UNIT-1

Transient Analysis

When a network is switched from one condition to another by change it's applied voltage or by change in one of the circuit elements during the period of time, branch currents and voltages are change from perivous values to new values. This time intervals is called transition Period the response of output of the circuit. During transition period is called Triasient Response

A network in which branch currents and node voltages are not changing with resepct to time is said to be studystate

Initial Conditions:

Let us consider an n^{th} order differential eqn

$$a_0 \frac{d^n i}{dt^n} + a_1 \frac{d^{n-1} i}{dt^{n-1}} + \cdots + a_{n-1} \frac{di}{dt} + a_n i = v(t) \quad \text{— (1)}$$

Initial conditions of the element in the network must be calculated to evaluate arbitary constants in general from of differential eqn

where

$a_0, a_1, a_2 \cdots a_{n-1} + a_n$ are the constants

$i(t)$ is dependent variable w.r to circuit

$v(t)$ is Independent variable w.r to circuit

$V(t)$ is a voltage called input or forcing function or exitation

The solution of above equation is known as respons of the system

Ex

If we applying Kirchhoff's laws to the network conting energy storing elements like inductor and capacitors and energy descipating elements like resistor, resuils in a differential equation to Solutions contists of two parts they are complementary function and Particular Solution

* Complementary function represents transient response which decays w.r to time.
* Particular solution represents study state part of the solution.

first order

$$a_0 \frac{di}{dt} + a_1 i(t) \ldots\ldots = v(t)$$

The above equation can be classified in-to two ways

$$a_0 \frac{di(t)}{dt} + a_1 i(t) = v(t)$$

* Homogenous equation
* Non-Homogenous equation

Homogenous equation having zero (0) response

Now

$$a_0 \frac{di(t)}{dt} + a_1 i(t) = 0 \text{ — Homogenous}$$

Non-Homogenous equation is a linear equation which involves in the function $v(t)$

Now

$$a_0 \frac{di(t)}{dt} + a_1 i(t) = v(t) \text{ — Non-Homogenous}$$

Solution of Homogeneous equation:

Finding Solution means to get the expression of $i(t)$

$$a_0 \cdot \frac{di(t)}{dt} = -a_1 \, i(t)$$

$$a_0 \frac{di(t)}{i(t)} = -a_1 \cdot dt$$

$$\frac{di(t)}{i(t)} = \frac{-a_1}{a_0} dt$$

Integration on b.s

$$\int \frac{di(t)}{i(t)} = \int \frac{-a_1}{a_0} dt$$

$$\int \frac{1}{i(t)} di(t) = \frac{-a_1}{a_0} \int 1 dt + K$$

$$\log i = \frac{-a_1}{a_0} t + K$$

where 'K' is the Integration constant

Take log on b.s

$$\log i = \log\left(\frac{-a_1}{a_0}\right) t + \log K$$

$$\log i = \log \left[e^{-a_1/a_0 t}\right] + \log K$$

$$\log i = + \log \left[e^{-\frac{a_1}{a_0} t} K\right]$$

If integration constant is unknown then the solution is general solution

If some initial information is provied about the network for which constant of integration is evoluted then the solution is called as particular solution

Ex:

If we know that at $t=0$ the value of $i(t)$ is a_2 in particular solution

* for higher order differential equation the no. of orbitary constants are equal to order of the equation.

* If this unknows are to be evoluled for the paticular solution all conditions must be known.

* lets us assume that referance time t=0, network condition is changed by switching action, Assume that switches opparate at zero time network condition at this instant called initial condition of network.

* To distiduigish the time Just before and just immedtly after the condition they will use negitive and Positive signs this $t(0)^-$ and $t(0)^+$

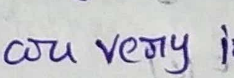
where $t(0)^-$ is instant at which the condition of the network is not at change but it is above to the change

$t(0)^+$ is the instant at which the condition of the network is just changed

Initial conditions of the network depends upon past condition before instant $(t=0^-)$: This conditions are at $t=0^-$ are given by voltage across the capacitor and current to the inductor. This are very important value because after switching $(t=0^+)$ new current's and voltages are appered in the network.

Initial Conditions of Ideal Elements

Resistor:



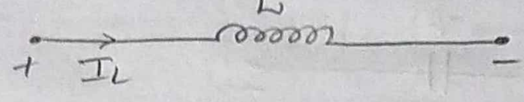
A Resistor both current and voltages are Proportional to with each other and they are linear relationship and also both are Independent of time

Hence, Resistoor allows instantanuous change of currents and voltages

$$V = IR \Rightarrow I = \frac{V}{R}$$

$$\uparrow\downarrow V \propto I \downarrow\uparrow$$

<u>Inductoor:</u>



The voltage equation of inductoor is given by

$$V_L = L \cdot \frac{di_L}{dt}$$

If DC current thorough the inductoor $\frac{di_L}{dt} = 0$. AS DC current is constant w.r. to time hence voltage across the inductoor $V_L = 0$. so foor the dc quanties are consideor in study state inductoor acts as shoort ~~ckt~~ cioncuit.

Inductoor current is given by $I_L = \frac{1}{L}\int v_L dt$

Limits of integration are decided by the porevious histoory that is i.e; $-\infty$ to $t(0^-)$

Let switching takes place at $t=0$. we can spilts the limits into $-\infty$ to 0^- and 0 to ∞

$$I_L = \frac{1}{L}\int_{-\infty}^{t} v_L dt$$

$$I_L = \frac{1}{L}\int_{-\infty}^{0^-} v_L dt + \frac{1}{L}\int_{0^+}^{t} v_L dt$$

In the above equation $\frac{1}{L}\int_{-\infty}^{0} v_L dt$ oreporesents porevious history of the inductoor current $I_L = i(0^-) + \frac{1}{L}\int_{0}^{t} v_L dt$

At $t = 0^+$ Inductoor current can be oreturn as

$$I_L(0^+) = I_L(0) - \frac{1}{L}\int_{0}^{0^+} v_L dt$$

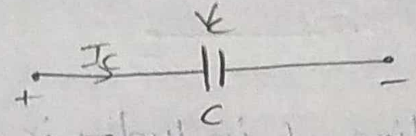
$I_L(0^-) = I_L(0^+)$ current through the inductor can

change instantanous.

i.e, current through the inductor before

and after is same

Capacitor:



Current through the capacitor is given by $I_c = C \cdot \frac{dv_c}{dt}$.

If dc voltage is applied to capacitor $\frac{dv_c}{dt}$ is becomes zero

as DC voltage constant w.r.to time.

Hence current through the capacitor becomes zero

As far as DC quntities are consider capacitor acts as open circuit

voltage across the capacitor is given $V_c = \frac{1}{C}\int i_c dt$

$$V_c = \frac{1}{C}\int_{-\infty}^{t} i_c dt$$

The limits can be splited in to $-\infty$ to 0^- and 0^- to t

$$V_c = \frac{1}{C}\int_{-\infty}^{0^-} i_c dt + \frac{1}{C}\int_{0^-}^{t} i_c dt$$

$$V_c = V_c(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i_c dt$$

$$V_c(0^+) = V_c(0^-) + 0$$

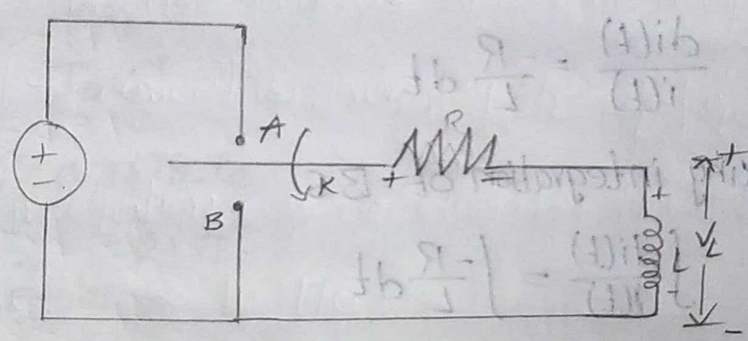
$$V_c(0^+) = V_c(0^-)$$

It doesn't allows sudden change

i.e, current through the capacitor before and after switching same.

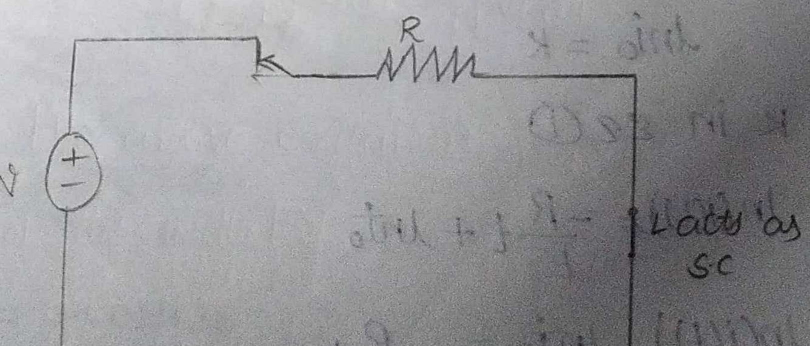
S.No	Element	Behaviour in study state	Behaviour at $t \to \infty$
1	Resistor	Resistor	Resistor
2.	L (inductor)	SC	OC
3.	I, V_c (capacitor), + −	OC	SC
4	$\to I_0$ (inductor), + −	I (current source), + −	(current source), + −
5.	k, V_0, C, + −	V (voltage source), + −	V (voltage source), + −

Response of source free (or) and revene series RL circuit:



Consider a series RL circuit. Let switch is kept at position 'A'. Before transition network is in study state assume at $t=0$. Now switch is move to position 'B'.

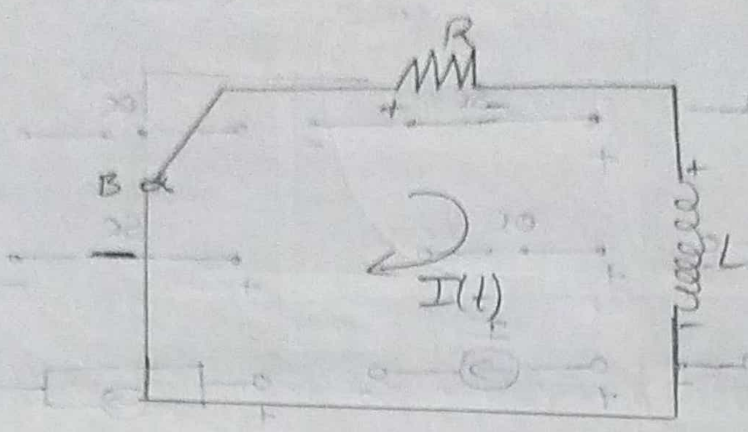
At $t = 0^-$ (at Position A)



$$I(0^-) = I(0^+) \Rightarrow I_0 \Rightarrow \frac{V}{R}$$

Because current to the inductor can not change instantly

at $t \geq 0$ at position B



Applying KVL

$$-R\cdot I(t) + L\frac{(di(t))}{dt} = 0$$

$$-L\frac{di(t)}{dt} = RI(t)$$

$$i(t) = -\frac{L}{R}\frac{di(t)}{dt}$$

$$\frac{di(t)}{i(t)} = -\frac{R}{L}dt$$

Taking integration of B.S

$$\int \frac{di(t)}{i(t)} = \int -\frac{R}{L}dt$$

$$\ln(i(t)) = -\frac{R}{L}t + K \quad (1)$$

$$\cancel{i(t) = e^{-R/L\,t + K}}$$

Initially condition $t=0$

$$\ln i_0 = -\frac{R}{L}0 + K$$

$$\ln i_0 = K$$

sub K in eq (1)

$$\ln(i(t)) = -\frac{R}{L}t + \ln i_0$$

$$\ln(i(t)) - \ln I_0 = -\frac{R}{L}t$$

$$\ln \frac{i(t)}{I_0} = \frac{-R}{L} t$$

$$\frac{i(t)}{I_0} = e^{-R/L \cdot t}$$

I_0 = maximum value of initial current

$$i(t) = I_0 \cdot e^{-R/L \cdot t}$$

From the above equation in source we RL series circuit the current through the inductor is exponitialy decreses

let us substitute different values of T as T, 2T, 4T, 6T ---

where T → Time constant

$$\therefore T = \frac{L}{R}$$

Time = 1T

$$i(t) = I_0 \cdot e^{-R/L \cdot T}$$

$$= I_0 e^{(-\frac{R}{L}) \cdot (L/R)}$$

$$= I_0 e^{-1}$$

$$i(t) = 0.3678 I_0$$

t = 2T

$$i(t) = I_0 e^{-R/L \, 2T}$$

$$= I_0 e^{-R/L \cdot 2(L/R)}$$

$$= I_0 e^{-2}$$

$$i(t) = 0.1353 I_0$$

t = 4T

$$i(t) = I_0 e^{-R/L \cdot 4T}$$

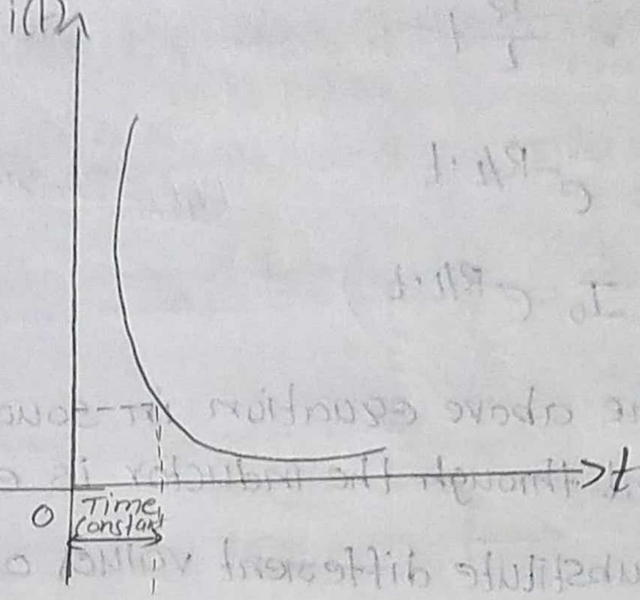
$$= I_0 e^{-R/L (L/R) 4}$$

$$= I_0 e^{-4}$$

$$i(t) = 0.0183 I_0$$

t = 6T

$$i(t) = 0.0024 I_0$$



Initial the current through the inductoor decreses rapidly to $0.3678 I_0$ times after that the rate of decaying slow down and reaches to study state and appoormently t=

Voltage across the inductoor is given by $v_L = L \cdot \frac{di(t)}{dt}$

$$v_L = L \cdot \frac{d\,i(t)}{dt}$$

$$i(t) = I_0 e^{-R/L t}$$

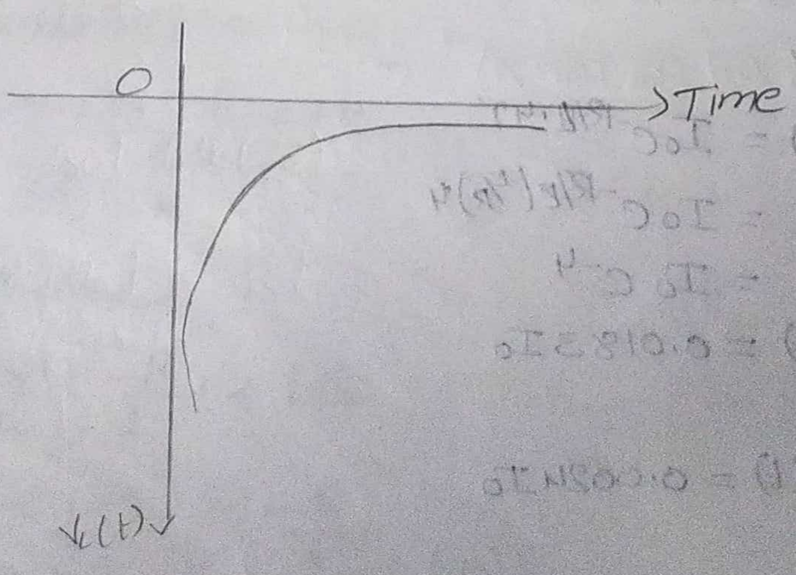
$$v_L = L \frac{d}{dt}\left[I_0 e^{-R/L\, t}\right]$$

$$v_L = L \cdot I_0 \left(-\frac{R}{L}\right) \cdot e^{-R/L\, t}$$

$$v_L = -I_0 R \cdot e^{-R/L\, t}$$

But we known that $V = I_0 R$

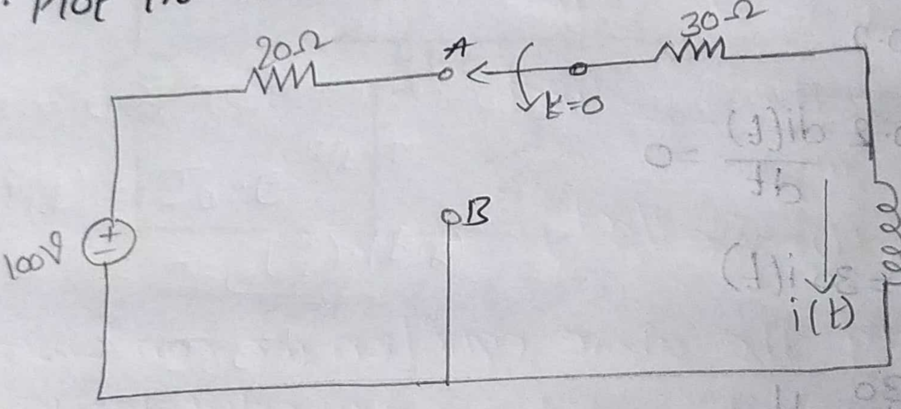
$$v_L(t) = -V \cdot e^{-R/L \cdot t}\ v$$



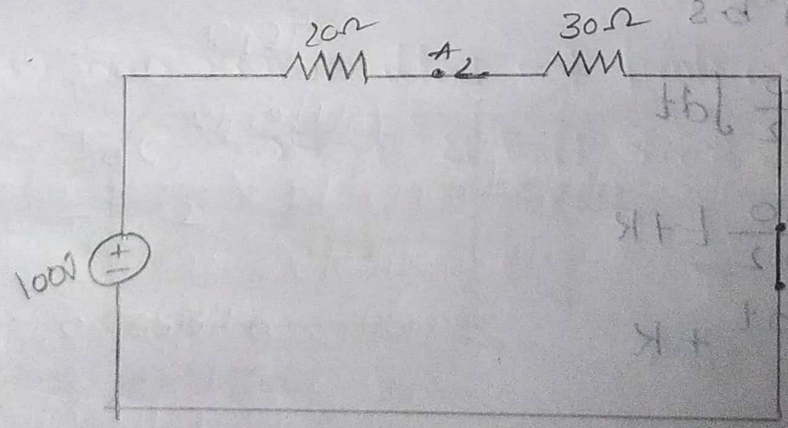
* For physicaly (or) general interpretation before switch is move to position B the stored energy in the inductor is $\frac{1}{2}LI^2$ and energy dissipated in the resistor is I^2R.

* After source removed energy stored in the inductor is totally dissipated by the resistor

The circuit shown in the fig, intially the switch is kept at position A for long time. At $t=0$ switch is moved to position B. Find the expression for the current for $t>0$. find the t point 3334 m/s, the value of current at $t = 6.6667$ m/s, the variation of current through the inductor 20 m/s. Plot the



At position A

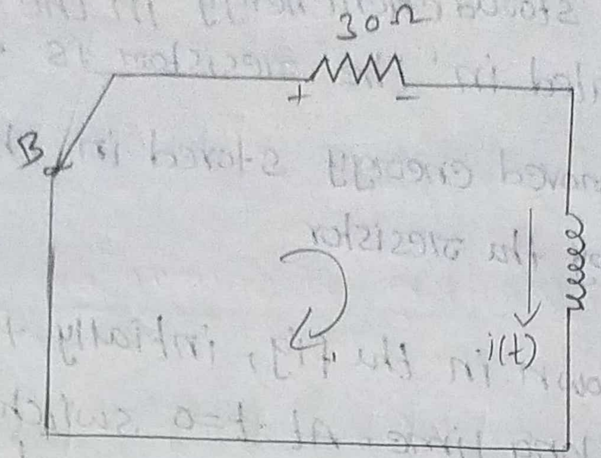


$t=0^-$

$$i(0^-) = i(0^+) = I_0 = \frac{V}{R}$$

$$i(0^-) = I_0 = \frac{100}{20+30} = 2A$$

At Position B



By Applying KVL

$$-i(t)\cdot R\ L\cdot\frac{di(t)}{dt}=0$$

$$-i(t)30-0.2$$

$$=-30i(t)-0.2\frac{di(t)}{dt}=0$$

$$=-0.2\frac{di(t)}{dt}=30\,i(t)$$

$$\frac{di(t)}{i(t)}=\frac{-30}{0.2}dt$$

Integrat on b.s

$$\int\frac{di(t)}{i(t)}=\frac{-30}{0.2}\int dt$$

$$\ln[i(t)]=\frac{-30}{0.2}t+k$$

$$i(t)=e^{-150t}+k$$

At $t=0$

$$\ln[i(t)]=-150t+\ln(I_0)$$

$$=\ln[i(t)]-\ln[I_0]=-150t$$

$$=\ln[i(t)/I_0]=e^{-150t}$$

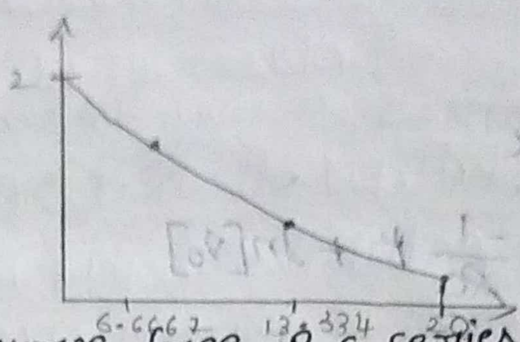
$$i(t)=I_0e^{-150t}$$

at $t = 6.6667\ ms$

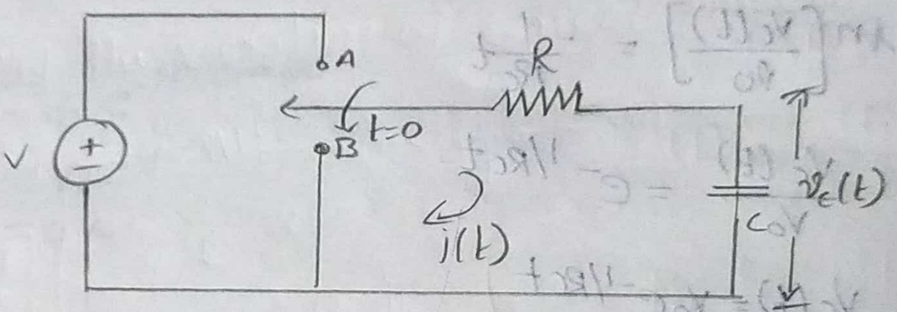
$i(6.6667) = 2 \times e^{-150 \times 6.6667 \times 10^{-3}} = 0.7357$

$i(13.3334) = 2 \times e^{-150 \times 13.3334 \times 10^{-3}} = 0.2706$

$i(20) = 2 \times e^{-150 \times 20 \times 10^{-3}} = 0.095$



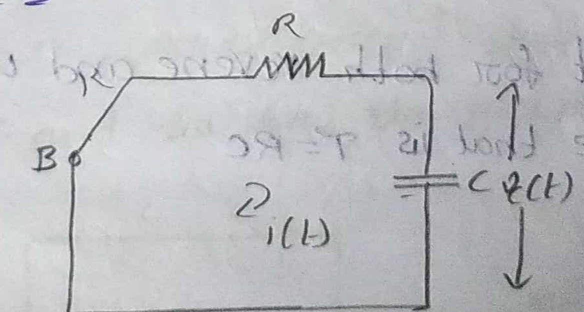
Response of source free R-C series circuit:



$v(0) = V_0 = v_c(0^+)$

capacitor doesn't allow the sudden change

At position B



Applying ~~KVL~~ KVL

$-Ri(t) - v_c(t) = 0$

$i(t)$

$i_c(t) = C\frac{dv_c(t)}{dt}$

$RC\frac{dv_c(t)}{dt} = -v_c\, dt$

$\frac{dv_c(t)}{v(t)} = \frac{-1}{RC}dt$

Integration on bs

$$\int \frac{dV(t)}{dt} = \int -\frac{1}{RC} \int dt$$

$$\ln[V_c(t)] = \frac{-1}{RC} t + K$$

At $t=0$

$$\ln[V_0] = K$$

$$\ln[V_c(t)] = \frac{-1}{RC} t + \ln[V_0]$$

$$\ln[V_c(t)] - \ln[V_0] = \frac{-1}{RC} t$$

$$\ln\left[\frac{V_c(t)}{V_0}\right] = \frac{-1}{RC} t$$

$$\frac{V_c(t)}{V_0} = e^{-1/RC\, t}$$

$$V_c(t) = V_0 e^{-1/RC\, t}$$

By observing the above equation we can observe that to voltage across the capacitor is expoential decayed by V_0 times. Th.

The time constant for both revene and unrevene series RC circuits is same that is $T = RC$

Now

$v(t)$ at $t = T$

$$V_c(t) = V_0 e^{-1/RC\, T}$$

$$= V_0 e^{-1/RC \cdot RC}$$

$$= V_0 e^{-1}$$

$$= 0.367 V_0$$

$t = 2T$

$$V_c(t) = V_0 e^{-1/RC\, 2T}$$

$$= V_0 e^{-1/RC\, 2RC}$$

$$= V_0 e^{-2}$$

$$= 0.135 V_0$$

$t = 4\tau$

$$V_c(t) = V_0 e^{-1/RC \, 4\tau}$$
$$= V_0 e^{-1/RC \cdot 4RC}$$
$$= V_0 e^{-4}$$
$$= 0.0183 V_0$$

$t = 6\tau$

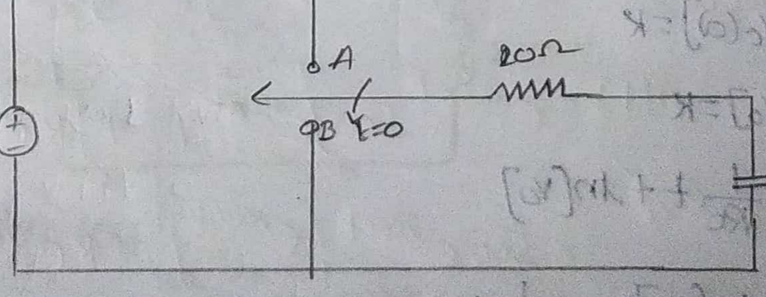
$$V_c(t) = V_0 e^{-1/RC \, 6\tau}$$
$$= V_0 e^{-1/RC \, 6(RC)}$$
$$= V_0 e^{-6}$$
$$= 2.478 V_0$$

$t = \infty$

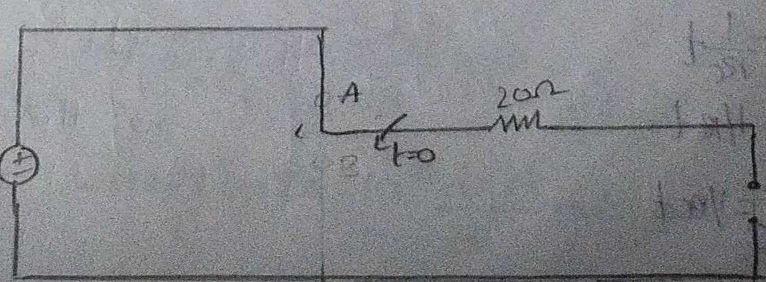
$$V_c(t) = V_0 e^{-1/RC \, \infty RC}$$
$$= V_0 e^{-\infty}$$
$$= 0 V_0$$

The voltage across the capacitor reaches to study state it is equal approximent at 6τ (or) 8τ and becomes zero at ∞

for a given RC-series circuit shown in fig where $R = 20\Omega$ and $C = 0.5f$ and the find the response $V_c(t)$

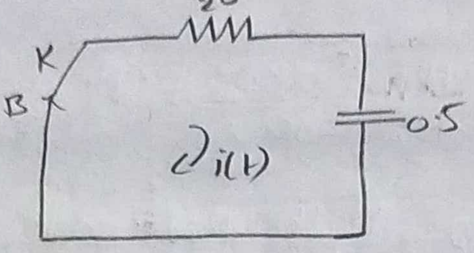


At position A



$V(0^-) = V_0 = V_0(0^+)$

At Position B



Applying KVL

$-Ri(t) - V_c(t) = 0$

$-Rc \dfrac{dV_c(t)}{dt} = V_c(t)$

$\dfrac{dV_c(t)}{V_c(t)} = \dfrac{-1}{Rc} dt$

I.O.B.S

$\displaystyle\int \frac{dV_c(t)}{V_c(t)} = \frac{-1}{Rc}\int dt$

$\ln[V_c[t]] = \dfrac{-1}{Rc} t + K$

At $t=0$

$\ln[V_c(0)] = K$

$\ln[V_0] = K$

$\ln[V_c(t)] = \dfrac{-1}{Rc} t + \ln[V_0]$

$\ln[V_c(t)] - \ln[V_0] = -\dfrac{1}{Rc} t$

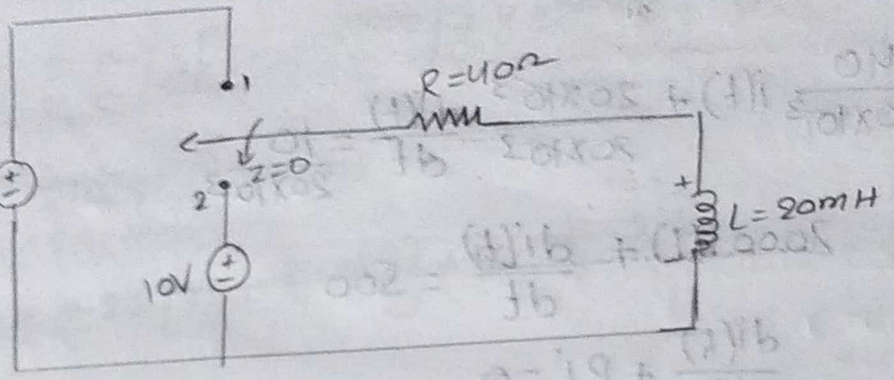
$\ln[V_c(t)/V_0] = \dfrac{-1}{Rc} t$

$V_c(t) = V_0 e^{-1/Rc\, t}$

$V_c(t) = 100 e^{-1/Rc\, t}$

Non-Homogenous : (RL circuit)

The network shown in the fig is under study state condition with switch K is at Position 1. find the express for $i(t)$ if switch is moved to Position 2. Draw the variation of $i(t)$



At position 1

At $t=0^-$

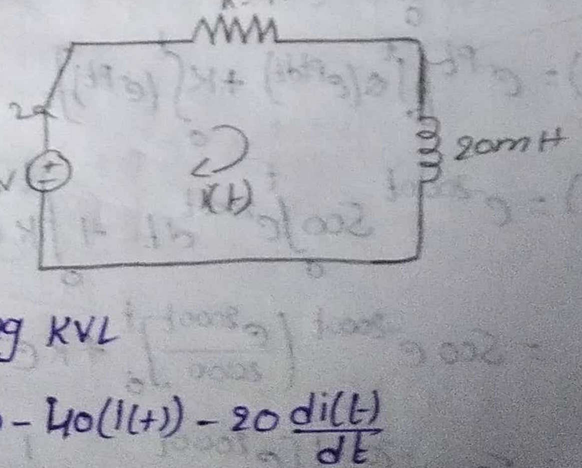
$$i(0^-) = I_0 = \frac{V}{R} = i(0^+)$$

$$= I_0 = \frac{40}{40} = 1A$$

At Position 2

At $t \geq 0^+$

At switch is moved to position 2



Applying KVL

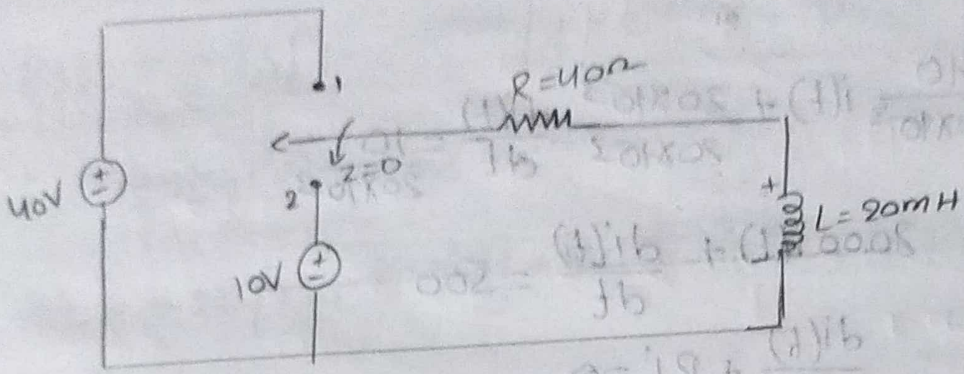
$$10 - 40(i(t)) - 20\frac{di(t)}{dt}$$

$$-40i(t) - 20^{mH}\frac{di(t)}{dt} = 10$$

$$-40i(t) - 20\times10^{-3}\frac{di(t)}{dt} + 10 = 0$$

Non-Homogenous (RL circuit)

The network shown in the fig is under study state condition with switch K is at Position 1. Find the express for $i(t)$ if switch is moved to position 2. Draw the variation of $i(t)$



At position 1

At $t=0^-$

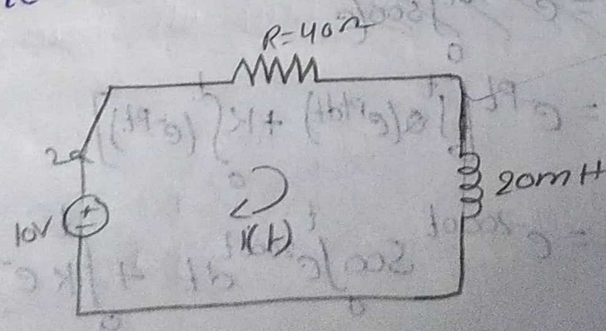
$$i(0^-) = I_0 = \frac{V}{R} = i(0^+)$$

$$= I_0 = \frac{40}{40} = 1A$$

At Position 2

At $t \geq 0^+$

At switch is moved to position 2



Applying KVL

$$10 - 40(i(t)) - 20\frac{di(t)}{dt}$$

$$-40i(t) - 20^{mH}\frac{di(t)}{dt} = 10$$

$$-40i(t) - 20\times10^{-3}\frac{di(t)}{dt} + 10 = 0$$

$$40\, i(t) + 20\times10^{-3}\frac{di(t)}{dt} = 10$$

Standed form of first order non-homogenous differential equ is given by

$$\frac{di(t)}{dt} + Pi = Q$$

$$\frac{40}{20\times10^{-3}} i(t) + \frac{20\times10^{-3}}{20\times10^{-3}} \frac{di(t)}{dt} = \frac{10}{20\times10^{-3}}$$

$$2000\, i(t) + \frac{di(t)}{dt} = 500$$

$$\frac{di(t)}{dt} + Pi' = Q$$

Compare the above equation with standar form of first order non-homogenous D.E

$$P = 2000$$
$$Q = 500$$

The solution for first order non-homogenous D.E is given by

$$i(t) = e^{-Pt}\int_0^t Q(e^{Pt}dt + K(e^{-Pt}))$$

$$\cancel{i(t) = e^{-2000t}\int_0^t 500(e}$$

$$i(t) = e^{-Pt}\left[\int_0^t Q(e^{Pt}dt) + K\int_0^t (e^{-Pt})\right]$$

$$i(t) = e^{-2000t}\, 500\int_0^t e^{2000t}dt + \int_0^t Ke^{-2000t}$$

$$= 500e^{-2000t}\left(\frac{e^{2000t}}{2000}\right]_0^t + Ke^{-2000t}$$

$$= 500e^{-2000t}\left[\frac{e^{2000t}}{2000} - \frac{1}{2000}\right] + Ke^{-2000t}$$

$$= 500e^{-2000t}\frac{1}{2000}\left[e^{2000t} - 1\right] + Ke^{-2000t}$$

$$= \frac{500}{2000} e^{-2000t}\left[e^{2000} - 1\right] + Ke^{-2000t}$$

$$= 0.25\, e^{-2000t}\left[e^{2000} - 1\right] + Ke^{-2000t}$$

$$= 0.25\left[\frac{e^{-2000}}{e^{-2000t}} - e^{-2000t}\right] + Ke^{-2000t}$$

$$i(t) = 0.25\left[1 - e^{-2000t}\right] + Ke^{-2000t} \quad -①$$

At $t=0 \Rightarrow$ intial conditions

$$i(0) = 0.25\left[1 - e^{-2000(0)}\right] + Ke^{-2000(0)}$$

$$I_0 = 0 + K$$

$$I_0 = K$$

$$\boxed{K=1}$$

sub in eq ①

$$i(t) = 0.25\left(\left[1 - e^{-2000t}\right] + e^{-2000t}\right)$$

$$\cancel{i(t) = 0.25(e^{-2000t})(1 + e^{-2000t})}$$

$$i(t) = 0.25 + 0.75e^{-2000t}$$

$t = T$

$t = 1T$

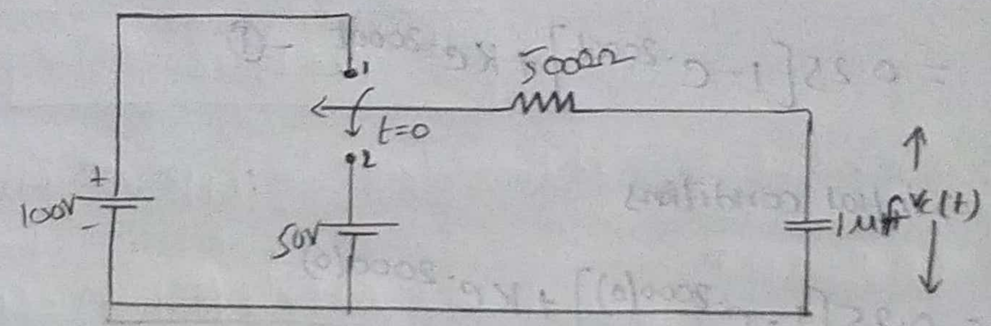
$$i(t) = 0.25 + 0.75e^{-2000(1)}$$

$$i(t) = 0.25 + 0$$

$$i(t) = 0.25$$

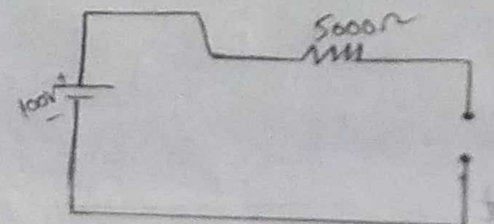
Non-Homogenous (RC-circuit)

The switch is moved from position 1 to position 2 and at t and find the voltages $V_c(t)$ and $V_r(t)$ for $t>0$



At Position 1

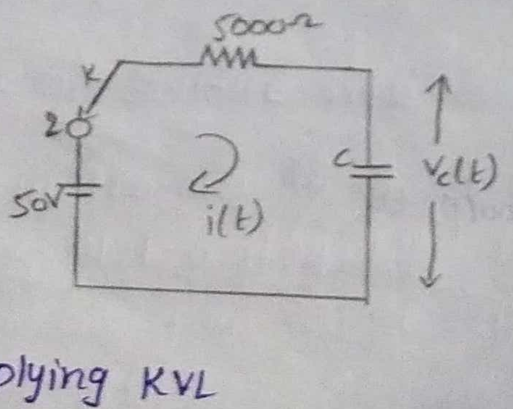
At $t=0^-$ position switch is connected to position



$$V(0^-) = V_0 = V_0(0^+) = 100V$$

At Position 2

At $t \geq 0^+$



The current through the resistor and capacitor both are same

$$V_i(t) = i(t)$$

Applying KVL

$$-5000\, i(t) - V_c(t) + 50 = 0$$

$$-5000\, c \frac{dV_c(t)}{dt} - V_c(t) + 50 = 0$$

$$-5000(1\times10^{-6}) \frac{dV_c(t)}{dt} - V_c(t) + 50 = 0$$

$$-(5\times10^{-3})(1\times10^{-6})\frac{dv_c(t)}{dt} - v_c(t)+50=0$$

$$\cancel{(5\times10^{-3})}(5\times10^{-3}\frac{dv_c(t)}{dt}+v_c(t)=50$$

$$=\frac{dv_c(t)}{dt}+200v_c(t)=10000$$

(Applying KVL

$$-5000\,i(t) - v_c(t)+50=0$$

$$-5000\,i(t) - C\cdot\frac{dv_c(t)}{dt}+50=0$$

$$-5000\,i(t)-(1\times10^{-6})\cdot\frac{dv_c(t)}{dt}=-50)$$

The standard form of the first order non-homogenous DE

$$\frac{dv_c}{dt}+Pv_c=Q$$

$$\frac{dv_c(t)}{dt}+200v_c(t)=10,000$$

Compare this in above equation

$$P=200 \quad Q=10000$$

$$v_c(t)=e^{-Pt}\int_0^t Q\,e^{Pt}dt+K\cdot e^{-Pt}$$

$$v_c(t)=e^{-200t}\int_0^t 10000\,e^{200t}dt+K\cdot e^{-200t}$$

$$v_c(t)=e^{-200t}\cdot 10,000\left[\frac{e^{200t}}{200}\right]_0^t+K\cdot e^{-200t}$$

$$v_c(t)=e^{-200t}\cdot 10000\left[\frac{e^{200t}}{200}-\frac{e^0}{200}\right]+K\cdot e^{-200t}$$

$$v_c(t)=e^{-200t}\cdot\frac{10,000}{200}\left[e^{200t}-1\right]+K\cdot e^{-200t}$$

$$v_c(t)=e^{-200t}\cdot 50\left[1-e^{-200t}\right]+K\cdot e^{-200t}$$

At $t=0$

$$v_c(t)=e^{-200(0)}\cdot 50\left[1-e^{-200(0)}\right]+K e^{-200(0)}$$

$$v_c(t)=K \Rightarrow K=100$$

$$v_c(t) = 50(1 - e^{-200t}) + 100e^{-200t}$$

$$v_c(t) = 50 - 50(e^{-200t}) + 100e^{-200t}$$

$$v_c(t) = 50 + 50e^{-200t}$$

$$\boxed{v_c(t) = 50(1 + e^{-200t})}$$

$$v_r(t) = R \times i(t)$$

$$I(t) = I_c(t) = I_R(t)$$

$$v_r(t) = (1\times10^{-6})\frac{dv_c(t)}{dt}$$

$$I_c(t) = C \cdot \frac{dv_c(t)}{dt}$$

$$= (1\times10^{-6})\frac{d}{dt}\left[50(1 + e^{-200t})\right]$$

$$v_r(t) = R(-0.01e^{-200t})$$

$$= 5000(-0.01e^{-200t})$$

$$= (1\times10^{-6})\left(50\frac{d}{dt} + 50\frac{d}{dt}e^{-200t}\right)$$

$$v_r(t) = -50e^{-200t}$$

$$= (1\times10^{-6})\left[0 + 50(-200)e^{-200t}\right]$$

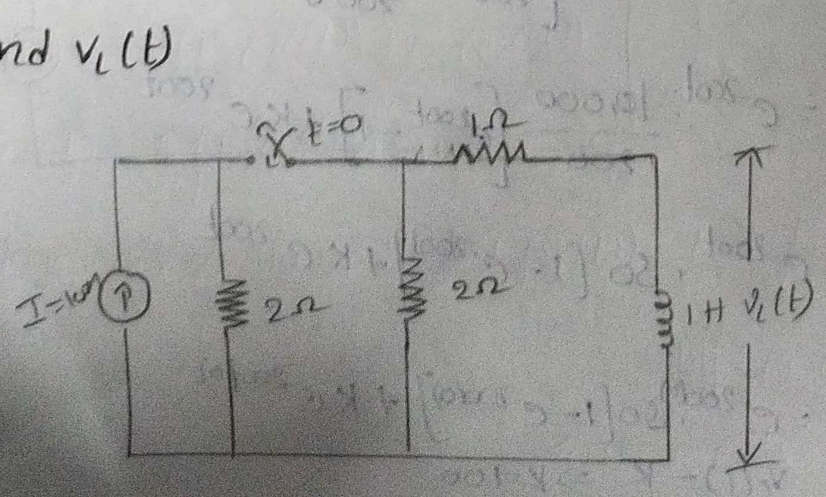
$$= (1\times10^{-6})(1\times10^{-3})e^{-200t}$$

$$= (1\times10^{-3})e^{-200t}$$

$$= 0.01e^{-200t}$$

Response of source free series R-L-C circuit:

Intially in the circuit the switch 'K' is kept open for very long time. At $t=0$ it is closed find the expression for $i(t)$ for $t>0$ and also find voltage across the inductor $v_L(t)$ sketch $i(t)$ and $v_L(t)$

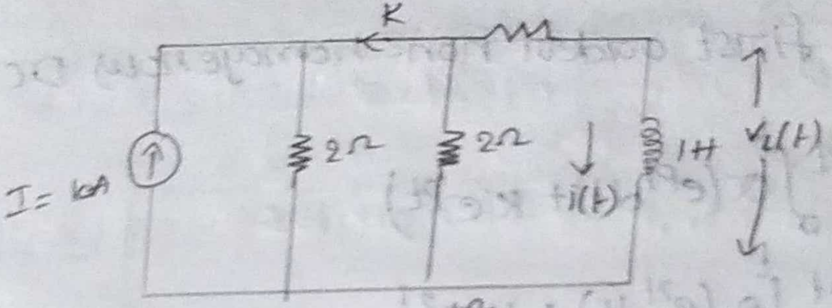


At Position A $t=0^-$

$$i(0^-) = I_0 = I(0^+) = 0$$

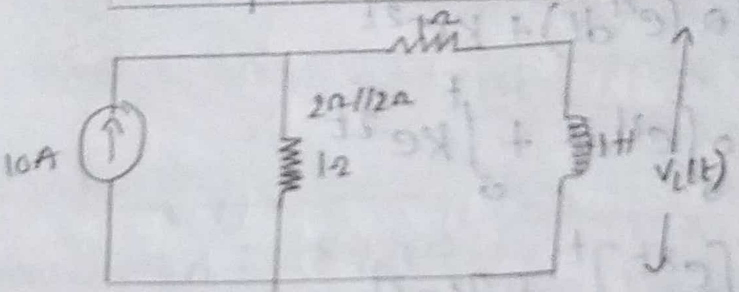
At Position B

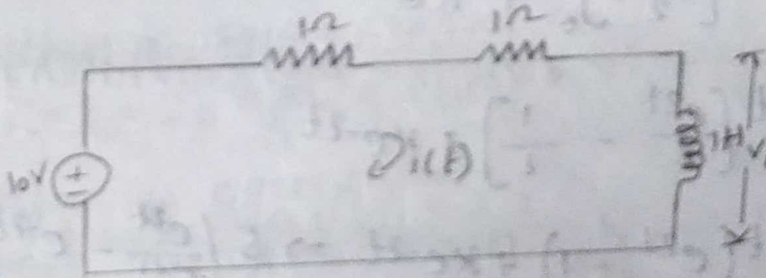
$t>0$



$2\Omega // 2\Omega$

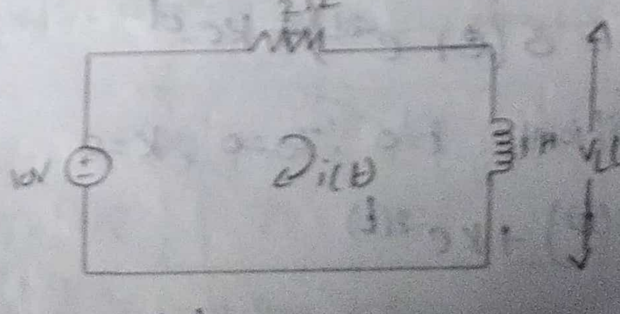
$$\frac{2 \cdot 2}{2+2} = \frac{4}{4} = 1\Omega$$





Series $R_1 + R_2$

$1\Omega + 1\Omega = 2\Omega$



By Applying KVL

$$-2i(t) - \frac{di(t)}{dt} + 10 = 0$$

$$+\left(\frac{di(t)}{dt} + 2i(t)\right) = +10$$

$$\frac{di(t)}{dt} + 2i(t) = 10$$

The standarded form

$$\frac{di(t)}{dt} + Pi(t) = Q$$

compare this in above equation

$$P = 2 \quad Q = 10$$

The solution for first order non-homogenous DE is given by

$$i(t) = e^{-Pt} \int_0^t Q(e^{Pt} dt) + Ke^{-Pt})$$

$$i(t) = e^{-2t} \int_0^t Q(e^{2t} dt) + Ke^{-2t}$$

$$= e^{-2t}\, 10 \int_0^t e^{2t} dt + \int_0^t Ke^{-2t}$$

$$= 10e^{-2t} \left[\frac{e^{2t}}{2}\right]_0^t + Ke^{-2t}$$

$$= 10e^{-2t} \left[\frac{e^{2t}}{2} - \frac{1}{2}\right] + Ke^{-2t}$$

$$= \frac{10}{2} e^{-2t}(e^{2t} - 1) + Ke^{-2t} \Rightarrow 5\left(\frac{e^{2t}}{e^{2t}} - e^{-2t}\right) + Ke^{-2t}$$

$$= 5(1 - e^{-2t}) + Ke^{-2t}$$

At $t=0$ intial conditions $t=0$, $I_0 = 0$, $K = 0$

$$i(t) = 5(1 - e^{-2(t)}) + Ke^{-2(t)}$$

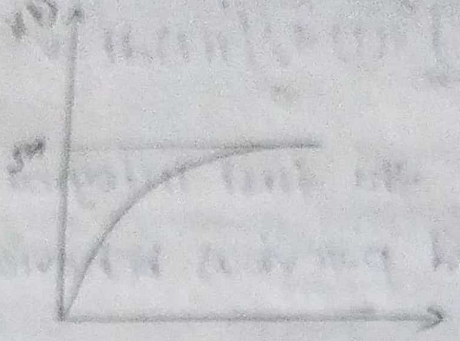
$$i(t) = 5(1 - e^{-2(t)}) + 0 \cdot (e^{-2(t)})$$

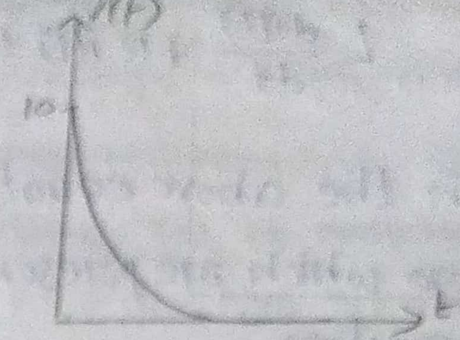
$$i(t) = 5(1 - e^{-2t}) A$$

Voltage Across the inductor $(V_L(t)) = L \frac{di(t)}{dt}$

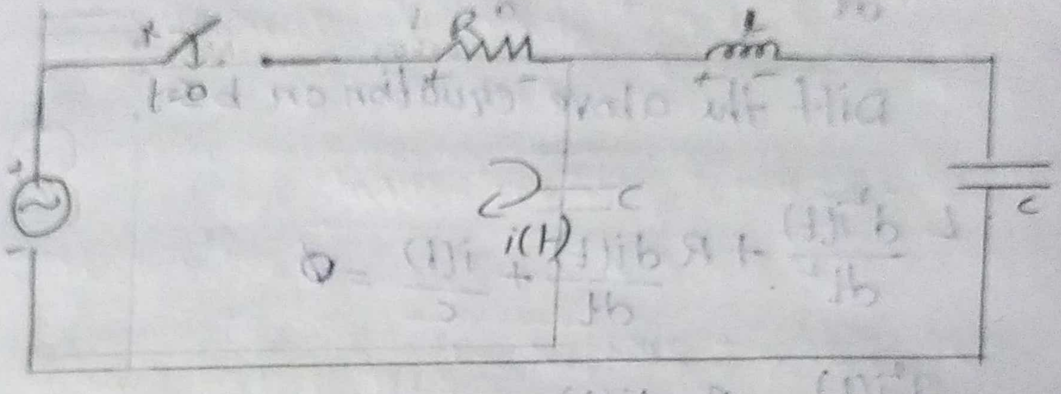
$$= 1 \frac{d}{dt}[5(1 - e^{-2t})] = [5(0 - e^{-2t}(-2))]$$

$$V_L(t) = 10e^{-2t} V$$





Response of source free series RLC circuit:



At position A : at $t(0^-)$:

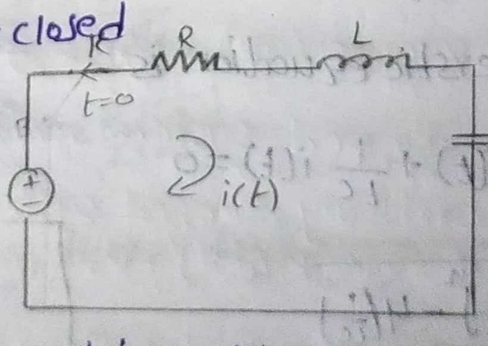
Inductor is short circuit

$i(0^-) = I_0 = i(0^+) = 0$

capacitor is open circuit

$v(0^-) = v_0 = v(0^+) = 0$

$t \geq 0$ switch is closed



By Applying KVL

$$-R(i(t)) - L\frac{di(t)}{dt} - \frac{1}{C}\int i(t)dt + V = 0$$

$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)dt = V$$

$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)dt = V$$

$$L\frac{di(t)}{dt} + R\,i(t) + \frac{1}{C}\int_{-\infty}^{0^-} i(t) + \frac{1}{C}\int_{0^-}^{t} i(t)\,dt = V$$

In the above equation the first integral part becomes zero which represented previous behaviour of the capacitor

Now,

$$L\frac{di(t)}{dt} + R\,i(t) + \frac{1}{C}\int_{0}^{t} i(t)\,dt = V$$

Diff the above equation on b.s

$$L\frac{d^2 i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{i(t)}{C} = 0$$

$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}\,i(t) = 0$$

To get the charactestic equation (or) auxiallary equation replace

$\frac{d^2}{dt^2}$ with s^2 and $\frac{d}{dt}$ with s

Now the characterstic equation is

$$s^2 i(t) + \frac{R}{L} s\,i(t) + \frac{1}{LC}\,i(t) = 0$$

$$\frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\left(\frac{1}{LC}\right)}}{2}$$

$$\therefore Ax^2 + Bx + C \qquad \frac{-B \pm \sqrt{B^2 - 4ac}}{2A}$$

$$s_{1,2} = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

Respons are depended on roots nature

$$s_{1,2} = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}$$

Trans = $\frac{LLO}{LLI}$ in initial value o

Before going to analysis the response of RLC circuit based on the roots of characteristic equations. we should difine the following terms

* Critical Resistence : which is the value of resistance used to reduce the square root value to zero. This is denoted by R_{cr}

$$\frac{R_{cr}}{2L} = \frac{1}{\sqrt{LC}}$$

$$R_{cr} = \frac{2L}{\sqrt{LC}}$$

$$R_{cr} = \frac{2\sqrt{L} \times \sqrt{L}}{\sqrt{L} \times \sqrt{C}}$$

$$\boxed{R_{cr} = 2\sqrt{L/C}}$$

* Damping ratio: It is denoted by ξ

It is difine as actual resistance to the critical resistance

$$\xi = \frac{R}{R_{cr}}$$

$$\xi = \frac{R}{2(\sqrt{L/C})}$$

$$\xi = \frac{R}{2}(\sqrt{C/L})$$

* Un damped natural frequence : It is denoted by wn

It is difined as the opposes due to the circuit becomes '0' then the circuit is oprating with natural frequence it is also called un damped natural frequence. otherwise the system is oprating with damping frequence denoted with wd

$$w_n = \text{natural frequency} = \frac{1}{\sqrt{LC}}$$

damping frequency (w_d) $= w_n\sqrt{1-\xi^2}$

The standard from of 2nd order transfer functions is given by $s^2 + 2\xi w_n s + w_n^2 = 0$

The denominator polynomial of closed looper transfer function of 2nd order system when equaling to zero is nothing but characteristic eqn

characteristic eqn is $s^2 + 2\xi w_n s + w_n^2 = 0$

Let s_1, s_2 are the roots of characteristic eqn then

$$\frac{-2\xi w_n \pm \sqrt{(-2\xi w_n)^2 - 4(1)w_n^2)}}{2(1)}$$

$$\Rightarrow \frac{-2\xi w_n \pm \sqrt{4\xi^2 w_n^2 - 4w_n^2}}{2}$$

$$\Rightarrow \frac{-2\xi w_n \pm 2w_n\sqrt{\xi^2 - 1}}{2}$$

$$\Rightarrow \frac{\cancel{2}(-\xi w_n \pm w_n\sqrt{\xi^2-1})}{\cancel{2}}$$

$$\Rightarrow -\xi w_n \pm w_n\sqrt{\xi^2-1}$$

$s_1 = -\xi w_n + w_n\sqrt{\xi^2-1}$; $s_2 = -\xi w_n - w_n\sqrt{\xi^2-1}$

where damping frequence $(w_d) = w_n\sqrt{1-\xi^2}$

If $\xi = 0$ system is undamped system roots are purely imaginary reponse is $K_1 \times \cos wt + K_2 \times \sin wt$

when $\xi = 1$ system is criticaly damped system roots are real and -ve equal ~~$0<\xi<1$~~ Response $= K_1(e^{-\alpha t}) + K_2 t(e^{-\alpha t})$

when $\xi > 1$ system is overdamped system roots are real and unequal response is $K_1 e^{-s_1 t} + K_2 e^{-s_2 t}$

when $0 < \xi < 1$ system is underdamped system roots are complex and conjugate. reponse $= K_1(e^{-\alpha t}\cos wt) + K_2(e^{-\alpha t}\sin wt)$

The general eqn of characteristic eqn is given by

$$i(t) = K_1 e^{(-\alpha + jw_d)t} + K_2 e^{(-\alpha - jw_d)t}$$

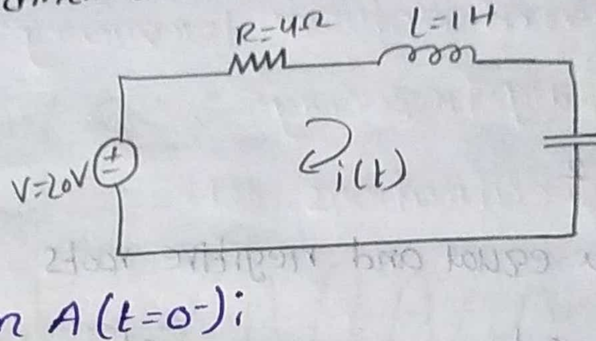
The response of RLC circuit can be written in another form

$i(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$

where $\alpha = \zeta\omega_n$; $\omega_d = \sqrt{1-\zeta^2}$

$\boxed{\text{Note: } e^{j\theta} = \cos\theta + j\sin\theta}$

Find the expression for current in RLC series circuit fed by DC voltage of 20V with $R = 4\Omega$, $L = 1H$, $C = \frac{1}{4}F$. Assume initial condintions to be zero



At position A (t=0⁻):

$I(0^-) = I_0 = I(0^+) = 0$

$V(0^-) = V_0 = V(0^+) = 0$

Applying KVL to the above circuit

$$-4i(t) - 1\frac{di(t)}{dt} - 4\int i(t)dt + 20 = 0$$

$$\frac{di(t)}{dt} + 4i(t) + 4\int i(t)dt = 20$$

$$\frac{di(t)}{dt} + 4i(t) + 4\int_{-\infty}^{t} i(t)\,dt = 20$$

$$\frac{di(t)}{dt} + 4i(t) + \int_{-\infty}^{0^-} i(t)dt + 4\int_{0^-}^{t} i(t)\,dt = 20$$

This is in the form of integro differential eqn

$$\frac{di(t)}{dt} + 4i(t) + 4\int_{0^-}^{t} i(t)dt = 20 \quad -①$$

differentiate the above eqn

$$\frac{d^2i(t)}{dt} + 4\frac{di(t)}{dt} + 4\,i(t) = 0$$

To get the characteristic eqn (or) auxilary eqn replace d^2/dt with s^2 and $\frac{d}{dt}$ with s

Now, the characteristic eqn is

$$s^2 + 4s + 4 = 0$$

This is in the form of $Ax^2 + Bx + C = 0$

$$s^2 i(t) + 4 s i(t) + 4 i(t) = 0$$

$$s^2 i(t) + 4 s i(t) + 4 i(t) = 0$$

$$s_{12} = \frac{-4 \pm \sqrt{4^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{0}}{2}$$

$$= -2$$

$$s_1 = -2 \quad s_2 = -2$$

~~i(t)~~ = This roots are equal and negitive roots

$$\cancel{i(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}}$$

$$\cancel{= K_1 e^{-2t} + K_2 e^{-2t}}$$

~~initial condition $t=0$~~

$$\cancel{i(0) = K_1 e^{-2(0)} + K_2 e^{-2(0)}}$$

$$\cancel{i(0) = K_1 e^0 + K_2 e^0}$$

$$\cancel{i(0) = K_1 + K_2}$$

$$\cancel{K_1 = -K_2}$$

$$\cancel{K_2 = -K_1}$$

$$i(t) = K_1 e^{-s_1 t} + K_2 \cdot t e^{-s_2 t}$$

Taking initial condition $t=0$

$$i(0) = K_1 e^{-2(0)} + K_2 \cdot 0 \, e^{-2(0)}$$

$$i(0) = K_1 e^{-0} + 0$$

$$K_1 = 0$$

$$i(t) = K_1 e^{-s_1 t} + K_2 \cdot t e^{-s_2 t}$$

$$i(t) = 0 + K_2 t e^{-2t}$$

$$i(t) = K_2 t e^{-2t}$$

sub intial values in eq ①

$$\frac{di(0)}{dt} + 4 i(0) + 4 \int_0^t i(0) dt = 20$$

$$\frac{di(0)}{dt} = 20$$

$$\frac{di(t)}{dt} = K_2 \left[t(-2) e^{-2t} + e^{-2t}(1) \right]$$

Taking intialy conditions $i(t) = 0$ & $t = 0$

$$\frac{di(0)}{dt} = K_2 \left[0(-2) e^{-2(0)} + e^{-2(0)} \right]$$

$$20 = K_2$$

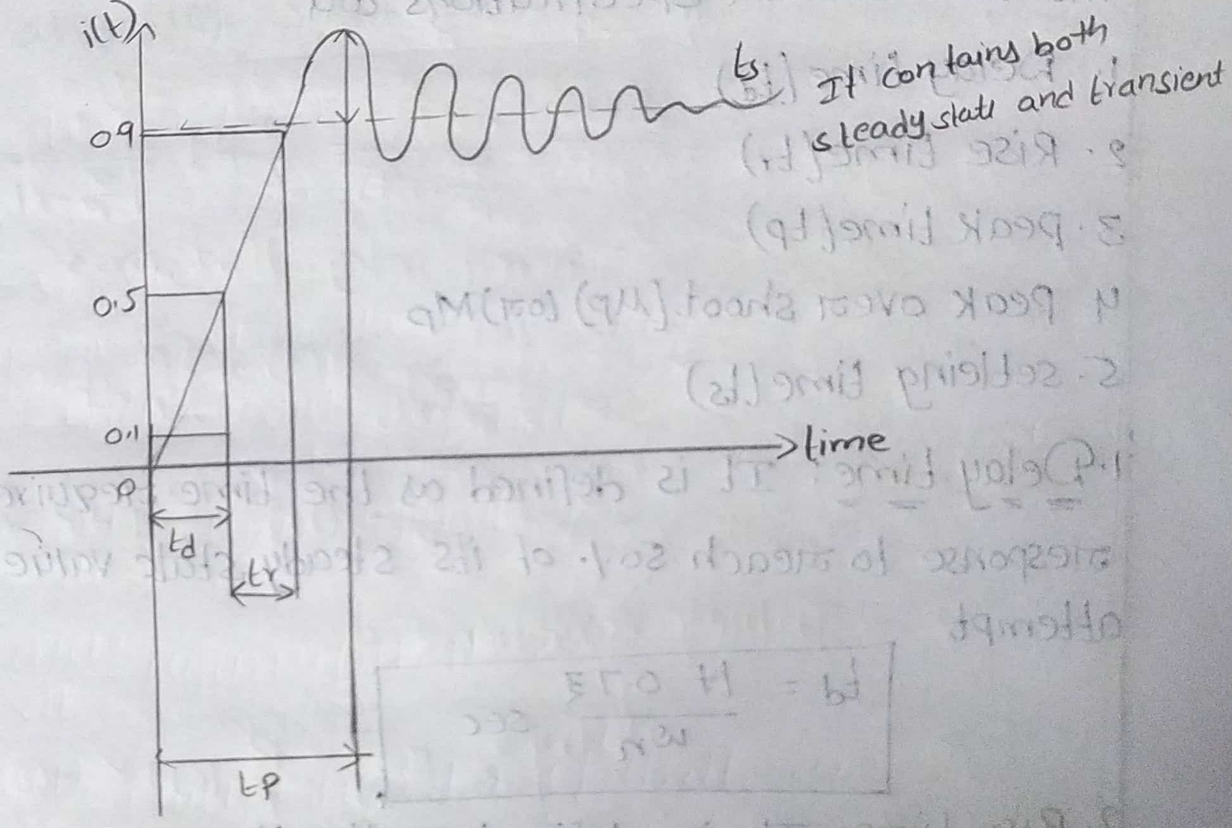
$$i(t) = K_1 e^{-s_1 t} + K_2 e^{-s_2 t}$$

$$= 0 + 20e^{-2t}$$

$$i(t) = 20e^{-2t}$$

~~Time Domaine~~

Response of RLC second order circuit:



Specifications:

→ Delay time (t_d)

→ Rise time (t_r)

→ peak time (t_p)

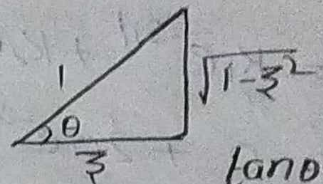
→ peak over shoot (M_p)

→ selteling time (t_s)

The response of second order RLC circuit for unit step input is given by

$$i(t) = i_{ss} - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\omega_d t + \theta\right]$$

where $\theta = \tan^{-1}\left[\frac{\sqrt{1-\zeta^2}}{\zeta}\right]$



$\tan\theta = \frac{\sqrt{1-\zeta^2}}{\zeta}$

where ζ is the damping ratio

ω_n is undamped natural frequency

ω_d is damping frequency

The time domain specifications are

1. Delay time (t_d)
2. Rise time (t_r)
3. peak time (t_p)
4. Peak over shoot (M_p) (or) M_p
5. setleing time (t_s)

1. **Delay time**: It is defined as the time required by the response to reach 50% of its steady state value in 1st attempt

$$\boxed{t_d = \frac{1+0.7\zeta}{\omega_n} \text{ sec}}$$

2. **Rise time**: It is defined as the time required by the responce to rise from 10% to 90% of it final value

$$\boxed{t_r = \frac{\pi-\theta}{\omega_d}}$$

$$t_r = \frac{\pi - \tan^{-1}\sqrt{1-\zeta^2}/\omega_d}{\omega_d}$$

3. **peak time**: It is defined as the time at which 1st peak over shoot occurs is known as peak time

$$\boxed{t_p = \frac{\pi}{\omega_d}}$$

4. Peak over shoot: The amount by which the response over shoots its final value is known as peak over shoot expressed in percentage form.

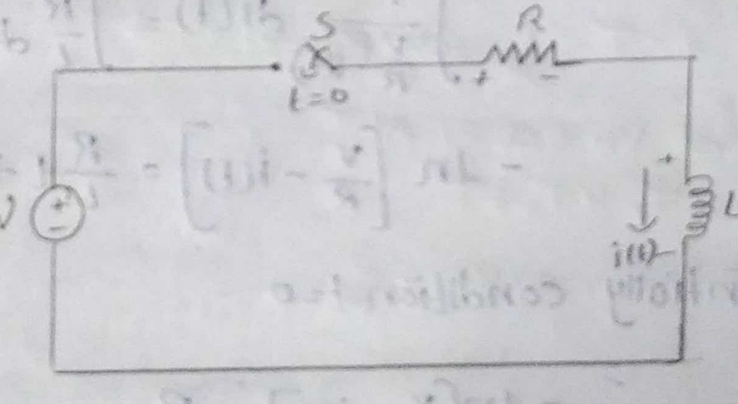
$$\boxed{M_p \text{ (or) } M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}}$$

$$\% M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100$$

5. Settling time: The time required for the response to decrease and becomes steady at its steady state value is known as settling time

$$\boxed{t_s = \frac{4}{\zeta \omega_n} \text{ sec}}$$

Response of RL series circuit with source:



At $t=0^-$

$i(0^-) = i(0^+) = \frac{V}{R}$

$I(0^-) = I_0 = I_0(0^+) = \frac{V}{R}$

$I_0 \rightarrow$ Max value of intial current

At $t \geq 0^+$; switch is closed

By applying KVL:

$$-i(t)R - L \cdot \frac{di(t)}{dt} + V = 0$$

In this condition voltage source is ~~indrodutce~~ introduce now

Applying KVL

$$-i(t)R - L \cdot \frac{di(t)}{dt} + V = 0$$

4. Peak over shoot: The amount by which the response over shoots its final value is known as peak over shoot expressed in percentage form

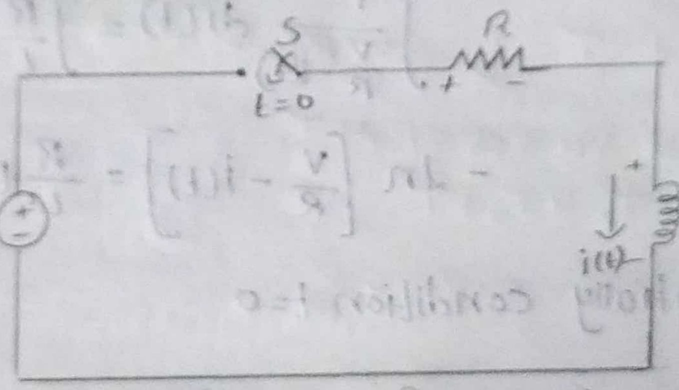
$$\boxed{Mp\ (or)\ Mp = e^{-3\pi/\sqrt{1-\zeta^2}}}$$

$$\%Mp = e^{-3\pi/\sqrt{1-\zeta^2}} \times 100$$

5. Settling time: The time required for the response to decrease and becomes steady at its steady state value is known as settling time

$$\boxed{t_s = \frac{4}{\zeta \omega n}\ sec}$$

Response of RL series circuit with Source:



At $t=0^-$

$i(0^-) = i(0^+) = \frac{V}{R}$

$I(0^-) = I_0 = I_0(0^+) = \frac{V}{R}$

$I_0 \rightarrow$ Max value of intial current

At $t \geq 0^+$; switch is closed

By applying KVL;

$$-i(t)R - L \cdot \frac{di(t)}{dt} + V = 0$$

In this condition voltage source is ~~indroduce~~ introduce now

Applying KVL

$$-i(t)R - L \cdot \frac{di(t)}{dt} + V = 0$$

$$i(t)R + L\frac{di(t)}{dt} = V$$

Divide R in b.s

$$\frac{R}{L}i(t) + \frac{di(t)}{dt} = \frac{V}{L}$$

$$\frac{R}{R}i(t) + \frac{L}{R}\frac{di(t)}{dt} = \frac{V}{R}$$

$$i(t) + \frac{L}{R}\frac{di(t)}{dt} = \frac{V}{R}$$

$$\frac{L}{R}\frac{di(t)}{dt} = \frac{V}{R} - i(t)$$

$$\frac{di(t)}{\frac{V}{R}-i(t)} = \frac{R}{L}dt = \left[\frac{1}{\frac{V}{R}-i(t)}\right]di(t) = \frac{R}{L}dt$$

Taking integration on b.s

$$\int \frac{1}{\frac{V}{R}-i(t)}di(t) = \int \frac{R}{L}dt$$

$$-\ln\left[\frac{V}{R} - i(t)\right] = \frac{R}{L}t + K \quad (1)$$

Initially condition $t=0$

$$-\ln\left[\frac{V}{R} - i(0)\right] = \frac{R}{L}(0) + K$$

$$K = -\ln\left(\frac{V}{R}\right)$$

sub K in eq (1)

$$-\ln\left[\frac{V}{R} - i(t)\right] = \frac{R}{L}t - \ln\left(\frac{V}{R}\right)$$

$$-\left[\ln\left[\frac{V}{R} - i(t)\right] - \ln\left(\frac{V}{R}\right)\right] = \frac{R}{L}t$$

$$-\ln\left[\frac{V/R - i(t)}{V/R}\right] = \frac{R}{L}t$$

$$\frac{V/R - i(t)}{V/R} = e^{-R/Lt}$$

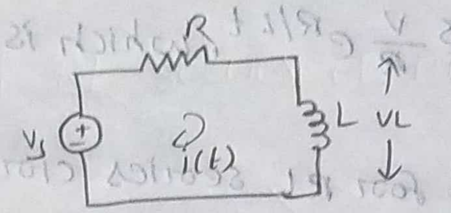
$$i(t) = \left(\frac{V}{R}\right) - \left(\frac{V}{R}\right) e^{-R/L\, t}$$

steady component   exponential component

In the above equation we can observe that, it is a first order non-homogenous D.E and it is a combination of both steady state response and Transisent response. Steady state response is also known as force response it is denoted by $\frac{V}{R}$ similarly transisent response is denoted by $\frac{V}{R} e^{-R/L\, t}$ which is also known as '0' input Response.

Time constant $\tau = \frac{L}{R}$ for RL series circuit. For different values of τ ($t = \tau, 2\tau, 4\tau, 6\tau \ldots$) the response $i(t) = 0.99$

~~$i(t) = 0.99$~~



$$i(t) = \left(\frac{V}{R}\right) - \left(\frac{V}{R}\right) e^{-R/L\, t}$$
$$= \left(\frac{V}{R}\right) - \left(\frac{V}{R}\right) e^{-8}$$
$$= 0.9966\frac{V}{R}$$

By applying KVL

$$V_S - Ri(t) - L\frac{di(t)}{dt} = 0$$

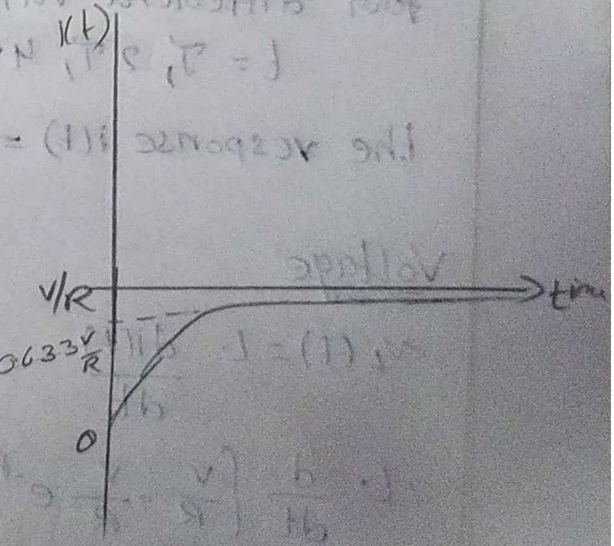
$$V_S - L\frac{di(t)}{dt} = Ri(t)$$

Divide the both sides with 'R'

$$\frac{V}{R} - \frac{L}{R}\frac{di(t)}{dt} = i(t)$$

$$-\frac{L}{R}\frac{di(t)}{dt} = \frac{-V}{R} + i(t)$$

$$\frac{di(t)}{i(t) - \frac{V}{R}} = -\frac{R}{L}dt$$



Integrating on b.s

$$\int \frac{di(t)}{i(t) - \frac{V}{R}} = \int -\frac{R}{L}dt$$

$$\ln\left(i(t) - \frac{V}{R}\right) = -\frac{R}{L}t + K$$

$$\ln \frac{i(t) - V/R}{R/V} = \frac{-R}{L}t$$

$$\frac{i(t) - \frac{V}{R}}{R/V} = e^{-R/L\,t}$$

$$i(t) - \frac{V}{R} = \frac{R}{V} e^{-R/L\,t}$$

$$i(t) = \frac{R}{V} e^{-R/L\,t} + \frac{V}{R}$$

$$\boxed{i(t) = \frac{V}{R} + e^{-R/L\,t}\,\frac{R}{V}}$$

In the above eqn we can observe that it is a first order Non-Homogeneous D.E and it is a combination of both steady state response and transition response

Steady state response is also a forced response is $\frac{V}{R}$

Illy transient response is $\frac{V}{R} e^{R/L\,t}$, which is also known as zero input response

at $t = T = \frac{L}{R}$ for RL series circuit

for different values of NT

$t = T, 2T, 4T, 6T$

the response $i(t) = 0.9975 \frac{V}{R}$

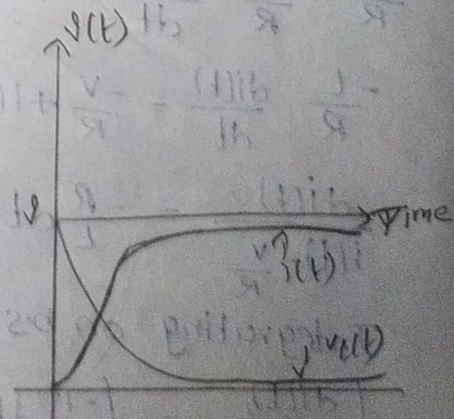
Voltage

$$V_L(t) = L \cdot \frac{di(t)}{dt}$$

$$L \cdot \frac{d}{dt}\left[\frac{V}{R} - \frac{V}{R} e^{-R/L\,t}\right]$$

$$L$$

$$V_L(t) = V \cdot e^{-R/L\,t} \text{ volts}$$



If you consider I_0 (intali values of max current through the inductor before switching the expression of $i(t)$ can be written as $\frac{V}{R} - \left(\frac{V}{R} - I_0\right) e^{-R/L\,t}$

$$\frac{i(t) - \frac{V}{R}}{R/V} = e^{-R/L\, t}$$

$$i(t) - \frac{V}{R} = \frac{R}{V} e^{-R/L\, t}$$

$$i(t) = \frac{R}{V} e^{-R/L\, t} + \frac{V}{R}$$

$$\boxed{i(t) = \frac{V}{R} + e^{-R/L\, t} \frac{R}{V}}$$

In the above eqn we can observe that it is a first order Non-Homogeneous D.E and it is a combination of both steady state response and transition response.

steady state response is also a forced response is $\frac{V}{R}$

||ly transient response is $\frac{V}{R} e^{R/L\, t}$, which is also known as zero input response

at $t = T = \frac{L}{R}$ for RL series circuit

for different values of T

$t = T, 2T, 4T, 6T$

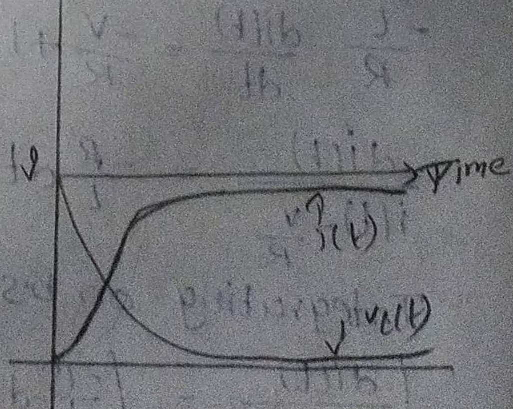
the response $i(t) = 0.9975 \frac{V}{R}$

Voltage

$$V_L(t) = L \cdot \frac{di(t)}{dt}$$

$$L \cdot \frac{d}{dt}\left[\frac{V}{R} - \frac{V}{R} e^{-R/L\, t}\right]$$

$$L$$



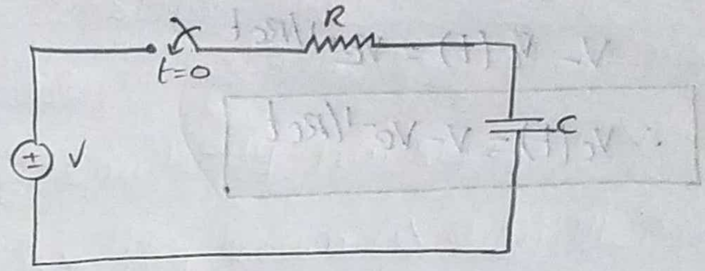
$$V_L(t) = V \cdot e^{-R/L\, t} \text{ volts}$$

If you consider I_0 (intali values of max current through

$$V_L(t) = (V - I_0 R)e^{-R/L t}$$

voltage across the current

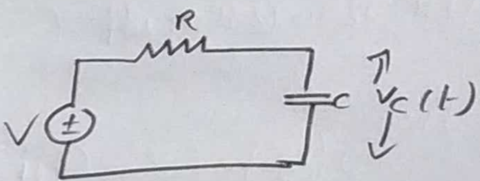
Response of Rc circuit with source:



At $t=0^-$:-

switch is in open condition

$$V(0^-) = V_0 = V(0^+)$$



At $t \geq 0$:

switch is at closed condition

By applying KVL

$$-i(t)R - V_C(t) + V = 0$$

where $i_C(t) = C \cdot \frac{dV_C(t)}{dt}$

$$-C\frac{dV_C(t)}{dt}R - V_C(t) + V = 0$$

$$\Rightarrow C\frac{dV_C(t)}{dt}R = V - V_C(t)$$

$$\frac{dV_C(t)}{V - V_C(t)} = \frac{1}{RC}dt$$

Integrating on B.S

$$\int \frac{dV_C(t)}{V - V_C(t)} = \int \frac{1}{RC} dt$$

$$-\ln[V - V_C(t)] = \frac{1}{RC}t + K \quad -①$$

Appling Intial condition $t=0$;

$$-\ln[V - V_C(0)] = \frac{1}{RC}(0) + K$$

$$① \Rightarrow -\ln[V - V_C(t)] = \frac{1}{RC}t - \ln[V]$$

$$-\ln[V - V_C(t)] + \ln[V] = \frac{1}{RC}t$$

$$-\left[\ln\left(\frac{V - V_c(t)}{V}\right)\right] = \frac{1}{RC}t$$

$$\frac{V - V_c(t)}{V} = e^{-1/RC\,t}$$

$$V - V_c(t) = Ve^{-1/RC\,t}$$

$$\boxed{\therefore V_c(t) = V - Ve^{-1/RC\,t}}$$