Unit - 5 Time varying Fields

Time Invarient fields (or) static fields:

If the Electrostatic and magnetic fields do not change with respect to time, such fields are called static field (or) Time invarient fields.

Time variant fields (or) Dynamic fields :

If the electrostatic field & magnetic fields changes with respect to time such fields are called Dynamic fields (or) Time varient fields.

→ The time varying fields are produced due to time varying currents.
In dynamic fields, the time varying electric field can be produced by time varying magnetic fields. and viceversa.

→ In time varying fields, the electric fields and magnetic fields are interdependent.

→ **Faradays law of Electromagnetic Induction :- (or)**

Faradays law and Lenz's law

Statement of Faradays law : The electromotive force (emf) induced in a closed path is proportional to the rate of change of magnetic flux enclosed by the closed path.

$$\boxed{e = -N \frac{d\phi}{dt}} \longrightarrow ①$$

where N = No. of turns in the circuit
e = Emf Induced

For $N=1$, $e = -\frac{d\phi}{dt} \rightarrow ②$ (for single turn circuit)

– ve sign indicates the direction of induced emf produced current which will produce magnetic field which will oppose original field.

↳ Statement of Lenz's law: The direction of induced emf is such that it opposes the cause producing it. i.e, changes in the magnetic flux.

$$e = -N \frac{d\phi}{dt} \text{ volts}$$

↳ Let us consider Faradays law. The emf induced is a scalar measured in volts.

The induced emf is given by, $e = \oint \bar{E} \cdot d\bar{L} \longrightarrow (3)$

The induced emf in Eq (3) indicates a voltage about a closed path such that any part of the path is changed, emf also will change.

The magnetic flux ϕ passing through specified area is given by,

$$\phi = \int_S \bar{B} \cdot d\bar{S} \longrightarrow (4)$$

where B = magnetic flux density

. Using above Eq (4), Eq (2) is rewritten as,

$$e = -\frac{d}{dt} \int_S \bar{B} \cdot d\bar{S} \longrightarrow (5)$$

from Eq (3) & (5), we get

$$e = \oint \bar{E} \cdot d\bar{L} = -\frac{d}{dt} \int_S \bar{B} \cdot d\bar{S} \longrightarrow (6)$$

$$\therefore e = \oint \bar{E} \cdot d\bar{L} = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S} \longrightarrow (6)$$

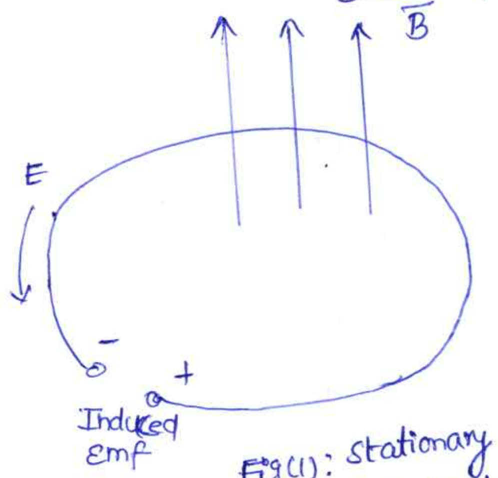
→ The variation of the flux ϕ with respect to time t can be caused due to any one of them:

1) By having a stationary closed path in a time varying $\bar{B}$ field
2) By having a time varying closed path in static $\bar{B}$ field.
3) By having a time varying closed path in time varying field $\bar{B}$.

1) A stationary closed path in a Time Varying $\bar{B}$ field (or) Statically Induced Emf (or) Transformer Emf: (or) Maxwell's Fourth Equation:-

Statically Induced Emf: When an emf is induced in a stationary closed path due to the time varying $\bar{B}$ field, the emf is called statically Induced emf.

→ The condition in which a closed path is stationary and $\bar{B}$ field is varying w.r.to time is shown in fig(1).



Fig(1): Stationary closed path in time varying $\bar{B}$ field

→ The closed circuit in which emf induced is stationary and magnetic flux is sinusoidally varying with time.

Hence $\oint \bar{E} \cdot d\bar{l} = - \int_S \frac{\partial}{\partial t} \cdot d\bar{S}$ ⟶ (7)

Eq (7) is integral form of Faraday's law.
This is similar to Transformer equation and emf is called "Transformer emf" or "statically Induced emf."

→ Using Stokes Theorem, a line integral can be converted to surface integral as,

$$\oint_S (\nabla \times \bar{E}) \cdot d\bar{S} = - \int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S} \longrightarrow (8)$$

Assuming that both the surface integrals taken over identical surfaces.

$$(\nabla \times \bar{E}) \cdot d\bar{s} = -\frac{\partial \bar{B}}{\partial t} \cdot d\bar{s}$$

Hence finally,

$$\boxed{\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}} \quad \text{——— (9)}$$

Eq (9) represents one of the Maxwell's Equation (in point form) point form of Faradays law.

Eq (9) is the differential form (or) point form of Faradays law.

Eq (8) & (9) are Maxwell 4th equation.

2) A moving closed path in static $\bar{B}$ field (or) Dynamically Induced Emf (or) Motional Emf :-

Dynamically Induced Emf : when the emf is induced in time varying closed path due to static $\bar{B}$ field, then the emf is called dynamically induced emf (or) motional emf

A moving closed path in static $\bar{B}$ field is shown in Fig (2). Here the field is stationary and closed path is moving to get a relative motion between them.

This action is similar to Generator action. Hence the induced emf is called generator emf (or) motional emf (or) Dynamically Induced emf.

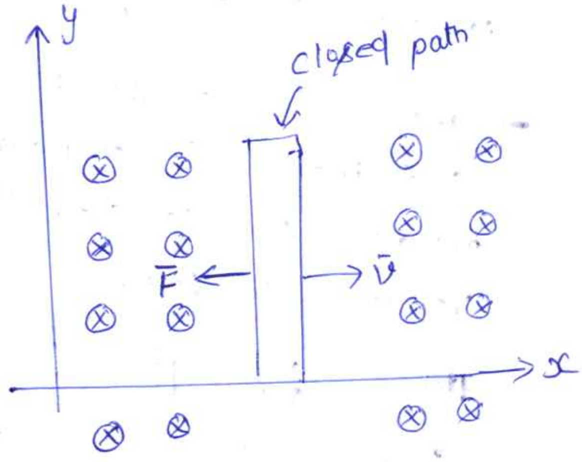


Fig (2) : Closed path moving in static $\bar{B}$ field with velocity $\bar{v}$.

Consider a charge Q is moved in a magnetic field $\bar{B}$ at a velocity $\bar{v}$. The force acting on charge is given by,

$$\bar{F} = Q\bar{v} \times \bar{B} \longrightarrow (10)$$

But motional electric field Intensity is defined as force per unit charge. It is given by

$$\bar{E} = \frac{\bar{F}}{Q} = \bar{v} \times \bar{B}$$

$$\bar{E} = \bar{v} \times \bar{B} \longrightarrow (11)$$

Thus the dynamical induced Emf is

$$\oint \bar{E} \cdot d\bar{L} = \oint (\bar{v} \times \bar{B}) \cdot d\bar{L} \longrightarrow (12)$$

The above Eq (12) is Integral form of Faradays law.

$$\nabla \times \bar{E} = \nabla \times (v \times \bar{B}) \longrightarrow (13)$$

The above Eq (13) is differential form or point form of Faradays law.

3) ~~By~~ ~~ba~~ A time varying closed path in time varying field $\bar{B}$:(or) General case :

A moving closed path in a time varying $\bar{B}$ field represents a general case in which both Emf's i.e., transformer Emf and motional Emf are present.

Thus induced emf for a moving closed path in Time varying field can be written as

Total Induced Emf = Transformer Emf + Motional Emf

$$\oint \bar{E} \cdot d\bar{L} = -\int \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s} + \oint (\bar{v} \times \bar{B}) \cdot d\bar{L} \rightarrow (14)$$

Eq (14) is in Integral form.

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} + \nabla \times (\bar{v} \times \bar{B}) \rightarrow (15)$$

Eq (15) is in differential form.

Faraday's Law

Integral Form | Point form

1) Transformer emf :

$$\oint \bar{E} \cdot d\bar{L} = -\int \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S} \qquad \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}.$$

2) Motional Emf :

$$\oint \bar{E} \cdot d\bar{L} = \oint (\bar{v} \times \bar{B}) \cdot d\bar{L} \qquad \nabla \times \bar{E} = \nabla \times (\bar{v} \times \bar{B})$$

3) General case :

$$\oint \bar{E} \cdot d\bar{L} = \oint - \int \frac{\partial \bar{B}}{\partial t} ds + \oint (\bar{v} \times \bar{B}) \cdot d\bar{L} \qquad \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} + \nabla \times (\bar{v} \times \bar{B})$$

→ **Inconsistance of Ampere's circuit law - Modification in Equation of continuity :- (or) Derivation of Displacement current Density (or) Modified Ampere's circuit law : (or) Modified Maxwell's Equation for Time varying field :-**

Consider Ampere's circuit law in point or differential form as,

$$\nabla \times \bar{H} = \bar{J} \longrightarrow (1)$$

Taking divergence on both sides

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot \bar{J} \longrightarrow (2)$$

According to vector identity, "divergence of curl of vector is zero". (i.e, $\nabla \cdot \nabla \times \bar{H} = 0$)

But $\nabla \cdot \bar{J} = 0$ is valid for only static fields.

The above result is not consistent and needs some modification.

In otherwords, Ampere circuit law is not consistent and needs modification.

let us consider some unknown Term J_D. We can modify the Ampere circuit law for time varying fields as,

$$\nabla \times \bar{H} = \bar{J} + \bar{J}_D \longrightarrow (3)$$

Taking divergence on both sides,

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot (\bar{J} + \bar{J}_D). \longrightarrow (4)$$

But $\nabla \cdot (\nabla \times \bar{H}) = 0$, then we get

$$\nabla \cdot (\bar{J} + \bar{J}_D) = 0.$$

i.e., $\nabla \cdot \bar{J} + \nabla \cdot \bar{J}_D = 0$

$$\nabla \cdot \bar{J}_D = -\nabla \cdot \bar{J} \longrightarrow (5)$$

But from continuity eq for Time varying fields,

$$\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}. \longrightarrow (6)$$

Substitute eq (6) in eq (5),
Hence we write

$$\nabla \cdot \bar{J}_D = -\left[-\frac{\partial \rho_v}{\partial t}\right] = \frac{\partial \rho_v}{\partial t}. \longrightarrow (7)$$

from Gauss law, in point form,

$$\nabla \cdot \bar{D} = \rho_v \longrightarrow (8)$$

differentiating on both sides

$$\nabla \cdot \frac{\partial \bar{D}}{\partial t} = \frac{\partial \rho_v}{\partial t} \longrightarrow (9)$$

from eq (7) & (9),

$$\therefore \nabla \cdot \bar{U} = \nabla \cdot \frac{\partial \bar{D}}{\partial t}.$$

i.e., $\bar{J}_D = \frac{\partial \bar{D}}{\partial t}$ = displacement current density $\longrightarrow$ (10)

Hence for time varying field, Ampere circuit law is written as from eq (3),

$$\therefore \nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}. \longrightarrow (11)$$

Eq (11) is modified Ampere circuit law in point form.

$$\oint \bar{H} \cdot d\bar{L} = I + \int_S \frac{\partial \bar{D}}{\partial t} \cdot d\bar{S} \longrightarrow (12)$$

Eq (12) modified Ampere circuit law in Integral form.

Equation of Displacement current :-

from Eq (10), we can write, $\boxed{J_D = \frac{\partial \bar{D}}{\partial t}}$

J_D = Displacement current density

From Eq (11),

$$\nabla \times \bar{H} = \bar{J} + \bar{J}_D$$
$$= J_C + J_D.$$

$\bar{J} = \bar{J}_C$ = conduction current density.

Maxwell's Equations :-

Maxwell's Equations for static field :-

Maxwell Eq. Number	significance	Integral form	Point form
1	Gauss law for Electrostatic field	$\oint \bar{D} \cdot d\bar{S} = \int_V \rho_V \, dv$	$\nabla \cdot \bar{D} = \rho_V$
2.	Gauss law for magnetostatic field	$\oint \bar{B} \cdot d\bar{S} = 0$	$\nabla \cdot \bar{B} = 0$
3.	Ampere circuit Law	$\oint \bar{H} \cdot d\bar{L} = \int_S \bar{J} \cdot d\bar{S}$	$\nabla \times \bar{H} = \bar{J}$
4.	Faraday's Law	$\oint \bar{E} \cdot d\bar{L} = 0$	$\nabla \times \bar{E} = 0$

→ Maxwell's Equations for Time Varying Fields :-

(1) maxwell's Equation Derived from Gauss's law for Electric field :-

A/c to Gauss law,

$$\int \bar{D} \cdot d\bar{s} = Q_{enclosed} \quad \text{——} \quad (1)$$

we ~~arrange Qenclosed~~ Eq (1) as.

$$\int \bar{D} \cdot d\bar{s} = \int_V \rho_v \, dv \quad \text{——} \quad (2)$$

This Eq (2) is called Maxwell's equation for Electric fields derived from Gauss law in Integral form.

statement : The total flux leaving out of a closed surface is equal to the total charge enclosed by a finite volume.

In point form, $\nabla \cdot \bar{D} = \rho_v$ —— (3)

(2) max well's Equation Derived from Gauss law for magnetic fields :

for magnetic fields,

$$\int_S \bar{B} \cdot d\bar{s} = 0 \quad \longrightarrow \quad (4)$$

Eq (4) is maxwell's equation for magnetic field derived from Gauss law in Integral form.

Statement : The surface integral of magnetic flux density over a closed surface is always equal to zero.

In point form, $\nabla \cdot \bar{B} = 0$ ⟶ (5)

(3) maxwell's Equation derived from Ampere's circuit law :-

A/c to Ampere circuit law,

$$\oint \bar{H} \cdot d\bar{L} = I_{enclosed} \quad \text{——} \quad (6)$$

~~the~~ above Eq is written as

$$\oint \bar{H} \cdot d\bar{L} = \int_S \bar{J} \cdot d\bar{s} \quad \text{--- (7)}$$

The above equation can be made further modification by adding displacement current density to the conduction current density as

$$\oint \bar{H} \cdot d\bar{L} = \int_S \left[\bar{J} + \frac{\partial \bar{D}}{\partial t} \right] \cdot d\bar{s} \quad \text{--- (8)}$$

Eq (8) is maxwell's Equation derived from Ampere circuit law in integral form.

Statement : The total magnetomotive force around any closed path is Equal to surface integral of the conduction and displacement current densities over the entire surface bounded by same closedpath.

In point form, $\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$. —— (9)

(4) Maxwell's Equation derived from Faradays law :-

According to faraday's law,

$$\oint \bar{E} \cdot d\bar{L} = - \int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s} \quad \text{--- (10)}$$

Eq (10) is maxwell's Equation derived from Faradays law in Integral form.

Statement : The total Emf induced in a closed path is Equal to negative surface integral of rate of change of flux density with respect to time over an entire surface bounded by same closed path.

In point form, $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$.

Maxwell's Equations in Tabular form.

Maxwell's Equation number	Significance	Integral form	Point form
1	Gauss law for Electric fields	$\oint \bar{D}\cdot d\bar{s} = \int_S \rho_v \, dV$	$\nabla\cdot\bar{D} = \rho_v$
2	Gauss law for magnetic fields	$\oint \bar{B}\cdot d\bar{s} = 0$	$\nabla\cdot\bar{B} = 0$
3.	Ampere circuital law	$\oint \bar{H}\cdot d\bar{L} = I + \int_S \frac{\partial \bar{D}}{\partial t}\cdot d\bar{s}$	$\nabla\times\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$.
4.	Faraday's law	$\oint \bar{E}\cdot d\bar{L} = -\int_S \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s}$	$\nabla\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}$

→ **Maxwell's Equations for Free space :-**

Let us consider a free space as a medium in which Electric & magnetic fields are present. Freespace is a non conducting medium in which volume charge density ρ_v is zero & conductivity σ is also zero.

Maxwell's Equations in free space are mentioned below:

Point form	Integral form
1) $\nabla\cdot\bar{D} = 0$	1) $\oint_S \bar{D}\cdot d\bar{s} = 0$
2) $\nabla\cdot\bar{B} = 0$	2) $\oint_S \bar{B}\cdot d\bar{s} = 0$
3) $\nabla\times\bar{H} = \frac{\partial \bar{D}}{\partial t}$	3) $\oint \bar{H}\cdot d\bar{L} = \int_S \frac{\partial \bar{D}}{\partial t}\cdot d\bar{s}$
4) $\nabla\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}$	4) $\oint \bar{E}\cdot d\bar{L} = -\int_S \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s}$.

→ Maxwell's Equations for Good conductor :-

The maxwell's equations for ~~cond~~ Good conductors are mentioned below

for Good conductors, $\bar{J} >> \frac{\partial \bar{D}}{\partial t}$ and $\rho_v = 0$.

Point form

1) $\nabla \cdot \bar{D} = 0$

2) $\nabla \cdot \bar{B} = 0$

3) $\nabla \times \bar{H} = \bar{J}$

4) $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$

Integral form

1) $\oint \bar{D} \cdot d\bar{s} = 0$

2) $\oint \bar{B} \cdot d\bar{s} = 0$

3) $\oint \bar{H} \cdot d\bar{l} = I = \int_s \bar{J} \cdot d\bar{s}$

4) $\oint \bar{E} \cdot d\bar{l} = -\int_s \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s}$.

→ Maxwells Equations for Harmonically varying fields :-

Let us assume that Electric field & magnetic fields are varying harmonically with time.

Then the Electric flux density $\bar{D}$ can be written as

$$\bar{D} = \bar{D}_0 \, e^{j\omega t}$$

Similarly magnetic flux density, $\bar{B} = \bar{B}_0 \, e^{j\omega t}$

Taking partial derivatives with respect to time,

$$\frac{\partial \bar{D}}{\partial t} = j\omega \bar{D}_0 e^{j\omega t} = j\omega \bar{D}.$$

$$\frac{\partial \bar{B}}{\partial t} = j\omega \bar{B}_0 e^{j\omega t} = j\omega \bar{B}.$$

Point form

1) $\nabla \cdot \bar{D} = \rho_v$

2) $\nabla \cdot \bar{B} = 0$

3) $\nabla \times \bar{H} = \bar{J} + j\omega \bar{D} = \sigma \bar{E} + j\omega(\varepsilon \bar{E}) = (\sigma + j\omega\varepsilon)\bar{E}$

4) $\nabla \times \bar{E} = -j\omega \bar{D} = -j\omega\mu \bar{H}$

Integral form :

1) $\oint_S \bar{D} \cdot d\bar{s} = \oint \rho_v \, dv$

2) $\oint \bar{B} \cdot d\bar{s} = 0$

3) $\oint \bar{H} \cdot d\bar{l} = I + \int_S j\omega \bar{D} \cdot d\bar{s} = \int_S \bar{J} \cdot d\bar{s} + \int_S j\omega \varepsilon \bar{E} \cdot d\bar{s}$

$$= \int_S \sigma \bar{E} \cdot d\bar{s} + \int_S j\omega \varepsilon \bar{E} \cdot d\bar{s}$$

$$= (\sigma + j\omega \varepsilon) \int_S \bar{E} \cdot d\bar{s}$$

4) $\oint \bar{E} \cdot d\bar{l} = - \int_S j\omega \bar{B} \cdot d\bar{s} = - \int_S j\omega \mu \bar{H} \cdot d\bar{s}$

$$= - j\omega \mu \int_S \bar{H} \cdot d\bar{s}$$

Significance of Displacement current :-

The current through the capacitor may be called as displacement current. In capacitor, the current does not flow through the capacitor, but it looks as if, since as much current flows out of one plate as flows into opposite one.

It is Apparent current which is necessary to bridge the gap where there is no conduction current.

For a capacitor connected to ac Voltage source as shown in fig

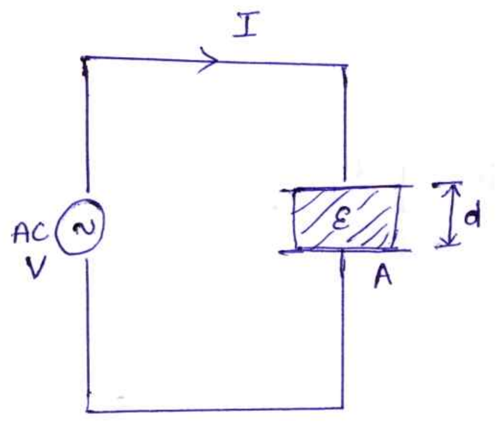


Fig : A capacitor connected to ac voltage sourse

$$\text{Current } I = \frac{dQ}{dt} = C \frac{dv}{dt}$$

where C = capacitance $= \frac{\varepsilon A}{d}$.

A = Area of plates

ε = permittivity of medium

d = separation between the plates.

$$I = \frac{\varepsilon A}{d} \frac{dv}{dt}.$$

since $E = \frac{V}{d}$.

$$I = \varepsilon A \frac{d\bar{E}}{dt}$$

$$\frac{I}{A} = \bar{J} = \varepsilon \frac{d\bar{E}}{dt}$$

$$\bar{J} = \frac{d\bar{D}}{dt} \quad (\because \bar{D} = \varepsilon \bar{E})$$

$$\bar{J}_d = \frac{d\bar{D}}{dt}$$

Vector $\bar{D}$ may vary with space, we may write

Displacement current density,
$$\boxed{\bar{J}_d = \frac{\partial \bar{D}}{\partial t}}$$

Here $\bar{D}$ = Electric displacement

J_d = displacement current density

→ Derive Maxwell's Fourth Equation, $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$:-

According to Faradays law,

$$e = -N\frac{d\phi}{dt} \longrightarrow (1)$$

for single turn circuit, $N=1$,

$$e = -\frac{d\phi}{dt} \longrightarrow (2)$$

We can write, $e = \oint \bar{E}\cdot d\bar{L} \longrightarrow (3)$

The magnetic flux ϕ passing through specified area is given by

$$\phi = \int_S \bar{B}\cdot d\bar{S} \longrightarrow (4).$$

where $\bar{B}$ = magnetic flux density

$\bar{S}$ = surface area.

substitute Eq (4) in Eq (2),

$$e = -\frac{d}{dt}\int_S \bar{B}\cdot d\bar{S} \longrightarrow (5)$$

from Eq (3) & Eq (5),

$$e = \oint \bar{E}\cdot d\bar{L} = -\frac{d}{dt}\int_S \bar{B}\cdot d\bar{S}$$

The above Eq can be written as

$$e = \oint \bar{E}\cdot d\bar{L} = -\int_S \frac{\partial \bar{B}}{\partial t}\cdot d\bar{S} \longrightarrow (6)$$

$$\Rightarrow \oint \bar{E}\cdot d\bar{L} = -\int_S \frac{\partial \bar{B}}{\partial t}\cdot d\bar{S} \longrightarrow (7)$$

using stokes theorem, convert line integral to surface integral as

$$\oint (\nabla \times \bar{E})\cdot d\bar{S} = -\int_S \frac{\partial \bar{B}}{\partial t}\cdot d\bar{S} \longrightarrow (8)$$

Assuming both surface integrals taken over identical surfaces,

$$(\nabla \times \bar{E}) \cdot d\bar{s} = -\frac{\partial \bar{B}}{\partial t} \cdot d\bar{s}$$

$$\Rightarrow \boxed{\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}} \quad \text{—— (9)}$$

Eq (8) & Eq (9) are Maxwell's Fourth Equation.

Eq (8) maxwell Fourth Eq in Integral form

Eq (9) is maxwell Fourth Eq in Point form.

Electromagnetic wave Propogation :

In general waves are means of transporting energy or information. Electromagnetic waves are created as a result of vibrations between an electric field and a magnetic field.

EM waves are propogated by changing the electric field and magnetic field that are kept normal to one another.

Typical examples of EM waves include radio waves, TV signals, radar beams and light rays.

→ ~~Our~~ Applications of Maxwell's Equation gives wave equation. Our major goal is to solve Maxwell's Equations and describe EM wave motion in the following media :

1) Free space : ($\sigma = 0$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$)

2) Lossless dielectrics (perfect dielectric) : ($\sigma = 0$, $\varepsilon = \varepsilon_r \varepsilon_0$, $\mu = \mu_0 \mu_r$, (or) $\sigma << \omega\varepsilon$)

3) Lossy dielectric (conducting medium) : ($\sigma \neq 0$, $\varepsilon = \varepsilon_0 \varepsilon_r$, $\mu = \mu_0 \mu_r$)

4) Good conductors : ($\sigma \simeq 0$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0 \mu_r$, or $\sigma >> \omega\varepsilon$)

→ Uniform Plane wave :-

Production of Electromagnetic waves by an oscillating dipole :-

→ When a charged particle undergoes acceleration, it radiates Electromagnetic Energy. Two metal rods are connected to an ac generator, which causes the charge to oscillate between two rods as shown in Fig(1).

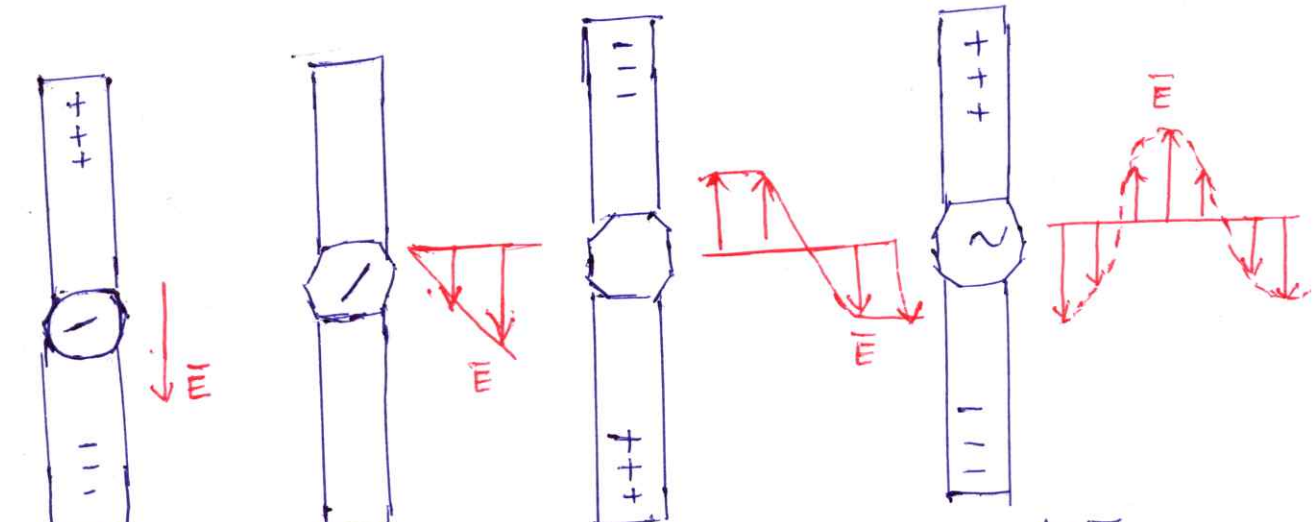
→ At $t = 0$, upper rod is given the maximum positive charge & bottom rod an Equal negative charge. The electric field near the oscillator is as shown in fig (1a)

→ As the charges oscillates, the rod become less charged, the field near the rod decreases its strength and Electric field moves away from the rod. when charges are neutralized as shown in Fig(1b), the Electric field becomes zero and this happens at time $t = \frac{T}{4}$.

→ This process continues, the upper rod soon obtains a maximum negative charge and lower rod becomes positive as shown in Fig(1c), resulting an electric field in upward direction and it occurs at $t = \frac{T}{2}$.

→ The oscillations continues as shown in Fig(1d). Note that the electric field points downward when upper rod is positive and field points upward when upper rod is negative.

→ The magnitude of field at any instant depends on the amount of charge on the rods at that instant. The oscillating charges create a current in the rods, a magnetic field is also generated when the current in rod is upwards as shown in Fig(2).

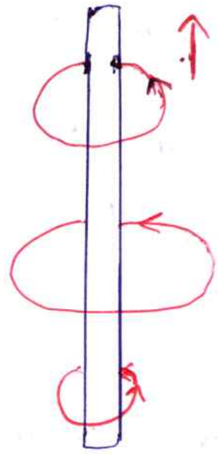


$t=0$ Fig(1a) | $t=\frac{T}{4}$ Fig(1b) | $t=\frac{T}{2}$ Fig(1c) | $t=T$ Fig(1d)

Fig(1): Production of Electromagnetic wave by oscillating Electric charges.

- Time changing electric field and magnetic fields produce electromagnetic waves as predicted by Maxwell.

- The induced electric field and magnetic fields are in phase at any point two fields reach their maximum at the same instant as shown in fig (3).

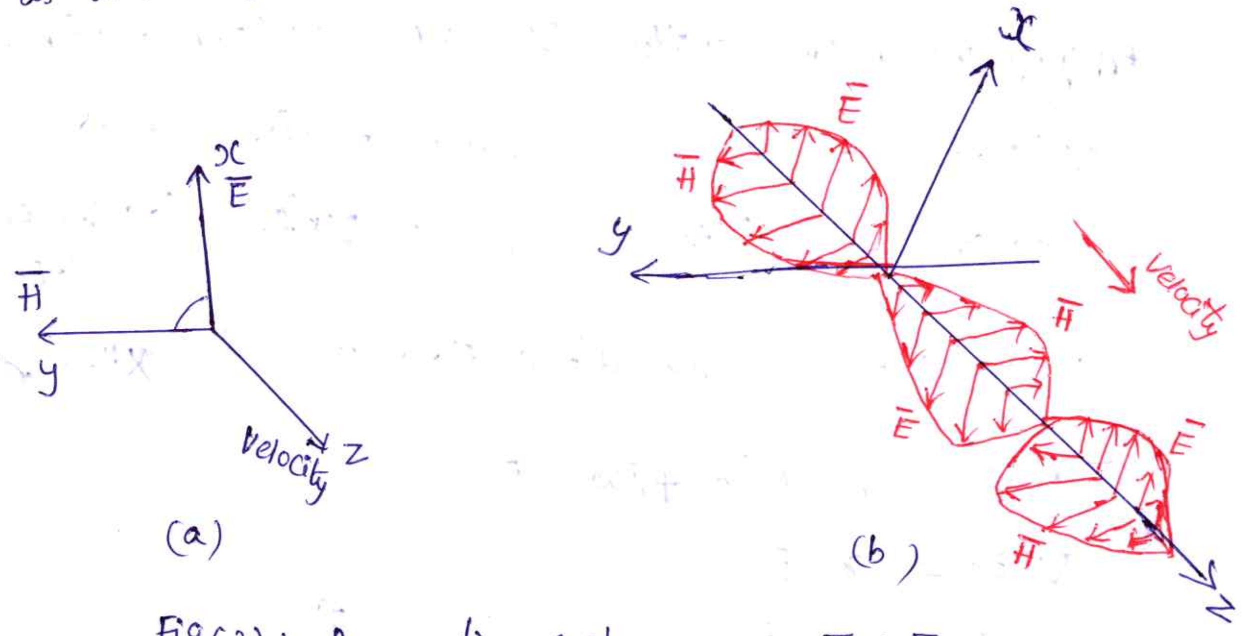


Fig(2): magnetic field associated with a current element.

- If the electric field is in x-direction and magnetic field is in y direction then the wave is travelling in Z-direction.

- If the phase in a wave is the same for all the points on a plane surface it is called a plane wave. If amplitude is also constant over the plane surface it is called "Uniform plane wave".

- Uniform plane waves do not exist in practice. At large distance from physical antennas and ground, however the waves can be approximated as uniform plane waves.



Fig(3): Propogation of plane wave, $\bar{E}$ & $\bar{H}$ vectors.

→ Wave Propogation in Lossy dielectrics (conducting medium) :-

→ A lossy dielectric is a medium in which electromagnetic wave as it propogates, loses power owing to imperfect dielectric with $\sigma \neq 0$.

→ consider a linear, isotrophic, homogeneous, lossy dielectric medium that is charge free ($\rho_v = 0$).

Assuming and suppressing the time factor $e^{j\omega t}$, Maxwell equation becomes,

$$\nabla \cdot \bar{E}_s = 0 \longrightarrow (1)$$

$$\nabla \cdot \bar{H}_s = 0 \longrightarrow (2)$$

$$\nabla \times \bar{E}_s = -j\omega\mu \bar{H}_s \longrightarrow (3)$$

$$\nabla \times \bar{H}_s = (\sigma + j\omega\varepsilon)\bar{E}_s . \longrightarrow (4)$$

Taking curl on both sides of Eq (3),

$$\nabla \times \nabla \times \bar{E}_s = -j\omega\mu(\nabla \times \bar{H}_s) \longrightarrow (5)$$

Applying the vector identity, $\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A$.

$$\nabla(\nabla \cdot \bar{E}_s) - \nabla^2 \bar{E}_s = -j\omega\mu(\nabla \times \bar{H}_s) \longrightarrow (6)$$

$$\nabla(0) - \nabla^2 \bar{E}_s = -j\omega\mu(\nabla \times \bar{H}_s) \quad (\because \text{from Eq (1) } \nabla \cdot E_s = 0).$$

$$\therefore -\nabla^2 \bar{E}_s = -j\omega\mu(\sigma + j\omega\varepsilon)\bar{E}_s \quad (\because \text{from Eq (4), } \nabla \times H_s = (\sigma + j\omega\varepsilon)E_s$$

$$-\nabla^2 \bar{E}_s = -j\omega\mu(\sigma + j\omega\varepsilon)\bar{E}_s$$

$$\nabla^2 \bar{E}_s = \gamma^2 \bar{E}_s$$

$$\Rightarrow \nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s = 0 \longrightarrow (7)$$

where $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon) \longrightarrow (8)$

γ is called propogation constant of the medium.

Similarly, for $\bar{H}$ field,

$$\nabla^2 \bar{H}_s - \gamma^2 \bar{H}_s = 0 \longrightarrow (9)$$

Eq (7) & Eq (9) are known as Homogenous vector "Helmholtz's Equation" or simply "wave Equation."

Since γ in Eq (7) & Eq (9) is a complex quantity, we may let

$$\gamma = \alpha + j\beta \longrightarrow (10)$$

We obtain α and β from Eqs (8) & (10) by noting that.

$$-\text{Re}\,\gamma^2 = \beta^2 - \alpha^2 = \omega^2 \mu \varepsilon \longrightarrow (11)$$

and $$|\gamma^2| = \beta^2 + \alpha^2 = \omega \mu \sqrt{\sigma^2 + \omega^2 \varepsilon^2} \longrightarrow (12)$$

From Eq (11) & Eq (12), we obtain,

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} \longrightarrow (13)$$

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]} \longrightarrow (14)$$

without loss of generality, if we assume that wave propogates, along $+a_z$ and that E_s has only an x- component, then.

$$E_s = E_{xs}(z)\, a_x \longrightarrow (15)$$

Substitute Eq (15) in Eq (7).

$$(\nabla^2 - \gamma^2)\bar{E}_s = 0$$

$$(\nabla^2 - \gamma^2) E_{xs}(z) = 0. \longrightarrow (16)$$

Hence, $$\left[\cancel{\frac{\partial^2 E_{xs}(z)}{\partial x^2}}_{\to 0} + \cancel{\frac{\partial^2 E_{xs}(z)}{\partial y^2}}_{\to 0} + \frac{\partial^2 E_{xs}(z)}{\partial z^2} - \gamma^2\right] E_{xs}(z) = 0$$

The above Eq is written as.

$$\left[\frac{\partial^2}{\partial z^2} - \gamma^2\right] E_{xs}(z) = 0. \longrightarrow (17)$$

This is a scalar wave Eq, a linear homogenous differential eq, with solution

$$E_{xs}(z) = E_0 e^{-\gamma z} + E_0' e^{\gamma z}. \longrightarrow (18)$$

where E_0 & E_0' are constant. since $e^{\gamma z}$ denotes a wave travelling along $-\bar{a}_z$. whereas we assume wave propogation along $\bar{a}_z$, $E_0' = 0$.. Inserting time factor $e^{j\omega t}$ in to Eq (18), and using eq (10), we get,

$$E(z,t) = Re\left[E_0 e^{-\alpha z} e^{j(\omega t - \beta z)} a_x\right] \longrightarrow (19)$$

$$\text{Similarly } H(z,t) = Re\left[H_0 e^{-\alpha z} e^{j(\omega t - \beta z)} a_x\right] \longrightarrow (20)$$

Eq (19) can be written as, $E(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \bar{a}_x$ — (19)

$$H_0 = \frac{E_0}{\eta}. \longrightarrow (21)$$

η is complex quantity, known as "Intrinsic Impedance" in ohms of the medium.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta| \angle\theta_\eta = |\eta| e^{j\theta_\eta} \longrightarrow (22)$$

$$\text{with } |\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{4}}} \qquad \tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} \longrightarrow (23)$$

where $0 \leq \theta_\eta \leq 45°$.

Sub. Eqs (21) & (22) in Eq (20).

$$H = Re\left[\frac{E_0}{|\eta| e^{j\theta_n}} e^{-\alpha z} e^{j(\omega t - \beta z)} a_y\right].$$

(or) $$H = \frac{E_0}{|\eta| e^{j\theta_n}} \cos(\omega t - \beta z - \theta_n)\, a_y. \rightarrow (24)$$

From eq (19) & (24), it is observed that wave propogates along a_z, it decreases or attenuates in amplitude by a factor $e^{-\alpha z}$. Hence α is known as "attenuation constant" (or) attenuation Co-efficient of medium. It is a measure of spatial rate of decay of wave in medium, measured in nepers per meter (Np/m).

$$1\ Np = 20 \log_{10} e = 8.686\ dB.$$

→ β is a measure of phase shift per unit length in radians per meter and is called phase constant (or) wave number.

wave velocity, $u = \frac{\omega}{\beta}$

wave length, $\lambda = \frac{2\pi}{\beta}$.

loss Tangent, $\boxed{\tan\theta = \frac{\sigma}{\omega\varepsilon}.}$

θ is called loss angle of medium.

$\tan\theta$, loss Tangent is used to determine whether a medium is classified by lossy dielectric, perfect dielectric, or good conductor

→ if $\tan\theta$ is very small ($\sigma \ll \omega\varepsilon$) a medium is good dielectric. (lossless or perfect dielectric)

→ if $\tan\theta$ is very large ($\sigma \gg \omega\varepsilon$) a medium is said to be Good conductor.

loss Tangent :

The ratio of magnitude of conduction current density, J_c to the displacement current density J_d in a lossy medium is

$$\frac{|\bar{J}_{cs}|}{|\bar{J}_{ds}|} = \frac{|\sigma \bar{E}_s|}{|j\omega\varepsilon \bar{E}_s|} = \frac{\sigma}{\omega\varepsilon} = \tan\theta, \text{ loss Tangent.}$$

→ Plane waves in loss less Dielectric (Perfect dielectric):-

In lossless dielectric, $\sigma << \omega\varepsilon$,

$$\sigma = 0, \quad \varepsilon = \varepsilon_0 \varepsilon_r, \quad \mu = \mu_0 \mu_r$$

1) attenuation constant $\alpha = 0$.

2) phase shift constant $\beta = \omega\sqrt{\mu\varepsilon}$.

3) characteristic wave Impedance (or) Intrinsic Impedance,

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \angle 0^\circ \ \Omega$$

4) Velocity $u = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}}$

5) wavelength $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{f\sqrt{\mu\varepsilon}}$

Definition:

Intrinsic Impedance or Characterstic Wave Impedance:

Wave impedance, also known as intrinsic impedance, is a measure of the impedance that a medium offers to the propagation of electromagnetic waves.

It is defined as the ratio of the electric field strength to the magnetic field strength in the medium and is expressed in units of ohms.

$$\eta = \frac{E}{H} \quad \text{ohms}$$

→ Plane waves in Free Space :-

plane waves ~~in~~ a free space comprise a special case of lossless dielectric medium

$$\sigma = 0, \quad \varepsilon = \varepsilon_0, \quad \mu = \mu_0$$

1) attenuation constant $\alpha = 0$.

2) phase shift constant $\beta = \omega\sqrt{\mu_0 \varepsilon_0} = \dfrac{\omega}{c}$.

3) velocity $u = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = c. = 3 \times 10^8\ m/s$:= speed of light in vaccuum.

4) wavelength $\lambda = \dfrac{2\pi}{\beta}$.

5) Characteristic Impedance (or) Intrinsic Impedance.

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi = 377$$

→ plane waves in Good conductors :-

A good conductor has high conductivity and large conduction currents.

For perfect or good conductor, $\dfrac{\sigma}{\omega\varepsilon} >> 1$; $\sigma >> \omega\varepsilon$.

$$\sigma \simeq \infty, \quad \varepsilon = \varepsilon_0, \quad \mu = \mu_0 \mu_r.$$

1) Attenuation constant $\alpha = \beta = \sqrt{\pi \mu \sigma f}$ (or) $\sqrt{\dfrac{\omega \mu \sigma}{2}}$

2) Phase shift constant $\beta = \sqrt{\pi \mu \sigma f}$.

3) velocity $u = \frac{\omega}{\beta} = \omega\sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2\omega}{\mu\sigma}} = \sqrt{\frac{4\pi f}{\mu\sigma}}$

$$u = \frac{4\pi f}{\mu\sigma}$$

4) Characteristic wave Impedance (or) Intrinsic impedance,

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

Skin depth (or) penetration depth in good conductors :-

Skin depth is a measure of the depth to which an Electro magnetic wave can penetrate the medium.

Skin depth, $\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$, metre.

→ If E_o is the amplitude of Electric field Intensity at the surface $z=0$, the amplitude will have a value $E_o e^{-\alpha z}$ at a distance z inside the surface through which the wave amplitude decreases by a factor e^{-1} (about 37%) is called skin depth or depth of penetration of medium.

→ The skin depth decreases with increasing frequency. Thus E & H can hardly propograte through good conductors.

→ The phenomenon of whereby field Intensity in a conductor rapidly decreases is known as skin effect.

The characteristic wave impedance,

$$\eta = \frac{1+j}{\alpha\delta}$$

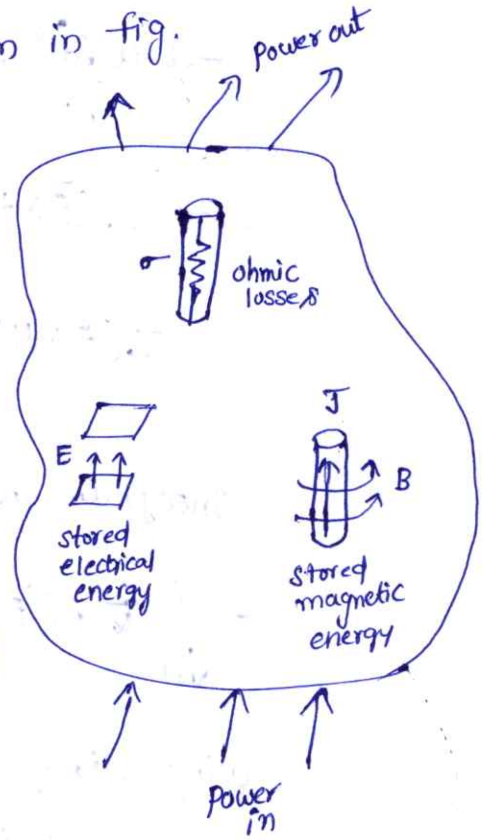
→ Poynting Theorem :- This theorem is based on law of conservation of energy in electromagnetism.

Statement: The ~~Net~~ Net power flowing out of a given volume V is equal to the time rate of decrease in the energy stored within volume V minus the ohmic power dissipated.

Total Power leaving the Volume = rate of decrease in Energy stored in Electric field & magnetic field. − ohmic power dissipated.

$$\oint_S (\bar{E} \times \bar{H})\, ds = -\frac{\partial}{\partial t}\int_V \left[\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right] dV - \int_V \sigma E^2\, dV.$$

Proof :- This theorem is illustrated as shown in fig.



Fig(1): Illustration of power balance for EM field.

From Maxwell's Equation,

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}. \quad \text{——} \; (1)$$

$$\nabla \times \bar{H} = \sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t}. \quad \text{——} \; (2)$$

Dotting both sides of Eq (2) with E gives,

$$E \cdot (\nabla \times H) = \sigma E^2 + \bar{E} \cdot \varepsilon \frac{\partial \bar{E}}{\partial t} \quad \text{——} \; (3)$$

For any vector fields A & B, we have

$$\nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A \cdot (\nabla \times B)$$

Apply this to Eq (3), (let A = E & B = H)

$$\nabla \cdot (\bar{E} \times \bar{H}) = \bar{H} \cdot (\nabla \times \bar{E}) - \bar{E} \cdot (\nabla \times \bar{H})$$

$$\bar{E} \cdot (\nabla \times \bar{H}) = \bar{H} \cdot (\nabla \times \bar{E}) - \nabla \cdot (\bar{E} \times \bar{H})$$

$$\bar{H}\cdot(\nabla\times\bar{E}) - \nabla\cdot(\bar{E}\times\bar{H}) = \sigma\bar{E}^2 + \bar{E}\cdot\varepsilon\frac{\partial\bar{E}}{\partial t}.$$

from maxwell eq, $\nabla\times\bar{E} = -\mu\frac{\partial\bar{H}}{\partial t}$

$$\bar{H}\cdot\left(-\mu\frac{\partial\bar{H}}{\partial t}\right) - \nabla\cdot(\bar{E}\times\bar{H}) = \sigma E^2 + E\cdot\varepsilon\frac{\partial E}{\partial t}.$$

$$-\mu\bar{H}\cdot\frac{\partial\bar{H}}{\partial t} - \nabla\cdot(\bar{E}\times\bar{H}) = \sigma E^2 + \varepsilon E\cdot\frac{\partial E}{\partial t}$$

Here $-\mu\bar{H}\cdot\frac{\partial H}{\partial t} = -\frac{1}{2}\mu\frac{\partial H^2}{\partial t}$, $\varepsilon\bar{E}\cdot\frac{\partial\bar{E}}{\partial t} = \frac{1}{2}\varepsilon\frac{\partial E^2}{\partial t}$

From above eq.

$$-\frac{1}{2}\mu\frac{\partial\bar{H}^2}{\partial t} - \nabla(E\times\bar{H}) = \sigma E^2 + \frac{1}{2}\varepsilon\frac{\partial E^2}{\partial t}.$$

$$-\nabla\cdot(\bar{E}\times\bar{H}) = \frac{1}{2}\varepsilon\frac{\partial E^2}{\partial t^2} + \frac{1}{2}\mu\frac{\partial H^2}{\partial t^2} + \sigma E^2$$

$$-\nabla\cdot(\bar{E}\times\bar{H}) = \frac{\partial}{\partial t}\left[\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right] + \sigma E^2$$

Integrate the above eq through out the volume,

$$-\int_{Vol}\nabla\cdot(\bar{E}\times\bar{H})\,dv = \frac{\partial}{\partial t}\int_{Vol}\left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right)dv + \int_{Vol}(\sigma E^2)\,dv$$

using divergence theorem, change the volume integral on LHS to surface integral,

$$-\oint_S(\bar{E}\times\bar{H})\,d\bar{S} = \frac{\partial}{\partial t}\int_{Vol}\left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right)dv + \int_{Vol}\sigma E^2\,dv$$

$$\Rightarrow \oint_S(\bar{E}\times\bar{H})\,d\bar{S} = -\frac{\partial}{\partial t}\int_V\left[\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right]dv - \int_V\sigma E^2\,dv.$$

Hence the above eq is known as Poynting Theorem.

→ Poynting vector ($\bar{P}$) :

By means of Electromagnetic waves, an Energy can be Transported from transmitter to receiver. The energy stored in an electric field and magnetic field is transmitted at a certain rate of energy flow which can be calculated with the help of poynting Theorem.

→ The cross product of E and $\bar{H}$, gives the powerdensity. power density is given by $\boxed{\bar{P} = \bar{E} \times \bar{H}}$

where $\bar{P}$ is called poynting vector.
$\bar{E}$ is Electric field Intensity measured in V/m
$\bar{H}$ is magnetic field Intensity measured in A/m.

→ Average power density (P_{avg}) :-

From poynting vector $\bar{P} = \bar{E} \times \bar{H}$.

let $\bar{E} = E_x a_x$, $\bar{H} = H_y a_y$.

$$\bar{P} = \bar{E} \times \bar{H} = (E_x a_x) \times (H_y a_y) = E_x H_y \bar{a}_z = P_z \bar{a}_z.$$

The above Eq indicates that $\bar{E}$, $\bar{H}$ & $\bar{P}$ are mutually perpendicular to Each other.

Consider the electric field propogates in free space given by,

$$\bar{E} = [E_m \cos(\omega t - \beta z)]\ \bar{a}_x.$$

Intrinsic impedance $\eta = \eta_0 = \dfrac{E_m}{H_m}$

$$\bar{H} = [H_m \cos(\omega t - \beta z)]\ \bar{a}_y.$$

$$\bar{H} = \left[\frac{E_m}{\eta_0} \cos(\omega t - \beta z)\right] \bar{a}_y.$$

A/c to poynthing Theorem,

$$\bar{P} = \bar{E} \times \bar{H} = \left(E_m \cos(\omega t - \beta z)\, \bar{a}_x \right] \times \left[\frac{E_m}{\eta_0} \cos(\omega t - \beta z)\bar{a}_y \right]$$

$$\bar{P} = \frac{E_m^2}{\eta_0} \cos^2(\omega t - \beta z)\, \bar{a}_z \ , \ \omega/m^2$$

This is nothing but power density measured in ~~w/m²~~ (watt/m²)

Average power density,

$$P_{avg} = \frac{1}{T}\int_0^T P \, dt$$

$$= \frac{1}{T}\int_0^T \frac{E_m^2}{\eta} \cos^2(\omega t - \beta z)\, dt.$$

$$= \frac{1}{T}\int_0^T \frac{E_m^2}{\eta}\left[\frac{1 + \cos 2(\omega t - \beta z)}{2}\right] dt$$

$$= \frac{E_m^2}{T\eta}\left[\int_0^T \left(\frac{1 + \cos 2(\omega t - \beta z)}{2}\right)\right] dt$$

$$= \frac{E_m^2}{T\eta}\left[\frac{t}{2} + \frac{\sin 2(\omega t - \beta z)}{2(2\omega)}\right]_0^T$$

$$= \frac{E_m^2}{T\eta}\left[\frac{t}{2} + \frac{\sin(2\omega t - 2\beta z)}{4\omega}\right]_0^T$$

$$= \frac{E_m^2}{T\eta}\left[\frac{T}{2} + \frac{\sin(2\omega T - 2\beta z)}{4\omega} - \frac{\sin(-2\beta z)}{4\omega}\right]$$

But $\omega T = 2\pi$.

$$P_{avg} = \frac{E_m^2}{T\eta}\left[\frac{T}{2} + \frac{\sin(4\pi - 2\beta z)}{4\omega} + \frac{\sin 2\beta z}{4\omega}\right]$$

$$= \frac{E_m^2}{T\eta}\left[\frac{T}{2} - \cancel{\frac{\sin 2\beta z}{4\omega}} + \cancel{\frac{\sin 2\beta z}{4\omega}}\right]$$

$$P_{avg} = \frac{E_m^2}{T\eta}\left[\frac{T}{2}\right]$$

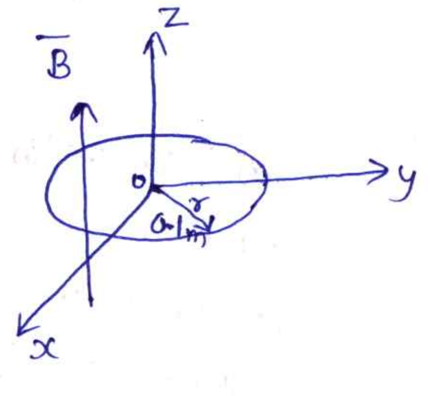
$$\therefore \boxed{P_{avg} = \frac{E_m^2}{2\eta}}$$

1) A circular loop conductor lies in plane $z=0$ and has a radius of 0.1 m and resistance of 5Ω. Given $\bar{B} = 0.2 \sin 10^3 t \, \bar{a}_z$ T, determine the current in the loop.

Sol: Given radius $r = 0.1$ m.

Resistance $= 5\Omega$

$\bar{B} = 0.2 \sin 10^3 t \, \bar{a}_z$



A circular loop is in $z=0$ plane. $\bar{B}$ is in z-direction, perpendicular to the loop.

Total flux $\phi = \int_S \bar{B} \cdot d\bar{s}$

for cylindrical coordinate system, $d\bar{s} = (r \, dr \, d\phi) \bar{a}_z$

$$\phi = \int_{\phi=0}^{2\pi} \int_{r=0}^{0.1} [0.2 \sin 10^3 t \, \bar{a}_z] \cdot [r \, dr \, d\phi \, \bar{a}_z].$$

$$\phi = \int_{\phi=0}^{2\pi} \int_{r=0}^{0.1} 0.2 \sin 10^3 t \, r \, dr \, d\phi.$$

$$= (0.2 \sin 10^3 t) \int_{\phi=0}^{2\pi} d\phi \int_{r=0}^{0.1} r \, dr$$

$$= (0.2 \sin 10^3 t) [\phi]_0^{2\pi} \left[\frac{r^2}{2}\right]_0^{0.1}$$

$$= 0.2 \sin 10^3 t \, [2\pi] \left[\frac{0.1}{2}\right]^2$$

$$\phi = 6.283 \times 10^{-3} \sin 10^3 t.$$

$$\text{Emf Induced, } e = -\frac{d\phi}{dt} = -\frac{d}{dt}[6.283 \times 10^{-3} \sin 10^3 t]$$

$$e = -6.283 \times 10^{-3} \times 10^3 \times \cos 10^3 t$$

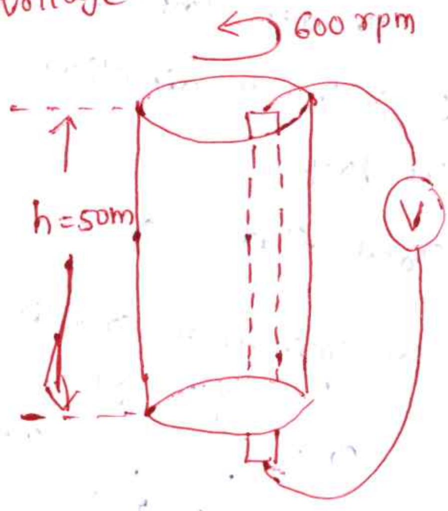
$$e = -6.283 \cos 10^3 t.$$

Current in conductor, $i = \dfrac{\text{Induced Emf}}{\text{Resistance}}$

$$i = \frac{-6.283 \cos 10^3 t}{5}$$

$$i = -1.2567 \cos 10^3 t \text{ , A}$$

2) A conducting cylinder of radius 7 cm and height 50 cm. rotates at 600 rpm in a radial field $\bar{B} = 0.10\, \bar{a}_r$ T. Sliding contacts at the top and bottom are used to connect a voltmeter. as shown in fig. Calculate Induced voltage.



Sol: A conducting cylinder rotates in the direction as shown in fig.

→ It rotates at 600 rpm, means in 1 sec there are 10 revolutions.

→ The radius of Cylinder ~~7~~ 7 cm = 0.07 m.

In 1 revolution, distance travelled by cylinder $= 2\pi r$, mtrs
$= 2\pi (0.07)$ m

for 10 revolutions, distance travelled $= (2\pi r) \times 10$
$= 2\pi \times 0.07 \times 10$ m

linear velocity $\bar{v} = (2 \times \pi \times 0.07 \times 10)\, \bar{a}_\phi$, m/s
$= 4.398\, \bar{a}_\phi$.

Electric field Intensity $\bar{E} = \bar{v} \times \bar{B}$
$= (4.398\, \bar{a}_\phi) \times (0.20\, \bar{a}_r)$

$\bar{E} = 0.8796(-\bar{a}_z)$ $\left(\because\ a_\phi \times \bar{a}_r = -\bar{a}_z\right)$

Emf Induced

$$e = \int \bar{E} \cdot d\bar{l} = \int_{z=0}^{0.5} 0.8796(\bar{a}_z) \cdot (dz\,\bar{a}_z)$$

$$= -0.8796\,[z]_0^{0.5} \qquad (\because \bar{a}_z \cdot \bar{a}_z = 1)$$

$$= -0.4398 \text{ V}$$

3) For a lossy dielectric, $\sigma = 5\,S/m$, $\varepsilon_r = 1$. The electric field intensity is $E = 100 \times \sin 10^{10} t$. Find J_c, J_D. and frequency at which both have equal magnitudes.

Sol: Conduction current density, $J_c = \sigma E$

$$J_c = 5\,[100 \sin 10^{10} t]$$

$$J_c = 500 \sin 10^{10} t. \quad A/m^2.$$

Displacement current density $J_D = \frac{\partial D}{\partial t} = \frac{\partial(\varepsilon E)}{\partial t}$

$$J_D = \frac{\partial}{\partial t}[\varepsilon_0 \varepsilon_r E]$$

$$= \varepsilon_0 \varepsilon_r \frac{\partial(E)}{\partial t}$$

$$= 8.854 \times 10^{-12} \times 1 \times \frac{\partial(100 \sin 10^{10} t)}{\partial t}.$$

$$= 8.854 \times 10^{-12} \times 100 \times \cos 10^{10} t \times 10^{10}$$

$$J_D = 8.854 \times \cos 10^{10} t. \quad A/m^2$$

For two current densities, the condition for equal magnitudes is given by

$$\frac{|\bar{J}_c|}{|J_D|} = \frac{\sigma}{\varepsilon\omega} = 1.$$

$$\omega = \frac{\sigma}{\varepsilon}$$

$$\omega = \frac{\sigma}{\varepsilon_0 \varepsilon_r} = \frac{5}{8.854 \times 10^{-12} \times 1}$$

$$\omega = 5.647 \times 10^{11}$$

But $\omega = 2\pi f$

$$f = \frac{\omega}{2\pi} = \frac{5.647 \times 10^{10}}{2\pi}$$

$$\therefore f = 89.877 \text{ GHZ}$$

4) Find the frequency at which conduction current density and displacement current density are equal in a medium with $\sigma = 2 \times 10^{-4}$ ℧/m, and $\varepsilon_r = 81$.

Sol:- Given

$$\sigma = 2 \times 10^{-4} \text{ m}$$

$$\varepsilon_r = 81.$$

The ratio of amplitudes of Two current densities is given as 1.

$$\frac{|\bar{J}_c|}{|\bar{J}_D|} = \frac{\sigma}{\omega \varepsilon} = 1$$

$$\omega = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0 \varepsilon_r}$$

$$\omega = \frac{2 \times 10^{-4}}{(8.854 \times 10^{-12})(81)} = 0.2788 \times 10^{6} \text{ rad/sec.}$$

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi} = \frac{0.2788 \times 10^{6}}{2\pi}$$

$$f = 44.372 \text{ K HZ.}$$

5) Find displacement current density within a parallel plate capacitor where $\varepsilon = 100\,\varepsilon_0$, $a = 0.01\ m^2$, $d = 0.05\ mm$ and the capacitor voltage $100 \sin 200\pi t$ volts.

Sol.

Given $\varepsilon = 100\,\varepsilon_0$,

Area $= a = 0.01\ m^2$.

distance $= d = 0.05\ mm = 0.05 \times 10^{-3}\ m$

voltage, $V = 100 \sin 200\pi t$.

current through capacitor $i_c = C \dfrac{dv}{dt}$

$$i_c = \left(\frac{\varepsilon A}{d}\right) \frac{dv}{dt}.$$

$$i_c = \frac{(100\,\varepsilon_0)(0.01)}{(0.05 \times 10^{-3})} \frac{d}{dt}[100 \sin 200\pi t]$$

$$i_c = \frac{100 \times 8.854 \times 10^{-12} \times 0.01}{0.05 \times 10^{-3}} \times 100 \times \cos 200\pi t \times 200\pi$$

$$i_c = 11.1262 \times 10^{-3} \cos 200\pi t \ , A$$

But for parallel plate capacitor $i_c = i_D$ = displacement current.

$\therefore$ Displacement current density $\bar{J}_D = \dfrac{i_D}{A} = \dfrac{i_c}{A}$

$$\bar{J}_D = \frac{11.1262 \times 10^{-3} \cos 200\pi t}{0.01}$$

$$\bar{J}_D = 1.1126 \cos 200\pi t \ , A/m^2$$

6) A conductor of length 100 cm moves at right angles to uniform field of strength 10000 lines per cm^2, with a velocity of 50 m/s. calculate Emf induced it when the conductor moves at a angle 30° to direction of field.

Sol: Given length l = 100 cm = 100×10^{-2} m = 1 m

magnetic field strength B = 1000 lines / cm^2

$= \frac{10000}{(10^{-2})^2 m} = 10^8$ lines

$= 1\ wb/m^2$

Note that $1\ Wb/m^2 = 10^8$ lines of force.

(i) The induced Emf in the conductor, $e = B.lv \sin\theta$

~~e~~ but here conductor moves at right angles to the field.

$\theta = 90°$.

Induced Emf $\Rightarrow e = (1) \times (1) \times (50) \sin 90° = 50\ V$

(ii) when conductor moves at angle 30°, to the direction of field,

$e = Blv \sin\theta$

$= (1)(1)(50) \sin 30°$

$e = 25\ V$