EM wave characterstics

- wave Equations for conducting & perfect Dielectric media
- uniform plane wave — Definition.
- All relations b/w E & H
- Sinusoidal variations.
- wave propagation in loseless & conducting medium
- conductors & Dielectrics — characterization
- wave propagation in Good conductors & Good dielectrics
- Polarization.
- Reflection & Refraction of plane wave.
  - Normal Incidences
  - oblique Incidences

  For both perfect conductors & dielectric
- Brewster & critical angle & total Internal reflection.
- surface Impedence
- poyting vector & poynting theorem; Applications
- powerloss in plane conductor
- problems.

Unit IV

EM wave characterstices

→* Frist application of maxwell Equation will be in relation to EM wave propagation

→* **wave :-**

→ waves are means of transporting energy (or) information.

→ wave is a function of both space & time.

→* **EM wave :-**

→ The wave produced by the variation in electric & magnetic fields aligned ⊥ lar to each other

→ EM waves examples

→ Radio waves

→ TV signals

→ Radar beams

→ Light rays

→ In this chapater our major goal is to solve Maxwell's Eq's & describe EM wave motion in the following medium

@ Free space	Dielectric $\sigma << \omega\varepsilon$ Lossless	Dielectric Lossy	Good conductors $\sigma >> \omega\varepsilon$
$\sigma = 0$	$\sigma = 0$	$\sigma \neq 0$	$\sigma \simeq \infty$
$\varepsilon = \varepsilon_0$	$\varepsilon = \varepsilon_0 \varepsilon_r$	$\varepsilon = \varepsilon_0 \varepsilon_r$	$\varepsilon = \varepsilon_0 \varepsilon_r$
$\mu = \mu_0$	$\mu = \mu_0 \mu_r$	$\mu = \mu_0 \mu_r$	$\mu = \mu_0$

*> Wave Equation :—

ⓐ $\nabla^2 \cdot \vec{E} = \mu\varepsilon \frac{\partial^2 E}{\partial t^2} + \mu\sigma \frac{\partial \vec{E}}{\partial t}$
ⓑ $\nabla^2 \cdot \vec{H} = \mu\varepsilon \frac{\partial^2 H}{\partial t^2} + \mu\sigma \frac{\partial \vec{H}}{\partial t}$
→ For conducting medium

ⓐ $\nabla^2 \cdot \vec{E} = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$
ⓑ $\nabla^2 \cdot \vec{H} = \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$
Perfect Dielectric medium.

*> General EM wave Equation :—

*→ Let us assume that electric & magnetic field exists in a linear, homogenous & isotrophic medium with parameters σ, μ, ε

*> Also assume that the medium is charge free medium hence we can write $\rho_v = 0$

*> Maxwell's Equation :— (Time varying fields)

(i) $\nabla \cdot \vec{D} = 0$
$\nabla \cdot \vec{E} = 0$

(ii) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow -\mu \frac{\partial \vec{H}}{\partial t}$

(iii) $\nabla \cdot \vec{B} = 0$

(iv) $\nabla \times \vec{H} = J_c + J_D$
$= \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$

$\because \vec{B} = \mu \vec{H}$
$\vec{D} = \varepsilon \vec{E}$
$\vec{J_c} = \sigma \vec{E}$

(i) Wave Equation for a perfect Dielectric

$\rightarrow$ Free space

→ consider wave propagation in free space medium

→ Free space is perfect dielectric

$\rho_v = 0 \rightarrow \varepsilon = \varepsilon_0 \varepsilon_r$

$J_c = 0 \quad \varepsilon = \varepsilon_0$

$\sigma = 0 \rightarrow \mu = \mu_0 \mu_r$

$\varepsilon_r = \mu_r = 1 \quad \mu = \mu_0$

→ $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow (1)$ | $\nabla \times \vec{H} = J_c + \frac{\partial \vec{D}}{\partial t} \rightarrow (2)$

Apply differentiation on both sides for Eq (1) & (2)

$$\nabla \times \frac{\partial \vec{E}}{\partial t} = -\frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\boxed{\nabla \times \frac{\partial \vec{E}}{\partial t} = -\mu_0 \frac{\partial^2 H}{\partial t^2}}$$

$$\nabla \times \frac{\partial \vec{H}}{\partial t} = \sigma E + \frac{\partial^2 \vec{D}}{\partial t^2} \quad [\because \text{—} = 0]$$

$$= 0 + \frac{\partial^2 \vec{D}}{\partial t^2}$$

$$\boxed{\nabla \times \frac{\partial \vec{H}}{\partial t} = \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \quad [\because \vec{D} = \varepsilon \vec{E}]$$

→ Taking curl on both sides of Eq's (1) & (2)

$$\nabla \times \nabla \times \vec{E} = -\nabla \times \frac{\partial \vec{B}}{\partial t}$$

$$= -\nabla \times \mu\left(\frac{\partial \vec{H}}{\partial t}\right)$$

$$= -\mu_0 \nabla \times \frac{\partial \vec{H}}{\partial t}$$

$$-\nabla^2 E = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

$$\nabla \times \nabla \times H = 0 + \nabla \times \frac{\partial \vec{D}}{\partial t}$$

$$= \nabla \times \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$= \varepsilon_0 \nabla \times \frac{\partial E}{\partial t}$$

$$-\nabla^2 H = -\varepsilon_0 \mu_0 \frac{\partial^2 H}{\partial t^2}$$

$$\boxed{\nabla^2 H = \varepsilon_0 \mu_0 \frac{\partial^2 H}{\partial t^2}}$$

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla^2 E$$

$$= \nabla(\nabla \varepsilon \vec{D}) - \nabla^2 E$$

$$= \nabla(\varepsilon \nabla \cdot \vec{D}) - \nabla^2 E$$

$\nabla \cdot \vec{D} = 0$

$$= 0 - \nabla^2 E$$

(i) $\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$

(ii) $\nabla^2 H = \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2}$

partial differential Eq

The wave velocity in free space $\boxed{C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$

(ii) Wave Equation for conducting medium

→ Conducting medium ($J \neq 0$, $\rho_v = 0$)

→ For conducting medium maxwell's Eqⁿs are

(i) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$

(ii) $\nabla \times \vec{H} = J_C + J_D$

$= \sigma E + \frac{\partial D}{\partial t} \Rightarrow \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \longrightarrow (1)$ $\quad \nabla \times H = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \longrightarrow (2)$

Taking differentiation on both side Eq (1) & (2)

$\nabla \times \frac{\partial \vec{E}}{\partial t} = -\frac{\partial^2 \vec{B}}{\partial t^2}$ $\quad \boxed{\nabla \times \frac{\partial \vec{H}}{\partial t} = \sigma \frac{\partial \vec{E}}{\partial t} + \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}}$

$\boxed{\nabla \times \frac{\partial \vec{E}}{\partial t} = -\mu \frac{\partial^2 \vec{H}}{\partial t^2}}$

Taking curl on both side Eqⁿs (1) & (2)

$\nabla \times \nabla \times \vec{E} = -\mu \, \nabla \times \frac{\partial \vec{H}}{\partial t}$

$= -\mu \left[\sigma \frac{\partial \vec{E}}{\partial t} + \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}\right]$

$= -\left[\mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}\right]$

$\cancel{-}\nabla^2 E = \cancel{-}\left[\mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}\right]$

$\boxed{\nabla^2 E = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}}$

$\nabla \times \nabla \times \vec{H} = \sigma(\nabla \times \vec{E}) + \varepsilon \, \nabla \times \frac{\partial \vec{E}}{\partial t}$

$= \sigma\left(-\mu \frac{\partial H}{\partial t}\right) + \varepsilon\left(-\mu \frac{\partial^2 H}{\partial t^2}\right)$

$= -\mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$

$\cancel{-}\nabla^2 H = \cancel{-}\left[\mu\sigma \frac{\partial H}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}\right]$

$\boxed{\nabla^2 H = \mu\sigma \frac{\partial H}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}}$

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(iii) Time Harmonic wave Equation (phasor Notation)

Harmonic → It is a slg | wave whose freq is an integral multiple of the freq of some reference signal(or) wave

*→ The electric & magnetic fields in phasor form.

Assume that the medium is

→ Homogeneous
→ Linear
→ isotropic
→ source free.

*→ $\vec{E} = |E| e^{j\omega t}$

$\frac{\partial \vec{E}}{\partial t} = j\omega \vec{E}$

$\frac{\partial^2 \vec{E}}{\partial t^2} = j^2 \omega^2 \vec{E} = -\omega^2 \vec{E}$

$\vec{H} = |H| e^{j\omega t}$

$\frac{\partial \vec{H}}{\partial t} = j\omega \vec{H}$

$\frac{\partial^2 \vec{H}}{\partial t^2} = -\omega^2 \vec{H}$

where ω – wave angular freq.

$\nabla^2 E = +\mu\varepsilon \frac{\partial^2 E}{\partial t^2} + \mu\sigma \frac{\partial E}{\partial t}$

$= \mu\varepsilon(-\omega^2 \vec{E}) + \mu\sigma j\omega \vec{E}$

$\boxed{\nabla^2 E = j\mu\sigma\omega \vec{E} - \mu\varepsilon\omega^2 \vec{E}}$ → conducting medium

$\boxed{\nabla^2 E = -\mu\varepsilon\omega^2 \vec{E}}$ → Free Space

$\nabla^2 H = \mu\varepsilon \frac{\partial^2 H}{\partial t^2} + \mu\sigma \frac{\partial E}{\partial t}$

$= \mu\varepsilon(-\omega^2 \vec{H}) + \mu\sigma j\omega \vec{H}$

$= -\omega^2\mu\varepsilon \vec{H} + j\mu\omega\sigma \vec{H}$

$\boxed{\nabla^2 H = j\mu\omega\sigma \vec{H} - \omega^2 \mu\varepsilon \vec{H}}$ → conducting Medium.

$\boxed{\nabla^2 H = -\omega^2 \mu\varepsilon \vec{H}}$ — Free Space

Wave Equation

By arranging the Eq's in matrix for we get a three dimensional EM wave Eq.

(a) For conducting Medium :—

$$\nabla^2 \begin{bmatrix} \vec{E} \\ \vec{D} \\ \vec{H} \\ \vec{B} \end{bmatrix} = \mu\sigma \frac{\partial}{\partial t} \begin{bmatrix} \vec{E} \\ \vec{D} \\ \vec{H} \\ \vec{B} \end{bmatrix} + \mu\varepsilon \frac{\partial^2}{\partial t^2} \begin{bmatrix} \vec{E} \\ \vec{D} \\ \vec{H} \\ \vec{B} \end{bmatrix}$$

$$\nabla^2 \cdot \vec{E} = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

multiply both side with ε

$$\nabla^2 \cdot \varepsilon \vec{E} = \mu\sigma \frac{\partial \varepsilon E}{\partial t} + \mu\varepsilon \frac{\partial^2 \varepsilon E}{\partial t^2}$$

$$\boxed{\nabla^2 D = \mu\sigma \frac{\partial D}{\partial t} + \mu\varepsilon \frac{\partial^2 D}{\partial t^2}}$$

$$\nabla^2 \cdot H = \mu\sigma \frac{\partial H}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

multiply both sides by μ

$$\nabla^2 \cdot \mu H = \mu\sigma \frac{\partial \mu H}{\partial t} + \mu\varepsilon \frac{\partial^2 \mu H}{\partial t^2}$$

$$\boxed{\nabla^2 \cdot B = \mu\sigma \frac{\partial B}{\partial t} + \mu\varepsilon \frac{\partial^2 B}{\partial t^2}}$$

(b) For free space

$\sigma = 0$, $\mu = \mu_0$ & $\varepsilon = \varepsilon_0$.

$$\nabla^2 \cdot \begin{bmatrix} \vec{E} \\ \vec{D} \\ \vec{H} \\ \vec{B} \end{bmatrix} = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \begin{bmatrix} \vec{E} \\ \vec{D} \\ \vec{H} \\ \vec{B} \end{bmatrix}$$

v_0 – velocity of propagation of EM wave in free space

$$v_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{1}{36\pi} \times 10^{-9}}} = \frac{1}{\sqrt{1/9 \times 10^{-16}}} = \frac{1}{\frac{1}{3} \times 10^{-8}} = 3 \times 10^{8}\ m/sec$$

Plane wave :-

* It is a wave of constant freq & amplitude with wavefronts that are an infinit long straight line.

* plane wave travels in the directions $\perp^{lar}$ to the wavefront.

$$\boxed{\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}}$$

where

$$\nabla^2 E = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2}$$

If propagation in x-direction

$$\nabla^2 E = \frac{\partial^2 E}{\partial x^2} + 0 + 0$$

$$\nabla^2 E = \frac{\partial^2 E}{\partial x^2}$$

$$\boxed{\therefore \frac{\partial^2 E}{\partial x^2} = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}}$$

uniform plane wave :-

It is defined as an EM wave in which electric & magnetic field intensities directed in fixed directions in space & constant on infinite planes $\perp^{lar}$ to the direction of propagation. i.e

(i) The plane wave has no electric & magnetic field components along the direction of propagation.

(ii) E & H have constant amplitude & phase on infinite planes $\perp^{lar}$ to the direction of propagation.

(or)

*> uniform plane wave means the $\vec{E}$ & $\vec{H}$ fields exists in the same plane more over the amplitude & phase of field vector $\vec{E}$ & $\vec{H}$ is constant over the entire plane.

*> An EM wave propagating in x-direction is said to be a uniform plane wave if its field $\vec{E}$ & $\vec{H}$ are independent of Y & Z directions & a uniform plane wave propagating in x-direction has no 'x'-component of $\vec{E}$ & $\vec{H}$

i.e $E_x = 0$; $H_x = 0$

**>uniform plane wave Eq

consider, the wave Eq/s for $\vec{E}$ & $\vec{H}$ fields as

$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mu\sigma \frac{\partial E}{\partial t} \quad \text{--- (1)}$$

$$\nabla^2 H = \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} + \mu\sigma \frac{\partial H}{\partial t} \quad \text{--- (2)}$$

* For free space condition.

$\sigma = 0$; $\mu = \mu_0$; $\varepsilon = \varepsilon_0$ $\because$ ε_r & $\mu_r = 1$

Eq (1) & (2) becomes

$$\nabla^2 E = \mu_0\varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 H = \mu_0\varepsilon_0 \frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 E = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2}$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

EM wave propagating in X-direction so, $\vec{E}$ is independent of Y & Z

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 E}{\partial x^2}$$

$$\boxed{\frac{\partial^2 E}{\partial t^2} = v^2 \frac{\partial^2 E}{\partial x^2}}$$

$$\boxed{\frac{\partial^2 H}{\partial t^2} = v^2 \frac{\partial^2 H}{\partial x^2}}$$

**→ General Solution of uniform plane wave Equation

*→ we can't generate uniform plane wave in practice, Since it isn't possible to keep constant E & H fields on infinite plane with infinite energy. However many practical waves can be approximated as uniform wave

*→ Eg:- The radiation rxed by a small antenna at at far distances can be approximated as a plane wave, ∵ the wavefront becomes almost spherical & a very small portion of the sphere acts like a plane at the Rxing Ax.

- → consider uniform plane wave travelling in x-direction in free space. Assume that in cartesian co-ordinates system.
(i) The electric field vector is oriented along y-direction
(ii) The magnetic " " " " z-direction.

The fields are independent in y & z direction & depends on x & t.

→ In free space EM wave Eq is given.

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \longrightarrow ①$$

$$\boxed{\frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

$\therefore \vec{E} \neq f(y)$; $\vec{E} \neq f(z)$

$\vec{H} \neq f(y)$; $\vec{H} \neq f(z)$

ie

$$\left.\begin{aligned} \frac{\partial \vec{E}}{\partial y} = 0 ; \quad \frac{\partial^2 \vec{E}}{\partial y^2} = 0 \\ \frac{\partial \vec{E}}{\partial z} = 0 ; \quad \frac{\partial^2 \vec{E}}{\partial z^2} = 0 \end{aligned}\right\} \longrightarrow ②$$

Substitute Eq ② in Eq ①

$$\frac{\partial^2 \vec{E}}{\partial x^2} + 0 + 0 = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\frac{\partial^2 \vec{E}}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \longrightarrow ③$$

$$\vec{E} = E_x \vec{a}_x + E_y \vec{a}_y + \vec{E}_z \vec{a}_z$$

$$\frac{\partial^2}{\partial x^2}\left[E_x \vec{a}_x + E_y \vec{a}_y + E_z \vec{a}_z\right] = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2}\left[E_x \vec{a}_x + E_y \vec{a}_y + E_z \vec{a}_z\right]$$

comparing both sides.

$$\frac{\partial^2 E_x}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} \longrightarrow (4)$$

$$\frac{\partial^2 E_y}{\partial y^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \longrightarrow (5)$$

$$\frac{\partial^2 E_z}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_z}{\partial t^2} \longrightarrow (6)$$

consider (4) Eqs: $\boxed{\frac{\partial^2 E_x}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$

* A/c to maxwells Eq in free space

$$\nabla \cdot \vec{D} = 0$$

$$\nabla \cdot \varepsilon \vec{E} = 0$$

$$\nabla \cdot \vec{E} = 0.$$

i.e $\nabla \cdot \vec{D} = \frac{\partial \vec{D}_x}{\partial x} + \frac{\partial \vec{D}_y}{\partial y} + \frac{\partial \vec{D}_z}{\partial z}$

$$0 = \frac{\partial \vec{D}_x}{\partial x} + \frac{\partial \vec{D}_y}{\partial y} + \frac{\partial \vec{D}_z}{\partial z}$$

$$0 = \frac{\partial \vec{E}_x}{\partial x} + \frac{\partial \vec{E}_y}{\partial y} + \frac{\partial \vec{E}_z}{\partial z}$$

from the eq (2) we can say

$$\frac{\partial E_y}{\partial y} = 0 ; \quad \frac{\partial^2 E_y}{\partial y^2} = 0$$

$$\frac{\partial E_z}{\partial z} = 0 ; \quad \frac{\partial^2 E_z}{\partial z^2} = 0$$

$$\therefore \frac{\partial \vec{E}_x}{\partial x} + 0 + 0 = 0$$

$$\boxed{\frac{\partial \vec{E}_x}{\partial x} = 0 ; \quad \frac{\partial^2 \vec{E}}{\partial x^2} = 0} \longrightarrow (7)$$

Put Eq ⑦ in ④

$$\frac{\partial^2 E_x}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$0 = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\boxed{\frac{\partial^2 E_x}{\partial t^2} = 0.}$$

The above eq is true only when $E_x = 0$ & $E_x = K$ (or) Kt

$|||^{ly}$ by solving $\nabla^2 \vec{H} = \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2}$ we get $H_x = 0$.

**→ when the uniform plane wave is propagating in the x-direction, $\vec{E}$ & $\vec{H}$ may have components in Z & Y directions but not in the X-direction. i.e

$$E_x = H_x = 0.$$

*→ Let the electric field be in the y-direction with the component E_y, then the wave eq becomes.

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

Let $E_y = E_0 e^{j\omega t}$

$$\frac{\partial E_y}{\partial t} = j\omega E_0 e^{j\omega t}$$

$$\frac{\partial E_y}{\partial t} = j\omega E_y$$

$$\frac{\partial^2 E_y}{\partial t^2} = -\omega^2 E_y \longrightarrow ⑧$$

Put eq ⑧ in eq ⑤

$$\frac{\partial^2 E_y}{\partial x^2} = -\mu_0 \varepsilon_0 \omega^2 E_y$$

$$\boxed{\frac{\partial^2 E_y}{\partial x^2} - \mu_0 \varepsilon_0 \omega^2 E_y = 0} \longrightarrow (9)$$

→ This is a second order differential Eq which represents the uniform plane wave propagating in the x-direction & the electric field in the Y- direction.

The General solution of this Eq is.

$$\boxed{E_y(x) = E^{+} e^{-j\beta x} + E^{-} e^{j\beta x}} \longrightarrow (10)$$

put $\frac{\partial}{\partial x} = D$ in eq (9)

$$D^2 E_y + \omega^2 \mu_0 \varepsilon_0 E_y = 0$$

$$E_y (D^2 + \omega^2 \mu_0 \varepsilon_0) = 0$$

$$D^2 + \omega^2 \mu_0 \varepsilon_0 = 0$$

$$\mu_0 \varepsilon_0 \omega^2 = \beta^2$$

$$\beta = \omega \sqrt{\mu_0 \varepsilon_0}$$

$$D^2 = \omega^2 \mu_0 \varepsilon_0$$

$$D = \sqrt{\omega^2 \mu_0 \varepsilon_0}$$

$$D = \pm \omega \sqrt{\mu_0 \varepsilon_0}$$

$$\boxed{D = \pm j\beta}$$

where β - phase shift constant of EM wave in free space & measured in "rad/m"

$$\frac{\partial^2 E_y}{\partial x^2} - \beta E_y = 0$$

Solution to above second order differential Eq.

$$\boxed{E_y(x) = E^+ e^{-j\beta x} + E^- e^{j\beta x}}$$ Solution

$$= E_0^+ e^{j\omega t} \cdot e^{-j\beta x} + E_0^- e^{j\omega t} \cdot e^{j\beta x}$$

$$= E_0^+ \left[e^{j(\omega t - \beta x)} \right] + E_0^- e^{j(\omega t + \beta x)}$$

$$= E_0^+ \left[(\cos j(\omega t - \beta x) + j \sin j(\omega t - \beta x)) \right] + E_0^- \left[(\cos j(\omega t + \beta x) + j \sin j(\omega t + \beta x)) \right]$$

**→ The wave thus consists one component of field travelling in

+ve x-axis direction having amplitude E_0^+

−ve x-axis direction having amplitude E_0^-

∴ By considering only real terms we get

$$\boxed{E_y = E_0^+ \cos(\omega t - \beta x) + E_0^- (\cos \omega t + \beta x)}$$

**→ In time domain, the wave can be expressed as.

$$E_y(x,t) = R_e \left[E_y(x) \cdot e^{j\omega t} \right]$$

$$= R_e \left[(E^+ e^{-j\beta x} + E^- e^{-j\beta x}) e^{j\omega t} \right]$$

$$= R_e \left[E^+ e^{j(\omega t - \beta x)} + E^- e^{-j(\omega t + \beta x)} \right]$$

$$E_y(x,t) = R_e \left[E^+ e^{-j\beta(x - v_0 t)} + E^- e^{-j\beta(x - v_0 t)} \right]$$

$$= f(x - v_0 t) + f(x + v_0 t)$$

$$\boxed{E_y(x,t) = f(x - v_0 t)}$$

$e^{-j\beta(x - v_0 t)}$

$e^{-j\beta x + j\beta v_0 t}$

$\beta v_0 t = \omega \sqrt{\mu_0 \varepsilon_0} \cdot \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$\beta v_0 t = \omega \cdot \sqrt{\mu_0 \varepsilon_0} \cdot \frac{t}{\sqrt{\mu_0 \varepsilon_0}}$

$$\boxed{\beta v_0 = \omega}$$

* If the wave propagates in free space without bounds it travels only in forward direction. The electric field components is

$$E_y(x,t) = f(x - v_0 t)$$

$$E_z(x,t) = f(x - v_0 t)$$

* Solution for ~~the~~ uniform wave is nothing but wave propagation in free space

Similary

$$\boxed{\begin{aligned} H_x(x,t) &= 0 \\ H_y(x,t) &= f(x - v_0 t) \\ H_z(x,t) &= f(x - v_0 t) \end{aligned}}$$

All Relations b/w $\vec{E}$ & $\vec{H}$:–

~~Relatio Ratio~~

*) The relation b/w $\vec{E}$ & $\vec{H}$ are

(i) $\frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

(ii) $\vec{E}\cdot\vec{H} = 0$

(iii) $\vec{E} \times \vec{H}$ gives the direction of wave propagation.

(i) $\frac{\vec{E}}{\vec{H}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

The above (Eq) relation can be proved from maxwell's equation assuming $\sigma = 0$

$$\nabla \times H = \frac{\partial D}{\partial t}$$

$$= \varepsilon \frac{\partial E}{\partial t}$$

$$\nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \vec{a}_x\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right] - \vec{a}_y\left[\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right]$$
$$+ \vec{a}_z\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]$$

where $H_x = 0$ b/c wave propagating in x-direction

y-components are not present in H_z

$$\nabla \times H = \vec{a}_x\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right] - \vec{a}_y\frac{\partial H_z}{\partial x} + \vec{a}_z\frac{\partial H_y}{\partial x}$$

$$= \vec{a}_x[0-0] - \vec{a}_y\frac{\partial H_z}{\partial x} + \vec{a}_z\, \partial H_y/\partial x$$

$$\nabla \times H = -\frac{\partial H_z}{\partial x} \cdot \vec{a_y} + \frac{\partial H_y}{\partial x} \vec{a_z}$$

$$\nabla \times H = \varepsilon_0 \frac{\partial E}{\partial t}$$

$$-\frac{\partial H_z}{\partial x} \vec{a_y} + \frac{\partial H_y}{\partial x} \vec{a_z} = \varepsilon_0 \frac{\partial}{\partial t}\left[E_x \vec{a_x} + E_y \vec{a_y} + E_z \vec{a_z}\right]$$

$$= \varepsilon_0 \frac{\partial}{\partial t}\left[E_y \vec{a_y} + E_z \vec{a_z}\right] \quad \left[\because E_x = 0\right]$$

wave is in x-direction

$$-\frac{\partial H_z}{\partial x} \vec{a_y} + \frac{\partial H_y}{\partial x} \vec{a_z} = \varepsilon_0 \left[\frac{\partial E_y}{\partial t} \vec{a_y} + \frac{\partial E_z}{\partial t} \vec{a_z}\right]$$

compare both sides

$$-\frac{\partial H_z}{\partial x} = \varepsilon_0 \frac{\partial E_y}{\partial t} \qquad \frac{\partial H_y}{\partial x} = \varepsilon_0 \frac{\partial E_z}{\partial t} \longrightarrow (2)$$

$$\frac{\partial H_z}{\partial x} = -\varepsilon_0 \frac{\partial E_y}{\partial t} \longrightarrow (1)$$

From general solution of uniform plane wave

$$E_y = f_1(x - v_0 t)$$

$$E_z = f_2(x - v_0 t)$$

Assume

~~x = ax~~ $u = x - v_0 t$

$$\frac{\partial u}{\partial t} = -v_0.$$

$$\frac{\partial u}{\partial x} = 1. \qquad \therefore \quad E_y = f_1(u) \qquad E_z = f_2(u)$$

$$\frac{\partial E_y}{\partial t} = \frac{\partial f_1(u)}{\partial t}$$

$$= \frac{\partial f_1(u)}{\partial u} \cdot \frac{\partial u}{\partial t}$$

$$= -v_0 \frac{\partial f_1(u)}{\partial u}$$

$$= -v_0 \, f_1'(u) \longrightarrow (a_1)$$

$$\frac{\partial E_y}{\partial x} = \frac{\partial f_1(u)}{\partial x}$$

$$= \frac{\partial f_1(u)}{\partial u} \cdot \frac{\partial u}{\partial x}$$

$$\frac{\partial E_y}{E_x} = f_1'(u) \cdot 1 \longrightarrow (b_1)$$

sublitute (b_1) in (a_1) we get

$$\frac{\partial E_y}{\partial t} = -v_0 \frac{\partial E_y}{\partial x} \longrightarrow (c_1)$$

Subtitute Eq (c_1) in Eq (1)

$$\frac{\partial H_z}{\partial x} = +\varepsilon_0 v_0 \frac{\partial E_y}{\partial x}$$

Integration on both sides w.r.t. x

$$H_z = +\varepsilon_0 v_0 E_y$$

$$= \frac{+\varepsilon_0}{\sqrt{\mu_0 \varepsilon_0}} E_y = \sqrt{\frac{\varepsilon_0}{\mu_0}} E_y$$

$$\therefore \; H_z = \sqrt{\frac{\varepsilon_0}{\mu_0}} E_y \Rightarrow \boxed{E_y = \sqrt{\frac{\mu_0}{\varepsilon_0}} H_z}$$

$$\frac{\partial E_z}{\partial t} = \frac{\partial f_2(u)}{\partial t}$$

$$= \frac{\partial f_2(u)}{\partial u} \cdot \frac{\partial u}{\partial t}$$

$$= -v_0 \frac{\partial f_2(u)}{\partial u}$$

$$= -v_0 \, f_2'(u) \longrightarrow (a_2)$$

$$\frac{\partial E_z}{\partial x} = \frac{\partial f_2(u)}{\partial x}$$

$$= \frac{\partial f_2(u)}{\partial u} \cdot \frac{\partial u}{\partial x}$$

$$\frac{\partial E_z}{\partial x} = f_2'(u) \longrightarrow (b_2)$$

Sublitue Eq (b_2) in (a_2)

$$\frac{\partial E_z}{\partial t} = -v_0 \frac{E_z}{\partial x} \longrightarrow (c_2)$$

Sublitute eq (c_2) in Eq (2)

$$\frac{\partial H_y}{\partial x} = -\varepsilon_0 v_0 \frac{\partial E_z}{\partial x}.$$

$$\boxed{E_z = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot H_y}$$

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

$$= \sqrt{0^2 + \left(\sqrt{\frac{\mu_0}{\varepsilon_0}} H_z\right)^2 + \sqrt{\frac{\mu_0}{\varepsilon_0}} H_y^2}$$

$$= \sqrt{\frac{\mu_0}{\varepsilon_0} H_z^2 + \frac{\mu_0}{\varepsilon_0} H_y^2}$$

$$= \sqrt{\frac{\mu_0}{\varepsilon_0} (H_z^2 + H_y^2]}$$

$$= \sqrt{\frac{\mu_0}{\varepsilon_0} (\vec{H}^2 |}$$

$$\vec{E} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \vec{H}$$

$\mu_0 = 4\pi \times 10^{-7}$

$\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$

$$= \sqrt{\frac{4\pi \times 10^{-7}}{\frac{1}{36\pi} \times 10^{-9}}} \cdot \vec{H}$$

$$\frac{\vec{E}}{\vec{H}} = \sqrt{4\pi \times 36\pi \times 10^{-7} \times 10^{9}}$$

$$= \sqrt{4 \times 36 \times \pi^2 \times 10^2}$$

$$= 2 \times 6 \times 10 \times \pi$$

$$\boxed{\frac{\vec{E}}{\vec{H}} = 120\pi \,/\, 377\,\Omega}$$

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(ii) $\vec{E}\cdot\vec{H} = 0$

$$\vec{E}\cdot\vec{H} = (E_y\vec{a_y} + E_z\vec{a_z})\cdot(H_y\vec{a_y} + H_z\vec{a_z})$$

$$= E_y H_y \vec{a_y}\cdot\vec{a_y} + E_z H_y \vec{a_z}\cdot\vec{a_y} + E_z\vec{a_z}\cdot H_y\vec{a_y} + E_z H_z \vec{a_z}\cdot\vec{a_z}$$

$$= E_y H_y (1) + E_y H_z(0) + E_z H_y(0) + E_z H_z(1)$$

$$= E_y H_y + E_z H_z$$

$$= H_z\sqrt{\frac{\mu_0}{\varepsilon_0}} + \left(\frac{-\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\right)H_z$$

$$\boxed{\vec{E}\cdot\vec{H} = 0}$$

(iii) $\vec{E}\times\vec{H} = \eta H\vec{a_z}$

$$E\times\vec{H} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ E_x & E_y & E_z \\ H_x & H_y & H_z \end{vmatrix}$$

$|E| = \sqrt{E_x^2 + E_y^2 + E_z^2}$

$|E| = \sqrt{E_y^2 + E_z^2}$

$H = H_y + H_z$

if H is unit magnitude vector

$\hat{H} = \vec{H}$

$$= \vec{a_x}[E_y H_z - E_z H_y] - \vec{a_y}[E_x H_z - E_z H_x] + a_z[E_x H_y - E_y H_x]$$

$$= \vec{a_x}[E_y H_z - E_z H_y] - 0 + 0$$

$$\nabla\times E = \vec{a_x}\sqrt{\frac{\mu_0}{\varepsilon_0}}\,H$$

$$= \vec{a_x}[E_y H_z - E_z H_y]$$

$$= \vec{a_x}\left[\sqrt{\frac{\mu_0}{\varepsilon_0}}\,H_z\cdot H_z + \sqrt{\frac{\mu_0}{\varepsilon_0}}\,H_y\cdot H_y\right]$$

$$= \vec{a_x}\sqrt{\frac{\mu_0}{\varepsilon_0}}\,(H_z^2 + H_y^2) \Rightarrow a_x\sqrt{\frac{\mu_0}{\varepsilon_0}}\,\vec{H}$$

Sinusoidal variations in EM wave

- The electric & magnetic field vectors in EM waves vary with time sinusoidally. This sinusoidal variations can be written as.

$$E = E\cos\omega t \quad \text{(or)} \quad E = Ee^{j\omega t}$$
$$H = H\sin\omega t \qquad\quad H = He^{j\omega t}$$

- The phasor notation for maxwell's are written as

$$\nabla \times H = j\omega D + J_c$$

$$\nabla \times E = -j\omega\mu H$$

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot B = 0$$

$$\nabla^2 \cdot \vec{E} = -\omega^2\mu\varepsilon\vec{E} \Rightarrow \nabla^2 \cdot H = -\omega^2\mu\varepsilon\vec{H} \rightarrow \text{Lossless medium}$$

$$\left.\begin{aligned}\nabla^2 \cdot \vec{E} &= j\omega\mu\sigma\vec{E} - \omega^2\mu\varepsilon\vec{E} \\ \nabla^2 \cdot H &= j\omega\mu\sigma\vec{H} - \omega^2\mu\varepsilon\vec{H}\end{aligned}\right\} \rightarrow \text{conduction medium}$$

Proof:-

$$\nabla \times H = \frac{\partial D}{\partial t} + J_c$$

$$= \frac{\varepsilon\partial E}{\partial t} + \sigma E$$

$$\nabla \times He^{j\omega t} = \frac{\varepsilon\partial Ee^{j\omega t}}{\partial t} + \sigma Ee^{j\omega t}$$

$$= j\omega\varepsilon\vec{E} + \sigma\vec{E}$$

$$\boxed{\nabla \times H = j\omega\varepsilon\vec{E} + \sigma\vec{E}}$$

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(ii) $\nabla \times E = -\frac{\partial B}{\partial t}$

$$\nabla \times E e^{j\omega t} = -\frac{\partial \mu H e^{j\omega t}}{\partial t}$$

$$\nabla \times E = -\mu j\omega H e^{j\omega t}$$

$$\nabla \times E = -j\mu\omega H$$

(iii) $\nabla \cdot D = \rho_v$

(iv) $\nabla \cdot B = 0$

→ Lossless medium :—

(v) $\nabla^2 E = \frac{\partial^2 E}{\partial t^2} \mu\varepsilon$

$$\nabla^2 \cdot E e^{j\omega t} = \frac{\partial^2 E e^{j\omega t}}{\partial t^2} \cdot \mu\varepsilon$$

$$= j^2\omega^2 \mu\varepsilon E e^{j\omega t}$$

$$= -\omega^2 \mu\varepsilon E$$

$$\boxed{\nabla^2 \cdot E = -\omega^2 \mu\varepsilon E}$$

(vii) $\nabla^2 \cdot H = \frac{\partial^2 H}{\partial t^2} \mu\varepsilon$

$$\nabla^2 \cdot H e^{j\omega t} = \frac{\partial^2 H e^{j\omega t}}{\partial t^2} \cdot \mu\varepsilon$$

$$= j^2\omega^2 \mu\varepsilon H e^{j\omega t}$$

$$= -\omega^2 \mu\varepsilon H$$

$$\boxed{\nabla^2 \cdot H = -\omega^2 \mu\varepsilon H}$$

→ Conducting Medium :—

(vi) $\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$

$$\nabla^2 \cdot E e^{j\omega t} = \mu\sigma \frac{\partial E \cdot e^{j\omega t}}{\partial t} + \mu\varepsilon \frac{\partial^2 E e^{j\omega t}}{\partial t^2}$$

$$\boxed{\nabla^2 E = j\omega\mu\sigma \vec{E} - \omega^2 \mu\varepsilon E}$$

(viii) $\nabla^2 H = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$

$$\nabla^2 \cdot H \cdot e^{j\omega t} = \mu\sigma \frac{\partial H e^{j\omega t}}{\partial t} + \mu\varepsilon \frac{\partial^2 H e^{j\omega t}}{\partial t}$$

$$\boxed{\nabla^2 \cdot H = j\omega\sigma \vec{H} - \omega^2 \mu\varepsilon H}$$

→ Wave propagation in Lossless medium / conducting medium, conductors / dielectrics

→ Wave propagation

→ Wave propagation basically refers to the ways in which wave can travel.

→ The propagation of wave can be generally illustrated for a lossy dielectric medium where in the progress of wave in a direction leads to loss of power.

→ To understand wave propagation in any kind of medium, few parameters are needed which can be determined i.e

v_p, λ, γ, β, α

- α → Attenuation
- β → phase constant
- γ → propagation constant
- λ → wave length
- v_p → phase velocity / velocity of propagation / wave velocity.

→ Phase velocity:—

The phase velocity of plane wave is defined as the velocity with which the phase of the wave propagates

It is denoted by v or v_p → m/sec

$$\boxed{v_p = \frac{dx}{dt} = \frac{\omega}{\beta}} \text{ m/s}$$

For free space. it is denoted as v_0

$$\boxed{v_0 = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\mu_0 \epsilon_0}\,\omega} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}}$$

Wavelength (λ) :—

The distance b/w successive crests of a wave, especially points in a electro magnetic wave.

(or)

→ The distance that must be travelled by the wave to change phase by 2π radians is called wavelength of the wave

It is denoted by λ — meters

→ In general the wave repeats itself after 2π radians

$$\therefore \boxed{\text{wavelength } (\lambda) = \frac{2\pi}{\beta}} \text{ meters}$$

→ For free space

$$\boxed{\lambda = \frac{2\pi}{\beta}}$$

$$= \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}}$$

$$= \frac{2\cancel{\pi}}{2\cancel{\pi} f\sqrt{\mu_0\varepsilon_0}}$$

$$\lambda = \frac{v_0}{f}$$

$$\lambda = \frac{c}{f}$$

$$\boxed{v = \lambda f}$$

Wave Propagation along conducting Medium :-

→ consider a conducting medium for which EM wave equation is given as.

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{H} = \mu\sigma \frac{\partial \vec{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

Let $\vec{E} = E_0 e^{j\omega t}$; $\vec{H} = H_0 e^{j\omega t}$

$$\therefore \left.\begin{aligned} \frac{\partial \vec{E}}{\partial t} &= j\omega \vec{E} & \frac{\partial^2 E}{\partial t^2} &= -\omega^2 \vec{E} \\ \frac{\partial H}{\partial t} &= j\omega \vec{H} & \frac{\partial^2 H}{\partial t^2} &= -\omega^2 \vec{H} \end{aligned}\right\} \text{Subtitute this in above Eq.}$$

$$\nabla^2 E = \mu\sigma j\omega \vec{E} + \mu\varepsilon (j\omega)^2 \vec{E} = j\omega \vec{E}\mu\sigma + \mu\varepsilon j\omega \vec{E}$$

$$= j\omega \vec{E}\mu(\sigma + j\omega\varepsilon)$$

$$\nabla^2 H = \mu\sigma j\omega \vec{H} + \mu\varepsilon (j\omega)^2 \vec{H} = j\omega\mu \vec{H} (\sigma + j\omega\varepsilon)$$

$$\left.\begin{aligned} \boxed{\nabla^2 E = \gamma^2 \vec{E}} \\ \boxed{\nabla^2 H = \gamma^2 \vec{H}} \end{aligned}\right\} \rightarrow \text{Eq's are called as Helm-holtz Equation.}$$

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

Propagation constant. $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon}$

**→ Propagation constant (γ)

*→ Propagation constant (γ) is a complex quantity made up of real & imaginary terms & it is measured in "1/m (or) m^{-1}".

$$\boxed{\begin{aligned} \therefore\ \gamma &= \alpha + j\beta \\ \alpha + j\beta &= \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} \end{aligned}}$$

*→ In general when the EM wave along conducting medium (i) it gets attenuated that means the amplitude of EM wave reduces.

α → represented real part of propagation constant
→ called attenuation constant
→ Measured in Nepers/meter
→ Expressed as decibels.

(ii) Phase change occurs

β → represents Imaginary part of propagation constant.
→ phase shift constant / phase constant (or) wave no.
→ Measured in rad/m.

**→ Determination of α & β

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} \quad —①$$

$$\gamma = \alpha + j\beta \quad —②$$

Equating ① & ②

$$\alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$= \sqrt{-\omega^2\mu\varepsilon + j(\omega\mu\sigma)}$$

$j\omega\mu(\sigma + j\omega\varepsilon)$
$j\omega\mu\sigma + j^2\omega^2\mu\varepsilon$
$j\omega\mu\sigma - \omega^2\mu\varepsilon$

$$\alpha + j\beta = \sqrt{-\omega^2 \mu \varepsilon + j(\omega\mu\sigma)}$$

squaring on both sides.

$$(\alpha + j\beta)^2 = \left(\sqrt{-\omega^2\mu\varepsilon + j\omega\mu\sigma}\right)^2$$

$$\alpha^2 - \beta^2 + j2\alpha\beta = -\omega^2\mu\varepsilon + j\omega\mu\sigma$$

$$(\alpha^2 - \beta^2) + j(2\alpha\beta) = -\omega^2\mu\varepsilon + j(\omega\mu\sigma)$$

compare real & Imaginary parts on both Sides

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon = -a \longrightarrow (3)$$

$$2\alpha\beta = \omega\mu\sigma = +b \longrightarrow (4)$$

$$(3)^2 + (4)^2 = (\alpha^2 - \beta^2)^2 + 4\alpha^2\beta^2 = (-a)^2 + b^2$$

$$\alpha^4 + \beta^4 - 2\alpha^2\beta^2 + 4\alpha^2\beta^2 = a^2 + b^2$$

$$\alpha^4 + \beta^4 + 2\alpha^2\beta^2 = a^2 + b^2$$

$$(\alpha^2 + \beta^2)^2 = a^2 + b^2$$

$$\alpha^2 + \beta^2 = \sqrt{a^2 + b^2} \longrightarrow (5)$$

from (5) + (3)

$$\alpha^2 + \cancel{\beta^2} = \sqrt{a^2 + b^2}$$

$$\alpha^2 - \cancel{\beta^2} = -a$$

$$2\alpha^2 = \sqrt{a^2 + b^2} - a$$

$$\alpha = \sqrt{\frac{1}{2}\sqrt{a^2 + b^2} - a}$$

$$\alpha = \frac{1}{\sqrt{2}}\sqrt{\sqrt{(\omega^4\mu^2\varepsilon^2 + \omega^2\mu^2\sigma^2)} - \omega^2\mu\varepsilon}$$

$$\alpha = \sqrt{\frac{1}{2}\left(\sqrt{\omega^4\mu^2\varepsilon^2 + \omega^2\mu^2\sigma^2} - \omega^2\mu\varepsilon\right)}$$

$$= \sqrt{\frac{1}{2}\left(\sqrt{\omega^4\mu^2\varepsilon^2\left(1 + \frac{\sigma^2}{\omega^2\varepsilon^2}\right)} - \omega^2\mu\varepsilon\right)}$$

$$= \sqrt{\frac{1}{2}\left(\omega^2\mu\varepsilon\sqrt{\left(1 + \frac{\sigma^2}{\omega^2\varepsilon^2}\right)} - \omega^2\mu\varepsilon\right)}$$

$$= \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left(\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} - 1\right)}$$

$$= \omega\sqrt{\frac{\mu\varepsilon}{2}\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1}$$

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\sqrt{1 - \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1}}$$

⑤ − ③

$$2\beta^2 = \sqrt{a^2 + b^2} + a$$

$$\beta = \sqrt{\frac{1}{2}\left(\sqrt{a^2 + b^2} + a\right)}$$

$$= \sqrt{\frac{1}{2}\left(\sqrt{\omega^4\mu^2\varepsilon^2 + \omega^2\mu^2\sigma^2} + \omega^2\mu\varepsilon\right)}$$

$$= \sqrt{\frac{1}{2}\left[\omega^2\mu\varepsilon\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} + \omega^2\mu\varepsilon\right]}$$

$$= \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left(\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} + 1\right)}$$

$$\boxed{\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} + 1\right)}}$$

Intrinsic Impedence for conducting Medium :-

→ A/c to maxwell's for conducting medium

$$\nabla \times H = J_c + J_D$$

$$= \sigma E + \frac{\partial D}{\partial t}$$

$$\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t}$$

Take

$$H = H e^{j\omega t - \gamma x} \Rightarrow H = (H e^{j\omega t}) e^{-\gamma x} \rightarrow \text{time wave representation}$$

$$E = E e^{j\omega t - \gamma x} \Rightarrow E = (E e^{j\omega t}) e^{-\gamma x}$$

$$\begin{bmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{bmatrix} = \sigma \vec{E} + j\omega\varepsilon \vec{E}$$

$$= (\sigma + j\omega\varepsilon)\vec{E}$$

$$= (\sigma + j\omega\varepsilon)(E_x \vec{a_x} + E_y \vec{a_y} + E_z \vec{a_z})$$

Here E_x & $H_x = 0$ b/c wave propagating in x- direction.

$$\begin{bmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & H_y & H_z \end{bmatrix} = (\sigma + j\omega\varepsilon)(E_y \vec{a_y} + E_z \vec{a_z})$$

$$\underbrace{\vec{a_x}\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right]}_{0 \text{ b/c no wave propagates in } y \& z \text{ direction}} - a_y\left[\frac{\partial H_z}{\partial x}\right] + a_z\left[\frac{\partial H_y}{\partial x}\right] = (\sigma + j\omega\varepsilon)\, E_y \vec{a_y} + E_z \vec{a_z}$$

$$-a_y\left(\frac{\partial H_z}{\partial x}\right) + \left(\frac{\partial H_y}{\partial x}\right)a_z = (\sigma + j\omega\varepsilon)E_y\vec{a_y} + (\sigma + j\omega\varepsilon)E_z\vec{a_z}$$

$$-\frac{\partial H_z}{\partial x} = (\sigma + j\omega\varepsilon)E_y$$

$$\frac{\partial H_z}{\partial x} = -(\sigma + j\omega\varepsilon)E_y$$

$$-\gamma H_z = -(\sigma + j\omega\varepsilon)E_y$$

$$\frac{E_y}{H_z} = \frac{\gamma}{\sigma + j\omega\varepsilon}$$

$$= \frac{\sqrt{j\omega\mu(\sigma + j\omega\mu)}}{\sigma + j\omega\varepsilon}$$

$$E_y = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \cdot H_z$$

$$\frac{\partial}{\partial x} H_z e^{+j\omega t - \gamma x}$$

$$H_z e^{j\omega t - \gamma x} (-\gamma)$$

$$-\gamma H_z$$

$\|\|\|$ for $\frac{\partial H_y}{\partial x} = -\gamma H_y$

$$\frac{\partial H_y}{\partial x} = (\sigma + j\omega\varepsilon)E_z$$

$$-\gamma H_y = (\sigma + j\omega\varepsilon)E_z$$

$$E_z = \frac{-\gamma}{\sigma + j\omega\varepsilon} H_y$$

$$= -\frac{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}}{\sigma + j\omega\varepsilon}$$

$$E_z = -\sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}\, H_y$$

$$\vec{E} = E_y\vec{a_y} + E_z\vec{a_z} \quad \because E_x = 0$$

$$\vec{E} = \sqrt{E_y^2 + E_z^2}$$

$$= \sqrt{\left(\sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} H_z\right)^2 + \left(\sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} H_y\right)^2}$$

$$= \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon} H_z^2 + \frac{j\omega\mu}{\sigma + j\omega\varepsilon} H_y^2}$$

$$= \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon} (H_z^2 + H_y^2)} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}\, \vec{H}$$

$$\boxed{\frac{\vec{E}}{\vec{H}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \eta}$$

Summary

Propagation characteristics of EM wave

(a) conducting Medium

1. propagation constant $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$ $1/m$.

2. phase shift constant $\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right]}$ rad/m

3. Attenuation constant, $\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right]}$ ~~dB~~/m

4. Velocity of propagation of EM wave

$$v_p = f\lambda = \frac{\omega}{\beta} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right]}}$$

$$v_p = \left[\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right]\right]^{-1} \text{ m/sec}$$

5. Intrinsic Impedence $\eta = \frac{E}{H} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$ Ω

(b) Free space medium :— (Prefect dielectric (or) Lossless dielectric)

$\sigma = 0$

$\mu = \mu_0 \mu_r \Rightarrow \mu = \mu_0$

$\varepsilon = \varepsilon_0 \varepsilon_r \Rightarrow \varepsilon = \varepsilon_0$

1. propagation constant $\gamma = \alpha + j\beta = \sqrt{j\omega\mu\,(\sigma + j\omega\varepsilon)}$

$$= \sqrt{j\omega\mu(0) + j^2\omega^2\mu\varepsilon}$$

$$\gamma = j\omega\sqrt{\mu\varepsilon} \Rightarrow j\omega\sqrt{\mu_0\varepsilon_0}\ m^{-1}$$

2) Phase shift constant $\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right]}$

$$= \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1}+1\right)}$$

$$= \omega\sqrt{\frac{\mu_0\varepsilon_0}{\cancel{2}}\cancel{2}}$$

$$\beta = \omega\sqrt{\mu_0\varepsilon_0}\ \text{rad/m}$$

3. Attenuation constant $(\alpha) = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right]}$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\sqrt{1}-1}$$

$$= \omega(0)$$

$$\alpha = 0\ \text{dB/m}$$

4. velocity of Propagation $v_0 = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ m/sec

5. velocity of propagation of an EM wave is the same as phase velocity $v_p = \frac{\omega}{\beta} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{\mu_0\varepsilon_0}} = v_0$

6. Intrensic Impedence $\frac{E}{H} = 120\pi\,\Omega = \sqrt{\mu_0/\varepsilon_0}$

(c) conductors & dielectrics

→ As some media behaves like

(i) Good conductors at one freq's range

(ii) Good dielectrics at some other freq range.

(i) wave propagation in Good dielectrics :—

(i) Attenuation constant α, is given by

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right]}$$

→ for dielectrics $\frac{\sigma}{\omega\varepsilon} >> 1$.

→ Expanding $\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}$ by Binomial series, higher order terms can be neglected.

$$\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} = 1+\frac{\sigma^2}{2\omega^2\varepsilon^2}$$

$$\alpha \simeq \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\left(\cancel{1}+\frac{\sigma^2}{2\omega^2\varepsilon^2}\right)-\cancel{1}\right]}$$

$$= \omega\sqrt{\frac{\mu\varepsilon\sigma^2}{4\omega^2\varepsilon^2}}$$

$$= \omega\sqrt{\frac{\mu\sigma^2}{4\omega^2\varepsilon}}$$

$$= \cancel{\omega}\frac{\sigma}{2\cancel{\omega}}\sqrt{\frac{\mu}{\varepsilon}} \Rightarrow \boxed{\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}}$$

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(ii) The phase shift constant (β) :—

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right]}$$

$$\simeq \omega\sqrt{\frac{\mu\varepsilon}{2}\left[1+\frac{\sigma^2}{2\omega^2\varepsilon^2}+1\right]}$$

$$\simeq \omega\sqrt{\frac{\mu\varepsilon}{2}\left[2+\frac{\sigma^2}{2\omega^2\varepsilon^2}\right]}$$

$$\simeq \omega\sqrt{\frac{\mu\varepsilon}{2}\cdot 2\left[1+\frac{\sigma^2}{4\omega^2\varepsilon^2}\right]}$$

$$\simeq \omega\sqrt{\mu\varepsilon}\sqrt{1+\frac{\sigma^2}{4\omega^2\varepsilon^2}}$$

$$\simeq \omega\sqrt{\mu\varepsilon}\left[1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right]$$

$$\boxed{\beta \simeq \omega\sqrt{\mu\varepsilon}\left[1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right]}$$

$\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} = 1+\frac{\sigma^2}{2\omega^2\varepsilon^2}$

a/c to binomial expansion

$\sqrt{1+\frac{\sigma^2}{4\omega^2\varepsilon^2}} = 1+\frac{\sigma^2}{8\omega^2\varepsilon^2}$

(iii) Intrinsic (or) characterstic impedence of General medium:

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\varepsilon}} = \sqrt{\frac{(j\omega\mu)}{(\sigma+j\omega\varepsilon)}}$$

$$= \sqrt{\frac{j\omega\mu}{j\omega\varepsilon\left(1+\frac{\sigma}{j\omega\varepsilon}\right)}} \Rightarrow \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}}\sqrt{\frac{1}{1+\frac{\sigma}{j\omega\varepsilon}}}$$

$\frac{\sigma}{\omega\varepsilon} \ll 1$ so should apply Binomial expansn.

$$= \sqrt{\frac{\mu}{\varepsilon}}\sqrt{\frac{1}{1+\frac{\sigma}{j\omega\varepsilon}}} = \sqrt{\frac{\mu}{\varepsilon}}\left[1+j\frac{\sigma}{2\omega\varepsilon}\right]$$

$$\boxed{\eta = \sqrt{\mu/\varepsilon}\cdot\left[1+j\frac{\sigma}{2\omega\varepsilon}\right]}$$

negative will be

$\sqrt{\frac{\mu}{\varepsilon}}\left(\sqrt{1+\frac{\sigma}{j\omega\varepsilon}}\right)^{-1/2}$

a/c to Binomial expansion $\because \frac{1}{j} = -j$

$\sqrt{\frac{\mu}{\varepsilon}}\left(1-\frac{\sigma}{j2\omega\varepsilon}\right)$

$\sqrt{\frac{\mu}{\varepsilon}}\left(1+\frac{j\sigma}{2\omega\varepsilon}\right)$

** Wave propagation in Good conductors :—

(i) The propagation constant, γ is given by.

$$\gamma = \sqrt{j^2\omega^2\mu\varepsilon + j\omega\mu\sigma}$$

$$= \sqrt{j\omega\mu(j\omega\varepsilon + \sigma)}$$

$$= \sqrt{j\omega\mu}\,\sqrt{j\omega\varepsilon + \sigma}$$

$$= \sqrt{j\omega\mu}\,\sqrt{\sigma\left(1 + \frac{j\omega\varepsilon}{\sigma}\right)}$$

$$= \sqrt{j\omega\mu\sigma}\,\sqrt{\left(1 + \frac{j\omega\varepsilon}{\sigma}\right)}$$

as $\frac{\sigma}{\omega\varepsilon} >> $ (or) $\frac{\omega\varepsilon}{\sigma} << 1$ for good conductors.

$$= \sqrt{j\omega\mu\sigma}\,\sqrt{1}$$

$$= \cancel{\sqrt{j\omega\mu\sigma}\,\angle 90^\circ}$$

$$= \sqrt{\omega\mu\sigma}\,\sqrt{j}$$

$$= \sqrt{\omega\mu\sigma}\,\angle 45^\circ$$

$$\gamma = \sqrt{\omega\mu\sigma}\,\angle 45^\circ$$

$$\gamma = \alpha + j\beta = \sqrt{\omega\mu\sigma}\,\angle 45^\circ$$

$$\alpha + j\beta = \sqrt{\omega\mu\sigma}\,\frac{1+j}{\sqrt{2}}$$

$j = \cos\pi/2 + j\sin\pi/2 = e^{j\pi/2} = 90^\circ$

$\sqrt{j} = (j)^{1/2}$

$\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}$

$= e^{j\pi/4}$

$= \angle 45^\circ$

$\sqrt{j}\ \cos 45^\circ + j\sin 45^\circ$

$\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

$\frac{1+j}{\sqrt{2}}$

$$\alpha + j\beta = \sqrt{\frac{\omega\mu\sigma}{2}}\,(1+j) \Rightarrow \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\alpha + j\beta = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}} \qquad \boxed{\therefore\ \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}}$$

Interinsic Impedence Good conductors :—

4.19

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$$\frac{\sigma}{j\omega\varepsilon} >> 1$$

$$= \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}\left[\frac{1}{1+\frac{\sigma}{j\omega\varepsilon}}\right]}$$

$$= \sqrt{\frac{\mu}{\varepsilon}\left(\frac{1}{\sigma/j\omega\varepsilon}\right)}$$

$$= \sqrt{\frac{\mu}{\cancel{\varepsilon}} \cdot \frac{j\omega\cancel{\varepsilon}}{\sigma}}$$

$$\boxed{\eta = \sqrt{\frac{j\omega\mu}{\sigma}}}$$

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	Conducting Medium	Free space	Good conductors $\frac{\sigma}{\omega\varepsilon} >> 1$	Good dielectrics $\frac{\sigma}{\omega\varepsilon} << 1$
1) Attenuation constant (α)	$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} - 1\right)}$	$\alpha = 0$	$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$ dB/m	$\alpha \simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$
2) Phase shift constant (β)	$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} + 1\right)}$	$\beta = \omega\sqrt{\mu_0\varepsilon_0}$ rad/m	$\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$ dB/m	$\beta = \omega\sqrt{\mu\varepsilon}\left[1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right]$
3) Intrinsic Impedence η	$\sqrt{\frac{j\omega\mu}{\sigma+j\omega\varepsilon}}$ Ω	$\sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\,\Omega$ (or) $377\,\Omega$	$\eta = \sqrt{\frac{j\omega\mu}{\sigma}}$ Ω	$\eta = \sqrt{\frac{\mu}{\varepsilon}}\left[1+j\frac{\sigma}{2\omega\varepsilon}\right]$
4) Phase velocity	$\frac{\omega}{\beta} = \left[\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} + 1\right)\right]^{-1/2}$ m/s.	$v_p = v_0$ $v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu_0\varepsilon_0}}$ m/s $v_p = v_0$	$v = \sqrt{\frac{2\omega}{\mu\sigma}}$ m/sec	$v = \frac{\omega}{\beta} \simeq \frac{1}{\sqrt{\mu\varepsilon}}\left[1-\frac{\sigma^2}{8\omega^2\varepsilon^2}\right]$
5. Propagation constant	$\gamma = \sqrt{-\omega^2\mu\varepsilon + j\omega\mu\sigma}$ m^{-1}	$\gamma = j\omega\sqrt{\mu_0\varepsilon_0}$	$\gamma = \sqrt{\omega\mu\sigma}\,\angle 45^\circ$	

Depth Of Penetration, δ (m)

4-20

→ The depth of penetration is defined as that depth at which the wave attenuates to $1/e$ (or) $\simeq 37\%$ of its Original amplitude.

→ It is also called skin depth

→ skin depth is a measure of depth to which an EM wave can penetrate the medium.

→ The depth of penetration δ

$$\boxed{\delta = \frac{1}{\alpha}}$$

where α — attenuation constant

→ Let the wave attenuation be represented by

$$E = E_0 e^{-\alpha z}$$

z- being the direction of propagation.

at $z = \delta$

$$\therefore E = E_0 e^{-\alpha\delta}$$

A/c to the definition of 'δ' (depth of penetration)

$$E = E_0 e^{-1} \quad \text{at } z = \delta$$

$$\therefore E_0 e^{-\alpha\delta} = E_0 e^{-1}$$

$$\alpha\delta = 1$$

$$\boxed{\delta = 1/\alpha}$$

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Q: Find the depth of penetration δ of the EM wave in copper $f = 60 Hz$ & $f = 100 MHz$

For copper:

$\sigma = 5.8 \times 10^{7}$ mho/m

$\mu_r = 1$

$\varepsilon_r = 1$

Sol: (i) For copper at $f = 60 Hz$ Cu is good conductor

$\frac{\sigma}{\omega\varepsilon} >> 1$

$\sigma = \omega\varepsilon$

$$\frac{\sigma}{\omega\varepsilon} = \frac{5.8 \times 10^{7}}{2\pi \times 60 \times 8.854 \times 10^{-12}}$$

$$= 174 \times 10^{4} >> 1$$

$\therefore$ At $f = 60 Hz$, Cu is very good conductor.

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$= \sqrt{\frac{2}{2\pi \times 60 \times 4\pi \times 10^{-7} \times 5.8 \times 10^{7}}} = 8.53 \times 10^{-3} m.$$

(ii) For copper at $f = 100 MHz$

$$\frac{\sigma}{\omega\varepsilon} = \frac{5.8 \times 10^{7}}{2\pi \times 8.854 \times 10^{-12} \times 100 \times 10^{6}} = 10.425 \times 10^{9} >> 1$$

$\therefore$ At $f = 100 Hz$, Cu is very good conductor.

$$\delta = \frac{1}{\alpha} = \frac{\sqrt{2}}{\sqrt{\omega\mu\sigma}}$$

$$= \sqrt{\frac{2}{2\pi \times 100 \times 10^{6} \times 4\pi \times 10^{-7} \times 5.8 \times 10^{7}}} = 6.608 \times 10^{-6} m$$

→ Polarization :— Polarization of a wave refers to the orientation of $\vec{E}$ at a given point w.r.t time

(or)

→ Polarization of a wave is defined as the direction of the electric field at a given point as a function of time.

→ The polarization of a composite wave is the direction of the electric field at a given point as a function of time.

→ **Types of Polarisations :—**

(a) Linear Polarization

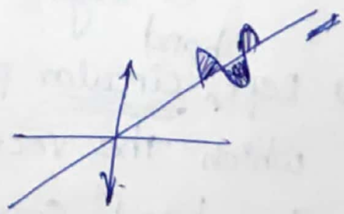
(b) circular polarization

(c) Elliptical polarization.

(a) Linear polarization :—

→ A wave is said to be linearly polarised if the electric field remains along a straight line as a function of time at some point in the medium.

→ Linear polarization of a wave is again of three types namely.



(a) Horizontal Polarization

(b) vertical polarization

(c) Theta polarization.

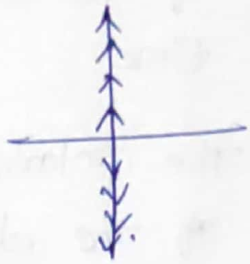
(a) Horizontal polarization :—

The Polarization in which electric field is arranged $\parallel^{el}$ to the horizontal plane is known as Horizontal plane.

The arrangment of electricfield vector in horizontal Polarization

(b) Vertical polarization

→ The polarization in which the electric field is arranged in ||el to the vectical plane



(ii) circular polarization :—

→ The polarization in which, $\vec{E}$/$\vec{H}$ of EM wave has constant length & rotates in a circular path

(or)

→ polarization of EM wave in which either the $\vec{E}$ & $\vec{H}$ executes a circle $\perp^{lar}$ to the path of propagation with a freq equal to that of the wave.

→ Depending on the direction in which $\vec{E}$ | $\vec{H}$ rotates it classified into

(i) Left hand circular polarization

(ii) Right hand circular polarization

(i) Left hand circular polarization in which the vector rotates in a left hand sense / anticlockwise direction w.r.t the direction of propagation.

(ii) Right hand circular polarization in which the vector rotates in a Right-hand sense / CLK wise direction w.r.t the direction of propagation.

(iii) Elliptical polarization :—

→ In Elliptical polarization it as to satisfies two conditions.

(i) In which the $\vec{E}$ in x & y directions i.e E_x & E_y aren't in phase & has a constant phase difference equal to 90°

(ii) In which the ratio of amplitudes of E_x & E_y is constant (except 1)

→ consider a wave travelling in z-direction, having x & y components of diff amplitudes & seperated by a phase of 90°

The electric field can be written as

$$E_0 = A\cos\omega t + B\sin\omega t \quad \text{—①}$$

$$E_0 = E_x + E_y \quad \text{—②}$$

Eq ① & ②

$$A\cos\omega t\,\vec{a_x} + B\sin\omega t\,\vec{a_y} = E_x\,\vec{a_x} + E_y\,\vec{a_y}$$

fields in x direction ⇒ $E_x = a\cos\omega t$ $\qquad$ $E_y = B\sin\omega t$ ⇒ fields in y direction.

Squaring on both sides

$$E_x^2 = a^2\cos^2\omega t \qquad E_y^2 = B^2\sin^2\omega t$$

$$\frac{E_x^2}{a^2} = \cos^2\omega t \rightarrow ⓐ \qquad \frac{E_y^2}{B^2} = \sin^2\omega t \quad ⓑ$$

sum ⓐ & ⓑ

$$\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} = \cos^2\omega t + \sin^2\omega t$$

(i) If $a \neq b$

$$\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} = \cos^2\omega t + \sin^2\omega t$$

$$\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} = 1$$

Above Eq. represents an ellipse, thus the wave is polarised elliptically.

(ii) If $a = b$

$$\frac{E_x^2}{a^2} + \frac{E_y^2}{b} = 1$$

$$\frac{E_x^2}{a^2} + \frac{E_y^2}{a^2} = 1$$

$$E_x^2 + E_y^2 = a^2$$

above Eq. represents a circle. Thus $a = b$ & phase d/nce is 90°, the polarization encountered by wave is circular polarization.

→ Reflection & Refraction of plane wave :-

→ In this topic, wave properties when the wave travel from one medium to another. properties like

→ Direction of propagation.

→ polarization

→ Reflection.

→ Refraction.

of the wave at the interface depends on the material involved.

→ The propagation of an EM wave through diff material having different constant parameters such as.

$\varepsilon, \mu, \sigma, \eta \ldots$ etc

→ Ex:- when a plane wave in air is incident normally on the surface of a perfect conductor, the wave completely reflects back.

→ The combination of Incident (forward) wave & reflected (backward) wave results in standing wave.

→ Depending upon the direction of incident wave two cases can be considered.

(a) **Normal incidence** :- The incident wave is normal to the plane of the boundary surface.

ⓑ Oblique incidence :— The incident wave make an angle 'θ' with the normal to the plane of the boundary surface.

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→ when the wave is incident obiquely at the boundary b/w two materials, the following terms are considered

ⓐ Incidence angle (θ_i) :—

The angle at which the incident wave makes with the normal to the interface (Boundary)

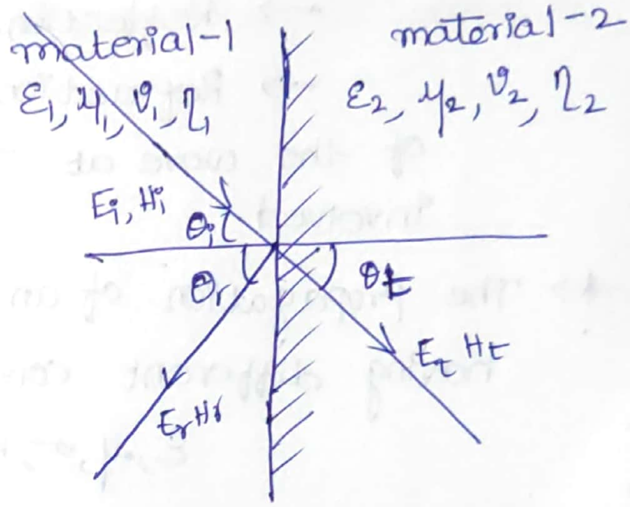


Fig: Reflection & Refraction of plane wave.

(b) Reflection angle (θ_r) :—

The angle at which the reflected wave makes with the normal to the interface (Boundary)

ⓒ Transmission angle (or) Refracted angle :— (θ_t)

The angle at which the transmitted wave makes with the normal to the interface (Boundary)

(d) Law of reflections :—

It states that a wave incident upon a reflecting surface will be reflected at an angle equal to the incident angle

$$\boxed{\theta_i = \theta_r}$$

(e) Reflection co-efficient :—

→ It is the ratio of the amplitudes of reflected & incident waves.

→ It gives the fraction of the incident wave i.e reflected back from the interface

→ It is dimensionless quantity & exists in the range −1 to 1

$$\boxed{\rho = \frac{E_r}{E_i} \text{ (or) } \rho = \frac{H_r}{H_i}}$$

(f) Snell's law :—

→ It is also called law of refraction.

→ It is used to determine the direction of wave travelling through refractive media with varying indices of refraction.

→ It states that the ratio of the sine angles of incidence & refraction is equivalent to the ratio of phase velocities in the two media (or) equivalent to the reciprocal of the ratio of the refractive indices

i.e $$\frac{\sin\theta_i}{\sin\theta_t} = \frac{V_1}{V_2} = \frac{\eta_2}{\eta_1}$$

for dielectric

$$\mu_1 = \mu_2 = \mu_0 \quad [\because \mu_{r_1} = \mu_{r_2} = 1]$$

$$\therefore V_1 = \frac{1}{\sqrt{\varepsilon_1 \mu_0}} ; \quad V_2 = \frac{1}{\sqrt{\varepsilon_2 \mu_0}}$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{\frac{1}{\sqrt{\varepsilon_1 \cancel{\mu_0}}}}{\frac{1}{\sqrt{\varepsilon_2 \cancel{\mu_0}}}} = \frac{\frac{1}{\sqrt{\varepsilon_1}}}{\frac{1}{\sqrt{\varepsilon_2}}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\boxed{\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1}}$$

(a) Normal incidence :—

(i) Reflection ~~of~~ by a perfect conductor :—

*→ when a wave in air is incident on a perfect conductor normally, it is entirely reflected.

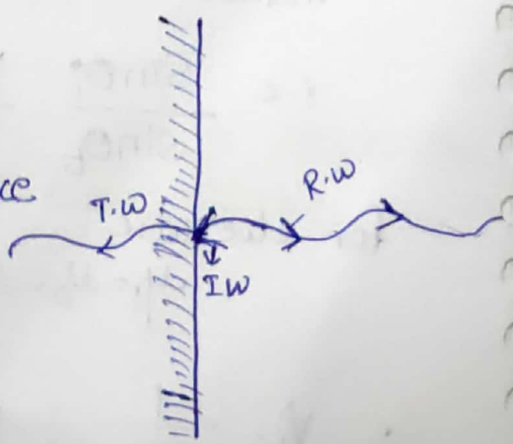
*→ As neither E nor H can exist in a perfect conductor, none of the energy transmitted through it.

As no losses within a perfect conductor no energy is absorbed in it.

*→ when an EM wave travelling in one medium is incident upon a second medium, it is partially reflected & partially transmitted.

*→ Let an EM wave is propagation along +ve direction hence the Electric field intensity of incidence EM wave can be written as

$$\vec{E_i} = E_i \, e^{j(\omega t - \beta z)} \longrightarrow ①$$



perfect conductor

4.05

→* Let Assume that EM wave is incidence on perfect conductor normally

Hence $\vec{E}$ of reflected wave is

$$\vec{E_r} = E_r\, e^{j(\omega t + \beta z + \theta)} \longrightarrow (2)$$

$\theta \rightarrow$ phase dfnce b/w incident & reflected wave.

A/c to the boundary condition.

$$E_{tan1} = E_{tan2} = 0 \quad \text{at } x = 0$$

b/c boundary b/w conductor & freespace (dielectric).

$$E_{tan} = \vec{E_i} + \vec{E_r} \Rightarrow$$

$E_{tan1} = E_{tan2}$

$E_{tan1} = 0$ b/c other medium is conductor.

$$E_{tan} = 0$$

$$\vec{E_i} + \vec{E_r} = 0$$

$$\vec{E_i} = -\vec{E_r}$$

i.e $|E_i| = |E_r|$, $\angle E_i = -\angle E_r$ } magnitudes are same but phase reversal on reflection i.e

$$\theta = 180^\circ \text{ (or) } \pi$$

∴ The resultant electric field intensity is given as

$$\vec{E}(x,t) = \vec{E_i} \cdot \vec{E_r}$$

$$= \vec{E_i}\, e^{j(\omega t - \beta x)} + \vec{E_r}\, e^{j(\omega t + \beta x + \theta)}$$

$$= \vec{E_i}\, e^{j(\omega t - \beta x)} + \vec{E_i}\, e^{j(\omega t + \beta x + \pi)}$$

$e^{j\pi} = \cos\pi + j\sin\pi$
$= (-1) + 0$
$= (-1)$

$$= \vec{E_i}\left[e^{j(\omega t - \beta x)} + e^{j(\omega t + \beta x)} \cdot e^{j\pi}\right]$$

$$= \vec{E_i}\left[e^{j(\omega t - \beta x)} + e^{j(\omega t + \beta x)} (-1)\right]$$

$$= \vec{E_i}\left[e^{j(\omega t - \beta x)} - e^{j(\omega t + \beta x)}\right]$$

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$$= E_i e^{j\omega t}\left[e^{-j\beta x} - e^{j\beta x}\right]$$

$$= E_i(\cos\omega t + j\sin\omega t)\left[\cancel{\cos\beta x} - j\sin\beta x - \cancel{\cos\beta x} - j\sin\beta x\right]$$

$$= E_i\left[\cos\omega t + j\sin\omega t\right]\left[-2j\sin\beta x\right]$$

$$= E_i\left[(-2j\cos\omega t\cdot\sin\beta x) - 2j^2\sin\beta x\,\sin\omega t\right]$$

we get, considering only real terms

$$\boxed{\vec{E}(x,t) = 2E_i\sin\beta x\cdot\sin\omega t}$$

*→ A/c to boundary condition for magnetic field

$$H_i - H_r = 0$$

From above Eq magnitudes are same & phase difference is zero. i.e $\theta = 0^\circ$

$$\vec{H}(x,t) = \vec{H}_i + \vec{H}_r$$

$$= H_i e^{j(\omega t - \beta x)} + H_r e^{j(\omega t + \beta x + \theta)} \quad \because H_r = H_i\ \&$$

$$= H_i e^{j(\omega t - \beta x)} + H_i e^{j(\omega t + \beta x + 0^\circ)}$$

$$= H_i\left[e^{j(\omega t - \beta x)} + e^{j(\omega t + \beta x)}\right]$$

$$= H_i e^{j\omega t}\left[e^{-j\beta x} + e^{j\beta x}\right]$$

$$= H_i e^{j\omega t}\left[\cos\beta x - \cancel{j\sin\beta x} + \cos\beta x + \cancel{j\sin\beta x}\right]$$

$$= 2H_i\left[\cos\omega t + j\sin\omega t\right]\left[\cos\beta x\right]$$

$$= 2H_i\left[\cos\omega t\cos\beta x + j\cos\beta x\sin\omega t\right]$$

consider only real terms

$$\boxed{\vec{H}(x,t) = 2H_i\left[\cos\omega t\cdot\cos\beta x\right]}$$

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→* The Reflection co-efficient

$$\rho = \frac{E_r}{E_i}$$

$$= \frac{-E_i}{E_i}$$

$$\boxed{\rho = -1}$$

→* Transmission co-efficient

$$\boxed{T = \frac{E_t}{E_i} = 0}$$

(ii) Reflection by a perfect Dielectric :-

* consider a uniform plane wave travelling along the x-direction & incident normally on the surface of a perfect dielectric media.

* Assume that $\vec{E}$ - y direction
$\vec{H}$ - z direction.

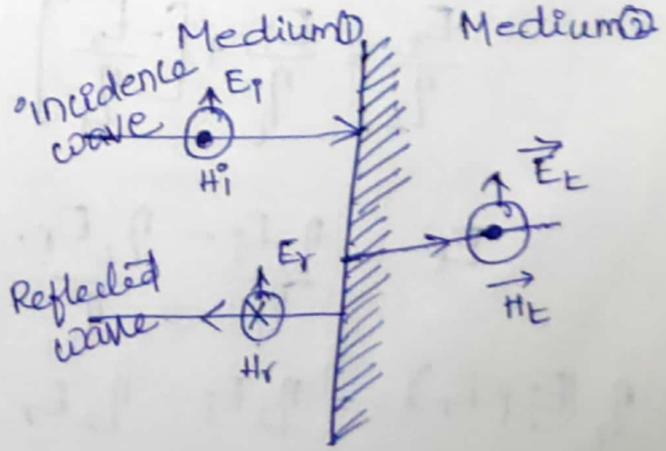
Medium ① properties → $\varepsilon_1, \mu_1, \sigma_1, \eta_1$
Medium ② properties → $\varepsilon_2, \mu_2, \sigma_2, \eta_2$

$\sigma = 0$ b/c lossless dielectric

* The field vector for the incidence wave in phasor notation are

$$\vec{E_i} = E_i e^{-j\beta x} \vec{a_y}$$

$$\vec{H_i} = H_i e^{-j\beta x} \vec{a_z}$$



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→ Field vector for the reflected wave

$$\vec{E_r} = E_r e^{j\beta x} \vec{a_y}$$
$$\vec{H_r} = H_r e^{j\beta x} \vec{a_z}$$

→ Field vector for the transmitted wave

$$\vec{E_t} = E_t e^{-j\beta x} \vec{a_y}$$
$$\vec{H_t} = H_t e^{-j\beta x} \vec{a_z}$$

where E_i, H_i ; H_r, E_r ; H_t, E_t } Amplitudes of incidence, reflected & transmission wave.

We know that

$$E_i = \eta_1 H_i \quad ; \quad E_t = \eta_2 H_t$$
$$E_r = -\eta_1 H_r$$

At the boundary, the tangential components of the field are continues

$$E_t = E_i + E_r$$
$$H_t = H_i + H_r.$$

∴ Sublitute 'H' value to get E

$$H_t = H_i + H_r$$

$$\frac{E_t}{\eta_2} = \frac{E_i}{\eta_1} + \left[-\frac{E_r}{\eta_1}\right]$$

$$E_t = \frac{\eta_2 E_i}{\eta_1} - \frac{\eta_2 E_r}{\eta_1}.$$

$$\eta_1 (E_i + E_r) = \eta_2 E_i - \eta_2 E_r$$

$$\eta_1 E_i + \eta_1 E_r = \eta_2 E_i - \eta_2 E_r$$

$$\eta_1 E_r + \eta_2 E_r = \eta_2 E_i - \eta_1 E_i$$

$$E_r(\eta_2 + \eta_1) = (\eta_2 - \eta_1) E_i$$

$$\Gamma = \frac{E_r}{E_i} = \left[\frac{\eta_2 + \eta_1}{\eta_2 + \eta_1}\right]$$

∴ Reflection co-efficient of electric fields

$$\boxed{\Gamma_E = \frac{E_r}{E_i} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}}$$

∴ Transmission co-efficient of electric fields

$$T_E = \frac{E_t}{E_i} = \frac{E_i + E_r}{E_i}$$

$$= \frac{E_i}{E_i} + \frac{E_r}{E_i}$$

$$= 1 + \frac{E_r}{E_i}$$

$$= 1 + \frac{\eta_2-\eta_1}{\eta_2+\eta_1}$$

$$= \frac{\eta_2 + \cancel{\eta_1} + \eta_2 - \cancel{\eta_1}}{\eta_2+\eta_1}$$

$$\boxed{T_E = \frac{2\eta_2}{\eta_2+\eta_1}}$$

→** Reflection co-efficient of Magnetic fields

$$\Gamma_H = \frac{H_r}{H_i} = \frac{-E_r/\eta_1}{E_i/\eta_1} \Rightarrow -\left[\frac{\eta_2-\eta_1}{\eta_2+\eta_1}\right] = \left[\frac{\eta_1-\eta_2}{\eta_1+\eta_2}\right]$$

→** Transmission co-efficient of magnetic field.

$$T = \frac{H_t}{H_i} = \frac{E_t/\eta_2}{E_i/\eta_1} = \frac{\eta_1}{\eta_2}\left[\frac{E_t}{E_i}\right] \Rightarrow \left[\frac{2\cancel{\eta_2}}{\eta_2+\eta_1}\right]\frac{\eta_1}{\cancel{\eta_2}}$$

$$\boxed{T = \frac{2\eta_1}{\eta_2+\eta_1}}$$

(b) **Oblique Incidence of a plane wave on a Boundary plane:—**

→ Reflection & transmission of a wave depend on
(i) The type of polarisation of a wave → $\parallel^{el}$ / $\perp^{lar}$
(ii) The medium of the boundary.

1) **Parallel polarization:—**

→ In which the electric field of the wave is $\parallel^{el}$ to the plane of incidence

→ It is also called vertical polarization.

2) **Perpendicular Polarisation:—**

→ In which the electric field of the wave is $\perp^{lar}$ to the plane of incidence

→ It is also called Horizontal polarization.

→ **Oblique Incidence**

① **Reflection by perfect conductor:—**

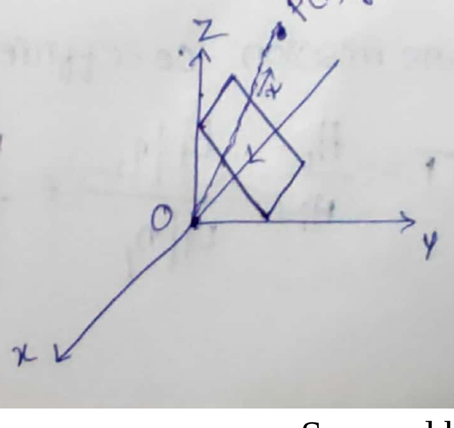
→ when a wave is incident on a perfect conductor, it is reflected back into the same medium. The resultant fields depends on the type of polarization.

(a) **~~parallel polarization~~:—**

→ consider wave is travelling along the x direction $\vec{E}$ can be written as

$$\vec{E} = E \cdot e^{-j\beta x}.$$

→ consider a plane which is normal to the direction of wave propagation.



→ consider a point $p(x,y,z)$ on the normal plane having distance vector $\vec{r}$

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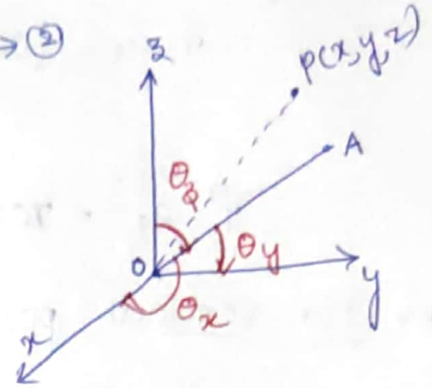
$$\therefore \vec{r} = x\vec{a_x} + y\vec{a_y} + z\vec{a_z} \longrightarrow (2)$$



x- component of $\vec{r}$ is given as

$$x = \vec{r} \cdot \vec{a_r} \longrightarrow (3)$$

Put (3) in (1)

$$\vec{E} = E e^{j\beta(\vec{r} \cdot \vec{a_x})}$$

→ consider a wave travelling in x-direction along 'OA' which makes an angle of $\theta_x, \theta_y \& \theta_z$ with x, y, z direction respectively.

$$\vec{a_n} = \cos\theta_x \vec{a_x} + \cos\theta_y \vec{a_y} + \cos\theta_z \vec{a_z} \rightarrow$$ A/c to direction cosines.

[cosine of angles made by vector with the required co-ordinate axis)

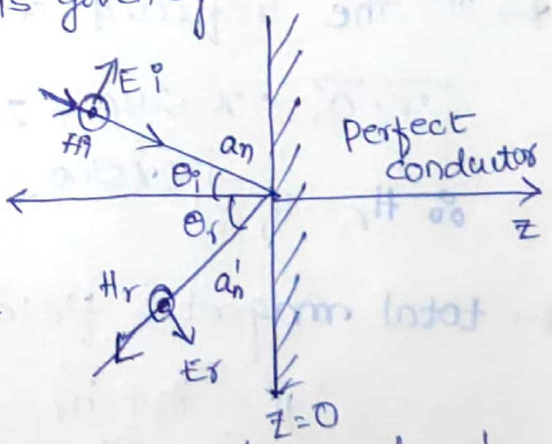
$$\vec{r} = \vec{a_x} x + y\vec{a_y} + z\vec{a_z}$$

$$\vec{r} \cdot \vec{a_n} = x\cos\theta_x + y\cos\theta_y + z\cos\theta_z.$$

(i) parallel polarization :—

→ The incident magnetic field is given by

$$H_i = H_1 e^{-j\beta(a_n \cdot r)}.$$



a_n — unit vector normal to the plane

$r = (x,y,z)$ radius of the vector plane

→ Assume that the wave is propagating obliquely along any arbitrary direction with unit vector $\vec{a_r}$

→ Let the position vector $\vec{r}$ be such that

$$r \cdot \vec{a_r} \cdot \text{ constant}$$

→ The incident wave in phasor notation can be expressed as

$$\vec{H_i} = H_i e^{-j\beta(\vec{r} \cdot \vec{a_r})} \vec{a_r}$$

→* using direction cosines

$$\vec{r}\cdot\vec{a_r} = (\cos\theta_x \vec{a_x}) + \cos\theta_y \vec{a_y} + \cos\theta_z \vec{a_z}) \cdot (x\vec{a_x} + y\vec{a_y} + z\vec{a_z}).$$

$$\vec{r}\cdot\vec{a_r} = x\cos\theta_x + y\cos\theta_y + z\cos\theta_z$$

→* <u>For vertical polarization.</u>

<u>magnetic vector</u> is along <u>the y-direction</u>, i.e normal to the plane of incidence (x-z plane) & direction of propagation.

→* If θ is the incident angle <u>with the</u> z-axis.

$$\theta_z = \theta; \quad \theta_y = \pi/2; \quad \theta_x = \theta - \pi/2$$

$$\vec{r}\cdot\vec{a_r} = x\cos(\theta - \pi/2) + y\cos(\pi/2) + z\cos\theta$$

$$\vec{r}\cdot\vec{a_r} = x\sin\theta + z\cos\theta$$

$$\therefore H_i = H_i e^{-j\beta(x\sin\theta + z\cos\theta)} \cdot \vec{a_y}$$

→* $|||^{ar}$ the reflected wave in the (−Z) direction

$$\vec{r}\cdot\vec{a_r} = x\sin\theta - z\cos\theta$$

$$\therefore H_r = H_r e^{-j\beta(x\sin\theta - z\cos\theta)} \cdot \vec{a_y}$$

→* total magnetic field vector of the standing wave

$$\vec{H} = \vec{H_i} + \vec{H_r}$$

$$= H_i e^{-j\beta(x\sin\theta + z\cos\theta)} + H_r e^{-j\beta(x\sin\theta - z\cos\theta)}$$

$\therefore$ $\boxed{H_i = H_r}$ → $\because$ magnetic field vector is $||^{el}$ to the boundary, it will be reflected without phase reversal.

$$\vec{H} = H_i \left[e^{-j\beta(x\sin\theta + z\cos\theta)} + e^{-j\beta(x\sin\theta - z\cos\theta)} \right]$$

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