

Free Electron theory

* classical free electron theory :-

Merits :-

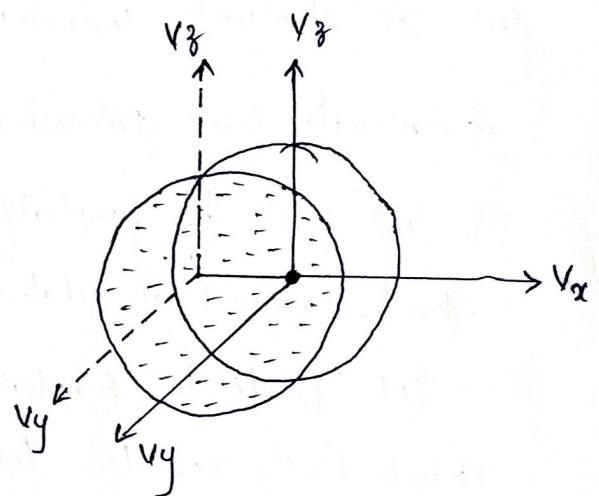
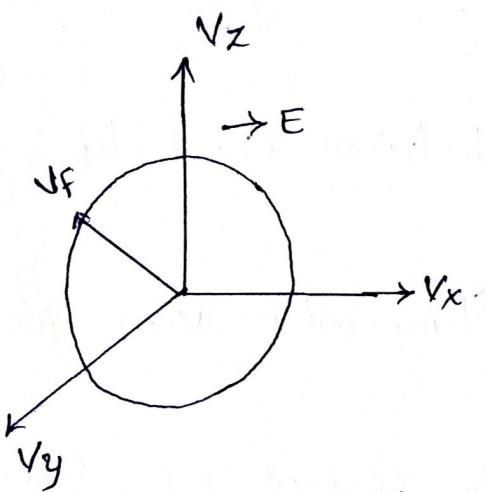
- (1) It Explains the electrical resistivity in a metal.
- (2) It Explains the electrical conductivity in a metal.
- (3) A relationship between resistivity and conductivity with temperature.
- (4) It Explains optical properties of metals.
- (5) It verifies ohm's law.
- (6) It derives Wiedemann - Franz law.

Demerits (or) Drawbacks :-

- (1) It fails to Explain the electrical conductivity of Semiconductors and insulators.
- (2) It fails to Explain the temperature variation of electrical conductivity at low temperature.
- (3) It fails to Explain the concept of specific heat of metals.
- (4) It fails to Explain the mean free path of the electrons.
- (5) The phenomenon like photo electric effect, Compton effect, black body radiation, could not be explained by classical free electron theory.
- (6) It fails to Explain temperature dependence of magnetic susceptibility and ferromagnetism.

Quantum free electron theory :-

To overcome the drawbacks of classical free electron theory, Sommerfeld proposed quantum free electron theory. He treated electron as quantum particle. The electrons follows the principles of quantum theory. Under this, the velocity of free electrons are plotted in the velocity space with dots from the origin of sphere. The maximum velocity of Electron at highest occupied level, i.e. fermi level is known as fermi velocity v_F and it is represented by the radius of Fermi sphere.



In the absence of electric field, these velocities cancel each other and net velocity in all directions is zero.

If the electric field E is applied along x -direction, then the electrons in the sphere experiences present near the fermi can occupy the higher energy levels as a result the sphere slightly displaces.

The relation between momentum p and wave vector $[k = \frac{2\pi}{\lambda}]$ is given by $p = \hbar k$
 $mv = \hbar k$

$$v = \frac{\hbar k}{m} \rightarrow ①$$

Differentiating the above eq w.r.t 't' we get.

$$a = \frac{dv}{dt} = \frac{\hbar}{m} \frac{dk}{dt}$$

Force on the electron in the applied field E is given by

$$F = eE$$

$$ma = eE$$

$$m \times \frac{\hbar}{m} \frac{dk}{dt} = eE$$

$$\frac{\hbar}{m} \frac{dk}{dt} = eE$$

$$dk = \frac{eE}{\hbar} dt$$

Integrating the above Equation, we get

$$\int dk = \int \frac{eE}{\hbar} dt$$

$$k(t) - k(0) = \frac{eE}{\hbar} t$$

Let τ_F and λ_F are the mean collision time and mean free path of Electron at fermi Surface then,

$$T_F = \frac{\lambda_F}{V_F}$$

on fermi surface $t = \tau_F$ and $k(t) - k(0) = \Delta k$.

$$\Delta k = \frac{eE}{\hbar} T_F \rightarrow ②$$

$$\Delta k = \frac{eE}{\hbar} \frac{\lambda_F}{V_F}$$

Current density (J) = $n e \Delta V$.

Here ΔV is the change in velocity.

From Eq ① $\Delta V = \frac{e}{m} \Delta k$ then

$$J = \frac{n e \Delta k}{m}$$

Substituting Eq ② in J

$$J = \frac{n e \Delta k}{m} \frac{e E}{\tau_F} T_F$$

$$J = \frac{n e^2 E}{m} \tau_F \rightarrow ③$$

$$J = \sigma E \rightarrow ④$$

By comparing Eq ③ & ④, we get

$$\sigma = \frac{n e^2 \tau_F}{m}$$

The above Eq represents electrical conductivity

Merits :-

- (1) It successfully explains the electrical thermal conductivity of metals.
- (2) Temperature dependence of conductivity of metals can be explained by this theory.
- (3) It explains the specific heat of metals.
- (4) It explains the magnetic susceptibility of metals.

Demerits :-

- (i) It is unable to explain the metallic properties exhibited by only certain crystals.
- (ii) It is unable to explain why the atomic arrays in metallic crystals should prefer certain structures only.

Fermi - Dirac distribution:-

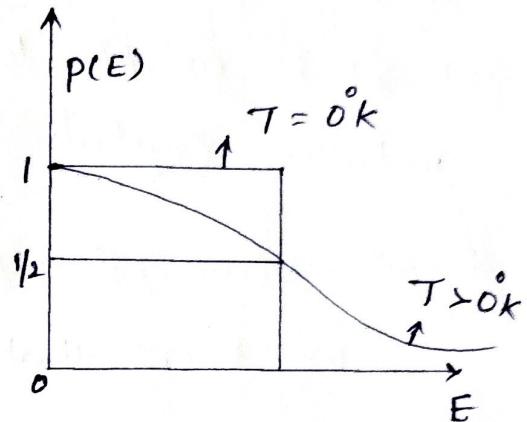
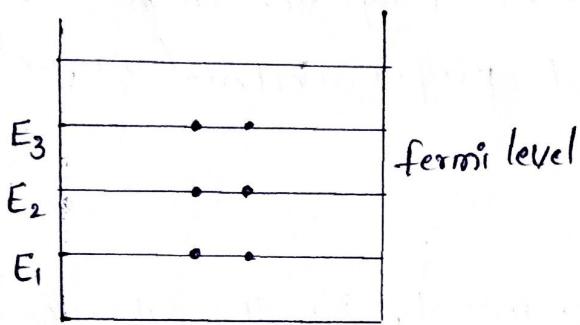
We know that in a metal, in the absence of an electric field, free electrons move at random similar to the behaviour of the electron gas. Since electrons are indistinguishable particles they are known as fermions (or) fermi particles. Hence, such an electron gas obeys Fermi - Dirac distribution.

The fermi Dirac distribution describes the behaviour of free electron gas, ~~the~~ taking into account the quantum theory and pauli's exclusion principle.

According to Quantum theory, the electrons will have discrete energy states and the occupation of electrons among these energy levels will be governed by pauli's exclusion principle. At absolute 0°C , two electrons occupy the ground state and two into each state of next higher energy levels. The highest energy level occupied by electron at absolute zero is known as Fermi Energy level which divides the occupied states from the unoccupied states. The energy of Fermi level is E_F . The probability of the occupation of an energy level E of an electron at temperature T is

$$F(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}}$$

Effect of temperature on Fermi Dirac distribution functions



Case 1 :- At temperature $T = 0^\circ\text{K}$

- (a) For $E < E_F$, $F(E) = 1$
- (b) For $E > E_F$, $F(E) = 0$

At absolute zero temperature $F(E)$ is a step function. All the states with energies up to E_F are filled with electrons and the states with Energies greater than E_F are Empty. Thus the fermi level may be the top most level filled with electrons at absolute temperature.

Case 2 :- At temperature $T > 0^\circ\text{K}$.

- (a) For $E < E_F$, $F(E) = 1$
- (b) For $E > E_F$, $F(E) = 0$
- (c) For $E = E_F$, $F(E) = 1/2$

Density of States :-

The number of electronic states per unit Energy range is called density of states of $g(E)$. Let us consider a spherical system of radius \vec{r} and it represents a vector to a point n_x, n_y and n_z ($n^2 = n_x^2 + n_y^2 + n_z^2$) in 3-dimensional space.

(4)

consider a sphere of radius n and another sphere of radius $(n+dn)$ in which Energy values are E and $E+dE$ respectively.

Number of Energy States available in sphere of radius n is

$$\frac{1}{8} \left[\frac{4}{3} \pi n^3 \right] \rightarrow ①$$

Number of Energy States in sphere of radius $(n+dn)$ is

$$\frac{1}{8} \left[\frac{4}{3} \pi (n+dn)^3 \right] \rightarrow ②$$

E and dE are Energy values of above two regions respectively from Eq ① & ② we can write number of Energy states available in dn or dE region as.

$$g(E) dE = \frac{1}{8} \left[\frac{4}{3} \pi (n+dn)^3 \right] - \frac{1}{8} \left[\frac{4}{3} \pi n^3 \right]$$

$$g(E) dE = \frac{\pi}{6} (n+dn)^3 - \frac{\pi}{6} n^3$$

$$g(E) dE = \frac{\pi}{6} [n^3 + dn^3 + 3n^2 dn + 3ndn^2 - n^3]$$

Neglecting higher order terms we get

$$g(E) dE = \frac{\pi}{6} (3n^2 dn)$$

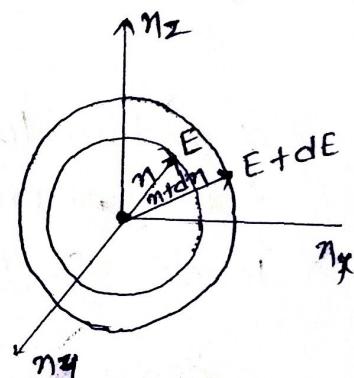
$$g(E) dE = \frac{\pi}{2} (n(dn)) \rightarrow (3)$$

The Expression for the n^{th} Energy level can be written as.

$$E = \frac{n^2 h^2}{8ma^2}$$

$$n^2 = \left[\frac{8ma^2}{h^2} \right] E \rightarrow (4)$$

$$n = \left[\frac{8ma^2}{h^2} E \right]^{1/2} \rightarrow (5)$$



Differentiating Eq. ④, taking n and E are variables, we get

$$2ndn = \frac{8ma^3}{h^2} dE$$

$$ndn = \frac{1}{2} \left[\frac{8ma^3}{h^2} \right] dE \rightarrow (6)$$

Substituting Eq. ⑤ and Eq. ⑥ in Eq. ③, we get

$$g'(E) dE = \frac{\pi}{2} \left[\frac{8ma^3}{h^2} \right]^{1/2} E^{1/2} \frac{1}{2} \left[\frac{8ma^3}{h^2} \right] dE$$

$$g'(E) dE = \frac{\pi}{4} \left[\frac{8ma^3}{h^2} \right]^{3/2} E^{1/2} dE$$

According to pauli's exclusion principle, Each Energy level contains two Electrons i.e Each Energy level will have two Sub Energy levels so above Equation should be multiplied by 2.

$$g'(E) dE = 2 \frac{\pi}{4} \left[\frac{8ma^3}{h^2} \right]^{3/2} E^{1/2} dE$$

$$= \frac{\pi}{2} \left[\frac{8ma^3}{h^2} \right]^{3/2} E^{1/2} dE$$

$$= \frac{\pi}{2} \left[\frac{8m}{h^2} \right]^{3/2} a^3 E^{1/2} dE.$$

Density of States $g(E) dE = \frac{g'(E)}{V} dE$ if length of Energy level is a then its volume $V = a^3$, so Density of states can be written as.

$$g(E) dE = \frac{g'(E)}{V} = \frac{\frac{\pi}{2} \left[\frac{8m}{h^2} \right]^{3/2} a^3 E^{1/2} dE}{V} \quad [\because a^3 = V]$$

$$g(E) dE = \frac{\pi}{2} \left[\frac{8m}{h^2} \right]^{3/2} a^3 E^{1/2} dE$$

4. Quantum Mechanics

Matter Waves :-

An Electromagnetic waves behaves like a particle. The particles like electrons, photons, neutrons etc. all behaves like waves are called "matter waves".

These matter waves are suggested by De-broglie are called De-broglie matter waves.

(1) Debroglie Hypothesis (or) De broglie-wavelength :-

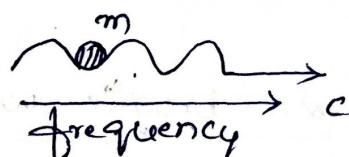
(or)

Dual nature of matter :-

Consider a photon of mass "m" moving with velocity of light 'c'.

Then the Energy of photon is given by

$$E = h\nu \rightarrow ①$$

where 'h' is planck's constant and ν is  frequency

From Einstein mass energy relation

$$E = mc^2 \rightarrow ②$$

from Eq ① & Eq ②, we get

$$h\nu = mc^2$$

$$\text{But } \nu = \frac{c}{\lambda}, \text{ then } \frac{hc}{\lambda} = mc^2$$

$$\frac{h}{\lambda} = mc$$

$$\lambda = \frac{h}{mc}$$

When a particle of mass m moves with a velocity ' v ' then it is associated with a wave of wavelength ' λ ' known as De-broglie wavelength is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

De broglie wavelength in terms of k.E :-

The k.E of a moving particle

$$E = \frac{1}{2}mv^2$$

Multiply and divide with 'm' on R.H.S

$$E = \frac{1}{2}mv^2 \times \frac{m}{m} = \frac{m^2v^2}{2m} = \frac{P^2}{2m} \quad [P = mv]$$

$$E = \frac{P^2}{2m}$$

$$P^2 = 2mE$$

$$P = \sqrt{2mE}$$

De broglie wavelength $\lambda = \frac{h}{P}$

$$\lambda = \frac{h}{\sqrt{2mE}}$$

De-broglie wavelength in terms of electrons :-

Consider an Electron of mass 'm' and charge 'e' that accelerated through a potential difference of 'V' volts. Then the k.E is equal to the loss of potential energy.

$$\frac{1}{2}mv^2 = eV$$

$$mv^2 = 2eV$$

Multiply with 'm' on both sides

$$m^2v^2 = 2meV$$

$$P^2 = 2meV$$

$$P = \sqrt{2meV}$$

De broglie wavelength $\lambda = \frac{h}{P} = \frac{h}{\sqrt{2meV}}$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}}$$

$$\lambda = \frac{12.26 \times 10^{-10}}{\sqrt{V}} \text{ m}$$

$$\lambda = \frac{12.26}{\sqrt{V}} \text{ Å}$$

Properties of matter waves:-

$$\text{DeBroglie wavelength } \lambda = \frac{h}{mv}$$

- (1) Lighter is the particle, greater is the wavelength associated with it.
- (2) Smaller is the velocity of the particle, greater the wavelength associated with it.
- (3) when $v = 0$, $\lambda = \infty$ i.e., waves become indeterminate and if $v = \infty$, $\lambda = 0$ this shows that matter waves are generated by the motion of the particles.
- (4) These waves are not electromagnetic waves.
- (5) The wave nature of matter introduces an uncertainty in the location of the position of particle.
- (6) matter waves travel faster than velocity of light.

Heisenberg uncertainty principle :-

Consider a particle of mass m moving with velocity v along x -direction. According to classical theory, this moving particle will have specific position and momentum at any time t i.e. its position is given

by $x = vt$ and its momentum $p = mv$. Relating the position and momentum, we get

$$x = \frac{p}{m} t$$

From this it is clear that anytime t particle position and momentum can be measured accurately. According to De-broglie hypothesis, this moving particle is associated with a matter wave. As this wave has some spread in the moving region and even the particle is somewhere with in the wave spread region, it is difficult to locate its exact position. Therefore, there is an uncertainty Δx in its position. As a result, the momentum of the particle, cannot be determined precisely.

It means that the position and momentum of a matter wave associated particle cannot be simultaneously determined with accuracy. Any attempt to determine these parameters will lead to uncertainties in each of the parameter. This is known as Heisenberg uncertainty principle for position and momentum.

Statement:- It is not possible to find simultaneously with the exact accuracy of both the position and momentum of a moving particle associated with matter wave.

If Δx and Δp are the uncertainties in the position and momentum of a moving particle then according to Heisenberg uncertainty principle.

$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad (\text{or}) \quad \Delta x \Delta p \geq \frac{\hbar}{4\pi}$$

From this it is clear that, if one parameter is measured accurately then the other associated parameter can not be measured accurately.

The uncertainty relation for the simultaneous measurement of Energy and Time is

$$\Delta E \Delta t \geq \frac{\hbar}{2} \quad (00) \quad \Delta E \Delta t \geq \frac{\hbar}{4\pi}$$

(2) Schrodinger's one dimensional Time-independent wave equation :-

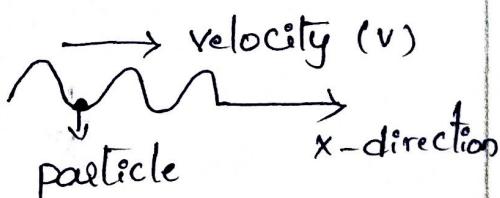
Consider a particle of mass 'm' moving with velocity 'v' along the x-direction. It is associated with a wave. The displacement of the wave is given by the wave function ψ .

Since the wavefunction depends upon the x-coordinate of the moving particle and time 't', it is given by

$$\psi(x, t) = A e^{i(kx - \omega t)} \rightarrow ①$$

where 'A' is the amplitude

Differentiate Eq ① w.r.t 'x', we get



$$\frac{d\psi}{dx} = A ik e^{i(kx - \omega t)}$$

Again differentiate, we have

$$\frac{d^2\psi}{dx^2} = A (ik)(ik) e^{i(kx - \omega t)}$$

$$\frac{d^2\psi}{dx^2} = -k^2 A e^{i(kx - \omega t)}$$

From Eq ① we know that $\psi = A e^{i(kx - \omega t)}$, we get

$$\frac{d^2\psi}{dx^2} = -k^2\psi, \text{ But } k = \frac{\omega\pi}{\lambda}$$

$$\frac{d^2\psi}{dx^2} = -\left(\frac{\omega\pi}{\lambda}\right)^2\psi$$

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi^2}{\lambda^2}\psi \rightarrow (2)$$

de-broglie wavelength associated with a particle is

$$\lambda = \frac{h}{mv}$$

$$\frac{1}{\lambda} = \frac{mv}{h}, \quad \frac{1}{\lambda^2} = \frac{m^2v^2}{h^2}$$

Now multiply Eq divide the numerator with ω^2 .

$$\frac{1}{\lambda^2} = \frac{\frac{1}{\omega}m^2v^2 \times 2}{h^2}$$

$$\frac{1}{\lambda^2} = \frac{\left(\frac{1}{\omega}mv^2\right)\omega m}{h^2}$$

$$\frac{1}{\lambda^2} = \frac{(K.E)\omega m}{h^2} \rightarrow (3)$$

Let E be the total Energy of the particle and V be the potential Energy of the particle, then

$$T.E = K.E + P.E$$

$$E = KE + V$$

$$K.E = E - V \rightarrow (4)$$

Substitute Eq (4) in Eq (3)

$$\frac{1}{\lambda^2} = (E - V) \frac{2m}{\hbar^2} \rightarrow ⑤$$

Substitute Eq ⑤ in Eq ②, we get

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi^2\psi}{\lambda^2}$$

Now substitute Eq ⑤ in the above Equation

$$\frac{d^2\psi}{dx^2} = -4\pi^2 \cdot \frac{2m}{\hbar^2} (E - V) \psi$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{\hbar^2} (E - V) \psi = 0$$

$$\left[\frac{1}{\lambda^2} = \frac{4\pi^2}{\hbar^2} \right]$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0 \rightarrow ⑥$$

$$\left[\because \frac{1}{\lambda^2} = \frac{\hbar^2}{4\pi^2} \right]$$

Eq ⑥ represents Schrodinger's one dimensional time-independent wave Equation.

For 3 dimensional motion of the particle the above Equation becomes

$$\nabla^2\psi + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

$$\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}$$

④ Schrodinger's one-dimensional Time dependent wave Equation :-

The displacement wave function of a particle is

$$\psi = A e^{i(kx - \omega t)} \rightarrow ①$$

Differentiate the above Equation w.r.t 't', we get.

$$\frac{d\psi}{dt} = A(-i\omega) e^{i(kx-\omega t)}$$

$$\text{but time period } T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T}$$

$$\frac{d\psi}{dt} = A(-i\omega\gamma) e^{i(kx-\omega t)} \quad \omega = 2\pi\gamma \quad [\because \gamma = \frac{1}{T}]$$

$$\frac{d\psi}{dt} = -i\omega\gamma A e^{i(kx-\omega t)}$$

$$\frac{d\psi}{dt} = -i\omega\gamma \psi \rightarrow \textcircled{1}$$

$$\text{Energy of a wave } E = h\gamma$$

$\gamma = \frac{E}{h}$, substitute this value in Eq \textcircled{1}

$$\frac{d\psi}{dt} = -i\omega\gamma \psi$$

$$\frac{d\psi}{dt} = -i\omega\frac{E}{h} \psi$$

$$\frac{d\psi}{dt} = -i\frac{E}{\hbar} \psi \quad [\because \gamma = \frac{E}{2\pi}]$$

mul & dev with i on R.H.S Side

$$\frac{d\psi}{dt} = -i\frac{E}{\hbar} \times \frac{i}{i} \psi$$

$$\frac{d\psi}{dt} = -i^2 \frac{E}{\hbar} \psi$$

$$\frac{d\psi}{dt} = \frac{E}{i\hbar} \psi \quad [i^2 = -1]$$

$$E\psi = i\hbar \frac{d\psi}{dt} \rightarrow \textcircled{2}$$

(5)

From Schrodinger's time - independent wave Equation

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E\psi - V\psi) = 0$$

Substitute Eq (2) in the above Eq, we get

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} \left[i\hbar \frac{d\psi}{dt} - V\psi \right] = 0$$

$$\frac{d^2\psi}{dx^2} = - \frac{2m}{\hbar^2} \left[i\hbar \frac{d\psi}{dt} - V\psi \right]$$

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = i\hbar \frac{d\psi}{dt} - V\psi$$

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V\psi = i\hbar \frac{d\psi}{dt} \rightarrow (3)$$

Eq (3) represents Schrodinger's time dependent wave equation in one - dimension.

For three dimensional motion, the above Equation becomes

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = i\hbar \frac{d\psi}{dt}$$

(5) physical significance of the wave function ψ :-

- (i) the wave function ψ has no direct physical meaning. It is a complex quantity representing the variation of matter wave. It connects the particle nature and its associated with wave nature statistically.

$$(2) \psi = a + ib$$

The complex conjugate of ψ is

$$\psi^* = a - ib$$

$$\psi\psi^* = (a+ib)(a-ib) = a^2 + b^2$$

$\psi\psi^*$ (or) $|\psi|^2$ is the probability density function.

$\psi\psi^* dx dy dz$ gives the probability of finding the electron in the region of space between x and $x+dx$, y and $y+dy$ and z and $z+dz$.

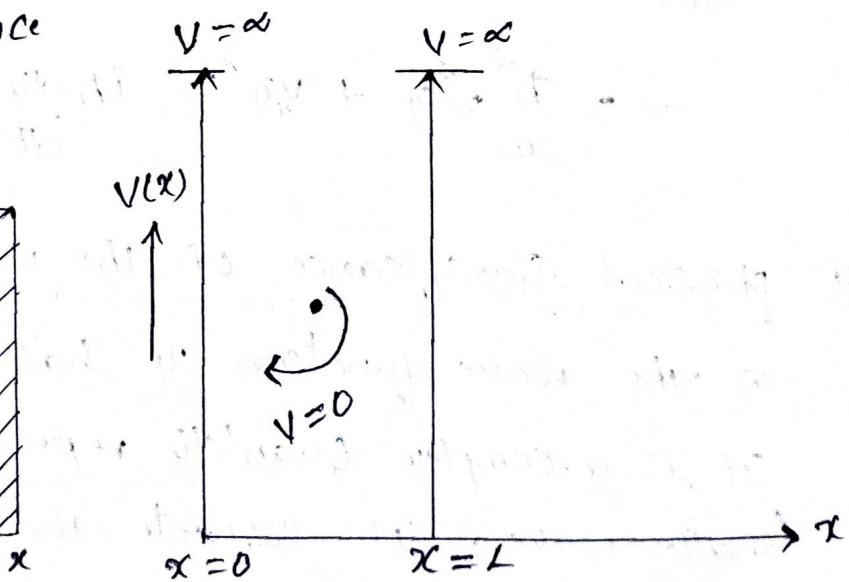
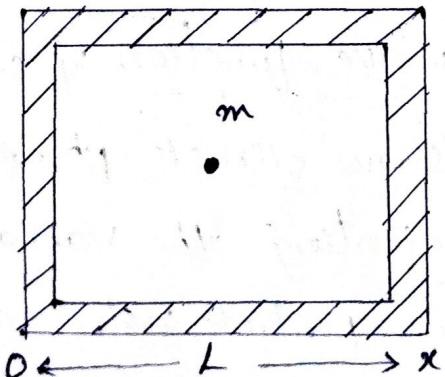
If the particle is present

$$\int_{-\infty}^{\infty} \psi\psi^* dx dy dz = 1$$

It is the probability density function. It is used to find the location of the particle.

⑥ particle in one dimensional infinite potential well :-

consider a particle of mass m in bouncing back and forth between infinitely high one-dimensional potential walls (box). Separated by a distance



Consider 'L' as shown in fig.

As the particle only inside the box hence ψ must be zero when $0 > x > L$

The potential $V = \infty$ at $x=0$ and $x=L$.

The potential $V = \infty$ at $x=0$ and $x=L$

The potential V is uniform throughout the well i.e.

$V=0$ for $0 < x < L$.

The one dimensional Schrodinger wave Equation for Stationary States :

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2} \psi = 0 \quad \text{for } (0 < x < L) \quad V=0.$$

$$\frac{d^2\psi}{dx^2} + k^2 \psi = 0 \rightarrow (1) \quad \text{where } k^2 = \frac{2mE}{\hbar^2} \rightarrow (2)$$

The general solution for Eq(1) can be written as.

$$\psi(x) = A \sin kx + B \cos kx \rightarrow (3)$$

Here 'k' represents wave vector.

where A & B are arbitrary constants and can be found by applying boundary condition of the problem.

(i) at $x=0$, $\psi(x)=0$

$$\therefore 0 = A \sin k(0) + B \cos k(0)$$

$$0 = 0 + B$$

Eq(3) becomes $\psi(x) = A \sin kx$

(ii) at $x=L$, $\psi(x)=0$

$$0 = A \sin kL$$

In the above Equation $A \neq 0$, because there would not be any solution.

$$\therefore \sin kL = 0$$

$$kL = n\pi \text{ where } n=1, 2, 3, 4, \dots$$

$$\therefore k = \frac{n\pi}{L} \rightarrow (4)$$

Hence the motion of the particle in the given region $0 < x < L$ is described by the Equation.

$$\psi(x) = A \sin \frac{n\pi x}{L} \rightarrow (5)$$

Eigen values of Energy :-

Equating Eq (5) & (4), we get.

$$\frac{\partial^2 E}{\partial x^2} = \frac{n^2 \pi^2}{L^2}$$

$$E_n = \frac{n^2 \pi^2}{L^2} \times \frac{\hbar^2}{2m}$$

$$E_n = \frac{n^2 \pi^2}{L^2} \times \frac{\hbar^2}{4\pi^2 m}$$

$$E_n = \frac{n^2 \hbar^2}{8mL^2} \rightarrow (6)$$

where n is the order. $n = 1, 2, 3, \dots$

L = width of the potential box.

From the above Equation.

The lowest Energy of the particle is obtained by putting $n = 1$.

The lowest energy of the particle is obtained by putting $n=1$.

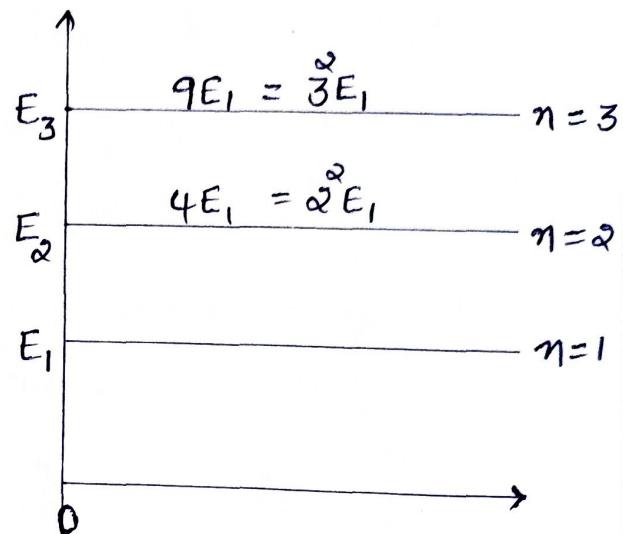
$$\text{i.e } E_1 = \frac{h^2}{8mL^2}$$

$$E_2 = \frac{\omega^2 h^2}{8mL^2} = 4E_1$$

$$E_3 = \frac{3\omega^2 h^2}{8mL^2} = 9E_1$$

and

$$E_n = n^2 E_1$$



fig(a) Energy level diagram of particle.

The difference between two consecutive energy states is given by $(n+1)^2 E_1 - n^2 E_1 = (2n+1) E_1$

These values of E_n are known as Eigen values of Energy and the corresponding wave function is known as ψ_n as Eigen function of the particle. The integer 'n' is the quantum number of Energy level E_n .

* Dual nature of Radiation and matter:-

wave theory of E.M radiation explained the interference, diffraction and polarisation.

Quantum theory of E.M radiation successfully explained the photoelectric effect, Crompton effect, black body radiation, X-ray spectra etc. Thus radiations have dual nature i.e wave and particle nature.

DeBroglie suggested that the particles like electrons, protons, neutrons etc have also dual nature i.e they also can have particle as well as wave nature.