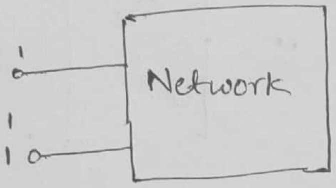
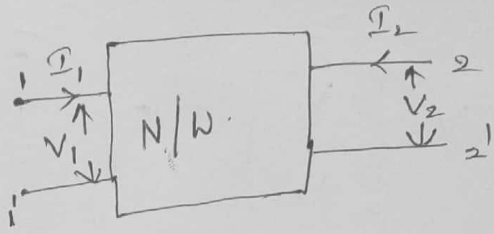
Two Port Network

Generally any Network may be represented Schematically by a rectangular box. A Network may be used for representing either Source (or) Load.



(a)



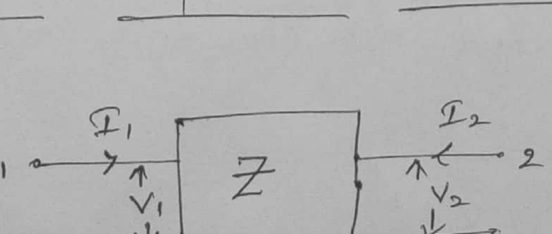
(b)

A pair of terminals at which a signal may enter (or) leave a N/W is called a port. A Port is defined as any pair of terminals into which energy is supplied, (or) from which energy is withdrawn, (or) where the n/w variables may be measured. One such N/W having only one pair of terminals (1-1') is shown in Fig (a).

The two port N/W has only two pairs of accessible terminals. Usually one pair represents the input & other represents the output. Such a building block is very common in electronic systems, communication s/m's, transmission & distribution systems.

Fig (b) shows a two-port N/W. in which the four terminals have been paired into ports 1-1' & 2-2'

⇒ **Open Circuit Impedance (Z) Parameters.:**



The Z parameters of a two port for the positive directions of voltages & currents may be defined by expressing the port voltages V_1 & V_2 in terms of the currents I_1 & I_2. Here V_1 & V_2 are dependent variables, and I_1, I_2 are Independent variables.

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (1)} \qquad V_1 - Z_{11} I_1 - Z_{12} I_2 = 0$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (2)} \qquad V_2 - Z_{21} I_1 - Z_{22} I_2 = 0.$$

we may write the matrix equation

$$[V] = [Z][I]$$

$$V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \qquad I = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}.$$

Here $Z_{11}, Z_{12}, Z_{21}, Z_{22}$ are the N/W functions and are called impedance (Z) parameters.

Suppose port $2-2'$ is left open circuited means $I_2 = 0$.

from Eq (1) $V_1 = Z_{11} I_1$

$$\Rightarrow Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}.$$

$Z_{11} \rightarrow$ open circuit input impedance.

from Eq (2) $\Rightarrow V_2 = Z_{21} I_1$

$$\Rightarrow Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}.$$

$Z_{21} \rightarrow$ open circuit forward transfer impedance

Suppose port $1-1'$ is left open circuited, means $I_1 = 0$.

from Eq (1) $\Rightarrow V_1 = Z_{12} I_2$

$$\Rightarrow Z_{12} = \frac{V_1}{I_2}$$

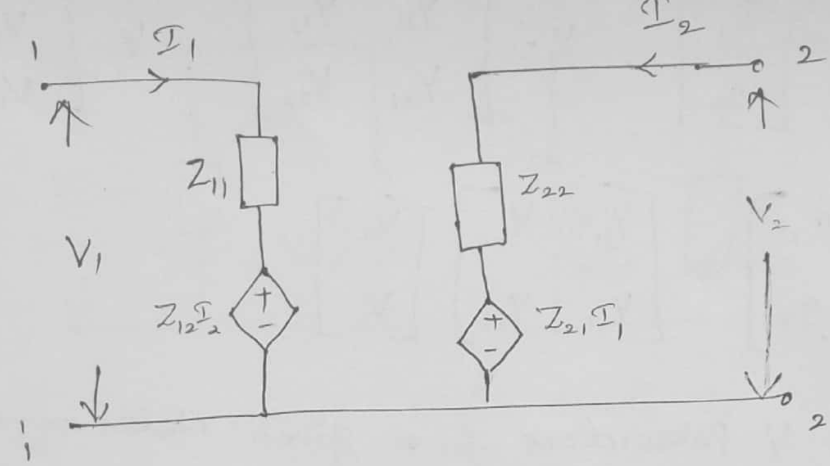
$Z_{12} \rightarrow$ open circuit reverse transfer impedance

from Eq (2) $\Rightarrow V_2 = Z_{22} I_2$

$$\Rightarrow Z_{22} = \frac{V_2}{I_2}.$$

$Z_{22} \rightarrow$ open circuit output impedance.

The equivalent circuit of the two-port N/w's governed by the equations ① & ②.

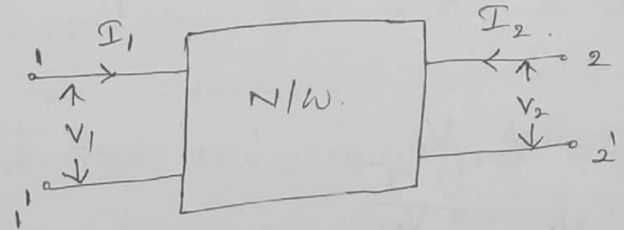


if the N/w under study is reciprocal (or) bilateral.

$$\boxed{Z_{21} = Z_{12}}$$

Symmetrical $\boxed{Z_{11} = Z_{22}}$.

⇒ Short Circuit Admittance (Y) Parameters :



The Y parameters of a two-port for the positive directions of voltages and currents may be defined by expressing the port currents I_1 & I_2 in terms of the voltages V_1 & V_2

Here I_1, I_2 are dependent variables

V_1, V_2 are independent variables.

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \longrightarrow ①$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \longrightarrow ②$$

$Y_{11}, Y_{12}, Y_{21}, Y_{22}$ are the network functions and are also called the admittance (Y) parameters.

The parameters can be represented by matrices as follows

$$I = Y\,V.$$

where $I = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$, $Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$, $V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

thus
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

The individual Y parameters for a given Network can be defined by setting each port voltage to zero. If we let V_2 be zero by short circuiting port 2-2', then

from Eq (1) $\Rightarrow$ $I_1 = Y_{11} V_1$

$$\Rightarrow Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}.$$

$Y_{11} \rightarrow$ short circuit input admittance.

and also
from Eq (2) $\quad I_2 = Y_{21} V_1$

$$\Rightarrow Y_{21} = \frac{I_2}{V_1}$$

$Y_{21} \Rightarrow$ short circuited forward transfer admittance.

If we let V_1 be zero by short circuiting port 1-1', then

from Eq (1) $\Rightarrow$ $I_1 = Y_{\cancel{2}12} V_2$

$$\Rightarrow Y_{\cancel{2}12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}.$$

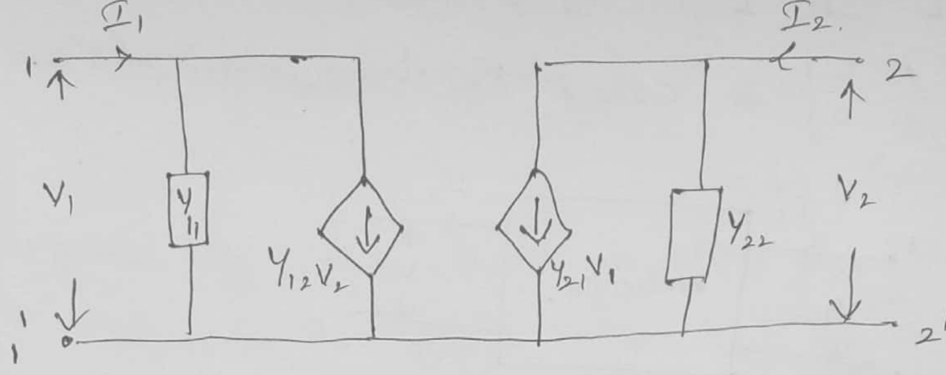
$Y_{\cancel{2}12} \Rightarrow$ short circuited reverse transfer admittance.

and also
from Eq (2) $\quad I_2 = Y_{22} V_2$

$$\Rightarrow Y_{22} = \frac{I_2}{V_2}.$$

$Y_{22} \rightarrow$ short circuit output admittance.

The equivalent circuit of the N/w governed by Eq ① & ② is shown in fig. below.



If the Network under study is reciprocal (or) bilateral

$$\boxed{Y_{12} = Y_{21}}$$

and also symmetry $\boxed{Y_{11} = Y_{22}}$.

⇒ **Transmission (ABCD) Parameters :**

Transmission parameters (or) ABCD parameters are widely used in transmission line theory and cascade Networks. In describing the transmission parameters the input variables V_1 and I_1 at port 1-1', usually called the sending end, are expressed in terms of the output variables V_2 and I_2 at port 2-2', called the receiving end. The transmission parameters provide a direct relationship between input and output.

They are defined by

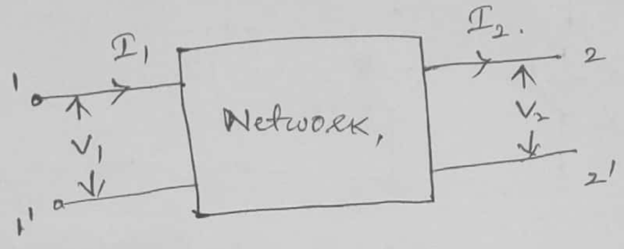
$$V_1 = AV_2 - BI_2 \quad \text{—— ①}$$

$$I_1 = CV_2 - DI_2 \quad \text{—— ②}$$

The -ve sign is used with I_2, and not for the parameter B and D. Both the port currents I_1 and $-I_2$ are directed to the right. i.e, with a -ve sign in Eq ① & ② the current at port 2-2', which leaves the port is designated as positive.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called the transmission matrix.



with port 2-2' open i.e, $I_2=0$. applying a voltage V_1 at the port 1-1', we have.

$$\text{Eq} \; ① \Rightarrow V_1 = A V_2 \Rightarrow A = V_1/V_2 \Big|_{I_2=0}$$

$$\text{Eq} \; ② \Rightarrow I_1 = C V_2 \Rightarrow C = \frac{I_1}{V_2} \Big|_{I_2=0}$$

with port 2-2' short circuited i.e with $V_2=0$, applying Voltage V_1 at port 1-1'. we have.

$$\text{from Eq} ① \Rightarrow V_1 = -B I_2 \Rightarrow -B = V_1/I_2 \Big|_{V_2=0}$$

$$\text{fom Eq} ② \Rightarrow I_1 = -D I_2 \Rightarrow -D = \frac{I_1}{I_2} \Big|_{V_2=0}.$$

⇒. HYBRID (h) Parameters :

Hyblid (or) h. parameters find extensive use in transistor circuits. The hybrid matrices describe a two-port, when the ~~volt~~ voltage of the input port and the current of the output port are expressed in terms of the current of the input port and the voltage of the ~~p~~ output port.

$$\text{Here} \quad \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}.$$

(or)

$$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow ①$$

$$I_2 = h_{21} I_1 + h_{22} V_2, \rightarrow ②$$

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Assuming Short Circuit Conditions at the output $V_2 = 0$, This gives from Eq (1) $V_1 = h_{11} I_1 \Rightarrow h_{11} = \frac{V_1}{I_1}$ → input impedance

and also from Eq (2) $I_2 = h_{21} I_1 \Rightarrow h_{21} = \frac{I_2}{I_1}$.

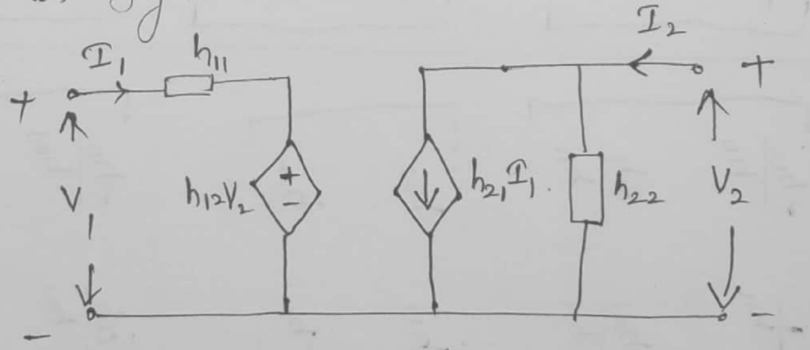
↳ forward Current gain.

in a Similar way, open circuit condition at input $I_1 = 0$.

from Eq (1) $V_1 = h_{12} V_2 \Rightarrow h_{12} = \frac{V_1}{V_2}$ → reverse Voltage gain.

Eq (2) $\Rightarrow I_2 = h_{22} V_2 \Rightarrow h_{22} = \frac{I_2}{V_2}$. → output admittance.

The equivalent circuit of the hybrid parameters representation is shown in fig. below.



parameter	Condition for reciprocity	condition for symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
h	$h_{12} = -h_{21}$	$h_{11}h_{22} - h_{21}h_{12} = 1$
ABCD	$AD - BC = 1$	$A = D$.

	$\lvert Z \rvert$	$\lvert Y \rvert$	$\lvert h \rvert$	$\lvert T \rvert$
$\lvert Z \rvert$	$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$
$\lvert Y \rvert$	$\begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$	$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta T}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$
$\lvert h \rvert$	$\begin{bmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{Y_{11}} & \frac{-Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta T}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{bmatrix}$
$\lvert T \rvert$	$\begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-Y_{22}}{Y_{21}} & \frac{-1}{Y_{21}} \\ -\frac{\Delta Y}{Y_{21}} & \frac{-Y_{11}}{Y_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$

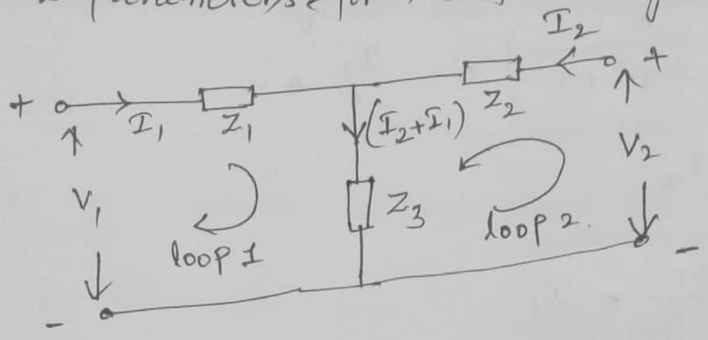
$\Delta Y = Y_{11} Y_{22} - Y_{21} Y_{12}$

$\Delta h = h_{11} h_{22} - h_{12} h_{21}$

$\Delta T = AD - BC$.

Relationship between the different types of two-port Networks.

(3) Find the Z-parameters for the following Network.



sol: Applying KVL to loop 1.

$$V_1 - I_1 Z_1 - Z_3(I_1 + I_2) = 0$$

$$V_1 - I_1 Z_1 - Z_3 I_1 - Z_3 I_2 = 0$$

$$V_1 - I_1(Z_1 + Z_3) - I_2 Z_3 = 0$$

$$V_1 = I_1(Z_1 + Z_3) + I_2 Z_3 \longrightarrow (1)$$

applying KVL to loop 2.

$$V_2 - I_2 Z_2 - (I_1 + I_2) Z_3 = 0$$

$$V_2 - I_2 Z_2 - I_1 Z_3 - I_2 Z_3 = 0$$

$$V_2 - Z_3 I_1 - I_2(Z_2 + Z_3) = 0$$

$$V_2 = I_1 Z_3 + I_2(Z_2 + Z_3) \longleftarrow (2).$$

comparing the Eq (1) & Eq (2) with the standard Z-parameter equations like as

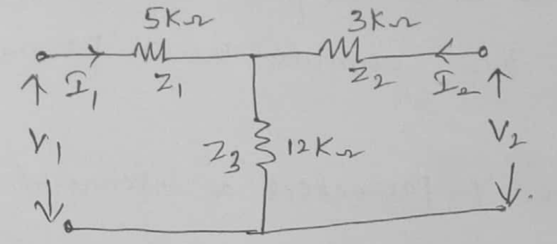
$$V_1 = I_1 Z_{11} + I_2 Z_{12}$$

$$V_2 = I_1 Z_{21} + I_2 Z_{22}.$$

We get $Z_{11} = (Z_1 + Z_3)$, $Z_{21} = Z_3$

$Z_{12} = Z_3$, $Z_{22} = (Z_2 + Z_3)$.

④ Find the Z-parameters for the Network shown in fig. below.



Sol: The Network looking to the previous problem.

$$Z_{11} = (Z_1 + Z_3) = (5+12)\,K\Omega = 17\,K\Omega.$$

$$Z_{12} = Z_3 = 12\,K\Omega$$

$$Z_{21} = Z_3 = 12\,K\Omega$$

$$Z_{22} = (Z_2 + Z_3) = (3+12)\,K\Omega = 15\,K\Omega$$

⑤ The following readings are obtained experimentally for an unknown two-port Network. compute Z-Parameters.

	V_1	V_2	I_1	I_2
output open	100V	60V	10A	0
input open	30V	40V	0	3A.

Sol: From the theory of Z-parameters, we know

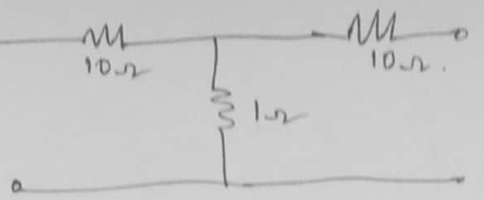
$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{100}{10} = 10\,\Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{60}{10} = 6\,\Omega$$

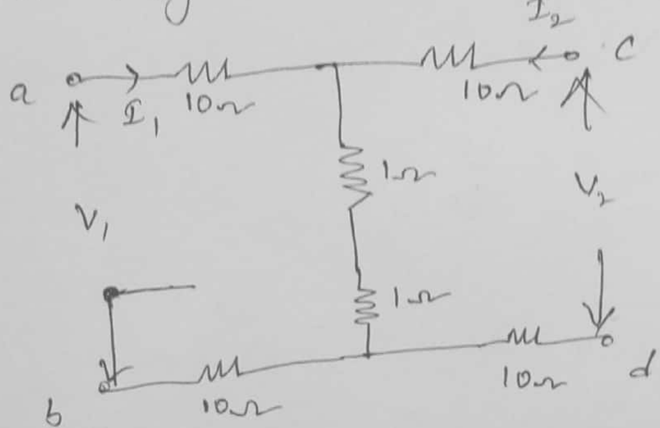
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{30}{3} = 10\,\Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{40}{3} = 13.33\,\Omega$$

① Find the Z-parameters of the combination of two identical T-sections as shown in fig. below. are connected in cascade.



Sol: The Series connection of the two identical T-sections has been shown in fig. below.



first the output is open circuited ($I_2 = 0$) (port 2-2'). and V_1 is applied at a-b terminals (port 1-1'). Let I_1 be the current entering into the network.

$$R_{eq\,1}(a\text{-}b) = 10+1+1+10 = 22\,\Omega.$$

$$V_1 = I_1 R_{eq\,1}$$

$$V_1 = I_1 \times 22 \Rightarrow Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 22\,\Omega$$

as output is open, $V_2 = I_1(1+1) = 2I_1$

$$\therefore Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 2\,\Omega$$

Next it is assumed that terminal a-b is open (port 1-1'). and applied voltage V_2 at c-d terminals (port 2-2'). when current I_2 enters into the network.

$$R_{eq(c-d)} = 10+1+1+10 = 22\,\Omega.$$

$$V_2 = I_2 R_{eq(c-d)} = 22 I_2.$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 22\,\Omega$$

as a-b terminals are open.

$$V_1 = (1+1) I_2$$

$$V_1 = 2 I_2.$$

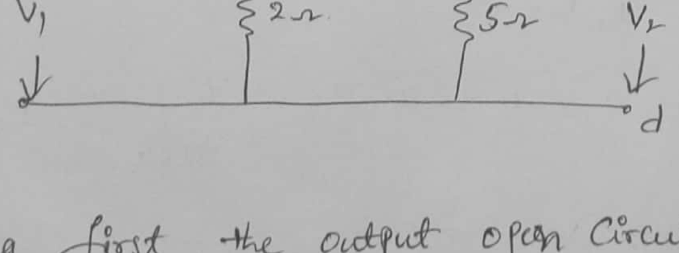
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 2\,\Omega$$

∴ z-parameters

$$Z_{11} = 22\,\Omega, \quad Z_{21} = 2\,\Omega$$
$$Z_{12} = 2\,\Omega, \quad Z_{22} = 22\,\Omega.$$

∴ Hence $Z_{11} = Z_{22} = 22\,\Omega$ & $Z_{12} = Z_{21} = 2\,\Omega$

(2) Find the z-parameters for the circuit shown in fig. below.



Sol: Assuming first the output open circuited (c-d), $I_2 = 0$

looking back into the circuit.

$$R_{eq(a-b)} = \left[(3+5) \| 2\right] + 1$$

$$= \frac{16}{10} + 1 = 2.6\,\Omega.$$

$$V_1 = I_1 R_{eq(a-b)} = I_1 (2.6)$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 2.6\,\Omega$$

⑥ The impedance parameters of a two port Network are $Z_{11} = 6\Omega$, $Z_{22} = 4\Omega$, $Z_{12} = Z_{21} = 3\Omega$. compute the Y-parameters of Network.

Soln:

in this problem the Y-parameters are interms of the Z-parameters

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 3 & 4 \end{bmatrix}$$

$$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$$

$$= 6 \times 4 - 3 \times 3$$

$$= 15.$$

$$Y = \begin{bmatrix} \frac{4}{15} & \frac{-3}{15} \\ \frac{-3}{15} & \frac{6}{15} \end{bmatrix}$$ //.

⑦ Following measurements are obtained on a two-terminal Network.

ⓐ when a Voltage of $100\angle 0^\circ$ V applied at input port with output port is open, $I_1 = 20\angle 0^\circ$ A. and $V_2 = 25\angle 0^\circ$ V.

ⓑ when a Voltage of $100\angle 0^\circ$ V applied at output port with input port is open, $I_2 = 10\angle 0^\circ$ A and $V_1 = 50\angle 0^\circ$ V.

write the loop equations for the N/W and determine the driving point (Z_{11} & Z_{22}) and transfer impedance (Z_{12} & Z_{21}).

Let us now assume the current through the $5\,\Omega$ resistor be

$$I_x = I_1 \times \left(\frac{2}{2+3+5}\right) = \frac{I_1}{5}\ A.$$

$$V_2 = 5 \times I_x$$

$$= 5 \times \frac{I_1}{5} = I_1$$

$$\therefore\ Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 1\ \text{ohm}$$

Next, assuming the input (a-b) to be open, $I_1 = 0$.

looking back into the circuit

$$R_{eq}\,(c\text{-}d) = (3+2)\,\|\,5$$

$$= 2.5\ \text{ohm}$$

$$V_2 = I_2\, R_{eq}(c\text{-}d) = 2.5\, I_2$$

$$\therefore\ Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 2.5\,\Omega$$

Let I_y be the current through the $2\,\Omega$ resistor.

$$I_y = I_2 \times \left(\frac{5}{3+5+2}\right) = \frac{I_2}{2}\ A.$$

$$\therefore\ V_1 = I_y \times 2$$

$$V_1 = \frac{I_2}{2} \times 2 \Rightarrow Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 1\,\Omega$$

Z-parameters.

$$\boxed{\begin{array}{ll} Z_{11} = 2.6\,\Omega, & Z_{21} = 1\,\Omega \\ Z_{12} = 1\,\Omega, & Z_{22} = 2.5\,\Omega \end{array}}$$

Hence the circuit is in Reciprocal (or) bilateral

i.e $\boxed{Z_{12} = Z_{21} = 1\,\Omega}$

EXAMPLE 13.3 *In a T network of Fig. E13.3, $Z_1 = 2\angle 0°$, $Z_2 = 5\angle -90°$, $Z_3 = 3\angle 90°$, find the Z-parameters.*

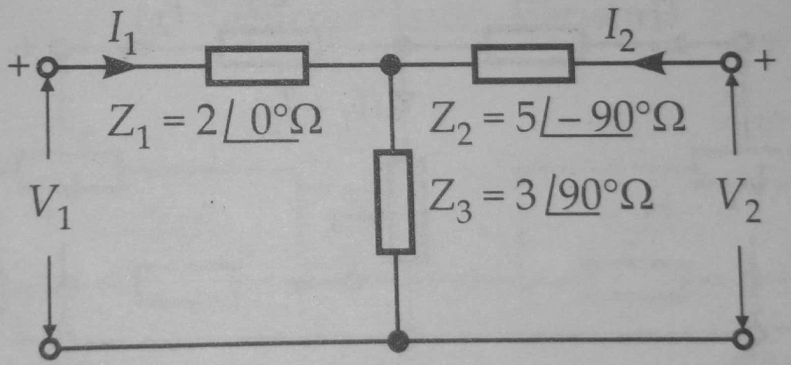


Fig. E13.3

d

EXAMPLE 13.14 *Find the open circuit parameter of the two port network shown in Fig. E13.22.*

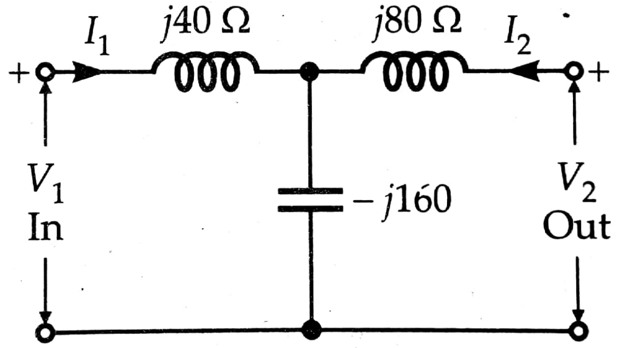


Fig. E13.22

determ

S

EXAMPLE 13.19 *Find Y-parameters of network shown in Fig. E13.29 from Z-parameters.*

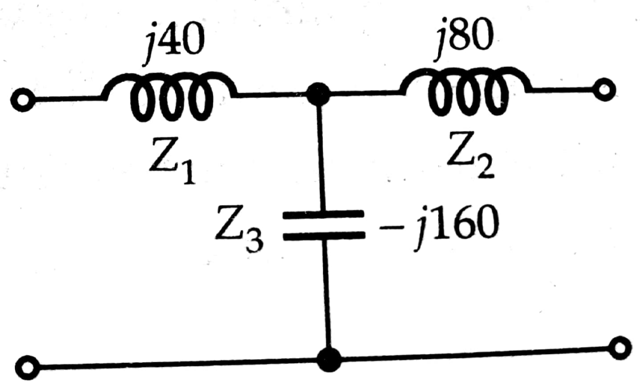
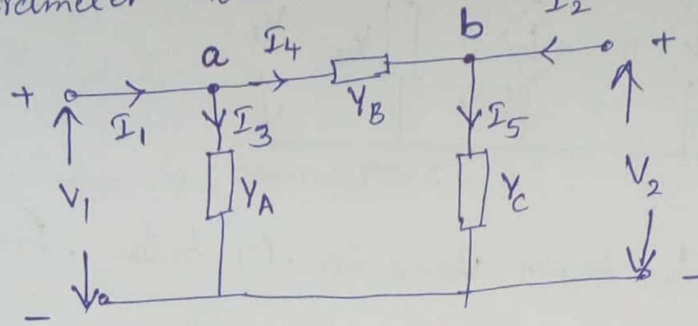


Fig. E13.29

① Find the Y-parameters of the following π circuit and draw the Y-parameter model.

sol:



sol: Applying KCL at node 'a'.

$$I_1 = I_3 + I_4$$

$$= V_1 Y_A + (V_1 - V_2) Y_B$$

$$= V_1 Y_A + V_1 Y_B - V_2 Y_B$$

$$I_1 = V_1 (Y_A + Y_B) + V_2 (-Y_B) \longrightarrow ①$$

Applying KCL at node "b".

$$I_2 + I_4 = I_5$$

$$I_2 = I_5 - I_4$$

$$= V_2 Y_C - (V_1 - V_2) Y_B$$

$$= V_2 Y_C - V_1 Y_B + V_2 Y_B$$

$$I_2 = V_1 (-Y_B) + V_2 (Y_B + Y_C) \longrightarrow ②$$

comparing the Eq ① & Eq ② with the Y-parameter standard equations i.e

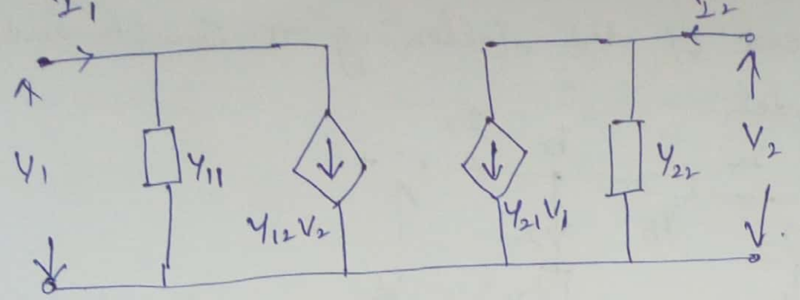
$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

We get;

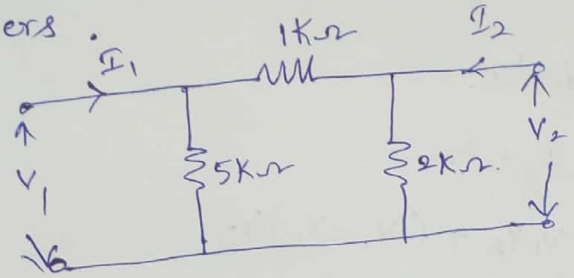
$$Y_{11} = (Y_A + Y_B), \quad Y_{21} = -Y_B$$

$$Y_{12} = -Y_B, \quad Y_{22} = (Y_B + Y_C)_{\,\shortparallel}$$



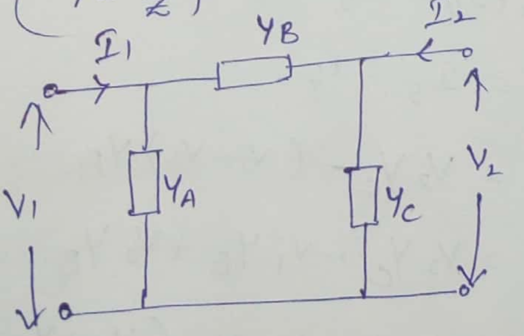
equivalent Circuit

(2) An π attenuator has beem shown in fig. below. find the Y- parameters.



sol: in the circuit values are in impedance of π Network. are given in ohms. First we converted into admittance values in mho's and redrawn the circuit.

$\left(Y = \frac{1}{Z}\right)$



$$Y_A = \frac{1}{5K\Omega} = 0.2 \times 10^{-3} \mho$$

$$Y_B = \frac{1}{1K\Omega} = 10^{-3} \mho$$

$$Y_C = \frac{1}{2K\Omega} = 0.5 \times 10^{-3} \mho.$$

Now the Y - Palameters

$$Y_{11} = (Y_A + Y_B) = (0.2 + 1)10^{-3} = 1.2 \times 10^{-3} \mho$$

$$Y_{12} = -Y_B = -10^{-3} \mho$$

$$Y_{21} = -Y_B = -10^{-3} \mho$$

$$Y_{22} = (Y_B + Y_C) = (1 + 0.5) \times 10^{-3} = 1.5 \times 10^{-3} \mho$$

(03) On short circuit test, the Currents and Voltages were determined experimentally for an unknown two-port Network as

$I_1 = 1mA$
$I_2 = -0.5mA$
$V_1 = 25V$ | at $V_2 = 0$.

and

$I_1 = -1mA$
$I_2 = -10mA$
$V_2 = 50V$ | at $V_1 = 0$.

Determine the Y- parameters.

sol: from the theory of Y parameters as,

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1\times10^{-3}}{25} = 40\,\mu\mho.$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-0.5\times10^{-3}}{25} = -20\,\mu\mho.$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-1\times10^{-3}}{50} = -20\,\mu\mho.$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{-10\times10^{-3}}{50} = -200\,\mu\mho.$$

//

---

(3) Following short circuit currents and voltages are obtained experimentally for a two-port network.

(a) With output short circuited ($V_2 = 0$)

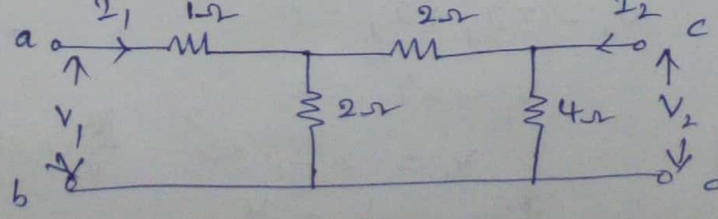
$I_1 = 5mA$, $I_2 = -0.3mA$, $V_1 = 25V$.

(b) With input short circuited. ($V_1 = 0$)

$I_1 = -5mA$, $I_2 = 10mA$, $V_2 = 30V$

Determine the Y- parameters.

(4) Find the Y- parameters for the N/w at fig. below.



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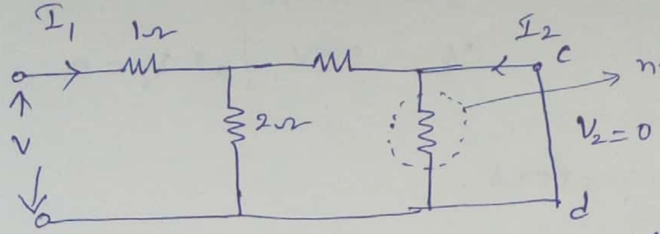
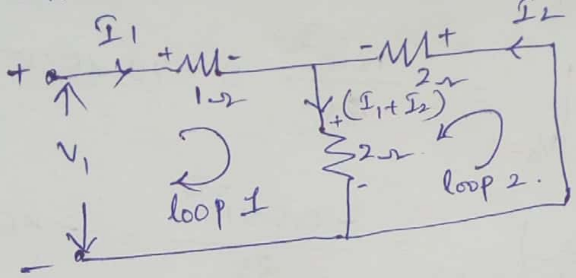
Sol: Here to find the $Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$ & $Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$. the output is short circuited.



again redraw the above circuit.



Applying KVL to loop 1.

$$V_1 - I_1\times 1 - (I_1 + I_2)2 = 0.$$

$$V_1 - I_1 - 2I_1 - 2I_2 = 0$$

$$V_1 - 3I_1 - 2I_2 = 0 \rightarrow (1).$$

also applying KVL to loop 2.

$$-2I_2 - 2(I_1 + I_2) = 0$$

$$-2I_2 - 2I_1 - 2I_2 = 0$$

$$-2I_1 - 4I_2 = 0 \longrightarrow (2)$$

$$-2I_1 = 4I_2$$

$$\cancel{I_1 = \frac{4I_2}{(-2)} = -2I_2}$$

$$I_2 = \frac{-2I_1}{4} = \frac{-I_1}{2}.$$

the above I_2 Value substitute in Eq (1), we get.

$$V_1 - 3I_1 - \cancel{2}\left(\frac{-I_1}{\cancel{2}}\right) = 0$$

$$V_1 - 3I_1 + I_1 = 0$$

$$V_1 - 2I_1 = 0$$

$$V_1 = 2I_1 \Rightarrow \cancel{\frac{V_1}{I_1} = 2}$$

$$\frac{I_1}{V_1} = \frac{1}{2} = Y_{11}$$

Scanned with CamScanner

again from Eq (2)

$$-2I_1 - 4I_2 = 0$$

$$-2I_1 = 4I_2$$

$$I_1 = \frac{4I_2}{(-2)} = -2I_2.$$

the above I_1 value substitute in Eq (1), we get.

$$V_1 - 3(-2I_2) - 2I_2 = 0$$

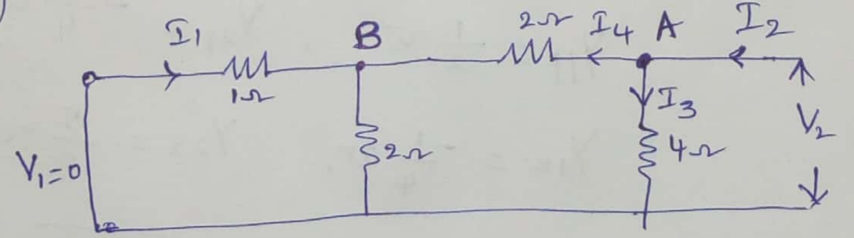
$$V_1 + 6I_2 - 2I_2 = 0$$

$$V_1 + 4I_2 = 0$$

$$V_1 = -4I_2.$$

$$\frac{I_2}{V_1} = \frac{1}{-4} \Rightarrow Y_{21} = \frac{I_2}{V_1} = -\frac{1}{4} \mho$$

For finding Y_{12} & Y_{22}, the short circuit to the input. $(V_1 = 0)$



$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0}.$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$$

Here first findout the total equivalent resistance

$$R_{eq} = \{(1 \| 2) + 2\} \| 4$$

$$= \frac{32}{20} = \frac{8}{5}$$

$$\frac{1 \times 2}{3} + 2$$

$$\frac{2+6}{3} = \frac{8}{3}$$

$$\frac{\frac{8}{3} \times 4}{\frac{8}{3} + 4} \quad \frac{32}{20}$$

$$V_2 = I_2 R_{eq}.$$

$$\frac{1}{R_{eq}} = \frac{I_2}{V_2} = Y_{22} = \frac{1}{\frac{8}{5}} = \frac{5}{8} \mho$$

apply KCL at node "A"

$$I_2 = I_3 + I_4$$

$$I_4 = I_2 - I_3 = \frac{V_2}{(8/5)} - \frac{V_2}{4} = \frac{5V_2}{8} - \frac{V_2}{4}$$

$$= V_2\left(\frac{5}{8} - \frac{1}{4}\right)$$

$I_4 = \frac{3}{8} V_2$.

at node B Current division rule is apply. & Here I_4 is the total current. find the current in 1Ω.

$$\therefore I_1 = -I_4 \times \frac{2}{2+1} \quad (\because \text{ why here} - \text{sign is opp. direction})$$

$$= -\frac{3V_2}{8} \times \frac{2}{3}$$

$$= -\frac{2V_2}{8}$$

$$I_1 = -\frac{1}{4} V_2.$$

$$\frac{I_1}{V_2} = -\frac{1}{4} = Y_{12}$$

We get the Y- parameters are

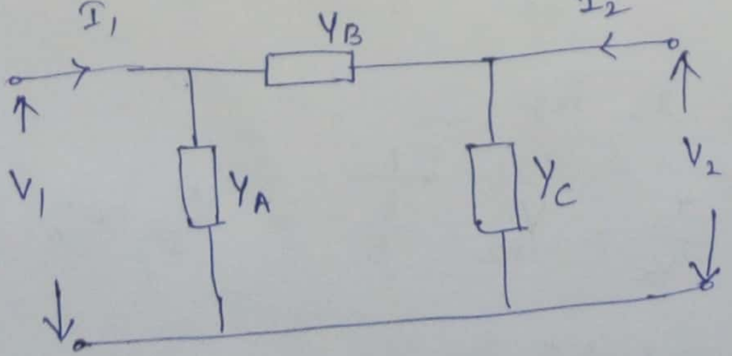
$$Y_{11} = \frac{1}{2} \mho, \quad Y_{21} = -\frac{1}{4} \mho$$

$$Y_{12} = -\frac{1}{4} \mho, \quad Y_{22} = \frac{5}{8} \mho.$$

x

(5) In a π-network, the Series arm ~~impedance~~ admittance is $0.05 \times 10^{-3} \angle -90°$ mho & shunt arm admittances are $0.1 \times 10^{-3} \angle 0° \mho$ and $0.2 \times 10^{-3} \angle 90° \mho$. Find the Y- parameters.

sol:



in this problem Series arm is $Y_B = 0.05 \times 10^{-3} \angle -90° \mho$

$$= -j\, 0.05 \times 10^{-3} \text{ mho}.$$

Shunt arm is $Y_A = 0.1 \times 10^{-3} \angle 0° = 0.1 \times 10^{-3}$ mho.

$$Y_C = 0.2 \times 10^{-3} \angle 90° = j\, 0.2 \times 10^{-3} \text{ mho}.$$

For the π Network, representing in Y-parameter form, as

$$Y_{11} = (Y_A + Y_B), \qquad Y_{21} = -Y_B$$

$$Y_{12} = -Y_B, \qquad Y_{22} = (Y_B + Y_C).$$

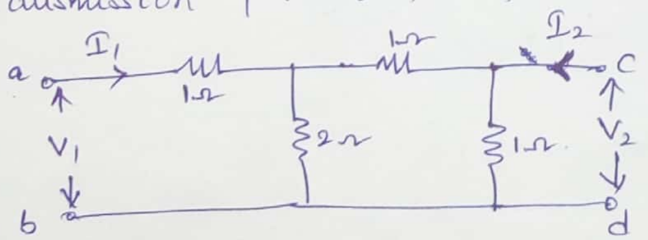
$$Y_{11} = (Y_A + Y_B) = (0.1 \times 10^{-3} - j\,0.05 \times 10^{-3}) = (0.1 - j0.05) \times 10^{-3} \text{ mho.}$$

$$Y_{12} = Y_{21} = -Y_B = -(-j\,0.05 \times 10^{-3}) = j\,0.05 \times 10^{-3} \text{ mho.}$$

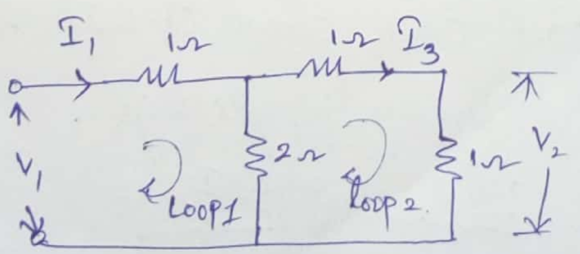
$$Y_{22} = (Y_B + Y_C) = (-j\,0.05 \times 10^{-3} + j\,0.2 \times 10^{-3})$$

$$= j\,0.15 \times 10^{-3} \text{ mho.}$$

⇒ Obtain transmission parameters of the network shown in fig. below



<u>Sol:</u> Let us first keep terminals c-d open circuit and apply Voltage V_1 at input port (a-b). The circuit has been shown in fig. below.



apply KVL to loop 1.

$$V_1 - I_1 \times 1 - 2(I_1 - I_3) = 0$$

$$V_1 - I_1 - 2I_1 + 2I_3 = 0$$

$$V_1 = 3I_1 - 2I_3 \longrightarrow (1)$$

apply KVL to loop 2. $\quad -2(I_3 - I_1) + I_3(1+1) = 0$

$$-2I_3 + 2I_1 - 2I_3 = 0$$

$$4I_3 - 2I_1 = 0$$

$$\Rightarrow I_3 = \frac{2}{4} I_1 = \frac{1}{2} I_1 \rightarrow ②$$

Substitute Eq ② in Eq ①, we get

$$V_1 = 3I_1 - 2\left(\frac{1}{2}\right) I_1$$

$$V_1 = 3I_1 - I_1$$

$$V_1 = 2I_1 \quad \text{—— ③}$$

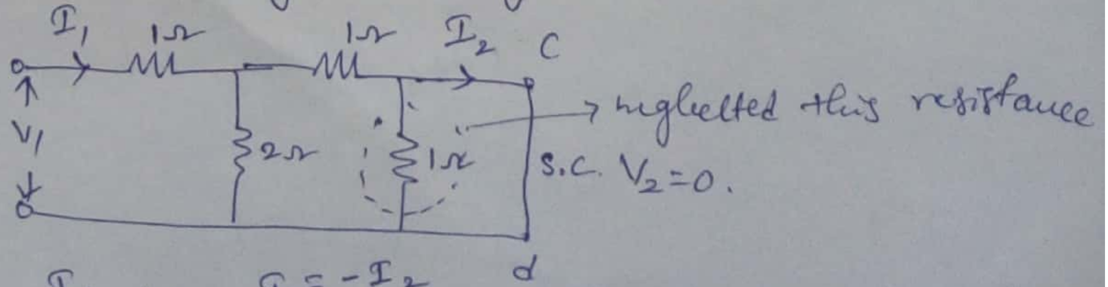
again. $V_2 = 1 \times I_3 = \frac{1}{2} I_1$. —④

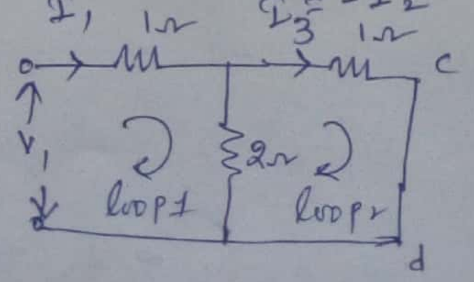
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 2\,\mho.$$

Dividing Eq ③ by Eq ④

$$\left.\frac{V_1}{V_2}\right|_{I_2=0} = 4 = A$$

Now at output port c-d, short circuit is applied and input port a-b, apply V_1 voltage.





Applying KVL to loop 1.

$$V_1 - I_1 - 2[I_1 - (-I_2)] = 0$$

$$V_1 - I_1 - 2(I_1 + I_2) = 0$$

$$V_1 - I_1 - 2I_1 - 2I_2 = 0$$

$$V_1 = 3I_1 + 2I_2 \rightarrow (5)$$

again apply KVL to loop 2

$$\boxed{\cancel{-2(I_1 + I_2) - 1 \times (-I_2) = 0} \\ \cancel{-2I_1 - 2I_2 + I_2 = 0}}$$

$$-2(-I_3 - I_1) - 1 I_3 = 0$$

put $I_3 = -I_2$.

$$-2(-I_2 - I_1) - 1 \times (-I_2) = 0$$

$$2I_2 + 2I_1 + I_2 = 0$$

$$3I_2 + 2I_1 = 0$$

$$I_1 = -\frac{3}{2} I_2 \rightarrow (6)$$

Substitute Eq (6) in Eq (5), we get

$$V_1 = 3I_1 + 2I_2$$

$$= 3\left(-\frac{3}{2} I_2\right) + 2I_2$$

$$= -\frac{9}{2} I_2 + 2I_2$$

$$V_1 = -\frac{5}{2} I_2.$$

$$\left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{5}{2} \text{ ohm} = B$$

from Eq (6) $\left.\dfrac{I_1}{-I_2}\right|_{V_2=0} = \dfrac{3}{2} = D$

Hence the transmission parameters are.

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 4 & \frac{5}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

Here $AD - BC = 1$ and $A \neq D$.

thus the circuit is reciprocal but not Symmetrical.

⇒ The impedance parameters of a two-port Network are $Z_{11} = 3\Omega = Z_{22}$ & $Z_{12} = 1\Omega = Z_{21}$. determine the ABCD parameters of the Network.

sol:

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$$
$$= 3 \times 3 - 1 \times 1$$
$$= 9 - 1 = 8$$

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 1 & 3 \end{bmatrix}$$

$$A = \frac{Z_{11}}{Z_{21}} = \frac{3}{1} = 3.$$

$$B = \frac{\Delta Z}{Z_{21}} = \frac{8}{1} = 8\Omega$$

$$C = \frac{1}{Z_{21}} = \frac{1}{1} = 1 \text{ mho.}$$

$$D = \frac{Z_{22}}{Z_{21}} = \frac{3}{1} = 3.$$

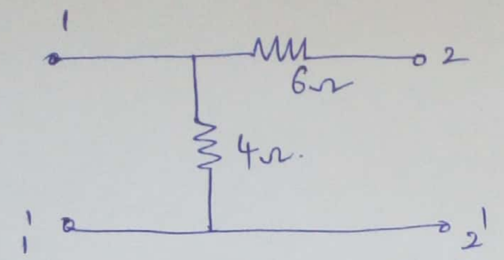
$AD - BC = 1$

&

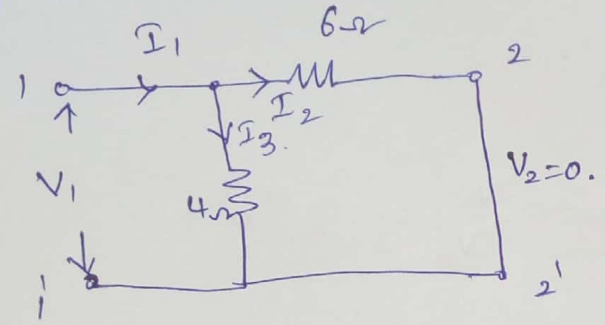
$A = D$.

∴ Thus the circuit is reciprocal and also Symmetrical.

⇒ Find the h-parameters for the network shown below.



sol: Let us first keep terminals 2-2' short circuit and apply Voltage V_1 at 1-1'.



$$R_{eq} = (4 \| 6) = \frac{4\times 6}{4+6} = \frac{24}{10} = 2.4\,\Omega.$$

$$V_1 = I_1 R_{eq}.$$

we know that

$$\Rightarrow h_{11} = \frac{V_1}{I_1} = R_{eq} = 2.4\,\Omega$$

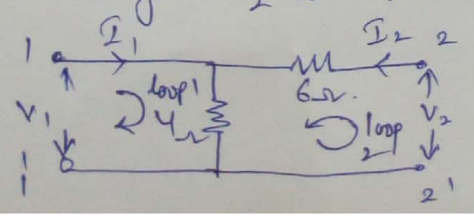
According to current division rule. find current in 6Ω.

$$I_2 = -I_1 \times \frac{4}{4+6}$$

$$\frac{I_2}{I_1} = \frac{4}{10} = -0.4$$

we know that $h_{21} = \frac{I_2}{I_1} = -0.4$

in Similar way that, open circuit to terminals 1-1', i.e. $I_1 = 0$. and apply voltage V_2 at terminals 2-2'.



from loop 1, $V_1 = 4 I_2 \rightarrow (1)$

from loop 2,

$$V_2 = 10 I_2 \Rightarrow (2)$$

from Eq (1) $I_2 = \frac{V_1}{4}$, Value in Eq (2), we get

$$V_2 = 10 \cdot \frac{V_1}{4}$$

$$V_2 = \frac{5}{2} V_1.$$

we know that $h_{12} = \frac{V_1}{V_2} = \frac{2}{5} = 0.4$ //

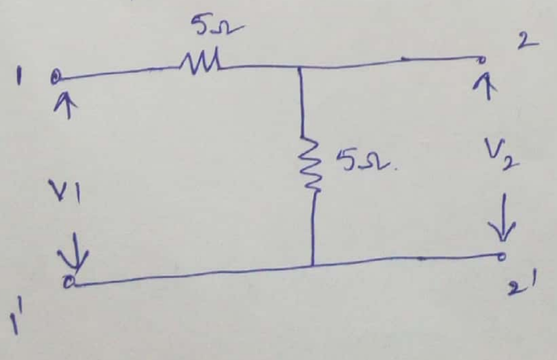
from Eq (2) ⇒ $V_2 = 10 I_2$

$$h_{22} = \frac{I_2}{V_2} = \frac{1}{10} = 0.1 \text{ mho.}$$

∴ h-parameters are

$$h = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 2.4 & 0.4 \\ -0.4 & 0.1 \end{bmatrix}$$

⇒ For the h-parameter for the Network shown below.



...symmetrical.

t open-

13.11 INTER-RELATIONSHIPS BETWEEN PARAMETERS OF TWO PORT NETWORK

13.11.1 Z-parameters in Terms of Y-parameters

Z being the impedance and Y being the admittance,

$$[Z]=[Y]^{-1}$$

or

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

Thus
$$Z_{11} = \frac{Y_{22}}{\Delta Y}, Z_{12} = -\frac{Y_{12}}{\Delta Y}$$
$$Z_{21} = -\frac{Y_{21}}{\Delta Y} \text{ and } Z_{22} = \frac{Y_{11}}{\Delta Y} \quad ...(13.60)$$
$$\left[\text{Here } \Delta Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = Y_{11}\,Y_{22} - Y_{12}\,Y_{21}\right]$$

13.11.2 *Z*-parameters in Terms of *ABCD* Parameters

From the governing equation

$I_1 = CV_2 - DI_2$, we can write
$$V_2 = \frac{1}{C} \cdot I_1 + \frac{D}{C} \cdot I_2 \quad ...(13.61)$$

Similarly, from another governing equation
$$V_1 = AV_2 - BI_2,$$
$$V_1 = \left[\frac{1}{C} \cdot I_1 + \frac{D}{C} \cdot I_2\right] A - BI_2$$
$$= \frac{A}{C} \cdot I_1 + \frac{AD - BC}{C} \cdot I_2 \quad ...(13.62)$$

Comparing these two equations with the governing equations of Z-parameter network, we can write
$$Z_{11} = \frac{A}{C}, Z_{12} = \frac{AD - BC}{C}$$
$$Z_{21} = \frac{1}{C}, Z_{22} = \frac{D}{C}.$$

13.11.3 *Z*-parameters in Terms of Hybrid Parameters

The governing equations of the *h*-parameter network are given by
$$V_1 = h_{11}\,I_1 + h_{12}\,V_2 \quad ...(13.63)$$
$$I_2 = h_{21}\,I_1 + h_{22}\,V_2 \quad ...(13.64)$$

From equation (13.64),
$$V_2 = -\frac{h_{21}}{h_{22}} \cdot I_1 + \frac{1}{h_{22}} \cdot I_2 \quad ...[13.64(a)]$$

From equation (13.63),
$$V_1 = h_{11} I_1 + h_{12}\,V_2$$
$$= h_{11}\,I_1 + h_{12}\left[-\frac{h_{21}}{h_{22}}\,I_1 + \frac{1}{h_{22}} \cdot I_2\right]$$
$$= \frac{\Delta h}{h_{22}}\,I_1 + \frac{h_{12}}{h_{22}} \cdot I_2 \quad ...(13.65)$$

Comparing equations (13.64) and (13.65) with the governing equations of Z-parameter network, here,
$$Z_{11} = \frac{\Delta h}{h_{22}}, \quad Z_{12} = \frac{h_{12}}{h_{22}}$$
$$Z_{21} = -\frac{h_{21}}{h_{22}}, Z_{22} = \frac{1}{h_{22}} \quad ...(13.66)$$
$$[\text{Here } \Delta h = h_{11}\,h_{22} - h_{21}\,h_{12}]$$

13.11.4 *Y*-parameters in Terms of *Z*-parameter

We know, $[I] = [Y][V] = [Y][Z][I]$

$(\because [V] = [Z][I])$

or $[Y] = [Z]^{-1}$

or
$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}$$

Thus,
$$Y_{11} = \frac{Z_{22}}{\Delta Z}, \quad Y_{12} = -\frac{Z_{12}}{\Delta Z}$$
$$Y_{21} = -\frac{Z_{21}}{\Delta Z}, Y_{22} = \frac{Z_{11}}{\Delta Z} \quad ...(13.67)$$
$$[\text{ Here, } \Delta Z = Z_{11}\,Z_{22} - Z_{12}\,Z_{21}]$$

13.11.5 *Y*-parameters in Terms of Transmission parameters (*ABCD* parameters)

Utilising the *ABCD* parameter representation, the governing equations are
$$V_1 = AV_2 - BI_2$$
giving
$$I_2 = \left(-\frac{1}{B}\right)V_1 + \left(\frac{A}{B}\right)V_2 \quad ...(13.68)$$
and
$$I_1 = CV_2 - DI_2$$
$$= CV_2 - D\left[\left(-\frac{1}{B}\right)V_1 + \frac{A}{B}\,V_2\right]$$
$$= \left(\frac{D}{B}\right)\,V_1 - \frac{AD - BC}{B} \cdot V_2 \quad ...(13.69)$$

Comparing equations (13.68) and (13.69) with the governing equations of Y-parameter network,
$$Y_{11} = \frac{D}{B}, Y_{12} = -\frac{AD - BC}{B}$$
$$Y_{21} = -\frac{1}{B}, Y_{22} = \frac{A}{B} \quad ...(13.70)$$

13.11.6 *Y*-parameter in Terms of *h*-parameter

The governing equations of h-parameter network are given by

$$V_1 = h_{11} I_1 + h_{12} V_2$$

giving $$I_1 = \left(\frac{1}{h_{11}}\right) V_1 + \left(-\frac{h_{12}}{h_{11}}\right) V_2 \quad ...(13.71)$$

and $$I_2 = h_{21} I_1 + h_{22} V_2$$

or, $$I_2 = h_{21} \left[\left(\frac{1}{h_{11}}\right) V_1 - \left(\frac{h_{12}}{h_{11}}\right) V_2\right] I_1 + h_{22} V_2$$

$$= \left(\frac{h_{21}}{h_{11}}\right) V_1 + \left(\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}\right) V_2 \quad ...(13.72)$$

Comparing these two equations with the governing equations of Y-parameter network,

$$Y_{11} = \frac{1}{h_{11}}; Y_{12} = -\frac{h_{12}}{h_{11}};$$
$$Y_{21} = \frac{h_{21}}{h_{11}}; Y_{22} = \frac{h_{11}\, h_{22} - h_{12}\, h_{21}}{h_{11}}.$$

13.11.7 *ABCD* parameter in Terms of *Z*-parameter

The Z-parameter equations are given by

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

which gives $$I_1 = \left(\frac{1}{Z_{21}}\right) V_2 + \left(\frac{Z_{22}}{Z_{21}}\right)(-I_2) ...(13.73)$$

and $$V_1 = Z_{11} I_1 + Z_{12} I_2$$

giving $$V_1 = Z_{11}\left\{\left(\frac{1}{Z_{21}}\right) V_2 + \left(-\frac{Z_{22}}{Z_{21}}\right) I_2\right\} + Z_{12} I_2$$

$$= \left(\frac{Z_{11}}{Z_{21}}\right) V_2 + (-I_2)\left[\frac{Z_{22}Z_{11} - Z_{12}Z_{21}}{Z_{21}}\right] \quad ...(13.74)$$

Comparing these two equations (13.73) and (13.74) with the governing equations of $ABCD$ parameters, we find that

$$A = \frac{Z_{11}}{Z_{21}}; \quad B = \frac{Z_{22}Z_{11} - Z_{12}\, Z_{21}}{Z_{21}}$$
$$C = \frac{1}{Z_{21}}; \quad D = \frac{Z_{22}}{Z_{21}} \quad ...(13.75)$$

13.11.8 *ABCD* parameters in Terms of *Y*-parameters

Y-parameter equations are given by

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

giving $$V_1 = \left(-\frac{Y_{22}}{Y_{21}}\right) V_2 + \left(-\frac{1}{Y_{21}}\right)(-I_2)$$

or, $$V_1 = \left(-\frac{Y_{22}}{Y_{21}}\right) V_2 - \left(-\frac{1}{Y_{21}}\right) I_2 \quad ...(13.76)$$

and $$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$= Y_{11}\left[\left(-\frac{Y_{22}}{Y_{21}}\right) V_2 + \left(\frac{1}{Y_{21}}\right) I_2\right] + Y_{12} V_2$$

$$= \left[-\frac{Y_{11}\, Y_{22} - Y_{12}\, Y_{21}}{Y_{21}}\right] V_2 + \left(-\frac{Y_{11}}{Y_{21}}\right)(-I_2)$$

$$= \left[-\frac{Y_{11}\, Y_{22} - Y_{12}\, Y_{21}}{Y_{21}}\right] V_2 - \left(-\frac{Y_{11}}{Y_{21}}\right) I_2 \quad ...(13.77)$$

Comparing equations (13.74) and (13.77) with the governing equations of $ABCD$ parameters,

$$A = \left(-\frac{Y_{22}}{Y_{21}}\right); \quad B = \left(-\frac{1}{Y_{21}}\right)$$
$$C = \left[-\frac{Y_{11}\, Y_{22} - Y_{12}\, Y_{21}}{Y_{21}}\right]; D = \left(-\frac{Y_{11}}{Y_{21}}\right) \quad ...(13.78)$$

13.11.9 *ABCD* parameters in Terms of *h*-parameters

From the governing equations of h-parameter network, we get

$$I_2 = h_{21} I_1 + h_{22} V_2$$

giving $$I_1 = \left(-\frac{h_{22}}{h_{21}}\right) V_2 + \left(-\frac{1}{h_{21}}\right)(-I_2) \quad ...(13.79)$$

and $$V_1 = h_{11} I_1 + h_{12} V_2$$

giving $$V_1 = h_{11}\left[\left(-\frac{h_{22}}{h_{21}}\right) V_2 + \left(-\frac{1}{h_{21}}\right)(-I_2)\right] h_{12} V_2$$

CHAPTER 13

or $$V_1 = \left(-\frac{h_{11}\,h_{22} - h_{12}h_{21}}{h_{21}} \right) V_2 + \left(-\frac{h_{11}}{h_{21}} \right) (-I_2)$$

or $$V_1 = \left(-\frac{h_{11}\,h_{22} - h_{12}\,h_{21}}{h_{21}} \right) V_2 - \left(-\frac{h_{11}}{h_{21}} \right) (I_2)$$

...(13.80)

Comparing equations (13.79) and (13.80) with the original *ABCD* parameter equations, we observe

$$A = \left(-\frac{h_{11}\,h_{22} - h_{12}\,h_{21}}{h_{21}} \right), \; B = -\left(\frac{h_{11}}{h_{21}} \right)$$
$$C = \left(-\frac{h_{22}}{h_{21}} \right), \; D = \left(-\frac{1}{h_{21}} \right) \quad ...(13.81)$$

13.11.10 *h*-parameter in Terms of *Z*-parameters

Utilising the governing equation

$$V_2 = Z_{21}\,I_1 + Z_{22}\,I_2$$

or the *Z*-parameter representation, we can write

$$I_2 = \left(-\frac{Z_{21}}{Z_{22}} \right) I_1 + \left(\frac{1}{Z_{22}} \right) V_2 \quad ...(13.82)$$

Also from the other governing equation

$V_1 = Z_{11}\,I_1 + Z_{12}\,I_2$, which can be written as

$$V_1 = Z_{11}\,I_1 + Z_{12} \left[\left(-\frac{Z_{21}}{Z_{22}} \right) I_1 + \left(\frac{1}{Z_{22}} \right) V_2 \right]$$
$$= \left(\frac{Z_{11}\,Z_{22} - Z_{12}\,Z_{21}}{Z_{22}} \right) I_1 + \left(\frac{Z_{12}}{Z_{22}} \right) V_2$$

...(13.83)

Comparing these two equations (13.82) and (13.83) with the governing equations of the *h*-parameter network,

$$h_{11} = \frac{Z_{11}\,Z_{22} - Z_{21}\,Z_{12}}{Z_{22}}$$
$$h_{12} = \frac{Z_{12}}{Z_{22}}$$
$$h_{21} = -\frac{Z_{21}}{Z_{22}} \quad ...(13.84)$$

and $$h_{22} = \frac{1}{Z_{22}}.$$

13.11.11 *h*-parameters in Terms of the *Y*-parameters

The governing equations of the *Y*-parameter equations are

$$I_1 = Y_{11}\,V_1 + Y_{12}\,V_2$$

and $$I_2 = Y_{21}\,V_1 + Y_{22}\,V_2$$

The first equation gives

$$V_1 = \left(\frac{1}{Y_{11}} \right) I_1 + \left(-\frac{Y_{12}}{Y_{11}} \right) V_2 \quad ...(13.85)$$

Thus the second equation becomes

$$I_2 = Y_{21}\,V_1 + Y_{22}\,V_2$$
$$= Y_{21} \left[\left(\frac{1}{Y_{11}} \right) I_1 - \left(\frac{Y_{12}}{Y_{11}} \right) V_2 \right] + Y_{22} V_2$$
$$= \left(\frac{Y_{21}}{Y_{11}} \right) I_1 + \left(\frac{Y_{11}\,Y_{22} - Y_{12}\,Y_{21}}{Y_{11}} \right) V_2$$

...(13.86)

Comparing equations (13.85) and (13.86) with the original *h*-parameter equations,

$$h_{11} = \frac{1}{Y_{11}}$$
$$h_{12} = -\frac{Y_{12}}{Y_{11}}$$
$$h_{21} = \frac{Y_{21}}{Y_{11}} \quad ...(13.87)$$
$$h_{22} = \frac{Y_{11}\,Y_{22} - Y_{12}\,Y_{21}}{Y_{11}}$$

13.11.12 *h*-parameter in Terms of *ABCD* parameters

The *ABCD* parameter equations are given by

$$I_1 = CV_2 - DI_2$$

giving $$I_2 = \left(-\frac{1}{D} \right) I_1 + \left(\frac{C}{D} \right) V_2 \quad ...(13.88)$$

and $$V_1 = AV_2 - BI_2$$

giving $$V_1 = AV_2 - B \left[\left(-\frac{1}{D} \right) I_1 + \left(\frac{C}{D} \right) V_2 \right]$$
$$= \left(\frac{B}{D} \right) I_1 + \left(\frac{AD - BC}{D} \right) V_2 \quad ...(13.89)$$

Comparing these two equations (13.88 and 13.89) with the h-parameter equations,

$$h_{11} = \left(\frac{B}{D}\right)$$

$$h_{12} = \left(\frac{AD - BC}{D}\right)$$

$$h_{21} = \left(-\frac{1}{D}\right)$$

$$h_{22} = \left(\frac{C}{D}\right)$$

two port
mine the
nd input

$[\because Z_L = \infty]$

13.13 DIFFERENT TYPES OF INTERCONNECTIONS OF TWO PORT NETWORKS

13.13.1 Series Connection

The open circuit impedance parameter is highly useful in characterising the *series connected* two port networks. Though here only two numbers of two port networks are shown to be connected in series, this result can be generalised for any number of two port network in series.

Let network A and B be the two port networks connected in series (Ref. Fig. 13.27).

All input and output currents and voltages have been shown in Fig. 13.27.

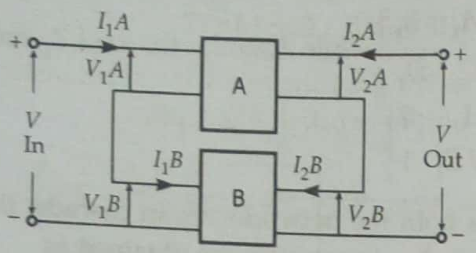


Fig. 13.27 Series connection of two 2-port networks.

For network A,

$$V_{1A} = Z_{11A}\ I_{1A} + Z_{12A}\ I_{2A}$$
$$V_{2A} = Z_{21A}\ I_{1A} + Z_{22A}\ I_{2A}$$

and for network B,

$$V_{1B} = Z_{11B}\ I_{1B} + Z_{12B}\ I_{2B}$$
$$V_{2B} = Z_{21B}\ I_{1B} + Z_{22B}\ I_{2B}$$

Referring to Fig. 13.27, the interconnection results

$$I_1 \equiv I_{1A} \equiv I_{1B}$$
$$I_2 \equiv I_{2A} \equiv I_{2B}$$
$$V_2 = V_{2A} + V_{2B}$$

However,

$$V_1 = V_{1A} + V_{1B}$$
$$= (Z_{11A} I_{1A} + Z_{12A} I_{2A}) + Z_{11B} I_{1B} + Z_{12B} I_{2B}$$
$$= I_1\,(Z_{11A} + Z_{11B}) + I_2\,(Z_{12A} + Z_{12B})$$

and $V_2 = V_{2A} + V_{2B}$

$$= (Z_{21A} I_{1A} + Z_{22A} I_{2A}) + (Z_{21B} I_{1B} + Z_{22B} I_{2B})$$

or $V_2 = I_1\,(Z_{21A} + Z_{21B}) + I_2\,(Z_{22A} + Z_{22B})$

Thus we get, for the series connected two numbers two port networks,

$$V_1 = (Z_{11A} + Z_{11B})\,I_1 + (Z_{12A} + Z_{12B})\,I_2$$
$$V_2 = (Z_{21A} + Z_{21B})\,I_1 + (Z_{22A} + Z_{22B})\,I_2$$

~~or, in matrix form,~~

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11A} + Z_{11B} & Z_{12A} + Z_{12B} \\ Z_{21A} + Z_{21B} & Z_{22A} + Z_{22B} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad ..(13.109)$$

Thus it has been observed that the overall Z parameter matrix for series connected two port networks is simply the sum of Z matrices of each individual network.

13.13.2 Cascade Connection

ABCD parameters are highly useful in characterising *cascaded* two port networks.

In Fig. 13.28 let X and Y be two networks connected in cascade. The results may be generalised for any number of networks.

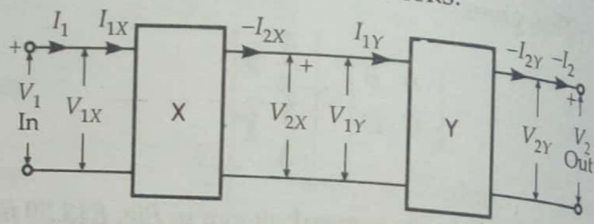


Fig. 13.28 Cascade connected two numbers two port network.

For network X,

$$V_{1X} = A_X\ V_{2X} - B_X I_{2X}$$
$$I_{IX} = C_X\ V_{2X} - D_X\ I_{2X}$$

and for network Y,

$$V_{1Y} = A_Y\ V_{2Y} - B_Y I_{2Y}$$
$$I_{1Y} = C_Y V_{2Y} - D_Y I_{2Y}$$

For the cascade connection,

$$I_1 = I_{1X}\ ; -I_{2X} = I_{1Y}\ ; I_2 = I_{2Y}$$
$$V_1 = V_{IX}\ ; V_{2X} = V_{1Y}\ ; V_2 = V_{2Y}$$

The overall transmission parameters for the combined networks as shown in Fig. 13.28, in matrix form become

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1X} \\ I_{1X} \end{bmatrix} = \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} V_{2X} \\ -I_{2X} \end{bmatrix}$$
$$= \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} V_{1Y} \\ I_{1Y} \end{bmatrix}$$
$$= \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} A_Y & B_Y \\ C_Y & D_Y \end{bmatrix} \begin{bmatrix} V_{2Y} \\ -I_{2Y} \end{bmatrix}$$
$$= \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} A_Y & B_Y \\ C_Y & D_Y \end{bmatrix} \begin{bmatrix} V_Y \\ -I_Y \end{bmatrix}$$
$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_Y \\ -I_Y \end{bmatrix} \qquad ...(13.110)$$

where, $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} A_Y & B_Y \\ D_Y & D_Y \end{bmatrix}$

The overall ABCD parameter network matrix for cascaded network is then the matrix product of ABCD matrices of individual network.

13.13.3 Parallel Connection

Let network A and B be connected in *parallel* as shown in Fig. 13.29. Y-parameter representation is very much useful.

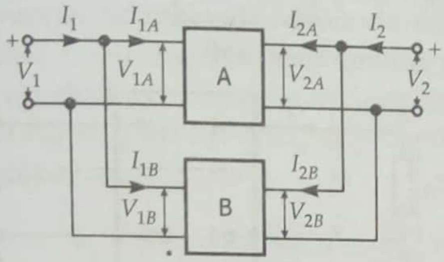


Fig. 13.29 Parallel connection of two 2-port networks.

For network A,

$$I_{1A} = Y_{11A}\, V_{1A} + Y_{12A}\, V_{2A}$$
$$I_{2A} = Y_{21A}\, V_{1A} + Y_{22A}\, V_{2A}$$

and for network B,

$$I_{1B} = Y_{11B}\, V_{1B} + Y_{12B}\, V_{2B}$$
$$I_{2B} = Y_{21B}\, V_{1B} + Y_{22B}\, V_{2B}$$

For the parallel connection as shown in the figure,

$$V_1 = V_{1A} = V_{1B};\ V_2 = V_{2A} = V_{2B}$$
$$I_1 = I_{1A} + I_{1B}\ ;\ I_2 = I_{2A} + I_{2B}$$

Thus,

$$\begin{aligned} I_1 &= I_{1A} + I_{1B} \\ &= (Y_{11A}V_{1A} + Y_{12A}V_{2A}) + (Y_{11B}V_{1B} + Y_{12B}V_{2B}) \\ &= (Y_{11A} + Y_{11B})\, V_1 + (Y_{12A} + Y_{12B})\, V_2 \end{aligned}$$

$$\begin{aligned} I_2 &= I_{2A} + I_{2B} \\ &= (Y_{21A}V_{1A} + Y_{22A}V_{1B}) + (Y_{21B}V_{1B} + Y_{22B}V_{2B}) \\ &= (Y_{21A} + Y_{21B})\, V_1 + (Y_{22A} + Y_{22B})\, V_2 \end{aligned}$$

Thus we finally obtain

$$I_1 = (Y_{11A} + Y_{11B})\, V_1 + (Y_{12A} + Y_{12B})\, V_2$$
$$I_2 = (Y_{21A} + Y_{21B})\, V_1 + (Y_{22A} + Y_{22B})\, V_2$$

Thus, in matrix form, the combined network parameter equations become,

$$\begin{bmatrix} I_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} Y_{11A} + Y_{11B} & Y_{12A} + Y_{12B} \\ Y_{21A} + Y_{21B} & Y_{22A} + Y_{22B} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad ...(13.111)$$

This result may be generalised for any number of Y parameter network paralleling. *The overall Y parameter matrix is then simply the summation of Y matrices of each individual two port network.*

13.19 CIRCUIT MODELLING OF AN IDEAL TRANSFORMER

The coefficient of coupling of a transformer being defined by K (refer to the chapter : Coupled Circuits),

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

[With reference to Fig. 13.36,

L_1 = Self induction of the primary coil

L_2 = Self induction of the secondary coil

M = mutual inductance between the primary and secondary coils]

[Refer to Fig. 13.36].

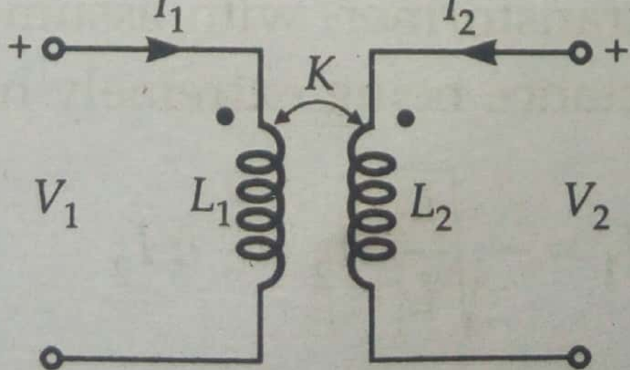


Fig. 13.36 Circuit model of a transformer.

An ideal transformer has perfect coupling with $K=1$.

Here, $V_1 = j\omega L_1\, I_1 + j\omega M I_2$

i.e., $$I_1 = \frac{V_1 - j\omega M I_2}{j\omega L_1} \qquad \text{...(13.146)}$$

and also, $V_2 = j\omega M I_1 + j\omega L_2\, I_2$

giving $$V_2 = j\omega M\left(\frac{V_1 - j\omega M I_2}{j\omega L_1}\right) + j\omega L_2\, I_2$$

$$= \frac{MV_1}{L_1} - \frac{j\omega M^2\, I_2}{L_1} + j\omega L_2\, I_2$$

However, with perfect coupling ($K=1$),

$$M = \sqrt{L_1\, L_2}$$

$$\therefore \quad V_2 = \frac{\sqrt{L_1\, L_2}\, V_1}{L_1} - \frac{j\omega L_1\, L_2\, I_2}{L_1} + j\omega L_2\, I_2$$

$$= \sqrt{\frac{L_2}{L_1}}\, V_1$$

$$= nV_1 \left[\text{Say } \sqrt{\frac{L_2}{L_1}} = n \text{ the turn ratio}\right]$$

However, n_1 and n_2 being the number of turns of primary and secondary from the theory of basic electrical engineering,

$$L_1 = Kn_1^2 \text{ and } L_2 = Kn_2^2$$

This gives,

$$n = \sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{Kn_2^2}{Kn_1^2}} = \frac{n_2}{n_1}$$

Thus, from equation (13.146),

$$I_1 = \frac{V_1}{j\omega L_1} - \frac{MI_2}{L_1} = \frac{V_1}{j\omega L_1} - \frac{\sqrt{L_1\, L_2}\, I_2}{L_1}$$

$$= \frac{V_1}{j\omega L_1} - \sqrt{\frac{L_2}{L_1}}\, I_2$$

In ideal transformer, with assumption that L_1, the self inductance being extremely high,

$$I_1 = -\sqrt{\frac{L_2}{L_1}}\, I_2 = -n\, I_2$$

or, $$I_2 = -\frac{I_1}{n}$$

Thus we finally obtain, for an ideal transformer,

$$\left.\begin{aligned} V_2 &= nV_1 \\ I_2 &= -I_1/n \end{aligned}\right\} \qquad \text{...(13.147)}$$

Figures 13.37 (*a*) and 13.37 (*b*) represent the circuit models.

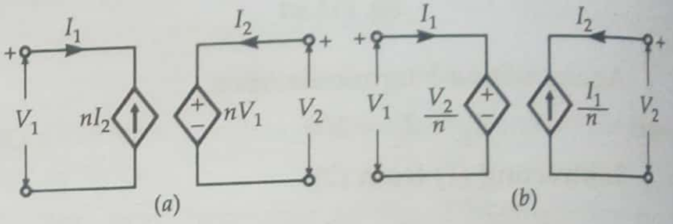


Fig. 13.37 Current and voltage source equivalents of transformer.

Total instantaneous power absorbed by the transformer

$$= p_1 + p_2 = v_1\, i_1 + v_2\, i_2$$
$$= v_1\, i_1 + (nv_1)(-i_1/n) = 0.$$

Thus ideal transformer is a lossless device.

Let us now rewrite the voltage equations of the ideal transformer again—

$$V_1 = j\omega L_1\, I_1 + j\omega M I_2$$

and $$V_2 = j\omega M I_1 + j\omega L_2\, I_2$$

Laplace transformation of these equations give

$$V_1(s) = L_1\,[sI_1(s) - i_1(0+)] + M[sI_2(s) - i_2(0+)] \qquad \text{...(13.148)}$$

$$V_1(s) = M[sI_1(s) - i_1(0+)] + L_2\,[sI_2(s) - i_2(0+)]$$

Reorientation of these equations gives

$$V_1(s) = L_1\, sI_1(s) + Ms\, I_2(s) - [L_1\, i_1(0+) + M i_2(0+)] \qquad \text{...(13.149)}$$

$$V_2(s) = Ms\, I_1(s) + L_2\, sI_2(s) - [Mi_1(0+) + L_2\, i_2(0+)]$$

These two circuit equations can then be diagrammatically represented as shown below (Fig. 13.38).

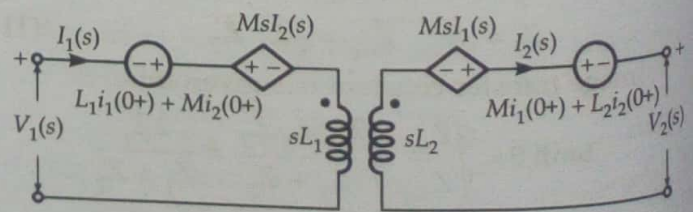


Fig. 13.38 Representation of circuit model of an ideal transformer.

However, assuming zero initial condition, the equations of $V_1(s)$ and $V_2(s)$ reduce to

$$V_1(s) = L_1 sI_1(s) + Ms\, I_2(s) \qquad \text{...(13.150)}$$
$$V_2(s) = Ms\, I_1(s) + L_2 sI_2(s)$$

The diagrammatic representation is then shown in Fig. 13.39.

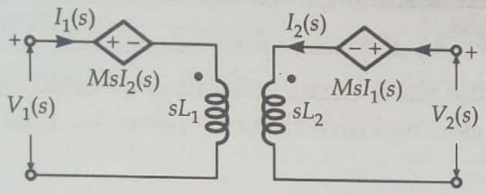


Fig. 13.39 Diagrammatic representation of the circuit model of a transformer.

13.20 MODELING OF NETWORK COMPONENTS

1. Transformer

The governing equations for an ideal transformer are given by (as shown in equation 13.147)

$$V_2 = nV_1$$
$$I_2 = -I_1 / n.$$

For our convenience we can rewrite these two equations as

$$V_1 = \frac{1}{n} . V_2$$
$$I_2 = \left(-\frac{1}{n}\right) . I_1$$

In matrix form, these equations are shown as

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 1/n \\ -1/n & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \qquad \text{...(13.151)}$$

Equation (13.151) exhibits the modeling form in terms of *h*-parameter concept.

We can also write the governing equations of an ideal transformer in the following forms (as shown in Art 13.19)

$$V_1(s) = sL_1 . I_1(s) + sM . I_2(s)$$
$$V_2(s) = sM . I_1(s) + sL_2 . I_2(s)$$

Thus, in *Z*-parameter form, these equations can be shown as

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} sL_1 & sM \\ sM & sL_2 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} \qquad \text{...(13.152)}$$