

# GATE-ES.47

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**Question:** Second order ordinary differential equation  $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$  has values  $y = 2$  and  $\frac{dy}{dx} = 1$  at  $x = 0$ . The value of  $y$  at  $x = 1$  is? (round off to three decimal places)

**Solution:**

We convert given second order differential equation to s domain using Laplace transform and solve for  $Y(s)$  and take inversion to get  $y(x)$ .

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y \xleftrightarrow{\mathcal{L}} s^2Y(s) - sy(0) - y'(0) - sY(s) + y(0) - 2Y(s) \quad (1)$$

$$y(0) = 2, y'(0) = 1. \quad (2)$$

$$Y(s)(s^2 - s - 2) = 2s - 1 \quad (3)$$

$$Y(s) = \frac{2s - 1}{s^2 - s - 2} \quad (4)$$

$$Y(s) = \frac{1}{s - 2} + \frac{1}{s + 1} \quad (5)$$

For inversion of  $Y(s)$  in partial fractions-

$$\frac{b}{s + a} \xleftrightarrow{\mathcal{L}^{-1}} be^{ax} \quad (6)$$

Where b, a are real numbers, we invert  $Y(s)$  to get  $y(x)$ :-

From (6)

$$Y(s) \xleftrightarrow{\mathcal{L}^{-1}} y(x) \quad (7)$$

$$y(x) = e^{-2x} + e^x \quad (8)$$

$$y(1) = e^{-2} + e \quad (9)$$

$$y(1) = 10.107 \quad (10)$$

Symbol	Values	Description
$Y(s)$	$\frac{2s-1}{s^2-s-2}$	y in s domain
$y(x)$	$e^{-2x} + e^x$	y in x domain

TABLE I  
PARAMETERS