NCERT DISCRETE 11.9.2.15

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Question: If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is A.M between a and b, then find value of n.
Solution: We represent a, b, A.M of those two in an A.P
The general term of A.P used is $x(n) = a + nd$. where

$$d = \frac{b-a}{k+1}\{k \text{ is no of A.M's inserted between } a, b\}$$

$$\Rightarrow d = \frac{b-a}{2} \tag{1}$$

$$x(0) = a \tag{2}$$

$$x(1) = A.M \tag{3}$$

$$x(2) = b \tag{4}$$

$$\Rightarrow x(1) = \frac{x(0)^n + x(2)^n}{x(0)^{n-1} + x(2)^{n-1}} \tag{5}$$

Convolution of $x(n)$ with $u(n)$.

$$x(n) * u(n) \overset{Z}{\longleftrightarrow} X(Z) \tag{6}$$

$$X(Z) = \frac{a}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \tag{7}$$

$$\Rightarrow X(Z) = \frac{az}{z-1} + \frac{d \cdot z}{(z-1)^2} \tag{8}$$

From contour integration method

$$x(n) = \frac{1}{2\pi j}\oint X(Z) z^{n-1}\, dz \tag{9}$$

$$\Rightarrow x(1) = \frac{1}{2\pi j}\oint X(Z)\, dz \tag{10}$$

According to Cauchy's Residue Theorm:
for a y(n) such that, N is no of poles of Y(Z)

$$y(n) = \frac{1}{2\pi j}\oint Y(Z)\, dz = \sum_{i=1}^{N} \text{RES}(Y, a_k) \tag{11}$$

$$\text{RES}(Y, a_k) = \frac{1}{(m-1)!}\lim_{z\to a_k}\frac{d^{m-1}}{dz^{m-1}}[Y(Z)\cdot(z-a_k)^m] \tag{12}$$

From (8)

$$x(1) = \lim_{z\to 1}\frac{az}{z-1}(z-1) + \lim_{z\to 1}\frac{1}{1!}\frac{d}{dz}\left(\frac{d\cdot z}{(z-1)^2}(z-1)^2\right) \tag{13}$$

$$\Rightarrow x(1) = a + d \tag{14}$$

From (1)

$$\Rightarrow x(1) = \frac{a+b}{2} \tag{15}$$

From (5)

$$\frac{x(0)^n + x(2)^n}{x(0)^{n-1} + x(2)^{n-1}} = \frac{x(0)+x(2)}{2} \tag{16}$$

$$\Rightarrow x(0)^n + x(2)^n = x(2)x(0)^{n-1} + x(0)x(2)^{n-1} \tag{17}$$

$$\Rightarrow x(0)^{n-1}(x(0)-x(2)) = x(2)^{n-1}(x(0)-x(2)) \tag{18}$$

$$\Rightarrow n \begin{cases} = 1 & \text{if } x(0) \neq x(2) \\ \in R & \text{if } x(0) = x(2) \end{cases} \tag{19}$$

S/No	Symbol	Values	Description
1	$x(0)$	a	First term of A.P
2	$x(1)$	$\frac{a+b}{2}$	A.M of first and third terms of A.P
3	$x(2)$	b	Third term of A.P

TABLE I
PARAMETERS

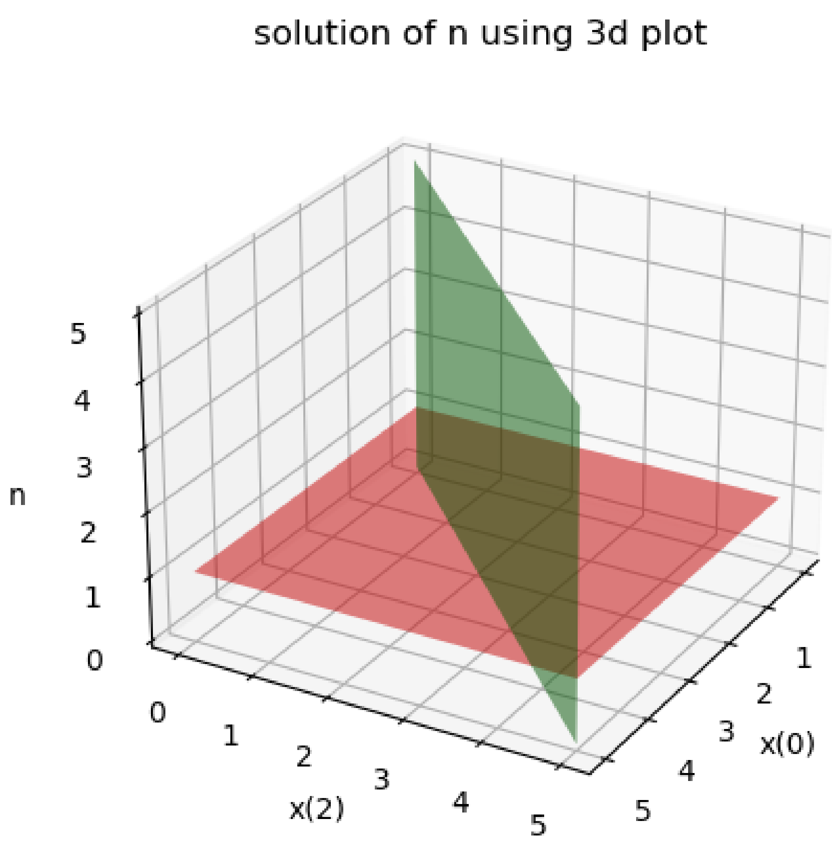

Fig. 1. Plot of n in planes