Unit 2: Static Magnetic Feilds

Introduction:-

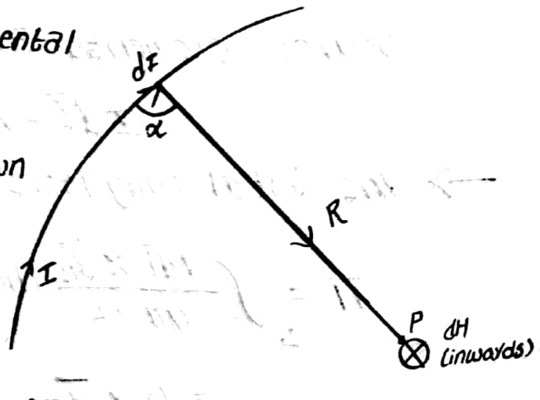
The electro static feild is characteriazed by electric feild intensity $\bar{E}$ and electric flux Density $\bar{D}$ are related as $\bar{D} = \epsilon\bar{E}$. similarly magneto static fields are obtain by magnetic feild intensity $\bar{H}$ and magnetic flux density $\bar{B}$ and they are related as $\bar{B} = \mu\bar{H}$.

Static electric feilds are obtained by static charges. Static magnetic feild's are obtained when static charges are moving with constant velocity (or) static magnetic fields are generated by constant current flow.

→ Electric feilds are discribed by columb's law and Gauss law. Similarity in magneto static feilds have Biot-Savart's law and Ampere's circuit law.

BIOT-SAVART'S LAW :-

It states that the elemental magnetic feild Intensity dH produced at point 'P', as shown in the figure.



magnetic feild due to current elements.

By a differential current equation Idl is proportional to the product Idl and sine the angle 'α' between the Idl and line joining the element to point 'P' and inversely proportional to square of the distance between element & point.

That is $dH \propto \frac{I dl \sin\alpha}{R^2}$

$$dH = K \frac{I dl \sin\alpha}{R^2}$$

→ where K is the proportionality constant

In SI units, $K = 1/4\pi$, so

$$dH = \frac{I dl \sin\alpha}{4\pi R^2}$$

→ From definition of cross product the above eqn

$$dH = \frac{I dl \times \bar{a}_R}{4\pi R^2} = \frac{I dl \times \bar{R}}{4\pi R^3}$$

* where $R = |\bar{R}|$ and $\bar{a}_R = \bar{R}/R$

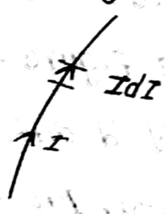
* we have different current distributions such as: line current, surface current and volume current as shown.

* If we define K as the surface current density and J as the volume current density, the source elements are related as

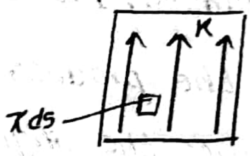
$$I d\bar{l} = \bar{K} ds = \bar{J} dv$$

→ The total magnetic feild intensity is given by

$$\bar{H} = \int_L \frac{I d\bar{l} \times \bar{a}_R}{4\pi R^2} \text{ (line current)}$$



$$\bar{H} = \int_S \frac{\bar{K} ds \times \bar{a}_R}{4\pi R^2} \text{ (surface current)}$$



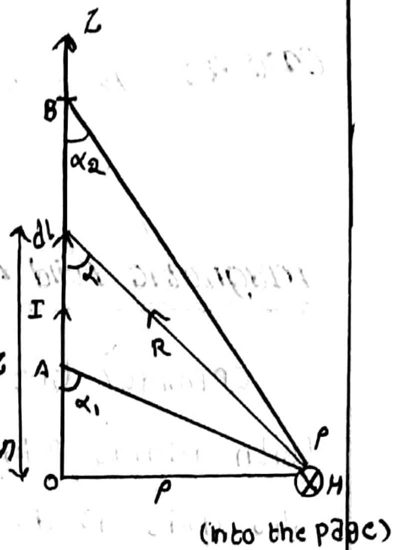
$$\bar{H} = \int_V \frac{\bar{J} dv \times \bar{a}_R}{4\pi R^2} \text{ (volume current)}$$

Magnetic feild intensity due stright conductor of finite length :-

→ consider a straight currents carrying conductors of finite length AB along z-axis with its upper and lower ends substending angles α_2 and α_1 and at P respectively.

→ It carriers direct current I. consider currents element $I\bar{dl}$ on the z-axis at distance from origin. According to biot - savart's law dH at P due to an element dI at (0,0,Z) is given by

$$d\bar{H} = \frac{I\bar{dl}\times\bar{R}}{4\pi R^3} \quad \text{--- (1)}$$



Finite straight conductor

But
$$d\bar{l} = dz\,\bar{a}_z$$
$$\bar{R} = \rho\bar{a}_\rho - z\bar{a}_z$$
$$R = \sqrt{\rho^2+z^2}$$
$$d\bar{l}\times R = \rho\, dz\, \bar{a}_\phi \quad \text{--- (2)}$$

→ Substitute equ (2) in equation (1), then

$$d\bar{H} = \frac{I\rho\, dz\, \bar{a}_\phi}{4\pi[\rho^2+z^2]^{3/2}}$$

→ The total magnetic feild intensity is given by

$$\bar{H} = \int \frac{I\rho\, dz}{4\pi[\rho^2+z^2]^{3/2}}\,\bar{a}_\phi$$

→ Let $z=\rho\cot\alpha$ and $dz=-\rho\,\mathrm{cosec}^2\alpha$, then above equation becomes

$$\bar{H} = \int \frac{I\rho\, dz}{4\pi[\rho^2+\rho^2\cot^2\alpha]^{3/2}}\,\bar{a}_\phi$$

$$\bar{H} = -\frac{1}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \operatorname{cosec}^2\alpha \, d\alpha}{\rho^3 \operatorname{cosec}^3\alpha} \bar{a}_\phi$$

$$= -\frac{1}{4\pi\rho} \int_{\alpha_1}^{\alpha_2} \sin\alpha \cdot d\alpha \cdot \bar{a}_\phi$$

$$= -\frac{1}{4\pi\rho} [-\cos\alpha]_{\alpha_1}^{\alpha_2} \bar{a}_\phi$$

$$\bar{H} = \frac{1}{4\pi\rho} (\cos\alpha_2 - \cos\alpha_1) \bar{a}_\phi$$

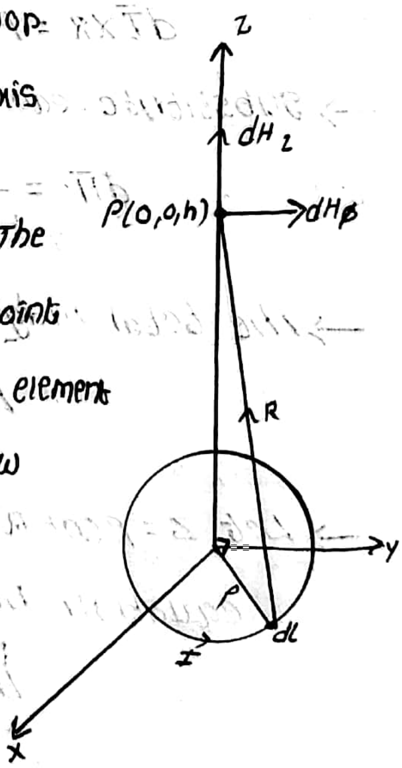
Case 1 :- If conductor is semi infinite then $\alpha_1 = 90°$ & $\alpha_2 = 0°$.

$$\bar{H} = \frac{1}{4\pi\rho} \bar{a}_\phi$$

Case 2 :- If conductor is infinite then $\alpha_1 = 180°$ & $\alpha_2 = 0°$

$$\bar{H} = \frac{1}{2\pi\rho} \bar{a}_\phi$$

Magnetic feild intensity on axis of current (circular) loop:-



Consider the circular current loop with radius ρ in xy plane with z-axis its axis is as shown in figure.

It carries direct current I. The magnetic feild intensity dH at point $P(0,0,h)$ contributed by current element $Id\bar{l}$ is given by Biot-Savart's law

$$d\bar{H} = \frac{Id\bar{l} \times \bar{R}}{4\pi R^3} \quad \text{—} (1)$$

where $d\bar{l} = \rho d\phi \, \bar{a}_\phi$

$$\bar{R} = -\rho\bar{a}_\rho + h\bar{a}_z$$

$$R = \sqrt{\rho^2 + h^2}$$

$$d\bar{I} \times \bar{R} = \rho d\phi\, \bar{a}_\phi \times (-\rho \bar{a}_\rho + h\bar{a}_z)$$

$$d\bar{I} \times \bar{R} = \rho h d\phi\, \bar{a}_\rho + \rho^2 d\phi\, \bar{a}_z$$

Substitute the above in eqn ①, we get

$$d\bar{H} = \frac{1}{4\pi[\rho^2+h^2]^{3/2}} (\rho h d\phi\, \bar{a}_\rho + \rho^2 d\phi\, \bar{a}_z)$$

Total magnetic feild is given

$$\bar{H} = \int \frac{1}{4\pi[\rho^2+h^2]^{3/2}} (\rho h d\phi\, \bar{a}_\rho + \rho^2 d\phi\, \bar{a}_z)$$

Due to symmetry all radial components get cancel

then, $$\bar{H} = \int_0^{2\pi} \frac{I\rho^2 d\phi}{4\pi[\rho^2+h^2]^{3/2}} \bar{a}_z$$

$$= \frac{I\rho^2}{4\pi[\rho^2+h^2]^{3/2}} \int_0^{2\pi} d\phi\, \bar{a}_z$$

$$= \frac{I\rho^2}{4\pi[\rho^2+h^2]^{3/2}} 2\pi \bar{a}_z$$

The magnetic feild intensity on axis of the circular current loop is given by

$$\boxed{\bar{H} = \frac{I\rho^2}{2[\rho^2+h^2]^{3/2}} \bar{a}_z}$$

Note :-

→ If $h=0$, then the magnetic feild intensity at center of the circular current loop is given by

$$\boxed{\bar{H} = \frac{I}{2\rho} \bar{a}_z}$$

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Ampere's circuit law :-

→ The Ampere's circuit law states that the line integral of H around a closed path is the same as the net current I enclosed by path.

→ In other words, the circulation of H equals I, that is

$$\boxed{\oint \bar{H} \cdot d\bar{l} = I} \quad \text{---} ①$$

→ By applying stoke's theorem to the left-hand side of equ ①, we obtain

$$\oint \bar{H} \cdot d\bar{l} = \int_S (\nabla \times \bar{H}) \cdot d\bar{s} \quad \text{---} ②$$

$$I = \int_S \bar{J} \cdot d\bar{s} \quad \text{---} ③$$

Subs the equ ② and ③ in equ ①, then

$$\int_S (\nabla \times \bar{H}) \cdot d\bar{s} = \int_S \bar{J} \cdot d\bar{s}$$

$$\boxed{\nabla \times \bar{H} = \bar{J}}$$

→ This is ampere law in point form. This is the third Maxwell's equation.

Applications of Ampere's law :-

→ Ampere's circuit law is used to determine H for symmetrical current distributions.

(i) Infinite line current

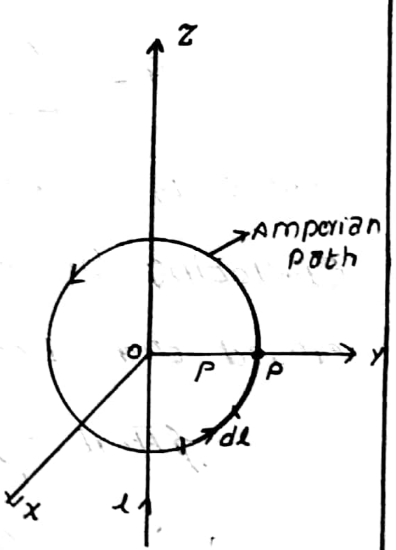
(ii) Infinite sheet current

(iii) Infinitely long coaxial transmission line

Infinite line current:-

→ let us consider an infinite line conductor that carries current I and lies along z-axis in figure.

→ To determine H at a point P. construct a closed path passing through P around the conductor. It is also called as an Amperian path.



Ampere's law applied to an infinite line current.

According to Ampere's law

$$\oint \bar{H}\cdot d\bar{l} = I \quad \text{——} ①$$

H has only H_ϕ component and

$$d\bar{l} = \rho\, d\phi\, \bar{a}_\phi$$

Therefore $\int_0^{2\pi} H_\phi \cdot \bar{a}_\phi \cdot \rho\, d\phi\, \bar{a}_\phi = I$

$$\int_0^{2\pi} H_\phi\, \rho\, d\phi = I$$

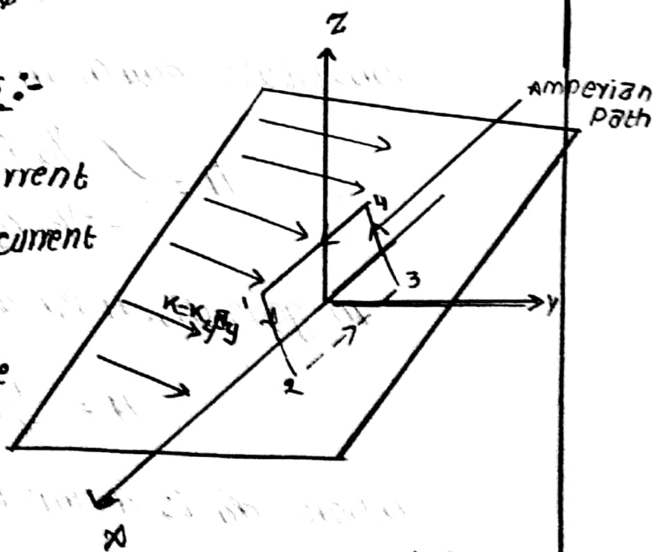
$$H_\phi \rho \int_0^{2\pi} d\phi = I \Rightarrow H_\phi \rho\, 2\pi = I$$

$$H_\phi = \frac{I}{2\pi\rho}$$

The magnetic feild intensity is given by

$$\bar{H} = \frac{I}{2\pi\rho}\, \bar{a}_\phi$$

Infinite Sheet of Current:-



→ consider an infinate current sheet carrying a uniform current density

$\bar{K} = K_y\, \bar{a}_y$ A/m in the $z=0$ plane as shown in figure. Let us construct a rectangular closed path of length a & width b in around the infinity current sheet as shown.

According to Ampere's law,

$$\oint \bar{H}\cdot d\bar{l} = I = K_y b \quad \text{——} (1)$$

$\bar{H}$ has only x-Component and remaining are get cancelled to each other due to symmetry.

$$\bar{H} = \begin{cases} H_0 \bar{a}_x & z>0 \\ -H_0 \bar{a}_x & z<0 \end{cases} \quad \text{——} (2)$$

where H_0 is the component along the x-direction

Evaluating the line integral of H in eqn (1) along the closed that $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$ gives

$$\oint \bar{H}\cdot d\bar{l} = \int_1^2 \bar{H}\cdot d\bar{l} + \int_2^3 \bar{H}\cdot d\bar{l} + \int_3^4 \bar{H}\cdot d\bar{l} + \int_4^1 \bar{H}\cdot d\bar{l}$$

$$= 0(-a) + (-H_0)(-b) + 0(a) + H_0(b)$$

$$= 2H_0 b \quad \text{——} (3)$$

Comparing eqn's (1) and (3), we get

$$2H_0 b = K_y b$$

$$H_0 = \frac{1}{2} K_y \quad \text{——} (4)$$

substitute eqn (4) in eqn (2), we get

$$\bar{H} = \begin{cases} \frac{1}{2} K_y \bar{a}_x & z>0 \\ -\frac{1}{2} K_y \bar{a}_x & z<0 \end{cases}$$

In general, $\bar{H}$ for an infinite current sheet is

$$\bar{H} = \frac{1}{2} \bar{K} \times \bar{a}_n$$

where $\bar{a}_n$ is a unit normal vector directed from the current sheet to the point of interest.

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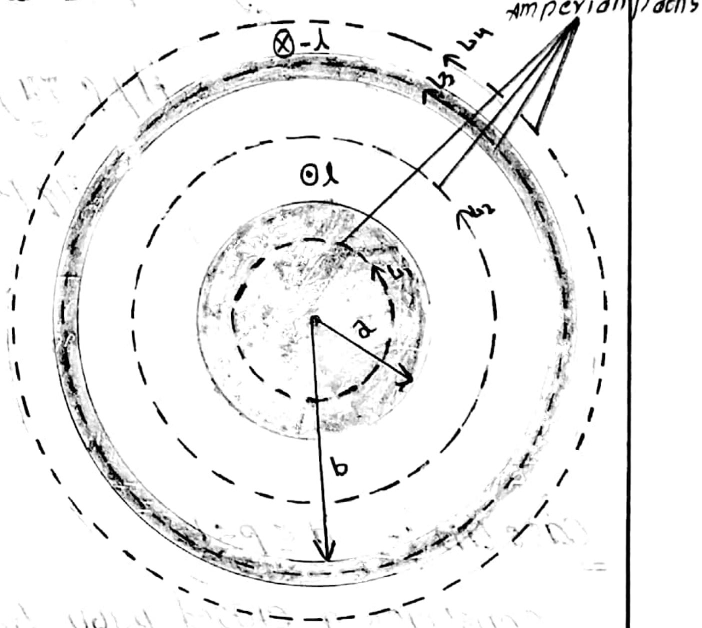
Infinitely long coaxial Transmission line :-

Consider an indefinitely long coaxial transmission line consisting of two concentric cylinders having their axes along z-axis. The cross section of line is shown in figure. The inner conductor has radius a and carries current I while the outer conductor has inner radius b and thickness t and carriers return current -I.



Cross section of the transmission lines

We can find $\bar{H}$ by considering four cases as follows

(i) $0 \le \rho \le a$ (ii) $a \le \rho \le b$

(iii) $b \le \rho \le b+t$ (iv) $\rho \ge b+t$

Case (i) :- $0 \le \rho \le a$

Construct a closed path L_1 of radius ρ as shown in figure.

According to Ampere law $\oint \bar{H} \cdot d\bar{l} = I_{enc}$ ——(1)

The current enclosed of closed path is given by

$$I_{enc} = I \frac{\text{Area of closed path}}{\text{Area of the inner conductor}}$$

$$= I \frac{\rho^2}{a^2}$$

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we know that $\overline{H} = H_\phi \overline{a}_\phi$ and $d\overline{l} = \rho d\phi \overline{a}_\phi$

Therefore $\int_0^{2\pi} H_\phi \cdot \overline{a}_\phi \cdot \rho d\phi \overline{a}_\phi = I \frac{\rho^2}{a^2}$

$$\int_0^{2\pi} H_\phi \cdot \rho d\phi = I \frac{\rho^2}{a^2}$$

$$H_\phi \rho \int_0^{2\pi} d\phi = I \frac{\rho^2}{a^2}$$

$$H_\phi \rho 2\pi = I \frac{\rho^2}{a^2}$$

$$H_\phi = \frac{I\rho}{2\pi a^2}$$

$$\boxed{\overline{H} = \frac{I\rho}{2\pi a^2} \overline{a}_\phi}$$

Case (ii) :- $a \le \rho \le b$

Construct a closed path L_2 of radius ρ as shown

According to Ampere's law $\oint \overline{H} \cdot d\overline{l} = I_{enc}$ —— (1)

The current enclosed by the closed path is I

we know that $\overline{H} = H_\phi \overline{a}_\phi$ and $d\overline{l} = \rho d\phi \overline{a}_\phi$

Therefore $\int_0^{2\pi} H_\phi \overline{a}_\phi \cdot \rho d\phi \overline{a}_\phi = I$

$$\int_0^{2\pi} H_\phi \rho d\phi = I$$

$$H_\phi \rho \int_0^{2\pi} d\phi = I$$

$$H_\phi \rho 2\pi = I$$

$$H_\phi = \frac{I}{2\pi\rho}$$

The magnetic feild intensity is given by

$$\boxed{\overline{H} = \frac{I}{2\pi\rho} \overline{a}_\phi}$$

Case (iii) :- $b \le \rho \le b+t$

Construct a closed path L_3 of radius ρ as shown in the figure. According to Ampere's law $\Rightarrow \oint \bar{H} \cdot d\bar{l} = I_{enc}$ — ①

The current enclosed by a closed path is given by

$$I_{enc} = I - I\frac{\pi[\rho^2 - b^2]}{\pi[(b+t)^2 - b^2]} = I\left[1 - \frac{(\rho^2 - b^2)}{(t^2 + 2tb)}\right]$$

we know that $\bar{H} = H_\phi \bar{a}_\phi$ and $\bar{dl} = \rho d\phi \bar{a}_\phi$

Therefore $\int_0^{2\pi} H_\phi \bar{a}_\phi \cdot \rho d\phi \bar{a}_\phi = I\left[1 - \frac{(\rho^2 - b^2)}{(t^2 + 2tb)}\right]$

$$\int_0^{2\pi} H_\phi \rho d\phi = I\left[1 - \frac{(\rho^2 - b^2)}{(t^2 + 2tb)}\right]$$

$$H_\phi \rho \int_0^{2\pi} d\phi = I\left[1 - \frac{(\rho^2 - b^2)}{(t^2 + 2tb)}\right]$$

$$H_\phi \rho 2\pi = I\left[1 - \frac{(\rho^2 - b^2)}{(t^2 + 2tb)}\right]$$

$$H_\phi = \frac{1}{2\pi\rho}\left[1 - \frac{(\rho^2 - b^2)}{(t^2 + 2tb)}\right]$$

$$\boxed{\bar{H} = \frac{1}{2\pi\rho}\left[1 - \frac{(\rho^2 - b^2)}{(t^2 + 2tb)}\right]\bar{a}_\phi}$$

Case (iv) :- $\rho \ge b+t$

Construct a closed path L_4 of radius ρ as shown in figure. According to ampere's law $\Rightarrow \oint \bar{H} \cdot d\bar{l} = I_{enc}$ — ①

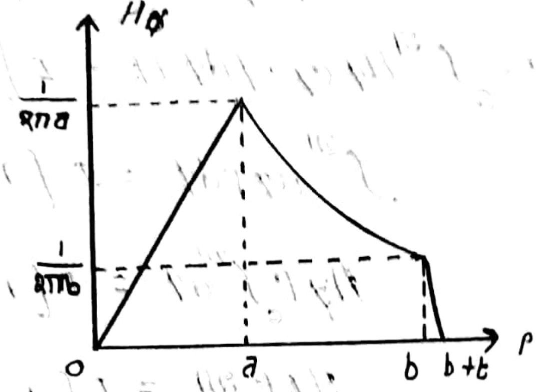
The current enclosed by the closed path is zero.

$$\oint \bar{H} \cdot d\bar{l} = I - I = 0$$

Therefore $\bar{H} = 0$.

The magnetic field intensity due to long coaxial transmission line is given by

$$H = \begin{cases} \dfrac{I\rho}{2\pi a^2}\,\bar{a}_\phi & 0 \le \rho \le a \\[2ex] \dfrac{1}{2\pi \rho}\,\bar{a}_\phi & a \le \rho \le b \\[2ex] \dfrac{1}{2\pi a}\left[1 - \dfrac{(\rho^2 - b^2)}{(t^2 + 2tb)}\right]\bar{a}_\phi & b \le \rho \le b+t \\[2ex] 0 & \rho \ge b+t \end{cases}$$



Magnetic Flux :-

The total number of magnetic lines of force passing through a given surface is called magnetic flux. It is denoted by ψ and units are weber.

→ The magnetic flux through a surface S is given by

$$\psi = \int \bar{B} \cdot d\bar{s} .$$

Magnetic flux density :-

The total magnetic flux passing through unit normal area is called magnetic flux density. It is denoted by B. units are Wb/m².

$$B = \frac{d\psi}{ds}\,\bar{a}_n \quad (Wb/m^2)$$

Magnetic feild Intensity :-

Magnetic feild Intensity at any point in the magnetic feild is defined as magnetic force that experienced by a unit north pole at a point. It is denoted by H and units are A/m.

→ The magnetic flux density and magnetic feild intensity are related to each other as

$$\bar{B} = \mu_0 \bar{H}$$

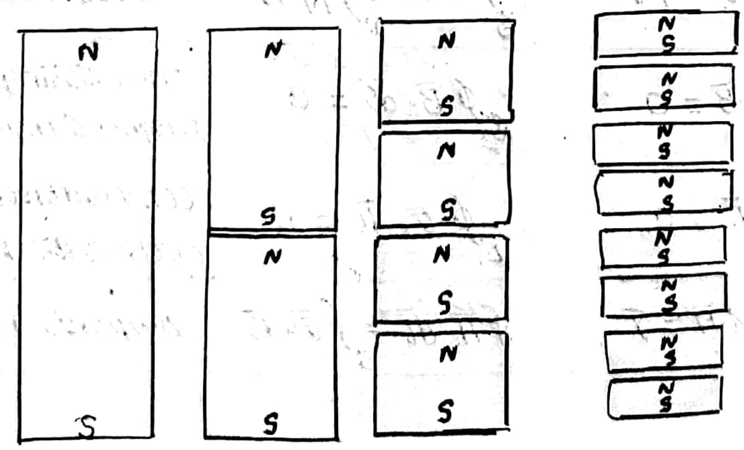
→ where μ_0 is constant known as permeability of free space and its value is

$$\mu_0 = 4\pi \times 10^{-7} H/m$$

Non Existing of isolated magnetic poles :-

→ The magnetic flux lines always form closed loop as shown in figure. This is due to the fact that it is not possible to have isolated poles.

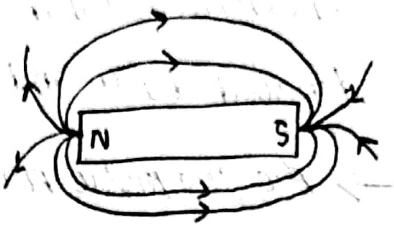
→ Let us take a magnetic bar, which have its individual poles. If we split into half, it again have individual poles for each part. so we can't separate the poles. Therefore an insolated pole doesn't exist.



→ The above figure shows successive division of a bar magnet in peices with north and south pole.

→ Thus total flux through a closed surface in a magnetic feild must be zero. That is

$$\boxed{\oint \bar{B} \cdot d\bar{s} = 0} \quad \text{—— (1)}$$



magnetic flux

→ This equation refers to law of conservation of magnetic flux.

→ By applying the divergence theorem to equ (1), we get

$$\oint \bar{B} \cdot d\bar{s} = \int_v \nabla \cdot \bar{B} \, dv = 0$$

$$\boxed{\nabla \cdot \bar{B} = 0}$$

→ The above equation is fourth maxwell equation.

Maxwell's equation for static EM feilds :-

→ maxwell's four equations for static electromagnetic feilds are given by.

Differntial form	Integral form	Remarks
$\nabla \cdot \bar{D} = \rho v$	$\oint_s \bar{D} \cdot d\bar{s} = \int_v \rho_v \, dv$	Gauss law
$\nabla \cdot \bar{B} = 0$	$\oint_s \bar{B} \cdot d\bar{s} = 0$	Non existing of magnetic monopole
$\nabla \times \bar{E} = 0$	$\oint_L \bar{E} \cdot d\bar{l} = 0$	conservativenes of electrostatic feild
$\nabla \times \bar{H} = \bar{J}$	$\oint_L \bar{H} \cdot d\bar{l} = \int_s \bar{J} \cdot d\bar{s}$	Ampere's law

Magnetic Scalar and Vector properties :-

→ Electrostatic fields have only scalar potential V, which is related to electric field intensity as

$$\bar{E} = -\nabla V.$$

→ where as magnetic fields have both scalar and vector magnetic potentials.

Magnetic Scalar potential :-

→ The magnetic scalar potential V_m is defined as its negative gradient gives the magnetic field intensity.

Therefore , $\boxed{\bar{H} = -\nabla V_m}$, if $\bar{J} = 0$

→ we know that

$$\bar{J} = \nabla \times \bar{H} = \nabla \times (-\nabla V_m) = 0$$

→ Because the curl of a gradient is zero. Thus the magnetic scalar potential between two points a and b is $\boxed{V_{m a.b} = \int_a^b \bar{H} \cdot d\bar{l}}$

Laplace equation for magnetic scalar potential :-

→ we know that $\nabla \cdot \bar{B} = 0$

that is $\nabla \cdot \bar{H} = 0$

But $\bar{H} = -\nabla V_m$

so $\nabla(-\nabla V_m) = 0$

$\therefore \nabla^2 V_m = 0$

The magnetic scalar potential V_m satisfy laplace.

Vector magnetic potentials :-

→ The vector magnetic potential A is defined as its curl gives magnetic flux density

$$\bar{B} = \nabla \times \bar{A}.$$

→ The vector magnetic potential can also expressed as

$$\boxed{\bar{A} = \int_L \frac{\mu_0 I d\bar{l}}{4\pi R}}$$ for line current.

→ The magnetic flux passing through a given area is

$$\boxed{\psi = \oint_L \bar{A} \cdot d\bar{L}}$$

Poission's equation for vector magnetic potential :-

→ From Ampere's circuit law

$$\nabla \times \bar{H} = \bar{J}$$

→ multiply both sides with μ_0, then we get

$$\nabla \times \mu_0 \bar{H} = \mu_0 \bar{J} \quad (\text{or}) \quad \nabla \times \bar{B} = \mu_0 \bar{J}$$

$$\text{But} \quad \bar{B} = \nabla \times \bar{A}$$

$$\text{Therefore} \quad \nabla \times \nabla \times \bar{A} = \mu_0 \bar{J}$$

$$\nabla (\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu_0 \bar{J}$$

$$\text{But} \quad \nabla \cdot \bar{A} = 0$$

$$\text{Therefore} \quad -\nabla^2 \bar{A} = \mu_0 \bar{J}$$

The possion expression for vector magnetic potential is given by $\nabla^2 \bar{A} = -\mu_0 \bar{J}$.

Forces due to magnetic feild :-

there are three ways in which force due to the magnetic feild can be experienced. Force can be due to a moving charge particle in magnetic feild $\bar{B}$.

(a) Force due to moving charge particle :-

According to columbs law of electric feild / force on a stationary (or) moving charge Q in an electric feild is given by

$$\bar{F_e} = Q\bar{E} \quad \text{--- (1)}$$

→ The direction of force is same as electric feild intensity.

→ A magnetic feild can exert force only on a moving charge.

→ The magnetic force experienced by a charge Q is F_m which is moving with a velocity $\bar{u}$ in a magnetic feild is given by

$$\bar{F_m} = Q\bar{u} \times \bar{B} \quad \text{--- (2)}$$

→ This is clear that $\bar{F_m}$ is perpendicular to both $\bar{u}$ & $\bar{B}$.

For a moving charge Q in presence of both electric and magnetic feild, the total force on charge is

$$\bar{F} = \bar{F_e} + \bar{F_m} = Q(\bar{E} + \bar{u} \times \bar{B}) \quad \text{--- (3)}$$

→ This is known as the Lorentz force equation. It relates mechanical force to electric force.

→ If the mass of charged particle moving in $\bar{E}$ and $\bar{B}$ feilds is 'm', by newton's second law of motion.

$$\bar{F} = m\frac{d\bar{u}}{dt} = Q(\bar{E} + \bar{u} \times \bar{B}) \quad \text{--- (4)}$$

→ This determines motion of charge particles in $\bar{E}$ & $\bar{B}$.

(b) Force on a current element :-

Consider a differential charge dQ moving with the velocity $\bar{u}$ in magnetic feild $\bar{B}$.

The differential magnetic force on differential charge carriers is given by

$$d\bar{F} = dQ\,\bar{u} \times \bar{B} \quad \text{--- (1)}$$

considering a current element $I d\bar{l}$ of a current-carrying conductor in magnetic feild.

$$I d\bar{l} = \frac{dQ}{dt} d\bar{l} = \frac{dQ}{dt}\frac{d\bar{l}}{dt} = dQ\,\bar{u} \quad \text{--- (2)}$$

Thus the force on a current element $I d\bar{l}$ in a magnetic feild is $d\bar{F} = I d\bar{l} \times \bar{B}$

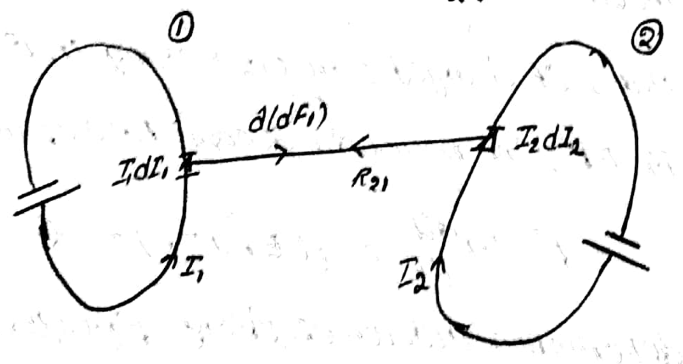
If the current I is passing through a linear conductor of length L, the force on conductor is

$$F = \int_L I d\bar{l} \times \bar{B}$$

force on that stright line in magnetic feild is given by $F = BIL \sin\theta\, \bar{a}_n$.

(c) Force between two current Elements :-

Let us consider two current elements $I_1 dI_1$ and $I_2 dI_2$ of conductors 1 and 2.



The distance between two elements is R_{21}.

According to Biot-Savart's law, both current elements produce magnetic fields.

Therefore the force $d(dF_1)$ on element $I_1 d\bar{l}_1$ due to field dB_2 produced by element $I_2 d\bar{l}_2$ as shown in figure is given by $d(d\bar{F}_1) = I_1 d\bar{l}_1 \times d\bar{B}_2$ —— ①

From Biot-Savart's law

$$d\bar{B}_2 = \frac{\mu_0 I_2 \, d\bar{l}_2 \times \bar{a}_{R_{21}}}{4\pi R_{21}^2}$$

From above equations

$$d(dF_1) = \frac{\mu_0 I_1 d\bar{l}_1 \times (I_2 d\bar{l}_2 \times \bar{a}_{R_{21}})}{4\pi R_{21}^2}$$

The total force F_1 on current loop 1 due to loop 2 is

$$F_1 = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1} \oint_{L_2} \frac{d\bar{l}_1 \times (d\bar{l}_2 \times \bar{a}_{R_{21}})}{R_{21}^2}$$

Similarly the force F_2 on loop 2 due to magnetic field B_1 from loop 1 is $\bar{F}_2 = -\bar{F}_1$

This is also called Ampere's force law.

Magnetic Torque and moment:-

→ The torque T (or mechanical moment of force) on the conductor loop is also defined as the cross product of force F and the moment arm r.

$$\boxed{\bar{T} = \bar{r} \times \bar{F}} \quad N\text{-}m$$

→ The torque (T) due to force on the conductor loop is equal to the cross product of magnetic dipole moment $\bar{m}$ and magnetic flux density $\bar{B}$.

$$\boxed{\bar{T} = \bar{m} \times B} \quad N\text{-}m$$

* magnetic dipole :-

A bar magnet (or) a small filamentary current loop is usually refered as magnetic dipole .

magnetic dipole $m = IS$

$= Q_m L$

where Q_m is pole strength

l is length.

→ The magnetic dipole moment is the product of current and area of loop, its direction is normal to loop.

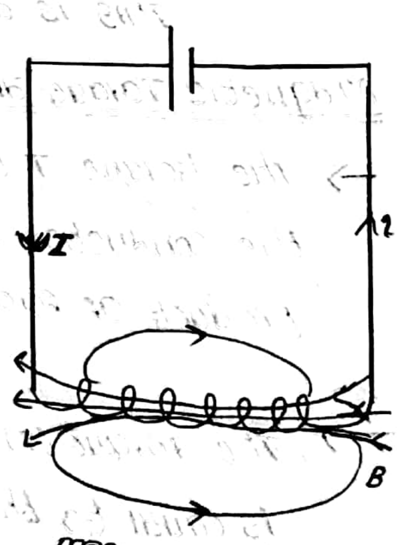
$$\boxed{m = IS\,\bar{a}_n} \quad A\text{-}m^2$$

Inductor :-

A circuit (or) part of circuit that has inductance is called an inductor.

Inductance :-

A circuit carrying current I produces a magnetic feild B which causes a flux ψ to pass through each turn of circuit as shown in figure.



magnetic feild produced by a circuit

→ If the circuit has N turns, then flux linkage λ is

$$\boxed{\lambda = N\psi} \quad (1)$$

→ The flux linkage λ is proportional to curren I produced,

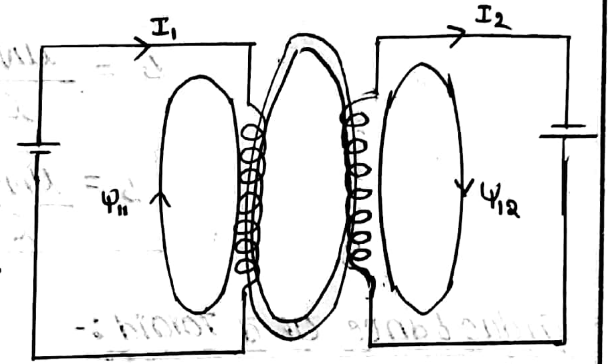
i.e $\lambda \propto I \Rightarrow \boxed{\lambda = LI} \quad (2)$

→ The inductance L of a linkage or an inductor is defined as ratio of magnetic flux linkage λ to current through the inductor, that is

$$\boxed{L = \lambda / I = \frac{N\psi}{I}}$$

→ The unit of inductance is Henry (H). This inductance is also called self inductance.

Mutual Induction :-



Mutual inductance

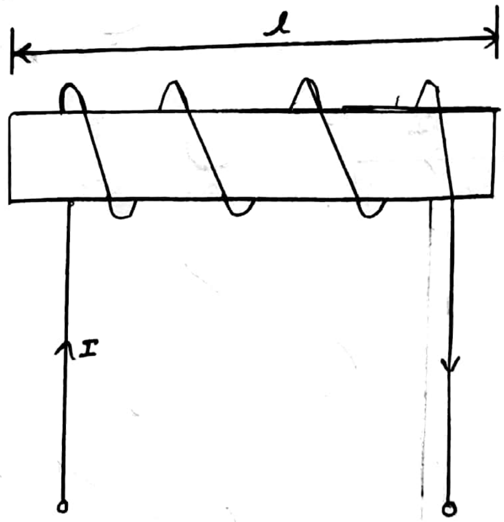
→ The mutual inductance M_{12} is the ratio of the flux linkage in one circuit to current in other circuit.

$$\boxed{M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1 \psi_{12}}{I_2}}$$

similarly

$$\boxed{M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_2 \psi_{21}}{I_1}}$$

Inductance of a solenoid :-



Solenoidal with N turns.

→ Consider a solenoid of N-turns as shown.

→ Let the current flowing through the solenoid be I. Let the length of the solenoid be L and cross section Area be A.

According to Ampere's law

$$\oint H\,dL = NI$$

$$HL = NI$$

The magnetic feild intensity inside solenoid is given by $H = \frac{NI}{l}$

The total flux linkage is given

Total flux linkage $= N\psi = NBA = N\mu HA$

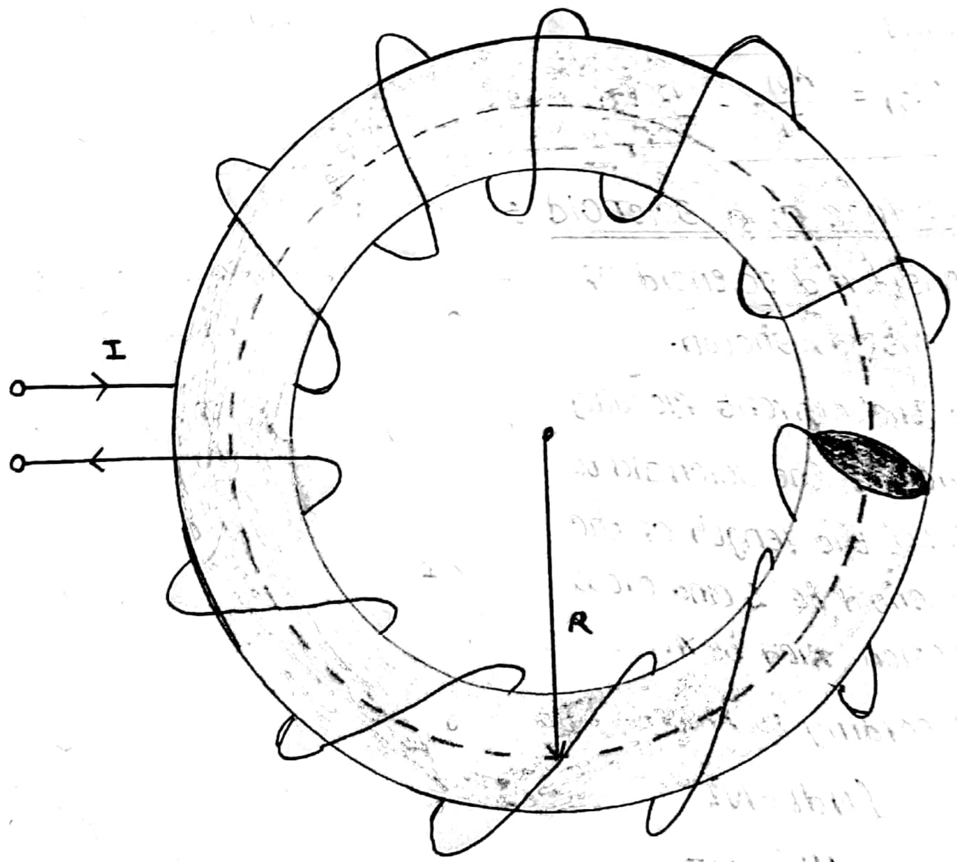
$$= N\mu\left[\frac{NI}{l}\right]A = \frac{\mu N^2 IA}{l}$$

The inductance of solenoid is $L = \frac{\text{Total flux linkage}}{\text{Total current}}$

$$L = \frac{\mu N^2 IA}{lI}$$

$$\boxed{L = \frac{\mu N^2 A}{l}}$$

Inductance of a Toroid :-



Toroidal Ring

Consider the torodial ring with N turns and carrying curren I. Let the radius of the toroid be R as shown in figure.

The magnetic flux density inside the toroidial ring is given by $B = \frac{\mu N I}{2\pi R}$

The total flux linkage of toroidal ring of N turns is given by Total flux linkage $= N\psi = NBA$

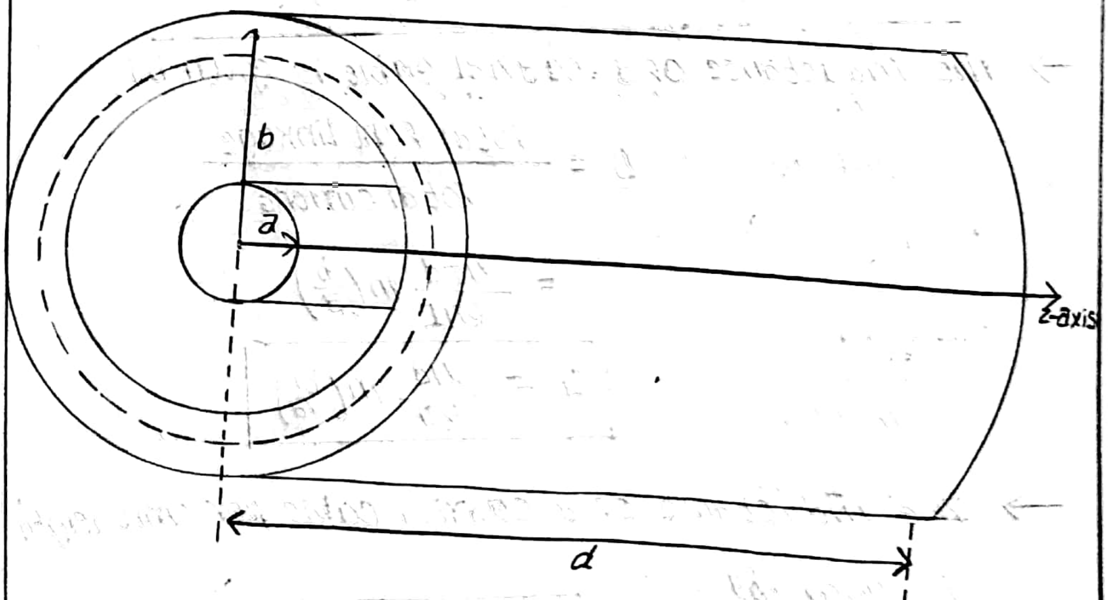
$$= N\left[\frac{\mu N I}{2\pi R}\right]A = \frac{\mu N^2 I A}{2\pi R}.$$

The inductance of a solenoid is given by

$$L = \frac{\text{Total flux linkage}}{\text{Total current}} = \frac{\mu N^2 I A}{2\pi R I}$$

$$\boxed{L = \frac{\mu N^2 A}{2\pi R}}$$

Inductance of a coaxial cable :-



Coaxial cable carrying current I

→ Consider a coaxial cable with inner radius a and outer conductor radius b as shown above.

→ Let the current through coaxial cable be I.

→ The magnetic feild intensity at any point between inner and outer conductor is given by

$$\bar{B} = \frac{\mu I}{2\pi \rho} \bar{a}_\phi .$$

→ The total magnetic flux linkage is given by

$$\psi = \int_s \bar{B} \cdot d\bar{s} .$$

$$d\bar{s} = d\rho \, dz \, \bar{a}_\phi .$$

where
$$\psi = \int_0^d \int_a^b \left(\frac{\mu I}{2\pi \rho} \bar{a}_\phi \right) \cdot (d\rho \, dz \, \bar{a}_\phi)$$

$$= \frac{\mu I}{2\pi} \int_0^d dz \int_a^b \frac{d\rho}{\rho}$$

$$= \frac{\mu I}{2\pi} d \, [\ln \rho]_a^b$$

$$= \frac{\mu I d}{2\pi} \ln\left(\frac{b}{a}\right)$$

→ The inductance of a coaxial cable is given by

$$L = \frac{\text{Total flux linkage}}{\text{Total current}}$$

$$= \frac{\mu I d}{2\pi I} \ln\left(\frac{b}{a}\right)$$

$$\boxed{L = \frac{\mu d}{2\pi} \ln(b/a)}$$

→ The inductance of a coaxial cable per unit length is given by

$$\boxed{L = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right)}$$

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Energy stored in the magnetic feild :-

→ If a current I is applied on a coil having N turns, it produces a magnetic feild with flux ψ.

→ The voltage induced in coil is $V = L\frac{dI}{dt}$.

→ we know that power $= VI = LI\frac{dI}{dt}$.

→ Then the energy stored is

$$W_m = \int P dt = \int LI \frac{dI}{dt} dt$$

$$= \int LI\, dI = \frac{1}{2}LI^2$$

$$\boxed{W_m = \frac{1}{2}LI^2 \text{ Joules}}$$

Energy Density stored in magnetic feild :-

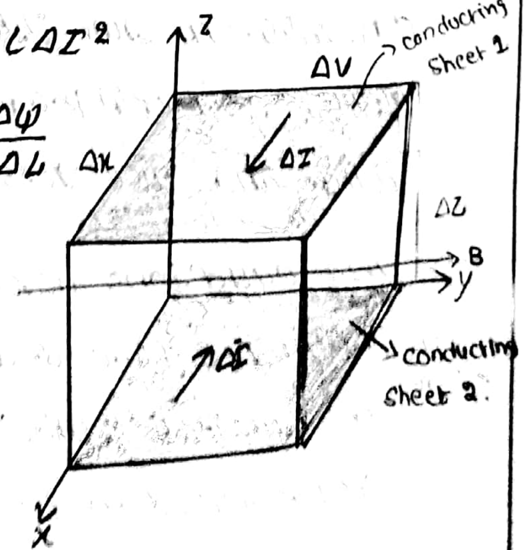
→ The energy stored in magnetic feild is $W_m = \frac{1}{2}LI^2$.

→ Consider a differential volume in a magnetic feild as shown in figure.

→ Let the volume be covered with conducting sheets at the top and bottom surfaces with current ΔI.

→ The energy stored in the differential volume is

$$\Delta W_m = \frac{1}{2}\Delta L \Delta I^2$$



we know that $\Delta L = \frac{\Delta\psi}{\Delta I}$

Therefore $\Delta W_m = \frac{1}{2}\Delta\psi\Delta I$

where $\Delta\psi = B\Delta x \Delta z$

$\Delta I = H\Delta y$

Then

$\Delta W_m = \frac{1}{2}B\Delta x \Delta z H\Delta y$

$\Delta W_m = \frac{1}{2}BH\Delta V$

The energy density stored in the magnetic feild is

$$w_m = \lim_{\Delta v \to 0} \frac{\Delta W_m}{\Delta v} = \frac{1}{2} BH$$

$$\boxed{w_m = \frac{1}{2} BH = \frac{1}{2} \mu H^2}$$

Magneto motive Force :-

→ It is the force that sets up magnetic flux within and around the object.

→ The units are amp-turns.

→ The magneto motive force = NI.

→ It is also called magnetic potential.

→ Then magneto motive force is given by

$$V_{ma,b} = \int_a^b \bar{H} \cdot d\bar{l} .$$

Maxwell's Equations For Time varying Feilds

→ The time varying feilds are produced due to time varying currents. In time varying feilds, the time varying electric feild will produce the time varying magnetic feild and vice versa. They are interdependent.

→ The equations describing relation between time varying electric and magnetics feilds are known as maxwell equations. These equations can be represented in integral and differential form.

Faraday's law :-

→ It states that the induced emf V_{emf} in any closed circuit is equal to rate of change of magnetic flux linkage by the circuit.

$$V_{emf} = -N\frac{d\psi}{dt}.$$

→ where N is the number of turns in circuit and ψ is the flux through each turn. The negative sign shows that the induced voltage acts in such a way as to oppose the flux producing it.

$$V_{emf} = -\frac{d\psi}{dt} \quad \text{——— (1)}$$

→ Induced emf in closed circuit is given by

$$V_{emf} = \oint_L \bar{E}\cdot d\bar{l}.$$

→ The magnetic flux ψ passing through a particular area is given by $\psi = \int_S \bar{B}\cdot d\bar{s}$. From above equation we have

$$\oint_L \bar{E}\cdot d\bar{l} = -\frac{d}{dt}\int_S \bar{B}\cdot d\bar{s}$$

$$\boxed{\oint_L \bar{E}\cdot d\bar{l} = -\int_S \frac{\partial B}{\partial t}\cdot d\bar{s}}$$

→ This is one of maxwell equations for time-varying fields in integral form as

$$\int_S \nabla\times\bar{E}\cdot d\bar{s} = -\int_S \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s}$$

→ By eliminating the surface integrals

$$\boxed{\nabla\times\bar{E} = -\frac{\partial B}{\partial t}}$$

→ This is one of maxwell equation in differential form.

→ This is one of the maxwell's equation for time-varying in differential form.

→ The variation of ~~flux~~ flux with time in 3 ways.

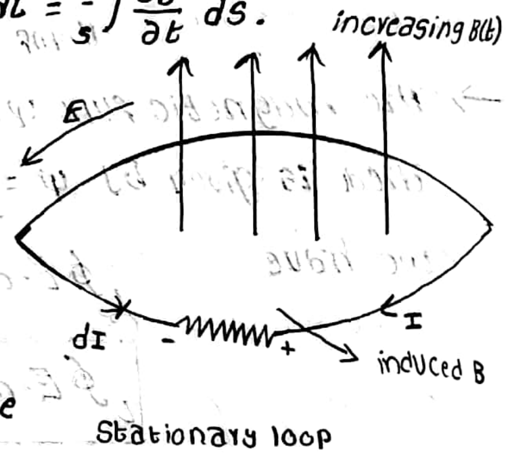
1) A stationary loop in a time varying B feild.
2) A moving loop in a static B feild.
3) A moving loop in a time varying B feild.

Transformer e.m.f :-

→ Consider a stationary conducting loop placed in the time varying magnetic feild B. as shown in figure.

According to Faraday's law

$$V_{emf} = \oint_{L} \bar{E}\cdot d\bar{L} = -\int_{S} \frac{\partial \bar{B}}{\partial t}\, d\bar{S}.$$



→ when emf is induced in a stationary conducting loop by time-varying B feild is called transformer emf. due to it is formed by the transformer action.

Motional EMF :-

→ When emf is induced in a moving loop by the static B feild is called motional EMF.

$$\bar{F} = Q(\bar{u}\times\bar{B}) \text{ and } \bar{E} = F/Q = (\bar{u}\times\bar{B})$$

$$V_{emf} = \oint_{L} \bar{E}\cdot d\bar{l} = \oint_{L} (\bar{u}\times\bar{B})\cdot d\bar{l}$$

Inconsistence of Ampere's law and displacement current Density :-

→ According to Ampere's law, for static E.M's feilds

$$\nabla \times (\bar{H}) = \bar{J} \quad \text{——} (1)$$

Taking the divergence on both sides of eqn (1), then

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot \bar{J}$$

But the divergence of curl of any vector is identically zero.

$$\nabla \cdot (\nabla \times \bar{H}) = 0 = \nabla \cdot \bar{J} \quad \text{——} (2)$$

But continuity equation for time varying feilds is given as

$$\nabla \cdot \bar{J} = \frac{\partial \rho v}{\partial t} \neq 0 \quad \text{——} (3)$$

Thus eqn (2) and (3) are incompactable for time-varying conditions. In other words Ampere law is do not consistent with time varying feilds and needs the modification to make it any time. To do this, we add a term $\bar{J_d}$ to eqn (1), then

$$\nabla \times (\bar{H}) = \bar{J} + \bar{J_d} \quad \text{——} (4)$$

Taking divergence on both sides, we get

$$\nabla \cdot (\nabla \times H) = \nabla \cdot (\bar{J} + \bar{J_d})$$

The divergence of curl of any vector is zero. Hence

$$\nabla \cdot (\bar{J} + \bar{J_d}) = 0$$

$$\nabla \cdot \bar{J} + \nabla \cdot \bar{J_d} = 0$$

$$\nabla \cdot \bar{J_d} = -\nabla \bar{J}.$$

But from continuity equation $\nabla \cdot \bar{J} = -\frac{\partial \rho v}{\partial t}$

Therefore $\nabla \cdot \bar{J}_d = \frac{\partial \rho_v}{\partial t}$

From Gauss law $\nabla \cdot \bar{D} = \rho_v$

$$\nabla \cdot \bar{J}_d = \frac{\partial}{\partial t}(\nabla \cdot \bar{D})$$

$$\nabla \cdot \bar{J}_d = \nabla \cdot \frac{\partial \bar{D}}{\partial t}$$

removing div on both sides

$$\bar{J}_d = \frac{\partial \bar{D}}{\partial t} \quad \text{—⑤}$$

Subs ⑤ in ④, we get

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

This is maxwell equation (based on Ampere circuit law).

The term $\bar{J}_d = \frac{\partial D}{\partial t}$ is known as displacement current density and $\bar{J} = \sigma \bar{E}$ is conduction current density.

Displacement Current :-

Displacement current is the flow of charge in dielectrics which results due to time varying feilds. Based on displacement current density, we can define the displacement current as

$$I_d = \int_s \frac{\partial \bar{D}}{\partial t} \cdot d\bar{s} = \frac{d}{dt} \int_s \bar{D} \cdot d\bar{s} = \frac{dQ}{dt} = I.$$

Gauss law :-

It states that the total electric flux ψ which is passing through any closed surface is equal to total charge enclosed by surface.

$$\psi = Q$$

Total flux $\psi = \oint_S \bar{D} \cdot d\bar{s}$

Total charge enclosed $Q = \int_V \rho_v dv$

$$\boxed{\oint_S \bar{D} \cdot d\bar{s} = \int_V \rho_v dv}$$

By applying divergence theorem, to find first term in above equation

$$\oint_S \bar{D} \cdot d\bar{s} = \int_V \nabla \cdot \bar{D} dv.$$

$$\int_V \nabla \cdot \bar{D} dv = \int_V \rho_v dv$$

$$\boxed{\nabla \cdot \bar{D} = \rho_v}$$

This is another maxwell equation.

Non-Existing of Isolated magnetic poles :-

→ The magnetic flux lines always from closed loops as shown in figure. This is due to fact that it is not possible to have isolated magnetic poles.

→ Thus the total flux through a closed surface in a magnetic feild must be zero as shown in figure.

That is $\boxed{\oint \bar{B} \cdot d\bar{s} = 0}$ —— ①

→ This equation refers to the law of conservation of magnetic flux. By applying the divergence theorem to eqn ①, we obtain

$$\oint \bar{B} \cdot d\bar{s} = \int_V \nabla \cdot \bar{B} dv = 0$$

(or) $\boxed{\nabla \cdot \bar{B} = 0}$

→ This equation is another maxwell's equation.

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General form of Maxwell's equations:-

S.No	Differential form	Integral form	Remarks
1.	$\nabla \cdot \bar{D} = \rho_V$	$\oint_S \bar{D} \cdot d\bar{s} = \int_V \rho_V \, dv$	Gauss's law
2.	$\nabla \cdot \bar{B} = 0$	$\oint_S \bar{B} \cdot d\bar{s} = 0$.	Non-existance of magnetic monopole
3.	$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$	$\oint_L \bar{E} \cdot d\bar{L} = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s}$	Faraday's law
4.	$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$	$\oint_L \bar{H} \cdot d\bar{L} = \int_S \left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right) \cdot d\bar{s}$	Ampere circuit law

Maxwell equations :-

1st equation :-

The total electric flux ψ passing through any closed surface is equal to total charge enclosed by the surface.

$$\oint_S \bar{D} \cdot d\bar{s} = \int_V \rho_V \, dv.$$

2nd equation :-

"The surface integral of magnetic flux density over a closed surface is always equal to zero."

$$\oint_S \bar{B} \cdot d\bar{s} = 0$$

3rd equation :-

"The line integral of electromotive force (emf) around any closed path is equal to negative surface integral of time rate of change of magnetic flux density over the surface bounded by path".

$$\oint \bar{E} \cdot d\bar{l} = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s}.$$

4th equation :-

"The line integral of magneto motive force (mmf) around any closed path is equal to the surface integral of conduction and displacement current densities over the surface bounded by the path".

Maxwell's equations for free space (or) good dielectric :-

For free space $\rho_v = 0$ and $\sigma = 0$.

S.No	Differential Form	Integral Form	Remarks
1.	$\nabla \cdot \bar{D} = 0$	$\oint_S \bar{D} \cdot d\bar{s} = 0$	Gauss's law
2.	$\nabla \cdot \bar{B} = 0$	$\oint_S \bar{B} \cdot d\bar{s} = 0$	Non existence of magnetic
3.	$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$	$\oint_L \bar{E} \cdot d\bar{l} = -\int_S \frac{\partial \bar{B}}{\partial t} d\bar{s}$	Faraday's law
4.	$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t}$	$\oint_L \bar{H} \cdot d\bar{l} = \int_S \frac{\partial \bar{D}}{\partial t} d\bar{s}$	Ampere's circuit law

Maxwell's equations for good conductor :-

For good conductor $\rho_v = 0$ and $J_d = 0$.

S.No	Differential form	Integral form	Remarks
1.	$\nabla \cdot \bar{D} = 0$	$\oint_S \bar{D} \cdot d\bar{s} = 0$	Gauss law
2.	$\nabla \cdot \bar{B} = 0$	$\oint_S \bar{B} \cdot d\bar{s} = 0$	Non-existence of magnetic monopole
3.	$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$	$\oint_L \bar{E} \cdot d\bar{l} = -\int_S \frac{\partial \bar{B}}{\partial t} d\bar{s}$	Faraday's law
4.	$\nabla \times \bar{H} = \bar{J}$	$\oint_L \bar{H} \cdot d\bar{l} = \int_S \bar{J} \cdot d\bar{s}$	Ampere's circuit law

Maxwell's equations for time - harmoic feilds form:-

A time - harmonic feild is the one that varies periodically (or) sinusoidally with time. For time harmonic fields.

$$\bar{D} = D_0 e^{j\omega t}$$

$$\frac{\partial \bar{D}}{\partial t} = j\omega e^{j\omega t} = j\omega \bar{D}$$

S. NO	Differential (Form)	Integral Form	Remarks
1.	$\nabla \cdot \bar{D} = \rho_v$	$\oint_S \bar{D} \cdot d\bar{S} = \int_V \rho_v dv$	Gauss law
2.	$\nabla \cdot \bar{B} = 0$	$\oint_S \bar{B} \cdot d\bar{S} = 0$	Non existance of magnetic monopole
3.	$\nabla \times \bar{E} = -j\omega \bar{B}$	$\oint_L \bar{E} \cdot d\bar{l} = -j\omega \int_S \bar{B} \cdot d\bar{s}$	Faraday's law
4.	$\nabla \times \bar{H} = \bar{J} + j\omega \bar{D}$	$\oint_L \bar{H} \cdot d\bar{l} = \int_S (\bar{J} + j\omega \bar{D}) \cdot d\bar{s}$	Ampere's circuit law

Boundary Conditions of electro magnetic feilds :-

→ If the feild exists in a region consisting of two different media then the conditions that feild must satisfy at the interface seperating the media are called boundary conditions.

→ These conditions are helpful in determining the feild on one side of the boundary if the feild on the other side is known.

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Electric Feild boundary conditions :-

→ There are three types of boundary conditions for the electric fields. They are :-

1. Dielectric (ϵ_{r_1}) and dielectric (ϵ_{r_2})
2. conductor and dielectric
3. conductor and free space.

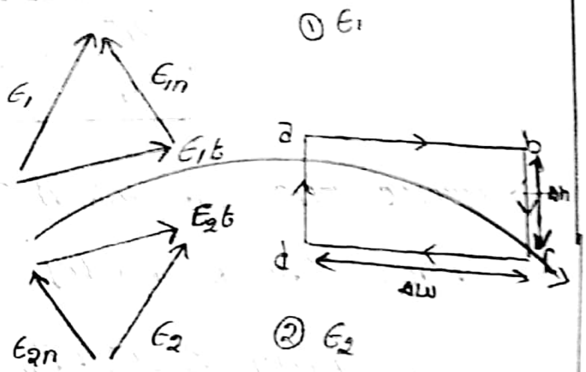
1. Dielectric (ϵ_{r_1}) and Dielectric (ϵ_{r_2}) boundary conditions:-

→ Consider the Electric feild Existing in a region consisting of two different dielectrics characterized by

$\epsilon_1 = \epsilon_0 \epsilon_{r_1}$

$\epsilon_2 = \epsilon_0 \epsilon_{r_2}$

Both are as shown in figure.



→ E_1 and E_2 in media ① and ②, can be expressed as

$$\bar{E_1} = \bar{E}_{1t} + \bar{E}_{1n}$$
$$\bar{E_2} = \bar{E}_{2t} + \bar{E}_{2n}$$

→ Apply $\oint E.\bar{dl} = 0$ to the closed path abcda, of figure, assuming that the path is very small wrt the variation of E.

→ we obtain $\int_a^b \bar{E}.\bar{dl} + \int_c^d \bar{E}.\bar{dl} + \int_b^c \bar{E}.\bar{dl} + \int_d^a \bar{E}.\bar{dl} = 0$

$$E_{1t}\,\Delta w - E_{1n}\frac{\Delta b}{2} - E_{2A}\frac{\Delta h}{2} - E_{2t}\,\Delta w + E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2} = 0$$

→ As $\Delta h \to 0$, above equation becomes,

$$E_{1t}\,\Delta W - E_{2t}\,\Delta W = 0$$

$$E_{1t}\,\Delta W = E_{2t}\,\Delta W$$

$$\boxed{E_{1t} = E_{2t}}$$

* E_t - Electric Feild tangential component.

→ Therefore the tangential component of $\bar{E}$ is continous across the boundary.

$$\bar{D} = \epsilon_0 \bar{E} \Rightarrow \bar{E} = \frac{\bar{D}}{\epsilon_0}$$

$$\boxed{\frac{D_{1t}}{\epsilon_{r_1}} = \frac{D_{2t}}{\epsilon_{r_2}}}$$

→ Hence the tangential component of D is discontinous across the boundary.

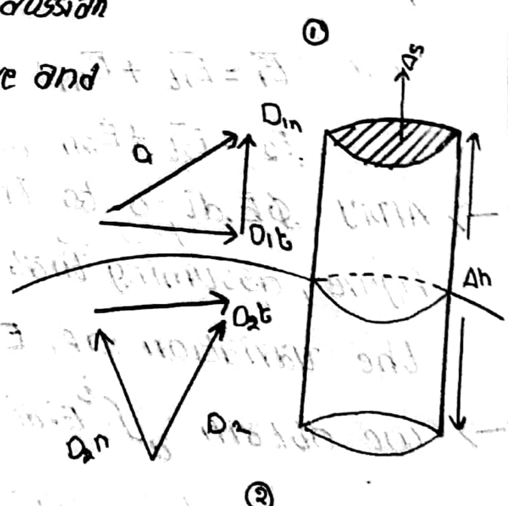
Similarly $\bar{D} = \bar{D_t} + \bar{D_n}$.

→ Apply $\oint \bar{D}\cdot d\bar{s} = Q$ to the Gaussian surface as shown in figure and allowing $\Delta h \to 0$ gives

$$\int_{top} \bar{D}\cdot d\bar{s} + \int_{bottom} \bar{D}\cdot d\bar{s} = \rho_s \Delta S$$

$$D_{1n}\Delta S - D_{2n}\Delta S = \rho_s \Delta S$$

$$\boxed{D_{1n} - D_{2n} = \rho_s}$$



a) dielectric - dielectric boundary

b) electric flux density components

→ If no free charges exist at the interface, $\rho_s = 0$ and above equation becomes

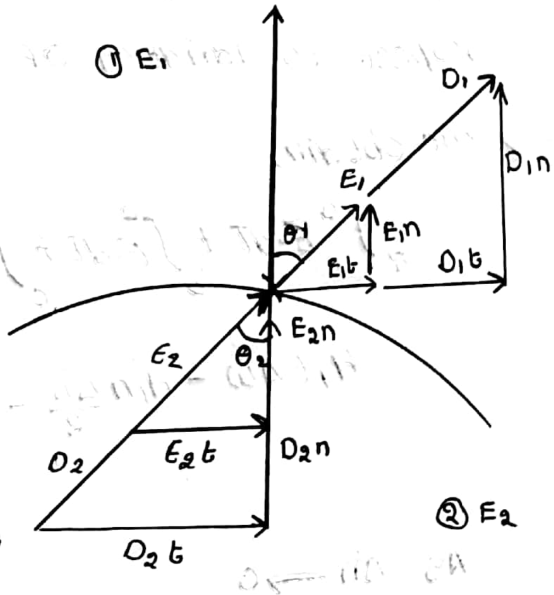
$$\boxed{D_{1n} = D_{2n}}$$

→ Therefore the normal component of D is continous across boundary,

$$\boxed{\epsilon_{r_1} E_{1n} = \epsilon_{r_2} E_{2n}}$$

→ Hence, the normal component of E is discontinous at the boundary.

The law of Reflection :-



→ If the feilds make an angle θ with the normal to be interface then

$$E_1 t = E_2 t$$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2 \quad \text{—①}$$

$$D_1 n = D_2 n$$

$$\epsilon_1 E_1 \cos\theta_1 = \epsilon_2 E_2 \cos\theta_2 \quad \text{—②}$$

From above equations

$$\frac{\cancel{E_1} \sin\theta_1}{\epsilon_1 \cancel{E_1} \cos\theta_1} = \frac{\cancel{E_2} \sin\theta_2}{\epsilon_2 \cancel{E_2} \cos\theta_2} \Rightarrow \frac{\tan\theta_1}{\epsilon_1} = \frac{\tan\theta_2}{\epsilon_2}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r_1}}{\epsilon_{r_2}}}$$

→ This is the law of refraction of the electric feild at a boundary free of charge.

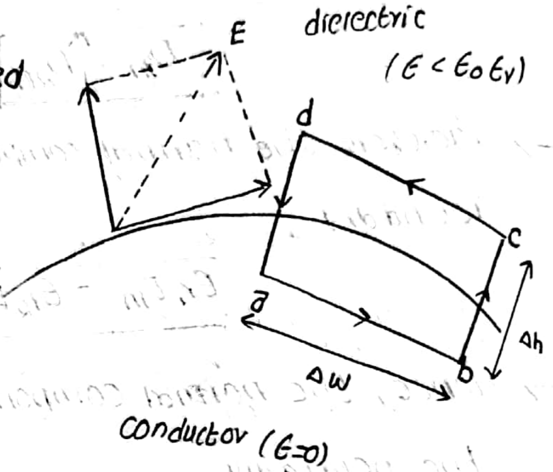
2. Conductor and dielectric boundary conditions :-

→ This case is shown in the below figure.

→ The conductor is assumed to be perfect.

(i.e $\sigma \to \infty$ and $\rho_v \to 0$)

and $E = 0$ inside the perfect conductor.



→ Apply $\oint \bar{E} \cdot d\bar{l} = 0$ to the closed path abcda of the figure assuming that the path is very small with respect to variation of E.

→ we obtain,

$$\int_a^b \bar{E} \cdot d\bar{l} + \int_b^c \bar{E} \cdot d\bar{l} + \int_c^d \bar{E} \cdot d\bar{l} + \int_d^a \bar{E} \cdot d\bar{l} = 0$$

$$H_{1t} \Delta w - H_{1n} \frac{\Delta h}{2} - H_{2n} \frac{\Delta h}{2} - H_{2t} \Delta w + H_{2n} \frac{\Delta h}{2} + H_{1n} \frac{\Delta h}{2} = K \Delta w$$

As $\Delta h \to 0$

$$H_{1t} \Delta w - H_{2t} \Delta w = K \Delta w$$

$$H_{1t} - H_{2t} = K$$

$$(H_{1t} - H_{2t}) \times \bar{a}_{n12} = \bar{K}$$

3. Conductor - free space boundary conditions :-

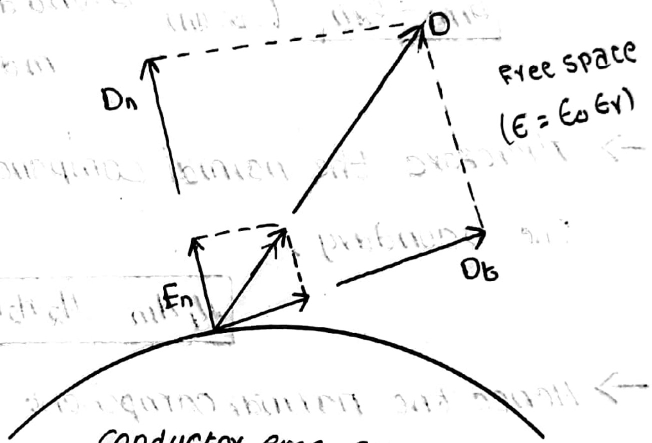
* Special case of conductor - dielectric boundary condition by replacing $\epsilon_r = 1 \Rightarrow \epsilon = \epsilon_0$.

→ This is a special case of the conductor - dielectric conditions and illustrated in figure.

→ The boundary conditions at the interface between a conductor and free space can be obtained by replacing by $\epsilon_r = 1$ in the conductor - dielectric conditions.

→ $D_t = \epsilon_0 E_t = 0$

$D_n = \epsilon_0 E_n = \rho_s$



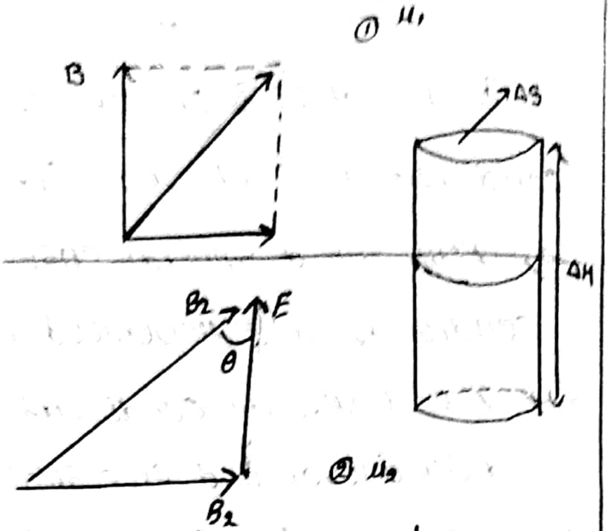
Conductor free space boundary conditions

Magnetic feild Boundary conditions :-

→ The magnetic feild boundary conditions are defined as the conditions that magnetic feild must satisfy at the boundary between two different media.

→ consider the boundary conditions between two magnetic media 1 and media 2 characterized by μ_1 and μ_2 as shown in the figure.

→ Apply $\oint \bar{B} \cdot d\bar{S} = 0$ to the Gaussian surface as shown in figure and allowing $\Delta h \to 0$ gives



$$\int_{top} \bar{B} \cdot d\bar{S} + \int_{bottom} \bar{B} \cdot d\bar{S} = 0$$

$$B_{1n} \Delta S - B_{2n} \Delta S = 0$$

$$\boxed{B_{1n} = B_{2n}} \quad (\because B = \mu H)$$

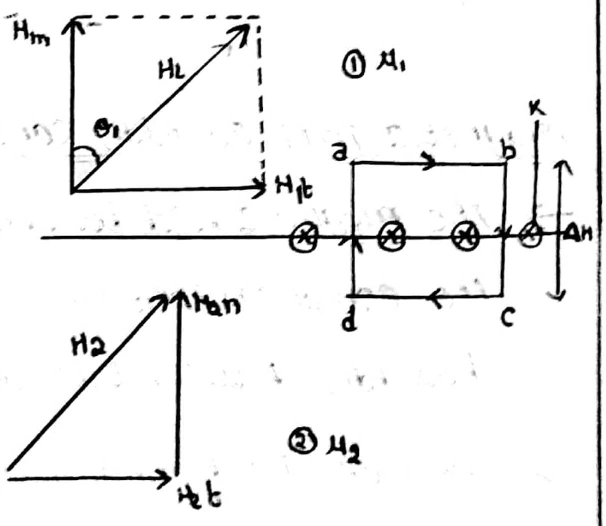
Boundary conditions between two magnetic media for B.

→ Therefore the normal component of B is continous at the boundary.

$$\boxed{\mu_1 H_{1n} = \mu_2 H_{2n}}$$

→ Hence the normal component of H is discontinous of the boundary.

→ similarly, apply $\oint \bar{H} \cdot d\bar{l} = I$ to the closed path abcda of the figure. where the surface current K on the boundary is assumed normal to path.



→ we obtain

$$\int_a^b \bar{H} \cdot d\bar{l} + \int_b^c \bar{H} \cdot d\bar{l} + \int_c^d \bar{H} \cdot d\bar{l} + \int_d^a \bar{H} \cdot d\bar{l} = K \Delta w$$

$$H_{1t} \Delta w - H_{1n} \frac{\Delta h}{2} - H_{2n} \frac{\Delta h}{2} - H_{2t} \Delta w + H_{2n} \frac{\Delta h}{2} + H_{1n} \frac{\Delta h}{2}$$

$= K \Delta w$

→ As $\Delta h \to 0$, above equation becomes

$$H_{1t} \Delta w - H_{2t} \Delta w = K \Delta w$$

$$H_{1t} - H_{2t} = K$$

$$(H_{1t} - H_{2t}) \times \bar{a}_{n12} = \bar{K}$$

* where a_{n12} is a unit vector normal to the interface and is decreted from medium 1 to medium 2

→ If the boundary is free ~~the~~ of current ($K \to 0$) then the above equation becomes

$$\boxed{H_{1t} = H_{2t}}$$

→ Therefore the tangential component of H is continous across the boundary.

$$\boxed{\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}}$$

→ Hence the tangential components of B is discontinous across the boundary.

* If the feilds make an angle θ with the normal to interface then $B_{1n} = B_{2n}$

$$B_1 \cos\theta_1 = B_2 \cos\theta_2 \quad \text{——— ① (From fig)}$$

$$\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}$$

$$\frac{B_1 \sin\theta_1}{\mu_1} = \frac{B_2 \sin\theta_2}{\mu_2} \quad \text{——— ②}$$

Divide equ ② wrt equ ①, then

$$\frac{\frac{\cancel{B_1}\sin\theta_1}{\mu_1}}{\cancel{B_1}\cos\theta_1} = \frac{\frac{\cancel{B_2}\sin\theta_2}{\mu_2}}{\cancel{B_2}\cos\theta_2}$$

$$\Rightarrow \frac{\tan\theta_1}{\mu_1} = \frac{\tan\theta_2}{\mu_2}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}}$$

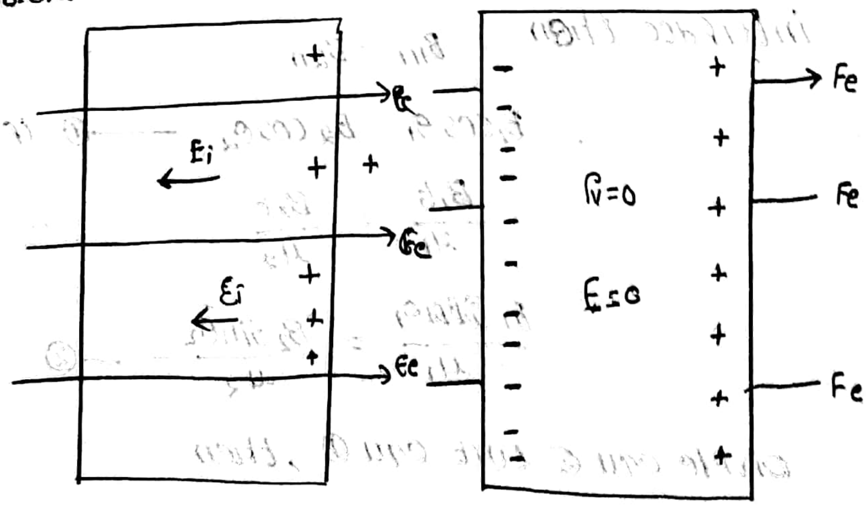
→ which is the law of ~~reflection~~ refraction for magnetic flux lines at a boundary with no surface current.

**** imp Topic :-**

→ In a conducting medium do the static, electric and magnetic feilds both exists ? Explain .

A) Consider an isolated conductor as shown in figure (a).

→ when an external electric feild E_e is applied, all the positive ~~force~~ free charges are moved in applied feild, while all the negative free charges move in opposite direction.



→ This leads to an important property of a conductor.

→ A perfect conductor cannot contain an electrostatic feild within it, under static conditions.

→ That means $E=0$, $\rho_v=0$, $V_{ab}=0$ inside a conductor.

→ A conductor is called an equipotential body, implying that potential is same.

→ This is based on the fact that

$$J=\sigma E \text{ as } \sigma\rightarrow\infty \text{ then } E\rightarrow 0.$$

A parallel-plate capacitor with plate area of 5 cm^2 and plate separation of 3 mm has a voltage $50 \sin 10^3 t$ V applied to its plates. Calculate the displacement current assuming $\varepsilon = 2\varepsilon_o$.

Solution:

$$D = \varepsilon E = \varepsilon \frac{V}{d}$$

$$J_d = \frac{\partial D}{\partial t} = \frac{\varepsilon}{d}\frac{dV}{dt}$$

Hence,

$$I_d = J_d \cdot S = \frac{\varepsilon S}{d}\frac{dV}{dt} = C\frac{dV}{dt}$$

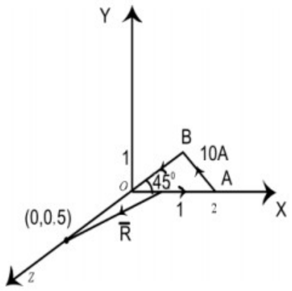
which is the same as the conduction current, given by

$$I_c = \frac{dQ}{dt} = S\frac{d\rho_s}{dt} = S\frac{dD}{dt} = \varepsilon S\frac{dE}{dt} = \frac{\varepsilon S}{d}\frac{dV}{dt} = C\frac{dV}{dt}$$

$$I_d = 2 \cdot \frac{10^{-9}}{36\pi} \cdot \frac{5 \times 10^{-4}}{3 \times 10^{-3}} \cdot 10^3 \times 50 \cos 10^3 t$$
$$= 147.4 \cos 10^3 t \text{ nA}$$

Problem:2.1

Find $\bar{H}$ at (0, 0, 5) due to side OA and side OB of the triangular current loop shown in Fig:2.3.



Solution:

Magnetic field intensity due to OA

$$d\bar{H}_{OA} = \frac{Id\bar{l} \times \bar{R}}{4\pi R^3} \quad \text{where} \quad \bar{R} = (0,\ 0,\ 5) - (x,\ 0,\ 0)$$

$$= -x\bar{a}_x + 5\bar{a}_z$$

$$d\bar{l} = dx\,\bar{a}_x$$

$$d\bar{H}_{OA} = \frac{10dx\,\bar{a}_x \times \left(-x\bar{a}_x + 5\bar{a}_z\right)}{4\pi\left(x^2+5^2\right)^{3/2}}$$

$$d\bar{H}_{OA} = \frac{10\left(-5dx\bar{a}_y\right)}{4\pi\left(x^2+5^2\right)^{3/2}}$$

$$\bar{H}_{OA} = -\int_0^2 \frac{50\bar{a}_y}{4\pi\left(x^2+25\right)^{3/2}}dx$$

$$= \frac{-50}{4\pi}\bar{a}_y\left[\frac{x/25}{\left(x^2+25\right)^{1/2}}\right]_0^2$$

$$= \frac{-50}{4\pi}\frac{1}{25}\bar{a}_y\frac{2}{(29)^{1/2}}$$

$$= -59.1\bar{a}_y \text{ mA/m}$$

Magnetic field intensity due to OB

$$\bar{R} = (0,\ 0,\ 5) - (1,\ 1,\ z) = -\bar{a}_x - \bar{a}_y + (5-z)\bar{a}_z$$

$$d\bar{l} = dz\,\bar{a}_z$$

$$d\bar{H}_{OB} = \frac{(10)\,dz\,\bar{a}_z \times \left(-\bar{a}_x - \bar{a}_y + (5-z)\bar{a}_z\right)}{4\pi\left[2+(5-z)^2\right]^{3/2}}$$

$$= \frac{(10)\left(-\bar{a}_y dz + \bar{a}_x dz\right)}{4\pi\left[2+(5-z)^2\right]^{3/2}}$$

$$\bar{H}_{OB} = \frac{10}{4\pi}\int_{\sqrt{2}}^{0}\frac{\left(-\bar{a}_y dz + \bar{a}_x dz\right)}{\left[2+(5-z)^2\right]^{3/2}}$$

$$= \frac{10}{4\pi}\left(-\bar{a}_y + \bar{a}_x\right)\int_{\sqrt{2}}^{0}\frac{dz}{\left(2+(5-z)^2\right)^{3/2}}$$

$$= \frac{10}{4\pi}\left(\bar{a}_y - \bar{a}_x\right)\left[\frac{(5-z)/2}{\left(2+(5-z)^2\right)^{1/2}}\right]_{\sqrt{2}}^{0}$$

$$= \frac{5}{4\pi}\left(\bar{a}_y - \bar{a}_x\right)\left[\frac{5}{\sqrt{27}} - \frac{\left(5-\sqrt{2}\right)}{\left(2+\left(5-\sqrt{2}\right)^2\right)^{1/2}}\right]$$

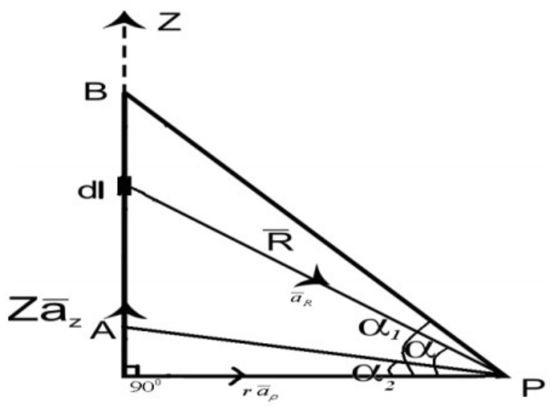
$$= \frac{5}{4\pi}\left(\bar{a}_y - \bar{a}_x\right)(0.9623 - 0.9303) = -12.73\bar{a}_x + 12.73\bar{a}_y \ mA/m$$

***Problem:2.2**

Show that the magnetic field due to a finite current element along Z-axis at point 'P', 'r' distance away along Y-axis is given by $\bar{H} = (I/4\pi r)(\sin\alpha_1 - \sin\alpha_2)\bar{a}_\phi$ where I is the current through the conductor, α_1 and α_2 are the angles made by the tips of the conductor at 'P'.

Solution:

Consider Fig:,



From Fig: , $\bar{R} = r\bar{a}_\rho - z\bar{a}_z$

and $d\bar{l} = dz\,\bar{a}_z$

$$d\bar{H} = \frac{I\,dz\,\bar{a}_z \times \left(r\,\bar{a}_\rho - z\,\bar{a}_z\right)}{4\pi\left(r^2+z^2\right)^{3/2}}$$

$$d\bar{H} = \frac{I\,dz\,r\,\bar{a}_\phi - 0}{4\pi\left(r^2+z^2\right)^{3/2}}$$

From Fig:2.4, $Tan\alpha = \frac{z}{r}$

$\therefore z = rTan\alpha$

$dz = rsec^2\alpha \quad d\alpha$

$$d\bar{H} = \frac{I\,r\,\bar{a}_\phi\left(rsec^2\alpha\ d\alpha\right)}{4\pi\left(r^2 + r^2Tan^2\alpha\right)^{3/2}}$$

$$= \frac{I\,r^2\,sec^2\alpha\ \bar{a}_\phi\,d\alpha}{4\pi\,r^3 sec^3\alpha}$$

$$= \frac{I\ \bar{a}_\phi\,d\alpha}{4\pi\,rsec\alpha}$$

$$\bar{H} = \frac{I\,\bar{a}_\phi}{4\pi\,r}\int_{\alpha_2}^{\alpha_1} \cos\alpha\,d\alpha$$

$$= \frac{I\,\bar{a}_\phi}{4\pi\,r}\left[\sin\alpha\right]_{\alpha_2}^{\alpha_1}$$

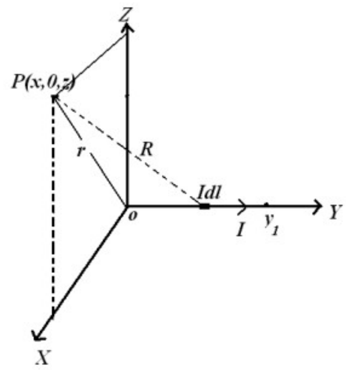
$$= \frac{I\,\bar{a}_\phi}{4\pi\,r}\left(\sin\alpha_1 - \sin\alpha_2\right)$$

$$\bar{H} = \frac{I\,\bar{a}_\phi}{4\pi\, r}\left(\sin\alpha_1 - \sin\alpha_2\right)$$

***Problem:2.3**

Derive an expression for magnetic field strength H, due to a current carrying conductor of finite length placed along the Y-axis, at a point P in X-Z plane and 'r' distance from the origin. Hence deduce expressions for H due to semi-infinite length of the conductor.

Solution:



The geometry of the given problem is shown in Fig: with the finite length(y_1 m)current carrying conductor lying along Y-axis.

Since the point P lies in the XZ plane, for all values of X and Z the line (OP=r) makes 90^0 with Y-axis

Where $\bar{r} = x\bar{a}_x + z\bar{a}_z$

The above figure can be modified as shown in Fig:2.6

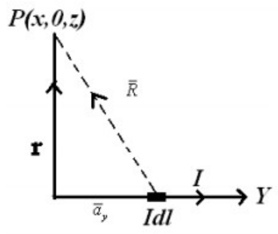


Fig: 2.6

From Fig:2.5, $\bar{R} = (x,0,z) - (0,y,0)$

$$\bar{R} = x\bar{a}_x - y\bar{a}_y + z\bar{a}_z$$

$$r^2 = x^2 + z^2$$

Consider a small differential current element Idl along Y-axis

According to Biot-Savart's law $d\bar{H} = \dfrac{Id\bar{l} \times \bar{R}}{4\pi R^3}$

Here $d\bar{l} = dy\bar{a}_y$

$$d\bar{H} = \frac{Idy\bar{a}_y \times (x\bar{a}_x + z\bar{a}_z - y\bar{a}_y)}{4\pi\left(x^2 + z^2 + y^2\right)^{3/2}}$$

$$d\bar{H} = \frac{Idy}{4\pi\left(r^2 + y^2\right)^{3/2}}\left(z\bar{a}_x - x\bar{a}_z\right)$$

Integrating w.r.t. y from y=0 to y_1, we get total magnetic field strength

$$\therefore \bar{H} = \int_{y=0}^{y_1} \frac{Idy}{4\pi\left(r^2 + y^2\right)^{3/2}}\left(z\bar{a}_x - x\bar{a}_z\right)$$

$$= \frac{I\left(z\bar{a}_x - x\bar{a}_z\right)}{4\pi}\int_{y=0}^{y_1} \frac{dy}{\left(r^2 + y^2\right)^{3/2}}$$

$$= \frac{I\left(z\bar{a}_x - x\bar{a}_z\right)}{4\pi}\left[\frac{y/r^2}{\sqrt{r^2+y^2}}\right]_0^{y_1} \quad \because \int \frac{dx}{\left(a^2+x^2\right)^{3/2}} = \frac{x/a^2}{\sqrt{a^2+x^2}}$$

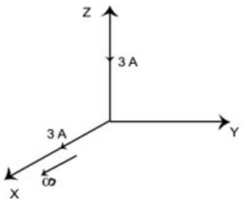
$$\bar{H} = \frac{I}{4\pi r^2\sqrt{\frac{r^2}{y_1^2}+1}}\left(z\bar{a}_x - x\bar{a}_z\right)$$

For a semi-infinite length conductor, $y_1=\infty$

$$\therefore \bar{H} = \frac{I}{4\pi r^2\sqrt{0+1}}\left(z\bar{a}_x - x\bar{a}_z\right) = \frac{I}{4\pi r^2}\left(z\bar{a}_x - x\bar{a}_z\right)$$

Problem:2.4

Find $\bar{H}$ at (-3, 4, 0) due to the current filament shown in Fig:



Solution:

For the element along X-axis is

$$\bar{R} = (-3, 4, 0) - (x, 0, 0)$$

$$d\bar{l} = dx\,\bar{a}_x$$

$$\bar{H}_x = \int_0^{\infty} \frac{3\,dx\,\bar{a}_x \times \left[(-3-x)\bar{a}_x + 4\bar{a}_y\right]}{4\pi\left[(-3-x)^2+16\right]^{3/2}}$$

$$= \frac{3}{4\pi}4\bar{a}_z\int_0^{\infty}\frac{dx}{\left[16+(-3-x)^2\right]^{3/2}}$$

$$= \frac{3}{\pi}\bar{a}_z\int_0^{\infty}\frac{dx}{\left[16+(3+x)^2\right]^{3/2}}$$

$$= \frac{3\bar{a}_z}{\pi}\left[\frac{(3+x)/16}{\left[16+(3+x)^2\right]^{1/2}}\right]_0^{\infty}$$

$$= \frac{3\bar{a}_z}{16\pi}\left[1-\left(\frac{3}{5}\right)\right] = 23.88\bar{a}_z \quad mA/m$$

For the element along Z-axis is

$$\bar{R} = (-3, 4, 0) - (0, 0, z)$$

$$d\bar{l} = dz\,\bar{a}_z$$

$$\bar{H}_z = \int_{\infty}^{0} \frac{3\,dz\,\bar{a}_z \times \left[\left(-3\bar{a}_x + 4\bar{a}_y - Z\bar{a}_z\right)\right]}{4\pi\left[9+16+z^2\right]^{3/2}}$$

$$= \frac{3}{4\pi}\int_0^{\infty}\frac{\left(3\bar{a}_y + 4\bar{a}_x\right)dz}{\left(25+z^2\right)^{3/2}}$$

$$= \frac{3\left(3\bar{a}_y + 4\bar{a}_x\right)}{4\pi}\int_0^{\infty}\frac{dz}{\left(25+z^2\right)^{3/2}}$$

$$= \frac{3\left(3\bar{a}_y + 4\bar{a}_x\right)}{4\pi}\left[\frac{z/25}{\left(25+z^2\right)^{1/2}}\right]_0^{\infty}$$

$$= \frac{3\left(3\bar{a}_y + 4\bar{a}_x\right)}{100\pi}(1-0) = 38.2\bar{a}_x + 28.65\bar{a}_y \quad mA/m$$

$$\bar{H} = \bar{H}_x + \bar{H}_z = 38.2\bar{a}_x + 28.65\bar{a}_y + 23.88\bar{a}_z \quad mA/m$$

Problem:2.5

The +Ve Y-axis (semi infinite line w.r.t origin) carries a filamentary current of 2A in the $-\bar{a}_y$ direction. Find $\bar{H}$ at (a) A(2, 3, 0) (b) B(3, 12, -4).

Solution:

(a) $\bar{R}$ = (2, 3, 0) – (0, y, 0)

$d\bar{l} = dy\,\bar{a}_y$

Since 2A is along $-\bar{a}_y$, limits are ∞ to 0.

$$\bar{H}_A = \int_{\infty}^{0} \frac{2\,dy\,\bar{a}_y \times \left[\left(2\bar{a}_x + (3-y)\bar{a}_y\right)\right]}{4\pi\left[4+(3-y)^2\right]^{3/2}}$$

$$\bar{H}_A = \frac{\bar{a}_z}{\pi}\int_0^{\infty}\frac{dy}{\left[4+(3-y)^2\right]^{3/2}}$$

$$= -\frac{\bar{a}_z}{\pi}\left[\frac{(3-y)/4}{\left[4+(3-y)^2\right]^{1/2}}\right]_0^{\infty}$$

$$= -\frac{\bar{a}_z}{4\pi}\left(-1-\frac{3}{\sqrt{13}}\right) = 145.8\bar{a}_z mA/m$$

(b) $\bar{R}$ = (3, 12, -4) – (0, y, 0)

$$\therefore\ \bar{H}_B = \int_{\infty}^{0}\frac{2dy\,\bar{a}_y \times \left[3\bar{a}_x + (12-y)\bar{a}_y - 4\bar{a}_z\right]}{4\pi\left[25+(12-y)^2\right]^{3/2}}$$

$$\bar{H}_B = \frac{6\bar{a}_z + 8\bar{a}_x}{4\pi}\int_0^{\infty}\frac{dy}{\left[25+(12-y)^2\right]^{3/2}}$$

$$= -\frac{6\bar{a}_z + 8\bar{a}_x}{4\pi}\left[\frac{(12-y)/25}{\left[25+(12-y)^2\right]^{1/2}}\right]_0^{\infty}$$

$$= -\frac{6\bar{a}_z + 8\bar{a}_x}{100\pi}\left(-1-\frac{12}{13}\right) = 48.97\bar{a}_x + 36.73\bar{a}_z \quad mA/m$$

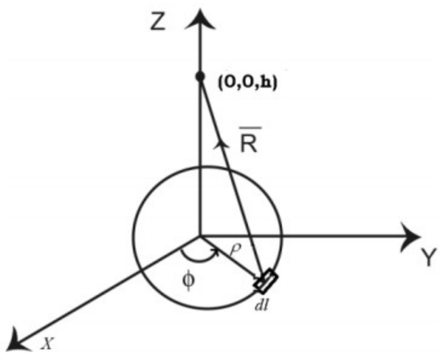
Problem:2.6

Show that the magnetic field intensity $\bar{H}$ at (0, 0, h) due to a circle which lies on XY plane with radius 'ρ' carries a current I as

$$\bar{H} = \frac{I\rho^2 \bar{a}_z}{2\left[\rho^2 + h^2\right]^{3/2}}$$

Solution:

Consider the circular loop shown in Fig:



$P(r, \theta, \phi)$

From Fig: $\bar{R} = -\rho\bar{a}_\rho + h\bar{a}_z$

and $d\bar{l} = \rho d\phi \bar{a}_\phi$

We have $d\bar{H} = \dfrac{Id\bar{l} \times \bar{R}}{4\pi R^3}$

$$d\bar{H} = \frac{I\rho d\phi \bar{a}_\phi \times (-\rho\bar{a}_\rho + h\bar{a}_z)}{4\pi(\rho^2 + h^2)^{3/2}}$$

$$d\bar{H} = \frac{I\rho d\phi(\rho\bar{a}_z + h\bar{a}_\rho)}{4\pi(\rho^2 + h^2)^{3/2}}$$

due to symmetry of circle the components in ρ direction will get cancelled.

$$d\bar{H} = \frac{I\rho^2 d\phi \bar{a}_z}{4\pi(\rho^2 + h^2)^{3/2}}$$

Integrating

$$\bar{H} = \frac{I}{4\pi}\int_0^{2\pi}\left[\frac{\rho^2 \bar{a}_z d\phi}{\left(\rho^2 + h^2\right)^{3/2}}\right]$$

$$\bar{H} = \frac{I\rho^2 \bar{a}_z}{2\left[\rho^2 + h^2\right]^{3/2}}$$

Problem:2.7

A circular loop located on $x^2+y^2 = 9$, z=0 carries a direct current of 10A along $\bar{a}_\phi$ direction. Determine $\bar{H}$ at (0, 0, 4) and (0, 0, -4).

Solution:

Here $\rho = 3$, h=4 and I=10A

$$\therefore \bar{H} = \frac{10}{4\pi}\frac{3^2}{\left(3^2 + 4^2\right)^{3/2}}\bar{a}_z \int_0^{2\pi} d\phi$$

$$= 0.36\bar{a}_z \text{ A/m}$$

Similarly $\bar{H}_{(0,0,-4)} = \bar{H}_{(0,0,4)} = 0.36\bar{a}_z A/m$

***Problem:2.8**

Find the field at the centre of a circular loop of radius 'a', carrying current I along ϕ direction in Z=0 plane.

Solution:

We have $\bar{H} = \dfrac{I\rho^2 \bar{a}_z}{2\left[\rho^2 + h^2\right]^{3/2}}$

Here ρ=a, and h=0

$$\bar{H} = \int_{\phi=0}^{2\pi} \frac{I a^2 d\phi \bar{a}_z}{4\pi\left[a^2 + 0\right]^{3/2}} = \frac{I\,\bar{a}_z}{2a} A/m$$

Problem:2.9

A thin ring of radius 5cm is placed on plane z=1cm so that its center is at (0, 0, 1 cm). If the ring carries 50mA along $\bar{a}_\phi$. Find $\bar{H}$ at

(a) (0, 0, -1cm) (b) (0, 0, 10cm)

Solution:

(a) Here ρ=5cm, h=2cm and I=50mA

We have $\bar{H} = \dfrac{I\rho^2 \bar{a}_z}{2\left[\rho^2 + h^2\right]^{3/2}}$

$$\bar{H} = \frac{50\times10^{-3}(5\times10^{-2})^2 \bar{a}_z}{2\left[(5\times10^{-2})^2 + (2\times10^{-2})^2\right]^{3/2}}$$

$$\bar{H} = \frac{125\,\bar{a}_z}{2[29]^{3/2}}$$

$$\overline{H} = 400\bar{a}_z mA/m$$

(b) Here ρ=5cm, h=9cm and I=50mA

We have $\bar{H} = \dfrac{I\rho^2 \bar{a}_z}{2\left[\rho^2 + h^2\right]^{3/2}}$

$$\bar{H} = \frac{50\times10^{-3}(5\times10^{-2})^2 \bar{a}_z}{2\left[(5\times10^{-2})^2 + (9\times10^{-2})^2\right]^{3/2}}$$

$$\bar{H} = \frac{125\,\bar{a}_z}{2[106]^{3/2}}$$

$$\overline{H} = 57.3\bar{a}_z mA/m$$

Problem:2.10

A square conducting loop 2a m on each side carries a current of I amp. Calculate the magnetic field intensity at the center of the loop.

Solution:

Consider a square loop with each side 2a m as shown in the Fig:.

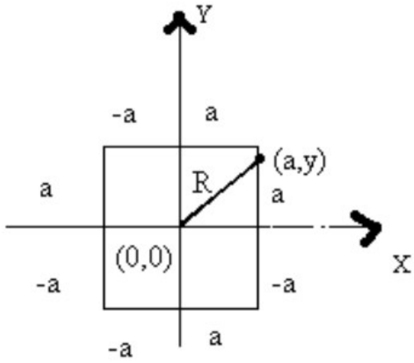


Fig: A square loop

We need to find magnetic field intensity at (0,0) due to elemental current flowing in one side of square loop at (a,y)

According to Biot Savart's law, we have

$$d\bar{H} = \frac{Id\bar{l} \times \bar{R}}{4\pi R^3}$$

From Fig.2.9, $d\bar{l} = dy\bar{a}_y$

$$\bar{R} = (0,0) - (a, y) = -a\bar{a}_x - y\bar{a}_y$$

$$\therefore d\bar{H}_{oneside} = \frac{Idy\bar{a}_y \times (-a\bar{a}_x - y\bar{a}_y)}{4\pi(a^2 + y^2)^{3/2}}$$

$$\therefore d\bar{H}_{oneside} = \frac{Iady\bar{a}_z}{4\pi(a^2 + y^2)^{3/2}}$$

$$\bar{H}_{oneside} = \int_{-a}^{a} \frac{Iady\bar{a}_z}{4\pi(a^2 + y^2)^{3/2}}$$

$$\bar{H}_{oneside} = \frac{Ia}{4\pi}\bar{a}_z \int_{-a}^{a} \frac{dy}{(a^2 + y^2)^{3/2}}$$

$$\bar{H}_{oneside} = \frac{Ia}{4\pi}\bar{a}_z \left[\frac{y / a^2}{(a^2 + y^2)^{1/2}}\right]_{-a}^{a}$$

$$\bar{H}_{oneside} = \frac{Ia}{4\pi a^2}\bar{a}_z \left[\frac{a}{(a^2 + a^2)^{1/2}} + \frac{a}{(a^2 + a^2)^{1/2}}\right]$$

$$\bar{H}_{oneside} = \frac{I}{4\pi a}\bar{a}_z \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$$

$$\bar{H}_{oneside} = \frac{I\sqrt{2}}{4\pi a}\bar{a}_z$$

Magnetic field intensity at (0,0) due to four sides is

$$\bar{H} = 4\bar{H}_{oneside} = \frac{4I\sqrt{2}}{4\pi a}\bar{a}_z$$

$$\bar{H} = \frac{I\sqrt{2}}{\pi a}\bar{a}_z$$

Problem:2.11

A square conducting loop 3cm on each side carries a current of 10A. Calculate the magnetic field intensity at the center of the loop.

Solution:

We have $\bar{H} = \frac{I\sqrt{2}}{\pi a}\bar{a}_z$

Here a=1.5x10^{-2}m and I=10A

$$\therefore \bar{H} = \frac{10\sqrt{2}}{\pi \times 1.5 \times 10^{-2}}\bar{a}_z = 300.105\bar{a}_z A/m$$

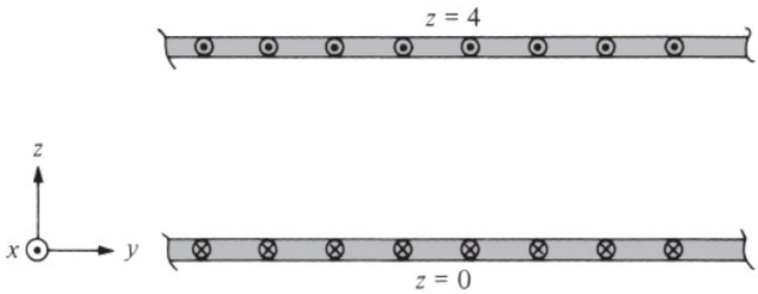
$H_\phi = 0$

Problem:2.12

Planes z=0 and z=4 carry current $\bar{K} = -10\bar{a}_x$ A/m and $\bar{K} = 10\bar{a}_x$ A/m respectively. Determine $\bar{H}$ at (a) (1, 1, 1) (b) (0, -3, 10)

Solution:

(a) We have $\bar{H} = \frac{1}{2}\bar{K} \times \bar{a}_n$



here $\bar{K} = -10\bar{a}_x A/m$ on Z=0 plane and Since point (1,1,1) is lying above the plane Z=0, $\bar{a}_n = \bar{a}_z$

$$\therefore \bar{H}_0 = \frac{1}{2}(-10)\bar{a}_x \times \bar{a}_z$$

$$= -5(-\bar{a}_y)$$

$$= 5\bar{a}_y$$

here $\bar{K} = 10\bar{a}_x A/m$ on Z=4 plane and Since point (1,1,1) is lying below the plane Z=4, $\bar{a}_n = -\bar{a}_z$

$$\therefore \bar{H}_4 = \frac{1}{2}10\bar{a}_x \times \left(-\bar{a}_z\right)$$

$$= -5(-\bar{a}_y)$$

$$= 5\bar{a}_y$$

(b) We have $\bar{H} = \frac{1}{2}\bar{K} \times \bar{a}_n$

here $\bar{K} = -10\bar{a}_x A/m$ on Z=0 plane and Since point (0,-3,10) is lying above the plane Z=0, $\bar{a}_n = \bar{a}_z$

$$\therefore \bar{H}_0 = \frac{1}{2}(-10\bar{a}_x) \times \bar{a}_z$$

$$= -5(-\bar{a}_y)$$

$$= 5\bar{a}_y$$

here $\bar{K} = 10\bar{a}_x A/m$ on Z=4 plane and Since point (0,-3,10) is lying above the plane Z=4, $\bar{a}_n = \bar{a}_z$

$$\therefore \bar{H}_4 = \frac{1}{2}10\bar{a}_x \times \bar{a}_z$$

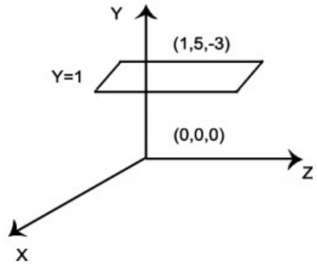
$$= 5(-\bar{a}_y)$$

$$= -5\bar{a}_y$$

$$\bar{H} = \bar{H}_0 + \bar{H}_4 = 5\bar{a}_y - 5\bar{a}_y = 0$$

Problem:2.13

Plane Y = 1 carries current $\bar{K} = 50\bar{a}_z$ mA/m. Find $\bar{H}$ at (a) (0, 0, 0) (b) (1, 5, -3)



Solution:

(a) here $\bar{K} = 50\bar{a}_z mA/m$ on Y=1 plane and Since point (0,0,0) is lying below the plane Y=1, $\bar{a}_n = -\bar{a}_y$

$$\therefore \bar{H} = \frac{1}{2}50\bar{a}_z \times (-\bar{a}_y)$$

$$= -25\left(-\bar{a}_x\right)$$

$$= 25\bar{a}_x \text{ mA/m}$$

(b) here $\bar{K} = 50\bar{a}_z mA/m$ on Y=1 plane and Since point (1,5,-3) is lying above the plane Y=1, $\bar{a}_n = \bar{a}_y$

$$\bar{H} = \frac{1}{2}50\bar{a}_z \times \bar{a}_y$$

$$= 25\left(-\bar{a}_x\right)$$

$$= -25\bar{a}_x \text{ mA/m}$$

***Problem:2.14**

A long coaxial cable has an inner conductor carrying a current of 1mA along +ve Z direction, its axis coinciding with Z-axis. Its inner conductor diameter is 6mm. If its outer conductor has an inside diameter of 12mm and thickness of 2mm, determine $\bar{H}$ at (0,0,0), (0,4.5mm,0) and (0,1cm,0). (No derivations).

Solution:

As per the derivations in the section "Infinitely long co-axial transmission line"
Given I=1mA, a=3mm,b=6mm and t=2mm.

Let the given points be P_1(0,0,0), P_2(0,4.5mm,0) and P_3(0,1cm,0).

Given points are in rectangular coordinate system $\therefore \rho = \sqrt{x^2 + y^2}$

For P_1, ρ=0, i.e.ρ<a, Hence case(i) formula from the section "**Infinitely long co-axial transmission line**"

$$\bar{H} = \frac{I\rho}{2\pi a^2}\bar{a}_\phi = 0A/m$$

For P_2, ρ=4.5mm, i.e a<ρ<b, Hence case(ii) formula from the section **"Infinitely long co-axial transmission line"**

$$\bar{H} = \frac{I}{2\pi\rho}\bar{a}_\phi = \frac{1\times10^{-3}}{2\pi\times4.5\times10^{-3}}\bar{a}_\phi = 0.03537\bar{a}_\phi A/m$$

For P_3, ρ=1cm=10mm, i.e. ρ>b+t, Hence case(iv) formula from the section **"Infinitely long co-axial transmission line"**

$$\bar{H} = 0A/m$$

Problem:2.15

A solenoid of length '*l*' and radius 'a' consists of 'N' turns of wire carrying current 'I'. Show that at point p along it's axis $\bar{H} = \frac{nI}{2}(\cos\theta_2 - \cos\theta_1)\bar{a}_z$ where $n = \frac{N}{l}$, θ_1, θ_2 are the angles subtended at 'p' by the end turns as illustrated in Fig:2.15. Also Show that if l >>a at the center of the solenoid $\bar{H} = nI\,\bar{a}_z$

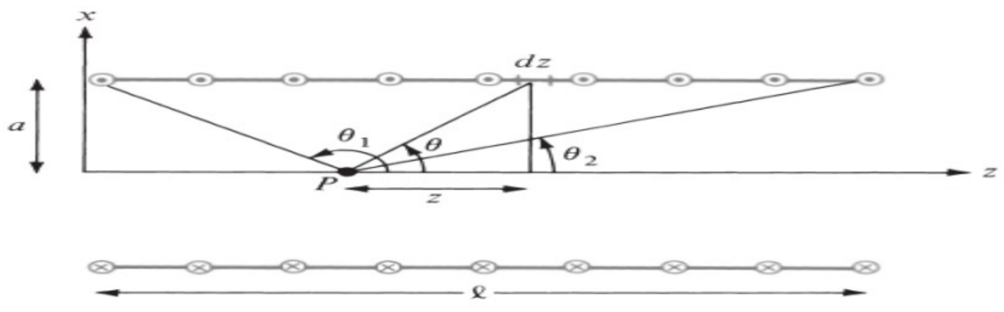


Fig: 2.15 cross section of a solenoid

Solution:

We know the elemental magnetic field intensity dH_z due to one turn (circle) at point p is

$$dH_z = \frac{I\,a^2 d\phi}{4\pi\left(a^2+z^2\right)^{3/2}}\bar{a}_z$$

$\therefore H_z = \frac{I\,a^2}{2\left(a^2+z^2\right)^{3/2}}\bar{a}_z$ which is the magnetic field intensity at point 'p' due to one turn. As the solenoid contains 'N' number of turns by considering the elemental length dl, the elemental magnetic field intensity due to a solenoid of length 'L' and having 'N' number of turns at point 'P' is

$$dH_z = \frac{I\,a^2 dl\,\bar{a}_z}{2\left(a^2+z^2\right)^{3/2}}$$

where dl = ndz = $\frac{N}{l}dz$

from Fig:2.15 $Tan\theta = \frac{a}{z}$

z = a cot θ

dz = -a cosec²θ dθ

$$dH_z = \frac{I\,a^2 n\left(-a\cos ec^2\theta\,d\theta\right)}{2\left(a^2+a^2\cot^2\theta\right)^{3/2}}\bar{a}_z$$

$$= \frac{I a^2 n\left(-a \operatorname{cosec}^2\theta \, d\theta\right)}{2 a^3 \operatorname{cosec}^3\theta} \bar{a}_z$$

$$= -\frac{nI}{2} \sin\theta \, d\theta \, \bar{a}_z$$

$$H_z = -\frac{nI}{2} \int_{\theta_2}^{\theta_1} \sin\theta \, d\theta \, \bar{a}_z$$

$$= -\frac{nI}{2} (\cos\theta_1 - \cos\theta_2) \bar{a}_z$$

$$= \frac{nI}{2} (\cos\theta_2 - \cos\theta_1) \bar{a}_z$$

At the center of the solenoid we can write

$$\cos\theta_2 = \frac{l/2}{\sqrt{a^2 + (l/2)^2}} = -\cos\theta_1$$

as l >>a

$$H_z = \frac{nI}{2} 2\cos\theta_2 \, \bar{a}_z$$

$$= nI \frac{l/2}{\sqrt{a^2 + (l/2)^2}} \bar{a}_z$$

$$= nI \frac{l/2}{l/2} \bar{a}_z = nI \, \bar{a}_z$$

Problem:2.16

A Toroid whose dimensions are shown in Fig: has 'N' turns and carries current I. Determine $\bar{H}$ inside and outside the Toroid.

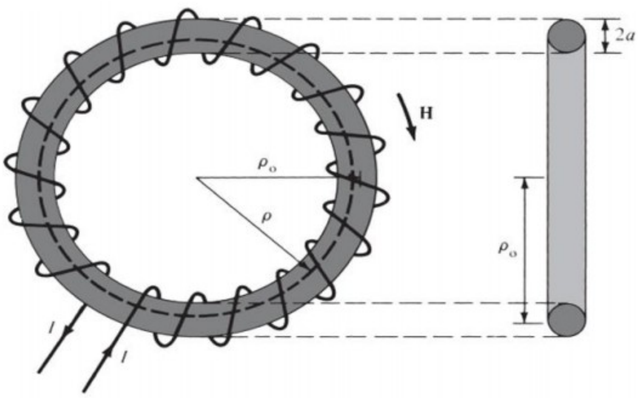


Fig: a toroid with a circular cross section

Solution:

Inside the toroid consider the closed path L_1, According to Ampere's circuit law

$$\oint_{L_1} \bar{H} \cdot d\bar{l} = I_{enc}$$

$$\Rightarrow H_\phi 2\pi \rho = nI$$

$$H_\phi = \frac{nI}{2\pi\rho}$$

$$\bar{H} = \frac{nI}{2\pi\rho} \bar{a}_\phi$$

outside the Toroid $\oint_L \bar{H} \cdot d\bar{l} = I_{enc}$

= nI - nI

$$\Rightarrow H_\phi = 0 \Rightarrow \bar{H} = 0$$

NOTE: By bending a solenoid in to a form of circle we get a toroid.

***Problem:2.17**

A long straight conductor with radius 'a' has a magnetic field strength $\bar{H} = \frac{Ir}{2\pi a^2}\bar{a}_\phi$ within the conductor (r<a) and $\bar{H} = \frac{I}{2\pi r}\bar{a}_\phi$ outside the conductor (r>a). Find the current density $\bar{J}$ in both the regions(r<a and r>a).

Solution:

we have $\bar{J} = \nabla \times \bar{H}$

Given $\bar{H} = \frac{Ir}{2\pi a^2}\bar{a}_\phi$ within the conductor (r<a)

Which has cylindrical symmetry, here ρ=r,

$$\bar{J} = \nabla \times \bar{H} = \frac{1}{r}\begin{vmatrix} \bar{a}_r & r\bar{a}_\phi & \bar{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ H_r & rH_\phi & H_z \end{vmatrix}$$

$$= \frac{1}{r}\begin{vmatrix} \bar{a}_r & r\bar{a}_\phi & \bar{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & r\frac{Ir}{2\pi a^2} & 0 \end{vmatrix} = \frac{I}{\pi a^2}\bar{a}_z A/m^2$$

And also given $\bar{H} = \frac{I}{2\pi r}\bar{a}_\phi$ outside the conductor (r>a)

$$\therefore \bar{J} = \frac{1}{r}\begin{vmatrix} \bar{a}_r & r\bar{a}_\phi & \bar{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & r\frac{I}{2\pi r} & 0 \end{vmatrix} = 0 A/m^2$$

***Problem:2.18**

A conducting plane at y=1 carries a surface current of $10\bar{a}_z$ mA/m. Find H and B at(a) (0,0,0) and (b) (2,2,2).

Solution:

(a)

We have $\bar{H} = \frac{1}{2}\bar{K} \times \bar{a}_n$

here $\bar{K} = 10\bar{a}_z mA/m$.

Since the point (0,0,0) is lying below the plane Y=1, $\bar{a}_n = -\bar{a}_y$

$$\therefore \bar{H} = \frac{1}{2}10\bar{a}_z \times (-\bar{a}_y)$$

$$= -5(-\bar{a}_x)$$

$= 5\bar{a}_x$ mA/m

$\bar{B} = \mu_0 \bar{H} = 4\pi \times 10^{-7} \times 5\bar{a}_x = 62.83 \times 10^{-10} \bar{a}_x T$

(b)

We have $\bar{H} = \frac{1}{2} \bar{K} \times \bar{a}_n$

here $\bar{K} = 10\bar{a}_z mA/m$.

Since the point (2,2,2) is lying above the plane Y=1, $\bar{a}_n = \bar{a}_y$

$$\therefore \bar{H} = \frac{1}{2} 10\bar{a}_z \times \bar{a}_y$$

$$= 5(-\bar{a}_x)$$

$$= -5\bar{a}_x \text{ mA/m}$$

$$\bar{B} = \mu_0 \bar{H} = 4\pi \times 10^{-7} \times -5\bar{a}_x = -62.83 \times 10^{-10} \bar{a}_x T$$

***Problem:2.19**

An infinitely long straight conducting rod of radius 'a' carries a current of I in +ve Z-direction. Using Ampere's Circuital Law, find $\bar{H}$ in all regions and sketch the variation of H as a function of radial distance. If I=3mA and a=2cm, find $\bar{H}$ and $\bar{B}$ at (0,1cm,0) and (0,4cm,0).

Solution:

Consider cylindrical co-ordinate system

Case(i): inside the conductor(ρ<a)

According to Ampere's circuit law

$$\oint_L \bar{H} \cdot d\bar{l} = I_{enc} = \int_s \bar{J} \cdot d\bar{s}$$

$$\oint_L \bar{H} \cdot d\bar{l} = \int_0^{2\pi} H_\phi \bar{a}_\phi \cdot \rho d\phi \, \bar{a}_\phi = H_\phi 2\pi\rho$$

$\bar{J}$ of the internal conductor is $\frac{I}{\pi a^2}\bar{a}_z$ and $d\bar{s} = \rho d\phi d\rho \bar{a}_z$

$$\therefore I_{enc} = \int_s \bar{J} \cdot d\bar{s}$$

$$= \iint \frac{I}{\pi a^2} \bar{a}_z . \rho \, d\phi \, d\rho \, \bar{a}_z$$

$$= \int_0^{2\pi} \int_0^{\rho} \frac{I}{\pi a^2} \rho \, d\phi \, d\rho$$

$$= \frac{I}{\pi a^2} \int_0^{\rho} \rho (2\pi) \, d\rho = \frac{2\pi I}{\pi a^2} \left(\frac{\rho^2}{2} \right) = \frac{I \rho^2}{a^2}$$

$$\therefore 2\pi H_\phi \rho = \frac{I\rho^2}{a^2}$$

$$H_\phi = \frac{I\rho}{a^2 2\pi} \Rightarrow \bar{H} = \frac{I\rho}{2\pi a^2}\bar{a}_\phi$$

Case(ii): outside the conductor($\rho>a$)

According to Ampere's circuit law

$$\oint_L \bar{H}\cdot d\bar{l} = I_{enc}$$

$$\Rightarrow \int_0^{2\pi} H_\phi \bar{a}_\phi . \rho\, d\phi\, \bar{a}_\phi = I_{enc}$$

$$\Rightarrow 2\pi H_\phi \rho = I$$

$$H_\phi = \frac{I}{2\pi\rho}$$

$$\bar{H} = \frac{I}{2\pi\rho}\bar{a}_\phi$$

Given points let be $P_1(0,1cm,0)$ and $P_2(0,4cm,0)$

For P_1 radial distance $\rho = \sqrt{0^2+1^2+0^2} = 1cm$

Also given a=2cm and I=3mA i.e. $\rho<a$ (inside the conductor)

$$\therefore \bar{H} = \frac{I\rho}{2\pi a^2}\bar{a}_\phi = \frac{3\times10^{-3}\times1\times10^{-2}}{2\pi(2\times10^{-2})^2}\bar{a}_\phi = 0.0119\bar{a}_\phi A/m \text{ and } \bar{B} = \mu_0\bar{H}$$

$$\therefore \bar{B} = 4\pi\times10^{-7}\times0.0119\bar{a}_\phi = 0.15\times10^{-7}\bar{a}_\phi T$$

For P_2 radial distance $\rho = \sqrt{0^2+4^2+0^2} = 4cm$

Here $\rho>a$ (outside the conductor)

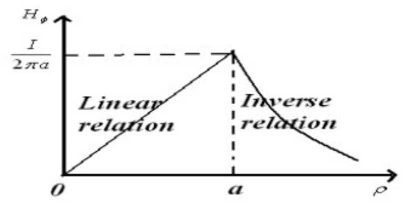
$$\therefore \bar{H} = \frac{I}{2\pi\rho}\bar{a}_\phi = \frac{3\times10^{-3}}{2\pi\times4\times10^{-2}}\bar{a}_\phi = 0.0119\bar{a}_\phi A/m \text{ and } \bar{B} = \mu_0\bar{H}$$

$$\therefore \bar{B} = 4\pi\times10^{-7}\times0.0119\bar{a}_\phi = 0.15\times10^{-7}\bar{a}_\phi T$$

Sketch of H_ϕ:

We have $H_\phi = \frac{I\rho}{2\pi a^2}\ for\ \rho<a$ i.e. $H_\phi \propto \rho$

And $H_\phi = \frac{I}{2\pi\rho}\ for\ \rho>a$ i.e. $H_\phi \propto 1/\rho$



***Problem:2.20**

Determine the magnetic flux, for the surface described by

(a) ρ=1m., $0 \le \phi \le \pi/2, 0 \le z \le 2m$

(b) a sphere of radius 2m.

If the magnetic field is of the form $\bar{H} = \frac{1}{\rho}\cos\phi \bar{a}_\rho A/m$

Solution:

We have magnetic flux $\psi = \int_s \bar{B}.\bar{ds} = \mu_0 \int_s \bar{H}.\bar{ds}$

(a) here $\bar{ds} = \rho d\phi dz \bar{a}_\rho$ (cylindrical symmetry)

$$\psi = \mu_0 \int_s \frac{1}{\rho}\cos\phi \bar{a}_\rho.\rho d\phi dz \bar{a}_\rho = \mu_0 \int_{z=0}^{2} dz \int_{\phi=0}^{\pi/2} \cos\phi d\phi = 2\mu_0 = 25.13\times 10^{-7} Wb$$

(c) here $\bar{ds} = r^2 \sin\theta d\theta d\phi \bar{a}_r$ (spherical symmetry)

$$\psi = \mu_0 \int_s \frac{1}{r}\cos\phi \bar{a}_r.r^2 \sin\theta d\theta d\phi \bar{a}_r = \mu_0 r \int_{\theta=0}^{\pi} \sin\theta d\theta \int_{\phi=0}^{2\pi} \cos\phi d\phi = 0Wb$$

***Problem:2.21**

In a conducting medium $\bar{H} = y^2 z \bar{a}_x + 2(x+1) yz \bar{a}_y - (x+1)z^2 \bar{a}_z A/m$. Find the current density at (1,0,-3) and calculate the current passing through Y=1 plane, $0 \le x \le 1, 0 \le z \le 1$.

Solution:

We have current density $\bar{J} = \nabla \times \bar{H}$

$$\bar{J} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 z & 2(x+1)yz & -(x+1)z^2 \end{vmatrix}$$

$$\bar{J} = -2(x+1)y\bar{a}_x + (z^2 + y^2)\bar{a}_y A/m^2$$

$$\bar{J}_{(1,0,-3)} = 9\bar{a}_y A/m^2$$

Current is $I = \int_s \bar{J}.\bar{ds}$, here $\bar{ds} = dxdz\bar{a}_y$

$$\therefore I = \int_s \left(-2(x+1)y\bar{a}_x + (z^2+y^2)\bar{a}_y\right).dxdz\bar{a}_y = \int_{x=0}^{1} dx \int_{z=0}^{1} (z^2+y^2)dz = 1.33A$$

Problem:2.22

Find the flux density at the center of a square loop of 10 turns carrying a current of 10A. The loop is in air and has a side of 2m.

Solution:

We have $\bar{H} = \frac{I\sqrt{2}}{\pi a}\bar{a}_z$

$\therefore$ magnetic flux density is $\bar{B} = \frac{\mu_0 I\sqrt{2}}{\pi a}\bar{a}_z$

Here no. of turns N=10

Total current I=N X current in each turn

I=10 X 10=100A

And a=1m

$$\therefore \overline{B} = \frac{\mu_0 100\sqrt{2}}{\pi \times 1}\overline{a}_z$$

$$\therefore \overline{B} = \frac{4\pi \times 10^{-7} \times 100\sqrt{2}}{\pi \times 1}\overline{a}_z = 56.569 \mu \overline{a}_z Tesla$$

Problem:2.22

Given the magnetic vector potential $\overline{A} = \frac{-\rho^2}{4}\overline{a}_z$ Wb/m. Calculate the total magnetic flux crossing the surface $\phi = \frac{\pi}{2}$, $1 \le \rho \le 2$m, $0 \le z \le 5$m.

Solution:

We have

$$\nabla \times \overline{A} = \frac{1}{\rho}\begin{vmatrix} \overline{a}_\rho & \rho\overline{a}_\phi & \overline{a}_Z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_Z \end{vmatrix}$$

$$\nabla \times \overline{A} = \frac{1}{\rho}\begin{vmatrix} \overline{a}_\rho & \rho\overline{a}_\phi & \overline{a}_Z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ 0 & 0 & -\frac{\rho^2}{4} \end{vmatrix}$$

$$= \frac{1}{\rho}\left[\overline{a}_\rho(0) - \rho\overline{a}_\phi \frac{\partial}{\partial\rho}\left(\frac{-\rho^2}{4}\right) + 0\right]$$

$$= \frac{1}{\rho}\left[-\rho\overline{a}_\phi\left(-\frac{1}{4}2\rho\right)\right] = \frac{\rho}{2}\overline{a}_\phi$$

$\because$ ϕ is constant and , ρ and z are varying, $d\overline{s} = d\rho\, dz\overline{a}_\phi$

The magnetic flux crossing the surface is $\psi = \int_S (\nabla \times \overline{A}) \cdot d\overline{s}$

$$= \int_{\rho=1}^{2} \int_{z=0}^{5} \frac{\rho}{2} d\rho dz$$

= 3.75 Wb

Problem:2.23

A current distribution gives rise to the vector magnetic potential $\overline{A} = x^2 y\overline{a}_x + y^2 x\overline{a}_y - 4xyz\overline{a}_z$ Wb/m. Calculate (a) $\overline{B}$ $at\,(-1,2,5)$ (b) The flux through the surface defined by Z=1, $0 \le x \le 1$, $-1 \le y \le 4$.

Solution:

(a) $\bar{B} = \nabla \times \bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & y^2 x & -4xyz \end{vmatrix}$

$$\bar{B} = \bar{a}_x\left(-4xz - 0\right) - \bar{a}_y\left(-4yz - 0\right) + \bar{a}_z\left(y^2 - x^2\right)$$

$$\bar{B} = -4xz\bar{a}_x + 4yz\bar{a}_y + \left(y^2 - x^2\right)\bar{a}_z$$

$$\bar{B}_{(-1,2,5)} = 20\bar{a}_x + 40\bar{a}_y + 3\bar{a}_z \text{ Wb/m}^2$$

(b)

Here $d\bar{s} = dx\,dy\,\bar{a}_z$

$$\psi = \int_S \nabla \times \bar{A} \cdot d\bar{s}$$

$$\psi = \int_{x=0}^{1}\int_{y=-1}^{4} \left(-4xz\bar{a}_x + 4yz\bar{a}_y + \left(y^2 - x^2\right)\bar{a}_z\right) \cdot dxdy\bar{a}_z$$

$$\psi = \int_{x=0}^{1}\int_{y=-1}^{4} \left(y^2 - x^2\right)dxdy$$

$$\psi = \int_{y=-1}^{4}\left[y^2 x - \frac{x^3}{3}\right]_0^1 dy = \int_{y=-1}^{4}\left[y^2 - \frac{1}{3}\right]dy$$

$$\psi = \frac{1}{3}\left[y^3 - y\right]_{-1}^{4} = \frac{1}{3}\left[64 - 4 + 1 - 1\right] = 20Wb$$

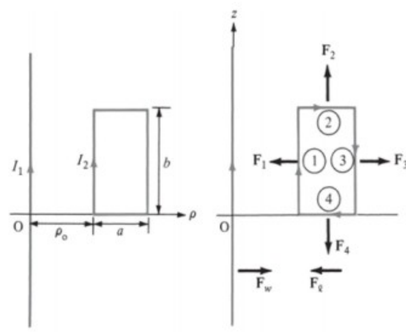
Problem:2.24

A rectangular loop carrying current I_2 is placed parallel to an infinitely long filamentary wire carrying current I_1 as shown in Fig.2.21. Show that the force experienced by the loop is given by

$$\bar{F} = -\frac{\mu_0 I_1 I_2 b}{2\pi}\left[\frac{1}{\rho_0} - \frac{1}{\rho_0 + a}\right]\bar{a}_\rho N$$

Solution:

Fig:



Let $\bar{F}_\ell$ be the force on the loop

$$\bar{F}_\ell = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4$$

$$= I_2 \oint d\bar{l}_2 \times \bar{B}_1$$

Where $\bar{F}_1, \bar{F}_2, \bar{F}_3$ *and* $\bar{F}_4$ are the forces exerted on sides of the loop.

We know for infinitely long wire $\bar{B} = \frac{\mu_0 I}{2\pi\rho}\bar{a}_\phi$

To evaluate $\bar{F}_1$, $d\bar{l}_2 = dz\bar{a}_z$, z ranges from 0 to b, I=I₁ and $\rho = \rho_0$

$$\bar{F}_1 = I_2 \int d\bar{l}_2 \times \bar{B}_1 = I_2 \int_{z=0}^{b} dz\bar{a}_z \times \frac{\mu_0 I_1}{2\pi\rho_0}\bar{a}_\phi$$

$$\therefore \bar{F}_1 = -\frac{\mu_0 I_1 I_2 b}{2\pi\rho_0}\bar{a}_\rho \qquad (attractive)$$

To evaluate $\bar{F}_3$, $d\bar{l}_2 = dz\bar{a}_z$, z ranges from b to 0, I=I$_1$ and $\rho = \rho_0 + a$

$$\bar{F}_3 = I_2 \int d\bar{l}_2 \times \bar{B}_1 = I_2 \int_{z=b}^{0} dz\bar{a}_z \times \frac{\mu_0 I_1}{2\pi(\rho_0 + a)}\bar{a}_\phi$$

$$\therefore \bar{F}_3 = \frac{\mu_0 I_1 I_2 b}{2\pi(\rho_0 + a)}\bar{a}_\rho \qquad (repulsive)$$

To evaluate $\bar{F}_2$, $d\bar{l}_2 = d\rho\bar{a}_\rho$, ρ ranges from ρ_0 to $\rho_0 + a$, I=I$_1$ and $\rho = \rho$

$$\bar{F}_2 = I_2 \int_{\rho=\rho_0}^{\rho_0+a} d\rho\bar{a}_\rho \times \frac{\mu_0 I_1}{2\pi\rho}\bar{a}_\phi$$

$$\therefore \bar{F}_2 = \frac{\mu_0 I_1 I_2}{2\pi}\ln\left(\frac{\rho_0 + a}{\rho_0}\right)\bar{a}_z \qquad (parallel)$$

To evaluate $\bar{F}_4$, $d\bar{l}_2 = d\rho\bar{a}_\rho$, ρ ranges from $\rho_0 + a$ to ρ_0, I=I$_1$ and $\rho = \rho$

$$\bar{F}_4 = I_2 \int_{\rho=\rho_0+a}^{\rho_0} d\rho\bar{a}_\rho \times \frac{\mu_0 I_1}{2\pi\rho}\bar{a}_\phi$$

$$\therefore \bar{F}_4 = -\frac{\mu_0 I_1 I_2}{2\pi}\ln\left(\frac{\rho_0 + a}{\rho_0}\right)\bar{a}_z \qquad (parallel)$$

Then the total force $\bar{F}_\ell$ on the loop is

$$\bar{F}_\ell = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4$$

$$\bar{F}_\ell = -\frac{\mu_0 I_1 I_2 b}{2\pi}\left[\frac{1}{\rho_0} - \frac{1}{\rho_0 + a}\right]\bar{a}_\rho N$$

and

$$\bar{F}_w = -\bar{F}_\ell$$

Problem:2.25

A charged particle of mass 2kg and 1C starts at origin with velocity $3\bar{a}_y$ m/s and travels in a region of uniform magnetic field $\bar{B} = 10\bar{a}_z$ Wb/m^2 at t=4sec. Calculate (a) velocity and acceleration of particle (b) the magnetic force on it.

Solution:

(a)We have $\bar{F} = m\frac{d\bar{u}}{dt} = Q\bar{u} \times \bar{B}$

Acceleration is

$$\bar{a} = \frac{d\bar{u}}{dt} = \frac{Q}{m}\bar{u} \times \bar{B}$$

Hence $\quad \frac{d}{dt}(u_x\bar{a}_x + u_y\bar{a}_y + u_z\bar{a}_z) = \frac{1}{2}\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ u_x & u_y & u_z \\ 0 & 0 & 10 \end{vmatrix} = 5(u_y\bar{a}_x - u_x\bar{a}_y)$

By equating components, we get

$$\frac{du_x}{dt} = 5u_y, \quad \frac{du_y}{dt} = -5u_x, \quad \frac{du_z}{dt} = 0 \Rightarrow u_z = c_0$$

u_x or u_y an be eliminated in the above equations by taking second derivative of one equation and utilizing the other. Thus

$$\frac{d^2u_x}{dt^2} = 5\frac{du_y}{dt} = -25u_x$$

or

$$\frac{d^2u_x}{dt^2} + 25u_x = 0$$

Which is a linear differential equation whose solution is

$$u_x = c_1 \cos 5t + c_2 \sin 5t$$

$$\frac{du_x}{dt} = 5u_y = -5c_1 \sin 5t + 5c_2 \cos 5t$$

or

$$u_y = -c_1 \sin 5t + c_2 \cos 5t$$

Let us determine c_0,c_1 and c_2 using initial conditions. At t=0, $\bar{u} = 3\bar{a}_y$.

Hence,

$$u_x = 0 \Rightarrow 0 = c_1.1 + c_2.0 \Rightarrow c_1 = 0$$
$$u_y = 3 \Rightarrow 3 = -c_1.0 + c_2.1 \Rightarrow c_2 = 3$$
$$u_z = 0 \Rightarrow 0 = c_0$$

Substituting the values of c_0,c_1 and c_2, gives velocity as

$$\bar{u} = (u_x, u_y, u_z) = (3\sin 5t, 3\cos 5t, 0)$$

Hence, velocity at t=4sec is

$$\bar{u} = (3\sin 20, 3\cos 20, 0)$$
$$= 2.739\bar{a}_x + 1.224\bar{a}_y \quad m/s$$

Acceleration is $\bar{a} = \frac{d\bar{u}}{dt} = (15\cos 5t, -15\sin 5t, 0)$

Hence, acceleration at t=4sec is

$$\bar{a} = 6.101\bar{a}_x - 13.703\bar{a}_y \quad m/s^2$$

(b) $\quad \bar{F} = m\frac{d\bar{u}}{dt} = m\bar{a} = 12.2\bar{a}_x - 27.4\bar{a}_y \quad N$.

Problem:2.26

A flux density of $0.05\bar{a}_y$ tesla in a material having magnetic susceptibility 2.5, find magnetic field current density and magnetization.

Solution:

Given $\bar{B} = 0.05\bar{a}_y$ and $\chi_m = 2.5$

Relative permiability $\mu_r = 1 + \chi_m = 1 + 2.5 = 3.5$

Permiability of material $\mu = \mu_0\mu_r = 4\pi \times 10^{-7} \times 3.5 = 4.398 \times 10^{-6}$ H/m

Magnetic field intensity $\bar{H} = \dfrac{\bar{B}}{\mu} = \dfrac{0.05\bar{a}_y}{4.398 \times 10^{-6}} = 11368.8\bar{a}_y$ A/m

Magnetic field current density $\bar{J} = \nabla \times \bar{H} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & 11368.8 & 0 \end{vmatrix} = 0$ A/m²

Magnetization $\bar{M} = \chi_m \bar{H} = 2.5 \times 11368.8\bar{a}_y = 28422\bar{a}_y$ A/m

Problem:2.27

In a certain material, $\chi_m = 4.2$ and $\bar{H} = 0.2x\bar{a}_y$ A/m. Determine: (a) μ_r , (b) μ , (c) $\bar{M}$,(d) $\bar{B}$,(e) $\bar{J}$, (f) $\bar{J}_b$.

Solution:

(a) $\mu_r = 1 + \chi_m = 1 + 4.2 = 5.2$

(b) $\mu = \mu_0\mu_r = 4\pi \times 10^{-7} \times 5.2 = 6.534 \times 10^{-6}$ H/m

(c) $\bar{M} = \chi_m \bar{H} = 4.2(0.2x\bar{a}_y) = 0.84x\bar{a}_y$ A/m

(d) $\bar{B} = \mu\bar{H} = 6.534 \times 10^{-6} \times 0.2x\bar{a}_y = 1.307x\bar{a}_y\ \mu\text{Wb/m}^2$

(e) $\bar{J} = \nabla \times \bar{H} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & 0.2x & 0 \end{vmatrix} = 0.2\bar{a}_z$ A/m²

(f) $\bar{J}_b = \chi_m \bar{J} = 4.2 \times 0.2\bar{a}_z = 0.84\bar{a}_z$ A/m²

Problem:2.28

Determine the self inductance of a co-axial cable of inner radius ‘a’ and outer radius ‘b’ and length of the co-axial cable is ‘l’.

Solution:

Assume inner conductor carries a current I in Z-direction and outer conductor carries a current I in opposite direction as shown in Fig:2.26.

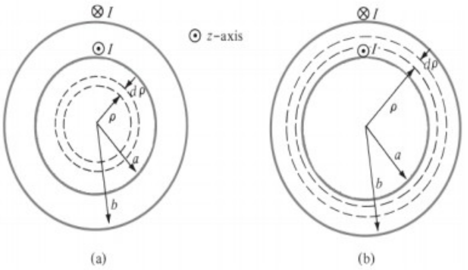


Fig: 2.26 (a) For finding L_{in} (b) For finding L_{out}

Inner Conductor:

We know for inner conductor $\bar{H} = \dfrac{I\rho}{2\pi a^2}\bar{a}_\phi$

Since it is a cylinder, we can use the cylindrical coordinate system to solve the problem

We have

$$W_m = \frac{1}{2}\int_v \mu H^2 dv$$

$$W_m = \frac{1}{2}\int_v \mu \frac{I^2\rho^2}{4\pi^2 a^4}\rho\, d\rho\, d\phi\, dz$$

$$W_m = \frac{1}{2}\int_{z=0}^{l}\int_{\rho=0}^{a}\int_{\phi=0}^{2\pi} \mu \frac{I^2\rho^2}{4\pi^2 a^4}\rho d\rho\, d\phi\, dz$$

$$L_{in} = \frac{2W_m}{I^2} = \frac{\mu}{4\pi^2}\int_{z=0}^{l}\int_{\varphi=0}^{2\pi} \frac{1}{a^4}\frac{a^4}{4} d\phi\, dz \qquad L_{in} = \frac{\mu}{4\pi^2}2\pi\, l = \frac{\mu\, l}{8\pi}$$

Outer conductor:

We know for outer conductor $\bar{H} = \dfrac{I}{2\pi\rho}\bar{a}_\phi$

$$L_{out} = \frac{2W_m}{I^2} = \frac{2}{I^2}\frac{1}{2}\int_{z=0}^{l}\int_{\rho=a}^{b}\int_{\phi=0}^{2\pi} \mu\left(\frac{I}{2\pi\rho}\right)^2 \rho\, d\rho\, d\phi\, dz$$

$$L_{out} = \frac{\mu}{4\pi^2}\times 2\pi \times l \ln\frac{b}{a} = \frac{\mu l}{2\pi}\ln\frac{b}{a}$$

The self inductance of co-axial cable is

$$L = L_{in} + L_{out} = \frac{\mu\, l}{2\pi}\left(\frac{1}{4} + \ln\frac{b}{a}\right)$$