UNIT-4

Push down Automata

Pushdown Automata (PDA):

Pushdown Automata is a finite automata with extra memory called Stack, which helps to recognize the context-free grammar like how finite automata helps to recognize the regular grammar. It is designed to remove the limitations on finite Automata.

Push down Automata (PDA) = Finite Automata + Stack.

Example:

$L=\{a^n b^n, n\geq 0\}$ is not recognized by finite automata due to the limitations of memory. But push down automata recognize it by in emerging number of a's and number of b's with the help of stack.

PDA is defined as seven tuple representation.

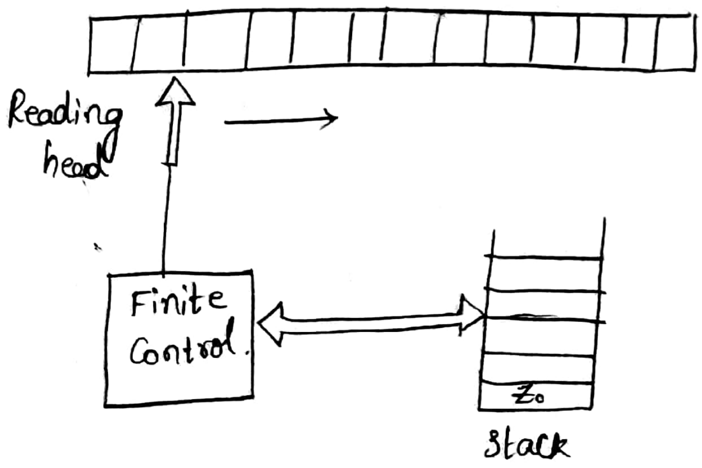
$M=\{Q,\Sigma,\Gamma,\delta,q_0,Z_0,F\}$

where
- Q is finite set of states
- Σ is finite set of input symbols
- Γ is finite set of stack symbols.
- δ is transitional function
- q_0 is initial state
- Z_0 is stack bottom symbol.
- F is final state.

Model:

PDA has four components:

Input tape, reading head, finite control and stack.



Input tape:

Input tape Consists of input symbols. Tape is divided into number of squares. Each square contains single input character. String is traversed from left to right.

Reading head:

Reading head scans each square from left to right in input tape, then reads input and send it to the finite Control of PDA.

Finite Control:

Finite Control is Considered as Control unit of PDA. A two way head is added with finite Control and stack. Depending on the input taken from the input tape and top of the stack, finite Control decides in which state of PDA will move and which stack symbol will push to the stack (or) pop from the stack (or) do nothing on the stack.

Stack:

Stack is a temporary storage of stack symbols. Stack symbol (Z_0) always denotes the bottom of the stack. Every move on PDA indicates one of the following operation to the stack.

1. one stack symbol may be added to the stack (PUSH).
2. one stack symbol may be deleted from the top of the stack (POP).

Graphical notation of PDA:

PDA is defined as

$$M = \{Q, \Sigma, F, \Gamma, \delta, q_0, Z_0\}$$

Transitional function δ Consists of three tuples:

* present state, present input and top of the stack which generates next state and stack symbols if symbol is pushed into the stack (or) ε if the top of the stack is pop.

Example:

$\delta(q_0, a, Z_0) \rightarrow (q_0, aZ_0)$

q_0 → Present state, a → present Input, Z_0 → Present top of the stack, q_0 → next state, aZ_0 → next top of the stack symbols.

In graphical notation of the PDA,

starting state is represented as $\rightarrow q_0$

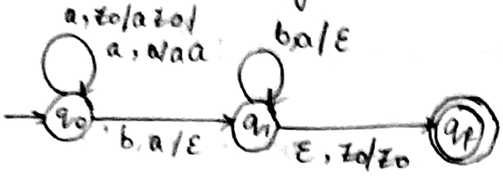
Final state is represented as q_f

Transition denoted by $\rightarrow$

Label of transition Consists of input Symbol, top of the Stack (present) and top of the Stack (next) which added after the transition (or) null Symbols if we pop the stack element.

Example:

Graphical Notation of given PDA is $L = \{a^n b^n, n \geq 0\}$



- Instantaneous Description:

Instantaneous description is called as an Informal Notation that explains how push down automata Computes the given input String and make decision that the given string is accepted (or) not.

Instantaneous Description (ID) is a triple (q, w, α), where q is Current State and $q \in Q$.

w is a remaining input and $w \in \Sigma$,

α is Contents of stack and $\alpha \in \Gamma$.

Turn stile ($\vdash$) Symbol is used for Connecting pairs of IDs that represent on (or) more moves of PDA.

Example:

Show the ID's of input string W = "aa bb" of given PDA.

$M = \{\{q_0, q_1, q_f\}, \{a, b\}, \{a, b, Z_0\}, \delta, q_0, Z_0, \{q_f\}\}$

Where δ is defined as

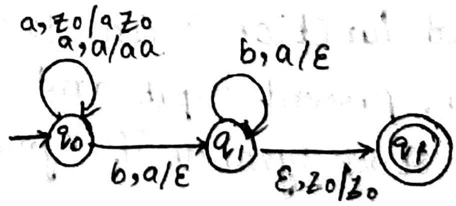
$\delta(q_0, a, Z_0) \rightarrow \{(q_0, aZ_0)\}$

$\delta(q_0, a, a) \rightarrow \{(q_0, aa)\}$

$\delta(q_0, b, a) \rightarrow \{(q_1, \varepsilon)\}$

$\delta(q_1, b, a) \rightarrow \{(q_1, \varepsilon)\}$

$\delta(q_1, \varepsilon, Z_0) \rightarrow \{(q_f, \varepsilon)\}$



Instantaneous Description for the string W is

ID $(q_0, aabb, Z_0) \vdash (q_0, abb, aZ_0)$

$\vdash (q_0, bb, aaZ_0)$

$\vdash (q_0, b, aZ_0)$

$\vdash (q_1, \varepsilon, Z_0)$

$\vdash (q_f, Z_0)$ (or) $\vdash (q_1, \varepsilon)$

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Acceptance of pushdown Automata:

There are two ways to declare that string is accepted by pushdown automata.

1. Accepted by empty stack.
2. Accepted by final state.

Accepted by empty stack:

Pushdown Automata accepts a string, after reading the entire string the stack is empty. It is Called Accepted by empty stack.

Example:

$W \in \Sigma^* \ (q_0, w, z_0) \vdash^* (q, \varepsilon, \varepsilon)$

Accepted by final state:

Pushdown Automata accepts a string, after reading the entire string PDA is in final state. It is called Accepted by final state

Example:

$W \in \Sigma^* \ (q_0, w, z_0) \vdash^* (q_f, \varepsilon, z_0)$

Deterministic pushdown Automata and Non-Deterministic push Down Automata:

Pushdown Automata are two types:

1. Deterministic pushdown Automata (DPDA)
2. Non-deterministic push Down Automata (NPDA)

Deterministic pushdown Automata (DPDA):

Pushdown automata is said to be deterministic pushdown automata if all the derivations in the design gives only single move.

If a pushdown automata being in a state with single input and single stack symbol gives a single move then that pushdown automata is called Deterministic pushdown automata.

DPDA can be represented as

$M_{DPDA} = \{Q, \Sigma, \Gamma, \delta, q_0, z_0, F\}$

where, δ is transitional function,

mapping $Q \times (\Sigma \cup \{\varepsilon\}) \times T \rightarrow (Q, T)$, where $|Q| = 1$

Example:

Construct PDA for the given language $L = \{wcw^R, w \in (a,b)^*\}$

$wcw^R, w \in (a,b)^*$

$a \rightarrow aca, abcba, aacaa$

$b \rightarrow bcb, bacab, bbcbb$

Transitions : (PUSH operation)

abbcbba

a → aa → ab

$\delta(q_0, a, z_0) \longrightarrow (q_0, a z_0)$

$\delta(q_0, a, a) \longrightarrow (q_0, aa)$

$\delta(q_0, b, a) \longrightarrow (q_0, ba)$

b → bb → ba

$\delta(q_0, b, z_0) \longrightarrow (q_0, b z_0)$

$\delta(q_0, b, b) \longrightarrow (q_0, bb)$

$\delta(q_0, a, b) \longrightarrow (q_0, ab)$

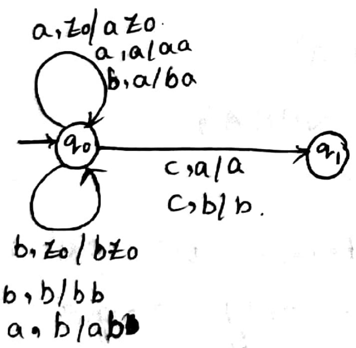
a → aca

b → bcb

Transitions : (when c is unknown) (skip operation then move to other state).

$(q_0, c, a) \longrightarrow (q_1, a)$

$(q_0, c, b) \longrightarrow (q_1, b)$

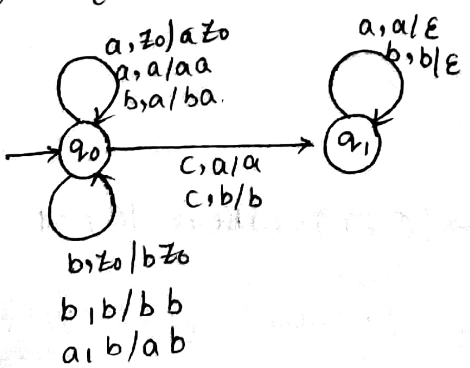


Transitions : (pop operation)

$\delta(q_1, a, a) \longrightarrow (q_1, \varepsilon)$

$\delta(q_1, b, b) \longrightarrow (q_1, \varepsilon)$

w = abcba



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String acceptance:

$|W| = abcba$

$ID = (q_0, abcba, Z_0) \vdash (q_0, bcba, aZ_0)$
$\vdash (q_0, cba, baZ_0)$
$\vdash (q_1, ba, baZ_0)$
$\vdash (q_1, a, aZ_0)$
$\vdash (q_1, \epsilon, Z_0)$

Construct PDA for the given language $L = \{a^n c b^n \mid n \geq 0\}$

Sol: Given $L = \{a^n c b^n\}$

$L = \{acb, aacbb, aaacbbb, aaaacbbbb \ldots\ldots\}$

Transitions: (PUSH operation)

a $\delta(q_0, a, Z_0) \rightarrow (q_0, aZ_0)$

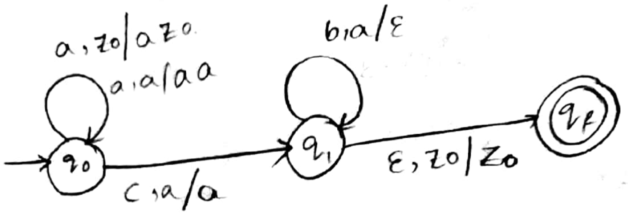
↓

aa $\delta(q_0, a, a) \rightarrow (q_0, aa)$

c $\delta(q_0, c, a) \rightarrow (q_1, a)$

b $\delta(q_1, b, a) \rightarrow (q_1, \epsilon)$

$\delta(q_1, \epsilon, Z_0) \rightarrow (q_f, Z_0)$



$|W| = aacbb$

$ID = (q_0, aacbb, Z_0) \vdash (q_0, acbb, aZ_0)$
$\vdash (q_0, cbb, aaZ_0)$
$\vdash (q_1, bb, aaZ_0)$
$\vdash (q_1, b, aZ_0)$
$\vdash (q_1, \epsilon, Z_0)$

3) Construct PDA for the given language $L=\{a^n b^n c^m d^m\ ;\ n\geq 1;\ m\geq 1\}$

Sol: $\delta(q_0, a, Z_1, Z_2) \longrightarrow (q_0, aZ_1, Z_2)$

$\delta(q_0, a, a, Z_2) \longrightarrow (q_0, aa, Z_2)$

$\delta(q_0, c, a, Z_2) \longrightarrow (q_0, aa, cZ_2)$

$\delta(q_0, c, a, c) \longrightarrow (q_0, aa, cc)$

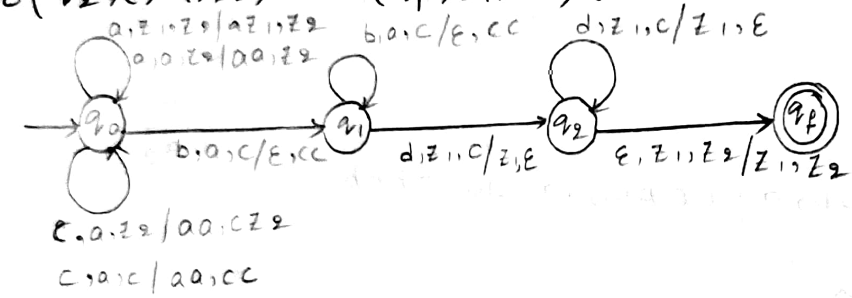
$\delta(q_0, b, a, c) \longrightarrow (q_1, \varepsilon, cc)$

$\delta(q_1, b, a, c) \longrightarrow (q_1, \varepsilon, cc)$

$\delta(q_1, d, Z_1, c) \longrightarrow (q_2, Z_1, \varepsilon)$

$\delta(q_2, d, Z_1, c) \longrightarrow (q_2, Z_1, \varepsilon)$

$\delta(q_2, \varepsilon, Z_1, Z_2) \longrightarrow (q_f, Z_1, Z_2)$ (or) $(q_2, \varepsilon, \varepsilon)$



4. Construct PDA for the given language $L=\{a^n b^m c^m d^n\ ;\ n\geq 1;\ m\geq 1\}$

Sol: $\delta(q_0, a, Z_1, Z_2) \longrightarrow (q_0, aZ_1, Z_2)$

$\delta(q_0, a, a, Z_2) \longrightarrow (q_0, aa, Z_2)$

$\delta(q_0, b, a, Z_2) \longrightarrow (q_0, aa, bZ_2)$

$\delta(q_0, b, a, b) \longrightarrow (q_0, aa, bb)$

$\delta(q_0, c, a, b) \longrightarrow (q_1, aa, \varepsilon)$

$\delta(q_1, c, a, b) \longrightarrow (q_1, aa, \varepsilon)$

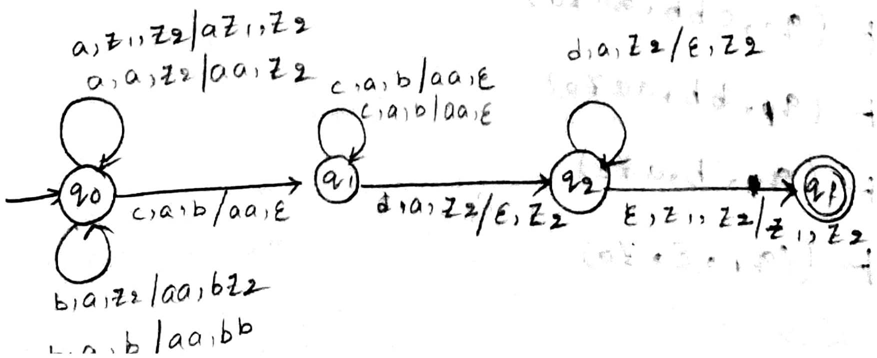
$\delta(q_1, d, a, Z_2) \longrightarrow (q_2, \varepsilon, Z_2)$

$\delta(q_2, d, a, Z_2) \longrightarrow (q_2, \varepsilon, Z_2)$

$\delta(q_2, \varepsilon, Z_1, Z_2) \longrightarrow (q_f, Z_1, Z_2)$ ~~(or) (q_2, ε, Z_2)~~

(or)

$(q_2, \varepsilon, \varepsilon)$



Nondeterministic pushdown automata:

A pushdown automata is said to be non-deterministic pushdown automata if one of the derivation generates more than one move. |

* If a pushdown automata being in a state with a single input & single stack symbol gives more than one move then the push down automata is called non- deterministic push down automata.

* NPDA can be represented as

$$M_{NPDA} = \{Q, \Sigma, \Gamma, \delta, q_0, z_0, F\}$$

Where δ is transition function mapping

$$Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \longrightarrow 2^Q \times T; \text{ where } |Q| = 1$$

Example:

1. Construct a PDA for given language $L = \{ww^R,\ w \in (a,b)^*\}$.

Sol:

$a = \{aa, abba, abaaba, abbbba, \ldots\ldots\}$

$b = \{bb, baab, babbab, baaaab, \ldots\ldots\}$

a — $\delta(q_0, a, z_0) \longrightarrow \delta(q_0, a z_0)$

aa $\quad \delta(q_0, a, a) \longrightarrow \delta(q_0, aa)$

ab $\quad \delta(q_0, b, a) \longrightarrow \delta(q_0, ba)$

pop $\quad \delta(q_0, a, a) \longrightarrow \delta(q_1, \epsilon)$

pop $\quad \delta(q_1, a, a) \longrightarrow (q_1, \epsilon)$

b $\quad \delta(q_0, b, z_0) \longrightarrow (q_0, b z_0)$

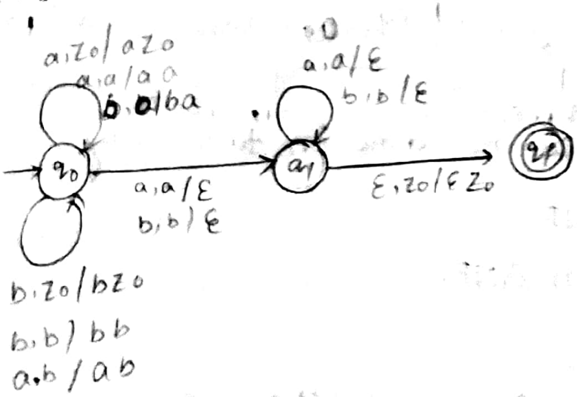
bb $\quad \delta(q_0, b, b) \longrightarrow (q_0, bb)$

ba $\quad \delta(q_0, a, b) \longrightarrow (q_0, ab)$

pop $\quad \delta(q_0, b, b) \longrightarrow (q_1, \epsilon)$

pop $\quad \delta(q_1, b, b) \longrightarrow (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) \longrightarrow (q_f, z_0)$ (or) (q_1, ϵ)



String acceptance:

$|w| = abba$

$ID = (q_0, abba, z_0) \vdash (q_0, bba, a z_0)$

$\vdash (q_0, ba, ba z_0)$

$\vdash (q_1, a, a z_0)$ (or) $\vdash (q_0, a, bba z_0)$

$\vdash (q_1, \epsilon, z_0)$ (or) $(q_1, \epsilon, aaz_0) \vdash (q_0, \epsilon, abba z_0)$ ✗

(q_f, z_0) (or) (q_1, ϵ) ✗

For odd palindrome:

$a = \{abbba, aba, aabaa, \ldots\}$
$b = \{bab, baaab, bbabb, \ldots\}$

a $\delta(q_0, a, z_0) \rightarrow (q_0, az_0)$ b $\delta(q_0, b, z_0) \rightarrow (q_0, bz_0)$

aa $\delta(q_0, a, a) \rightarrow (q_0, aa)$ ba $\delta(q_0, a, b) \rightarrow (q_0, ab)$

ab $\delta(q_0, b, a) \rightarrow (q_0, ba)$ bb $\delta(q_0, b, b) \rightarrow (q_0, bb)$

$\delta(q_0, a, b) \rightarrow (q_1, b)$

$\delta(q_0, b, b) \rightarrow (q_1, b)$

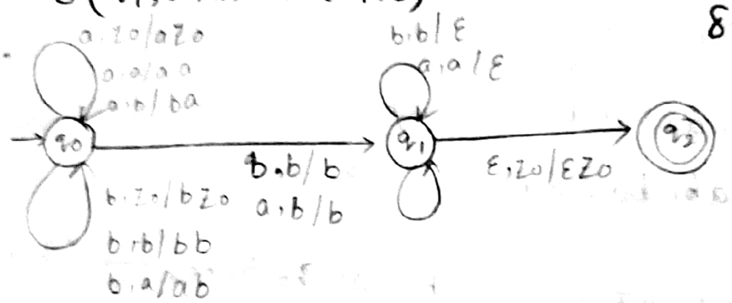
$\delta(q_1, a, a) \rightarrow (q_1, \varepsilon)$

$\delta(q_1, b, b) \rightarrow (q_1, \varepsilon)$

$\delta(q_1, b, b) \rightarrow (q_1, \varepsilon)$

$\delta(q_1, a, a) \rightarrow (q_1, \varepsilon)$

$\delta(q_1, \varepsilon, z_0) \rightarrow (q_f, z_0)$
(q_1, ε)



String acceptance $w = abbba$.

ID $\vdash (q_0, abbba, z_0)$
$\vdash (q_0, bbba, az_0)$
$\vdash (q_0, bba, baz_0)$
$\vdash (q_1, ba, baz_0)$ (or) $(q_0, ba, bbaz_0)$
$\vdash (q_1, a, az_0)$ (or) $(q_1, a, bbaz_0)$ | $(q_0, a, bbbaz_0)$ | (q_1, a, baz_0)
$\vdash (q_1, \varepsilon, z_0)$ (or) $(q_1, \varepsilon, aaz_0)$ | $(q_1, \varepsilon, abbbaz_0)$ | $(q_1, \varepsilon, baz_0)$
(q_f, z_0) (q_1, ε)

Conversion of CFG to PDA:

Conversion of CFG to PDA can be done in two ways.

1. If the given grammar is in GNF
2. If the given grammar is not in GNF.

If the given grammar is in GNF:

If the given grammar is in GNF then conversion of CFG to PDA

Procedure:

1. If the given CFG is in GNF then follow the steps. If it is not in GNF convert it into GNF then follow the steps
2. For Starting symbol 'S' of CFG is put on the stack by transition function

$$\delta(q_0, \varepsilon, z_0) \rightarrow (q_1, Sz_0)$$

3. For production in the form $A \rightarrow aB$ the transition function is

$$\delta(q_1, a, A) \rightarrow (q_1, B)$$

4. For production in the form $A \rightarrow a$ then the transition function is

$$\delta(q_1, a, A) \rightarrow (q_1, \epsilon)$$

5. For accepting a string two transitional functions are added.

$\delta(q_1, \epsilon, z_0) \rightarrow (q_f, z_0) \rightarrow$ final state acceptance

$(q_1, \epsilon) \rightarrow$ Empty stack acceptance.

1. Convert equivalent PDA for the following context free grammar and check the acceptance of string abbaadbbbbab.

Sol: $S \rightarrow aAB/bBA$

$A \rightarrow bS/a$

$B \rightarrow aS/b$

Sol: Given $S \rightarrow aAB$

$S \rightarrow bBA$

$A \rightarrow bS$

$A \rightarrow a$

$B \rightarrow aS$

$B \rightarrow b$

$$\delta(q_0, \epsilon, z_0) \rightarrow (q_1, Sz_0)$$

For production $S \rightarrow aAB$, the transition function is

$$\delta(q_1, a, S) \rightarrow (q_1, AB)$$

For production $S \rightarrow bBA$, the transition function is

$$\delta(q_1, b, S) \rightarrow (q_1, BA)$$

For production $A \rightarrow bS$, the transition function is

$$\delta(q_1, b, A) \rightarrow (q_1, S)$$

For production $A \rightarrow a$, the transition function is

$$\delta(q_1, a, A) \rightarrow (q_1, \epsilon)$$

For production $B \rightarrow aS$, the transition function is

$$\delta(q_1, a, B) \rightarrow (q_1, S)$$

For production $B \rightarrow b$ the transition function is

$$\delta(q_1, b, B) \rightarrow (q_1, \epsilon)$$

For string acceptance adding two transitional functions

$$\delta(q_1, \epsilon, z_0) \rightarrow (q_f, z_0) \text{ (or) } (q_1, \epsilon)$$

String acceptance of $W = abbaaabbbbab$.

$\delta(q_0, \varepsilon, Z_0) \rightarrow (q_1, S Z_0)$	$S \mid Z_0$
$ID = \delta(q_1, abbaaabbbbab, S) \longrightarrow$	
$\vdash (q_1, bbaaabbbbab, AB)$	$A \mid B \mid Z_0$
$\vdash (q_1, baaabbbbab, S)$	$S \mid B \mid Z_0$
$\vdash (q_1, aaabbbbab, BA)$	$B \mid \cancel{B} \mid B \mid Z_0$
$\vdash (q_1, aabbbbab, S)$	$S \mid A \mid B \mid Z_0$
$\vdash (q_1, abbbbab, AB)$	$\cancel{A} \mid B \mid A \mid B \mid Z_0$
$\vdash (q_1, bbbbab, B)$ (Top of the stack)	$\cancel{B} \mid A \mid B \mid Z_0$
$\vdash (q_1, bbbab, A)$	$A \mid B \mid Z_0$
$\vdash (q_1, bbab, S)$	$S \mid B \mid Z_0$
$\vdash (q_1, bab, BA)$	$\cancel{B} \mid A \mid B \mid Z_0$
$\vdash (q_1, ab, A)$	$\cancel{A} \mid B \mid Z_0$
$\vdash (q_1, b, B)$	$\cancel{B} \mid Z_0$
$\vdash (q_1, \varepsilon, Z_0)$	Z_0

(q_f, Z_0) $\quad$ (q_1, ε)

2. Convert PDA for the following context free grammar and check the acceptance of String 010000

$S \rightarrow 0BB$

$B \rightarrow 0S / 1S / 0$

Sol: $S \longrightarrow 0BB$

$B \longrightarrow 0S$

$B \longrightarrow 1S$

$B \longrightarrow 0$

$\delta(q_0, \varepsilon, Z_0) \longrightarrow (q_1, S Z_0)$

For production $S \rightarrow 0BB$, the transition function is.

$\delta(q_1, 0, S) \rightarrow (q_1, BB)$

For production $B \rightarrow 0S$, the transition function is.

$\delta(q_1, 0, B) \longrightarrow (q_1, S)$

For production $B \rightarrow 1S$, the transition function is

$\delta(q_1, 1, B) \rightarrow (q_1, S)$

For production $B \rightarrow 0$ the transition function is

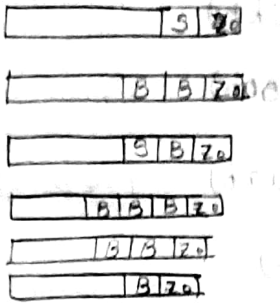
$$\delta(q_1, 0, B) \longrightarrow (q_1, \varepsilon)$$

For string ~~ap~~ acceptance adding two transitional functions,

$$\delta(q_1, \varepsilon, Z_0) \longrightarrow (q_f, Z_0) \text{ (or) } (q_1, \varepsilon)$$

String acceptance $W = 010000$

$$\begin{aligned}
ID &\vdash \delta(q_1, 010000, S) \\
&\vdash (q_1, 10000, BB) \\
&\vdash (q_1, 0000, S) \\
&\vdash (q_1, 000, BB) \\
&\vdash (q_1, 00, BB) \\
&\vdash (q_1, 0, B) \\
&\vdash (q_1, \varepsilon, Z_0) \left\langle \begin{matrix} (q_f, Z_0) \\ (q_1, \varepsilon) \end{matrix} \right.
\end{aligned}$$



If the given grammar is not in GNF:

If the grammar is not in GNF then follow the steps.

1. If the grammar is not in GNF then follow the steps.
2. The starting symbol 'S' of CFG is put into stack by transition function
   $$\delta(q_0, \varepsilon, Z_0) \longrightarrow (q_1, S Z_0)$$
3. For the production in the form $S \rightarrow aAb$ the transition function as
   $$\delta(q_1, \varepsilon, S) \longrightarrow (q_1, aAb)$$
4. For each terminal in CFG, the transition function is
   $$\delta(q_1, a, a) \rightarrow (q_1, \varepsilon)$$
5. For accepting a string two transitional functions are added
   $$\delta(q_1, \varepsilon, Z_0) \longrightarrow \delta(q_f, Z_0)$$
   $$q_1, \varepsilon$$

• Construct equivalent PDA for the following CFG

$S \longrightarrow aSbb / abb$. check the acceptance of string aaabbb

Sol: $S \rightarrow aSbb$

$S \rightarrow abb$ (not in GNF)

$\delta(q_0, \varepsilon, Z_0) \longrightarrow (q_1, S Z_0)$

For production $S \rightarrow aSbb$, the transition function is

$$\delta(q_1, \varepsilon, S) \longrightarrow (q_1, aSbb)$$

For production $S \rightarrow abb$, the transition function is

$$\delta(q_1, \varepsilon, S) \longrightarrow (q_1, abb)$$

For each terminal, transition function is

$$\delta(q_1, a, a) \rightarrow (q_1, \varepsilon)$$

$$\delta(q_1, b, b) \rightarrow (q_1, \varepsilon)$$

$$(q_1, \varepsilon, Z_0) \longrightarrow (q_f, Z_0)$$

$$\longrightarrow (q_1, \varepsilon)$$

String acceptance $|W| = aaabbb$

$ID = \delta(q_1, \varepsilon aaabbb, S)$

$\vdash (q_1, aaabbb, aSbb)$

$\vdash (q_1, aabbb, Sbb)$

$\vdash (q_1, \varepsilon aabbb, aab)$

$\vdash (q_1, aabbb, aab)$

$\vdash (q_1, abbb, ab)$

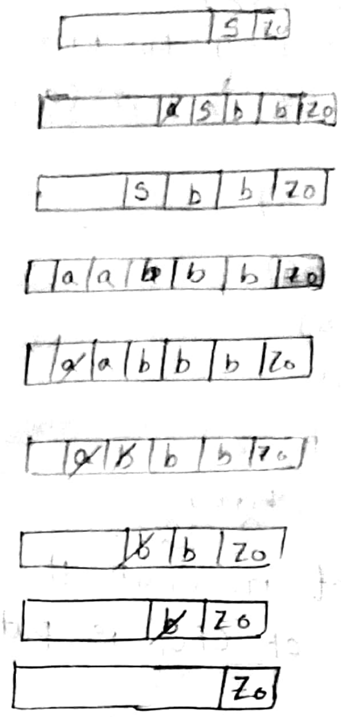
$\vdash (q_1, bbb, bb)$

$\vdash (q_1, bb, bb)$

$\vdash (q_1, b, b)$

$\vdash (q_1, \varepsilon, z_0)$

$(q_f, z_0) \quad (q_1, \varepsilon)$



Conversion of PDA to CFG:

consider a PDA $M = \{Q, \Sigma, \Gamma, \delta, q_0, z_0, F\}$ which accepts a language L. A CFG, $G = (V, \Sigma, P, S)$, equivalent to M. can be constructed using following procedure.

Procedure:

1. For non terminals,

$[q, x, q]$ where q is states in Q

x is in stack symbol in Γ.

2. For productions

$S \rightarrow [q_0, z_0, q]$

where q is a states in Q.

3. A PDA production of the form

$$\delta(q, a, x) \rightarrow (\gamma, Y_1 Y_2)$$

state, i/p, Top of stack, next state, Stack symbol.

4. PDA of the ~~form~~ then productions are

$$[q, X, r_k] \rightarrow a[\gamma, Y_1, r_1]\,[r_1, Y_2, r_k]$$

Example:

$$[q, X, q_0] \rightarrow a[r, Y_1, {}^{q_0}_{q_1}{}_{q_2}]\,[{}^{q_0}_{q_1}{}_{q_2}, Y_2, q_0]$$

$$[q, X, q_1] \rightarrow a[r, Y_1, {}^{q_0}_{q_1}{}_{q_2}]\,[{}^{q_0}_{q_1}{}_{q_2}, Y_2, q_1]$$

$[q, X, q_2] \longrightarrow a[r, Y_1, q_0][q_?, Y_2, q_2]$

4. A PDA of the form $[q, a, X] \rightarrow (r, \varepsilon)$, [POP operation] production are.

$[q, X, r] \rightarrow a$

ii $\delta(q, \varepsilon, X) \longrightarrow (r, \varepsilon)$

$\delta(q, X, r) \longrightarrow \varepsilon$

5. A PDA of the form $\delta(q, a, X) \rightarrow (r, X)$ (skip operation) productions are

$[q, X, r_k] \longrightarrow a[r, X, r_k]$

all states in Q.

6. Then replace all productions non-terminals into Capital Alphabets.

7. Finally we got CFG, (in that eliminate useless symbols) then Simplify the CFG.

1. Convert the following PDA into equivalent CFG. $L = \{a^n b^{n+m} c^m ; m, n \geq 1\}$

$\delta(q_0, a, Z_0) \rightarrow (q_0, aZ_0)$

$\delta(q_0, a, a) \longrightarrow (q_0, aa)$

$\delta(q_0, b, a) \longrightarrow (q_1, \varepsilon)$

$\delta(q_1, b, a) \longrightarrow (q_1, \varepsilon)$

$\delta(q_1, b, Z_0) \rightarrow (q_1, bZ_0)$

$\delta(q_1, b, b) \rightarrow (q_1, bb)$

$\delta(q_1, c, b) \rightarrow (q_2, \varepsilon)$

$\delta(q_2, c, b) \rightarrow (q_2, \varepsilon)$

$\delta(q_2, \varepsilon, Z_0) \rightarrow (q_2, \varepsilon)$

Sol: $M = \{(q_0, q_1, q_2), \{a, b, c\}, \{Z_0, a, b\}, \delta, q_0, Z_0, q_2\}$.

For non terminals

$\underbrace{[q_0, Z_0, q_0]}_{A}, \underbrace{[q_0, Z_0, q_1]}_{B}, \underbrace{[q_0, Z_0, q_2]}_{C}, \underbrace{[q_1, Z_0, q_0]}_{D}, \underbrace{[q_1, Z_0, q_1]}_{E}, \underbrace{[q_1, Z_0, q_2]}_{F}$

$\underbrace{[q_2, Z_0, q_0]}_{G}, \underbrace{[q_2, Z_0, q_1]}_{H}, \underbrace{[q_2, Z_0, q_2]}_{I}, \underbrace{[q_0, a, q_0]}_{J}, \underbrace{[q_0, a, q_1]}_{K}, \underbrace{[q_0, a, q_2]}_{L}$

$\underbrace{[q_1, a, q_0]}_{M}, \underbrace{[q_1, a, q_1]}_{N}, \underbrace{[q_1, a, q_2]}_{O}, \underbrace{[q_2, a, q_0]}_{P}, \underbrace{[q_2, a, q_1]}_{Q}, \underbrace{[q_2, a, q_2]}_{R}$

$\underbrace{[q_0, b, q_0]}_{S}, \underbrace{[q_0, b, q_1]}_{U}, \underbrace{[q_0, b, q_2]}_{V}, \underbrace{[q_1, b, q_0]}_{W}, \underbrace{[q_1, b, q_1]}_{X}, \underbrace{[q_1, b, q_2]}_{Y}$

$\underbrace{[q_2, b, q_0]}_{Z}, \underbrace{[q_2, b, q_1]}_{A'}, \underbrace{[q_2, b, q_2]}_{B'}$.

For productions

$S \longrightarrow [q_0, Z_0, q_0] \mid [q_0, Z_0, q_1] \mid [q_0, Z_0, q_2]$

Consider a PDA in the form

$\delta(q_0, b, a) \rightarrow (q_1, \varepsilon)$

$[q_0, a, q_1] \rightarrow b$

$\delta(q_1, b, a) \rightarrow (q_1, \varepsilon)$

$[q_1, a, q_1] \rightarrow b$

$\delta(q_1, c, b) \rightarrow (q_2, \epsilon)$

$[q_1, b, q_2] \rightarrow c$

$\delta(q_2, c, b) \rightarrow (q_2, \epsilon)$

$[q_2, b, q_2] \rightarrow c$

$\delta(q_2, \epsilon, Z_0) \rightarrow (q_2, \epsilon)$

$[q_2, Z_0, q_2] \rightarrow \epsilon.$

Consider PDA of the form

$\delta(q_0, a, Z_0) \longrightarrow (q_0, aZ_0)$ then productions are.

$[q_0, Z_0, q_0] \longrightarrow a[q_0, a, q_0][q_0, Z_0, q_0]$

$[q_0, Z_0, q_0] \longrightarrow a[q_0, a, q_1][q_1, Z_0, q_0]$

$[q_0, Z_0, q_0] \longrightarrow a[q_0, a, q_2][q_2, Z_0, q_0]$

$[q_0, Z_0, q_1] \longrightarrow a[q_0, a, q_0][q_0, Z_0, q_1]$

$[q_0, Z_0, q_1] \longrightarrow a[q_0, a, q_1][q_1, Z_0, q_1]$

$[q_0, Z_0, q_1] \longrightarrow a[q_0, a, q_2][q_2, Z_0, q_1]$

$[q_0, Z_0, q_2] \longrightarrow a[q_0, a, q_0][q_0, Z_0, q_2]$

$[q_0, Z_0, q_2] \longrightarrow a[q_0, a, q_1][q_1, Z_0, q_2]$

$[q_0, Z_0, q_2] \longrightarrow a[q_0, a, q_2][q_2, Z_0, q_2]$

PDA of the form

$\delta(q_0, a, a) \longrightarrow (q_0, aa)$

$[q_0, \underset{J}{a}, q_0] \rightarrow a[q_0, \underset{J}{a}, q_0][q_0, \underset{J}{a}, q_0]$

$[q_0, \underset{J}{a}, q_0] \rightarrow a[q_0, \underset{K}{a}, q_1][q_1, \underset{M}{a}, q_0]$

$[q_0, \underset{J}{a}, q_0] \rightarrow a[q_0, \underset{L}{a}, q_2][q_2, \underset{P}{a}, q_0]$

$[q_0, \underset{K}{a}, q_1] \rightarrow a[q_0, \underset{J}{a}, q_0][q_0, \underset{K}{a}, q_1]$

$[q_0, \underset{K}{a}, q_1] \rightarrow a[q_0, \underset{K}{a}, q_1][q_1, \underset{N}{a}, q_1]$

$[q_0, \underset{K}{a}, q_1] \rightarrow a[q_1, \underset{L}{a}, q_2][q_2, \underset{Q}{a}, q_1]$

$[q_0, \underset{L}{a}, q_2] \rightarrow a[q_0, \underset{J}{a}, q_0][q_0, \underset{L}{a}, q_2]$

$[q_0, \underset{L}{a}, q_2] \rightarrow a[q_0, \underset{K}{a}, q_1][q_1, \underset{O}{a}, q_2]$

$[q_0, \underset{L}{a}, q_2] \rightarrow a[q_0, \underset{L}{a}, q_2][q_2, \underset{R}{a}, q_2]$

PDA of the form

$\delta(q_1, b, Z_0) \longrightarrow (q_1, bZ_0)$

$\underset{D}{[q_1, Z_0, q_0]} \longrightarrow a\underset{W}{[q_1, b, q_0]}\underset{A}{[q_0, Z_0, q_0]}$

$\underset{D}{[q_1, Z_0, q_0]} \longrightarrow a\underset{X}{[q_1, b, q_1]}\underset{D}{[q_1, Z_0, q_0]}$

$\underset{D}{[q_1, Z_0, q_0]} \longrightarrow a\underset{Y}{[q_1, b, q_2]}\underset{G}{[q_2, Z_0, q_0]}$

$\underset{E}{[q_1, Z_0, q_1]} \longrightarrow a\underset{W}{[q_1, b, q_0]}\underset{B}{[q_0, Z_0, q_1]}$

$\underset{E}{[q_1, Z_0, q_1]} \longrightarrow a\underset{X}{[q_1, b, q_1]}\underset{E}{[q_1, Z_0, q_1]}$

$\underset{E}{[q_1, Z_0, q_1]} \longrightarrow a\underset{Y}{[q_1, b, q_2]}\underset{H}{[q_2, Z_0, q_1]}$

$\underset{F}{[q_1, Z_0, q_2]} \longrightarrow a\underset{W}{[q_1, b, q_0]}\underset{C}{[q_0, Z_0, q_2]}$

$\underset{F}{[q_1, Z_0, q_2]} \longrightarrow a\underset{X}{[q_1, b, q_1]}\underset{F}{[q_1, Z_0, q_2]}$

$\underset{F}{[q_1, Z_0, q_2]} \longrightarrow a\underset{Y}{[q_1, b, q_2]}\underset{I}{[q_2, Z_0, q_2]}$

PDA of the form

$\delta(q_1, b, b) \longrightarrow (q_1, bb)$

$\underset{W}{[q_1, b, q_0]} \longrightarrow a\underset{W}{[q_1, b, q_0]}\underset{T}{[q_0, b, q_0]}$

$\underset{W}{[q_1, b, q_0]} \longrightarrow a\underset{X}{[q_1, b, q_1]}\underset{W}{[q_1, b, q_0]}$

$\underset{W}{[q_1, b, q_0]} \longrightarrow a\underset{Y}{[q_1, b, q_2]}\underset{Z}{[q_2, b, q_0]}$

$\underset{X}{[q_1, b, q_1]} \longrightarrow a\underset{W}{[q_1, b, q_0]}\underset{U}{[q_0, b, q_1]}$

$\underset{X}{[q_1, b, q_1]} \longrightarrow a\underset{X}{[q_1, b, q_1]}\underset{X}{[q_1, b, q_1]}$

$\underset{X}{[q_1, b, q_1]} \longrightarrow a\underset{Y}{[q_1, b, q_2]}\underset{A'}{[q_2, b, q_1]}$

$\underset{Y}{[q_1, b, q_2]} \longrightarrow a\underset{W}{[q_1, b, q_0]}\underset{V}{[q_0, b, q_2]}$

$\underset{Y}{[q_1, b, q_2]} \longrightarrow a\underset{X}{[q_1, b, q_1]}\underset{Y}{[q_1, b, q_2]}$

$\underset{Y}{[q_1, b, q_2]} \longrightarrow a\underset{Y}{[q_1, b, q_2]}\underset{B'}{[q_2, b, q_2]}$

Now

~~$A \longrightarrow aJA$~~

~~$A \longrightarrow aKD$~~

$S \longrightarrow A|B|C$

$K \longrightarrow b$

$N \longrightarrow b$

$Y \longrightarrow c$

$B' \longrightarrow c$

$I \longrightarrow \varepsilon$

$A \longrightarrow aJA$

$A \longrightarrow aKD$

$A \longrightarrow aLG$

$B \longrightarrow aJB$

$B \longrightarrow aKF$

$B \longrightarrow aLH$

$C \longrightarrow aJC / aKF / aLI$

{a, b, c, K, N, Y, B'

$J \rightarrow aJJ / aKM / aLP$

$K \rightarrow aJK / aKN / aLQ$

$L \rightarrow aJL / aKO / aLR$

$D \rightarrow aWA / axD / aYG$

$E \rightarrow aWB / axE / aYH$

$F \rightarrow aWC / axF / aYI$

$W \rightarrow aWT / axW / aYZ$

$X \rightarrow aWU / axX / aYA'$

$Y \rightarrow aWV / axY / aYB'$

Q. Convert the following PDA into CFG

$M = \{\{P,Q\}, \{0,1\}, \{X,Z\}, \delta, q_0, Z\}$

$\delta(q_0, 1, Z) \rightarrow (q_0, XZ)$ push

$\delta(q, 1, X) \rightarrow (q, XX)$ push

$\delta(q, \varepsilon, X) \rightarrow (q, \varepsilon)$ pop

$\delta(q, 0, X) \rightarrow (P, X)$ skip

$\delta(P, 1, X) \rightarrow (P, \varepsilon)$ pop

$\delta(P, 0, Z) \rightarrow (q, Z)$ skip

2). Convert the following PDA into CFG.

$\delta(q_0, a, Z_0) \rightarrow (q_1, Z_1 Z_0)$ $\quad L = ww^R$

$\delta(q_0, b, Z_0) \rightarrow (q_1, Z_2 Z_0)$

$\delta(q_1, a, Z_1) \rightarrow (q_1, Z_1 Z_1)$ $\rightarrow$

$\delta(q_1, b, Z_1) \rightarrow (q_1, \lambda)$

$\delta(q_1, b, Z_2) \rightarrow (q_1, Z_2 Z_2)$

$\delta(q_1, a, Z_2) \rightarrow (q_1, \lambda)$

$\delta(q_1, \lambda, Z_0) \rightarrow (q_1, \lambda)$ // accepted by empty stack

Sol: $M = \{\{q_0, q_1\}, \{a, b\}, \{Z_0, Z_1, Z_2\}, \delta, q_0, Z_0, q_1\}$

For non terminals

$\underbrace{[q_0, Z_0, q_0]}_{A}, \underbrace{[q_0, Z_0, q_1]}_{B}, \underbrace{[q_1, Z_0, q_0]}_{C}, \underbrace{[q_1, Z_0, q_1]}_{D}, \underbrace{[q_0, Z_1, q_0]}_{E}, \underbrace{[q_0, Z_1, q_1]}_{F}$

$\underbrace{[q_1, Z_1, q_0]}_{G}, \underbrace{[q_1, Z_1, q_1]}_{H}, \underbrace{[q_0, Z_2, q_0]}_{I}, \underbrace{[q_0, Z_2, q_1]}_{J}, \underbrace{[q_1, Z_2, q_0]}_{K}, \underbrace{[q_1, Z_2, q_1]}_{L}$

For productions

$S \rightarrow [q_0, Z_0, q_0] \mid [q_0, Z_0, q_1]$

Consider PDA of the

$\delta(q_1, b, Z_1) \rightarrow (q_1, \varepsilon)$

$[q_1, Z_1, q_1] \rightarrow b$

$\delta(q_1, a, Z_2) \rightarrow (q_1, \varepsilon)$

$[q_1, Z_2, q_1] \rightarrow a$

$\delta(q_1, \varepsilon, Z_0) \rightarrow (q_1, \varepsilon)$

$[q_1, Z_0, q_1] \rightarrow \varepsilon$.

Consider PDA of the form

$$\delta(q_0, a, Z_0) \longrightarrow (q_1, Z_1 Z_0)$$

$$\underset{A}{[q_0, Z_0, q_0]} \longrightarrow a\,\underset{G}{[q_1, Z_1, q_0]}\,\underset{A}{[q_0, Z_0, q_0]}$$

$$\underset{A}{[q_0, Z_0, q_0]} \longrightarrow a\,\underset{H}{[q_1, Z_1, q_1]}\,\underset{C}{[q_1, Z_0, q_0]}$$

$$[q_0, Z_0, q_1] \longrightarrow a\,[q_1, Z_1, q_0]\,\underset{B}{[q_0, Z_0, q_1]}$$

$$\underset{B}{[q_0, Z_0, q_1]} \longrightarrow a\,\underset{H}{[q_1, Z_1, q_1]}\,\underset{D}{[q_1, Z_0, q_1]}$$

Consider PDA of the form

$$\rightarrow \delta(q_0, b, Z_0) \longrightarrow (q_1, Z_2 Z_0)$$

$$\underset{A}{[q_0, Z_0, q_0]} \longrightarrow b\,\underset{K}{[q_1, Z_2, q_0]}\,\underset{A}{[q_0, Z_0, q_0]}$$

$$\underset{A}{[q_0, Z_0, q_0]} \longrightarrow b\,\underset{L}{[q_1, Z_2, q_1]}\,\underset{C}{[q_1, Z_0, q_0]}$$

$$\underset{B}{[q_0, Z_0, q_1]} \longrightarrow b\,\underset{K}{[q_1, Z_2, q_0]}\,\underset{B}{[q_0, Z_0, q_1]}$$

$$\underset{B}{[q_0, Z_0, q_1]} \longrightarrow b\,\underset{L}{[q_1, Z_2, q_1]}\,\underset{D}{[q_1, Z_0, q_1]}$$

$$\longrightarrow \delta(q_1, a, Z_1) \longrightarrow (q_1, Z_1 Z_1)$$

$$\underset{G}{[q_1, Z_1, q_0]} \longrightarrow a\,\underset{G}{[q_1, Z_1, q_0]}\,\underset{E}{[q_0, Z_1, q_0]}$$

$$\underset{G}{[q_1, Z_1, q_0]} \longrightarrow a\,\underset{H}{[q_1, Z_1, q_1]}\,\underset{G}{[q_1, Z_1, q_0]}$$

$$\underset{H}{[q_1, Z_1, q_1]} \longrightarrow a\,\underset{G}{[q_1, Z_1, q_0]}\,\underset{F}{[q_0, Z_1, q_1]}$$

$$\underset{H}{[q_1, Z_1, q_1]} \longrightarrow a\,\underset{H}{[q_1, Z_1, q_1]}\,\underset{H}{[q_1, Z_1, q_1]}$$

$$\longrightarrow \delta(q_1, b, Z_2) \longrightarrow (q_1, Z_2 Z_2)$$

$$\underset{K}{[q_1, Z_2, q_0]} \longrightarrow b\,\underset{K}{[q_1, Z_2, q_0]}\,\underset{I}{[q_0, Z_2, q_0]}$$

$$\underset{K}{[q_1, Z_2, q_0]} \longrightarrow b\,\underset{L}{[q_1, Z_2, q_1]}\,\underset{K}{[q_1, Z_2, q_0]}$$

$$\underset{L}{[q_1, Z_2, q_1]} \longrightarrow b\,\underset{K}{[q_1, Z_2, q_0]}\,\underset{J}{[q_0, Z_2, q_1]}$$

$$\underset{L}{[q_1, Z_2, q_1]} \longrightarrow b\,\underset{L}{[q_1, Z_2, q_1]}\,\underset{L}{[q_1, Z_2, q_1]}$$

Now

{H, L, D,

$S \longrightarrow A / B$

$H \longrightarrow b$

$L \longrightarrow a$

$D \longrightarrow \varepsilon$

$A \longrightarrow aGA / aHC / bKA / bLC$

$B \longrightarrow aGB / aHD / bKB / bLD$

$G \rightarrow aGE / aHG$

$H \rightarrow aGF / aHH.$

$K \rightarrow bKI / bLK$

$L \rightarrow bKJ / bLL$

Given $M = \{\{p, q\}, \{0, 1\}, \{x, Z\}, \delta, q, Z\}$ (q_1, q_0; Z_1, Z_0)

1: For non terminals

$[q_0, z_0, q_0]$ $[q_0, z_0, q_1]$ $[q_0, z_0, q_0]$ $[q_1, z_0, q_1]$

$[q_1, z_1, q_0]$ $[q_0, z_1, q_1]$ $[q_1, z_1, q_0]$ $[q_1, z_1, q_1]$

~~[p, x, p]~~ . ~~[p, x, q]~~

~~[q, z, q]~~ [q, z, p] ~~[p, x, p]~~ [p, Z, q]

$\underbrace{[q, Z, q]}_{A}, \underbrace{[q, Z, p]}_{B}, \underbrace{[p, Z, q]}_{C}, \underbrace{[p, Z, p]}_{D}, \underbrace{[q, x, q]}_{E}, \underbrace{[q, x, p]}_{F}, \underbrace{[p, x, q]}_{G}, \underbrace{[p, x, p]}_{H}$

For production

$S \longrightarrow [q, Z, q] / [q, Z, p]$

Consider PDA of the form

$\delta(p, 1, x) \longrightarrow (p, \varepsilon)$

$(p, x, p) \longrightarrow 1$

$\delta(q, \varepsilon, x) \longrightarrow (q, \varepsilon)$

$(q, x, q) \longrightarrow \varepsilon$

Consider PDA of the form

$\delta(q, 1, Z) \longrightarrow (q, xZ)$

$[q, Z, q] \longrightarrow 1[q, x, q][q, Z, q]$

$[q, Z, q] \longrightarrow 1[q, x, p][p, Z, q]$

$[q, z, p] \rightarrow 1[q, x, q][q, z, p]$

$[q, z, p] \rightarrow 1[q, x, p][p, z, p]$

For production

$\delta(q, 1, x) \rightarrow (q, xx)$

$[q, x, q] \rightarrow 1[q, x, q][q, x, q]$

$[q, x, q] \rightarrow 1[q, x, p][p, x, q]$

$[q, x, p] \rightarrow 1[q, x, q][q, x, p]$

$[q, x, p] \rightarrow 1[q, x, p][p, x, p]$

Consider PDA of the form

$\delta(q, 0, x) \rightarrow (p, x)$

$[q, x, q] \rightarrow 0[p, x, q]$

$[q, x, p] \rightarrow 0[p, x, p]$

Consider PDA of the form

$\delta(p, 0, z) \rightarrow (q, z)$

$[p, z, q] \rightarrow 0[q, z, q]$

$[p, z, p] \rightarrow 0[q, z, p]$

$S \rightarrow A / B$

$H \rightarrow 1$

$G \rightarrow \epsilon / 0G$

$A \rightarrow 1EA / 1FC$

$B \rightarrow 1EB / 1FD$

$E \rightarrow 1EE / 1FG$

$F \rightarrow 1EF / 1FH / 0H$

$C \rightarrow 0A$

$D \rightarrow 0B$

Two Stack PDA:

Finite automata recognizes regular languages such as $\{a^n / n \geq 1\}$

Adding one stack to finite automata it become PDA recognizes Context free languages $\{a^n b^n / n \geq 1\}$

In case of Context sensitive language such as $\{a^n b^n c^n / n \geq 1\}$ the PDA is help less because it has only one stack memory. To overcome this two stack PDA is introduced, Not only two stack more than two stacks are also added to the PDA if required.

Two stack PDA define as 9 tuples P.

$$M = \{Q, \Sigma, \Gamma, \Gamma', \delta, q_0, z_1, z_2, F\}$$

Where

Q is finite set of states

Σ is finite set of input symbols

Γ is finite set of stack one symbol.

Γ' is finite set of stack two symbol.

δ is transition function.

q_0 is starting symbol

z_1 is bottom of the stack one

z_2 is bottom of the stack two

F is final state

Transition function δ is in the form

$$Q(\Sigma \cup \{\varepsilon\}) \Gamma \times \Gamma' \rightarrow (Q, \Gamma, \Gamma')$$

Construct two stack PDA for language $L = \{a^n b^n c^n / n \geq 1\}$

Sol: $L = \{\varepsilon, abc, aabbcc, aaabbbccc \ldots\ldots\ldots\}$

$(q_0, a, z_1, z_2) \longrightarrow (q_0, az_1, z_2)$

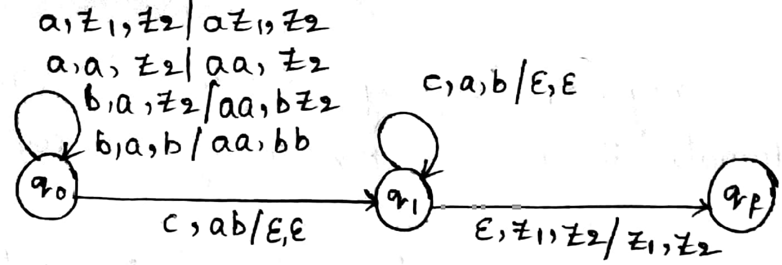
$(q_0, a, a, z_2) \longrightarrow (q_0, aa, z_2)$

$(q_0, b, a, z_2) \longrightarrow (q_0, aa, bz_2)$

$(q_0, b, a, b) \longrightarrow (q_0, aa, bb)$

$(q_0, c, a, b) \longrightarrow (q_1, \varepsilon, \varepsilon)$

$(q_1, c, a, b) \longrightarrow (q_1, \varepsilon, \varepsilon)$



IInd method: for $(n \geq 0)$

$\delta(q_0, \varepsilon, Z_1, Z_2) \longrightarrow (q_0, Z_1, Z_2)$

$\delta(q_0, a, Z_1, Z_2) \longrightarrow (q_0, aZ_1, Z_2)$

$\delta(q_0, a, a, Z_2) \longrightarrow (q_0, aa, Z_2)$

$\delta(q_0, b, a, Z_2) \longrightarrow (q_1, a, bZ_2)$

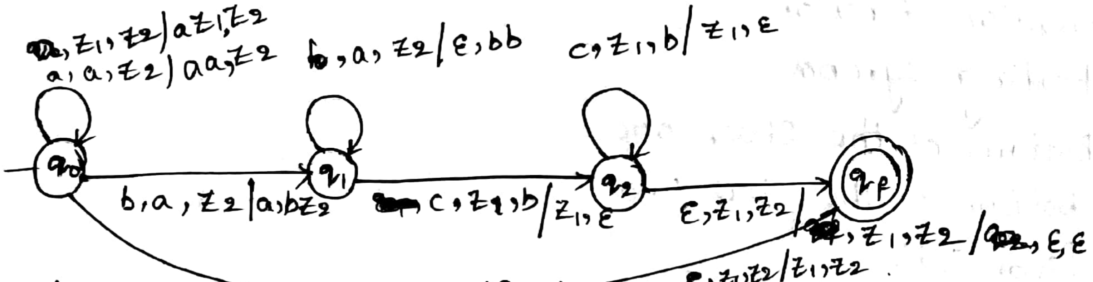
$\delta(q_1, b, a, b) \longrightarrow (q_1, \varepsilon, bb)$

$\delta(q_1, c, Z_1, b) \longrightarrow (q_2, Z_1, \varepsilon)$

$\delta(q_2, c, Z_1, b) \longrightarrow (q_2, Z_1, \varepsilon)$

$\delta(q_2, \varepsilon, Z_1, Z_2) \longrightarrow (q_f, Z_1, Z_2)$

$(q_2, \varepsilon, \varepsilon)$



2) Construct two stack PDA for language $L = \{a^n b^n c^n d^n \mid n \geq 1\}$

Sol: $\delta(q_0, a, Z_1, Z_2) \longrightarrow (q_0, aZ_1, Z_2)$

$\delta(q_0, a, a, Z_2) \longrightarrow (q_0, aa, Z_2)$

$\delta(q_0, b, a, Z_2) \longrightarrow (q_0, aa, bZ_2)$

$\delta(q_0, b, a, b) \longrightarrow (q_0, aa, bb)$

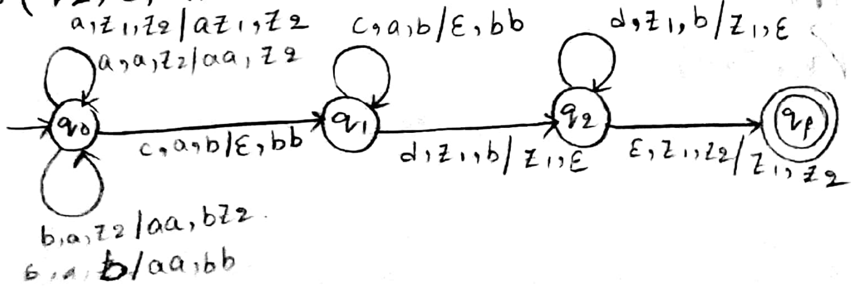
$\delta(q_0, c, a, b) \longrightarrow (q_1, \varepsilon, bb)$

$\delta(q_1, c, a, b) \longrightarrow (q_1, \varepsilon, bb)$

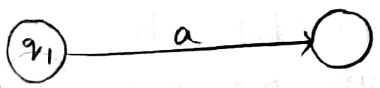
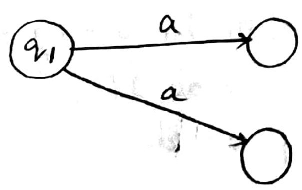
$\delta(q_1, d, Z_1, b) \longrightarrow (q_2, Z_1, \varepsilon)$

$\delta(q_2, d, Z_1, b) \longrightarrow (q_2, Z_1, \varepsilon)$

$\delta(q_2, \varepsilon, Z_1, Z_2) \longrightarrow (q_f, Z_1, Z_2)$ (or) $(q_2, \varepsilon, \varepsilon)$



Difference between NPDA and DPDA:

DPDA Deterministic push down Automata	NPDA Non deterministic push down Automata
1. It is less powerful than NPDA	1. It is more powerful than DPDA
1. It is possible to Convert every DPDA to a Corresponding NPDA	2. It is not possible to Convert every NPDA to a Corresponding DPDA
. The language accepted by DPDA is a Subset of the language accepted by NPDA.	3. The language accepted by NPDA is not a subset of the language accepted by DPDA.
4. The language accepted by DPDA is called DCFL (Deterministic Context-free Language) which is a Subset of NCFL (Non-deterministic-Context free Language) accepted by NPDA.	4. The language accepted by NPDA is called NCFL (Non-deterministic Context free Language).
5. For every input with the current State, there is only one move. $M=(Q,\Sigma,\Gamma,q_0,Z,F,\delta)$ $\delta: Q*\Sigma*\Gamma \rightarrow Q*T$ 	5. For every input with the current State, we can have multiple moves. $M=(Q,\Sigma,\Gamma,q_0,Z,F,\delta)$ $\delta: Q*\{\Sigma\cup\varepsilon\}*\Gamma \rightarrow 2^{Q*\Gamma*}$ 

Applications of PDA:

* For designing the parsing phase of a compiler. (Syntax analysis).
* For implementation of stack applications.
* For evaluating the arithmetic expressions.
* For solving the Towers of Hanoi problem.