Lap appendix

EE23BTECH11046 - Poluri Hemanth*

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1. Laplace transform of f(t):

$$f(t)u(t) \xleftrightarrow{\mathcal{L}} \int_0^\infty f(t)e^{-st}\,dt \tag{1}$$
$$= F(s) \tag{2}$$

2. Laplace transform of powers of t
Let $f(t) = t^n u(t)$
From (2),and considering $h = st$

$$F(s) = \frac{1}{s^{n+1}} \int_0^\infty h^n e^{-h}\,dh \tag{3}$$
$$(n-1)! = \int_0^\infty e^{-t} t^{n-1}\,dt \text{ (Gamma function)} \tag{4}$$

From (3),(4)

$$F(s) = \frac{n!}{s^{n+1}} \tag{5}$$
$$t^n u(t) \xleftrightarrow{\mathcal{L}} \frac{n!}{s^{n+1}} \tag{6}$$

3. Frequency shift property:
Let $f(t) = y(t)e^{-at}u(t)$
From(2),

$$F(s) = \int_0^\infty y(t)e^{-(s+a)t}\,dt \tag{7}$$
$$y(t)e^{-at}u(t) \xleftrightarrow{\mathcal{L}} Y(s+a) \tag{8}$$

4. Inverse Laplace for partial fractions
From (7),(9) we get

$$\frac{b}{(s+a)^n} \xleftrightarrow{\mathcal{L}^{-1}} \frac{b}{(n-1)!} \cdot t^{n-1} e^{-at} \cdot u(t) \tag{9}$$

5. Laplace transform of derivatives:
   Let $f(t) = y'(t)u(t)$
   From (2), integration by parts,recursion

$$
\begin{aligned}
F(s) &= \int_0^\infty e^{-st}\ dy && (10)\\
&= [y(t)e^{-st}]_0^\infty + s\int_0^\infty y(t)e^{-st}dt && (11)\\
&= -y(0) + sY(s) && (12)
\end{aligned}
$$

   From(13),recursion

$$y'(t)u(t) \xleftrightarrow{\mathcal{L}} sY(s) - \int y'(t)\ dt|_{t=0} \tag{13}$$

$$y^{(n)}(t)u(t) \xleftrightarrow{\mathcal{L}} s^nY(s) - \sum_{k=0}^{n-1} s^{(n-1-k)}y^{(k)}(0) \tag{14}$$