

Components of automatic control system

1. The basic components of an automatic control system are error detector, Amplifier and controller, Actuator (Power Actuator), plant and sensor or a feedback system. The block diagram of an automatic control system is shown in below figure.

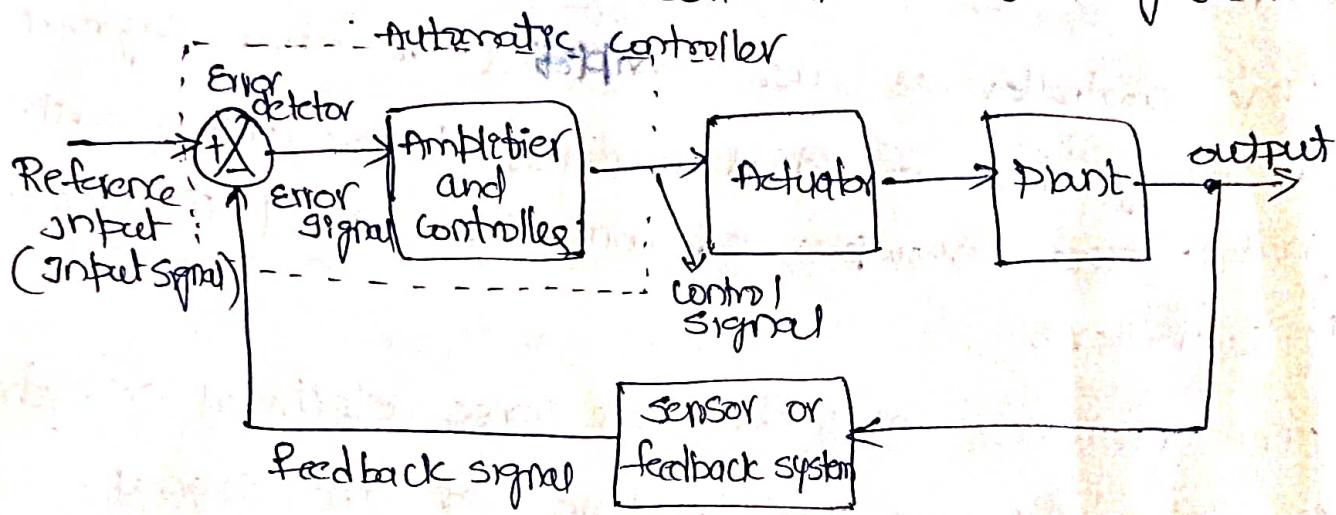


Fig :- Block diagram of automatic control system.

2. The plant is the open loop system whose output is controlled by closed loop system.
3. The combination of error detector, amplifier and controller is called automatic controller. Without this unit the system becomes open loop system.
4. In automatic control systems, the reference input is proportional to the desired output and the feedback signal is proportional to the current output of the system.

5. The error detector compares the reference input and feedback signal and produces the error signal. As the error signal is a weak signal it is further amplified by the amplifier and modified by the controller for better control action.

6. The error signal is used to correct the output if there is a deviation from the desired value. The examples of error detector are potentiometer, LVDT.

7. The controller is also amplifies the error signal and integrates or differentiates to produce a control signal. The examples of controllers are P, PI, PD and PID controllers.

8. The controllers may be electronic, electrical or hydraulic depending on the nature of the signal.

9. The actuator is a power amplifying device which amplifies the controller output and converts to the suitable form of energy for the plant.

10. This process is continues till we get the difference between reference input and feedback signal is zero as it indicates the output settles to the desired value.

11. The actuator may be pneumatic motor / valve, hydraulic motor or electric motor. Examples of electric motors are employed as actuator are DC servomotor, AC servomotor, Stepper motor.

- (2) The feedback system samples the output to produce a feedback signal which is proportional to current or p.
- (3) The feedback system converts the output variable to another variable such as displacement, pressure or voltage.
- (4) Transducers, Tachogenerators are used as a feedback systems.

Electromagnetic field Motors:-

These motors are economical for higher power ratings, i.e., above 1kW. This type of servo motors are similar to conventional DC motors. It has the following features.

1. The number of slots and commutator segments is large to improve commutation.
2. Compensating and compensating windings are provided to eliminate sparking.
3. The diameter to length ratio is kept low to reduce inertia.
4. Ovate shafts are employed to withstand the high torque stresses.
5. Eddy currents are reduced by complete laminations of the magnetic circuit and by using low loss steel.

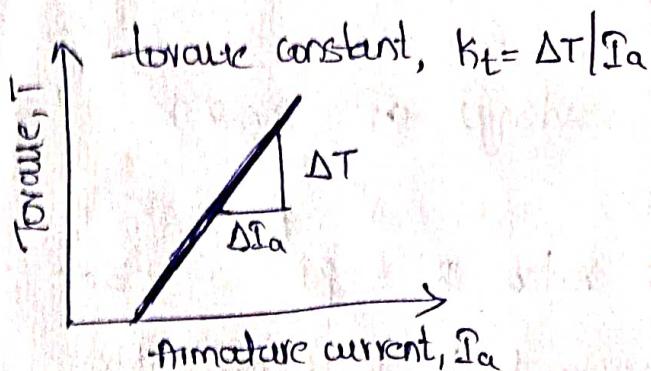
In this type of motor, the torque and speed are controlled by varying the armature current and the field current.

In armature controlled mode of operation, the field current is held constant and the armature current is varied to control the torque.

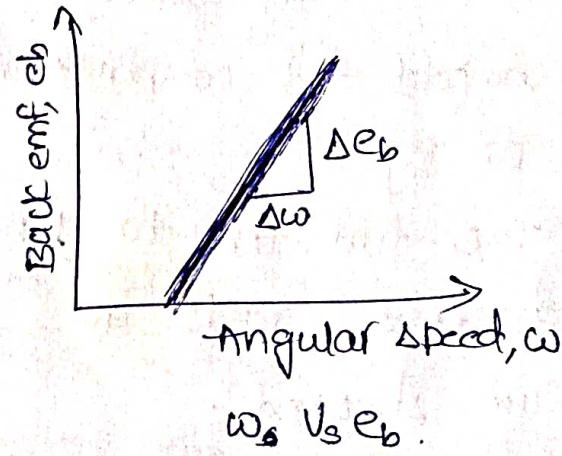
In the field controlled mode, the armature current is maintained constant and the field current is varied to control the torque.

Armature controlled DC servomotor :- It is a DC shunt motor. The field is excited by a constant DC supply. If the field current is constant then speed is directly proportional to armature current and torque is directly proportional to armature current.

In small motors, the armature voltage is controlled by a variable resistance but in large motors the armature voltage is controlled by thyristors in order to reduce power loss. The steady state operating characteristics of an armature controlled DC servomotor are shown in below figure.



I_a Vs T .



ω Vs T .

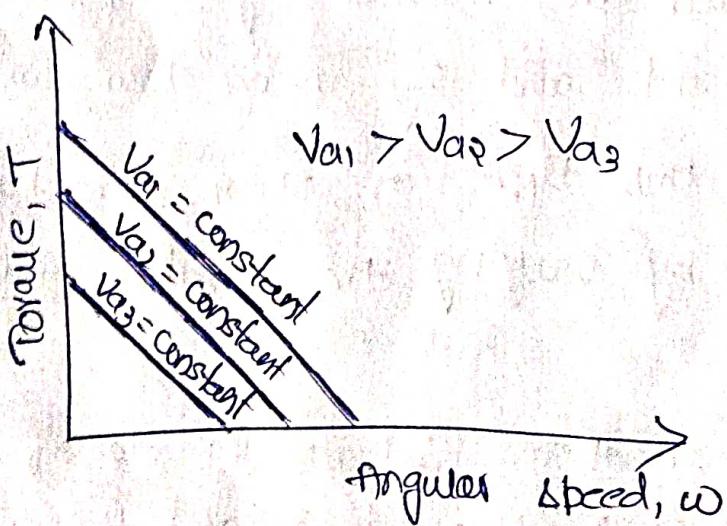
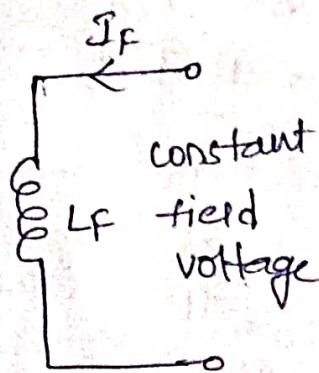
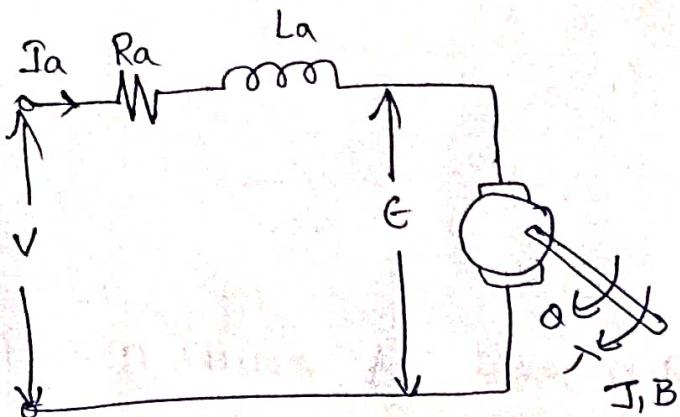


fig. characteristics of armature controlled DC servomotor

Frasstki function :-



R_a = armature resistance

L_a = armature self inductance

i_a = armature current

i_f = field current

e = induced emf in armature

V = applied voltage

T = Torque

θ = angular displacement

J = equivalent moment of inertia

B = coefficient of friction.

APPLY KVL TO ABOVE CIRCUIT

$$V = R_a i_a + L \frac{di_a}{dt} + e \rightarrow (1)$$

when armature is rotating, the induced emf is

$$e \propto \phi \omega \Rightarrow e = k_b \phi \omega$$

where ω is angular frequency.

As field current is constant replace k_bφ with a constant

$$e = k_b \omega$$

$$e = k_b \frac{d\theta}{dt} \rightarrow (2) \quad [\because \omega = \frac{d\theta}{dt}]$$

(3)

servo motor.

(13)

Torque is given

$$T \propto \phi^{\circ} a$$

$$T = K_1 \phi^{\circ} a = K^{\circ} a$$

$$T = K^{\circ} a \rightarrow (3)$$

$$T = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} \rightarrow (4)$$

Take the Laplace transform of eqns (1), (2), (3), (4)

$$V(s) = R_a I_a(s) + L_a \dot{I}_a(s) + E(s) \rightarrow (a)$$

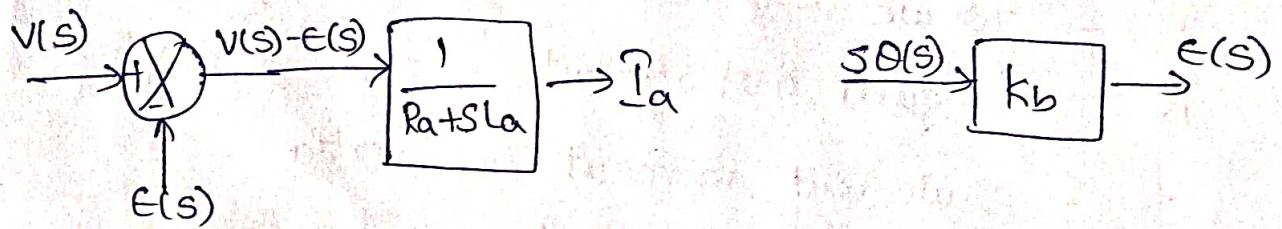
$$V(s) - E(s) = I_a(s) [R_a + s L_a] \rightarrow (a)$$

$$E(s) = K_b s \theta(s) \rightarrow (b)$$

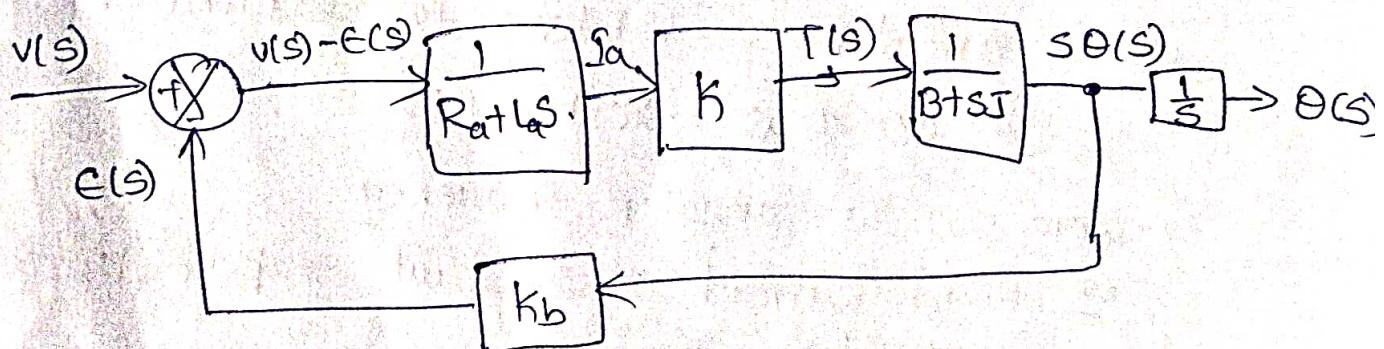
$$T(s) = K^{\circ} I_a(s) \rightarrow (c)$$

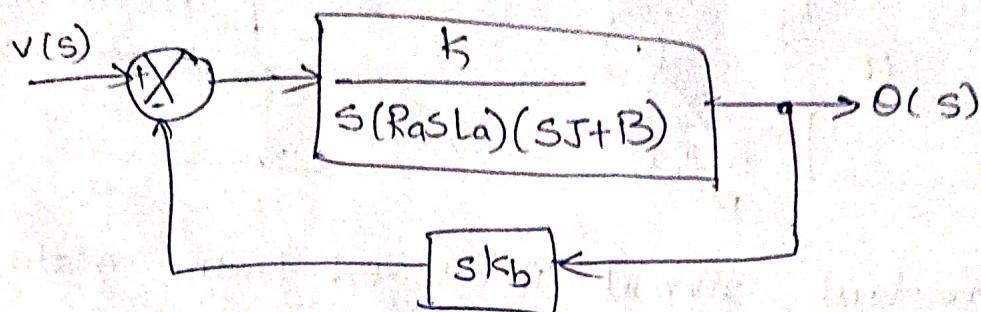
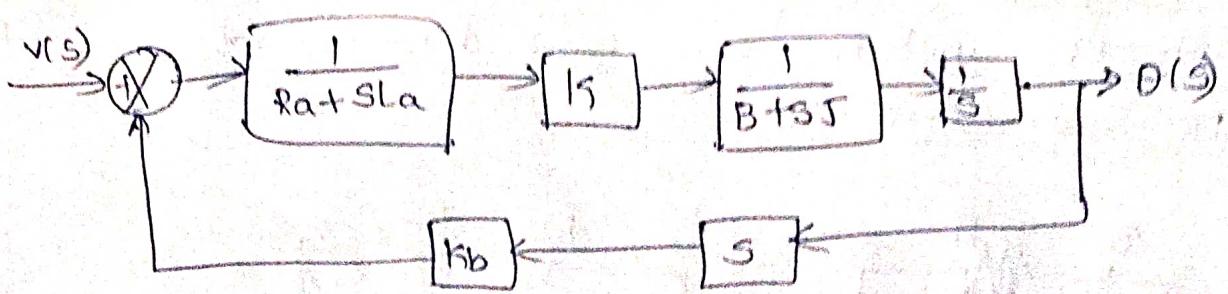
$$T(s) = s^2 J \cancel{\theta}(s) + B s \theta(s) \rightarrow$$

$$T(s) = s \theta(s) [s J + B] \rightarrow (d)$$



We know that $V(s)$ is input and $\theta(s)$ is output





$$\frac{O(s)}{V(s)} = \frac{\frac{1}{s}}{1 + \frac{K}{s(RaSLa)(SJ+B)} sK_b} = \frac{\frac{K}{s(RaSLa)(SJ+B)}}{s(RaSLa)(SJ+B) + sK_b}$$

$$\frac{O(s)}{V(s)} = \frac{K}{(RaSLa)(SJ+B)s + Kk_b s}$$

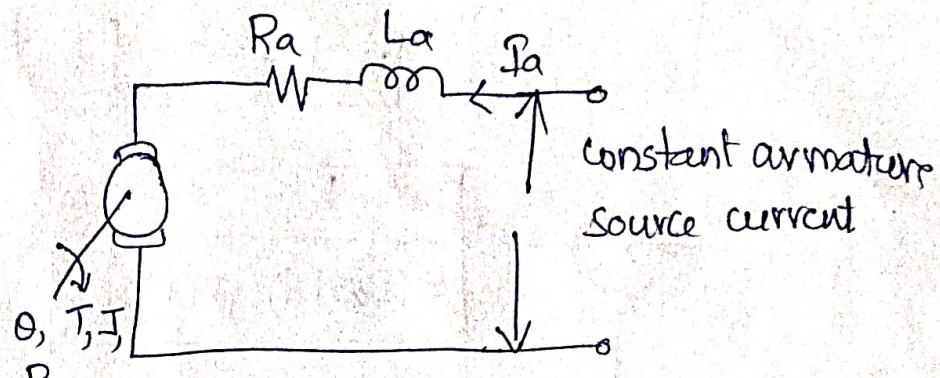
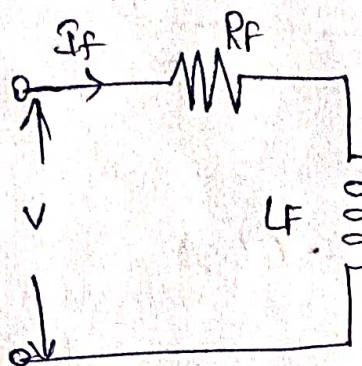
Field controlled DC servomotor :- It is a DC shunt motor. In this motor the armature is applied or supplied with a constant current voltage.

When the armature voltage is constant the torque is directly proportional to the field flux but the field current is proportional to flux, therefore the torque of the motor is controlled by controlling the field current.

(4)

(5)

Transfer function and Block diagram of field controlled DC servo motor :-



1. A constant current I_a is fed to the armature
2. Flux is proportional to the field current

$$\phi \propto I_f$$

$$\phi = K_f I_f \rightarrow (1)$$
3. Apply KVL to above circuit

$$V_f = R_f I_f + L_f \frac{dI_f}{dt} \rightarrow (2)$$
4. Torque is proportional to flux and armature current

$$T \propto \phi I_a$$

$$T = K' \phi I_a$$

$$= K' K_f I_f I_a$$

$$T = K K_f I_f I_a \rightarrow (3) \quad [\because K' \text{ is constant} \Rightarrow I_a]$$

$$5. \quad T = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} \rightarrow (4) \quad \begin{aligned} K &= K' I_a \\ \theta &= \text{angular displacement} \end{aligned}$$

J = Moment of Inertia

B = coefficient of friction

• After laplace transform, we get

$$\phi(s) = k_f I_f(s)$$

$$V_f(s) = R_f I_f(s) + s L_f I_F(s)$$

$$= I_f(s) [R_f + s L_f]$$

$$I_f(s) = \frac{V_f(s)}{s L_f + R_f} \rightarrow (1)$$

$$T(s) = K I_f(s) k_f \rightarrow (2)$$

$$T(s) = J s^2 \theta(s) + B s \theta(s)$$

$$T(s) = \theta(s) [J s^2 + s B] \rightarrow (3)$$

Here output is angular displacement and input $V(s)$

Sub eqn (1) in eqn (2)

$$T(s) = K k_f \frac{V_f(s)}{R_f + s L_f} \rightarrow (4)$$

Sub eqn (4) in eqn (3)

$$\theta(s) [J s^2 + s B] = K \frac{R_f k_f}{R_f + s L_f} \frac{V_f(s)}{R_f + s L_f}$$

$$\frac{\theta(s)}{V_f(s)} = \frac{K k_f}{s (J s + B) (R_f + s L_f)}$$

$$= \frac{K k_f}{R_f B s \left(1 + s \frac{1}{B} \right) \left(1 + s \frac{L_f}{R_f} \right)}$$

$$\boxed{\frac{\theta(s)}{V_f(s)} = \frac{K k_f}{R_f B s \left(1 + s T_m \right) \left(1 + s T_f \right)}}$$

where $T_m = \frac{J}{B}$ = mechanical time constant

$T_f = \frac{L_f}{R_f}$ = time constant for field circuit

Ac Servomotor :- (1) The representation of Ac servomotor as a control system is shown in figure.

2. The reference winding is excited by a constant voltage source with a frequency range 50 to 1000Hz.
3. The system can be made less susceptible to low frequency noise we when we operate at more than 400Hz.
4. Due to this feature, ac devices are used in aircraft and missile control system.
5. The control windings is excited by the modulated control signal as the control signal is low frequency signal ranges from 0 to 50 Hz.

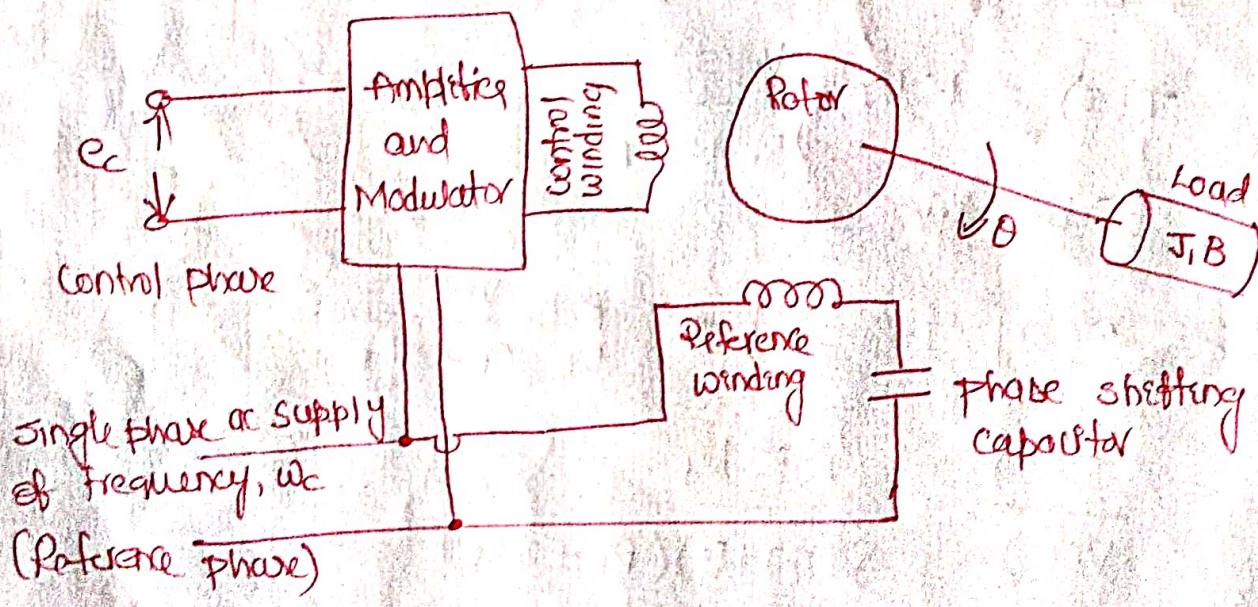


fig : Symbolic representation of an ac servomotor.

6. In order to produce rotating magnetic field, the control phase voltage must and reference phase voltage must have same frequency and two voltages must be in time quadrature.
7. Hence the control signal is modulated by a carrier whose frequency is equal to the reference voltage frequency and this modulated signal is applied to control winding.

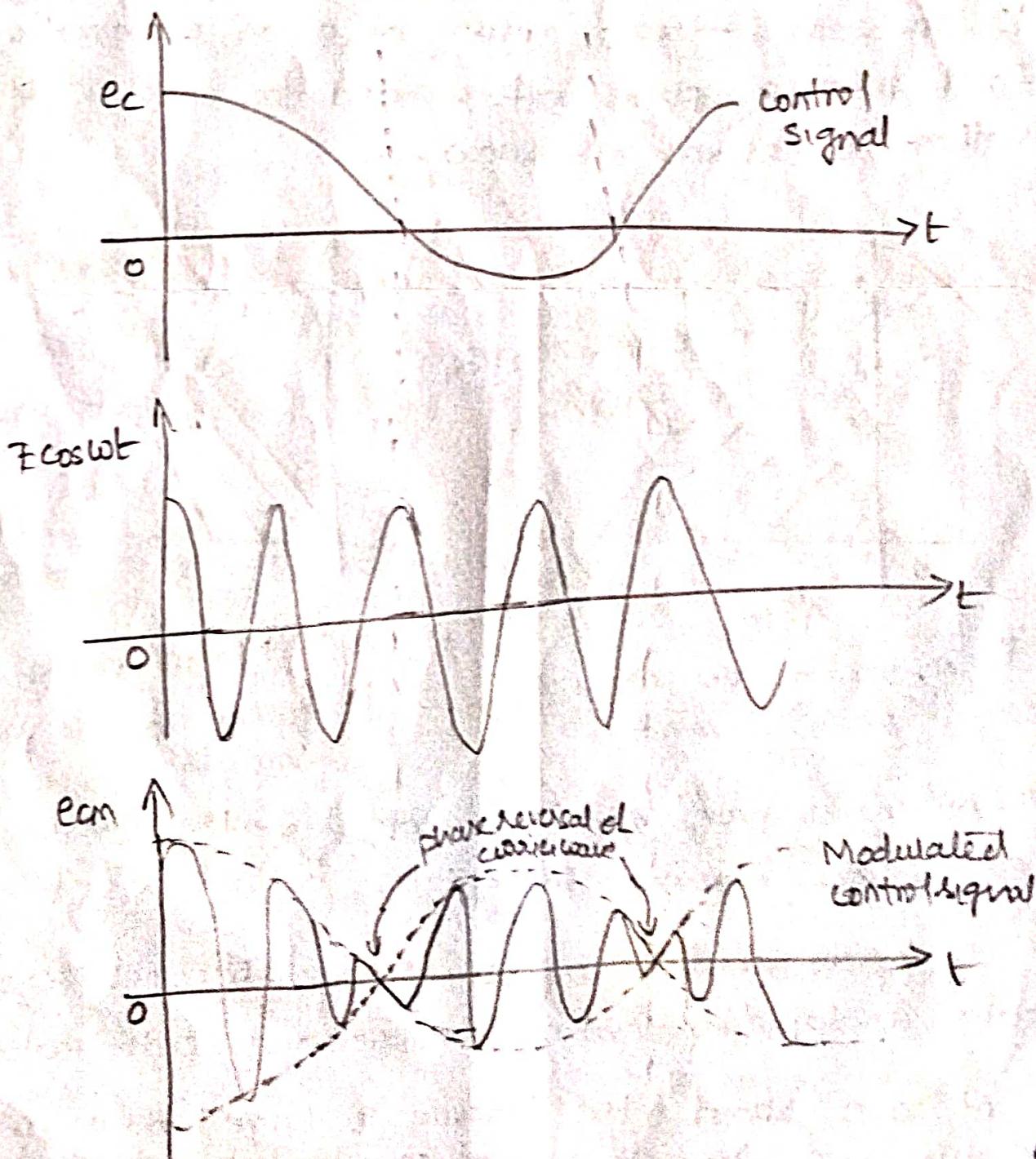
8. Here we are performing amplitude Modulation and the information is available on the envelope of the modulated signal.

9. The 90° phase difference between control signal and reference signal is obtained by the insertion of a capacitor in reference winding.

10. In modulation process control signal is modulating signal : e_c ; e_c = control signal.

$$e_{car} = \text{carrier signal} = E \cos \omega t$$

$$e_{cm} = \text{Modulated control signal.}$$



11. In above figure, if e_c is positive then phases of e_{cm} and e_{car} are same. On the other hand if e_c is negative then, there is 180° phase difference.

$$\text{i.e., } e_{cm} = |E + e_c| \cos \omega t \text{ for } e_c > 0$$

$$|E + e_c| \cos(\omega t + \pi) \text{ for } e_c < 0$$

12. This phase reversal changes the direction of rotation of the magnetic field and hence reversal in the direction of rotation of the Motor shaft.

13. The speed-torque curves of a ac servomotor for fixed reference phase voltage and for constant control voltage is shown in below.

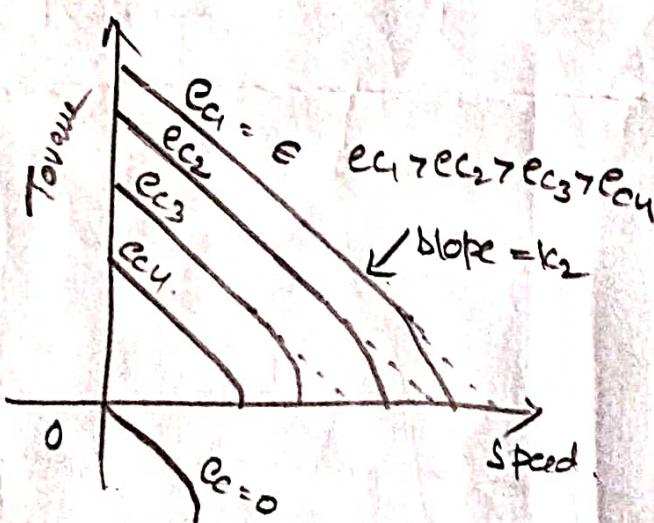


fig a) speed torque curves of an ac servomotor.

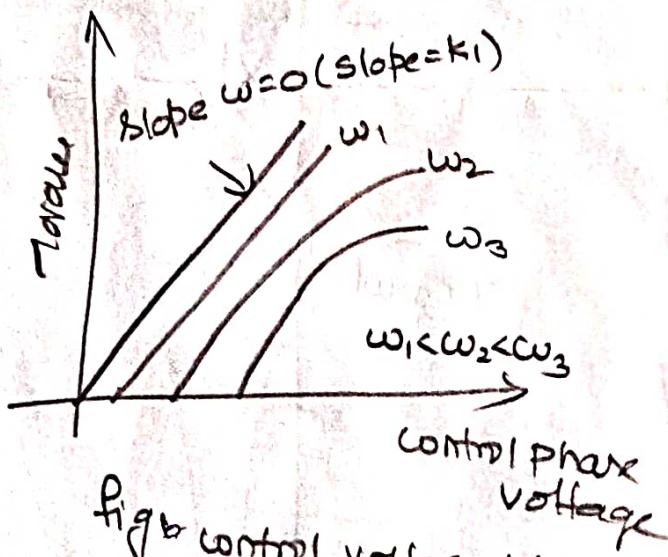


fig b) control voltage Vs Torque curves of an ac servomotor.

14. All curves have negative slope but the curves for $e_c = 0$ goes through the origin.

15. So when control phase voltage becomes zero, the motor develops decelerating torque and so the motor stops.

16. At zero speed, the torque is large and we need very less requirement for a servomotor in order to provide rapid acceleration.

(17) The speed-torque curves of ac servomotor is linear at low speed regions and non linear at high speed regions.

(18) In order to derive transfer function for the motor, we will take linearizing approximations.

(19) So we are extending linear portions of speed-torque curves to high speed regions as shown by dashed lines.

(20) Even with this extension, the curves are not still parallel to each other which indicates that the torque does not vary linearly with output voltage E_c for constant speed as shown in fig (b).

Transfer function of ac servomotor:

Let, T_m = Torque developed by servomotor

θ = Angular displacement of rotor.

ω = Angular speed = $\frac{d\theta}{dt}$.

T_L = Torque required by the load.

J = Moment of inertia of load and the rotor

B = Viscous frictional coefficient of load and the rotor.

K_1 = Slope of control phase voltage Vs Torque characteristics.

K_2 = Slope of speed-torque characteristic.

Torque developed by the motor, $T_m = K_1 E_c - K_2 \frac{d\theta}{dt}$

→ (7)

The rotating part of the motor and the load can be modelled by equation

$$\text{Load torque, } T_L = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} \rightarrow (2)$$

$$J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = k_1 E_c - k_2 \frac{d\theta}{dt}$$

on taking Laplace transforms with zero initial conditions, we get

$$J s^2 \Theta(s) + B s \Theta(s) = k_1 E_c(s) - k_2 s \Theta(s)$$

$$[J s^2 + B s + k_2 s] \Theta(s) = k_1 E_c(s)$$

$$\frac{\Theta(s)}{E_c(s)} = \frac{k_1}{s(Js + B + k_2)} = \frac{\frac{k_1}{B + k_2}}{s(\frac{1}{B + k_2}s + 1)} = \frac{k_m}{s(\tau_m s + 1)}$$

where, $k_m = \frac{k_1}{B + k_2}$ = Motor gain constant

$$\tau_m = \frac{J}{B + k_2} = \text{Motor time constant}$$

Potentiometer: 1. A potentiometer is a device which can be used to convert a linear or angular displacement into a voltage.

2. A potentiometer is a variable resistance whose value varies according to angular displacement or linear displacement of the wiper contact.

3. The resistance element can be constructed by depositing a conducting material on a plastic base.

4. The potentiometer has an output shaft to which a wiper attached.

5. The potentiometer is excited by a dc or ac voltage. The output voltage is measured at wiper contact with respect to reference.

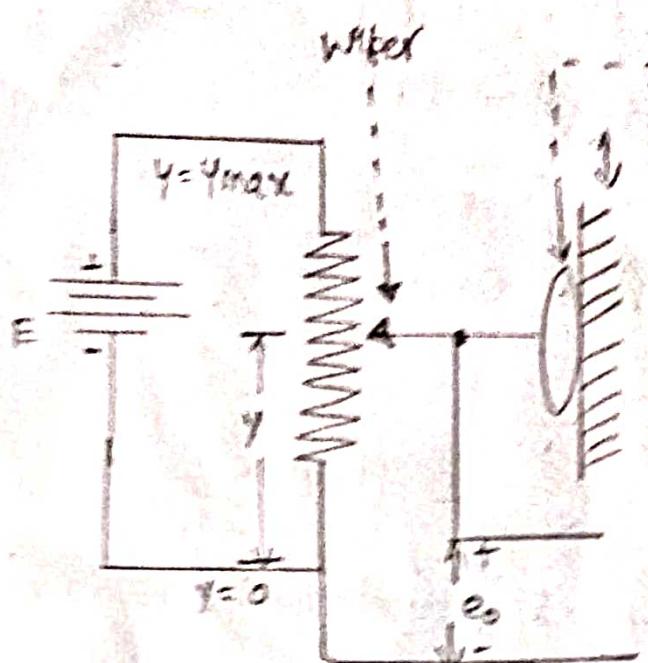


Fig: Linear displacement potentiometer

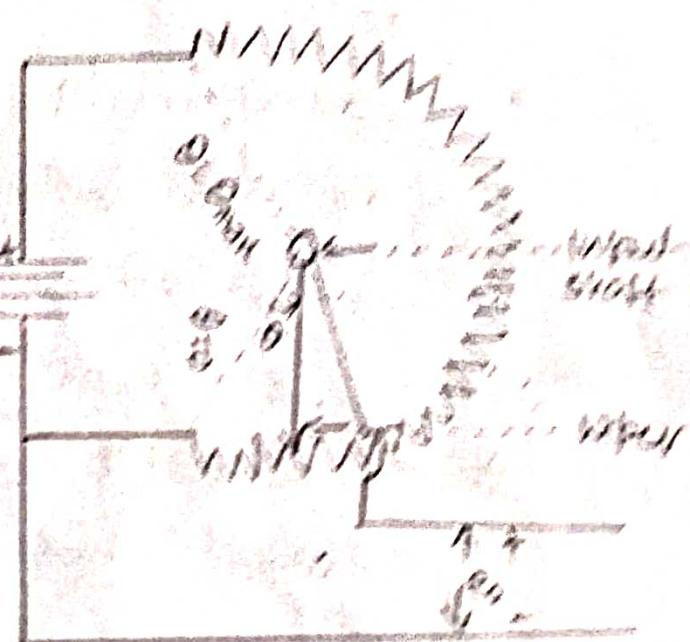


Fig: angular displacement potentiometer

Linear displacement potentiometer

y = Displacement of the wiper contact from reference
- ($y=0$)

y_{max} = Maximum displacement of wiper contact

E = Excitation voltage of potentiometer

e_o = Output voltage

If $y=0$, then $e_o=0$, and if $y=y_{max}$, then e_o is displacement y .

$$\text{Output voltage, } e_o = \frac{E}{y_{max}} y = K_p y$$

where, $K_p = \frac{E}{y_{max}}$ = sensitivity of potentiometer
in volt/mm

Angular displacement potentiometer:-

θ = angular displacement of wiper arm from reference ($\theta=0$)

θ_{\max} = Maximum displacement of wiper arm

E = Excitation voltage of potentiometer

e_o = Output voltage

If $\theta=0$, then $e_o=0$ and if $\theta=\theta_{\max}$ then $e_o=E$

for displacement θ ,

$$\text{Output voltage, } e_o = \frac{E}{\theta_{\max}} \theta = k_p \theta$$

where $k_p = \frac{E}{\theta_{\max}}$ = sensitivity of potentiometer in volt/deg

Applications of potentiometer:-

1. Potentiometers can be used either to convert a mechanical motion to proportional voltage or as an error signal.
2. A potentiometer excited by dc or ac voltage is used to produce an output voltage proportional to displacement of the input shaft.
3. When potentiometers are used as error detector, two identical potentiometers are required as shown in figure.
4. Both potentiometers are excited by the same source with same voltage
5. If the wiper arm of both the potentiometers are in the same position then the voltage between two wiper arms is zero.

6. The position of one wiper arm is kept as reference output and the displacement is applied to another arm of potentiometer.

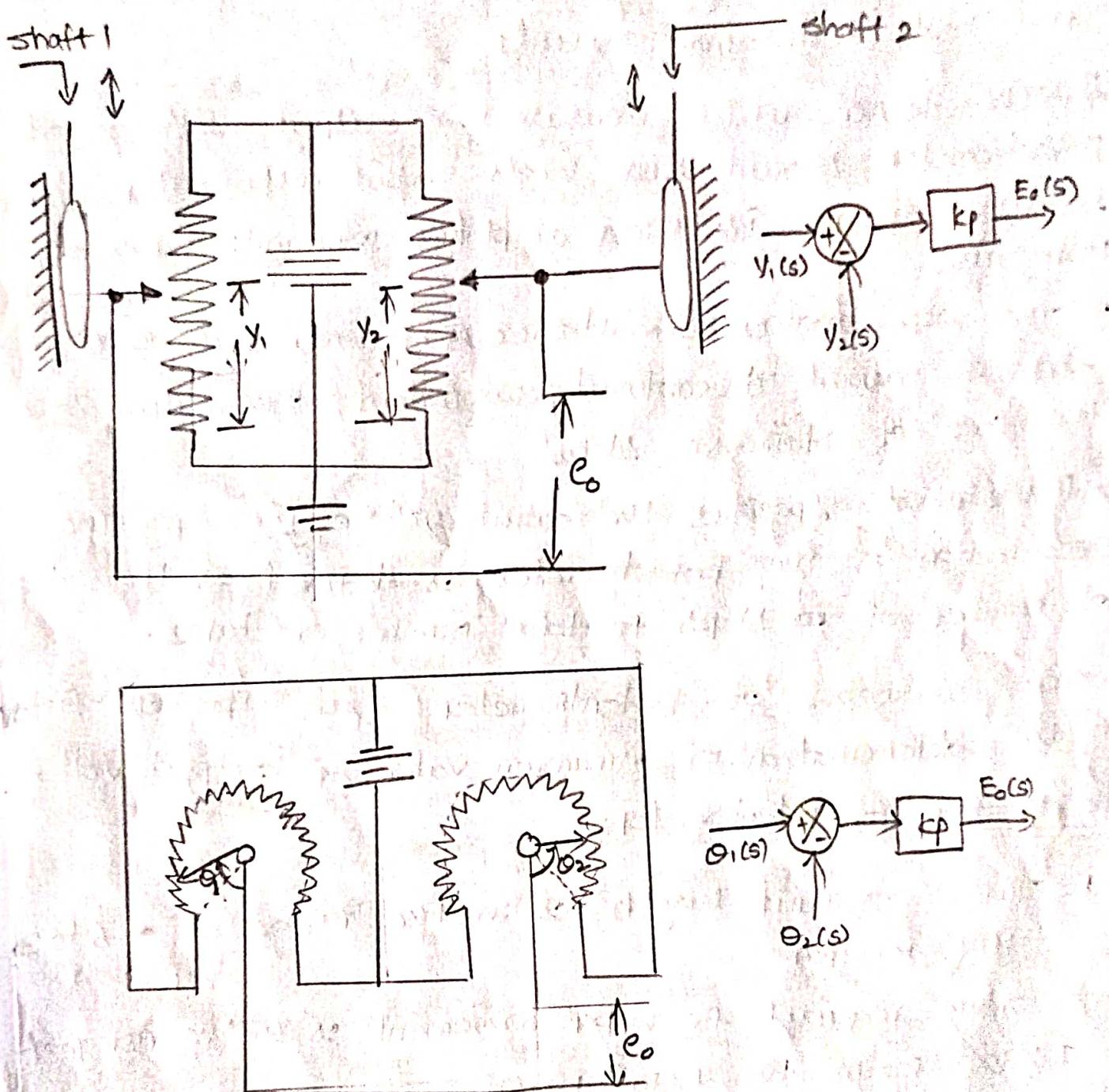


fig:- potentiometer as error detector

7. Hence the output voltage which is measured between two arms is proportional to the difference between the displacement of both the wiper arms.

Characteristics of potentiometer:-

1. In ideal characteristics of potentiometer, the resistance is varied linearly with displacement. This can be achieved by having large radius, more number of turns and high resistance elements.
2. The device which measure the output voltage of Potentiometer should have high input impedance to avoid loading error. Isolation amplifier works with high input impedance.
3. The wiper makes simultaneous contacts with adjacent turns to avoid discontinuity in output. The output is in the form of staircase steps.
4. Resolution specifies the output voltage per step. The resolution of the potentiometer is defined as the ratio of number of the steps to total number of turns.

5. The resolution of potentiometer is an important factor in the determination of minimum value of output voltage.

Specification of potentiometer:-

1. Turns per unit length is in the range of 6 to 30 turns per mm.
2. Torque required for wiper movement is in the range of 1×10^{-3} kg-m to 1×10^{-2} kg-m.
3. The total resistance of the potentiometer is in the range of 5 ohms to 1-mega ohms.
4. Power rating is 1 to 10 watts.
5. Heat dissipation is 1/2 watts per cm².

6. Generator voltage is supplied with
a voltage gradient in order to meet with the objective
synchro = in synchronous system is a full connection of
primary transformer and the synchronous control transformer
elements and related to an synchro pulse

The synchro pulse comprises fine angular displacement
and provides the output which is proportional to
angular difference of the ends of both the shafts
this can be used in two ways

- (i) for control the angular position of load from a remote
place long distance.
- (ii) for automatic correction of changes after disturbance
in the angular position of the load.

Synchro Transmitter

construction = synchro transmitter consists of stator and
rotor. Stator is made of laminated silicon steel and
stotted on inner periphery to achieve three phase winding.
With three coils 120° apart. The stator winding is star
connected.

The rotor is of dc motor construction with a
single winding and windings are terminated on two slip
rings. The single phase ac. voltage is applied to
rotor through slip windings.

Working principle = when the rotor is excited by ac
voltage then current flows in rotor produces magnetic
field due to this field an emf is induced on stator
coils.

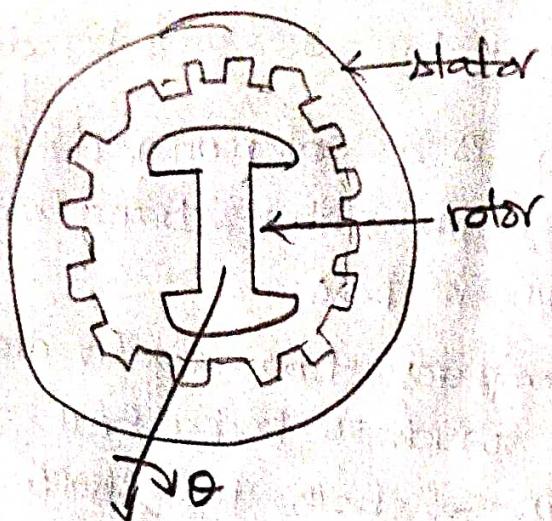


fig:- Constructional features

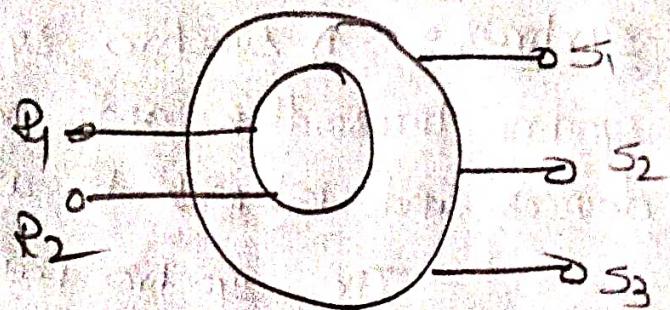


fig:- Schematic symbol of a synchro transmitter

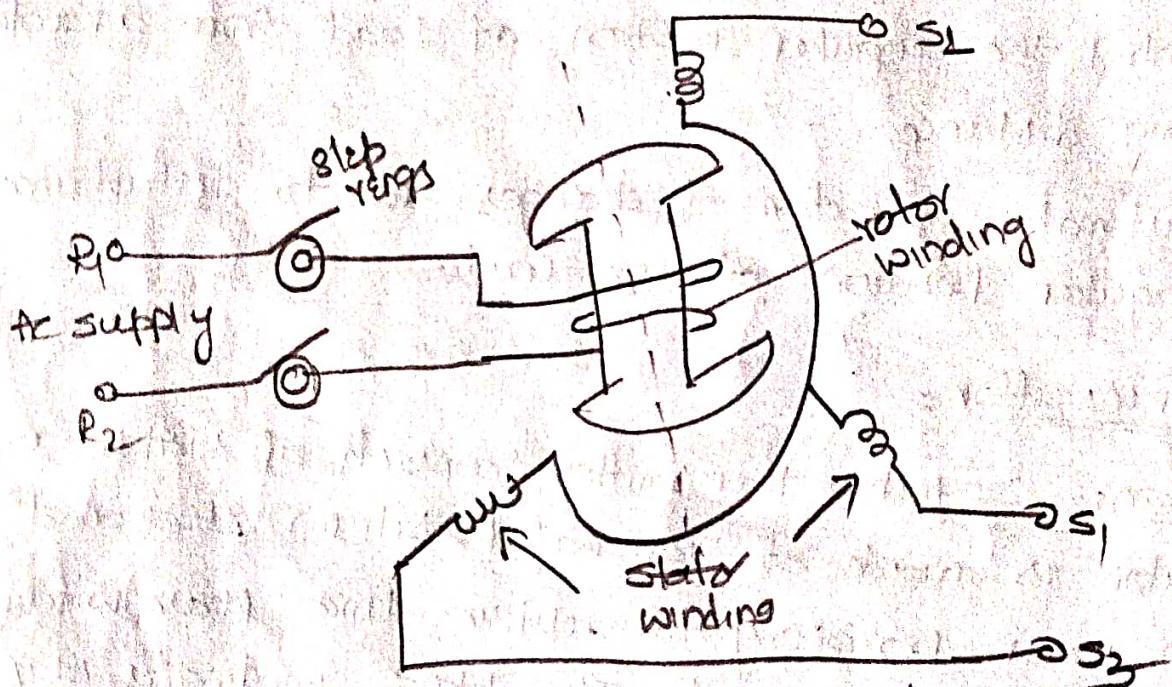


fig :- electrical circuit

fig :- synchro transmitter

Let E_R = Instantaneous value of ac voltage applied to rotor.

E_{S1}, E_{S2}, E_{S3} = Instantaneous value of emf induced in stator coils $S1, S2, S3$ with respect to neutral respectively.

E_R = Maximum value of rotor excitation voltage.

The voltage induced in stator coil depend on the angular position of the coils with respect to rotor axis. When the rotor rotates in anti clockwise direction by angle θ , emfs are induced in stator coils. The frequency of induced emf is same as that of rotor frequency. The magnitude of induced emfs are proportional to the turns ratio k_t turns ratio and coupling coefficient. The turns ratio k_t is a constant but coupling coefficient K_c is a function of rotor angular position.

$$\text{induced emf in star coil} = k_t k_c E_r \sin \omega t$$

Let e_{s2} be the reference vector.

When $\theta=0$, the flux linkage of coil s_2 is maximum and when $\theta=90^\circ$ the flux linkage of coil s_2 is zero. Hence the flux linkage of coil s_2 is a function of $\cos \theta$. The flux linkage of coil s_3 will be maximum after a rotation of 120° in anti clockwise direction and that of s_1 after a rotation of 240° .

Coupling coefficient, K_c for coil $-s_2 = k_c \cos \theta$.

Coupling coefficient, K_c for coil $-s_3 = k_c \cos(\theta - 120)$

Coupling coefficient, K_c for coil $-s_1 = k_c \cos(\theta - 240)$.

Hence emfs of stator coils with respect to neutral

$$e_{s2} = k_t k_c \cos \theta E_r \sin \omega t = k E_r \cos \theta \sin \omega t \rightarrow (1)$$

$$e_{s3} = k_t k_c \cos(\theta - 120) E_r \sin \omega t = k E_r \cos(\theta - 120) \sin \omega t$$

$$e_{s1} = k_t k_c \cos(\theta - 240) E_r \sin \omega t = k E_r \cos(\theta - 240) \sin \omega t$$

By KVL, the coil-to-coil emf can be expressed as (11)

$$e_{S1S2} = e_{S1} - e_{S2} = \sqrt{3} K E_x \sin(\theta + 240^\circ) \sin \omega t \rightarrow (2)$$

$$e_{S2S3} = e_{S2} - e_{S3} = \sqrt{3} E_x k \sin(\theta + 120^\circ) \sin \omega t$$

$$e_{S3S1} = e_{S3} - e_{S1} = \sqrt{3} K E_x \sin \theta \sin \omega t$$

When $\theta = 0$, in eqn (1) the maximum emf is induced in coil S_2 but in eqn (2) the coil-to-coil voltage e_{S3S1} is zero. This position of the rotor is defined as the electrical zero of the transmitter. The electrical zero position is used as reference for specifying the angular positions of the rotor.

Here the input to the synchro transmitter is the angular position of st. rotor shaft and the output is a set of three stator coil-to-coil voltages.

Synchro control transformer :- The constructional features of synchro control transformer is similar to that of synchro transmitter, except the shape of rotor. Here the rotor is made cylindrical so that the air gap is uniform. Due to this feature, we can minimize the changes in the rotor impedance.

Working :- The generated emf of the synchro transmitter is applied as input to the stator coils of control transformer. The rotor shaft is connected to the load whose position has to be maintained constant at the desired value.

Depending on the current position of the rotor and applied emf on the stator, an emf

an emf is induced in the rotor winding. This emf is used to drive a motor so that the position of the load is corrected.

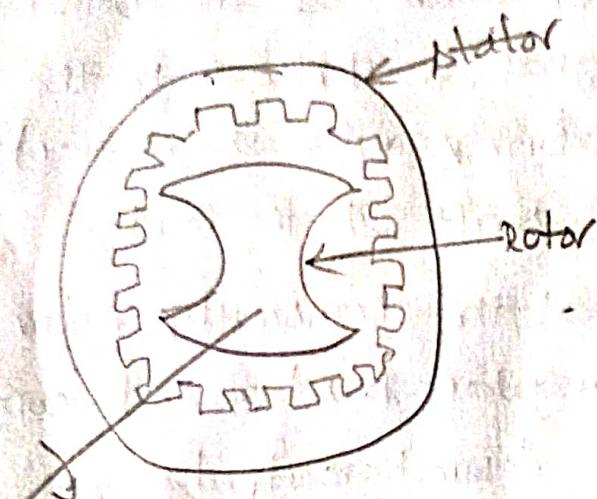


fig:- constructional feature

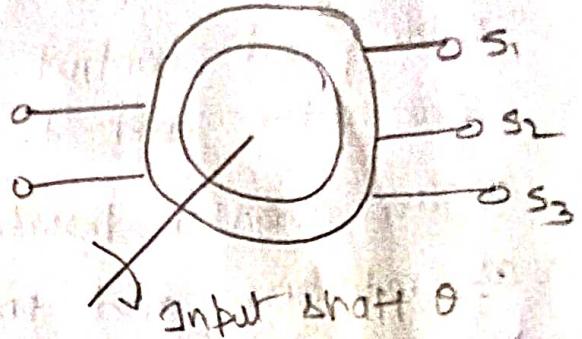


fig:- Schematic symbol of a synchro control transformer

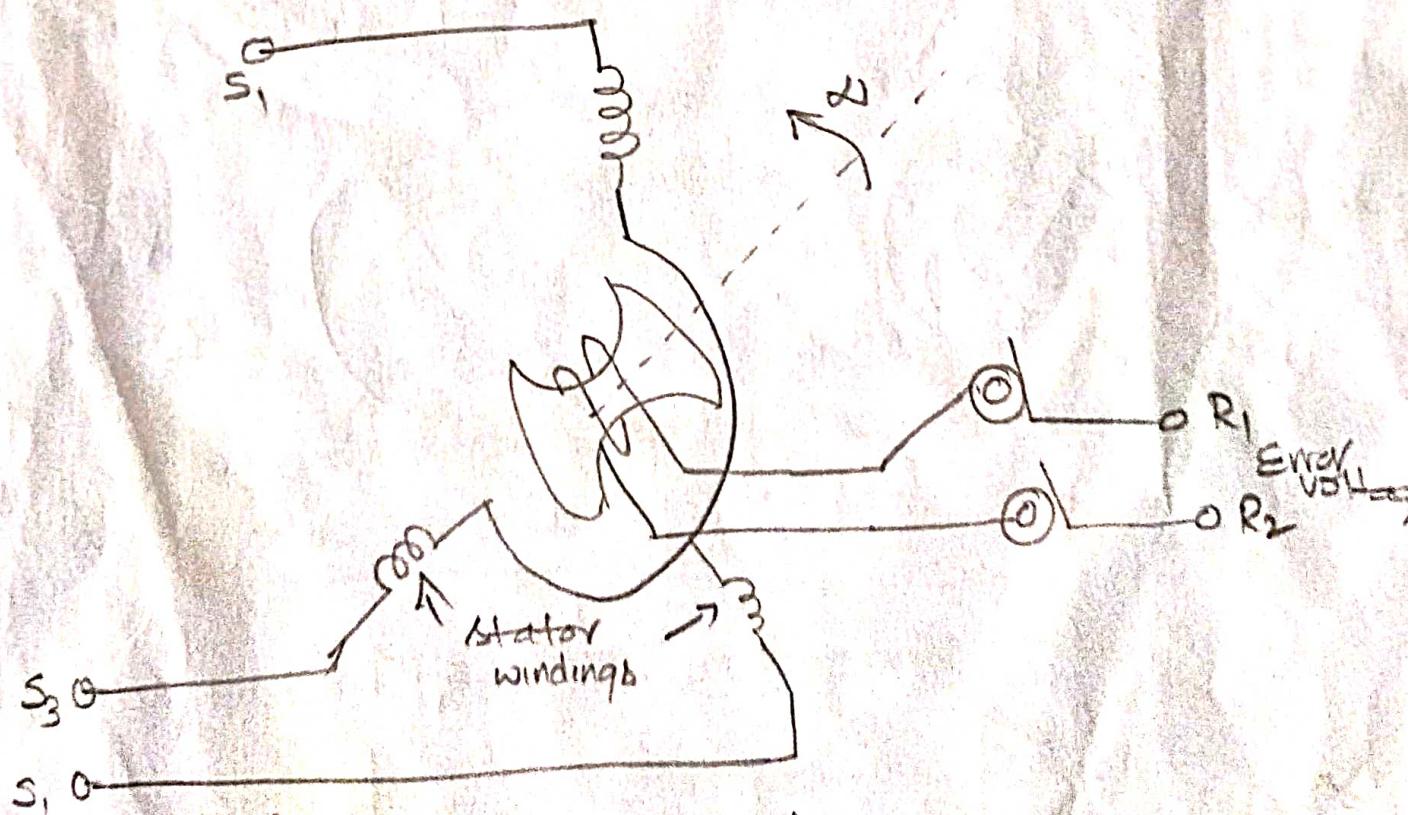


fig Electrical circuit

Fig : synchro control transformer

Synchro as error detector :-

1. The synchro error detector is an interconnection of a synchro transmitter and synchro control transformer.
2. The stator leads of of transmitter are directly connected to the stator leads of the control transformer.
3. Here the input ac signal is applied to the transmitter rotor and control transformer rotor is connected to a servo motor and to the shaft of the load.
4. The angular position of the transmitter rotor is used as reference and the angular position of control transformer rotor is the desired output.

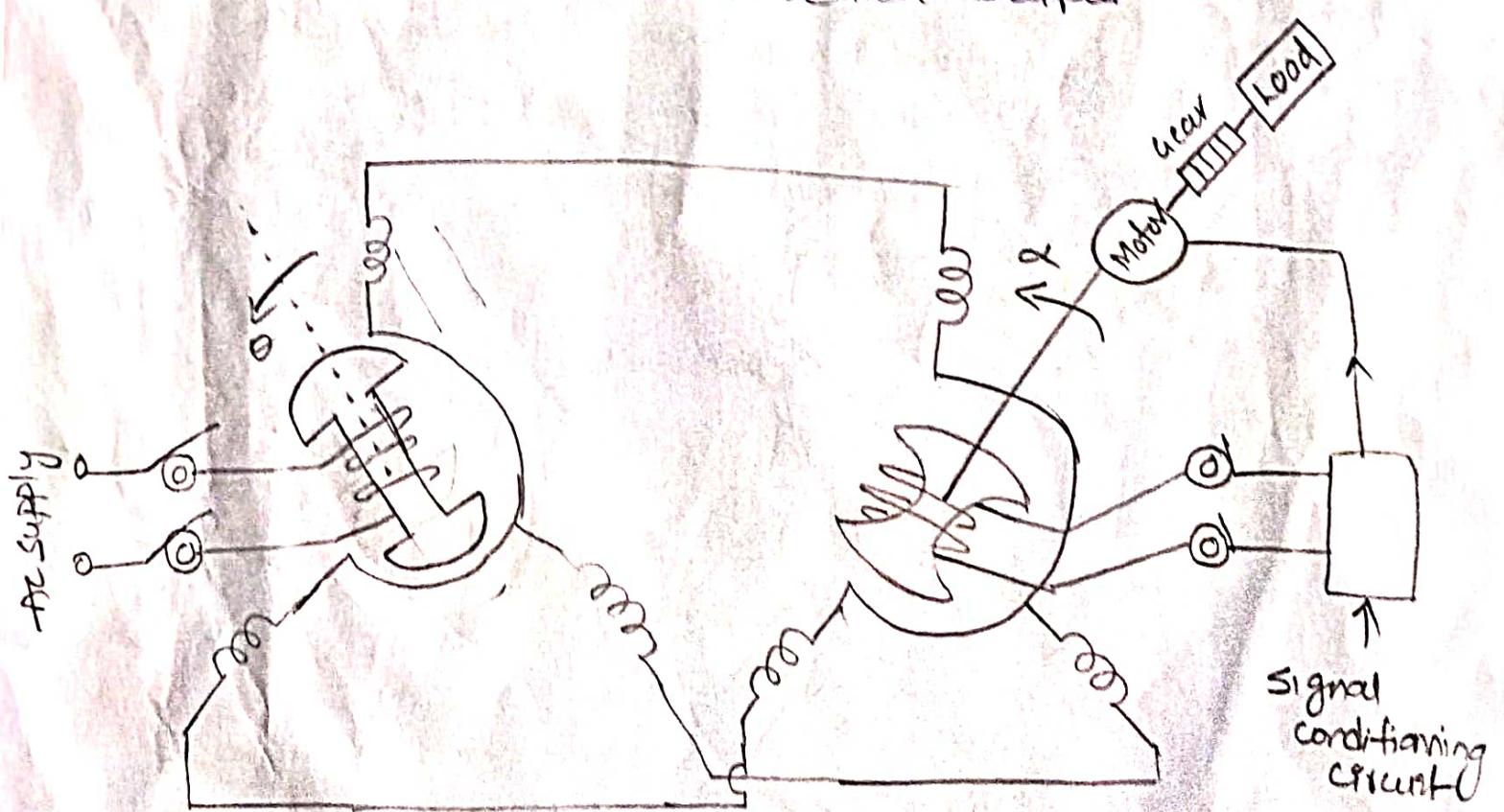


fig servosystem using synchro error detector.

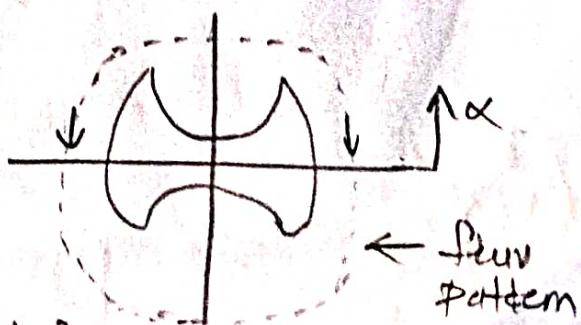
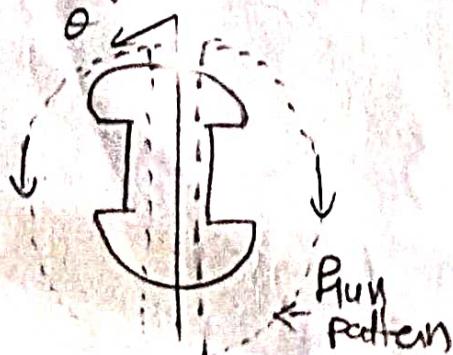


fig. Rotor positions and flux pattern

5. When the transmitted rotor is excited by ac voltage flux is produced in rotor thereby producing emfs are induced in stator coils.
6. The induced emf available across the rotor step rings of control transformer is measured by a signal conditioning circuit.
7. The output of signal conditioning circuit is used to drive motor to find the position of load.
8. Initially ^{rotor} shafts of transmitter and control transformer are in aligned position.
9. In this case the transmitter rotor is in electrical zero position and control transformer rotor is in null position. The angular difference of both rotor axis is 90° .
10. The angular difference of both rotor axis is 90° i.e., aligned position.
11. The null position of a control transformer rotor indicates the position of rotor at which the output of rotor winding is zero.
12. From when the transmitter rotor rotates through an angle θ from its electrical position then the synchronous control transformer also rotates in the same direction by an angle α .
13. The angular difference between two rotor axis is equal to $(90 - \theta + \alpha)$ and the voltage v_m induced in control transformer rotor is proportional to cosine of this ^{above} angle.
14. The error voltage is amplified and used to drive a servo motor. (13)

(15) voltage across slip rings of control transformer

$$\begin{aligned} e_m &= k' E_r \cos(90 - \theta + \alpha) \sin \omega t \\ &= k' E_r \cos(90 - (\theta - \alpha)) \sin \omega t \\ &= k' E_r \sin(\theta - \alpha) \sin \omega t \end{aligned}$$

where k' is proportional constant.

$$\text{let } \phi(t) = \theta - \alpha$$

for small values of $\phi(t)$, $\sin(\theta - \alpha) = \sin \phi(t) \approx \phi(t)$

$$e_m = k' E_r \phi(t) \sin \omega t$$

(16) from above equation we can say that output voltage of the synchro error detector is a modulated signal with carrier frequency.

(17) The signal conditioning circuit demodulates the voltage across slip rings and develops a demodulated signal and amplified error voltage to drive the motor.

The demodulated error voltage, $e = k_s \phi(t)$

where k_s = sensitivity of synchro error detector volt/dyn

on taking Laplace transform to above equation we get

$$E(s) = k_s \phi(s)$$

$$\therefore k_s = \frac{E(s)}{\phi(s)}$$

The above equation is the transfer function of synchro error detector.

Tachometer :- 1. Tachometer used for measuring the rotational speed and/or angular velocity of the machine.

2. It works on the principle of relative motion between the magnetic field and shaft of the machine.

3. This relative motion induces emf in the conducting coil called armature which is placed between two permanent magnets.

4. The induced emf is directly proportional to the speed of the shaft.

5. Basically we are having mechanical tachometers and electrical tachometers.

6. Mechanical tachometer measures speed of shaft in revolution per minute.

7. The electrical tachometer converts the angular velocity into an electrical voltage.

8. Depends on the nature of induced voltage the electrical tachometer are of two types.

(i) AC Tachometer Generator.

(ii) DC Tachometer generator.

DC Tachometer Generator :-

1. It contains permanent magnet, armature commutator, brushes, variable resistor and the moving coil voltmeter.

2. The machine whose speed is to be measured
is coupled with the shaft of the AC or DC tachometer generator.

3. When the armature conductor moves in the magnetic field, an emf induced in the conductor and it depends on flux link with the conductor and the speed of the shaft.

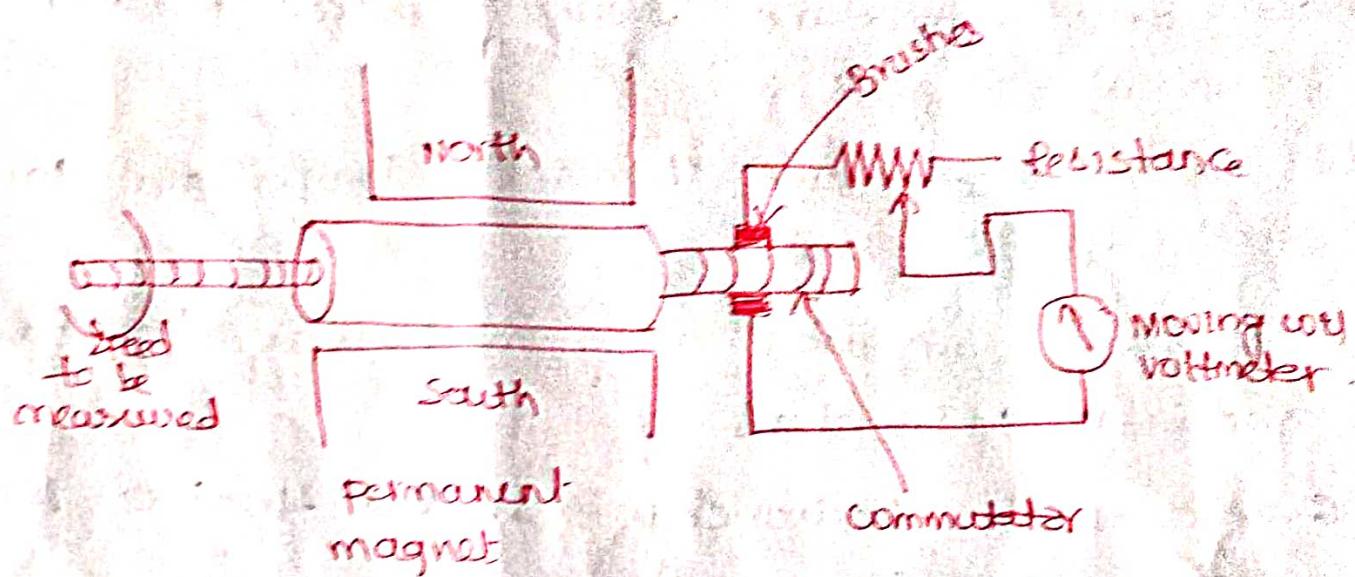


Fig: DC Tachometer generator.

4. The commutator converts the alternating current of the armature coil to the direct current with the help of brushes.

5. The resistance is connected in series with the voltmeter for conducting heavy current of the armature.

6. The moving coil voltmeter measures the induced emf with polarity indicating the direction of motion of the shaft.

The induced emf is given as

$$E = \frac{\phi PN}{60} \times \frac{z}{a}$$

where, E = generated voltage

ϕ - flux per pole in weber

P - number of poles.

N - speed in revolution per minutes.

Z - number of the conductor in armature windings.

a - number of parallel paths in the armature windings.

$$E \propto N$$

$$E = KN$$

$$K = \text{constant} = \frac{\phi P}{60} \times \frac{z}{a}$$

Advantages :-

1. The commutator and brushes The polarity of the induced voltage indicates the direction of rotation of the shaft

2. The conventional DC type voltmeter is used for measuring the induced voltage.

Disadvantages of DC generator :-

1. The commutator and brushes require frequent maintenance.

2. The output resistance of the DC tachometer is kept high as compared to input resistance.

AC Tachometer generator

1. The DC tachometer generator uses the commutator and brushes which have many disadvantages.
2. We can overcome two, above by having AC tachometer generator.
3. The AC tachometer has stationary armature and rotating magnetic field.
4. The rotating magnetic field produces emf in the stationary coil of stator.
5. The amplitude and frequency of the induced emf are proportional to speed of the shaft.

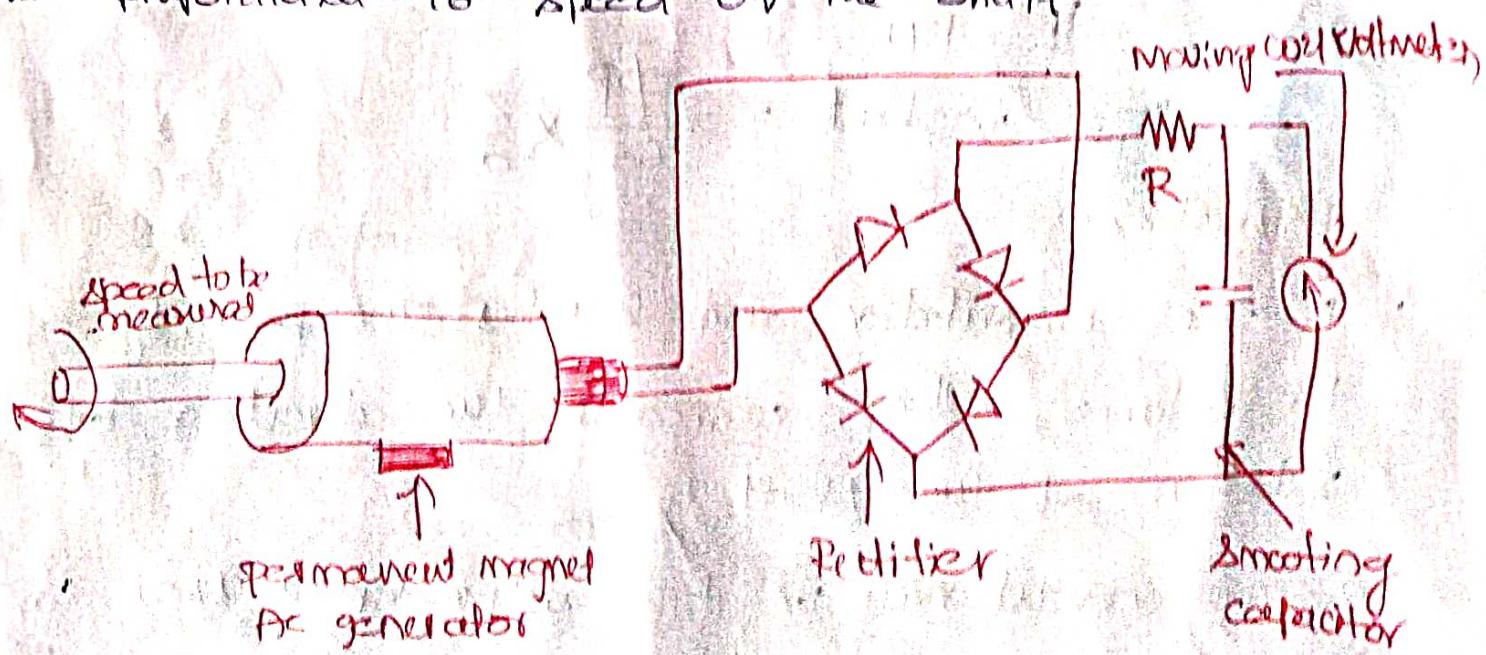


fig AC Tachometer generator

6. The above circuit is used to measure the speed of the machine (rotor) by taking the amplitude of the induced voltage.

7. The induced voltage are rectified and then passes through the capacitor filter for smoothening the ripples of rectified voltages.

AC position control system:

① Servosystems have wide range of applications in automated industries, space crafts, missiles, robots etc.

② An AC servomotor is a two phase AC motor. It consists of a reference winding and control winding which are separated by 90° and supplied with AC voltage phases shifted by 90° .

③ The torque produced in the motor is adjusted controlled by adjusting the control winding voltage.

④ In AC position control system, AC servomotor is controlled in closed loop system made by using either position or velocity feedback or the combination of both and this mechanism is called servo mechanism.

⑤ An error voltage with amplitude e corresponding to position of synchro transmitter rotor is produced.

⑥ In position feedback this error voltage is amplified and given to control winding i.e.,

$$V_c = k_1 e$$

⑦ In position plus velocity feedback the error voltage and its derivative is used for feedback. i.e.,

$$V_c = k_1 e + k_2 \frac{d}{dt} e$$

⑧ Torque produced by AC servomotor is directly proportional to control winding voltage.

⑨ Hence the motor of servomotor start move towards the reference position.

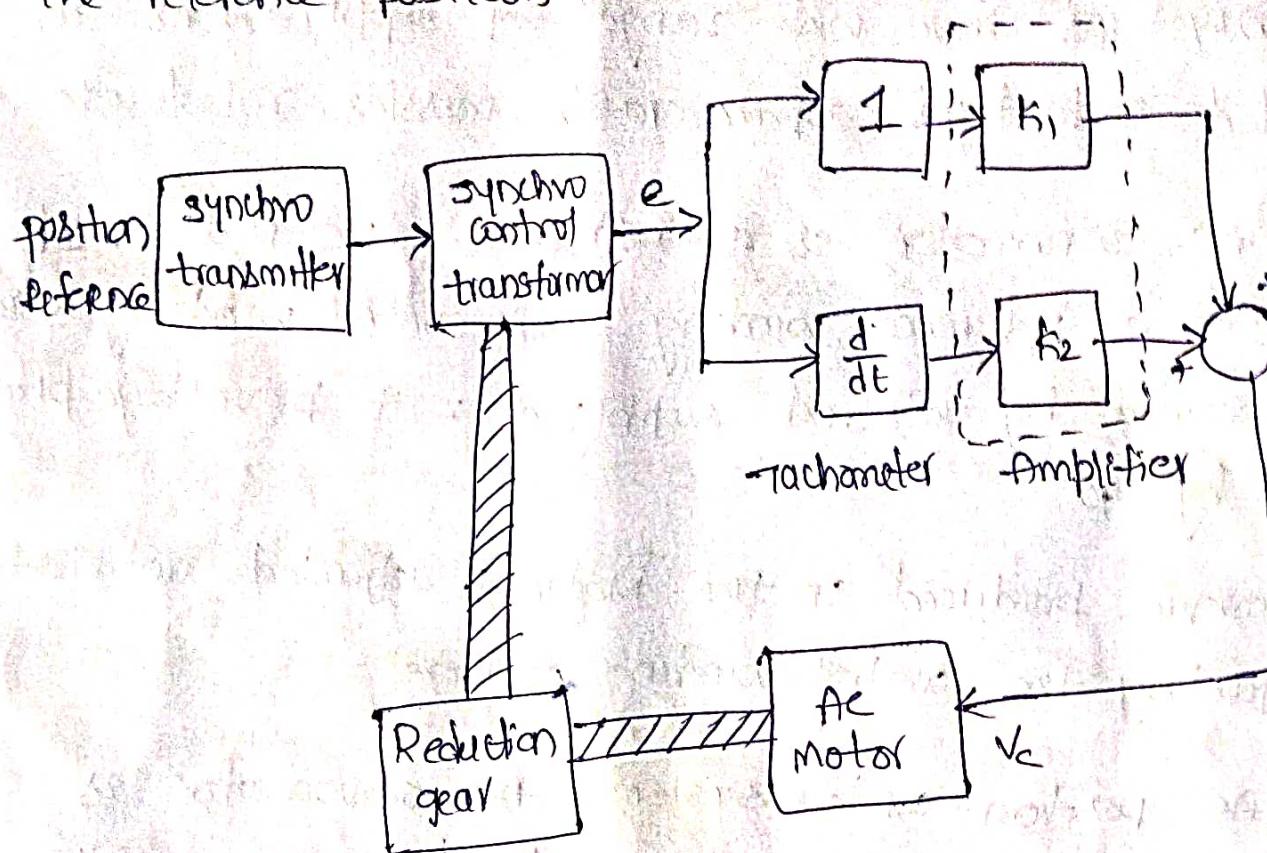


fig: AC position control system.

Controllers :-

• controllers will control the steady state error and transient response of the system.

⑩ P-controller :- It is called as proportional controller. It is a device which produces output which is proportional to a input. It improves steady state error and also stability. It decreases sensitivity of the system to parameter variations. It produces constant steady state error.

$$\text{Transfer function} = G_c(s) = K_p$$

② P-I controller :- It is known as proportional integral controller. The output of the device proportional to input and also integral of input. It increases order of system by due to this steady state error reduces. The system is less stable compared with original system.

$$\text{Transfer function} = G_c(s) = K_p + \frac{K_i}{s}$$

where, K_p = proportionality constant.

K_i = integral constant

③ P-D controller :- It is known as proportional derivative controller. The output of the device contains two terms, one term is proportional to input signal and another term proportional to derivative of input. It increases the damping ratio but there is a reduction in peak overshoot.

$$\text{Transfer function} = G_c(s) = K_p + K_d s$$

where K_p = proportionality constant

$K_d s$ = Derivative constant.

④ P.I.D controller :- It is known as proportional, integral derivative controller. The output of the system contains three terms. First term is proportional to input, second term proportional integral of input and third term proportional to derivative of input. It stabilizes the gain and reduces the steady state error and also reduces the peak overshoot.

$$\text{Transfer function} = G_c(s) = K_p + \frac{K_i}{s} + K_d s$$

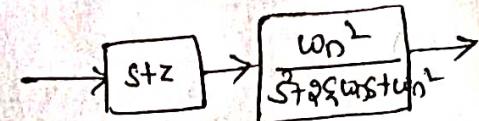
Effect of adding a zero to a system :-

Let us consider a second order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta \text{const} + \omega_n^2} \quad (\text{original system})$$

Now add a zero at $s = -z$ to the above second order closed loop system.

$$\frac{C_z(s)}{R(s)} = \frac{(s+z)\omega_n^2}{s^2 + 2\zeta \text{const} + \omega_n^2}$$



After adding a zero to the transfer function affects the gain of the system. In order to make the gain unaffected, divide the numerator by z .

$$\frac{C_z(s)}{R(s)} = \frac{(s+z) \cdot \frac{\omega_n^2}{z}}{s^2 + 2\zeta \text{const} + \omega_n^2} \quad (\text{modified system})$$

To analyse the effect of adding a zero to the system, we have to find time domain specifications.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} + \frac{s}{z} \cdot \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$$

If $C(t)$ is the step response of the system $\frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$, then step response of $\frac{s}{z} \left(\frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \right)$ is $\frac{1}{z} \frac{d}{dt}[C(t)]$

$$\therefore C_z(t) = C(t) + \frac{1}{z} \frac{d}{dt} C(t).$$

$$L\{f(t)\} = F(s)$$

where $C_z(t)$ is the response of the system with added zero and $C(t)$ is the response of original system.

$C(t)$ is the response of original system.

From above equation, we can observe the following two points

1. The rise time of $G_z(t)$ is less than that of $C(t)$. Hence the peak occurs earlier in $C_z(s)$ than $C(s)$.

2. The maximum peak overshoot of $G_z(t)$ is greater than that of $C(t)$.

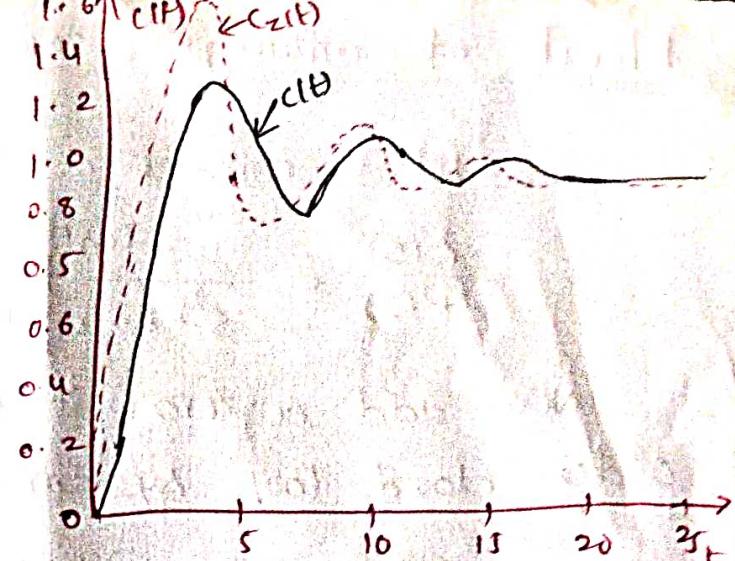
The addition of a zero to a system decreases rise time and increases maximum peak overshoot.

The values of rise time and peak overshoot depend on the selection of z . The smaller values of zero i.e., the zero close to origin produces large overshoot whereas the large values of zero i.e., the zero away from the origin produces negligible effect on transient response.

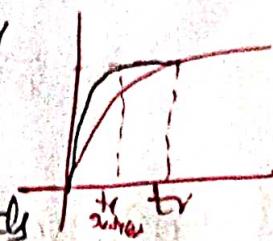
Therefore, selecting the zero near the origin will avoid large overshoots.

Performance indices :- When we design a control system, we are having parameters. By handling all these parameters, it becomes difficult to design a control system. So we are taking one optimized parameter or metric a function to a design a ^{control system} ~~parameters~~, here we are taking one optimized parameter instead of taking ξ , M_p , t_r , t_p , t_s etc to design a control system.

So we are introducing the following errors while designing a control system.



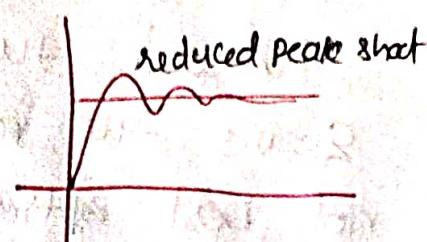
① Integral square error (ISE): Integral square error is given by $ISE = \int_0^\infty e^2(t) dt$. It by minimizing ISE results in the reduction of rise-time to limit large initial error. By minimizing ISE results in reduction of peak overshoot to limit the effect of small error lasting for a long time.



The above two requirements depends on ξ value.

Let us consider 2nd order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$



Best damping ratio for undamped case ($\xi < 1$) to minimize ISE. If input is $R(s) = Y_s$.

$$\begin{aligned} E(s) &= R(s) - C(s) \\ &= \frac{1}{s} - \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \end{aligned}$$

$$E(s) = \frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$ISE = \int_0^\infty e^2(t) dt$$

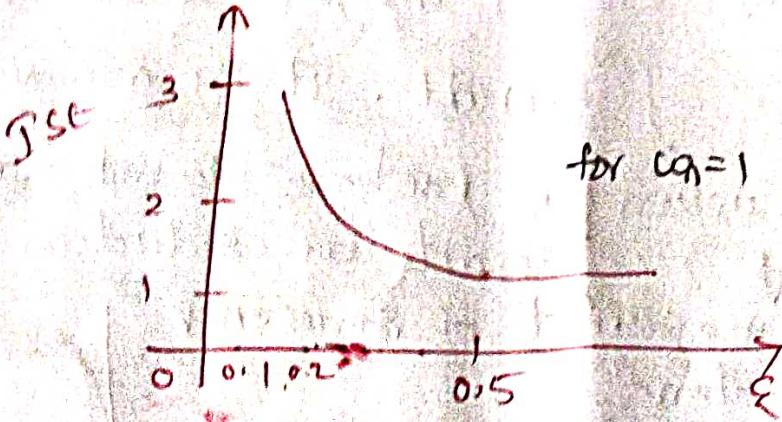
$$e(t) = r(t) - c(t)$$

$$= Y - Y_t + \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\sqrt{1-\xi^2}\omega_n t + \theta)$$

$$e(t) = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\sqrt{1-\xi^2}\omega_n t + \theta)$$

Squaring and integrating from 0 to ∞ yields

$$ISE = \frac{1}{2\omega_n} \left(\frac{1}{2\xi} + 2\xi \right)$$



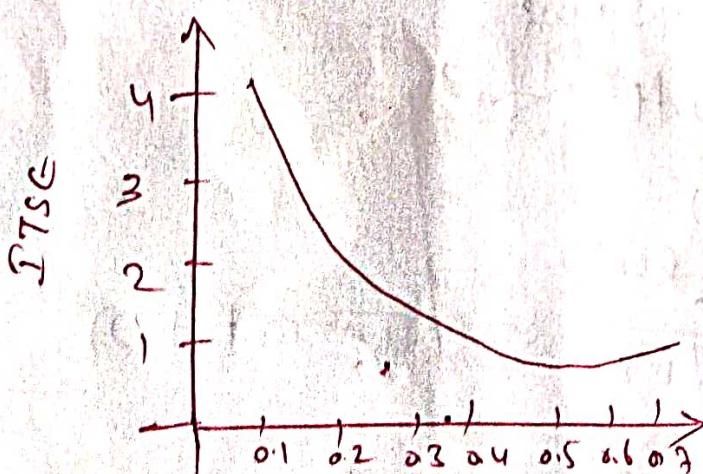
differentiate the TSE w.r.t ξ and make it equal to zero then error minimize and $\xi = 0.5$. After the $\xi = 0.5$ the response $\overset{\text{TSE response}}{y}$ is constant that's why TSE is not best optimizing metric to optimize the control system. So we move for $\mathcal{I}TSE$.

③ Integral time multiple small end of TSE :-

$$\mathcal{I}TSE = \int_0^\infty t e^2(t) dt$$

Solving the above using $e^2(t)$ yields

$$\mathcal{I}TSE = \frac{1}{\omega_n^2} \left(\xi^2 + \frac{1}{8\xi^2} \right)$$



When we solve $\xi = 0.7$

③ Integral absolute error: $(IAE)^o$

$$IAE = \int_0^\infty |e(t)| dt$$

④ Integral of time multiplied absolute error (ITAE)

$$ITAE = \int_0^\infty t |e(t)| dt$$

⑤ Selecting of ISE metric

is poor because it is flat and also minimum at 0.5

⑥ IAE is minimized at 0.7 and gives slightly better selectivity than ISE

⑦ ITAE performance index or metric is minimized at $\xi = 0.707$ and it is most sensitive among the three above indices.

⑧ ITSE is somewhat less sensitive to variations.

