Close coiled helical spring with axial load

Consider the close - coiled spring as shown in fig.

The spring is subjected to an axial load P.

d - diameter of spring wire

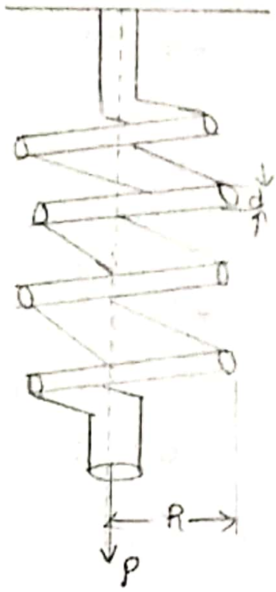
R - mean radius of the coil

n - number of coils

L - Length of the wire of the spring $= 2\pi R n$

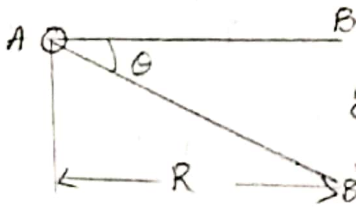
δ - deflection of the spring caused by load P

G - Modulus of rigidity

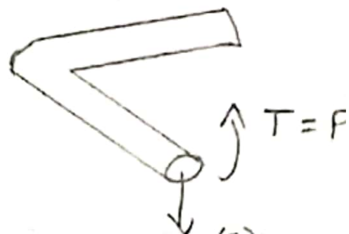


(a)

The circular section of the wire is subjected to a vertical shear due to a load P and a torque $T = PR$ in the direction shown in fig.



(b)



(c)

The vertical shear stress

$$\tau_1 = \frac{P}{A}$$

$$= \frac{P}{\frac{\pi d^2}{4}}$$

$$\tau_1 = \frac{4P}{\pi d^2}.$$

The cross - section of the wire is twisted as shown in fig.

A line AB is distorted to AB'.

The torque PR causes stress.

The maximum value of that stress at outer fibre will be

As we know that $\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\Rightarrow \tau_2 = \frac{Tr}{J} = \frac{PR \times \frac{d}{2}}{\frac{\pi d^4}{32}}$$

$$\Rightarrow \tau_2 = \frac{16 PR}{\pi d^3}$$

The maximum stress which occurs in outer most fibre of the wire will be

$$\tau_{max} = \tau_1 + \tau_2 = \frac{4P}{\pi d^2} + \frac{16 PR}{\pi d^3}$$

$$\tau_{max} = \frac{16 PR}{\pi d^3}\left[1 + \frac{d}{4R}\right]$$

If d is neglected as compared to mean coil radius then

$$\tau_{max} = \frac{16 PR}{\pi d^3}$$

Deflection of the spring

The axial load tends to elongate the spring. If the point where the load is attached to the spring radial arm moves down by an amount δ, Then δ can be approximated as $R\theta$.

The angle of twist θ is obtained from the torsion formula

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\theta = \frac{TL}{GJ}$$

$$\theta = \frac{PR \times 2\pi Rn}{G \times \frac{\pi d^4}{32}}$$

$$\theta = \frac{64 PR^2 n}{Gd^4}$$

deflection $\delta = R\theta$

$$\delta = R\theta = \frac{64 PR^3 n}{Gd^4}$$

The stiffness of the spring is the load required to produce unit deflection and is denoted by 'K'.

$$K = \frac{P}{\delta} = \frac{Gd^4}{64 R^3 n}$$

Strain Energy stored in the spring is due to axial load

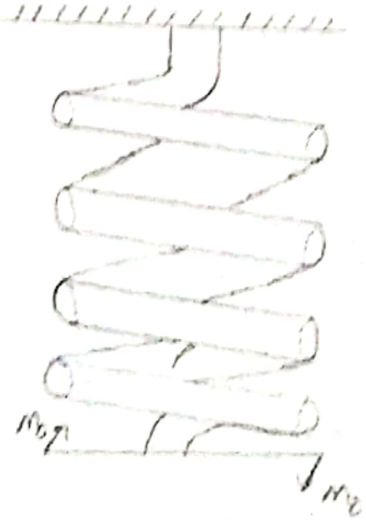
$$U = \frac{1}{2} \times T \times \theta$$

$$= \frac{1}{2} \times PR \times \frac{64 PR^2 n}{Gd^4}$$

$$U = \frac{32 P^2 R^3 n}{Gd^4}$$

Close coiled helical spring with axial twist

Let us consider a closely coiled helical spring subjected to an axial couple M_e as shown in fig.



Due to the couple the coils curvature increases (or) decreases.

-> Before the axial couple is applied

R_1 = Initial radius of curvature

n_1 = initial number of coils

L = length of the spring

→ After the axial couple is applied

R_2 = Changed radius of curvature

n_2 = Changed number of coils

Change of curvature $= \frac{1}{R_2} - \frac{1}{R_1} = \frac{M_0}{EI}$ —— (1)

∴ The change in curvature and bending moment can be related by the bending equation.

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\frac{1}{R} = \frac{M}{EI}$$

Now $L = 2\pi R_1 n_1 = 2\pi R_2 n_2$

$$\frac{1}{R_1} = \frac{2\pi n_1}{L}$$

$$\frac{1}{R_2} = \frac{2\pi n_2}{L}$$

$$\frac{M_0}{EI} = \frac{2\pi n_2}{L} - \frac{2\pi n_1}{L}$$

$$\frac{M_0}{EI} = \frac{2\pi}{L}[n_2 - n_1]$$

$$M_0 = \frac{2\pi EI}{L}[n_2 - n_1] \quad \text{—— (2)}$$

$$M_0 = \frac{2\pi EI}{2\pi Rn}[n_2 - n_1] \qquad \therefore L = 2\pi Rn$$

$$M_0 = \frac{EI}{Rn}[n_2 - n_1]$$

$$n_2 = \frac{M_0 nR}{EI} + n_1$$

$\therefore$ The number of coils decreases or Increases to n_2 from the initial value of n_1.

For a circular section of wire forming the spring.

$$n_2 = \frac{M_0 nR}{E \times \frac{\pi d^4}{64}} + n_1 \qquad \therefore I = \frac{\pi d^4}{64}$$

$$n_2 = \frac{64 M_0 nR}{E\pi d^4} + n_1$$

From above eq. (2)

$$M_0 = \frac{2\pi EI}{L}[n_2 - n_1]$$

The total angular rotation or angular twist is given by the change in angular distance between the coil ends. This will be equal to

$$\phi = 2\pi n_2 = 2\pi n_1$$

$$\phi = 2\pi [n_2 - n_1]$$

$$M_0 = \frac{EI}{L} 2\pi [n_2 - n_1]$$

$$M_0 = \frac{EI}{L}\phi$$

$$\phi = \frac{M_0 L}{EI} \qquad \therefore M_0 = T$$

$$\phi = \frac{TL}{EI}$$

$$\phi = \frac{T \times 2\pi R n}{E \times \frac{\pi d^4}{64}}$$

$$\phi = \frac{128\,\pi R n T}{E\,\pi d^4}$$

$$\phi = \frac{128\,R n T}{E d^4}$$

For a circular section of diameter d, the maximum stress in the spring due to constant Bending moment M is given by

$$\frac{M_0}{I} = \frac{\sigma}{y}$$

$$\sigma_{max} = \frac{M_0 y}{I}$$

$$\sigma_{max} = \frac{M_0 \frac{d}{2}}{\frac{\pi d^4}{64}}$$

$$\sigma_{max} = \frac{32\,M_0}{\pi d^3}$$

Strain Energy due to axial twist

The sections of the wire of a spring subjected to an axial twist are subjected to a constant Bending moment.

$$U = \frac{1}{2} \times M_0 \times \phi$$

$$= \frac{1}{2} \times T \times \frac{128 T R n}{E d^4}$$

$$U = \frac{64 T^2 R n}{E d^4}$$

Problems

A close-coiled helical spring has a mean diameter of 105 mm and 18 coils of wire of diameter 10 mm. Find the maximum stress in the wire, the increase in the number of turns and the total rotation when the coil is subjected to an axial twist of 1.2 Nm. E = 200 GPa.

Given data:-

Dia of wire d = 10 mm

wire subjected to a constant B.m equal to the axial torque

$M_0 = 1.2$ N-m

$M_0 = 1200$ N-mm

The maximum stress is

$$\sigma_{max} = \frac{32 M_0}{\pi d^3}$$

$$\sigma_{max} = \frac{32 \times 1200}{\pi (10)^3}$$

$$\sigma_{max} = 12.2 \ N/mm^2$$

number of turns = 18

Mean dia of coil = 105

If circular section

$$I = \frac{\pi d^4}{64}$$

$$= \frac{\pi (10)^4}{64}$$

$$I = 490.9 \ mm^4$$

Change of curvature $= \frac{1}{R_2} - \frac{1}{R_1} = \frac{M_0}{EI}$

$$\frac{2\pi (n_2 - n_1)}{L} = \frac{M_0}{EI}$$

$$(n_2 - n_1) = \frac{\cancel{2\pi} M_0 L}{2\pi EI}$$

$$(n_2 - n_1) = \frac{M_0 \times 2\pi R n}{2\pi EI}$$

$$n_2 - n_1 = \frac{M_0 R n}{EI}$$

$$n_2 - n_1 = \frac{1200 \times \frac{105}{2} \times 18}{2 \times 10^5 \times 490.9}$$

$\therefore\ n_2 - n_1 = 0.011$

Increase in number of turns $= 0.011$

Total angular shift $= 2\pi\,(n_2 - n_1)$

$$\phi = 2\pi\,(0.011)$$

$$\phi = 0.069 \text{ radians}$$

Find the axial and torsional stiffnesses of a spring made of a wire diameter 6mm with 20 turns of mean diameter 50 mm. Determine the maximum stress in the wire when subjected to an axial twist of 2 N-m G = 80 Gpa and E = 200GPa

Given data:-

wire dia = 6 mm

mean diameter = 50 mm

number of turns = 20

$G = 80$ GPa

$E = 200$ GPa

$M_0 = 20$ N-m

$M_0 = 2000$ N-mm

The maximum stress in the wire

$$\sigma_{max} = \frac{32\,M_0}{\pi d^3}$$

$$= \frac{32 \times 2000}{\pi \times (6)^3}$$

$\sigma_{max} = 94.31 \ N/mm^2$

The axial stiffness is given by

$$K = \frac{P}{\delta} = \frac{Gd^4}{64nR^3}$$

$$K = \frac{80 \times 10^3 \times (6)^4}{64 \times 20 \times (25)^3}$$

$\therefore D = 2R$

$R = \frac{D}{2}$

$R = \frac{50}{2}$

$$K = 5.18 \ N/mm$$

Torsional stiffness is given by

$$\phi = \frac{128 \, TnR}{Ed^4}$$

$$\frac{T}{\phi} = \frac{Ed^4}{128 \, nR}$$

Torsional stiffness $q = \frac{T}{\phi}$

$$q = \frac{Ed^4}{128 \, nR}$$

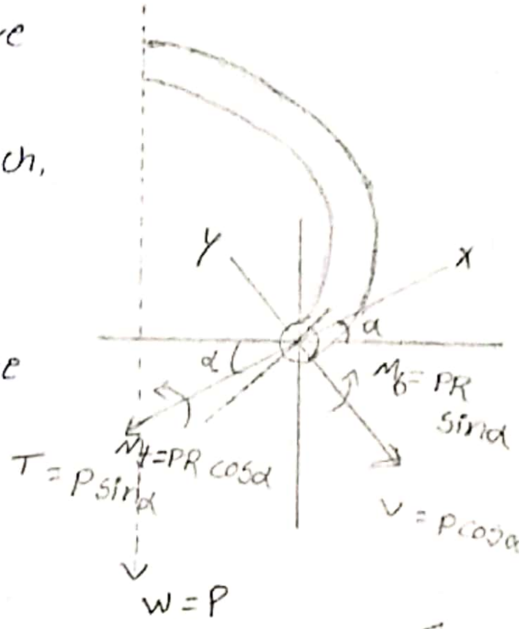
$$q = \frac{200 \times 10^3 \times (6)^4}{128 \times 20 \times 25}$$

$$q = 4050 \ N/rad.$$

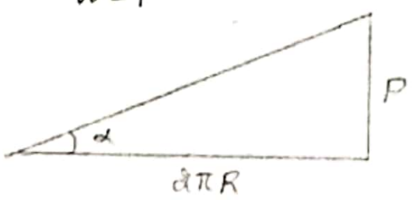
Open - coiled spring subjected to axial load

Consider an open - coiled spring subjected to an axial load.

If α is the angle made by the helical centre line to the horizontal. from the traingle showing the length of the wire and the pitch,



$$\tan\alpha = \frac{P}{2\pi R}$$



If we consider a diametrical plane cutting the coil, the axial load produces a moment $M = PR$, as in the case of a close - coiled spring. However, this moment is at an angle α to the helical central line of the coil. The vertical load P is also inclined to the plane of the section at an angle α. This leads to the following four effects on the section of the coil.

* Bending moment $M_b = M\sin\alpha = PR\sin\alpha$
* Torsional moment $M_t = M\cos\alpha = PR\cos\alpha$
* Axial tension $T = P\sin\alpha$
* Transverse shear $V = P\cos\alpha$

M_b and M_t are components of M acting at an angle α to the helical center line. The axial tension and transverse shear are components of the vertical load P acting obliquely to the normal section of the wire.

The stresses due to these effects can be determined as follows.

1) Maximum bending stress

$$\sigma_b = \frac{32 M_b}{\pi d^3}$$

$$\sigma_b = \frac{32\ PR\sin\alpha}{\pi d^3}$$

ii) Maximum shear stress

$$\tau = \frac{16\ m_t}{\pi d^3}$$

$$= \frac{16\ PR\cos\alpha}{\pi d^3}$$

iii) Uniform tensile stress due to axial tension T

$$\sigma_t = \frac{4P\sin\alpha}{\pi d^2}$$

iv) The shear stress due to transverse shear V is maxing having a value of $\tau_v = \frac{4}{3}$ x average shear stress of a circular section

$$\tau_v = \frac{4}{3} \times \frac{V}{A}$$

$$= \frac{4}{3} \times \frac{P\cos\alpha}{\pi d^2/4}$$

$$= \frac{4}{3} \times \frac{4P\cos\alpha}{\pi d^2}$$

$$\tau_v = \frac{16\ P\cos\alpha}{3\pi d^2}$$

v) The maximum tensile stress σ at A is given by

$$\sigma_{max} = \sigma_b + \sigma_t$$

$$= \frac{32\ PR\sin\alpha}{\pi d^3} + \frac{4P\sin\alpha}{\pi d^2}$$

$$\sigma_{max} = \frac{32 PR \sin\alpha}{\pi d^3}\left[1 + \frac{d}{8R}\right]$$

Note :- The direct tensile stress is very small compared to the bending stress.

vis The maximum shear stress

$$\tau_{max} = \tau_t + \tau_v$$

$$= \frac{16 PR \cos\alpha}{\pi d^3} + \frac{16 P \cos\alpha}{3\pi d^2}$$

$$\tau_{max} = \frac{16 PR \cos\alpha}{\pi d^3}\left[1 + \frac{d}{3R}\right]$$

Note :- The stress due to transverse shear is small compared to that of due to torsional moment.

-> The combined effect of these stresses can be easily derived from the mohr circle.

The major principal stress is

$$\sigma_1 = \frac{\sigma_{max}}{2} + \sqrt{\left[\frac{\sigma_{max}}{2}\right]^2 + \left[\tau_{max}\right]^2}$$

$$= \frac{16PR \sin\alpha}{\pi d^3}\left[1 + \frac{d}{8R}\right] + \sqrt{\left[\frac{16 PR \sin\alpha}{\pi d^3}\left[1 + \frac{d}{8R}\right]\right]^2 + \left[\frac{16 PR \cos\alpha}{\pi d^3}\left[1 + \frac{d}{3R}\right]\right]^2}$$

Neglecting the effects of axial tension and transverse shear

$$\sigma_1 = \frac{16 PR \sin\alpha}{\pi d^3} + \sqrt{\left[\frac{16 PR \sin\alpha}{\pi d^3}\right]^2 + \left[\frac{16 PR \cos\alpha}{\pi d^3}\right]^2}$$

$$\sigma_1 = \frac{16PR}{\pi d^3}\left[\sin\alpha + \sqrt{\sin^2\alpha + \cos^2\alpha}\right]$$

$$\sigma_1 = \frac{16PR}{\pi d^3}\left[\sin\alpha + 1\right] \qquad \therefore \sin^2 x + \cos^2 x = 1$$

The maximum shear stress

$$(\tau_{max})_{max} = \pm\sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \left(\tau_{max}\right)^2}$$

$$= \pm\sqrt{\left[\frac{16PR\sin\alpha}{\pi d^3}\left[1+\frac{d}{8R}\right]\right]^2 + \left[\frac{16PR\cos\alpha}{\pi d^3}\left[1+\frac{d}{3R}\right]\right]^2}$$

Neglecting the effects of transverse shear and axial tension

$$(\tau_{max})_{max} = \pm\frac{16PR}{\pi d^3}\sqrt{\sin^2\alpha + \cos^2\alpha}$$

$$(\tau_{max})_{max} = \pm\frac{16PR}{\pi d^3}$$

Deformation of the spring

under the action of the axial load, the spring undergoes deformation, which can be measured as a vertical deflection of the spring or as a total angular rotation of the lower point with respect to the fixed upper point.

Deflection due to BM M_b

Length of the spring $L = \frac{2\pi R n}{\cos\alpha}$

$$L = 2\pi R n \sec\alpha$$

W.K.T $\tan\alpha = \frac{P}{2\pi R}$

Along ox, torque $M_t = PR\cos\alpha$

Along oy, B.M $M_b = PR\sin\alpha$

Now $\frac{M}{I} = \frac{\sigma}{y}$

$$\sigma = \frac{M_b y}{I}$$

$$\sigma = \frac{PR\sin\alpha \times \frac{d}{2}}{\frac{\pi d^4}{64}}$$

$$\sigma = \frac{32\,PR\sin\alpha}{\pi d^3}$$

From torque equation.

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\theta = \frac{TL}{GJ}$$

$T = M_t$

$$\theta = \frac{M_t L}{GJ}$$

$$\theta = \frac{PR \cos\alpha \, L}{GJ}$$

From Bending eq

$$\frac{M}{I} = \frac{E}{R}$$

$\therefore \quad L = \phi R$

$$\phi = \frac{L}{R}$$

$$\frac{1}{R} = \frac{\phi}{L}$$

$$\frac{M}{I} = \frac{E\phi}{R}$$

$$\phi = \frac{ML}{EI}$$

work done $= \frac{1}{2} \times P \times \delta$

Strain energy stored in the spring under torsion & bending is

$$U = \frac{1}{2} \times T \times \theta + \frac{1}{2} \times M_b \times \phi$$

$$U = \frac{1}{2} \times M_t \times \theta + \frac{1}{2} M_b \phi$$

$$P\delta = M_t \theta + M_b \phi$$

$$P\delta = PR \cos\alpha \times \frac{M_t L}{GJ} + PR \sin\alpha \times \frac{M_b L}{EI}$$

$$P\delta = PR \cos\alpha \times \frac{PR \cos\alpha \, L}{GJ} + PR \sin\alpha \times \frac{PR \sin\alpha \, L}{EI}$$

$$\delta = PR^2 L \left[\frac{\cos^2\alpha}{GJ} + \frac{\sin^2\alpha}{EI} \right]$$

$$\delta = PR^2 \times 2\pi R n \sec\alpha \left[\frac{\cos^2\alpha}{G \times \frac{\pi d^4}{32}} + \frac{\sin^2\alpha}{E \frac{\pi d^4}{64}} \right]$$

$$= 2 PR^3 \pi n \sec\alpha \left[\frac{\cos^2\alpha}{G \times \frac{\pi d^4}{32}} + \frac{\sin^2\alpha}{E \frac{\pi d^4}{64}} \right]$$

$$= 2 PR^3 \pi n \sec\alpha \left[\frac{32 \cos^2\alpha}{G \pi d^4} + \frac{64 \sin^2\alpha}{E \pi d^4} \right]$$

$$= \frac{2 \times 32 \times PR^3 \pi n \sec\alpha}{\pi d^4} \left[\frac{\cos^2\alpha}{G} + \frac{2 \sin^2\alpha}{E} \right]$$

$$\delta = \frac{64 PR^3 n \sec\alpha}{d^4} \left[\frac{\cos^2\alpha}{G} + \frac{2 \sin^2\alpha}{E} \right]$$

Deflection due to axial twist M_t

$$\delta = \frac{64 PR^3 n \sec\alpha}{d^4} \left[\frac{\cos^2\alpha}{G} + \frac{2 \sin^2\alpha}{E} \right]$$

$\therefore \alpha = 0$

$$\delta = \frac{64 PR^3 n}{d^4}$$ //.

Angular rotation

Let β be the resultant of the rotations θ and ϕ

$$\beta = \theta \sin\alpha - \phi \cos\alpha$$

$$\beta = \frac{M_t L}{GJ} \sin\alpha - \frac{M_b L}{EI} \cos\alpha$$

$$= \frac{PR\cos\alpha \times 2\pi R n}{G \frac{\pi d^4}{32} \times \sec\alpha} \sin\alpha - \frac{PR\sin\alpha \times 2\pi R n}{E \frac{\pi d^4}{64}} \cos\alpha \times \sec\alpha$$

$$= \frac{PR \times 2\pi R n \sin\alpha}{G \frac{\pi d^4}{32}} - \frac{PR \times 2\pi R n \times \sin\alpha}{E \frac{\pi d^4}{64}}$$

$$= \frac{64\, PR^2 n \sin\alpha}{G d^4} - \frac{64 \times PR^2 \times 2n \sin\alpha}{E d^4}$$

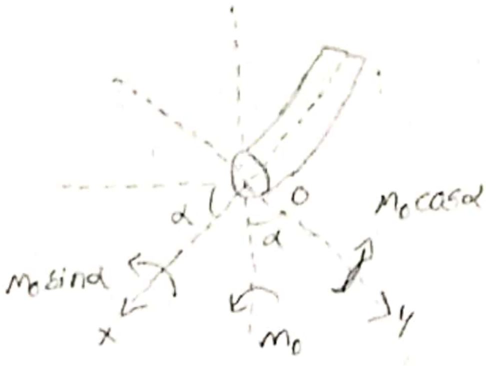
$$\beta = \frac{64\, PR^2 n \sin\alpha}{d^4} \left[\frac{1}{G} + \frac{2}{E} \right]$$

$\therefore$ The relative rotation b/w the ends of the springs is

$$\theta = \frac{64\, PR^2 n \sin\alpha}{d^4} \left[\frac{1}{G} - \frac{2}{E} \right]$$

open - coiled helical spring subjected to axial torque

consider an open coiled helical spring under the action of axial Torque M_0. The axial torque M_0 can be split into two components.



component along OX, $M_t = M_0 \sin\alpha$

which at all sections produces torsion In the spring wire

component along oy $M_b = M_0 \cos\alpha$

which at all sections produces bending moment in the spring wire and tends to change the curvature of the coils.

work done by applied torque $= \frac{1}{2} M_0 \beta$

strain energy stored In the spring $U = \frac{1}{2} T\theta + \frac{1}{2} M\phi$

$$\frac{1}{2} M_0\beta = \frac{1}{2} T\theta + \frac{1}{2} M\phi$$

$\therefore T = M_t$

$M = M_b$

$$M_0\beta = M_t \times \frac{M_t L}{GJ} + M_b \times \frac{M_b L}{EI}$$

$$M_0\beta = \frac{M_t^2 L}{GJ} + \frac{M_b^2 L}{EI}$$

$$= \left[\frac{M_0^2 \sin^2\alpha}{G \times \frac{\pi d^4}{32}} + \frac{M_0^2 \cos^2\alpha}{E \times \frac{\pi d^4}{64}}\right] \times 2\pi R n \sec\alpha$$

$$M_0\beta = \left[\frac{32 M_0^2 \sin^2\alpha}{G\pi d^4} + \frac{64 M_0^2 \cos^2\alpha}{E\pi d^4}\right] \times 2\pi R n \sec\alpha$$

$$\beta = \frac{64 M_0 R n \sec\alpha}{d^4}\left[\frac{\sin^2\alpha}{G} + \frac{2\cos^2\alpha}{E}\right]$$

If $\alpha = 0$

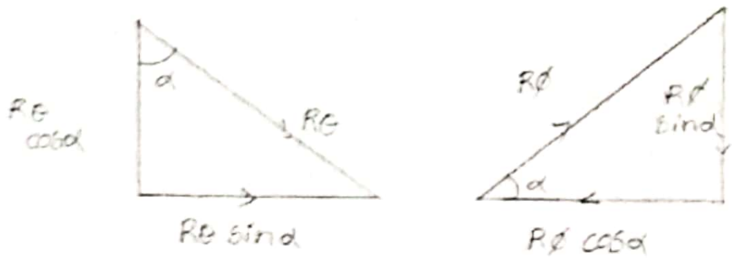
$$\beta = \frac{64\, M_b R n}{d^4}\left[\frac{2}{E}\right]$$

$$\beta = \frac{128\, M_b R n}{E d^4}$$

Axial deflection due to axial torque

Let δ = axial deflection

$$\delta = R\theta \cos\alpha - R\phi \sin\alpha$$



$$\delta = R \times \frac{M_t L}{GJ} \cos\alpha - R \times \frac{M_b L}{EI} \sin\alpha$$

$$= \left[R \times \frac{M_b \sin\alpha \cos\alpha}{G \frac{\pi d^4}{32}} - \frac{R\, M_b \cos\alpha \sin\alpha}{E \frac{\pi d^4}{64}} \right] \times L$$

$$= \left[\frac{32\, R\, M_b \sin\alpha \cos\alpha}{G \pi d^4} - \frac{R \times 64 \times M_b \cos\alpha \sin\alpha}{E \pi d^4} \right] \times 2\pi R n \sec\alpha$$

$$\delta = \frac{64\, R^2 n \sin\alpha\, M_b}{d^4}\left[\frac{1}{G} - \frac{2}{E}\right]$$

Strain energy $U = \frac{1}{2} T \theta$

$$U = \frac{32\, M_b^2 R n \sec\alpha}{d^4}\left[\frac{\sin^2\alpha}{G} + \frac{2\cos^2\alpha}{E}\right]$$

An open - coiled helical spring is made of a wire of diameter 10 mm, a[illegible] carries an axial load of 150 N. If the permissible normal and shear stresses are 100 MPa and 70 MPa, Find the mean radius of the coil, inclination α of the helical axis, and the pitch. If G = 80 GPa and E = 200 GPa an axial stiffness of 4 N/mm is required, Find the number of coils required.

Given data :-

shear stress $\tau_{max} = \frac{16PR}{\pi d^3}$

$$\tau_{max} = 70 \text{ MPa}$$

$$70 = \frac{16 \times 150 \times R}{\pi (10)^3}$$

$$R = 91.6 \text{ mm}$$

permissible normal stress $\sigma_{max} = 100$ MPa

$$\sigma_{max} = \frac{16PR}{\pi d^3} [\sin\alpha + 1]$$

$$100 = 70 [\sin\alpha + 1]$$

$$100 = 70 \sin\alpha + 70$$

$$70 \sin\alpha = 30$$

$$\sin\alpha = \frac{30}{70}$$

$$\sin\alpha = 0.428$$

$$\alpha = 25.34°$$

Pitch $P = 2\pi R \tan\alpha$

$$P = 2\pi \times 91.6 \times \tan(25.34)$$

$$P = 272.54 \text{ mm}$$

$$\text{Axial stiffness} = \frac{P}{\delta} = 4 \text{ N/mm}$$

$$P = 4\delta \text{ N/mm}$$

W.K.T $$\delta = \frac{64 PR^3 n \sec\alpha}{d^4} \left[\frac{\cos^2\alpha}{G} + \frac{2\sin^2\alpha}{E} \right]$$

$$\frac{P}{4} = \frac{64 PR^3 n \sec\alpha}{d^4} \left[\frac{\cos^2\alpha}{G} + \frac{2\sin^2\alpha}{E} \right]$$

$$\sin\alpha = 0.428$$

$$\cos\alpha = 0.904$$

$$\tan\alpha = \frac{P}{2\pi R}$$

$$= \frac{272.54}{2\pi \times 91.6}$$

$$\tan\alpha = \cancel{0.260}\ 0.473$$

$$n = \frac{d^4 \cos\alpha}{4 \times 64 \times (91.6)^3} \times \frac{1}{\frac{\cos^2\alpha}{G} + \frac{2\sin^2\alpha}{E}}$$

$$n = \frac{(10)^4 \times 0.904}{4 \times 64 \times (91.6)^3} \times \frac{1}{\frac{(0.904)^2}{80 \times 10^3} + \frac{2 \times (0.428)^2}{200 \times 10^3}}$$

$$n = 3.65$$

An open - coiled helical spring has 20 coils of wire of diameter 10 mm at a pitch of 80 mm, the coils having a mean radius of 120 mm. If the spring is subjected to an axial twist of 5 Nm, Find the maximum normal and shear stress in the section of the wire. if $E = 200$ GPa and $G = 80$ GPa. determine the axial Extension of the spring and the relative rotation b/w the ends. Find the strain energy stored in the spring.

Given data

dia of wire = 10 mm

pitch p = 80mm

mean coil radius = 120 mm

Axial twist M_0 = 5 N-m

Inclination of helical axis

$$\tan\alpha = \frac{P}{2\pi R}$$

$$\tan\alpha = \frac{80}{2\pi \times 120}$$

$$\alpha = 0.105$$

Twisting moment $M_t = M_0 \sin\alpha$

$$= 5000 \times \sin(0.105)$$

$$M_t = 524.03 \text{ N-mm}$$

Bending moment $M_b = M_0 \cos\alpha$

$$= 5000 \times \cos(0.105)$$

$$M_b = 4972.46$$

Shear stress due to M_t

$$\tau = \frac{16 M_t}{\pi d^3}$$

$$\tau = \frac{16 \times 524.03}{\pi (10)^3}$$

$$\tau = 2.668 \ N/mm^2$$

Normal stress due to M_b

$$\sigma = \frac{32 M_b}{\pi d^3}$$

$$\sigma = \frac{32 \times 4972.46}{\pi (10)^3}$$

$$= 50.64 \ N/mm^2$$

major principle stress

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \frac{50.64}{2} + \sqrt{\left[\frac{50.64}{2}\right]^2 + (2.668)^2}$$

$$\sigma_1 = 50.78 \ N/mm^2$$

maximum shear stress

$$\tau_{max} = \sqrt{\left[\frac{\sigma}{2}\right]^2 + \tau^2}$$

$$= 25.46 \ N/mm^2$$

Axial deflection $\delta = \dfrac{64 R^2 n \sin\alpha \, M_o}{d^4} \left[\dfrac{1}{G} - \dfrac{2}{E}\right]$

$$= \frac{64 \times (120)^2 \times 5000 \times 20 \times \sin(0.105)}{(10)^4} \left[\frac{1}{80 \times 10^3} - \frac{2}{200 \times 10^3}\right]$$

$$\delta = 2.41 \ mm$$

Relative rotation

$$\theta = \frac{64\, M_0\, R\, n\, \sec\alpha}{d^4}\left[\frac{\sin^2\alpha}{G} + \frac{2\cos^2\alpha}{E}\right]$$

$$= \frac{64 \times 5000 \times 120 \times 20 \times 1.005}{(10)^4}\left[\frac{(0.104)^2}{80 \times 10^3} + \frac{2 \times (0.994)^2}{200 \times 10^3}\right]$$

$$\theta = 0.773$$

Strain energy $U = \frac{1}{2} M_0\, \theta$

$$= \frac{1}{2} \times 5000 \times 0.773$$

$$U = 1932.5$$