COURSE MATERIAL	
SUBJECT	NUMERICAL METHODS AND PROBABILITY THEORY (20A54402)
UNIT	V
COURSE	II B. TECH
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8. LECTURE NOTES

5.1 INTRODUCTION

In day-to-day life, we come across many random variables such as ------

1.Number of male children in a family having three children.

2.Number of passengers getting into a bus at the bus stand.

3.I.Q. of children

4.Number of stones thrown successively at a mango on the tree until the mango in hit

5.Marks scored by a candidate in the P.U.E. examination.

For a quick analysis of distributions of such random variables, we consider their theoretical equivalents. These equivalent distributions are originated according to certain theoretical assumptions and restrictions. Such theoretically designed distributions are called theoretical distributions.

There are many types (families) of theoretical distributions. Some of them

(i) Binomial distribution

(ii) Poisson distribution

(iii) Normal distribution

(iv) Uniform distribution

(v) Exponential distribution.

BERNOULLI EXPERIMENT:

Suppose a random experiment has two outcomes, namely, "Success" and "Failure". Let probability of Success be p and let probability of Failure be q=(1-p). Such an experiment is called Bernoulli experiment or Bernoulli trial.

Let a Bernoulli experiment be conducted (repeated) n times. Let the variable X_i (i=1,2, 3,…n) take values 1 and 0 according as the i th experiment is a Success or a Failure. Then, X_i is a Bernoulli variate with parameter p. It denotes the number of success in the i^{th} experiment.

Let $X = X_1+X_2+.....+X_n$. Then, X denotes the number of success in these n repetitions.

Example:

1. Let coin be tossed 3 times. Let X_i (i=1,2,3) be a variate which takes values 1 and 0 according as the i^{th} toss result in "Head" or "Tail". Then, $X = X_1+X_2+X_3$ denotes "the number of head" obtained in the 3 tosses.

Result:

If $X_1, X_2, \ldots X_n$ are independently and identically distributed (i,i,d) Bernoulli variates with common parameter p, their sum $X = X_1+X_2 + \ldots.+X_n$ is a Binomial variate with parameters n and p

5.2 BINOMIAL DISTRIBUTION

A Probability distribution which has the following probability mass function (p.m.f) is called Binomial distribution.

$$p(x) = nC_x\, p^x\, q^{n-x},\ x=0,1,2\ldots\ldots n.$$

$$0<p<1;\ q=1-p$$

Here n, p are parameters the variable X is discrete and it is called Binomial variate.

Note1: Binomial p.m.f. has two independent constants, namely, n and p. These two constants are the parameters of binomial distribution

Note2: A binomial distribution with parameter n and p is denoted by b(x;n,p) or B(n,p).

EXAMPLES FOR BINOMIAL VARIATE

1.Number of heads obtained in 3 tosses of a coin.

2.Number of male children in a family of 5 children

3.Number of bombs hitting a bridge among 8 bombs which are dropped on it.

4.Number of defective articles in a random sample of 5 articles drawn from a manufactured lot

5.Number of seeds germinating among 10 seeds which were sown

MEAN & VARIANCE OF BINOMIAL DISTIBUTION

Mean is E(X) = np

Variance is Var(X) = np(1-p) = npq

Standard deviation is S.D.(X) = $\sqrt{npq}$

RECURRENCE RELATION BETWEEN SUCCESSIVE TERMS

We have $p(x) =^n n_x p^x q^{n-x}$

and $p(x-1) =^n C_{x-1} p^{x-1} q^{n-x+1}$

Therefore,

$$\frac{p(x)}{p(x-1)} = \frac{nC_x p^x q^{n-x}}{nC_{x-1} p^{x-1} q^{n-x+1}} = \frac{(n-x+1)}{x}$$

$$p(x) = \frac{(n+1-x)}{x} . \frac{p}{q} . p(x-1)$$

Thus, this is the recurrence relation between success probabilities of binomial distribution when p(x-1) is known, this relation can be used to obtain p(x).

1. The incidence of an occupational disease in an industry is such that the worker has 25% change of suffering from it. What is the probability that out of 5 workers, at the most two contacts that disease?

Solutions:

X: number of workers contracting the diseases among 5 workers

Then, X is a binomial variate with parameter n=5 and

P= P [a worker contracts the disease] = 25/100 = 0.25

The probability mass function (p.m.f) is –

P(x) = 5Cx $(0.25)^x$ $(0.75)^{5-x}$, x=0,1, 2….5

The probability that at the most two workers contact the disease is ---

$$P[X \leq 2] = p(x=0) + p(x=1) + p(x=2)$$
$$=^5 C_0 (0.25)^0 (0.75)^5 +^5 C_1 (0.25)^1 (0.75)^4 +^5 C_2 (0.25)^2 (0.75)^3$$
$$= 0.2373 + 0.3955 + 0.2637$$
$$= 0.8965$$

2. In a large consignment of electric lamps, 5% are defective. A random sample of 8 lamps is taken for inspection. What is the probability that it has one or more defectives?

Solution:

X: number of defective lamps

Then, X is B (n=8, p=5/100 = 0.05)

The p.m.f. is –

p(x) = $8C_x(0.05)^x(0.95)^{8-x}, x = 0,1,2,\ldots 8$

P [sample has one or more defectives] = 1-P [no defectives]

= 1-p (x=0)

$= 1 - 8C_0(0.05)^0(0.95)^8$

$= 1 - 0.6634 = 0.3366$

3. In a Binomial distribution the mean is 6 and the variance is 1.5. Then, find (i) P[X=2] and (ii) P[X≤2].

Solution:

Let n and p be the parameters. Then,

Mean = np = 6

Variance = npq = 1.5

$$\frac{Variance}{Mean} = \frac{npq}{np} = \frac{1.5}{6} = \frac{1}{4}$$

Therefore, q = ¼ and p = ¾

Therefore, Mean = n*3/4 = 6

That is, n = 24/3 = 8

The p.m.f is -------

p(x) = $8Cx\ (3/4)^x\ (1/4)^{8-x}$, x=0,1, 2…….8

(i) P[X=2] = $8C_2\ (3/4)^2\ (1/4)^6$ =252/65536=0.003845

(ii) P[X≤2] = p(x=0) +p(x=1) +p (x=2)

= $8C_0\ (3/4)^0\ (1/4)^8 + 8C_1\ (3/4)^1\ (1/4)^7 + 8C_2\ (3/4)^2\ (1/4)^6$

= 277/65536 = 0.004227

5.3 POISSON DISTRIBUTION

A probability distribution which has the following probability mass function (p.m.f) is called Poisson distribution

$$p(x) = \frac{e^{-\lambda}\ \lambda^{x}}{x!}, \qquad x = 0, 1, 2 \dots \dots \dots \dots$$

Here, the variable X is discrete and it is called Poisson variate.

Note 1: λ is the parameter of Poisson, Poisson distribution has only one parameter

Note 2: Poisson distribution may be treated as limiting form of binomial distribution under the following conditions. (Binomial distribution tends to Poisson distribution under the following conditions.)

(i) p is very small ($p \to 0$)

(ii) n is very large ($n \to \infty$) and

(iii) $np=\lambda$ is fixed

EXAMPLES FOR POISSON VARIATE:

Many variables which occur in nature vary according to the Poisson law. Some of them are

1. Number of deaths occurring in a city in a day
2. Number of road accidents occurring in a city in a day
3. Number of incoming telephone calls at an exchange in one minute
4. Number of vehicles crossing a junction in one minute.

1. The number of accidents occurring in a city in a day is a Poisson variate with mean 0.8. Find the probability that on a randomly selected day

(i)There are no accidents

(ii)There are accidents

Solution:

Let X: number of accidents per day.

Then, X is $P(\lambda=0.8)$.

The p.m.f. is –

$$p(x) = \frac{e^{-0.8}(0.8)^x}{x!}, x = 0,1,2,3, \ldots.$$

(i) Probability that a particular day there are no accidents is ------------

P [no accidents] = P[X=0]

$$= \frac{e^{-0.8}(0.8)^0}{0!} = e^{-0.8} = 0.449$$

(ii) P [accidents occur] = 1-P [no accidents]

= 1-p(0) = 1-0.449 = 0.551

2. The number of persons joining a cinema queue in a minute has Poisson distribution with parameter 5.8. Find the probability that (i) no one joins the queue in a particular minute (ii)2 or more persons join the queue in the minute.

Solution:

Let X: number of persons joining the queue in a minute Then, is P(λ=5.8).

The p.m.f is -------------------

$$p(x) = \frac{e^{-5.8}(5.8)^x}{x!}, x = 0,1,2,3, \ldots.$$

(i) P [no one joints the queue] = P[X=0] = p(0)

$$= \frac{e^{-5.8}(5.8)^0}{0!}$$

$= e^{-5.8}$=0.003

(ii) P [two or more join] = $P[X \geq 2]$

$$= 1 - P[X < 2]$$

$$= 1 - \{p(0) + p(1)\}$$

$$= 1 - \left\{\frac{e^{-5.8}(5.8)^0}{0!} + \frac{e^{-5.8}(5.8)^1}{1!}\right\}$$

$$= 1 - e^{-5.8}\{1 + 5.8\}$$

$$= 1 - 0.003 \times 6.8$$

$$= 1 - 0.0204 = 0.9796$$

3. The average number of telephone calls booked at an exchange between 10-00 A.M. and 10-10 A.M. is Find the probability that on a randomly selected day 2 or more calls are booked between 10-00 A.M. and 10-10 A.M. On how many days of a year, would you expect booking of 2 or more calls during those times gap.

Solution:

Let X: number of telephone calls booked at the exchange during 10-00 A.M. to 10-10 A.M. Then, X is P(λ=4).

The p.m.f is ---

$$p(x) = \frac{e^{-4}4^x}{x!}, x = 0,1,2,3,\ldots$$

P [2 or more calls] = 1-P [less than 2 calls]

= 1 – [p (0) +p (1)]

= 1 – e^{-4}[(40)/0! +(41)/1!]

= 1 – 0.0183[1+4]

= 1 – 0.0915 = 0.9085

A year has 365 days. Out of these N = 365 days, the number of days on which there will be 2 or more calls is ---

N *P [2 or more calls] = 365 * 0.9085 = 332

4. 2 percent of the fuses manufactured by a firm are expected to be defective, Find the probability that a box containing 200 fuses contains

(i) defective fuses

(ii) 3 or more defective fuse

Solution:

2 percent of the fuses are defective. Therefore, probability that a fuse is defective is p = 2/100 = 0.02

Let X denote the number of defective fuses in the box of 200 fuses.

Here, p is very small and n is very large. Therefore, X can be treated as Poisson variate with parameter λ=np = 200 * 0.02 = 4.

The p.m.f. is ----

$p(x) = \frac{e^{-4}4^x}{x!}, x = 0,1,2,3, \ldots$

P [box has defective fuses] = 1-P [no defective fuses]

= 1 – p (0)

= 1 – e^{-4}[(4^0)/0!]

= 1 – 0.0183 =0.9817

P [3 or more defective fuses] = 1-P [less than 3 defective fuses]

= 1 – [p (0) +p (1) +p (2)]

= 1 – e^{-4}[1+4+8]

= 1 – 0.0183*13

= 1 – 0.2379 =0.7621

5. The probability that a razor blade manufactured by a firm is defective is 1/500. Blades are supplied in packets of 5 each. In a lot of 10,000 packets, how many packets would

(i)Be free defective blades?

(ii) Contains exactly one defective blade? ($e^{-0.0}$1=0.99)

Solution:

Let X be the number of defective blades in a packet of 5 blades. p = 1/500

Since p is very small and n is sufficiently large, X is treated as Poisson variate with parameter λ=np = 5*(1/500) = 0.01

$$p(x) = \frac{e^{-0.01}(0.01)^x}{x!}, x = 0,1,2,3, \ldots$$

(i) P [no defective blades] = p (0)

= $e^{-0.01}$[$(0.01)^0$/0!] = 0.99

The number of packets which will be free of defective blades is -----

N * P [no defective blades] = 10000*0.99 = 9900

(ii) P [one defective blade] = p (1)

$= e^{-0.01}[(0.01)^{1}/1!] = 0.0099$

The number pf packets which will have one defectives blade is ----

N * P [one defective blade] = 10000*0.0099 = 99

6. On an average, a typist mistakes while typing one page. What is the probability that a randomly observed page in free of mistakes? Among 200 pages, in how many pages would you expect mistakes?

Solution:

Let X: number of mistakes in a page.

Then, X is P(λ=3).

The p.m.f. is ---

$$p(x) = \frac{e^{-3}3^{x}}{x!}, x = 0,1,2,3, \ldots.$$

P [page is free of mistakes] = p(0)

$$= \frac{e^{-3}3^{0}}{0!} = e^{-3} = 0.0498$$

P [page has mistakes] = 1-P [Page has no mistakes]

= 1-0.0498=0.9502

Among 200 pages, the expected number of pages containing mistakes is ---

N*P [page has mistakes] = 200*0.9502=190

7. In a Poisson distribution P[X=2] = P[X=3]. Find P[X=4].

Solution:

Let λ be the parameter

Here, P [X = 2] = P[X=3]

$$\Rightarrow 3 \times \frac{e^{-\lambda}\lambda^{2}}{2!} = \frac{e^{-\lambda}\lambda^{3}}{3!}$$

$$\Rightarrow 1 = \frac{\lambda}{3}$$

And so, λ=3

The p.m.f. is ----

$$p(x) = \frac{e^{-3}3^x}{x!}, x = 0,1,2,\ldots.$$

P[X=4] = p(4) = $\frac{e^{-3}3^2}{4!}$

$= \frac{0.0498 \times 81}{24} = 0.1681$

8. For a Poisson variable 3 * P[X=2] = P[X=4]. Find standard deviation.

Solution:

Here, 3*P [X = 2] = P[X=4]

$\Rightarrow 3 \times \frac{e^{-\lambda}\lambda^2}{2!} = \frac{e^{-\lambda}\lambda^4}{4!}$

$\Rightarrow 3 = \frac{\lambda^2}{3 \times 4}$

And so, $\lambda^2=36$

That is, λ=6

Thus, the parameter is λ=6

The standard deviation S.D.(X) = $\sqrt{\lambda} = \sqrt{6} = 2.449$

5.4 NORMAL DISTRIBUTION

A probability distribution which has the following probability density function (p.d.f) is called Normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}, -\infty < x < \infty, \qquad \sigma > 0$$

Here, the variable X is continuous and it is called Normal variate.

Note 1: The distribution has two parameters, namely, μ and σ.

(Here, Π=3.14 and e=2.718)

Note 2: This normal distribution has Mean E(X) = μ and

Variance = V(X) = σ^2. S.D.(X)=σ.

Note 3: A normal variate with parameters μ and σ is denoted by N (μ, σ^2)

EXAMPLES FOR NORMAL VARIATE

Many of the variables which occur in nature have normal distribution. Some examples are –

1.Height of students of a college

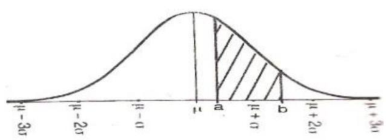
2.Weight of apples grown in an orchard

3.I.Q(Intelligence Quotient) of a large group of children.

4.Marks scored by students in an examination

PROPERTIES OF NORMAL DISTRIBUTION (Properties of normal Curve):

A normal distribution with parameters μ and σ has the following properties.



1.The curve is Bell –shaped

a. It is symmetrical (Non-skew).

That is $\beta_1 = 0$

b. The mean, media and mode are equal

2. The curve is asymptotic to the X-axis. That is, the curve touches the X-axis only at $-\infty$ and $+\infty$.

3. The curve has points of inflexion at $\mu - \sigma$ and $\mu + \sigma$.

4. For the distribution ….

a. Standard deviation = σ

b. Quartile deviation = 2/3 σ (approximately)

c. Mean deviation = 4/5 σ (approximately)

5. The distribution is mesokurtic. That is, $\beta_2 = 3$.

6. Total area under the curve is unity.

$P[a < X \le b]$ = Area bounded by the curve and the ordinates at a and b

a. $P[\mu - \sigma < X \le \mu + \sigma] = 0.6826 = 68.26\%$

b. $P[\mu - 2\sigma < X \le \mu + 2\sigma] = 0.9544 = 95.44\%$

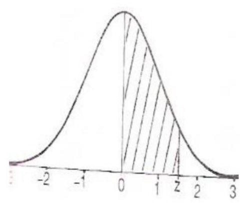
c. $P[\mu - 3\sigma < X \le \mu + 3\sigma] = 0.9974 = 99.74\%$

STANDARD NORMAL VARIATE:

A normal variate with mean μ=0 and standard deviation σ =1 is called Standard Normal Variate. It is denoted by Z. Its probability density function is –

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{-z^2}{2}}, -\infty < z < \infty$$

The graph of standard normal distribution is shown in the figure



The shaded area in the figure represents the probability that the variate takes a value between 0 and z. This area can be read from the table of areas under Standard Normal Curve. Corresponding to nay positive z, the area from 0 to z can be read from this table.

Let X be a normal variate with mean μ and standard deviation σ. Then $Z = \frac{X-\mu}{\sigma}$ is a Standard Normal Variate

Therefore, to find any probability regarding X, the Standard Normal Variate can be made use of.

Note: The Standard Normal Variate is denoted by N (0,1).

1. X is a normal variate with mean 42 and standard deviation 4. Find the probability that a value taken by X is

   (i)less than 50 (ii) greater than 50

   (iii) less than 40 (iv) greater than 40

   (v) between 40 and 44 (vi) between 37 and 41.

Solution:

X is a normal variate with parameters μ=42 and σ=4

Therefore,

$z = \frac{x-\mu}{\sigma} = \frac{x-42}{4}$ is a Standard Normal Variate.

(i) $p(x < 50) = p(\frac{x-42}{4} < \frac{50-42}{4})$

= P[Z<2]

= area from (-∞) to 2

= [area from (-∞) to 0] + [area from 0 to 2]

= 0.5 + 0.4772(from the table)

= 0.977200

(ii) $p(x > 50) = p(\frac{x-42}{4} > \frac{50-42}{4})$

= P[Z<2]

= area from 2 to ∞

= [area from 0 to ∞] - [area from 0 to 2]

= 0.5 - 0.4772(from the table)

= 0.0228

(iii) $p(x < 40) = p(\frac{x-42}{4} < \frac{40-42}{4})$

= P [Z < -0.5]

= area from (- ∞) to (-0.5)

= area from 0.5 to ∞

= [area from 0 to ∞] - [area from 0 to 0.5]

= 0.5 - 0.1915

= 0.3085

(iv) $p(x > 40) = p(\frac{x-42}{4} > \frac{40-42}{4})$

= P [Z > -0.5]

= area from (-0.5) to ∞

= area from (-0.5) to 0] + [area from 0 to ∞]

= [area from 0 to 0.5] + [area from 0 to ∞]

= 0.1915 + 0.5

= 0.6915

(v) $p(40 < x < 44) = p(\frac{40-42}{4} < \frac{x-42}{4} < \frac{44-42}{4})$

= P [-0.5 < Z < 0.5]

= area from -0.5 to 0.5

= area from -0.5 to 0] + [area from 0 to 0.5]

= area from 0 to 0.5] + [area from 0 to 0.5]

= 0.1915 + 0.1915

= 0.3830

(vi) $p(37 < x < 41) = p(\frac{37-42}{4} < \frac{x-42}{4} < \frac{41-42}{4})$

= P [-1.25 < Z <-0.25]

= area from –1.25 to 0.25

= area from 0.25 to 1.25

= area from 0 to 1.25] - [area from 0 to 0.25]

= 0.3944 – 0.0987

= 0.2957.

2. Height of students is normally distributed with mean 165 cms and standard deviation 5 cms. Find the probability that height of a students is

(i) more than 177 cms.

(ii) less than 162 cms.

Solution:

Let X denote height. Then, X is a normal variate with parameters μ = 165 cms and σ=5

Then $z = \frac{x-\mu}{\sigma} = \frac{x-165}{5}$

(i) Probability that the student is more than 177 cms tall is –

$p(x > 177) = p(\frac{x-165}{5} > \frac{177-165}{5})$

= P [Z > 2.4]

= area from 2.4 to ∞

= [area from 0 to ∞] – [area from 0 to 2.4]

= 0.5 – 0.4918

= 0.0082

(ii) Probability that the student is less than 162 cms. Tall is -----

$p(x < 162) = p(\frac{x-165}{5} < \frac{162-165}{5})$

= P [Z < -0.6]

= area from (-∞) to (-0.6)

= area from 0.6 to ∞

= [area from 0 to ∞] – [area from 0 to 0.6]

= 0.5 – 0.2258

= 0.2742

3. Mean life of electric bulbs manufactured by a firm is 1200 hrs. The standard deviation is 200 hrs.

(i) In a lot of 10,000 bulbs, how many bulbs are expected have life 1050 hrs. or more?

(ii) What is the percentage of bulbs which are expected to find before 1500 hrs. of service?

Solution:

Let X denotes the life of the bulbs. Then, X is a normal variate with parameters

μ=1200hrs σ=200 hrs

Then $z = \frac{x-\mu}{\sigma} = \frac{x-1200}{200}$

(i) Probability that life of a bulb is 1050 hrs. or more is ---

$p(x \geq 1050) = p(\frac{x-1200}{200} \geq \frac{1050-1200}{200})$

=P [Z ≤ -0.75]

=0.2734 + 0.5

=0.7734

In a lot of n=10,000 bulbs, expected number of bulbs with life 1050 hrs. or more is ---

N * P[X≥1050] =10000*0.7734=7734

(ii) Probability that life of a bulb is before 1500 hrs is ---

$$p(x \leq 1050) = p(\frac{x-1200}{200} \leq \frac{1500-1200}{200})$$

= P [Z < 1.5]

=0.5 + 0.4332

=0.9332

The percentage of bulbs with life less than 1500 hrs is ---

100 * P[X<1500] =100*0.9332=93.32

4. The mean and standard deviation of marks scored by a group of students in an examination are 47 and 10 respectively. If only 20% of the students have to be promoted, which should be the marks limits for promotion?

Solution:

Let X denotes marks. Then, X is a normal variate with parameters μ=47 and σ=10.

Then $z = \frac{x-\mu}{\sigma} = \frac{x-47}{10}$

Let a be the marks above which if a student scores he would be promoted. Then, since only 20% of the students have to be promoted the probability of a student getting promotion should be 20/100=0.2

Therefore,

$$p(x \geq a) = 0.2$$

$$p\left(\frac{x-47}{10} \geq \frac{a-47}{10}\right) = 0.2$$

$$p\left(z \geq \frac{a-47}{10}\right) = 0.2$$

And so, P[Z≥.z] = 0.2 where z= $\frac{a-47}{1}$

That is, [area from z to ∞] =0.2

That is, [area from 0 to z] =0.3

From the table of areas, the value of z for which [area from 0 to z] = 0.3 is z=0.84

Therefore, z=0.84.

And so, $\frac{a-47}{10} = 0.84$

$$a - 47 = 8.4$$

$$a = 55.4$$

Thus, the marks limit for promotion is a = 55.4

9. Practice Quiz

1. The mean of binomial distribution is…………………….. [B]

   A) 0 B) np C) npq D) $np(1+q)$

2. The probability of getting 2 heads in tossing 5 coins [A]

   A) $5_{C_2}\left(\frac{1}{2}\right)^5$ B) $5_{C_2}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)$ C) $5_{C_3}\left(\frac{1}{2}\right)^2$ D) None

3. The mean of binomial distribution is 4 and variance is 2 then P = [B]

   A) 1 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) None

4. If $n = 400P = 0.2$ of binomial distribution then standard deviation is [A]

   A) 8 B) 9 C) 10 D) None

5. The condition for binomial distribution approaches to poison distribution when. [C]

   A) $P \to$ Very small B) $n \to$ Very large value, $np = \lambda$constant

   C) Both A and B D) None

6. In …………………. distribution mean, variance are equal [C]

   A) Normal B) Binomial C) Poisson D) None

7. If X is a Poisson variate with λ= 3 then $P(x=0) =$……………….. [A]

   A) e^{-3} B) $\frac{e^{-3}}{-3}$ C) $-3e^{-3}$ D) None

8. The shape normal curve is [B]

   A) parabola B) bell shape C) Hyperbola D) None

9. The area under the whole normal curve is [B]

   A) 0 B) 1 C) 0.5 D) None

10. If $f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}$ then mean, variances are [A]

A) $(0,1)$ B) $(1,0)$ C) $(0,0)$ D) None

11. The mean, mode, median of a normal distribution are …………… [A]

A) Equal B) unequal C) Both A and B D) None

12. The mean of Poisson distribution is 8, then its variance is---- [D]

A) 2 B) 1 C) 6 D) 8

13. If P (1) =P (2), then the mean of the Poisson distribution is---- [C]

A) 1 B) 0 C) 2 D) -1

14. If x is a normal variate find $P(-0.8 \le z \le 1.53)$ [C]

A) 0.5 B) 1 C) 0.7251 D) None

15. If $\mu = 1, \sigma = 3$ find $P(3.43 \le x \le 6.19)$ [D]

A) 1.5 B) 0.5 C) 0.75 D) 0.1672

10. Assignments

S.No	Question	BL	CO
1	Out of 800 families with 5 children each, how many would you expect to have (a) 3 boys (b) 5 girls (c) either 2 or 3 boys, assume equal probabilities for boys and girls.	L3	CO5
2	2% of the items of a factory are defective. The items are packed in boxes. What is the probability that there will be (i) 2 defective items (ii) at least three defective items in a box of 100 items.	L3	CO5
3	If X is a Poisson variate such that $3P(x=4)=\frac{1}{2}P(x=2)+P(x=0)$ find (i) the mean of X (ii) $P(x \leq 2)$	L3	CO5
4	Suppose the weights of 800 male students are normally distributed with $\mu = 140$ pounds and standard deviation 10 pounds find the number of students whose weight are (i) between 138 and 148 pounds (ii) more than 152 pounds.	L3	CO5
5	In a normal distribution 7% of the items are under 35, 89% are under 63. Determine the mean and variance of the distribution.	L3	CO5

11. Part A- Question & Answers

S.No	Question& Answers	BL	CO
1	Define Binomial distribution? Sol: A Probability distribution which has the following probability mass function (p.m.f) is called Binomial distribution. $p(x) = nC_x\, p^x\, q^{n-x}$, x=0,1, 2……n.	L1	5
2	What are the mean and variance of binomial distribution? Sol: Mean is E(X) = np Variance is Var(X) = np(1-p) = npq	L1	5
3	If a coin is tossed 5 times, find the probability of exactly 2 heads? Sol: Number of trials n=5 Probability of head p= 1/2 and hence the probability of tail q =1/2 For exactly two heads: x=2 $P(x=2) = 5C_2\, p^2\, q^{5-2} = 10 \times (½)^2 \times (½)^3 = 5/16$	L1	5
4	If 75% of all purchases at a certain store are made with a credit card and X is the number among ten randomly selected purchases made with a credit card, then find mean and variance. Sol: Here n=10 p=75%=0.75, q=0.25. It follows Binomial distribution. E(X) = np = (10) (.75) = 7.5, V(X) = npq = 10(.75) (.25) = 1.875	L3	5
5	Write the probability mass function for poison distribution? Sol: A probability mass function for Poisson distribution $p(x) = \frac{e^{-\lambda}\, \lambda^x}{x!}, \quad x = 0, 1, 2 \ldots\ldots\ldots\ldots$	L1	5
6	A random variable X has a Poisson distribution with parameter λ such that P (X = 1) = (0.2) P (X = 2). Find λ value. Sol: For the Poisson distribution, the probability function is defined as: $p(x) = \frac{e^{-\lambda}\, \lambda^x}{x!}$ where λ is a parameter. Given that, P (x = 1) = (0.2) P (X = 2) $\frac{e^{-\lambda}\, \lambda^1}{1!} = (0.2)\, \frac{e^{-\lambda}\, \lambda^2}{2!} \Rightarrow \lambda = \lambda^2/10 \Rightarrow \lambda = 10$	L3	5

7	Define standard normal distribution? Sol: A normal variate with mean μ=0 and standard deviation σ =1 is called Standard Normal Variate. It is denoted by Z. Its probability density function is $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{-z^2}{2}}, -\infty < z < \infty$	L1	5
8	Define Normal distribution. Sol: A probability distribution which has the following probability density function (p.d.f) is called Normal distribution $$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}, -\infty < x < \infty, \quad \sigma > 0$$	L1	5
9	X is a normal variate with mean 42 and standard deviation 4. Find the probability that a value taken by X is greater than 40. Sol: $p(x > 40) = p(\frac{x-42}{4} > \frac{40-42}{4})$ = P [Z > -0.5] = [area from 0 to 0.5] + [area from 0 to ∞] = 0.1915 + 0.5 = 0.6915	L3	5
10	Write down any two properties of Normal distribution. Sol: 1. The curve is Bell –shaped 2. The curve is asymptotic to the X-axis. That is, the curve touches the X-axis only at -∞ and +∞	L1	5

12. Part B- Questions

S.No	Question	BL	CO
1	a) Find mean, variance of binomial distributions b) The mean and variance of binomial distribution are 4 and $\frac{4}{3}$ respectively find $P(x \geq 1)$	L2	**5**
2	a) Find mean, variance of a Poisson distribution b) If the variance of a Poisson variate is 3, then find the probability that (i) x = 0, (ii) $0 < x \leq 3$ (iii) $1 < x < 4$	L3	**5**
3	Out of 800 families with 5 children each, how many would you expect to have (a) 3 boys (b) 5 girls (c) either 2 or 3 boys, assume equal probabilities for boys and girls.	L3	**5**
4	The marks obtained in statistics in a certain examination found to be normally distributed. If 15% of the students' marks>=60, 40% <= 30marks, Find the mean and standard deviation.	L3	**5**
5	The heights of 1000 students are normally distributed with a mean of 174.5 cm and a standard deviation of 6.9 cm. Assuming that the heights are recorded to the nearest half-cm, how many of these students would you expect to have heights: (i) Less than 160.0 cms? (ii) Between 171.5 and 182.0 cms inclusive? (iii) Greater than or equal to 188.0 cms.	L3	**5**

13. Supportive Online Certification Courses

1. Introduction to Probability and Statistics by Prof. G. Srinivasan
   IIT Madras – 4 weeks

14. Real Time Applications

S. No	Application	CO
1	Probability Theory used in the earthquake risk assessment and its consequent possible reduction	4

15. Contents Beyond the Syllabus

1. Probability theory with applications to engineering problems such as the reliability of circuits and systems.

16. Prescribed Text Books & Reference Books

Text Book

1. B. S. Grewal, Higher Engineering Mathematics, Khanna Publishers.
2. Erwin Kreyszig, Advanced Engineering Mathematics, John Wiley & Sons.

References:

1. B. V. Ramana, Higher Engineering Mathematics, McGraw Hill Education
2. Alan Jeffrey, "Advanced Engineering Mathematics", Elsevier Publishers