7-01-20

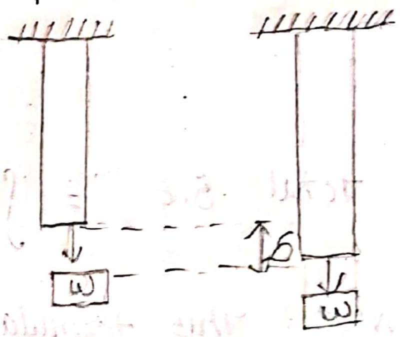
UNIT - I

ENERGY THEOREMS

Strain Energy

When a force is applied on a body the internal energy stored in a body to regain it's original shape is called strain energy or resilience.

Strain Energy due to axial loading.



* load is applied gradually
* work done $= \frac{1}{2} W \delta$
* Strain energy = workdone

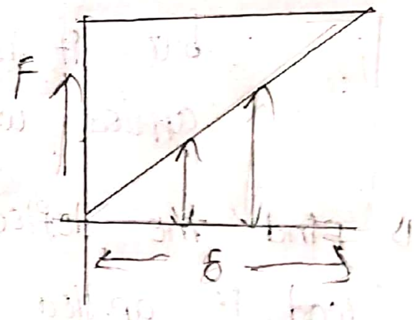
$$S.E = \frac{1}{2} W \delta$$

$$= \frac{1}{2} \times W \times \frac{WL}{AE}$$

$$= \frac{1}{2} \frac{W^2 L}{AE}$$

$$= \frac{1}{2} \frac{W^2}{A^2} \times \frac{AL}{E}$$

$$= \frac{1}{2} \sigma^2 \times \frac{AL}{E}$$

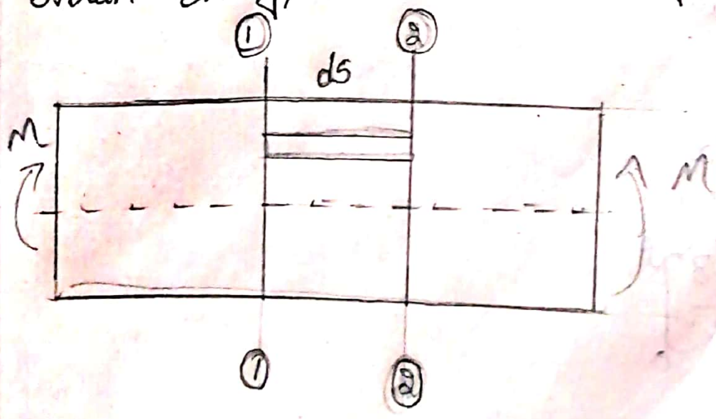


$$S.E = \frac{\sigma^2}{2E} \times \text{volume}$$

Strain energy per unit volume $= \frac{\sigma^2}{2E}$

Torsion $S.E = \frac{\tau^2}{4C} \times V$

Strain Energy due to Bending.



$S.E = \frac{\sigma^2}{2E} \times volume$

Bending equation is $\frac{\sigma}{y} = \frac{M}{I} \Rightarrow \sigma = \frac{My}{I}$

$S.E$ in element $= \frac{\sigma^2}{2E} \times da \times ds$

$$= \frac{M^2 y^2}{2EI^2} \, da \, ds$$

$S.E$ in total elemental strip $= \frac{M^2}{2EI^2} \int y^2 \, da \; ds$

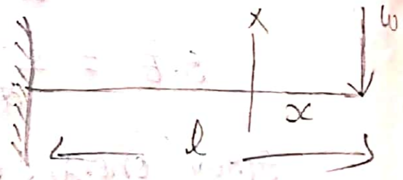
$$= \frac{M^2 \cancel{I}}{2EI^{\cancel{2}}} \, ds$$

$$= \frac{M^2}{2EI} \, ds$$

Total $S.E = \int \frac{M^2}{2EI} \, ds$

Note: This formula can be applied for calculating deflections when loads are applied on beams but it is laborious when the beams are applied with combination of loads.

1) Find the deflection for a cantiliver beam when the load is applied at free end. let it be w. find



$M_x = -wx$

$$S.E = \int \frac{M^2}{2EI} \, dx$$

$$= \int_0^l \frac{(-wx)^2}{2EI} \, dx$$

$$= \int_0^l \frac{w^2 x^2}{2EI} \, dx$$

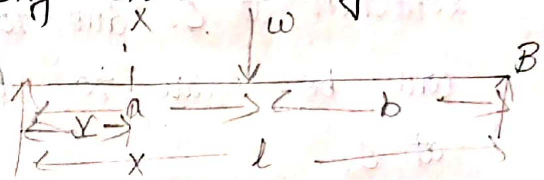
$$= \frac{w^2}{2EI} \left[\frac{x^3}{3} \right]_0^l$$

$$S.E = \frac{w^2 l^3}{6EI}$$

$$\frac{1}{2} \times w \times \delta = \frac{w^2 l^3}{6EI}$$

$$\delta = \frac{w l^3}{3EI}$$

Find out the deflection for a S.S.B ~~whose~~ AB whose length is 'l' for which the point load w is applied at a distance 'a' from the left end & 'b' from right end. (useing strain energ Eq).



$$M_x = \frac{wb}{l} x$$

$$M_x = \frac{wa}{l} x$$

$$S.E = \int \frac{M^2 \, dx}{2EI}$$

$$= \int_0^a \frac{w^2 b^2}{l^2 (2EI)} x^2 \, dx + \int_0^b \frac{w^2 a^2}{l^2 (2EI)} x^2 \, dx$$

$$= \frac{w^2 b^2}{2EI l^2} \int_0^a x^2 \, dx + \frac{w^2 a^2}{2EI l^2} \int_0^b x^2 \, dx$$

$$= \frac{w^2 b^2}{2EI l^2} \cdot \frac{a^3}{3} + \frac{w^2 a^2}{2EI l^2} \cdot \frac{b^3}{3}$$

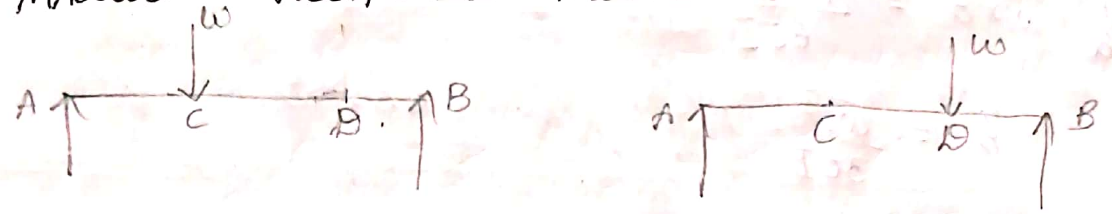
$$= \frac{w^2 a^2 b^2}{6EI l^2} [a+b]$$

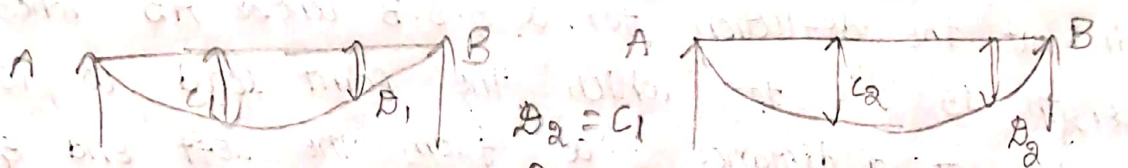
$$= \frac{w^2 a^2 b^2}{6EI l}$$

$$\frac{1}{2} \times w \times \delta = \frac{w^2 a^2 b^2}{6EI l}$$

$$\delta = \frac{w a^2 b^2}{3EI l}$$

MAXWELL Reciprocal Theorem





$D_2 = C_1$

$D_1 = C_2$

Deflection at C due to the load 'w' applied at D will be equal to deflection at D due to load 'w' at C.

One end of a semicircular arch hinged and other end is roller and horizontal load P is applied at the roller end. Find the deflection at the roller end (Horizontal displacement). Consider 'r' radius of curvature of the arch:

$$S.E = \int \frac{m^2}{2EI} ds$$

$$= \int \frac{m^2}{2EI} r\,d\theta$$

$$= \int \frac{P^2 r^2 \sin^2\theta}{2EI} r\,d\theta$$

$$= \frac{P^2 r^3}{2EI} \int \sin^2\theta \, d\theta$$

$$= \frac{P^2 r^3}{2EI} \int_0^{\pi/2} \frac{(1 + \cos 2\theta)}{2} d\theta$$

$$= \frac{P^2 r^3}{2EI} \left[\frac{1}{2} \times \frac{\pi}{2}\right] \times 2$$

$$= \frac{P^2 r^3}{2EI} \left[\frac{\pi}{4}\right] \times 2$$

$$= \pi P^2 r^3 / 4EI$$

$$\frac{1}{2} \times W \times \delta = \frac{P^2 r^3 \pi}{4EI}$$

$$\delta = \frac{\pi P r^3}{2EI}$$

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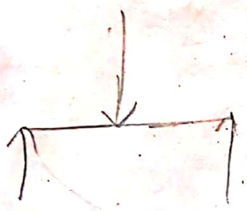
Castigliano first theorem :

If any beam or trust subjected to any system of loading then deflection at any point 'r' is given by partial differentiation of total strain energy in the beam with respect to force at that point.

$$\delta_r = \frac{\partial U}{\partial P_r}$$

where U = Total strain energy

P_r = Force on the point 'r'

Beam	Strain energy	δ
	$\frac{W^2 l^3}{6EI}$	$\frac{Wl^3}{3EI}$
	$\frac{W^2 a^2 b^2}{6EIl}$	$\frac{Wa^2 b^2}{3EIl}$
	$\frac{\pi P^2 r^3}{4EI}$	$\frac{\pi P r^3}{2EI}$

Assuptions for castigliano first theorem :

I) material should be elastic and obeys hook's law

II) Temperature remains constant

III) Supports are ~~o~~ unyeild.

iv) when a beam is subjected to both axial & bending then strain energy is given by

$$S.E = \Sigma \frac{P^2 L}{2AE} + \Sigma \int \frac{M^2 ds}{2EI}$$

v) when a couple is applied on a beam then rotation is given by

$$\theta = \frac{\partial U}{\partial M_0}$$

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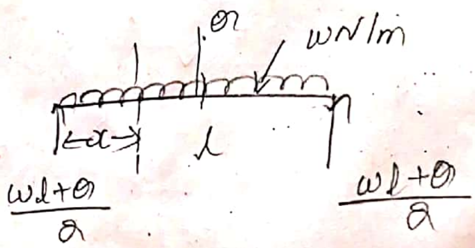
Note:

To find out the deflection at any point for a beam or trust subjected U.D.L introduce ficitious force and wherever we need the deflection and then partial differentiate w.r.to the ficitious force.

Find the deflection for a simple supported beam subjected to U.D.L



$$M_x = \left[\left(\frac{wl+Q}{2}\right)x - wx \times \frac{x}{2}\right]$$

$$\delta = \frac{\partial U}{\partial Q}$$

$$= \frac{\partial}{\partial Q}\left[\int \frac{M^2 dx}{2EI}\right]$$

$$= \frac{1}{2EI}\int \left[\frac{\partial}{\partial Q} M^2\right] dx$$

$$\therefore \frac{\partial (M_x)^2}{\partial Q} = \frac{\partial}{\partial Q}\left[\left(\frac{wl+Q}{2}\right)x - \frac{wx^2}{2}\right]^2$$

$$= 2\left[\left(\frac{wl+Q}{2}\right)x - \frac{wx^2}{2}\right] \times \frac{\partial}{\partial Q}\left[\left(\frac{wl+Q}{2}\right)x - \frac{wx^2}{2}\right]$$

$$= 2\left[\frac{(wl+Q)x}{2} - \frac{wx^2}{2}\right] x \frac{x}{2}$$

$$= (wl+Q)\frac{x^2}{2} - \frac{wx^3}{2}$$

$$\int \frac{\partial m^2}{\partial Q}\,dx = \int \left((wl+Q)\frac{x^2}{2} - \frac{wx^3}{2} \right) dx$$

$$= (wl+Q)\frac{x^3}{6} - \frac{wx^4}{8}$$

$$\delta = \frac{1}{2EI} \times 2 \int_0^{l/2} \left[\frac{\partial}{\partial Q} m^2 \right] dx$$

$$= \frac{1}{EI}\left[(wl+Q)\frac{x^3}{6} - \frac{wx^4}{8} \right]_0^{l/2}$$

$$\therefore Q = 0$$

$$= \frac{1}{EI}\left[wl \times \frac{l^3}{48} - \frac{wl^4}{128} \right]$$

$$\therefore \delta = \frac{5}{384}\frac{wl^4}{EI}$$

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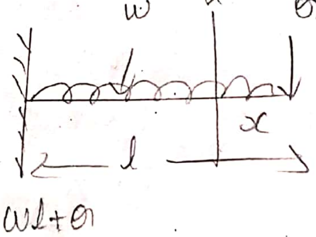
Find the deflection for a cantilever beam subjected to U.D.L



$$M_x = -Qx - w\cdot x\cdot\frac{x}{2}$$

$$= -Qx - \frac{wx^2}{2}$$

$$= -\left[Qx + \frac{wx^2}{2} \right]$$

$$S.E = \int \frac{m^2}{2EI}\,dx$$

$$= \frac{1}{2EI}\int \left(Qx + \frac{wx^2}{2} \right)^2 dx$$

$$\delta = \frac{\partial U}{\partial Q}$$

$$= \frac{\partial}{\partial Q}\left[\frac{1}{2EI}\int \left(Qx + \frac{wx^2}{2} \right)^2 dx \right.$$

$$\delta = \frac{1}{2EI}\int\left[\frac{\partial}{\partial\theta}\left[\theta x + \frac{wx^2}{2}\right)^2\right]dx$$

$$= \frac{1}{\cancel{2}EI}\int \cancel{2}\cdot\left(\theta x + \frac{wx^2}{2}\right)\cdot x\, dx$$

$$\therefore \frac{\partial}{\partial x} f(y) = f'(y)\frac{\partial y}{\partial x}$$

$$= \frac{1}{EI}\int\left(\theta x^2 + \frac{wx^3}{2}\right)dx$$

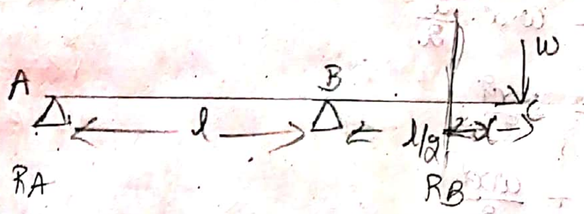
$$= \frac{1}{EI}\left[\theta\frac{x^3}{3} + \frac{wx^4}{8}\right]_0^l$$

Here $\theta = 0$

$$= \frac{1}{EI}\left[\frac{wx^4}{8}\right]_0^l$$

$$= \frac{1}{EI}\frac{wl^4}{8}$$

$$\delta = \frac{wl^4}{8EI}$$



$R_A = -\frac{w}{2}$

$R_B = \frac{3w}{2}$

Total S.E = S.E AB + S.E BC

$$= \int_{-l/2}^{l/2}\left(-\frac{w}{2}x\right)^2\frac{dx}{2EI} + \int_0^{l/2}(-wx)^2\frac{dx}{2EI}$$

$$= \frac{2}{2EI}\int_0^{l/2}\frac{w^2x^2}{4}dx + \int_0^{l/2}\frac{w^2x^2}{2EI}dx$$

$$= \frac{w^2}{4EI}\int_0^{l/2} x^2\,dx + \frac{w^2}{2EI}\int_0^{l/2} x^2\,dx$$

$$= \frac{w^2}{4EI}\left[\frac{x^3}{3}\right]_0^{l/2} + \frac{w^2}{2EI}\left[\frac{x^3}{3}\right]_0^{l/2}$$

$$= \frac{w^2}{4EI} \times \frac{l^3}{24} + \frac{w^2}{2EI} \times \frac{l^3}{24}$$

$$= \frac{w^2 l^3}{96EI} + \frac{w^2 l^3}{48EI}$$

$$S.E = \frac{w^2 l^3}{32EI}$$

Deflection of trusses (using strain energy theorem).

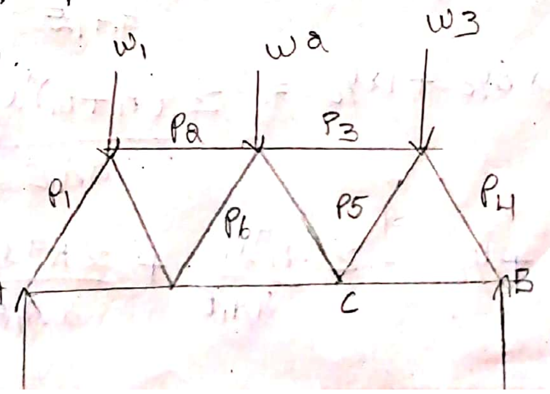
* It is assumed that loads are transfered axially in the members. i.e There won't be any moments.

* Deflection in trusses can be found using the basic strain energy method. ($S.E = \frac{1}{2} w \delta$), But If the member of trusses increases it will be very difficult to calculate the deflection. So a reciprocal theorem is introduced. (maxwell)

* Let the truss subjected to External load system w_1, w_2, w_3 ---

* Let the deflection at joint C be y_c.

* Let P_1, P_2, P_3, P_4 be the forces in various number of forces due to external loading system.



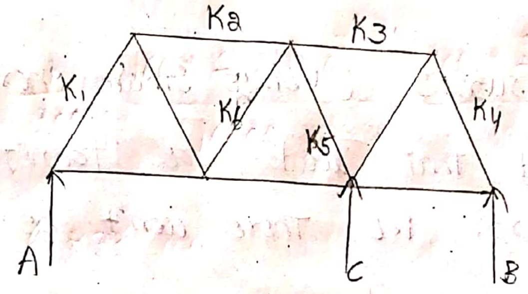
External work done $= w_e$

Total $SE = \dfrac{\Sigma P_1^2 l_1}{2A_1E}$

$$w_e = \frac{\Sigma P_1^2 l_1}{2A_1E} \quad \text{— (1)}$$

2. Let the external load system will be removed and unit load be applied, forces generated in members due to the unique load be K_1, K_2, K_3 ---

Let the deflection at C due to unique load be 'δy_c'



work done $= \frac{1}{2} \times 1 \times \delta y_c$

$$S.E \text{ Stored} = \Sigma \frac{K_1^2 l_1}{2A_1E}$$

$$\frac{1}{2} \times 1 \times \delta y_c = \Sigma \frac{K_1^2 l_1}{2A_1E}$$

If we apply both unique load and External load then the forces generated in the members are given by $P_1K_1 + P_2K_2$ ---

Total work done $= w_e + \frac{1}{2} \times 1 \times \delta y_c + 1 \times y_c$

$$\text{Total } S.E = \Sigma \frac{(P_1 + K_1)^2 l_1}{2A_1E}$$

$$w_e + \frac{1}{2} \times 1 \times \delta y_c + 1 \times y_c = \Sigma \frac{(P_1 + K_1)^2 l_1}{2A_1E}$$

$$w_e + \frac{1}{2}\delta y_c + y_c = \Sigma \frac{P_1^2 l_1}{2A_1E} + \frac{\Sigma K_1^2 l_1}{2A_1E} + \Sigma \frac{2P_1K_1l_1}{2A_1E}$$

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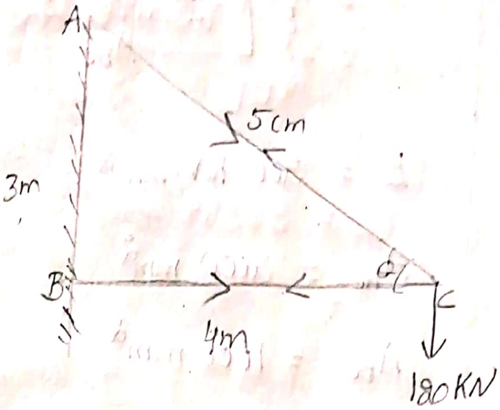
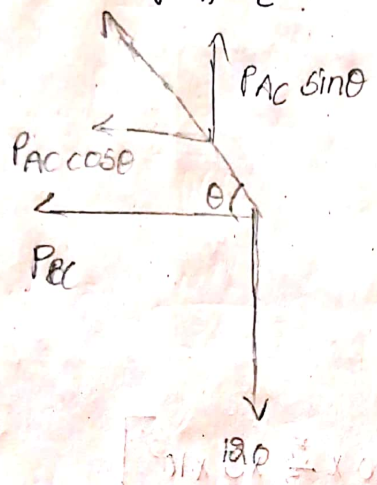
$$y_c = \sum \frac{P_1 K_1 l_1}{A_1 E}$$

Find out the deflection for the following truss.



Consider joint C :-



$\therefore \sin\theta = \frac{3}{5}$

$\cos\theta = 4/5$

$\tan\theta = \frac{3}{4}$

$$y = \sum \frac{P_1 K_1 l_1}{A_1 E}$$

$$y = \frac{1}{E} \sum \frac{P_1 K_1 l_1}{A_1}$$

$$= \frac{1}{E} \sum \sigma_1 K_1 l$$

$\sum V = 0$

$P_{AC} \sin\theta = 180$

$P_{AC} = 180 \times \frac{5}{3}$

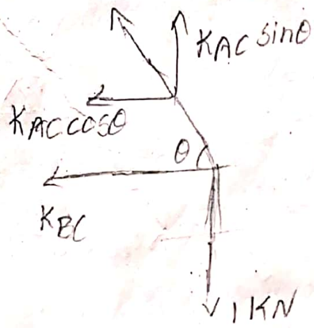
$= 300$ KN

$\sum H = 0$

$P_{BC} + P_{AC} \cos\theta = 0$

$P_{BC} = -P_{AC} \cos\theta$

$= -160$ KN



$\sum V = 0$

$K_{AC} \sin\theta = 1$

$K_{AC} = 1.66$ KN

$\sum H = 0$

$K_{BC} + K_{AC} \cos\theta = 0$

$K_{BC} = -K_{AC} \times \frac{4}{5}$

$K_{BC} = -1.328$ KN

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$$Y_c = \frac{1}{E} \sum \frac{P_1 K_1 L_1}{A_1}$$

$$= \frac{1}{E}\left[\frac{P_{BC} K_{BC} L_{BC}}{A_{BC}} + \frac{P_{AC} K_{AC} L_{AC}}{A_{AC}}\right]$$

$E = 200 \; KN/mm^2$

$A_{AC} = 2000 \; mm^2$

$A_{BC} = 1600 \; mm^2$

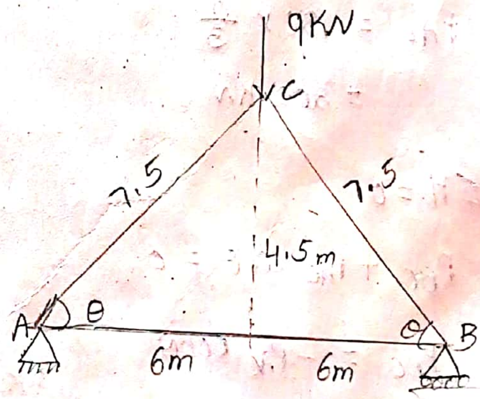
$L_{BC} = 4 \times 10^3 \; mm$

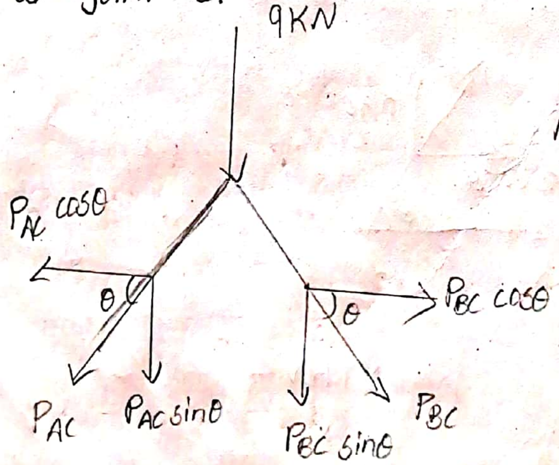
$L_{AC} = 5 \times 10^3 \; mm$

$$Y_c = \frac{1}{200}\left[\frac{160 \times \frac{4}{3} \times 4 \times 10^3}{1600} + \frac{200 \times \frac{5}{3} \times 5 \times 10^3}{2000}\right]$$

$Y_c = 6.833 \; mm$

Find out the deflection at joint c.





$\sin\theta = \frac{4.5}{7.5}$

$\cos\theta = \frac{6}{7.5}$

$\Sigma V = 0$

$P_{AC} \sin\theta + P_{BC} \sin\theta + 9 = 0$

$(P_{AC} + P_{BC}) \sin\theta = -9$

$P_{AC} + P_{BC} = -15$

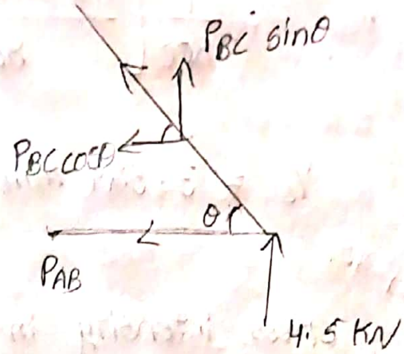
$\Sigma H = 0$

$P_{AC} \cos\theta = P_{BC} \cos\theta$

$P_{AC} = P_{BC}$

$\therefore \; P_{AC} = P_{BC} = -7.5$

Consider joint B :-

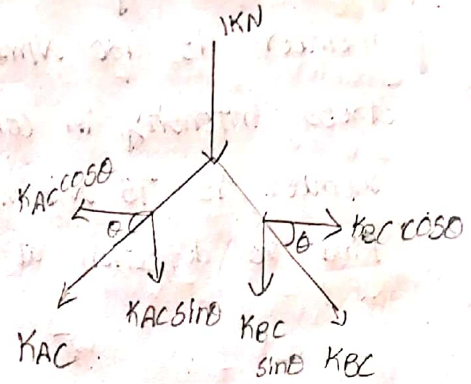


$\Sigma H = 0$

$P_{AB} + P_{BC} \cos\theta = 0$

$P_{AB} = -(-7.5) \times \frac{6}{7.5}$

$P_{AB} = 6 \text{ KN}$



$\Sigma V = 0$

$K_{AC} \sin\theta + K_{BC} \sin\theta + 1 = 0$

$(K_{AC} + K_{BC}) \sin\theta = -1$

$K_{AC} + K_{BC} = -\frac{5}{3}$

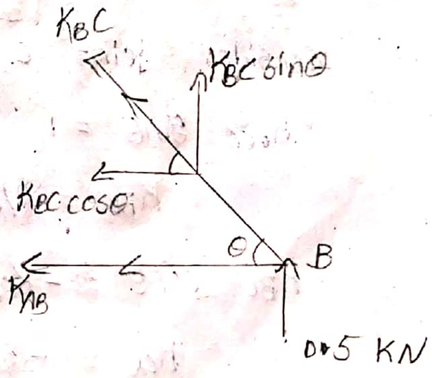
$\Sigma H = 0$

$K_{AC} \cos\theta = K_{BC} \cos\theta$

$K_{AC} = K_{BC}$

$2 K_{AC} = -\frac{5}{3}$

$\therefore \; K_{AC} = K_{BC} = -0.83$



Consider joint at B :-

$\Sigma H = 0$

$K_{BC} \cos\theta + K_{AB} = 0$

$K_{AB} = -K_{BC} \cos\theta$

$= -(-0.83)\left(\frac{6}{7.5}\right)$

$= 0.664$

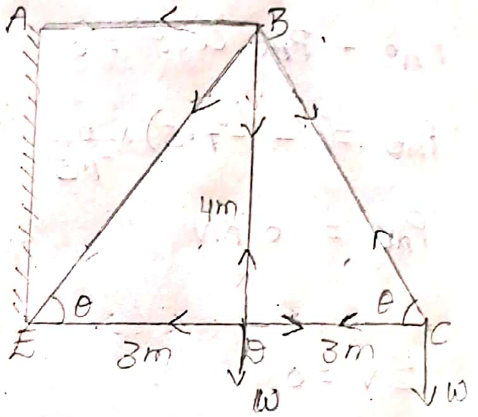
Area for all member $= 200 \text{ mm}^2$

$E = 200 \text{ KN/mm}^2$

$$Y_C = \frac{1}{E}\left[\frac{P_{BC}\,K_{BC}\,L_{BC}}{A_{BC}} + \frac{P_{AC}\,K_{AC}\,L_{AC}}{A_{AC}}\right] + \frac{P_{AB}\,K_{AB}\,L_{AB}}{A_{AB}}$$

$$= \frac{1}{200}\left[\frac{7.5\times0.83\times7.5\times10^3}{200} + \frac{7.5\times0.83\times7.5\times10^3}{200} + \frac{6\times0.664\times12}{200}\right]$$

$$Y_C = 3.529\ mm$$



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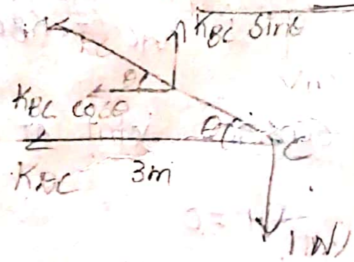
Stress intensity in tension member is 120 N/mm².
Stress intensity in compression member is 75 N/mm².
Find the deflection at C.

member	σ	K	L	$\sigma KL/E$
AB	120	3/2	3	54×10^4
BC	120	5/4	5	75×10^4
CD	-75	-3/4	3	168750
DE	-75	-3/4	3	168750
BD	0	0	4	0
EB	-75	-5/4	5	468750
				10.48

Consider joint C



$K_{BC}\sin\theta = 1$

$K_{BC} = 5/4$

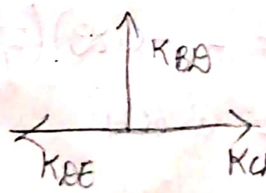
$\sin\theta = 4/5$

$\cos\theta = 3/5$

$K_{BC}\cos\theta = -K_{CD}$

$K_{CD} = -\frac{5}{4}\times\frac{3}{5}$

$K_{CD} = \frac{-3}{4}$



$K_{BD} = 0$

$K_{DE} = K_{CD}$

$K_{DE} = \frac{-3}{4}$

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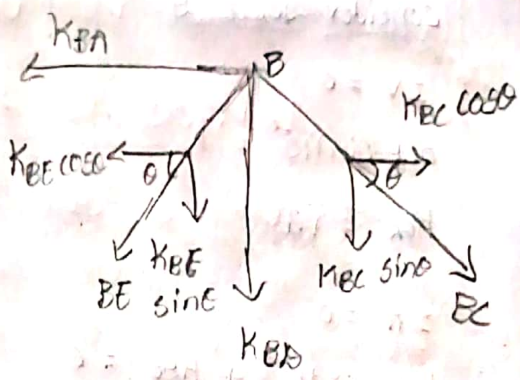
$\Sigma V = 0$



$K_{BD} + K_{BE}\sin\theta + K_{BC}\sin\theta = 0$

$K_{BE} \times \frac{4}{5} = -\frac{5}{4} \times \frac{4}{5}$

$K_{BE} = -\frac{5}{4}$

$\Sigma H = 0$

$K_{BE}\cos\theta + K_{AB} = K_{BC}\cos\theta$

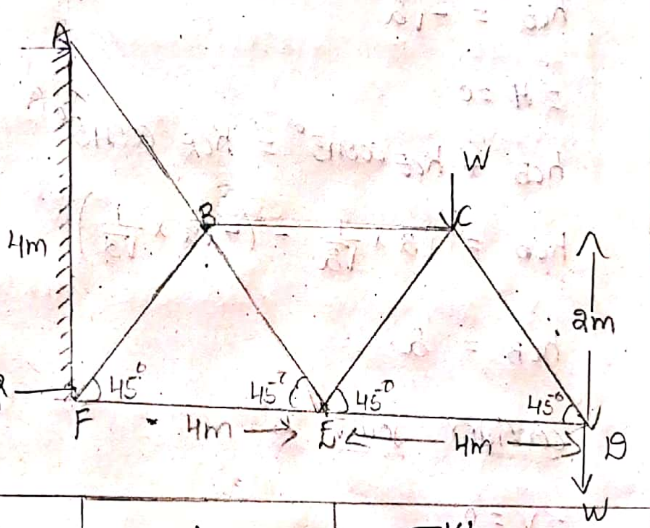
$\left(-\frac{5}{4}\right)\left(\frac{3}{5}\right) + K_{AB} = \left(\frac{5}{4}\right)\left(\frac{3}{5}\right)$

$K_{AB} = \frac{3}{4} + \frac{3}{4}$

$= \frac{6}{4}$

$= \frac{3}{2}$

Stress intensity in tension member is 140 N/mm². Stress intensity in compression member is 80 N/mm². Determine deflection at D. $E = 2\times10^5$ N/mm²



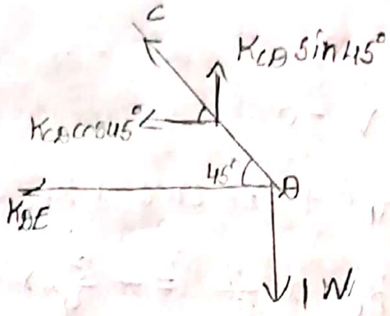
member	σ	K	L	σKL
AB	140	$2\sqrt{2}$	$2\sqrt{2}\times10^3$	1.12×10^6
BC	140	2	4×10^3	1.12×10^6
CD	140	$\sqrt{2}$	$2\sqrt{2}\times10^3$	56×10^4
DE	−80	−1	4×10^3	32×10^4
EF	−80	−3	4×10^3	96×10^4
FB	140	$\sqrt{2}$	$2\sqrt{2}\times10^3$	56×10^4
BE	140	$\sqrt{2}$	$2\sqrt{2}\times10^3$	56×10^4
EC	−80	$-\sqrt{2}$	$2\sqrt{2}\times10^3$	32×10^4

Consider joint D

$\Sigma V = 0$

$K_{CD} \sin 45^\circ = 1$

$K_{CD} = \sqrt{2}$



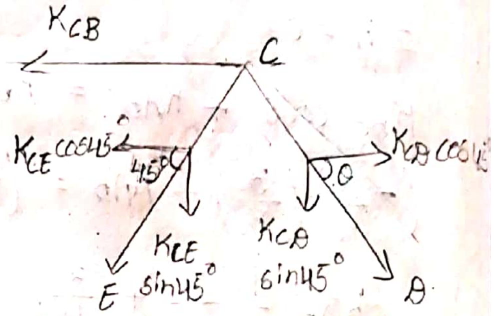
$\Sigma H = 0$

$K_{CD} \cos 45^\circ + K_{DE} = 0$

$\sqrt{2} \times \frac{1}{\sqrt{2}} + K_{DE} = 0$

$K_{DE} = -1$

Consider joint C



$\Sigma V = 0$

$K_{CE} \sin 45^\circ + K_{CD} \sin 45^\circ = 0$

$K_{CE} \times \frac{1}{\sqrt{2}} = -\sqrt{2} \times \frac{1}{\sqrt{2}}$

$K_{CE} = -\sqrt{2}$

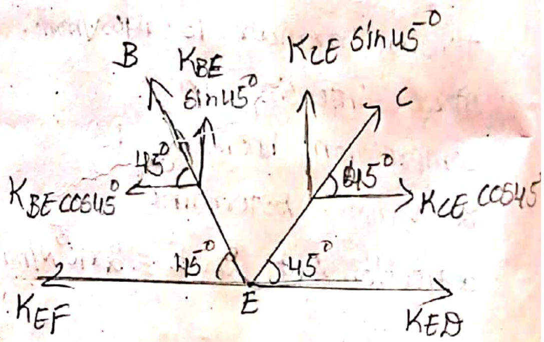
$\Sigma H = 0$

$K_{CB} + K_{CE} \cos 45^\circ = K_{CD} \cos 45^\circ$

$K_{CB} = \sqrt{2} \times \frac{1}{\sqrt{2}} - \left(-\sqrt{2} \times \frac{1}{\sqrt{2}}\right)$

$K_{CB} = 2$

Consider joint E



$\Sigma V = 0$

$K_{BE} \sin 45^\circ + K_{CE} \sin 45^\circ = 0$

$K_{BE} \times \frac{1}{\sqrt{2}} = -K_{CE}\left(\frac{1}{\sqrt{2}}\right)$

$= -\left(-\sqrt{2} \times \frac{1}{\sqrt{2}}\right)$

$K_{BE} = \sqrt{2}$

$\Sigma H = 0$

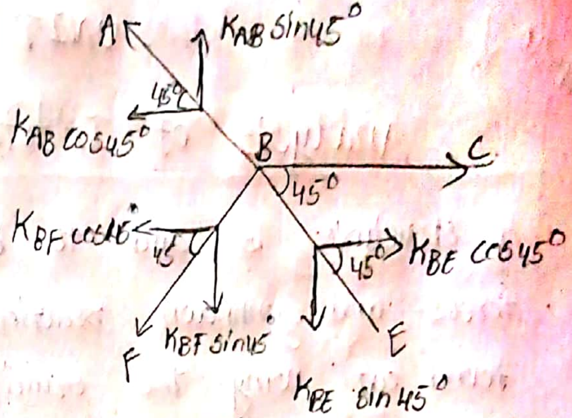
$K_{BE} \cos 45^\circ + K_{EF} = K_{CE} \cos 45^\circ + K_{ED}$

$\sqrt{2} \times \frac{1}{\sqrt{2}} + K_{EF} = -\sqrt{2} \times \frac{1}{\sqrt{2}} + (-1)$

$K_{EF} = -1 - 1 - 1$

$K_{EF} = -3$

Consider joint B



$\Sigma V = 0$

$$K_{AB} \times \frac{1}{\sqrt{2}} = K_{BF} \times \frac{1}{\sqrt{2}} + K_{BE} \times \frac{1}{\sqrt{2}}$$

$$\frac{K_{AB}}{\sqrt{2}} - \frac{K_{BF}}{\sqrt{2}} = \sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$\frac{K_{AB}}{\sqrt{2}} - \frac{K_{BF}}{\sqrt{2}} = 1$$

$$K_{AB} - K_{BF} = \sqrt{2}$$

$\Sigma H = 0$

$$K_{AB} \times \frac{1}{\sqrt{2}} + K_{BF} \times \frac{1}{\sqrt{2}} = K_{BC} + K_{BE} \times \frac{1}{\sqrt{2}}$$

$$\frac{K_{AB}}{\sqrt{2}} + \frac{K_{BF}}{\sqrt{2}} = 2 + \sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$\frac{K_{AB}}{\sqrt{2}} + \frac{K_{BF}}{\sqrt{2}} = 2+1$$

$$K_{AB} + K_{BF} = 3\sqrt{2}$$

solving both eq.

$$K_{AB} + \cancel{K_{BF}} = 3\sqrt{2}$$
$$K_{AB} - \cancel{K_{BF}} = \sqrt{2}$$

$$2K_{AB} = 3\sqrt{2} + \sqrt{2}$$

$$2K_{AB} = (3+1)\sqrt{2}$$

$$2K_{AB} = 4\sqrt{2}$$

$$K_{AB} = 2\sqrt{2}$$

$$2\sqrt{2} + K_{BF} = 3\sqrt{2}$$

$$K_{BF} = 3\sqrt{2} - 2\sqrt{2}$$

$$K_{BF} = \sqrt{2}$$

$$\therefore \Sigma \frac{\sigma KL}{E} = \frac{5.52 \times 10^6}{2 \times 10^5} = 27.6\,mm$$