NCERT DISCRETE 11.9.2.15

EE23BTECH11046 - Poluri Hemanth*

Question: If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is A.M between a and b, then find value of n.

Solution:

S/No	Symbol	Values	Description
1	$x(0)$	a	First term of A.P
2	$x(1)$	$\frac{a+b}{2}$	A.M of first and third terms of A.P
3	$x(2)$	b	Third term of A.P

TABLE I
PARAMETERS

$x(n) = a + nd$. Where,

$$d = \frac{b-a}{k+1}\{k \text{ is no of A.M's inserted between } a, b\} \tag{1}$$

$$= \frac{b-a}{2} \tag{2}$$

$$x(0) = a \tag{3}$$

$$x(1) = A.M \tag{4}$$

$$x(2) = b \tag{5}$$

$$x(1) = \frac{x(0)^n + x(2)^n}{x(0)^{n-1} + x(2)^{n-1}} \tag{6}$$

Using Z transform.

$$x(n) * u(n) \overset{Z}{\longleftrightarrow} X(Z) \tag{7}$$

$$X(Z) = \frac{a}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \tag{8}$$

$$\tag{9}$$

From contour integration method

$$x(n) = \frac{1}{2\pi j}\oint X(Z)z^{n-1}\,dz \tag{10}$$

$$\implies x(1) = \frac{1}{2\pi j}\oint X(Z)\,dz \tag{11}$$

According to Cauchy's Residue Theorm:
For a $y(n)$ such that,

$$y(n) = \frac{1}{2\pi j}\oint Y(Z)\,dz \tag{12}$$

$$= \sum_{i=1}^{N} \text{RES}(Y, a_k)\ [N \text{ is no of poles of } Y(Z)] \tag{13}$$

where,

$$\text{RES}(Y, a_k) = \frac{1}{(m-1)!}\lim_{z\to a_k}\frac{d^{m-1}}{dz^{m-1}}[Y(Z)\cdot(z-a_k)^m] \tag{14}$$

From (8),(13)

$$x(1) = \lim_{z\to 1}\frac{a}{1-z^{-1}}(z-1) + \lim_{z\to 1}\frac{1}{1!}\frac{d}{dz}\left(\frac{d\cdot z^{-1}}{(1-z^{-1})^2}(z-1)^2\right) \tag{15}$$

$$\implies x(1) = a + d \tag{16}$$

From (2)

$$x(1) = \frac{a+b}{2} \tag{17}$$

From (6)

$$\frac{x(0)^n + x(2)^n}{x(0)^{n-1} + x(2)^{n-1}} = \frac{x(0)+x(2)}{2} \tag{18}$$

$$\implies x(0)^n + x(2)^n = x(2)x(0)^{n-1} + x(0)x(2)^{n-1} \tag{19}$$

$$\implies x(0)^{n-1}(x(0)-x(2)) = x(2)^{n-1}(x(0)-x(2)) \tag{20}$$

$$\implies n\begin{cases} = 1 & \text{if } a \neq b \\ \in R & \text{if } a = b \end{cases} \tag{21}$$

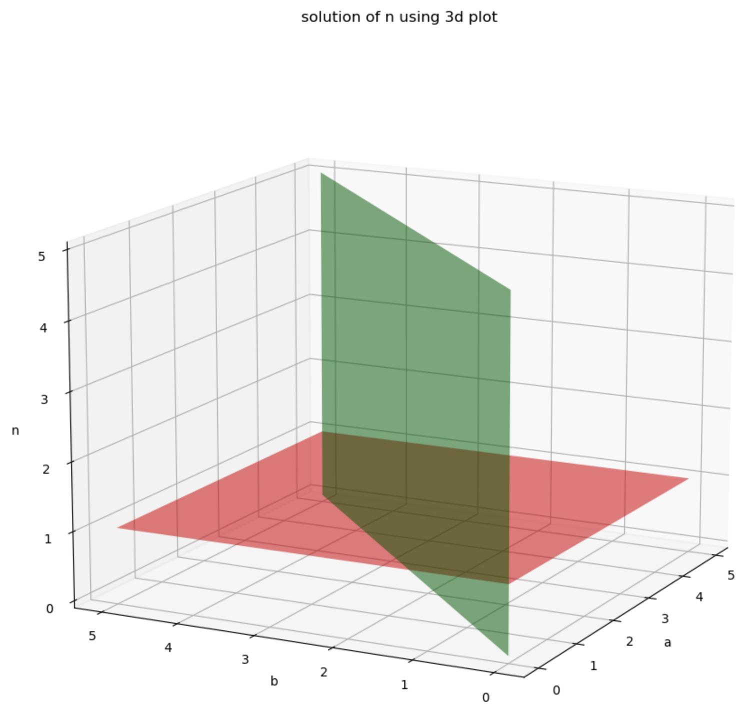

Fig. 1. Plot of n in planes