THIN CYLINDER

INTRODUCTION

* The vessels such as boilers, compressed air receivers etc... are of cylinderical and spherical forms.

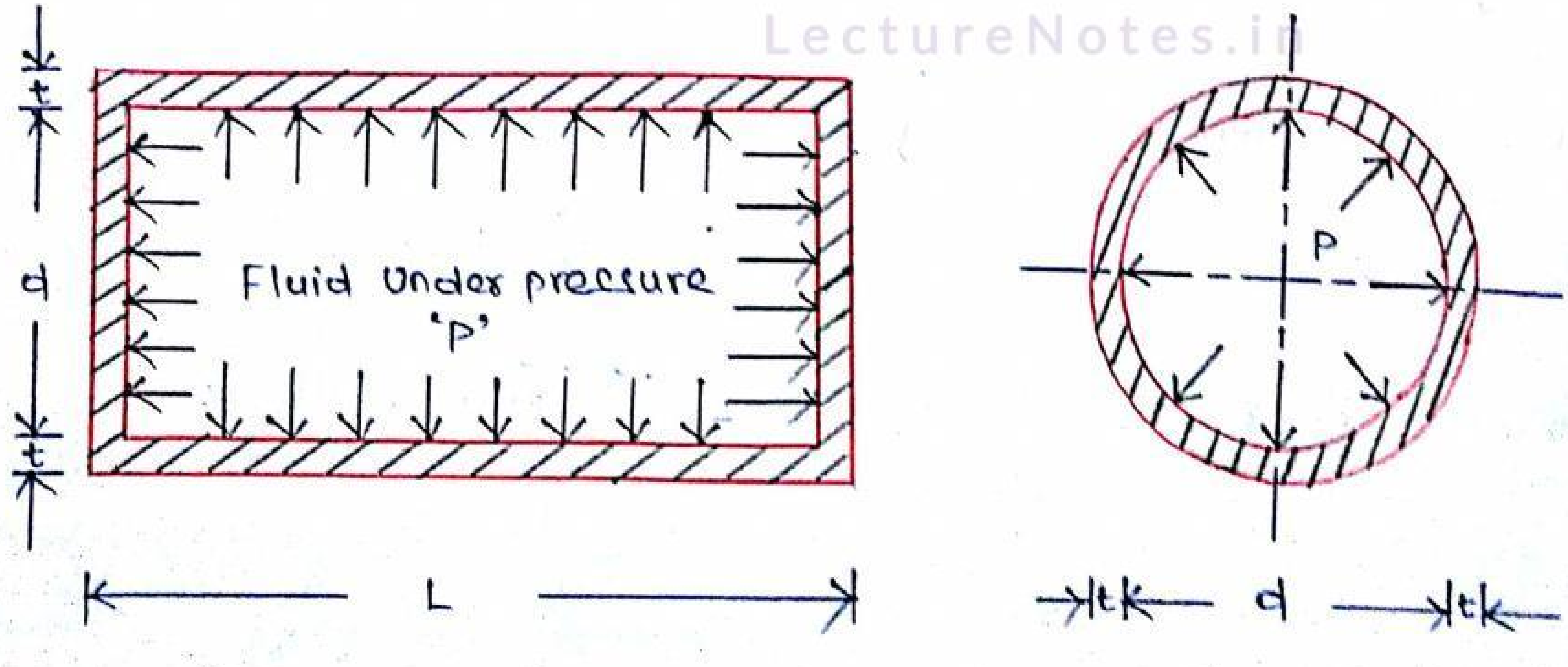
* These vessels are generally used for storing fluids (Liquid or Gas) under pressure.

* The walls of such vessels are thin as compared to their diameters.

* If the thickness of the wall of the cylinderical vessel is less than $\frac{1}{15}$ to $\frac{1}{20}$ of its internal diameter, the cylindrical vessel is known as Thin cylinder.

THIN CYLINDRICAL VESSEL SUBJECTED TO INTERNAL PRESSURE

Thin cylindrical vessel in which a fluid under pressure is stored.



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Let,

- d - Internal diameter of the Thin cylinder

t - Thickness of the wall of the cylinder

p - Internal pressure of the Fluid.

L - Length of the cylinder

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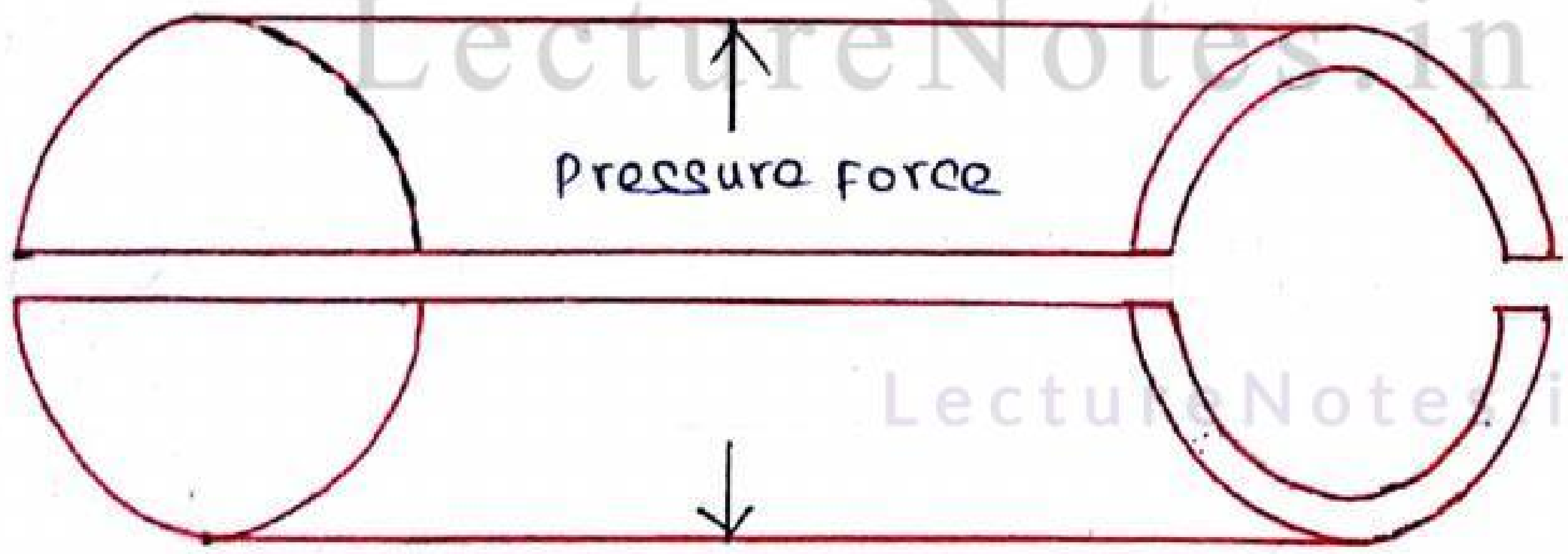
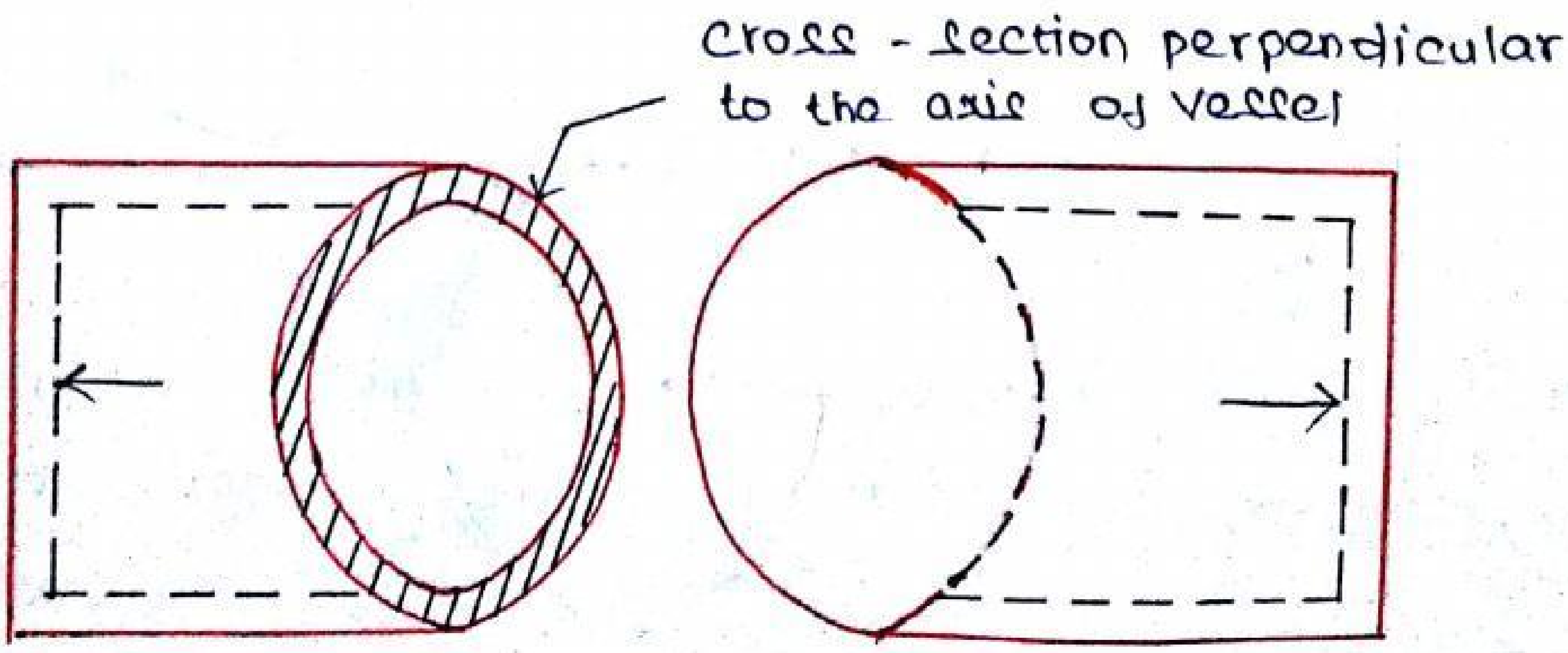
* The Internal pressure "p", the cylindrical vessel may fail by splitting up in any one of the two ways

* The forces, due to pressure of the fluid acting vertically upwards and downwards on the thin cylinder, tend to burst the cylinder.

* The forces, due to pressure of the Fluid, acting at the ends of thin cylinder, tend to burst the thin cylinder





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STRESSES IN A THIN CYLINDRICAL VESSEL SUBJECTED TO INTERNAL PRESSURE

When a thin cylindrical vessel is subjected to internal fluid pressure, the stresses in the wall of the cylinder on the cross-section along the axis and on the cross-section perpendicular to the axis are set up. These stresses are tensile and are known as

1. Circumferential stress (or) Hoop stress
2. Longitudinal stress.

* The stress acting along the circumference of the cylinder is called circumferential stress
* The stress acting along the length of the cylinder is called longitudinal direction (Longitudinal stress)
* The circumferential stress is also known as Hoop stress

DERIVATION OF FORMULA FOR CIRCUMFERENTIAL AND LONGITUDINAL STRESS

Circumferential Stress

* Consider a thin cylindrical vessel subjected to an internal fluid pressure.
* The circumferential stress will be set up in the material of the cylinder, if the bursting of the cylinder takes place.

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The expression for hoop stress or circumferential stress (σ_c)

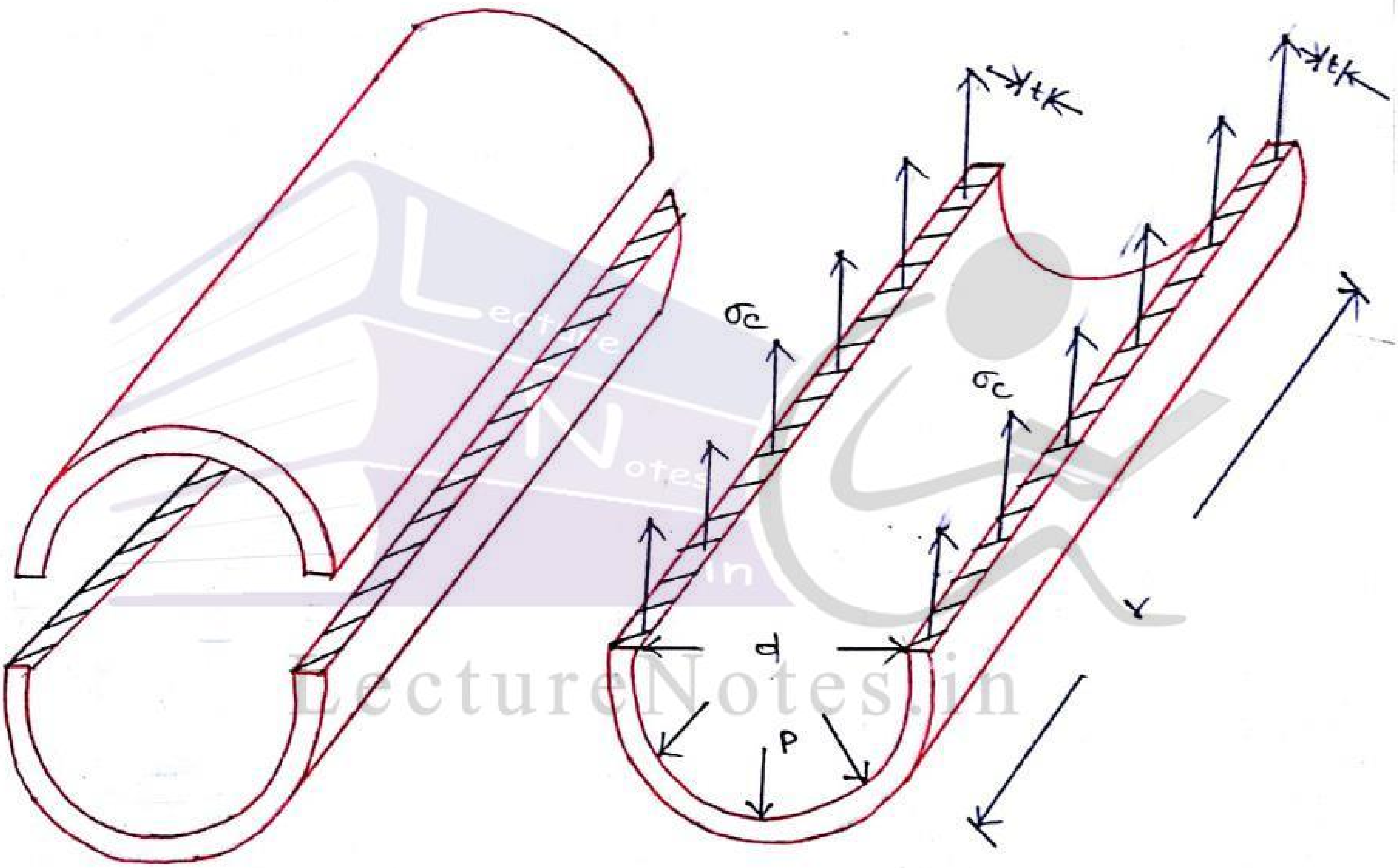
let,

P - Internal pressure of fluid

d - Internal diameter of the cylinder

t - Thickness of the wall of the cylinder

σ_c - Circumferential or Hoop stress in the material



* The bursting will takes place if the force due to fluid pressure is more than the resistance force due to circumferential stress set up in the material.

* In the limiting case, the two forces should be equal.

Force due to fluid pressure $= P \times$ Area on which P is acting

$= P \times (d \times L) \rightarrow (1)$

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$\therefore$ P is acting projected area

$(d \times L)$

Force due to circumferential stress

$= \sigma_c \times$ Area on which σ_c is acting

$= \sigma_c \times (L \times t + L \times t)$

$= \sigma_c \times 2Lt$

$= 2\sigma_c \times L \times t \rightarrow (2)$

Equating (1) & (2)

$$P \times d \times \cancel{L} = 2\sigma_c \times \cancel{L} \times t$$

$$\boxed{\sigma_c = \frac{Pd}{2t}}$$

$\therefore$ The stress is Tensile

Longitudinal Stress

Consider a thin cylindrical vessel subjected to internal fluid pressure. The longitudinal stress will be set up in the material of the cylinder, if the bursting of the cylinder takes place along the section AB

The longitudinal stress (σ_L) developed in the material

let,

p - Internal pressure of fluid stored in thin cylinder

d - Internal diameter of cylinder

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t - Thickness of the cylinder

σ_L - Longitudinal stress in the material

* The bursting will take place if the force due to fluid pressure acting on the ends of the cylinder is more than the resisting force due to longitudinal stress (σ_L) developed in the material

* In the limiting case, both the forces should be equal

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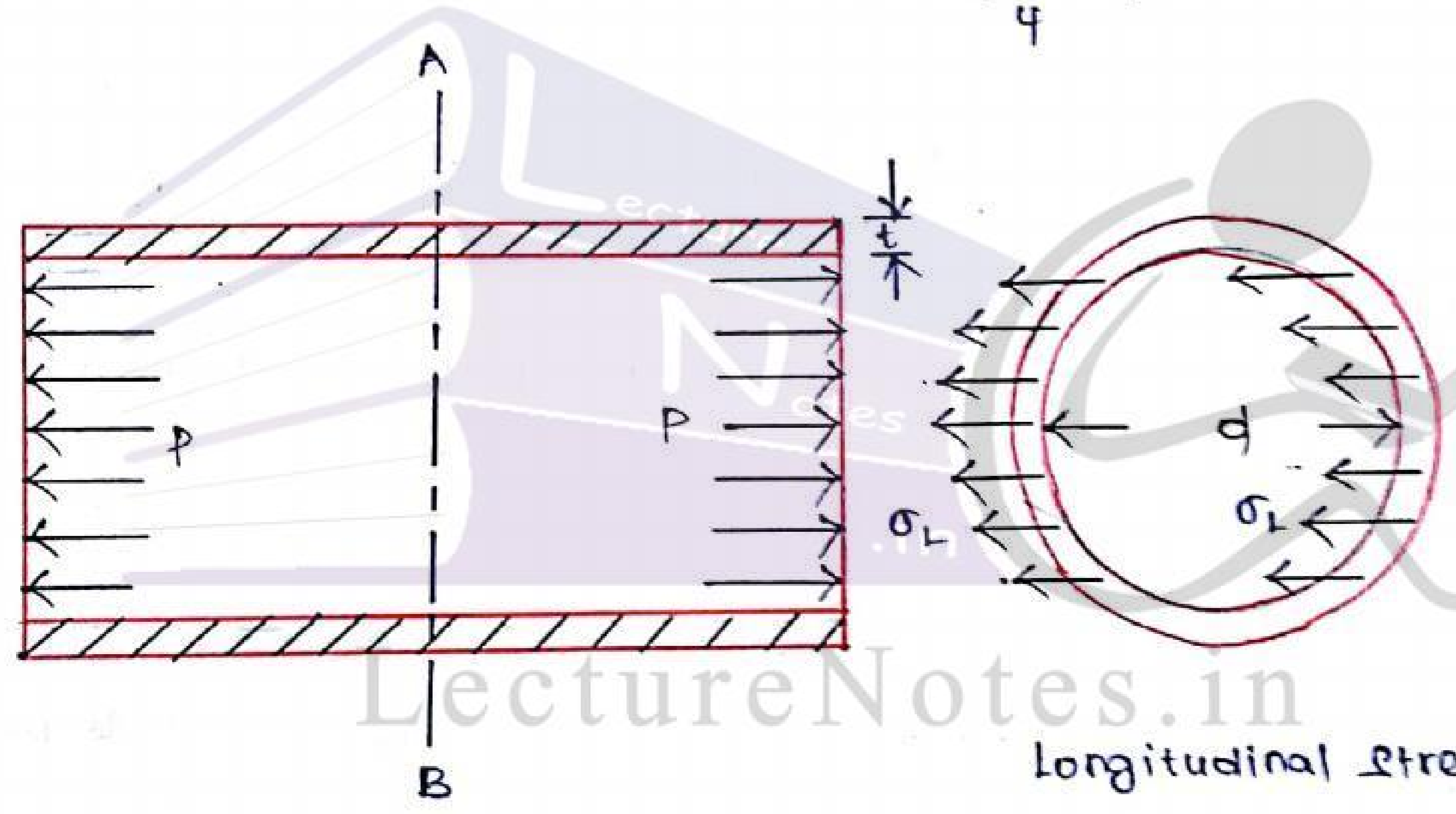
Force due to fluid pressure = $p \times$ Area on which p is acting

$$= p \times \frac{\pi}{4} d^2$$



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Longitudinal stress (σ_2) develop

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Resisting force = $\sigma_L \times$ Area on which σ_L is acting

$$= \sigma_L \times \pi d \times t$$

Force due to fluid pressure = Resisting force

$$p \times \frac{\pi}{4} d^2 = \sigma_L \times \pi d \times t$$

$$\sigma_L = \frac{p \times \frac{\pi}{4} d^2}{\pi d \times t}$$

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$$\boxed{\sigma_L = \frac{Pd}{4t}}$$

The stress σ_L is also tensile

$$\sigma_c = \sigma_L$$

$$\frac{Pd}{2t} = \frac{Pd}{4t}$$

$$\frac{1}{2} \times \frac{Pd}{2t} = \frac{Pd}{2 \times 2t}$$

Half of circumferential stress } = longitudinal stress

circumferential stress (σ_c) is two times the longitudinal stress(σ_L)

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Problm.no : 1

A cylindrical pipe of diameter 1.5m and thickness 1.5cm is subjected to an internal fluid pressure of 1.2 N/mm^2.

Determine

(i) longitudinal stress developed in the pipe

(ii) circumferential stress developed in the pipe

Given data

Dia. of pipe (d) = 1.5m

Thickness (t) = 1.5 cm ⇒ 1.5×10^{-2} m

Internal fluid pressure (P) = 1.2 N/mm^2

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Solution

$$\frac{d}{t} = \frac{1.5}{1.5 \times 10^{-2}}$$

$$= 100$$

$100 > 20$ ∴ Thin cylinder

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$$\frac{d}{t} > 20$$

Longitudinal stress (σ_L)

$$\sigma_L = \frac{p \times d}{4t}$$

$$= \frac{1.2 \times 1.5}{4 \times (1.5 \times 10^{-2})}$$

$$\boxed{\sigma_L = 30\ N/mm^2} \rightarrow \text{Ans (i)}$$

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Circumferential stress (σ_c)

$$\sigma_c = \frac{pd}{2t}$$

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$$= \frac{1.2 \times 1.5}{2 \times (1.5 \times 10^{-2})}$$

$$\boxed{\sigma_c = 60\ N/mm^2} \rightarrow \text{Ans (ii)}$$

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Problm no: 2

A cylinder of Internal diameter 2.5m and of Thickness 5cm contains a gas. If the tensile stress in the material is not to exceed 80 N/mm², Determine the Internal pressure of the gas.

Given data

Internal dia. of cylinder (d) = 2.5 m

Thickness of cylinder (t) = 5cm ⇒ 5×10^{-2} m

Maximum permissible stress = 80 N/mm²

let, p- Internal pressure of Gas

Solution

- Maximum permissible stress is given
- Hence this should be equal to circumferential stress (σ_c)

$$\sigma_c = 80\ N/mm^2$$

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$$\sigma_c = \frac{Pd}{2t}$$

$$P = \frac{2t\times\sigma_c}{d}$$

$$= \frac{2\times5\times10^{-2}\times80}{2.5}$$

$$\boxed{P = 3.2\ N/mm^2}$$

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Prblm.no:3

A Thin cylinder of Internal diameter 1.25m contains a fluid at an internal pressure of $2 N/mm^2$. Determine the maximum thickness of the cylinder

(i) The longitudinal stress is not to exceed $20 N/mm^2$

(ii) The circumferential stress is not to exceed $45 N/mm^2$

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Given data

Internal diameter of cylinder $(d) = 1.25 m$

Internal pressure of fluid $(P) = 2 N/mm^2$

longitudinal stress $(\sigma_L) = 20 N/mm^2$

circumferential stress $(\sigma_c) = 45 N/mm^2$

Solution

Circumferential stress

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$$\sigma_c = \frac{Pd}{2t}$$

$$t = \frac{P \times d}{2 \times \sigma_c}$$

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$$= \frac{2 \times 1.25}{2 \times 45}$$

$$\boxed{t = 0.0277m}$$

$$= 2.77cm$$

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Longitudinal stress

$$\sigma_L = \frac{pd}{4t}$$

$$t = \frac{pd}{4 \times \sigma_L}$$

$$= \frac{2 \times 1.25}{4 \times 30}$$

$$\boxed{t = 0.0208\,m}$$

$$= 2.08\,cm$$

Maximum Thickness of cylinder

should not be less than

$$\boxed{t = 2.77\,cm}$$

Problm. no: 4

A Water main 80 cm diameter contains water at a pressure head of 100 m. If the weight density of water is 9810 N/m^3. Find the thickness of the metal required for the water main. Give the permissible stress as 20 N/mm^2

Given data

Dia. of main (d) = 80 cm

pressure head of water (h) = 100 m

Weight density of water (w) = $\rho \times g$

$= 1000 \times 9.81 \Rightarrow 9810\ N/m^3$

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Permissible stress = 20 N/mm^2

Pressure of water inside the water main

$$p = \rho \times g \times h$$

$$= w \times h$$

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$$= 9810 \times 100 \ N/m^2$$

$$p = \frac{9810 \times 100}{1000} \ N/mm^2$$

$$= 0.981 \ N/mm^2$$

Circumferential stress

$$\sigma_c = \frac{p \times d}{2 \times t}$$

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$$t = \frac{p \times d}{2 \times \sigma_c}$$

$$= \frac{0.981 \times 80}{2 \times 20}$$

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$$\boxed{t = 2cm}$$

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EFFECT OF INTERNAL PRESSURE ON THE DIMENSIONS OF A THIN CYLINDRICAL SHELL

When a fluid having internal pressure (p) is stored in a thin cylindrical shell, due to internal pressure of the fluid the stresses set up at any point of the material of the shell.

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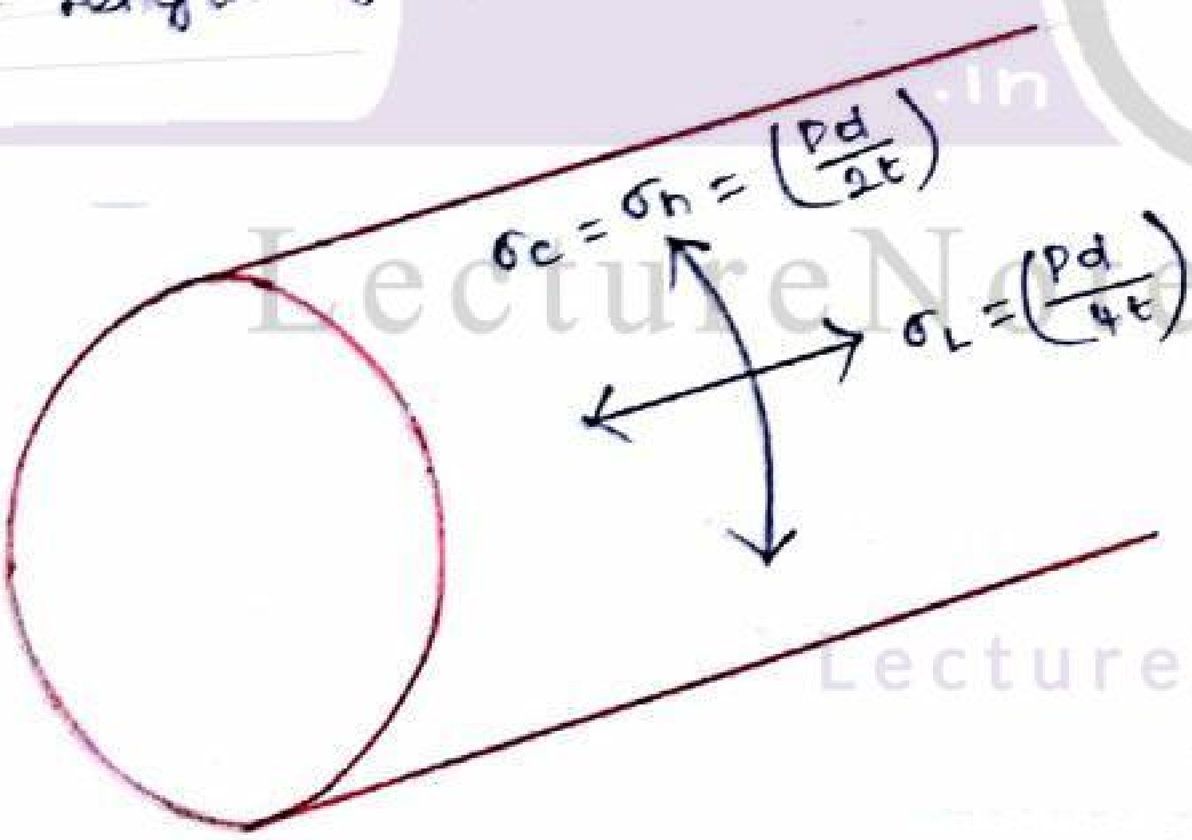
(i) Hoop or circumferential stress (σ_c) acting on longitudinal section

(ii) Longitudinal stress (σ_L) acting on the circumferential section

Let,

p - Internal pressure of fluid

l - length of cylindrical shell



d - Diameter of the cylindrical shell

t - Thickness of the cylindrical shell

E - Modulus of elasticity for the material of the shell

σ_c - Hoop stress in the material

σ_L - Longitudinal stress in the material

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μ - poisson's ratio

σ_h - Major principal stress (σ_1)

σ_l - Minor principal stress (σ_2)

$$\boxed{\sigma_1 = \sigma_c = \sigma_h = \frac{pd}{2t}}$$

$$\boxed{\sigma_2 = \sigma_L = \frac{pd}{4t}}$$

- let,

ϵ_c - Circumferential strain

ϵ_l - Longitudinal strain

CIRCUMFERENTIAL STRAIN

$$\epsilon_c = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E}$$

$$= \frac{pd}{2tE} - \frac{\mu pd}{4tE}$$

$$= \frac{pd}{2tE}\left[1 - \frac{\mu}{2}\right] \rightarrow (1)$$

Circumferential strain $= \dfrac{\text{change in circumferential}}{\text{Original circumferential}}$

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$$= \frac{\text{Final circumference} - \text{Original circumference}}{\text{Original Circumference}}$$

let,

δd - Increase in diameter

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d_i - Initial dia. of cylinder

d_f - Final dia. of cylinder

Initial or Original circumference

$$= \pi d_i$$

Final circumference

$$= \pi d_f \Rightarrow \pi(d_i + \delta d)$$

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$$\text{Circumferential strain} = \frac{\pi(d_i + \delta d) - \pi d_i}{\pi d_i}$$

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$$= \frac{\cancel{\pi d_i} + \cancel{\pi}\,\delta d - \cancel{\pi d_i}}{\cancel{\pi} d_i}$$

$$= \frac{\delta d}{d_i}$$

$$\epsilon_c = \epsilon_h = \frac{\delta d}{d} \rightarrow (2)$$

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Equating the two values of ϵ_c (1) & (2)

$$\frac{\delta d}{d} = \frac{Pd}{2tE}\left[1 - \frac{\mu}{2}\right]$$

Change in diameter

$$\boxed{\delta d = \frac{Pd^2}{2tE}\left(1 - \frac{\mu}{2}\right)}$$

LONGITUDINAL STRAIN

$$\epsilon_L = \frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E}$$

$$= \frac{Pd}{4tE} - \frac{\mu Pd}{2tE}$$

$$\boxed{\epsilon_L = \frac{Pd}{2tE}\left(\frac{1}{2} - \mu\right)} \rightarrow (3)$$

Longitudinal strain = $\dfrac{\text{Change in length due to pressure}}{\text{Original length}}$

Let,

L_o - Original length $\Rightarrow \pi L_i$

L_f - Final length $\Rightarrow \pi L_f \Rightarrow \pi(L_i + \delta L)$

δL - Increase in length

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$$\text{longitudinal strain} = \frac{l_f - l_i}{l_i}$$

$$\boxed{\epsilon_L = \frac{\delta L}{L}} \rightarrow (4)$$

Equating the two values of ϵ_L (3) & (4)

$$\frac{\delta L}{L} = \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right)$$

change in length

$$\boxed{\delta L = \frac{p \times d \times L}{2tE}\left(\frac{1}{2} - \mu\right)}$$

VOLUMETRIC STRAIN (ϵ_V)

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$$\epsilon_V = \frac{\delta V}{V}$$

δV - change in Volume

V_i - Initial volume $\Rightarrow \frac{\pi}{4} d^2 L$

V_f - Final volume $\Rightarrow \frac{\pi}{4} (d + \delta d)^2 \cdot (L + \delta L)$

$$\frac{\delta V}{V} = \frac{F.V - I.V}{I.V}$$

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$$\frac{\delta V}{V} = \frac{\frac{\pi}{4}(d+\delta d)^2(L+\delta L) - \frac{\pi}{4}d^2 L}{\frac{\pi}{4}d^2 L}$$

$$\frac{\delta V}{V} = \frac{\left[\frac{\pi}{4}\; d^2 + 2d\,\delta d + \delta d^2\right]\left[L + \delta L\right] - \frac{\pi}{4}d^2 L}{\frac{\pi}{4}d^2 L}$$

$$\frac{\delta V}{V} = \frac{\frac{\pi}{4}d^2 L + d^2\delta L + 2d\,\delta d\,L + 2d\,\delta d\,\delta L + \delta d^2 L + \delta d^2 \delta L - d^2 L}{\frac{\pi}{4}d^2 L}$$

$$\frac{\delta V}{V} = \frac{d^2 L + d^2\delta L + 2d\,\delta d\,L - d^2 L}{d^2 L}$$

$$= \frac{d^2\delta L + 2d\,\delta d\,L}{d^2 L}$$

$$= \frac{\delta L}{L} + 2\frac{\delta d}{d}$$

$$\epsilon_V = \epsilon_L + 2\epsilon_c$$

$$\epsilon_c = \frac{\delta d}{d} \quad ; \quad \epsilon_L = \frac{\delta L}{L}$$

$$\epsilon_c = \frac{\sigma_c}{E} - \mu\frac{\sigma_L}{E}$$

$$= \frac{Pd}{2tE} - \mu\frac{Pd}{4tE}$$

$$\boxed{\epsilon_c = \frac{Pd}{2tE}\left[1 - \frac{\mu}{2}\right]}$$

$$\epsilon_L = \frac{\sigma_L}{E} - \mu\frac{\sigma_c}{E}$$

$$= \frac{Pd}{4tE} - \mu\frac{Pd}{2tE}$$

$$\boxed{\epsilon_L = \frac{Pd}{2tE}\left[\frac{1}{2} - \mu\right]}$$

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Prblm. no: 5

Calculate

(i) The change in diameter

(ii) change in length

(iii) change in volume of a Thin cylindrical shell 100 cm diameter, 1cm thick and 5m long when subjected to internal pressure of 3 N/mm^2

Take the value of $E = 2\times10^5$ N/mm^2 and poisson's ratio $\mu = 0.2$

Given data

Diameter of shell (d) = 100 cm

Thickness of shell (t) = 1cm

length of shell (L) = 5m $\Rightarrow$ 5 x 100 cm $\Rightarrow$ 500 cm

Internal pressure (E) = 2×10^5 N/mm^2

poisson's ratio (μ) = 0.20

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Solution

Change in diameter (δd)

$$\delta d = \frac{Pd^2}{2tE}\left[1 - \frac{\mu}{2}\right]$$

$$= \frac{3\times100^2}{2\times1\times(2\times10^5)}\left[1-\frac{1}{2}\times0.20\right]$$

$$= \frac{3}{40}\left[1-0.15\right]$$

$$\boxed{\delta d = 0.06375 \text{ cm}}$$

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(ii) change in Length

$$\delta L = \frac{PdL}{2tE}\left[\frac{1}{2} - \mu\right]$$

$$= \frac{3 \times 100 \times 500}{2 \times 1 \cdot (2 \times 10^5)}\left[\frac{1}{2} - 0.20\right]$$

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$$= \frac{15}{40} \times 0.20$$

$$\boxed{\delta L = 0.075 \text{ cm}}$$

(iii) change in Volume

$$\delta V = V\left[2\epsilon_c + \epsilon_L\right]$$

$$= V\left[2\frac{\delta d}{d} + \frac{\delta L}{L}\right]$$

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$$= V\left[2 \times \frac{0.06275}{100} + \frac{0.075}{500}\right]$$

$$= V\left[0.001275 + 0.00015\right]$$

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$$= 0.001425 V$$

$$\delta V = 0.001425 \times 3926990.817$$

V - Original volume

$$\boxed{\delta V = 5595.96 \text{ cm}^3}$$

$$\therefore \left(V = \frac{\pi}{4} d^2 \cdot L\right)$$

$$= \frac{\pi}{4} \times 100^2 \times 500$$

$$V = 3926990.817 \text{ cm}^3$$

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Problm. no: 6.

A cylindrical thin drum 80 cm in diameter and 2m long has a shell thickness of 1 cm. If the drum is subjected to an internal pressure of 2.5 N/mm^2, Determine

(i) Change in diameter

(ii) change in length

(iii) change in volume

Take $E = 2 \times 10^5$ N/mm^2 ; poisson's ratio $= 0.25$

Given data

Diameter of drum $(d) = 80$ cm

Length of drum $(l) = 2m \Rightarrow 2 \times 100 \Rightarrow 200$ cm

Thickness of drum $(t) = 1$ cm

Internal pressure $(P) = 2.5$ N/mm^2

Young's modulus $(E) = 2 \times 10^5$ N/mm^2

poisson's ratio $(\mu) = 0.25$

Solution

(i) Change in diameter (δd)

$$d = \frac{Pd^2}{2tE}\left(1 - \frac{1}{2} \times \mu\right)$$

$$= \frac{2.5 \times 80^2}{2 \times 1 \times 2 \times 10^5}\left[1 - \frac{1}{2} \times 0.25\right]$$

$$= 0.04\left[1 - 0.125\right]$$

$$\boxed{\delta d = 0.035\ cm}$$

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(ii) Change in Length (δL)

$$\delta L = \frac{pdL}{2tE}\left(\frac{1}{2} - \mu\right)$$

$$= \frac{2.5 \times 80 \times 200}{2 \times 1 \times (2 \times 10^5)}\left[\frac{1}{2} - 0.25\right]$$

$$\delta L = 0.0375 \text{ cm}$$

(iii) Volumetric Strain

$$\frac{\delta V}{V} = 2\frac{\delta d}{d} + \frac{\delta L}{L}$$

$$= 2 \times \frac{0.035}{80} + \frac{0.0375}{200}$$

$$= 0.000875 + 0.000125$$

$$= 0.001 \times V$$

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$$\text{Volume} = \frac{\pi}{4} \times d^2 \times L$$

$$= \frac{\pi}{4} \times 80^2 \times 200$$

$$V = 1507964.473 \text{ cm}^3$$

$$= 0.001 \times 1507964.473$$

$$\boxed{\delta V = 1507.96 \text{ cm}^3}$$

Prblm. no: 7

A Cylindrical Shell 90cm long 20cm internal diameter having thickness of metal as 8mm is filled with fluid at atmospheric pressure. If an additional 20 cm³ of fluid

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is pumped into the cylinder.

Find

(i) The pressure exerted by the fluid on the cylinder

(ii) The hoop stress induced.

Take $E = 2 \times 10^5 \ N/mm^2$ and $\mu = 0.2$

Given data

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length of cylinder $(L) = 90$ cm

Diameter of cylinder $(d) = 20$ cm

Thickness of cylinder $(t) = 8 \text{ mm} \Rightarrow 0.8$ cm

Volume of additional fluid $= 20 \ cm^3$

volume of cylinder $(V) = \frac{\pi}{4} \times d^2 \times L$

$$= \frac{\pi}{4} \times 20^2 \times 90$$

$$V = 28274.33 \ cm^3$$

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Increase in Volume (δV) = Volume of additional fluid

$$= 20 \ cm^3$$

$$E = 2 \times 10^5 \ N/mm^2$$

$$\mu = 0.2$$

Solution

(i) Pressure exerted by fluid on the cylinder

Volumetric strain

$$\frac{\delta V}{V} = 2\epsilon_c + \epsilon_L$$

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$$\frac{20}{28274.33} = 2\,\epsilon_c + \epsilon_L \rightarrow (1)$$

Circumferential Strain

$$\epsilon_c = \frac{Pd}{2Et}\left[1 - \frac{1}{2}\mu\right]$$

Longitudinal Strain

$$\epsilon_L = \frac{Pd}{2tE}\left[\frac{1}{2} - \mu\right]$$

Sub. these values in equtn (1), we get

$$\frac{20}{28274.33} = \frac{2Pd}{2tE}\left[1 - \frac{1}{2}\times\mu\right] + \frac{Pd}{2tE}\left[\frac{1}{2} - \mu\right]$$

$$= \frac{2P\times 20}{2\times 2\times 10^5 \times 0.8}\left[1 - \frac{1}{2}\times 0.3\right] + \frac{P\times 20}{0.8\times 2\times 10^5}\left[\frac{1}{2} - 0.3\right]$$

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$$= \frac{P}{8000}\times 0.85 + \frac{P}{8000}\times 0.20$$

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$$0.000707 = \frac{1.05P}{8000}$$

$$= \frac{0.000707\times 8000}{1.05}$$

$$\boxed{P = 5.386\ N/mm^2}$$

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Hoop Stress (σ_h)

$$\sigma_h = \frac{Pd}{2t}$$

$$= \frac{5.386 \times 20}{2 \times 0.8}$$

$$\boxed{\sigma_h = 67.32 \ N/mm^2}$$

Prblm.no: 8

A cylinderical vessel whose ends are closed by means of rigid flange plates, is made of steel plate 3mm thick. The length and the internal diameter of the vessel are 50 cm and 25 cm respectively. Determine the longitudinal and hoop stresses in the cylinderical shell due to an internal fluid pressure of 3 N/mm². Also calculate the increase in length, diameter and volume of the vessel. Take $E = 2 \times 10^5$ N/mm² and $\mu = 0.3$

Given data

Thickness $(t) = 3mm \Rightarrow 0.3 cm$

Length of the cylindrical vessel $(L) = 50$ cm

Internal diameter $(d) = 25$ cm

Internal fluid pressure $(P) = 3 \ N/mm^2$

Young's modulus $(E) = 2 \times 10^5 \ N/mm^2$

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Poisson's ratio $\mu = 0.3$

Let's

σ_h - Hoop stress

σ_L - Longitudinal stress

δd - Increase in diameter

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δL - Increase in Length

δV - Increase in Volume

Solution

Hoop Stress

$$\sigma_h = \frac{P \times d}{2t}$$

$$= \frac{3 \times 25}{2 \times 0.3}$$

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$$\boxed{\sigma_h = 125 \ N/mm^2}$$

Longitudinal stress

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$$\sigma_L = \frac{P \times d}{4t}$$

$$= \frac{3 \times 25}{4 \times 0.3}$$

$$\boxed{\sigma_L = 62.5 \ N/mm^2}$$

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For Circumferential Strain

$$\epsilon_c = \frac{\sigma_c}{E} - \mu \frac{\sigma_L}{E}$$

$$= \frac{1}{E}\left[\sigma_c - \mu \times \sigma_L\right]$$

$$= \frac{1}{2\times10^5}\left[125 - 0.3\times 62.5\right]$$

$$= \frac{1}{2\times10^5}(125 - 18.75)$$

$$\epsilon_c = 53.125\times10^{-5}$$

But Circumferential Strain

$$\epsilon_c = \frac{\delta d}{d}$$

Equating the two values of circumferential strain ϵ_c

$$\frac{\delta d}{d} = 53.125\times10^{-5}$$

$$\delta d = 53.125\times10^{-5}\times d$$

$$= 53.125\times10^{-5}\times 25$$

Increase in diameter $\Rightarrow$ $\boxed{\delta d = 0.0133 \text{ cm}}$

For Longitudinal Strain

$$\epsilon_L = \frac{\sigma_L}{E} - \mu\times\frac{\sigma_c}{E}$$

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$$= \frac{1}{E}\left[\sigma_L - \mu \times \sigma_h\right]$$

$$= \frac{1}{2\times10^5}\left[62.5 - 125 \times 0.3\right]$$

$$= \frac{1}{2\times10^5}\left[62.5 - 37.5\right]$$

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$$= \frac{25}{2\times10^5}$$

$$\boxed{\epsilon_L = 12.5 \times 10^{-5}}$$

Longitudinal Strain

$$\epsilon_L = \frac{\delta L}{L}$$

$$\delta L = \epsilon_L \times L$$

$$= 12.5\times10^{-5} \times 50$$

$\therefore$ Increase in length $\Rightarrow$ $\boxed{\delta L = 0.00625 \text{ cm}}$

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Volumetric Strain

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$$\frac{\delta V}{V} = 2\frac{\delta d}{d} + \frac{\delta L}{L}$$

$$= 2 \times 53.125\times10^{-5} + 12.5\times10^{-5}$$

$$= 106.25\times10^{-5} + 12.5\times10^{-5}$$

$$= 118.75\times10^{-5} \times V$$

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Increase in volume

$$\delta V = 118.75 \times 10^{-5} \times V$$

$$= 118.75 \times 10^{-5} \times \frac{\pi}{4} \times 25^2 \times 50$$

$$\boxed{\delta V = 29.13 \ cm^2}$$

Prblm.no: 9

A cylindrical vessel is 1.5m diameter and 4m long is closed at ends by rigid plates. It is subjected to an internal pressure of 3 N/mm². If the maximum principal stress is not to exceed 150 N/mm², find the thickness of the shell. Assume E = 2×10⁵ N/mm² and poisson's ratio = 0.25. Find the changes in diameter, length and volume of the shell.

Given data

Diameter (d) = 1.5m ⇒ 1500 mm

Length (L) = 4m ⇒ 4000 mm

Internal pressure (P) = 3 N/mm²

Max. principal stress = 150 N/mm²

Max. principal stress means the circumferential stress

∴ circumferential stress (σ_c) = 150 N/mm²

$E = 2 \times 10^5 \ N/mm^2$

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Poisson's ratio (μ) = 0.25

let,

t - Thickness of the shell

δd - change in diameter

δL - change in length

δV - change in volume

Solution

Circumferential stress (σ_c)

$$\sigma_c = \frac{P \times d}{2t}$$

$$t = \frac{P \times d}{2 \times \sigma_c}$$

$$= \frac{3 \times 1500}{2 \times 150}$$

$$\boxed{t = 15\,mm}$$

Change in diameter

$$\delta d = \frac{Pd^2}{2t \times E}\left(1 - \frac{1}{2} \times \mu\right)$$

$$= \frac{3 \times 1500}{2 \times 15 \times 2 \times 10^5}\left(1 - \frac{1}{2} \times 0.25\right)$$

$$\boxed{\delta d = 0.984\,mm}$$

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Change in length (δL)

$$\delta L = \frac{P \times d \times L}{2t \times E}\left(\frac{1}{2} - \mu\right)$$

$$= \frac{2 \times 1500 \times 4000}{2 \times 15 \times (2 \times 10^5)}\left(\frac{1}{2} - 0.25\right)$$

$$\boxed{\delta L = 0.75\,mm}$$

Change in Volume

$$\frac{\delta V}{V} = 2\frac{\delta d}{d} + \frac{\delta L}{L}$$

$$= 2 \times \frac{0.984}{1500} + \frac{0.75}{4000}$$

$$= 1.4995 \times 10^{-3} \times V$$

$$= 1.4995 \times 10^{-3} \times \left(\frac{\pi}{4} \times 1500^2 \times 4000\right)$$

$$= 10599340.91\ mm^3$$

Prblm. no: 10

A closed cylindrical vessel made of steel plates 4 mm thick with plane ends, carries fluid under a pressure of 3 N/mm² . The dia of cylinder is 25 cm and length is 75 cm, calculate the longitudinal and hoop stresses in the cylinder wall and determine the change in diameter

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length and Volume of the cylinder. Take $E = 2.1 \times 10^5\ N/mm^2$ and $\mu = 0.286$

Given data

Thickness $(t) = 4mm$

Fluid pressure $(P) = 2\ N/mm^2$

Diameter $(d) = 25\ cm \Rightarrow 25 \times 10\ mm \Rightarrow 250\ mm$

length $(L) = 75\ cm \Rightarrow 750\ mm$

$(E) = 2.1 \times 10^5\ N/mm^2$

Poisson's ratio $(\mu) = 0.286$

let,

σ_1 = Hoop stress

σ_2 = Longitudinal stress

δd = change in diameter

δL = change in length

δV = change in volume

Solution

(i) Longitudinal stress

$$\sigma_L = \frac{P \times d}{4 \times t}$$

$$= \frac{2 \times 250}{4 \times 4}$$

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$$\sigma_L = 46.875 \ N/mm^2$$

(ii) Hoop stress

$$\sigma_h = \frac{P \times d}{2 \times t}$$

$$= \frac{3 \times 250}{2 \times 4}$$

$$\boxed{\sigma_h = 93.75 \ N/mm^2}$$

(iii) change in diameter

$$\delta d = \frac{P \times d^2}{2t \times E}\left(1 - \frac{1}{2} \times \mu\right)$$

$$= \frac{3 \times 250^2}{2 \times 4 \times 2.1 \times 10^5}\left(1 - \frac{1}{2} \times 0.286\right)$$

$$\boxed{\delta d = 0.0956 \ mm}$$

(iv) change in length

$$\delta L = \frac{P \times d \times L}{2E \times t}\left(\frac{1}{2} - \mu\right)$$

$$= \frac{3 \times 250 \times 750}{2 \times 2.1 \times 10^5 \times 4}\left(\frac{1}{2} - 0.286\right)$$

$$\boxed{\delta L = 0.0716 mm}$$

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(v) change in volume

$$\frac{\delta V}{V} = 2\frac{\delta d}{d} + \frac{\delta l}{l}$$

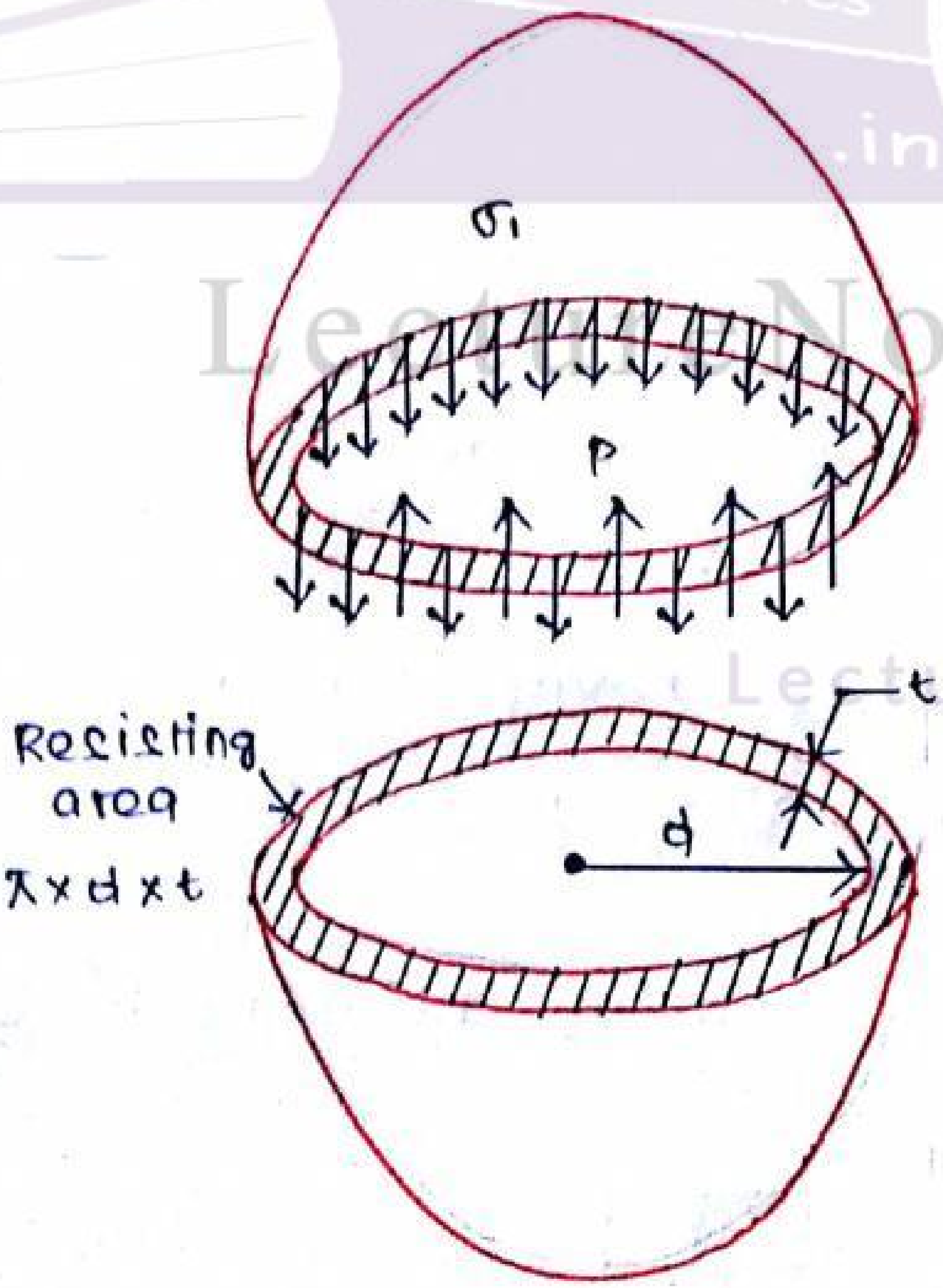
$$= 2 \times \frac{0.0956}{250} + \frac{0.0716}{750}$$

$$= 8.6026 \times 10^{-4} \times V$$

$$= 8.6026 \times 10^{-4} \times \left(\frac{\pi}{4} \times 250^2 \times 750\right)$$

$$\boxed{\delta V = 31671.18\ mm^3}$$

THIN SPHERICAL SHELLS



Thin spherical shell of internal diameter 'd' and thickness 't' and subjected to an internal fluid pressure 'p'.

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* The Fluid Inside the shell has a tendency to split the shell into two hemisphere along x-x axis

The Force (P) which has a tendency to split the shell

$$= p \times \frac{\pi}{4} d^2$$

The area resisting this force $= \pi \times d \times t$

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∴ Hoop or circumferential stress (σ_c) induced in the material of the shell is given by

$$\sigma_1 = \frac{\text{Force 'p'}}{\text{Area resisting the force 'p'}}$$

$$= \frac{p \times \frac{\pi}{4} d^2}{\pi \cdot d \cdot t}$$

$$\boxed{\sigma_1 = \frac{p \cdot d}{4t}}$$

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The stress σ_1 is tensile in nature.

The fluid inside the shell is also having tendency to split the shell into two hemispheres along y-y axis.

Tensile hoop stress will also be equal to $\frac{p \cdot d}{4t}$

$$\boxed{\sigma_2 = \frac{p \times d}{4t}}$$

The stress σ_2 will be at right angles to σ_1

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Change in dimension of a thin spherical shell due to an internal pressure

The stresses σ_1 and σ_2 at any point are equal

$$\sigma_1 = \frac{Pd}{4t} \quad ; \quad \sigma_2 = \frac{Pd}{4t}$$

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There is no shear stress at any point in the shell

$$\text{Maximum shear stress} = \frac{\sigma_1 - \sigma_2}{2}$$

$$= \frac{P \times d}{4t} - \frac{P \times d}{4t}$$

$$= 0$$

The stresses σ_1 and σ_2 are acting at right angle to each others

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Strain in any one direction

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$$e = \frac{\sigma_1}{E} - \frac{\mu \times \sigma_1}{E}$$

$$= \frac{\sigma_1}{E}(1 - \mu)$$

$$e = \frac{Pd}{4tE}(1 - \mu)$$

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$$e_c = \frac{\delta d}{d}$$

$$\frac{\delta d}{d} = \frac{P \times d}{4tE}(1-\mu) \rightarrow (1)$$

Volumetric Strain

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$$\text{Volumetric Strain} = \frac{\text{Change in volume}}{\text{Original volume}}$$

$$= \frac{dv}{v}$$

Taking differential of the above equation

Sphere (volume)

$$V = \frac{4}{3}\pi r^3$$

(or)

$$V = \frac{\pi}{6} d^3$$

$$dv = \frac{\pi}{6} \times 3d^2 \times d(d)$$

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$$\frac{dv}{v} = \frac{\cancel{\frac{\pi}{6}} \times 3\cancel{d^2} \times d(d)}{\cancel{\frac{\pi}{6}} \times \cancel{d^3}}$$

$$= 3\frac{d(d)}{d} \rightarrow (2)$$

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From equation (1)

$$\frac{\delta d}{d} = \frac{P \times d}{4tE}(1-\mu)$$

Sub. this value in equation (2)

$$\boxed{\frac{dv}{v} = \frac{3 \times P \times d}{4tE}(1-\mu)}$$

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Prblm.no: 9

A Spherical Shell of Internal diameter 0·9m and of thickness 10mm is subjected to an internal pressure of 1·4 N/mm². Determine the Increase in diameter and Increase in volume. Take $E = 2\times10^5$ N/mm² and $\mu = \frac{1}{3}$

Given data

Internal diameter $(d) = 0\cdot9\text{m} \Rightarrow 0\cdot9\times10^3$ mm

Thickness of shell $(t) = 10$ mm

Fluid pressure $P = 1\cdot4$ N/mm²

$E = 2\times10^5$ N/mm²

$\mu = \frac{1}{3}$

Solution

$$\frac{\delta d}{d} = \frac{P\times d}{4tE}(1-\mu)$$

$$= \frac{1\cdot4\times(0\cdot9\times10^3)}{4\times10\times(2\times10^5)}\left(1-\frac{1}{3}\right)$$

$$= 105\times10^{-6}\times\overset{(d)}{0\cdot9\times10^3}$$

$$= 94\cdot5\times10^{-3}\text{ mm}$$

Increase in diameter

$$\boxed{\delta d = 0\cdot0945\text{ mm}}$$

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$$\text{Volumetric strain} = 3 \times \frac{\delta d}{d}$$

$$= 3 \times 105 \times 10^{-6}$$

$$= 315 \times 10^{-6} \times V$$

$$= 315 \times 10^{-6} \times \left(\frac{\pi}{6} d^3\right)$$

$$= 315 \times 10^{-6} \times \frac{\pi}{6} \times (0.9 \times 10^3)^3$$

$$\boxed{\delta V = 120228.5 \text{ mm}^3}$$

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THICK CYLINDERS

Introduction

* The Hoop stress and Longitudinal stress are constant over the thickness and the radial stress is small and can be neglected.

* If the ratio of thickness to internal diameter is more than $\frac{1}{20}$, then cylindrical shell is known as thick cylinders

* The Hoop stress in case of a thick cylinder will not be uniform across the thickness.

* Actually the hoop stress will vary from a maximum value at the inner circumference to a minimum value at the outer circumference.

LAME'S THEOREM - THICK CYLINDERS

Thick cylinder subjected to an internal fluid pressure

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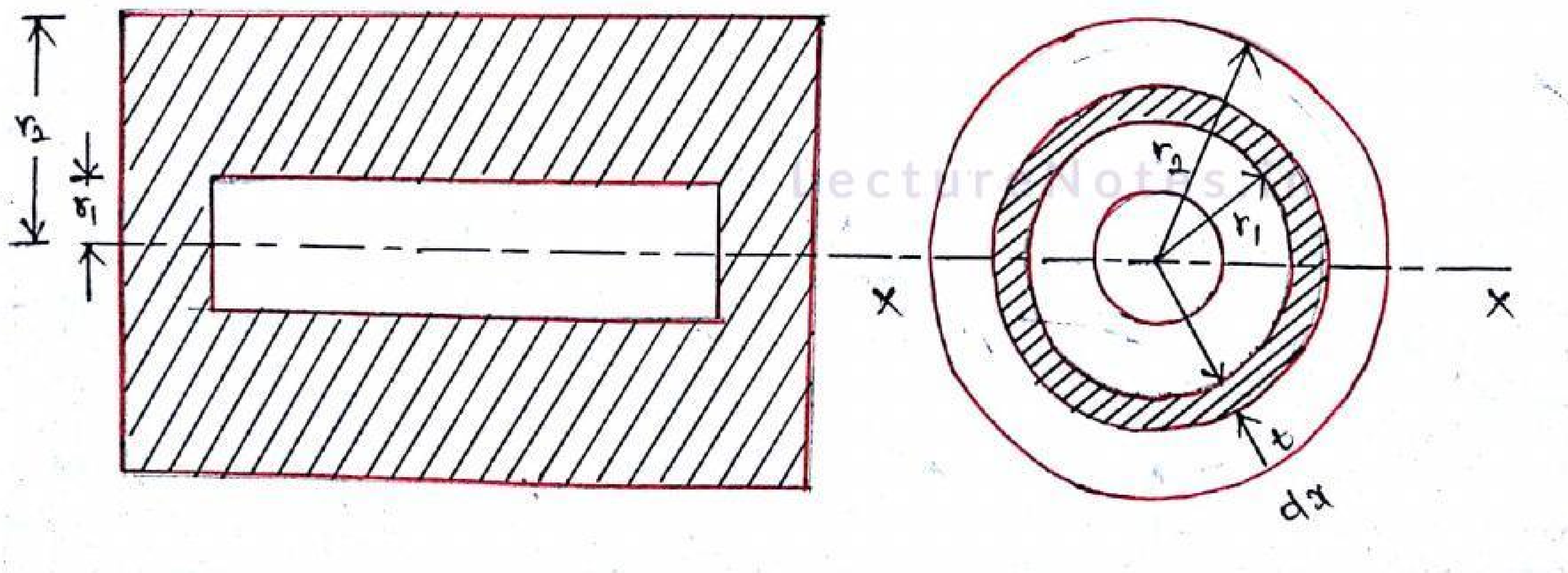

Let,

r - External radius of the cylinder

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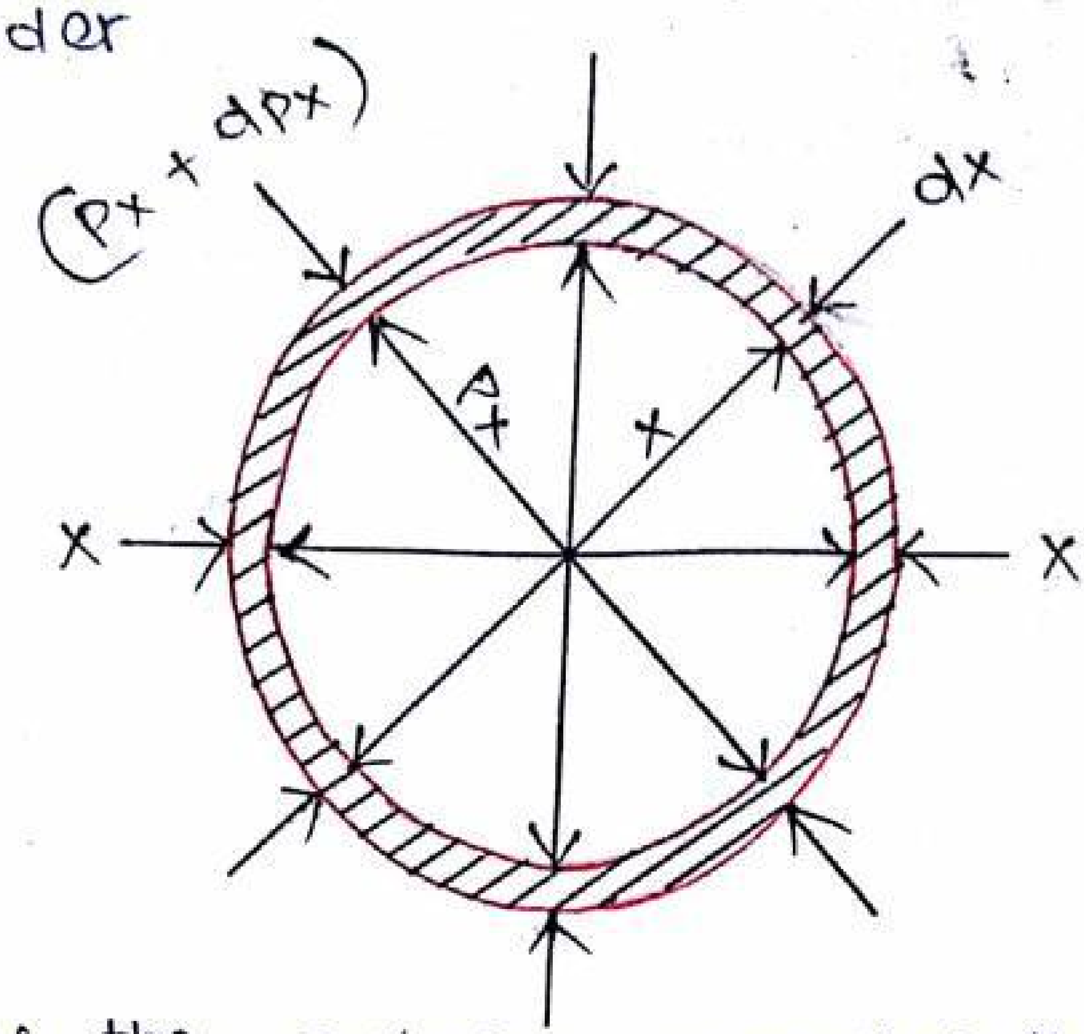
r - Internal radius of the cylinder

L - length of cylinder



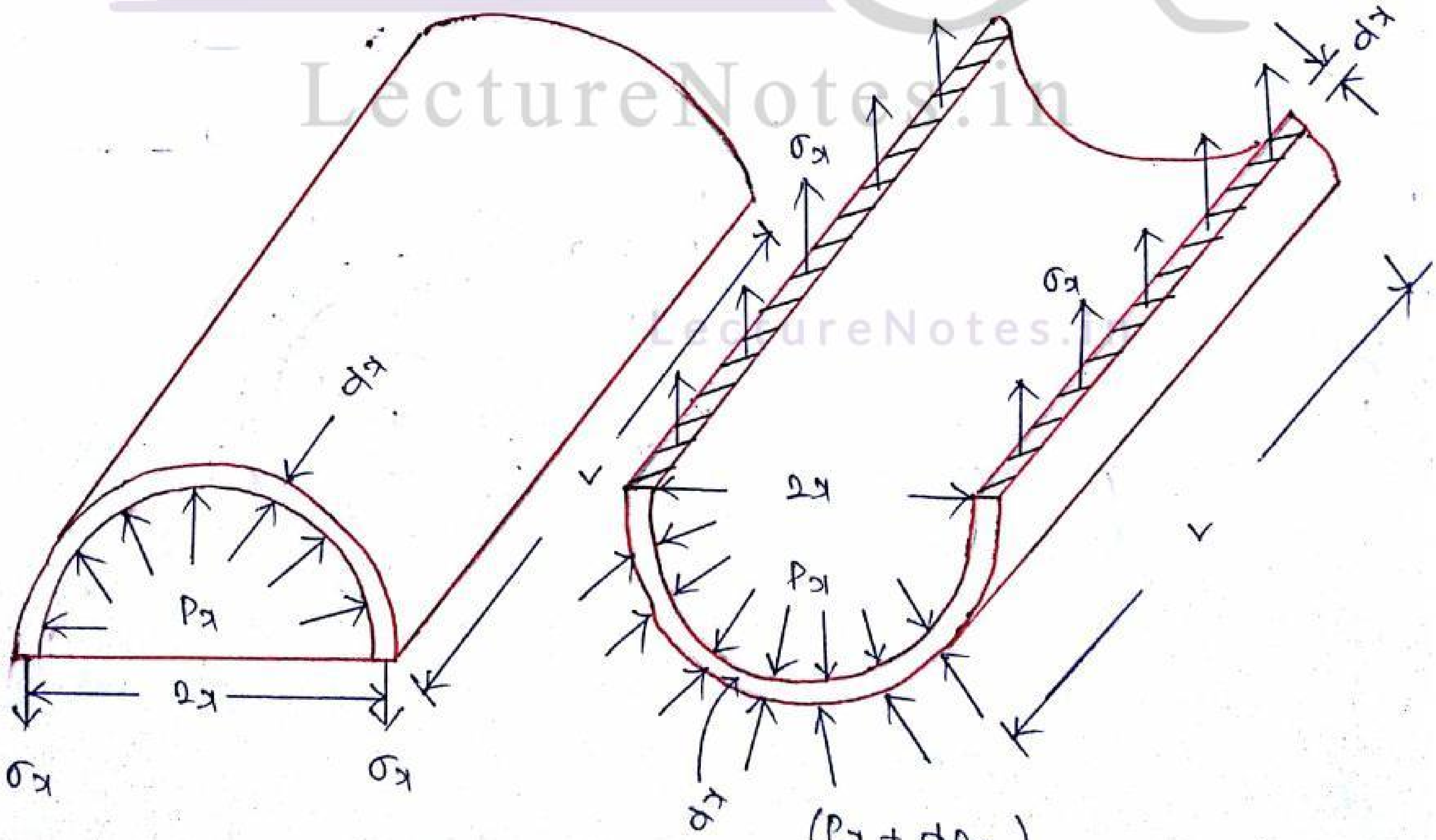
Consider an elementary ring of the cylinder of radius x and thickness dx

Let,

p_x - Radial pressure on the Inner Surface of the ring

$p_x + dp_x$ - Radial pressure on the outer Surface of the ring

σ_x - Hoop Stress induced in the ring



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The value of 'a' and 'b' are sub. in the hoop stress

Hoop Stress

At any radius 'r'

$$\sigma_r = \frac{b}{r^2} + a$$

$$= \frac{576000}{r^2} + 6.4$$

At $r = 200$ mm, $\sigma_{200} = \frac{576000}{200^2} + 6.4$

$$= 14.4 + 6.4$$

$$= 20.8 \text{ N/mm}^2$$

At $r = 200$ mm, $\sigma_{200} = \frac{576000}{800^2} + 6.4$

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$$= 6.4 + 6.4$$

$$= 12.8 \text{ N/mm}^2$$

Radial pressure and hoop stress distribution diagram

* Radial pressure and hoop stress distribution across the section.
* AB is taken a horizontal line AC = 8 N/mm²
* The variation between B and C is parabolic
* The curve BC show the variation of radial pressure. across AB
* The curve DE which is also parabolic

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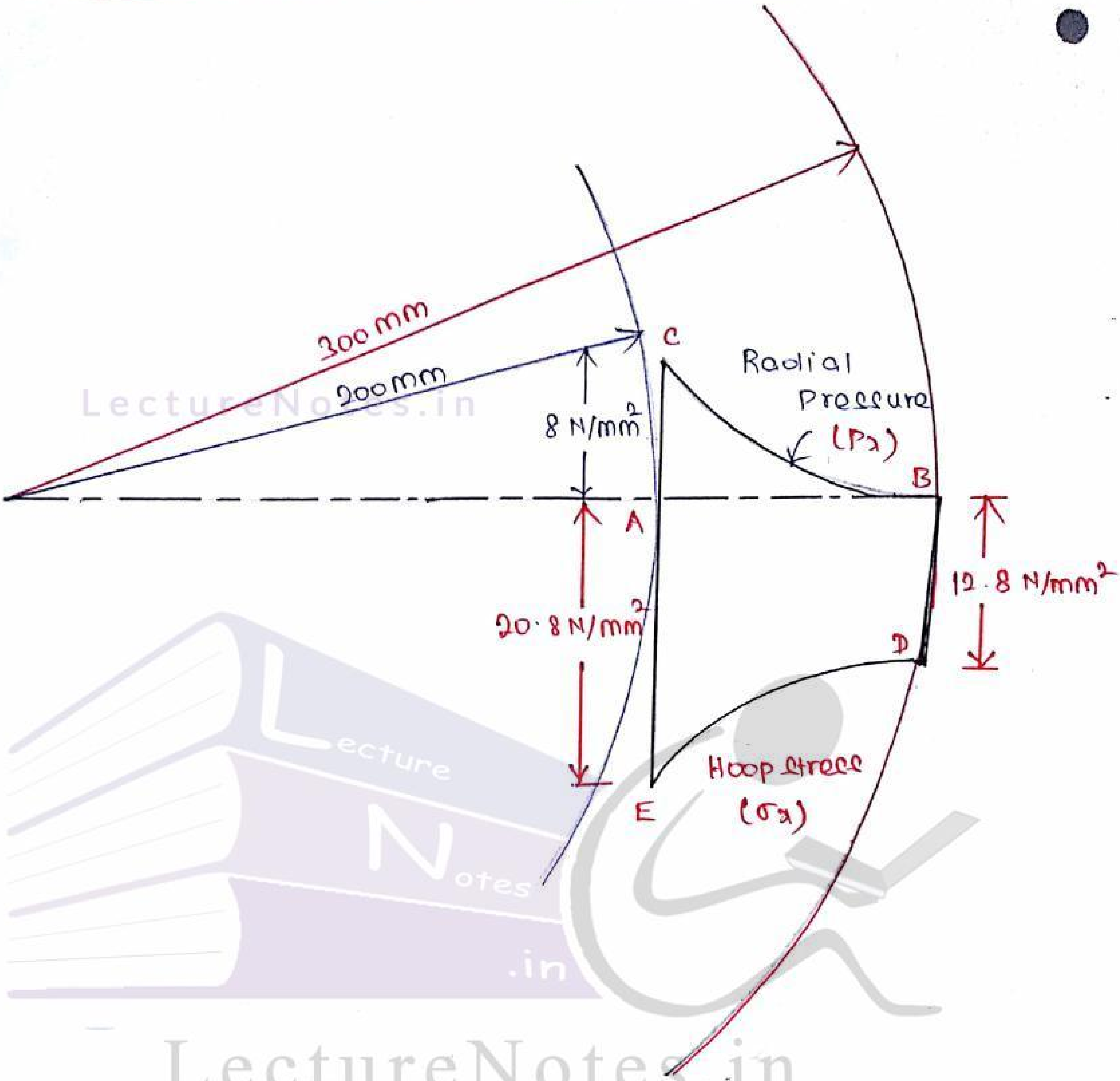
HOOP STRESS AND RADIAL PRESSEURE DISTRIBUTION



* The Radial pressure is compressive

* The hoop stress in Tensile.

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Thickness $(t) = 100\,mm$

$\therefore$ External dia $= D + 2 \times t$

$= 400 + 2 \times 100$

$= 600\,mm$

External radius $(r_2) = \frac{600}{2}$

$= 300\,mm$

fluid pressure $p_0 = 8\,N/mm^2$

At

$x = r_1$, $p_x = p_0 = 8\,N/mm^2$

Solution

Radial pressure (p_x)

$$p_x = \frac{b}{x^2} - a \rightarrow (1)$$

Apply the Boundry condition

* At $x = r_1 = 200\,mm$, $p_x = 8\,N/mm^2$
* At $x = r_2 = 300\,mm$, $p_x = 0$

Sub. these boundary condition in equtn (1)

$$8 = \frac{b}{200^2} - a$$

$$= \frac{b}{40000} - a \rightarrow (2)$$

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$$0 = \frac{b}{300^2} - a$$

$$0 = \frac{b}{90000} - a \rightarrow (2)$$

Subtracting equation (2) from equation (1)

$$8 = \frac{b}{40000} - a - \left(\frac{b}{90000} - a\right)$$

$$= \frac{b}{40000} - \cancel{a} - \frac{b}{90000} + \cancel{a}$$

$$= \frac{b}{40000} - \frac{b}{90000}$$

$$= \frac{9b - 4b}{360000}$$

$$= \frac{5b}{360000}$$

$$b = \frac{360000 \times 8}{5}$$

$$b = 576000$$

Sub this value in equn (2)

$$0 = \frac{576000}{90000} - a$$

$$a = \frac{576000}{90000}$$

$$a = 6.4$$

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$$\sigma_r = p_r + 2a \rightarrow (4)$$

Equating two values of σ_r given by equation (3) and (4)

$$p_r + 2a = -p_r - r \cdot \frac{dp_r}{dr}$$

$$r \cdot \frac{dp_r}{dr} = -p_r - p_r - 2a$$

$$r \cdot \frac{dp_r}{dr} = -2p_r - 2a$$

$$\frac{dp_r}{dr} = -\frac{2p_r}{r} - \frac{2a}{r}$$

$$= \frac{-2(p_r + a)}{r}$$

$$\frac{dp_r}{(p_r + a)} = -\frac{2\,dr}{r}$$

Integrating the above equation

$$\log_e (p_r + a) = -2\log_e r + \log_e b$$

Where,

$\log_e b$ is a constant of Integration

$$\log_e (p_r + a) = -\log_e r^2 + \log_e b$$

$$= \log_e \frac{b}{r^2}$$

$$p_r + a = \frac{b}{r^2}$$

$\Rightarrow$ Radial pressure $\Rightarrow$ $\boxed{p_r = \frac{b}{r^2} - a}$

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Substituting the values of P_r in equation (ii)

$$\sigma_r = \frac{b}{r^2} - a + 2a$$

Hoop stress $\Rightarrow$ $\boxed{\sigma_r = \frac{b}{r^2} + a}$

The Radial pressure (P_r)

The Hoop stress (σ_r)

At any point radius r.

$\therefore$ These two equation are called Lame's equation

a & b - (Constant) From boundary conditions

* At $r = r_1$, $P_r = P_0$ (Pressure of fluid inside the cylinder)
* At $r = r_2$, $P_r = 0$ (Atmosphere pressure)

Problm. no: 10

Determine the maximum and minimum hoop stress across the section of a pipe of 400 mm internal diameter and 100mm thick, when the pipe contains a fluid at a pressure of 8 N/mm². Also sketch the radial pressure distribution and Hoop stress distribution across the section

Given data

Internal dia = 400 mm

Internal radius (r_1) = $\frac{400}{2} \Rightarrow 200$ mm

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Bursting force

$$= p_x(2x \cdot L) - (p_x + dp_x) \times 2(x + dx) \cdot L$$

Resisting force

$$= \sigma_x \times 2dx \cdot L$$

For safe design

Resisting force = Bursting force

$$\sigma_x \times 2dx \cdot L = p_x(2x \cdot L) - (p_x + dp_x) \times 2(x + dx) \cdot L$$

$$\sigma_x \cdot dx \cdot L = p_x \cdot \frac{2}{2} x \times L - p_x \cdot x + p_x \cdot dx \times \frac{2}{2} dp_x\, x + dp_x \cdot dx \cdot L$$

$$\sigma_x \cdot dx \cdot \cancel{L} = \cancel{L}(p_x \cdot \cancel{x}) - [p_x \cdot \cancel{x} + p_x \cdot dx + dp_x \cdot x + dp_x \cdot dx]$$

$$\sigma_x \cdot dx = -p_x \cdot dx - dp_x \cdot x + dp_x \cdot dx$$

$$\sigma_x = -p_x \frac{dx}{dx} - \frac{x \cdot dp_x}{dx}$$ $(dp_x \cdot dx)$ Negligible which is small Quantity

$$\sigma_x = -p_x - \frac{x \cdot dp_x}{dx} \rightarrow (1)$$

* The longitudinal strain at any point in the section is constant and is independent of the radius

* This means cross section remain plane after straining and this is true for section remote from any end fixing.

* Longitudinal strain is constant, hence longitudinal stress will also be constant.

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σ_2 - longitudinal stress

Hence at any point at a distance x from the centre, three principal stresses are acting

They are :

* Radial compressive stress (P_x)
* Hoop (or circumferential) tensile stress (σ_x)
* longitudinal tensile stress (σ_2)

The longitudinal strain e_L at this point

$$e_L = \frac{\sigma_2}{E} - \frac{\mu \sigma_x}{E} + \frac{\mu P_x}{E}$$

But longitudinal strain is constant

$$\frac{\sigma_2}{E} - \frac{\mu \sigma_x}{E} + \frac{\mu P_x}{E} = \text{constant}$$

σ_2 = Constant

E, μ = Material of the cylinder (Constant)

$\sigma_x - P_x$ = constant

= $2a$

where 'a' is constant

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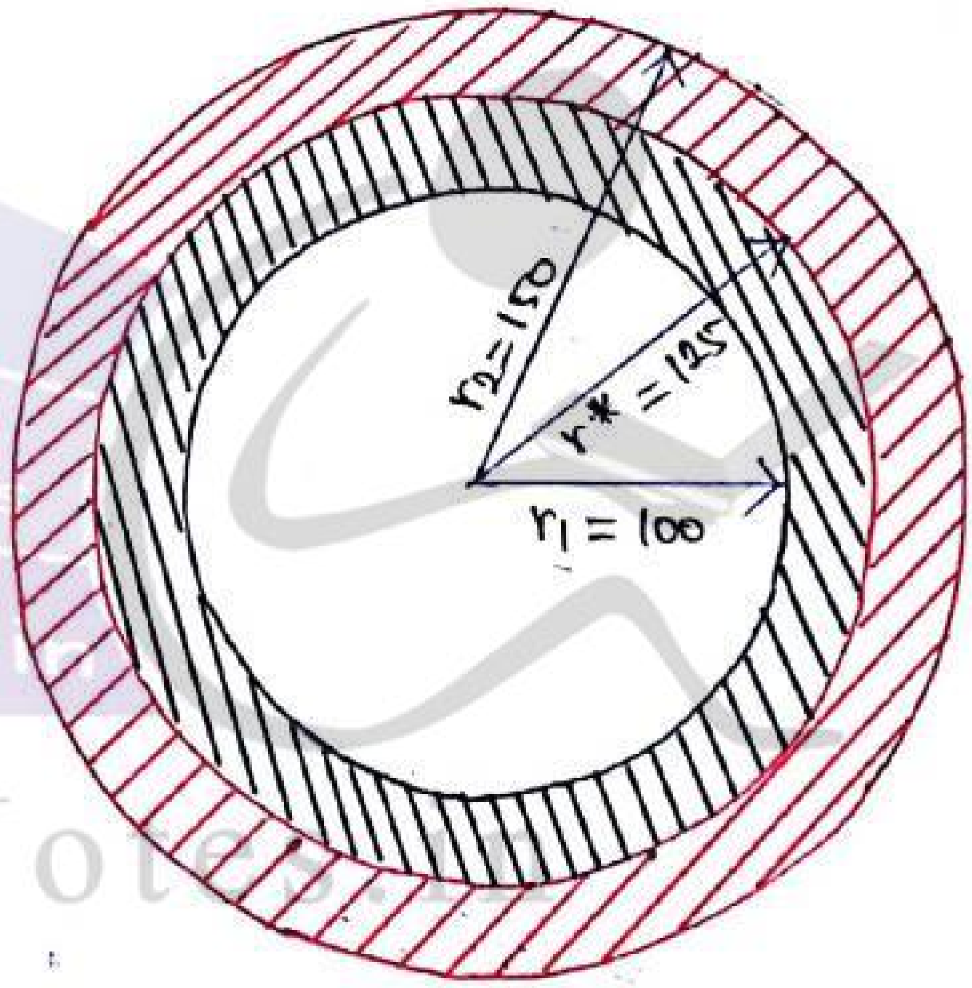
COMPOUND THICK CYLINDERS

Problem on Compound Thick cylinder

A Compound cylinder is made by shrinking a cylinder of external diameter 300mm and internal diameter 250mm over another cylinder of external diameter 250 mm and internal diameter 200mm. The radial pressure at the junction after shrinking is 8 N/mm². Find the final stresses set up across the section when the compound cylinder is subjected to an internal fluid pressure of 84.5 N/mm²

Given data



For Outer Cylinder

External diameter = 300mm

External radius

$$r_2 = \frac{300}{2} = 150\,mm$$

Internal diameter = 250mm

Radius at the Junction

$$r^* = \frac{250}{2} = 125\,mm$$

For Inner Cylinder

Internal diameter = 200mm

Internal radius

$$r_1 = \frac{200}{2} = 100\,mm$$

Radial pressure due to shrinking at the junction

$$p^* = 8\ N/mm^2$$

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Fluid pressure in the compound cylinder

$$P = 84.5 \text{ N/mm}^2$$

(i) Stresses due to shrinking in the outer and Inner cylinders before the fluid pressure is admitted.

(a) Lame's equation for outer cylinders are :

$$P_x = \frac{b_1}{x^2} - a_1 \rightarrow (1)$$

$$\sigma_x = \frac{b_1}{x^2} + a_1 \rightarrow (2)$$

$x = 150 \text{ mm}$, $P_x = 0$

Sub. these values in equation (1)

$$0 = \frac{b_1}{150^2} - a_1$$

$$= \frac{b_1}{22500} - a_1 \rightarrow (3)$$

$x = r^* = 125 \text{ mm}$, $P_x = p^* = 8 \text{ N/mm}^2$

Sub. these values in equation (1)

$$8 = \frac{b_1}{125^2} - a_1$$

$$= \frac{b_1}{15625} - a_1 \rightarrow (4)$$

Subtracting equation (3) from equation (4) we get

$$0 = \frac{b_1}{22500} - a_1 - 8\left(\frac{b_1}{15625} - a_1\right)$$

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$$0 = \frac{b_1}{22500} - a_1 - 8 \frac{b_1}{15625} + a_1$$

$$8 = -\frac{b_1}{22500} + \frac{b_1}{15625}$$

$$= \frac{-15625 b_1 + 22500 b_1}{22500 \times 15625}$$

$$= \frac{(-15625 + 22500) b_1}{22500 \times 15625}$$

$$b_1 = \frac{8 \times 22500 \times 15625}{(-15625 + 22500)}$$

$$\boxed{b_1 = 409090.9}$$

Sub. the value of b_1 in equtn (3)

$$0 = \frac{409090.9}{22500} - a_1$$

$$a_1 = \frac{409090.9}{22500}$$

$$\boxed{a_1 = 18.18}$$

Sub the values of a_1 and b_1 in equtn (2)

$$\sigma_r = \frac{409090.9}{r^2} + 18.18$$

$r = 150\,mm$; $r = 125\,mm$

Hoop stress at the outer and inner surface of the outer cylinder

$$\sigma_{150} = \frac{409090.9}{150^2} + 18.18$$

$$\boxed{\sigma_{150} = 36.36\ N/mm^2} \text{ (Tensile)}$$

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$$\sigma_{125} = \frac{407090.9}{125^2} + 18.18$$

$$\sigma_{125} = 44.36 \ N/mm^2 \text{ (tensile)}$$

(b) Lame's equation for the inner cylinder

$$P_x = \frac{b_2}{x^2} - a_2 \rightarrow (5)$$

$$\sigma_x = \frac{b_2}{x^2} + a_2 \rightarrow (6)$$

$x = r_1 = 100mm$, $P_x = 0$ (There is no Fluid pressure)

Sub. these value in equtn (5)

$$0 = \frac{b_2}{100^2} - a_2$$

$$= \frac{b_2}{10000} - a_2 \rightarrow (7)$$

$x = r^* = 125mm$, $P_x = p^* = 8 \ N/mm^2$

Sub these value in equtn (5)

$$8 = \frac{b_2}{125^2} - a_2$$

$$= \frac{b_2}{15625} - a_2 \rightarrow (8)$$

$$0 = \frac{b_2}{10000} - a_2 - 8\left(\frac{b_2}{15625} - a_2\right)$$

$$= \frac{b_2}{10000} - a_2 - 8\frac{b_2}{15625} + a_2$$

Subtracting equation (7) from equtn (8) →

$$8 = \frac{b_2}{15625} - \frac{b_2}{10000} \rightarrow \frac{10000 b_2 - 15625 b_2}{15625 \times 10000}$$

$$= \frac{b_2(10000 - 15625)}{15625 \times 10000}$$

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$$= \frac{-5625\, b_2}{15625 \times 10000}$$

$$b_2 = -\frac{8 \times 15625 \times 10000}{5625}$$

$$\boxed{b_2 = -222222.2}$$

Sub. the value of b_2 in equtn (7)

$$0 = -\frac{222222.2}{10000} - a_2$$

$$\boxed{a_2 = -22.22}$$

Sub. the values of a_2 and b_2 in equtn (6)

$$\sigma_x = -\frac{222222.2}{x^2} - 22.22$$

Hoop stress for the Inner cylinder

$x = 125$, $x = 100\,mm$

$$\sigma_{125} = -\frac{222222.2}{125^2} - 22.2$$

$$= -14.22 - 22.22$$

$$\boxed{\sigma_{125} = -36.44\ N/mm^2}$$ (compressive)

$$\sigma_{100} = -\frac{222222.2}{100^2} - 22.22$$

$$= -22.22 - 22.22$$

$$\boxed{\sigma_{100} = -44.44\ N/mm^2}$$ (compressive)

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(ii) Stresses due to Fluid pressure alone

- When the Fluid under pressure is admitted Inside the Compound Cylinder,

$$P_x = \frac{B}{x^2} - A \rightarrow (9)$$

$$\sigma_x = \frac{B}{x^2} + A \rightarrow (10)$$

Where A and B are constant

$x = 100mm$, $P_x = P = 84.5\ N/mm^2$

Sub the values in equtn (9)

$$84.5 = \frac{B}{100^2} - A$$

$$= \frac{B}{10000} - A \rightarrow (11)$$

$x = 150mm$, $P_x = 0$

Sub these values in equtn (9)

$$0 = \frac{B}{150^2} - A$$

$$= \frac{B}{22500} - A \rightarrow (12)$$

Subtracting equtn (12) from equtn (11)

$$84.5 = \frac{B}{10000} - A - \left(\frac{B}{22500} - A\right)$$

$$= \frac{B}{10000} - \cancel{A} - \frac{B}{22500} + \cancel{A}$$

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$$84.5 = \frac{B}{10000} - \frac{B}{22500}$$

$$= \frac{22500B - 10000B}{10000 \times 22500}$$

$$= \frac{(22500-10000)B}{10000 \times 22500}$$

$$= \frac{12500 \times B}{10000 \times 22500}$$

$$B = \frac{84.5 \times 10000 \times 22500}{12500}$$

$$\boxed{B = 1521000}$$

Sub this value in equtn (12)

$$0 = \frac{1521000}{22500} - A$$

$$A = \frac{1521000}{22500}$$

$$\boxed{A = 67.6}$$

Sub the values of A and B in equtn (10)

$$\sigma_x = \frac{1521000}{x^2} + 67.6$$

Hence the Hoop stress due to Internal fluid pressure alone

$$\sigma_{100} = \frac{1521000}{100^2} + 67.6$$

$$\boxed{\sigma_{100} = 219.7 \ N/mm^2}$$ (Tensile)

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$$\sigma_{125} = \frac{1521000}{125^2} + 67.6$$

$$= 97.344 + 67.6$$

$$\boxed{\sigma_{125} = 164.94\ N/mm^2}$$

$$\sigma_{150} = \frac{1521000}{150^2} + 67.6$$

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$$= 67.6 + 67.6$$

$$\boxed{\sigma_{150} = 135.2\ N/mm^2}$$

The Resultant Stress will be the algebraic sum of the Initial Stresses due to Shrinkage and those due to Internal fluid pressure.

Inner cylinder

$F_{100} = \sigma_{100}$ Due to Shrinkage + σ_{100} Due to Internal Fluid pressure

$= -44.44 + 219.7$

$\boxed{F_{100} = 175.26\ N/mm^2}$ (Tensile)

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$F_{125} = \sigma_{125}$ Due to Shrinkage + σ_{125} Due to Internal Fluid pressure

$= -36.44 + 164.94$

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$\boxed{F_{125} = 128.5\ N/mm^2}$ (Tensile)

Outer Cylinder

$F_{125} = \sigma_{125}$ Due to Shrinkage + σ_{125} Due to Internal Fluid pressure

$= 44.36 + 164.94$

$\boxed{F_{125} = 209.3\ N/mm^2}$ (Tensile)

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