

GATE-ES.47

EE23BTECH11046 - Poluri Hemanth*

Question: Second order ordinary differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$ has values $y = 2$ and $\frac{dy}{dx} = 1$ at $x = 0$. The value of y at $x = 1$ is? (round off to three decimal places)

Solution:

We convert given second order differential equation to s domain using Laplace transform and solve for $Y(s)$ and take inversion to get $y(x)$.

Symbol	Values	Description
$Y(s)$	$\frac{2s-1}{s^2-s-2}$	y in s domain
$y(x)$	$e^{2x} + e^{-x}$	y in x domain
$y(0)$	2	y at $x = 0$
$y'(0)$	1	$y'(x)$ at $x = 0$

TABLE I
PARAMETERS

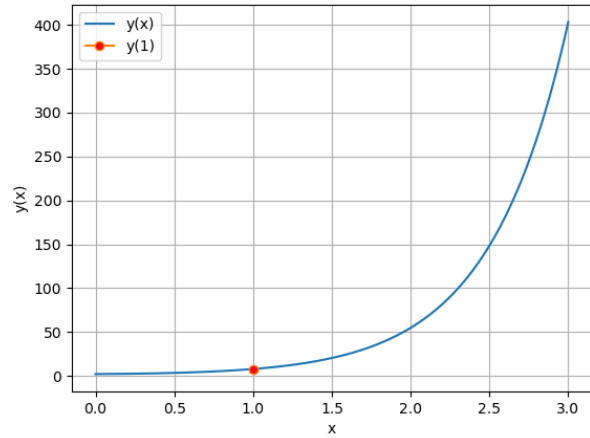


Fig. 1. Plot of $y(x)$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y \xleftrightarrow{\mathcal{L}} s^2Y(s) - sy(0) - y'(0) - sY(s) + y(0) - 2Y(s) \quad (1)$$

$$Y(s)(s^2 - s - 2) = 2s - 1 \quad (2)$$

$$\Rightarrow Y(s) = \frac{2s - 1}{s^2 - s - 2} \quad (3)$$

$$\Rightarrow Y(s) = \frac{1}{s - 2} + \frac{1}{s + 1} \quad (4)$$

For inversion of $Y(s)$ in partial fractions-

$$\frac{b}{s + a} \xleftrightarrow{\mathcal{L}^{-1}} be^{-ax} \quad (5)$$

Where b, a are real numbers, we invert $Y(s)$ to get $y(x)$:-

From (5)

$$Y(s) \xleftrightarrow{\mathcal{L}^{-1}} y(x) \quad (6)$$

$$y(x) = e^{2x} + e^{-x} \quad (7)$$

$$\Rightarrow y(1) = 7.757 \quad (8)$$