Lap appendix

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1. Laplace transform of f(t):

$$f(t)u(t) \xleftrightarrow{\mathcal{L}} \int_0^\infty f(t)\, dt = F(s) \tag{1}$$

2. Laplace transform of powers of t
   Let $f(t) = t^n u(t)$
   From (1),$h = st$

$$F(s) = \frac{1}{s^{n+1}} \int_0^\infty h^n e^{-h}\, dh \tag{2}$$

$$(n-1)! = \int_0^\infty e^{-t} t^{n-1}\, dt \text{ (Gamma function)} \tag{3}$$

$$\Rightarrow F(s) = \frac{n!}{s^{n+1}} \tag{4}$$

$$\Rightarrow t^n u(t) \xleftrightarrow{\mathcal{L}} \frac{n!}{s^{n+1}} \tag{5}$$

3. Frequency shift property:
   Let $f(t) = y(t)e^{-at}u(t)$
   From(1),

$$F(s) = \int_0^\infty y(t)e^{-(s+a)t}\, dt \tag{6}$$

$$\Rightarrow y(t)e^{-at}u(t) \xleftrightarrow{\mathcal{L}} Y(s+a) \tag{7}$$

4. Inverse Laplace for partial fractions
   From (5),(7) we get

$$\frac{b}{(s+a)^n} \xleftrightarrow{\mathcal{L}^{-1}} \frac{b}{(n-1)!} \cdot t^{n-1} e^{-at} \cdot u(t) \tag{8}$$

5. Laplace transform of derivatives:
   Let $f(t) = y'(t)u(t)$

From (1), integration by parts,recursion

$$F(s) = \int_0^\infty e^{-st}\ dy \tag{9}$$

$$= [y(t)e^{-st}]_0^\infty + s\int_0^\infty y(t)e^{-st}dt \tag{10}$$

$$= -y(0) + sY(s) \tag{11}$$

From(11),recursion

$$y'(t)u(t) \xleftrightarrow{\mathcal{L}} sY(s) - \int y'(t)\ dt|_{t=0} \tag{12}$$

$$y^{(n)}(t)u(t) \xleftrightarrow{\mathcal{L}} s^nY(s) - \sum_{k=0}^{n-1} s^{(n-1-k)}y^{(k)}(0) \tag{13}$$