UNIT-1

TORSION OF CIRCULAR SHAFT

TORSION

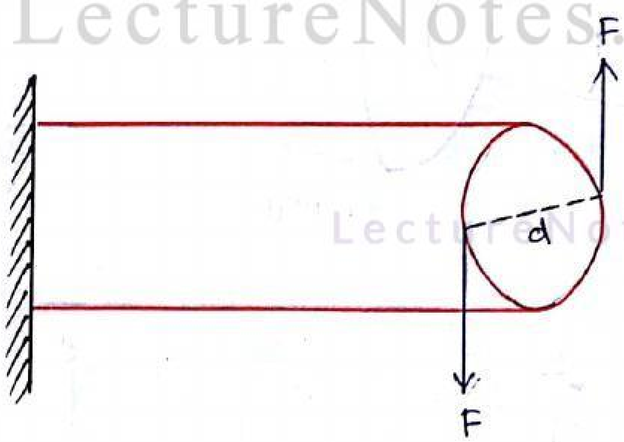
Twisting or Turning moment

When a pair of forces of equal magnitude but opposite directions acting on body, it tend to twist the body.

It is known as Twisting moment (or) Torsional moment (or) Simply as Torque.

Torque is equal to the product of the force applied and the distance between the point of application of the force and the axis of the shaft

One end is fixed and the other end is free



$$\boxed{T = F \times d}$$

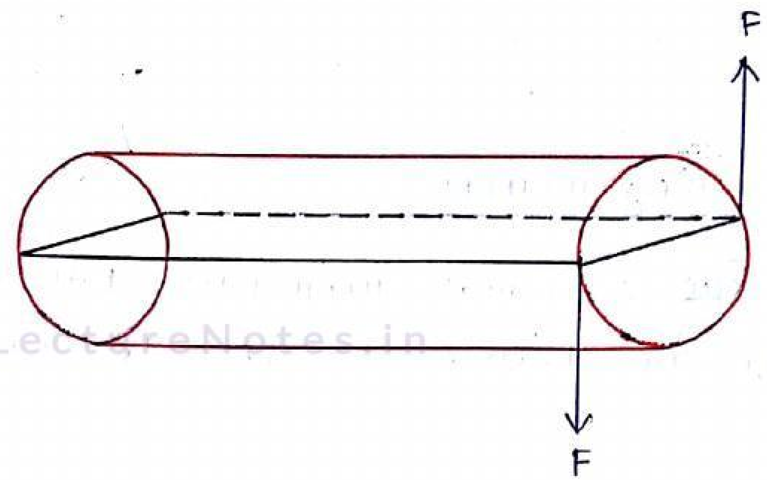
Where,

T - Torque (Nm)

F - Tangential Force (N)

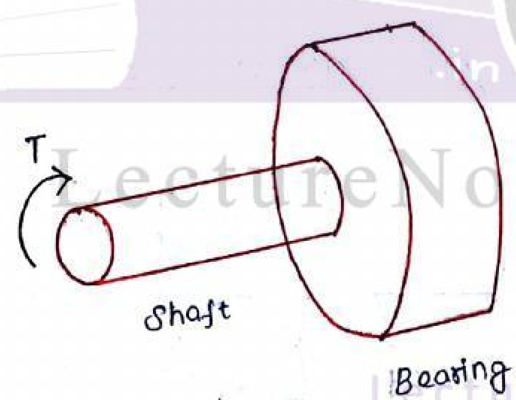
d - Diameter of the shaft (m)

Both the end of the Shaft are Free



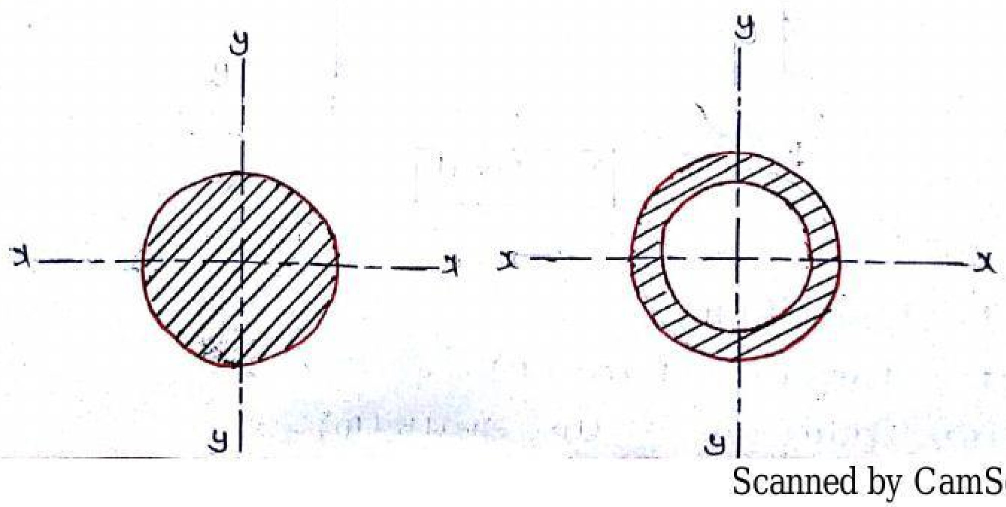
SHAFT

Shaft is a rotating machine element which are used to Transmit power in machines.

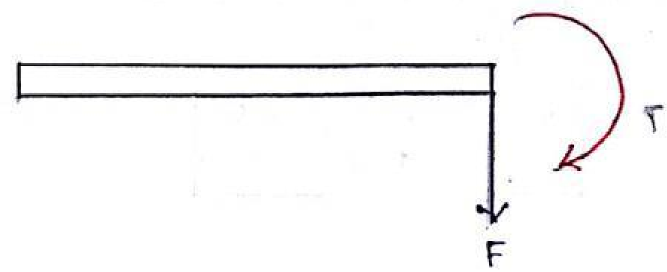


They may be

* Solid
* Hollow



TORQUE

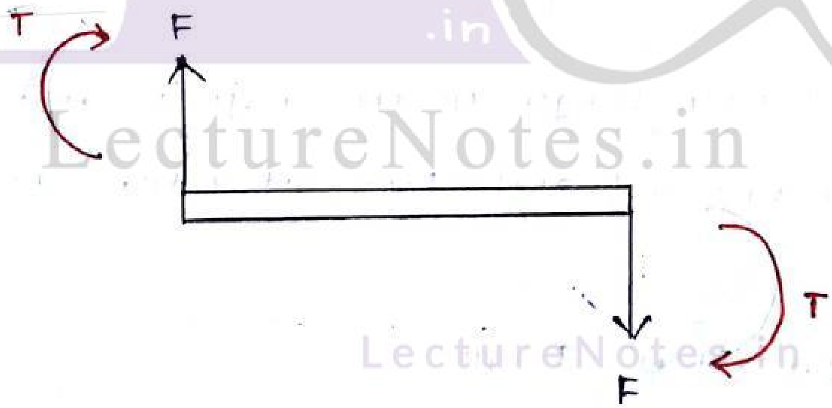


Torque is produced due to a single force. It is the Turning effect of a force.

Example: We Rotate a spanner in a clockwise direction to Tighten a Nut.

The Turning effect produced is called the Torque or Moment of Force.

COUPLE



Couple is produced due to Two Forces that are equal magnitude but in opposite direction (Unlike parallel Forces) but do not have the same Line of action.

Example: Double arm spanner, Steering wheel etc.,

DERIVATION OF TORSION EQUATION

Torsion equation for circular shaft

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}}$$

T - Torque (or) Twisting moment (Nmm)

J - Polar moment of Inertia (mm^4)

τ - Shear stress (N/mm^2)

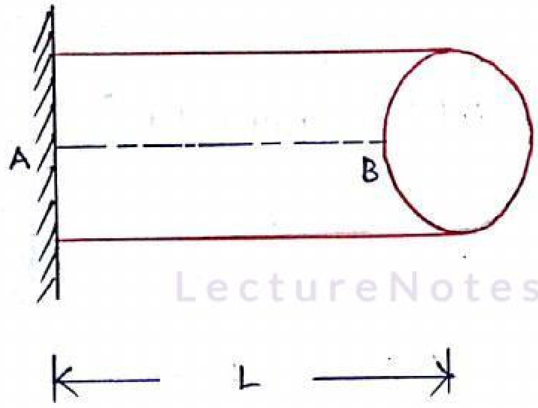
R - Radius of the circular shaft (mm)

C - Modulus of Rigidity (N/mm^2)

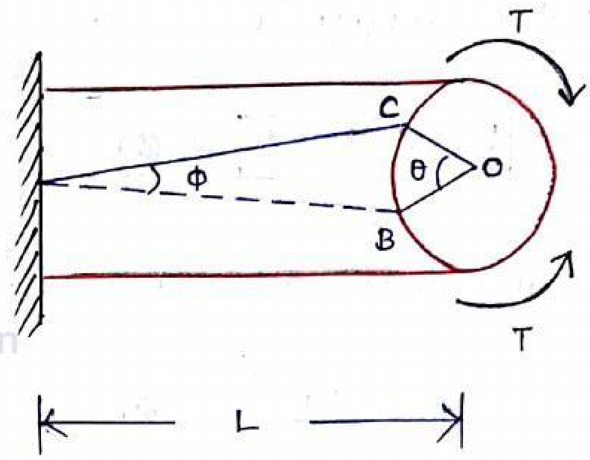
θ - Angle of Twist

L - Length of Shaft

* Let us consider length of the shaft 'L' and Radius
* Fixed at one end and free at other end as shown in fig.
* It is subjected 'Torque'
* 'AB' is a line drawn parallel to the axis of shaft.
* As a result of Torque the free end will rotate in clockwise direction.
* The point 'B' will shifted to 'C'



BEFORE TWISTING

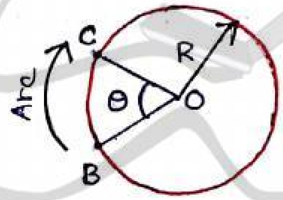


AFTER TWISTING

Trignomentry property

$$\tan\phi = \frac{OPP}{Adj}$$

$$= \frac{BC}{AB}$$



CROSS SECTION

$$\tan\phi = \frac{BC}{L}$$

Every cross section of the shaft will be subjected to shear stress

let,

$BAC = \phi$

$BOC = \theta$

Since ϕ is very small angle

$$\tan\phi = \phi$$

Scanned by CamScanner

$$\boxed{\phi = \frac{BC}{L}} \rightarrow (1)$$

Also the length of the arc 'BC'

$$\boxed{BC = R \times \theta} \rightarrow (2) \qquad \therefore (\text{Arc length} = R\theta)$$

From equation (1) & (2)

$$\boxed{\phi = \frac{R\theta}{L}} \rightarrow (3)$$

We know that,

$$\text{Modulus of Rigidity} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

From the Triangle, BAC

$$C = \frac{\tau}{\phi}$$

$$\boxed{\tau = C\phi} \rightarrow (4)$$

From equation (3) & (4)

$$\tau = C \times \frac{R\theta}{L}$$

$$\boxed{\frac{\tau}{R} = \frac{C\theta}{L}} \rightarrow (5)$$

For a given shaft subjected to given Torque (T)

The value of C, θ, L are constant

$$\therefore \frac{\tau}{R} = \text{Constant}$$

* Therefore shear stress at any point in the shaft is proportional to the distance of the point from the axis of the shaft.

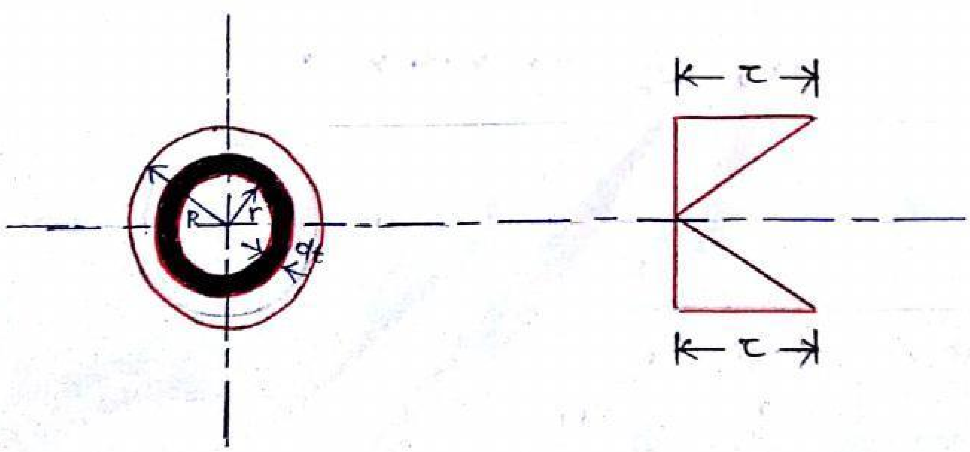
* Hence, the shear stress is maximum at the outer surface and shear is zero at the axis of the shaft.

* As we discussed earlier, whenever a Torque is applied on a circular shaft, Internal shear stresses are induced in the cross section of the shaft.

* The Resultant of these shear stresses from a couple about the longitudinal axis of the shaft.

* This couple, which is numerically equal to the applied external Torque, and is termed as Torsion moment of resistance.

* Now, let us consider an element area 'dA' on the cross section of a solid circular shaft, which is at a radius of 'r' from the centre 'O' of the section as shown in fig.



let,

dr - Thickness of the element

τ_r - Shear stress at the radius 'r'

Also, we know that

$$\frac{\tau}{R} = \text{constant}$$

$$\frac{\tau}{R} = \frac{\tau_r}{r}$$

$$\tau_r = r \times \frac{\tau}{R}$$

Shear resistance offered by the element

$$dF = \tau_r \times dA$$

$$= \left(\frac{\tau}{R}\right) \times r \cdot dA$$

$$\text{stress} = \frac{\text{force}}{\text{Area}}$$

force = stress × Area

$$dF = \tau_r \times dA$$

Moment of resistance offered by the element

$$dT = \frac{\tau}{R} \times r \times dA \times r$$

$$dT = \frac{\tau}{R} \times r^2 \times dA$$

Total Torque,

$$T = \int_0^R dT$$

$$= \left(\int_0^R \frac{\tau}{R} \times r^2 \right) \times dA$$

$$T = \frac{\tau}{R} \int_0^R r^2 \cdot dA$$

∴ We know that $\int_0^R r^2 \cdot dA$ = Moment of Inertia of the Circular section about an axis perpendicular to the plane of the section i.e., Polar Moment of Inertia.

$$T = \frac{\tau}{R} \times J$$

$$\frac{T}{J} = \frac{\tau}{R} \qquad \therefore \left(\frac{\tau}{R} = \frac{C\theta}{L} \right)$$

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}}$$

MAXIMUM TORQUE TRANSMITTED BY A CIRCULAR SOLID SHAFT

The maximum Torque transmitted by a Circular Solid Shaft, is obtained from the maximum stress induced at the outer surface of the solid shaft.

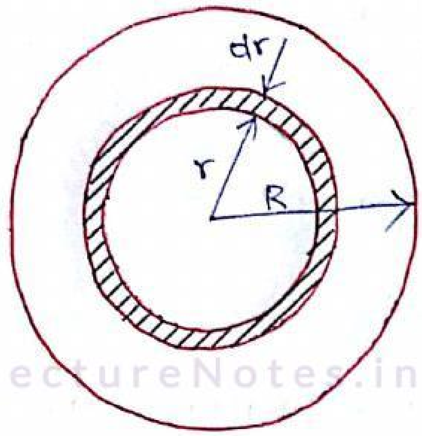
Consider a shaft subjected a Torque 'T' as shown in fig.

let,

τ - Maximum shear stress induced at the outer face

R - Radius of the shaft.

τ_r - Shear stress at a radius 'r' from the centre



$\therefore$ (Circumference of circle $= 2\pi r$)

Consider an elementary circular ring of Thickness 'dr' at a distance 'r' from the centre as shown in fig. Then the area of the ring.

$$dA = 2\pi r \cdot dr.$$

From equation (6) we have

$$\frac{\tau}{R} = \frac{\tau_r}{r}$$

$\therefore$ Shear stress at the radius 'r'

$$\tau_r = \frac{\tau}{R} \cdot r$$

$$\tau_r = \tau \cdot \frac{r}{R}$$

$\therefore$ Turning force on the Elementary circular ring (dF)

dF = Shear stress acting on the ring x Area of ring

$\therefore$ (Force = Stress x Area)

$= \tau_r \times dA$

$\therefore \left(\tau_r = \tau \cdot \frac{r}{R}\right)$

$= \tau \times \frac{r}{R} \times 2\pi r \cdot dr$

$$dF = \frac{\tau}{R} \times 2\pi r^2 \cdot dr$$

Now Turning moment due to turning force on the elementary ring (dT)

dT = Turning force on the ring x Distance of the ring from the axis

$\therefore$ (Moment = Force x distance)

$$= \frac{\tau}{R} \times 2\pi r^2 \cdot dr \times r$$

$$dT = \frac{\tau}{R} \times 2\pi r^3 \cdot dr$$

$\therefore$ The total Turning moment or Total torque is obtained by Integrating the above equation between the Limit 0 and R

$$T = \int_0^R dT = \int_0^R \frac{\tau}{R} \times 2\pi r^3 \cdot dr$$

$$= \frac{\tau}{R} \int_0^R 2\pi r^3 \cdot dr$$

$$= \frac{\tau}{R} \times 2\pi \int_0^R r^3 \cdot dr$$

$$= \frac{\tau}{R} \times 2\pi \left[\frac{R^4}{4}\right] - \frac{\tau}{R} \times 2\pi \left[\frac{0^4}{4}\right]$$

$$= \frac{\tau}{R} \times 2\pi \times \frac{R^4}{4}$$

$$= \tau \times \frac{\pi}{2} \times R^3$$

$$= \tau \times \frac{\pi}{2} \times \left(\frac{D}{2}\right)^3$$

$$= \tau \times \frac{\pi}{2} \times \frac{D^3}{8} \qquad \therefore \left(R = \frac{D}{2}\right)$$

$$= \tau \times \frac{\pi D^3}{16}$$

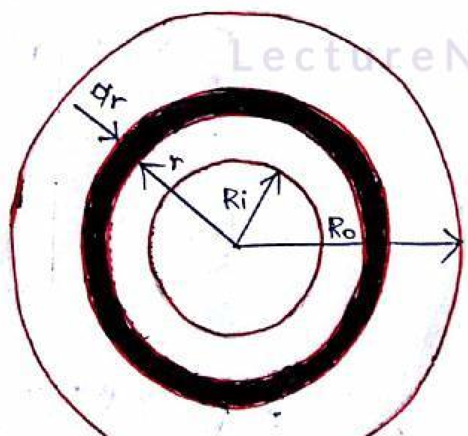
$$\boxed{T = \frac{\pi}{16} \times \tau D^3}$$

TORQUE TRANSMITTED BY A HOLLOW CIRCULAR SHAFT

Torque transmitted by a hollow circular shaft is obtained in the same way as for a solid shaft.

Consider a hollow shaft let it subjected to a Torque 'T' as shown in fig.

Take an elementary circular ring of Thickness 'dr' at a distance 'r' from the centre as shown in fig.



Hollow Shaft

Let,

R_o - Outer radius of the shaft

R_i - Inner radius of the shaft

r - Radius of elementary circular ring

dr - Thickness of the ring

τ - Maximum shear stress induced at outer surface of the shaft.

τ_r - Shear stress induced on the elementary ring

dA - Area of the elementary circular ring $\Rightarrow (2\pi r \times dr)$

Shear stress at the elementary ring is obtained

From equation (6)

$$\frac{\tau}{R_o} = \frac{\tau_r}{r}$$

$$\tau_r = \frac{\tau}{R_o} \times r$$

Turning force on the ring $(dF) = \tau_r \times dA$

$$= \left(\frac{\tau}{R_o} \times r\right) \times 2\pi r \cdot dr$$

$$= 2\pi \frac{\tau}{R_o} r^2 \cdot dr$$

Turning Moment (dT)

dT = Turning force $\times$ Distance of the ring from centre

$$= 2\pi \frac{\tau}{R_o} r^2 \cdot dr \times r$$

$$dT = 2\pi \frac{\tau}{R_o} r^3 . dr$$

The total Turning moment or Total torque T is obtained by Integrating the above equation between the Limit R_i and R_o

$$T = \int_{R_i}^{R_o} dT = \int_{R_i}^{R_o} 2\pi \frac{\tau}{R_o} r^3 . dr$$

$$= 2\pi \frac{\tau}{R_o} \int_{R_i}^{R_o} r^3 . dr$$

$$= 2\pi \frac{\tau}{R_o} \left[\frac{r^4}{4}\right]$$

$$= 2\pi \frac{\tau}{R_o} \left[\frac{R_o^4}{4} - \frac{R_i^4}{4}\right]$$

$$= 2\pi \frac{\tau}{R_o} \left[\frac{R_o^4 - R_i^4}{4}\right]$$

$$= \frac{2\pi}{4} . \tau \left[\frac{R_o^4 - R_i^4}{4}\right]$$

$$T = \frac{\pi}{2} \tau \left[\frac{R_o^4 - R_i^4}{R_o}\right]$$

Let,

D_o - Outer diameter of the shaft

D_i - Inner diameter of the shaft

Then,

$$R_o = \frac{D_o}{2} \quad \text{and} \quad R_i = \frac{D_i}{2}$$

Sub. the values of R_o and R_i in equation

$$T = \frac{\pi}{2} \cdot \tau \left[\frac{\left(\frac{D_o}{2}\right)^4 - \left(\frac{D_i}{2}\right)^4}{\left(\frac{D_o}{2}\right)} \right]$$

$$= \frac{\pi}{2} \cdot \tau \left[\frac{\frac{D_o^4}{16} - \frac{D_i^4}{16}}{\frac{D_o}{2}} \right]$$

$$= \frac{\pi}{2} \cdot \tau \left[\frac{D_o^4 - D_i^4}{\cancel{16}\,8} \times \frac{\cancel{2}}{D_o} \right]$$

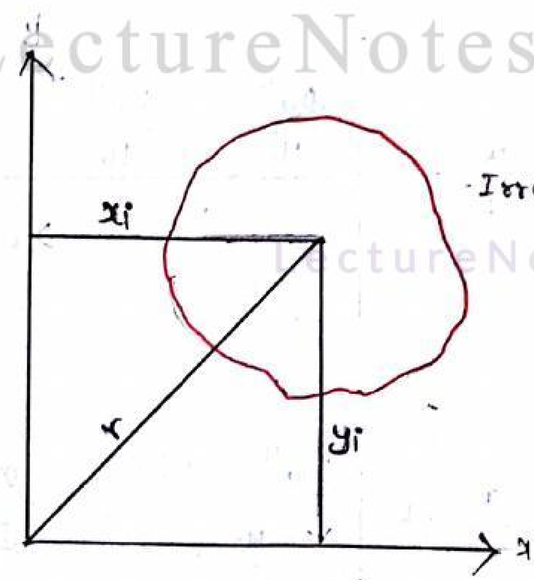
$$\boxed{T = \frac{\pi}{16} \cdot \tau \left[\frac{D_o^4 - D_i^4}{D_o} \right]}$$

ASSUMPTION MADE IN TORSION EQUATION

The derivation of shear stress produced in a circular shaft subjected to torsion, is based on the following assumption

* The material of the shaft is uniform throughout
* The twist along the shaft is uniform
* The shaft is of uniform circular section throughout.
* Cross section of the shaft, which are plane before twist remain plane after twist.
* All radii which are straight before twist remain straight after twist.

POLAR MOMENT OF INERTIA



Moment of Inertia about x axis

$$I_{xx} = A \times (y_c)^2$$

$$I_{xx} = \int_i y_i^2 \cdot dA_i$$

Moment of Inertia about y axis

$$I_{yy} = \int_i x_i^2 \cdot dA_i$$

Where,

I_{xx} - MOI of a body about x axis

I_{yy} - MOI of a body about y axis

A - Area of Total body

y_c - Equal to the distance of the centroid from x axis and y axis

i - Any point of the body

Moment of Inertia about z axis

$$I_{zz} = \int_i r^2 . dA_i$$

$$= \int_i x_i^2 dA_i + \int_i y_i^2 \cdot dA_i$$

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

MOI about z axis is equal to the sum total of MOI x-axis and y axis

POLAR MOMENT OF INERTIA

The polar moment of Inertia of a

* Solid shaft
* Hollow shaft

The Moment of Inertia of any section relative to axis perpendicular to the plane of cross section is known as polar moment of Inertia.

It is denoted by 'J'

The axis perpendicular to the plane of cross section called polar axis.

The polar moment of Inertia is equal to the sum of the moment of Inertia about any two mutually perpendicular axis in its plane which intersect on the polar axis

let,

I_{xx} - Moment of Inertia about x-axis

I_{yy} - Moment of Inertia about y-axis

I_{zz} - Moment of Inertia about z-axis

$$I_{zz} = I_{xx} + I_{yy}$$

Polar moment of Inertia is Indicated by the letter 'J'

$$\boxed{J = I_{xx} + I_{yy}}$$

The Units are 'mm^4'

$$I_{xx} = \int_i y_i^2 \cdot dA_i$$

Moment of Inertia about y axis

$$I_{yy} = \int_i x_i^2 \cdot dA_i$$

Where,

I_{xx} - MOI of a body about x axis

I_{yy} - MOI of a body about y axis

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y_c - Equal to the distance of the centroid from x axis and y axis

i - Any point of the body

Moment of Inertia about z axis

$$I_{zz} = \int_i r^2 \cdot dA_i$$

$$= \int_i x_i^2 \, dA_i + \int_i y_i^2 \cdot dA_i$$

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

MOI about z axis is equal to the sum total of MOI x-axis and y axis

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The polar moment of Inertia is equal to the sum of the moment of Inertia about any two mutually perpendicular axis in its plane which intersect on the polar axis

let,

I_{xx} - Moment of Inertia about x-axis

I_{yy} - Moment of Inertia about y-axis

I_{zz} - Moment of Inertia about z-axis

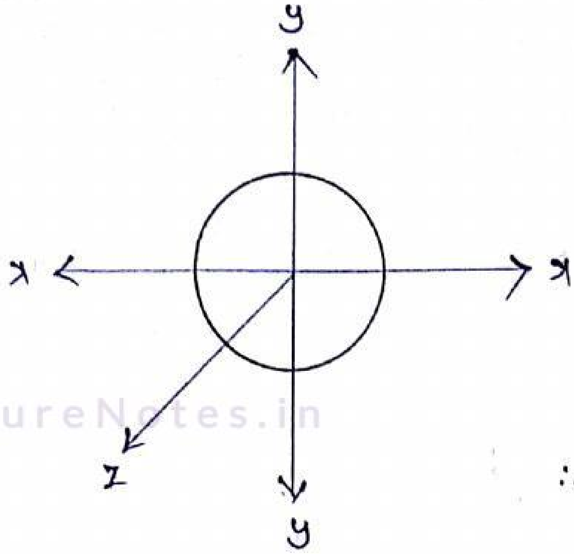
$$I_{zz} = I_{xx} + I_{yy}$$

Polar moment of Inertia is indicated by the letter 'J'

$$\boxed{J = I_{xx} + I_{yy}}$$

The units are 'mm^4'

Polar Section modulus of a Solid Shaft



∴ (Due to Symmetry)

Where,

d – diameter of solid shaft

For Circular section

$$J = I_{xx} + I_{yy}$$

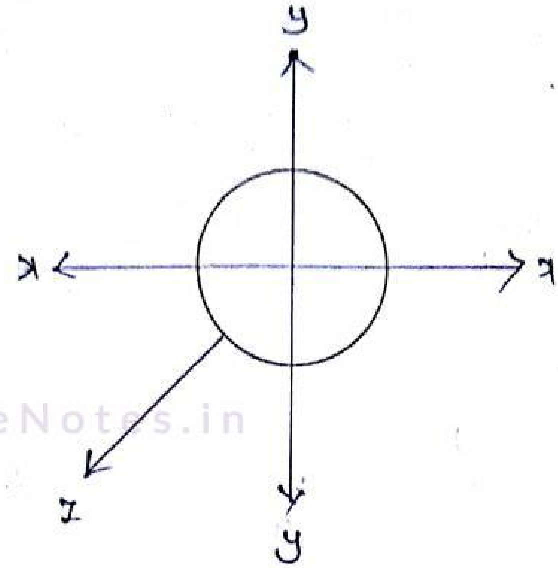
$$= \frac{\pi}{64} d^4 + \frac{\pi}{64} d^4$$

$$= \frac{\pi d^4 + \pi d^4}{64}$$

$$= \frac{\cancel{2}^{1} \pi d^4}{\cancel{64}_{32}}$$

$$\boxed{J = \frac{\pi d^4}{32}}$$

Polar Section modulus of a Hollow shaft



∴ (Due to symmetry)

D_o - External diameter of hollow shaft

D_i - Internal diameter of hollow shaft

Then,

Polar moment of inertia

$$J = I_{xx} + I_{yy}$$

$$= \frac{\pi}{64}(D_o^4 - D_i^4) + \frac{\pi}{64}(D_o^4 - D_i^4)$$

$$= \frac{\pi(D_o^4 - D_i^4) + \pi(D_o^4 - D_i^4)}{64}$$

$$= \frac{\cancel{2}^{1}\pi(D_o^4 - D_i^4)}{\cancel{64}\,32}$$

$$\boxed{J = \frac{\pi(D_o^4 - D_i^4)}{32}}$$

POLAR MODULUS

- Polar modulus is defined as the ratio of the polar moment of Inertia to the radius of the shaft. It is also called Torsional section modulus. It is denoted by Z_p

$$\text{Polar modulus} = \frac{\text{Polar moment of Inertia}}{\text{Radius}}$$

$$\boxed{Z_p = \frac{J}{R}}$$

(i) For a Solid Shaft

$$J = \frac{\pi}{32} D^4$$

$$Z_p = \frac{\frac{\pi}{32} D^4}{R} \qquad \therefore \left(R = \frac{D}{2}\right)$$

$$= \frac{\frac{\pi}{32} D^4}{D/2}$$

$$= \frac{\pi}{\cancel{32}_{16}} D^{\cancel{4}\,3} \times \frac{2}{D}$$

$$\boxed{Z_p = \frac{\pi}{16} D^3}$$

(ii) For a Hollow Shaft

$$J = \frac{\pi}{32} \left(D_o^4 - D_i^4\right)$$

$$I_p = \frac{\frac{\pi}{32}\left[D_o^4 - D_i^4\right]}{R}$$

Where,

R - Outer radius

$$= \frac{\frac{\pi}{32}\left[D_o^4 - D_i^4\right]}{D_o/2} \quad \therefore \left(R = \frac{D_o}{2}\right)$$

$$= \frac{\pi}{32}\left[D_o^4 - D_i^4\right] \times \frac{2}{D_o}$$

$$\boxed{I_p = \frac{\pi}{16 D_o} \times \left[D_o^4 - D_i^4\right]}$$

POWER TRANSMITTED BY SHAFT

$$P = \text{Torque} \times \text{Angular velocity}$$

$$= T \times \omega$$

$$= T \times \frac{2\pi N}{60} \quad \therefore \left(\omega = \frac{2\pi N}{60}\right)$$

$$\boxed{P = \frac{2\pi N T}{60} \text{ watts}}$$

Where,

P - Power (KW)

N - No. of Rotation (rpm)

T - Mean torque transmitted (KN.m)

ω - Angular speed of shaft

Problem on Torsion

Prblm. no: 1

Two shafts of the same material and of same length are subjected to the same torque. if the first shaft is of a solid circular section and the second shaft is of hollow circular section, whose internal diameter is 2/3 of the outside diameter and the maximum shear stress developed in each shaft is the same, compare the weights of the shafts.

Given data

Two shaft of the same material and same length (one is solid and other is Hollow) transmit the same torque and develops the same maximum stress.

Let,

T - Torque transmitted by each shaft

τ - Max. shear stress developed in each shaft

D - Outer diameter of the solid shaft

D_o - Outer diameter of the Hollow shaft

D_i - Inner diameter of the hollow shaft $= \frac{2}{3} D_o$

W_s - Weight of the solid shaft

W_h - Weight of the hollow shaft

L - length of each shaft

ω - Weight density of the material of each shaft

Solution

Torque transmitted by the solid shaft

$$\boxed{T = \frac{\pi}{16} \tau D^3} \rightarrow (1)$$

Torque transmitted by the Hollow shaft

$$T = \frac{\pi}{16} \tau \left[\frac{D_o^4 - D_i^4}{D_o} \right]$$

$$= \frac{\pi}{16} \tau \left[\frac{D_o^4 - \left(\frac{2}{3} D_o\right)^4}{D_o} \right] \therefore \left(D_i = \frac{2}{3} D_o \right)$$

$$= \frac{\pi}{16} \tau \left[\frac{D_o^4 - \frac{(2)^4}{(3)^4} D_o^4}{D_o} \right]$$

$$= \frac{\pi}{16} \tau \left[\frac{D_o^4 - \frac{16}{81} D_o^4}{D_o} \right]$$

$$= \frac{\pi}{16} \tau \left[\frac{81 D_o^4 - 16 D_o^4}{81 \times D_o} \right]$$

$$= \frac{\pi}{16} \tau \times \frac{65 D_o^{\cancel{4}\,3}}{81 \times \cancel{D_o}}$$

$$\boxed{T = \frac{\pi}{16} \tau \times \frac{65 D_o^3}{81}} \rightarrow (2)$$

Torque transmitted by solid and hollow shaft are equal hence equating equation ① & ②

$$\frac{\pi}{16} \tau D^3 = \frac{\pi}{16} \tau \times \frac{65}{81} D_o^3$$

$\therefore$ (Cancelling $\frac{\pi}{16} \tau$ from both sides)

$$D^3 = \frac{65}{81} D_o^3$$

$$D = \left[\frac{65}{81} D_o^3\right]^{\frac{1}{3}}$$

$$= \left(\frac{65}{81}\right)^{\frac{1}{3}} \times D_o$$

$$\boxed{D = 0.929\, D_o} \rightarrow ③$$

Now,

Weight of solid shaft (W_s) = Weight density × Volume of solid shaft

= ω × (Area of cross section × Length)

$$\boxed{W_s = \omega \times \frac{\pi}{4} D^2 \times L} \rightarrow ④$$

Weight of hollow shaft (W_h) = Weight density × Volume of Hollow shaft

= ω × Area of cross section × Length of hollow shaft

$$= \omega \times \frac{\pi}{4} [D_o^2 - D_i^2] \times L \qquad \therefore \left(D_i = \frac{2}{3} D_o\right)$$

$$= \omega \times \frac{\pi}{4} [D_o^2 - (2/3\, D_o)^2] \times L$$

$$= \omega \times \frac{\pi}{4} \left[D_o^2 - \frac{(2)^2}{(3)^2} D_o^2\right] \times L$$

$$= \omega \times \frac{\pi}{4} \left[D_o^2 - \frac{4}{9} D_o^2\right] \times L$$

$$W_h = \omega \times \frac{\pi}{4} \left[\frac{9D_o^2 - 4D_o^2}{9}\right] \times L$$

$$\boxed{W_h = \omega \times \frac{\pi}{4} \times \frac{5}{9} D_o^2 \times L} \rightarrow (5)$$

Dividing equation (4) by equation (5)

$$\frac{W_s}{W_h} = \frac{\omega \times \frac{\pi}{4} D^2 \times L}{\omega \times \frac{\pi}{4} \times \frac{5}{9} D_o^2 \times L}$$

$$= \frac{D^2}{\frac{5}{9} D_o^2}$$

$$= \frac{9D^2}{5D_o^2}$$

$$= \frac{9}{5} \times \frac{(0.929 D_o)^2}{D_o^2} \qquad \therefore (D = 0.929 D_o)$$

From equation (1)

$$= \frac{9}{5} \times (0.929)^2 \times \frac{D_o^2}{D_o^2}$$

$$= \frac{9}{5} \times (0.929)^2$$

$$\frac{\text{Weight of Solid Shaft}}{\text{Weight of hollow Shaft}} = \frac{1.55}{1}$$

Prblm. no:2

A Solid Circular Shaft and a hollow Circular Shaft whose Inside diameter is $\left(\frac{3}{4}\right)$ of the outside diameter, are of the same material, of equal length and are required to transmit a given torque. Compare the weight of these two shaft if the maximum shear stress developed in the two shafts are equal.

Given data

Diameter of hollow shaft $(D_i) = \frac{3}{4}$ Dia. at Outside

$$\boxed{D_i = \frac{3}{4} D_o}$$

Let,

L - Length of both Shaft (equal length)

T - Torque transmitted by each Shaft (equal Torque)

τ - Maximum Shear Stress developed in each shaft (equal max. Shear stress)

D - Diameter of Solid Shaft

W_s - Weight of Solid shaft

W_h - Weight of hollow Shaft

$$= \frac{9}{5} \times (0.929)^2 \times \frac{D_o^2}{D_o^2}$$

$$= \frac{9}{5} \times (0.929)^2$$

$$\frac{\text{Weight of Solid Shaft}}{\text{Weight of hollow Shaft}} = \frac{1.55}{1}$$

Prblm. no:2

A Solid Circular Shaft and a hollow Circular shaft whose Inside diameter is $\left(\frac{3}{4}\right)$ of the outside diameter, are of the same material, of equal length and are required to transmit a given torque. Compare the weight of these two shaft if the maximum shear stress developed in the two shafts are equal.

Given data

Diameter of hollow shaft (D_i) = $\frac{3}{4}$ Dia. at Outside

$$\boxed{D_i = \frac{3}{4} D_o}$$

Let,

L - Length of both Shaft (equal length)

T - Torque transmitted by each Shaft (equal Torque)

τ - Maximum Shear Stress developed in each shaft (equal max. Shear Stress)

D - Diameter of Solid Shaft

W_s - Weight of Solid shaft

W_h - Weight of hollow Shaft

Solution

Torque transmitted by a solid shaft

$$\boxed{T = \frac{\pi}{16} \times \tau \times D^3} \rightarrow (1)$$

Torque transmitted by a hollow shaft

$$T = \frac{\pi}{16} \times \tau \times \left[\frac{D_o^4 - D_i^4}{D_o}\right]$$

$$\therefore \left(D_i = \frac{3}{4} D_o\right)$$

$$= \frac{\pi}{16} \times \tau \times \left[\frac{D_o^4 - \left(\frac{3}{4} D_o\right)^4}{D_o}\right]$$

$$= \frac{\pi}{16} \times \tau \times \left[\frac{D_o^4 - \frac{(3)^4}{(4)^4} D_o^4}{D_o}\right]$$

$$= \frac{\pi}{16} \times \tau \times \left[\frac{D_o^4 - \frac{81}{256} D_o^4}{D_o}\right]$$

$$= \frac{\pi}{16} \times \tau \times \frac{256 D_o^4 - 81 D_o^4}{256 D_o}$$

$$= \frac{\pi}{16} \times \tau \times \frac{175 D_o^{\cancel{4}\,3}}{256 \cancel{D_o}}$$

$$\boxed{T = \frac{\pi}{16} \times \tau \times \frac{175}{256} D_o^3} \rightarrow (2)$$

$$\begin{pmatrix}\text{Torque transmitted by} \\ \text{Solid Shaft}\end{pmatrix} = \begin{pmatrix}\text{Torque transmitted by} \\ \text{Hollow Shaft}\end{pmatrix}$$

Hence equating equation ① and ②

$$\cancel{\frac{\pi}{16}} \times \cancel{\tau} \times D^3 = \cancel{\frac{\pi}{16}} \times \cancel{\tau} \times \frac{175}{256} D_o^3$$

$$D^3 = \frac{175}{256} D_o^3$$

$$D = \left[\frac{175}{256} D_o^3\right]^{\frac{1}{3}}$$

$$= \left(\frac{175}{256}\right)^{\frac{1}{2}} \times D_o$$

$$\boxed{D = 0.880\, D_o} \rightarrow ③$$

Now, Weight of Solid Shaft

W_s = Weight density × Volume of Solid Shaft

= ω × Area of cross section × Length

$$\boxed{W_s = \omega \times \frac{\pi}{4} D^2 \times L} \rightarrow ④$$

Weight of Hollow Shaft

W_h = ω × Area of cross section × Length

$$= \omega \times \frac{\pi}{4}\left[D_o^2 - D_i^2\right] \times L$$

$$= \omega \times \frac{\pi}{4}\left[D_o^2 - \left(\frac{3}{4} D_o\right)^2\right] \times L$$

$$= w \times \frac{\pi}{4}\left[D_0^2 - \frac{(3)^2}{(4)^2} D_0^2\right] \times L$$

$$= w \times \frac{\pi}{4}\left[D_0^2 - \frac{9}{16} D_0^2\right] \times L$$

$$= w \times \frac{\pi}{4} \times \frac{16 D_0^2 - 9 D_0^2}{16} \times L$$

$$\boxed{W_h = w \times \frac{\pi}{4} \times \frac{7}{16} D_0^2 \times L} \rightarrow (5)$$

Diff equation (4) and (5)

$$\frac{W_s}{W_h} = \frac{\cancel{w} \times \frac{\cancel{\pi}}{\cancel{4}} D^2 \times \cancel{L}}{\cancel{w} \times \frac{\cancel{\pi}}{\cancel{4}} \times \frac{7}{16} D_0^2 \times \cancel{L}}$$

$$= \frac{D^2}{(7/16) D_0^2}$$

$$= \frac{16 D^2}{7 D_0^2}$$

$$= \frac{16}{7} \times \frac{(0.880 D_0)^2}{D_0^2}$$

From equtn (1)

$\therefore (D = 0.880 D_0)$

$$= \frac{16}{7} \times (0.880)^2 \times \frac{\cancel{D_0^2}}{\cancel{D_0^2}}$$

$$= 1.77$$

(OR)

$$\frac{W_s}{W_h} = \frac{\cancel{\phi} \times \cancel{\frac{\pi}{4}} D^2 \times \cancel{l}}{\cancel{\phi} \times \cancel{\frac{\pi}{4}} \times \frac{7}{16} D_o^2 \times \cancel{l}}$$

$$= \frac{D^2}{\left(7/16\right) D_o^2}$$

$$= \frac{(0.880 D_o)^2}{0.4375 D_o^2}$$

$$= \frac{(0.880)^2 \times D_o^2}{0.4375 D_o^2}$$

$$\boxed{\frac{W_s}{W_h} = \frac{0.774 D_o^2}{0.4375 D_o^2}}$$

Prblm.no: 3

A Hollow circular shaft 20mm thick transmits 300 kW power at 200 rpm. Determine the external diameter of the shaft if the shear strain due to torsion is not to exceed 0.00086. Take modulus of rigidity $= 0.8 \times 10^5$ N/mm^2

Given data

Thickness (t) = 20 mm $\quad \therefore$ (1 kW = 1000 W)

Power transmitted (P) = 300 kW $\Rightarrow$ 300,000 W

Speed (N) = 200 rpm

Shear strain (ϕ) = 0.00086

Modulus of Rigidity (C) = 0.8×10^5 N/mm^2

Let,

D_o - External dia. of shaft

D_i - Internal dia. of shaft

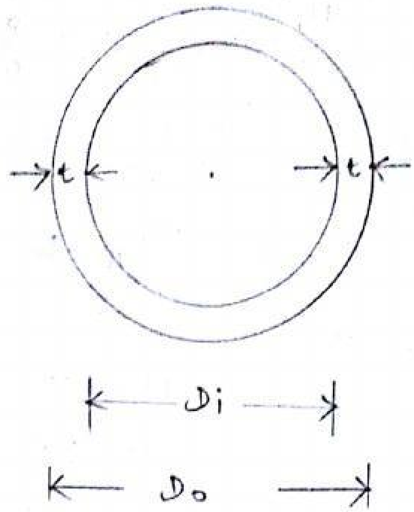
Solution

$$D_o = D_i + 2t$$

$$= D_i + (2 \times 20)$$

$$D_o = D_i + 40$$

$$D_i = D_o - 40 \rightarrow (1)$$



Power transmitted by shaft

$$P = \frac{2\pi N T}{60}$$

$$300 \times 10^3 = \frac{2\pi \times 200 \times T}{60}$$

$$T = \frac{(300 \times 10^3) \times 60}{2\pi \times 200}$$

$$= 14323.9 \text{ Nm} \Rightarrow 14323.9 \times 1000 \text{ Nmm}$$

$$\boxed{T = 14323900 \text{ Nmm}}$$

$$\text{Modulus of Rigidity } (C) = \frac{\text{Shear stress}}{\text{Shear Strain}}$$

$$0.8 \times 10^5 = \frac{\text{Shear Stress}}{0.00086}$$

$$\text{Shear stress} = 0.8 \times 10^5 \times 0.00086$$

$$\boxed{\tau = 68.8\ N/mm^2}$$

Now,

Using Hollow Circular Shaft equation

$$T = \frac{\pi}{16} \times \tau \times \frac{D_o^4 - D_i^4}{D_o}$$

$$14322900 = \frac{\pi}{16} \times 68.8 \times \frac{D_o^4 - (D_o - 40)^4}{D_o}$$

$$\therefore (D_i = D_o - 40)$$

$$\boxed{D_o = 107.94\ mm}$$

$$D_i = D_o - 40$$

$$= 107.94 - 40$$

$$\boxed{D_i = 67.94\ mm}$$

Prblm.no: 4

A hollow shaft of external diameter 120 mm transmits 300 kW power at 200 rpm. Determine the maximum Internal diameter if the maximum stress in the shaft is not to exceed 60 N/mm^2

Given data

External diameter $(D_o) = 120\ mm$

Power $(P) = 300\ kW \Rightarrow 300 \times 10^3\ W$

Speed $(N) = 200\ rpm$

Max. shear stress $(\tau) = 60\ N/mm^2$

$$\boxed{\tau = 68.8\ N/mm^2}$$

Now,

Using Hollow Circular Shaft equation

$$T = \frac{\pi}{16} \times \tau \times \frac{D_o^4 - D_i^4}{D_o}$$

$$14323900 = \frac{\pi}{16} \times 68.8 \times \frac{D_o^4 - (D_o - 40)^4}{D_o}$$

$$\boxed{D_o = 107.94\ mm} \qquad \therefore (D_i = D_o - 40)$$

$$D_i = D_o - 40$$

$$= 107.94 - 40$$

$$\boxed{D_i = 67.94\ mm}$$

Prblm.no: 4

A hollow shaft of external diameter 120 mm transmits 300 kW power at 200 rpm. Determine the maximum internal diameter if the maximum stress in the shaft is not to exceed 60 N/mm²

Given data

External diameter $(D_o) = 120\ mm$

Power $(P) = 300\ kW \Rightarrow 300 \times 10^3\ W$

Speed $(N) = 200\ rpm$

Max. shear stress $(\tau) = 60\ N/mm^2$

Let,

D_i = Internal dia. of Shaft

Solution

Power transmitted by Shaft

$$P = \frac{2\pi N T}{60}$$

$$800 \times 10^3 = \frac{2\pi \times 200 \times T}{60}$$

$$T = \frac{800 \times 10^3 \times 60}{2\pi \times 200}$$

$$= 14323.9 \text{ Nm} \Rightarrow 14323.9 \times 1000 \text{ Nmm}$$

$$\boxed{T = 14323900 \text{ Nmm}}$$

Now,

Using Hollow Circular Shaft equation

$$T = \frac{\pi}{16} \times \tau \times \frac{(D_o^4 - D_i^4)}{D_o}$$

$$14323900 = \frac{\pi}{16} \times 60 \times \frac{(120^4 - D_i^4)}{120}$$

$$\frac{14323900 \times 16 \times 120}{\pi \times 60} = 120^4 - D_i^4$$

$$145902000 = 207360000 - D_i^4$$

$$D_i^4 = 207360000 - 145902000$$

$$D_i = (61458000)^{1/4}$$

$$\boxed{D_i = 88.5\ mm}$$

Prblm.no: 5

Find the maximum shear stress induced in a solid circular shaft of diameter 15 cm when the shaft transmits 150 KW power at 180 rpm.

Given data

Diameter of shaft $(D) = 15cm \Rightarrow 150\ mm$

Power transmitted $(P) = 150\ KW \Rightarrow 150 \times 10^3\ W$

Speed of shaft $(N) = 180\ rpm$

Let,

τ - Maximum shear stress induced in the shaft

Solution

Power transmitted by shaft

$$P = \frac{2\pi NT}{60}$$

$$150 \times 10^3 = \frac{2\pi \times 180 \times T}{60}$$

$$T = \frac{150 \times 10^3 \times 60}{2\pi \times 180}$$

$$= 7957.7\ Nm \Rightarrow 7957.7 \times 1000\ Nmm$$

$$\boxed{T = 7957700\ Nmm}$$

Now,

Using Solid Shaft equation

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$7957700 = \frac{\pi}{16} \times \tau \times 150^3$$

$$\tau = \frac{16 \times 7957700}{\pi \times 150^3}$$

$$\boxed{\tau = 12\ N/mm^2}$$

Prblm.no:6

A Solid Cylindrical Shaft is to transmit 300 KW power at 100 rpm

(a) If the shear stress is not to exceed 80 N/mm^2, find its diameter

(b) What percent saving in weight would be obtained if this shaft is replaced by a hollow one whose internal diameter equal to 0.6 of the external diameter, the length, the material and maximum shear stress being the same?

Given data

$$\text{Power } (P) = 300\,KW \Rightarrow 300 \times 10^3\ W$$

$$\text{Speed } (N) = 100$$

$$\text{Max. shear stress } (\tau) = 80\ N/mm^2$$

Let, D = Dia. of Solid Shaft

Solution

- Power transmitted by shaft

$$P = \frac{2\pi NT}{60}$$

$$800 \times 10^3 = \frac{2\pi \times 100 \times T}{60}$$

$$T = \frac{800 \times 10^3 \times 60}{2\pi \times 100}$$

$$= 28647.8 \text{ Nm} \Rightarrow 28647.8 \times 1000 \text{ Nmm}$$

$$T = 28647800 \text{ Nmm}$$

Now,

Using solid shaft equation

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$28647800 = \frac{\pi}{16} \times 80 \times D^3$$

$$D = \left(\frac{16 \times 28647800}{\pi \times 80}\right)^{1/3}$$

$$= 121.8 \text{mm}$$

$$D = \text{say } 122.0 \text{mm}$$

(b) percent saving in weight

Let,

D_o = External dia. of hollow shaft

D_i = Internal dia. of hollow shaft

$$D_i = 0.6 \times D_o$$

The length, material and maximum shear stress in solid and hollow shafts are given the same.

* Torque, transmitted by solid shaft is equal to the torque transmitted by hollow shaft.

Torque transmitted by hollow shaft

$$T = \frac{\pi}{16} \times \tau \times \frac{(D_o^4 - D_i^4)}{D_o}$$

$$= \frac{\pi}{16} \times \overset{5}{\cancel{80}} \times \frac{[D_o^4 - (0.6 D_o)^4]}{D_o}$$

$$T = \frac{\pi}{16} \times 5 \times \frac{[D_o^4 - (0.6 D_o)^4]}{D_o}$$

Torque transmitted by solid shaft $\quad \therefore (D_i = 0.6 \times D_o)$

$$T = 28647800 \text{ Nmm}$$

$\therefore$ Equating the two Torque, we get

$$28647800 = \pi \times 5 \times \left(\frac{0.8704 D_o^{\cancel{4}\,3}}{\cancel{D_o}}\right)$$

$$= \pi \times 5 \times 0.8704 D_o^3$$

$$D_o^3 = \left(\frac{28647800}{\pi \times 5 \times 0.8704}\right)$$

$$= \left(\frac{28647800}{\pi \times 5 \times 0.8704}\right)^{1/3}$$

$$D_o = 127.96 \text{ mm}$$

Say 128 mm

$\therefore$ Internal diameter $(D_i) = 0.6 \times D_o$

$= 0.6 \times 128$

$$\boxed{D_i = 76.8 \text{ mm}}$$

Now,

Let

W_s - Weight of Solid Shaft

W_h - Weight of Hollow Shaft

W_s = Weight density $\times$ Area of Solid shaft $\times$ Length

$= \omega \times \frac{\pi}{4} D^2 \times L$

Where,

ω - Weight density

Similarly

W_h = Weight density $\times$ Area of hollow shaft $\times$ length

$= \omega \times \frac{\pi}{4} [D_o^2 - D_i^2] \times L$

$\therefore$ (Both shafts are of same length and of same material)

Now,

Percent saving in weight

$$= \frac{W_s - W_h}{W_s} \times 100$$

$$= \frac{10 \times \frac{\pi}{4} D^2 \times L - 10 \times \frac{\pi}{4} [D_o^2 - D_i^2] \times L}{10 \times \frac{\pi}{4} D^2 \times L} \times 100$$

$$= \frac{\cancel{10} \times \cancel{\frac{\pi}{4}} \times \cancel{L} \left[D^2 - (D_o^2 - D_i^2)\right]}{\cancel{10} \times \cancel{\frac{\pi}{4}} D^2 \times \cancel{L}} \times 100$$

$$= \frac{D^2 - (D_o^2 - D_i^2)}{D^2} \times 100$$

$$= \frac{(122)^2 - (128^2 - 76.8^2)}{(122)^2} \times 100$$

$$= \frac{14884 - (16284 - 5898)}{14884} \times 100$$

$$= \frac{14884 - 10486}{14884} \times 100$$

$$\left.\begin{array}{l}\text{Percentage} \\ \text{saving in} \\ \text{Weight}\end{array}\right\} = 29.55\%$$

Prblm. no : 7

A Solid Steel Shaft has to transmit 75 kW at 200 rpm. Taking allowable Shear stress as 70 N/mm², Find suitable diameter for the shaft, if the maximum torque transmitted at each revolution exceeds the means by 30%.

Given data

Power transmitted $(P) = 75\ KW \Rightarrow 75 \times 10^3\ W$

R.P.M of the Shaft $(N) = 200$

Shear Stress $(\tau) = 70\ N/mm^2$

Let,

T - Mean torque transmitted

T_{max} - Maximum torque transmitted $\Rightarrow 1.2 \times T$

D - Suitable diameter of the Shaft

Solution

Power transmitted by Shaft

$$P = \frac{2\pi N T}{60}$$

$$(75 \times 10^3) = \frac{2\pi \times 200 \times T}{60}$$

$$T = \frac{75 \times 10^3 \times 60}{2\pi \times 200}$$

$$= 3580.98\ Nm$$

$$\boxed{T = 3580980\ Nmm} \rightarrow \text{Ans (i)}$$

$$T_{max} = 1.3 \times T$$

$$= 1.3 \times 3580980$$

$$\boxed{T_{max} = 4655274 \text{ N.mm}} \longrightarrow \text{Ans (ii)}$$

Maximum Torque transmitted by a Solid Shaft

$$T_{max} = \frac{\pi}{16} \times \tau \times D^3$$

$$4655274 = \frac{\pi}{16} \times 70 \times D^3$$

$$D^3 = \frac{16 \times 4655274}{\pi \times 70}$$

$$= \left(\frac{16 \times 4655274}{\pi \times 70} \right)^{1/3}$$

$$\boxed{D = 69.57 \text{mm}} \simeq 70 \text{mm} \longrightarrow \text{Ans (iii)}$$

Prblm. no : 8

A Hollow Shaft is to transmit 300 KW power at 80 rpm If the Shear Stress is not to exceed 60 N/mm^2 and the Internal diameter is 0.6 of the external diameter, find the external and Internal diameter assuming that the maximum torque is 1.4 times the mean

Given data

Power transmitted $(P) = 300 \text{ KW} \Rightarrow 300 \times 10^3 \text{ W}$

Speed of the Shaft $(N) = 80 \text{ rpm}$

Maximum shear stress (τ) = 60 N/mm^2

Internal diameter (D_i) = $0.6 \times$ External diameter

Maximum Torque (T_{max}) = $1.4 \times$ Times the mean torque

$= 1.4 \times T$

Let,

Internal diameter (D_i) = ?

External diameter (D_o) = ?

Solution

Power transmitted by Shaft

$$P = \frac{2\pi NT}{60}$$

$$300 \times 10^3 = \frac{2\pi \times 80 \times T}{60}$$

$$T = \frac{(300 \times 10^3) \times 60}{2\pi \times 80}$$

$$\boxed{T = 35809.8 \text{ Nm}}$$

$T_{max} = 1.4 \times T$

$= 1.4 \times 35809.8$

$= 50133.7$ Nm

$$\boxed{T_{max} = 50133700 \text{ Nmm}}$$

Maximum Torque transmitted by a Hollow shaft

$$T_{max} = \frac{\pi}{16} \times \tau \times \left[\frac{D_o^4 - D_i^4}{D_o}\right]$$

$$50133700 = \frac{\pi}{16} \times 60 \times \left[\frac{D_o^4 - (0.6 \times D_o)^4}{D_o}\right]$$

$$= \frac{\pi}{16} \times 60 \times \left[\frac{D_o^4 - 0.6^4 \times D_o^{\cancel{4}\,3}}{\cancel{D_o}}\right]$$

$$= \frac{\pi}{16} \times 60 \times D_o^4 - 0.1296$$

$$= \frac{\pi}{16} \times 60 \times 0.8704 D_o^3$$

$$D_o^3 = \frac{16 \times 50133700}{\pi \times 60 \times 0.8704}$$

$$= \left(\frac{16 \times 50133700}{\pi \times 60 \times 0.8704}\right)^{1/3}$$

$$= 169.2 \text{ mm}$$

$$\boxed{D_o \simeq 170 \text{ mm}} \longrightarrow \text{Ans (i)}$$

$$D_i = 0.6 \times D_o$$

$$= 0.6 \times 169.2$$

$$\boxed{D_i = 102 mm} \longrightarrow \text{Ans (ii)}$$

Prblm.no: 9

A Hollow shaft having an inside diameter 60% of its outer diameter, is to replace a solid shaft transmitting the same power at the same speed. calculate the percentag saving in material, if the material to be used is also the same.

Given data

Let,

D_o - Outer diameter of the Hollow Shaft

D_i - Inner diameter of the Hollow Shaft $\Rightarrow$ 60% of D_o

$$= \frac{60}{100} \times D_o$$

$$D_i = 0.6 \times D_o$$

D - Diameter of the Solid Shaft

P - Power transmitted by hollow shaft or by Solid shaft

N - Speed of each shaft

τ - Maximum shear stress induced in each shaft.

Since material of both shaft is same and hence shear stress will be same.

Solution

Power transmitted by Solid or Hollow shaft

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{60 \times P}{2\pi N} \Rightarrow (\text{Constant})$$

$\therefore$ (P and N are same for solid and hollow shaft)

$\therefore$ Torque transmitted by solid shaft is the same as the torque transmitted by hollow shaft.

Torque transmitted by solid shaft

$$\boxed{T = \frac{\pi}{16} \tau D^3} \rightarrow (1)$$

Torque transmitted by Hollow shaft

$$T = \frac{\pi}{16} \tau \left[\frac{D_o^4 - D_i^4}{D_o}\right]$$

$\therefore (D_i = 0.6 \times D_o)$

$$= \frac{\pi}{16} \tau \left[\frac{D_o^4 - (0.6 \times D_o)^4}{D_o}\right]$$

$$= \frac{\pi}{16} \tau \left[\frac{D_o^{\cancel{4}^3} - 0.1296 D_o^{\cancel{4}^3}}{\cancel{D_o}}\right]$$

$$\boxed{T = \frac{\pi}{16} \tau \times 0.8704 D_o^3} \rightarrow (2)$$

∴ Since the Torque transmitted is the same

Hence equating ① and ②

$$\cancel{\frac{\pi}{16}\tau} D^3 = \cancel{\frac{\pi}{16}\tau} \times 0.8704\, D_o^3$$

$$D^3 = 0.8704\, D_o^3$$

$$= (0.8704)^{1/3} D_o$$

$$\boxed{D = 0.9548\, D_o}$$

Area of Solid Shaft $= \frac{\pi}{4} D^2$

$$= \frac{\pi}{4} \times (0.9548\, D_o)^2$$

$$= 0.716\, D_o^2$$

Area of hollow Shaft $= \frac{\pi}{4}\left[D_o^2 - D_i^2\right]$

$$= \frac{\pi}{4}\left[D_o^2 - (0.6 \times D_o)^2\right]$$

$$= \frac{\pi}{4}\left[D_o^2 - 0.6^2 \times D_o^2\right]$$

$$= \frac{\pi}{4}\left[D_o^2 - 0.36 \times D_o^2\right]$$

$$= \frac{\pi}{4} \times 0.64\, D_o^2$$

$$= 0.502\, D_o^2$$

For the shaft of the same material, the weight of the shaft is proportional to the areas.

$\therefore$ Saving in material = Saving in area

$$= \frac{\text{Area of Solid Shaft} - \text{Area of hollow Shaft}}{\text{Area of Solid Shaft}}$$

$$= \frac{0.716 D_0^2 - 0.502 D_0^2}{0.716 D_0^2}$$

$= 0.2988$

$\therefore$ percentage saving in material $= 0.2988 \times 100$

$= 29.88$

COMBINED BENDING AND TORSION

* When a shaft is transmitting torque or power, it is subjected to shear stresses.
* At the same time the shaft is also subjected to bending moments due to gravity or Inertia loads.
* Due to bending moment, bending stresses are also set up in the shaft.
* Hence each shaft in a particle is subjected to shear stress and bending stress.

For the shaft of the same material, the weight of the shaft is proportional to the areas.

$\therefore$ Saving in material = Saving in area

$$= \frac{\text{Area of Solid Shaft} - \text{Area of hollow Shaft}}{\text{Area of Solid Shaft}}$$

$$= \frac{0.716 D_0^2 - 0.502 D_0^2}{0.716 D_0^2}$$

$= 0.2988$

$\therefore$ percentage saving in material $= 0.2988 \times 100$

$= 29.88$

COMBINED BENDING AND TORSION

* When a shaft is transmitting torque or power, it is subjected to shear stresses.

* At the same time the shaft is also subjected to bending moments due to gravity or inertia loads.

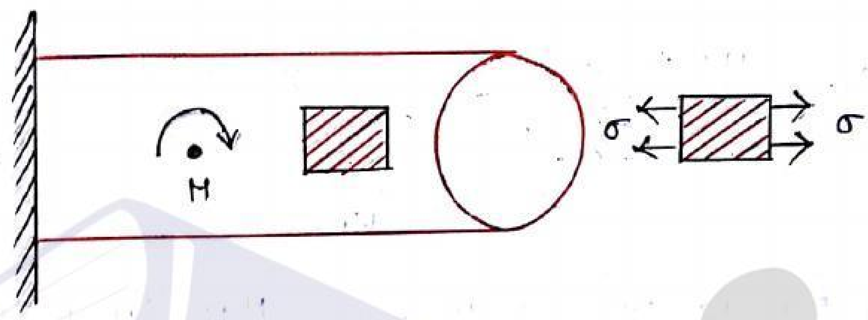
* Due to bending moment, bending stresses are also set up in the shaft.

* Hence each shaft in a particle is subjected to shear stress and bending stress.

* For design purposes it is necessary to find the principal stresses, maximum shear stress and strain energy.

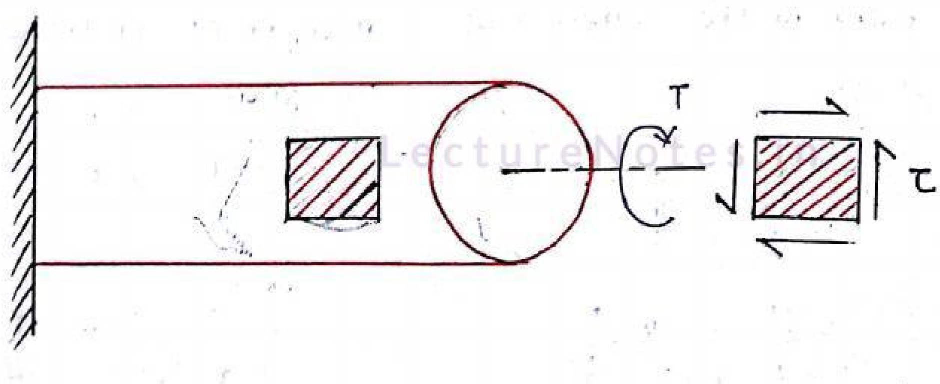
* The principal stresses and maximum shear stress when a shaft is subjected to bending and Torsion

Bending Moment



If a Bending moment is applied on the beam axial stresses (Tension / Compression) are Induced.

Twisting Moment



If a Twisting moment is applied on the beam shear stress are Induced.

Bending equation

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma_b = \frac{M \times y}{I}$$

Where,

σ_b - Max. Bending stress on the surface of the shaft

M - Bending Moment at the section

y -

I - Moment of Inertia

The bending stress is maximum at a point on the surface of the shaft

$$\sigma_b = \frac{M}{I} \times \frac{D}{2}$$

$$= \frac{M}{\frac{\pi}{64} D^{4 \cdot 3}} \times \frac{D}{2}_{16}$$

$$\boxed{\sigma_b = \frac{32M}{\pi D^3}}$$

Torsion equation

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\tau = \frac{T}{J} \times R$$

Where,

τ - Max. Shear stress on the surface of the shaft

T - Torque at the section

J - Polar moment of Inertia

R - Radius

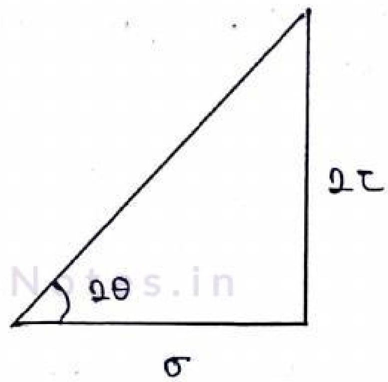
The shear stress is maximum at a point on the surface of the shaft

$$\tau = \frac{T}{J} \times R$$

$$= \frac{T}{\frac{\pi}{32} D^4} \times \frac{D}{2}_{16}$$

$$\boxed{\tau = \frac{16T}{\pi D^3}}$$

We know that the angle θ made by the plane of maximum shear with the Normal cross section is given by



$$\tan 2\theta = \frac{2\tau}{\sigma}$$

$$= \frac{2 \times \frac{16T}{\pi D^3}}{\frac{32M}{\pi D^3}}$$

$$= \frac{32T/\pi D^3}{32M/\pi D^3}$$

$$\tan 2\theta = \frac{T}{M}$$

Major principal stress $= \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$

$$= \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$= \frac{\cancel{32}^{16}M}{\cancel{2} \times \pi D^3} + \sqrt{\left(\frac{\cancel{32}^{16}M}{\cancel{2} \times \pi D^3}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2}$$

$$= \frac{16M}{\pi D^3} + \sqrt{\left(\frac{16M}{\pi D^3}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2}$$

Major principal stress $= \frac{16}{\pi D^3}\left(M + \sqrt{M^2 + T^2}\right)$

Minor principal stress $= \frac{\sigma}{2} - \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$

$$= \frac{\sigma_b}{2} - \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$= \frac{32M}{2 \times \pi D^3} - \sqrt{\left(\frac{32M}{2 \times \pi D^3}\right)^2 + \left(\frac{16T}{\pi D^3}\right)}$$

Minor principal stress $= \frac{16}{\pi D^3}\left(M - \sqrt{M^2 + T^2}\right)$

Maximum shear stress $= \frac{\text{Major principal stress} - \text{Minor principal stress}}{2}$

$$= \frac{\frac{16}{\pi D^3}\left(M + \sqrt{M^2 + T^2}\right) - \frac{16}{\pi D^3}\left(M - \sqrt{M^2 + T^2}\right)}{2}$$

$$= \frac{16}{\pi D^3} \times \frac{\cancel{M} + \sqrt{M^2 + T^2} - \cancel{M} + \sqrt{M^2 + T^2}}{2}$$

$$= \frac{16}{\pi D^3} \times \frac{2\sqrt{M^2 + T^2}}{2}$$

$$\text{Maximum shear stress} = \frac{16}{\pi D^3} \times \sqrt{M^2 + T^2}$$

For a Hollow Shaft

$$\text{Major principal stress} = \frac{16 D_o}{\pi [D_o^4 - D_i^4]} \left(M + \sqrt{M^2 + T^2} \right)$$

$$\text{Minor principal stress} = \frac{16 D_o}{\pi [D_o^4 - D_i^4]} \left(M - \sqrt{M^2 + T^2} \right)$$

$$\text{Maximum shear stress} = \frac{16 D_o}{\pi [D_o^4 - D_i^4]} \left(\sqrt{M^2 + T^2} \right)$$

Prblm. no: 10

A Solid Shaft of diameter 80 mm is subjected to a Twisting moment of 8 MN-mm and a bending moment of 5 MN-mm at a point. Determine

(i) Principal stresses

(ii) Position of the plane on which they act.

Given data

Diameter of Shaft $(D) = 80\,mm$

Twisting moment $(T) = 8\,MN\text{-}mm \Rightarrow 8 \times 10^6\,N\text{-}mm$

Bending moment $(M) = 5\,MN\text{-}mm \Rightarrow 5 \times 10^6\,N\text{-}mm$

Solution

Major principal stress

$$= \frac{16}{\pi D^3} \left(M + \sqrt{M^2 + T^2} \right)$$

$$= \frac{16}{\pi \times 80^2}\left(5\times10^6 + \sqrt{(5\times10^6)^2 + (8\times10^6)^2}\right)$$

$$= \frac{16\times10^6}{\pi\times80^2}\left(5+\sqrt{5^2+8^2}\right)$$

$$= 9.947\left(5+\sqrt{25+64}\right)$$

$$= 142.57 \text{ N/mm}^2$$

Minor principal stress

$$= \frac{16}{\pi D^2}\left(M - \sqrt{M^2 + T^2}\right)$$

$$= \frac{16}{\pi\times80^2}\left(5\times10^6 - \sqrt{(5\times10^6)^2 + (8\times10^6)^2}\right)$$

$$= \frac{16\times10^6}{\pi\times80^2}\left(5-\sqrt{5^2+8^2}\right)$$

$$= 9.947\times(-4.42)$$

$$= -44.1 \text{ N/mm}^2$$

$$= 44.1 \text{ N/mm}^2 \text{ (Tensile)}$$

Position of plane

$$\tan 2\theta = \frac{T}{M}$$

$$= \frac{8\times10^6}{5\times10^6}$$

$\tan 2\theta = 1.6$

$1 \sec = \frac{1}{60} \times 50.21$

$2\theta = \tan^{-1} \times 1.6$

$= \frac{50.21}{60}$

$\theta = \frac{\tan^{-1} \times 1.6}{2}$

$= 0.8285$

$= 28^\circ 59' 50.21''$

$\Rightarrow 59 + 0.8285$

$\theta = 28^\circ 59.84'$

$\Rightarrow 59.84$

Prblm. no: 11

The maximum allowable shear stress in a hollow shaft of external diameter equal to twice the internal diameter is 80 N/mm^2. Determine the diameter of the shaft if it is subjected to a torque of 4×10^6 N-mm and a bending moment of 3×10^6 N-mm

Given data

Maximum shear stress = 80 N/mm^2

Torque (T) = 4×10^6 N-mm

Bending moment (M) = 3×10^6 N-mm

Let,

D_o - External diameter of shaft

$D_o = 2D_i$

D_i - Internal diameter of shaft

$\therefore \left(D_i = \frac{D_o}{2} \right)$

Solution

Maximum shear stress

$$= \frac{16 D_o}{\pi (D_o^4 - D_i^4)} \left(\sqrt{M^2 + T^2} \right)$$

$$80 = \frac{16 D_o}{\pi\left[D_o^4 - \left(\frac{D_o}{2}\right)^4\right]}\left(\sqrt{(3\times10^6)^2 + (4\times10^6)^2}\right)$$

$$= \frac{16 D_o \times 10^6\left(\sqrt{3^2 + 4^2}\right)}{\pi\left[D_o^4 - \frac{D_o^4}{16}\right]}$$

$$= \frac{16 \cancel{D_o} \times 10^6\left(\sqrt{9 + 16}\right)}{\pi D_o^{\cancel{4}}\left[1 - \frac{1}{16}\right]}$$

$$= \frac{(16 \times 10^6)\times 5}{\pi D_o^3\left[\frac{16-1}{16}\right]}$$

$$80 = \frac{(16\times10^6)\times 5}{\pi D_o^3 \times \frac{15}{16}}$$

$$D_o^3 = \frac{(16\times10^6)\times5\times16}{\pi\times15\times80}$$

$$= (0.3395\times10^6)^{1/3}$$

$$\therefore$$

$$\boxed{D_o = 69.76\ mm}$$

$$D_i = \frac{D_o}{2}$$

$$= \frac{69.78}{2}$$

$$\boxed{D_i = 34.89\,mm}$$

DESIGN OF SHAFT ACCORDING TO THEORIES OF FAILURE

THEORIES OF FAILURES

There are Five Theories of Failures.

1. Maximum principal stress theory
2. Maximum principal strain theory
3. Maximum shear stress theory
4. Maximum strain energy theory
5. Maximum shear strain energy theory.

For a Three-dimensional stress system subjected to principal stresses σ_1, σ_2 and σ_3 the equation for criterion of failure and for design according to

1. Maximum Principal Stress Theory

OR

Rankine Theory

CRITERION OF FAILURE

$$\sigma_1 \geq \sigma_t^*$$

$$|\sigma_3| \geq \sigma_c^*$$

Where,

σ_t^* - Elastic limit In simple tension

σ_c^* - Elastic limit In Simple Compression

FOR DESIGN

$$\sigma_1 = \sigma_t$$

Where,

σ_t = Permissible Stresses in Simple Tension

$$= \frac{\sigma_t^*}{\text{Safety Factor}}$$

2. Maximum Principal Strain Theory

(OR)

Saint Vanant Theory

CRITERION FAILURE

$$\frac{1}{E}\left[\sigma_1 - \mu(\sigma_2 + \sigma_3)\right] \geq \frac{1}{E}\sigma_t^*$$

(or)

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) \geq \sigma_t^*$$

$$\left|\left[\sigma_3 - \mu(\sigma_1 + \sigma_2)\right]\right| \geq \sigma_c^*$$

FOR DESIGN

- $$\sigma_1 - \mu(\sigma_2 + \sigma_3) = \sigma_t$$

$$\sigma_3 - \mu(\sigma_1 + \sigma_2) = \sigma_c$$

3. Maximum Shear Stress theory

(OR)

Guest and Tresca Theory

AT FAILURE

$$(\sigma_1 - \sigma_3) \geq \sigma_t^*$$

FOR DESIGN

$$(\sigma_1 - \sigma_3) = \sigma_t$$

Where,

$$\sigma_t = \frac{\sigma_t^*}{\text{Safety Factor}}$$

4. Maximum Strain Energy Theory

(OR)

Haigh's Theory

AT FAILURE

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \geq \frac{1}{2E}(\sigma_t^*)^2$$

For Two-Dimensional Stress-System

AT FAILURE

$$\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1 \times \sigma_2) \geq (\sigma_t^*)^2$$

FOR DESIGN

- $$\sigma_1 - \mu(\sigma_2 + \sigma_3) = \sigma_t$$

$$\sigma_3 - \mu(\sigma_1 + \sigma_2) = \sigma_c$$

3. Maximum Shear Stress theory

(OR)

Guest and Tresca Theory

AT FAILURE

$$(\sigma_1 - \sigma_3) \geq \sigma_t^*$$

FOR DESIGN

$$(\sigma_1 - \sigma_3) = \sigma_t$$

Where,

$$\sigma_t = \frac{\sigma_t^*}{\text{Safety Factor}}$$

4. Maximum Strain Energy Theory

(OR)

Haigh's Theory

AT FAILURE

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \geq \frac{1}{2E}(\sigma_t^*)^2$$

For Two - Dimensional Stress - System

AT FAILURE

$$\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1 \times \sigma_2) \geq (\sigma_t^*)^2$$

FOR DESIGN

$$\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1 \times \sigma_2) = \sigma_t$$

5. MAXIMUM SHEAR STRAIN ENERGY THEORY

(OR)

Mises and Henky Theory (or) Energy of distortion theory

CRITERION OF FAILURE

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \geq 2 \times (\sigma_t^*)^2$$

FOR DESIGN

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \geq 2 \times \sigma_t^2$$

For two dimensional stress system in case of design

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_t^2$$

Prblm.no:12

In a mettalic body the principal stresses are $+35\ MN/m^2$ and $-95\ MN/m^2$, the third principal stress being zero. The elastic limit zero in simple tension as well as in simple compression is equal and is $220\ MN/m^2$. Find the factor of safety based on the elastic limit if the criterior of failure for the material is the maximum principal stress theory

Given data:

$\sigma_1 = +35\ MN/m^2$

$\sigma_2 = 0$

$$\sigma_3 = -95 \ MN/m^2$$

$$\sigma = \sigma^* = \sigma^* = 220 \ MN/m^2$$

Where,

σ_{et} = Elastic limit in simple tension

σ_{ec} = Elastic limit in simple compression

Solution

$\sigma_1 = \sigma_t$ (Working stress in tension)

$$\sigma_1 = \frac{\sigma_{et}}{\text{Factor of safety}}$$

$$F.O.S = \frac{\sigma}{\sigma_1}$$

$$= \frac{220}{35}$$

$$\boxed{F.O.S = 6.28}$$

$|\sigma_3| = \sigma_c$ (Working stress in compression)

$$|\sigma_3| = \frac{\sigma_{ec}}{F.O.S} \Rightarrow |-95| = \frac{\sigma_{ec}}{F.O.S}$$

$$F.O.S = \frac{200}{95}$$

$$\boxed{F.O.S = 2.3}$$

The material according to the Maximum principal stress theory will fail due to the compressive principal stress.

Prblm. no: 13

In a cast-iron body the principal stresses are 40 MN/m^2 and − 100 MN/m^2 the third principal stress being zero. The elastic limit stresses in simple tension and in simple compression are 80 MN/m^2 and 400 MN/m^2 respectively. Find the factor of safety based on the elastic limit if the criterion of failure is the maximum principal stress theory.

Given data

$\sigma_1 = 40\ MN/m^2$

$\sigma_2 = 0$

$\sigma_3 = -100\ MN/m^2$

$\sigma = 80\ MN/m^2$ (Elastic limit in simple tension)

$\sigma = 400\ MN/m^2$ (Elastic limit in simple compression)

Solution

$\sigma_1 = \sigma_t$ (Working stress in Tension)

$$\sigma_1 = \frac{\sigma_{et}}{F.O.S}$$

$$40 = \frac{80}{F.O.S}$$

$$F.O.S = \frac{80}{40}$$

$$\boxed{F.O.S = 2}$$

$|\sigma_2| = \sigma_c$ (Working stress in Compression)

$$|\sigma_2| = \frac{\sigma_{ec}}{F.O.S}$$

$$F.O.S = \frac{\sigma_{ec}}{|\sigma_2|}$$

$$= \frac{400}{100}$$

$$\boxed{F.O.S = 4}$$

The material will fail due to Tensile principal stress

Prblm.no:14

A Shaft is Subjected to a maximum torque of 10 kN/m and a maximum bending moment of 7.5 kNm at a particular section. If the allowable equivalent stress in simple tension is 160 MN/m². Find the diameter of the shaft according to the maximum shear stress theory.

Given data

Maximum Torque (T) = 10 kNm

Maximum bending moment (M) = 7.5 kN.m

Allowable equivalent stress in simple tension

$$\sigma_t = 160\ MN/m^2$$

Let,

Dia. of the shaft

Solution

We know that,

$$M = \frac{\pi}{32} d^3 \cdot \sigma_b$$

σ_b - Maximum bending stress

$$\sigma_b = \frac{32M}{\pi d^3}$$

$$T = \tau \times \frac{\pi}{16} d^3$$

$$\tau = \frac{16T}{\pi d^3}$$

Principal stress are given by

$$\sigma_1, \sigma_2 = \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$= \frac{1}{2}\left[\sigma_b \pm \sqrt{\sigma_b^2 + 4\tau^2}\right]$$

$$= \frac{1}{2}\left[\frac{32M}{\pi d^3} \pm \sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + \left(\frac{32T}{\pi d^3}\right)}\right]$$

$$= \frac{16}{\pi d^3}\left[M \pm \sqrt{M^2 + T^2}\right]$$

$$\sigma_1 = \frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right]$$

$$\sigma_2 = 0$$

$$\sigma_3 = \frac{16}{\pi d^3}\left[M - \sqrt{M^2 + T^2}\right]$$

Maximum shear stress theory

$$\sigma_t = \sigma_1 - \sigma_3$$

$$= \frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right] - \frac{16}{\pi d^2}\left[M - \sqrt{M^2 + T^2}\right]$$

$$= \frac{16}{\pi d^3}\left(\cancel{M} + \sqrt{M^2 + T^2} - \cancel{M} + \sqrt{M^2 + T^2}\right)$$

$$= \frac{16}{\pi d^2} \times 2\left(\sqrt{M^2 + T^2}\right)$$

$$= \frac{32}{\pi d^2}\sqrt{M^2 + T^2}$$

$$d^2 = \frac{32}{\pi \sigma_t}\sqrt{M^2 + T^2}$$

$$= \frac{32 \times 10^2}{\pi \times 160 \times 10^6} \times \sqrt{5^2 + 10^2}$$

$$= 7.957 \times 10^{-4}$$

$$= 0.0926 \text{ m}$$

$$\boxed{d = 92.6 \text{ m}}$$

Prblm.no:15

Solve example prblm.no:14 using the strain energy. Take poisson's ratio $\frac{1}{m} = 0.24$

Solution

From prblm.no:14

Max. Torque (T) = 10 KNm

Max. BM (M) = 7.5 KNm

$$\sigma_1 = \frac{16}{\pi d^3}\left(M + \sqrt{M^2 + T^2}\right)$$

$$\sigma_2 = 0$$

$$\sigma_3 = \frac{16}{\pi d^3}\left(M - \sqrt{M^2 + T^2}\right)$$

Now according to strain energy theory

$$\sigma_t^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \frac{2}{m}\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)$$

$$= \sigma_1^2 + \sigma_3^2 - \frac{2}{m} \times \sigma_3\sigma_1 \qquad \therefore (\sigma_2 = 0)$$

$$= \left(\frac{16}{\pi d^3}\right)^2 \left(M + \sqrt{M^2+T^2}\right)^2 + \left(\frac{16}{\pi d^3}\right)^2 \left(M - \sqrt{M^2+T^2}\right)^2$$

$$- \frac{2}{m} \times \left(\frac{16}{\pi d^3}\right)^2 \left(M + \sqrt{M^2+T^2}\right)\left(M - \sqrt{M^2+T^2}\right)$$

$$= \left(\frac{16}{\pi d^3}\right)^2 \left[2\left(M^2 + M^2 + T^2\right) - \frac{2}{m}\left(\cancel{M^2} - \cancel{M^2} - T^2\right)\right]$$

$$= \left(\frac{16}{\pi d^3}\right)^2 4M^2 + 2T^2 + \frac{2T^2}{m}$$

$$= \frac{16}{\pi d^3}\sqrt{4M^2 + 2\left(1 + \frac{1}{m}\right)T^2}$$

$$= \frac{16 \times 2}{\pi d^3} \sqrt{M^2 + \left(\frac{1 + 1/m}{2}\right) T^2} \quad \therefore \left(\text{Poisson's ratio}\right)$$

$$\frac{1}{m} = 0.24$$

$$= \frac{32}{\pi d^3} \sqrt{M^2 + \left(\frac{1 + 0.24}{2}\right) T^2}$$

$$= \frac{32}{\pi d^3} \sqrt{M^2 + 0.62 T^2}$$

$$d^3 = \frac{32}{\pi \sigma_t} \sqrt{M^2 + 0.62 T^2}$$

$\therefore$ (Max. B.M = 7.5 kN.m)

$\therefore$ (Max. Torque = 10 kN.m)

$$= \frac{32 \times 10^3}{\pi \times 160 \times 10^6} \sqrt{7.5^2 + 0.62 \times 10^2}$$

$$= 6.922 \times 10^{-4}$$

$$= 0.0885 \text{ m}$$

$$\boxed{d = 88.5 \text{ mm}}$$

Prblm.no : 16

Solve example prblm.no : 14 using shear strain energy

Solution

Max. Torque (T) = 10 kN.m

Max. BM (M) = 7.5 kN.m

Allowable equivalent stress in simple tension (σ_t) = 160 MN/m²

Let,

Diameter of the shaft $(d) = ?$

$$\sigma_1 = \frac{16}{\pi d^3}\left(M + \sqrt{M^2 + T^2}\right)$$

$$\sigma_2 = 0$$

$$\sigma_3 = \frac{16}{\pi d^3}\left(M - \sqrt{M^2 + T^2}\right)$$

Now,

According to shear strain energy theory

$$2\sigma_t^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

$$= \sigma_1^2 - 2\sigma_1\sigma_2 + \sigma_2^2 + \sigma_2^2 - 2\sigma_2\sigma_3 + \sigma_3^2 \overset{+\sigma_3^2}{\wedge} - 2\sigma_3\sigma_1 + \sigma_1^2$$

$$= \cancel{2}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - \cancel{2}(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$$

$$= \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$$

$$= \sigma_1^2 + \sigma_3^2 - \sigma_3\sigma_1 \qquad \therefore (\sigma_2 = 0)$$

$$= \left(\frac{16}{\pi d^3}\right)^2\left(M + \sqrt{M^2 + T^2}\right)^2 + \left(\frac{16}{\pi d^3}\right)^2\left(M - \sqrt{M^2 + T^2}\right)^2$$

$$- \left(\frac{16}{\pi d^3}\right)^2\left(M + \sqrt{M^2 + T^2}\right)\left(M - \sqrt{M^2 + T^2}\right)$$

$$= \left(\frac{16}{\pi d^3}\right)^2\left[2(M^2 + M^2 + T^2) - (M^2 - M^2 - T^2)\right]$$

$$= \left(\frac{16}{\pi d^3}\right)^2 4M^2 + 2T^2 - \cancel{M^2} + \cancel{M^2} + T^2$$

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Let,

Diameter of the shaft $(d) = ?$

$$\sigma_1 = \frac{16}{\pi d^3}\left(M + \sqrt{M^2 + T^2}\right)$$

$$\sigma_2 = 0$$

$$\sigma_3 = \frac{16}{\pi d^3}\left(M - \sqrt{M^2 + T^2}\right)$$

Now,

According to shear strain energy theory

$$2\sigma_t^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

$$= \sigma_1^2 - 2\sigma_1\sigma_2 + \sigma_2^2 + \sigma_2^2 - 2\sigma_2\sigma_3 + \sigma_3^2 + \sigma_3^2 - 2\sigma_3\sigma_1 + \sigma_1^2$$

$$= \cancel{2}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - \cancel{2}(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$$

$$= \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$$

$$= \sigma_1^2 + \sigma_3^2 - \sigma_3\sigma_1 \qquad \therefore (\sigma_2 = 0)$$

$$= \left(\frac{16}{\pi d^3}\right)^2\left(M + \sqrt{M^2 + T^2}\right)^2 + \left(\frac{16}{\pi d^3}\right)^2\left(M - \sqrt{M^2 + T^2}\right)^2$$

$$-\left(\frac{16}{\pi d^3}\right)^2\left(M + \sqrt{M^2 + T^2}\right)\left(M - \sqrt{M^2 + T^2}\right)$$

$$= \left(\frac{16}{\pi d^3}\right)^2\left[2(M^2 + M^2 + T^2) - (M^2 - M^2 - T^2)\right]$$

$$= \left(\frac{16}{\pi d^3}\right)^2 4M^2 + 2T^2 - \cancel{M^2} + \cancel{M^2} + T^2$$

$$= \left(\frac{16}{\pi d^3}\right)^2 \left(4M^2 + 3T^2\right)$$

$$= \left(\frac{16 \times 2}{\pi d^3}\right)^2 \left[M^2 + \frac{3}{4} T^2\right]$$

$$= \left(\frac{32}{\pi d^3}\right)^2 \left[M^2 + \frac{3}{4} T^2\right]$$

$$= \frac{32}{\pi d^3} \sqrt{M^2 + \frac{3}{4} T^2}$$

$$d^3 = \frac{32}{\pi \sigma_t} \sqrt{M^2 + \frac{3}{4} T^2}$$

$$= \frac{32 \times 10^3}{\pi \times 160 \times 10^6} \sqrt{7.5^2 + \frac{3}{4} \times 10^2}$$

$$= 7.29 \times 10^{-4}$$

$$= 0.09 m$$

$$\boxed{d = 90 mm}$$

Prblm. no: 17

A Hollow mild steel shaft having 100mm external diameter and 50mm Internal diameter is subjected to a twisting moment of 8 KNm and a bending moment of 2.5 KNm. Calculate the principal stresses and find direct stress which, acting alone, would produce the same

(i) Maximum elastic strain energy

(ii) Maximum elastic shear strain energy as that produced by the principal stresses acting together.

Take poisson's ratio = 0.25

Given data

External diameter (D) = 100mm ⇒ 0.1 m

Internal diameter (d) = 50 mm ⇒ 0.05 m

Twisting moment (T) = 8 kNm ⇒ 8000 Nm

Bending moment (M) = 2.5 kNm ⇒ 2500 Nm

Poisson's ratio (μ) = 0.25

let,

* Bending stress due to Bending moment
* Shear stress due to Twisting moment

Solution

Using Bending equation

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

Bending stress (σ_b)

$$\sigma_b = \frac{M}{I} \times y$$

Sectional modulus (Z)

$$\therefore \left(Z = \frac{I}{y} \right)$$

$$= \frac{M}{(I/y)}$$

$$= \frac{2500}{\frac{\pi}{64}\left[D^4 - d^4\right] \Big/ \frac{D}{2}}$$

$$= \frac{2500}{\frac{\pi}{64}\left[D^4 - d^4\right] \times \frac{2}{D}}$$

$$= \frac{2500}{\frac{\pi}{64}\left[0.1^4 - 0.05^4\right] \times \frac{2}{0.1}}$$

$$= \frac{2500}{\frac{\pi}{64}\left[0.0001 - 0.00000625\right] \times \frac{2}{0.1}}$$

$$= 27.17 \times 10^6 \ N/m^2$$

$$\boxed{\sigma_b = 27.17 \ MN/m^2}$$

Shear stress (τ)

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\tau = \frac{T}{J} \times R$$

$$= \frac{8000}{\frac{\pi}{32}\left[D^4 - d^4\right]} \times \frac{D}{2}$$

$$= \frac{8000 \times 32}{\pi(0.1^4 - 0.05^4)} \times \frac{0.1}{\cancel{2}}$$

$$= \frac{8000 \times 16 \times 0.1}{\pi(0.0001 - 0.00000625)}$$

$$= 43.46 \times 10^6 \ N/m^2$$

$$\boxed{\tau = 43.46 \ MN/m^2}$$

Principle Stress (σ_1 and σ_2)

$$\sigma_1 \text{ and } \sigma_2 = \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + C^2}$$

$$= \frac{27.17}{2} \pm \sqrt{\left(\frac{27.17}{2}\right)^2 + 43.46^2}$$

$$= 13.585 \pm 45.53$$

$$= 13.585 + 45.53$$

$$= 59.115$$

$$\boxed{\sigma_1 = 59.12 \ MN/m^2}$$

$$\sigma_2 = 13.585 - 45.53$$

$$= -31.945$$

$$\boxed{\sigma_2 = -31.95 \ MN/m^2}$$

(ii) Single direct stress which would produce the same maximum elastic strain energy as produced by the principal stress acting together.

let,

σ - single direct stress

Strain energy due to single direct stress $= \frac{1}{2E} \times \sigma^2$

Max. strain energy produced by principal stresses

$$= \frac{1}{2E}\left(\sigma_1^2 + \sigma_2^2 - 2\mu \sigma_1 \times \sigma_2\right)$$

Equating the two strain energies ① & ②

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1 \times \sigma_2\right] = \frac{1}{2E} \times \sigma^2$$

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1 \times \sigma_2 = \sigma^2$$

$$(59.12)^2 + (-31.95)^2 - (2 \times 0.25 \times 59.12) \times (-31.95) = \sigma^2$$

$$3495 + 1020.8 + 944.4 = \sigma^2$$

$$5460.2 = \sigma^2$$

$$\sigma = \sqrt{5460.2}$$

$$= 73.89$$

$$\boxed{\sigma \simeq 73.9 \ MN/m^2}$$

(iii) Single direct stress which would produce the same maximum elastic shear strain energy as produced by the principal stresses acting together.

Shear strain energy by single direct stress

$$= \frac{1}{12C}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

$$= \frac{1}{12C}\left[(\sigma - 0)^2 + (0 - 0)^2 + (0 - \sigma)^2\right]$$

∴ single stress is uniaxial stress system

$$= \frac{1}{12C}\left[\sigma^2 + \sigma^2\right]$$

$$= \frac{1}{12C} \times 2\sigma^2 \longrightarrow ③$$

Maximum shear strain energy due to principal stresses $\sigma_1, \sigma_2, 0$

$$= \frac{1}{12C}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-0)^2+(0-\sigma_1)^2\right]$$

$$= \frac{1}{12C}\left[(\sigma_1-\sigma_2)^2+\sigma_2^2+\sigma_1^2\right]$$

$$= \frac{1}{12C}\left[(\sigma_1^2+\sigma_2^2-2\sigma_1\sigma_2)+\sigma_2^2+\sigma_1^2\right]$$

$$= \frac{1}{12C}\left[2(\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2)\right] \rightarrow 4$$

Equating the Two shear energies given by equation 3 and 4

$$\cancel{\frac{1}{12C}\times 2}\sigma^2 = \cancel{\frac{1}{12C}\times 2}\times(\sigma_1^2 \times \sigma_2^2-\sigma_1\sigma_2)$$

$$\sigma^2 = \sigma_1^2+\sigma_2^2-\sigma_1\sigma_2$$

$$=(59.12)^2+(-21.95)^2-(59.12)(-21.95)$$

$$= 3495 + 1020.8 + 1888.8$$

$$\sigma^2 = 6404.6$$

$$=\sqrt{6404.6}$$

$$\boxed{\sigma = 80.02\ MN/m^2}$$

Prblm.no: 18

A steel shaft is subjected to an end thrust producing a stress of 90 MPa and the Minimum shearing stress on the surface arising from torsion is 60 MPa. The yield point of the material in simple tension was found to be 300 MPa. Calculate the factor of safety of the shaft according to the following theories

- (i) Maximum shear stress theory

  (ii) Maximum distortion energy theory

Given data

$\sigma_1 = \phi$

$\sigma_2 = 0$

$\sigma_3 = -90 \ MN/m^2$

$\tau_{max} = 60 \ MN/m^2$

$\sigma_{et} = 200 \ MN/m^2$

Solution

(i) Maximum shear stress theory

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$60 = \frac{\sigma_1 - (-90)}{2}$$

$$60 \times 20 = \sigma_1 + 90$$

$$120 - 90 = \sigma_1$$

$$30 = \sigma_1$$

$$\sigma_1 = 30 \ MN/m^2$$

$$\sigma_1 - \sigma_3 = \sigma_t$$

$$30 - (-90) = \sigma_t$$

$$\boxed{\sigma_t = 120\ MN/m^2}$$

$$F.O.S = \frac{\sigma_{et}}{\sigma_t}$$

$$= \frac{300}{120}$$

$$= 2.5$$

(ii) Maximum distortion energy theory

$$\sigma_t^2 = \sigma_1^2 + \sigma_3^2 - \sigma_3\sigma_1$$

$$= 30^2 + (-90)^2 - (-90)(30)$$

$$= \sqrt{11700}$$

$$\sigma_t = 108.17\ MN/m^2$$

$$F.O.S = \frac{\sigma_{et}}{\sigma_t}$$

$$= \frac{300}{108.17}$$

$$\boxed{F.O.S = 2.77}$$

Springs

Introduction

Springs are elastic members which distort under under load and regain their original shape when load is removed

They are used in

* Railway carriages
* Motor cars
* Scooters
* Motorcycles
* Rickshaws
* Governor

According to their uses, the spring perform the following function

(i) To absorb shock or impact loading as in carriage springs

(ii) To store energy as in clock springs.

(iii) To apply forces to and to control motions as in brakes and clutches.

(iv) To measure forces as in spring balances.

(v) To change the variations characteristics of a member as in flexible mounting of motors.

TYPES OF SPRINGS

1. Helical spring

* closed - coiled helical spring
* open - coiled helical spring
* Tension helical spring
* Compression helical spring

2. Leaf springs

(i) Full - elliptic (ii) Semi - elliptic (iii) cantilever

3. Torsion springs
4. circular springs
5. Belleville springs
6. Flat springs

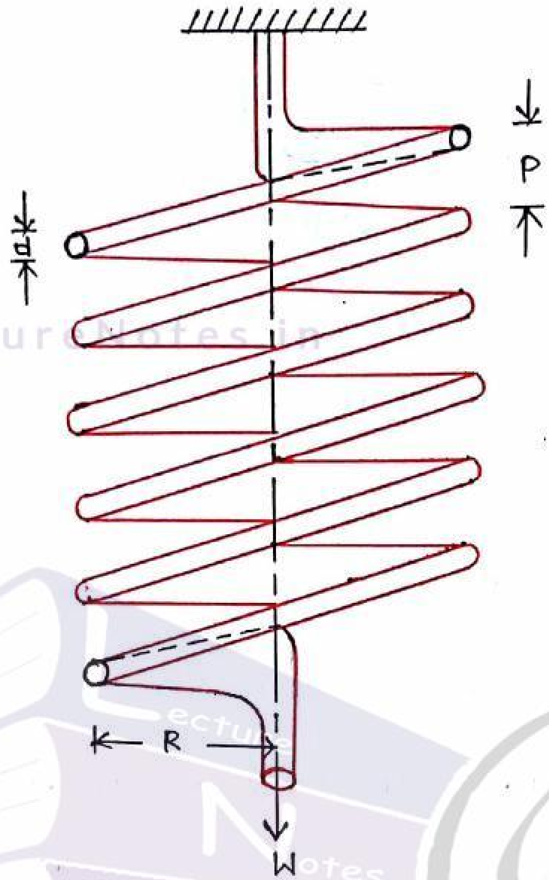
HELICAL SPRINGS

Helical springs are the thick spring wires coiled into a helix.

They are of two types

* closed - coiled helical springs
* open - coiled helical springs

CLOSED-COILED HELICAL SPRINGS



* Closed - Coiled Helical Springs are the springs in which helix angle is very small or in other words the pitch between two adjacent turns is small.

* A closed-coiled helical spring carrying an axial load.

* As the helix angle in case of closed-coiled helical springs are small, hence the bending effect on the spring is ignored

* We assume that the coils of a closed-coiled helical springs are to stand purely torsional stresses

Expression for max. Shear Stress induced in wire

closed - Coiled helical spring subjected to an axial load.

Let,

d - Diameter of spring wire

p - pitch of the helical spring

n - Number of coils

R - Mean radius of spring coil

W - Axial load on spring

C - Modulus of Rigidity

τ - Max. shear stress induced in the wire

θ - Angle of twist in spring due to axial load

δ - Deflection of spring due to axial load

L - Length of wire.

Twisting moment on the wire

$$T = W \times R \quad \rightarrow (1)$$

Twisting moment is also given by

$$T = \frac{\pi}{16} \tau d^3 \quad \rightarrow (2)$$

Equation (1) & (2) equating

$$W \times R = \frac{\pi}{16} \tau d^3$$

$$\boxed{\tau = \frac{16W \times R}{\pi d^3}}$$

Maximum shear stress induced in the wire

Expression for Deflection of Spring

Length of one coil $= \pi D$ (or) $2\pi R$

$\therefore$ Total length of the wire $=$ Length of one coil $\times$ No. of coils

$$l = 2\pi R \times n$$

Every section of the wire is subjected to Torsion, hence the strain energy stored by the spring due to Torsion

$\therefore$ Strain energy stored by the spring

$$U = \frac{\tau^2}{4C} \times \text{volume}$$

$$= \left(\frac{16W \times R}{\pi d^3}\right)^2 \times \frac{1}{4C} \times \left(\frac{\pi}{4} d^2 \times 2\pi R \cdot n\right)$$

$\therefore \left(\tau = \frac{16WR}{\pi d^3}\right)$ $\therefore \left(\text{volume} = \frac{\pi}{4} d^2 \times \text{Total length of wire}\right)$

$$= \frac{16 \times 16 \times W^2 \times R^2}{\pi^2 \cdot d^{(3+3)}} \times \frac{1}{4C} \times \frac{\pi}{4} d^2 \times 2\pi R \cdot n$$

$$= \frac{\cancel{256} \times W^2 \times R^2}{\pi \cancel{^2} \cdot d^{\cancel{6}\,4}} \times \frac{1}{\cancel{4}C} \times \frac{\cancel{\pi}}{\cancel{4}} d^{\cancel{2}} \times \cancel{2\pi} R \cdot n$$

$$= \frac{(16 \times 2) \times W^2 \times R^3}{d^4} \times \frac{1}{C} \cdot n$$

$\therefore$ Strain energy $\boxed{(U) = \dfrac{32 W^2 R^3 . n}{C d^4}}$

Work done on the Spring = Average load × Deflection

$$= \frac{1}{2} W \times \delta \qquad \therefore (\text{Deflection} = \delta)$$

Equating the work done on spring to the energy stored, we get

$$\frac{1}{2} W \cdot \delta = \frac{32 W^2 R^3 . n}{C d^4}$$

$$\boxed{\delta = \frac{64 W R^3 n}{C d^4}}$$

Expression for Stiffness of Spring

Stiffness of Spring

$$S = \text{Load per unit deflection}$$

$$= \frac{W}{\delta}$$

$$= \frac{W}{\dfrac{64 . W R^3 . n}{C d^4}}$$

$$= \cancel{W} \times \frac{C d^4}{64 . \cancel{W} R^3 . n}$$

$$\boxed{S = \frac{C d^4}{64 . R^3 . n}}$$

Solid length of the Spring

* The Solid length of the spring means the distance between the coils when the coils are touching each other.

* There is no gap between the coils

* The solid length is given by

$$\text{Solid length} = \text{Number of Coils} \times \text{Dia. of wire}$$

$$\boxed{\text{Solid length} = n \times d}$$

Prblm.no: 19

A closed coiled helical spring is to carry a load of 500N. It mean coil diameter is to be 10 times that of the wire diameter. Calculate these diameters if the maximum shear stress in the material of the spring is to be 80 N/mm²

Given data

Load on Spring $(W) = 500\,N$

Maximum shear stress $(\tau) = 80\,N/mm^2$

Let,

d - Diameter of wire

D - Mean diameter of coil

$D = 10\,d$

Solution

Maximum shear stress induced in the wire

$$\tau = \frac{16WR}{\pi d^3}$$

$$80 = \frac{16 \times 500 \times \left(\frac{D}{2}\right)}{\pi d^3} \quad \therefore \left(R = \frac{D}{2}\right)$$

$$= \frac{8000 \times \frac{\cancel{10}^{5} d}{2}}{\pi d^3}$$

$$= \frac{8000 \times 5d}{\pi d^3}$$

$$80 \times \pi d^{\cancel{3}\,2} = 8000 \times 5\cancel{d}$$

$$d^2 = \frac{8000 \times 5}{80 \times \pi}$$

$$d^2 = 159.25$$

$$= \sqrt{159.25}$$

$$= 12.6\,mm \Rightarrow 12.6/10 \Rightarrow 1.26\,cm$$

$$\boxed{d = 1.26\,cm} \rightarrow \text{Ans (i)}$$

Mean diameter of coil

$$D = 10 \times d$$

$$= 10 \times 1.26$$

$$\boxed{D = 12.6\,cm} \rightarrow \text{Ans (ii)}$$

Prblm.no:20

- In problem.no: 19, if the stiffness of the spring is 20N per mm deflection and modulus of rigidity = 8.4×10^4 N/mm^2. Find the Number of coil in the closed coiled helical spring.

Given data

Stiffness (s) = 20 N/mm

Modulus of Rigidity (C) = 8.4×10^4 N/mm^2

From prblm.no: 19

$$W = 500\,N$$

$$\tau = 80\ N/mm^2$$

$$d = 12.6\,mm$$

$$D = 126\,mm$$

$$R = \frac{D}{2} \Rightarrow \frac{126}{2}$$

$$R = 63\,mm$$

Let,

n - Number of coils in the spring

Solution

$$\text{Stiffness} = \frac{\text{Load}}{\text{Deflection}}$$

$$= \frac{W}{\delta}$$

$$20 = \frac{500}{\delta}$$

$$\delta = \frac{500}{20}$$

$$\boxed{\delta = 25mm}$$

$$\delta = \frac{64WR^3 \cdot n}{C \cdot d^4}$$

$\therefore (R = 62mm)$

$$25 = \frac{64 \times 500 \times (62)^3 \times n}{8.4 \times 10^4 \times 12.6^4}$$

$$n = \frac{25 \times 8.4 \times 10^4 \times 12.6^4}{64 \times 500 \times (62)^3}$$

$$= 6.6 \text{ say } 7.0$$

$$\boxed{n = 7}$$

Prblm.no: 21

A closed coiled helical spring of round steel wire 10mm in diameter having 10 complete turns with a mean diameter of 12cm is subjected to an axial load of 200N

Determine

(i) The deflection of the spring

(ii) Maximum shear stress in the wire

(iii) Stiffness of the spring.

Take, $C = 8 \times 10^4 \ N/mm^2$

Given data

Diameter of wire $(d) = 10mm$

No. of Turns $(n) = 10$

Mean diameter of coil $(D) = 12cm \Rightarrow 12 \times 10 \Rightarrow 120mm$

$\therefore$ Radius of coil (R) = $\frac{D}{2} \Rightarrow \frac{120}{2}$

$$R = 60mm$$

Axial load (W) = 200N

Modulus of Rigidity (C) = 8×10^4 N/mm^2

let,

δ = Deflection of the Spring

τ = Max. Shear Stress in the wire

S = Stiffness of the Spring.

Solution

Deflection of the Spring

$$\delta = \frac{64 WR^3 \times n}{Cd^4}$$

$$= \frac{64 \times 200 \times 60^3 \times 10}{8 \times 10^4 \times 10^4}$$

$$\boxed{\delta = 34.5 \text{ mm}} \rightarrow \text{Ans(i)}$$

Max. Shear Stress in the wire

$$\tau = \frac{16WR}{\pi d^3}$$

$$= \frac{16 \times 200 \times 60}{\pi \times 10^3}$$

$$\boxed{\tau = 61.1 \text{ N/mm}^2} \rightarrow \text{Ans(ii)}$$

Stiffness of the spring

$$S = \frac{W}{\delta}$$

$$= \frac{200}{34.5}$$

$$\boxed{\delta = 5.8 \text{ N/mm}} \rightarrow \text{Ans (iii)}$$

Prblm. no: 22

A closed coiled helical spring of 10cm mean diameter is made up of 1cm diameter rod and has 20 turns. The spring carries an axial load of 200 N. Determine the shearing stress. Taking the value of modulus of rigidity = 8.4×10^4 N/mm², Determine the deflection when carrying this load. Also calculate the stiffness of the spring and the frequency of free vibration for a mass hanging from it.

Given data

Mean diameter of coil $(D) = 10$ cm $\Rightarrow 10 \times 10$ mm $\Rightarrow 100$ mm

Mean radius of coil $(R) = 5$ cm $\Rightarrow 5 \times 10$ mm $\Rightarrow 50$ mm

Diameter of rod $(d) = 1$ cm $\Rightarrow 1 \times 10$ mm $\Rightarrow 10$ mm

Number of turns $(n) = 20$

Axial load $(W) = 200$ N

Modulus of Rigidity $(C) = 8.4 \times 10^4$ N/mm²

Let,

τ - Shear stress in the material of the spring

δ - Deflection of the spring due to axial load

s - Stiffness of spring

f - Frequency of free vibration

Solution

- Shear stress in the material of the spring

$$\tau = \frac{16WR}{\pi d^3}$$

$$= \frac{16 \times 200 \times 50}{\pi \times 10^3}$$

$$\boxed{\tau = 50.92\ N/mm^2} \rightarrow \text{Ans(i)}$$

Deflection of the Spring.

$$\delta = \frac{64 WR^3 \times n}{C d^4}$$

$$= \frac{64 \times 200 \times 50^3 \times 20}{8.4 \times 10^4 \times 10^4}$$

$$\boxed{\delta = 38.095\ mm} \rightarrow \text{Ans(ii)}$$

Stiffness of the spring

$$\text{Stiffness} = \frac{\text{Load on Spring}}{\text{Deflection of Spring}}$$

$$= \frac{200}{38.095}$$

$$\boxed{S = 5.25\ N/mm} \rightarrow \text{Ans (iii)}$$

Frequency of free vibration

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{981}{3.8095}}$$

$$\boxed{f = 2.55 \text{ cycles/sec}} \rightarrow \text{Ans (iv)}$$

Prblm no: 23

A closed coiled helical spring of mean diameter 20cm is made of 3cm diameter rod and has 16 turns. A weight of 3 kN is dropped on this spring. Find the height by which the weight should be dropped before striking the spring so that the spring may be compressed by 18cm. Take $C = 8 \times 10^4$ N/mm²

Given data

Mean diameter of coil $(D) = 20\text{cm} \Rightarrow 20 \times 10\text{mm} \Rightarrow 200\text{mm}$

Mean radius of coil $(R) = \frac{200}{2} \Rightarrow 100\text{mm}$

Dia. of spring rod $(d) = 3\text{cm} \Rightarrow 3 \times 10\text{ mm} \Rightarrow 30\text{mm}$

Number of Turns $(n) = 16$

Weight dropped $(W) = 3\text{kN} \Rightarrow 3 \times 1000\text{ N} \Rightarrow 3000\text{ N}$

Compression of the spring $(\delta) = 18\text{ cm} \Rightarrow 18 \times 10\text{mm} \Rightarrow 180\text{mm}$

Modulus of Rigidity $(C) = 8 \times 10^4$ N/mm²

Let,

h - Height through which the weight W is dropped

W - Gradually applied load which produces the

Compression of spring equal to 180 mm

Solution

$$\delta = \frac{64 W \cdot R^3 \cdot n}{C d^4}$$

$$180 = \frac{64 \times W \times 100^3 \times 16}{8 \times 10^4 \times 30^4}$$

$$W = \frac{180 \times 8 \times 10^4 \times 30^4}{64 \times 100^3 \times 16}$$

$$\boxed{W = 11390 \text{ N}}$$

Work done by the falling weight on spring

$= \text{Weight falling } (h + \delta)$

$= 2000 (h + 180) \text{ N-mm}$

Energy stored in the spring

$= \frac{1}{2} W \times \delta$

$= \frac{1}{2} \times 11390 \times 180$

$= 1025100 \text{ N-mm}$

Equating the work done by the falling weight on the spring to the energy stored in the spring.

$2000 (h + 180) = 1025100$

$h + 180 = 1025100 / 2000$

$= 241.7$

$= 241.7 - 180$

$$\boxed{h = 161.7 \text{ mm}}$$

Problem no: 24

The stiffness of a closed - coiled helical spring is 1.5 N/mm of compression under of maximum load of 60N. The maximum shearing stress produced in the wire of the spring is 125 N/mm². The solid length of the spring (when the coils are touching) is given as 5 cm. Find.

(i) Diameter of wire

(ii) Mean diameter of the coil

(iii) Numbers of coils required

Take $C = 4.5 \times 10^4$ N/mm²

Given data

Stiffness of spring $(S) = 1.5$ N/mm

Load on spring $(W) = 60$ N

Maximum shear stress $(\tau) = 125$ N/mm²

Solid length of spring $= 5$ cm $\Rightarrow 5 \times 10$ mm $\Rightarrow 50$ mm

Modulus of Rigidity $(C) = 4.5 \times 10^4$ N/mm²

Let,

d - Diameter of wire

D - Mean dia. of coil

R - Mean radius of coil

$$\therefore \left(R = \frac{D}{2}\right)$$

n - Number of coils

Solution

$$\delta = \frac{cd^4}{64 \cdot R^3 \cdot n}$$

$$1.5 = \frac{4.5 \times 10^4 \times d^4}{64 \times R^3 \times n}$$

$$d^4 = \frac{1.5 \times 64 \times R^3 \times n}{4.5 \times 10^4}$$

$$= 0.002133\, R^3 \times n \rightarrow (1)$$

$$\tau = \frac{16 W \times R}{\pi d^3}$$

$$125 = \frac{16 \times 60 \times R}{\pi d^3}$$

$$R = \frac{125 \times \pi d^3}{16 \times 60}$$

$$\boxed{R = 0.40906\, d^3} \rightarrow (2)$$

Substituting the value of R in equation (1)

$$d^4 = 0.002133 \times (0.40906 \cdot d^3)^3 \times n$$

$$= 0.002133 \times (0.40906^3) \times d^9 \times n$$

$$= 0.00014599 \times d^9 \times n$$

$$\frac{d^{\cancel{9}\,5} \cdot n}{d^{\cancel{4}}} = \frac{1}{0.00014599}$$

$$d^5 \cdot n = \frac{1}{0.00014599} \rightarrow (3)$$

Solid length

$$\text{Solid length} = n \times d$$

$$50 = n \times d$$

$$n = \frac{50}{d} \rightarrow (4)$$

Sub. this value of n in equation (3)

$$d^{\cancel{5}\,4} \times \frac{50}{\cancel{d}} = \frac{1}{0.00014599}$$

$$d^4 = \frac{1}{0.00014599} \times \frac{1}{50}$$

$$= (136.99)^{1/4}$$

$$\boxed{d = 3.42\,mm} \rightarrow \text{Ans(i)}$$

Sub. this value in equation (4)

$$n = \frac{50}{d}$$

$$= \frac{50}{3.42}$$

$$= 14.62 \text{ say } 15$$

$$\boxed{n = 15} \rightarrow \text{Ans (ii)}$$

Also from equation ②

$$R = 0.40906 \times d^2$$

$$= 0.40906 \times (2.42)^2$$

$$\boxed{R = 16.36 \text{ mm}} \rightarrow \text{Ans (iii)}$$

Mean. diameter of coil

$$D = 2R$$

$$= 2 \times 16.36$$

$$\boxed{D = 32.72 \text{mm}} \rightarrow \text{Ans (iv)}$$

Prblm. no: 25

A Closed - coiled helical spring has a stiffness of 10 N/mm. Its length when fully compressed, with adjacent coils touching each other is 40 cm. The modulus of Rigidity of the material of the spring is 0.8×10^5 N/mm^2

(i) Determine the wire diameter and mean coil diameter if their ratio is $\frac{1}{10}$

(ii) If the gap between any two adjacent coil is 0.2 cm, what maximum load can be applied before the spring becomes solid., adjacent coils touch?

(iii) What is the corresponding maximum shear stress in the spring?

Given data

Stiffness of Spring $(S) = 10\ N/mm$

length of Spring when fully compressed., (Solid length)

$= 40\ cm \Rightarrow 40 \times 10\ mm \Rightarrow 400\ mm$

Modulus of Rigidity $(C) = 0.8 \times 10^5\ N/mm^2$

let,

d - Diameter of wire of Spring

D - Mean coil diameter

n - Number of Turns

W - Maximum load applied when Spring becomes Solid

τ - Maximum Shear Stress Induced in the wire.

Solution

$$\boxed{\frac{d}{D} = \frac{1}{10}}$$

Gap between any Two adjacent coil $= 0.2\ cm \Rightarrow 0.2 \times 10\ mm$

$= 2.0\ mm$

Total Gap in coils = Gap between two adjacent coil × Number of Turns

$= 2 \times n$

When the Spring is Fully compressed, there is no gap in the coils and hence maximum compression of the coil will be equal to the total gap in the coil.

Maximum Compression (δ) = $2 \times n$ mm

$$\delta = \frac{Cd^4}{64 \cdot R^3 \cdot n}$$

$$= \frac{0.8 \times 10^5 \times d^4}{64 \cdot R^3 \cdot n}$$

$$d^4 = \frac{16 \times 64 \times R^3 \times n}{0.8 \times 10^{\cancel{5}\,4}}$$

$$= \frac{1\cancel{6} \times 64 \times R^3 \times n}{(0.8 \times 1\cancel{0}) \times 10^3}$$

$$= \frac{64 \times R^3 \times n}{\cancel{8} \times 10^3}$$

$$d^4 = \left(\frac{8}{10^3}\right) R^3 \times n \rightarrow (1)$$

Solid length

$$\text{Solid length} = n \times d$$

$$400 = n \times d$$

$$n = \frac{400}{d} \rightarrow (2)$$

Sub. the value of n in equation (1)

$$d^4 = \left(\frac{8}{10^3}\right) \times R^3 \times \left(\frac{400}{d}\right)$$

$$= 3.2 \times \frac{R^3}{d}$$

$$d^5 = 3.2 \times R^3$$

Mean coil radius

$$R = \frac{D}{2}$$

$$d^5 = 3.2 \times \left(\frac{D}{2}\right)^3$$

$$= \frac{3.2 \times D^3}{8}$$

$$= 0.4 D^3$$

$$\frac{d^5}{D^3} = 0.4$$

$$\frac{d^3}{D^3} \cdot d^2 = 0.4$$

$$\left(\frac{1}{10}\right)^3 \cdot d^2 = 0.4 \qquad \therefore \left(\frac{d}{D} = \frac{1}{10}\right)$$

$$d^2 = 0.4 \times 10^3$$

$$= 400$$

$$= \sqrt{400}$$

$$\boxed{d = 20\,mm} \Rightarrow 20 \div 10\,cm \Rightarrow 2\,cm$$

$$\frac{d}{D} = \frac{1}{10}$$

$$D = 10 \times d$$

$$= 10 \times 2$$

$$\boxed{D = 20 \text{ cm}}$$

Let us find first number of Turns

From equation 2

$$n = \frac{400}{d}$$

$$= \frac{400}{20} \qquad \therefore (d = 20 \text{ mm})$$

$$\boxed{n = 20}$$

$$\delta = 2 \times n$$

$$= 2 \times 20$$

$$\boxed{\delta = 40 \text{ mm}}$$

Stiffness of Spring

$$S = \frac{W}{\delta}$$

$$10 = \frac{W}{40}$$

$$W = 10 \times 40$$

$$\boxed{W = 400 \text{ N}}$$

$$\tau = \frac{16 \cdot W \cdot R}{\pi d^3}$$

$R = \frac{D}{2} \Rightarrow \frac{20}{2} \Rightarrow 10cm$

$$= \frac{16 \times 400 \times 100}{\pi \times 20^3}$$

$10\ cm \Rightarrow 10 \times 10 mm$

$\therefore (R = 100\ mm)$

$$\boxed{\tau = 25.465\ N/mm^2}$$

Prblm.no: 26

Two closed - coiled concentric helical springs of the same length, are wound out of the same wire, circular in cross - section and supports a compressive load 'p'. The Inner spring consists of 20 turns of mean diameter 16 cm and the outer spring has 18 turns of mean diameter 20 cm. calculate the maximum stress produced in each spring if the diameter of wire = 1 cm and P = 1000 N

Given data

Total load supported (P) = 1000 N

- Both the spring are of the same length of the same material and having same dia. of wire. Hence values of L, C and 'd' will be same.

For Inner Spring

No. of Turns $n_i = 20$

Mean dia $D_i = 16\ cm \Rightarrow 16 \times 16\ mm \Rightarrow 160\ mm$

Dia. of wire $d_i = 1\ cm \Rightarrow 1 \times 10\ mm \Rightarrow 10mm$

For Outer Spring

- No. of Turns $(n_o) = 12$

Mean dia $(D_o) = 20\,cm \Rightarrow 20 \times 10\,mm \Rightarrow 200\,mm$

Dia. of wire $(d_o) = 1\,cm \Rightarrow 10\,mm$

$$R_o = \frac{D_o}{2} = \frac{200}{2}$$

$$R_o = 100\,mm$$

Let,

W_i - Load carried by Inner Spring

W_o - Load carried by Outer Spring

τ_i - Max. Shear Stress produced in Inner Spring

τ_o - Max. Shear Stress produced in outer Spring

Now

$W_i + W_o =$ Total load carried

$W_i + W_o = 1000 \rightarrow (1)$

∴ Since there are two closed - coiled concentric helical springs, hence deflection of both the springs will be same.

$$\delta_o = \delta_i$$

Where,

δ_o - Deflection of outer Spring

δ_i - Deflection of Inner Spring

Solution

The deflection of closed-coiled spring

$$\delta = \frac{64 W \times R^3 \times n}{C \times d^4}$$

For Outer Spring

$$\delta_o = \frac{64 W_o \times R_o^3 \times n_o}{C \times d_o^4}$$

$$= \frac{64 W_o \times 100^3 \times 18}{C \times 10^4}$$

$$\therefore (R_o = 100), (d_o = 10)$$

Similarly for Inner Spring

$$\delta_i = \frac{64 W_i \times R_i \times n_i}{C \times d_i^4}$$

$$= \frac{64 W_i \times 80^3 \times 20}{C \times 10^4}$$

Material of wires is same. Hence value of 'C' will be same

$$\delta_o = \delta_i$$

$$\frac{64 W_o \times 100^3 \times 18}{\cancel{C \times 10^4}} = \frac{\cancel{64} W_i \times 80^3 \times 20}{\cancel{C \times 10^4}}$$

$$W_o \times 100^3 \times 18 = W_i \times 80^3 \times 20$$

$$W_o = \frac{W_i \times 80^2 \times 20}{100^2 \times 18}$$

$$W_o = 0.569\, W_i$$

Sub. the value of W_o in equation (1)

$$W_i + 0.569 W_i = 1000$$

$$1.569\, W_i = 1000$$

$$W_i = \frac{1000}{1.569}$$

$$\boxed{W_i = 637.3\, N}$$

From equtn (1)

$$W_i + W_o = 1000$$

$$W_o = 1000 - W_i$$

$$= 1000 - 637.3$$

$$\boxed{W_o = 362.7\, N}$$

Maximum shear stress produced

$$\tau = \frac{16 W R}{\pi d^3}$$

For outer spring (Maximum shear stress)

$$\tau_o = \frac{16 W_o \times R_o}{\pi d_o^3}$$

$$= \frac{16 \times 262.7 \times 100}{\pi \times 10^3}$$

$$\boxed{\tau_o = 184.72 \ N/mm^2}$$

Similarly for Inner spring (Maximum shear stress)

$$\tau_i = \frac{16 \times W_i \times R_i}{\pi \times d_i^3}$$

$$= \frac{16 \times 637.3 \times 80}{\pi \times 10^3}$$

$$\boxed{\tau_i = 259.66 \ N/mm^2}$$

Prblm.no: 27

A closed coiled helical spring made of 10mm diameter steel wire has 150 coils of 100mm mean diameter. The spring is subjected to an axial load of 100N. Calculate

(i) The maximum shear stress induced

(ii) The deflection

(iii) Stiffness of the spring

Take Modulus of Rigidity $C = 8.16 \times 10^4 \ N/mm^2$

Given data

Dia. of wire $(d) = 10 \ mm$

Number of coils $(n) = 15$

Mean dia. of coil $(D) = 100 \ mm$

Axial load $(W) = 100\ N$

Modulus of Rigidity $(C) = 8.16 \times 10^4\ N/mm^2$

Solution

Maximum shear stress Induced (τ)

$$\tau = \frac{16WR}{\pi d^3}$$

$$= \frac{16 \times 100 \times 50}{\pi \times 10^3}$$

$$\boxed{\tau = 24.46\ N/mm^2}$$

Deflection (δ)

$$\delta = \frac{64 W \times R^3 \times n}{C \times d^4}$$

$$= \frac{64 \times 100 \times 50^3 \times 15}{8.16 \times 10^4 \times 10^4}$$

$$\boxed{\delta = 14.7mm}$$

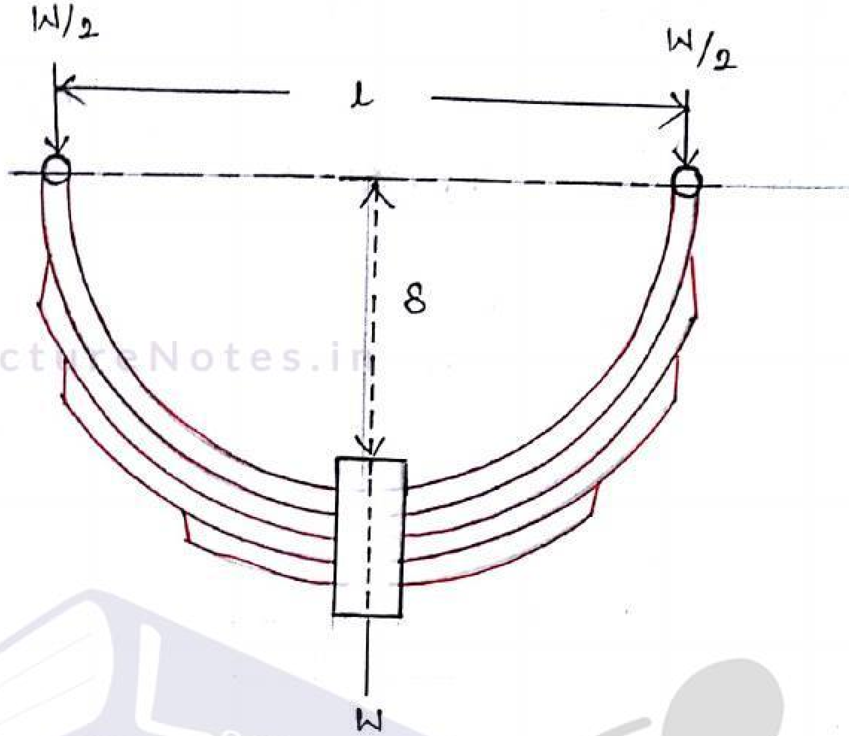
Stiffness of the Spring

$$\text{Stiffness} = \frac{\text{Load on Spring}}{\text{Deflection of Spring}}$$

$$= \frac{100}{14.7}$$

$$\boxed{\text{Stiffness} = 6.802\ N/mm}$$

LAMINATED OR LEAF SPRING



* The Laminated springs are used to absorb shocks in railway wagons, coaches and road vehicles (such as cars, Lorries, etc...)

* The Laminated spring which consists of a number of parallel Strips of a metal having different length and same width placed one over the other

* Initially all the plates are bent to the same radius and are free to Slide one over the other.

* The initial position of the spring, which is having some Central deflection 'δ.'

* The spring rest on the axis of th- vehicle and its top Plate is pinned at the ends to the chassis of the vehicle.

* When the spring is loaded to the designed load W, all the plates become flat and the central deflection (δ) disappears.

let,

b - Width of each plate

n - Number of plates

l - Span of spring

σ - Maximum bending stress developed in the plates

t - Thickness of each plate.

W - Point load acting at the centre of the spring.

δ - original deflection of the top spring.

Expression for Maximum bending stress developed in the plate

The Load W acting at the centre of the lowermost plate, will be shared equally on the two ends of the top plate

Bending moment at the centre $\Big\}$ = Load at one end $\times \frac{l}{2}$

$$M = \frac{W}{2} \times \frac{l}{2}$$

$$\boxed{M = \frac{Wl}{4}} \rightarrow (1)$$

The Moment of Inertia of each plate

$$\boxed{I = \frac{bt^3}{12}}$$

The Relation among bending stress (σ), bending moment (M) and Moment of Inertia

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\therefore \left(y = \frac{t}{2}\right)$$

$$M = \frac{\sigma}{y} \times I$$

$$= \frac{\sigma \times bt^3/\cancel{12}}{\frac{t}{\cancel{2}}\,6}$$

$$\boxed{M = \frac{\sigma \cdot bt^2}{6}}$$

Total Resisting moment by 'n' plates

$$= n \times M$$

$$= \frac{n \times \sigma \cdot bt^2}{6} \rightarrow (2)$$

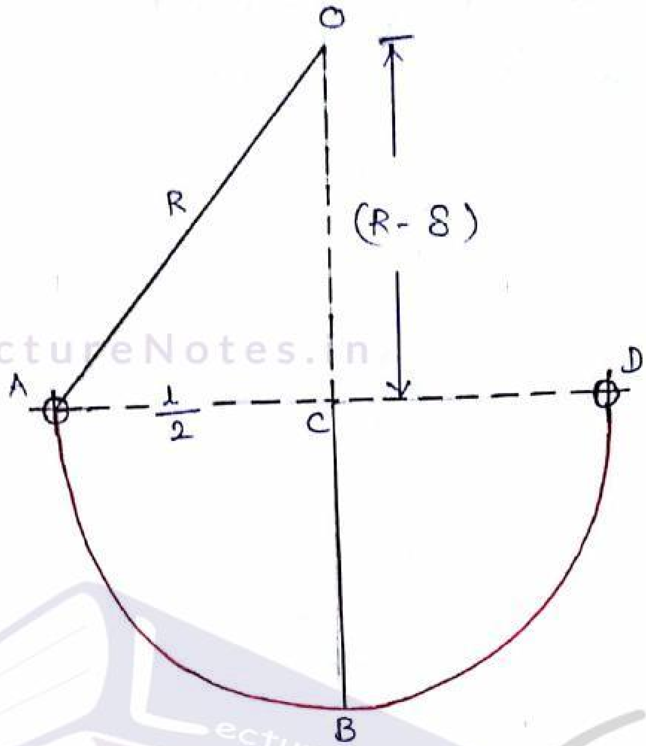
The Maximum B.M due to load is equal to the resisting moment, therefore equating (1) & (2)

$$\frac{W \cdot l}{4} = \frac{n\sigma \cdot bt^2}{6}$$

$$\sigma = \frac{\overset{3}{\cancel{6}}W \cdot l}{\underset{2}{\cancel{4}} \cdot n \cdot b \cdot t^2}$$

$$\boxed{\sigma = \frac{3Wl}{2nbt^2}}$$

Expression for central deflection of the Leaf Spring



Now

R - Radius of the plate to which they are bent

From Triangle ACO

$$AO^2 = AC^2 + CO^2$$

$$R^2 = \left(\frac{l}{2}\right)^2 + \left(R - \delta\right)^2$$

$$= \frac{l^2}{4} + R^2 + \delta^2 - 2R\delta$$

$$= \frac{l^2}{4} + R^2 - 2R\delta$$

$$\cancel{R^2} - \cancel{R^2} + 2R\delta = \frac{l^2}{4}$$

$$\delta = \frac{l^2}{4 \times 2R}$$

$$\delta = \frac{l^2}{8R} \rightarrow (2)$$

The relation between bending stress, Modulus of elasticity and Radius of curvature (R) is given by

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$R = \frac{E \times y}{\sigma}$$

$$= \frac{E \times \frac{t}{2}}{\sigma} \qquad \therefore \left(y = \frac{t}{2}\right)$$

$$\boxed{R = \frac{E \times t}{2\sigma}}$$

Sub. this value of R in equation (2)

$$\delta = \frac{l^2 \times \cancel{2}\sigma}{\cancel{8}_{4} \times E \times t}$$

$$\boxed{\delta = \frac{\sigma . l^2}{4Et}}$$

Prblm.no:28

A leaf spring carries a central load of 3000 N. The leaf spring is to be made of 10 steel plates 5 cm wide and 6 mm thick. If the bending stress is limited to 150 N/mm^2

Determine

(i) Length of the Spring

(ii) Deflection at the centre of the Spring.

Take $E = 2 \times 10^5\ N/mm^2$

Given data

Central load $(W) = 3000\ N$

No. of plates $(n) = 10$

Width of each plates $(b) = 5\ cm \Rightarrow 5 \times 10\ mm = 50\ mm$

Thickness $(t) = 6\ mm$

Bending stress $(\sigma) = 150\ N/mm^2$

Modulus of Elasticity $(E) = 2 \times 10^5\ N/mm^2$

let,

l - Length of Spring

δ - Deflection at the centre of spring

Solution

$$\sigma = \frac{3Wl}{2nbt^2}$$

$$150 = \frac{3 \times 3000 \times l}{2 \times 10 \times 50 \times 6^2}$$

$$l = \frac{150 \times 2 \times 10 \times 50 \times 6^2}{3 \times 3000}$$

$$\boxed{l = 600\ mm}$$

$$\delta = \frac{\sigma \cdot l^2}{4Et}$$

$$= \frac{150 \times 600^2}{4 \times 2 \times 10^5 \times 6}$$

$$\boxed{\delta = 11.25\ mm}$$

Problm.no:29

A laminated spring 1m long is made up of plates each 5cm wide and 1cm thick. If the bending stress in the plate is limited to 100 N/mm^2, how many plates would be required to enable the spring to carry a central point load of 2KN? If the $E = 2.1\times10^5\ N/mm^2$, what is the deflection under the load ?

Given data

length of spring $(l) = 1m \Rightarrow 1000\ mm$

Width of each plate $(b) = 5cm \Rightarrow 50\ mm$

Thickness of each plate $(t) = 1cm \Rightarrow 10\ mm$

Bending stress $(\sigma) = 100\ N/mm^2$

central load on spring $W = 2KN \Rightarrow 2000\ N$

young's modulus $(E) = 2.1\times10^5\ N/mm^2$

Let,

n - Number of plates

δ - Deflection under the load

Solution

$$\sigma = \frac{3Wl}{2nbt^2}$$

$$100 = \frac{3\times2000\times1000}{2\times n\times50\times10^2}$$

$$n = \frac{8 \times 2000 \times 1000}{100 \times 2 \times 50 \times 100}$$

$$\boxed{n = 6}$$

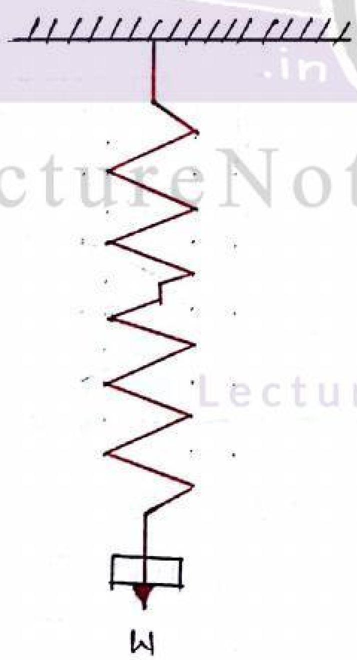
Deflection under load

$$\delta = \frac{\sigma \times l^2}{4E \times t}$$

$$= \frac{100 \times 1000^2}{4 \times 2.1 \times 10^5 \times 10}$$

$$\boxed{\delta = 11.9\,mm}$$

SPRINGS IN SERIES

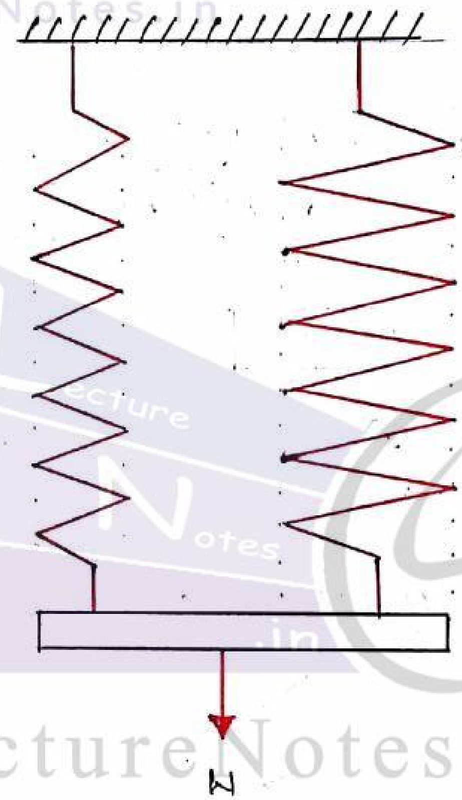


* Two springs are connected

* Each spring is subjected to the same load applied at the end of one spring.

* A little consideration will show that the total extension of the assembly is equal to the algebric sum of the extensions of the two springs.

SPRINGS IN PARALLEL



* Two springs are connected in parallel
* The Extension of each spring is the same.
* A little consideration will show that the load applied on the assembly is shared by two springs.

Problm.no:30

Two - closed coiled helical springs wound from the same wire, but with different core radii having equal no. of coils are compressed between rigid plates at their ends. calculate the maximum shear stress induced in each spring, if the wire diameter is 10mm and the load applied between the rigid plates is 500 N. The core radii of the springs 100 mm and 75 mm respectively.

Given data

No. of coils in the outer spring $(n_1) = n_2$

n_2 - Number of coil in the Inner spring

Diameter of spring wire $(d) = 10\text{mm}$

Load $(W) = 500\text{ N}$

Radius of outer spring $(R_1) = 100\text{mm}$

Radius of Inner spring $(R_2) = 75\text{mm}$

Let,

τ_1 - Shear stress developed in the outer spring

W_1 - Load shared by the outer spring

τ_2, W_2 - Corresponding values for the Inner spring.

Solution

Deflection of the outer spring

$$\delta_1 = \frac{64 W_1 R_1^3 n_1}{Cd^4}$$

$$= \frac{64 \times W_1 \times (100)^{3} \times n_1}{C \times (10)^4} \Rightarrow \frac{64 \times 10^{6} \times W_1 \times n_1}{C \times 10^4}$$

$$= \frac{6400\, W_1 n_1}{C} \rightarrow (1)$$

$$\delta_2 = \frac{64\, W_2 R_2^3 n_2}{Cd^4}$$

$$= \frac{64 \times W_2 \times (75)^3 \times n_2}{C \times (10)^4} \Rightarrow \frac{27 \times 10^{6} \times W_2 \times n_2}{C \times 10^4}$$

$$\delta_2 = \frac{2700\, W_2 n_2}{C} \rightarrow (2)$$

Spring are held b/w two Rigid plates

∴ Deflection in both the springs must equal

Now equating (1) & (2)

$$\frac{6400\, W_1 n_1}{C} = \frac{2700\, W_2 n_2}{C}$$

$$\frac{64\cancel{00}\, W_1 \cancel{n_1}}{\cancel{C}} = \frac{27\cancel{00}\, W_2 \cancel{n_2}}{\cancel{C}} \quad \therefore (n_1 = n_2)$$

$$W_1 = \frac{27}{64} W_2$$

We know that

$$W_1 + W_2 = 500$$

$$\frac{27 \times W_2}{64} + W_2 = 500 \Rightarrow \frac{27 W_2 + 64 W_2}{64} = 500$$

$$\frac{91\, W_2}{64} = 500$$

$$W_2 = \frac{500 \times 64}{91}$$

$$\boxed{W_2 = 351.6 N}$$

$$W_1 = 500 - W_2$$

$$= 500 - 351.6$$

$$\boxed{W_1 = 148.4 \text{ N}}$$

Relation for Torque

$$W_1 R_1 = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$148.4 \times 100 = \frac{\pi}{16} \times \tau_1 \times (10)^3$$

$$\tau_1 = \frac{148.4 \times 100 \times 16}{\pi (10)^3}$$

$$\tau_1 = 75.6 \text{ N/mm}^2$$

$$\boxed{\tau_1 = 75.6 \text{ MPa}}$$

Similarly

$$\tau_2 = \frac{351.6 \times 75 \times 16}{\pi \times (10)^3}$$

$$= 134.3 \text{ N/mm}^2$$

$$\boxed{\tau_2 = 134.4 \text{ MPa}}$$