

NCERT DISCRETE 11.9.2.15

EE23BTECH11046 - Poluri Hemanth*

Question: If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is A.M between a and b , then find value of n .

Solution:

Symbol	Values	Description
$x(0)$	a	First term of A.P
$x(1)$	$\frac{a+b}{2}$	A.M of first and third terms of A.P
$x(2)$	b	Third term of A.P
k	1	No of A.M's inserted between a, b

TABLE I
PARAMETERS

$x(n) = a + nd$ Where,

$$d = \frac{b-a}{k+1}, \quad (1)$$

$$= \frac{b-a}{2} \quad (2)$$

$$x(1) = \frac{x(0)^n + x(2)^n}{x(0)^{n-1} + x(2)^{n-1}} \quad (3)$$

Using Z transform.

$$x(n) * u(n) \xleftrightarrow{Z} X(Z) \quad (4)$$

$$X(Z) = \frac{a}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \quad (5)$$

$$(6)$$

From contour integration method

$$x(n) = \frac{1}{2\pi j} \oint X(Z) z^{n-1} dz \quad (7)$$

$$\Rightarrow x(1) = \frac{1}{2\pi j} \oint X(Z) dz \quad (8)$$

According to Cauchy's Residue Theorem:
For a $y(n)$ such that,

$$y(n) = \frac{1}{2\pi j} \oint Y(Z) dz \quad (9)$$

$$= \sum_{k=1}^N \text{RES}(Y, a_k), (N \text{ is no of poles of } Y(Z)), \quad (10)$$

Where,

$$\text{RES}(Y, a_k) = \frac{1}{(m-1)!} \lim_{z \rightarrow a_k} \frac{d^{m-1}}{dz^{m-1}} [Y(Z) \cdot (z - a_k)^m] \quad (11)$$

From (5),(10)

$$x(1) = \lim_{z \rightarrow 1} \frac{a}{1-z^{-1}}(z-1) + \lim_{z \rightarrow 1} \frac{1}{1!} \frac{d}{dz} \left(\frac{d \cdot z^{-1}}{(1-z^{-1})^2} (z-1)^2 \right) \quad (12)$$

$$\Rightarrow x(1) = a + d \quad (13)$$

From (2)

$$x(1) = \frac{a+b}{2} \quad (14)$$

From (3)

$$\frac{x(0)^n + x(2)^n}{x(0)^{n-1} + x(2)^{n-1}} = \frac{x(0) + x(2)}{2} \quad (15)$$

$$\Rightarrow x(0)^n + x(2)^n = x(2)x(0)^{n-1} + x(0)x(2)^{n-1} \quad (16)$$

$$\Rightarrow x(0)^{n-1}(x(0) - x(2)) = x(2)^{n-1}(x(0) - x(2)) \quad (17)$$

$$\Rightarrow n \begin{cases} = 1 & \text{if } a \neq b \\ \in R & \text{if } a = b \end{cases} \quad (18)$$

solution of n using 3d plot

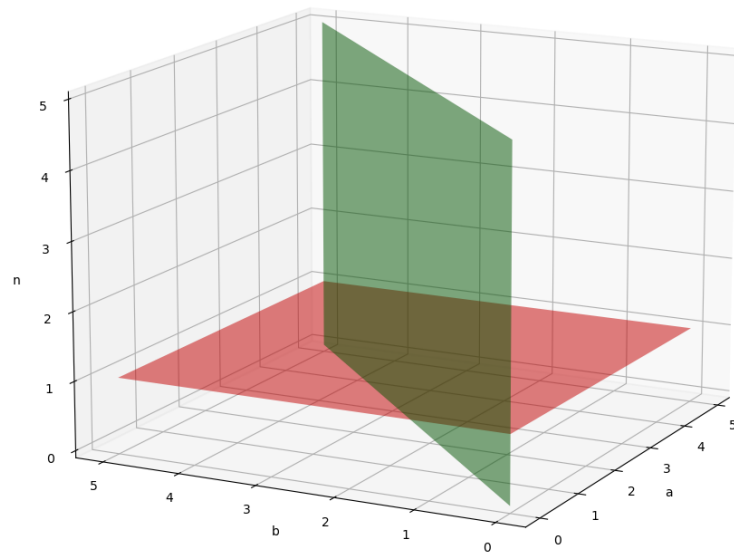


Fig. 1. Plot of n in planes