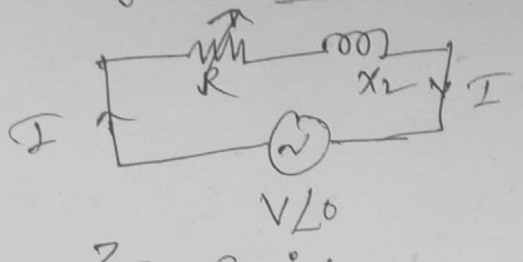
Locus diagram of Series RL Circuit with variable Resistance



$Z = R + jX_L$

polar form of $Z = |Z|\angle\theta$. $|Z| = \sqrt{R^2 + X_L^2}$ & $\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$

Variation in R	$\lvert Z\rvert = \sqrt{R^2+X_L^2}$	$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$
$R \to 0$	$\lvert Z\rvert = X_L$	$\theta = 90^\circ$
$R \to \infty$	$\lvert Z\rvert = \infty$	$\theta = 0^\circ$.

Current in the circuit

$$I = \frac{V\angle 0}{|Z|\angle\theta} = \frac{V}{|Z|}\angle\phi = \frac{V}{\sqrt{R^2 + X_L^2}} \angle -\tan\left(\frac{X_L}{R}\right).$$

$$\Rightarrow I = (I_x + jI_y)$$

$$V\angle 0 = I.Z = (I_x + jI_y)(R + jX_L)$$

equating Real & Imaginary parts of both sides

$$V = I_x R - I_y X_L \quad -①$$

$$0 = I_x X_L + I_y R \quad -②$$

Now eliminate variable (In this case R) from the two equations and combine them into one equation

$$0 = I_x X_L + I_y\left(\frac{V + I_y X_L}{I_x}\right).$$

$$I_x^2 X_L + I_y^2 X_L + V I_y = 0$$

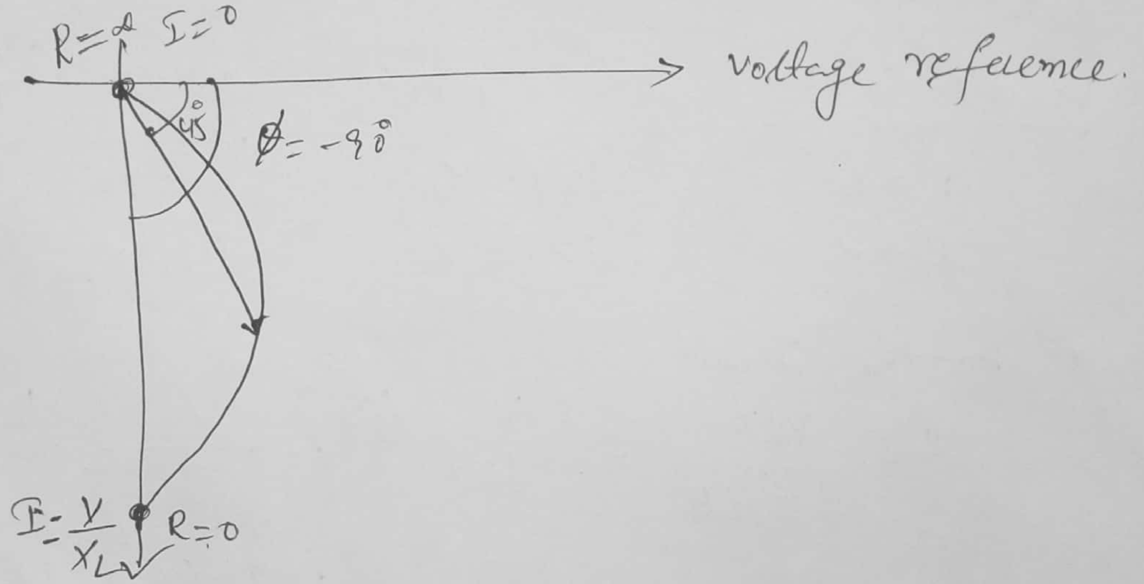
$$I_x^2 + I_y^2 + \frac{V}{X_L}.I_y = 0$$

adding $\left(\frac{V}{2X_L}\right)^2$ to both sides of the above equation we can written as

$$I_x^2 + I_y^2 + 2 \cdot \frac{V}{2X_L} \cdot I_y + \left(\frac{V}{2X_L}\right)^2 = \left(\frac{V}{2X_L}\right)^2$$

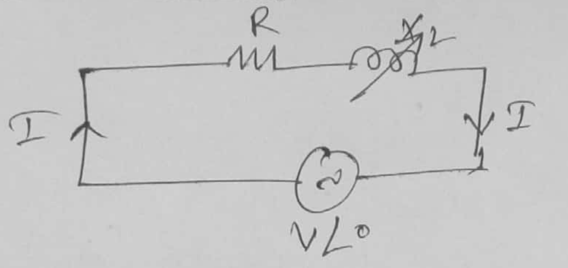
$$\left(I_x - 0\right)^2 + \left(I_y + \frac{V}{2X_L}\right)^2 = \left(\frac{V}{2X_L}\right)^2.$$

From Equation a Circle with a radius of $\left(\frac{V}{2X_L}\right)$ and with it co.ordinates of the Centre as $\left(0, \frac{V}{2X_L}\right)$.



Variation in R	$\lvert I \rvert = \frac{V}{\sqrt{R^2 + X_L^2}}$.	$\phi = -\tan^{-1}\left(\frac{X_L}{R}\right)$
$R \to 0$	$I = \frac{V}{X_L}$ (max current)	$\phi = -90°$
$R = X_L$	$I = \frac{V}{\sqrt{2} X_L}$	$\phi = -45°$.
$R \to \infty$	$I = 0$ (min. current)	$\phi = 0°$.

Series RL Circuit - Variable Reactance



$Z = R + jX_L$

polar form $Z = |Z| \angle \theta$, $|Z| = \sqrt{R^2 + X_L^2}$ & $\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$

Variation in X_L	$\lvert Z \rvert = \sqrt{R^2+X_L^2}$	$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$
$X_L \to 0$	$\lvert Z \rvert = R$	$\theta = 0$
$X_L \to \infty$	$\lvert Z \rvert = \infty$	$\theta = 90^\circ$

Current in the circuit is $I = \frac{V\angle 0}{|Z|\angle \theta}$.

$$I = \frac{V \angle 0}{\sqrt{R^2+X_L^2} \angle \tan^{-1}\left(\frac{X_L}{R}\right)} = |I| \angle \phi = (I_x + jI_y)$$

apply KVL to the circuit.

$$V\angle 0 = I.Z$$
$$= (I_x + jI_y)(R + jX_L)$$

equating Real & Imaginary parts on both sides

$$V = I_x R - I_y X_L \rightarrow ①$$

$$0 = I_x X_L + I_y R \text{ —— } ②$$

Now eliminating the variable (In this case X_L) from the above two equations and combine them into one equation

$$0 = I_x \left(\frac{I_x R - V}{I_y}\right) + I_y . R$$

$$0 = I_x^2 R - I_x . V + I_y^2 . R$$

$$0 = I_x^2 - \frac{V}{R} I_x + I_y^2 .$$

adding $\left(\frac{V}{2R}\right)^2$ to both sides of the above equation, we can written as

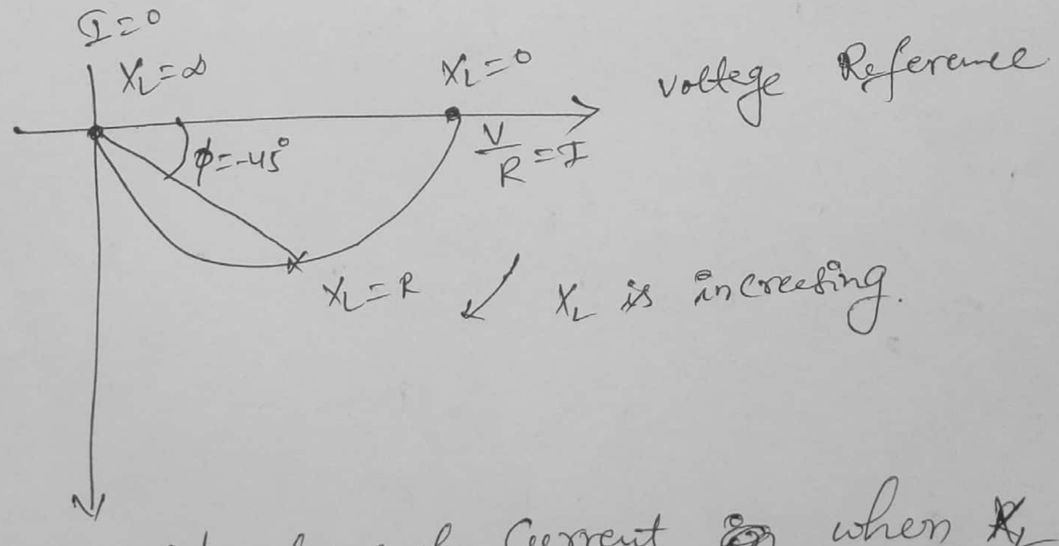
$$I_x^2 - \frac{V}{R}\cdot I_x + I_y^2 + \left(\frac{V}{2R}\right)^2 = \left(\frac{V}{2R}\right)^2$$

$$\left(I_x - \frac{V}{2R}\right)^2 + \left(I_y - 0\right)^2 = \left(\frac{V}{2R}\right)^2.$$

The above equation shows a circle with a radius of $\left(\frac{V}{2R}\right)$ and with it co-ordinates of the Centre $\left(\frac{V}{2R}, 0\right)$.

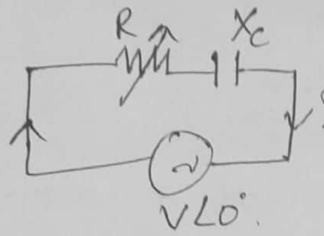
<u>Variation in X_L.</u> $\quad |I| = \frac{V}{\sqrt{R^2 + X_L^2}}$. $\quad \phi = -\tan^{-1}\left(\frac{X_L}{R}\right)$

$X_L \to 0$	$I = \frac{V}{R}$ (max current)	$\phi = 0°$
$X_L = R$	$I = \frac{V}{\sqrt{2}R}$	$\phi = -45°$.
$X_L \to \infty$	$I = 0$ (min current)	$\phi = -90°$



RL circuit locus of Current when X_L is Variable

Series RC Circuit — with Resistance is Variable.



Consider impedance $Z = R - jX_c$.

polar form of $Z = |Z|\angle\theta$, $|Z| = \sqrt{R^2 + X_c^2}$, $\theta = \tan^{-1}\left(\frac{-X_c}{R}\right)$

Variation in R	$\|Z\| = \sqrt{R^2+X_c^2}$	$\theta = \tan^{-1}\left(\frac{-X_c}{R}\right)$
$R \to 0$	$\|Z\| = X_c$	$\theta = -90^\circ$
$R \to \infty$	$\|Z\| = \infty$	$\theta = 0^\circ$

Current in the Circuit

$$I = \frac{V\angle 0}{Z\angle\theta} = \frac{V}{\sqrt{R^2+X_c^2}} \angle + \tan^{-1}\left(\frac{X_c}{R}\right) = |I|\angle\phi = (I_x + jI_y)$$

apply KVL to the circuit

$$V\angle 0 = I.Z = (I_x + jI_y)(R - jX_c)$$

Equating Real and Imaginary parts on both sides

$$V = I_x R + I_y X_c \rightarrow (1)$$

$$0 = -I_x X_c + I_y R \rightarrow (2)$$

Now eliminate the Variable (In this case it is R) from the two equations and combine them into one equation

$$0 = -I_x X_c + I_y\left(\frac{V - I_y X_c}{I_x}\right)$$

$$-I_x^2 X_c - I_y^2 X_c + V I_y = 0$$

$$I_x^2 + I_y^2 - \frac{V}{X_c} I_y = 0$$

adding $\left(\frac{V}{2X_c}\right)^2$ to both sides of the above equation we can written as

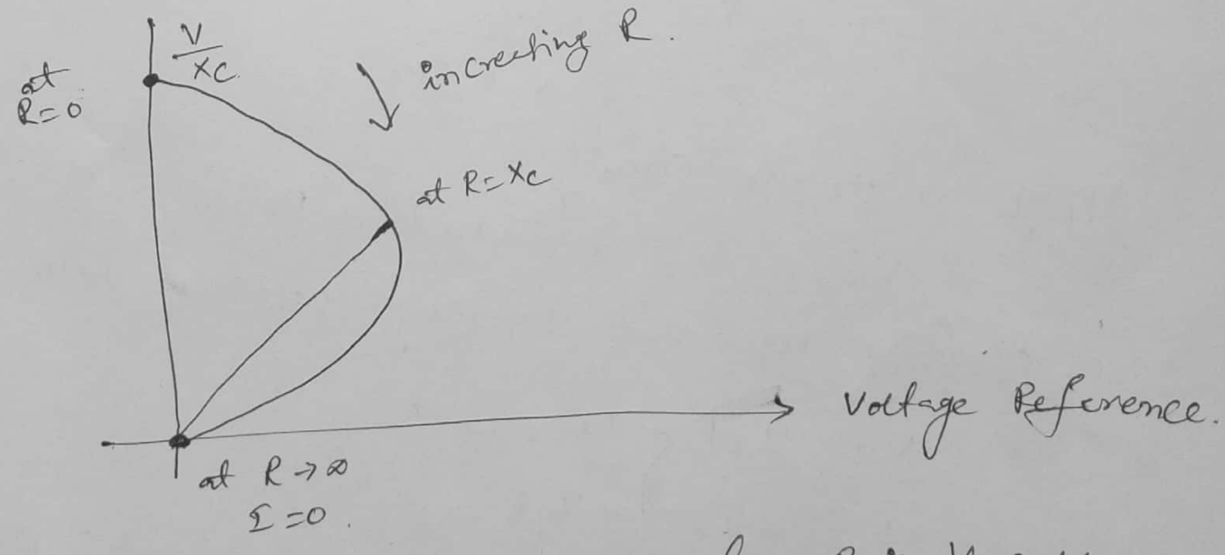
$$I_x^2 + I_y^2 - 2\frac{V}{2X_C} I_y + \left(\frac{V}{2X_C}\right)^2 = \left(\frac{V}{2X_C}\right)^2$$

$$(-I_x - 0)^2 + \left(I_y - \frac{V}{2X_C}\right)^2 = \left(\frac{V}{2X_C}\right)^2$$

from the above equation a circle with a radius of $\left(\frac{V}{2X_C}\right)$ and with it co-ordinates of the centre as $\left(0, \frac{V}{2X_C}\right)$

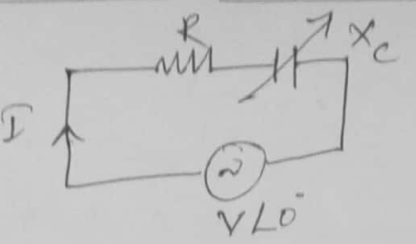
Variation in R. $\quad |I| = \frac{V}{\sqrt{R^2 + X_C^2}}$. $\quad \phi = \tan^{-1}\left(\frac{X_C}{R}\right)$.

$R \to 0$	$I = \frac{V}{X_C}$ (max current)	$\phi = 90^\circ$
$R = X_C$	$I = \frac{V}{\sqrt{2} X_C}$	$\phi = 45^\circ$
$R \to \infty$	$I = 0$ (min. current)	$\phi = 0^\circ$.



RC circuit locus of current when R is Variable.

Series RC circuit with Reactance is Variable.



consider impedance $Z = R - jX_C$

polar form of $Z = |Z|\angle\theta$ $\quad |Z| = \sqrt{R^2 + X_C^2}$, $\theta = \tan^{-1}\left(\frac{-X_C}{R}\right)$

Variation in X_C	$\|Z\| = \sqrt{R^2+X_C^2}$	$\theta = \tan^{-1}\left(\frac{-X_C}{R}\right)$
$X_C \to 0$	$\|Z\| = R$	$\theta = 0^\circ$
$X_C \to \infty$	$\|Z\| = \infty$	$\theta = -90^\circ$

Current in the Circuit.

$$I = \frac{V\angle 0}{|Z|\angle\theta} = \frac{V}{\sqrt{R^2+X_C^2}} \angle \tan^{-1}\left(\frac{X_C}{R}\right) = |I|\angle\phi = (I_x + jI_y)$$

apply KVL to the circuit.

$$V\angle 0 = I.Z = (I_x + jI_y)(R - jX_C)$$

equating Real and Imaginary parts on the both sides

$$V = I_x R + I_y X_C \quad —①$$

$$0 = -I_x X_C + I_y R \quad —②$$

Now eliminate the variable (in this case it is X_C) from the above two equations and combine them into one equation.

$$0 = -I_x\left(\frac{V - I_x R}{I_y}\right) + I_y R$$

$$I_x^2 R + I_y^2 R - V I_x = 0 \Rightarrow I_x^2 + I_y^2 - \frac{V}{R} I_x = 0$$

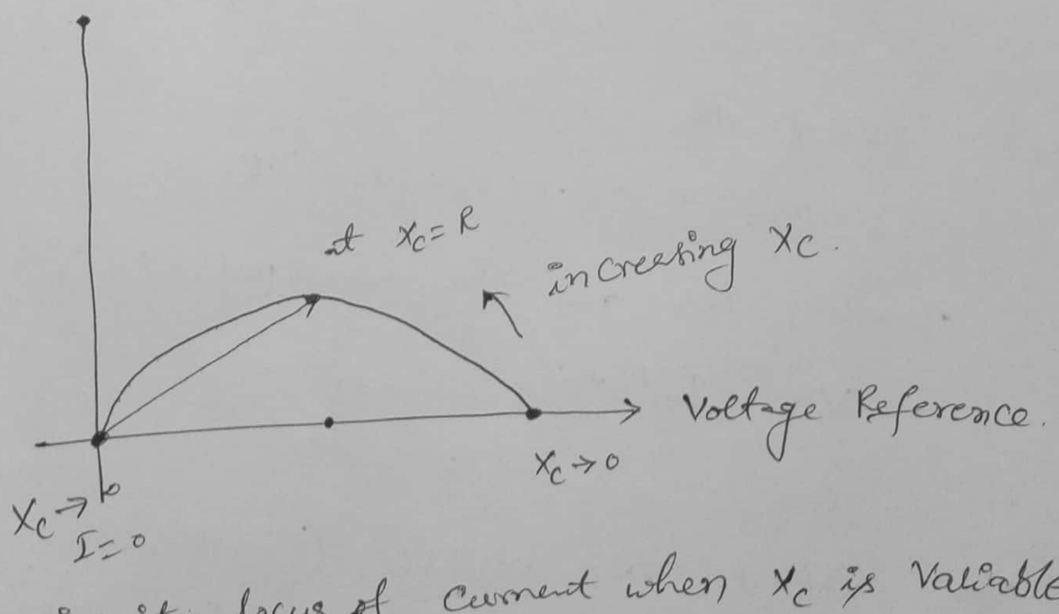
adding $\left(\frac{V}{2R}\right)^2$ to both sides of the above equation we can written as

$$I_x^2 + I_y^2 - 2\frac{V}{2R} I_x + \left(\frac{V}{2R}\right)^2 = \left(\frac{V}{2R}\right)^2$$

$$\left(I_x - \frac{V}{2R}\right)^2 + \left(I_y - 0\right)^2 = \left(\frac{V}{2R}\right)^2$$

from the above equation a circle with a raldius of $\left(\frac{V}{2R}\right)$ and with it co.ordinates of the centre as $\left(\frac{V}{2R}, 0\right)$.

Variation in X_C	$\lvert I \rvert = \frac{V}{\sqrt{R^2 + X_C^2}}$	$\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$
$X_C \to 0$	$I = \frac{V}{R}$ (max current)	$\phi = 0^\circ$
$X_C = R$	$I = \frac{V}{\sqrt{2} R}$	$\phi = 45^\circ$
$X_C \to \infty$	$I = 0$ (min. current)	$\phi = 90^\circ$



RC circuit - locus of current when X_C is Variable.

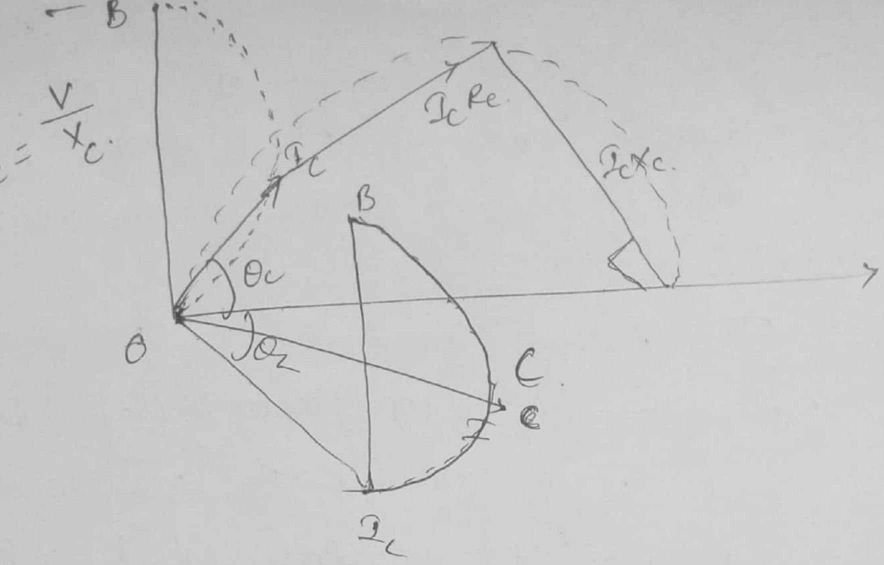


fig ©.

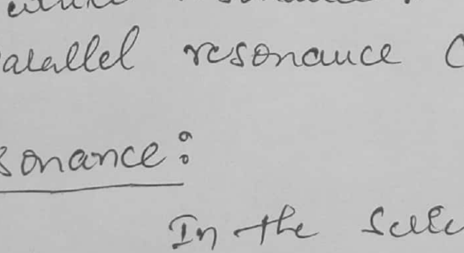
⇒ Resonance:

An AC circuits comprising of R, L and C is said to be in resonance when applied voltage (or) source Voltage and Source current are in phase. At resonance the powerfactor of the circuit is unity and circuit acts as purely Resistive $(\because Z = R)$

Resonance in Series circuit is referred to as Series resonance while resonance is in parallel circuit is referred to as a parallel resonance (or) anti-resonance.

Series resonance:

In the series RLC circuit as shown in fig. below



The Circuit Current I is given by $I = \frac{V}{Z}$ A.

Here Z is the equivalent impedance of the circuit.

$$Z = R + jX_L - jX_C.$$

$$Z = R + j(X_L - X_C)$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) \quad (\because \omega = 2\pi f)$$

$\cos\phi = \frac{R}{Z}$

at Resonance, the powerfactor being unity. (Pf = 1) Pf = 1.

$$\boxed{Z = R}$$

Therefore, reactive part of the complex impedance must to be zero. i.e $X_L - X_C = 0$. The frequency at which the resonance ~~is~~ occurs is called resonant frequency. (f_r (or) f_0)

$$X_L - X_C = 0$$

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$f_0 \text{ (or) } \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}.$$

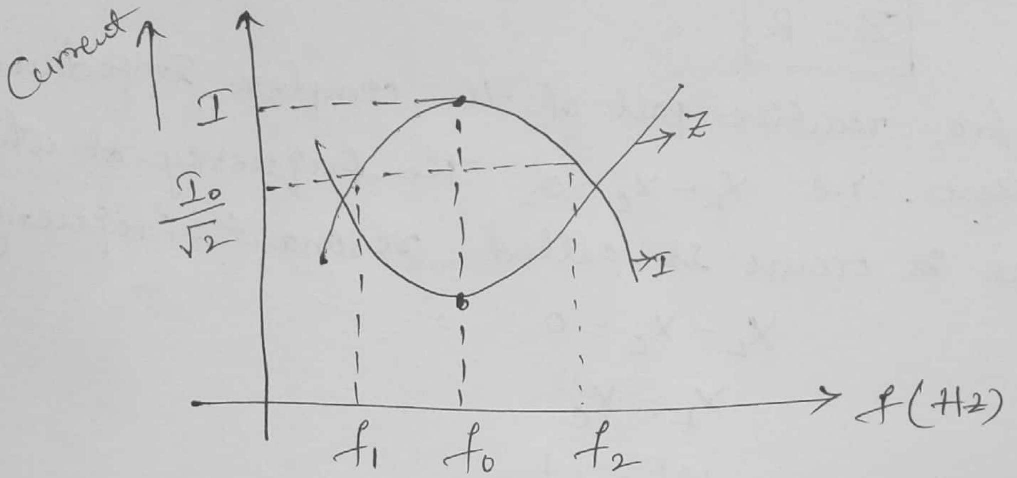
Effects of Series Resonance:

- at resonance condition $X_L = X_C$: the impedance of the ckt is minimum. and the current in the circuit is maximum. $\left(I = \frac{V}{Z} = \frac{V}{R}\right)$.

Variation of frequency:

Here two frequencies f_1 and f_2 near resonant frequency have been identified and represent the two points of ~~intersection~~ interception of current curve at $1/\sqrt{2}\, I_0$, where I_0 is being the current at resonance.

As the product of $I_0/\sqrt{2}$ and $V/\sqrt{2}$ represent rms power which becomes $\frac{P_0}{2}$. hence the points f_1 and f_2 are also called half power points f_1 being the lower half power frequeny and f_2 being the upper half power frequency.



→ Quality factor (Q) of Series Resonating Circuit:

In a series resonating circuit, Q factor is defined as the ratio of the voltage across the inductor (or) capacitor to the applied voltage. It is also the voltage magnification in the circuit at resonance.

$$Q = \frac{V_L}{V} \text{ (or) } \frac{V_c}{V}. \qquad \left(\because I_0 = \frac{V}{R}\right).$$

$$Q = \frac{V_L}{V} = \frac{I_0 X_L}{I_0 R} = \frac{X_L}{R} = \frac{\omega_0 L}{R}$$

and also.

$$Q = \frac{V_c}{V} = \frac{I_0 X_c}{I_0 R} = \frac{X_c}{R} = \frac{1}{\omega_0 C R}. \quad \left(\because I_0 = \frac{V}{R}\right).$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

and also

$$Q = \frac{1}{\omega_0 C R} = \frac{1}{\frac{1}{\sqrt{LC}} \cdot CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Quality factor (Q) ⇒ $Q = \frac{\text{Resonant frequency}}{\text{Bandwidth}}$

$$Q = \frac{f_0}{B.W} = \frac{f_0}{(f_2 - f_1)} = \frac{\omega_0}{(\omega_2 - \omega_1)}$$

⇒ Selectivity: The selectivity of a resonating circuit is defined by the ratio of f_0 (Resonant frequency) to the band width of the circuit.

i.e selectivity $= \frac{f_0}{(f_2 - f_1)}$.

Quality factor (Q) ⇒ $Q = \frac{\text{Resonant frequency}}{\text{Bandwidth}}$

$$Q = \frac{f_0}{B.W} = \frac{f_0}{(f_2 - f_1)} = \frac{\omega_0}{(\omega_2 - \omega_1)}$$

⇒ Selectivity: The selectivity of a resonating circuit is defined by the ratio of f_0 (Resonant frequency) to the band width of the circuit.

i.e selectivity $= \frac{f_0}{(f_2 - f_1)}$.

Some important formulas

$$f_2 - f_1 = BW = \frac{R}{2\pi L}.$$

$$\omega_0^2 = \omega_1 \omega_2 \quad (\text{or}) \quad \omega_0 = \sqrt{\omega_1 \omega_2}.$$

↳ resonance frequency is a geometric mean of cut off frequency.

$$f_2 - f_0 = f_0 - f_1 = \frac{R}{4\pi L}.$$

$$f_2 - f_0 = \frac{R}{4\pi L}$$

$$f_2 = f_0 + \frac{R}{4\pi L} \quad (\text{or}) \quad f_2 = \frac{R + \sqrt{R^2 + 4L/C}}{4\pi L}.$$

↳ upper cut off frequency

$$f_0 - f_1 = \frac{R}{4\pi L}$$

$$f_1 = f_0 - \frac{R}{4\pi L} \quad (\text{or}) \quad f_1 = \frac{-R + \sqrt{R^2 + 4L/C}}{4\pi L}$$

↳ lower cut off frequency.

→ The frequency at which maximum Voltage occur across Capacitor is

$$\boxed{f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}}$$

→ The frequency at which maximum Voltage occur across inductor is

$$\boxed{f_L = \frac{1}{2\pi\sqrt{LC - \frac{R^2C^2}{2}}}}$$

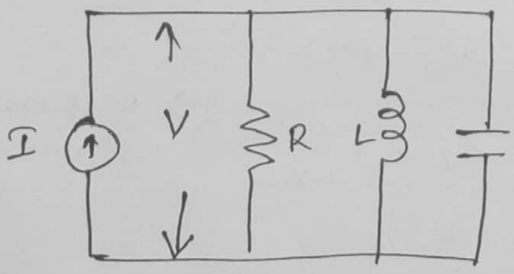
parallel Resonance : (or) admittance Resonance :

In parallel Circuits, it is easier to analyse if the admittance is calculated. At resonance, the P.f being unity, reactive part of Z and reactive part of Y must be zero. Equating the reactive part of Y to zero, we can get the expression for resonance frequency. At resonance the admittance is minimum and hence impedance is maximum, In other words current is minimum.

Case (a) :

Resonance in parallel RLC circuit (ideal circuit)

Consider the parallel RLC circuit shown in fig below.



The total admittance $Y = Y_R + Y_L + Y_C$

$$= \frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$= \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$= \frac{1}{R} + \frac{j}{j \times jX_L} + \frac{1}{-jX_C}\frac{j}{j}$$

$$= \frac{1}{R} - \frac{j}{X_L} + \frac{j}{X_C}$$

$$Y = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

At resonance, the imaginary part of Y is Zero.

$$\frac{1}{X_C} - \frac{1}{X_L} = 0$$

Scanned with CamScanner

$$\omega_0 C - \frac{1}{\omega_0 L} = 0$$

$$\omega_0 C = \frac{1}{\omega_0 L}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad (\because \omega_0 = 2\pi f_0).$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

The Resonance frequency is same as that of Series Resonance frequency.

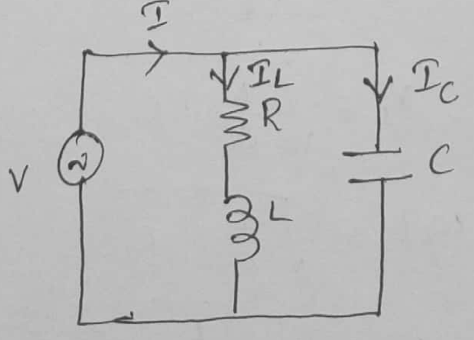
$$\text{Quality factor } (Q) = \frac{I_C \text{ (or) } I_L}{I}.$$

$$Q = \frac{I_C}{I} = \frac{\left(\frac{V}{X_C}\right)}{\left(\frac{V}{R}\right)} = \frac{R}{X_C} = \frac{R}{\frac{1}{\omega_0 C}} = \omega_0 RC$$

$$Q = \frac{I_L}{I} = \frac{\left(\frac{V}{X_L}\right)}{\left(\frac{V}{R}\right)} = \frac{R}{X_L} = \frac{R}{\omega_0 L}$$

Case (5):

Resonance in a parallel circuit



total admittance of the Circuit

$$Y = Y_C + Y_L$$

$$= \frac{1}{Z_C} + \frac{1}{Z_L}$$

$$= \frac{1}{-jX_C} + \frac{1}{(R+jX_L)}$$

$$= \frac{j}{-jX_C\, j} + \frac{R-jX_L}{(R+jX_L)(R-jX_L)}$$

$$= \frac{j}{X_C} + \frac{R-jX_L}{R^2+X_L^2}$$

$$= \frac{R}{R^2+X_L^2} + \frac{j}{X_C} - \frac{jX_L}{R^2+X_L^2}$$

$$Y = \frac{R}{R^2+X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right) \longrightarrow (1)$$

at resonance, the reactive part of Y must be Zero.

$$\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2} = 0$$

$$\frac{1}{X_C} = \frac{X_L}{R^2+X_L^2}$$

$$R^2+X_L^2 = X_L \cdot X_C.$$

$$= (\omega_0 L)\left(\frac{1}{\omega_0 C}\right)$$

$$R^2+X_L^2 = \frac{L}{C}$$

$$X_L^2 = \frac{L}{C} - R^2.$$

$$X_L = \sqrt{\frac{L}{C} - R^2}.$$

$$\omega_0 L = \sqrt{\frac{L}{C} - R^2}.$$

$$\omega_0 = \frac{1}{L}\sqrt{\frac{L}{C} - R^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad (\because \omega_0 = 2\pi f_0)$$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

at resonance, the admittance is denoted by Y_0

$$\omega_0 C - \frac{1}{\omega_0 L} = 0$$

$$\omega_0 C = \frac{1}{\omega_0 L}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad (\because \omega_0 = 2\pi f_0)$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

The Resonance frequency is same as that of Series Resonance frequency.

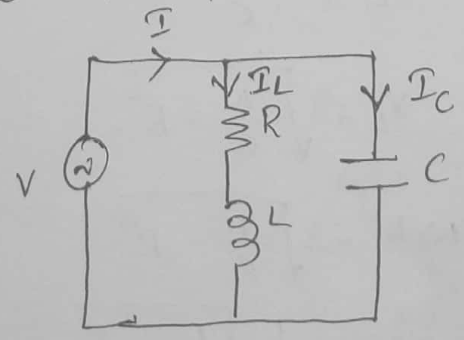
Quality factor $(Q) = \dfrac{I_C \text{ (or) } I_L}{I}$.

$$Q = \frac{I_C}{I} = \frac{\left(\frac{V}{X_C}\right)}{\left(\frac{V}{R}\right)} = \frac{R}{X_C} = \frac{R}{\frac{1}{\omega_0 C}} = \omega_0 RC$$

$$Q = \frac{I_L}{I} = \frac{\left(\frac{V}{X_L}\right)}{\left(\frac{V}{R}\right)} = \frac{R}{X_L} = \frac{R}{\omega_0 L}$$

Case (b):

Resonance in a parallel circuit



total admittance of the Circuit

$$Y = Y_C + Y_L$$

$$= \frac{1}{Z_C} + \frac{1}{Z_L}$$

$$= \frac{1}{-jX_C} + \frac{1}{(R+jX_L)}$$

$$= \frac{j}{-jX_C\, j} + \frac{R-jX_L}{(R+jX_L)(R-jX_L)}$$

$$= \frac{j}{X_C} + \frac{R-jX_L}{R^2+X_L^2}$$

$$= \frac{R}{R^2+X_L^2} + \frac{j}{X_C} - \frac{jX_L}{R^2+X_L^2}$$

$$Y = \frac{R}{R^2+X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right) \longrightarrow (1)$$

at resonance, the reactive part of Y must be zero.

$$\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2} = 0$$

$$\frac{1}{X_C} = \frac{X_L}{R^2+X_L^2}$$

$$R^2+X_L^2 = X_L \cdot X_C$$

$$= (\omega_0 L)\left(\frac{1}{\omega_0 C}\right)$$

$$R^2+X_L^2 = \frac{L}{C}$$

$$X_L^2 = \frac{L}{C} - R^2$$

$$X_L = \sqrt{\frac{L}{C} - R^2}$$

$$\omega_0 L = \sqrt{\frac{L}{C} - R^2}$$

$$\omega_0 = \frac{1}{L}\sqrt{\frac{L}{C} - R^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \qquad (\because \omega_0 = 2\pi f_0)$$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

at resonance, the admittance is denoted by Y_0

from Eq ① $Y_0 = \frac{R}{R^2+X_L^2}$

$$Z_0 = \frac{1}{Y_0}$$

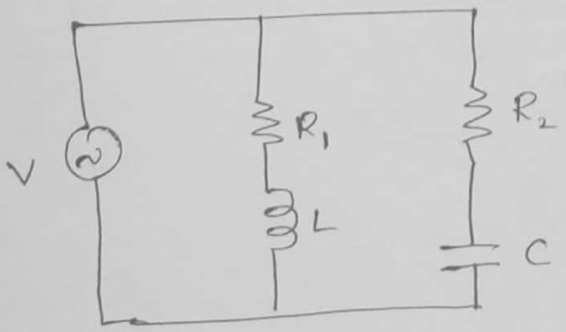
$$= \frac{R^2+X_L^2}{R}$$

$$= \frac{L/C}{R}$$

$$\boxed{Z_0 = \frac{L}{RC}}$$

Case ⓒ:

Parallel Resonance two branch circuit
one branch consists of R_1-L
another " " of R_2-C.



The total admittance $Y = Y_L + Y_C$

$$= \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$= \frac{1}{(R_1+jX_L)} + \frac{1}{(R_2-jX_C)}$$

$$= \frac{R_1-jX_L}{R_1^2+X_L^2} + \frac{R_2+jX_C}{R_2^2+X_C^2}$$

Separate the Real & imaginary parts.

$$Y = \left(\frac{R_1}{R_1^2+X_L^2} + \frac{R_2}{R_2^2+X_C^2}\right) + j\left(\frac{X_C}{R_2^2+X_C^2} - \frac{X_L}{R_1^2+X_L^2}\right)$$

at resonance, the reactive part of Y is Zero.

i.e $\frac{X_C}{R_2^2+X_C^2} - \frac{X_L}{R_1^2+X_L^2} = 0$

by solving the above expression, we get.

$$\omega_0^2 = \frac{1}{LC}\left[\frac{(R_1^2 - L/C)}{(R_2^2 - L/C)}\right]$$

$$\omega_0 = \frac{1}{\sqrt{LC}}\left\{\sqrt{\frac{(R_1^2 - L/C)}{(R_2^2 - L/C)}}\right\}. \qquad (\because \omega_0 = 2\pi f_0)$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}\left\{\sqrt{\frac{(R_1^2 - L/C)}{(R_2^2 - L/C)}}\right\}.}$$

from the above Expression.

(i) if $R_1 = R_2$, $\omega_0 = \frac{1}{\sqrt{LC}}$.

and $Z = \frac{1}{Y} = \sqrt{\frac{C}{L}}$.

(ii) if $R_1 = R_2 = \sqrt{\frac{L}{C}}$ then the value of ω_0 is indeterminate.

i.e resonance occurs at all frequencies, the circuit is purely resistive.

in this case impedance $(Z) = \sqrt{\frac{L}{C}}$.

(iii) The circuit will never be at resonance if $R_1^2 < \frac{L}{C}$

and $R_2^2 > \frac{L}{C}$ (or) Vice-Versa.

On the other hand, if Z is large in the same circuit, the impedance Z_L will be small compared to Z and the current in this circuit is approximately E/Z at all frequencies. The circuit is not selective. Then a series circuit is only *selective* if supplied from a voltage source having a low internal impedance.

In a parallel resonating circuit let E_C be the voltage across the capacitor (*i.e.*, across the parallel combination). If Z is small at all frequencies in comparison to the impedance of the parallel circuit Z_L, E_C will be almost same to E and the circuit is not selective. If, however, Z is large compared to the impedance Z_L of the parallel circuit, the current flow will be equal to $\frac{E}{Z}$.

Here E_C is proportional to the impedance of the parallel circuit and hence the circuit becomes selective.

Thus a parallel circuit attains *selectivity* provided Z is large.

EXAMPLE 5.1 *A RLC tank circuit is composed of components having values as $R = 0.2\,\Omega$; $L = 100$ mH and $C = 50\,\mu F$. Determine the resonance frequency and the corresponding input current at 24 V.*

SOLUTION. $f_0 = \dfrac{1}{2\pi\sqrt{100\times10^{-3}\times50\times10^{-6}}}$ Hz

$= 71.21$ Hz

At resonance $I = \dfrac{V}{R} = \dfrac{24}{0.2} = \mathbf{120\ A}$.

EXAMPLE 5.2 *A series LCR circuit has inductance of 10 mH and resistance of 2 ohms. What is the value of capacitance that will produce resonance. Also find the current at resonance frequency and the maximum instantaneous energy stored in the inductance at resonance. Assume the supply as 230 V, 10000 Hz.*

SOLUTION. As $\quad f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

[f being the resonance frequency]

$\therefore$

$$C = \frac{1}{f_0^2\times4\pi^2\times L}$$

$$= \frac{1}{(10{,}000)^2\times4\pi^2\times10\times10^{-3}}$$

$= \mathbf{0.025\ \mu F}$

Again, $I_0 = \dfrac{V}{R}$

[$\because$ I_0 being the circuit current during series resonance *i.e.*, when $Z = R$]

$$= \frac{230\angle0^\circ}{2} = \mathbf{115\angle0^\circ\ A}$$

Energy in inductor (instantaneous, maximum)

$$= \frac{1}{2}LI_{max}^2 = \frac{1}{2}L\times(\sqrt{2}\,I_{rms})^2$$

$$= LI_{rms}^2 = 10\times10^{-3}\times(115)^2$$

$= \mathbf{132.25\ J}$.

EXAMPLE 5.3 *What is the resonance frequency of a series RLC circuit where $R = 10\,\Omega$, $L = 25$ mH and $C = 100\,\mu F$? Evaluate the Q factor also.*

SOLUTION. $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{25\times10^{-3}\times100\times10^{-6}}}\ \text{Hz}$$

$\simeq \mathbf{100.71\ Hz}$.

We know, Q factor of a series LRC circuit can be expressed as any of the two following relations

$$Q = \frac{\omega_0 L}{R} \quad \text{or} \quad \frac{1}{\omega_0 RC}$$

Using $\quad Q = \dfrac{\omega_0 L}{R}$,

$$Q = \frac{2\pi\times100.71\times25\times10^{-3}}{10} = \mathbf{1.58}$$

[Using $\quad Q = \dfrac{1}{\omega_0 RC}$

$$Q = \frac{1}{2\pi\times100.71\times10\times100\times10^{-6}} = 1.58$$]

i.e., both the formulae yield same result.

EXAMPLE 5.4 *A coil is at resonance at 10 kHz with a capacitor. If the resistance and inductance of the coil are 200 Ω and 5 H, find the Q factor of the coil.*

SOLUTION. $f_0 = 10000$ Hz; $L = 5$ H; $R = 200\,\Omega$

$$Q = \frac{2\pi f_0 L}{R} = \frac{2\pi\times10000\times5}{200} = \mathbf{1570}\ \text{(very high)}.$$

EXAMPLE 5.5 *A coil has a resistance of 20 Ω and inductance of 80 mH and is connected in series with a 100 pF capacitor. Determine, at resonance, the circuit*

CHAPTER 5

impedance and also find the resonant frequency. If the supply to the circuit is a 50 V source having an internal impedance of 2 ohms, find also the circuit current and the voltage across the capacitor.

SOLUTION.

$$f_0 = \frac{1}{2\pi\sqrt{80\times10^{-3}\times100\times10^{-12}}}$$

[f_0 = frequency of resonance]

$$= 56.298 \text{ kHz}$$

$$Z = R = 20\,\Omega \qquad \text{[at resonance, } Z = R]$$

$$I_0 = \frac{V}{R} = \frac{50\text{ V}}{20\,\Omega} = 2.5 \text{ A}$$

[I_0 = current at resonance]

$$V_C = I X_C = 2.5\times\frac{1}{2\pi f_0 C}$$

[V_C = voltage across the capacitor]

$$= \frac{2.5}{2\pi\times56.298\times10^3\times100\times10^{-12}}$$

$$= \mathbf{70.71\ kV}.$$

EXAMPLE 5.6 *A coil having an inductance and resistance of 50 mH and 100 ohms is connected in series with a capacitor and a 100 V, 1 kHz source. Obtain the value of capacitance that will cause resonance in the circuit. Find the circuit current at resonance frequency.*

SOLUTION.

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

or, $$1000 = \frac{1}{2\pi\sqrt{50\times10^{-3}\times C}}$$

$$\therefore \quad C = \frac{1}{4\pi^2\times50\times10^{-3}\times(1000)^2} = 0.5\ \mu\text{F}$$

$$|I_0| = \frac{|V|}{|Z|}$$

[Z_0 being the circuit impedance at resonance and is equal to the value of resistance]

$$= \frac{100}{10} = \mathbf{10\ A}$$

EXAMPLE 5.7 *A 5 μF condenser is connected in series with a coil having inductance of 50 mH. Determine the frequency of resonance, the resistance of the coil if a 50 V source operating at resonance frequency causes a circuit current of 10 mA. What is the Q factor of the coil ?*

SOLUTION.

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{50\times10^{-3}\times5\times10^{-6}}}$$

$$= 318.47 \text{ Hz}$$

Also, $$I_0 = \frac{V}{R} \qquad \text{[at resonance } |Z| = |R|]$$

or, $$10\times10^{-3} = \frac{50}{R} \qquad [\because I_0 = 10 \text{ mA, given}]$$

$$\therefore \quad R = \frac{50}{10\times10^{-3}} = 5000\,\Omega$$

$$Q_{\text{factor}} = \frac{\omega_0 L}{R} = \frac{2\pi\times318.47\times50\times10^{-3}}{5000}$$

$$= \mathbf{0.02} \quad \text{[very low]}.$$

EXAMPLE 5.8 *A 50 μF capacitor, when connected in series with a coil having 40 Ω resistance, resonates at 1000 Hz. Find the inductance of the coil. Also obtain the circuit current if the applied voltage is 100 V. Also calculate the voltage across the capacitor and the coil at resonance.*

SOLUTION. At resonance,

$$X_L = X_C \quad \text{or,} \quad 2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$\therefore \quad L = \frac{1}{4\pi^2 f_0^2 C}\ \text{H}$$

$$= \frac{1}{4\pi^2\times(1000)^2\times50\times10^{-6}} = \mathbf{0.5\ mH}$$

$$|I_0| = \frac{V}{Z} = \frac{V}{R} \text{ at resonance } = \frac{100}{40} = \mathbf{2.5\ A}$$

[Power loss of the coil

$$= I_0^2 R = (2.5)^2\times40 = 250 \text{ watts}]$$

Again, $$V_C = I_0 X_C = 2.5\times\frac{1}{2\pi f_0 C}$$

$$= 2.5\times\frac{1}{2\pi\times1000\times50\times10^{-6}} = 7.96 \text{ V}$$

Again,

$$X_L = \omega L = 2\pi\times1000\times0.5\times10^{-3}$$

$$= 3.14\,\Omega$$

$$\therefore \quad V_{\text{coil}} = I_0 Z \text{ coil (at resonance)}$$

$$= 2.5\sqrt{(40)^2+(3.14)^2} = \mathbf{100.31\ V}.$$

Substitution of equation (*i*) in (*f*), we get

$$\frac{fr^2}{f_1} - f_1 = \text{BW} \quad \text{or} \quad f_1^2 + f_1\,\text{BW} - fr^2 = 0$$

$$\therefore \quad f_1 = \frac{-\text{BW} \pm \sqrt{(\text{BW})^2 + 4 f_r^2}}{2}$$

$$= -\frac{\text{BW}}{2} + \sqrt{\frac{(\text{BW})^2}{4} + fr^2}$$

[–ve sign has not been taken into account]

However, as $\frac{(\text{BW})^2}{4}$ is so small, it can be neglected. This gives

$$f_1 \simeq f_r - \frac{\textbf{BW}}{2} \qquad ...(j)$$

In a similar way,

$$f_2 \simeq f_r + \frac{\textbf{BW}}{2}. \qquad ...(k)$$

EXAMPLE 5.12 *Calculate half-power frequencies of a series resonant circuit where the resonance frequency is 150×10^3 Hz and the bandwidth is 75 kHz.*

SOLUTION. We know $f_2 - f_1 = \text{BW}$

and $\sqrt{f_1 f_2} = fr$

(*fr* being the frequency of resonance)

Here, $f_2 - f_1 = 75$, $\sqrt{f_1 f_2} = f_r = 150$

Solving these two equations,

$$f_1 = \textbf{117 kHz} \;; \qquad f_2 = \textbf{192 kHz}.$$

EXAMPLE 5.13 *A 220 V, 100 Hz ac source supplies a series LCR circuit with a capacitor and a coil. If the coil has 50 mΩ resistance and 5 mH inductance, find, at a resonance frequency of 100 Hz what is the value of capacitor? Also calculate the Q factor and half power frequencies of the circuit.*

SOLUTION. We know,

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

or, $$100 = \frac{1}{2\pi\sqrt{5 \times 10^{-3} \times C}}$$

or, $$C = \frac{1}{4\pi^2 \times 100^2 \times 5 \times 10^{-3}} = \textbf{507 μF}$$

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi \times 100 \times 5 \times 10^{-3}}{50 \times 10^{-3}} = 62.83$$

$$\text{BW(bandwidth)} = \frac{f_0}{Q} = \frac{100}{62.83} = 1.59 \text{ Hz}$$

f_2, higher half power frequency

$$= f_0 + \frac{\text{BW}}{2} = 100 + \frac{1.59}{2}$$

$$= \textbf{100.795 Hz}$$

f_1, lower half power frequency

$$= f_0 - \frac{\text{BW}}{2} = 100 - \frac{1.59}{2}$$

$$= \textbf{99.205 Hz}.$$

EXAMPLE 5.14 *A current source, having internal resistance as 50 kΩ feeds a tank circuit. If the constituents of the tank circuits are a 500 nF capacitor and a coil of resistance 50 ohms and inductance 250 mH, find the frequency of resonance and Q factor.*

SOLUTION. $$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

[for the tank circuit]

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.25 \times 500 \times 10^{-9}} - \left(\frac{50}{0.25}\right)^2}$$

$$= \textbf{449.26 Hz}$$

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi \times 449.26 \times 0.25}{50} = \textbf{14.16}.$$

EXAMPLE 5.15 *A coil with resistance of 10 ohms and inductance of 0.5 H is connected in parallel with a 400μF capacitor. Calculate the frequency at which the circuit will act as a non-inductive resistance. Find its value.*

SOLUTION. For a *R-L* and *C* parallel combination, at resonance, the resonance frequency is given by

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2} \quad \text{or} \quad \omega_0^2 = \frac{1}{LC} - \left(\frac{R}{L}\right)^2$$

$$= \frac{1}{0.5 \times 400 \times 10^{-6}} - \left(\frac{10}{0.5}\right)^2$$

or, $\omega_0 = 67.82$ rad/sec

[ω_0 being the frequency of resonance]

$$\therefore f_0 = \frac{67.82}{2\pi} = \textbf{10.80 Hz}.$$

We have also seen in the text that in parallel circuit, at resonance, the circuit offers only a resistance (known as dynamic resistance) the value of which is given by $\frac{L}{CR}$ ohms

$$= \frac{0.5}{400 \times 10^{-6} \times 10} = \textbf{125 Ω}.$$

The Bandwidth

$$= f_2 - f_1$$

$$= \frac{R}{2\pi L} = \frac{2}{2 \times 3.14 \times 2 \times 10^{-3}} = \mathbf{159.23\ Hz}$$

(**Ans.** of (*ii*))

The half power frequencies are given by

$$f_1 = \frac{-R + \sqrt{R^2 + 4\,L/C}}{4\pi L}$$

$$= \frac{-2 + \sqrt{4 + 4 \times 2 \times 10^{-3} / 10 \times 10^{-6}}}{4\pi \times 2 \times 10^{-3}}$$

$$= \mathbf{1049.16\ Hz}$$

$$f_2 = \frac{R + \sqrt{R^2 + 4\,L/C}}{4\pi L}$$

$$= \frac{2 + \sqrt{4 + 4 \times 10^{-3} \times 2 / 10 \times 10^{-6}}}{4\pi \times 2 \times 10^{-3}}$$

$$= \mathbf{1208.4\ Hz.} \quad \textbf{(Ans. of (iv))}$$

Example 5.26 *In a RLC series circuit, the resistance, inductance and capacitance are 10Ω, 100 mH and 10 microfarad. Calculate ω_0, ω_1 and ω_2. Also find BW and selectivity.*

Solution.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100 \times 10^{-3} \times 10 \times 10^{-6}}}$$

$$= \mathbf{1000\ rad/sec.}$$

$$\omega_1 = \frac{-R + \sqrt{R^2 + 4\,L/C}}{2L}$$

$$= \frac{-10 + \sqrt{100 + 4 \times 100 \times 10^{-3} / 10 \times 10^{-6}}}{2 \times 100 \times 10^{-3}}$$

$$= \mathbf{951.25\ rad/sec.}$$

$$\omega_2 = \frac{+R + \sqrt{R^2 + 4\,L/C}}{2L}$$

$$= \frac{10 + \sqrt{100 + 4 \times 100 \times 10^{-3} / 10 \times 10^{-6}}}{2 \times 100 \times 10^{-3}}$$

$$= \mathbf{1051.25\ rad/sec.}$$

$$\text{BW} = f_2 - f_1 = \frac{R}{2\pi L}$$

$$= \frac{10}{2\pi \times 100 \times 10^{-3}} = 15.92\ \text{Hz}$$

$$\text{Selectivity} = \frac{1}{Q},$$

But $\quad Q = \dfrac{\omega_0 L}{R} = \dfrac{1000 \times 100 \times 10^{-3}}{10} = 10$

$$\therefore\ \text{Selectivity} = \frac{\mathbf{1}}{\mathbf{10}} = \mathbf{0.1.}$$

Example 5.27 *Obtain the values of R, L, C in a series RLC circuit that resonates at 1.5 kHz and consumes 50 W from a 50 V a.c. source operating at the resonance frequency. The bandwidth is 0.75 kHz.*

Solution. For the series circuit, at resonance,

$$V_R = V_{supply} = 50\ \text{V}$$

But $\quad P_{loss} = V_R^2 / R \quad$ or, $\quad \dfrac{50^2}{R}$

$\therefore \quad$ **$R = 50\,\Omega$.** $\quad [\because P_{loss} = 50\ \text{W}]$

The Q factor is $\quad \dfrac{f_0}{\text{BW}} = \dfrac{1.5}{0.75} = 2.$

Again,

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi \times 1.5 \times 10^3 L}{50}$$

or, $\quad L = \dfrac{50 \times 2}{2\pi \times 1.5 \times 10^3} = \mathbf{0.0106\ H.}$

Again,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ in series resonating circuit.}$$

$$\therefore\ C = \frac{1}{4\pi^2 f_0^2 L}$$

$$= \frac{1}{4 \times 3.14^2 \times (1.5 \times 10^3)^2 \times 0.0106}$$

$$= 1.06 \times 10^{-6}\ \text{F.}$$

Thus $\quad R = \mathbf{50\,\Omega}$; $L = \mathbf{0.0106\ H}$; $C = \mathbf{1.06\,\mu\,F}$.

Example 5.28 *A series resonating circuit has $R = 1\,k\Omega$, half power frequencies of 10 and 90 kHz. Determine the bandwidth and the resonant frequency. Calculate the inductance and capacitance of the circuit.*

Solution. Bandwidth $(= \text{BW}) = f_2 - f_1$

$$= 90 - 10 = 80\ \text{kHz}$$

In series circuit,

$$f_0 = \sqrt{f_2 f_1} = \sqrt{90 \times 10} = 30\ \text{kHz}$$

The Q factor of the coil is given by

$$Q = \frac{f_0}{\text{BW}} = \frac{30}{80} = \frac{3}{8} \text{ only.}$$

$$Q=\frac{2\pi f_0\ L}{R}$$

Also,

$$\frac{3}{8}=\frac{2\ \pi\times 30\times 10^3\times L}{1000}$$

$$\therefore\quad L=\frac{3\times 1000}{8\times 2\times \pi\times 30\times 10^3}=2\text{ mH}$$

At series resonance,

$$X_L=X_C$$

$$\omega_0\ L=\frac{1}{\omega_0\ C}$$

$$C=\frac{1}{(2\pi\ f_0)^2\ L}$$

$$=\frac{1}{4\pi^2\times(30\times 10^3)^2\times 0.002}$$

$$=0.014\ \mu\text{F}.$$

Thus, BW = **80 kHz** ; f_0 = **30 kHz** ;
L = **2 mH** ; C = **0.014 μF.**

EXAMPLE 5.29 *Find the magnitude of the frequency when the drop across the capacitor in series RLC circuit is maximum.*

SOLUTION. Circuit impedance

$$Z=\sqrt{R^2+(X_L-X_C)^2}.$$

Circuit current, $I=\dfrac{V}{Z}$ and drop across C is

$$V_C=IX_C=\frac{V}{Z}\cdot X_C$$

$$V_C^2=\frac{V^2\times X_C^2}{Z^2}$$

$$V_C^2=\frac{V^2}{(\omega C)^2\left[R^2+\left(\omega L-\dfrac{1}{\omega C}\right)^2\right]}$$

To find $V_{C_{max}}$, the above expression is to be differentiated w.r.t. ω and the result is to be equated to zero. [*the reader may do this as an exercise*]. The solution gives

$$\omega_C=\sqrt{\frac{\mathbf{1}}{\mathbf{LC}}-\frac{\mathbf{R^2}}{\mathbf{2L^2}}}\ \textbf{rad / sec.}$$

EXAMPLE 5.30 *Find the value of R_1 such that the circuit given in Fig. E5.7 is resonant.*

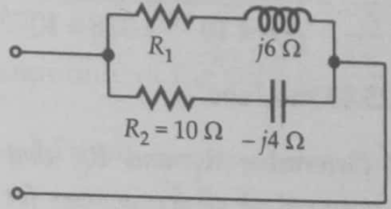


Fig. E5.7

SOLUTION. The equivalent admittance of the given circuit is

$$Y=\frac{1}{R_1+j\,6}+\frac{1}{10-j\,4}$$

$$=\frac{R_1-j\,6}{R_1^2+36}+\frac{10+j\,4}{10^2+16}$$

$$=\frac{R_1}{R_1^2+36}-j\frac{6}{R_1^2+36}+\frac{10}{10^2+16}+j\frac{4}{10^2+16}$$

$$=\frac{R_1}{R_1^2+36}+\frac{10}{10^2+16}+j\left(\frac{4}{10^2+16}-\frac{6}{R_1^2+36}\right)$$

During resonance, the imaginary part of Y must be zero.

i.e.,
$$\frac{4}{10^2+16}=\frac{6}{R_1^2+36}$$

or,
$$R_1^2+36=6\times 116/4=174$$

$$\therefore\quad \mathbf{R_1=11.75\ \Omega.}$$

EXAMPLE 5.31. *A series RLC circuit has 10 Ω resistance, 60 mH inductance and C farad capacitance. At a frequency of 25 Hz, the p.f. of the circuit is 45° lead. At what frequency will the circuit be resonant ?*

SOLUTION. $f=25$ Hz

$$\therefore\quad \omega=2\pi\times 25=157\text{ rad / sec.}$$

Again, since $\tan\phi=\dfrac{X_C-X_L}{R}$

$$\therefore\quad \tan\phi=\frac{(1/\omega C)-\omega L}{R}$$

or
$$\tan 45°=\frac{1-\omega^2\ LC}{R\omega C}=1$$

$$\therefore\quad 1-\omega^2\ LC=R\omega C$$

or,
$$1-(157)^2\times 60\times 10^{-3}\times C=10\times 157\times C$$

or,
$$1=C[1570+157^2\times 60\times 10^{-3}]=C\times 3049$$

$$\therefore\quad C=328\ \mu\text{F}$$

CHAPTER 5

$$Q=\frac{2\pi f_0\ L}{R}$$

Also,

$$\frac{3}{8}=\frac{2\ \pi\times 30\times 10^3\times L}{1000}$$

$$\therefore\quad L=\frac{3\times 1000}{8\times 2\times \pi\times 30\times 10^3}=2\text{ mH}$$

At series resonance,

$$X_L=X_C$$

$$\omega_0\ L=\frac{1}{\omega_0\ C}$$

$$C=\frac{1}{(2\pi\ f_0)^2\ L}$$

$$=\frac{1}{4\pi^2\times(30\times 10^3)^2\times 0.002}$$

$$=0.014\ \mu\text{F}.$$

Thus, BW = **80 kHz** ; f_0 = **30 kHz** ;
L = **2 mH** ; C = **0.014 μF.**

EXAMPLE 5.29 *Find the magnitude of the frequency when the drop across the capacitor in series RLC circuit is maximum.*

SOLUTION. Circuit impedance

$$Z=\sqrt{R^2+(X_L-X_C)^2}.$$

Circuit current, $I=\dfrac{V}{Z}$ and drop across C is

$$V_C=IX_C=\frac{V}{Z}\cdot X_C$$

$$V_C^2=\frac{V^2\times X_C^2}{Z^2}$$

$$V_C^2=\frac{V^2}{(\omega C)^2\left[R^2+\left(\omega L-\dfrac{1}{\omega C}\right)^2\right]}$$

To find $V_{C_{max}}$, the above expression is to be differentiated w.r.t. ω and the result is to be equated to zero. [*the reader may do this as an exercise*]. The solution gives

$$\omega_C=\sqrt{\frac{1}{LC}-\frac{R^2}{2L^2}}\ \textbf{rad/sec.}$$

EXAMPLE 5.30 *Find the value of R_1 such that the circuit given in Fig. E5.7 is resonant.*

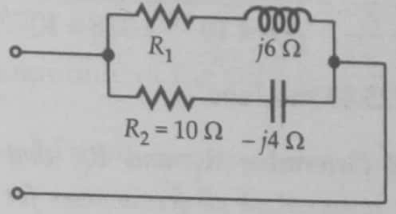


Fig. E5.7

SOLUTION. The equivalent admittance of the given circuit is

$$Y=\frac{1}{R_1+j\,6}+\frac{1}{10-j\,4}$$

$$=\frac{R_1-j\,6}{R_1^2+36}+\frac{10+j\,4}{10^2+16}$$

$$=\frac{R_1}{R_1^2+36}-j\frac{6}{R_1^2+36}+\frac{10}{10^2+16}+j\frac{4}{10^2+16}$$

$$=\frac{R_1}{R_1^2+36}+\frac{10}{10^2+16}+j\left(\frac{4}{10^2+16}-\frac{6}{R_1^2+36}\right)$$

During resonance, the imaginary part of Y must be zero.

i.e., $$\frac{4}{10^2+16}=\frac{6}{R_1^2+36}$$

or, $$R_1^2+36=6\times 116/4=174$$

$$\therefore\quad \mathbf{R_1=11.75\ \Omega.}$$

EXAMPLE 5.31. *A series RLC circuit has 10 Ω resistance, 60 mH inductance and C farad capacitance. At a frequency of 25 Hz, the p.f. of the circuit is 45° lead. At what frequency will the circuit be resonant ?*

SOLUTION. $f=25$ Hz

$$\therefore\quad \omega=2\pi\times 25=157\text{ rad/sec.}$$

Again, since $\tan\phi=\dfrac{X_C-X_L}{R}$

$$\therefore\quad \tan\phi=\frac{(1/\omega C)-\omega L}{R}$$

or $$\tan 45°=\frac{1-\omega^2\ LC}{R\omega C}=1$$

$$\therefore\quad 1-\omega^2\ LC=R\omega C$$

or, $$1-(157)^2\times 60\times 10^{-3}\times C=10\times 157\times C$$

or, $$1=C[1570+157^2\times 60\times 10^{-3}]=C\times 3049$$

$$\therefore\quad C=328\ \mu\text{F}$$

Thus, the resonant frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{60 \times 10^{-3} \times 328 \times 10^{-6}}}$$

$$= \mathbf{225.43\ rad/sec.}$$

EXAMPLE 5.32 *Determine R_L and R_C that causes the circuit to be resonant at all frequencies for the circuit shown in Fig. E5.8.*

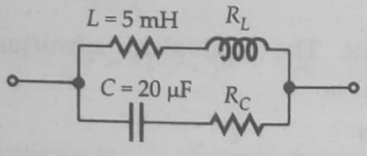


Fig. E5.8

SOLUTION. It has been shown in the text that for a R_L-Lare R_C-C parallel circuit, the condition of all time resonance is that

$$R_L^2 = R_C^2 = \frac{L}{C}$$

i.e., $$R_L = R_C = \sqrt{\frac{L}{C}} = \sqrt{\frac{5 \times 10^{-3}}{20 \times 10^{-6}}} = \mathbf{15.8\ \Omega}.$$

EXAMPLE 5.33 *A series resonating circuit has a source frequency of 5 kHz and source impedance of $(2 + j\,4)\,\Omega$. The load impedance being $(10 - j\,X_C)\,\Omega$, find the value of C provided the power consumed by the resistor is maximum.*

SOLUTION. $f_0 = 5000$ Hz

The total impedance of the given circuit is

$$Z_T = Z_{source} + Z_{load} = 2 + j\,4 + 10 - j\,X_C$$

$$= [12 + j\,(4 - X_C)]\,\Omega$$

At resonance the imaginary part of Z_T is zero i.e., at resonance, $X_L = X_C$

$$\therefore \quad X_C = 4\,\Omega = \frac{1}{\omega_0\,C}$$

$$\therefore \quad C = \frac{1}{\omega_0\,X_C} = \frac{1}{2\,\pi \times 5000 \times 4} = \mathbf{8\ \mu F.}$$

At resonance, $Z_T = 12\,\Omega$ (resistive)

i.e., $$I = \frac{V}{Z_T} = \frac{100}{12} = 8.33\text{ A}$$

Power in the resistor

$$= I^2 R = (8.33)^2 \times 10 = \mathbf{694.44\ W.}$$

[It may be noted that as at resonance I is maximum, hence, the power transfer is maximum].

EXAMPLE 5.34 *For a series resonant circuit, $R = 5\,\Omega$, $L = 1$ H and $C = 0.25\ \mu F$. Find the resonance frequency and Bandwidth.*

SOLUTION. ω_0 (resonance frequency of RLC circuit)

$$= \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{1 \times 0.25 \times 10^{-6}}} = 2000\text{ rad/sec}$$

i.e., $$f_0 = \frac{\omega_0}{2\pi} = \mathbf{318.47\ Hz.}$$

$$\text{BW (Bandwidth)} = \frac{R}{L} = \frac{5}{1} = 5\text{ rad/sec.} = \mathbf{0.8\ Hz.}$$

EXAMPLE 5.35 *The Q factor of a RLC circuit is 5 at its resonance frequency of 1 kHz. Assuming the power dissipation of 250 W when the current drawn is 1 A, find the circuit parameters. Determine the BW of the circuit.*

SOLUTION.

$$W = I^2 R$$

or, $$R = \frac{W}{I^2} = \frac{250}{1^2} = 250\text{ ohm}$$

Also, $$Q\text{ factor} = \frac{\omega_0\,L}{R} = \frac{2\pi \times 1000 \times L}{250} = 5$$

$[\because\ Q = 5$, as given$]$

$$\therefore \quad L = \frac{250 \times 5}{2\,\pi \times 1000} = 0.2\text{ H}$$

However, f_0 (resonant frequency) $= \dfrac{1}{2\pi\sqrt{LC}}$

$$\therefore \quad 1000 = \frac{1}{2\pi\sqrt{0.2 \times C}}$$

or, $$10^6 = \frac{1}{4\,\pi^2 \times 0.2 \times C}$$

$$\therefore \quad C = \frac{1}{4\pi^2 \times 0.2} \times 10^{-6} = 0.127\ \mu F.$$

Thus, the parameters of LRC circuit are

$$R = \mathbf{250\ \Omega}\ ;\ L = \mathbf{0.2\ H}\ ;\ C = \mathbf{0.127\ \mu F}$$

Again, $$Q = \frac{f_0}{\text{BW}} = \frac{1000}{\text{BW}} = 5$$

$$\therefore \quad \text{BW} = \mathbf{200\ Hz.}$$

EXAMPLE 5.36 *In Fig. E5.9, $R = R_L = 1\,\Omega$, $L = 1$ H, $C = 0.5$ F. Find the resonance frequency and the admittance at the resonant frequency.*

$$= \frac{1}{\sqrt{LC}} \sqrt{\frac{2Q_0}{2Q_0 - \frac{1}{Q_0}}}$$

$$\left(\text{Since } Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} \right)$$

$$\therefore \quad \omega = \frac{1}{\sqrt{LC}} \sqrt{\frac{2\,Q_0^2}{2Q_0^2 - 1}}.$$

Example 5.38 *Find the resonant frequency for the parallel circuit shown in Fig. E5.10.*

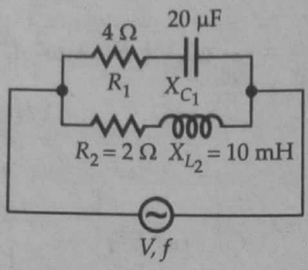


Fig. E5.10

Solution. Given that $R_1 = 4\Omega,\ C_1 = 20\,\mu\text{F},$
$R_2 = 2\,\Omega,\ L_2 = 10\text{ mH}$

$$\therefore \quad \omega_0 = \frac{1}{\sqrt{L_2 C_1}} \left[\frac{R_1^2 - L_2/C_1}{R_2^2 - L_2/C_1} \right]^{1/2}$$

$$= \frac{1}{\sqrt{10 \times 10^{-3} \times 20 \times 10^{-6}}} \left[\frac{4^2 - (10 \times 10^{-3})/(20 \times 10^{-6})}{2^2 - (10 \times 10^{-3})/(20 \times 10^{-6})} \right]^{1/2}$$

$$= 2236 \left[\frac{16 - 0.5 \times 10^3}{4 - 0.5 \times 10^3} \right]^{1/2}$$

$$= 2209 \text{ rad/sec.}$$

$$\therefore \quad f_0 = \frac{2209}{2\pi} = \mathbf{351.75\ Hz.}$$

Example 5.39 *In a series RLC circuit, instantaneous voltage and currents are $v = 100 \sin(314\,t + 30°)$ V and $i = 4 \sin(314\,t + 30°)$. Find the values of R and C provided L=0.5 H.*

Solution. We observe that in the series RLC circuit, the instantaneous voltage and currents are in phase. This indicates that the circuit is at resonance, the net circuit impedance being R.

$$\therefore \quad R = \frac{V_m}{I_m} = \frac{100}{4} = 25\,\Omega$$

At resonance,

$$X_L = X_C \quad \text{or,} \quad \omega_0 L = \frac{1}{\omega_0 C}$$

i.e.,
$$C = \frac{1}{\omega^2 L} = 20.2 \times 10^{-6}\ \text{F} = \mathbf{20.2\ \mu F.}$$

Example 5.40 *A resonating series circuit has 10 Ω resistance. If the supply is 10 V, obtain the power at half power frequency.*

Solution. I_0 (= current at resonance)

$$= \frac{V}{R} = \frac{10}{10} = 1\text{ A.}$$

We know, power at half power frequency is 1/2 P_{max};

$$\therefore \quad P_{1/2\,\text{power}} = \frac{1}{2} P_{max}$$

[P_{max} = power at the time of resonance]

$$= \frac{1}{2}(I)^2 \times R = \frac{1}{2} \times 1^2 \times 10 = \mathbf{5\ W.}$$

Example 5.41 *A RLC circuit operates at a frequency of ω and the ratio of ω to the resonant frequency is α. Obtain an expression of admittance of the circuit in terms of R, Q and α.*

Solution. $Z = R + j(X_L - X_C)$ in the series circuit.

$$\therefore \ Y = \frac{1}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{1/R}{1 + j\left(\alpha\omega_0 L/R - \frac{1}{\alpha\,\omega_0\,RC}\right)}$$

$$\left[\because \text{ given that } \frac{\omega}{\omega_0} = \alpha\right]$$

$$= \frac{1}{R\left[1 + j\left(\alpha Q - \frac{Q}{\alpha}\right)\right]}$$

$$\left[\because Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}\right]$$

$$\text{or,} \quad Y = \frac{1}{R\left[1 + jQ\left(\alpha - \frac{1}{\alpha}\right)\right]}$$

Thus, the admittance is given by

$$\mathbf{Y} = \frac{\mathbf{1}}{\mathbf{R}\left[\mathbf{1} + \boldsymbol{j}\,\mathbf{Q}\left(\boldsymbol{\alpha} - \frac{\mathbf{1}}{\boldsymbol{\alpha}}\right)\right]} \ \textbf{mho.}$$

$$f = f_0 \pm \delta f_0$$
$$= 75 \pm 0.0102 \times 75$$
$$\simeq \mathbf{75.765\ Hz}\ \text{or}\ \mathbf{74.24\ Hz.}$$

EXAMPLE 5.46 *Show that the circuit shown in Fig. E5.14 can have more than one resonant condition. Assume C to be variable.*

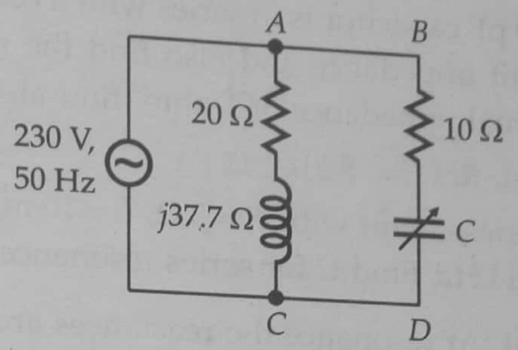


Fig. E5.14

SOLUTION. Let the admittance of branch AC be Y_1 and that of branch BD be Y_2

$$\therefore \quad Y_1 = \frac{1}{20 + j\,37.7} \quad \text{and} \quad Y_2 = \frac{1}{10 - j\,X_C}$$

i.e., $$Y_1 = 0.011 - j\,0.0207$$

and $$Y_2 = \frac{10 + j\,X_C}{100 + X_C^2}$$

At resonance, the imaginary parts of Y_1 and Y_2 must balance each other.

$$\therefore \quad 0.0207 = \frac{X_C}{100 + X_C^2}$$

or, $$0.0207\,X_C^2 - X_C + 2.07 = 0.$$

or, $$X_C = 46.14 \ \text{or}\ 2.17$$

At 50 Hz, $$C = \frac{1}{314\,X_C}$$

$$= \text{either } 69\ \mu\text{F} \ \text{or}\ \mathbf{1.46\ \mu F.}$$

As C can be either 69 μF or 1.46 μF to have resonance, it is evident that the given circuit is *resonant under both the conditions.*

Solution. Z_{in} [as given in the previous example]

$$= 6.83\,\Omega$$

[$\because$ we had found that Z is cancelling out with imaginary part of the parallel circuit hence, $Z_{in} = 6.83$ as evident from the previous example]

$$\therefore \qquad I = \frac{400}{6.83} = 58.56\text{ A}\,[V = 400\text{ V}]$$

$$\text{Power} = V.I\,[\text{at resonance } \cos\phi = 1]$$

$$= 400 \times 58.56 = \mathbf{23.42\ kW.}$$

Example 5.22 *Show that no value of R_L in the circuit shown in Fig. E5.4 will make it resonant.*

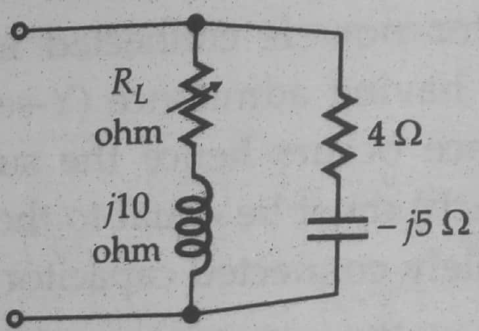


Fig. E5.4

Solution. At resonance, in a parallel *RLC* circuit, the imaginary part of the total admittance (Y_T) must be zero.

Let the inductance of the inductive and capacitive circuits be given as Y_L and Y_C.

$$\therefore \qquad Y_L = \frac{1}{R_L + j\,10} = \frac{R_L - j\,10}{R_L^2 + 100}\text{ mho}$$

$$Y_C = \frac{1}{4 - j\,5} = \frac{4 + j\,5}{4^2 + 5^2}\text{ mho}$$

Thus imaginary part of Y_T is

$$\left(\frac{-10}{R_L^2+100}+\frac{5}{41}\right) \text{ mho at resonance,}$$

$$\therefore \quad \frac{10}{R_L^2+100}=\frac{5}{41} \quad \text{or} \quad R_L^2=-18$$

Thus, $\quad \boldsymbol{R_L = j4.24}$

This value of R_L is physically impossible.

Example 5.23 *Calculate the value of R_C in the circuit shown in Fig. E5.5 to yield resonance.*

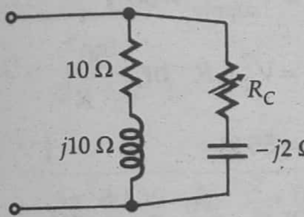


Fig. E5.5

Solution. In circuit of Fig. E5.5,

$$Y_L=\frac{1}{10+j\,10} \text{ mho}=\frac{10-j\,10}{100+100}=\frac{10}{200}-j\frac{10}{200}$$

and $$Y_C=\frac{1}{R_C-j\,2}=\frac{R_C+j\,2}{R_C^2+4} \text{ mho}$$

at resonance, imaginary part of Y_L must be equal to that of Y_C

i.e., $$-j\frac{10}{200}+j\frac{2}{R_C^2+4}=0$$

This gives $\boldsymbol{R_C = 6}$ **ohm.**

Example 5.24. *A circuit has been shown in Fig. E5.6. Find the frequency at which this circuit will be at resonance. If the capacitor and inductors are interchanged, what would be the value of the resonance frequency ?*

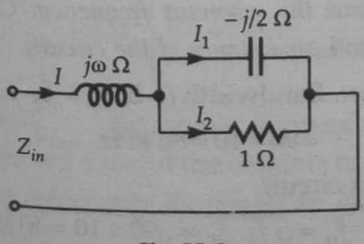


Fig. E5.6

Solution. At resonance, imaginary part of input impedance must be zero.

However,

$$Z_{in}=j\omega+\frac{(-j/2\omega)\times 1}{-j/2\,\omega+1}$$

$$\therefore \quad Z_{in}=j\,\omega-\frac{j}{2\,\omega-j}=j\,\omega-\frac{j\,(2\omega+j)}{1+4\,\omega^2}$$

Thus the imaginary part of Z_{in} is

$$\left(\omega-\frac{2\,\omega}{1+4\,\omega^2}\right)\Omega$$

At resonance, $\quad \omega-\dfrac{2\,\omega}{1+4\,\omega^2}=0.$

This gives, $\omega = \mathbf{0.5\ rad/sec.}$

If the capacitor and the inductor of the circuit are interchanged,

$$Z_{in}=-\frac{j}{2\,\omega}+\frac{j\,\omega}{1+j\,\omega}=-\frac{j}{2\,\omega}+\frac{j\,\omega(1-j\,\omega)}{1+\omega^2}$$

$$=-\frac{j}{2\,\omega}+\frac{j\,\omega}{1+\omega^2}+\frac{\omega^2}{1+\omega^2}$$

Thus $$-\frac{1}{2\,\omega}+\frac{\omega}{1+\omega^2}=0.$$

[Since at resonance, imaginary part of the input impedance is zero.]

This gives $\omega = \mathbf{1\ rad/sec.}$

5.7 ADDITIONAL EXAMPLES

Example 5.25 *A series RLC circuit has $R = 2\,\Omega$, $L = 2.0$ mH, $C = 10\,\mu$F. Calculate (i) Q factor of the circuit, (ii) the bandwidth, (iii) the resonant frequency and (iv) the half power frequency f_1 and f_2.*

Solution. $R = 2\,\Omega$, $L = 2$ mH, $C = 10\,\mu$F.

$\therefore f_0$ (resonant frequency of the series circuit)

$$=\frac{1}{2\pi\sqrt{LC}}$$

$$=\frac{1}{2\,\pi\sqrt{2\times10^{-3}\times10\times10^{-6}}}=\mathbf{1126.13\ Hz.}$$

(**Ans.** of (*iii*))

Q factor of the circuit

$$=\frac{\omega_0\,L}{R}=\frac{2\,\pi\times1126.13\times2\times10^{-3}}{2}=\mathbf{7.07.}$$

(**Ans.** of (*i*))