Unit - 3
Transformers

3. Principle and operation of Transformer :-

Transformer :-

- Transformer is a device used to transfer the Electrical power from one circuit to another circuit with out change in Frequency.
- It is used to vary the voltage levels such that to vary the current levels.
- It works with the principle of Electromagnetic Induction (or) Mutual induction i.e, Faradays laws of Electromagnetic Induction.
- (It is a static device hence the efficiency is higher than other machines (since there will be no rotational losses).)
- (Another property of Transformer is to maintain constant flux density hence it is called "constant Flux machine.")

working principle of Transformer :-

- The working principle of Transformer is based on Mutual Induction between two coupled coils.
- According to this principle a changing flux creates an induced Emf in each turn equal to the derivative of Flux, so that the total induced Emf across N turns is

$$E = N \frac{d\phi}{dt}$$

This can be seen in Transformer shown in Figure (1) as Explained below

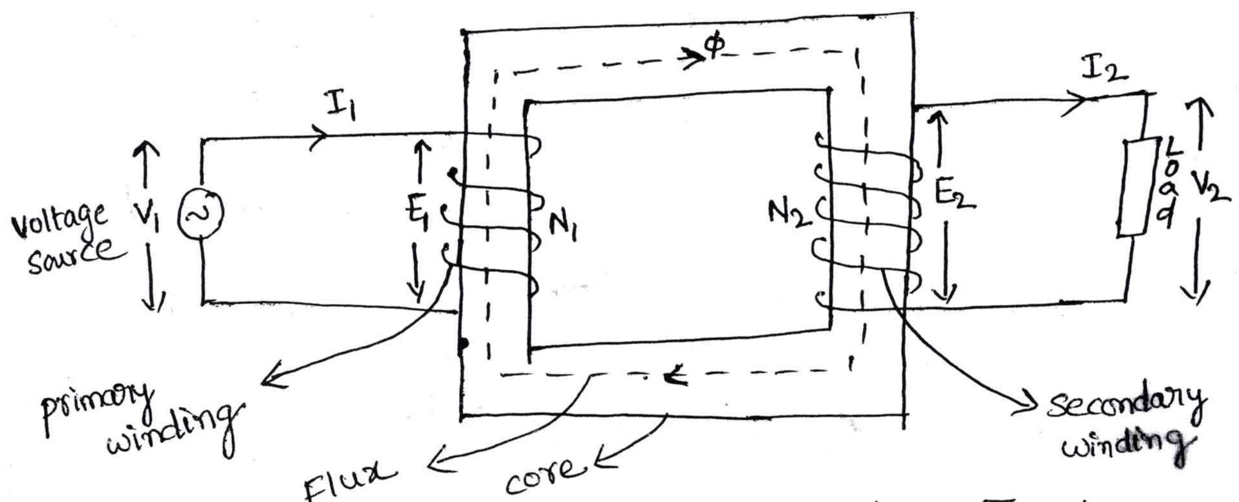


Fig (1) :- General arrangement of a Transformer.

→ Transformer core is wound with two windings having N_1 and N_2 number of Turns.

→ The winding with N_1 turns is known as primary winding as it is fed by alternating voltage source.

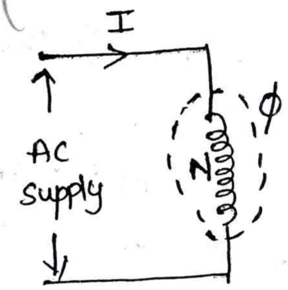
→ This voltage source forms a closed path in the primary so that alternating currents starts flowing through the primary.

→ Alternating Flux set up by alternating mmf $N_1 I_1$ creates an induced Emf E_1 in the primary winding. This Emf E_1 is self Induced Emf.

→ During the path completion of alternating flux through the core, induced Emf's are also created in other than the primary winding.

→ The winding in which emfs are induced mutually is known as secondary winding. This Emf is mutually induced emf. E_2 3

→ Self Inductance :-



→ when ever the current flows then mmf (magneto-motive force) will developed.

→ mmf is nothing but ampere turns.

→ This mmf will produce Flux ϕ.

→ This flux will induces an emf across winding.

i.e, mmf = amp Turns $= NI$

⇓

Flux linkage $= \psi = N\phi$

Emf induced across winding $e = -\frac{d\psi}{dt}$

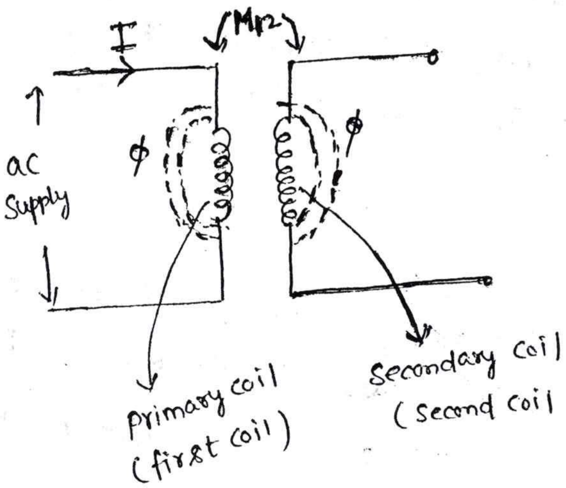
$$= -\frac{dN\phi}{dt}$$

$$e = -N\frac{d\phi}{dt}$$

The above Eq can be written as $e = -L\frac{dI}{dt}$.

where L = self inductance of coil.

Mutual Inductance:-



Here we will have two induced emf's e_1 and e_2.

e_1 → self induced emf due to the current flowing in own circuit

→ It is induced in primary coil, hence called primary induced emf.

e_2 → It is emf induced in the secondary coil due to the current I in first coil and hence called Mutually Induced emf.

Main Parts of a Transformer:- (or) constructional details of a Transformer:-

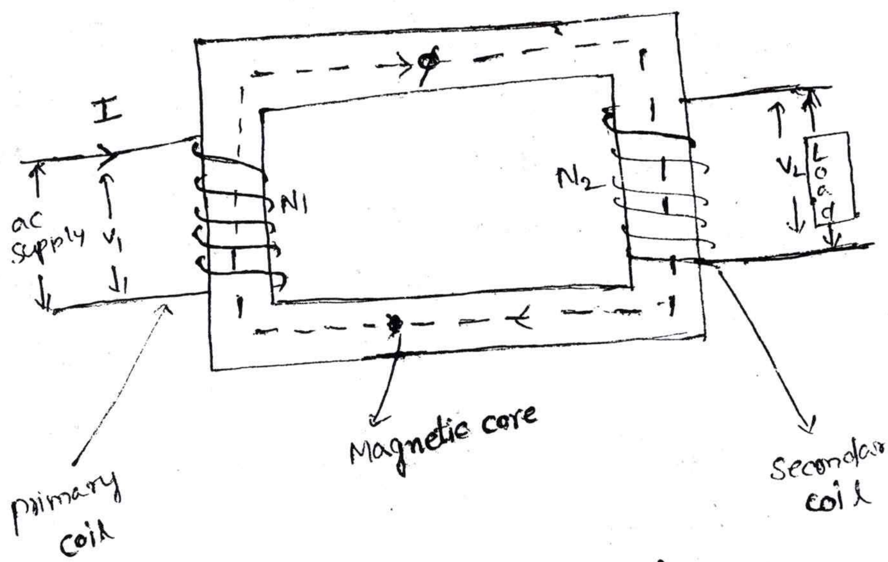


fig (1): General representation of Transformer

(1) Primary winding :-

Primary winding is one which gets the ac ~~supply~~ power supply from the External source. For any Transformer the winding which receives the supply from External source is called Primary winding.

(2) Secondary winding :-

This winding is the second winding of the Transformer and through which the Transformer connects to the External load.

→ These two windings have different no. of Turns for reliable operation.

(3) Magnetic Core :-

The magnetic core is the medium through which the Transformer can Transfer the Electrical Energy from ~~one~~ one circuit to another circuit. The both windings of the Transformer are wound on the same magnetic core. This magnetic core provides the mechanical strength to the Transformer. The core is made up of with silicon steel laminations. Due to this magnetic core, the permeability will increase. The permeability is one which represents the strength of the magnetic core with which it allows more magnetic flux lines through it. Hence we get the reliable & optimum operation.

Functions of magnetic core :-

→ It is used to produce the magnetic flux.

→ It acts as path to the flow of magnetic flux lines.

→ It connects primary and Secondary windings magnetically.

Hence we can say that in Transformers the primary and secondary windings are magnetically connected but not Electrically i.e, These Two windings are Electrically isolated

→ It provides mechanical strength to the Transformer.

(4) Insulating medium:

The insulating medium is provided in between the primary and Secondary windings. This is normally dielectric medium. This medium avoids the electrical contacts between primary & Secondary (LV & HV) windings of the Transformer. Due to the property of dielectric strength, this medium avoids the conductive channels in between LV & HV windings. i.e, it avoids the ionization in between windings. The rating of Transformer is designed according to the type of insulating medium, the dielectric strength of the medium Generally oil and air are used as insulating medium for Transformers

Types of Transformers :-

~~Constructional Features of 1-ϕ Transformer~~ ;

According to the constructions of core, the Transformers can be classified in to

1. Core Type 2. shell Type 3. Berry Type.

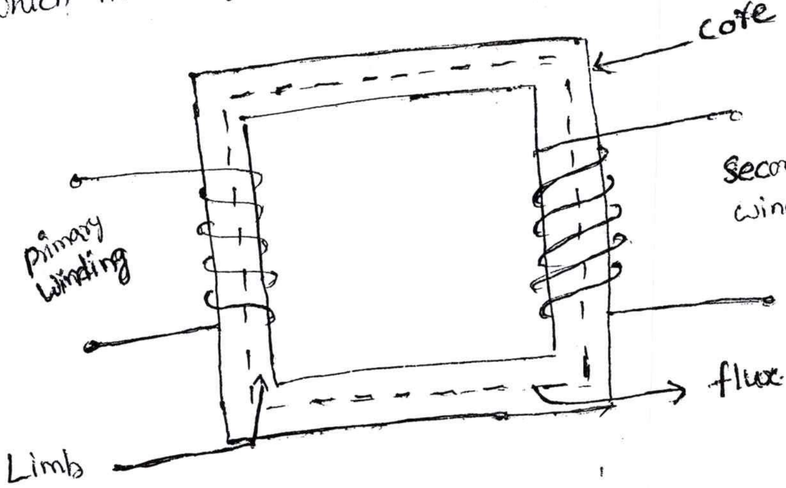
1) Core Type :

→ In this type of construction, transformer has Two Limbed magnetic circuit as a core and these limbs are sorrounded by Transformer windings

→ Each Low Voltage (LV) and high voltage (HV) windings of Transformer have two sections, one is on each of The two limbs of the core, insulated each other by paper (or) mica.

→ At first LV winding is placed on the core after that the HV windings as shown in figure (2).

→ This type of winding arrangement is known as concentric winding, which minimizes the insulation requirement and leakage flux.



core type → fig (1a): representation

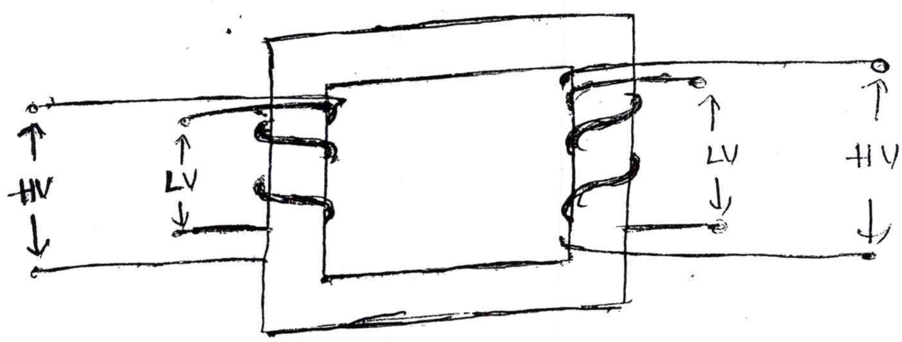


fig (1b): construction

fig (1) : core type construction

Fig (1): Core type construction

Advantages:

1) core type of Transformers are suitable for Extra high Voltage (EHV) and high power levels.
2) It requires less core material compared to shell type construction, for a given output and voltage ratings.

Disadvantages:

1) For a specified transformer ratings core construction requires more conducting material compared to shell type.

2) **SHELL Type Transformer :-**

In this shell type construction, transformer has three limbed magnetic circuit as a core.

In this case both the windings (LV & HV) are wound on central portion of the magnetic core.

In other words, the magnetic core sorrounds the major portion of the windings.

LV and HV windings of Transformer are sub divided into subsections and are placed on central limb of the core.

This type of arrangement minimizes the amount of leakage flux. in shell type construction.

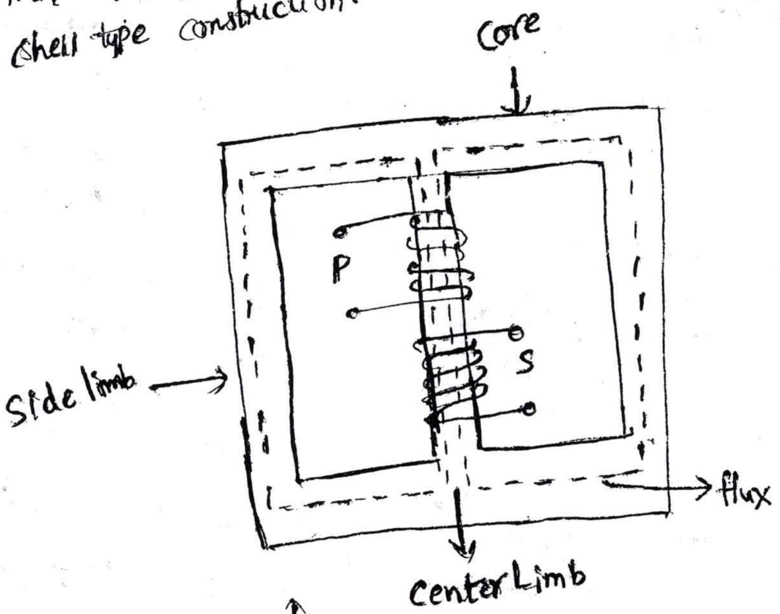
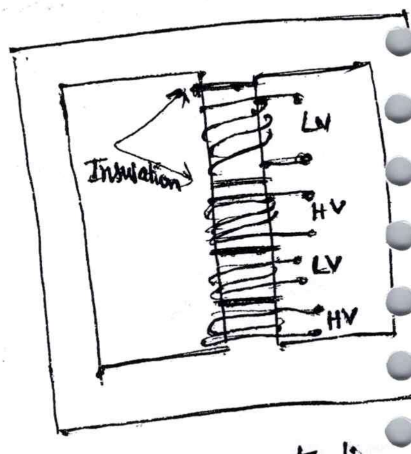


fig:(2a) Representation



fig(2b): Construction.

fig 2: SHELL TYPE construction

Advantages:

1. Shell Type Transformers are suitable for Low voltage and low power Levels.
2. It requires less conducting material compared to core type construction for a specified output and voltage ratings.

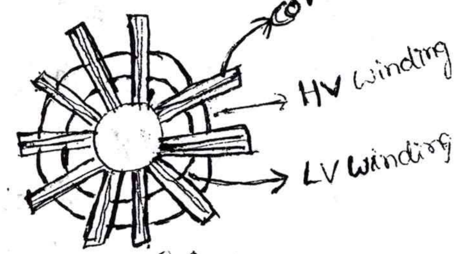
Disadvantages:

For a given Transformer ratings, shell type construction requires more core material compared to core type.

3). Berry Type Transformers :-

This has distributed magnetic circuit. The number of independent magnetic circuits are more than Two. Its core construction is like spokes of a wheel. otherwise it is symmetrical To that of shell type.

Diagramatically it can be shown as in fig.



Comparison of core and Shell Type Transformers :-

Core Type	Shell Type
1. The winding encircles the core	1. The core encircles most part of the winding
2. Cylindrical type of coils are used	2. Generally multilayer disc type or sandwid coils are used.
3. As windings are distributed, The natural cooling is more effective	3. As windings are sorrounded by the core The natural cooling does not exist.
4. The coils can be easily removed from maintenance point of view	4. For removing any winding for the maintenance, large number of limi-ations are required to be removed. This is difficult.
5. It has a single magnetic circuit	5. It has double magnetic circuit
6. In single phase type, the core has two limbs	6. In single phase type, Core has Three limbs.

Why The Transformer rating is in KVA not in KW?

→ When Electrical power is Transformed from primary winding to secondary, there are few power losses in between. These power losses appear in the form of heat which increase the temperature of the device.

→ ~~As Seen,~~ It is known that copper loss of a Transformer depends on current and core or iron loss depends on voltage. Hence total Transformer loss depends on volt-ampere (VA) and not on phase angle between voltage and current i.e, it is independent of load powerfactor. That is why rating of transformers is in KVA and not in KW.

→ Thus the transformer rating is specified as the product of voltage and current and called VA rating.

on bothsides, primary and Secondary VA rating remains same. This rating is generally expressed in KVA (kilo voltampere rating)

Can DC supply be used for Transformer?

Transformer is a static device which transforms AC power from one circuit to another without changing in frequency. This means it is a pure AC device which cannot be operated on DC.

→ The transformer works on the principle of Mutual Induction for which current in one coil must change uniformly. If DC supply is given, the current will not change due to constant supply and transformer will not work.

→ Practically winding resistance is very small. For dc the inductive reactance X_L is zero as dc has no frequency. So total impedance of winding is very low for dc. Thus winding will draw very high current if dc supply is given to it. This may cause the burning of windings due to extra heat generated and may cause permanent damage to the Transformer

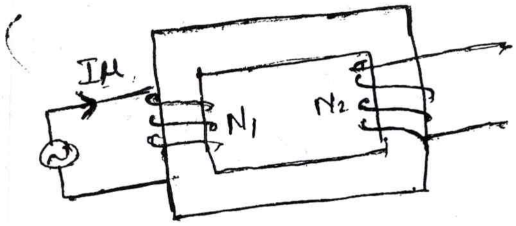
→ There can be saturation of the core due to which transformer draws very large current from the supply when connected to dc.

→ Thus dc supply should not be connected to the Transformers.

→ Operation of Ideal Transformer :-

→ An Ideal Transformer is one whose windings are not having resistances

- having no magnetic leakage
- having no copper losses and core losses
- having the primary and Secondary windings on loss free core.
- Ideal Transformer is one which is having two purely inductive coils on loss free core.



Consider an ideal Transformer whose primary is connected to an alternating voltage source V_1. The primary draws a current I_μ which is called magnetising current whose function is to magnetise the core. This current I_μ lags $90°$ with respect to V_1 since the ideal Transformer having purely inductive coils.

This magnetising current I_μ establishes the magnetic flux in the core. The magnetic flux value depends upon the magnitude of I_μ. since I_μ is alternating, the magnetic flux is also (varying) alternating one.

Therefore the magnetic flux is inphase with I_μ. The rate of change of this magnetic flux produces an induced emf, e_1 across primary winding. This emf is called as primary induced emf (or) self induced emf. This emf e_1 is exactly same in magnitude as applied voltage V_1 but opposite in nature ($180°$ out of phase). The rate of change of flux links with the secondary winding called secondary produces an emf e_2 across secondary winding (or) mutually induced emf

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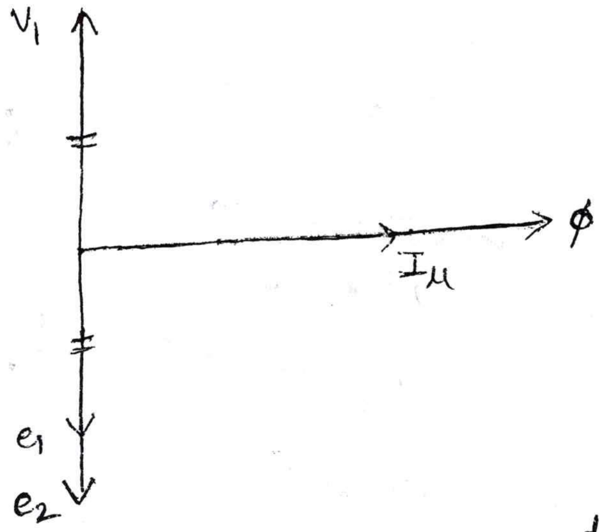
This emf e_2 having the magnitude depends up on the no. of Turns on secondary winding N_2. This induced emf e_2 varies same as that of e_1 i.e, e_2 and e_1 are in phase with each other, i.e, e_1 and e_2 are opposite to V_1.

~~Fig (a) gives the representation~~ ~~wave form~~ ~~representation~~ of operation of ~~Ideal Transformer~~: The vector diagram representation is given below :



This type of Transformer is not appeared in realistic conditions i.e, in practice. To get an idea about an actual Transformer first we consider only the operation of an ideal Transformer.

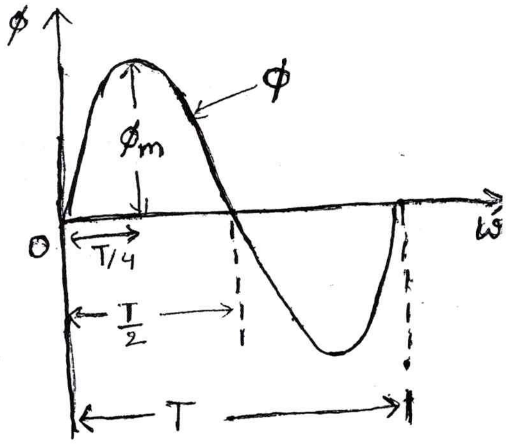
EMF Equation of Transformer:-

Let N_1 be the no. of Turns on primary winding.

N_2 be the no. of Turns on Secondary winding.

ϕ_m be the maximum value of flux

f be the frequency of ac Supply voltage.



The alternating supply voltage produces the magnetic flux in the core which is also alternating with a frequency of f Hz.

Consider the waveform of the alternating flux. From the figure, it is observed that the flux reaches it's maximum value during the time $T/4$ seconds.

$\therefore$ Average rate of change of Flux $= \frac{d\phi}{dt} = \frac{\phi_m}{T/4} = \frac{4\phi_m}{T}$

we Know that Time period, $T = \frac{1}{\text{frequency}}$. i.e, $T = \frac{1}{f}$

$\therefore$ Average rate of change of flux $= 4f\phi_m$.

This is the average rate of change of flux for 1 Turn.

Also we Know that, the rate of change of flux for 1 turn is nothing but an induced Emf per Turn.

$\therefore$ An induced Emf per turn = Average rate of change of flux per Turn

$= 4f\phi_m$ Volts / Turn

Since the magnetic flux Varies sinusoidally we should consider the rms value of induced Emf.

The rms (root mean Square) value can be obtained by multiplying the average value with form factor.

Form factor for sinusoidal quantity $= \frac{\text{RMS value}}{\text{Average value}} = 1.11$

$\therefore$ Rms value of induced Emf = form factor × Average value

$= 1.11 \times 4 f \phi_m$

$= 4.44 f \phi_m$ volts/Turn.

Since primary has N_1 no. of Turns.

Rms value of induced Emf across primary winding = (induced emf/turn) × No of Turns.

i.e, Primary induced Emf $e_1 = 4.44 f \phi_m \times N_1$

$= 4.44 f \phi_m N_1$ volts.

Since $\phi_m = B_m \times A$.

where B_m = Max value of flux density in Tesla

A = Area of the core

$\therefore$ Primary induced Emf $e_1 = 4.44 f B_m A N_1$

Similarly,

Secondary induced Emf $e_2 = 4.44 f B_m A N_2$.

where N_2 = No. of Turns in Secondary coil

The Emf Equations of Transformers are

$$\boxed{\begin{aligned} e_1 &= 4.44 f B_m A N_1 \quad \text{— (1)} \\ e_2 &= 4.44 f B_m A N_2 \quad \text{— (2)} \end{aligned}}$$

From the above Equations, $\frac{e_1}{N_1} = 4.44 f B_m A$ — (3)

$\frac{e_2}{N_2} = 4.44 f B_m A$ — (4)

from eq's (3) & (4), $\boxed{\frac{e_1}{N_1} = \frac{e_2}{N_2} = 4.44 f B_m A}$

= induced Emf/Turn.

From the above, it is concluded that the induced emf per Turn in primary and as well as secondary is same.
Hence the magnitudes of induced emf' in primary and secondary must depends on no. of Turns on primary and secondary

From ① & ② $\frac{②}{①} \Rightarrow \frac{e_2}{e_1} = \frac{4.44 f B_m A N_2}{4.44 f B_m A N_1}$

$$\boxed{\frac{e_2}{e_1} = \frac{N_2}{N_1} = K.}$$

where K = Transformation ratio.

→ **Use of Transformation ratio (K) :-**

→ We have to obtain the induced emf in one winding or no. of turns on that winding with the help of this Transformation ratio.
We have to Transfer the induced emf in one winding to other side.
It is also called voltage Transformation ratio.

→ If $N_2 > N_1$ then $K>1$, ∴ $E_2 > E_1$ i.e, no. of Turns on secondary are more than that of primary winding, the secondary induced emf becomes more than that of primary winding. This type of Transformer is known as 'step up Transformer'.

→ Similarly if $N_1 > N_2$ Then $K<1$, ∴ $E_2 < E_1$ - This type of Transformer is known as step down Transformer.

→ The main principle of ideal Transformer is input power = output power
i.e, input volt-ampere = output volt-ampere
Let V_1 be the input voltage, I_1 be the input current
V_2 be the output voltage, I_2 be the output current.
Then $V_1 I_1 = V_2 I_2$

$\frac{e_1}{N_1} = \frac{e_2}{N_2}$

$e_1 = \frac{e_2}{N_2} \times N_1$

$\frac{e_1}{e_2} = \frac{N_1}{N_2} = K$

$\frac{e_1}{N_1} = \frac{e_2}{N_2} = K$

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$e_1 = \frac{N_1}{N_2} = K$

$\frac{e_1}{e_2}$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$e_1 \times N_2$

Voltage Regulation :

It is defined as the change in Secondary terminal voltage from no load to full load and is expressed as no load secondary voltage or rated voltage..

Let V_{02} be the rated or no load Secondary voltage

V_2 be the secondary terminal voltage at Full load

$$\therefore \text{Voltage Regulation} = \frac{\text{No load Voltage} - \text{Full load voltage}}{\text{No load Voltage (or) Full load voltage}}$$

$$\%\ \text{Voltage Regulation} = \%\ \varepsilon = \frac{V_{02} - V_2}{V_{02}} \times 100.$$

i.e, when the voltage regulation is expressed as no load secondary voltage, then the regulation is termed as "Regulation down"

$$\%\ \text{Voltage Regulation} = \%\ \varepsilon = \frac{V_{02} - V_2}{V_2} \times 100$$

i.e, when the regulation is expressed as the full load secondary voltage then it is termed as "Regulation up"

Normally we should consider the voltage Regulation down.

$$V_{02} - V_2 = I_2 Z_2 = I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi.$$

Ref to Sec. Side

$$\therefore \text{Voltage Regulation},\ \%\ \varepsilon = \frac{I_2 Z_2}{V_2} \times 100 \Rightarrow \frac{I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi}{V_2} \times 100$$

V.R Ref to

$$= \frac{I_2 R_{02}}{V_2} \times 100 \cos\phi \pm \frac{I_2 X_{02}}{V_2} \times 100 \sin\phi$$

$$= V_r \cos\phi \pm V_x \sin\phi.$$

$$\%\ \varepsilon = V_r \cos\phi + V_x \sin\phi \quad \text{(for Lagging power factor.)}$$

$$\%\ \varepsilon = V_r \cos\phi - V_x \sin\phi \quad \text{(For leading powerfactor).}$$

V.R. Ref to pri

$$= \frac{I_1 R_{01} \cos\phi \pm I_1 X_{01} \sin\phi}{V_1} \times 100$$

where V_r = % resistance drop = $\frac{I_2 R_{02}}{V_2} \times 100$

V_x = % reactance drop = $\frac{I_2 X_{02}}{V_2} \times 100$

let $\frac{I_2 R_{02}}{V_2} \times 100 = \% R$ = percentage Resistance

$\frac{I_2 X_{02}}{V_2} \times 100 = \% X$ = Percentage Reactance.

$$\% \epsilon = \% R \cos\phi \pm \% X \sin\phi .$$

$$= \epsilon_r \pm \epsilon_x .$$

where ϵ_r = % resistive drop = $\% R \cos\phi$

ϵ_x = % reactance drop = $\% X \sin\phi .$

ii) → Condition For Zero Voltage Regulation:

We know that the voltage regulation can be expressed as

$$\% \epsilon = V_r \cos\phi \mp V_x \sin\phi .$$

where V_r is the percentage Resistance drop

V_x is the percentage reactance drop.

$\cos\phi$ is the powerfactor.

For the zero voltage regulation, $\% \epsilon = 0$.

i.e, $V_r \cos\phi - V_x \sin\phi = 0$

$$\Rightarrow V_r \cos\phi = V_x \sin\phi$$

$$\Rightarrow \frac{\sin\phi}{\cos\phi} = \frac{V_r}{V_x}.$$

$$\Rightarrow \tan\phi = \frac{V_r}{V_x}.$$

Various losses that occur in a Transformer:

In a Transformer there exists two types of losses.

1) core loss or Iron losses 2) copper loss.

1) Core loss or Iron losses:

a) **Hysterisis Losses:** This loss is produced in magnetic core of the Transformer due to alternating flux, it undergoes a cycle of magnetisation and demagnetisation. Due to hysterisis effect there is loss of energy in this process which is called Hysterisis loss.

It is given by, Hysterisis Loss $= K_h B_m^{1.6} f \cdot V$ watts.

where K_h = Hysterisis constant,

B_m = maximum flux density,

f = Frequency

V = Volume of the core.

b) **Eddy Current Losses:** The induced emf in the core tries to setup eddy current in the core and hence responsible for the eddy current losses.

The eddy current loss is given by,

$$\text{Eddy current Loss} = K_e B_m^2 f^2 t^2 \text{ watts.}$$

where K_e = Eddy current constant

t = thickness of the core.

The Flux in the core is almost constant as supply voltage V_1 at rated frequency 'f' is always constant. Hence the flux density B_m in the core and hence both the hysterisis and eddy current losses are constant at all the loads. Hence the core or Iron losses are also called Constant losses.

2) **Copper losses:** The copper losses are due to power wasted in the form of I^2R losses due to the resistance of the primary and secondary windings. The copper loss depends on the magnitude of the currents flowing through the windings.

$$\text{Total Cu loss} = I_1^2 R_1 + I_2^2 R_2 = I_1^2 (R_1 + R_2') = I_2^2 (R_2 + R_1')$$

$$= I_1^2 R_{01} = I_2^2 R_{02}.$$

The cu losses are called variable losses.

Total losses = Iron losses + Copper losses.
= W_i + W_{cu}.

→ ## Efficiency:

The efficiency is defined as the ratio of power output to power input.

$$\eta = \frac{\text{Power output}}{\text{Power Input}} \times 100$$

$$= \frac{\text{output}}{\text{output + losses}} = \frac{\text{output}}{\text{output + Iron loss + Copper loss}}$$

$$= \frac{V I \cos\phi}{V I \cos\phi + W_i + W_{cu}}.$$

$$\eta = \frac{\text{VA rating} \times \text{Powerfactor}}{\text{VA rating} \times \text{Powerfactor} + W_i + W_{cu}}$$

For any load,

$$\eta = \frac{x \times \text{FL KVA} \times \cos\phi}{x \times \text{FLKVA} \times \cos\phi + W_i + x^2 W_{FL\,cu}} \times 100$$

where x = ~~Fraction~~ $\frac{\text{Actual load}}{\text{Full load}}$

→ ## Condition For maximum Efficiency :-

The efficiency is function of load i.e, load current I_2 assuming $\cos\phi_2$ constant. The secondary Terminal voltage V_2 is also assumed constant.

So for maximum efficiency, $\frac{d\eta}{dI_2} = 0$.

$$\frac{d}{dI_2}\left[\frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + W_i + x^2 \cancel{I_2^2} R_{02}}\right] = 0$$

$$\therefore \left(V_2 I_2 \cos\phi_2 + W_i + x^2 \cdot I_2^2 R_{02}\right) \frac{d}{dI_2}\left[V_2 I_2 \cos\phi_2\right) \cancel{=0}$$

$$\left(V_2 I_2 \cos\phi_2\right) \frac{d}{dI_2}\left(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02}\right] = 0$$

$$\therefore \left(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02}\right) V_2 \cos\phi_2 - \left(V_2 I_2 \cos\phi_2\right)\left(V_2 \cos\phi_2 + 2 I_2 R_{02}\right) = 0$$

cancelling $V_2 \cos\phi_2$ from both ~~terms~~ we get,

$$V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02} - V_2 I_2 \cos\phi_2 - 2 I_2^2 R_{02} = 0$$

$$\therefore \quad W_i - I_2^2 R_{02} = 0$$

$$W_i = I_2^2 R_{02}$$

$$W_i = W_{cu}.$$

$\therefore$ So condition to achieve maximum efficiency is $\boxed{\text{Copper loss} = \text{Iron losses}}$

→ <u>KVA supplied at maximum efficiency</u> :-

Load at which maximum efficiency occurs

$$\text{KVA at } \eta_{max} = \text{FL KVA rating} \times \sqrt{\frac{\text{Iron loss}}{\text{FL Copper loss}}}$$

$$= \text{FL KVA} \times \sqrt{\frac{W_i}{x^2 W_{FLcu}}}$$

Transformer Test (or) OC and SC Test :-

→ The performance of a Transformer can be evaluated knowing the parameters of its Equivalent circuit.

→ Two simple tests help to determine the constants of Equivalent circuit and power losses in transformer.

→ These two tests enable the Efficiency and voltage regulation to be calculated without actually loading the transformer.

1) Open circuit Test (or) OC Test (or) No load Test :-

→ OC Test gives the no-load iron loss or core loss of the transformer.

→ It also gives the no-load current I_o which is used to calculate the parameters R_o and X_m of magnetising circuit.

→ The transformer is connected as shown in Fig (1).

→ One of the windings usually the low voltage winding is connected to supply voltage source while the high voltage winding is kept open.

→ When rated voltage is applied to the transformer using an autotransformer the ~~armature~~ ammeter gives no load current I_o, the wattmeter gives the total power loss. ~~but~~ since copper loss is very small so it is neglected. hence the wattmeter reading can be taken as only the iron loss of transformer.

Tabular column : OC Test

V_1	I_1 (or) I_o	W_i (or) W_o

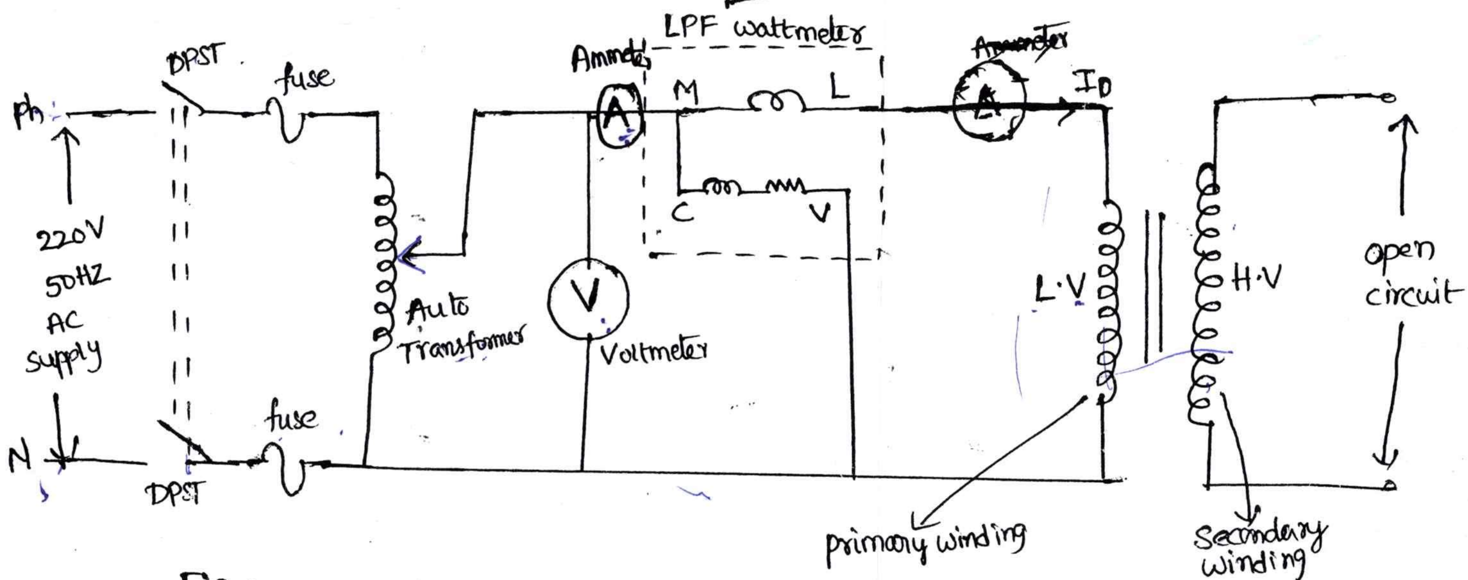


Fig : OC Test of a Transformer.

The wattmeter reading can be expressed as, $W_o = V_1 I_o \cos\phi_o$

· We have $I_c = I_o \cos\phi_o$ and $I_m = I_o \sin\phi_o$.

$$\cos\phi_o = \frac{W}{V_1 I_o} = \frac{I_c}{I_o}$$

and $R_o = \dfrac{V_1}{I_c}$ and $X_m = \dfrac{V_1}{I_m}$.

The total iron loss depends on the frequency (f) and max. Flux density (B_m) used.
The total iron loss has two components which are given as

a) hysterisis loss $\Rightarrow W_h = K_1 B_m^{1.6} f$.

b) Eddy current loss $\Rightarrow W_e = K_2 B_m^2 f^2$

The total iron loss $\Rightarrow W_i = W_h + W_e$.

Knowing K_1 and K_2, the hysterisis and eddy losses can be calculated.

Short circuit Test (or) SC Test :-

→ In this test one winding usually the low voltage winding is short circuited as shown in Fig (2).

→ A small voltage is applied to the primary and is increased carefully till the current I_1 in the primary winding reaches the rated full load current value.

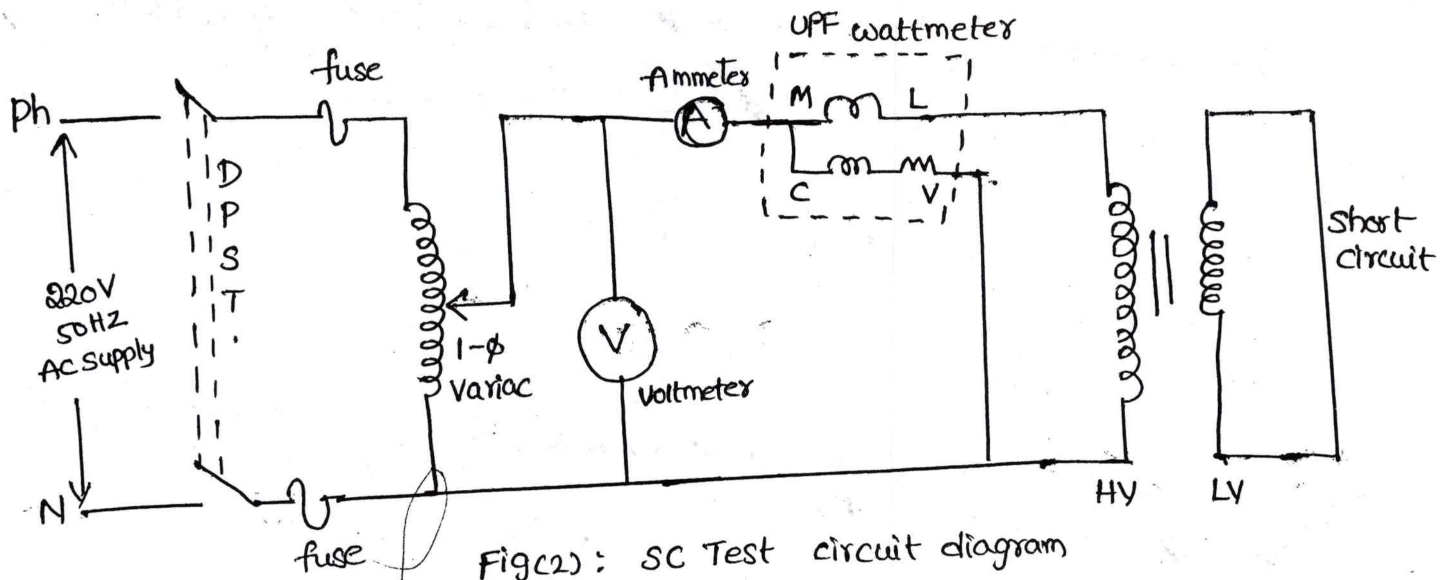
→ under these conditions the copper loss in the windings is the same as that on full load.

→ The reading of the wattmeter gives the copper loss in the winding only since iron loss is negligibly small.

~~Copper loss~~ ~~W_{sc}~~

Tabular column : SC Test

V_{sc}	I_{sc} (or) I_1	W_{sc}



Fig(2): SC Test circuit diagram

copper loss $W_{sc} = I_1^2 R_{01}$

$$Z_{01} = \frac{V_{sc}}{I_1} \quad \& \quad R_{01} = \frac{W_{sc}}{I_1^2}$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

where Z_{01}, R_{01} & X_{01} are equivalent impedance, resistance and leakage reactance of Transformer.

1) A $1-\phi$ transformer has to be designed to work with a primary voltage 11kV and a secondary voltage of 440V. If the maximum flux density is $1.2 wb/m^2$ and the no. of primary turns is 1400. Calculate a) No. of secondary turns

b) Area of cross section of the core. The supply voltage frequency is 50Hz's.

Sol:- Given, $f = 50Hz's$

primary voltage, $V_1 = 11KV$

secondary voltage, $V_2 = 440V$

max. flux density, $B_m = 1.2 wb/m^2$

$N_1 = 1400$

$N_2 = ?$

$A = ?$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{I_1}{I_2} = K$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$N_2 = \frac{V_2}{V_1} \times N_1$$

$$N_2 = \frac{440}{11 \times 10^3} \times 1400$$

$$N_2 = 56$$

$$V_1 = E_1 = 4.44 f B_m A N_1$$

$$A = \frac{E_1}{4.44 f B_m N_1}$$

$$= \frac{11 \times 10^3}{4.44 \times 50 \times 1.2 \times 1400}$$

$$= 294.9 cm^2$$

Q) 100KVA, 11000V/400V, 50HZ, 1-ϕ transformer has 40 secondary turns calculate no. of primary turns and primary current and secondary current.
(June 2015, II ECE)

Sol: Given,

$V_1 = 11000V$

$V_2 = 400V$

$f = 50HZ$

rating of transformer is KVA = 100KVA

$N_2 = 40$

$N_1 = ?$

$I_1 = ?$

$I_2 = ?$

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{I_1}{I_2} = K$$

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} \Rightarrow N_1 = \frac{V_1}{V_2} \times N_2 \Rightarrow \frac{11000}{400} \times 40$$

$$\cancel{N_2 = \frac{V_2}{V_1} \times N_2 \Rightarrow N_1 = \frac{400}{11000} \times 40}$$

$= 1100$ turns

$$\frac{V_2}{V_1} = \frac{I_1}{I_2}$$

$$KVA = V_2 I_2 = V_1 I_1$$

$$\Rightarrow KVA = V_2 I_2$$

$$100\,KVA = 400(I_2)$$

$$I_2 = \frac{100\,KVA}{400}$$

$$I_2 = \frac{100 \times 10^3}{400} = 250A$$

$$I_1 = \frac{V_2}{V_1} \times I_2$$

$$I_1 = \frac{400}{11000} \times 250$$

$$I_1 = 9.09A$$

3) A 1-ϕ transformer has 400 primary and 1000 secondary turns the net cross-sectional area of the core is $60cm^2$. If the primary winding connected to 50Hz supply & 520V. calculate
a) peak value of flux density of core b) voltage secondary winding.

Sol:- Given,

$N_1 = 400$

$N_2 = 1000$

$A = 60cm^2 = 60 \times 10^{-4}\,m^2$

$f = 50Hz$

$E_1 = V_1 = 520V$

$B_m = ?$

$E_2 = V_2 = ?$

$E_1 = 4.44 f B_m A N_1$

$$B_m = \frac{E_1}{4.44 f A N_1}$$

$$= \frac{520}{4.44 \times 50 \times 60 \times 10^{-4} \times 400}$$

$$= 0.975 \ wb/m^2$$

$$\frac{N_2}{N_1} = \frac{V_2}{V_1}$$

$$V_2 = \frac{N_2}{N_1} \times V_1$$

$$= \frac{1000}{400} \times 520$$

$$V_2 = 1300V$$

4) Emf/turn for a 1-ø 2310/220 V, 50Hz transformer is approximately 13V. Calculate a) No. of primary & secondary turns b) Net cross-sectional area of the core for $B_m = 1.4T$

Sol:- Given,

$E_1 = V_1 = 2310V$

$E_2 = V_2 = 220V$

$f = 50Hz$

Emf / turn = 13 V

$N_1 = ?$, $N_2 = ?$

$B_m = 1.4$

$A = ?$

$E_1 = \text{Emf/Turn} \times N_1$

$2310 = 13 \times N_1$

$$N_1 = \frac{2310}{13}$$

$N_1 = 177.69 \simeq 178$ turns

$$\frac{N_2}{N_1} = \frac{V_2}{V_1}$$

$$N_2 = \frac{V_2}{V_1} \times N_1$$

$$= \frac{220}{2310} \times 178$$

$= 16.95$

$\simeq 17$ turns

$E_1 = 4.44 f B_m A N_1$

$$A = \frac{E_1}{4.44 f B_m N_1}$$

$$A = \frac{2310}{4.44 \times 50 \times 1.4 \times 178}$$

$A = 0.0417 \text{ m}^2$

The no. load current of transformer is 5A at 0.45 power factor when supplied at 230V, 50HZ. The No. of turns and primary winding is 200. Calculate maximum value of flux, core loss, I_μ & I_w

Sol:- Given,

No load current, $I_0 = 5A$

power factor $\cos\phi_0 = 0.45$

No load voltage $E_0 = V_0 = 230V$

$f = 50HZ$

$N_1 = 200$

$\phi_m = ?$

$W_0 = ?$

$I_\mu = ?, I_w = ?$

$E_1 = 4.44 f B_m A N_1$

$E_1 = 4.44 f \phi_m N_1$

$$\phi_m = \frac{E_1}{4.44 f N_1}$$

$$= \frac{230}{4.44 \times 50 \times 200}$$

$= 5.18 mwb$

ii) Core loss :-

$W_0 = V_0 I_0 \cos\phi_0$

$= 230 \times 5 \times 0.45$

$= 517.5W$

iii) I_μ & I_w

$I_\mu = I_0 \sin\phi_0$

$\cos\phi_0 = 0.45$

$\phi_0 = \cos^{-1}(0.45)$

$= 63.26^\circ$

$I_\mu = I_0 \sin\phi_0$

$= 5 \times \sin(63.26)$

$I_\mu = 4.465A$

$I_\omega = I_0 \cos\phi_0$

$= 5 \times 0.45$

$= 2.25A$

6) The no load current of transformer 4A at 0.25 power factor when supplied at 250V, 50Hz. 1-ϕ AC supply the no. of turns on primary windings is 200. calculate 1) rms value of flux and maximum value of flux. 2) core loss 3) I_μ & I_ω.

Sol:- Given,

$I_0 = 4A$

$\cos\phi = 0.25$

$V_1 = E_1 = V_0 = 250V$

$f = 50Hz$

$N_1 = 200$

$rms = \frac{\phi_m}{\sqrt{2}}$

$\omega_0 = ?$

$I_\mu = ?$

$I_\omega = ?$

$E_1 = 4.44 f \phi_m N_1$

$\phi_m = \frac{E_1}{4.44 f N_1}$

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(20)

$$= \frac{250}{4.44 \times 50 \times 200}$$

$$= 5.63\,mwb$$

rms value $= \frac{\phi_m}{\sqrt{2}}$

$$= \frac{5.63m}{\sqrt{2}} = 3.981 \times 10^{-3}$$

ii) <u>Core loss:-</u>

$$W_0 = V_0 I_0 \cos\phi_0$$

$$= 250 \times 4 \times 0.25 = 250\,W$$

iii) I_μ & I_w

$$I_\mu = I_0 \sin\phi_0$$

$$= 4 \sin(75.52)$$

$$= 3.87A$$

$\cos\phi_0 = 0.25$

$\phi_0 = \cos^{-1}(0.25)$

$= 75.52$

$$I_w = I_0 \cos\phi_0$$

$$= 4 \times 0.25$$

$$= 1A$$

→ Problems on Efficiency :

1.) In a 25 KVA, 2000/200 V transformer Iron and copper losses are 350 and 400 W respectively. calculate the efficienc on U.P.F. ($\cos\phi = 1$) at (i.) full load (ii.) half full load. Determine the load for maximum efficiency, iron and copper losses in this case.

Sol^n:- Given, t/f rating, $S = 25$ KVA

primary voltage, $V_1 = 2000V$

Secondary voltage, $V_2 = 200V$

Iron losses, $W_i = 350W = 0.35$ KW

Copper losses, $W_{cu} = 400W = 0.4$ KW

At UPF, $\% \eta_{FL} = ?$, $\% \eta_{\frac{1}{2}FL} = ?$, $S\eta_{max} = ?$

(1.) η on UPF at FL :

w.k.t, $$\% \eta_{FL} = \frac{x S \cos\phi}{x S \cos\phi + W_i + x^2 W_{FLcu}} \times 100$$

Here

$x = 1$, $S = 25$ KVA

$\cos\phi = UPF = 1$, $W_i = 0.35$ KW

$W_{cu} = 0.4$ KW

$$\% \eta_{FL} = \frac{1 \times 25 \times 1}{1 \times 25 \times 1 + 0.35 + (1)^2 \times 0.4} \times 100 = \frac{25}{25 + 0.35 + 0.4} \times 100$$

$= 97.08\%$

(2.) η on UPF at half FL :

w.k.t, $$\% \eta_{FL} = \frac{x S \cos\phi}{x S \cos\phi + W_i + x^2 W_{FLcu}} \times 100$$

Here $x = \frac{\frac{1}{2} FL}{FL} = \frac{1}{2}$, $\cos\phi = UPF = 1$

$S = 25$ KVA

$W_i = 0.35$ KW

$W_{cu} = 0.4$ KW

$$\% \eta_{\frac{1}{2}FL} = \frac{\frac{1}{2} \times 25 \times 1}{\frac{1}{2}\times 25 \times 1 + 0.35 + \left(\frac{1}{2}\right)^2 \times 0.4} \times 100 = \frac{12.5}{12.5 + 0.35 + 0.1} \times 100$$

$$= 96.52\ \%$$

→ Load for which maximum efficiency occurs:

$$\begin{pmatrix}\text{Load KVA} \\ \text{at which} \\ \eta_{max} \text{ occurs}\end{pmatrix} = FL_{KVA} \times \sqrt{\frac{\text{Iron loss}}{\text{FL copper loss}}}$$

$$S_{\eta_{max}} = S_{FL} \times \sqrt{\frac{W_i}{W_{FLcu}}}$$

$$= 25 \times \sqrt{\frac{0.35}{0.4}}$$

$$= 23.38\ KVA$$

→ At maximum efficiency we know $\boxed{\text{Cu loss} = \text{Iron loss}}$

From given data, Cu loss, $W_{cu} = 350\,W$

$\Rightarrow$ Iron loss, $W_i = 350\,W$

2) A 100 KVA 1-ϕ transformer has an iron loss 1 KW and FL copper loss 1.5 KW. Find the maximum efficiency at power factor of 0.8 lagging and the corresponding KVA load.

Solⁿ: Given, rating of t/f, S = 100 KVA

Iron loss, $W_i = 1\ KW$ Power factor = 0.8

FL Copper loss, $W_{cu_{FL}} = 1.5\ KW$

$\%\ \eta$ at 0.8 p.f. = ?

KVA load at η_{max} = ?

→ At 0.8 lagging pf full load efficiency :

$$\% \eta_{FL} = \frac{xS\cos\phi}{xS\cos\phi + W_i + x^2 W_{FLcu}} \times 100$$

Here $x = 1$, $S = 100\text{ KVA}$, $\cos\phi = 0.8$

$W_i = 1\text{KW}$ and $W_{FLcu} = 1.5\text{ KW}$

$$\% \eta_{FL} = \frac{1 \times 100 \times 0.8}{1 \times 100 \times 0.8 + 1 + (1)^2 \times 1.5} \times 100 = \frac{80}{80+1+1.5} \times 100$$

$$= 96.96\%$$

→ Load for which maximum efficiency occurs :

$$\begin{pmatrix}\text{Load KVA} \\ \text{at which} \\ \eta_{max}\text{ occurs}\end{pmatrix} = \text{FL KVA} \times \sqrt{\frac{\text{Iron loss}}{\text{FL Copper loss}}}$$

$$S\eta_{max} = S_{FL} \times \sqrt{\frac{W_i}{W_{FLcu}}}$$

$$= 100 \times \sqrt{\frac{1}{1.5}}$$

$$= 81.64\text{ KVA}$$

3) A 2300/230V, 1-ϕ transformer has the primary and secondary winding resistances of 2Ω and 0.02Ω respectively. Its iron loss at normal supply voltage is 600W. calculate the secondary current at which maximum efficiency occurs and also the maximum efficiency when the powerfactor of the load is 0.9.

V_1 V_2 R_1 R_2 W_i 0.6k V_2=? pf

Sol^n: Given, primary winding resistance, $R_1 = 2\Omega$

Secondary winding resistance, $R_2 = 0.02\Omega$

primary voltage, $V_1 = 2300V$

secondary voltage, $V_2 = 230V$

Iron loss, $W_i = 600 W = 0.6 KW$

$I_2 = ?$ $\qquad \eta_{max} = ?$

→ At maximum efficiency, $\boxed{\text{cu loss} = \text{Iron loss}}$

$$\Rightarrow I_2^2 R_{02} = W_i$$

w.k.t., $R_{02} = R_2 + R_1'$

$$= R_2 + K^2 R_1$$

Here $K = \frac{V_2}{V_1} = \frac{2300}{230} = 0.1$

$$R_{02} = R_2 + K^2 \times R_1 = 0.02 + (0.1)^2 2$$

$$= 0.04 \Omega$$

At maximum efficiency, $I_2^2 R_{02} = W_i$

$$I_2^2 (0.04) = 600$$

$$\Rightarrow I_2 = \sqrt{\frac{600}{0.04}} = 122.47 A$$

→ $\% \eta_{max} = \frac{O/P}{I/P} \times 100 = \frac{O/P}{O/P + \text{loss}} \times 100$

$$= \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + \text{Iron loss} + \text{Cu loss}} \times 100$$

At maximum efficiency, $W_{cu} = W_i$

$$\therefore W_i = W_{cu} = 600 W$$

$V_2 = 230 V$, $I_2 = 122.47 A$, $\cos\phi = 0.9$

$$\% \eta_{max} \text{ at } 0.9 \text{ PF} = \frac{230 \times 122.47 \times 0.9}{230 \times 122.47 \times 0.9 + 600 + 600} \times 100$$

$$= 95.55 \%$$

4) A 600 KVA 1-ϕ transformer has an efficiency of 92% both at FL and half load at UPF. Determine its efficiency at 60% of FL at 0.8 power-factor lag.

Sol^n: Given,

rating of t/f, $S = 600$ KVA

$\eta_{FL} = \eta_{\frac{1}{2}FL} = 92\%$ at UPF

$\eta_{60\% FL}$ at 0.8 p.f. = ?

→ At full load, UPF :

$$\% \eta_{FL} = \frac{xS\cos\phi}{xS\cos\phi + W_i + x^2 W_{FLcu}} \times 100$$

Here $x = 1$, $S = 600$ KVA, $\cos\phi = 1$, $\eta = 92\%$

$$\Rightarrow 92 = \frac{1 \times 600 \times 1}{1 \times 600 \times 1 + W_i + (1)^2 W_{cu}} \times 100$$

$$600 + W_i + W_{cu} = 600/92 \times 100$$

$$\Rightarrow W_i + W_{cu} = 52.174 \text{ KW} \longrightarrow (1)$$

→ At half FL, UPF :

$x = \frac{1}{2} = 0.5$, $S = 600$, $\cos\phi = 1$, $\eta = 92\%$

$$\% \eta_{\frac{1}{2}FL} = \frac{xS\cos\phi}{xS\cos\phi + W_i + x^2 W_{cu}} \times 100$$

$$= \frac{0.5 \times 600 \times 1}{0.5 \times 600 \times 1 + W_i + (0.5)^2 W_{cu}} \times 100$$

$$\Rightarrow W_i + 0.25 W_{cu} = 26.086 \text{ KW} \longrightarrow (2)$$

Solving eqn^s (1) & (2), we get

$$\cancel{W_i} + W_{cu} = 52.174$$

$$\cancel{W_i} + 0.25 W_{cu} = 26.086$$

(−)

$$0.75 W_{cu} = 17.394$$

$W_i + 34.78 = 52.174$

$W_i = 52.174$

$$\Rightarrow W_{cu} = \frac{17.394}{0.75} = 34.78\ KW$$

sub above value in equ^n (1), we get

$$W_i = 17.39\ KW$$

~~$W_i + 34.78 = 52.174$~~
~~$W_i = -34.78 + 52.174$~~
~~$W_i = 34.78 + 52.174$~~

→ At 60% FL, 0.8 P.f. :

$$\%\ \eta_{60\%FL} = \frac{xS\cos\phi}{xS\cos\phi + W_i + x^2 W_{cu}} \times 100$$

Here, $x = 60\%\ FL = \frac{60}{100} = 0.6$, $S = 600\ KVA$

$W_i = 17.39\ KW$, $W_{cu} = 34.78\ KW$

$\cos\phi = 0.8$

$$\%\ \eta_{60\%FL} = \frac{0.6 \times 600 \times 0.8}{0.6 \times 600 \times 0.8 + 17.39 + (0.6)^2 \times 34.78} \times 100$$

$= \cancel{85.9\%}\ 90.59\ \%$

→ 5) A 100 KVA, 1000/10,000V, 50 Hz 1-ϕ transformer has an iron loss of 1100 W. The copper loss with 5A in this High voltage winding is 400W. calculate the efficiencies at (i.) 25% (ii.) 50% of normal load for power factor of (a.) 1.0 (b.) 0.8

The output terminal voltage being maintained at 10,000 V. (Set -3, Mech NOV-2010)

<u>**Sol^n**</u> : Given,

rating of t/f, $S = 100\ KVA$

Primary voltage, $V_1 = 1000V$

Secondary voltage, $V_2 = 10,000V$ $\eta_{25\%FL} = ?$

Iron loss, $W_i = 1100W = 1.1\ KW$ $\eta_{50\%FL} = ?$

Copper loss at 5A, $W_{cu} = 400W$

rating, $S = V_2 I_{2FL}$

$$I_{2FL} = \text{rating}/V_2 = \frac{100 \times 10^3}{10,000} = 10\,A.$$

w.k.t, $W_{cu} = I_2^2 R_{02}$

$\Rightarrow W_{cu} \propto I_2^2$

$$\frac{W_{cu} \text{ on any load}}{W_{cu} \text{ at full load}} = \left[\frac{I_2 \text{ on any load}}{I_2 \text{ on Fullload}}\right]^2$$

$$\Rightarrow \frac{400}{W_{cuFL}} = \left(\frac{5}{10}\right)^2$$

$$\Rightarrow W_{cuFL} = \left(\frac{10}{5}\right)^2 \times 400$$

$$= 1600\,W = 1.6\,KW$$

(i) <u>At 25% of load</u> :

(a) at 1.0 P.f. (UPf)

$$\%\eta_{25\%FL} = \frac{x S \cos\phi}{x S \cos\phi + W_i + x^2 W_{cu}} \times 100$$

Here, $x = \frac{25}{100} = 0.25$, $S = 100\,KVA$

$\cos\phi = 1$, $W_i = 1.1\,KW$, $W_{cu} = 1.6\,KW$

$$\%\eta_{25\%FL} = \frac{0.25 \times 100 \times 1}{0.25 \times 100 \times 1 + 1.1 + (0.25)^2 \times 1.6} \times 100$$

$$= \frac{25}{25 + 1.1 + (0.25)^2 \times 1.6} \times 100$$

$$= 95.41\,\%$$

(b) at 0.8 pf

$$\% \eta_{25\% FL} = \frac{x \times FL\ KVA \times \cos\phi}{x \times FL\ KVA \times \cos\phi + W_i + x^2 W_{FLcu}} \times 100$$

Here, $x = 0.25$, $\cos\phi = 0.8$

FL KVA, $S = 100$ KVA , $W_i = 1.1$ KW, $W_{cu} = 1.6$ KW.

$$\% \eta_{25\% FL} = \frac{0.25 \times 100 \times 0.8}{0.25 \times 100 \times 0.8 + 1.1 + (0.25)^2 \times 1.6} \times 100$$

$$= 94.33\%$$

(ii) At 50% load :

(a) at 1.0 pf (UPF) :-

$$\% \eta_{50\% FL} = \frac{x \times S \times \cos\phi}{x S \cos\phi + W_i + x^2 W_{FLcu}} \times 100$$

Here, $x = 50\% = \frac{50}{100} = 0.5$, $\cos\phi = 1$

$S = 100$ KVA , $W_i = 1.1$ KW , $W_{FLcu} = 1.6$ KW.

$$\% \eta_{50\% FL} = \frac{0.5 \times 100 \times 1}{0.5 \times 100 \times 1 + 1.1 + (0.5)^2 \times 1.6} \times 100$$

$$= 97.08\%$$

(b) at 0.8 pf :-

$$\% \eta_{50\% FL} = \frac{x S \cos\phi}{x S \cos\phi + W_i + x^2 W_{FLcu}} \times 100$$

Here, $x = 0.5$, $\cos\phi = 0.8$, $S = 100$ KVA , $W_i = 1.1$ KW
$W_{cu} = 1.6$ KW

$$\% \eta_{50\% FL} = \frac{0.5 \times 100 \times 0.8}{0.5 \times 100 \times 0.8 + 1.1 + (0.5)^2 \times 1.6} \times 100$$

$$= 96.38\%$$

6.) A single phase transformer working at UPF has an efficiency of 90% at both half load and at full load of 500 KVA. Determine the efficiency at 75% of full load.

(Set-4 Mech NOV-20..)

Sol^n: Given,

$$\text{at UPF, } \%\eta_{\frac{1}{2}FL} = \%\eta_{FL} = 90\%$$

$$\text{Rating of t/f, } S = 500 \text{ KVA}$$

$$\%\eta_{75\% \text{ of } FL} = ?$$

→ At $\frac{1}{2}$FL, UPF :

$$\%\eta_{\frac{1}{2}FL} = 90\%, \quad \cos\phi = 1$$

$$S = 500 \text{ KVA}, \quad x = \frac{1}{2} = 0.5$$

$$\%\eta_{\frac{1}{2}FL} = \frac{xS\cos\phi}{xS\cos\phi + W_i + x^2 W_{cuFL}} \times 100$$

$$\Rightarrow 90 = \frac{0.5 \times 500 \times 1}{0.5 \times 500 \times 1 + W_i + (0.5)^2 \times W_{cu}} \times 100$$

$$0.5 \times 500 \times 1 + W_i + 0.25 W_{cu} = \frac{0.5 \times 500 \times 100}{90}$$

$$\Rightarrow 250 \qquad = \cancel{277.78} - 250$$

$$\Rightarrow W_i + 0.25 W_{cu} = 55.55 \text{ KW} \longrightarrow (1)$$

→ At FL, UPF :

$$\%\eta_{FL} = 90\%, \quad \cos\phi = 1$$

$$S = 500 \text{ KVA}, \quad x = 1$$

$$\%\eta_{FL} = \frac{xS\cos\phi}{xS\cos\phi + W_i + x^2 W_{FLcu}} \times 100$$

$$\Rightarrow 90 = \frac{1 \times 500 \times 1}{1 \times 500 \times 1 + W_i + (1)^2 W_{cu}} \times 100$$

$$\Rightarrow W_i + W_{cu} = 27.77 \text{ KW} \longrightarrow (2)$$

Solving eqns ① & ②, we get

$W_i = 18.51$ K.W.

$W_{cu} = 37.04$ K.W.

→ At 75% of FL, UPF :-

$$\%\eta_{75\%\,FL} = \frac{xS\cos\phi}{xS\cos\phi + W_i + x^2 W_{FLcu}} \times 100$$

Here, $x = 75\% = \frac{75}{100} = 0.75$

$\cos\phi = 1$, $S = 500$ KVA

$W_i = 18.51$ KW

$W_{cu} = 37.04$ KW

$$\%\eta_{75\%\,of\,FL} = \frac{0.75 \times 500 \times 1}{0.75 \times 500 \times 1 + 18.51 + (0.75)^2 \times (37.04)} \times 100$$

$$= 90.5\%$$

Regulation problems :-

At 20KVA 2000/200 V 1-ϕ Transformer has the following parameters

HV winding : $R_1 = 3\Omega$, $X_1 = 5.3\Omega$, LV winding : $R_2 = 0.05\Omega$, $X_2 = 0.1\Omega$

Find the Voltage Regulation at i) At Power factor 0.8 lagging ii) UPF &

iii) 0.707 power factor Leading

Sol: Given rating of Transformer = 20 KVA

primary voltage $V_1 = E_1 = 2000$ V

Secondary voltage $V_2 = E_2 = 200$ V

Primary resistance $R_1 = 3\Omega$, Primary reactance = X_1 = ~~0.05Ω~~ 5.3Ω

secondary resistance $R_2 = 0.05\Omega$, Secondary reactance = $X_2 = 0.1\Omega$

Transformer voltage regulation : at 0.8 pf lag = ?

at UPF = ?

at 0.707 pf lead = ?

~~Equivalent~~ transformation ratio = $K = \frac{V_2}{V_1} = \frac{200}{2000} = 0.1$

Equivalent resistance refered to secondary $R_{02} = R_2 + K^2 R_1$

$= 0.05 + (0.1)^2 (3)$

$R_{02} = 0.08\Omega$

Equivalent reactance refered to ~~primary~~ Secondary $X_{02} = X_2 + K^2 X_1$

$= 0.1 + (0.1)^2 (5.3)$

$X_{02} = 0.153\ \Omega$

we know, rating $= V_2 I_2 \Rightarrow I_2 = \frac{\text{rating}}{V_2} = \frac{20\times10^3}{200}$

$\therefore I_2 = 100$ A

we have $\%\ Reg = \frac{I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi}{V_2} \times 100$

where '+' for Lagging pf

'−' for leading pf.

i) at 0.8 pf lagging:- $\%\ Reg = \frac{I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi}{V_2} \times 100$

Here $I_2 = 100$ A, $R_{02} = 0.08\Omega$, $X_{02} = 0.153\Omega$, $V_2 = 200$V

$\Rightarrow \cos\phi = 0.8 \Rightarrow \phi = \cos^{-1} 0.8$

$\Rightarrow \sin\phi = \sin(\cos^{-1} 0.8) = 0.6$

$$\% \text{ Reg} = \frac{100 \times 0{\cdot}08 \times 0{\cdot}8 + 100 \times 0{\cdot}153 \times 0{\cdot}6}{200} \times 100$$

$$\% \text{ Reg.} = 7{\cdot}79\ \%$$

ii) <u>At unity pf</u> :-   Here $\cos\phi = 1$, ~~& sinϕ =~~ 0.

$$\phi = \cos^{-1}(1)$$

$$\sin\phi = \sin(\cos^{-1}(1)) \Rightarrow \sin\phi = 0.$$

$$\% \text{ Reg} = \frac{I_2 R_{02} \cos\phi}{V_2} \times 100 = \frac{100 \times 0{\cdot}08 \times 1}{200} \times 100$$

$$\% \text{ Reg.} = \cancel{8}\,4\ \%$$

iii) <u>At 0·707 lead Pf</u> :

$$\% \text{ Reg} = \frac{I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi}{V_2} \times 100$$

Here $\Rightarrow \cos\phi = 0{\cdot}707$

$$\phi = \cos^{-1}(0{\cdot}707)$$

$$\sin\phi = \sin\left[\cos^{-1} 0{\cdot}707\right]$$

$$\Rightarrow \sin\phi = 0{\cdot}707$$

$$\% \text{ Reg} = \frac{100 \times 0{\cdot}08 \times 0{\cdot}707 - 100 \times 0{\cdot}153 \times 0{\cdot}707}{200} \times 100$$

$$\% \text{ Reg} = -2{\cdot}58\ \%$$

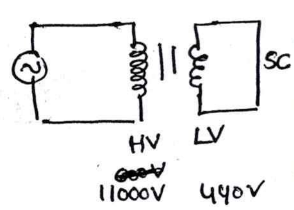
→ A 300 KVA, 11000/440 V, single phase, 50 HZ, Transformer gave the following test results, open circuit test on LV side a normal voltage and frequency input 1.3 KW, 21.1 amp, short circuit test HV side with voltage 600 V, input 2.80 kW, 15 amp. Calculate the efficiency & regulation for full load at 0.8 pf lagging.

Sol: Given Transformer 300 KVA, 11000/440 V, 50 HZ.

OC Test: Iron loss $W_0 = 1.3$ kW

SC Test: Copper loss $W_{sc} = 2.80$ kW

$V_{sc} = 600$ V, $I_{sc} = 15$ A.



$$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{2.80\times10^3}{(15)^2} = 12.44\,\Omega$$

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{600}{15} = 40\,\Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{(40)^2-(12.44)^2}$$

$$= 38.01\,\Omega$$

rating VA = $V_1 (I_1)_{FL}$

$$I_1 = \frac{\text{VA rating}}{V_1} \Rightarrow I_1 = \frac{300\times10^3}{11000}$$

$$I_1 = 27.27\text{ A}$$

Full load copper loss $W_{cu(FL)} = (I_1)_{FL}^2 R_{01}$

$$= (27.27)^2 \times 12.44$$

$$= 9252.89\text{ kW}.$$

Efficiency (η) at any load :

$$\eta = \frac{x\, S \cos\phi}{x\, S\cos\phi + W_o + x^2 W_{cuFL}} \times 100$$

$x = FL = 1$

rating $S = 300\,KVA = 300\times10^3$

$\cos\phi = 0.8$, $W_o = 1.3\,KW$, $W_{cu(FL)} = 9252.89\,W$

$$\eta = \frac{1\times300\times10^3\times0.8}{1\times300\times10^3\times0.8 + 1.3\times10^3 + 9252.89}\times100$$

$\eta = 95.78\,\%$

Regulation : $(I_1)_{FL} = 27.27\,A$

$\cos\phi = 0.8$, lagg.

$\sin\phi = 0.6$

$$\%R = \frac{I_1(R_{01}\cos\phi + X_{01}\sin\phi)}{V_1}\times100$$

$$= \frac{27.27\,(12.44\times0.8 + 38.015\times0.6)}{11000}\times100$$

$\%R = 8.12\,\%$

→ A 11000/400V distribution Transformer takes a no load primary current of 1A at a powerfactor of 0.24 lagging. Find (I) coreloss Current (II) magnetizing current (III) Iron loss.

(II ECE, Dec-2014)

Sol:

Given No load Current = $I_0 = 1A$.

powerfactor $\cos\phi = 0.24$

$\phi = \cos^{-1} 0.24$

$\phi =$

$\sin\phi =$

(V_1) primary voltage = No load voltage (V_0)

$V_1 = V_0 = 11000$ Volts

Coreloss current $I_w = ?$

magnetizing current $I_\mu = ?$

Iron loss (or) core loss $W_0 = ?$

(i) Core loss current (or) working current $I_w = I_0 \cos\phi_0$

$I_w = 1 \times 0.24 = 0.24A$

$\boxed{I_w = 0.24A}$

(ii) magnetizing current $I_\mu = I_0 \sin\phi_0$

$= 1 \times 0.97$

$\boxed{I_\mu = 0.97\ A}$

(iii) Iron loss (or) Core loss $W_0 = V_0 I_0 \cos\phi_0$

$= 11000 \times 1 \times 0.24$

$\boxed{W_0 = 2640 \text{ watts}}$

Previous Question paper problems

→ Calculate the flux in the core of single phase Transformer having a primary voltage of 230V at 50 HZ and 50 turns. If the flux density in the core is 1 Tesla. calculate the net cross sectional area of the core. (CSE, NOV-2012, set-1)

Sol:- Given Primary voltage $V_1 = 230\,V = E_1$.

Frequency $f = 50\,Hz$

No. of Turns in primary, $N_1 = 50$

Flux densinty, $B_m = 1$ Tesla or wb/m^2.

flux $\phi = ?$

Cross section area $A = ?$

we Know, $V_1 = E_1 = 4.44\, f\, B_m\, A\, N_1$.

$$230 = 4.44 \times 50 \times 1 \times A \times 50$$

$$\Rightarrow A = \frac{230}{4.44 \times 50 \times 1 \times 50} = 0.0207\ m^2$$

$$\Rightarrow A = 0.0207\,m^2$$

we Know $\phi = B_m \times A$.

$$\phi = 1 \times 0.0207 = 0.0207\ wb$$

$$\Rightarrow \phi = 0.02\ wb$$

→ The maximum flux density in the core of 250/3000 Volts 50 Hz single phase transformer is 1.2 weber per meter square. If the emf per turn is 8 V. determine primary and secondary turns and area of the core. (CSE, Nov 2012, set 2)

Sol: Given Primary voltage $V_1 = E_1 = 250\,V$

Secondary voltage $V_2 = E_2 = 3000\,V$

Frequency $f = 50\,Hz$.

max Flux density $B_m = 1.2\ wb/m^2$ (or) Tesla.

emf / Turn $= 8\,V$.

Primary Turns $N_1 = ?$

Secondary Turns $N_2 = ?$

Area of core $A = ?$

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We know $E_1 = \text{Emf/Turn} \times N_1$.

$250 = 8 \times N_1$

$\Rightarrow N_1 = \dfrac{\cancel{250}^{31.2}}{\cancel{8}} = 31.2 \simeq 31$ Turns

we know $\dfrac{N_2}{N_1} = \dfrac{V_2}{V_1}$.

$\Rightarrow N_2 = \dfrac{V_2}{V_1} \times N_1$

$\Rightarrow N_2 = \dfrac{3000}{250} \times 31 = 372$

$\Rightarrow N_2 = 372$ Turns

we know $E_1 = 4.44\, f\, B_m\, A\, N_1$.

$\Rightarrow A = \dfrac{E_1}{4.44 \times f \times B_m \times N_1}$.

$= \dfrac{250}{4.44 \times 50 \times 1.2 \times 31}$

Area, $A = 0.0302\ m^2$

→ A 25 KVA, 2500/250V, single phase Transformer gave the following figures:

OC Test (LV side)	250V	1.4A	105 W
SC Test (HV side)	105 V	8A	320 W

compute the Equivalent circuit parameters refered to LV side and HV side. obtain the percentage Regulation at full load with 0.8 Pf lagging.

Sol: Given Transformer 25 KVA $\underbrace{2500}_{\substack{Pri\\HV}} / \underbrace{250}_{\substack{sec\\LV}}$ V

(i) OC Test : In OC Test ~~is~~, meters on LV side. i.e., secondary

Hence R_o & X_o will be refer to secondary.

given $V_o = 250V$, $I_o = 1.4A$, $W_o = 105 W$.

$$\cos\phi_o = \frac{W_o}{V_o I_o} = \frac{105}{250 \times 1.4} = 0.3$$

$$\cos\phi_o = 0.3$$
$$\phi_o = \cos^{-1}(0.3)$$

$$\sin\phi_o = 0.9539$$

working current $I_w = I_o \cos\phi_o = 1.4 \times 0.3$
$= 0.42A$

magnetising current $I_\mu = I_o \sin\phi_o = 1.4 \times 0.9539$

$I_\mu = 1.3354$ A

$$R_o = \frac{V_o}{I_w} = \frac{250}{0.42}$$
$$= 595.2\ \Omega$$

$$X_o = \frac{V_o}{I_m} = \frac{250}{1.3354}$$
$$= 187.209\ \Omega.$$

(2) SC Test : The meters are on HV side i.e, primary side

Hence we get R_{01}, X_{01}, Z_{01}

given $V_{sc} = 104V$, $I_{sc} = 8A$, $W_{sc} = 320\ W$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{320}{(8)^2} = 5\,\Omega$$

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{104}{8} = 13\,\Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{(13)^2 - (5)^2}$$

$$= 12\,\Omega$$

Transformation ratio, $K = \frac{V_2}{V_1} = \frac{250}{2500} = 0.1$

~~Transformation ratio, is K~~ $\Rightarrow K = 0.1$

$$R_{02} = K^2 R_{01} = (0.1)^2 \times 5 = 0.05\,\Omega$$

$$X_{02} = K^2 X_{01} = (0.1)^2 \times 12 = 0.12\,\Omega$$

$$Z_{02} = K^2 \times Z_{01} = (0.1)^2 \times 13 = 0.13\,\Omega$$

we know $V_1 (I_1)_{FL} = VA$

$$I_1 = \frac{VA}{V_1} = \frac{25 \times 10^3}{2500}$$

$$\Rightarrow I_1 = 10A$$

Regulation : Regulation at Full load with $\cos\phi = 0.8$ lag.

$$\%R = \frac{I_1 (R_{01} \cos\phi + X_{01} \sin\phi)}{V_1}$$

$\cos\phi = 0.8 \Rightarrow \phi = \cos^{-1} 0.8$

$\sin\phi = 0.6$

$$\%R = \frac{10(5 \times 0.8 + 12 \times 0.6)}{2500} = 4.48\%$$

$\%R = 4.48\%$

→ A 7·5 KVA, 2400V/120 V Transformer was tested by short circuiting low voltage side and applying 100V to high voltage side. The measured power input was 145 W. Determine the regulation when the load has 0·8 Powerfactor lagging. (II ECE, June-2015)

Sol :-

Given Transformer 7·5 KVA, 2400 V / 120 V
HV LV
pri sec

SC Test ; $V_{sc} = 100V$

$W_{sc} = 145W.$

To calculate I_{sc}, calculate FL rated current.

rating VA $= V_1(I_1)_{FL}$

$7.5\,KVA = 2400\,(I_1)_{FL}$

$$I_1 = \frac{7.5\times10^3}{2400} = 3.1\,A$$

$(I)_{rated\,FL} = 3.1\,A$

$$\Rightarrow \boxed{I_{sc} = (I_1)_{FL} = 3.1A}$$

$$\Rightarrow R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{145}{(3.1)^2} = \frac{145}{9.61} = 15.08\,\Omega$$

$$\Rightarrow Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{100}{3.1} = 32.25\,\Omega$$

$$\Rightarrow X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{(32.25)^2 - (15.08)^2}$$

$= 28.5\,\Omega$

⇒ Full load copper loss $W_{cu(FL)} = (I_1)_{FL}^2 R_{01}$

$= (3.1)^2(15.08)$

$W_{cu(FL)} = 144.91$ watt

Regulation :- at 0.8 powerfactor lagging :

$$\%R = \frac{I_1 (R_{01} \cos\phi + X_{01} \sin\phi)}{V_1}$$

$\cos\phi = 0.8 \Rightarrow \phi = \cos^{-1} 0.8$

$\Rightarrow \sin\phi = 0.6$

$V_1 = 2400 \text{ V}$

$I_1 = 3.1 \text{ A}$

$$\%R = \frac{3.1 (15.08 \times 0.8 + 28.5 \times 0.6)}{2400} \times 100$$

$\%R = 3.79 \%$

Construction Details of 3-ϕ Induction motor :-

Basically, the Induction motor consists of two parts, namely

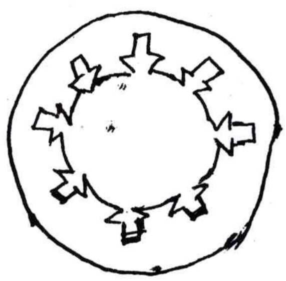
1) STator : Three phase winding which is stationary called stator.

2) RoTor: The part which rotates and is connected to mechanical load through shaft, called rotor.

STator :-

The stator has a laminated type of construction made up of stamping which are 0·4 to 0·5 mm thick. The stampings are slotted on its periphery to carry the stator winding. The stampings are insulated from each other. The no. of stampings are stamped together to build stator core. The built up core is then fitted in a cast or fabricated steel frame.

→ The slots on the periphery of stator core carries a 3-phase winding connect either in star or delta. This three phase winding is called stator winding.



Fig(1) : STator lamination.

RoTor :- The rotor is placed inside the stator. The rotor core is also laminated in construction and uses cast iron. It is cylindrical, with slots on its periphery. The rotor conductors is placed in the rotor slots.

There are two types of rotor constructions in the Induction motor:

They are:

(1) squirrel cage rotor

(2) slip ring (or) wound rotor.

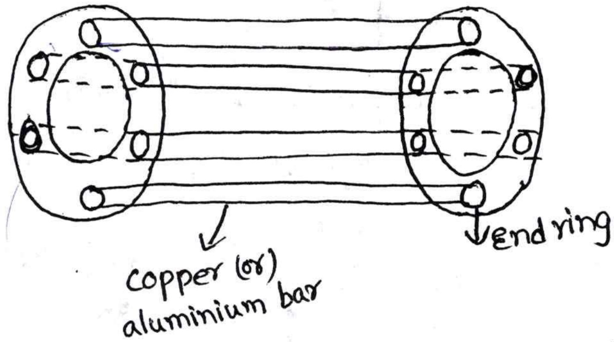
(1) Squirrel cage Rotor:

The rotor core is cylindrical and slotted on its periphery. The rotor consists of uninsulated copper or aluminium bars called rotor conductors. The bars are placed in the slots. These bars are permanently shorted at each End with the help of conducting copper ring called End ring. The entire structure looks like a cage, Forming a closed Electrical circuit. So the rotor is called squirrel cage rotor.

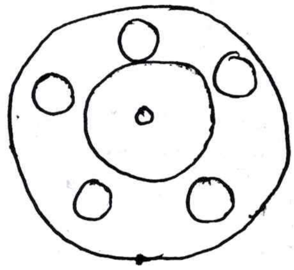
→ This rotor is also called short circuited rotor.

→ In this type of rotor, the slots are not arranged parallel to the shaft axis but are skewed as shown in Fig.

Fig(2a): squirrel cage rotor



Fig(2b): symbolic representation



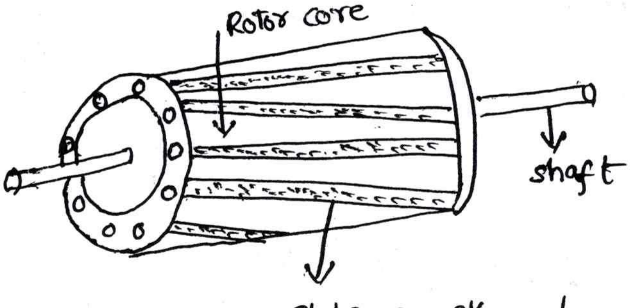


Fig (2C): skewing in rotor construction.

(2) Slip ring rotor (or) wound rotor :-

→ In this type of construction, rotor winding is exactly similar to the stator. The rotor carries a 3-phase star or delta connected, distributed winding.

→ The rotor construction is laminated and slotted. The slots contains rotor winding. The three ends of 3-phase winding are permanently connected to slip rings.

→ slips rings are used to connect external stationary circuit to internal rotating circuit.

→ So in this type of rotor, the external resistances can be added with the help of brushes.

→ This arrangement is shown in fig.

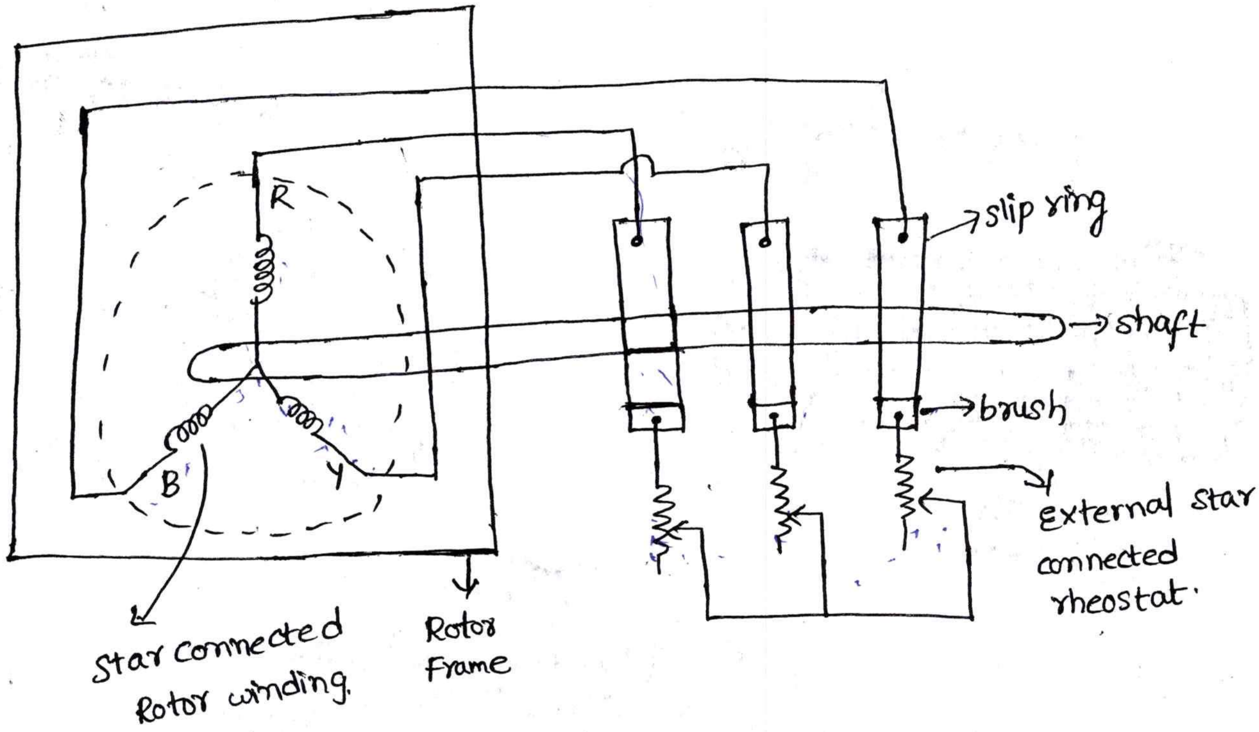


Fig: slip ring (or) wound Rotor

→ Comparison of Squirrel cage and wound rotor :-

Slip ring or wound rotor	Squirrel cage rotor
1. Rotor consists of 3-phase winding	1. Rotor consists of bars which are shorted at the ends with the help of endrings.
2. construction is complicated	2. construction is very simple.
3. Resistance can be added externally	3. External resistance cannot be added.
4. Slip rings & brushes are present to add external resistance	4. slip rings & brushes are absent.
5. construction is delicate & due to brushes, frequent maintenance is necessary	~~5. Due to simple~~ robust ~~construction~~, 5. construction is robust & maintenance free.
6. Rotors are very costly	6. Due to simple construction, rotors are cheap.
7. only 5% of induction motors in industry, use slip ring rotor	7. Very common and almost 95% induction motors use this type of rotor.
8. High starting torque can be obtained	8. Moderate starting torque cannot be controlled.
9. speed control by rotor resistance is possible	9. Speed control by rotor resistance is not possible.
10. Rotor copper losses are high, hence efficiency is less.	10. Rotor copper losses are less, hence efficiency is high.
11. used for lifts, hoists, cranes, elevators, compressors etc.	11. used for lathes, drilling machin fans, blowers, waterpumps, grinders, printing machine etc.

(by 120°, has constant amplitude of $1.5\,\phi_m$, where ϕ_m is maximum amplitude of an individual flux.

(2) The resultant always keeps rotating with a certain speed in space.)

4 [principle and operation of 3-ϕ Induction MOTOR :-

principle: Induction motor works on the principle of Faradays law of Electromagnetic Induction.

operation:

→ consider a 3-ϕ Induction motor as shown in Fig(1), with the stator and rotor wound for 3-phases and having identical no. of poles.

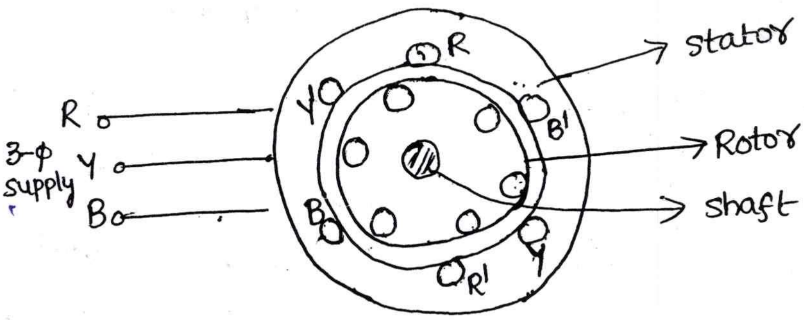
→ Fig(2) shows stator winding and Fig(3) shows rotor winding.

→ when 3-phase stator winding is energized from 3-phase supply a rotating magnetic field is set up, which rotates at synchronous speed Ns. $\left(N_s = \frac{120f}{P}\right)$

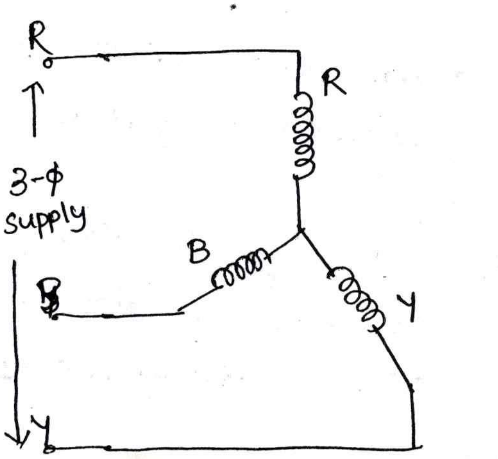
→ The rotating magnetic field passes through airgap and cuts the rotor conductors which are yet stationary.

→ ~~Due to~~

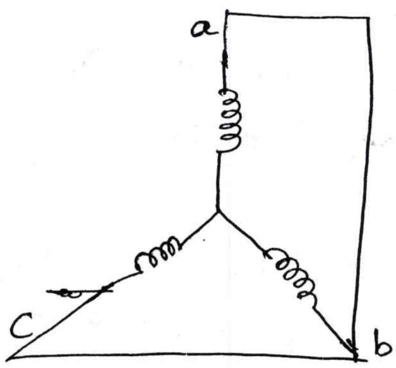
~~Fig(1)~~



Fig(1): Induction motor.



Fig(2): stator winding



Fig(3): short circuited rotor winding

→ Due to relative speed between the rotating flux and stationary rotor, emf's are induced in rotor conductors.

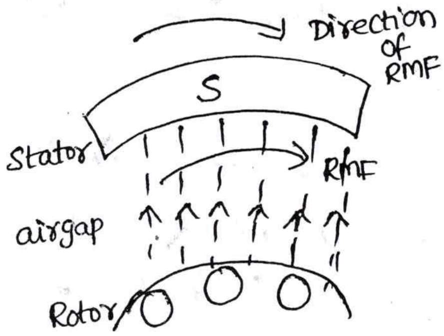
→ The rotor winding carry the 3-phase currents, if the rotor is short circuited as shown in Fig (3).

→ Now the rotor behaves like a current carrying conductor placed in rotating magnetic field of Stator.

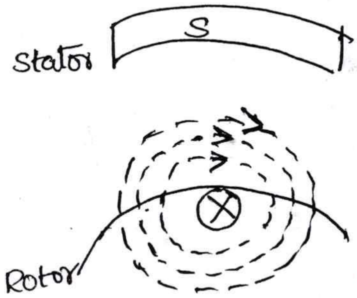
→ Consequently a Mechanical force acts on the rotor conductors.

→ The sum of mechanical forces on all the rotor conductors produce a torque, which tends to move the rotor in the same direction as rotating magnetic field.

→ Fig(4) shows the operation of 3-phase induction motor.



Fig(4a): production of RMF



Fig(4b): production rotor flux due to rotor current

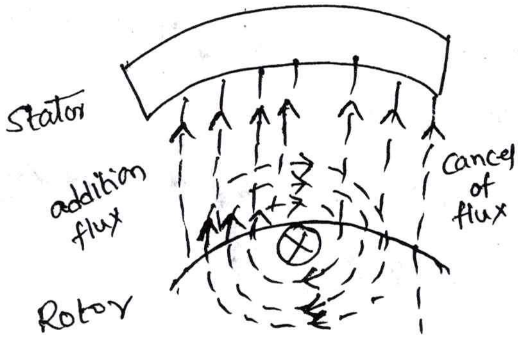


Fig (4c): superimposition of two flux

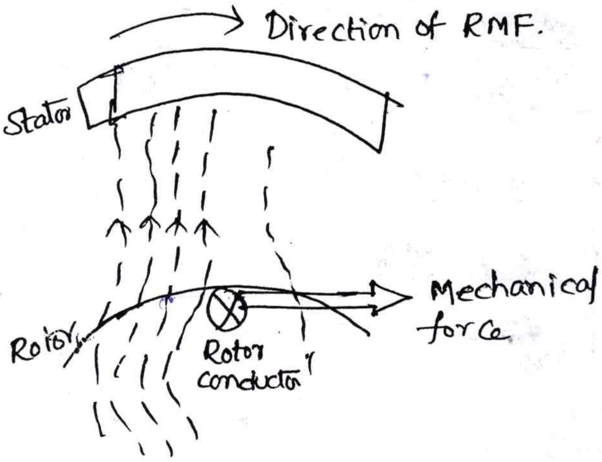


Fig (4d): direction of force

→ According to Lenz law, the direction of rotor current opposes the cause producing it.

→ The cause of rotor current is induced emf which is induced because of relative motion present b/w RMF & rotor conductors.

→ Hence to oppose the relative motion i.e., to reduce relative speed, rotor experience a torque in the same direction as RMF and tries to catch the speed of RMF.] 4.

→ **Definitions:**

→ **Synchronous speed (N_s):-**

when 3-phase balanced currents flows through stator winding of a 3-phase induction motor, a Rotating Magnetic Field is produced in the airgap ~~of~~ between stator & rotor.

→ The speed in which the rotating magnetic field rotates in space is known as synchronous speed N_s.

$$N_s = \frac{120f}{P}$$

where N_s = Synchronous speed
f = frequency
P = No. of poles on stator

→ **slip (S):**

→ The Rotor of an Induction motor runs at speed N, which is always less than synchronous speed (N_s) of rotating magnetic field.

→ The Relative speed between the rotor & the rotating magnetic field is known as "slip" in an induction motor.

*→ The difference between the synchronous speed N_s of rotating magnetic field and actual speed N of rotor is called "slip".

$$\text{slip, } S = (N_s - N) \text{ rpm}$$

It is usually expressed as percentage of synchronous speed.

$$\% \text{ slip, } S = \frac{N_s - N}{N_s} \times 100$$

~~Not~~

(i) The quantity $N_s - N$ is called slip speed.

(ii) when rotor is stationary i.e., $N = 0$ then slip $s = 1$ or 100%.

(iii) In an induction motor, the change in slip from No load to full load is hardly 0.1% to 3%. so that it is essentially a constant speed motor.

Effect of slip on Rotor Parameters :

Let us study the effect of slip on the following rotor parameters:

1) Rotor Frequency 2) magnitude of rotor induced emf

3) Rotor Reactance 4) rotor powerfactor 5) Rotor Current.

1) **Effect of slip on Rotor frequency :-**

In Induction motor, the speed of rotating magnetic field is

$$N_s = \frac{120 f}{P} \longrightarrow (1)$$

where f = frequency of supply in Hz.

Let f_r is the frequency of rotor in running condition.

$N_s - N$ be the slip speed.

we can write for rotor in running condition,

$$N_s - N = \frac{120 f_r}{P} \longrightarrow (2)$$

rotor poles = stator pole = P.

divide (2) by (1) eq

$$\frac{N_s - N}{N_s} = \frac{\frac{120 f_r}{P}}{\frac{120 f}{P}}$$

$$s = \frac{f_r}{f} \qquad \left(\because \frac{N_s - N}{N_s} = s = \text{slip}\right)$$

$$\boxed{f_r = s f}$$

Torque Equation

The Torque produced in induction motor depends on the following

1) part of rotating magnetic field which reacts with rotor (ϕ)

2) magnitude of rotor current in running condition. (I_{2r})

3) powerfactor of rotor circuit in running condition. ($\cos\phi_{2r}$)

Mathematically,

$$T \propto \phi \, I_{2r} \cos\phi_{2r}$$

Here $E_2 \propto \phi$.

$$T \propto E_2 \, I_{2r} \cos\phi_{2r} \longrightarrow (1)$$

we have $I_{2r} = \dfrac{E_{2r}}{Z_{2r}} = \dfrac{S E_2}{\sqrt{R_2^2+(S X_2)^2}}$

$$\cos\phi_{2r} = \frac{R_2}{Z_{2r}} = \frac{R_2}{\sqrt{R_2^2+(SX_2)^2}}$$

From Eq (1)

$$\therefore \quad T \propto E_2 \cdot \frac{SE_2}{\sqrt{R_2^2+(SX_2)^2}} \cdot \frac{R_2}{\sqrt{R_2^2+(SX_2)^2}}$$

$$T \propto \frac{S E_2^2 R_2}{R_2^2+(SX_2)^2} \quad \text{N-m.}$$

$$\boxed{T = K \frac{SE_2^2 R_2}{R_2^2+(SX_2)^2} \quad \text{N-m.}}$$

where K = constant of proportionality

$K = \dfrac{3}{\omega_s} = \dfrac{3}{2\pi n_s}$, where n_s = synchronous speed in 'RPS'

$$n_s = \frac{N_s}{60}$$

$$\boxed{T = \frac{3}{2\pi n_s} \frac{SE_2^2 R_2}{R_0^2+(SX_2)^2} \quad \text{N-m}}$$

→ Starting Torque T_{st} :-

The starting Torque is nothing but the torque produced by an induction motor at start.

At start, $N = 0$ and slip $s = 1$.

so putting $s=1$ in Torque equation, we get T_{st}.

starting Torque $\boxed{T_{st} = \frac{3}{2\pi n_s} \cdot \frac{E_2^2 R_2}{(R_2^2 + X_2^2)}}$ or $\boxed{T_{st} = \frac{K E_2^2 R_2}{R_2^2 + X_2^2}}$

→ Condition For Maximum Torque

Torque developed by an induction motor $T = \frac{K s E_2^2 R_2}{R_2^2 + (s X_2)^2}$

for maximum Torque, $\frac{dT}{ds} = 0$.

$$\Rightarrow \frac{d}{ds}\left[\frac{K \cdot s E_2^2 R_2}{R_2^2 + s^2 X_2^2}\right] = 0. \qquad \left(\because \frac{d}{dt}\frac{u}{v} = \frac{u dv - v du}{v^2}\right)$$

$$\Rightarrow \frac{K s E_2^2 R_2 \frac{d}{ds}(R_2^2 + s^2 X_2^2) - (R_2^2 + s^2 X_2^2)\frac{d}{ds}(K s E_2^2 R_2)}{(R_2^2 + s^2 X_2^2)^2} = 0$$

$$\Rightarrow K s E_2^2 R_2 [2 s X_2^2] - (R_2^2 + s^2 X_2^2)[K E_2^2 R_2] = 0$$

$$\Rightarrow 2 K s^2 E_2^2 R_2 X_2^2 - K R_2^3 E_2^2 - K s^2 X_2^2 E_2^2 R_2 = 0$$

$$\Rightarrow K s^2 E_2^2 X_2^2 R_2 - K E_2^2 R_2^3 = 0$$

$$\Rightarrow K E_2^2 R_2 [s^2 X_2^2 - R_2^2] = 0$$

$$s^2 X_2^2 - R_2^2 = 0 \quad \Rightarrow \quad s^2 X^2 = R_2^2$$

$$\boxed{s = \frac{R_2}{X_2}}$$

This is the slip at which Torque is maximum, $\boxed{S_m = \frac{R_2}{X_2}}$

* → Expression of Maximum Torque :-

Torque developed in Induction motor $T = \dfrac{K\, S\, E_2^2 R_2}{R_2^2 + (S X_2)^2}$

Maximum Torque developed $T_m = \dfrac{K\, S_m\, E_2^2\, R_2}{R_2^2 + (S_m X_2)^2}$

condition for maximum Torque, $S_m = \dfrac{R_2}{X_2}$ substitute in above Eq.

$$T_m = \frac{K\left(\frac{R_2}{X_2}\right) E_2^2 R_2}{R_2^2 + \left[\left(\frac{R_2}{X_2}\right) X_2\right]^2}$$

$$\boxed{T_m = \frac{K\, E_2^2}{2\, X_2} \text{ N-m.}} \quad (\text{or}) \quad \boxed{T_m = \frac{3}{2\pi n_s} \cdot \frac{E_2^2}{2X_2}} \text{ N-m.}$$

→ Torque - slip characteristics of an Induction motor :-

Torque developed by Induction motor $T = \frac{K\, S\, E_2^2\, R_2}{R_2^2 + (S X_2)^2}$ → ①

case - 1 : when slip is small :

when slip 's' is small, $(SX_2)^2$ in above Eq, becomes very small compared to R_2^2 in the denominator. so neglect $(SX_2)^2$.

$$T = \frac{K\, S\, E_2^2\, R_2^2}{R_2} \Rightarrow T = K\, S\, E_2^2 \longrightarrow ②$$

$$\Rightarrow \boxed{T \propto S.} \longrightarrow ②$$

From above Eq ②, Torque is directly proportional to slip. Hence the graph is straight line in nature in low slip region.

case - 2 : when slip is large :

when slip is large, X_2 is larger than R_2 .. so neglect R_2^2 in denominat

Torque Eq ①, becomes

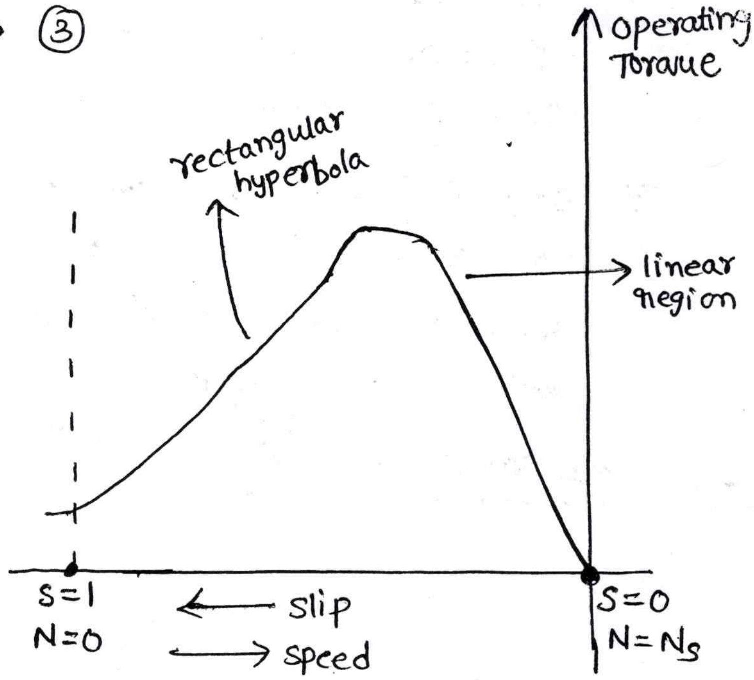
$$T = \frac{K\, S\, E_2^2\, R_2}{(SX_2)^2} = \frac{K\, S E_2^2 R_2}{S^2 X_2^2} = \frac{K\, E_2^2 R_2}{S X_2^2}$$

$$T = \frac{K\, E_2^2\, R_2}{S\, X_2^2} \longrightarrow ③$$

$$\Rightarrow \boxed{T \propto \frac{1}{S}}$$

From above Eq ③, Torque is inversely proportional to slip.

Hence the graph is rectangular hyperbola, in high slip region.



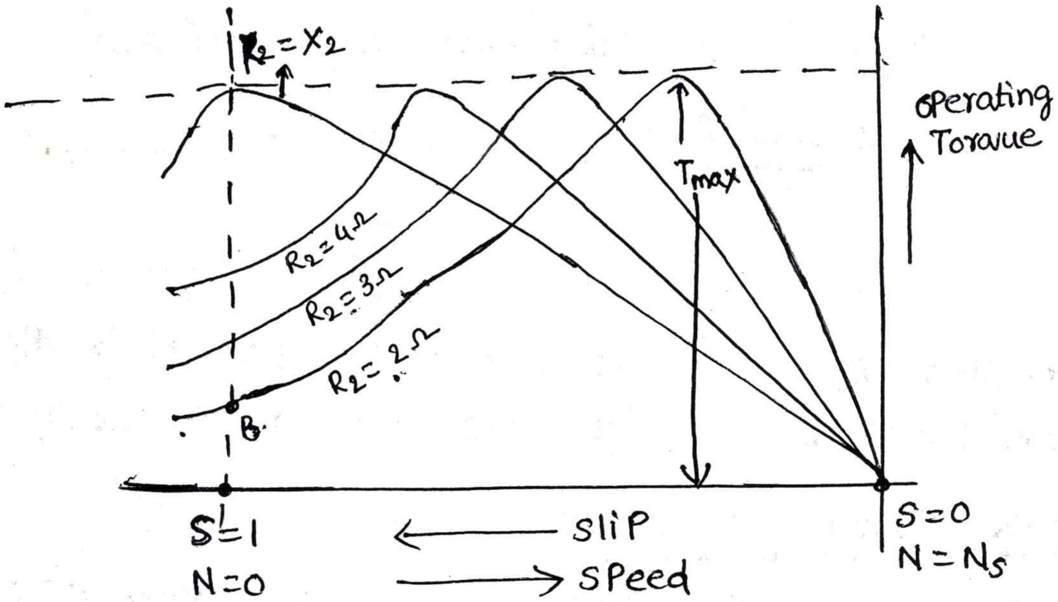
Fig(1): Torque - slip characteristics

→ Torque - slip characteristics has two parts

1) straight line (or) linear region called stable region of operation.

2) Rectangular hyperbola called unstable region of operation.

→ Torque-slip characteristics of Induction motor at different rotor resistance is shown in Fig(2).



Fig(2): Torque-slip characterestics of Induction motor at different rotor resistances.

→ Applications of Induction motor :-

1) 1-ϕ Induction motors are used for very small commercial applications such as household appliances.

2) 3-ϕ Induction motors are used where speed is maintained constant. They are used in Lift irrigation, for driving Floar mills, as prime movers in motor-generator sets, for driving lathes, in Rice mills and as voltage regulators.

→ advantages of Induction motor :-
1) It has simple & rugged construction.
2) It is relatively cheap
3) It requires little maintenance
4) It has high efficiency & good powerfactor
5) It has self starting Torque.

→ dis advantages:-

1) It is a constant speed motor and its speed cannot be changed easily

2) Its starting Torque is inferior to dc shunt motor.

Problems

1) A 6-pole 3-ϕ Induction motor is connected to 50 HZ supply. If it runs at 970 rpm, Find the % slip, and Frequency of rotor currents.

(II ECE, June 2015)

Sol:

Given pole $P = 6$

frequency $f = 50$ HZ

speed $N = 970$ rpm

slip $s = ?$

synchronous speed $N_s = \frac{120f}{P}$

$= \frac{120 \times 50}{6}$

$N_s = 1000$ rpm

% slip $= \% s = \frac{N_s - N}{N_s} \times 100 = \frac{1000 - 970}{1000} \times 100 = 3\%$

$\boxed{\% s = 3\%} \Rightarrow$ ~~0.03~~ $\boxed{s = 0.03}$

Frequency of rotor emf, $f_r = sf = 0.03 \times 50 = 1.5$ HZ $\Rightarrow \boxed{f_r = 1.5 \text{ HZ}}$

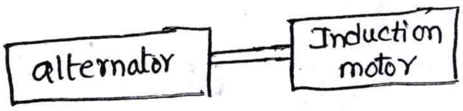
2) A 6 pole alternator running at 1000 rpm supplies an 8-pole induction motor. Find the actual speed of Motor if the slip is 2.5%.

Sol:

~~Given pol~~ Given $s = 2.5\%$

$P = 8$

$N = ?$



The frequency of 3-ϕ supply fed to induction motor is determined from the speed of alternator and its no. of poles.

alternator; $N_s = \frac{120f}{P} \Rightarrow f = \frac{N_s P}{120} = \frac{1000 \times 6}{120} = 50$

$\boxed{f = 50 \text{ HZ}}$

Induction motor;

synchronous speed of motor $N_s = \frac{120f}{P}$

$N_s = \frac{120 \times 50}{8} = 750$

$\boxed{N_s = 750 \text{ rpm}}$

$$\% \text{ slip}, s = \frac{N_s - N}{N_s} \times 100$$

$$2.5 = \frac{750 - N}{750} \times 100$$

$$750 - N = \frac{750 \times 2.5}{100}$$

Actual speed, $\boxed{N = 731.25 \text{ rpm}}$

3) A 3-ϕ Induction motor is wound for 4 poles and is supplied from 50 HZ. calculate (i) synchronous speed (ii) speed of motor when slip is 4%. (iii) The current frequency when motor runs at 600 rpm (iv) frequency of rotor current at standstill.
(II ECE, Dec 2014)

Sol:

Given pole $P = 4$

frequency $f = 50$ HZ

slip $\% s = 4\%$

(i) synchronous speed $N_s = \frac{120f}{P} = \frac{120 \times 50}{4}$

$$\boxed{N_s = 1500 \text{ rpm}}$$

(ii) speed of motor, N

$$\% s = \frac{N_s - N}{N_s} \times 100$$

$$4 = \frac{1500 - N}{1500} \times 100 \Rightarrow N = 1400 \text{ rpm}$$

speed of motor (or) actual speed $\boxed{N = 1400 \text{ rpm}}$

(iii) Rotor current frequency at running speed 600 rpm.

$$f_r = sf$$

slip at 600 rpm is to be find out.

$N = 600$ rpm.

$$\% s = \frac{N_s - N}{N_s} \times 100 = \frac{1500 - 600}{1500} \times 100 = 60\% \Rightarrow \% s = 60\%$$

$$s = \frac{60}{100} = 0.6$$

$\Rightarrow S = 0.6$

$f_r = sf$

$= 0.6 \times 50$

$\boxed{f_r = 30 \text{ HZ}}$

(iv) Rotor ~~current~~ current frequency at standstill condition, (rest)

At standstill, speed $N = 0$

slip $s = 1$.

$\therefore$ Rotor current frequency at stand still, $f_r = sf$

$f_r = (1) \times 50$

$\boxed{f_r = 50 \text{ HZ}}$

4) A 4-pole 3-ϕ Induction motor runs at 1740 rpm. what is the frequency of rotor current if supply frequency is 60 HZ.

Sol: Given pole $P = 4$

actual speed $N = 1740$ rpm.

supply frequency $f = 60$ HZ

rotor frequency at running condition $f_r = ?$

synchronous speed $N_s = \frac{120f}{P}$

$= \frac{120 \times 60}{4}$

$\boxed{N_s = 1800 \text{ rpm}}$

$\% \text{ slip} = \frac{N_s - N}{N_s} \times 100 = \frac{1800 - 1740}{1800} \times 100$

$\% S = 3.33 \%$

$S = \frac{3.33}{100} = 0.033 \Rightarrow \boxed{S = 0.033}$

Rotor frequency at running, $f_r = sf$

$= 0.033 \times 60$

$\boxed{f_r = 2 \text{ HZ}}$

5) A 12 pole 3-ϕ Induction motor runs at 485 rpm on a 50Hz supply. calculate (1) slip and (2) Frequency of rotor emf.

(II ECE, Dec 2014)

Sol:

Given No. of poles $P = 12$

actual speed $N = 485$ rpm.

frequency $f = 50$ Hz.

Slip $S = ?$

Frequency of rotor emf $f_r = ?$

Synchronous speed $N_s = \frac{120 f}{P}$

$N_s = \frac{120 \times 50}{12} = 500 \Rightarrow \boxed{N_s = 500 \text{ rpm}}$

~~slip S = Ns - N = 500 - 485 = 15~~

1) % Slip = % S = $\frac{N_s - N}{N_s} \times 100 = \frac{500 - 485}{500} \times 100 = 3\%$

% S = 3 %

$\boxed{S = 0.03}$

2) Frequency of rotor emf $f_r = s f$.

$f_r = 0.03 \times 50$

$\boxed{f_r = 1.5 \text{ Hz}}$

Alternators:

5 [Principle and operation of alternators :-

The working principle of an alternator or AC generator is similar to the basic working principle of a DC generator.

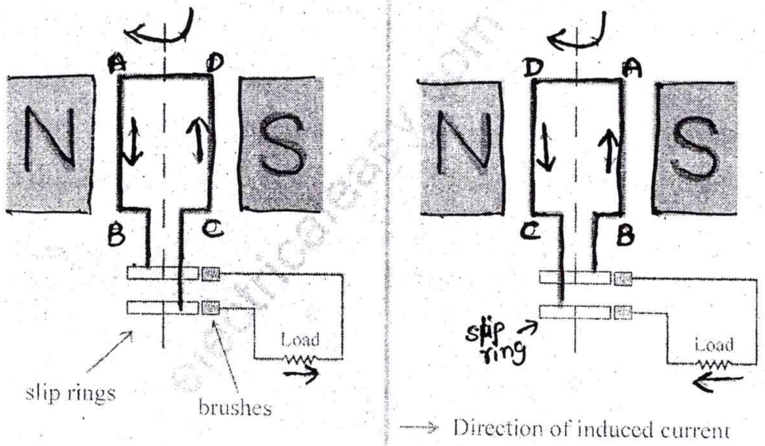


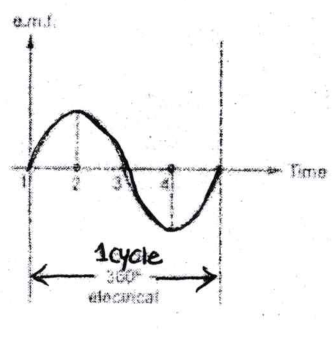
fig: case 1 fig: case 2

Above figure helps you understanding **how an alternator or AC generator works**. According to the **Faraday's law of electromagnetic induction**, whenever a conductor moves in a magnetic field EMF gets induced across the conductor. If the close path is provided to the conductor, induced emf causes current to flow in the circuit.

Now, see the above figure. Let the conductor coil ABCD is placed in a magnetic field. The direction of magnetic flux will be form N pole to S pole. The coil is connected to slip rings, and the load is connected through brushes resting on the slip rings.

Now, consider the case 1 from above figure. The coil is rotating clockwise, in this case the direction of induced current can be given by **Fleming's right hand rule**, and it will be along A-B-C-D.

As the coil is rotating clockwise, after half of the time period, the position of the coil will be as in second case of above figure. In this case, the direction of the induced current according to Fleming's right hand rulewill be along D-C-B-A. It shows that, the direction of the current changes after half of the time period, that means we get an alternating current. shown in fig below (3).



Fig(3): AC emf.] 5

Unit-5

Synchronous Generators

The Machine which produces 3-ϕ electrical power from Mechanical power is called an alternator (or) synchronous generator.

→ **Principle**:-

An alternator works on the same principle of faraday's law of electromagnetic induction as a dc generator i.e; when the flux linking a conductor changes an EMF is induced in the conductor.

→ **Construction**:-

Similar to other rotating machines, an alternator consists of two main parts namely stator and rotor.

The stator is stationary part of the machine, which carries the armature winding in which the output voltage is generated. The o/p of the machine is taken from the stator.

The rotor is the rotating part of the machine, which produces the main field flux.

The frequency of o/p ac voltage of a synchronous generator is directly proportional to the rotor speed. To maintain the frequency constant, the rotor must always moves at synchronous speed.

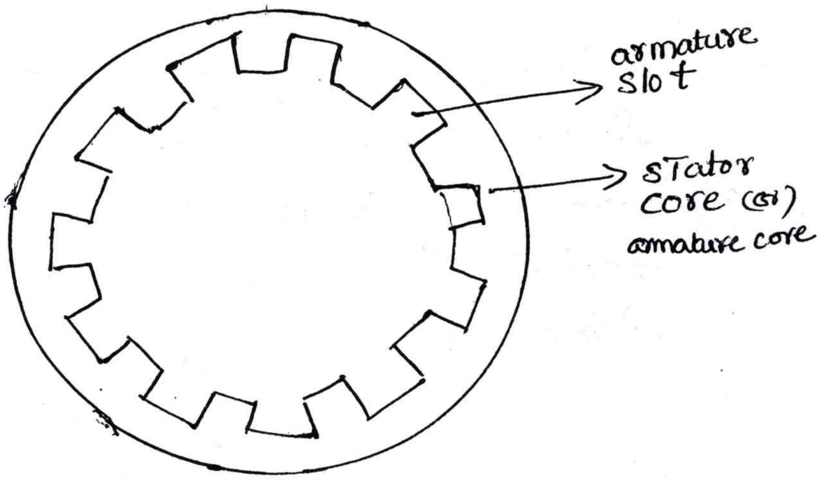
→ Advantages of stationary armature

1. Better insulation
2. Ease of current collection
3. Increased armature tooth strength
4. Reduced armature leakage reactance.
5. Lesser no. of slip ring
6. Lesser rotor weight & inertia
7. Improved ventilation & heat dissipation

→ construction of Alternator

(1) Stator construction:-

It is the stationary part of the machine and is built up of sheet steel laminations having slots on its inner periphery. 3-ϕ winding is placed in these slots and serves as the armature winding of alternator. The armature winding is always connected in star and the neutral is connected to ground.



(2) Rotor:-

There are two types of rotor constructions namely:-

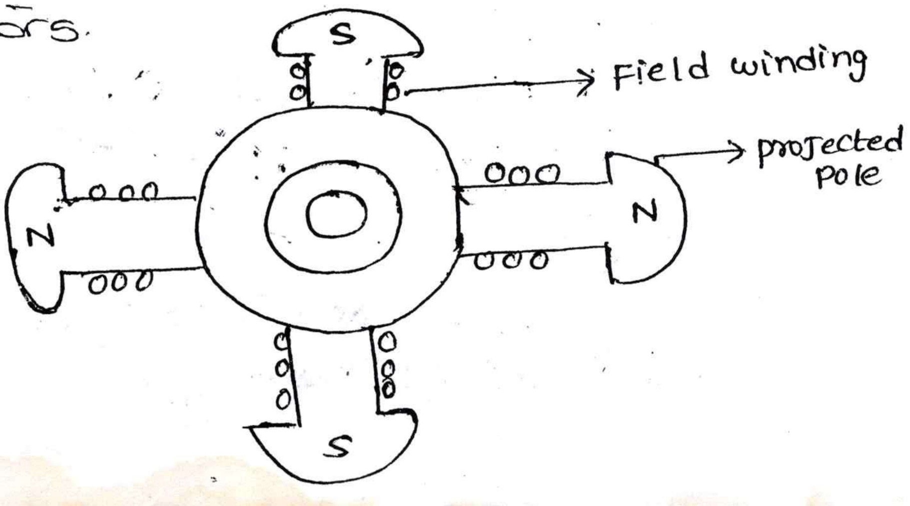
1. salient - pole type
2. cylindrical rotor type (or) Non - salient pole type

↳(1) Salient - pole Rotor (concentrated)

The term salient means "projecting". Thus, a salient - pole rotor consists of poles projecting out from the surface of the rotor core.

A salient pole rotor have concentrated winding on the poles. A salient pole synchronous machine has a non - uniform air gap. The air gap is minimum under the pole centres and it is maximum in between the poles. The pole faces are so shaped that the radial air gap length increases from pole centre to the pole tips. so that the flux distribution in the air gap is sinsoidal. This will help the machines to generate sinusoidal EMF.

"Salient - pole" generators have a large no. of poles, and operator at lower speeds. The rotor of this type is used almost entirely for slow and moderate speed alternators.



The field coils are placed on the pole pieces and connected in series. The ends of field winding are connected to DC supply. Through slip rings mounted on shaft of the field structure.

↳ (2) <u>Non-salient poles [distributed]</u> :-

It has its rotor constructed that it forms a smooth cylinder. The construction is such that there are no physical poles to be seen as in the salient pole construction.

The rotor of this type are used in very high speed alternators driven by steam turbines. The cylindrical rotor type alternator has two or four poles on the rotor such a construction provides a greater mechanical strength and permits more accurate dynamic balancing. The smooth rotor of the machine makes less winding losses and the operation is less noisy because of uniform airgap.

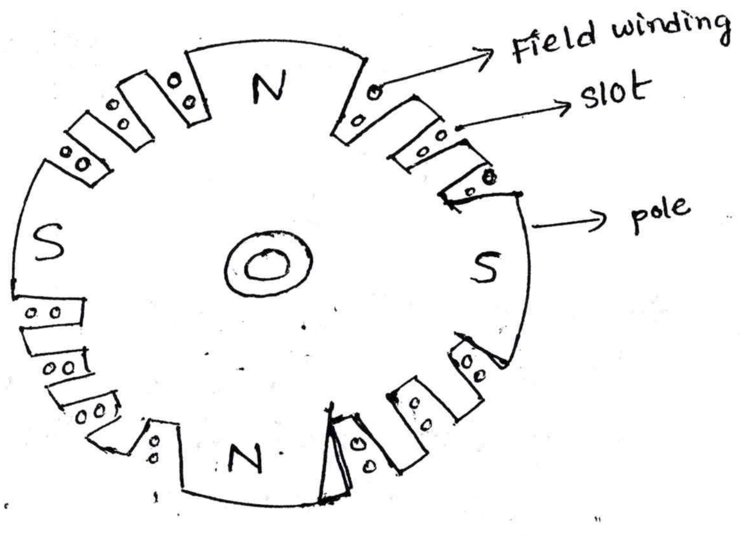


Fig: Non-salient pole (or) smooth cylindrical rotor.

↳ Frequency of Induced EMF :-

$$f = \frac{N \cdot P}{120}$$

N → Syn. Speed
P → no. of poles

→ Comparison :

Salient pole	cylindrical (or) Non-salient
→ It has projecting Poles	→ No. projecting poles
→ Large diameter & short axial length	→ small diameter and long axial length
→ used for low speed alternators	→ used for high speed turbo alternators
→ There is no mechanical balance.	→ Mechanical balance is there

Problem :

→ A 415V, 4pole, 3-ϕ alternator is driven at 1500rpm. calculate frequency of operation

Sol :- $P = 4$, $N = 1500$

$$f = \frac{P \cdot N}{120} = \frac{4 \times 1500}{120}$$

$$\boxed{f = 50\,Hz}$$

→ An alternator on open-circuit generates 360V at 60Hz when the field current is 3.6A. Neglecting saturation, determine the open circuit emf when the frequency is 40Hz and field current is 2.4A.

Sol :- Given: open ckt generated voltage $E_1 = 360V$
frequency $f_1 = 60\,Hz$
open ckt Emf $E_2 = ?$

Frequency $f_2 = 40\,Hz$

we have, Induced Emf Equation

Sol $\rightarrow E_{ph} = 4.44 f \phi T K_p K_d$

$\rightarrow E \propto \phi f$

$\rightarrow \frac{E_1}{E_2} = \frac{\phi_1 f_1}{\phi_2 f_2} \Rightarrow \frac{360}{E_2} = \frac{(3.6)\times 60}{(2.4)\times 40}$

$\boxed{E_2 = 160V}$

→ A 16 pole, 3-ϕ alternator has a star-connected winding with 144 slots and 10 conductors per slot. The flux per pole is 0.03 Wb distributed sinusoidally and the speed is 375 rpm. Find the line voltage.

Sol:

$\phi = 0.03\,Wb, \quad f = \frac{PN}{120} = \frac{16\times 375}{120} \Rightarrow \boxed{f = 50\,Hz}$

$Z_{ph} = 144\times 10 = \frac{1440}{3(3\text{-}\phi)} = 480$

$T_{ph} = \frac{Z_{ph}}{2} = \frac{480}{2} = 240$

$m = \frac{\text{Slots}}{\text{Pole}\times ph} = \frac{144}{16\times 3} = 3$

$\beta = 180\times \frac{\text{Poles}}{\text{Slots}} = \frac{180\times 16}{144} = 20^\circ$

$K_d = \frac{\sin m\beta/2}{m\sin \beta/2} = \frac{\sin\left[\frac{3\times 20}{2}\right]}{3\sin 20/2} = \frac{\sin 30}{3\sin 10} = 0.9598$

assume coil to be full pitched $\boxed{K_p = 1}$

$E_{ph} = 4.44\times 1\times 0.9598\times 50\times 0.03\times 240$

$\Rightarrow \boxed{E_{ph} = 1543.44V}$

$E_{Line} = \sqrt{3}\cdot E_{ph}$

$\boxed{E_{line} = 2657.21\,\text{volts}}$

→ Induced EMF Equation of an Alternator :-

consider an alternator with 'p' poles. Let the flux per pole be 'ϕ'

Rotor speed be N rpm

No of turns of stator winding per phase be T'

average emf induced emf in one conductor

$$\Rightarrow \quad E \propto \frac{d\phi}{dt}$$

$$\Rightarrow \quad E_{avg} = \frac{P.\phi}{(60/N)} = \frac{NP\phi}{60}$$

$$\therefore f = \frac{NP}{120} \Rightarrow 2f = \frac{NP}{60}$$

$$\cancel{\left(\therefore E_{avg} = \frac{P\phi}{(60/N)} = \frac{NP\phi}{60}\right)}$$

$$\Rightarrow \quad \therefore E_{avg} = 2f\phi \text{ volts}$$

Where 'f' is the frequency for 'T' turns

[1 Turn = 2 conductor]

8

$$E_{avg} = 2f\phi(2T)$$

$$E_{avg} = 4f\phi T$$

rms voltage $E = avg \times$ form factor

$$E = 4f\phi T \times 1.11$$

$$\boxed{E = 4.44 f\phi T}$$

$\because$ since, formfactor $= \frac{\text{rms value}}{\text{avg value}}$

$1.11 = \frac{rms}{avg}$

∴ Here assumed as winding is concentrated. In practice they are distributed over the entire Periphery of the armature and hence the emf's in all the turns cannot be simply added.

→ To compensate for this, voltage is multiplied by the distribution factor 'kd' [kd < 1].

The windings are generally short pitched. When full pitches, we multiply the voltage by pitch factor 'kp', which is also less than unity.

$$\therefore E = 4.44\, f \phi T K_d K_p \text{ volts}$$

rms Emf/ph

pitch factor

distributed factor

flux per pole

No. of Turns/ph

frequency

$$\therefore K_d = \frac{\text{Emf with a distributed winding}}{\text{Emf with a concentrated winding}}$$

$$\Rightarrow \boxed{K_d = \frac{\sin m\beta/2}{m \sin \beta/2}}$$

m = no of slots/pole × phases, β = slot angle

$= 180/n$

$$n = \frac{\text{no. of slots}}{\text{Poles}}$$

$$\therefore K_p = \frac{\text{Emf with a short pitch coil}}{\text{Emf with a full pitch coil}}$$

$$\boxed{K_p = \cos \alpha/2}$$

$\alpha = 180° - \text{coil span}$