NCERT DISCRETE 11.9.2.15

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Question: If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is A.M between a and b, then find value of n.

Solution:

Symbol	Values	Description
$x(0)$	a	First term of A.P
$x(1)$	$\frac{a+b}{2}$	A.M of first and third terms of A.P
$x(2)$	b	Third term of A.P
k	1	No of A.M's inserted between a,b

TABLE I
PARAMETERS

A.M of two numbers a,b is $\frac{a+b}{2}$.

$$x(n) = x(0) + n \cdot d \cdot u(n) \tag{1}$$

Where,

$$x(1) = \frac{x(0)^n + x(2)^n}{x(0)^{n-1} + x(2)^{n-1}} \tag{2}$$

$$= \frac{a+b}{2} \tag{3}$$

$$\Rightarrow \frac{x(0)^n + x(2)^n}{x(0)^{n-1} + x(2)^{n-1}} = \frac{x(0)+x(2)}{2} \tag{4}$$

$$\Rightarrow x(0)^{n-1}(x(0) - x(2)) = x(2)^{n-1}(x(0) - x(2)) \tag{5}$$

From (5)

$$\Rightarrow n \begin{cases} = 1 & \text{if } a \neq b \\ \in R & \text{if } a = b \end{cases} \tag{6}$$

From (1)

$$d = x(1) - x(0) \tag{7}$$

$$= \frac{a+b}{2} - a \tag{8}$$

$$= \frac{b-a}{2} \tag{9}$$

Using Z transform.

$$x(n) * u(n) \overset{Z}{\longleftrightarrow} X(z) \tag{10}$$

$$X(z) = \frac{a}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \tag{11}$$

From (9)

$$X(z) = \frac{a}{1-z^{-1}} + \frac{(b-a)z^{-1}}{2(1-z^{-1})^2} \tag{12}$$

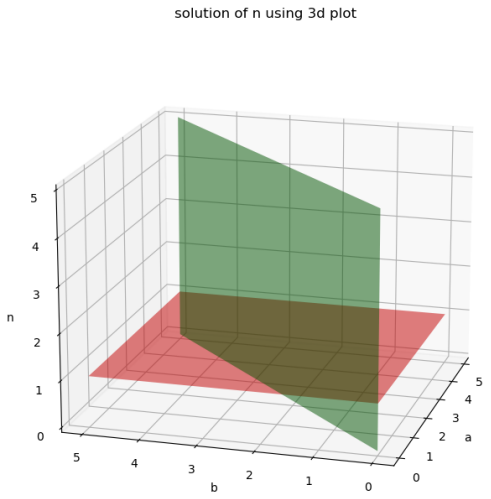

Fig. 1. Plot of n in planes