

NCERT DISCRETE 11.9.2.15

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Question: If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is A.M between a and b, then find value of n.

Solution: Lets insert an A.M between a, b and represent a, b, A.M of those two in an A.P

The general term of A.P used is $x(n) = a + nd$. where

$$d = \frac{b-a}{k+1} \{k \text{ is no of A.M's inserted between } a, b\}$$

$$\Rightarrow d = \frac{b-a}{2} \quad (1)$$

$$x(0) = a \quad (2)$$

$$x(1) = A.M \quad (3)$$

$$x(2) = b \quad (4)$$

$$\Rightarrow x(1) = \frac{x(0)^n + x(2)^n}{x(0)^{n-1} + x(2)^{n-1}} \quad (5)$$

Take Z transform of $x(n)$ after convolution with $u(n)$ as there are no $x(n)$ where n is negative.

$$x(n) * u(n) \xleftrightarrow{Z} X(Z) \quad (6)$$

$$X(Z) = \frac{a}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \quad (7)$$

To find $x(1)$ we take inverse Z transform using contour integration and Cauchy's residue theorem.

$$x(n) = \frac{1}{2\pi j} \oint X(Z) z^{n-1} dz \quad (8)$$

$$\Rightarrow X(Z) z^{n-1} = \frac{az}{z-1} + \frac{d \cdot z}{(z-1)^2} \quad (9)$$

$$(10)$$

According to Cauchy's Residue Theorem:
for a $y(n)$ such that, N is no of poles of $Y(Z)$

$$y(n) = \frac{1}{2\pi j} \oint Y(Z) dz = \sum_{i=1}^N \text{RES}(Y, a_k) \quad (11)$$

$$\text{RES}(Y, a_k) = \frac{1}{(m-1)!} \lim_{z \rightarrow a_k} \frac{d^{m-1}}{dz^{m-1}} [Y(Z) \cdot (z - a_k)^m] \quad (12)$$

$$x(1) = \lim_{z \rightarrow 1} \frac{az}{z-1} (z-1) + \lim_{z \rightarrow 1} \frac{1}{1!} \frac{d}{dz} \left(\frac{d \cdot z}{(z-1)^2} (z-1)^2 \right) \quad (13)$$

$$\Rightarrow x(1) = a + d \quad (14)$$

From (1)

$$\Rightarrow x(1) = \frac{a+b}{2} \quad (15)$$

From (5)

$$\frac{x(0)^n + x(2)^n}{x(0)^{n-1} + x(2)^{n-1}} = \frac{x(0) + x(2)}{2} \quad (16)$$

$$\Rightarrow x(0)^n + x(2)^n = x(2)x(0)^{n-1} + x(0)x(2)^{n-1} \quad (17)$$

$$\Rightarrow x(0)^{n-1}(x(0) - x(2)) = x(2)^{n-1}(x(0) - x(2)) \quad (18)$$

$$\Rightarrow n \begin{cases} = 1 & \text{if } x(0) \neq x(2) \\ \in R & \text{if } x(0) = x(2) \end{cases} \quad (19)$$

S/No	Symbol	Values	Description
1	$x(0)$	a	First term of A.P
2	$x(1)$	$\frac{a+b}{2}$	A.M of first and third terms of A.P
3	$x(2)$	b	Third term of A.P

TABLE I
PARAMETERS

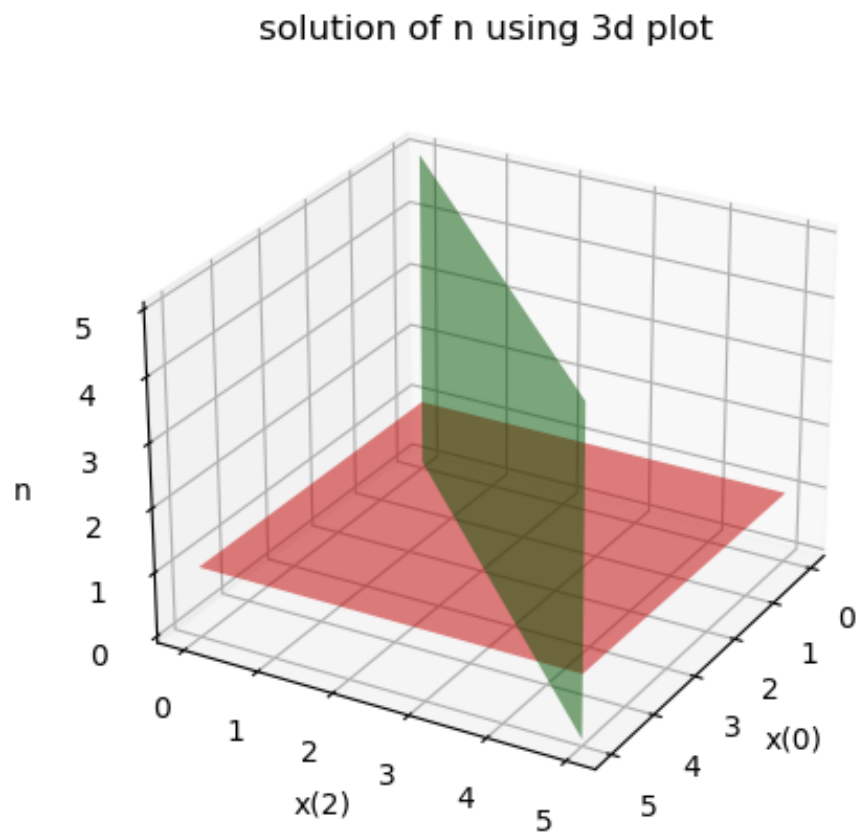


Fig. 1. Plot of n in planes