UNIT-2

FREQUENCY RESPONSE

Resonance : Resonance is the phenomenon at which both Voltage and Current are inphase.

→ At resonance total Circuit acts as resistive Circuit.

→ power factor of the Circuit = 1 i.e., Unity power factor(UPF).

Resonant frequency : The frequency with which resonance Occurs are Known as Resonant frequency. (or)

The frequency at which Capacitive reactance (X_C) is equal to inductive reactance (X_L) i.e., $X_C = X_L$ is Called resonant frequency. Units : Hz

Applications : It is Used in Radio station at recieving Sides. Basically we are having two types.

(i) Series Resonance.

(ii) parallel Resonance.

(i) Series Resonance : Consider a Circuit having Resistance, Inductance, Capacitance Connected in Series, Excited with an alternating Voltage 'V'.

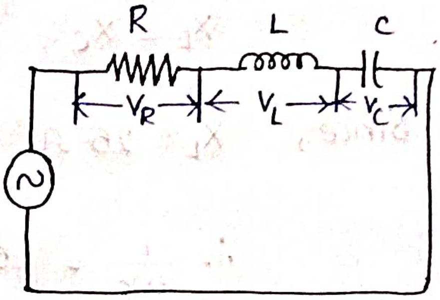
Here total impedance of Circuit $Z = R + j(X_L - X_C)$

By Varying Supply frequency at Some frequency the reactance of the Circuit is equal to Zero.

Capacitive Reactance (X_c) is equal to Inductive reactance (X_L) this is Called Resonant Condition.

$$\boxed{\therefore Z = R}$$

∴ Total Circuit act as Resistive Ckt



$$\therefore X_L - X_C = 0$$

$$\boxed{X_L = X_C}$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 LC = 1$$

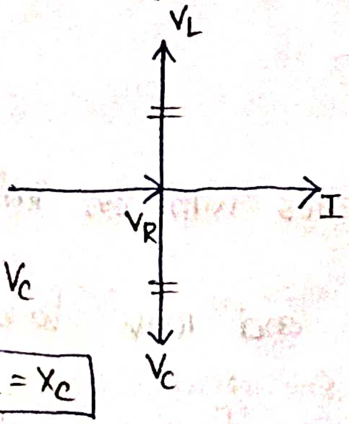
$$\omega^2 = \frac{1}{LC} \Rightarrow (2\pi f)^2 = \frac{1}{LC}$$

$$f^2 = \frac{1}{4\pi^2 LC}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

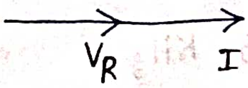
'f_r' is Known as Resonant frequency.

Phasor Diagram :

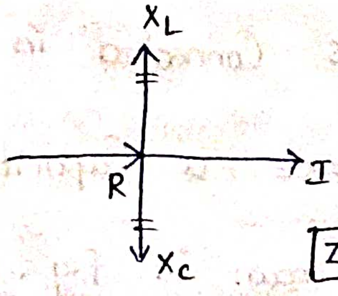


$V_L = V_C$

$\boxed{\therefore X_L = X_C}$

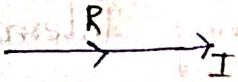


At Resonance



$\boxed{Z = R}$

$\boxed{\therefore X_L = X_C}$

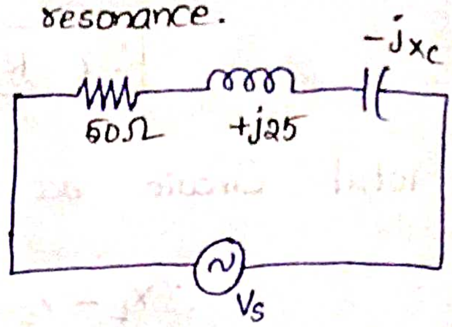


Z = R at Resonance.

$X_L = X_C$, So the Voltage drop across the L and C is zero and Current is Maximum and 'z' is minimum.

Problems:

1. For the Circuit shown in Fig 4.18 determine the Value of Capacitive reactance and impedance at resonance.



Sol:- At resonance

$$X_L = X_C$$

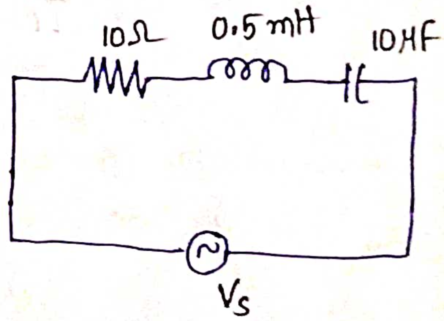
Since, $X_L = 25\,\Omega$

$$\therefore \frac{1}{\omega C} = 25$$

The Value of impedance at resonance is $Z = R$

$$\boxed{\therefore Z = 50\,\Omega}$$

2. Determine the resonant frequency for the circuit shown in Figure.



Sol:- The resonant frequency is

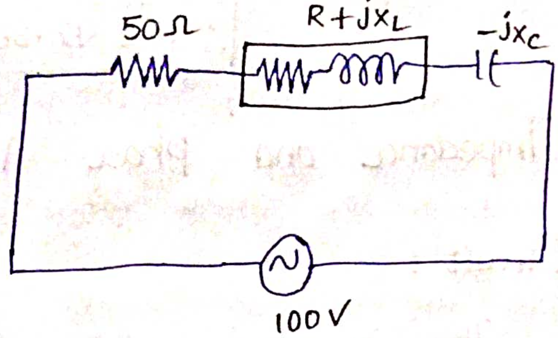
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{10\times10^{-6}\times 0.5\times10^{-3}}}$$

$$f_r = 2.25\ KHz$$

3. A $50\,\Omega$ resistor is Connected in Series with an inductor having Internal resistance, a Capacitor and 100V Variable frequency Supply as shown in Fig. At a frequency of

200 Hz, a maximum Current of 0.7 A flows through the Circuit and voltage across the Capacitor is 200 V. Determine the Circuit Constants.



Sol:- At resonance, Current in the Circuit is maximum.

$$I = 0.7 \text{ A}$$

Voltage across Capacitor is $V_c = IX_c$

Since, $V_c = 200$ $I = 0.7$

$$X_c = \frac{1}{\omega C}$$

$$\omega C = \frac{0.7}{200}$$

$$C = \frac{0.7}{200 \times 2\pi \times 200}$$

$$C = 2.785 \ \mu F$$

At resonance $X_L - X_C = 0 \Rightarrow X_L = X_C$

Since, $X_C = \frac{1}{\omega C} = \frac{200}{0.7} = 285.7\ \Omega$

$$X_L = \omega L = 285.7\ \Omega$$

$$\therefore L = \frac{285.7}{2\pi \times 200} = 0.23 \text{ H}$$

At resonance, the total impedance

$$Z = R + 50$$

$$\therefore R + 50 = \frac{V}{I} = \frac{100}{0.7}$$

$R + 50 = 142.86\ \Omega$

$$\boxed{\therefore R = 92.86\ \Omega}$$

Impedance and phase Angle of a Series resonant Circuit :

The Impedance of a Series RLC Circuit is

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The Variation of X_C and X_L with frequency is shown in figure.

At Zero frequency, both X_C and Z are infinitely large, and X_L is Zero because at Zero frequency the Capacitor act as an Open Circuit and the inductor act as a short Circuit. As the frequency increases, X_C decreases and X_L increases. Since X_C is larger than X_L, at frequencies below the resonant frequency f_r, Z decreases along with X_C. At resonant frequency $X_C = X_L$, and $Z = R$. At frequencies above the resonant frequency f_r, X_L is larger than X_C, Causing Z to increase. The phase angle as a function of frequency is shown in fig 4.2

At a frequency below the resonant frequency, Current leads the Source Voltage because the Capacitive reactance is greater than the inductive reactance. The phase angle decreases as the frequency approaches the resonant value, and is 0° at resonance. At frequencies above resonance, the Current lags behind the Source voltage, because the inductive reactance is greater than Capacitive reactance. As the frequency goes higher, the phase angle approaches 90°.

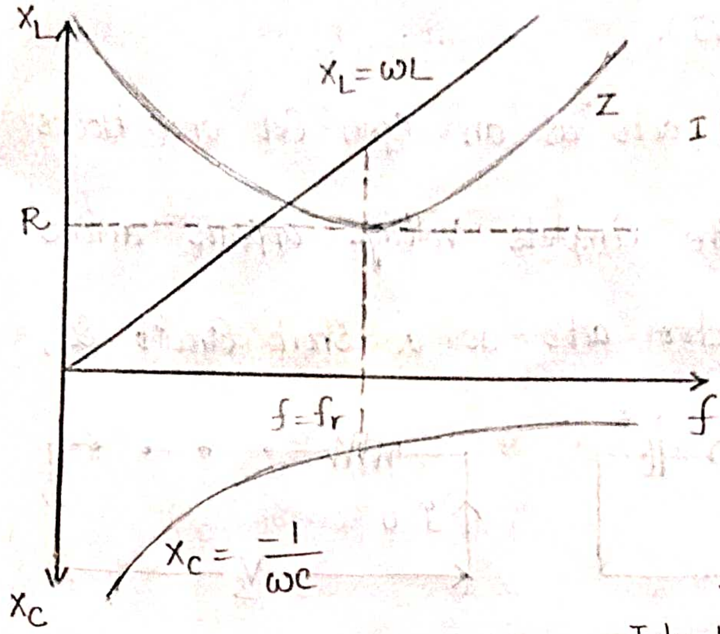


Fig 4.21

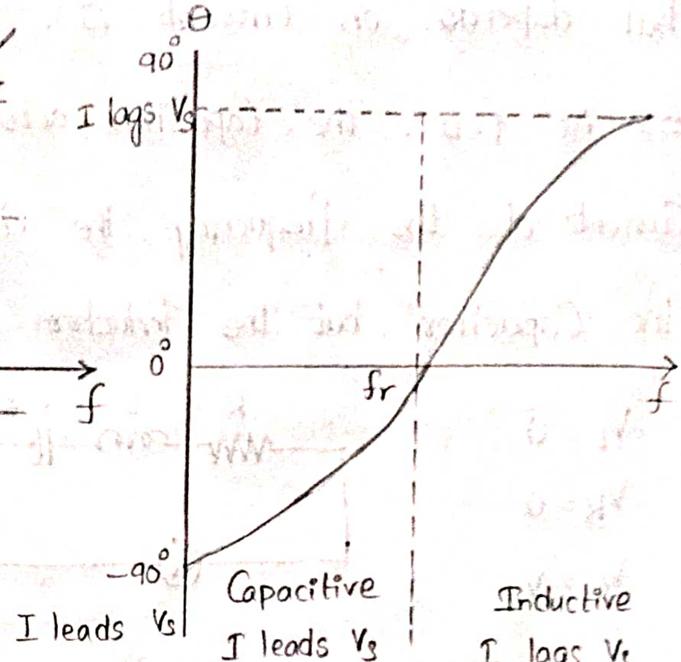
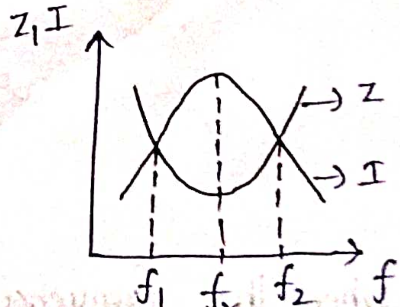


Fig 4.22

Voltages and Currents in a Series Resonant Circuit:

The Variation of impedance and Current with frequency is shown in figure.

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$



$$Z = R + j(X_L - X_C)$$

⇒ At resonant frequency, the Capacitive reactance is equals to inductive reactance, and hence the impedance is minimum. Because of minimum impedance, Maximum Current flows through the Circuit. The Current Variation with frequency is plotted in figure.

→ The Voltage drop across resistance is $V_R = I \cdot R$

The Voltage drop across Inductance is $V_L = I \cdot X_L = I \cdot 2\pi f_L$

The Voltage drop across Capacitance is $V_C = I \cdot X_C = I \cdot \frac{1}{2\pi f_C}$

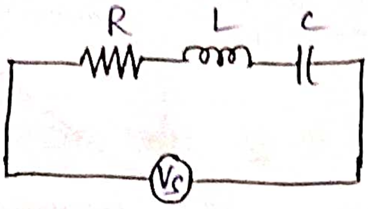
From the above drops V_R is independent of frequency but depends on Current (I).

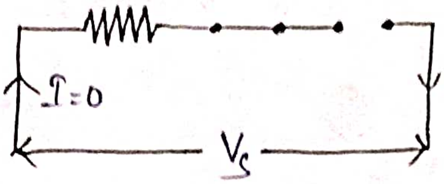
→ At f=0, the Capacitor acts as an Open Ckt and blocks Current at the frequency, the Complete Voltage appears across the Capacitor, but the inductor acts as a Short Circuit. So,

$V_L = 0$

$V_R = 0$

$V_C = V_S$





→ As the frequency increases, X_C decreases and X_L increases. Causing total reactance $(X_C - X_L)$ to decrease. As a result the impedance decreases and the Current increases. As the Circuit Current Increases.

V_R is ↑ $= IR$

V_L is ↑ $= IX_L$

V_C is ↑ $= IX_C$

→ when the frequency reaches its resonant value 'f_r', the impedance is equal to 'R'. i.e., $\boxed{Z = R}$. At this frequency, Current reaches its Maximum Value and V_R is at Maximum Value.

$V_R = IR$ (Maximum)

$\boxed{V_L = V_C}$ (at resonance)

⇒ As the frequency is increased above resonance, X_L continuous to (↑) and X_C continuous to (↓), causing the total reactance $(X_L - X_C)$ to increase. As a result there is (↑) in Z and decrease in current

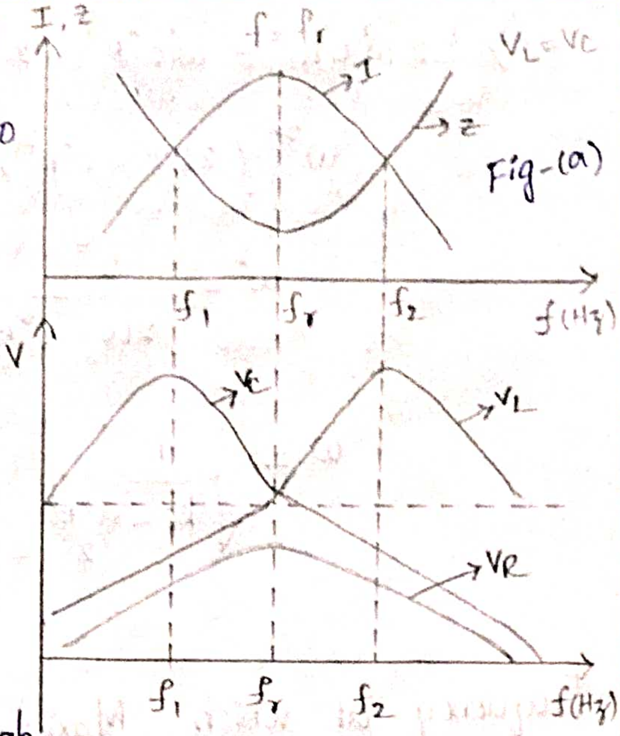
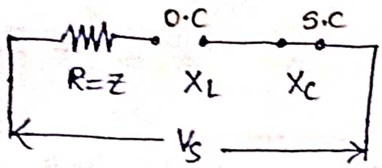


Fig-(b)

⇒ As frequency becomes very high, I reaches zero. both V_R and V_C reaches zero and V_L reaches V_S.



Frequency at which Maximum Value of V_L Occurs :

The Voltage across an inductor is given by $V_L = I X_L$

$= I \cdot \omega L$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Voltage drop across inductor

$$V_L = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \cdot \omega L$$

Squaring on Both Sides

$$V_L^2 = \frac{V^2 \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \frac{V^2 \omega^4 \cdot L^2 C^2}{\omega^2 C^2 R^2 + (\omega^2 L C - 1)^2}$$

By differentiating V_L^2 w.r.t ω and setting $\frac{dV_L^2}{d\omega} = 0$, we get

$$\Rightarrow 2\omega^2 CL - \omega^2 C^2 R^2 - 2 = 0$$

$$\omega^2 (2LC - C^2R^2) = 2$$

$$\omega^2 = \frac{2}{2LC - C^2R^2} = \frac{1}{LC - \frac{C^2R^2}{2}}$$

$$\omega = \frac{1}{\sqrt{LC - \frac{C^2R^2}{2}}} \Rightarrow \boxed{f_L = \frac{1}{2\pi\sqrt{LC - \frac{C^2R^2}{2}}}}$$

Frequency at which Maximum Value of V_C Occurs:

The Voltage across a Capacitor is given by $V_C = I X_C$

$$= I \cdot \frac{1}{\omega C}$$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$V_C = I \cdot X_C = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} \cdot \frac{1}{\omega C}$$

Squaring on B.S

$$V_C^2 = \frac{V^2}{\omega^2 C^2 \cdot \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

$$= \frac{V^2}{\omega^2 C^2 \left[R^2 + \frac{(\omega^2 LC - 1)^2}{\omega^2 C^2}\right]}$$

$$= \frac{V^2}{R^2\omega^2C^2 + (\omega^2 LC - 1)^2}$$

By differentiating V_C^2 w.r to 'w' and Equating to zero

$$\frac{dV_C^2}{d\omega} = 0 = \frac{-V^2\left[2\omega C^2R^2 + 2(\omega^2 LC - 1)\cdot 2\omega LC\right]}{\left[R^2\omega^2C^2 + (\omega^2LC - 1)^2\right]^2}$$

$$2\omega_0^2 C R^2 + 2(\omega^2 LC - 1)\, 2\omega LC = 0$$

$$2\omega^2 L^2 C + CR^2 - 2L = 0$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

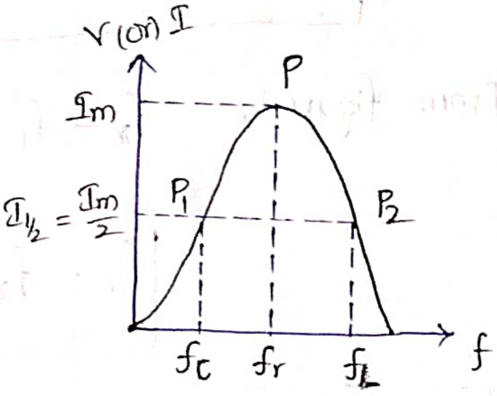
$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$\boxed{f_c = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L}}}$$

Band width of Series Resonant Circuit:

Here 'f_1' is Called "lower Cut-off frequency" at which Current is 0.707 times the Current of resonant Value, 'f_2' is Called "Upper Cut-off frequency"

Equating ① and ②



$$\frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$$

$$L\,\cancel{(\omega_1 + \omega_2)} = \frac{1}{C}\left[\frac{\cancel{\omega_1 + \omega_2}}{\omega_1 \omega_2}\right]$$

$$\omega_1 \cdot \omega_2 = \frac{1}{LC}$$

We Know, $\omega_r^2 = \frac{1}{LC}$ So $\omega_r^2 = \omega_1 \omega_2$

$$\boxed{\begin{aligned} \omega_r &= \sqrt{\omega_1 \cdot \omega_2} \\ f_r &= \sqrt{f_1 f_2} \end{aligned}}$$

∴ The resonant frequency 'f_r' is the geometric mean of the half-power frequencies f_1 and f_2

By adding ① and ②

$$\frac{1}{\omega_1 C} - \omega_1 L + \omega_2 L - \frac{1}{\omega_2 C} = 2R$$

$$L(\omega_2 - \omega_1) + \frac{1}{C}\left[\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right] = 2R$$

We Know, $\omega_r^2 = \frac{1}{LC}$

$$\Rightarrow C = \frac{1}{L\omega_r^2}$$

$$(\omega_2 - \omega_1) L + \frac{\cancel{\omega_r^2} L (\omega_2 - \omega_1)}{\cancel{\omega_r^2}} = 2R$$

$$\cancel{2}L(\omega_2 - \omega_1) = \cancel{2}R$$

$$(\omega_2 - \omega_1) = \frac{R}{L}$$

$$\boxed{\text{Band width (B.W)} = f_2 - f_1 = \frac{R}{2\pi L}} \rightarrow (1)$$

From figure, $f_r - f_1 = \frac{R}{4\pi L}$

$$\boxed{f_1 = f_r - \frac{R}{4\pi L}}$$

from figure, $f_2 - f_r = \frac{R}{4\pi L}$

$$\boxed{f_2 = f_r + \frac{R}{4\pi L}}$$

Divide equation 1 on B.S with f_r.

$$f_2 - f_1 = \frac{R}{2\pi L} \Rightarrow \frac{f_2 - f_1}{f_r} = \frac{R}{2\pi f_r \cdot L}$$

$$\frac{f_2 - f_1}{f_r} = \frac{1}{Q} \Rightarrow \frac{BW}{f_r} = \frac{1}{Q}$$

$$\boxed{Q = \frac{f_r}{B.W}} \quad (\text{or}) \quad \boxed{Q = \frac{f_r}{f - f_1}}$$

Quality factor: It is Ratio of Reactance to resistance

$$Q = \frac{X}{R} = \frac{X_L}{R} = \frac{X_C}{R} \quad [\text{OR}]$$

Quality factor is defined as ratio of $\frac{\text{Reactive power}}{\text{Active power}}$

$$Q = \frac{\text{Reactive power}}{\text{Resistive (or) Active power}} = \frac{\cancel{I^2} X}{\cancel{I^2} R} = \frac{X}{R}$$

$$Q = 2\pi \times \frac{\text{Maximum Energy stored}}{\text{Energy dissipated Per Cycle}}$$

For inductor: Maximum Energy stored $= \frac{1}{2} L I_m^2$

$$\text{Energy dissipated per cycle} = \frac{I^2 R}{f} = \frac{\left(\frac{I_m}{\sqrt{2}}\right)^2 \times R}{f}$$

$$= \frac{\frac{I_m^2}{2} \times R}{f}$$

$$\therefore Q = 2\pi \times \frac{\cancel{\frac{1}{2}} \times L \cancel{I_m^2} \times f}{\frac{\cancel{I_m^2}}{\cancel{2}} \times R} = \frac{2\pi L f}{R}$$

$$\boxed{Q = \frac{2\pi f L}{R} = \frac{\omega L}{R} = \frac{X_L}{R}}$$

For Capacitor: Maximum Energy stored $= \frac{1}{2} C . V_m^2$

$$= \frac{1}{2} C \left(\frac{I_m}{\omega C}\right)^2$$

$$= \frac{1}{2} . C . \frac{I_m^2}{\omega^2 C^2}$$

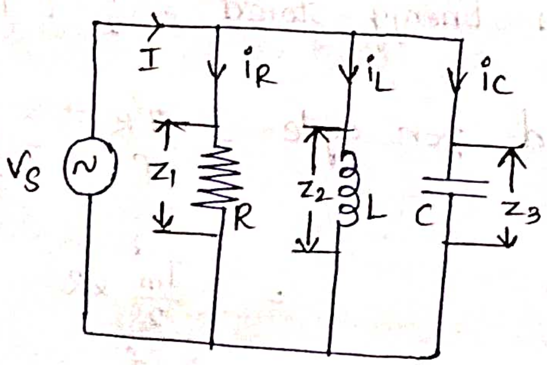
Energy dissipated per Cycle $= \frac{I^2 R}{f} = \frac{\frac{I_m}{\sqrt{2}}^2 \cdot R}{f}$

$$= \left(\frac{I_m}{\sqrt{2}}\right)^2 \times \frac{R}{f} = \frac{I_m^2}{2} \cdot \frac{R}{f}$$

$$Q = \frac{2\pi \times \cancel{\frac{1}{2}} \cdot \frac{\cancel{C} I_m^2}{\omega^2 \cdot C^{\cancel{2}}}}{\frac{\cancel{I_m^2}}{\cancel{2}} \times \frac{R}{f}} = \frac{2\pi f \times \frac{1}{\omega^2 C}}{R}$$

$$\boxed{Q = \frac{\frac{1}{\omega C}}{R} = \frac{X_C}{R}}$$

Parallel Resonance: Consider a parallel Connection of ideal Elements R, L, C.



Parallel Resonance (or) Anti Resonance is a Condition in Parallel Ckt in which the effect of inductive and Capacitive reactance (Susceptance) Completely, neutralize, The impedance is increases to Maximum Value $[Z=R]$ and the Current falls to minimum value $\left[I = \frac{V}{R}\right]$ with a Unity Power factor

If we Vary Supply frequency at particular frequency Net Susceptance of the Circuit Equals to Zero. The frequency at which this Condition Occurs is Known as Resonant frequency.

$$Y = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$Y = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

$$Y = G + j\,(B_C - B_L)$$

where G is Conductance

B_C is Capacitive Susceptance

$\omega C = 2\pi f C$.

B_L is inductive Susceptance $\frac{1}{\omega L} = \frac{1}{2\pi f L}$

At resonance $B_C = B_L$ i.e., $B_C - B_L = 0$

$$\frac{1}{X_C} - \frac{1}{X_L} = 0$$

$$\frac{1}{X_C} = \frac{1}{X_L}$$

$$\omega C = \frac{1}{\omega L}$$

$$\omega^2 = \frac{1}{LC}$$

$$(2\pi f)^2 = \frac{1}{LC}$$

$$f^2 = \frac{1}{4\pi^2 LC}$$

$$\boxed{f = f_r = \frac{1}{2\pi\sqrt{LC}}}$$

Variation of Capacitive and Inductive Susceptance, Impedance and Current with frequency :

$$Y = G + jB = G + j\,(B_C - B_L)$$

$$= \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

$$= \frac{1}{R} + j\left(\omega c - \frac{1}{\omega L}\right)$$

$$= \frac{1}{R} + j\left(2\pi f c - \frac{1}{2\pi f L}\right)$$

(i) At $f = 0$, $Y = \infty$, $Z = 0$ and $I \uparrow$

$f = \infty$, $Y = 0$, $Z = \infty$ and $I \downarrow$

$f = f_r$, $Y = G$, $Z = \frac{1}{G} = R$ and I min

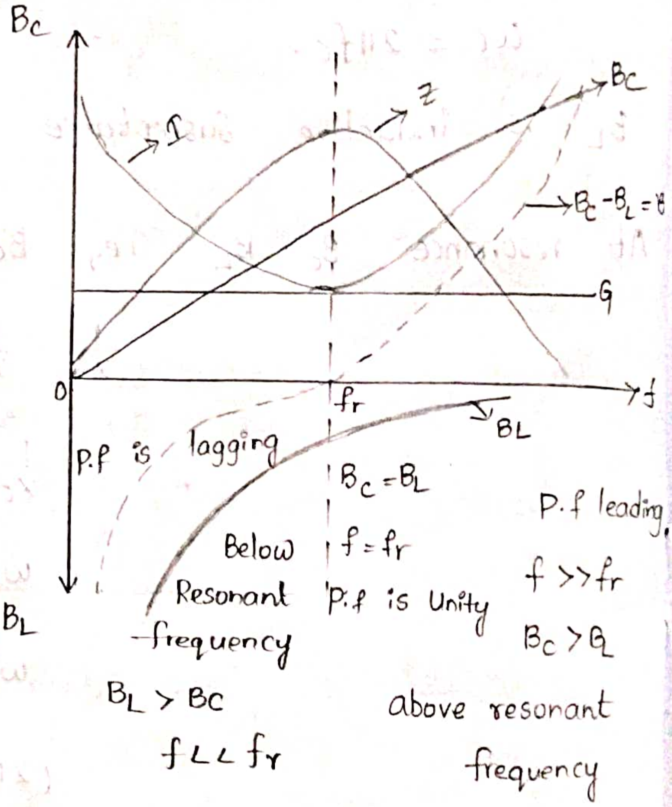
(ii) $B_L = \frac{1}{X_L} = \frac{1}{\omega L} = \frac{1}{2\pi f L}$

$B_L \propto \frac{1}{f}$

(iii) $B_C = \frac{1}{X_C} = \omega c = 2\pi f c$

$B_C \propto f$

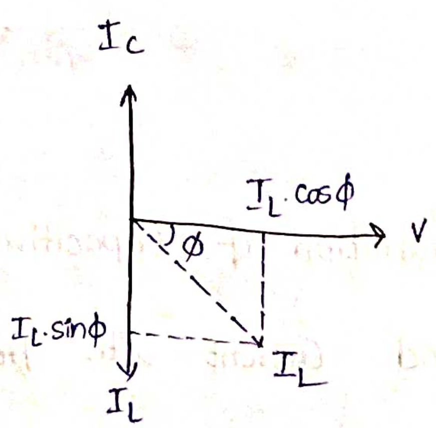
(iv) G is independent of 'f'.



$I = I_L \cdot \cos\phi_L$

$I_L = \frac{V}{Z_L}$

$\cos\phi_L = \frac{R}{Z_L}$



$$I = \frac{V}{Z_L} \cdot \frac{R}{Z_L} = \frac{VR}{Z_L^2} = \frac{VR}{L/c}$$

$$= \frac{V/L}{Rc}$$

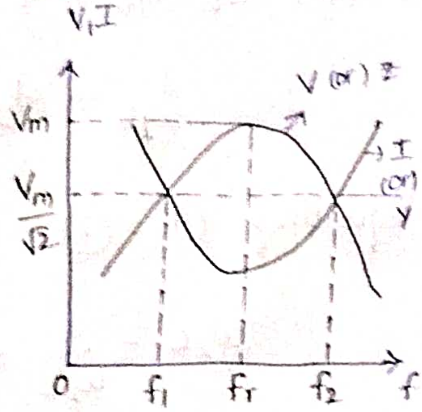
$\boxed{I = \frac{V}{Z}}$ $\boxed{\therefore Z_L = \frac{L}{C}}$

$\boxed{Z = \frac{L}{RC} \text{ is dynamic impedance.}}$

Band width (BW) of parallel Resonant Circuit:

So, Band width is given by

$\therefore BW = f_2 - f_1$



where, f_1 = lower Cut-off frequency

f_2 = higher Cut-off frequency

So, Power $P = \frac{V_m^2}{R} = \boxed{V_m^2 G} \longrightarrow (1)$

at $P_1 = \left(\frac{V_m}{\sqrt{2}}\right)^2 * G = 0.5\ P$

$P_2 = 0.5\ P$

Now, power delivered to the resistance at the f_1 and f_2 is equal to 0.5 times of power at Resonance frequency.

$\therefore$ Power of half power frequencies of f_1 & $f_2 = 0.5\ V_m^2 G$

$$(V_m^2 \cdot G)_{1/2} = 0.5\ V_m^2 \cdot G$$

$$\left(\frac{I}{Y}\right)^2 \cdot G = \frac{1}{2}\left(\frac{I}{G}\right)^2 \times G$$

$$\left(\frac{1}{\sqrt{G^2 + (B_C - B_L)^2}}\right)^2 = \frac{1}{2G^2} \Rightarrow \frac{1}{G^2 + (B_C - B_L)^2} = \frac{1}{2G^2}$$

$$(B_C - B_L)^2 = 2G^2 - G^2$$

$$(B_C - B_L)^2 = G^2$$

$$B_C - B_L = \pm G$$

$$(B_C - B_L)^2 = G^2 \Rightarrow B_C - B_L = \pm G$$

At f_2 $\quad B_C - B_L = G \longrightarrow (1)$

$$\omega_2 C - \frac{1}{\omega_2 L} = 1/R \longrightarrow (2)$$

f_1 (lower Cut off frequency), $\quad B_C - B_L = -G$

(or) $B_L - B_C = G$ $\rightarrow$ (2)

$$\frac{1}{\omega_1 L} - \omega_1 C = 1/R \rightarrow (3)$$

Equating (1) = (2)

$$\omega_2 C - \frac{1}{\omega_2 L} = \frac{1}{\omega_1 L} - \omega_1 C$$

$$C\left[\omega_2 + \omega_1\right] = \frac{1}{L}\left[\frac{\omega_2 + \omega_1}{\omega_1 \cdot \omega_2}\right]$$

$$\omega_1 \cdot \omega_2 = \frac{1}{LC} \rightarrow (4)$$

We Know, $\omega_r^2 = \frac{1}{LC}$

Compare with equation (4)

$$\omega_1 \cdot \omega_2 = \omega_r^2 = \omega_r = \sqrt{\omega_1 \cdot \omega_2}$$

$$\boxed{f_r = \sqrt{f_1 \cdot f_2}}$$

Adding (1) + (2), $\frac{1}{\omega_1 L} - \omega_1 C + \omega_2 C - \frac{1}{\omega_2 L} = \frac{1}{R} + \frac{1}{R}$

$$C\left[\omega_2 - \omega_1\right] + \frac{1}{L}\left[\frac{1}{\omega_1} - \frac{1}{\omega_2}\right] = \frac{2}{R}$$

$$C\left[\omega_2 - \omega_1\right] + \frac{1}{L}\left[\frac{\omega_2 - \omega_1}{\omega_1 \cdot \omega_2}\right] = \frac{2}{R}$$

$$C\left[\omega_2 - \omega_1\right] + C\cancel{\omega_r^2}\left[\frac{\omega_2 - \omega_1}{\cancel{\omega_1 \omega_2}}\right] = \frac{2}{R}$$

$$\cancel{2}C\left[\omega_2 - \omega_1\right] = \frac{\cancel{2}}{R}$$

$$\omega_2 - \omega_1 = \frac{1}{RC}$$

$$\boxed{f_2 - f_1 = \frac{1}{2\pi RC}}$$

So, we know that,

$$Q = \frac{f_r}{B.W} = \frac{1}{\cancel{2\pi}\sqrt{LC}} \times \cancel{2\pi} RC$$

$$\boxed{Q = R.\sqrt{\frac{C}{L}}}$$

Quality factor for parallel Resonant Circuit:

$$Q = 2\pi \left[\frac{\text{Energy stored in Inductor}}{\text{Energy dissipated Per Cycle}}\right]$$

For Inductor:

$$= 2\pi \left[\frac{\frac{1}{\cancel{2}} L I^2}{\frac{I^2}{\cancel{2}} R \times \frac{1}{f}}\right] \qquad \text{where} \quad I = \frac{V}{WL}, \quad I = \frac{V}{R}$$

$$= 2\pi \left[\frac{L\left(\frac{V}{WL}\right)^2}{\left(\frac{V}{R}\right)^2 . R . \frac{1}{f}}\right] = \cancel{2\pi}\left[\frac{1}{(\cancel{2\pi} f L)} \times R \times \cancel{f}\right]$$

$$= \frac{R}{2\pi f L}$$

$$\boxed{Q = \frac{R}{2\pi f L} = \frac{R}{X_L}}$$

For Capacitor:

$$Q = 2\pi \left[\frac{\frac{1}{2} C V_m^2}{\frac{V_{Rms}^2}{R} . T}\right] = \pi \left[\frac{C . V_m^2}{\left(\frac{V_m^2}{V2}\right) . \frac{1}{R} . T}\right]$$

$$= 2\pi \left[\frac{C . R}{T}\right]$$

$$= 2\pi f . C R \qquad = W_c . R$$

$$\boxed{Q = \frac{R}{X_c}}$$