1. Matrices

Introduction :

We are familiar with the concept of Matrix, the matrix is an important tool in the study of many subjects like physics, Mechanics, statistics, electronic circuits and computer applications.

The definition of Matrix is: ~~matri~~

The system of mxn numbers (elements) [real (or) complex] arranged in the form of an orderd set of m rows, each row consisting of an orderd set of n numbers b/w [ ] (or) ( ) (or) ‖ ‖ is called a matrix of order (or) type of mxn i.e Let A be the matrix of order mxn Then

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

$A = [a_{ij}]_{m \times n}$

i = rows = m

j = columns = n

Find the Rank of Matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

To find rank, the determinant of 3x3 is Minor of order 3

$$|A| = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

$$= 1(6-8) - 2(4-0) + 3(4-0)$$

$$= -2 - 8 + 12 = 2 \neq 0$$

The Non-vanishing ($\neq 0$) order of the minor is called rank of the matrix

$|A| \neq 0$

Rank of the Matrix is 3

Find the rank of the Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{vmatrix}$ To find rank of the 3rd order Minor is $|A|_{3\times3}$

$= 1(24-25) - 2(18-20) + 3(15-16)$

$= -1 - 2(-2) + 3(-1)$

$= -1 + 4 - 3 = 0$

$|A| = 0$

consider 2nd order Minor $= |A|_{2\times2}$

$|A| = \begin{vmatrix} 3 & 4 \\ 4 & 5 \end{vmatrix}$

$= (15-16) = -1$

$|A| \neq 0$

2nd order Minor $\neq 0$

Rank of Matrix is 2

3. Find the Rank of the Matrix.

$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -4 & 6 \\ 3 & -6 & 9 \end{bmatrix}$

Ans:- $|A| = \begin{vmatrix} 1 & -2 & 3 \\ 2 & -4 & 6 \\ 3 & -6 & 9 \end{vmatrix}$

$= 1(-36+36) + 2(18-18) + 3(-12+12)$

$|A| = 0$

consider $|A|_{2\times2} = \begin{vmatrix} 1 & -2 \\ 2 & -4 \end{vmatrix} = -4 + 4 = 0$

Rank of the Matrix = 1

4. Find the Rank of the Matrix, $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 0 & 5 & 4 \end{bmatrix}$.

Ans:- To find Minor (determinant) consider the Matrix rows = Minimum

Min (m, n) = Min(2, 4)

consider $A_{2\times2}$ sub Matrix.

consider 2nd order Minor $= |A|_{2\times2}$

$|A| = \begin{vmatrix} 1 & 2 \\ -2 & 0 \end{vmatrix} = 0 - (-4) = 4 \neq 0$

Rank of the

5. Find the ...
$\begin{bmatrix} 1 & 2 & 3 \\ 2 & K & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2

The given Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & K & 7 \\ 3 & 6 & 10 \end{bmatrix}$ has rank = 2

A: $|A| = \begin{bmatrix} 1 & 2 & 3 \\ 2 & K & 7 \\ 3 & 6 & 10 \end{bmatrix}$ $|A|_{3\times 3} = 0$ 3rd order minor = 0

$= 1[10K-42] - 2[20-21] + 3[12-3K] = 0$

$= 10K - 42 - 40 + 42 + 36 - 9K = 0$

$= K - 4 = 0$

$= \boxed{K = 4}$

zero row and non-zero row of a Matrix:
If all the elements in a row of a Matrix are zeros then it is called a zero row and if there is atleast one non-zero element is present in a row then it is called a non-zero row.

Methods to find rank of a Matrix:
i. Echelon form:
A Matrix is said to be in an echlon form, if it satisfies the following Condition:
1. All rows with zero elements will below the non-zero rows
2. The no. of zeroes in the first non-zero row is less than the no. of such zeroes in the next row.
3. Every Matrix can be transformed into echelon form by applying row transformations only.
4. In the echelon form the rank of the Matrix is equal to the no. of non zero rows.

1. Reduce the matrix $A = \begin{bmatrix} 2 & 3 & 7 \\ 3 & -2 & 4 \\ 1 & -3 & -1 \end{bmatrix}$ in the echelon form.

A: To reduce into echelon form, Apply row transformations and make below diagnol elements zero.

$A = \begin{bmatrix} 2 & 3 & 7 \\ 3 & -2 & 4 \\ 1 & -3 & -1 \end{bmatrix}$

$R_2 \to 2R_2 - 3R_1$, $R_3 \to 2R_3 - R_1$

$$A \simeq \begin{bmatrix} 2 & 3 & 7 \\ 0 & -13 & -13 \\ 0 & -9 & -9 \end{bmatrix}$$

$\frac{R_2}{-13}$, $\frac{R_3}{-13}$

$$A \simeq \begin{bmatrix} 2 & 3 & 7 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$\begin{bmatrix} 2 & 3 & 7 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is under Echelon form

Rank of the Matrix is '2'

Q. Reduce the matrix into echelon form and find it's Rank where $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$

A:- The given matrix is $\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$

To reduce the above matrix into echelon form, apply row transformations and make below diagonals zero

$R_2 \to R_2 + R_1$

$R_3 \to R_3 + 2R_1$

$R_4 \to R_4 - R_1$

$\begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & -11 & 8 & -5 \\ 0 & 4 & -3 & 2 \end{bmatrix}$ $R_3 \to 2R_3 - 11R_2$

$R_4 \to 2R_4 + 2R_2$

$$\begin{bmatrix} 0 & -2 & 2 & -1 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_4 \to 6R_4 + R_3$

$$\begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

∴ Rank of the matrix is 4

3. Define the rank of the matrix and hence find the rank of $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$

Ans- Rank: The number 'r' is said to be rank of a matrix if

i. There exists atleast one minor of 'A' which is non-zero.

ii. All the minors of order $(r+1)$ and above if they exist are zeros.

The given Matrix $A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$

To reduce the above Matrix into echelon form, apply row transformation and make below diagonal elements zero.

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 4R_1$

$R_4 \to R_4 - 4R_1$

$$A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & -15 & -21 \end{bmatrix}$$

$R_4 \to R_4 - 3R_3$

$$\begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

R_3, R_2 has same no. of zeros, increase one zero in R_3

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of the matrix is '2'.

4. For what value of 'k' the matrix $\begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix}$ has rank '3'

A:- The given matrix

$$A = \begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix}$$ has rank $\rho(A) = r = 3$

To find 'k' value, reduce the above matrix into echelon form.

$R_2 \to 4R_2 - R_1$

$R_3 \to 4R_3 - kR_1$

$R_4 \to 4R_4 - 9R_1$

$$A \simeq \begin{bmatrix} 4 & 4 & -3 & 1 \\ 0 & 0 & -1 & -1 \\ 0 & (8-4k) & (8+3k) & (8-k) \\ 0 & 0 & (4k+27) & 3 \end{bmatrix}$$

$R_2 \rightleftarrows R_3$

$$\simeq \begin{bmatrix} 4 & 4 & -3 & 1 \\ 0 & (8-4k) & (8+3k) & (8-k) \\ 0 & 0 & -1 & -1 \\ 0 & 0 & (4k+27) & 3 \end{bmatrix}$$

$R_4 \to R_4 + (4k+27) R_3$

$$\simeq \begin{bmatrix} 4 & 4 & -3 & 1 \\ 0 & (8-4k) & (8+3k) & (8-k) \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & (-4k-24) \end{bmatrix}$$ is under echelon form.

Given matrix A has rank (r=3)

i.e $-4K-24=0$

$\boxed{K=-6}$

Reduction to Normal form (first canonical form):

Every $m \times n$ matrix of rank 'r' can be reduced into the form I_r (or) $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ (or) $\begin{bmatrix} I_r & 0 \end{bmatrix}$ (or) $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$ by using row and column transformations. I_r = order of the unit matrix gives rank 'r'.

For Square matrix:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad I_r = I_4 \qquad A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

(Row transfer) column transformation

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

For Non-square matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} I_3 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} I_2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} I_3 \\ 0 \end{bmatrix} \qquad \begin{pmatrix} I_3 \\ 0 \end{pmatrix}$$

• Reduce the following matrix into Normal form, hence find the rank where $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

Sol- The give matrix

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

Apply row transformations make below diagonals zero

$R_2 \to 2R_2 - R_1$

$R_3 \to 2R_3 - 3R_1$

$R_4 \to R_4 - 3R_1$

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 0 & -5 & -3 & -7 \\ 0 & -7 & 9 & -1 \\ 0 & -6 & 3 & -4 \end{bmatrix}$$

$R_3 \to 5R_3 - 7R_2$

$R_4 \to 5R_4 - 6R_2$

$$\approx \begin{bmatrix} 2 & 3 & -1 & -1 \\ 0 & -5 & -3 & -7 \\ 0 & 0 & 66 & 44 \\ 0 & 0 & 33 & 22 \end{bmatrix}$$

$R_4 \to 2R_4 - R_3$

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 0 & -5 & -3 & -7 \\ 0 & 0 & 66 & 44 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now apply column transformation and make upper triangle elements zero.

$C_2 \to 2C_2 - 3C_1$, $C_3 \to 2C_3 + C_1$, $C_4 \to 2C_4 + C_1$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -10 & -6 & -14 \\ 0 & 0 & 132 & 88 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

~~$C_3 \to 5C_3 - 3C_2$, $C_4 \to 5C_4 - 7C_2$~~

$\frac{R_2}{2}, \frac{R_3}{44}$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -5 & -3 & -7 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -5 & 0 & 0 \\ 0 & 0 & 66 & 44 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to 5C_3 - 3C_2$, $C_4 \to 5C_4 - 7C_2$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -5 & 0 & 0 \\ 0 & 0 & 15 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to \frac{R_3}{5}$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -5 & 0 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to 3C_4 - 2C_3$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -5 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\frac{C_1}{2}, \frac{C_2}{-5}, \frac{C_3}{3}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of the matrix $= 3$

$$\begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

2. Find the rank of the Matrix by reducing into normal form for the matrix
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & -4 \\ 2 & 3 & 5 & -5 \\ 3 & -4 & -5 & 8 \end{bmatrix}$$

A:- Given matrix
$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & -4 \\ 2 & 3 & 5 & -5 \\ 3 & -4 & -5 & 8 \end{bmatrix}$$

Apply row transformations make below the diagonal elements zero.

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 2R_1$
$R_4 \to R_4 - 3R_1$
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -5 \\ 0 & 1 & 3 & -7 \\ 0 & -7 & -8 & 5 \end{bmatrix}$$

$R_3 \to R_3 - R_2$
$R_4 \to R_4 + 7R_2$
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & -6 & -30 \end{bmatrix}$$
$R_4 \to R_4 + 6R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -42 \end{bmatrix}$$
$\frac{R_4}{-42}$
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Apply column transformations make above diagonals zero
$C_2 \to C_2 - C_1$ $C_3 \to C_3 - C_1$, $C_4 \to C_4 - C_1$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$C_3 \to C_3 - C_2$, $C_4 \to C_4 + 5C_2$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$C_4 \to C_4 + 3C_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_4 = I_r$$

Rank of the Matrix $= 4$

3. Reduce the matrix
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$
into Normal Form; hence Find it's Rank?

A:- Given Matrix
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$

To reduce into normal form, apply row and column transformations to make unit

$R_2 \to R_2 - 2R_1$; $R_3 \to R_3 - 3R_1$

$$\approx \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -5 \\ 0 & -6 & -4 & -22 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\approx \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -5 \\ 0 & 0 & 0 & -12 \end{bmatrix}$$

$C_2 \to C_2 - 2C_1$, $C_3 \to C_3 - 3C_1$, $C_4 \to C_4 - 4C_1$

$$\approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -3 & -2 & -5 \\ 0 & 0 & 0 & -12 \end{bmatrix}$$

$C_3 \to 3C_3 - 2C_2$ & $C_4 \to 3C_4 - 5C_2$

$$\approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & -12 \end{bmatrix} \quad \frac{C_2}{-3}, \frac{C_4}{-12}$$

$$\approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad C_4 \leftrightarrow C_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad [I_3, 0]$$

Rank of the matrix is 3!

4. Reduce the matrix $\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ into normal form, hence find the rank of the matrix.

A:- Given Matrix $\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$

To reduce into Normal form, apply row and column transformations to make unit Matrix.

$C_2 \leftrightarrow C_1$

$$\begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 4 & 2 & 6 \\ 1 & 2 & 3 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_1$ $\begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 4 & 2 & 6 \\ 0 & 2 & 1 & 3 \end{bmatrix}$ $R_3 \to 2R_3 - R_4$

$$\begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 4 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 - 2C_1,\ C_4 \to C_4 + 2C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to 2C_3 - C_2,\ C_4 \to 4C_4 - 6C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{C_2}{4}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$[I_2, 0] = [I_r, 0]$

The rank of the matrix $= 2$

Normal form of PAQ:

$$A_{m\times n} = I_{m\times m}\cdot A\cdot I_{n\times n}$$

$$A_{3\times4} = I_{3\times3}\cdot A\cdot I_{4\times4}$$

column

$$\begin{bmatrix} - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\cdot A\cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \text{Post factor}$$

(Row) (Row) Pre-factor (Column)

Normal form = P.A.Q

1. Find the Non-Singular Matrices 'P' and 'Q' such that PAQ is the Normal form of A and hence find it's ra

Where i. $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ ii. $A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 2 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

A. The given matrix $A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 2 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

The Normal form of PAQ

$$A_{m\times n} = I_{m\times m}\cdot A\cdot I_{n\times n} \qquad m=3$$

$$A_{3\times4} = I_{3\times3}\cdot A\cdot I_{4\times4} \qquad n=4$$

$$\begin{bmatrix} 2 & 1 & -3 & -6 \\ 2 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\cdot A\cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Apply row transformations on L.H.S of 'A' and Pre-factor of 'A' on R.H.S.

Apply column transformations on same L.H.S of 'A' and post-factor of 'A' on R.H.S.

$R_2 \to R_2 - R_1$, $R_3 \to 2R_3 - R_1$

$$\begin{bmatrix} 2 & 1 & -3 & -6 \\ 0 & -4 & 4 & 8 \\ 0 & 1 & 5 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}\cdot A\cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to 4R_3 + R_2$

$$\begin{bmatrix} 2 & 1 & -3 & -6 \\ 0 & -4 & 4 & 8 \\ 0 & 0 & 24 & 48 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -5 & 1 & 8 \end{bmatrix} \cdot A \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to 2C_2 - C_1$
$C_3 \to 2C_3 + 3C_1$
$C_4 \to C_4 + 3C_1$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -8 & 8 & 8 \\ 0 & 0 & 48 & 48 \end{bmatrix} \begin{array}{l} \cancel{C_3 \to C_3 + C_2} \\ \cancel{C_4 \to C_4 + C_2} \end{array} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ -5 & 1 & 8 \end{bmatrix} \cdot A \cdot \begin{bmatrix} 1 & -1 & 3 & 3 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_3 \to C_3 + C_2$, $C_4 \to C_4 + C_2$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -8 & 0 & 0 \\ 0 & 0 & 48 & 48 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -5 & 1 & 8 \end{bmatrix} \cdot A \cdot \begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_4 \to C_4 - C_3$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -8 & 0 & 0 \\ 0 & 0 & 48 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -5 & 1 & 8 \end{bmatrix} \cdot A \cdot \begin{bmatrix} -1 & -1 & 2 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\frac{R_1}{2}, \frac{R_2}{-8}, \frac{R_3}{48}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{1}{8} & \frac{-1}{8} & 0 \\ \frac{-5}{48} & \frac{1}{48} & \frac{1}{6} \end{bmatrix} \cdot A \cdot \begin{bmatrix} -1 & -1 & 2 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$[I_3, 0]$

$$P = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{1}{8} & \frac{-1}{8} & 0 \\ \frac{-5}{48} & \frac{1}{48} & \frac{1}{6} \end{bmatrix} \qquad Q = \begin{bmatrix} -1 & -1 & 2 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find two Non-singular Matrices P and Q such that PAQ the Normal form where $A = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$

Given matrix

S:- $A = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & \dots \end{bmatrix}$

$$\begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 4R_1$

$$\begin{bmatrix} 1 & -1 & 2 & -3 \\ 0 & 5 & -8 & 14 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to 5R_3 - 3R_2$

$R_4 \to 5R_4 - R_2$

$$\begin{bmatrix} 1 & -1 & 2 & -3 \\ 0 & 5 & -8 & 14 \\ 0 & 0 & 24 & -22 \\ 0 & 0 & 8 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ 12 & -1 & 0 & 0 \\ 4 & -1 & 0 & 5 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_4 \to 3R_4 - R_3$

$$\begin{bmatrix} 1 & -1 & 2 & -3 \\ 0 & 5 & -8 & 14 \\ 0 & 0 & 24 & -22 \\ 0 & 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ 12 & -1 & 0 & 0 \\ 0 & 4 & 0 & 15 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 + C_1$, $C_3 \to C_3 - 2C_1$, $C_4 \to C_4 + 3C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & -8 & 14 \\ 0 & 0 & 24 & -22 \\ 0 & 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ 12 & -1 & 0 & 0 \\ 0 & 4 & 0 & 15 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_3 \to 5C_3 + 8C_2$, $C_4 \to 5C_4 - 14C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 120 & -110 \\ 0 & 0 & 0 & 50 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ 12 & -1 & 0 & 0 \\ 0 & 4 & 0 & 15 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & 1 & 8 & -14 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$\frac{C_3}{10}, \frac{C_4}{10}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 12 & 11 \\ 0 & 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ 12 & -1 & 0 & 0 \\ 0 & 4 & 0 & 15 \end{bmatrix} A \begin{bmatrix} 1 & 1 & \frac{-1}{5} & \frac{1}{10} \\ 0 & 1 & \frac{4}{5} & \frac{-7}{5} \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$C_4 \to 12C_4 - 11$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & 60 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ 12 & -1 & 0 & 0 \\ 0 & 4 & 0 & 15 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -1/5 & 17/5 \\ 0 & 1 & 4/5 & 128/5 \\ 0 & 0 & 1/2 & -11/2 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$\frac{R_2}{5}, \frac{R_3}{12}, \frac{R_4}{60}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{-4}{5} & 1/5 & 0 & 0 \\ 1 & -1/12 & 0 & 0 \\ 0 & 1/15 & 0 & \frac{1}{4} \end{bmatrix} A \begin{bmatrix} 1 & 1 & -1/5 & 17/5 \\ 0 & 1 & 4/5 & 128/5 \\ 0 & 0 & 1/2 & -11/2 \\ \end{bmatrix}$$

The given

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$A_{3\times3} = I_{3\times3} \cdot A \cdot I_{3\times3}$$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to 2R_2 - R_1, \; R_3 \to 2R_3 - R_1$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 3 & -1 \\ 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix} \cdot A \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to 3R_3 + R_2$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 3 & -1 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ -4 & 2 & 6 \end{bmatrix} \cdot A \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to 2C_2 + C_1, \; C_3 \to 2C_3 - 3C_1$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 6 & -2 \\ 0 & 0 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ -4 & 2 & 6 \end{bmatrix} \cdot A \begin{bmatrix} 1 & 1 & -3 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$C_3 \to 3C_3 + C_2$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & -24 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ -4 & 2 & 6 \end{bmatrix} \cdot A \cdot \begin{bmatrix} 1 & 1 & -8 \\ 0 & 2 & 2 \\ 0 & 0 & 6 \end{bmatrix}$$

$\frac{R_1}{2}, \frac{R_2}{6}, \frac{R_3}{-24}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-1}{6} & \frac{1}{3} & 0 \\ \frac{-1}{6} & \frac{-1}{12} & \frac{-1}{4} \end{bmatrix} \cdot A \cdot \begin{bmatrix} 1 & 1 & -8 \\ 0 & 2 & 2 \\ 0 & 0 & 6 \end{bmatrix}$$

Cauchy - Binet formula : Let $A_{m\times n}$ matrix and $B_{n\times m}$ Matrix, the product AB becomes an $m\times m$ matrix. The Cauchy Binet formula shows how to express the determinant of AB interms of 'A' and 'B' matrices. When m = n it reduces

$$\det(AB) = \det(A).\det(B)$$

Statement (theorem): Let 'A' be $m\times n$ matrix and 'B' be an $n\times m$ matrix then

$$\det(AB) = \sum_{J} \det A(J).\det B(J)$$

where $J = J_1 J_2 J_3 \dots J_m$ and $1 \le J_1 \le J_2 \le \dots \le J_m \le J_n$

Here A(J) denotes the matrix formed from 'A' using columns and B(J) denotes the sub-matrix formed from 'B' using rows $(J_1, J_2, \dots J_m)$,

~~verify cauchy-Binet for~~

verify cauchy-Binet formula for the Matrices

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & -1 \end{bmatrix} ; \quad B = \begin{bmatrix} 1 & 1 \\ 3 & 1 \\ 0 & 2 \end{bmatrix}.$$

- The given matrices

$$A = \begin{array}{c} \begin{array}{ccc} J_1 & J_2 & J_3 \end{array} \\ \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & -1 \end{bmatrix} \end{array} \quad B = \begin{array}{cc} \begin{bmatrix} 1 & 1 \\ 3 & 1 \\ 0 & 2 \end{bmatrix} & \begin{array}{c} J_1 \\ J_2 \\ J_3 \end{array} \end{array}$$

By Cauchy-Binet formula

$$\det(AB) = \det(A).\det(B) \quad \text{--- (1)}$$

$$(AB) = \begin{bmatrix} 1+3+0 & 1+1+4 \\ 3+3+0 & 3+1-2 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 6 & 2 \end{bmatrix}$$

$$|AB| = (8-36) = -28$$

Select columns from matrix 'A' | select rows from Matrix 'B'.

$$A_{J_1 J_2} = A_{12} = \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix} = 1-3 = -2$$

$$B_{12} = \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = 1-3 = -2$$

$$|A_{13}| = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} = -1-6 = -7$$

$$B_{13} = \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = 2-0 = 2$$

$$|A_{23}| = \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} = -1-2 = -3$$

$$B_{23} = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} = 6-0 = 6$$

∴ From - 1

$$\det(AB) = \det(A).\det(B)$$

$\text{Det}(AB) = (\det A_{12} \times \det B_{12}) + (\det A_{13} \times \det B_{13}) + (\det A_{23} \times \det B_{23})$

$-28 = ((-2)(-2)) + (-7 \times 2) + (-3 \times 6)$

$-28 = 4 - 14 - 18$

$-28 = -10 - 18 = -28$

$\therefore$ LHS = R.H.S

Cauchy-Binet formula is verifyed.

2. Verify Cauchy-Binet formula for the Matrices

i. $A = \begin{bmatrix} 1 & -1 & 0 \\ -3 & 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 2 & 2 \end{bmatrix}$

ii. $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$

Ans:- Given matrix

i. $A = \begin{bmatrix} 1 & -1 & 0 \\ -3 & 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 2 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 1-2+0 & 1+1+0 \\ -3+2+2 & -3-1+2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix}$

$|AB| = 2 - 2 = 0$

$|A_{12}| = \begin{vmatrix} 1 & -1 \\ -3 & 1 \end{vmatrix} = (1-3) = -2$ | $|B_{12}| = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -1-2 = -3$

$|A_{13}| = \begin{vmatrix} 1 & 0 \\ -3 & 1 \end{vmatrix} = 1 - 0 = 1$ | $|B_{13}| = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 2-2 = 0$

$|A_{23}| = \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix} = -1 - 0 = -1$ | $|B_{23}| = \begin{vmatrix} 2 & -1 \\ 2 & 2 \end{vmatrix} = 4+2 = 6$

$\text{Det}(AB) = (A_{12} \times B_{12}) + (A_{13} \times B_{13}) + (A_{23} \times B_{23})$

$0 = (-2 \times -3) + (1 \times 0) + (-1 \times 6)$

$0 = 6 + 0 - 6 = 0$

$\boxed{0 = 0}$

ii. $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ $B = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}$

$(AB) = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix} = \begin{bmatrix} 2+1 & 0-1 \\ 3+4 & 0-4 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 7 & -4 \end{bmatrix}$

$|AB| = -12 + 7 = -5$

$|A| = (8-3) = 5$

$|B| = \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = -1$

$|AB| = |A| \cdot |B|$

$-5 = (5)(-1)$

$\boxed{-5 = -5}$

Inverse of a Matrix by Gauss-Jordan Method:

We can find the inverse of a Matrix by elementary row transformations is called Gauss-Jordan Method.

Write the given Matrix $A = I_m \cdot A$ — (1)

Apply row transformations on eq-(1) upto the Matrix 'A' becomes unit Matrix

$$I_m = B \cdot A \quad -(2)$$

i.e 'B' is called inverse of the Matrix 'A':

$$I = B \cdot A$$

$$B = \frac{I}{A} = A^{-1}$$

A = Non-Singular Matrix

1. Find the inverse of the Matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ by Gauss-Jordan Method.

A:- The given Matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ By Gauss-Jordan Method the inverse of a Matrix we obtain

By applying elementary row transformations only

$$A_{3\times3} = \underset{\text{(Pre-factor)}}{I_{3\times3}} \cdot A$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_2 \to R_2 - R_1, \quad R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & -6 \\ 0 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \cdot A$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix}1 & 1 & 3\\0 & 2 & -6\\0 & 0 & -4\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\-1 & 1 & 0\\1 & 1 & 1\end{bmatrix}$$

$R_1 \to 2R_1 - R_2$

$$\begin{bmatrix}2 & 0 & 12\\0 & 2 & -6\\0 & 0 & -4\end{bmatrix} = \begin{bmatrix}3 & -1 & 0\\-1 & 1 & 0\\1 & 1 & 1\end{bmatrix} \cdot A$$

$R_1 \to R_1 + 3R_3$, $R_2 \to 4R_2 - 6R_3$

$$\begin{bmatrix}2 & 0 & 0\\0 & 8 & 0\\0 & 0 & -4\end{bmatrix} = \begin{bmatrix}6 & 2 & 3\\-10 & -2 & -6\\1 & 1 & 1\end{bmatrix} \cdot A$$

$\frac{R_1}{2}, \frac{R_2}{8}, \frac{R_3}{-4}$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}3 & 1 & 3/2\\\frac{-5}{4} & -1/4 & -3/4\\\frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4}\end{bmatrix} \cdot A$$

$I_{3\times3} = B.A$

i.e 'B' is called inverse of 'A'

$$A^{-1} = B = \begin{bmatrix}3 & 1 & 3/2\\\frac{-5}{4} & \frac{-1}{4} & -3/4\\\frac{-1}{4} & \frac{-1}{4} & -1/4\end{bmatrix}$$

2. Find the inverse of the Matrix by using gauss-jordon Method of the Matrix $\begin{bmatrix}-1 & -3 & 3 & -1\\1 & 1 & -1 & 0\\2 & -5 & 2 & -3\\-1 & 1 & 0 & 1\end{bmatrix}$

A:- The given Matrix $\begin{bmatrix}-1 & -3 & 3 & -1\\1 & 1 & -1 & 0\\2 & -5 & 2 & -3\\-1 & 1 & 0 & 1\end{bmatrix}$

Apply Gauss-Jordon Method and reduce the given matrix into unit Matrix

$A_{4\times4} = I_{4\times4} \cdot A$

$$\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 + R_1$, $R_3 \to R_3 + 2R_1$, $R_4 \to R_4 - R_1$

$$\begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & -11 & 8 & -5 \\ 0 & 4 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \to 2R_3 - 11R_2$, $R_4 \to R_4 + 2R_2$

$$\begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -7 & -11 & 2 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix} A$$

$R_4 \to 6R_4 + R_3$

$$\begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -7 & -11 & 2 & 0 \\ -1 & 1 & 2 & 6 \end{bmatrix} A$$

$R_1 \to 2R_1 - 3R_2$

$$\begin{bmatrix} -2 & 0 & 0 & 1 \\ 0 & -2 & 2 & -1 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -3 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -7 & -11 & 2 & 0 \\ -1 & 1 & 2 & 6 \end{bmatrix} A$$

$R_2 \to 3R_2 + R_3$

$$\begin{bmatrix} -2 & 0 & 0 & 1 \\ 0 & -6 & 0 & -2 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -3 & 0 & 0 \\ -4 & -8 & 2 & 0 \\ -7 & -11 & 2 & 0 \\ -1 & 1 & 2 & 6 \end{bmatrix} A$$

$R_1 \to R_1 - R_4$, $R_2 \to R_2 + 2R_4$, $R_3 \to R_3 - R_4$

$$\begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 \\ 0 & 0 & -6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -4 & -2 & -6 \\ -6 & -6 & 6 & 12 \\ -6 & -12 & 0 & -6 \\ -1 & 1 & 2 & 6 \end{bmatrix} A$$

$\frac{R_1}{-2}$, $\frac{R_2}{-6}$, $\frac{R_3}{-6}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 & 3 \\ 1 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \end{bmatrix} A$$

$I_{4\times4} = B.A$

'B' is called Inverse of 'A'

$$B = A^{-1} = \begin{bmatrix} 0 & 2 & 1 & 3 \\ 1 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 6 \end{bmatrix}$$

2. Compute the inverse of the Matrix $A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{bmatrix}$ by Gauss - Jordon Method.

Given Matrix

$$A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{bmatrix}$$

By apply Gauss Jordon Method we need to convert unit matrix.

$$A_{4\times4} = I_{4\times4} \cdot A$$

$$\begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_3 \to R_3 - 2R_1, \; R_4 \to R_4 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -2 & -3 \\ 0 & 1 & -1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix} \cdot A$$

$R_4 \to R_4 - R_2$

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -3 & -5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ -1 & -2 & 0 & 1 \end{bmatrix} \cdot A$$

$R_4 \to 2R_4 - 3R_3$

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ -2 & 2 & -3 & 2 \end{bmatrix} \cdot A$$

$R_1 \to R_1 - R_2$,

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ -2 & 2 & -3 & 2 \end{bmatrix} \cdot A$$

$R_2 \to R_2 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & -2 & 1 & 0 \\ -2 & 2 & -3 & 2 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + R_4$, $R_2 \to R_2 - R_4$, $R_3 \to R_3 - 3R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 2 & -3 & 2 \\ 3 & -4 & 4 & -2 \\ -6 & -8 & 10 & -6 \\ -2 & 2 & -3 & 2 \end{bmatrix} \cdot A$$

$\frac{R_3}{-2}$, $\frac{R_4}{-1}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 & -3 & 2 \\ 3 & -4 & 4 & -2 \\ 3 & 4 & -5 & 3 \\ -2 & 2 & -3 & 2 \end{bmatrix} \cdot A$$

$I_{4\times4} = B.A$

'B' is called inverse of 'A'

$$B = A^{-1} = \begin{bmatrix} -3 & 2 & -3 & 2 \\ 3 & -4 & 4 & -2 \\ 3 & 4 & -5 & 3 \\ -2 & 2 & -3 & 2 \end{bmatrix}$$

ystem of linear equations:-

Gaussian elimination Method ;

The system of linear equations are two types.

1. Non-homogeneous system of linear equations

2. Homogeneous system of linear equations.

1. Non-Homogeneous (AX=B) :

The given system of Non-homogeneous linear equati are

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n = b_3$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots a_{mn}x_n = b_n$$

Non-homogeneous system in Matrix form

$$\begin{bmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ a_{31} & a_{32} \cdots & a_{3n} \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$AX = B$

A = Coefficient Matrix

X = Set of unknown

B = Constants

Solution for Non-homogeneous :

- Consider the Non-homogeneous system of linear equations and write in the matrix form $AX = B$
- Write the Augmented matrix $[AB]$ (or) $[A|B]$, apply row transformations on $[AB]$ and reduce it into echelon form
- Divide the constants in echelon form write $Ax = B$,
- Observe Rank of A, Rank of AB and Number of unknowns $(n) = (x, y, z)$

Consistancy : [having solution]

$\rho(A) = \rho(AB) \rightarrow (=n)$ unique solution will exist

$\searrow (<n)$ Infinitely Many solutions.

Inconsistancy : [No solution]

$\rho(A) \neq \rho(AB)$

Prove that the following set of linear equations are consistent and solve them

$$3x + 3y + 2z = 1$$
$$x + 2y = 4$$
$$10y + 3z = -2$$
$$2x - 3y - z = 5$$

Given Non-homogeneous system of Linear equation is

$$3x + 3y + 2z = 1$$
$$x + 2y = 4$$
$$10y + 3z = -2$$
$$2x - 3y - z = 5$$

Write the above system into Matrix form

$AX = B$

$$\begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 0 \\ 0 & 10 & 3 \\ 2 & -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -2 \\ 5 \end{bmatrix}$$

Consider the Augmeted matrix [AB] such that adding constants as a last column in the coeffitient matrix.

$$\begin{bmatrix} 3 & 3 & 2 & 1 \\ 1 & 2 & 0 & 4 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{bmatrix} = [AB]$$

Apply row transformations on (AB) and reduce it into echelon form.

$R_2 \rightarrow 3R_2 - R_1, \quad R_4 \rightarrow 3R_4 - 2R_1$

$$\begin{bmatrix} 3 & 3 & 2 & 1 \\ 0 & 3 & -2 & 11 \\ 0 & 10 & 3 & -2 \\ 0 & -15 & 7 & [illegible] \end{bmatrix}$$

$R_3 \rightarrow 3R_3 - 10R_2$ $\quad -6 - 110 \quad 116$

$R_4 \rightarrow R_4 + 5R_2$

$$\begin{bmatrix} 3 & 3 & 2 & 1 \\ 0 & 3 & -2 & 11 \\ 0 & 0 & 29 & -116 \\ 0 & 0 & -17 & 68 \end{bmatrix} \quad R_4 \to \quad \frac{R_3}{29}, \frac{R_4}{17}$$

$$\begin{bmatrix} 3 & 3 & 2 & 1 \\ 0 & 3 & -2 & 11 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & -1 & 4 \end{bmatrix} \quad R_4 \to R_4 + R_3$$

$$\begin{bmatrix} 3 & 3 & 2 & 1 \\ 0 & 3 & -2 & 11 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ is under Echelon form

$$A = \begin{bmatrix} 3 & 3 & 2 \\ 0 & 3 & -2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 11 \\ -4 \\ 0 \end{bmatrix}$$

$\rho(A) = 3 = r$ $\quad$ $\rho(AB) = 3 = r$

4×3

But no. of unknowns $\quad n = 3 \quad (x, y, z)$

$\rho(A) = \rho(AB) = 3 =$ no. of unknown

The given system is consistant and it follows unique Solution.

To find solution, write $Ax = B$

$$\begin{bmatrix} 3 & 3 & 2 \\ 0 & 3 & -2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \\ -4 \\ 0 \end{bmatrix}$$

$3x + 3y + 2z = 1 \quad -(1)$

$3y - 2z = 11 \quad -(2)$

$\boxed{z = -4} \quad -(3)$

from -(2)

$3y - 2(-4) = 11$

$3y + 8 = 11 \qquad 3y = 3$

$\boxed{y = \cancel{3}} \qquad \boxed{y = 1}$

From - (1)

$3x+3(-1)+2(-4)=1$

$3x+3-8=1$

$3x=6$

$\boxed{x=2}$

unique solution $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$

Test for consistency, if it is consistent solve the System

$$5x+3y+7z=4$$
$$3x+26y+2z=9$$
$$7x+2y+10z=5$$

- Given System

$$5x+3y+7z=4$$
$$3x+26y+2z=9$$
$$7x+2y+10z=5$$

Write the given system into Matrix form $AX=B$

$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

Write above Matrix into Augumented Matrix.

$$[AB] = \begin{bmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{bmatrix}$$

Apply row transformations and reduce it into echelon form

$R_2 \to 5R_2 - 3R_1$, $R_3 \to 5R_3 - 7R_1$

$$[AB] \sim \begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{bmatrix} \quad R_3 \to 11R_3 + R_2$$

$$\sim \begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \frac{R_2}{11}$$

$$\sim \begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ is under echelon form

$\rho(A) = 2$

$\rho(AB) = 2$

No. of unknows = 3

$\rho(A) = \rho(AB) \neq n$

$\rho(A) = \rho(AB) = 2 < n(=3)$

The given system is consistent $\rho(A) = \rho(AB)$ and it follows infinite solution.

$AX = B$

$$\begin{bmatrix} 5 & 3 & 7 \\ 0 & 11 & -1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$$

$n - r = 3-2$
$= 1$

we have to consider only '1' variable

Let $z = k$

$11y - z = 3$

$11y - k = 3$

$11y = 3+k$

$\boxed{y = \frac{3+k}{11}}$

$5x+3y+7z = 4$

$5x = 4-3y-7z$

$5x = 4-3\left(\frac{3+k}{11}\right)-7k$

$5x = \frac{44-9-3k-7k}{11}$

$5x = \frac{35-80k}{11} = \frac{5(7-16k)}{11}$

$x = \frac{7-16k}{11}$

Solution set $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$= \begin{bmatrix} \frac{7-16k}{11} \\ \frac{3+k}{11} \\ k \end{bmatrix}$$

$k = 0, 1, 2, \ldots \infty$

Homogeneous linear equation: $(Ax=0)$

Let $\rho(A)=r$, no.of unknowns $=n$ then

i. If $r=n$ then the system is consistent and havin trivial solution (or) zero solution

ii. If $r<n$ the system is consistent and having infinite no.of solution in this case $n-r$ ~~value~~ variables to constan

Solve the system
$$x+y+w=0$$
$$y+z=0$$
$$x+y+z+w=0$$
$$x+y+2z=0$$

:- $x+y+w=0$
$y+z=0$
$x+y+z+w=0$
$x+y+2z=0$

Write the ~~above~~ matrix ~~form of~~ the above matrix i.e AX

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - R_1 \quad R_4 \to R_4 - 1$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & -1 \end{bmatrix} \quad R_4 \to R_4 - 2R_3$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$ it is under echelon form

$\rho(A)=4$, no.of unknowns $=4$

$\rho(A)=n=r$

It is consistancy it has trivial solution

$x=0, y=0, z=0, w=0$

$$X = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

• Solve the system $x+2y+3z=0$
$3x+4y+4z=0$
$7x+10y+12z=0$

Ans:- Given System $x+2y+3z=0$
$3x+4y+4z=0$
$7x+10y+12z=0$

write the above system into matrix form
$AX=0$

$$\begin{bmatrix}1 & 2 & 3\\3 & 4 & 4\\7 & 10 & 12\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$R_2 \to R_2-3R_1 \quad R_3 \to R_3-7R_1$

$$\begin{bmatrix}1 & 2 & 3\\0 & -2 & -5\\0 & -4 & -9\end{bmatrix} \quad R_3 \to R_3-2R_2$$

$$\begin{bmatrix}1 & 2 & 3\\0 & -2 & -5\\0 & 0 & 1\end{bmatrix}$$ It is under echelon form.

$\rho(A)=3$, no. of unknowns $=3$

$\rho(A)=n=r$

It is consistancy it has trivial solution

$x=0, y=0, z=0$

$$X=\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}.$$

3. Solve the system $x+2y-z=0$;
$2x+y+z=0$
$x-4y+5z=0$

Ans:- Given system $x+2y-z=0$
$2x+y+z=0$
$x-4y+5z=0$

Write the above system into matrix form
$AX=0$

$$\begin{bmatrix}1 & 2 & -1\\2 & 1 & 1\\1 & -4 & 5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}4&2&1&3\\6&3&4&7\\2&1&0&1\end{bmatrix}\begin{bmatrix}x\\y\\z\\w\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$R_2 \to 2R_2 - 3R_1$

$R_3 \to 2R_3 - R_1$

$$\begin{bmatrix}4&2&1&3\\0&0&5&5\\0&0&-1&-1\end{bmatrix} \quad \frac{R_2}{5}$$

$R_3 \to 5R_3 + R_2$

$$\begin{bmatrix}4&2&1&3\\0&0&5&5\\0&0&0&0\end{bmatrix}$$ It is under echelon form.

$\rho(A) = 2 \quad n = 4$

$n - r = 4 - 2 = 2$ variables are constant.

It is a consistant, but it has infinite values, convert the matrix into equations.

$4x + 2y + z + 3w = 0$

$5z + 5w = 0$ $\quad \therefore$ Put $\boxed{z = k_1}$

$5(z + w) = 0$

$z + w = 0$

$k_1 + w = 0$ $\quad$ Put $\boxed{y = k_2}$

$w = -k_1$

$4x + 2(k_2) + k_1 + 3(-k_1)$

$4x + 2k_2 - 2k_1$

$4x = 2k_1 - 2k_2$

$x = \frac{2(k_1 - k_2)}{4_2}$

$x = \frac{k_1 - k_2}{2}$

$$X = \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-\frac{1}{2}\\1\\0\\0\end{bmatrix} + \begin{bmatrix}\frac{1}{2}\\0\\1\\-1\end{bmatrix}$$

2. Solve the system $x+3y-2z=0,\ 2x-y+4z=0,\ x-11y+14z=0$

A:- Given system
$$x+3y-2z=0$$
$$2x-y+4z=0$$
$$x-11y+14z=0$$

Write the above system into Matrix form i.e $AX=0$

$$\begin{bmatrix}1 & 3 & -2\\ 2 & -1 & 4\\ 1 & -11 & 14\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$R_2 \to R_2-2R_1,\ R_3 \to R_3-R_1$

$$\begin{bmatrix}1 & 3 & -2\\ 0 & -7 & 8\\ 0 & -14 & 16\end{bmatrix}$$

$\frac{R_3}{2}$
$$\begin{bmatrix}1 & 3 & -2\\ 0 & -7 & 8\\ 0 & -7 & 8\end{bmatrix}$$

~~$R_3 \to 7R_3-13R_2$~~

$R_3 \to R_3-R_2$
$$\begin{bmatrix}1 & 3 & -2\\ 0 & -7 & 8\\ 0 & 0 & 0\end{bmatrix}$$

It is under echelon form.

$\rho(A)=2$ no.of unknowns $=3$

$\rho(A)<n$

It is consistancy, but it has infinite solution. Convert it into equations

$$x+3y-2z=0 \quad -(1)$$
$$-7y+8z=0 \quad -(2)$$

$n-r=3-2=1$ variable is constant.

$\boxed{z=k_1}$

'z' value in eq(2)

$$-7y+8k_1=0$$
$$-7y=-8k_1$$
$$\boxed{y=\frac{8k_1}{7}}$$

'y', 'z' values in eq-(1)

$$x+3\left(\frac{8k_1}{7}\right)-2(k_1)=0$$
$$x+\frac{24k_1}{7}-2k_1=0$$
$$7x+24k_1-14k_1=0$$

$7x+10k_1=0$

$$\boxed{x = \frac{-10k_1}{7}}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-10}{7} \\ \frac{8}{7} \\ 1 \end{bmatrix}$$

Non-homogeneous system of linear equations:

1. Solve the System $5x+3y+7z=4$, $3x+26y+2z=9$, $7x+2y+10z=5$

A:- Given

$$5x+3y+7z=4$$
$$3x+26y+2z=9$$
$$7x+2y+10z=5$$

Write the above system into Matrix $AX=B$

$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

Assume Aggumented form $[AB]$

$$\begin{bmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{bmatrix}$$

$R_2 \to 5R_2 - 3R_1$

$R_3 \to 5R_3 - 7R_1$

$$\begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{bmatrix}$$

$\frac{R_2}{11}$

$$\begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & -11 & 1 & -3 \end{bmatrix} \quad R_3 \to R_3 + R_2$$

$$\begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ It is under echelon form.

$\rho(A) = 2 \qquad \rho(AB) = 2 \qquad n = 3$

$\rho(A) = \rho(AB) < n$

Now, it is consistancy but it has infinite solution, convert it into equations.

$n - r = 3 - 2 = 1$ only variable is constant.

$5x + 3y + 7z = 4 \quad — (1)$

$11y - z = 3 \quad — (2)$

Now, take $\boxed{z = k_1}$

$11y - k_1 = 3$

$11y = 3 + k_1$

$\boxed{y = \frac{3+k_1}{11}}$

Put z, y values in eq − (1)

$5x + 3\left(\frac{3+k_1}{11}\right) + 7(k_1) = 4$

$5x + \frac{9+3k_1}{11} + 7k_1 = 4$

$\cancel{9 + 3k_1 = 4 - 5x}$

$5x = 4 - \left(\frac{9+3k_1}{11}\right) - 7k_1$

$5x = \frac{44 - 9 - 3k_1 - 77k_1}{11} = \frac{35 - 80k_1}{11} = \frac{5(7 - 16k_1)}{11} = 5x$

$x = \frac{7 - 16k_1}{11}$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{7-16k_1}{11} \\ \frac{3+k_1}{11} \\ k_1 \end{bmatrix} = \begin{bmatrix} \frac{7}{11} \\ \frac{3}{11} \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{-16}{11} \\ \frac{1}{11} \\ 0 \end{bmatrix}$$

• Investigate for what values of AB the $x + 2y + 3z = 4$, $x + 3y + 4z = 5$, $x + 3y + az = b$ have i) No solution ii) a unique solution and iii) infinite solution.

A: Given $x + 2y + 3z = 4$

$x + 3y + 4z = 5$

$x + 3y + az = b$

Write the above system into Matrix form. $AX = B$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 3 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ b \end{bmatrix}$$

Assum...

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 5 \\ 1 & 3 & a & b \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & a-3 & b-4 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & a-4 & b-5 \end{bmatrix}$$ Now it is under echelon form.

$a \neq 4, b \neq 5$

ii) a unique Solution

$\rho(A) = 3$ No. of unknown $= 3$

$\rho(AB) = 3$

$\rho(A) = \rho(AB) = n = r$

The given system consistence and it has unique Solution.

i) No Solution

If $a = 4, b \neq 5$

$\rho(A) = 2$, $\rho(AB) = 3$

$\rho(A) \neq \rho(AB)$

iii) Infinite Solution

If $a = 4, b = 5$

$\rho(A) = 2$ $\rho(AB) = 2$ $n = 3$

$\rho(A) = \rho(AB) < n$

Now, it is consistency, it has infinite solution.

3. Discuss for what values of λ, μ the equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ have i) No Solution ii) Unique Solution, infinite Solution.

A:- Write the above systems into ~~echelon~~ matrix Form $AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

Assume Auggumented Matrix $[AB]$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

$R_2 \to R_2 - R_1, \; R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}$$ Now it is under echelon form.

ii) An unique solution $\lambda \neq 3 \quad \mu \neq 10$

$\rho(A) = 3 \qquad n = 3$

$\rho(AB) = 3$

$\rho(A) = \rho(AB) = n = r$

The given system is consistency and it has unique solution

i) No solution $\lambda = 3 \quad \mu \neq 10$

$\rho(A) = 2, \; \rho(AB) = 3$

$\rho(A) \neq \rho(AB)$

iii) Infinite Solution

If $\lambda = 3 \quad \mu = 10$

$\rho(A) = 2 \quad \rho(AB) = 2 \qquad n = 3$

$\rho(A) = \rho(AB) < n$

Now, it is Consistency, it has infinite Solution.

4. Solve the System $x+y+z=3, \; 3x-5y+2z=8, \; 5x-3y+4z$

A:- Write the above System into Matrix form $AX=B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -5 & 2 \\ 5 & -3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 14 \end{bmatrix}$$

Assume Auggomented Matrix $[AB]$

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$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 3 & -5 & 2 & 8 \\ 5 & -3 & 4 & 14 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1,\ R_3 \to R_3 - 5R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -8 & -1 & -1 \\ 0 & -8 & -1 & -1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -8 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$ It is under echloon form.

$\rho(A) = 2 \quad \rho(AB) = 2 \quad n = 3$

It is consistancy, but it has infinite solutions, convert it into ~~echelon form~~ equations

$x + y + z = 3 \quad -(1)$

$-8y - z = -1 \quad -(2)$

$n - r = 3 - 2 = 1$

only one variable is constant

$\boxed{z = k_1}$

'z' value in '2'

$-8y - k_1 = -1$

$-8y = k_1 - 1$

$y = \frac{k_1 - 1}{-8}$

$\boxed{y = \frac{1 - k_1}{8}}$

'z', 'y' values in eq -(1)

$x + \frac{1-k_1}{8} + k_1 = 3$

$8x + 1 - k_1 + 8k_1 = 24$

$8x + 7k_1 = 23$

$8x = 23 - 7k_1$

$x = \frac{23 - 7k_1}{8}$

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} \frac{-7}{8} \\ \frac{-1}{8} \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{23}{8} \\ \frac{1}{8} \\ 0 \end{bmatrix}$$

Inverse of the matrix by gauss Jordan Method.

1. $\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & 5 \\ 1 & 3 & 2 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

A:- $\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & 5 \\ 1 & 3 & 2 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot A$

$R_3 \to R_3 - R_1, \quad R_4 \to R_4 - R_1$

$\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & 5 \\ 0 & 3 & 5 & -2 \\ 0 & 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \cdot A$

$R_3 \to R_3 - 3R_2, \quad R_4 \to R_4 - R_2$

$\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & -7 & -17 \\ 0 & 0 & -3 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & -3 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{bmatrix} \cdot A$

$R_1 \to R_1 - R_4$

$\cancel{R_1 \to 7R_1 - 3R_3}$ $\quad R_4 \to 7R_4 - 3R_3$

$\begin{bmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & -7 & -17 \\ 0 & 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ -1 & -3 & 1 & 0 \\ 4 & 2 & -3 & 7 \end{bmatrix} A$

$R_1 \to 2R_1 - 9R_4 \quad R_2 \to 7R_2 + 4R_3$

$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 7 & 0 & 33 \\ 0 & 0 & -7 & -17 \\ 0 & 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} -32 & -16 & 27 & -65 \\ -4 & -5 & 4 & 0 \\ -1 & -3 & 1 & 0 \\ 4 & 2 & -3 & 7 \end{bmatrix}$

$R_2 \to 2R_2 + 33R_4 \quad R_3 \to 2R_3 - 17R_4$

$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 14 & 0 & 0 \\ 0 & 0 & -14 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} -32 & -16 & 27 & -65 \\ -41 & -109 & 41 & 0 \\ -70 & -40 & 53 & -119 \\ 4 & 2 & -3 & 7 \end{bmatrix}$

$\frac{R_1}{2} \quad \frac{R_2}{14} \quad \frac{R_3}{-14} \quad \frac{R_4}{-2}$

$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -16 & -8 & \frac{27}{2} & \frac{-65}{2} \\ \frac{-41}{14} & \frac{-109}{14} & \frac{41}{14} & 0 \\ 5 & \frac{20}{7} & \frac{53}{\ldots} & \frac{-119}{\ldots} \end{pmatrix}$

2. $\begin{bmatrix} 1 & -2 & -3 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

A:- Given

$$\begin{bmatrix} 1 & -2 & -3 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_2/2$, $R_3/3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix} \cdot A$$

Inverse of $(A^{-1}) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$

*Iterative methods of solution: The preceding methods of solving simultaneously linear are known as direct methods as they obtain exact solutions on the other hand on iterative methods, that in which we start from an approximation to true solution and obtain better and better approx from a competition cycle repeated upto desired aquring of solution.

Simple Iterative methods such as

1. Jacobi's Iterative methods
2. Gauss seidel Iterative methods.

1. Jacobi's Iterative method:

Consider the eq
$$\left.\begin{array}{l} a_1x+b_1y+c_1z=d_1 \\ a_2x+b_2y+c_2z=d_2 \\ a_3x+b_3y+c_3z=d_3 \end{array}\right\} \rightarrow (1)$$

Here, the coefficient of leading diagonals a_1, b_2, c_3 are large then the other coeffients,

write, $x=\frac{1}{a_1}[d_1-b_1y-c_1z]$

$y=\frac{1}{b_2}[d_2-a_2x-c_2z]$

$z=\frac{1}{c_3}[d_3-a_3x-b_3y]$

Start from initial approximations $x_0, y_0, z_0=0$

First Iteration:

$$x_1=\frac{1}{a_1}[d_1-b_1y_0-c_1z_0]=\frac{1}{a_1}[d_1-0-0]=\frac{d_1}{a_1}$$

$$y_1=\frac{1}{b_2}[d_1-a_2x_0-c_2z_0]=\frac{1}{b_2}[d_2]=\frac{d_2}{b_2}$$

$$z_1=\frac{1}{c_3}[d_3-a_3x_0-b_3y_0]=\frac{d_3}{c_3}$$

Second Iteration:

$$x_2=\frac{1}{a_1}[d_1-b_1y_1-c_1z_1]$$

$$y_2=\frac{1}{b_2}[d_2-a_2x_1-c_2z_1]$$

$$z_3=\frac{1}{c_3}[d_3-a_3x_1-b_3y_1]$$

Repeate like this upto the values x, y, z are equal to succesive iteration.

1. Solve by the jacobi's Iteration Method:

$20x+y-2z=17$
$3x+20y-z=-18$
$2x-3y-20z=25$

A:- Given

$20x+y-2z=17 \Rightarrow x=\frac{1}{20}[17-y+2z]$ — (1)
$3x+20y-z=-18 \Rightarrow y=\frac{1}{20}[-18-3x+z)$ — (2)
$2x-3y-20z=25 \Rightarrow z=\frac{1}{20}[25-2x+3y]$ — (3)

Initial values $x,y,z=0$

First Iteration:
$y=0, z=0$ in (1) we get

$x_1=\frac{1}{20}[17-0+0]=17/20=0.85$

$x=0, z=0$ in (2) we get

$y_1=\frac{1}{20}[-18-0+0]=-18/20=-0.9$

$x=0, y=0$ in (3) we get

$z_1=\frac{1}{20}[25-0+0]=25/20=1.25$

Second Iteration:

$y_1=-0.9,\ z_1=1.25$ in (1) we get

$x_2=\frac{1}{20}[17-(-0.9)+2(1.25)]$
$=\frac{1}{20}(20.4)$
$x_2=1.02$

$x_1=0.85,\ z_1=1.25$ in (2) we get

$y_2=\frac{1}{20}[-18-3(0.85)+1.25)$

$=\frac{1}{20}[-19.3]$

$y_2=-0.965$

$x_1=0.85,\ y_1=-0.9$ in (3)

$$z_2 = \frac{1}{20}[25 - 2(0.85) + 3(-0.9)]$$
$$= \frac{1}{20}(20.6)$$
$z_2 = 1.03$

$x_2 = 1.02$
$y_2 = -0.965$
$z_2 = 1.03$

3rd iteration:

$y_2 = -0.965, \ z_2 = 1.03$

$$x_3 = \frac{1}{20}(17 - (-0.965) + 2(1.03))$$
$$= 1.00125$$

$$y_3 = \frac{1}{20}(-18 - 3(1.02) + 1.03))$$
$$= -1.00185$$

$$z_3 = \frac{1}{20}(25 - 2(1.02) + \cancel{3(1.03)})^{-0.965} = \cancel{1.199} = 1.00325$$

4th iteration:

$10x + y - z = 11.19$
$x + 10y + z = 28.08$
$-x + y + 10z = 35.61$

3. $10x - 5y - 2z = 3$
$4x - 10y + 3z = -3$
$x + 6y + 10z = -3$

Sol:- Given

$10x + y - z = 11.19 \Rightarrow x = \frac{1}{10}(11.19 - y + z)$
$x - 10y + z = 28.08 \Rightarrow y = \frac{1}{10}(28.08 - x - z)$
$-x + y + 10z = 35.61 \Rightarrow z = \frac{1}{10}(35.61 + x - y)$

Initial values $x, y, z = 0$

First iteration :

$y=0, z=0$

$x_1 = \frac{1}{10}[11.19-0+0] = \frac{11.19}{10} = 1.119$

$x=0, y=0$

$y_1 = \frac{1}{10}[28.08-0-0] = \frac{28.08}{10} = 2.808$

$x=0, y=0$

$z_1 = \frac{1}{10}(35.61+0-0) = \frac{35.61}{10} = 3.561$

$x_1 = 1.119, y_1 = 2.808, z_1 = 3.561$

Second Iteration

$x_2 = \frac{1}{10}[11.19-2.808+3.561] = \frac{11.943}{10} = 1.1943$

$y_2 = \frac{1}{10}[28.08-1.119-3.561] = \frac{23.4}{10} = 2.34$

$z_2 = \frac{1}{10}[35.61+1.119-2.808) = \frac{33.921}{10} = 3.3921$

Third Iteration

$x_3 = \frac{1}{10}[11.19-2.34+3.3921] = \frac{11.943}{10} = 1.1943$

$y_3 = \frac{1}{10}[28.08-1.1943-3.3921] = 2.34936$

$z_3 = \frac{1}{10}[35.61+1.1943-2.343] = 3.44643$

3. $10x - 5y - 2z = 3$
$4x - 10y + 3z = -3$
$x + 6y + 10z = -3$

Ans:- $10x - 5y - 2z = 3 \longrightarrow x = \frac{1}{10}[3 + 5y + 2z]$
$4x - 10y + 3z = -3 \longrightarrow y = \frac{1}{10}[-3 - 4x - 3z]$
$x + 6y + 10z = -3 \longrightarrow z = \frac{1}{10}[-3 - x - 6y]$
$x, y, z = 0$

First Iteration:
$y = 0, z = 0$
$x_1 = \frac{1}{10}[3 + 0 + 0] = \frac{3}{10} = 0.3$
$x = 0, z = 0$
$y_1 = \frac{1}{10}(-3) = -3/10 = -0.3$
$y = 0, x = 0$
$z_1 = \frac{1}{10}(-3) = -3/10 = -0.3$

Second Iteration
$x_2 = \frac{1}{10}[3 + 5(-0.3) + 2(-0.3)] = \frac{1}{10}[0.9] = 0.09$
$y_2 = \frac{1}{10}[-3 - 4(0.3) - 3(-0.3)] = \frac{1}{10}[-3.3] = -0.33$
$z_2 = \frac{1}{10}[-3 - 0.3 - 6(-0.3)) = \frac{1}{10}[-1.5] = -0.15$

Third Iteration
$x_3 = \frac{1}{10}(3 + 5(-0.33) + 2(-0.15))] = \frac{1}{10}[1.05) = 0.105$
$y_3 = \frac{1}{10}[-3 - 4(0.09) - 3(-0.15)] = \frac{-2.91}{10} = -0.291$
$z_3 = \frac{1}{10}[-3 - 0.09 - 6(-0.33))] = \frac{-1.11}{10} = -0.111$

auss-Seidel Iteration Method: This is the modification - jacobie's iteration method. The given system of linear quation.

$a_1x+b_1y+c_1z=d_1$

$a_2x+b_2y+c_2z=d_2$

$a_3x+b_3y+c_3z=d_3$

Here the coefficients of leading diagonals a_1, b_2, c_3 are ge than the other coefficients.

Write $x=\frac{1}{a_1}[d_1-b_1y-c_1z]$

$y=\frac{1}{b_2}[d_2-a_2x-c_2z]$.

$z=\frac{1}{c_2}[d_3-a_3x-b_3y]$

ake initial values $x=x_0=0$

$y=y_0=0$

$z=z_0=0$

$x_1=\frac{1}{a_1}[d_1-b_1(0)-c_1(0)] = \frac{d_1}{a_1}$

$y_1=\frac{1}{b_2}[d_2-a_2(x_1)-c_1(0)]$

$z_1=\frac{1}{c_3}[d_2-a_3x_1-b_3y_1]$

epeat like this upto the values of x, y, z are equal to wo Successive iteration.

Use gauss-seidel iteration method to solve the system $10x+y+z=12$, $2x+10y+z=13$, $2x+2y+10z=14$

: Given system

$10x+y+z=12 \rightarrow x=\frac{1}{10}[12-y-z]$

$2x+10y+z=13 \rightarrow y=\frac{1}{10}[13-2x-z]$

$2x+2y+10z=14 \rightarrow z=\frac{1}{10}[14-2x-2y]$

irst iteration for finding x_1, y_1, z_1

Taking $x_0, y_0, z_0=0$

r x_1 taking $y_0=0$, $z_0=0$ in weget

$x=\frac{1}{10}[12-0-0]=\frac{12}{10}=1.2$

For 'y_1' taking $x_1 = 1.2$ and $z_0 = 0$

$$y_1 = \frac{1}{10}[13-2(1.2)-0] = \cancel{\frac{1.6}{10}} = \cancel{1.06}\ \frac{10.6}{10} = 1.06$$

for 'z_1' taking $y_1 = 1.06$ and $x_1 = 1.2$

$$z_1 = \frac{1}{10}[14-2(1.2)-2(1.06)] = \frac{9.48}{10} = 0.946$$

Second Iteration Taking $y_1 = 1.06$, $z_1 = 0.946$

$$x_2 = \frac{1}{10}[12-1.06-0.948] = 0.9992 \simeq 1$$

$$y_2 = \frac{1}{10}[13-2(0.9992)-0.948] = 1.005$$

$$z_2 = \frac{1}{10}[14-2(0.9992)-2(1.005)] = 0.99916 \simeq 1$$

third iteration

$$x_3 = \frac{1}{10}[12-1.00-0.99] = 1.00$$

$$y_3 = \frac{1}{10}[13-2(1.00)-0.99] = 1.00$$

$$z_3 = \frac{1}{10}[14-2(0.90)-2(1.00)] = 1.00$$

Similarly we find the fourth approximation of x, y, z and we get as same values as $x_4 = 1.00$, $y_4 = 1.00$, $z_4 = 1.00$

So the ~~given~~ solution of given system of linear equation is -'1'.

2. Solve the system of linear equation using gauss sciedel iterations

$$27x+6y-z=85$$
$$6x+15y+2z=72$$
$$x+y+54z=110$$

1.926

A:- Given system

$$27x+6y-z=85 \rightarrow x = \frac{1}{27}[85-6y+z] \quad \text{--- (1)}$$
$$6x+15y+2z=72 \rightarrow y = \frac{1}{15}[72-6x-2z] \quad \text{--- (2)}$$
$$x+y+54z=110 \rightarrow z = \frac{1}{54}[110-x-y] \quad \text{--- (3)}$$

Taking $x_0, y_0, z_0 = 0$

First Iteration

$$x_1 = \frac{1}{27}[85-6(0)+0] = \frac{85}{27} = 3.148$$

$$y_1 = \frac{1}{15}[72 - 6(3.14) - 2(0)] = \frac{53.16}{15} = 3.544$$

$$z_1 = \frac{1}{54}[110 - 3.148 - 3.544] = \frac{103.308}{54} = 1.913$$

Second Iteration:

$$x_2 = \frac{1}{27}[85 - 6(3.544) + 1.913] = \frac{65.64}{27} = 2.43$$

$$y_2 = \frac{1}{15}[72 - 6(2.43) - 2(1.91)] = \frac{53.6}{15} = 3.57$$

$$z_2 = \frac{1}{54}[110 - 2.43 - 3.57] = \frac{104}{54} = 1.92$$

Third Iteration:

$$x_3 = \frac{1}{27}[85 - 6(3.57) + 1.92] = 2.40$$

$$y_3 = \frac{1}{15}[72 - 6(2.40) - 2(1.92)] = 3.584$$

$$z_3 = \frac{1}{54}[110 - 2.40 - 3.58] = 1.92$$

Fourth Iteration

$$x_4 = \frac{1}{27}[85 - 6(3.58) + 1.92] = 2.42$$

$$y_4 = \frac{1}{15}[72 - 6(2.42) - 2(1.92)] = 3.57$$

$$z_4 = \frac{1}{54}[110 - 2.42 - 3.57] = 1.92$$