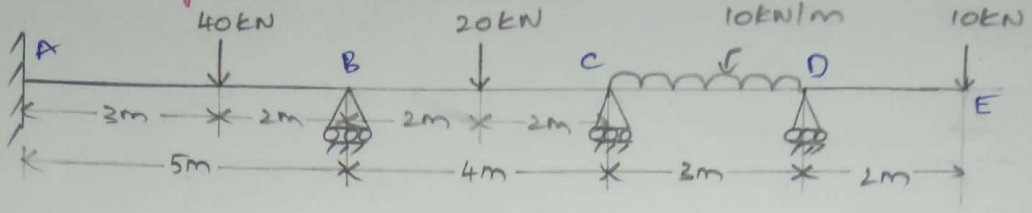
UNIT-3

MOMENT DISTRIBUTION METHOD

Analysize beam as shown in figure by moment distribution method and draw the B.M. Assume the EI is constant throughout the beam.



$$\bar{M}_{AB} = -\frac{wab^2}{l^2} \Rightarrow \frac{-40\times3\times2^2}{5^2} \Rightarrow -19.2\ KN.m$$

$$\bar{M}_{BA} = \frac{wa^2b}{l^2} \Rightarrow \frac{40\times3^2\times2}{5^2} \Rightarrow 28.8\ KN.m$$

$$\bar{M}_{BC} = -\frac{wl}{8} \Rightarrow \frac{-20\times4}{8} \Rightarrow -10\ KN.m$$

$$\bar{M}_{CB} = \frac{wl}{8} \Rightarrow \frac{20\times4}{8} \Rightarrow 10\ KN.m$$

$$\bar{M}_{CD} = -\frac{wl^2}{12} \Rightarrow \frac{-10\times3^2}{12} \Rightarrow -7.5\ KN.m$$

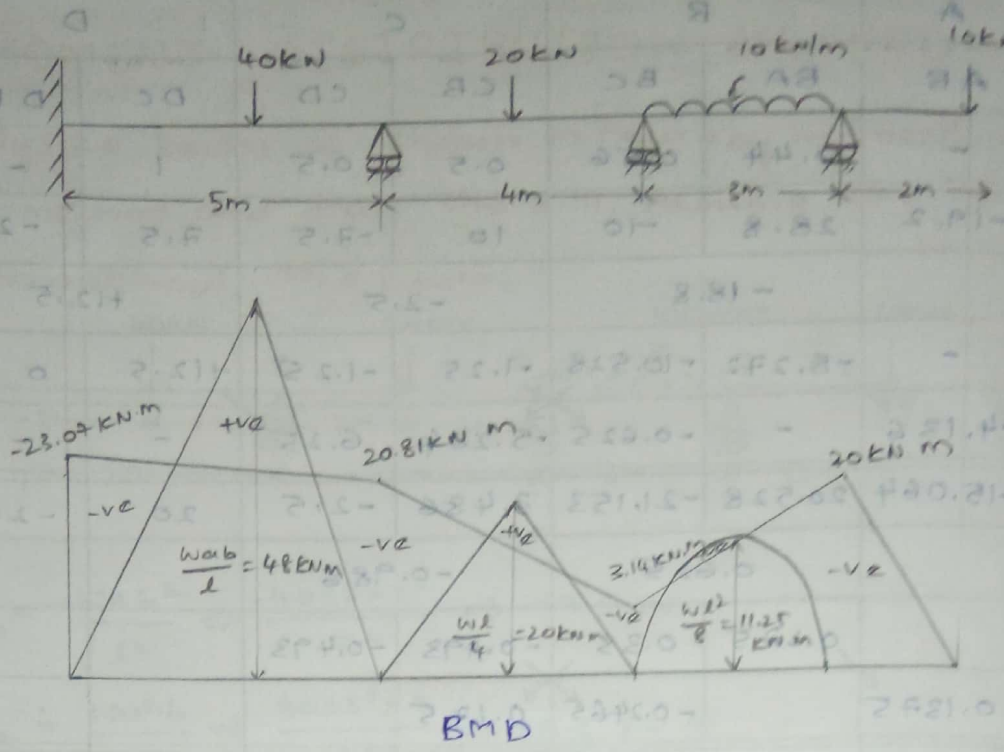
$$\bar{M}_{DC} = \frac{wl^2}{12} \Rightarrow \frac{10\times3^2}{12} \Rightarrow 7.5\ KN.m$$

$$\bar{M}_{DE} = -10\times2 \Rightarrow -20\ KN.m$$

$$\bar{M}_{ED} = 10\times2 \Rightarrow 20\ KN.m$$

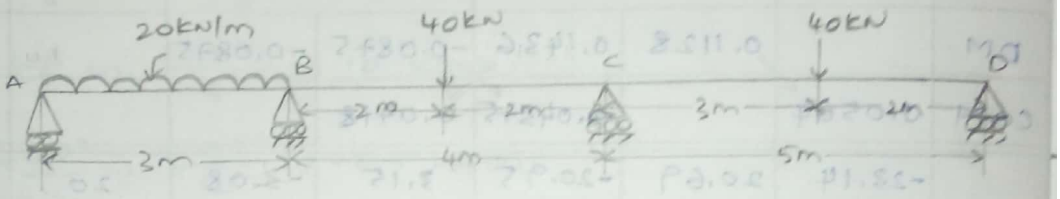
Joint	Member	K	Σk	Distribution factor $= \frac{k}{\Sigma k}$
B	BA	$\frac{4EI}{L} \Rightarrow \frac{4}{5}$	$\frac{9}{5}$	0.44
	BC	$\frac{4EI}{L} \Rightarrow 1$		0.56
C	CB	$\frac{4EI}{L} \Rightarrow 1$	2	0.5
	CD	$\frac{3EI}{L} = 1$		0.5
D	DC	$\frac{4EI}{L} = \frac{4}{3}$	$\frac{4}{3}$	1

Joint	A		0.44 B	0.56	0.5 C	0.5		D
FEM	-19.2	28.8	-10	10	-7.5	7.5		-20
					6.25	+12.5		
	-19.2	28.8	-10	10	-1.25	20		-20
		-18.8			-2.5			
		-8.272	-10.528	-1.25	-1.25			
COM	-4.136		-0.625	-5.264				
	-23.336	20.528	-21.153	3.486	-2.5	20		-20
		0.625		-0.986				
		0.275	0.35	-0.493	-0.493			
COM	0.1375		-0.246	0.175				
	-23.19	20.803	-21.049	3.168	-2.993	20		-20
		0.246		-0.175				
		0.108	0.137	-0.087	-0.087			
COM	0.054		-0.043	0.0685				
	-23.13	20.91	-20.95	3.149	-3.08	20		-20
		0.04		-0.069				
		0.017	0.022	-0.0345	-0.0345			
COM	0.0085		-0.017	0.011				
	-23.12	20.92	-20.94	3.12	-3.11	20		-20
		0.02		-0.01				
		0.008	0.011	-0.005	-0.005			
COM	0.004		-0.0025	0.0056				
Final moment	-23.11	20.92	-20.93	3.12	-3.12	20		-20



BMD

2.



Analysize the continuous beam by Moment Distribution Method. Draw SFD & BMD. Assume EI is constant throughout the section.

So:- $\bar{M}_{AB} = \frac{-wl^2}{12} \Rightarrow -\frac{20\times 3^2}{12} \Rightarrow -15\ KN.m$

$\bar{M}_{BA} = \frac{wl^2}{12} \Rightarrow \frac{20\times 3^2}{12} \Rightarrow 15\ KN.m$

$\bar{M}_{BC} = \frac{-wl}{8} \Rightarrow -\frac{40\times 4}{8} \Rightarrow -20\ KN.m$

$\bar{M}_{CB} = \frac{wl}{8} \Rightarrow \frac{40\times 4}{8} \Rightarrow 20\ KN.m$

$\bar{M}_{CD} = \frac{-wab^2}{l^2} \Rightarrow -\frac{40\times 3\times 2^2}{5^2} \Rightarrow -19.2\ KN.m$

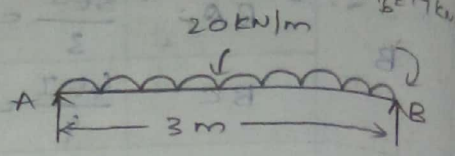
$\bar{M}_{DC} = \frac{wa^2b}{l^2} \Rightarrow \frac{40\times 3^2\times 2}{5^2} \Rightarrow 28.8\ KN.m$

Joint	Member	$K = \frac{I}{L}$	Σk	$D.F = \frac{k}{\Sigma k}$
B	BA	$\frac{3EI}{3} = 1$	2	1/2
	BC	$\frac{4EI}{4} = 1$		1/2
C	CB	$\frac{4EI}{4} = 1$	$\frac{8}{5}$	5/8
	CD	$\frac{3EI}{5} = \frac{3}{5}$		3/8

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	-	1/2	1/2	5/8	3/8	-
F.E.M	-15	15	-20	20	-19.2	28.8
	+15	7.5			-14.4	-28.8
	0	22.5	-20	20	-33.6	0
		-2.5		13.6		
DM		-1.25	-1.25	8.5	5.1	
COM			4.25	-0.625		
	0	21.25	-17	27.875	-28.5	0
		-4.25		0.625		
DM		-2.125	-2.125	0.390	0.234	
COM			0.195	-1.062		
	0	19.125	-18.93	27.203	-28.266	0
		-0.195		1.063		
DM		-0.0975	-0.0975	0.664	0.398	
COM			0.332	-0.048		
	0	19.027	-18.995	27.819	-27.868	0
		-0.032		0.049		
DM		-0.016	-0.016	0.0306	0.0183	
COM			0.0153	-0.008		
	0	19.011	-18.995	27.841	-27.849	0
		-0.016		0.008		
DM		-0.008	-0.008	0.005	0.003	
COM			0.0025	-0.004		
Final moments	0	19.003	-19.000	27.842	-27.846	0

Reactions :-

$R_A + R_{B1} = 60\,KN$



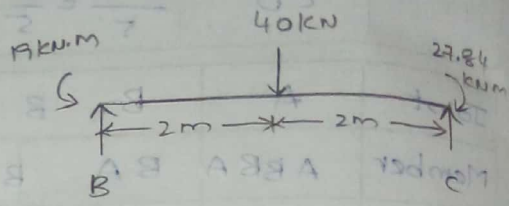
$\Sigma M_A = 0$

$R_B \times 3 - 20 \times 3 \times \frac{3}{2} - 19 = 0$

$R_{B1} = 36.33\,KN$

$R_A = 23.67\,KN$

$R_{B2} + R_{C1} = 40\,KN$



$\Sigma M_B = 0$

$R_{C1} \times 4 - 40 \times 2 - 27.84 + 19 = 0$

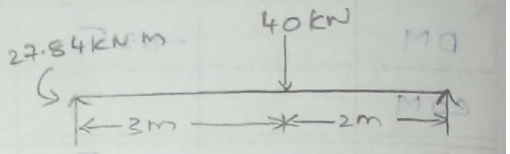
$R_{C1} = 22.21\,KN$

$R_{B2} = 17.79\,KN$

$R_B = R_{B1} + R_{B2} = 36.33 + 17.79$

$R_B = 54.12\,KN$

$R_{C2} + R_D = 40\,KN$



$\Sigma M_C = 0$

$R_D \times 5 - 40 \times 3 + 27.84 = 0$

$R_D = +18.43\,KN$

$R_{C2} = 21.57\,KN$

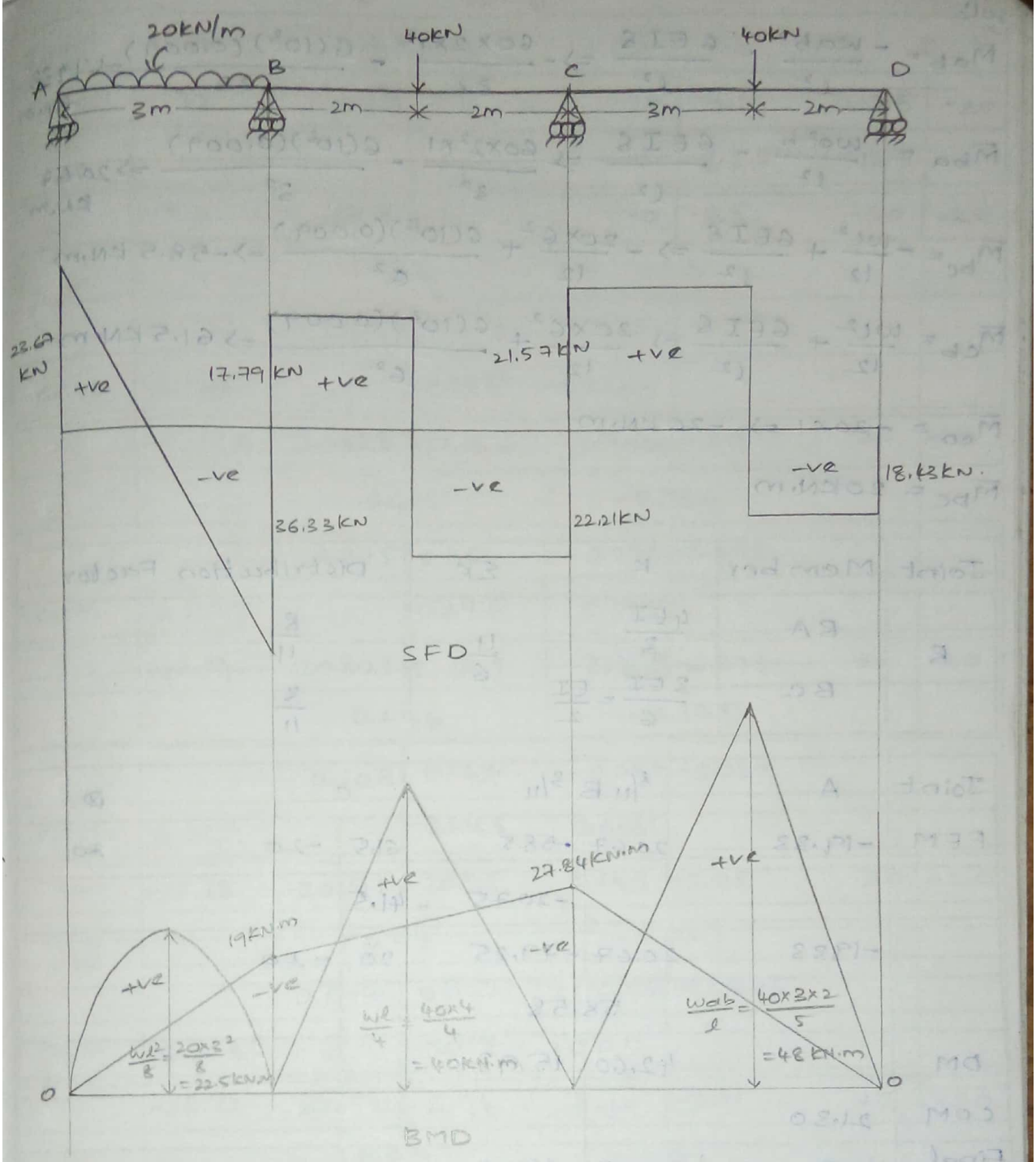
$R_C = R_{C1} + R_{C2} = 22.21 + 21.57$

$R_C = 43.78$

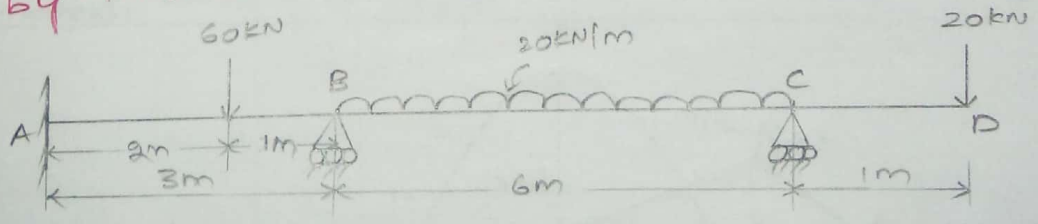
$R_A + R_B + R_C + R_D = 140\,KN$

$23.67 + 54.12 + 43.78 + 18.43 = 140$

$140 = 140$



3. Analysize the continuous beam as shown in fig by moment distribution method. Support B sinks by 9mm. Take EI = 1×10^{12} N.mm^2. Draw BMD



$EI = 1\times10^{12}$ N.mm^2

$EI = 1\times10^{12} \times (10^{-3})^2$

$EI = 1\times10^{6}$ N.m^2

$EI = 1\times10^{6}\times10^{-3}$ KN.m^2

$EI = 1\times10^{3}$ KN.m^2

Sol:

$$\bar{M}_{ab} = -\frac{wab^2}{l^2} - \frac{6EI\delta}{l^2} \Rightarrow -\frac{60\times2\times1^2}{3^2} - \frac{6(10^3)(0.009)}{3^2} \Rightarrow -19.33 \text{ KNM}$$

$$\bar{M}_{ba} = \frac{wa^2b}{l^2} - \frac{6EI\delta}{l^2} \Rightarrow \frac{60\times2^2\times1}{3^2} - \frac{6(10^3)(0.009)}{3^2} \Rightarrow 20.67 \text{ KN.m}$$

$$\bar{M}_{bc} = -\frac{wl^2}{12} + \frac{6EI\delta}{l^2} \Rightarrow -\frac{20\times6^2}{12} + \frac{6(10^3)(0.009)}{6^2} \Rightarrow -58.5 \text{ KN.m}$$

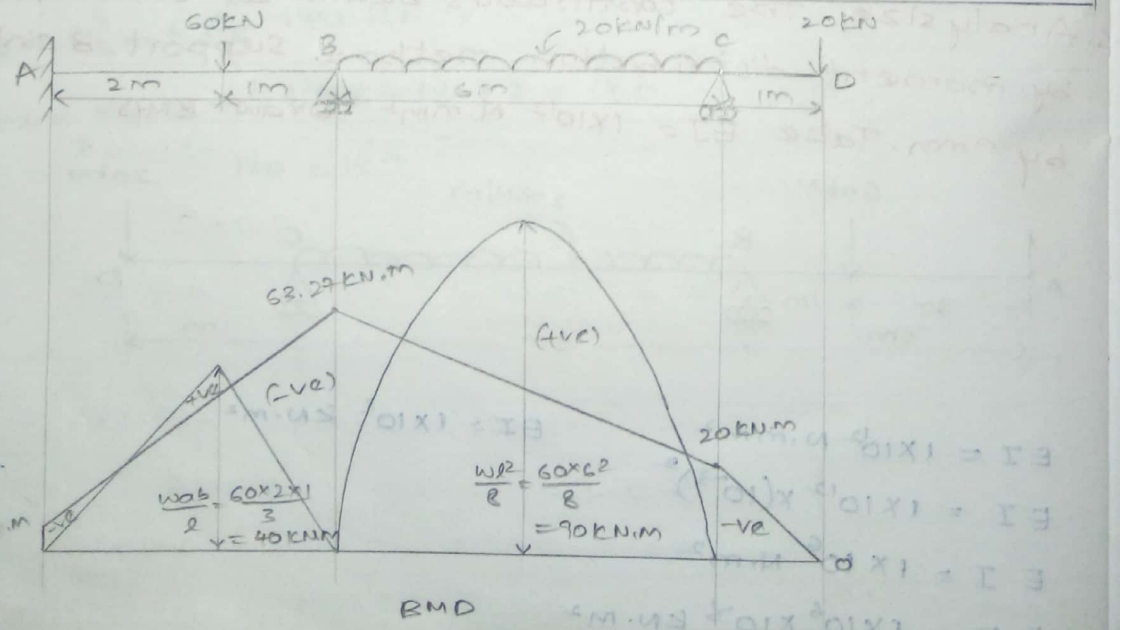
$$\bar{M}_{cb} = \frac{wl^2}{12} + \frac{6EI\delta}{l^2} \Rightarrow \frac{20\times6^2}{12} + \frac{6(10^3)(0.009)}{6^2} \Rightarrow 61.5 \text{ KN.m}$$

$$\bar{M}_{cd} = -20\times1 \Rightarrow -20 \text{ KN.m}$$

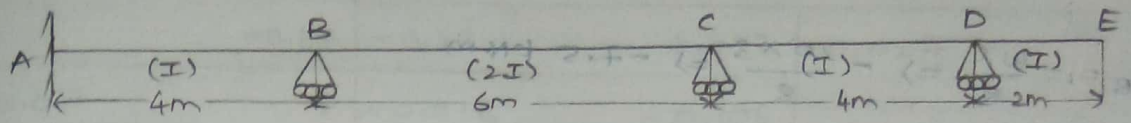
$$\bar{M}_{DC} = 20 \text{ KN.m}$$

Joint	Member	K	ΣK	Distribution Factor
B	BA	$\frac{4EI}{3}$	$\frac{11}{6}$	$\frac{8}{11}$
	BC	$\frac{3EI}{6} = \frac{EI}{2}$		$\frac{3}{11}$

Joint	A	8/11 B	3/11	C	D
FEM	-19.33	20.67	-58.5	61.5	-20 ~~20~~
			-20.75	-41.5	
	-19.33	20.67	-79.25	20	-20
		58.58			
DM		42.60	15.97		
COM	21.30				
Final moments	1.97	63.27	-63.28	20	-20



4. Analysize the continuous beam by MDM. If support B sinks by 12mm. Given $E = 200\ KN/mm^2$ $I = 20\times10^6\ mm^4$. Draw the BMD.



Sol:- $\bar{M}_{AB} = -\frac{6EI\delta}{l^2} \Rightarrow -\frac{6\times(200\times10^6)\times(2\times10^{-5})\times0.012}{4^2}$

$\bar{M}_{AB} = -18\ KN.m$

$\bar{M}_{BA} = -\frac{6EI\delta}{l^2} \Rightarrow +\frac{6\,(200\times10^6)\,(2\times10^{-5})\times0.012}{4^2}$

$\bar{M}_{BA} = +18\ KN.m$

$\bar{M}_{BC} = \frac{6EI\delta}{l^2} \Rightarrow \frac{6\,(200\times10^6)2(2\times10^{-5})\times0.012}{6^2}$

$\bar{M}_{BC} = 1.6\ KN.m$

$\bar{M}_{CB} = \frac{6EI\delta}{l^2} \Rightarrow \frac{6(200\times10^6)2(2\times10^{-5})\times0.012}{6^2}$

$\bar{M}_{CB} = 16\ KN.m$

$\bar{M}_{CD} = \bar{M}_{DC} = 0$

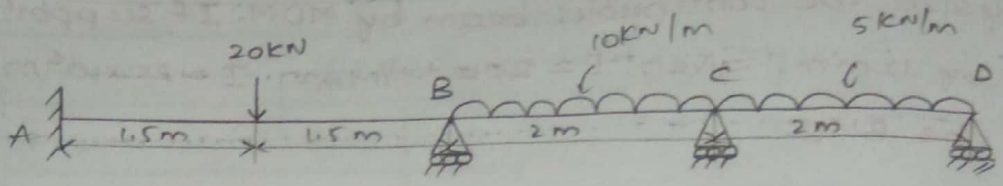
Joint	Member	K	ΣK	D.F = $\frac{K}{\Sigma K}$
B	BA	$\frac{4EI}{4} = 1$	$\frac{7}{3}$	$\frac{3}{7}$
	BC	$\frac{6EI}{6} = \frac{4}{3}$		$\frac{4}{7}$
C	CB	$\frac{6EI}{6} = \frac{4}{3}$	$\frac{25}{12}$	$\frac{16}{25}$
	CD	$\frac{3EI}{4} = \frac{3}{4}$		$\frac{9}{25}$

A		$\frac{3}{7}$ B $\frac{4}{7}$		$\frac{16}{25}$ C $\frac{9}{25}$	D
-18	-18	16	16	0	0
	2		-16		
0	0.857	1.142	-10.24	-5.76	
0.428		5.12	0.571		
-17.572	-17.143	22.262	6.331	-5.76	0
	-5.12		-0.571		
0	-2.19	-2.92	-0.365	-0.205	
-1.09		-0.182	1.46		

To be continued.... ☹

A	3/7 B	4/7	16/25 C	9/25	D
-18.662	-19.33	19.16	7.426	-5.965	0
	0.17		-1.461		
	0.072	0.097	-0.935	-0.526	
0.036		-0.467	0.048		
-18.62	-19.26	18.79	6.539	-6.491	0
	0.47		-0.048		
	0.201	0.268	-0.030	-0.017	
0.1		-0.015	0.134		
-18.52	-19.06	19.043	6.643	-6.508	0
	0.017		-0.135		
	0.007	0.0097	-0.0864	-0.0486	
0.0036		-0.0432	0.0048		
-18.51	-19.05	+19.00	6.56	-6.45	0
	0.05		-0.11		
	0.021	0.028	-0.07	-0.039	
0.010		-0.035	0.014		
-18.5	-19.03	19.01	6.50	6.49	0

5.



$\bar{M}_{AB} = -\frac{wl}{8} \Rightarrow -\frac{20\times3}{8} \Rightarrow -7.5$ KN.m

$\bar{M}_{BA} = \frac{wl}{8} \Rightarrow \frac{20\times3}{8} \Rightarrow 7.5$ KN.m

$\bar{M}_{BC} = -\frac{wl^2}{12} \Rightarrow -\frac{10\times2^2}{12} \Rightarrow -3.33$ KN.m

$\bar{M}_{CB} = \frac{wl^2}{12} \Rightarrow \frac{10\times2^2}{12} \Rightarrow 3.33$ KN.m

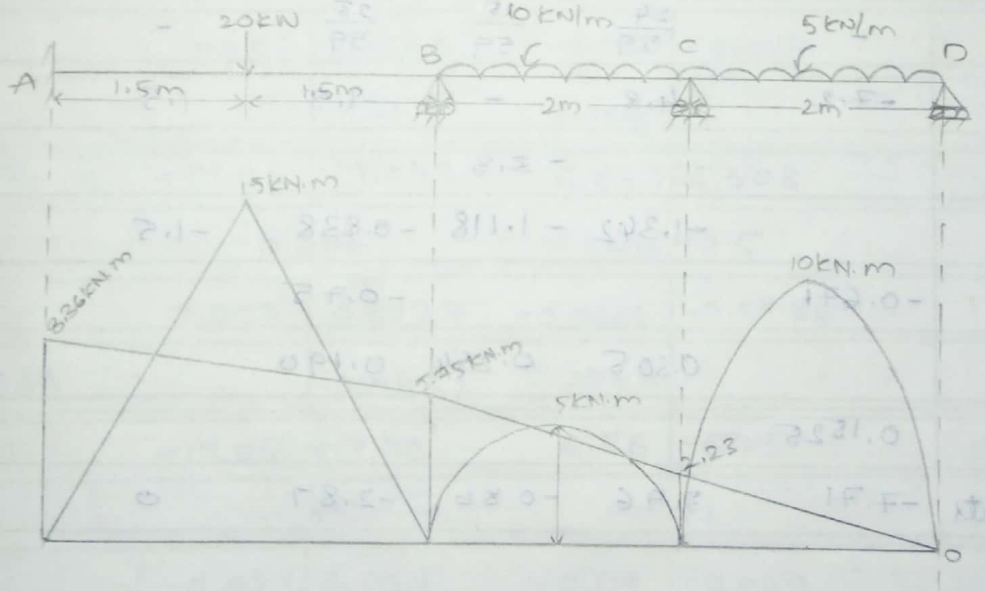
$\bar{M}_{CD} = -\frac{wl^2}{12} \Rightarrow -\frac{5\times2^2}{12} \Rightarrow -1.67$ KN.m

$\bar{M}_{DC} = \frac{wl^2}{12} \Rightarrow \frac{5\times2^2}{12} \Rightarrow 1.67$ KN.m

Joint	Member	K	ΣK	D.F = $\frac{K}{\Sigma K}$
B	BA	$\frac{4IE}{L} \Rightarrow \frac{4}{3}$	$\frac{10}{3}$	$\frac{2}{5}$
	BC	$\frac{4EI}{L} = \frac{4}{2} = 2$		$\frac{3}{5}$
C	CB	$\frac{4EI}{L} = \frac{4}{2} = 2$	$\frac{7}{2}$	$\frac{4}{7}$
	CD	$\frac{3EI}{2} = \frac{3}{2}$		$\frac{3}{7}$

A		2/5 B	3/5	4/7 C	3/7	D
-7.5		7.5	-3.33	3.33	-1.67	1.67
					-0.835	-1.67
-7.5		7.5	-333	3.33	-2.505	0
		-4.17		-0.825		
		-1.66	-2.50	-0.471	+0.353	
-0.83			-0.235	-1.25		
-8.33		5.84	-6.06	1.609	-2.858	0
		0.22		1.249		
		0.088	0.132	0.713	0.535	
0.044			0.356	0.066		
-8.28		5.92	-5.57	2.388	-2.323	0

A	2/5 B	3/5	4/9 C	3/9	D
-8.28	5.92	-5.57	2.388	-2.323	0
	-0.35		-0.065		
	-0.14	-0.21	-0.037	-0.027	
-0.07		-0.018	-0.105		
-8.35	5.78	-5.79	2.246	-2.35	0
	0.01		0.104		
	0.004	0.006	0.059	0.044	
0.002		0.029	0.003		
-8.348	5.784	-5.755	+2.184	-2.306	
	-0.029		0.122		
	-0.011	-0.017	0.069	0.052	
-0.0055		0.034	-0.0085		
-8.3535	5.773	-5.732	2.244	-2.254	
	-0.041		0.01		
	-0.016	-0.024	0.005	0.00428	
-0.0082		0.002	-0.012		
-8.36	5.75	-5.75	2.237	-2.249	



6 A continuous beam ABC fixed at A, is supported on an elastic column BD loaded as shown in figure. Analyse frame and draw BMD. AB = 2I, BC = BD = I

Sol: $\bar{M}_{AB} = -\frac{wab^2}{l^2} \Rightarrow -\frac{10\times2\times3^2}{5^2}$

$\bar{M}_{AB} = -7.2\ kN.m$

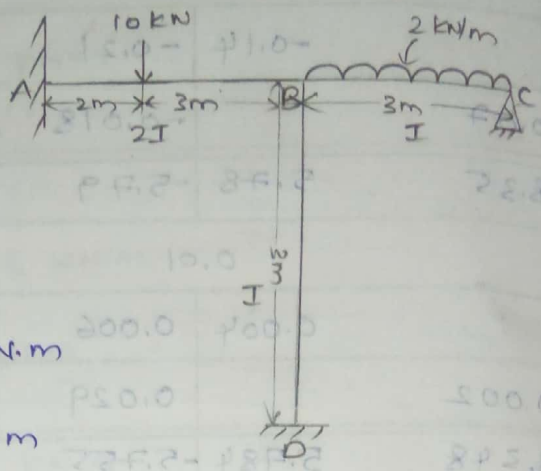
$\bar{M}_{BA} = \frac{wa^2b}{l^2} \Rightarrow \frac{10\times2^2\times3}{5^2} \Rightarrow$

$\bar{M}_{BA} = 4.8\ kN.m$

$\bar{M}_{BC} = -\frac{wl^2}{12} \Rightarrow -\frac{2\times3^2}{12} \Rightarrow -1.5\ kN.m$

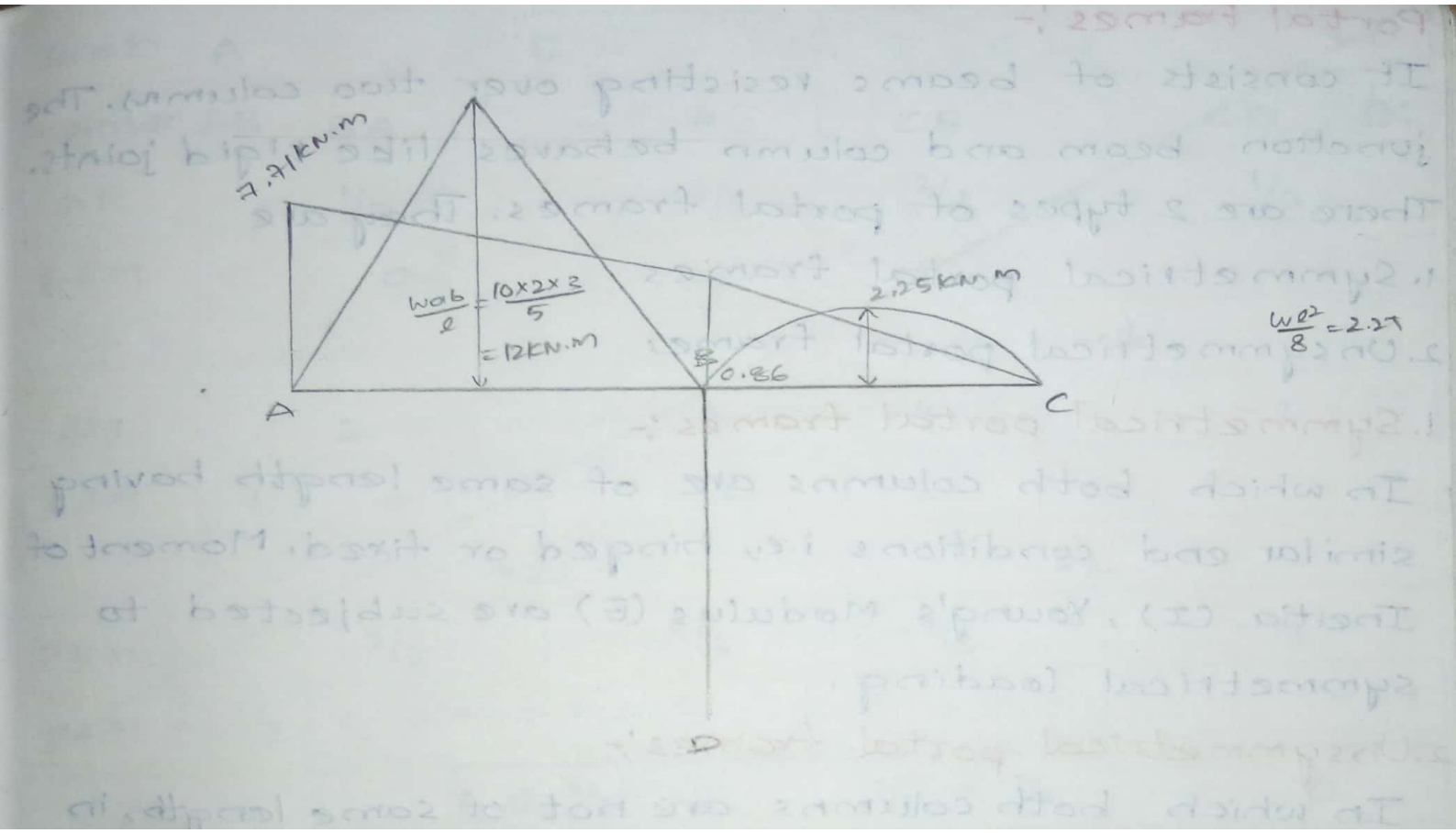
$\bar{M}_{CB} = \frac{wl^2}{12} \Rightarrow \frac{2\times3^2}{12} \Rightarrow 1.5\ kN.m$

$\bar{M}_{BD} = \bar{M}_{DB} = 0$



Joint	Member	K	ΣK	DF = K/ΣK
B	BA	$\frac{4EI}{L} \Rightarrow \frac{4(2EI)}{5}$	$\frac{59}{15}$	$\frac{24}{59}$
	BC	$\frac{3EI}{L} = \frac{3EI}{3}$		$\frac{15}{59}$
	BD	$\frac{4EI}{L} = \frac{4EI}{3}$		$\frac{20}{59}$

Joint	A	B			C	D
Member	AB	BA	BD	BC	CB	DB
D.F	-	$\frac{24}{59}$	$\frac{20}{59}$	$\frac{15}{59}$	-	-
FEM	-7.2	4.8	-	-1.5	1.5	-
			-3.3			
BM		-1.342	-1.118	-0.838	-1.5	
COM	-0.671			-0.75		
BM		0.305	0.254	0.190		
COM	0.1525					
Final moments	-7.71	3.76	-0.86	-2.89	0	0



Portal Frames :-

It consists of beams resisting over two columns. The junction beam and column behaves like rigid joints. There are 2 types of portal frames. They are

1. Symmetrical portal frames
2. Unsymmetrical portal frames

1. Symmetrical portal frames :-

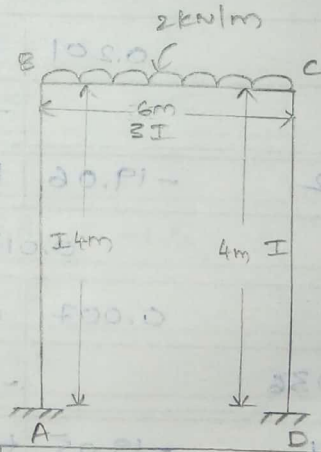
In which both columns are of same length having similar end conditions i.e, hinged or fixed. Moment of Inertia (I), Young's Modulus (E) are subjected to symmetrical loading.

2. Unsymmetrical portal frames :-

In which both columns are not of same length, in this frame it is subjected to horizontal moment known as sway to one side or other.

So in rigid joint both columns and beams have motion of translated.

1. A portal frame ABCD is loaded as shown in figure is loaded with a UDL of 2KN/m on the horizontal member. The MOI of AB, CD = I and BC = 3I. Find the support reactions and BM in frame by MDM and draw BMD.



Sol: $\overline{M}_{AB} = \overline{M}_{BA} = \overline{M}_{CD} = \overline{M}_{DC} = 0$

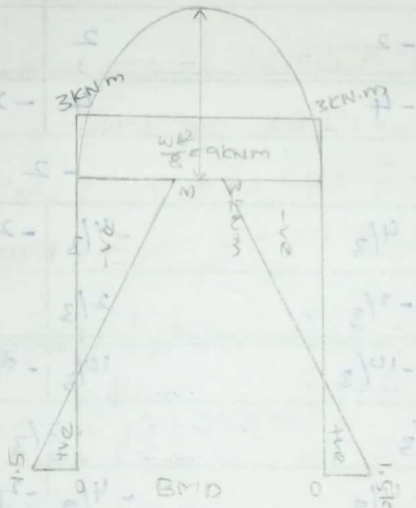
$\overline{M}_{BC} = -\frac{wL^2}{12} \Rightarrow -\frac{2 \times 6^2}{12} \Rightarrow -6\ KN.m$

$\overline{M}_{CB} = \frac{wL^2}{12} \Rightarrow \frac{2 \times 6^2}{12} \Rightarrow 6\ KN.m$

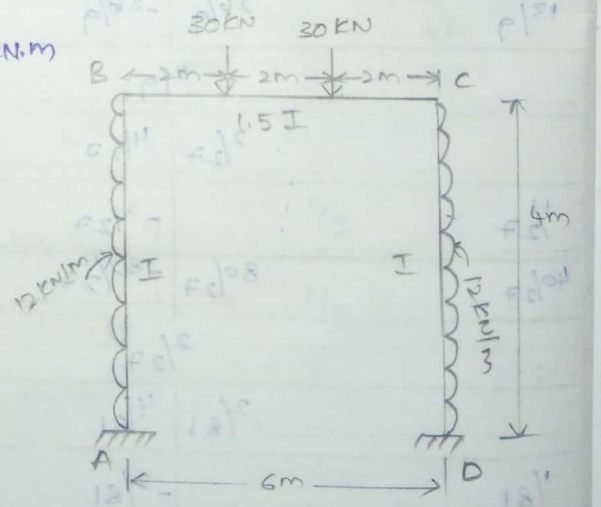
Joint	Member	K	ΣK	D.F = $\frac{K}{\Sigma K}$
B	BA	$\frac{4EI}{L} = \frac{4EI}{4}$	3EI	1/3
	BC	$\frac{4E(I3)}{L} = \frac{12EI}{6}$		2/3
C	CB	$\frac{4E(I3)}{L} = \frac{12EI}{6}$	3EI	2/3
	CD	$\frac{4EI}{L} = \frac{4EI}{4}$		1/3

A	1/3 B 2/3		2/3 C 1/3	D
0	0	-6	6	0
	6		-6	
	2	4	-4	-2
1		-2	2	-1
1	2	-4	4	-2 -1
	2		-2	
	2/3	4/3	-4/3	-2/3
1/3		-2/3	2/3	-1/3
4/3	8/3	-10/3	10/3	-8/3 -2/3
	2/3		-2/3	
	2/9	4/9	-4/9	-2/9
1/9		-2/9	2/9	-1/9
13/9	26/9	-28/9	28/9	-26/9 -7/9
	2/9		-2/9	
	2/27	4/27	-4/27	-2/27
1/27		-2/27	2/27	-1/27
40/27	80/27	-82/27	82/27	80/27 -22/27
	2/27		-2/27	
	2/81	4/81	-4/81	-2/81
1/81		-2/81	2/81	-1/81
1.5	242/81	-244/81	244/81	242/81 -67/81

A	1/3 B	2/3	2/3 C	1/3 D
$\frac{121}{81}$	$\frac{242}{81}$ $-\frac{244}{81}$		$+\frac{244}{81}$	$-\frac{242}{81}$ $-\frac{67}{81}$
	$\frac{2}{81}$		$-\frac{2}{81}$	
	$\frac{2}{243}$	$\frac{4}{243}$	$-\frac{4}{243}$	$-\frac{2}{243}$
$\frac{1}{243}$		$-\frac{2}{243}$	$\frac{2}{243}$	$-\frac{1}{243}$
$\frac{364}{243}$	2.995	-3.004	+3.004	-2.995 $-\frac{364}{243}$
	0.009		-0.009	
	0.003	0.006	-0.006	-0.003
0.0015		-0.003	0.003	-0.0015
1.449	2.998	3.001	-3.007	2.998 -1.449
	0.003		-0.003	
	0.001	0.002	-0.002	-0.001
+0.0001		-0.001	0.001	-0.0001
+1.50	3	-3	3	-3 1.50



2. Analysize the symmetric portal frame as shown in figure by MDM. Draw SFD and BMD.



Sol: $\overline{M}_{AB} = -\frac{wL^2}{12} \Rightarrow -\frac{12\times4^2}{12} \Rightarrow -16\ KN.m$

$\overline{M}_{BA} = \frac{wL^2}{12} \Rightarrow \frac{12\times4^2}{12} \Rightarrow 16\ KN.m$

$\overline{M}_{BC} = -\frac{wab^2}{l^2} + -\frac{wab^2}{l^2}$

$= -\frac{30\times2\times4^2}{6^2} + \left(-\frac{30\times4\times2^2}{6^2}\right)$

$\overline{M}_{BC} = -40\ KN.m$

$\overline{M}_{CB} = \frac{wa^2b}{l^2} + \frac{wa^2b}{l^2}$

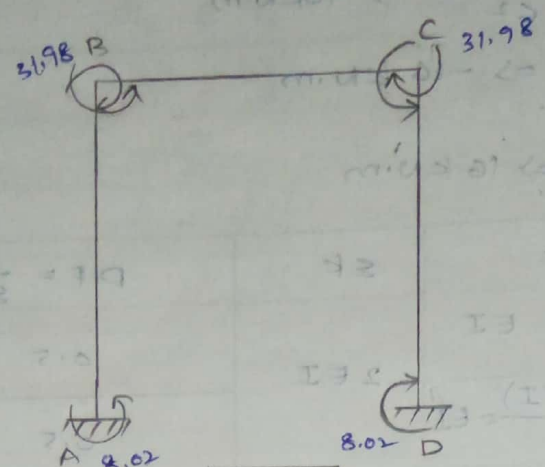
$= \frac{30 \times 2^2 \times 4}{6^2} + \frac{30 \times 4^2 \times 2}{6^2} \Rightarrow 40\,kN.m$

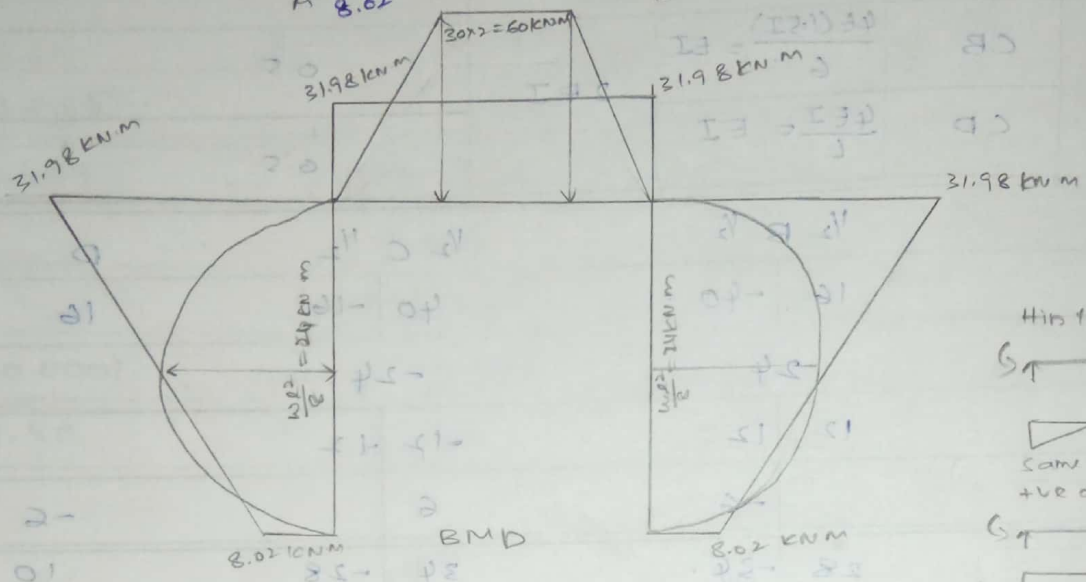
$M_{CD} = \frac{-wL^2}{12} \Rightarrow \frac{-12 \times 4^2}{12} \Rightarrow -16\,KN.m$

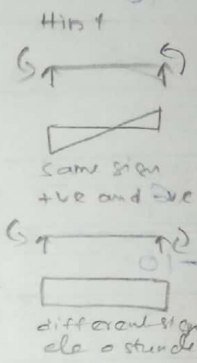
$M_{DC} = \frac{wL^2}{12} \Rightarrow \frac{12 \times 4^2}{12} \Rightarrow 16\,KN.m$

Joint	Member	K	ΣK	D.F = $\frac{K}{\Sigma K}$
B	BA	$\frac{4EI}{L} = EI$	2EI	0.5
	BC	$\frac{4E(1.5I)}{6} = EI$		0.5
C	CB	$\frac{4E(1.5I)}{6} = EI$	2EI	0.5
	CD	$\frac{4EI}{L} = EI$		0.5

A	½ B	½	½ C	½	D
-16	16	-40	40	-16	16
	24		-24		
	12	12	-12	-12	
6		-6	6		-6
-10	28	-34	34	-28	10
	6		-6		
	3	3	-3	-03	
1.5		-1.5	1.5		-1.5
-8.5	31	-32.5	32.5	-31	8.5
	1.5		-1.5		
	0.75	0.75	-0.75	-0.75	
0.375		-0.375	0.375		-0.375
-8.125	31.75	-32.125	32.125	-31.75	8.125
	0.375		-0.375		
	0.1875	0.1875	-0.1875	-0.1875	
0.09375		-0.09375	0.09375		-0.09375
-8.03	31.93	-32.03	32.03	-31.93	8.03
	0.1		-0.1		
	0.05	0.05	-0.05	-005	
0.025		-0.025	0.025		-0.025
-8	31.98	-31.98	31.98	-31.98	8.02







S.F :-

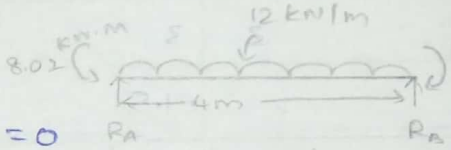
$R_A + R_B = 48\,KN$

$\Sigma M_A = 0$

$R_B \times 4 + 8.02 - 12 \times 4 \times 4/2 - 31.98 = 0$

$R_B = 29.99\,KN$

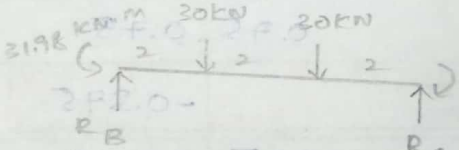
$R_A = 18.01\,KN$



$R_B + R_C = 60\,KN$

$\Sigma M_B = 0$

$R_B \times 6 - 30 \times 4 - 30 \times 2 - 31.98 + 31.98 = 0$



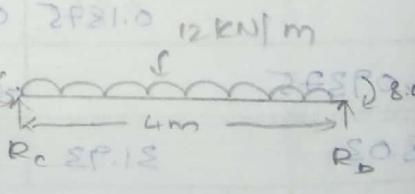
$R_C = 30\,KN$

$R_B = 30\,KN$

$R_C + R_D = 48\,KN$

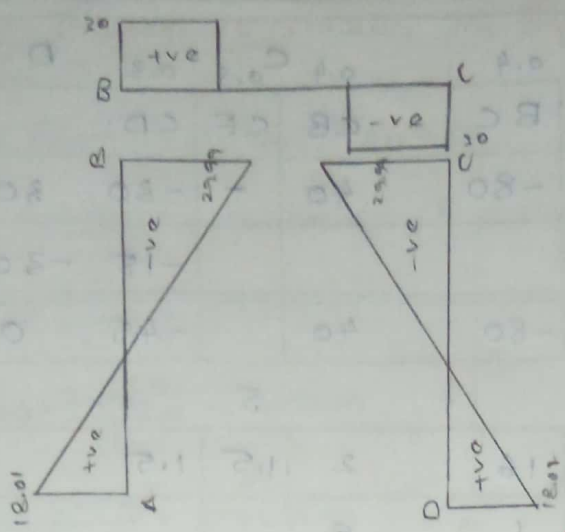
$\Sigma M_C = 0$

$R_D \times 4 - 8.02 - 12 \times 4 \times \frac{4}{2} + 31.98 = 0$

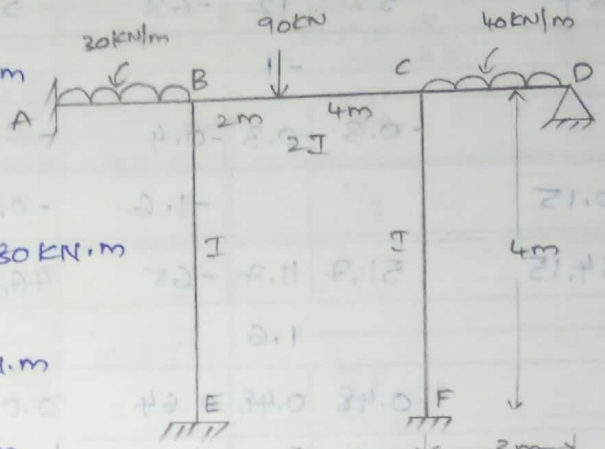


$R_D = 18.01\,KN$

$R_B = 29.99\,KN$



3. Analyze the frame as shown in figure by MDM and draw SFD and BMD.



Sol: $\bar{M}_{AB} = \frac{-wl^2}{12} \Rightarrow \frac{-30\times4^2}{12} \Rightarrow -40\ KN.m$

$\bar{M}_{BA} = \frac{wl^2}{12} = \frac{30\times4^2}{12} \Rightarrow 40\ KN.m$

$\bar{M}_{BC} = \frac{-wab^2}{l^2} \Rightarrow \frac{-90\times2\times4^2}{6^2} \Rightarrow -80\ KN.m$

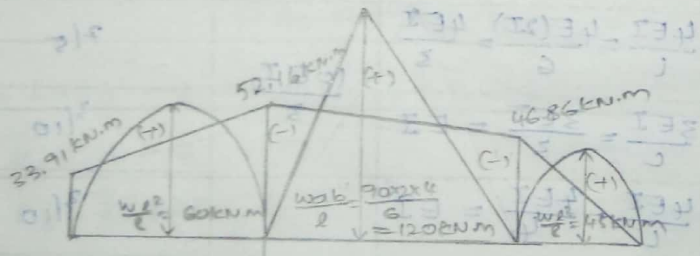
$\bar{M}_{CB} = \frac{wa^2b}{l^2} \Rightarrow \frac{90\times2^2\times4}{6^2} \Rightarrow 40\ KN.m$

$\bar{M}_{CD} = \frac{-wl^2}{12} = \frac{-40\times3^2}{12} \Rightarrow -30\ KN.m$

$\bar{M}_{DC} = \frac{wl^2}{12} = \frac{40\times3^2}{12} \Rightarrow 30\ KN.m$ $\quad \bar{M}_{BE} = \bar{M}_{EB} = \bar{M}_{CF} = \bar{M}_{FC} = 0$

Joint	Member	K	ΣK	D.F = $\frac{K}{\Sigma K}$
B	BA	$\frac{4EI}{L} = \frac{4EI}{4} = EI$	$\frac{10EI}{3}$	3/10
	BC	$\frac{4EI}{L} = \frac{4E(2I)}{6} = \frac{4EI}{3}$		2/5
	BE	$\frac{4EI}{L} = \frac{4EI}{4} = EI$		3/10
C	CB	$\frac{4EI}{L} = \frac{4E(2I)}{6} = \frac{4EI}{3}$	$\frac{10EI}{3}$	2/5
	CD	$\frac{3EI}{L} = \frac{3EI}{3} = EI$		3/10
	CF	$\frac{4EI}{L} = \frac{4EI}{4} = EI$		3/10

A		$B_{0.3}$ 0.3	0.4	0.4	$C_{0.3}$ 0.3	D	EB	FC
	BA	BE	BC	CB	CF	CD		
-40	40	-	-80	40	-	-30	30	-	-
						-15	-30		
-40	40	-	-80	40		-45	0	-	-
		40			5				
	12	12	16	2	1.5	1.5			
6			1	8			6	0.75	
-34	52	12	-63	50	1.5	-43.5		6	0.75
		-1			-8				
	-0.3	-0.3	-0.4	-3.2	-2.4	-2.4			
-0.15			-1.6	-0.2				-0.15	-1.2
-34.15	51.7	11.7	-65	46.6	-0.9	-45.9		+5.85	-0.45
		1.6			0.2				
	0.48	0.48	0.64	0.08	0.06	0.06			
0.24			0.04	0.32				0.24	0.03
-33.91	52.18	12.18	-64.32	47	-0.84	-45.84		6.09	-0.42
		-0.04			-0.32				
	-0.012	-0.012	-0.016	-0.128	-0.096	-0.096			
-0.006			-0.064	-0.008				-0.006	-0.048
-33.91	52.16	12.16	-64.4	46.86	-0.93	-45.93			
-33.91	52.16	-52.24		46.86	-46.86			6.08	-0.46



Analyze the frame as shown in figure. Draw SFD and BMD.

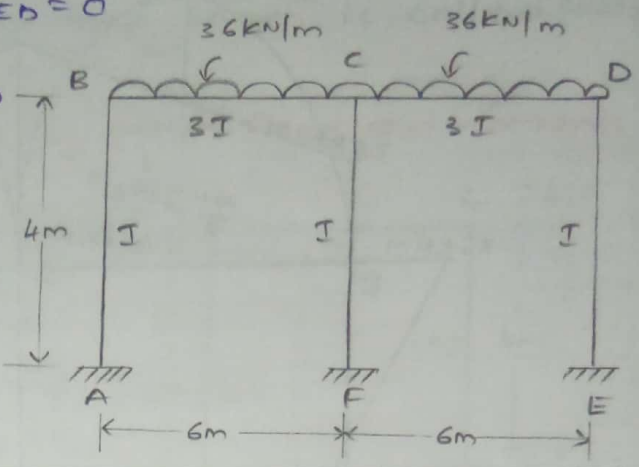
Sol: $\bar{M}_{AB} = \bar{M}_{BA} = \bar{M}_{CF} = \bar{M}_{FC} = \bar{M}_{DE} = \bar{M}_{ED} = 0$

$\bar{M}_{BC} = -\frac{wL^2}{12} \Rightarrow -\frac{36\times6^2}{12} = -108\,KN.m$

$\bar{M}_{CB} = \frac{wL^2}{12} \Rightarrow \frac{36\times6^2}{12} = 108\,KN.m$

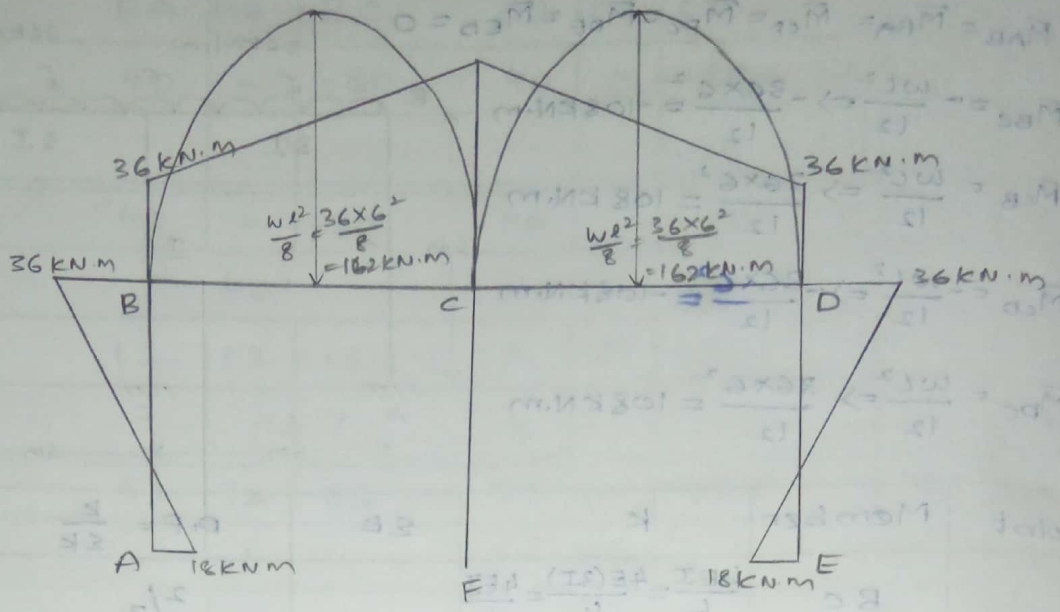
$\bar{M}_{CD} = -\frac{wL^2}{12} \Rightarrow -\frac{36\times6^2}{12} = -108\,KN.m$

$\bar{M}_{DC} = \frac{wL^2}{12} \Rightarrow \frac{36\times6^2}{12} = 108\,KN.m$



Joint	Member	K	ΣK	D.F = K/ΣK
B	BC	$\frac{4EI}{L}=\frac{4E(3I)}{6}=\frac{4EI}{2}$	3EI	2/3
	BA	$\frac{4EI}{L}=\frac{4EI}{4}=EI$		1/3
C	CB	$\frac{4EI}{L}=\frac{4E(3I)}{6}=\frac{4EI}{2}$	5EI	2/5
	CD	$\frac{4EI}{L}=\frac{4E(3I)}{6}=\frac{4EI}{2}$		2/5
	CF	$\frac{4EI}{L}=\frac{4EI}{4}=EI$		1/5
D	DC	$\frac{4EI}{L}=\frac{4E(3I)}{6}=\frac{4EI}{2}$	3EI	2/3
	DE	$\frac{4EI}{L}=\frac{4EI}{4}=EI$		1/3

A	1/3 B 2/3		2/5 C 1/5	2/5	2/3 D 1/3		CF
	BA	BC	CB	CF	CD	DC	DE	ED
0	0	-108	108	0	-108	108	0	0
	108			0		-108		
	36	72	0	0	0	-72	-36	
18		0	36		-36	0		-18
18	36	-36	144	0	-144	36	-36	-18



BMD

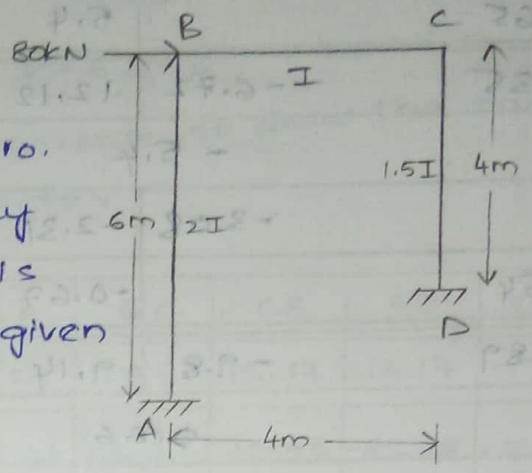
Sway :-

When the joints of the beam are having lateral displacement / deflections then the frame is called sway frame.

Analyze the portal frame by MDM. Fixed end moments in all the

d: Fixed end moments in all the members is equa to zero. Hence there is no non-sway moments only sway force is calculated but here they given force equal to 80 KN.

∴ Sway force = 80 KN



Non sway Analysis :-

Since there is no fixed end moments non sway analysis is zero.

Sway Force :-

S = 80 KN

Sway Analysis :- $\frac{\bar{M}_{AB}}{\bar{M}_{CD}} = \frac{Mf_1}{Mf_2} = \frac{\frac{-6E_1I_1\Delta}{L_1^2}}{\frac{-6E_2I_2\Delta}{L_2^2}} \Rightarrow \frac{-6E(2I)\Delta}{6^2} \times \frac{4^2}{-6E(1.5I)\Delta}$

$= \frac{2\times 4^2}{6^2\times 1.5} \Rightarrow \frac{16}{27}$

Let us assume fixed end moments developed in AB & BA = -16 KN.m and fixed end moments developed in CD & DC = 27 KN.m

For these orbitary FEM moment distribution is carried out.

Joint	Member	K	ΣK	D.F = $\frac{K}{\Sigma K}$
B	BA	$\frac{4EI}{L} = \frac{4E(2I)}{6} = \frac{4EI}{3}$	$\frac{7}{3}$	4/7
	BC	$\frac{4EI}{L} = \frac{4EI}{4} = EI$		3/7
C	CB	$\frac{4EI}{L} = \frac{4EI}{4} = EI$	$\frac{5}{2}$	2/5
	CD	$\frac{4E(1.5I)}{4} \Rightarrow 1.5EI$		3/5

A	4/7 B	3/7	2/5 C	3/5 D
-16	-16		-27	-27
	16		27	
	9.28	6.72	10.8	16.2
4.65		5.4	3.36	8.1
-11.35	-6.72	12.12	14.16	-10.8 -18.9
	-5.4		-3.36	
	-3.08	-2.31	-1.34	-2.01
-1.54		-0.67	-1.157	-1.00
-12.89	-9.8	9.14	11.66	-12.81 -19.9
	0.66		1.15	
	0.377	0.282	0.46	0.69
0.188		0.23	0.141	0.345
-12.70	-9.42	9.65	12.26	-12.12 -19.85
	-0.23		-0.14	
	-0.131	-0.098	-0.056	-0.084
-0.065		-0.028	-0.049	-0.042
-12.76	-9.55	9.52	12.15	-12.2 -19.59
	0.03		0.05	
	0.017	0.012	0.02	0.03
0.008		0.01	0.006	0.015
-12.75	-9.53	9.54	12.17	-12.17 -19.57

$\Sigma M_B = 0$

$H_A \times 6 - 12.75 - 9.53 = 0$

$H_A = +3.71$ KN

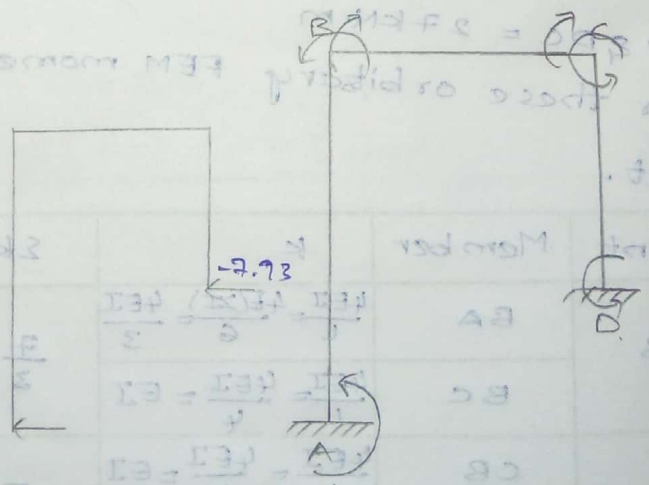
$H_B = -3.71$ KN

$\Sigma M_C = 0$

$H_D \times 4 - 12.17 - 19.57 = 0$

$H_D = 7.93$ KN

$H_C = -7.93$ KN



$S' = H_A + H_D$

$= 2.74 + 7.95$

$S' = 11.69$

sway correction factor, $k = \frac{S}{S'}$

$= \frac{80}{11.69}$

$k = 6.843$

Final End moments:-

By finding the final end moments to draw the table with the sway correction factor.

Member	AB	BA	BC	CB	CD	DC
Assumed sway moments	-12.75	-9.53	9.53	12.17	-12.17	-19.57
Actual sway moments (C.S.M x S.C.F)	-87.24	-65.21	65.21	83.28	-83.28	-133.91
Non sway moments	-	-	-	-	-	-
Final end moments	-87.24	-65.21	65.21	83.28	-83.28	-133.91

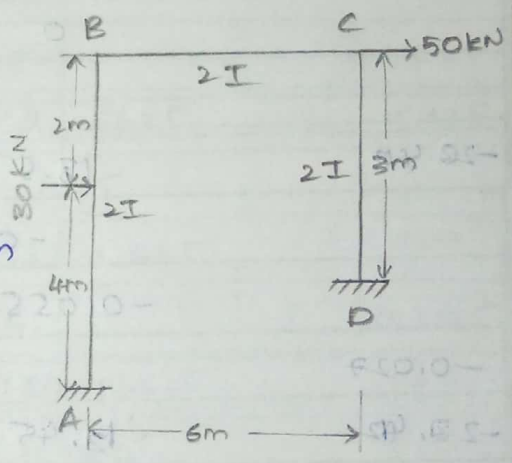
2. Analyze the frame as shown in figure by MDM and draw BMD.

Sol:- Non-sway Analysis:-

$M_{AB} = -\frac{wab^2}{l^2} \Rightarrow -\frac{30 \times 4 \times 2^2}{6^2} \Rightarrow -13.33 \text{ kN.m}$

$M_{BA} = \frac{wa^2b}{l^2} \Rightarrow \frac{30 \times 4^2 \times 2}{6^2} \Rightarrow 26.67 \text{ kN.m}$

$M_{BC} = M_{CB} = M_{CD} = M_{DC} = 0$



Joint	Member	K	ΣK	D.F = K/ΣK
B	BA	$\frac{4EI}{L} = \frac{4E(2I)}{6} = \frac{4EI}{3}$	$\frac{8EI}{3}$	1/2
	BC	$\frac{4EI}{L} = \frac{4E(2I)}{6} = \frac{4EI}{3}$		1/2
C	CB	$\frac{4EI}{L} = \frac{4E(2I)}{6} = \frac{4EI}{3}$	4EI	1/2
	CD	$\frac{4EI}{3} = \frac{4E(2I)}{3} = \frac{8EI}{3}$		1/2

A	½ B ½		½ C ½	D
-13.33	26.67			
	-26.67			
	-13.335	-13.335		
-6.6675			-6.6675	
-19.99	13.33	-13.33	-6.6675	0
	0		6.6675	
	-	-	3.33	3.33
-		1.66		1.66
-19.99	13.33	-11.67	-3.33	3.33	1.66
	-1.66		0	
	-0.83	-0.83	-	-	-
-0.415		-	-0.415		-
-20.40	12.5	-12.5	-3.74	3.33	1.66
	0		0.41	
	0	0	0.205	0.205	
		0.102			0.102
-20.40	12.5	-12.39	-3.535	3.535	1.762
	-0.11		0	
	-0.055	-0.055	-	-	
-0.027			-0.027		-
-20.42	12.45	-12.45	-3.55	3.535	1.762

$\Sigma M_B = 0$

$6H_A + 20.412 + 30 \times 2 - 12.45 = 0$

$H_A = -11.327\ kN$

$\Sigma M_C = 0$

$3H_D - 1.774 - 3.542 = 0$

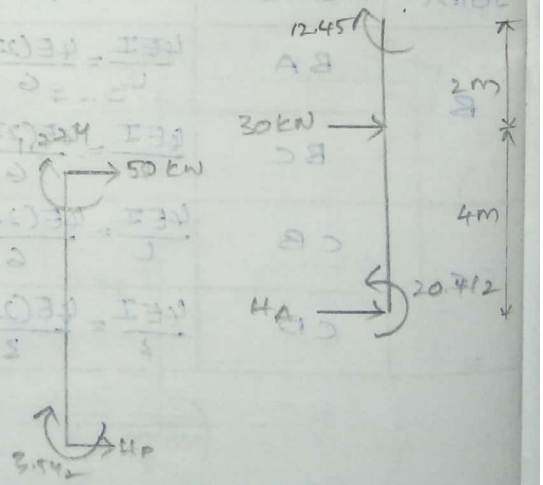
$H_D = 1.772$

$\Sigma H = 0$

$-S + 30 + H_A + H_D + 50 = 0$

$-S + 30 - 11.327 + 1.772 + 50 = 0$

$S = 70.445\ kN$



Sway Analysis :-

$$\frac{M_{AB}}{M_{CD}} = \frac{\frac{-6EI_1\Delta_1}{L^2}}{\frac{-6EI_2\Delta_2}{L_2^2}} \Rightarrow \frac{6(2I)}{6^2} \times \frac{3^2}{6EI}$$

$$\frac{M_{AB}}{M_{CD}} = \frac{1}{2}$$

Let us assume the FEM in column AB & BA = $-1 \times 10 = -10$ kN.m

Let us assume the FEM in column CD & DC = $-2 \times 10 = -20$ kN.m

A	½ B	½	½ C	½	D
-10	-10		-20		-20
	10		20		
	5	5	10	10	
2.5		5	2.5		5
-7.5	-5	10	12.5	-10	-15
	-5		-2.5		
	-2.5	-2.5	-1.25	-1.25	
-1.25		-0.625	-1.25		-0.625
-8.75	-7.5	6.875	10	-11.25	-15.625
	0.625		1.25		
	0.3125	0.3125	0.625	0.625	
0.156		0.3125	0.156		0.3125
-8.59	-7.18	7.5	10.781	-10.625	-15.31
	-0.32		-0.156		
	-0.16	-0.16	-0.078	-0.078	
-0.08		-0.039	-0.08		-0.039
-8.67	-7.34	7.309	10.62	-10.70	-15.349
	0.03		0.08		
	0.015	0.015	0.04	0.04	
0.0075		0.02	0.0075		0.02
-8.68	-7.32	7.33	10.66	-10.66	-15.33

$\Sigma M_B = 0$

$H_A' \times 6 + 8.66 + 7.33 = 0$

$H_A' = -2.66$ kN

$\Sigma M_C = 0$

$H_D' \times 3 + 15.23 + 10.66 = 0$

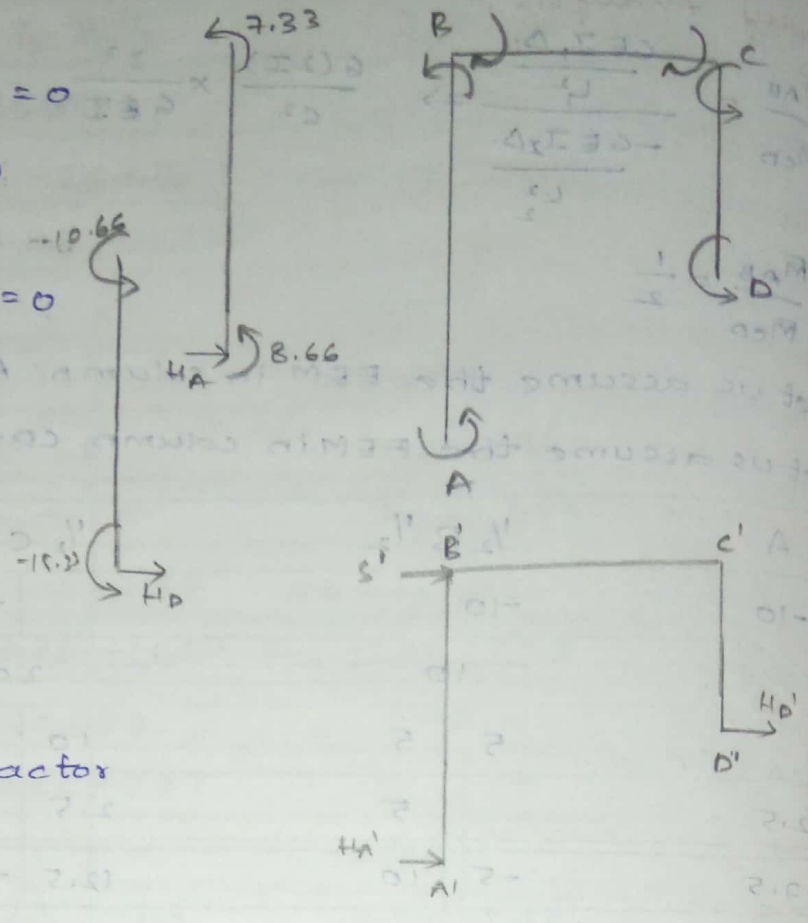
$H_D' = -8.63$ kN

$\Sigma H = 0$

$S' + H_A' + H_D' = 0$

$S' - 2.66 - 8.63 = 0$

$S' = 11.29$ kN

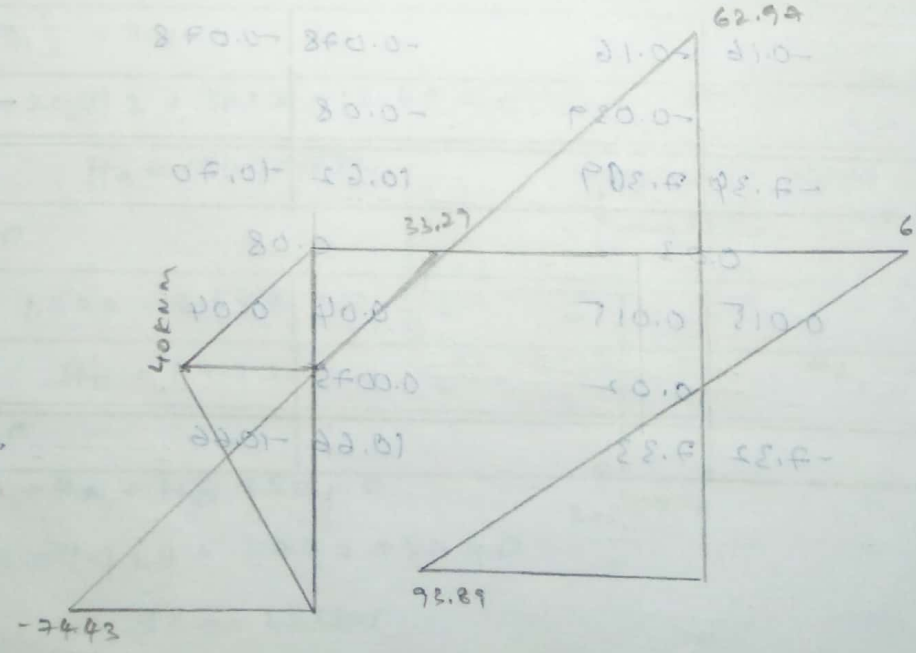


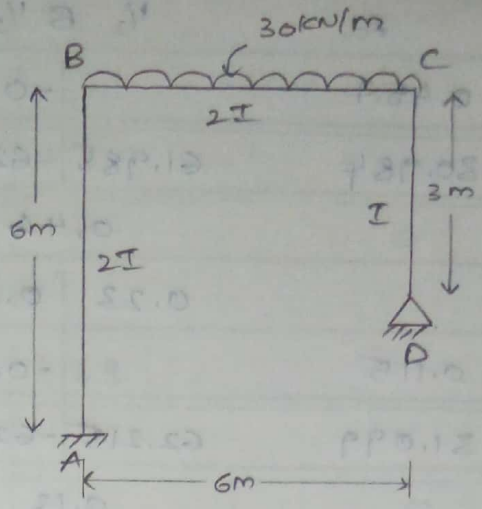
Sway correction factor

$$k = \frac{S}{S'} = \frac{70445}{11.29}$$

$k = 6.24$

Member	AB	BA	BC	CB	CD	DC
Assumed sway moments	-8.66	-7.32	7.33	10.66	-10.66	-15.33
Actual sway moments (ASM x k)	-54.02	-45.67	45.74	66.51	-66.51	-95.66
Non sway moments	-20.412	12.45	-12.45	-3.54	3.54	1.77
Final Moments	-74.43	-33.22	33.29	62.97	-62.97	-93.89





Non-sway Analysis :-

$\bar{M}_{AB} = \bar{M}_{BA} = 0$

$\bar{M}_{BC} = -\frac{wL^2}{12} = -\frac{30 \times 6^2}{12} \Rightarrow -90\,KN.m$

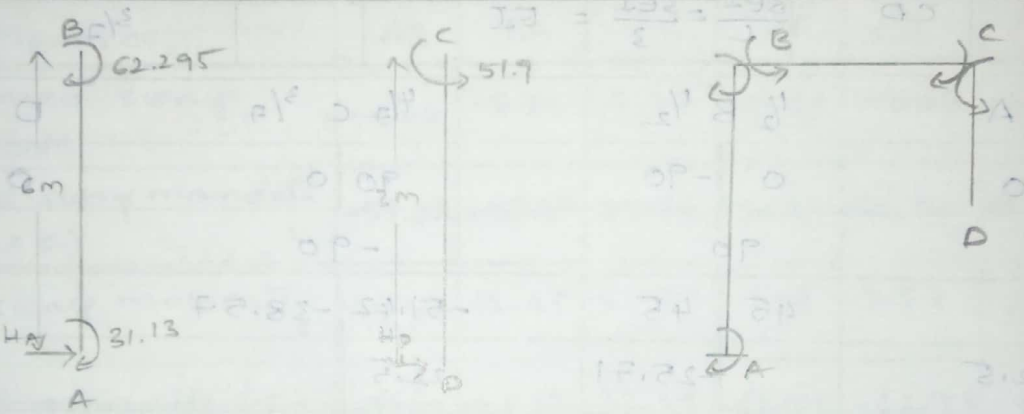
$\bar{M}_{CB} = \frac{wL^2}{12} = \frac{30 \times 6^2}{12} \Rightarrow 90\,KNm$

$\bar{M}_{CD} = \bar{M}_{DC} = 0$

Joint	Member	k	Σk	D.F = $\frac{k}{\Sigma k}$
B	BA	$\frac{4EI}{L} = \frac{4E(2I)}{6} = \frac{8EI}{6} = \frac{4EI}{3}$	$\frac{8EI}{3}$	1/2
	BC	$\frac{4EI}{L} = \frac{4E(2I)}{6} = \frac{8EI}{6} = \frac{4EI}{3}$		1/2
C	CB	$\frac{4EI}{L} = \frac{4E(2I)}{6} = \frac{8EI}{6} = \frac{4EI}{3}$	$\frac{7EI}{3}$	4/7
	CD	$\frac{3EI}{L} = \frac{3EI}{3} = EI$		3/7

A	1/2 B	1/2	4/7 C	3/7	D
0	0	-90	90	0	0
	90		-90		
	45	45	-51.42	-38.57	
22.5		-25.71	22.5		
22.5	45	-70.71	61.08	-38.57	
	25.71		-22.51		
	12.855	12.855	-12.86	-9.647	
6.427		-6.431	6.427		
28.927	57.855	-64.286	54.647	-48.217	
	6.431		-6.43		
	3.215	3.215	-3.67	-2.75	
1.60		-1.83	1.60		
30.527	61.07	-62.90	52.57	-50.96	
	1.83		-1.61		
	0.915	0.915	-0.92	-0.69	

A	1/2	B	1/2	4/7	C	3/7	D
0.457		-0.46		0.457			
30.984	61.985	-62.445		52.107	-51.65		0
	0.46			-0.457			
	0.23	0.23		-0.261	-0.195		
0.115		-0.130		0.115			
31.099	62.215	-62.345		51.961	-51.845		0
	0.13			-0.116			
	0.065	0.065		-0.066	-0.0497		
0.0325		-0.033		0.0325			
31.13	62.28	-62.31		51.92	-51.89		0
	0.03			-0.03			
	0.015	0.015		-0.0171	-0.0128		
0.0075		-0.0085		0.0075			
31.13	62.295	-62.303		51.91	-51.90		



$\Sigma M_B = 0$

$6H_A - 31.13 - 62.295 = 0$

$H_A = 15.57\,kN$

$\Sigma M_C = 0$

$3H_D + 51.9 = 0$

$H_D = -17.3\,kN$

$\Sigma H = 0$

$S = -15.57 + 17.3$

$S = 1.73$

Sway Analysis :-

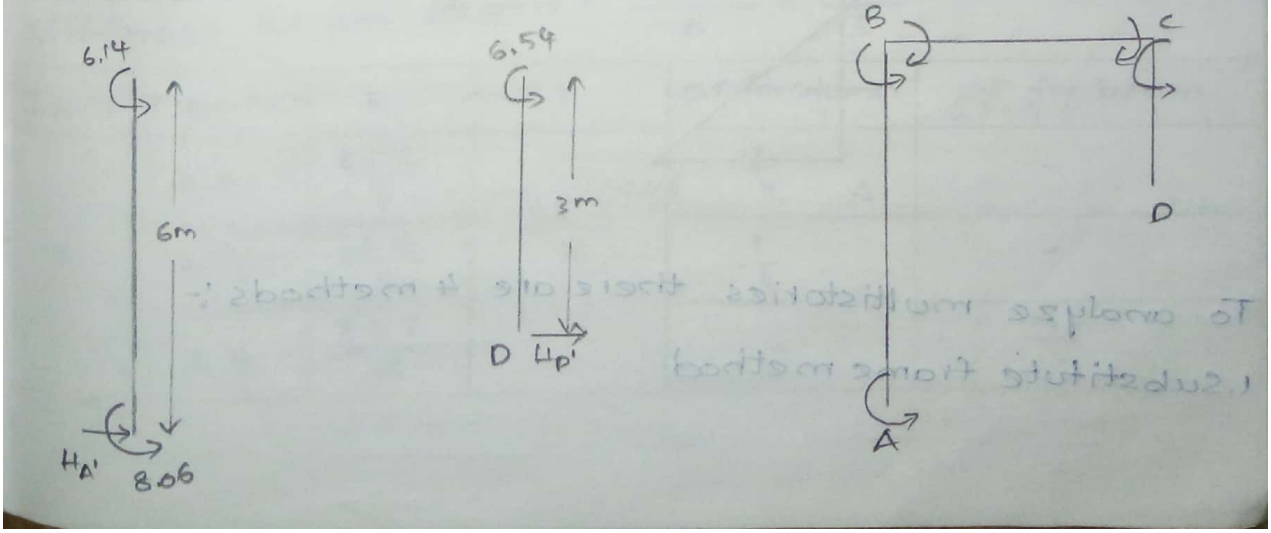
$$\frac{M_{f1AB}}{M_{f2CD}} = \frac{\frac{-6E(2I_1)\Delta_1}{L_1^2}}{\frac{-6E(I)\Delta_2}{L_1^2}} = \frac{\frac{-12}{6^2}}{\frac{-6}{3^2}} \Rightarrow \frac{1}{2}$$

Fixed end moments :-

$M_{AB} = M_{BA} = -1 \times 10 = -10\,kN.m$

$M_{CD} = M_{DC} = -2 \times 10 = -20\,kN.m$

A	1/2 B 1/2		4/7 C 3/7	D
-10	-10		-20	-20
			+10	+20
-10	-10		-10	0
	10		10	
	5	5	5.71	4.28
2.5		2.85	2.5	
-7.5	-5	7.85	8.21	-5.72 0
	-2.85		-2.49	
	-1.425	-1.425	-1.422	-1.06
-0.7125		-0.711	-0.7125	
-8.212	-6.425	5.714	6.07	-6.78 0
	+0.711		0.71	
	0.355	0.355	0.405	0.304
0.1775		0.202	0.1775	
-8.03	-6.07	6.271	6.652	-6.476 0
	-0.201		-0.176	
	-0.1	-0.1	-0.100	-0.075
-0.05		-0.05	-0.05	
-8.08	-6.17	6.121	6.502	-6.551 0
	+0.049		0.049	
	0.0245	0.0245	0.028	0.021
0.0122		0.014	0.0122	
-8.06	-6.14	6.15	6.54	-6.53 0



$\Sigma M_B = 0$

$6H_A' + 8.06 + 6.14 = 0$

$H_A' = -2.367\ kN$

$\Sigma M_C = 0$

$3H_D' + 6.23 = 0$

$H_D' = 2.07\ kN$

$\Sigma H = 0$

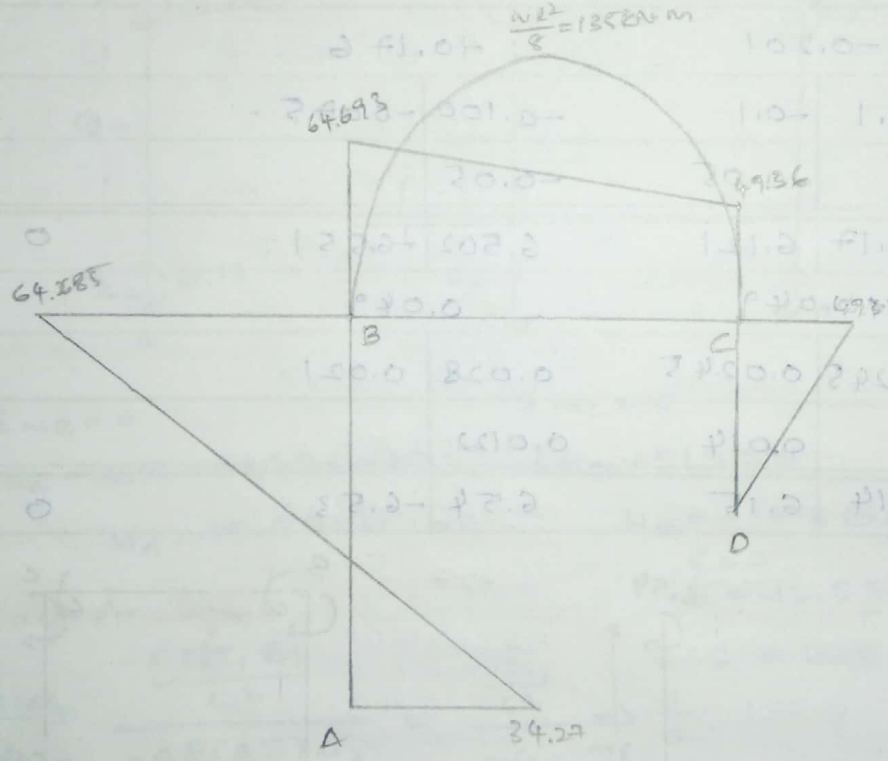
$S' + H_A' + H_D' = 0$

$S' = 2.367 + 2.07$

$S' = 4.437\ kN$

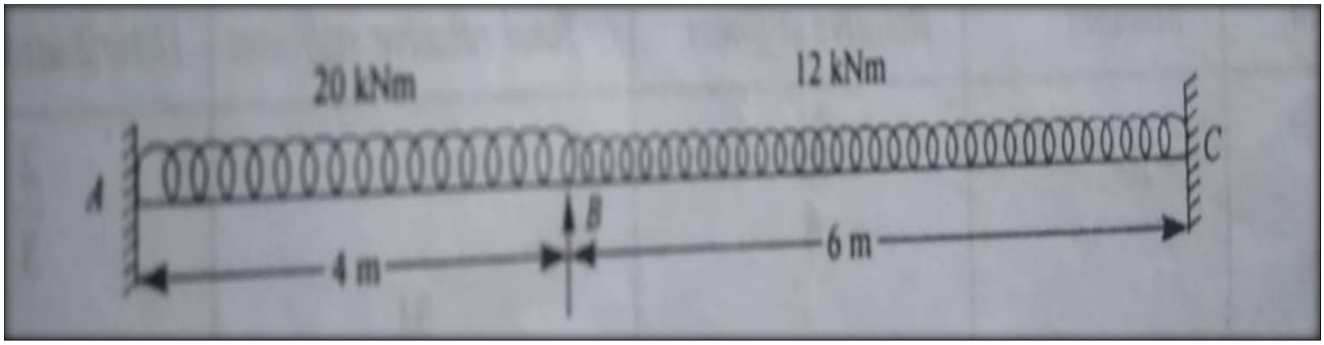
$K = \frac{S}{S'} = -\frac{1.75}{4.437} = -0.39$

Member	AB	BA	BC	CB	CD	DC
Assumed sway moment	-8.06	-6.14	6.15	6.54	-6.53	-
Actual sway moment (ASM x K)	3.14	2.39	-2.39	-2.55	2.55	-
Non sway moment	31.13	62.295	-62.303	51.91	-51.90	-
Final moment	34.27	64.685	-64.693	49.36	-49.35	-



To analyze multistories there are 4 methods :-

1. Substitute frame method

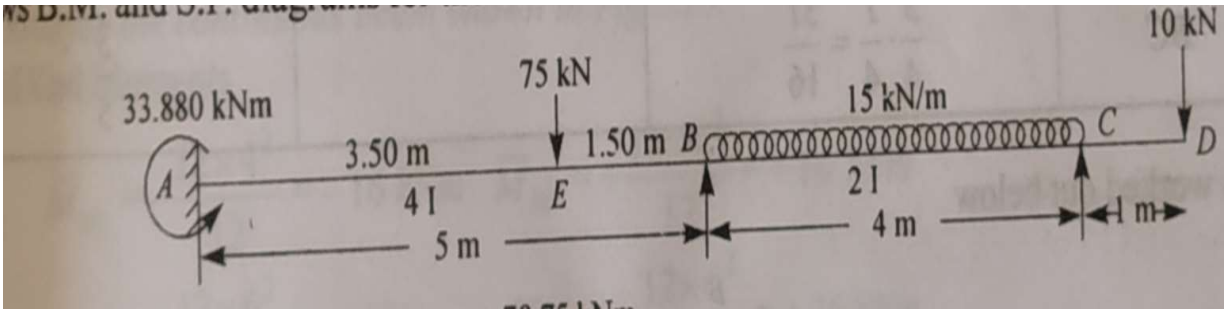


FIXED END MOMENTS	
MFAB	-26.67
MFBA	26.67
MFBC	-36
MFCB	36

DISTRUBUTION TABLE			
JOINT	MEMBER	K	ΣK/K
B	BA	4EI/4	0.60
	BC	4EI/6	0.4

BALANCE → SUM (add negative sign) → DISTRIBUTION → CARRYOVER (half of the moment) → NET MOMENT (I.M+D.M) → REPEAT

JOINT	A		B			C
DF			0.60	0.4		
F.E.M	-26.67		26.67	-36		36
BALANCE						
I.M	-26.67		26.67	-36		36
SUM			9.333333333			
DISTRIBUTION			5.6	3.733333		
CARRY OVER	2.8					1.866667
NET	-23.87		32.26667	-32.2667		37.86667
SUM						
DISTRIBUTION						
CARRY OVER						
NET						
SUM						
DISTRIBUTION						
CARRY OVER						



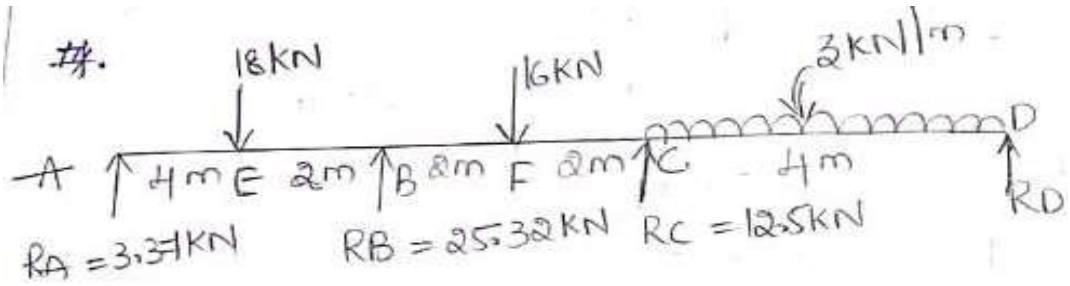
FIXED END MOMENTS	
MFAB	-23.63
MFBA	55.13
MFBC	-20.00
MFCB	20.00

AS PER PROBLEM	
MCD	10

DISTRUBUTION TABLE							
JOINT	MEMBER	joint condn	E	M.O.I	LENGTH	K	ΣK/K
B	BA	FIXED	4	4	5	3.20	0.68
	BC	hinged	3	2	4	1.50	0.32

BALANCE → SUM (add negative sign) → DISTRIBUTION → CARRYOVER (half of the moment) → NET MOMENT (I.M+D.M) → REPEAT

JOINT	A		B			C
DF			0.68	0.32		
F.E.M	-23.63		55.13	-20.00		20.00
BALANCE				-5.00		-10.00
I.M	-23.63		55.13	-25.00		10.00
SUM			-30.13			
DISTRIBUTION			-20.51	-9.61		
CARRY OVER	-10.26					
NET	-33.88		34.61	-34.61		10.00
SUM						
DISTRIBUTION						
CARRY OVER						
NET						
SUM						
DISTRIBUTION						
CARRY OVER						



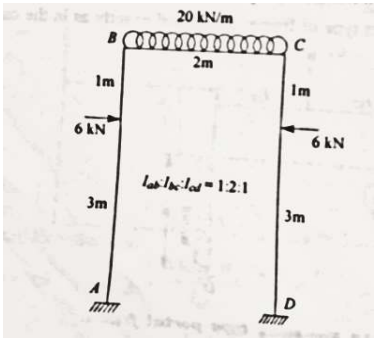
FIXED END MOMENTS	
MFAB	-8.00
MFBA	16.00
MFBC	-8.00
MFCB	8.00
MFCD	-4.00
MFDC	4.00

DISTRUBUTION TABLE							
JOINT	MEMBER	joint condn	E	M.O.I	LENGTH	K	ΣK/K
B	BA	HINGED	3	1	6	0.50	0.40
	BC	hinged	3	1	4	0.75	0.60
C	CB	HINGED	3	1	4	0.75	0.50
	CD	hinged	3	1	4	0.75	0.50

BALANCE → SUM (add negative sign) → DISTRIBUTION → CARRYOVER (half of the moment) → NET MOMENT (I.M+D.M) → REPEAT

JOINT	A		B			C			D
DF			0.40	0.60		0.50	0.50		
F.E.M	-8.00		16.00	-8.00		8.00	-4.00		4.00
BALANCE	8.00	→	4.00				-2.00	←	-4.00
I.M	0.00		20.00	-8.00		8.00	-6.00		0.00
SUM			-12.00			-2.00			
DISTRIBUTION			-4.80	-7.20		-1.00	-1.00		
CARRY OVER	0.00			-0.50		-3.60			
NET	0.00		15.20	-15.70		3.40	-7.00		
SUM			0.50			3.60			
DISTRIBUTION			0.20	0.30		1.80	1.80		
CARRY OVER				-0.90		-0.15			
NET			15.40	-15.40		5.05	-5.20		
SUM									
DISTRIBUTION									
CARRY OVER									

FIXED END MOMENTS	
MFAB	-1.12
MFBA	3.37
MFBC	-6.67
MFCB	6.67
MFCD	-3.37
MFDC	1.12



DISTRUBUTION TABLE							
JOINT	MEMBE	joint condr	E	M.O.I	LENGTH	K	ΣK/K
B	BA	FIXED	4	1	4	1.00	0.20
	BC	FIXED	4	2	2	4.00	0.80
C	CB	FIXED	4	2	2	4.00	0.80
	CD	FIXED	4	1	4	1.00	0.20
AS PER PROBLEM							
MCD		10					

BALANCE → SUM (add negative sign) → DISTRIBUTION → CARRYOVER (half of the moment) → NET MOMENT (I.M+D.M) → REPEAT

JOINT	A		B			C			D
DF			0.20	0.80		0.80	0.20		
F.E.M	-1.12		3.37	-6.67		6.67	-3.37		1.12
BALANCE	1.12		0.56						
I.M	-1.12		3.37	-6.67		6.67	-3.37		1.12
SUM			3.30			-3.30			
DISTRIBUTION			0.66	2.64		-2.64	-0.66		
CARRY OVER	0.33			-1.32		1.32			-0.33
NET	-0.79		4.03	-5.35		5.35	-4.03		0.79
SUM			1.32			-1.32			
DISTRIBUTION			0.26	1.06		-1.06	-0.26		
CARRY OVER	0.13			-0.90		-0.15			-0.13
NET	-0.66		4.29	-5.19		4.14	-4.29		0.66
SUM			0.90			0.15			
DISTRIBUTION			0.18	0.72		0.12	-0.86		
CARRY OVER	0.09			0.06		0.36			-0.43
NET	-0.57		4.47	-4.41		4.62	-5.15		
SUM									
DISTRIBUTION									
CARRY OVER									

WE BALANCE
1.FAR END IS HINGED
2. FAR END HAS OVERHANG
WE CARRY OVER
1.FAR END IS FIXED