Unit 3

Image Restoration Degradation & Restoration

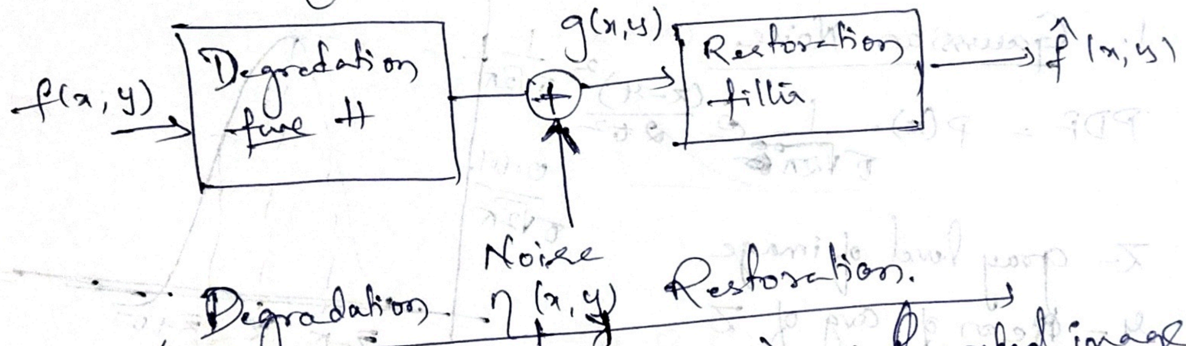
→ Image Restoration is used to Reconstruct or Recover an image that has been degraded using prior knowledge of the degradation phenomenon.

→ Image Restoration technique is used for Modelling the degradation & applying inverse process in order to recover the original image.

→ Image Restoration techniques are best formulated in Spatial domain, & in frequency domain.

* Spatial domain is applicable when the degradation is additive Noise.
* frequency domain filter applicable in Image blur.

A Model of image degradation & Restoration process.



g (x, y) → degraded image

f(x,y) → i/p image

H – Degradation func

Take H → as linear & position invariant process then the degraded image in spatial domain can be given as

$$g(x, y) = h(x, y) \underset{\text{Convolution}}{*} f(x, y) + \eta(x, y) \quad \text{--- (1)}$$

Taking FT

$$G(u, v) = H(u, v) \cdot F(u, v) + N(u, v) \quad \text{--- (2)}$$

$\hat{f}(x, y)$ → Recoved image By using Restoration filter

Import Noise probability Density func / Noise Models

Noise – in an image will arise during acquisition or during transmission.

During image acquisition the performance of image sensors are affected due to environmental cod^n etc.

During image transmittion due to interference in the channel during transmission Noise will be added into the image.

— The Spatial properties of Noise Refers to Spatial characteristics of Noise

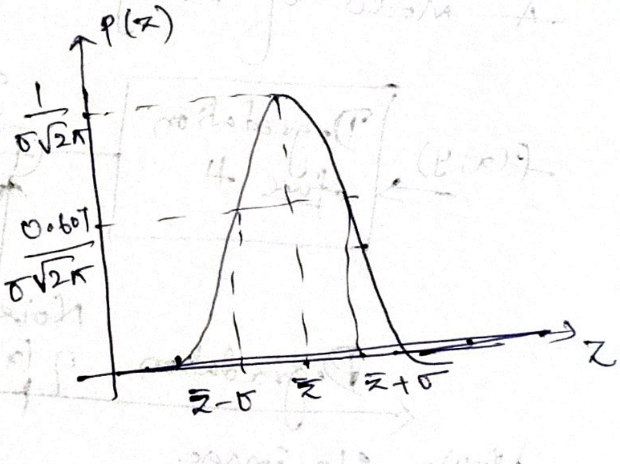
The Frequency properties of Noise in the F.T.

PDF → defines the distribution of Noise in an image

Most Common PDF

1. Gaussian Noise

$$PDF = p(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$$



z – gray level of image

μ – Mean of avg of z

σ – Standard deviation

σ^2 – variance

70% of values in the range $(\mu-\sigma, \mu+\sigma)$

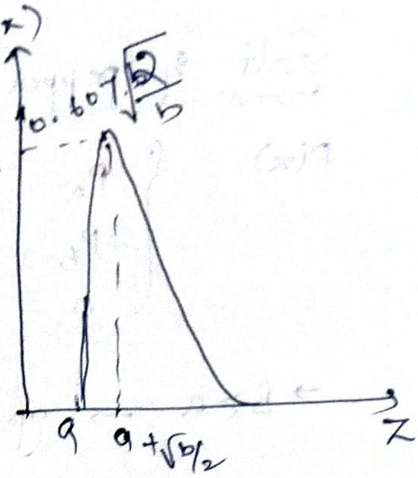
95% $(\mu-2\sigma, \mu+2\sigma)$

* Most commonly used as it is easy to calculate

* It arises due to electronic circuit Noise / Sensor Noise due to poor illumination or high temperature

Rayleigh Noise

$$P(z) = \begin{cases} \frac{2}{b}(z-a)\, e^{-\frac{(z-a^2)}{b}} & ; z \geq a \\ 0 & ; z < a \end{cases}$$



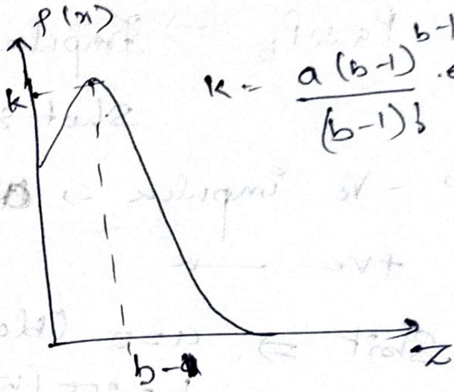
Mean = $\mu = a + \sqrt{\pi b/4}$

Variance $\sigma^2 = b\,\frac{(4+\pi)}{4}$

* It is helpfull in rang imaging - is used in digital camera

Erlang [Gamma] Noise :

$$P(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & ; z \geq 0 \\ 0 & ; z < 0 \end{cases}$$



$$K = \frac{a(b-1)^{b-1}}{(b-1)!} \cdot e^{(b-1)}.$$

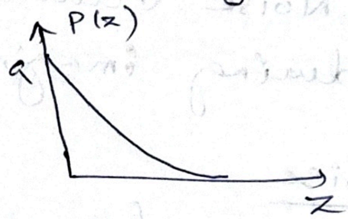
Mean $= \mu = b/a$

Variance $\sigma^2 = b/a^2$

* It find application in laser image

Exponential Noise

$$P(z) = \begin{cases} a e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$



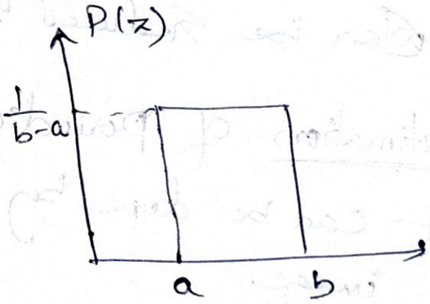
$\mu = 1/a$

$\sigma^2 = 1/a^2$

It is a spatial case of $b = 0$ in

Uniform Noise model

$$P(z) = \begin{cases} \frac{1}{b-a} & ; a \leq z \leq b \\ 0 & ; \text{otherwise.} \end{cases}$$



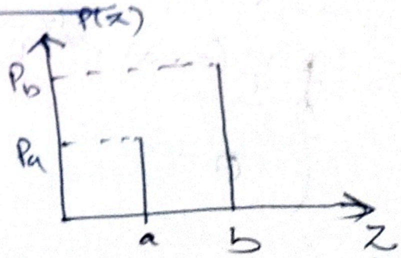
$\mu = \frac{a+b}{2}$

$\sigma^2 = \frac{(b+a)^2}{12}$

* Usefull in numerous random no generation that a used in simulations

Salt & pepper Noise [Impulse Noise]

$$P(z) = \begin{cases} P_a & ; z = a \\ P_b & ; z = b \\ 0 & ; \text{Otherwise} \end{cases}$$



→ $b > a$ → graylevel b → will appear as light dot
graylevel a ———"——— dark dot

→ $P_a = 0$ $P_b = 0$ → unipolar

$P_a \approx P_b$ ⇒ Impulse Noise known as Salt & pepper Noise
Shot & spike Noise.

→ -ve impulse → a & b are saturated & appear as black point
- +ve ——"—— ———"——— Salt point

Shot ⇒ $a = 0$ (black)
$b = 255$ (white)

- Impulse Noise occurs when faulty switching takes place during imaging.

Periodic Noise

→ It arises typically from electrical & electromechanical interference during image acquisition.
→ Periodic Noise is the only type of spatial dependent Noise
→ Can be Reduced by using frequency domain filtering

Estimation of Periodic Noise parameters

* It can be done by inspection of Fourier spectrum of an image.
* Fourier spectrum of an image produces the frequency spikes and can be detected by visual analysis once we perform the fourier analysis of the image

* Automated Analysis can be also be used if Knowledge about a general location of freq component of an image is known.

* If Image system are available. One of the simplest way to study characteristics of system noise is to capture the set of images in a flat environment.

* Use the data from image strips to calculate mean & Variance of the gray level.

* Consider the strip known as subimage → S

$$\mu = \sum_{z_i \in S} z_i \, p(z_i) \quad \text{—} ①$$

$$\sigma^2 = \sum_{z_i \in S} (z_i - \mu)^2 \, p(z_i) \quad \text{—} ②$$

z_i → gray level values of pixels.

$p(z_i)$ → Normalized histogram values.

→ Histogram shape decides the shape (kind) of the noise

Restoration in Presence of Noise Only ←

Spatial Filtering

We know that $g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$ in spatial domain $h(x,y) = 0$

$h(x,y)$ is the degradation func

$g(x,y) = f(x,y) + \eta(x,y)$

$G(u,v) = F(u,v) + N(u,v)$

1. ~~N(u,v)~~ = $G(u,v) - N(u,v) = F(u,v)$

Spatial filtering is used in presence of additive noise

There are 2 methods
1. Mean filter
2. Order statistics filter

I Mean filter

I Arthmetic Mean filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) S_{xy}} g(s,t) \quad \text{--- (1)}$$

S_{xy} → Set of co-ordinates in a rectangular Subimage window of size ~~n×m~~ Centered at point (x,y)

→ Avg of Corrupted image $g(x,y)$ in the area designed by S_{xy} is processed by Arthmetic mean filter.

* It smoothens the local variations
* Noise is Reduced because of blurring.

II Geometric Mean filter.

Operation is Represented by $\hat{f}(x,y) = \left[\prod_{(s,t) S_{xy}}^{\text{Product}} g(s,t) \right]^{1/mn}$ — (1)

* ~~product~~ Each Restored image is given by product of pixels in the subimage window ~~raised~~ to the power of $\frac{1}{mn}$
* Smoothing similar to arithmetic mean filter
* lose less image details.

III Harmonic Mean filter

Operation is Represented by $\hat{f}(x,y) = \dfrac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}}$ — (3)

* works well for salt noise & fails for pepper noise
* also works well for Gaussian Noise.

IV Contraharmonic mean filter

$$\hat{f}(x,y) = \frac{\sum_{(s,t) \in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t) \in S_{xy}} g(s,t)^{Q}}$$

Q - order of the filter.

- Reduces the effect of salt & pepper noise

Q → +ve it eliminates pepper Noise

Q → -ve it eliminates Salt noise

Cannot do both simultaneously

Reduces to arithmetic mean filter if Q = -1

(2) Order statistics filter for restoration of image in.

The Response of order statistics filter depends on ordering (ranking) of the pixels.

1. Median filter.

operation is represented as $\hat{f}(x,y) = \underset{(s,t) \in S_{xy}}{\text{median}} \{g(s,t)\}$ —(1)

→ Replaces the value of pixel by the median of gray levels

→ Excellent noise reduction with less blurring.

→ effective in presence of → bipolar, unipolar impulse noise

2. Min & Max filter.

$\hat{f}(x,y) = \underset{(s,t) \in S_{xy}}{\max} \{g(s,t)\}$ (1)

$\hat{f}(x,y) = \underset{(s,t) \in S_{x,y}}{\min} \{g(s,t)\}$ (2)

* Finding the brightest point
* Reduces pepper Noise

* Finding the darkest point
* Reduces salt noise

(3) Mid point filter → Computes the midpoint b/n max & min values.

$$\hat{f}(x,y) = \frac{1}{2}\left[\underset{(s,t) \in S_{xy}}{\max}\{g(s,t)\} + \underset{(s,t) \in S_{xy}}{\min}\{g(s,t)\}\right] \quad (1)$$

* Combines order statistics & averaging.
* works best for Randomly distributed noise. Gaussian Uniform Noise.

4. Alpha - trimmed mean filter.

$\frac{d}{2}$ - lowest, $d/2$ highest graylevel values.

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

When $d=0$ → Reduced to arithmetic mean funct

$d = \frac{mn-1}{2}$ → Median filter. d-(0 to mn-1)

[$g_r(s,t)$ Represents the Remaining pixel values from $mn - \left(\frac{d}{2}+\frac{d}{2}\right) = \underline{mn-d}$ pixels.

for other values → it is usefull in multiple type of noise Reduction

Adaptive filter in

There ~~are~~ types of adaptive filters.

1. Adaptive, local Noise Reduction.
2. Adaptive Median filter

1. Adaptive, local Noise Reduction.

Mean & Variance → measure of avg contrast.

↓

measure of avg graylevel

Adaptive filter operates on a local Region specified by S_{xy}. → is a region on the image.

This filter is based on 4 quantities

ⓐ $g(x,y)$ → value of noisy image (x,y)

ⓑ σ_n^2 → variance of the noise Corrupting $f(x,y)$ to form $g(x,y)$

③ m_L → local mean of the pixel in S_{xy}

④ σ_L^2 = local variance of the pixel in S_{xy}

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_n^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

2. Adaptive Median filter.

→ Median filter performs well only when the spatial density of impulse noise is very less. ~~[illegible]~~

So this disadvantage can be overcome by adaptive median filter.

- The adaptive median filter can handle impulse noise of larger values.

* It preserves the details of the image while smoothing non impulse noise.

* Let Z_{min} = Min value of gray level in S_{xy}

Z_{max} = Max

Z_{med} = Median

S_{max} = Max allowed size of S_{xy}

→ Algorithm works in two levels.

Level A → $A_1 = Z_{med} - Z_{min}$

$A_2 = Z_{med} - Z_{max}$.

if $A_1 > 0$ & $A_2 < 0$ go to level B.

else increase the window size.

if window $\leq S_{max}$ Repeat level A

else o/p Z_{xy}.

Level B → $B_1 = Z_{xy} - Z_{min}$

$B_2 = Z_{xy} - Z_{max}$

if $B_1 > 0$ AND $B_2 < 0$ o/p Z_{xy}.

else o/p Z_{med}

It is used for 3 main purpose

1. To remove salt & pepper noise
2. To provide smoothing.
3. To reduce distortion.

Periodic Noise Reduction using freq domain filtering.

* Periodic Noise is added to the image due electrical or electromechanical interference during the image acquisition
* Periodic Noise ~~an~~ Reduction using freq domain filtering to Remove particular Range of frequency.

1. Band Reject filters :-

~~Attenuate~~ a band of frequencies above the origin of the FT

ideal

$$H(u,v) = \begin{cases} 1 & ; D(u,v) < D_0 - \frac{w}{2} \\ 0 & ; D_0 - \frac{w}{2} \le D(u,v) \le D_0 + \frac{w}{2} \\ 1 & ; D(u,v) > D_0 + \frac{w}{2} \end{cases}$$

$D(u,v) \rightarrow$ distance from the origin of the centred freq. band.

$w \rightarrow$ width of the band

$D_0 \rightarrow$ radial center / cut off freq.

~~Ideal~~ Butterworth.

$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)\, w}{D^2(u,v) - D_0^2}\right]^{2n}} \quad \text{--- (2)} \quad n - \text{order of the filter}$$

& Gaussian Band reject filter

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D^2(u,v) - D_0^2}{D(u,v)\, w}\right]^2} \quad \text{--- (3)}$$

2. Band pass filter -

opposite operation of a band reject.

$$H_{bp}(u,v) = 1 - H_{br}(u,v) \quad \text{--- (1)}$$

3. Notch filter.

→ passes frequencies in predefined neighborhoods at the center of the fre

Ideal notch reject filter:

of radius D_0 & centre at (u_0, v_0) with symmetry at $(-u_0, -v_0)$ is.

$$H(u,v) = 0 \; ; \; D_1(u,v) \le D_0 \text{ or } D_2(u,v) \le D_0$$

$$1 \quad \text{otherwise}$$

where $D_1(u,v) = \left[\left(\frac{u-M}{2} - u_0\right)^2 + \left(\frac{v-N}{2} - v\right)^2\right]^{1/2}$

$$D_2(u,v) = \left[\left(\frac{u-M}{2} + u_0\right)^2 + \left(\frac{v-N}{2} + v_0\right)^2\right]^{1/2}$$

$$H(u,v) = \frac{1}{1 + \left[\frac{D_0^2}{D_1(u,v)\, D_2(u,v)}\right]^n} \quad — (2)$$

Butterworth ~~Gaussian~~.

$$H(u,v) = \frac{1}{1 + \left[\frac{D_0^2}{D_1(u,v)\, D_2(u,v)}\right]^n} \quad — (2)$$

Gaussian

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D_1(u,v)\, D_2(u,v)}{D_0^2}\right]} \quad — (3)$$

HPF $\Rightarrow u_0 = v_0 = 0$

$$H_{np}(u,v) = 1 - H_{nr}(u,v) \quad — (4)$$

Linear position - Invariant Degradation:

- A linear spatially - invariant degration system with additive noise can be modelled in spatial domain as the convolution of the degration (point spread) function with an image followed by the additiona of the noise

or degration model = $g(x,y) = H[f(x,y)] + \eta(x,y)$.

In the absence of additive noise $\eta(x,y)=0$

$$\therefore g(x,y) = H[f(x,y)]$$

For scalar values of a & b H is linear if

$$H[a f_1(x,y) + b f_2(x,y)] = aH[f_1(x,y)] + bH[f_2(x,y)]$$

H is position-Invariant if

$$g(x,y) = H[f(x,y)] = H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta)$$

indicates the response is dependent on the value of the i/p, not its position.

- f(x,y) can be expressed as

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha, y-\beta)\, d\alpha\, d\beta.$$

Using the defn. for $\eta(x,y)=0$

$$g(x,y) = H[f(x,y)] = H[f(x,y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\,\delta(x-\alpha, y-\beta)\, d\alpha$$

Assume H is linear, using the integral linearity.

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H[f(\alpha,\beta)\delta(x-\alpha, y-\beta)]\, d\alpha\, d\beta.$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\, H[\delta(x-\alpha, y-\beta)]\, d\alpha\, d\beta$$

* The term $h(x, \alpha, y, \beta) = H[\delta(x-\alpha, y-\beta)]$ is called the impulse Response

$\eta(x,y)=0$ then $h(x, \alpha, y, \beta)$ is the Response of H to the impulse 1 at Co-ordinates x, y. In optics an impulse ~~become~~ a point of light, $h(x, \alpha, y, \beta)$ is commonly Reffered to as the point spread func or spf

All physical optical system blur the point to some degree Blurring being determined by the quality of optical components

$$g(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta)\, h(x, \alpha, y, \beta)\, d\alpha\, d\beta$$

is called as Superposition integral of 1st kind

If the Response to an impulse is known, the Response to any input f can be calculated by this equation.

If H is position invariant then

$$g(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta)\, h(x-\alpha, y-\beta)\, d\alpha\, d\beta$$

Which is Convolution integral. - tells us that Impulse Response of any linear system allows to compute its Response at any i/p f. the Result is simply the convolution of impulse Response & i/p function

In the presence of Noise.

$$g(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta)\, h(x, \alpha, y, \beta) + \eta(x,y)$$

If H is position invariant then

$$g(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta),\, h(x-\alpha, y-\beta)\, d\alpha\, d\beta + \eta(x,y)$$

Using the notation of convolution.

$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$ → Taking FT.

$G(u,v) = H(u,v)\,F(u,v) + N(u,v)$

Because degradations are modelled as convolution, image restoration is often called image deconvolution.

* Filters used in Restoration process — deconvolution filters

Estimation of degradation model.

1. Estimation by Image observation
2. Estimation by experiment
3. Estimation by modelling.

$$g(x,y) = f(x,y) * h(x,y) + \eta(x,y)$$

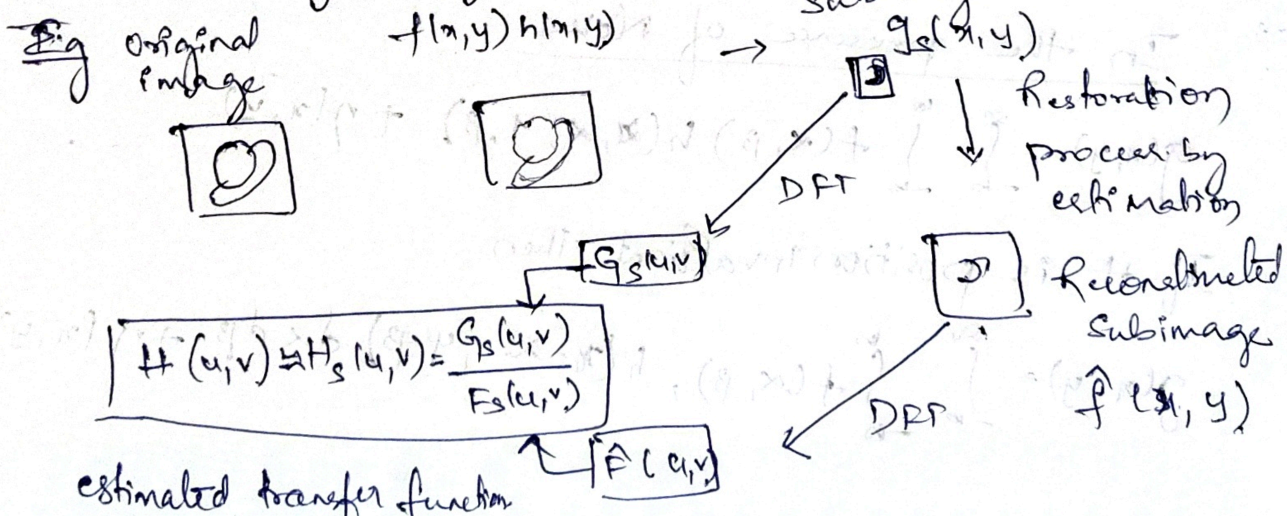
$$G(u,v) = F(u,v) \cdot H(u,v) + N(u,v)$$

Phase to estimate is to estimation $H(u,v)$.

if exact $h(x,y)$ is known regardless of noise we can do deconvolution to get $f(x,y)$ back from $g(x,y)$

To estimate – Methods
1. Estimation by Image observation
2. Estimation by Experiment
3. Estimation by Modelling

1. Estimation by Image observation



$$\boxed{H(u,v) \approx H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}}$$

estimated transfer function

(1) The ~~degraded~~ of the image is degraded by an unknown degradation function H.

(2). Assume that H is linear & position invariant

(3) Gather information from the image.

(4) Look at the small rectangular section of blurred image, which contain sample structure

5. To reduce the noise effect, look at the strong signal content area

6. Now process the subimage to arrive at a result that it a unblurred as possible.

7. Let the observed subimage be denoted by $g_s(x,y)$. & processed image $f_s(x,y)$. Assuming negligible noise effect we have

$$H_s(u,v) = \frac{G_s(u,v)}{F_s(u,v)}$$

2. Estimation by experiment :-

→ If equipment similar to that used to acquire the degraded image is available, it is possible to obtain an accurate estimate of the degradation.

↳ Image similar to the degraded image can be acquired with various system settings untill they are degraded as closely as possible to the image we wish to restore.

↳ Then the idea is to obtain the impulse response of the degradation by imaging an impulse (small dot of light) using the same system settings.

↳ An impulse is simulated by a light dot of light as bright as possible to reduce the effect of noise.

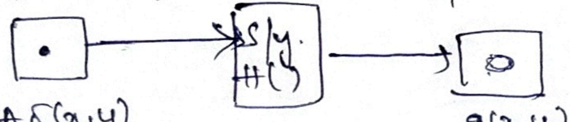
Then recalling that the FT of an impulse is constant it follows that

$$H(u,v) = \frac{G(u,v)}{A}$$

A → intensity of light source.

$G(u,v)$ is the observed spectrum

i/p impulse image   Response image from the s/y



$A\delta(x,y)$ $g(x,y)$

↓DFT   ↓DFT

$DFT\{A\delta(x,y)\} = A$ $G(u,v)$

↳ $H(u,v) = \frac{G(u,v)}{A}$ ↵

Estimation by Modelling

- A physical model is often used to obtain.

In many situations, the point spread function $h(x,y)$ is known explicitly prior to the image restoration process. In these cases, the recovery of $f(x,y)$ is known as the classical linear image restoration problem.

This problem has been thoroughly studied & a long list restoration methods for this situation includes numerous well-known techniques such as

- Inverse filtering
- Wiener filtering etc

Inverse filtering.

From degradation model $G(u,v) = F(u,v)\,H(u,v) + N(u,v)$.
after obtaining $H(u,v)$, from the estimating models.
we can also estimate $F(u,v)$, by inverse filter

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}.$$

Here Noise is enhanced when $H(u,v)$ is small.

To avoid the side effect of enhancing noise, we can apply the formulation to frequency component (u,v) with in a radius (D_0) from the center of $H(u,v)$.

In practice, the inverse filter is not popularly used.

Limitations:

- Even if $H(u,v)$ is known, $F(u,v)$ cannot be recovered exactly because $N(u,v)$ is the random function, which is not known.
- If the $H(u,v)$ has '0' or small value, the ratio easily dominates the estimate $\hat{F}(u,v)$

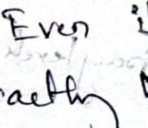
Approach to overcome the limitation.

- To get rid of '0' or small value problem is to limit the filter frequencies to values near the origin ~~the origin~~.

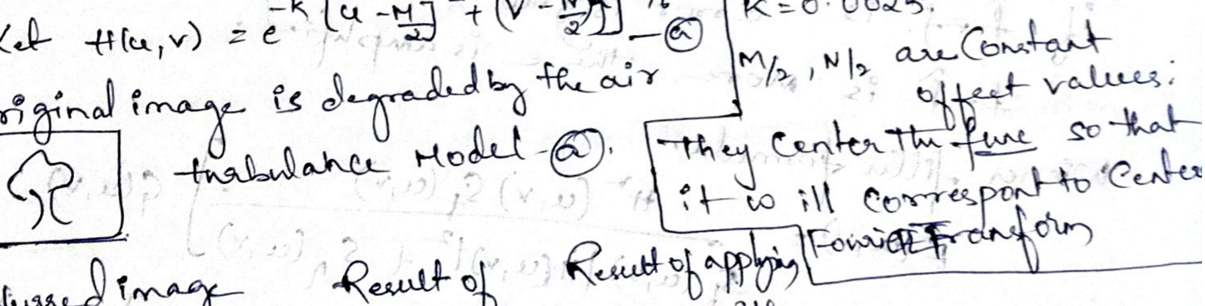
Eg Let $H(u,v) = e^{-k\left[\left(u-\frac{M}{2}\right)^2 + \left(v-\frac{N}{2}\right)^2\right]^{5/6}}$ — ⓐ

$K = 0.0025$. $M/2$, $N/2$ are constant offset values: They center the func so that it will correspond to center Fourier transform

Eg original image is degraded by the air turbulance model ⓐ.



Blurred image due to turbulance — Result of applying the full filter — Result of applying the filter with $D_0 = 40$ — $D_0 = 70$ — $D_0 = 85$.



$D_0 = 70$ suits perfectly.

D_0 - distance from the origin, center is zero frequency. try to fine the window & then apply to particular window.

D_0 - the window

Minimum Mean Square error (wiener) filtering

- The problem with inverse filter is that it has no explicit provision for handling Noise
- Wiener filter incorporates both degradation function & statistical characteristics of Noise in Restoration process

Objective of the wiener filter is to find the estimate of uncorrupted image f, such that the mean square error is minimum.

This error measure is given by $e^2 = E\{(f-\hat{f})^2\}$

The wiener filter is basically an optimum filter.

Coditions.

- Noise & image are uncorrelated
- One or the other has zero mean.
- Gray levels in $\hat{f}$ are linear function of gray levels in g.

Objective is to optimize mean square error

$e^2 = E\{(f-\hat{f})^2\}$ $E\{\cdot\}$ is the expected value of argument

$f \to$ original image

$\hat{f} \to$ Estimate of original image

here optimization means try ~~to~~ to find the optimum value for e^2

if $\hat{f} = f$ is error is zero

$f-\hat{f}$ is more the e is amplified.

Wiener filter formula.

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)\,S_f(u,v)}{S_f(u,v)\,|H(u,v)|^2 + S_\eta(u,v)}\right] G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right] G(u,v)$$

$S_f \to$

$$= \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \underbrace{S_\eta(u,v)}_{\text{constant}} / \underbrace{S_f(u,v)}_{\text{unknown}}}\right] G(u,v)$$

$H(u,v)$ - Degradation function.

$S_\eta(u,v)$ = power spectrum of noise

$S_f(u,v)$ = power spectrum of the undegraded image

*. No problem with zero unless $H(u,v)$ & $S_f(u,v)$ are both zero & when noise is zero. wiener filter ~~=~~ Inverse filter.

Approximation of wiener filter.

Since $S_\eta(u,v) = |N(u,v)|^2$ & $S_f(u,v) = |F(u,v)|^2$ are seldom known, the wiener filter is frequently approximated.

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + k}\right] G(u,v)$$

Practically k is chosen manually to obtain the best visual Result.

Advantages

1. The wiener filter doesnot have zero value problem.
2. The Result obtained by wiener filter is more closer to the original image than inverse filter.

Constrained least Sequare filter or CLS filter

* The Wiener filter
  1. is not optimal when we do not have information on the power spectra P.
  2. Performance depends upon the correct estimation of the value of k.
* Constrained least Square filter.
  1. ~~It~~ does not make any assumption about the original undegraded image
  2. It makes use of only the mean & variance of the noise

Wiener filter is optimal in average sense.

CLS yields optimal Result to each image to which it is applied

Degradation model. $g(x,y) = f(x,y) * h(x,y) + \eta(x,y)$

written in matrix form $g = Hf + \eta$

Objective: to find the minimum of a Criterion function.

$$C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\nabla^2 f(x,y)]^2$$

Subject to the Constraint

$\|g - H\hat{f}\|^2 = \|\eta\|^2$ — ①   where $\|w\|^2 = w^T w$ → Euclidean vector norm

we get a Constrained least square filter.

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2}\right] G(u,v)$$

where $P(u,v)$ = Fourier transform of $p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

Laplacian operator

γ is a parameter so the constrained in eqn ① is satisfied

* When the noise is high CLS performs better
* when the noise is low performance of CLS is similar to wiener filter
* It is possible to adjust the γ to achieve optimality satisfying the Constrained in Eqn ①.

Procedure to adjust.

Define $r = g - H\hat{f}$

It can shown that $\phi(\gamma) = r^T r = \|r\|^2$.

we want to adjust gamma so that $\|r\|^2 = \|\eta\|^2 \pm a$ — ①   a - a accuracy factor

1. Specify an initial value of γ
2. Compute $\|r\|^2$
3. Stop if ① is satisfied.

   Otherwise return step 2 after ~~after~~ ~~increasing~~ γ

increasing γ if $\|r\|^2 < \|\eta\|^2 - a$

or decreasing γ if $\|r\|^2 > \|\eta\|^2 + a$

Use the new value of γ to recompute

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma\, |P(u,v)|^2}\right] G(u,v)$$

Geometric Mean filter

It is the generalized form of wierner filter given by

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2}\right]^{\alpha} \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \beta\, \frac{S_\eta(u,v)}{S_f(u,v)}}\right]^{1-\alpha} G(u,v)$$

α, β being positive, real constants.

It consists of 2 expressions in the brackets raised to the power of α, $1-\alpha$.

when $\alpha = 1$, the filter reduces to inverse filter.

$\alpha = 0$. the filter is called as parametric wiernerfilter

$\alpha = 0$, $\beta = 1$ the filter reduces to standard wiernerfilter.

$\alpha = 1/2$, The filter becomes the product of 2 quantities raised to the same power. acts as geometric mean.

$\alpha < 1/2$ $\beta = 1$ the filter tend to more towards inversefilter

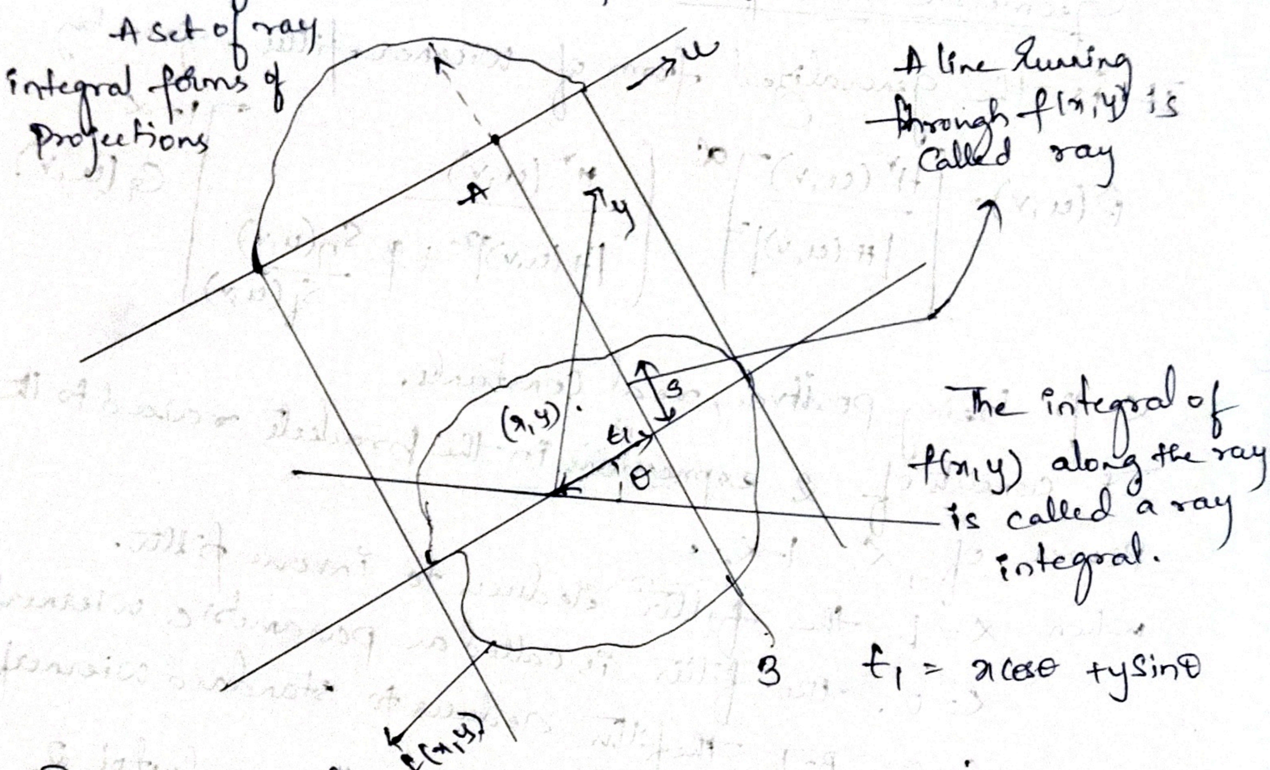
$\alpha > 1/2$ $\beta = 1$. - wierner filter

$\alpha = 1/2$ $\beta = 1$ - Spectrum equalization filter.

Image Reconstruction from projections.

Image Reconstruction from projection is a special class of Image Restoration process. where a 2-D object is reconstructed from several 1D-projections. Each projection is obtained by projecting a parallel x-rays (or other penetrating radiation) beam through the object.

~~Image Reconstruction from projection is a special class of~~



$t_1 = x\cos\theta + y\sin\theta$

Projections under all the angle θ will have 2 dimensional representation of the image under which one co-ordinate is position in the projection profile t and the other is the angle θ.

There are 2 methods to obtain projection data

1. Parallel projection
2. Fan Beam projection.

Parallel projection transforms the image into another 2-D representation.

The integral of the function $f(x,y)$ along the lines AB is called the ray integers & mathematically given by the function.

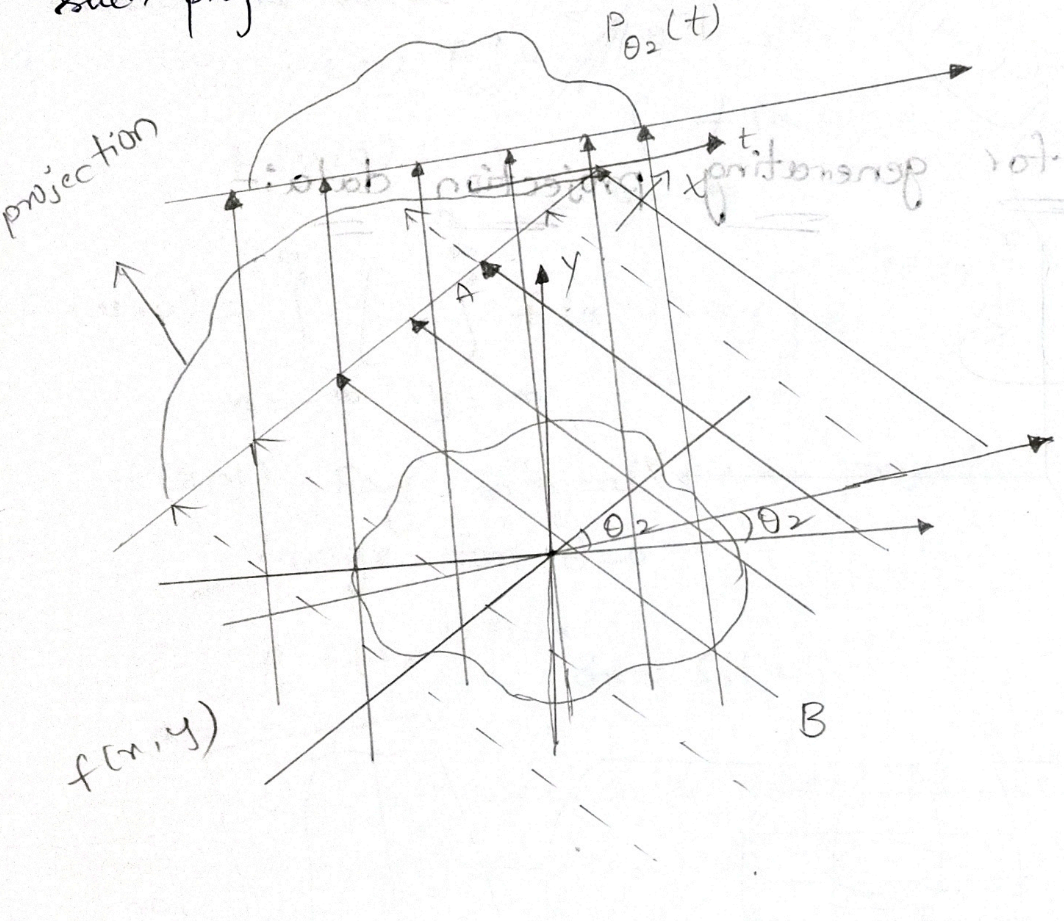
$$P_\theta(t_1) = \int_{ray\,AB} f(x,y)\, ds.$$

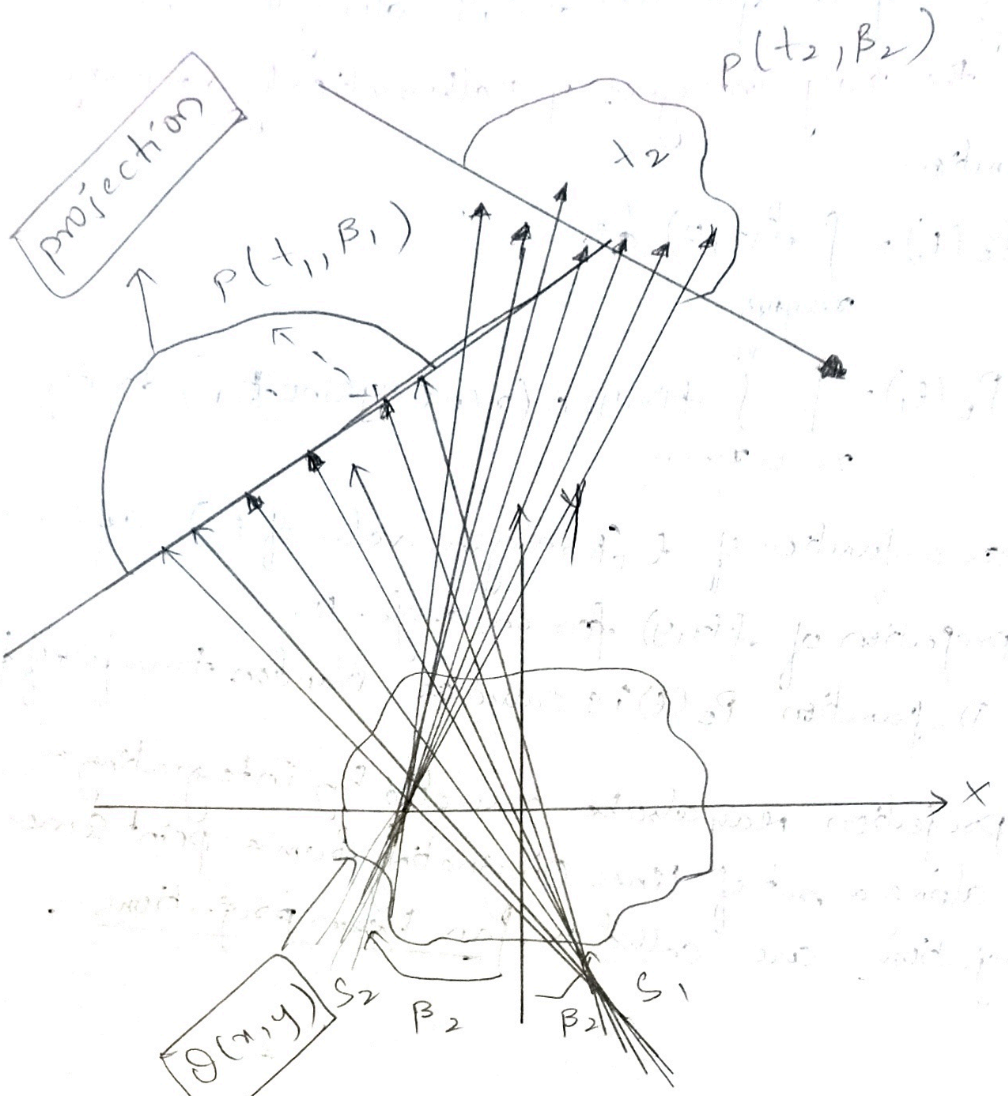
$$P_\theta(t_1) = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} f(x,y)\, \delta(x\cos\theta + y\sin\theta - t_1)\, dx\, dy.$$

$P_\theta(t)$ as a function of t (for a given value of θ) defines the Parallel projection of $f(x,y)$ for an angle θ.

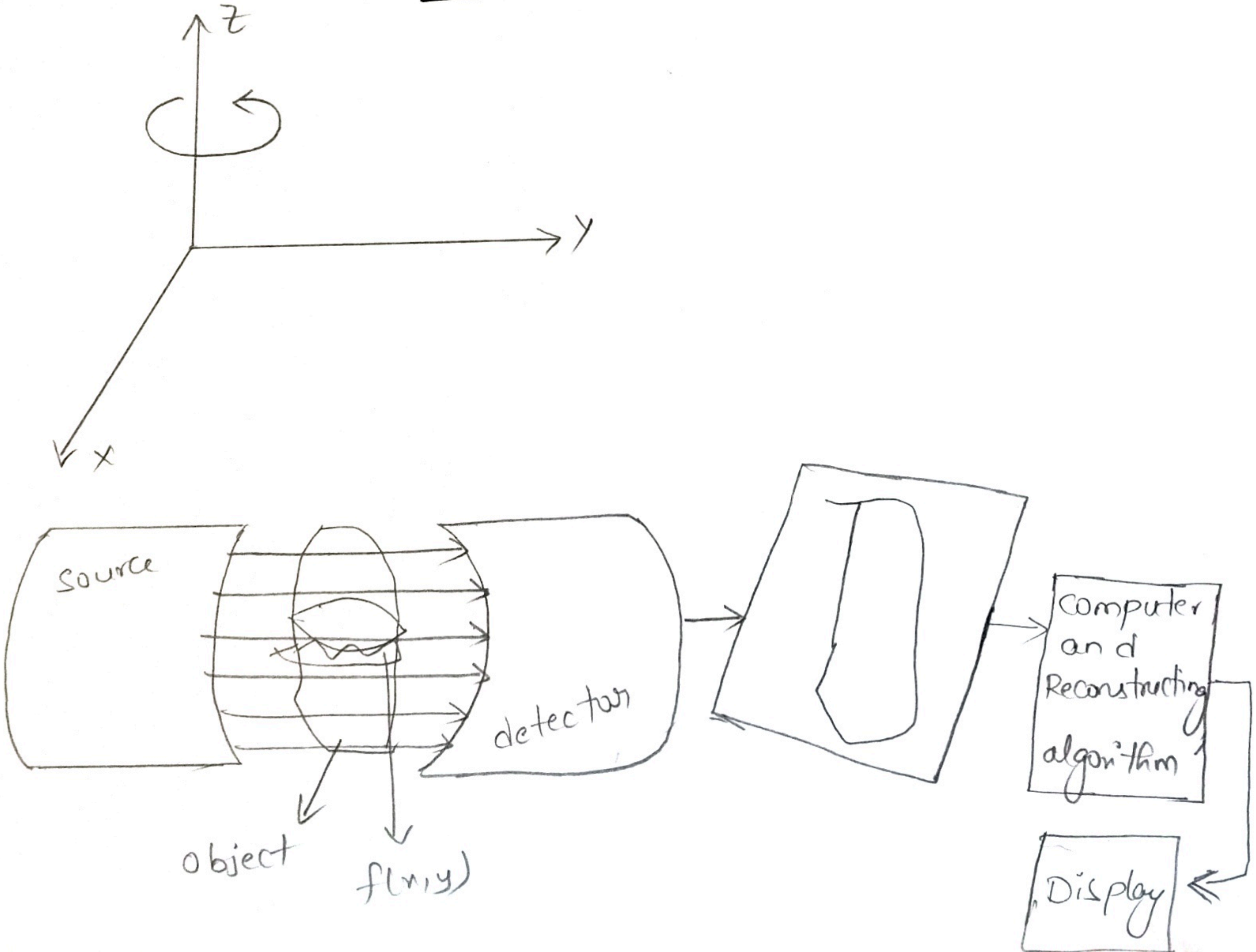
The 2D function $P_\theta(t)$ is called as Random transform of $f(x,y)$

* The projection may also be generated by integrating a function along a set of lines Originating from a point source, such projections are called as fan beam projections.





Methods for generating projection data:-



Fourier slice theorem:-

* The fourier slice theorem relates the 1D fourier transform of a projection of a function to the 2D fourier transform of f(x,y)

Let F(u,v) be the fourier transform of the image f(x,y)

$$F(u,v) = \int_{x=-\infty}^{+\infty} \int_{y=-\infty}^{+\infty} f(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$

Let $S_\theta(\omega)$ = The fourier transform of the projection $P_\theta(t)$

$$S_\theta(\omega) = F[P_\theta(t)] = \int_{t=-\infty}^{\infty} P_\theta(t)\, e^{-j2\pi\omega t}\, dt$$

Consider the value of F(u,v) on the line v=0 is given by i.e., F(u,0)

$$F(u,0) = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} f(x,y)\, e^{-j2\pi ux}\, dx\, dy$$

$$F(u,0) = \int_{x=-\infty}^{\infty} \left[\int_{y=-\infty}^{\infty} f(x,y)\, dy \right] e^{-2j\pi ux}\, dx$$

f(x,y) = Ray integral i.e., projection of the image for $\theta = 0$

$$F(u,0) = \int_{x=-\infty}^{\infty} P_0(t)\, e^{-j2\pi ux}\, dx = S_0(\omega)$$

The above result indicates that the values of the fourier transform F(u,v) on the line defined by v=0 can be obtained by fourier transforming the vertical projection of the image. This results can be generalized to show that, if $F(\omega,\theta)$ denotes the values of F(u,v) along a line at an angle 'θ'

with 'w' axis, then $F(\omega, \theta) = S_{\theta}(\omega)$