Unit-2 Conductors and Dielectrics

→ Behaviour of conductors in an Electric field :-

→ A conductor is a material in which large number of free Electrons possess.

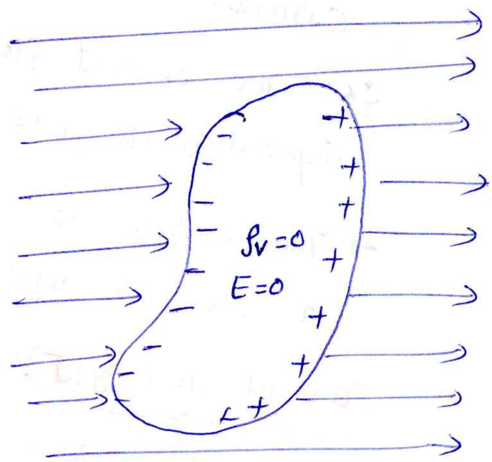
→ Under the effect of applied Electric field, the available free Electrons start moving.

→ If the conductor is placed in an Electro static field, There are no. of Electrons appeared in conductors. The free Electrons are detached from the nucleus and atoms. These atoms becomes positively charged. Both electrons and +vely charged atoms being placed in Electro static field experience some forces.

→ However These forces are opposite in nature and hence the electrons and positively charged atoms travels in opposite direction.

→ If we place an isolated conductor in an Electric field as shown in fig(1).

→ Due to the force exerted by the External applied electric field, the free Electrons in The conductor move in a direction opposite to the Electric field and arranged on The surface conductor, one side -Ve Charges, other side +Ve charges.



Fig(1): Isolated conductor in static field.

Such a separation of charges reforced as induced Charges. These induced charges effect to produce an electric field. The Net Electric field inside a conductor is zero, when the steady state is reached.

$$E = 0, \quad \rho_v = 0$$

→ Thus it implies that, the potential is same at all points in the conductor of material.

→ Thus neither Volume charge density (ρ_v) ~~and~~ nor Electric field intensity ($\bar{E}$) can be maintained with an isolated conductor under static conditions.

→ Current and current Density :-

↳ The current (I) is defined as the rate of flow of charge and is measured in amperes.

↳ **Drift current:** The current which exists in the conductors, due to drifting of electrons, under the influence of the applied voltage is called drift current.

Displacement current or convection current :- In dielectrics, flow of charges under the influence of Electric field Intensity, such a current is called the displacement current (or) convection current.

Ex: The current flowing across the capacitor through the dielectrics separating its plates is an example of convection current.

→ The analysis of such currents in field theory is based on defining a current density.

Current density (J): It is defined as the current passing through the unit surface area, when the surface is held normal to the direction of the current. J is measured in ampere per square metre (A/m^2).

→ Relation between $\bar{J}$ and ρ_v :-

$$\bar{J} = \rho_v \bar{v}.$$

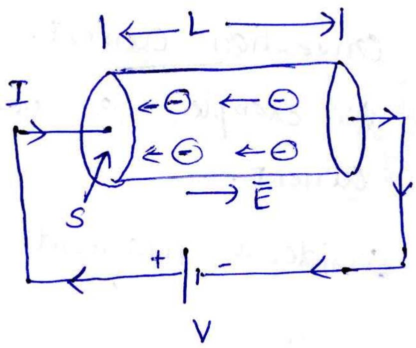
where $\bar{J}$ = current density, ρ_v = current density of volume charge

$\bar{v}$ = velocity vector

→ Conduction current Density and convection current Density

a) Conduction current & conduction current density :-

Let I be the conduction current in the conductor of sectional area 'S.'



If V is the potential difference between two points, The Electric field applied is uniform & its magnitude is given by

$$E = \frac{V}{L} \Rightarrow V = EL = IR \longrightarrow (1)$$

The Conductor has uniform cross section 'S' and hence we can write, its Resistance is given as

$$R = \frac{L}{\sigma S} \longrightarrow (2)$$

$$\cancel{I = \oint_S \bar{J}\cdot d\bar{S}}$$

where σ = conductivity
S = cross section area
L = length of conductor

sub eq (2) in eq (1);

$$EL = I\left(\frac{L}{\sigma S}\right)$$

$$E = \frac{I}{\sigma S} = \frac{J}{\sigma} \qquad \left(\because J = \frac{I}{S}\right)$$

$$\therefore E = \frac{J}{\sigma} \Rightarrow \boxed{J = \sigma E}$$

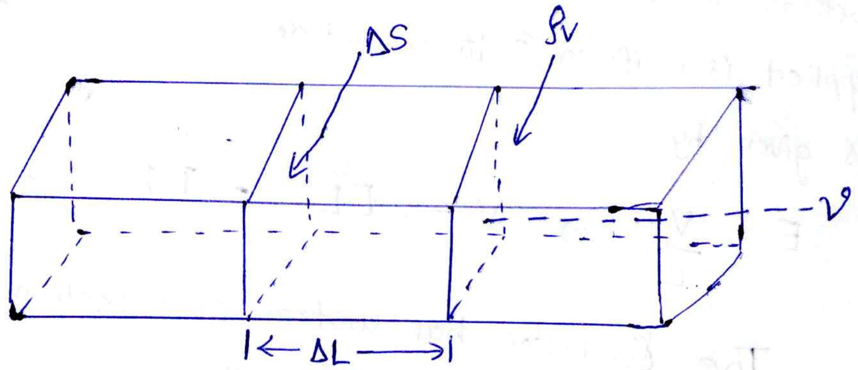
where J = conduction current density.

b) Convection current & convection current density :-

In dielectrics, flow of charges under the influence of Electric field intensity. such a current is called displacement current (or) convection current.

for Example, a beam of Electrons in a Vacuum tube is a convection current.

Consider a filament as shown in fig(2).



Fig(2):- Filament with flow of charge

If the flow of charge of density ρ_v at velocity $v_y = a$

Then current flow through the filament is.

$$\Delta I = \frac{\Delta Q}{\Delta t}$$

but $\Delta Q = \rho_v \, \Delta S \, \Delta L$

$$\therefore \Delta I = \rho_v \, \Delta S \, \frac{\Delta L}{\Delta t} = \rho_v \, \Delta S \, v_y$$

Then the y-directed current density J_y is given by

$$J_y = \frac{\Delta I}{\Delta S} = \frac{\rho_v \, \Delta S \, v_y}{\Delta S} = \rho_v \, v_y$$

Hence in generalized form $\boxed{J = \rho_v \bar{v}}$

where J = convection current density

$\bar{v}$ = velocity vector

Continuity Equation in Integral form and Point form :-

The continuity equation of the current is based on the principle of conservation of charge. The principle states that,

The charges can neither be created nor be destroyed.

→ consider a closed surface S, with a current density $\bar{J}$, then

The total current I crossing the surface S is given by,

$$I = \oint_S \bar{J} \cdot d\bar{s} \longrightarrow (1)$$

The current flows outwards from the closed surface. It is known that the current means the flow of positive charges.

A/c to principle of Conservation of charge, there must be decrease of an equal amount of positive charges inside the surface.

Hence the outward rate of flow of positive charges gets balanced by The rate of decrease of charge inside the surface.

Let Q_i = Charge within the closed surface

$-\frac{dQ_i}{dt}$ = Rate of decrease of charge inside the closed surface.

-ve sign indicates decrease in charge.

$$\therefore \boxed{I = \oint_S \bar{J} \cdot d\bar{s} = -\frac{dQ_i}{dt}} \longrightarrow (2)$$

Eq (2) is Integral form of continuity Equation of current.

'-ve' sign indicates the outward flow of current from closed surface.

for Point form :

using divergence theorem, convert the surface integral in integral form to volume integral.

$$\oint_S \bar{J} \cdot d\bar{s} = \int_{Vol} (\nabla \cdot \bar{J})\, dv.$$

$$-\frac{dQ_i}{dt} = \int_{Vol} (\nabla\cdot\bar{J})\, dv$$

But $Q_i = \int_{Vol} \rho_v \, dv$, where ρ_v = Volume charge density

$$\therefore \int_{Vol} (\nabla\cdot\bar{J})\, dv = -\frac{d}{dt}\left[\int_{Vol} \rho_v \, dv\right] = -\int_{Vol} \frac{\partial \rho_v}{\partial t}\, dv.$$

For constant surfaces, the derivative becomes partial derivative.

$$\int_{Vol} (\nabla\cdot\bar{J})\, dv = \int_{Vol} -\frac{\partial \rho_v}{\partial t}\, dv.$$

for incremental volume ΔV,

$$\oint_{Vol} (\nabla\cdot\bar{J})\, \Delta V = -\frac{\partial \rho_v}{\partial t}\, \Delta V.$$

$$\boxed{\nabla\cdot\bar{J} = -\frac{\partial \rho_v}{\partial t}} \longrightarrow (3)$$

This is the point form (or) differential form of continuity equation of the current.

→ For steady currents, $\boxed{\nabla\cdot\bar{J} = 0}$ ⟶ (4) (since $\frac{\partial \rho_v}{\partial t} = 0$)

→ **Classification of ~~Conductor~~ Materials : (or)**

Conductors and Dielectrics (or) Insulators :-

On the basis of the behaviour under the influence of External electric field, we divide the Materials as far as their Electric properties are concerned, into two groups :

1) conductors and 2) Insulators (or) dielectrics.

(1) Conductors : A conductor is a material in which a large number of free electrons possesses. Such materials in which an electric field produces a steady drift current through them are called Conductors. Examples are metals, carbon etc.

(2) Insulators (or) Dielectrics : The materials in which the electrons are strongly bound to the atoms and cannot be detached by the application of an electric field. such materials are called Insulators or Dielectrics.

examples : sulphur, porcelain, mica etc.

(3) Semiconductors : semiconductors is the third class of materials, and have properties intermediate between conductors and Insulators.

→ Point form of ohm's law :-

. under the effect of applied electric field, the available free electrons start moving. After some time, the electrons attain the constant average velocity is called drift velocity (V_d). The current constituted due to drifting of such electrons in metallic conductors is called drift current.

→ The drift velocity is directly proportional to applied electric field.

$$V_d \propto \bar{E} \longrightarrow (1)$$

$$V_d = -\mu_e \bar{E} \longrightarrow (2)$$

constant of proportionality is called mobility of electrons.

→ According to relation between $\bar{J}$ and $\bar{V}$, we can write,

$$\bar{J} = \rho_v \bar{V} \longrightarrow (3)$$

The above relation can be expressed as

$$\bar{J} = \rho_e \bar{V}_d \longrightarrow (4)$$

where ρ_e = charge density due to free electrons.

$= ne$, where n is no. of free electrons

Substitute Eq (2) in eq (4), we get

$$\bar{J} = -\rho_e \mu_e \bar{E} \longrightarrow (5)$$

$$\boxed{\bar{J} = \sigma \bar{E}} \longrightarrow (6)$$

where σ = conductivity of material $= -\rho_e \mu_e$.

Eq (6) is called Point form of ohm's law.

$\bar{J}$ = conduction current density.

→ Resistance of a Conductor :-

Consider that Voltage V is applied to a conductor of length L having uniform cross section S, as shown in fig.

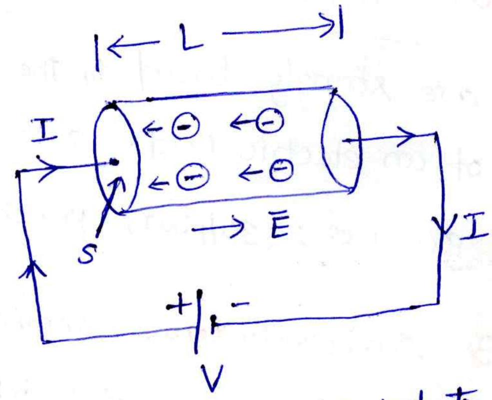


Fig: conductor subjected to Voltage V.

The applied Electric field is uniform and its magnitude is

$$E = \frac{V}{L} \longrightarrow (1)$$

The conductor has uniform cross section S, and hence

$$I = \int_S \bar{J} \cdot d\bar{S} = JS. \longrightarrow (2).$$

$$\text{Thus, } J = \frac{I}{S} = \sigma E \longrightarrow (3).$$

substitute eq (1) in (3),

$$J = \sigma \frac{V}{L}, \quad \text{where } \sigma = \text{conductivity of the material.}$$

$$V = \frac{JL}{\sigma} = \frac{IL}{S\sigma} = \left(\frac{L}{\sigma S}\right) I.$$

$$\therefore \frac{V}{I} = \frac{L}{\sigma S}$$

$$\therefore \boxed{R = \frac{V}{I} = \frac{L}{\sigma S}.} \longrightarrow (4)$$

Thus the ratio of potential difference between two ends of conductors to the current flowing is the resistance of conductor.

eq (4) is ohm's law in normal form, and it is true for uniform fields. and resistance is measured in ohms (Ω).

For Non-uniform fields,

$$R = \frac{V_{ab}}{I} = \frac{-\int_b^a \bar{E} \cdot d\bar{L}}{\int_S \bar{J} \cdot d\bar{S}} = \frac{-\int_b^a \bar{E} \cdot d\bar{L}}{\int_S \sigma \bar{E} \cdot d\bar{S}}.$$

The numerator is a line integration giving potential difference across two ends, while denominator is a surface integration giving current flowing through the material.

The resistance also be expressed as,

$$\boxed{R = \frac{L}{\sigma S} = \frac{\rho_c L}{S}} \ \Omega.$$

where $\rho_c = \frac{1}{\sigma}$ = Resistivity of conductor, Ωm

Properties of a conductor:

1) Under static conditions, no charge and no electric field can exist at any point within the conducting material.
2) The charge can exist on the surface of the conductor giving rise to surface charge density.
3) Within a conductor, the charge density is always zero.
4) The charge distribution on the surface depends on the shape of surface
5) The conductivity of an ideal conductor is infinite
6) The conductor surface is an Equipotential surface.

→ Relaxation Time (τ):

Consider a conducting material which is linear & homogeneous.

The current density for such a material is,

$$\bar{J} = \sigma \bar{E} \quad , \quad \text{where } \sigma = \text{conductivity}$$

$$\bar{D} = \varepsilon \bar{E} \quad , \quad \text{for linear material}$$

$$\bar{E} = \frac{\bar{D}}{\varepsilon}$$

$$\therefore \quad \bar{J} = \sigma \frac{\bar{D}}{\varepsilon} = \frac{\sigma}{\varepsilon} \bar{D}$$

The point form of continuity equation states that

$$\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}$$

$$\nabla \cdot \left(\frac{\sigma}{\varepsilon} \bar{D} \right) = -\frac{\partial \rho_v}{\partial t}$$

$$\frac{\sigma}{\varepsilon} \nabla \cdot \bar{D} = -\frac{\partial \rho_v}{\partial t}$$

$$\frac{\sigma}{\varepsilon} \rho_v = -\frac{\partial \rho_v}{\partial t} \qquad (\because \rho_v = \nabla \cdot \bar{D})$$

$$\therefore \quad \frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon} \rho_v = 0$$

This is differential equation in ρ_v, whose solution is given by,

$$\rho_v = \rho_0 \, e^{-(\sigma/\varepsilon)t} = \rho_0 \, e^{-t/\tau}$$

where ρ_0 = charge density at $t=0$.

$\tau = \frac{\varepsilon}{\sigma}$ sec. = relaxation time.

∴ Relaxation Time (τ): is defined as time required by charge density to decay to 36.8 % of its initial value.

$$\boxed{\tau = \text{relaxation time} = \frac{\varepsilon}{\sigma} \ \text{sec}}$$

Note: Homogeneous: When the physical characteristics of medium donot vary from point to point then the medium is called homogeneous.

Linear: If flux density $\bar{D}$ is directly proportional to electric field $\bar{E}$, the material is linear.

→ Electric field Inside a dielectric (or) Polarization of dielectric :-

The charges in dielectrics are bound by finite forces called bound charges. They cannot contribute to conduction process.

→ The application of an Electric field will shift the positive and negative charges in opposite directions within the dielectric material gives rise to 'dipole'.

→ when the dipole results from the displacement of bound charges the dielectric is said to be 'polarized'.

→ ∴ The process due to which separation of bound charges results to produce Electric dipoles, under the influence of Electric field $\bar{E}$ is called polarization.

Polarization is defined as the total dipole moment per unit volume.

Concept of polarization:

Consider an atom of dielectric, which consists of a nucleus with positive charge and negative charges in the form of revolving electrons in the orbit. The negative charge is considered to be in the form of cloud of Electrons as shown in fig (1).

Here, applied $\bar{E}$ is Zero, so the no. of +ve charges is same as -ve charges and hence atom is electrically neutral. Both the charges are coinciding at the centre.

Hence there cannot exist an electric dipole. This is unpolarized atom.

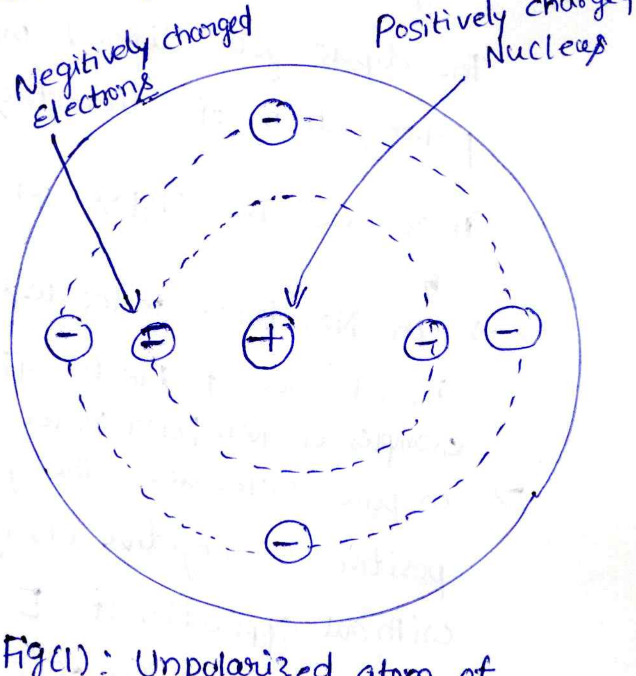
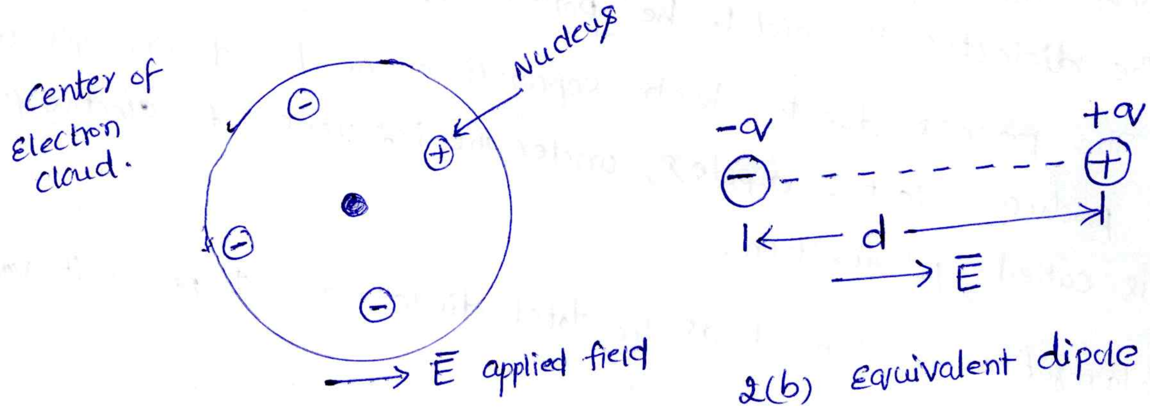


Fig (1): Unpolarized atom of dielectric.

→ When Electric field $\bar{E}$ is applied, the symmetrical distribution of charges gets disturbed. The +ve charges experience a force $\bar{F} = Q\bar{E}$ and -ve charges experience a force $\bar{F} = -Q\bar{E}$ in the opposite direction.

→ Now there is separation b/w the nucleus and the centre of electron cloud. as shown in fig (2). Such an atom is called Polarized atom.

→



2(a) Polarized atom.

2(b) Equivalent dipole

Fig(2): Polar Dielectric

→ It can be seen that an Electron Cloud has a Centre separated from the nucleus. This forms the electric dipole. The equivalent dipole is shown in fig (2b).

The dipole gets aligned with the applied field. This process is called polarization of dielectrics.

There are Two types of dielectrics : 1) Non Polar 2) Polar.

→ In Non Polar molecules, the dipole arrangement is totally absent in absence of Electric field $\bar{E}$.

examples of Non Polar molecules are Hydrogen, oxygen and rare gases.

→ In polar molecules, the permanent displacements between centres of positive & negative charges exist. Thus dipole arrangement exist without application of $\bar{E}$ and such dipoles are randomly oriented. under the application of $\bar{E}$, the dipole experiences Torque & align with the direction of $\bar{E}$. This is called polarization of polar molecules.

Examples of polar molecule are water, sulphur dioxide, HCL etc.

Mathematical Expression for polarization :-

When dipole is formed due to polarization, there exists an electric dipole moment $\bar{P}$.

$$\bar{P} = Q\,\bar{d} \longrightarrow (1)$$

where $\bar{P}$ = Dipole moment
Q = magnitude of one of the two charges.
$\bar{d}$ = Distance vector from negative to positive charge.

Let n = No. of dipoles per unit volume.
ΔV = Total Volume of the dielectric.
N = Total dipoles = $n\,\Delta V$.

Then The Total dipole moment is to be obtained using Superposition principle as,

$$\bar{P}_{total} = Q_1\bar{d}_1 + Q_2\bar{d}_2 + \cdots + Q_n\bar{d}_n = \sum_{i=1}^{n} Q_i d_i \longrightarrow (2)$$

If dipoles are randomly oriented, then $\bar{P}_{total}$ is zero. But if dipoles are aligned in the direction of applied $\bar{E}$, then $\bar{P}_{total}$ has a significant value.

The Polarization $\bar{P}$ is defined as the total dipolemoment per unit Volume.

$$\therefore \quad \bar{P} = \lim_{\Delta V \to 0} \frac{\sum_{i=1}^{n} Q_i d_i}{\Delta V} \quad , \quad C/m^2 \longrightarrow (3)$$

Flux density in dielectric is written as

$$\bar{D} = \varepsilon_0 \bar{E} + \bar{P} \longrightarrow (4)$$

For isotropic and linear medium, the $\bar{P}$ and $\bar{E}$ are parallel to each other at every point and related to each other as,

$$\bar{P} = \chi_e\,\varepsilon_0\,\bar{E} \longrightarrow (5)$$

where χ_e = Dimensionless quantity called electric susceptibility of the material.

Substitute Eq (5) in Eq (4)

$$\bar{D} = \varepsilon_0 \bar{E} + \chi_e \varepsilon_0 \bar{E}$$

$$\bar{D} = (\chi_e + 1) \varepsilon_0 \bar{E} \longrightarrow (6).$$

we know, $\bar{D} = \varepsilon \bar{E}$, where $\varepsilon = \varepsilon_R \varepsilon_0$.

$$\bar{D} = \varepsilon_R \varepsilon_0 \bar{E} \longrightarrow (7).$$

from Eq (6) & (7), we can write

$$\therefore \varepsilon_R = \chi_e + 1 \longrightarrow (8).$$

The quantity $\chi_e + 1$ is defined as relative permitivity or dielectric constant of dielectric material.

Properties of dielectric materials :-

1) The dielectrics do not contain any free charges but contain bound charges.
2) Bound charges are under the internal molecular & atomic forces and cannot contribute to the conduction.
3) When subjected to an external field $\bar{E}$, the bound charges shift their relative positions. Due to this small electric dipoles gets induced inside the dielectric. This is called polarization.
4) Due to the polarization, the dielectrics can store the energy
5) Due to polarization, flux density of dielectric increases by amount equal to polarization.
6) The induced dipoles produce their own electric field & align in the direction of applied electric field.

7) When polarization occurs, the volume charge density is formed inside the dielectric while the surface charge density is formed over the surface of dielectric.

8) The Electric field outside and inside the dielectric gets modified due to induced electric dipoles.

→ Dielectric strength and Dielectric constant :-

Dielectric constant : Dielectric constant (or) Relative permitivity ε_r is the ratio of permitivity of dielectric to that of free space

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = 1 + \chi_e$$

Dielectric strength(E): It is the maximum Electric field that a dielectric can tolerate or withstand without electrical breakdown. The dielectric strength is measured in V/m.

→ Boundary conditions :-

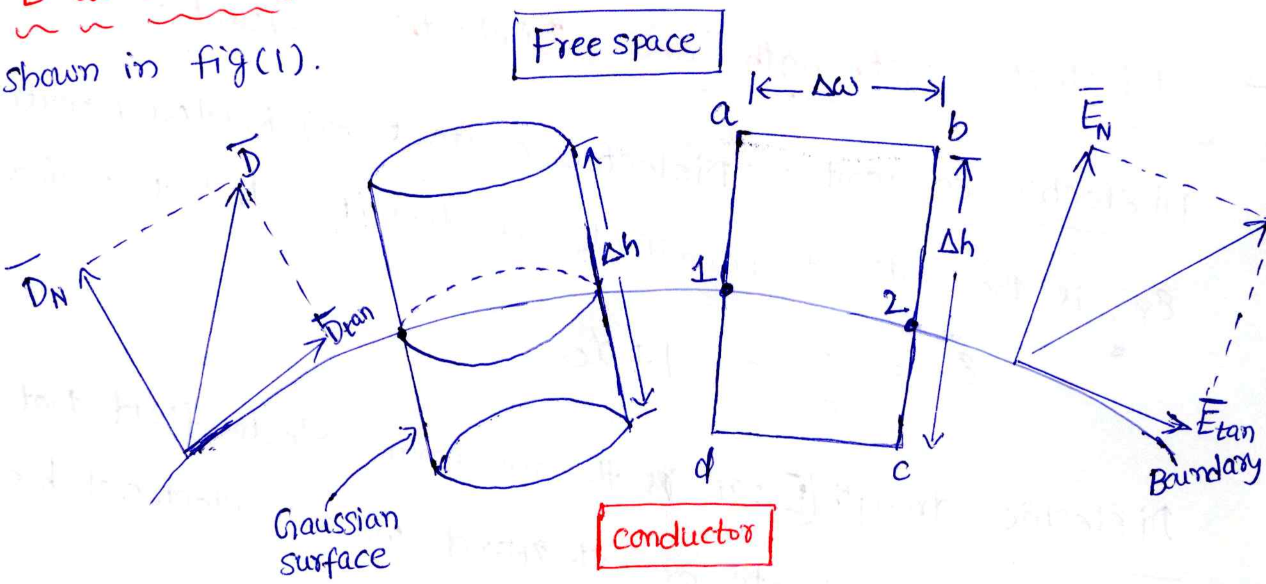
The conditions existing at the boundary of two mediums when field passes from one medium to other are called boundary conditions.

1) Boundary between conductor and free space
2) Boundary between conductor and dielectric
3) Boundary between two dielectrics.

1) Boundary conditions between conductor and Freespace :-

Consider a boundary between conductor and free space. The conductor is ideal having infinite conductivity. Such conductors are copper, silver etc.

$\bar{E}$ at boundary : Consider the conductor & free space boundary as shown in fig(1).



Fig(1) : Boundary between conductor and Freespace.

Let $\bar{E}$ be the Electric field Intensity in the direction shown in fig(1), making some angle with the boundary.

This $\bar{E}$ can be resolved into two components :

(1) the component tangential to the surface ($\bar{E}_{tan}$).

(2) The Component normal to the surface ($\bar{E}_N$).

It is Known that $\oint \bar{E}\cdot d\bar{L} = 0$. ⟶ (1)

i.e, Workdone in moving a unit positive charge along closed path is zero.

→ Consider a rectangular closed path 'abcda' as shown in fig (1). It is traced in clockwise direction as a-b-c-d-a and hence $\oint \bar{E}\cdot d\bar{L}$ can be divided into four parts.

$$\oint \bar{E}\cdot d\bar{L} = \int_a^b \bar{E}\cdot d\bar{L} + \int_b^c \bar{E}\cdot d\bar{L} + \int_c^d \bar{E}\cdot d\bar{L} + \int_d^a \bar{E}\cdot d\bar{L} = 0 \longrightarrow (2).$$

The closed contour is placed in such a way that its two sides a-b and c-d are parallel to the tangential direction to the surface while the other two are normal to the surface at the boundary.

→ The rectangle is an elementary rectangle with elementary height Δh and elementary width Δw.

→ The rectangle is placed in such a way that half of it is in the conductor and remaining half is in free space. Thus $\frac{\Delta h}{2}$ is in conductor and $\frac{\Delta h}{2}$ is in free space.

→ Now the boundary portion c-d is in the conductor, where $\bar{E} = 0$, hence the corresponding integral is zero.

$$\therefore \int_a^b \bar{E}\cdot d\bar{L} + \int_b^c \bar{E}\cdot d\bar{L} + \int_a^d \bar{E}\cdot d\bar{L} = 0. \longrightarrow (3)$$

As the width Δw is very small, $\bar{E}$ over it can be assumed constant & hence can be taken out integration.

$$\therefore \int_a^b \bar{E}\cdot d\bar{L} = \bar{E}\int_a^b d\bar{L} = \bar{E}(\Delta w). \longrightarrow (4).$$

But Δw is along tangential direction to the boundary in which direction $\bar{E} = \bar{E}_{tan}$.

$$\therefore \int_a^b \bar{E}\cdot d\bar{L} = E_{tan}(\Delta w), \quad \text{where } E_{tan} = |\bar{E}_{tan}|. \longrightarrow (5).$$

Now b-c is parallel to the normal component, so we have $\bar{E} = \bar{E}_N$ along this direction.

let $E_N = |\bar{E}_N|$.

over the small height Δh, E_N can be assumed constant & can be taken out of integration.

$$\therefore \int_b^c \bar{E}\cdot d\bar{L} = \bar{E}\int_b^c d\bar{L} = E_N\int_b^c d\bar{L}. \longrightarrow (6).$$

But out of b-c, b-2 is in free space and 2-c is in the conductor where $\bar{E} = 0$.

$$\therefore \int_b^c d\bar{L} = \int_b^2 d\bar{L} + \int_2^c d\bar{L} = \frac{\Delta h}{2} + 0 = \frac{\Delta h}{2} \longrightarrow (7).$$

$$\therefore \int_b^c \bar{E} \cdot d\bar{L} = E_N\left(\frac{\Delta h}{2}\right) \longrightarrow (8)$$

Similarly for path d-a, the condition is same as for the path b-c, only direction is opposite.

$$\therefore \int_d^a \bar{E} \cdot d\bar{L} = -E_N\left(\frac{\Delta h}{2}\right) \longrightarrow (9).$$

Substitute eq (4), eq (8) & eq (9) in eq (3), we get.

$$\therefore E_{tan}\,\Delta W + E_N\left(\frac{\Delta h}{2}\right) - E_N\left(\frac{\Delta h}{2}\right) = 0. \longrightarrow (10)$$

$E_{tan}\,\Delta W = 0.$ but $\Delta W \neq 0$ as finite.

$$\therefore \boxed{E_{tan} = 0} \longrightarrow (11).$$

Thus the Tangential component of the electric field Intensity is zero at the boundary between conductor & free space.

Now $\bar{D} = \varepsilon_0 \bar{E}$ for free space

$$\boxed{D_{tan} = \varepsilon_0 E_{tan} = 0} \longrightarrow (12).$$

Thus the Tangential Component of Electric flux density is zero at the boundary between conductor and free space.

D_N at the boundary :

→ To find normal component of $\bar{D}$, select a closed Gaussian surface in the form of right circular cylinder as shown in fig(1).

→ Its height is Δh and is placed in such a way that $\frac{\Delta h}{2}$ is in the conductor and remaining $\frac{\Delta h}{2}$ is in the freespace. Its axis is in the normal direction to the surface.

→ According to Gauss's law, $\oint_S \bar{D}\cdot d\bar{s} = Q$.

The surface integral must be evaluated over three surfaces,
(i) Top (ii) Bottom and (iii) Lateral.

Let the area of top and bottom is same equal to ΔS.

$$\therefore \int_{top} \bar{D}\cdot d\bar{s} + \int_{bottom} \bar{D}\cdot d\bar{s} + \int_{lateral} \bar{D}\cdot d\bar{s} = Q. \longrightarrow (13).$$

The bottom surface is in the conductor where $\bar{D}=0$, hence corresponding integral is zero.

$$\therefore \int_{top} \bar{D}\cdot d\bar{s} + \int_{lateral} \bar{D}\cdot d\bar{s} = Q. \longrightarrow (14).$$

The lateral surface is $2\pi r \Delta h$ where r is the radius of cylinder. But as $\Delta h \to 0$, this lateral surface area reduces to zero & corresponding integral is zero.

→ Only component of $\bar{D}$ present is the normal component having magnitude D_N. The top surface is very small over which D_N can be assumed constant and can be taken out of integration.

$$\therefore \int_{top} \bar{D}\cdot d\bar{s} = D_N \int_{top} d\bar{s} = D_N \Delta S. \longrightarrow (15)$$

From Gauss's law,

$$D_N \Delta S = Q. \longrightarrow (16)$$

But at the boundary, the charge exists in the form of surface charge density ρ_s C/m².

$$Q = \rho_s \, \Delta S. \longrightarrow (17).$$

Equating eq (16) and eq (17).

$$D_N \, \Delta S = \rho_s \, \Delta S$$

$$\therefore \boxed{D_N = \rho_s.} \longrightarrow (18)$$

Thus the flux leaves the surface normally and the normal component of flux density is equal to the surface charge density.

$$D_N = \varepsilon_0 E_N = \rho_s \longrightarrow (19)$$

$$\therefore \boxed{E_N = \frac{\rho_s}{\varepsilon_0}} \longrightarrow (20).$$

2) Boundary conditions between Conductor and Dielectric :-

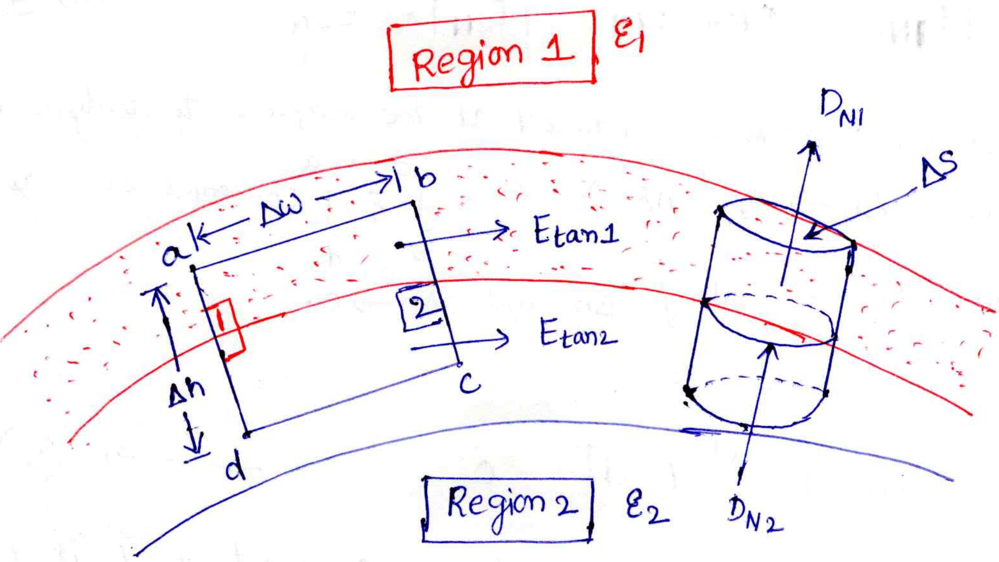
The free space is a dielectric with $\varepsilon = \varepsilon_0$.
Thus if the boundary is between conductor and dielectric with $\varepsilon = \varepsilon_0 \varepsilon_r$.

$$\therefore E_{tan} = D_{tan} = 0.$$

$$D_N = \rho_s.$$

and $$E_N = \frac{\rho_s}{\varepsilon} = \frac{\rho_s}{\varepsilon_0 \varepsilon_r}.$$

→ Boundary conditions between Two perfect Dielectrics :-



Fig(1): Boundary between Two perfect dielectrics.

Let us consider the boundary between two perfect dielectrics one dielectric has permitivity ε_1 while the other has permitivity ε_2.

The interface is shown in fig(1).

$\bar{E}$ & $\bar{D}$ are to be obtained by resolving Each into two components, tangential to the boundary and normal to surface.

consider a closed path abcda rectangular in shape having elementary height Δh & Elementary width Δw, as shown in fig(1).

It is placed in such a way that $\frac{\Delta h}{2}$ is in dielectric 1 & the remaining in dielectric 2.

Let us evaluate the integral of $\bar{E}\cdot d\bar{l}$ along this path, tracing it in clockwise direction as a-b-c-d-a.

$$\oint \bar{E}\cdot d\bar{l} = 0. \longrightarrow (1)$$

$$\therefore \int_a^b \bar{E}\cdot d\bar{l} + \int_b^c \bar{E}\cdot d\bar{l} + \int_c^d \bar{E}\cdot d\bar{l} + \int_d^a \bar{E}\cdot d\bar{l} = 0. \longrightarrow (2)$$

Now $\bar{E}_1 = \bar{E}_{1t} + \bar{E}_{1N} \longrightarrow (3)$

and $\bar{E}_2 = \bar{E}_{2t} + \bar{E}_{2N} \longrightarrow (4)$

Let $|\bar{E}_{1t}| = E_{tan1}$, $|\bar{E}_{2t}| = E_{tan2}$ → (3)

$|\bar{E}_{1N}| = \cancel{E_{tan2}}\ E_{1N}$ $|\bar{E}_{2N}| = E_{2N}$ → (4)

Now for rectangle to be reduced at the surface to analyse boundary conditions, $\Delta h \to 0$, As $\Delta h \to 0$, $\int_b^c$ & $\int_d^a$ becomes zero as these are line integrals, along Δh and $\Delta h \to 0$.

Hence eq (2) reduces to

$$\int_a^b \bar{E} \cdot d\bar{l} + \int_c^d \bar{E} \cdot d\bar{l} = 0. \longrightarrow (5)$$

Now a-b is in dielectric 1, hence component of $\bar{E}$ is E_{tan1} as a-b direction is tangential to the surface.

$$\int_a^b \bar{E} \cdot d\bar{l} = E_{tan1} \int_a^b d\bar{l} = E_{tan1}\ \Delta\omega. \longrightarrow (6)$$

while c-d is in dielectric 2, hence component of $\bar{E}$ is E_{tan2} as c-d direction is also tangential to the surface. But c-d is opposite to a-b hence the corresponding integral is negative of the integral obtained for path a-b.

$$\int_c^d \bar{E} \cdot d\bar{l} = -E_{tan2}\ \Delta\omega \longrightarrow (7)$$

Substituting eq (6) & eq (7) in eq (5), we get,

$$E_{tan1}\ \Delta\omega - E_{tan2}\ \Delta\omega = 0. \longrightarrow (8)$$

$$\therefore \boxed{E_{tan1} = E_{tan2}} \longrightarrow (9)$$

Thus the Tangential components of field intensity at boundary in both dielectrics remain same. i.e., Electric field intensity is continuous across boundary.

The relation between $\bar{D}$ and $\bar{E}$ is

$$\bar{D} = \varepsilon \bar{E} . \longrightarrow (10)$$

Hence if D_{tan1} and D_{tan2} are magnitudes of Tangential components of $\bar{D}$ in dielectric 1 & 2 respectively, then

$$D_{tan1} = \varepsilon_1 E_{tan1} \quad \text{and} \quad D_{tan2} = \varepsilon_2 E_{tan2} . \longrightarrow (11)$$

$$\frac{D_{tan1}}{\varepsilon_1} = E_{tan1} \quad \text{and} \quad \frac{D_{tan2}}{\varepsilon_2} = E_{tan2}$$

from eq (9),

$$\therefore \frac{D_{tan1}}{\varepsilon_1} = \frac{D_{tan2}}{\varepsilon_2} \longrightarrow (12)$$

$$\therefore \boxed{\frac{D_{tan1}}{D_{tan2}} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}} \longrightarrow (13)$$

Thus Tangential components of $\bar{D}$ undergoes some change across the interface the boundary. hence tangential $\bar{D}$ is said to be discontinuous across the boundary.

→ To find Normal components :

Let us use Gauss's law. Consider the Gaussian surface in the form of right circular cylinder, placed in such a way that half of it lies in dielectric 1 while the remaining half in dielectric 2.

→ The height $\Delta h \to 0$ hence flux leaving from its lateral surface is zero. The surface area at its top & bottom is ΔS.

$$\therefore \oint \bar{D} \cdot d\bar{S} = Q . \longrightarrow (14).$$

$$\left[\int_{top} + \int_{bottom} + \int_{lateral} \right] \bar{D} \cdot d\bar{S} = Q . \longrightarrow (15)$$

$$\text{But} \int_{lateral} \bar{D} \cdot d\bar{S} = 0 \quad \text{as} \quad \Delta h \to 0 . \longrightarrow (16)$$

$$\int_{top} \bar{D} \cdot d\bar{S} + \int_{bottom} \bar{D} \cdot d\bar{S} = Q . \longrightarrow (17)$$

The flux leaving normal to boundary is normal to the top & bottom surfaces.

$\therefore |\bar{D}| = D_{N1}$ for dielectric 1 & D_{N2} for dielectric 2.

As top & bottom surfaces are elementary, flux density can be assumed as constant and can be taken out of integration.

$$\therefore \int_{top} \bar{D} \cdot d\bar{s} = D_{N1} \int_{top} d\bar{s} = D_{N1} \Delta S. \longrightarrow (18)$$

for Top surfaces, the direction of D_N is entering the boundary while for bottom surface, the direction of D_N is leaving the boundary. Both are in opposite direction at the boundary.

$$\therefore \int_{bottom} \bar{D} \cdot d\bar{s} = - D_{N2} \int_{bottom} d\bar{s} = -D_{N2} \Delta S \longrightarrow (19)$$

$$\therefore D_{N1} \Delta S - D_{N2} \Delta S = Q. \longrightarrow (20)$$

But $Q = \rho_s \Delta S. \longrightarrow (21)$

$$\therefore D_{N1} - D_{N2} = \rho_s. \longrightarrow (22)$$

At ideal diectric media, boundary, the surface charge density is assumed as zero.

$$\rho_s = 0.$$

$$D_{N1} - D_{N2} = 0$$

$$\boxed{D_{N1} = D_{N2}} \longrightarrow (23)$$

Hence The Normal component of flux density $\bar{D}$ is continuous at the boundary between two perfect dielectrics.

Now $D_{N1} = \varepsilon_1 E_{N1}$ and $D_{N2} = \varepsilon_2 E_{N2}$.

$$\frac{D_{N1}}{D_{N2}} = \frac{\varepsilon_1 E_{N1}}{\varepsilon_2 E_{N2}} = 1. \quad \text{as } D_{N1} = D_{N2}$$

$$\therefore \boxed{\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}} \longrightarrow (24).$$

$\therefore$ The Normal components of electric field intensity $\bar{E}$ are inversly proportional to relative Permitivities of two media.

→ Refraction of $\overline{D}$ at the Boundary :

The directions of $\overline{D}$ & $\overline{E}$ change at the boundary between two dielectrics.

Let $\overline{D_1}$ & $\overline{E_1}$ make an angle θ_1 with the normal to the surface. $\overline{D_1}$ & $\overline{E_1}$ direction is same as $\overline{D_1} = \varepsilon \overline{E_1}$. This is shown in fig(2)

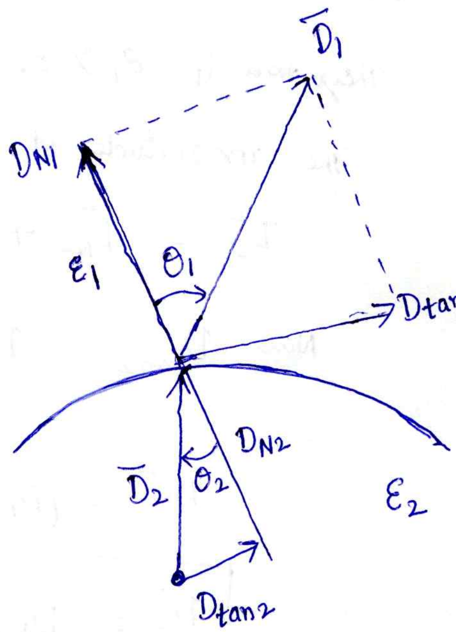


Fig (2): Refraction of $\overline{D}$.

Let $|\overline{D_1}| = D_1$ & $|\overline{D_2}| = D_2$

$$\therefore \quad D_{N1} = D_1 \cos\theta_1 \longrightarrow (25)$$

$$D_{N2} = D_2 \cos\theta_2 \longrightarrow (26)$$

but $D_{N1} = D_{N2}$.

$$\therefore \quad D_1 \cos\theta_1 = D_2 \cos\theta_2 \longrightarrow (27)$$

while $\dfrac{D_{tan1}}{D_{tan2}} = \dfrac{\varepsilon_1}{\varepsilon_2}$

But $D_{tan1} = D_1 \sin\theta_1$ & $D_{tan2} = D_2 \sin\theta_2$.

$$\therefore \quad \frac{D_1 \sin\theta_1}{D_2 \sin\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{D_{tan1}}{D_{tan2}} \longrightarrow (28)$$

Now, $\tan\theta_1 = \dfrac{D_{tan1}}{D_{N1}}$ & $\tan\theta_2 = \dfrac{D_{tan2}}{D_{N2}}$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{D_{tan1}}{D_{tan2}} \frac{D_{N2}}{D_{N1}}$$

but $D_{N1} = D_{N2}$.

$$\therefore \quad \frac{\tan\theta_1}{\tan\theta_2} = \frac{D_{tan1}}{D_{tan2}} = \frac{\varepsilon_1}{\varepsilon_2} \longrightarrow (29)$$

$$\therefore \quad \boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}} \longrightarrow (30)$$

This is called Law of Refraction.

Thus the angles θ_1 and θ_2 are dependent on permitivities of two media and not on $\bar{D}$ or $\bar{E}$.

Thus if $\varepsilon_1 > \varepsilon_2$, then $\theta_1 > \theta_2$.

The magnitude of $\bar{D}$ in region 2 can be obtained as

$$D_2^2 = D_{N2}^2 + D_{tan2}^2 = (D_1 \cos\theta_1)^2 + D_{tan2}^2 \longrightarrow (31)$$

Now $D_{tan2} = D_2 \sin\theta_2 = \dfrac{D_1 \sin\theta_1 \varepsilon_2}{\varepsilon_1}$, from eq (28)

$$\therefore D_2^2 = (D_1 \cos\theta_1)^2 + \left(D_1 \sin\theta_1 \frac{\varepsilon_2}{\varepsilon_1}\right)^2$$

$$\boxed{D_2 = D_1 \sqrt{\cos^2\theta_1 + \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 \sin^2\theta_1}} \longrightarrow (32)$$

Similarly, magnitude of E_2 can be obtained as

$$\boxed{E_2 = E_1 \sqrt{\sin^2\theta_1 + \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 \cos^2\theta_1}\,.} \longrightarrow (33)$$

Thes Equation shows that

1) $\bar{D}$ is larger in the region of larger permittivity
2) $\bar{E}$ is larger in the region of smaller permittivity
3) $|\bar{D}_1| = |\bar{D}_2|$ if $\theta_1 = \theta_2 = 0^\circ$
4) $|\bar{E}_1| = |\bar{E}_2|$ if $\theta_1 = \theta_2 = 90^\circ$.

To find the angles θ_1 & θ_2, with respect to normal use the dot product if normal direction to the boundary is Known.

→ Concept of capacitance :-

Consider two conducting materials M_1 & M_2 which are placed in a dielectric medium having permittivity ε.

→ The material M_1 carries a positive charge Q, material M_2 carries a negative charge Q.

→ The total charge of system is zero.

In conductors, charges cannot reside within conductor, it resides only on surface. It is shown in fig.

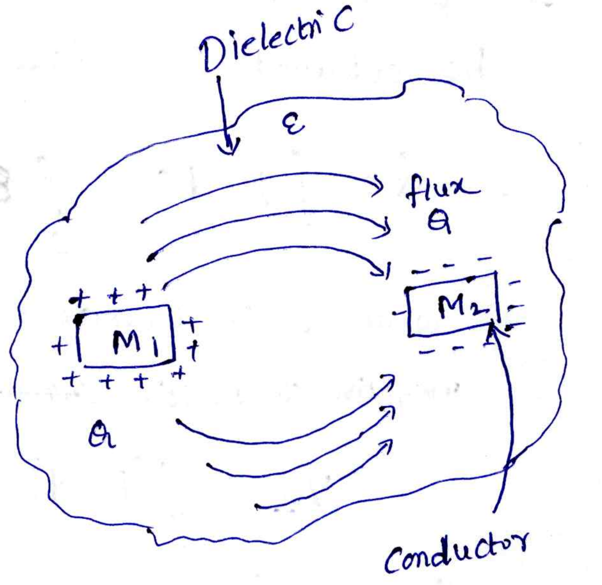


Fig: Concept of capacitance.

→ Such a system which has two conducting surfaces carrying equal and opposite charges, separated by dielectric is called capacitive system giving rise to a capacitance.

→ The Electric field is normal to conductor surface and the Electric flux is directed from M_1 towards M_2 in such a system. There exist a potential difference between the two surfaces M_1 & M_2. Let this potential is V_{12}.

→ The ratio of magnitudes of the total charge on any one of the two conductors and potential difference between the conductors is called the Capacitance of two conductor system, denoted as C.

$$C = \frac{Q}{V_{12}} \longrightarrow (1)$$

In general, $$C = \frac{Q}{V} \longrightarrow (2)$$

where Q = charge in Coulomb

V = potential difference in volts

Capacitance is measured in Farad (F).

→ As charge Q resides only on the surface of conductor, it can be obtained from Gauss's law. as,

$$Q = \oint_S \bar{D} \cdot d\bar{s} = \oint_S \varepsilon \bar{E} \cdot d\bar{s} = \oint_S \varepsilon_0 \varepsilon_r \bar{E} \cdot d\bar{s} \longrightarrow (3)$$

→ while V is workdone in moving unit positive charge from negative to positive ~~charge~~ surface and can be obtained as,

$$V = -\int_L \bar{E} \cdot d\bar{L} = -\int_-^+ \bar{E} \cdot d\bar{L} \longrightarrow (4)$$

Hence capacitance $C = \frac{Q}{V} = \frac{\oint_S \varepsilon \bar{E} \cdot d\bar{S}}{-\int_-^+ \bar{E} \cdot d\bar{L}}$, F $\longrightarrow (5)$

Capacitors in Series :-

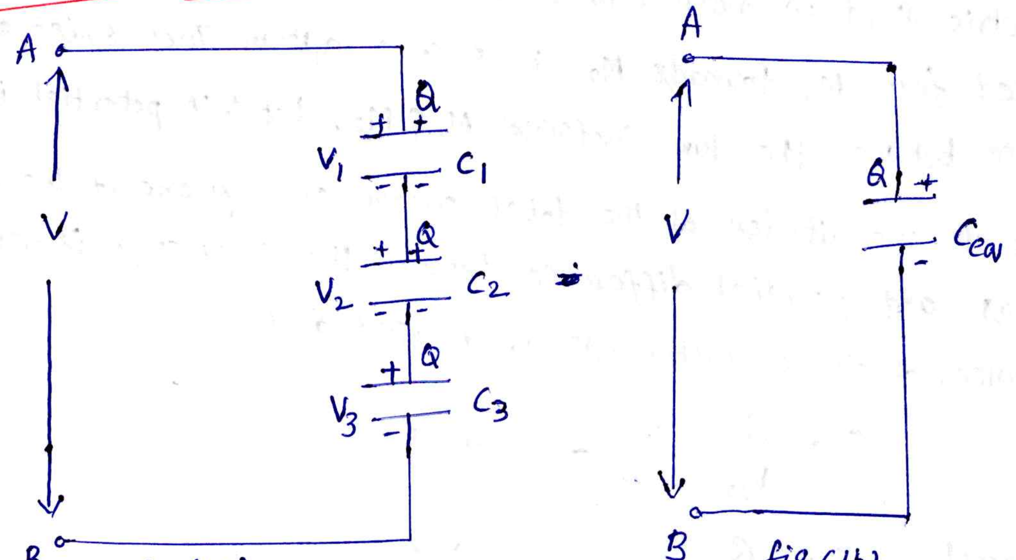


fig (1a) fig (1b)

Fig(1): Capacitors in series.

Consider the three capacitors in series connected across the applied voltage V as shown in fig.

$$Q = C_1 V_1 = C_2 V_2 = C_3 V_3 .$$

$$V_1 = \frac{Q}{C_1}, \quad V_2 = \frac{Q}{C_2}, \quad V_3 = \frac{Q}{C_3}$$

$$C_{eq} = \frac{Q}{V} \quad (or) \quad V = \frac{Q}{C_{eq}}.$$

But $V = V_1 + V_2 + V_3$.

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$

$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

For 'n' capacitors in series, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$.

For capacitors in series, charge is same but voltage across is different.

Capacitors in Parallel :-

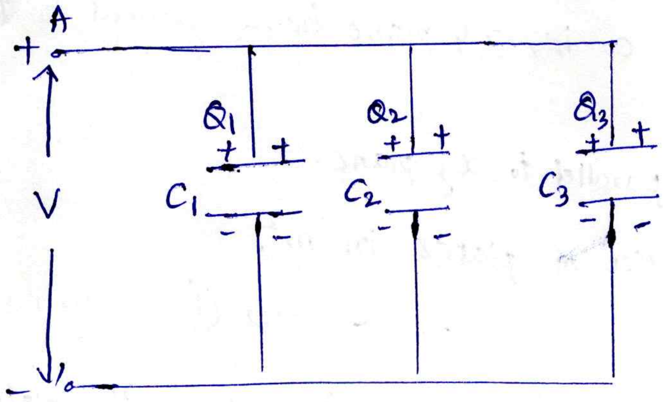


fig 2(a)

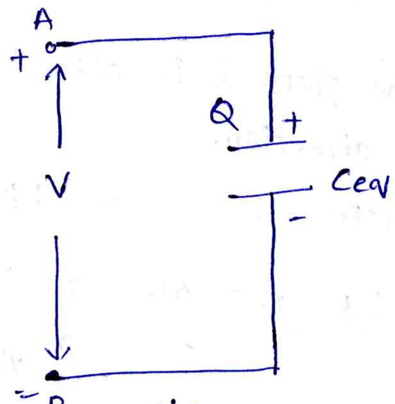


fig 2(b)

Fig (2): Capacitors in parallel.

When capacitors are in parallel, the same voltage exists across them but charges are different.

$$Q_1 = C_1 V, \quad Q_2 = C_2 V, \quad Q_3 = C_3 V \text{ and } Q = C_{eq} V$$

Total charge, $Q = Q_1 + Q_2 + Q_3$

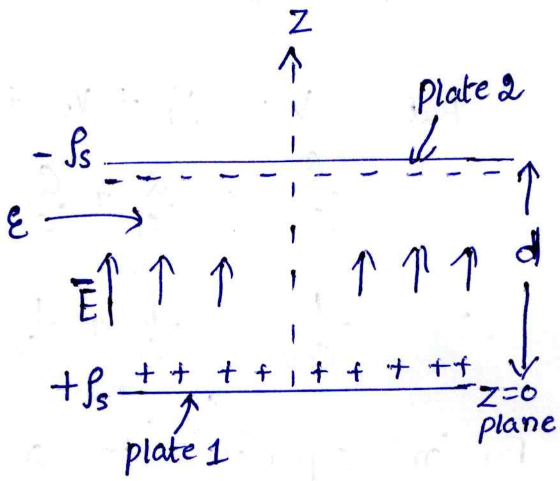
$$C_{eq} V = C_1 V + C_2 V + C_3 V$$

$$\therefore \quad C_{eq} = C_1 + C_2 + C_3$$

For 'n' capacitors in parallel, $C_{eq} = C_1 + C_2 + \cdots + C_n$.

Parallel plate capacitor :-

A parallel plate capacitor is shown in fig. It consists of two parallel metallic plates separated by distance 'd'.



Fig(1): parallel plate capacitor.

→ The space between plates is filled with dielectric of permittivity ε.

→ The lower plate, plate 1 carries positive charge with charge density $+\rho_s$.

→ The upper plate, Plate 2 carries negative charge with charge density $-\rho_s$.

→ The plate 1 is placed in $z=0$ i.e., xy plane hence normal to the Z direction.

→ Plate 2 is in $z=d$ plane, parallel to xy plane.

Let A = Area of cross section of plates in m^2.

$$Q = \rho_s A \longrightarrow (1)$$

To find potential difference, let us obtain $\bar{E}$ between the plates.

$$\bar{E}_1 = \frac{\rho_s}{2\varepsilon}\bar{a}_N = \frac{\rho_s}{2\varepsilon}\bar{a}_z \quad V/m \longrightarrow (2)$$

$\bar{E}_1$ is normal at the boundary between conductor & diectric without any tangential component.

$$\bar{E}_2 = -\frac{\rho_s}{2\varepsilon}\bar{a}_N = \frac{\rho_s}{2\varepsilon}(-\bar{a}_z) \quad V/m \longrightarrow (3)$$

The direction of $\bar{E}_2$ is downwards i.e., in $-\bar{a}_z$ direction.

In between the plates,

$$\bar{E} = \bar{E}_1 + \bar{E}_2 = \frac{\rho_s}{2\varepsilon}\bar{a}_z + \frac{\rho_s}{2\varepsilon}\bar{a}_z = \frac{\rho_s}{\varepsilon}\bar{a}_z \longrightarrow (4)$$

The potential difference is given by,

$$V = -\int_{-}^{+} \bar{E} \cdot d\bar{L} = -\int_{upper}^{lower} \frac{\rho_s}{\varepsilon} \bar{a}_z \cdot d\bar{L}$$

Now $d\bar{L} = dx\, \bar{a}_x + dy\, \bar{a}_y + dz\, \bar{a}_z$ in Cartesian system.

$$V = -\int_{z=d}^{z=0} \frac{\rho_s}{\varepsilon} \bar{a}_z \cdot \left[dx\, \bar{a}_x + dy\, \bar{a}_y + dz\, \bar{a}_z \right]$$

$$= -\int_{z=d}^{z=0} \frac{\rho_s}{\varepsilon} dz$$

$$= -\frac{\rho_s}{\varepsilon} \left[z \right]_d^0$$

$$= -\frac{\rho_s}{\varepsilon} \left[0 - d \right]$$

$$V = \frac{\rho_s d}{\varepsilon} \quad \text{Volt}$$

∴ The capacitance is the ratio of charge Q to Voltage V.

$$C = \frac{Q}{V} = \frac{\rho_s A}{\frac{\rho_s d}{\varepsilon}} = \frac{\varepsilon A}{d} \quad \text{Farad}$$

Thus if $\varepsilon = \varepsilon_0 \varepsilon_r$.

$$\boxed{C = \frac{\varepsilon_0 \varepsilon_r A}{d}} \quad \text{Farad}$$

It can be seen that value of capacitance depends on,

1) The permittivity of the dielectric used
2) The area of cross section of the plates
3) The distance of separation of the plates

It is not dependent on the charge or the potential difference between the plates.

→ Capacitance of a Co-axial Cable :-

consider a co axial cable or co axial capacitor as shown in fig.

Let $a =$ Inner radius.

$b =$ outer radius.

→ The two concentric conductors are separated by dielectric of permittivity ε.

→ The length of cable is L, m.

→ The inner conductor carries a charge density $+\rho_L$ c/m on its surface then equal and opposite charge density $-\rho_L$ c/m exists on the outer conductor.

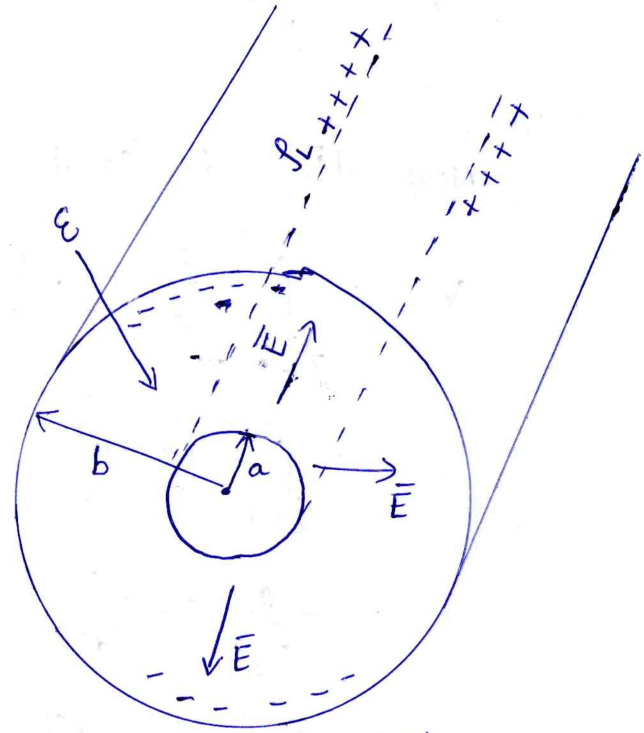


Fig : co axial cable

$$Q = \rho_L \times L \longrightarrow ①$$

Assuming ~~spherical~~ cylindrical co ordinate system, $\bar{E}$ will be radial from inner to outer conductor and for finite line charge it is given by

$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon r}\,\bar{a}_r \longrightarrow ②$$

The potential difference is workdone in moving unit charge against $\bar{E}$ i.e., from $r=b$ to $r=a$.

$$V = -\int_{-}^{+} \bar{E}\cdot d\bar{l} = -\int_{r=b}^{r=a} \left(\frac{\rho_L}{2\pi\varepsilon r}\,\bar{a}_r\right)\cdot(dr\,\bar{a}_r)$$

$$= -\frac{\rho_L}{2\pi\varepsilon}\int_{r=b}^{r=a} \frac{1}{r}\,dr$$

$$= -\frac{\rho_L}{2\pi\varepsilon}\left[\ln r\right]_b^a = -\frac{\rho_L}{2\pi\varepsilon}(\ln a - \ln b)$$

$\left(\because \int \frac{1}{x}dx = \log x\right)$

$\left(\because \log a - \log b = \log a/b\right)$

$$= -\frac{\rho_L}{2\pi\varepsilon}\ln\left(\frac{a}{b}\right)$$

$$V = \frac{\rho_L}{2\pi\varepsilon}\ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{V} = \frac{\rho_L \times L}{\frac{\rho_L}{2\pi\varepsilon} \ln\left(\frac{b}{a}\right)}$$

$$\therefore \boxed{C = \frac{2\pi\varepsilon L}{\ln\left(\frac{b}{a}\right)}} \text{ farad}$$

4) Capacitance of a spherical capacitor :-

Consider a spherical capacitor formed of two concentric spherical conducting shells of radius a and b. The capacitor is shown in fig.

→ The radius of outer sphere is 'b' and inner sphere is 'a'. Thus $b > a$.

→ The region between two spheres is filled with a dielectric of permittivity ε.

→ The inner sphere has positive charge $+Q$. and outer sphere has negative charge $-Q$.

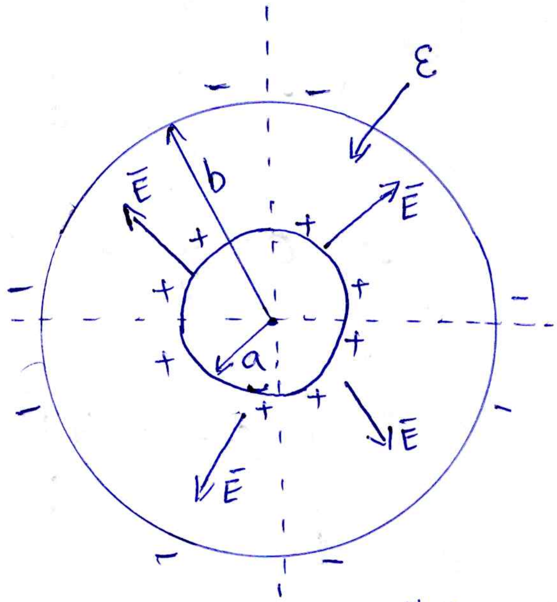


Fig: spherical capacitor.

→ Consider a Gaussian surface as a sphere of radius r, it can be obtained that $\bar{E}$ is in radial direction and given by

$$\bar{E} = \frac{Q}{4\pi\varepsilon r^2} \bar{a}_r \quad V/m. \longrightarrow (1)$$

The potential difference is workdone in moving unit positive charge against the direction of $\bar{E}$ i.e, from $r=b$ to $r=a$.

$$V = -\int_{-}^{+} \bar{E} \cdot d\bar{L} = -\int_{r=b}^{r=a} \frac{Q}{4\pi\varepsilon r^2} \bar{a}_r \cdot d\bar{L} \longrightarrow (2)$$

Now $d\bar{L} = dr\ \bar{a}_r$, in radial direction.

$$V = -\int_{r=b}^{r=a} \frac{Q}{4\pi\varepsilon r^2} \bar{a}_r \cdot dr\, \bar{a}_r$$

$$= -\int_{r=b}^{r=a} \frac{Q}{4\pi\varepsilon r^2} dr.$$

$$= -\frac{Q}{4\pi\varepsilon}\left[\int_{r=b}^{a} \frac{1}{r^2} dr\right]$$

$$= -\frac{Q}{4\pi\varepsilon}\left[-\frac{1}{r}\right]_{r=b}^{r=a}$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{a}-\frac{1}{b}\right]. \text{ Volt} \longrightarrow (3)$$

$$\therefore C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon}\left[\frac{1}{a}-\frac{1}{b}\right]}$$

$$\boxed{C = \frac{4\pi\varepsilon}{\left[\frac{1}{a}-\frac{1}{b}\right]}} \text{ farad} \longrightarrow (4)$$

Capacitance of Single Isolated Sphere :-

Consider a single isolated sphere of radius 'a', given a charge $+Q$.

It forms a capacitance with an outer plate which is infinitely large.

Hence $b = \infty$.

The capacitance of such a single isolated spherical conductor can be obtained by substituting $b = \infty$ in above Eqn (4).

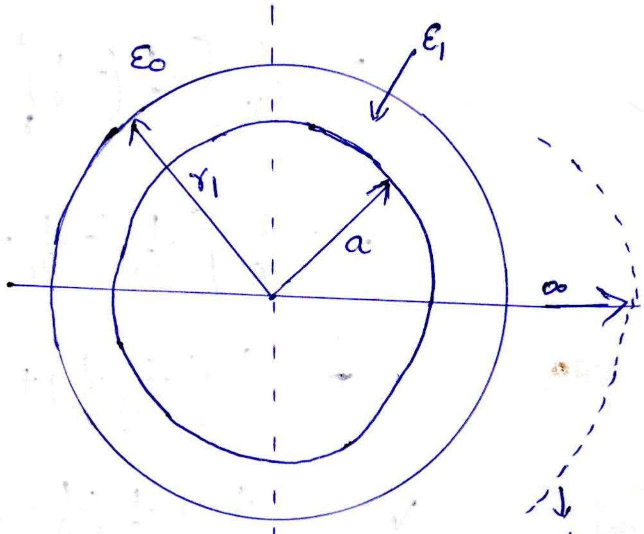
$$C = \frac{4\pi\varepsilon}{\left[\frac{1}{a}-\frac{1}{\infty}\right]}$$

$$C = \frac{4\pi\varepsilon}{\left(\frac{1}{a}-0\right)}$$

$$\boxed{C = 4\pi\varepsilon a} \text{ farad.} \longrightarrow (5).$$

This is the case of spherical conductor at a large distance from other conductors.

Isolated Sphere Coated with Dielectric :-



Fig(1): Isolated Sphere

- Consider a single isolated sphere coated with a dielectric having permittivity ε_1 upto radius r_1.
- The radius of inner sphere is 'a' as shown in fig(1). It is placed in a free space so outside sphere $\varepsilon = \varepsilon_0$. It carries a charge $+Q$.

So for $a < r < r_1$, $\varepsilon = \varepsilon_1$.

and for $r > r_1$, $\varepsilon = \varepsilon_0$.

The Potential difference is work done in bringing unit positive charge from outer sphere $r = \infty$ to inner sphere $r = a$ against $\bar{E}$.

This is to be splitted in two as,

$$V = -\int_{-}^{+} \bar{E} \cdot d\bar{L} = -\int_{r=\infty}^{r=a} \bar{E} \cdot d\bar{L}$$

$$V = -\int_{r=\infty}^{r=r_1} \bar{E} \cdot d\bar{L} - \int_{r=r_1}^{r=a} \bar{E} \cdot d\bar{L} \longrightarrow (6)$$

Now for $a < r < r_1$,

$$\bar{E}_1 = \frac{Q}{4\pi\varepsilon_1 r^2}\,\bar{a}_r \longrightarrow (7)$$

Now for $r_1 < r < \infty$,

$$\bar{E}_2 = \frac{Q}{4\pi\varepsilon_0 r^2}\,\bar{a}_r \longrightarrow (8)$$

while $d\bar{L} = dr\,\bar{a}_r$, as $\bar{E}_1$ & $\bar{E}_2$ are in radial direction.

$$V = -\int_{r=\infty}^{r=r_1} \frac{Q}{4\pi\varepsilon_0 r^2}\,\bar{a}_r \cdot dr\,\bar{a}_r - \int_{r=r_1}^{r=a} \frac{Q}{4\pi\varepsilon_1 r^2}\,\bar{a}_r \cdot dr\,\bar{a}_r$$

$$V = -\frac{Q}{4\pi}\left[\frac{1}{\varepsilon_0}\int_{r=\infty}^{r=r_1}\frac{1}{r^2}\,dr + \frac{1}{\varepsilon_1}\int_{r=r_1}^{r=a}\frac{1}{r^2}\,dr\right]$$

$$= -\frac{Q}{4\pi}\left[\frac{1}{\varepsilon_0}\left(-\frac{1}{r}\right)_{r=\infty}^{r=r_1} + \frac{1}{\varepsilon_1}\left(-\frac{1}{r}\right)_{r=r_1}^{r=a}\right]$$

$$= -\frac{Q}{4\pi}\left[\frac{1}{\varepsilon_0}\left(-\frac{1}{r_1}+\frac{1}{\infty}\right) + \frac{1}{\varepsilon_1}\left(-\frac{1}{a}+\frac{1}{r_1}\right)\right]$$

$$= \frac{Q}{4\pi}\left[\frac{1}{\varepsilon_0}\left(\frac{1}{r_1}\right) + \frac{1}{\varepsilon_1}\left(\frac{1}{a}\right) - \frac{1}{\varepsilon_1}\left(\frac{1}{r_1}\right)\right]$$

$$V = \frac{Q}{4\pi}\left[\frac{1}{\varepsilon_1}\left(\frac{1}{a}-\frac{1}{r_1}\right) + \frac{1}{\varepsilon_0 r_1}\right] \longrightarrow (9)$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi}\left[\frac{1}{\varepsilon_1}\left(\frac{1}{a}-\frac{1}{r_1}\right) + \frac{1}{\varepsilon_0 r_1}\right]}$$

$$C = \left[\frac{4\pi}{\frac{1}{\varepsilon_1}\left(\frac{1}{a}-\frac{1}{r_1}\right) + \frac{1}{\varepsilon_0 r_1}}\right] \text{F.} \longrightarrow (10)$$

Now $$\frac{1}{C} = \frac{\frac{1}{\varepsilon_1}\left(\frac{1}{a}-\frac{1}{r_1}\right) + \frac{1}{\varepsilon_0 r_1}}{4\pi} \longrightarrow (11)$$

$$\frac{1}{C} = \frac{\left(\frac{1}{a}-\frac{1}{r_1}\right)}{4\pi\varepsilon_1} + \frac{1}{4\pi\varepsilon_0 r_1} \longrightarrow (12)$$

Now $C_1 = \frac{4\pi\varepsilon_1}{\left(\frac{1}{a}-\frac{1}{r_1}\right)}$ = capacitance of spherical conductor

$C_2 = 4\pi\varepsilon_0 r_1$ = Capacitance of isolated sphere

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \longrightarrow (13)$$

$$C = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} \longrightarrow (14).$$

Thus Total capacitance can be treated to be two capacitors C_1 & C_2 in series.

$$V = \frac{D_1}{\varepsilon_1} d_1 + \frac{D_2}{\varepsilon_2} d_2 \longrightarrow (3)$$

The magnitude of surface charge is same as on each plate.

Hence $\rho_s = D_1 = D_2 . \longrightarrow (4).$

substitute eq (4) in eq (3),

$$V = \rho_s \left[\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right] \longrightarrow (5)$$

Now, $C = \frac{Q}{V} = \frac{Q}{\rho_s \left[\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right]}$

But $Q = \rho_s \times A$

$$\therefore \quad C = \frac{\rho_s \times A}{\rho_s \left[\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right]} = \frac{A}{\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}} \longrightarrow (6).$$

$$C = \frac{1}{\frac{d_1}{\varepsilon_1 A} + \frac{d_2}{\varepsilon_2 A}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \longrightarrow (7)$$

where $C_1 = \frac{\varepsilon_1 A}{d_1}$ and $C_2 = \frac{\varepsilon_2 A}{d}.$

Thus the result can be generalized for a capacitor with n dielectrics as,

$$\therefore \quad \boxed{C = \frac{A}{\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} + \frac{d_3}{\varepsilon_3} + \dots + \frac{d_n}{\varepsilon_n}}} \longrightarrow (8)$$

→ Composite Parallel Plate Capacitor :

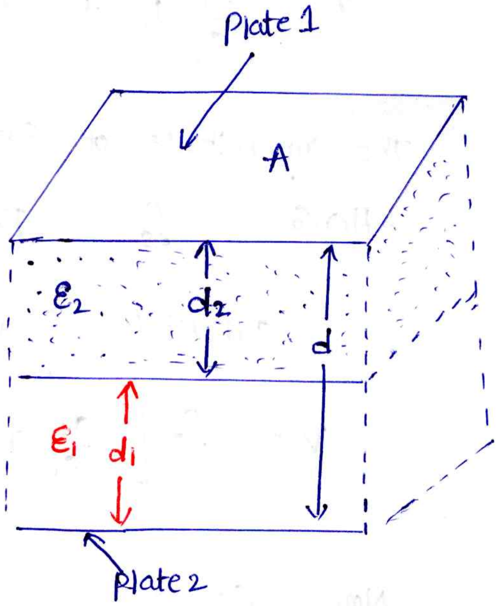


Fig: Composite Parallel Plate capacitor.

→ The composite parallel plate capacitor is one in which the space between the plates is filled with more than one dielectric.

→ Consider a composite capacitor with space filled with two separate dielectrics for the distances d_1 and d_2.

→ This is shown in fig. The dielectric interface is parallel to the conducting plates.

→ The space d_1 is filled with dielectric having permittivity ε_1 and space d_2 is filled with dielectric having permittivity ε_2.

Let Q = charge on each plate

E_1 = Electric field intensity in region d_1.

E_2 = Electric field intensity in region d_2.

Both intensities are uniform.

$\therefore V_1 = E_1 d_1$

$V_2 = E_2 d_2$

where E_1 & E_2 are the magnitudes of the two intensities.

$$V = V_1 + V_2 = E_1 d_1 + E_2 d_2 \longrightarrow (1)$$

At a dielectric - dielectric Interface, the normal components of flux densities are equal. i.e., $D_{N1} = D_{N2}$

Now $D_1 = \varepsilon_1 E_1$ and $D_2 = \varepsilon_2 E_2$.

~~Substituting~~ $E_1 = \frac{D_1}{\varepsilon_1}$ & $E_2 = \frac{D_2}{\varepsilon_2}$. $\longrightarrow (2)$

Substituting in Eq (2) in Eq (1).

↳ Dielectric Boundary Normal to the Plates in composite parallel plate capacitor :-

→ Consider the composite capacitor in which dielectric boundary is normal to the conducting plates.

→ The dielectric ε_1 occupying area A_1 of the plates while the dielectric ε_2 occupying area A_2 as shown in fig (2).

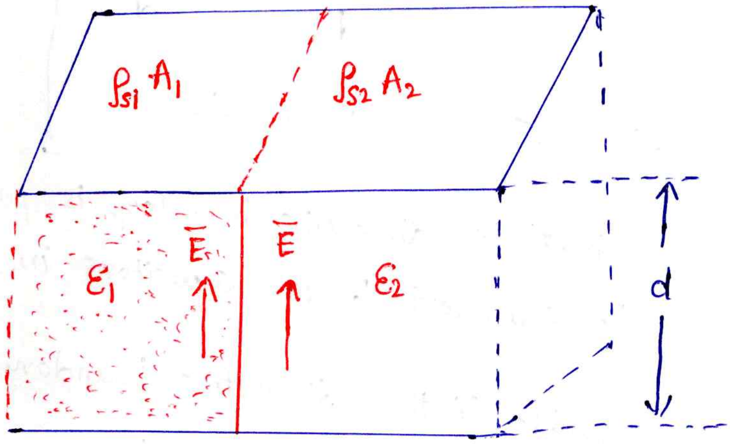


Fig : Dielectric boundary normal to the plates.

→ The Total potential across two plates is V and distance between the plates is d.

→ Hence magnitude of $\bar{E}$ is, $E = \dfrac{V}{d} \longrightarrow (9)$

→ At the boundary, both $\bar{E}_1$ and $\bar{E}_2$ are Tangential and for dielectric - dielectric interface tangential Components are Equal.

$$E_{tan1} = E_{tan2} = E_1 = E_2 = \frac{V}{d} \longrightarrow (10)$$

Now $D_1 = \varepsilon_1 E_1$ and $D_2 = \varepsilon_2 E_2$

$$D_1 = \frac{\varepsilon_1 V}{d} \quad \text{and} \quad D_2 = \frac{\varepsilon_2 V}{d} \longrightarrow (11)$$

on the plate the charge is divided into two parts.

on Area A_1, the charge density $\rho_{s1} = D_1$

on Area A_2, The charge density $\rho_{s2} = D_2$

$$Q = Q_1 + Q_2$$

$$= \rho_{s1} A_1 + \rho_{s2} A_2$$

$$Q = D_1 A_1 + D_2 A_2 \longrightarrow (12)$$

$$Q = \frac{\varepsilon_1 V A_1}{d} + \frac{\varepsilon_2 V A_2}{d} \longrightarrow (13)$$

Now $$C = \frac{Q}{V} = \frac{\dfrac{\varepsilon_1 V A_1}{d} + \dfrac{\varepsilon_2 V A_2}{d}}{V}$$

$$\boxed{\begin{aligned} C &= \frac{\varepsilon_1 A_1}{d} + \frac{\varepsilon_2 A_2}{d} \longrightarrow (14) \\ \therefore\ C &= C_1 + C_2 \end{aligned}}$$

Thus if dielectric boundary is parallel to the plates, the arrangement is equivalent to two capacitors in series for which $C_{eq} = \dfrac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$

while if the dielectric boundary is normal to the plates, the arrangement is equivalent to two capacitors in parallel for which $C_{eq} = C_1 + C_2$.

→ Energy stored in a capacitor :-

Consider a parallel plate capacitor as shown in fig.

It is supplied with voltage V.

Let $\bar{a}_N$ is the direction normal to the plates.

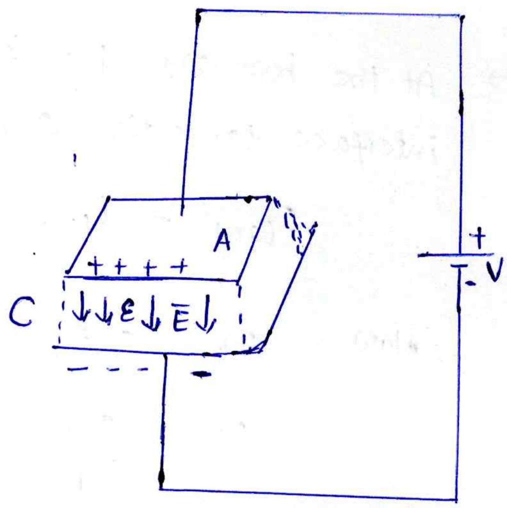


Fig: parallel plate capacitor.

$$\bar{E} = \frac{V}{d}\,\bar{a}_N \longrightarrow (1)$$

The energy stored is given by,

$$W_E = \frac{1}{2}\int_{Vol} \bar{D}\cdot\bar{E}\ dv$$

$$= \frac{1}{2}\int_{Vol} \varepsilon\bar{E}\cdot\bar{E}\ dv \quad , \text{ but } \bar{E}\cdot\bar{E} = |\bar{E}|^2$$

$$= \frac{1}{2}\int_{Vol} \varepsilon\,|\bar{E}|^2\ dv \quad , \text{ but } |\bar{E}| = \frac{V}{d}.$$

$$W_E = \frac{1}{2}\,\varepsilon\,\frac{V^2}{d^2}\int_{Vol} dv \quad , \text{ but } \int_{Vol} dv = \text{Volume} = A\times d$$

$$W_E = \frac{1}{2} \varepsilon \frac{V^2}{d^2} Ad = \frac{1}{2} \frac{\varepsilon A}{d} V^2$$

$$\boxed{W_E = \frac{1}{2} C V^2}$$ Joule where $C = \frac{\varepsilon A}{d}$

If the dielectric is freespace then there is increase in stored energy if the free space is repaced by other dielectric having $\varepsilon_r > 1$.

Energy Density

Energy Density is the energy stored per unit volume as volume tends to zero.

$$W_E = \frac{1}{2} \varepsilon \int_{vol} |\bar{E}|^2 \, dv$$

$$W_E = \frac{1}{2} \varepsilon |\bar{E}|^2 \quad J/m^3 = \text{Energy density.}$$

using $|\bar{D}| = \varepsilon |\bar{E}|$.

$$\boxed{W_E = \frac{1}{2} \frac{|\bar{D}|^2}{\varepsilon}} \quad J/m^3$$

$$= \frac{1}{2} \frac{|D||D|}{\varepsilon}$$

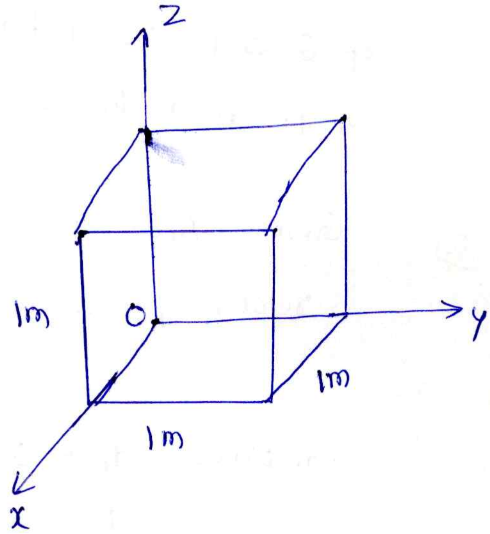
$$\boxed{W_E = \frac{1}{2} |\bar{D}| |\bar{E}|} \quad J/m^3$$

Problems

1) Find the total current in outward direction from a cube of 1m, with one corner at the origin and edges parallel to the coordinate axes if $J = 2x^2\,\bar{a}_x + 2xy^3\,\bar{a}_y + 2xy\,\bar{a}_z$ A/m².

Sol: Given $J = 2x^2\,\bar{a}_x + 2xy^3\,\bar{a}_y + 2xy\,\bar{a}_z$.

The cube is shown in fig.



According to continuity equation,

$$I = \oint \bar{J}\cdot d\bar{s} = \int_{Vol} (\nabla\cdot\bar{J})\, dv$$

The cube is a volume hence use volume integral

$dV = dx\,dy\,dz.$

$$\nabla\cdot\bar{J} = \left(\frac{\partial}{\partial x}\bar{a}_x + \frac{\partial}{\partial y}\bar{a}_y + \frac{\partial}{\partial z}\bar{a}_z\right)\cdot\left(2x^2\bar{a}_x + 2xy^3\,\bar{a}_y + 2xy\,\bar{a}_z\right)$$

$$= \frac{\partial}{\partial x}(2x^2) + \frac{\partial}{\partial y}2xy^3 + \frac{\partial}{\partial z}2xy$$

$$= 4x + 6xy^2 + 0$$

$$\nabla\cdot\bar{J} = 4x + 6xy^2$$

$$I = \int_{Vol}(4x + 6xy^2)\, dx\,dy\,dz$$

$$I = \int_{z=0}^{1}\int_{y=0}^{1}\left[\int_{x=0}^{1}(4x+6xy^2)\,dx\right]dy\,dz$$

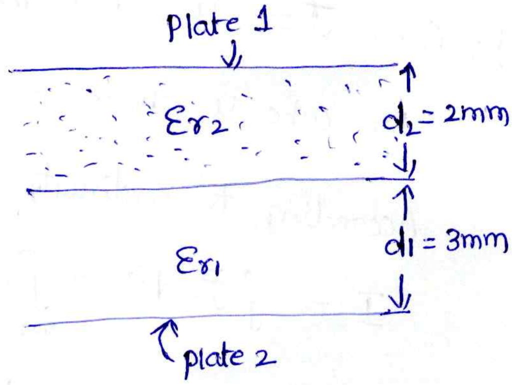
$$= \int_{z=0}^{1}\int_{y=0}^{1}\left[\frac{4x^2}{2} + \frac{6x^2y^2}{2}\right]_0^1 dy\,dz = \int_{z=0}^{1}\int_{y=0}^{1}(2+3y^2)\,dy\,dz$$

$$= \int_{z=0}^{1}\left[2y + \frac{3y^3}{3}\right]_0^1 dz = \int_{z=0}^{1}(2+1)\,dz = \int_{z=0}^{1} 3\,dz$$

$$= 3\,[z]_0^1 = 3\,[1-0]$$

$$I = 3 \text{ A}$$

2) A parallel plate capacitor has a plate area of $1.5\ m^2$ and a plate separation of 5 mm. There are two dielectrics in between the plates. The first dielectric has a thickness of 3 mm with a relative permittivity of 6 and second has a thickness of 2 mm with a relative permittivity of 4. find the capacitance.



Sol: Given Area $A = 1.5\ m^2$

relative permittivity $\varepsilon_{r1} = 6$

$\varepsilon_{r2} = 4$

distance, $d_1 = 2\ mm = 2 \times 10^{-3}\ m$

$d_2 = 3\ mm = 3 \times 10^{-3}\ m$

$$C_1 = \frac{\varepsilon_1 A}{d_1} = \frac{\varepsilon_0 \varepsilon_{r1} A}{d_1} = \frac{8.854 \times 10^{-12} \times 6 \times 1.5}{2 \times 10^{-3}}$$

$$C_1 = 26.562\ nF$$

$$C_2 = \frac{\varepsilon_2 A}{d_2} = \frac{\varepsilon_0 \varepsilon_{r2} A}{d_2} = \frac{8.854 \times 10^{-12} \times 4 \times 1.5}{3 \times 10^{-3}}$$

$$C_2 = 26.562\ nF$$

The two capacitors are in series,

$$\therefore C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(26.562 \times 10^{-9})(26.562 \times 10^{-9})}{(26.562 \times 10^{-9}) + (26.562 \times 10^{-9})}$$

$$C_{eq} = 13.281\ nF$$

This is the required capacitance.

(or)

Alternative,
$$C = \frac{A}{\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}} = \frac{1.5}{\frac{3 \times 10^{-3}}{\varepsilon_0 \times 6} + \frac{2 \times 10^{-3}}{\varepsilon_0 \times 4}}$$

$$C = 13.281\ nF$$

3) A parallel plate capacitor consists of two square metal plates with 500 mm side and separated by 10 mm. A slab of sulphur ($\varepsilon_r = 4$) 6 mm thick is placed on the lower plate and air gap of 4 mm. Find the capacitance of capacitor.

Sol: This is composite capacitor.

$d_1 = 6mm = 6 \times 10^{-3}$ m , $\varepsilon_{r1} = 4$

$d_2 = 4mm = 4 \times 10^{-3}$ m. $\varepsilon_{r2} = 1$.

$A = 500 \times 500 \ mm^2 = 2500 mm^2 = 0.25 \ m^2$

$$C = \frac{1}{\frac{d_1}{\varepsilon_1 A} + \frac{d_2}{\varepsilon_2 A}} = \frac{A}{\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}} = \frac{A}{\frac{d_1}{\varepsilon_0 \varepsilon_{r1}} + \frac{d_2}{\varepsilon_0 \varepsilon_{r2}}}$$

$$= \frac{0.25}{\frac{6 \times 10^{-3}}{8.854 \times 10^{-12} \times 4} + \frac{4 \times 10^{-3}}{8.854 \times 10^{-12} \times 1}}$$

$$C = 402.4545 \ pF$$

4) The polarization within a region having $\varepsilon_R = 2.7$ has the uniform value of $\bar{P} = -0.2\,\bar{a}_x + 0.7\,\bar{a}_y + 0.3\,\bar{a}_z \ \mu C/m^2$. Find i) $\bar{E}$ ii) $\bar{D}$ and iii) Magnitude of voltage gradient.

Sol: Polarization $\bar{P} = \chi_e \varepsilon_0 \bar{E}$.

$\varepsilon_R = \chi_e + 1$

$\chi_e = \varepsilon_R - 1$

$\chi_e = 2.7 - 1 = 1.7$

Given $\bar{P} = -0.2\,\bar{a}_x + 0.7\,\bar{a}_y + 0.3\,\bar{a}_z \ \mu C/m^2 = \chi_e \varepsilon_0 \bar{E}$.

$$(-0.2\,\bar{a}_x + 0.7\,\bar{a}_y + 0.3\,\bar{a}_z) \times 10^{-6} = 1.7 \times 8.854 \times 10^{-12} \times \bar{E}$$

$$\bar{E} = \frac{(-0.2\,\bar{a}_x + 0.7\,\bar{a}_y + 0.3\,\bar{a}_z)}{1.7 \times 8.854 \times 10^{-12}}$$

$$\bar{E} = -13.2874\,\bar{a}_x + 46.5060\,\bar{a}_y + 19.9311\,\bar{a}_z \ KV/m$$

ii) $\bar{D} = \varepsilon_0 \varepsilon_R \bar{E}$

$= (8.854 \times 10^{-12}) \times 2.7 \times (-13.2874\, \bar{a}_x + 46.5060\, \bar{a}_y + 19.9311\, \bar{a}_z)$

$= -0.3176\, \bar{a}_x + 1.111\, \bar{a}_y + 0.4764\, \bar{a}_z\ \mu C/m^2$

iii) $-\nabla V = \bar{E}$, where ∇V = Voltage gradient.

magnitude of voltage gradient $= |\bar{E}|$

$\therefore |\bar{E}| = \sqrt{(-0.3176)^2 + (1.111)^2 + (0.4764)^2}$

$= 52.3125\ KV/m$

5) The co axial cable is required to transmit electric power. The p.d. between the inner and outer counductors is to be filled mainly with nitrogen gas under pressure whose dielectric strength is 25×10^{-6} V/m. The radius of outer conductor is double that of inner conductor.

(i) Determine the capacitance of cable

(ii) Determine the energy stored in the electric field of this cable when p.d. is 2×10^5 V.

Sol: (i) Capacitance per unit length is given by

$$\frac{C}{L} = \frac{2\pi \varepsilon_0 \varepsilon_r}{\ln\left(\frac{b}{a}\right)}$$

Given, $b = 2a$.

$$\frac{C}{L} = \frac{2\pi \times 8.854 \times 10^{-12} \times 25 \times 10^{-6}}{\ln\left(\frac{2a}{a}\right)}$$

$$\frac{C}{L} = 2 \times 10^{-15}\ F/m$$

So, for $L = 1m$, $C = 2 \times 10^{-15}\ F/m$

$$E = \frac{1}{2} CV^2$$

$$= \frac{1}{2} \times 2\times10^{-15} \times (2\times10^{5})^2$$

$$E = 40.129 \ \mu J$$

6) A spherical condensor has a capacity of 54 pF. It consist of two concentric sphere differing in radii by 4 cm. and having air as dielectric. Find their radii.

Sol: Given $C = 54\, pF = 54 \times 10^{-12}\, F$

difference in radii $= b - a = 4\, cm = 4\times10^{-2}\, m.$

for air, $\varepsilon_r = 1$ & $\varepsilon_0 = 8.854 \times 10^{-12}$.

The capacitance of spherical capacitor is

$$C = \frac{4\pi\varepsilon}{\left(\frac{1}{a} - \frac{1}{b}\right)} \quad \text{where } b > a$$

$$54\times10^{-12} = \frac{4\pi \times 8.854\times10^{-12} \times 1}{\left(\frac{1}{a} - \frac{1}{b}\right) \times 10^{-12}}$$

$$= \frac{4\pi \times 8.854\times10^{-12} \cdot (ab)}{(b-a)}$$

$$ab = 1.9413 \times 10^{-2} \longrightarrow (1)$$

Given $b - a = 4\times10^{-2}\, m$

$$b = 4\times10^{-2} + a \longrightarrow (2)$$

Substitute Eq (2) in (1)

$$a(4\times10^{-2} + a) = 1.9413 \times 10^{-2}$$

$$a^2 + 0.04a - 0.019413 = 0.$$

Here $a = 1,\ b = 0.04,\ c = -0.019413.$

Quadric Equation

$$x^2 + 2xy + y = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \frac{-0.04 \pm \sqrt{(0.04)^2 - 4(1 \times -0.019413)}}{2(1)}$$

$$\Rightarrow a = 0.1207 \text{ m.}$$

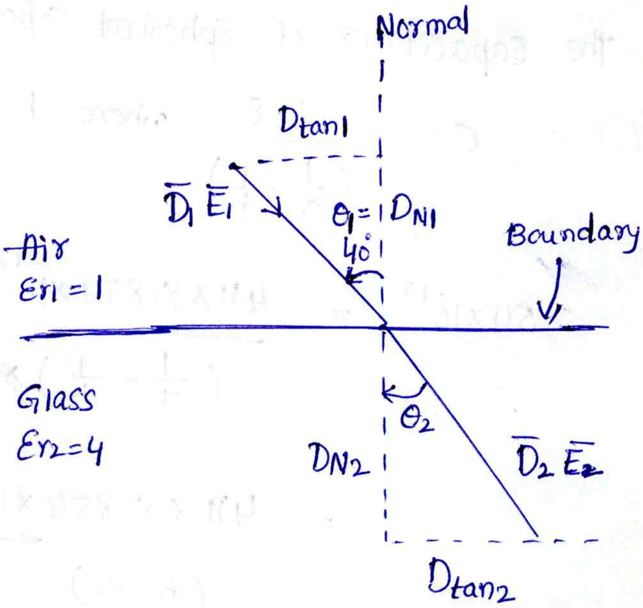
$$b = (4 \times 10^{-2}) + a$$

$$= (4 \times 10^{-2}) + 0.1207$$

$$\Rightarrow b = 0.1607 \text{ m.}$$

7) At the boundary between the glass ($\varepsilon_r = 4$) and air, the lines of electric field make an angle of $40°$ with normal to the boundary. If the electric flux density in the air is $0.25\ \mu C/m^3$. Determine the orientation and magnitude of electric flux density in the glass.

Sol: The arrangement is shown in fig.



For the boundary between two dielectrics,

$$D_{N1} = D_{N2}$$

and $$\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1}$$

Now $D_1 = 0.25\ \mu C/m^2$

$$\cos\theta_1 = \frac{D_{N1}}{D_1}$$

$$\cos 40° = \frac{D_{N1}}{0.25}$$

$$D_{N1} = \cos 40° \times 0.25$$

$$= 0.1915\ \mu C/m^2$$

$D_{N2} = D_{N1} = 0.1915\ \mu C/m^2.$

Now $D_1 = \sqrt{(D_{N1})^2 + (D_{tan1})^2}$

$0.25 = \sqrt{(0.1915)^2 + (D_{tan1})^2}$

$D_{tan1} = 0.1607\ \mu C/m^2.$

But $\dfrac{D_{tan1}}{D_{tan2}} = \dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{\varepsilon_0 \varepsilon_{r1}}{\varepsilon_0 \varepsilon_{r2}} = \dfrac{\varepsilon_{r1}}{\varepsilon_{r2}}$

$\therefore\ D_{tan2} = \dfrac{\varepsilon_{r2}}{\varepsilon_{r1}} \times D_{tan1}$

$= \dfrac{4}{1} \times 0.1607$

$= 0.6428\ \mu C/m^2.$

$D_2 = \sqrt{(D_{N2})^2 + (D_{tan2})^2}$

$= 0.6707\ \mu C/m^2.$

$\cos\theta_2 = \dfrac{D_{N2}}{D_2} = \dfrac{0.1915}{0.6707} = 0.2855$

$\theta_2 = 73.41^\circ$

8) what is the capacitance of capacitor consisting of two parallel plates 30 cm by 30 cm, separated by 5 mm in air? what is the energy stored by the capacitor if it is charged to a potential difference of 500 V.

Sol: Given Area $A = 30 \times 30\ cm^2 = 30 \times 30 \times 10^{-4}\ m$

distance $d = 5\ mm = 5 \times 10^{-3}\ m$

For air, $\varepsilon_r = 1$

Capacitance of capacitor, $C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d}$

$$C = \frac{8.854 \times 10^{-12} \times 1 \times 30 \times 30 \times 10^{-4}}{5 \times 10^{-3}}$$

$$C = 1.5937 \times 10^{-10} \text{ F}$$

Given Voltage $V = 500\text{ V}$

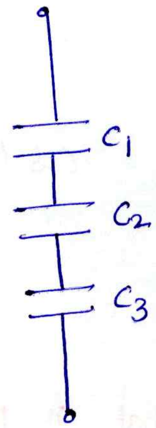
Energy stored by capacitor $E = \frac{1}{2} C V^2$

$$= \frac{1}{2} \times 1.5937 \times 10^{-10} \times (500)^2$$

$$= 20\ \mu J$$

9) A parallel plate capacitor has three dielectrics as $\varepsilon_{r1} = 1$, $d_1 = 0.4\text{ mm}$, $\varepsilon_{r2} = 2$, $d_2 = 0.6\text{ mm}$, and $\varepsilon_{r3} = 1$, $d_3 = 0.8\text{ mm}$ while area of cross section is 20 cm^2. find the capacitance.

Sol The arrangement is shown in fig.



$$C_1 = \frac{\varepsilon_1 A}{d_1} = \frac{\varepsilon_0 \varepsilon_{r1} A}{d_1}$$

$$C_2 = \frac{\varepsilon_2 A}{d_2} = \frac{\varepsilon_0 \varepsilon_{r2} A}{d_2}$$

$$C_3 = \frac{\varepsilon_3 A}{d_3} = \frac{\varepsilon_0 \varepsilon_{r3} A}{d_3}$$

Given Area $A = 20 \times 10^{-4}\text{ m}^2$.

$d_1 = 0.4 \times 10^{-3}\text{ m}$, $\varepsilon_{r1} = 1$

$d_2 = 0.6 \times 10^{-3}\text{ m}$, $\varepsilon_{r2} = 2$

$d_3 = 0.8 \times 10^{-3}\text{ m}$, $\varepsilon_{r3} = 1$.

$$C_1 = \frac{8.854 \times 10^{-12} \times 1 \times 20 \times 10^{-4}}{0.4 \times 10^{-3}} = 4.427 \times 10^{-11} \, F$$

$$C_2 = \frac{8.854 \times 10^{-12} \times 2 \times 20 \times 10^{-4}}{0.6 \times 10^{-3}} = 2.9513 \times 10^{-11} \, F$$

$$C_3 = \frac{8.854 \times 10^{-12} \times 1 \times 20 \times 10^{-4}}{0.8 \times 10^{-3}} = 2.2135 \times 10^{-11} \, F$$

~~Ceq~~ when three capacitors are connected in Series, then Equivalent Capacitance is $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

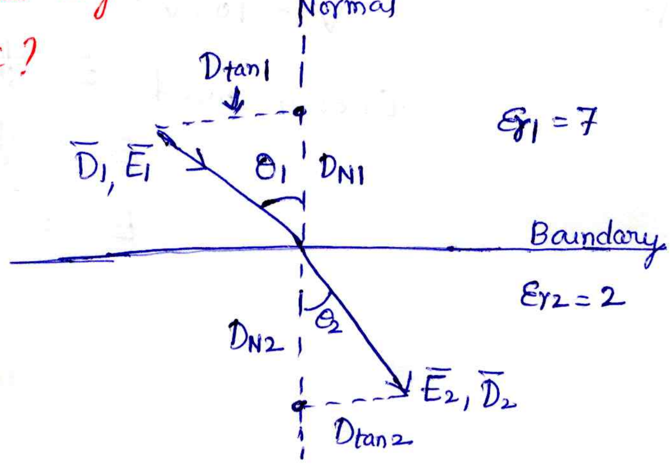
$$= \frac{1}{\frac{1}{4.427 \times 10^{-11}} + \frac{1}{2.9513 \times 10^{-11}} + \frac{1}{2.2135 \times 10^{-11}}}$$

$$= \frac{1}{2.2588 \times 10^{10} + 3.3883 \times 10^{10} + 4.5177 \times 10^{10}}$$

$$= 9.8378 \, pF$$

10) An Electric field in medium whose relative permittivity is 7 passes into a medium of relative permittivity 2. If $\bar{E}$ makes an angle of $60°$ with the boundary normal, what angle does the field makes with the normal in the second dielectric?

Sol: The arrangement is shown in fig.



From the boundary conditions of two dielectrics, $D_{N1} = D_{N2}$

$$\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1}$$

Given $\theta_1 = 60°$

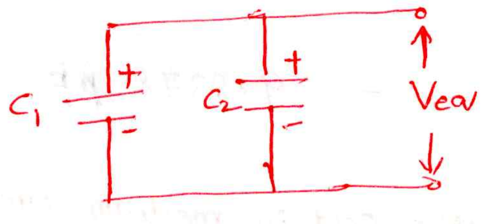
Now, $$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_0 \varepsilon_{r1}}{\varepsilon_0 \varepsilon_{r2}} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

$$\frac{\tan 60°}{\tan\theta_2} = \frac{7}{2}$$

$$\Rightarrow \quad \theta_2 = 26.329°$$

11) A $2\,\mu F$ capacitor is charged by connecting it across a 100 V dc supply. It is now disconnected and then it is connected across another $2\,\mu F$ capacitor. Assuming no leakage, determine the p.d. between the plates of each capacitor and energy stored.



Soo

Sol:

Given $C_1 = 2\,\mu F$.

$V = 100\,V$.

Energy stored $E = \frac{1}{2} C_1 V^2$

$$E = \frac{1}{2} \times 2 \times 100^2$$

$$E = 0.01\,J.$$

The total energy stored when two capacitors are connected in parallel must remain same as before.

$E = 0.01 \cdot J.$

$E = \frac{1}{2} C_1 V_{eq}^2 + \frac{1}{2} C_2 V_{eq}^2$

$0.01 = \left(\frac{1}{2} \times 2 \times 10^{-6} \times V_{eq}^2\right) + \left(\frac{1}{2} \times 2 \times 10^{-6} \times V_{eq}^2\right)$

$0.01 = 2 \times 10^{-6} V_{eq}^2$

$V_{eq} = 70.7106 \text{ V}$

$\therefore$ Voltage across each capacitor, $V_{eq} = 70.7106 \text{ V}$

12) A paper capacitor is made up of aluminium foil of 100 cm^2 placed on both sides of paper of thickness 0.03 mm. If the dielectric constant of paper is 3 and its dielectric breakdown strength is 200 KV/cm. what is the value of capacitor and rating of capacitor.

Sol: Given Area $A = 100 \text{ cm}^2 = 100 \times 10^{-4} \text{ m}^2$

distance $d = 0.03 \text{ mm} = 0.03 \times 10^{-3} \text{ m}$

$\varepsilon_r = 3.$

dielectric break down strength, $E = 200 \text{ KV/cm} = \frac{200 \times 10^3}{10^{-2}} \text{ V/m}$

$C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d}$

$= \frac{8.854 \times 10^{-12} \times 3 \times 100 \times 10^{-4}}{0.03 \times 10^{-3}}$

$C = 8.854 \times 10^{-9} \text{ F}$

Voltage rating of capacitor, $V = E \times d$

$= 2 \times 10^7 \times 0.03 \times 10^{-3}$

$V = 600 \text{ Volt}$

13) Two parallel conducting plates 3 cm apart and situated in air are connected to a source of constant potential difference of 72 KV. Find the electric field intensity between the plates. Is it within permissible value? If a mica sheet of $\varepsilon_r = 4$ of thickness 1cm, is introduced between the plates, determine the field intensities in air and mica. Given the dielectric strengths of air & mica as 30 and 1000 kV/cm respectively.

Sol:

Voltage $V = 72$ KV.

distance $d = 3$ cm. $= 3\times10^{-2}$ m

for air, $\varepsilon_r = 1$.

Electric field Intensity $E = \frac{V}{d}$

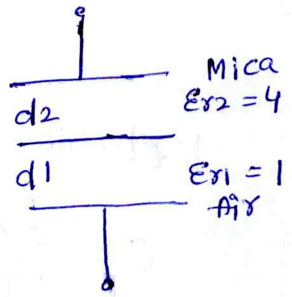
$= \frac{72\times10^{3}}{3\times10^{-2}}$

$= 2.4\times10^{6}$ V/m

$E = 24$ KV/m.

for air, The Given dielectric strength is 30 KV/cm. Hence E is within the permissible Limits.

Now the capacitor is modified as shown in fig.



$d_1 = 2$ cm, $\quad d_2 = 1$ cm.

$C_1 = \frac{\varepsilon_0 \varepsilon_{r1} A}{d_1}$ & $C_2 = \frac{\varepsilon_0 \varepsilon_{r2} A}{d_2}$

$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{A}{\frac{d_1}{\varepsilon_0\varepsilon_{r1}} + \frac{d_2}{\varepsilon_0 \varepsilon_{r2}}}$

$$C_{eq} = \frac{A\,\varepsilon_0}{\frac{2\times10^{-2}}{1} + \frac{1\times10^{-2}}{1}}$$

$$= \frac{A\,\varepsilon_0 \cancel{8.854\times10^{-12}}}{2\times10^{-2} + 1\times10^{-2}}$$

$$C_{eq} = 44.44\,\varepsilon_0 A \quad F$$

$$C_1 = \cancel{50\,\varepsilon_0 A}. \quad \frac{\varepsilon_0\,\varepsilon_{r1}\,A}{d_1} = \frac{\varepsilon_0\times1\times A}{2\times10^{-2}}$$

$$C_1 = 50\,\varepsilon_0 A.$$

$$C_2 = \frac{\varepsilon_0\,\varepsilon_{r2}\,A}{d_2} = \frac{\varepsilon_0\times4\times A}{1\times10^{-2}}$$

$$C_2 = 400\,\varepsilon_0 A$$

Note: The charge on C_1 & C_2 is same as capacitors are in Series.

$Q = C_1 V_1 = C_2 V_2 = C_{eq} V.$,

Where $V = 72$ KV given.

$$C_1 V_1 = C_{eq} V$$

$$V_1 = \frac{C_{eq} V}{C_1} = \frac{44.44\,\varepsilon_0 A}{50\,\varepsilon_0 A}\times 72 = 64 \text{ KV}$$

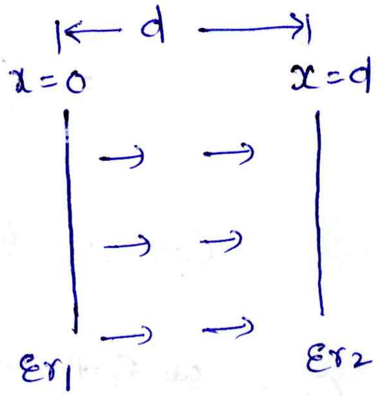
$$V_2 = \frac{C_{eq} V}{C_2} = \frac{44.44\,\varepsilon_0 A}{400\,\varepsilon_0 A}\times 72 = 8 \text{ KV}.$$

Electric field Intensity of air, $E_1 = \frac{V_1}{d_1} = \frac{64}{2} = 32$ KV/cm.

Electric field Intensity of Mica, $E_2 = \frac{V_2}{d_2} = \frac{8}{1} = 8$ KV/cm.

While given permission values are 30 KV/cm and 1000 KV/cm respectively. So for air it is not within permissible value. While for mica it is within permissible value.

14) The permittivity of the dielectric of parallel plate capacitor increases uniformly from one plate to other. If A is the surface areas of the plate and d is the thickness of dielectric, derive an expression for capacitance.



Sol: The arrangement is shown in fig.

The ε_r varies linearly from ε_{r1} to ε_{r2}

The Equation for this linear behaviour is

$$\varepsilon_r = Kx + A.$$

At $x=0$, $\varepsilon_r = \varepsilon_{r1}$

$$\therefore A = \varepsilon_{r1}.$$

At $x=d$, $\varepsilon_r = \varepsilon_{r2}$

$$\therefore \varepsilon_{r2} = Kd + \varepsilon_{r1} \quad \text{i.e,} \quad K = \frac{\varepsilon_{r2}-\varepsilon_{r1}}{d}$$

$$\varepsilon_r = \left[\frac{\varepsilon_{r2}-\varepsilon_{r1}}{d}\right]x + \varepsilon_{r1}.$$

Let the plate at $x=0$ carries positive charges.

$$\bar{E}_1 = \frac{+\rho_s}{2\varepsilon}\bar{a}_x.$$

$$\bar{E}_2 = -\frac{\rho_s}{2\varepsilon}(-\bar{a}_x)$$

$$E = \bar{E}_1 + \bar{E}_2 = \frac{\rho_s}{2\varepsilon}\bar{a}_x + \frac{\rho_s}{2\varepsilon}\bar{a}_x = \frac{2\rho_s\bar{a}_x}{2\varepsilon}$$

$$\bar{E} = \frac{\rho_s}{\varepsilon}\bar{a}_x$$

$$V = -\int_{-}^{+} E\, d\bar{l} = -\int_{x=d}^{x=0} \frac{\rho_s}{\varepsilon}\bar{a}_x \cdot dx\, \bar{a}_x$$

$$= -\int_{x=d}^{x=0} \frac{\rho_s}{\left[\left(\frac{\varepsilon_{r2}-\varepsilon_{r1}}{d}\right)x + \varepsilon_{r1}\right]\varepsilon_0}\,dx \quad \left(\text{since } \bar{a}_x\cdot\bar{a}_x = 1\right)$$

$$V = -\frac{\rho_s}{\varepsilon_0} \ln\left\{\left(\frac{\varepsilon_{r2}-\varepsilon_{r1}}{d}\right)x + \varepsilon_{r1}\right\}_{x=d}^{x=0} \times \frac{d}{(\varepsilon_{r2}-\varepsilon_{r1})}$$

$$= -\frac{\rho_s\, d}{\varepsilon_0(\varepsilon_{r2}-\varepsilon_{r1})}\left[\ln(\varepsilon_{r1}) - \ln(\varepsilon_{r2}-\varepsilon_{r1}+\varepsilon_{r1})\right]$$

$$= -\frac{\rho_s\, d}{\varepsilon_0(\varepsilon_{r2}-\varepsilon_{r1})} \ln\left[\frac{\varepsilon_{r1}}{\varepsilon_{r2}}\right]$$

$$V = \frac{\rho_s\, d}{\varepsilon_0(\varepsilon_{r2}-\varepsilon_{r1})} \ln\left[\frac{\varepsilon_{r2}}{\varepsilon_{r1}}\right]$$

And, $Q = \rho_s A$

$$C = \frac{Q}{V} = \frac{\rho_s A}{\frac{\rho_s\, d}{\varepsilon_0(\varepsilon_{r2}-\varepsilon_{r1})} \ln\left(\frac{\varepsilon_{r2}}{\varepsilon_{r1}}\right)}$$

$$\boxed{C = \frac{\varepsilon_0(\varepsilon_{r2}-\varepsilon_{r1})A}{d \ln\left(\frac{\varepsilon_{r2}}{\varepsilon_{r1}}\right)}}, \text{ Farad.}$$