GATE-ES.47

EE23BTECH11046 - Poluri Hemanth*

Question: Second order ordinary differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$ has values $y = 2$ and$\frac{dy}{dx} = 1$ at $x = 0$.The value of y at $x = 1$ is?(*round off to three decimal places*)

Solution:
Let y be:-

$$y = A_1 \cdot e^{s_1 x} + A_2 \cdot e^{s_2 x} \quad (1)$$

where s_1 s_2 *are roots of*$-$

$$s^2 - s - 2 = 0 \quad (2)$$

$$\Rightarrow (s-2)(s+1) = 0 \quad (3)$$

$$s_1 = 2 \ \ and \ \ s_2 = -1 \quad (4)$$

$$y = A_1 \cdot e^{2x} + A_2 \cdot e^{-x} \quad (5)$$

$As \ \ y = 2 \ \ at \ \ x = 0$

$$2 = A_1 + A_2 \quad (6)$$

$$As \ \ \frac{dy}{dx} = 1 \ \ atx = 0$$

$$1 = 2A_1 - A_2 \quad (7)$$

$From \ \ 6 \ \ and \ \ 7$

$$A_1 = 1 \ , A_2 = 0 \quad (8)$$

$$\Rightarrow y = e^{2x} \quad (9)$$

$$\Rightarrow y \ at \ x = 1 \ is \ e^2 \quad (10)$$

$$\Rightarrow y = 7.389$$