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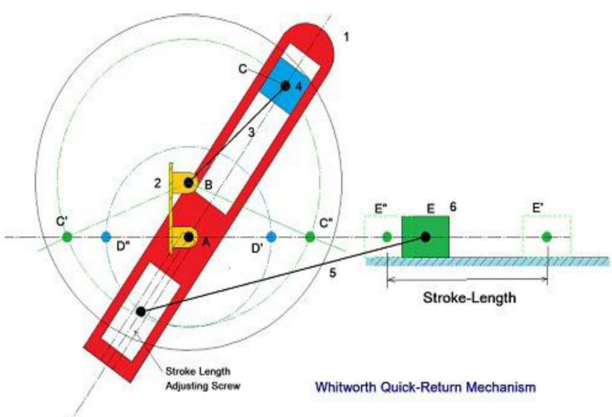
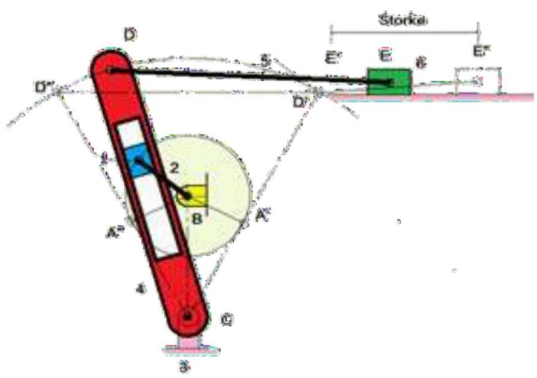
Machines and Mechanisms



Whitworth Quick-Return Mechanism



Crank-Rocker Quick Return Mechanism for Shaping Machine

Course Contents

1.1 Machine and Mechanism:

- **Mechanism:**

– If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others, it is known as a *mechanism*.

- **Machine:**

– A *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work.

- **Analysis:**

– *Analysis* is the study of motions and forces concerning different parts of an existing mechanism.

- **Synthesis:**

– *Synthesis* involves the design of its different parts.

1.2 Types of constrained motion:

1.2.1 Completely constrained motion:

– When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion.

– For example, the piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (*i.e.* it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank.

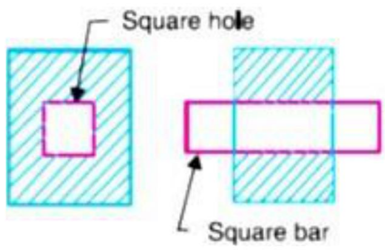


Fig. 1.1

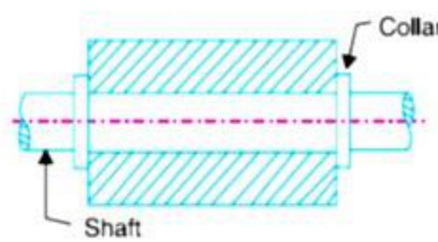


fig. 1.2

– The motion of a square bar in a square hole, as shown in Fig. 1.1, and the motion of a shaft with collars at each end in a circular hole, as shown in Fig. 1.2, are also examples of completely constrained motion.

1.2.2 Incompletely constrained motion:

– When the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained motion. The change in the direction of impressed force may alter the direction of relative motion between the pair. A circular bar or shaft in a circular hole, as shown in Fig. 1.3, is an example of an incompletely constrained motion as it may either rotate or slide in a hole. These both motions have no relationship with the other.

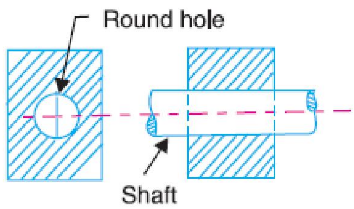


Fig. 1.3

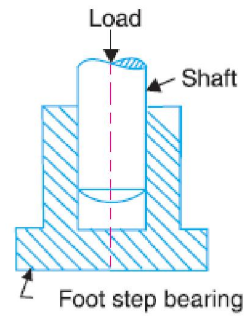


FIG. 1.4

1.2.3 Successfully constrained motion:

- When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion. Consider a shaft in a foot-step bearing as shown in Fig. 1.4.
- The shaft may rotate in a bearing or it may move upwards. This is a case of incompletely constrained motion. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is said to be successfully constrained motion. The motion of an I.C. engine

1.3 Types of Links:

- A mechanism is made of a number of resistant bodies out of which some may have motions relative to the others. A resistant body or a group of resistant bodies with rigid connections preventing their relative movements is known as a link.
- A link may also define as a member or a combination of members of a mechanism, connecting other members and having motion relative to them.
- Links may be classified into binary, ternary and quaternary.

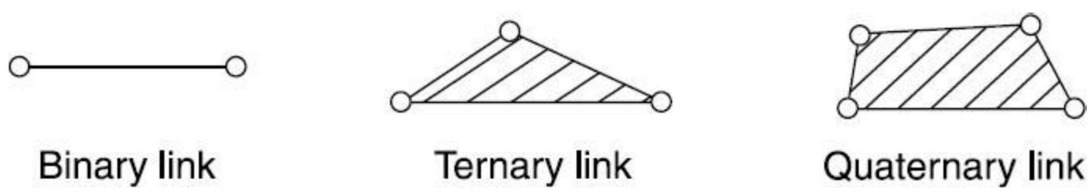


FIG. 1.4 Types of link

1.4 Kinematic Pair:

- When two kinematic links are connected in such a way that their motion is either completely or successfully constrained, these two links are said to form a kinematic pair.
- Kinematic pairs can be classified according to:

1.4.1 Kinematic pairs according to nature of contact:

a. Lower Pair:

- A pair of links having surfaced or area contact between the members is known as a lower pair. The contact surfaces of two links are similar.
- Examples: Nut turning on a screw, shaft rotating in a bearing.

b. Higher Pair:

- When a pair has a point or line contact between the links, it is known as a higher pair. The contact surfaces of two links are similar.
- Example: Wheel rolling on a surface, Cam and Follower pair etc.

1.4.2 Kinematic pairs according to nature of contact:

a. Closed Pair:

- When the elements of a pair are held together mechanically, it is known as a closed pair. The two elements are geometrically identical; one is solid and full and the other is hollow or open. The latter not only envelops the former but also encloses it. The contact between the two can be broken only by destruction of at least one of the members.

b. Unclosed Pair:

- When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair. In this, the links are not held together mechanically, e.g. cam and follower pair.

1.4.3 Kinematic pairs according to Nature of Relative Motion:

a. Sliding pair:

- When two links have a sliding motion relative to another; the kinematic pair is known as sliding pair.

b. Turning pair:

- When one link is revolve or turn with respect to the axis of first link, the kinematic pair formed by two links is known as turning pair.

c. Rolling pair:

- When the links of a pair have a rolling motion relative to each other, they form a rolling pair.

d. Screw pair:

- If two mating links have a turning as well as sliding motion between them, they form a screw pair.

e. Spherical pair:

- When one link in the form of sphere turns inside a fixed link, it is a spherical pair.

1.5 Types of Joint:

– The usual types of joints in a chain are:
  - Binary Joint
  - Ternary Joint
  - Quaternary Joint

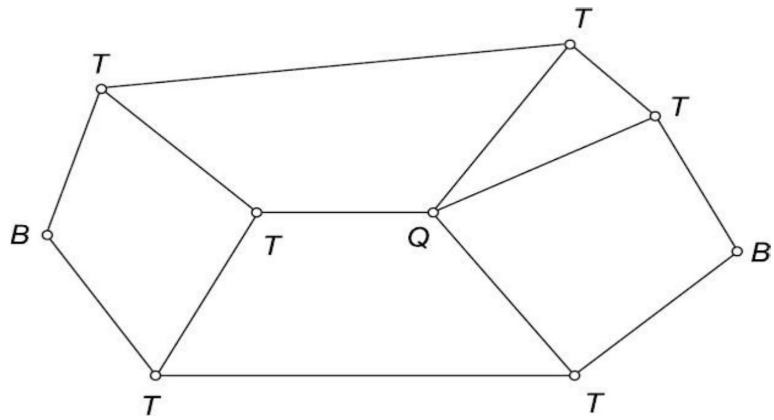


Fig1.5. Types of joint

a. **Binary Joint:**
   - If two links are joined at the same connection, it is called a binary joint. For example, in fig. at joint B

b. **Ternary Joint:**
   - If three links joined at a connection, it is known as a ternary link. For example point T in fig.

c. **Quaternary Joint:**
   - If four links joined at a connection, it is known as a quaternary link. For example point Q in fig.

1.6 Degrees of Freedom:

- An unconstrained rigid body moving in space can describe the following independent motion:
  a. Translational motion along any three mutually perpendicular axes x, y and z.
  b. Rotational motion about these axes

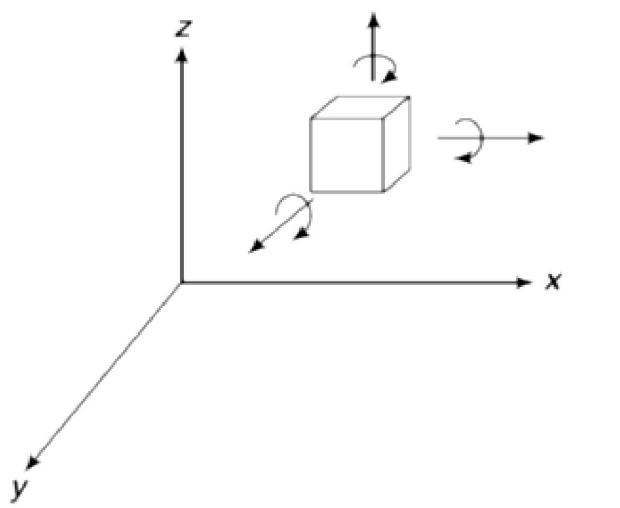


Fig.1.6 Degrees of freedom

- A rigid body possesses six degrees of freedom.

- Degrees of freedom of a pair is defined as the number of independent relative motions, both translational and rotational, a pair can have.
- DOF = 6 – Number of Restraints

1.7 Kinematic chain

- Kinematic chain is defined as the combination of kinematic pairs in which each links forms a part of two kinematic pairs and the relative motion between the links is either completely constrained or successfully constrained.
- Examples: slider-crank mechanism
- For a kinematic chain

$$N = 2P - 4 = 2(j + 2)/3$$

- Where N = no. of links, P = no. of Pairs and j = no. of joints
- When,

LHS > RHS, then the chain is locked
LHS = RHS, then the chain is constrained
LHS < RHS, then the chain is unconstrained

1.8 Kutzbach Criterion

- DOF of a mechanism in space can be determined as follows:
- In mechanism one link should be fixed. Therefore total no. of movable links are in mechanism is (N-1)
- Any pair having 1 DOF will impose 5 restraints on the mechanism, which reduces its total degree of freedom by 5P1.
- Any pair having 2 DOF will impose 4 restraints on the mechanism, which reduces its total degree of freedom by 4P2
- Similarly, the other pairs having 3, 4 and 5 degrees of freedom reduce the degrees of freedom of mechanism. Thus,
- Thus,

$$F = 6(N-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - 1P_5 - 0P_6$$

- Hence,

$$F = 6(N-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - 1P_5$$

- The above equation is the general form of **Kutzbach criterion.** This is applicable to any type of mechanism including a spatial mechanism.

1.9 Grubler's criterion

- If we apply the Kutzbach criterion to planer mechanism, then equation of Kutzbach criterion will be modified and that modified equation is known as Grubler's Criterion for planer mechanism.
- Therefore in planer mechanism if we consider the links having 1 to 3 DOF, the total number of degree of freedom of the mechanism considering all restraints will becomes,

$$F = 3(N-1) - 2P_1 - 1P_2$$

- The above equation is known as **Grubler's criterion** for planer mechanism.
- Sometimes all the above empirical relations can give incorrect results, e.g. fig (a) has 5 links, 6 turning pairs and 2 loops. Thus, it is a structure with zero degree of freedom.

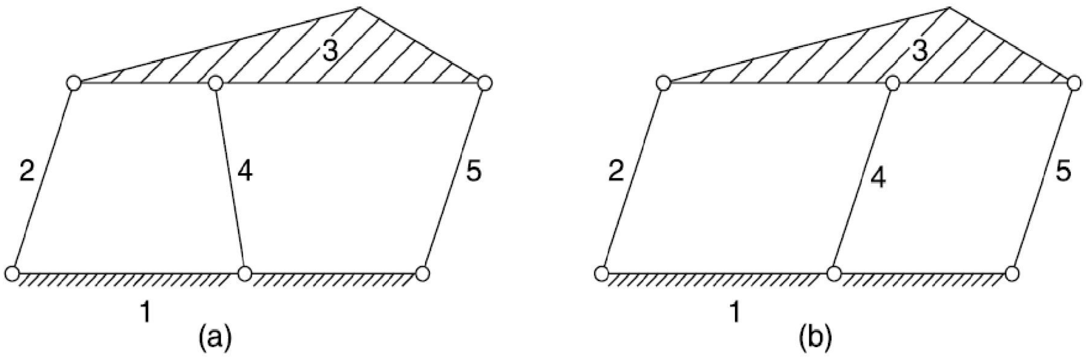


Fig. 1.7

- However, if the links are arranged in such a way as shown in fog. (b), a double parallelogram linkage with one degree of freedom is obtained. This is due to the reason that the lengths of links or other dimensional properties are not considered in these empirical relations.
- Sometimes a system may have one or more link which does not introduce any extra constraint. Such links are known as redundant links and should not be counted to find the degree of freedom. For example fig. (B) has 5 links, but the function of the mechanism is not affected even if any one of the links 2, 4 and 5 are removed. Thus, the effective number of links in this case is 4 with 4 turning pairs, and thus 1 degree of freedom.
- In case of a mechanism possessing some redundant degree of freedom, the effective degree of freedom is given by,

$$\mathbf{F = 3\,(N - 1) - 2\,P_1 - 1P_2 - F_r}$$

- Where Fr = no. of redundant degrees of freedom

1.10 The Four-Bar chain

- A four bar chain is the most fundamental of the plane kinematic chains. It is a much proffered mechanical device for the mechanisation and control of motion due to its simplicity and versatility. Basically, it consists of four rigid links which are connected in the form of a quadrilateral by four pin-joints.
- When one of the link fixed, it is known as mechanism or linkage. A link that makes complete revolution is called the crank. The link opposite to the fixed link is called coupler, and the forth link is called a lever or rocker if it oscillates or another crank if it rotates.
- It is impossible to have a four-bar linkage if the length of one of the link is greater than the sum of other three. This has been shown in fig.

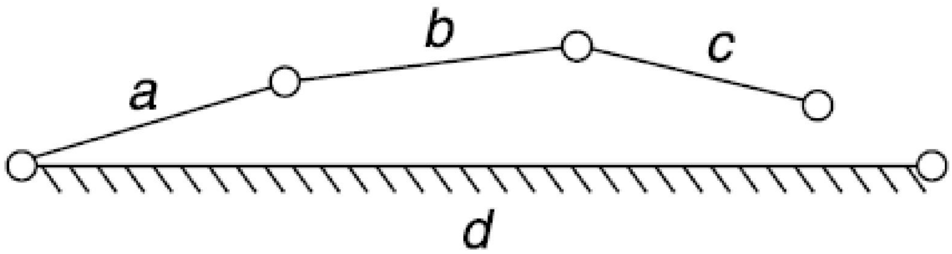


Fig. 1.7 Four bar chain

1.11 Grashof's law:

- We have already discussed that the kinematic chain is a combination of four or more kinematic pairs, such that the relative motion between the links or elements is completely constrained The simplest and the basic kinematic chain is a four bar chain or quadric cycle chain, as shown in Fig. 5.18. It consists of four links, each of them forms a turning pair at A, B, C and D. The four links may be of different lengths.
- According to Grashof's 's law for a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links.

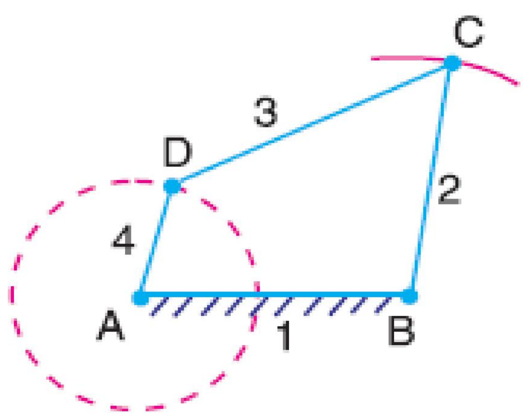


Fig. 1.8 Grashof's law

- A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to the other links. The mechanism in which no link makes a complete revolution will not be useful. In a four bar chain, one of the links, in particular the shortest link, will make a complete revolution relative to the other three links, if it satisfies the Grashof's law. Such a link is known as crank or driver. In Fig.5.18, AD (link 4) is a crank.
- The link BC (link 2) which makes a partial rotation or oscillates is known as lever or rocker or follower and the link CD (link 3) which connects the crank and lever is called connecting rod or coupler. The fixed link AB (link 1) is known as frame of the mechanism.

1.12 Inversion of Mechanism:

- When the number of links in kinematic chain is more than three, the chain is known as mechanism. When one link of the kinematic chain at a time is fixed, give the different mechanism of the kinematic chain. The method of generating different mechanism by fixing a link is called the inversion of mechanism.
- The number of inversion is equal to the numbers of links in the kinematic chain.
- The inversion of mechanism may be classified as:
  - **a.** Inversion of four-bar chain
  - **b.** Inversion of single slider crank chain
  - **c.** Inversion of double slider crank chain

1.13 Inversion of Four-Bar chain

1.13.1 First inversion: coupled wheel of locomotive

- The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links is shown in Fig.

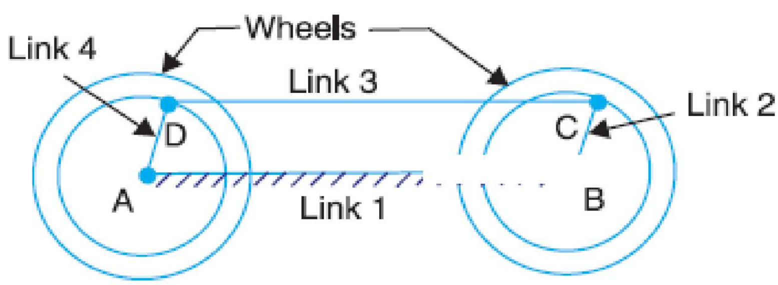


Fig. 1.9 coupled wheel of locomotive

- In this mechanism, the links *AD* and *BC* (having equal length) act as cranks and are connected to the respective wheels. The link *CD* acts as a coupling rod and the link *AB* is fixed in order to maintain a constant centre to Centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

1.13.2 Second inversion: Beam Engine

- A part of the mechanism of a beam engine (also known as cranks and lever mechanism) which consists of four links is shown in Fig. 1.10.
- In this mechanism, when the crank rotates about the fixed centre A, the lever oscillates about a fixed centre D. The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank.

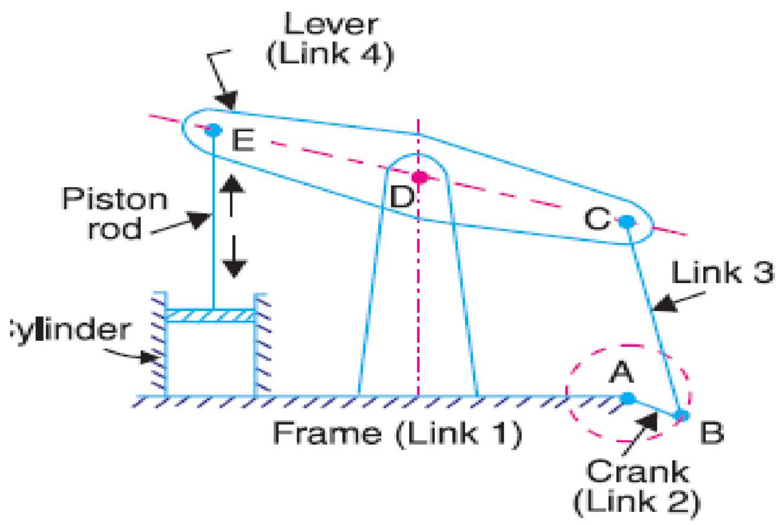


Fig. 1.10 beam engine

- In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.

1.13.3 Third inversion: watts indicator mechanism

- A Watt's indicator mechanism (also known as Watt's straight line mechanism or double lever mechanism) which consists of four links is shown in Fig.
- The four links are: fixed link at A, link AC, link CE and link BFD. It may be noted that BF and FD form one link because these two parts have no relative motion between them. The links CE and BFD act as levers.
- The displacement of the link BFD is directly proportional to the pressure of gas or steam which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point E at the end of the link CE traces out approximately a straight line.

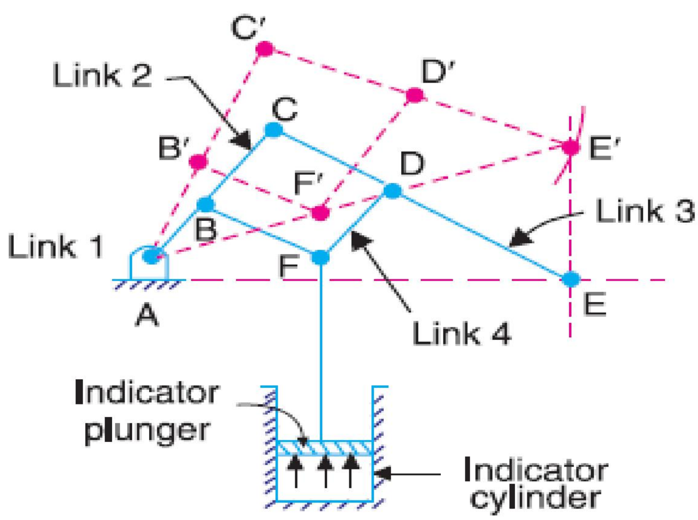


Fig. 1.11 watts indicator mechanism

1.14 The slider-crank chain

- When one of the turning pairs of a four-bar chain is replaced by a sliding pair, it becomes a single slider-crank chain or simply a slider-crank chain.
- It is also possible to replace two sliding pairs of a four-bar chain to get a double slider-crank chain. In a slider-crank chain, the straight line path of the slider may be passing through the fixed pivot O or may be displaced.
- The distance e between the fixed pivot O and the straight line path of the slider is called the offset and the chain so formed an offset slider-crank chain.
- Different mechanisms obtained by fixing different links of a kinematic chain are known as its inversions.

1.14.1 First inversion

- This inversion is obtained when link 1 is fixed and links 2 and 4 are made the crank and slider respectively. (fig.a)
- **Applications:**
  - **a** Reciprocating engine
  - **b** Reciprocating compressor

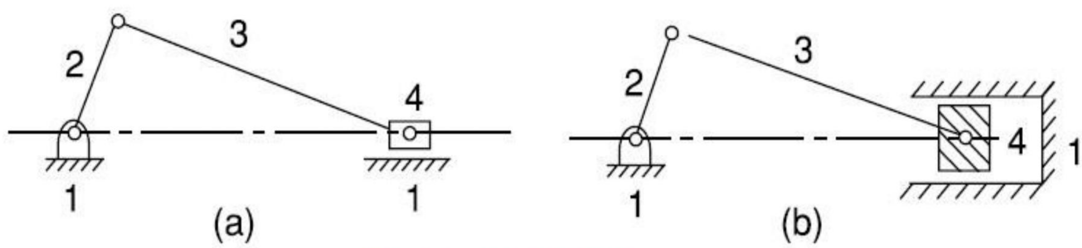


Fig. 1.12 first inversion

1.14.2 Second inversion

- Fixing of the link 2 of a slider-crank chain results in the second inversion.
- **Applications:**
  - **a** Whitworth quick-return mechanism
  - **b** Rotary engine

1.14.3 Third Inversion

- By Fixing of the link 3 of the slider-crank mechanism, the third inversion is obtained. Now the link 2 again acts as a crank and the link 4 oscillates.
- **Applications:**
  - **a** **Oscillating cylinder engine**
  - **b** Crank and slotted-lever mechanism

1.14.4 <u>Fourth Inversion</u>

- If the link 4 of the slider-crank mechanism is fixed, the fourth inversion is obtained. Link 3 can oscillates about the fixed pivot B on the link 4. This makes the end A of the link 2 to oscillate about B and the end O to reciprocate along the axis of the fixed link 4.

– **Application: Hand Pump**

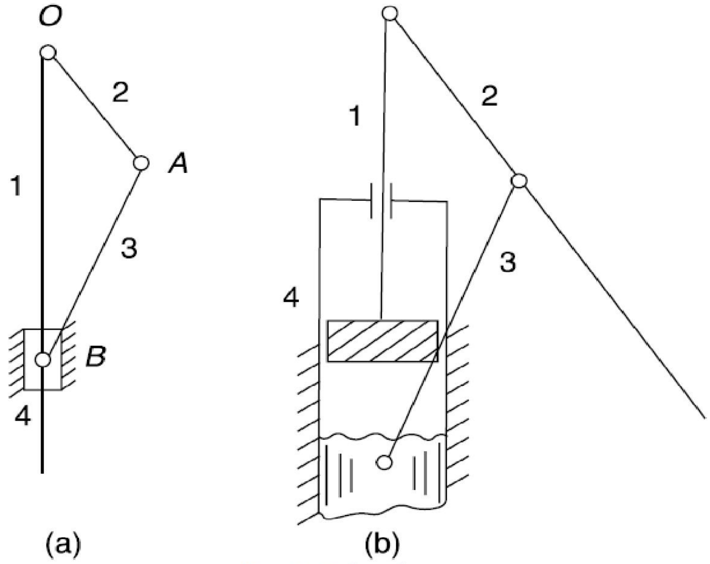


Fig. 1.13 hand pump

– Fig.1.13 shows a hand-pump. Link 4 is made in the form of a cylinder and a plunger fixed to the link 1 reciprocates in it.

1.15 Whitworth Quick-Return Mechanism:

– This mechanism used in shaping and slotting machines.
– In this mechanism the link CD (link 2) forming the turning pair is fixed; the driving crank CA (link 3) rotates at a uniform angular speed and the slider (link 4) attached to the crank pin at a slides along the slotted bar PA (link 1) which oscillates at D.
– The connecting rod PR carries the ram at R to which a cutting tool is fixed and the motion of the tool is constrained along the line RD produced.

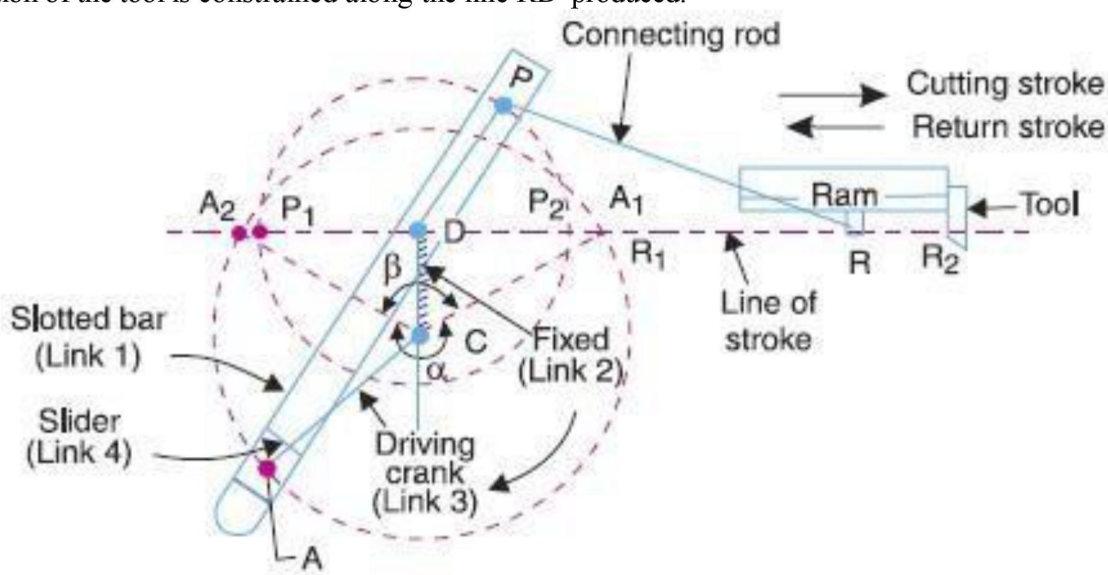


Fig. 1.14 Whitworth quick returns mechanism

– The length of effective stroke = 2 PD. And mark P1R1 = P2 R2 = PR.

$$\frac{\text{□□□□ □□ □□□□□□□□ □}}{\textit{time of return}} = \frac{}{\beta} = \frac{\alpha}{360° - \alpha} = \frac{360° - \beta}{\beta}$$

1.16 Rotary engine

– Sometimes back, rotary internal combustion engines were used in aviation. But now- a-days gas turbines are used in its place.

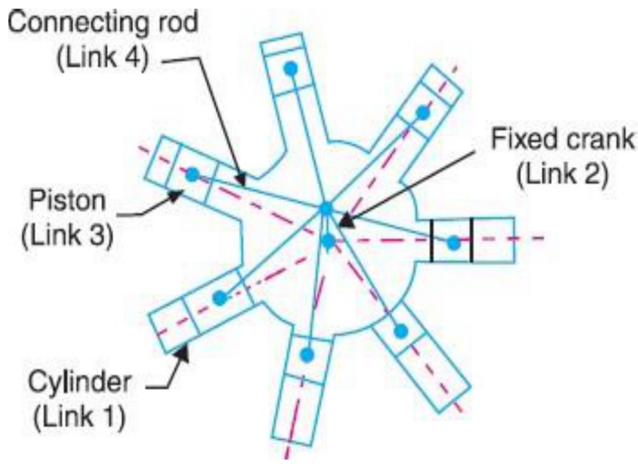


Fig. 1.15 rotary engine

– It consists of seven cylinders in one plane and all revolves about fixed center D, as shown in Fig. 5.25, while the crank (link 2) is fixed. In this mechanism, when the connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinders forming link 1.

1.17 Oscillating cylinder engine

– The arrangement of oscillating cylinder engine mechanism, as shown in Fig. Is used to convert reciprocating motion into rotary motion.

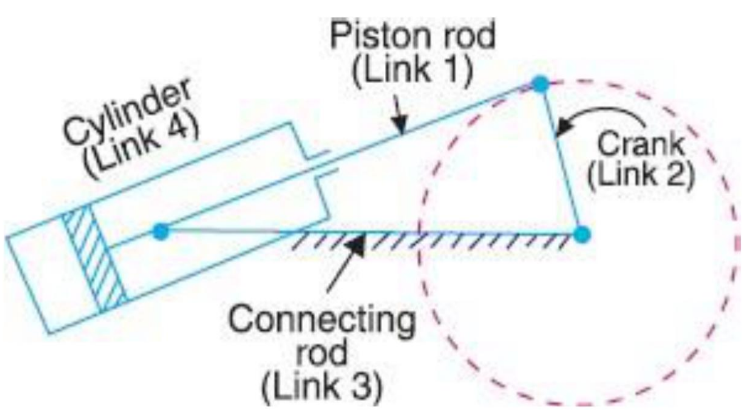


Fig. 1.16 oscillating cylinder engine

- In this mechanism, the link 3 forming the turning pair is fixed. The link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism. When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at *A*.

1.18 Crank and slotted-lever Mechanism

- This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.
- In this mechanism, the link AC (i.e. link 3) forming the turning pair is fixed, as shown in Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed center C. A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A.
- A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R1R2. The line of stroke of the ram (i.e. R1R2) is perpendicular to AC produced.

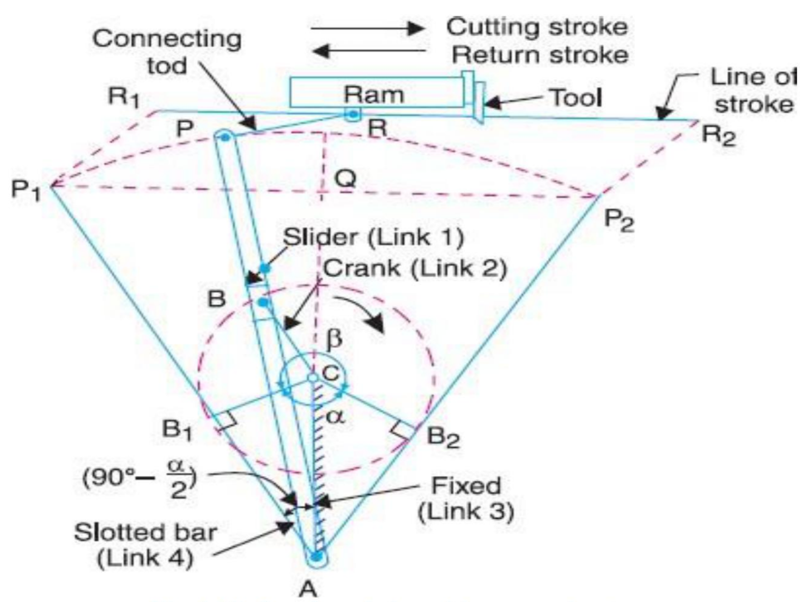


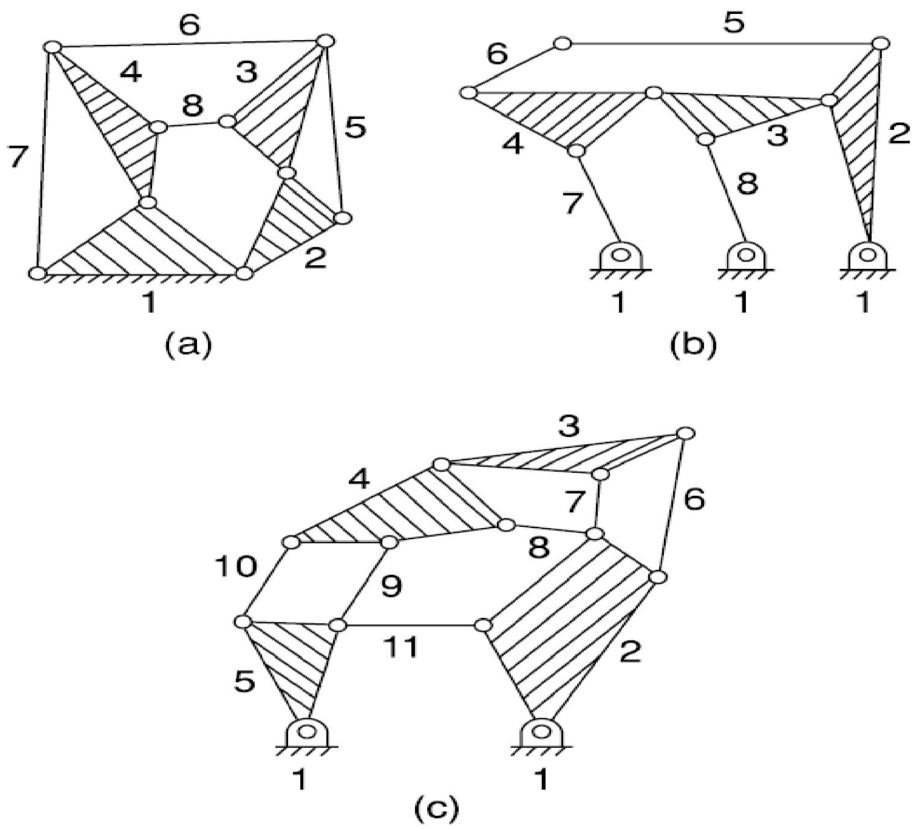
Fig.*1.17 Crank and slotted lever mechanism*

- In the extreme positions, *AP*1 and *AP*2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position *CB*1 to *CB*2 (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position *CB*2 to *CB*1 (or through angle α) in the clockwise direction. Since the crank has uniform angular speed, therefore,

$$\frac{\text{□□□□ □□ □□□□□□□□ □}}{time\ of\ return} = \frac{}{\alpha} = \frac{\beta}{360° - \beta} = \frac{360° - \alpha}{\alpha}$$

1.19 Example based on Degrees of Freedom:

1 **For the kinematic linkages shown in following fig. calculate the following:**
The numbers of binary links (N_b)
The numbers of ternary links (Nt)
The numbers of other (quaternary) links (No)
The numbers of total links (n)
The numbers of loops (L)
The numbers of joints or pairs (P1)
The numbers of degrees of freedom (F)



a $N_b = 4$; $N_t = 4$; $N_0 = 0$; $N = 8$; $L = 4$; $P_1 = 11$ (by counting) $P_1 = (N + L – 1) = 11$
$F = 3 (N – 1) – 2P_1$
$F = 3 (8 – 1) – 2\times11 = -1$ or,
v $F = N – (2 L + 1)$
$F = 8 – (2 \times4 + 1) = -1$

b $N_b = 4$; $N_t = 4$; $N_0 = 0$; $N = 8$; $L = 3$; $P_1 = 10$ (by counting) $P_1 = (N + L – 1) = 10$

$F = 3\ (N - 1) - 2P_1$
$F = 3\ (8 - 1) - 2 \times 10 = 1$
or, $F = N - (2\ L + 1)$
$F = 8 - (2 \times 3 + 1) = 1$

c $N_b = 7$; $N_t = 2$; $N_0 = 2$; $N = 11$; $L = 5$; $P_1 = 15$ (by counting) $F = N - (2\ L + 1)$
$F = 11 - (2 \times 5 + 1) = 0$
Therefore the linkage is a structure.

References

1. Theory of Machines by S.S.Rattan, Tata McGraw Hill
2. Theory of Machines by R.S. Khurmi & J.K.Gupta,S.Chan d
3. Theory of machines and mechanisms by P.L.Ballaney by Khanna Publication