GATE-ES.47

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Question: Second order ordinary differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$ has values $y = 2$ and $\frac{dy}{dx} = 1$ at $x = 0$.The value of y at $x = 1$ is?($round\ off\ to\ three\ decimal\ places$)

Solution:

We convert given second order differential equation to s domain using Laplace transform and solve for $Y(s)$ and take inversion to get $y(x)$.

Symbol	Values	Description
$Y(s)$	$\frac{2s-1}{s^2-s-2}$	y in s domain
$y(x)$	$e^{-2x} + e^x$	y in x domain
$y(0)$	2	y at $x = 0$
$y'(0)$	1	$y'(x)$ at $x = 0$

TABLE I
PARAMETERS

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y \overset{\mathcal{L}}{\longleftrightarrow} s^2Y(s) - sy(0) - y'(0) - sY(s) + y(0) - 2Y(s) \tag{1}$$

$$Y(s)\left(s^2 - s - 2\right) = 2s - 1 \tag{2}$$

$$Y(s) = \frac{2s-1}{s^2 - s - 2} \tag{3}$$

$$Y(s) = \frac{1}{s-2} + \frac{1}{s+1} \tag{4}$$

For inversion of $Y(s)$ in partial fractions-

$$\frac{b}{s+a} \overset{\mathcal{L}^{-1}}{\longleftrightarrow} be^{ax} \tag{5}$$

Where b, a are real numbers, we invert $Y(s)$ to get $y(x)$:-

From (5)

$$Y(s) \overset{\mathcal{L}^{-1}}{\longleftrightarrow} y(x) \tag{6}$$

$$y(x) = e^{-2x} + e^x \tag{7}$$

$$y(1) = e^{-2} + e \tag{8}$$

$$y(1) = 2.854 \tag{9}$$