Lap appendix

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1. Laplace transform of any function in time domain

$$f(t)u(t) \xleftrightarrow{\mathcal{L}} \int_0^\infty f(t)\, dt = F(s) \tag{1}$$

2. Laplace transform of $y(t) = t^n$ From (1) Let $h = st$

$$Y(s) = \frac{1}{s^{n+1}} \int_0^\infty h^n e^{-h}\, dh \tag{2}$$

From Gamma function:

$$(n-1)! = \int_0^\infty e^{-t} t^{n-1}\, dt \tag{3}$$

From (3),(2)

$$Y(s) = \frac{n!}{s^{n+1}} \tag{4}$$

3. Laplace transform of nth order derivative of y(t) is
From (1)

$$\frac{dy}{dt} u(t) \xleftrightarrow{\mathcal{L}} \int_0^\infty e^{-st}\, dy \tag{5}$$

From integration by parts

$$\frac{dy}{dt} u(t) \xleftrightarrow{\mathcal{L}} [y(t)e^{-st}]_0^\infty + s \int_0^\infty y(t)e^{-st} dt \tag{6}$$

From(1)

$$\frac{dy}{dt} u(t) \xleftrightarrow{\mathcal{L}} -y(0) + sY(s) \tag{7}$$

From (9),from recuression,

$$y^{(n)}(t)u(t) \xleftrightarrow{\mathcal{L}} s^n Y(s) - \sum_{k=0}^{n-1} s^{(n-1-k)} y^{(k)}(0) \tag{8}$$

4. Frequency shift property

$$y(t)e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \int_0^\infty y(t)e^{-(s+a)t}\, dt \tag{9}$$

Replace t with $(s+a)t$,From (1)

$$y(t)e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} Y(s+a) \tag{10}$$

5. The Inverse Laplace for partial fractions
From (4),(13) we get

$$\frac{b}{(s+a)^n} \overset{\mathcal{L}^{-1}}{\longleftrightarrow} \frac{b}{(n-1)!} \cdot t^{n-1}e^{-at} \cdot u(t) \tag{11}$$