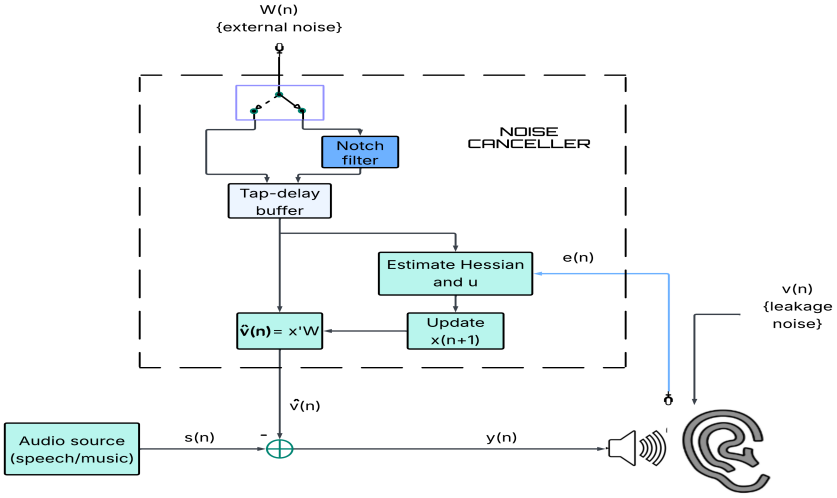
Block Diagram



Justifications And Tradeoffs

Parameter	Recommended Range	Reason / Theory	Out-of-Range Effects	Values
N (filter length)	$5 \leq N \leq 32$	Covers autocorrelation span of $w(n)$; low N = fast, high N = precise	N too small $\Rightarrow$ underfitting, poor noise capture; too large $\Rightarrow$ slow, unstable	$N = 8$
γ (forgetting)	$0.9 \leq \gamma < 1$	Controls memory depth in H_n update: $H_n = \gamma H_{n-1} + \ldots$	Low γ loses correlation info; high $\gamma \to 1$ reacts slowly to non-stationarity	$\gamma = 0.91$
$\mu = \frac{2}{\lambda_{\max}}$ (step size)	$0.01 \leq \mu \leq 1$ (typically)	Ensures convergence of LMS update: $\mu < \frac{2}{\lambda_{\max}}$	Too small μ: slow convergence; too large μ: divergence, unstable weights	$\mu = \frac{2}{\lambda_{\max}}$
ϵ (reg.)	$10^{-8} \leq \epsilon \leq 10^{-3}$	Ensures $H^{-1} \succ 0$, avoids singularity	Too low $\Rightarrow$ ill-conditioned inverse; too high $\Rightarrow$ biased direction	$\epsilon = 10^{-6}$
H^{-1} update	Dynamic via Broyden-like rule	Approximates Hessian inverse for Quasi-Newton adaptation	Poor conditioning slows or destabilizes convergence	Broyden-like update

Table: Recommended parameter ranges, theory, and the impact of deviation in adaptive filtering.

- **Convexity Guarantees Global Minimum:** The MSE cost surface is convex, ensuring that gradient-based methods like LMS converge to the global minimum, avoiding local minima.
- **Smoothness Enables Stability:** The differentiable and continuous nature of MSE ensures stable and gradual updates in adaptive filtering.

Pros And Cons

Criterion	LMS + Hessian	NLMS	RLS
Complexity	$\mathcal{O}(N^2)$	$\mathcal{O}(N)$	$\mathcal{O}(N^3)$
Memory	Hessian approx. ($\sim N^2$)	Minimal ($\sim N$)	High — inv. corr. matrix ($\sim N^2$)
SNR Gain	High (MSE-optimal path)	Moderate	High (near-optimal est.)

Table: Comparison of LMS + Hessian, NLMS, and RLS algorithms across various metrics.

- **LMS with Hessian Preconditioning:**
  - **Pros:**
  - Flexibility in Cost Functions: Allows the use of various cost functions beyond MSE, unlike RLS which is limited to specific forms.

- **Partial Suppression with Second-Order Notch Filter:**
  - **Cons:**
    - Increasing filter order improves frequency selectivity but introduces more phase shift, potentially distorting the signal.
    - When tonal frequencies are close (e.g., $\pm$70Hz), some side frequencies may also be retained, reducing suppression effectiveness.

References

- Paulo S. R. Diniz, "Adaptive Filtering: Algorithms and Practical Implementation"
- Machine Learning (EE2802) Logistic Regression Slides
- Sherman-Morrison Formula