Unit - I

1. Define rank of a Matrix?

2. State Cauchy - Binet formula?

3. Find the rank of (i) $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$ (iv) $\begin{bmatrix} 3 & -2 & 4 & 1 \\ 2 & 2 & 1 & 0 \\ 4 & -1 & 0 & 3 \end{bmatrix}$ (v) $\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$

4. Find the Inverse of the matrix by elementary row operations (i) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ (ii) $\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$

5. Solve the System of equations using Gauss elemination Method

(i) $x+y+z=6$, $x-y+2z=5$, $3x+y+z=8$, $2x-2y+3z=7$...

(ii) $3x+4y+5z=18$, $2x-y+8z=13$, $5x-2y+7z=20$

6. Solve the System of equations using Gauss - Seidel iteration method

(i) $27x+6y-z=85$, $6x+15y+2z=72$, $x+y+54z=100$

(ii) $10x+y+z=12$, $2x+10y+z=13$, $2x+2y+10z=14$.

7. (i) Define Echelon form & Normal form of a Matrix?

(ii) verify Cauchy - Binet formula for $A=\begin{bmatrix} 1 & 2 & 3 & 0 \\ 3 & 4 & -1 & 6 \end{bmatrix}_{2\times 4}$

and $B=\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 1 \\ 2 & 5 \end{bmatrix}_{4\times 2}$

Unit - 2

1. (i) Define Eigen value & eigen vector of a matrix?

   (ii) State Cayley - Hamilton Th?

   (iii) Define Modal matrix & spectral matrices!

2. Find the eigen values & the corresponding eigen vectors of the matrix (i) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. (ii) $\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$

3. (i) A Square matrix 'A' and its transpose 'A^T', have the Same eigen values

   (ii) prove that Similar matrices have the Same eigen values

4. verify Cayley - Hamilton Th for the matrices

   (i) $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Also find A^{-1}.

5. (i) Define Quadratic form?

   (ii) Find the Q.F. corresponding to the matrix $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

   (iii) Find the Symmetric matrix corresponding to the Q.F. $x^2 + 2y^2 + 3z^2 + 4xy + 5yz + 6zx$

6. Reduce the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}$ to the Diagonal form.

7. (i) Define canonical form of a Q.F?

   (ii) Define Index, Signature, Nature of a Q.F.

   (iii) Define Sylvester's law of Inertia

8. Reduce the Q.F. $3x^2+5y^2+3z^2-2xy-2yz+2zx$ to the canonical form by orthogonal reduction.

Unit - 3

1. Statements of (i) Rolle's Th (ii) Lagrange's MVT (iii) Cauchy's MVT
   (iv) Taylor's Th with Lagrange's form of remainder
   (v) Maclaurin's Th with Lagrange's form of remainder.

2. Verify Rolle's Th for (i) $f(x) = \tan x$ in $[0, \pi]$
   (ii) $f(x) = (x-a)^m (x-b)^n$ where m, n are +ve integers in $[a,b]$
   (iii) $f(x) = \frac{\sin x}{e^x}$ in $[0, \pi]$ (vi) $f(x) = |x|$ in $[-1,1]$
   (iv) $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$
   (v) $f(x) = \sqrt{4-x^2}$ in $[-2, 2]$

3. Show that $g(x) = 8x^3 - 6x^2 - 2x + 1$ has a zero between '0' & '1'.

4. Verify Lagrange's MVT for
   (i) $f(x) = x^3 - x^2 - 5x + 3$ in $[0,4]$ (ii) $f(x) = \log_e x$ in $[1, e]$
   (iii) $f(x) = x(x-1)(x-2)$ in $[0, 0.5]$ (iv) $f(x) = e^{-x}$ in $[-1, 1)$

5. Using mean value Th P.T (i) $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1}\left(\frac{3}{5}\right) > \frac{\pi}{3} - \frac{1}{8}$
   (ii) For any $x > 0$, $1 + x < e^x < 1 + xe^x$

6. If $f(x) = \log x$ and $g(x) = x^2$ in $[a,b]$ with $b > a > 1$, using Cauchy's theorem P.T. $\frac{\log b - \log a}{b - a} = \frac{a+b}{2c^2}$

7. Verify Taylor's Th for $f(x) = x(x-1)(x-2)$ in $[0, \frac{1}{2}]$ with Lagrange's remainder upto '2' terms.

8. Verify Maclaurin's Th for $f(x) = (1-x)^{5/2}$ with Lagrange's form of remainder upto 3 terms when $x = 1$.

9. (i) Expand '$\log x$' in powers of $(x-1)$
   (ii) Using Maclaurin's series expand '$\tan x$' & hence find the series for $\log(\sec x)$
   (iii) Find the Maclaurin's series for e^x, $\cos x$ & $\sinh x$

10. S.T. $\log(1+e^x) = \log 2 + x/2 + x^2/8 - \frac{x^4}{192} + \cdots$ and hence deduce that $\frac{e^x}{1+e^x} = 1/2 + x/4 - \frac{x^2}{48} + \cdots$

Unit - 4

1. (i) Define a function of two variables?

   (ii) Define Total Derivative & chain rule of a function of two variables?

   (iii) Define Jacobian Transformation?

   (iv) Define functional Dependence

2. (i) verify $JJ' = 1$ for $x = uv$, $y = \frac{u}{v}$

   (ii) Find f_x & f_y for $f(x,y) = \log\sqrt{x^2+y^2}$

   (iii) If $u = (x^2+y^2+z^2)^{-1/2}$ then S.T. $(u_x)^2+(u_y)^2+(u_z)^2 = u^4$

3. Find the volume of greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$.

4. Find the maximum & minimum distances of the point $(3,4,12)$ from the sphere $x^2+y^2+z^2=1$.

5. Find the points on the surface $z^2 = xy+1$, nearest to the origin.

6. Find the maximum & minimum values of $\sin x \sin y \sin(x+y)$.

7. (i) If $x+y+z = u$, $y+z = uv$, $z = uvw$, find $J\left(\frac{x,y,z}{u,v,w}\right)$

   (ii) If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ then P.T. $J = 4$.

8. Examine the function for extreme values

   (i) $x^4+y^4-2x^2+4xy-2y^2$ (ii) $x^3y^2(1-x-y)$

9. Find the shortest distance from origin to the surface $xyz^2 = 2$

10. P.T. $u = \frac{x^2-y^2}{x^2+y^2}$, $v = \frac{2xy}{x^2+y^2}$ are functionally dependent & also find the relation b/w them.

11. Find the dimensions of a rectangular parallelopiped box open at the top of max capacity whose surface area is 108 sq. inches.

12. Define a saddle point of a function.

Unit - 5

1. Evaluate (i) $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\,dx\,dy$ (b) $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy\,dx}{1+x^2+y^2}$

(iii) $\int_0^1 \int_0^{x^2} e^{y/x}\,dy\,dx$ (iv) $\int_0^1 \int_0^2 \sqrt{xy}\,dx\,dy$

2. Evaluate $\iint_R y\,dx\,dy$ where 'R' is the region bounded by the parabolas $y^2=4x$ & $x^2=4y$.

3. Evaluate $\iint_R xy\,dx\,dy$ where 'R' is the region bounded by x-axis, ordinate $x=2a$ and the curve $x^2=4ay$.

4. By changing the order of Integration, evaluate

(a) $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$ (b) $\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$ (c) $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\,dx\,dy$

(d) $\int_0^{\infty} \int_0^{\infty} \frac{e^{-y}}{y}\,dx\,dy$

5. Evaluate (a) $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\,dx\,dy\,dz$ (b) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$

(c) $\int_1^e \int_0^{\log y} \int_1^{e^x} \log z\,dz\,dx\,dy$ (d) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$

6. Evaluate $\iint (x+y)^2\,dx\,dy$ over the area bounded by the ellipse $x^2/a^2 + \frac{y^2}{b^2} = 1$.

7. Evaluate $\int_0^1 \int_x^{\sqrt{x}} x^2y^2(x+y)\,dy\,dx$

8. Define Double & Triple Integrals?

Unit- II

2 marks Questions:

1) Statements of Rolle's Theorem, Lagrange's mean value theorem, Cauchy's mean value theorem

2) The conditions of Rolle's theorem are sufficient but not necessary. Justify this statement with an example.

Ex: (1) $f(x) = |\sin x|$ is not differentiable at $x = \pi$ in $(0, 2\pi)$

$f(x) = \sin x, \ \forall x \in [0, \pi]$

$= -\sin x, \ \forall x \in [\pi, 2\pi]$.

But $f'(x) = \cos x = 0$, when $x = \frac{\pi}{2} \in (0, \pi)$

and $f'(x) = -\cos x = 0$ when $x = \frac{3\pi}{2} \in (\pi, 2\pi)$

(2) let $f(x) = \sin x$ in $[0, \frac{3\pi}{4}]$

clearly f is continuous in $[0, \frac{3\pi}{4}]$ and derivable in $(0, \frac{3\pi}{4})$.

But $f(0) = 0$ & $f(\frac{3\pi}{4}) = \sin\frac{3\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\therefore$ condition (3) of Rolle's theorem is not satisfied.

But $f'(x) = \cos x = 0$ at $x = \frac{\pi}{2} \in (0, \frac{3\pi}{4})$

(3) Examine if Rolle's theorem is applicable for $f(x) = |x|$ in $[-1, 1]$.

(4) verify Rolle's theorem for $f(x) = \frac{x^2 - x - 6}{x - 1}$ in the interval $(-2, 3)$.

(5) verify whether Rolle's theorem can be applied to the function $f(x) = \tan x$ in $[0, \pi]$

(6) Find 'c' using Lagrange's mean value theorem for the function $f(x) = \cos x$ in $[0, \pi/2]$

(7) Show that for any $x > 0$, $1 + x < e^x < 1 + xe^x$

(8) Show that $x < \sin^{-1} x < \frac{x}{\sqrt{1-x^2}}$ for $x > 0$

(9) Show that $|\cos b - \cos a| < |b - a|$, $\forall a, b$.

(10) Show that $\frac{x}{1+x} < \log(1+x) < x$

(11) Find the regions in which $f(x) = 1-4x-x^2$ is increasing and the region in which $f(x)$ is decreasing using L-M-T.

Sol: Given $f(x) = 1-4x-x^2$

Since $f(x)$ is a polynomial in x, it is continuous on $[a,b]$ and derivable on (a,b), $\forall a,b \in R$.

$\therefore$ f satisfies the two conditions of L-M-V-T.

Now $f'(x) = -4-2x$, $\forall x \in R$.

and $f'(x) = 0$ at $x = -2$.

$\therefore f'(x) > 0$, if $x < -2$ and $f'(x) < 0$, if $x > -2$

Hence f is strictly increasing on $(-\infty, -2)$ and strictly decreasing on $(-2, \infty)$.

(12) Find 'c' using L-M-V-T for $f(x) = x - x^3$ in $[-2,1]$

Ans: $c = \pm 1$

(13) Find 'c' using mean value theorem for the functions $f(x) = \frac{1}{x^2}$, $g(x) = \frac{1}{x}$ on $[a,b]$

Ans: $c = \frac{2ab}{a+b} \in (a,b)$

(14) Find 'c' using Cauchy's mean value theorem for $f(x) = \sin x$ and $g(x) = \cos x$ on $[0, \frac{\pi}{2}]$

Ans: $c = \frac{\pi}{4} \in (0, \frac{\pi}{2})$

(15) Obtain Taylor's series expansion of e^x about $x = -1$.

(16) Obtain Maclaurin's series expansion of $\log_e(1+x)$.

(17) Expand $(1+x)^n$ in powers of x.

(18) Expand $\sqrt{x}$ in powers of x

(19) Expand $\sin x$ in powers of $(x - \frac{\pi}{2})$.

(20) Geometrical interpretation of Rolle's theorem & L-M-V-T.

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2 Marks Questions:

1) verify whether Rolle's theorem can be applied to the function $f(x) = \frac{1}{x^2}$ in $[-1,1]$

Sol: Given $f(x) = \frac{1}{x^2}$ in $[-1,1]$

$\Rightarrow f'(x) = \frac{-2}{x^3}$

$f'(x)$ does not exists when $x=0$.

$\therefore f'(x)$ does not exists in $(-1,1)$

$\Rightarrow f(x)$ is not differentiable in $(-1,1)$.

Hence Rolle's theorem can not be applicable to $f(x) = \frac{1}{x^2}$ in $[-1,1]$.

2) verify Rolle's theorem for $f(x) = \frac{x^2-x-6}{x-1}$ in the interval $(-2,3)$.

Sol: Given $f(x) = \frac{x^2-x-6}{x-1}$ in $(-2,3)$.

$$\Rightarrow f'(x) = \frac{(x-1)(2x-1) - (x^2-x-6)(1)}{(x-1)^2}$$

$$= \frac{2x^2 - x - 2x + 1 - x^2 + x + 6}{(x-1)^2} = \frac{x^2-2x+7}{(x-1)^2}$$

$f'(x)$ does not exist at $x=1$ when $-2<x<3$.

$\therefore f'(x)$ does not exist in $(-2,3)$.

$\Rightarrow f(x)$ is not derivable

3) Give an example of a function which is continuous on $[-1,1]$ and for which mean value theorem does not hold with explation.

Sol: The function $f(x) = |x|$ is continuous in $(-1,1]$

But it is not differentiable in at $x=0 \in (-1,1)$

$\therefore$ Lagrange's mean value theorem can not be applicable to $f(x)$.

4) Find 'c' using Lagrange mean value theorem for the function $f(x) = \cos x$ in the interval $[0, \pi/2]$.

Sol: Given $f(x) = \cos x$ in $[0, \pi/2]$

we know that $f(x)$ is continuous and differentiable in $[0, \frac{\pi}{2}]$.

$\therefore$ By L-M-T, we have $f'(c) = \frac{f(b)-f(a)}{b-a}$, $a<c<b$. — ①

Now $f(x) = \cos x \Rightarrow f'(x) = -\sin x \Rightarrow f'(c) = -\sin c$

$f(b) = \cancel{f}(\frac{\pi}{2}) = \cos\frac{\pi}{2} = 0$

$f(a) = f(0) = \cos 0 = 1$

Sub all these in ①, we get $-\sin c = \frac{0-1}{\pi/2 - 0}$

$\Rightarrow -\sin c = -\frac{2}{\pi} \Rightarrow c = \sin^{-1}\frac{2}{\pi} \in (0, \pi/2)$

4) Find the region in which $f(x) = 1-4x-x^2$ is increasing and the region in which it is decreasing using M-V-T.

Sol: Given $f(x) = 1-4x-x^2$

since $f(x)$ is a polynomial in x, it is continuous on $[a,b]$ and derivable on (a,b).

$\therefore$ f satisfies the conditions of L-M-T.

~~By L-M-T, we have f(c)=~~

Now $f'(x) = -4-2x = -2(2+x)$

and $f'(x) = 0$ when $x = -2$.

$\therefore f'(x) > 0$, for $x < -2$ and $f'(x) < 0$ for $x > -2$.

Hence $f(x)$ is strictly increasing on $(-\infty, -2)$ and decreasing on $(-2, \infty)$.

5) Explain why mean value theorem does not hold for $f(x) = x^{2/3}$ in $[-1,1]$

Sol. Given $f(x) = x^{2/3}$ in $[-1,1]$

Then $f'(x) = \frac{2}{3}x^{\frac{2}{3}-1} = \frac{2}{3}x^{-1/3} = \frac{2}{3}\cdot\frac{1}{x^{1/3}}$

$f'(x) \nexists$ exists when $x=0$. in $(-1,1)$

$\therefore f'(x)$ does not exists in $(-1,1)$

$\Rightarrow f(x)$ is not derivable in $(-1,1)$

Hence L-M-T can not be applicable

Unit – II

5 Marks Questions:

① verify Rolle's theorem for the function
$f(x) = \frac{\sin x}{e^x}$ in $(0, \pi)$

② verify Rolle's theorem for $f(x) = \log\left[\frac{x^2+ab}{(a+b)x}\right]$ in $[a,b]$

③ verify Rolle's theorem for $f(x) = x(x+3)\, e^{-\frac{x}{2}}$ in $[-3,0]$

④ verify Rolle's theorem for $f(x) = e^x(\sin x - \cos x)$ in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

⑤ verify Lagrange's mean value theorem for $f(x) = \log_e x$ in the interval $[1,e]$

⑥ verify Lagrange's mean value theorem for the function $f(x) = e^x$ in $[0,1]$.

⑦ Using mean value theorem, show that
$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\left(\frac{4}{3}\right) < \frac{\pi}{4} + \frac{1}{6}$$

⑧ Using Lagrange's mean value theorem prove that
$$\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1}\left(\frac{3}{5}\right) > \frac{\pi}{3} - \frac{1}{8}$$

⑨ Using Lagrange's mean value theorem to prove the inequality $x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}$ if $x > 0$.

⑩ Using L-M-V-T to prove the inequality
$$x \le \sin^{-1}x \le \frac{x}{\sqrt{1-x^2}}, \quad 0 \le x \le 1.$$

⑪ Find c of Cauchy's mean value theorem for $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$ in $(a,b]$, where $0 < a < b$.

⑫ verify Cauchy's mean value theorem for the functions
(i) $f(x) = e^{-x}$ & $g(x) = e^x$ in $[2,6]$.
(ii) $f(x) = \sin x$, $g(x) = \cos x$ in (a,b)

⑬ Obtain the Taylor's series expansion of $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$ upto the power $\left(x-\frac{\pi}{2}\right)^4$.

⑭ verify Taylor's theorem for $f(x) = (1-x)^{5/2}$ with Lagrange's form of remainder upto 2 terms with $x=1$.

(15) Show that $\frac{\sin^{-1}x}{\sqrt{1-x^2}} = x + 4\frac{x^3}{3!} + \cdots$

(16) Show that $\log(1+e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \cdots$ and hence deduce that $\frac{e^x}{e^x+1} = \frac{1}{2} + \frac{x}{4} - \frac{x^3}{48} + \cdots$

(17) Find the approximate value of $\sqrt{10}$ using Taylor's series.

(18) Obtain the Maclaurin's series expansion of $\log_e(1+x)$.

(19) Expand $\tan x$ upto five derivatives using Taylor's series.

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old question papers

Unit - III

2M:

1) Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, if $z = x^2y - x\sin(xy)$
2) If $x = u(1-v)$, $y = uv$ find $J = \frac{\partial(x,y)}{\partial(u,v)}$
3) Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = \log(x^2+y^2)$
4) If $u = x^2 - 2y^2$, $v = 2x^2 - y^2$ and $J = \frac{\partial(u,v)}{\partial(x,y)}$
5) Let $z = \log(x^2+y^2)$ Evaluate $\frac{\partial^2 z}{\partial x^2}$ and $\frac{\partial^2 z}{\partial x \partial y}$
6) Let $u = x+3y^2$, $v = 4x^2y$. Evaluate $\frac{\partial(u,v)}{\partial(x,y)}$ at $(1,-1)$
7) Let $u = 3x^2+y$, $v = 4xy^2$ Evaluate $\frac{\partial(u,v)}{\partial(x,y)}$ at $(1,-1)$
8)

5M

1) If $y_1 = \frac{x_2x_3}{x_1}$, $y_2 = \frac{x_3x_1}{x_2}$, $y_3 = \frac{x_1x_2}{x_3}$ show that the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 is 4.
2) Divide 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum.
3) Determine whether the following functions are functionally dependent or not. If functionally dependent, find the functional relation between them
$$u = \sin^{-1}x + \sin^{-1}y, \quad v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$$
4) Find the maximum and minimum values of
$$f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$$

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5) If $u = x+y+z$, $uv = y+z$, $uvw = z$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

6) Discuss the maxima and minima of $f(x,y) = x^3y^2(1-x-y)$

7) Determine whether the following functions are functionally dependent or not. If functionally dependent, find the functional relation between them
$u = x^2+y^2+2xy+2x+2y$, $v = e^x e^y$

8) Find the maximum and minimum distances of the point $(3,4,12)$ from the sphere $x^2+y^2+z^2=1$

9) Find the value of $\frac{\partial^2 u}{\partial t^2}$ given $u = x^2 - 4ay$,
$x = at^2$, $y = 2at$.

10) ~~Find~~ Let $u = x+3y^2-z^2$, $v = 4x^2yz$, $w = 2z^2-xy$
Evaluate the Jacobian at $(1,-1,0)$.

11) Find the minimum distance of the point $(3,4,12)$ from the sphere $x^2+y^2+z^2=4$.

12) Find the value of $\frac{d^2u}{dt^2}$ given $u = x^2-4ay$,
$x = at^2$, $y = 2at$.

13) Prove that $u = \frac{x^2y^2}{x^2+y^2}$, $v = \frac{2xy}{x^2+y^2}$ are functionally dependent and find the relation between them.

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$$\therefore\ x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{x}{\sqrt{y^2-x^2}}-\frac{xy}{x^2+y^2}+\frac{-x}{\sqrt{y^2-x^2}}+\frac{xy}{x^2+y^2}$$

$$=0=R.H.S$$

Hence Euler's Th. is verified.

(3) verify Euler's theorem for the function $xy+yz+zx$.

(4) If $u=f(x,y,z)=(x^2+y^2+z^2)^{-1/2}$ S.T. $\sum x\frac{\partial u}{\partial x}=-u$.

(5) If u is a homogeneous function of x & y of degree 'n', P.T

(i) $x\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial x\partial y}=(n-1)\frac{\partial u}{\partial x}$ (or) $xu_{xx}+yu_{xy}=(n-1)u_x$

(ii) $x\frac{\partial^2 u}{\partial x\partial y}+y\frac{\partial^2 u}{\partial y^2}=(n-1)\frac{\partial u}{\partial y}$ (or) $xu_{xy}+yu_{yy}=(n-1)u_y$

Sol: By Euler's Th., we have

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu \quad -(1)$$

Diffg (1) Partially w.r.t x, we get

$$\left(x\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial x}\right)+y\frac{\partial^2 u}{\partial x\partial y}=n\frac{\partial u}{\partial x}$$

$$x\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial x\partial y}=n\frac{\partial u}{\partial x}-\frac{\partial u}{\partial x}=(n-1)\frac{\partial u}{\partial x}=(n-1)u_x$$

Similarly we can prove (ii).

(6) If $u(x,y)=\log\left(\frac{x^4+y^4}{x+y}\right)$ S.T. $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3$

Sol: Given $u(x,y)=\log\left(\frac{x^4+y^4}{x+y}\right)$ $-(1)$

The function (1) is not a homogeneous function.

$$\therefore\ e^u=\frac{x^4+y^4}{x+y}=f(x,y),\ \text{Say}.$$

Now $f(x,y)$ is a homogeneous function of degree '3'.

$\therefore$ e^u is a homogeneous function of degree '3'.

Hence by Euler's theorem,

$$x \frac{\partial(e^u)}{\partial x} + y \frac{\partial}{\partial y}(e^u) = 3e^u$$

ie, $x\, e^u \frac{\partial u}{\partial x} + y\, e^u \cdot \frac{\partial u}{\partial y} = 3e^u$

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3.$$

(7) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, $x+y \neq 0$, P.T $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$

Take $\sin u = \frac{x^2+y^2}{x+y} = f(x,y)$ and proceed as the above Problem.

(8) If $u = \frac{y}{z} + \frac{z}{x}$ then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$

Ans = 0

9) Find $\frac{du}{dt}$, if $u = \sin\left(\frac{x}{y}\right)$ where $x = e^t$, $y = t^2$

10) If $z = u^2 + v^2$, $u = r\cos\theta$, $v = r\sin\theta$

then find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$

[Hint: $\frac{\partial z}{\partial r} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial r} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial r}$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial \theta} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial \theta}.$$

Unit-IV

2 Marks Questions

1) Evaluate $\int_0^2 \int_0^x e^{x+y}\, dy\, dx$.

2) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\, dy\, dx$.

3) Evaluate $\int_0^1 \int_0^{x^2} e^{y/x}\, dy\, dx$.

4) Evaluate $\int_0^a \int_{x/a}^{\sqrt{x/a}} x^2y^2(x+y)\, dy\, dx$.

5) Evaluate $\iint (x^2+y^2)\, dx\, dy$ in the Positive quadrant for which $x+y \le 1$.

6) Evaluate $\iint x^2\, dx\, dy$ over the region bounded by hyperbola $xy=4$, $y=0$, $x=1$, $x=4$.

7) Evaluate $\int_0^\infty \int_0^{\pi/2} e^{-r^2}\, r\, d\theta\, dr$.

8) Evaluate $\int_0^\pi \int_0^{a\sin\theta} r\, dr\, d\theta$.

9) Find the area of the cardioid $r=a(1+\cos\theta)$.

10) Evaluate $\int_0^1 \int_y^1 \int_0^{1-x} x\, dz\, dx\, dy$

11) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} e^x\, dz\, dy\, dx$.

12) Change the order of integration and evaluate $\int_{-a}^{a} \int_0^{\sqrt{a^2-y^2}} f(x,y)\, dx\, dy$.

Unit IV

2 marks questions.

1) Evaluate $\int_1^a \int_1^b \frac{dy\,dx}{xy}$

2) Evaluate $\int_0^1 dx \int_1^2 dy \int_1^3 xyz\,dz$

3) $\iint_D (x^2+y^2)\,dx\,dy = $ ______ where $D: y=x,\ y^2=x$.

4) Evaluate $\int_0^2 \int_0^x (x+y)\,dy\,dx$, evaluate $\int_0^1 \int_0^x e^{x+y}\,dy\,dx$.

5) Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\,dx\,dy\,dz$.

6) Evaluate $\int_0^1 \int_0^1 \frac{dx\,dy}{\sqrt{1-x^2}\cdot\sqrt{1-y^2}}$.

7) Evaluate $\int_0^3 \int_0^2 (4-y)^2\,dy\,dx$

8) Evaluate $\int_{-1}^1 \int_{-2}^2 \int_{-3}^3 dx\,dy\,dz$.

9) Evaluate $\int_0^a \int_0^{\sqrt{ay}} xy\,dx\,dy$

10) Evaluate $\int_0^\pi \int_0^{a\cos\theta} r\sin\theta\,dr\,d\theta$

11) Evaluate $\int_0^1 \int_x^1 (x^2+y^2)\,dy\,dx$.

12) Evaluate $\iint_A xy\,dx\,dy$ where A is the domain bounded by x-axis, ordinate $x=2a$ and curve $x^2=4ay$.

13) Evaluate $\int_0^1 \int_0^x e^{x+y}\,dy\,dx$

14) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\,dx\,dy$

15) Evaluate $\int_0^{\pi/2} \int_{-1}^1 x^2y^2\,dx\,dy$

16) Evaluate $\int_0^1 dx \int_0^x e^{y/x}\,dy$.

5 marks questions

1) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\,dx\,dy$. (Ans: $\frac{3}{35}$)

2) Evaluate $\int_0^5 \int_0^{x^2} x(x^2+y^2)\,dx\,dy$ (Ans: $\frac{29\times 5^6}{24}$)

3) Evaluate $\int_0^1 \int_0^{x^2} e^{y/x}\,dy\,dx$ (Ans: $\frac{1}{2}$)

4) Evaluate $\iint_R y\,dx\,dy$ where R is the region bounded by the parabolas $y^2=4x$ and $x^2=4y$. (Ans: $\frac{48}{5}$)

5) Evaluate $\iint_R xy\,dx\,dy$ where R is the region bounded by the x-axis, ordinate $x=2a$ and $x^2=4ay$. (Ans: $\frac{a^4}{3}$).

6) Evaluate $\iint_R (x+y)^2\,dx\,dy$ over the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
(Ans: $\frac{\pi}{4}ab(a^2+b^2)$)

7) Evaluate $\iint_R (x^2+y^2)\,dx\,dy$ in the positive quadrant for which $x+y\le 1$. (Ans: $\frac{1}{6}$)

8) Evaluate $\int_0^{\pi/4} \int_0^{a\sin\theta} \frac{r\,dr\,d\theta}{\sqrt{a^2-r^2}}$. (Ans: $a\left(\frac{\pi}{4}-\frac{1}{\sqrt{2}}\right)$.

9) Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z\,dz\,dx\,dy$.

10) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \cancel{xyz\,dz\,dy\,dx}\ \frac{1}{\sqrt{1-x^2-y^2-z^2}}\,dz\,dy\,dx$.

11) Evaluate $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2+y^2+z^2)\,dx\,dy\,dz$ (Ans: $\frac{8}{3}abc(a^2+b^2+c^2)$

12) Evaluate $\iiint_V dx\,dy\,dz$ where V is the finite region bounded by the co-ordinate planes $x=0, y=0, z=0$ and $2x+3y+4z=12$

13) Find the area of the loop of the curve $r=a(1+\cos\theta)$.

14) Find the area bounded b/w the curves $y^2=4ax$ and $x^2=4ay$.

15) Evaluate $\iiint_D xyz\,dx\,dy\,dz$ where D is the region bounded by the positive octant of the sphere $x^2+y^2+z^2=a^2$.

16) Evaluate $\iiint_R (x+y+z)\,dz\,dy\,dx$ where R is the region bdd by $x=0, x=1, y=0, y=1, z=0, z=1$.

17) Find the whole area of the lemniscate $r^2 = a^2\cos 2\theta$.

18) Evaluate $\int_0^1 \int_x^{\sqrt{x}} x^2y^2(x+y)\,dy\,dx$.

19) Evaluate $\iiint xy^2z\,dx\,dy\,dz$ taken over the positive octant of the sphere.

20) Evaluate $\int_0^1 \int_y^1 \int_0^{1-x} x\,dz\,dx\,dy$.

~~All problems in change of order of integration~~.

21) Find the area of the quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

22) Find the area enclosed by curves $y = x^2$ and $y = x^3$.

23) Find the area of circle using double integral.

24) Change the order of integration and evaluate the following:

a) $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$

b) $\int_0^a \int_{x/a}^{\sqrt{x/a}} (x^2+y^2)\,dx\,dy$

c) $\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$

d) $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\,dx\,dy$

e) $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xy\,dx\,dy$

f) $\int_{-a}^a \int_0^{\sqrt{a^2-y^2}} f(x,y)\,dx\,dy$.

(g) $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$.

10 Marks Questions:

1) Evaluate $\iint_R y^2\,dx\,dy$ where R is bounded by the Parabolas $y^2=4x$ and $x^2=$,

2) Evaluate $\iint(x^2+y^2)\,dx\,dy$ over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

3) Evaluate $\int_0^3\int_1^2 xy(1+x+y)\,dy\,dx$.

4) Evaluate $\iint r\sin\theta\,dr\,d\theta$ over the cardioid $r=a(1+\cos\theta)$ above the initial line.

5) Evaluate $\iint r^3\,dr\,d\theta$ over the area included between the circles $r=2\sin\theta$ and $r=4\sin\theta$.

6) Evaluate the double integral $\int_0^a\int_0^{\sqrt{a^2-y^2}}(x^2+y^2)\,dy\,dx$ by changing into Polar Coordinates.

7) Evaluate $\int_0^a\int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}}\frac{dx\,dy}{\sqrt{a^2-x^2-y^2}}$ by changing into Polar Coordinates.

8) Evaluate $\int_{y=0}^1\int_{x=y}^a\frac{x}{x^2+y^2}\,dx\,dy$ by changing into Polar Coordinates.

9) Change the order of integration and evaluate $\int_0^1\int_{x^2}^{2-x}xy\,dy\,dx$.

10) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}y^2\,dy\,dx$ by changing the order of integration.

11) Change the order of integration and evaluate $\int_0^a\int_{x/a}^{\sqrt{x/a}}(x^2+y^2)\,dx\,dy$.

12) Evaluate by changing the order of integration $\int_0^1\int_x^{\sqrt{2-x^2}}\frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$

13) Evaluate $\int_0^\infty\int_0^\infty x\,e^{-\frac{x^2}{y}}\,dy\,dx$ by changing the order of integration.

14) Find by double integration the area lying inside the circle $r=a\sin\theta$ and outside the cardioid $r=a(1-\cos\theta)$.

15) Find by double integral the volume common to the cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$.

10 Marks Questions:

1) Evaluate $\iint_R y^2\,dx\,dy$ where R is bounded by the Parabolas $y^2=4x$ and $x^2=$.

2) Evaluate $\iint (x^2+y^2)\,dx\,dy$ over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

3) Evaluate $\int_0^3\int_1^2 xy(1+x+y)\,dy\,dx$.

4) Evaluate $\iint r\sin\theta\,dr\,d\theta$ over the cardioid $r=a(1+\cos\theta)$ above the initial line.

5) Evaluate $\iint r^3\,dr\,d\theta$ over the area included between the circles $r=2\sin\theta$ and $r=4\sin\theta$.

6) Evaluate the double integral $\int_0^a\int_0^{\sqrt{a^2-y^2}}(x^2+y^2)\,dy\,dx$ by changing into Polar Coordinates.

7) Evaluate $\int_0^a\int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}}\frac{dx\,dy}{\sqrt{a^2-x^2-y^2}}$ by changing into Polar Coordinates.

8) Evaluate $\int_{y=0}^{1}\int_{x=y}^{a}\frac{x}{x^2+y^2}\,dx\,dy$ by changing into Polar coordinates.

Imp 9) Change the order of integration and evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dy\,dx$.

10) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}} y^2\,dy\,dx$ by changing the order of integration.

11) Change the order of integration and evaluate $\int_0^a\int_{x/a}^{\sqrt{x/a}}(x^2+y^2)\,dx\,dy$.

Imp 12) Evaluate by changing the order of integration $\int_0^1\int_x^{\sqrt{2-x^2}}\frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$

13) Evaluate $\int_0^\infty\int_0^\infty x e^{-\frac{x^2}{y}}\,dy\,dx$ by changing the order of integration.

14) Find by double integration the area lying inside the circle $r=a\sin\theta$ and outside the cardioid $r=a(1-\cos\theta)$.

15) Find by double integral the volume common to the cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$.

Imp

(16) Evaluate $\int_0^1 \int_{e^x}^{e} \frac{dy\,dx}{\log y}$ by changing the order of integration.

Imp

(17) Change the order of integration in $I = \int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$ and hence evaluate.

(18) Change the order of integration and hence evaluate

$$I = \int_0^a \int_{\sqrt{ax}}^{a} \frac{y^2\,dx\,dy}{\sqrt{y^4 - a^2x^2}}$$

(19) Evaluate $\int_{-1}^{1} \int_0^x \int_{x-z}^{x+z} (x+y+z)\,dx\,dy\,dz$

(20) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dx\,dy\,dz$.

Imp.

(21) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$ by changing to spherical polar coordinates.

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LA & C

Unit-I

1) Determine the eigen values of $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

2) Obtain the rank of the matrix $\begin{bmatrix} -1 & 3 \\ 0 & 2 \end{bmatrix}$

3) Find the rank of the matrix (i) $A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 1 & -9 & 6 \\ 4 & 8 & 5 \\ 7 & 9 & 4 \end{bmatrix}$

4) Test the system for consistency and if consistent solve it

(i) $u+2v+2w=1$, $2u+v+w=2$, $3u+2v+2w=3$, $v+w=0$

5 (ii) $2x+y+5z=4$, $3x-2y+2z=2$; $5x-8y-4z=1$

(iii) $2x-y+4z=12$, $3x+2y+z=10$, $x+y+z=6$.

(iv) $x+y+z+t=4$, $x-z+2t=2$, $y+z-3t=-1$, $x+2y-z+t=3$

5) P.T. the eigen values of an orthogonal matrix are of unit modulus.

6) Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ using Caley-Hamilton Th.

7) write the condition for the system $AX=B$ is consistent.

8) Find the rank of the matrix $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by reducing it to normal form.

9) what is symmetric matrix? Give proper example.

2 Marks Questions:

10) Define symmetric matrix and if $\begin{bmatrix} 3 & a & b \\ -2 & 2 & 4 \\ 7 & 4 & 5 \end{bmatrix}$ is symmetric, then $(a,b) =$ ______

11) If $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & b & c \end{bmatrix}$ is orthogonal then $(|a|,|b|,|c|) =$ —

12) Find the rank of $\begin{bmatrix} 0 & 3 & 1 \\ 2 & 3 & 5 \\ 2 & 1 & 2 \end{bmatrix}$

13) What is the rank of the matrix $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & -2 & 3 & 0 \\ 0 & 0 & 4 & 8 \\ 2 & 4 & 0 & 6 \end{bmatrix}$

14) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$

15) S.T the matrix $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ is skew symmetric.

16) Obtain a real symmetric matrix of the Q.F

$$3x_1^2 - 2x_2^2 - 2x_3^2 - 4x_1x_2 + 12x_2x_3 + 8x_1x_3.$$

17) Find the rank of $\begin{bmatrix} 4 & -2 & 2 \\ 5 & 3 & 2 \\ 2 & 4 & 1 \end{bmatrix}$

18) Find the eigen values of the matrix $\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$

~~19) P.T the eigen values of her~~

19) Define the rank of a matrix

20) State C-H T.

21) Find the rank of the matrix $\begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$

22) " the eigen values of $A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$

10 (or) 5 Marks Questions

1) Solve the system of homogeneous equations given by $2x+y+2z=0$, $x+y+3z=0$, $4x+3y+8z=0$

2) Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$ to canonical form and hence find its rank.

3) Diagonalize the following matrix by an orthogonal transformation and also find the matrix of transformation. $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 3 \end{bmatrix}$

4) Reduce the matrix $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$

5) Test for Consistency and if consistent solve the system $5x+3y+7t=4$, $3x+26y+2t=9$, $7x+2y+10t=5$

6) Diagonalize the following matrix by an orthogonal transformation and also find the matrix of transformation (i) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 7 & 0 & -2 \\ 0 & 5 & -2 \\ -2 & -2 & 6 \end{bmatrix}$

7) Reduce the matrix $\begin{bmatrix} 1 & -2 & 1 & 2 \\ 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \end{bmatrix}$ into the normal form and hence find its rank.

8) Reduce the matrix $\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & 5 \\ 1 & 3 & 2 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ into normal form and hence find its rank.

9) If $A = \begin{bmatrix} 1 & -1 & -1 & 2 \\ 4 & 2 & 2 & -1 \\ 2 & 2 & 0 & -2 \end{bmatrix}$ find two non-singular matrices P and Q such that PAQ is in the normal form.

5)

10) Determine the nature, index and signature of the Q.F: $-4x_1^2 - 2x_2^2 - 13x_3^2 - 4x_1x_2 - 8x_2x_3 - 4x_3x_1$

11) Reduce the Q.F $3x^2+5y^2+3z^2-2yz+2zx-2xy$ to the canonical form. Also specify the matrix of transformation.

12) Reduce the Q.F $5x^2+26y^2+10z^2+4yz+6xy+14xz$ to canonical form by orthogonalization.

13) Determine the rank of $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 0 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

14) Find the characteristic equation of the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ hence find A^{-1}.

15) Find a matrix P which transform the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ to diagonal form.

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16) Determine A^{-1}, A^{-2} if $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ using C-H-T.

17) Reduce the Q.F $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 3x$ to the canonical form also find rank, index and signature.

18) Reduce the Q.F $x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ by orthogonal transformation. Find rank, signature and nature

19) If $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ then find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ and also find A^{-1}

20) Reduce the Q.F $2xy + 2xz - 2yz$ to a Canonical form and also find its nature.

21) Find the C.E of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ and hence find its inverse, using Cayley Hamilton Th..

22) Find a matrix P which transforms the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ to diagonal form.

23) Diagonalise the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$

24) Reduce the Q.F $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ to Canonical form and also find its corresponding linear transform.

25) Determine the eigen values & corresponding eigen vectors of the following system.
$10x_1 + 2x_2 + x_3 = \lambda x_1$, $2x_1 + 10x_2 + x_3 = \lambda x_2$; $2x_1 + x_2 + 10x_3 = \lambda x_3$

26) Reduce the Q.F $Q = x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ to a Canonical form.

27) Determine A^{-1}, A^{-2} if $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ using C-H-T.

28) Using C-H-T find the inverse of $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ and also find A^3.

29) ~~Reduce to Q.F~~

Determine the values of λ for which the following system of equations has non-trivial solutions. Find them.

$$(\lambda-1)x + (3\lambda+1)y + 2\lambda z = 0$$
$$(\lambda-1)x + (4\lambda-2)y + (\lambda+3)z = 0$$
$$2x + (3\lambda+1)y + 3(\lambda-1)z = 0$$

30) Find the eigen vectors of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ and hence reduce the Q.F $x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ to a sum of squares. Also write its nature.

31) Test for consistency and solve the following system of eqn.

$x+2y+z=3$, $2x+3y+2z=5$, $3x-5y+5z=2$, $3x+9y-z=4$

32) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$, $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

33) Test for consistency and solve $x+2y+3z=14$, $4x+5y+7z=35$, $3x+3y+4z=21$

34) Find A^{-1} of $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}$ using C-H-T.

35) Investigate for what values of λ & μ the simultaneous equations:

$x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$

have (i) no solution (ii) unique soln. (iii) infinite no. of solns

36) Q.F.: $x^2+2y^2+3z^2-2yz-2zx-2xy$

37) P.T. eigen values of an orthogonal matrix are unit modulus

38) Test for consistency and solve

$5x+3y+7z=4$, $3x+5y+2z=9$, $7x+2y+3z=4$

39) ~~Prd~~ state C-H-T and if

$$A=\begin{pmatrix} 1 & 2 & -2 \\ 1 & -1 & 2 \\ -1 & -4 & -3 \end{pmatrix}$$ evaluate A^{-1}, A^{-2}

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