}{4\pi R^2}$$

$$\bar{H} = \frac{I \sin\theta \, \bar{a}_N}{4\pi R^2} \oint dL \longrightarrow (3)$$

But $\oint dL$ = Circumference of circle = $2\pi R \longrightarrow (4)$

$$\bar{H} = \frac{I \sin\theta \, 2\pi R \, \bar{a}_N}{4\pi R^2} = \frac{I \sin\theta \, \bar{a}_N}{2R} \longrightarrow (5)$$

As $I d\bar{L}$ is Tangential to the circle, & R_{12} is radius, angle θ must be 90°.

$$\bar{H} = \frac{I \sin 90^\circ \, \bar{a}_N}{2R} = \frac{I}{2R} \bar{a}_N \quad A/m. \longrightarrow (6)$$

$\bar{a}_N = \bar{a}_z$ if the circular loop is placed in xy plane.

Now $\bar{B} = \mu \bar{H}$

$= \mu_0 \mu_r \bar{H}$

$\bar{B} = \mu_0 \bar{H}$ ($\mu_r = 1$ for free space) $\longrightarrow (7)$

∴ Magnetic flux density $\bar{B}$ at centre of circular conductor carrying current I placed in free space is

$$\bar{B} = \frac{\mu_0 I}{2R} \bar{a}_N \quad wb/m^2 \longrightarrow (8).$$

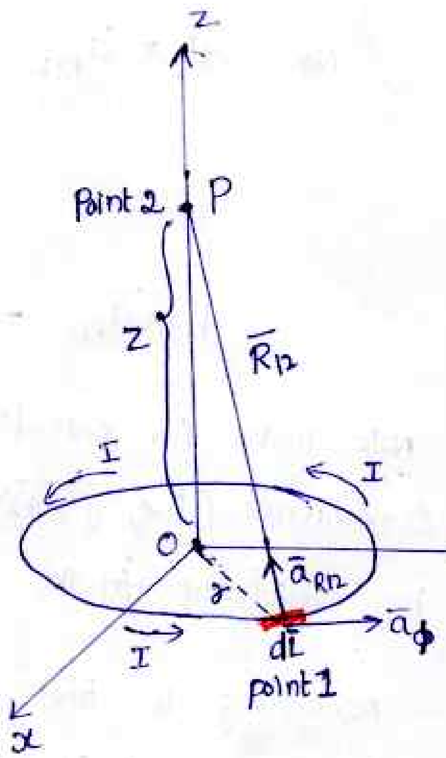
→ Magnetic field Intensity $\bar{H}$ on the axis of a circular loop:

→ Consider a circular loop carrying a direct current I, placed in xy plane, with Z axis as its axis as shown in fig(1).

→ The magnetic field Intensity $\bar{H}$ at point P is to be obtained.

→ The point P is at a distance z from the plane of circular loop, along its axis.

→ The radius of circular loop is r. Consider the differential length $d\bar{L}$ of the circular loop as shown in fig.



Fig(1): Circular loop.

→ In cylindrical co-ordinate system,

$$d\bar{L} = dr\ \bar{a}_r + r\, d\phi\ \bar{a}_\phi + dz\ \bar{a}_z$$

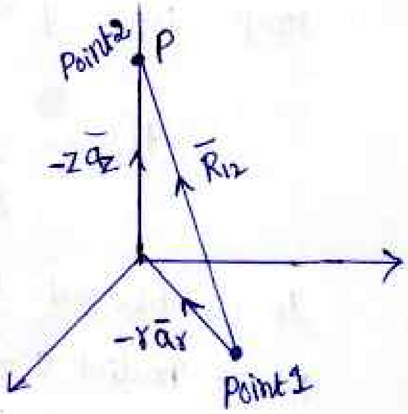
But $d\bar{L}$ is in the plane for which r is constant and z=0 constant plane.

The $Id\bar{L}$ is tangential at point 1 in $\bar{a}_\phi$ direction.

$$I\, d\bar{L} = I r\, d\phi\ \bar{a}_\phi \longrightarrow (1)$$

unit vector $\bar{a}_{R12}$ is in the direction along the line joining differential current element to point P.

$$\bar{a}_{R12} = \frac{\bar{R}_{12}}{|\bar{R}_{12}|}$$



Fig(2)

from fig(2),

$$\bar{R}_{12} = -r\,\bar{a}_r + z\, a_z$$

$$|\bar{R}_{12}| = \sqrt{(-r)^2 + (z)^2} = \sqrt{r^2+z^2}$$

$$\therefore \quad \bar{a}_{R12} = \frac{-r\,\bar{a}_r + z\,\bar{a}_z}{\sqrt{r^2+z^2}} \quad \text{——— (3)}$$

Now
$$d\bar{L} \times \bar{a}_{R12} = \begin{vmatrix} \bar{a}_r & \bar{a}_\phi & \bar{a}_z \\ 0 & r\,d\phi & 0 \\ -r & 0 & z \end{vmatrix}$$

$$d\bar{L} \times \bar{a}_{R12} = z\,r\,d\phi\,\bar{a}_r + r^2\,d\phi\,\bar{a}_z \longrightarrow (4).$$

Note that for calculating cross product, $|\bar{R}_{12}|$ is neglected for convenience (i.e, $\sqrt{r^2+z^2}$ term in denominator of $\bar{a}_{R12}$) and it must be considered in the further calculations.

According to Biot Savart law, the differential field strength $d\bar{H}$ at point P is given by,

$$d\bar{H} = \frac{I\,dL \times \bar{a}_{R12}}{4\pi(R_{12}^2)} = \frac{I\,[z r\,d\phi\,\bar{a}_r + r^2\,d\phi\,\bar{a}_z]}{4\pi\left(\sqrt{r^2+z^2}\right)^2\left(\sqrt{r^2+z^2}\right)}$$

Note ~~that~~ $d\bar{H} = \dfrac{I\,[z r\,\bar{a}_r + r^2\,\bar{a}_z]\,d\phi}{4\pi\,(r^2+z^2)^{3/2}} \longrightarrow (5).$

Total $\bar{H}$ can be obtained by integrating $d\bar{H}$ over circular loop. i.e, $\phi = 0$ to 2π:

$$\bar{H} = \int_{\phi=0}^{2\pi} \frac{I\,[z r\,\bar{a}_r + r^2\,\bar{a}_z]\,d\phi}{4\pi\,(r^2+z^2)^{3/2}} \longrightarrow (6)$$

It is observed that $d\bar{H}$ consists of two components $\bar{a}_r$ and $\bar{a}_z$. Due to radial symmetry all $\bar{a}_r$ components are going to cancel each other. So $\bar{H}$ exists only along the axis in $\bar{a}_z$ direction.

$$\text{so,} \quad \bar{H} = \frac{I}{4\pi} \int_{\phi=0}^{2\pi} \frac{r^2 \, d\phi}{(r^2+z^2)^{3/2}} \, \bar{a}_z$$

$$= \frac{I r^2 \bar{a}_z}{4\pi (r^2+z^2)^{3/2}} \int_{\phi=0}^{2\pi} d\phi$$

$$= \frac{I r^2 \bar{a}_z}{4\pi (r^2+z^2)^{3/2}} \left[\phi\right]_0^{2\pi}$$

$$= \frac{I r^2 \, 2\pi \, \bar{a}_z}{4\pi (r^2+z^2)^{3/2}}$$

$$\bar{H} = \frac{I r^2}{2(r^2+z^2)^{3/2}} \, \bar{a}_z \quad A/m \longrightarrow (7)$$

where r = radius of circular loop

z = Distance of point P along the axis.

Note : If point P is shifted at the centre of circular loop i.e., $Z=0$, we get

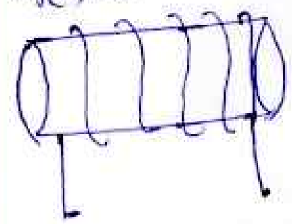
$$\bar{H} = \frac{I r^2}{2(r^2)^{3/2}} \, \bar{a}_z$$

$$\bar{H} = \frac{I}{2r} \, \bar{a}_z \quad A/m.$$

→ Magnetic field Intensity $\bar{H}$ due to Solenoid :-

Solenoid : A solenoid is one in which Each turn of wire looks like a circular current carrying wire.

Fig(a) Solenoid.



from Fig(1);

Let n = No. of Turns / unit length.

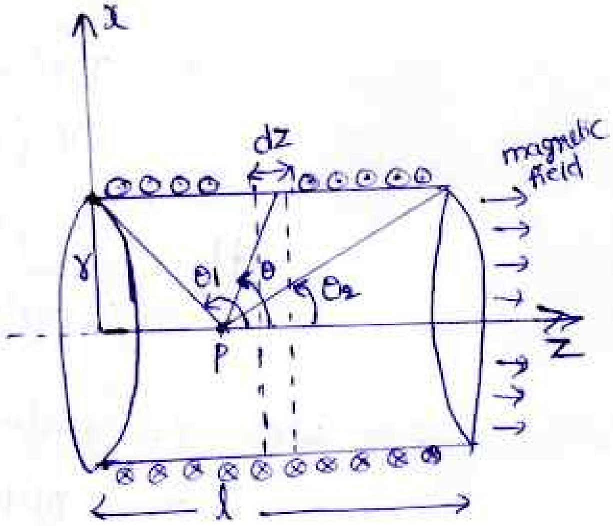
N = Total No. of turns.

l = length of the Solenoid.

$n = N/l$.

r = radius of solenoid.

Now $\bar{H}$ at point P which is on the axis of solenoid is to be obtained.



Fig(1)

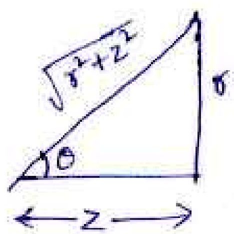
→ For a length of dz, the no. of turns are $n\,dz$.

→ ~~Mag~~ field Intensity $\bar{H}$ due to circular loop is given as

$$\bar{H} = \frac{I r^2}{2(r^2+z^2)^{3/2}}$$

→ $\bar{H}$ due to no. of turns Extended for a length of dz is

$$d\bar{H} = \frac{I r^2}{2(r^2+z^2)^{3/2}} (n\,dz)\, \bar{a}_z$$



Fig(2)

$$\tan\theta = \frac{r}{z}$$

$$z = \frac{r}{\tan\theta} = r\cot\theta$$

$$z = r\cot\theta$$

$$dz = -r\,\text{cosec}^2\theta\, d\theta$$

$$dH = \frac{I r^2 n\left(-r\,\mathrm{cosec}^2\theta\right) d\theta}{2\left(r^2 + r^2\cot^2\theta\right)^{3/2}}\,\bar{a}_z$$

$$= -\frac{I n\, r^3\,\mathrm{cosec}^2\theta\, d\theta}{2r^3\left(1+\cot^2\theta\right)^{3/2}}\,\bar{a}_z$$

$$= \frac{-In\,\cancel{r^3}\,\cancel{\mathrm{cosec}^2}\theta}{2\cancel{r^3}\,\mathrm{cosec}^{\cancel{3}}\theta}\,\bar{a}_z$$

$$d\bar{H} = -\frac{nI}{2}\sin\theta\, d\theta\,\bar{a}_z$$

Integrating on both sides,

$$\bar{H} = -\frac{nI}{2}\int_{\theta=\theta_1}^{\theta=\theta_2}\sin\theta\, d\theta\,\bar{a}_z$$

$$= -\frac{nI}{2}\left[-\cos\theta\right]_{\theta_1}^{\theta_2}\bar{a}_z$$

$$\bar{H} = \frac{nI}{2}\left[\cos\theta_2 - \cos\theta_1\right].\bar{a}_z,\quad A/m$$

Case 1: If point P is exactly at the centre :

$$\bar{H} = \frac{NI}{2}\,\frac{1}{\sqrt{\frac{l^2}{4}+r^2}}\,\bar{a}_z.$$

Case 2: If the solenoid is a long solenoid $(l >> r)$.

$$\bar{H} = \frac{N}{l}I\,\bar{a}_z \quad (\text{or})$$

$$\bar{H} = nI\,\bar{a}_z$$

18 → **Relationship between magnetic flux and Flux Density (or) Maxwell's Second Equation :-**

Magnetic flux density $\bar{B}$ and Magnetic flux intensity $\bar{H}$ are related through the property of medium called permeability μ.
The relation is given by,

$$\bar{B} = \mu \bar{H} \longrightarrow ①$$

For free space, $\mu_r = 1$.

$$\bar{B} = \mu_0 \mu_r \bar{H}$$

$$\therefore \bar{B} = \mu_0 \bar{H} \longrightarrow ②$$

Magnetic flux density $\bar{B}$ is defined as the flux passing through unit area in a plane at right angles to the direction of flux.

$$\bar{B} = \frac{\phi}{S}, \quad \text{weber}/m^2.$$

If the flux passing through the unit area is not exactly at right angles to the plane, but making some angle with the plane, then the flux crossing the area is given by.

$$\phi = \int_S \bar{B} \cdot d\bar{S}, \quad \text{wb.} \longrightarrow ③$$

where ϕ = magnetic flux in webers
$\bar{B}$ = magnetic flux density in wb/m^2 (or) Tesla
$d\bar{S}$ = open surface through which flux is passing.

If now consider a closed surface, which is defining a certain volume. The magnetic flux lines exist in the form of closed loop. Thus for a closed surface the no. of magnetic flux lines entering must be equal to no. of flux lines leaving. No magnetic flux can reside in a closed surface. Hence the integral $\bar{B} \cdot d\bar{S}$ evaluated over closed surface is always zero.

$$\therefore \boxed{\oint_S \bar{B} \cdot d\bar{S} = 0} \longrightarrow ④$$

The above Eq (4) is called law of conservation of magnetic flux (or) Gauss law in Integral form for magnetic field.

Applying divergence theorem, to Eq (4),

$$\oint \bar{B}\cdot d\bar{s} = \int_{Vol} \nabla\cdot\bar{B}\; dv = 0 \longrightarrow (5)$$

where dv = Volume enclosed by closed surface.

In Eq (5), dv is not zero, then we can write

$$\boxed{\nabla\cdot\bar{B} = 0} \longrightarrow (6)$$

The divergence of magnetic flux density is always zero.

This is Gauss's law in differential form (or) point form for magnetic field.

Eq (4) and Eq (6) are called "Maxwell's Second Equation."

Ampere's Circuital Law :-

In electrostatics, Gauss's law is useful to obtain $\bar{E}$ in case of complex problems. Similarly in magnetostatics the complex problems can be solved using a law called Ampere's circuital law.

Statement of Ampere's circuital law (or) Ampere's work law:

It states that, The line integral of magnetic field intensity $\bar{H}$ around a closed path is exactly equal to the direct current enclosed by that path.

$$\boxed{\oint \bar{H}\cdot d\bar{L} = I}$$

This law is very helpful to determine $\bar{H}$ when current distribution is symmetrical.

Proof of Ampere's circuital law :

→ Consider a long straight conductor carrying direct current I placed along Z axis as shown in fig.

→ consider a closed circular path of radius 'r' which encloses the straight conductor carrying direct current I.

→ The point P is at a perpendicular distance r from the conductor.

→ consider $d\bar{L}$ at point P which is in $\bar{a}_\phi$ direction, tangential to circular path at Point P.

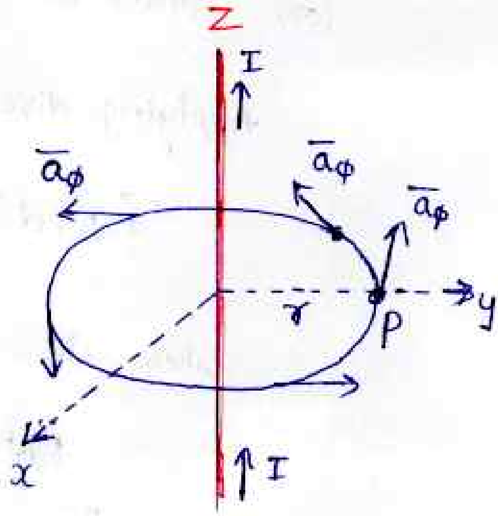


Fig.

$$d\bar{L} = r\, d\phi\ \bar{a}_\phi \longrightarrow (2)$$

From Biot savart law, $\bar{H}$ at point P due to infinitely long conductor is

$$\bar{H} = \frac{I}{2\pi r}\bar{a}_\phi .$$

$$\bar{H}\cdot d\bar{L} = \left(\frac{I}{2\pi r}\bar{a}_\phi\right)\cdot\left(r\, d\phi\ \bar{a}_\phi\right)$$

$$\bar{H}\cdot d\bar{L} = \frac{I}{2\pi r} r\, d\phi = \frac{I}{2\pi} d\phi \longrightarrow (3) \quad (\because \bar{a}_\phi\cdot\bar{a}_\phi = 1)$$

Integrating $\bar{H}\cdot d\bar{L}$ over the entire closed path,

$$\oint \bar{H}\cdot d\bar{L} = \int_{\phi=0}^{2\pi}\frac{I}{2\pi} d\phi = \frac{I}{2\pi}\int_{\phi=0}^{2\pi} d\phi$$

$$= \frac{I}{2\pi}\left[\phi\right]_{\phi=0}^{\phi=2\pi} = \frac{I(2\pi)}{2\pi}$$

$$\Rightarrow \oint \bar{H}\cdot d\bar{L} = I = \text{current carried by conductor.}$$

∴ This proves that the integral $\bar{H}\cdot d\bar{L}$ along closed path gives the direct current enclosed by that path.

Applications of Ampere's circuital law :-

mag. field Intensity $\bar{H}$ can be obtained due to infinitely long straight conductor, Infinite sheet of charge, coaxial cable etc.

1) Magnetic field Intensity due to Infinite sheet of current (or) surface charge :

→ Consider an infinite sheet of current in $z=0$ plane. The surface current density is $\bar{K}$. The current flowing in positive y direction

hence $\bar{K} = K_y \bar{a}_y$. ⟶ ①

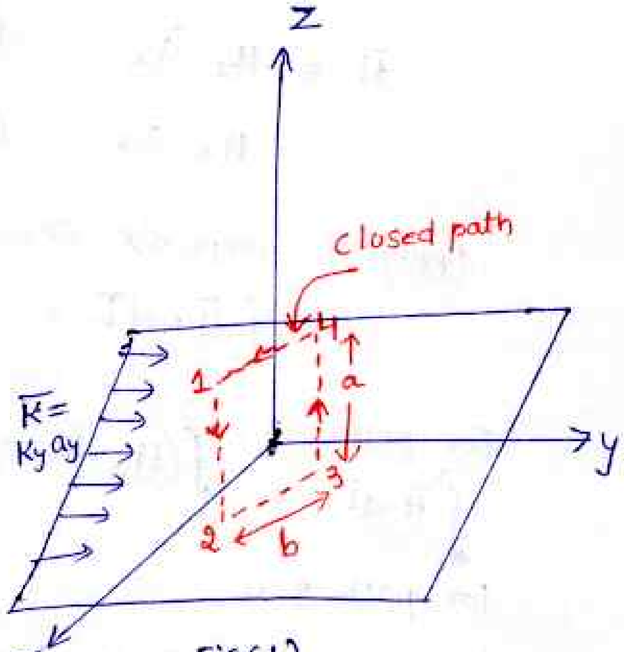
→ consider a closed path 1-2-3-4 as shown in fig(1).

→ The width of the path is 'b', and height is 'a'.

→ It is perpendicular to the direction of current, hence in xz plane.

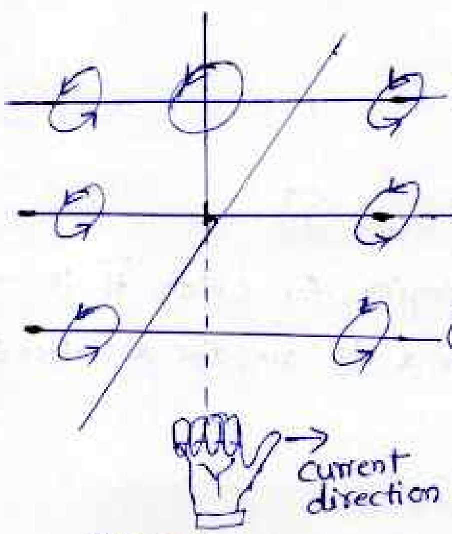
→ The current flowing across the distance b is given by $K_y b$.

$$I_{enc} = K_y b. \longrightarrow ②$$

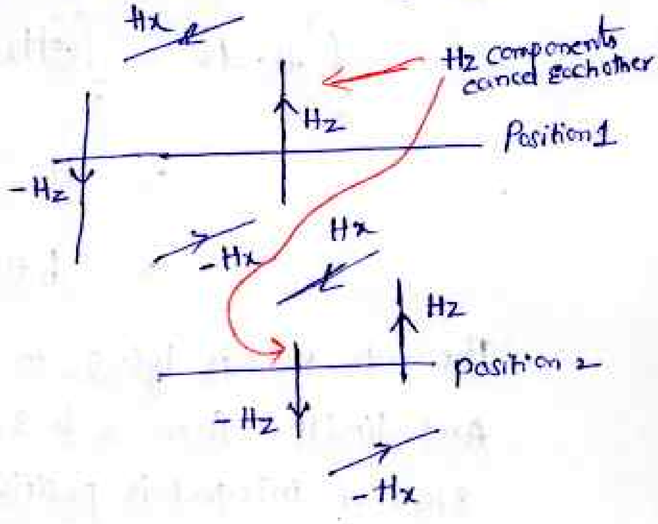


Fig(1)

consider the magnetic lines of forces due to current in $\bar{a}_y$ direction, according to Right hand Thumb rule. These are shown Fig (2).



Fig(2a)



Fig(2b).

→ The components of $\bar{H}$ in z direction directions are oppositely directed ($-H_z$ for position 1 & $+H_z$ for position 1, between two positions). All such components are cancelled each other.

→ Hence $\bar{H}$ cannot have any component in z direction.

→ As current flowing in y-direction, $\bar{H}$ cannot have component in y-direction.

So $\bar{H}$ has only component in x direction.

$$\bar{H} = \begin{cases} H_x \bar{a}_x & \text{for } z > 0 \\ -H_x \bar{a}_x & \text{for } z < 0 \end{cases} \quad (3)$$

Applying Ampere's circuital law,

$$\oint \bar{H} \cdot d\bar{L} = I_{encl.} \longrightarrow (4).$$

For path 1-2,

$$\int_1^2 \bar{H} \cdot d\bar{L} = \int_1^2 (H_x \bar{a}_x) \cdot (dz\, \bar{a}_z) = 0 \rightarrow (5) \quad (\because \bar{a}_x \cdot \bar{a}_z = 0)$$

For path 3-4,

$$\int_3^4 \bar{H} \cdot d\bar{L} = \int_3^4 (H_x \bar{a}_x) \cdot (dz\, \bar{a}_z) = 0 \rightarrow (6) \quad (\because \bar{a}_x \cdot \bar{a}_z = 0)$$

For path 2-3,

$$\int_2^3 H \cdot d\bar{L} = \int_2^3 (-H_x \bar{a}_x)(dx\, \bar{a}_x) \qquad (\because \bar{a}_x \cdot \bar{a}_x = 1)$$

$$= H_x \int_2^3 dx$$

$$= b H_x \longrightarrow (7)$$

The path 2-3 is lying in $z < 0$ region, for which $\bar{H}$ is $-H_x a_x$. And limits from 2 to 3, positive x to negative x. Hence effective sign of integral is positive for path 4-1,

$$\int_4^1 \bar{H}\cdot d\bar{L} = \int_4^1 (H_x \bar{a}_x)\cdot(dx\,\bar{a}_x) = H_x \int_4^1 dx \qquad (\because \bar{a}_x\cdot\bar{a}_x = 1)$$

$$= b\,H_x \longrightarrow (8)$$

$$\therefore \oint \bar{H}\cdot d\bar{L} = \int_1^2 \bar{H}\cdot d\bar{L} + \int_2^3 \bar{H}\cdot d\bar{L} + \int_3^4 \bar{H}\cdot d\bar{L} + \int_4^1 \bar{H}\cdot d\bar{L}$$

$$= 0 + b\,H_x + 0 + b\,H_x.$$

$$\oint \bar{H}\cdot d\bar{L} = 2\,b\,H_x \longrightarrow (9).$$

Eq (8) & (9).

$$2b\,H_x = I_{enc} \longrightarrow (10)$$

Eq (10) and Eq (2)

$$2\cancel{b}\,H_x = K_y \cancel{b}$$

$$\Rightarrow 2H_x = K_y$$

$$H_x = \frac{1}{2}K_y$$

Hence
$$\boxed{\begin{aligned}\bar{H} &= \frac{1}{2}K_y\,\bar{a}_x, \quad \text{for } z > 0. \longrightarrow 11(a)\\ &= -\frac{1}{2}K_y\,\bar{a}_x, \quad \text{for } z < 0. \longrightarrow 11(b).\end{aligned}}$$

$\therefore$ In general, for Infinite sheet of current density $\bar{K}$, we can write,

$$\boxed{\bar{H} = \frac{1}{2}\bar{K}\,\bar{a}_N}$$

where $\bar{a}_N$ = Unit Vector normal from the current sheet to point at which $\bar{H}$ is to be obtained.

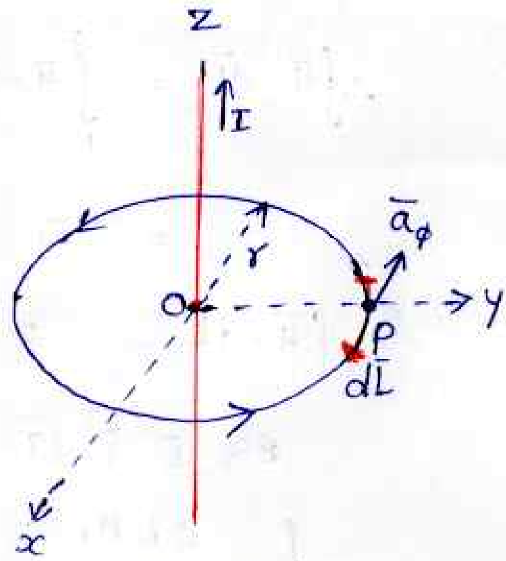
(2) Magnetic field Intensity ($\bar{H}$) due to Infinitely long straight conductor (or) long current carrying Filament :-

→ consider a Infinitely long straight conductor placed along z-axis, carrying a direct current I as shown in fig(1)

→ consider amperian closed path, enclosing conductor as shown in fig.

→ consider point P on closed path at which $\bar{H}$ is to be obtained.

→ The radius of path is r and hence 'P' is at a perpendicular distance 'r' from the conductor.



Fig(1).

→ The magnitude of $\bar{H}$ depends on r and direction is always tangential to closed path i.e., $\bar{a}_\phi$.

So $\bar{H}$ has only component in $\bar{a}_\phi$ direction, say H_ϕ.

→ Consider elementary length $d\bar{L}$ at point P and in cylindrical coordinate it is $r\,d\phi$ in $\bar{a}_\phi$ direction.

$$\bar{H} = H_\phi \bar{a}_\phi \quad \& \quad d\bar{L} = r\,d\phi\,\bar{a}_\phi.$$

$$\bar{H}\cdot d\bar{L} = (H_\phi \bar{a}_\phi)\cdot(r\,d\phi\,\bar{a}_\phi)$$

$$= H_\phi\, r\, d\phi. \qquad (\because \bar{a}_\phi \cdot \bar{a}_\phi = 1) \rightarrow (1)$$

According to Ampere circuital law,

$$\oint \bar{H}\cdot d\bar{L} = I$$

From Eq (1), $\oint H_\phi r\, d\phi = I$

$$\therefore \int_{\phi=0}^{2\pi} H_\phi \, r \, d\phi = I.$$

$$H_\phi \, r \int_{\phi=0}^{2\pi} d\phi = I.$$

$$H_\phi \, r \, [\phi]_0^{2\pi} = I.$$

$$H_\phi \, r \, [2\pi] = I$$

$$H_\phi = \frac{I}{2\pi r}$$

Hence $\bar{H}$ at point P is given by

$$\boxed{\bar{H} = H_\phi \, \bar{a}_\phi = \frac{I}{2\pi r} \bar{a}_\phi \, , \quad A/m}$$

→ **Point form of Ampere's Circuital Law (or) Maxwell's Third Equation :-**

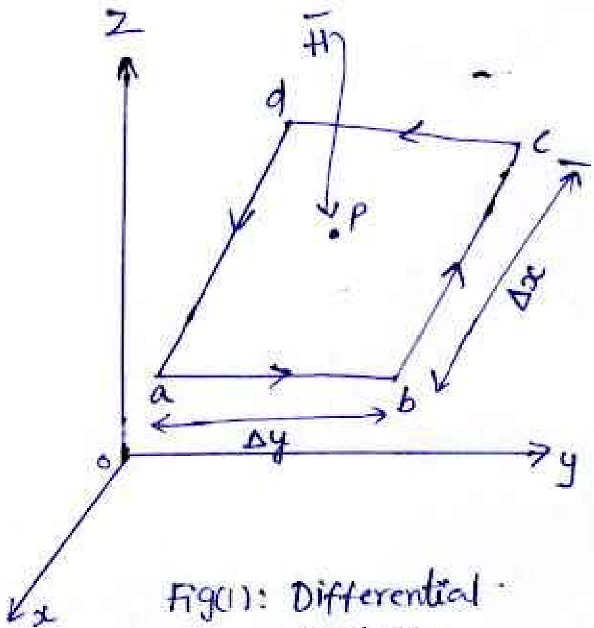
Ampere circuital law is

$$\oint \bar{H} \cdot d\bar{l} = I.$$

→ Consider the differential surface elements having sides Δx & Δy plane as shown in Fig.

→ The unknown current has produced $\bar{H}$ at the centre of this incremental closed path.

The total magnetic field Intensity $\bar{H}$ at point P which is centre of the small rectangle is



Fig(1): Differential surface element.

$$\bar{H} = H_{x0} \, \bar{a}_x + H_{y0} \, \bar{a}_x + H_{z0} \, \bar{a}_z \longrightarrow (1)$$

The total current density is

$$\bar{J} = J_x \bar{a}_x + J_y \bar{a}_y + J_z \bar{a}_z \longrightarrow (2)$$

To apply Ampere's circuital law to this closed path a-b-c-d-a, let us evaluate the closed line integral of $\bar{H}$.

According to right hand thumb rule, the current is in $\bar{a}_z$ direction.

→ Along path a-b, $\bar{H} = H_y a_y$ and $d\bar{L} = \Delta y \, \bar{a}_y$.

$$\therefore \left(\bar{H} \cdot d\bar{L}\right)_{a-b} = (H_y \bar{a}_y) \cdot (\Delta y \, \bar{a}_y) = H_y \Delta y \longrightarrow (3).$$

H_y can be expressed in terms of H_{yo} existing at P and rate of change of H_y in x-direction with x. The distance in x direction of a-b from point P is $\left(\frac{\Delta x}{2}\right)$.

$$\therefore H_y = \left[H_{yo} + \frac{\Delta x}{2} \frac{\partial H_y}{\partial x}\right]$$

$$\left(\bar{H} \cdot d\bar{L}\right)_{a-b} = \left[H_{yo} + \frac{\Delta x}{2} \frac{\partial H_y}{\partial x}\right] \Delta y \longrightarrow (4).$$

→ Along path b-c, $\bar{H} = -H_x \bar{a}_x$ and $d\bar{L} = \Delta x \, \bar{a}_x$.

Here $\bar{H}$ is in $-\bar{a}_x$ direction.

$$\left(\bar{H} \cdot d\bar{L}\right)_{b-c} = (-H_x \bar{a}_x) \cdot (\Delta x \, \bar{a}_x) = -H_x \Delta x.$$

But H_x can be expressed as

$$H_x = H_{xo} + \frac{\Delta y}{2} \frac{\partial H_x}{\partial y}$$

$$\left(\bar{H} \cdot d\bar{L}\right)_{b-c} = -\left[H_{xo} + \frac{\Delta y}{2} \frac{\partial H_x}{\partial y}\right] \Delta x. \longrightarrow (5)$$

→ Along path c-d, $\bar{H} = -H_y \bar{a}_y$ and $d\bar{L} = \Delta y \, \bar{a}_y$

$$\left(\bar{H} \cdot d\bar{L}\right)_{c-d} = (-H_y \bar{a}_y) \cdot (\Delta y \, \bar{a}_y) = -H_y a_y$$

H_y can be expressed as

$$H_y = H_{yo} - \frac{\Delta x}{2} \frac{\partial H_y}{\partial x}.$$

$$\left(\bar{H}\cdot d\bar{L}\right)_{c-d} = -\left[H_{y0} - \frac{\Delta x}{2}\frac{\partial H_y}{\partial x}\right]\Delta y \longrightarrow (6)$$

→ for path d-a , $\bar{H} = H_x\,\bar{a}_x$ and $d\bar{L} = \Delta x\,\bar{a}_x$.

$$\left(\bar{H}\cdot d\bar{L}\right)_{d-a} = \left(H_x\bar{a}_x\right)\cdot\left(\Delta x\,\bar{a}_x\right) = H_x\,\bar{a}_x$$

H_x can be expressed as

$$H_x = H_{x0} - \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}.$$

$$\left(\bar{H}\cdot d\bar{L}\right)_{d-a} = \left[H_{x0} - \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}\right]\Delta x \longrightarrow (7).$$

Total $\bar{H}\cdot d\bar{L}$ can be obtained by adding eq (4), (5), (6) & (7).

$$\bar{H}\cdot d\bar{L} = \left(\bar{H}\cdot d\bar{L}\right)_{a-b} + \left(\bar{H}\cdot d\bar{L}\right)_{b-c} + \left(\bar{H}\cdot d\bar{L}\right)_{c-d} + \left(\bar{H}\cdot d\bar{L}\right)_{d-a}.$$

$$= \cancel{H_{y0}\,\Delta y} + \frac{\Delta x\,\Delta y}{2}\frac{\partial H_y}{\partial x} - \cancel{H_{x0}\,\Delta x} - \frac{\Delta x\,\Delta y}{2}\frac{\partial H_x}{\partial y}$$

$$- \cancel{H_{y0}\,\Delta y} + \frac{\Delta x\,\Delta y}{2}\frac{\partial H_y}{\partial x} + \cancel{H_{x0}\,\Delta x} - \frac{\Delta x\,\Delta y}{2}\frac{\partial H_x}{\partial y}$$

$$= \cancel{2}\,\frac{\Delta x\,\Delta y}{\cancel{2}}\frac{\partial H_y}{\partial x} - \cancel{2}\,\frac{\Delta x\,\Delta y}{\cancel{2}}\frac{\partial H_x}{\partial y}$$

$$\bar{H}\cdot d\bar{L} = \Delta x\,\Delta y\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]$$

$$\therefore \oint \bar{H}\cdot d\bar{L} = \Delta x\,\Delta y\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]. \longrightarrow (8)$$

The current enclosed = (current density normal to closed path) x (Area of closed path)

$$I_{enc} = J_z\,\Delta x\,\Delta y \longrightarrow (9)$$

where J_z = current density in $\bar{a}_z$ direction

from eq (8) & (9), A/c to Ampere's circuital law. we can write,

$$\frac{\oint \bar{H} \cdot d\bar{L}}{\Delta x \ \Delta y} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z . \longrightarrow (10)$$

∴ This gives accurate result as the closed path shrinks to point i.e, $\Delta x \ \Delta y$ area tends to zero.

$$\lim_{\Delta x \Delta y \to 0} \frac{\oint \bar{H} \cdot d\bar{L}}{\Delta x \ \Delta y} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z . \longrightarrow (11)$$

→ Similarly, considering incremental closed path in yz plane, we get J_x.

$$\lim_{\Delta y \Delta z \to 0} \frac{\oint \bar{H} \cdot d\bar{L}}{\Delta y \ \Delta z} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x . \longrightarrow (12).$$

→ Similarly considering incremental closed path in zx plane, we get J_y.

$$\lim_{\Delta z \Delta x \to 0} \frac{\oint \bar{H} \cdot d\bar{L}}{\Delta z \ \Delta x} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J . \longrightarrow (13).$$

Total $\bar{J}$ can be obtained as

$$\bar{J} = J_x \bar{a}_x + J_y \bar{a}_y + J_z \bar{a}_z .$$

$$= \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right] \bar{a}_x + \left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right] \bar{a}_y + \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right] \bar{a}_z$$

$$\Rightarrow \quad \bar{J} = \text{curl } \bar{H} = \nabla \times \bar{H}$$

$$\therefore \boxed{\text{Curl } \bar{H} = \nabla \times \bar{H} = \bar{J}} \longrightarrow (14).$$

Eq (14) is called Point form of Ampere's Circuit law.

This eq (14) is called Maxwell's Third Equation.

Magnetic forces

→ **Magnetic force (F_m):** The magnetic force exerted on a charge moving with velocity $\bar{v}$ in a steady magnetic field $\bar{B}$ is given as

$$\bar{F}_m = Q\,\bar{v} \times \bar{B} \text{ . Newton.}$$

→ **Lorentz Force (or) force on a moving point charge :-**

→ A static electric field $\bar{E}$ exerts a force on a static or moving charge Q. Thus according to coulomb's law, the Force $\bar{F}_e$ exerted on electric charge can be obtained as,

$$\bar{F}_e = Q\bar{E} \text{ , Newton} \longrightarrow (1)$$

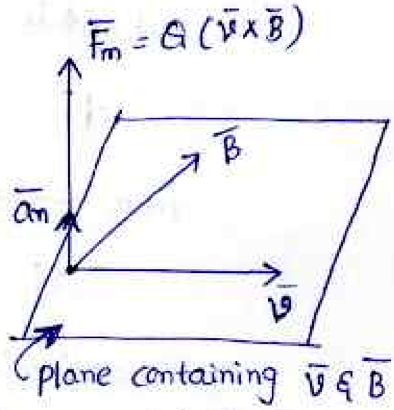
where $\bar{F}_e$ = Electric force.

→ Now consider that a charge is placed in a steady magnetic field. It experiences a force only if it is moving. Then a magnetic force ($\bar{F}_m$) exerted on a charge Q moving with a velocity $\bar{v}$ in a steady magnetic field $\bar{B}$ is given as

$$\bar{F}_m = Q\,\bar{v} \times \bar{B} \text{ Newton} \longrightarrow (2)$$

→ The magnitude of magnetic force $\bar{F}_m$ is directly proportional to magnitudes of Q, $\bar{v}$ and $\bar{B}$ and also sine of the angle between $\bar{v}$ and $\bar{B}$.

→ The direction of $\bar{F}_m$ is perpendicular to plane containing $\bar{v}$ and $\bar{B}$ both as shown in fig.



Here $\bar{a}_n$: Normal to plane

Fig (1) : Magnetic force on a moving charge in magnetic field.

The total force on moving charge in the presence of both electric and magnetic fields is

$$\bar{F} = \bar{F_e} + \bar{F_m}$$

$$= Q\bar{E} + Q\bar{v} \times \bar{B}$$

$$\therefore \bar{F} = Q(\bar{E} + \bar{v} \times \bar{B}) \longrightarrow ③$$

The above Eq ③ is called Lorentz force equation, which relates mechanical force to the electrical force.

~~If~~ the mass of charge is $\bar{m}$, then

$$\therefore \bar{F} = ma = m\frac{dv}{dt} = Q(\bar{E} + \bar{v} \times \bar{B}), \text{ Newton}$$

→ Force on a current Element in a magnetic Field :-

The Force Exerted on a differential Element of charge dQ moving in a steady magnetic field is given by

$$d\bar{F} = dQ\, \bar{v} \times \bar{B} \longrightarrow ①$$

The current density $\bar{J}$ interms of velocity of Volume charge density is

$$\bar{J} = \rho_v \bar{v} \longrightarrow ②$$

But $dQ = \rho_v\, dV \longrightarrow ③$

Substitute Eq ③ in Eq ①

$$d\bar{F} = \rho_v\, dV\, \bar{v} \times \bar{B}$$

~~from Eq ②~~,

$$d\bar{F} = \bar{J} \times \bar{B}\, dV \rightarrow ④ \quad \left(\because \overset{\text{From Eq ②}}{\bar{J} = \rho_v \bar{v}} \right)$$

The relationship between current element & $\bar{J}$ is

$$\bar{J}\, dV = \bar{K}\, dS = I\, d\bar{L} \longrightarrow ⑤$$

Then force exerted on surface current density is given by

$$d\bar{F} = \bar{K} \times \bar{B} \; ds \longrightarrow (6)$$

Similarly the force exerted on differential current element is given by

$$d\bar{F} = I d\bar{L} \times \bar{B} \longrightarrow (7)$$

Integrating eq (4), over a volume, the force is given by

$$\bar{F} = \int_{vol} \bar{J} \times \bar{B} \; dv \longrightarrow (8)$$

Integrating eq (6) over either open or closed surface, the force is

$$\bar{F} = \int_{S} \bar{K} \times \bar{B} \; ds \longrightarrow (9)$$

Integrating eq (7) over a closed path, the force is

$$\bar{F} = \oint I d\bar{L} \times \bar{B} \longrightarrow (10)$$

→ Force on a straight and Long current carrying conductors :-

If the conductor is straight long carrying current I and field $\bar{B}$ is uniform along it, then integrating differential force $d\bar{F}$, we get

$$\oint d\bar{F} = \oint I d\bar{L} \times \bar{B}$$

$$\Rightarrow \bar{F} = I d\bar{L} \times \bar{B}$$

$$\boxed{\bar{F} = I \bar{L} \times \bar{B}}$$

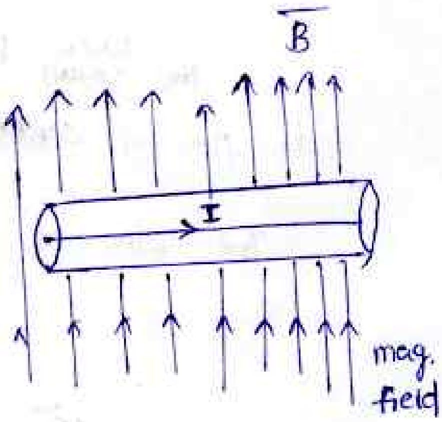


Fig: Straight & long current carrying conductor.

The magnitude of force is given by

$$\boxed{F = I L B \sin\theta}$$

$$\Rightarrow \boxed{F = BIL \sin\theta}$$

→ Force between Two straight long and Parallel conductors carrying currents :-

→ Consider two straight long & parallel conductors which are separated by distance d.

→ Let two conductors have length 'l' carrying currents I_1 & I_2 shown in fig.

→ Let us assume the current in conductor 1 is moving out and current in conductor 2 is moving in. Thus the two currents are in opposite directions.

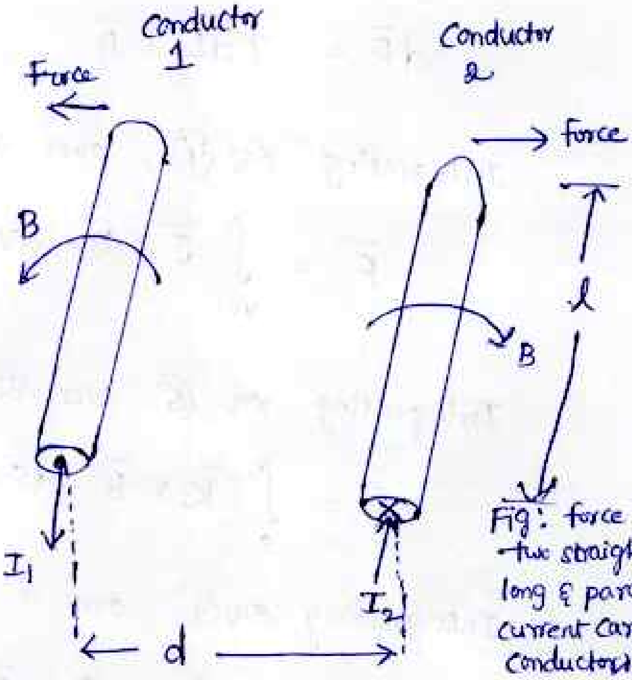


Fig: force b/w two straight long & parallel current carrying conductors.

→ Hence the conductors carrying current in opposite directions are repelled.

The force on conductor 2 of length l is given by

$$d\bar{F_2} = I_2 \left(dl_2 \times \bar{B}_{21} \right).$$

where B_2 is magnetic flux density at conductor 2 due to flow of current I_1 in conductor 1.

The value of $\bar{B}_{21} = \mu_0 H_{21}$

$$\bar{B}_{21} = \frac{\mu_0 I_1}{2\pi d}. \qquad \left(\because H = \frac{I}{2\pi d}\right)$$

$$d\bar{F_2} = I_2 \left(dl \times \frac{\mu_0 I_1}{2\pi d} \right)$$

$$\int d\bar{F_2} = \int I_2 \left(dl \times \frac{\mu_0 I_1}{2\pi d} \right)$$

$$\bar{F_2} = \frac{\mu_0 I_1 I_2}{2\pi d} \int dl$$

$$\Rightarrow \boxed{F_2 = \frac{\mu_0 I_1 I_2 l}{2\pi d}}$$

where μ_0 = permeability of vacuum or air
$= 4\pi \times 10^{-7}$ H/m.

d = separation of conductor.

→ Magnetic Dipole and Dipole moment :

Magnetic Dipole : A bar magnet or a small filamentary current loop is usually referred as a magnetic dipole.

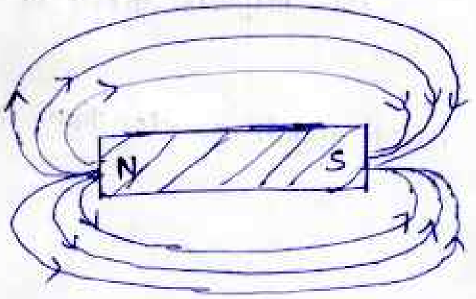


Fig (a): Magnetic field due to small bar magnet.

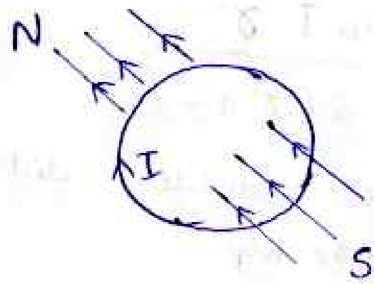


Fig (b): Magnetic field due to small current loop.

magnetic Dipole moment ($\bar{m}$) :- The magnetic dipole moment ($\bar{m}$) is defined as the product of current through the loop and the area of the loop, directed normal to the current loop.

$\bar{m}$ = current through loop × Area of loop

$$= (I \times \pi r^2)\, \bar{a}_n$$

$$= (I \times S)\, \bar{a}_n$$

$$\boxed{\bar{m} = (I S)\bar{a}_n} \quad \text{Ampere - m}^2$$

Using definition of magnetic dipole moment, The Torque can be Expressed as

$$\text{Torque} \quad \boxed{\bar{T} = \bar{m} \times \bar{B}} \quad \text{N-m}$$

→ A differential current loop as a magnetic dipole :-

→ consider a circular current loop with radius 'r', which is carrying a current 'I' in anticlockwise direction.

→ Consider a point P which is 'Z' distance apart from the centre of the loop.

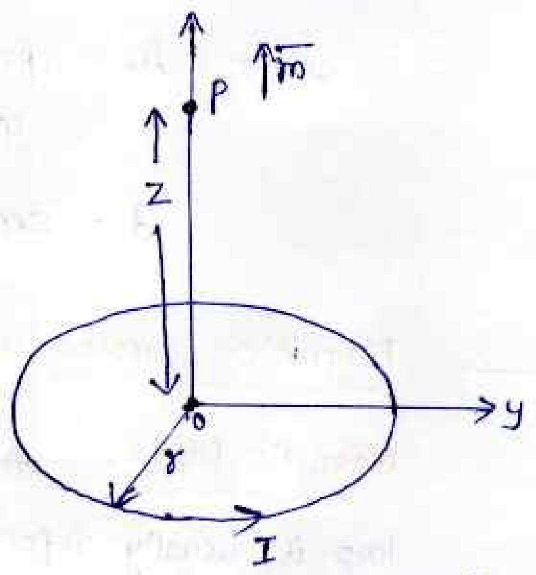


Fig: Magnetic field of current loop.

→ The magnetic field at P is

$$\vec{B} = \frac{\mu_0 I r^2}{2(r^2+z^2)^{3/2}}$$

If now consider distance Z is much much greater than radius r of the loop.

$$B = \frac{\mu_0 I r^2}{2\left[Z^2\left(\left(\frac{r}{Z}\right)^2+1\right)\right]^{3/2}}$$

$$= \frac{\mu_0 I r^2}{2Z^3\left[\left(\frac{r}{Z}\right)^2+1\right]^{3/2}}$$

As $Z >> r$, then above eq written as

$$B = \frac{\mu_0 I r^2}{2Z^3}$$

$$= \frac{\mu_0 I r^2}{2Z^3} \times \frac{2\pi}{2\pi}$$

$$= \frac{\mu_0 2I(\pi r^2)}{4\pi Z^3}$$

Surface Area $S = \pi r^2$

$$B = \frac{\mu_0 2IS}{4\pi z^3} = \frac{\mu_0 2m}{4\pi} \qquad (\because m = IS)$$

Dipole moment m is a vector quantity, the above eq written as

$$\Rightarrow \bar{B} = \frac{\mu_0}{4\pi}\frac{2\bar{m}}{z^3}$$

The above is similar to $\bar{E} = \frac{1}{4\pi\epsilon_0}\frac{2\bar{p}}{r^3}$

$\therefore$ The comparing above two eq's $\bar{B}$ & $\bar{E}$, we can say that $\bar{m}$ is magnetic dipole moment.

→ Torque on a current Loop placed in magnetic field :-

→ consider a differential current loop placed in x-y plane in magnetic field $\bar{B}$.

→ The loop is placed in plane, such that the sides of loop are parallel to axes respectively.

→ Let dx & dy are the lengths of sides of the loop as shown in Fig(1).

→ Assume current in loop flows in anti clock wise direction.

→ Let magnetic field at centre of loop be $\bar{B}_0$.

→ As per the concept of force, the Total force on closed loop is zero.

→ Assume that the origin for the torque is at the centre of the loop.

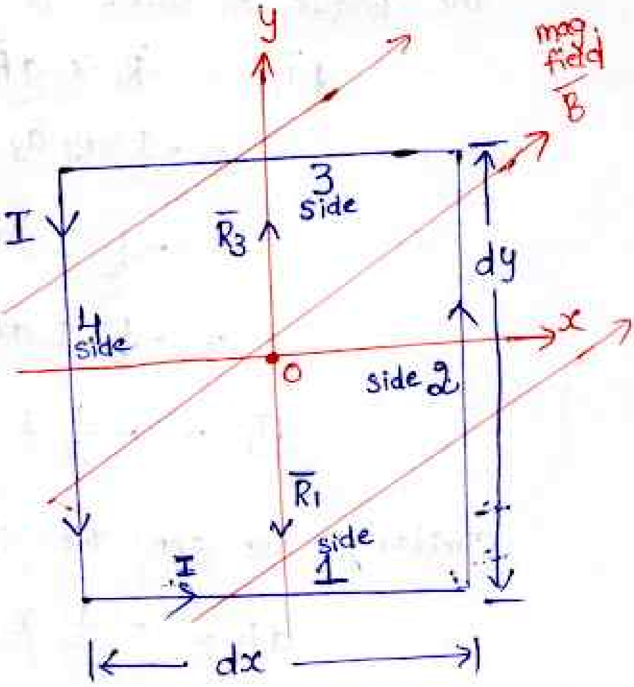


Fig1: A differential current loop in a magnetic field.

Consider side 1 of the differential loop. The differential force exerted on side 1 is given by

$$d\bar{F}_1 = I\, d\bar{L}_1 \times \bar{B}_0 = I\, dx\, \bar{a}_x \times \bar{B}_0 \quad \text{—— (1)}$$

Let $\bar{B}_0 = B_{0x}\,\bar{a}_x + B_{0y}\,\bar{a}_y + B_{0z}\,\bar{a}_z$ —— (2)

On simplification, $d\bar{F}_1$ can be written as

$$d\bar{F}_1 = I\,dx\left[(\bar{a}_x \times \bar{a}_x) B_{0x} + (\bar{a}_x \times \bar{a}_y) B_{0y} + (\bar{a}_x \times \bar{a}_z) B_{0z}\right]$$

$$= I\,dx\left[(0) B_{0x} + a_z \bar{B}_{0y} + (-\bar{a}_y) B_{0z}\right]$$

$$dF_1 = I\,dx\left[B_{0y}\,\bar{a}_z - B_{0z}\,\bar{a}_y\right] \longrightarrow (3)$$

The moment arm for this side 1 is

$$\bar{R}_1 = -\frac{1}{2}\,dy\,\bar{a}_y \longrightarrow (4)$$

The Torque on side 1 is

$$d\bar{T}_1 = \bar{R}_1 \times d\bar{F}_1$$

$$= -\frac{1}{2}\,dy\,\bar{a}_y \times I\,dx\,(B_{0y}\,\bar{a}_z - B_{0z}\,\bar{a}_y)$$

$$= -\frac{1}{2}\,I\,dx\,dy\left[B_{0y}(\bar{a}_y \times \bar{a}_z) - B_{0z}(\bar{a}_y \times \bar{a}_y)\right]$$

$$= -\frac{1}{2}\,I\,dx\,dy\left[B_{0y}\,\bar{a}_x - B_{0z}(0)\right]$$

$$dT_1 = -\frac{1}{2}\,I\,dx\,dy\,B_{0y}\,\bar{a}_x \longrightarrow (5)$$

Similarly we get the Torque on side 3 as

$$dT_3 = -\frac{1}{2}\,I\,dx\,dy\,B_{0y}\,\bar{a}_x \longrightarrow (6)$$

∴ From eq (5) & (6) it is clear that Torque contributions of side 1 & side 3 are same.

Now consider side 2, The torque contributions is

$$d\bar{T}_2 = \bar{R}_2 \times d\bar{F}_2$$

$$= \left(\tfrac{1}{2}dx\,\bar{a}_x\right) \times \left[(I\,dy\,\bar{a}_y) \times (B_{0x}\bar{a}_x + B_{0y}\bar{a}_y + B_{0z}\bar{a}_z) \right]$$

$$= \left(\tfrac{1}{2}dx\,\bar{a}_x\right) \times \left[-I\,dy\,(B_{0x}\bar{a}_z - B_{0z}\bar{a}_x) \right]$$

$$= -\tfrac{1}{2}dx\,dy\;I\left[B_{0x}(\bar{a}_x \times \bar{a}_z) - B_{0z}(\bar{a}_x \times \bar{a}_x) \right]$$

$$= -\tfrac{1}{2}dx\,dy\;I\left[B_{0x}(-\bar{a}_y) - B_{0z}(0) \right]$$

$$d\bar{T}_2 = +\tfrac{1}{2}dx\,dy\;I\,B_{0x}\,\bar{a}_y \longrightarrow (7)$$

Similarly, Torque contribution on side 4 is

$$d\bar{T}_4 = \tfrac{1}{2}dx\,dy\;I\,B_{0x}\,\bar{a}_y \longrightarrow (8)$$

∴ It is observed that Torque contributions on side 2 & side 4 are same

Hence Total torque,

$$d\bar{T} = d\bar{T}_1 + d\bar{T}_2 + d\bar{T}_3 + d\bar{T}_4$$

$$= -\tfrac{1}{2}I\,dx\,dy\,B_{0y}\bar{a}_x + \tfrac{1}{2}dx\,dy\,I\,B_{0x}\bar{a}_y - \tfrac{1}{2}I\,dx\,dy\,B_{0y}\bar{a}_x + \tfrac{1}{2}dx\,dy\,I\,B_{0x}\bar{a}_y$$

$$= -I\,dx\,dy\,B_{0y}\bar{a}_x + I\,dx\,dy\,B_{0x}\bar{a}_y$$

$$d\bar{T} = I\,dx\,dy\,[B_{0x}\bar{a}_y - B_{0y}\bar{a}_x] \longrightarrow (9)$$

But $B_{0x}\bar{a}_y - B_{0y}\bar{a}_x = \bar{a}_z \times (B_{0x}\bar{a}_x + B_{0y}\bar{a}_y + B_{0z}\bar{a}_z)$

$$= \bar{a}_z \times \bar{B}_0 \longrightarrow (10)$$

Substitute Eq (10) in Eq (9)

$$d\bar{T} = I\,dx\,dy\,[\bar{a}_z \times \bar{B}_0] \longrightarrow (11)$$

But we can write, $dx\,dy\,\bar{a}_z = d\bar{S}$, & $\bar{B}_0 = \bar{B}$ ($\bar{B}_0$ is same on every point of loop).

$$d\bar{T} = I\,d\bar{S} \times \bar{B}$$

$$\Rightarrow \bar{T} = I\bar{S} \times \bar{B}$$

$$\boxed{\bar{T} = \bar{m} \times \bar{B}} \longrightarrow (12) \qquad (\because \bar{m} = I\bar{S})$$

→ Classification of magnetic materials :-

On the basis of magnetic behaviour, the magnetic materials are classified as diamagnetic, paramagnetic, ferromagnetic, antiferromagnetic, ferrimagnetic and supermagnetic.

1) **Dia magnetic materials :** The magnetic materials in which the orbital magnetic moment and electron spin magnetic moment cancel each other, making net permanent magnetic moment of each atom zero are called diamagnetic materials.

Ex: Bismuth, lead, copper, silicon, diamond, graphite, sulphur etc

2) **Para magnetic materials :** The magnetic materials in which the orbital and spin magnetic moments donot cancel other, resulting in a net magnetic moment of an atom are called paramagnetic materials.

ex: potassium, tungston, oxygen, rare earth metals.

3) **Ferro magnetic materials:** The materials in which the atoms have large dipole moment due to electron spin magnetic moments are called ferromagnetic materials.

Ex: Iron, nickel, Cobalt

4) **Antiferro magnetic materials :** The materials in which dipole moments of an adjacent atoms line up in antiparallel fashion are called antiferro magnetic materials.

Ex: oxides, chlorides & sulphides at low temperature.

5) **Ferrimagnetic materials :** The materials in which the magnetic dipole moments are lined up in antiparallel fashion, but net magnetic moment is non-zero are called ferrimagnetic materials.

Ex: Nickel ferrite, nickel-zinc-ferrite, iron-oxide-magnetite

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6) Supermagnetic materials: In supermagnetic materials, the ferro magnetic materials are suspended in the dielectric matrix.

ex: magnetic tapes, used for audio, video and data recordings.

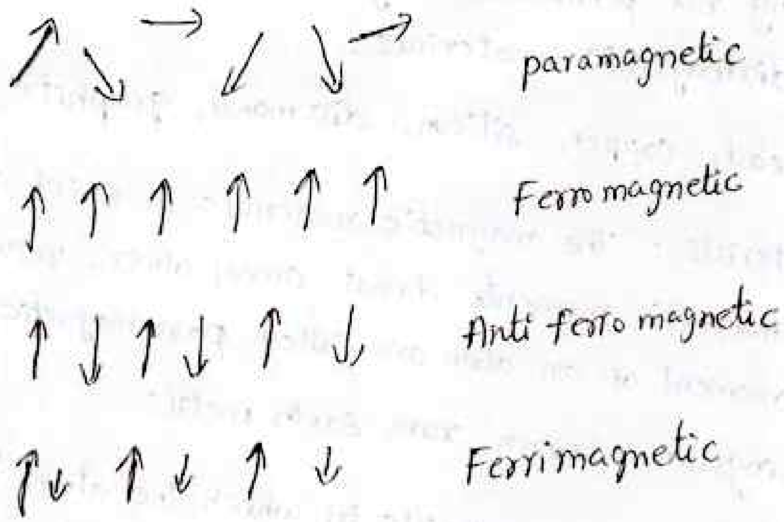
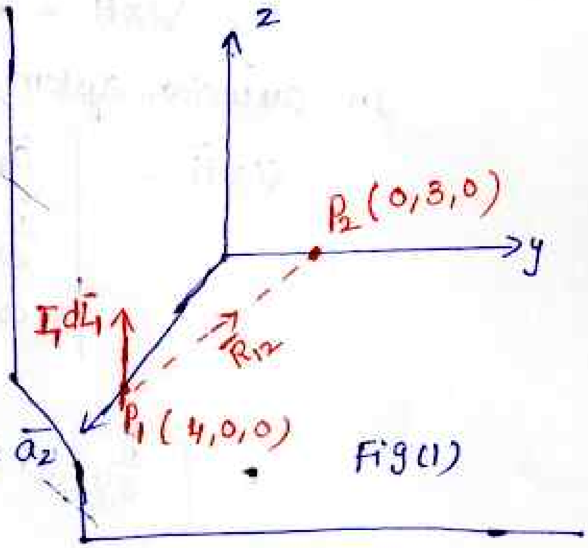


fig: Dipole arrangement in different types of magnetic materials.

Problems :

1) Find the incremental field strength at P_2 due to current element of $2\pi \bar{a}_z$ μAm at P_1. The coordinates of P_1 and P_2 are $(4,0,0)$ and $(0,3,0)$ respectively.

Sol: Two points P_1 & P_2 along with $I_1 d\bar{L}_1$ current element at P_1 is shown in Fig(1).



Fig(1)

According to Biot Savart Law,

$$d\bar{H}_2 = \frac{I_1\, d\bar{L}_1 \times \bar{a}_{R12}}{4\pi R_{12}^2}$$

$$\bar{R}_{12} = \bar{R}_{P2} - \bar{R}_{P1}$$

$$= (0-4)\,\bar{a}_x + (3-0)\,\bar{a}_y + 0\cdot \bar{a}_z$$

$$= -4\,\bar{a}_x + 3\,\bar{a}_y$$

$$\therefore \bar{a}_{R12} = \frac{\bar{R}_{12}}{|\bar{R}_{12}|} = \frac{-4\,\bar{a}_x + 3\,\bar{a}_y}{\sqrt{(4)^2+(3)^2}} = \frac{-4\,\bar{a}_x + 3\,\bar{a}_y}{5}$$

Given $I_1\, d\bar{L}_1 = 2\pi\, \bar{a}_z$

$$\therefore I_1\, d\bar{L}_1 \times \bar{a}_{R12} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 0 & 0 & 2\pi \\ -\frac{4}{5} & \frac{3}{5} & 0 \end{vmatrix}$$

$$= -\frac{4}{5}\, 2\pi\, \bar{a}_y - \frac{3}{5}\, 2\pi\, \bar{a}_x$$

$$= -\frac{2\pi}{5}\left[3\,\bar{a}_x + 4\,\bar{a}_y\right]$$

$$\therefore d\bar{H}_2 = \frac{-\frac{2\pi}{5}\left[3\,\bar{a}_x + 4\,\bar{a}_y\right]}{4\pi(5)^2} = -4\times10^{-3}\left[3\,\bar{a}_x + 4\,\bar{a}_y\right]$$

$$d\bar{H}_2 = -12\,\bar{a}_x - 16\,\bar{a}_y \ \ nA/m.$$

2) A $\bar{H}$ due to current source is given by $\bar{H} = [y \cos x(\alpha x)]\bar{a}_x + (y+e^{x})\,\bar{a}_z$. Describe the current density over yz plane.

Sol: From the point of Ampere's circuital law.

$$\nabla \times \bar{H} = \bar{J}.$$

In cartesian system,

$$\nabla \times \bar{H} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y\cos(\alpha x) & 0 & y+e^{x} \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}\left(y+e^{x}\right)\right]\bar{a}_x + \left[\frac{\partial y\cos(\alpha x)}{\partial z} - \frac{\partial(y+e^{x})}{\partial x}\right]\bar{a}_y$$

$$+ \left[-\frac{\partial}{\partial y}\, y\cos(\alpha x)\right]\bar{a}_z$$

$$\nabla \times \bar{H} = (1)\,\bar{a}_x + (0 - e^{x})\,\bar{a}_y + (-\cos\alpha x)\,\bar{a}_z.$$

$$\Rightarrow \bar{J} = \bar{a}_x - e^{x}\,\bar{a}_y - \cos\alpha x\,\bar{a}_z.$$

on yz plane, $x = 0$

$\therefore \bar{J}$ on yz plane $\Rightarrow \bar{a}_x - e^{0}\,\bar{a}_y - \cos\alpha(0)\,\bar{a}_z$

$$\Rightarrow \bar{a}_x - (1)\bar{a}_y - \cos 0\,\bar{a}_z$$

$$= \bar{a}_x - \bar{a}_y - \bar{a}_z$$

3) ~~Given~~ Find the flux passing the portion of the plane $\phi = \frac{\pi}{4}$ defined by $0.01 < r < 0.05$ m and $0 < z < 2$ m. A current filament of 2.5 A is along the z axis in the $\bar{a}_z$ direction, in free space.

Sol: The arrangement is shown in Fig.

Due to current carrying conductor in free space along z axis $\bar{H}$ is given by,

$$\bar{H} = \frac{I}{2\pi r} \bar{a}_\phi$$

Given $I = 2.5$ A

$$\bar{H} = \frac{2.5}{2\pi r} \bar{a}_\phi$$

$$\bar{H} = \frac{0.3978}{r} \bar{a}_\phi \text{ A/m}$$

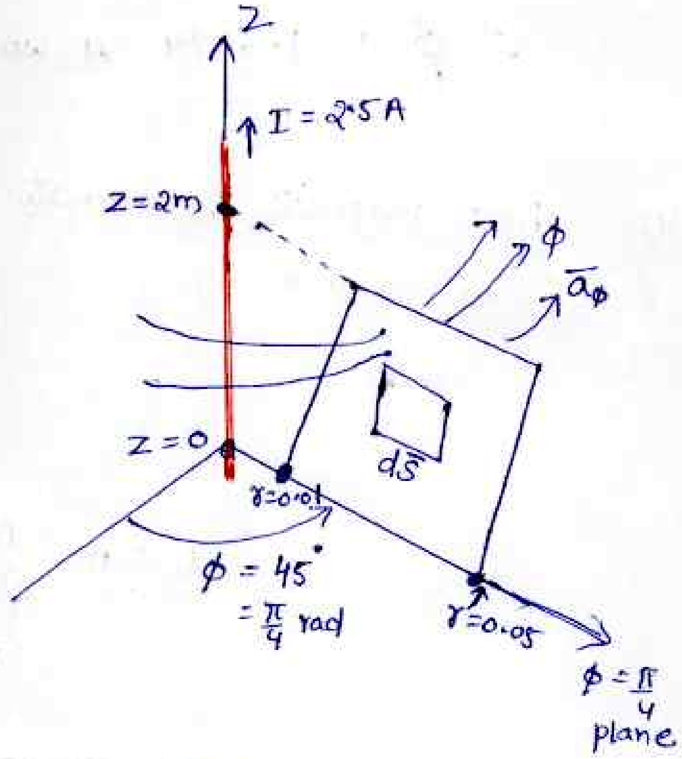


Fig (1).

Magnetic flux density $\bar{B}$ is

$$\bar{B} = \mu_0 \bar{H}$$

$$= 4\pi \times 10^{-7} \times \frac{0.3978}{r} \bar{a}_\phi$$

$$\bar{B} = \frac{5\times10^{-7}}{r} \bar{a}_\phi$$

The flux crossing the surface is $\phi = \int_S \bar{B} \cdot d\bar{s}$

Now $d\bar{s} = dr\, dz\, \bar{a}_\phi$ normal to $\bar{a}_\phi$ direction.

$$\therefore \phi = \int_{z=0}^{2} \int_{r=0.01}^{0.05} \left(\frac{5\times10^{-7}}{r} \bar{a}_\phi \right) \cdot \left(dr\, dz\, \bar{a}_\phi \right)$$

$$= \int_{z=0}^{2} \int_{r=0.01}^{0.05} \frac{5\times10^{-7}}{r}\, dr\, dz$$

$$= 5\times10^{-7} \int_{z=0}^{2} dz \int_{r=0.01}^{0.05} \frac{1}{r}\, dr$$

$$= 5\times10^{-7} \left[z \right]_0^2 \left[\ln r \right]_{0.01}^{0.05}$$

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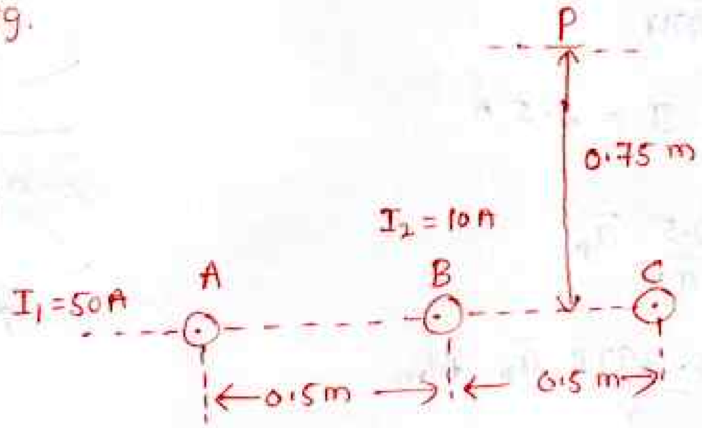
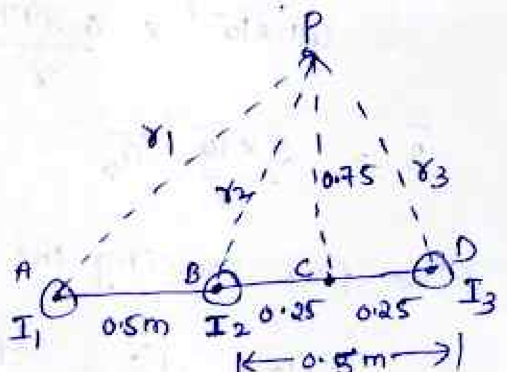
$$= 5\times10^{-7} \ln\left[\frac{0.05}{0.01}\right] [2]$$

$$\therefore \phi = 1.6094 \ \mu wb$$

4) Find magnetic flux density at Point P due to current I_1, I_2 & I_3 shown in fig.



Sol: The magnetic field strength due to straight conductor carrying current I at a point at a distance r from it is given by $\bar{H} = \frac{I}{2\pi r}\bar{a}_\phi$



from Δ^{le} ACP,

$$r_1 = \sqrt{(0.5+0.25)^2+(0.75)^2}$$

$$= \sqrt{(0.75)^2+(0.75)^2}$$

$$= 1.0606$$

from Δ^{le} BCP,

$$r_2 = \sqrt{(0.25)^2+(0.75)^2} = 0.7905$$

from Δ^{le} PCD,

$$r_3 = \sqrt{(0.25)^2+(0.75)^2} = 0.7905$$

All the currents are coming out of paper hence according to right hand Thumb rule, produce $\bar{H}$ in the same direction at P.

$$\bar{H}_p = \bar{H}_1 + \bar{H}_2 + \bar{H}_3$$

$$= \frac{I_1}{2\pi r_1}\bar{a}_\phi + \frac{I_2}{2\pi r_2}\bar{a}_\phi + \frac{I_3}{2\pi r_3}\bar{a}_\phi$$

$$= \left[\frac{50}{2\pi \times 1.0606} + \frac{10}{2\pi \times 0.7905} + \frac{40}{2\pi \times 0.7905}\right]\bar{a}_\phi$$

$$\bar{H}_p = 17.5697\ \bar{a}_\phi .$$

5) A circular loop located on $x^2+y^2=9$, $z=0$ carries a current of 10 A. Determine H at (0, 0, 5) and (0, 0, -5). Taken the direction of current in anticlockwise direction.

Sol: $\bar{H}$ on the axis of a circular loop is given by

$$\bar{H} = \frac{I r^2}{2(r^2+z^2)^{3/2}}\bar{a}_z \quad A/m$$

Here given, $r = 3$, from $x^2+y^2 = (3)^2$

$I = 10$ A.

$z = \pm 5$

for (0, 0, 5), $\bar{H} = \frac{10(3)}{2(3^2+5^2)^{3/2}}\bar{a}_z$

$= 0.227\ \bar{a}_z$

for (0, 0, -5), $\bar{H} = -\frac{10(3)}{2(3^2+(-5)^2)^{3/2}}\bar{a}_z$

$= -0.227\ \bar{a}_z$

6) In cylindrical region $0 < r < 0.5\,m$ the current density is $\bar{J} = 4.5 e^{-2r} \bar{a}_z\ A/m^2$, & $\bar{J} = 0$ elsewhere. Use ampere's law to find $\bar{H}$.

Sol: Given $\bar{J} = 4.5\, e^{-2r}\, \bar{a}_z$

$$ds = r\, dr\, d\phi\ \bar{a}_z$$

$$\int \bar{J}\cdot d\bar{s} = \int_{\phi=0}^{2\pi}\int_{r=0}^{r} (4.5\, e^{-2r}\, \bar{a}_z)\cdot(r\, dr\, d\phi\ \bar{a}_z)$$

$$= \int_{\phi=0}^{2\pi}\int_{r=0}^{r} 4.5\ e^{-2r}\ r\, dr\, d\phi \qquad (\because \bar{a}_z\cdot\bar{a}_z = 1)$$

$$= 4.5 \int_{\phi=0}^{2\pi} d\phi \int_{r=0}^{r} r\ e^{-2r}\ dr$$

$$= 4.5\, [\phi]_0^{2\pi} \int_{r=0}^{r} r\, e^{-2r}\, dr$$

$$= 4.5 \times 2\pi \left\{ r \times \frac{e^{-2r}}{-2} - \int 1 \times \frac{e^{-2r}}{-2}\, dr \right\}_{r=0}^{r}$$

$$= 9\pi \left\{ -\frac{r e^{-2r}}{2} + \frac{1}{2}\left[\frac{e^{-2r}}{-2}\right] \right\}_{r=0}^{r}$$

put $r = 0.5$,

$$= 9\pi \left\{ -\frac{0.5 e^{-1}}{2} - \frac{1}{4} e^{-1} + 0 + \frac{1}{4} \right\}$$

$$= 1.8678$$

But $\int \bar{J}\cdot d\bar{s} = \bar{I} = \int \bar{H}\cdot d\bar{L}$

$\bar{H} = H_\phi\, a_\phi$ and $d\bar{L} = r\, d\phi\, \bar{a}_\phi$

$$\Rightarrow \int \bar{J}\cdot d\bar{s} = \int H_\phi \, r \, d\phi$$

$$\Rightarrow 1.8678 = \int_{\phi=0}^{2\pi} H_\phi \, r \, d\phi$$

$$1.8678 = r H_\phi \int_{\phi=0}^{2\pi} d\phi$$

$$1.8678 = r H_\phi \, [\phi]_0^{2\pi}$$

$$1.8678 = 2\pi r H_\phi$$

$$\Rightarrow H_\phi = \frac{1.8678}{2\pi r}$$

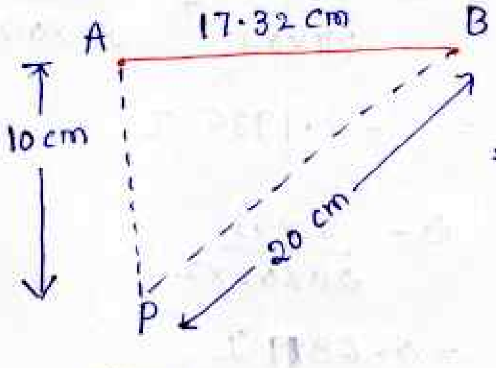
$$\therefore \bar{H} = \frac{0.2972}{r}\,\bar{a}_\phi \quad \text{at} \quad 0 < r < 0.5 \, , \ A/m$$

(or)

$$= 0 \, , \quad \text{Elsewhere} \, , \ A/m.$$

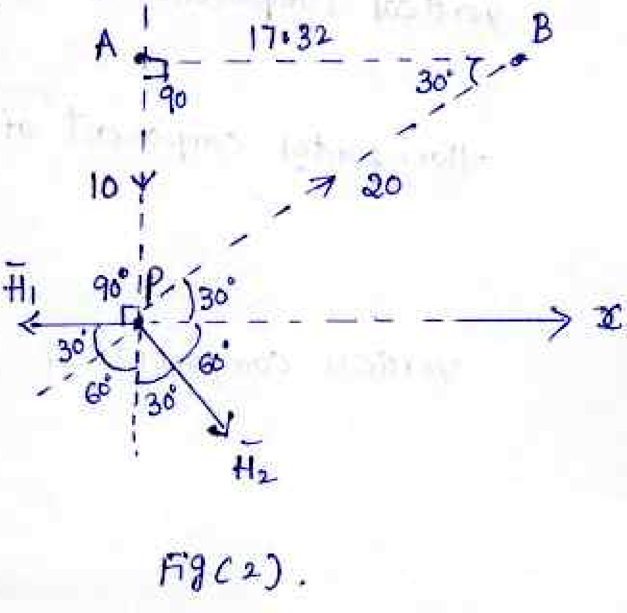
7) Two Infinitely long and parallel wires carry currents $I_1 = I_2 = I$ A in opposite direction. They are separated by a distance of 17.32 cm. If the magnetic field intensity at a point, 10 cm from one wire and 20 cm from other is 13.783 A/m, determine the value of I.

Sol :



Fig(1)

$\Rightarrow$



Fig(2).

The arrangement is shown in fig(1). The two long infinite parallel wires be at A and B. The point P is at a distance of 10 cm from A and 20 cm from B where total magnetic field intensity is $H_t = 13.783$ A/m.

$$H_1 = \frac{I}{2\pi R_1} \quad \& \quad H_2 = \frac{I_2}{2\pi R_2}$$

where $R_1 = 10$ cm, $R_2 = 20$ cm.

The directions of $\bar{H}_1$ & $\bar{H}_2$ are perpendicular to AP & BP respectively as shown in fig(2)., decided by right hand screw rule for the opposite directions of currents.

$$\bar{H}_1 = H_1 \bar{a}_1 \quad \& \quad \bar{H}_2 = H_2 \bar{a}_2$$

where $\bar{a}_1$ & $\bar{a}_2$ are unit vectors along H_1 & H_2.

$$\bar{H}_t = \bar{H}_1 + \bar{H}_2$$

Horizontal component of $\bar{H}_1 = -H_1 = -\frac{I}{2\pi \times 0.1}$

Horizontal component of $\bar{H}_2 = H_2 \cos 60^\circ = \frac{I}{2\pi \times 0.2} \times 0.5$

vertical component of $\bar{H}_1 = 0$.

vertical component of $\bar{H}_2 = -H_2 \sin 60 = -\frac{I}{2\pi \times 0.2} \times \frac{\sqrt{3}}{2}$

Horizontal component of $\bar{H}_t = -\frac{I}{2\pi \times 0.1} + \frac{I \times 0.5}{2\pi \times 0.2}$

$$= -1.1936\, I$$

Vertical component of $\bar{H}_t = 0 - \frac{I \times \sqrt{3}}{2\pi \times 0.2 \times 2}$

$$= -0.6891\, I$$

H_t = Horizontal component of $\bar{H}_t$ + Vertical component of $\bar{H}_t$

$= -1.1936\, I - 0.6819\, I.$

$$|\bar{H}_t| = \sqrt{(1.1936 I)^2 + (0.6819 I)^2} = 13.783 \text{ (given)}$$

$$1.4246\, I^2 + 0.4649\, I^2 = (13.783)^2$$

$$I^2 = 100.53$$

$$I = 10.026.$$

8) If $\bar{H} = x^2 \bar{a}_x + 2yz\, \bar{a}_y + (-x^2)\, \bar{a}_z$ A/m then find the current density at a) 2, 3, 4 b) $r = 6$, $\phi = 45°$, $z = 3$.
c) $r = 3$, $\theta = 60°$, $\phi = 90°$.

Sol: current density, $\bar{J} = \nabla \times \bar{H}$

a) $x = 2$, $y = 3$, $z = 4$.

$$\nabla \times \bar{H} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & 2yz & -x^2 \end{vmatrix}$$

$$= \bar{a}_x \left[\frac{\partial(-x^2)}{\partial y} - \frac{\partial}{\partial z}(2yz) \right] - \bar{a}_y \left[\frac{\partial}{\partial x}(-x^2) - \frac{\partial}{\partial y}(x^2) \right]$$

$$+ \bar{a}_z \left[\frac{\partial}{\partial x}(2yz) - \frac{\partial}{\partial y}(x^2) \right]$$

$$= \bar{a}_x [0 - 2y] - \bar{a}_y [-2x] + \bar{a}_z [0 - 0].$$

$$\bar{J} = -2y\, \bar{a}_x + 2x\, \bar{a}_y \longrightarrow (1)$$

$$\bar{J} = -2(3)\, \bar{a}_x + 2(2)\, \bar{a}_y$$

$$\Rightarrow \bar{J} = -6\, \bar{a}_x + 4\, \bar{a}_y \quad \text{A/m}^2 \longrightarrow (A)$$

b) $r = 6$, $\phi = 45°$, $z = 3$.

Already $\bar{J}$ is obtained in cartesian system in Eq ①, so convert the given cylindrical coordinates to cartesian.

$$x = r\cos\phi = 6 \times \cos 45° = 4.2426$$

$$y = r\sin\phi = 6 \times \sin 45° = 4.2426$$

$$z = z = 3.$$

From Eq ①, $\bar{J} = -2 \times 4.2426\,\bar{a}_x + 2 \times 4.2426\,\bar{a}_y$

$\bar{J} = -8.4852\,\bar{a}_x + 8.4852\,\bar{a}_y$

c) $r = 3$, $\theta = 60°$, $\phi = 90°$.

Convert the given spherical coordinates to cartesian.

$$x = r\sin\theta\cos\phi = 3 \times \sin 60° \cos 90° = 0$$

$$y = r\sin\theta\sin\phi = 3 \times \sin 60 \sin 90° = 2.598$$

$$z = r\cos\theta = 3 \times \cos 60° = 1.5$$

using Eq ①,

$$\bar{J} = -2(2.598)\,\bar{a}_x + 2 \times 0\,\bar{a}_y$$

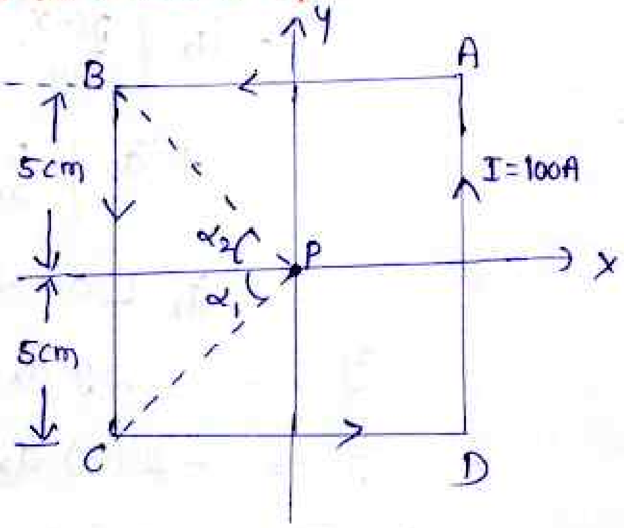
$$= -5.196\,\bar{a}_x \quad A/m^2.$$

9) A wire carrying a current of 100 A is bent into a square of 10 cm side. Calculate the field at the centre of square.

Sol: Consider one side of the square shown in fig., in xy plane.

Consider BC segment, which is finite length of wire.

As B is above point P, α_1 is negative & α_2 is positive.



$$\alpha_1 = -\tan^{-1}\frac{5}{5} = -45^\circ$$

$$\alpha_2 = +\tan^{-1}\frac{5}{5} = +45^\circ$$

$$\bar{H}_1 = \frac{I}{4\pi r}\left[\sin\alpha_2 - \sin\alpha_1\right].$$

$$= \frac{100}{4\pi \times 5\times 10^{-2}}\left[\sin 45^\circ - \sin(-45^\circ)\right]$$

$$= 225.079 \text{ A/m}$$

As the square is in x-y plane, the direction of $\bar{H}$ is normal to x-y plane i.e, $\bar{a}_z$.

$$\bar{H}_1 = 225.079\,\bar{a}_z \text{ A/m}$$

Such 4 sides contribute $\bar{H}$ at the centre of the square i.e, at P.

Hence $\bar{H}_p = 4\bar{H}_1 = 4\left(225.079\,\bar{a}_z\right]$

$$\therefore \bar{H}_p = 900.3163\,\bar{a}_z$$

10) A long straight wire carries a current of $I = 1$ amp. At what distance is magnetic field $H = 1$ A/m.

Sol: Given Current $I = 1$ amp

magnetic field Intensity $H = 1$ A/m.

For Infinite long wire carrying current, the magnetic field Intensity is $\bar{H} = \frac{I}{2\pi r}\bar{a}_\phi$

$$H = \frac{I}{2\pi r}$$

$$r = \frac{I}{2\pi H}$$

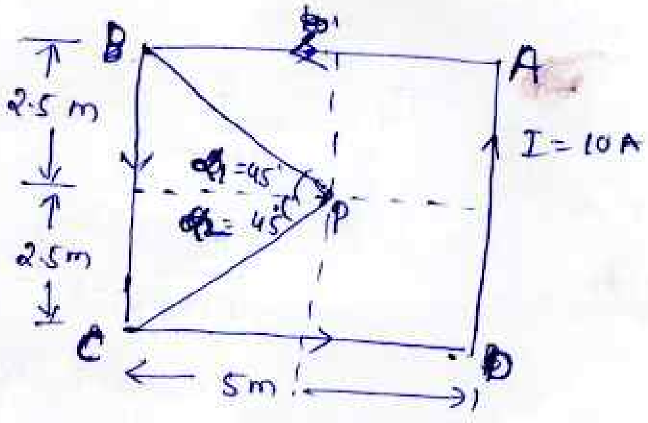
$$\Rightarrow r = \frac{1}{2\pi} \text{ m}$$

11) find the magnetic field Intensity at the Centre of Square loop of 5 m carrying 10 A of current.

Sol: Given $I = 10\ A$

~~distan~~ side of square = 5

$r = \frac{5}{2} = 2.5\ m.$



Consider one side of square as shown in fig.

Consider BC segment which is finite length of wire.

As B is above point P, α_1 is negative, & α_2 is positive

$$\alpha_1 = -\tan^{-1}\frac{2.5}{2.5} = -45^\circ$$

$$\alpha_2 = -\tan^{-1}\frac{2.5}{2.5} = +45^\circ$$

$$\bar{H}_1 = \frac{I}{4\pi r}[\sin\alpha_2 - \sin\alpha_1]$$

$$= \frac{10}{4\pi \times 2.5}[\sin 45^\circ - \sin(-45^\circ)]$$

$\bar{H}_1 =$

As the square is in xy plane, the direction of $\bar{H}$ is normal to $\bar{a}_z$ plane. i-e, $\bar{a}_z$.

$\bar{H}_1 =$ $\bar{a}_z$ A/m

Such 4 sides contribute $\bar{H}$ at the centre of square loop at 'P'

Hence $\bar{H}_P = 4\bar{H}_1$

$= 4[\qquad]$

$\bar{H}_P =$

12) Two long parallel wires separated by 7m apart carry current of 55 A & 105 A respectively in the same direction. Determine the magnitude & direction of force b/w them per unit length.

Sol. force exerted on conductor is $F = \frac{\mu I_1 I_2 l}{2\pi d}$

Given d = distance of separation = 7 m

current I_1 = 55 A

current I_2 = 105 A

l = length of conductors

$$\frac{F}{l} = \frac{\mu_0 \mu_r I_1 I_2}{2\pi d}$$

Assume $\mu_r = 1$, for air.

$$\frac{F}{l} = \frac{4\pi \times 10^{-7} \times 1 \times 55 \times 105}{2\pi \times 7}$$

$$\frac{F}{l} = 1.65 \times 10^{-4} \text{ N/m}$$

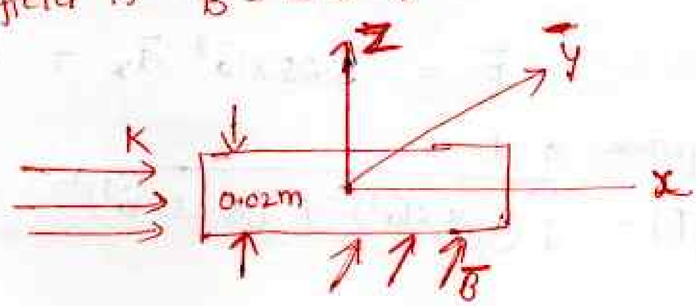
Hence force b/w conductors per unit length is given by

$$\frac{F}{l} = 1.65 \times 10^{-4} \text{ N/m}$$

As the currents in both the conductors are in same direction, The conductors will experience force of attraction.

13) A current strip 2cm wide carries a current of 15A in $\bar{a}_z$ direction, as shown in fig. find the force on the strip of unit length if uniform field is $\bar{B} = 0.2\, \bar{a}_y$ tesla.

S.



Sol: Given Current $I = 15\,\bar{a}_z$

$dL = 2\text{ cm} = 2 \times 10^{-2}\text{ m}$

$\bar{B} = 0.2\,\bar{a}_y\ T$

Force on a strip, $\bar{F} = I\,d\bar{L} \times \bar{B}$

$= 15(2\times10^{-2})\,\bar{a}_z \times 0.2\,\bar{a}_y$

$= 30\times10^{-2} \times 0.2\,(-\bar{a}_x)$ $\left(\because \bar{a}_z \times \bar{a}_y = -\bar{a}_x\right)$

$= 0.06\,(-\bar{a}_x)$

$\therefore \bar{F} = -60\,\bar{a}_x\text{ mN}$

14) Calculate the force on straight conductor of length 30 cm apart carrying current of 5A in $\bar{a}_z$ direction & magnetic field $\bar{B} = 3.5\times10^{-3}(\bar{a}_x - \bar{a}_y)$ Tesla where $\bar{a}_x$ & $\bar{a}_y$ are unit vectors.

Sol: Given $I = 5\text{ A}$

$\bar{B} = 3.5\times10^{-3}(\bar{a}_x - \bar{a}_y)$

$L = 30\text{ cm} = 30\times10^{-2}\text{ m} = 0.3\text{ m}$

$d\bar{L} = 0.3\,\bar{a}_z$

force on a straight conductor, $\bar{F} = I\,d\bar{L}\times\bar{B}$

$\bar{F} = 5(0.3\,\bar{a}_z)\times[3.5\times10^{-3}(\bar{a}_x - \bar{a}_y)]$

$= 5.25\times10^{-3}[\bar{a}_z \times (\bar{a}_x - \bar{a}_y)]$

$= 5.25\times10^{-3}[(\bar{a}_z\times\bar{a}_x) - (\bar{a}_z\times\bar{a}_y)]$

$= 5.25\times10^{-3}[\bar{a}_y - (-\bar{a}_x)]$

$\bar{F} = 5.25\times10^{-3}\,\bar{a}_x + 5.25\times10^{-3}\,\bar{a}_y$

magnitude of force

$\therefore |\bar{F}| = \sqrt{(5.25\times10^{-3})^2 + (5.25\times10^{-3})^2} = 7.4246\times10^{-3}\text{ N}$

15) What is the maximum torque on a square loop of 1000 turns in a field of uniform flux density B Tesla ? The loop has 10 cm side and carries a current of 3A. What is the magnetic moment of the loop ?

Sol: Given $N =$ no. of Turns $= 1000$.

$a =$ side of loop of square shape $= 10\ cm$
$= 10 \times 10^{-2}\ m$

$I =$ current through loop $= 3A$.

1) The magnetic Torque on a single turn loop

$T = BIS$, where $S =$ Area of loop of square shape

For N turns loop, The maximum torque exerted is given by

$$T_{max} = NBIS$$

let $B = 1$ testla.

$$T_{max} = 1000 \times 1 \times 3 \times (10 \times 10^{-2} \times 10 \times 10^{-2}) \quad (\because \text{Area}, S = a^2)$$

$$= 30\ N\ m$$

2) The magnetic moment of a loop is

$m = (IS)\,\bar{a}_n$, where $\bar{a}_n =$ unit vector in direction normal to current loop

$$= 3 \times (10 \times 10^{-2} \times 10 \times 10^{-2})\,\bar{a}_n$$

$$= 0.03\,\bar{a}_n$$

16) A conductor of 5m long lies along z-direction with a current of 2A in $\bar{a}_z$ direction. Find the force experienced by conductor if $\bar{B} = 0.06\,\bar{a}_x$ Tesla.

Sol- Given length of conductor $l = 5m$

current $I = 2A$.

$\bar{B} = 0.06\,\bar{a}_x$

$\therefore$ force Exerted on current carrying conductor in magnetic field,

$\bar{F} = I\,d\bar{L} \times \bar{B}$

$= 2(5\bar{a}_z) \times 0.06\,\bar{a}_x$

$= 10\,(0.06)\,(\bar{a}_z \times \bar{a}_x)$

$\bar{F} = 0.6\,\bar{a}_y$

17) What is the force per meter length between two long parallel wires separated by 10 cm in air and carrying a current of 100 A. in the same direction.

Sol: Given distance $d = 10\text{ cm} = 10 \times 10^{-2}\text{ m}$.

current $I_1 = I_2 = 100\text{ A}$.

for air, $\mu_r = 1$.

Force per meter length is given by

$$\frac{F}{l} = \frac{\mu I_1 I_2}{2\pi d}$$

$$= \frac{\mu_0 \mu_r I_1 I_2}{2\pi d}$$

$$= \frac{4\pi \times 10^{-7} \times 1 \times 100 \times 100}{2\pi \times 10 \times 10^{-2}}$$

$$\therefore \frac{F}{l} = 2000 \times 10^{-5} \quad N/m.$$

$=$