UNIT-4

DESIGN OF CLUTCHES, BRAKES AND SPRINGS

CLUTCHES AND BRAKES

Clutches:

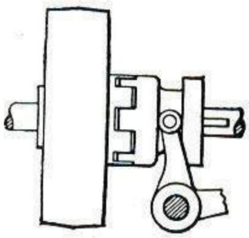
A Clutch is a mechanical device which is used to connect or disconnect the source of power from the remaining parts so the power transmission system at the will of the operator. The flow of mechanical power is controlled by the clutch.

Types of Clutches
(i) Positive Clutches (ii) Friction clutches

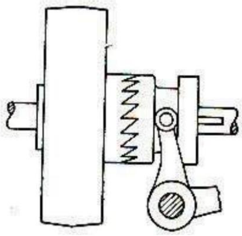
Positive Clutches: In this type of clutch, the engaging clutch surfaces interlock to produce rigid joint they are suitable for situations requiring simple and rapid disconnection, although they must be connected while shafts are stationery and unloaded, the engaging surfaces are usually of jaw type. The jaws may be square jaw type or spiral jaw type. They are designed empirically by considering compressive strength of the material used.

The merits of the positive clutches are

. (i) Simple (ii) No slip (iii) No heat generated compact and low cost.



Square-jaw clutch



Spiral-jaw clutch

Friction Clutches: Friction Clutches work on the basis of the frictional forces developed between the two or more surfaces in contact. Friction clutches are usually – over the jaw clutches due to their better performance. There is a slip in friction clutch. The merits are

(i) They friction surfaces can slip during engagement which enables the driver topickup and accelerate the load with minimum shock.
(ii) They can be used at high engagement speeds since they do not have jaw or teeth
(iii) Smooth engagement due to the gradual increase in normal force.

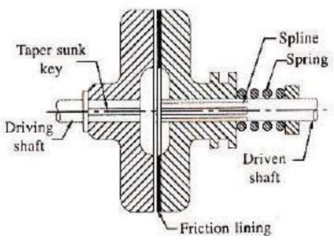
The major types of friction clutches are

(i) Plate clutch (Single plate) (multiple plate)
(ii) Cone clutch
(iii) Centrifugal clutch
(iv) Dry
(v) Magnetic current clutches
(vi)Eddy current clutches

We will be studying about single plate multi-plate and cone clutches.

Single plate clutch:

A single plate friction clutch consisting of two flanges shown in fig 2. One flange is rigidly keyed in to the driving shaft, while the other is free to move along the driven shaft due to spliced connection. The actuating force is provided by a spring, which forces the driven flange to move towards the driving flange. The face of the drive flange is linked with friction material such as cork, leather or ferodo



Torque transmitted by plate or disc clutch

A friction disk of a single plate clutch is shown in above fig

The following notations are used in the derivation

D_o = Outer diameter of friction disc (mm)

D_i = Inna diameter of friction disc (mm)

P = pressure of intensity N/mm^2

F = Total operating force (N) (Axial force)

T = torque transmitted by friction (N-mm)

Consider an elemental ring of radius r and radial thickness dr

Area of elemental length = $2\pi r.\ dr$

Axial force length = $2\pi r\ r.\ P$

(μ or f) friction force = 2πr. drp μ

Friction torque = 2πr dr Pμ * r

Total axial force Fa = ------------ (1)

$$F_a = \int_{D_i/2}^{D_0/2} 2\pi\, r\, dr p$$

Torque Transmitted by friction T = $\int_{D1/2}^{D_0/2} 2\pi r^2\, dr\ \mu p$ ----------------- (2)

There are two criteria to obtain the torque capacity – uniform pressure and uniform wear

1. Uniform pressure Theory:

In case of new clutches, un playing assumed to be uniformly distributed over the entire surface area of the friction disc. With this assumption, P is regarded as constant.

Equation – 1 becomes

$$F_a = \int_{D_i/2}^{D_0/2} 2\pi\, r\, dr p$$

$$F_a = 2\pi\, p \int_{D_i/2}^{D_0/2} r\, dr =$$

$$2\pi\, p \left[\frac{r^2}{2}\right]_{D1/2}^{D_0/2}$$

$$F_a = \frac{2\pi}{2} p \left[\frac{D_0^2}{2} - \frac{D_1^2}{2}\right]$$

$$F_a = \frac{1}{4}\pi\, p\left(D_0^2 - D_1^2\right)$$

$$\text{or} \qquad P = \frac{4\,Fa}{\pi\left[D_0^2 - D_1^2\right]}$$

From Equation -2

$$T = \int_{D1/2}^{D_0/2} 2\pi\ \mu\ pr^2 dr$$

$$T = 2\pi \mu p \int_{D_1/2}^{D_0/2} r^2 dr$$

$$T = 2\pi\,\mu\,p\left[\frac{r^3}{3}\right]_{D_i/2}^{D_0/2}$$

$$T = \frac{2}{3}\pi\mu\,p\left[\frac{Do^3}{2} - \frac{Di^3}{2}\right]$$

$$T = \frac{1}{3}\pi\,\mu\,p\left(Do^3 - D_1^3\right)$$

Substituting the value of p from equation 3

$$T = \frac{\pi\mu\left(D^3 - D_1^3\right)}{12} \qquad \frac{4Fa}{\pi\left(D_o^3 - D_1^3\right)}$$

$$T = \frac{1}{3}\,\mu\,Fa\,\frac{\left(D_0^3 - D_i^3\right)}{\left(D_0^2 - D_i^2\right)}$$

$$T = \frac{\mu\,Fa\,Dm}{2}$$

Where $\frac{2}{3}\left[\frac{D_0^3 - D_i^3}{D_0^2 - D_i^2}\right] = Dm$ mean diameter

Torque transmitted by n- friction surfaces

$$T = \frac{n^1\mu Fa Dm}{2}$$

Axial force = $\pi P\left(R_2^2 - R_1^2\right) = \frac{\pi}{4}P\left(D_0^2 - D_i^2\right)$

Uniform Wear Theory:

According to this theory, it is assumed that the wear is uniformly distributed over the entire surface --- of the friction disc. This assumption is used for workout clutches. The axial wear of the friction disc is import ional to the frictional work. The work done by the frictional force (μ P) and subbing velocity (2πrN) where 'N' is speed in rpm. Assuming speed N and coefficient of friction 'μ' is constant for given configuration

Wear $\propto$ Pr

Pr = constant C

When clutch plate is new and rigid. The wear at the outer radius will be more, which will release the pressure at the outer edge due to the rigid pressure plate this will change the pressure distribution. During running condition, the pressure distribution is adjusted in such a manner that the product pressure is constant, C.

From equation - (1)

$$\text{Axial force } Fa = 2\pi \int_{D_1/2}^{D_0/2} Pr\, dr$$

$$Fa = 2\pi\, c \int_{D_1/2}^{D_0/2} dr$$

$$Fa = 2\pi\, c[r]_{D_1/2}^{D_0/2}$$

$$Fa = 2\pi\, c \left[\frac{D_0}{2} - \frac{D_1}{2}\right]$$

$$\therefore\ C = \frac{Fa}{\frac{2\pi\ [D_0 - D_1]}{2}} \qquad - \quad (7)$$

From equation - (2)

$$T = \int_{D_1/2}^{D_0/2} 2\pi\ \mu\ Pr^2 dr = 2\pi\ \mu c \int_{D_1/2}^{D_0/2} r\, dr$$

$$= 2\pi\ \mu c \left[\frac{r^2}{2}\right]_{D_1/2}^{D_0/2}$$

$$= 2\pi\ \mu c \left[\frac{D_0/2}{8} - \frac{D_1/2}{8}\right]$$

$$T = \frac{\pi\ \mu c}{8}\left[D_0^2 - D_1^2\right]$$

Substitute the value of C from equation – (7)

$$T = \frac{\pi\ \mu}{4}\left[D_0^2 - D_1^2\right]\frac{Fa}{\pi\left[D_0 - D_1\right]}$$

$$T = \mu\ Fa\left[D_0 - D_1\right]$$

2 2

$$T = \frac{\mu\, Fa\ Dm}{2} \quad - \quad (8)$$

$$\text{Where Dm} = \frac{D_0 + D_1}{2} \quad - \quad (9)$$

Torque transmitted by "n" friction plates

$$T = \frac{n^1 \mu Fa\ Dm}{2}$$

Axial force Fa =

Average pressure occurs at mean radius $(r_m = D_m / 2)$

$$\therefore \text{ Fa} = \frac{\pi\ b\ Dm (D_0 - D_1)}{2} \quad - \quad (10)$$

Maximum pressure occurs at inner radius

$$\therefore \text{ Fa} = \frac{\pi\ p\ D_i}{2} (D_0 - D_i) \quad - \quad (11)$$

Note: The major portion of the life of friction lining comes under the uniform wear friction lining comes under the uniform wear criterion in design of clutches uniform wear theory is justified.

Problems:

1. A single plate friction clutch of both sides effective has 300 mm outer diameter and 160 mm inner diameter. The coefficient of friction o.2 and it runs at 1000 rpm. Find the power transmitted for uniform wear and uniform pressure distributions cases if allowable maximum pressure is 0.08 Mpa.

Given:

$N^1 = I = 2$, $D_0 = 300$ mm $D_1 = 160$mm $\mu = 0.2$

N = 1000 rpm p = 0.08 Mpa = 0.08 N /mm^2

Solution:

i. Uniform wear theory

$$\text{Mean Diameter } D_{m} = \frac{D_0 - D_1}{2} = \frac{300+160}{2} = 230\,mm$$

Axial Force

$$\text{Fa} = \frac{1}{2}\pi\ b\ D_1 (D_0 - D_1)$$

From DDH 13.32 or Equation - (11)

$$\therefore \ Fa = \frac{1}{2}\pi \times 0.08 \times 160\ (300 - 160)$$

$$= 2814.87\ N$$

Torque transmitted

$$T = \frac{1}{2} \mu\, n^1\, Fa\, Dm$$

$$T = \frac{1}{2} 0.2 \times 2 \times 2814.87 \times 230$$

$$T = 129484\ N - mm$$

Power transmitted

$$P = \frac{2\pi\ M\,T}{60 \times 10^6}$$

$$P = \frac{2\pi \times 1000 \times 129484}{60 \times 10^6} =$$

$$P = 13.56\ kW$$

ii. Uniform wear theory

Mean Diameter $D_m = \frac{2}{3} = \left(\frac{D_0^3 - D_1^3}{D_0^2 - D_1^2} \right)$

$$D_m = \frac{2}{3} = \left(\frac{300^3 - 160^3}{300^2 - 160^2} \right)$$

$$D_m = 237.1\ mm$$

Axial Force $\quad Fa = \dfrac{\pi\ p\ (D_0^2 - D_1^2)}{4}$

$$Fa = \frac{\pi\ 0.08\ (300^2 - 160^2)}{4}$$

$$Fa = 4046.4\ N$$

Torque transmitted

$$T = n^1 \frac{1}{2} \mu\ \ Fa\, Dm$$

$$T = 2\frac{1}{2}0.2 \times 4046.4 \times 237.1$$

$$T = 191880.3 \ N - mm$$

Power transmitted

$$P = \frac{2\pi \ nT}{60 \times 10^6}$$

$$P = \frac{2 \ \times \pi \ \times 1000 \times 191880.3}{60 \times 10^6} =$$

$$P = 20.1 \ kW$$

2. A car engine develops maximum power of 15 kW at 1000 rpm. The clutch used is single plate clutch both side effective having external diameter 1.25 times internal diameter μ = 0.3. Mean axial pressure is not to exceed 0.085 N/ mm^2. Determine the dimension of the friction surface and the force necessary to engage the plates. Assumeuniform pressure condition.

Given p = 15 kW, n – 1000rpm, I =2 both sides are effective $D_0 = 1.25 \, D_1$, $\mu = 0.3$, $p = 0.085 \ N/mm^2$

Torque transmitted

$$P = \frac{2\pi \ nT}{60 \times 10^6}$$

$$T = \frac{P \times 60 \times 10^6}{2\pi \, n}$$

$$T = \frac{15 \times 60 \times 10^6}{2\pi \ 1000}$$

$$= 143.239 \ N - mm$$

Mean Diameter D_m

$$D_m = \frac{2}{3}\left(\frac{D_0^3 - D_i^3}{D_0^2 - D_i^2}\right) = \frac{2}{3}\left(\frac{(1.25D_i)^3 - D_i^3}{(1.25D_i)^2 - D_i^2}\right)$$

$= 1.13\ D_i$

Axial Force $\quad Fa = \frac{\pi}{4} p \left(D_0^2 - D_i^2\right)$

$$Fa = \frac{\pi}{4} 0.085\ \left(1.25D_i^2 - D_i^2\right)$$

$$Fa = 0037552\ D_i^2$$

Torque transmitted $\quad T = i\ \frac{1}{2}\ \mu\ Fa\ Dm$

$$143239 = 2 \times \frac{1}{2} 0.3 \times 0.037552 \times D_i^2 \times 1.13 D_i$$

$\therefore \quad D_i = 224$ mm

$D_0 = 280$ mm

$D_m = 253$ mm

$Fa = 1884.21$ N

Thickness of disc h = 2 mm

3. Design a single plate clutch consist of two pairs of contacting surfaces for a torque capacity of 200 N-m. Due to space limitation the outside diameter of the clutch is to be 250mm

Given:
Single plate clutch, Torque = 2 x 10^5 N-mm, D0 = 250mm I = 2 (since two pairs of contacting surfaces)

Solution:

Assume suitable friction material – leather $\mu = 0.3$ to 0.5 P = varies from 0.07 to 0.29 Mpa select $\mu = 0.4$, P = 0.135 Mpa – N /mm^2

1. Torque transmitted= 2 x 10^5 N-mm
2. Mean diameter

Assuming uniform wear theory

$$D_m = \frac{D_i + D_0}{2} = \frac{D_i + 250}{2}$$

3. Axial force :

For uniform, wear condition

$$Fa = \frac{1}{2}\pi\, p D_i\, (D_i - D_i) =$$

$$= \frac{1}{2}\pi\, 0.135 \times D_i\, (250 - D_i)$$

$$= 0.212\, D_i\, (250 - D_i)$$

Torque transmitted

$$T = \frac{1}{2}\mu\, Fa\, D_m\, i$$

$$\text{i.e } 2 \times 10^5 = \frac{1}{2} 0.4 \times 0.212\, D_i\, (250 - D_i) = \frac{1}{2}(250 + D_i) \times 2$$

$$= 62500\, D_1 - D_1^3 - 4716981.132 = 0$$

By trial and error method

Inner dia D_i = 85.46 mm is 86 mm

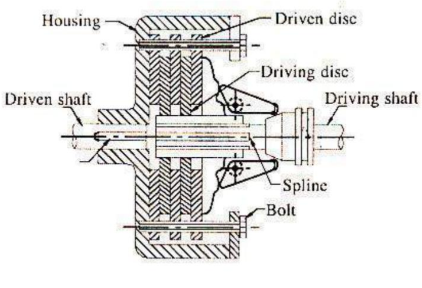
Outer dia D_0 = 250 mm given, mean m = dia of friction surface

Dm = 168 mm

Fa = 0.212 x 86 (250 – 86) = 2990 N

Multiple plate clutch

Fig. shows a multiple plate clutch. The driving discs are splined to the driving shaft so that they are free to slip along the shaft but must rotate with it. The driven discs drive the housing by means of bolts along which they are free to slide. The housing is keyed to the driven shaft by a sunk key. In the clutch shown there are five pairs of friction surfaces. The driving discs may be pressed against the driven discs by a suitable mechanism so that the torque may be transmitted by friction between the discs.



Multi disc Clutch:

Equations derived for torque transmitting velocity of single plate are modified to account for the number of pairs of contacting surfaces in the following way.

For uniform pressure $T = \frac{1}{2} i \mu_1 Fa\ Dm = \frac{\pi}{4} p (D_2^2 - D_i^2)$

Uniform wear $T = (D_0^2 - D_i^2)$

Where I = number of pairs of contacting surfaces.

For uniform pressure theory

$$T = \frac{1}{2} i \mu_1 Fa\ Dm$$

$$D_m = \frac{2}{3} \frac{(D_0^3 - D_i^3)}{(D_0^2 - D_i^2)}$$

$$Fa = \frac{\pi}{4} p (D_0^2 - D_i^2)$$

Where I = number of friction surfaces

Uniform wear theory

$$T = \frac{1}{2} i \mu_1 Fa\ Dm$$

$$D_m = \frac{(D_2 + D_i)}{2}$$

$$Fa = \frac{1}{2} \pi b D_m (D_0 - D_i)$$

Maximum pressure occurs at inner radius

$$Fa = \frac{1}{2} \pi_1 p D_1 (D_0 - D_i)$$

Problem: A multi plate clutch having effective diameter 250mm and 150mm has to transmit 60 kW at 1200 rpm. The end thrust is 4.5 kN and coefficient of friction is 0.08 calculate the number of plates easuming (i) Uniform wear and (ii) uniform pressure distribution on the plates

Given: D_0 = 250 mm D_i = 150mm, P = 60 kW, N = 1200 rpm, Fa = 4.5 kN = 4500N, μ = 0.08

$$P = \frac{2\pi\ NT}{60 \times 10^6}$$

$$\text{Torque } T = \frac{P \times 60 \times 10^6}{2\ \pi\ 1200} = 477500\ N - mm$$

(i) Uniform wear theory

$$\text{Mean diameter Dm} = \frac{D_0 + D_i}{2} = \frac{250 + 150}{2} = 200\ mm$$

$$T = \frac{1}{2}\, i\ \mu\ Fa\ Dm$$

$$477500 = \frac{1}{2}\, i\ 0.08 \times 4500 \times 200$$

$\therefore$ Number of friction plates, I = 13.26 ≅ 1.4 (even numbers)

Total number of plates i + 1 = 14 + 1 = 15

Uniform pressure

$$D_m = \frac{2}{3}\frac{(D_0^3 - D_i^3)}{(D_0^2 - D_i^2)} = 204.17 \text{ mm}$$

$$T = i \times \frac{1}{2}\, Fa \times \mu \times Dm$$

$$477500 = i\ \frac{1}{2} \times 4500 \times 0.08 \times 204.17$$

$\therefore$ i = 12.99 ≅ 14 (even number)

Total number of plates = 14 + 1 = 15

Problem 4:

A multi plate clutch of alternate bronze and steel plates is to transmit 6 kW power at 800 rpm. The inner radius is 38 mm and outer radius is 70 mm. The coefficient of friction is 0.1 and maximum allowable pressure is 350 kN /m^2 determine

(i) Axial force required
(ii) Total number of discs
(iii) Average pressure and
(iv) Actual maximum pressure

Given: $P = 60$ kW, $N = 800$ rpm, $R_1 = 38$mm, $D_i= 76$ mm $R_0 = 70$, $D_0 = 140$mm, $\mu = 0.1$,
$P = 350$ kN / m^2 = 0.35 N / mm^2

1. Axial force

$$Fa = \frac{1}{2}\pi_1 \; p\, D_1 \; (D_0 - D_1) \text{ - (13.32 DDH)}$$

$$Fa = \frac{1}{2}\pi \; 0.35 \times 76\; (140 - 76)$$

$$= 2674.12 \text{ N}$$

Torque to be transmitted

$$P = \frac{2\pi\; NT}{60 \times 10^6}$$

$$T = \frac{P \times 60 \times 10^6}{2\,\pi\; N} = \frac{6 \times 60 \times 10^6}{2 \times \pi \times 800} = 71625\; N - mm$$

Assuming uniform wear theory

Mean Diameter $\quad D_m = \frac{(D_2 + D_1)}{2} = \frac{140 + 76}{2} = 108\, mm$

Torque transmitted $T = \frac{1}{2}\, i\, \mu_1\, Fa\; Dm$

$$71625 = n\frac{1}{2} \times 0.1 \times 2674.12 \times 108$$

$$n = 4.96 \cong 6 \text{ (even number)}$$

Number of driving (steel) discs = $n_1 = \frac{n}{2} = \frac{6}{2} = 3$

Number of driven (bronze) discs n_2

$$= n_1 + 1 = 3 + 1 = 4$$

3. Average pressure occurs at mean diameter

Axial force $Fa = \frac{1}{2}\pi_1 \; p\, D_m \; (D_0 - D_i)$

$$2674.12 = \frac{1}{2}\pi_1 \; p\; 108\; (140 - 76) \text{ -}$$

$\therefore$ Average pressure p = 0.246 N/ mm^2

4. For 6 friction surface, torque transmitted

$$T = \frac{1}{2} i \mu_1 Fa\ Dm$$

$$71625 = 6 \frac{1}{2} 0.1_1\ Fa \times 108 -$$

$$\therefore\ Fa = 2210.6\ N$$

Maximum pressure occur at inner radius

$$\text{Axial force} = \frac{1}{2} \pi_1\ p\, D_1\ (D_0 - D_i)$$

$$2210.6 = \frac{1}{2} \pi_1\ p\ 76\ (140 - 76) -$$

$$\therefore\ \text{Actual maximum pressure } P = 0.289\ N/mm^2$$

Problem 5:
In a maultilate clutch radial width of the friction material is to be 0.2 of maximum radius. The coefficient of friction is 0.25. The clutch is 60KW at 3000 rpm. Its maximum diameter is 250mm and the axial force is limited is to 600N. Determine (i) Number of driving and driven plates (ii) mean unit pressure on each contact surface assume uniform wear

Given: Radial width = 0.2 Ro, $\mu = 0.25$, P = 60KW, N = 3000rpm, $D_0 = 250$mm,
$\therefore$Ro = 125mm, Fa = 600N uniform wear condition.

Solution b = Ro- Ri
0.2 Ro =Ri
Ri= 0.8 Ro = 0.8x 125 = 100mm
$\therefore$ Inner diameter 2x100 = 200mm

i) Number of disc

$$\text{Torque transmitted } T = \frac{P \times 60 \times 10^6}{2\pi N}$$

$$\frac{60 \times 60 \times 10^6}{2\pi\, 3000} = 191000 N.mm$$

For uniform wear condition

$$\text{Mean diameter } D_m = \frac{1}{2}(D_o + D_i)) = \frac{1}{2}(250 + 200) = 225mm$$

Torque Transmitted

$$T = \frac{1}{2} \mu\, n\, FaDm$$

$$19100 = \frac{1}{2} 0.25 \times n, 600 \times 225$$

i.e $n = 11.32$

Number of active surfaces n =12 ($\therefore$ n must be even number)

Number of disc on the driver shaft

$$n_1 = \frac{n}{2} = \frac{12}{2} = 6$$

Number disc on the driven shaft

$$n_1 = \frac{n}{2} + 1 = \frac{12}{2} + 1 = 7$$

$\therefore$ Total number of plates : n1 + n2 = 617 = 13

ii) Mean unit pressure

$$Fa = \frac{1}{2} \pi p D_m (D_o - D_i)$$

$$600 = \frac{1}{2} \pi p \, 225 (250 - 200)$$

$\therefore$ P = 0.34 N/ mm^2

iii) for actual mean unit pressureActual axial force

$$Fa = \frac{2T}{\mu n D_m} \quad \therefore T = \frac{1}{2} \mu fa D_m n$$

$$= \frac{2 \times 191000}{0.25 \times 12 \times 225} = \pi 565.926 N / \backslash$$

$$565.926 = \frac{1}{2} \pi PD_m \, 225 (250 - 200)$$

Actual mean unit pressure P = $0.032 N / mm^2$

A Multiple plate clutch has steel on bronze is to transmit 8 KW at 1440 rpm. The inner diameter of the contact is 80mm and outer diameter of contact is140 mm. The clutch operates in oil with coefficient of friction of 0.1. The overage allowable pressure is 0.35Mpa. Assume uniform wear theory and determine the following.

a) Number of steel and bronze plates
b) Axial force required
c) Actual maximum pressure

Given P = 8KW, N = 1440 rpm, D1 = 80mm, Do = 140mm, $\mu = 0.1$, P = 0.35 N / mm^2
Uniform Wear Theory.

Solution:

1) Number of steel and bronze platesFor uniform wear theory

Axial force $Fa = \frac{1}{2} \pi\ p\ (D_o - D_i)$

$$= \frac{1}{2} \pi\ 0.35\ 80(140 - 80)$$

$Fa = 2638.9\ \mu\ N$

Mean diameter $Dm = \frac{1}{2}(D_2 + D_i) = \frac{1}{2}(140 + 80) = 110mm$

Torque transmitted $T = \frac{P \times 60 \times 10^6}{2\pi\ N}$

$$T = \frac{8 \times 60 \times 10^6}{2\pi - 1440} = 53055.556\ N - mm$$

Also $T = \frac{1}{2}\mu\ FaDmn$

$$53055.556 = \frac{1}{2} \times 2638.94 \times 110 \times n$$

$$\therefore n = 3.655$$

No. of Active surface is 4

Number discs on the driver shaft (Bronze)

$$m_1 = \frac{n}{2} = \frac{4}{2} = 2$$

Number disc on the driven shaft (Steel)

$$n_2 = \frac{n}{2} + 1 = \frac{4}{2} + 1 = 3$$

Total No. of disc $n_1\ n_2 = 2 + 3 = 5$

b) Axial force required

$$Fa = \frac{2T}{\mu\ n\ D\ m} = \frac{2 \times 53055.552}{0.1 \times 04\ 110} = 2411.62$$

c) Actual maximum pressure since maximum presser occur at inner diameter

$$Fa = \frac{1}{2} \pi\ P\ D_i(D_o$$

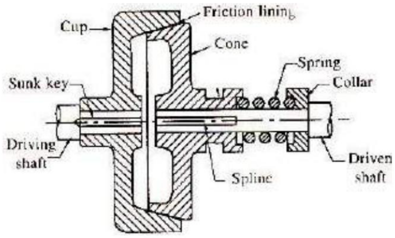
$- D_{i1})$

$$2411.62 = \frac{1}{2} P .80 (140 - 80)$$

$\therefore$ P = 0.32 N / mm^2

Cone clutch

A simple form of a cone clutch is shown in fig. it consists of ;i driver or cup and the follower or cone. The cup is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits the outside conical surface of the cone. The slope of the cone face is made small enough to give a high normal force. The cone is fitted to the driven shaft by a feather key. The follower may be shifted along the shaft by a forked shifting lever in order to engage the clutch by bringing the two conical surfaces in contact.



Advantages and disadvantages of cone clutch:Advantages:

1. This clutch is simple in design.
2. Less axial force is required to engage the clutch.

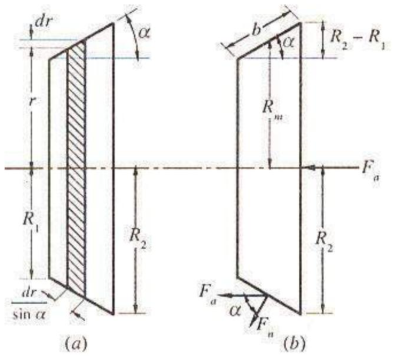
Disadvantages :

1. There is a tendency to grab.
2. There is some reluctance in disengagement.

Strict requirements made to the co-axiality of the shafts being connected

Torque transmitted by the cone clutch:-

Let Di = Inner diameter of cone, Ri = inner radius of cone,
Do = Outer diameter of cone, Ro = Outer radius of cone,
Rm = mean radius of cone, Dm = mean diameter of cone,
$\propto$ = Semi cone angle or pitch cone angle or face angle,
P = Intensity of normal pressure at contact surface,
μ = Coefficient of friction,
Fa = Axial force
Fn=Normal force = $\frac{Fa}{\sin \alpha}$



Consider an elemental ring of radius 'r' and thickness 'dr' as shown in the figure

The sloping length $= \dfrac{dr}{\sin\alpha}$

Area of elementary ring $= 2\pi r \dfrac{dr}{\sin\alpha}$

Normal force on the ring $= P\, 2\pi r \dfrac{dr}{\sin\alpha}$

Axial component of the above force $\dfrac{p 2\pi r.dr}{Sin\alpha} \times Sin\alpha$

$= 2\pi\ pr\, dr$

Total axial force Fa $= \int_{Ri}^{Ro} 2\pi\ pr\, dr$ ------------ (1)

Frictional force outer ring $= \mu\ p\ 2\pi r \dfrac{dr}{Sin\,\alpha}$

Moment of friction force about the axial

$= \mu\ P 2\pi\ r \dfrac{dr}{Sin\,\alpha} \times r$

$= 2\pi\ p\ \mu\ r^2 \dfrac{dr}{Sin\,\alpha}$

Total torque T $= \int_{Ri}^{Ro} 2\pi\ \mu \Pr^2 \dfrac{dr}{Sin\,\alpha}$ ---------- (2)

Uniform pressure theory: p constant

Equation (1) becomes

$$Fa = \int_{Ri}^{Ro} 2\pi\ pr\, dr = 2\pi\ p \int_{Ri}^{Ro} dr$$

$$Fa = \pi\ P \left(R_o^2 - R_i^2\right)$$

$$p = \frac{Fa}{\pi\left(R_o^2 - R_i^2\right)} \quad \text{----------} \quad (3)$$

Equation 2 becomes $\quad {}^{Ro}$

$^2($ ^{2}

$\frac{dr}{2}$) $\frac{2\pi\ P\ \mu}{Ro}$

$$T = \int_{Ri} 2\pi \ p\ \mu\ r \quad R_o - r_i \qquad \frac{}{Sin\alpha} = \frac{2}{Sin\alpha} \int_{Ri} r\ dr$$

$$T = \frac{2\pi \mu p}{Sin\alpha} = \frac{R_a^3 - R_i^3}{3}$$

Substitutes the value of P from equation ---------- (3)

$$T = \frac{2\pi \mu P}{Sin\alpha} = \frac{Fa}{\pi\left(R_o^2 - R_1^2\right)} \times \frac{R_a^3 - R_i^3}{3}$$

$$T = \frac{2\pi Fa}{3 Sin\alpha} = \frac{Fa}{\pi\left(R_o^2 - R_1^2\right)} \times \frac{R_a^3 - R_i^3}{\left(R_o^2 - R_i^2\right)}$$

$$T = \frac{2\pi Fa}{2 Sin\alpha} \times \frac{2}{3} \times \left(\frac{D_o^3 - D_i^3}{D_o^2 - D_{io}^2}\right) = \frac{\mu Fa}{2 Sin \alpha} . Dm \quad \text{---------- (4)}$$

Where $D m = \frac{2}{3}\left(\frac{D_o^3 - D_i^3}{D_o^2 - D_{i0}^2}\right)$

Axial force $Fa = \pi P \left(D_o^2 - D_i^2\right)$ ---------- (5)

Uniform wear: For uniform wear condition Pr = C Constant

Equation (1) become Fa = $\int_{Ri}^{Ro} 2\pi P r d r = 2\pi P \int_{Ri}^{Ro} dr$

$$Fa = 2\pi c \left(R_o - R_i\right) \text{ or}$$

$$C = \frac{Fa}{2\pi \left(R_o - R_i\right)}$$

Equation (2) become

$$T = \int_{Ri}^{Ro} 2\pi P \mu r^2 \frac{dr}{} = \frac{2\pi \mu C}{} \int^{Ro} r d r$$

$\underline{2\pi \mu c}$

$Sin\alpha$ $\quad Sin\alpha \quad _{Ri}$

$\left(R^2 - R^2\right)$

T = $\quad = \quad _a \quad ^i$

$Sin\alpha \quad 2$

Substitute for C

$$T = \frac{2\pi \ \mu \left(R_a^2 - R_i^2\right)}{Sin\alpha} \times \frac{Fa}{2\pi \left(R_o - R_i\right)}$$

$$= \frac{\mu\, Fa}{2\, Sin\alpha} \times \frac{(D_o + D_i)}{2} \cdot \frac{\mu\, Fa\, D\, m}{2\, Sin\,\alpha}$$

Where Dm = Mean diameter $$Dm = \frac{D_o + D_i}{2}$$

If the clutch is engaged when one member is stationary and other rotating, then the cone faces will tend to slide on each other in the direction of an element of the cone. This will resist the engagement and then force

Axial load $$Fa^i = Fa\,(Sin\alpha + cos\alpha)$$

Force width

$$b = \frac{D_o - D_i}{2\, Sin\,\alpha}$$

Outer diameter $$D_o = Dm + b\, Sin\alpha$$

Inner diameter $$D_i = Dm - b\, Sin\alpha$$

Problem:
A cone clutch is to transmit 7.5 KW at 600 rpm. The face width is 50mm, mean diameter is 300mm and the face angle 15°. Assuming co efficient of friction as 0.2, determine the axial force necessary to hold the clutch parts together and the normal pressure on the cone surface.

Given P = 7.5 KW, N = 600 rpm, b = 50mm,
Dm = 300mm, ∝ = 15° μ = 0.2

Solution:

$$T = \frac{P \times 60 \times 10^6}{2\,\pi\, N} = \frac{7.5 \times 60 \div 10^6}{2\,\pi\, 600} = 119375\ N - mm$$

Torque transmitted $$T = \frac{\mu\, Fa\, Dm}{2\, Sin\,\alpha}$$

$$119375 = \frac{0.2\, Fa \times 300}{2\, Sin\, 15}$$

$$\therefore Fa = 1029.88\ N$$

Also $Fa = \pi\ Dm\ Pb\ Sin\,\alpha$ ---------- Equation 13.37 DDH

$$1029.88 = \pi \times 300\, P \times 50\ Sin\ 15$$

$$P = 0.0844 N\,/\,mm^2$$

A friction cone clutch has to transmit a torque of 200 N-m at 1440 rpm. The longer diameter of the cone is 350mm. The cone pitch angle is 6.25°the force width is 65mm. the coefficient of friction is 0.2. Determine i) the axial force required to transmit the torque. ii) The average normal pressure on the contact surface when maximum torque is transmitted.

Data $T = 200$ N-m, 2×10^5 N-mm $N = 1440$ rpm
$Do = 350$, $\propto = 6.25°$ $b = 65mm$, $\mu = 0.2$

Solution

I) Axial force

Outer diameter $D_o = D_m + b \; Sin \; \alpha$

$$350 = D_m + 65 \; Sin \; 6.25$$

$$\therefore D_m = 342.92mm$$

Torque transmitted $$T = \frac{1}{2} \frac{\mu \, Fa \; D_m}{Sin \; \alpha}$$

$$2 \times 10^5 = \frac{1}{2} \times \frac{0.2 \times Fa \; D_m}{Sin \; 6.25}$$

$\therefore$ Axial force required $Fa = 634.934 \; N$

ii) Average normal pressure

$$Fa = \pi \; D_m \; Pb \; Sin \; \alpha$$

$$634.934 = \pi . \; 342.92 \times 65 Sin \; 6.25° \; P$$

$\therefore$ Average Normal pressure

$$P = 0.0833 \; N / mm^2$$

An engine developing 30 KW at 1250 rpm is filted with a cone clutch. The cone face angle of 12.5°. The mean diameter is 400 rpm $\mu = 0.3$ and the normal pressure is not to exceed 0.08 N / mm². Design the clutch

Date: $P = 30KW$, $N = 1250$ rpm, $\propto = 12.5°$, $Dm = 400mm$, $\mu = 0.3$, $P = 0.08$ N/ mm²

Solution

i) Torque transmitted

$$T = \frac{p \times 60 \; 10^6}{2\pi \; N} = \frac{30 \times 60 \times 10^6}{2\pi \; 1250}$$

$T = 229200\ N - mm$

ii) Axial force Fa

$$T = \frac{1}{2}\frac{\mu\ Fa\ D_m}{Sin\ \alpha}, \quad 229200 = \frac{1}{2} \times \frac{0.3\ Fa \times 400}{Sin\ 12.5}$$

Fa= 826.8 N

Dimensions

$$Fa = \pi\ D_m\ Pb\ Sin\ \alpha$$

$$826.8 = \pi . \times 400 \times 0.08 \times b \times Sin\ 12.5$$

$$b = 38mm$$

Inner diameter $D_m = D_m - b\ Sin\alpha = 400 - 38\ Sin\ 12.5 = 392mm$

Outer diameter $D_m = D_m + b\ Sin\alpha = 400 + 38\ Sin\ 12.5 = 408mm$

BRAKES

A brake is defined as a machine element used to control the motion by absorbing kinetic energy of a moving body or by absorbing potential energy of the objects being lowered by hoists, elevators, etc. The obsorbed energy appears as heat energy which should be transferred to cooling fluid such as water or surrounding air. The difference between a clutch and a brake is that whereas in the former both the members to be engaged are in motion, the brake connects a moving member to a stationary member.

Block or shoe brake

A single-block brake is shown in fig. It consists of a short shoe which may be rigidly mounted or pivoted to a lever. The block is pressed against the rotating wheel by an effort Fat one end of the lever. The other end of the lever is pivoted on a fixed fulcrum O. The friclional force produced by the block on the wheel will retard the rotation of the wheel. This type of brake is commonly used in railway trains. When the brake is applied, the lever with the block can be considered as a free body in equilibrium under the action of the following forces.

1. Applied force F at the end of the lever.
2. Normal reaction F_n between the shoe and the wheel.
3. Frictional or tangential braking force F_θ between the shoe and the wheel.
4. Pin reaction.

Let F = Operating force

M_1 = Torque on the wheel

r = Radius of the wheel

2θ = Angle of contact surface of the block

μ = Coefficient of friction

F_θ = Tangential braking force = $\frac{M_t}{r}$

F_n = Normal force = $\frac{F_\theta}{\mu}$

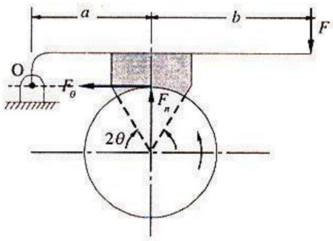
a = Distance between the fulcrum pin and the center of the shoe

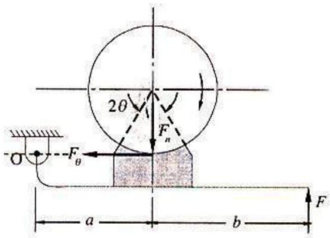
b = Distance between the center of the shoe to the end of lever where the effort is applied

c = Distance between the fulcrum pin and the line of action of F_g

Consider the following three cases;

(i) Line of action of tangential force F_0 passes through fulcrum





Taking moments about O,

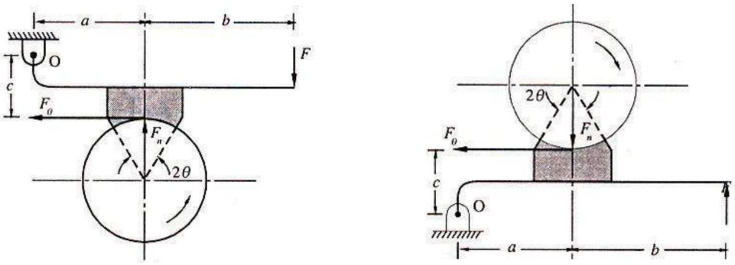
$$F\ (a+b) = F_n\, a = \frac{F_\theta}{\mu}\, a \qquad \left[F_n = \frac{F_\theta}{\mu} \right]$$

$$\therefore \text{Actuating force } F = \frac{F_\theta\, a}{\mu(a+b)}$$

In this case the actuating force is the same whether the direction of tangential force is towards or away from the fulcrum.

(ii) Line of action of tangential force F_θ is in between the center of the drum and the fulcrum

(a) Direction of F_θ is towards the fulcurm :



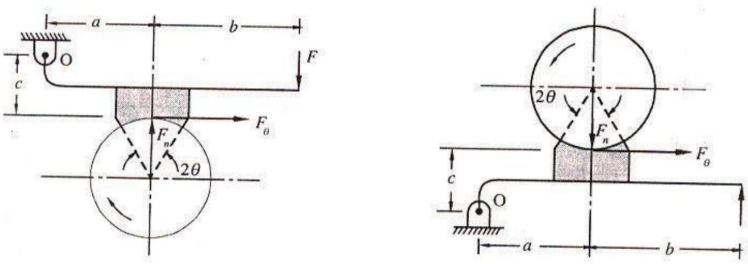
Taking moments about O,

$$F\ (a+b) = F_n\, c = F_n\, a$$

$$F\ (a+b) = F_n\, a - F_\theta\, c = \frac{F_\theta\, a}{\mu} - F_\theta\, c \qquad \left[F_n = \frac{F_\theta}{\mu} \right]$$

$$\therefore \text{Actuating force } F = \frac{F_\theta\, a}{(a+b)} \left[\frac{1}{\mu} - \frac{c}{a} \right]$$

(b) Direction of F_θ is away from the fulcrum :



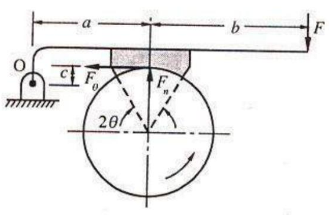
Taking moments about O,

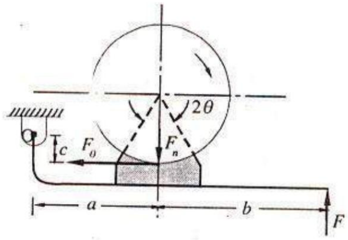
$$F\,(a+b) = F_n\,a - F_\theta\,c = \frac{F_\theta}{\mu}\,a + F_\theta\,c \qquad \left[F_n = \frac{F_\theta}{\mu}\right]$$

$$\therefore \text{Actuating force } F = \frac{F_\theta\,a}{(a+b)}\left[\frac{1}{\mu} - \frac{c}{a}\right]$$

(iii) Line of action of tangential force F_θ is above the center of the drum and the fulcrum:

(a) Direction of F_θ is towards the fulcrum :



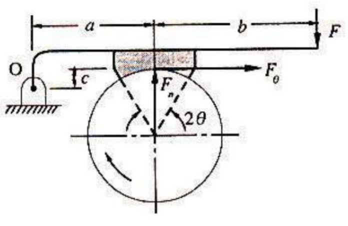


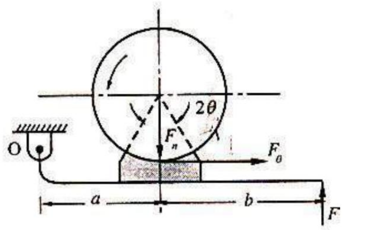
Taking moments about O,

$$F\,(a+b) = F_n\,a - F_\theta\,c = \frac{F_\theta\,a}{\mu} + F_\theta\,c \qquad \left[F_n = \frac{F_\theta}{\mu}\right]$$

$$\therefore \text{Actuating force } F = \frac{F_\theta\,a}{(a+b)}\left[\frac{1}{\mu} - \frac{c}{a}\right]$$

(b) Direction of F_θ is away from the fulcrum :





Taking moments about O,

$$F\,(a+b) + F_\theta\,c = F_\theta\,a = \frac{F_\theta}{\mu}\,a \qquad \left[F_n = \frac{F_\theta}{\mu}\right]$$

$$F\,(a+b) = \frac{F_\theta\,a}{\mu} - F_\theta\,c$$

$$\therefore \text{Actuating force } F = \frac{F_\theta\,a}{(a+b)}\left[\frac{1}{\mu} - \frac{c}{a}\right]$$

Note: If the direction of F_0 is towards the fulcrum, use the clockwise rotation formula and if the direction of $F_{\&}$ is away from the fulcrum, use counter clockwise formula from the data handbook.

When the angle of contact between the block and the wheel is less than 60°, we assume that the normal pressure is uniform between them. But when the angle of contact 2θ is more than 60°, we assume that the unit pressure normal to the surface of contact is less at the ends than at the center and the wear in the direction of applied force is uniform. In such case we employ the equivalent coefficient of friction μ', which is given by.

$$\text{Equivalent coefficient of friction } \mu^1 = \mu \times \frac{4\sin\theta}{2\theta + \sin 2\theta}$$

Where μ = Actual coefficient of friction

θ = Semi block angle

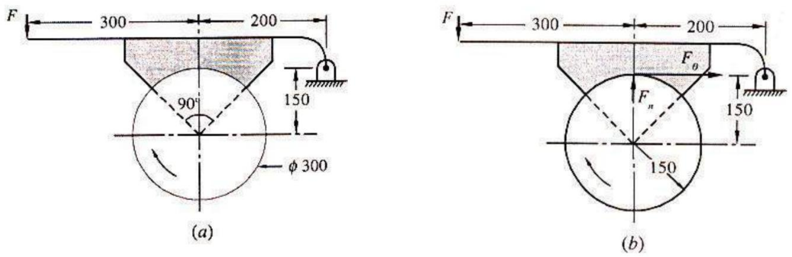
For the given value of θ. The value of $\frac{4\sin\theta}{2\theta + \sin 2\theta}$ can be found by using the chart given in

The brake is self -energizing when the friction force helps to apply the brake. If this effect is great enough to apply the brake with zero external force, the brake is called self-locking i.e., the brake is self locking when the applied force *F* is zero or negative.

$$\text{Normal pressure on the shoe p } P = \frac{\textit{Normal load}}{\textit{Projected area of shoe}} = \frac{F_n}{2wr \sin\theta}$$

Where w = Width of the shoe

Example: The block type hand brake shown in fig. 3.1 la has a face width of 45 mm. The friction material permits a maximum pressure of 0.6 MPa and a coefficient of friction of 0.24. Determine; 1. Effort F, 2. Maximum torque, 3. Heat generated if the speed of the drum is 100 rpm and the brake is applied for 5 sec. at full capacity to bring the shaft to stop.



Data: w = 45 mm, p = 0.6 MPa = 0.6 N/mm^2, μ = 0.24, 2θ = 90°, θ = 45^{e}, d = 300 mm, r = 150 mm, n = 100 rpm

Solution:

Since $2\theta > 60°$, equivalent coefficient of friction $\mu^1 = \mu \times \dfrac{4\sin\theta}{2\theta + \sin 2\theta}$

$$= 0.24 \times \frac{4\sin 45}{\frac{90 \times \pi}{180} + \sin 90} = 0.264$$

Allowable pressure $p = \dfrac{F_n}{2\,wr\sin\theta}$

$$\text{i.e., } 0.6 = \frac{F_n}{2 \times 45 \times 150 \sin 45}$$

$\therefore$ Normal force $F_n = 5727.56$ N

Tangential force $F_\theta = \mu' F_n = 0.264 \times 5727.56 = 1512.1$ N
The various forces acting on the shoe are shown in fig. 3.11b.
From the figure, a = 200 mm, b = 300 mm, c = 0
The tangential force F_θ, passes through the fulcurm.

$\therefore$ Effort $F = \dfrac{F_\theta\, a}{\mu' (a + b)}$

$$= \frac{1512.1 \times 200}{0.264\,(200 + 300)} = 2291.1\ N$$

Torque on the drum $M_I = F_\theta\, r = 1512.1 \times 150 = 226815$ N-mm = 226.815 N-m

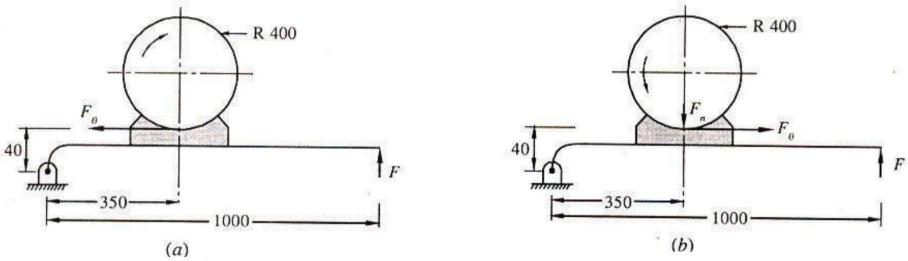
Power absorbed $N = \dfrac{M_1 n}{9550} = \dfrac{226.815 \times 100}{9550} = 2.375\,kW = 2.375\ kJ/\sec$

Heat generated during 5 sec = 5 x 2.375 = 11.875 kJ

Example:

A 400 mm radius brake drum contacts a single shoe as shown in fjg.3.12a, and sustains 200 N-m torque at 500 rpm. For a coefficient of friction 0.25, determine:

1. Normal force on the shoe.
2. Required force F to apply the brake for clockwise rotation.
3. Required force F to apply the brake for counter clockwise rotation.
4. The dimension c required to make the brake self-locking, assuming the other dimensionsremains the same.
5. Heat generated.



Data; r = 400 mm, M_1- 200 N-m = 200 x 10^3 N-mm, n = 500 rpm, μ=0.25,
a = 350 mm, i = 1000 mm, b = l - a = 1000 - 350 = 650 mm, c = 40 mm

Solution:

Tangential friction force $$F_\theta = \frac{M_1}{r} = \frac{200 \times 10^3}{400} = 500\ N$$

Normal force on the $$F_\theta = \frac{F_\theta}{\mu} = \frac{500}{0.25} = 2000\ N$$

From the figure, the tangential force F_θ lies between the fulcrum and the center of drum.. When the force F_θ acts towards the fulcrum (clockwise rotation),

Actuating force $$F_\theta = \frac{F_0 a}{a+b}\left[\frac{1}{\mu} - \frac{c}{a}\right]$$

$$= \frac{500 \times 350}{350+650}\left[\frac{1}{0.25} - \frac{40}{350}\right] = 680\,N$$

For anticlockwise rotation of the drum, the tangential force F_s acts away from the fulcrum as shown in figure.

∴ Actuating force $$F_\theta = \frac{F_0 a}{a+b}\left[\frac{1}{\mu} + \frac{c}{a}\right]$$

$$= \frac{500 \times 350}{350+650}\left[\frac{1}{0.25} + \frac{40}{350}\right] = 720\,N$$

When the drum rotates in clockwise direction, self locking will occur. For self locking effort $F \leq 0$.

$$i.e., = \frac{F_0 a}{a+b}\left[\frac{1}{\mu} - \frac{c}{a}\right] \le 0 \quad \text{or} \quad \frac{c}{a} \ge \frac{1}{\mu}, \quad c \ge \frac{a}{\mu}$$

$$\therefore \quad c \ge \frac{350}{0.25} \ge 1200mm$$

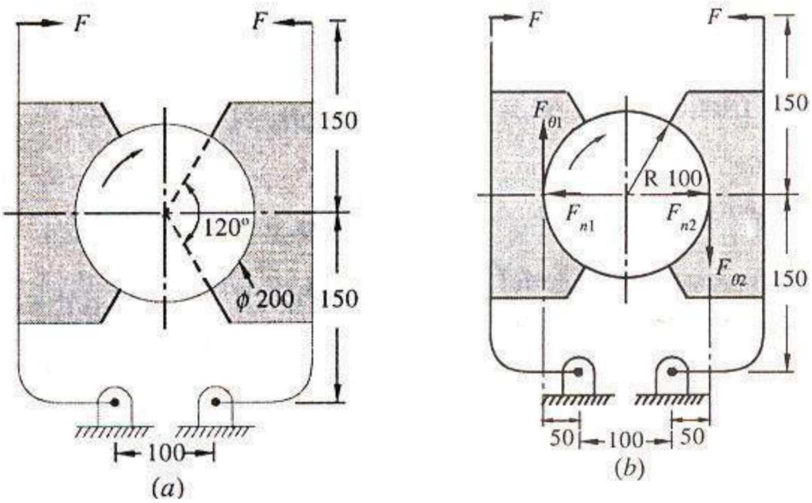
Heat generated $\quad H_g = \mu \; p \; A_c v = F_n \; v$

$$= 0.25 \times 2000 \times \frac{\pi \times 800 \times 500}{60 \times 1000} = 1071.98\ W$$

$$= 10.472\ kW = 10.472\ kJ/s$$

Example: The layout of a brake to be rated at 250 N-m at 600 rpm is shown in figure. The drum diameter is 200 mm and the angle of contact of each shoe is 120°. The coefficient of friction may be assumed as 0.3 Determine.

1. Spring force F required to set the brake.
2. Width of the shoe if the value of pv is 2 N-m/mm^2-sec



Data: $M_t = 250$ N-m $= 250 \times 10^3$ N-mm, $\quad n = 600$ rpm, $\quad d = 200$ mm, $\quad r = 100$ mm,
$2\theta = 120°$, $\quad 0 = 60°$, $\quad \mu = 0.3$, $\quad pv = 2$ N-m/mm^2-sec

Solution:

Since 26 > 60°, equivalent coefficient of friction $\mu' = \mu \times \dfrac{4Sin\theta}{2\theta + Sin2\theta}$

$$= 0.3 \times \frac{Sin\ 60}{\frac{120 + \pi}{180} + \sin 120} = 0.351$$

The various forces acting on the shoes are shown in figure.

Left hand shoe:

For left hand shoe, a = 150 mm, b = 150 mm, c = 100 – $\dfrac{100}{2}$ = 50 *mm*

The tangential force $F_{\theta 1}$ lies above the center of the drum and fulcrum, and acting away from the fulcrum (counter clockwise).

$\therefore$ Spring force $F = \dfrac{F_{01}\, a}{a + b}\left[\dfrac{1}{\mu} - \dfrac{c}{a}\right]$

$$F = \frac{F_{01} \times 150}{150 + 150}\left[\frac{1}{0.351} - \frac{50}{150}\right]$$

$\therefore$ Tangential force F_{gi} = 0.795 F

Right hand shoe:

The tangential force $F_{\theta 2}$ lies below the center of the drum and the fulcrum, and acting towards the fulcrum (clockwise).

$\therefore$ Spring force $F = \dfrac{F_{02}\, a}{a + b}\left[\dfrac{1}{\mu} - \dfrac{c}{a}\right]$

$$F = \frac{F_{02} \times 150}{150 + 150}\left[\frac{1}{0.351} - \frac{50}{150}\right]$$

$\therefore$ Tangential force $F_{02} = 0.6285\, F$

Torque $\qquad M_1 = (F_{01} + F_{01})\, r$

i.e., $\qquad 250 \times 10^3 = (0.795\ F + 0.6285\ F) \times 100$

$\therefore$ Spring force F = 1756.2 N

The maximum load occurs on left hand shoe.

∴ Tangential force $F_{01} = 0.795 \times 1756.2 = 1396.2$ N

Normal force on left hand shoe $F_{01} = \dfrac{F_{01}}{\mu} = \dfrac{1396.2}{0.351} = 3977.8\ N$

Surface velocity of drum $v = \dfrac{\pi\ dn}{60 \times 1000} = \dfrac{\pi \times 200 \times 600}{60 \times 1000} = 6.283\ m/\sec$

By data pv = 2 N-m/mm^2 – sec

∴ Normal pressure $p = \dfrac{2}{v} = \dfrac{2}{6.283} = 0.3183\ N/mm^2$

Also pressure $p = \dfrac{F_{n1}}{2wr \sin\theta}$

1.e., $0.3183 = \dfrac{3977.8}{2wr \times 100 \sin 60}$

∴ Width of the shoe w = 72.15mm

Band brakes

A band brake consists of a band, generally made of metal, and embracing a part of the circumference of the drum. The braking action is obtained by tightenting the band. The difference in the tensions at each end of the band determines the torque capacity.

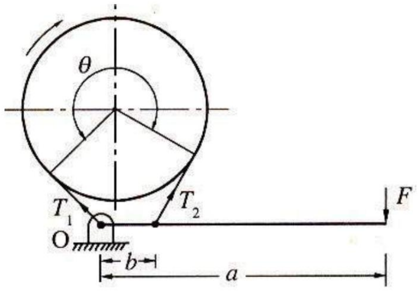
Simple band brakes: When one end of the band is connected to the fixed fulcrum, then the band brake is called simple band brake as shown in figure

Let
T_1 = Tight side tension in N
T_2 = Slack side tension in N
θ = Angle of lap in radians
μ = Coefficient of friction
D = Diameter of brake drum in mm
M_1 = Torque on the drum in N-mm

Ratio of tensions $\frac{T_1}{T_2} = e^{\mu\theta}$

Braking force $F_\theta = T_1 - T_2 = \frac{2M_t}{D}$

Clockwise rotation of the drum:



Taking moments about the fulcrum O,

$$F \times a = T_2 \times b$$

$\therefore$ Force at the end of lever $F = \frac{T_2 b}{a}$

Braking force $F_\theta = T_1 - T_2 = T_2 e^{\mu\theta} - T_2 = T_2(e^{\mu\theta} - 1)$ $\left[\frac{T_1}{T_2} = e^{\mu\theta}\right]$

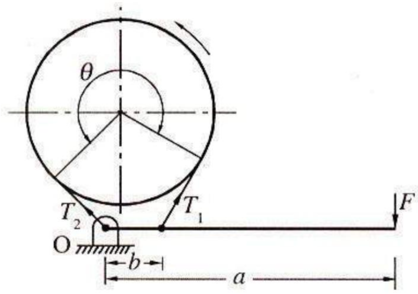
:. Slack side tension $T_2 = \frac{F_\theta}{e^{\mu\theta} - 1}$

Substitute the value of T_2 in equation (1), we get

$$F = \frac{T_2\, b}{a}\left[\frac{1}{e^{\mu\theta} - 1}\right]$$

Counter clockwise rotation of the drum:

For counter clockwise rotation of the drum, the tensions T_1 and T_2 will exchange their places and for the same braking torque, a larger force *F* is required to operate the brake.



Taking moments about the fulcrum O,

$$F \times a = T_1 \times b$$

$\therefore$ Force at the end of lever $F = \dfrac{T_1\, b}{a}$

Braking force $F_\theta = T_1 - T_2 = T_1 - \dfrac{T_1}{e^{\mu\theta}} = T_1\left(1 - \dfrac{1}{e^{\mu\theta}}\right)$ $\quad \left[\dfrac{T_1}{T_2} = e^{\mu\theta}\right]$

Tight side tension $T_2 = \dfrac{F_\theta\, e^{\mu\theta}}{e^{\mu\theta} - 1}$

Substitute the value of T_1, in equation (i), we get

$$F = \frac{F_\theta\, b}{a}\left[\frac{e^{\mu\theta}}{e^{\mu\theta} - 1}\right]$$

This type of brake does not have any self-energizing and self-locking properties.

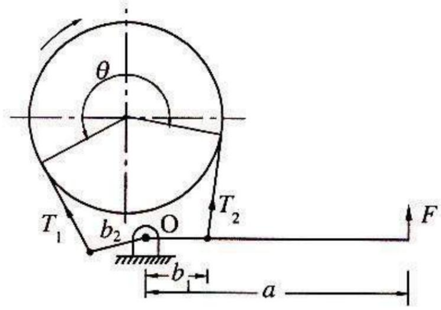
Thickness of the band h = 0.005 D

Width of band $w = \dfrac{T_1}{h\, \sigma_d}$

where σ_d is the allowable tensile stress in the band.

Differential band brakes:

In differential band brake the two ends of the band are attached to pins on the lever at a distance of b_1 and b_2 from the pivot pin as shown in figure. It is to be noted that when $b_2 > b_1$, the force F must act upwards in order to apply the brake. When $b_2 < b_1$, the force F must act downwards to apply the brake.



Clockwise rotation of the drum:

Taking moments about the fulcrum O,

$$T_1 \times b_2 = T_2 \times b_1 + F \times a$$

$$T_2 e^{\mu\theta} b_2 - T_2 b_1 = Fa \qquad (T_1 = T_2 e^{\mu\theta})$$

Braking force $F_\theta = T_1 - T_2 = T_2 e^{\mu\theta} - T_2 = T_2 (e^{\mu\theta} - 1) \qquad (T_1 = T_2 e^{\mu\theta})$

Slack side tension $T_2 = \dfrac{F_\theta}{e^{\mu\theta} - 1}$

Substitute the value of T_2 in equation (1), we get

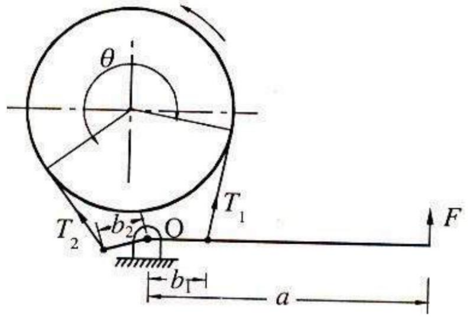
$$F = \frac{F_\theta}{a} \left[\frac{b_2 e^{\mu\theta} - b_1}{e^{\mu\theta} - 1} \right]$$

For the brake to be self locking, the force F at the end of the lever must be equal to zero or negative.

$$\therefore \frac{F_\theta}{a} \left[\frac{b_2 e^{\mu\theta} - b_1}{e^{\mu\theta} - 1} \right] \le 0$$

The condition for self-locking is $b_2 e^{\mu\theta} \le b_1$

Counter clockwise rotation of the drum:



Taking moments about the fulcrum O,

$$T_2 \times b_2 = T_1 \times b_1 + F \times a$$

$$T_2 b_2 - T_2 e^{\mu\theta} b_1 = Fa \qquad (\because T_1 = T_2 e^{\mu\theta})$$

Or $$F = \frac{T_2 (b_2 - b_1 e^{\mu\theta})}{a}$$

Braking force $F_\theta = T_1 - T_2 = T_2 e^{\mu\theta} - T_2 = T_2 (e^{\mu\theta} - 1) \qquad (\because T_1 = T_2 e^{\mu\theta})$

Slack side tension $T_2 = \dfrac{F_\theta}{e^{\mu\theta} - 1}$

Substitute the value of T_2 in equation (1), we get

$$F = \frac{F_\theta}{a} \left[\frac{b_2 - b_1 e^{\mu\theta}}{e^{\mu\theta} - 1} \right]$$

For self-brake locking, the force F must be zero or negative.

i.e., $$\frac{F_\theta}{a} \left[\frac{b_2 - b_1 e^{\mu\theta}}{e^{\mu\theta} - 1} \right] \le 0$$

The condition for self-locking is

$$b_2 \le b_1 e^{\mu\theta}$$

If $b_1 = b_2 = b$, the brake is called two way brake and is shown in figure. This type of brake can be used in either direction of rotation with same effort.

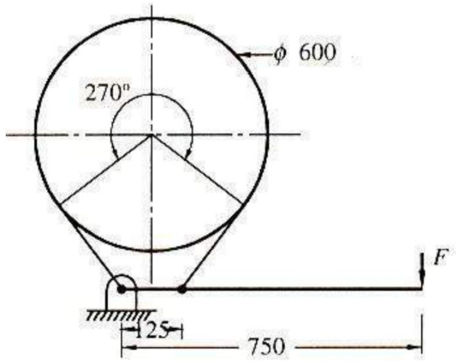
$\therefore$ Force at the end of lever $F = \dfrac{F_\theta\, b}{a} \left[\dfrac{e^{\mu\theta} + 1}{e^{\mu\theta} - 1} \right]$

Example:

A simple band brake operates on a drum 0.6 m in diameter rotating at 200 rpm. The coefficient of friction is 0.25 and the angle of contact of the band is 270°. One end of the band is fastened to a fixed pin and the other end to 125 mm from the fixed pin. The brake arm is 750 mm long.

(i) What is the minimum pull necessary at the end of the brake arm to .stop the wheel if 35kW is being absorbed? What is the direction of rotation for minimum pull?

(ii) Find the width of 2.4 mm thick steel band if the maximum tensile stress is not to exceed 55N/mm².



Data: D = 0.6 m = 600 mm, n = 200 rpm, p = 0.25, 0= 270°, a = 750 mm, b = 125 mm, N = 35kW, h = 2.4 mm, $\sigma_d = 55\ N/mm^2$

Solution:

Frictional torque $M_1 = \dfrac{9550\,N}{n}$

$$= \frac{9550 \times 35}{200} = 1671.25\ N-m = 1671.25 \times 10^3\ N-mm$$

Braking for $F_\theta = \dfrac{M_1}{R} = \dfrac{2M_1}{D} = \dfrac{2 \times 1671.25 \times 10^3}{600} = 5570.83\ N$

Ratio of tensions $\dfrac{T_1}{T_2} = e^{\mu\theta} = e^{0.25 \times 270 \times \pi 180} = 3.248$

Clockwise rotation:

Force $F = \dfrac{F_\theta\, b}{a} \left[\dfrac{1}{e^{\mu\theta} - 1} \right]$

$$= \frac{5570.83 \times 125}{750}\left[\frac{1}{3.248 - 1}\right] = 413.02\ N$$

Counter clockwise rotation:

Force $F = \frac{F_\theta\, b}{a}\left[\frac{e^{\mu\theta}}{e^{\mu\theta} - 1}\right]$

$$= \frac{5570.83 \times 125}{750}\left[\frac{3248}{3.248 - 1}\right] = 1341.5\ N$$

Therefore the minimum pull $F = 413.02$ N

The direction of rotation is clockwise.

Braking force $F_\theta = T_1 - T_2 = T_1 - \frac{T_1}{e^{\mu\theta}} = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right)$

i.e.,

$$5570.83 = T_1\left[1 - \frac{1}{3.248}\right]$$

$\therefore$ Tight side tension $T_1 = 8048.96\ N$

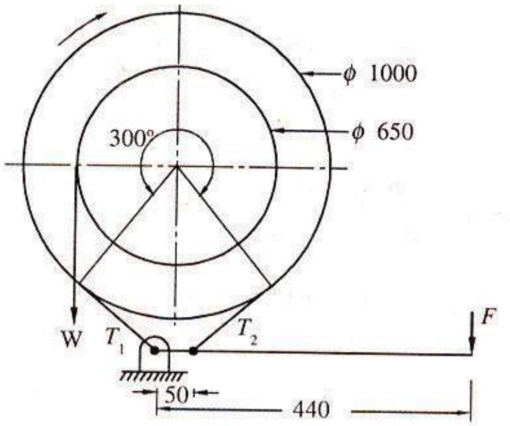
Width of band $w = \frac{T_1}{h\,\sigma_d}$

$$= \frac{8048.96}{2.4 \times 55} = 60.98\ mm = 62\ mm$$

Problem:

In a simple band brake, the length of lever is 440 mm. The tight end of the band is attached to the fulcrum of the lever and the slack end to a pin 50 mm from the fulcrum. The diameter of the brake drum is 1 m and the arc of contact is 300°. The coefficient of friction between the band and the drum is 0.35. The brake drum is attached to a hoisting drum of diameter 0.65 m that sustains a load of 20 kN. Determine;

1. Force required at the end of lever to just support the load.
2. Required force when the direction of rotation is reversed.
3. Width of stee! band if the tensile stress is limited to 50 N/mnr.



Data: a = 440 mm, b = 50 mm, D= 1 m = 1000 mm, D_b = 0.65 m = 650 mm,
θ = 300°, μ = 0.35, W = 20kN = 20x 10^3 N, σ_d = 50N/mm^2

Solution:

Torque on hoisting drum $M_1 = W\ R_b = \frac{WD_b}{2}$

$$= \frac{20 \times 10^3 \times 650}{2} = 6.5 \times 10^6\ N - mm$$

The hoisting drum and the brake drum are mounted on same shaft.

$\therefore$ Torque on brake drum M1 = 6.5 x 10^6 *N-mm*

Braking for $F_\theta = \frac{M_1}{R} = \frac{2M_1}{D}$

$$= \frac{2 \times 6.5 \times 10^6}{1000} = 13000\ N$$

Ratio of tensions $\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.35 \times 300 \times \pi 180} = 6.25$

Clockwise rotation:

Force at the end of lever $F = \frac{F_\theta\ b}{a} \left[\frac{1}{e^{\mu\theta} - 1}\right]$

$$= \frac{13000 \times 50}{440} \left[\frac{1}{6.25 - 1}\right] = 281.38\ N$$

Counter clockwise rotation:

Force $$F = \frac{F_\theta\, b}{a}\left[\frac{e^{\mu\theta}}{e^{\mu\theta}-1}\right]$$

$$= \frac{13000 \times 50}{440}\left[\frac{6.25}{6.25-1}\right] = 1758.66\ N$$

Thickness of band h = 0.005 D

$$= 0.005 \times 1000 = 5 \text{ mm}$$

Braking force $$F_\theta = T_1 - T_2 = T_1 - \frac{T_1}{e^{\mu\theta}} = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right)$$

i.e.,

$$13000 = T_1\left[1 - \frac{1}{6.25}\right]$$

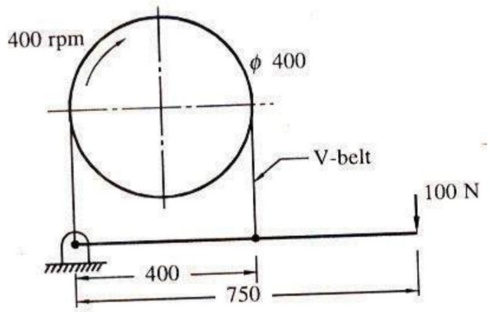
$\therefore$ Tight side tension $T_1 = 15476.2\ N$

Width of band $$w = \frac{T_1}{h\,\sigma_d}$$

$$= \frac{15476.2}{5 \times 50} = 61.9\ mm = 62\ mm$$

Problem:

A band brake shown in figure uses a V-belt. The pitch diameter of the V-grooved pulley is 400 mm. The groove angle is 45° and the coefficient of friction is 0.3. Determine the power rating.



Data: D - 400 mm, 2a = 45°, α =22.5°, μ -0.3, F = 100N, b = 400 mm,
a = 750 mm, θ = 180° = π rad, n = 400 rpm

Solution:

Ratio of tensions for V-belt, $\frac{T_1}{T_2} = e^{\mu\theta / \sin\alpha}$

$$= e^{0.3 \times \pi / \sin 22.5} = 11.738$$

For clockwise rotation,

Force $F = \frac{F_\theta\, b}{a}\left[\frac{1}{e^{\mu\theta} - 1}\right]$

i.e., $100 = \frac{F_\theta \times 400}{750}\left[1 - \frac{1}{11.738 - 1}\right]$

$\therefore$ Braking force $F_\theta = 2013.4\, N$

Torque on the drum $M_1 = F_\theta R = \frac{F_\theta\, D}{2}$

$$= \frac{2013.4 \times 400}{2} = 402680\ N - mm = 402.68\ N - m$$

Power rating

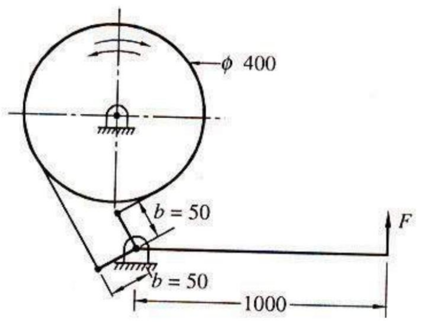
$$N = \frac{M_1\, n}{9550}$$

$$= \frac{402.68 \times 400}{9550} = 16.87\ kW$$

Problem

Figure shows a two way band brake. It is so designed that it can operate equally well in both clockwise and counter clockwise rotation of the brake drum. The diameter of the drum is 400 mm and the coefficient of friction between the band and the drum is 0.3. The angle of contact of band brake is 270° and the torque absorbed in the band brake is 400 N-m. Calculate;

1. Force F required at the end of the lever.
2. Width of the band if the allowable stress in the band is 70 MPa.



Data: D = 400 mm, μ = 0.3, θ = 270°, M_t = 400 N-m = 400 x 10^3 N-mm, σ_d = 70 MPa = 70 N/mm2, b = b_l = b_2 = 50mm, a = 1000mm

Solution:

Braking force $F_\theta = T_1 - T_2 = \frac{2M_t}{D} = \frac{2 \times 400 \times 10^3}{400} = 2000\ N$

Ratio of tensions $\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times 270 \times \pi 180} = 4.1112$

Force at the end of lever $F = \frac{F_\theta b}{a} \left[\frac{e^{\mu\theta} + 1}{e^{\mu\theta} - 1}\right]$

$= \frac{2000 \times 50}{1000} \left[\frac{4.1112 + 1}{4.112 - 1}\right] = 164.28\ N$

Band thickness h = 0.005 D

= 0.005 x 400 = 2 mm

Braking force $F_\theta = T_1 - T_2 = T_1 (1 - \frac{1}{e^{\mu\theta}})$ $\left[\frac{T_1}{T_2} = e^{\mu\theta}\right]$

i.e., $\left[\frac{1}{} \right. = 2000$

$T_1 \left|1 - \frac{}{4.1112}\right|]$

$\therefore$ Tight side tension $T_1 = 2642.84\ N$

Width of band $w = \frac{T_1}{h\,\sigma_d}$

$= \frac{2642.84}{2 \times 70} = 18.88\ mm = 20\ mm$

Introduction

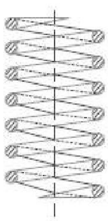
A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows:

To cushion, absorb or control energy due to either shock or vibration as inn car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.

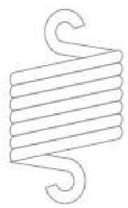
1. To apply forces, as in brakes, clutches and spring loaded valves.
2. To control motion by maintaining contact between two elements as in cams and followers.
3. To measure forces, as in spring balances and engine indicators.
4. To store energy, as in watches, toys, etc.

Types of springs:

1. Helical springs. The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads.



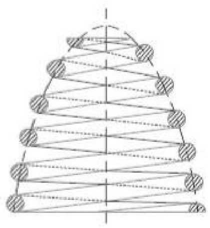
(*a*) Compression helical spring.



(*b*) Tension helical spring.

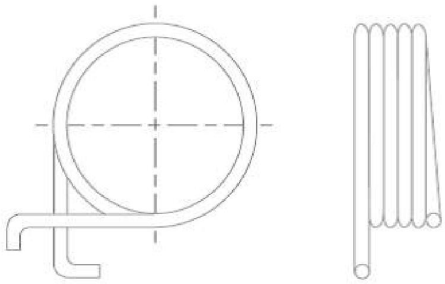
2. Conical and volute springs. The conical and volute springs, as shown in Fig. 23.2, are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired

(*a*) Conical spring.



(*b*) Volute spring.

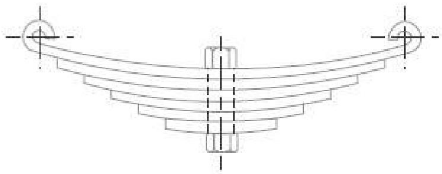
3. Torsion springs. These springs may be of **helical** or **spiral** type as shown in Fig. The **helical type** may be used only inn applications where the load tends to wind up the spring and are used in various electrical mechanisms.



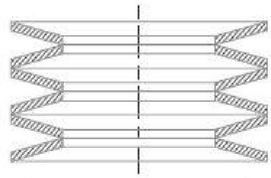
(a) Helical torsion spring.

(b) Spiral torsion spring.

4. Laminated or leaf springs. The laminated or leaf spring (also known as **flat spring** or **carriage spring**) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts.



Laminated or leaf springs.



Disc or bellevile springs.

5. Disc or Belleville springs. These springs consist of a number of conical discs held together against slipping by a central bolt or tube.

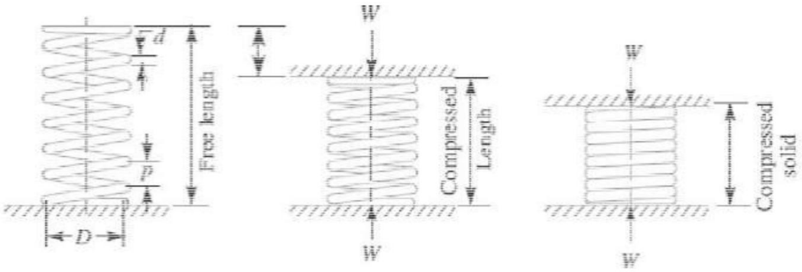
6. Special purpose springs. These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring. These springs are used for special types of application only.

<u>**Terms used in Compression Springs**</u>

1. Solid length. When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be **solid.**

Solid length of the spring, $L_S = n'.d$ where n' = Total number of coils, and d = Diameter of the wire.

2.**Free length.** The free length of a compression spring, as shown in Fig., is the length of the spring in the free or unloaded condition.



Free length of the spring,

L_F = Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance) $n'.d + \delta_{max} + 0.15\, \delta_{max}$

2. Spring index. The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Spring index, C = D / d where D = Mean diameter of the coil, and d = Diameter of the wire.

3. Spring rate. The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically, Spring rate, k = W / δ where W = Load, and δ = Deflection of the spring.

4. Pitch. The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically, Pitch of the coil,

$$p \quad \frac{\text{Free Length}}{n' \quad 1}$$

Stresses in Helical Springs of Circular Wire

Consider a helical compression spring made of circular wire and subjected to an axial load *W*, as shown in Fig.(*a*).

Let D = Mean diameter of thee spring coil,

d = Diameter of the spring wire,

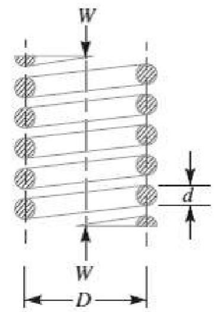
n = Number of active coils,

G = Modulus of rigidity for the spring material,

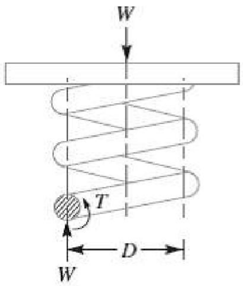
W = Axial load on the spring,

τ = Maximum shear stress induced in the wire,
C = Spring index = D/d, p
= Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load W.



(a) Axially loaded helical spring.



(b) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Now consider a part of the compression spring as shown in Fig. (*b*). The load *W* tends to rotate the wire due to the twisting moment (*T*) set up in the wire. Thus Torsional shear stress is induced in the wire.

A little consideration will show that part of the spring, as shown in Fig.(*b*), is in equilibrium under the action of two forces *W* and the twisting moment *T*. We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\tau_1 = \frac{8W.D}{\pi\, d^3} \qquad \text{...}(i)$$

The Torsional shear stress diagram is shown in Fig. (*a*).

In addition to the Torsional shear stress (τ_1) induced in the wire, the following stresses also act on the wire:

1. Direct shear stress due to the load *W*, and

2. Stress due to curvature of wire.
We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi\, d^3} \pm \frac{4W}{\pi\, d^2}$$

Maximum shear stress induced in the wire,

= Torsional shear stress + Direct shear stress

$$= \frac{8W.D}{\pi\, d^3} + \frac{4W}{\pi\, d^2} = \frac{8W.D}{\pi\, d^3}\left(1 + \frac{d}{2D}\right)$$

Deflection of Helical Springs off Circular Wire

Total active length of the wire,

$$l = \text{Length of one coil} \times \text{No. of active coils} = \pi D \times n$$

Let θ = Angular deflection of the wire when acted upon by the torque T.

$\therefore$ Axial deflection of the spring,

$$\delta = \theta \times D/2 \qquad ...(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T.l}{J.G} \qquad ...\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

$= \frac{\pi}{32} \times d^4$, d being the diameter of spring wire.

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T.l}{J.G} = \frac{\left(W \times \frac{D}{2}\right)\pi\, D.n}{\frac{\pi}{32} \times d^4\, G} = \frac{16 W.D^2.n}{G.d^4} \qquad ...(ii)$$

Substituting this value of θ in equation (i), we have

$$\delta = \frac{16 W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\, W.D^3.n}{G.d^4} = \frac{8\, W.C^3.n}{G.d} \qquad ... (\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\, D^3.n} = \frac{G.d}{8\, C^3.n} = \text{constant}$$

Buckling of Compression Springs

It has been found experimentally that when the free length of the spring (L_F) is more than four times the mean or pitch diameter (D), then the spring behaves like a column and may fail by buckling at a comparatively low load.

$$W_{cr} = k \times K_B \times L_F$$

where k = Spring rate or stiffness of the spring = W/δ,

L_F = Free length of the spring, and

K_B = Buckling factor depending upon the ratio L_F / D.

Surge in springs

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end.

$$f_n = \frac{d}{2\pi D^2 . n}\sqrt{\frac{6\,G.g}{\rho}} \text{ cycles/s}$$

Where d = Diameter of the wire,

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity, and

ρ = Density of the material of the spring.

Problem: A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm^2, find the axial load which the spring can carry and the deflection per active turn.

Solution. Given : $d = 6$ mm ; $D_o = 75$ mm ; $\tau = 350$ MPa $= 350$ N/mm^2 ; $G = 84$ kN/mm^2 $= 84 \times 10^3$ N/mm^2

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

∴ Spring index, $$C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let W = Axial load, and

δ / n = Deflection per active turn.

1. *Neglecting the effect of curvature*

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_S \times \frac{8\,W.D}{\pi\,d^3} = 1.043 \times \frac{8\,W \times 69}{\pi \times 6^3} = 0.848\,W$$

∴ $W = 350 / 0.848 = 412.7$ N **Ans.**

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 412.7\,(69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm Ans.}$$

2. *Considering the effect of curvature*

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8W.C}{\pi\,d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\,W$$

∴ $W = 350 / 0.913 = 383.4$ N **Ans.**

and deflection of the spring,

Problem: Design a spring for a balance to measure 0 to 1000 N over a scale of length 80 mm. The spring is to be enclosed in a casing of 25 mm diameter. The approximate number of turns is 30. The modulus of rigidity is 85 kN/mm^2. Also calculate the maximum shear stress induced.

Solution:

Design of spring

Let D = Mean diameter of the spring coil,

d = Diameter of the spring wire, and

C = Spring index = D/d.

Since the spring is to be enclosed in a casing of 25 mm diameter, therefore the outer diameter of the spring coil ($D_o = D + d$) should be less than 25 mm.

We know that deflection of the spring (δ),

$$80 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 1000 \times C^3 \times 30}{85 \times 10^3 \times d} = \frac{240\,C^3}{85\,d}$$

$$\therefore \quad \frac{C^3}{d} = \frac{80 \times 85}{240} = 28.3$$

Let us assume that d = 4 mm. Therefore

$$C^3 = 28.3\,d = 28.3 \times 4 = 113.2 \quad \text{or} \quad C = 4.84$$

and $D = C.d = 4.84 \times 4 = 19.36$ mm Ans.

We know that outer diameter of the spring coil,

$$D_o = D + d = 19.36 + 4 = 23.36 \text{ mm Ans.}$$

Since the value of D_o = 23.36 mm is less than the casing diameter of 25 mm, therefore the assumed dimension, d = 4 mm is correct.

Maximum shear stress induced

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 4.84 - 1}{4 \times 4.84 - 4} + \frac{0.615}{4.84} = 1.322$$

$\therefore$ Maximum shear stress induced,

$$\tau = K \times \frac{8\,W.C}{\pi\,d^2} = 1.322 \times \frac{8 \times 1000 \times 4.84}{\pi \times 4^2}$$
$$= 1018.2 \text{ N/mm}^2 = 1018.2 \text{ MPa Ans.}$$

Problem: Design a helical compression spring for a maximum load of 1000 N for a deflection of 25 mm using the value of spring index as 5. The maximum permissible shear stress for spring wire is 420 MPa and modulus of rigidity is 84 kN/mm^2.

Take Wahl's factor, K

Solution. Given : $W = 1000$ N ; $\delta = 25$ mm ; $C = D/d = 5$; $\tau = 420$ MPa $= 420$ N/mm^2 ; $G = 84$ kN/mm$^2 = 84 \times 10^3$ N/mm^2

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

and maximum shear stress (τ),

$$420 = K \times \frac{8\,W.C}{\pi\,d^2} = 1.31 \times \frac{8 \times 1000 \times 5}{\pi\,d^2} = \frac{16\,677}{d^2}$$

$$\therefore \quad d^2 = 16\,677 / 420 = 39.7 \quad \text{or} \quad d = 6.3 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 3 having diameter (d) = 6.401 mm.

$\therefore$ Mean diameter of the spring coil,

$$D = C.d = 5\,d = 5 \times 6.401 = 32.005 \text{ mm } \textbf{Ans.} \qquad \ldots (\because C = D/d = 5)$$

and outer diameter of the spring coil,

$$D_o = D + d = 32.005 + 6.401 = 38.406 \text{ mm } \textbf{Ans.}$$

2. *Number of turns of the coils*

Let n = Number of active turns of the coils.

We know that compression of the spring (δ),

$$25 = \frac{8W.C^3.n}{G.d} = \frac{8 \times 1000\,(5)^3\,n}{84 \times 10^3 \times 6.401} = 1.86\,n$$

$$\therefore \quad n = 25 / 1.86 = 13.44 \text{ say } 14 \textbf{ Ans.}$$

For squared and ground ends, the total number of turns,

$$n' = n + 2 = 14 + 2 = 16 \textbf{ Ans.}$$

3. *Free length of the spring*

We know that free length of the spring

$$= n'.d + \delta + 0.15\,\delta = 16 \times 6.401 + 25 + 0.15 \times 25$$

$$= 131.2 \text{ mm } \textbf{Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{131.2}{16 - 1} = 8.75 \text{ mm } \textbf{Ans.}$$

Problem: Design a close coiled helical compression spring for a service load ranging from 2250 N to 2750 N. The axial deflection of the spring for the load range is 6 mm. Assume a spring index of 5. The permissible shear stress intensity is 420 MPa and modulus of rigidity, G = 84 kN/mm^2. Neglect the effect of stress concentration. Draw a fully dimensioned sketch of the spring, showing details of the finish of the end coils.

Solution. Given : $W_1 = 2250$ N ; $W_2 = 2750$ N ; $\delta = 6$ mm ; $C = D/d = 5$; $\tau = 420$ MPa $= 420$ N/mm^2 ; $G = 84$ kN/mm$^2 = 84 \times 10^3$ N/mm^2

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil for a maximum load of $W_2 = 2750$ N, and

d = Diameter of the spring wire.

We know that twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2750 \times \frac{5d}{2} = 6875\ d \qquad \ldots\left(\because C = \frac{D}{d} = 5\right)$$

We also know that twisting moment (T),

$$6875\ d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 420 \times d^3 = 82.48\ d^3$$

$$\therefore \quad d^2 = 6875 / 82.48 = 83.35 \quad \text{or} \quad d = 9.13 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 3/0 having diameter (d) = 9.49 mm.

$\therefore$ Mean diameter of the spring coil,

$$D = 5d = 5 \times 9.49 = 47.45 \text{ mm } \textbf{Ans.}$$

We know that outer diameter of the spring coil,

$$D_o = D + d = 47.45 + 9.49 = 56.94 \text{ mm } \textbf{Ans.}$$

and inner diameter of the spring coil,

$$D_i = D - d = 47.45 - 9.49 = 37.96 \text{ mm } \textbf{Ans.}$$

2. *Number of turns of the spring coil*

Let n = Number of active turns.

It is given that the axial deflection (δ) for the load range from 2250 N to 2750 N (*i.e.* for $W = 500$ N) is 6 mm.

We know that the deflection of the spring (δ),

$$6 = \frac{8\ W.C^3.n}{G.d} = \frac{8 \times 500\ (5)^3\ n}{84 \times 10^3 \times 9.49} = 0.63\ n$$

$$\therefore \quad n = 6 / 0.63 = 9.5 \text{ say } 10 \textbf{ Ans.}$$

For squared and ground ends, the total number of turns,

$$n' = 10 + 2 = 12 \textbf{ Ans.}$$

3. *Free length of the spring*

Since the compression produced under 500 N is 6 mm, therefore maximum compression produced under the maximum load of 2750 N is

$$\delta_{max} = \frac{6}{500} \times 2750 = 33 \text{ mm}$$

We know that free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\ \delta_{max}$$
$$= 12 \times 9.49 + 33 + 0.15 \times 33$$
$$= 151.83 \text{ say } 152 \text{ mm } \textbf{Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{152}{12 - 1} = 13.73 \text{ say } 13.8 \text{ mm } \textbf{Ans.}$$

Energy Stored in Helical Springs of Circular Wire

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let W = Load applied on the spring, and

δ = Deflection produced in the spring due to the load W.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W.\delta$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} \text{ or } W = \frac{\pi\,d^3.\tau}{8K.D}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times \pi\,d^3.\tau}{8K.D} \times \frac{D^3.n}{G.d^4} = \frac{\pi\,\tau.D^2.n}{K.d.G}$$

Substituting the values of W and δ in equation **(*i*)**, we have

$$U = \frac{1}{2} \times \frac{\pi\,d^3.\tau}{8\,K.D} \times \frac{\pi\,\tau.D^2.n}{K.d.G}$$

$$= \frac{\tau^2}{4\,K^2.G}(\pi\,D.n)\left(\frac{\pi}{4} \times d^2\right) = \frac{\tau^2}{4\,K^2.G} \times V$$

Where V = Volume of the spring wire

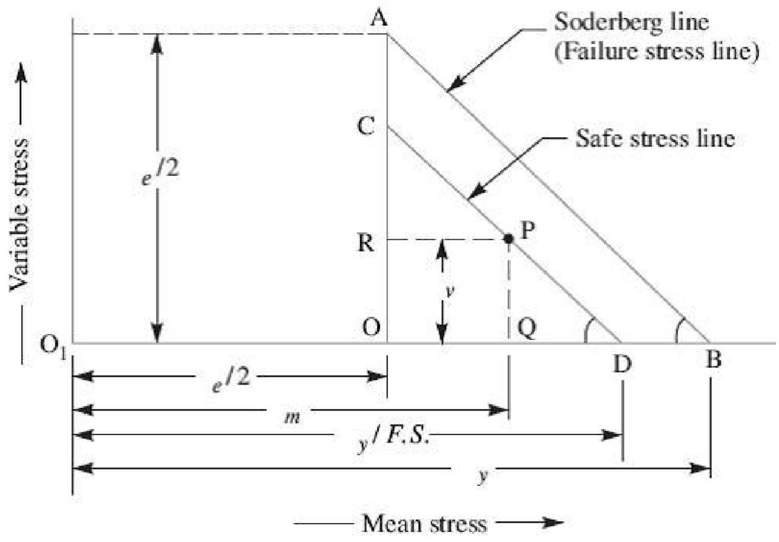
= Length of spring wire × Cross-sectional area of spring wire

Helical Springs Subjected to Fatigue Loading

The helical springs subjected to fatigue loading are designed by using the Soderberg line method. The spring materials are usually tested for Torsional endurance strength under a repeated stress that varies from zero to a maximum. Since the springs are ordinarily loaded in one direction only (the load in springs is never reversed in nature), therefore a modified Soderberg diagram is used for springs, as shown in Fig.

The endurance limit for reversed loading is shown at point A where the mean shear stress is equal to τe / 2 and the variable shear stress is also equal to τe / 2. A line drawn from A to B (the yield point in shear, τy) gives the Soderberg's failure stress line. If a suitable factor of safety (F.S.) is applied to the yield strength (τy), a safe stress line CD may be drawn

parallel to the line AB, as shown in Fig. Consider a design point P on the line CD. Now the value of factor of safety may be obtained as discussed below:



From similar triangles PQD and AOB, we have

$$\frac{PQ}{QD} = \frac{OA}{OB} \quad \text{or} \quad \frac{PQ}{O_1D - O_1Q} = \frac{OA}{O_1B - O_1O}$$

$$\frac{\tau_v}{\frac{\tau_y}{F.S.} - \tau_m} = \frac{\tau_e/2}{\tau_y - \frac{\tau_e}{2}} = \frac{\tau_e}{2\tau_y - \tau_e}$$

or
$$2\,\tau_v \,.\, \tau_y - \tau_v .\, \tau_e = \frac{\tau_e \,.\, \tau_y}{F.S.} - \tau_m . \tau_e$$

$$\therefore \quad \frac{\tau_e \,.\, \tau_y}{F.S.} = 2\,\tau_v . \tau_y - \tau_v . \tau_e + \tau_m . \tau_e$$

Dividing both sides by $\tau_e . \tau_y$ and rearranging, we have

$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\tau_v}{\tau_e}$$

Springs in Series

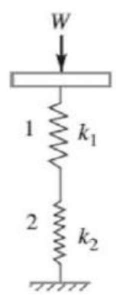
Total deflection of the springs,

$$\delta - \delta_1 + \delta_2$$

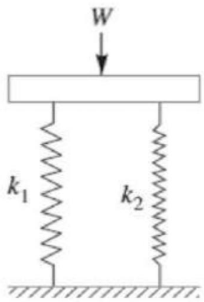
$$\frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2}$$

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

k = Combined stiffness of the springs.



Springs in Parallel



$$W = W_1 + W_2$$
$$\delta k = \delta . k_1 + \delta . k_2$$
$$k = k_1 + k_2$$

k = Combined stiffness of the springs, and

δ = Deflection produced.

Surge in Springs or finding nattural frequency of a helical spring:

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end.

This wave of compression travels along the spring indefinitely. If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur. This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon is called ***surge.***

It has been found that the natural frequency of spring should be at least twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies up to twentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 . n}\sqrt{\frac{6\,G.g}{\rho}} \text{ cycles/s}$$

Where d = Diameter of the wire,

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity, and

ρ = Density of the material of the spring

The surge in springs may be eliminated by using the following methods:

1. By using friction dampers on the centre coils so that the wave propagation dies out.
2. By using springs of high natural frequency.
3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

Energy Stored in Helical Springs of Circular Wire

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let W = Load applied on the spring, and

δ = Deflection produced in the spring due to the load W.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W.\delta$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8\,W.D}{\pi\, d^3} \text{ or } W = \frac{\pi\, d^3.\tau}{8\,K.D}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times \pi\, d^3.\tau}{8\,K.D} \times \frac{D^3.n}{G.d^4} = \frac{\pi\,\tau.D^2.n}{K.d.G}$$

Substituting the values of W and δ in equation ***(i)***, we have

$$U = \frac{1}{2} \times \frac{\pi\, d^3.\tau}{8\,K.D} \times \frac{\pi\,\tau.D^2.n}{K.d.G}$$

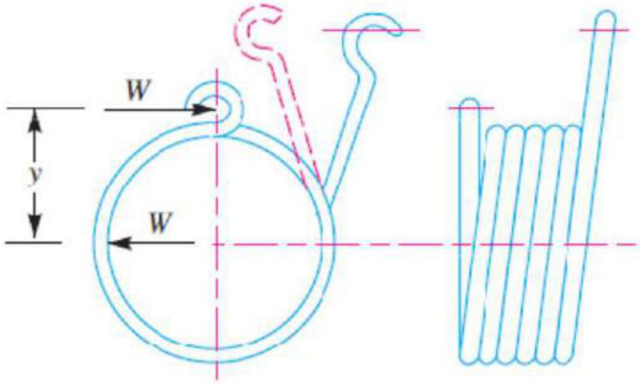
$$= \frac{\tau^2}{4\,K^2.G}(\pi\, D.n)\left(\frac{\pi}{4} \times d^2\right) = \frac{\tau^2}{4\,K^2.G} \times V$$

Where V = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

Helical Torsion Springs

The helical torsion springs as shown in Fig., may be made from round, rectangular or square wire. These are wound in a similar manner as helical compression or tension springs but the ends are shaped to transmit torque. The primary stress in helical torsion springs is bending stress whereas in compression or tension springs, the stresses are Torsional shear stresses. The helical torsion springs are widely used for transmitting small torques as in door hinges, brush holders in electric motors, automobile starters etc. A little consideration will show that the radius of curvature of the coils changes when the twisting moment is applied to the spring. Thus, the wire is under pure bending. According to A.M. Wahl, the bending stress in a helical torsion spring made of round wire is



$$\sigma_b = K \times \frac{32\,M}{\pi\, d^3} = K \times \frac{32\,W.y}{\pi\, d^3}$$

Where K = Wahl's stress factor = $\frac{4C^2 \;\; C \;\; 1}{4C^2 \;\; 4C}$

C = Spring index,

M = Bending moment = W × y,

W = Load acting on the spring,

y = Distance of load from the spring axis, and

d = Diameter of spring wire.

And Total angle of twist or angular deflection,

$$*\theta = \frac{M.l}{E.I} = \frac{M \times \pi D.n}{E \times \pi\, d^4 / 64} = \frac{64\, M.D.n}{E.d^4}$$

Where l = Length of the wire = π.D.n,

D = Diameter of the spring, and

n = Number of turns. And deflection,

$$\delta = \theta \times y = \frac{64\, M.D.n}{E.d^4} \times y$$

When the spring is made of rectangular wire having width b and thickness t, then

$$\sigma_b = K \times \frac{6\,M}{t.b^2} = K \times \frac{6\,W \times y}{t.b^2}$$

Where

$$K = \frac{3C^2 \quad C \quad 0.8}{3C^2 - 3C}$$

Angular deflection,

$$\theta = \frac{12\ \pi M.D.n}{E.t.b^3}; \quad \text{and} \quad \delta = \theta.y = \frac{12\ \pi M.D.n}{E.t.b^3} \times y$$

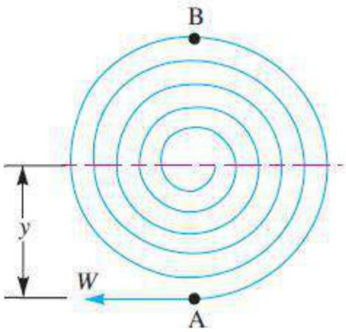
In case the spring is made of square wire with each side equal to b, then substituting t = b, in the above relation, we have

$$\sigma_b = K \times \frac{6\ M}{b^3} = K \times \frac{6W \times y}{b^3}$$

$$\theta = \frac{12\ \pi M.D.n}{E.b^4}; \quad \text{and} \quad \delta = \frac{12\ \pi M.D.n}{E.b^4} \times y$$

Flat Spiral Spring

A flat spring is a long thin strip of elastic material wound like a spiral as shown in Fig. These springs are frequently used in watches and



gramophones etc. When the outer or inner end of this type of spring is wound up in such a way that there is a tendency in the increase of number of spirals of the spring, the strain energy is stored into its spirals. This energy is utilized in any useful way while the spirals open out slowly. Usually the inner end of spring is Clamped to an arbor while the outer end may be Pinned or clamped. Since the radius of curvature of every spiral decreases when the spring is wound up, therefore the material of the spring is in a state of pure bending. Let W = Force applied at the outer end A of the spring,

y = Distance of centre of gravity of the spring from,
l = Length of strip forming the spring,

b = Width of strip,

t = Thickness of strip,

I = Moment of inertia of the spring section = $b.t^3/12$,
and Z = Section modulus of the spring section = $b.t^2/6$

When the end A of the spring is pulled up by a force W, then the bending moment on the spring, at a distance y from the line of action of W is given by

$$M = W \times y$$

The greatest bending moment occurs in the spring at B which is at a maximum distance from the application of W.

Bending moment at B,

$$M_B = M_{max} = W \times 2y = 2W.y = 2M$$

Maximum bending stress induced in the spring material,

$$\sigma_b = \frac{M_{max}}{Z} = \frac{2W \times y}{b.t^2/6} = \frac{12W.y}{b.t^2} = \frac{12M}{b.t^2}$$

Assuming that both ends of the spring are clamped, the angular deflection (in radians) of the spring is given by $\theta = \frac{M.l}{E.I} = \frac{12\, M.l}{E.b.t^3}$

And the deflection,

$$\delta = \theta \times y = \frac{M.l.y}{E.I}$$

$$= \frac{12\, M.l.y}{E.b.t^3} = \frac{12W.y^2.l}{E.b.t^3} = \frac{\sigma_b.y.l}{E.t}$$

The strain energy stored in the spring

$$= \frac{1}{2} M.\theta = \frac{1}{2} M \times \frac{M.l}{E.I} = \frac{1}{2} \times \frac{M^2.l}{E.I}$$

$$= \frac{1}{2} \times \frac{W^2.y^2.l}{E \times bt^3/12} = \frac{6\, W^2.y^2.l}{E.b.t^3}$$

$$= \frac{6\, W^2.y^2.l}{E.b.t^3} \times \frac{24bt}{24bt} = \frac{144\, W^2 y^2}{Eb^2t^4} \times \frac{b\,t\,l}{24}$$

... **(Multiplying the numerator and denominator by 24 bt)**

$$= \frac{(\sigma_b)^2}{24\, E} \times b\,t\,l = \frac{(\sigma_b)^2}{24\, E} \times \text{Volume of the spring}$$

Problem: A helical torsion spring of mean diameter 60 mm is made of a round wire of 6 mm diameter. If a torque of 6 N-m is applied on the spring, find the bending stress induced and the angular deflection of the spring in degrees. The spring index is 10 and modulus of elasticity for the spring material is 200kN/mm^2. The number of effective turns may be taken as 5.5.

Solution. Given : $D = 60$ mm ; $d = 6$ mm ; $M = 6$ N-m $= 6000$ N-mm ; $C = 10$; $E = 200$ kN/mm^2 $= 200 \times 10^3$ N/mm^2 ; $n = 5.5$

Bending stress induced

We know that Wahl's stress factor for a spring made of round wire,

$$K = \frac{4C^2 - C - 1}{4C^2 - 4C} = \frac{4 \times 10^2 - 10 - 1}{4 \times 10^2 - 4 \times 10} = 1.08$$

$\therefore$ Bending stress induced,

Problem: A spiral spring is made of a flat strip 6 mm wide and 0.25 mm thick. The length of the strip is 2.5 meters. Assuming the maximum stress of 800 MPa to occur at the point of greatest bending moment, calculate the bending moment, the number of turns too winds up the spring and the strain energy stored in the spring. Take E = 200 kN/mm^2.

Bending moment in the spring

Let M = Bending moment in the spring.

We know that the maximum bending stress in the spring material (σ_b),

$$800 = \frac{12\,M}{b.t^2} = \frac{12\,M}{8\,(0.25)^2} = 32\,M$$

$\therefore$ $M = 800 / 32 = 25$ N-mm **Ans.**

Number of turns to wind up the spring

We know that the angular deflection of the spring,

$$\theta = \frac{12\,M.l}{E.b.t^3} = \frac{12 \times 25 \times 2500}{200 \times 10^3 \times 6\,(0.25)^3} = 40 \text{ rad}$$

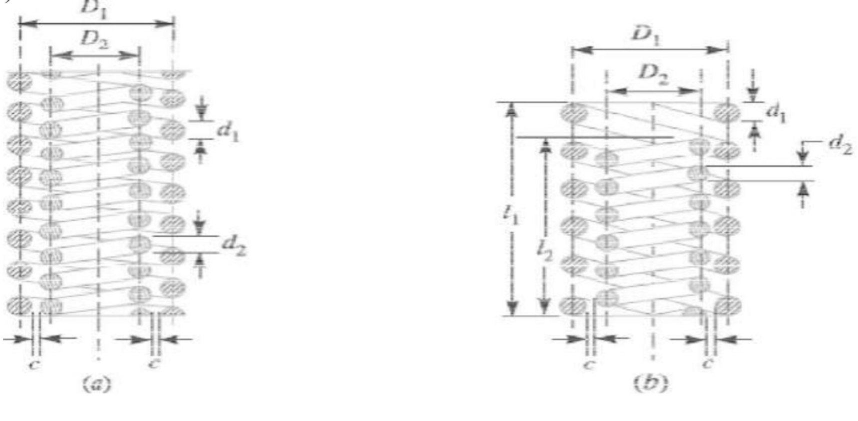
Concentric or Composite Springs or coaxial springs or nested springs

A concentric or composite spring is used for one of the following purposes:

1. To obtain greater spring force within a given space

2. To insure the operation of a mechanism in the event of failure of one of the springs.

The concentric springs for the above two purposes may have two or more springs and have the same free lengths as shown in Fig. (*a*) And are compressed equally. Such springs are used in automobile clutches; valve springs in aircraft, heavy duty diesel engines and rail-road car suspension systems. Sometimes concentric springs are used to obtain a spring force which does not increase in a direct relation to the deflection but increases faster. Such springs are made of different lengths as shown in Fig. (*b*). the shorter spring begins to act only after the longer spring is compressed to a certain amount. These springs are used in governors of variable speed engines to take care of the variable centrifugal force. The adjacent coils of the concentric spring are wound in opposite directions to eliminate any tendency to bind. If the same material is used, the concentric springs are designed for the same stress. In order to get the same stress factor (*K*), it is desirable to have the same spring index (*C*).



Consider a concentric spring as shown in Fig. (*a*).

Let W = Axial load,

W_1 = Load shared by outer spring,

W_2 = Load shared by inner spring,

d_1= Diameter of spring wire of outer spring,

d_2 = Diameter of spring wire of inner spring,

D_1 = Mean diameter of outer spring,

D_2 = Mean diameter of inner spring,

δ_1 = Deflection of outer spring,

δ_2 = Deflection of inner spring,

n_1 = Number of active turns of outer spring, and

n_2 = Number of active turns of inner spring.

Assuming that both the springs are made of same material, then the maximum shear stress induced in both the springs is approximately same, *i.e*.

$$\tau_1 = \tau_2$$

$$\frac{8\,W_1 . D_1 . K_1}{\pi\,(d_1)^3} = \frac{8\,W_2 . D_2 . K_2}{\pi\,(d_2)^3}$$

When stress factor, $K_1 = K_2$, then $\dfrac{W_1 . D_1}{(d_1)^3} = \dfrac{W_2 . D_2}{(d_2)^3}$.

If both the springs are effective throughout their working range, then their free length and deflection are equal, *i.e*.

$$\delta_1 = \delta_2$$

$$\frac{8W_1\,(D_1)^3\,n_1}{(d_1)^4\,G} = \frac{8W_2\,(D_2)^3\,n_2}{(d_2)^4\,G} \quad \text{or} \quad \frac{W_1\,(D_1)^3\,n_1}{(d_1)^4} = \frac{W_2\,(D_2)^3\,n_2}{(d_2)^4} \qquad ...(ii)$$

When both the springs are compressed until the adjacent coils meet, then the solid length of both the springs is equal, *i.e*s $n_1.d_1 = n_2.d_2$

The equation ***(ii)*** may be written as

$$\frac{W_1\,(D_1)^3}{(d_1)^5} = \frac{W_2\,(D_2)^3}{(d_2)^5} \qquad ...(iii)$$

Now dividing equation ***(iii)*** by equation ***(i)***, we have

$$\frac{(D_1)^2}{(d_1)^2} = \frac{(D_2)^2}{(d_2)^2} \quad \text{or} \quad \frac{D_1}{d_1} = \frac{D_2}{d_2} = C, \text{ the spring index} \qquad ...(iv)$$

i.e. the springs should be designed in such a way that the spring index for both the springs is same. From equations ***(i)*** and ***(iv)***, we have

$$\frac{W_1}{(d_1)^2} = \frac{W_2}{(d_2)^2} \quad \text{or} \quad \frac{W_1}{W_2} = \frac{(d_1)^2}{(d_2)^2} \qquad ..(v)$$

From Fig. 23.22 (*a*), we find that the radial clearance between the two springs,

$$*c = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right)$$

Usually, the radial clearance between the two springs is taken as

$$\frac{d_1 - d_2}{2}$$

$$\therefore \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) = \frac{d_1 - d_2}{2}$$

or $$\frac{D_1 - D_2}{2} = d_1 \qquad \text{----- (vi)}$$

From equation ***(iv)***, we find that $D_1 = C.d_1$, and $D_2 = C.d_2$

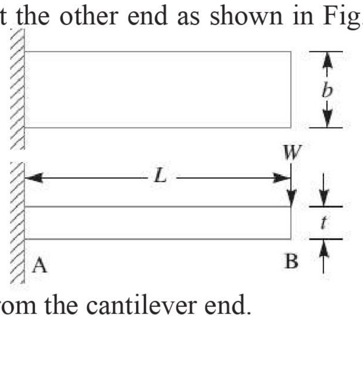
Substituting the values of D_1 and D_2 in equation ***(vi)***, we have

$$\frac{C.d_1 - C.d_2}{2} = d_1 \quad \text{or} \quad C.d_1 - 2\,d_1 = C.d_2$$

$$d_1\,(C-2) = C.d_2 \quad \text{or} \quad \frac{d_1}{d_2} = \frac{C}{C-2}$$

Leaf Springs

Leaf springs (also known as **flat springs**) are made out of flat plates. The advantage of leaf spring over helical spring is that the ends of the spring may be guided along a definite path as it deflects to act as a structural member in addition to energy absorbing device. Thus the leaf springs may carry lateral loads, brake torque, driving torque etc., in addition to shocks. Consider a single plate fixed at one end and loaded at the other end as shown in Fig. This plate may be used as a flat spring.



Let t = Thickness of plate,

b = Width of plate, and

L = Length of plate or distance of the load W from the cantilever end.

We know that the maximum bending moment at the cantilever end A,

$$M = W.L$$

And section modulus,

$$Z = \frac{I}{y} = \frac{b\,t^3/12}{t/2} = \frac{1}{6} \times b.t^2$$

Bending stress in such a spring,

$$\sigma = \frac{M}{Z} = \frac{W.L}{\frac{1}{6} \times b.t^2} = \frac{6\,W.L}{b.t^2}$$

We know that the maximum deflection for a cantilever with concentrated load at the free end is given by

$$\delta = \frac{W.L^3}{3E.I} = \frac{W.L^3}{3E \times b.t^3/12} = \frac{4\,W.L^3}{E.b.t^3}$$

$$= \frac{2\,\sigma.L^2}{3\,E.t}$$

If the spring is not of cantilever type but it is like a simply supported beam, with length $2L$ and load $2W$ in the centre, as shown in Fig. then Maximum bending moment in the centre,

$M=W.L$

Section modulus, $Z = b.t2 / 6$

Bending stress,

$$\sigma = \frac{M}{Z} = \frac{W.L}{b.t^2/6}$$

$$= \frac{6\,W.L}{b.t^2}$$

We know that maximum deflection of a simply supported beam loaded in the centre is given by

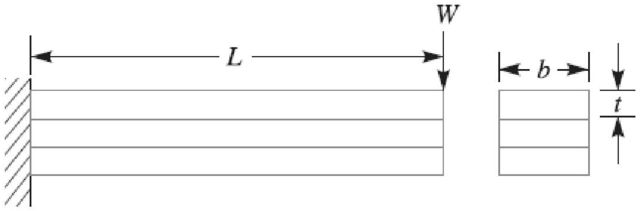
$$\delta = \frac{W_1\,(L_1)^3}{48\,E.I} = \frac{(2W)\,(2L)^3}{48\,E.I} = \frac{W.L^3}{3\,E.I}$$

From above we see that a spring such as automobile spring (semi-elliptical spring) with length $2L$ and loaded in the centre by a load $2W$, may be treated as a double cantilever. If the plate of cantilever is cut into a series of n strips of width b and these are placed as shown in Fig., then equations ***(i)*** and ***(ii)*** may be written as

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad ...(iii)$$

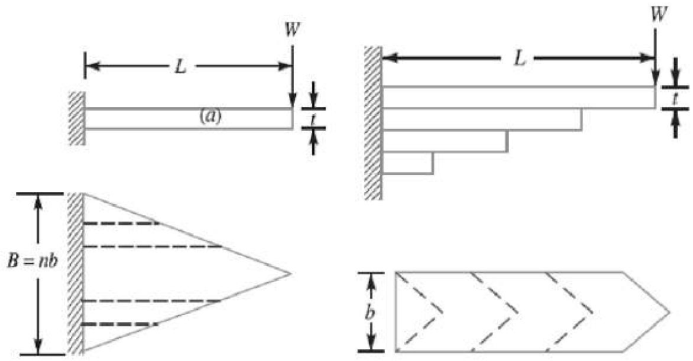
$$\delta = \frac{4\,W.L^3}{n.E.b.t^3} = \frac{2\,\sigma.L^2}{3\,E.t} \qquad ...(iv)$$

And



The above relations give the stress and deflection of a leaf spring of uniform cross section.

The stress at such a spring is maximum at the support.



If a triangular plate is used as shown in Fig., the stress will be uniform throughout. If this triangular plate is cut into strips of uniform width and placed one below the other, as shown in Fig. to form a graduated or laminated leaf spring, then

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad ...(v)$$

$$\delta = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{\sigma.L^2}{E.t} \qquad ...(vi)$$

Where n = Number of graduated leaves.

A little consideration will show that by the above arrangement, the spring becomes compact so that the space occupied by the spring is considerably reduced.

When bending stress alone is considered, the graduated leaves may have zero width at the loaded end. But sufficient metal must be provided to support the shear. Therefore, it becomes necessary to have one or more leaves of uniform cross-section extending clear to the end. We see from equations ***(iv)*** and ***(vi)*** that for the same deflection, the stress in the uniform cross-section leaves (*i.e.* full length leaves) is 50% greater than in the graduated leaves, assuming that each spring element deflects according to its own elastic curve. If the suffixes

F and G are used to indicate the full length (or uniform cross section) and graduated leaves, then

$$\sigma_F = \frac{3}{2}\sigma_G$$

$$\frac{6W_F.L}{n_F.b.t^2} = \frac{3}{2}\left[\frac{6\,W_G.L}{n_G.b.t^2}\right] \quad \text{or} \quad \frac{W_F}{n_F} = \frac{3}{2}\times\frac{W_G}{n_G}$$

$$\frac{W_F}{W_G} = \frac{3\,n_F}{2\,n_G} \qquad \text{...}(vii)$$

Adding 1 to both sides, we have

$$\frac{W_F}{W_G} + 1 = \frac{3\,n_F}{2\,n_G} + 1 \quad \text{or} \quad \frac{W_F + W_G}{W_G} = \frac{3\,n_F + 2\,n_G}{2\,n_G}$$

$$W_G = \left(\frac{2\,n_G}{3\,n_F + 2\,n_G}\right)(W_F + W_G) = \left(\frac{2\,n_G}{3\,n_F + 2\,n_G}\right)W \qquad \text{...}(viii)$$

where W = Total load on the spring = $W_G + W_F$

W_G = Load taken up by graduated leaves, and

W_F = Load taken up by full length leaves.

From equation **(*vii*)**, we may write

$$\frac{W_G}{W_F} = \frac{2\,n_G}{3\,n_F}$$

or

$$\frac{W_G}{W_F} + 1 = \frac{2\,n_G}{3\,n_F} + 1$$

$$\frac{W_G + W_F}{W_F} = \frac{2\,n_G + 3\,n_F}{3\,n_F}$$

$$\therefore \quad W_F = \left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right)(W_G + W_F) = \left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right)W \qquad \text{...}(ix)$$

Bending stress for full length leaves,

$$\sigma_F = \frac{6\,W_F.L}{n_F.b\,t^2} = \frac{6\,L}{n_F.b.t^2}\left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right)W = \frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}$$

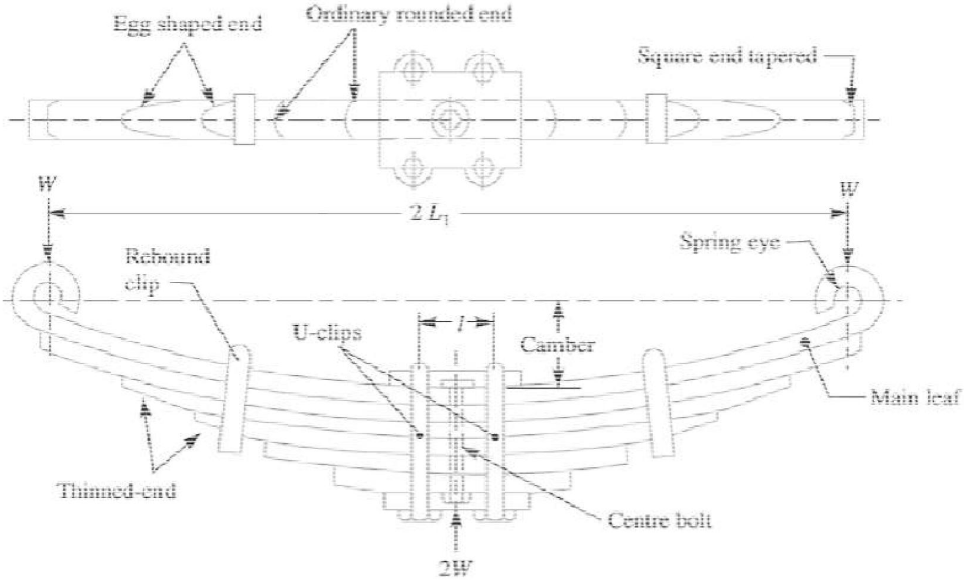
Since

$$\sigma_F = \frac{3}{2}\sigma_G, \text{ therefore}$$

$$\sigma_G = \frac{2}{3}\sigma_F = \frac{2}{3}\times\frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)} = \frac{12\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}$$

The deflection in full length and graduated leaves is given by equation **(*iv*)**, *i.e*.

$$\delta = \frac{2\,\sigma_F \times L^2}{3\,E.t} = \frac{2\,L^2}{3\,E.t}\left[\frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}\right] = \frac{12\,W.L^3}{E.b.t^3\,(2\,n_G + 3\,n_F)}$$

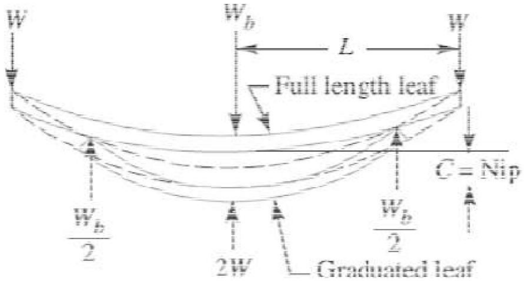


Equalized Stress in Spring Leaves (Nipping)

We have already discussed that the stress in the full length leaves is 50% greater than the stress in the graduated leaves. In order to utilize the material to the best advantage, all the leaves should be equally stressed. This condition may be obtained in the following two ways:

1. By making the full length leaves of smaller thickness than the graduated leaves. In this way, the full length leaves will induce smaller bending stress due to small distance from the neutral axis to the edge of the leaf.

2. By giving a greater radius of curvature to the full length leaves than graduated leaves, as shown in Fig. before the leaves are assembled to form a spring. By doing so, a gap or clearance will be left between the leaves. This initial gap, as shown by C in Fig, is called ***nip.***



Consider that under maximum load conditions, the stress in all the leaves is equal. Then at maximum load, the total deflection of the graduated leaves will exceed the deflection of the full length leaves by an amount equal to the initial gap C. In other words,

$$\delta_G = \delta_F + C$$

$$C = \delta_G - \delta_F = \frac{6\,W_G\,.\,L^3}{n_G\,E.b.t^3} - \frac{4\,W_F.L^3}{n_F.E.b.t^3} \qquad ...(i)$$

Since the stresses are equal, therefore

$$\sigma_G = \sigma_F$$

$$\frac{6\,W_G.L}{n_G\,b.t^2} = \frac{6\,W_F\,L}{n_F\,b.t^2} \quad \text{or} \quad \frac{W_G}{n_G} - \frac{W_F}{n_F}$$

$$\therefore \qquad W_G = \frac{n_G}{n_F} \times W_F = \frac{n_G}{n} \times W$$

$$W_F = \frac{n_F}{n_G} \times W_G = \frac{n_F}{n} \times W$$

Substituting the values of WG and WF in equation **(*i*)**, we have

$$C = \frac{6W.L^3}{n.E.b.t^3} - \frac{4W.L^3}{n.E.b.t^3} = \frac{2W.L^3}{n.E.b.t^3} \qquad ...(ii)$$

The load on the clip bolts (W_b) required to close the gap is determined by the fact that the gap is equal to the initial deflections of full length and graduated leaves.

$$\therefore \qquad C = \delta_F + \delta_G$$

$$\frac{2W.L^3}{n.E.b.t^3} = \frac{4L^3}{n_F.E.b.t^3} \times \frac{W_b}{2} + \frac{6\,L^3}{n_G.E.b.t^3} \times \frac{W_b}{2}$$

Or

$$\frac{W}{n} = \frac{W_b}{n_F} + \frac{3\,W_b}{2\,n_G} = \frac{2\,n_G.W_b + 3\,n_F.W_b}{2\,n_F.n_G} = \frac{W_b\,(2\,n_G + 3\,n_F)}{2\,n_F.n_G}$$

$$W_b = \frac{2\,n_F.n_G.W}{n\,(2\,n_G + 3\,n_F)} \qquad ...(iii)$$

The final stress in spring leaves will be the stress in the full length leaves due to the applied load ***minus*** the initial stress.

Final stress,

$$\sigma = \frac{6\,W_F.L}{n_F.b.t^2} - \frac{6\,L}{n_F.b.t^2} \times \frac{W_b}{2} = \frac{6\,L}{n_F.b.t^2}\left(W_F - \frac{W_b}{2}\right)$$

$$= \frac{6\,L}{n_F.b.t^2}\left[\frac{3n_F}{2n_G + 3n_F} \times W - \frac{n_F.n_G.W}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6W.L}{b.t^2}\left[\frac{3}{2n_G + 3n_F} - \frac{n_G}{n(2n_G + 3n_F)}\right]$$

$$= \frac{6\,W.L}{b.t^2}\left[\frac{3n - n_G}{n(2n_G + 3n_F)}\right]$$

$$= \frac{6\,W.L}{b.t^2}\left[\frac{3(n_F + n_G) - n_G}{n(2n_G + 3n_F)}\right] = \frac{6\,W.L}{n.b.t^2} \qquad ...(iv)$$

Length of Leaf Spring Leaves

The length of the leaf spring leaves may be obtained as discussed below:

Let $2L_1$ = Length of span or overall length of the spring,

l = Width of band or distance between centres of *U*-bolts. It is the in effective length of the spring,

n_F = Number of full length leaves,

n_G = Number of graduated leaves, and

n = Total number of leaves = $n_F + n_G$.

We have already discussed that the effective length of the spring,

$2L = 2L_1 - l$ ... (When band is used)

Problem: Design a leaf spring for the following specifications:

Total load = 140 kN ; Number of springs supporting the load = 4 ; Maximum number of leaves = 10; Span of the spring = 1000 mm ; Permissible deflection = 80 mm. Take Young's modulus, E = 200 kN/mm2 and allowable stress in spring material as 600 MPa.

Solution. Given : Total load = 140 kN ; No. of springs = 4; n = 10 ; $2L$ = 1000 mm or L = 500 mm ; δ = 80 mm ; E = 200 kN/mm^2 = 200 × 10^3 N/mm^2 ; σ = 600 MPa = 600 N/mm^2

We know that load on each spring,

$$2W = \frac{\text{Total load}}{\text{No. of springs}} = \frac{140}{4} = 35 \text{ kN}$$

$\therefore$ $W = 35 / 2 = 17.5 \text{ kN} = 17\,500 \text{ N}$

Let t = Thickness of the leaves, and

b = Width of the leaves.

We know that bending stress (σ),

$$600 = \frac{6\,W.L}{n.b.t^2} = \frac{6 \times 17\,500 \times 500}{n.b.t^2} = \frac{52.5 \times 10^6}{n.b.t^2}$$

$\therefore$ $n.b.t^2 = 52.5 \times 10^6 / 600 = 87.5 \times 10^3$ ...(*i*)

and deflection of the spring (δ),

$$80 = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{6 \times 17\,500\,(500)^3}{n \times 200 \times 10^3 \times b \times t^3} = \frac{65.6 \times 10^6}{n.b.t^3}$$

$\therefore$ $n.b.t^3 = 65.6 \times 10^6 / 80 = 0.82 \times 10^6$ ...(*ii*)

Dividing equation (*ii*) by equation (*i*), we have

$$\frac{n.b.t^3}{n.b.t^2} = \frac{0.82 \times 10^6}{87.5 \times 10^3} \quad \text{or} \quad t = 9.37 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

Now from equation (*i*), we have

$$b = \frac{87.5 \times 10^3}{n.t^2} = \frac{87.5 \times 10^3}{10\,(10)^2} = 87.5 \text{ mm}$$

and from equation (*ii*), we have

$$b = \frac{0.82 \times 10^6}{n.t^3} = \frac{0.82 \times 10^6}{10\,(10)^3} = 82 \text{ mm}$$

Taking larger of the two values, we have width of leaves,

b = 87.5 say 90 mm **Ans.**

Problem:

A truck spring has 12 number of leaves, two of which are full length leaves.

The spring supports are 1.05 m apart and the central band is 85 mm wide. The central load is to be 5.4 kN with a permissible stress of 280 MPa. Determine the thickness andd width of the steel spring leaves. The ratio of the total depth to the width of the spring is 3. Also determine the deflection of the spring.

Solution. Given : $n = 12$; $n_F = 2$; $2L_1 = 1.05$ m $= 1050$ mm ; $l = 85$ mm ; $2W = 5.4$ kN $= 5400$ N or $W = 2700$ N ; σ_F 280 MPa = 280 N/mm^2

Thickness and width of the spring leaves

Let t = Thickness of the leaves, and

b = Width of the leaves.

Since it is given that the ratio of the total depth of the spring ($n \times t$) and width of the spring (b) is 3, therefore

$$\frac{n \times t}{b} = 3 \quad \text{or} \quad b = n \times t / 3 = 12 \times t / 3 = 4\,t$$

We know that the effective length of the spring,

$$2L = 2L_1 - l = 1050 - 85 = 965 \text{ mm}$$

$$\therefore \quad L = 965 / 2 = 482.5 \text{ mm}$$

and number of graduated leaves,

$$n_G = n - n_F = 12 - 2 = 10$$

Assuming that the leaves are not initially stressed, therefore maximum stress or bending stress for full length leaves (σ_F),

$$280 = \frac{18\,W.L}{b.t^2\,(2n_G + 3n_F)} = \frac{18 \times 2700 \times 482.5}{4\,t \times t^2\,(2 \times 10 + 3 \times 2)} = \frac{225\,476}{t^3}$$

$$\therefore \quad t^3 = 225\,476 / 280 = 805.3 \quad \text{or} \quad t = 9.3 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

and $\quad b = 4\,t = 4 \times 10 = 40$ mm **Ans.**

Deflection of the spring

We know that deflection of the spring,

$$\delta = \frac{12\,W.L^3}{E.b.t^3\,(2n_G + 3n_F)}$$

$$= \frac{12 \times 2700 \times (482.5)^3}{210 \times 10^3 \times 40 \times 10^3\,(2 \times 10 + 3 \times 2)} \text{ mm}$$

$= 16.7$ mm **Ans.** ... (Taking $E = 210 \times 10^3$ N/mm^2)