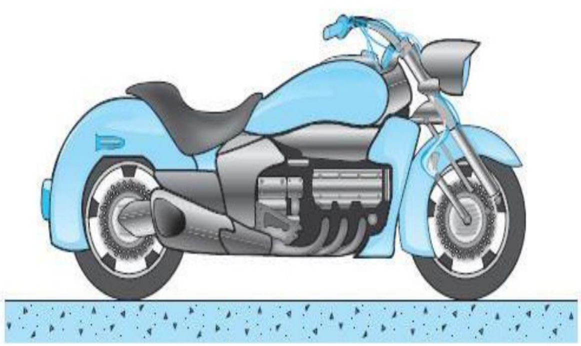
# 3

# Velocity and Acceleration Analysis



## Course Contents

## 3.1 Introduction

– There are many methods for determining the velocity of any point on a link in a mechanism whose direction of motion (i.e. path) and velocity of some other point on the same link is known in magnitude and direction, yet the following two methods :

1. Instantaneous centre method
2. Relative velocity method

– The instantaneous centre method is convenient and easy to apply in simple mechanisms, whereas the relative velocity method may be used to any configuration diagram.

## 3.2 Velocity Of Two Bodies Moving In Straight Lines

– Here we shall discuss the application of vectors for the relative velocity of two bodies moving along parallel lines and inclined lines, as shown in Fig. 2.1 (a) and 2.2 (a) respectively.

– Consider two bodies A and B moving along parallel lines in the same direction with absolute velocities $v_A$ and $v_B$ such that $v_A > v_B$ , as shown in Fig. 2.1 (a). The relative velocity of A with respect to B,

$$\boldsymbol{v_{AB} = vector\ difference\ of\ v_A\ and\ v_B = \vec{v_A} - \vec{v_B}}$$

– From Fig. 2.1 (b), the relative velocity of A with respect to B (i.e. $v_{AB}$) may be written in the vector form as follows :

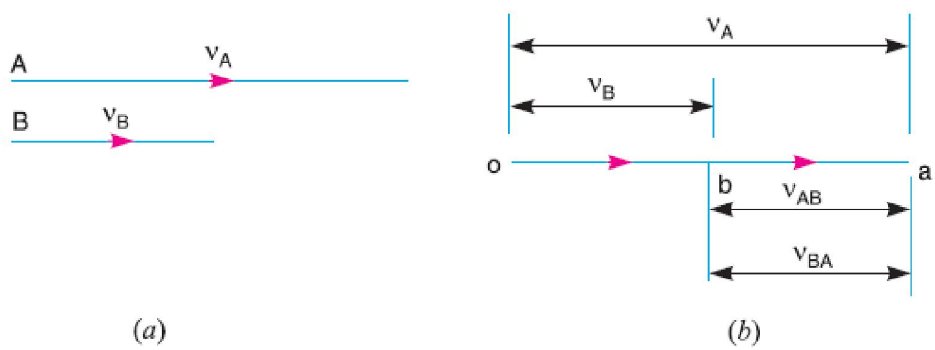



*Fig. 3.1 relative velocity of two bodies moving along parallel line*

– Similarly, the relative velocity of B with respect to A,

$$\boldsymbol{v_{AB} = vector\ difference\ of\ v_A\ and\ v_B}$$

– Now consider the body B moving in an inclined direction as shown in Fig. 2.2 (a). The relative velocity of A with respect to B may be obtained by the law of parallelogram of velocities or triangle law of velocities. Take any fixed point o and draw vector oa to represent $v_A$ in magnitude and direction to some suitable scale. Similarly, draw vector ob to represent $v_B$ in magnitude and direction to the same scale. Then vector ba represents the relative velocity of A with respect to B as shown in Fig. 7.2 (b). In the similar way as discussed above, the relative velocity of A with respect to B,

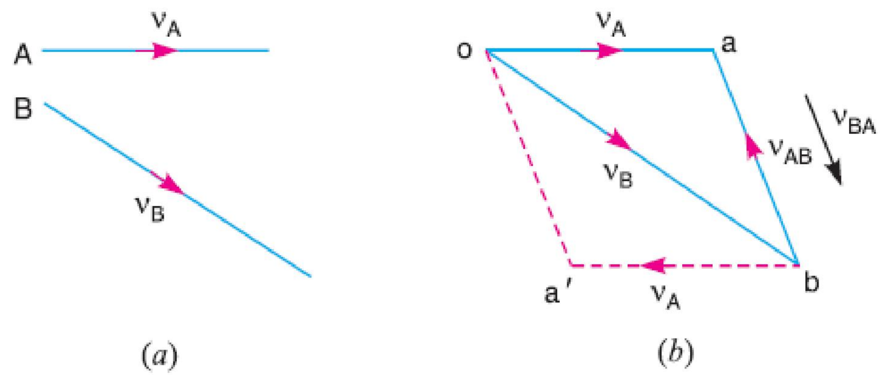



*Fig. 3.2 relative velocity of two bodies moving along inclined line*

$$v_{AB} = vector\ differece\ of\ v_A\ and\ v_B$$

- Similarly, the relative velocity of B with respect to A

$$v_{BA} = vector\ differece\ of\ v_B\ and\ v_A$$

- From above, we conclude that the relative velocity of a point A with respect to B ($v_{AB}$) and the relative velocity of point B with respect to A ($v_{BA}$) are equal in magnitude but opposite in direction

$$v_{AB} = -v_{BA}$$

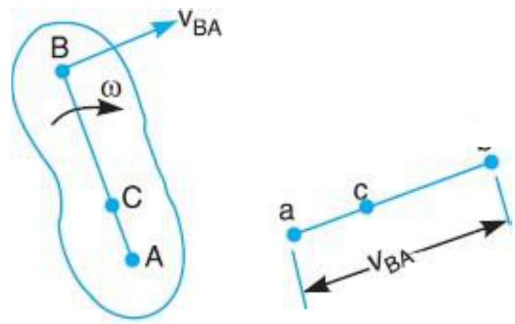
## 3.3 Motion Of A Link

- Consider two points A and B on a rigid link A B, as shown in Fig. 2.3 (a). Let one of the extremities (B) of the link move relative to A, in a clockwise direction. Since the distance from A to B remains the same, therefore there can be no relative motion between A and B, along the line AB. It is thus obvious, that the relative motion of B with respect to A must be perpendicular to AB.
- Hence velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.
- The relative velocity of B with respect to A (i.e. $v_{BA}$) is represented by the vector ab and is perpendicular to the line A B as shown in Fig. 2.3 (b).
- We know that the velocity of the point B with respect to A

$$v_{BA} = \omega \times AB \ \ldots\ldots \ldots \ldots \ldots\ldots \ldots \ldots .. (i)$$

- Similarly the velocity of the point C on AB with respect to A

$$v_{CA} = \omega \times AC \ \ldots \ldots \ldots \ldots \ldots .. \ldots \ldots \ldots (ii)$$




(a) (b)

*Fig. 3.3 Motion of a Link*

– Form equation (i) and (ii),

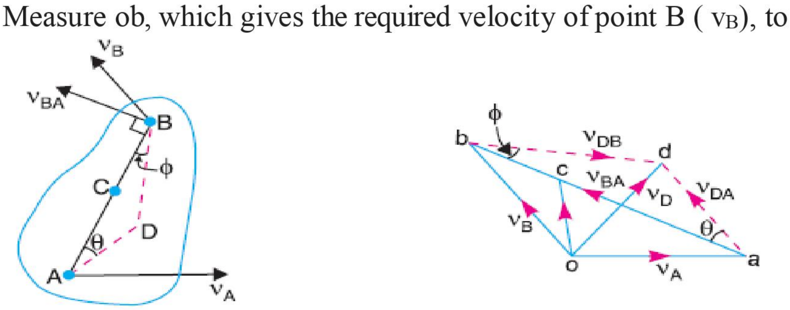
$$\frac{v_{CA}}{v_{BA}} = \frac{\omega \times AC}{\omega \times AB} = \frac{AC}{AB} \dots\dots\dots\dots\dots\dots\dots (iii)$$

– Thus, we see from equation (iii), that the point c on the vector ab divides it in the same ratio as C divides the link AB.

## 3.4 Velocity Of A Point On A Link By Relative Velocity Method

– Consider two points A and B on a link as shown in Fig. 2.4 (a). Let the absolute velocity of the point A i.e. $v_A$ is known in magnitude and direction and the absolute velocity of the point B i.e. $v_B$ is known in direction only. Then the velocity of B may be determined by drawing the velocity diagram as shown in Fig. 2.4 (b). The velocity diagram is drawn as follows :

1 Take some convenient point o, known as the pole.
2 Through o, draw oa parallel and equal to $v_A$, to some suitable scale.
3 Through a, draw a line perpendicular to AB of Fig. 2.4 (a). This line will represent the velocity of B with respect to A, i.e. $v_{BA}$.
4 Through o, draw a line parallel to vB intersecting the line of vBA at b
5 Measure ob, which gives the required velocity of point B ( $v_B$), to the scale

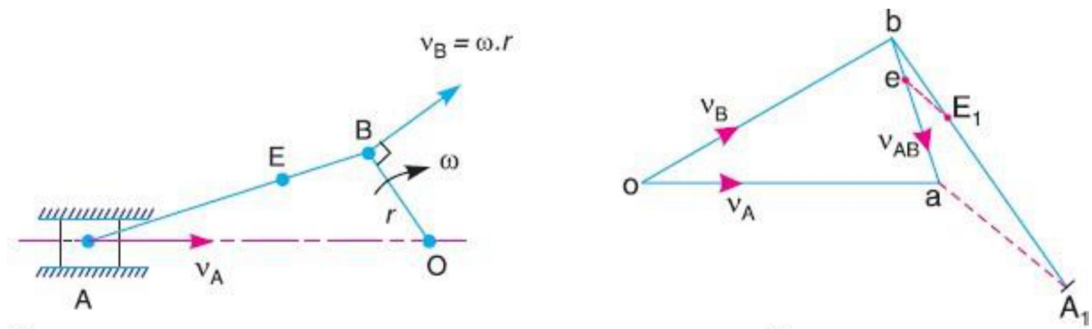



(a) Motion of points on a link. (b) Velocity diagram.

*Fig. 3.4*

## 3.5 Velocities In Slider Crank Mechanism

- In the previous article, we have discussed the relative velocity method for the velocity of any point on a link, whose direction of motion and velocity of some other point on the same link is known. The same method may also be applied for the velocities in a slider crank mechanism.
- A slider crank mechanism is shown in Fig. 2.5 (a). The slider A is attached to the connecting rod AB. Let the radius of crank OB be r and let it rotates in a clockwise direction, about the point O with uniform angular velocity ω rad/s. Therefore, the velocity of B i.e. $v_B$ is known in magnitude and direction. The slider reciprocates along the line of stroke AO.




(a) Slider crank mechanism. (b) Velocity diagram.

*Fig. 3.5*

- The velocity of the slider A (i.e. $v_A$) may be determined by relative velocity method as discussed below :
  1. From any point o, draw vector ob parallel to the direction of $v_B$ (or perpendicular to OB) such that ob = $v_B = \omega.r$, to some suitable scale, as shown in Fig. 2.5 (b).
  2. Since AB is a rigid link, therefore the velocity of A relative to B is perpendicular to AB. Now draw vector ba perpendicular to A B to represent the velocity of A with respect to B i.e. $v_{AB}$.
  3. From point o, draw vector oa parallel to the path of motion of the slider A (which is along AO only). The vectors ba and oa intersect at a. Now oa represents the velocity of the slider I.e. $v_A$, to the scale.
- The angular velocity of the connecting rod A B ($\boldsymbol{\omega}_{AB}$) may be determined as follows:

$$\boldsymbol{\omega_{AB}} = \frac{\boldsymbol{v_{BA}}}{\boldsymbol{AB}} = \frac{\boldsymbol{ab}}{\boldsymbol{AB}}$$

## 3.6 Rubbing Velocity At A Pin Joint

- The links in a mechanism are mostly connected by means of pin joints. The rubbing velocity is defined as the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.
- Consider two links OA and OB connected by a pin joint at O as shown in fig.

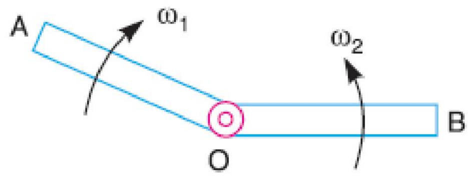



*Fig. 3.6 Links connected by pin joints*

- Let,
  ω1 = angular velocity of link OA
  ω2 = angular velocity of link OB
- According to the definition,
- Rubbing velocity at the pin joint O

$$= (\omega_1 - \omega_1) \times r \text{ if the links move in the same direction}$$
$$= (\omega_1 + \omega_1) \times r \text{ if the links move in the same direction}$$

## 3.7 Examples Based On Velocity

### 3.7.1 In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about

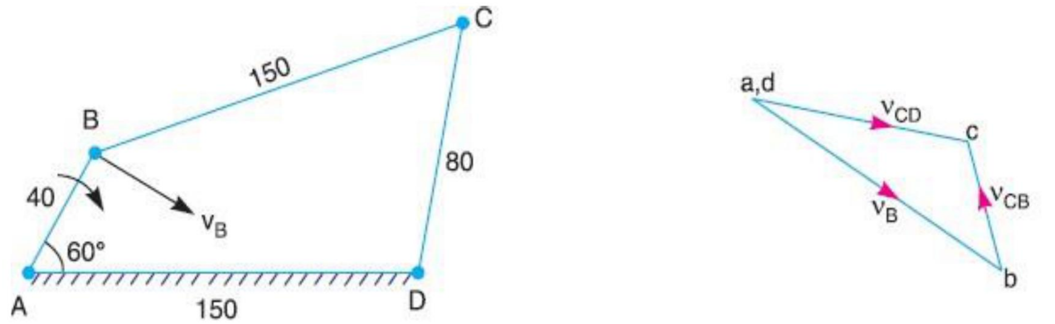
**D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°.**

- **Given :** $N_{BA}$ = 120 r.p.m. or $\omega_{BA} = 2\ \pi \times 120/60 = 12.568$ rad/s
- Since the length of crank A B = 40 mm = 0.04 m, therefore velocity of B with respect to A or velocity of B, (because A is a fixed point),
- Since the length of crank A B = 40 mm = 0.04 m, therefore velocity of B with respect to A or velocity of B, (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 12.568 \times 0.04 = 0.503 \text{ m/s}$$

- Since the link AD is fixed, therefore points a and d are taken as one point in the velocity diagram. Draw vector ab perpendicular to B A, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B (i.e. $v_{BA}$ or $v_B$) such that

$$\text{Vector ab} = v_{BA} = v_B = 0.503 \text{ m/s}$$




(a) Space diagram (All dimensions in mm).

(b) Velocity diagram.

*Fig. 3.7*

- Now from point b, draw vector bc perpendicular to CB to represent the velocity of C with respect to B (i.e. $v_{CB}$) and from point d, draw vector dc perpendicular to CD to represent the velocity of C with respect to D or simply velocity of C (i.e. $v_{CD}$ or $v_C$). The vectors bc and dc intersect at c.

  By measurement, we find that

  $$V_{CD} = v_C = \text{vector } dc = 0.385 \text{ m/s}$$

- Angular velocity of link CD,

  $$\boldsymbol{\omega_{CD} = \frac{v_{CD}}{CD} = \frac{0.385}{0.08} = 4.8 \ rad/s}$$

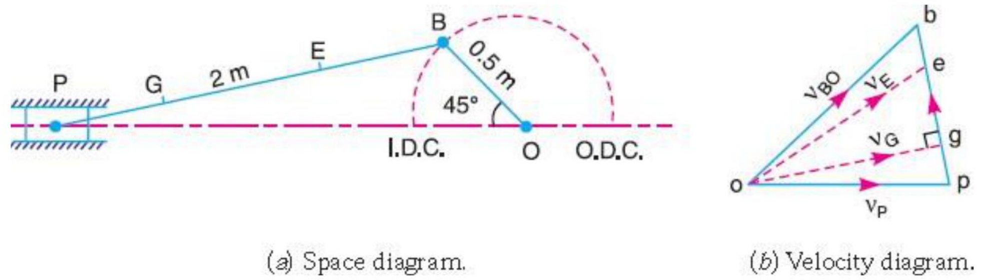
**3.7.2 The crank and connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centre position, determine:**

**1. Velocity of piston, 2. Angular velocity of connecting rod, 3. Velocity of point E on the connecting rod 1.5 m from the gudgeon pin, 4. velocities of rubbing at the pins of the crank shaft, crank and crosshead when the diameters of their pins are 50 mm, 60 mm and 30 mm respectively, 5. Position and linear velocity of any point G on the connecting rod which has the least velocity relative to crank shaft.**

- **Given:**
- $N_{BO} = 180$ r.p.m. or $\omega_{BO} = 2\pi \times 180/60 = 18.852$ rad/s
- Since the crank length $OB = 0.5$ m, therefore linear velocity of $B$ with respect to $O$ or velocity of $B$ (because $O$ is a fixed point),

  $$v_{BO} = v_B = \omega_{BO} \times OB = 18.852 \times 0.5 = 9.426 \text{ m/s}$$

- First of all draw the space diagram and then draw the velocity diagram as shown in fig.




(a) Space diagram. (b) Velocity diagram.

*Fig. 3.8*

- By measurement, we find that velocity of piston P,

$$v_P = vector\ op = 8.15\ m/s$$

- From the velocity diagram, we find that the velocity of P with respect to B

$$v_{PB} = \text{vector } bp = 6.8 \text{ m/s}$$

- Since the length of connecting rod PB is 2 m, therefore angular velocity of the connecting rod,

$$\omega_{PB} = \frac{v_{PB}}{PB} = \frac{6.8}{2} = 3.4\ rad/s$$

$$v_E = vector\ oe = 8.5\ m/s$$

- We know that velocity of rubbing at the pin of crank-shaft

$$= \frac{d_0}{2} \times \omega_{BO} = 0.47\ m/s$$

- Velocity of rubbing at the pin of crank

$$= \frac{d_B}{2}(\omega_{BO} + \omega_{PB}) = 0.6675\ m/s$$

- Velocity of rubbing at the pin of crank

$$= \frac{d_c}{2} \times \omega_{PB} = 0.051\ m/s$$

- By measurement we find that

$$vector\ bg = 5\ m/s$$

- By measurement we find linear velocity of point G

$$v_G = vector\ og = 8\ m/s$$

**3.7.3 In Fig. , the angular velocity of the crank OA is 600 r.p.m. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are: OA = 28 mm; AB = 44 mm; BC 49 mm; and BD = 46 mm. The centre distance between the canters of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider moves along a horizontal path and OC is vertical.**

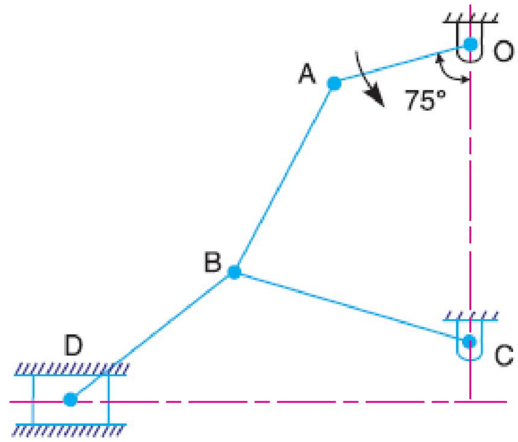



*Fig. 3.9*

– **Given :**

– $N_{AO}$ = 180 r.p.m. or $\omega_{BO} = 2\,\pi \times 180/60 = 18.852$ rad/s

– OA = 28 mm

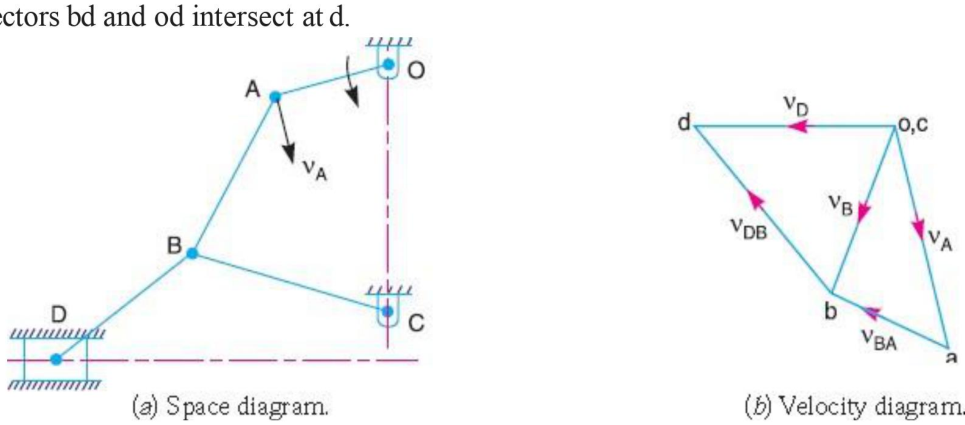
$$v_{OA} = v_A = \omega_{AO} \times AO = \mathbf{1.76\ m/s}$$

– Since the points O and C are fixed, therefore these points are marked as one point, in the velocity diagram. Now from point o, draw vector oa perpendicular to O A, to some suitable scale, to represent the velocity of A with respect to O or simply velocity of A such that

$$vector\ oa = v_{OA} = v_A = \mathbf{1.76\ m/s}$$

– From point a, draw vector ab perpendicular to A B to represent the velocity of B with respect A (i.e. $v_{BA}$) and from point c, draw vector cb perpendicular to CB to represent the velocity of B with respect to C or simply velocity of B (i.e. $v_{BC}$ or $v_B$). The vectors ab and cb intersect at b.

– From point b, draw vector bd perpendicular to BD to represent the velocity of D with respect to B (i.e. $v_{DB}$) and from point o, draw vector od parallel to the path of motion of the slider D which is horizontal, to represent the velocity of D (i.e. $v_D$). The vectors bd and od intersect at d.




(a) Space diagram. (b) Velocity diagram.

*Fig.3.10*

– By measurement, we find that velocity of slider D,

$$v_D = vector\ od = 1.6\ m/s$$

– By measurement from velocity diagram, we find that velocity of D with respect to B,

$$v_{DB} = vector\ bd = 1.7\ m/s$$

– Therefore angular velocity of link BD

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96\ rad/s$$

### 3.7.4 The mechanism, as shown in Fig. 7.11, has the dimensions of various links as follows :

**AB = DE = 150 mm; BC = CD = 450 mm; EF = 375 mm. The crank AB makes an angle of**
**45° with the horizontal and rotates about A in the clockwise direction at a uniform speed of 120 r.p.m. The lever DC oscillates about the fixed point D, which is connected to AB by the coupler BC.**
**The block F moves in the horizontal guides, being driven by the link EF. Determine: 1. velocity of the block F, 2. angular velocity of DC, and 3. rubbing speed at the pin C which is 50 mm in diameter.**

– Given :

– $N_{BA}$ = 120 r.p.m. or $\omega_{BA} = 2\pi \times 120/60 = 4\pi$ rad/s

– Since the crank length A B = 150 mm = 0.15 m, therefore velocity of B with respect to A or simply velocity of B (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 4\pi \times 0.15 = 1.885\ m/s$$

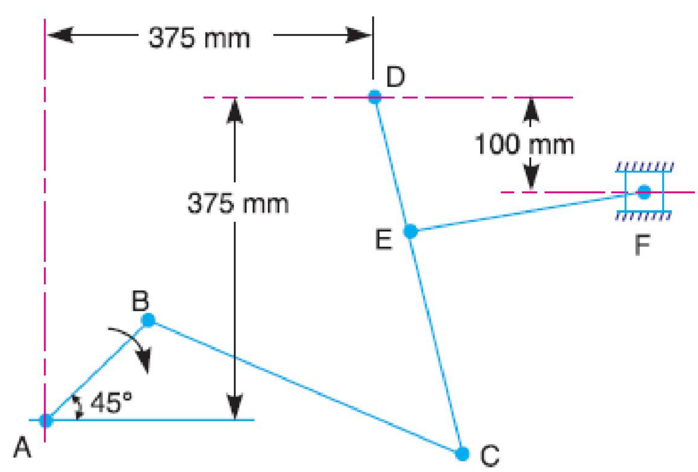
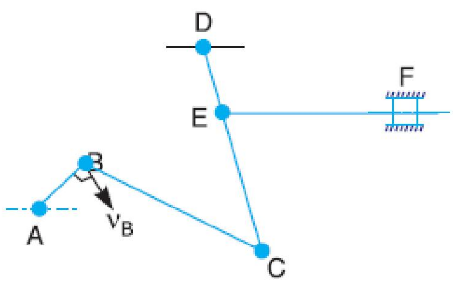



*Fig.3.11*

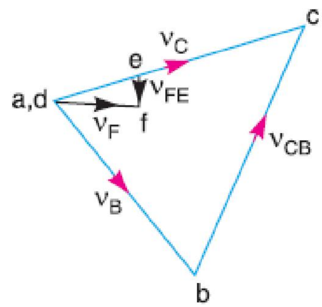
– Since the points A and D are fixed, therefore these points are marked as one point as shown in Fig. (b). Now from point a, draw vector ab perpendicular to A B, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B, such that

Vector ab = $v_{BA} = v_B$ = 1.885 m/s

– The point C moves relative to B and D, therefore draw vector bc perpendicular to BC to represent the velocity of C with respect to B (i.e. vCB), and from point d, draw vector dc perpendicular to DC to represent the velocity of C with respect to D or simply velocity of C (i.e. vCD or vC). The vectors bc and dc intersect at c.




(a) Space diagram.




(b) Velocity diagram.

*Fig. 3.12*

– Since the point E lies on DC, therefore divide vector dc in e in the same ratio as E divides CD in Fig. (a). In other words

ce/cd = CE/CD

– From point e, draw vector ef perpendicular to EF to represent the velocity of F with respect to E (i.e. $v_{FE}$) and from point d draw vector df parallel to the path of motion of F, which is horizontal, to represent the velocity of F i.e. $v_F$. The vectors ef and df intersect at f.

$v_F$ = vector df = 0.7 m/s

– By measurement from velocity diagram, we find that velocity of C with respect to D,

$v_{CD}$ = vector *dc* = 2.25 m/s

$$\omega_{DC} = \frac{v_{CD}}{DC} = 5\ \frac{rad}{s}$$

– From velocity diagram, we find that velocity of C with respect to B,

$v_{CB}$ = vector *bc* = 2.25 m/s

– Angular velocity of BC,

$$\omega_{CD} = \frac{v_{CD}}{BC} = \frac{2.25}{0.45} = 5\ rad/s$$

## 3.8 Velocity Of A Point On A Link By Instantaneous Centre Method

- The instantaneous centre method of analyzing the motion in a mechanism is based upon the concept that any displacement of a body (or a rigid link) having motion in one plane, can be considered as a pure rotational motion of a rigid link as a whole about some centre, known as instantaneous centre or virtual centre of rotation.

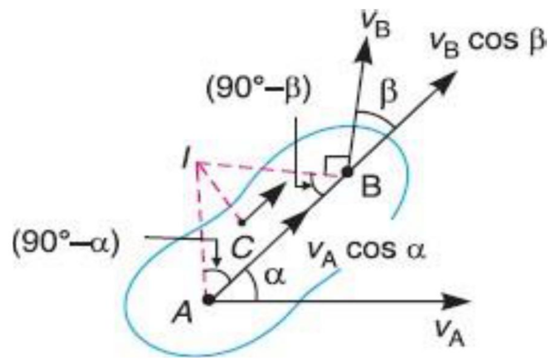



*Fig. 3.13 velocity of a point on a link*

- The velocities of points A and B, whose directions are given a link.by angles $\alpha$ and $\beta$ as shown in Fig. If vA is known in magnitude and direction and vB in direction only, then the magnitude of vB may be determined by the instantaneous centre method as discussed below :
- Draw AI and BI perpendiculars to the directions vA and vB respectively. Let these lines intersect at I, which is known as instantaneous centre or virtual centre of the link. The complete rigid link is to rotate or turn about the centre I.
- Since A and B are the points on a rigid link, therefore there cannot be any relative motion between them along line AB.
- Now resolving the velocities along AB,

$$v_A \times \cos\alpha = v_B \times \cos\beta$$

$$\frac{v_A}{v_B} = \frac{\cos\beta}{\cos\alpha} = \frac{\sin(90-\beta)}{\sin(90-\alpha)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

- Applying Lami's theorem to triangle ABI,

$$\frac{AI}{\sin(90-\beta)} = \frac{BI}{\sin(90-\alpha)}$$

$$\frac{AI}{BI} = \frac{\sin(90-\beta)}{\sin(90-\alpha)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

- Hence,

$$\frac{v_A}{v_B} = \frac{AI}{BI}$$

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \omega \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (iii)$$

– If C is any other point on link, then

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \frac{v_C}{CI} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (iv)$$

## 3.9 Properties Of Instantaneous Method

– The following properties of instantaneous centre are important :

1. A rigid link rotates instantaneously relative to another link at the instantaneous centre for the configuration of the mechanism considered.
2. The two rigid links have no linear velocity relative to each other at the instantaneous centre. At this point (i.e. instantaneous centre), the two rigid links have the same linear velocity relative to the third rigid link. In other words, the velocity of the instantaneous centre relative to any third rigid link will be same whether the instantaneous centre is regarded as a point on the first rigid link or on the second rigid link

## 3.10 Number Of Instantaneous Centre In A Mechanism:

– The number of instantaneous centres in a constrained kinematic chain is equal to the number of possible combinations of two links. The number of pairs of links or the number 3 of instantaneous centres is the number of combinations of n links taken two at a time. Mathematically, number of instantaneous centres

$$N = \frac{n\,(n-1)}{2}, where\ n = Number\ of\ Link$$

## 3.11 Location of Instantaneous centres:

– The following rules may be used in locating the instantaneous centres in a mechanism :

1. When the two links are connected by a pin joint (or pivot joint), the instantaneous centre lies on the centre of the pin as shown in Fig. (a). such an instantaneous centre is of permanent nature, but if one of the links is fixed, the instantaneous centre will be of fixed type.
2. When the two links have a pure rolling contact (i.e. link 2 rolls without slipping upon the fixed link 1 which may be straight or curved), the instantaneous centre lies on their point of contact, as shown in Fig.(b). The velocity of any point A on the link 2 relative to fixed link 1 will be perpendicular to I12 A and is proportional to I12 A.
3. When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact. We shall consider the following three cases :

a. When the link 2 (slider) moves on fixed link 1 having straight surface as shown in Fig.(c), the instantaneous centre lies at infinity and each point on the slider have the same velocity.

b. When the link 2 (slider) moves on fixed link 1 having curved surface as shown in Fig.(d),the instantaneous centre lies on the centre of curvature of the curvilinear path in the configuration at that instant.

c. When the link 2 (slider) moves on fixed link 1 having constant radius of curvature as shown in Fig. 6.6 (e), the instantaneous centre lies at the centre of curvature i.e. the centre of the circle, for all configuration of the links.

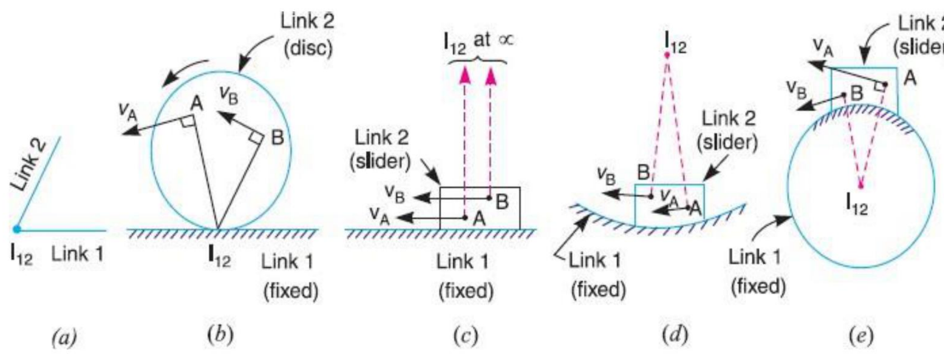



*Fig. 3.14 Location of Instantaneous centres*

## 3.12 Kennedy's Theorem

– The Aronhold Kennedy's theorem states that "if three bodies move relatively to each other, they have three instantaneous centres and lie on a straight line."

– Consider three kinematic links A , B and C having relative plane motion. The number of instantaneous centres (N) is given by

$$N = \frac{n\,(n-1)}{2} = \frac{3\,(3-1)}{2} = 3$$

– The two instantaneous centres at the pin joints of B with A , and C with A (i.e. $I_{ab}$ and $I_{ac}$) are the permanent instantaneous centre According to Aronhold Kennedy's theorem, the third instantaneous centre $I_{bc}$ must lie on the line joining $I_{ab}$ and $I_{ac}$. In order to prove this let us consider that the instantaneous centre $I_{bc}$ lies outside the line joining $I_{ab}$ and $I_{ac}$ as shown in Fig. The point $I_{bc}$ belongs to both the links B and C. Let us consider the point $I_{bc}$ on the link B. Its velocity $v_{BC}$ must be perpendicular to the line joining $I_{ab}$ and $I_{bc}$. Now consider the point $I_{bc}$ on the link C. Its velocity $v_{BC}$ must be perpendicular to the line joining $I_{ac}$ and $I_{bc}$.

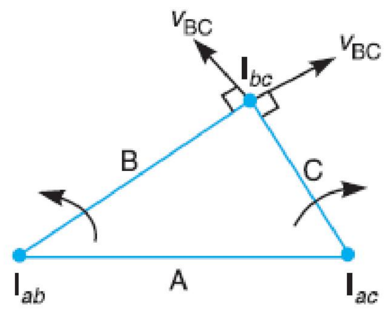
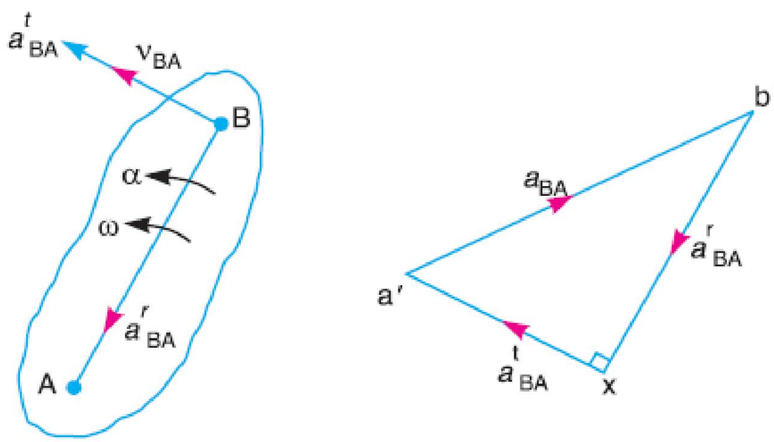



*Fig. 3.15 Aronhold Kennedy's theorem*

– We have already discussed that the velocity of the instantaneous centre is same whether it is regarded as a point on the first link or as a point on the second link. Therefore, the velocity of the point Ibc cannot be perpendicular to both lines Iab Ibc and Iac Ibc unless the point Ibc lies on the line joining the points Iab and Iac. Thus the three instantaneous centres (Iab, Iac and Ibc) must lie on the same straight line. The exact location of Ibc on line Iab Iac depends upon the directions and magnitudes of the angular velocities of B and C relative to A.

## 3.13 Acceleration Diagram for a Link

– Consider two points A and B on a rigid link as shown in Fig. (a). Let the point B moves with respect to A, with an angular velocity of **ω** rad/s and let **α** rad/s2 be the angular acceleration of the link AB.




(a) Link. (b) Acceleration diagram.

*Fig. 3.16 Acceleration of a link*

– We have already discussed that acceleration of a particle whose velocity changes both in magnitude and direction at any instant has the following two components .

1 The centripetal or radial component, which is perpendicular to the velocity of the particle at the given instant.

2 The tangential component, which is parallel to the velocity of the particle at the given instant.

- Thus for a link A B, the velocity of point B with respect to A (i.e. vBA) is perpendicular to the link A B as shown in Fig.(a). Since the point B moves with respect to A with an angular velocity of $\boldsymbol{\omega}$ rad/s, therefore centripetal or radial component of the

  acceleration of B with respect to A

$$a^r_{BA} = \omega^2 \times Length\ of\ link\ AB = \omega^2 \times AB = {v^2_{BA}}/{AB}$$

- This radial component of acceleration acts perpendicular to the velocity vBA, In other words, it acts parallel to the link AB.

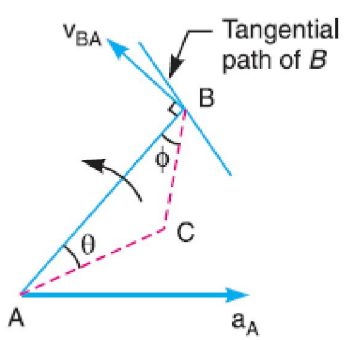
  We know that tangential component of the acceleration of B with respect to A ,

$$a^t_{BA} = \alpha \times Length\ of\ link\ AB = \alpha \times AB$$

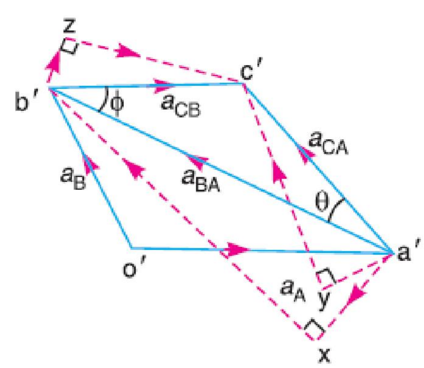
- This tangential component of acceleration acts parallel to the velocity vBA. In other words, it acts perpendicular to the link AB.
- In order to draw the acceleration diagram for a link A B, as shown in Fig. 8.1 (b), from any point b', draw vector b'x parallel to BA to represent the radial component of acceleration of B with respect to A.

## 3.14 Acceleration of a Point on a Link

- Consider two points A and B on the rigid link, as shown in Fig. (a). Let the acceleration of the point A i.e. aA is known in magnitude and direction and the direction of path of B is given. The acceleration of the point B is determined in magnitude and direction by drawing the acceleration diagram as discussed below.




(a) Points on a Link.




(b) Acceleration diagram.

*Fig. 3.17 acceleration of a point on a link*

- From any point o', draw vector o'a' parallel to the direction of absolute acceleration at point A i.e. $a_A$ , to some suitable scale, as shown in Fig. 8.2 (b).
- We know that the acceleration of B with respect to A i.e. aBA has the following two components:

1 Radial component of the acceleration of B with respect to A i.e. $a^r{}_{BA}$
2 Tangential component of the acceleration B with respect to A i.e. $a^t{}_{BA}$

- Draw vector a'x parallel to the link AB such that,

$$vector\ a'x = a^r_{BA} = v^2_{BA}/AB$$

- From point x, draw vector xb' perpendicular to AB or vector a'x and through o' draw a line parallel to the path of B to represent the absolute acceleration of B i.e. $a_B$
- By joining the points a' and b' we may determine the total acceleration of B with respect to A i.e. $a_{BA}$. The vector a' b' is known as acceleration image of the link AB.
- For any other point C on the link, draw triangle a' b' c' similar to triangle ABC. Now vector b' c' represents the acceleration of C with respect to B i.e. $a_{CB}$, and vector a' c' represents the acceleration of C with respect to A i.e. $a_{CA}$. As discussed above, $a_{CB}$ and $a_{CA}$ will each have two components as follows :
  a. $a_{CB}$ has two components; $a^r_{CB}$ and $a^t_{CB}$ as shown by triangle b'zc' in fig.b
  b. $a_{CA}$ has two components; $a^r_{CA}$ and $a^t_{CA}$ as shown by triangle a'yc'
- The angular acceleration of the link AB is obtained by dividing the tangential component of acceleration of B with respect to A to the length of the link.

$$\alpha_{AB} = a^t_{BA}/AB$$

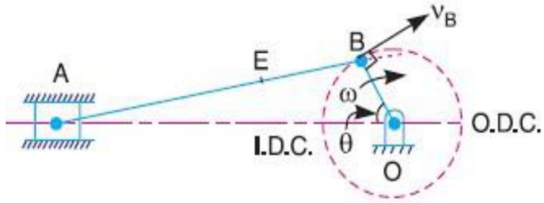
## 3.15 Acceleration in Slider Crank Mechanism

- A slider crank mechanism is shown in Fig. 8.3 (a). Let the crank OB makes an angle **θ** with the inner dead centre (I.D.C) and rotates in a clockwise direction about the fixed point O with uniform angular velocity $\omega_{BO}$ rad/s
- Velocity of B with respect to O or velocity of B (because O is a fixed point),

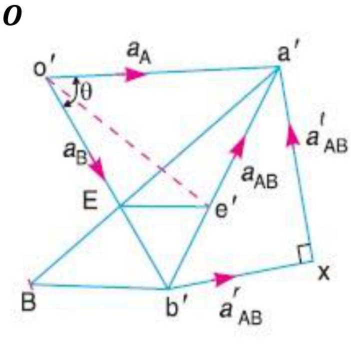
$$v_{BO} = v_B = \omega_{BO} \times OB\ acting\ tangentially\ at\ B$$

- We know that centripetal or radial acceleration of B with respect to O or acceleration of B (Because O is a fixed point)

$$a^r_{BO} = a_B = \omega^2_{BO} \times OB = \frac{v^2_{BO}}{BO}$$




(a) Slider crank mechanism.




(b) Acceleration diagram.

*Fig. 3.18 acceleration in the slider crank mechanism*

- The acceleration diagram, as shown in Fig. 8.3 (b), may now be drawn as discussed below:

1 Draw vector o' b' parallel to BO and set off equal in magnitude of a=a, to some BO suitable scale.

2 From point b', draw vector b'x parallel to BA. The vector b'x represents the radial component of the acceleration of A with respect to B whose magnitude is given by :

$$a^r_{AB} = v^2_{AB} / BA$$

3 From point x, draw vector xa' perpendicular to b'x. The vector xa' represents the tangential components of the acceleration of . ct to B.

4 Since the point A reciprocates along AO, the eleration must be parallel to velocity. Therefore from o', draw l to A O, intersecting the vector xa' at a'.

5 The vector b' a', which is the sum of the ve d x a', represents the total acceleration of A with respect to B i.e. vector b'a' represents the acceleration of the connecting rod AB.

6 The acceleration of any other point on A B ay be obtained by dividing the vector b' a' at e' in the same ratio as E 3 in Fig. 8.3 (a). In other words

$$a'e'/a'b' = $$

7 The angular acceleration of the connecting rc be obtained by dividing the tangential component of the acceleration of A t to B to the length of AB. In other words, angular acceleration of AB,

$$\alpha_{AB} = a^t_{AB}$$

## 3.16 Examples Based on Acceleration

### 3.16.1 The crank of the slider crank mec otates clockwise at a constant speed of 300 r.p.m. The 150 mm and the connecting rod is 600 mm long. Deter

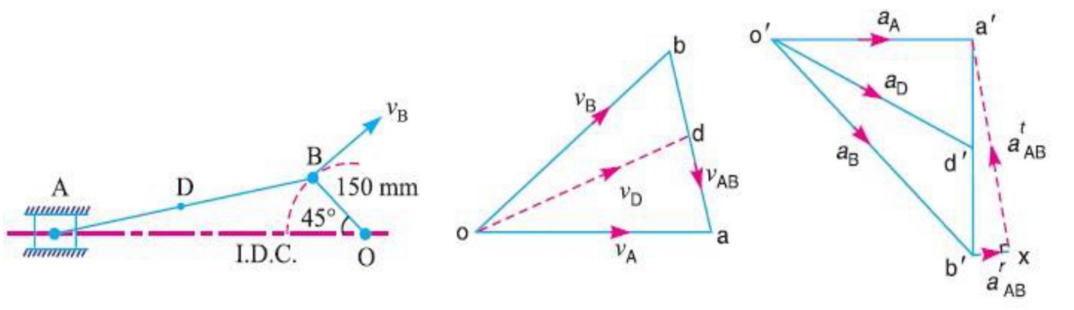
1. **Linear velocity and acceleration of the m he connecting rod, and**
2. **Angular velocity and angular acceleratio nnecting rod, at a crank angle of 45° from inner dead centre position**

- **Given:**
- $N_{BO}$ = 300 r.p.m. or $\omega_{BO}$ = 2 π × 300/60 = 31.42 rad/s; OB = 150 mm = 0.15 m ; B A = 600 mm = 0.6 m
- We know that linear velocity of B with respect to O or velocity of B,

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713\ m/s$$

- Draw vector ob perpendicular to BO, to some suitable scale, to represent the velocity of B with respect to O or simply velocity of B i.e. $v_{BO}$ or vB, such that

vector ob = $v_{BO}$ = $v_B$ = 4.713 m/s




(a) Space diagram. (b) Velocity diagram. (c) Acceleration diagram.

*Fig. 3.19*

- From point b, draw vector ba perpendicular to BA to represent the velocity of A with respect to B i.e. $v_{AB}$ , and from point o draw vector oa parallel to the motion of A (which is along AO) to represent the velocity of A i.e. $v_A$. The vectors ba and oa intersect at a.
- By measurement we find the velocity A with respect to B,

$$v_{AB} = vector\ ba = 3.4\ m/s$$
$$v_A = \text{vector oa} = 4\ \text{m/s}$$

- In order to find the velocity of the midpoint D of the connecting rod A B, divide the vector ba at d in the same ratio as D divides A B, in the space diagram. In other words,

$$bd/ba = BD/BA$$

- By measurement, we find that

$$\text{v}_\text{D} = \text{vector od} = 4.1\ \text{m/s}$$

- We know that the radial component of the acceleration of B with respect to O or the acceleration of B,

$$a_{BO}^r = a_B = \frac{v_{BO}^2}{OB} = \frac{(4.713)^2}{0.15} = 148.1\ m/s^2$$

- And the radial component of the acceleration of A with respect to B,

$$a_{\frac{A}{B}}^r = \frac{v_{AB}^2}{BA} = \frac{(3.4)^2}{0.6} = 19.3\ m/s^2$$
$$vector\ o'b' = a^r_{BO} = a_B = 148.1\ m/s^2$$

- By measurement, we find that

$$a = vector\ o'd' = 117\ m/s^2$$

- We know that angular velocity of the connecting rod AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67\ rad/s^2$$

- From the acceleration diagram, we find that

$$\text{a}^\text{t}_\text{AB} = 103\ \text{m/s}^2$$

- We know that angular acceleration of the connecting rod AB,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67\ rad/s^2$$

**3.16.2 An engine mechanism is shown in Fig. 8.5. The crank CB = 100 mm and the connecting rod BA = 300 mm with centre of gravity G, 100 mm from B. In the position shown, the crankshaft has a speed of 75 rad/s and an angular acceleration of 1200 rad/s². Find:**

**1. Velocity of G and angular velocity of AB, and**

**2. Acceleration of G and angular acceleration of AB.**

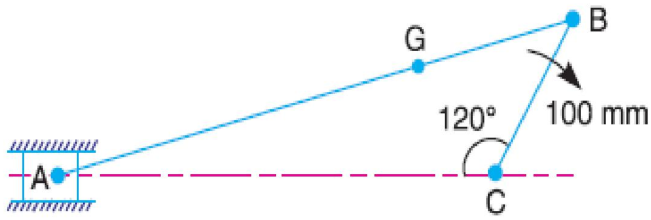



*Fig. 3.20*

- **Given :**
- $\omega_{BC}$ = 75 rad/s ; $\alpha_{BC}$ = 1200 rad/s², CB = 100 mm = 0.1 m; B A = 300 mm = 0.3 m
- We know that velocity of B with respect to C or velocity of B

$$v_{BC} = v_B = \omega_{BC} \times CB = 75 \times 0.1 = 7.5\ m/s$$

- Since the angular acceleration of the crankshaft, $\alpha_{BC}$ = 1200 rad/s², therefore tangential component of the acceleration of B with respect to C,

$$a^t_{BC} = \alpha_{BC} \times CB = 1200 \times 0.1 = 120\ m/s^2$$

$$vector\ cb = v_{BC} = v_B = 7.5\ \frac{m}{s}$$

- By measurement, we find that velocity of G,

$$v_G = ector\ cg = 6.8\ m/s$$

- From velocity diagram, we find that the velocity of A with respect to B,

$$v_{AB} = vector\ ba = 4\ m/s$$

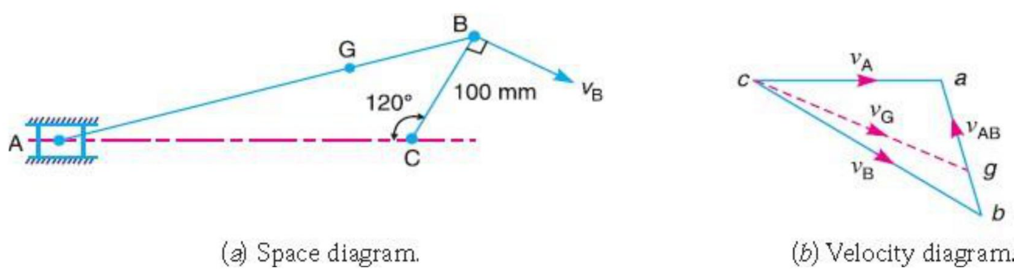
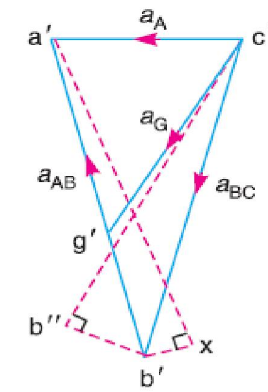



*Fig. 3.21*

- We know that angular velocity of AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{4}{0.3} = 13.3\ rad/s$$




(c) Acceleration diagram.

*Fig. 3.22*

- We know that radial component of the acceleration of B with respect to C

$$a^r_{BC} = \frac{v_{BC}^2}{CB} = \frac{(7.5)^2}{0.1} = 562.5\ m/s^2$$

- And radial component of the acceleration of A with respect to B,

$$a^r_{AB} = \frac{v_A^2}{CB} = \frac{(4)^2}{0.3} = 53.3\ m/s^2$$

$$vector\ c'b'' = {}^r{}_{BC} = 562.5\ m/s^2$$

$$vecto\ ''b' = a^t{}_{BC} = 120\ m/s^2$$

$$vecto\ 'x = a^r{}_{AB} = 53.3\ m/s^2$$

- By measurement we find that acceleration of G,

$$a_G = vector\ xa' = 414\ m/s^2$$

- From acceleration diagram, we find that tangential component of the acceleration of A with respect to B,

$$a^t_{AB} = ector\ xa' = 546\, m/s^2$$

- Angular acceleration of AB

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{546}{0.3} = 1820\ rad/s^2$$

**3.16.3 In the mechanism shown in Fig. 8.7, the slider C is moving to the right with a velocity of 1 m/s and an acceleration of 2.5 m/s2.The dimensions of various links are AB = 3 m inclined at 45° with the vertical and BC = 1.5 m inclined at 45° with the horizontal. Determine: 1. the magnitude of vertical and horizontal component of the acceleration of the point B, and 2. the angular acceleration of the links AB and BC.**

- **Given:**
- $v_C$ = 1 m/s ; $a_C$ = 2.5 m/s2; AB = 3 m ; BC = 1.5 m
- Here,

$$vector\ d = v_{CD} = v_c = 1 m/s$$

- By measurement, we find that velocity of B with respect to A

$$v_{BA} = vector\ ab = 0.72\ m/s$$

- Velocity of B with respect to C

$$v_{BC} = vector\ cb = 0.72\ m/s$$

- We know that radial component of acceleration of B with respect to C,

$$a^r_{\frac{B}{C}} = \frac{v^2_{BC}}{CB} = \frac{(0.72)^2}{1.5} = 0.346\ m/s^2$$

- And radial component of acceleration of B with respect to A,

$$a^r_{\frac{B}{A}} = \frac{v^2_{BA}}{AB} = \frac{(0.72)^2}{3} = 0.173\ m/s^2$$

$$vectro\ d'c' = a_{cd} = a_c = 2.5\ m/s^2$$

$$vecto\ 'x = a^r_{BC} = 0.346\ \frac{m}{s^2}$$

$$vecto\ 'y = a^r_{BA} = 0.173\ m/s^2$$

- By measurement,

$$vector\ b'b'' = 1.13\ m/s^2$$

- By measurement from acceleration diagram, we find that tangential component of acceleration of the point B with respect to A

$$a^t_{BA} = ector\ yb' = 1.41\ m/s^2$$

- And tangential component of acceleration of the point B with respect to C,

$$a^t_{BC} = vector\ xb' = 1.94\ m/s^2$$

- we know that angular velocity of AB,

$$\alpha_{AB} = \frac{v^t_{BA}}{AB} = 0.47\ rad/s^2$$

- And aglular acceleration of BC,

$$\alpha_{BC} = \frac{a^t_{BC}}{CB} = \frac{1.94}{1.5}\ rad/s^2$$

# References

1. Theory of Machines by S.S.Rattan, Tata McGraw Hill publication.
2. Theory of Machines by R.S. Khurmi & J.K.Gupta,S.Chand publication.