5

GEARS

Course Contents

5.1 Introduction

- If power transmitted between two shafts is small, motion between them may be obtained by using two plain cylinders or discs 1 and 2 as shown in fig.
- If there is no slip of one surface relative to the other, a definite motion of 1 can be transmitted to 2 and vice-versa. Such wheels are termed as "**friction wheels**". However, as the power transmitted increases, slip occurs between the discs and the motion no longer remains definite.
- Assuming no slipping of the two surfaces, the following kinematic relationship exists for their linear velocity:
- To transmit a definite motion of one disc to the other or to prevent slip between the surfaces, projection and recesses on the two discs can be made which can mesh with each other. This leads to formation of teeth on the discs and the motion between the surfaces changes from rolling to sliding. The discs with the teeth are known as **gears** or **gear wheels**.
- It is to be noted that if the disc I rotates in the clockwise direction, 2 rotates in the counter clockwise direction and vice-versa.

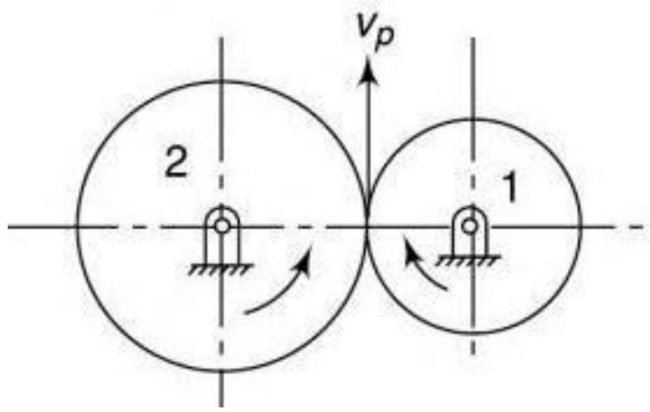


Fig. 5.1

5.2 Advantages and Disadvantages of Gear Drive

Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

Disadvantages

1. The manufacture of gears required special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.
3. They are costly.

5.3 Classification of Gears

5.3.1. According to the position of axes of the shafts

A. The axes of the two shafts between which the motion is to be transmitted, may be Parallel shaft,
B. Intersecting (Non parallel) shaft
C. Non-intersecting and non-parallel shaft.

A. Parallel shaft

- **Spur gear**

– The two parallel and co-planar shafts connected by the gears are called *spur gears.* These gears have teeth parallel to the axis of the wheel.
– They have straight teeth parallel to the axes and thus are not subjected to axial thrust due to tooth load.
– At the time of engagement of the two gears, the contact extends across the entire width on a line parallel to the axis of rotation. This results in sudden application of the load, high impact stresses and excessive noise at high speeds.
– If the gears have external teeth on the outer surface of the cylinders, the shaft rotate in the opposite direction.
– In an internal spur gear, teeth are formed on the inner surface of an annulus ring. An internal gear can mesh with an external pinion (smaller gear) only and the two shafts rotate in the same direction.

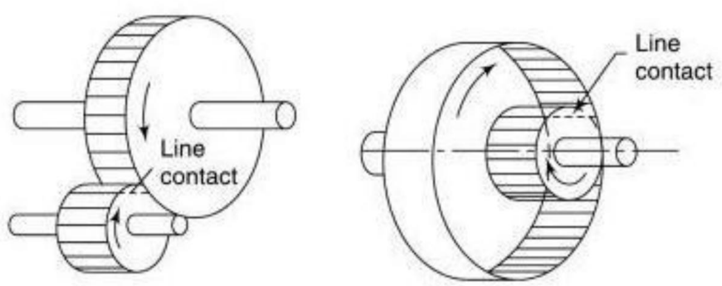


Fig.5.3 (a) Spur Gear

- *Spur rack and pinion*

– Spur rack is a special case of a spur gear where it is made of infinite diameter so that the pitch surface is plane.
– The spur rack and pinion combination converts rotary motion into translator motion, or vice-versa.
– It is used in a lathe in which the rack transmits motion to the saddle.

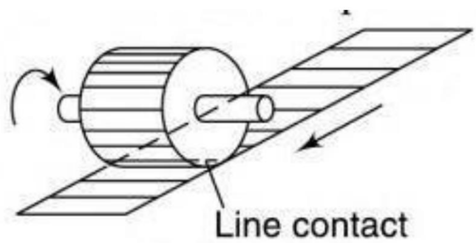


Fig. 5.3(b) Rack and pinion

- *Helical Spur Gears*

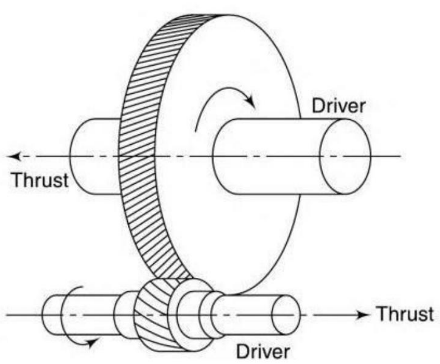
– In helical gears, the teeth are curved, each being helical in shape. Two mating gears have the same helix angle, but have teeth of opposite hands.

– At the beginning of engagement, contact occurs only at the point of leading edge of the curved teeth. As the gears rotate, the contact extends along a diagonal line across the teeth. Thus, the load application is gradual which results in low impact stresses and reduction in noise. Therefore, the helical gear can be used at higher velocities than the spur gears and have greater load-carrying capacity.

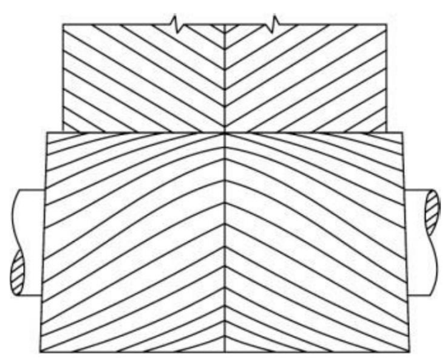
– Helical gears have the **disadvantage** of having end thrust as there is a force component along the gear axis. The bearing and assemblies mounting the helical gears must be able to withstand thrust loads.

– **Double helical:** A double-helical gear is equivalent to a pair of helical gears secured together, one having a right hand helix and other left hand helix.
  - The teeth of two rows are separated by groove used for tool run out.
  - Axial thrust which occurs in case of single-helical gears is eliminated in double-helical gears.
  - This is because the axial thrusts of the two rows of teeth cancel each other out. These can be run at high speeds with less noise and vibrations.

– **Herringbone gear**: If the left and the right inclinations of a double-helical gear meet at a common apex and there is no groove in between, the gear is known as Herringbone gear.



Helical gear
(c)



Herringbone gear
(d)

Fig. 5.3

B. Intersecting Shafts

– The two non-parallel or intersecting, but coplanar shafts connected by gears are called **bevel gears**
– When teeth formed on the cones are straight, the gears are known as bevel gears when inclined, they are known as **spiral** or **helical bevel**.

- **Straight Bevel Gears (**

– The teeth are straight, radial to the point of intersection of the shaft axes and vary in cross section throughout their length.
– Usually, they are used to connect shafts at right angles which run at low speeds
– Gears of the same size and connecting two shafts at right angles to each other are known as "Mitre" gears.

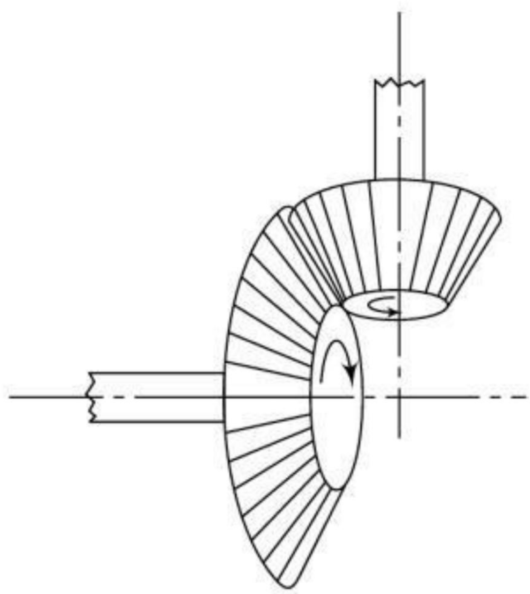


Fig. 5.3(e) Straight Bevel Gears

- *Spiral Bevel Gears*

– When the teeth of a bevel gear are inclined at an angle to the face of the bevel, they are known as spiral bevels or helical bevels.
– They are smoother in action and quieter than straight tooth bevels as there is gradual load application and low impact stresses. Of course, there exists an axial thrust calling for stronger bearings and supporting assemblies.
– These are used for the drive to the differential of an automobile.

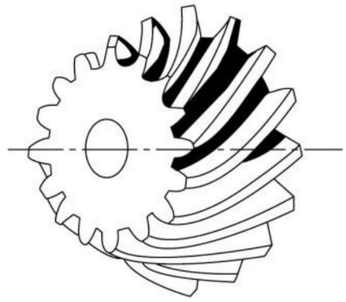


Fig. 5.3(f) Spiral Bevel Gear

- *Zero Bevel Gears*

– Spiral bevel gears with curved teeth but with a zero degree spiral angle are known as zero bevel gears.

– Their tooth action and the end thrust are the same as that of straight bevel gears and, therefore, can be used in the same mountings.

– However, they are quieter in action than the straight bevel type as the teeth are curved.

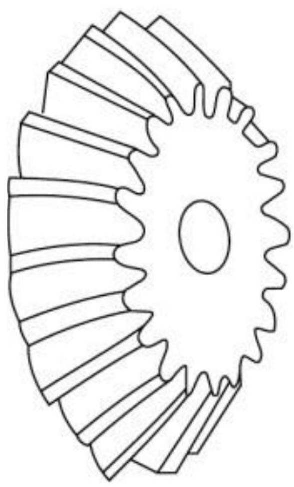


Fig. 5.3(g) Zero Bevel Gears

C. *Non-intersecting and non-parallel shaft(Skew shaft)*

– The two non-intersecting and non-parallel i.e. non-coplanar shaft connected by gears are called skew bevel gears or spiral gears and the arrangement is known as skew bevel gearing or spiral gearing.

– In these gears teeth have a point contact.

– These gears are suitable for transmitting small power.

– **Worm gear** is as special case of a spiral gear in which the larger wheel, usually, has a hollow shape such that a portion of the pitch diameter of the other gear is enveloped on it.

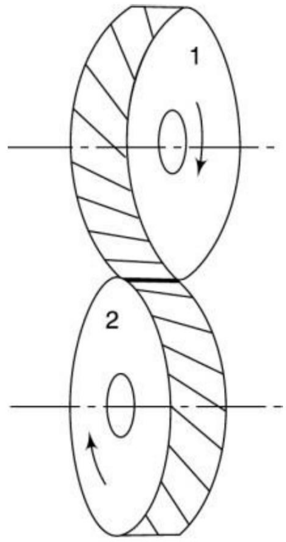


Fig.5.3 (h)Non-intersecting and non-parallel shaft

5.3.2. According to the peripheral velocity of the gears

(a) Low velocity $V < 3$ m/sec

(b) Medium velocity $3 < V < 15$ m/sec

(c) High velocity $V > 15$ m/sec

5.3.3. According to position of teeth on the gear surface

(a) Straight,

(b) Inclined, and

(c) Curved.

5.4 Terms Used in Gears

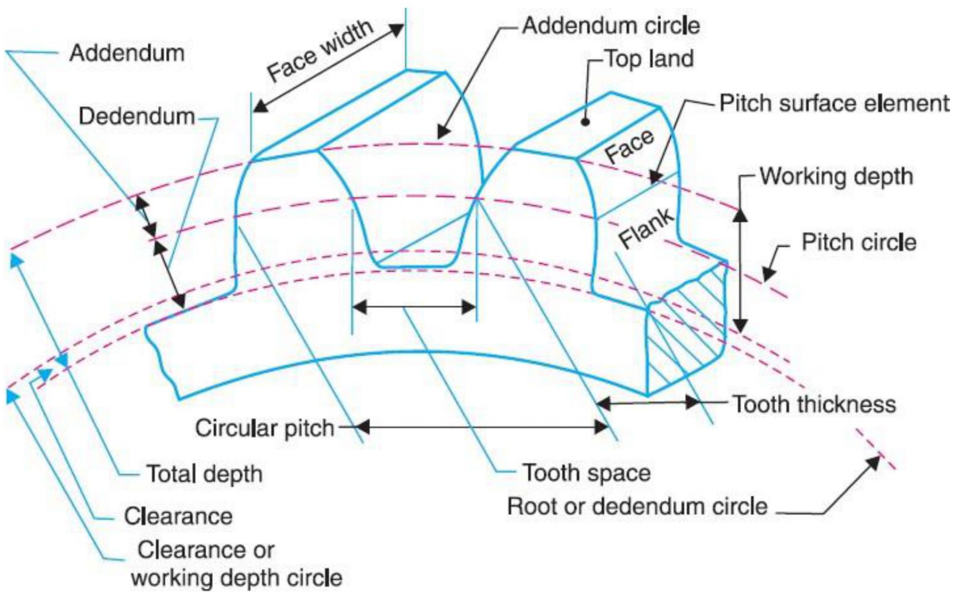


Fig.5.4 Terms used in gears.

1. Pitch circle. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2. Pitch circle diameter. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as **pitch diameter**.

3. Pitch point. It is a common point of contact between two pitch circles.

4. Pitch surface. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. Pressure angle or angle of obliquity. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point.

- For more power transmission lesser pressure on the bearing and pressure angle must be kept small.
- It is usually denoted by ø.
- The standard pressure angles are 20° and 25°.Gears with pressure angle has become obsolete.

6. Addendum. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

- Standard value = 1 module

7. Dedendum. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

- Standard value = 1.157 module

8. Addendum circle. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. Dedendum circle. It is the circle drawn through the bottom of the teeth. It is also called root circle.

10. Clearance. It is the radial difference between the addendum and the Dedendum of a tooth.

Addendum circle diameter $= d + 2m$

Dedendum circle diameter $= d - 2 \times 1.157m$

Clearance $= 1.157m - m$

$= 0.157m$

11. Full depth of Teeth It is the total radial depth of the tooth space.

Full depth= Addendum + Dedendum

12. Working Depth of Teeth The maximum depth to which a tooth penetrates into the tooth space of the mating gear is the working depth of teeth.

- Working depth = Sum of addendums of the two gears.

15. Working depth. It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. Tooth thickness. It is the width of the tooth measured along the pitch circle.

17. Tooth space. It is the width of space between the two adjacent teeth measured along the pitch circle.

18. Backlash. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

19. Face of tooth. It is the surface of the gear tooth above the pitch surface.
20. Flank of tooth. It is the surface of the gear tooth below the pitch surface.
21. Top land. It is the surface of the top of the tooth.
22. Face width. It is the width of the gear tooth measured parallel to its axis.

20. Fillet It is the curved portion of the tooth flank at the root circle.

21. *Circular pitch*. It is the distance measured on the circumference of the pitch circle from point of one tooth to the corresponding point on the next tooth.

- It is usually denoted by p_c.

  Mathematically,

  $$\text{Circular pitch, } p_c = \frac{\pi d}{T}$$

  Where *d* = Diameter of the pitch circle, and

  T = Number of teeth on the wheel.

- The angle subtended by the circular pitch at the center of the pitch circle is known as the **pitch angle**.

22. Module (m). It is the ratio of the pitch diameter in mm to the number of teeth.

$$m = \frac{d}{T}$$

$$\text{Also} \quad p_c = \frac{\pi d}{T} = \pi m$$

- Pitch of two mating gear must be same.

23. Diametral Pitch (P) It is the number of teeth per unit length of the pitch circle diameter in inch.

OR

It is the ratio of no. of teeth to pitch circle diameter in inch.

$$P_d = \frac{T}{d}$$

- The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, and 20. The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14 and 18 are of second choice.

24. Gear Ratio (G). It is the ratio of the number of teeth on the gear to that on the pinion.

$$G = \frac{T}{t}$$ Where T= No of teeth on gear

t = No. of teeth on pinion

25. Velocity Ratio (VR) The velocity ratio is defined as the ratio of the angular velocity of the follower to the angular velocity of the driving gear.

$$VR = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}$$

26. Length of the path of contact. It is the length of the common normal cut-off by the Addendum circles of the wheel and pinion.

OR

The locus of the point of contact of two mating teeth from the beginning of engagement to the end of engagement is known as the contact.

a. **Path of Approach** Portion of the path of contact from the beginning of the engagement to the pitch point.
b. **Path of Recess** Portion of the path of contact from the pitch point to the end of engagement.

27. Arc of Contact The locus of a point on the pitch circle from the beginning to the end of engagement of two mating gears is known as the arc of contact.

a. **Arc of Approach** It is the portion of the arc of contact from the beginning of engagement to the pitch point.
b. **Arc of Recess** The portion of the arc of contact from the pitch point to the end of engagements the arc of recess.

28. Angle of Action ( ) It is the angle turned by a gear from the beginning of engagement to the end of engagement of a pair of teeth, i.e., the angle turned by arcs of contact of respective gear wheels.

$$\delta = \alpha + \beta \quad \text{Where } \alpha = \text{Angle of approach}$$

$$\beta = \text{Angle of recess}$$

29. Contact ratio .It is the angle of action divided by the pitch angle

$$\text{Contact ratio} = \frac{\delta}{\gamma} = \frac{\alpha+\beta}{\gamma}$$

OR

$$\text{Contact ratio} = \frac{\text{Arcofcontact}}{\text{Circularpitch}}$$

5.5 Condition for Constant Velocity Ratio of Toothed Wheels –Law of Gearing

- To understand the theory consider the portions of two gear teeth gear 1 and gear 2 as shown in figure 1.5.
- The two teeth come in contact at point C and the direction of rotation of gear 1 is anticlockwise & gear 2 is clockwise.
- Let TT be the common tangent & NN be the common normal to the curve at the point of contact C. From points O, &O_2, draw O_1 A & O_2 B perpendicular to common normal NN.
- When the point D is consider on gear 1, the point C moves in the direction of "CD" & when it is consider on gear 2. The point C moves in direction of "CE".
- The relative motion between tooth surfaces along the common normal NN must be equal to zero in order to avoid separation.
- So, relative velocity

$$V_1 \cos\alpha = V_2 \cos\theta$$

$$(\omega_1 \times O_1 C)\cos\alpha = (\omega_2 \times O_2 C) \qquad (\because v = r\omega) \qquad ....(1)$$

$\cos\alpha$

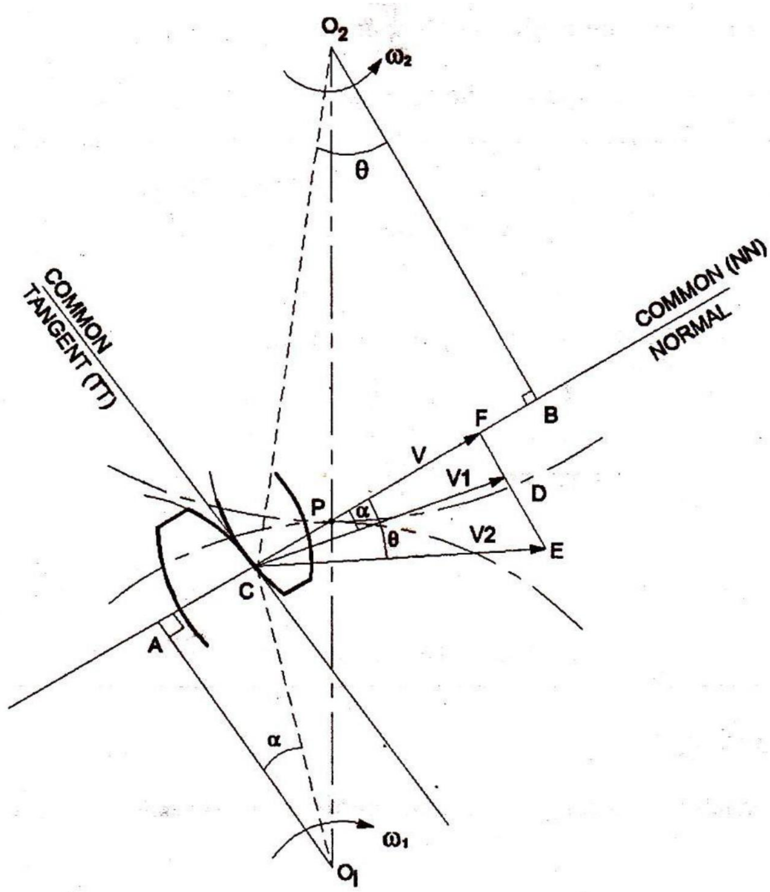


Fig.5.5 Law of gearing

- But from $\Delta\, O_1 AC$, $\cos\alpha = \dfrac{O_1 A}{O_1 C}$

  and from $\Delta\, O_2 BC$, $\cos\theta = \dfrac{O_2 B}{O_2 C}$
- Putting above value in equation (1) it become

$$(\omega_1 \times O_1 C)\frac{O_1 A}{O_1 C} = (\omega_2 \times O_2 C)\frac{O_2 B}{O_2 C}$$

$$\omega_1 \times O_1 A = \omega_2 O_2 B$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2 B}{O_1 A} \qquad \text{.......} (2)$$

- From the similar triangle $\Delta\, O_1 AP$ & $\Delta\, O_2 BP$

$$\frac{O_2 B}{O_1 A} = \frac{O_2 P}{O_1 P} \qquad \text{.......} (3)$$

- Now equating equation (2) & (3)

$$\frac{\omega_1}{\omega_2} = \frac{O_2 B}{O_1 A} = \frac{O_2 P}{O_1 P} = \frac{PB}{AP}$$

- From the above we can conclude that the angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the central O_1& O_2.
- If it is desired that the angular velocities of two gear remain constant, the common normal at the point of contact of two teeth always pass through a fixed point P. This fundamental condition is called as law of gearing. Which must be satisfied while designing the profiles of teeth for gears.

5.6 Standard Tooth Profiles or Systems

Following four types of tooth profiles or systems are commonly used in practice for interchangeability:

a) $14\frac{1}{2}^\circ$ composite system.

b) $14\frac{1}{2}^\circ$ full depth involute system.

c) 20°full depth involute system.

d) 20°stub involute system.

a) $14\frac{1}{2}^\circ$ *composite system:*

- This type of profile is made with circular arcs at top and bottom portion and middle portion is a straight line as shown in Fig. 1.6(a).
- The straight portion corresponds to the involute profile and the circular arc portion corresponds to the cycloidal profile.
- Such profiles are used for general purpose gears.

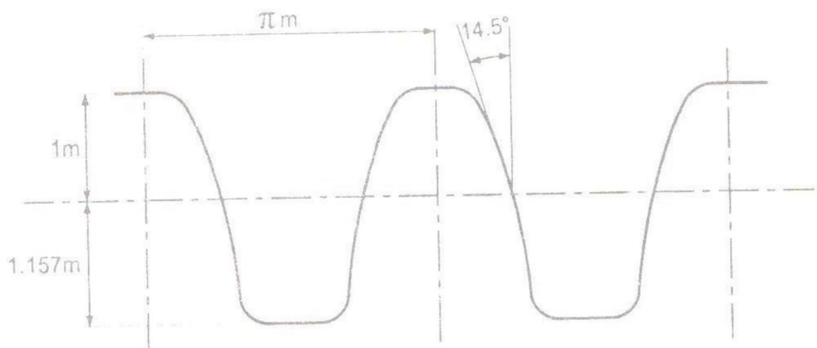


Fig.5.6(a) $14\frac{1}{2}^{\circ}$ *composite system*

b) $14\frac{1}{2}^{\circ}$ ***full depth involute system:***

- This type of profile is made straight line except for the fillet arcs.
- The whole profile corresponds to the involute profile. Therefore manufacturing of such profile is easy but they have interface problem.

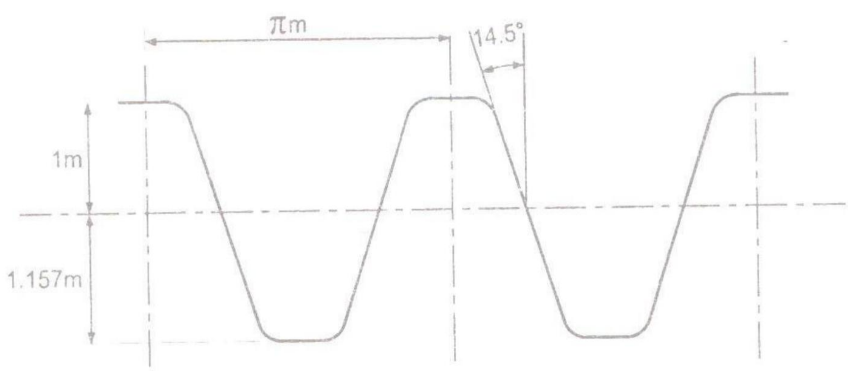


Fig.5.6(b) $14\frac{1}{2}^{\circ}$ *full depth involute system*

c) 20°full depth involute system:

- This type of profile is same as $14\frac{1}{2}^{\circ}$ full depth involute system except the pressure angle.

- The increase of pressure angle from $14\frac{1}{2}^\circ$ to 20° results in a stronger tooth, since the tooth acting as a beam is wider at the base.
- This type of gears also have interference problem if number of teeth is less.

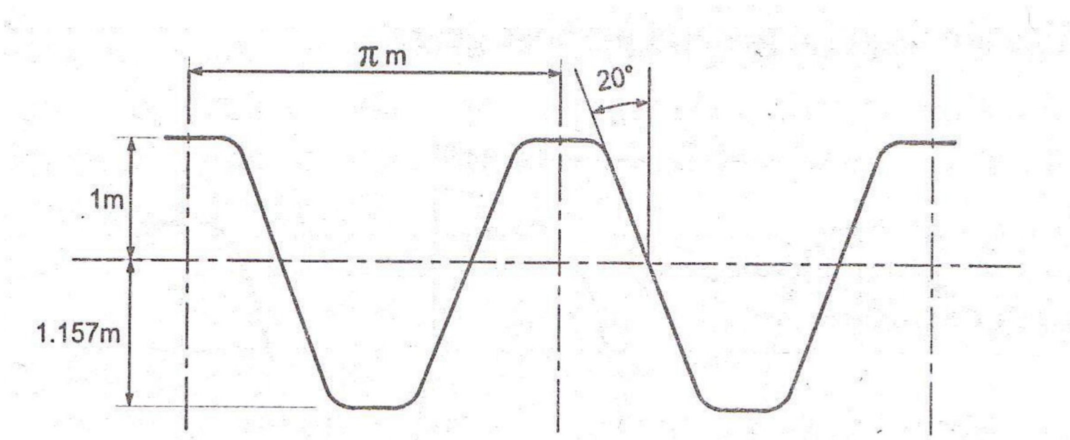


Fig.5.6(c) 20 *full depth involute system*

d) 20°stub involute system:

- The problem of interference in 20 full depth involute system is minimized by removing extra addendum of gear tooth which causes interference.
- Such modified tooth profile is called "Stub tooth profile".
- This type of gears are used for heavy load.

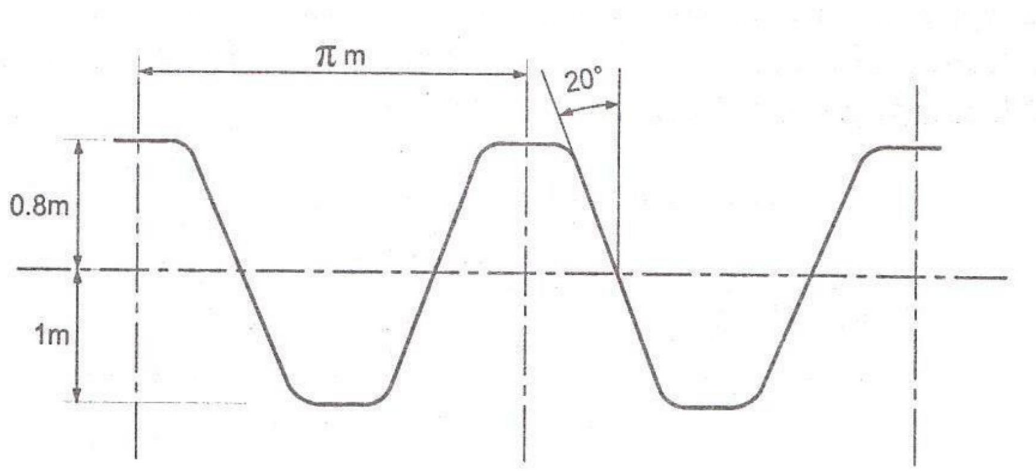


Fig.5.6(d) 20° *stub involute system*

5.7 Length of Path of Contact And Length of Arc of Contact

5.7.1 Length of Path of Contact

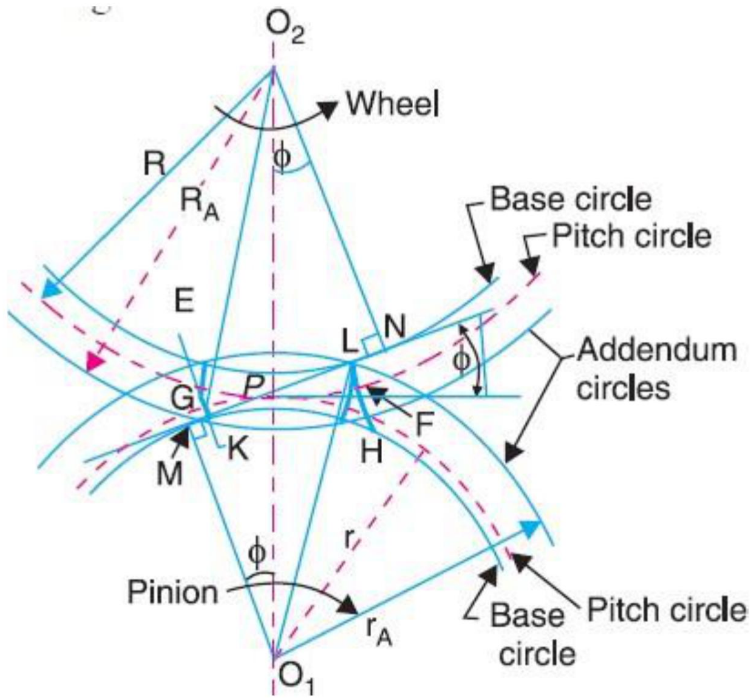


Fig.5.7 Length of path of contact

- When the pinion rotates in clockwise direction, the contact between a pair of involute teeth begins at *K* (on the flank near the base circle of pinion or the outer end of the tooth face on the wheel) and ends at L (on the flank near the base circle of wheel).
- MN is the common normal at the point of contacts and the common tangent to the base circles.
- The point K is the intersection of the addendum circle of wheel and the common tangent.
- The point L is the intersection of the addendum circle of pinion and common tangent.
- **Length of path of contact** is the length of common normal cutoff by the addendum circles of the wheel and the pinion.
- Thus the length of path of contact is KL which is the sum of the parts of the path of contacts KP and PL. The part of the path of contact KP is known as **path of approach** and the part of the path of contact PL is known as **path of recess.**

L.P.C =KL

=KP+PL

Where, KP = path of approach

PL = path of recess

Let

$R = O_2P$ = pitch circle radius of wheel
$R_A = O_2K$ = addendum circle radius of wheel
$r = O_1P$ = pitch circle radius of pinion
$r_A = O_1L$ = addendum circle radius of pinion

Length of the path of contact = Path of approach + path of recess

$$= KP + PL$$

$$= (KN - PN) + (ML - MP)$$

$$= \left(\sqrt{O_2K^2 - O_2} - PN\right) + \left(\sqrt{O_1L^2 - O_1} - MP\right)$$

$$= \left(\sqrt{R_A^2 - (R\cos)^2} - R\sin ø\right) + \left(\sqrt{r_A^2 - (r\cos ø)^2} - r\sin ø\right)$$

5.7.2 Length of Arc of Contact

- The arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth.
- The arc of contact is EPF or GPH.
- Considering the arc of contact GPH, it is divided into two parts i.e. arc GP and arc PH. The arc GP is known as **arc of approach** and the arc PH is called **arc of recess**.
- The angles subtended by these arcs at O1 are called **angle of approach** and **angle of recess** respectively.

Length of the arc of contact GPH $= (GP + PH)$

= Arc of approach+ Arc of recess

$$= \frac{KP}{\cos ø} + \frac{PL}{\cos ø}$$

$$= \frac{KP + PL}{\cos\cos ø}$$

$$= \frac{KL}{\cos ø}$$

$$= \frac{\textit{Length of path of contact}}{\cos \varnothing}$$

Contact Ratio (or Number of Pairs of Teeth in Contact)

- The contact ratio or the number of pairs of teeth in contact is defined as the ratio of the length of the arc of contact to the circular pitch.

  Mathematically,
  Contact ratio or number of pairs of teeth in contact

$$= \frac{\text{Length of arc of contact}}{\text{Circular pitch}}$$

$$= \frac{\text{Length of arc of contact}}{\pi \text{ m}}$$

Note:

- For continuous transmission of the motion, at least one tooth of any one wheel must be in contact with another tooth of second wheel so 'n' must be greater than unity.
- If 'n' lies between 1& 2, no. of teeth in contact at any time will not be less than one and will never mate two.
- If 'n' lies between 2 & 3,it is never less than two pair of teeth and not more than three pairs and so on.
- If 'n' is 1.6, one pair of teeth are always in contact where as two pair of teeth are in contact for 60% of the time

5.8 Interference in Involute Gears

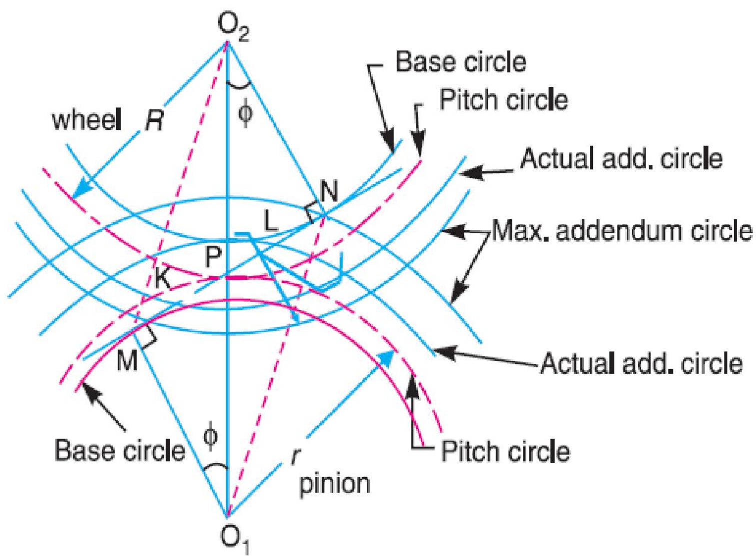


Fig.5.8 Interference in involute gears

- Fig. shows a pinion with centreO_1, in mesh with wheel or gear with centreO_2. MN is the common tangent to the base circles and KL is the path of contact between the two mating teeth.
- A little consideration will show that if the radius of the addendum circle of pinion is increased to O_1N, the point of contact L will move from L to N. When this radius is further increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel. The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as **interference,** and occurs when the teeth are being cut. In brief, the phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.

- Similarly, if the radius of the addendum circles of the wheel increases beyond O_2M, then the tip of tooth on wheel will cause interference with the tooth on pinion.
- The points M and N are called **interference points.** Interference may be avoided if the path of contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is O_1N and of the wheel is O_2M.

How to avoid interference?

- The interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth.

OR

- Interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.

- When interference is just avoided, the maximum length of path of contact is MN

$$\text{Maximum length of path of contact} = MN$$

$$= MP + PN$$

$$= r\sin\phi + R\sin\phi$$

$$= (r + R)\sin\phi$$

$$\text{Maximum length of arc of contact} = \frac{(r + R)\sin\phi}{\cos\phi}$$

Note:

In case the addenda on pinion and wheel is such that the path of approach and path of recess are half of their maximum possible values, then

- Path of approach, $KP = \frac{1}{2}$

$$\left(\sqrt{R_A^2 - (R\cos\ )^2} - R\sin\phi\right) = \frac{1}{2} r\sin\phi$$

- Path of recess, $PL = \frac{1}{2} PN$

$$\sqrt{r_A^2 - (r\cos\phi)^2} - r\sin\phi = \frac{1}{2} R\sin\phi$$

- Length of the path of contact $= KP + PL$

$$= \frac{1}{2} MP + \frac{1}{2} PN$$

$$= \frac{(r + R)\sin\phi}{2}$$

5.9 Minimum Number of Teeth on the Pinion in Order to Avoid Interference

– In order to avoid interference, the addendum circles for the two mating gears must cut the common tangent to the base circles between the points of tangency.
– The limiting condition reaches, when the addendum circles of pinion and wheel pass through points *N* and *M* (see Fig.) respectively.

Let t = Number of teeth on the pinion,
T = Number of teeth on the wheel,
m = Module of the teeth,
r = Pitch circle radius of pinion = $mt / 2$
G = Gear ratio = $T / t = R / r$
ø = Pressure angle or angle of obliquity.

From O_1NP,

$$O_1N^2 = O_1P^2 + PN^2 - 2OP \times PN\cos(O_1PN)$$

$$\therefore O_1N^2 = r^2 + (R\sin\phi)^2 - 2r(R\sin\phi)\times\cos(90+\phi)$$

$$\therefore O_1N^2 = r^2 + (R\sin\phi)^2 - 2r(R\sin\phi)\times\cos(90+\phi)$$

$$\therefore O_1N^2 = r^2 + R^2\sin^2\phi + 2rR\sin^2\phi$$

$$\therefore O_1N^2 = r^2\left[1 + \frac{R^2\sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right]$$

$$\therefore O_1N^2 = r^2\left[1 + \frac{R^2\sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right]$$

$$\therefore O_1N^2 = r^2\left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$

$$\therefore O_1N = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi}$$

$$\therefore O_1N = \frac{mt}{2}\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi}$$

Let $A_p \cdot m$ = Addendum of the pinion, where A_P is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

Addendum of the pinion = $O_1N - O_1P$

$$A_P.m = \frac{mt}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - \frac{mt}{2}$$

$$\therefore A_P.m = \frac{mt}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - \frac{mt}{2}$$

$$\therefore A_P.m = \frac{mt}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$\therefore A_P.m = \frac{mt}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2 ø}-1\right]$$

$$\therefore A_P. = \frac{t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2 ø}-1\right]$$

$$\therefore A_P = \frac{t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2 ø}-1\right]$$

$$\therefore A_P = \frac{t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2 ø}-1\right]$$

$$\therefore t = \frac{2A_P}{\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2 ø}-1\right]}$$

$$\therefore t = \frac{2A_P}{\left[\sqrt{1+G(G+2)\sin^2 ø}-1\right]}$$

Note:

– If the pinion and wheel have equal teeth, then $G = 1$.

$$\therefore t = \frac{2A_P}{\left[\sqrt{1+3\sin^2 ø}-1\right]}$$

Min. no of teeth on pinion

Sr. no	System of gear teeth	Min. no of teeth on pinion
1	Composite	12
2	Full depth involute	32
3	Full depth involute	18
4	Stub involute	14

5.10 Minimum Number of Teeth on the Wheel in Order to Avoid Interference

Let T = Minimum number of teeth required on the wheel in order to avoid interference,

$A_w \cdot m$ = Addendum of the wheel, where A_W is a fraction by which the standard Addendum for the wheel should be multiplied.

From O_2MP

$$O_2M^2 = O_2P^2 + PM^2 - 2O_2P \times PM\cos(O_2PM)$$

$$\therefore O_2M^2 = R^2 + (r\sin\phi)^2 - 2r(R\sin\phi)\times\cos(90+\phi)$$

$$\therefore O_2M^2 = R^2 + r^2\sin^2\phi + 2rR\sin^2\phi$$

$$\therefore O_2M^2 = R^2\left[1 + \frac{r^2\sin^2\phi}{R^2} + \frac{2r\sin^2\phi}{R}\right]$$

$$\therefore O_2M^2 = R^2\left[1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi\right]$$

$$\therefore O_2M = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi}$$

$$\therefore O_2M = \frac{mT}{2}\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi}$$

Addendum of the wheel = $O_2M - O_2P$

$$A_w m = \frac{mT}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - \frac{mT}{2}$$

$$\therefore A_w m = \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$\therefore A_w m = \frac{mT}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2 ø}-1\right]$$

$$\therefore A_w = \frac{T}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2 ø}-1\right]$$

$$\therefore T = \frac{2A_w}{\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2 ø}-1\right]}$$

$$\therefore T = \frac{2A_w}{\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2 ø}-1\right]}$$

Note:

- From the above equation, we may also obtain the minimum number of teeth on pinion. Multiplying both sides by t/T,

$$T \times \frac{t}{T} = \frac{2A_w \times \frac{t}{T}}{\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2 ø}-1\right]}$$

$$\therefore t = \frac{2A_w}{G\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2 ø}-1\right]}$$

- If wheel and pinion have equal teeth, then $G = 1$,

$$\therefore T = \frac{2A_w}{\left[\sqrt{1+3\sin^2 ø}-1\right]}$$

5.11 Minimum Number of Teeth on a Pinion for Involute Rack in Order to Avoid Interference

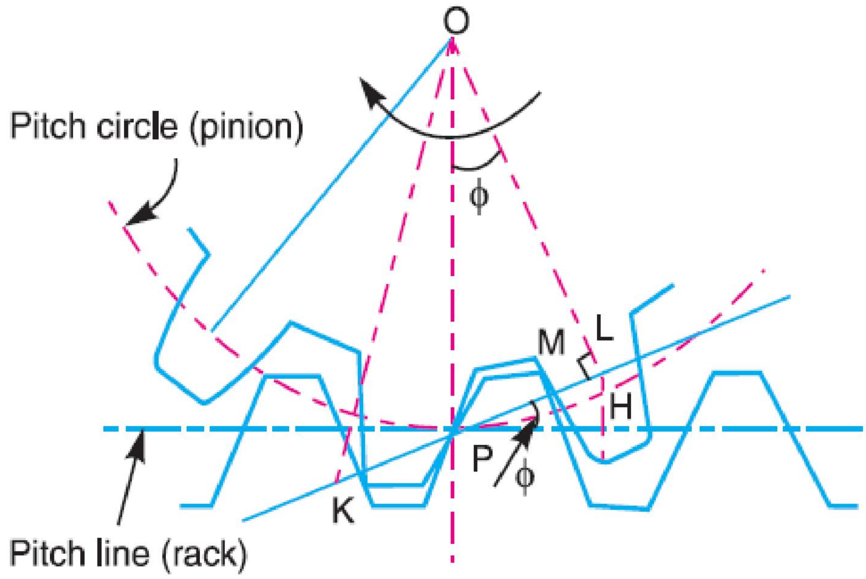


Fig.5.11 Rack and pinion in mesh

Let t = Minimum number of teeth on the pinion,

r = Pitch circle radius of the pinion = $\frac{m \cdot t}{2}$ and

ø= Pressure angle or angle of obliquity, and

$A_R \cdot m$= Addendum for rack, where A_R is the fraction by which the standard addendum of one module for the rack is to be multiplied.

Addendum for rack, $A_R \cdot m = LH$

$$\therefore A_R \cdot m = PL \sin \phi$$

$$\therefore A_R \cdot m = r \sin \varphi \times \sin \varphi$$

$$\therefore A_R \cdot m = r \sin^2 \varphi$$

$$\therefore A_R \cdot m = \frac{mt \sin^2 \varphi}{2}$$

$$\therefore t = \frac{2A_R}{\sin^2 \varphi}$$

Note:

- In case of pinion, max. value of addendum radius to avoid interference if AF

$$= O_2M^2 + AF^2$$

$$= (r\cos ø)^2 + (R\sin ø + r\sin ø)^2$$

- Max value of addendum of pinion is

$$(A_p)_{max} = r\left[\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2 ø} - 1\right]$$

$$= \frac{mt}{2}\left[\sqrt{1 + G(G+2)\sin^2 ø} - 1\right]$$

5.12 Comparison of Cycloidal and Involute tooth forms

Cycloidal teeth	Involute teeth
Pressure angle varies from maximum at the beginning of engagement, reduce to zero at the pitch point and again increase to maximum at the end of the engagement resulting in smooth running of gears.	Pressure angle is constant throughout the engagement of teeth. This result in smooth running of the gears.
It involves double curves for the teeth, epicycloid and hypocycloid. This complicates the manufacturer.	It involves the single curves for the teeth resulting in simplicity of manufacturing and of tool
Owing to difficulty of manufacturer, these are costlier	These are simple to manufacture and thus are cheaper.
Exact center distance is required to transmit a constant velocity ratio.	A little variation in a centre distance does not affect the velocity ratio.
Phenomenon of interference does not occur at all.	Interference can occur if the condition of minimum no. of teeth on a gear is not followed.
The teeth have spreading flanks and thus are stronger.	The teeth have radial flanks and thus are weaker as compared to the Cycloidal form for the same pitch.
In this a convex flank always has contact with a concave face resulting in less wear.	Two convex surfaces are in contact and thus there is more wear.

5.13 HELICAL AND SPIRAL GEARS

- In helical and spiral gears, the teeth are inclined to the axis of a gear. They can be right handed or left-handed, depending upon the direction in which the helix slopes away from the viewer when a gear is viewed parallel to the axis of the gear.

- In Fig. Gear1 is a right-handed helical gear whereas 2 are left handed. The two mating gears have parallel axes and equal helix angle α OR ψ. The contact between two teeth on the two gears is first made at one end which extends through the width of the wheel with the rotation of the gears.
- Figure (a) shows the same two gears when looking from above. Now, if the helix angle of the gear 2 is reduced by a few degrees so that the helix angle of the gear 1 is ψ_1, and that of gear 2 is ψ_2 and it is desired that the teeth of the two gears still mesh with each other tangentially, it is essential to rotate the axis of gear 2 through some angle as shown in Fig. (b).

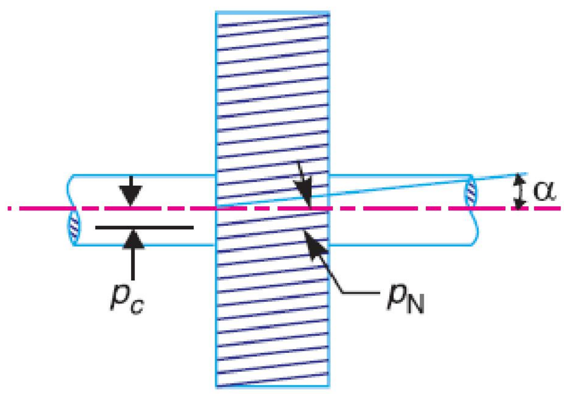


Fig.5.13(a) Helical Gear

- The following definitions may be clearly understood in connection with a helical gear as shown in Fig.

1. Normal pitch. It is the distance between similar faces of adjacent teeth, along a helix on the pitch cylinder normal to the teeth. It is denoted by p_N.
2. Axial pitch. It is the distance measured parallel to the axis, between similar faces of adjacent teeth. It is the same as circular pitch and is therefore denoted by pc. If α is the helix angle, then

$$\text{Circular pitch}, p_c = \frac{p_N}{\cos \alpha}$$

Note: The **helix angle** is also known as **spiral angle** of the teeth.

Efficiency of Spiral Gears

- A pair of spiral gears 1 and 2 in mesh is shown in Fig. . Let the gear 1 be the driver and the gear 2 the driven. The forces acting on each of a pair of teeth in contact are shown in Fig.
- The forces are assumed to act at the center of the width of each teeth and in the plane tangential to the pitch cylinders

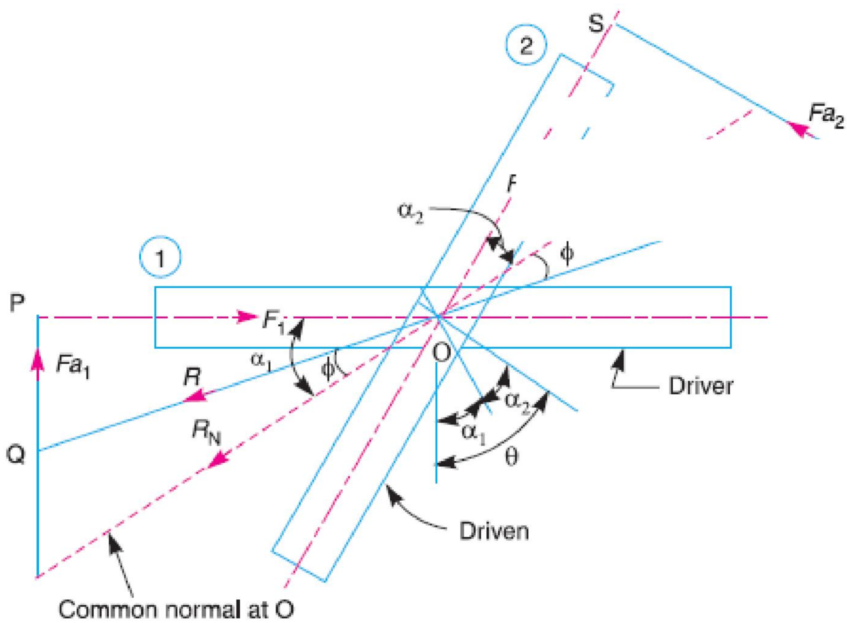


Fig.5.13 (b)

Let F_1= Force applied tangentially on the driver,

F_2 = Resisting force acting tangentially on the driven, F_{a1} = Axial or end thrust on the driver,

F_{a2} = Axial or end thrust on the driven,

R_N = Normal reaction at the point of contact

ϕ = Angle of friction,

R = Resultant reaction at the point of contact, and

θ = Shaft angle $=\alpha_1+\alpha_2$

...($\because$ Both gears are of the same hand)

From triangle OPQ, $F1 = R\cos(\alpha_1-\phi)$

$\therefore$ Work input to the driver $= F_1 \times \pi d_1 \cdot N_1 = R\cos(\alpha_1-\phi)\times \pi d_1 \cdot N_1$

From triangle OST, $F_2 = R\cos(\alpha_2+\phi)$

$\therefore$ Work output of the driven $= F_2 \times \pi d_2 \cdot N_2 = R\cos(\alpha_2 + \phi) \times \pi d_2 \cdot N_2$

$\therefore$ Efficiency of spiral gears,

$$\eta = \frac{\text{Work output}}{\text{Work input}} = \frac{R\cos(\alpha_2 + \phi) \times \pi d_2 \cdot N_2}{R\cos(\alpha_1 - \phi) \times \pi d_1 \cdot N_1}$$

$$= \frac{\cos(\alpha_2 + \phi) \times d_2 \cdot N_2}{\cos(\alpha_1 - \phi) \times d_1 \cdot N_1}$$

Pitch circle diameter of gear 1,

$$d_1 = \frac{p_{c1} \times T_1}{\pi} = \frac{p_N}{\text{Cos}\alpha_1} \times \frac{T_1}{\pi}$$

Pitch circle diameter of gear 2,

$$d_2 = \frac{p_{c2} \times T_2}{\pi} = \frac{p_N}{\text{Cos}\alpha_2} \times \frac{T_2}{\pi}$$

$$\therefore \frac{d_2}{d_1} = \frac{T_2 \text{Cos}\alpha_1}{T_1 \text{Cos}\alpha_2} \quad \cdots\cdots\cdots(2)$$

$$\frac{N_2}{N_1} = \frac{T_1}{T_2} \quad \cdots\cdots\cdots(3)$$

Multiplying equation (2) and (3) we get

$$\frac{d_2 N_2}{d_1 N_1} = \frac{\cos\alpha_1}{\cos\alpha_2}$$

Substituting this value in equation (1)

$$\eta = \frac{\cos(\alpha_2 + \phi) \times \cos\alpha_1}{\cos(\alpha_1 - \phi) \times \cos\alpha_2} \quad \cdots\cdots(4)$$

$$= \frac{\cos(\alpha_1 + \alpha_2 + \phi) + \cos(\alpha_1 - \alpha_2 - \phi)}{\cos(\alpha_1 + \alpha_2 - \phi) + \cos(\alpha_1 - \alpha_2 + \phi)}$$

$$\left(\because \cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]\right)$$

$$= \frac{\cos(\theta+\phi) + \cos(\alpha_1 - \alpha_2 - \phi)}{\cos(\theta-\phi) + \cos(\alpha_1 - \alpha_2 + \phi)} \quad \cdots\cdots\cdots(5)$$

$$(\because \theta = \alpha_1 + \alpha_2)$$

Since the angle θ and ϕ are constants, therefore the efficiency will be maximum, when $\cos(\alpha_1 - \alpha_2 + \phi)$ is maximum *i.e.*

$\cos(\alpha_1 - \alpha_2 + \phi) = 1$

$\therefore \alpha_1 - \alpha_2 + \phi = 0$

$\therefore \alpha_1 = \alpha_2 + \phi$ and $\alpha_2 = \alpha_1 - \phi$

Since $\alpha_1 + \alpha_2 = \theta$ therefore

$\alpha_1 = \theta - \alpha_2 = \theta - \alpha_1 + \phi$ OR $\alpha_1 = \dfrac{\theta + \phi}{2}$

Similarly $\alpha_2 = \dfrac{\theta - \phi}{2}$

Substituting $\alpha_1 = \alpha_2 + \phi$ and $\alpha_2 = \alpha_1 - \phi$ in equation (5) we get

$$\eta_{max} = \frac{\cos(\theta+\phi) + 1}{\cos(\theta-\phi) + 1}$$

EXAMPLES

Example 5.1: Two spur gears have a velocity ratio of 1/3 the driven gear has 72 teeth of 8 mm module and rotates at 300 rpm. Calculate the number of teeth and Speed of driver. What will be the pitch line velocity?

Solution:

Given data	Find
$VR = 1/3$	$V_p = ?$
$T_2 = 72$ teeth	$T_1 = ?$
$m = 8$ mm	
$N_2 = 300$	

$$VR = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

$$\therefore \frac{1}{3} = \frac{300}{N_1} = \frac{T_1}{72}$$

$$\therefore T_1 = 24 \ \& \ N_1 = 900 \text{ rpm}$$

Pitch line velocity

$$V_P = r_1\omega_1 = r_2\omega_2$$

$$= \frac{2\pi N_1}{60} \times \frac{d_1}{2}$$

$$= \frac{2\pi N_1}{60} \times \frac{mT_1}{2}$$

$$= \frac{2\pi \times 900}{60} \times \frac{8 \times 24}{2}$$

$$= 9047.78 \text{ mm / sec}$$

Example 5.2: The number of teeth of a spur gear is 30 and it rotates at 200 rpm. What will be its circular pitch and the pitch line velocity if it has a module of 2 mm?

Solution:

Given data

$T = 30$

$N = 200$ rpm

$m = 2$ mm

Find:

$P_c = ?$

$V_p = ?$

Circular pitch $P_c = \pi \cdot m$

$= \pi \cdot 2$

$= 6.28$mm

Pitch line velocity $V_p = \omega \cdot r$

$$= \frac{2\pi N}{60} \times \frac{d}{2}$$

$$= \frac{2\pi \times 200}{60} \times \frac{2 \times 30}{2}$$

$= 628.3$mm/ s

Example 5.3: The following data relate to two meshing gears velocity ratio = 1/3, module = 1mm, Pressure angle 20°, center distance= 200 mm. Determine the number of teeth and the base circle radius of the gear wheel.

Solution:

Given data

$VR = 1/3$

$Ø = 20°$

$C = 200$ mm

$m = 4$ mm

Find:

$T_1 = ?$

$T_2 = ?$

Base circle radius of gear wheel = ?

(1) $$VR = \frac{N_2}{N_1} = \frac{1}{3} = \frac{T_1}{T_2}$$

$$\therefore T_2 = 3T_1 \qquad \cdots\cdots\cdots(1)$$

Centre distance $C = \frac{d_1 + d_2}{2}$

$$\therefore 200 = \frac{m(T_1 + T_2)}{2} \qquad \left(\because m = \frac{d}{2}\right)$$

$$\therefore 200 = \frac{4(T_1 + T_2)}{2}$$

$$\therefore T_1 + T_2 = 100 \qquad \cdots\cdots\cdots(2)$$

By solving equation (1) & (2)

$$T_1 = 25$$

$$T_2 = 75$$

(2) No of teeth of gear wheel $T_2 = 75$

But $m = \frac{d_2}{T_2}$

$$\therefore d_2 = mT_2$$

$$\therefore d_2 = 300\text{mm}$$

Base circle radius $d_{b2} = \frac{d_2}{2}\cos\varphi$

$$= \frac{300}{2} \times \cos 20^\circ$$

$$= 141\text{mm}$$

Example 5.4: Each of the gears in a mesh has 48 teeth and a module of 8 mm. The teeth are of 20° involute profile. The arc of' contact is 2.25 times the circular pitch. Determine the addendum.

Solution:

Given data Find:

$T = t = 48$ Addendum = ?

$m = 8$ mm

$ø = 20°$

Arc of contact = $2.25\ P_c$

Arc of contact = $2.25P_c$

$= 2.25 \times \pi m$

$= 2.25 \times \pi \times 8$

$= 56.55mm$

Let $m = \frac{d}{t} = \frac{2r}{T}$

$\therefore R = r = \frac{mT}{2} = \frac{8 \times 48}{2}$

$\therefore R = r = 192mm$

Also $R_a = r_a$ ($\because$ toothsizesame)

$L.A.C = \frac{L.P.C}{COS\ \varphi}$

$\therefore 56.55 = \frac{L.P.C}{COS20^\circ}$

$\therefore L.P.C = 53.14\ mm$

$$L.P.C = \left(\sqrt{R_A^2 - (R\cos ø)^2} - R\sin ø\right) + \left(\sqrt{r_A^2 - (r\cos ø)^2} - r\sin ø\right)$$

$$\therefore 53.14 = 2\left[\sqrt{R_A^2 - (R\cos\cos ø)^2}\right] - (R + r)\sin ø \quad (\because R_A = r_A)$$

$$\therefore 53.14 = 2\left[\sqrt{R_A^2 - (192\cos 20)^2}\right] - (192 + 192)\sin 20^\circ$$

$$\therefore 53.14 = 2\left[\sqrt{R_A^2 - 32551.73}\right] - 131.33$$

$$\therefore \sqrt{R_A^2 - 32551.73} = 92.23\text{mm}$$

$$\therefore R_A = 202.63\text{mm}$$

Now $R_A = R + \text{Addendum}$

$$\therefore \text{Addendum} = R_A - R$$

$$\therefore \text{Addendum} = 10.63\text{mm}$$

Example 5.5: Two involute gears in mesh have 20° pressure angle. The gear ratio is 3 and the number of teeth on the pinion is 24. The teeth have a module of 6 mm. The pitch line velocity is 1.5 m/s and the addendum equal to one module. Determine the angle of action of pinion (the angle turned by the pinion when one pair of teeth is in the mesh) and the maximum velocity of sliding.

Solution:

Given data

$\text{Ø} = 20°$

$G = T/t = 3$

$t = 24$

$m = 6$ mm

$V_p = 1.5$ m/s

Addendum = 1 module

Find:

Angle of action of the pinion = ?

Max. velocity of sliding = ?

$$r = \frac{mt}{2} = \frac{6 \times 24}{2} = 72\text{mm} \qquad R = \frac{mT}{2} = \frac{6 \times 72}{2} = 216\text{mm}$$

$$(\because T = 24 \times 3 = 72)$$

$$r_a = r + \text{Add.} = 72 + (1 \times 6) = 78\text{mm} \qquad R_A = R + \text{Add.} = 216 + (1 \times 6) = 222\text{mm}$$

Let the length of path of contact KL = KP + PL

$$KP = \left(\sqrt{R_A^2 - (R\cos ø)^2} - R\sin ø\right)$$

$$=\left(\sqrt{222^2-\left(216\cos 20^\circ\right)^2}-216\sin 20^\circ\right)$$

$$=16.04\text{mm}$$

$$PL=\left(\sqrt{r_A^{\ 2}-(r\cos\varnothing)^2}-r\sin\varnothing\right)$$

$$=\left(\sqrt{78^2-\left(72\cos 20^\circ\right)^2}-72\sin 20^\circ\right)$$

$$=14.18\text{mm}$$

$$\text{Arcofcontact}=\frac{\text{Pathofcontact}}{\cos\varphi}$$

$$=\frac{16.04+14.18}{\cos 20^\circ}$$

$$=32.16\text{mm}$$

$$\text{Angleturnedthroughbypinion}(\theta)=\frac{\text{Lengthofarcofcontact}\times 360^\circ}{\text{circumferenceofpinion}}$$

$$=\frac{32.16\times 360}{2\pi\times 72}$$

$$=25.59$$

$$\text{Max.velocityofsliding}=(\omega_p+\omega_g)\times KP$$

$$=\left(\frac{V}{r}+\frac{V}{}\right)\times KP \qquad (\because V=r\omega)$$

$$=\left(\frac{1500}{72}+\frac{1500}{216}\right)\times 16.04$$

$$=445.6\text{mm/ sec}$$

Example 5.6: Two involute gears in a mesh have a module of 8mm and pressure angle of 20°. The larger gear has 57 while the pinion has 23 teeth. If the addendum on pinion and gear wheels are equal to one module, Determine

i. Contact ratio(No. of pairs of teeth in contact)
ii. Angle of action of pinion and gear wheel

iii. Ratio of sliding to rolling velocity at the
   a. Beginning of the contact.
   b. Pitch point.
   c. End of the contact.

Solution:

Given data

$\text{Ø} = 20°$

$m = 8$ mm

$T = 57$

$t = 23$

Addendum = 1 module = 8 mm

Find:

1. Contact ratio = ?
2. Angle of action of pinion and gear = ?
3. Ratio of sliding to rolling velocity at the
   a. Beginning of contact
   b. Pitch point
   c. End of contact

i. Let the length of path of contact KL = KP+PL

$$KP = \left(\sqrt{R_A^{\,2} - (R\cos ø)^2} - R\sin ø\right)$$

$$= \left(\sqrt{236^2 - \left(228\cos 20^\circ\right)^2} - 228\sin 20^\circ\right)$$

$$= 20.97\text{mm}$$

$$PL = \left(\sqrt{r_A^{\,2} - (r\cos ø)^2} - r\sin ø\right)$$

$$= \left(\sqrt{100^2 - \left(92\cos 20^\circ\right)^2} - 92\sin 20^\circ\right)$$

$$=18.79\text{mm}$$

$$\text{Arcofcontact} = \frac{\text{Pathofcontact}}{\cos\varphi}$$

$$= \frac{KP + KP}{\cos\varphi}$$

$$= \frac{20.97 + 18.79}{\cos 20^\circ}$$

$$= 42.29\text{mm}$$

$$\text{Contactratio} = \frac{\text{Lengthofarcofcontact}}{P_c}$$

$$= \frac{42.21}{\pi m} = 1.68 \quad \text{say } 2$$

i.

$$\text{Angleofactionofpinion}(\delta_p) = \frac{\text{Lengthofarcofcontact} \times 360^\circ}{\text{circumferenceofpinion}}$$

$$= \frac{42.31 \times 360^\circ}{2\pi \times 92}$$

$$= 26.34^\circ$$

$$\text{Angleofactionofpinion}(\delta_g) = \frac{\text{Lengthofarcofcontact} \times 360^\circ}{\text{circumferenceof gear}}$$

$$= \frac{42.31 \times 360^\circ}{2\pi \times 228}$$

$$= 10.63$$

i. Ratio of sliding to rolling velocity:

a. Beginning of contact

$$\frac{\text{Slidingvelocity}}{\text{Rollingvelocity}} = \frac{(\omega_p + \omega_g) KP}{\omega_p r}$$

$$= \frac{\left(\omega_p + \frac{92}{228}\omega_p\right) \times 20.97}{\omega_p \times 92}$$

$$= 0.32$$

b. Pitch point

$$\frac{\text{Sliding velocity}}{\text{Rolling velocity}} = \frac{(\omega_p + \omega_g)KP}{\omega_p r}$$

$$= \frac{(\omega_p + \omega_g) \times 0}{\omega_p r}$$

$$= 0$$

c. End of contact

$$\frac{\text{Sliding velocity}}{\text{Rolling velocity}} = \frac{(\omega_p + \omega_g)PL}{\omega_p r}$$

$$= \frac{\left(\omega_p + \frac{92}{228}\omega_p\right) \times 18.79}{\omega_p \times 92}$$

$$= 0.287$$

Example 5.7: Two 20° gears have a module pitch of 4 mm. The number of teeth on gears 1and 2 are 40 and 24 respectively. If the gear 2 rotates at 600 rpm, determine the velocity of sliding when the contact is at the tip of the tooth of gear 2. Take addendum equal to one module. Also, find the maximum velocity of sliding.

Solution:

Given data

$\text{Ø} = 20°$

$m = 4$ mm

$N_p = 600$ rpm

$T = 40$

$t = 24$

Addendum = 1 module = 4 mm

Find:

Velocity of sliding = ?

Max. velocity of sliding = ?

$$r = \frac{mt}{2} = \frac{4 \times 24}{2} = 48\text{mm}$$

$$R = \frac{mT}{2} = \frac{4 \times 40}{2} = 80\text{mm}$$

$$r_a = r + \text{Add.} = 48 + (1 \times 4) = 52\text{mm}$$

$$R_A = R + \text{Add.} = 80 + (1 \times 4) = 84\text{mm}$$

(Note: The tip of driving wheel is in contact with a tooth of driving wheel at the end of engagement. So it is required to find path of recess.)

Path of recess

$$PL = \left(\sqrt{r_A^2 - (r\cos ø)^2} - r\sin ø\right)$$

$$= \left(\sqrt{52^2 - \left(48\cos 20^\circ\right)^2} - 48\sin 20^\circ\right)$$

$$= 9.458\text{mm}$$

Velocity of sliding

$$= (\omega_p + \omega_g) \times PL$$

$$= \frac{2\pi}{60}(600 + 360) \times 9.458$$

$$\left(\because \frac{N_g}{N_p} = \frac{t}{T} \Rightarrow N_g = 600 \times \frac{24}{40} = 360\text{rpm}\right)$$

$$= 956.82\text{mm/ sec}$$

Path of recess

$$\therefore KP = \left(\sqrt{R_A^2 - (R\cos ø)^2} - R\sin ø\right)$$

$$= \left(\sqrt{84^2 - \left(80\cos 20^\circ\right)^2} - 80\sin 20^\circ\right)$$

$$= 10.108\text{mm}$$

Max. Velocity of sliding

$$= \left(\omega_p + \omega_g\right) \times KP$$

$$= \frac{2\pi}{60}(600 + 360) \times 10.108$$

$$= 1016.16\text{mm/ sec}$$

Example 5.8: Two 20° involute spur gears mesh externally and give a velocity ratio of 3. The module is 3 mm and the addendum is equal to 1.1 module. If the pinion rotates at the 120 rpm, determine

I. Minimum no of teeth on each wheel to avoid interference

II. Contact ratio

Solution:

Given data

$Ø = 20°$

$VR = 3$

$m = 3$

$N_P = 120$

Addendum $= 1.1$ module

Find:

t_{min} & $T_{min} = ?$

Contact ratio = ?

I.

$$T = \frac{2A_w}{\left[\sqrt{1 \quad \frac{1}{G}\left(\frac{1}{\ } + 2\right)\sin^2 ø} - 1\right]}$$

$$\therefore T = \frac{2\times 1.1}{\left[\sqrt{1+\frac{1}{3}\left(\frac{1}{3} + 2\right)\sin^2 20^\circ} - 1\right]}$$

$$\therefore T = 49.44 \text{teeth}$$

$$\therefore T = 51 \text{teeth} \qquad \text{And} \qquad t = \frac{T}{3} = \frac{51}{3} = 17 \text{teeth}$$

II.

$$r = \frac{mt}{2} = \frac{3\times 17}{2} = 25.5\text{mm} \quad R = \frac{mT}{2} = \frac{3\times 51}{2} = 76.5\text{mm}$$

$$r_a = r + \text{Add.} = 25.5 + (1.1\times 3) = 28.8\text{mm} \quad R_A = R + \text{Add.} = 76.5 + (1.1\times 3) = 28.8\text{mm}$$

$$\text{Contactratio} = \frac{\text{Lengthofpathofcontact}}{\cos\varphi \times P_c}$$

$$= \frac{\left(\sqrt{R_A^{\,2} - (R\cos\phi)^2} - R\sin\phi\right) + \left(\sqrt{r_A^{\,2} - (r\cos\phi)^2} - r\sin\phi\right)}{\cos 20^\circ \times \pi \times 3}$$

$$= \frac{\left(\sqrt{79.8^2 - \left(76.5\cos 20^\circ\right)^2} - 76.5\sin 20^\circ\right) + \left(\sqrt{28.8^2 - \left(25.5\cos 0^\circ\right)^2} - 25.5\sin 20^\circ\right)}{\cos 20^\circ \times \pi \times 3}$$

$$= 1.78$$

Thus 1 pair of teeth will always remain in contact whereas for 78 % of the time, 2 pairs of teeth will be in contact.

Example 5.9: Two involute gears in a mesh have a velocity ratio of 3. The arc of approach is not to be less than the circular pitch when the pinion is the driver The pressure angle of the involute teeth is 20°.Determine the least no of teeth on the each gear. Also find the addendum of the wheel in terms of module.

Solution:

Given data

$Ø = 20°$

$VR = 3$

Find:

least no of teeth on the each gear = ?

Addendum = ?

$$\text{Arc of approach} = \text{circular pitch}$$

$$= \pi \cdot m$$

$$\therefore \text{Path of approach} = \text{Arc of approach} \times \cos 20^\circ$$

$$= \pi \cdot m \cdot \cos 20^\circ$$

$$= 2.952m \qquad \cdots\cdots\cdots(1)$$

Let the max length of path of approach $= r\sin\varphi$

$$= \frac{mt}{2}\sin 20^\circ$$

$$= 0.171mt \qquad \cdots\cdots(2)$$

From eq. 1. And 2.

$$\therefore 0.171mt = 0.2952m$$

$$\therefore t = 17.26 \cong 18\,\text{teeth}$$

$$T = 18\times3 = 54\text{teeth}$$

Max. Addendum of the wheel

$$A_{wmax} = \frac{mt}{2}\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi}-1\right]$$

$$= \frac{m\times 54}{2}\left[\sqrt{1+\frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20^\circ}-1\right]$$

$$= 1.2m$$

Example 5.10: Two 20° involute spar gears have a module of 10 mm. The addendum is equal to one module. The larger gear has 40 teeth while the pinion has 20 teeth will the gear interfere with the pinion?

Solution:

Given data

Ø=20°

m = 10 mm

Addendum = 1 module

= 1×10

= 10mm

Find:

Interference or not?

Let the pinion is the driver

t = 20 teeth

T = 40 teeth

$$r = \frac{mt}{2} = \frac{10\times20}{2} = 100\text{mm} \qquad R = \frac{mT}{2} = \frac{10\times40}{2} = 200\text{mm}$$

$$r_a = r + \text{Add.} = 100 + 10 = 110\text{mm} \qquad R_A = R + \text{Add.} = 200 + 10 = 210\text{mm}$$

$$\text{Path of approach} = \left(\sqrt{R_A^2 - (\quad)^2} - R\sin ø\right)$$

$$= \left(\sqrt{210^2 - (200\cos20^\circ)^2} - 200\sin20^\circ\right)$$

$$= 25.29\text{mm}$$

To avoid the interference…….

$$\text{Max length of path of approach} = r\sin\varphi$$

$$= 100 \times \sin 20$$

$$= 34.20\text{mm} > 25.29\text{mm}$$

So Interference will **not occur.**

Example 5.11: Two 20° involute spur gears have a module of 10 mm. The addendum is one module. The larger gear has 50 teeth and the pinion has 13 teeth. Does interference occur? If it occurs, to what value should the pressure angle be change to eliminate interference?

Solution:

Given data

Ø=20°
m=10 mm
Addendum = 1 module = 10
mm T = 50 and t = 13

$$r = \frac{mt}{2} = \frac{10\times13}{2} = 65\text{mm} \qquad R = \frac{mT}{2} = \frac{10\times50}{2} = 250\text{mm}$$

$r_a = r + \text{Add.} = 65 + 10 = 75\text{mm}$ $R_A = R + \text{Add.} = 250 + 10 = 260\text{mm}$

$$R_{a\max} = \sqrt{(R\cos\varphi)^2 + (R\sin\varphi + r\sin\varphi}$$

$$= \sqrt{(250\cos 20^\circ)^2 + (250\sin 20^\circ + 65\sin 20^\circ)}$$

$$= 258.45\text{mm}$$

Here actual addendum radius R_a (260 mm) > $R_{a\ max}$ value

So interference will

occur. The new value of ø can be found by

comparing

$$R_{a\max} = R_a$$

$$\therefore R_a = R_{a\max}$$

$$\therefore R_a = \sqrt{(RCos\varphi)^2 + (RSin\varphi + rSin\varphi}$$

$$\therefore 260 = \sqrt{(250Cos\varphi)^2 + (250Sin\varphi + 65Sin\varphi}$$

$$\therefore 260^2 = (250Cos\varphi)^2 + (250Sin\varphi + 65Sin\varphi)^2$$

$$\therefore Cos^2\varphi = 0.861$$

$$\therefore \varphi = 21.88^{\circ}$$

Note: If pressure angle is increased to 21.88° interference can be avoided

Example 5.12: The following data related to meshing involute gears:
No. of teeth on gear wheel = 60
Pressure angle = 20°
Gear ratio =1.5
Speed of gear wheel = 100 rpm
Module = 8 mm

The addendum on each wheel is such that the path of approach and path of recess on each side are 40 % of the maximum possible length each. Determine the addendum for the pinion and the gear and the length of arc of contact.

Solution:

Given data	Find:
T = 60	Addendum for gear and pinion=?
Ø= 20°	Length of arc of contact=?
G= 1.5	
N_g=100	
rpm m=8	
mm	

Let pinion is driver…

Max. Possible length of path of approach $= r\sin\phi$

$\therefore$ Actual length of path of approach $= 0.4\, r\sin\phi$ (Given in data)

Same way…

Actual length of path of recess = 0.4 Rsinϕ (Given in data)

$$\therefore 0.4\, r\sin\phi = \left(\sqrt{R^2 - (R\cos\phi)^2} - R\sin\phi\right)$$

$$\therefore 0.4 \times 160 \sin 20 = \left(\sqrt{R_A^2 - (240\cos 20)^2} - 240\sin 20\right)$$

$$\therefore R_a = 248.33 \text{ mm}$$

$$\therefore \text{Addendum of wheel} = 248.3 - 240 = 8.3\text{mm}$$

Also

$$0.4\, R\sin\phi = \sqrt{r_A^2 - (r\cos\phi)^2} - r\sin\phi$$

$$\therefore 0.4 \times 240 \times \sin 20 = \sqrt{r_A^2 - (160\cos 20)^2} - 160\sin 20$$

$$\therefore r_a = 173.98 = 174 \text{ mm}$$

$$\therefore \text{Addendum of pinion} = 174 - 160 = 14 \text{ mm}$$

$$\text{Length of Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi}$$

$$= \frac{(r\sin\varphi + R\sin\phi) \times 0.4}{\cos\phi}$$

$$= \frac{(160 + 240) \times \sin 20 \times 0.4}{\cos 20}$$

$$= 58.2 \text{ mm}$$

Example 5.13: A pinion of 20 ° involute teeth rotating at 274 rpm meshes with a gear and provides a gear ratio of 1.8. The no. of teeth on the pinion is 20 and the module is 8mm .If interference is just avoided

Determine:
1. Addendum on wheel and pinion
2. Path of contact
3. Max. Velocity of sliding on both side of pitch point

Solution:

Given data

$\varnothing = 20°$

$m = 8$ mm

$N_p = 275$ rpm

$T = 36$

$t = 20$

Find:

1. Addndum on wheel and pinion = ?
2. Path of contact = ?
3. Max. velocity of sliding on both side of pitchpoint = ?

Max. Addendum on wheel

$$\therefore A_{w\,max} = R\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2 \varnothing} - 1\right]$$

$$\therefore A_{w\,max} = 144\left[\sqrt{1 + \frac{1}{1.8}\left(\frac{1}{1.8} + 2\right)\sin^2 20} - 1\right]$$

$$= 11.5 \text{ mm}$$

Max. Addendum on pinion

$$\therefore A_{p\,max} = r\left[\sqrt{1 + G(G+2)\sin^2 \varnothing} - 1\right]$$

$$\therefore A_{p\,max} = 80\left[\sqrt{1 + 1.8(1.8+2)\sin^2 20} - 1\right]$$

$$= 27.34 \text{ mm}$$

Path of contact when interference is just avoided ….

= Max. path of approach + Max. path of recess

$= r\sin\phi + R\sin\phi$

$= 80\sin20 + 144\sin20$

$= 27.36 + 49.25$

$= 76.6$ mm

Velocity of sliding ononesideof approach

$$= \left(\omega_p + \omega_g\right) \text{Path of approach} \qquad \left(\begin{aligned} \omega_p &= \frac{2\pi \times 275}{60} = 28.8 \text{ rad / sec} \\ \omega_g &= \frac{28.8}{1.8 - G} = 16 \text{ rad / sec} \end{aligned} \right)$$

$$= (28.8 + 16) \times 27.36$$

$$= 1225.72 \text{ mm / sec}$$

Velocity of sliding on side of path of recess

$$= \left(\omega_p + \omega_g\right) \text{Path of recess}$$

$$= (28.8 + 16) \times 49.25$$

$$= 2206 \text{ mm/ sec}$$

Example 5.14: A pinion of 20 involute teeth and 125 mm pitch circle diameter drives a rack. The addendum of both pinion and rack is 6.25 mm. What is the least pressure angle which can be used to avoid interference? With this pressure angle, find the length of the arc of contact and the minimum number of teeth in contact at a time.

Solution:

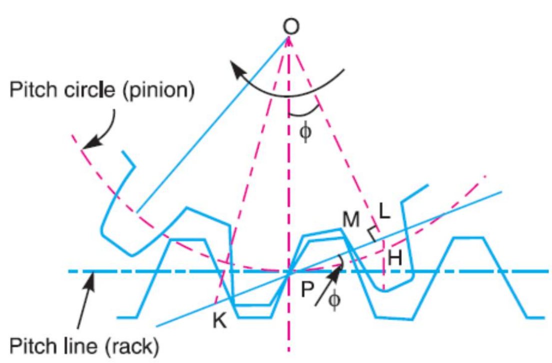
Given data Find:

$T = 20$ 1. Least pressure angle to avoid interference = ?

$d = 125$ mm 2. Length of arc of contact = ?

$r = OP = 62.5$ mm 3. Min. no. of teeth in contact = ?

Addendum for rack / pinion, LH = 6.25 mm



Least pressure angle to avoid interference

Let ø = Least pressure angle to avoid interference.

We know that for no interference, rack addendum,

From fig..... $LH = PL\sin\phi$

$$= r\sin\phi \times \sin\phi$$
$$= r\sin^2\phi$$
$$\therefore \sin^2\phi = \frac{LH}{r} = \frac{6.25}{62.5}$$
$$\therefore \phi = (18.4349)^\circ$$

Length of arc of contact

Now, $KL = \sqrt{OK^2 -}$

$$= \sqrt{(OP + 6.25)^2 - (r\cos}$$
$$= \sqrt{(62.5 + 6.25)^2 - (62.5 \times \cos 18.439^\circ)^2}$$
$$= 34.8 \text{ mm}$$

$$\text{Length of Arc of Contact} = \frac{KL}{\cos\phi} = \frac{34.8}{\cos 18.439^\circ} = 36.68 \text{ mm}$$

Min. No. of teeth in contact

$$\text{Min. no. of teeth in contact} = \frac{\text{Length of arc of contact}}{p_c}$$
$$= \frac{\text{Length of arc of contact}}{\pi \cdot m}$$
$$= \frac{36.68}{19.64}$$
$$= 1.87$$
$$\cong 2$$

Example 5.15: In a spiral gear drive connecting two shafts, the approximate center distance is 400 mm and the speed ratio = 3. The angle between the two shafts is 50° and the normal pitch is 18 mm. The spiral angles for the driving and driven wheels are equal.

Find : 1. Number of teeth on each wheel,
2. Exact center distance, and
3. Efficiency of the drive, if friction angle = 6°.
4. Maximum efficiency.

Solution:

Given data:

$L = 400$ mm $\quad \theta = 50^\circ \quad G = \frac{T_2}{T_1} = 3$

$\phi = 6 \quad P_N = 18$ mm

***1.** No. of teeth on wheel:*

$$L = \frac{P_N \cdot T_1}{2\pi}\left[\frac{1}{\cos\alpha_1} + \frac{G}{\cos\alpha_2}\right]$$

$$\therefore 400 = \frac{P_N \cdot T_1}{2\pi} \times \frac{1+G}{\cos\alpha_1}$$

$$\therefore 400 = \frac{18 \cdot T_1}{2\pi} \times \frac{1+3}{\cos 25^\circ} \qquad \begin{pmatrix} \because \alpha_1 = \alpha_2 \\ \theta = \alpha_1 + \alpha_2 \\ \therefore 50 = 2\alpha_1 \\ \therefore \alpha_1 = 25^\circ \end{pmatrix}$$

$$\therefore T_1 = 31.64 \approx 32$$

$$\therefore T_2 = 3T_1 = 96$$

***2.** Exact center distance (L):*

$$L = \frac{P_N \cdot T_1}{2\pi}\left[\frac{1}{\cos\alpha_1} + \frac{G}{\cos\alpha_2}\right]$$

$$= \frac{P_N \cdot T_1}{2\pi}\left[\frac{1+G}{\cos\alpha_1}\right] \qquad (\because \alpha_1 = \alpha_2)$$

$$= \frac{18 \cdot 32}{2\pi}\left[\frac{1+3}{\cos 25^\circ}\right]$$

$$= 404.600 \text{ mm}$$

***3.** Efficiency of drive:*

$$\eta = \frac{\cos(\alpha_2 + \phi) \times \cos\alpha_1}{\cos(\alpha_1 - \phi) \times \cos\alpha_2}$$

$$= \frac{\cos(\alpha_1 + \phi)}{\cos(\alpha_1 - \phi)} \qquad (\because \alpha_1 = \alpha_2)$$

$$= \frac{\cos(25+6)}{\cos(25-6)}$$

$$= 90.655\%$$

***4.** Maximum efficiency:*

$$\eta_{max} = \frac{\cos(\theta+\phi)+1}{\cos(\theta-\phi)+1}$$

$$= \frac{\cos(50+6)+1}{\cos(50-6)+1}$$

$$= 90.685\ \%$$

Example 5.16: A drive on a machine tool is to be made by two spiral gear wheels, the spirals of which are of the same hand and has normal pitch of 12.5 mm. The wheels are of equal diameter and the center distance between the axes of the shafts is approximately 134 mm. The angle between the shafts is 80° and the speed ratio 1.25.

Determine : 1. the spiral angle of each wheel,
2. The number of teeth on each wheel,
3. The efficiency of the drive, if the friction angle is 6°, and
4. The maximum efficiency.

Solution:

Given data:

$P_N = 12.5$ mm

$L = 134$ mm

$G = 1.25$

$\theta = 80^\circ$

__1.__ Spiral angle of each wheel

Weknow that........

$$\therefore \frac{d_2}{d_1} = \frac{T_2 Cos\alpha_1}{T_1 Cos\alpha_2}$$

$\therefore T_1 Cos\alpha_2 = T_2 Cos\alpha_1$ $\quad (\because d_1 = d_2)$

$\therefore Cos\alpha_1 = 1.25 Cos\alpha_2$ $\quad (\because \frac{T_1}{T_2} = G = 1.25)$

$\therefore Cos\alpha_1 = 1.25\ Cos(\theta - \alpha_1)$ $\quad (\because \alpha_1 + \alpha_2 = \theta)$

$\therefore Cos\alpha_1 = 1.25\ Cos(80 - \alpha_1)$

$\therefore Cos\alpha_1 = 1.25(Cos80 \cdot Cos\alpha_1 + Sin80 \cdot Sin\alpha_1)$

$(\because Cos(A - B) = CosA \cdot CosB + SinA \cdot SinB)$

By solving...............

$$\tan\alpha_1 = 0.636$$
$$\therefore \alpha_1 = 32.46^\circ$$

and $\alpha_2 = 80^\circ - 32.46^\circ = 47.54^\circ$

***2.** No. of teeth on wheel:*

$$L = \frac{d_1 + d_2}{2}$$
$$\therefore 134 = \frac{2d_1}{2} \qquad (\because d_1 = d_2)$$
$$\therefore d_1 = 134 \text{ mm}$$

Let $p_{c1} = \frac{\pi d_1}{T_1} \Rightarrow d_1 = \frac{p_{c1} \cdot T_1}{\pi}$

$$\therefore d_1 = \frac{P_N}{\cos\alpha_1} \times \frac{T_1}{\pi}$$
$$\therefore T_1 = \frac{d_1 \cdot \cos\alpha_1 \cdot \pi}{P_N}$$
$$\therefore T_1 = \frac{134 \times \cos 32.24 \times \pi}{12.5}$$
$$\therefore T_1 = 28.4 \approx 30 \text{ nos.}$$

Now, $G = \frac{T_1}{T_2} = 1.25 \Rightarrow T_2 = \frac{T_1}{G} = \frac{30}{1.25}$

$$T_2 = 24 \text{ nos.}$$

***3.** Efficiency of drive:*

$$\eta = \frac{\cos(\alpha_2 + \phi) \times \cos\alpha_1}{\cos(\alpha_1 - \phi) \times \cos\alpha_2}$$
$$= \frac{\cos(47.24 + 6) \times \cos 32.46}{\cos(32.46 - 6) \times \cos 47.24}$$
$$= 83\ \%$$

***4.** Maximum efficiency:*

$$\eta_{max} = \frac{\cos(\theta + \phi) + 1}{\cos(\theta - \phi) + 1}$$
$$= \frac{\cos(80 + 6) + 1}{\cos(80 - 6) + 1}$$
$$= 83.8\ \%$$

Example 5.17: Find the minimum no. of teeth on gear wheel and the arc of contact(in term of module) to avoid the interference in the following cases:

I. The gear ratio is unity
II. The gear ratio is 3
III. Pinion gear with a rack

Addendum of the teeth is 0.84 module and the power component is 0.95 times the normal thrust.

Solution:

Here $A_w = 0.84$

$$\cos\phi = 0.95 \Rightarrow \phi = 18.19$$

$$\therefore \sin\phi = 0.3122$$

I. *Gear ratio is unity*

- Let min. no of teeth on gear wheel T

$$\therefore T = \frac{2A_w}{\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varnothing}-1\right]}$$

$$= \frac{2A_w \cdot G}{\left[\sqrt{G^2+(1+2G)\sin^2\varnothing}-G\right]}$$

$$= \frac{2\times 0.84\times 1}{\left[\sqrt{1^2+(1+2)(0.3123)^2}-1\right]}$$

$$= 12.73$$

$$\therefore T \cong 13 \text{ teeth}$$

$$\therefore t \cong 13 \text{ teeth}$$

- Length of arc of contact:

$$L.P.C = \left(\sqrt{R_A^{\,2}-(R\cos\varnothing)^2}-R\sin\varnothing\right)+\left(\sqrt{r_A^{\,2}-(r\cos\varnothing)^2}-r\sin\varnothing\right)$$

$$= 2\left(\sqrt{r_A^{\,2}-(r\cos\varnothing)^2}-r\sin\varnothing\right) \qquad \left(\begin{aligned} \because r &= \frac{m\cdot t}{2} = \frac{m\cdot 13}{2} = 6.5m \\ r_A &= r + \text{addendum} \\ &= 6.5m + 0.84m \\ &= 7.34m \end{aligned}\right)$$

$$= 2m\left(\sqrt{(7.34)^2-(6.5\times 0.95)^2}-(6.5\times 0.3123)\right)$$

$$= 3.876m$$

$$\text{L.A.C} = \frac{3.876\text{m}}{\cos \phi} = \frac{3.876\text{m}}{0.95}$$

$$\therefore \text{L.A.C} = 4.08\text{m}$$

II. Gear ratio G = 3

- Let min. no of teeth on gear wheel T

$$\therefore T = \frac{2A_w}{\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right]}$$

$$= \frac{2A_w \cdot G}{\left[\sqrt{G^2+(1+2G)\sin^2\phi} - G\right]}$$

$$= \frac{2\times 0.84\times 3}{\left[\sqrt{3^2+(1+2\times 3)(0.3123)^2} - 3\right]}$$

$$= 45.11$$

$$\therefore T \cong 45 \text{ teeth}$$

$$\therefore t \cong 15 \text{ teeth}$$

- Length of arc of contact:

$$\text{L.P.C} = \left(\sqrt{R_A^2 - (R\cos\phi)^2} - R\sin\phi\right) + \left(\sqrt{r_A^2 - (r\cos\phi)^2} - r\sin\phi\right) \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

$$\left(\begin{array}{l}
\because r = \dfrac{m\cdot t}{2} = \dfrac{m\cdot 15}{2} = 7.5\text{m} \\
r_A = r + \text{addendum} \\
\quad = 7.5\text{m} + 0.84\text{m} \\
\quad = 8.34\text{m} \\
R = \dfrac{mT}{2} = \dfrac{m\times 45}{2} = 22.5\text{m} \\
R_A = R + \text{addendum} \\
\quad = 22.5\text{m} + 0.84\text{m} \\
\quad = 23.34\text{m}
\end{array}\right)$$

putting all values in equation (1)

$$= \left(\sqrt{(23.34\text{m})^2 - (22.5\text{m} \times 0.95)^2} - 22.5\text{m} \times 0.3122\right) + \left(\sqrt{(8.34\text{m})^2 - (7.5\text{m} \times 0.95)^2} - 7.5\text{m} \times 0.3122\right)$$

$$= 4.343\text{m}$$

$$\text{L.A.C} = \frac{4.343\text{m}}{\cos\phi} = \frac{3.876\text{m}}{0.95}$$

$$\therefore \text{L.A.C} = 4.57\text{m}$$

III. *Pinion gear with a rack*

- Min. no. of teeth on pinion t

$$t = \frac{2\,A_R}{\sin^2\phi} = \frac{2 \times 0.84}{(0.3123)^2}$$

$$\therefore t = 17.23$$

$$\therefore t \cong 18$$

- Length of arc of contact:

$$\text{L.P.C} = \left(\sqrt{R_A^2 - (R\cos\phi)^2} - R\sin\phi\right) + \left(\sqrt{r_A^2 - (r\cos\phi)^2} - r\sin\phi\right)$$

$$= 2\left(\sqrt{r_A^2 - (r\cos\phi)^2} - r\sin\phi\right)$$ (∵ assume rack and pinion same dimension)

$$= 2\left(\sqrt{(9.84)^2 - (9\text{m} \times 0.95)^2} - 9\text{m} \times 0.3123\right)$$

$$\left(\begin{aligned} \because r &= \frac{mt}{2} = \frac{18\text{m}}{2} = 9\text{m} \\ r_A &= r + \text{addendum} \\ &= 9\text{m} + 0.84\text{m} \\ &= 9.84\text{m} \end{aligned}\right)$$

$$= 4.12\text{m}$$

$$\text{L.A.C} = \frac{4.12\text{m}}{\cos\phi} = \frac{4.12\text{m}}{0.95}$$

$$\therefore \text{L.A.C} = 4.337\text{m}$$

Gear Train

Introduction

Definition

- When two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear train or train of toothed wheels.
- The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

Types of Gear Trains

1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Epicyclic gear train
5. Compound epicyclic gear train

Simple gear train.

- When there is only one gear on each shaft, as shown in Fig. , it is known as **simple gear train.** The gears are represented by their pitch circles.
- When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig.
- Since the gear 1 drives the gear 2, therefore gear 1 is called the **driver** and the gear 2 is called the **driven** or **follower.** It may be noted that the motion of the driven gear is opposite to the motion of driving gear.

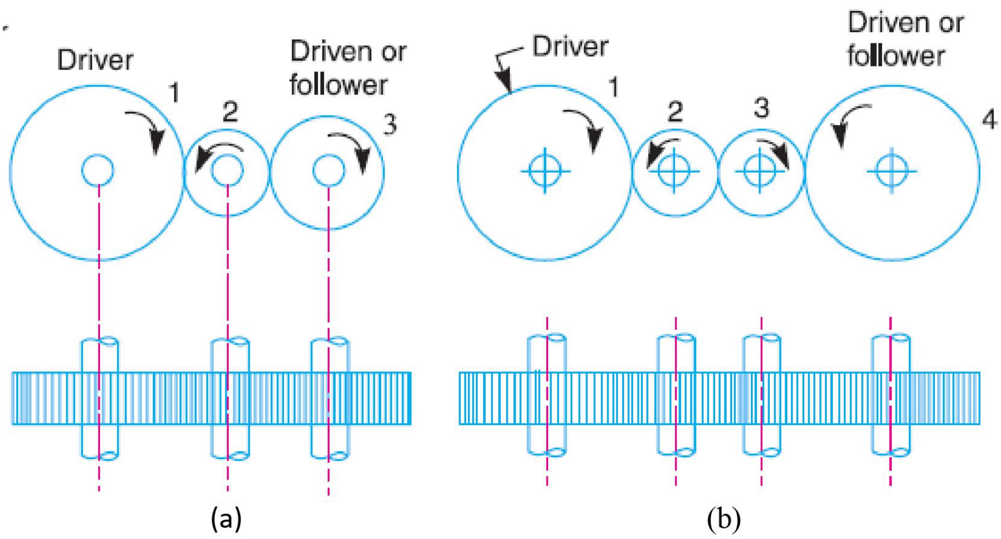


Fig.5.2.1 Simple gear train

Let

N_1 = Speedof driver rpm
N_2 = Speedof intermediatewheel rpm
N_3 = Speedof follower rpm
T_1 = Number of teethon driver
T_2 = Number of teethonintermediatewheel T_3
= Number of teethonfollower

– Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \qquad \cdots\cdots\cdots(1)$$

– Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

$$\frac{N_2}{N_3} = \frac{T_3}{T_2} \qquad \cdots\cdots\cdots(2)$$

– The speed ratio of the gear train as shown in Fig. (a) Is obtained by multiplying the equations (1) and (2).

$$\frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2}$$
$$\therefore \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

– Sometimes, the distance between the two gears is large. The motion from one gear to another, in such a case, may be transmitted by either of the following two methods:

1. By providing the large sized gear, or
   - A little consideration will show that this method (i.e. providing large sized gears) is very inconvenient and uneconomical method.

2. By providing one or more intermediate gears.
   - This method (i.e. providing one or more intermediate gear) is very convenient and economical.

– It may be noted that when the number of intermediate gears are **odd**, the motion of both the gears (i.e. driver and driven or follower) is like as shown in Fig. (a).
– If the numbers of intermediate gears are **even**, the motion of the driven or follower will be in the **opposite direction** of the driver as shown in Fig (b).

- **speed ratio** (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth.

$$\text{Speedratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

- **Train value** of the gear train is the ratio of the speed of the driven or follower to the speed of the driver.

$$\text{Trainvalue} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

Compound Gear Train

– When there is more than one gear on a shaft, as shown in Fig., it is called a **compound train of gear.**

– The idle gears, in a simple train of gears do not affect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven.

– But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts.

– In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in Fig.

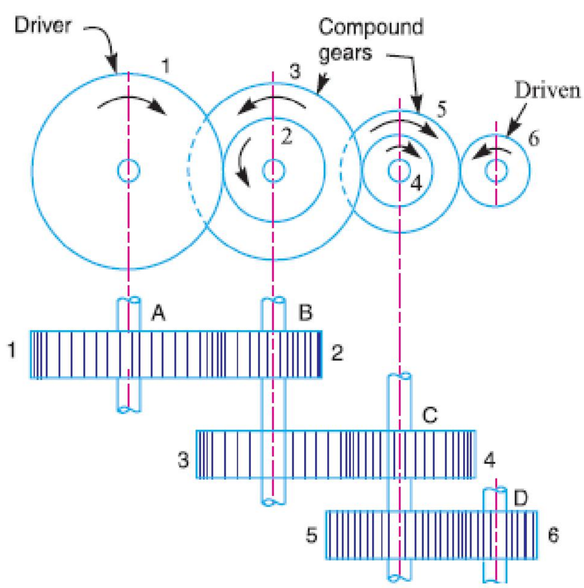


Fig. 5.2.2 compound gear train

– In a compound train of gears, as shown in Fig., the gear 1 is the driving gear mounted on shaft A; gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let

N1 = Speed of driving gear 1,
T1 = Number of teeth on driving gear 1,
N2 ,N3 ..., N6 = Speed of respective gears in r.p.m.,
and T2 ,T3..., T6 = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \qquad \cdots\cdots\cdots(1)$$

Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \qquad \cdots\cdots\cdots(2)$$

And for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \qquad \cdots\cdots\cdots(3)$$

The speed ratio of compound gear train is obtained by multiplying the equations (1), (2) and (3),

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

– The **advantage** of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears.
– If a simple gear train is used to give a large speed reduction, the last gear has to be very large.
– Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed.

Reverted Gear Train

- When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as **reverted gear train.**
- Gear 1 (i.e. first driver) drives the gear 2 (i.e. first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (i.e. the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is **like.**

Let

T1 = Number of teeth on gear 1,

r1 = Pitch circle radius of gear 1, and

N1 = Speed of gear 1 in r.p.m.

Similarly,

T2, T3, T4 = Number of teeth on respective gears,

r2, r3, r4 = Pitch circle radii of respective gears, and

N2, N3, N4 = Speed of respective gears in r.p.m.

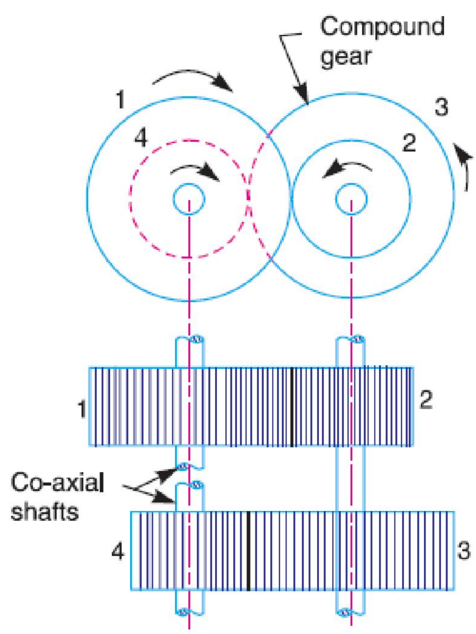


Fig. 5.2.3 Reverted gear train

- Since the distance between the centers of the shafts of gears 1 and 2 as well as gears 3 and 4 is same, therefore

$$r_1 + r_2 = r_3 + r_4$$

– Also, the circular pitch or module of all the gears is assumed to be same; therefore number of teeth on each gear is directly proportional to its circumference or radius.

$$T_1 + T_2 = T_3 + T_4$$

$$\text{Speedratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}$$

Application

– The reverted gear trains are used in automotive transmissions**,** lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).

Epicyclic Gear Train

– In an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in Fig. where a gear A and the arm C have a common axis at O1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O2, about which the gear B can rotate.

– If the arm is fixed, the gear train is simple and gear A can drive gear B or **vice- versa**, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O1), then the gear B is forced to rotate **upon** and **around** gear A. Such a motion is called **epicyclic** and the gear trains arranged in such a manner that one or more of their members move upon and around another member is known as **epicyclic gear trains** (**epi.** means upon and **cyclic** means around). The epicyclic gear trains may be **simple** or **compound.**

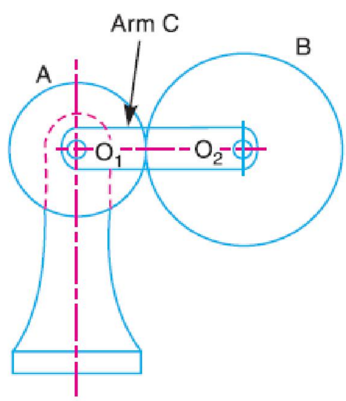


Fig. 5.2.4 Epicyclic gear train

Sr. No.	Condition of motion	Revolution of element		
		Arm C	Gear A	Gear B
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_A}{T_B}$
2	Arm fixed gear A rotates through $+x$ revolutions	0	$+x$	$-x\frac{T_A}{T_B}$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	y- $x\frac{T_A}{T_B}$

Application

- The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

Compound Epicyclic Gear Train—Sun and Planet Gear

- A compound epicyclic gear train is shown in Fig. It consists of two co-axial shafts S1 and S2, an annulus gear A which is fixed, the compound gear (or planet gear) B-C, the sun gear D and the arm H. The annulus gear has internal teeth and the compound gear is carried by the arm and revolves freely on a pin of the arm H. The sun gear is co-axial with the annulus gear and the arm but independent of them.
- The annulus gear A meshes with the gear B and the sun gear D meshes with the gear C. It may be noted that when the annulus gear is fixed, the sun gear provides the drive and when the sun gear is fixed, the annulus gear provides the drive. In both cases, the arm acts as a follower.

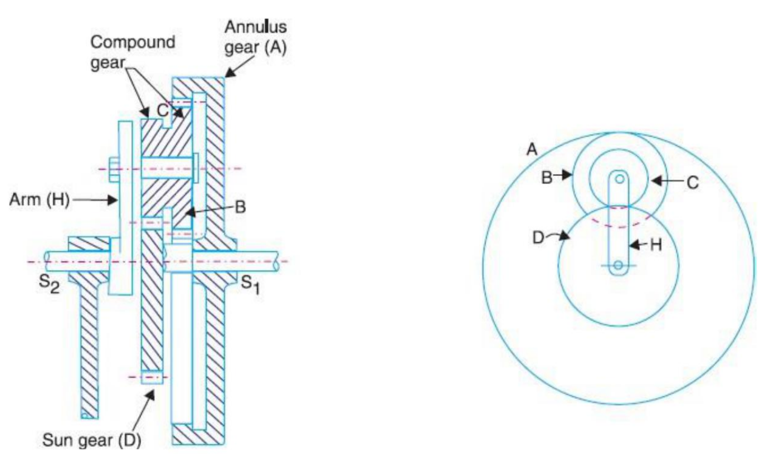


Fig. 5.2.5 Compound epicyclic gear train.

Note: The gear at the center is called the **sun gear** and the gears whose axes move are called **planet gears.**

Let T_A, T_B, T_C, and T_D be the teeth and N_A, N_B, N_C and N_D be the speeds for the gears *A*, *B*, *C* and *D* respectively. A little consideration will show that when the arm is fixed and the sun gear *D* is turned anticlockwise, then the compound gear *B*-*C* and the annulus gear A will rotate in the clockwise direction.

The motion of rotations of the various elements is shown in the table below.

Table of motions

Sr. No.	Condition of motion	Revolution of motion			
		Arm	**Gear D**	**Compound Gear (B-C)**	**Gear A**
1	Arm fixe, gear D rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_D}{T_C}$	$-\frac{T_D}{T_B} \times \frac{T_C}{T_A}$
2	Arm fixed gear D rotates through + *x* revolutions	0	+x	$x \frac{T_D}{T_C}$	$-x \frac{T_D}{T_C} \times \frac{T_B}{T_A}$
3	Add + *y* revolutions to all elements	+y	+y	+y	+y
4	Total motion	+y	x + y	$y - x \frac{T_D}{T_C}$	$y - x \frac{T_D}{T_C} \times \frac{T_B}{T_A}$