17.3 EQUATIONS OF FILTER NETWORKS

The study of the behaviour of any filter requires the calculation of its propagation constant γ, attenuation α, phase shift β and its characteristic impedance Z_0.

T-Network

Consider a symmetrical *T*-network as shown in Fig. 17.4.

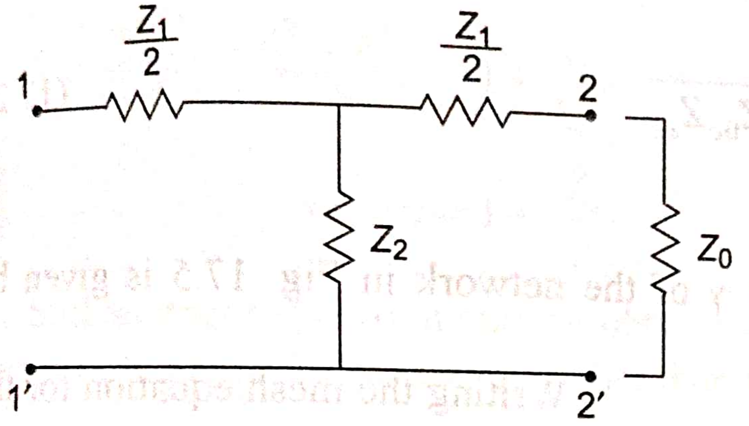


Fig. 17.4

As has already been mentioned in Section 16.13, if the image impedances at port 1-1′ and port 2-2′ are equal to each other, the image impedance is then called the characteristic, or the iterative impedance, Z_0. Thus, if the network in Fig. 17.4 is terminated in Z_0, its input impedance will also be Z_0. The value of input impedance for the *T*-network when it is terminated in Z_0 is given by

$$Z_{in} = \frac{Z_1}{2} + \frac{Z_2\left(\frac{Z_1}{2} + Z_0\right)}{\frac{Z_1}{2} + Z_2 + Z_0}$$

also $\quad Z_{in} = Z_0$

$$\therefore \quad Z_0 = \frac{Z_1}{2} + \frac{2Z_2\left(\frac{Z_1}{2} + Z_0\right)}{Z_1 + 2Z_2 + 2Z_0}$$

$$Z_0 = \frac{Z_1}{2} + \frac{(Z_1Z_2 + 2Z_2Z_0)}{Z_1 + 2Z_2 + 2Z_0}$$

$$Z_0 = \frac{Z_1^2 + 2Z_1Z_2 + 2Z_1Z_0 + 2Z_1Z_2 + 4Z_0Z_2}{2(Z_1 + 2Z_2 + 2Z_0)}$$

$$4Z_0^2 = Z_1^2 + 4Z_1Z_2$$

$$Z_0^2 = \frac{Z_1^2}{4} + Z_1Z_2$$

The characteristic impedance of a symmetrical T-section is

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2} \tag{17.1}$$

Z_{0T} can also be expressed in terms of open circuit impedance Z_{0c} and short circuit impedance Z_{sc} of the T-network. From Fig. 17.4, the open circuit impedance $Z_{0c} = \frac{Z_1}{2} + Z_2$ and

$$Z_{sc} = \frac{Z_1}{2} + \frac{\frac{Z_1}{2} \times Z_2}{\frac{Z_1}{2} + Z_2}$$

$$Z_{sc} = \frac{Z_1^2 + 4Z_1Z_2}{2Z_1 + 4Z_2}$$

$$Z_{0c} \times Z_{sc} = Z_1Z_2 + \frac{Z_1^2}{4}$$

$$= Z_{0T}^2 \quad \text{or} \quad Z_{0T} = \sqrt{Z_{0c}Z_{sc}} \tag{17.2}$$

Propagation Constant of T-Network

By definition the propagation constant γ of the network in Fig. 17.5 is given by $\gamma = \log_e I_1/I_2$

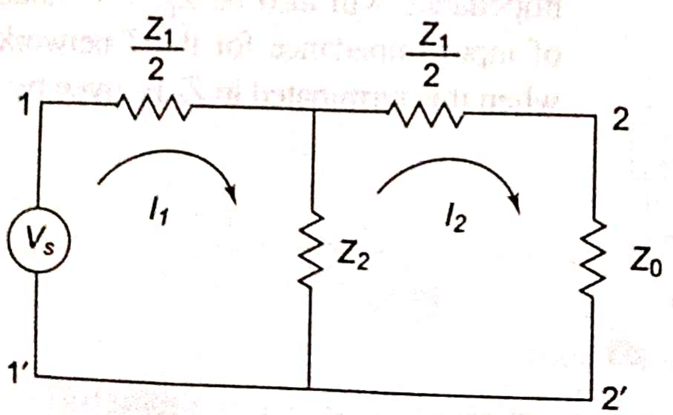


Fig. 17.5

Writing the mesh equation for the 2nd mesh, we get

$$I_1Z_2 = I_2\left(\frac{Z_1}{2} + Z_2 + Z_0\right)$$

$$\frac{I_1}{I_2} = \frac{\frac{Z_1}{2} + Z_2 + Z_0}{Z_2} = e^\gamma$$

$\therefore$

$$\frac{Z_1}{2} + Z_2 + Z_0 = Z_2e^\gamma$$

$$Z_0 = Z_2(e^\gamma - 1) - \frac{Z_1}{2} \tag{17.3}$$

The characteristic impedance of a T-network is given by

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} \tag{17.4}$$

Squaring Eqs 17.3 and 17.4 and subtracting Eq. 17.4 from Eq. 17.3, we get

$$Z_2^2(e^{\gamma}-1)^2 + \frac{Z_1^2}{4} - Z_1Z_2(e^{\gamma}-1) - \frac{Z_1^2}{4} - Z_1Z_2 = 0$$

$$Z_2^2(e^{\gamma}-1)^2 - Z_1Z_2(1+e^{\gamma}-1) = 0$$

$$Z_2^2(e^{\gamma}-1)^2 - Z_1Z_2e^{\gamma} = 0$$

$$Z_2(e^{\gamma}-1)^2 - Z_1e^{\gamma} = 0$$

$$(e^{\gamma}-1)^2 = \frac{Z_1e^{\gamma}}{Z_2}$$

$$e^{2\gamma}+1-2e^{\gamma} = \frac{Z_1}{Z_2e^{-\gamma}}$$

Rearranging the above equation, we have

$$e^{-\gamma}(e^{2\gamma}+1-2e^{\gamma}) = \frac{Z_1}{Z_2}$$

$$(e^{\gamma}+e^{-\gamma}-2) = \frac{Z_1}{Z_2}$$

Dividing both sides by 2, we have

$$\frac{e^{\gamma}+e^{-\gamma}}{2} = 1+\frac{Z_1}{2Z_2}$$

$$\cosh\gamma = 1+\frac{Z_1}{2Z_2} \tag{17.5}$$

Still another expression may be obtained for the complex propagation constant in terms of the hyperbolic tangent rather than hyperbolic cosine.

$$\sinh\gamma = \sqrt{\cos h^2\gamma - 1}$$

$$= \sqrt{\left(1+\frac{Z_1}{2Z_2}\right)^2 - 1} = \sqrt{\frac{Z_1}{Z_1} + \left(\frac{Z_1}{2Z_2}\right)^2}$$

$$\sinh\gamma = \frac{1}{Z_2}\sqrt{Z_1Z_2+\frac{Z_1^2}{4}} = \frac{Z_{0T}}{Z_2} \tag{17.6}$$

Dividing Eq. 17.6 by Eq. 17.5, we get

$$\tanh\gamma = \frac{Z_{0T}}{Z_2+\frac{Z_1}{2}}$$

But $$Z_2 + \frac{Z_1}{2} = Z_{0c}$$

Also from Eq. 17.2, $Z_{0T} = \sqrt{Z_{0c} Z_{sc}}$

$$\tanh \gamma = \sqrt{\frac{Z_{sc}}{Z_{0c}}}$$

Also $$\sinh \frac{\gamma}{2} = \sqrt{\frac{1}{2}(\cosh \gamma - 1)}$$

Where $$\cosh \gamma = 1 + (Z_1 / 2Z_2)$$

$$= \sqrt{\frac{Z_1}{4Z_2}} \qquad (17.7)$$

π-Network

Consider asymmetrical π-section shown in Fig. 17.6. When the network is terminated in Z_0 at port 2-2′, its input impedance is given by

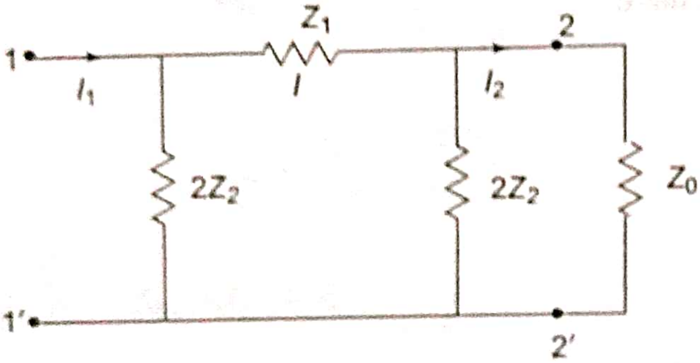


Fig. 17.6

$$Z_{in} = \frac{2Z_2\left[Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}\right]}{Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0} + 2Z_2}$$

By definition of characteristic impedance, $Z_{in} = Z_0$

$$Z_0 = \frac{2Z_2\left[Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}\right]}{Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0} + 2Z_2}$$

$$Z_0 Z_1 + \frac{2Z_2 Z_0^2}{2Z_2 + Z_0} + 2Z_0 Z_2 = \frac{2Z_2(2Z_1 Z_2 + Z_0 Z_1 + 2Z_0 Z_2)}{(2Z_2 + Z_0)}$$

$$2Z_0 Z_1 Z_2 + Z_1 Z_0^2 + 2Z_0^2 Z_2 + 4Z_2^2 Z_0 + 2Z_2 Z_0^2$$

$$= 4Z_1 Z_2^2 + 2Z_0 Z_1 Z_2 + 4Z_0 Z_2^2$$

$$Z_1 Z_0^2 + 4Z_2 Z_0^2 = 4Z_1 Z_2^2$$

$$Z_0^2 (Z_1 + 4Z_2) = 4Z_1 Z_2^2$$

$$Z_0^2 = \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2}$$

Rearranging the above equation leads to

$$Z_0 = \sqrt{\frac{Z_1 Z_2}{1 + Z_1 / 4Z_2}} \qquad (17.8)$$

which is the characteristic impedance of a symmetrical π-network,

$$Z_{0\pi} = \frac{Z_1 Z_2}{\sqrt{Z_1 Z_2 + Z_1^2/4}}$$

From Eq. 17.1 $Z_{0T} = \sqrt{\dfrac{Z_1^2}{4} + Z_1 Z_2}$

$$\therefore \quad Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} \tag{17.9}$$

$Z_{0\pi}$ can be expressed in terms of the open circuit impedance Z_{0c} and short circuit impedance Z_{sc} of the π network shown in Fig. 17.6 exclusive of the load Z_0.

From Fig. 17.6, the input impedance at port 1-1′ when port 2-2′ is open is given by $Z_{0C} = \dfrac{2Z_2(Z_1 + 2Z_2)}{Z_1 + 4Z_2}$

Similarly, the input impedance at port 1-1′ when port 2-2′ is short circuited is given by $Z_{sc} = \dfrac{2Z_1 Z_2}{2Z_2 + Z_1}$

Hence $\quad Z_{0c} \times Z_{sc} = \dfrac{4Z_1 Z_2^2}{Z_1 + 4Z_2} = \dfrac{Z_1 Z_2}{1 + Z_1/4Z_2}$

Thus from Eq. 17.8

$$Z_{0\pi} = \sqrt{Z_{0c}\, Z_{sc}} \tag{17.10}$$

Propagation Constant of π-Network

The propagation constant of a symmetrical π-section is the same as that for a symmetrical T-section.

i.e. $\quad \cosh \gamma = 1 + \dfrac{Z_1}{2Z_2}$

17.4 CLASSIFICATION OF PASS BAND AND STOP BAND

π sections.

17.6 CONSTANT—K LOW PASS FILTER

A network, either T or π, is said to be of the constant-k type if Z_1 and Z_2 of the network satisfy the relation

$$Z_1 Z_2 = k^2 \tag{17.20}$$

where Z_1 and Z_2 are impedances in the T and π sections as shown in Fig. 17.8. Equation 17.20 states that Z_1 and Z_2 are inverse if their product is a constant, independent of frequency. k is a real constant, that is the resistance. k is often termed as design impedance or nominal impedance of the constant k-filter.

The constant k, T or π type filter is also known as the *prototype* because other more complex networks can be derived from it. A prototype T and π-sections are shown in

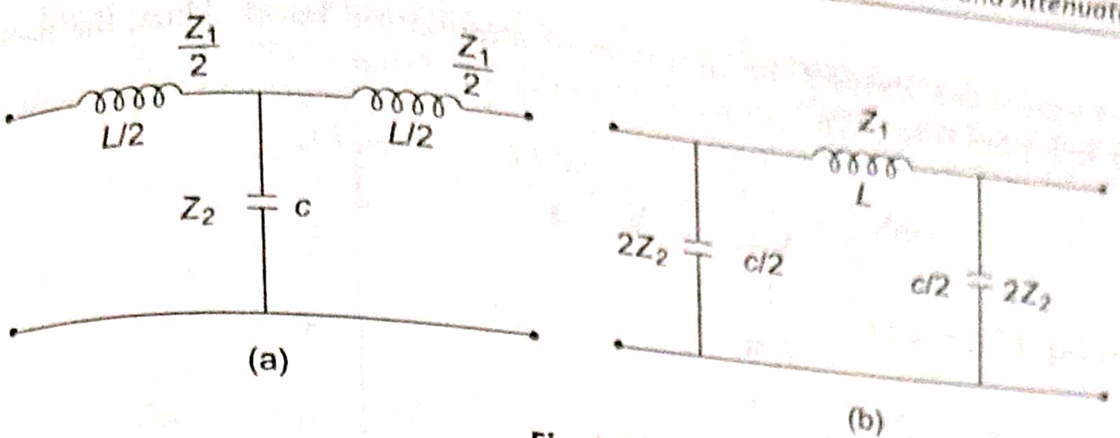


Fig. 17.8

Fig. 17.8 (a) and (b), where $Z_1 = j\omega_L$ and $Z_2 = 1/j\omega_C$. Hence $Z_1Z_2 = \frac{L}{C} = k^2$ which is independent of frequency.

$$Z_1Z_2 = k^2 = \frac{L}{C} \quad \text{or} \quad k = \sqrt{\frac{L}{C}} \tag{17.21}$$

Since the product Z_1 and Z_2 is constant, the filter is a constant-k type. From Eq. 17.18(a) the cut-off frequencies are $Z_1/4Z_2 = 0$,

i.e. $$\frac{-\omega^2 LC}{4} = 0$$

i.e. $$f = 0 \text{ and } \frac{Z_1}{4Z_2} = -1$$

$$\frac{-\omega^2 LC}{4} = -1$$

or $$f_c = \frac{1}{\pi\sqrt{LC}} \tag{17.22}$$

The pass band can be determined graphically. The reactances of Z_1 and $4Z_2$ will vary with frequency as drawn in Fig. 17.9. The cut-off frequency at the intersection of the curves Z_1 and $-4Z_2$ is indicated as f_c. On the X-axis as $Z_1 = -4Z_2$ at cut-off frequency, the pass band lies between the frequencies at which $Z_1 = 0$, and $Z_1 = -4Z_2$.

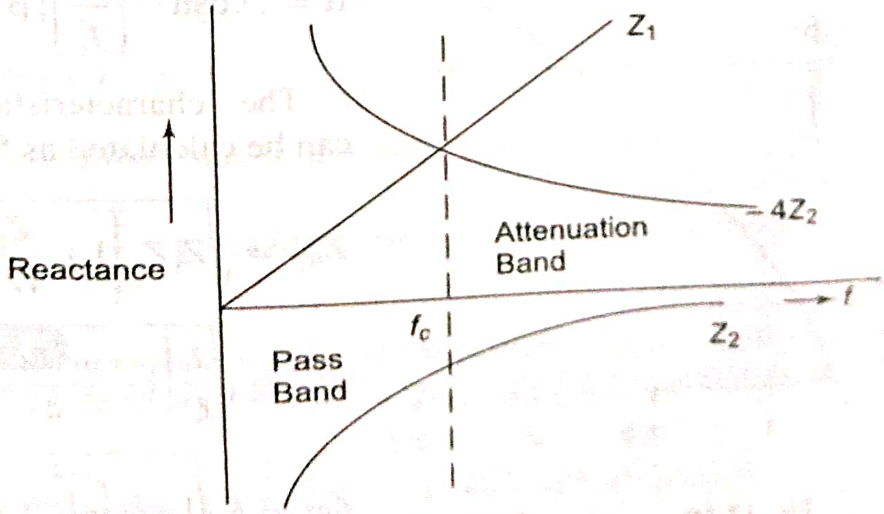


Fig. 17.9

All the frequencies above f_c lie in a stop or attenuation band. Thus, the network is called a low-pass filter. We also have from Eq. 17.7 that

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{-\omega^2 LC}{4}} = \frac{J\omega\sqrt{LC}}{2}$$

From Eq. 17.22 $\sqrt{LC} = \dfrac{1}{f_c\pi}$

$$\therefore \quad \sinh\frac{\gamma}{2} = \frac{j2\pi f}{2\pi f_c} = j\frac{f}{f_c}$$

We also know that in the pass band

$$-1 < \frac{Z_1}{4Z_2} < 0$$

$$-1 < \frac{-\omega^2 LC}{4} < 0$$

$$-1 < -\left(\frac{f}{f_c}\right)^2 < 0$$

or
$$\frac{f}{f_c} < 1$$

and
$$\beta = 2\sin^{-1}\left(\frac{f}{f_c}\right); \alpha = 0$$

In the attenuation band,

$$\frac{Z_1}{4Z_2} < -1, \text{ i.e.} \frac{f}{f_c} < 1$$

$$\alpha = 2\cosh^{-1}\left[\frac{Z_1}{4Z_2}\right] = 2\cosh^{-1}\left(\frac{f}{f_c}\right); \beta = \pi$$

The plots of α and β for pass and stop bands are shown in Fig. 17.10.

Thus, from Fig. 17.10, $\alpha = 0, \beta = 2\sinh^{-1}\left(\dfrac{f}{f_c}\right)$ for $f < f_c$

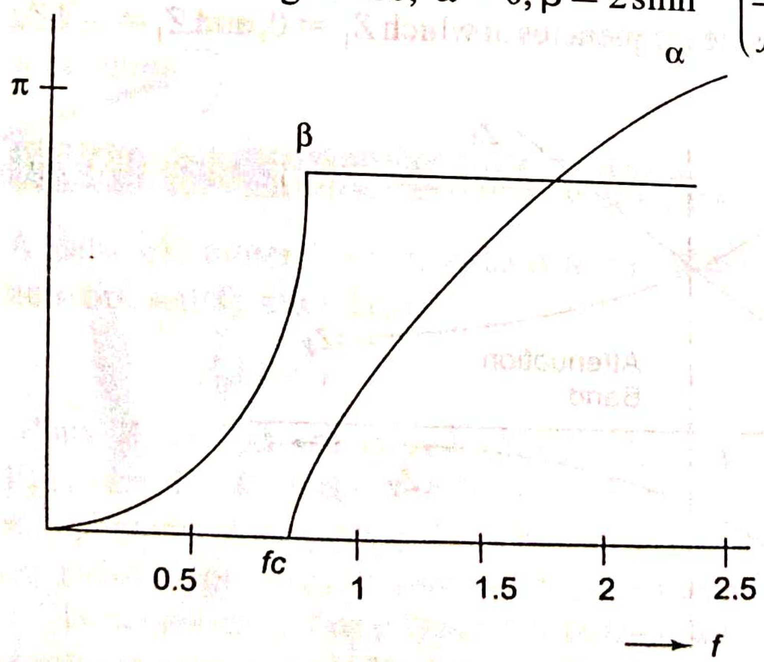


Fig. 17.10

$$\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right); \beta = \pi \text{ for } f > f_c$$

The characteristic impedance can be calculated as follows

$$Z_{0T} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)}$$

$$= \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2 LC}{4}\right)}$$

$$Z_{0T} = k\sqrt{1 - \left(\frac{f}{f_c}\right)^2} \tag{17.23}$$

From Eq. 17.23, Z_{0T} is real when $f < f_c$, i.e. in the pass band at $f = f_c$, $Z_{0T} = 0$; and for $f > f_c$, Z_{0T} is imaginary in the attenuation band, rising to infinite reactance at infinite frequency. The variation of Z_{0T} with frequency is shown in Fig. 17.11.

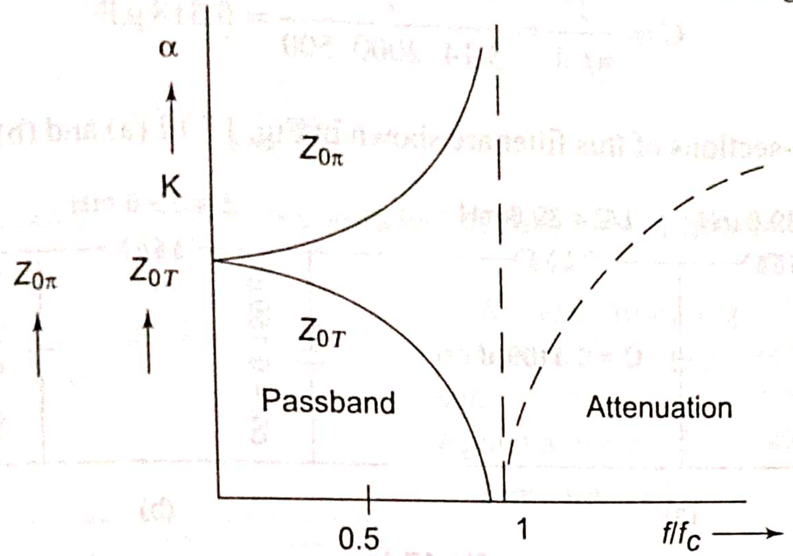


Fig. 17.11

Similarly, the characteristic impedance of a π-network is given by

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{k}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}} \quad (17.24)$$

The variation of $Z_{0\pi}$ with frequency is shown in Fig. 17.11. For $f < f_c$, $Z_{0\pi}$ is real; at $f = f_c$, $Z_{0\pi}$ is infinite, and for $f > f_c$, $Z_{0\pi}$ is imaginary. A low pass filter can be designed from the specifications of cut-off frequency and load resistance.

At cut-off frequency, $Z_1 = -4Z_2$

$$j\omega_c L = \frac{-4}{j\omega_c C}$$

$$\pi^2 f_c^2 LC = 1$$

Also we know that $k = \sqrt{L/C}$ is called the design impedance or the load resistance

$$\therefore \quad k^2 = \frac{L}{C}$$

$$\pi^2 f_c^2 k^2 C^2 = 1$$

$C = \dfrac{1}{\pi f_c k}$ gives the value of the *shunt capacitance*

and $L = k^2 C = \dfrac{k}{\pi f_c}$ gives the value of the series inductance.

Example 17.1 *Design a low pass filter (both π and T-sections) having a cut-off frequency of 2 kHz to operate with a terminated load resistance of 500 Ω.*

Solution It is given that $k = \sqrt{\dfrac{L}{C}} = 500\ \Omega$, and $f_c = 2000$ Hz

We know that $L = \dfrac{k}{\pi f_c} = \dfrac{500}{3.14 \times 2000} = 79.6 \text{ mH}$

$$C = \frac{1}{\pi f_c k} = \frac{1}{3.14 \cdot 2000 \cdot 500} = 0.318\,\mu\text{F}$$

The T and π-sections of this filter are shown in Fig. 17.12 (a) and (b) respectively.

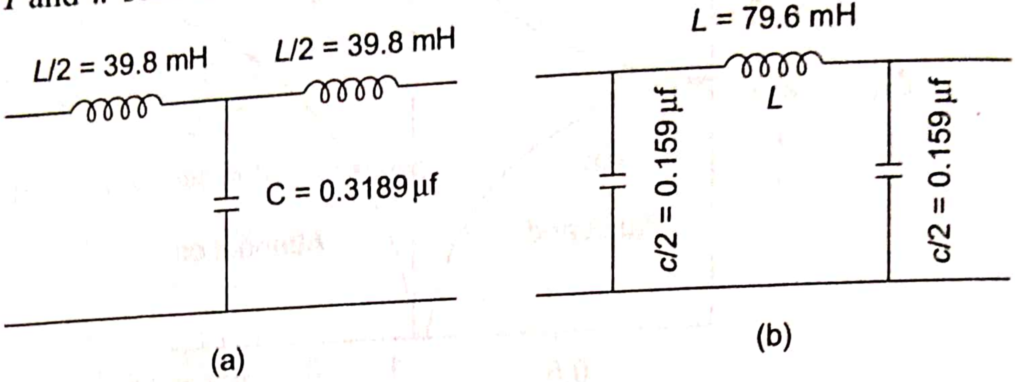


Fig. 17.12

17.7 CONSTANT K-HIGH PASS FILTER

Constant K-high pass filter can be obtained by changing the positions of series and shunt arms of the networks shown in Fig. 17.8. The prototype high pass filters are shown in Fig. 17.13, where $Z_1 = -j/\omega_C$ and $Z_2 = j\omega L$.

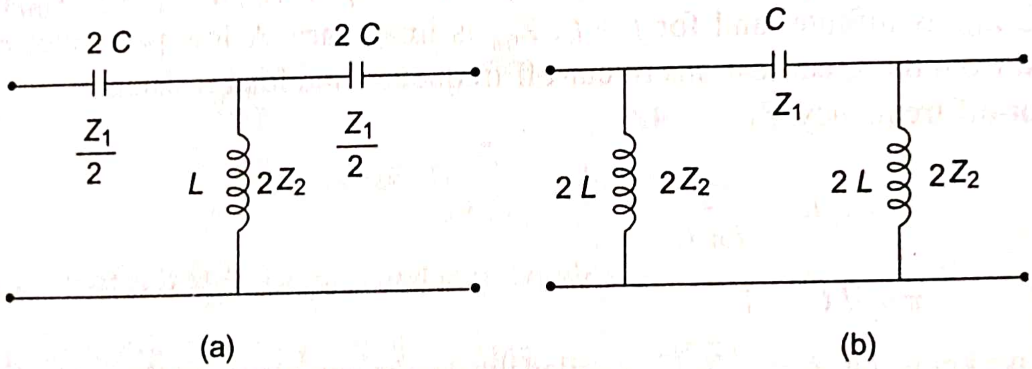


Fig. 17.13

Again, it can be observed that the product of Z_1 and Z_2 is independent of frequency, and the filter design obtained will be of the constant k type. Thus, Z_1Z_2 are given by

$$Z_1 Z_2 = \frac{-j}{\omega C} j\omega L = \frac{L}{C} = k^2$$

$$k = \sqrt{\frac{L}{C}}$$

The cut-off frequencies are given by $Z_1 = 0$ and $Z_1 = -4Z_2$.

$$Z_1 = 0 \text{ indicates } \frac{j}{\omega C} = 0, \text{ or } \omega \to \propto$$

From $Z_1 = -4Z_2$

$$\frac{-j}{\omega C} = -4\,j\omega L$$

$$\omega^2 LC = \frac{1}{4}$$

or
$$f_c = \frac{1}{4\pi\sqrt{LC}} \tag{17.25}$$

The reactances of Z_1 and Z_2 are sketched as functions of frequency as shown in Fig. 17.14.

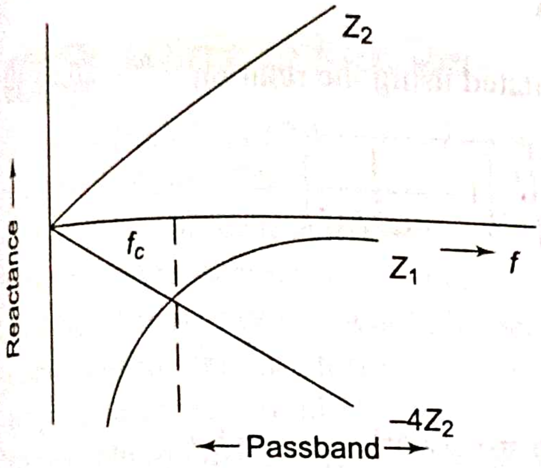


Fig. 17.14

As seen from Fig. 17.14, the filter transmits all frequencies between $f = f_c$ and $f = \propto$. The point f_c from the graph is a point at which $Z_1 = -4Z_2$.

From Eq. 17.7,

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{-1}{4\omega^2 LC}}$$

From Eq. 17.25, $f_c = \dfrac{1}{4\pi\sqrt{LC}}$

$$\therefore \qquad \sqrt{LC} = \frac{1}{4\pi f_c}$$

$$\therefore \qquad \sinh\frac{\gamma}{2} = \sqrt{\frac{-(4\pi)^2(f_c)^2}{4\omega^2}} = j\frac{f_c}{f}$$

In the pass band, $-1 < \dfrac{Z_1}{4Z_2} < 0$, $\alpha = 0$ or the region in which $\dfrac{f_c}{f} < 1$ is a pass band

$$\beta = 2\sin^{-1}\left(\frac{f_c}{f}\right)$$

In the attenuation band $\dfrac{Z_1}{4Z_2} < -1$, i.e. $\dfrac{f_c}{f} > 1$

$$\alpha = 2\cosh^{-1}\left[\frac{Z_1}{4Z_2}\right]$$

$$= 2\cos^{-1}\left(\frac{f_c}{f}\right); \quad \beta = -\pi$$

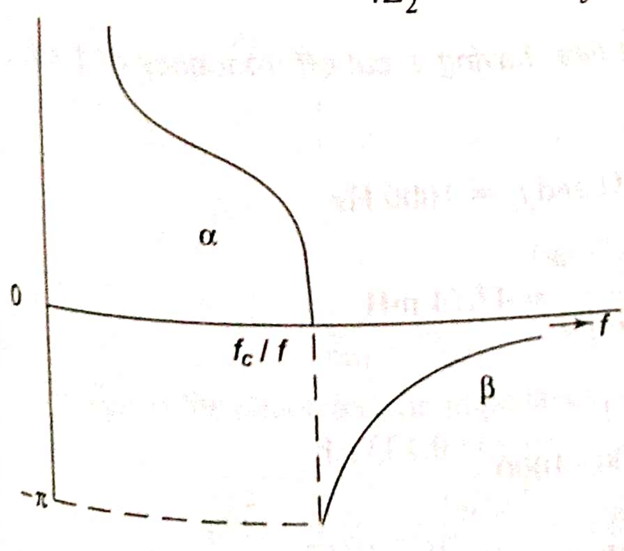


Fig. 17.15

The plots of α and β for pass and stop bands of a high pass filter network are shown in Fig. 17.15.

A high pass filter may be designed similar to the low pass filter by choosing a resistive load r equal to the constant k, such that $R = k = \sqrt{L/C}$

$$f_c = \frac{1}{4\pi\sqrt{L/C}}$$

$$f_c = \frac{k}{4\pi L} = \frac{1}{4\pi Ck}$$

Since $\sqrt{C} = \frac{L}{k}$,

$$L = \frac{k}{4\pi f_c} \text{ and } C = \frac{1}{4\pi f_c k}$$

The characteristic impedance can be calculated using the relation

$$Z_{0T} = \sqrt{Z_1Z_2\left(1+\frac{Z_1}{4Z_2}\right)} = \sqrt{\frac{L}{C}\left(1-\frac{1}{4\omega^2 LC}\right)}$$

$$Z_{0T} = k\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

Similarly, the characteristic impedance of a π-network is given by

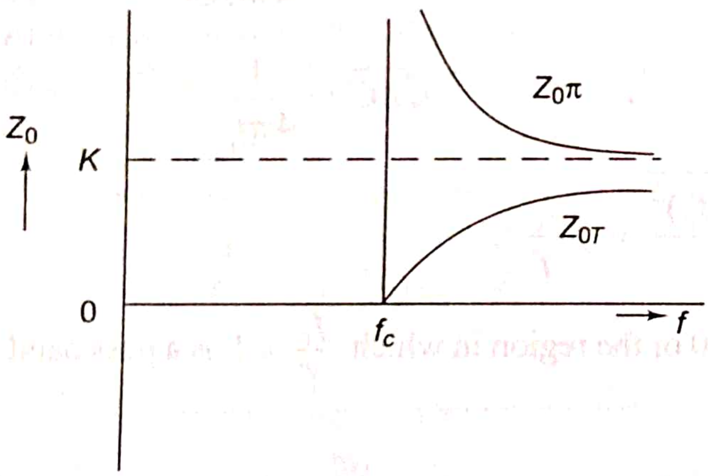


Fig. 17.16

$$Z_{0\pi} = \frac{Z_1Z_2}{Z_{0T}} = \frac{k^2}{Z_{0T}}$$

$$= \frac{k}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \tag{17.26}$$

The plot of characteristic impedances with respect to frequency is shown in Fig. 17.16.

Example 17.2 *Design a high pass filter having a cut-off frequency of 1 kHz with a load resistance of 600 Ω.*

Solution It is given that $R_L = K = 600\ \Omega$ and $f_c = 1000$ Hz

$$\therefore \quad L = \frac{K}{4\pi f_c} = \frac{600}{4\times\pi\times 1000} = 47.74 \text{ mH}$$

$$C = \frac{1}{4\pi k f_c} = \frac{1}{4\pi\times 600\times 1000} = 0.133\ \mu\text{F}$$

The T and π-sections of the filter are shown in Fig. 17.17.

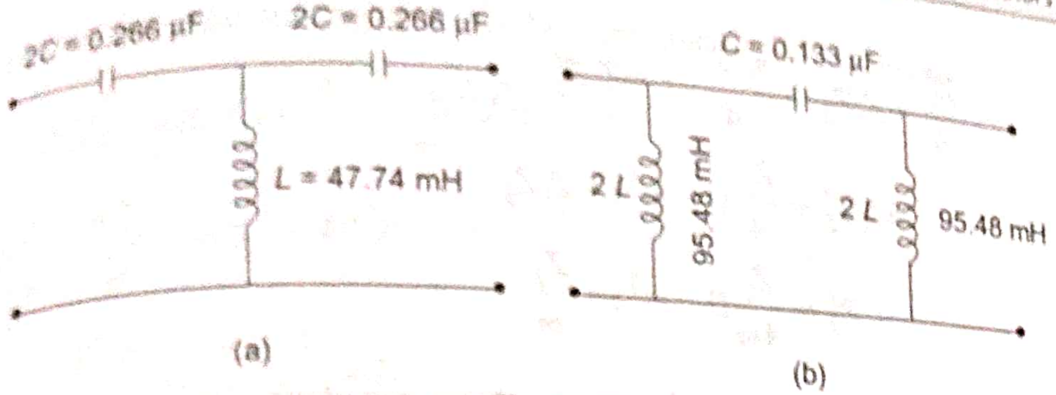


Fig. 17.17

17.8 m-DERIVED T-SECTION

It is clear from Figs 17.10 and 17.15 that the attenuation is not sharp in the stop band for *k*-type filters. The characteristic impedance, Z_0 is a function of frequency and varies widely in the transmission band. Attenuation can be increased in the stop band by using ladder section, i.e. by connecting two or more identical sections. In order to join the filter sections, it would be necessary that their characteristic impedances be equal to each other at all frequencies. If their characteristic impedances match at all frequencies, they would also have the same pass band. However, cascading is not a proper solution from a practical point of view. This is because practical elements have a certain resistance, which gives rise to attenuation in the pass band also. Therefore, any attempt to increase attenuation in stop band by cascading also results in an increase of 'α' in the pass band. If the constant *k* section is regarded as the prototype, it is possible to design a filter to have rapid attenuation in the stop band, and the same characteristic impedance as the prototype at all frequencies. Such a filter is called *m-derived filter*. Suppose a prototype T-network shown in Fig. 17.18 (a) has the series arm modified as shown in Fig. 17.18 (b), where *m* is a constant. Equating the characteristic impedance of the networks in Fig. 17.18, we have

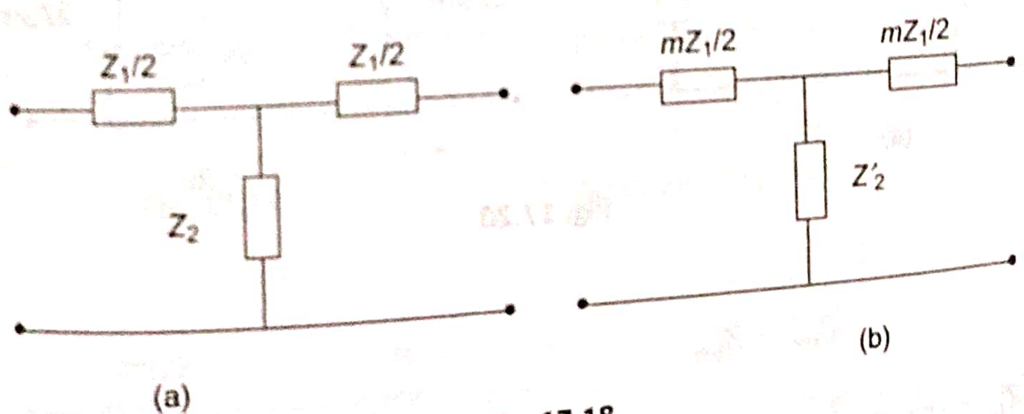


Fig. 17.18

$$Z_{0T} = Z_{0T'}$$

where $Z_{0T'}$ is the characteristic impedance of the modified (*m-derived*) T-network.

$$\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{\frac{m^2 Z_1^2}{4} + m Z_1 Z_2'}$$

$$\frac{Z_1^2}{4} + Z_1Z_2 = \frac{m^2Z_1^2}{4} + mZ_1Z_2'$$

$$mZ_1Z_2' = \frac{Z_1^2}{4}(1-m^2) + Z_1Z_2$$

$$Z_2' = \frac{Z_1}{4m}(1-m^2) + \frac{Z_2}{m} \tag{17.27}$$

It appears that the shunt arm Z_2' consists of two impedances in series as shown in Fig. 17.19.

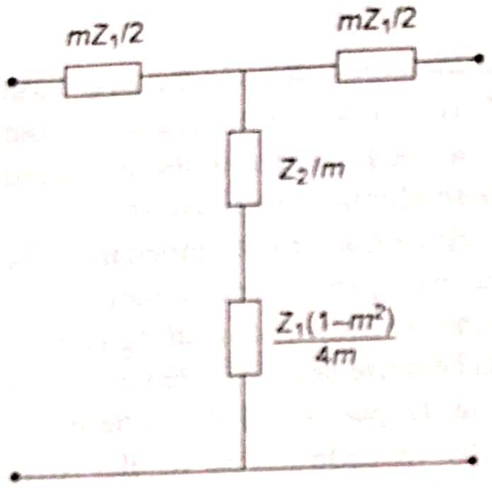


Fig. 17.19

From Eq. 17.27, $\frac{1-m^2}{4m}$ should be positive to realize the impedance Z_2' physically, i.e. $0 < m < 1$. Thus m-derived section can be obtained from the prototype by modifying its series and shunt arms. The same technique can be applied to π section network. Suppose a prototype π-network shown in Fig. 17.20 (a) has the shunt arm modified as shown in Fig. 17.20 (b).

The characteristic impedances of the prototype and its modified sections have to be equal for matching.

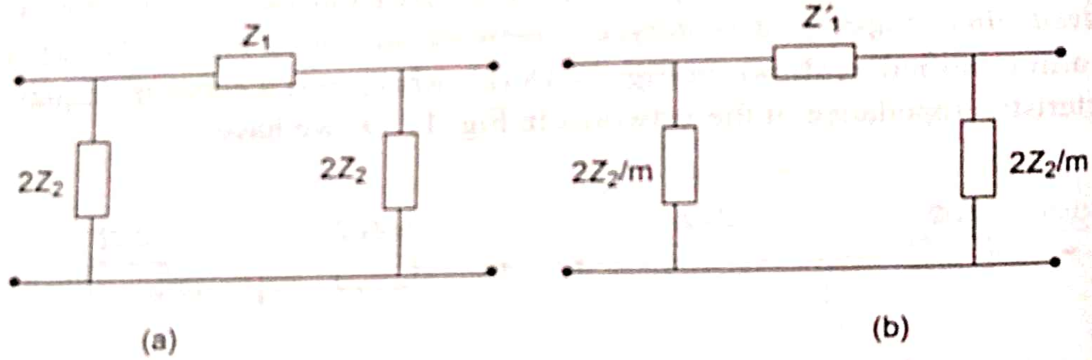


Fig. 17.20

$$Z_{0\pi} = Z_{0\pi}'$$

where $Z_{0\pi}'$ is the characteristic impedance of the modified (m-derived) π-network.

$$\therefore \quad \sqrt{\frac{Z_1Z_2}{1+\frac{Z_1}{4Z_2}}} = \sqrt{\frac{Z_1'\frac{Z_2}{m}}{1+\frac{Z_1'}{4\cdot Z_2/m}}}$$

Squaring and cross multiplying the above equation results as under.

$$(4Z_1Z_2 + mZ_1'Z_1) = \frac{4Z_1'Z_2 + Z_1Z_1'}{m}$$

$$Z_1'\left(\frac{Z_1}{m} + \frac{4Z_2}{m} - mZ_1\right) = 4Z_1Z_2$$

or
$$Z_1' = \frac{Z_1Z_2}{\frac{Z_1}{4m} + \frac{Z_2}{m} - \frac{mZ_1}{4}}$$

$$= \frac{Z_1Z_2}{\frac{Z_2}{m} + \frac{Z_1}{4m}(1-m^2)}$$

$$Z_1' = \frac{Z_1Z_2\frac{4m^2}{(1-m^2)}}{\frac{Z_2 4m^2}{m(1-m^2)} + Z_1 m} = \frac{mZ_1\frac{Z_2 4m}{(1-m^2)}}{mZ_1 + \frac{Z_2 4m}{(1-m^2)}} \tag{17.28}$$

It appears that the series arm of the m-derived π section is a parallel combination of mZ_1 and $4mZ_2 / 1 - m^2$. The derived m section is shown in Fig. 17.21.

m-Derived Low Pass Filter

In Fig. 17.22, both m-derived low pass T and π filter sections are shown. For the T-section shown in Fig. 17.22 (a), the shunt arm is to be chosen so that it is resonant at some frequency f_∞ above cut-off frequency f_c.

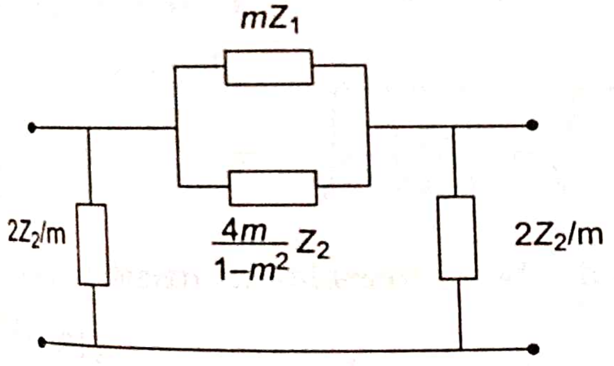


Fig. 17.21

If the shunt arm is series resonant, its impedance will be minimum or zero. Therefore, the output is zero and will correspond to infinite attenuation at this particular frequency. Thus, at f_∞

$$\frac{1}{m\omega_r C} = \frac{1-m^2}{4m}\omega_r L,$$ where ω_r is the resonant frequency

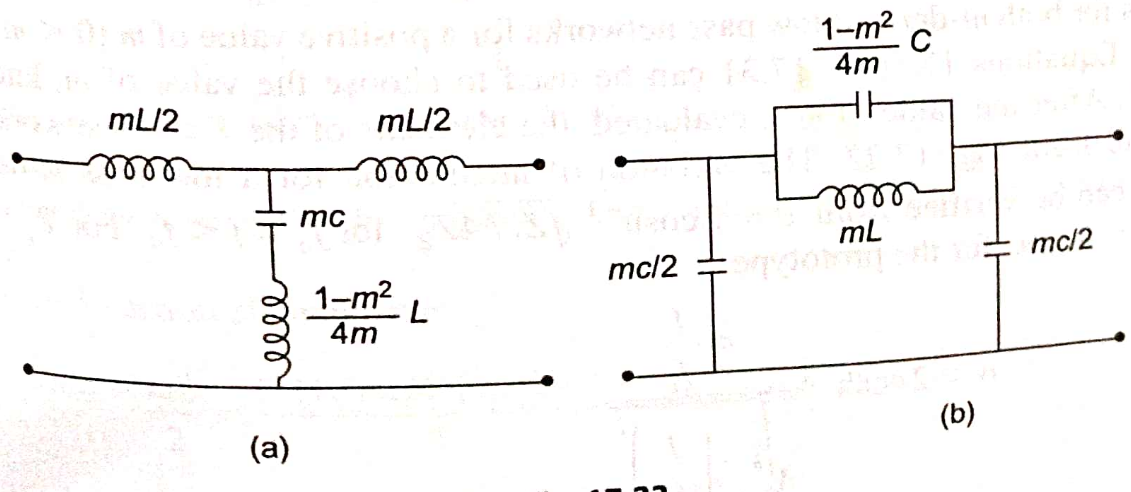


Fig. 17.22

$$\omega_r^2 = \frac{4}{(1-m^2)LC}$$

$$f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}} = f_\infty$$

Since the cut-off frequency for the low pass filter is $f_c = \dfrac{1}{\pi\sqrt{LC}}$

$$f_\infty = \frac{f_c}{\sqrt{1-m^2}} \tag{17.29}$$

or
$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2} \tag{17.30}$$

If a sharp cut-off is desired, f_∞ should be near to f_c. From Eq. 17.29, it is clear that for the smaller the value of m, f_∞ comes close to f_c. Equation 17.30 shows that if f_c and f_∞ are specified, the necessary value of m may then be calculated. Similarly, for m-derived π section, the inductance and capacitance in the series arm constitute a resonant circuit. Thus, at f_∞ a frequency corresponds to infinite attenuation, i.e. at f_r

$$m\omega_r L = \frac{1}{\left(\frac{1-m^2}{4m}\right)\omega_r C}$$

$$\omega_r^2 = \frac{4}{LC(1-m^2)}$$

$$f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}}$$

Since,
$$f_c = \frac{1}{\pi\sqrt{LC}}$$

$$f_r = \frac{f_c}{\sqrt{1-m^2}} = f_\infty \tag{17.31}$$

Thus for both m-derived low pass networks for a positive value of m $(0 < m < 1)$, $f_\infty > f_c$. Equations 17.30 or 17.31 can be used to choose the value of m, knowing f_c and f_r. After the value of m is evaluated, the elements of the T or π-networks can be found from Fig. 17.22. The variation of attenuation for a low pass m-derived section can be verified from $\alpha = 2\cosh^{-1}\sqrt{Z_1/4Z_2}$ for $f_c < f < f_\infty$. For $Z_1 = j\omega L$ and $Z_2 = -j/\omega C$ for the prototype.

$$\therefore \qquad \alpha = 2\cosh^{-1}\frac{m\dfrac{f}{f_c}}{\sqrt{1-\left(\dfrac{f}{f_\infty}\right)^2}}$$

and
$$\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}} = 2\sin^{-1}\frac{m\dfrac{f}{f_c}}{\sqrt{1-\left(\dfrac{f}{f_c}\right)^2(1-m)^2}}$$

Figure 17.23 shows the variation of α, β and Z_0 with respect to frequency for an m-derived low pass filter.

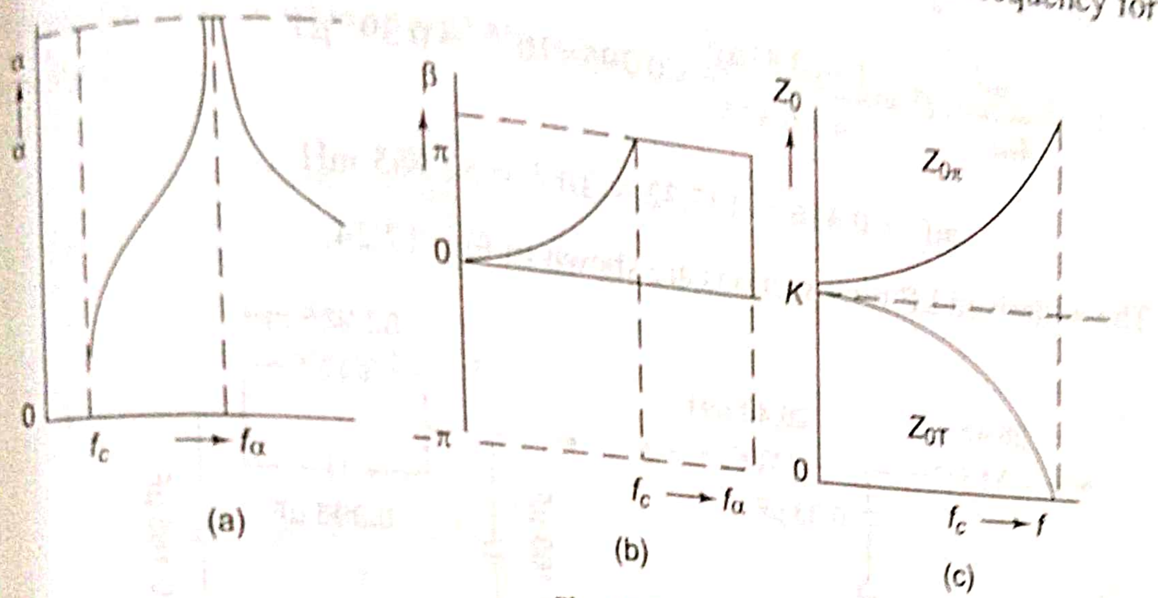


Fig. 17.23

Example 17.3 *Design a m-derived low pass filter having cut-off frequency of 1 kHz, design impedance of 400 Ω, and the resonant frequency 1100 Hz.*

Solution $k = 400\ \Omega, f_c = 1000\text{ Hz}; f_\alpha = 1100\text{ Hz}$

From Eq. 17.30

$$m = \sqrt{1-\left(\frac{f_c}{f_\alpha}\right)^2} = \sqrt{1-\left(\frac{1000}{1100}\right)^2} = 0.416$$

Let us design the values of L and C for a low pass, K-type filter (prototype filter). Thus,

$$L = \frac{k}{\pi f_c} = \frac{400}{\pi\times 1000} = 127.32\text{ mH}$$

$$C = \frac{1}{\pi k f_c} = \frac{1}{\pi\times 400\times 1000} = 0.795\ \mu\text{F}$$

The elements of m-derived low pass sections can be obtained with reference to Fig. 17.22.

Thus the T-section elements are

$$\frac{mL}{2} = \frac{0.416\times 127.32\times 10^{-3}}{2} = 26.48\text{ mH}$$

$$mC = 0.416\times 0.795\times 10^{-6} = 0.33\ \mu\text{F}$$

$$\frac{1-m^2}{4m}L = \frac{1-(0.416)^2}{4 \cdot 0.416} \times 127.32 \times 10^{-3} = 63.27 \text{ mH}$$

The π-section elements are

$$\frac{mC}{2} = \frac{0.416 \times 0.795 \times 10^{-6}}{2} = 0.165\ \mu\text{F}$$

$$\frac{1-m^2}{4m} \times C = \frac{1-(0.416)^2}{4 \times 0.416} \times 0.795 \times 10^{-6} = 0.395\ \mu\text{F}$$

$$mL = 0.416 \times 127.32 \times 10^{-3} = 52.965 \text{ mH}$$

The m-derived LP filter sections are shown in Fig. 17.24.

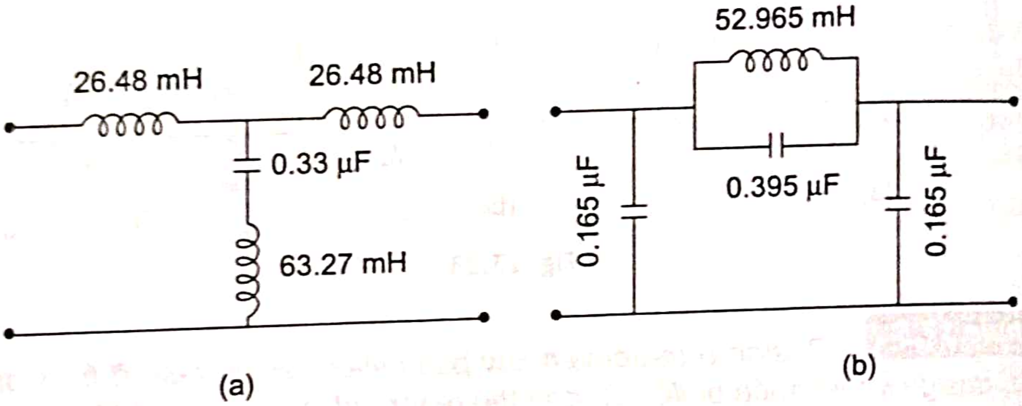


Fig. 17.24

m-derived High Pass Filter

In Fig. 17.25 both m-derived high pass T and π-sections are shown.

If the shunt arm in T-section is series resonant, it offers minimum or zero impedance. Therefore, the output is zero and, thus, at resonance frequency, or the frequency corresponds to infinite attenuation.

$$\omega_r \frac{L}{m} = \frac{1}{\omega_r \frac{4m}{1-m^2} C}$$

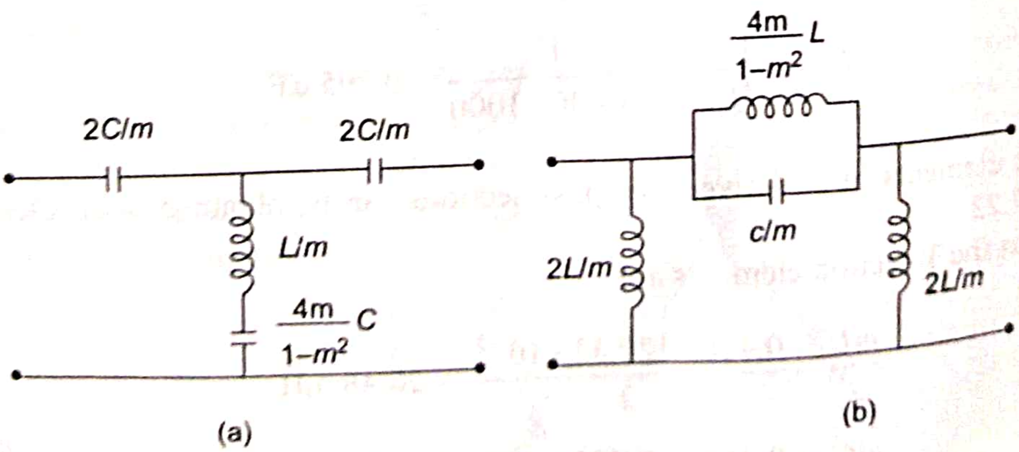


Fig. 17.25

$$\omega_r^2 = \omega_\infty^2 = \frac{1}{\frac{L}{m}\frac{4m}{1-m^2}C} = \frac{1-m^2}{4LC}$$

$$\omega_\infty = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \text{ or } f_\infty = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

rom Eq. 17.25, the cut-off frequency f_c of a high pass prototype filter is n by

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$f_\infty = f_c\sqrt{1-m^2} \tag{17.32}$$

$$m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2} \tag{17.33}$$

Similarly, for the m-derived π-section, the resonant circuit is constituted by the es arm inductance and capacitance. Thus, at f_∞

$$\frac{4m}{1-m^2}\omega_r L = \frac{1}{\frac{\omega_r}{m}C}$$

$$\omega_r^2 = \omega_\infty^2 = \frac{1-m^2}{4LC}$$

$$\omega_\infty = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \text{ or } f_\infty = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

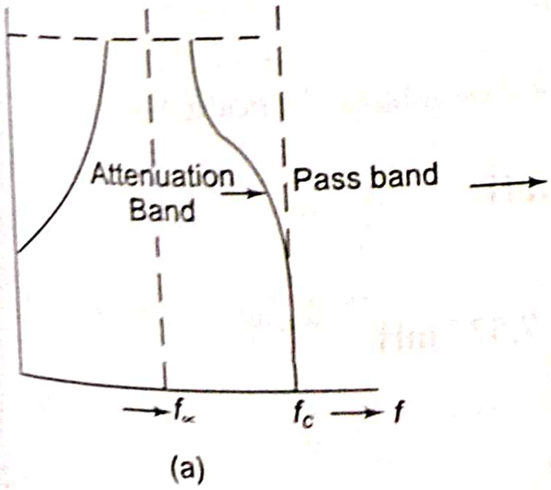


Fig. 17.26

Thus, the frequency corresponding to infinite attenuation is the same for both sections.

Equation 17.33 may be used to determine m for a given f_∞ and f_c. The elements of the m-derived high pass T or π-sections can be found from Fig. 17.25. The variation of α, β and Z_0 with frequency is shown in Fig. 17.26.

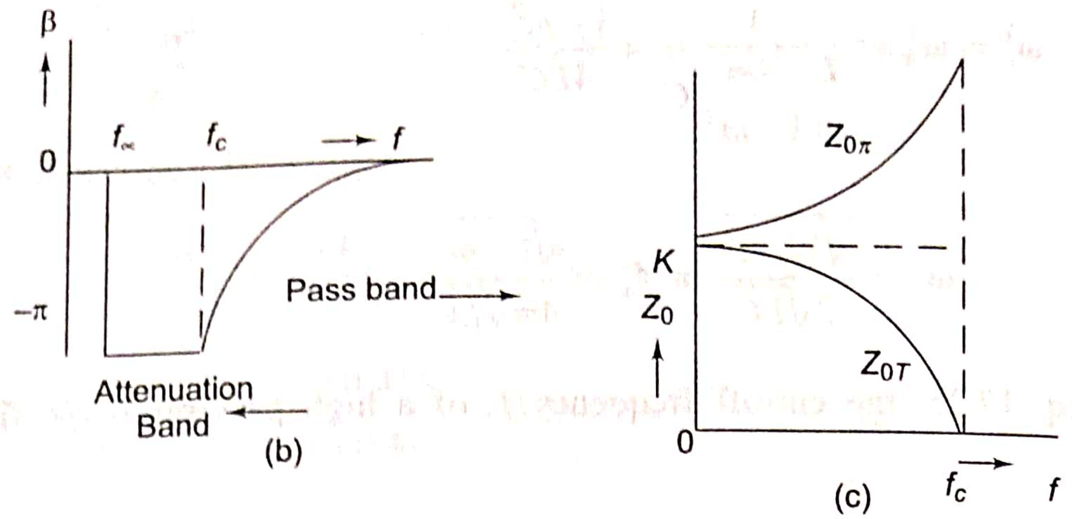


Fig. 17.26

Example 17.4 *Design a m-derived highpass filter with a cut-off frequency of 10 kHz; design impedance of 5 Ω and m = 0.4.*

Solution For the prototype high pass filter,

$$L = \frac{k}{4\pi f_c} = \frac{500}{4\times\pi\times10000} = 3.978 \text{ mH}$$

$$C = \frac{1}{4\pi k f_c} = \frac{1}{4\pi\times500\times10000} = 0.0159\ \mu\text{F}$$

The elements of m-derived high pass sections can be obtained with reference to Fig. 17.25. Thus, the T-section elements are

$$\frac{2C}{m} = \frac{2\times0.0159\times10^{-6}}{0.4} = 0.0795\ \mu\text{F}$$

$$\frac{L}{m} = \frac{3.978\times10^{-3}}{0.4} = 9.945 \text{ mH}$$

$$\frac{4m}{1-m^2}C = \frac{4\times0.4}{1-(0.4)^2}\times0.0159\times10^{-6} = 0.0302\ \mu\text{F}$$

The π-section elements are

$$\frac{2L}{m} = \frac{2\times0.0159\times10^{-3}}{0.4} = 19.89 \text{ mH}$$

$$\frac{4m}{1-m^2}\times L = \frac{4\times0.4}{1-(0.4)^2}\times3.978\times10^{-3} = 7.577 \text{ mH}$$

$$\frac{C}{m} = \frac{0.0159}{0.4}\times10^{-6} = 0.0397\ \mu\text{F}$$

T and π sections of the m-derived highpass filter are shown in Fig. 17.27.

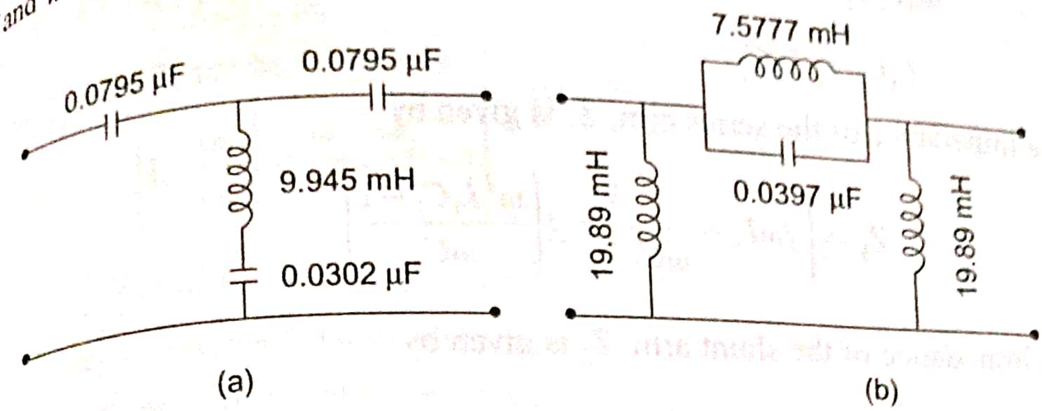


Fig. 17.27

17.9 BAND PASS FILTER

As already explained in Section 17.1, a band pass filter is one which attenuates all frequencies below a lower cut-off frequency f_1 and above an upper cut-off frequency f_2. Frequencies lying between f_1 and f_2 comprise the pass band, and are transmitted with zero attenuation. A band pass filter may be obtained by using a low pass filter followed by a high pass filter in which the cut-off frequency of the LP filter is above the cut-off frequency of the HP filter, the over lap thus allowing only a band of frequencies to pass. This is not economical in practice; it is more economical to combine the low and high pass functions into a single filter section.

Consider the circuit in Fig. 17.28, each arm has a resonant circuit with same resonant frequency, i.e. the resonant frequency of the series arm and the resonant frequency of the shunt arm are made equal to obtain the band pass characteristic.

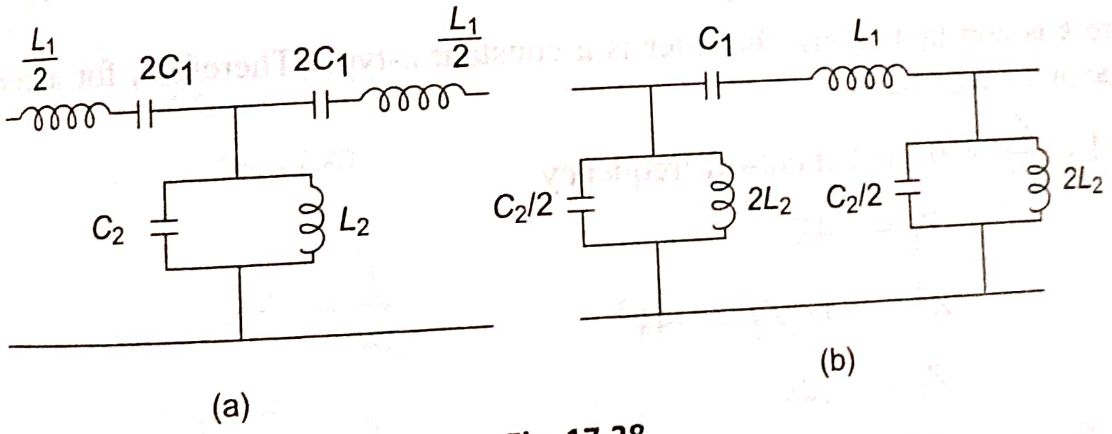


Fig. 17.28

For this condition of equal resonant frequencies.

$$\omega_0 \frac{L_1}{2} = \frac{1}{2\omega_0 C_1} \quad \text{for the series arm} \tag{17.34}$$

from which, $\omega_0^2 L_1 C_1 = 1$

and

$$\frac{1}{\omega_0 C_2} = \omega_0 L_2 \quad \text{for the shunt arm} \tag{17.35}$$

from which, $\omega_0^2 L_2 C_2 = 1$

$$\omega_0^2 L_1 C_1 = 1 = \omega_0^2 L_2 C_2$$

$$L_1 C_1 = L_2 C_2 \tag{17.36}$$

The impedance of the series arm, Z_1 is given by

$$Z_1 = \left(j\omega L_1 - \frac{j}{\omega C_1}\right) = j\left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right)$$

The impedance of the shunt arm, Z_2 is given by

$$Z_2 = \frac{j\omega L_2 \dfrac{1}{j\omega C_2}}{j\omega L_2 + \dfrac{1}{j\omega C_2}} = \frac{j\omega L_2}{1 - \omega^2 L_2 C_2}$$

$$Z_1 Z_2 = j\left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right)\left(\frac{j\omega L_2}{1 - \omega^2 L_2 C_2}\right)$$

$$= \frac{-L_2}{C_1}\left(\frac{\omega^2 L_1 C_1 - 1}{1 - \omega^2 L_2 C_2}\right)$$

From Eq. 17.36, $L_1 C_1 = L_2 C_2$

$$Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = k^2$$

where k is constant. Thus, the filter is a constant k-type. Therefore, for a constant k-type in the pass band.

$-1 < \dfrac{Z_1}{4Z_2} < 0$, and at cut-off frequency

$$Z_1 = -4Z_2$$

$$Z_1^2 = -4Z_1 Z_2 = -4k^2$$

$$\therefore \quad Z_1 = \pm j2k$$

i.e. the value of Z_1 at lower cut-off frequency is equal to the negative of the value of Z_1 at the upper cut-off frequency.

$$\therefore \quad \left(\frac{1}{j\omega_1 C_1} + j\omega_1 L_1\right) = -\left(\frac{1}{j\omega_2 C_1} + j\omega_2 L_1\right)$$

or

$$\left(\omega_1 L_1 - \frac{1}{\omega_1 C_1}\right) = \left(\frac{1}{\omega_2 C_1} - \omega_2 L_1\right)$$

$$(1 - \omega_1^2 L_1 C_1) = \frac{\omega_1}{\omega_2}(\omega_2^2 L_1 C_1 - 1) \tag{17.37}$$

From Eq. 17.34, $L_1C_1 = \frac{1}{\omega_0^2}$

Hence Eq. 17.37 may be written as

$$\left(1-\frac{\omega_1^2}{\omega_0^2}\right) = \frac{\omega_1}{\omega_2}\left(\frac{\omega_2^2}{\omega_0^2}-1\right)$$

$$(\omega_0^2-\omega_1^2)\omega_2 = \omega_1(\omega_2^2-\omega_0^2)$$

$$\omega_0^2\omega_2-\omega_1^2\omega_2 = \omega_1\omega_2^2-\omega_1\omega_0^2$$

$$\omega_0^2(\omega_1+\omega_2) = \omega_1\omega_2(\omega_2+\omega_1)$$

$$\omega_0^2 = \omega_1\omega_2$$

$$f_0 = \sqrt{f_1 f_2} \quad (17.38)$$

$$Z_1 = -\,2jk$$

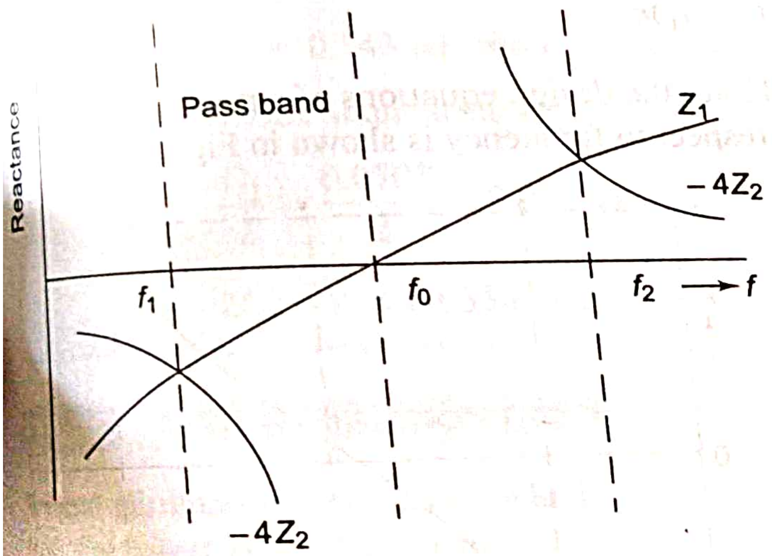


Fig. 17.29

Thus, the resonant frequency is the geometric mean of the cut-off frequencies. The variation of the reactances with respect to frequency is shown in Fig. 17.29.

Design If the filter is terminated in a load resistance $R = K$, then at the lower cut-off frequency

$$\left(\frac{1}{j\omega_1C_1}+j\omega_1L_1\right) = -2jk$$

$$\frac{1}{\omega_1C_1}-\omega_1L_1 = 2k$$

$$1-\omega^2{}_1C_1L_1 = 2k\omega_1C_1$$

Since

$$L_1C_1 = \frac{1}{\omega_0^2}$$

$$1-\frac{\omega_1^2}{\omega_0^2} = 2k\omega_1C_1$$

or

$$1-\left(\frac{f_1}{f_0}\right)^2 = 4\pi kf_1C_1$$

$$1-\frac{f_1^2}{f_1f_2} = 4\pi kf_1C_1 \quad (\because f_0 = \sqrt{f_1f_2})$$

$$f_2-f_1 = 4\pi kf_1f_2C_1 \quad (17.39)$$

$$C_1 = \frac{f_2-f_1}{4\pi kf_1f_2}$$

Since $$L_1 C_1 = \frac{1}{\omega_0^2}$$

$$L_1 = \frac{1}{\omega_0^2 C_1} = \frac{4\pi k f_1 f_2}{\omega_0^2 (f_2 - f_1)}$$

$$L_1 = \frac{k}{\pi(f_2 - f_1)} \tag{17.40}$$

To evaluate the values for the shunt arm, consider the equation

$$Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2} = k^2$$

$$\therefore \quad L_2 = C_1 k^2 = \frac{(f_2 - f_1)k}{4\pi f_1 f_2} \tag{17.41}$$

and $$C_2 = \frac{L_1}{k^2} = \frac{1}{\pi(f_2 - f_1)k} \tag{17.42}$$

Equations 17.39 through 17.42 are the design equations of a prototype band pass filter. The variation of α, β with respect to frequency is shown in Fig. 17.30.

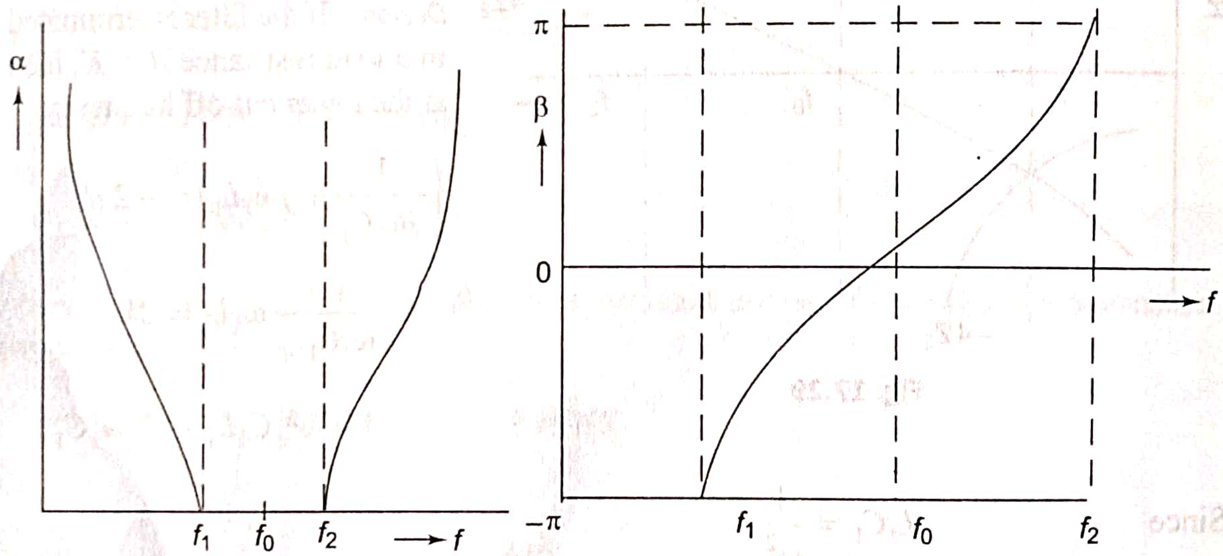


Fig. 17.30

Example 17.5 *Design k-type band pass filter having a design impedance of 500 Ω and cut-off frequencies 1 kHz and 10 kHz.*

Solution $k = 500\ \Omega; f_1 = 1000$ Hz; $f_2 = 10000$ Hz

From Eq. 17.40,

$$L_1 = \frac{k}{\pi(f_2 - f_1)} = \frac{500}{\pi 9000} = \frac{55.55}{\pi} \text{mH} = 16.68 \text{ mH}$$

From Eq. 17.39,

$$C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2} = \frac{9000}{4 \times \pi \times 500 \times 1000 \times 10000} = 0.143\ \mu\text{F}$$

From Eq. 17.41,

$$L_2 = C_1 k^2 = 3.57 \text{ mH}$$

From Eq. 17.42,

$$C_2 = \frac{L_1}{k^2} = 0.0707\,\mu\text{F}$$

Each of the two series arms of the constant k, T-section filter is given by

$$\frac{L_1}{2} = \frac{17.68}{2} = 8.84 \text{ mH}$$

$$2C_1 = 2 \times 0.143 = 0.286\ \mu\text{F}$$

And the shunt arm elements of the network are given by

$$C_2 = 0.0707\ \mu\text{F and } L_2 = 3.57 \text{ mH}$$

For the constant-k, π section filter the elements of the series arm are

$$C_1 = 0.143\ \mu\text{F and } L_1 = 16.68 \text{ mH}$$

The elements of the shunt arms are

$$\frac{C_2}{2} = \frac{0.0707}{2} = 0.035\ \mu\text{F}$$

$$2L_2 = 2 \times 0.0358 = 0.0716 \text{ H}$$

17.10 BAND ELIMINATION FILTER

A band elimination filter is one which passes without attenuation all frequencies less than the lower cut-off frequency f_1, and greater than the upper cut-off frequency f_2. Frequencies lying between f_1 and f_2 are attenuated. It is also known as band stop filter. Therefore, a band stop filter can be realized by connecting a low pass filter in parallel with a highpass section, in which the cut-off frequency of low pass filter is below that of a high pass filter. The configurations of T and π constant k band stop sections are shown in Fig. 17.31. The band elimination filter is designed in the same manner as is the band pass filter.

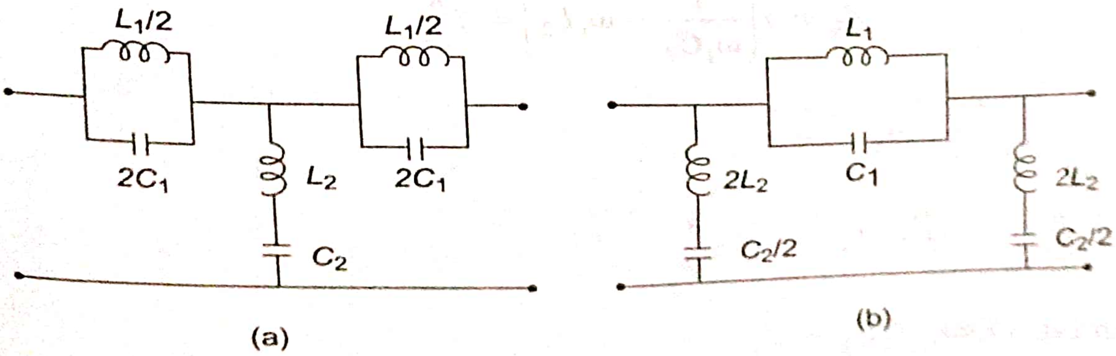


Fig. 17.31

As for the band pass filter, the series and shunt arms are chosen to resonate at the same frequency ω_0. Therefore, from Fig. 17.31 (a), for the condition of equal resonant frequencies

$$\frac{\omega_0 L_1}{2} = \frac{1}{2\omega_0 C_1} \quad \text{for the series arm}$$

or

$$\omega_0^2 = \frac{1}{L_1 C_1} \tag{17.43}$$

$$\omega_0 L_2 = \frac{1}{\omega_0 C_2} \quad \text{for the shunt arm}$$

$$\omega_0^2 = \frac{1}{L_2 C_2} \tag{17.44}$$

$$\frac{1}{L_1 C_1} = \frac{1}{L_2 C_2} = k$$

Thus

$$L_1 C_1 = L_2 C_2 \tag{17.45}$$

It can be also verified that

$$Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1} = k^2 \tag{17.46}$$

and

$$f_0 = \sqrt{f_1 f_2} \tag{17.47}$$

At cut-off frequencies, $Z_1 = -4Z_2$

Multiplying both sides with Z_2, we get

$$Z_1 Z_2 = -4Z_2^2 = k^2$$

$$Z_2 = \pm j\frac{k}{2} \tag{17.48}$$

If the load is terminated in a load resistance, $R = k$, then at lower cut-off frequency

$$Z_2 = j\left(\frac{1}{\omega_1 C_2} - \omega_1 L_2\right) = j\frac{k}{2}$$

$$\frac{1}{\omega_1 C_2} - \omega_1 L_2 = \frac{k}{2}$$

$$1 - \omega_1^2 C_2 L_2 = \omega_1 C_2 \frac{k}{2}$$

From Eq. 17.44, $L_2 C_2 = \dfrac{1}{\omega_0^2}$

$$1-\frac{\omega_1^2}{\omega_0^2}=\frac{k}{2}\omega_1 C_2$$

$$1-\left(\frac{f_1}{f_0}\right)^2=k\pi f_1 C_2$$

$$C_2=\frac{1}{k\pi f_1}\left[1-\left(\frac{f_1}{f_0}\right)^2\right]$$

Since $$f_0=\sqrt{f_1 f_2}$$

$$C_2=\frac{1}{k\pi}\left[\frac{1}{f_1}-\frac{1}{f_2}\right]$$

$$C_2=\frac{1}{k\pi}\left[\frac{f_2-f_1}{f_1 f_2}\right] \tag{17.49}$$

From Eq. 17.44, $$\omega_0^2=\frac{1}{L_2C_2}$$

$$L_2=\frac{1}{\omega_0^2C_2}=\frac{\pi k f_1 f_2}{\omega_0^2(f_2-f_1)}$$

Since $$f_0=\sqrt{f_1 f_2}$$

$$L_2=\frac{k}{4\pi(f_2-f_1)} \tag{17.50}$$

Also from Eq. 17.46,

$$k^2=\frac{L_1}{C_2}=\frac{L_2}{C_1}$$

$$\therefore \quad L_1=k^2C_2=\frac{k}{\pi}\left(\frac{f_2-f_1}{f_1 f_2}\right) \tag{17.51}$$

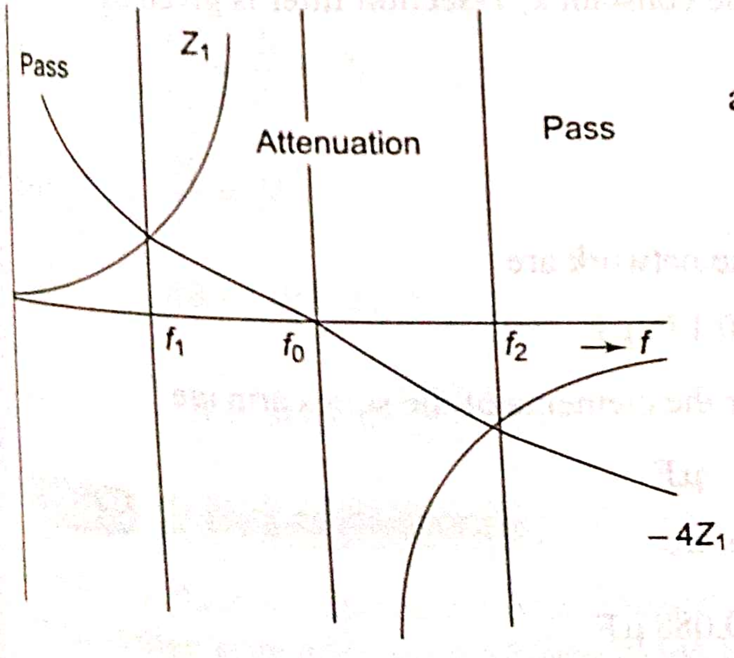


Fig. 17.32

$$\text{and } C_1=\frac{L_2}{k^2} \tag{17.52}$$

$$=\frac{1}{4\pi k(f_2-f_1)}$$

The variation of the reactances with respect to frequency is shown in Fig. 17.32.

Equation 17.49 through Eq. 17.52 are the design equations of a prototype band elimination filter. The variation of α, β with respect to frequency is shown in Fig. 17.33.

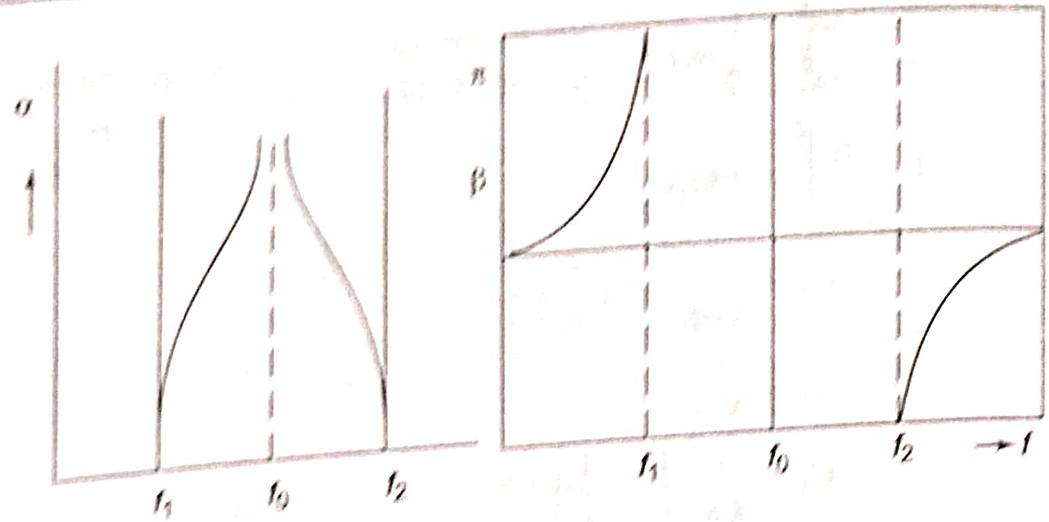


Fig. 17.33

Example 17.6 *Design a band elimination filter having a design impedance of 600 Ω and cut-off frequencies $f_1 = 2$ kHz and $f_2 = 6$ kHz.*

Solution $(f_2 - f_1) = 4$ kHz

Makinkg use of the Eqs 17.49 through 17.52 in Section 17.10, we have

$$L_1 = \frac{k}{\pi}\left(\frac{f_2 - f_1}{f_2 f_1}\right) = \frac{600 \times 4000}{\pi \times 2000 \times 6000} = 63 \text{ mH}$$

$$C_1 = \frac{1}{4\pi k(f_2 - f_1)} = \frac{1}{4 \times \pi \times 600(4000)} = 0.033\ \mu\text{F}$$

$$L_2 = \frac{1}{4\pi k(f_2 - f_1)} = \frac{600}{4\pi(4000)} = 12 \text{ mH}$$

$$C_2 = \frac{1}{k\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right] = \frac{1}{600 \times \pi}\left[\frac{4000}{2000 \times 6000}\right] = 0.176\ \mu\text{F}$$

Each of the two series arms of the constant k, T-section filter is given by

$$\frac{L_1}{2} = 31.5 \text{ mH}$$

$$2C_1 = 0.066\ \mu\text{F}$$

And the shunt arm elements of the network are

$$L_2 = 12 \text{ mH and } C_2 = 0.176\ \mu\text{F}$$

For the constant k, π-section filter the elements of the series arm are

$$L_1 = 63 \text{ mH},\ C_1 = 0.033\ \mu\text{F}$$

and the elements of the shunt arms are

$$2L_2 = 24 \text{ mH and } \frac{C_2}{2} = 0.088\ \mu\text{F}$$

17.11 ATTENUATORS

An attenuator is a two-port resistive network and is used to reduce the signal level by a given amount. In a number of applications, it is necessary to introduce a specified loss between source and a matched load without altering the impedance relationship. Attenuators may be used for this purpose. Attenuators may be symmetrical or asymmetrical, and can be either fixed or variable. A fixed attenuator with constant attenuation is called a *pad*. Variable attenuators are used as volume controls in radio broadcasting sections. Attenuators are also used in laboratory to obtain small value of voltage or current for testing circuits.

The increase or decrease in power due to insertion or substitution of a new element in a network can be conveniently expressed in decibels (dB), or in nepers. In other words, attenuation is expressed either in decibels (dB) or in nepers. Accordingly, the attenuation offered by a network in decibels is

$$\text{Attenuation in dB} = 10\log_{10}\left(\frac{P_1}{P_2}\right) \tag{17.53}$$

where P_1 is the input power and P_2 is the output power.

For a properly matched network, both terminal pairs are matched to the characteristic resistance, R_0 of the attenuator.

Hence,
$$\frac{P_1}{P_2} = \frac{I_1^2 R_0}{I_2^2 R_0} = \frac{I_1^2}{I_2^2} \tag{17.54}$$

where I_1 is the input current and I_2 is the output current leaving the port.

or
$$\frac{P_1}{P_2} = \frac{V_1^2}{V_2^2} \tag{17.55}$$

where V_1 is the voltage at port 1 and V_2 is the voltage at port 2

$$\text{Hence, attenuation in dB} = 20\log_{10}\left(\frac{V_1}{V_2}\right) \tag{17.56}$$

$$= 20\log_{10}\left(\frac{I_1}{I_2}\right) \tag{17.57}$$

If
$$\frac{V_1}{V_2} = \frac{I_1}{I_2} = N \tag{17.58}$$

then
$$\frac{P_1}{P_2} = N^2$$

and
$$\text{dB} = 20\log_{10} N \tag{17.59}$$

or
$$N = \text{antilog}\left(\frac{\text{dB}}{20}\right) \tag{17.60}$$

17.12 T-TYPE ATTENUATOR

Basically, there are four types of attenuators, T, π, lattice and bridged T-type. The basic design principles are discussed in the following Sections. Figure 17.34 shows the symmetrical T-attenuator. An attenuator is to be designed for desired values of characteristic resistance, R_0 and attenuation.

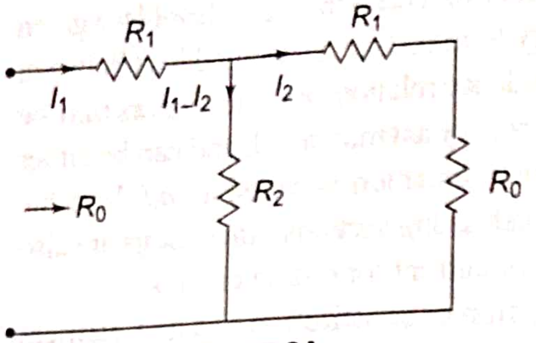


Fig. 17.34

The values of the arms of the network can be specified in terms of characteristic impedance, Z_0, and propagation constant, γ, of the network. The network in the figure is a symmetrical resistive circuit; hence $Z_0 = R_0$ and $\gamma = \alpha$. The design equations can be obtained by applying Kirchhoff's law to the network in Fig. 17.34.

$$R_2 (I_1 - I_2) = I_2 (R_1 + R_0)$$

$$I_2 (R_2 + R_1 + R_0) = I_1 R_2$$

$$\frac{I_1}{I_2} = \frac{R_1 + R_0 + R_2}{R_2} = N \tag{17.61}$$

The characteristic impedance of the attenuator is R_0 when it is terminated in a load of R_0

Hence, $$R_0 = R_1 + \frac{R_2(R_1 + R_0)}{R_1 + R_0 + R_2}$$

Substituting Eq. 17.61, we have

$$R_0 = R_1 + \frac{(R_1 + R_0)}{N}$$

$$NR_0 = NR_1 + R_1 + R_0$$

$$R_0(N - 1) = R_1 (N + 1)$$

$$R_1 = \frac{R_0(N-1)}{N+1} \tag{17.62}$$

From Eq. 17.61, we have

$$NR_2 = R_1 + R_0 + R_2$$

$$(N - 1)R_2 = (R_1 + R_0)$$

Substituting the value of R_1 from Eq. 17.62, we have

$$(N-1)R_2 = R_0 \frac{(N-1)}{N+1} + R_0$$

$$(N-1)R_2 = \frac{2NR_0}{(N+1)}$$

$$R_2 = \frac{2NR_0}{N^2 - 1} \tag{17.63}$$

Equations 17.62 and 17.63 are the design equations for the symmetrical T-attenuator.

Example 17.7 Design a T-pad attenuator to give an attenuation of 60 dB and to work in a line of 500 Ω impedance.

Solution
$$N = \frac{I_1}{I_2} = \text{antilog}\frac{D}{20}$$
$$= \text{antilog}\frac{60}{20} = 1000$$

Each of the series arm is given by
$$R_1 = \frac{R_0(N-1)}{N+1} = 500\frac{(1000-1)}{(1000+1)} = 499\ \Omega$$

The shunt arm resistor R_2 is given by
$$R_2 = \frac{2N}{N^2-1} = R_0 = \frac{2\times 1000}{(1000)^2-1}\times 500 = 1\ \Omega$$

17.13 π-TYPE ATTENUATOR

Figure 17.35 shows symmetrical attenuator. The series and shunt arm of the attenuator can be specified in terms of Z_0 and propagation constant γ. In this case also, the network is formed by resistors and is symmetrical, hence $Z_0 = R_0$ and $\gamma = \alpha$. From the fundamental equations, we have

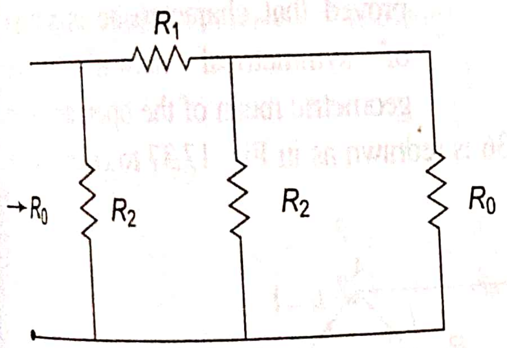


Fig. 17.35

$$R_1 = R_0 \sinh \alpha \tag{17.64}$$
$$R_2 = R_0 \coth \alpha/2 \tag{17.65}$$
$$\therefore\ R_1 = R_0\frac{e^{\alpha}-e^{-\alpha}}{2} \tag{17.66}$$

By definition of propagation constant
$$e^{\gamma} = \frac{I_1}{I_2} = N$$

Here $\qquad \gamma = \alpha$ and $e^{\alpha} = N$

Therefore, Eq. 17.66 can be written as
$$R_1 = R_0\frac{N-\frac{1}{N}}{2} = R_0\frac{N^2-1}{2N} \tag{17.67}$$

Similarly, from Eq. 17.65,
$$R_2 = R_0\frac{\cosh\alpha/2}{\sinh\alpha/2} = R_0\frac{e^{\alpha/2}+e^{-\alpha/2}}{e^{\alpha/2}-e^{-\alpha/2}}$$
$$R_2 = R_0\frac{e^{\alpha}+1}{e^{\alpha}-1} = R_0\frac{(N-1)}{(N+1)} \tag{17.68}$$

Equations 17.67 and 17.68 are the design equations for the symmetrical π-attenuator.

Example 17.8 *Design a π-type attenuator to give 20 dB attenuation and to have a characteristic impedance of 100 Ω.*

Solution Given $R_0 = 100\ \Omega$, $D = 20$ dB.

$$N = \text{Antilog}\ \frac{D}{20} = 10$$

$$R_1 = R_0 \frac{(N^2-1)}{2N} = 100\frac{(10^2-1)}{2+10} = 495\ \Omega$$

$$R_2 = R_0 \frac{(N+1)}{(N-1)} = 100\left(\frac{10+1}{10-1}\right) = 122.22\ \Omega$$

17.14 LATTICE ATTENUATOR

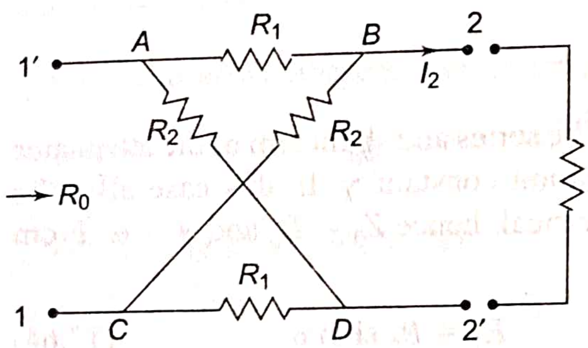


Fig. 17.36

A symmetrical resistance lattice is shown in Fig. 17.36. The series and the diagonal arm of the network can be specified in terms of the characteristic impedance Z_0 and propagation constant γ.

It has already been stated and proved that characteristic impedance of symmetrical network is the geometric mean of the open and short circuit impedance. The circuit in Fig. 17.36 is redrawn as in Fig. 17.37 to calculate the open and short circuit impedances.

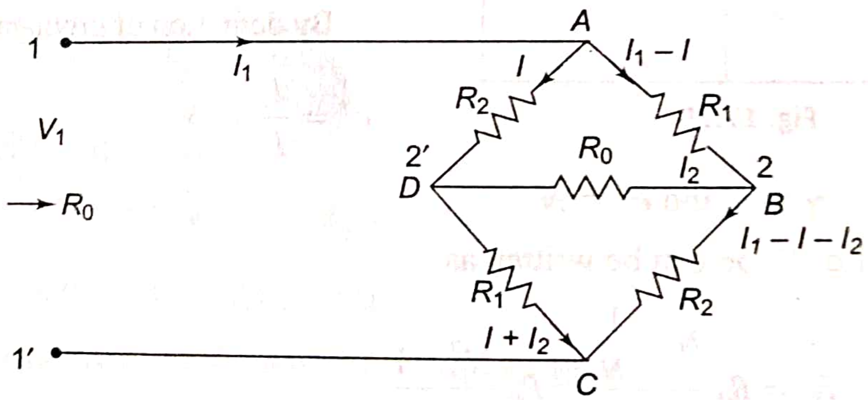


Fig. 17.37

Thus, $$Z_{sc} = \frac{2R_1R_2}{R_1+R_2}$$

$$Z_{0c} = \frac{R_1+R_2}{2}$$

Hence, $$Z_0 = R_0 = \sqrt{Z_{0c}Z_{sc}}$$

$$R_0 = \sqrt{R_1R_2}$$

In Fig. 17.37 the input impedance at 1-1′ is R_0 when the network is terminated in R_0 at 2-2′. By applying Kirchhoff's voltage law, we get

$$V_1 = I_1R_0 = (I_1 - I)R_1 + I_2R_0 + (1 + I_2)R_1$$

$$I_1R_0 = R_1(I_1 + I_2) + I_2R_0$$

$$I_1(R_0 - R_1) = I_2(R_1 + R_0)$$

$$\frac{I_1}{I_2} = \frac{R_1 + R_0}{R_0 - R_1} = \frac{1 + \dfrac{R_1}{R_0}}{1 - \dfrac{R_1}{R_0}} \tag{17.69}$$

$$N = e^{\alpha} = \frac{I_1}{I_2} = \frac{1 + \dfrac{R_1}{R_0}}{1 - \dfrac{R_1}{R_0}} \tag{17.70}$$

$$e^{\alpha} = \frac{1 + \sqrt{R_1 / R_2}}{1 - \sqrt{R_1 / R_2}}$$

The propagation constant $$\alpha = \log\left[\frac{1 + \sqrt{\dfrac{R_1}{R_2}}}{1 - \sqrt{\dfrac{R_1}{R_2}}}\right] \tag{17.71}$$

From Eq. 17.70

$$N\left(1 - \frac{R_1}{R_0}\right) = \left(1 + \frac{R_1}{R_0}\right)$$

$$R_1 = R_0\frac{(N-1)}{(N+1)} \tag{17.72}$$

Similarly, we can express $$R_2 = R_0\frac{(N+1)}{(N-1)} \tag{17.73}$$

Equations 17.72 and 17.73 are the design equations for lattice attenuator.

Example 17.9 *Design a symmetrical lattice attenuator to have characteristic impedance of 800 Ω and attenuation of 20 dB.*

Solution Given $R_0 = 800\ \Omega$ and $D = 20$ dB

$$N = \text{antilog}\frac{D}{20} = \text{antilog}\frac{20}{20} = 10$$

From the design equations of lattice attenuator

Series arm resistance $R_1 = R_0 \dfrac{(N-1)}{(N+1)}$

$$= 800\frac{(10-1)}{(10+1)} = 654.545\ \Omega$$

Diagonal arm resistance $R_2 = R_0 \dfrac{(N+1)}{(N-1)}$

$$= 800\frac{(10+1)}{(10-1)} = 977.777\ \Omega$$

The resulting lattice attenuator is shown in Fig. 17.38.

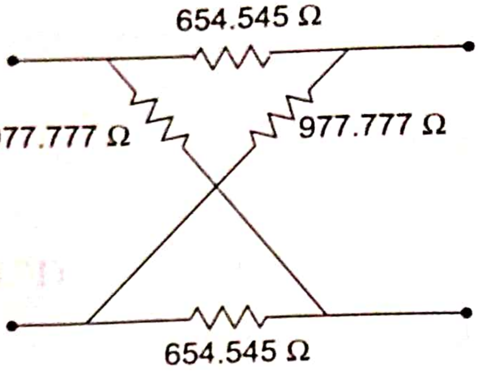


Fig. 17.38

17.15 BRIDGED-*T* ATTENUATOR

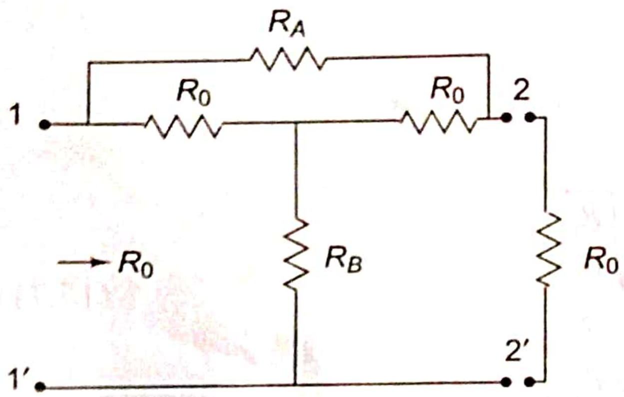


Fig. 17.39

A bridged-T attenuator is shown in Fig. 17.39. In this case also since the attenuator is formed by resistors only, $Z_0 = R_0$ and $\gamma = \alpha$. The bridged-T network may be designed to have any characteristic resistance R_0 and desired attenuation by making $R_A\ R_B = R^2{}_0$. Here R_A and R_B are variable resistances and all other resistances are equal to the characteristic resistance R_0 of the network.

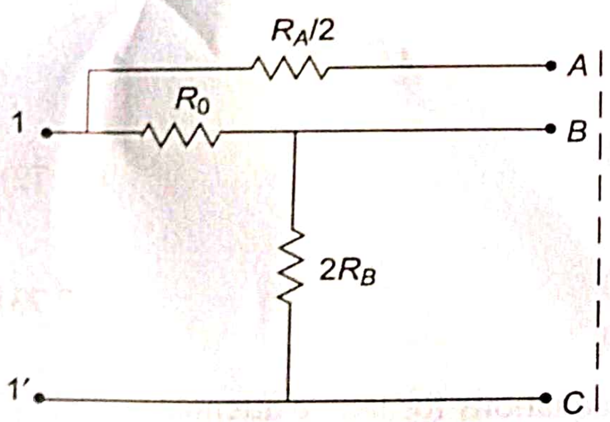


Fig. 17.40

A symmetrical resistance lattice network can be converted into an equivalent T, π or bridged-T resistance network using the bisection theorem. We can obtain the design equations of the bridged-T attenuator by bisection theorem. A bisected half sections is shown in Fig. 17.40. According to the bisection theorem, a network having mirror image symmetry can be reduced to an equivalent lattice structure. The series arm of the equivalent lattice is found by bisecting the given network into two parts, short circuiting all the cut wires and equating the series impedance of the lattice to the input impedance of the bisected network; the diagonal arm is equal to the input impedance of the bisected network when cut wires are open circuited.

From Fig. 17.40, when the cut wires A, B, C are shorted, the input resistance of the network is given by

$$R_{sc} = \frac{R_0 \times R_{A/2}}{R_0 + R_{A/2}} = \frac{R_0 R_A}{2R_0 + R_A} \qquad (17.74)$$

This resistance is equal to the series arm resistance of the lattice network shown in Fig. 17.36.

$$\therefore \quad \frac{R_0 R_A}{2R_0 + R_A} = R_1 \tag{17.75}$$

From Eq. 17.72, we have

$$R_1 = R_0 \frac{(N-1)}{(N+1)}$$

$$\text{Hence,} \quad \frac{R_0 R_A}{(2R_0 + R_A)} = R_0 \frac{(N-1)}{(N+1)}$$

From which $R_A = R_0 (N - 1)$ (17.76)

From Fig. 17.40, when the cut wires A, B, C are open, the input resistance of the network is given by

$$R_{0c} = (R_0 + 2R_B) \tag{17.77}$$

This resistance is equal to the diagonal arm resistance of the lattice network shown in Fig. 17.36.

$$\therefore \quad R_0 + 2R_B = R_2 \tag{17.78}$$

From Eq. 17.73, we have

$$R_2 = R_0 \frac{(N+1)}{(N-1)}$$

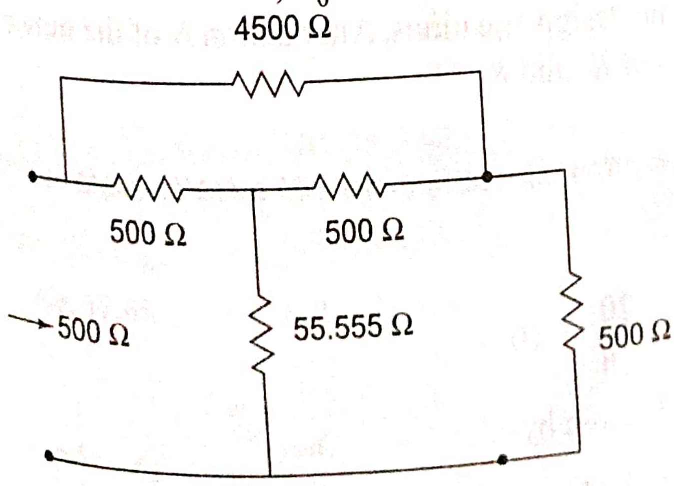
$$\text{Hence} \quad (R_0 + 2R_B) = R_0 \frac{(N+1)}{(N-1)}$$

$$\text{From which} \quad R_B = \frac{R_0}{N-1} \tag{17.79}$$

Equations 17.76 and 17.79 are the design equations for bridged-T attenuator.

Example 17.10 *Design a symmetrical bridged T - attenuator with an attenuation of 20 dB and terminated into a load of 500 Ω.*

Solution $D = 20\ dB;\ R_0 = 500\,\Omega$



$$N = \text{antilog}\frac{D}{20} = \text{antilog}\frac{20}{20} = 10$$

$$R_A = R_0(N-1) = 500(10-1) = 4500\,\Omega$$

$$R_B = \frac{R_0}{(N-1)} = \frac{500}{(10-1)} = 55.555\,\Omega$$

The desired configuration of the attenuator is shown in Fig. 17.41.

Fig. 17.41

17.16 L-TYPE ATTENUATOR

An L-type asymmetrical attenuator is shown in Fig. 17.42. The attenuator is connected between a source with source resistance $R_s = R_0$ and load resistance $R_L = R_0$.

The design equations can be obtained by applying simple laws.

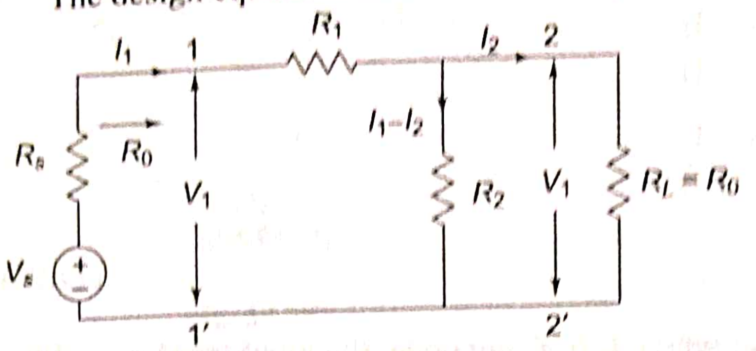


Fig. 17.42

$$V_2 = (I_1 - I_2)R_2 = I_2 R_L$$

or $I_1 R_2 = I_2(R_2 + R_L)$

$$\frac{I_1}{I_2} = \frac{R_2 + R_L}{R_2} = N \quad (17.80)$$

$$1 + \frac{R_L}{R_2} = N$$

$$R_2 = \frac{R_L}{N-1} \quad (17.81)$$

As $R_L = R_0$, Eq. 17.81 can be written as

$$R_2 = \frac{R_0}{N-1} \quad (17.82)$$

The resistance of the network as viewed from 1-1′ into the network is

$$R_0 = R_1 + \frac{R_2 R_0}{R_2 + R_0}$$

$$R_1 = \frac{R_0^2}{R_2 + R_0} \quad (17.83)$$

Substituting the value of R_2 from Eq. 17.82, we have

$$R_1 = \frac{R_0^2}{\dfrac{R_0}{N-1} + R_0} = \frac{R_0^2(N-1)}{R_0 + R_0(N-1)}$$

$$R_1 = R_0\frac{(N-1)}{N} \quad (17.84)$$

Equations 17.82 and 17.84 are the design equations. Attenuation N of the network can be varied by varying the values of R_1 and R_2.

Example 17.11 *Design a L-type attenuator to operate into a load resistance of 600 Ω with an attenuation of 20 dB.*

Solution $N = \text{antilog}\dfrac{\text{dB}}{20} = \text{antilog}\dfrac{20}{20} = 10$

The series arm of the attenuator is given by

$$R_1 = R_0\left(\frac{N-1}{N}\right) = 600\left(\frac{10-1}{10}\right) = 540\ \Omega$$

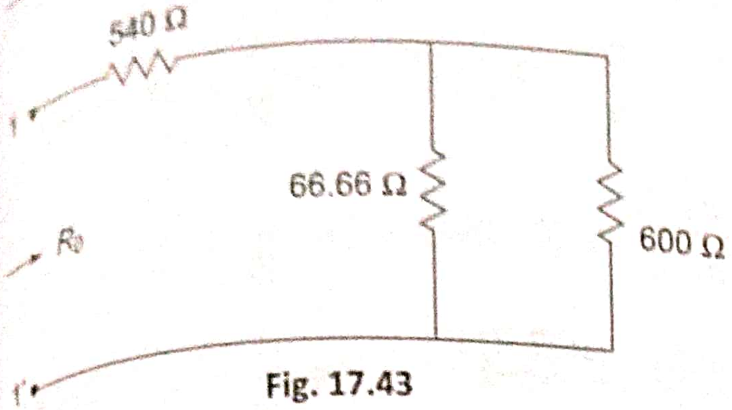


Fig. 17.43

The shunt arm of the attenuator is given by

$$R_2 = \frac{R_0}{N-1} = \frac{600}{9} = 66.66\,\Omega$$

The desired configuration of the network is shown in Fig. 17.43.

17.17 EQUALIZERS

Equalizers are networks designed to provide compensation against distortions that occur in a signal while passing through an electrical network. In general, any electrical network has attenuation distortion and phase distortion. Attenuation distortion occurs due to non-uniform attenuation against frequency characteristics. Phase distortion occurs due to phase delay against frequency characteristics. An attenuation equalizer is used to compensate attenuation distortion in any network. These equalizers are used in medium to high frequency carrier telephone systems, amplifiers, transmission lines and speech reproduction, etc. A phase equalizer is used to compensate phase distortion in any network. These equalizers are used in TV signal transmission lines and in facsimile.

17.18 INVERSE NETWORK

The geometrical mean of two impedances Z_1 and Z_2 is a real number and they are said to be inverse if

$$Z_1 Z_2 = R_0^2$$

where R_0 is a resistance

Consider $Z_1 = R_1$ and $Z_2 = R_2$

The product $Z_1\, Z_2$ is a real number

Therefore, the two impedances are said to be inverse if they satisfy the relation $Z_1 Z_2 = R_1 R_2 = R_0^2$.

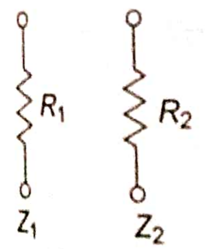


Fig. 17.44

In another case, consider $Z_1 = j\omega L$ and $Z_2 = \dfrac{1}{j\omega C}$

$$Z_1 Z_2 = \frac{j\omega L}{j\omega C} = \frac{L}{C}$$

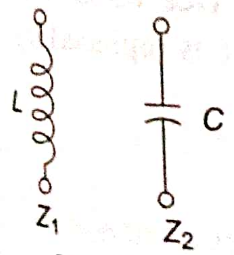


Fig. 17.45

The product $Z_1 Z_2$ is a real number

Therefore, the two impedances are inverse.

Similarly,

$$\text{Let}\quad Z_1 = R_1 + j\omega L \tag{17.85}$$

$$\text{and}\quad Z_2 = \frac{R_2 \dfrac{1}{j\omega C}}{R_2 + \dfrac{1}{j\omega C}} = \frac{-jR_2}{\omega C R_2 - j} \cdot \frac{\omega C R_2 + j}{\omega C R_2 + j} \tag{17.86}$$

$$= \frac{R_2 - j\omega C R_2^2}{1+\omega^2 C^2 R_2^2}$$

$$Z_1 Z_2 = (R_1 + j\omega L)\left(\frac{R_2 - j\omega C R_2^2}{1+\omega^2 C^2 R_2^2}\right)$$

$$= \frac{R_1 R_2 + \omega^2 R_2^2 LC + j(\omega L R_2 - \omega C R_1 R_2^2)}{1+\omega^2 C^2 R_2^2} \quad (17.87)$$

The imaginary part of the above equation must be zero.

Therefore, we get $\omega L R_2 = \omega C R_1 R_2^2$

$$\frac{L}{C} = R_1 R_2 = R_0^2 \quad (17.88)$$

The two impedances Z_1 and Z_2 are inverse, when the above condition is satisfied. An inverse network may be obtained by

(i) Converting each series branch into parallel branch and vice-versa.

(ii) Converting each resistance element R into a corresponding resistive element $\frac{R_0^2}{R}$.

(iii) Converting each inductance L into capacitance $C^1 = \frac{L}{R_0^2}$.

(iv) Converting each capacitance C into inductance $L^1 = CR_0^2$.

Example 17.12 *Obtain the inverse network of the network shown in Fig. 17.46.*

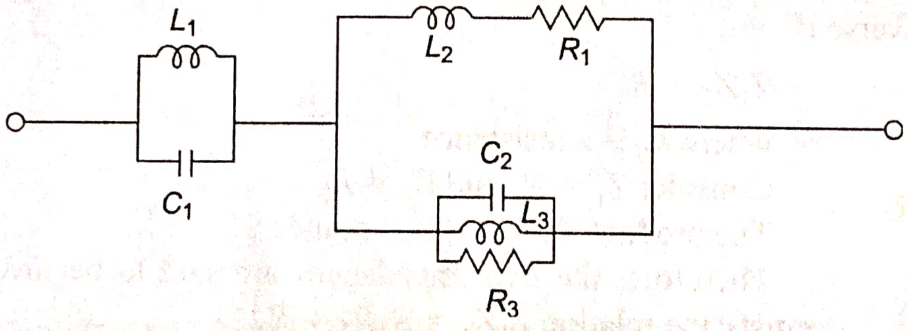
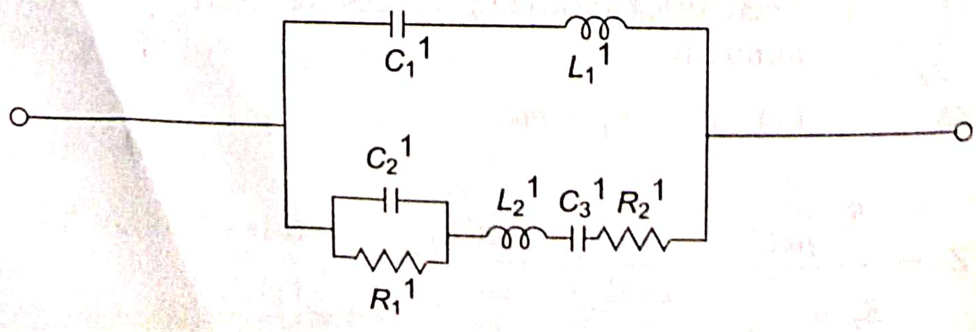


Fig. 17.46

Solution The parallel branch is converted into a series branch and vice-versa. The capacitance is replaced by inductance and vice-versa. The resistance is replaced by another resistance as shown in Fig. 17.47.



Where $C_1^1 = \frac{L_1}{R_0^2}, iL_1^1 = C_1 R_0^2, R_1^1 = \frac{R_0^2}{R_1}$

$$C_2^1 = \frac{L_2}{R_0^2}, iL_2^1 = C_2 R_0^2, R_2^1 = \frac{R_0^2}{R_2}$$

$$C_3^1 = \frac{L_3}{R_0^2} \text{ and } R_0 = \text{design impedance.}$$

17.19 SERIES EQUALIZER

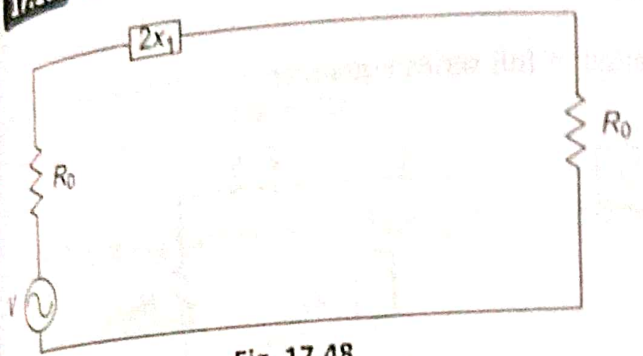


Fig. 17.48

The series equalizer is a two terminal network connected in series with a network to be corrected. (see Fig. 17.48)

Let N = Input to output power ratio of the load

D = Attenuation in decibels

R_0 = Resistance of the load as well as source

P_1 = Input power

P_1 = Load power

$2X_1$ = Reactance of the equalizer

V_{max} = Voltage applied to the network

Attenuation $D = \log_{10} N$

$$\text{or} \quad N = \text{antilog}\left(\frac{D}{10}\right) \tag{17.89}$$

$$N = \frac{\text{Maximum power delivered to the load when equlizer is not present}}{\text{Power delivered to the load when equalizer is present}}$$

$$N = \frac{P_i}{P_l}$$

$$P_i = \left(\frac{V_{max}}{2R_0}\right)^2 R_0 = \frac{V_{max}^2}{4R_0}$$

When the equalizer is connected,

$$I_1 = \frac{V_{max}}{\sqrt{(2R_0)^2 + (2X_1)^2}}$$

$$P_l = \left[\frac{V_{max}}{\sqrt{(2R_0)^2 + (2X_1)^2}}\right]^2 R_0$$

$$= \left[\frac{V_{max}^2}{4(R_0^2 + X_1^2)} \right] R_0 \tag{17.90}$$

$$\text{Therefore, } N = \frac{P_i}{P_l} = \frac{V_{max}^2 / 4R_0}{\dfrac{V_{max}^2 R_0}{4(R_0^2 + X_1^2)}} = 1 + \frac{X_1^2}{R_0^2} \tag{17.91}$$

By knowing the values of R_0 and N, X_1 can be determined.

17.20 FULL SERIES EQUALIZER

Figure 17.49 shows the configuration of full series equalizer.

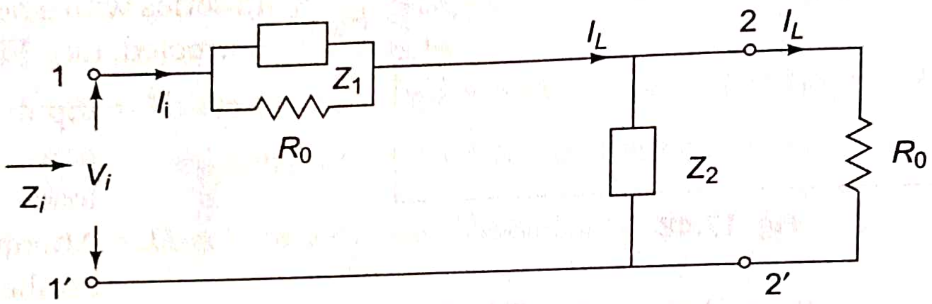


Fig. 17.49

The circuit is a constant resistance equalizer satisfying the relation $Z_1 Z_2 = R_0^2$. The input impedance is given by

$$Z_i = \frac{R_0 Z_1}{R_0 + Z_1} + \frac{R_0 Z_2}{R_0 + Z_2}$$

$$= \frac{R_0[2Z_1Z_2 + R_0(Z_1 + Z_2)]}{R_0^2 + R_0(Z_1 + Z_2) + Z_1 Z_2} \tag{17.92}$$

If we substitute $Z_1 Z_2 = R_0^2$ in the above equation

$$Z_i = R_0$$

$$|V_i| = I_i Z_i = I_i R_0$$

$$|V_l| = I_i Z_i = I_i \frac{R_0 Z_2}{R_0 + Z_2} \tag{17.93}$$

$$N = \left| \frac{V_i}{V_l} \right|^2 = \left| \frac{R_0 + Z_2}{Z_2} \right|^2 = 1 + \frac{R_0^2}{X_2^2} \tag{17.94}$$

$$= 1 + \frac{X_1^2}{R_0^2}$$

Since Z_1 and Z_2 are pure reactances and $X_1 X_2 = R_0^2$

(i) When $X_1 = \omega L$,

$$X_2 = \frac{1}{\omega C_1} \text{ since both are inverse}$$

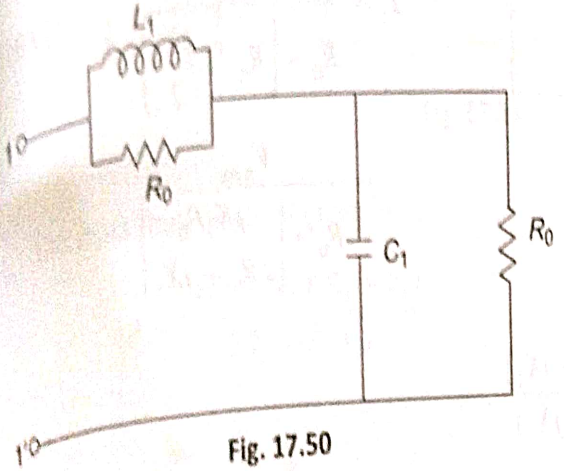


Fig. 17.50

The full series equalizer is shown in Fig. 17.50.

Where $\frac{L_1}{C_1} = R_0^2$

From the equation

$$N = 1 + \frac{X_1^2}{R_0^2}$$

$$= 1 + \frac{\omega^2 L_1^2}{R_0^2}$$

By knowing the values of N and R_0, the elemental values of L_1, C_1 may be obtained.

(ii) When $X_1 = \frac{1}{\omega C_1}$,

$$X_2 = \omega L_1$$

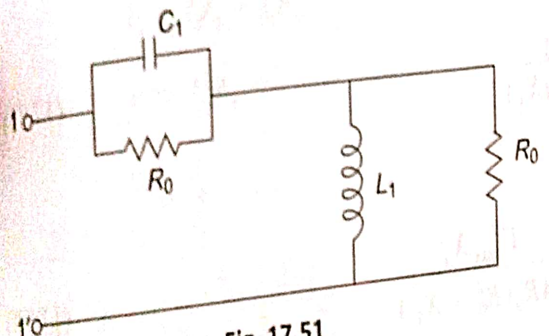


Fig. 17.51

The full series equalizer is shown in Fig. 17.51.

Here $\frac{L_1}{C_1} = R_0^2$

From the equation

$$N = 1 + \frac{R_0^2}{X_2^2} = 1 + \frac{R_0^2}{\omega^2 L_1^2}$$

By knowing the values of N and R_0, the values of L_1, C_1 may be obtained.

17.21 SHUNT EQUALIZER

The shunt equalizer is a two terminal network connected in shunt with a network to be corrected.

Let N = Input to output power ratio
D = Attenuation in decibels
R_0 = Source resistance/load resistance
I_s = Source current
I_l = Load current
P_i = Input power
P_l = Load power
$\frac{X_1}{2}$ = Reactance of shunt equalizer

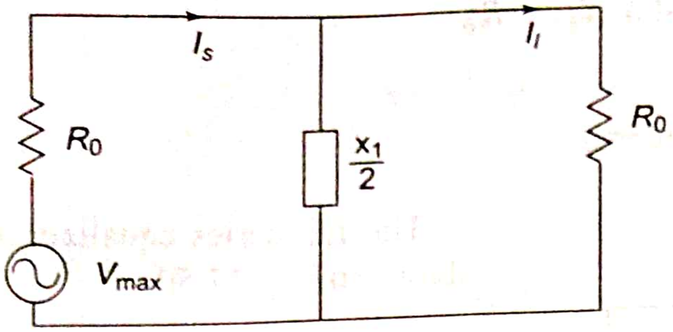


Fig. 17.52

The shunt equalizer connected to the network is shown in Fig. 17.52.

Source current

$$I_s = \frac{V_{max}}{R_0 + \left(R_0 \,//\, \frac{jX_1}{2}\right)} \tag{17.95}$$

$$= \frac{V_{max}}{R_0 + \left[\frac{jX_1 R_0}{2R_0 + jX_1}\right]}$$

$$= \frac{V_{max}[2R_0 + jX_1]}{2R_0(R_0 + jX_1)}$$

Load current $I_l = I_s \dfrac{jX_1/2}{R_0 + \dfrac{jX_1}{2}} = I_s \dfrac{jX_1}{2R_0 + jX_1}$ (17.96)

Substituting I_s in the above equation

$$I_l = \frac{V_{max}\, jX_1}{2R_0(R_0 + jX_1)} \tag{17.97}$$

Power delivered to the load

$$P_l = |I_l|^2 R_0 = \frac{V_{max}^2 X_1^2}{4R_0(R_0^2 + X_1^2)} \tag{17.98}$$

and $\quad P_i = V_{max}^2 / 4R_0$

Therefore, $\quad N = \dfrac{P_i}{P_l} = \dfrac{\dfrac{V_{max}^2}{4R_0}}{\dfrac{V_{max}^2 X_1^2}{4R_0(R_0^2 + X_1^2)}}$

$$\therefore \quad N = 1 + \left(\frac{R_0}{X_1}\right)^2 \tag{17.99}$$

By knowing the values of R_0 and N, X_1 can be determined.

17.22 FULL SHUNT EQUALIZER

Figure 17.53 shows the full shunt equalizer. It is also a constant resistance equalizer which satisfies the equation $Z_1 Z_2 = R_0^2$.

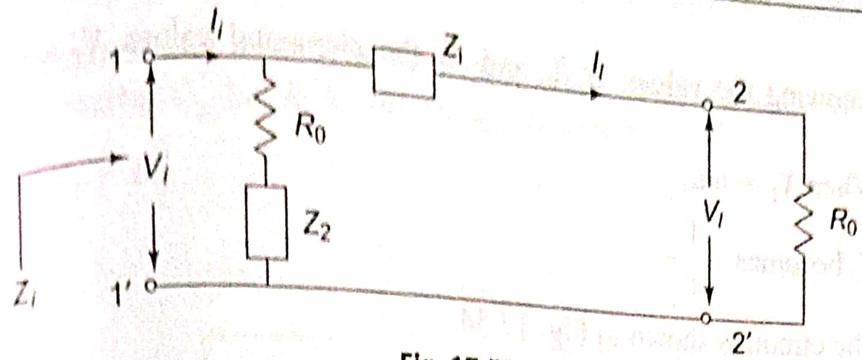


Fig. 17.53

The input impedance is given by

$$Z_i = \frac{(R_0 + Z_2)(R_0 + Z_1)}{2R_0 + Z_1 + Z_2} \tag{17.100}$$

$$= \frac{Z_1 Z_2 + R_0^2 + R_0(Z_1 + Z_2)}{2R_0 + Z_1 + Z_2}$$

$$Z_l = R_0$$

Since $Z_1 Z_2 = R_0^2$,

$$V_i = I_i Z_i = I_i R_0$$

$$V_l = I_l R_0$$

$$\frac{V_i}{V_l} = \frac{I_i}{I_l} \tag{17.101}$$

But $I_l = I_i \dfrac{(R_0 + Z_2)}{2R_0 + Z_1 + Z_2}$ (17.102)

$$\frac{I_i}{I_l} = \frac{Z_1 + Z_2 + 2R_0}{R_0 + Z_2}$$

Multiplying both numerator and denominator by Z_1, we get

$$\frac{I_i}{I_l} = \frac{Z_1^2 + Z_1 Z_2 + 2R_0 Z_1}{Z_1 R_0 + Z_1 Z_2}$$

$$\frac{I_i}{I_l} = \frac{(Z_1 + R_0)^2}{R_0(Z_1 + R_0)} = \frac{Z_1 + R_0}{R_0}$$

Therefore, $N = \left|\dfrac{V_i}{V_l}\right|^2 = \left|\dfrac{I_i}{I_l}\right|^2 = \left|\dfrac{R_0 + Z_1}{R_0}\right|^2$ (17.103)

$$N = 1 + \frac{X_1^2}{R_0^2} = 1 + \frac{R_0^2}{X_2^2}$$

since Z_1 and Z_2 are pure reactances and are equal to X_1 and X_2 respectively.

By knowing the values or R_0 and N_1 the elemental values X_1 and X_2 can be obtained.

(i) When $X_1 = \omega L_1$

X_2 becomes $\dfrac{1}{\omega C_1}$

The circuit is shown in Fig. 17.54.

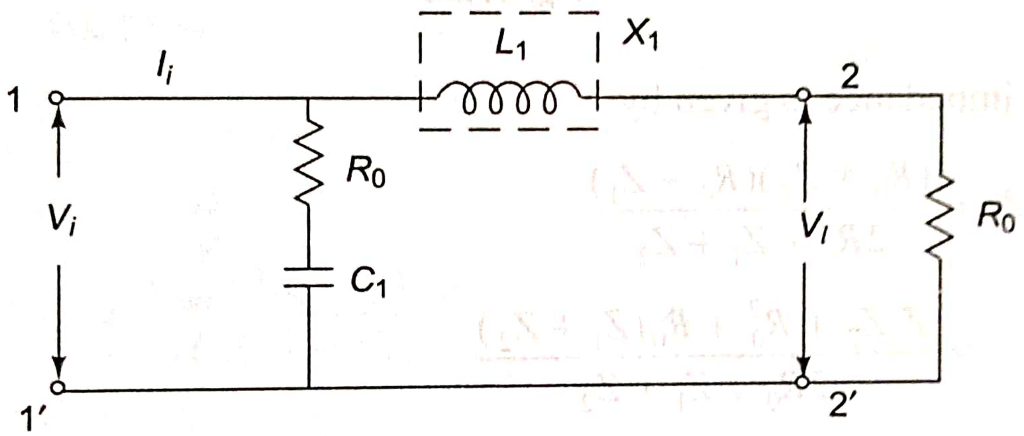


Fig. 17.54

(ii) When $X_1 = \dfrac{1}{\omega C_1}$

X_2 becomes ωL_1

The circuit is shown in Fig. 17.55.

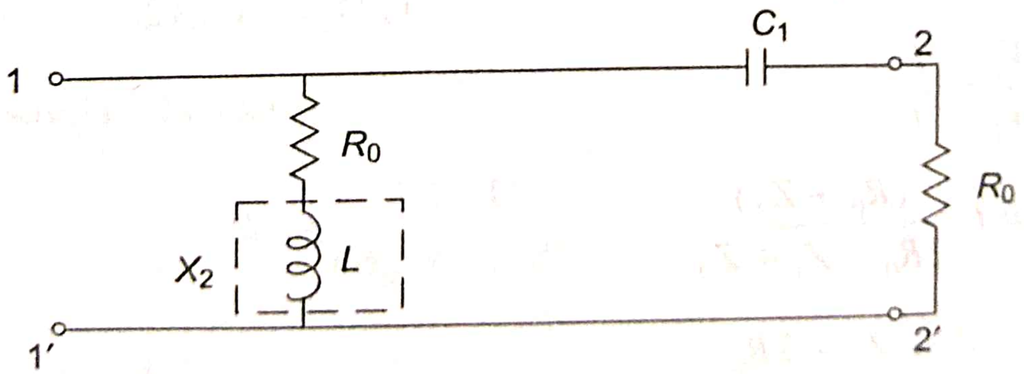


Fig. 17.55