AC & DC Circuits

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Current:- The flow of electrons in conductive materials.

Current is defined as rate of flow of electrons in a conductive (or) semi conductive materials.

Current is denoted by I where $I = \frac{\text{charge}}{\text{Time}} = \frac{Q}{t}$

The units for current is Amperes (A)

Voltage (V) or Potential difference (Pd) :-

The potential difference (or) Voltage are defined as the difference b/w the electric potentials at any two given points in a circuit.

The Units for Voltage (or) potential differce is Volts (V).

Electric potential:-

The ability of a charged particle to do work is called it's electric potential.

$$\text{Electric potential}(V) = \frac{\text{Workdone}}{\text{Charge}} = \frac{W}{Q}$$

Units is 'Volts'

Electro motive Force (EMF):-

EMF is the force which causes an electric current to flow in an electrical circuit. Units for EMF is 'Volts'.

Power:-

Power is defined as rate of change of energy

$$P = \frac{\text{Energy}}{\text{Time}} = \frac{W}{t}$$

(or)

Power is the product of Voltage and Current

$P = V \times I$, Units is Watt's (W)

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$P = VI$

$P = (IR)I$ (Ohms law $V = IR$, $I = V/R$)

$= I^2 R$

$P = V(V/R) = V^2/R$

Energy:-

The capacity to do work is called energy.

Electrical energy is the electrical work when there is transfer of charge.

It is also called as electrical energy and electrical work done.

Electrical energy (W) = Power x Time

$W = P \times T$

$W = VIt$

Charge:-

The current can be measured by measu[ring] how many electrons are passing through material per second.

This can be expressed in terms of charge carried by those electrons in the material per second.

Charge $(Q) = I \times t$

Units for charge is Coloumbs (C)

Source:-

Source is one which produces energy.

Ex:- Generator, Battery

Load:-

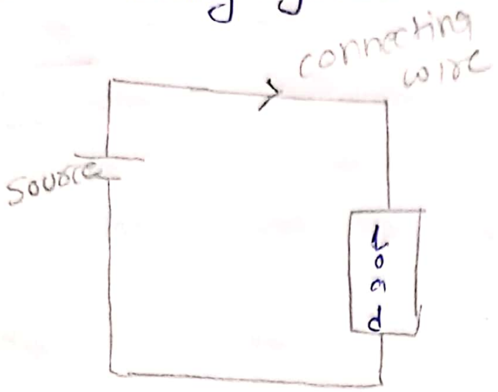
Load is that which consumes energy.

Ex:- Fans, bulbs, motors, TV's.

Electrical circuit:

Any electrical circuit consists of three basic parts:-

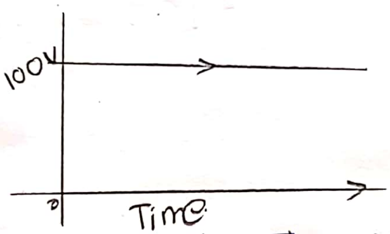
① Energy source such as battery (or) generator
② Load (or) sink such as bulbs or motors etc
③ Connecting system such as wires etc.

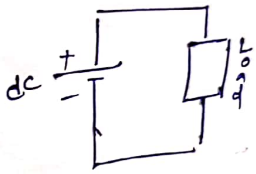


Types of supply:-

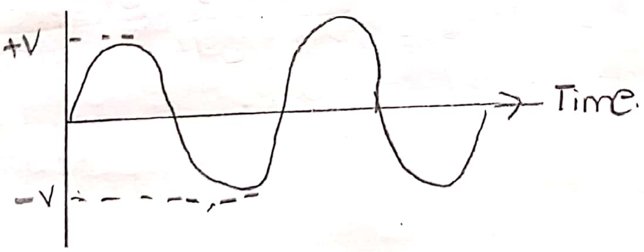
There are two types of supply
They are:- ① DC
② AC

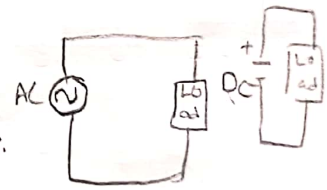
* **Direct current:-** The voltage (or) current output of a DC source is constant with respect to time



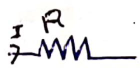


* **Alternating current:-** The voltage or current is called alternating current when it periodically changes it's direction and magnitude with respect to time





Basic electrical circuit elements:- (RLC) :

1) * **Resistance:-**
Electrical resistance is a property of material by virtue of which it opposes the flow of current through it.
The symbol for resistance is 
The unit for resistor is Ω (ohm's)

A/c to ohm's law voltage "current" relation is $V \propto I$
$V = IR$

where R = constant, R = resistance.

$$R = V/I$$

Power obseaved by the resistor

$$P = V \times I$$
$$P = IR \times I$$
$$P = I^2 R$$
$$P = VI$$
$$= V(V/R)$$
$$P = \frac{V^2}{R}.$$

Energy lossed in a resistance is given by

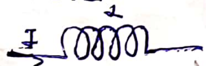
$$W = \int_0^t P\,dt.$$
$$= P \int_0^t dt.$$
$$\Rightarrow W = P.t$$
$$= VI.t$$
$$= I^2 Rt$$
$$= \frac{V^2}{R} t$$

2) Inductance (L) :

Inductance is the property of material by virtue of which it opposes the change in magnitude (or) direction of AC current passing through it.

(or)

Inductance is the element in which energy is stored in the form of magnetic field.

The symbol for inductance is 

It is denoted by letter "L" where L = constant Inductance

For inductance voltage is proportional to the rate of change of current $(V \propto \frac{dI}{dt}) \Rightarrow \boxed{V = L\frac{dI}{dt}}$

Unit : Henry (H).

Power taken by the Inductor.

$$P = VI$$

$$P = \left(L \cdot \frac{dI}{dt}\right) I$$

$$\boxed{P = LI \frac{dI}{dt}}$$

$$W = \int_0^t P\, dt$$

$$= \int_0^t VI\, dt.$$

$$= \int_0^t \left(L \frac{dI}{dt}\right) I\, dt$$

$$= \int_0^t L\, dI \cdot I$$

$$W = \int_0^t LI\, dI.$$

$$W = L \int_0^t I\, dI = L \frac{I^2}{2}$$

$$W = \frac{1}{2} LI^2$$

3) Capacitance (C) :-

Capacitance is the property of element by virtue of which it opposes the sudden change in voltages accross it (or)

Capacitance is the element which stores energy in the form of electric feild.

The symbol for capacitor is $\xrightarrow{I} \vert\vert^{C}$ — c : constant capacitance

Unit :- Farad (F)

A/c to coloumb's law $Q \propto V$

$$Q = CV$$

$$\boxed{C = Q/V}$$

we know

$q = it$

$i = \frac{dq}{dt}$

Power taken by the capacitance.

$$P = Vi \Rightarrow V\left(C\frac{dv}{dt}\right) \Rightarrow CV\frac{dv}{dt}$$

$$W = \int_0^t P\,dt$$

$$W = \int_0^t CV \cdot \frac{dV}{dt} \cdot dt$$

$$W = C \int_0^t V \cdot dV$$

$$W = C\,\frac{V^2}{2}$$

$$W = \frac{1}{2} CV^2$$

Ohm's law:-

In 19th century a german philosopher Gorge Sim Ohm stated ohm's law.

Statement:-

At constant temperature conditions, the current through a conductor is directly propotional to voltage applied across it

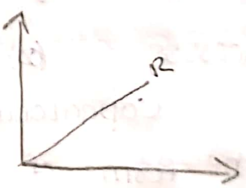
$$I \propto V$$

$$I = \frac{V}{R}$$

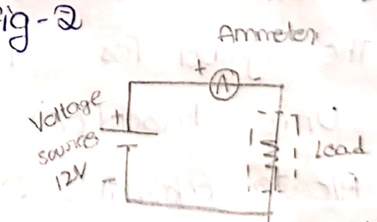
where R is called constant & it is called resistance.

$$V = IR$$

Explanation:-



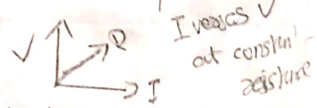
The ohm's law can be demonstrated by connecting a voltage source 'V', and Ammeter 'A' and a load as shown in fig-2



Case:-1

If the resistance remains constant R= 3Ω

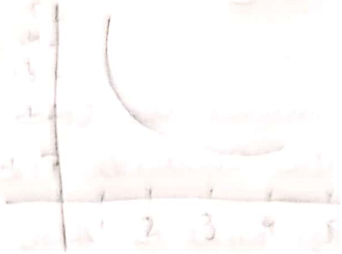
A/c to ohms law



The current where is directly with the supply voltage. The plot (or) Graph drawn between V and I is a straigh line as shown in fig-3.

case 2:-

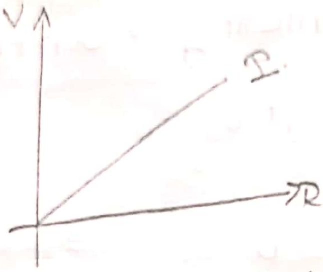
If the supply voltage V is constant say V = 12 Volts then the current is inversely proportional to resistance 'R'. The graph drawn b/w current and resistance is a hyperbola curve as shown in fig 4



fig(4) I Versus R at constant V

case 3:-

Say I = 4A then the voltage is proportional to resistance. The graph drawn b/w voltage & resistance is a straigh line as shown in fig 5.



Fig(5) V versus R at constant I.

* Ohm's law ⇒ Draw back's :

* It is not applicable to the non-linear devices such as diodes, Zener diodes. Voltage Regulators etc.
* It does not hold good for non-metallic conductors such as silicon carbide
* Electrolytes where enormous gases are produced on either electrode
* Metals which gets heated up due to the flow of current
* Arc lamps

→ Kirchoff's laws:-

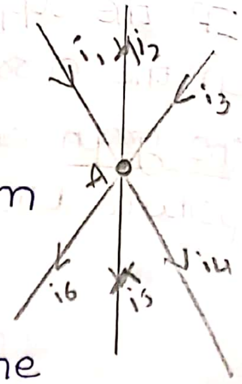
There are two types of Kirchoff's laws.

① Kirchoff Current law

② Kirchoff's Voltage law

Kirchoff's first law :- (or)

① KCL (Kirchoff's current law) :



KCL states that sum of current entering into a node is equal to sum of current leaving that node.

(or)

KCL can also be stated as the algebraic sum of all the currents meeting at any Junction (or) node of any kind is zero

Explanation :-

Consider a simple system in which six currents are meeting at junction A (or) node A. In this current entering at node 'A' are I_1, I_3, I_5 and current leaving at node 'A' are I_2, I_4, I_6

A/c to KCL

Sum of Current Entering = Sum of Current's leaving.

$$i_1 + i_3 + i_5 = i_2 + i_4 + i_6.$$

(or)

$$i_1 - i_2 + i_3 - i_4 + i_5 - i_6 = 0$$

$$\Sigma i = 0.$$

Kirchoff's second law (or)

② KVL (Kirchoff's Voltage law) :

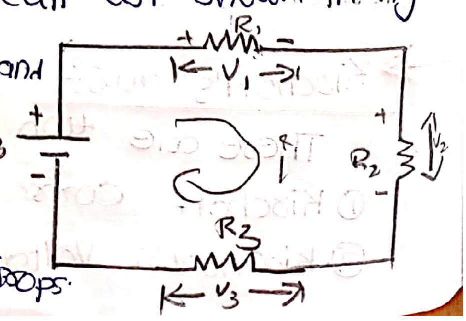
KVL states that the algebraic sum of all branch voltages around any closed path in a circuit is equal to zero

(or)

KVL also states that in any closed loop the algebraic sum of voltage raises = voltage drops.

Explanation :

Consider a simple circuit as shown in fig 2.



Here V_s is supply voltage and V_1, V_2 & V_3 are voltage drops

A/c to KVL Sum of Voltage raise = Sum of Voltage drops.

$$V_S = V_1 + V_2 + V_3$$

(or)

$$V_S - V_1 - V_2 - V_3 = 0$$

$$\Sigma V = 0$$

Types of connections of networks

① Series network

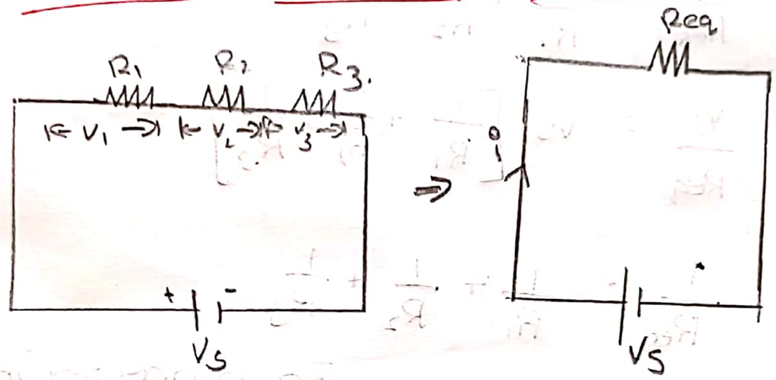
② Parallel network

③ Series - parallel network

④ Complex networks

Resistance network:-

1) * Resistance connected in series:-



A/c to KVL

$$V_S = V_1 + V_2 + V_3$$

$$iR_{eq} = iR_1 + iR_2 + iR_3$$

$$iR_{eq} = i(R_1 + R_2 + R_3)$$

$$\rightarrow R_{eq} = R_1 + R_2 + R_3.$$

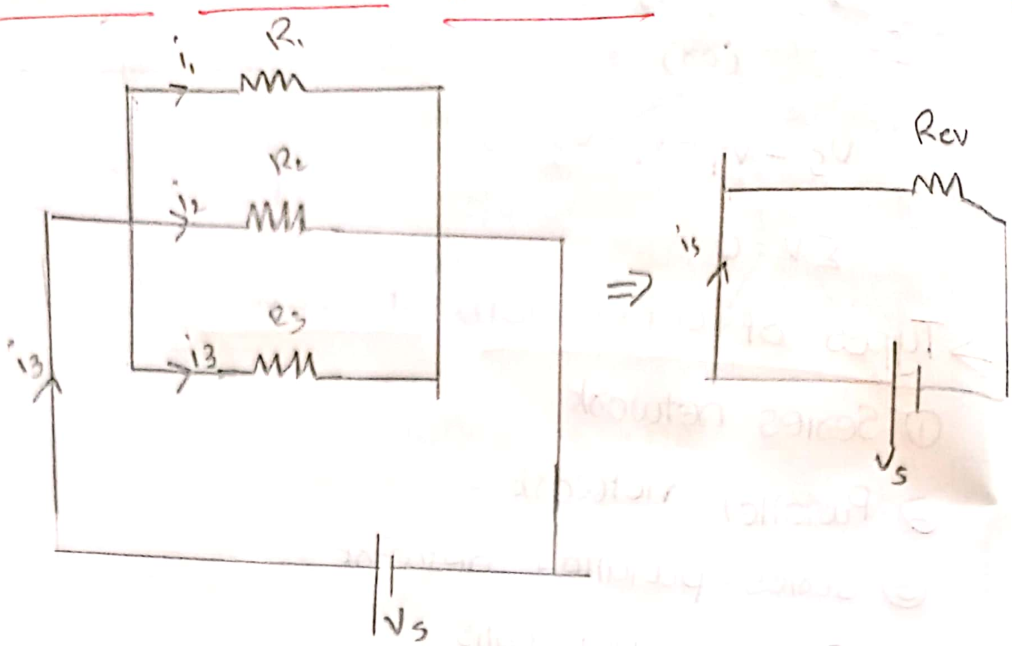
If 'n' no. of resistan $R_1, R_2, R_3 \ldots R_n$ are connected in series then the equalent resistance is

$$R_{eq} = R_1 + R_2 + R_3 + \ldots + R_n$$

Note:-

The current flowing through all the resistors is connected in series is same where, as the voltage differs.

2) Resistance connected in Parallel :-



A/c to KCL $i_s = i_1 + i_2 + i_3$

$$\frac{V_s}{R_{eq}} = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3}$$

$$\frac{V_s}{R_{eq}} = V_s \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

If n no. of resistances are connected in parallel, then

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

Note:-

If two resistances are connected in parallel then

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R_{ev} = \frac{R_1 R_2}{R_1 + R_2}$$

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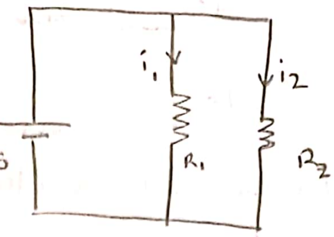
Current Division Rule:-

Current through Particular Branch = Total Current × $\frac{\text{opposite Resistance}}{\text{Total resistance of the loops}}$

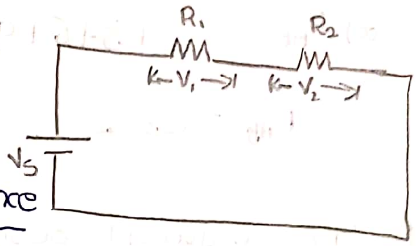


ex $i_1 = I \times \frac{R_2}{R_1 + R_2}$

$i_2 = I \times \frac{R_1}{R_1 + R_2}$

Voltage Division Rule:-

Voltage across particular element = Total voltage × $\frac{\text{Same element resistance}}{\text{Total resistance}}$



$V_1 = V_S \times \frac{R_1}{R_1 + R_2}$

$V_2 = V_S \times \frac{R_2}{R_1 + R_2}$

Factors affecting resistance:-

The factor which affect the resistances are the length of material, area of cross-section of materials resistance (r) is directly propotional to length and inversly propotional to area of crosssection of material. $R \alpha \frac{l}{A}$

$R = \rho \frac{l}{A}$

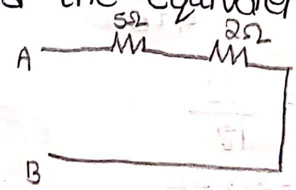
where ρ = Specific resistance
Resistivity, ohm-m (Ω-m)

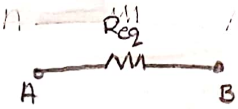
l = length of the material.

A = Area of cross section, m^2

Problems:

1) Find the equivalent resistance b/w A and B



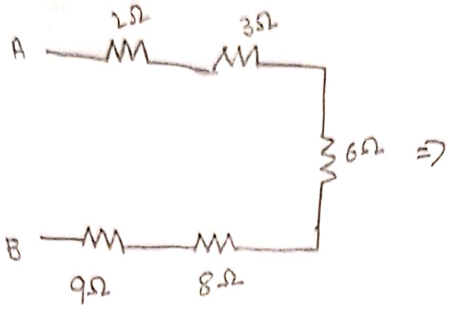


$R_{eq} = R_1 + R_2$

$5+2$

$R_{eq} = 7\Omega$

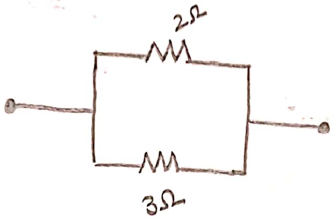
2) Find equivalent resistance b/w A and B.



R_{eq} (or) $R_{AB} = 2+3+6+9+8$

$R_{AB} = 28\Omega$

3) Find the equivalent resistance b/w A and B.



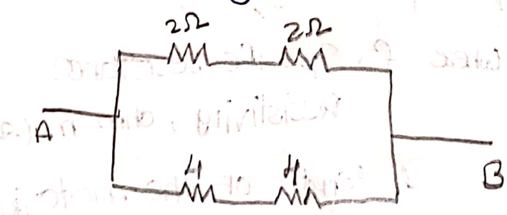
$R_1 \| R_2$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{2} + \frac{1}{3} \Rightarrow \frac{3+2}{6} \Rightarrow 5/6$$

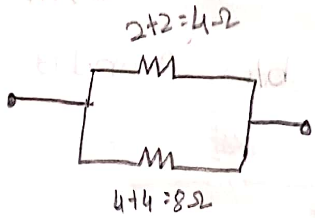
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \Rightarrow \frac{2 \times 3}{2+3} = \frac{6}{5}\Omega$$

$$R_{eq} = \frac{6}{5}\Omega$$

4)



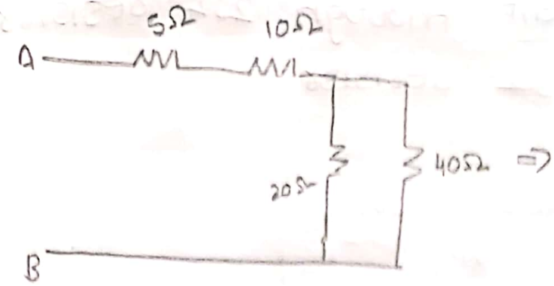
Find R_{AB}?

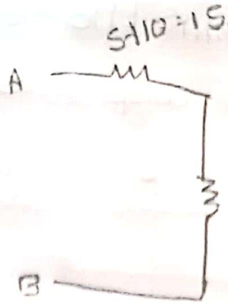


$$R_{eq} = 4 \| 8 = \frac{4 \times 8}{4+8}$$

$$= \frac{32}{12}$$

$$= 8/3\ \Omega$$





$20 \parallel 40$

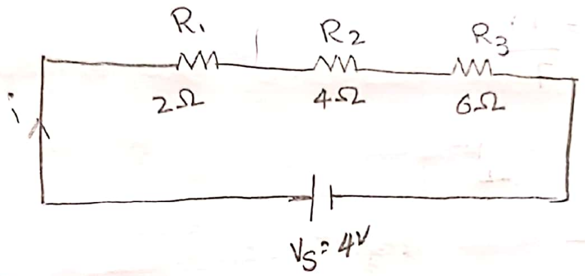
$= \frac{20 \times 40}{20+40}$

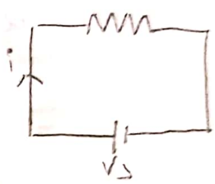
$= \frac{800}{60} = \frac{40}{3}$

$R_{eq} = 15 + \frac{40}{3} = \frac{85}{3}\ \Omega$

5) Three resistors R_1, R_2, R_3 are connected in series with a constant voltage source 4 volts. Find the equivalent resistance and the current flow in the circuit.

Here R_1, R_2, R_3 are in series





$R_{eq} = R_1 + R_2 + R_3$

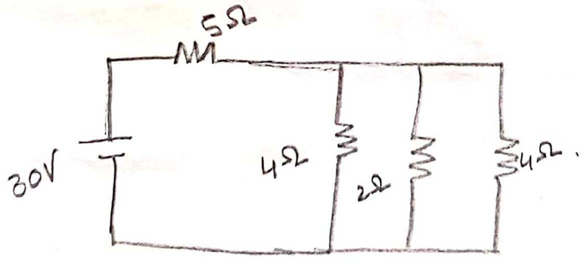
$= 2+4+6$

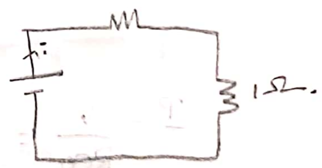
$R_{eq} = 12\Omega$

$i = \frac{V}{R_{eq}}$

$= \frac{4}{12} = 1/3\ A$

6) Determine the total current in the circuit shown





Here $4\Omega \parallel 2\Omega \parallel 4\Omega$

$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{2} + \frac{1}{4}$

$= 0.25 + 0.5 + 0.25$

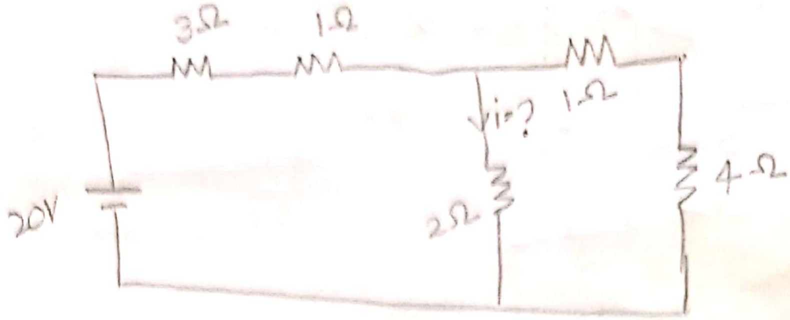
$\frac{1}{R_{eq}} = 1 \Rightarrow \boxed{R_{eq} = 1}$

$R_{eq} = 5+1 = 6\Omega$

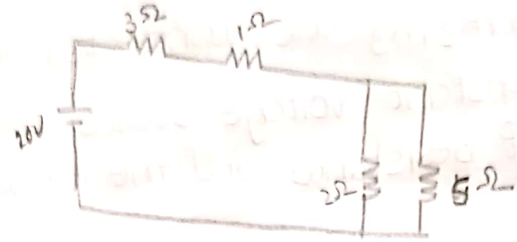
$i = \frac{V}{R_{eq}} = \frac{30}{6} = 5 Amp$

Determine the current through 2Ω resistor an voltage drop across 3Ω resistor

6)

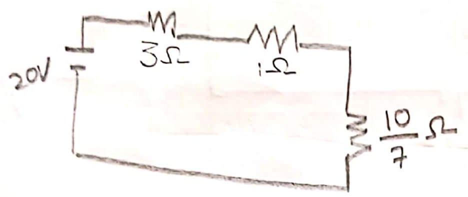


Here 4Ω and 1Ω is in Series



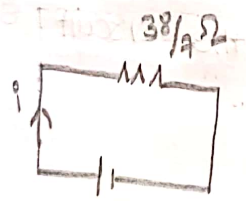
Here 2Ω // 5Ω

$$\Rightarrow \frac{2\times 5}{2+5} = \frac{10}{7}$$



Here 3Ω, 1Ω, $\frac{10}{7}\Omega$ are in series.

$$R_{eq} = 3+1+\frac{10}{7} = \frac{21+7+10}{7} = \frac{38}{7}$$



$$I = \frac{V}{R_{eq}} = \frac{20}{38/7}$$

Total Current $I = 3.68\,A$

Current Through 2Ω :-

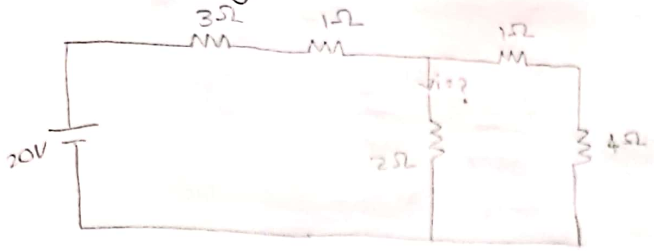


Fig (1)

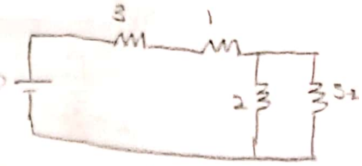


Fig (2)

By current division Rule.

$$I_{2\Omega} = \text{Total Current} \times \frac{\text{opposite Resistance}}{\text{Sum of resistance of that loop.}}$$

$$I_{2\Omega} = 3.68 \times \frac{5}{2+5}$$

$$I_{2\Omega} = 2.629\ A$$

Voltage across 3Ω :-

$$V_{3\Omega} = IR$$

$$= 3.68 \times 3$$

$$\boxed{V_{3\Omega} = 11.04\ \Omega}$$

8) Determine the total Current in the circuit and find current through 2Ω.

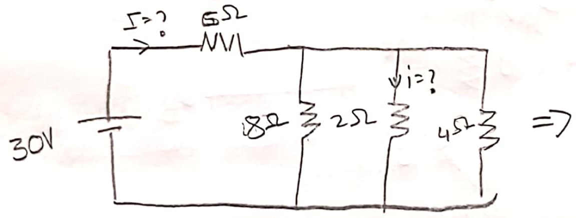
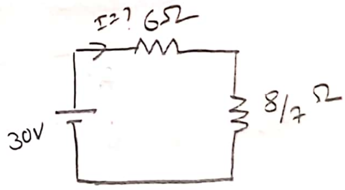


Fig (1)



Here $8 // 2 // 4$

$$\frac{1}{Req} = \frac{1}{8} + \frac{1}{2} + \frac{1}{4}$$

$$= \frac{2+8+4}{16}$$

$$= \frac{14}{16}$$

$$\boxed{Req = 8/7}$$

Here 6Ω and $8/7\,\Omega$ are in series.

$$Req = 6 + \frac{8}{7} = \frac{42+8}{7}$$

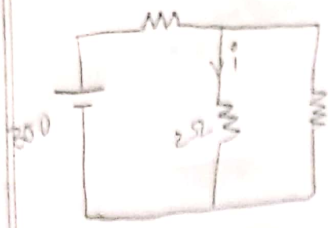
$$= \frac{50}{7}$$

Total resistance $Req = \frac{50}{7}\,\Omega$

Total Current $I = \frac{V}{Req} = \frac{30}{\frac{50}{7}} = 4.2$

$I = 4.2A$

Current through 2Ω



$8 \parallel 4 = \frac{8 \times 4}{8+4} = \frac{32}{12}$

$= 8/3\ \Omega$

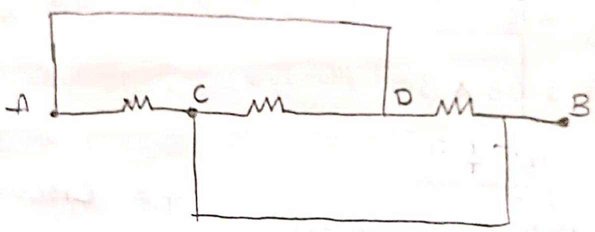
By current division rule,

$I_{2\Omega} = \text{Total Current} \times \frac{\text{opp Resistance}}{\text{Sum of total resistance of that loop}}$

$= 4.2 \times \frac{8/3}{2+8/3}$

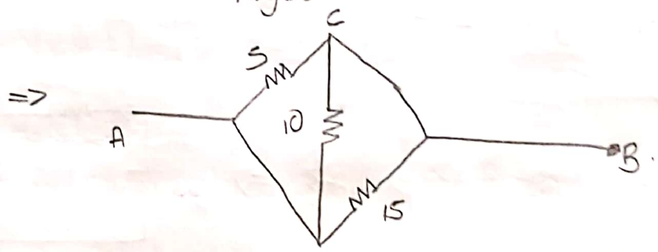
$I_{2\Omega} = 2.4\ A$

9) Find the equivalent resistance b/w A and B terminals



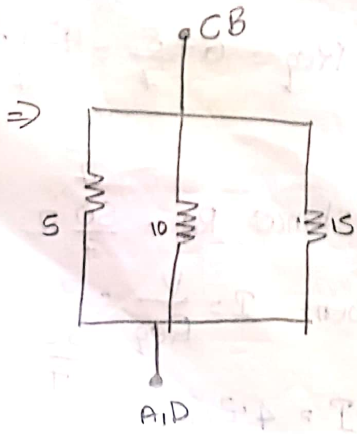
Fig(1)

$\Rightarrow$



(Fig)(2).

$\Rightarrow$



Here $5 \parallel 10 \parallel 15$

$\frac{1}{R_{eq}} = \frac{1}{5} + \frac{1}{10} + \frac{1}{15}$

$= 0.2 + 0.1 + 0.06$

$\frac{1}{R_{eq}} = 0.36$

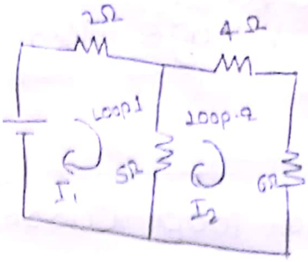
$R_{eq} = \frac{1}{0.36}$

$\boxed{R_{eq} = 2.72\ \Omega}$

KVL:-

8 Loop Analysis (or) Mesh Analysis :- KVL analysis :-

In the network shown below find all branch currents and voltage drops across all resistors. Apply loop analysis



Algebric sum of all voltages

Loop 1:-

$$-12 + 2I_1 + 5(I_1 - I_2) = 0.$$

$$2I_1 + 5I_1 - 5I_2 = 12.$$

$$7I_1 - 5I_2 = 12 \rightarrow (1)$$

Loop 2:-

$$5(I_2 - I_1) + 4I_2 + 6I_2 = 0.$$

$$5I_2 - 5I_1 + 4I_2 + 6I_2 = 0.$$

$$-5I_1 + 15I_2 = 0 \rightarrow (2)$$

By solving (1) & (2) Eqn's we get

$I_1 = 2.25$ A.

$I_2 = 0.75$ A.

Current through 2Ω, $I_{2\Omega} = I_1 = 2.25$ A.

Voltage drop consists 2Ω, $V_{2\Omega} = I_{2\Omega} R = 2.25 \times 2$

$= 4.5$ V

Current through 5Ω, $I_{5\Omega} = I_1 - I_2 = 2.25 - 0.75$

$= 1.5$ A

Voltage, $V_{5\Omega} = I_{5\Omega} \times R = 1.5 \times 5 = 7.5$ V.

Current through 4Ω, $I_{4\Omega} = I_2 = 0.75$ A.

Voltage: $V_{4\Omega} = I_{4\Omega} \times R = 0.75 \times 4$

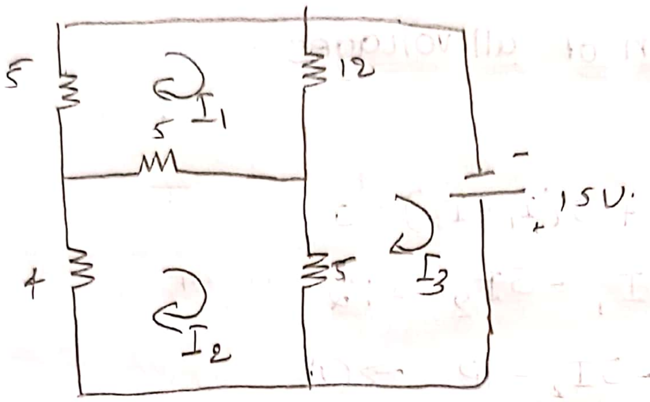
$V_{4\Omega} = 3$ V

Current through 6Ω $I_{6\Omega} = I_2 = 0.75$

Voltage $V_{6\Omega} = I_{6\Omega} \times R = 0.75 \times 6$

$V_{6\Omega} = 4.5V$

2) Find loop currents for the following.



→ Super position Theorem:-

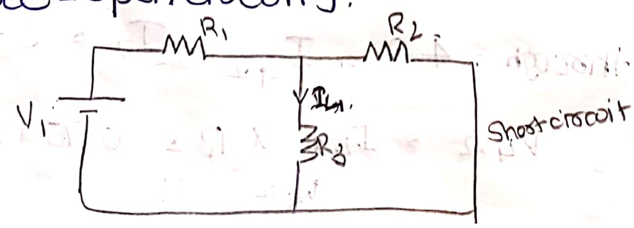
Statement:- Super position Theorem states that in any linear network containing two (or) more sources then the response in any element is equal to the algebric sum of responses produced by individual sources acting alone while the other source are non-operative.

Limitations:-

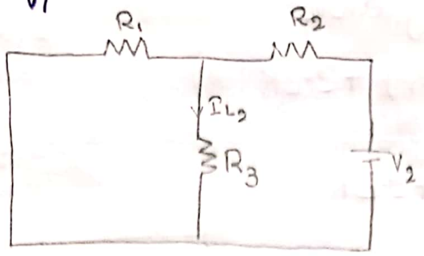
* It is applicable only to linear network.
* It is applicable when two (or) more sources are available in the network.
* Power cannot be determined using this theorem

Explanation:-

① Keep voltage source V_1 as active and deactivate voltage source (V_2) (voltage - short circuit, current source - open circuit).

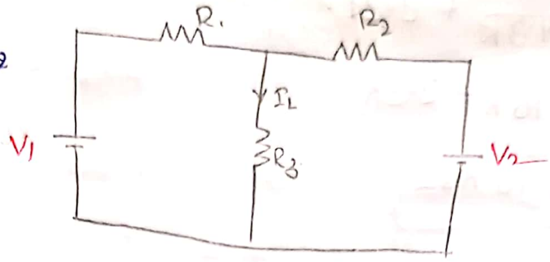


② When Keep voltage source V_2 an in active and deactivate V_1

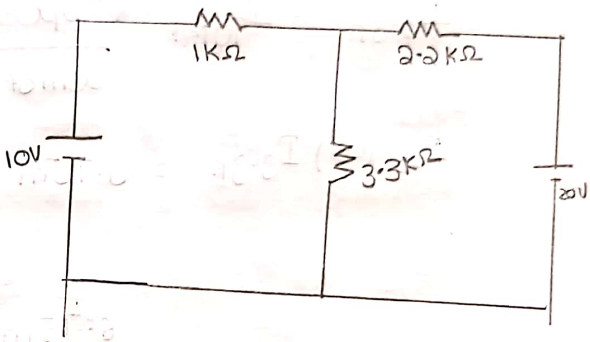


③ Keep both sources V_1 and V_2 active

$\Rightarrow I_L = I_{L_1} + I_{L_2}$

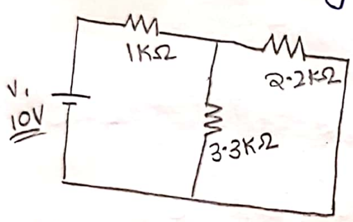


Q 1) Find the current through 3·3 kΩ by using Superposition theorem.



case ①

When $V_1 = 10V$ acting alone



Here 2·2 is parallell 3·3

$R_{eq} = 1K\Omega + (2{\cdot}2K\Omega \,//\, 3{\cdot}3K\Omega)$

$= 1K + \dfrac{2{\cdot}2K + 3{\cdot}3K}{2{\cdot}2K + 3{\cdot}3K}$

$= 1K + \dfrac{7{\cdot}26K^2}{5{\cdot}5K}$

$R_{eq} = 1K + 1{\cdot}32K$

$R_{eq} = 2{\cdot}32K\Omega$

$I_T = V/R_{eq}$

$I_T = \dfrac{10}{2{\cdot}32K}$

$I_T = 4{\cdot}31\,mA$

By current distant role

Current through 3·3KΩ

$I_{3\cdot3K} = I_{total} \times \dfrac{\text{Op/distance}}{\text{Sum of the resistance of that loop}}$

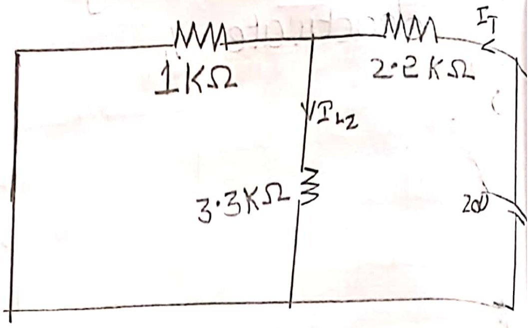
I_4 (or) $I_{3\cdot3K} = 4{\cdot}31m \times \dfrac{2{\cdot}2K}{2{\cdot}2 + 3{\cdot}3K}$

$I_4 = 4{\cdot}31m \times \dfrac{2{\cdot}2K}{5{\cdot}5K}$

$I_4 = 1{\cdot}72mA$

case ②

When $V_2 = 20V$ acting alone.



$$R_{eq} = (1K\Omega \,//\, 3.3K) + 2.2K$$

$$= \frac{1K + 3.3K}{1K + 3.3K} + 2.2K.$$

$$= \frac{3.3K^2}{4.3K} + 2.2K.$$

$$= 0.76K + 2.2K$$

$$\boxed{R_{eq} = 2.96\,K\Omega}$$

By current division role

Current through 3.3KΩ

$$I_{3.3k} = I_{Total} \times \frac{\text{opposite resistance}}{\text{Sum of total resistance of that loop.}}$$

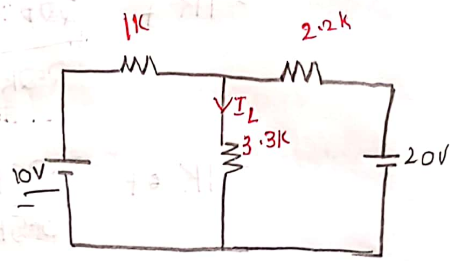
$$I_{L2} \text{ (or) } I_{3.3K} = 6.75m \times \frac{1K}{1K+3.3K.}$$

$$6.75m \times \frac{1K}{4.3K}$$

$$\boxed{I_{L2} = 1.56\,mA}$$

case ③:-

When Both sources are acting



$$I_L = I_{L1} + I_{L2}.$$

$$= 1.72mA + 1.56mA$$

$$\boxed{I_L = 3.28\,mA}$$

We can solve above problem by loop Analasis.

Loop 1:-

$$-10 + 1K I_1 + 3.3K(I_1 - I_2) = 0$$

$$4.3K\,I_1 - 3.3K I_2 = 10 \rightarrow ①$$

Loop 2:-

$$3.3K(I_2 - I_1) + 2.2K I_2 + 20 = 0 \rightarrow ②$$

Scanned with CamScanner

Solving eqn ① and ② we get

$I_1 = -0.86\ mA$

$I_2 = -4.15\ mA$

$I_{3\Omega} = I_1 - I_2$

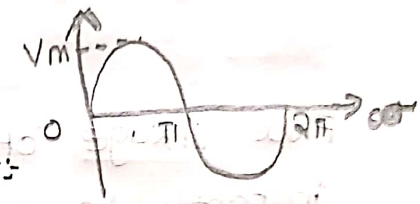
$= -0.86 - (-4.15)$

$\boxed{I_{3\Omega} = 3.29mA}$

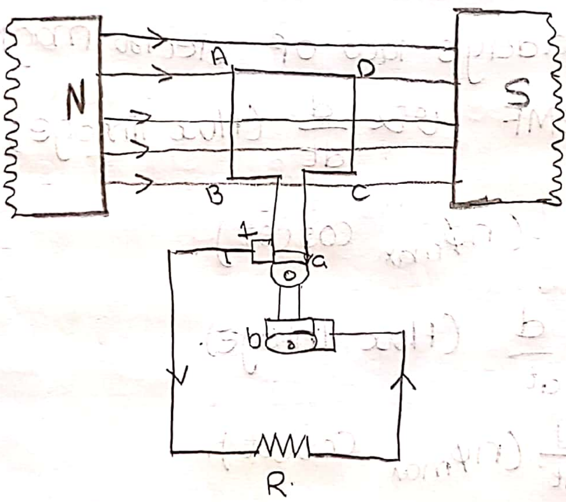
AC Fundamentals

Alternating Voltage:-

A voltage which changes its polarity of regular intervals of time is called an Alternating voltage.



Equation of voltage and current:-

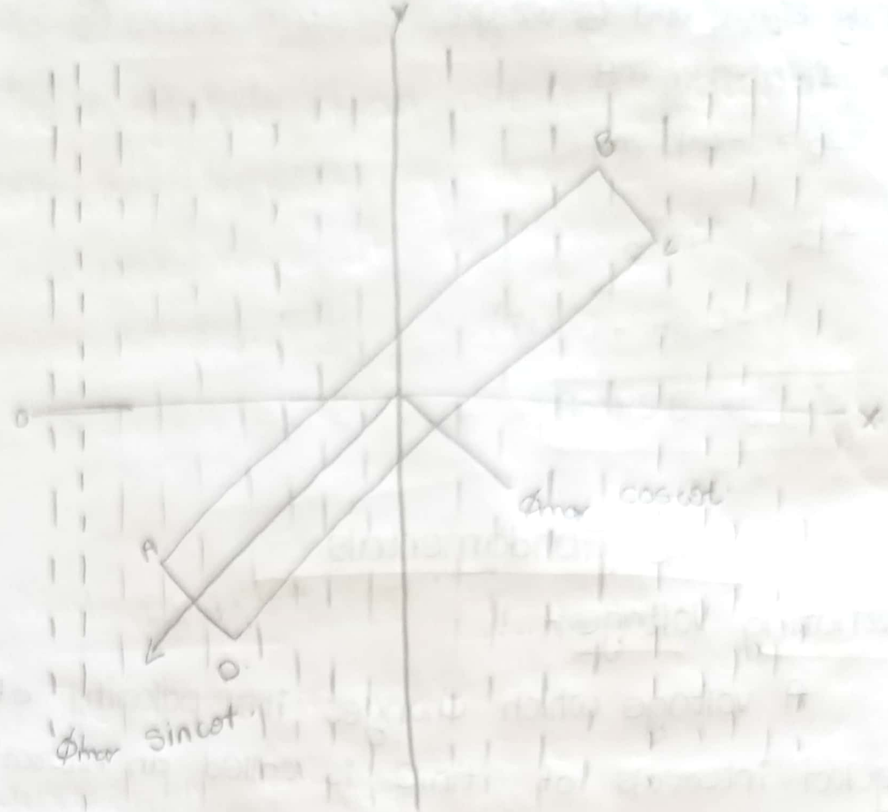


Eqn of voltage and current:-

Consider rectangular coil ABCD rotates about its own axis in a magnetic field. At any angle θ (or) ωt. If the coil moves in clock wise direction. If the maximum Flux linking is divided into two perpendicular components

① $\phi_{max} \sin \omega t$ which is ||el to plane of the coil. This Component induces no emf in the coil.

② $\phi_{max} \cos \omega t$ which is ⊥r to plane of the coil, This component induces emf in the coil.



fig(2).

Flux linkage of the coil at the considered instant = No of turns x Flux linking

A/c to Faraday's law of electro magnetic induction EMF $\vartheta \propto \frac{d}{dt}$ (Flux linkage)

Lenz's law $\vartheta = -\frac{d}{dt}(n\phi_{max} \cos\omega t)$.

Emf $\vartheta \propto \frac{d}{dt}$ (Flux linkage).

$$\vartheta = -\frac{d}{dt}(n\phi_{max} \cos\omega t).$$

$$\vartheta = -n\phi_{max} \frac{d}{dt}(\cos\omega t).$$

$$= -n\phi_{max}(-\sin\omega t \cdot \omega).$$

When the coil is turned through 90° in clockwise direction then $\vartheta_{max} = n\phi_{max}\,\omega \sin\omega t$ → ①

Max Emf $V_m = n\phi_{max}\,\omega$.

From Eq ①

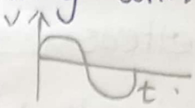
$$\Rightarrow \boxed{\vartheta = V_m \sin\omega t} - ②$$

Similarly AC current eqn is given as $\boxed{i = I_m \sin \omega t}$ → (3)

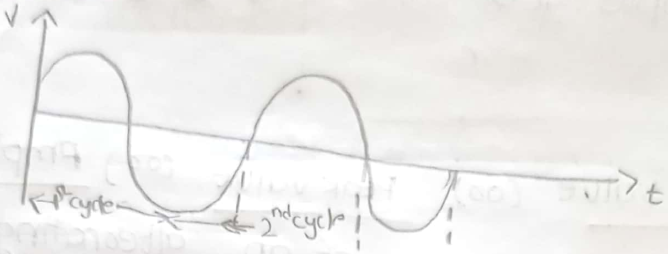
Definitions :-

1) Wave Form :-

The wave form (or) wave shape is a graphical plot of variation of alternating quantity with respect to time (or) time angle.



2) Cycle :-



Each repitition of time varying quantity reccuring at regular intervals of time is called as cycle.

In general

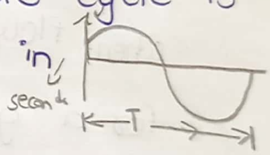
1 cycle = 360° electrical

(or)

1 cycle = 2π radians electrical.

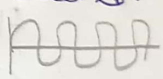
3) Time period :

The time taken for one complete cycle is called as timeperiod. It is expressed in seconds



4) Frequency :-

Frequency is defined as no of cycles per second.



It is measured in Hertz's.

It is denoted by 'f'. $\boxed{f = \frac{1}{T}}$ where T = Timeperiod.

5) Angular velocity :-

It is the angle traced out per unit time.

It is measured in degree/sec.

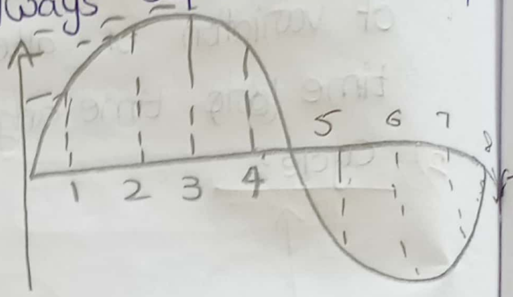
It is denoted by ω.

$$\omega = \frac{2\pi}{T} \cdot \frac{1}{T}$$

$$\omega = \frac{2\pi}{T} f$$

The magnitude of an alternating quantity at a particular instant of time is known as instantaneous value. It is always represented with small letters.

For example : i(t).



6) Maximum value (or) Peak value (or) Amplitude:-

The maximum value of an alternating quantity in one cycle is called as maximum value (or) Peak value.

7) RMS value :- (Root mean square) or (Effective value) or (virtual value) :-

The RMS value of an alternating current is that the value of steady DC current which when flowing through a given circuit ~~the value~~ for a given time produces the heating effect as produced by the given alternating current flowing through the same circuit for the same time.

8) Average value:-

The average value of an alternating current is defined as that value of steady current which transfer the same amount of charge across any cycle as is transfered by the given AC current in the same circuit for the same time.

(or)

The average value is the total area under the complete curve divided by distance of curve.

9) Form Factor:-

Form Factor is defined as the ratio of RMS value to the average value

$$\text{Form Factor} = \frac{\text{RMS value}}{\text{Average value}}$$

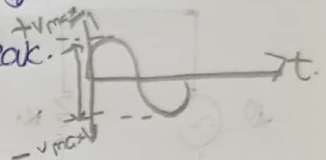
10) Peak Factor:- (or) Crest Factor (or) Amplitude Factor:-

Peak Factor is defined as the ratio of maximum value to the RMS value.

$$\text{Peak factor} = \frac{\text{max value}}{\text{Rms value}}$$

11) Peak to Peak value:-

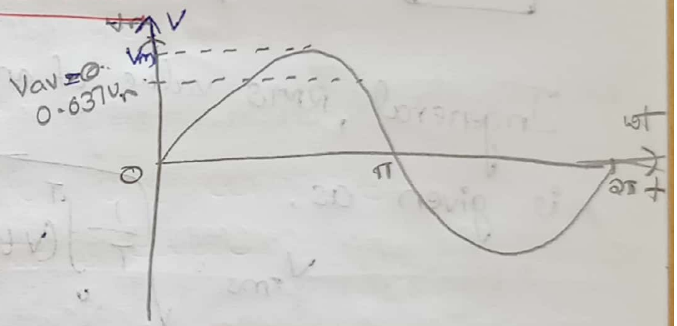
The peak to peak value of a sine wave is the value from positive to negative weak.



Derivation :

12) Average Value of Sine value:-

In general, Avg value of any function v(t) with timeperiod T is given by.



$$V_{av} = \frac{1}{T}\int_0^T v(t)\,dt$$

Avg value of sine wave is Total area under half cycle curve divided by distance of curve

$$V_{avg} = \frac{1}{\pi}\int_0^\pi v(t)\;d(\omega t).$$

where $v(t) = V_m \sin\omega t$.

$$V_{avg} = \frac{1}{\pi}\int_0^\pi V_m \sin\omega t \; d\omega t$$

$$= \frac{V_m}{\pi}\int_0^\pi \sin\omega t \; d\omega t.$$

$$= \frac{V_m}{\pi}\left(-\cos\omega t\right)_0^\pi$$

$$V_{avg} = \frac{V_m}{\pi}\left[-(\cos\pi - \cos 0)\right]$$

$$= \frac{V_m}{\pi}\left[-((-1)-1)\right]$$

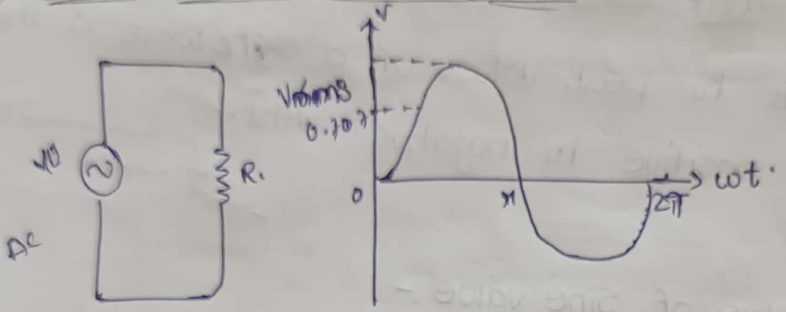
$$= \frac{V_m}{\pi}\left[-(-2)\right]$$

$$V_{avg} = \frac{2V_m}{\pi}$$

$$\boxed{V_{avg} = 0.637\ V_m} \text{ volts}$$

13 Derivation

RMS value of Sine wave:-



In general, RMS value of any function with time period is given as,

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (v(t))^2\, d\omega t}$$

for sin wave, $v(t) = V_m \sin\omega t$.

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (V_m \sin\omega t)^2\, d\omega t}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m \sin^2\omega t\, d\omega t}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \sin^2\omega t\, d\omega t}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int \frac{1-\cos 2\omega t}{2}\, d\omega t}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{4\pi}\left\{\int_0^{2\pi} 1\, d\omega t - \int_0^{2\pi} \cos 2\omega t\, d\omega t\right.}$$

$\boxed{\int 1\,dx = x}$

$$= \sqrt{\frac{V_m^2}{4\pi}\left\{[\omega t]_0^{2\pi} - \left[\frac{\sin 2\omega t}{2}\right]_0^{2\pi}\right\}}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left\{(2\pi - 0) - \frac{1}{2}\left[\sin 2(2\pi) - \sin 2(0)\right]\right\}}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left\{2\pi - \frac{1}{2}\left[\sin 4\pi - 0\right]\right\}}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left\{2\pi - \frac{1}{2}(0 - 0)\right\}}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{\cancel{4}_2\pi}\left\{\cancel{2}\pi\right.}$$

$$= \sqrt{\frac{V_m^2}{2}}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707 \cdot V_m}$$

Note:- If the function consists of no. of Sinusoidal terms then,

$$V_{(t)} = V_0 + (V_{c_1}\cos\omega t + V_{c_2}\cos 2\omega t + \ldots) + V_{s_1}\sin\omega t + V_{s_2}\sin 2\omega t + \ldots)$$

The Rms value is given for above equation is

$$V_{rms} = \sqrt{V_m^2 + \frac{1}{2}(V_{c_1}^2 + V_{s_2}^2 + \ldots) + \frac{1}{2}(V_{s_1}^2 + V_{s_2}^2 + \ldots)}$$

form factor of sine wave:-

$$\text{factor} = \frac{\text{Rms value}}{\text{Avg value}}$$

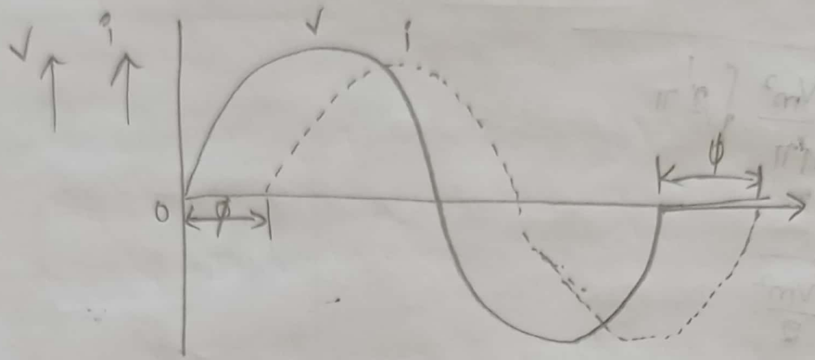
$$= \frac{0.707\, V_m}{0.637\, V_m}$$

$$= 1.11$$

form factor = 1.11.

14) Phase :- phase of a sinwave is an angular measurement that specifies the position of the sin wave relative to the refrence.

15) Phase difference :- when two alternating to the same frequency different zero points, they are said to have a phase difference.



Here $v = V_m \sin\omega t$.

$i = I_m (\sin \omega t - \phi)$

i' lags behind voltage v by phase difference ϕ

→ Voltage and current relationship with phaser diagram in R, L and C circuit :

1) A C circuit diagram containting resistance only :-

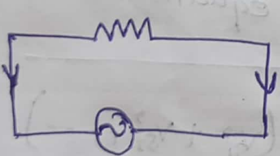


fig (1) $V = V_m \sin\omega t$

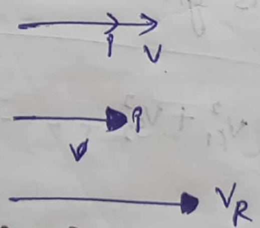


fig (2)

phase diagram

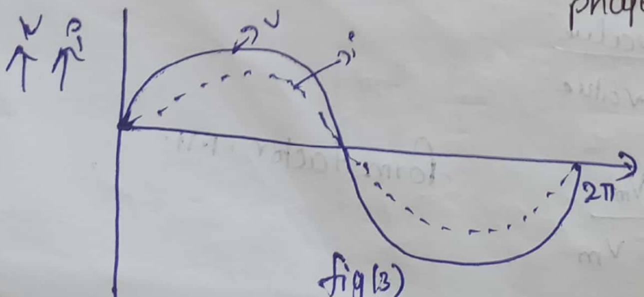


fig (3)

Consider a circuit containing a pure resistance R Ω, connected across ac supply as shown in fig 1.

ac voltage Equation is

$$V = V_m \sin\omega t$$

ac current equation is

$$i = I_m \sin\omega t$$

Power consumed $= P = VI = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$

where $V = V_{rms} = \frac{V_m}{\sqrt{2}}$

$I = I_{rms} = \frac{I_m}{\sqrt{2}}$

2) AC circuit Containing Inductance only

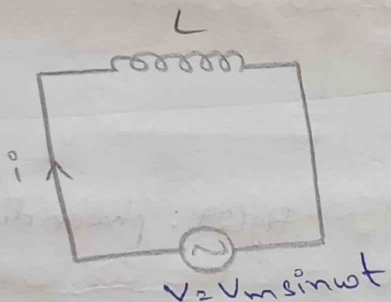
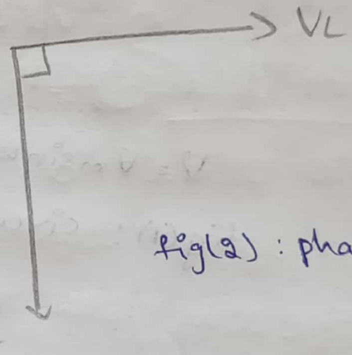


fig (1) = ~~ac~~ circuit



fig(2) : phasor Diagram

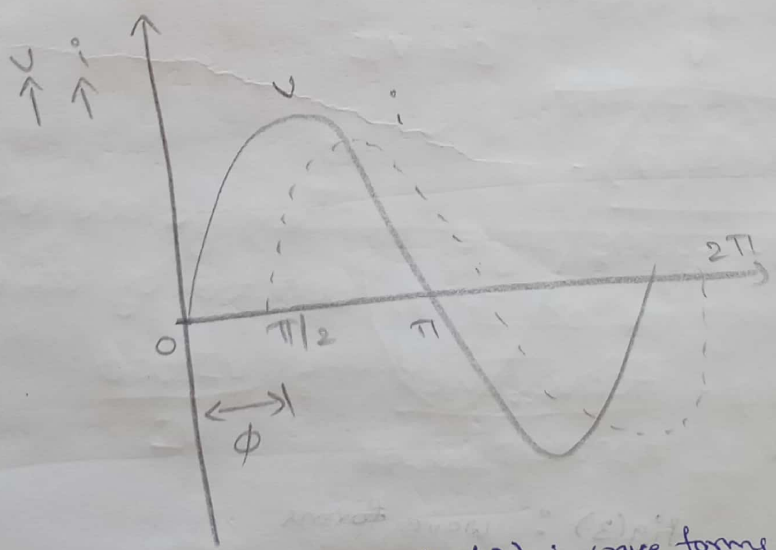


fig (3) : wave forms

Consider an alternating voltage applied to a pure Inductance L, as shown in fig (1).

The ac voltage Equation is given as

$$v = V_m \sin \omega t$$

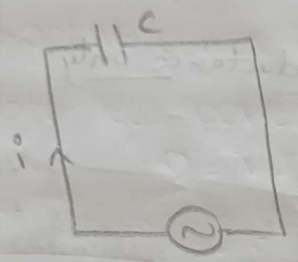
ac current Equation is

$$i = I_m \sin(\omega t - \phi).$$

Here current 'i' lags behind by voltage with a phase difference $\phi = \frac{\pi}{2}$

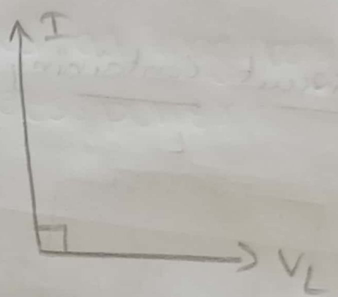
so, $i = I_m \sin(\omega t - \frac{\pi}{2})$

3) <u>Ac circuit containing capacitance only</u> :-

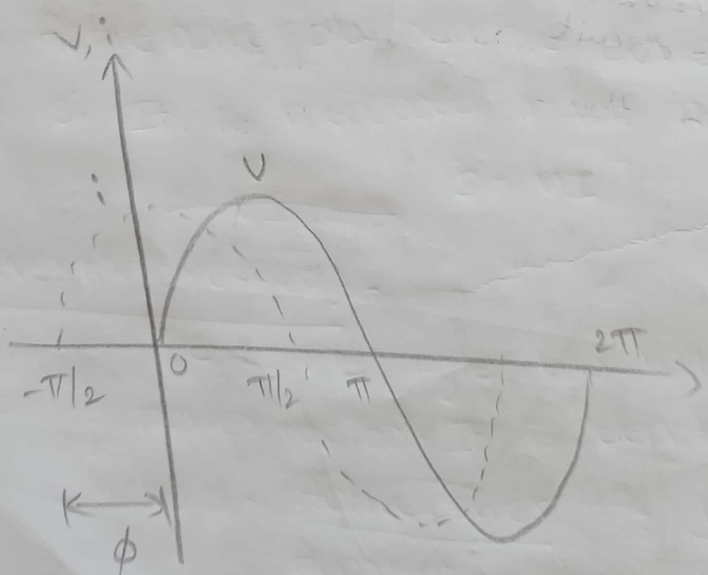


$V = V_m \sin \omega t$

fig (1): circuit



fig(2) : phaser diagram.



fig(3) : wave forms

Consider an alternating voltage applied to a pure Inductance L, as shown in fig(1).

The ac voltage equation is given as

$$V = V_m \sin \omega t$$

ac current equation is

$$i = I_m \sin(\omega t + \phi)$$

Here current 'i' ~~lags~~ Leads by voltage V with a phase difference $\phi = \frac{\pi}{2}$

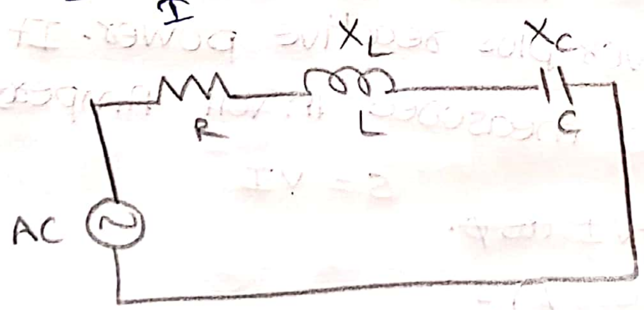
So, $i = I_m \sin(\omega t + \frac{\pi}{2})$

Impedance :-

Impedance is defined as the measure of opposition to the current flow. It is denoted by 'Z' and it's units are :- ohms (Ω)

Impedance (Z) is the ratio of voltage to the current.

$$\Rightarrow Z = \frac{V}{I}$$



$$\Rightarrow Z = R + J(X_L - X_C)$$

where R = Resistance

X_L = Inductive Reactance

X_C = Capacitive Reactance

In short,

$$Z = R + JX$$

Active Power (P):-

Active power is the power that continuously give flows from source to load in an electric circuit. It is denoted by 'P'. It is measured in watts. Active power is also called as ~~activ~~ real power, true power (or) actual power (or) Watt-Full power.

$$P = VI\cos\phi$$

Reactive power (Q);

Reactive power is the power that continuously flows from source to load and returns back to the source in an electric circuit. It is denoted by Q. It is measured in volt Ampere reactive (VAR)

Re-active power is called as imaginary power (or) Watt-less power (or) Useless power

$$Q = VI\sin\phi$$

Apparent power (S):-

The product of voltage (V) and current I is called Apparent power and it is the combination of Active power plus reactive power. It is denoted by 'S'. It is measured in volt Ampers.

$$S = VI$$

~~P = VI cos φ.~~

Power Factor (cos φ):-

The cosine angle b/w voltage and current is called power factor. It is denoted by cos φ.

$$\cos\phi = \frac{\text{Active power}}{\text{Apparent power}} = \frac{VI\cos\phi}{VI}$$

$$\cos\phi = \frac{R}{Z} = \frac{\text{Resistance}}{\text{Impedence}}.$$

Note:- The phase angle φ is calculated by $\tan\phi = \frac{X}{R}$.

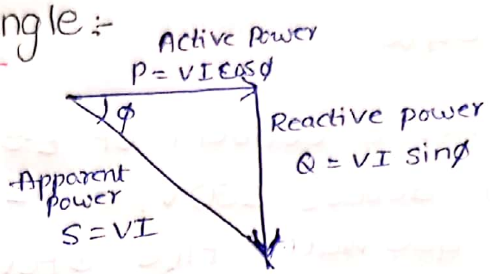
* Power factor can never have a value greater than 1.

* For inductive circuits the current lags the voltage and the powerfactor is said to be lagging.

For capacitive circuits the current leads the voltage. & the powerfactor is said to be leading in nature.

For resistive circuits the power factor is unity.

Power Triangle :-



Problems !

Q) A coil having a resistance of 7Ω and an inductance of 31.8 mH is connected to 230V, 50Hz supply. Calculate ① Circuit current ② phase Angle ③ Power factor ④ Power consumed.

Resistance $R = 7\Omega$

Inductance $L = 31.8\ mH$

Voltage $V = 230V$

Frequency $F = 50Hz$

$X_L = 2\pi f L$

$= 2 \times 3.14 \times 50 \times 31.8 \times 10^{-3}$

Inductive reactance $= X_L = 10\Omega$

$z = R + jX_L$

$z = \sqrt{R^2 + X_L^2}$

$= \sqrt{(7)^2 + (10)^2}$

$z = 12.2\ \Omega$

① Current $I = \frac{V}{Z} = \frac{230}{12\Omega} = 18.85\ A$

② Phase angle $\phi : \tan\phi = \frac{X_L}{R}$

$\phi = \tan^{-1}\left(\frac{10}{7}\right) \Rightarrow \phi = 55°$ lag.

3) Power Factor $= \cos\phi$

$= \cos 55^{\circ}$ lag

$= 0.573$ lag

4) Power Consumed

$P = VI\cos\phi$

$= 230 \times 18.85 \times 0.573$

$P = 2484.24W$

2) A choke coil takes a current of 2.5 Amperes when connected across 250 V, 50 Hz supply and consumed 400 watts. Find ① Power Factor ② Resistance of the coil ③ Inductance of the coil.

Ans Current $I = 2.5A$

Voltage $V = 250V$

Frequency $f = 50Hz$

Active power $P = 400W$

Power Factor :-

$$\cos\phi = \frac{\text{Active Power}}{\text{Apparent power}} = \frac{400}{VXI}$$

$$\cos\phi = \frac{400}{250 \times 2.5} = 0.64 \text{ lag}.$$

ii) Resistance :-

Impedance $Z = \frac{V}{I} = \frac{250}{2.5} = 100\Omega$

$\cos\phi = \frac{R}{Z} \Rightarrow R = Z\cos\phi.$

$= 100 \times 0.64$

$R = 64\Omega$

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(iii) Inductance (L) :-

$$Z = R + jXS$$

$$Z^2 = R_1^2 + X_L^2$$

$$X_L^2 = Z^2 - R^2$$

$$X_L = \sqrt{Z^2 - R^2}$$

$$= \sqrt{(100)^2 - (64)^2}$$

$$X_L = 76.84\ \Omega$$

$$X_L = 2\pi f L$$

$$\Rightarrow L = \frac{X_L}{2\pi f}$$

$$L = \frac{76.84}{2 \times 3.14 \times 50}$$

$$\boxed{L = 0.245\ H}$$