GATE-CS.51

EE23BTECH11046 - Poluri Hemanth*

Question: Consider the differential equation $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$ and the boundary conditions $y(0) = 1$ and $\frac{dy}{dx}(0) = 0$. The solution to equation is: **Solution:**

Symbol	Values	Description
$Y(s)$	$\frac{s+8}{s^2+8s+16}$	y in s domain
$y(x)$	$(1+4x)e^{-4x}u(x)$	y in x domain
$y(0)$	1	y at $x = 0$
$y'(0)$	0	$y'(x)$ at $x = 0$
$u(x)$	$= \begin{cases} 1 & \text{if } x > 0 \\ 0 & o.w \end{cases}$	unit step function

TABLE I
PARAMETERS

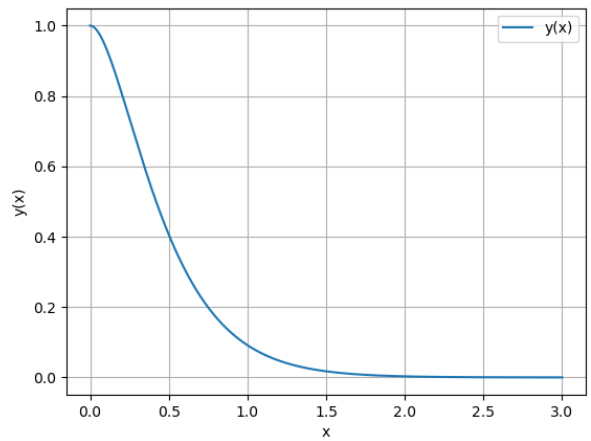

Fig. 1. Plot of y(x)

We use Laplace transorm in order to find solution of a second order differential equation

$$\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y \overset{\mathcal{L}}{\longleftrightarrow} s^2Y(s) - sy(0) - y'(0) + 8sY(s) - 8y(0) + 16Y(s) \tag{1}$$

$$Y(s)(s^2 + 8s + 16) = s + 8 \tag{2}$$

$$\Rightarrow Y(s) = \frac{s+8}{s^2+8s+16} \tag{3}$$

$$= \frac{1}{s+4} + \frac{4}{(s+4)^2} \tag{4}$$

For inversion of $Y(s)$ in partial fractions-

$$\frac{b}{(s+a)^n} \overset{\mathcal{L}^{-1}}{\longleftrightarrow} \frac{b}{(n-1)!} \cdot x^{n-1}e^{-ax} \cdot u(x) \tag{5}$$

Where b, a are real numbers, we invert $Y(s)$ to get $y(x)$:-

$$Y(s) \overset{\mathcal{L}^{-1}}{\longleftrightarrow} y(x) \tag{6}$$

From (4),(5)

$$y(x) = \frac{1}{0!}e^{-4x} \cdot u(x) + \frac{4}{1!}x \cdot e^{-4x} \cdot u(x) \tag{7}$$

$$= (1+4x)e^{-4x}u(x) \tag{8}$$