

# Unit - III

## Steam Power Cycles

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- 15.1. Carnot cycle. 15.2. Rankine cycle. 15.3. Modified Rankine cycle. 15.4. Regenerative cycle.  
15.5. Reheat cycle. 15.6. Binary vapour cycle—Additional/Typical Worked Examples—Highlights—  
Objective Type Questions—Theoretical Questions—Unsolved Examples.
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### 15.1. CARNOT CYCLE

Fig. 15.1 shows a Carnot cycle on  $T$ - $s$  and  $p$ - $V$  diagrams. It consists of (i) two constant pressure operations (4-1) and (2-3) and (ii) two frictionless adiabatics (1-2) and (3-4). These operations are discussed below :

1. **Operation (4-1).** 1 kg of boiling water at temperature  $T_1$  is heated to form wet steam of dryness fraction  $x_1$ . Thus heat is absorbed at constant temperature  $T_1$  and pressure  $p_1$  during this operation.

2. **Operation (1-2).** During this operation steam is expanded isentropically to temperature  $T_2$  and pressure  $p_2$ . The point '2' represents the condition of steam after expansion.

3. **Operation (2-3).** During this operation heat is rejected at constant pressure  $p_2$  and temperature  $T_2$ . As the steam is exhausted it becomes wetter and cooled from 2 to 3.

4. **Operation (3-4).** In this operation the wet steam at '3' is compressed isentropically till the steam regains its original state of temperature  $T_1$  and pressure  $p_1$ . Thus cycle is completed.

Refer  $T$ - $s$  diagram :

Heat supplied at constant temperature  $T_1$  [operation (4-1)] = area 4-1-b-a =  $T_1(s_1 - s_4)$  or  $T_1(s_2 - s_3)$ .

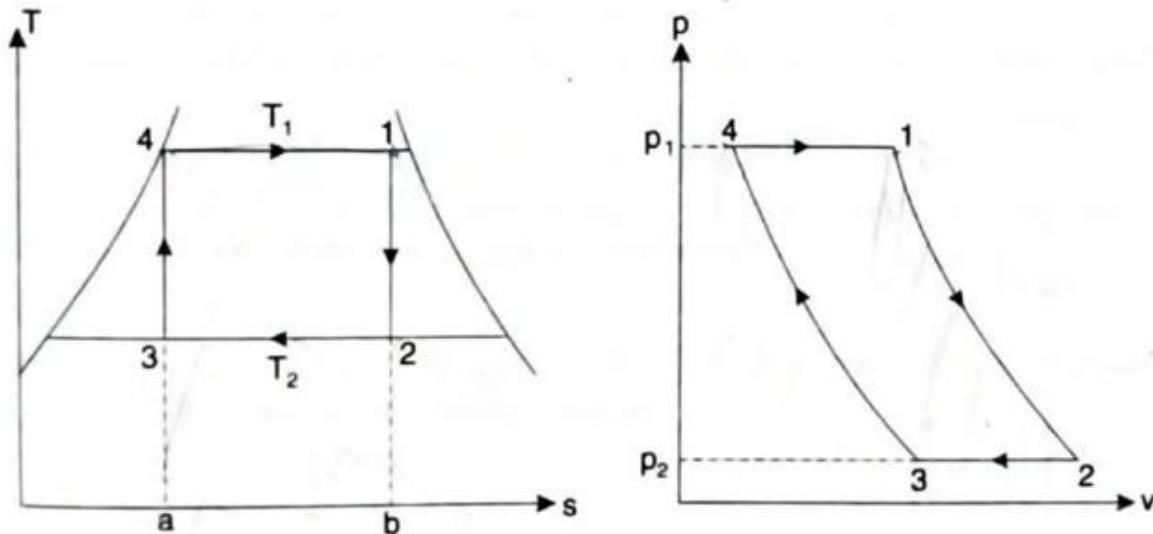


Fig. 15.1. Carnot cycle on  $T$ - $s$  and  $p$ - $V$  diagrams.

Heat rejected at constant temperature  $T_2$  (operation 2-3) = area 2-3-a-b =  $T_2(s_2 - s_3)$ .

Since there is no exchange of heat during isentropic operations (1-2) and (3-4)  
 Net work done = Heat supplied - heat rejected

$$= T_1(s_2 - s_3) - T_2(s_2 - s_3)$$

$$= (T_1 - T_2)(s_2 - s_3).$$

Carnot cycle

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{(T_1 - T_2)(s_2 - s_3)}{T_1(s_2 - s_3)} = \frac{T_1 - T_2}{T_1} \quad \dots(15.1)$$

### Limitations of Carnot cycle

Though Carnot cycle is simple (thermodynamically) and has the *highest thermal efficiency* for given values of  $T_1$  and  $T_2$ , yet it is *extremely difficult to operate in practice* because of the following reasons :

1. It is difficult to compress a wet vapour isentropically to the saturated state as required by the process 3-4.
2. It is difficult to control the quality of the condensate coming out of the condenser so that the state '3' is exactly obtained.
3. The efficiency of the Carnot cycle is greatly affected by the temperature  $T_1$  at which heat is transferred to the working fluid. Since the critical temperature for steam is only  $374^\circ\text{C}$ , therefore, if the cycle is to be operated in the *wet region*, the maximum possible temperature is severely limited.
4. The cycle is still more difficult to operate in practice with superheated steam due to the necessity of supplying the superheat at constant temperature instead of constant pressure (as it is customary).

• In a practical cycle, limits of pressure and volume are far more easily realised than limits of temperature so that at present no practical engine operates on the Carnot cycle, although all modern cycles aspire to achieve it.

## 15.2. RANKINE CYCLE

**Rankine cycle** is the theoretical cycle on which the steam turbine (or engine) works.

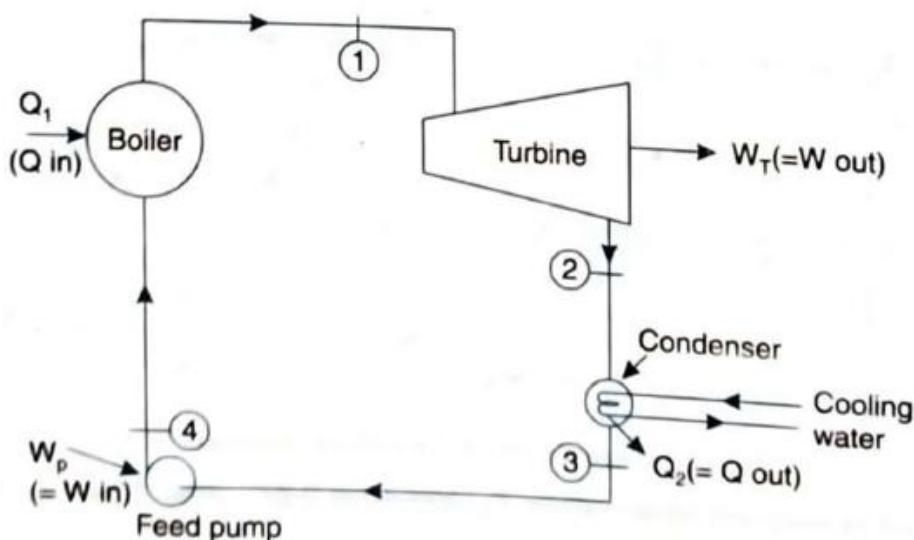
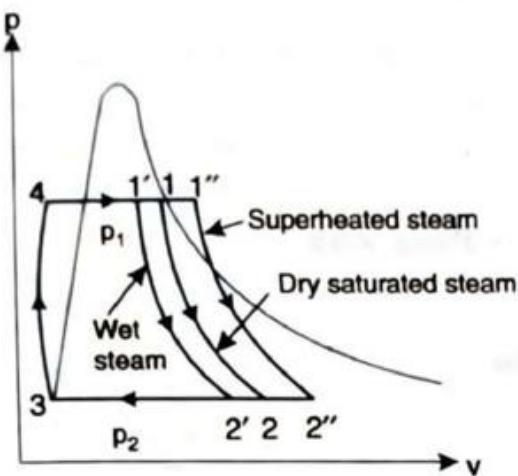
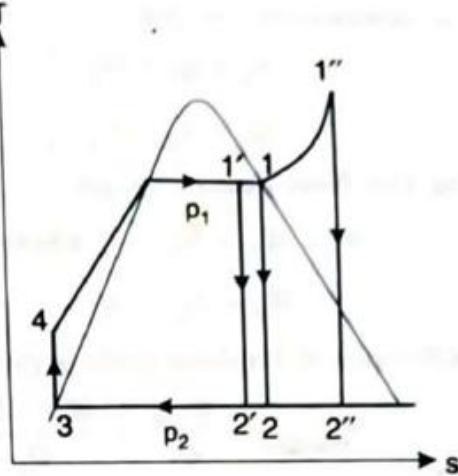


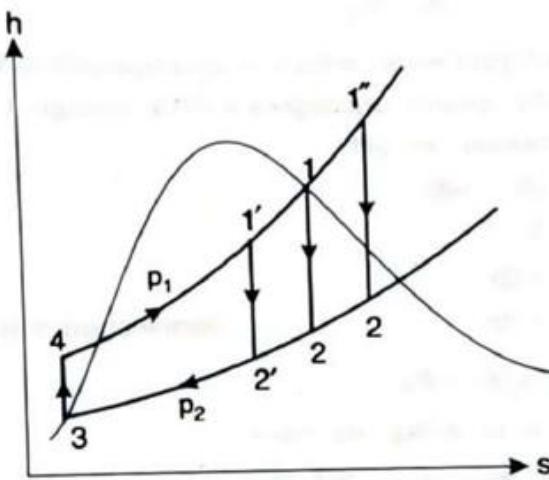
Fig. 15.2. Rankine cycle.



(a)



(b)



(c)

Fig. 15.3. (a)  $p$ - $v$  diagram ; (b)  $T$ - $s$  diagram ; (c)  $h$ - $s$  diagram for Rankine cycle.

The Rankine cycle is shown in Fig. 15.2. It comprises of the following processes :

**Process 1-2** : Reversible adiabatic expansion in the turbine (or steam engine).

**Process 2-3** : Constant-pressure transfer of heat in the condenser.

**Process 3-4** : Reversible adiabatic pumping process in the feed pump.

**Process 4-1** : Constant-pressure transfer of heat in the boiler.

Fig. 15.3 shows the Rankine cycle on  $p$ - $v$ ,  $T$ - $s$  and  $h$ - $s$  diagrams (when the saturated steam enters the turbine, the steam can be wet or superheated also).

**Considering 1 kg of fluid :**

Applying *steady flow energy equation* (S.F.E.E.) to boiler, turbine, condenser and pump :

(i) **For boiler** (as control volume), we get

$$h_{f_4} + Q_1 = h_1 \quad \dots(15.2)$$

$$Q_1 = h_1 - h_{f_4}$$

(ii) **For turbine** (as control volume), we get

$$h_1 = W_T + h_2, \text{ where } W_T = \text{turbine work}$$

$$W_T = h_1 - h_2$$

$$\dots(15.3)$$

(iii) For condenser, we get

$$h_2 = Q_2 + h_{f_3} \quad \dots(15.4)$$

$$Q_2 = h_2 - h_{f_3}$$

(iv) For the feed pump, we get

$$h_{f_3} + W_P = h_{f_4}, \quad \text{where } W_P = \text{Pump work}$$

$$W_P = h_{f_4} - h_{f_3}$$

Now, efficiency of Rankine cycle is given by

$$\begin{aligned}\eta_{\text{Rankine}} &= \frac{W_{\text{net}}}{Q_1} = \frac{W_T - W_P}{Q_1} \\ &= \frac{(h_1 - h_2) - (h_{f_4} - h_{f_3})}{(h_1 - h_{f_3})} \quad \dots(15.5)\end{aligned}$$

The feed pump handles liquid water which is incompressible which means with the increase in pressure its density or specific volume undergoes a little change. Using general property relation for reversible adiabatic compression, we get

$$Tds = dh - vdp$$

$$ds = 0$$

$$dh = vdp$$

or

$$\Delta h = v \Delta p \quad \dots \text{(since change in specific volume is negligible)}$$

or

$$h_{f_4} - h_{f_3} = v_3 (p_1 - p_2)$$

When  $p$  is in bar and  $v$  is in  $\text{m}^3/\text{kg}$ , we have

$$h_{f_4} - h_{f_3} = v_3 (p_1 - p_2) \times 10^5 \text{ J/kg}$$

The feed pump term ( $h_{f_4} - h_{f_3}$ ) being a small quantity in comparison with turbine work,  $W_T$ , is usually neglected, especially when the boiler pressures are low.

$$\text{Then, } \eta_{\text{Rankine}} = \frac{h_1 - h_2}{h_1 - h_{f_4}} \quad \dots[15.5(a)]$$

### Comparison Between Rankine Cycle and Carnot Cycle

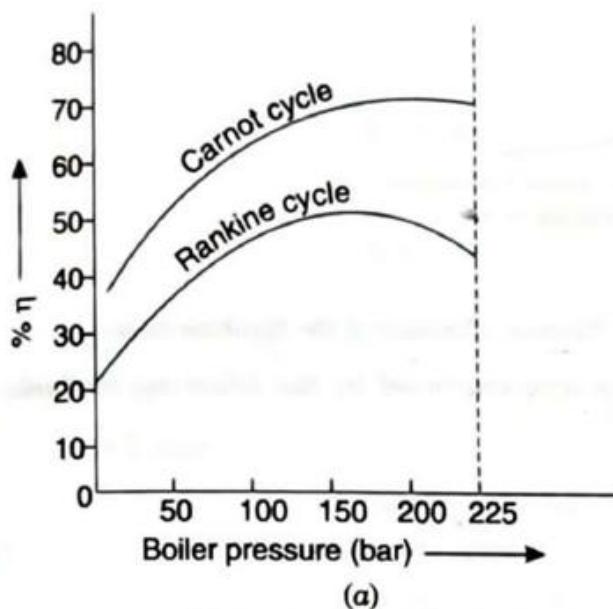
The following points are worth noting :

- (i) Between the same temperature limits Rankine cycle provides a higher specific work output than a Carnot cycle, consequently Rankine cycle requires a smaller steam flow rate resulting in smaller size plant for a given power output. However, Rankine cycle calls for higher rates of heat transfer in boiler and condenser.
  - (ii) Since in Rankine cycle only part of the heat is supplied isothermally at constant higher temperature  $T_1$ , therefore, its efficiency is lower than that of Carnot cycle. The efficiency of the Rankine cycle will approach that of the Carnot cycle more nearly if the superheat temperature rise is reduced.
  - (iii) The advantage of using pump to feed liquid to the boiler instead to compressing a wet vapour is obvious that the work for compression is very large compared to the pump.
- Fig. 15.4 shows the plots between efficiency and specific steam consumption against boiler pressure for Carnot and ideal Rankine cycles.

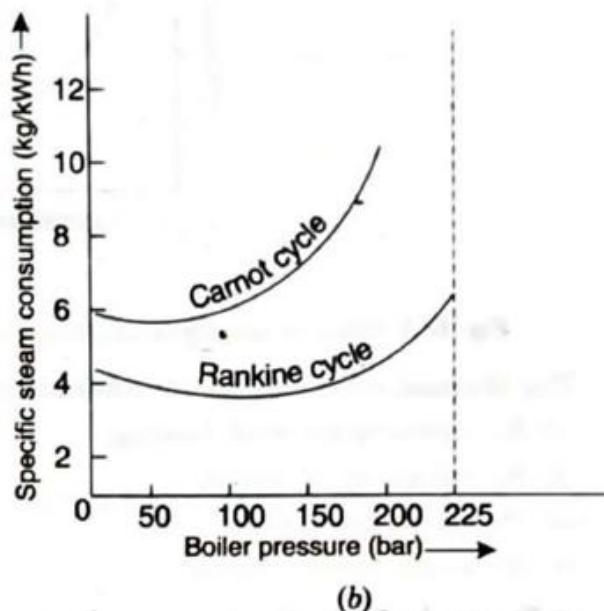
## Effect of Operating Conditions on Rankine Cycle Efficiency

The Rankine cycle efficiency can be *improved* by :

- Increasing the average temperature at which heat is supplied.*
- Decreasing/reducing the temperature at which heat is rejected.*



(a)



(b)

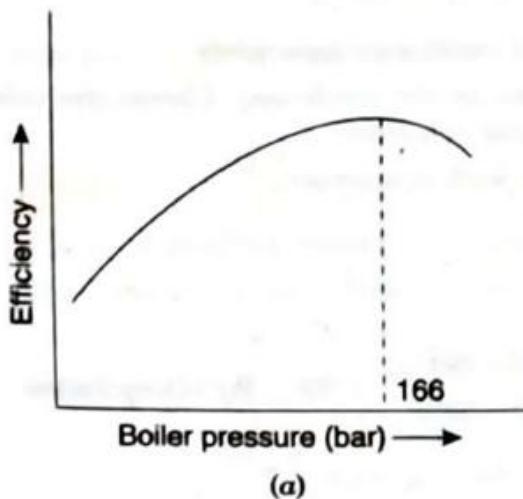
Fig. 15.4

This can be achieved by making suitable changes in the conditions of steam generation or condensation, as discussed below :

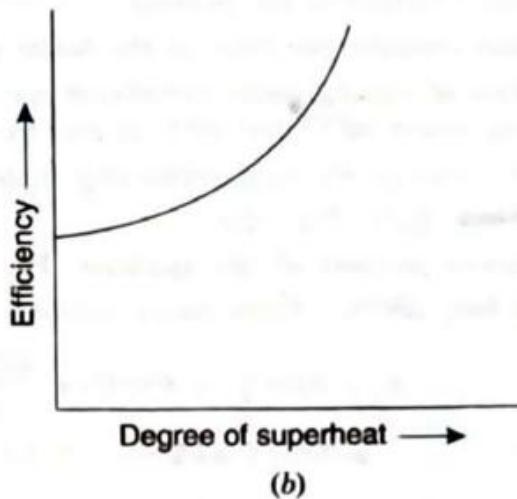
1. **Increasing boiler pressure.** It has been observed that by increasing the boiler pressure (other factors remaining the same) the cycle tends to rise and reaches a maximum value at a boiler pressure of about 166 bar [Fig. 15.5 (a)].

2. **Superheating.** All other factors remaining the same, if the steam is superheated before allowing it to expand the Rankine cycle efficiency may be increased [Fig. 15.5 (b)]. The use of superheated steam also ensures longer turbine blade life because of the absence of erosion from high velocity water particles that are suspended in wet vapour.

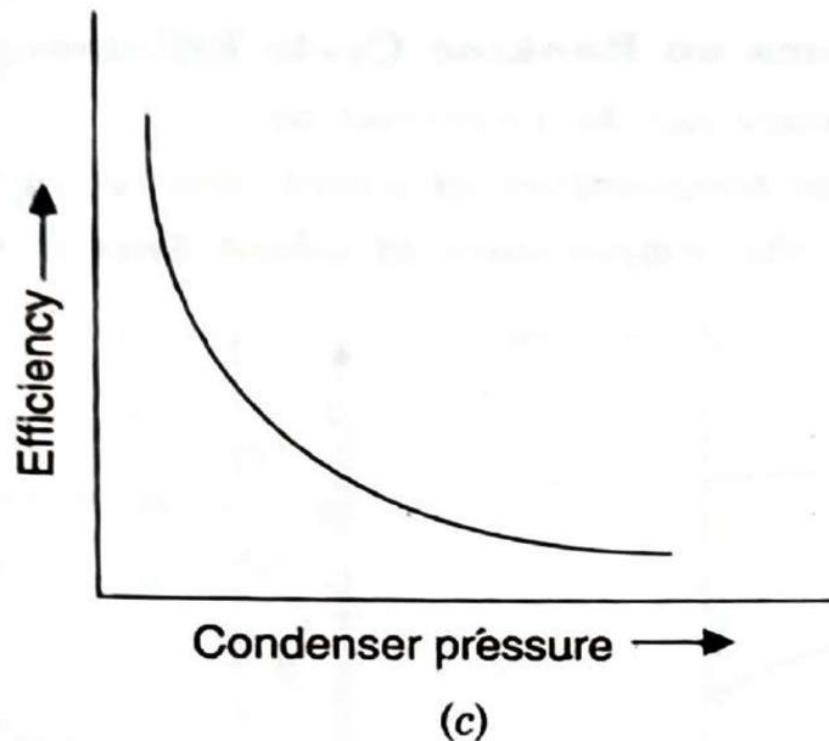
3. **Reducing condenser pressure.** The thermal efficiency of the cycle can be amply improved by reducing the condenser pressure [Fig. 15.5 (c)] (hence by reducing the temperature at which heat is rejected), especially in high vacuums. But the increase in efficiency is obtained at the *increased cost of condensation apparatus*.



(a)



(b)



**Fig. 15.5.** Effect of operating conditions on the thermal efficiency of the Rankine cycle.

The thermal efficiency of the Rankine cycle is also *improved* by the following methods :

- (i) *By regenerative feed heating.*
- (ii) *By reheating of steam.*
- (iii) *By water extraction.*
- (iv) *By using binary-vapour.*

**Example 15.2.** In a steam power cycle, the steam supply is at 15 bar and dry and saturated. The condenser pressure is 0.4 bar. Calculate the Carnot and Rankine efficiencies of the cycle. Neglect pump work.

**Solution.** Steam supply pressure,  $p_1 = 15 \text{ bar}$ ,  $x_1 = 1$

Condenser pressure,  $p_2 = 0.4 \text{ bar}$

**Carnot and Rankine efficiencies :**

From steam tables :

**At 15 bar :**  $t_s = 198.3^\circ\text{C}$ ,  $h_g = 2789.9 \text{ kJ/kg}$ ,  $s_g = 6.4406 \text{ kJ/kg K}$

**At 0.4 bar :**  $t_s = 75.9^\circ\text{C}$ ,  $h_f = 317.7 \text{ kJ/kg}$ ,  $h_{fg} = 2319.2 \text{ kJ/kg}$ ,

$$s_f = 1.0261 \text{ kJ/kg K}, s_{fg} = 6.6448 \text{ kJ/kg K}$$

$$T_1 = 198.3 + 273 = 471.3 \text{ K}$$

$$T_2 = 75.9 + 273 = 348.9 \text{ K}$$

$$\eta_{\text{Carnot}} = \frac{T_1 - T_2}{T_1} = \frac{471.3 - 348.9}{471.3}$$

$$= 0.259 \text{ or } 25.9\%. \text{ (Ans.)}$$

$$\eta_{\text{Rankine}} = \frac{\text{Adiabatic or isentropic heat drop}}{\text{Heat supplied}} = \frac{h_1 - h_2}{h_1 - h_{f_2}}$$

where  $h_2 = h_{f_2} + x_2 h_{fg_2} = 317.7 + x_2 \times 2319.2$

...(i)

**Value of  $x_2$  :**

As the steam expands isentropically,

$$s_1 = s_2$$

$$6.4406 = s_{f_2} + x_2 s_{fg_2} = 1.0261 + x_2 \times 6.6448$$

$$\therefore x_2 = \frac{6.4406 - 1.0261}{6.6448} = 0.815$$

$$h_2 = 317.7 + 0.815 \times 2319.2 = 2207.8 \text{ kJ/kg}$$

[From eqn. (i)]

Hence,  $\eta_{\text{Rankine}} = \frac{2789.9 - 2207.8}{2789.9 - 317.7} = 0.2354 \text{ or } 23.54\%. \text{ (Ans.)}$

**Example 15.3.** In a steam turbine steam at 20 bar, 360°C is expanded to 0.08 bar. It then enters a condenser, where it is condensed to saturated liquid water. The pump feeds back the water into the boiler. Assume ideal processes, find per kg of steam the net work and the cycle efficiency.

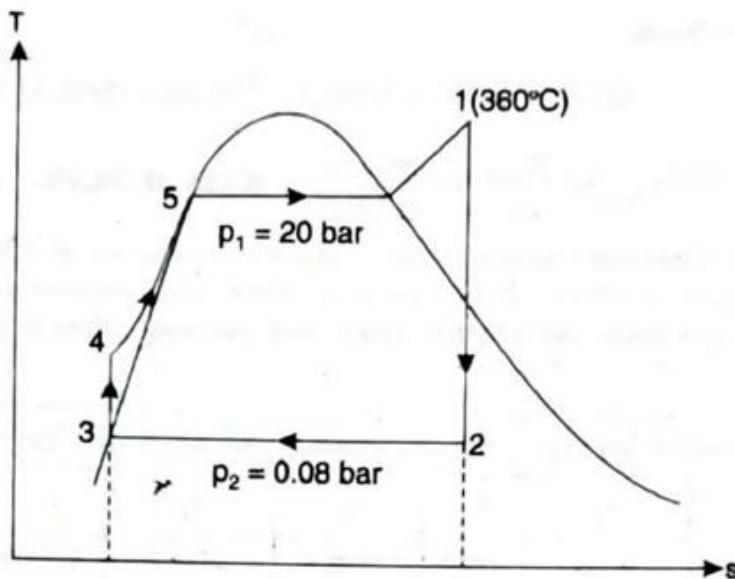


Fig. 15.7

**Solution.** Boiler pressure,

$$p_1 = 20 \text{ bar (360°C)}$$

Condenser pressure,

$$p_2 = 0.08 \text{ bar}$$

From steam tables :

**At 20 bar ( $p_1$ ), 360°C :**

$$h_1 = 3159.3 \text{ kJ/kg}$$

$$s_1 = 6.9917 \text{ kJ/kg K}$$

**At 0.08 bar ( $p_2$ ) :**

$$h_3 = h_{f(p_2)} = 173.88 \text{ kJ/kg},$$

$$s_3 = s_{f(p_2)} = 0.5926 \text{ kJ/kg K}$$

$$h_{fg(p_2)} = 2403.1 \text{ kJ/kg}, \quad s_{g(p_2)} = 8.2287 \text{ kJ/kg K}$$

$$v_{f(p_2)} = 0.001008 \text{ m}^3/\text{kg} \quad \therefore \quad s_{fg(p_2)} = 7.6361 \text{ kJ/kg K}$$

Now

$$s_1 = s_2$$

$$6.9917 = s_{f(p_2)} + x_2 s_{fg(p_2)} = 0.5926 + x_2 \times 7.6361$$

$$\therefore x_2 = \frac{0.69917 - 0.5926}{7.6361} = 0.838$$

$$h_2 = h_{f(p_2)} + x_2 h_{fg(p_2)}$$

$$= 173.88 + 0.838 \times 2403.1 = 2187.68 \text{ kJ/kg.}$$

**Net work,  $W_{\text{net}}$  :**

$$W_{\text{net}} = W_{\text{turbine}} - W_{\text{pump}}$$

$$W_{\text{pump}} = h_{f_1} - h_{f(p_2)} (= h_{f_3}) = v_{f(p_2)} (p_1 - p_2)$$

$$= 0.00108 (\text{m}^3/\text{kg}) \times (20 - 0.08) \times 100 \text{ kN/m}^2$$

$$= 2.008 \text{ kJ/kg}$$

$$\text{[and } h_{f_1} = 2.008 + h_{f(p_1)} = 2.008 + 173.88 = 175.89 \text{ kJ/kg}]$$

$$W_{\text{turbine}} = h_1 - h_2 = 3159.3 - 2187.68 = 971.62 \text{ kJ/kg}$$

$$W_{\text{net}} = 971.62 - 2.008 = 969.61 \text{ kJ/kg. (Ans.)}$$

**Cycle efficiency,  $\eta_{\text{cycle}}$ :**

$$Q_1 = h_1 - h_{f_1} = 3159.3 - 175.89 = 2983.41 \text{ kJ/kg}$$

$$\eta_{\text{cycle}} = \frac{W_{\text{net}}}{Q_1} = \frac{969.61}{2983.41} = 0.325 \text{ or } 32.5\%. \text{ (Ans.)}$$

**Example 15.4.** A Rankine cycle operates between pressures of 80 bar and 0.1 bar. The maximum cycle temperature is 600°C. If the steam turbine and condensate pump efficiencies are 0.9 and 0.8 respectively, calculate the specific work and thermal efficiency. Relevant steam table extract is given below.

$p(\text{bar})$	$t(^{\circ}\text{C})$	Specific volume ( $\text{m}^3/\text{kg}$ )		Specific enthalpy ( $\text{kJ/kg}$ )			Specific entropy ( $\text{kJ/kg K}$ )		
		$v_f$	$v_g$	$h_f$	$h_g$	$h_g$	$s_f$	$s_g$	$s_g$
0.1	45.84	0.0010103	14.68	191.9	2392.3	2584.2	0.6488	7.5006	8.1494
80	295.1	0.001385	0.0235	1317	1440.5	2757.5	3.2073	2.5351	5.7424

80 bar, 600°C	$v$	0.486 $\text{m}^3/\text{kg}$
Superheat table	$h$	3642 $\text{kJ/kg}$
	$s$	7.0206 $\text{kJ/kg K}$

(GATE)

**Solution.** Refer Fig. 15.8

**At 80 bar, 600°C :**

$$h_1 = 3642 \text{ kJ/kg};$$

$$s_1 = 7.0206 \text{ kJ/kg K}.$$

$$\text{Since } s_1 = s_2,$$

$$\therefore 7.0206 = s_{f_1} + x_2 s_{fg_2}$$

$$= 0.6488 + x_2 \times 7.5006$$

$$\text{or } x_2 = \frac{7.0206 - 0.6488}{7.5006} = 0.85$$

$$\text{Now, } h_2 = h_{f_2} + x_2 h_{fg_2}$$

$$= 191.9 + 0.85 \times 2392.3$$

$$= 2225.36 \text{ kJ/kg}$$

**Actual turbine work**

$$= \eta_{\text{turbine}} \times (h_1 - h_2)$$

$$= 0.9 (3642 - 2225.36) = 1275 \text{ kJ/kg}$$

$$\text{Pump work} = v_{f(p_1)} (p_1 - p_2)$$

$$= 0.0010103 (80 - 0.1) \times \frac{10^5}{10^3} \text{ kN/m}^2 = 8.072 \text{ kJ/kg}$$

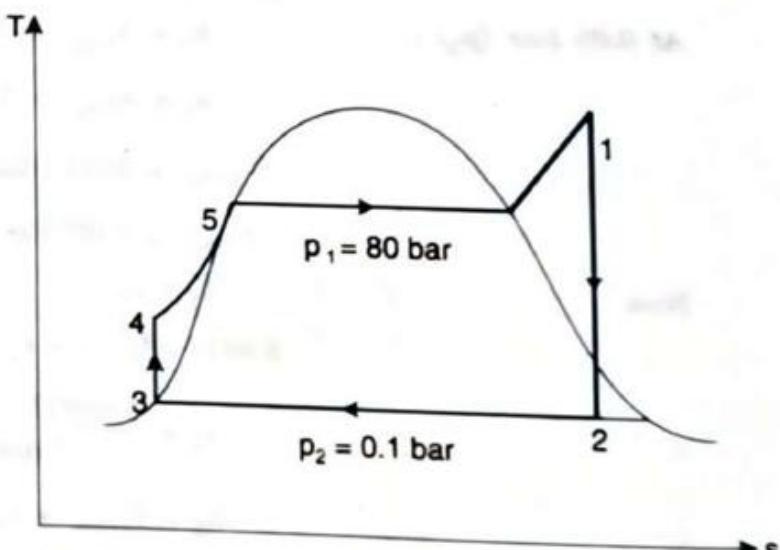


Fig. 15.8

Actual pump work

$$= \frac{8.072}{\eta_{\text{pump}}} = \frac{8.072}{0.8} = 10.09 \text{ kJ/kg}$$

**Specific work**  $(W_{\text{net}}) = 1275 - 10.09 = 1264.91 \text{ kJ/kg. (Ans.)}$

Thermal efficiency

$$= \frac{W_{\text{net}}}{Q_1}$$

where,

$$Q_1 = h_1 - h_{f_4}$$

But  $h_{f_4} = h_{f_3} + \text{pump work} = 191.9 + 10.09 = 202 \text{ kJ/kg}$

$\therefore$  Thermal efficiency,  $\eta_{\text{th}} = \frac{1264.91}{3642 - 202} = 0.368 \text{ or } 36.8 \%. \text{ (Ans.)}$

### 13.4. REGENERATIVE CYCLE

In the Rankine cycle it is observed that the condensate which is fairly at low temperature has an irreversible mixing with hot boiler water and this results in decrease of cycle efficiency. Methods are, therefore, adopted to heat the feed water from the hot well of condenser irreversibly by interchange of heat within the system and thus improving the cycle efficiency. This heating method is called regenerative feed heat and the cycle is called *regenerative cycle*.

The principle of regeneration can be practically utilised by extracting steam from the turbine at several locations and supplying it to the regenerative heaters. The resulting cycle is known as *regenerative or bleeding cycle*. The heating arrangement comprises of : (i) For medium capacity turbines—not more than 3 heaters ; (ii) For high pressure high capacity turbines—not more than 5 to 7 heaters ; and (iii) For turbines of super critical parameters 8 to 9 heaters. The most advantageous condensate heating temperature is selected depending on the turbine throttle conditions and this determines the number of heaters to be used. The final condensate heating temperature is kept 50 to 60°C below the boiler saturated steam temperature so as to prevent evaporation of water in the feed mains following a drop in the boiler drum pressure. The conditions of steam bled for each heater are so selected that the temperature of saturated steam will be 4 to 10°C higher than the final condensate temperature.

Fig. 15.15 (a) shows a diagrammatic layout of a condensing steam power plant in which a surface condenser is used to condense all the steam that is not extracted for feed water heating. The turbine is double extracting and the boiler is equipped with a superheater. The cycle diagram (*T-s*) would appear as shown in Fig. 15.15 (b). This arrangement constitutes a *regenerative cycle*.

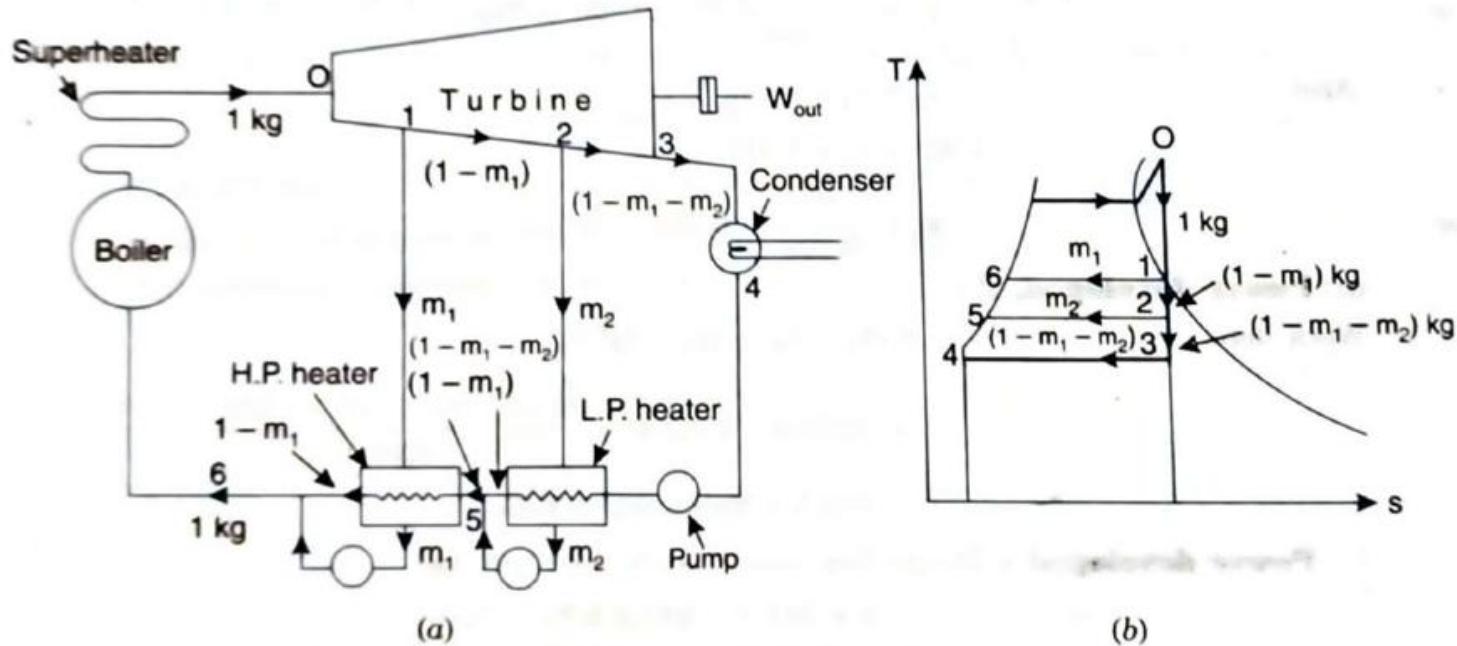


Fig. 15.15. Regenerative cycle.

Let,

$m_1$  = kg of high pressure (H.P.) steam per kg of steam flow,

$m_2$  = kg of low pressure (L.P.) steam extracted per kg of steam flow, and

$(1 - m_1 - m_2)$  = kg of steam entering condenser per kg of steam flow.

**Energy/Heat balance equation for H.P. heater :**

$$m_1(h_1 - h_{f_6}) = (1 - m_1)(h_{f_6} - h_{f_5})$$

or  $m_1[(h_1 - h_{f_6}) + (h_{f_6} - h_{f_5})] = (h_{f_6} - h_{f_5})$

or  $m_1 = \frac{h_{f_6} - h_{f_5}}{h_1 - h_{f_5}}$  ... (15.8)

**Energy/Heat balance equation for L.P. heater :**

$$m_2(h_2 - h_{f_5}) = (1 - m_1 - m_2)(h_{f_5} - h_{f_3})$$

or  $m_2[(h_2 - h_{f_5}) + (h_{f_5} - h_{f_3})] = (1 - m_1)(h_{f_5} - h_{f_3})$

or  $m_2 = \frac{(1 - m_1)(h_{f_5} - h_{f_3})}{(h_2 - h_{f_3})}$  ... (15.9)

All enthalpies may be determined ; therefore  $m_1$  and  $m_2$  may be found. The maximum temperature to which the water can be heated is dictated by that of bled steam. The condensate from the bled steam is added to feed water.

**Neglecting pump work :**

The heat supplied externally in the cycle

$$= (h_0 - h_{f_6})$$

Isentropic work done  $= m_1(h_0 - h_1) + m_2(h_0 - h_2) + (1 - m_1 - m_2)(h_0 - h_3)$

The thermal efficiency of regenerative cycle is

$$\begin{aligned}\eta_{\text{thermal}} &= \frac{\text{Work done}}{\text{Heat supplied}} \\ &= \frac{m_1(h_0 - h_1) + m_2(h_0 - h_2) + (1 - m_1 - m_2)(h_0 - h_3)}{(h_0 - h_{f_6})} \quad \dots (15.10)\end{aligned}$$

[The work done by the turbine may also be calculated by summing up the products of the steam flow and the corresponding heat drop in the turbine stages.

i.e., Work done  $= (h_0 - h_1) + (1 - m_1)(h_1 - h_2) + (1 - m_1 - m_2)(h_2 - h_3)$

**Advantages of Regenerative cycle over Simple Rankine cycle :**

1. The heating process in the boiler tends to become reversible.
2. The thermal stresses set up in the boiler are minimised. This is due to the fact that temperature ranges in the boiler are reduced.
3. The thermal efficiency is improved because the average temperature of heat addition to the cycle is increased.
4. Heat rate is reduced.
5. The blade height is less due to the reduced amount of steam passed through the low pressure stages.
6. Due to many extractions there is an improvement in the turbine drainage and it reduces erosion due to moisture.
7. A small size condenser is required.

**Disadvantages :**

1. The plant becomes more complicated.
2. Because of addition of heaters greater maintenance is required.
3. For given power a large capacity boiler is required.
4. The heaters are costly and the gain in thermal efficiency is not much in comparison to the heavier costs.

**Note.** In the absence of precise information (regarding actual temperature of the feed water entering and leaving the heaters and of the condensate temperatures) the following assumption should always be made while doing calculations :

1. Each heater is ideal and bled steam just condenses.
2. The feed water is heated to saturation temperature at the pressure of bled steam.
3. Unless otherwise stated the work done by the pumps in the system is considered negligible.
4. There is equal temperature rise in all the heaters (usually  $10^{\circ}\text{C}$  to  $15^{\circ}\text{C}$ ).

**Example 15.12.** A steam turbine is fed with steam having an enthalpy of  $3100 \text{ kJ/kg}$ . It moves out of the turbine with an enthalpy of  $2100 \text{ kJ/kg}$ . Feed heating is done at a pressure of 3.2 bar with steam enthalpy of  $2500 \text{ kJ/kg}$ . The condensate from a condenser with an enthalpy of  $125 \text{ kJ/kg}$  enters into the feed heater. The quantity of bled steam is  $11200 \text{ kg/h}$ . Find the power developed by the turbine. Assume that the water leaving the feed heater is saturated liquid at 3.2 bar and the heater is direct mixing type. Neglect pump work.

**Solution.** Arrangement of the components is shown in Fig. 15.16.

At 3.2 bar,

$$h_{f_2} = 570.9 \text{ kJ/kg.}$$

Consider  $m \text{ kg}$  out of  $1 \text{ kg}$  is taken to the feed heater (Fig. 15.16).

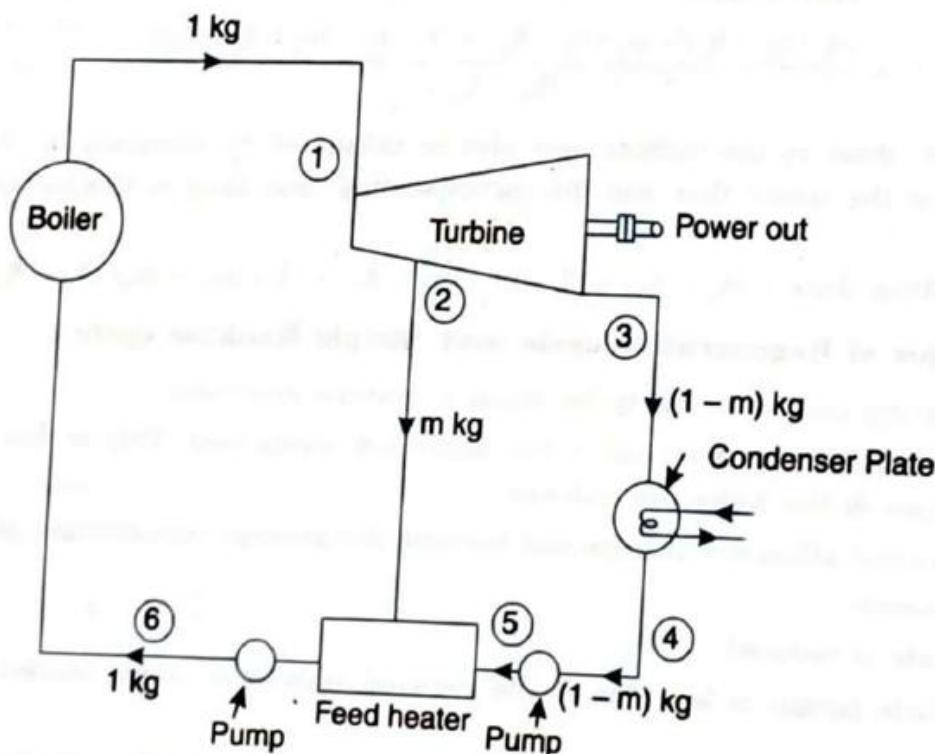


Fig. 15.16

*Energy balance for the feed heater is written as :*

$$m h_2 + (1 - m) h_{f_1} = 1 \times h_{f_2}$$

$$m \times 2100 + (1 - m) \times 125 = 1 \times 570.9$$

$$2100 m + 125 - 125 m = 570.9$$

$$1975 m = 570.9 - 125$$

$$\therefore m = 0.226 \text{ kg per kg of steam supplied to the turbine}$$

*∴ Steam supplied to the turbine per hour*

$$= \frac{11200}{0.226} = 49557.5 \text{ kg/h}$$

*Net work developed per kg of steam*

$$= (h_1 - h_2) + (1 - m) (h_2 - h_3)$$

$$= (3100 - 2500) + (1 - 0.226) (2500 - 2100)$$

$$= 600 + 309.6 = 909.6 \text{ kJ/kg}$$

*∴ Power developed by the turbine*

$$= 909.6 \times \frac{49557.5}{3600} \text{ kJ/s}$$

$$= 12521.5 \text{ kW. (Ans.)}$$

$$(\because 1 \text{ kJ/s} = 1 \text{ kW})$$

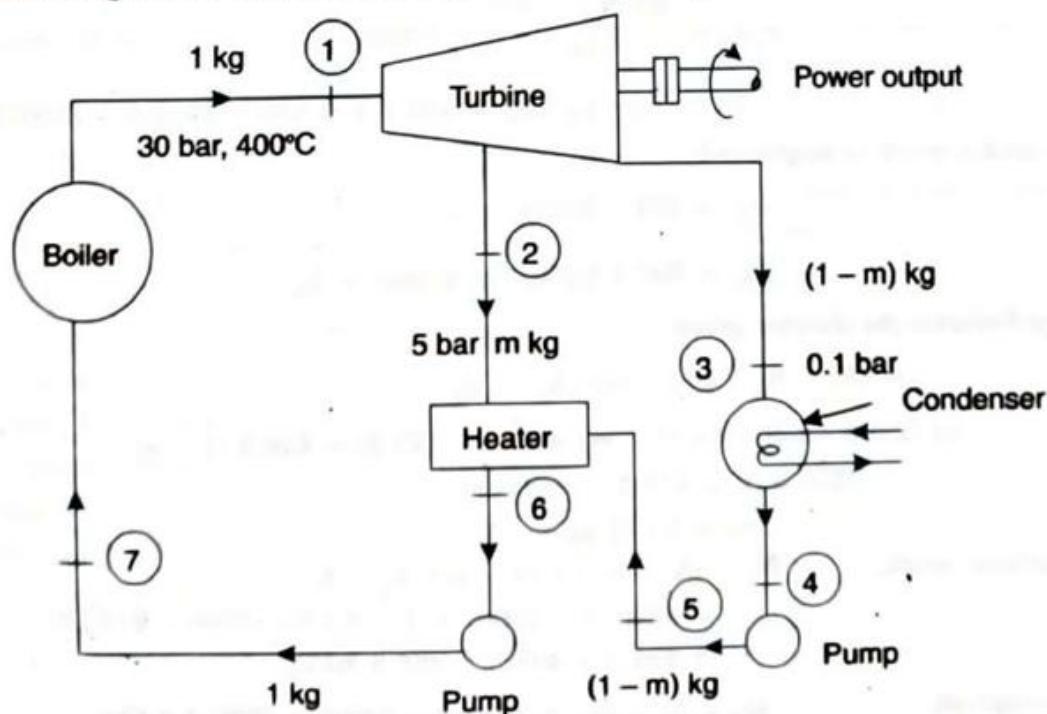
**Example 15.13.** In a single-heater regenerative cycle the steam enters the turbine at 30 bar, 400°C and the exhaust pressure is 0.10 bar. The feed water heater is a direct contact type which operates at 5 bar. Find :

(i) The efficiency and the steam rate of the cycle.

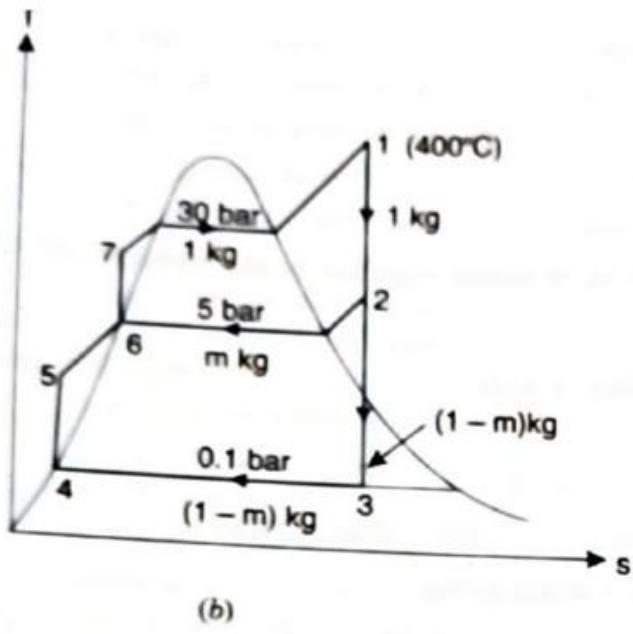
(ii) The increase in mean temperature of heat addition, efficiency and steam rate as compared to the Rankine cycle (without regeneration).

Pump work may be neglected.

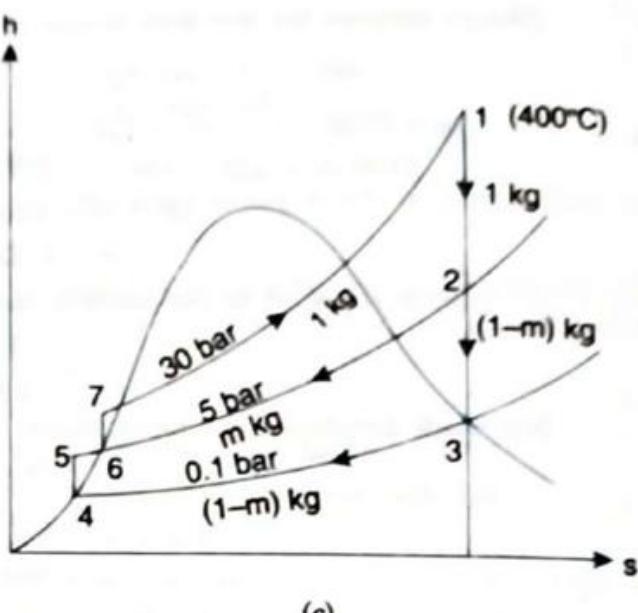
**Solution.** Fig. 15.17 shows the flow, T-s and h-s diagrams.



(a)



(b)



(c)

Fig. 15.17

From steam tables :

**At 30 bar, 400°C :**

$$h_1 = 3230.9 \text{ kJ/kg}, s_1 = 6.921 \text{ kJ/kg} \quad K = s_2 = s_3,$$

**At 5 bar :**

$$s_f = 1.8604, s_g = 6.8192 \text{ kJ/kg} \quad K, h_f = 640.1 \text{ kJ/kg}$$

Since  $s_2 > s_g$ , the state 2 must lie in the superheated region. From the table for superheated steam  $t_2 = 172^\circ\text{C}$ ,  $h_2 = 2796 \text{ kJ/kg}$ .

**At 0.1 bar :**

$$s_f = 0.649, s_{fg} = 7.501, h_f = 191.8, h_{fg} = 2392.8$$

Now,

$$s_2 = s_3$$

$$6.921 = s_{f_3} + x_3 s_{fg_3} = 0.649 + x_3 \times 7.501$$

$$x_3 = \frac{6.921 - 0.649}{7.501} = 0.836$$

$$h_3 = h_{f_3} + x_3 h_{fg_3} = 191.8 + 0.836 \times 2392.8 = 2192.2 \text{ kJ/kg}$$

Since pump work is neglected

$$h_{f_4} = 191.8 \text{ kJ/kg} = h_{f_5}$$

$$h_{f_6} = 640.1 \text{ kJ/kg} \text{ (at 5 bar)} = h_{f_7}$$

Energy balance for heater gives

$$m(h_2 - h_{f_6}) = (1-m)(h_{f_6} - h_{f_5})$$

$$m(2796 - 640.1) = (1-m)(640.1 - 191.8) = 448.3(1-m)$$

$$2155.9m = 448.3 - 448.3m$$

$$m = 0.172 \text{ kg}$$

Turbine work,

$$W_T = (h_1 - h_2) + (1-m)(h_2 - h_3)$$

$$= (3230.9 - 2796) + (1 - 0.172)(2796 - 2192.2)$$

$$= 434.9 + 499.9 = 934.8 \text{ kJ/kg}$$

Heat supplied,

$$Q_1 = h_1 - h_{f_6} = 3230.9 - 640.1 = 2590.8 \text{ kJ/kg.}$$

(i) Efficiency of cycle,  $\eta_{\text{cycle}}$  :

$$\eta_{\text{cycle}} = \frac{W_T}{Q_1} = \frac{934.8}{2590.8} = 0.3608 \text{ or } 36.08\%. \quad (\text{Ans.})$$

$$\text{Steam rate} = \frac{3600}{934.8} = 3.85 \text{ kg/kWh.} \quad (\text{Ans.})$$

(ii)  $T_{m_1} = \frac{h_1 - h_{f_7}}{s_1 - s_7} = \frac{2590.8}{6.921 - 1.8604} = 511.9 \text{ K} = 238.9^\circ\text{C.}$

$T_{m_1}$  (without regeneration)

$$= \frac{h_1 - h_{f_4}}{s_1 - s_4} = \frac{3230.9 - 1918}{6.921 - 0.649} = \frac{3039.1}{6.272} = 484.5 \text{ K} = 211.5^\circ\text{C.}$$

Increase in  $T_{m_1}$  due to regeneration

$$= 238.9 - 211.5 = 27.4^\circ\text{C.} \quad (\text{Ans.})$$

$W_T$  (without regeneration)

$$= h_1 - h_3 = 3230.9 - 2192.2 = 1038.7 \text{ kJ/kg}$$

Steam rate without regeneration

$$= \frac{3600}{1038.7} = 3.46 \text{ kg/kWh}$$

∴ Increase in steam rate due to regeneration

$$= 3.85 - 3.46 = 0.39 \text{ kg/kWh.} \quad (\text{Ans.})$$

$$\eta_{\text{cycle}} \text{ (without regeneration)} = \frac{h_1 - h_3}{h_1 - h_{f_4}} = \frac{1038.7}{3230.9 - 1918} = 0.3418 \text{ or } 34.18\%. \quad (\text{Ans.})$$

Increase in cycle efficiency due to regeneration

## 15.5. REHEAT CYCLE

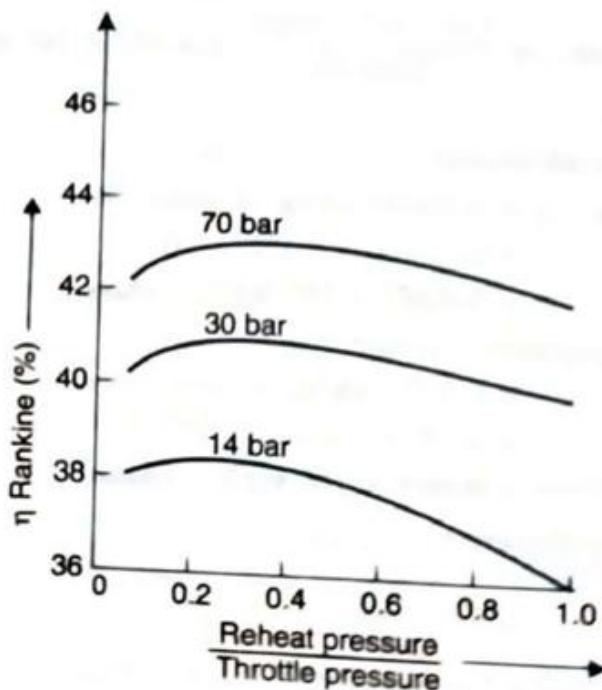
For attaining greater thermal efficiencies when the initial pressure of steam was raised beyond 42 bar it was found that resulting condition of steam after expansion was increasingly wetter and exceeded in the safe limit of 12 per cent condensation. It, therefore, became necessary to *reheat* the steam after part of expansion was over so that the resulting condition after complete expansion fell within the region of permissible wetness.

The reheating or resuperheating of steam is now universally used when high pressure and temperature steam conditions such as 100 to 250 bar and 500°C to 600°C are employed for throttle. For plants of still higher pressures and temperatures, a double reheating may be used.

In actual practice reheat improves the cycle efficiency by about 5% for a 85/15 bar cycle. A second reheat will give a much less gain while the initial cost involved would be so high as to prohibit use of two stage reheat except in case of very high initial throttle conditions. The cost of reheat equipment consisting of boiler, piping and controls may be 5% to 10% more than that of the conventional boilers and this additional expenditure is justified only if gain in thermal efficiency is sufficient to promise a return of this investment. Usually a plant with a base load capacity of 50000 kW and initial steam pressure of 42 bar would economically justify the extra cost of reheating.

The improvement in thermal efficiency due to reheat is greatly dependent upon the reheat pressure with respect to the original pressure of steam.

Fig. 15.23 shows the reheat pressure selection on cycle efficiency.



Condenser pressure : 12.7 mm Hg  
Temperature of throttle and heat : 427°C  
Fig. 15.23. Effect of reheat pressure selection on cycle

Fig. 15.24 shows a schematic diagram of a theoretical single-stage reheat cycle. The corresponding representation of ideal reheating process on  $T$ - $s$  and  $h$ - $s$  chart is shown in Fig. 15.22 (a and b).

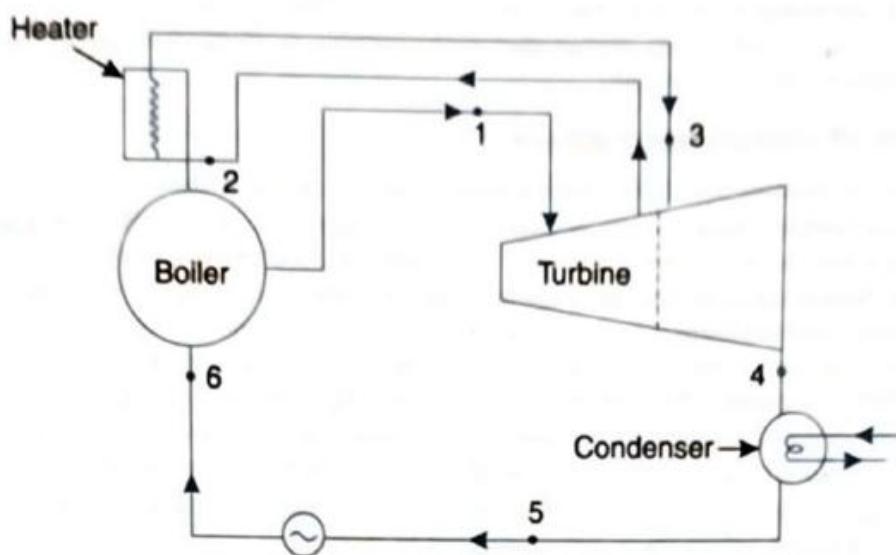


Fig. 15.24. Reheat cycle.

Refer Fig. 15.24, (a). 5-1 shows the formation of steam in the boiler. The steam as at state point 1 (i.e., pressure  $p_1$  and temperature  $T_1$ ) enters the turbine and expands isentropically to a certain pressure  $p_2$  and temperature  $T_2$ . From this state point 2 the whole of steam is drawn out of the turbine and is reheated in a reheat器 to a temperature  $T_3$ . (Although there is an *optimum pressure* at which the steam should be removed for reheating, if the highest return is to be obtained, yet, for simplicity, the whole steam is removed from the high pressure exhaust, where the pressure is about *one-fifth* of boiler pressure, and after undergoing a 10% pressure drop, in circulating through the heater, it is returned to intermediate pressure or low pressure turbine). This reheated steam is then readmitted to the turbine where it is expanded to condenser pressure isentropically.

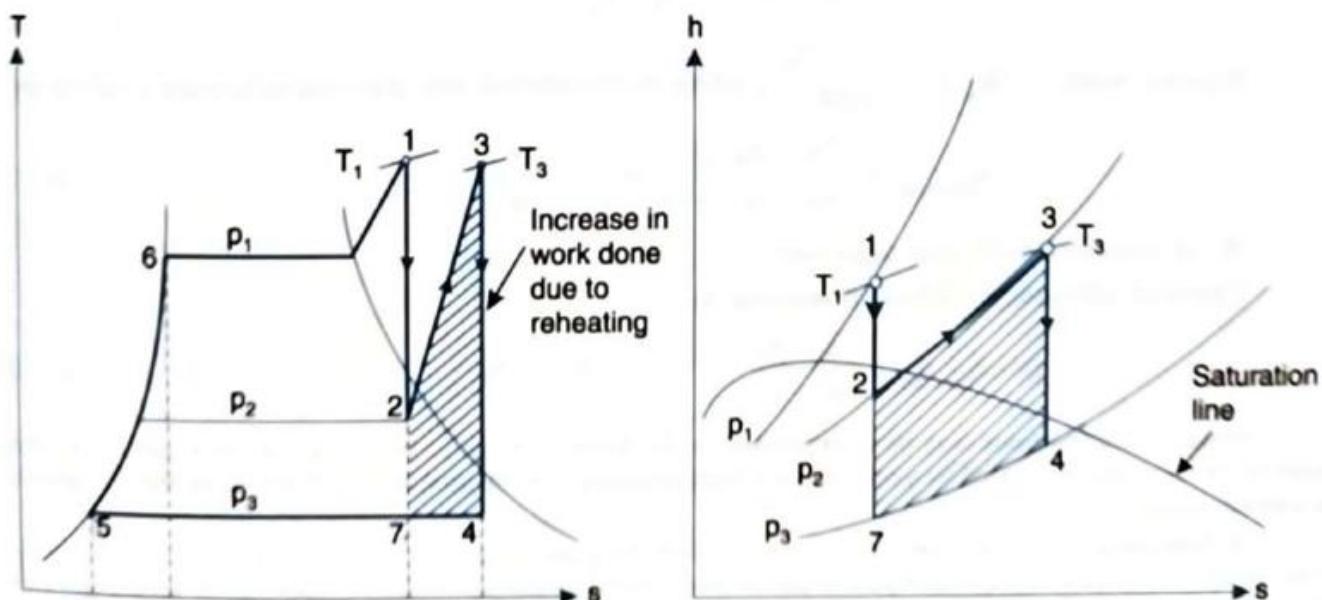


Fig. 15.25. Ideal reheating process on  $T$ - $s$  and  $h$ - $s$  chart.

### Advantages of 'Reheating' :

1. There is an increased output of the turbine.
2. Erosion and corrosion problems in the steam turbine are eliminated/avoided.
3. There is an improvement in the thermal efficiency of the turbines.
4. Final dryness fraction of steam is improved.
5. There is an increase in the nozzle and blade efficiencies.

### Disadvantages :

1. Reheating requires more maintenance.
2. The increase in thermal efficiency is not appreciable in comparison to the expenditure incurred in reheating.

**Example 15.18.** Steam at a pressure of 15 bar and 250°C is expanded through a turbine at first to a pressure of 4 bar. It is then reheated at constant pressure to the initial temperature of 250°C and is finally expanded to 0.1 bar. Using Mollier chart, estimate the work done per kg of steam flowing through the turbine and amount of heat supplied during the process of reheat. Compare the work output when the expansion is direct from 15 bar to 0.1 bar without any reheat. Assume all expansion processes to be isentropic.

**Solution.** Refer Fig. 15.26.

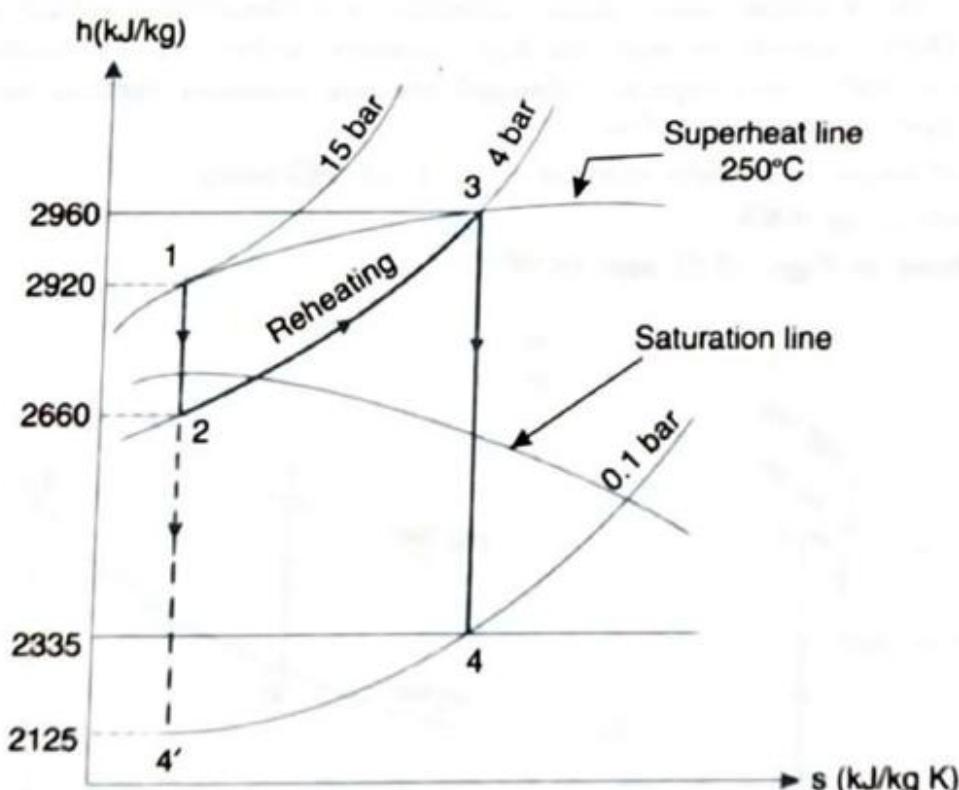


Fig. 15.26

Pressure,

$$p_1 = 15 \text{ bar} ;$$

$$p_2 = 4 \text{ bar} ;$$

$$p_4 = 0.1 \text{ bar}.$$

Work done per kg of steam,

$$W = \text{Total heat drop}$$

$$= [ (h_1 - h_2) + (h_3 - h_4) ] \text{ kJ/kg}$$

... (i)

Amount of heat supplied during process of reheat,

$$h_{\text{reheat}} = (h_3 - h_2) \text{ kJ/kg} \quad \dots(ii)$$

From Mollier diagram or *h-s* chart,

$$h_1 = 2920 \text{ kJ/kg}, h_4 = 2660 \text{ kJ/kg}$$

$$h_3 = 2960 \text{ kJ/kg}, h_4 = 2335 \text{ kJ/kg}$$

Now, by putting the values in eqns. (i) and (ii), we get

$$W = (2920 - 2660) + (2960 - 2335)$$

$$= 885 \text{ kJ/kg. (Ans.)}$$

Hence work done per kg of steam = 885 kJ/kg. (Ans.)

Amount of heat supplied during reheat,

$$h_{\text{reheat}} = (2960 - 2660) = 300 \text{ kJ/kg. (Ans.)}$$

If the expansion would have been continuous without reheating i.e., 1 to 4', the work output is given by

$$W_1 = h_1 - h_4'$$

From Mollier diagram,

$$h_{4'} = 2125 \text{ kJ/kg}$$

$$W_1 = 2920 - 2125 = 795 \text{ kJ/kg. (Ans.)}$$

**Example 15.19.** A steam power plant operates on a theoretical reheat cycle. Steam at boiler at 150 bar, 550°C expands through the high pressure turbine. It is reheated at a constant pressure of 40 bar to 550°C and expands through the low pressure turbine to a condenser at 0.1 bar. Draw *T-s* and *h-s* diagrams. Find :

(i) Quality of steam at turbine exhaust ; (ii) Cycle efficiency ;

(iii) Steam rate in kg/kWh.

(AMIE)

Solution. Refer to Figs. 15.27 and 15.28

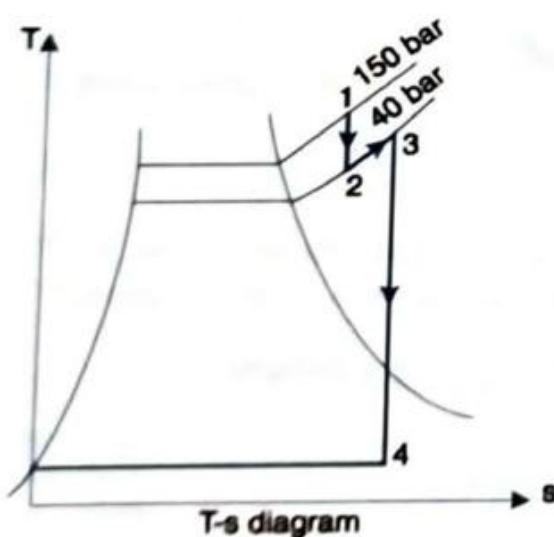


Fig. 15.27

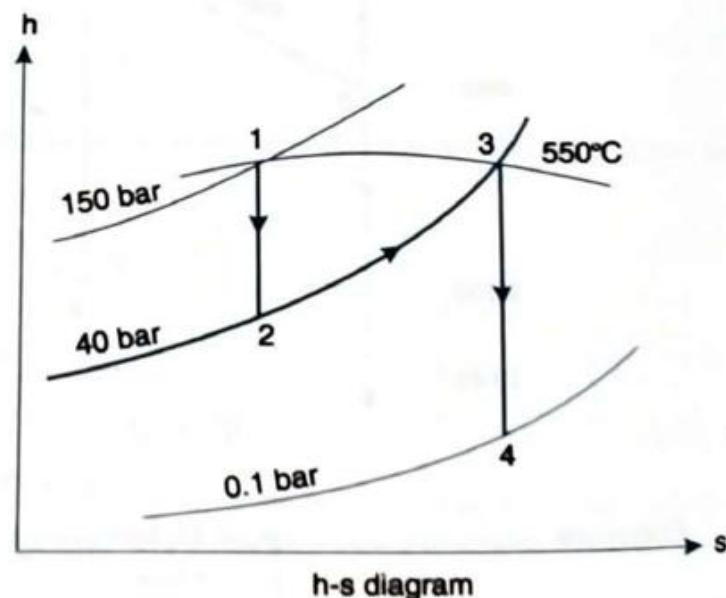


Fig. 15.28

From Mollier diagram (*h-s* diagram) :

$$h_1 = 3450 \text{ kJ/kg}; h_2 = 3050 \text{ kJ/kg}; h_3 = 3560 \text{ kJ/kg}; h_4 = 2300 \text{ kJ/kg}$$

$$h_{f_4} \text{ (from steam tables, at 0.1 bar)} = 191.8 \text{ kJ/kg}$$

(i) **Quality of steam at turbine exhaust,  $x_4$  :**

$$x_4 = 0.88 \text{ (From Mollier diagram)}$$

(ii) **Cycle efficiency,  $\eta_{\text{cycle}}$  :**

$$\eta_{\text{cycle}} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_{f_4}) + (h_3 - h_2)}$$

$$= \frac{(3450 - 3050) + (3560 - 2300)}{(3450 - 191.8) + (3560 - 3050)} = \frac{1660}{3768.2} = 0.4405 \text{ or } 44.05\%. \quad (\text{Ans.})$$

(iii) **Steam rate in kg/kWh :**

$$\text{Steam rate} = \frac{3600}{(h_1 - h_2) + (h_3 - h_4)} = \frac{3600}{(3450 - 3050) + (3560 - 2300)}$$

$$= \frac{3600}{1660} = 2.17 \text{ kg/kWh.} \quad (\text{Ans.})$$

Amount of heat suppl

From Mollier diagram

Now, by putting the v

# Gas Turbines

25.1. Gas turbines—general aspects. 25.2. Classification of gas turbines. 25.3. Merits of gas turbines. 25.4. Constant pressure combustion gas turbines—Open cycle gas turbines—Methods for improvement of thermal efficiency of open cycle gas turbine plant—Effect of operating variables on thermal efficiency—Closed cycle gas turbine—Merits and demerits of closed cycle gas turbine over open cycle gas turbine. 25.5. Constant volume combustion turbines. 25.6. Uses of gas turbines. 25.7. Gas turbine fuels. 25.8. Jet propulsion—Turbo-jet—Turbo-prop—Ram-jet—Pulse-jet engine—Rocket engines—Highlights—Objective Type Questions—Theoretical Questions—Unsolved Examples.

## 25.1. GAS TURBINES—GENERAL ASPECTS

Probably a wind-mill was the first turbine to produce useful work, wherein there is no pre-compression and no combustion. The characteristic features of a gas turbine as we think of the name today include a *compression process* and a *heat addition* (or combustion) *process*. The gas turbine represents perhaps the most satisfactory way of producing very large quantities of power in a self-contained and compact unit. The gas turbine may have a future use in conjunction with the oil engine. For smaller gas turbine units, the inefficiencies in compression and expansion processes become greater and to improve the thermal efficiency it is necessary to use a heat exchanger. In order that a small gas turbine may compete for economy with the small oil engine or petrol engine it is necessary that a compact effective heat exchanger be used in the gas turbine cycle. The thermal efficiency of the gas turbine alone is still quite modest 20 to 30% compared with that of a modern steam turbine plant 38 to 40%. It is possible to construct combined plants whose efficiencies are of order of 45% or more. Higher efficiencies might be attained in future.

The following are the major fields of application of gas turbines :

- |                         |                       |
|-------------------------|-----------------------|
| 1. Aviation             | 2. Power generation   |
| 3. Oil and gas industry | 4. Marine propulsion. |

The efficiency of a gas turbine is not the criteria for the choice of this plant. A gas turbine is used in aviation and marine fields because it is *self contained, light weight not requiring cooling water and generally fit into the overall shape of the structure*. It is selected for power generation because of its *simplicity, lack of cooling water, needs quick installation and quick starting*. It is used in oil and gas industry because of *cheaper supply of fuel and low installation cost*.

The gas turbines have the following limitations : (i) *They are not self starting ;* (ii) *low efficiencies at part loads ;* (iii) *non-reversibility ;* (iv) *higher rotor speeds and* (v) *overall efficiency of the plant low.*

## 25.2. CLASSIFICATION OF GAS TURBINES

The gas turbines are mainly divided into two groups :

1. **Constant pressure combustion gas turbine**
  - (a) Open cycle constant pressure gas turbine
  - (b) Closed cycle constant pressure gas turbine.

## 2. Constant volume combustion gas turbine

In almost all the fields open cycle gas turbine plants are used. Closed cycle plants were introduced at one stage because of their ability to burn cheap fuel. In between their progress remained slow because of availability of cheap oil and natural gas. Because of rising oil prices, now again, the attention is being paid to closed cycle plants.

### MERITS OF GAS TURBINES

#### (i) Merits over I.C. engines :

1. The mechanical efficiency of a gas turbine (95%) is quite high as compared with I.C. engine (85%) since the I.C. engine has a large number of sliding parts.
2. A gas turbine does not require a flywheel as the torque on the shaft is continuous and uniform. Whereas a flywheel is a must in case of an I.C. engine.
3. The weight of gas turbine per H.P. developed is less than that of an I.C. engine.
4. The gas turbine can be driven at a very high speeds (40000 r.p.m.) whereas this is not possible with I.C. engines.
5. The work developed by a gas turbine per kg of air is more as compared to an I.C. engine. This is due to the fact that gases can be expanded upto atmospheric pressure in case of a gas turbine whereas in an I.C. engine expansion upto atmospheric pressure is not possible.
6. The components of the gas turbine can be made lighter since the pressures used in it are very low, say 5 bar compared with I.C. engine, say 60 bar.
7. In the gas turbine the ignition and lubrication systems are much simpler as compared with I.C. engines.
8. Cheaper fuels such as paraffine type, residue oils or powdered coal can be used whereas special grade fuels are employed in petrol engine to check knocking or pinking.
9. The exhaust from gas turbine is less polluting comparatively since excess air is used for combustion.
10. Because of low specific weight the gas turbines are particularly suitable for use in aircrafts.

#### Demerits of gas turbines

1. The thermal efficiency of a simple turbine cycle is low (15 to 20%) as compared with I.C. engines (25 to 30%).
2. With wide operating speeds the fuel control is comparatively difficult.
3. Due to higher operating speeds of the turbine, it is imperative to have a speed reduction device.
4. It is difficult to start a gas turbine as compared to an I.C. engine.
5. The gas turbine blades need a special cooling system.
6. One of the main demerits of a gas turbine is its *very poor thermal efficiency at part loads*, as the quantity of air remains same irrespective of load, and output is reduced by reducing the quantity of fuel supplied.
7. Owing to the use of nickel-chromium alloy, the manufacture of the blades is difficult and costly.
8. For the same output the gas turbine produces five times exhaust gases than I.C. engine.
9. Because of prevalence of high temperature (1000 K for blades and 2500 K for combustion chamber) and centrifugal force the life of the combustion chamber and blades is short/small.

- 1-2 represents : Ideal isentropic compression.
- 3-4 represents : Ideal isentropic expansion.

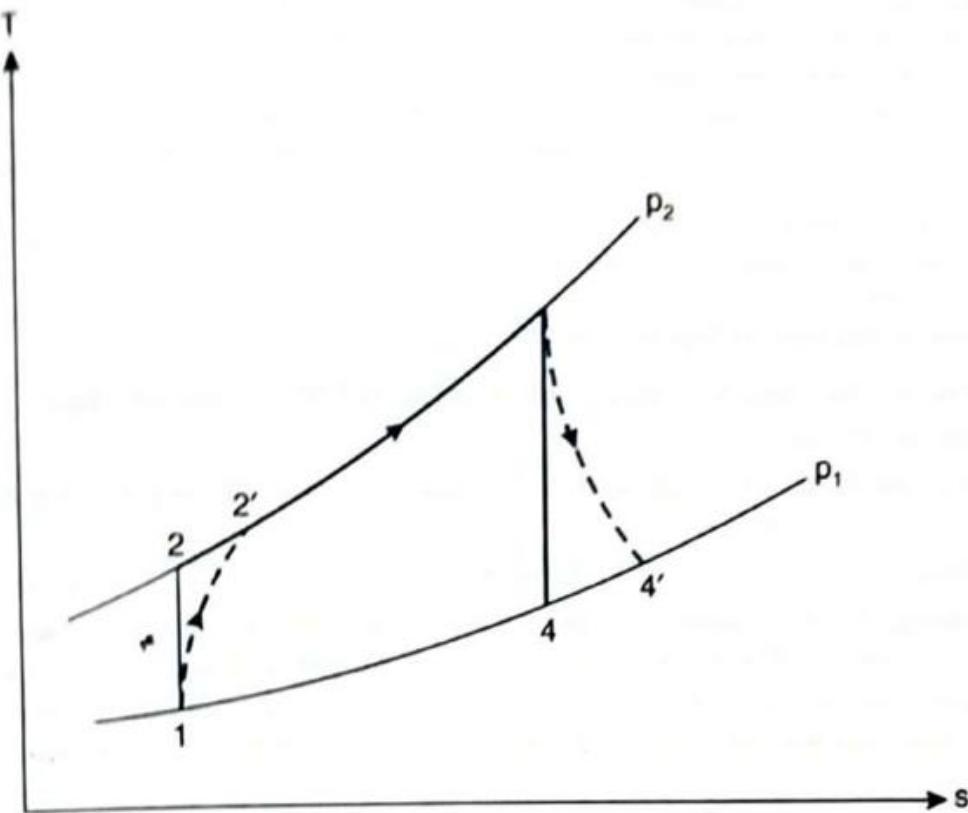


Fig. 25.2

Assuming change in kinetic energy between the various points in the cycle to be negligibly small compared with enthalpy changes and then applying the flow equation to each part of cycle, for unit mass, we have

$$\begin{aligned} \text{Work input (compressor)} &= c_p (T_2' - T_1) \\ \text{Heat supplied (combustion chamber)} &= c_p (T_3 - T_2') \\ \text{Work output (turbine)} &= c_p (T_3 - T_4') \\ \therefore \text{Network output} &= \text{Work output} - \text{Work input} \\ &= c_p (T_3 - T_4') - c_p (T_2' - T_1) \end{aligned}$$

and

$$\begin{aligned} \eta_{\text{thermal}} &= \frac{\text{Network output}}{\text{Heat supplied}} \\ &= \frac{c_p (T_3 - T_4') - c_p (T_2' - T_1)}{c_p (T_3 - T_2')} \end{aligned}$$

Compressor isentropic efficiency,  $\eta_{\text{comp}}$

$$\begin{aligned} &= \frac{\text{Work input required in isentropic compression}}{\text{Actual work required}} \\ &= \frac{c_p (T_2 - T_1)}{c_p (T_2' - T_1)} = \frac{T_2 - T_1}{T_2' - T_1} \quad \dots(25.1) \end{aligned}$$

Turbine isentropic efficiency,  $\eta_{\text{turbine}}$

$$= \frac{\text{Actual work output}}{\text{Isentropic work output}}$$

The ideal cycle for this arrangement is 1-2-3-4-5-6 ; the compression process without intercooling is shown as 1-L' in the actual case, and 1-L in the ideal isentropic case.

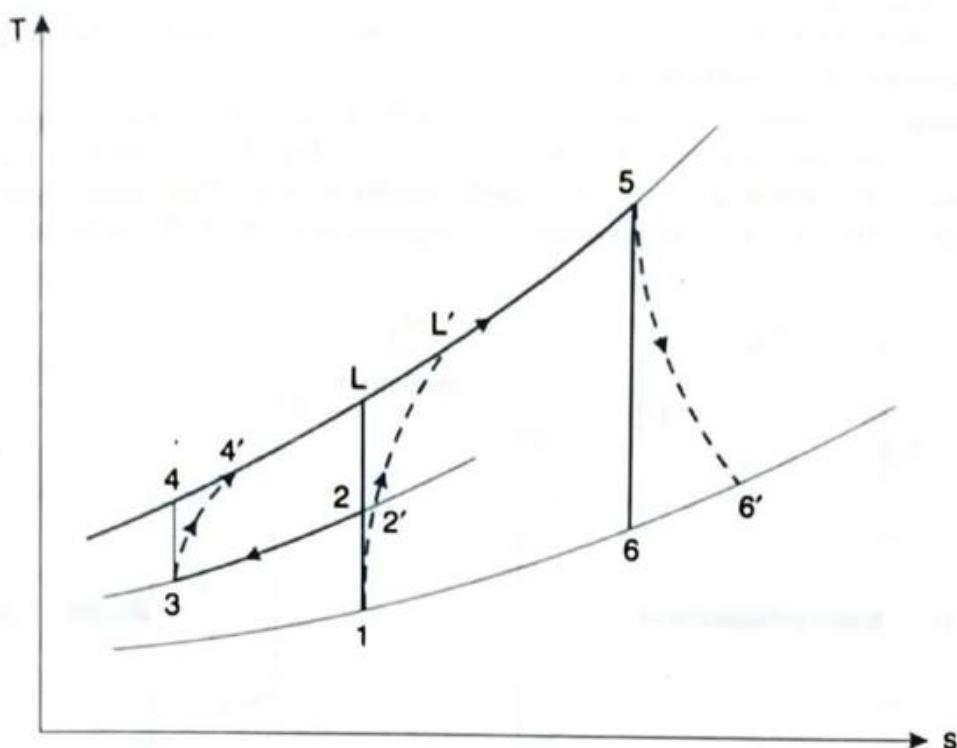


Fig. 25.4. T-s diagram for the unit.

Now,

Work input (*with intercooling*)

$$= c_p(T_2' - T_1) + c_p(T_4' - T_3) \quad \dots(25.3)$$

Work input (*without intercooling*)

$$= c_p(T_L' - T_1) = c_p(T_2' - T_1) + c_p(T_L' - T_2') \quad \dots(25.4)$$

By comparing equation (25.4) with equation (25.3) it can be observed that the *work input with intercooling is less than the work input with no intercooling*, when  $c_p(T_4' - T_3)$  is less than  $c_p(T_L' - T_2')$ . This is so if it is assumed that isentropic efficiencies of the two compressors, operating separately, are each equal to the isentropic efficiency of the single compressor which would be required if no intercooling were used. Then  $(T_4' - T_3) < (T_L' - T_2')$  since the pressure lines diverge on the T-s diagram from left to the right.

Again, work ratio

$$\begin{aligned} &= \frac{\text{Network output}}{\text{Gross work output}} \\ &= \frac{\text{Work of expansion} - \text{Work of compression}}{\text{Work of expansion}} \end{aligned}$$

From this we may conclude that *when the compressor work input is reduced then the work ratio is increased*.

However, the heat supplied in the combustion chamber when intercooling is used in the cycle, is given by,

$$\text{Heat supplied with intercooling} = c_p(T_5 - T_4')$$

Also the heat supplied when intercooling is not used, with the same maximum cycle temperature  $T_5$ , is given by

$$\text{Heat supplied without intercooling} = c_p(T_5 - T_{L'})$$

Thus, the heat supplied when intercooling is used is greater than with no intercooling. Although the network output is increased by intercooling it is found in general that the increase in heat to be supplied causes the thermal efficiency to decrease. When intercooling is used supply of cooling water must be readily available. The additional bulk of the unit may offset the advantage to be gained by increasing the work ratio.

**2. Reheating.** The output of a gas turbine can be amply improved by expanding the gas in two stages with a *reheater* between the two as shown in Fig. 25.5. The H.P. turbine drives the compressor and the L.P. turbine provides the useful power output. The corresponding *T-s* diagram is shown in Fig. 25.6. The line 4'-L' represents the expansion in the L.P. turbine if reheating is not employed.

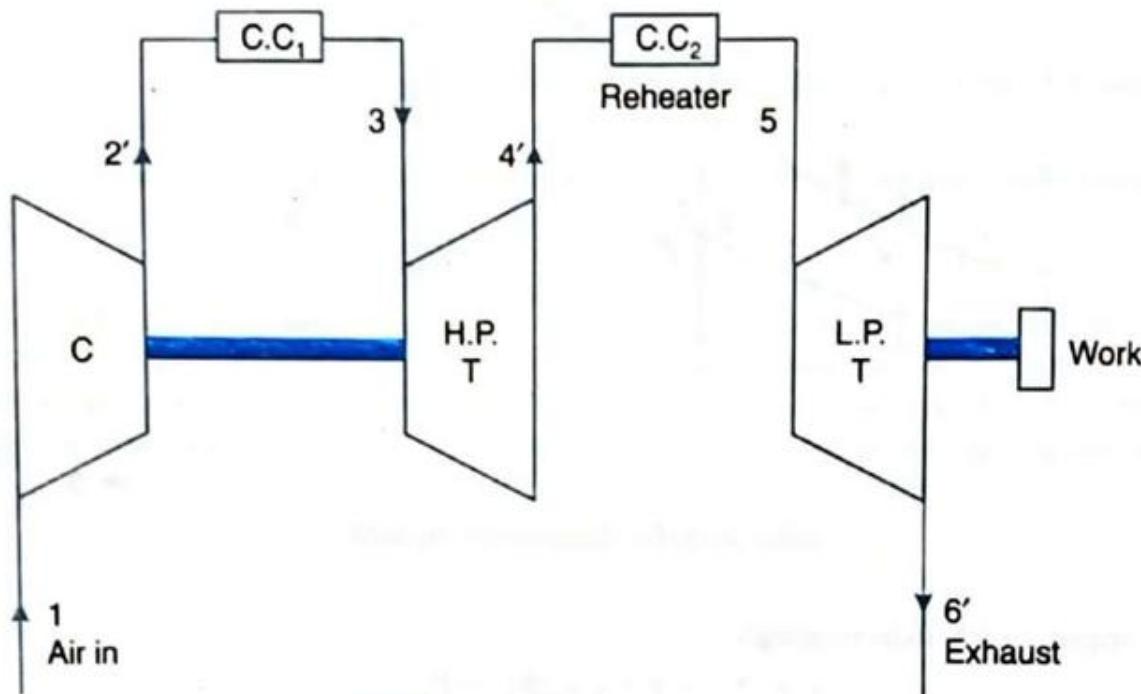


Fig. 25.5. Gas turbine with reheat.

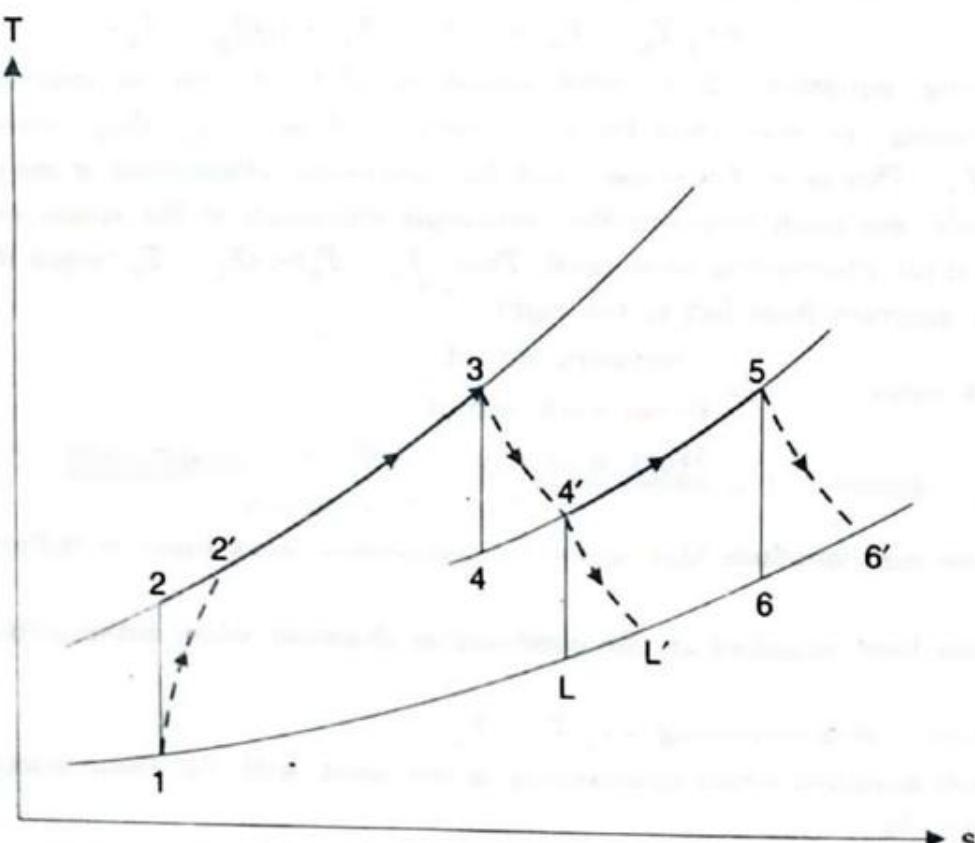


Fig. 25.6. *T-s* diagram for the unit.

Neglecting mechanical losses the work output of the H.P. turbine must be exactly equal to the work input required for the compressor i.e.,  $c_{pa}(T_2' - T_1) = c_{pg}(T_3 - T_4')$

The work output (net output) of L.P. turbine is given by,

$$\text{Network output (with reheating)} = c_{pg}(T_5 - T_6')$$

and Network output (without reheating)  $= c_{pg}(T_4' - T_L')$

Since the pressure lines diverge to the right on  $T-s$  diagram it can be seen that the temperature difference  $(T_5 - T_6')$  is always greater than  $(T_4' - T_L')$ , so that reheating increases the network output.

Although network is increased by reheating the heat to be supplied is also increased, and the net effect can be to reduce the thermal efficiency

$$\text{Heat supplied} = c_{pg}(T_3 - T_2') + c_{pg}(T_5 - T_4').$$

Note.  $c_{pa}$  and  $c_{pg}$  stand for specific heats of air and gas respectively at constant pressure.

**3. Regeneration.** The exhaust gases from a gas turbine carry a large quantity of heat with them since their temperature is far above the ambient temperature. They can be used to heat the air coming from the compressor thereby reducing the mass of fuel supplied in the combustion chamber. Fig. 25.7 shows a gas turbine plant with a regenerator. The corresponding  $T-s$  diagram is shown in Fig. 25.8.  $2'-3$  represents the heat flow into the compressed air during its passage through the heat exchanger and  $3-4$  represents the heat taken in from the combustion of fuel. Point 6 represents the temperature of exhaust gases at discharge from the heat exchanger. The maximum temperature to which the air could be heated in the heat exchanger is ideally that of exhaust gases, but less than this is obtained in practice because a temperature gradient must exist for an unassisted transfer of energy. The effectiveness of the heat exchanger is given by :

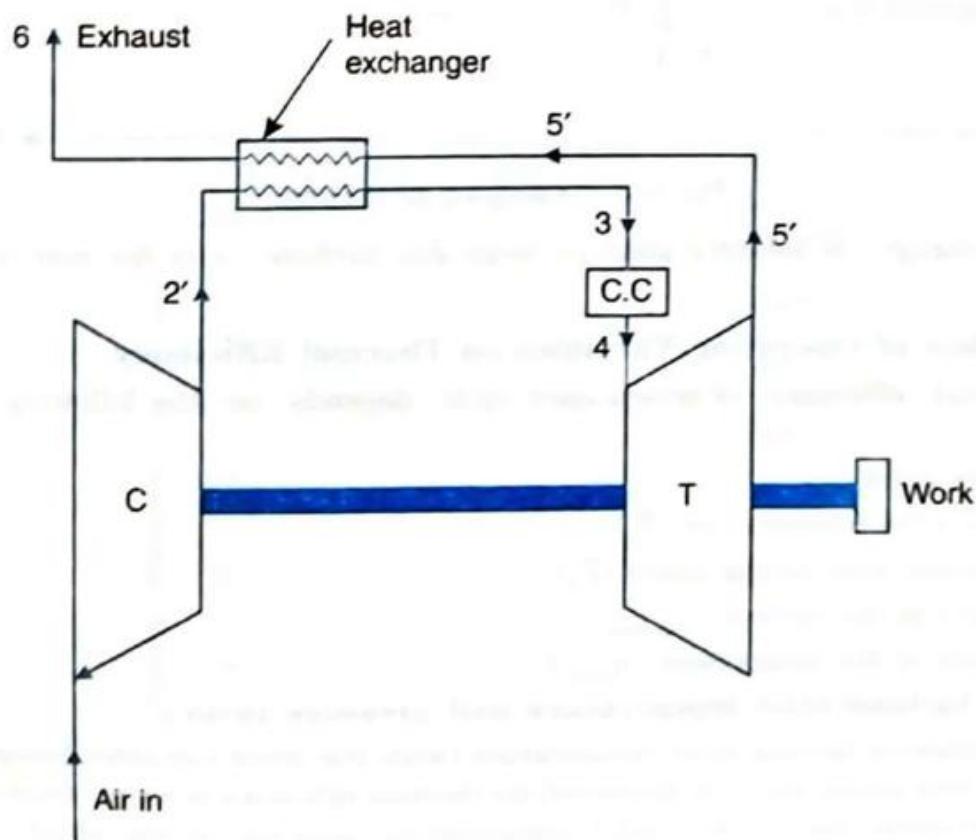


Fig. 25.7. Gas turbine with regenerator.

Effectiveness,

$$\epsilon = \frac{\text{Increase in enthalpy per kg of air}}{\text{Available increase in enthalpy per kg of air}}$$

$$= \frac{(T_3 - T_2')}{(T_5' - T_2')}$$

(assuming  $c_{pa}$  and  $c_{pg}$  to be equal)

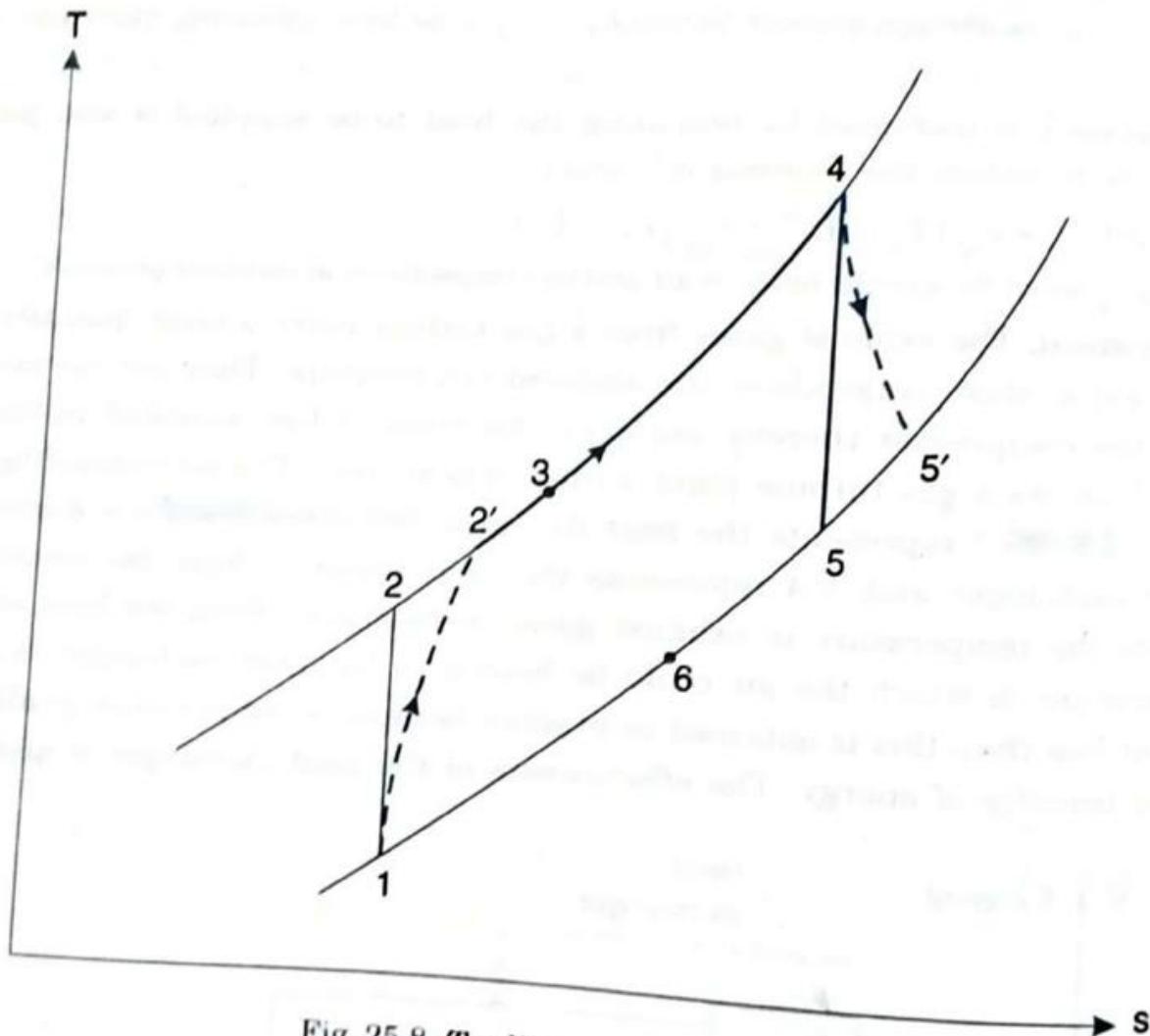


Fig. 25.8. T-s diagram for the unit.

A heat exchanger is usually used in large gas turbine units for marine propulsion.

#### 25.4.3. Effect of Operating Variables

#### 25.4.4. Closed Cycle Gas Turbine (Constant pressure or joule cycle).

Fig. 25.13 shows a gas turbine operating on a constant pressure cycle in which the closed system consists of air behaving as an ideal gas. The various operations are as follows : Refer Figs. 25.14 and 25.15.

**Operation 1-2 :** The air is compressed isentropically from the lower pressure  $p_1$  to the upper pressure  $p_2$ , the temperature rising from  $T_1$  to  $T_2$ . No heat flow occurs.

**Operation 2-3 :** Heat flow into the system increasing the volume from  $V_2$  to  $V_3$  and temperature from  $T_2$  to  $T_3$  whilst the pressure remains constant at  $p_2$ .  
Heat received =  $mc_p(T_3 - T_2)$ .

**Operation 3-4 :** The air is expanded isentropically from  $p_2$  to  $p_1$ , the temperature falling from  $T_3$  to  $T_4$ . No heat flow occurs.

**Operation 4-1 :** Heat is rejected from the system as the volume decreases from  $V_4$  to  $V_1$  and the temperature from  $T_4$  to  $T_1$  whilst the pressure remains constant at  $p_1$ . Heat rejected =  $mc_p(T_4 - T_1)$

$$\begin{aligned}\eta_{air-standard} &= \frac{\text{Work done}}{\text{Heat received}} \\ &= \frac{\text{Heat received/cycle} - \text{Heat rejected/cycle}}{\text{Heat received/cycle}} \\ &= \frac{mc_p(T_3 - T_2) - mc_p(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}\end{aligned}$$

Now, from isentropic expansion

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

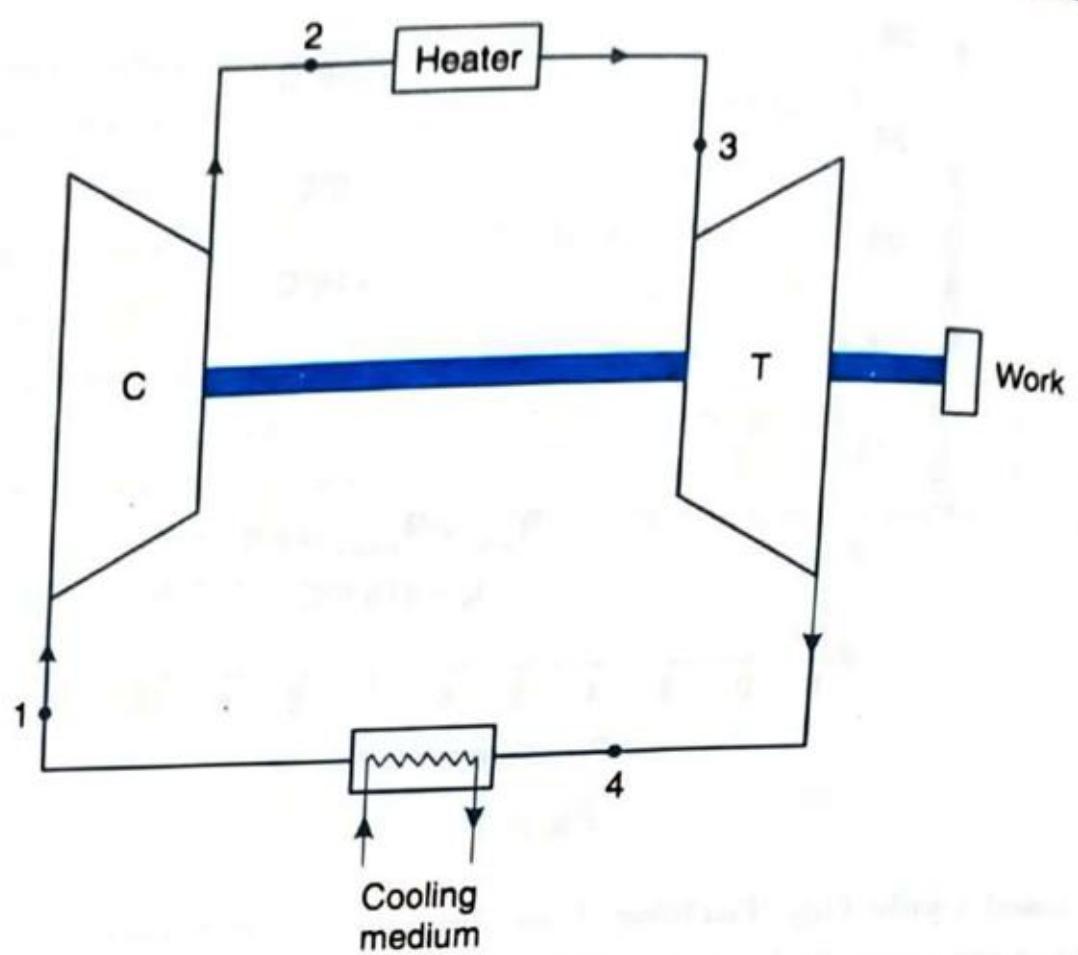


Fig. 25.13. Closed cycle gas turbine.

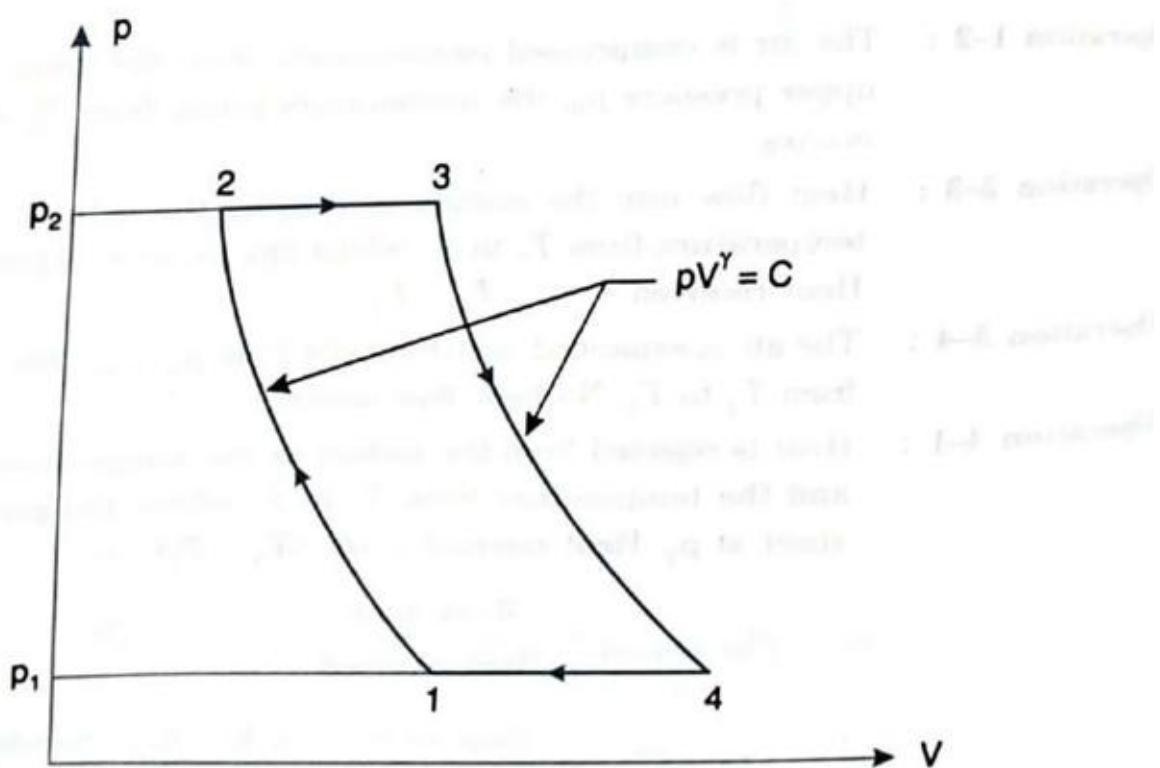


Fig. 25.14.  $p$ - $V$  diagram.

$$T_2 = T_1 (r_p)^{\frac{\gamma-1}{\gamma}}, \text{ where } r_p = \text{Pressure ratio}$$

Similarly,

$$\frac{T_3}{T_4} = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \text{ or } T_3 = T_4 (r_p)^{\frac{\gamma-1}{\gamma}}$$

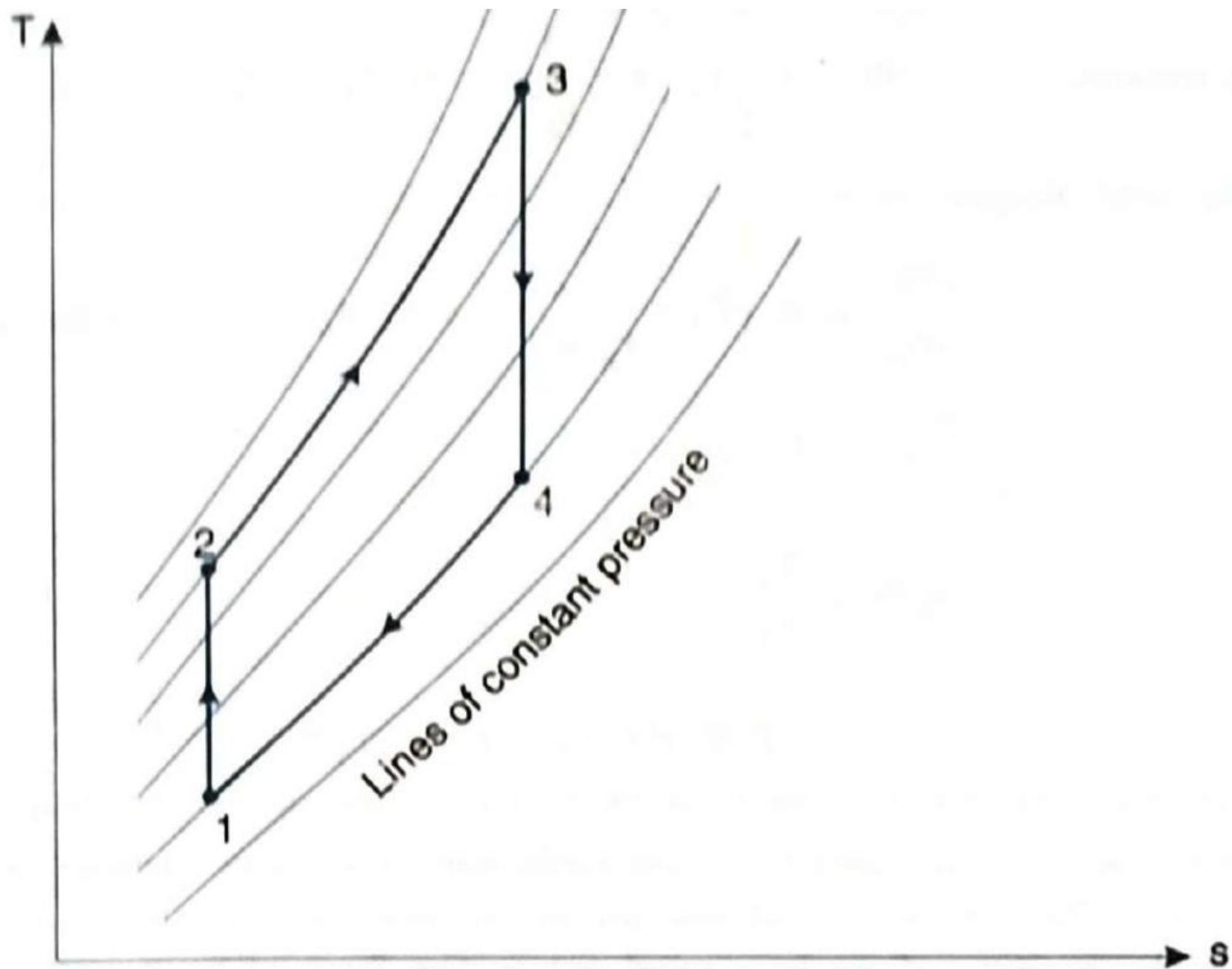


Fig. 25.15.  $T$ - $s$  diagram.

**Example 25.2.** A gas turbine unit has a pressure ratio of 6 : 1 and maximum cycle temperature of 610°C. The isentropic efficiencies of the compressor and turbine are 0.80 and 0.82 respectively. Calculate the power output in kilowatts of an electric generator geared to the turbine when the air enters the compressor at 15°C at the rate of 16 kg/s.

Take  $c_p = 1.005 \text{ kJ/kg K}$  and  $\gamma = 1.4$  for the compression process, and take  $c_p = 1.11 \text{ kJ/kg K}$  and  $\gamma = 1.333$  for the expansion process.

**Solution.**  $T_1 = 15 + 273 = 288 \text{ K}$ ;  $T_3 = 610 + 273 = 883 \text{ K}$ ;  $\frac{p_2}{p_1} = 6$ ,

$$\eta_{\text{compressor}} = 0.80; \eta_{\text{turbine}} = 0.82; \text{Air flow rate} = 16 \text{ kg/s}$$

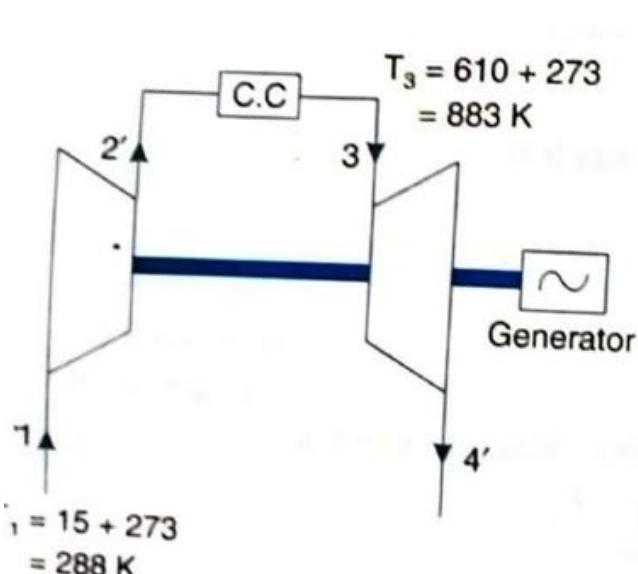
For compression process :  $c_p = 1.005 \text{ kJ/kg K}$ ,  $\gamma = 1.4$

For expansion process :  $c_p = 1.11 \text{ kJ/kg K}$ ,  $\gamma = 1.333$

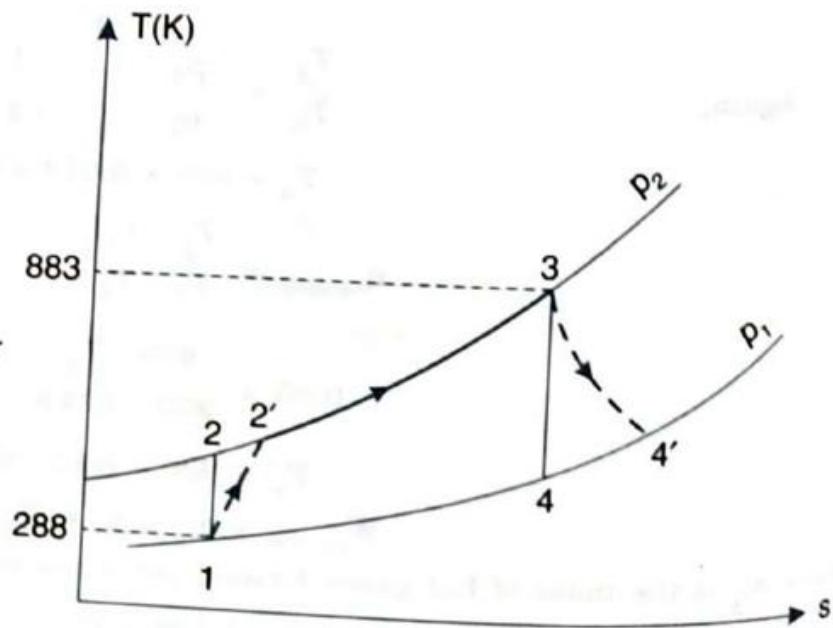
In order to evaluate the network output it is necessary to calculate temperatures  $T_2'$  and  $T_4'$ . To calculate  $T_2'$  we must first calculate  $T_2$  and then use the isentropic efficiency.

For an isentropic process,

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.4-1}{1.4}} = 1.67$$



(a)



(b)

Fig. 25.20

$$T_2 = 288 \times 1.67 = 481 \text{ K}$$

Also,

$$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.8 = \frac{481 - 288}{T_2' - T_1}$$

$$\therefore T_2' = \frac{481 - 288}{0.8} + 288 = 529 \text{ K}$$

Similarly for the turbine,

$$\frac{T_3}{T_4} = \left( \frac{p_3}{p_4} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.333-1}{1.333}} = 1.565$$

$$\therefore T_4 = \frac{T_3}{1.565} = \frac{883}{1.565} = 564 \text{ K}$$

Also,

$$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4} = \frac{883 - T_4'}{883 - 564}$$

$$\therefore 0.82 = \frac{883 - T_4'}{883 - 564}$$

$$\therefore T_4' = 883 - 0.82 (883 - 564) = 621.4 \text{ K}$$

Hence,

$$\begin{aligned} \text{Compressor work input, } W_{\text{compressor}} &= c_p (T_2' - T_1) \\ &= 1.005 (529 - 288) = 242.2 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Turbine work output, } W_{\text{turbine}} &= c_p (T_3 - T_4') \\ &= 1.11 (883 - 621.4) = 290.4 \text{ kJ/kg} \end{aligned}$$

∴ Network output,

$$\begin{aligned} W_{\text{net}} &= W_{\text{turbine}} - W_{\text{compressor}} \\ &= 290.4 - 242.2 = 48.2 \text{ kJ/kg} \end{aligned}$$

$$\text{Power in kilowatts} \quad = 48.2 \times 16 = 771.2 \text{ kW. (Ans.)}$$

**Example 25.3.** A gas turbine unit receives air at 1 bar and 300 K and compresses it adiabatically to 6.2 bar. The compressor efficiency is 88%. The fuel has a heating value of 44186 kJ/kg and the fuel-air ratio is 0.017 kJ/kg of air.

The turbine internal efficiency is 90%. Calculate the work of turbine and compressor per kg of air compressed and thermal efficiency.

For products of combustion,  $c_p = 1.147 \text{ kJ/kg K}$  and  $\gamma = 1.333$ . (U.P.S.C. 1997)

Solution. Given :  $p_1 (= p_4) = 1 \text{ bar}$ ,  $T_1 = 300 \text{ K}$ ;  $p_2 (= p_3) = 6.2 \text{ bar}$ ;  $\eta_{\text{compressor}} = 88\%$ ;

$C = 44186 \text{ kJ/kg}$ ; Fuel-air ratio = 0.017 kJ/kg of air,  $\eta_{\text{turbine}} = 90\%$ ;

$c_p = 1.147 \text{ kJ/kg K}$ ;  $\gamma = 1.333$ .

For isentropic compression process 1-2 :

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{6.2}{1} \right)^{\frac{1.333-1}{1.333}} = 1.684$$

$$\therefore T_2 = 300 \times 1.684 = 505.2 \text{ K}$$

Now,

$$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2' - T_1}$$

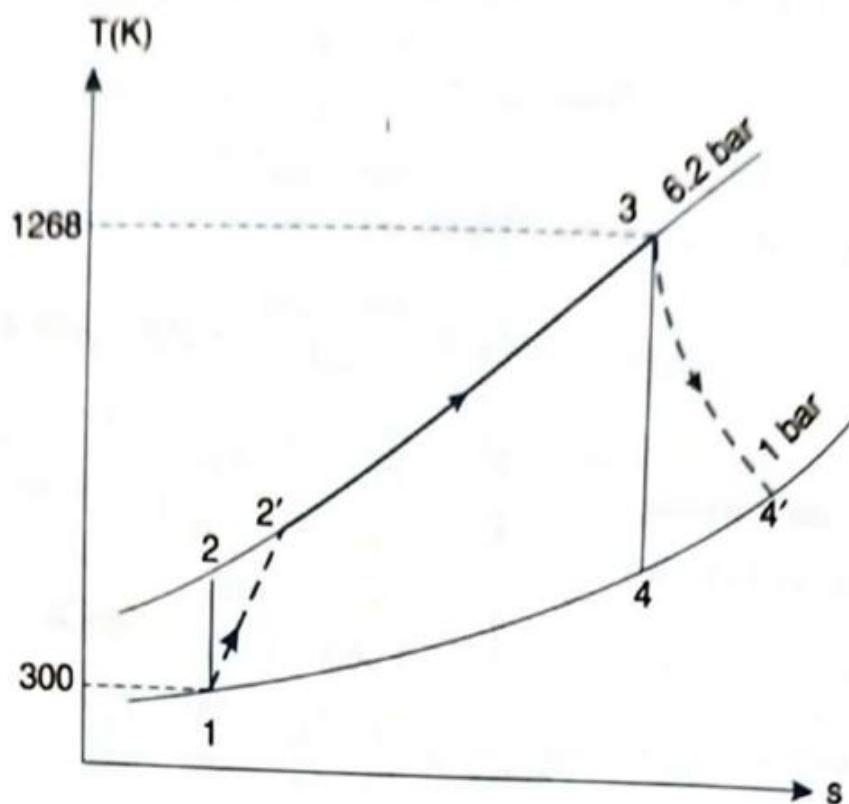


Fig. 25.21

$$0.88 = \frac{505.2 - 300}{T_{2'} - 300}$$

$$T_{2'} = \left( \frac{505.2 - 300}{0.88} + 300 \right) = 533.2 \text{ K}$$

Heat supplied

$$= (m_a + m_f) \times c_p(T_3 - T_{2'}) = m_f \times C$$

or

or

$$(1 + 0.017) \times 1.005(T_3 - 533.2) = 0.017 \times 44186$$

$$\therefore T_3 = \frac{0.017 \times 44186}{(1 + 0.017) \times 1.005} + 533.2 = 1268 \text{ K}$$

For isentropic expression process 3-4 :

$$\frac{T_4}{T_3} = \left( \frac{p_4}{p_3} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{1}{6.2} \right)^{\frac{1.333-1}{1.333}} = 0.634$$

$$T_4 = 1268 \times 0.634 = 803.9 \text{ K}$$

Now,

$$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$0.9 = \frac{1268 - T_4'}{1268 - 803.9}$$

$$T_4' = 1268 - 0.9(1268 - 803.9) = 850.3 \text{ K}$$

$$W_{\text{compressor}} = c_p(T_2' - T_1) = 1.005(533.2 - 300) = 234.4 \text{ kJ/kg}$$

$$W_{\text{turbine}} = c_{pg}(T_3 - T_4') = 1.147(1268 - 850.3) = 479.1 \text{ kJ/kg}$$

$$\text{Network} = W_{\text{turbine}} - W_{\text{compressor}} \\ = 479.1 - 234.4 = 244.7 \text{ kJ/kg}$$

Heat supplied per kg of air  
 $= 0.017 \times 44186 = 751.2 \text{ kJ/kg}$

$$\text{Thermal efficiency, } \eta_{\text{th}} = \frac{\text{Network}}{\text{Heat supplied}} \\ = \frac{244.7}{751.2} = 0.3257 \text{ or } 32.57\%. \text{ (Ans.)}$$

**Example 25.4.** Find the required air-fuel ratio in a gas turbine whose turbine and compressor efficiencies are 85% and 80%, respectively. Maximum cycle temperature is 875°C. The working fluid can be taken as air ( $c_p = 1.0 \text{ kJ/kg K}$ ,  $\gamma = 1.4$ ) which enters the compressor at 1 bar and 27°C. The pressure ratio is 4. The fuel used has calorific value of 42000 kJ/kg. There is a loss of 10% of calorific value in the combustion chamber. (GATE 1998)

**Solution.** Given :  $\eta_{\text{turbine}} = 85\%$ ;  $\eta_{\text{compressor}} = 80\%$ ;  $T_3 = 273 + 875 = 1148 \text{ K}$ ,  $T_1 = 27 + 273 = 300 \text{ K}$ ;  $c_p = 1.0 \text{ kJ/kg K}$ ;  $\gamma = 1.4$ ,  $p_1 = 1 \text{ bar}$ ,  $p_2 = 4 \text{ bar}$  (Since pressure ratio is 4);  $C = 42000 \text{ kJ/kg K}$ ,  $\eta_{\text{cc}} = 90\%$  (since loss in the combustion chamber is 10%)

For isentropic compression 1-2 :

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = (4)^{\frac{14-1}{14}} = 1.486$$

$$T_2 = 300 \times 1.486 = 445.8 \text{ K}$$

$$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_{2'} - T_1}$$

$$0.8 = \frac{445.8 - 300}{T_{2'} - 300}$$

$$T_{2'} = \frac{445.8 - 300}{0.8} + 300 = 482.2 \text{ K}$$

or  
or

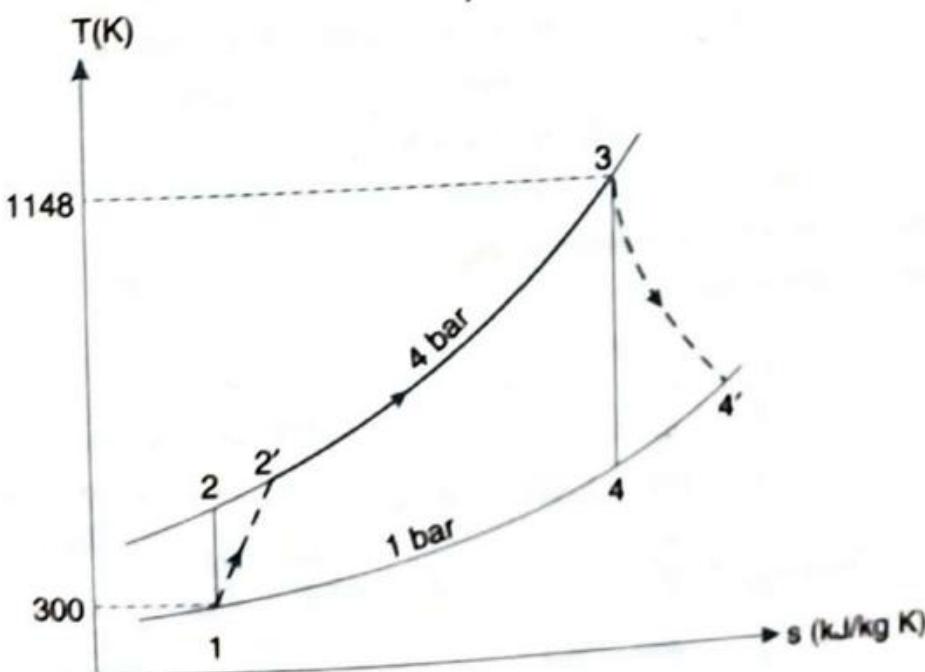


Fig. 25.22

**Example 25.8.** In an air-standard regenerative gas turbine cycle the pressure ratio is 5. Air enters the compressor at 1 bar, 300 K and leaves at 490 K. The maximum temperature in the cycle is 1000 K. Calculate the cycle efficiency, given that the efficiency of the regenerator and the adiabatic efficiency of the turbine are each 80%. Assume for air, the ratio of specific heats is 1.4. Also, show the cycle on a T-s diagram. (GATE, 1997)

**Solution.** Given :  $p_1 = 1 \text{ bar}$ ;  $T_1 = 300 \text{ K}$ ,  $T_2' = 490 \text{ K}$ ;  $T_3 = 1000 \text{ K}$

$$\frac{p_2}{p_1} = 5, \eta_{\text{turbine}} = 80\%, \epsilon = 80\% = 0.8; \gamma = 1.4$$

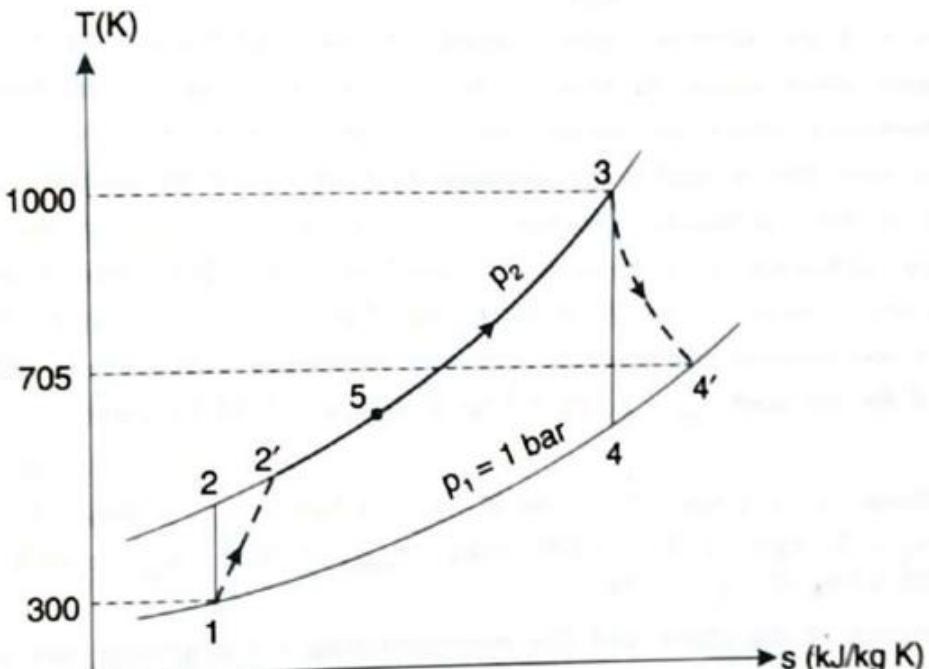


Fig. 25.25

Now,

$$\frac{T_3}{T_4} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = (5)^{\frac{1.4-1}{1.4}} = 1.5838$$

$$\therefore T_4 = \frac{T_3}{1.5838} = \frac{1000}{1.5838} = 631.4 \text{ K}$$

Also,

$$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4}$$

or

$$0.8 = \frac{1000 - T_4'}{1000 - 631.4}$$

$$\therefore T_4' = 1000 - 0.8(1000 - 631.4) = 705 \text{ K}$$

Effectiveness of heat exchanger,  $\epsilon = \frac{T_5 - T_2'}{T_4' - T_2'}$

or

$$0.8 = \frac{T_5 - 490}{705 - 490}$$

$$T_5 = 0.8(705 - 490) + 490 = 662 \text{ K}$$

$$\begin{aligned}\text{Work consumed by compressor} &= c_p (T_2' - T_1) \\ &= 1.005 (490 - 300) = 190.9 \text{ kJ/kg}\end{aligned}$$

$$\begin{aligned}\text{Work done by turbine} &= c_p (T_3 - T_4') \\ &= 1.005 (1000 - 705) = 296.5 \text{ kJ/kg}\end{aligned}$$

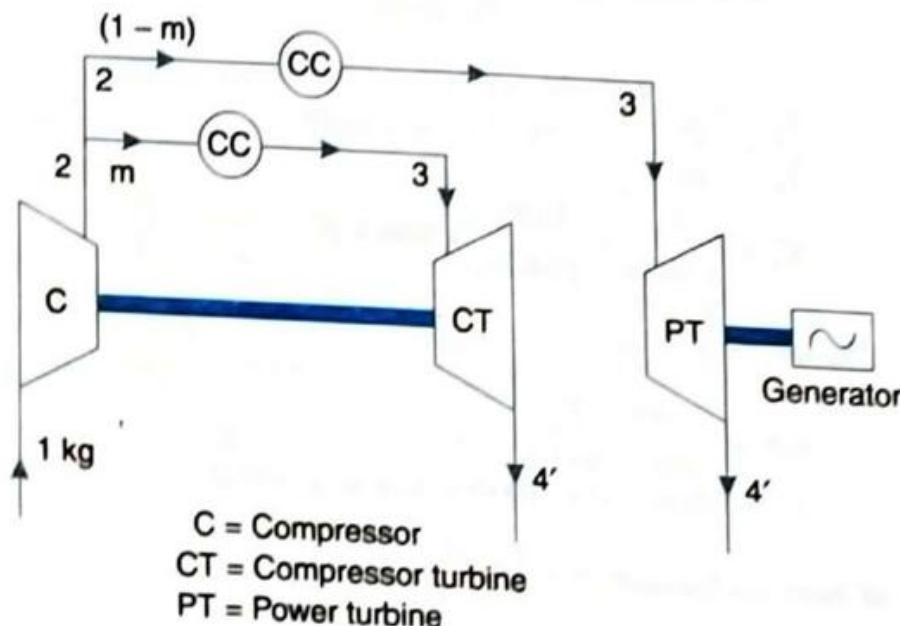
$$\begin{aligned}\text{Heat supplied} &= c_p (T_3 - T_5) \\ &= 1.005 (1000 - 662) = 339.7 \text{ kJ/kg}\end{aligned}$$

$$\begin{aligned}\therefore \text{Cycle efficiency, } \eta_{\text{cycle}} &= \frac{\text{Network}}{\text{Heat supplied}} \\ &= \frac{\text{Turbine work} - \text{Compressor work}}{\text{Heat supplied}} \\ &= \frac{296.5 - 190.9}{339.7} = 0.31 \text{ or } 31\%. \quad (\text{Ans.})\end{aligned}$$

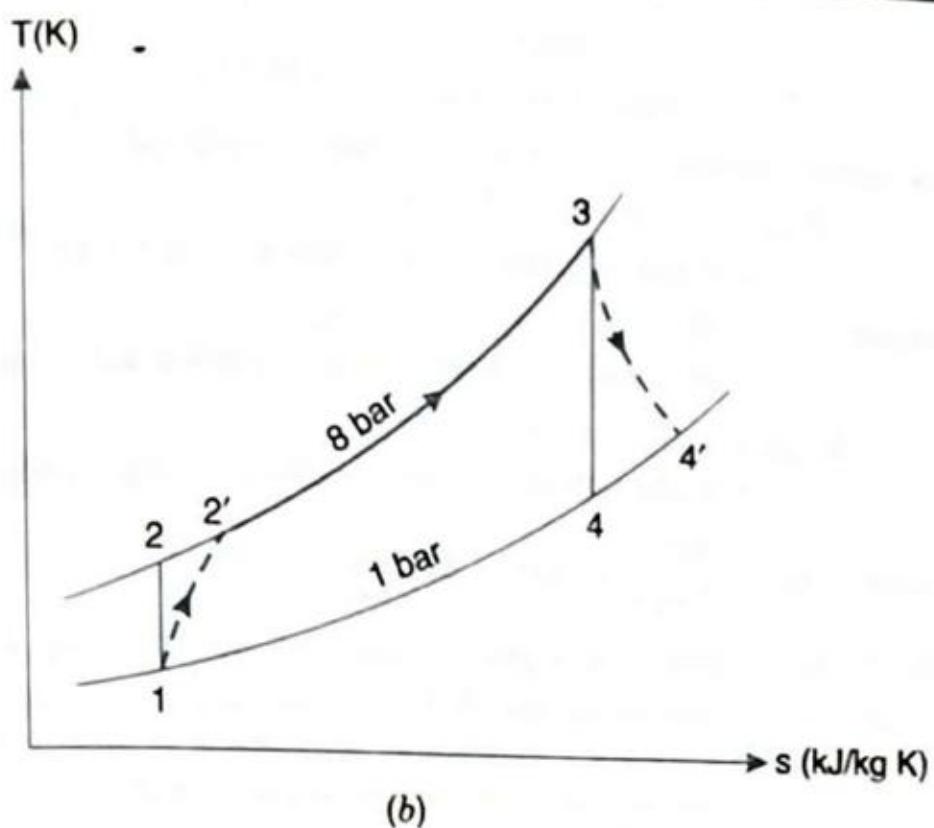
**Example 25.9.** A gas turbine plant consists of two turbines. One compressor turbine to drive compressor and other power turbine to develop power output and both are having their own combustion chambers which are served by air directly from the compressor. Air enters the compressor at 1 bar and 288 K and is compressed to 8 bar with an isentropic efficiency of 76%. Due to heat added in the combustion chamber, the inlet temperature of gas to both turbines is 900°C. The isentropic efficiency of turbines is 86% and the mass flow rate of air at the compressor is 23 kg/s. The calorific value of fuel is 4200 kJ/kg. Calculate the output of the plant and the thermal efficiency if mechanical efficiency is 95% and generator efficiency is 96%. Take  $c_p = 1.005 \text{ kJ/kg K}$  and  $\gamma = 1.4$  for air and  $c_{pr} = 1.128 \text{ kJ/kg K}$  and  $\gamma_g = 1.34$  for gases.

**Solution.** Given :  $p_1 = 1 \text{ bar}$ ;  $T_1 = 288 \text{ K}$ ;  $p_2 = 8 \text{ bar}$ ,  $\eta_{(\text{isen})} = 76\%$ ;  $T_3 = 900^\circ\text{C}$  or  $1173 \text{ K}$ ,  $\eta_{T(\text{isen.})} = 86\%$ ,  $m_a = 23 \text{ kg/s}$ ; C.V. =  $4200 \text{ kJ/kg}$ ;  $\eta_{\text{mech.}} = 95\%$ ;  $\eta_{\text{gen.}} = 96\%$ ;  $c_p = 1.005 \text{ kJ/kg K}$ ;  $\gamma_a = 1.4$ ;  $c_{pr} = 1.128 \text{ kJ/kg K}$ ;  $\gamma_g = 1.34$ .

The arrangement of the plant and the corresponding  $T-s$  diagram are shown in Fig. 25.26 (a), (b) respectively.



(a)



(b)

Fig. 25.26

Considering *isentropic compression process 1–2*, we have

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{8}{1} \right)^{\frac{1.4-1}{1.4}} = 1.811$$

$$\therefore T_2 = 288 \times 1.811 = 521.6 \text{ K}$$

Also,  $\eta_{C(isen.)} = \frac{T_2 - T_1}{T_2' - T_1}$

or  $0.76 = \frac{521.6 - 288}{T_2' - 288}$

or  $T_2' = \frac{521.6 - 288}{0.76} + 288 = 595.4 \text{ K}$

Considering *isentropic expansion process 3–4*, we have

$$\frac{T_4}{T_3} = \left( \frac{p_4}{p_3} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{1}{8} \right)^{\frac{1.34-1}{1.34}} = 0.59$$

$$\therefore T_4 = 1173 \times 0.59 = 692.1 \text{ K}$$

Also,  $\eta_{T(isen.)} = \frac{T_3 - T_4'}{T_3 - T_4}$

or  $0.86 = \frac{1173 - T_4'}{1173 - 692.1}$

$$\therefore T_4' = 1173 - 0.86 (1173 - 692.1) = 759.4 \text{ K}$$

Consider 1 kg of air-flow through compressor

$$W_{\text{compressor}} = c_p (T_2' - T_1') = 1.005 (595.4 - 288) = 308.9 \text{ kJ}$$

This is equal to work of compressor turbine.

$$\therefore 308.9 = m_1 \times c_{pg} (T_3 - T_4'), \text{ neglecting fuel mass}$$

$$\text{or } m_1 = \frac{308.9}{1.128(1173 - 759.4)} = 0.662 \text{ kg}$$

$$\text{and flow through the power turbine} = 1 - m = 1 - 0.662 = 0.338 \text{ kg}$$

$$\therefore W_{PT} = (1 - m) \times c_{pg}(T_3 - T_4') \\ = 0.338 \times 1.128(1173 - 759.4) = 157.7 \text{ kJ}$$

$$\therefore \text{Power output} = 23 \times 157.7 \times \eta_{\text{mech.}} \times \eta_{\text{gen.}} \\ = 23 \times 157.7 \times 0.95 \times 0.96 = 3307.9 \text{ kJ. (Ans.)}$$

$$Q_{\text{input}} = c_{pg}T_3 - c_{pa}T_2' \\ = 1.128 \times 1173 - 1.005 \times 595.4 = 724.7 \text{ kJ/kg of air}$$

$$\text{Thermal efficiency, } \eta_{\text{th}} = \frac{157.7}{724.7} \times 100 = 21.76\%. \text{ (Ans.)}$$

**Example 25.10.** Air is drawn in a gas turbine unit at 15°C and 1.01 bar and pressure ratio is 7 : 1. The compressor is driven by the H.P. turbine and L.P. turbine drives a separate power shaft. The isentropic efficiencies of compressor, and the H.P. and L.P. turbines are 0.82, 0.85 and 0.85 respectively. If the maximum cycle temperature is 610°C, calculate :

(i) The pressure and temperature of the gases entering the power turbine.

(ii) The net power developed by the unit per kg/s mass flow.

(iii) The work ratio.

(iv) The thermal efficiency of the unit.

Neglect the mass of fuel and assume the following :

For compression process  $c_{pa} = 1.005 \text{ kJ/kg K}$  and  $\gamma = 1.4$

For combustion and expansion processes ;  $c_{pg} = 1.15 \text{ kJ/kg}$  and  $\gamma = 1.333$ .

**Solution.** Given :  $T_1 = 15 + 273 = 288 \text{ K}$ ,  $p_1 = 1.01 \text{ bar}$ , Pressure ratio  $= \frac{p_2}{p_1} = 7$ ,

$$\eta_{\text{compressor}} = 0.82, \eta_{\text{turbine (H.P.)}} = 0.85, \eta_{\text{turbine (L.P.)}} = 0.85,$$

$$\text{Maximum cycle temperature, } T_3 = 610 + 273 = 883 \text{ K}$$

(i) Pressure and temperature of the gases entering the power turbine,  $p_4'$  and  $T_4'$ :

Considering isentropic compression 1-2, we have

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = (7)^{\frac{14-1}{14}} = 1.745$$

$$\therefore T_2 = 288 \times 1.745 = 502.5 \text{ K}$$

$$\text{Also } \eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.82 = \frac{502.5 - 288}{T_2' - 288}$$

$$\therefore T_2' = \frac{502.5 - 288}{0.82} + 288 = 549.6 \text{ K}$$

$$W_{\text{compressor}} = c_{pa}(T_2' - T_1) = 1.005 \times (549.6 - 288) = 262.9 \text{ kJ/kg}$$

Now, the work output of H.P. turbine = Work input to compressor

$$\therefore c_{pg}(T_3 - T_4') = 262.9$$

$$1.15(883 - T_4') = 262.9$$

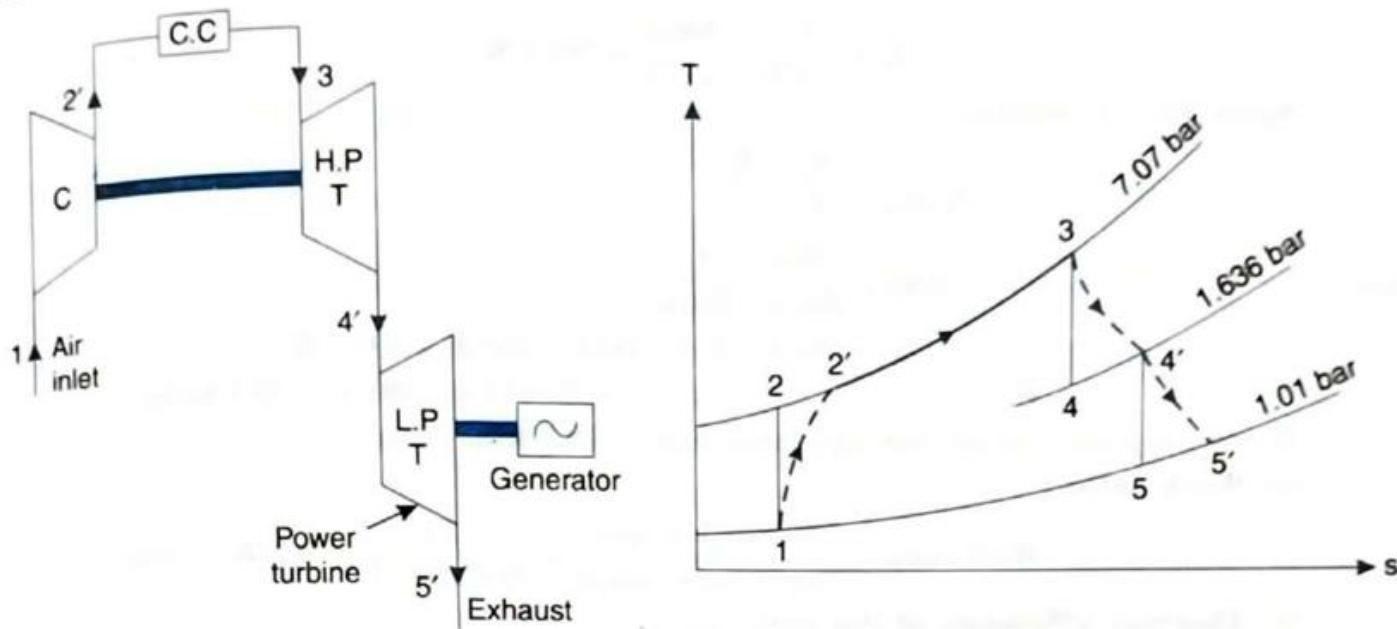


Fig. 25.27

$$\therefore T_4' = 883 - \frac{262.9}{1.15} = 654.4 \text{ K}$$

i.e., Temperature of gases entering the power turbine = 654.4 K. (Ans.)

Again, for H.P. turbine :

$$\eta_{turbine} = \frac{T_3 - T_4'}{T_3 - T_4} \quad \text{i.e., } 0.85 = \frac{883 - 654.4}{883 - T_4}$$

$$\therefore T_4 = 883 - \left( \frac{883 - 654.4}{0.85} \right) = 614 \text{ K}$$

Now, considering isentropic expansion process 3-4, we have

$$\frac{T_3}{T_4} = \left( \frac{p_3}{p_4} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\text{or } \frac{p_3}{p_4} = \left( \frac{T_3}{T_4} \right)^{\frac{1}{\gamma-1}} = \left( \frac{883}{614} \right)^{\frac{1.33}{0.33}} = 4.32$$

$$\text{i.e., } p_4 = \frac{p_3}{4.32} = \frac{7.07}{4.32} = 1.636 \text{ bar}$$

i.e., Pressure of gases entering the power turbine = 1.636 bar. (Ans.)

(ii) Net power developed per kg/s mass flow, P :

To find the power output it is now necessary to calculate  $T_5'$ .

The pressure ratio,  $\frac{p_4}{p_5}$ , is given by  $\frac{p_4}{p_3} \times \frac{p_3}{p_5}$

$$\text{i.e., } \frac{p_4}{p_5} = \frac{p_4}{p_3} \times \frac{p_3}{p_1} = \frac{7}{4.32} = 1.62 \quad (\because p_2 = p_3 \text{ and } p_5 = p_1)$$

$$\text{Then, } \frac{T_4'}{T_5} = \left( \frac{p_4}{p_5} \right)^{\frac{\gamma-1}{\gamma}} = (1.62)^{\frac{0.33}{1.33}} = 1.127$$

$$\therefore T_5' = \frac{T_4'}{1.127} = \frac{654.4}{1.127} = 580.6 \text{ K.}$$

Again, for L.P. turbine

$$\eta_{\text{turbine}} = \frac{T_4' - T_5'}{T_4' - T_0}$$

$$\text{i.e., } 0.85 = \frac{654.4 - T_5'}{654.4 - 580.6}$$

$$\therefore T_5' = 654.4 - 0.85(654.4 - 580.6) = 591.7 \text{ K}$$

$$W_{\text{L.P. turbine}} = c_{pg}(T_4' - T_5') = 1.15(654.4 - 591.7) = 72.1 \text{ kJ/kg}$$

Hence net power output (per kg/s mass flow) = 72.1 kW. (Ans.)

### (iii) Work ratio :

$$\text{Work ratio} = \frac{\text{Network output}}{\text{Gross work output}} = \frac{72.1}{72.1 + 262.9} = 0.215. \text{ (Ans.)}$$

### (iv) Thermal efficiency of the unit, $\eta_{\text{thermal}}$ :

$$\text{Heat supplied} = c_{pg}(T_3 - T_2) = 1.15(883 - 549.6) = 383.4 \text{ kJ/kg}$$

$$\therefore \eta_{\text{thermal}} = \frac{\text{Network output}}{\text{Heat supplied}} = \frac{72.1}{383.4} = 0.188 \text{ or } 18.8\%. \text{ (Ans.)}$$

**Example 25.11.** The pressure ratio of an open-cycle gas turbine power plant is 5.6. Air is taken at 30°C and 1 bar. The compression is carried out in two stages with perfect intercooling in between. The maximum temperature of the cycle is limited to 700°C. Assuming the isentropic efficiency of each compressor stage as 85% and that of turbine as 90%, determine the power developed and efficiency of the power plant, if the air-flow is 1.2 kg/s. The mass of fuel may be neglected, and it may be assumed that  $c_p = 1.02 \text{ kJ/kg K}$  and  $\gamma = 1.41$ . (P.U.)

**Solution.** Refer Fig. 25.28.

Pressure ratio of the open-cycle gas turbine = 5.6

Temperature of intake air,  $T_1 = 30 + 273 = 303 \text{ K}$

Pressure of intake air,  $p_1 = 1 \text{ bar}$

Maximum temperature of the cycle,  $T_5 = 700 + 273 = 973 \text{ K}$

Isentropic efficiency of each compressor,  $\eta_{\text{comp.}} = 85\%$

Isentropic efficiency of turbine,  $\eta_{\text{turbine}} = 90\%$

Rate of air-flow,  $\dot{m}_a = 1.2 \text{ kg/s}$

$c_p = 1.02 \text{ kJ/kg K}$  and  $\gamma = 1.41$ .

### Power developed and efficiency of the power plant :

Assuming that the pressure ratio in each stage is same, we have

$$\frac{p_2}{p_1} = \frac{p_4}{p_3} = \sqrt{\frac{p_4}{p_1}} = \sqrt{5.6} = 2.366$$

Since the pressure ratio and the isentropic efficiency of each compressor is the same then the work input required for each compressor is the same since both the compressors have the same inlet temperature (perfect intercooling) i.e.,  $T_1 = T_3$  and  $T_2' = T_4'$ :

$$\text{Now, } \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = (2.366)^{\frac{1.41-1}{1.41}} = 1.2846 \text{ or } T_2 = 303 \times 1.2846 = 389.23 \text{ K}$$

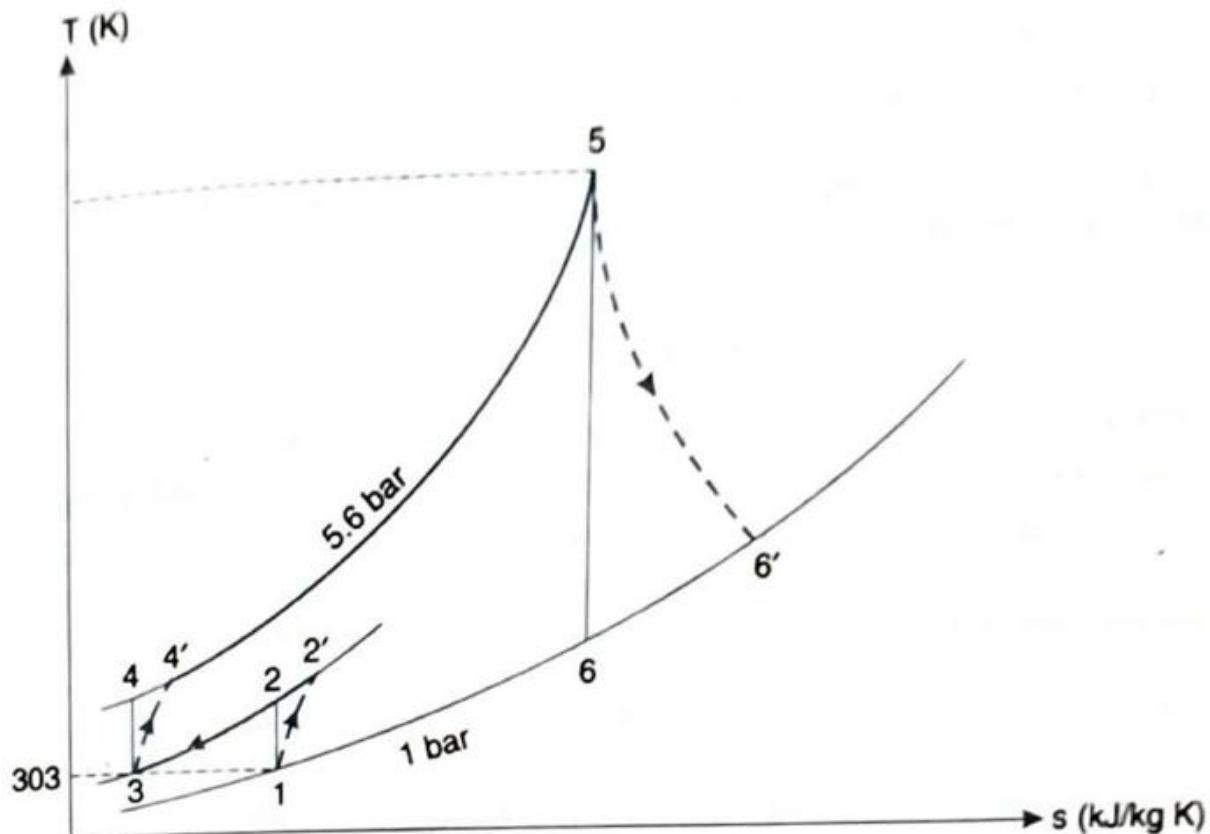


Fig. 25.28

Also,  $\eta_{comp.} = \frac{T_2 - T_1}{T_2' - T_1}$  or  $0.85 = \frac{389.23 - 303}{T_2' - 303}$

or  $T_2' = \frac{389.23 - 303}{0.85} + 303 = 404.44 \text{ K}$

Work input to 2-stage compressor,  $W_{comp.} = 2 \times m \times c_p (T_2' - T_1)$   
 $= 2 \times 1.2 \times 1.02 (404.44 - 303) = 248.32 \text{ kJ/s}$

For turbine, we have

$$\frac{T_5}{T_6} = \left( \frac{P_5}{P_6} \right)^{\frac{\gamma-1}{\gamma}} = (5.6)^{\frac{1.41-1}{1.41}} = 1.65 \quad \text{or} \quad T_6 = \frac{T_5}{1.65} = \frac{973}{1.65} = 589.7 \text{ K}$$

Also,  $\eta_{turbine} = \frac{T_5 - T_6'}{T_5 - T_6}$

or  $0.9 = \frac{973 - T_6'}{973 - 589.7} \quad \text{or} \quad T_6' = 973 - 0.9 (973 - 589.7) = 628 \text{ K}$

∴ Work output of turbine,  $W_{turbine} = m \times c_p (T_5 - T_6')$   
 $= 1.2 \times 1.02 (973 - 628) = 422.28 \text{ kJ/s}$

Network output,  $W_{net} = W_{turbine} - W_{comp.}$   
 $= 422.28 - 248.32 = 173.96 \text{ kJ/s or kW}$

Hence power developed  $= 173.96 \text{ kW. (Ans.)}$

Heat supplied,  $Q_s = m \times c_p \times (T_5 - T_4')$   
 $= 1.2 \times 1.02 \times (973 - 404.44) = 695.92 \text{ kJ/s}$

∴ Power plant efficiency,  $\eta_{th} = \frac{W_{net}}{Q_s} = \frac{173.96}{695.92} = 0.25 \quad \text{or} \quad 25\%. \quad (\text{Ans.})$

## Steam Nozzles

Steam turbines, water turbines and gas turbines produce power by utilising the kinetic energy of the jets produced by passing high pressure steam, water and gas through the devices called nozzles. Corresponding to the fluids used, the nozzles are called steam nozzles, water nozzles and gas nozzles.

- A nozzle is a device, a duct or passage of varying cross-section in which a steadily flowing fluid can be made to accelerate by a pressure drop along the duct.
- So when a fluid flows through a nozzle, its velocity increases continuously and pressure decreases continuously. In the nozzle, the working fluid expands from a higher pressure to a lower pressure.
- The main use of a steam nozzle in steam turbines is to produce a jet of steam with a high velocity.

**These nozzles serve two purposes:**

1. To convert pressure energy and thermal energy into kinetic energy and thereby increases the velocity of the flowing fluid
2. To direct the fluid jet at the specific angle known as nozzle angle.

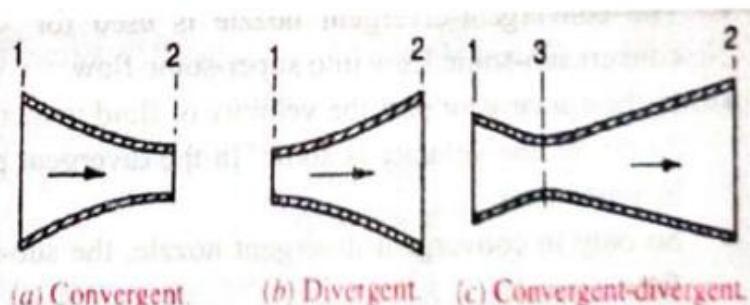
The energy conversion is brought about in the following ways:

1. The high-pressure, high-temperature steam first expands in the nozzles emanates as a high-velocity fluid stream.
2. The high-velocity steam coming out of the nozzles impinges on the blades mounted on a wheel. The fluid stream suffers a loss of momentum while flowing past the blades that is absorbed by the rotating wheel entailing the production of torque.
3. The moving blades move as a result of the impulse of steam (caused by the change of momentum) and also as a result of the expansion and acceleration of the steam relative to them. In other words they also act as the nozzles.

**Note:** Since the mass of steam which is passed through any section of the nozzle remains constant, the variation of steam pressure in the nozzle depends upon the velocity, specific volume, and dryness fraction of steam.

### Types of Nozzles:

1. Convergent Nozzle
2. Divergent Nozzle
3. Convergent-Divergent Nozzle



### Convergent Nozzle

In a convergent nozzle, the cross-sectional area decreases continuously from its entrance to exit. It is used in a case where the back pressure is equal to or greater than the critical pressure ratio.

## Steam Nozzles

### Divergent Nozzle

The cross-sectional area of the divergent nozzle increases continuously from its entrance to exit. It is used in a case, where the back pressure is less than the critical pressure ratio.

### Convergent-Divergent Nozzle

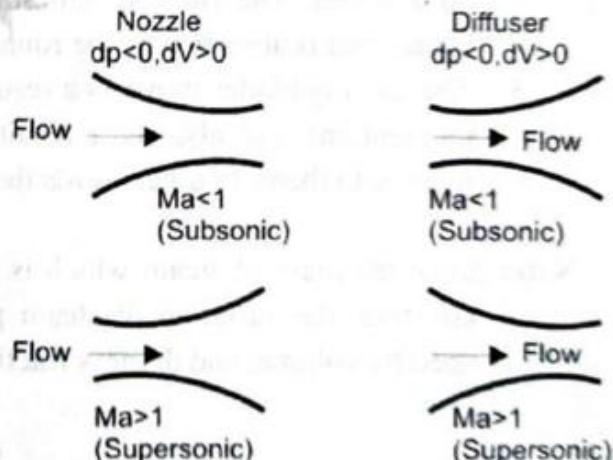
In this case, the cross-sectional area first decreases from its entrance to the throat and then increases from throat to exit. The section where cross-sectional area is minimum is called 'throat' of the nozzle. It is widely used in many types of steam turbines.

### Flow-Through Nozzles

- A **nozzle** is a duct that increases the velocity of the flowing fluid at the expense of pressure drop.
- A duct which decreases the velocity of a fluid and causes a corresponding increase in pressure is a **diffuser**.
- The same duct may be either a nozzle or a diffuser depending upon the end conditions across it.
- A fluid is said to be **compressible** if its density changes with the change in pressure brought about by the flow
- If the density does not change or changes very little, the fluid is said to be incompressible. Usually the gases and vapors are compressible, whereas liquids are incompressible.

### Significance of Mach number:

- If the Mach number is less than one, flow is sub-sonic, and the nozzle is convergent. By using this convergent nozzle, the flow of the fluid can be increased to sonic velocity. But by using convergent nozzle we cannot obtain super-sonic flow.
- If Mach number is equal to one, flow is sonic.
- If Mach number is greater than one, flow is supersonic and the nozzle is divergent.
- The convergent-divergent nozzle is used for convert sub-sonic flow into super-sonic flow.
- In the convergent part the velocity of fluid is increased from sub-sonic to sonic condition. At throat, the velocity is sonic. In the divergent part, the velocity is increased from sonic to super-sonic.
- So only in convergent-divergent nozzle, the sub-sonic flow is converted into super-sonic flow.



## **Effect of friction on Nozzle**

When the steam flows through a nozzle, the final velocity of steam for a given pressure drop is reduced due to the frictional effects;

**The frictional effects are due to:**

- (i) Friction between sides of nozzle (wall of nozzle) and fluid.
- (ii) Internal fluid friction and
- (iii) Due to eddies in the flow.

If the friction is neglected, the expansion of steam (the flow of steam) in the nozzle is assumed to be isentropic. The Mollier Chart shows the isentropic flow (1-t-2) of steam through a convergent-divergent nozzle. The enthalpy drop ( $h_1 - h_2$ ) is known as isentropic enthalpy drop.

But in actual case, the friction losses occur. Most of the friction in convergent-divergent nozzle is assumed to occur between the throat and exit. The expansion upto throat is taken to be isentropic. This is due to low initial velocity. In Mollier Chart, 1-t-2' is the actual expansion of steam through nozzle. Figure shows the actual expansion of steam through nozzle.

So the actual enthalpy drop is ( $h_1 - h_2'$ )

**Effects:**

1. Reduction in actual enthalpy drop.
2. Reduction in exit velocity of fluid.
3. Increase in the dryness-fraction of steam.
4. Increase in the specific volume and
5. Decrease in the mass-flow rate.

**Note:** The actual expansion of steam in the nozzle is expressed by the curve 1-2' (adiabatic expansion) instead of 1-2 (isentropic expansion).

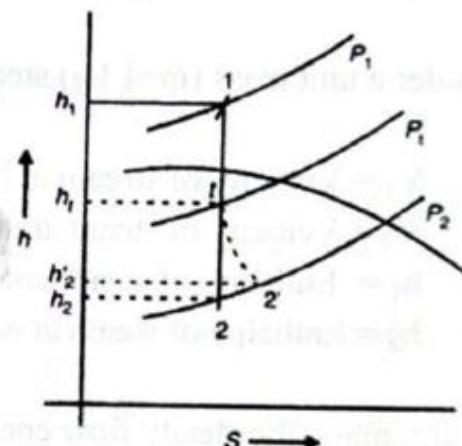


Fig. 19.5. *h-S diagram with friction*

## **Nozzle efficiency:**

It is the ratio of useful heat drop to the isentropic heat drop. It is denoted by K

$$K = \frac{\text{Useful heat drop}}{\text{Isentropic heat drop}} = \frac{h_1 - h_2'}{h_1 - h_2}$$

The efficiency of a nozzle generally varies from 0.85 to 0.95.

## **Factors affecting nozzle efficiency**

1. Material and finish of the nozzle.
2. Size and shape of the nozzle.
3. Angle of divergence.
4. Nature of the fluid and its state
5. Friction
6. Fluid velocity.
7. Turbulence in the flow passages

## Application Of Steam Nozzle:

- Steam and Gas turbine, Jet Engine and Rocket Motors
- It is used to measure the discharge of fluid - e.g. Venturimeter
- Injectors for pumping feed water to boilers.
- The supersonic gas turbine engine: for the air intake when the air requirement of the engine is high.
- Rockets: for providing sufficient thrust to move upwards.
- For removing air from the condenser using the injector.
- Spray painting
- Steam jet refrigeration system

## Velocity of steam

Steam enters the nozzle with high pressure and low velocity, and leaves with high velocity and low pressure.

Consider a unit mass ( $m=1 \text{ kg}$ ) steady flow of steam through a nozzle.

Let  $V_1$  = Velocity of stream at the entrance of nozzle – m/s

$V_2$  = Velocity of steam at any section considered – m/s

$h_1$  = Enthalpy of steam entering the nozzle – kJ/kg

$h_2$  = Enthalpy of steam at any section considered – kJ/kg

For unit mass, the steady flow energy equation is,

$$q - w = \Delta h + \Delta PE + \Delta KE$$

For a horizontal nozzle,  $\Delta PE = 0$ ,  $Q=0$  and  $W=0$ .

$$h_1 + \frac{1}{1000} \times \frac{1}{2} m V_1^2 = h_2 + \frac{1}{1000} \times \frac{1}{2} m V_2^2 + \text{losses} \quad \text{kJ/kg} \quad (\because Q = 0 \text{ & } W = 0)$$

$$h_1 + \frac{V_1^2}{2000} = h_2 + \frac{V_2^2}{2000} \quad (\because \text{for unit mass, } m = 1 \text{ & by neglecting losses in nozzle})$$

$$h_1 - h_2 = \frac{V_2^2 - V_1^2}{2000}$$

$$V_2^2 - V_1^2 = 2000(h_1 - h_2)$$

$$V_2 = \sqrt{V_1^2 + 2000(h_1 - h_2)}$$

$$= \sqrt{2000(h_1 - h_2)}$$

$$= 44.72 \sqrt{(h_1 - h_2)}$$

( $\because$  Inlet velocity  $V_1$  is negligible as compared to outlet velocity  $V_2$ )

### **Mass of the steam discharged through nozzle**

The isentropic process in nozzle may be approximately expressed by,

$$pv^n = \text{Constant}$$

where,  $n = 1.135$  for saturated steam

$= 1.3$  for superheated steam

Let  $P_1$  = initial pressure of steam

$P_2$  = pressure of steam at the throat or exit

$v_1$  = specific volume of steam at entry

$v_2$  = specific volume of steam at pressure  $p_2$

$V_1$  = Velocity of steam at entry

$V_2$  = velocity of steam at exit or throat

**Note:** As the steam passes through the nozzle, its pressure is dropped. So the enthalpy is also dropped. This reduction of enthalpy must be equal to the increase in kinetic energy, Hence the work done by the steam upon itself is equal to the enthalpy drop.

For the process  $pv^n = \text{constant}$ , the work done is  $\frac{n}{n-1} (P_1 v_1 - P_2 v_2)$

#### **Additional reading:**

The flow of steam through a nozzle may be regarded as either an ideal adiabatic (isentropic) flow, or adiabatic flow modified by friction and supersaturation.

If friction is negligible, three steps are essential in the process of expansion from pressure  $P_1$  to  $p_2$ :

(i) Driving of steam upto the nozzle inlet from the boiler. The 'flow-work' done on the steam is  $p_1 v_1$  and results in similar volume of steam being forced through the exit to make room for fresh charge (steam).

(ii) Expansion of steam through the nozzle while pressure changes from  $p_1$ , to  $p_2$ , the work done being  $\frac{1}{n-1} (P_1 v_1 - P_2 v_2)$

Where,  $n$  is the index of the isentropic expansion,

$v_1$  = volume occupied by 1 kg of steam at entrance to nozzle, and

$v_2$  = volume occupied by 1 kg of steam as it leaves the nozzle.

Alternatively, this work done is equal to the change of internal energy,  $u_1 - u_2$  as during isentropic expansion work is done at the cost of internal energy.

(iii) Displacement of the steam from the low pressure zone by an equal volume discharged from the nozzle. This work amounts to  $P_2 V_2$  which is equal to the final flow work spent in forcing the steam out to make room for fresh charge (steam).

Thus, the new work done in increasing kinetic energy of the steam,

$$W = p_1 v_1 + \frac{1}{n-1} (p_1 v_1 - p_2 v_2) - p_2 v_2$$

$$= \frac{n}{n-1} (p_1 v_1 - p_2 v_2)$$

This is same as the work done during Rankine cycle.

$$\text{Alternatively, } W = p_1 v_1 + (u_1 - u_2) - p_2 v_2 \\ = (u_1 + p_1 v_1) - (u_2 + p_2 v_2) \\ = h_1 - h_2$$

Gain in Kinetic energy = work done during isentropic process

$$\frac{V_2^2}{2} - \frac{V_1^2}{2} = \frac{n}{n-1} (p_1 v_1 - p_2 v_2)$$

$$\frac{V_2^2}{2} = \frac{n}{n-1} (p_1 v_1 - p_2 v_2) \quad (\because \text{As } V_1 \ll V_2, V_1 \text{ can be neglected})$$

$$= \frac{n}{n-1} p_1 v_1 \left( 1 - \frac{p_2 v_2}{p_1 v_1} \right)$$

$$= \frac{n}{n-1} p_1 v_1 \left( 1 - \frac{p_2}{p_1} \left( \frac{p_1}{p_2} \right)^{\frac{1}{n}} \right) \quad \left( \because p_1 v_1^n = p_2 v_2^n \Rightarrow \frac{v_2}{v_1} = \left( \frac{p_1}{p_2} \right)^{\frac{1}{n}} \right)$$

$$= \frac{n}{n-1} p_1 v_1 \left( 1 - \frac{p_2}{p_1} \left( \frac{p_2}{p_1} \right)^{-\frac{1}{n}} \right)$$

$$= \frac{n}{n-1} p_1 v_1 \left( 1 - \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right)$$

$$V_2 = \sqrt{\frac{2n}{n-1} p_1 v_1 \left( 1 - \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right)}$$

We know that,

Mass of steam discharged through nozzle per second,

$$m = \frac{\text{Volume of steam flowing per second}}{\text{Specific volume of steam}}$$

$$= \frac{\text{Area} \times \text{velocity}}{\text{Specific volume of steam}}$$

$$= \frac{A \times V_2}{v_2}$$

$$= \frac{A}{v_2} \sqrt{\frac{2n}{n-1} p_1 v_1 \left( 1 - \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right)}$$

$$= \frac{A}{v_1 \left( \frac{p_1}{p_2} \right)^{\frac{1}{n}}} \sqrt{\frac{2n}{n-1} p_1 v_1 \left( 1 - \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right)} \quad \left( \because p_1 v_1^n = p_2 v_2^n \Rightarrow \frac{v_2}{v_1} = \left( \frac{p_1}{p_2} \right)^{\frac{1}{n}} \Rightarrow v_2 = v_1 \left( \frac{p_1}{p_2} \right)^{\frac{1}{n}} \right)$$

$$= \frac{A}{v_1} \left( \frac{p_2}{p_1} \right)^{\frac{1}{n}} \sqrt{\frac{2n}{n-1} p_1 v_1 \left( 1 - \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right)}$$

$$= \frac{A}{v_1} \sqrt{\frac{2n}{n-1} p_1 v_1 \left( \left( \frac{p_2}{p_1} \right)^{\frac{2}{n}} - \left( \frac{p_2}{p_1} \right)^{\frac{2}{n} + \frac{n-1}{n}} \right)}$$

$$= A \sqrt{\frac{2n}{n-1} \frac{p_1 v_1}{v_1^2} \left( \left( \frac{p_2}{p_1} \right)^{\frac{2}{n}} - \left( \frac{p_2}{p_1} \right)^{\frac{n+1}{n}} \right)}$$

$$m = A \sqrt{\frac{2n}{n-1} \frac{p_1}{v_1} \left( \left( \frac{p_2}{p_1} \right)^{\frac{2}{n}} - \left( \frac{p_2}{p_1} \right)^{\frac{n+1}{n}} \right)}$$

## Condition for maximum discharge

Mass of steam discharged through the nozzle,

$$m = A \sqrt{\frac{2n}{n-1} \frac{p_1}{v_1} \left( \left( \frac{p_2}{p_1} \right)^{\frac{2}{n}} - \left( \frac{p_2}{p_1} \right)^{\frac{n+1}{n}} \right)}$$

There is only one value of the ratio (called critical pressure ratio)  $\frac{p_2}{p_1}$  which will produce the maximum discharge.

This can be obtained by differentiating 'm' with respect to  $\frac{p_2}{p_1}$  and equating it to zero.

*all other values are constant*

$$\frac{dm}{d[\frac{p_2}{p_1}]} = 0$$

$$\frac{d}{d[\frac{p_2}{p_1}]} \left( \left( \frac{p_2}{p_1} \right)^{\frac{2}{n}} - \left( \frac{p_2}{p_1} \right)^{\frac{n+1}{n}} \right) = 0$$

$$\frac{2}{n} \left( \frac{p_2}{p_1} \right)^{\frac{2}{n}-1} - \frac{n+1}{n} \left( \frac{p_2}{p_1} \right)^{\frac{n+1}{n}-1} = 0$$

$$\frac{2}{n} \left( \frac{p_2}{p_1} \right)^{\frac{2-n}{n}} = \frac{n+1}{n} \left( \frac{p_2}{p_1} \right)^{\frac{1}{n}}$$

$$\frac{\left( \frac{p_2}{p_1} \right)^{\frac{2-n}{n}}}{\left( \frac{p_2}{p_1} \right)^{\frac{1}{n}}} = \frac{n+1}{n} \times \frac{n}{2} = \frac{n+1}{2}$$

$$\left( \frac{p_2}{p_1} \right)^{\frac{2-n}{n}-\frac{1}{n}} = \frac{n+1}{2}$$

$$\left( \frac{p_2}{p_1} \right)^{\frac{1-n}{n}} = \frac{n+1}{2}$$

$$\left[ \left( \frac{p_2}{p_1} \right)^{\frac{1-n}{n}} \right]^{-1} = \left[ \frac{n+1}{2} \right]^{-1}$$

$$\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \frac{2}{n+1} \quad (i)$$

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \quad (ii)$$

This is the condition for maximum discharge.

### Maximum discharge,

$$\begin{aligned} m_{\max} &= A \sqrt{\frac{2n}{n-1} \frac{p_1}{v_1} \left( \left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}} \right)} \\ &= A \sqrt{\frac{2n}{n-1} \frac{p_1}{v_1} \left( \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \right)^{\frac{2}{n}} - \left(\frac{2}{n+1}\right)^{\frac{n+1}{n}}} \\ &= A \sqrt{\frac{2n}{n-1} \frac{p_1}{v_1} \left( \left(\frac{2}{n+1}\right)^{\frac{2}{n-1}} - \left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}} \right)} \end{aligned}$$

$$\text{For dry saturated steam, } n=1.135, \quad m_{\max} = 0.6356 A \sqrt{\frac{p_1}{v_1}}$$

$$\text{For super heated steam, } n=1.13, \quad m_{\max} = 0.667 A \sqrt{\frac{p_1}{v_1}}$$

$$\text{For super heated steam, } n=\gamma=1.4, \quad m_{\max} = 0.685 A \sqrt{\frac{p_1}{v_1}}$$

### Critical Pressure Ratio

There is only one value of the ratio (called critical pressure ratio)  $\frac{p_2}{p_1}$  which will produce the maximum discharge from the nozzle. This ratio is called critical pressure.

Where  $p_1$  = inlet pressure

$p_2$  = outlet pressure

- (i) For saturated steam,  $n=1.135$ , Critical pressure ratio,

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} = \left(\frac{2}{1.135+1}\right)^{\frac{1.135}{1.135-1}} = 0.577$$

(ii) For super heated steam,  $n=1.13$ , Critical pressure ratio,

$$\frac{p_2}{p_1} = \left( \frac{2}{n+1} \right)^{\frac{n}{n-1}} = \left( \frac{2}{1.13+1} \right)^{\frac{1.13}{1.13-1}} = 0.546$$

(iii) For gases,  $n=\gamma=1.4$ , Critical pressure ratio,

$$\frac{p_2}{p_1} = \left( \frac{2}{n+1} \right)^{\frac{n}{n-1}} = \left( \frac{2}{1.4+1} \right)^{\frac{1.4}{1.4-1}} = 0.5282$$

### Super saturated or Metastable flow through nozzle

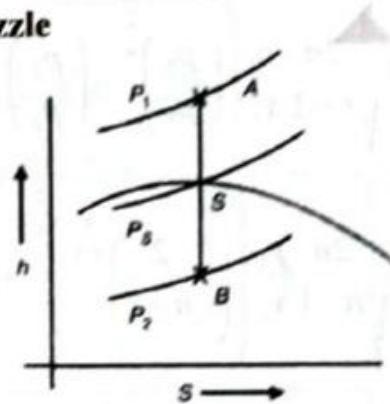
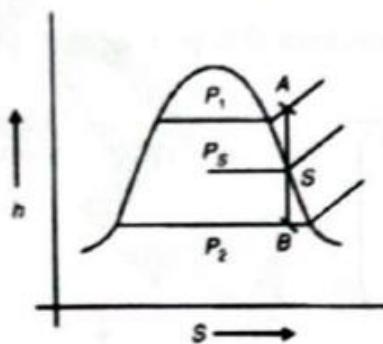


Fig. 19.6

Ideal Expansion of superheated steam in an nozzle is isentropic, and is shown by a vertical line on Mollier diagram. During the expansion, at point S, where the pressure is  $P_S$ , steam becomes dry saturated and condensation (a change of phase) should begin occur.

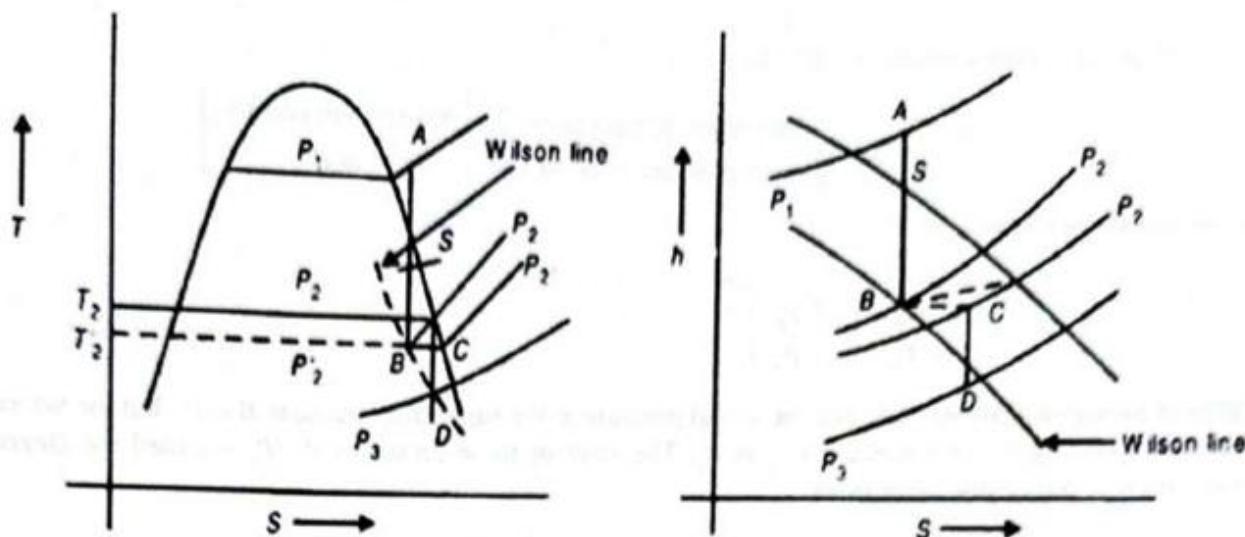
But from practical point of view, the steam has a great velocity (sometimes sonic and even supersonic). Thus, in nozzles, the phenomenon of condensation does not occur at the expected rate at point S, as the time available is very short (about 0.001 sec) due to the very high velocity of steam passing through the nozzle (near sonic).

As a result of this, equilibrium between the liquid and the vapour phase is delayed and the steam continues to expand in a dry state. The steam in such a set of conditions between states S and B, is said to be **supersaturated** or in **metastable** state. It is also called **supercooled** steam, as its temperature at any pressure is less than the saturation temperature corresponding to the pressure. The flow of supersaturated steam, through the nozzle is called supersaturated flow or metastable flow.

The steam in states between S and B is supersaturated or a metastable state.

**Note 1:** The steam during the expansion AS remains dry and condensation is suppressed.

**Note 2:** Thus condensation does not start immediately after S is passed, no drops of liquid are formed until some state B is reached, where condensation suddenly occurs, a phenomenon sometimes called Condensation Shock.

**Fig. 19.7**

A limit to the supersaturated state was observed by Wilson and a line drawn on the chart through the observed points is known as **Wilson line**. This line becomes the saturation line for all practical purposes.

The flow is also called **super cooled flow** because at any pressures between  $p_2$  and  $p_3$  the temperature of the vapour is always lower than the saturation temperature corresponding to that pressure. The difference in this temperature is known as the **degree of under – cooling**.

When the expansion reaches the Wilson line (as shown by point B), the condensation occurs at constant enthalpy, the pressure remaining constant. This is shown by BC. Further isentropic expansion to the exit pressure represented by CD.

The ratio of saturation pressures corresponding to the temperatures at S and B is known as the degree of supersaturation. During the process BC, the partial condensation of steam releases sufficient heat to raise the temperature of the steam back to the saturation temperature.

The problem on superheated flow cannot be solved by using Mollier chart unless Wilson line is drawn on it.

#### **Wilson's Line:**

There is generally a limit to super-saturation. It is up to 96% dryness and beyond it, steam condensation occurs suddenly and irreversibly at constant enthalpy and remains in stable condition thereafter. This limit line is known as Wilson's line.

$$\therefore \text{Degree of under-cooling} = T_2 - T_2'$$

$$= \left[ \begin{array}{l} \text{Saturation temperature} \\ \text{corresponding to point } B \end{array} \right] - \left[ \begin{array}{l} \text{Actual temperature} \\ \text{at } B \end{array} \right]$$

For this purpose we have

$$\frac{T_B}{T_A} = \left( \frac{P_B}{P_A} \right)^{\frac{1}{\gamma}}$$

**Degree of Supersaturation:** Note that the actual pressure at the supersaturated state  $B$  is  $P_2$ , but the saturation pressure corresponding to the temperature  $t_s$  is  $P_2'$ . The ratio of these pressures  $P_2/P_2'$  is called the *Degree of Supersaturation or Supersaturation ratio*.

$$\text{Degree of supersaturation} = \frac{\text{Actual pressure}}{\text{Saturation pressure corresponding to actual temperature } (T_2')} \\ = \frac{P_2}{P_2'}$$

**Effects of Supersaturated Flow:** The following effects in the nozzle, in which super saturation occurs are,

1. **Increase in discharge by 2 to 5% due to increase in density due to super cooling:** Since the condensation does not take place during super saturation expansion, so the temperature at which the super saturation occurs will be less than the saturation temperature corresponding to the pressure. Therefore, the density of supersaturated steam will be more than for the equilibrium conditions, which gives the increase in mass of steam discharged.
2. **Decrease in exit velocity:** The super saturation reduces the heat drop below that for thermal equilibrium. Hence the exit velocity of the steam is reduced.
3. The super saturation **increases dryness fraction** of the steam and increase in Enthalpy.
4. The super saturation increases the entropy and specific volume of the steam.

### Ref:

<https://www.engineeringnotes.com/thermal-engineering/nozzle/nozzle-applications-general-flow-analysis-velocity-pressure-and-phenomenon-thermodynamics/50082>

- 19.1. Introduction. 19.2. Classification of steam turbines. 19.3. Advantages of steam turbine over the steam engines. 19.4. Description of common types of turbines. 19.5. Methods of reducing wheel or rotor speed. 19.6. Difference between impulse and reaction turbines. 19.7. Impulse turbines—Velocity diagram for moving blade—Work done on the blade—Blade velocity co-efficient—Expression for optimum value of the ratio of blade speed to steam speed (for maximum efficiency) for a single stage impulse turbine—Advantages of velocity compounded impulse turbine. 19.8. Reaction turbines—Velocity diagram for reaction turbine blade—Degree of reaction ( $R_d$ )—Condition for maximum efficiency. 19.9. Turbines efficiencies. 19.10. Types of power in steam turbine practice. 19.11. "State point locus" and "Reheat factor". 19.12. Reheating steam. 19.13. Bleeding. 19.14. Energy losses in steam turbines. 19.15. Steam turbine governing and control. 19.16. Special forms of steam turbines—Highlights—Objective Type Questions—Theoretical Questions—Unsolved Examples.

### 19.1. INTRODUCTION

The **steam turbine** is a prime-mover in which the potential energy of the steam is transformed into kinetic energy, and latter in its turn is transformed into the mechanical energy of rotation of the turbine shaft. The turbine shaft, directly or with the help of a reduction gearing, is connected with the driven mechanism. Depending on the type of the driven mechanism a steam turbine may be utilised in most diverse fields of industry, for power generation and for transport. Transformation of the potential energy of steam into the mechanical energy of rotation of the shaft is brought about by different means.

### 19.2. CLASSIFICATION OF STEAM TURBINES

There are several ways in which the steam turbines may be classified. The most important and common division being with respect to the *action of the steam*, as :

- (a) Impulse.
- (b) Reaction.
- (c) Combination of impulse and reaction.

Other classification are :

#### 1. According to the number of pressure stages :

- (i) *Single stage turbines* with one or more velocity stages usually of small power capacities ; these turbines are mostly used for driving centrifugal compressors, blowers and other similar machinery.
- (ii) *Multistage impulse and reaction turbines* ; they are made in a wide range of power capacities varying from small to large.

#### 2. According to the direction of steam flow :

- (i) *Axial turbines* in which steam flows in a direction parallel to the axis of the turbine.

### ADVANTAGES OF STEAM TURBINE OVER STEAM ENGINES

The following are the *principal advantages of steam turbine over steam engines* :

1. The thermal efficiency of a steam turbine is much higher than that of a steam engine.
2. The power generation in a steam turbine is at a uniform rate, therefore necessity to use a flywheel (as in the case of steam engine) is not felt.
3. Much higher speeds and greater range of speed is possible than in case of a steam engine.
4. In large thermal stations where we need higher outputs, the steam turbines prove very suitable as these can be made in big sizes.
5. With the absence of reciprocating parts (as in steam engine) the balancing problem is minimised.
6. No internal lubrication is required as there are no rubbing parts in the steam turbine.
7. In a steam turbine there is no loss due to initial condensation of steam.
8. It can utilise high vacuum very advantageously.
9. Considerable overloads can be carried at the expense of slight reduction in overall efficiency.

### DESCRIPTION OF COMMON TYPES OF TURBINES

The common types of steam turbines are :

1. Simple impulse turbine.
2. Reaction turbine.

The main difference between these turbines lies in the way in which the steam is expanded while it moves through them. In the former type steam expands in the nozzles and its pressure does not alter as it moves over the blades while in the latter type the steam expands continuously as it passes over the blades and thus there is gradual fall in the pressure during expansion.

#### 1. Simple impulse turbines

Fig. 19.1 shows a simple impulse turbine diagrammatically. The top portion of the figure exhibits a longitudinal section through the upper half of the turbine, the middle portion shows one set of nozzles which is followed by a ring of moving blades, while lower part of the diagram indicates approximately changes in pressure and velocity during the flow of steam through the turbine. This turbine is called '*simple*' impulse turbine since the expansion of the steam takes place in *one set of the nozzles*.

As the steam flows through the nozzle its pressure falls from steam chest pressure to condenser pressure (or atmospheric pressure if the turbine is non-condensing). Due to this relatively higher ratio of expansion of steam in the nozzles the steam leaves the nozzle with a very high velocity. Refer Fig. 19.1, it is evident that the velocity of the steam leaving the moving blades is a large portion of the maximum velocity of the steam when leaving the nozzle. The loss of energy due to this higher exit velocity is commonly called the "carry over loss" or "leaving loss".

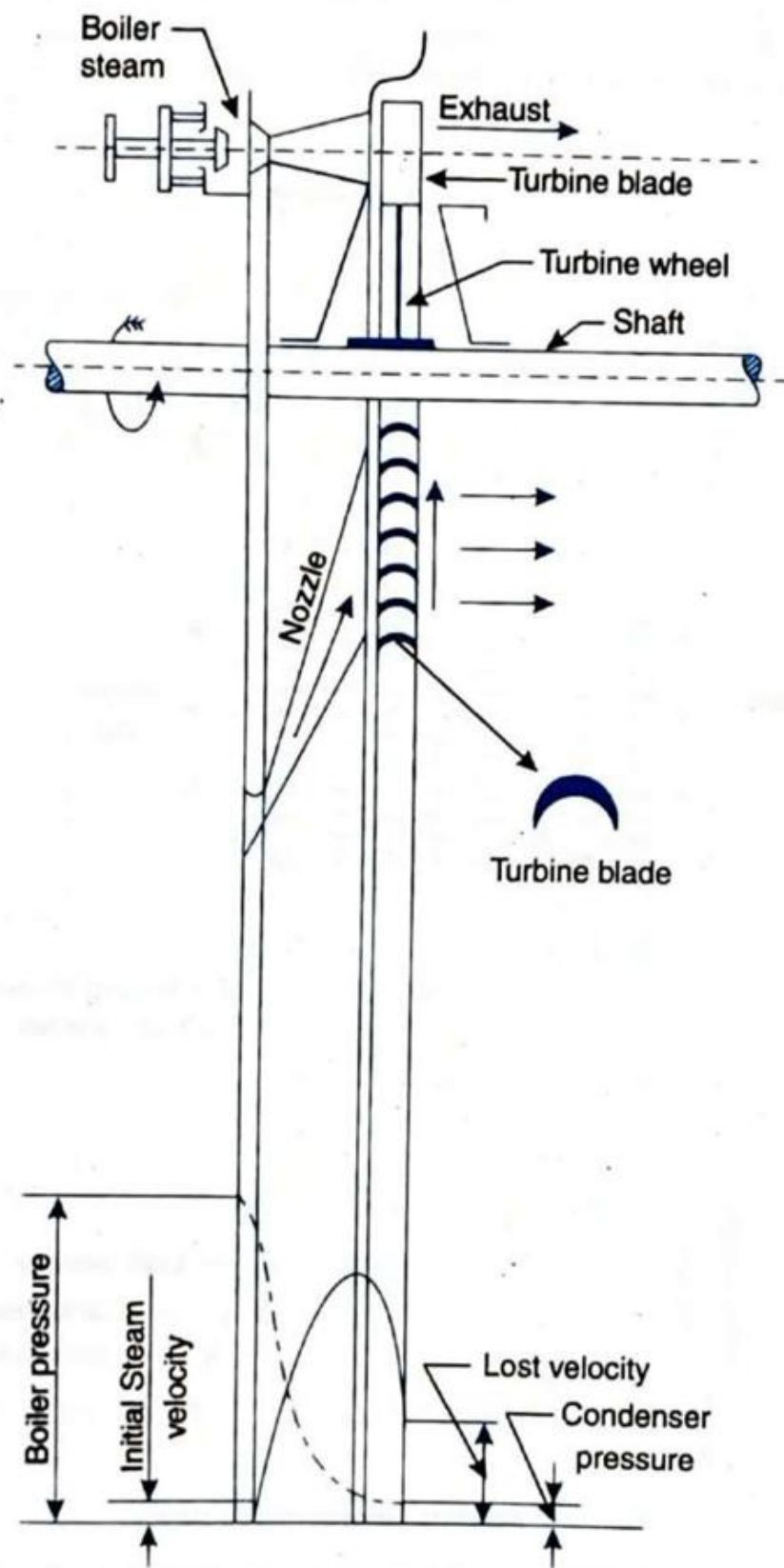


Fig. 19.1. Simple impulse turbine.

The principal example of this turbine is the well known "De Laval turbine" and in this turbine the 'exit velocity' or 'leaving velocity' or 'lost velocity' may amount to 3.3 per cent of the nozzle outlet velocity. Also since all the kinetic energy is to be absorbed by one ring of the moving

blades only, the velocity of wheel is too high (varying from 25000 to 30000 r.p.m.). This wheel or rotor speed however, can be reduced by different methods (discussed in the following article).

## 2. Reaction turbine

In this type of turbine, *there is a gradual pressure drop and takes place continuously over the fixed and moving blades*. The function of the fixed blades is (the same as the nozzle) that they alter the direction of the steam as well as allow it expand to a larger velocity. As the steam passes over the moving blades its kinetic energy (obtained due to fall in pressure) is absorbed by them. Fig. 19.2. shows a three stage reaction turbine. The changes in pressure and velocity are also shown there in.

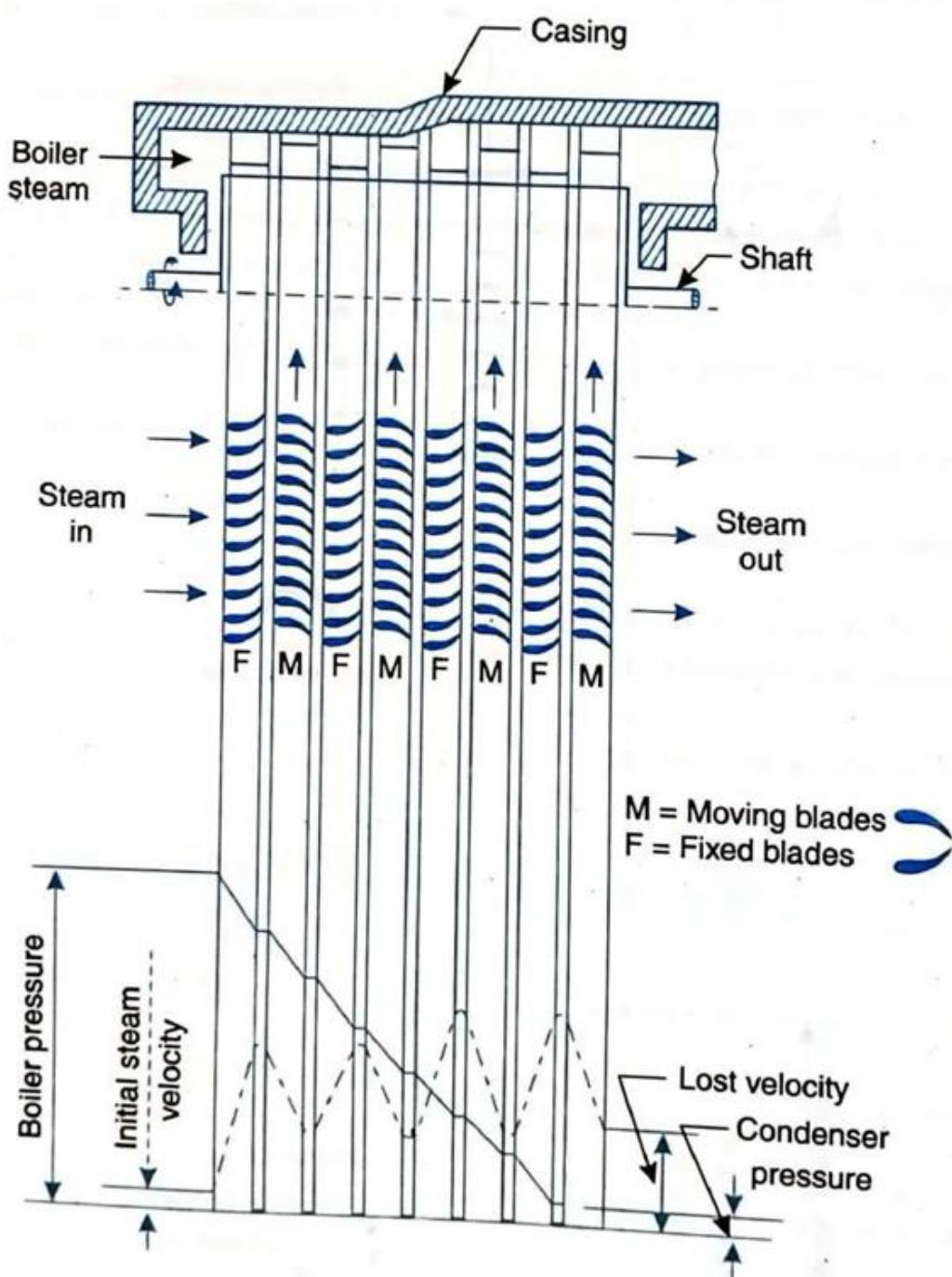


Fig. 19.2. Reaction turbine (three stage).

As the volume of steam increases at lower pressures therefore, the diameter of the turbine must increase after each group of blade rings. It may be noted that in this turbine since the pressure drop per stage is small, therefore the number of stages required is much higher than an impulse turbine of the same capacity.

## 19.6. DIFFERENCE BETWEEN IMPULSE AND REACTION TURBINES

S. No.	Particulars	Impulse turbine	Reaction turbine
1.	<i>Pressure drop</i>	Only in nozzles and not in moving blades.	In fixed blades (nozzles) as well as in moving blades.
2.	<i>Area of blade channels</i>	Constant.	Varying (converging type).
3.	<i>Blades</i>	Profile type.	Aerofoil type.
4.	<i>Admission of steam</i>	Not all round or complete.	All round or complete.
5.	<i>Nozzles /fixed blades</i>	Diaphram contains the nozzle.	Fixed blades similar to moving blades attached to the casing serve as nozzles and guide the steam.
6.	<i>Power</i>	Not much power can be developed.	Much power can be developed.
7.	<i>Space</i>	Requires less space for same power.	Requires more space for same power.
8.	<i>Efficiency</i>	Low.	High.
9.	<i>Suitability</i>	Suitable for small power requirements.	Suitable for medium and higher power requirements.
10.	<i>Blade manufacture</i>	Not difficult.	Difficult.