3-2-20

UNIT - II

Analysis of Indeterminate Structures

A structure is said to be Indeterminate if the unknown forces and unknown reactions in the structure can be found out using the equilibrium conditions and compatibility equations.

Ex :- simply supported beam, cantilever beam.

* A structure is said to be indeterminate if the unknown forces and unknown reactions in the structure can't be found out using the equilibrium conditions and compatibility equations.

Ex :- Fixed beams, contineous beams.

Degree of Indeterminace

Degree of indeterminace is are two types.

1) Degree of Static Indeterminacies

2) Degree of Kinematic indeterminacies

Degree of Static Indeterminace is also classified into two types.

1) Degree of External static indeterminacy

2) Degree of Internal Static Indeterminacies.

Degree of static

structures which cannot be analysed by the equilibrium conditions are statically Indeterminate structures. (denoted by D_s)

* D_s is given by sum of degree of External static indeterminace D_{se} and degree of

Internal static indeterminace D_{SI} :

Degree of Kinematic Indeterminace

A structure is said to be kinematically indeterminate if the displacement components can't be found out using the compatability equations.

Structure		D_S		D_K
		D_{Se}	D_{SI}	
Beams	2D	$r-3$	-	-
Frames	2D	$r-3$	$3C$	$3j-r$
	3D	$r-6$	$6C$	$6j-r$
Truss	2D	$r-3$	$m-(2j-r)$	$2j-r$
	3D	$r-6$	$m-(3j-r)$	$3j-r$

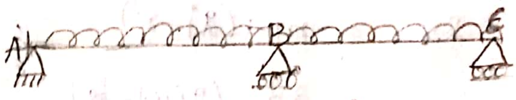
Equilibrium conditions for two-dimensional structure

For two dimensional structure, equilibrium conditions

$\{\Sigma F_x = 0, \ \Sigma F_y = 0, \ \Sigma M_z = 0\}$ - Three

For space structure $\{\Sigma F_x = 0, \ \Sigma F_y = 0, \ \Sigma F_z = 0, \ \Sigma M_x = 0$

$\Sigma M_y = 0, \ \Sigma M_z = 0\}$ six



Total no. of reactions

$2+1+1 = 4$

no. of unknowns greater than no. of equilibrium condition

-s. Therefore this is indeterminate structure.

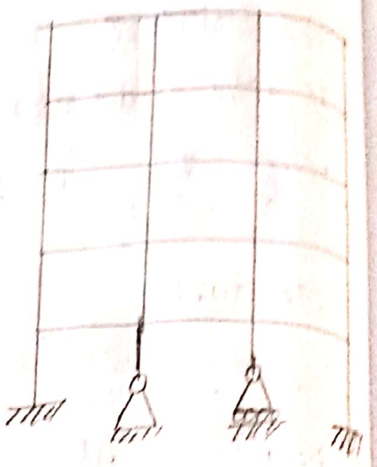
$D_{Se} = 4-3 = 1$

r = Total no. of reactions for the given structure

m = no. of members.

j = no. of joints

c = closed no. of closed loops.



$$D_{se} = r - 3$$
$$= 9 - 3$$
$$= 6$$

$$D_{si} = 3 \times 12$$
$$= 36$$

$$D_s = D_{se} + D_{si}$$
$$= 6 + 36$$
$$= 42$$

$$D_k = 3(24) - 9$$
$$= 63.$$

$r = 3$

$m = 23.$

$j = 16$

$$D_{se} = r - 3$$
$$= 3 - 3$$
$$= 0$$

$$D_{si} = m - (2j - r)$$
$$= 23 - (2(16) - 3)$$
$$=$$

$$D_s = D_{se} + D_{si}$$
$$= 1 +$$

$$D_k = 2j - r$$
$$= 2(16) - 4$$
$$=$$

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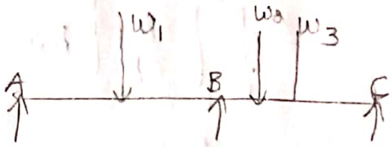
Castigliano second theorem

In any and every case of statical indetermination where in indefinate no. of different values of resultant force satisify the condition of statical equilibrium.

There actual values are given by those tended by total strain energy to a minimum.



* If the reaction at A is 'X' then the equilibrium conditions are not sufficient to find the 'X'.

* For ~~instantance~~ instance assuming any value X the reactions V_B and V_C can be determine.

* Hence infinite values of X with corresponding values of V_B and V_C along with the given loads satisify the conditions of equilibrium.

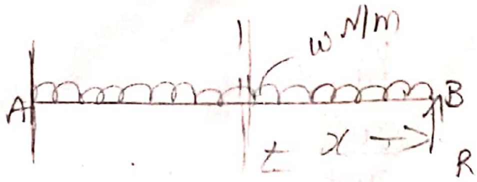
* Hence our problem is findout the value of X. which can be determined by using the fact that deflection at 'A' is zero.

* But by the first theorem of castigliano deflection at A is given by $\frac{\partial U}{\partial x}$

$$\rightarrow \frac{\partial U}{\partial x} = 0$$

using $\frac{\partial U}{\partial x} = 0$ we can findout actual value of X.

Find the reactions at simple supported end, R.



Let reaction at B

$$M_x = Rx - \frac{wx^2}{2}$$

$$U = \int_0^l \frac{M^2\,dx}{2EI}$$

$$U = \frac{1}{2EI}\int_0^l \left[Rx - \frac{wx^2}{2}\right]^2 dx$$

$$\frac{\partial U}{\partial X} = 0$$

$$\frac{\partial}{\partial R}\left[\frac{1}{2EI}\int_0^l \left[Rx - \frac{wx^2}{2}\right)^2 dx\right] = 0$$

$$= \int_0^l \left[\frac{\partial}{\partial R}\left[Rx - \frac{wx^2}{2}\right]^2\right] dx$$

$$= \int_0^l 2\left[Rx - \frac{wx^2}{2}\right] x \, dx$$

$$= 2\int_0^l Rx^2 - \frac{wx^3}{2} \, dx$$

$$= 2\left[R\frac{x^3}{3} - \frac{wx^4}{8}\right]_0^l$$

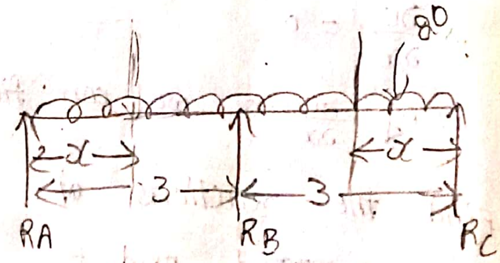
$$\Rightarrow 2R\frac{l^3}{3} - \frac{wl^4}{4} = 0$$

$$\Rightarrow 2R\frac{l^3}{3} = \frac{wl^4}{4}$$

$$2R = \frac{3wl}{4}$$

$$R = \frac{3wl}{8}$$

$$U = \int \frac{m^2 dx}{2EI}$$



$$Y_A = 0$$

$$\frac{\partial U}{\partial RA} = 0$$

$$\Rightarrow \frac{\partial}{\partial RA}\left[\int \frac{m^2 dx}{2EI}\right] = 0$$

$$\frac{\partial}{\partial RA}\left[\int m^2 dx\right] = 0$$

$$\frac{\partial}{\partial R_A}\left[\int (M_{x\,AB})^2 + (M_{x\,BC})^2\right] dx = 0$$

$$\frac{\partial}{\partial R_A}\left[\int \left(R_A x - 20 \cdot \frac{x^2}{2}\right)^2 + \left(R_C x - \frac{20 x^2}{2}\right)^2\right] dx = 0$$

$$\frac{\partial}{\partial R_A}\left[\int (R_A x - 10x^2)^2 dx\right] + \frac{\partial}{\partial R_A}\left[\int (R_C x - 10x^2)^2 dx\right] = 0$$

$$\frac{\partial}{\partial R_A}\left[\int (R_A x - 10x^2)^2 dx\right] = 0$$

$$\int 2\,(R_A x - 10x^2)\, x\, dx = 0$$

$$2\int_0^{3/2} 2\left(R_A x^2 - 10x^3\right) dx = 0$$

$$\int_0^{3/2} (R_A x^2 - 10x^3)\, dx = 0$$

$$\left[R_A \times \frac{x^3}{3} - 10\frac{x^4}{4}\right]_0^{3/2} = 0$$

$$R_A \times \frac{(3/2)^3}{3} - 10 \times \frac{(3/2)^4}{4} = 0$$

$$R_A \times \frac{(3/2)^3}{3} = 10 \times \frac{(3/2)^4}{4}$$

$$\frac{R_A}{3} = 3.75$$

$$R_A = 11.25$$

Taking moments about C

$$R_A \times 6 + R_B \times 3 - 20 \times 6 \times \frac{6}{2} = 0$$

$$R_B \times 3 = -67.5 + 360$$

$$R_B = 97.5$$

$R_A + R_B + R_C = 120$

$R_C = 120 - 97.5 - 11.25$

$R_C = 11.25$

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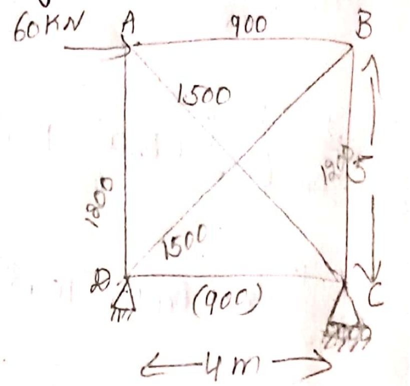
Deflection of Truss (using castigliano second theorem



$D_{se} = r - 3$

$= 3 - 3$

$= 0$

$D_{si} = m - (2j - r)$

$= 6 - (2(4) - 3)$

$= 1$

$D_s = D_{se} + D_{si}$

$= 1$

let BD be the '0' force member.

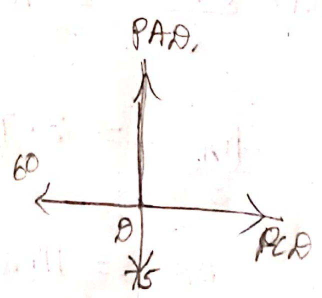
$\Sigma V = 0 \quad V_D + V_C = 0$

$\Sigma H = 0 \quad H_D - 60 = 0 \quad H_D = 60$

$\Sigma M_B = 0 \quad V_D \times 4 + 60 \times 5 = 0 \quad V_D = -75$ KN

$\therefore V_C = 75$ KN

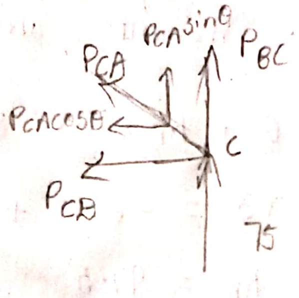
Consider joint D



$P_{AD} = 75$

$P_{CD} = 60$

Consider joint C



$P_{BC} + 75 + P_{CA} \times \frac{5}{\sqrt{41}} = 0$

$P_{BC} = -75 + \frac{5}{\sqrt{41}} \times 15\sqrt{41}$

~~PBC~~ $P_{BC} = 0$

member	P	K	L	A	$\frac{PKL}{A}$	$\frac{K^2L}{A}$	$S = P + XK$
AB	0	$-\frac{4}{\sqrt{41}}$	4	9×10^{-4}	0	1.73×10^{3}	-30.04
BC	0	$-\frac{5}{\sqrt{41}}$	5	1.2×10^{-3}	0	2.54×10^{3}	-37.55
CD	60	$-\frac{4}{\sqrt{41}}$	4	9×10^{-4}	-166.58×10^{3}	1.73×10^{3}	29.95
DA	75	$-\frac{5}{\sqrt{41}}$	5	1.2×10^{-3}	-244.02×10^{3}	2.54×10^{3}	37.44
AC	$-15\sqrt{41}$	1	$\sqrt{41}$	1.5×10^{-3}	-410×10^{3}	4.26×10^{3}	-47.94
BD	0	1	$\sqrt{41}$	1.5×10^{-3}	0	4.26×10^{3}	48.10
					-820.62×10^{3}	17060	

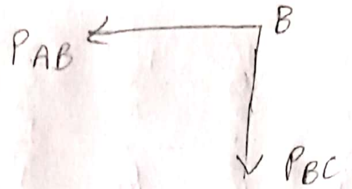
$$X = -\frac{\sum \frac{PKL}{A}}{\sum \frac{K^2L}{A}}$$

$X = 48.10$

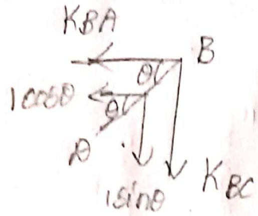
$P_{CD} + P_{CA} \times \frac{4}{\sqrt{41}} = 0$

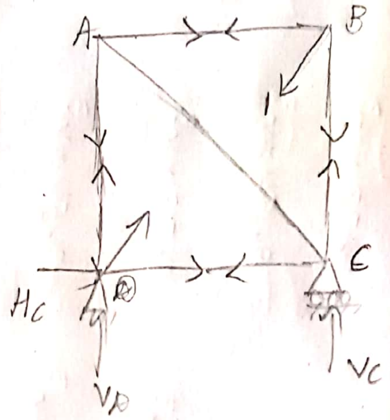
$P_{CA} = -5\sqrt{41}$

Consider joint B



$P_{AB} = 0$





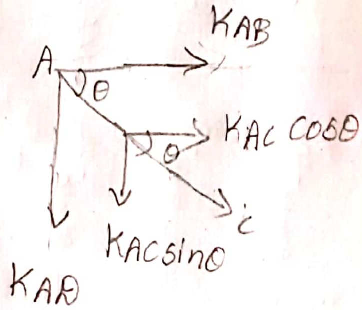
Joint B

$K_{AB} + \cos\theta = 0$

$K_{AB} = \frac{-4}{\sqrt{41}}$

$K_{BC} + \sin\theta = 0$

$K_{BC} = \frac{-5}{\sqrt{41}}$



Joint A

$K_{AB} + K_{AC}\cos\theta = 0$

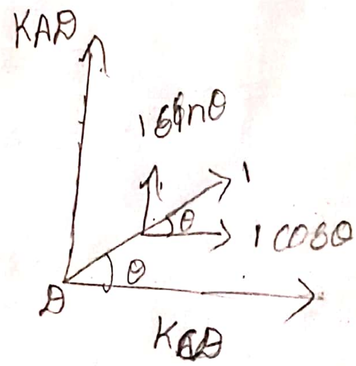
$K_{AC} \times \frac{4}{\sqrt{41}} = \frac{4}{\sqrt{41}}$

$K_{AC} = \frac{4}{\sqrt{41}} \times \frac{\sqrt{41}}{4}$

$K_{AC} = 1$

$K_{AD} + K_{AC}\sin\theta = 0$

$K_{AD} = \frac{-5}{\sqrt{41}}$



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$$Kc\theta + \cos\theta = 0$$

$$Kc\theta = \frac{-4}{\sqrt{41}}$$