

# GATE-ES.47

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**Question:** Second order ordinary differential equation  $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$  has values  $y = 2$  and  $\frac{dy}{dx} = 1$  at  $x = 0$ . The value of  $y$  at  $x = 1$  is? (round off to three decimal places)  
[GATE-ES 2023]

**Solution:**

We convert given second order differential equation to s domain using Laplace transform and solve for  $Y(s)$  and take inversion to get  $y(x)$ .

Symbol	Values	Description
$Y(s)$	$\frac{2s-1}{s^2-s-2}$	y in s domain
$y(x)$	$e^{2x} + e^{-x}$	y in x domain
$y(0)$	2	y at $x = 0$
$y'(0)$	1	$y'(x)$ at $x = 0$

TABLE I  
PARAMETERS

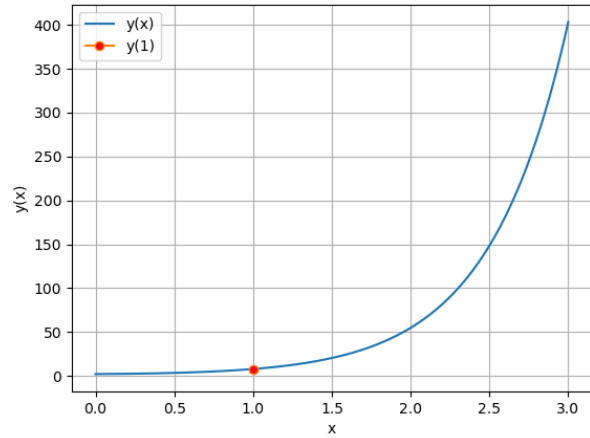


Fig. 1. Plot of  $y(x)$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y \xleftrightarrow{\mathcal{L}} s^2Y(s) - sy(0) - y'(0) - sY(s) + y(0) - 2Y(s) \quad (1)$$

$$Y(s)(s^2 - s - 2) = 2s - 1 \quad (2)$$

$$\Rightarrow Y(s) = \frac{2s - 1}{s^2 - s - 2} \quad (3)$$

$$\Rightarrow Y(s) = \frac{1}{s - 2} + \frac{1}{s + 1} \quad (4)$$

For inversion of  $Y(s)$  in partial fractions-

$$\frac{b}{s + a} \xleftrightarrow{\mathcal{L}^{-1}} be^{-ax} \quad (5)$$

Where  $b, a$  are real numbers, we invert  $Y(s)$  to get  $y(x)$ :-

From (5)

$$Y(s) \xleftrightarrow{\mathcal{L}^{-1}} y(x) \quad (6)$$

$$y(x) = e^{2x} + e^{-x} \quad (7)$$

$$\Rightarrow y(1) = 7.757 \quad (8)$$