STRUCTURAL ANALYSIS - II

- → M.D.M [Moment distribution method]
- → Kani's method.
- → Flexibility method
- → Stiffness Matrix Method
- → Conjugate beam method

Basics :-

Beam :- It is a horizontal structural member

* The main function of beam is load is transferred from slab to column.

Types of Beam :-

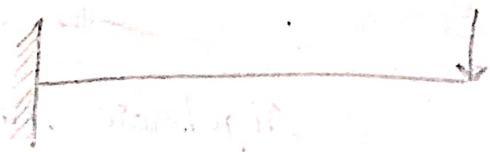
* Cantilever beam
* Simply supported beam
* Overhanging beam
* Continuous beam
* fixed beam
* Propped Cantilever beam (→ simple support)

~~Types of load~~ :-

Cantilever Beam :-

Cantilever Beam is defined as the a beam is fixed at one end free at another end is known as Cantilever beam.

Ex:-



Simply Supported Beam :-

S.S.B is defined as the a beam is supported at both ends is known as S.S.B.

UNIT-1 :- Moment Distribution Me...

2 Marks :-

- → M.D.M
- → Distribution factor
- → Carryover factor
- → Stiffness factor.
- → What is Sway and Non Sway?

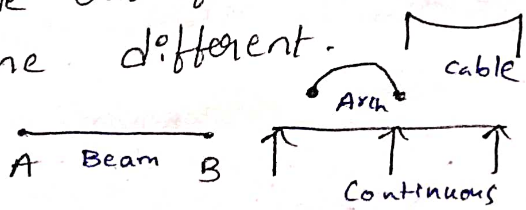
Date:- 26/9/22 ⟹ Introduction

Structural Analysis :-

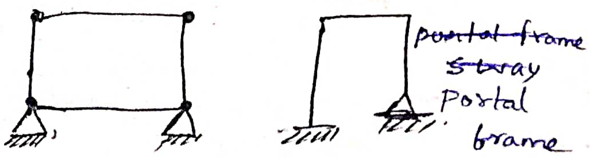
Civil Eng Structures

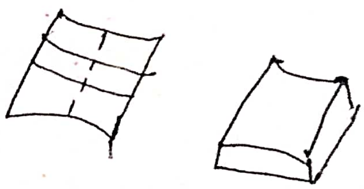
- **Discrete structure**
  Made out of one dimensional line different.
- **Continum structure**
  Made out of 2 (or) 3 dimensional line different
  1. Triangular
  2. Tetrahedral
  3. Hexahedral

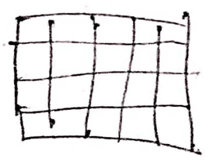
Ex:-



Truss







- Planer's Structure
  - Pinjointed
  - Ridged jointed
- Space structure
  - Pinjointed
  - Ridged Joint.

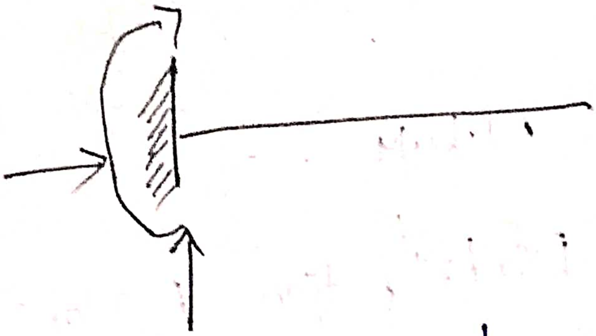
Analysis :-

- → External Support reaction
- → All Internal member due to force
- → Displacement function.

→ fixed Support

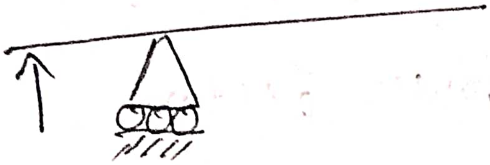
(H, V, M)

(3)



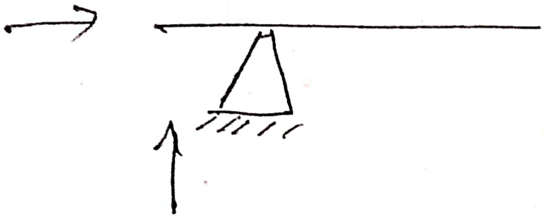
→ Roller Support

(V) (1)



→ Hinged Support

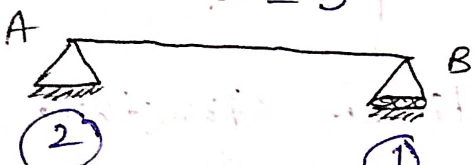
(H, V) (2)



Determinate structure

$R \leq E$

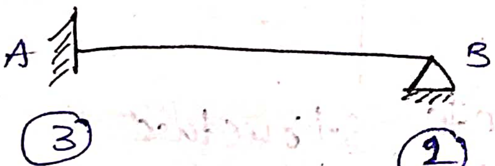
$3 \leq 3$



Indeterminate Structure

$R > E$

$5 > 3$



Date 27/9/2022

UNIT-1 :- Moment Distribution Method for frames

Moment Distribution Method

- Force method
  - Column Analogy Method
  - Three Moment equation
  - Strain energy theorem
  - flexibility Method
- Displacement method
  - Moment distribution method
  - Kan's method
  - Stiffness matrix method
  - Slope deflection method

Introduction :-

This method is widely used for the analysis of indeterminate structures. This system is most suitable for analysis of Continuous beams and portal frames.

This method was proposed by prof. Hardy Cross of USA in the year of 1929. This method Consists in Solving indirectly the equations of equilibrium as formulated in Slope-deflection ~~equation~~ (or) Method. Without finding the displacements.

This is an iterative procedures. This is also known as relaxation method.

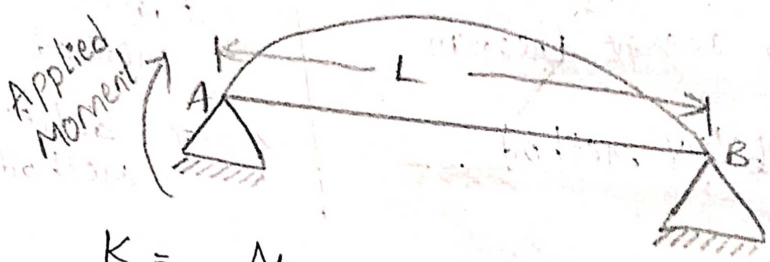
In this method is preffered for applications of Higher degree of indeterminate structures.

mportant basic terms :-

Stiffness :-

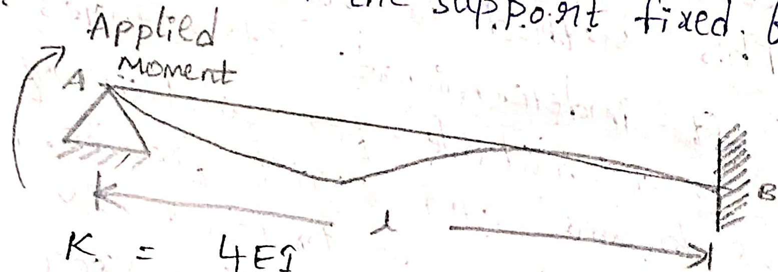
When the moment is required to produce deformation at simply supported beams an frames. Moment required to ~~produce~~ rotate an end by unit angle. When rotation is permit at the end is called Stiffness of the beam. It is denoted by the Symbol "K."

Case - i :- When the support is hinged



$$K = \frac{M_{AB}}{\theta_{AB}} = \frac{3EI}{L}$$

Case - ii :- When the support fixed for end



$$K = \frac{4EI}{L}$$

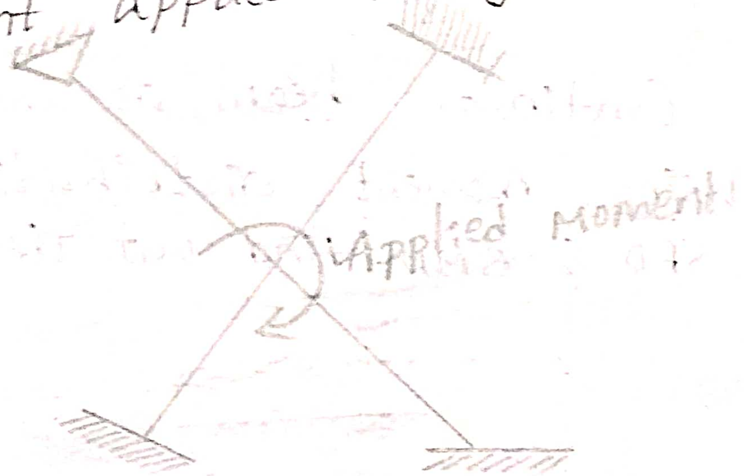
2. **Carryover Moment :-**

When a moment is applied at one end of a member allowing rotation of that end and fixing the far end some ~~M~~moment is develop at far end is known as carry over moment.

Carryover factor :- It is defined as the ratio of carryover moment to applied moment is called carry over factor.

3. Distribution factor :

* When a moment is applied to a rigid joint when a no. of member are meeting at the applied moment is shared by the member meeting at that point.
* Distribution factor is defined as the ratio of the moment shared by a member to the moment applied at joint.



$\therefore$ Distribution factor $= \dfrac{M_{AB}}{M}$

4. Joint Stiffness :-

The distribution factor for a member is relative stiffness to the total stiffness

$= \dfrac{K}{\Sigma K}$

Where, Σ is over various members meeting at Joint.

ΣK is called Joint stiffness.

Sign Conventions :-

1. fixed Support :-

The following sign conventions are using in fixed support are

All Clockwise Moments are Positive

All Anticlockwise Moments are Negative

2. Hinged Support :-

The following Sign Conventions are using in Hinged Support.

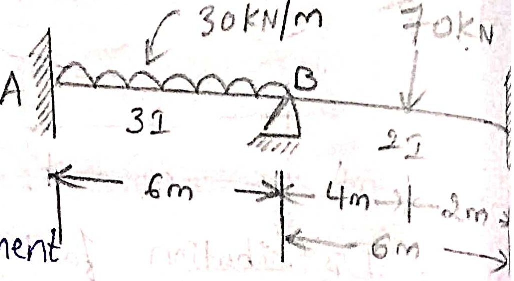
All Clockwise [+]

All Anticlockwise [-]

problem - 1 :-

1. Analyze the Continuous beam as Shown in figure by using moment distribution method and draw SFD & BMD. find out the reaction.

Sol:-



Step-1 :- fixed end Moment

for Span AB,

$$M_{FAB} = \frac{-WL^2}{12} = \frac{-30\times6^2}{12}$$

$$= -90 \text{ kN-m}$$

$$M_{FBA} = \frac{WL^2}{12} = \frac{30\times6^2}{12} = 90 \text{ kN-m}$$

for Span BC,

$$M_{FBC} = -\frac{Wab^2}{l^2} = -\frac{70\times4\times2^2}{6^2} = 31.1 \text{ kN-m}$$

$$M_{FCB} = \frac{Wa^2b}{l^2} = \frac{70\times4^2\times2}{6^2} = 62.2 \text{ kN-m}$$

Step-2 :- Distribution table

S.No	Joint	Member	Relative Stiffness Member (K)	Total Stiffness ΣK	Distribution factor
1.	B	BA	$\frac{4EI}{L} = \frac{4E(3I)}{6} = 2EI$	$2EI + 1.33EI = 3.33EI$	$\frac{K}{\Sigma K} = \frac{2EI}{3.33EI} = 0.60$
		BC	$\frac{4EI}{L} = \frac{4E(2I)}{6} = 1.33EI$	$3.33EI$	$\frac{K}{\Sigma K} = \frac{1.33EI}{3.33EI} = 0.40$

Step - 3 :- Moment Distribution Table

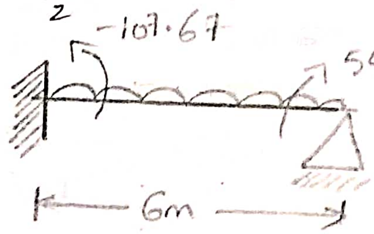
Joint	A	B		C
Members	AB	BA	BC	CB
DF		0.60	0.40	
fixed end Moment	-90	90	-31.1	62.2
Balance		-35.34	-23.56	
Carry over Moment (K)	-17.67			-11.78
final end Moment	-107.67	54.7	-54.7	50.4

Step - 4 :- Reactions

Span AB :-

$\Sigma f_x = 0$

$R_A \times 6 - 107.67 + 54.7 - 30 \times 6 \times \frac{6}{2} = 0$



$R_A \times 6 = 592.97$

$R_A = \frac{592.97}{6} = 98.82\,KN$

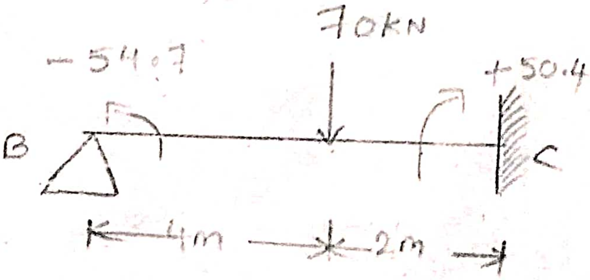
$\Sigma f_y = 0$

$R_A + R_{B1} = 30 \times 6$

$R_{B1} = 30 \times 6 - R_A$

$R_{B1} = 30 \times 6 - 98.82 = 81.18\,KN$

Span BC :-



$\Sigma f_x = 0$

$R_{B2} \times 6 - 54.7 + 50.4 - 70 \times 2 = 0$

$R_{B2} \times 6 = 144.3$

$R_{B2} = \frac{144.3}{6}$

$= 24.05\,KN$

$\Sigma f_y = 0$

$R_{B_2} + R_C = 70$

$R_C = 70 - R_{B_2}$

$= 70 - 24.05$

$= 45.95\ kN$

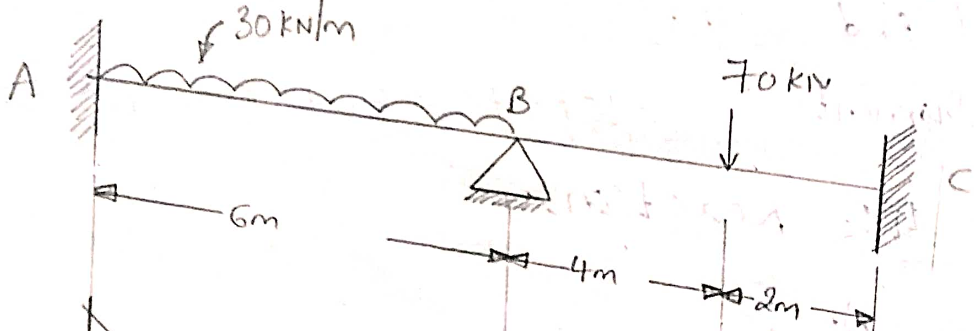
Step - 5 :- Bending Moment

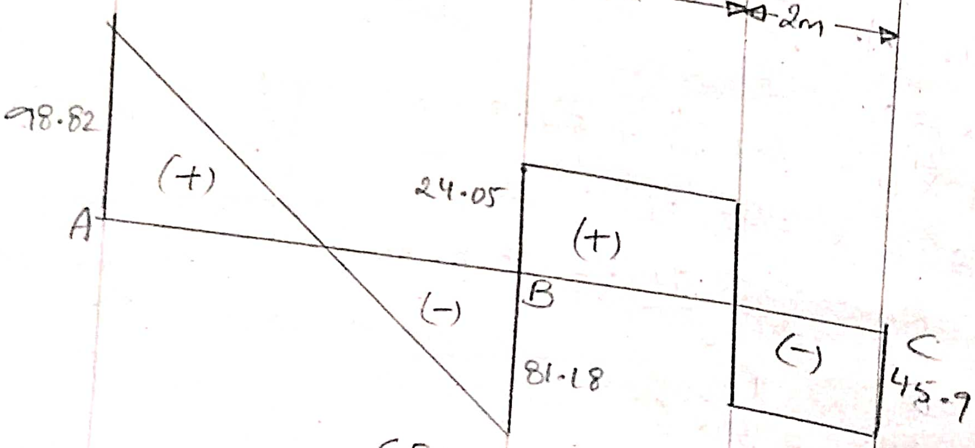
Span AB,

$$M = \frac{wl^2}{8} = \frac{30 \times 6^2}{8} = 135\ kNm$$

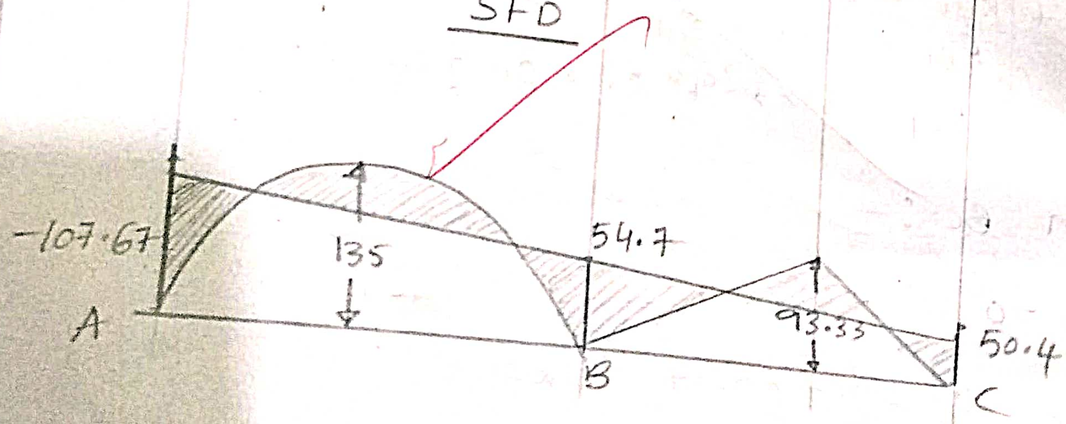
Span BC,

$$M = \frac{Wab}{l} = \frac{70 \times 4 \times 2}{6} = 93.33\ kNm$$



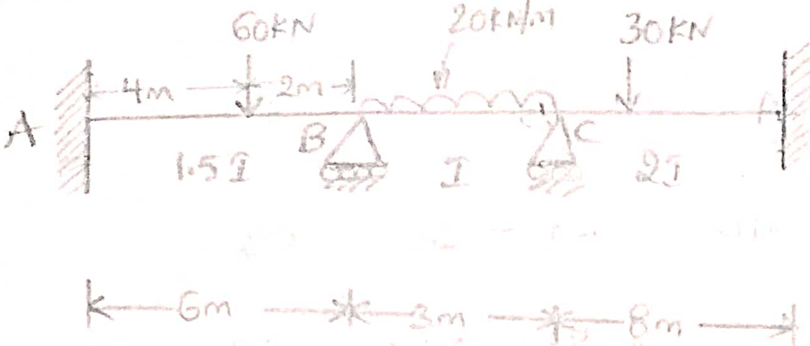


SFD



BMD

② Analyze the Continuous beam as shown in figure and draw the S.F.D and B.M.D



Sol:- Step - 1 :- fixed end Moments

for Span AB :-

$$M_{FAB} = -\frac{Wab^2}{l^2} = \frac{-60\times4\times2^2}{6^2} = -26.6 \text{ KN-m}$$

$$M_{FBA} = +\frac{Wa^2b}{l^2} = +\frac{60\times4^2\times2}{6^2} = 53.33 \text{ KN-m}$$

for Span BC :-

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{20\times3^2}{12} = -15 \text{ KN-m}$$

$$M_{FCB} = +\frac{wL^2}{12} = \frac{20\times3^2}{12} = +15 \text{ KN-m}$$

for Span CD :-

$$M_{FCD} = -\frac{WL}{8} = -\frac{30\times8}{8} = -30 \text{ KN-m}$$

$$M_{FDC} = +\frac{WL}{8} = +\frac{30\times8}{8} = +30 \text{ KN-m}$$

Step - 2 :- Distribution factor

Joint	Member	Relative Stiffness factor (K)	Total Stiffness ΣK	Distribution factor
B	BA	$\frac{4EI}{l} = \frac{4E(1.5I)}{6} = 1EI$	$EI + 1.33EI = 2.33EI$	$\frac{1EI}{2.33EI} = 0.43$
	BC	$\frac{4EI}{l} = \frac{4EI}{3} = 1.33EI$	$2.33EI$	$\frac{1.33EI}{2.33EI} = 0.57$
C	CB	$\frac{4EI}{l} = \frac{4EI}{3} = 1.33EI$	$1.33EI + EI = 2.33EI$	$\frac{1.33EI}{2.33EI} = 0.57$
	CD	$\frac{4EI}{L} = \frac{4E(2I)}{8} = EI$	$2.33EI$	$\frac{EI}{2.33EI} = 0.43$

Step-3 :- Moment Distribution

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF		0.43	0.57	0.57	0.43	
F.E.M	-26.66	53.33	-15	+15	-30	30
Balance		-16.48	-21.84	8.55	6.45	
Carry over Moment	-8.24		4.275	-10.92		3.22
Balance		-1.81	-2.41	6.22	4.69	
COM	-0.90		3.11	-1.20		2.34
Balance		-1.33	-1.61	0.68	0.516	
COM	-0.45		0.34	-0.80		0.25
Balance		-0.146	-0.19	0.456	0.344	
COM	-0.73		0.228	-0.095		0.172
Balance		-0.09	-0.129	0.054	0.040	
COM.	-37.13	33.99	-33.99	17.94	-17.94	35.98

Step-4 :- Bending Moment Diagram

Span AB,

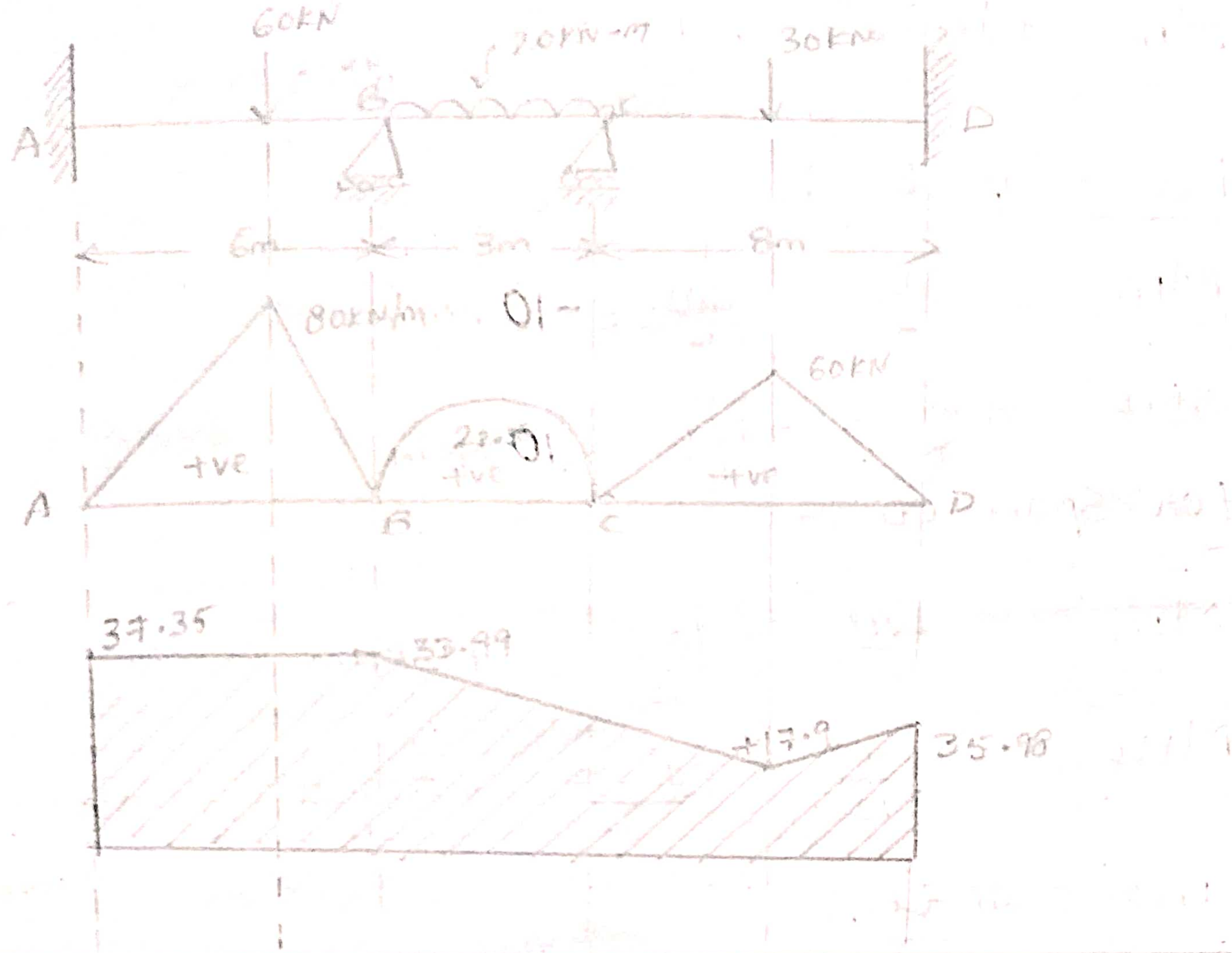
$$M = \frac{Wab}{l} = \frac{60 \times 4 \times 2}{6} = 80 \text{ kN-m}$$

Span BC,

$$M = \frac{Wl^2}{8} = \frac{20 \times 3^2}{8} = 22.5 \text{ kN-m}$$

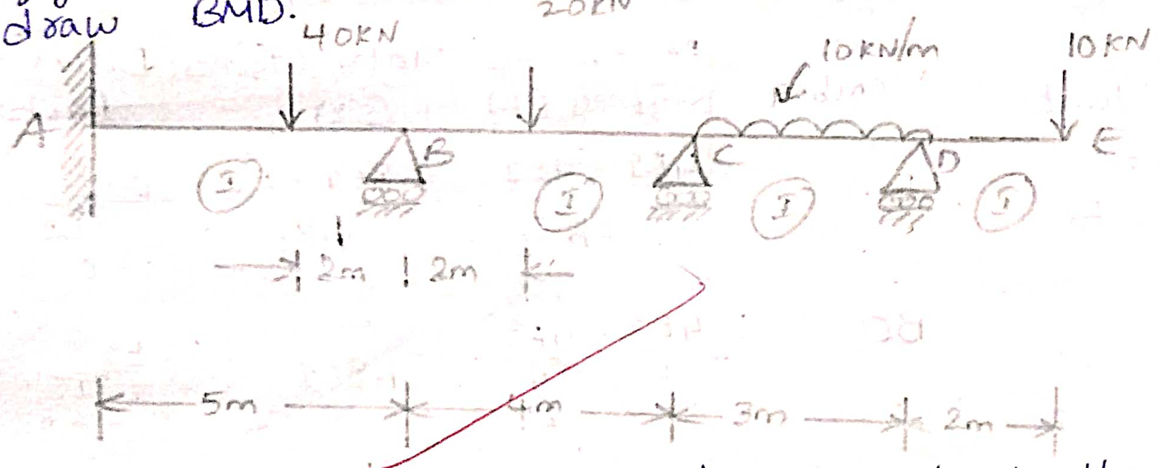
Span CD,

$$M = \frac{WL}{4} = \frac{30 \times 8}{4} = 60 \text{ kN-m}$$



Continuous Beam with Simply Support ends

1. Analyze the Continuous beam as Shown in figure by using Moment distribution method. and draw BMD.



Assume EI is Constant throughout the Section.

Sol:- **Step-1 :- fixed end Moment**

for Span AB,

$$M_{FAB} = -\frac{Wab^2}{l^2} = -\frac{40\times3\times2^2}{5^2} = -19.2 \text{ KN-m.}$$

$M_{FBA} = +\frac{Wa^2b}{l^2} = +\frac{40\times3^2\times2}{5^2} = 28.8\ kN\text{-}m$

For Span BC :-

$M_{FBC} = -\frac{WL}{8} = \frac{-20\times2}{8} = -10\ kN\text{-}m$

$M_{FCB} = +\frac{WL}{8} = \frac{+20\times2}{8} = +10\ kN\text{-}m$

For Span CD :-

$M_{FCD} = -\frac{WL^2}{12} = -\frac{10\times3^2}{12} = -7.5\ kNm$

$M_{FDC} = +\frac{WL^2}{12} = +\frac{10\times3^2}{12} = +7.5\ kNm$

For Span DE :-

$M_{FDE} = -WL = -10\times2 = -20\ kNm$

~~Mf~~

Step-2 :- Distribution factor

Sl. NO	Joint	Member	Relative Stiffness (K)	Total Stiffness (ΣK)	Distribution factor
	B	BA	$\frac{4EI}{L} = \frac{4EI}{5} = 0.8EI$	$0.8EI + EI = 1.8EI$	$\frac{0.8EI}{1.8EI} = 0.44$
		BC	$\frac{4EI}{L} = \frac{4EI}{4} = EI$	$1.8EI$	$\frac{EI}{1.8EI} = 0.555 \approx 0.56$
	C	CB	$\frac{4EI}{L} = \frac{4EI}{4} = EI$	$EI + EI = 2EI$	$\frac{EI}{2EI} = 0.5$
		CD	$\frac{3EI}{L} = \frac{3EI}{3} = EI$	$2EI$	$\frac{EI}{2EI} = 0.5$
	D	DC	$\frac{3EI}{L} = \frac{3EI}{3} = EI$	$EI + 0 = EI$	$\frac{EI}{EI} = 1$
		DE	0	EI	

Step - 3 :- Moment Distribution table.

Joint	A	B		C		D	E
Member	AB	BA	BC	CB	CD	DC	DE
D.F		0.44	0.56	0.5	0.5	1	0
F.E.M	-19.2	+28.8	-10	+10	-7.55	+7.55	-20
Balance		-8.272	-10.528	-1.227	+1.227	12.45	0
COM	-4.136		-0.613	-5.264	6.225	-0.613	0
Balance		0.269	0.343	-0.480	-0.480	0.613	
COM	0.134		-0.24	0.171	0.306	-0.24	
Balance		+0.105	+0.1344	-0.238	-0.238	+0.24	
COM	+0.052		-0.119	0.067	0.12	-0.119	
final end Moment Balance	-23.15	20.329	-21.023	3.029	-2.844	17.89	

Step - 4 :- Final End Moments

M_{AB} = - 23.152 KNm

M_{BA} = + 20.903

M_{BC} = - 20.765 KNm

M_{CB} = 3.029 KNm

M_{CD} = - 3 KNm

M_{DC} = + 20 KNm

M_{DE} = - 20 KNm

M_{ED} = 0 KNm

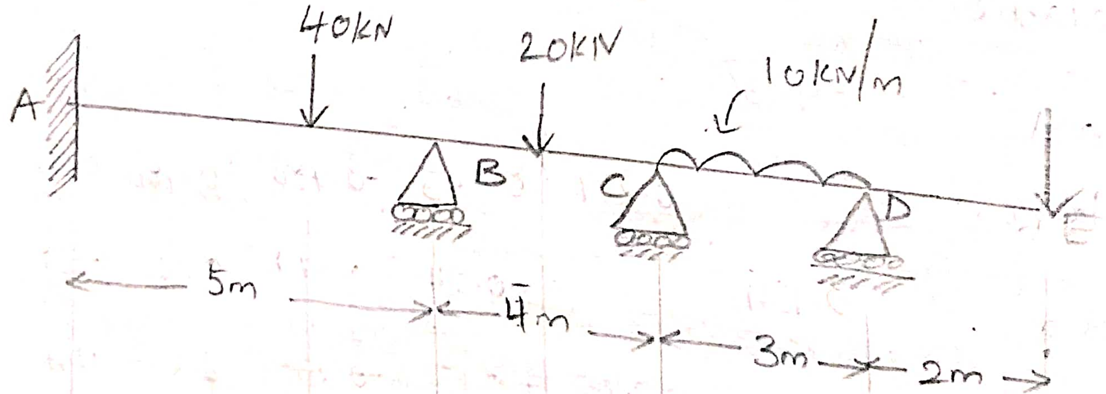
Step-5:- Bending Moment

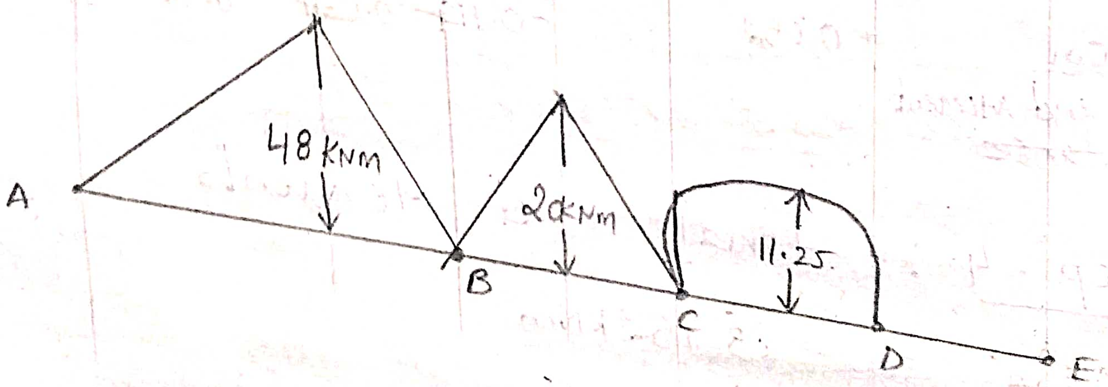
Span AB $= M = \frac{Wab}{l} = \frac{40 \times 3 \times 2}{5} = 48\,kNm$

Span BC $= M = \frac{WL}{4} = \frac{20 \times 4}{4} = 20\,kNm$

Span CD $= M = \frac{wl^2}{8} = \frac{10 \times 3^2}{8} = 11.25\,kNm$

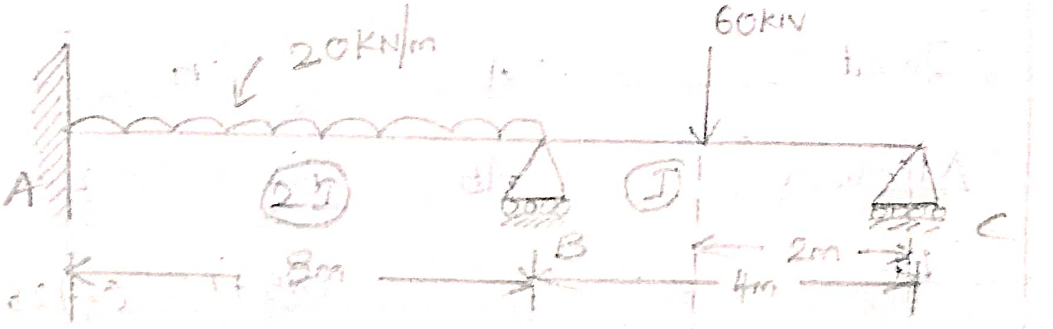
Step-6 :- BMD

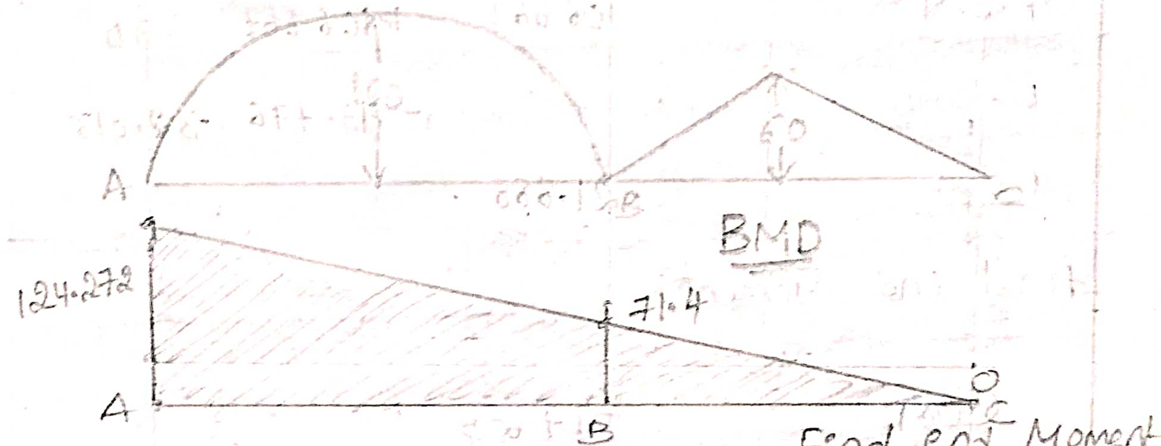




BMD

2. Analyze the Continuous beam as shown in figure by using moment distribution method. Calculate the BMD. & Draw BMD.





Find end Moment

Sol:- Step-1 :- fixed end Moments

Span AB,

$$M_{FAB} = -\frac{WL^2}{12} = -\frac{20\times 8^2}{12} = -106.667 \text{ KNm}$$

$$M_{FBA} = +\frac{WL^2}{12} = +\frac{20\times 8^2}{12} = +106.667 \text{ KNm}$$

Span BC,

$$M_{FBC} = -\frac{WL}{8} = -\frac{60\times 4}{8} = -30 \text{ KNm}$$

$$M_{FCB} = +\frac{WL}{8} = +\frac{60\times 4}{8} = +30 \text{ KNm}$$

Step-2 :- Distribution factor

Joint	Member	Relative Stiffness (K)	Total Stiffness (ΣK)	Distribution factor
B	BA	$\frac{4EI}{L} = \frac{4E(2I)}{8} = EI$	$EI + 0.75EI = 1.75EI$	$\frac{EI}{1.75EI} = 0.571$
	BC	$\frac{3EI}{L} = \frac{3EI}{4} = 0.75EI$	$1.75EI$	$\frac{0.75EI}{1.75EI} = 0.428$

Step-3:-

Moment Distribution table

Joint	A	B		C
Member	AB	BA	BC	CB
DF		0.571	0.428	
F.E.M	-106.667	+106.667	-30	+30
Balance	-106.667	+106.667	-15	-30
COM	-106.667	+106.667	-45	0
Balance		-35.211	-26.393	
COM	-17.605			
final end Moment	-124.272	+71.45	-71.4	

Step-4:-

final end Moments

$M_{AB} = -124.272$

$M_{BA} = +71.45$

$M_{BC} = -71.4$

Step-5:-

Bending Moment

Span AB,

$$M = \frac{wl^2}{8}$$

$$= \frac{20 \times 8^2}{8} = 160 \text{ kNm}$$

Span BC,

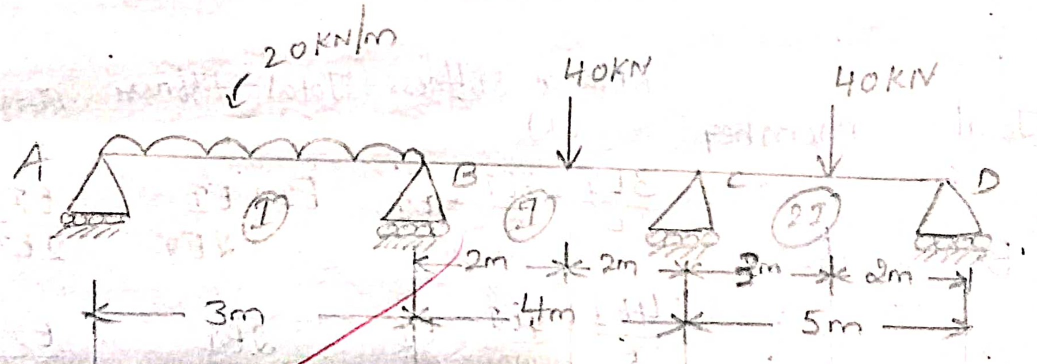
$$M = \frac{Wl}{4}$$

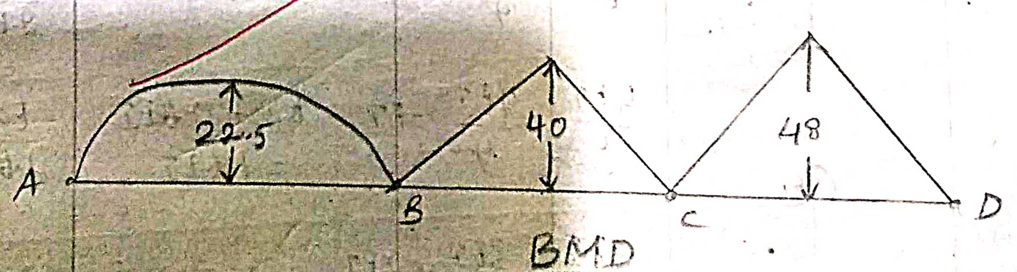
$$= \frac{60 \times 4}{4}$$

$$= 60 \text{ kNm}$$

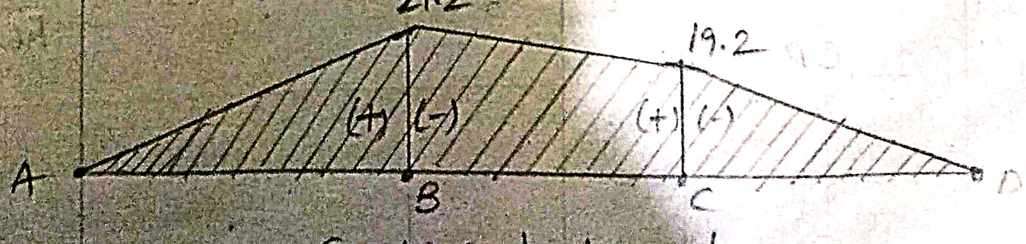
---

(4) Analyze the Continuous beam ABCD as shown in figure by Moment distribution method.





BMD



final end Moment

tep-1 :-

fixed end Moments

pan AB,

$M_{FAB} = -\frac{wl^2}{12} = -\frac{20\times3^2}{12} = -15$ KNm

$M_{FBA} = +\frac{wl^2}{12} = +\frac{20\times3^2}{12} = +15$ KNm

Span BC,

$M_{FBC} = -\frac{wL}{8} = -\frac{40\times4}{8} = -20$ kNm

$M_{FCB} = +\frac{wL}{8} = +\frac{40\times4}{8} = +20$ kNm

Span CD,

$M_{FCD} = -\cancel{\frac{wL}{8}} = -\frac{wa^2b}{L^2} = -\frac{40\times3^2\times2}{5^2} = -28.8$ KNm

$M_{FDC} = +\frac{wab^2}{L^2} = +\frac{40\times3\times2^2}{5^2} = 19.2$ kNm

Step-2 :-

Distribution factor

Joint	member	Relative Stiffness (K)	Total Stiffness Σk	Distribution factor
B	BA	$\frac{3EI}{L} = \frac{3EI}{3} = EI$	$EI + EI = 2EI$	$\frac{EI}{2EI} = 0.5$
	BC	$\frac{4EI}{L} = \frac{4EI}{4} = EI$	$2EI$	$\frac{EI}{2EI} = 0.5$
C	CB	$\frac{4EI}{L} = \frac{4EI}{4} = EI$	$EI + 0.6EI = 1.6EI$	$\frac{EI}{1.6EI} = 0.625$
	CD	$\frac{3EI}{L} = \frac{3EI}{5} = 0.6EI$	$1.6EI$	$\frac{0.6EI}{1.6EI} = 0.375$

Step-3 :-

Moment distribution table

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC.
DF		0.5	0.5	0.625	0.375	
F.E.M	-15	+15	-20	+20	-28.8	+19.2
	+15	+7.5			+9.6	-19.2
	0	+22.5	-20	+20	-19.2	0
Balance		-1.25	-1.25	-0.5	-0.3	
COM			-0.25	-0.625		
Balance		+0.125	+0.125	+0.390	+0.243	
COM			+0.195	+0.0625		
Balance		-0.097	-0.097	-0.039	-0.023	
COM			-0.0195	-0.048		
final end Moment		21.27	-21.2	+19.24	-19.2	

Step-4 :- final end Moment

$M_{BA} = +21.27$

$M_{BC} = -21.2$

$M_{CB} = +19.24$

$M_{CD} = -19.2$

Step-5 :- Bending Moment

for span AB, $M = \frac{WL^2}{8} = \frac{20 \times 3^2}{8} = 22.5\, kNm$

for span BC, $M = \frac{WL}{4} = \frac{40 \times 4}{4} = 40\, kNm$

for span CD, $M = \frac{Wab}{l} = \frac{40 \times 3 \times 2}{5} = 48\, kNm$

Moment Distribution Method

Relative Stiffness = k

The relative Stiffness of a member at a joint whose farther end is fixed is $\frac{I}{l}$

I = MI

L = Span (or) length.

Whose farther " " Hinged (or) S.S is

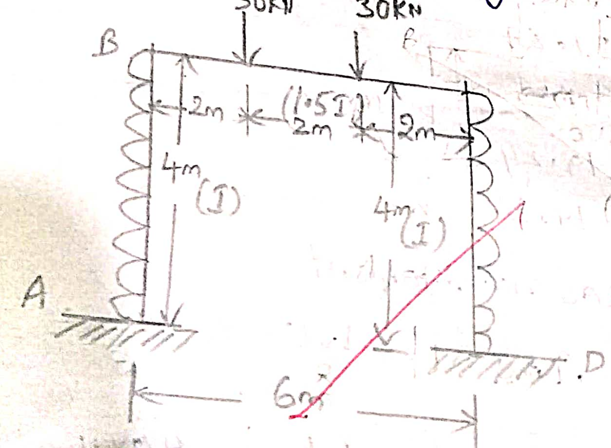
$\frac{3}{4} \times \frac{I}{L}$.

2.- Distribution factor

$$D.F = \frac{R.S \text{ of member}}{T.R.S \text{ @ Joint}}$$

The Sum of DF at a Joint is always equal to 1.

1) Solve the following Symmentrical ~~plane~~ portal frame by using by using M.D.M

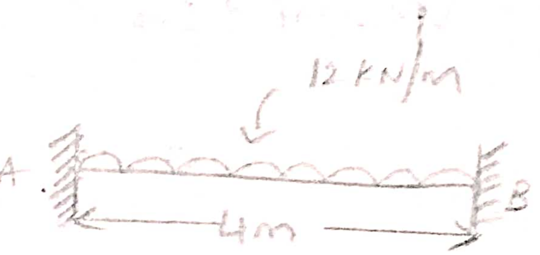


Analysis :- We find out the end Forces (or) Moments

Design :- Dimensions

1. fixed end Moment :-

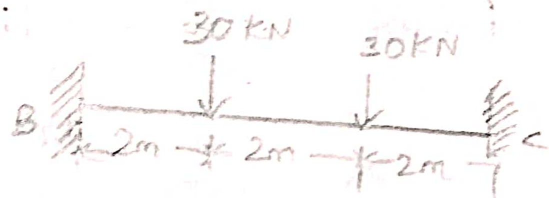
For Span AB,



$\overline{M}_{AB} = -\frac{WL^2}{12} = \frac{-12 \times 4^2}{12} = -16 \text{ kNm}$

$\overline{M}_{BA} = +\frac{WL^2}{12} = +16 \text{ kNm}$

For Span BC,



$\overline{M}_{BC} = -\frac{Wab^2}{L^2} + -\frac{Wab^2}{L^2}$

$= -\frac{30 \times 2 \times 4^2}{6^2} + -\frac{30 \times 4 \times 2^2}{6^2}$

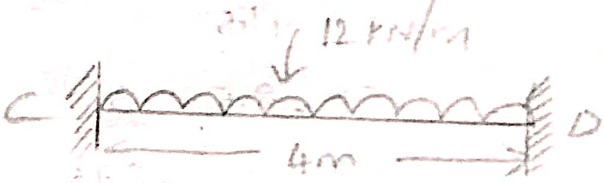
$= -40 \text{ kNm}$

$\overline{M}_{CB} = +\frac{Wa^2b}{L^2} + \frac{Wa^2b}{L^2}$

$= \frac{30 \times 2^2 \times 4}{6^2} + \frac{30 \times 4^2 \times 2}{6^2}$

$= +40 \text{ kNm}$

for span CD,



$\overline{M}_{CD} = -\frac{WL^2}{12} = \frac{-12 \times 4^2}{12} = -16 \text{ kNm}$

$\overline{M}_{DC} = +\frac{WL^2}{12} = +16 \text{ kNm}$

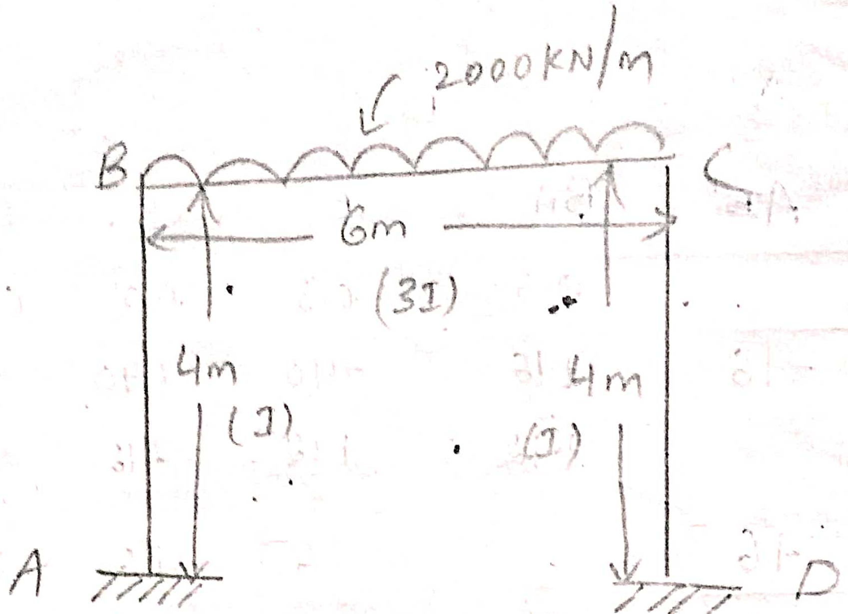
2. Distribution table :-

Joint	Member	R.S	T.R.S	D.F
B	BA	$\frac{I}{L} = \frac{I}{4}$	$\frac{I}{4} + \frac{I}{6} = \frac{1}{2}I = 0.5I$	$\frac{I}{4_2} \times 2I = 0.5$
	BC	$\frac{1.5I}{L} = \frac{1.5I}{6}$		$\frac{1.5I}{6_3} \times 2I = 0.5$
C	CB	$\frac{I}{L} = \frac{1.5I}{6}$	$\frac{1.5I}{6} + \frac{I}{4} = \frac{1}{2}I = 0.5I$	$\frac{1.5I}{6_3} \times 2I = 0.5$
	CD	$\frac{I}{L} = \frac{I}{4}$		$\frac{I}{4_2} \times 2I = 0.5$

3. Moment Distribution Table :-

	A	B		C		D
	AB	BA	BC	CB	CD	DC
		0.5	0.5	0.5	0.5	
F.E.M	-16	+16	-40	+40	-16	+16
		+12	+12	-12	-12	
	+6		-6	+6	$\boxed{+6}$	-6
	-10	+28	-34	+34	-28	+10
		+3	+3	-3	-3	
	+1.5		-1.5	+1.5		-1.5
	-8.5	+31	+32.5	+32.5	-31	+8.5
		+0.75	+0.75	-0.75	-0.75	
	+0.375		-0.375	+0.375		-0.375
	-8.125	31.75	-32.125	+32.125	-31.75	8.125
		+0.187	+0.187	-0.187	+0.187	
	+0.0935		-0.0935	+0.0935		-0.0935
	-8.031	+31.937	-32.031	32.031	-31.563	+8.031
		+0.047	+0.047	-0.234	-0.234	
	+0.0235		-0.117	+0.0235		-0.117
	-8.007	+31.984	-32.101	+31.820	-31.797	+7.914
		+0.0585	+0.0585	-0.0115	-0.0115	
	+0.0292		-0.00575	+0.0292		-0.0057
	-7.977	+32.03	-32.04	+31.837	-31.8	7.908

(2)



Step-1 :- fixed END Moment

$\bar{M}_{AB} = 0$, $\bar{M}_{BA} = 0$

$\bar{M}_{CD} = 0$, $\bar{M}_{DC} = 0$

Due to there is no loads.

for span BC, $\bar{M}_{BC} = -\frac{wl^2}{12} = -\frac{2000 \times 6^2}{12} = -6000$ KN.m

$\bar{M}_{CB} = +\frac{wl^2}{12} = +\frac{2000 \times 6^2}{12} = +6000$ KNm

Step-2 :- Distribution table,

Joint	Member	RS	T.R.S	DF
B	BA	$\frac{I}{L} = \frac{I}{4}$	$\frac{I}{4} + \frac{3I}{6} = \frac{3}{4} I$	$\frac{I}{4} \times \frac{4I}{3} = \frac{1}{3} = 0.33$
	BC	$\frac{I}{L} = \frac{3I}{6}$		$\frac{3I}{6} \times \frac{4I}{3} = \frac{2}{3} = 0.67$
C	CB	$\frac{I}{L} = \frac{3I}{6}$	$\frac{3I}{6} + \frac{I}{4} = \frac{3I}{4}$	$\frac{3I}{6} \times \frac{4I}{3} = \frac{2}{3} = 0.67$
	CD	$\frac{I}{L} = \frac{I}{4}$		$\frac{I}{4} \times \frac{4I}{3} = \frac{1}{3} = 0.33$

Step 3: Moment Distribution Table

	A		B		C	D
	AB	BA	BC	CB	CD	DC
		0.33	0.67	0.67	0.33	
F.E.M	0	0	-6000	+6000	0	0
		+1980	+4020	-4020	+1980	
	+990		-2010	+2010		+990
	+990	+1980	-3990	+3990	+1980	+990
		+663.3	+1346.7	-3999.9	-1970.1	
	+331.65		-1999.95	+673.35		-985.05
	+1321.65	+2643.3	-4643.25	+663.45	+9.9	+4.95
		+659.983	+1339.966	-451.144	-222.205	
	+329.991		-225.572	+669.983		-111.102
	+1651.641	+3303.28	-3528.856	+882.289	-212.305	-106.152
		+74.440	+151.135	-448.886	-221.093	
	+37.22		-224.443	+75.567		-110.546
	+1688.861	+3377.72	-3602.164	+508.97	-433.398	-216.698
		+74.066	+150.377	-50.633	-24.938	
	+37.033		-25.316	+75.188		-12.469
	+1725.894	+3451.786	-3477.103	+533.525	-458.336	-229.167
		+8.354	+16.962	-50.376	-24.812	
	+4.177		-25.188	+8.481		-12.406
	+1730.071	+3460.14	-3485.329	+491.63	-483.148	-241.573
		+8.312	+16.876	-5.646	-2.781	
	+4.156		-2.823	+8.438		-1.390
	+1734.227	+3468.452	-3471.276	+494.422	-485.929	-242.963
		+0.931	+1.892	-5.690	-2.802	
	+0.465		-2.845	+0.946		-1.401
	+1734.692	+3469.383	-3472.229	+489.678	-488.731	-244.364
		+0.936	+1.900	+0.634	-0.312	
	+0.468		-0.317	+0.95		-0.156
	+1735.16	+3470.319	-3470.646	+489.994	-489.043	-244.52
		+0.107	+0.219	-0.637	-0.313	
	+0.0535		-0.318	+0.109		+0.156
	+1735.213	+3470.42	-3470.74	+489.466	-489.356	-244.364

	+0.105	+0.214	-0.0737	-0.0363	
+0.0525		-0.0368	+0.107		-0.018
+1735.265	+3470.52	-3470.5	+489.49	-489.4	-244.3

	A	B		C		D
	AB	BA	BC	CB	CD	DC
		$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	
F.E.M	0	0	-6000	+6000	0	0
		+2000	+4000	-4000	-2000	
	+1000		-2000	+2000		-1000
	+1000	+2000	-4000	+4000	-2000	-1000
		+666.67	+1333.333	-1333.333	-666.67	
	+333.335		-666.665	+666.665		+333.335
	+1333.335	+2666.67	-3333.332	+3333.332	-2666.67	-1333.33
		+222.220	+444.441	-444.441	-222.220	
	+111.11		-222.220	+222.220		-111.11
	+1444.445	+2888.89	-3111.111	+3111.111	-2888.89	-1444.44
		+74.073	+148.147	+148.147	-74.073	
	+37.036		-74.073	+74.073		-37.036
	+1481.481	+2962.963	-3037.037	+3037.037	-2962.963	-1481.48
		+24.691	+49.382	-49.382	-24.691	
	+12.345		-24.691	+24.691		-12.345
	+1493.826	+2987.654	-3012.346	+3012.346	-2987.654	-1493.8
		+8.230	+16.461	-16.461	-8.230	
	+4.115		-8.230	+8.230		-4.115

+1497.941 +2995.884 -3004.115 +3004.115 -2995.884 -1497.941

+2.743 +5.487 -5.487 -2.743

+1.371 -2.743 +2.743 -1.371

+1499.312 +2998.627 -3001.371 +3001.371 -2998.627 -1499.312

+0.914 +1.829 -1.829 -0.914

+0.457 -0.914 +0.914 -0.457

+1499.771 +2999.541 -3000.456 +3000.456 -2999.541 -1499.769

+0.305 +0.61 -0.61 +0.305

+0.1525 -0.305 +0.305 -0.1525

+1499.923 +2999.846 -3000.151 +3000.151 -2999.846 -1499.921

+0.102 +0.204 -0.204 -0.102

+0.051 -0.102 +0.102 -0.051

+1499.974 +2999.948 -3000.049 +3000.049 -2999.948 -1499.972

+0.0336 +0.0673 -0.0673 -0.0336

+0.0168 -0.0336 +0.0336 -0.0168

+1499.990 +2999.981 -3000.015 +3000.015 -2999.981 -1499.988

+0.0113 +0.0226 -0.0226 -0.0113

+0.00565 -0.0113 +0.0113 -0.00565

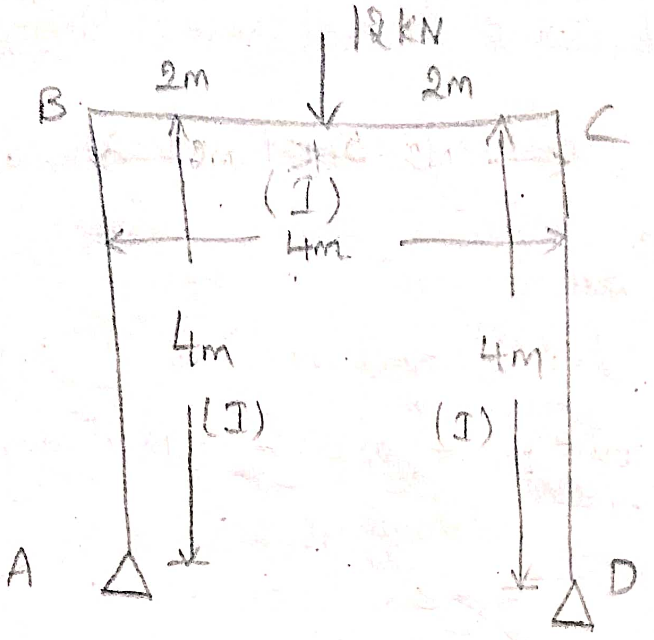
+1499.995 +2999.992 -3000.003 +3000.03 -2999.992 -1499.993

+0.00366 +0.00733 -0.00733 -0.00366

+0.00183 -0.00366 +0.00366 -0.00183

+1499.996 +2999.995 -2999.995 +3000.026 +3000 -1499.994

(4)



5 (or) 6 Iterations

Step-1 :- fixed end Moments

for Span AB,

$\bar{M}_{AB} = 0$

$\bar{M}_{BA} = 0$ [Due to there is no loads].

for Span BC,

$\bar{M}_{BC} = -\frac{WL}{8} = -\frac{12\times4}{8} = -6$ kN.m

$\bar{M}_{CB} = +\frac{WL}{8} = +\frac{12\times4}{8} = +6$ kN.m

for Span CD,

$\bar{M}_{CD} = 0$

$\bar{M}_{DC} = 0$

$\frac{12+16}{64} = \frac{28}{64}$

Step-2 :- Moment Distribution table.

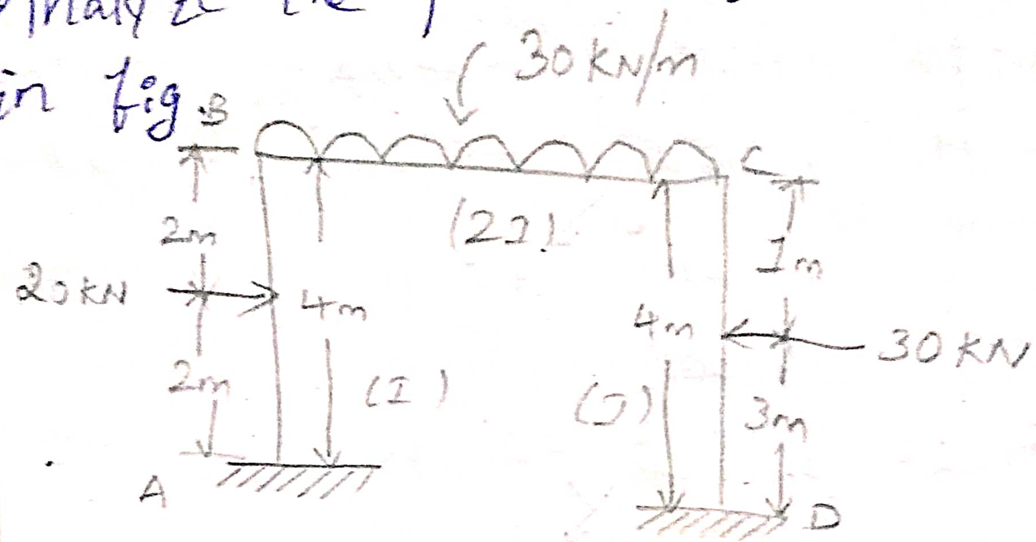
Joint	Member	R.S	T.R.S	D.F
B	BA	$\frac{3}{4}\times\frac{I}{L} = \frac{3}{4}\times\frac{I}{4} = \frac{3I}{16}$	$\frac{3}{16}I + \frac{I}{4} = \frac{7I}{16}$	$\frac{3I}{16}/\frac{7I}{16} = \frac{3I}{16}\times\frac{16}{7I} = \frac{3}{7}$
	BC	$\frac{I}{L} = \frac{I}{4}$		$\frac{I}{4}/\frac{7I}{16} = \frac{I}{4}\times\frac{16}{7I} = \frac{4}{7}$
C	CB	$\frac{I}{L} = \frac{I}{4}$	$\frac{I}{4} + \frac{3}{16}I = \frac{7I}{16}$	$I/4/\frac{7I}{16} = \frac{4}{7}$
	CD	$\frac{3}{4}\times\frac{I}{L} = \frac{3}{4}\times\frac{I}{4} = \frac{3I}{16}$		$\frac{3I}{16}/\frac{7I}{16} = \frac{3}{7}$

Step - 3 :- Moment Distribution table

	A	B		C		D
	AB	BA	BC	CB	CD	DC
~~FEM~~	0	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{4}{7}$	$\frac{3}{7}$	0
F.E.M	0	0	−6	+6	0	0
		+2.571	+3.428	−3.428	+2.571	
			−1.714	+1.714		
	0	+2.571	−4.286	+4.286	+2.571	0
		+0.735	+0.98	−0.98	−0.735	
			−0.49	+0.49		
	0	+3.306	−3.796	+3.796	−3.306	0
		+0.21	+0.28	−0.28	+0.21	
			−0.14	+0.14		
	0	+3.516	−3.656	+3.656	−3.516	0
		+0.06	+0.08	−0.08	−0.06	
			−0.04	+0.04		
	0	+3.576	−3.736	+3.736	−3.576	0
		+0.0685	+0.0914	−0.0914	~~−0.0171~~ −0.0685	
			−0.0114	~~+0.0457~~ +0.0114		
	0	+3.644	−3.656	+3.~~348~~656	~~−3.353~~ −3.644	0
		+0.0051	+0.0068	−0.0257 −0.0068	−0.0192 −0.0051	
			−0.0128	~~+0.0634~~ +0.0128		

0	+3.576	-3.616	+3.616	-3.576	0
	+0.0171	+0.022	-0.022	-0.0171	
		-0.011	+0.011		
0	+3.593	-3.605	+3.605	-3.593	0
	+0.0051	+0.0068	-0.0068	-0.00514	
		-0.0034	+0.0034		
0	+3.598	-3.601	+3.601	-3.598	0
	+0.0012	+0.00171	-0.00171	-0.00128	
		-0.00085	+0.00085		
0	~~+3.603~~ +3.599	-3.600	+3.600	-3.599	0
	+0.00042	+0.00057	-0.00057	-0.00042	
		-0.00028	+0.00028		
	+3.599	-3.602	+3.599	-3.599	

Analyze the portal frame by using as shown in fig.



Step-1 :- fixed end moments

Span AB, $M_{FAB} = -\frac{Wl}{8} = \frac{-20 \times 4}{8} = -10$ kN-m

$M_{FBA} = \frac{Wl}{8} = \frac{+20 \times 4}{8} = +10$ kN-m

Span BC, $M_{FBC} = -\frac{wl^2}{12} = -\frac{30 \times 4^2}{12} = -40$ kN-m

$M_{FCB} = +\frac{wl^2}{12} = +40$ kN-m

Span CD, $M_{FCD} = -\frac{Wab^2}{l^2} = -\frac{50 \times 1 \times 3^2}{4^2} = -28.125$ kN-m

$M_{FDC} = +\frac{Wa^2b}{l^2} = +\frac{50 \times 1^2 \times 3}{4^2} = +9.375$ kN-m

Step-2 :- Distribution table

Joint	Member	R.S (K)	T.R.S (ΣK)	D.F ($\frac{K}{\Sigma K}$)
B	BA	$\frac{I}{L} = \frac{I}{4}$	$\frac{I}{4} + \frac{2I}{4} = \frac{3I}{4}$	$\frac{I/4}{3I/4} = \frac{1}{3}$
	BC	$\frac{2I}{L} = \frac{2I}{4}$		$\frac{2I}{4} \times \frac{4}{3I} = 2/3$
C	CB	$\frac{2I}{L} = \frac{2I}{4}$	$\frac{2I}{4} + \frac{I}{4} = \frac{3I}{4}$	$\frac{2I}{4} \times \frac{4}{3I} = \frac{2}{3}$
	CD	$\frac{I}{L} = \frac{I}{4}$		$\frac{I}{4} \times \frac{4}{3I} = \frac{1}{3}$

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F		1/3	2/3	2/3	1/3	
F.E.M	-10	10	-40	+40	-28.125	9.37
Balance		10	20	-7.96	-3.92	
COM	5		-3.958	+10		+1.975
Balance		+1.319	+2.641	-6.67	-3.33	
COM	+0.6595	~~1.115~~	~~2.23~~ -3.335	~~-0.88~~ +1.320	-0.439	-1.665
Balance.		+1.111	+2.223	-0.88	-0.44	
COM	+0.555		-0.44	+1.111		-0.22
Balance.		+0.1466	+0.293	-0.740	-0.370	
COM	+0.0733		-0.370	+0.146		+0.185
Balance		+0.1233	+0.2466	-0.0973	-0.0486	
COM	+0.0615		-0.0486	+0.1233		-0.0243
Balance		+0.0162	+0.0324	-0.0822	-0.0411	
COM	+0.0081		-0.0411	+0.0162		-0.0205
Fixed end Moment	-3.642	+22.71	-22.75	+36.28	-36.27	+5.280

UNIT - 2 :- KANI'S METHOD

Introduction :-

Kani's method was derived by Dr. Jasper Kani in the year of 1947. This method is suitable to without approximate moment for the whole frame, beams (or) Statically indeterminate structures.

Kani's method is Similar to Moment-Distribution method & Slope-deflection equation. We use Gauss-Jordon method. with iterative procedure to Solve the Slope-deflection equation.

This method is very useful for analysis for Multistoreyed buildings.

The greatest advantage of this method is even if a mistake is Committed in distribution in one of the cycle.

In this method we use a factor is Called rotation factor.

Rotation factor :-

It is Mathematically expressed as as a

$$u = -\frac{1}{2} \cdot DF$$

(or)

$$u = -\frac{1}{2}\left[\frac{K}{\Sigma K}\right].$$

It is denoted by the symbol "u."

Sign Conventions :-

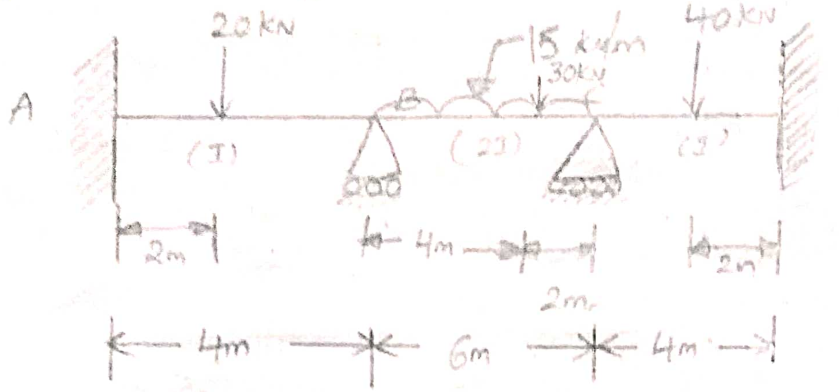
The following Sign Conventions are used.

(i) Clockwise end moments are positive.

(ii) Clockwise rotations are positive.

Kanis Method to Continuous Beam with Fixed ends :-

1. Analyze the Continuous beam as shown in fig by using Kani's method.



Sol:- **Step - 1 :-**

Fixed end Moments

For span, AB, =

$M_{FAB} = -\frac{WL}{8} = -\frac{20\times4}{8} = -10\,kN.m$

$M_{FBA} = +\frac{WL}{8} = +\frac{20\times4}{8} = +10\,kN.m$

For span BC,

$M_{FBC} = -\frac{wL^2}{12} - \frac{Wab^2}{L^2} = -\frac{15\times6^2}{12} - \frac{30\times4\times2^2}{6^2}$

$= -58.33\,kN.m$

$M_{FCB} = +\frac{wL^2}{12} + \frac{Wa^2b}{L^2} = +\frac{15\times6^2}{12} + \frac{30\times4^2\times2}{6^2}$

$= +71.68\,kN.m$

For span CD,

$M_{FCD} = -\frac{WL}{8} = \frac{-40\times4}{8} = -20\,kN.m$

$M_{FDC} = +\frac{WL}{8} = +\frac{40\times4}{8} = +20\,kN.m$

Step-2 :-

Rotation factor :-

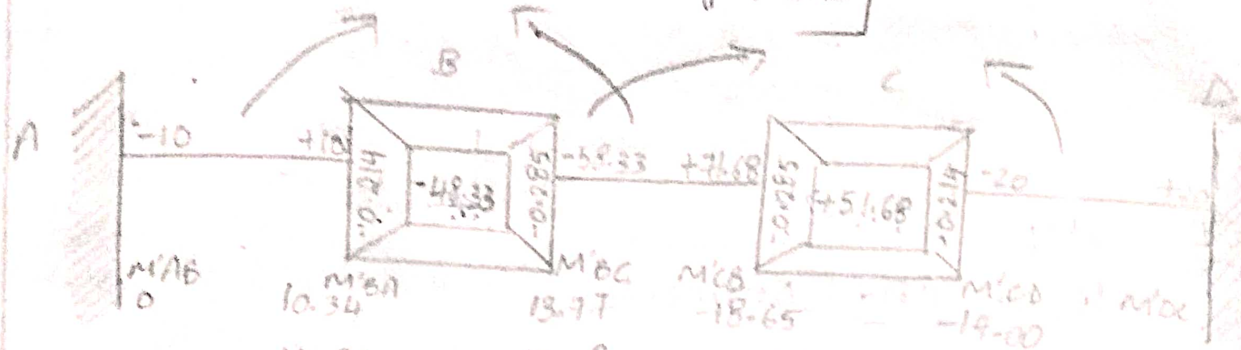
Joint	Member	R.S (K)	T.R.S (ΣK)	D.F	R.F
B	BA	$\frac{I}{L} = \frac{I}{4}$	$\frac{I}{4}+\frac{I}{3} = \frac{7I}{12}$	$\frac{I/4}{7I/12} = \frac{3}{7}$	$= -\frac{1}{2}\left(\frac{3}{7}\right) = -0.214$
	BC	$\frac{I}{L} = \frac{2I}{6} = \frac{I}{3}$		$\frac{I/3}{7I/12} = \frac{4}{7}$	$= -\frac{1}{2}\left(\frac{4}{7}\right) = -0.285$
C	CB	$\frac{I}{L} = \frac{2I}{6} = \frac{I}{3}$	$\frac{I}{3}+\frac{I}{4} = \frac{7I}{12}$	$\frac{I/3}{7I/12} = \frac{4}{7}$	$= -\frac{1}{2}\left(\frac{4}{7}\right) = -0.285$
	CD	$\frac{I}{L} = \frac{I}{4}$		$\frac{I/4}{7I/12} = \frac{3}{7}$	$-\frac{1}{2}\left(\frac{3}{7}\right) = -0.214$

Step-3 :-

KANI'S table

Fixed End Moment.

$M_{AB} = M_{BA} = R.f \times [\Sigma\, FEM$ at Join B $+ \Sigma$ Rotation Contribution from far end$]$



~~MBA~~	14.33	19.08	-20.16	-15.14
	14.656	19.519	-20.291	-15.236
	$\boxed{14.68}$	$\boxed{19.55}$	$\boxed{-20.30}$	$\boxed{-15.24}$

At Joint B = M_{FAB} = $10 - 58.33 = -48.33$

At Joint C = M_{FBC} = $+71.68 - 20 = +51.68$

$M'_{BA} = u \times$ (f.E.m @ B + Rotation Constribution far end)

$= -0.214\ (-48.33 + 0)$

$= 10.34$ KN.m

M'_{BC} = $u \times$ (f.E.M @ B + Rotation Constribution far end)

$= -0.285\ (-48.33 + 0)$

$= 13.78$ KNm

M'_{CB} = u (f.E.M @ + Rotation Contribution)

$= -0.285\ (51.68 + 13.77 + 0)$

$= -18.65$ KNm

M'_{CD} = u (F.E.M + Rotation Contribution)

$= -0.214\ (51.68 + 13.77 + 0)$

$= -14.00$ kN.m

Cycle - 2

$M'_{BA} = u[\text{FEM at } B + \text{Rotation contribution}]$

$= -0.214 [-48.33 + 0 - 18.65] = 14.33$ kNm

$M'_{BC} = u[M_{FAB} + \text{Rotation contribution}]$

$= -0.285 (-48.33 + 0 - 18.65) = 19.08$ kN.m

$M'_{CB} = u (M_{FAB} + \text{Rotation})$

$= -0.285 (51.68 + 19.08 + 0) = -20.16$ kNm

$M'_{CD} = -0.214 (51.68 + 19.08 + 0) = -15.14$ kNm

Cycle - 3 :-

$M'_{BA} = u (\text{f.E.M @ } B + \text{Rotation Contribution})$

$= -0.214 (-48.33 + 0 + (-20.16))$

$= +14.656$ kN.m

$M'_{BC} = u (\text{f.E.M @ } B + \text{Rotation Contribution}$

$= -0.285 (-48.33 + (-20.16))$

$= 19.519$ kN.m

$M'_{CB} = -0.285 (+51.68 + 19.\cancel{08}\,519)$

$= \cancel{20.296 \text{ kN.M}}\ -20.291$ kN.m

$M'_{CD} = -0.214 (51.68 + 19.519)$

$= -15.236$ kN.m

Cycle - 4 :-

$M'_{BA} = -0.214 (-48.33 + (-20.291))$

$= +14.684$ kN.m

$M'_{BC} = -0.285 (-48.33 + (-20.291))$

$= +19.55$ kN.m

$M'_{CB} = -0.285 (51.68 + 19.55)$

$= -20.300$ kNm.

$M'_{CD} = -0.214 (51.68 + 19.55)$

$= -15.243$

Step-4 :-

final end Moment $[M_{AB}]$

= fixed end Moment + 2(near end Contribution) + far end Contribution

$M_{AB} = -10 + 2 \times (0) + (14.68) = \cancel{29.36}\ +4.68.$

$M_{BA} = +10 + 2 \times (14.68) + (0) = \cancel{39.36}\ +39.36$

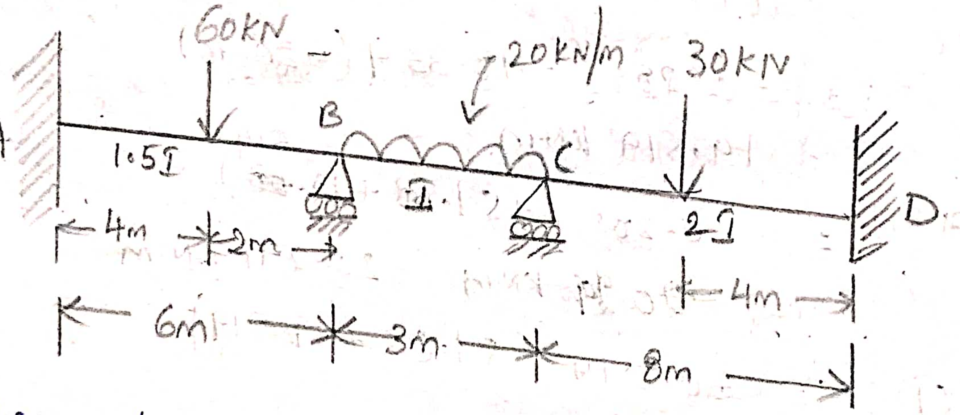
$M_{BC} = -58.33 + 2 \times (19.55) \cancel{+(14.68) =} + (-20.30) = -39.53$

$M_{CB} = +71.68 + 2 \times (-20.30) \cancel{+(19.55) =} + (19.55 = 50.63$

$M_{CD} = -20 + 2 \times (-15.24) \cancel{+(-20.30)} = -50.48$

$M_{DC} = +20 + 2 \times (0) + (-15.24) = +4.76$

(2) Analyze the Continuous beam as shown in fig. by using Kani's method.



Step-1 :-

Fixed end Moments.

For span AB,

$$M_{FAB} = -\frac{Wab^2}{L^2} = -\frac{60 \times 4 \times 2^2}{6^2} = -26.667\ kN.m$$

$$M_{FBA} = +\frac{Wa^2b}{L^2} = +\frac{60 \times 4^2 \times 2}{6^2} = +53.33\ kN.m$$

for Span BC,

$$M_{FBC} = \frac{-WL^2}{12} = -\frac{20 \times 3^2}{12} = -15\ kN.m$$

$M_{FCB} = +\frac{wl^2}{12} = +\frac{20 \times 3^2}{12} = +15$ kN·m

For span CD,

$M_{FCD} = -\frac{wl}{8} = -\frac{30 \times 8}{8} = -30$ kN·m

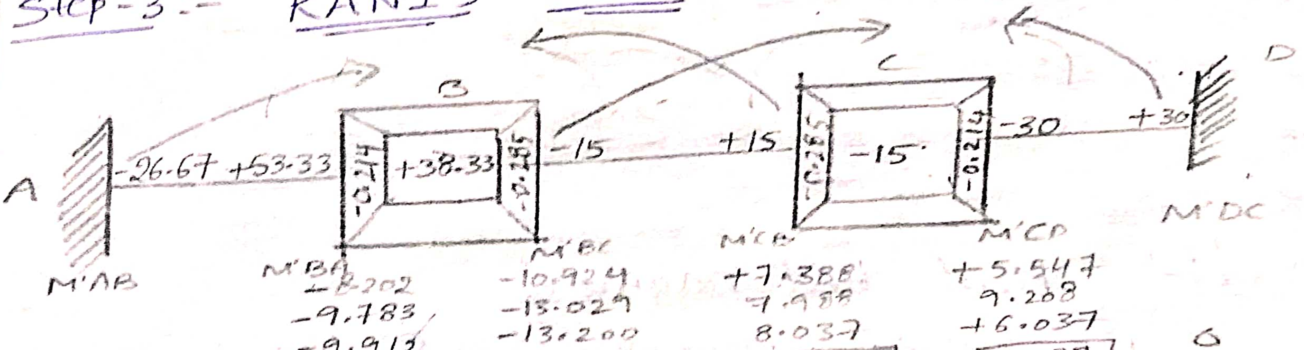
$M_{FDC} = +\frac{wl}{8} = +30$ kN·m

Step - 2 :-

Rotation factor.

Joint	Member	R.S (K)	T.R.S (ΣK)	D.F ($\frac{K}{\Sigma K}$)	R.F
B	BA	$\frac{I}{L} = \frac{1.5}{6} I$	$\frac{1.5}{6} I + \frac{I}{3} = \frac{7}{12} I$	$\frac{1.5 I}{6} \times \frac{12}{7 I} = \frac{3}{7}$	$= -\frac{1}{2} \times \frac{3}{7} = \frac{-3}{14} = -0.214$
	BC	$\frac{I}{L} = \frac{I}{3}$		$\frac{I}{3} \times \frac{12^4}{7 I} = \frac{4}{7}$	$-\frac{1}{2} \times \frac{4^2}{7} = -\frac{2}{7} = -0.285$
C	CB	$\frac{I}{L} = \frac{I}{3}$	$\frac{I}{3} + \frac{2I}{8} = \frac{7}{12} I$	$\frac{I}{3} \times \frac{12^4}{7 I} = \frac{4}{7}$	$-\frac{1}{2} \times \frac{4^2}{7} = -\frac{2}{7} = -0.285$
	CD	$\frac{I}{L} = \frac{2I}{8}$		$\frac{2I}{8_4} \times \frac{12^3}{7} I = \frac{3}{7}$	$-\frac{1}{2} \times \frac{3}{7} = \frac{-3}{14} = -0.214$

Step - 3 :- KANI'S TABLE



Cycle : - 1 : $\boxed{-9.922}$ $\boxed{-13.214}$ $\boxed{8.040}$ $\boxed{6.037}$

(Contribution far end)

$M'BA$ = $u \times$ (F.E.M @ B + Rotation) = -8.202 kN·m

$= -0.214 (+38.33 + 0)$

$M'BC$ $= -0.285 (+38.33 + 0) = -10.924$ kN·m

$M'CB$ $= -0.285 (-15 + -10.924) = +7.388$ KN.M

$M'CD$ $= -0.214 (-15 + -10.924) = +5.547$ KN·m

Cycle :- 2

$M'BA$ $= -0.214 (+38.33 + 7.388) = -9.783$ kN·m

$M'BC$ $= -0.285 (+38.33 + 7.388) = -13.029$ kNm

$M'_{CB} = -0.285(-15 + -13.029$
$= 7.988$

$M'_{CD} = -0.214(-15 + -13.029)$
$= +5.948$

Cycle-3 :-

$M'_{BA} = -0.214(+38.33 + 7.988)$
$= -9.912$

$M'_{BC} = -0.285(+38.33 + 7.988)$
$= -13.200$

$M'_{CB} = -0.285(-15 + -13.200)$
$= 8.037$

$M'_{CD} = -0.214(-15 + -13.200)$
$= +6.034$

Cycle-4 :-

$M'_{BA} = -0.214(+38.33 + 8.037) = -9.922$
$M'_{BC} = -0.285(+38.33 + 8.037) = -13.214$
$M'_{CB} = -0.285(-15 + -13.214) = 8.040$
$M'_{CD} = -0.214(-15 + -13.214) = 6.037$

Step-4 :- final End Moment

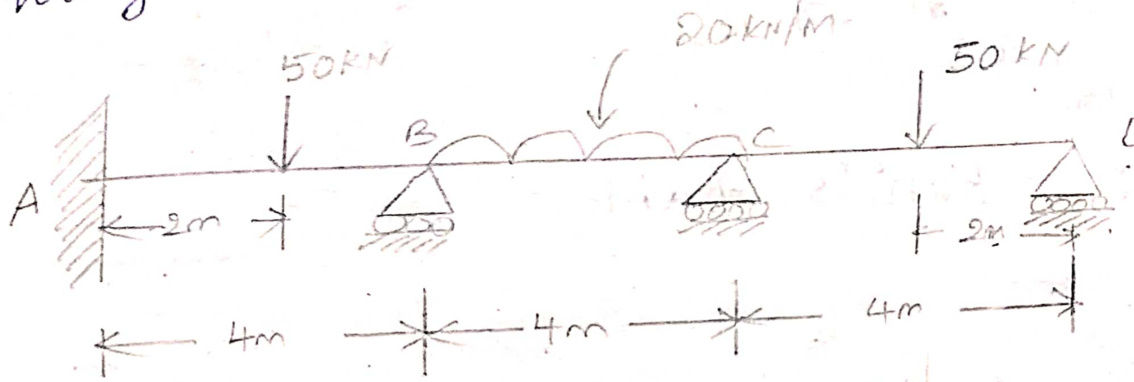
$M_{AB} = F.E.M + 2(\text{near end Contribution}) + \text{far end Contribution}$
$= -26.67 + 2(0) + (-9.922) = -36.592$
$M_{BA} = +53.33 + 2(-9.922) + (0) = +33.486$
$M_{BC} = -15 + 2(-13.214) + (8.040) = -33.388$
$M_{CB} = +15 + 2(8.040) + (-13.214) = +17.866$
$M_{CD} = -30 + 2(6.037) + (8.040) = -17.926$
$M_{DC} = +30 + 2(0) + (6.037) = 36.037$

~~portal frame :-~~

~~Using~~

Application to Continuous beams with Simply Support & overhanging ends :-

Analyze the Continuous beam as shown in fig. by using Kani's method. flexural rigidity is Constant throughout the Section.



Sol: Step - 1 :-

fixed end Moment

For Span AB,

$M_{FAB} = -\frac{WL}{8} = -\frac{50\times4}{8} = -25$ kNm

$M_{FBA} = +\frac{WL}{8} = +\frac{50\times4}{8} = +25$ kNm

for Span BC,

$M_{FBC} = -\frac{WL^2}{12} = -\frac{20\times4^2}{12} = -26.667$ kN.m

$M_{FCB} = +\frac{WL^2}{12} = +\frac{20\times4^2}{12} = +26.667$ kN.m

for Span CD,

$M_{FCD} = -\frac{WL}{8} = -\frac{50\times4}{8} = -25$ kNm

$M_{FDC} = +\frac{WL}{8} = +\frac{50\times4}{8} = +25$ kNm

Modified F.E.M

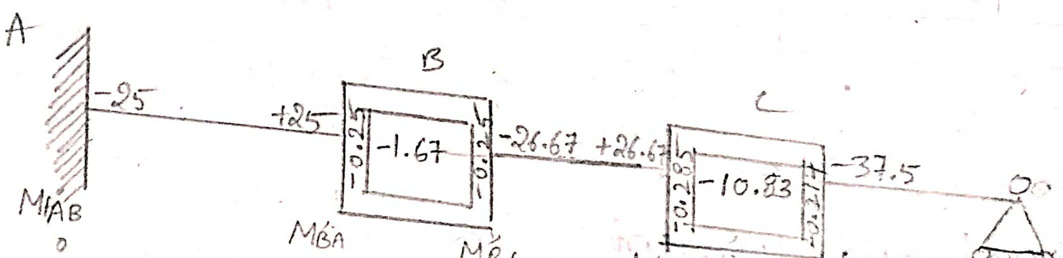
$M_{FCD} = -25 + \frac{1}{2}(+25) = -25 + 12.5) = -37.5$ kNm

$M_{FDC} = +25 + (-25) = 0$

Step-2 :- ~~Distributio~~ Rotation factor

Joint	Member	R.S (k)	T.R.S (Σk)	D.F $\left(\frac{k}{\Sigma k}\right)$	R.F $= \left(-\frac{1}{2}\text{ D.F}\right)$
B	BA	$\frac{I}{L} = \frac{I}{4}$	$\frac{I}{4} + \frac{I}{4} = \frac{I}{2}$	$\frac{I}{4} \times \frac{2}{I} = \frac{1}{2}$	$-\frac{1}{2} \times \frac{1}{2} = -0.25$
	BC	$\frac{I}{L} = \frac{I}{4}$		$\frac{I}{4} \times \frac{2}{I} = \frac{1}{2}$	$-\frac{1}{2} \times \frac{1}{2} = -0.25$
C	CB	$\frac{I}{L} = \frac{I}{4}$	$\frac{I}{4} + \frac{3I}{16} = \frac{7}{16}I$	$\frac{I}{4} \times \frac{16}{7I} = \frac{4}{7}$	$-\frac{1}{2} \times \frac{4}{7} = -\frac{2}{7} = -0.$
	CD	$\frac{3I}{4L} = \frac{3I}{4 \cdot 4} = \frac{3I}{16}$		$\frac{3I}{16} \times \frac{16}{7I} = \frac{3}{7}$	$-\frac{1}{2} \times \frac{3}{7} = -\frac{3}{14} = -0.21$
	DC				

Step-3 :- Kani's table.



M'BA	M'BC	M'CB	M'CD
+0.417	+0.417	+2.967	+2.229
-0.324	-0.324	+3.178	+2.386
-0.377	-0.377	+3.193	+2.398
$\boxed{-0.380}$	$\boxed{-0.380}$	$\boxed{+3.194}$	$\boxed{+2.398}$

F.E.M @ B

At Joint B = +25 − 26.67 = −1.67 kN.m

At Joint C = +26.67 − 37.5 = −10.83 kNm.

Cycle ÷ 1 :-

= u × (F.E.M @ B + Rotation Contribution far end)

$M'_{BA} = -0.25 \times (-1.67 + 0) = +0.4175$

$M'_{BC} = -0.25 \times (-1.67 + 0) = +0.4175$

$M'_{CB} = -0.285 \times (-10.83 + 0.4175) = +2.967$

$M'_{CD} = -0.214 \times (-10.83 + 0.417) = +2.228$

Cycle-2 :-

$M'_{BA} = -0.25 \times (-1.67 + 2.967) = -0.324$

$M'_{BC} = -0.25 \times (-1.67 + 2.967) = -0.324$

$M'_{CB} = -0.285 \times (-10.83 + -0.324) = +3.178$

$= -0.214 \times (-10.83 + -0.324) = +2.386$

Cycle-3:

$M'_{BA} = -0.25\times(-1.67+3.178) = -0.377$

$M'_{BC} = -0.25\times(-1.67+3.178) = -0.377$

$M'_{CB} = -0.285\times(-10.83+-0.377) = +3.193$

$M'_{CD} = -0.214\times(-10.83+-0.377) = +2.398.$

Cycle-4:-

$M'_{BA} = -0.25\times(-1.67+3.193) = -0.380$

$M'_{BC} = -0.25\times(-1.67+3.193) = \cancel{3.8}\ -0.380$

$M'_{CB} = -0.285\times(-10.83+-0.380) = +3.194$

$M'_{CD} = -0.214\times(-10.83+-0.380) = +2.398.$

Step-4:- final End Moments

$= f.E.M + 2\,(\text{Near end Contribution}) + \text{Rotation contribution far end.}$

$M'_{AB} = -25+2(0)+(-0.380) = -25.38\ kNm$

$M'_{BA} = +25+2(-0.380)+(0) = +24.24\ kNm$

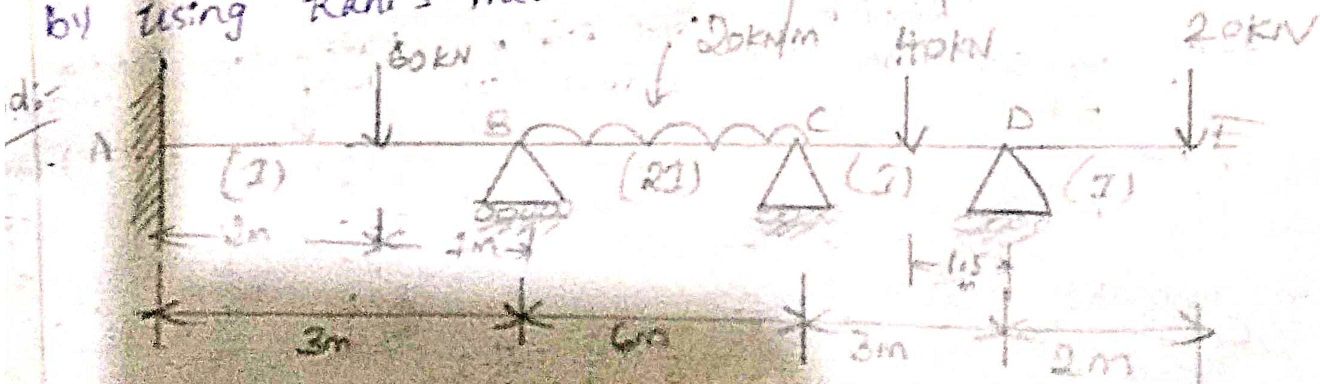
$M'_{BC} = -26.67+2(-0.380)+(+3.194) = -24.23\ kNm$

$M'_{CB} = +26.67+2(3.194)+(-0.380) = +32.678\ kNm$

$M'_{CD} = -37.5+2(+2.398)+(0) = -32.704\ kNm$

$M'_{DC} = 0$

Analyze the Continuous beam as shown in fig. by using Kani's method?



Step-1:- fixed end Moment

for span AB,

$$M_{FAB} = -\frac{wab^2}{l^2} = -\frac{60\times2\times1^2}{3^2} = -13.33 \text{ kNm}$$

$$M_{FBA} = +\frac{wa^2b}{l^2} = +\frac{60\times2^2\times1}{3^2} = +26.67 \text{ kN.m}$$

for span BC,

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{20\times6^2}{12} = -60 \text{ kNm}$$

$$M_{FCB} = +\frac{wl^2}{12} = +\frac{20\times6^2}{12} = +60 \text{ kNm}$$

for span CD,

$$M_{FCD} = -\frac{wl}{8} = -\frac{40\times3}{8} = -15 \text{ kNm}$$

$$M_{FDC} = +\frac{wl}{8} = +\frac{40\times3}{8} = +15 \text{ kNm}$$

for span DE,

$$M_{FDE} = -wl = -20\times2 = -40 \text{ kNm}$$

$$M_{FED} = 0$$

Modified F.E.M

$$M_{FDC} = +15+\frac{1}{2}\times40 \qquad M_{FDC} = +40 \text{ kNm}$$

$$= +35 \text{ kNm}.$$

$$M_{FCD} = -15+\frac{1}{2}(15-40) = -27.5 \text{ kN.m}$$

$$M_{FDE} = -40+40$$

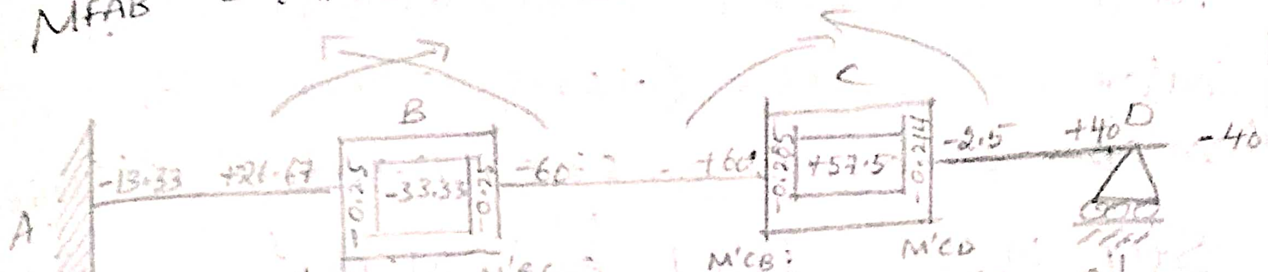
$$= 0$$

Step-2:- Rotation factor.

Joint	member	Relative stiffness (K)	Total Relative stiffness (ΣK)	Distribution factor $\left(\frac{K}{\Sigma K}\right)$	Rotation factor.
B	BA	$\frac{I}{L} = \frac{I}{3}$	$\frac{I}{3}+\frac{I}{3} = \frac{2}{3}I$	$\frac{I}{3}/\frac{2}{3}I = \frac{1}{2}$	$= -\frac{1}{2}\left(\frac{1}{2}\right) = -\frac{1}{4} = -0.$
	BC	$\frac{I}{L} = \frac{2I}{6_3} = \frac{I}{3}$		$I/3/\frac{2}{3}I = \frac{1}{2}$	$-\frac{1}{2}\left(\frac{1}{2}\right) = -\frac{1}{4}$
C	CB	$\frac{I}{L} = \frac{2I}{6_3} = \frac{I}{3}$	$\frac{I}{3}+\frac{I}{4} = \frac{7}{12}I$	$\frac{I}{3}/\frac{7I}{12_4} = \frac{4}{7}$	$-\frac{1}{2}\left(\frac{4}{7}\right) = -\frac{2}{7} = -0.2$
	CD	$\frac{3I}{4L} = \frac{3I}{4\times3} = \frac{3I}{12_4} = \frac{I}{4}$		$\frac{I}{4}/\frac{7I}{12_3} = \frac{3}{7}$	$-\frac{1}{2}\left(\frac{3}{7}\right) = -\frac{3}{14} = -0.$

$M_{FAB} = M_{FAB} + 0.5$ (Balancing Moment @ B)

$M_{FAB} = M_{FAB} - 0.5$ (unbalancing Moment @ B)



M_{AB}	M'_{BA}	M'_{BC}	M'_{CB}	M'_{CD}		
0	+8.33	+8.33	-18.761	-14.087	0	0
0	+13.02	+13.02	-20.098	-15.091	0	0
0	+13.357	+13.357	-20.194	-15.163	0	0
0	$\boxed{+13.381}$	$\boxed{+13.381}$	$\boxed{-20.201}$	$\boxed{-15.166}$	0	0

At Joint B = M_{FB} = +26.67 − 60 = −33.33 kNm

At Joint C = M_{FB} = +60 − 2.5 = +57.5 kNm

Cycle :- 1

= R.F × (F.E.M @ B + Rotation Contribution for end)

$= -0.25 \times (-33.33 + 0) = +8.33$

$M'_{BA} = -0.25 \times (-33.33 + 0) = +8.33$

$M'_{BC} = -0.25 \times (-33.33 + 0) = +8.33$

$M'_{CB} = -0.285 \times (+57.5 + 8.33) = -18.761$

$M'_{CD} = -0.214 \times (+57.5 + 8.33) = -14.087$

Cycle :- 2

$= -0.25 \times (-33.33 + -18.761) = +13.02$

$M'_{BA} = -0.25 \times (-33.33 + -18.761) = +13.02$

$M'_{BC} = -0.25 \times (-33.33 + -18.761) = +13.02$

$M'_{CB} = -0.285 \times (+57.5 + 13.02) = -20.098$

$M'_{CD} = -0.214 \times (+57.5 + 13.02) = -15.091$

Cycle :- 3

$= -0.25 \times (-33.33 + -20.098) = +13.357$

$M'_{BA} = -0.25 \times (-33.33 + -20.098) = +13.357$

$M'_{BC} = -0.25 \times (-33.33 + -20.098) = +13.357$

$M'_{CB} = -0.285 \times (+57.5 + 13.357) = -20.194$

$M'_{CD} = -0.214 \times (+57.5 + 13.357) = -15.163$

Cycle :- 4

$M'_{BA} = -0.25 \times (-33.33 + -20.194) = +13.381$

$M'_{BC} = -0.25 \times (-33.33 + -20.194) = +13.381$

$M'_{CB} = -0.285 \times (+57.5 + 13.381) = -20.201$

$M'_{CD} = -0.214 \times (+57.5 + 13.381) = -15.168$

Step-4 :- final end Moment

= f.E.M + 2 (Near end Contribution) + (Rotation Contribution far end)

$M'_{AB} = -13.33 + 2(0) + (13.381) = +0.051$

$M'_{BA} = +26.67 + 2(+13.381) + (0) = +53.432$

$M'_{BC} = -60 + 2(+13.381) + (-20.201) = -53.439$

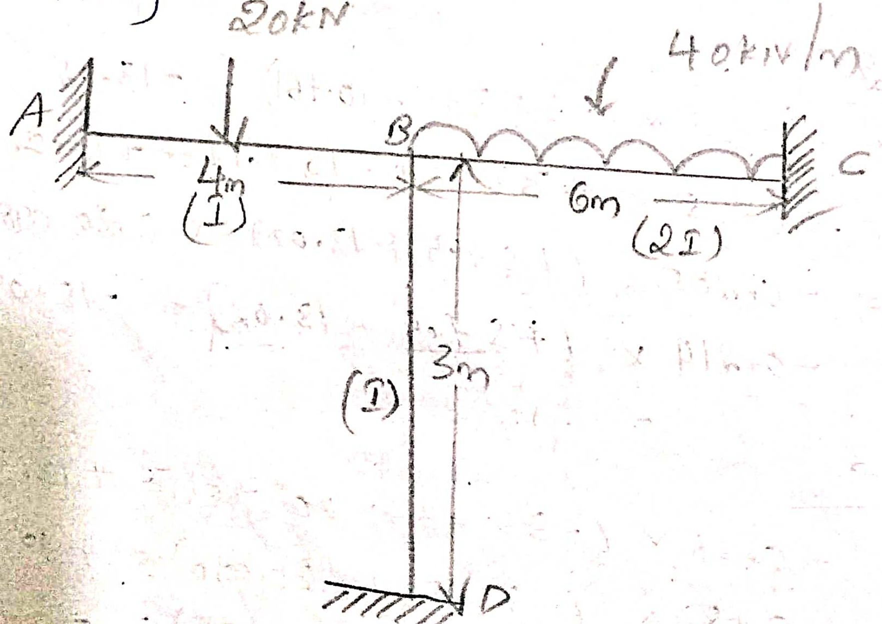
$M'_{CB} = +60 + 2(-20.201) + (13.381) = \cancel{+66.56)} +32.979$

$M'_{CD} = -2.5 + 2(-15.168) + 0 = -32.836$

$M'_{DC} = 0$

T- portal frames :-

1. Analyze the ridged ~~beam~~ frame as Shown in fig. by using the kani's method ?



Step-1:- fixed end Moment

for Span AB,

$M_{FAB} = -\frac{Wl}{8} = \frac{-20\times4}{8} = -10\,kNm$

$M_{FBA} = +\frac{Wl}{8} = +\frac{20\times4}{8} = +10\,kNm$

for span BC,

$M_{FBC} = -\frac{Wl^2}{12} = \frac{-40\times6^2}{12} = -120\,kNm$

$M_{FCB} = +\frac{Wl^2}{12} = +\frac{40\times6^2}{12} = +120\,kNm$

for Span BD,

$M_{FBD} = 0$

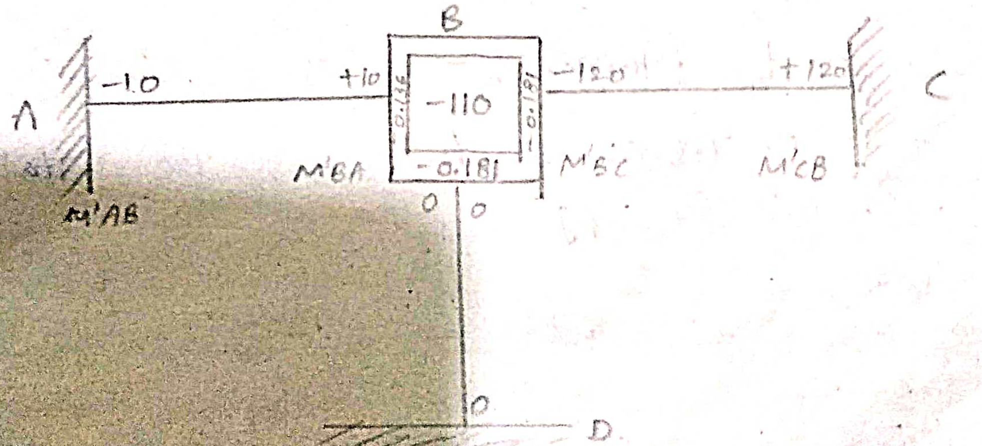
$M_{FDB} = 0$

Due to there is no loads acting.

Step-2:- Rotation table:

Joint	Member	R.S (R)	T.R.S (Σk)	D.F	R.F
B	BA	$\frac{I}{L} = \frac{I}{4}$	$\frac{I}{4}+\frac{I}{3}+\frac{I}{3} = \frac{11I}{12}$	$\frac{I}{4}\Big/\frac{11I}{12} = \frac{3}{11} = 0.272$	$-\frac{1}{2}\times\frac{3}{11} = \frac{-3}{22} = -0.136$
	BC	$\frac{I}{L} = \frac{2I}{6} = \frac{I}{3}$		$\frac{I}{3}\Big/\frac{11I}{12} = \frac{4}{11} = 0.363$	$-\frac{1}{2}\times\frac{4}{11} = -\frac{2}{11} = -0.181$
	BD	$\frac{I}{L} = \frac{I}{3}$		$\frac{I}{3}\Big/\frac{11I}{12} = \frac{4}{11} = 0.363$	$-\frac{1}{2}\times\frac{4}{11} = -\frac{2}{11} = -0.181$

Step-3:- Kanis table.



Cycle :- 1 :-

$= u \times (f.E.M @ B + \text{Rotation Contribution for} ...$

$M'_{AB} = 0$

$M'_{BA} = -0.136 \times (-110+0) = +14.96$

$M'_{BC} = -0.181 \times (-110+0) = +19.91$.

$M'_{BD} = -0.181 \times [-110+0+0+0] = +19.91$

Step - 4 :-

final end moments

$M_{AB} = M_{FAB} + 2M_{AB} + M_{BA}$

$= -10 + 2(0) + 14.96 = +4.96$ kN.m

$M_{BA} = +10 + 2(+14.96) + (0) = +39.92$ kN.m

$M_{BC} = -120 + 2(+19.91) + (\cancel{+14.96}\ 0) = -80.18$ kNm.

$M_{CB} = +120 + 2(0) + (+19.91) = +139.91$ kNm.

$M_{CD} = 0 + 2(19.9) + 0 = 39.8$ kNm

$M_{DB} = 0 + 2(0) + 19.9 = +19.9$ kNm

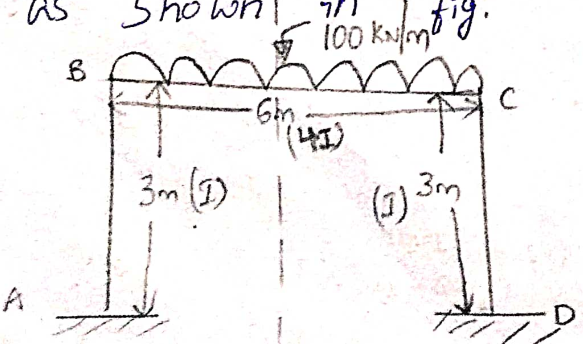
2. portal frames :-

portal frames are divided into two types :-

1. Non-Sway portal frames. → Symmetrical portal fra...
2. Sway portal frames. → unsymmetrical

NON-SWAY portal frames :-

Determine the moments at A, B, C, D for the portal frame as shown in fig.



Step-1 :- Fixed end Moments :-

for Span AB,

$M_{FAB} = M_{FBA} = 0$

for span BC,

$M_{FBC} = -\frac{wl^2}{12} = \frac{-100\times6^2}{12} = -300 \text{ KNm}$

$M_{FCB} = +\frac{wl^2}{12} = \frac{+100\times6^2}{12} = +300 \text{ KN.m}$

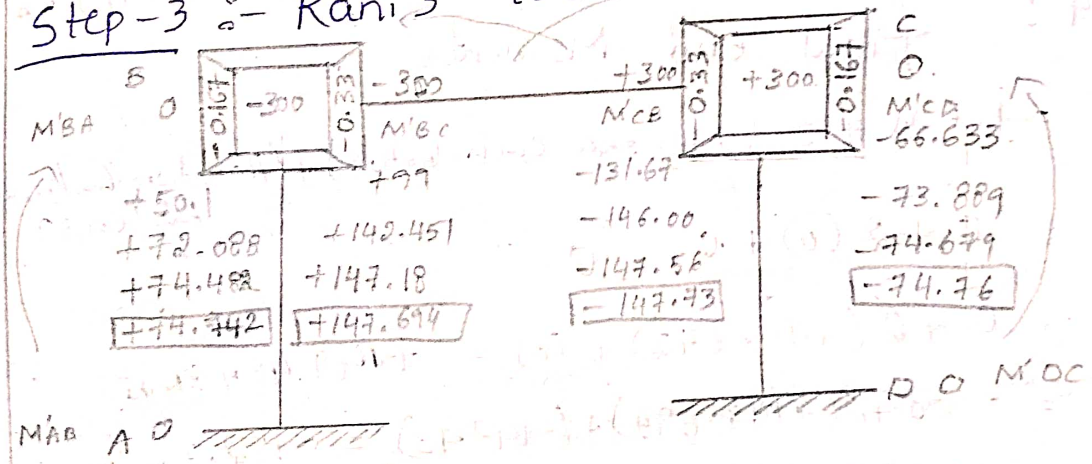
for Span CD,

$M_{FCD} = M_{FDC} = 0.$

Step-2 :- Rotation table

Joint	Member	R.S (k)	T.R.S (Σk)	Df	R.f
B	BA	$\frac{I}{L} = \frac{I}{3}$	$\frac{I}{3} + \frac{2}{3}I = 1I$	$\frac{I}{3}\times\frac{1}{I} = \frac{1}{3}$	$-\frac{1}{2}\times\frac{1}{3} = \frac{-1}{6} = -0.167$
	BC	$\frac{I}{L} = \frac{4I}{6} = \frac{2}{3}I$		$\frac{2}{3}I\times\frac{1}{I} = \frac{2}{3}$	$\frac{-1}{2}\times\frac{2}{3} = \frac{-1}{3} = -0.33$
C	CB	$\frac{I}{L} = \frac{4I}{6} = \frac{2}{3}I$	$\frac{2}{3}I + \frac{I}{3} = 1I$	$\frac{2}{3}I\times\frac{1}{I} = \frac{2}{3}$	$\frac{-1}{2}\times\frac{2}{3} = \frac{-1}{3} = -0.33$
	CD	$\frac{I}{L} = \frac{I}{3}$		$\frac{I}{3}\times\frac{1}{I} = \frac{1}{3}$	$\frac{-1}{2}\times\frac{1}{3} = \frac{-1}{6} = -0.167$

Step-3 :- Kani's table.



At Joint B = 0 − 300 = −300 KN.m

At Joint C = +300 − 0 = +300 KN.m

Cycle :-1 :- $u \times$ (f.E.M @ B + Rotation Contribution far end)

$M'_{BA} = -0.167\times(-300+0) = +50.1$ KN.m

$M'_{BC} = -0.333\times(-300+0) = +99.9$ KN.m $\simeq 100$ KN.m

$M'_{CB} = -0.333\times(+300+99) = -132.86$ KN.m

$M'_{CD} = -0.167\times(+300+99) = -66.633$ KN.m

Cycle :- 2 :-

$M'_{BA} = -0.167(-300 + -132.86) = +72.288$ kN.m

$M'_{BC} = -0.333(-300 + -132.86) = \overset{144.142}{+142.451}$ kN.m

$M'_{CB} = -0.333(+300 + 144.451) = \overset{-147.899\text{ kN.m}}{\cancel{-146.00\text{ kN.m}}}$

$M'_{CD} = -0.167(+300 + \overset{144.142}{\cancel{142.451}}) = \cancel{-73.889\text{ kN.m}}$ -74.171 kN.m.

Cycle :- 3 :-

$M'_{BA} = -0.167(-300 + -\overset{147.899}{\cancel{146.00}}) = +74.\overset{799}{\cancel{182}}$ kN.m

$M'_{BC} = -0.333(-300 + \overset{147.899}{\cancel{-146.00}}) = +149.15$ kNm

$M'_{CB} = -0.333(+300 + 149.15) = -149.56$ kNm

$M'_{CD} = -0.167(+300 + 149.15) = -\cancel{74.679\text{ kNm}}$ 75.00 kNm.

Cycle :- 4 :-

$M'_{BA} = -0.167(-300 + -149.56) = \overset{75.076\text{ kNm}}{+\cancel{74.742\text{ kNm}}}$

$M'_{BC} = -0.333(-300 + -149.56) = +149.\overset{70}{\cancel{694}}$ kNm

$M'_{CB} = -0.333(+300 + 149.\overset{70}{\cancel{694}}) = -149.75$ kNm

$M'_{CD} = -0.167(+300 + 149.\overset{70}{\cancel{694}}) = -\cancel{74.76\text{ kNm}}$ 75.099 kNm

Step-4 :- final end Moments

= f.E.M + 2 (Near end contribution) + (Rotation Contribution far end)

$M_{AB} = 0 + 2(0) + 0 = 0.$

$M_{BA} = 0 + 2(+74.742) + (0) = +149.484$ kN.m

$M_{BC} = -300 + 2(+147.694) + (-147.73) = -152.342$ kN.m.

$M_{CB} = +300 + 2(-147.73) + (+147.694) = +152.234$ kN.m.

$M_{CD} = 0 + 2(-74.76) + 0 = -149.52$ kNm

$M_{DC} = 0$

Cycl...

$M'_{BA} = -0.167(-300 + -149.75) = +75.108$ kNm

$M'_{BC} = -0.333(-300 + -149.75) = +149.76$ kNm

$M'_{CB} = -0.333(+300 + 149.76) = -149.770$ kNm

$M'_{CD} = -0.167(+300 + 149.76) = -75.109$ kNm

Step-4:-

final end Moments.

$M_{AB} = 0 + 2(0) + 74.60 = +74.62$ kNm

$M_{BA} = 0 + 2(74.62) + 0 = 149.24$ kNm

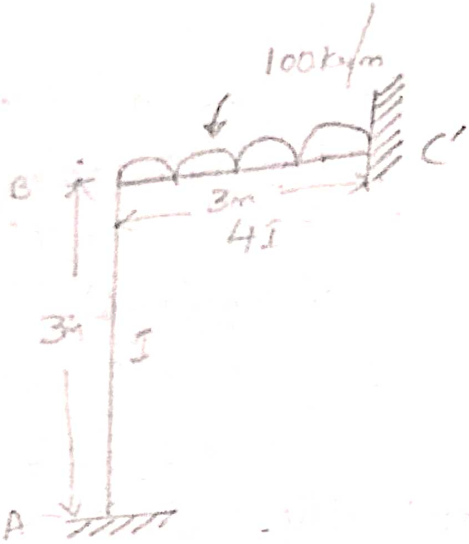
$M_{BC} = -300 + 2(149.76) - (149.77) = -150.25$ kNm

$M_{CB} = +300 + 2(-149.770) + (149.76) = -150.22$ kNm

$M_{CD} = 0 + 2(-75.109) + (0) = -150.21$ kNm

$M_{DC} = 0$

II nd Method :-



Step-1:- fixed end Moments

for span AB,

$M_{FAB} = M_{FBA} = 0$

for span BC,

$M_{FBC} = -\frac{wl^2}{12} = -\frac{100 \times 6^2}{12} = -300$ kN.m

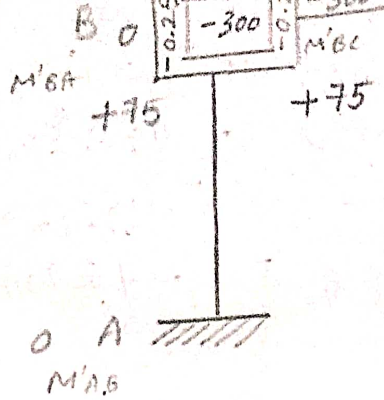
$M_{FCB} = +\frac{wl^2}{12} = +\frac{100 \times 6^2}{12} = +300$ kN.m

for span CD

$M_{FCD} = M_{FDC} = 0$

Step-2:- Rotation table

Joint	Member	Relative stiffness	TRS	DF	RF
B	BA	$\frac{I}{L} = \frac{I}{3}$	$\frac{I}{3} + \frac{I}{3} = \frac{2}{3}I$	$\frac{I}{3} / \frac{2}{3}I = \frac{1}{2}$	$= -\frac{1}{2} \times \frac{1}{2} = \frac{-1}{4} = -0.25$
	BC	$\frac{1}{2}\left(\frac{I}{L}\right) = \frac{1}{2}\left(\frac{4I}{6}\right) = \frac{I}{3}$		$\frac{I}{3} / \frac{2}{3}I = \frac{1}{2}$	$-\frac{1}{2} \times \frac{1}{2} = -1/4 = -0.2$



Cycle - 1 :-

$$M'_{BA} = -0.25 \left(-300 + 0 + 0\right] = +75 \text{ KNm}.$$

$$M'_{BC} = -0.25 \left[-300 + 0 + 0\right] = +75 \text{ KNm}.$$

~~Cycle - 2 :-~~

$$\cancel{M'_{BA} = -0.25 \left(-300 + 75\right] = +56.25 \text{ KNm}.}$$

$$\cancel{M'_{BC} =}$$

Step - 4 :-

fixed ~~end~~ Moments

$$= F.E.M + 2(\text{Near end Contribution}) + \text{far end Contrib.}$$

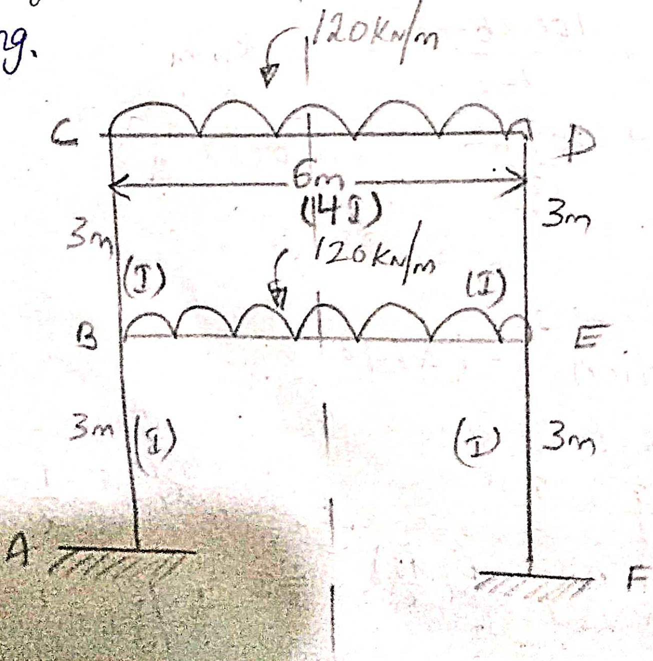
$$M_{AB} = 0 + 2(0) + 75 = +75 \text{ KNm}$$

$$M_{BA} = 0 + 2(+75) + (0) = +150 \text{ KNm}$$

$$M_{BC} = -300 + 2(+75) + 0 = -150 \text{ KNm}.$$

$$M_{CB} = 0 + 2(0) + 75 = +75 \text{ KNm}.$$

Analyze the ~~frame~~ (5m) as shown in fig. Taking advantage of Symmetry of the frame & Loading.



Step-1 :- Fixed end Moment

Span AB,

$$M_{FAB} = M_{FBA} = 0.$$

Span BC,

$$M_{FBC} = M_{FCB} = 0.$$

Span CD,

$$M_{FCD} = -\frac{wL^2}{12} = -\frac{120\times6^2}{12} = -360\ kN{\cdot}m$$

$$M_{FDC} = +\frac{wL^2}{12} = +\frac{120\times6^2}{12} = +360\ kN{\cdot}m.$$

Span DE,

$$M_{FDE} = M_{FED} = 0$$

Span BE,

$$M_{FBE} = -\frac{wL^2}{12} = -\frac{120\times6^2}{12} = -360\ kN{\cdot}m$$

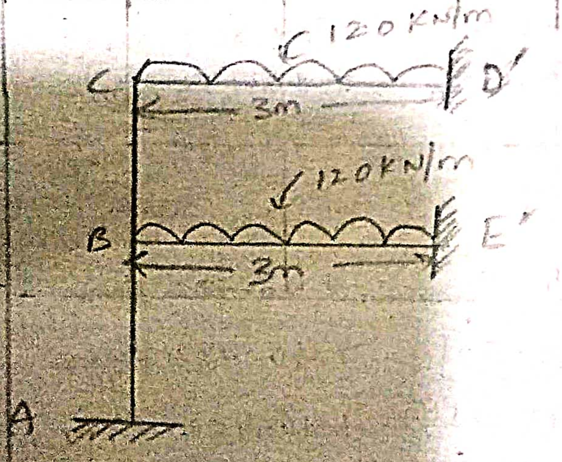
$$M_{FEB} = +\frac{wL^2}{12} = +\frac{120\times6^2}{12} = +360\ kN{\cdot}m$$

Span EF,

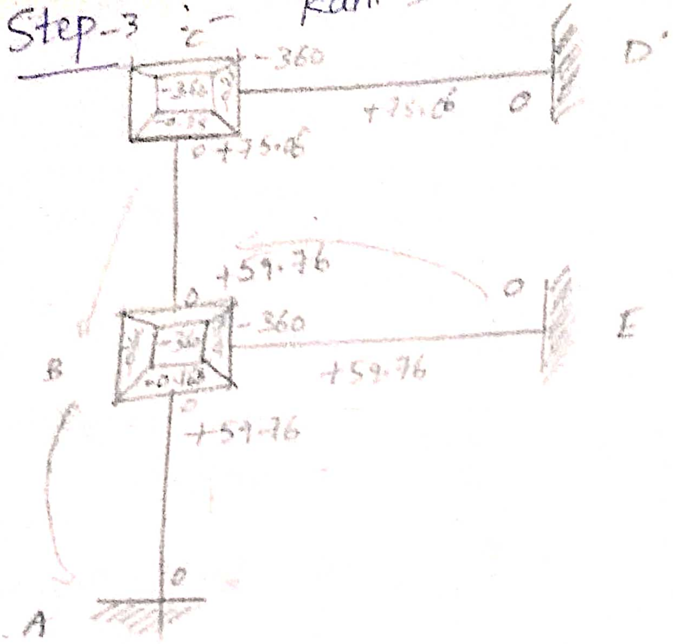
$$M_{FEF} = M_{FFE} = 0.$$

Step-2 :- Rotation factor table.

Joint	member	R.S	T.R.S	D.F	R.F
B	BA	$\frac{I}{L} = \frac{I}{3}$	$\frac{I}{3}+\frac{I}{3}+\frac{I}{3} = I$	1/3	-0.166
	BE	$\frac{1}{2}\left(\frac{I}{L}\right) = \frac{1}{\cancel{2}}\times\frac{\cancel{4}^{2}I}{\cancel{6}\,3} = \frac{I}{3}$		1/3	-0.166
	BC	$\frac{I}{L} = \frac{I}{3}$		1/3	-0.166
C	CB	$\frac{I}{L} = \frac{I}{3}$	$\frac{I}{3}+\frac{I}{3} = \frac{2I}{3}$	1/2	-0.25
	CD	$\frac{1}{2}\left(\frac{I}{L}\right) = \frac{1}{\cancel{2}}\times\frac{\cancel{4}^{2}I}{\cancel{6}\,3} = \frac{I}{3}$		1/2	-0.25.



Step-3 :-



Cycle :- I :-

$M_{BA} = u.[\Sigma B + \text{far end contribution}]$

$= -0.166(-360+0+0+0)$

$= +59.76$ kN.m

$M'_{BC} = -0.166(-360+0+0+0] = +59.76$ kN.m.

$M'_{BE} = -0.166[-360+0+0+0] = +59.76$ kNm.

$M'_{CB} = -0.25[-360+59.76+0] = +75.06$ kNm

$M_{CD} = -0.25(-360+59.76+0] = +75.06$ kNm.

Cycle :- II :-

$M_{BA} = -0.166[-360+75.06] = +47.30$ kN.m

$M_{BC} = -0.166[-360+75.06] = +47.30$ kNm

$M_{BE} = -0.166[-360+0+0+75.06] = +47.30$ kNm

$M_{CB} = -0.25[-360+47.30+0] = +78.175$ kNm.

$M_{CD} = -0.25(-360+47.30+0) = +78.175$ kNm

Cycle - III :-

$M_{BA} = -0.166[-360+0+0+78.175] = +46.782$ kN.m

$M_{BC} = -0.166[-360+0+0+78.175] = +46.782$ kNm

$M_{BE} = -0.166[-360+0+0+78.175] = +46.782$ kNm

$M_{CB} = \overset{-0.25}{\cancel{-0.166}}[-360+0+0+46.782] = +78.304$ kNm

$M_{CD} = -0.25(-360+0+0+46.782] = +78.304$ kNm

Cycle - IV

= $-0.166\,(-360+0+0+78.304] = +46.761$ KNm

M_{BA} = $-0.166\,(-360+0+0+78.304] = +46.761$ KNm

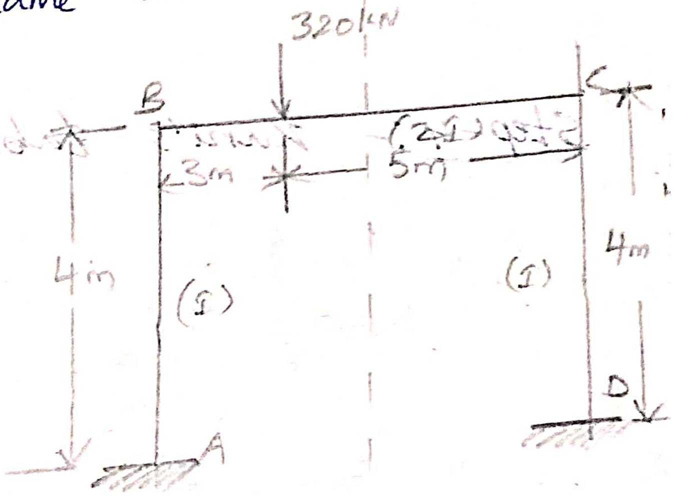
M_{BC} = $-0.166\,(-360+0+0+78.304] = +46.761$ KNm

M_{BE} = $-0.25\,(-360+0+0+46.761) = +78.309$ KNm

M_{CB} = $-0.25\,(-360+0+0+46.761) = +78.309$ KNm.

M_{CD} = -0.25

Analyze the portal frame as shown in fig. by using Kani's method ?



Step-1 :- fixed end moments

for AB Span,

$M_{AB} = M_{BA} = 0$

for BC Span,

$M_{FBC} = -\dfrac{Wab^2}{L^2}$

$= -\dfrac{320\times3\times5^2}{8^2} = -375$ kN.m

$M_{FCB} = +\dfrac{Wa^2b}{L^2}$

$= \dfrac{-320\times3^2\times5}{8^2} = +225$ KN.M.

For CD Span,

$M_{FCD} = M_{FDC} = 0$.

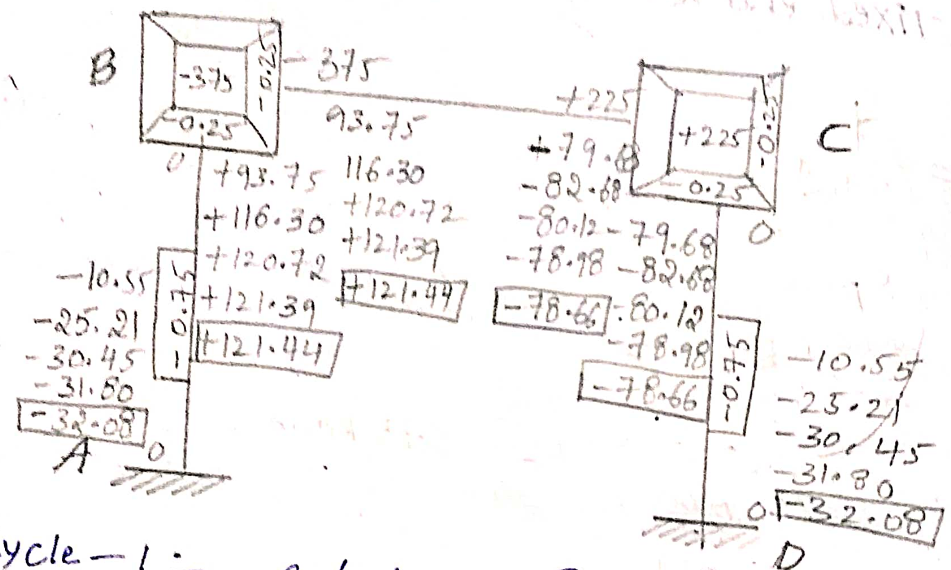
Step-2 :- Rotation factor table

Joint	Member	R.S (K)	T.R.S (Σk)	DF	R.F
B	BA	$\frac{I}{L} = \frac{I}{4}$	$\frac{I}{4}+\frac{I}{4} = \frac{I}{2}$	$\frac{I}{4}/\frac{I}{2} = \frac{1}{2}$	$\frac{-1}{2}\times\frac{1}{2} = \frac{-1}{4} = -0.25$
	BC	$\frac{I}{L} = \frac{2I}{8} = \frac{I}{4}$		$\frac{I}{4}/\frac{I}{2} = \frac{1}{2}$	$\frac{-1}{2}\times\frac{1}{2} = \frac{-1}{4} = -0.25$
C	CB	$\frac{I}{L} = \frac{2I}{8} = \frac{I}{4}$	$\frac{I}{4}+\frac{I}{4} = \frac{I}{2}$	$\frac{I}{4}/\frac{I}{2} = \frac{1}{2}$	$\frac{-1}{2}\times\frac{1}{2} = \frac{-1}{4} = -0.25$
	CD	$\frac{I}{L} = \frac{I}{4}$		$\frac{I}{4}/\frac{I}{2} = \frac{1}{2}$	$\frac{-1}{2}\times\frac{1}{2} = \frac{-1}{4} = -0.25$

Step-3 :- Kani's table Displacement factor

Story	member	RS	T.R.S	Distribution factor DF	Displacement factor D.F = u' = -1.5 Co
1	BA	$\frac{I}{l} = \frac{I}{4}$	$\frac{I}{4} + \frac{I}{4} = \frac{I}{2}$	$\frac{I/4}{I/2} = \frac{1}{2} = 0.5$	$-1.5 \times 0.5 = -0.$
	CD	$\frac{I}{L} = \frac{I}{4}$		$\frac{I/4}{I/2} = \frac{1}{2} = 0.5$	$-1.5 \times 0.5 = -0.$

Step-4 :- Kani's table



Cycle-1 :- Rotation Contribution

$M'_{BA} = u\left[\Sigma M_B + \text{far end Moments}\right]$

$= -0.25(-375+0+0+0]$

$M'_{BC} = -0.25(-375+0+0+0] = +93.75 \text{ kN.m}$

$M'_{CB} = -0.25\,(+225+93.75+0+0] = +93.75 \text{ kNm}$

$M'_{CD} = -0.25\,(+225+93.75+0+0] = -79.68 \text{ kNm}$

Displacement Contribution

$= -0.25\,(+225+93.75+0+0] = -79.68 \text{ kNm}$

$M''_{CD} = M''_{BA} = -0.75\,(93.75+0+(-79.68]+0+0$

$= -10.55 \text{ KNm.}$

Cycle :-2 :- Rotation Contribution

$M'_{BA} = -0.25(-375+0+-79.68+ - 10.55)$

$M'_{BC} = -0.25\ (-375 + 0 + -79.68 + -10.55) = +116.30$ KNm

$M'_{CB} = -0.25\ (+225 + 116.30 + 0 + (-10.55)) = -82.68$ KNm

$M'_{CD} = -0.25\ (+225 + 116.30 + 0 + (-10.55)) = -82.68$ KNm

Distribution Contribution

$M''_{CD} = M''_{BA} = -0.75\ (+116.30 + -82.68 + 0 + 0]$

$= -25.21$ KNm

Cycle :- 3 :- Rotation Contribution

$M'_{BA} = -0.25\ (-375 + 0 + -82.68 + -25.21) = +120.72$ KNm

$M'_{BC} = -0.25\ (-375 + 0 + -82.68 + -25.21) = +120.72$ KNm

$M'_{CB} = -0.25\ (+225 + 120.72 + 0 + \cancel{(-25.21)} \overset{-25.21}{}) = -80.12$ KNm

$M'_{CD} = -0.25 (+225 + 120.72 + 0 + -25.21) = -80.12$ KNm

Distribution Contribution

$M''_{CD} = M''_{BA}$

$= -0.75\ (+120.72 + -80.12 + 0 + 0]$

$= -30.45$ KNm.

Cycle :-4 :- Rotation Contribution

$M'_{BA} = -0.25\ (-375 + 0 + -80.12 + -30.45) = +121.39$ KNm

$M'_{BC} = -0.25\ (-375 + 0 + -80.12 + -30.45) = +121.39$ KNm

$M'_{CB} = -0.25\ (+225 + 121.39 + 0 + -30.45) = -78.98$ KNm

$M'_{CD} = -0.25\ (+225 + 121.39 + 0 + -30.45) = -78.98$ KNm

Distribution Contribution

$M''_{CD} = M''_{BA} = -0.75\ (+121.39 + -78.98 + 0 + 0]$

$= -31.80$ KNm.

Cycle :-5 :- Rotation Contribution

$M'_{BA} = -0.25\ (-375 + 0 + -78.98 + -31.80] = +121.44$ KNm

$M'_{BC} = -0.25\ (-375 + 0 + -78.98 + -31.80] = +121.44$ KNm

$M'_{CB} = -0.25\ (+225 + 121.44 + -31.80] = -78.66$ KNm

$M'_{CD} = -0.25\ (+225 + 121.44 + -31.80] = -78.66$ KNm

Distribution Contribution

$M''_{CD} = M''_{BA} = -0.75\ (+121.44 + -78.66 + 0 + 0]$

$= -32.08$ KNm

Step :- 5 :- Fixed end Moments

$M_{AB} = M_{FAB} + 2m'_{AB} + m'_{BA} + M''_{BA}$

$= 0 + 2(0) + 121.44 + -32.08 = +89.36$ kN

$M_{BA} = 0 + 2(121.44) + 0 + -32.08 = +210.8$ kNm

$M_{BC} = -375 + 2(+121.44) + -78.66 + 0 = -210.7$ kNm

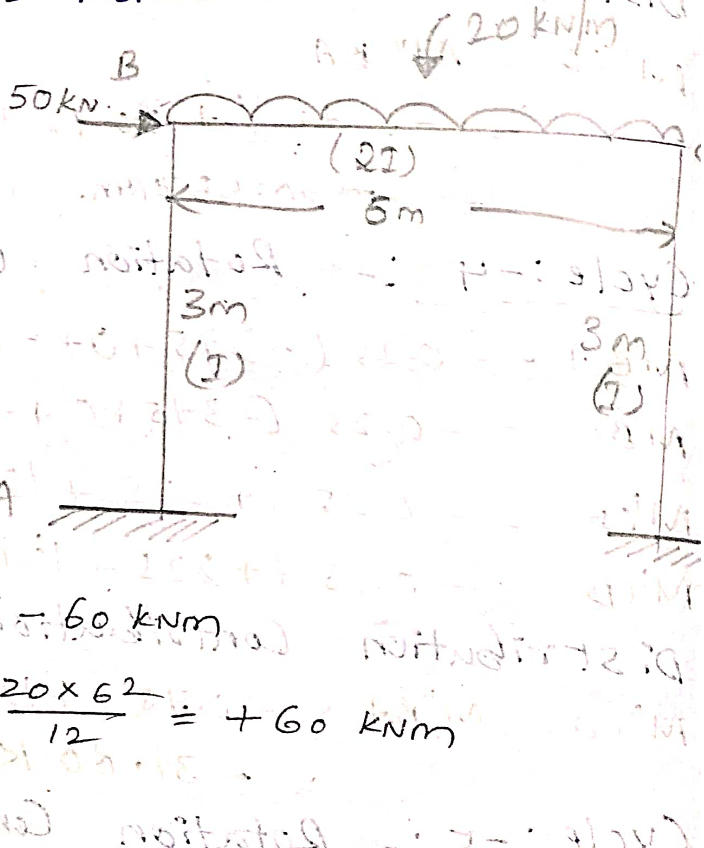
$M_{CB} = 225 + 2(-78.66) + 121.44 + 0 = +189.12$ kNm

$M_{CD} = \overset{0}{\cancel{225}} + 2(-78.66) + 0 + -32.08 = -189.4$ kNm

$M_{DC} = 0 + 2(0) + -78.66 + -32.08 = -110.74$ kNm

---

4) Analyze the Jointed frame as shown in fig. by using Kani's method



Sol:-

Step-1 :- Fixed end Moment

for span AB,

$M_{FAB} = M_{FBA} = 0$

for span BC,

$M_{FBC} = -\frac{wl^2}{12}$

$= -\frac{20\times6^2}{12} = -60$ kNm

$M_{FCB} = +\frac{wl^2}{12} = +\frac{20\times6^2}{12} = +60$ kNm

for span CD,

$M_{FCD} = M_{FDC} = 0$

Step-2 :- Rotation factor & Displacement factor

Joint	Member	R.S	T.R.S	D.f	R.f
B	BA	$\frac{I}{L} = \frac{I}{3}$	$\frac{I}{3} + \frac{I}{3} = \frac{2I}{3}$	$\frac{I/3}{2I/3} = \frac{1}{2}$	-0.25
	BC	$\frac{I}{L} = \frac{2I}{6} = \frac{I}{3}$		$\frac{I/3}{2I/3} = \frac{1}{2}$	-0.25
C	CB	$\frac{I}{L} = \frac{2I}{6} = \frac{I}{3}$	$\frac{I}{3} + \frac{I}{3} = \frac{2I}{3}$	$\frac{I/3}{2I/3} = \frac{1}{2}$	-0.25
	CD	$\frac{I}{L} = \frac{I}{3}$		$\frac{I/3}{2I/3} = \frac{1}{2}$	-0.25

Storey / Joint	Member	R.S	T.R.S	D.f	D.f = -1.5 DF
1	BA	$\frac{I}{L} = \frac{I}{3}$	$I/3 + I/3 = \frac{2I}{3}$	$\frac{I/3}{2I/3} = \frac{1}{2}$	$-1.5 \times \frac{1}{2} = -0.75$
	CB	$\frac{I}{L} = \frac{2I}{6/3} = I/3$		$\frac{I/3}{2I/3} = \frac{1}{2}$	$-1.5 \times \frac{1}{2} = -0.75$

Note :- Taking the Section through Columns of 1st storey & Considering the Horizontal equilibrium of the upper portion we get storey force $S = 50KN$.

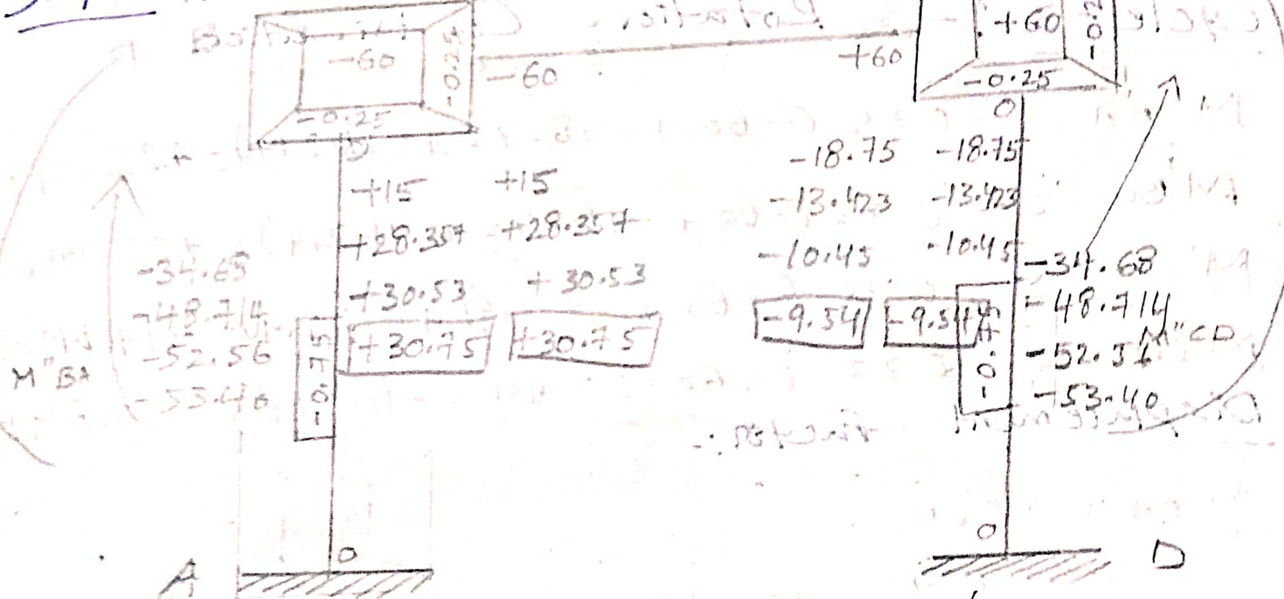
storey Moment $= \frac{S \cdot Length}{3}$

$= \frac{50 \times 3}{3}$

$= \frac{150KN}{3}$

$= 50KN$

Step-3 :- Kani's table.



Cycle :- 1 :- Rotation Contribution

$M'_{BA} = u\,[\Sigma M_B + \text{far end Moment}]$

$= -0.25\,[-60 + 0 + 0 + 0] = +15$ KN.M

$M'_{BC} = -0.25\,(-60 + 0] = +15$ KNm

$M'_{CB} = -0.25\,(+60 + 15 + 0] = -18.75$ KNm

$M'_{CD} = -0.25\,[60 + 15 + 0 + 0] = -18.75$ KNm

Displacement Contribution :-

$M''_{BA} = u'\,[\text{Top Moments} + \text{Bottom moments}]$

$M''_{CD} = M''_{BA} = -0.75\,(+15 - 18.75 + \frac{50}{50}]$ $= \cancel{-72.187 \text{ KNM}}$ -34.68 KNM

Disp...

$M''BA = M''CD = -0.75\,(+28.375 + -13.423 + 50)$

$= -48.714$ kNm

Cycle - 3 :- Rotation Contribution.

$M'BA = -0.25\,(+60 + -13.423 + -48.714) = +30.53$ kNm

$M'BC = -0.25\,(-60 + -13.423 + -48.714) = +30.53$ kNm

$M'CB = -0.25\,(+60 + 30.53 + -48.714) = -10.45$ kNm

$M'CD = -0.25\,(+60 + 30.53 + -48.714) = -10.45$ kNm

Displacement factor :-

$M''BA = M''CD = -0.75\,(+30.53 + -10.45 + 50] = -52.56$ kNm

Cycle-4 :- Rotation Contribution

$M'BA = -0.25\,(-60 + -10.45 + -52.56) = +30.75$ kNm

$M'BC = -0.25\,(-60 + -10.45 + -52.56) = +30.75$ kNm

$M'CB = -0.25\,(+60 + 30.75 + -52.56) = -9.54$ kNm

$M'CD = -0.25\,(+60 + 30.75 + -52.56) = -9.54$ kNm

Displacement factor :-

$M''BA = M''CD = -0.75\,(+30.75 - 9.54 + 50) = -53.40$ kNm

Step - 4 :- final end moments.

$M_{AB} = M_{FAB} + 2(m'AB) + M'BA +$ Displacement factor

$= 0 + 2(0) + 30.75 + (-53.4) = -22.65$ kNm

$M_{BA} = 0 + 2(30.75) + 0 + -53.4 = 8.1$ kNm

$M_{BC} = -60 + 2(30.75) + (-9.54) + 0 = -8.04$ kNm

$M_{CB} = -60 + 2(-9.55) + 30.75 + 0 = +71.65$ kNm

$M_{CD} = 0 + 2(-9.55) + 0 + (-53.4) = -72.5$ kNm

$M_{DC} = -0 + 2(0) + 9.55 + 0 + (-53.4) = -43.85$ kNm

Unit - IV :-

Stiffness Matrix Method

→ Determinancy

→ Indeterminancy

→ Degree of freedom.

1. What is meant by Matrix? What are the different types of matrix

Static indeterminancy & Kinematic indeterminancy

Stiffness matrix Method & flexibility Matrix method

Degree of Static indeterminancy :-

Statically indeterminate structures are those structures which can't be analysed with the help of equations of static equilibrium alone. This structures are also known as hyperstatic for the analysis of the structure it becomes necessary to consider the deformation of the structure because the equation of static alone (or) not sufficient for the solution of the problem

* In the case of statically indeterminate structures the no. of unknowns is greater than the no. of equations.
* No. of this additional equations necessary for the solution of the problem. is known as degree of static indeterminancy. (or) Degree of redundancy.

The total degree of static indeterminancy of the structure Ds may be considered as the

Sum of the following two types of indeterminate structures.
1. Degree of external indeterminancy [Dse].
2. Degree of internal indeterminancy [Dsi]
Mathematically expressed as,

Degree of Static indeterminancy = Dse + Dsi

$D_s = D_{se} + D_{si} \longrightarrow$ (1)

~~$D_{se} = [r - 6] \longrightarrow$ (2)~~ the degree of external indeterminancy for a
~~$D_{se} = [r - 3] \longrightarrow$ (3)~~

Space structure is given by the equation.

$D_{se} = [r - 6] \longrightarrow$ (2)

where, r = Reactions

for plane structure it is given by the equ

$D_{se} = [r - 3] \longrightarrow$ (3)

frames :- A pin-Jointed frame is statically indeterminant internally if it has just the min. no. of members is required to preserve its geometry. If the no. of members is more it is the pin-Jointed frame is internally indeterminant to the extend.

The total no. of members in a pin-Jointed plane frame with joints is given by the equations.

$M' = 2j - 3$

In case of pin-Jointed space frame the most elementary frame is tetrahydran having four joints and six members the total no. of members is required in a pin-Jointed space frame is given by equation.

$M' = 3j - 6.$

on the other hand if the actual no. of members M' is more than the requirement as per equations the frame is overstiff and consequently it is statically indeterminant

The degree of internal indeterminancy

→ The degree of statical indeterminacy for a plane frame is given by the equation

$$D_{si} = M - (2j - 3)$$

→ Space frame

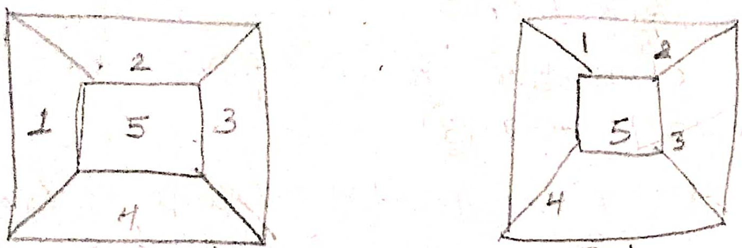
$$D_{si} = M - (3j - 6).$$

Where, M = Members

j = Joints

Hybrid frames :- Multi-storeys, no. of frames

Consider the Ridged-Jointed frame as shown in fig.[a] An open Configuration Can be obtained in these Case by introducing as shown in fig [b]



(a) (b)

→ The degree of internal indeterminancy of a ridged Jointed frame is 3C.

→ The degree of internal indeterminanacy of a plane frame with hybrid joints is given by the equation

$$D_{si} = 3C - \Sigma(m-1)$$

$$D_{si} = 6C - \Sigma 3[m-1].$$

Degree of kinematic indeterminancy :-

A skeletal structure is said to be kinematically indeterminancy if the displacement components of its Joints Can't be determined by a Compatibility equations alone.

In the Case of kinematic indeterminancy structures the no. of unknown displacements components is greater than the no. of Compatability equations for these structures, Additional equations based on the equilibrium must be written in order to obtain a sufficient no. of equations for the determination of all unknown displacement Components the no. of these additional equations necessary for the determinancy of all the independent displacement Components is known as degree of kinematic indeterminancy (or) Degree of freedom. of the Structure

The degree of kinematic displacement Components of kinematic indeterminancy of a pin - Jointed plane frame is given by the equation

$$D_K = (2j - r)$$

The degree of kinematic displacement Components of kinematic indeterminancy of a pin - Jointed space frame is given by the equation

$$D_K = (3j - e)$$

Where,

J = Joints

e = equations

Structural Analysis :-

The Main Aim of the structural analysis is to determine internal forces and displacements. The purpose building frames received Loads, Self weight and transfer them to foundation.

Classification of structure :-

The structures are classified into three types

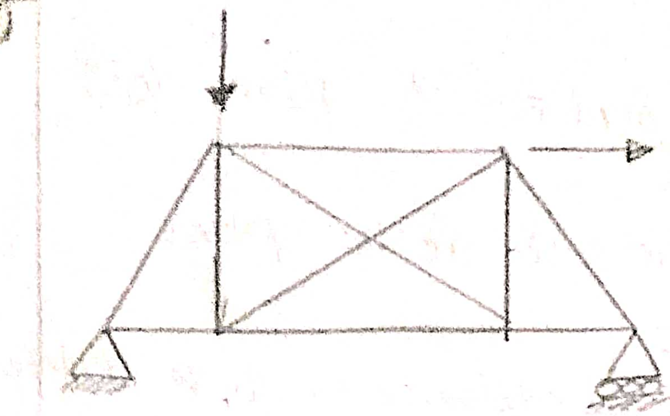
1) Skeletol Structure
2) Surface Structure
3) Ground structure

1) **Skeletol Structure** :- which can be indicated or idealized to a Series of Straight or Curved lines structure looks like a skeletol

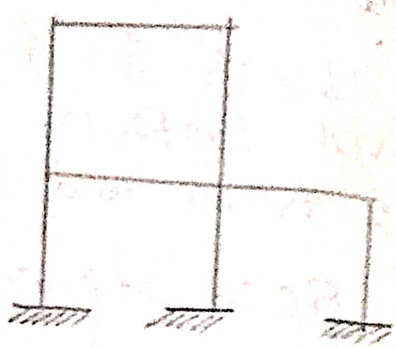
2. Surface Structure :-
Which Can be idealized to plane (or) Curved Surfaces.
Ex:- Beams and Schells & folder plans

Solid structure :-
Which Can be neither idealized to a Skeletol nor to a plane or Curved Surfaces.
In general only Skeletol structures Can be analysed by a elementory methods of Structural Mechanics.

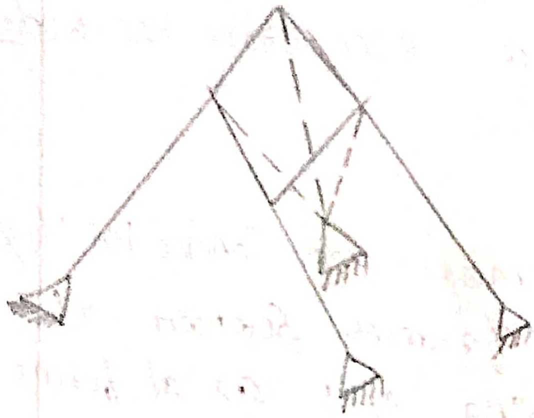
Skeletol Structure :-
Pin - Jointed frame :- Members are Connected by means of Joints. These frames Support the applied loads by developing only axial forces in Consistent members if external forces act at the joint & members are straight
Ridged joints :- The joint of ridged jointed frame are to be ridged So that the angle b/w the members meeting at a joint remains unchanged.
These frames resist external forces by developing B.M, S.F, Axial force and twisting mome
Skeletol Structures are Classified as :-
1. plane frames
2. Space frames.
1. plane frames :- All members as well as external Loads will be in one plane.
2. Space frames :- All members will not lie in one plane.



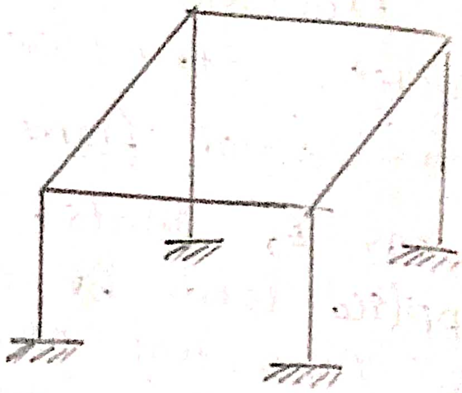
plane truss



plane frame



Space truss



Space frame

Determinancy :-

To keep a body in equilibrium certain no. of reactive components are necessary.

$\therefore$ $\Sigma f_x = \Sigma f_y = \Sigma f_z = 0$

$\Sigma M_x = \Sigma M_y = \Sigma M_z = 0$

If entire structure is in equilibrium, every part of it however small must also be in equilibrium.

Indeterminant Structures :-

In these structure. the no. of reactive compo-nents will be more than min i.e., necessary to keep the body in equilibrium

Classification of indeterminate structures :-

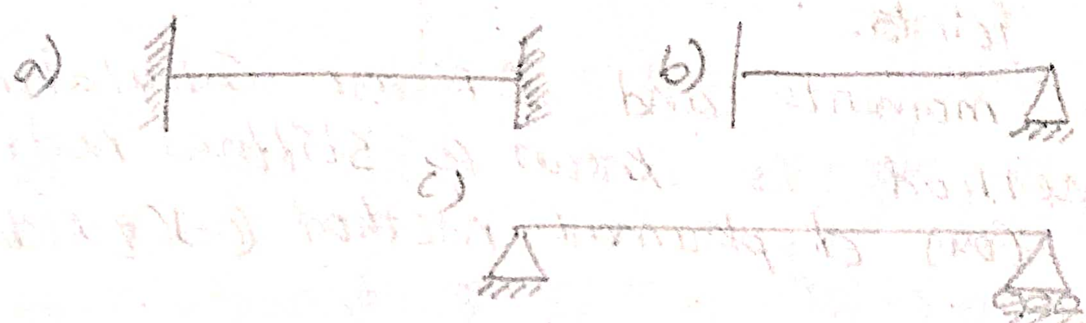
The following indeterminate structures are

1. Degree of static indeterminancy
2. Degree of kinematic indeterminancy

Degree of freedom (or) Degree of kinematic indeterminancy :-

for a beam the degree of freedom at an end is given below.

a) free end condition - 3
b) Simply Supported ~~Roller end~~ - 2
c) Hinged end/Roller end - 1
d) fixed end - 0.



Hence degree of kinematic indeterminancy D_K for
i) one end is hinged & other end roller - 3
ii) propped Cantilever - 2
iii) fixed beam - 0

Matrix Method for structural Analysis :- indeterminate structure

The Analysis of indeterminate structure is the major field in structural engineering there are several methods of the analysis the best among these studied so far is the Kani's method.

→ The analysis of high degree indeterminate structure and development of computers have given rise to new method called the matrix method.

Basically matrix methods of analysis are classified into 2 methods are;
1. Stiffness matrix method
2. flexibility matrix method

Stiffness matrix method:-

The Systematic development of Slope deflection method in the matrix form has given rise to Stiffness matrix method.

→ In this method the basic unknows are displacement of joints.

→ The equation of equilibrium are formed and Solved to get Slopes and deflections at the joints.

→ using moments and S.F are Calculated this method is known as Stiffness matrix method (or) displacement method (or) equilibrium method.

Different Approaches to matrix method:-

There are two Approaches to be solution of Matrix method are:

1. Direct method / Structure approach.
2. Transformation matrix approach / element approach

Direct Approach / Structure approach:-

In these method gives a Clear Concept of analysis.

Transformation matrix approach:-

1) In these method ideally Shooted for developing general purposes. Computer programes for Structural analysis.

2) Transformation matrix is used in matrix method are explained first then Stiffness method & flexibility method.

Stiffness Matrix M:

If a Structure is having n coordinates it's force response to the displacement is represent by

$$[K] = \begin{bmatrix} K_{11} & K_{12} & \cdots & k_{1n} \\ K_{21} & K_{22} & \cdots & K_{2n} \\ \vdots & \vdots & & \vdots \\ K_{m1} & K_{m2} & & K_{mn} \end{bmatrix}$$

And this method is known as Stiffness matrix.
The element of Stiffness matrix K_{ij} is the force at Coordinate i due to unit displacement at Coordinate. In this method the basic of unknows to be determine in the analysis are displacement components of various joints the degree of kinematic indeterminancy [Degree of freedom] is identified first Co-ordinate no is assigned to each one of the unknown displacement Components.

The basic unknowns are $\theta_A, \theta_B, \theta_C$ the displacement Co-ordinate direction 1, 2, n respectively.

Let the forces developed due to applied loads in the restrained structure in the Co-ordinate direction. (or) $P_1, P_2 \cdots P_n$ determine the Stiffness matrix of the structure by applying unit displacement in each of the Co-ordinate direction and find the forces developed.

Matrix formation.

$$\begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1n} \\ K_{21} & K_{22} & \cdots & K_{2n} \\ \vdots & \vdots & & \vdots \\ K_{n1} & K_{n2} & & K_{nn} \end{bmatrix} \begin{bmatrix} \theta_A \\ \theta_B \\ \vdots \\ \theta_n \end{bmatrix} \begin{bmatrix} P_{1L} \\ P_{2L} \\ \vdots \\ P_{nL} \end{bmatrix}$$

$$[P] = [P_L] + [K][\theta]$$

$$[K][\theta] = [P] - [P_L] = [P - P_L]$$

Step-by-step procedure for Stiffness Matrix method by structure approach :-

Step-1 :- Determine the degree of kinematic indeterminancy [Degree of freedom].

Step-2 :- Assign the co-ordinate numbers to the unknown displacement

Step-3 :- Determine the forces developed in each of the co-ordinate direction of a fully restrained structure.

Step-4 :- Determine the stiffness matrix [K] by unit displacement to the restrained structure in each of the co-ordinate structure are find the forces developed in all the co-ordinate ~~at~~ direction.

Step-5 :- Observing the final forces in various co-ordinate directions note the final force [P].

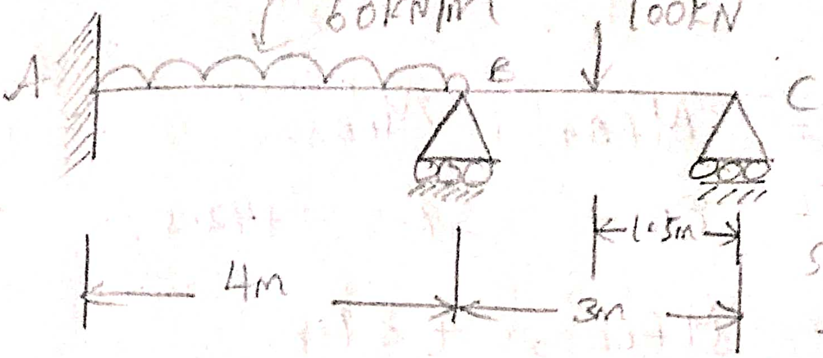
Step-6 :- Solve the stiffness equ^n

Step-7 :- Calculate the final end moments

Stiffness Matrix Method problems :-

1. Analyse the Continuous beam as shown in fig by using SMM. Take EI is Constant through the section.

Sol:-



fixed - 0
hins - 1
Simp/Roller - 2
free end condition - 3

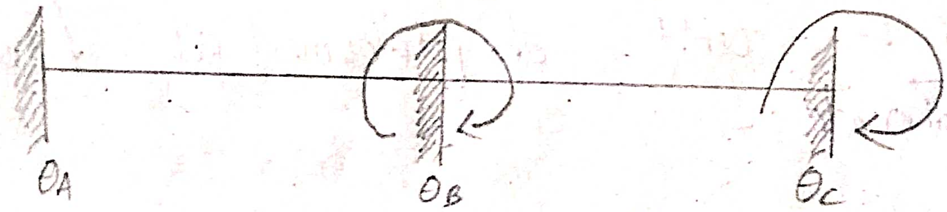
Step-1 :-

Degree of ^kinematic^ indeterminancy (or) Degree of freedom

$(D_f) = 0 + \cancel{1} 2 + \cancel{1} 2$

$= \cancel{4}\ 2$.

Step-2 :- Assigning Co-ordinates



Step-2 :- fixed end Moments.

for Span AB,

$M_{FAB} = -\frac{wl^2}{12} = -\frac{60 \times 4^2}{12} = -80$ kNm

$M_{FBA} = +\frac{wl^2}{12} = +\frac{60 \times 4^2}{12} = +80$ kNm

for Span BC,

$M_{FBC} = -\frac{wl^2}{8} = -\frac{100 \times 3^2}{8} = -37.5$ kNm

$M_{FCB} = +\frac{wl^2}{8} = +\frac{100 \times 3^2}{8} = +37.5$ kNm.

Step-3 :- force Matrix.

$$[P] = [P_1 - P_L] \quad \text{External forces}$$

$$\begin{bmatrix} P_{Li} - P_{2i} \end{bmatrix} = 0$$

$$P_{1L} = M_{FBA} + M_{FBC}$$

$$= 80 + -37.5 = +42.5.$$

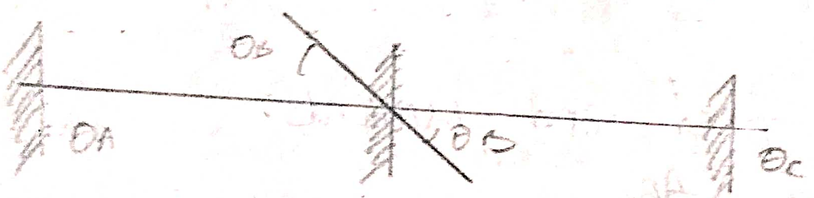
$$P_{2L} = M_{FCB} = +37.5$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 0 - 42.5 \\ 0 - 37.5 \end{bmatrix} = \begin{bmatrix} -42.5 \\ -37.5 \end{bmatrix}$$

Step-4 :- Stiffness matrix.

$$[k] = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

Case-i :- unit displacement at first Coordinate :-



$$k_{11} = \frac{2EI}{L}\left(2\theta_B + \theta_A - \cancel{\frac{3\delta}{l}}\right) + \frac{2EI}{L}\left(2\theta_B + \theta_C - \cancel{\frac{3\delta}{L}}\right)$$

$$= \frac{4EI\theta_B}{L} + \cancel{\frac{2EI\theta_A}{L}} + \frac{4EI\theta_B}{L} + \cancel{\frac{2EI\theta_C}{L}}$$

$$= \frac{4EI}{L} + \frac{4EI}{L} = \frac{4EI}{4} + \frac{4EI}{3}$$

$$= 2.33\ EI$$

$K_{11} = \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\delta}{L}\right)$ $\quad \theta_B = 1,\ \theta_C + \theta_B = 0$

$\boxed{\theta_B = 1,\ \theta_C = \theta_A = 0}$

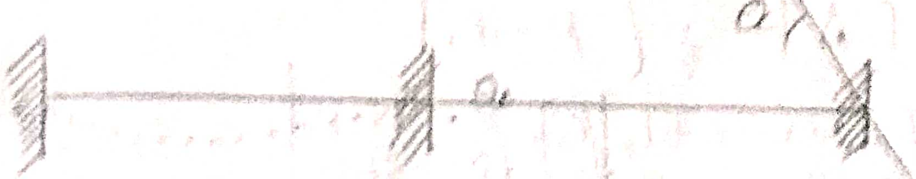
$= \frac{4EI\theta_C}{4} + \frac{2EI\theta_B}{L}$

$= \frac{2EI}{L}(\theta_B)$

$= \frac{2EI}{L} = \frac{2EI}{3} = 0.67\,EI$

~~K_{21}~~

Case – II :- unit displacement @ Co-ordinate



$K_{21} = \frac{2EI}{L}\left[2\theta_C + \theta_B - \cancel{\frac{3\delta}{L}}\right]$

$= \frac{2EI}{L}(\theta_C)$

$= \frac{2EI}{L} = \frac{2EI}{3} = 0.67\,EI$

$\cancel{K_{22}} = \frac{2EI}{L} = \frac{2EI}{3} = 0.67\,EI$

$K_{21} = \frac{2EI}{L}\left[2\theta_B + \theta_C - \frac{3\delta}{L}\right]$

$= \frac{2EI}{L}[\theta_C]$

$= \frac{2EI}{3} = 0.67\,EI$

$K_{22} = \frac{2EI}{L}\left(2\theta_C + \theta_B + \frac{3\delta}{L}\right]$

$= \frac{2EI}{3}[2\theta_C]$

$= \frac{4EI}{3}$

$= 1.33\,EI$

Stiffness matrix :-

$$[K] = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 2.33EI & 0.67EI \\ 0.67EI & 1.33EI \end{bmatrix}$$

$$= EI \begin{bmatrix} 2.33 & 0.67 \\ 0.67 & \cancel{\text{and}}\ 1.33 \end{bmatrix}$$

Step-5 :- Stiffness equ ~~(or)~~ (or) displacement equation

$$[K][\Delta] = [P - P_1]$$

$$EI \begin{bmatrix} 2.33 & 0.67 \\ 0.67 & 1.33 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 0 - 42.5 \\ 0 - 37.5 \end{bmatrix}$$

$$\cancel{-124.15}$$

$$EI \begin{bmatrix} 2.33 & 0.67 \\ 0.67 & 1.33 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} -42.5 \\ -37.5 \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 2.33 & 0.67 \\ 0.67 & 1.33 \end{bmatrix} \begin{bmatrix} -42.5 \\ -37.5 \end{bmatrix}$$

$$\cancel{\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} -124.15 \\ -78.35 \end{bmatrix}}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI} \left[\frac{1}{2.33 \times 1.33 - 0.67 \times 0.67} \right] = \cancel{0.3}\ 0.377$$

$$\cancel{\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 1.33 & -0.67 \\ -0.67 & 2.33 \end{bmatrix} \begin{bmatrix} -42.5 \\ -37.5 \end{bmatrix}}$$

$$\cancel{\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} -31.4 \\ -58.9 \end{bmatrix}}$$

$$\frac{1}{EI}(0.377)\begin{bmatrix} & \\ -0.67 & 2.33\end{bmatrix}\begin{bmatrix} \\ -37.5\end{bmatrix}$$

$$= \frac{1}{EI}(0.377)\begin{pmatrix}1.33\times -42.5 + -0.67\times -37.5\\ -0.67\times -42.5 + 2.33\times -37.5\end{pmatrix}$$

$$= \frac{1}{EI}(0.377)\begin{bmatrix}-31.4\\ -58.9\end{bmatrix}$$

$$= \begin{bmatrix}-11.83\\ -22.20\end{bmatrix}\times\frac{1}{EI} = \begin{bmatrix}\frac{-11.83}{EI}\\ \frac{-22.20}{EI}\end{bmatrix}$$

$$\boxed{\theta_B = \frac{-11.83}{EI}}$$

$$\boxed{\theta_C = \frac{-22.20}{EI}}$$

Step-6:- final end moments

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\delta}{L}\right]$$

$$= -80 + \frac{\cancel{2}EI}{\cancel{4}2}\left[-\frac{11.83}{EI}\right]$$

$$= -80 + \frac{EI}{2}\left(-\frac{11.83}{EI}\right)$$

$$= -80 + -5.91 = -85.91\ kNm$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right)$$

$$= 80 + \frac{\cancel{2}EI}{\cancel{4}2}\left(2\left(-\frac{11.83}{EI}\right) + 0\right)$$

$$= 80 + (-12.83] = 68.17\ kNm$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\delta}{L}\right)$$

$$= -37.5 + \frac{2EI}{3}\left(2(-11.83)2 + -22.83\right.$$

$$= -68.62\ kNm$$

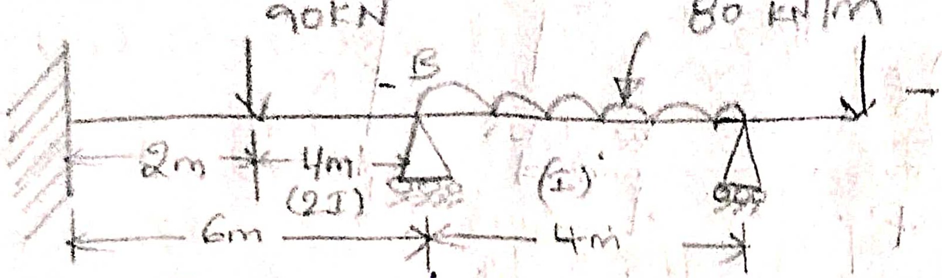
$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\delta}{L}\right)$$

$$= 37.5 + \frac{2}{3}EI\left(2\left(-\frac{22.20}{EI}\right) - \frac{11.83}{EI}\right)$$

$$= 37.5 + \frac{2}{3}EI\left[-\frac{11.83}{EI}\right)$$

$$= 37.5 + 0.67\ (-11.83)$$

2) Analyze the Continuous beam as shown in figure.



Sol:-

Step-1 :- Degree of kinematic indeterminancy (or) Degree of freedom

$$D_f = 0 + \circlearrowleft + \circlearrowleft$$
$$= \cancel{4}\ 2$$

Displacement matrix $= \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix}$

Step-2 :- fixed end Moments.

for Span AB,

$$M_{FAB} = -\frac{Wab^2}{l^2} = -\frac{90 \times 2 \times 4^2}{6^2} = -80\,KNm$$

$$M_{FBA} = +\frac{Wa^2b}{l^2} = +\frac{90 \times 2^2 \times 4}{6^2} = +40\,KNm$$

for Span BC,

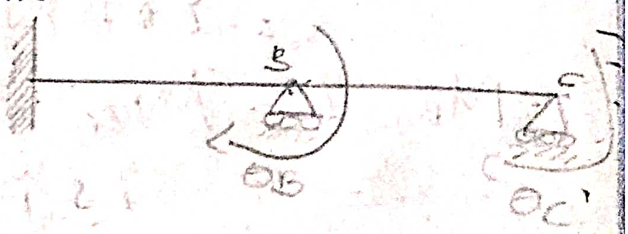
$$M_{FBC} = -\frac{wl^2}{12} = -\frac{80 \times 4^2}{12} = -106.667\,KNm$$

$$M_{FCB} = +\frac{wl^2}{12} = +\frac{80 \times 4^2}{12} = +106.667\,KNm$$

Step-3 :- force Matrix

$$[P] = [P_i - P_L]$$

$$[P_1 = P_2] = 0$$



$$P_{1L} = M_{FBA} + M_{FBC}$$
$$= +40 + -106.667 = -66.667\,KN.m$$

$$P_{2L} = M_{FCB} = +106.667$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 0 + 66.667 \\ 0 - 106.667 \end{bmatrix} = \begin{bmatrix} +66.667 \\ -106.667 \end{bmatrix}$$

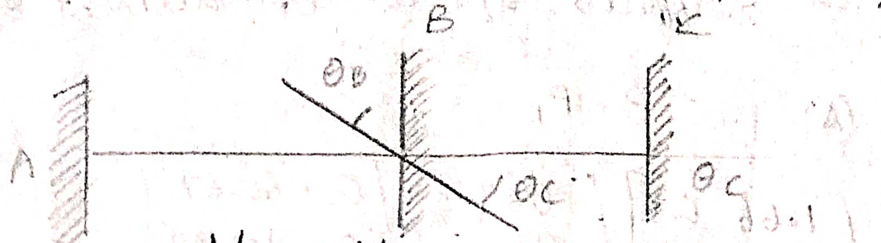
Step-4 :- Displacement matrix

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix}$$

Stiffness Matrix :- Step-3

$$[K] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

Case-i :- unit displacement @ 1^{st} Coordinate.



$\boxed{\theta_A = \theta_C = 0}$

$\theta_A = M_{BA} + M_{BC}$.

$$K_{11} = \frac{2EI}{L}\left(2\theta_B + \theta_A - \cancel{\frac{3\delta}{L}}\right) + \frac{2EI}{L}\left(2\theta_B + \theta_C - \cancel{\frac{3\delta}{L}}\right)$$

$$= \frac{4EI\theta_B}{L} + \cancel{\frac{2EI\theta_A}{L}} + \frac{4EI\theta_B}{L} + \cancel{\frac{2EI\theta_C}{L}}$$

$$= \frac{4EI}{L} + \frac{4EI}{L} = \frac{4EI}{6} + \frac{4EI}{4}$$

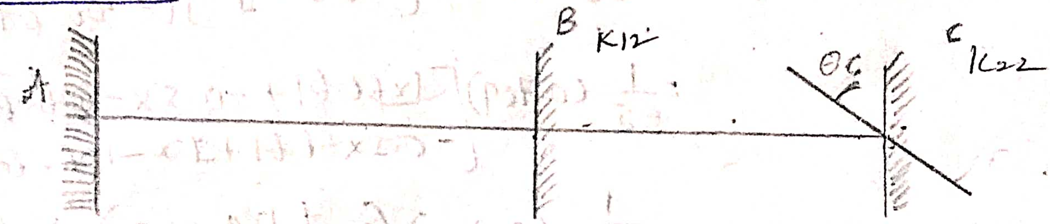
$$\cancel{= 1.66EI = 1.66E(2I)} = \frac{4E(2I)}{6} + \frac{4EI}{4} \cancel{= 2.33EI}$$

$\theta_B = 1 \longrightarrow$ 1

$$K_{21} = M_{CB} = \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\delta}{L}\right) \cancel{= \frac{8EI}{6} + EI = 1.33EI}$$

$$= \frac{2EI\theta_B}{L} = \frac{2EI}{4} = 0.5EI \longrightarrow 2$$

Case -ii :- unit displacement @ 2^{nd} coordinate



$\boxed{\theta_B = 0}$

$$K_{12} = \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\delta}{L}\right)$$

$$= \frac{2EI}{L}(\theta_C) = \frac{2EI}{4} = 0.5EI \longrightarrow 3$$

$$K_{22} = \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\delta}{L}\right)$$

$$= \frac{2EI}{L}(2\theta_C) = \frac{4EI}{L}\theta_C$$

$$= \frac{4EI}{4} = 1EI.$$

Stiffness matrix $= K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$

$$= \begin{bmatrix} 2.33EI & 0.5EI \\ 0.5EI & EI \end{bmatrix}$$

$$= EI \begin{bmatrix} 2.33 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

Step - 6 :- Stiffness equ (or) Displacement equ.

$$[K][\Delta] = [P - P_L]$$

$$EI \begin{bmatrix} 2.33 & 0.5 \\ 0.5 & 1 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} +66.67 \\ -106.67 \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 2.33 & 0.5 \\ 0.5 & 1 \end{bmatrix} \begin{bmatrix} +66.67 \\ -106.66 \end{bmatrix}$$

$$= \frac{1}{EI} \left(\frac{1}{2.33 \times 1 - 0.5 \times 0.5} \right)$$

$$= \frac{1}{EI} (0.480) = \begin{bmatrix} 2.33 & 0.5 \\ 0.5 & 1 \end{bmatrix} \begin{bmatrix} +66.67 \\ -106.66 \end{bmatrix}$$

$$= \frac{1}{EI} (0.480 \begin{bmatrix} 1 & -0.5 \\ -0.5 & 2.33 \end{bmatrix} \begin{bmatrix} 66.67 \\ -106.66 \end{bmatrix}$$

$$= \frac{1}{EI} (0.480) \begin{bmatrix} 1 \times 66.67 + -0.5 \times -106.66 \\ -0.5 \times 66.67 + 2.33 \times -106.66 \end{bmatrix}$$

$$= \frac{1}{EI} (0.480) \begin{bmatrix} +120 \\ -281.85 \end{bmatrix}$$

$$= \begin{bmatrix} +57.6 \\ -135.288 \end{bmatrix} \times \frac{1}{EI}$$

$$\boxed{\theta_B = \frac{+57.6}{EI}}$$

$$\boxed{\theta_C = \frac{-135.288}{EI}}$$

Step-7:- final end Moments

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)$$

$$= -80 + \frac{2E(2I)}{6}\left(\frac{57.6}{EI}\right) = -80 + 38.4$$

$$= \cancel{-60.8\ KNm} \qquad = -41.67\ KNm$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right)$$

$$= +40 + \frac{2E(2I)}{6}\left(2 \times \frac{57.6}{EI}\right)$$

$$= +40 + 76.8 = +116.8\ KNM$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\delta}{L}\right)$$

$$= -106.66 + \frac{2EI}{4}\left(2 \times 57.6 + -135.288\right)$$

$$= -106.66 + -10.044$$

$$= -116.70\ KNm$$

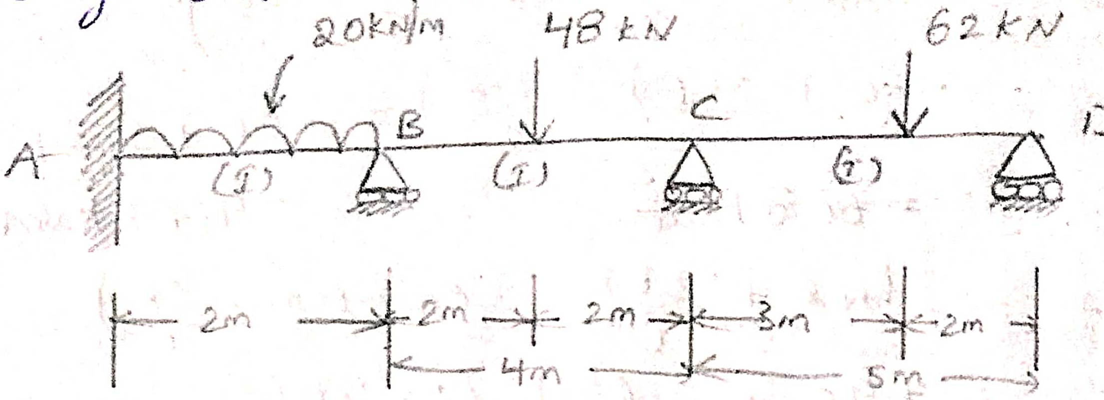
$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\delta}{L}\right)$$

$$= +106.67 + \frac{2EI}{4}\left(2 \times -135.288 + 57.6\right)$$

$$= +106.67 + -106.48$$

$$= -0.19\ KNm$$

3) Analyze the beam as shown in fig. by using stiffness matrix method?
H.W



Step-1:- Degree of freedom

$$D_f = 0+1+1+1$$
$$= 3$$

Step-2:- fixed end moments

for span AB,

$$M_{FAB} = -\frac{wl^2}{12} = \frac{-20\times 2^2}{12} = -6.667\ \text{kNm}$$

$$M_{FBA} = +\frac{wl^2}{12} = \frac{+20\times 2^2}{12} = +6.667\ \text{kNm}$$

for Span BC,

$$M_{FBC} = -\frac{WL}{8} = \frac{-48\times 4}{8} = -24\ \text{kNm}$$

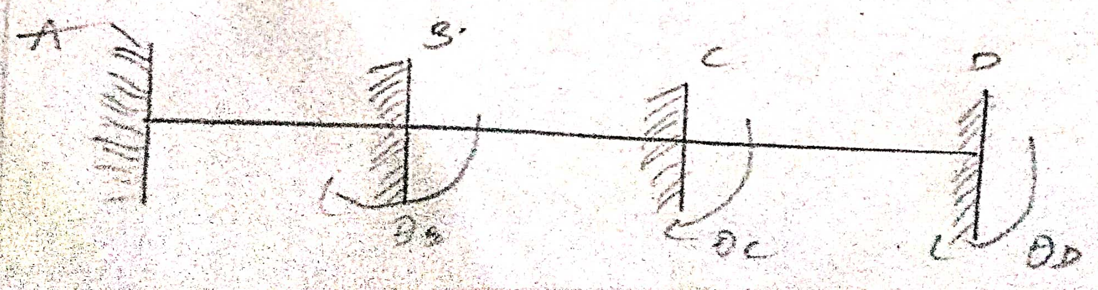
$$M_{FCB} = +\frac{WL}{8} = \frac{+48\times 4}{8} = +24\ \text{kNm}$$

for Span CD,

$$M_{FCD} = -\frac{Wab^2}{L^2} = \frac{-62\times 3\times 2^2}{5^2} = -29.76\ \text{kNm}$$

$$M_{FDC} = +\frac{Wa^2b}{L^2} = \frac{+62\times 3^2\times 2}{5^2} = +44.64\ \text{kNm}$$

Step-3:- force matrix



$[P] = P_i - P_L$

$P_{1i} = P_{2i} = P_{3i} = 0$

$P_{1L} = M_{FBA} + M_{FBC}$

$= 6.667 + -24 = -17.33 \text{ kNm}$

$P_{2L} = M_{FCB} + M_{FCD}$

$= +24 + -29.76 = -5.76 \text{ kNm}$

$P_{3L} = M_{FDC}$

$= +44.64 \text{ kNm.}$

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 0 - (-17.33) \\ 0 - (-5.76) \\ 0 - 44.64 \end{bmatrix} = \begin{bmatrix} +17.33 \\ +5.76 \\ -44.64 \end{bmatrix}.$$

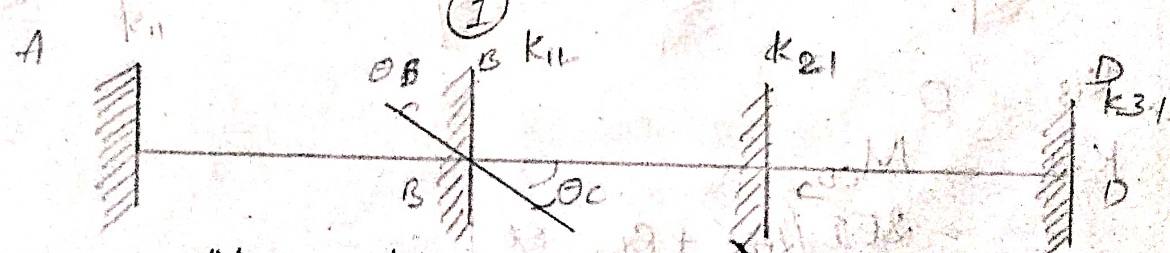
<u>Step - 4 :-</u> Displacement matrix

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix}$$

<u>Step - 5 :-</u> Stiffness matrix

$$[k] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$$

<u>Case - i</u> :- unit displacement @ 1st coordinate



$K_{11} = M_{BA} + M_{BC}$

$= \frac{2EI}{L}\left(2\theta_B + \cancel{\theta_A} - \cancel{\frac{3\delta}{L}}\right) + \frac{2EI}{L}\left(2\theta_B + \cancel{\theta_C} - \cancel{\frac{3\delta}{L}}\right)$ $(\theta_B = 1)$

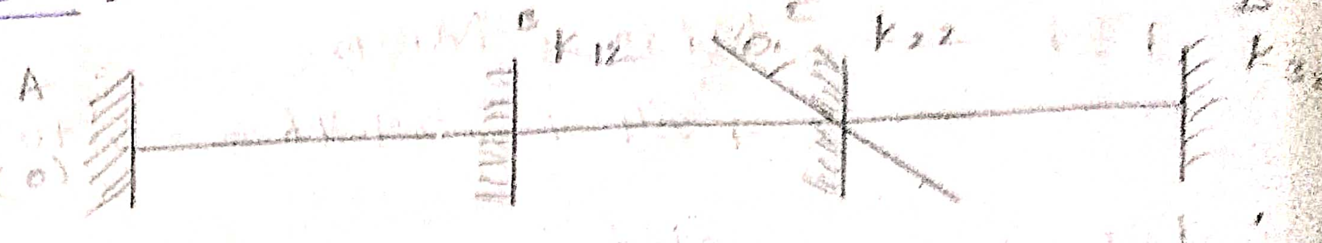
$= \frac{4EI}{L}\theta_B + \frac{4EI}{L}\theta_B = \frac{4EI}{8} + \frac{4EI}{4} = 3EI$

$$K_{21} = M_{CB} = \frac{2EI}{L}\left[2\theta_c + \cancel{\theta_b} - \cancel{\frac{3\delta}{L}}\right]$$

$$= \frac{2EI}{L} = \frac{2EI}{4} = 0.5EI$$

$K_{31} = 0$

Case-ii :- unit displacement at 2nd coordinate



$$K_{12} = M_{BC}$$

$$= \frac{2EI}{L}\left(2\theta_b + \theta_c - \cancel{\frac{3\delta}{L}}\right)$$

$$= \frac{2EI}{4}(\theta_c) = \frac{2EI}{4} = 0.5EI$$

$$K_{22} = M_{CB} + M_{CD} \qquad (\theta_c = 1)$$

$$= \frac{2EI}{L}\left(2\theta_c + \cancel{\theta_b} - \cancel{\frac{3\delta}{L}}\right) + \frac{2EI}{L}\left(2\theta_c + \cancel{\theta_D} - \cancel{\frac{3\delta}{L}}\right)$$

$$= \frac{4EI}{L} + \frac{4EI}{L} = \frac{4EI}{4} + \frac{4EI}{5}$$

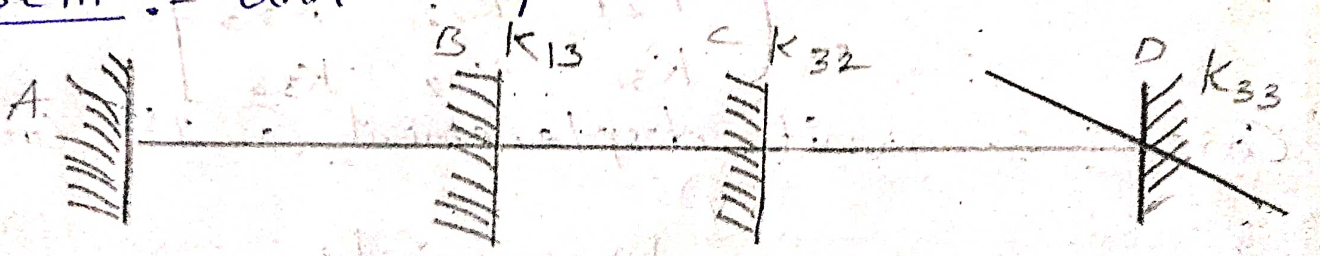
$$= 1.8EI$$

$$K_{32} = M_{DC}$$

$$= \frac{2EI}{L}\left(\cancel{2\theta_b} + \theta_c - \cancel{\frac{3\delta}{L}}\right)$$

$$= \frac{2EI}{L} = \frac{2EI}{5} = 0.4EI.$$

Case-iii :- unit displacement at 3rd coordinate.



$K_{13} = 0$

$$K_{32} = M_{CD}$$

$$= \frac{2EI}{L}\left(\cancel{2\theta_c} + \theta_D - \cancel{\frac{3\delta}{L}}\right)$$

$$= \frac{2EI}{5} = 0.4EI$$

$K_{33} = M_{DC}$

$= \frac{2EI}{L}\left(2\theta_D + \theta_C - \frac{3\delta}{L}\right) \quad (\theta_D = 1)$

$= \frac{4EI}{L} + \frac{4EI}{5} = 0.8EI.$

stiffness matrix $= K = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$

$= \begin{bmatrix} 3EI & 0.5EI & 0 \\ 0.5EI & 1.8EI & 0.4EI \\ 0 & 0.4EI & 0.8EI \end{bmatrix}$

$= EI \begin{bmatrix} 3 & 0.5 & 0 \\ 0.5 & 1.8 & 0.4 \\ 0 & 0.4 & 0.8 \end{bmatrix}$

step-6 :- stiffness equ (or) Displacement equ

$(K)\,[\Delta] = (P_i - P_L)$

$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$

$EI \begin{bmatrix} 3 & 0.5 & 0 \\ 0.5 & 1.8 & 0.4 \\ 0 & 0.4 & 0.8 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \begin{pmatrix} 17.33 \\ 5.76 \\ -44.64 \end{pmatrix}$

$\begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 3 & 0.5 & 0 \\ 0.5 & 1.8 & 0.4 \\ 0 & 0.4 & 0.8 \end{bmatrix} \begin{bmatrix} 17.33 \\ 5.76 \\ -44.64 \end{bmatrix}$

$= \frac{1}{EI} \begin{bmatrix} 1.28 & -0.4 & +0.2 \\ -0.4 & +2.4 & -1.2 \\ +0.2 & +1.2 & +5.15 \end{bmatrix} \times \frac{1}{3(1.8\times0.8-0.4\times0. \ldots) - 0.5(0.5\times0.8-0.4\times \ldots) + 0(0.5\times0.4)-(1.8\times0 \ldots)} \begin{pmatrix} 17.33 \\ 5.76 \\ -44.64 \end{pmatrix}$

$= \frac{1}{EI} \begin{pmatrix} 1.28 & -0.4 & +0.2 \\ -0.4 & +2.4 & -1.2 \\ 0.2 & -1.2 & 5.15 \end{pmatrix} \times 0.274 \begin{pmatrix} 17.33 \\ 5.76 \\ -44.64 \end{pmatrix}$

$= \frac{1}{EI} \begin{pmatrix} 0.350 & -0.109 & 0.0548 \\ -0.109 & +0.657 & -0.328 \\ +0.0548 & -0.328 & +1.411 \end{pmatrix} \begin{pmatrix} +17.33 \\ +5.76 \\ -44.64 \end{pmatrix}$

$\begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \frac{1}{EI} \begin{pmatrix} +2.991 \\ +16.537 \\ -63.926 \end{pmatrix}$

Step - 7 :-

final end moments

$M_{RAB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)$

$\phantom{M_{RAB}} \quad (0)$

$= -6.667 + \frac{2EI}{2}(2\times \cancel{2.991} + 2.991)$

$= -3.676\ kNm$

$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right)$

$= +6.667 + \frac{2EI}{2}(2\times 2.991)$

$= +12.649\ kNm$

$M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\delta}{L}\right)$

$= -24 + \frac{2EI}{4}(2\times 2.991 + 16.537)$

$= -12.74\ kNm$

$M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\delta}{L}\right)$

$= +24 + \frac{2EI}{4}(2\times 16.537 + 2.991)$

$= +42.032\ kNm$

$M_{CD} = M_{FCD} + \frac{2EI}{L}\left(2\theta_C + \theta_D - \frac{3\delta}{L}\right)$

$= -29.76 + \frac{2EI}{5}(2\times 16.537 + -63.926)$

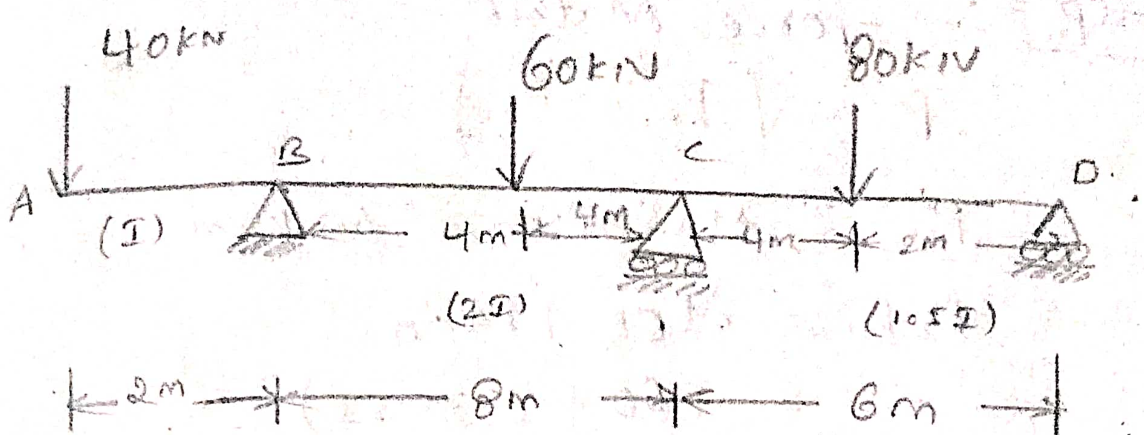
$= -42.100\ kNm$

$M_{DC} = M_{FDC} + \frac{2EI}{L}\left(2\theta_D + \theta_C - \frac{3\delta}{L}\right)$

$= +44.64 + \frac{2EI}{5}(2\times -63.926 + 16.537)$

$= +0.144\ kNm.$

4)



Analyze the Continuous beam as shown in fig. by using the displacement method or stiffness method?

. Overhanging portion being the determinate portion it may be dropped from the indeterminate structure analysis. after accounting it's effect on the rest of the beam. Hence, the beam considered for the analysis as shown in fig. in which the cantilever moment 80 kNm is the final moment at the end B. The final force vector is

$$P_i = \begin{bmatrix} -80 \\ 0 \\ 0 \end{bmatrix}$$

Step-1:- Degree of kinematic indeterminacy

$$D_F = 1+1+1 \qquad K = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$$

$$\therefore D_F = 3$$

Step-2 :- fixed end Moments

$$M_{FBC} = -\frac{WL}{8} = -\frac{60\times 8}{8} = -60\,kNm$$

$$M_{FCB} = +60\,kNm.$$

$$M_{FCD} = -\frac{wab^2}{L^2} = -\frac{80\times 4\times 2^2}{6^2} = -35.55\,kNm$$

$$M_{FDC} = +\frac{wa^2b}{L^2} = +\frac{80\times 4^2\times 2}{6^2} = +71.11\,kNm$$

Step 3 :- Force Matrix

$P = [P_1 \quad P_2]$

$P_{1L} = M_{BA} + M_{BC} = 60 \text{ kNm}$

$P_{2L} = M_{CB} + M_{CD}$

$= +60 + (-35.55)$

$= +24.45 \text{ kNm}$

$P_{3L} = M_{DC}$

$= +71.11 \text{ kNm}.$

$$[P] = \begin{bmatrix} -80+60 \\ 0-(24.45) \\ 0-(71.11) \end{bmatrix} = \begin{bmatrix} -20 \\ -24.45 \\ -71.11 \end{bmatrix}$$

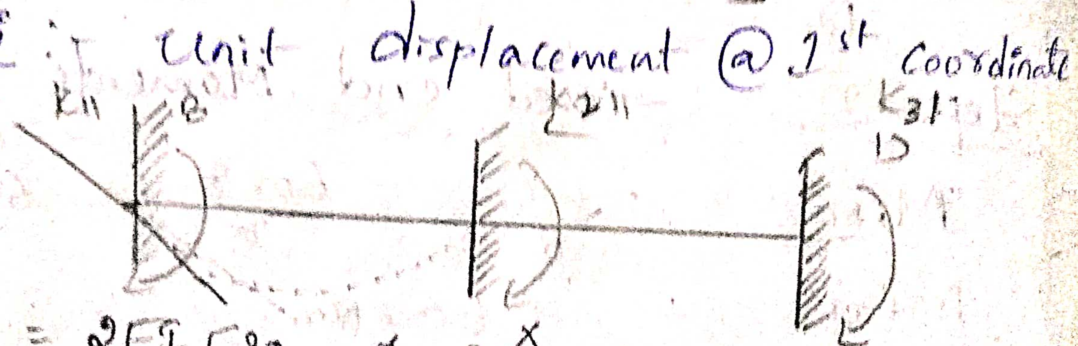
Step 4 :- Displacement matrix on factor.

$$\begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

Step 5 :- Stiffness matrix.

$$[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$$

Case 1 :- Unit displacement @ 1st coordinate



$$K_{11} = \frac{2EI}{L}\left(2\theta_B + \cancel{\theta_C} - \cancel{\frac{3\delta}{L}}\right) + \cancel{\frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\delta}{L}\right)}$$

$$= \frac{2E(2I)}{8}(2\theta_B) = \frac{4EI}{8} \times 2 = \frac{8EI}{8} = EI$$

$$K_{21} = \frac{2EI}{L}\left(2\theta_c + \theta_b - \frac{3\delta}{L}\right]$$

$$= \frac{2EI}{L}(\theta_b)$$

$$= \frac{2E(2I)}{8}(\theta_b) = \frac{4EI}{8_2} = 0.5EI$$

$$K_{31} = 0.$$

Case – ii :- unit displacement @ 2nd coordinate.

$$K_{12} = \frac{2EI}{L}\left[\cancel{2\theta_b} + \theta_c - \cancel{\frac{3\delta}{L}}\right]$$

$$= \frac{2EI}{L}(\theta_c) = \frac{2E(2I)}{8}(\theta_c) = \frac{4EI}{8_2} = 0.5EI$$

$$K_{22} = M_{CB} + M_{CD}$$

$$= \frac{2EI}{L}\left(2\theta_c + \cancel{\theta_b} - \cancel{\frac{3\delta}{L}}\right) + \frac{2EI}{L}\left(2\theta_c + \cancel{\theta_D} - \cancel{\frac{3\delta}{L}}\right)$$

$$= \frac{2EI}{L}(2\theta_c) + \frac{2EI}{L}(2\theta_c]$$

$$= \frac{4E(2I)}{8} + \frac{4E(1.5I)}{6}$$

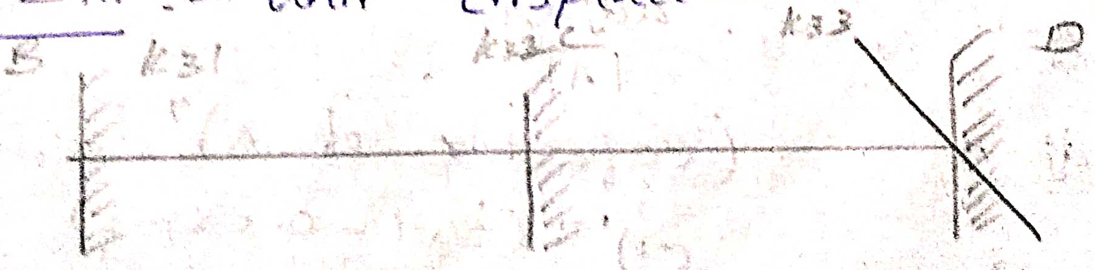
$$= \frac{8EI}{8} + \frac{6EI}{6} = EI + EI = 2EI$$

$$K_{23} = M_{DC}$$

$$= \frac{2EI}{L}\left(\cancel{2\theta_D} + \theta_c - \frac{3\delta}{L}\right)$$

$$= \frac{2EI}{L}(\theta_c) = \frac{3EI}{6_2} = 0.5EI.$$

Case – iii :- unit displacement @ 3rd coordinate



$$K_{31} = 0$$

$$K_{32} = \frac{2EI}{L}\left[2\cancel{\theta_c} + \theta_D\right] = \frac{2E(1.5I)}{6} = \frac{3EI}{6_2} = 0.5EI$$

$$K_{33} = \frac{2EI}{L}\left[2\theta_D + \cancel{\theta_c}\right] = \frac{4EI}{6} = \frac{4(1.5I)E}{6}$$

$$= \frac{6EI}{6} = EI$$

Step - 6 :- Compatability Equation

$$[k][\Delta] = [P - P_L]$$

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \begin{bmatrix} -20 \\ +24.45 \\ -71.11 \end{bmatrix}$$

$$\begin{bmatrix} EI & 0.5EI & 0 \\ 0.5EI & 2EI & 0.5EI \\ 0 & 0.5EI & EI \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \begin{bmatrix} -20 \\ -24.45 \\ -71.11 \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} EI \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 2 & 0.5 \\ 0 & 0.5 & 1 \end{bmatrix} = \begin{bmatrix} -20 \\ -24.45 \\ -71.11 \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 2 & 0.5 \\ 0 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} -20 \\ -24.45 \\ -71.11 \end{bmatrix}$$

$$= \frac{1}{EI}\Big(1[2\times1 - 0.5\times0.5] - 0.5(0.5\times1 - 0.5\times0) + 0(0.5\times0.5 - 0\ldots$$
$$0.5[0.5\times1 - 0.5\times0] - 2(1\times1 - 0\times0) + 0.5(1\times0.5 - 0\ldots$$
$$0(0.5\times0.5 - 2\times0) - 0.5(1\times0.5 - 0.5\times0) + 1(1\times2 - 0.5\ldots$$

$$= \frac{1}{EI} \begin{bmatrix} 1.75 & -0.25 & 0 \\ -0.25 & -2 & 0.25 \\ 0 & -0.25 & 1.75 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$\text{adj } A = (\text{Cofactor of } A)^T$$

$$a_{11} = (-1)^{1+1}(2\times1 - 0.5\times0.5)$$

$$= 1.75$$

$$a_{12} = (-1)^{1+2} \times (0.5\times1 - 0\times0.5)$$

$$= -0.5$$

$$a_{13} = (-1)^{1+3} \times (0.5\times0.5 - 2\times0] = 0.25$$

$a_{21} = (-1)^{2+1} (0.5\times1 - 0.5\times0) = 0.5$

$a_{22} = (-1)^{2+2} (1\times1 - 0) = 1$

$a_{23} = (-1)^{2+3} (1\times0.5 - 0.5\times0) = -0.5$

$a_{31} = (-1)^{3+1} (0.5\times0.5 - 0) = 0.25$

$a_{32} = -0.5$

$a_{33} = (-1)^{3+3} (1\times2 - 0.5\times0.5) = 1.75$

$$\begin{bmatrix}\theta_B\\ \theta_C\\ \theta_D\end{bmatrix} = \frac{1}{EI} \times \begin{pmatrix} 1.75 & -0.5 & 0.25 \\ -0.5 & 1 & -0.5 \\ 0.25 & -0.25 & 1.75 \end{pmatrix} \times \frac{1}{1(2\times1-0.5\times0.5)-0.5(0.5\times1-0.5\times0)} \begin{pmatrix}-20\\-24.45\\-71.11\end{pmatrix}$$

$$= \frac{1}{EI}\begin{pmatrix} 1.75 & -0.5 & 0.25 \\ -0.5 & 1 & -0.5 \\ 0.25 & -0.25 & 1.75 \end{pmatrix} \times 0.667 \begin{pmatrix}-20\\-24.45\\-71.11\end{pmatrix}$$

$$= \frac{1}{EI}\begin{pmatrix} 1.155 & -0.33 & 0.16 \\ -0.33 & 0.66 & -0.33 \\ +0.16 & -0.33 & 1.155 \end{pmatrix} \times \begin{pmatrix}-20\\-24.45\\-71.11\end{pmatrix}$$

$$= \frac{1}{EI}\begin{bmatrix} 1.155\times-20 + -0.33\times-24.45 + 0.16\times-71.11 \\ -0.33\times-20 + 0.66\times-24.45 + -0.33\times-71.11 \\ +0.16\times-20 + -0.33\times-24.45 + 1.155\times-71.11 \end{bmatrix}$$

$$\begin{pmatrix}\theta_B\\ \theta_C\\ \theta_D\end{pmatrix} = \frac{1}{EI}\begin{pmatrix}-26.409\\+13.409\\-77.263\end{pmatrix}$$

Step - 7 :- final end moments

$M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_b + \theta_c - \frac{3\delta}{L}\right)$

$= -60 + \frac{2E(2I)}{8}\left(2\times -\frac{26.409}{EI} + \frac{13.929}{EI}\right)$

$= -79.44$ KNm

$M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_c + \theta_b - \frac{3\delta}{L}\right)$

$= +60 + \frac{2E(2I)}{8}(2\times 13.929 + -26.409)$

$= +60.724$ KNm

$M_{CD} = M_{FCD} + \frac{2EI}{L}\left(2\theta_C + \theta_D - \frac{3\delta}{L}\right)$

$= -35.55 + \frac{2E(1.5I)}{6}\left(2\times 13.929 + -77.263\right)$

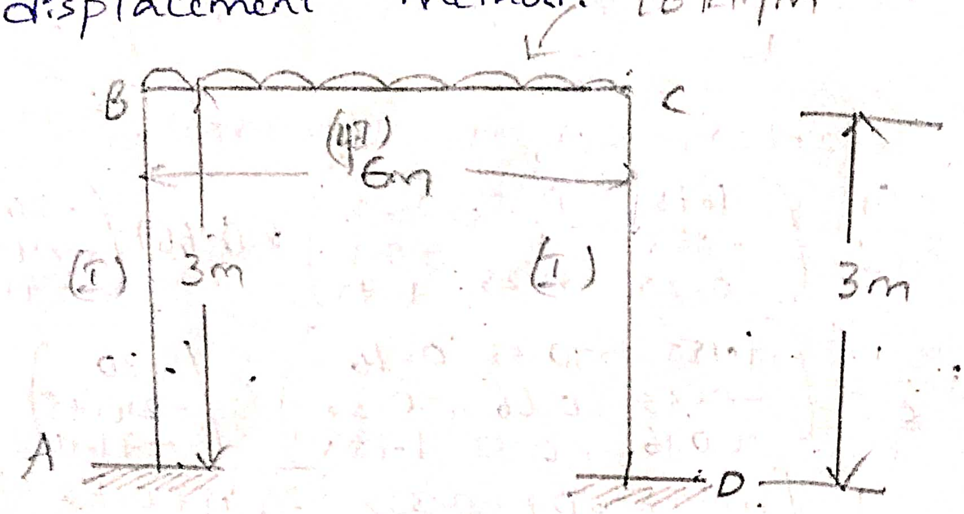
$= -60.252$ kNm

$M_{DC} = M_{FDC} + \frac{2EI}{L}\left(2\theta_D + \theta_C - \frac{3\delta}{L}\right)$

$= 71.11 + \frac{2E(1.5I)}{6}\left(2\times -77.263 + 13.929\right)$

$= +0.8115$ kNm.

* Analyze the portal frame by using the displacement method.



Sol:- **Step - 1 :-** Degree of kinematic indeterminancy

$D_K = 0+1+1+0 = 2.$

Step - 2 :- fixed end moments

$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$

$M_{FBC} = -\frac{WL^2}{\cancel{8}12} = -\frac{10\times 6^2}{\cancel{8}12} = \cancel{-45\text{ kNm}}$ -30 kNm

$M_{FCB} = +\frac{WL^2}{\cancel{8}12} = +\frac{10\times 6^2}{\cancel{8}12} = \cancel{+45\text{ kNm}}$ $+30$ kNm.

Step - 3 :- force matrix

$P = [P_i - P_L]$

$P_{1i} = P_{2i} = 0$

$P_{1L} = M_{FBA} + M_{FBC}$

$= 0 + -30 = -30$

$P_2 L = M_{FCB} + M_{FCD}$

$= +30 + 0 = +30$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} \cancel{P_1 - P_L} \end{bmatrix} = \begin{bmatrix} 0-(-30) \\ 0-(+30) \end{bmatrix} = \begin{bmatrix} +30 \\ -30 \end{bmatrix}$$

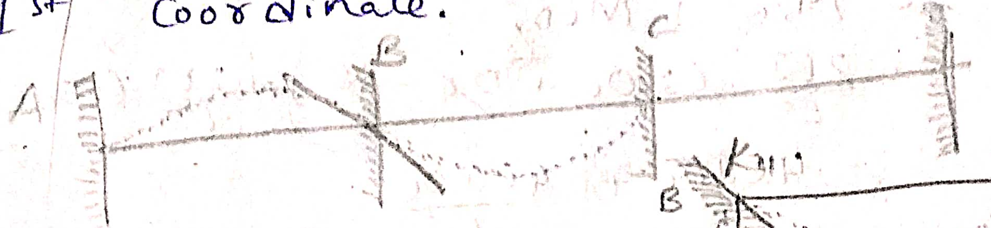
Step - 4 :- Displacement matrix

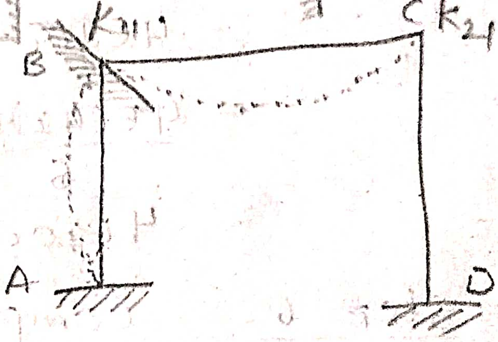
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix}$$

Step - 5 :- Stiffness matrix

$$[K] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

Case - i :- unit displacement @ 1st coordinate.





$K_{11} = M_{BA} + M_{BC}$

$= \frac{2EI}{L}(2\theta_B + \cancel{\theta_A} - \cancel{\frac{3\delta}{L}}) + \frac{2EI}{L}(2\theta_B + \cancel{\theta_C} - \cancel{\frac{3\delta}{L}})$

$= \frac{2EI}{L}(2\theta_B) + \frac{2EI}{L}(2\theta_B)$ $[\because \theta_B = 1]$

$= \frac{4EI}{L} + \frac{4EI}{L} = \frac{4EI}{\cancel{3}} + \frac{4E(4I)}{6} = \cancel{2EI}\ \frac{4EI \cdot 3 + 16EI}{6}$

$= 4EI\theta_B \longrightarrow (1)$

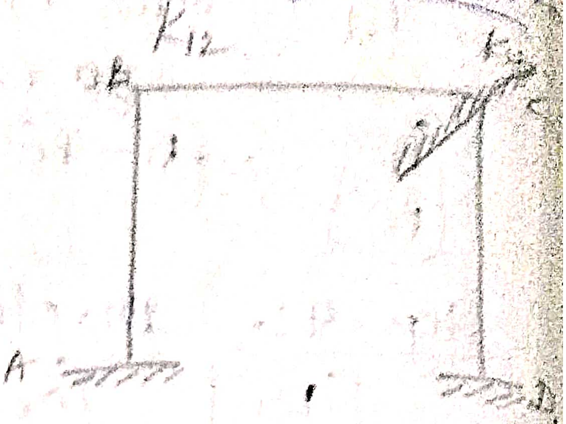
$K_{21} = M_{CB}$ $\boxed{\theta_C = 0}$ $\boxed{\theta_B = 1}$

$= \frac{2EI}{L}(2\theta_C + \theta_B - \cancel{\frac{3\delta}{L}})$

$= \frac{2E(4)}{6}\theta_B = \cancel{\frac{2EI}{6}}{}_3 = \frac{EI}{3} = 0.33\,EI$

$= \frac{8EI}{6} = 1.33\,EI\theta_B \longrightarrow (2)$

Unit displacement @ 2nd Coordinate:



$K_{12} = M_{BC}$

$= \frac{2EI}{L}\left(2\theta_B + \theta_C - \cancel{\frac{3\delta}{L}}\right)$

$= \frac{4EI\,\theta_C}{L}$

$= \frac{\cancel{4}E\cancel{I}(4I)\,\theta_C}{6\;3}$ $\cancel{= \frac{16\,EI\theta_C}{4} = 4EI\theta_C} \rightarrow 1$

$= 1.33\,EI\theta_C \longrightarrow (1)$

$K_{21} = M_{CD}$

$= \frac{2EI}{L}\left(2\theta_C + \cancel{\theta_D} - \cancel{\frac{3\delta}{L}}\right)$

$= \frac{4EI\theta_C}{L} = \frac{4EI\,\theta_C}{3} = 1.33EI\theta_C \longrightarrow (2)$

$= M_{CB} + M_{CD}$

$= \frac{2EI}{L}\left(2\theta_C + \cancel{\theta_B} - \cancel{\frac{3\delta}{L}}\right) + \frac{2EI}{L}\left(2\theta_C + \cancel{\theta_D} - \cancel{\frac{3\delta}{L}}\right)$

$= \frac{4EI\theta_C}{L} + \frac{4EI\theta_C}{L}$

$= \frac{4E(4I)\theta_C}{6} + \frac{4EI\theta_C}{3}$

$= 4EI\theta_C \longrightarrow (2)$

Step - 6 :- Compatability equation

$[K][\Delta] = [P_i - P_L]$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} +30 \\ -30 \end{bmatrix}$$

$$\begin{bmatrix} 4EI & 1.33EI \\ 1.33EI & 4EI \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} +30 \\ -30 \end{bmatrix}$$

$$EI\begin{bmatrix} 4 & 1.33 \\ 1.33 & 4 \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} +30 \\ -30 \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI}\left(\frac{1}{16 - (1.33 \times 1.33)}\right)\begin{bmatrix} 4 & -1.33 \\ -1.33 & 4 \end{bmatrix}\begin{bmatrix} 30 \\ -30 \end{bmatrix}$$

$$= \frac{1}{EI}(0.0702)\begin{bmatrix} 4 & -1.33 \\ -1.33 & 4 \end{bmatrix}\begin{bmatrix} +30 \\ -30 \end{bmatrix}$$

$$= \frac{1}{EI}\begin{pmatrix} 0.280 & -0.093 \\ -0.093 & 0.280 \end{pmatrix}\begin{pmatrix} 30 \\ -30 \end{pmatrix}$$

$$= \frac{1}{EI}\begin{bmatrix} 0.280 \times 30 + -0.093 \times -30 \\ 0.093 \times 30 + 0.280 \times -30 \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI}\begin{pmatrix} 11.19 \\ -11.19 \end{pmatrix}$$

$$\theta_B = \frac{+11.19}{EI}, \quad \theta_C = \frac{-11.19}{EI}$$

Step - 6 :- final end Moments

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)$$

$$= 0 + \frac{2EI}{3}\left(2(0) + 11.19\right) = +7.46\,kNm$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right)$$

$$= 0 + \frac{2EI}{3}\left(2 \times \frac{11.19}{EI} + 0\right)$$

$$= +14.92\,kN.m$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\delta}{L}\right)$$

$$= -30 + \frac{2E(4I)}{6}\left(2 \times \frac{11.19}{EI} + \frac{-11.19}{EI}\right)$$

$$= -15.08\,KNm.$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\delta}{L}\right)$$

$$= +30 + \frac{2E(4I)}{6}\left(-2 \times \frac{11.19}{EI} + \frac{+11.19}{EI}\right)$$

$$= +\cancel{44.92}\ 15.08\,KNm$$

$$M_{CD} = M_{FCD} + \frac{2EI}{L}\left(2\theta_C + \theta_D - \frac{3\delta}{L}\right)$$

$$= 0 + \frac{2EI}{3}\left(2 \times \frac{-11.19}{EI} + 0\right)$$

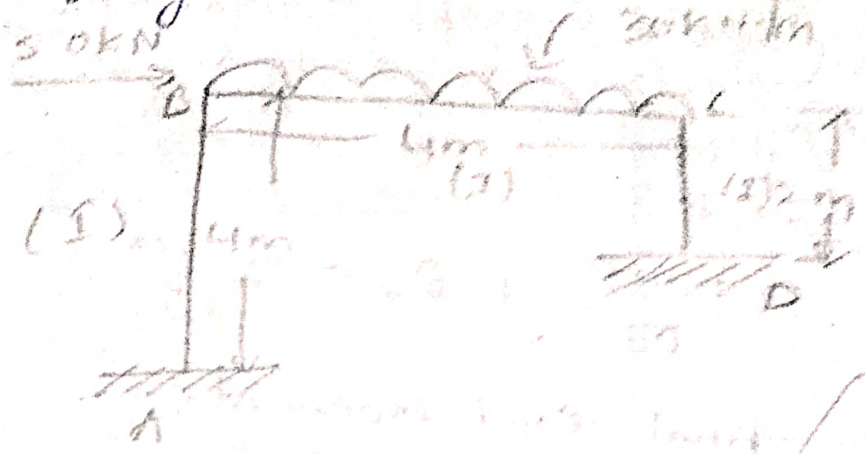
$$= -14.92\,KNm$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L}\left(2\theta_D + \theta_C - \frac{3\delta}{L}\right)$$

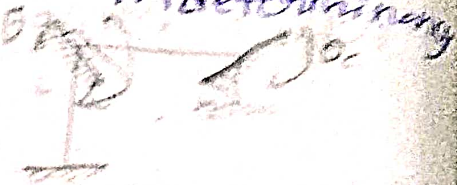
$$= 0 + \frac{2EI}{3}\left(2(0) + \frac{-11.19}{EI}\right)$$

$$= -7.46\,KNm.$$

* Analyze the portal frame as shown in fig. by using the stiffness matrix method?



Sol: **Step-1:-** Degree of kinematic indeterminacy



$$D_k = 0+1+1+1+0 = 3$$

Step-2:- ~~fixed end moments~~ Displacement Matrix

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} \Delta \\ \theta_B \\ \theta_C \end{bmatrix}$$

Step-3:- fixed end moments

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$$

$$M_{FBC} = -\frac{wL^2}{12} = -\frac{30(4)^2}{12} = -40\,kNm$$

$$M_{FCB} = +\frac{wL^2}{12} = +\frac{30(4)^2}{12} = +40\,kNm$$

Step-4:- force matrix

$$P = (P_i - P_L)$$

$P_{1i} = 50$ $\quad P_{1L} = 0$

$P_{2i} = 0$ $\quad P_{2L} = M_{BA} + M_{BC} = 0 - 40 = -40$

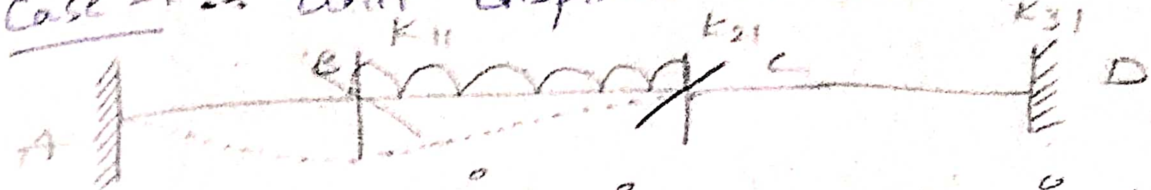
$P_{3i} = 0$ $\quad P_{3L} = M_{CB} + M_{CD} = 40 + 0 = 40$

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 50-0 \\ 0+40 \\ 0-40 \end{bmatrix} = \begin{bmatrix} 50 \\ 40 \\ -40 \end{bmatrix}$$

Step-5 :- Stiffness Matrix

$$[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$$

Case-i :- unit displacement @ 1st Coordinate



$$K_{11} = \frac{2EI}{L}\left(2\overset{0}{\theta_B} + \overset{0}{\theta_A} - \frac{3\delta}{L}\right) + \frac{2EI}{L}\left(2\overset{0}{\theta_B} + \overset{0}{\theta_C} - \frac{3\delta}{L}\right)$$

$$= \frac{2EI}{L}\left[-\frac{3\delta}{L}\right] + \frac{2EI}{L}\left(-\frac{3\delta}{L}\right)$$

$$= \frac{2EI}{4}\left(-\frac{3(1)}{4}\right) + \frac{2EI}{4}\left(-\frac{3(1)}{4}\right)$$

$$= -0.75\,EI$$

$$\frac{2EI}{L}\left(\frac{6EI\delta}{L^2}\right) + \frac{2EI}{L}\left(\frac{6EI\delta}{L^2}\right)$$

$$= \frac{12EI\delta}{L^3} + \frac{12EI\delta}{L^3}$$

$$= \frac{12EI(1)}{4^3} + \frac{12EI(1)}{4^3}$$

$$= +0.375EI\delta \longrightarrow (1)$$

K_{21} =

Unit - IV :- Flexibility matrix Method

The Systematic development of consistent deformation method in the matrix form has lead to flexibility matrix method. In these method is an analytical method it has similarities with numerical methods such as finite element method.

→ In the way it use matrices to solve big Linear Systems but the theory is analytical.

→ The matrix method of structural analysis is based on the elastic theory.

→ where it can be assumed that most structures behave like complex elastic springs and the load displacement, relationship is linear.

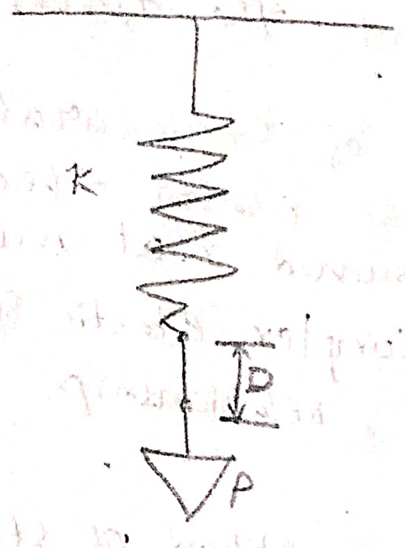
The classical method of structural analysis the two methods that from the direct basis for the matrix method or conjugate beam method & slope deflection method.

The basic unknowns are redunant forces has to first identify basic ~~in~~determinate structures and there by identify redunant forces. The number of redunant forces is equal to degree of static indeterminancy the displacement in basic determinate structure due to a given loading and redunant forces are found and the consistenty conditions.

are formed. The equations developed are in the matrix form since computers can be used for solving simultaneous equations there is no limit for the members of equations that can be handled. This method is known as force, compatability, flexibility method.

Concept of flexibility matrix :-

Consider an elastic spring element subjected to a load "p" causing a displacement "D" which is directly proportional to the load "p" as shown in fig.



$\cancel{P = \frac{1}{fd}}$

$\cancel{P = kd}$

$D \propto P$

$D = fP$

f = flexibility

$P = \frac{1}{f} \cdot D$

$P = k \cdot D$

where, k = stiffness $= \frac{1}{f}$

from equ ①, if $P = 1$ and then $D = f$.
Hence flexibility is defined as displacement D which is produced are developed an account of unit force. $P = 1$.

from equ ②, if $P = 1$ then $P = k$.
Hence stiffness k is defined as the force P developed an account of unit displacement.

$$D = [\overset{\to \text{force}}{f} \cdot P] \to \text{Loads.}$$

Flexibility matrix formation :-

It a structure has n number of coordinates it's displacement response to the forces is represented by

$$\delta = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} & \cdots & \delta_{1n} \\ \delta_{21} & \delta_{22} & \delta_{23} & \cdots & \delta_{2n} \\ \delta_{31} & \delta_{32} & \delta_{33} & \cdots & \delta_{3n} \end{bmatrix}$$

Above equations can be expressed as in the matrix form as

$\cancel{P = \Delta}$

$$[\Delta] = \Delta_L + [\delta][P]$$

$$[P] = [\delta]^{-1}\,[(\Delta) - (\Delta_L)]$$

The steps involved in the flexibility matrix method.

Step - 1 :- Determine the degree of static. ~~Kinematic~~ indeterminancy

Step - 2 :- Choose the redudance.

Step - 3 :- Assign the coordinates to the redunance force directions

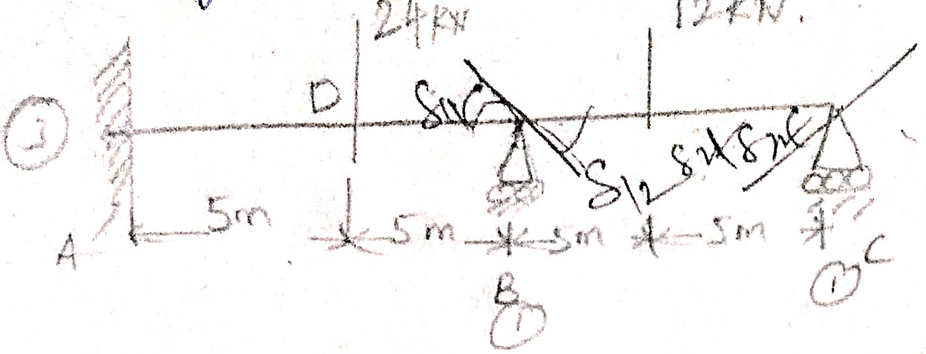
Step - 4 :- Remove restraints to redudance forces. on the get basic ~~redunance~~ determinate structures.

Step - 5 :- Determine the deflections in coordinate directions due to given loading conditions in the basic determinate structures.

Step - 6 :- Determine the flexibility matrix

Step - 7 :- Applying the compatability conditions.

2) Analyse the Continuous beam as shown in fig by using flexibility matrix method ?

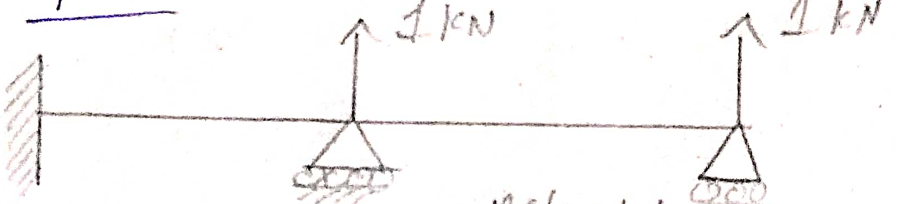


Sol:- **Step-1 :- Degree of static indeterminacy**

$$D_s = R - 3$$
$$= 5 - 3 = 2$$

$$(\delta) = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}$$

Step-2 :- Identify Redundancy factors.-

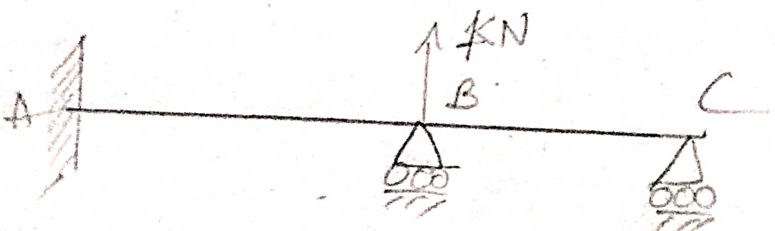


Released structure

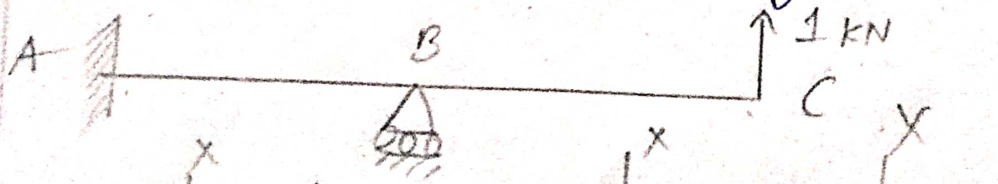
Step-3 :- flexibility matrix.

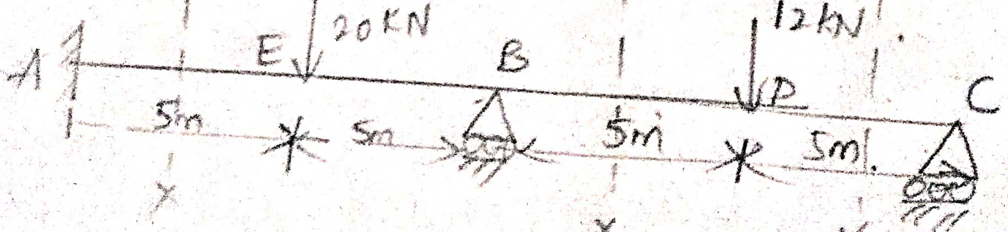
$$[F][\delta] = [\Delta] - [\Delta L]$$

Case -1 :- unit force applying 1st coordinate



Case -2 :- unit force applying 2nd coordinate.





δ or θ = $\int_0^l \frac{M m\, dx}{EI}$

portion	CD	DB	BF	EA
origin	C	D	B	E
Limits	(0,5)	(0,5)	(0,5)	(0,5)
M	0	$-12x$	$-12(5+x)$	$-12(10+x)+24x$
m_1	0	0	$1x$	$(x+5)$
m_2	x	$(x+5)$	$(x+10)$	$(x+15)$

Where,

M = Moment due to pure loading

m_1 = Moment due to unit force at 1^{st} coordinate without any given loading

m_2 = Moment due to unit force at 2nd coordinate without any given loading.

$$\delta_{11} = \int_0^l m_1 m_1 \frac{dx}{EI}$$

$$= \int_0^5 0 \cdot \frac{dx}{EI} + \int_0^5 0 \cdot \frac{dx}{EI} + \int_0^5 x^2 \cdot \frac{dx}{EI} + \int_0^5 (x+5)(x+5) \frac{dx}{EI}$$

$$= 0 + 0 + \int_0^5 x^2 \cdot \frac{dx}{EI} + \int_0^5 (x+5)^2 \cdot \frac{dx}{EI}$$

$$= \left[\frac{x^3}{3}\right]_0^5 \cdot \frac{dx}{EI} + \int_0^5 x^2 + 2(5)(x) + 25 \cdot \frac{dx}{EI}$$

$$= \frac{1}{EI}[41.67] + \left(\frac{x^3}{3} + 10 \cdot \frac{x^2}{2} + x \times 25\right]_0^5$$

$$= \frac{1}{EI}[41.67) + [41.67 + 125 + 125]$$

$$= \frac{333.33}{EI} \longrightarrow (1)$$

$$\delta_{12} = \int_0^l M_1 M_2 \cdot dx .$$

$$= \int_0^5 x(x+10)\frac{dx}{EI} + \int_0^5 (x+5)(x+5).\frac{dx}{EI}$$

$$= \int_0^5 x^2 + 10x \cdot \frac{dx}{EI} + \int_0^5 (x^2+15x+5x+75)\frac{dx}{EI}$$

$$= \int_0^5 x^2 + 10x \cdot \frac{dx}{EI} + \int_0^5 x^2 + 20x + 75 \cdot \frac{dx}{EI}$$

$$= \frac{1}{EI}\left(\frac{x^3}{3} + 10\frac{x^2}{2}\right]_0^5 + \left(\frac{x^3}{3} + 20\frac{x^2}{2} + 75\right]_0^5$$

$$= \frac{1}{EI}[41.67 + 125] + (41.67 + 250 + 75)$$

$$= \frac{833.33}{EI} \rightarrow (2)$$

$$\delta_{21} = \int_0^l M_2 \cdot M_1 \cdot dx$$

$$= \int_0^5 0 + \int_0^5 0 + \int_0^5 (x+10)(1 \times x) + \int_0^5 (x+15)(x+5)$$

$$= \int_0^5 (x^2+10x) + \int_0^5 (x^2+5x+15x+75)\frac{dx}{EI}.$$

$$= \int_0^5 (x^2+10x)\frac{dx}{EI} + \int_0^5 (x^2+20x+75)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left(\frac{x^3}{3} + \frac{10x^2}{2}\right]_0^5 + \left(\frac{x^3}{3} + 20 \cdot \frac{x^2}{2} + 75\right)_0^5$$

$$= \frac{1}{EI}[41.67 + 125) + (41.67 + 250 + 75)$$

$$= \frac{833.33}{EI} \rightarrow (3)$$

$$\delta_{22} = \int_0^l M_2 \cdot M_2 \cdot dx$$

$$= \int_0^5 x^2 + \int_0^5 (x+5)^2 + \int_0^5 (x+10)^2 +$$

$$= \int_0 (x+10)^2 \cancel{(x+5)^2} \cdot dx$$

$$= \int_0^5 \frac{x^3}{3} + \int_0^5 (x^2+25+2x(5)) + \int_0^5 (x^2+100+2(10)x) +$$

$$\int_0^5 (x^2+225+2(15)(x))$$

$$= 41.67 + \left(\frac{x^3}{3} + 25x + 10 \cdot \frac{x^2}{2}\right]_0^5 + \left(\frac{x^3}{3} + 100x + 20 \cdot \frac{x^2}{2}\right)_0^5$$

$$+ \left(\frac{x^3}{3}\right) + 225x + 30 \cdot \frac{x^2}{2}\Big)_0^5$$

$$= \cancel{(41.67 + 191.67 + 391.67 + 641.67)}$$

$$= 41.67 + 179.66 + 791.66 + 1541.67$$

$$= \frac{2554.16}{EI} \rightarrow (4)$$

$$\Delta_{1L} = \int_0 M \cdot m_1 \cdot \frac{dx}{EI}$$

$$= \int_0^5 0 + \int_0^5 -12x \times 0 + \int_0^5 (-12)(5+x) \cdot x +$$

$$\int_0^5 -12(5+x)(x+10) + \int_0^5 -12(10+x) - 24(x)(x+15)$$

$$= \int_0^5 (-60 + -12x)x + \int_0^5 (-60 + -12x)(x+10) +$$

$$\int_0^5 (-120 + -12x) - 24(x)(x+15)$$

$$= \frac{-1}{EI}\int_0^5 60x + 12x^2 - \int_0^5 12x^2 + 120x + 24x^2 + 60x + 600 + 120x)$$

$$= \frac{-1}{EI}\int_0^5 (60x + 12x^2 - \int_0^5 36x^2 + 300x + 600$$

$$= \frac{-1}{EI}\left(60\frac{x^2}{2} + 12\left(\frac{x^3}{3}\right)\right)_0^5 - \left(36 \cdot \frac{x^3}{3} + 300 \times \frac{x^2}{2} + 600x\right)_0^5$$

$$= -1250 - \cancel{4787.5} \; 8250 = \frac{9500}{EI}$$

$$\Delta_{21} = \int_0^{L} M_1 M_2 \frac{dx}{EI}$$

$$= \frac{-25750}{EI}$$

Step-6 :- Compatability equation

$$[F][\delta] = \{\Delta - \Delta_L\}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \begin{bmatrix} 333.33 & 833.33 \\ 833.33 & 2666.66 \end{bmatrix} = \begin{bmatrix} 0 - (9500) \\ 0 - (25750) \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \begin{bmatrix} 333.33 & 833.33 \\ 833.33 & 2666.66 \end{bmatrix} = \begin{bmatrix} 9500 \\ 25750 \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{pmatrix} 333.33 & 833.33 \\ 833.33 & 2666.66 \end{pmatrix} \left(\frac{1}{333.33 \times 2666.66 - 833.33 \times 833.33} \right)$$

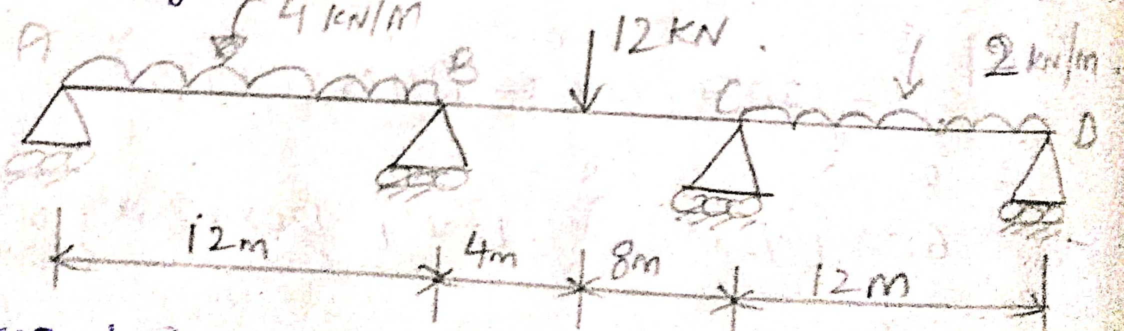
$$= \cancel{\begin{bmatrix} 333.33 & 833.33 \\ 833.33 & 2666.66 \end{bmatrix} (5.143 \times 10^{-6})}$$

$$= \begin{pmatrix} 2666.66 & -833.33 \\ -833.33 & 333.33 \end{pmatrix} (5.143 \times 10^{-6}) \begin{pmatrix} 9500 \\ 25750 \end{pmatrix}$$

$$= \frac{1}{EI} (5.143 \times 10^{-6}) \begin{pmatrix} 3875022.5 \\ 666612.5 \end{pmatrix}$$

$$= \frac{1}{EI} \begin{pmatrix} 19.929 \\ 3.428 \end{pmatrix}$$

3) Analyze the continuous beam as shown in fig. by using flexibility method or force method



Step-1 :- Degree of static indeterminancy

$R = 1 + 1 + 1 + 1$

$R = 4$

$$D_s = R - 2$$
$$= 4 - 2 = 2.$$

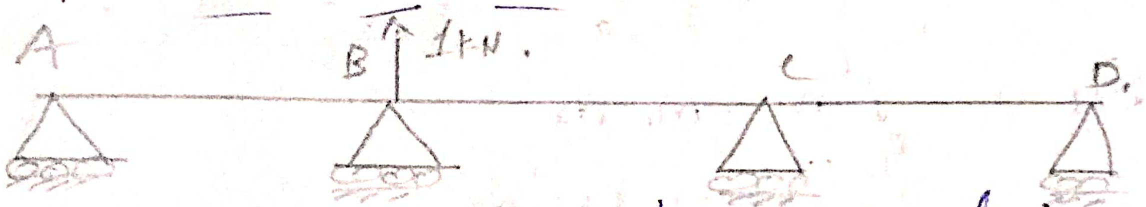
$$\delta = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}$$

Step- 2 :- Identify redundancy factor.

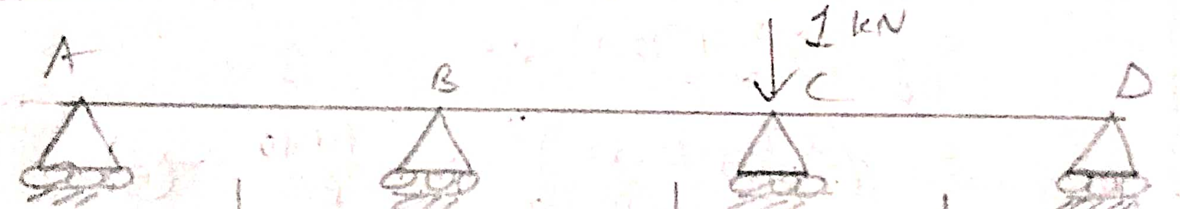
Step-3 :- flexibility matrix

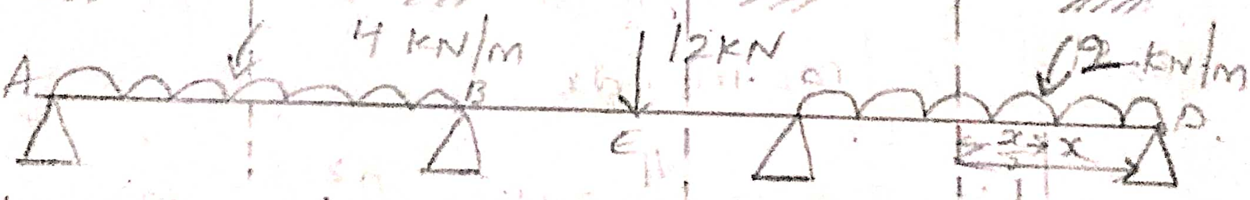
$$[\delta][P] = [\Delta - \Delta_L]$$

Step- unit force at 1st coordinate :-



unit force at 2nd coordinate :-





Portion	DC	CE	EB	BA
Origin	D	C	E	B
Limits	(0, 12)	(0, 8)	(0, 4)	(0, 12)
M	$-2 \times x \times \frac{x}{2} = -x^2$	$-2 \times 12 \times (x + \frac{12}{2}) = -24(x+6)$	$-2 \times 12 (x + \frac{12}{2} + 8) = -24(x+14) + 2x$	$-2 \times 12 (x + \frac{12}{2} + 8 + 4) + 12(x+4) + 8x \cdot \frac{x}{2}$
m_1	0	0	0	x
m_2	0	x	$(x+8)$	$(x+12)$

$$\delta_{11} = \int_0^L M_1 M_1 \cdot dx$$

$$= \int_0^{12} x^2 \cdot dx$$

$$= \left(\frac{x^3}{3}\right)_0^{12} = \frac{12^3}{3} = \frac{576}{EI} \rightarrow (1)$$

$$\delta_{12} = \int_0^L M_1 m_2 \cdot dx$$

$$= \int_0^{12} (x)(x+12) \cdot dx$$

$$= \int_0^{12} (x^2 + 12x) \cdot dx$$

$$= \left(\frac{x^3}{3} + 12 \cdot \frac{x^2}{2}\right)_0^{12} = \frac{1440}{EI} \rightarrow (2)$$

$$\delta_{21} = \int_0^L m_2 \cdot m_1 \cdot dx$$

$$= \int_0^{12} (x+12) \cdot (x) \cdot dx$$

$$= \int_0^{12} (x^2 + 12x) \cdot dx$$

$$= \left(\frac{x^3}{3} + 12 \cdot \frac{x^2}{2}\right)_0^{12} = \frac{1440}{EI} \rightarrow (3)$$

$$\delta_{22} = \int_0^L m_2 \cdot m_2 \cdot dx$$

$$= \int_0^8 x^2 + \int_0^{4} (x+8)^2 + \int_0^{12} (x+12)^2$$

$$= \int_0^8 x^2 + \int_0^4 (x^2 + 64 + 16x) + \int_0^{12} (x^2 + 144 + 24x)$$

$$= \left(\frac{x^3}{3}\right)_0^8 + \left(\frac{x^3}{3} + 64x + 16\frac{x^2}{2}\right)_0^4 + \left(\frac{x^3}{3} + 144x + 24\frac{x^2}{2}\right)_0^{12}$$

$$= 170.667 + \cancel{341.33}\ 5589.33 + \cancel{3168}\ 4032$$

$$= \cancel{\frac{3679.997}{EI}} \rightarrow (4)$$

$$= \frac{9791.997}{EI} \rightarrow (4)$$

$$\Delta_{1L} = \int_0 M \cdot m_1 \cdot \frac{dx}{EI}$$

$$= \int_0^{12} (-2 \times 12 \left(x + \frac{12}{2} + 8 + 4\right] + 12 (x+4)$$

$$+ 8 \cdot x \cdot \frac{x}{2} \times x \, dx$$

$$= \int_0^{12} -24\left(x + \frac{12}{2} + 8 + 4\right) + 12x + 48 + 8 \cdot \frac{x^2}{2} \cdot x \cdot dx$$

$$= \int_0^{12} (-24x + -144 + -192 + -96) + 12x + 48 + \frac{8x^3}{3} dx$$

$$= \left(-24 \cdot \frac{x^2}{2} + -144x + -192x + -96x\right) + 12\frac{x^2}{2} + 48 \cdot x$$

$$8 \cdot \frac{1}{3} \cdot \frac{x^4}{4} \Big)_0^{12}$$

$$= -6912 + 1440 + 13824$$

$$= \frac{8352}{EI}$$

$$= -24(x+8) + 12(x+4) + 2x^2.$$

$$= -(24x + 432 + 12x + 48 + 2x^2)$$

$$= -(36x + 2x^2 + 480)$$

$$= \int_0^{12} (2x^2 + 36x + 480) \, x \cdot dx$$

$$= \int_0^{12} 2x^3 + 36x^2 + 480x$$

$$= \left(2 \cdot \frac{x^4}{4} + 36 \times \frac{x^3}{3} + 480 \cdot \frac{x^2}{2}\right)_0^{12} = \frac{65664}{EI}$$

$$\Delta_{2L} = \int_0 M \cdot m_2 \cdot \frac{dx}{EI}$$

$$= \int_0^8 -24(x+6)(x) + \int_0^4 -2 \times 12 \left(x + \frac{12}{2} + 8\right) - 24(x+4$$

$$-12x)(x+8 + -2 \times 12 \left(x + \frac{12}{2} + 8 + 4\right) + 12(x+4)$$

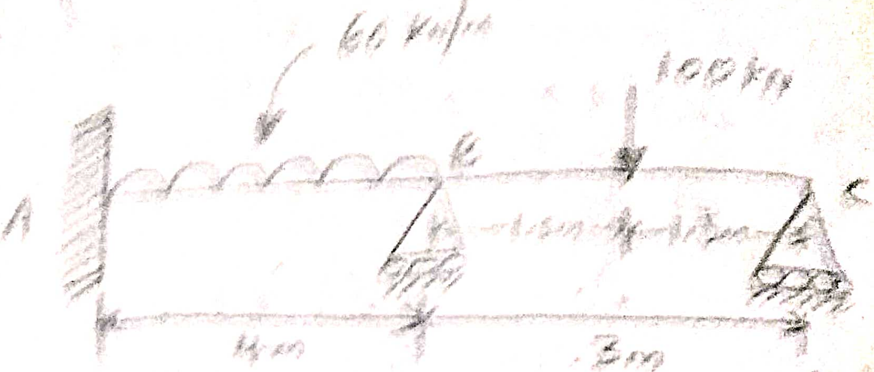
$$-x^2 \times \frac{x \times x}{2} \quad (x+12)$$

$$= -(24x + 144)x + -24(x+14) - 24x + 96 - 288x)$$

$$x + 8 + -24(x+18) + 12x + 48 - \frac{x^4}{2}(x+12)$$

Flexibility (or) force (or) Displacement

(1) Analyse the continuous beam as in fig by using the flexibility matrix method (or) force method as shown fig.



Step-1 :- Degree of ~~kinematic~~ static indeterminacy

$D_s = R - 3$

$x = 0$
$y = 0$
$z = 0$

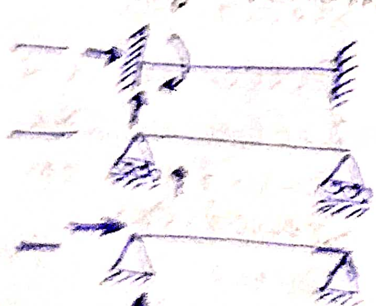
Total no. of reactions $= 3 + 1 + 1$
$= 5$

Note :- Degree of static indeterminacy =
$D_s = R - 3$

where,

r = Total no. of reactions

Fixed support = 3
Roller condition = 1
Hinged condition = 2.



Equilibrium conditions (or) equations

$\Sigma M = 0$, $\Sigma H = 0$, $\Sigma V = 0$.

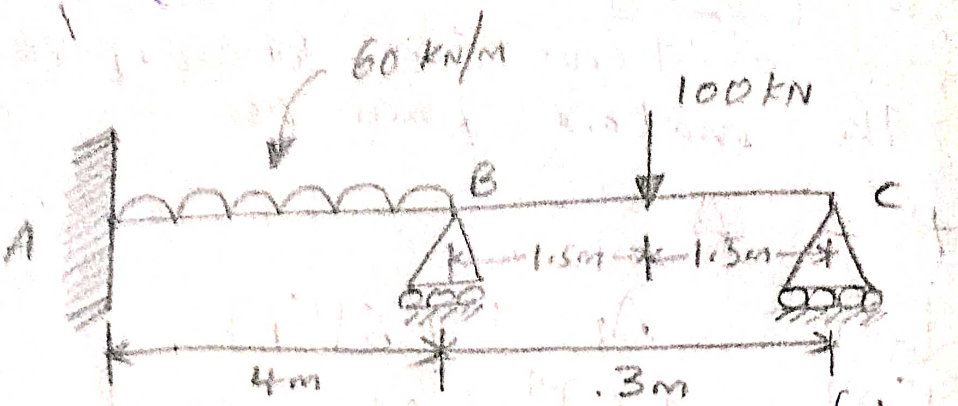
$\Sigma F_x, \Sigma F_y, \Sigma M_z = 3$

$D_s = 5 - 3$

$D_s = 2$.

$$\delta = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}$$

Analyse the continuous beam as shown in fig by using the flexibility matrix method (or) force method as shown in fig.



Step-1 :- Degree of ~~Kinematic~~ Static indeterminancy

$D_s = R - 3$

$x = 0$
$y = 0$
$z = 0$.

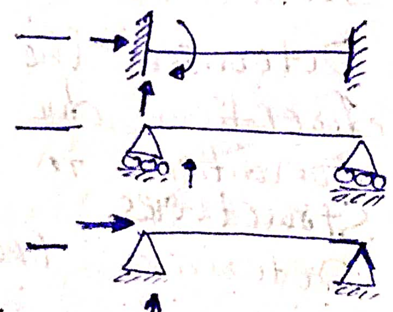
Total no. of reactions $= 3 + 1 + 1$
$= 5$

Note :- Degree of static indeterminancy =
$D_s = R - 3$

Where,

r = Total No. of reactions

Fixed Support = 3
Roller Condition = 1
Hinged Condition = 2.



Equilibrium Conditions (or) equations

$\Sigma M = 0$, $\Sigma H = 0$. $\Sigma V = 0$.

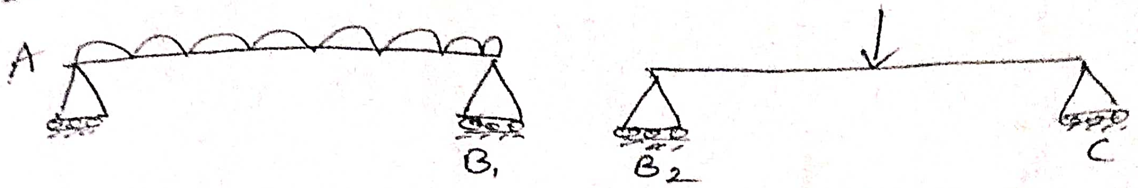
Σf_x, Σf_y, ΣM_z = ~~X~~ 3

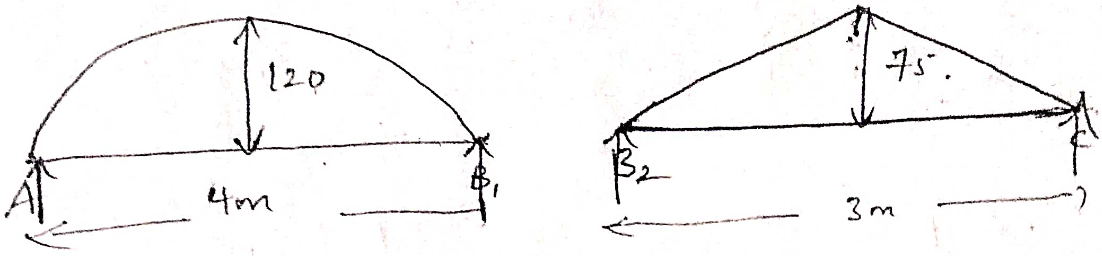
$D_s = 5 - 3$

$D_s = 2$.

$$\delta = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}$$

Step - 2 :- Degree Of Redudance





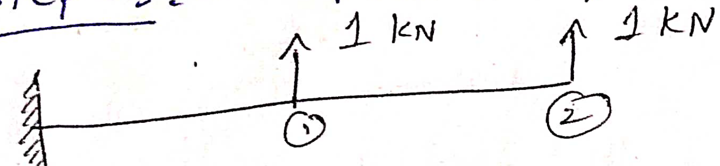
$$\Delta_{1L} = \frac{\text{Total Load}}{2}$$

$$\Delta_{1L} = \frac{\frac{2}{3} \times 4 \times \frac{120}{EI}}{2} = \frac{160}{EI}$$

$$\Delta_{2L} = \frac{\frac{2}{3} \times 4 \times \frac{120}{EI}}{2} + \frac{\frac{1}{2} \times 3 \times \frac{75}{EI}}{2}$$

$$= 216.25$$

Step -3 :- flexibility matrix.



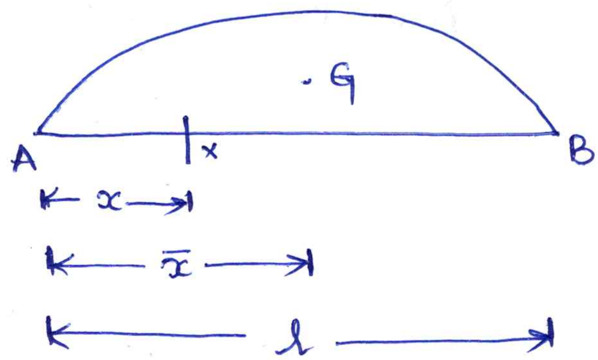
δ_{11} =

Conjugate Beam Method

Consider the bending moment diagram for a beam AB with supports at A and B.

taking A as origin.

Consider any section X distant x from A.



Let M be the bending moment at X.

$$\Rightarrow \quad EI \frac{d^2y}{dx^2} = M$$

Integrating from 0 to x we have

$$\int_0^x EI \frac{d^2y}{dx^2} = \int_0^x M$$

$$EI \left[\frac{dy}{dx}\right]_0^x = \int_0^x M\,dx$$

$$EI\,(i_x - i_a) = a_x$$

where a_x is the area of the BM diagram from 0 to x.

$$\therefore \quad i_x = i_a + \frac{a_x}{EI} \qquad \text{—— (a)}$$

Consider again $EI \dfrac{d^2y}{dx^2} = M$

multiplying by x we get

$$\Rightarrow \quad EI\, x \frac{d^2y}{dx^2} = Mx$$

integrating for the whole range from A to B, we get

$$\int_0^l EI x \frac{d^2y}{dx^2} = \int_0^l Mx\, dx$$

$$\Rightarrow \quad EI \int_0^l x \frac{d^2y}{dx^2} = \int_0^l Mx\, dx.$$

$\therefore \quad EI \left[x \frac{dy}{dx} - y \right]_0^l =$ moment of the whole BM diagram about A

$= a\bar{x}.$

where a is the area of BMD from A to B.

$\bar{x}$ - centroidal distance of this diagram from A.

Applying Boundary conditions

at $x = 0, \quad y = 0$

$x = l \quad y = 0 \quad \frac{dy}{dx} = i_b$

Substituting the limits, we get

$$EI l\, i_b = a\bar{x} \qquad \therefore \quad i_b = \frac{a\bar{x}}{EIl} \quad \text{—— (b)}$$

IIIly $i_a = -\dfrac{a(l-\bar{x})}{EIl}$ ——— (c)

again $EI\dfrac{d^2y}{dx^2} = M$

$\therefore EIx\dfrac{d^2y}{dx^2} = Mx$

integrating from 0 to x

$$EI\int_0^x x\frac{d^2y}{dx^2}\,dx = \int_0^x Mx\,dx$$

$$\therefore EI\left[x\frac{dy}{dx} - y\right]_0^x = a_x\bar{x}_x$$

= moment of area of BMD between A and ~~x~~ about A

where a_x - area of BMD between A and X

$\bar{x}_x$ - Centroidal distance of diagram from A.

substituting the limits we get

$$EI(x.i_x - y) = a_x\bar{x}_x$$

$$\Rightarrow y = xi_x - \frac{a_x\bar{x}_x}{EI}$$

but from eq (a) $i_x = i_a + \dfrac{a_x}{EI}$

$$\therefore y = x\left[i_a + \frac{a_x}{EI}\right] - \frac{a_x\bar{x}_x}{EI}$$

$$= xi_a + \frac{a_x}{EI}(x - \bar{x}_x)$$

It can be realized that

$a_x(x - \bar{x}_x)$ is the moment of the area a_x

about X.

$$\therefore\ y = x i_a + \frac{\text{moment of } a_x \text{ about } X}{EI}$$

<u>Summary of results:</u>

$$i_a = -\frac{a(l-\bar{x})}{EIl} = -\frac{a}{EI}\left[\frac{l-\bar{x}}{l}\right] \quad \text{--- (1)}$$

$$i_b = \frac{a\bar{x}}{EIl} = \frac{a}{EI}\left[\frac{\bar{x}}{l}\right] \quad \text{--- (2)}$$

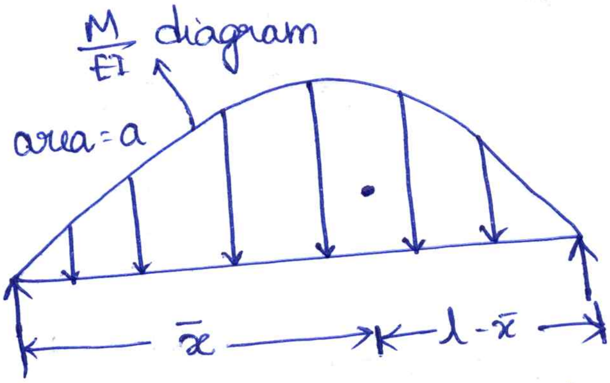
$$i_x = i_a + \frac{a_x}{EI} \quad \text{--- (3)}$$

$$y = x i_a + \text{Moment of } \left(\frac{a_x}{EI}\right) \text{ about } X \quad \text{--- (4)}$$

Suppose we consider the $\frac{M}{EI}$ diagram as a load diagram for the beam.

the reactions at the left end and right supports would be i_a, i_b

$\frac{a(l-\bar{x})}{l}$, $\frac{a\bar{x}}{l}$ numerically.



$i_a = \frac{a(l-\bar{x})}{l}$ $\qquad$ $i_b = \frac{a\bar{x}}{l}$

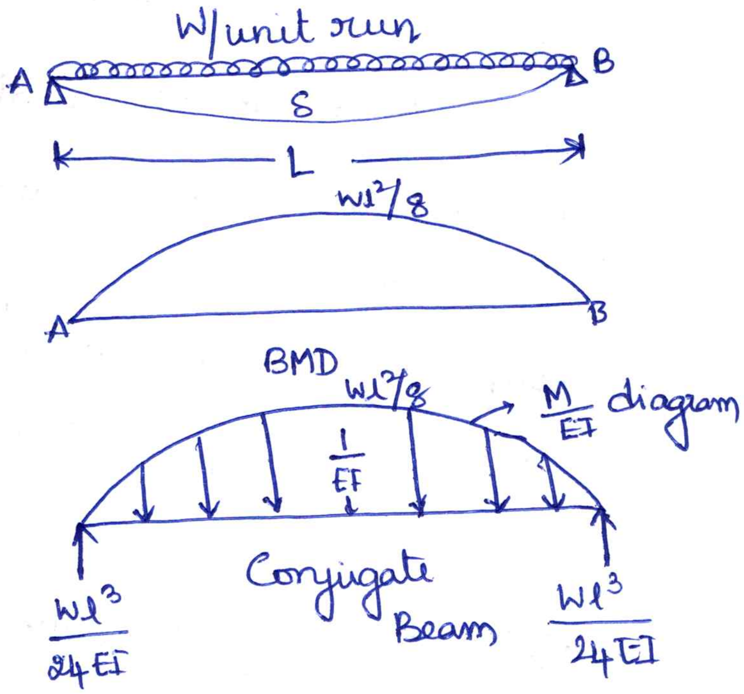
An imaginary beam for which the load diagram is the $\frac{M}{EI}$ diagram is called the <u>Conjugate Beam</u>.

The following observations are made from the summary of the results obtained above

* The slope at any section of the given beam is equal to the shear force at the corresponding section of the conjugate beam.
* The deflection at any section for the given beam is equal to the bending moment at corresponding section of the conjugate beam.

1 P-1] A simply supported beam of length l carries a udl of w per unit run over the whole span. Find the slope at each end and the deflection at the centre.

Sol:-



total load on conjugate beam = area of the load diagram on the conjugate beam.

$$= \frac{2}{3} \times \frac{wl^2}{8EI} \times l$$

$$= \frac{wl^3}{12EI}.$$

Reaction at each end of the conjugate beam

$$= SF \; \frac{wl^3}{24EI}.$$

Slope at each end of the beam

$$= \text{SF at each end of Conjugate Beam} = \frac{wl^3}{24EI}.$$

Deflection at the centre of the given beam

= BM at the centre of the conjugate beam.

$$= \left(R_A \times \frac{l}{2}\right) - \left[\left(\begin{array}{c}\text{Parabola area}\\ \text{(half)}\end{array}\right) \times \text{CG distance}\right]$$

$$= \left[\frac{Wl^3}{24EI} \times \frac{l}{2}\right] - \left[\left(\frac{1}{2} \times \frac{2}{3} \times l \times \frac{Wl^2}{8EI}\right) \times \frac{3}{8} \times \frac{l}{2}\right]$$

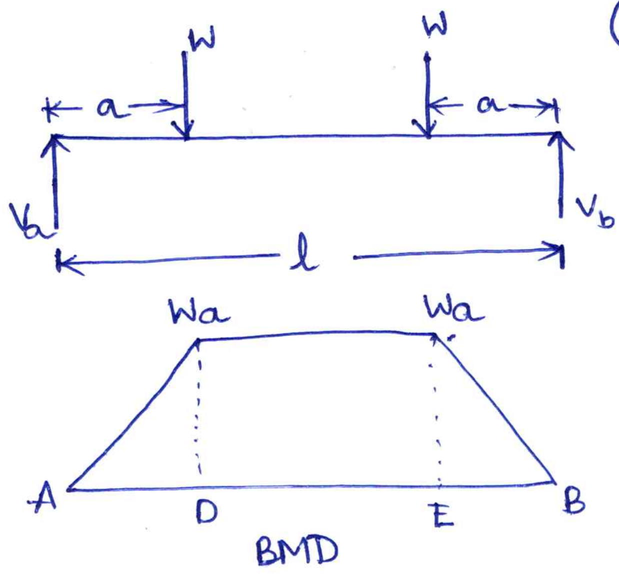
$$= \left[\frac{Wl^3}{24EI} \times \frac{l}{2}\right] - \left[\left(\frac{Wl^3}{24EI}\right) \times \frac{3}{8} \times \frac{L}{2}\right]$$

$$= \frac{Wl^3}{384EI}(8-3) = \frac{5Wl^3}{384\,EI}.$$

P-2] A beam of length l is simply supported at the ends and carries a concentrated load W at a distance 'a' from each end. Find the slope at each end and under each load. Find also the deflection under each load and at the centre.

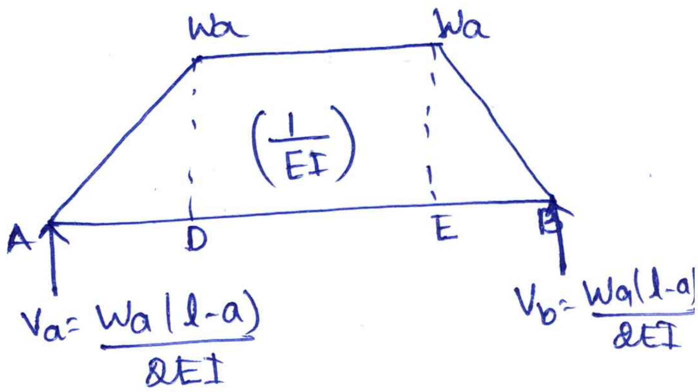
Sol:- The $\frac{M}{EI}$ diagram is practically the same as the M diagram except in the case of $\frac{M}{EI}$ diagram, we should imagine a thickness of $\frac{1}{EI}$.

Total load on the beam (conjugate) = area of load digrm.



$\therefore$ Reaction at each support for the conjugate beam = $\frac{1}{2}$ The total load

$$= \frac{Wa(l-a)}{2EI}$$



Slope at each end of the given beam

= Shear force at each end of the conjugate beam

$$= \frac{Wa(l-a)}{2EI}.$$

Slope under each load of the given beam

= Shear force under each load of conjugate beam

$$= \frac{Wa(l-a)}{2EI} - \frac{1}{2} a \frac{Wa}{EI}$$

$$= \frac{Wa(l-a)}{2EI} - \frac{Wa^2}{2EI} = \frac{Wa(l-2a)}{2EI}.$$

Deflection under each load of the given beam

= BM under each load of the Conjugate beam

$$= \frac{Wa(l-a)}{2EI} \times a - \frac{1}{2} \times a \times \frac{Wa}{EI} \times \frac{a}{3}$$

$$= \frac{Wa^2(l-a)}{2EI} - \frac{Wa^3}{6EI}$$

$$= \frac{Wa^2}{6EI}(3l-3a-a) = \frac{Wa^2(3l-4a)}{6EI}$$

Deflection at the centre of the given beam

= BM at the centre of the conjugate beam

$$= \frac{Wa(l-a)}{2EI} \times \frac{l}{2} - \frac{1}{2} \times a \times Wa \times \frac{1}{EI}\left[\frac{l}{2} - \frac{2}{3}a\right]$$

$$- \frac{(l-2a)Wa}{2EI} \times \frac{(l-2a)}{4}$$

$$= \frac{Wal(l-a)}{4EI} - \frac{Wa^2}{12EI}(3l-4a)$$

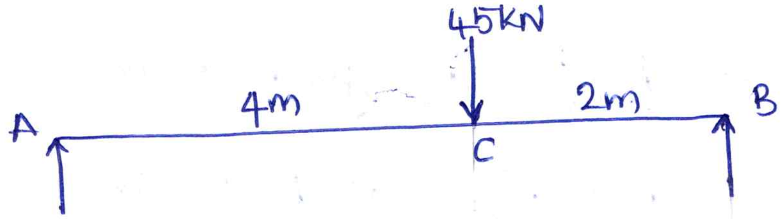
$$- \frac{Wa(l-2a)^2}{8EI}$$

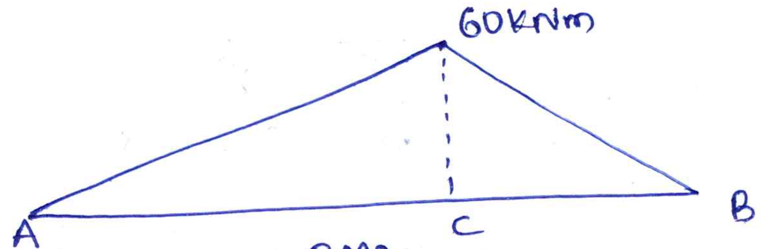
$$= \frac{Wa}{24EI} \cdot \left[6l(l-a) - 2a(3l-4a) - 3(l-2a)^2\right]$$

$$= \frac{Wa}{24EI}(3l^2 - 4a^2).$$

P-3] A beam AB of span 6m carries a point load of 45 KN at a distance of 4m from the left end A. Find (i) the slope at A (ii) the deflection under the load (iii) section where the deflection is maximum (iv) maximum deflection.

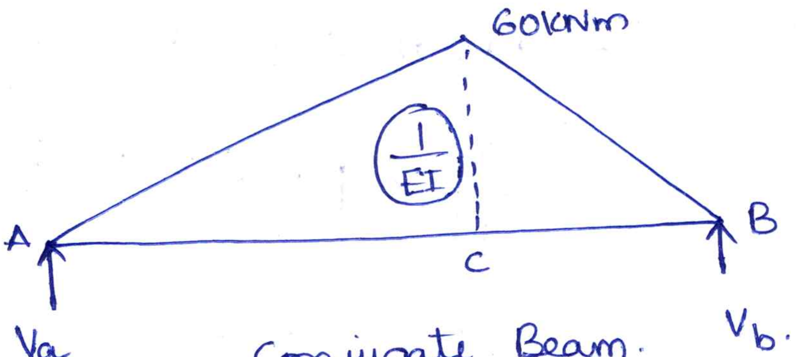
Sol:





BMD

$$\frac{Wab}{l} = \frac{45 \times 4 \times 2}{6} = 60$$



Conjugate Beam.

Total load on conjugate beam

$$= \frac{1}{2} \times 6 \times \frac{60}{EI} = \frac{180}{EI}$$

Let V_a and V_b be the reactions at A and B for the conjugate beam.

$$V_b \times 6 = \frac{180}{EI} \times \frac{10}{3}$$

$$V_b = \frac{100}{EI}$$

$$V_a = \frac{180}{EI} - \frac{100}{EI} = \frac{80}{EI}$$

Slope at A for the given beam

$= $ shear force at A for the Conjugate beam

$$= \frac{80}{EI} = \frac{80 \times 10^6}{200 \times 8.325 \times 10^7} = 4.8048 \times 10^{-3} \text{ rad}$$

deflection at C for the given beam

$=$ BM at C for the conjugate beam

$$= \frac{80}{EI} \times 4 - \frac{1}{2} \times 4 \times \frac{60}{EI} \times \frac{4}{3}$$

$$= \frac{160}{EI} = \frac{160}{200 \times 8.325 \times 10^7}$$

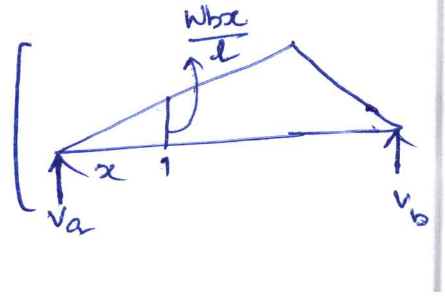
$$= \underline{9.6 \text{ mm}}$$

Maximum deflection:

This is the section is where the maximum BM occurs for the conjugate beam. This is the section where the SF is zero for the conjugate beam.

Let this section be at a distance x from A. Equating SF for the conjugate beam to zero

$$\frac{80}{EI} - \frac{1}{2} \times x \times \frac{15x}{EI} = 0$$



$$7.5x^2 = 80 \Rightarrow \underline{x = 3.26 \text{m}}$$

Maximum deflection for given beam

$= \text{BM at } x$

$= 3.26 \text{ m for Conjugate beam}$

$$= \frac{80}{EI} \times 3.266 - \frac{1}{2}(3.26)(15 \times 3.26)\left[\frac{3.26}{3}\right]$$

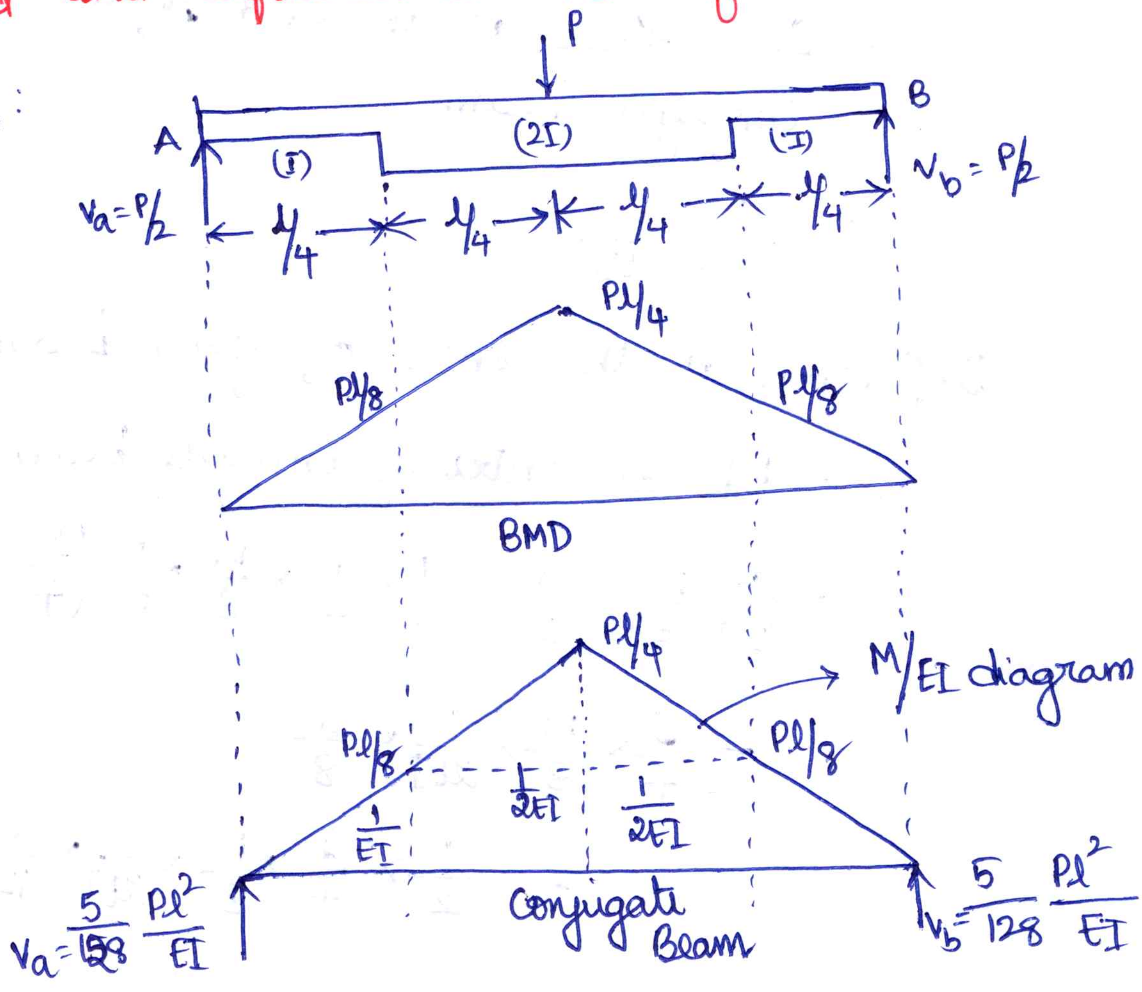
$$= \frac{261.28}{EI} - \frac{87.09}{EI}$$

$$= \frac{174.19 \times 10^{9}}{200 \times 8.325 \times 10^{7}}$$

$= \underline{10.5 \text{ mm}}$

P-4] Determine the slope & deflection at each end and deflection at centre of beam shown.

Sol:



Total load on conjugate beam

= area of load diagram on conjugate beam

$$= 2\left[\frac{1}{2}\times\frac{l}{4}\times\frac{Pl}{8}\times\frac{1}{EI}\right]$$

$$+ 2\left[\frac{l}{4}\times\frac{Pl}{8}\times\frac{1}{2EI} + \frac{1}{2}\times\frac{l}{4}\times\frac{Pl}{8}\times\frac{1}{2EI}\right]$$

$$= \frac{5}{64}\frac{Pl^2}{EI}.$$

Reaction at each support of conjugate beam

$$V_a = V_b = \frac{5}{128}\frac{Pl^2}{EI}.$$

Slope at each end of the given beam

= Shear force at the end of the conjugate beam

$$= \frac{5}{128}\frac{Pl^2}{EI}$$

Deflection at the centre of given beam

= BM at centre of conjugate beam

$$= \frac{5}{128}\frac{Pl^2}{EI}\times\frac{l}{2} - \frac{1}{2}\times\frac{l}{4}\times\frac{Pl}{8}\times\frac{1}{EI}\left(\frac{l}{4}+\frac{l}{12}\right)$$

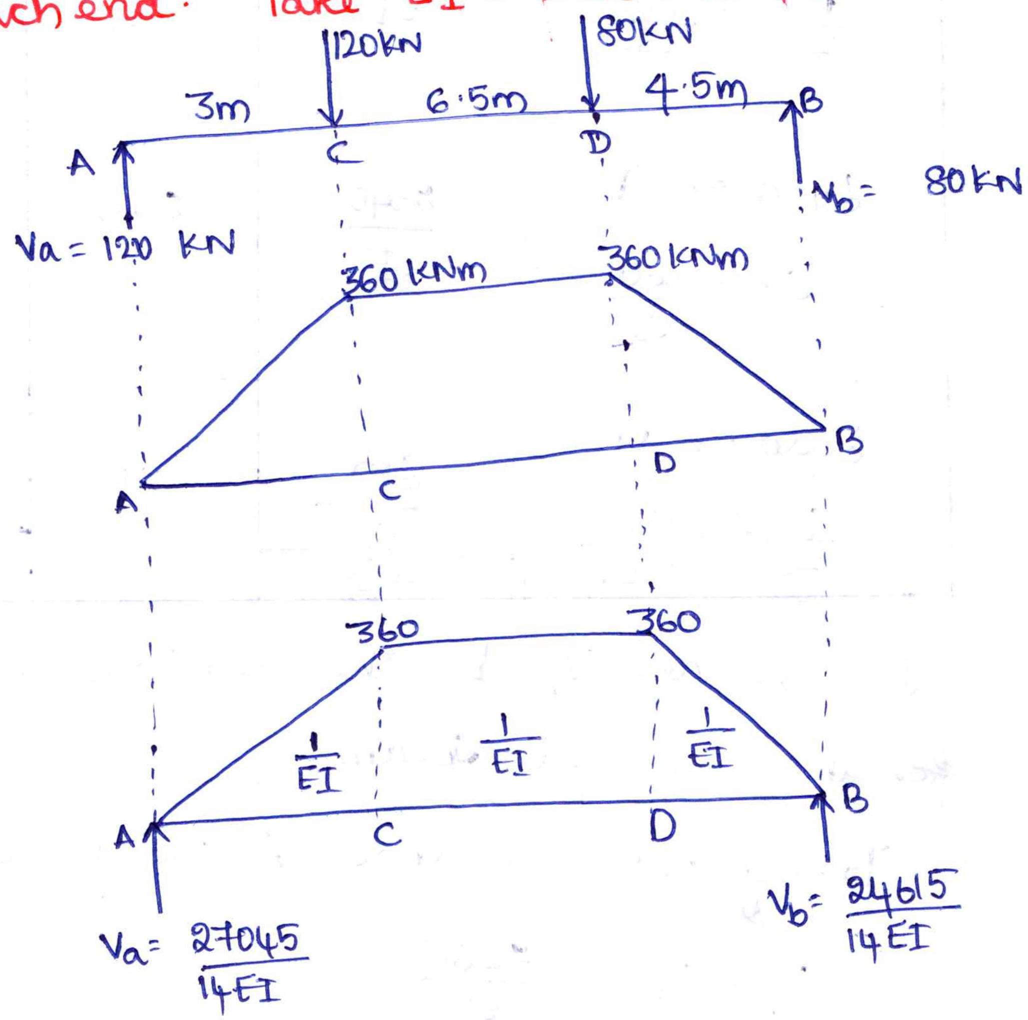
$$- \frac{l}{4}\times\frac{Pl}{8}\times\frac{1}{2EI}\times\frac{l}{8}$$

$$- \frac{1}{2}\times\frac{l}{4}\times\frac{Pl}{8}\times\frac{1}{2EI}\times\frac{l}{12}$$

$$= \frac{3}{256} \frac{Pl^3}{EI}.$$

P-5] A horizontal girder of steel having a uniform section is 14 m long and is simply supported at its end. it carries concentrated loads of 120 KN and 80 KN at sections 3m and 4.5m from left and right ends respectively. Find the slope and deflection under the loads and the slope at each end. Take $EI = 3.36 \times 10^{11}$ KN mm^2.

Sol:-



Taking moments about the left end A

$$V_b \times 14 = 120 \times 3 + 80 \times 9.5$$

$$V_b = 80 \text{ KN}$$

$$V_a = (120 + 80) - 80 = 120 \text{ KN}$$

BM at C = $120 \times 3 = 360$ KNm.

BM at D = $80 \times 4.5 = 360$ KNm.

The properties of the loading on the conjugate beam are given below

Load Component	Magnitude of load	Distance from A	Moment about A
Load on AC $\frac{1}{2} \times 3 \times \frac{360}{EI}$	$\frac{540}{EI}$	$\frac{2}{3} \times 3 = 2$	$\frac{1080}{EI}$
Load on CD $6.5 \times \frac{360}{EI}$	$\frac{2340}{EI}$	6.25	$\frac{14625}{EI}$
Load on DB $\frac{1}{2} \times 4.5 \times \frac{360}{EI}$	$\frac{810}{EI}$	11	$\frac{8910}{EI}$
	$\frac{3690}{EI}$		$\frac{24615}{EI}$

Let V_a and V_b be the reactions at A and B for the conjugate beam.

Taking moments about A we have

$$V_b \times 14 = \frac{24615}{EI}$$

$$V_b = \frac{24615}{14 EI}$$

$$V_a = \frac{3690}{EI} - \frac{24615}{14 EI} = \frac{27045}{EI}$$

Slope at A for given beam

$= $ SF at A for conjugate beam

$$= \frac{27045}{14EI}$$

$$= \frac{27045 \times 10^6}{14 \times 3.36 \times 10^{11}} = 0.005749 \text{ rad}$$

Slope at C for given beam

$= $ SF at C for Conjugate beam

$$= \frac{27045}{14EI} - \frac{540}{EI}$$

$$= \frac{19485}{EI} = \frac{19485 \times 10^6}{14 \times 3.36 \times 10^{11}}$$

$$= 0.004142 \text{ rad.}$$

Slope at D for given beam

$= $ SF at D for conjugate beam

$$= \frac{24615}{14EI} - \frac{810}{EI} = \frac{13275}{14EI}$$

$$= \frac{13275 \times 10^6}{14 \times 3.36 \times 10^{11}} = 0.002822 \text{ rad.}$$

Slope at B for given beam

$= $ SF at B for conjugate beam

$$= \frac{24615}{EI} = \frac{24615 \times 10^6}{14 \times 3.36 \times 10^{11}}$$

$$= 0.005232 \text{ rad.}$$

Deflection at C for given beam

= BM at C for conjugate beam

$$= \frac{27045}{14EI} \times 3 - \frac{540}{EI} \times \frac{3}{3}$$

$$= \frac{73575}{14\ EI}$$

$$= \frac{73575 \times 10^9}{14 \times 10^{11} \times 3.36} = 15.64 \text{ mm}$$

Deflection at D for the given beam

= BM at D for the conjugate beam

$$= \frac{24615}{14EI} \times 4.5 - \frac{810}{EI} \times \frac{4.5}{3}$$

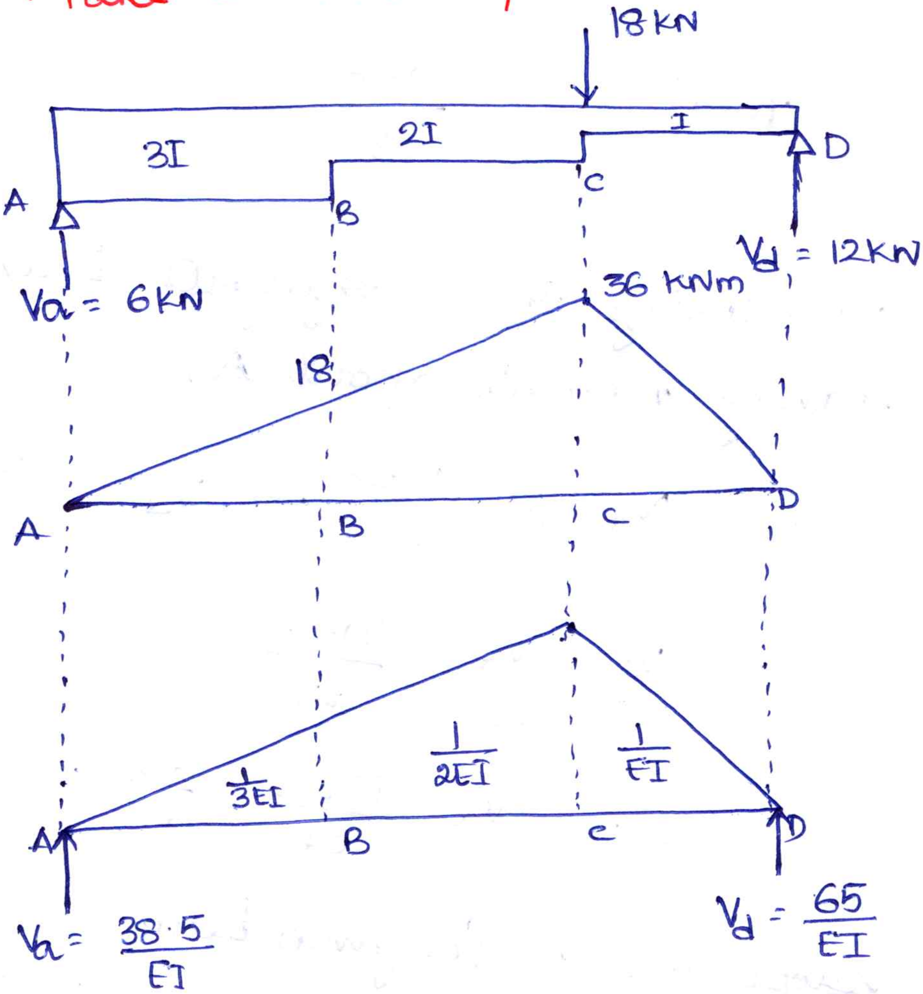
$$= \frac{93757.5}{14\ EI} = \frac{93757.5 \times 10^9}{14 \times 3.36 \times 10^{11}}$$

$$= \underline{\underline{19.93 \text{ mm}}}$$

P-6] A beam ABCD 9m long is simply supported at the ends A and D and carries a concentrated load of 18kN at C. The parts AB, BC and CD are each 3m long

The moment of inertia of the section for the parts AB, BC and CD are respectively 3I, 2I, I. Find (i) slopes at A, B, C, D (ii) Deflections at B and C. Take E = 200 kN/mm² and $I = 4.15 \times 10^7$ mm (9)

Sol:-



Properties of loading on the Conjugate beam :

Load Component	Magnitude of Load	Distance from A	Moment about A
Load on AB $\frac{1}{2} \times 3 \times \frac{18}{3EI}$	$\frac{9}{EI}$	2m $(\frac{2}{3} \times 3 = 2)$	$\frac{18}{EI}$
Load on BC $3 \times \frac{18}{2EI}$	$\frac{27}{EI}$	4.5m $(3 + \frac{3}{2} = 4.5)$	$\frac{121.5}{EI}$
$\frac{1}{2} \times 3 \times \frac{18}{2EI}$	$\frac{13.5}{EI}$	$3 + \frac{2}{3} \times 3$ = 5m	$\frac{67.5}{EI}$

Load on CD

$\frac{1}{2} \times 3 \times \frac{36}{EI}$ $\qquad \frac{54}{EI}$ $\qquad 3+3+\frac{1}{3}\times 3$ $\qquad \frac{378}{EI}$

7m

total $\qquad \frac{103.5}{EI}$ $\qquad \frac{585}{EI}$

considering the conjugate beam and taking moments about A.

$$V_d \times 9 = \frac{585}{EI} \Rightarrow V_d = \frac{65}{EI}$$

$$V_a + V_d = \frac{103.5}{EI}$$

$$\Rightarrow V_a = \frac{103.5}{EI} - \frac{65}{EI} = \frac{38.5}{EI}$$

Slope at A for given beam

= SF at A for the conjugate beam

$$= \frac{38.5}{EI} = \frac{38.5 \times 10^6}{200 \times 4.15 \times 10^7}$$

$$= 0.004639 \text{ rad.}$$

Slope at B for given beam

= SF at B for the conjugate beam

$$= \frac{38.5}{EI} - \frac{9}{EI} = \frac{29.5}{EI}$$

$$= \frac{29.5 \times 10^6}{200 \times 4.15 \times 10^7} = 0.003554 \text{ rad}$$

Slope at D for the given beam

$= \text{SF at D for conjugate beam}$

$$= -\frac{65}{EI} = \frac{65 \times 10^6}{200 \times 4.15 \times 10^7}$$

$$= -0.007831 \text{ rad.}$$

Deflection at B for the given beam

$= \text{BM at B for conjugate beam}$

$$= \frac{38.5}{EI} \times 3 - \frac{9}{EI} \times 1$$

$$= \frac{106.5}{EI} = \frac{106.5 \times 10^9}{200 \times 4.15 \times 10^7}$$

$$= 12.83 \text{ mm}$$

Deflection at C for the given beam

$= \text{BM at C for conjugate beam}$

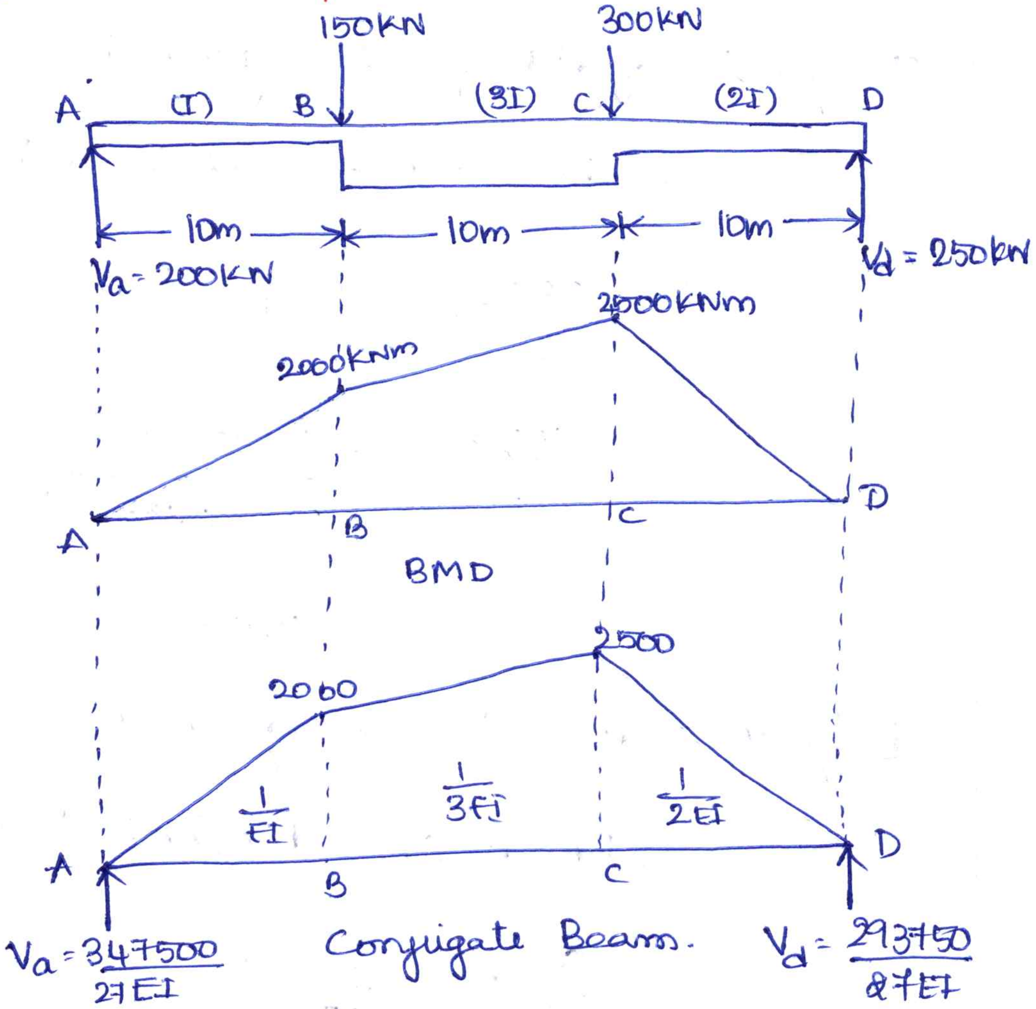
$$= \frac{65}{EI} \times 3 - \frac{54}{EI} \times 1$$

$$= \frac{141}{EI} = \frac{141 \times 10^9}{200 \times 4.15 \times 10^7}$$

$$= 16.99 \text{ mm.}$$

P-7] A beam ABCD is simply supported at its ends A and D over a span of 30 m. It is made up of three portions AB, BC, CD each 10 m in length. The moments of inertia of the section of these portions are I, 3I, 2I respectively where $I = 2\times10^{10}\,mm^4$ The beam carries a point load 150 kN at B and a point load of 300 kN at C. Neglecting the weight of the beam calculate the slopes and deflections at A, B, C, D. Take $E = 200\ kN/mm^2$.

Sol:-



Consider the given beam

$$V_a + V_d = 150 + 300$$

taking moments about A

$$V_d \times 30 = 150\times10 + 300\times20$$

$$V_d = 250\ KN$$

$$V_a = 450 - 250 = 200\ KN$$

BM at B = 200 × 10 = 2000 KNm.

BM at ~~D~~C = 250 × 10 = 2500 KNm.

The properties of the loads on the conjugate beam are given below.

Load component	Magnitude	Distance from A	Moment about A
Load on AB			
$\frac{1}{2} \times 10 \times \frac{2000}{EI}$	$\frac{10000}{EI}$	$\frac{2}{3} \times 10 = \frac{20}{3}$	$\frac{200000}{3EI}$
Load on BC			
$\frac{2000}{3EI} \times 10$	$\frac{20000}{3EI}$	$10 + \frac{10}{2} = 15$	$\frac{100000}{EI}$
$\frac{1}{2} \times 10 \times 500 \times \frac{1}{3EI}$	$\frac{2500}{3EI}$	$10 + \frac{2}{3} \times 10 = \frac{50}{3}$	$\frac{125000}{9EI}$
Load on CD			
$\frac{1}{2} \times 10 \times \frac{2500}{EI}$	$\frac{6250}{EI}$	$10 + 10 + \frac{10}{3} = \frac{70}{3}$	$\frac{437500}{3EI}$
total	$\frac{71250}{3EI}$		$\frac{2937500}{9EI}$

Let R_a & R_d be the reactions at A & D for the conjugate beam.

Taking moments about A we have

$$R_d \times 30 = \frac{2937500}{9EI}$$

$$\therefore R_d = \frac{2937500}{27EI}$$

$$R_a = \frac{71250}{3EI} - \frac{293750}{27EI}$$

$$= \frac{347500}{27EI}$$

Slope at A for given beam

= SF at A for ~~given~~ Conjugate beam

$$= \frac{347500}{27EI} = \frac{347500 \times 1000^2}{27 \times 200 \times 2 \times 10^{10}}$$

$$= 0.003218 \text{ rad}$$

Slope at B for given beam

= SF at B for conjugate beam

$$= \frac{347500}{27EI} - \frac{1000}{EI}$$

$$= \frac{77500}{27EI} = \frac{77500 \times 1000^2}{27 \times 200 \times 2 \times 10^{10}}$$

$$= 0.0007176 \text{ rad.}$$

Slope at C for given beam

= SF at C for conjugate beam

$$= \frac{293750}{27EI} - \frac{6250}{EI} = \frac{125000}{27EI}$$

$$= \frac{125000 \times 1000^2}{27 \times 200 \times 2 \times 10^{10}} = 0.001157 \text{ rad}$$

Slope at D for given beam

= SF at D for conjugate beam

$$= \frac{293750}{27EI}$$

$$= \frac{293750 \times 1000^2}{27 \times 200 \times 2 \times 10^{10}} = 0.00272 \text{ rad.}$$

Deflection at A for given beam = 0

Deflection at B for given beam

= BM at B for conjugate beam

$$= \frac{347500}{27EI} \times 10 - \frac{10000}{EI} \times \frac{10}{3}$$

$$= \frac{2575000}{27\ EI} = \frac{2575000 \times 1000^3}{27 \times 200 \times 2 \times 10^{10}}$$

$$= 23.84 \text{ mm}$$

Deflection at C for given beam

= BM at C for conjugate beam

$$= \frac{293750}{27EI} \times 10 - \frac{6250}{EI} \times \frac{10}{3}$$

$$= \frac{2375000}{27\ EI} = \frac{2375000 \times 1000^3}{27 \times 200 \times 2 \times 10^{10}}$$

$$= \underline{\underline{21.99 \text{ mm}}}$$

STRUCTURAL ANALYSIS – II

1. Define Carry over Moment and distribution Factor ?
2. Differentiate single bay and single storey portal frame ?
3. Define Relative Stiffness?
4. Mention the Situation where in sway will occur in portal frame?
5. State the reasons for sidesway in portal frame?
6. Write the advantages of Kanis Method?
7. What is rotation factor in kanis method?
8. What are the salient points in the kanis method of analysis?
9. Briefly explain the basic concept of kanis method analysis?

10 Write concepts of flexibility method?

11. Define stiffness and write the basic equation of stiffness method?

12. Differentiate between the statically determinate structures and statically indeterminate structures?

13. Explain the relation between the flexibility and stiffness matrix?

14.Write the flexibility and stiffness coefficients for flexural displacement?

15. Differentiate local and global coordinates? Illustrate with an examples?

16.Explain the difference between the local and global stiffness matrix?

17. Differentiate between the static indeterminancy and kinematic indeterminancy?

18. What is fixed end moment for beam subject to udl throughout?

19. Write the concept of conjugate beam method?

20. What is mean by Flexural Rigidity?

21. What is the use of conjugate beam method over other methods?