SLOPE DEFLECTION METHOD

PRESENTATION OVERVIEW:

1. INTRODUCTION.(Basic Idea of Slope Deflection Method)
2. ASSUMPTIONS IN THE SLOPE DEFLECTION METHOD .
3. APLICATION OF SLOPE DEFLECTION METHOD.
4. SIGN CONVENTION.
5. PROCEDURE.
6. SLOPE DEFLECTION EQUATION.
7. EXAMPLE.

INTRODUCTION:

- *This method was developed by axel bendexon in Germany in 1914. This method is applicable to all types of statically indeterminate beams & frames and in this method, we solve for unknown joint rotations, which are expressed in terms of the applied loads and the bending moments.Deflections due to shear and axial stresses are not considered as the effect are small.*
- ***Indeterminate structure:*** *the structure which can not be analyzed by the equations of static equilibriums alone are called indeterminate structures.*

ASSUMPTIONS IN THE SLOPE DEFLECTION METHOD:

This method is based on the following simplified assumptions:

- *All the joints of the frame are rigid,*
- *Distortion, due to axial and shear stresses, being very small, are neglected.*

APPLICATIONS OF SLOPE DEFLECTION METHOD:

1. Continuous Beams
2. Frames with out side sway
3. Frames with side sway

SIGN CONVENTION:-

(1)ROTATIONS:- Clockwise joint rotations are considered as

- (-ve).

(2) END MOMENTS:- clockwise end moments are considered as (+ve).

The procedure is as follows:

(1) Determine the fixed end moments at the end of each span due to applied loads acting on span by considering each span as fixed ended. Assign ± Signs w.r.t. above sign convention.

(2) Express all end moments in terms of fixed end moments and the joint rotations by using slope – deflection equations.

(3) Establish simultaneous equations with the joint rotations as the unknowns by applying the condition that sum of the end moments acting on the ends of the two members meeting at a joint should be equal to zero.

(4) Solve for unknown joint rotations.

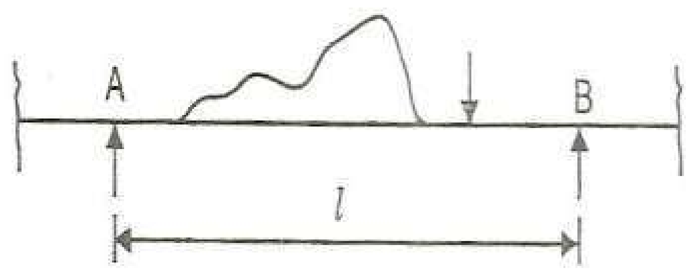
(5) Substitute back the end rotations in slope – deflection equations and compute the end moments.

(6) Determine all reactions and draw S.F. and B.M. diagrams and also sketch the elastic curve

SLOPE DEFLECTION EQUATION:

General slope-deflection equations:

$$M_{AB}=M_{FAB}+\frac{2EI}{l}\left(2\theta_A+\theta_B+\frac{3\Delta}{l}\right)$$



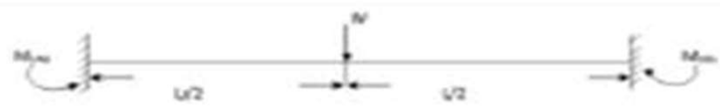
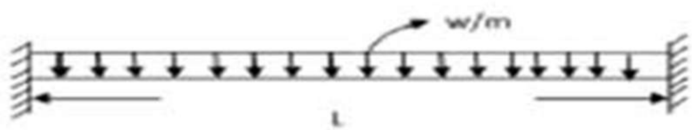
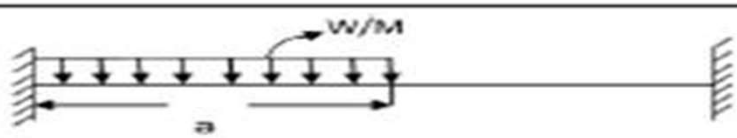
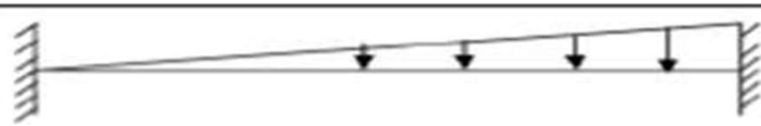
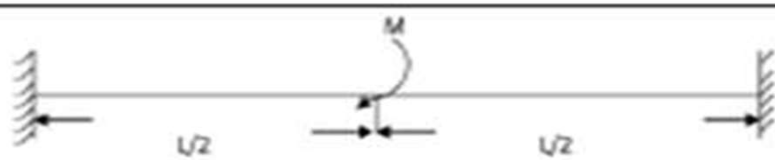
$$M_{BA}=M_{FBA}+\frac{2EI}{l}\left(2\theta_B+\theta_A+\frac{3\Delta}{l}\right)$$

M_{FAB}, M_{FBA} - Fixed end moments at A and B respectively due to the given loading

θ_A, θ_B - Slopes at A and B respectively

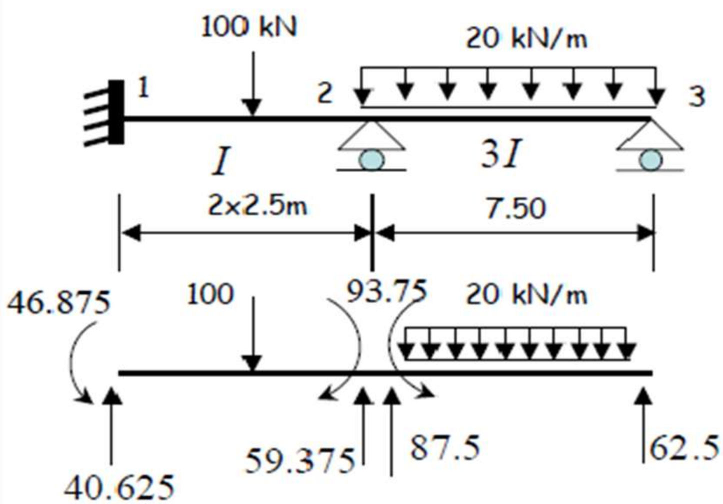
Δ - Sinking of support A with respect to B

EQUATION FOR FIXED END MOMENT:

S.No	CASES	BOTH ENDS FIXED	
		M_{FAB}	M_{FBA}
1		$-\frac{wl}{8}$	$+\frac{wl}{8}$
2		$-\frac{wl^2}{12}$	$+\frac{wl^2}{12}$
3		$-\frac{wab^2}{l^2}$	$+\frac{wab^2}{l^2}$
4		$-\frac{wa}{12l^2}(6l^2-8al+3a^2)$	$+\frac{wa^2}{12l^2}(4l^2-3a)$
5		$-\frac{wl^2}{30}$	$+\frac{wl^2}{20}$
6		$+\frac{m}{4}$	$+\frac{m}{4}$
7		$\frac{Mb}{l^2}(3a-l)$	$\frac{Ma}{l^2}(3b-l)$

EXAMPLE FOR BEAM:

Example: It is required to determine the support moments for the continuous beam.



$$-M_{12}^F = M_{21}^F = \frac{100*5}{8} = 62.5kNm$$

$$-M_{23}^F = M_{32}^F = \frac{20*7.5^2}{12} = 93.75kNm$$

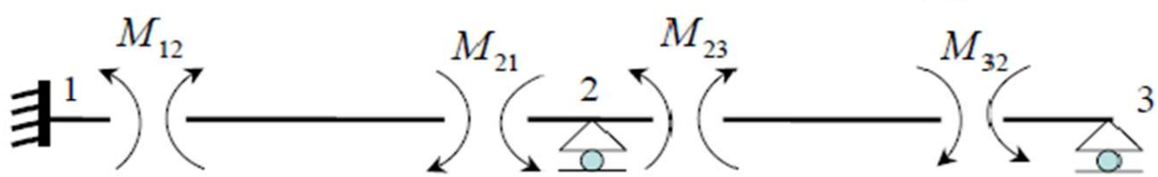
Slope − Deflection.Equations

$$M_{12} = \frac{2EI}{5}\theta_2 - 62.5 =$$

$$M_{21} = \frac{2EI}{5}2\theta_2 + 62.5 =$$

$$M_{23} = \frac{6EI}{7.5}(2\theta_2 + \theta_3) - 93.75 =$$

$$M_{32} = \frac{6EI}{7.5}(\theta_2 + 2\theta_3) + 93.75 =$$



Equilibrium.equations.of.joints

$$M_{21} + M_{23} = 0$$

$$M_{32} = 0$$

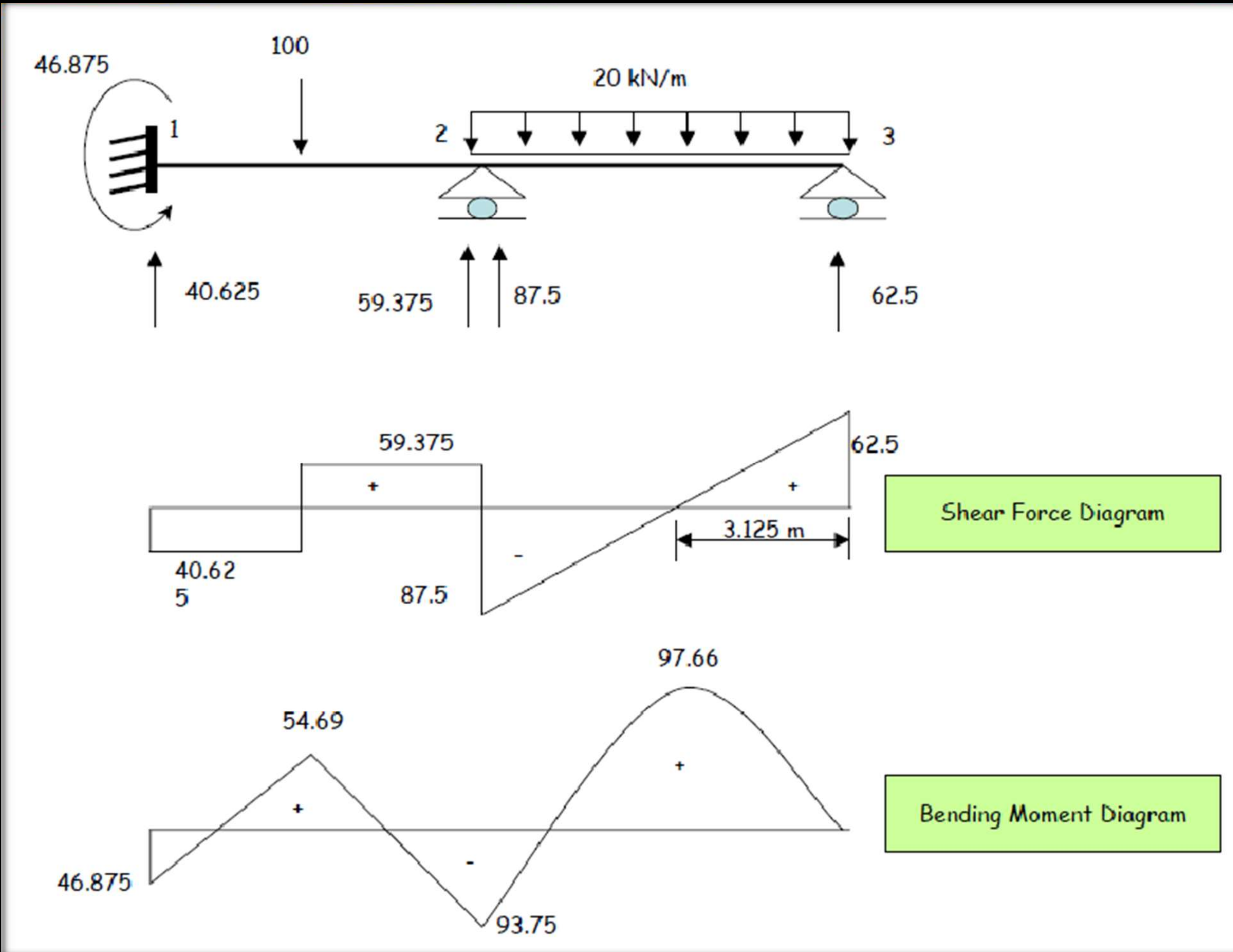
$$2.4EI\theta_2 + 0.8EI\theta_3 = 31.25$$

$$0.8EI\theta_2 + 1.6EI\theta_3 = -93.75 \rightarrow \theta_2 = \frac{39.0625}{EI} \rightarrow \theta_3 = \frac{-78.125}{EI}$$

Substitude.these.results.in.slope.deflection.equations

$$M_{12} = -46.875kNm, \rightarrow M_{21} = 93.75kNm$$

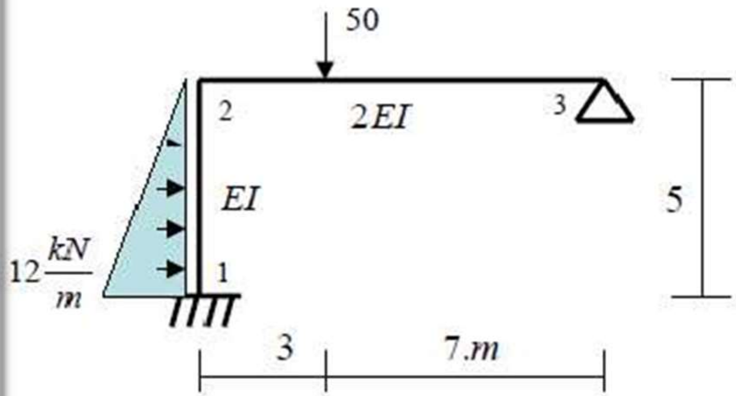
$$M_{23} = -93.75kNm, \rightarrow M_{32} = 0kNm$$



46.875
100
20 kN/m
1
2
3
40.625
59.375
87.5
62.5
59.375
62.5
3.125 m
40.62
5
87.5
Shear Force Diagram
97.66
54.69
46.875
93.75
Bending Moment Diagram

EXAMPLE FOR FRAME:

Example: Find member end moments and draw the diagrams of the frame



$$-M_{12}^{F}=\frac{q_0*L^2}{20}=\frac{12*5^2}{20}=15$$

$$M_{21}^{F}=\frac{q_0*L^2}{30}=\frac{12*5^2}{30}=10kNm$$

$$-M_{23}^{F}=\frac{Pab^2}{L^2}=\frac{50*3*7^2}{10^2}=73.5kNm$$

$$M_{32}^{F}=\frac{Pa^2b}{L^2}=\frac{50*3^2*7}{10^2}=31.5kNm$$

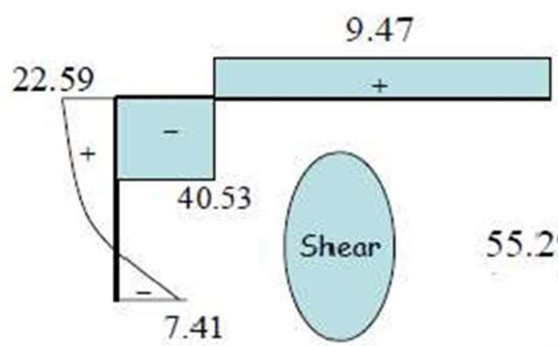
Slope – Deflection.Equations

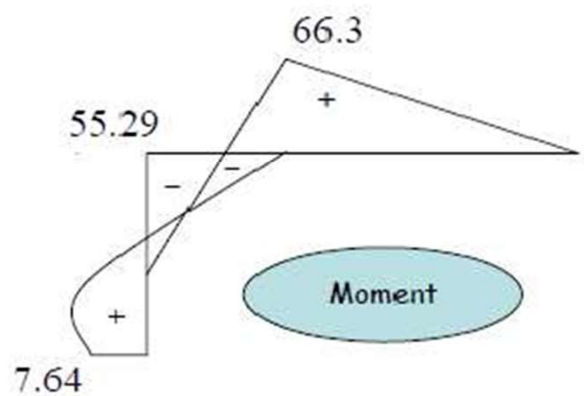
$$M_{12}=\frac{2EI}{5}\theta_2-15=$$

$$M_{21}=\frac{2EI}{5}2\theta_2+10=$$

$$M_{23}=\frac{4EI}{10}(2\theta_2+\theta_3)-73.5=$$

$$M_{32}=\frac{4EI}{10}(\theta_2+2\theta_3)+31.5=$$





$$M_{21}+M_{23}=0$$

$$M_{32}=0$$

$$\theta_2=\frac{56.61}{EI}\rightarrow\theta_3=\frac{-67.68}{EI}$$

Substitude.these.results.in.slope.deflection.equations

$$M_{12}=7.64kNm,\rightarrow M_{21}=55.29kNm$$

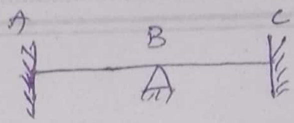
$$M_{23}=-55.29kNm,\rightarrow M_{32}=0kNm$$

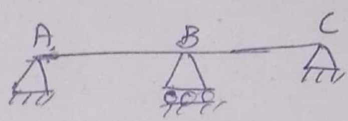
Steps of [SDM]

1) Find Fixed end Moment AB, BA ; BC, CB ;

2) Apply slope deflection Equation For each span

3) Apply condition of Equilibrium

$\boxed{M_{BA} + M_{BC} = 0}$ $\longrightarrow$ 

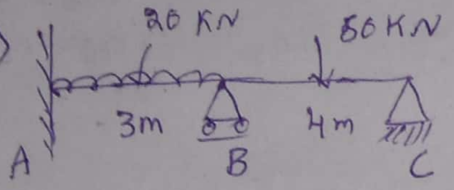
$\boxed{M_{AB} = 0 \quad M_{CB} = 0}$ $\longrightarrow$ 

4) Find Final Moment

B.M.D. { 5) Find support reaction by seperating AB, BC

6) Find final support reaction

7) Draw SFD & B.M.D

1) 

$\therefore \theta_A = 0$ (fixed end)

i) Find Fixed end moments

$M_{AB} = -\frac{wl^2}{12} = -\frac{20\times 9}{12} = -15$ KN-m

$M_{BA} = 15$ KN-m

$M_{BC} = -\frac{wL}{8} = \frac{-50\times 4}{8} = -25$

$M_{CB} = \frac{wL}{8} = 25$ KN-m

2) Apply S.D.E for span AB

$$M_{AB} = M_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$M_{AB} = -15 + \frac{2EI}{3}\left(2(0) + \theta_B - 0\right)$$

$$M_{AB} = -15 + \frac{2EI}{3}\theta_B \quad — ①$$

$$M_{BA} = 15 + \frac{2EI}{3}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)$$

$$= 15 + \frac{2EI}{3}(2\theta_B) \Rightarrow 15 + \frac{4EI}{3}\theta_B \quad — ②$$

Apply S.D.E for span BC

$$M_{BC} = M_{BC} + \frac{2EI}{L}\left[2\theta_B + \theta_C - \frac{3\Delta}{L}\right]$$

$$= -25 + \frac{2EI}{L}\left[2\theta_B + \theta_C - 0\right] \Rightarrow -25 + \frac{2EI}{4}(2\theta_B + \theta_C)$$

$$M_{BC} = -25 + \frac{EI}{2}(2\theta_B + \theta_C) \longrightarrow ③$$

$$M_{CB} = 25 + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\Delta}{L}\right)$$

$$= 25 + \frac{2EI}{4}\left[2\theta_C + \theta_B\right]$$

$$M_{CB} = 25 + \frac{EI}{2}\left[2\theta_C + \theta_B\right] \longrightarrow ④$$

3) Apply condition of Equilibrium

$M_{BA} + M_{BC} = 0$ $[M_{AB} = 0, M_{CB} = 0]$ → both ends are simply supported

[at simply supported end the bending moment is zero]

$15 + \frac{4EI}{3}\theta_B + \left(-25 + \frac{1}{2}EI\left[2\theta_B + \theta_C\right]\right)$

$15 + \frac{4}{3}EI\theta_B - 25 + EI\theta_B + \frac{1}{2}EI\theta_C = 0$

$\frac{7}{3}EI\theta_B + \frac{1}{2}EI\theta_C - 10 = 0$

$\frac{7}{3}EI\theta_B + \frac{1}{2}EI\theta_C = 10 \longrightarrow (1)$

Here $M_{CB} = 0$

$25 + \frac{1}{2}EI\theta_B + EI\theta_C = 0$

$\frac{1}{2}EI\theta_B + EI\theta_C = -25 \longrightarrow (2)$

Solve (1) and (2)

$EI\theta_B = 10.8$ $\theta_C = -30.4$

4) Find final moments Put $EI\theta_B$ & $EI\theta_C$ in eq (1) (2) (3) (4)

$EI\theta_B = 10.8$ $EI\theta_C = -30.4$

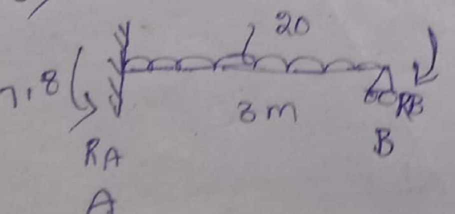
$M_{AB} = -15 + \frac{2}{3}(10.8) = -7.8$ KN-m

$M_{BA} = 15 + \frac{4}{3}(10.8) = 29.4$ KN-m

$M_{BC} = -25 + 10.8 + \frac{1}{2}(-30.4) = -29.4$ KN.m

$M_{CB} = 25 + \frac{1}{2}(10.8) + (-30.4) = 0$

5) Find support reaction

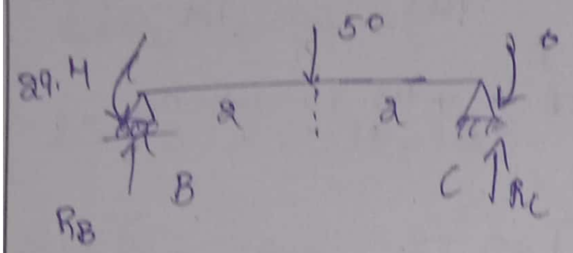


$\Sigma M_A = 0$ ↻+

$-7.8 + 20 \times 3 \times \frac{3}{2} + 29.4 - R_B \times 3 = 0$

$R_B = 37.2$ KN.

$R_A + R_B = 20 \times 3 \Rightarrow R_A = 60 - 37.2 \Rightarrow R_A = 22.8$ KN.



$\Sigma M_B = 0$

$-29.4 + 50 \times 2 - R_C \times 4 = 0 \Rightarrow R_C = 17.65$

$R_B + R_C = 50 \Rightarrow R_B = 32.35$ KN.

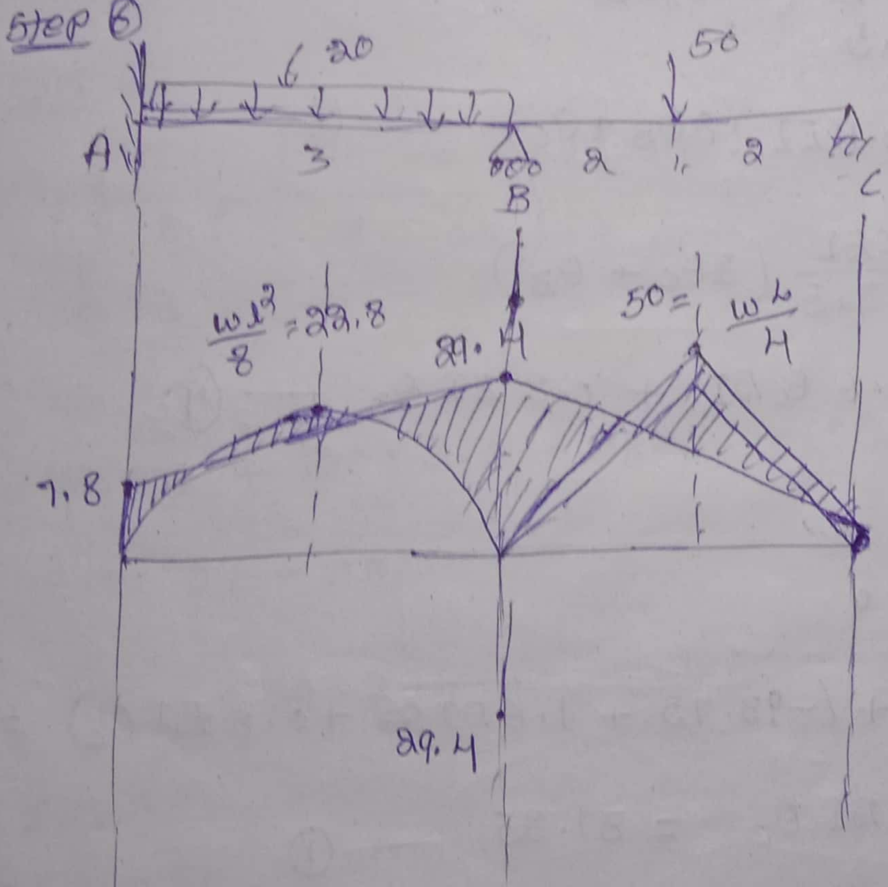
Final reaction

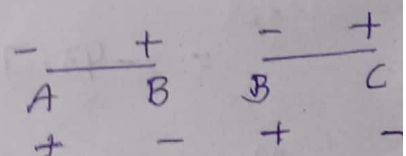
$R_A = 22.8$ KN

$R_B = 37.2 + 32.35 = 69.55$ KN

$R_C = 17.65$ KN.

Step 6

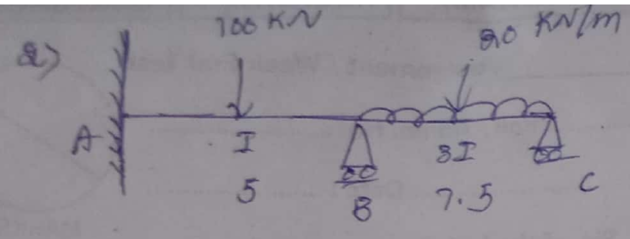




//// — Net B.M.D

/ — B.M.D

2)



Step (1)

$$M_{AB}^{F} = \frac{-wl}{8} = \frac{100 \times 5}{8} = -62.5 \qquad M_{BC}^{F} = \frac{-wl^2}{12} = \frac{20(7.5)^2}{12} = -93.75$$

$$M_{BA}^{F} = +62.5 \qquad M_{CB}^{F} = +93.75 \text{ KN-m}$$

Step (2)

$$M_{AB} = M_{AB}^{F} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$= -62.5 + \frac{2EI}{5}(0 + \theta_B - 0) \Rightarrow -62.5 + \frac{2EI}{5}\theta_B \quad \text{— (1)}$$

$$M_{BA} = M_{BA} + \frac{2EI}{L}\left[2\theta_B + \theta_A - \frac{3\Delta}{L}\right]$$

$$= 62.5 + \frac{2EI}{5}[2\theta_B] \Rightarrow 62.5 + \frac{4EI}{5}\theta_B \quad \text{— (2)}$$

$$M_{BC} = M_{BC} + \frac{2E(3I)}{L}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right)$$

$$= -93.75 + \frac{6EI}{7.5}(2\theta_B + \theta_C)$$

$$= -93.75 + 0.8EI(2\theta_B + \theta_C) \quad \text{— (3)}$$

$$M_{CB} = 93.75 + \frac{6EI}{7.5}(2\theta_C + \theta_B)$$

$$= 93.75 + 1.6\,\theta_C EI + 0.8\,EI\,\theta_B \quad \text{— (4)}$$

Step (3)

$$M_{BA} + M_{BC} = 0$$

$$62.5 + 0.8\,EI\,\theta_B + (-93.75 + 1.6EI\theta_B + 0.8EI\theta_C) = 0$$

$$2.4\,EI\theta_B + 0.8\,EI\,\theta_C = 31.25 \quad \text{— (1)}$$

Step (4) Here $M_{CB} = 0$

$$\cancel{93.75 + 1.6}\quad 0.8\,EI\theta_B + 1.6\,EI\theta_C = -93.75 \quad \text{— (2)}$$

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slove ① & ②

$EI\theta_B = 39.06$

$EI\theta_C = -78.125$

Step ④ sub. $EI\theta_B$, $EI\theta_C$ in eq ① ② ③ ④

$-62.5 + \frac{2}{5}(39.06) = M_{AB} = -46.876$

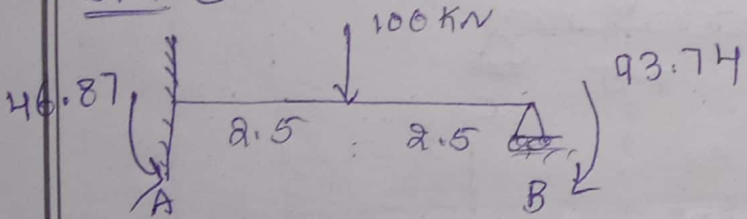
$62.5 + \frac{4}{5}(39.06) = M_{BA} = 93.748$

$-93.75 + 1.6EI\theta_B + 0.8EI\theta_C = M_{BC}$

$-93.75 + 1.6(39.06) + 0.8(-78.125) = M_{BC} = -93.754$

$M_{CB} = 93.75 + 1.6(-78.125) + 0.8(39.06) = 0.$

Step ⑤



$\Sigma M_A = 0$

$-46.87 + 100 \times 2.5 - R_B \times 5 + 93.74 = 0$

$296.87 - R_B \times 5 = 0$

$R_B = 59.374$

$\Sigma M = 0$

$R_A + R_B = 100$

$R_A = 40.626$

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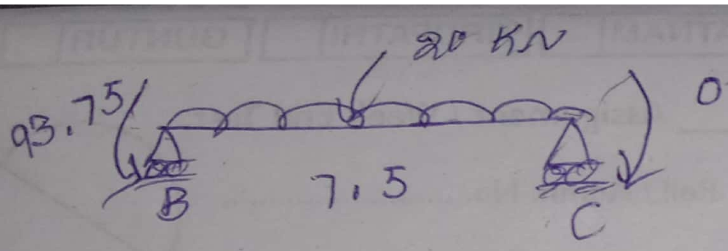

$\Sigma M_B = 0$

$-93.75 + 20 \times 7.5 \times \frac{7.5}{2} - R_C \times 7.5 = 0$

$468.75 - R_C \times 7.5 = 0$

$R_C = 62.5$

$\cancel{\Sigma} M = 0$

$R_B + R_C = 20 \times 7.5$

$R_B = 87.5$

$R_A = 40.626$

$R_B = 59.374 + 87.5 = 146.874$

$R_C = 62.5$

100

20 KN

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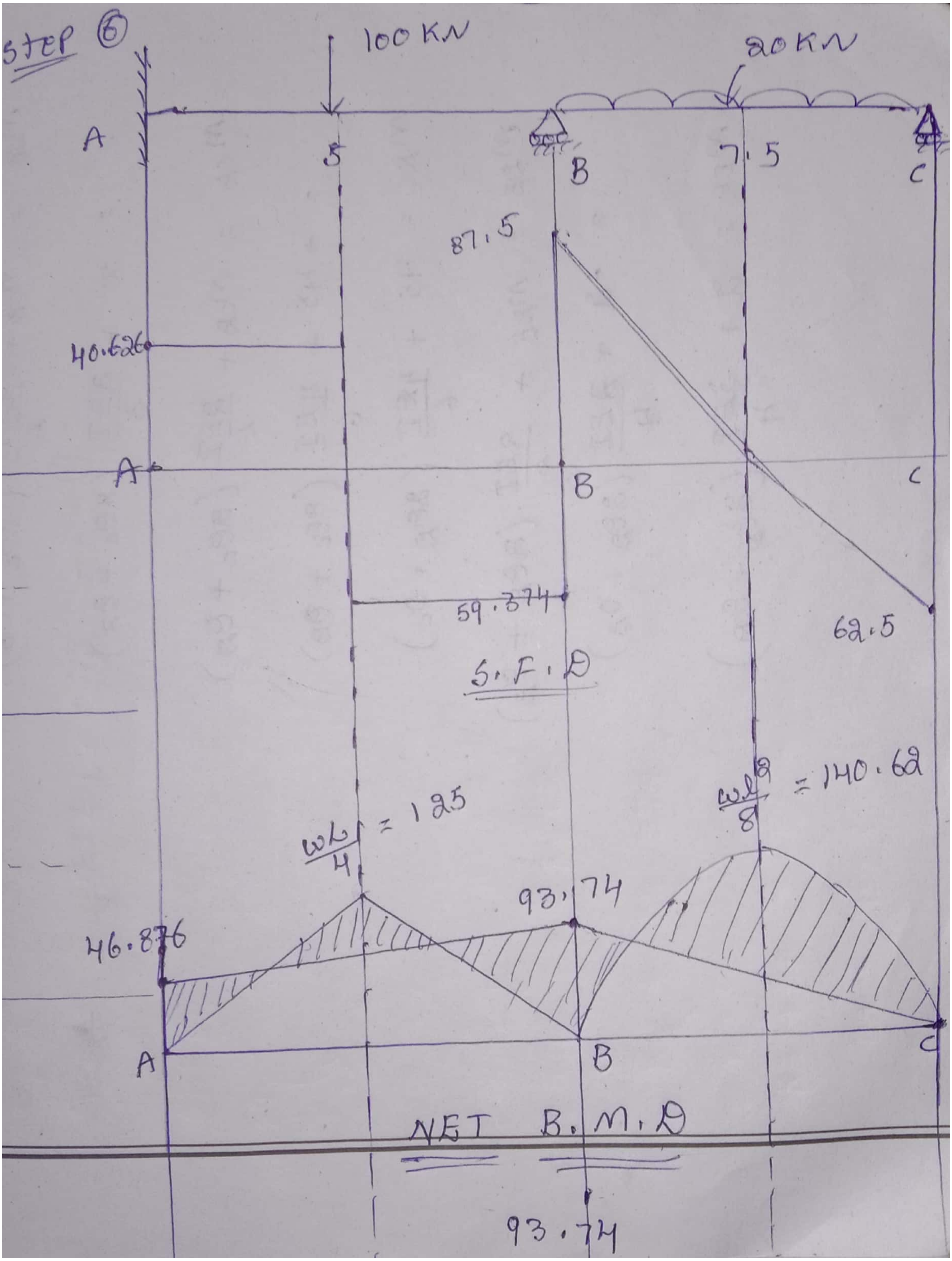
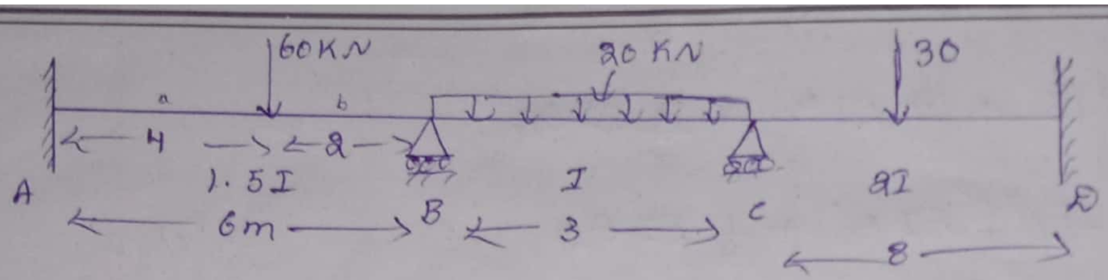
STEP ⑥


100 KN
20 KN
A
5
B
7.5
C
87.5
40.626
A
B
C
59.374
62.5
S.F.D
$\frac{wL}{4} = 125$
$\frac{wl^2}{8} = 140.62$
93.74
46.876
A
B
C
NET B.M.D
93.74

5)



Step (1)

$M_{AB} = -\frac{wab^2}{l^2} = -26.67$ $\quad M_{BA} = \frac{wa^2b}{l} = 53.33$

$M_{BC} = -\frac{wl^2}{12} = -15$ $\quad M_{CB} = 15$

$M_{CD} = -\frac{wl}{8} = -30$ $\quad M_{DC} = 30$

Step (2)

$M_{AB} = M_{AB} + \frac{2E(1.5I)}{L}(2\theta_A + \theta_B)$

$= -26.67 + \frac{3EI}{6}(2\theta_A + \theta_B)$

$M_{BA} = 53.33 + \frac{3EI}{6}(2\theta_B + \theta_A)$

$M_{BC} = M_{BC} + \frac{2EI}{L}(2\theta_B + \theta_C)$

$= -15 + \frac{2EI}{3}(2\theta_B + \theta_C)$

$M_{CB} = 15 + \frac{2EI}{3}(2\theta_C + \theta_B)$

$M_{CD} = -30 + \frac{2E(2I)}{8}(2\theta_C + \theta_D)$

$= -30 + \frac{4EI}{8}(2\theta_C + \theta_D)$

$M_{DC} = 30 + \frac{4EI}{8}(2\theta_D + \theta_C)$

$M_{AB} = -26.67 + EI\theta_A + \frac{1}{2} EI\theta_B$

$M_{BA} = 53.33 + EI\theta_B + \frac{1}{2} EI\theta_A$

$M_{BC} = -15 + \frac{4}{3} EI\theta_B + \frac{2}{3} EI\theta_C$

$M_{CB} = 15 + \frac{4}{3} EI\theta_C + \frac{2}{3} EI\theta_B$

$M_{CD} = -36 + EI\theta_C + \frac{1}{2} EI\theta_D$

$M_{DC} = 30 + EI\theta_D + \frac{1}{2} EI\theta_C$

Step 3

$M_{BA} + M_{BC} = 0$ $\qquad M_{CB} + M_{CD} = 0$; $\theta_A = 0$ $\quad \theta_D = 0$

$53.33 + EI\theta_B + \frac{1}{2} EI\theta_A - 15 + \frac{4}{3} EI\theta_B + \frac{2}{3} EI\theta_C = 0$

$\frac{7}{3} EI\theta_B + \frac{2}{3} EI\theta_C = -38.33$ —— 1

$15 + \frac{4}{3} EI\theta_C + \frac{2}{3} EI\theta_B - 30 + EI\theta_C + \frac{1}{2} EI\theta_D = 0$

$\frac{2}{3} EI\theta_B + \frac{7}{3} EI\theta_C = 15$ —— 2

$EI\theta_B = -19.88$

$EI\theta_C = 12.11$

$M_{AB} = -36.61$ $\qquad M_{BA} = 33.45$

$M_{BC} = -33.43$ $\qquad M_{CB} = 17.89$

$M_{CD} = -17.89$ $\qquad M_{DC} = 36.055$

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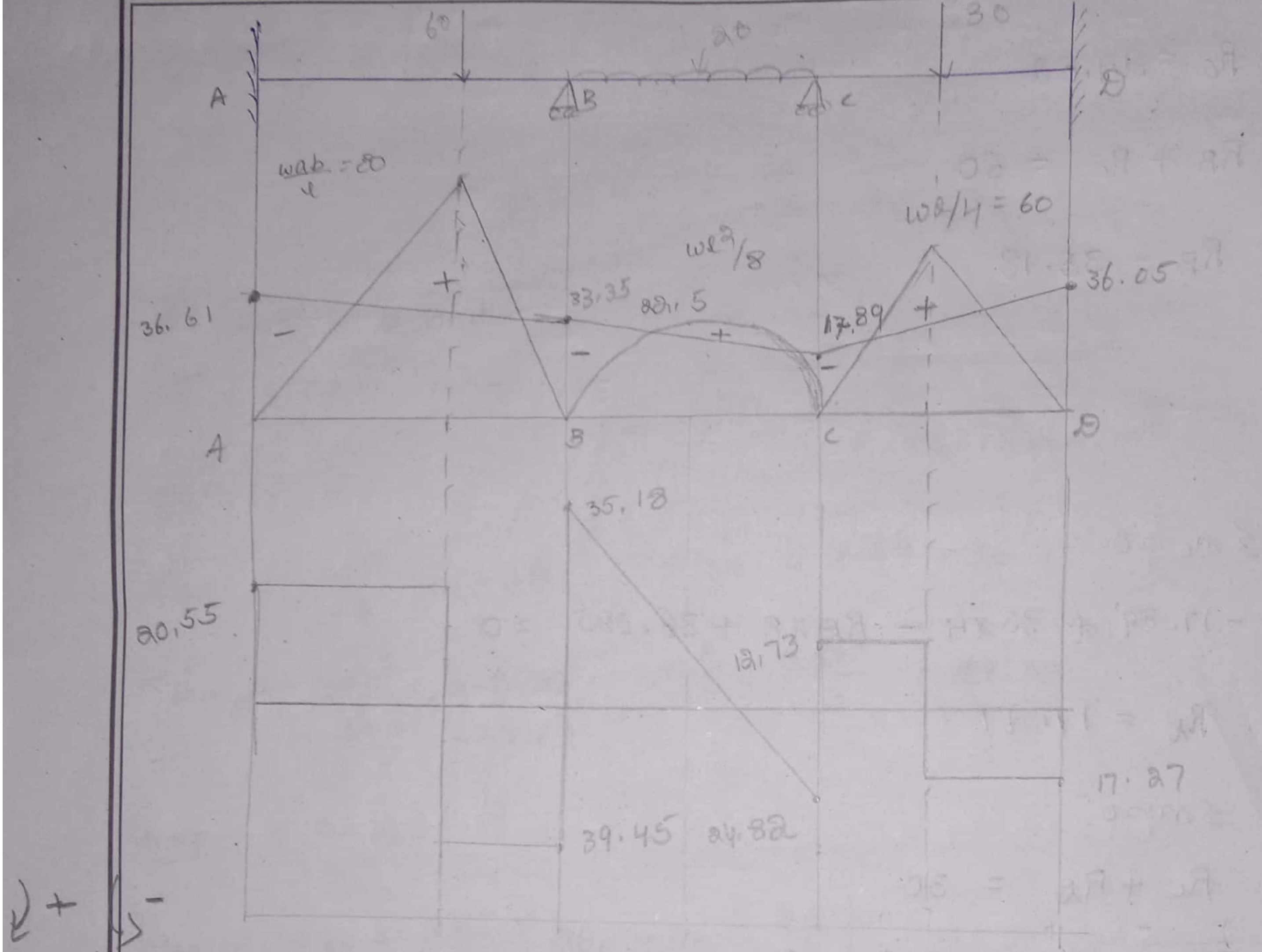
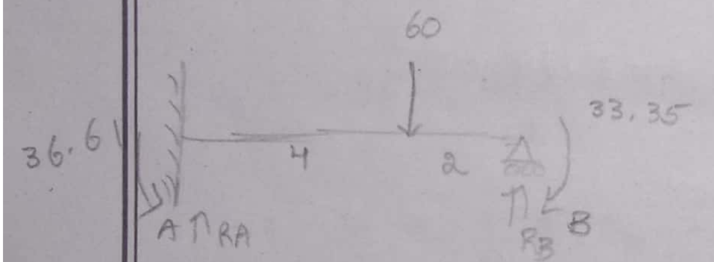




$\Sigma M_A = 0$

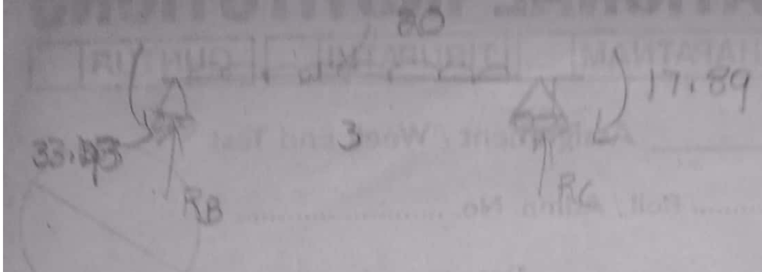
$-36.61 + 60 \times 4 - R_B \times 6 + 33.35 = 0$

$R_B = 39.45$

$\Sigma M = 0$

$R_A + R_B = 60$

$R_A = 20.55$



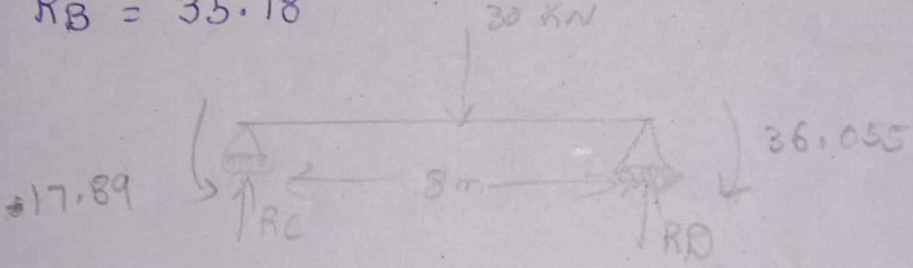
$\Sigma M_B = 0$

$-33.43 + 20 \times 3 \times \frac{3}{2} - R_C \times 3 + 17.89 = 0$

$R_C = 24.82$

$R_B + R_C = 60$

$R_B = 35.18$



$\Sigma M_C = 0$

$-17.89 + 30 \times 4 - R_D \times 8 + 36.055 = 0$

$R_D = 17.27$

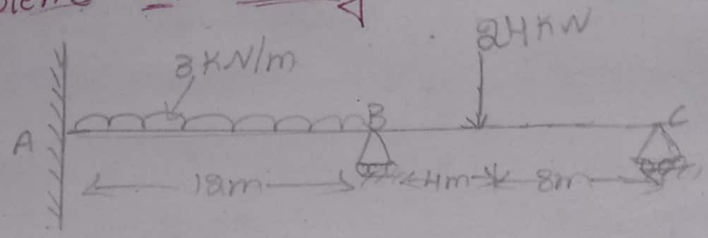
$\Sigma M = 0$

$R_C + R_D = 30$

$R_C = 12.73$

Problems on sinking

1)



The support B sinks by 0.03 m. $E = 200$ GPa

$= 200 \times 10^9$ N/m²

$I = 0.2 \times 10^{-3}$ m²

$= 200 \times 10^6$ kN/m²

Step :- 1

$M_{AB}^F = -\frac{wl^2}{12} = -36$ $\qquad M_{BA}^F = +36$

$M_{BC}^F = -\frac{wab^2}{l^2} = \cancel{-5.33}$ $\quad -42.67$ $\qquad M_{CB}^F = \frac{wa^2b}{l^2} = 21.33$

Step :- 2

$$M_{AB} = M_{AB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\Delta}{L}\right]$$

$$= -36 + \frac{2EI}{12}\left[2\theta_A + \theta_B - \frac{3(0.03)}{12}\right]$$

$\therefore \theta_A = 0$

$$= -36 + \frac{1}{6}EI\theta_B - 1.25 \times 10^{-3} EI \quad \text{— (1)}$$

$$M_{BA} = 36 + \frac{1}{3}EI\theta_B - 1.25 \times 10^{-3} EI \quad \text{— (2)}$$

$$M_{BC} = M_{BC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\Delta}{l}\right]$$

$$= -42.67 + \frac{2EI}{12}\left[2\theta_B + \theta_C - \frac{3(0.03)}{12}\right]$$

$$= -42.67 + \frac{1}{3}EI\theta_B + \frac{1}{6}EI\theta_C + 1.25 \times 10^{-3} EI \quad \rightarrow (3)$$

$$M_{CB} = M_{CB}^F + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\delta}{L}\right)$$

$$= 21.33 + \frac{2EI}{12}\left(2\theta_C + \theta_B - \frac{3(-0.03)}{12}\right)$$

$$= \frac{1}{3}EI\theta_C + \frac{1}{6}EI\theta_B + 71.33$$

Step :- 3

$$M_{BA} + M_{BC} = 0$$

$$36 + \frac{1}{3}EI\theta_B - 1.25\times10^{-3}(40\times10^3) + \frac{1}{3}EI\theta_B + \frac{1}{6}EI\theta_C$$

$$+ 1.25\times10^{-3}(40\times10^3) - 42.67 = 0$$

$$\frac{2}{3}EI\theta_B + \frac{1}{6}EI\theta_C = 6.67 \longrightarrow 1$$

$$M_{CB} = 0$$

$$\frac{1}{6}EI\theta_B + \frac{1}{3}EI\theta_C = -71.33$$

Solving 1 & 2

$EI\theta_B = 72.57$ $\qquad$ $EI\theta_C = -250.27$

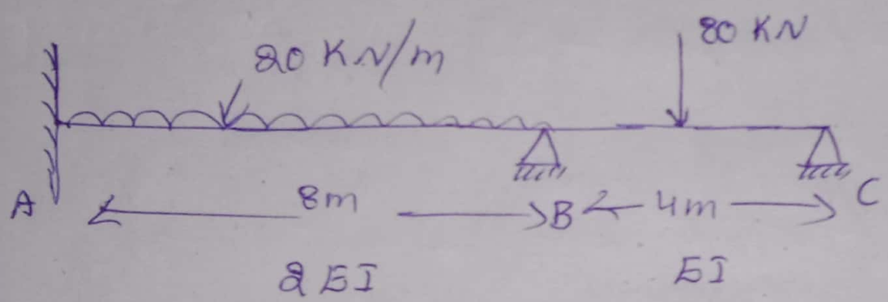
Substitue $EI\theta_B$ & $EI\theta_C$ values in above equations

weget :

$M_{AB} = -73.90$ $\qquad$ $M_{BA} = 10.19$

$M_{BC} = -10.19$ $\qquad$ $M_{CB} = 0.$

2)



$EI = 4000 \ KN m^2$ and joint B sinks by 0.01m.

Step :- 1

$$M_{AB} = -\frac{w \times l^2}{12} = -106.67 \text{ KNm}$$

$$M_{BA} = 106.67$$

$$M_{BC} = -\frac{wl}{8} = -40$$

$$M_{CB} = 40$$

Step :- 2

$$M_{AB} = M_{AB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$$

$$= -106.67 + \frac{2(2EI)}{8}\left[\theta_B - \frac{3 \times (0.01)}{8}\right]$$

$$= 0.5EI\theta_B - 114.17$$

$$M_{BA} = M_{BA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

$$= 106.67 + \frac{2(2EI)}{8}\left(2\theta_B - \frac{3(0.01)}{8}\right)$$

$$= EI\theta_B + 99.17$$

$$M_{BC} = -40 + \frac{2EI}{4}\left(2\theta_B + \theta_C - \frac{3(-0.01)}{4}\right]$$

$$= EI\theta_B + 0.5EI\theta_C - 25$$

$$M_{CB} = 40 + \frac{2EI}{l}\left[2\theta_c + \theta_B - \frac{3\delta}{l}\right]$$

$$= 40 + \frac{2EI}{4}\left[2\theta_c + \theta_B - \frac{3(-0.01)}{4}\right]$$

$$= EI\theta_c + 0.5\,EI\theta_B + 55$$

Step - 3

$$M_{BA} + M_{BC} = 0$$

$$EI\theta_B + 99.17 + EI\theta_B + 0.5\,EI\theta_c - 25 = 0$$

$$2EI\theta_B + 0.5\,EI\theta_c + 74.17 = 0 \quad \text{—} (1)$$

$$\Sigma M_{CB} = 0$$

$$0.5\,EI\theta_B + EI\theta_c + 55 = 0 \quad \text{—} (2)$$

solving (1) & (2)

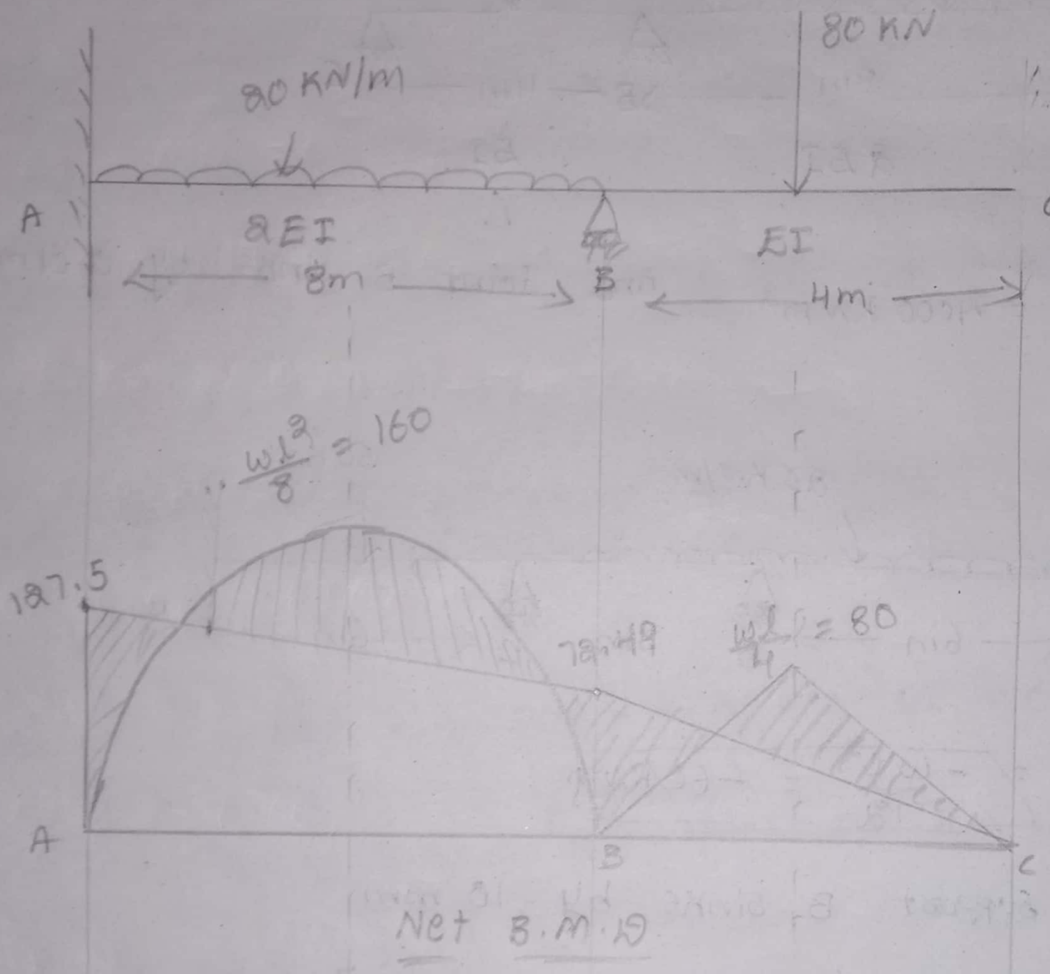
$EI\theta_B = -26.66$ $EI\theta_c = -41.66$

Step (4)

sub values in above equations we get.

$M_{AB} = -127.5$ $M_{BA} = 72.51$

$M_{BC} = -72.49$ $M_{CB} = 0$.



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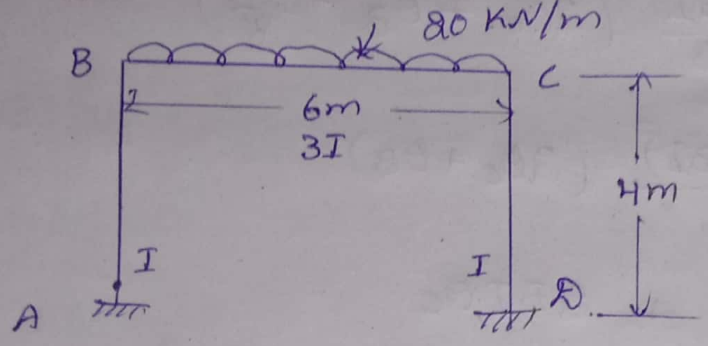
Problems on Frames

1) 

Step :- 1) Find Fixed moments

$M_{AB} = 0$ $\qquad$ $M_{BA} = 0$

$M_{BC} = -\frac{wl^2}{12} = -60$ $\qquad$ $M_{CB} = 60$

$M_{CD} = 0$ $\qquad$ $M_{DC} = 0$

Step 2 :- Apply slope deflection equations.

$$M_{AB} = M_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)$$

$$= \frac{2EI}{4}(\theta_B) \qquad \therefore \theta_A = 0$$

$$= 0.5\ EI\theta_B$$

$$M_{BA} = M_{BA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right)$$

$$= \frac{2EI}{4}(2\theta_B)$$

$$= EI\theta_B$$

$$M_{BC} = M^F_{BC} + \frac{2E(3I)}{6}\left(2\theta_B + \theta_C - \frac{3\delta}{L}\right)$$

$$= -60 + 2EI\theta_B + EI\theta_C$$

$$M_{CB} = M_{CB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\delta}{L}\right)$$

$$= 60 + \frac{2E(3I)}{6}(2\theta_C + \theta_B)$$

$$= 60 + EI\theta_B + 2EI\theta_C$$

$$M_{CD} = M_{CD} + \frac{2EI}{L}\left(2\theta_C + \theta_D - \frac{3\delta}{L}\right)$$

$$= 0 + \frac{2EI}{4}(2\theta_C)$$

$\therefore \theta_D = 0$

$$= EI\theta_C$$

$$M_{DC} = M_{DC} + \frac{2EI}{L}\left(2\theta_D + \theta_C - \frac{3\delta}{L}\right)$$

$$= 0 + \frac{2EI}{4}(\theta_C)$$

$$= 0.5\,EI\theta_C$$

Step :- 3

$$M_{BA} + M_{BC} = 0$$

$$EI\theta_B + 2EI\theta_B + EI\theta_C - 60 = 0$$

$$3EI\theta_B + EI\theta_C = 60 \quad \text{—— (1)}$$

$$M_{CB} + M_{CD} = 0$$

$$EI\theta_B + 2EI\theta_C + EI\theta_C + 60 = 0$$

$$EI\theta_B + 3EI\theta_C = -60 \quad \text{—— (2)}$$

solving ① & ②

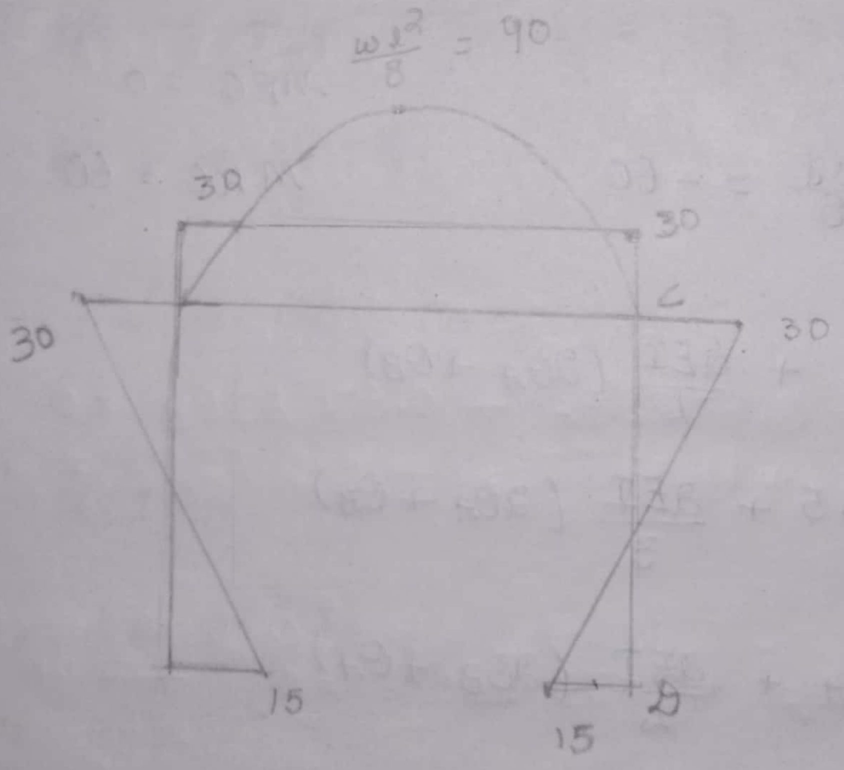
$EI\theta_B = 30$ $\quad$ $EI\theta_C = -30$.

Substitute $EI\theta_B$ & $EI\theta_C$ values in above equations.

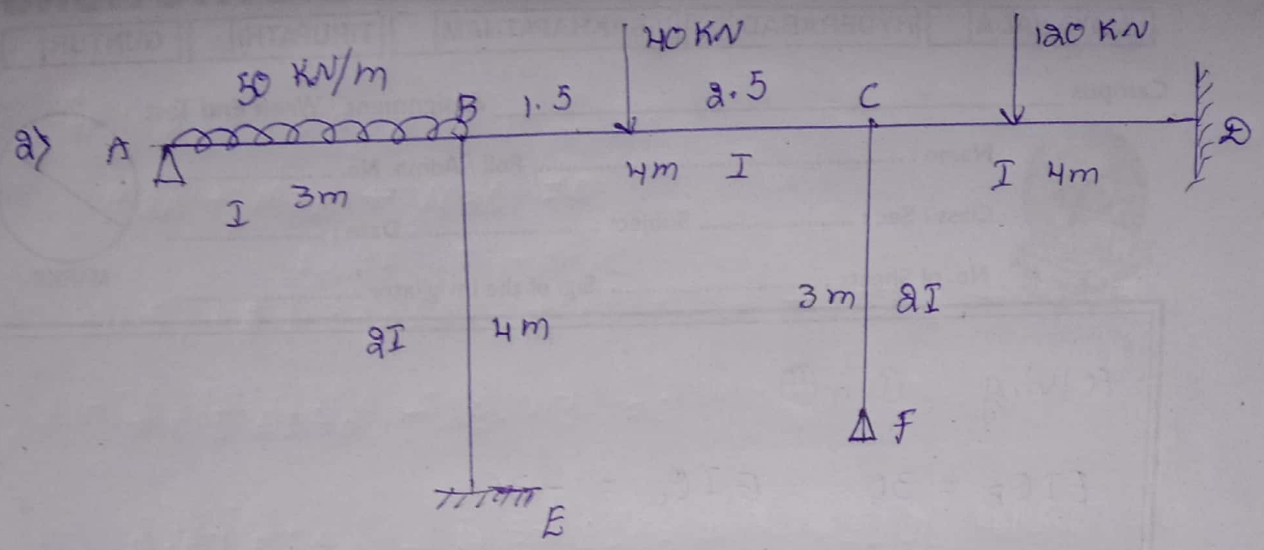
$M_{AB} = 15$ $\quad$ $M_{BA} = 30$

$M_{BC} = -30$ $\quad$ $M_{CB} = 30$

$M_{CD} = -30$ $\quad$ $M_{DC} = -15$



2)



STEP ①

$M_{AB} = -\frac{wl^2}{12} = -37.5$ $\quad$ $M_{BA} = 37.5$

$M_{BC} = -\frac{wab^2}{l^2} = -23.43$ $\quad$ $M_{CB} = \frac{wa^2b}{l^2} = 14.06$

$M_{BE} = 0$ $\quad$ $M_{EB} = 0$

$M_{CF} = 0$ $\quad$ $M_{FC} = 0$

$M_{CD} = -\frac{wl}{8} = -60$ $\quad$ $M_{DC} = 60$

STEP :- 2

$$M_{AB} = M_{AB} + \frac{2EI}{L}(2\theta_A + \theta_B)$$

$$= -37.5 + \frac{2EI}{3}(2\theta_A + \theta_B)$$

$$M_{BA} = M_{BA} + \frac{2EI}{L}(2\theta_B + \theta_A)$$

$$= 37.5 + \frac{2EI}{3}(2\theta_B + \theta_A)$$

$$= 37.5 + 1.33EI\theta_B + 0.67EI\theta_A$$

$$M_{BC} = -23.43 + \frac{2EI}{4}(2\theta_B + \theta_C)$$

$$= -23.43 + EI\theta_B + 0.5EI\theta_C$$

$$M_{CB} = 14.06 + \frac{2EI}{4}(2\theta_C + \theta_B)$$

$$= 14.06 + EI\theta_C + 0.5EI\theta_B$$

$$M_{CD} = -60 + \frac{2EI}{4}(2\theta_C + \theta_D) \qquad \therefore \theta_D = 0$$

$$= -60 + EI\theta_C + 0.5EI\theta_D$$

$$= -60 + EI\theta_C$$

$$M_{DC} = 60 + \frac{2EI}{4}(2\theta_D + \theta_C)$$

$$= 60 + 0.5EI\theta_C$$

$$M_{BE} = 0 + \frac{2E(2I)}{4}(2\theta_B + \theta_E) \qquad \therefore \theta_E = 0$$

$$= 2EI\theta_B$$

$$M_{EB} = 0 + \frac{2E(2I)}{4}(2\theta_E + \theta_B)$$

$$= EI\theta_B$$

$$M_{CF} = 0 + \frac{2E(2I)}{3}(2\theta_C + \theta_F)$$

$$= 2.67\,EI\theta_C + 1.33\,EI\theta_F$$

$$M_{FC} = 0 + \frac{2E(2I)}{3}(2\theta_F + \theta_C)$$

$$= 2.67\,EI\theta_F + 1.33\,EI\theta_C$$

Step :- 3

$M_{BA} + M_{BE} + M_{BC} = 0$

$37.5 + 1.33 EI\theta_B + 0.67 EI\theta_A + 2EI\theta_B + EI\theta_B$

$+ 0.5 EI\theta_C - 23.43 = 0$

$0.67 EI\theta_A + 4.33 EI\theta_B + 0.5 EI\theta_C + 14.07 = 0$ $\rightarrow$ ①

$M_{CB} + M_{CF} + M_{CD} = 0$

$14.06 + EI\theta_C + 0.5 EI\theta_B + 2.67 EI\theta_C + 1.33 EI\theta_F$

$+ EI\theta_C - 60 = 0$

$0.5 EI\theta_B + 4.67 EI\theta_C + 1.33 EI\theta_F - 45.94 = 0$ $\rightarrow$ ②

$M_{AB} = 0$

$-37.5 + 2.67 EI\theta_A + 0.67 EI\theta_B = 0$ $\longrightarrow$ ③

$M_{FC} = 0$

$2.67 EI\theta_F + 1.33 EI\theta_C = 0$ — ④

solving ①, ②, ③ & ④

$$\left|\begin{matrix} 0.67 & 4.33 & 0.5 & 0 \\ 0 & 0.5 & 4.67 & 1.33 \\ 2.67 & 0.67 & 0 & 0 \\ 0 & 0 & 1.33 & 2.67 \end{matrix}\right| \left|\begin{matrix} -14.07 \\ 45.94 \\ 37.5 \\ 0 \end{matrix}\right|$$

$R_2 \longleftrightarrow R_3$

$$\left[\begin{array}{cccc|c} 0.67 & 4.33 & 0.5 & 0 & -14.07 \\ 2.67 & 0.67 & 0 & 0 & 37.5 \\ 0 & 0.5 & 4.67 & 1.33 & 45.94 \\ 0 & 0 & 1.33 & 2.67 & 0 \end{array}\right]$$

$R_2 \rightarrow R_2 - 4R_1$

$$\left[\begin{array}{cccc|c} 0.67 & 4.33 & 0.5 & 0 & -14.07 \\ 0 & -16.65 & -2 & 0 & 93.78 \\ 0 & 0.5 & 4.67 & 1.33 & 45.94 \\ 0 & 0 & 1.33 & 2.67 & 0 \end{array}\right]$$

$R_3 \rightarrow R_3(-16.65) - 0.5\,R_2$

$$\left[\begin{array}{cccc|c} 0.67 & 4.33 & 0.5 & 0 & -14.07 \\ 0 & -16.65 & -2 & 0 & 93.78 \\ 0 & 0 & -76.75 & -22.14 & -811.79 \\ 0 & 0 & 1.33 & 2.67 & 0 \end{array}\right]$$

$R_4 \rightarrow (-76.75) \times R_4 - 1.33\,R_3$

$$\left[\begin{array}{cccc|c} 0.67 & 4.33 & 0.5 & 0 & -14.07 \\ 0 & -16.65 & -2 & 0 & 93.78 \\ 0 & 0 & -76.75 & -22.14 & -811.79 \\ 0 & 0 & 0 & -175.47 & +1079.68 \end{array}\right]$$

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$$-175.47\, EI\theta_F = 1079.68$$

$$EI\theta_F = -6.15$$

$$-76.75\, EI\theta_C - 22.14\, EI\theta_F = -811.79$$

$$-76.75\, EI\theta_C = -811.79 + 22.14(-6.15)$$

$$-76.75\, EI\theta_C = -947.95$$

$$EI\theta_C = 12.35$$

$$-16.65\, EI\theta_B - 2EI\theta_C = 93.78$$

$$-16.65\, EI\theta_B = 93.78 + 2\times(12.35)$$

$$-16.65\, EI\theta_B = 118.48$$

$$EI\theta_B = -7.115$$

$$0.67\, EI\theta_A + 4.33\, EI\theta_B + 0.5\, EI\theta_C = -14.07$$

$$0.67\, EI\theta_A = -14.07 - 4.33\times(-7.115) + 0.5\times(12.35)$$

$$0.67\, EI\theta_A = 10.555$$

$$\therefore\ EI\theta_A = 15.75$$

$$EI\theta_B = -7.115$$

$$EI\theta_C = 12.35$$

$$EI\theta_F = -6.15$$

//.

Substitute above values in momentum equations.

we get.

$M_{AB} = 0$ $M_{BA} = 38.58$

$M_{BC} = -24.37$ $M_{CB} = 22.85$

$M_{CD} = -47.65$ $M_{DC} = 66.175$

$M_{BE} = -14.23$ $M_{EB} = -7.115$

$M_{CF} = 24.79$ $M_{FC} = 0$

SLOPE DEFLECTION METHOD

Author :- Axel Bendixen (German Scientist) "1915"
G.A. Maney

* In slope and deflection method the unknown are rotations of joints [slopes]

* The slope deflection method comes under category of displacement method.

Slope Deflection equations:-

$$M_{AB} = \bar{M}_{ab} + \frac{2EI}{l_{ab}}\left(2\theta_a + \theta_b - \frac{3\delta}{l}\right)$$

$$M_{BA} = \bar{M}_{ba} + \frac{2EI}{l_{ba}}\left(2\theta_b + \theta_a - \frac{3\delta}{l}\right)$$

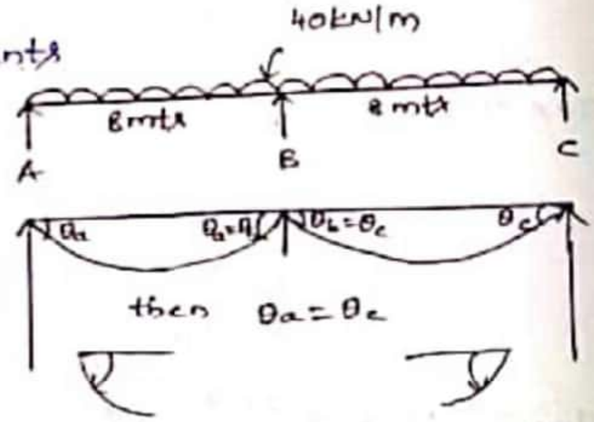
$\bar{M}_{ab}, \bar{M}_{ba}$ = fixed end moments

θ_a, θ_b = supports at A, B

δ deflection respect to span A, B

①

Calculating fixed end moments



$$\bar{M}_{ab} = -\frac{wl^2}{12} \Rightarrow \frac{-40\times 8^2}{12}$$

$$= -213.33 \text{ kN. m}$$

$$\bar{M}_{ba} = \frac{+wl^2}{12} \Rightarrow \frac{40\times 8^2}{12}$$

$$= 213.33 \text{ kN. m}$$

$$\bar{M}_{bc} = -\frac{wl^2}{12} = \frac{-40\times 8^2}{12}$$

$$= -213.33 \text{ kN m}$$

$$\bar{M}_{cb} = \frac{wl^2}{12} = \frac{40\times 8^2}{12}$$

$$= 213.33 \text{ kN m}$$

$M_{AB} = 0$

$M_{CB} = 0$

$\delta = 0$

$\theta_a, \theta_b = ?$

$M_{BA} + M_{BC} = 0$ CORS

$$213.33 + \frac{2EI}{8}(2\theta_b + \theta_a) + (-213.33) + \frac{2EI}{8}(2\theta_b + \theta_c) = 0$$

$$\frac{\cancel{4}EI\theta_b}{8_2} + \frac{2EI\theta_a}{8} + \frac{\cancel{4}EI\theta_b}{8_2} + \frac{2EI\theta_c}{8} = 0$$

$$\frac{EI\theta_b}{2} + \frac{EI\theta_b}{2} + \frac{EI\theta_a}{4} + \frac{EI\theta_c}{4} = 0$$

$$\frac{EI\theta_a}{4} + E\cancel{I}\theta_b + \frac{EI\theta_c}{4} = 0$$

$M_{AB} = 0$

$$-213.33 + \frac{EI}{4}(2\theta_a + \theta_b) \Rightarrow 0$$

$$\frac{EI}{2}\theta_a + \cancel{\frac{EI}{4}}\theta_b = 213.33$$

$M_{CB} = 0$

$$213.33 + \frac{EI}{4}(2\theta_c + \theta_b) = 0$$

$$213.33 + \frac{EI}{2}\theta_c + \frac{EI}{4}\theta_b = 0$$

$$\cancel{\frac{EI}{4}}\theta_b + \frac{EI}{2}\theta_c = -213.33$$

Due to symmetrical $\theta_b = 0$

$$EI\theta_c = -213.33 \times 2 \Rightarrow -426.66$$

$$EI\theta_c = -426.66$$

$$EI\theta_a = 213.33 \times 2$$

$$EI\theta_a = 426.66$$

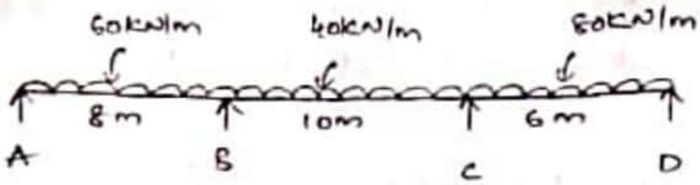
$$M_{BA} = 213.23 + \frac{2EI}{8}(426.66)$$

$M_{BA} = 319.99 \simeq 320\,kN.m$

$$M_{BC} = -213.33 + \frac{2EI}{8}(-426.66)$$

$M_{BC} = 319.99 \simeq 320\,kN.m$

(2)



calculating fixed end moments:-

$\bar{M}_{ab} = -\frac{wl^2}{12} = \frac{-60\times 8^2}{12} \Rightarrow -320$ kN m

$\bar{M}_{ba} = \frac{wl^2}{12} = \frac{60\times 8^2}{12} \Rightarrow 320$ kN.m

$\bar{M}_{BC} = \frac{-wl^2}{12} = -\frac{40\times 10^2}{12} \Rightarrow -333.33$ kN.m

$\bar{M}_{cb} = \frac{wl^2}{12} = \frac{40\times 10^2}{12} \Rightarrow 333.33$ kN.m

$\bar{M}_{cd} = \frac{-wl^2}{12} = -\frac{80\times 6^2}{12} \Rightarrow -240$ kN.m

$\bar{M}_{dc} = \frac{wl^2}{12} = \frac{80\times 6^2}{12} \Rightarrow 240$ kN.m

(i) $M_{AB} = 0$; $M_{DC} = 0$

$M_{ab} = +\bar{M}_{ab} + \frac{2EI}{l_{ab}}(2\theta_a + \theta_b) = 0$

$= -320 + \frac{2EI}{8}(2\theta_a + \theta_b) = 0$

$0.25EI(2\theta_a + \theta_b) = 320$

$0.5EI\theta_a + 0.25EI\theta_b = 320 \rightarrow (1)$

$M_{dc} = \bar{M}_{dc} + \frac{2EI}{l_{dc}}(2\theta_d + \theta_c) = 0$

$= 240 + \frac{2EI}{6}(2\theta_d + \theta_c) = 0$

$\frac{2}{3}EI\theta_d + \frac{1}{3}EI\theta_c = -240 \rightarrow (2)$

$M_{BA} + M_{BC} = 0$

$M_{BA} = M_{BA} + \frac{2EI}{l_{BA}}(2\theta_B + \theta_A) = 0$

$= 320 + \frac{2EI}{8}(2\theta_B + \theta_A) = 0$

$0.25EI(2\theta_B + \theta_a) = -320$

$0.5EI\theta_b + 0.25EI\theta_a = -320 \rightarrow (3)$

$M_{bc} = \bar{M}_{bc} + \frac{2EI}{l_{bc}}(2\theta_b + \theta_c) = 0$

$= -333.33 + \frac{2EI}{10}(2\theta_b + \theta_c) = 0$

$0.2EI(2\theta_b + \theta_c) = 333.33$

$0.4EI\theta_b + 0.2EI\theta_c = 333.33 \rightarrow (4)$

$M_{BA} + M_{BC} = 0$

$0.25EI\theta_a + 0.5EI\theta_b + 320 + 0.4EI\theta_b + 0.2EI\theta_c - 333.33 = 0$

$0.25EI\theta_a + 0.9EI\theta_b + 0.2EI\theta_c = 13.33 \rightarrow (5)$

$M_{CB} + M_{CD} = 0$

$M_{CB} = \bar{M}_{cb} + \frac{2EI}{l_{cb}}(2\theta_c + \theta_b)$

$= 333.33 + \frac{2EI}{10}(2\theta_c + \theta_b)$

$0.2EI(2\theta_c + \theta_b) = -333.33$

$0.4EI\theta_c + 0.2EI\theta_b = -333.33 \rightarrow (6)$

$M_{cD} = \bar{M}_{cd} + \frac{2EI}{l_{cd}}(2\theta_c + \theta_d) = 0$

$= -240 + \frac{2EI}{6}(2\theta_c + \theta_d) = 0$

$\frac{1}{3}EI(2\theta_c + \theta_d) = 240$

$\frac{2}{3}EI\theta_c + \frac{1}{3}EI\theta_d = 240 \rightarrow (7)$

$M_{CB} + M_{CD} = 0$

$0.2EI\theta_b + 0.4EI\theta_c + \frac{2}{3}EI\theta_c + \frac{1}{3}EI\theta_d + 333.33 - 240 = 0.$

$0.2EI\theta_b + 1.067EI\theta_c + 0.333EI\theta_d = -93.33 \rightarrow (8)$

Taking eq. 1 & eq. 3

$0.5EI\theta_a + 0.25EI\theta_b = 320 \rightarrow 1$

$2(0.25EI\theta_a + 0.5EI\theta_b = -320) \rightarrow 3$

multiplying eq. 3 with 2

$0.5EI\theta_a + 0.25EI\theta_b = 320$

$\underline{-0.5EI\theta_a \pm 1.00EI\theta_b = \mp 640}$

$-0.75EI\theta_b = 960$

$$EI\theta_b = -\frac{960}{0.75}$$

$$EI\theta_b = -1280 \rightarrow (9)$$

Taking eq. (3) & eq. (7)

$\cancel{0.4 EI \theta_c + 1.2 EI \theta_b = -3.27.73}$

$$\frac{2}{3} EI\theta_c + \frac{1}{3} EI\theta_d = 240 \rightarrow (7)$$

$$2\left(\frac{1}{3} EI\theta_c + \frac{2}{3} EI\theta_d = -240\right)^2 \rightarrow (2)$$

multiplying eq. (2) with (2)

$$\frac{2}{3} EI\theta_c + \frac{1}{3} EI\theta_d = 240$$

$$\frac{2}{3} EI\theta_c + \frac{4}{3} EI\theta_d = -480$$

$$-EI\theta_d = 720$$

$$EI\theta_d = -720 \rightarrow (10)$$

substituting eq. (9) & eq. (10) we get in eq. (8) is

$$0.2\, EI\left(-\frac{1280}{EI}\right) + 1.067\, EI\theta_c + 0.333\, EI\left(-\frac{720}{EI}\right) = 0$$

$$-256 + 1.067\, EI\theta_c - 239.76 = 0$$

$$1.067\, EI\theta_c = 495.7$$

$$EI\theta_c = 464.57$$

Taking eq (1) & eq (5) we get

$$0.25\, EI\theta_a + 0.9\, EI\theta_b + 0.2\, EI\theta_c = 13.33 \rightarrow (5)$$

$$0.5\, EI\theta_a + 0.25\, EI\theta_b = 320 \rightarrow (1)$$

multiplying eq (5) with (2)

$$0.5\, EI\theta_a + 1.8\, EI\theta_b + 0.4\, EI\theta_c = 26.66$$

$$0.5\, EI\theta_a + 0.25\, EI\theta_b = 320.00$$

$$1.55\, EI\theta_b + 0.4\, EI\theta_c = -293.34 \rightarrow (a)$$

Taking eq. (2) & eq. (8) we get

$$0.2\, EI\theta_b + 1.067\, EI\theta_c + 0.333\, EI\theta_d = -93.33 \rightarrow (8)$$

$$0.333\, EI\theta_c + 0.667\, EI\theta_d = -240 \rightarrow (2)$$

multiplying eq. (2) with 3.204

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$$\cancel{0.4\,EI\,\theta_b + 1.067\,EI\,\theta_c + 0.333\,E\ldots}$$

$$\cancel{\underline{-1.067\,EI\,\theta_c \pm 2.137\,EI\,\theta_d = \mp 768.96}}$$

$$\cancel{0.2\,EI\,\theta_b \qquad - 1.804\,EI\,\theta_d = 862.29}$$

$$0.4\,EI\,\theta_b + 2.134\,EI\,\theta_c + 0.666\,EI\,\theta_d = -186.66$$

$$\underline{-0.333\,EI\,\theta_c \pm 0.667\,EI\,\theta_d = \mp 240}$$

$$+0.4\,EI\,\theta_b + 1.801\,EI\,\theta_c = \cancel{406.66}\ 53.34 \rightarrow (b)$$

Solving (a) & (b) equations

$$1.55\,EI\,\theta_b + 0.4\,EI\,\theta_c = -293.34$$

$$\underline{-0.4\,EI\,\theta_b \pm 1.801\,EI\,\theta_c = \cancel{406.66}\ -53.34}$$

$$EI\,\theta_b = -208.83 \qquad EI\,\theta_c = 75.87.$$

substitute θ_b in eq. (1)

$$0.5\,EI\,\theta_a + 0.25\,EI\left(\frac{-208.83}{EI}\right) = 320$$

$$EI\,\theta_a = 744.415$$

substitute θ_c in eq. (2)

$$\frac{2}{3}EI\,\theta_d + \frac{1}{3}EI\left(\frac{75.87}{EI}\right) = -240$$

$$EI\,\theta_d = -397.93$$

$$M_{AB} = \bar{M}_{ab} + \frac{EI}{4}(2\theta_a + \theta_b) \Rightarrow -320 + \frac{EI}{4}\left(\frac{2\times 744.415}{EI}\right) + \frac{EI}{4}\left(\frac{-208.83}{EI}\right)$$

$$M_{AB} = 0$$

$$M_{BA} = \bar{M}_{ba} + \frac{EI}{4}(2\theta_b + \theta_a) \Rightarrow 320 + \frac{EI}{4}\left(\frac{2\times -208.83}{EI}\right) + \frac{EI}{4}\left(\frac{744.415}{EI}\right)$$

$$M_{BA} = 401.68\ kN\cdot m$$

$$M_{BC} = \bar{M}_{bc} + \frac{EI}{5}(2\theta_b + \theta_c) \Rightarrow -333.33 + \frac{EI}{5}\left(\frac{2\times -208.83}{EI}\right) + \frac{EI}{5}\left(\frac{75.87}{EI}\right)$$

$$M_{BC} = -401.68\ kN\,m$$

$$M_{CB} = \bar{M}_{cb} + \frac{EI}{5}(2\theta_c + \theta_b) \Rightarrow 333.33 + \frac{EI}{5}\left(\frac{2\times 75.87}{EI}\right) + \frac{EI}{5}\left(\frac{-208.83}{EI}\right)$$

$$M_{CB} = 321.91\ kN\cdot m$$

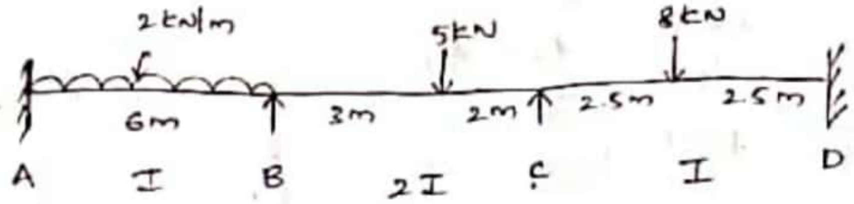
$$M_{CD} = \bar{M}_{cd} + \frac{EI}{3}(2\theta_c + \theta_d) \Rightarrow -240 + \frac{EI}{3}\left(\frac{2\times 75.87}{EI}\right) + \frac{EI}{3}\left(\frac{-397.93}{EI}\right)$$

$$M_{CD} = -322.0\ kN\cdot m$$

$$M_{DC} = \bar{M}_{dc} + \frac{EI}{3}(2\theta_d + \theta_c) \Rightarrow 240 + \frac{EI}{3}\left(\frac{2\times -397.93}{EI}\right) + \frac{EI}{3}\left(\frac{75.87}{EI}\right)$$

$$M_{DC} = 3.33\times 10^{-3} = 0.003 \simeq 0$$

(3)



Boundary condition

$\theta_A = 0 \quad \theta_d = 0$

because fixed support has no angle.

calculating fixed end moments

$\bar{M}_{ab} = \frac{-wl^2}{12} \Rightarrow \frac{-2 \times 6^2}{12} \Rightarrow -6 \text{ kN.m}$

$\bar{M}_{ba} = \frac{wl^2}{12} \Rightarrow \frac{2 \times 6^2}{12} \Rightarrow 6 \text{ kN.m}$

$\bar{M}_{bc} = \frac{-wab^2}{l^2} \Rightarrow -\frac{5 \times 3 \times 2^2}{5^2} \Rightarrow -2.4 \text{ kN.m}$

$\bar{M}_{cb} = \frac{+wa^2 b}{l^2} \Rightarrow \frac{5 \times 3^2 \times 2}{5^2} \Rightarrow \overset{3.6}{\cancel{2.4}} \text{ kN.m}$

$\bar{M}_{cd} = \frac{-wL}{8} \Rightarrow \frac{-8 \times 5}{8} \Rightarrow -5 \text{ kN.m}$

$\bar{M}_{dc} = \frac{wL}{8} \Rightarrow \frac{8 \times 5}{8} \Rightarrow 5 \text{ kN.m}$

$M_{ab} = \bar{M}_{ab} + \frac{2EI}{l_{ab}}(2\theta_a + \theta_b)$

$= -6 + \frac{2EI}{6}(2\theta_a + \theta_b)$

$= -6 + \frac{EI}{3}\theta_b \rightarrow (1)$

$M_{ba} = \bar{M}_{ba} + \frac{2EI}{l_{ab}}(2\theta_b + \theta_a)$

$= 6 + \frac{2EI}{6}(2\theta_b)$

$= 6 + \frac{2EI\theta_b}{3} \rightarrow (2)$

$M_{BC} + M_{BA} = 0.$

$M_{BC} = \bar{M}_{bc} + \frac{2EI}{l_{bc}}(2\theta_b + \theta_c) \qquad I = 2I$

$= -2.4 + \frac{2 \times 2EI}{5}(2\theta_b + \theta_c)$

$= -2.4 + \frac{8EI\theta_b}{5} + \frac{4EI\theta_c}{5} \rightarrow (3)$

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$M_{CB} + M_{CD} = 0$

$$M_{CB} = \bar{M}_{cb} + \frac{2EI}{l_{cb}}(2\theta_c + \theta_b)$$

$$= \cancel{2.4}\ 3.6 + \frac{4EI}{5}(2\theta_c + \theta_b)$$

$$= \cancel{2.4}\ 3.6 + \frac{8EI\theta_c}{5} + \frac{4EI\theta_b}{5} \rightarrow 4$$

$$M_{CD} = \bar{M}_{cd} + \frac{2EI}{l_{cd}}(2\theta_c + \theta_d)$$

$$= -5 + \frac{2EI}{5}(2\theta_c + \theta_d)$$

$$= -5 + \frac{4EI\theta_c}{5} + \cancel{\frac{2EI\overset{(0)}{\theta_d}}{5}} \rightarrow 5$$

$M_{BC} + M_{BA} = 0$

$$-2.4 + \frac{8EI\theta_b}{5} + \frac{4EI\theta_c}{5} + 6 + \frac{2EI\theta_b}{3} = 0.$$

$$2.26\,EI\theta_b + 0.8EI\theta_c + 3.6 = 0 \rightarrow 6$$

$M_{CB} + M_{CD} = 0$

$$\cancel{2.4}\ 3.6 + \frac{8EI\theta_c}{5} + \frac{4EI\theta_b}{5} - 5 + \frac{4EI\theta_c}{5} + \cancel{\frac{2EI\theta_d}{5}} = 0$$

$$2.4\,EI\theta_c + 0.8\,EI\,\theta_b - 1.4 = 0 \rightarrow 7$$

solving eq. 6 & eq. 7

$$2.26\,EI\theta_b + 0.8\,EI\,\theta_c = -3.6 \rightarrow 6$$

$$2.825\,(0.8\,EI\,\theta_b + 2.4\,EI\,\theta_c = 1.4) \rightarrow 7$$

multiplying eq. 7 with 2.825

$$2.26\,EI\,\theta_b + 0.8\,EI\,\theta_c = -3.6$$

$$2.26\,EI\,\theta_b + 6.78\,EI\,\theta_c = 3.955$$

$$-5.98\,EI\,\theta_c = -7.555$$

$$EI\,\theta_c = 1.263$$

substitute $EI\theta_c = 1.263$ in eq. ⑥

$$2.26 EI\theta_b + 0.8 EI\left(\frac{1.263}{EI}\right) = -3.6$$

$$2.26 EI\theta_b + 1.0104 = -3.6$$

$$2.26 EI\theta_b = -4.6104$$

$$EI\theta_b = -2.04$$

$$M_{AB} = \bar{M}_{ab} + \frac{2EI}{l_{ab}}(2\theta_a + \theta_b)$$

$$= -6 + \frac{2EI}{6}(-2.04)$$

$M_{AB} = -6.68$ kN.m

$$M_{BA} = \bar{M}_{ba} + \frac{2EI}{l_{ba}}(2\theta_b + \theta_a)$$

$$= 6 + \frac{2}{6}(2\times -2.04)$$

$M_{BA} = 4.64$ kN.m

$$M_{BC} = \bar{M}_{bc} + \frac{2EI}{l_{bc}}(2\theta_b + \theta_c) \Rightarrow -2.4 + \frac{4EI(2\theta_b)}{l_{bc}} + \frac{4EI\theta_c}{l_{bc}}$$

$$= -2.4 + \frac{4\times 2(-2.04)}{5} + \frac{4\times(1.263)}{5}$$

$M_{BC} = -4.65$ kN.m

$$M_{CB} = M_{cb} + \frac{2EI}{l_{cb}}(2\theta_c + \theta_b) \Rightarrow M_{cb} + \frac{4EI(2\theta_c)}{l_{cb}} + \frac{4EI\theta_b}{l_{cb}}$$

$$= 3.6 + \frac{4\times 2(1.263)}{5} + \frac{4(-2.04)}{5}$$

$M_{CB} = 3.98$ kN.m

$$M_{CD} = M_{cd} + \frac{2EI}{l_{cd}}(2\theta_c + \theta_d)$$

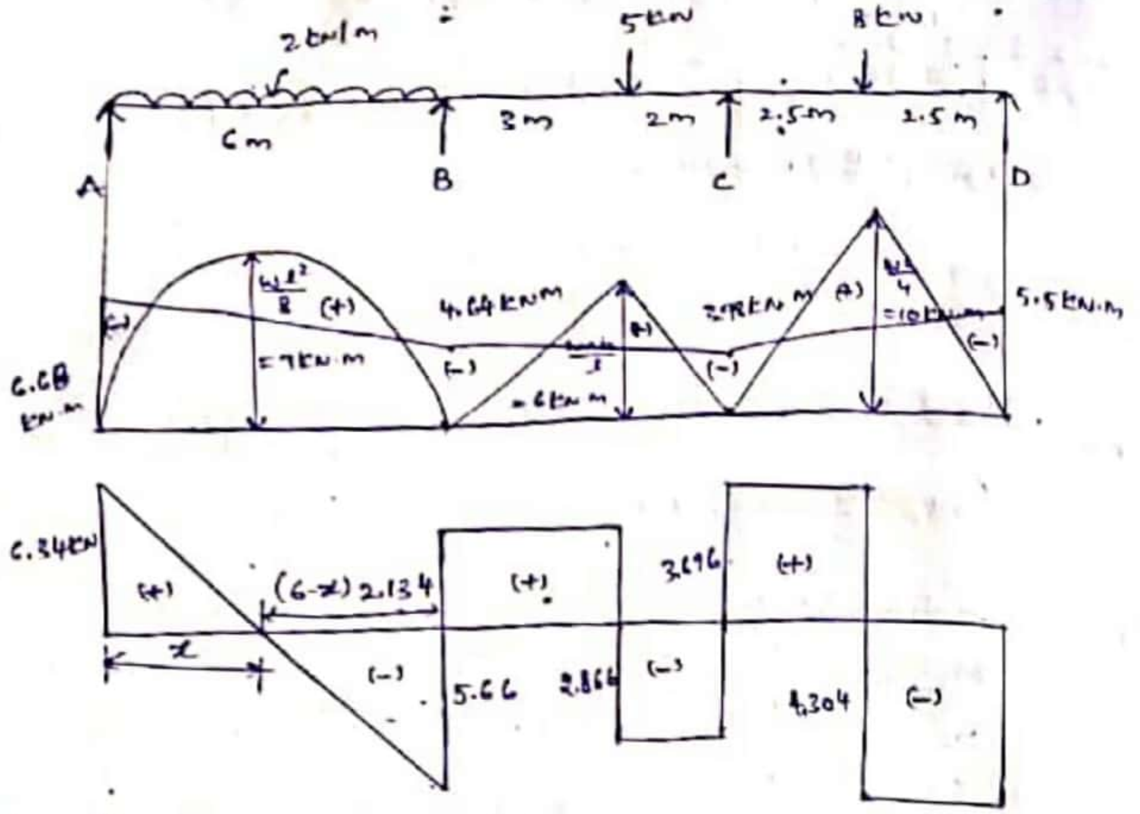
$$= -5 + \frac{2}{5}(2\times 1.263)$$

$M_{CD} = -3.98$ kN.m

$$M_{DC} = M_{dc} + \frac{2EI}{l_{dc}}(2\theta_d + \theta_c)$$

$$= 5 + \frac{2}{5}(1.263)$$

$M_{DC} = 5.50$ kN.m.



Reactions :

$R_A + R_B = 12$ kN

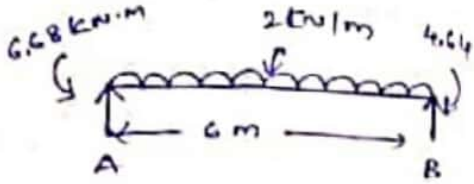
$\Sigma M_A = 0$

$R_{B1} \times 6 - 4.64 - 2 \times 6 \times 6/2 + 6.68 = 0$

$R_{B1} = \frac{33.96}{6}$

$R_{B1} = 5.66$ kN.

$R_A = 6.34$ kN



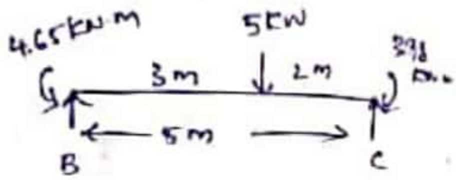
$R_{B2} + R_{C1} = 5$ kN

$\Sigma M_B = 0$

$R_C \times 5 - 5 \times 3 - 3.98 + 4.65 = 0$

$R_{C1} = 14.33/5$

$R_{C1} = 2.866$ kN



$R_{B2} = 2.134$ kN.

$R_{C2} + R_D = 8$ kN

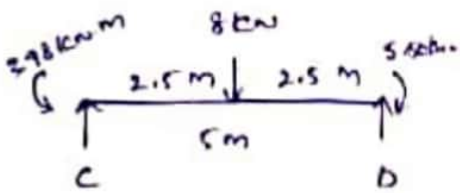
$\Sigma M_C = 0$

$R_D \times 5 - 5.50 - 8 \times 2.5 + 3.98 = 0$

$R_D = 21.52/5$

$R_D = 4.304$ kN

$R_{C2} = 3.696$ kN



$R_B = R_{B1} + R_{B2}$

$= 5.66 + 2.134$

$R_B = 7.794$ kN

$\therefore R_C = R_{C1} + R_{C2}$

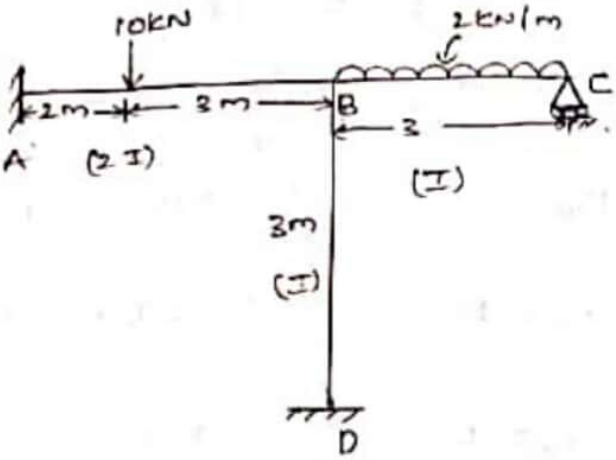
$= 2.866 + 3.696$

$R_C = 6.562$ kN.

$$\frac{x}{6.34} = \frac{6-x}{5.66}$$

$x = 3.17$ from A

(4)



Calculation of fixed end moments:-

$\cancel{\bar{M}_{ab} = -\frac{wl^2}{12} \Rightarrow \frac{-10\times 5^2}{12}}$

$\cancel{\bar{M}_{ab}}$

$\cancel{\bar{M}_{ba} = \frac{wl^2}{12} \Rightarrow \frac{10\times 5^2}{12}}$

$$\bar{M}_{ab} = \frac{-Wab^2}{l^2} \Rightarrow \frac{-10\times 2\times 3^2}{5^2}$$

$\bar{M}_{ab} = -7.2$ kN.m

$$\bar{M}_{ba} = \frac{Wa^2b}{l^2} \Rightarrow \frac{10\times 2^2\times 3}{5^2}$$

$\bar{M}_{ba} = 4.8$ kN.m

$$\bar{M}_{bc} = -\frac{wl^2}{12} \Rightarrow -\frac{2\times 3^2}{12} \Rightarrow -1.5 \text{ kN.m}$$

$$\bar{M}_{cb} = \frac{wl^2}{12} \Rightarrow \frac{2\times 3^2}{12} \Rightarrow 1.5 \text{ kN.m}$$

$\bar{M}_{bd} = 0$

$\bar{M}_{db} = 0$

* $\theta_A = \theta_D = 0$

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$$M_{AB} = \bar{M}_{ab} + \frac{2EI}{l_{ab}}(2\theta_a + \theta_b)$$

$$= -7.2 + \frac{2E(2I)}{5}(\theta_b)$$

$$M_{AB} = -7.2 + \frac{4EI}{5}[\theta_b]$$

$$M_{BA} = \bar{M}_{ba} + \frac{2EI}{l_{ba}}(2\theta_b + \theta_a]$$

$$= 4.8 + \frac{2E(2I)}{5}(2\theta_b)$$

$$M_{BA} = 4.8 + \frac{4EI}{5}(2\theta_b)$$

$$M_{BD} = \bar{M}_{bd} + \frac{2EI}{l_{bd}}(2\theta_b + \theta_d)$$

$$= 0 + \frac{2EI}{3}(2\theta_b)$$

$$M_{BD} = \frac{2EI}{3}(2\theta_b)$$

$$M_{DB} = \bar{M}_{db} + \frac{2EI}{l_{db}}(2\theta_d + \theta_b)$$

$$= 0 + \frac{2EI}{3}(\theta_b)$$

$$M_{DB} = \frac{2EI}{3}(\theta_b)$$

$$M_{BC} = \bar{M}_{bc} + \frac{2EI}{l_{bc}}(2\theta_b + \theta_c)$$

$$M_{BC} = -1.5 + \frac{2EI}{3}(2\theta_b + \theta_c)$$ Doubt.

$$M_{CB} = \bar{M}_{bc} + \frac{2EI}{l_{cb}}(2\theta_c + \theta_b)$$

$$M_{CB} = 1.5 + \frac{2EI}{3}(2\theta_c + \theta_b)$$

$$M_{BC} + M_{BA} + M_{BD} = 0$$

$$-1.5 + \frac{2EI}{3}(2\theta_b + \theta_c) + 4.8 + \frac{4EI}{5}(2\theta_b) + \frac{2EI}{3}(2\theta_b) = 0$$

$$-1.5 + \frac{4EI\theta_b}{3} + \frac{2EI\theta_c}{3} + 4.8 + \frac{8EI\theta_b}{5} + \frac{4EI\theta_b}{3} = 0.$$

$$\frac{4EI\theta_b}{3} + \frac{8EI\theta_b}{5} + \frac{4EI\theta_b}{3} + \frac{2EI\theta_c}{3} - 1.5 + 4.8 = 0$$

$$EI\theta_b\left[\frac{4}{3} + \frac{8}{5} + \frac{4}{3}\right] + \frac{2EI\theta_c}{3} + 3.3 = 0$$

$$\frac{64EI\theta_b}{15} + \frac{2EI\theta_c}{3} + 3.3 = 0 \rightarrow (1)$$

$$M_{CB} = 0$$

$$1.5 + \frac{2EI}{3}(2\theta_c + \theta_b) = 0$$

$$1.5 + \frac{4EI\theta_c}{3} + \frac{2EI\theta_b}{3} = 0.$$

$$\frac{2EI\theta_b}{3} + \frac{4EI\theta_c}{3} + 1.5 = 0 \rightarrow (2)$$

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solving eq. ① & ②

$$\frac{64EI\theta_b}{15} + \frac{2EI\theta_c}{3} + 3.3 = 0$$

$$\frac{2EI\theta_b}{3} + \frac{4EI\theta_c}{3} + 1.5 = 0 \rightarrow ②$$

multiplying eq. ② with $\frac{32}{15}$

$$\frac{64EI\theta_b}{15} + \frac{2EI\theta_c}{3} = -3.3$$

$$\frac{64EI\theta_b}{15} + \frac{128EI\theta_c}{15} = -\frac{16}{5} \left(\frac{32}{15}\right) -9.6$$

$$- \qquad - \qquad +$$

$EI\theta_b = -0.6483 \qquad \theta_b = \frac{-0.6483}{EI}$

$EI\theta_c = -0.8008 \qquad \theta_c = \frac{-0.8008}{EI}$

$$M_{AB} = -7.2 + \frac{4EI}{5}[\theta_b]$$

$$= -7.2 + \frac{4EI}{5}\left[\frac{-0.6483}{EI}\right]$$

$M_{AB} = -7.718$ kN.m

$$M_{BA} = 4.8 + \frac{4EI}{5}[2\theta_b]$$

$$= 4.8 + \frac{4EI}{5}\left[\frac{2\times -0.6483}{EI}\right]$$

$M_{BA} = 3.76$ kN m

$$M_{BC} = -1.5 + \frac{2EI}{3}[2\theta_b + \theta_c]$$

$$= -1.5 + \frac{2EI}{3}\left[\frac{2\times -0.6483}{EI} + \frac{-0.8008}{EI}\right]$$

$M_{BC} = -2.89$ kNm

$$M_{CB} = 1.5 + \frac{2EI}{3}(2\theta_c + \theta_b)$$

$$= 1.5 + \frac{2EI}{3}\left[\frac{2\times -0.8008}{EI} + \frac{-0.6483}{EI}\right]$$

$M_{CB} = 0.0000667 \simeq 0$

$$M_{BD} = \frac{2EI}{3}(2\theta_b)$$

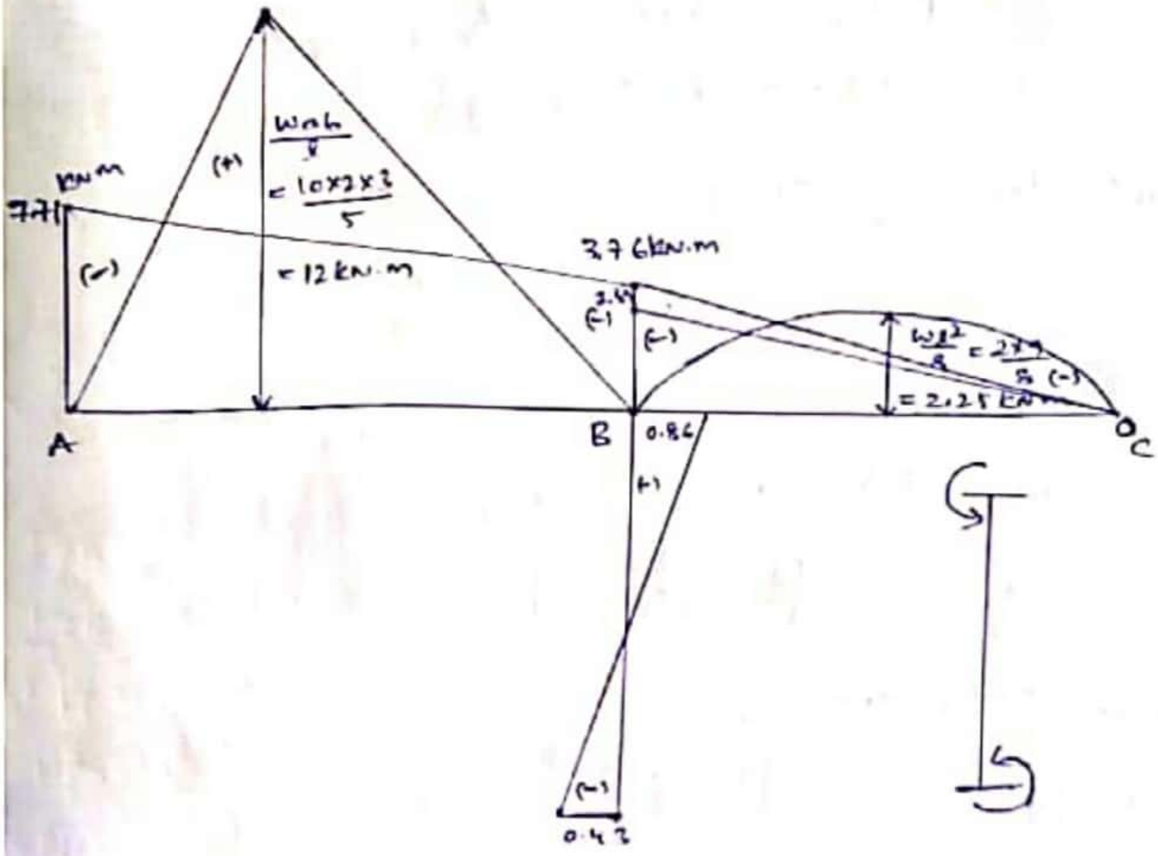
$$= \frac{2EI}{3}\left[\frac{2\times -0.6483}{EI}\right]$$

$$M_{BD} = -0.86 \text{ kN·m}$$

$$M_{DB} = \frac{2EI}{3}(\theta_b)$$

$$= \frac{2EI}{3}\left[\frac{-0.6483}{3}\right]$$

$$M_{DB} = -0.43 \text{ kN·m}$$



In structural analysis self weight of the beam or column are neglected only consider live loads to calculate moments.

If there is no loads no moment is developed.

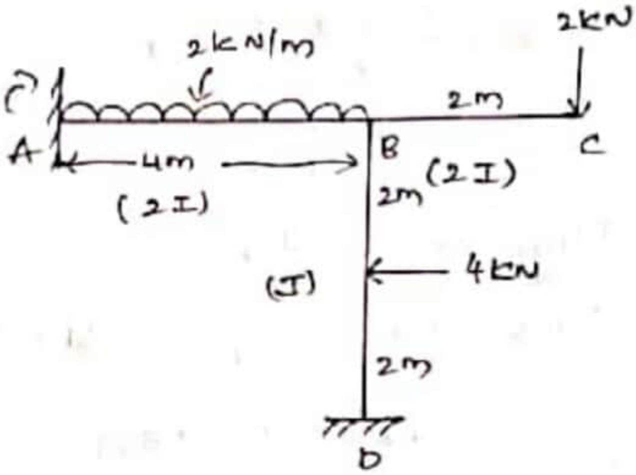
At fixed supports θ is zero

calculating fixed end moments

Imp

$\theta_A = \theta_D = 0$

$\theta_B = ?$

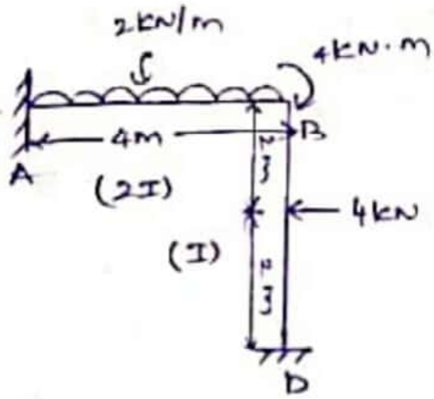


$\bar{M}_{AB} = -\frac{wl^2}{12} = \frac{-2\times 4^2}{12}$

$\bar{M}_{AB} = -2.67$ kN.m

$\bar{M}_{BA} = \frac{wl^2}{12} = \frac{2\times 4^2}{12}$

$\bar{M}_{BA} = 2.67$ kN.m



$\bar{M}_{BC} = 0$

=

$\bar{M}_{CB} = 0$

=

$\bar{M}_{BD} = \frac{-wl}{8} \Rightarrow \frac{+\cancel{4\times 4}}{\cancel{8}} \Rightarrow \frac{-4\times 4}{8}$

$\bar{M}_{BD} = \cancel{+2}\ -2$ kN.m

$\bar{M}_{DB} = -\frac{wl}{8} = \frac{+4\times 4}{8}$

$\bar{M}_{DB} = +2$ kN.m

$M_{BA} + M_{BD} = 4$

$M_{AB} = \bar{M}_{AB} + \frac{2EI}{l_{AB}}(2\theta_A + \theta_b)$

$= -2.67 + \frac{2E(2I)}{4}(\theta_b)$

$M_{AB} = -2.67 + EI\theta_b$

$M_{BA} = \bar{M}_{BA} + \frac{2EI}{l_{BA}}(2\theta_B + \theta_A)$

$= 2.67 + \frac{2E(2I)}{4}(2\theta_B)$

$M_{BA} = 2.67 + 2EI\theta_b$

$$M_{Bb} = \bar{M}_{bd} + \frac{2EI}{l_{bd}}(2\theta_b + \theta_d)$$

$$= -2 + \frac{2EI}{4}(2\theta_b)$$

$$= -2 + \frac{4EI}{4}\theta_b$$

$$M_{BD} = -2 + EI\theta_b$$

$$M_{DB} = \bar{M}_{db} + \frac{2EI}{l_{db}}(2\theta_d + \theta_b)$$

$$= +2 + \frac{2EI}{4}(\theta_b)$$

$$M_{DB} = +2 + \frac{EI}{2}\theta_b$$

$$M_{BA} + M_{BD} = 4$$

$$2.67 + 2EI\theta_b + (-2) + EI\theta_b = 4.$$

$$0.6 + 3EI\theta_b = 4$$

$$3EI\theta_b = 4 - 0.67$$

$$EI\theta_b = \frac{\cancel{+0.67}\ 3.33}{3}$$

$$EI\theta_b = \cancel{+0.2233}\ 1.11$$

$M_{AB} = -2.67 + (-0.2233)$

$M_{AB} = -2.89$ kN·m

$M_{BA} = 2.67 + 2(-0.2233)$

$M_{BA} = 2.22$ kN·m

$M_{BD} = 2 + (-0.2233)$

$M_{BD} = 1.72$ kN·m

$M_{DB} = -2 + 0.5(-0.2233)$

$M_{DB} = -2.111$ kN·m

$M_{AB} = -2.67 + 1.11$

$M_{AB} = -1.56$ kN·m

$M_{BA} = 2.67 + 2(1.11)$

$M_{BA} = 4.89$ kN·m

$M_{BD} = -2 + 1.11$

$M_{BD} = -0.89$ kN·m

$M_{DB} = +2 + 0.5(1.11)$

$M_{DB} = 2.55$ kN·m