UNIT - III

FIR Digital Filters

* Linear phase FIR filter
* characteristic response
* Location of zeros
* Design of FIR filter using windowing Technique
    * Rectangular
    * Hanning
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    * Blackman
* Design of FIR filter by Frequency Sampling technique.
* problems
* Realization of FIR Systems
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    * Direct form
    * Cascade form
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* Comparison of FIR & IIR filters

Introduction :

A filter is a frequency selective system. Depending on the form of the unit impulse response of the system digital filters are classified

In FIR system, the impulse response is of finite duration i.e. it has a finite no. of non zero terms. FIR filters are usually implemented using non-recursive structures. The response of FIR filter depends only on the present & past input samples.

In many digital processing applications FIR filters are preferred over IIR filters

Advantages of FIR filters over IIR filters

→ FIR filters are always stable

→ FIR filters with exactly linear phase can easily be designed

→ FIR filters can be realised in both recursive & non-recursive structures

→ FIR filters are free of limit cycle oscillations, when implemented on a finite word length digital system

→ Excellent design methods are available for various kinds of FIR filters

Disadvantages of FIR filters

→ The implementation of narrow transition band, FIR filter is very costly, as it requires considerably more arithmetic operations and hardware components such as multipliers adders and delay elements.

→ Memory requirement & execution time are very high.

* Linear phase FIR filter

The transfer function of a FIR Causal filter is given by

$$H(z) = \sum_{n=0}^{N-1} h[n] z^{-n} \quad \text{--- (1)}$$

where $h[n]$ is the impulse response of the filter

The F.T of $h[n]$ is

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h[n] e^{-j\omega n} \quad \text{--- (2)}$$

which is periodic with period 2π

i.e $H(e^{j\omega}) = H(e^{j(\omega+2\pi k)})$ $k=0,1,2 --$

$\therefore$ $H(e^{j\omega})$ is Complex it can be expressed as

$$H(e^{j\omega}) = \pm|H(e^{j\omega})| e^{j\theta(\omega)} \quad \text{--- (3)}$$

where $|H(e^{j\omega})| \rightarrow$ magnitude response

$\theta(\omega) \rightarrow$ phase response.

The phase delay τ_p & group delay τ_g of a filter is

$$\tau_p = -\frac{\theta(\omega)}{\omega} \quad \text{--- (4)}$$

$$\tau_g = -\frac{d}{d\omega}\theta(\omega) \quad \text{--- (5)}$$

For FIR filters with linear phase we can define

$$\theta(\omega) = -\tau_p \omega$$

$$\boxed{\theta(\omega) = -\alpha\omega} \quad ; -\pi \le \omega \le \pi \quad \text{--- (6)}$$

Substitute in τ_p & τ_g

where α is a Constant phase delay in samples,

$\therefore$ we've $\tau_p = \tau_g = \alpha$, which means that α is independent of frequency,

$$\therefore \sum_{n=0}^{N-1} h[n]\, e^{-j\omega n} = \pm |H(e^{j\omega})|\, e^{j\theta(\omega)} \quad \text{--- (7)}$$

$$\sum_{n=0}^{N-1} h[n] \cos\omega n = \pm |H(e^{j\omega})| \cos\theta(\omega) \quad \text{--- (8)}$$

$$\& \quad -\sum_{n=0}^{N-1} h[n] \sin\omega n = \pm |H(e^{j\omega})| \sin\theta(\omega) \quad \text{--- (9)}$$

$(\because \theta(\omega) = -\alpha\omega)$

$$\frac{(9)}{(8)} = \frac{\sum_{n=0}^{N-1} h[n] \sin\omega n}{\sum_{n=0}^{N-1} h[n] \cos\omega n} = \frac{\sin\alpha\omega}{\cos\alpha\omega}$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n) \sin[(\alpha - n)\omega] = 0 \quad \text{--- (10)}$$

The above eq^n (10) is zero when

$$\boxed{h[n] = h(N-1-n)} \quad , n = 0, 1, \dots N-1$$

&

$$\boxed{\alpha = \frac{N-1}{2}}$$

∴ FIR filters will have constant phase & group delays when the impulse response is symmetrical about $\alpha = \frac{N-1}{2}$

Thus the phase response of the filter will be linear. The IR for even & odd values of N

Whenever constant group delay alone is preferred the impulse response will be the form $h[n] = -h(N-1-n)$ and is antisymmetric about the centre of the impulse response sequence.

where $\theta(\omega) = \beta - \alpha\omega$

$$\therefore \quad H(e^{j\omega}) = \pm|H(e^{j\omega})| \, e^{j(\beta-\alpha\omega)}$$

$$\sum_{n=0}^{N-1} h[n] \, e^{-j\omega n} = \pm|H(e^{j\omega})| \, e^{j(\beta-\alpha\omega)}$$

i.e $$\sum_{n=0}^{N-1} h[n] \cos\omega n = \pm|H(e^{j\omega})| \cos(\beta-\alpha\omega)$$

& $$-\sum_{n=0}^{N-1} h[n] \sin\omega n = \pm|H(e^{j\omega})| \sin(\beta-\alpha\omega)$$ (10)

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$$\frac{(2)}{(1)} = \frac{-\sum_{n=0}^{N-1} h[n] \sin \omega n}{\sum_{n=0}^{N-1} h[n] \cos \omega n} = \frac{\sin(\beta - \alpha\omega)}{\cos(\beta - \alpha\omega)}$$

$$\Rightarrow \sum_{n=0}^{N-1} h[n] \sin[\beta - (\alpha - n)\omega] = 0$$

when $\beta = \pi/2$

$$\sum_{n=0}^{N-1} h(n) \cos(\alpha - n)\omega = 0$$

if

$$\Rightarrow h[n] = -h[N-1-n]$$

&

$$\alpha = \frac{N-1}{2}$$

$\therefore$ FIR filters have constant group delay τ_g and not constant phase delay when the impulse response is antisymmetrical about $\alpha = \frac{N-1}{2}$

$h\left(\frac{N-1}{2}\right) = 0$ for antisymmetric odd seque

* The length of an FIR filter is 7. If this filter has a linear phase, show that the eq^n

$$\sum_{n=0}^{N-1} h(n) \sin(\alpha - n)\omega = 0$$ is satisfied

Sol: Given the length of the filter 7

$\therefore$ For linear phase $\alpha = \frac{N-1}{2} = \frac{7-1}{2} = 3$

$\therefore$ The Condition for symmetry when N is odd

$$h(n) = h(N-1-n)$$

$\therefore$ filter coefficients are

$h(0) = h(6)$

$h(1) = h(5)$

$h(2) = h(4)$ &

$h(3)$

$$\therefore \sum_{n=0}^{N-1} h(n) \sin(\alpha - n)\omega = \sum_{n=0}^{6} h(n) \sin(3-n)\omega$$

$$= h(0)\sin 3\omega + h(1)\sin 2\omega + h(2)\sin\omega + h(3)\sin 0 + h(4)\sin(-\omega) + h(5)\sin(-2\omega) + h(6)\sin(-3\omega)$$

$$= 0$$

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* The following T.F characterizes an FIR filter (N=9). Determine the magnitude response and S.T the phase & group delays are consta

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

Sol: The T.F of the filter is given by

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-1}$$

$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4} + h(5) z^{-5}$$
$$+ h(6) z^{-6} + h(7) z^{-7} + h(8) z^{-8}$$

The phase delay $\alpha = \frac{N-1}{2} = 4$

$\therefore \alpha = 4$

$$\therefore H(z) = z^{-4} [h(0) z^{4} + h(1) z^{3} + h(2) z^{2} + h(3) z^{1}$$
$$+ h(4) z^{0} + h(5) z^{-1} + h(6) z^{-2} + h(7) z^{-3} + h(8) z^{-4}]$$

$\because h(n) = h(N-1-n)$

$$H(z) = z^{-4} [h(0)(z^{4} + z^{-4}) + h(1)(z^{3} + z^{-3})$$
$$+ h(2)(z^{2} + z^{-2}) + h(3)(z + z^{-1}) + h(4)]$$

$\therefore$ The frequency response

$$H(z)/_{z=e^{j\omega}} = e^{-j4\omega} [h(0)(e^{j4\omega} + e^{-j4\omega})$$
$$+ h(1)(e^{j3\omega} + e^{-j3\omega})$$
$$+ h(2)(e^{j2\omega} + e^{-j2\omega})$$
$$+ h(3)(e^{j\omega} + e^{-j\omega}) + h(4)]$$

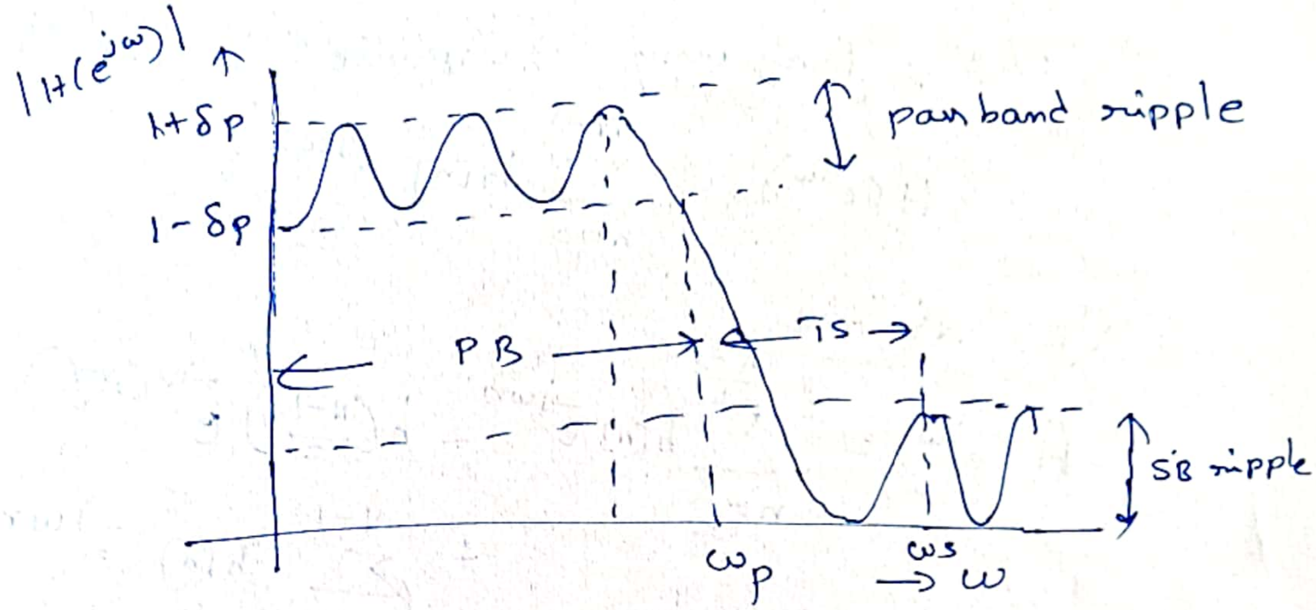
$$= e^{-j4\omega} [h(4) + 2 \sum_{n=0}^{3} h(n) \cos(4-n)\omega]$$

$$= e^{-j4\omega} |H(e^{j\omega})|$$

phase response

$$\theta(\omega) = -4\omega \Rightarrow \tau_p = \frac{-\theta(\omega)}{\omega} = 4$$
$$\tau_g = -\frac{d}{d\omega}\theta(\omega) = 4$$

Magnitude response of practical LPF



where $\delta_p \rightarrow$ PB ripple

$\delta_s \rightarrow$ SB ripple

$\omega_p \Rightarrow$ PB edge freq

$\omega_s \Rightarrow$ SB "

* Frequency response of Linear phase FIR filter

The frequency response is given by

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h[n]\, e^{-j\omega n}$$

If N is odd

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-3}{2}} h[n]\, e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=\frac{N+1}{2}}^{N-1} h[n]\, e^{-j\omega n}$$

Using symmetry property $h[n] = h(N-1-n)$

Consider

$$\sum_{n=\frac{N+1}{2}}^{N-1} h[n]\, e^{-j\omega n} = \sum_{n=\frac{N+1}{2}}^{N-1} h(N-1-n)\, e^{-j\omega n}$$

let $N-1-n = K$

$$= \sum_{K=0}^{\frac{N-3}{2}} h(K)\, e^{-j\omega(N-1-K)}$$

$$= \sum_{n=0}^{\frac{N-3}{2}} h[n]\, e^{-j\omega(N-1-n)}$$

$$\therefore H(e^{j\omega}) = h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h[n]\left[e^{-j\omega n} + e^{-j\omega(N-1-n)}\right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} h[n]\left\{\left[e^{-j\omega\left(n-\left(\frac{N-1}{2}\right)\right)}\right] + e^{j\omega\left(n-\left(\frac{N-1}{2}\right)\right)}\right\}\right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}} h[n] \cos\omega\left(\frac{N-1}{2}-n\right)\right]$$

If N is even

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N}{2}-1} h[n]\, e^{-j\omega n} + \sum_{n=\frac{N}{2}}^{N-1} h[n]\, e^{-j\omega n}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n)\, e^{-j\omega n} + \sum_{n=\frac{N}{2}}^{N-1} h[N-1-n]\, e^{-j\omega n}$$

Let $K = N-1-n$

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega n} + \sum_{K=0}^{\frac{N}{2}-1} h(K)\, e^{-j\omega(N-1-K)}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega n} + \sum_{n=0}^{\frac{N}{2}-1} h(n)\, e^{j\omega(n-N+1)}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n) \left[e^{-j\omega n} + e^{j\omega(n-N+1)} \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n) \left(e^{-j\omega\left(n-\left(\frac{N-1}{2}\right)\right)} + e^{j\omega\left(n-\left(\frac{N-1}{2}\right)\right)} \right)$$

$$= 2 e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n) \cos\omega\left(n-\left(\frac{N-1}{2}\right)\right)$$

$$= 2 e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n) \cos\omega\left[\frac{N-1}{2}-n\right]$$

→ Frequency response of Linear phase FIR filter when impulse response is antisymmetric & N is even i.e

$$H(e^{j\omega}) = e^{j\left[\frac{\pi}{2} - \omega\left(\frac{N-1}{2}\right)\right]} \sum_{n=1}^{N/2} 2h\left(\frac{N}{2}-n\right) \sin\left(\omega\left(n-\frac{1}{2}\right)\right)$$

For N is odd

$$H(e^{j\omega}) = e^{j\left[\frac{\pi}{2} - \omega\left(\frac{N-1}{2}\right)\right]} \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2}-n\right) \sin\omega n$$

* Determine frequency response of FIR filter defined by $y[n] = 0.25\,x[n] + x[n-1] + 0.25\,x[n-2]$
Calculate phase delay & Group delay

Sol: Given $y[n] = 0.25\,x[n] + x[n-1] + 0.25\,x[n-2]$

taking DTFT

$$Y(e^{j\omega}) = 0.25\,X(e^{j\omega}) + e^{-j\omega} X(e^{j\omega}) + 0.25\,e^{-j2\omega} X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = 0.25 + e^{-j\omega} + 0.25\,e^{-j2\omega}$$

$$= e^{-j\omega}\left[1 + 0.25\,e^{j\omega} + 0.25\,e^{-j\omega}\right]$$

$$H(e^{j\omega}) = e^{-j\omega}\left[1 + 0.5\cos\omega\right]$$

$$\theta(\omega) = -\omega$$

$$\therefore \quad \tau_p = -\frac{\theta(\omega)}{\omega} = 1$$

$$\tau_g = -\frac{d}{d\omega}\theta(\omega) = \frac{-d}{d\omega}(-\omega) = 1$$

* Location of the zeros of Linear phase FIR filter

The transfer function of a linear phase FIR filter is given by

$$H(z) = \sum_{n=0}^{N-1} h[n] z^{-n} \quad \text{—} (1)$$

If z_0 is a zero of $H(z)$, then

$$H(z)/_{z=z_0} = H(z_0) = \sum_{n=0}^{N-1} h[n] z_0^{-n} = 0$$

$$H(z_0) = h(0) + h(1) z_0^{-1} + \text{--} + h(N-1) z_0^{-(N-1)} = 0 \quad \text{—} (2)$$

For a linear phase filter

$h(n) = h(N-1-n)$, then we've

$$H(z_0) = h(N-1) + h(N-2) z_0^{-1} + \text{—} + h(1) z_0^{-(N-2)} + h(0) z_0^{-(N-1)} = 0$$

$$= z_0^{-(N-1)} \left[h(N-1) z_0^{N-1} + h(N-2) z_0^{N-2} + \text{--} + h(1) z_0^{1} + h(0) \right] = 0$$

$$= z_0^{-(N-1)} \sum_{n=0}^{N-1} h(n) z_0^{n} = 0$$

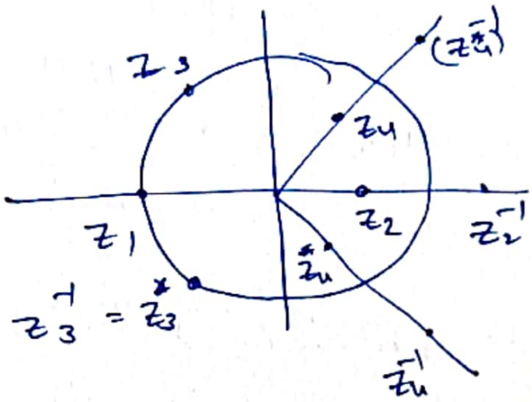
$$= z_0^{-(N-1)} \sum_{n=0}^{N-1} h(n) \left(z_0^{-1}\right)^{-n} = 0$$

$$H(z_0) = z_0^{-(N-1)} H(z_0^{-1}) = 0$$

$$\therefore H(z_0) = H(z_0^{-1}) = 0 \quad \text{—} (1)$$

From the above eq^n we find that if z_0 is a zero of $H(z)$ $\Rightarrow$ z_0^{-1} is also a zero

So we can find the location of zeros in a linear phase FIR filter



→ If $z_1 = -1$ then $z_1^{-1} = z_1$, the zero lies at $z_1 = -1$, this group contains only one zero on the unit circle

→ If z_2 is real zero with $|z_2| < 1$ then z_2^{-1} is also real zero and there are two zeros in this group

→ If z_3 is complex zero with $|z_3| = 1$ then $z_3^{-1} = z_3^*$ and there are two zeros in this group

→ If z_4 is a complex zero with $|z_4| \neq 1$ This group contains four zeros $z_4, z_4^{-1}, z_4^*, (z_4^*)^{-1}$.

Procedure

* Design of FIR filter using windowing Techniques

→ choose the desired frequency response of the filter $H_d(e^{j\omega})$.

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d[n]\, e^{-j\omega n}$$

→ Take inverse FT to obtain desired impulse response $h_d[n]$

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

→ Choose a window sequence $w(n)$

$$w(n) = \begin{cases} w(-n) \neq 0 \;; & |n| \le \left(\frac{N-1}{2}\right) \\ = 0 \;; & |n| > \left(\frac{N-1}{2}\right) \end{cases}$$

and multiply $h_d[n]$ by $w(n)$ to convert the infinite duration impulse response to a finite duration impulse response $h[n]$

$$h[n] = h_d[n]\, w(n)$$

$\left(H(e^{j\omega}) = H(e^{j\omega}) * W(e^{j\omega})\right)$

$\left(H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\theta})\, W(e^{j(\omega-\theta)})\, d\theta\right)$

→ The T.F of the filter by taking Z.T of $h[n]$

$$\text{i.e } H(z) = \sum_{n=-\infty}^{\infty} h[n]\, z^{-n}$$

The frequency response of any digital filter is periodic in frequency

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d[n] e^{-j\omega n} \quad — (1)$$

To implement the above eq^n (1) for designing a digital filter are

→ Impulse response is of infinite duration

→ Filter is non causal & unrealizable

The infinite impulse response can be converted to a finite duration impulse response by truncating the infinite into finite.

This results in undesirable oscillation in the passband & stopband of the digital filter

These undesirable oscillations can be reduced by using a set of time limited weighting function $w(n)$ refered to as window function

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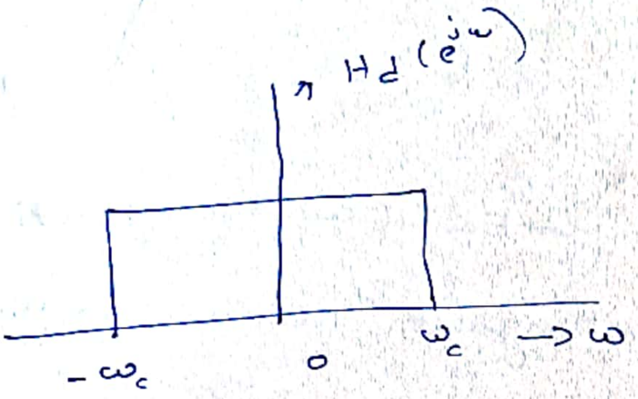
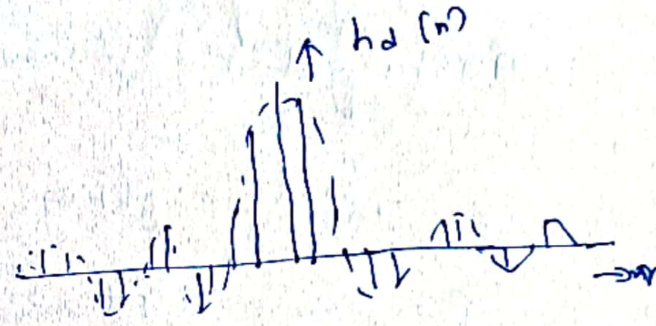
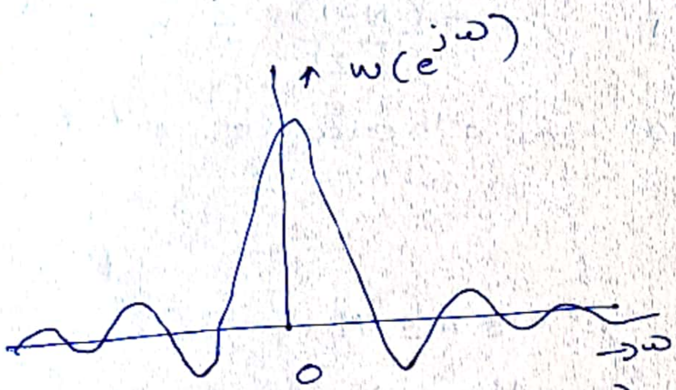
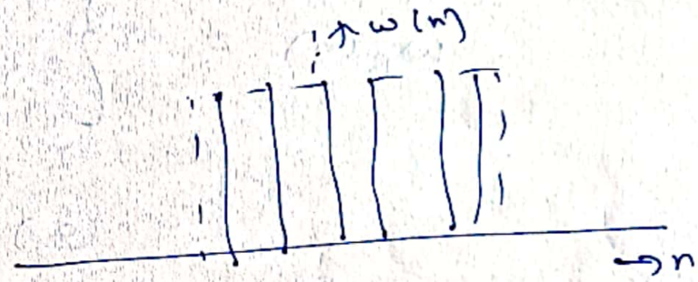
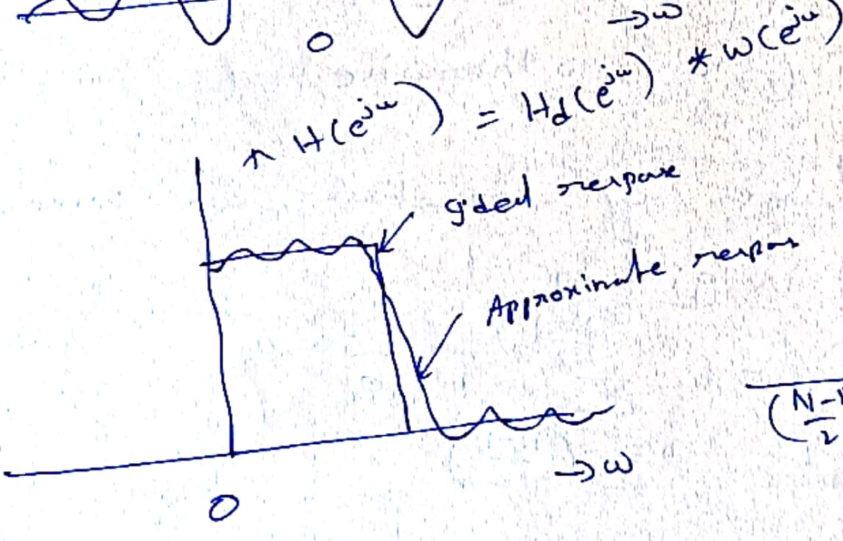
The windowing technique is shown in below

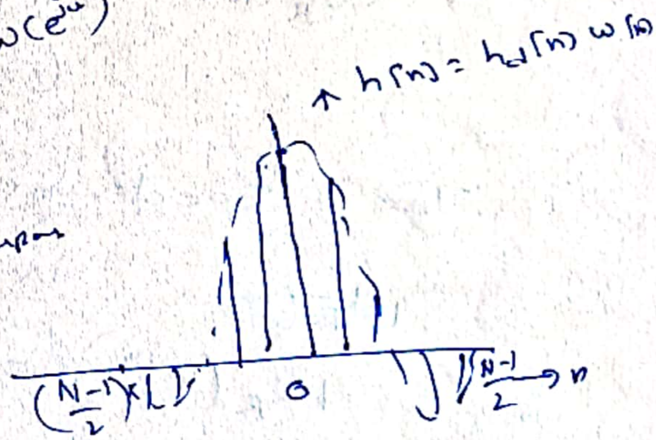


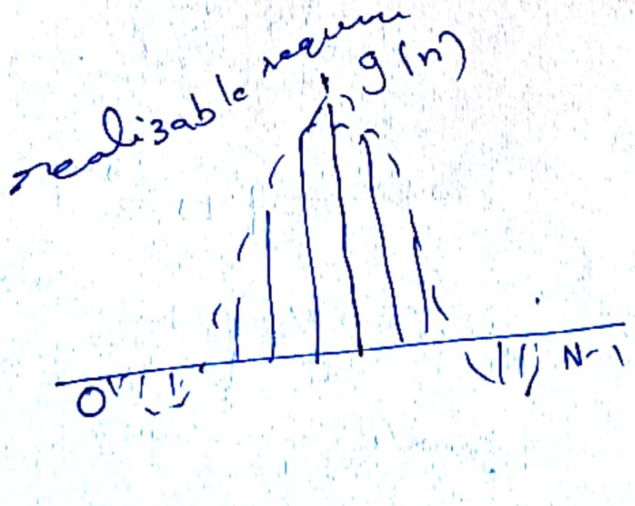












Type of windows

1. Rectangular window:

The weighting function (window function) for an N pt rectangular window is given by

$$w_R(n) = \begin{cases} 1 ; & -\frac{(N-1)}{2} \le n \le \frac{(N-1)}{2} \\ 0 ; & \text{otherwise} \end{cases}$$

(or)

$$\omega_R(n) = \begin{cases} 1 ; & 0 \le n \le N-1 \\ 0 ; & \text{otherwise} \end{cases}$$

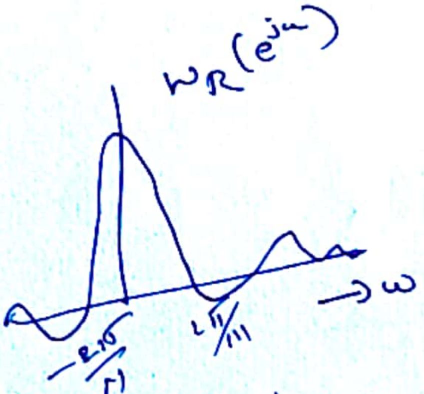
The Spectrum of rectangular window is given by

$$W_R(e^{j\omega}) = \sum_{n=-\left(\frac{N-1}{2}\right)}^{\left(\frac{N-1}{2}\right)} 1 \cdot e^{-j\omega n}$$

$$= e^{j\omega\left(\frac{N-1}{2}\right)} + \cdots + e^{j\omega} + 1 + e^{-j\omega} + \cdots + e^{-j\omega\left(\frac{N-1}{2}\right)}$$

$$= e^{j\omega\left(\frac{N-1}{2}\right)}\left[1 + e^{-j\omega} + \cdots + e^{-j\omega(N-1)}\right]$$

$$= e^{j\omega(N-1)/2}\left[\frac{1 - e^{-j\omega N}}{1 - e^{-j\omega}}\right] \quad \left(\because 1 + a + a^2 + \cdots a^{N-1} \ (\text{or}) \ \sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}\right)$$



$$= \frac{e^{j\omega N/2}\left(1 - e^{-j\omega N}\right)}{e^{j\omega/2}\left(1 - e^{-j\omega}\right)} = \frac{e^{j\omega N/2} - e^{-j\omega N/2}}{e^{j\omega/2} - e^{-j\omega/2}}$$

$$W_R(e^{j\omega}) = \sin\frac{\omega N}{2} \Big/ \sin\frac{\omega}{2}$$

The width of mainlobe is $\frac{4\pi}{N}$

The first side lobe will be 13dB down the peak of main lobe

2. Triangular or Bartlett window: The window function is defined as

$$w_T(n) = \begin{cases} 1 - \dfrac{2|n|}{N-1} & ; -\dfrac{(N-1)}{2} \le n \le \left(\dfrac{N-1}{2}\right) \\ 0 & ; \text{otherwise} \end{cases}$$

(or)

$$w_T(n) = \begin{cases} 1 - \dfrac{2|n - (N-1)/2|}{N-1} & ; 0 \le n \le N-1 \\ 0 & ; \text{otherwise.} \end{cases}$$

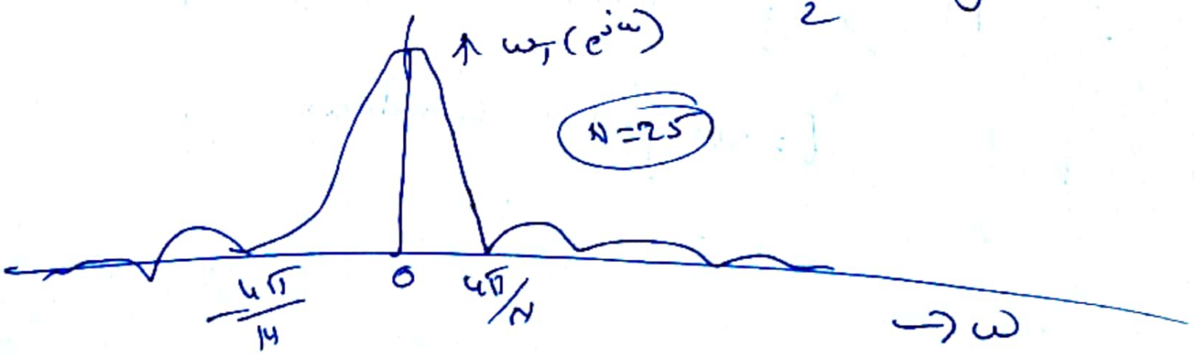
Main lobe width is $\underline{\underline{\dfrac{8\pi}{N}}}$; first side lobe at $\underline{\underline{-25dB}}$

Triangular window produces a smooth magnitude response in at both PB & SB
(disadv → compared to Rect window)
But in this window the transition region is more
the attenuation in SB is less

So Triangular window is not usually good choice

The F.T of the triangular window is

$$W_T(e^{j\omega}) = \left[\frac{\sin\left(\frac{N-1}{4}\right)\omega}{\sin\frac{\omega}{2}}\right]^2$$



* Hanning window

The Hanning window sequence is given by

$$\omega_{Hn}(n) = \begin{cases} 0.5 + 0.5\cos\left(\frac{2\pi n}{N-1}\right); & -\frac{(N-1)}{2} \le n \le \frac{(N-1)}{2} \\ 0 & ;\text{ otherwise} \end{cases}$$

The frequency response of Hanning window is

$$W_{Hn}(e^{j\omega}) = 0.5\,\frac{\sin\frac{\omega N}{2}}{\sin\omega/2} + 0.25\,\frac{\sin\left(\frac{\omega N}{2} - \pi N/(N-1)\right)}{\sin\left(\omega/2 - \pi/(N-1)\right)}$$
$$+ 0.25\,\frac{\sin\left(\omega N/2 + \pi N/(N-1)\right)}{\sin\left(\omega/2 + \pi/(N-1)\right)}$$

Width of main lobe is $\frac{8\pi}{N}$

The peak of the first side lobe is $-32\,dB$ relative to the max value

~~But hamming window generates less oscillation in the side~~
~~than hanning window. So the~~
~~Hamming window is generally preferred~~

∴ Hanning window is preferable to triangular window

* Hamming window

window sequence

$$w_H(n) = \begin{cases} 0.54 + 0.46 \cos\left(\frac{2\pi n}{N-1}\right) & ; -\frac{(N-1)}{2} \le n \le \frac{(N-1)}{2} \\ 0 & ; \text{otherwise.} \end{cases}$$

The frequency response of Hamming window is

$$W_H(e^{j\omega}) = 0.54 \frac{\sin \frac{\omega N}{2}}{\sin \omega/2} + 0.23 \frac{\sin\left(\omega N/2 - \pi N/(N-1)\right)}{\sin\left(\omega/2 - \pi/(N-1)\right)} + 0.23 \frac{\sin\left(\omega N/2 + \pi N/(N-1)\right)}{\sin\left(\omega/2 + \pi/(N-1)\right)}$$

width of main lobe is $\frac{8\pi}{N}$

The peak of the first side lobe is at 41 dB from the main lobe peak

Hamming window generates lesser oscillation in the side lobes than the Hanning window for the same main lobe width. So the Hamming window generally preferred

* Kaiser window

$$w(n) = \frac{I_0\left[\alpha \sqrt{\left(\frac{N-1}{2}\right)^2 - \left(n - \frac{N-1}{2}\right)^2}\right]}{I_0\left[\alpha\left(\frac{N-1}{2}\right)\right]} ; 0 \le n \le N-$$

* Blackman window

The Blackman window sequence is given by

$$w_B(n) = \begin{cases} 0.42 + 0.5 \cos \frac{2\pi n}{N-1} + 0.08 \cos \frac{4\pi n}{N-1} & ; -\frac{(N-1)}{2} \le n \le \frac{(N-1)}{2} \\ 0 & ; \text{otherwise} \end{cases}$$

(or)

$$w_B(n) = \begin{cases} 0.42 - 0.5 \cos \left(\frac{2\pi n}{N-1}\right) + 0.08 \frac{4\pi n}{N-1} & ; 0 \le n \le N-1 \\ 0 & ; \text{otherwise} \end{cases}$$

The width of the main lobe is $\frac{12\pi}{N}$

The peak of the first side lobe is at -58dB.

Type of window	Approximate transition width of main lobe	Minimum SB attenuation (dB)	Peak of first side lobe (dB)
Rect	$\frac{4\pi}{N}$	-21	-13
Bartlett	$\frac{8\pi}{N}$	-25	-25
Hanning	$\frac{8\pi}{N}$	-44	-31
Hamming	$\frac{8\pi}{N}$	-51	-41
Blackman	$\frac{12\pi}{N}$	-78	-58

* Design an Ideal LPF with $N = 11$ with freq respon

$$H_d(e^{j\omega}) = \begin{cases} 1 & ; -\pi/2 \leq \omega \leq \frac{\pi}{2} \\ 0 & ; \pi/2 \leq |\omega| \leq \pi \end{cases}$$

Sol: using window $\omega(n) = 1 \; ; -5 \leq n \leq 5$

Filter coefficients are

$$h_d[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} 1 \cdot e^{j\omega n} \, d\omega = \frac{1}{n\pi} \sin\frac{n\pi}{2} \quad ; n \neq 0$$

$\therefore \; h_d(n) = h_d(N-1-n)$

$h_d(0) = 1/2$

$h_d(1) = \frac{1}{\pi} = h_d(-1)$

$h_d(2) = h_d(-2) = 0$

$h_d(3) = \frac{-1}{3\pi} = h_d(-3)$

$h_d(4) = h_d(-4) = 0$

$h_d(5) = h_d(-5) = \frac{1}{5\pi}$

The desired filter coefficients are

$h[n] = h_d(n)\, \omega(n) = h_d(n) \; ; -5 \leq n \leq 5$

$\therefore$

$h(0) = 1/2$

$h(1) = h(-1) = \frac{1}{\pi}$

$h(2) = h(-2) = 0$

$h(3) = h(-3) = \frac{-1}{3\pi}$

$h(4) = h(-4) = 0$

$h(5) = h(-5) = \frac{1}{5\pi}$

The above coefficients correspond to noncausal filter which is not realizable

The realizable digital filter T.F $H(z)$ is given by

$$H(z) = z^{-\left(\frac{N-1}{2}\right)}\left[h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{-n} + z^{n}\right]\right]$$

$$= z^{-5}\left[h(0) + \sum_{n=1}^{5} h(n)\left[z^{-n} + z^{n}\right]\right]$$

$$= z^{-5}\left[h(0) + h(1)\left[z^{-1} + z^{1}\right] + h(3)\left[z^{-3} + z^{3}\right] + h(5)\left(z^{5} + z^{-5}\right)\right]$$

$$= h(5) + h(3)\,z^{-2} + h(1)\,z^{-4} + h(1)\,z^{-6} + h(3)\,z^{-8} + h(5)\,z^{-10}$$

$$H(z) = \frac{1}{5\pi} - \frac{1}{3\pi}z^{-2} + \frac{1}{\pi}z^{-4} + \frac{1}{2}z^{-5} + \frac{1}{\pi}z^{-6} - \frac{1}{3\pi}z^{-8} + \frac{1}{5\pi}z^{-10}$$

$\therefore$ Coefficients of the realizable digital filter are

$h(0) = \frac{1}{5\pi} = h(10)$

$h(1) = 0 = h(9)$

$h(2) = -\frac{1}{3\pi} = h(8)$

$h(3) = 0 \quad h(7)$

$h(4) = \frac{1}{\pi} = h(6)$

$h(5) = \frac{1}{2}$

* A LPF is to be designed with the following desired frequency response

$$H_d(e^{j\omega}) = \begin{cases} e^{-j2\omega} ; & -\pi/4 \le \omega \le \frac{\pi}{4} \\ 0 ; & \pi/4 \le |\omega| \le \pi \end{cases}$$

Determine the filter coefficients $h[n]$ if the window function is defined as

$$\omega(n) = \begin{cases} 1 ; & 0 \le n \le 4 \\ 0 ; & \text{otherwise.} \end{cases}$$

Also determine the frequency response $H(e^{j\omega})$ of the designed filter

Sol: For the given filter

$$H_d(e^{j\omega}) = \begin{cases} e^{-j2\omega} ; & -\frac{\pi}{4} \le \omega \le \pi/4 \\ 0 ; & \pi/4 \le \omega \le \pi \end{cases}$$

The filter coefficients are given by

$$\therefore h_d[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{-j2\omega} e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{j\omega(n-2)} d\omega$$

$$h_d(n) = \frac{1}{\pi(n-2)} \sin(n-2)\frac{\pi}{4} ; n \neq 2$$

$$\therefore h_d[n] = h_d(N-1-n)$$

$$h_d(0) = \frac{1}{2\pi}$$

$$h_d(1) = \frac{1}{\sqrt{2}\,\pi} = h_d(3)$$

$$h_d(2) = \frac{1}{4}$$

$$h_d(4) = \frac{1}{2\pi}$$

The filter Coefficients of the filter using rectangular window

$$h[n] = h_d[n]\,\omega(n) = h_d(n) \quad ; 0 \le n \le 4$$

$h(n) = h(N-1-n)$

$$\therefore \quad h(0) = \frac{1}{2\pi} = h(4)$$

$$h(1) = h(3) = \frac{1}{\sqrt{2}\,\pi}$$

$$h(2) = \frac{1}{4}$$

The realizable digital filter is

$$H(z) = \sum_{n=0}^{N-1} h[n]\, z^{-n}$$

$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4}$$

$$= \frac{1}{2\pi} + \frac{1}{\sqrt{2}\,\pi} z^{-1} + 0.25\, z^{-2} + \frac{1}{\sqrt{2}\,\pi} z^{-3} + \frac{1}{2\pi} z^{-4}$$

$$= z^{-2}\left[0.25 + \frac{1}{\sqrt{2}\,\pi}\left(z + z^{-1}\right) + \frac{1}{2\pi}\left(z^{2} + z^{-2}\right)\right]$$

The freq respons

$$\therefore \quad H(e^{j\omega}) = \sum_{n=0}^{4} h[n]\, e^{-j\omega n}$$

$$= h[0] + h[1] e^{-j\omega} + h(2) e^{-j2\omega} + h(3) e^{-j3\omega} + h(4) e^{-j4\omega}$$

$$= e^{-j2\omega}\left[h(0) e^{j2\omega} + h(1) e^{j\omega} + h(2) + h(3) e^{-j\omega} + h(4) e^{-j2\omega}\right]$$

$$= e^{-j2\omega}\left[h(2) + h(1)\left(e^{j\omega} + e^{-j\omega}\right) + h(0)\left(e^{j2\omega} + e^{-j2\omega}\right)\right]$$

$$= e^{-j2\omega}\left[\tfrac{1}{4} + \frac{1}{\pi}\cos 2\omega + \frac{\sqrt{2}}{\pi}\cos\omega\right]$$

* A filter is to be designed with the following desired frequency response.

$$H_d(e^{j\omega}) = \begin{cases} 0 & ; -\frac{\pi}{2} \le \omega \le \frac{\pi}{2} \\ e^{-j2\omega} & ; \pi/2 \le |\omega| \le \pi \end{cases}$$

determine the filter coefficient $h(n)$ if the window function is defined as $\omega(n) = \begin{cases} 1 ; 0 \le n \le 4 \\ 0, \text{otherwise} \end{cases}$

Also determine the frequency response $H(e^{j\omega})$ of the ~~dg~~ designed filter

Sol.

$$h_d[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{\pi/2} e^{-j2\omega} e^{j\omega n} d\omega + \int_{\pi/2}^{\pi} e^{-j2\omega} e^{j\omega n} d\omega \right]$$

$$h_d(n) = \frac{1}{\pi(n-2)} \left[\sin(n-2)\pi - \sin(n-2)\frac{\pi}{2} \right] ; n \ne 2$$

$N = 5$ $\quad h_d[n] = h_d(N-1-n)$

$h_d(0) = 0$

$h_d(1) = \frac{-1}{\pi}$

$h_d(2) = \frac{1}{2}$

$h_d(3) = \frac{-1}{\pi}$

$h_d(4) = 0$

$\because \omega(n) = 1 \quad ; 0 \le n \le 4$

$\therefore h(n) = h_d(n)\,\omega(n) = h_d(n) \;; 0 \le n \le 4$

$h(0) = h(4) = 0$

$h(1) = h(3) = -\frac{1}{\pi}$

$h(2) = \frac{1}{2}$

$\therefore$ The T-F of the filter

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= z^{-2}\left[\frac{1}{2} - \frac{1}{\pi}(z + z^{-1})\right]$$

$\therefore$ The frequency response

$$H(e^{j\omega}) = e^{-j2\omega}\left[\frac{1}{2} - \frac{1}{\pi}(e^{j\omega} + e^{-j\omega})\right]$$

$$= e^{-j2\omega}\left[\frac{1}{2} - \frac{2}{\pi}\cos\omega\right]$$

(or)

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}} h(n)\cos\omega\left(\frac{N-1}{2}-n\right)\right]$$

$$= e^{-j2\omega}\left[\frac{1}{2} - \frac{2}{\pi}\cos\omega\right]$$

* Design a filter with

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & ; -\frac{\pi}{4} \le \omega \le \frac{\pi}{4} \\ 0 & ; \frac{\pi}{4} \le |\omega| \le \pi \end{cases}$$

Using a Hamming window with $N = 7$

Sol: $\because h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{-j3\omega} e^{j\omega n} d\omega$$

$$= \frac{\sin \pi(n-3)/4}{\pi(n-3)} \quad ; \quad n \neq 3$$

$$h_d(0) = \frac{0.707}{3\pi} = h_d(6)$$

$$h_d(1) = h_d(5) = \frac{1}{2\pi}$$

$$h_d(2) = h(4) = \frac{0.707}{\pi}$$

$$h_d(3) = 1/4$$

Haming window function is

$$w(n) = 0.54 - 0.46 \cos \frac{2\pi n}{N-1} \quad ; \quad 0 \le n \le N-1$$

$$w(0) = w(6) = 0.08$$

$$w(1) = w(5) = 0.31$$

$$w(2) = w(4) = 0.77$$

$$w(3) = 1$$

∴ filter coefficient of the resultant filter

$h(n) = h_d(n)\,\omega(n)$

$h(0) = h(6) = 0.006$

$h(1) = h(5) = 0.049$

$h(2) = h(4) = 0.173$

$h(3) = 0.25$

∴ The frequency response.

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}} h(n)\cos\omega\left(\frac{N-1}{2} - n\right) \right]$$

(or)

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)\, e^{-j\omega n}$$

$$= e^{-j3\omega} \left[0.25 + 0.012\cos 3\omega + 0.098\cos 2\omega + 0.346\cos\omega \right]$$

* Design a digital FIR LPF using rectangular window by taking 9 samples of $\omega(n)$ and with a cut off frequency of 1.2 rad/sec.

Sol: Cut off frequency of given LPF $\omega_c = 1.2$ rad/sec and $N = 9$

For LPF

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & ; -\omega_c \le \omega \le \omega_c \; ; \alpha = \frac{N-1}{2} \\ 0 & ; \text{otherwise} \end{cases}$$

$\alpha = 4$

The desired impulse response

$$h_d[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{-j\omega\alpha} \cdot e^{j\omega n} d\omega$$

$$= \frac{1}{\pi(n-\alpha)} \sin(n-\alpha)\omega_c \quad \text{for } n \neq \alpha$$

$$= \frac{1}{\pi(n-4)} \sin(n-4)\omega_c \quad \text{for } n \neq 4$$

$h_d(0) = h_d(8) = -0.0793$

$h_d(1) = h_d(7) = -0.0470$

$h_d(2) = h_d(6) = 0.1075$

$h_d(3) = h_d(5) = 0.2967$

$h_d(4) = 0.382$

The frequency response is

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}} h(n)\cos\omega\left(\frac{N-1}{2} - n\right)\right]$$

$$= e^{-j4\omega}\big(0.382 - 0.1586\cos 4\omega - 0.094\cos 3\omega + 0.215\cos 2\omega + 0.5934\cos\omega\big)$$

(3)

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= z^{-4}\big[0.382 + 0.2967[z + z^{-1}] + 0.1075[z^2 + z^{-2}] - 0.4070[z^3 + z^{-3}] - 0.0793[z^4 + z^{-4}]\big]$$

$$H(z)\big/_{z=e^{j\omega}} = e^{-j4\omega}\big[0.382 - 0.1586\cos\omega - 0.094\cos 3\omega + 0.215\cos 2\omega + 0.5934\cos\omega\big]$$

$\therefore$ Magnitude response

$$|H(e^{j\omega})| = 0.382 + 0.5934\cos\omega + 0.215\cos 2\omega - 0.094\cos 3\omega - 0.1586\cos 4\omega$$

* Design a HPF using Hamming window with a cutoff frequency of 1.2 rad/sec & N=9

Sol: The desired frequency response for HPF is

$$H_d(e^{j\omega}) = \begin{cases} 0 & ; -\omega_c \le \omega \le \omega_c \\ e^{-j\omega\alpha} & ; \omega_c < |\omega| \le \pi \end{cases}$$

The desired impulse response

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega c} e^{-j\omega\alpha} e^{j\omega n} d\omega + \int_{\omega c}^{\pi} e^{-j\omega\alpha} e^{j\omega n} d\omega\right]$$

$$h_d(n) = \frac{1}{\pi(n-\alpha)}\left\{\sin(n-\alpha)\pi - \sin(n-\alpha)\right]\omega_c$$

$h_d(0) = h_d(8) = 0.0792$

$h_d(1) = h_d(7) = 0.0469$

$h_d(2) = h_d(6) = -0.1075$

$h_d(3) = h_d(5) = -0.2966$

$h_d(4) = 0.618$

The window sequence for Hamming window is given by

$$\omega_H(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right) ; 0 \le n \le N-1$$

$W_{H}(0) = w_{H}(8) = 0.08$

$w(1) = w(7) = 0.2147$

$w(2) = w(6) = 0.54$

$w(3) = w(5) = 0.8652$

$w(4) = 1$

$\therefore$ filter coefficients $h(n) = h_d(n)\, w_H(n)$

$h(n) = h(N-1-n)$

$h(0) = h(8) = 0.0063$

$h(1) = h(7) = 0.010$

$h(2) = h(6) = -0.058$

$h(3) = h(5) = -0.2566$

$h(4) = 0.618$

Frequency response is

$$H(e^{jw}) = e^{-jw\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}} h(n)\cos w\left(\frac{N-1}{2} - n\right)\right]$$

$$= e^{-j4w}\left[0.618 + 0.0216\cos 4w + 0.020\cos 3w - 0.116\cos 2w - 0.5132\cos w\right]$$

* Design a BPF to pass frequencies in the range 1 to 2 rad/sec Using Hanning window, $N=5$

Sol:

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & ; -\omega_{c_2} \le \omega \le -\omega_{c_1} \ \& \ \omega_{c_1} \le \omega \le \omega_{c_2} \\ 0 & ; \text{otherwise} \end{cases}$$

$$\omega_{c_1} = 1 \ \& \ \omega_{c_2} = 2 \ ; \ \alpha = \frac{N-1}{2}$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_{c2}}^{-\omega_{c1}} e^{-j\omega\alpha} \cdot e^{j\omega n}\, d\omega + \frac{1}{2\pi} \int_{\omega_{c1}}^{\omega_{c2}} e^{-j\omega\alpha}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{\pi(n-\alpha)} \left[\sin 2(n-\alpha) - \sin(n-\alpha)\right]$$

$$h_d(n) = \frac{1}{\pi(n-2)} \left[\sin 2(n-2) - \sin(n-2)\right]$$

$$h_d(0) = h_d(4) = -0.2651$$

$$h_d(1) = h_d(3) = 0.0215$$

$$h_d(2) = 0.3183$$

Hanning window sequence is given by

$$\omega(n) = 0.5 - 0.5 \cos\left(\frac{2\pi n}{N-1}\right) ; 0 \le n \le N-1$$

$$\therefore \quad \omega(0) = \omega(4) = 0$$

$$\omega(1) = \omega(2) = 0.5$$

$$\omega(2) = 1$$

$\therefore$ Filter Coefficients are $h(n) = h_d(n)\,\omega(n)$

$$h(0) = h(4) = 0$$

$$h(1) = h(3) = 0.0108$$

$$h(2) = 0.3183$$

Frequency response is

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left(h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}} h(n)\cos\omega\left(\frac{N-1}{2}-n\right)\right)$$

$$= e^{-j2\omega}\left[0.3183 + 0.216\cos\omega\right)$$

* Design a BSF to reject frequencies in the range 1 to 2 rad/sec using rectangular window, with N = 7.

Sol:

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha}; & -\pi \le \omega \le -\omega_{c2} \ \& \\ & -\omega_{c1} \le \omega \le \omega_{c1} \ \& \\ & \omega_{c2} \le \omega \le \pi \\ 0; & \text{otherwise} \end{cases}$$

; $\omega_{c1} = 1$ & $\omega_{c2} = 2$ & $\alpha = \frac{N-1}{2}$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_{c2}} e^{-j\omega\alpha} e^{j\omega n}\, d\omega + \int_{-\omega_{c1}}^{\omega_{c1}} e^{-j\omega\alpha} e^{j\omega n}\, d\omega + \int_{\omega_{c1}}^{\pi} e^{-j\omega\alpha} e^{j\omega n}\, d\omega\right.$$

$$h_d(n) = \frac{1}{\pi(n-\alpha)}\left[\sin(n-\alpha)\omega_{c_1} + \sin(n-\alpha)\pi - \sin(n-\alpha)\omega_{c_2}\right.$$

$$h_d(0) = h_d(6) = 0.0446$$

$$h_d(1) = h_d(5) = 0.2652$$

$$h_d(2) = h_d(4) = -0.0216$$

$$h_d(3) = 0.6817$$

$\therefore$ The rectangular window is given by

$$\omega(n) = 1 \; ; \; 0 \leq n \leq 6$$

$$\therefore h(n) = h_d(n)\omega(n) = h_d(n)$$

Frequency response is

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}} h(n)\cos\omega\left[\frac{N-1}{2}-n\right]\right]$$

$$= e^{-j3\omega}\left[0.6817 + 0.0892\cos 3\omega + 0.5304\cos 2\omega - 0.0432\cos\omega\right]$$

* Design a HP FIR filter for the following specification :

cutoff frequency = 500Hz

Sampling " = 2000 Hz & N = 11

Sol:

Given $f_c = 500Hz$

$f_s = 2000Hz$ & $N = 11$

$$\omega_c = \frac{2\pi f_c}{f_s} = \pi/2 \text{ rad/sec}$$

The desired frequency response of the filter

$$H_d(e^{j\omega}) = \begin{cases} 1; & \omega_c \leq |\omega| \leq \pi \\ 0; & \text{otherwise} \end{cases}$$

Impulse response of the filter

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} e^{j\omega n}\, d\omega + \int_{\omega_c}^{\pi} 1 \cdot e^{j\omega n}\, d\omega\right]$$

$$= \frac{\sin n\pi}{n\pi} - \frac{\sin n\frac{\pi}{2}}{n\pi}$$

$$= -\frac{\sin n\pi/2}{n\pi}$$

Assume rect window $\omega(n) = 1; 0 \leq n \leq 10$

* Design a BP FIR filter for the following specifications

cutoff freq's: 400 Hz & 800 Hz

Sampling freq : 2000 Hz

$N = 11$

Sol: Given Cutoff freq's

$f_{c1} = 400\,Hz$ & $f_{c2} = 800\,Hz$

$f_s = 2000\,Hz$

$$\omega_{c1} = \frac{2\pi f_{c1}}{f_s} = 0.4\pi$$

$$\omega_{c2} = \frac{2\pi f_{c2}}{f_s} = 0.8\pi$$

$$\therefore H_d(e^{j\omega}) = \begin{cases} 1; & -\omega_{c2} \le \omega \le -\omega_{c1} \ \& \ \omega_{c1} \le \omega \le \omega_{c1} \\ 0; & \text{otherwise} \end{cases}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-0.8\pi}^{-0.4\pi} 1 \cdot e^{j\omega n}\, d\omega + \int_{0.4\pi}^{0.8\pi} 1\, e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{n\pi}\left[\sin(0.8n\pi) - \sin(0.4n\pi)\right]$$

Assume window as rectangular $\omega(n) = 1 ; 0 \le n \le 10$

$h(n) = h_d(n)\,\omega(n)$

* Design an FIR BSF for the following specification.

cutoff freq = 400Hz & 800Hz

Sampling freq = 2000Hz, N = 11

$$\omega_{c1} = \frac{2\pi f_{c1}}{f_s} = 0.4\pi$$

$$\omega_{c2} = \frac{2\pi f_{c2}}{f_s} = 0.8\pi$$

$$H_d(e^{j\omega}) = \begin{cases} 1 ; & -\pi \le \omega \le -\omega_{c2} \ \& \ -\omega_{c1} \le \omega \le \omega_{c1} \ \& \ \omega_{c2} \le \omega \le \pi \\ 0 ; & \text{otherwise} \end{cases}$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{-0.8\pi} 1 \cdot e^{j\omega n} d\omega + \int_{-0.4\pi}^{0.4\pi} 1 \cdot e^{j\omega n} d\omega + \int_{0.8\pi}^{\pi} 1 \cdot e^{j\omega n} d\omega \right]$$

$$= \frac{1}{n\pi} \left[\sin(0.4n\pi) - \sin(0.8n\pi) \right]$$

Assume window sequence $\omega(n) = 1 ; 0 \le n \le 10$

$$h[n] = h_d[n]\,\omega(n)$$

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* Design of FIR filter by Frequency sampling Technique

In this method, the ideal frequency response is sampled at sufficient no. of points. These samples are the DFT coefficients of the impulse response of the filter.

Two design techniques are available

Type - I design

→ choose the ideal frequency response $H_d(e^{j\omega})$

→ Generate the sequence $H(K)$; $0 \le K \le N-1$

i.e. $H(K) = H_d(e^{j\omega})\Big/_{\omega = \frac{2\pi K}{N}}$

→ Compute the N samples of $h(n)$ using the equation

N → odd $\quad h[n] = \frac{1}{N}\left[H(0) + 2 \sum_{K=1}^{\frac{N-1}{2}} Re\left\{ H(K)\, e^{\frac{j2\pi nK}{N}} \right\}\right]$

N → even $\quad h[n] = \frac{1}{N}\left[H(0) + 2 \sum_{K=1}^{\frac{N}{2}-1} H(K)\, e^{\frac{j2\pi nK}{N}} \right]$

→ Take Z Transform of the impulse response $h(n)$ to get the T.F $H(z)$

$$H(z) = \sum_{n=0}^{N-1} h[n]\, z^{-n}$$

Type - II design

→ Choose the ideal frequency response $H_d(e^{j\omega})$

→ $H(k) = H_d(e^{j\omega})\Big/_{\omega = \frac{2\pi(k+1)}{2N}}$; $k = 0, \ldots N-1$

→ Compute N samples of h(n) using the following eq's

when N is odd : $h[n] = \frac{2}{N} \sum_{k=0}^{\frac{N-3}{2}} Re\left[H(k) e^{\frac{jn\pi(2k+1)}{N}} \right]$

when N is even : $h[n] = \frac{2}{N} \sum_{k=0}^{\frac{N}{2}-1} Re\left[H(k) e^{\frac{jn\pi(2k+1)}{N}} \right]$

→ Take z- Transform of the impulse response to get T.F $H(z)$

$$H(z) = \sum_{n=0}^{N-1} h[n] z^{-n}$$

Note : $\sum_{k=0}^{N-1} \cos k\theta = \frac{\sin \frac{N\theta}{2} \cos\left(\frac{N-1}{2}\right)\theta}{\sin \frac{\theta}{2}}$

$$\sum_{k=0}^{N-1} \sin k\theta = \frac{\sin \frac{N\theta}{2} \sin\left(\frac{N-1}{2}\right)\theta}{\sin \frac{\theta}{2}}$$

* Design a linear phase LP FIR filter with a cutoff frequency of $\pi/2$ rad/sec. Using frequency sampling technique. Take $N = 13$

Sol:

$$\because H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & ; 0 \leq |\omega| \leq \frac{\pi}{2} \\ 0 & ; \text{otherwise} \end{cases}$$

$$\text{where } \alpha = \frac{N-1}{2} = 6$$

Let us choose type-I design

$$\therefore \omega_K = \frac{2\pi k}{N} = \frac{2\pi k}{13}$$

$$H(K) = H_d(e^{j\omega}) \Big/ \omega = \frac{2\pi k}{13}$$

when $k=0 \Rightarrow \omega_k = \omega_0 = 0$

$$\omega_1 = \frac{2\pi}{13}, \omega_2 = 4\pi/13, \omega_3 = \frac{6\pi}{13}$$

$$\omega_4 = 8\pi/13, \omega_5 = \frac{10\pi}{13}, \omega_6 = \frac{12\pi}{13}$$

$$\omega_7 = \frac{14\pi}{13}, \omega_8 = \frac{16\pi}{13}, \omega_9 = \frac{18\pi}{13}$$

$$\omega_{10} = \frac{20\pi}{13}, \omega_{11} = \frac{22\pi}{13}, \omega_{12} = \frac{24\pi}{13}$$

$$\therefore H(K) = \begin{cases} e^{-j6\times\frac{2\pi k}{13}} & ; K = 0,1,2,3 \\ 0 & ; K = 4,5,6,7,8,9 \\ e^{-j6\times\frac{2\pi k}{3}} & ; K = 10,11,12 \end{cases}$$

$\therefore$ The sample of impulse response are given by

$$h[n] = \frac{1}{N}\left\{ H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left[H(k)\, e^{\frac{j2\pi nk}{N}}\right]\right.$$

$$h(n) = \frac{1}{13}\left[H(0) + 2\sum_{k=1}^{6} Re\left[H(k)\, e^{j\frac{2\pi nk}{13}}\right]\right.$$

$$H(0) = 0, \quad H(k) = 1 \quad ; k = 1, 2, 3$$

$$H(k) = 0 \quad ; k = 4, 5, 6$$

$$\therefore h(n) = \frac{1}{13}\left\{1 + 2\sum_{k=1}^{3} Re\left(e^{-j\frac{12\pi k}{13}} \cdot e^{j\frac{2\pi nk}{13}}\right)\right\}$$

$$= \frac{1}{13}\left\{1 + 2\sum_{k=1}^{3} Re\left[e^{\frac{-j2\pi k(6-n)}{13}}\right]\right\}$$

$$= \frac{1}{13}\left[1 + 2\sum_{k=1}^{3} \cos\frac{2\pi k}{13}(6-n)\right]$$

$$= \frac{1}{13}\left[1 + 2\sum_{k=0}^{3} \cos\frac{2\pi k}{13}(6-n) - 1\right]$$

$$= \frac{1}{13}\left[2\sum_{k=0}^{3} \cos\frac{2\pi k}{13}(6-n) - 1\right]$$

$$\therefore \sum_{k=0}^{M-1} \cos k\theta = \frac{\sin\frac{M\theta}{2}\cos\left(\frac{M-1}{2}\right)\theta}{\sin\frac{\theta}{2}}$$

Here $M - 1 = 3$

$$\theta = \frac{2\pi}{13}(6-n) \quad \therefore M = 4$$

$$h(n) = \frac{1}{13} \left\{ \frac{2 \sin \frac{4}{2} \times \frac{2\pi}{13}(6-n) \cos \frac{3}{2} \times \frac{2\pi}{13}(6-n)}{\sin \frac{\pi(6-n)}{13}} - 1 \right\}$$

$$= \frac{1}{13} \left\{ \frac{2 \sin \frac{4\pi}{13}(6-n) \cos \frac{3\pi}{13}(6-n) - \sin \frac{\pi(6-n)}{13}}{\sin \frac{\pi(6-n)}{13}} \right\}$$

$$= \frac{1}{13} \left\{ \frac{\sin\left[\frac{4\pi}{13}(6-n) + \frac{3\pi(6-n)}{13}\right] + \sin\left(\frac{4\pi}{13}(6-n) - \frac{3\pi(6-n)}{13}\right) - \sin \frac{\pi(6-n)}{13}}{\sin \frac{\pi(6-n)}{13}} \right\}$$

$$= \frac{1}{13} \left\{ \frac{\sin\left(\frac{7\pi(6-n)}{13}\right) + \sin\left(\frac{\pi(6-n)}{13}\right) - \sin\left(\frac{\pi(6-n)}{13}\right)}{\sin \frac{\pi(6-n)}{13}} \right\}$$

$$h(n) = \frac{1}{13} \left\{ \frac{\sin \frac{7\pi(6-n)}{13}}{\sin \frac{\pi(6-n)}{13}} \right\}; \quad n = 0, 1, 2 \ldots 12 \text{ except } n = 6$$

$\underline{n=6}$

$$h(6) = \lim_{n \to \infty} h[n] = \frac{7}{13} = 0.5384$$

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$h(0) = -0.0513$

$h(1) = 0.0677$

$h(2) = 0.0434$

$h(3) = -0.1084$

$h(4) = -0.0396$

$h(5) = 0.3190$

$h(6) = 0.5384$

$\therefore\ h(n) = h(N-1-n)$ & $\alpha = \frac{N-1}{2}$ for linear phase

$h(7) = h(5) = 0.3190$

$h(8) = h(4) = -0.0396$

$h(9) = h(3) = -0.1084$

$h(10) = h(2) = 0.0434$

$h(11) = h(1) = 0.0677$

$$\therefore\ \text{T.F}\ H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= \sum_{n=0}^{12} h(n) z^{-n}$$

$$H(z) = -0.0513\,[1 + z^{-12}] + 0.0677\,[z^{-1} + z^{-11}] + 0.0434\,[z^{-2} + z^{-10}] - 0.1084\,[z^{-3} + z^{-9}] - 0.0396\,[z^{-4} + z^{-8}] + 0.3190\,[z^{-5} + z^{-7}] + 0.5384\,z^{-6}$$

* Design a linear phase FIR filter of length $N = 11$. which has a symmetric unit sample response and a frequency response that satisfies the conditions.

$$H\left(\frac{2\pi k}{11}\right) = \begin{cases} 1 & ; \; k = 0, 1, 2 \\ 0.5 & ; \; k = 3 \\ 0 & ; \; k = 4, 5 \end{cases}$$

Sol :

$$\alpha = \frac{N-1}{2} = 5$$

$$\omega = \omega_k = \frac{2\pi k}{N} = \frac{2\pi k}{11}$$

$$\therefore H(k) = H_d(e^{j\omega}) = H(\omega) \Big/_{\omega = \frac{2\pi k}{N}}$$

$$= \begin{cases} 1.\, e^{-j\omega_k \alpha} & ; \; k = 0, 1, 2 \\ 0.5\, e^{-j\omega_k \alpha} & ; \; k = 3 \\ 0 & ; \; k = 4, 5 \end{cases}$$

For $k = 0 \Rightarrow$ $H(0) = e^{-j5\left(\frac{2\pi \times 0}{11}\right)} = 1$

$$H(1) = e^{-j10\pi/11}$$

$$H(2) = e^{-j20\pi/11}$$

$$H(3) = e^{-j30\pi/11}$$

$$H(4) = 0 \quad \& \quad H(5) = 0$$

$$h[n] = \frac{1}{N} \sum_{k=0}^{N-1} H(k)\, e^{j\frac{2\pi n k}{N}}$$

$$h[n] = \frac{1}{N}\left\{H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left[H(k)\, e^{\frac{j2\pi nk}{N}}\right]\right\}$$

$$= \frac{1}{11}\left\{1 + 2\sum_{k=1}^{5} Re\left[H(k)\, e^{\frac{j2\pi nk}{N}}\right]\right\}$$

$$= \frac{1}{11}\left[1 + 2\sum_{k=1}^{2} Re\left[e^{-j5\frac{(2\pi k)}{11}}\, e^{\frac{j2\pi nk}{11}}\right.\right.$$

$$\left.+ 2 Re\left[0.5\, e^{-j5\frac{(2\pi\times3)}{11}} \cdot e^{\frac{j2\pi n3}{11}}\right]\right]$$

$$= \frac{1}{11}\left[1 + 2\sum_{k=1}^{2} Re\left\{e^{j\frac{2\pi k}{11}(n-5)}\right\} + 2 Re\left[0.5\, e^{j\frac{6\pi}{11}(n-5)}\right]\right]$$

$$h[n] = \frac{1}{11} + \frac{2}{11}\cos\frac{2\pi}{11}(n-5) + \frac{2}{11}\cos\frac{4\pi}{11}(n-5) + \cos\frac{6\pi}{11}(n-5)$$

$$h[0] = \frac{1}{11} + \frac{2}{11}\cos\left(\frac{-10\pi}{11}\right) + \frac{2}{11}\cos\left(\frac{-20\pi}{11}\right) + \cos\left(\frac{-30\pi}{11}\right)$$

$= -0.5854$

$h[1] = 0.787$

$h(2) = 0.3059$

$h(3) = -0.9120$

$h(4) = 0.1770$

$h(5) = 1.4545$

For linear phase FIR filter

$h[n] = h(N-1-n)$

$\therefore h(6) = h(4) = 0.1770$

$h(7) = h(3) = -0.9120$

$h(8) = h(2) = 0.3590$

$h(9) = h(1) = 0.787$

$h(10) = h(0) = -0.5804$

$\therefore$ T.F. of the filter

$$H(z) = \sum_{n=0}^{10} h[n]\, z^{-n}$$

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* structures for FIR systems

In general, an FIR system is described by the difference equation

$$y[n] = \sum_{k=0}^{N-1} b_k \, x(n-k) \quad \text{--- (1)}$$

apply Z.T on both sides

$$Y(z) = \sum_{k=0}^{N-1} b_k z^{-k} X(z)$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{N-1} b_k z^{-k} \quad \text{--- (2)}$$

the unit impulse response of an FIR system is identical to the coefficients

i.e. $h(n) = \begin{cases} b_n & ; 0 \le n \le N-1 \\ 0 & ; \text{otherwise} \end{cases}$

$\therefore$ The system function of an FIR filter can be written as

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} \quad \text{--- (3)}$$

F h

* Direct - form realization (Transversal s/m)

The convolution sum relationship gives the system response as

$$y[n] = \sum_{k=0}^{N-1} h(k)\, x(n-k) \quad \text{--- (1)}$$

apply Z.T on bothsides

$$Y(z) = \sum_{k=0}^{N-1} b_k\, z^{-k} X(z) \quad (\because b_k = h(k))$$

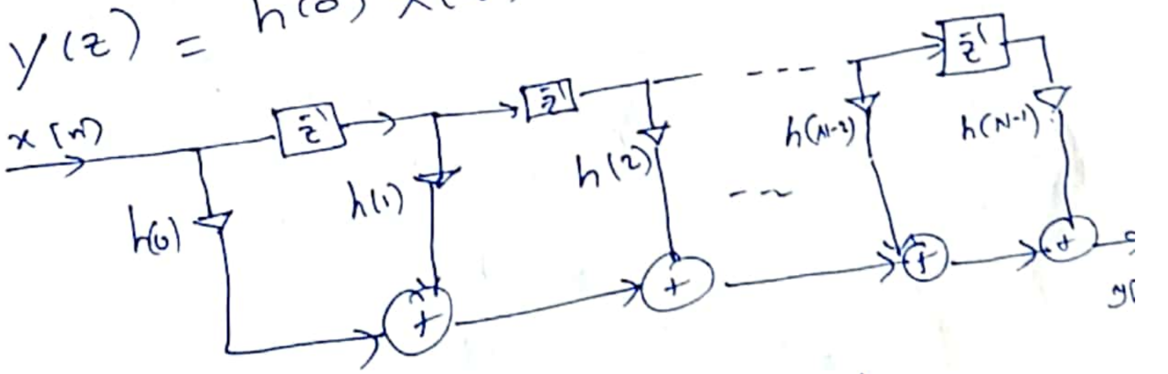
$$\frac{Y(z)}{X(z)} \doteq H(z) = \sum_{k=0}^{N-1} b_k\, z^{-k}$$

$\therefore$ The s/m function of an FIR filter can be written as

$$H(z) = \sum_{k=0}^{N-1} h(k)\, z^{-k}$$

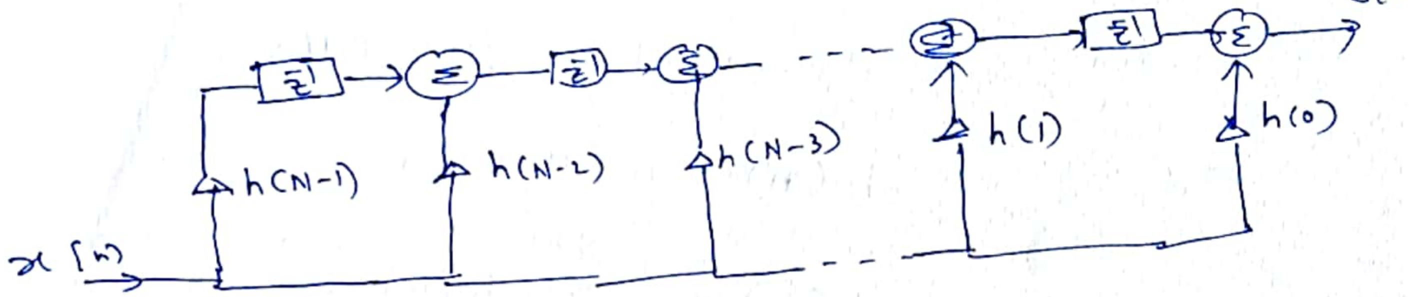
$$H(z) = h(0) + h(1) z^{-1} + \dots + h(N-1) z^{-(N-1)}$$

$$Y(z) = h(0) X(z) + h(1) z^{-1} X(z) + \dots + h(N-1) z^{-(N-1)} X(z)$$



Direct form realization

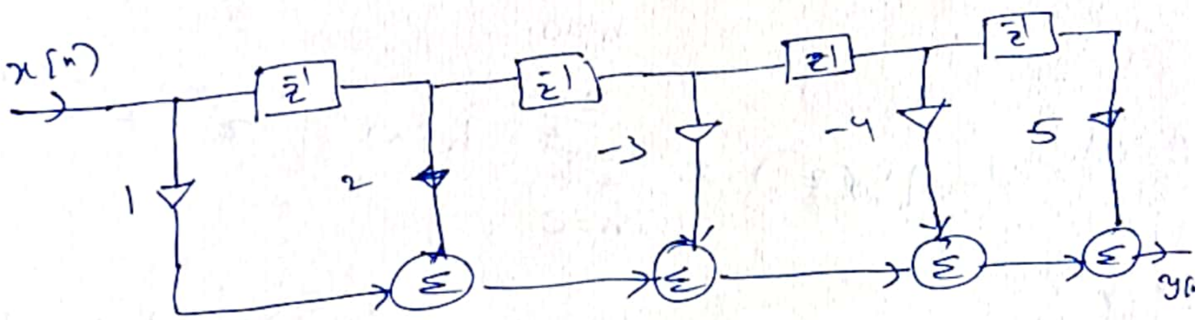
Transposed structure



* Determine the direct form realization of s/m function $H(z) = 1+2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$

sol: Given $H(z) = 1+2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$

$$Y(z) = X(z) + 2z^{-1}X(z) - 3z^{-2}X(z) - 4z^{-3}X(z) + 5z^{-4}X(z)$$



* Obtain direct form realization for the s/m function

$$H(z) = 1 + 3z^{-1} + 4z^{-2} + 4z^{-3} + 3z^{-4} + z^{-5}$$

* ~~Cascade~~

Direct form realisation of linear phase FIR sh

For a linear phase FIR filter

$$h(n) = h(N-1-n)$$

$$H(z) = \sum_{n=0}^{N-1} h[n]\, z^{-n}$$

For N even

$$H(z) = \sum_{n=0}^{\frac{N-2}{2}} h(n)\, z^{-n} + \sum_{n=N/2}^{N-1} h(n)\, z^{-n}$$

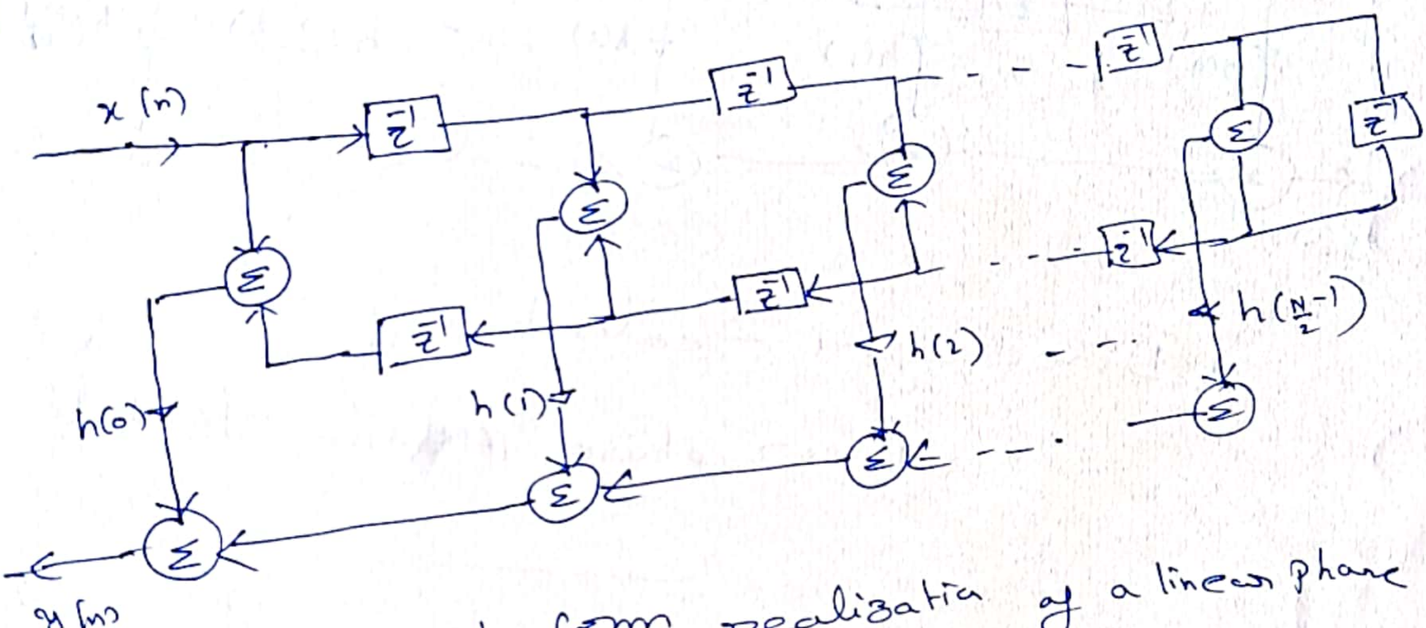
$$= \sum_{n=0}^{\frac{N-2}{2}} h(n)\, z^{-n} + \sum_{n=0}^{\frac{N-2}{2}} h(N-1-n)\, z^{-(N-1-n)}$$

$$= \sum_{n=0}^{\frac{N-2}{2}} h(n)\, z^{-n} + \sum_{n=0}^{\frac{N-2}{2}} h(n)\, z^{-(N-1-n)}$$

$$H(z) = \sum_{n=0}^{\frac{N-2}{2}} h(n) \left[z^{-n} + z^{-(N-1-n)} \right]$$

$$\frac{Y(z)}{X(z)} = h(0)\left[1 + z^{-(N-1)}\right] + h(1)\left[z^{-1} + z^{-(N-2)}\right] + \cdots h\left[\left(\frac{N}{2}-1\right)\right]\left[z^{-\left(\frac{N}{2}-1\right)} + z^{-\left(N-1-\frac{N}{2}+1\right)}\right]$$

$$Y(z) = h(0)\left[X(z) + z^{-(N-1)}X(z)\right] + h(1)\left[z^{-1}X(z) + z^{-(N-2)}X(z)\right] + \cdots + h\left(\tfrac{N}{2}-1\right)\left[z^{-\left(\frac{N}{2}-1\right)}X(z) + z^{-\frac{N}{2}}X(z)\right]$$



Direct form realization of a linear phase FIR s/m for N is even

For N odd

$$H(z) = \sum_{n=0}^{\frac{N-3}{2}} h(n)z^{-n} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n)z^{-n} + h\left(\frac{N-1}{2}\right)z^{-\left(\frac{N-1}{2}\right)}$$

$$= h\left(\frac{N-1}{2}\right)z^{-\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n)\left[z^{-n} + z^{-(N-1-n)}\right]$$

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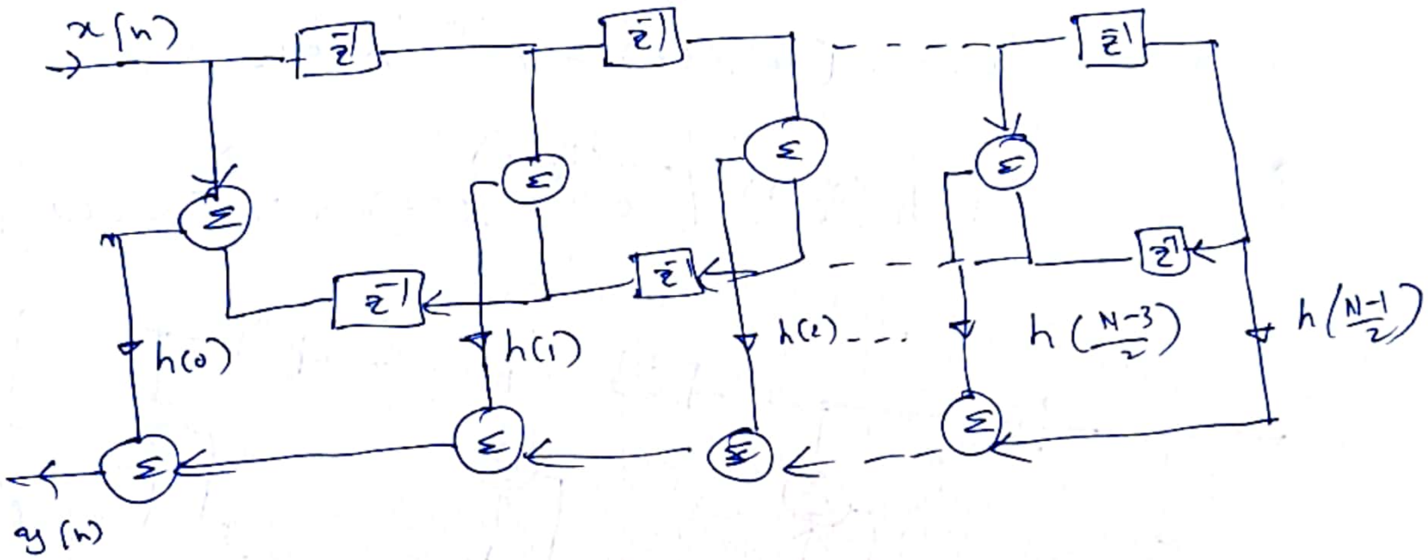


Direct-form realisation of a linear phase FIR Sm for N odd

* Cascade form realization

The s/m function

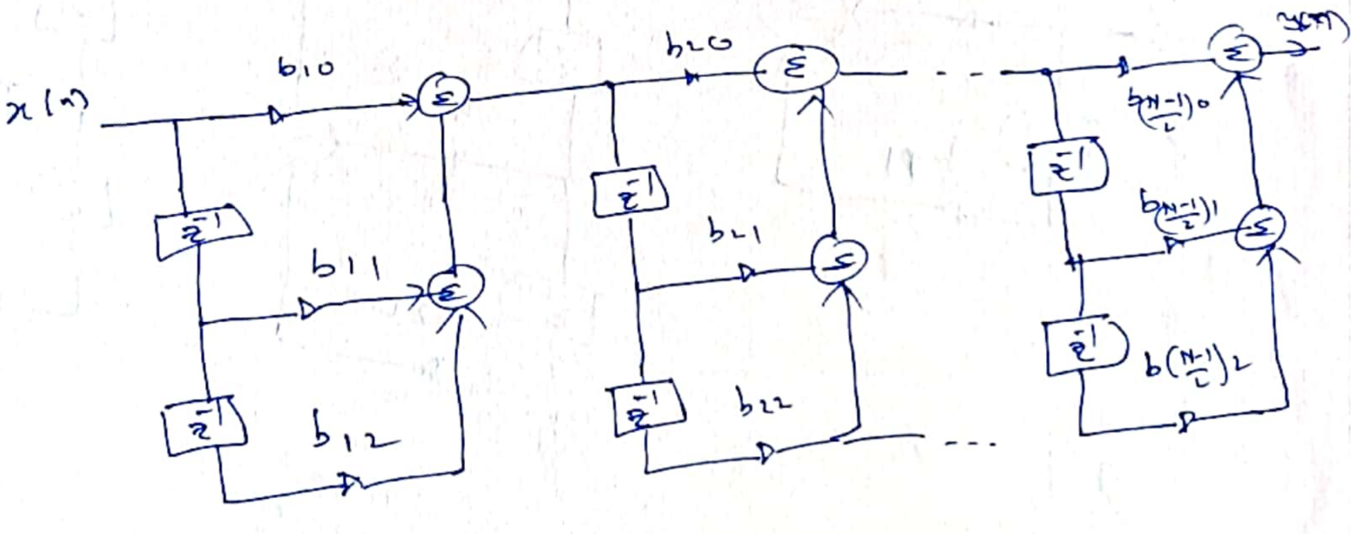
$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-1})$$

In this form convert $H(z)$ into factored form of $H(z)$

For N odd

i.e. $$H(z) = \prod_{k=1}^{\frac{N-1}{2}} (b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2})$$

$$= (b_{10} + b_{11} z^{-1} + b_{12} z^{-2})(b_{20} + b_{21} z^{-1} + b_{22} z^{-2}) \ldots$$
$$\times (b_{(\frac{N-1}{2})0} + b_{(\frac{N-1}{2})1} z^{-1} + b_{(\frac{N-1}{2})2} z^{-2})$$



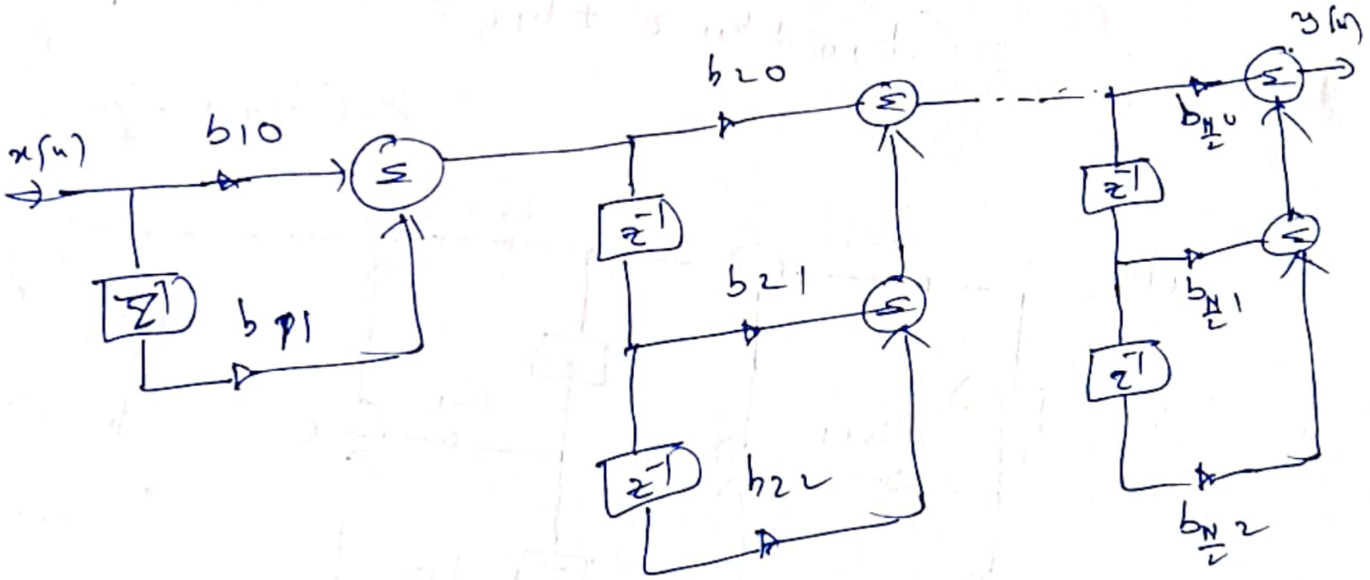
It will have $\frac{(N-1)}{2}$ 2nd order factors

For N even

$$H(z) = (b_{10} + b_{11} z^{-1}) \prod_{k=2}^{N/2} (b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2})$$

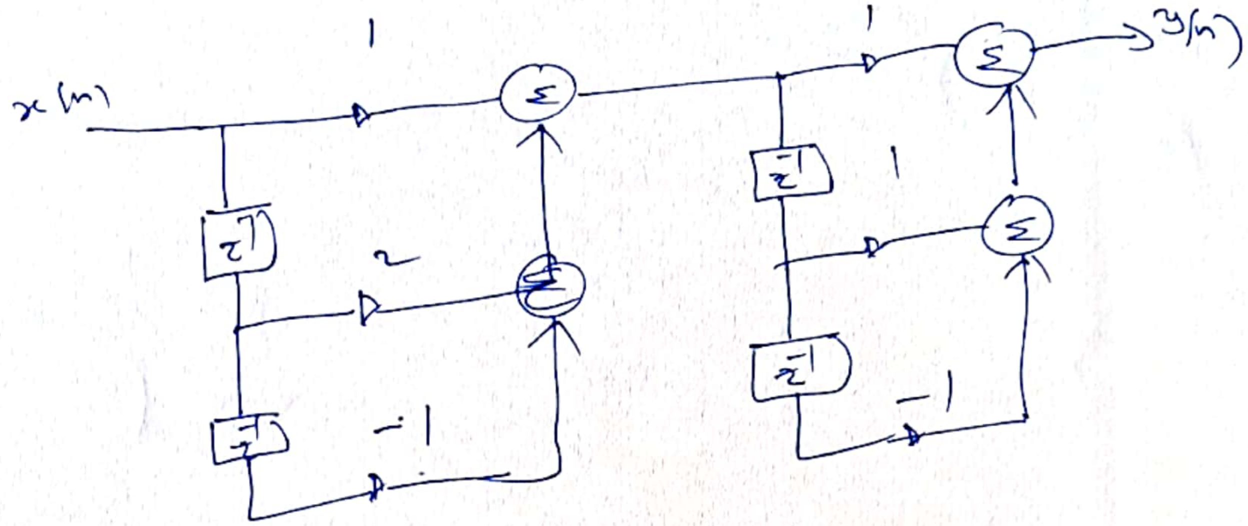
It will have one first order factor & $\frac{N-2}{2}$ 2nd order factors.

$$= (b_{10} + b_{11} z^{-1}) (b_{20} + b_{21} z^{-1} + b_{22} z^{-2}) \cdots \times (b_{\frac{N}{2}0} + b_{\frac{N}{2}1} z^{-1} + b_{\frac{N}{2}2} z^{-2})$$



* Obtain the Cascade realisation of S/m function

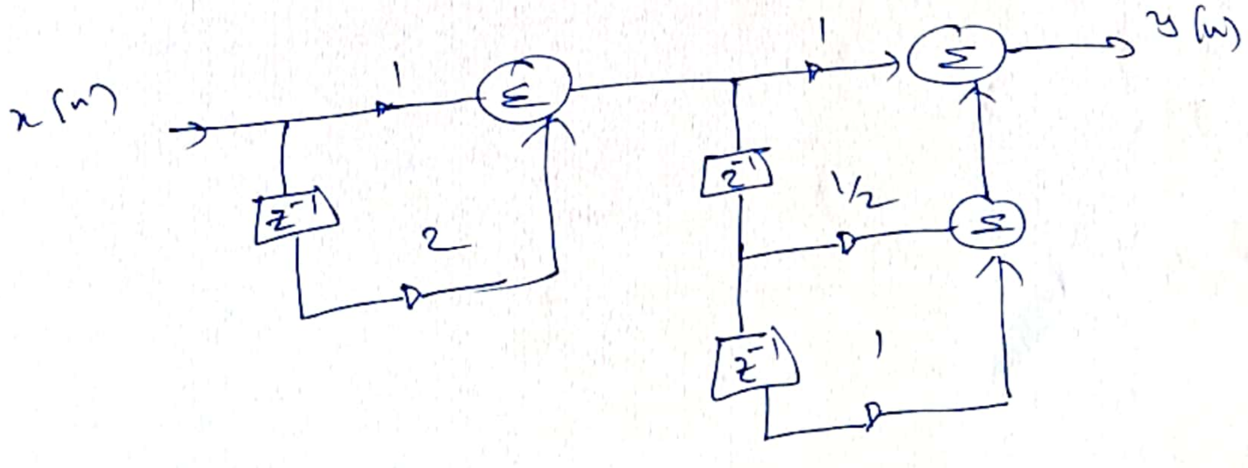
$$H(z) = (1 + 2z^{-1} - z^{-2})(1 + z^{-1} - z^{-2})$$



* Obtain the Cascade realisation of S/m function

$$H(z) = 1 + \frac{5}{2} z^{-1} + 2z^{-2} + 2z^{-3}$$

$$= (1 + 2z^{-1})\left(1 + \frac{1}{2} z^{-1} + z^{-2}\right)$$



*

Lattice structure

Lattice filters are used extensively in digital Speech processing and implementing adaptive filters.

Let us Consider FIR filter with s/m fuction

$$H(z) = A_m(z) = 1 + \sum_{K=1}^{m} \alpha_m(K)\, z^{-K} \;, m \geq 1$$

$\& \; A_0(z) = 1$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = 1 + \sum_{K=1}^{m} \alpha_m(K)\, z^{-K}$$

The impulse response of the mth filter

$h_m(0) = 1$

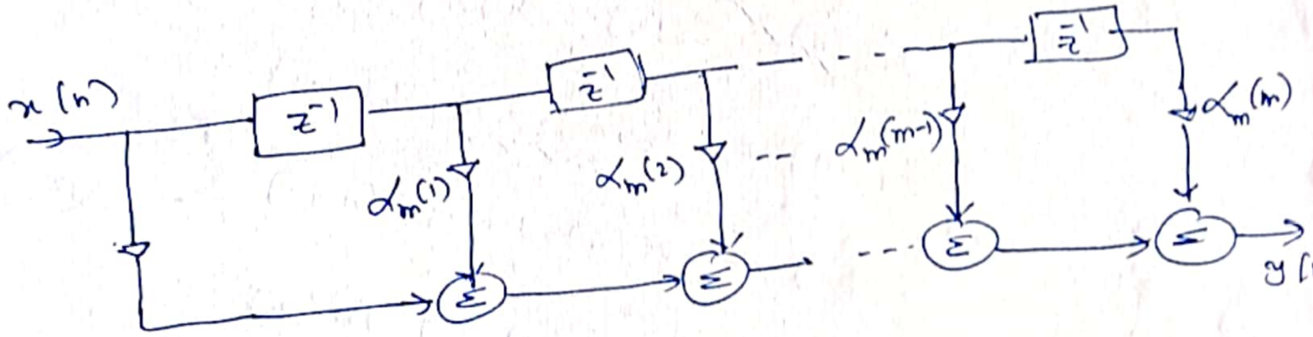
$\& \; h_m(K) = \alpha_m(K)$

let $\alpha_m(0) = 1$

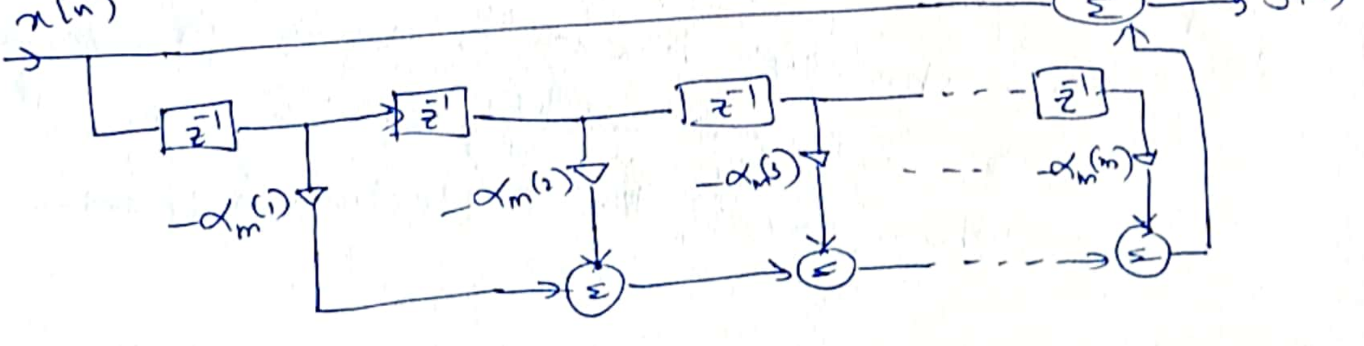
$$Y(z) = X(z) + \sum_{K=1}^{m} \alpha_m(K)\, z^{-K} X(z)$$

apply inverse Z.T

$$y[n] = x[n] + \sum_{K=1}^{m} \alpha_m(K)\, x(n-K)$$



Direct form structures of the FIR filter



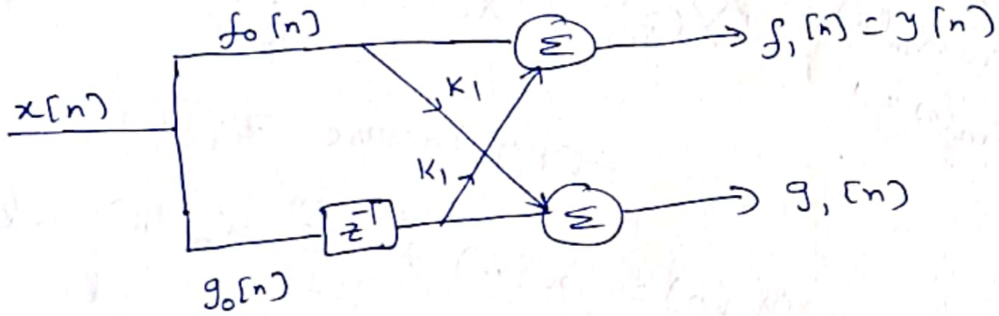
Let $\hat{x}(n) = -\sum_{k=1}^{m} \alpha_m(k)\, x(n-k)$ is one-step forward predicted value of $x[n]$ based on m past inputs

& $y[n] = x[n] - \hat{x}[n] \rightarrow$ prediction error sequence.

If $m=1$

then $y(n) = x[n] + \alpha_1(1)\, x(n-1)$ — (1)

This o/p can also be obtained from a first order or single stage lattice filter as shown below.



$f_0[n] = g_0[n] = x[n]$

$f_1[n] = f_0[n] + K_1 g_0[n-1]$

$\Rightarrow f_1(n) = x[n] + K_1 x[n-1]$ — (2)

$g_1[n] = K_1 f_0[n] + g_0[n-1]$

$\Rightarrow g_1(n) = K_1 x[n] + x(n-1)$

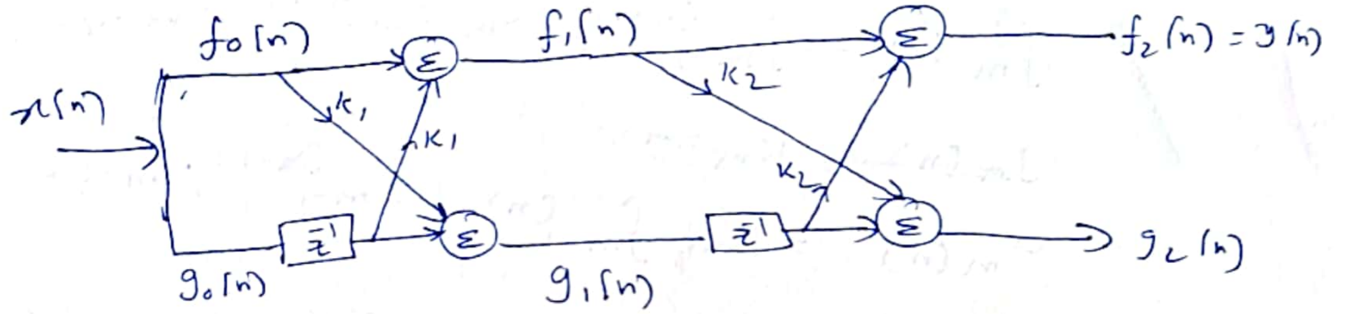
$\therefore$ The o/p is exactly same as obtained in eq(1) if we select $K_1 = \alpha_1(1)$

where K_1 is the reflection coefficient

If $m = 2$

$$y[n] = x[n] + \alpha_2(1)\, x(n-1) + \alpha_2(2)\, x(n-2) \quad \text{--- (3)}$$

by Cascading two lattice stages as shown in below figure it is possible to obtained the same o/p as in eqn (3)



$$f_2[n] = f_1[n] + K_2\, g_1[n-1]$$

$$= x[n] + K_1 x[n-1] + K_2 [K_1 x(n-1) + x(n-2)]$$

$$y[n] = x[n] + K_1(1+K_2)\, x(n-1) + K_2\, x(n-2) \quad \text{--- (4)}$$

by Comparing eqn (3) & (4)

$$\boxed{K_2 = \alpha_2(2)} \quad \& \quad K_1(1+K_2) = \alpha_2(1)$$

$$\Rightarrow K_1 = \frac{\alpha_2(1)}{1+K_2}$$

$$\boxed{K_1 = \frac{\alpha_2(1)}{1+\alpha_2(2)}}$$

$\therefore$ The reflection coefficients K_1 & K_2 of the lattice filter can be obtained from the coefficients $\{\alpha_m(k)\}$ of the direct form realisation.

* The lattice filter is generally described by the following set of order-recursive eq^ns

$$f_0[n] = g_0[n] = x[n]$$

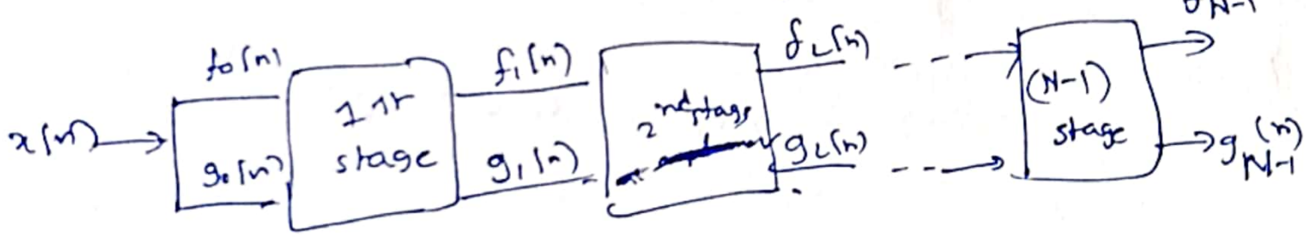
$$f_m(n) = f_{m-1}(n) + K_m\, g_{m-1}(n-1) \quad ; m = 1, 2, \dots N-1$$

$\cancel{g_m(n) = K_m(x-1)}$

$$g_m(n) = K_m\, f_{m-1}(n) + g_{m-1}(n-1) \quad ; m = 1, 2, \dots N-1$$

The o/p of N-1 stage filter corresponds to the o/p of a (N-1) order FIR filter

i.e. $y(n) = f_{N-1}(n)$



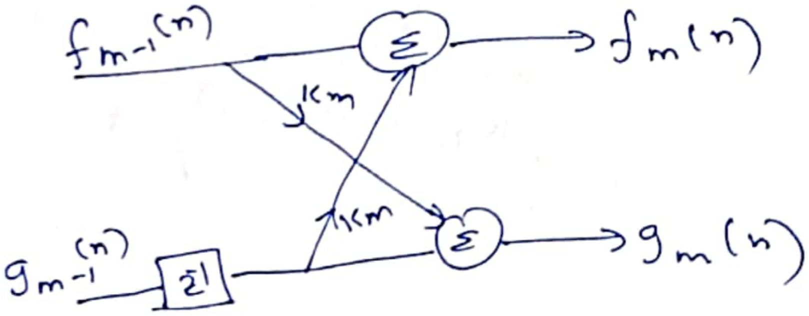


fig: (N-1) stage lattice filter

The o/p $f_m(n)$ of an m-stage lattice filter can be expressed as

$$f_m(n) = \sum_{k=0}^{m} \alpha_m(k)\, x(n-k), \quad \alpha_m(0) = 1$$

$\because$ The above eq^n is a conv sum, it follows that

app the Z.T

$$F_m(z) = A_m(z) X(z)$$

$$\Rightarrow A_m(z) = \frac{F_m(z)}{X(z)} = \frac{F_m(z)}{F_0(z)}$$

The other O/P from lattice can also be obtained

$$g_2(n) = K_2 f_1(n) + g_1(n-1)$$

$$= K_2 [x(n) + K_1 x(n-1)] + K_1 x(n-1) + x(n-2)$$

$$= K_2 x(n) + K_1 (1+K_2) x(n-1) + x(n-2)$$

$$g_2(n) = \alpha_2(2) x(n) + \alpha_2(1) x(n-1) + x(n-2)$$

$$\therefore g_m(n) = \sum_{k=0}^{m} \beta_m(k) x(n-k)$$

where filter coefficients $\{\beta_m(k)\}$ associated with a filter that produces $f_m(n) = y(n)$

$$\therefore \cancel{\beta_m(k) = \sum_{k=0}^{m} \beta_m(k) x(n-k)}$$

$$\beta_m(k) = \alpha_m(m-k) \; ; \; k = 0, 1 \ldots m$$

with $\beta_m(m) = 1$

$\because$ the data are run in reverse order through the predictor, the prediction performed using $\hat{x}(n-m) = -\sum_{k=0}^{m-1} \beta_m(k)\, x(n-k)$

is called backward prediction

But $A_m(z)$ is called forward predictor

$$G_m(z) = B_m(z)\, X(z)$$

$$\Rightarrow \cancel{G_m(z)}\ B_m(z) = \frac{G_m(z)}{X(z)}$$

where $B_m(z)$ represents the s/m function of an FIR filter

$$B_m(z) = \sum_{k=0}^{m} \beta_m(k)\, z^{-k}$$

$$\because \beta_m(k) = \alpha_m(m-k)$$

$$\therefore B_m(z) = \sum_{k=0}^{m} \alpha_m(m-k)\, z^{-k}$$

$$= \sum_{l=0}^{m} \alpha_m(l)\, z^{l-m}$$

$$= z^{-m} \sum_{l=0}^{m} \alpha_m(l)\, z^{+l}$$

$$B_m(z) = z^{-m} A_m(z^{-1})$$

The zeros of $B_m(z)$ are reciprocal of zeros of $A_m(z)$

$B_m(z)$ is called reciprocal or reverse polynomial of $A_m(z)$

relationship b/w direct form FIR filter & lattice structure

we have

$\because \quad F_0(z) = G_0(z) = X(z)$

$F_m(z) = F_{m-1}(z) + K_m z^{-1} G_{m-1}(z)$; $m = 1, 2 \ldots N-1$

$G_m(z) = K_m F_{m-1}(z) + z^{-1} G_{m-1}(z)$; $m = 1, 2 \ldots N-1$

Divide with $X(z)$; $\frac{F_0(z)}{X(z)} = \frac{G_0(z)}{X(z)} = 1$

$$\frac{F_m(z)}{X(z)} = \frac{F_{m-1}(z)}{X(z)} + K_m z^{-1} \frac{G_{m-1}(z)}{X(z)}$$

$$\frac{G_m(z)}{X(z)} = K_m \frac{F_{m-1}(z)}{X(z)} + z^{-1} \frac{G_{m-1}(z)}{X(z)}$$

$$A_0(z) = B_0(z) = 1$$

$$A_m(z) = A_{m-1}(z) + K_m z^{-1} B_{m-1}(z)$$

$$B_m(z) = K_m A_{m-1}(z) + z^{-1} B_{m-1}(z)$$

In the matrix form

$$\begin{bmatrix} A_m(z) \\ B_m(z) \end{bmatrix} = \begin{bmatrix} 1 & K_m \\ K_m & 1 \end{bmatrix} \begin{bmatrix} A_{m-1}(z) \\ z^{-1} B_{m-1}(z) \end{bmatrix}$$

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Conversion of lattice Coefficients to direct form filter coefficients

The direct form FIR filter coefficients $\{\alpha_m(k)\}$ can be obtained from the lattice coefficients $\{k_i\}$ by using the following relations.

$$A_0(z) = B_0(z) = 1$$

$$A_m(z) = A_{m-1}(z) + k_m z^{-1} B_{m-1}(z) \quad ; m = 1, 2 .. N-1$$

$$B_m(z) = z^{-m} A_m(z^{-1}) \quad ; m = 1, 2, .. N-1$$

The solution is obtained recursively beginning with $m=1$. Thus we obtain $(N-1)$ FIR filters for each value of m

Ex: Given a three stage lattice filter with coefficients $k_1 = 1/4$, $k_2 = 1/2$ & $k_3 = 1/3$. Determine the FIR filter coefficients for the direct form structure.

Sol: For $m = 1$

$$A_1(z) = A_0(z) + k_1 z^{-1} B_0(z)$$

$$= 1 + k_1 z^{-1} = 1 + 1/4 z^{-1}$$

$$=$$

Hence the coefficients of an FIR filter corresponding to the single stage lattice are

$$\alpha_1(0) = 1, \quad \alpha_1(1) = k_1 = \frac{1}{4}$$

$\because$ $B_m(z)$ is the reverse polynomial of $A_m(z)$

$$B_1(z) = \frac{1}{4} + z^{-1}$$

For $\underline{m=2}$

$$A_2(z) = A_1(z) + K_2 z^{-1} B_1(z)$$

$$= 1 + \frac{1}{4} z^{-1} + \frac{1}{2} z^{-1} \left(\frac{1}{4} + z^{-1}\right)$$

$$= 1 + \frac{3}{8} z^{-1} + \frac{1}{2} z^{-2}$$

Hence the FIR filter parameters corresponding to the two-stage lattice are

$$\alpha_2(0) = 1, \quad \alpha_2(1) = \frac{3}{8}, \quad \alpha_2(2) = \frac{1}{2}$$

$$\& \quad B_2(z) = \frac{1}{2} + \frac{3}{8} z^{-1} + z^{-2}$$

Finally the 3rd stage

$$A_3(z) = A_2(z) + K_3 z^{-1} B_2(z)$$

$$= 1 + \frac{13}{24} z^{-1} + \frac{5}{8} z^{-2} + \frac{1}{3} z^{-3}$$

$\therefore$ $\alpha_3(0) = 1$, $\alpha_3(1) = \frac{13}{24}$, $\alpha_3(2) = \frac{5}{8}$ & $\alpha_3(3) =$

Generally

$\therefore$ $\alpha_m(0) = 1$

$\alpha_m(m) = K_m$

$$\alpha_m(k) = \alpha_{m-1}(k) + K_m \alpha_{m-1}(m-k)$$

$$= \alpha_{m-1}(k) + \alpha_m(m)\, \alpha_{m-1}(m-k)$$

$1 \le k \le$

$m = 1, 2 \ldots N$

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\# Conversion of direct form FIR filter coefficients to lattice coefficients

$$\because A_m(z) = A_{m-1}(z) + K_m z^{-1} B_{m-1}(z)$$

$$= A_{m-1}(z) + K_m [B_m(z) - K_m A_{m-1}(z)]$$

$$A_{m-1}(z) = \frac{A_m(z) - K_m B_m(z)}{1 - K_m^2}, \quad m = N-1, N-2, \ldots 1$$

Eg: Determine the lattice coefficients corresponding to the FIR filter with Sm function $H(z) = A_3(z) = 1 + \frac{13}{24} z^{-1} + \frac{5}{8} z^{-2} + \frac{1}{3} z^{-3}$

Sol:

$$\because K_3 = \alpha_3(3) = \frac{1}{3}$$

$$B_3(z) = \frac{1}{3} + \frac{5}{8} z^{-1} + \frac{13}{24} z^{-2} + z^{-3}$$

$$A_2(z) = \frac{A_3(z) - K_3 B_3(z)}{1 - K_3^2}$$

$$A_2(z) = 1 + \frac{3}{8} z^{-1} + \frac{1}{2} z^{-2}$$

Hence $K_2 = \alpha_2(2) = \frac{1}{2}$ & $B_2(z) = \frac{1}{2} + \frac{3}{8} z^{-1} + z^{-2}$

$$A_1(z) = \frac{A_2(z) - K_2 B_2(z)}{1 - K_2^2} = 1 + \frac{1}{4} z^{-1}$$

$$\therefore K_1 = \alpha_1(1) = \frac{1}{4}$$

$\therefore \; K_m = \alpha_m(m) \;;\; \alpha_{m-1}(0) = 1$

$$\alpha_{m-1}(k) = \frac{\alpha_m(k) - K_m \beta_m(k)}{1 - K_m^2}$$

$$= \frac{\alpha_m(k) - \alpha_m(m)\,\alpha_m(m-k)}{1 - \alpha_m^2(m)} \;;\; 1 \le k \le m$$

* Consider an FIR filter with S/m function

$$H(z) = 1 + 2.88 z^{-1} + 3.4048 z^{-2} + 1.74 z^{-3} + 0.4 z^{-4}$$

sketch the direct form & lattice realisation of this filter. Is the S/m minimum phase

Sol: Given

$$A_4(z) = H(z) = 1 + 2.88 z^{-1} + 3.4048 z^{-2} + 1.74 z^{-3} + 0.4 z^{-4}$$

$$B_4(z) = 0.4 + 1.74 z^{-1} + 3.4048 z^{-2} + 2.88 z^{-3} + z^{-4}$$

$\therefore K_4 = 0.4$

$$A_3(z) = \frac{A_4(z) - K_4 B_4(z)}{1 - K_4^2} = 1 + 2.6 z^{-1} + 2.432 z^{-2} + 0.7 z^{-3}$$

$\therefore K_3 = 0.7$ & $B_3(z) = 0.7 + 2.432 z^{-1} + 2.6 z^{-2} + z^{-3}$

$$A_2(z) = \frac{A_3(z) - K_3 B_3(z)}{1 - K_3^2} = 1 + 1.76 z^{-1} + 1.2 z^{-2}$$

$$B_2(z) = 1.2 + 1.76 z^{-1} + z^{-2}$$

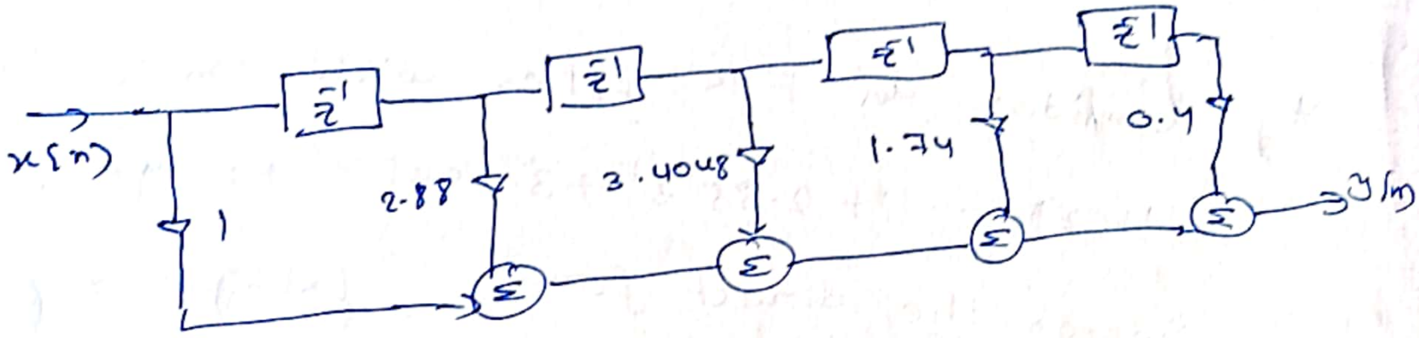
$\therefore K_2 = 1.2$

$$\because A_1(z) = \frac{A_2(z) - K_2 B_2(z)}{1 - K_2^2} = 1 + 0.8 z^{-1}$$

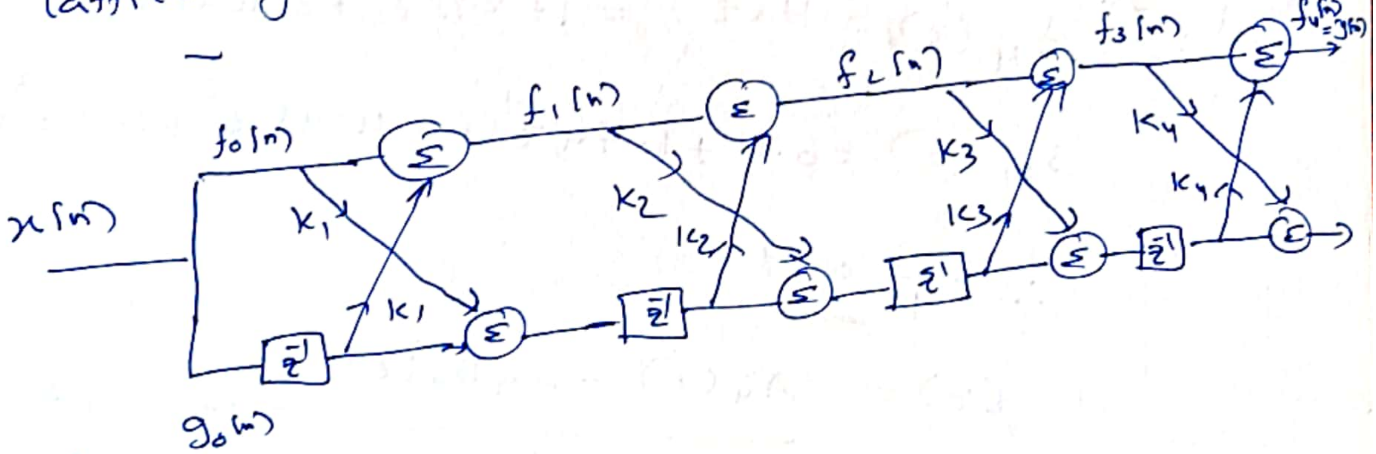
$$\therefore K_1 = 0.8$$

$\because K_2 > 1$, the s/m is not minimum phase

Direct-form



lattice form



* Realise the system with $H(z) = 5 + 3z^{-1}$ by using lattice structure

Sol: Given $H(z) = \frac{Y(z)}{X(z)} = 5 + 3z^{-1}$

$$Y(z) = 5X(z) + 3z^{-1}X(z)$$

apply inverse Z-T

$$y(n) = 5x(n) + 3x[n-1]$$

$$= 5\left[x(n) + \frac{3}{5}x[n-1]\right]$$

Comparing with standard

$$K_1 = \frac{3}{5}$$

$\therefore$

* Determine Filter T.F using the lattice params

$K_1 = 1/2$, $K_2 = 0.6$, $K_3 = -0.7$ & $K_4 = 1/3$

Sol: let $A_0(z) = B_0(z) = 1$

$$A_1(z) = A_0(z) + K_1 B_0(z) z^{-1} = 1 + \frac{1}{2}z^{-1}$$

$$B_1(z) = \frac{1}{2} + z^{-1}$$

$$A_2(z) = A_1(z) + K_2 B_1(z) z^{-1} = 1 + 0.8z^{-1} + 0.6z^{-2}$$

$$\therefore A_4(z) = 1 + 0.146z^{-1} + 0.053z^{-2} - 0.593z^{-3} + \frac{1}{3}z^{-4}$$

* Determine the lattice coefficients corresponding to the FIR s/m with the s/m function $H(z) = 1 + \frac{7}{9} z^{-1} + \frac{3}{5} z^{-2}$ and realise it

Sol: Given $H(z) = \frac{Y(z)}{X(z)} = 1 + \frac{7}{9} z^{-1} + \frac{3}{5} z^{-2}$

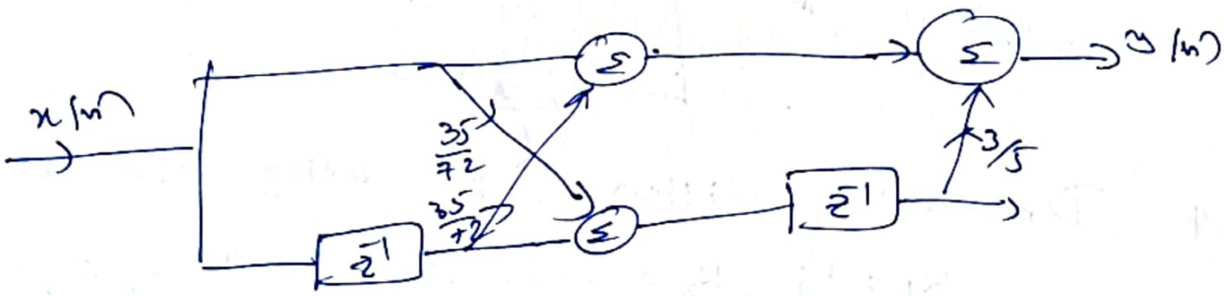
$$Y(z) = X(z) + \frac{7}{9} z^{-1} X(z) + \frac{3}{5} z^{-2} X(z)$$

Taking inverse z-T on bothsides

$$y(n) = x(n) + \frac{7}{9} x(n-1) + \frac{3}{5} x(n-2)$$

Compare $K_2 = \frac{3}{5}$

$$K_1 (1 + K_2) = \frac{7}{9} \Rightarrow K_1 = \frac{35}{72}$$



* Comparison of FIR & IIR filters

FIR	IIR
* Linear phase chars can be easily achieved	* Linear phase char's can not be achieved
* FIR filters can be realised recursively & non-recursively	* IIR filters are easily realized recursively
* The digital filter can be directly designed to achieve the desired specificaties	* The design involves design of analog filter and then transforming analog to digital filter
* Only a finite no. of samples of impulse response are considered	* All the infinite samples of impulse response are considered
* Errors due to round off noise less severe in FIR filter	* Round off noise in IIR filters are more
* Greater flexibility to control the shape of their response	* Less flexibility
* FIR filters are always stable	* IIR filters are not always stable