NCERT DISCRETE 11.9.2.15

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Question: if $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$is A.M between a and b, then find value of n.

Solution:A.M of two numbers is average of those two.Numbers a,b,and their A.M can be represented as three consecutive terms of an A.P $x(m)$.
So Let,

$$x(0) = a \tag{1}$$
$$x(1) = A.M \tag{2}$$
$$x(2) = b \tag{3}$$
$$Now\ common\ difference\ is\ \frac{b-a}{2} \tag{4}$$
$$\Rightarrow x(m) = a + m \cdot \frac{b-a}{2} \tag{5}$$

$$\frac{x(0)^n + x(2)^n}{x(0)^{n-1} + x(2)^{n-1}} = \frac{x(0) + x(2)}{2} \tag{6}$$
$$2(x(0)^n + x(2)^n) = x(0)^n + x(2)^n + x(2).x(0)^{n-1} + x(0).x(2)^{n-1} \tag{7}$$
$$x(0)^n + x(2)^n = x(2).x(0)^{n-1} + x(0).x(2)^{n-1} \tag{8}$$
$$x(0)^{n-1}.(x(0) - x(2)) = x(2)^{n-1}(x(0) - x(2)) \tag{9}$$

For $x(0) \neq x(2)$
$x(0)^{n-1}$=$x(2)^{n-1}$
$\Rightarrow$ n=1.
For $x(0) = x(2)$
$\Rightarrow n \in$ R i.e n is a real value.

Columns	Symbol	Values	Description
1	$x(m)$	$a + m \cdot \frac{b-a}{2}$	General term of A.P
2	$x(0)$	a	First term of A.P
3	$x(1)$	$\frac{a+b}{2}$	A.M of first and third terms of A.P
4	$x(2)$	b	Third term of A.P

TABLE I
PARAMETERS