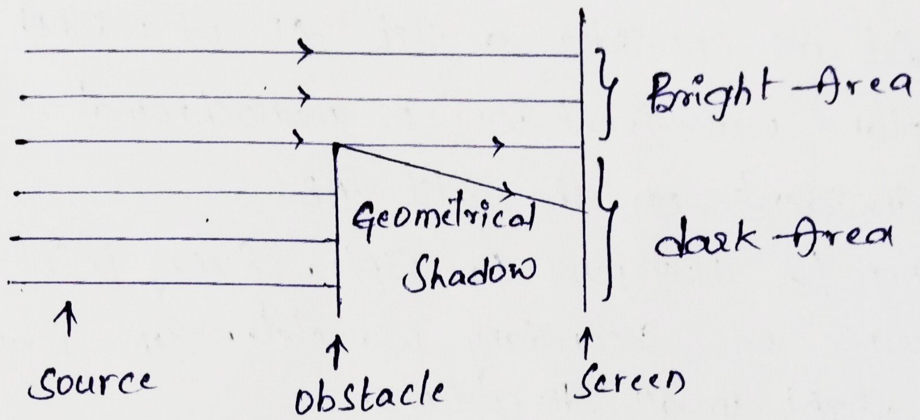
Diffraction



when the light falls on the obstacle (or) small aperture (hole). Then the light bends around the obstacle or small aperture, then the light rays Enters into the Geometrical Shadow. This bending of light is called diffraction

Types of diffraction :-

The diffraction phenomena are classified into two ways:

1) Fresnel Diffraction.
2) Fraunhofer Diffraction.

Fresnel Diffraction :-

In this diffraction the source and screen are seperated at finite distance. To study this diffraction lenses are not used because the source and screen seperated at finite distance. This diffraction can be studied in the direction of propagation of light. In this diffraction the incidence wavefront must be spherical (or) cylindrical.

Fraunhofer Diffraction :-

In this diffraction the source and screen are seperated at infinite distance. To study this diffraction lenses are used because the source and screen seperated at infinite distance. This diffraction can be studied in any direction. In this diffraction the incidence wavefront must be plane.

Fraunhofer Single Slit diffraction :-

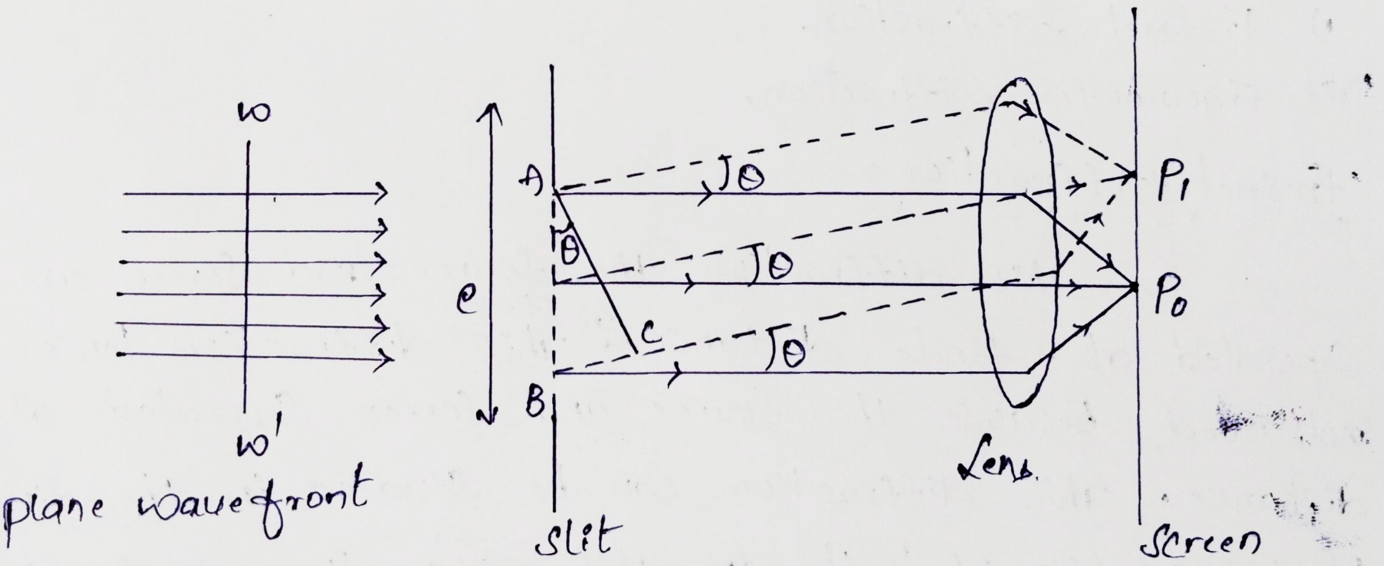
Let us consider a slit AB of width 'e'.

Let a plane wave-front ww' of monochromatic light of wavelength λ is incident on the slit AB.

According to Huygen's principle, Every point on the wavefront is a source of secondary wavelets. The wavelets spread out to the right in all directions.

The secondary wavelets which are travelling normal to the slit are brought to focus at point P_0 on the screen by using the lens.

These secondary wavelets have no path difference. Hence at point P_0 the intensity is maxima and is known as central maximum. The secondary wavelets travelling at an angle θ with the normal are focused at point P_1.



Intensity at point P_1 depends up on the path difference between the wavelets A and B reaching to point P_1.

To find the path difference, a perpendicular AC is drawn to B from A.

$$\text{path difference} \quad \sin\theta = \frac{BC}{AB}$$

$$AB \sin\theta = BC$$

path difference $BC = e\sin\theta$.

phase difference $= \frac{2\pi}{\lambda}$ (path difference)

$= \frac{2\pi}{\lambda}(e\sin\theta)$

Let the width of the slit is divided into 'n' Equal parts. Then the phase difference between any two successive waves from these parts would be

$$\frac{1}{n}(\text{phase difference}) = \frac{1}{n}\left[\frac{2\pi e\sin\theta}{\lambda}\right] = d$$

using the vector addition method, the resultant amplitude R is

$$R = \frac{a\sin\frac{nd}{2}}{\sin\frac{d}{2}} \Rightarrow \frac{a\sin\left[\frac{n}{2}\times\frac{2\pi}{n\lambda}e\sin\theta\right]}{\sin\frac{1}{2}\times\frac{2\pi}{n\lambda}e\sin\theta}$$

$$R = \frac{a\sin\left[\frac{\pi}{\lambda}e\sin\theta\right]}{\sin\left[\frac{\pi}{n\lambda}e\sin\theta\right]}$$

$$R = \frac{a\sin\alpha}{\sin\frac{\alpha}{n}} \Rightarrow \frac{a\sin\alpha}{\frac{\alpha}{n}} \Rightarrow \frac{na\sin\alpha}{\alpha}$$

$$R = \frac{A\sin\alpha}{\alpha} \quad \left[\because na = A \;\&\; \alpha = \frac{\pi e\sin\theta}{\lambda}\right]$$

Therefore Resultant Intensity $I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$

principal maximum :-

The Resultant amplitude R can be written as.

$$I = \frac{A^2\sin^2\alpha}{\alpha^2} = max, \quad \alpha = 0$$

we know that, $\alpha = \frac{\pi e\sin\theta}{\lambda}$

then, $\frac{\pi e\sin\theta}{\lambda} = 0$

$\sin\theta = 0$

$\theta = 0^\circ$

$$I = A^2 \lim_{\alpha \to 0} \frac{\sin^2\alpha}{\alpha^2}$$

$$I = A^2 (1)$$

$$I = R^2 = A^2 \quad [\because A^2 = R^2]$$

For $\theta = 0$ & $\alpha = 0$, the resultant intensity is max at P_0 and is known as principal maximum.

Minimum Intensity positions:

I will be minimum when $\sin\alpha = 0$

$$\alpha = \pm m\pi \quad , \quad m = 1, 2, 3 \ldots$$

$$\alpha = \frac{\pi e \sin\theta}{\lambda} = \pm m\pi$$

$$\Rightarrow e\sin\theta = \pm m\pi$$

Secondary Maximum :-

In between these minima secondary maxima positions are located. This can be obtained by differentiating the Expression of I w.r.t α and Equation to zero.

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[\frac{A^2 \sin^2\alpha}{\alpha^2}\right] = 0$$

$$A^2 \left[\frac{\alpha^2 (2\sin\alpha\cos\alpha) - \sin^2\alpha (2\alpha)}{\alpha^4}\right] = 0$$

$$\frac{A^2\, 2\alpha \sin\alpha}{\alpha^4} (\alpha\cos\alpha - \sin\alpha) = 0$$

$$\alpha\cos\alpha - \sin\alpha = 0$$

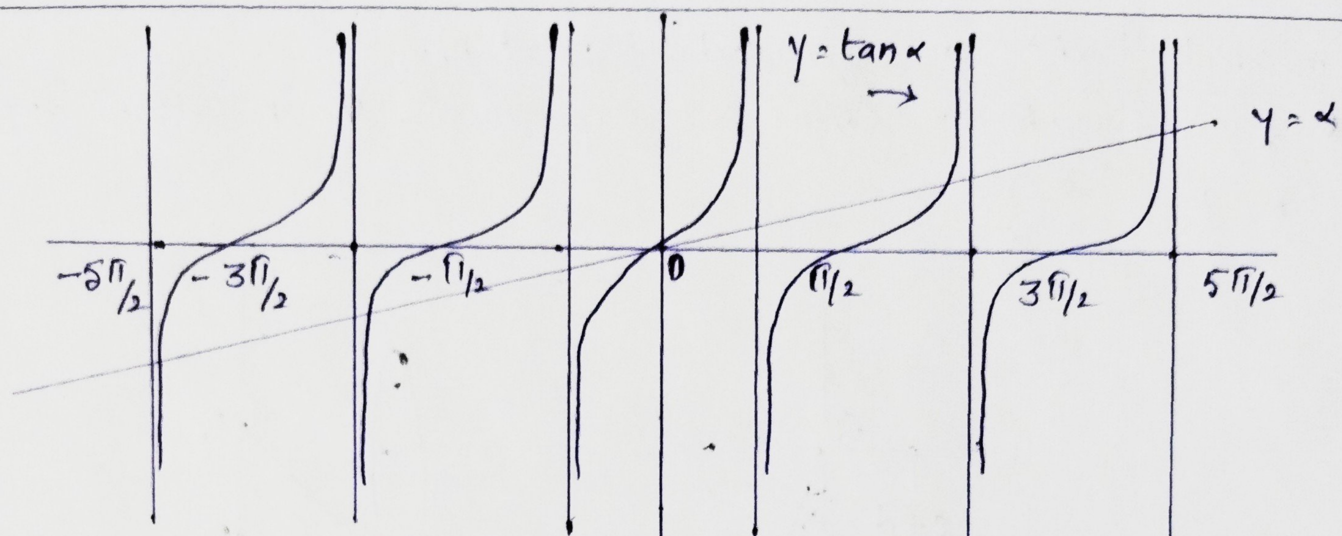
The condition for getting the secondary maxima is.

$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\alpha\cos\alpha = \sin\alpha$$

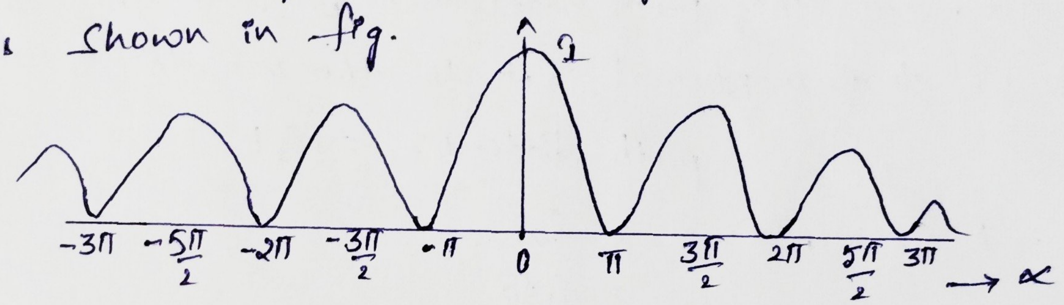
$$\alpha = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha.$$

The plots of $y = \alpha$ & $y = \tan\alpha$ as shown in fig.



In the graph the two curves gives the values of satisfying the above Equation. From the graph intersecting points are $\alpha = 0, \pm \pi/2 \pm 3\pi/2 \pm 5\pi/2$

From the above concepts the intensity distribution curve verses α is shown in fig.

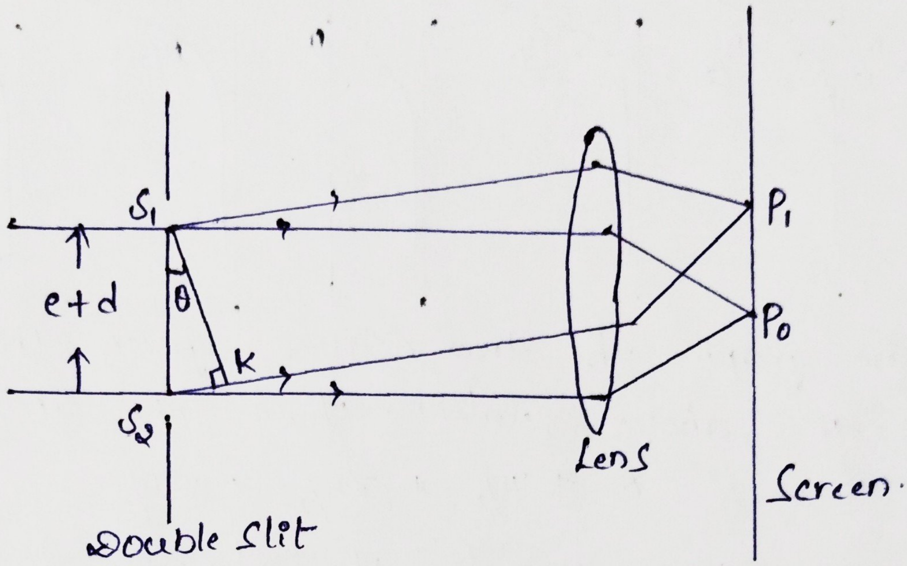


Fraunhofer double slit diffraction :-

* Let S_1 and S_2 be the two slits Equal width e and seperated by a distance 'd'.
* The distance between the two slits is (e + d)
* A monochromatic light of wavelength λ is incident on the two slits.
* The diffraction light from these slits is focused on the screen by using a lens
* The diffraction of a two slits is a combination of interference and diffraction.
* when the plane wavefront is incident on the two slits, the secondary wavelets from these slits travel in all directions.
* the wavelets travelling in the direction of incident light is focused at P_0. The wavelets travelling at an angle θ with the

incident light are focused at point P_1.

* From Fraunhofer single slit experiment, the resultant amplitude is $R = \frac{A \sin\alpha}{\alpha}$.



To find out the path difference between the two wavelets.
Let us draw a normal S_1K to the wavelet S_2.

$$\text{path difference} = S_2K$$

From $\Delta\, S_1S_2K$, $\sin\theta = \frac{S_2K}{S_1S_2}$

$$\sin\theta = \frac{S_2K}{e+d}$$

$$(e+d)\sin\theta = S_2K = \text{Path difference.}$$

$$\text{phase difference} = \frac{2\pi}{\lambda}(\text{path difference})$$

$$= \frac{2\pi}{\lambda}(e+d)\sin\theta = \delta$$

By using vector addition method, we can calculate the resultant amplitude at point P_1 by taking the resultant amplitudes of the two slits S_1 and S_2 as sides of the triangle. The third side gives resultant amplitude.

$$AC^2 = AB^2 + BC^2 + 2(AB)(BC)\cos\delta$$

$$R^2 = \left(A\frac{\sin\alpha}{\alpha}\right)^2 + \left(A\frac{\sin\alpha}{\alpha}\right)^2 + 2\left(\frac{A\sin\alpha}{\alpha}\right)\left(\frac{A\sin\alpha}{\alpha}\right)\cos\delta.$$

$$R^2 = \left[\frac{A \sin\alpha}{\alpha}\right]^2 (1+1+2\cos\delta)$$

$$R^2 = 2\left[\frac{A^2 \sin^2\alpha}{\alpha^2}\right](1+\cos\delta)$$

$$R^2 = \frac{2A^2\sin^2\alpha}{\alpha^2}\left(2\cos^2\frac{\delta}{2}\right) \quad \left[\because 1+\cos\theta = 2\cos^2\frac{\theta}{2}\right]$$

$$R^2 = 4\left(\frac{A^2\sin^2\alpha}{\alpha^2}\right)(\cos^2\beta) \quad \left[\because \frac{\delta}{2} = \beta\right]$$

The Resultant Intensity $I = R^2 = 4\left(\frac{A\sin\alpha}{\alpha}\right)^2 \cos^2\beta$.

From the above Eq, it is clear that the resultant intensity is a product of 2 factors i.e,

(1) $\left[\frac{A \sin\alpha}{\alpha}\right]^2$ represents the diffraction pattern due to single slit.

(2) $\cos^2\beta$ represents the interference pattern due to wavelets from double slit.

Diffraction effect :-

$\sin\alpha = 0$, but $\alpha \neq 0$.

$\sin\alpha = 0 = \sin(\pm n\pi)$

$\alpha = \pm n\pi$

we know that $\alpha = \frac{\pi e \sin\theta}{\lambda}$ value

From this value, we get $\frac{\pi e \sin\theta}{\lambda} = \pm n\pi$

$$\boxed{e\sin\theta = \pm n\lambda}$$

Interference Effect :-

'$\cos^2\beta$' represents interference pattern.

Interference maximum will occur for. $\cos^2\beta = 1$,

$\cos\beta = \pm 1 = \cos(\pm m\pi)$

where $\beta = \pm m\pi$, where $m = 0, 1, 2, 3 \dots$

We know that $\beta = \frac{\pi(e+d)\sin\theta}{\lambda}$

then, we get $\frac{\pi(e+d)\sin\theta}{\lambda} = m\pi$

$$(e+d)\sin\theta = m\lambda$$

Interference Effect :-

Interference minima will occur for $\cos^2\beta = 0$

$$\beta = \pm(2n+1)\pi/2$$

$$\frac{\pi(e+d)\sin\theta}{\lambda} = \pm(2n+1)\pi/2$$

$$\boxed{(e+d)\sin\theta = \pm(2n+1)\lambda/2}$$

Intensity distribution :-

Fig (a), (b) & (c) represents the intensity variations of due to diffraction, Interference and both Effects respectively. From fig (c) it is clear that the resultant minima are not Equal to zero.

fig (a)

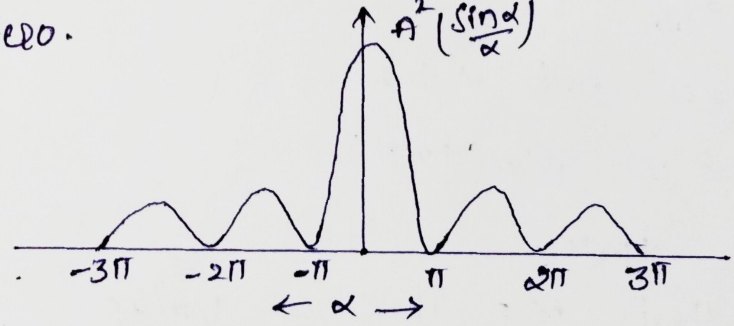


fig (b)

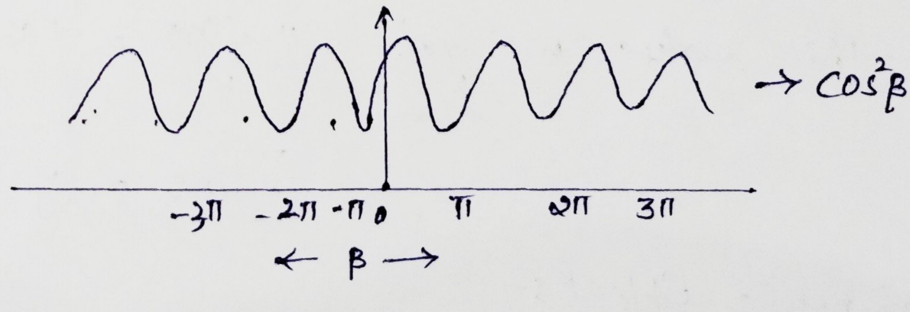
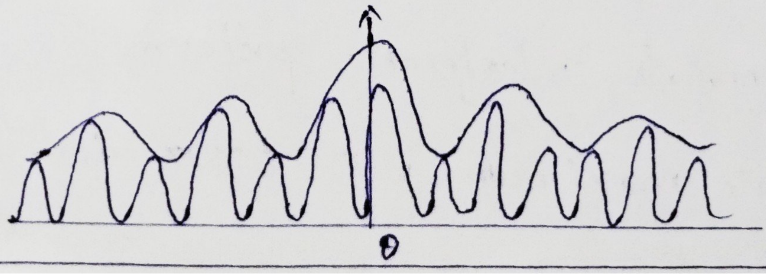


fig (c)



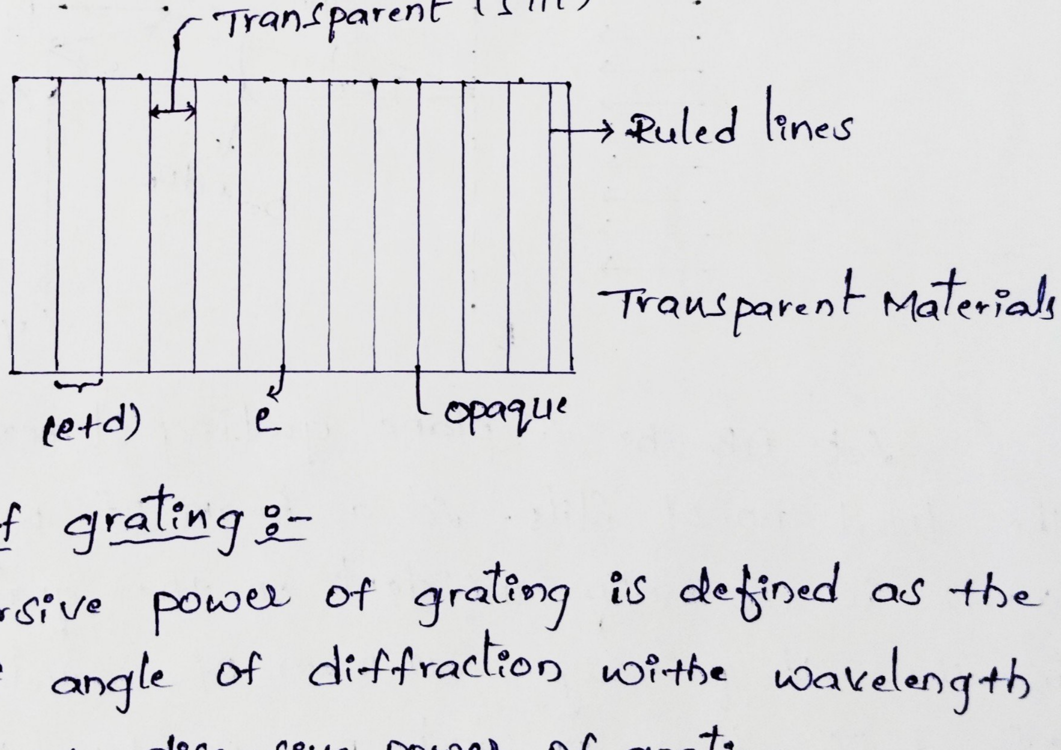
Diffraction grating :-

A set of large number of parallel slits of same width and seperated by opaque spaces is known as diffraction grating. Fraunhofer used the first grating consisting of a large number of parallel wires placed side by side very closely at regular seperation. Now the grating are constructed by ruling the equi-distant parallel lines on a transparent material such as glass. The ruled lines are opaque light while the space between the two lines is transparent to light and act as a ~~light~~. slit.

Let 'e' be the width of the line and 'd' be the width of the slit. then $(e+d)$ is known as grating Element. If N is the number of lines per inch on the grating, then.

$$N(e+d) = 1 \text{ inch} = 2.54 \text{ cm}$$

$$(e+d) = \frac{2.54}{N}$$



Dispersive power of grating :-

The dispersive power of grating is defined as the rate of change of angle of diffraction with the wavelength i.e. $\frac{d\theta}{d\lambda}$ is known as dispersive power of grating.

The condition for maxima is $(e+d)\sin\theta = n\lambda$

On differentiation we get $(e+d)\cos\theta \, d\theta = n \, d\lambda$

By using above Eq, we get: $\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$

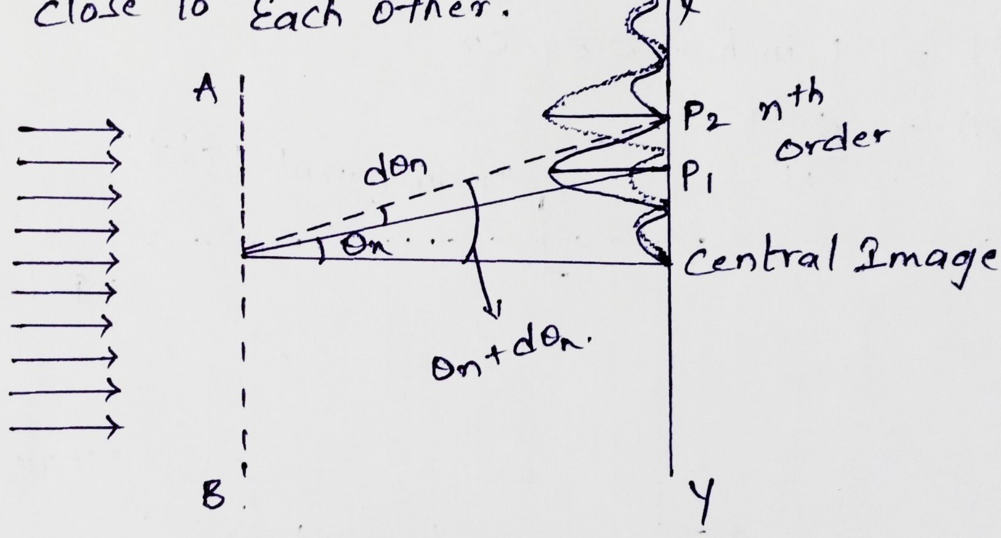
This is the expression for dispersive power of grating.

Conclusions :-

* The dispersive power is directly proportional to diffraction order 'n'.
* The dispersive power is inversely proportional to grating element $(e+d)$.
* The dispersive power is inversely proportional to $\cos\theta$.

Resolving power of Grating :-

The resolving power of grating is defined as the capacity to form seperate diffraction maxima of two wavelengths which are very close to each other.



Let AB be a plane grating element $(e+d)$ and N be the total no. of slits. Let a beam of wavelengths λ and $\lambda+d\lambda$ is normally incident on the grating in the fig P_1 is the n^{th} primary maximum of wavelength λ at an angle of diffraction θ_n and P_2 is the n^{th} primary maximum of wavelength $(\lambda+d\lambda)$ at an angle of diffraction $(\theta_n+d\theta_n)$

According to Rayleigh Criterion, the two wavelengths will be resolved if the principle maximum of one falls on the first minimum of the ~~order~~ other.

The principle maximum of λ in the direction θ_n is given by

$$(e+d)\sin\theta_n = \pm n\lambda \rightarrow (1)$$

The wavelength $(\lambda+d\lambda)$ form its n^{th} primary maxima in the direction $(\theta_n + d\theta_n)$

$$(e+d)\sin(\theta_n+d\theta_n) = \pm n(\lambda+d\lambda) \rightarrow (2)$$

The principle minima of Equation is

$$N(e+d)\sin\theta = m\lambda \quad [\text{where as } m = 0, N, 2N \ldots]$$

The first minimum of wavelength λ from in the direction $(\theta_n + d\theta_n)$ can be obtained by substituting m as $(Nn+1)$

$$N(e+d)\sin(\theta_n+d\theta_n) = m\lambda$$

$$N(e+d)\sin(\theta_n+d\theta_n) = (Nn+1)\lambda \rightarrow (3)$$

Multiplying Eq (2) with N.

$$N(e+d)\sin(\theta_n+d\theta_n) = \pm nN(\lambda+d\lambda) \rightarrow (4)$$

From Eq (3) & (4)

$$Nn(\lambda+d\lambda) = (nN+1)\lambda$$

$$Nn\lambda + Nnd\lambda = nN\lambda + \lambda$$

$$nNd\lambda = \lambda$$

$$\frac{\lambda}{d\lambda} = nN$$

But from Eq (1), we get $n = \frac{(e+d)\sin\theta_n}{\lambda}$

Substitute 'n' value in $\frac{\lambda}{d\lambda} = nN$

$\therefore$ Resolving power of Grating $\boxed{\frac{\lambda}{d\lambda} = \frac{N(e+d)\sin\theta_n}{\lambda}}$

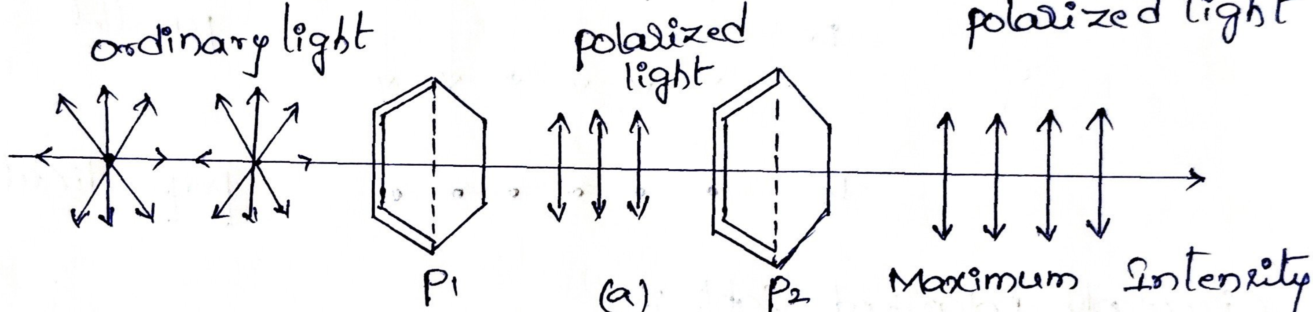
Polarization

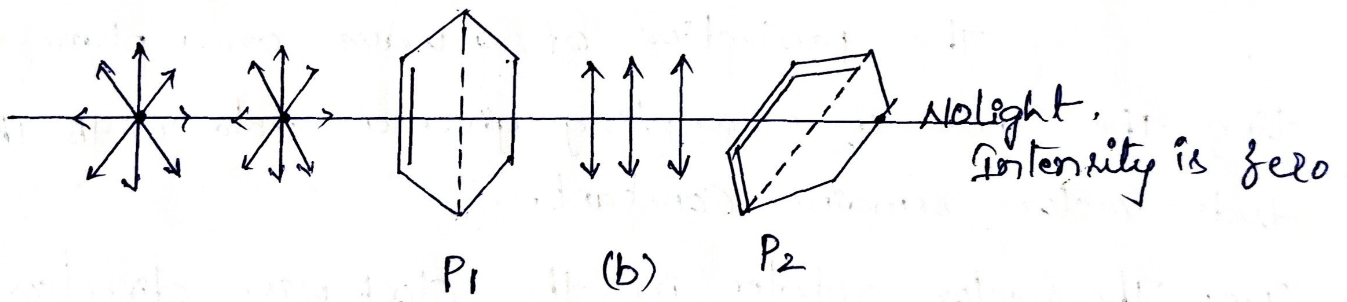
Polarization:- It is the process of converting ordinary light into polarized light.

Polarized wave:- The wave which is unsymmetrical about the direction of propagation is called polarized wave.

Polarized light:- The light which has acquired the property of one sidedness is called polarized light.

Polarization of light waves:-





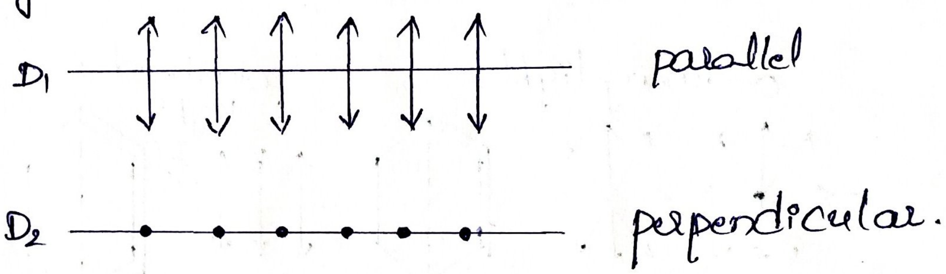
When an ordinary light is passed through a pair of tourmaline crystal plates with their planes parallel to each other, then the maximum intensity is obtained. When their planes perpendicular to each other, the intensity is zero. This shows that light is transverse wave motion.

Types of polarized light :-

There are three different types of polarized light

* plane polarized light
* Circularly polarized light
* Elliptically polarized light.

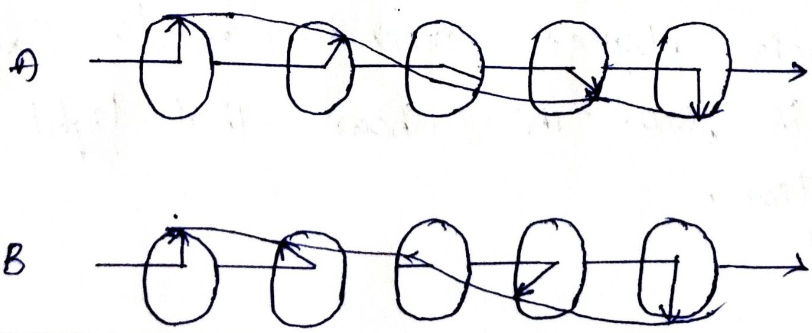
Plane polarized light :- when the vibrations of light are confined along a single direction, the light is said to be plane polarized light.

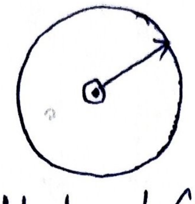


Circularly polarized light :-

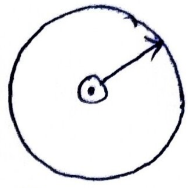
The projection of a wave on a plane intercepting the axis of propagating gives a circle with the amplitude vector remains constant.

I.e, The vector rotates in the clockwise direction with respect to the direction of propagation; its result in right, circularly polarized light while the rotation anti-clockwise direction results in left circularly polarized light.



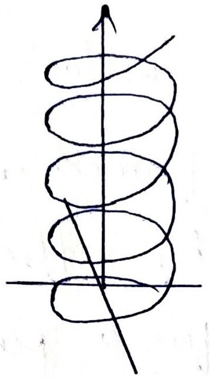


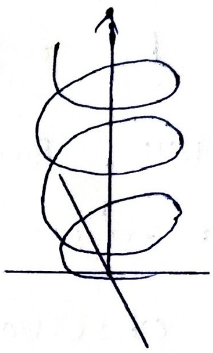
Left hand Circular



Right hand Circular

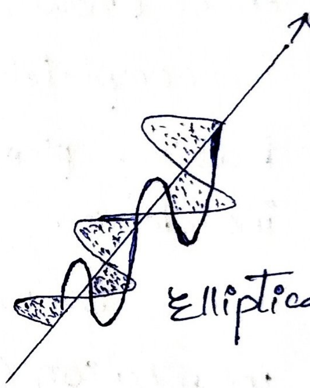
If the vibrations are along a Circle, The Light is Said to be Circularly polarized light.





Elliptically polarized light :-

The projection of a wave on a plane intercepting the axis of propagating gives a Ellipse and amplitude vector is not constant but various periodically.

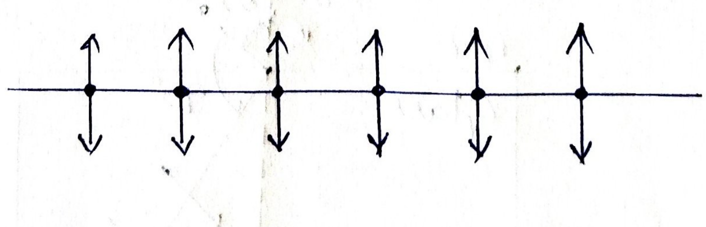


Elliptical

If the vibrations are along an Ellipse, the light is Said to be Elliptically polarized light.

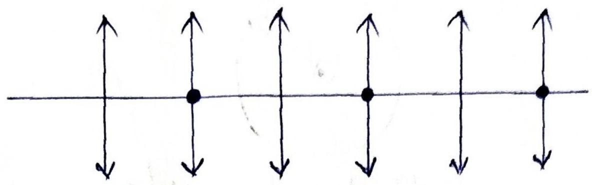
unpolarized Light :-

unpolarized light (or) ordinary light has vibrations both parallel and perpendicular to the plane of the paper.



Partially Polarized light :-

If the linearly polarized light contains Small additional component of unpolarized light it becomes partially polarized light.

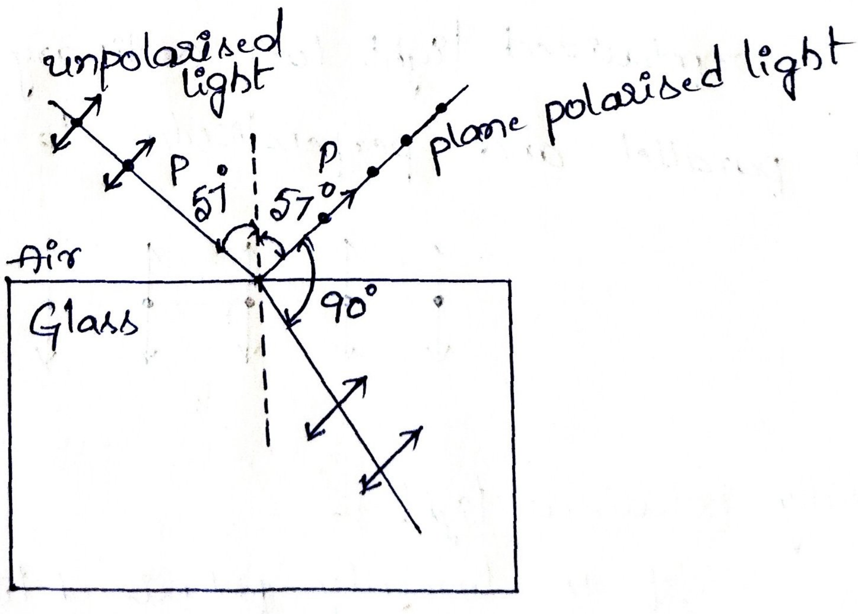


polarization by reflection (Brewster's Law):-

In 1811, Brewster performed a number of Experiments to study the polarization of light by reflection at different surfaces. He observed that for a particular angle of incidence known as angle of polarisation, the reflected light is completely polarised in the plane of incidence i.e, having plane of vibration perpendicular to the plane of incidence.

Brewster proved that the tangent of the angle of polarisation (p) is numerically Equal to the refractive index (μ) of the medium, i.e $\mu = \tan p$.

This is known as Brewster's law. He also proved that the reflected and refracted rays are perpendicular to Each other.



Suppose a beam of unpolarised light is incident on glass surface at polarising angle p. The polarising angle for air - glass is 57°. A part of the incident light is reflected while a part is refracted. Let r be the angle of refraction. From Brewster's law,

$$\mu = \tan p \rightarrow (1)$$

From Snell's law: $\mu = \frac{\sin p}{\sin r} \rightarrow (2)$

From (1) & (2), we get

$$\frac{\sin p}{\sin r} = \tan p$$

$$\frac{\sin p}{\sin r} = \frac{\sin p}{\cos p}$$

$$\cos p = \sin r$$

$$\cos p = \cos(90^\circ - r) \quad [\because \sin\theta = \cos(90-\theta)]$$

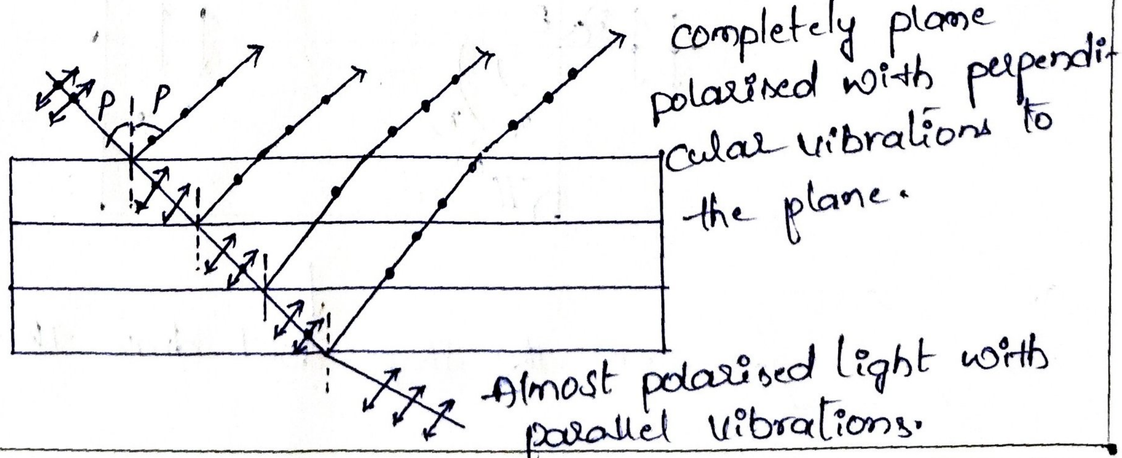
$$p = 90^\circ - r$$

$$p + r = 90^\circ$$

The angle between reflected and refracted ray is 90°.

Polarization by refraction (pile of plates):-

A pile of plates consists of about 9 or 10 glass plates arranged one above the other.



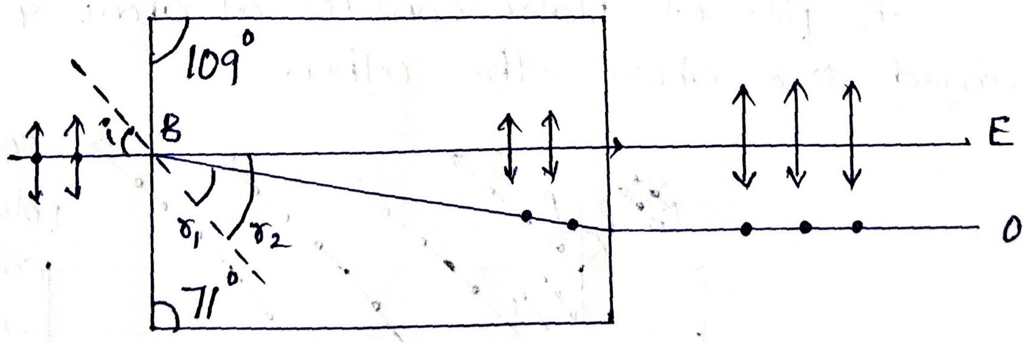
when ordinary light is incident on pile of plates at polarising angle, a few vibrations perpendicular to the plane of incidence are reflected by the first plate and rest are refracted through it. when this beam of light is refracted by the second plate, again some vibrations perpendicular to the plane of incidence are reflected by this plate and rest are transmitted.

It is important to mention here that each time, the light is incident on the surface of glass plate at Brewsters angle.

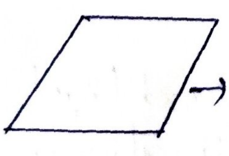
The above procedure continues for different glass plates. At the last plate, we get almost plane polarized light with vibrations parallel to the plane of incidence. So, the refracted light is plane polarized light.

Birefringence (polarization by Double refraction):-

The phenomenon of splitting of a light ray into two refracted rays is called Double Refraction.



don't draw this (square).



→ Draw this shape

When a beam of ordinary unpolarised light is passed through a calcite crystal, the refracted light is split up into two refracted rays. The one which always obeys the ordinary laws of refraction and having vibrations perpendicular to the principal section is known as ordinary ray. The other, in general, doesn't obey the laws of refraction and having vibrations parallel to the principal section is known as Extraordinary ray.

Both the rays are plane polarised light. This phenomenon are known as doubly refracting crystals.

There are 2 types of doubly refracting crystals.

1) Uniaxial crystal (2) Biaxial crystal.

Uniaxial crystal :- In this crystal there is only one direction along which the 2 refracted rays travel with the same velocity.

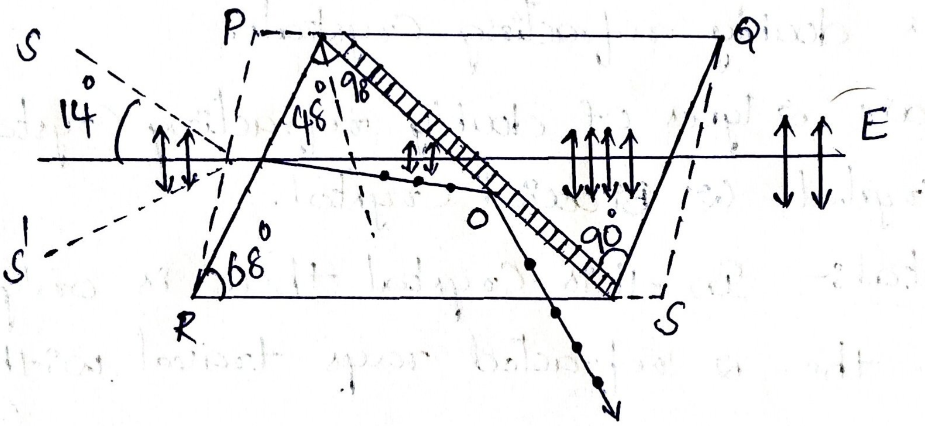
Biaxial crystals :- There are two such directions along which the velocities are the same.

Nicol prism :-

When an ordinary light is passed through a calcite crystal, it splits into ordinary and Extraordinary rays. Nicol Eliminated the ordinary beam by utilizing the phenomenon of total internal reflection at canada balsam seperating the two pieces of calcite. This device is called "Nicol prism".

Construction:- A calcite crystal whose length is 3 times as that of its width is taken. The end faces of this crystal are grounded in such a way that the angle in the principle section becomes 68° & 112°. Then calcite crystal cut into two pieces. The cut surfaces are grounded and polished optically flat and then cemented together by canada balsam. The refractive index of canada balsam lies between refractive indices of O-ray and E-ray i.e.

$$\mu_e < \mu_{cb} < \mu_o$$



Working:-

When an ordinary beam of light incident on the Nicol prism, it splits into ordinary and Extraordinary plane polarized light. From the values of refractive indices the canada balsam acts as rarer medium for ordinary ray and denser medium for Extraordinary ray. Moreover the dimensions of the crystal are so chosen that the angle of incidence of ordinary ray at the calcite-balsam surfaces becomes greater than the corresponding critical angle 69°. Under these conditions, the ordinary ray is completely reflected at calcite-balsam. The Extraordinary ray is not totally reflected because it travelling from a rarer medium to denser Medium and is thus transmitted with no appreciable loss in intensity.

Refractive index for ordinary $\mu_o = 1.658$

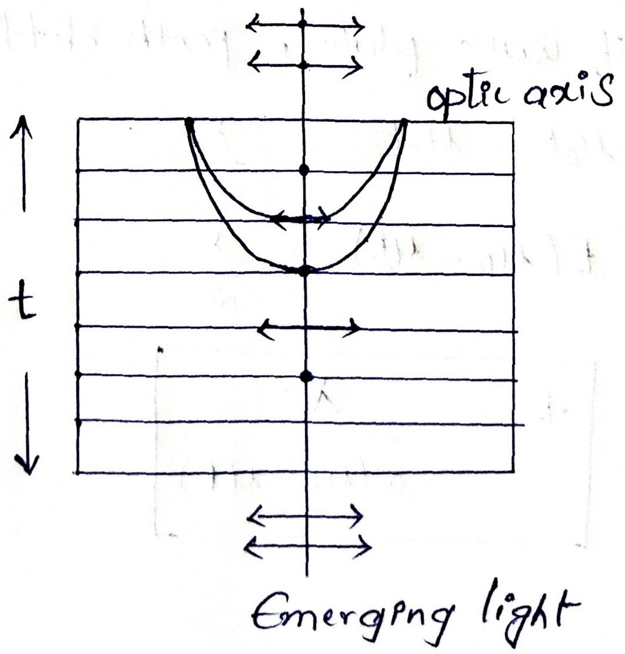
Refractive index for Canada balsam $\mu_{cb} = 1.55$

Refractive index for Extra-ordinary $\mu_e = 1.486$.

Thus, only the Extra ordinary ray is refracted. Since, Extra-ordinary ray is plane polarised having vibrations parallel to the principle plane, the light Emerging from the Nicol's prism is plane polarized.

Quarter wave plate :-

Consider the case of calcite crystal cut with optic axis parallel to the surface. It has been shown that when a plane polarised light falls normally on a thin plate of uniaxial crystal cuts parallel to its optic axis, the light splits up into ordinary and Extra-ordinary plane polarised light. They travel along with same path but different velocity. The velocity Extra-ordinary ray is greater than the ordinary ray.



If the thickness of a crystal is taken such that it introduces a path difference of $\frac{\lambda}{4}$ or phase difference of $\frac{\pi}{2}$, then that crystal is called Quarter wave plate.

Let, μ_0, μ_e are the refractive indices of ordinary, Extraordinary rays and t is the thickness of the calcite crystal.

Then the path difference between ordinary and Extra ordinary ray $= \mu_0 t - \mu_e t$

But for Quarter wave plate, path difference $= \frac{\lambda}{4}$

$$\mu_0 t - \mu_e t = \frac{\lambda}{4}$$

$$t(\mu_0 - \mu_e) = \frac{\lambda}{4}$$

$$\boxed{t = \frac{\lambda}{4(\mu_0 - \mu_e)}}$$

Half wave plate:-

Then the path difference between ordinary and Extra-ordinary ray $= \mu_0 t - \mu_e t$

But for Half wave plate, path difference $= \frac{\lambda}{2}$

$$\mu_0 t - \mu_e t = \frac{\lambda}{2}$$

$$t(\mu_0 - \mu_e) = \frac{\lambda}{2}$$

$$\boxed{t = \frac{\lambda}{2(\mu_0 - \mu_e)}}$$

Interference

Interference :-

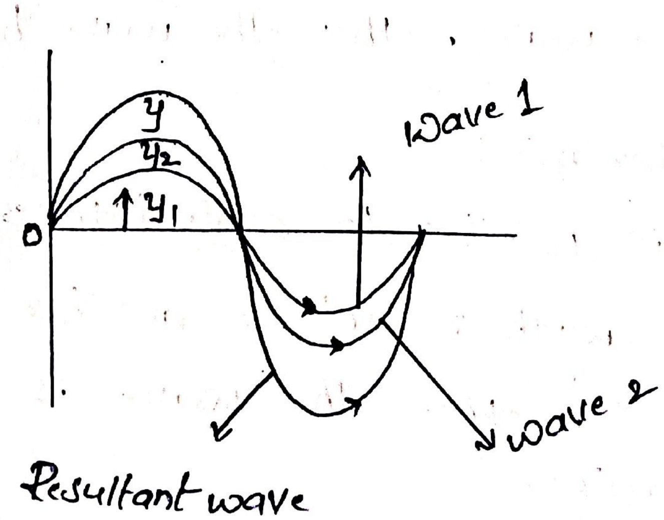
when two (or) more light waves. Superimposed in the medium then the resultant displacement is Equal to the algebric Sum of the displacements due to individual waves.

If y_1 and y_2 are the displacements due to two light waves, then the resultant displacement of the resultant wave is given by $y = y_1 + y_2$

The variation of the resultant displacement influences amplitude variation, which causes intensity variations. This modification in the distribution of intensity in the region of superposition is known as interference.

Constructive Interference :-

when the resultant amplitude is Equal to the sum of the 2 amplitudes due to two light waves, then the interference becomes constructive interference.

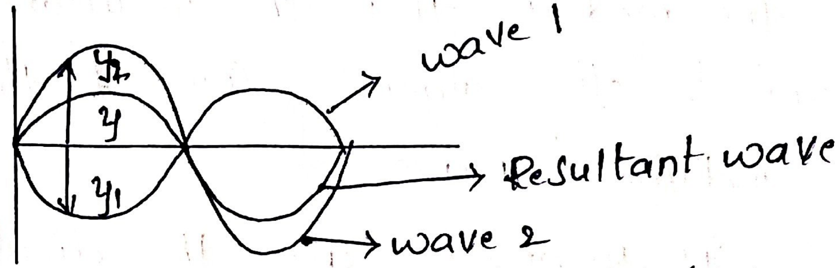


$$\boxed{y = y_1 + y_2}$$

Destructive interference :-

If the resultant amplitude is Equal to the difference of two amplitudes, then the interference becomes desctructive Interference.

$$\boxed{y = y_1 - y_2}$$



The intensity variations are studied as interference patterns (or) fringes (or) bright and dark fringes.

Coherence :-

To observe the interference, the two light waves should be coherent.

Def :- The two light waves are said to be coherent if they have same frequency, same wavelength, a constant phase and same amplitude.

coherence is of 2 types :-

1. Temporal coherence :-

If it is possible to control the phase at a point on the wave with respect to another point on the same wave, then the wave has temporal coherence

2. Spatial coherence :-

If it is possible to control the phase at a point on a wave with respect to another ~~wave~~ point on a second wave, then the wave is said to be spatial coherence.

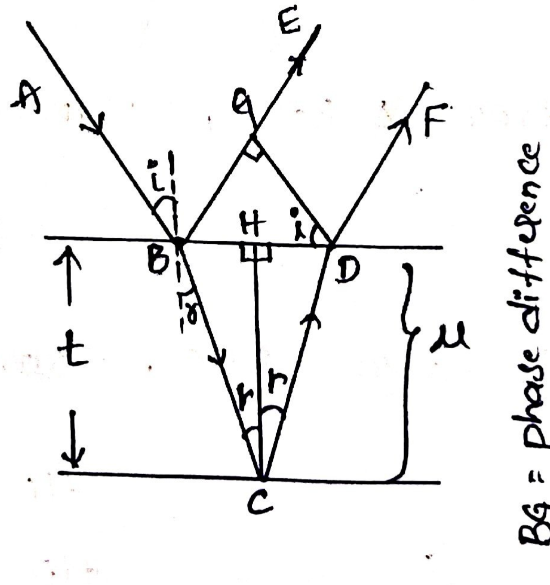
Conditions for Interference :-

1. The two light sources emitting light waves should be coherent.
2. The seperation between the two sources should be small.
3. The distance between the two sources and the screen should be large.
4. To view, interference fringes, the background should be dark.
5. The sources should be narrow i.e they must be small.
6. The sources should be monochromatic.

Interference in thin film by reflection :-

when light is incident on a plane parallel thin film, some portion gets reflected from the upper surface and the remaining portion is transmitted into the film. Again some portion of the transmitted light is reflected back into the film by the lower surface and emerges through the upper surface. These reflected light beams superimpose with each other, producing interference and forming interference patterns.

Consider a transparent plane parallel thin film of thickness 't' with refractive index "μ".



Let a monochromatic light ray "AB" be incident at an angle of incidence of "i".

on the upper Surface of the film. BE and BC are the reflected and transmitted light rays. Let the angle of refraction is "r". They ray "BC" will be reflected into the film and Emerge through the film in the form of the light ray "DF". These two light rays Superimpose and depending upon path difference between them, the produce interference patterns.

To know the path difference, Let us drawn the normal DG to BE. From the points D and G onwards, the light rays travel Equal distances. By the time the light ray travels from B to G, the transmitted ray has to travel from B to C and C to D.

The path difference between light rays (1) and (2) is. path difference $= \mu(BC + CD)$ in film $- BG$ in air.

Consider the ΔBCH, $\cos r = \frac{HC}{BC}$

$$BC = \frac{HC}{\cos r} = \frac{t}{\cos r} \rightarrow ①$$

$||^{ly}$ from ΔDCH, $\cos r = \frac{HC}{CD} \Rightarrow \frac{t}{\cos r} = CD. \rightarrow ②$

From ΔBHC, $\tan r = \frac{BH}{CH} = \frac{BH}{t}$

$$BH = t \tan r \rightarrow ③$$

$||^{ly}$ from ΔDHC, $\tan r = \frac{HD}{HC}$

$$HD = HC \tan r$$

$$HD = t \tan r \rightarrow ④$$

and $BD = BH + HD$

$BD = t\tan r + t\tan r$

$BD = 2t\tan r \rightarrow (5)$

from ΔBGD, $\sin i = \frac{BG}{BD}$

$BG = BD\sin i$

$BG = 2t\tan r\sin i$

From Snell's law, $\mu = \frac{\sin i}{\sin r}$

$\sin i = \mu\sin r$

$BG = 2t\tan r\,\mu\sin r$

$BG = 2\mu t\tan r\sin r \rightarrow (6)$

Substitute BC, CD and BG values in path difference

Then path difference $= \mu\left[\frac{t}{\cos r} + \frac{t}{\cos r}\right] - 2t\tan r\,\mu\sin r$

$= \mu\left[\frac{t+t}{\cos r}\right] - 2\mu t\tan r\sin r$

$= \frac{2\mu t}{\cos r} - 2\mu t\cdot\frac{\sin r}{\cos r}\cdot\sin r$

$= \frac{2\mu t}{\cos r} - \frac{2\mu t\sin^2 r}{\cos r}$

$= \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right]$

$= \frac{2\mu t}{\cancel{\cos r}}\cos^{\cancel{2}} r$

path difference $= 2\mu t\cos r$

At the point B, reflection occurs from the upper surface of the thin film. So first reflected light ray undergoes an additional path difference of $\lambda/2$.

Total path difference $= 2\mu t \cos r + \frac{\lambda}{2}$.

Condition for bright fringe :-

When the path difference is Equal to integral multiples of 'λ' then the reflected rays (1) and (2) meet in phase and undergo constructive interference.

$$2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2}, \quad \text{where } n = 0, 1, 2, 3 \ldots$$

Condition for dark fringe :-

When the path difference is Equal to half integral multiples of 'λ' then the rays (1) & (2) meet in out of phase and undergo destructive interference.

$$2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

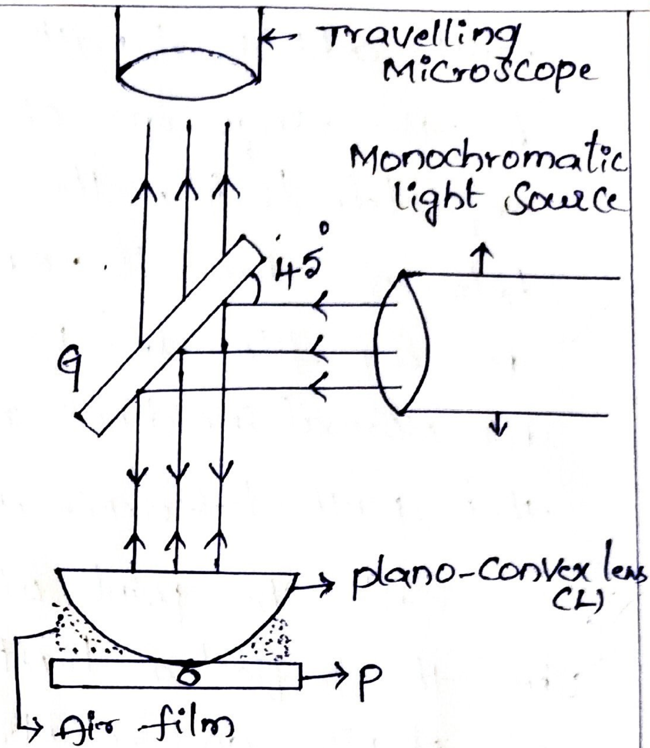
$$2\mu t \cos r = n\lambda, \quad \text{where } n = 0, 1, 2, 3 \ldots$$

Newton's rings :-

The Monochromatic light is allowed to fall normally and the film is viewed in the reflected light, alternate dark and bright concentric rings are observed in the film. These rings are known as Newton's rings.

Experimental Arrangement :-

The plano - convex Lens (L) of Large radius of curvature is placed with its convex surface on a plane glass plate (p). The lens makes the contact with the plate at 'o'. The monochromatic light falls on a glass plate "G" held at an angle of 45°. The glass plate 'G' reflects normally a part of the incident light towards the air film enclosed by the lens "L" and the glass plate "P". A part of the light is reflected by the curved surface of the Lens 'L'. and a part is transmitted which is reflected back from the plane surface of the plate. These reflected rays interfere and give rise to an interference pattern in the form of circular rings. These rings are seen near the upper surface of the air-film through the microscope.

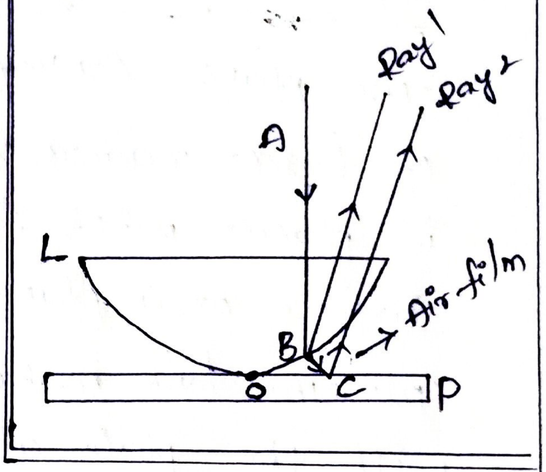


P → plane glass plate
G → Glass plate
O → point of contact

Explanation of Newton's Rings :-

Newton's rings are formed due to inteference between the light rays reflected from the top and bottom surfaces of air film between the plate and the lens.

A part of the incident monochromatic light 'AB' is reflected at 'B' in the form of the ray(1). The other part of the light is refracted along "BC". Then at "C" it is again reflected in the form of ray(2) with additional path difference of $\lambda/2$



As the rings are observed in the reflected light, the path difference between them is $2\mu t \cos r + \lambda/2$

for air film $\mu = 1$

for Normal incidence $r = 0$

then path difference is $2t + \frac{\lambda}{2}$

At the point of contact $t = 0$, path difference is $\lambda/2$ i.e the reflected light ~~at the~~ & incident rays are out of phase and destructive interference. Hence the central spot is dark.

Theory:- To determine the diameter of dark & bright fringes:

Condition for bright ring:

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = (2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3 \ldots$$

Condition for dark ring:

$$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2t = n\lambda, \text{ where } n = 1, 2, 3 \ldots$$

From the property of a Circle

$$NA \times NB = NO \times ND$$

After substituting the values

$$r \times r = t \times (2R - t)$$

$$r^2 = 2Rt - t^2$$

As 't' is small, t^2 will be negligible

$$r^2 = 2Rt$$

$$t = \frac{r^2}{2R}$$

for, bright ring, ∴ the condition is

$$2t = (2n-1)\frac{\lambda}{2}$$

$$2\left(\frac{r^2}{2R}\right) = (2n-1)\frac{\lambda}{2}$$

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

But $r = \frac{D}{2}$, then

$$\left(\frac{D}{2}\right)^2 = \frac{(2n-1)\lambda R}{2}$$

$$\frac{D^2}{4} = \frac{(2n-1)\lambda R}{2}$$

$$D^2 = \frac{\cancel{4}^{2}(2n-1)\lambda R}{\cancel{2}}$$

$$D^2 = (2n-1)\,2\lambda R.$$

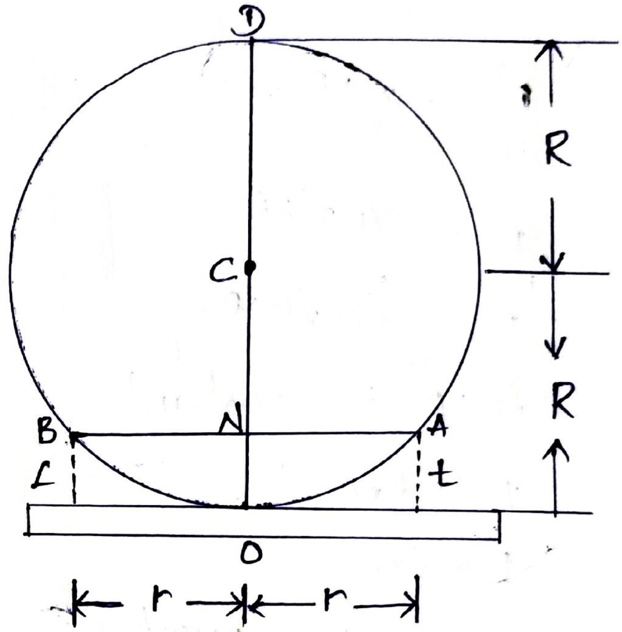
$$D = \sqrt{(2n-1)}\sqrt{2\lambda R}$$

Here $\sqrt{2\lambda R}$ is constant, so

$$D = k\sqrt{2n-1}$$

$$D \propto \sqrt{2n-1}$$

$$D \propto \sqrt{\text{odd natural numbers}}$$



Thus, the diameter of the bright rings are proportional to the square root of odd natural numbers.

for dark ring the condition is:

$$2t = n\lambda$$

$$2\left[\frac{r^2}{2R}\right] = n\lambda$$

$$r^2 = n\lambda R.$$

But $r = \frac{D}{2}$

$$\frac{D^2}{4} = n\lambda R.$$

$$D^2 = 4 n\lambda R$$

$$D = \sqrt{n}\sqrt{4\lambda R}.$$

But, $\sqrt{4\lambda R}$ is constant

$$D = k\sqrt{n}$$

$$D \propto \sqrt{n}$$

$D \propto \sqrt{\text{natural numbers}}$.

Thus, the diameters of dark rings are proportional to the square root of natural numbers.

Determination of wavelength of light :-

Let 'R' be the radius of curvature of a plano-convex lens, 'λ' be the wavelength of light.

Let D_m and D_n are the diameters of m^{th} and n^{th} dark rings respectively. then.

$$D_m^2 = 4m\lambda R, \quad D_n^2 = 4n\lambda R.$$

$$D_n^2 - D_m^2 = 4(n-m)\lambda R$$

$$\Rightarrow \boxed{\lambda = \frac{D_n^2 - D_m^2}{4(n-m)R}}$$

Determination of Radius of curvature of plano convex lens :

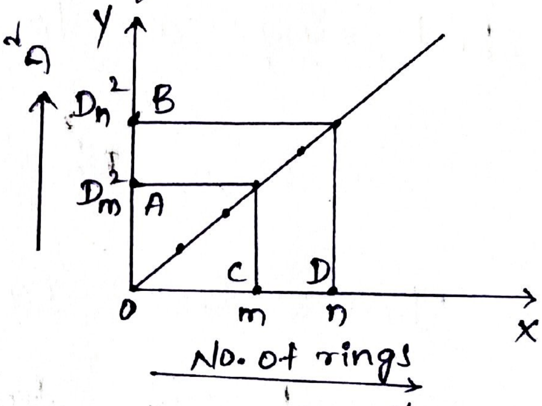
From $D_n^2 - D_m^2 = 4(n-m)\lambda R$.

$$\boxed{R = \frac{D_n^2 - D_m^2}{4(n-m)\lambda}}$$

Experiment:-

After forming the Newton's rings, the microscope is adjusted so that the centre of the cross wire is adjusted at the central dark spot of the ring by counting the number of rings, the microscope is moved to the extreme left of the pattern and the cross wire is adjusted tangentially in the middle of the nth dark ring. The reading of microscope is noted.

Now the microscope is moved to the right and its readings are noted at (n-3) (n-6) ... rings



Again crossing the central dark spot in the same direction, the readings corresponding to (n-6), (n-3) & nth rings are noted.

The difference between left and right readings gives the diameter of the particular rings.

From the Graph :

$$\frac{D_n^2 - D_m^2}{n-m} = \frac{AB}{CD}$$

By substituting the above value in the radius of curvature formula, we get 'R' value.

Determination of Refractive index of a liquid:

The experiment is performed when there is an air film between the glass plate and the plano convex lens. The diameter of m^{th} and n^{th} rings are determined with the help of a travelling microscope.

we have

$$D_n^2 - D_m^2 = 4(n-m)\lambda R \rightarrow (1)$$

The plano convex lens and plane glass plate placed into the container which consists of the liquid whose refractive index (μ) is to be determined.

Now, the air film is replaced by the liquid film. Again, the diameters of the same m^{th} and n^{th} dark rings are to be obtained. Then, we have.

$$D_n'^2 - D_m'^2 = \frac{4(n-m)\lambda R}{\mu} \rightarrow (2)$$

From (1) & (2) Equations

$$\frac{(1)}{(2)} \Rightarrow \frac{D_n^2 - D_m^2}{D_n'^2 - D_m'^2} = \frac{4(n-m)\lambda R}{\frac{4(n-m)\lambda R}{\mu}}$$

$$\boxed{\mu = \frac{D_n^2 - D_m^2}{D_n'^2 - D_m'^2}}$$