

## HYDRAULIC MACHINES

### **HYDRO-DYNAMIC FORCE OF JETS:**

The liquid comes out in the form of a jet from the outlet of a nozzle, the liquid is flowing under pressure. If some plate, which may be fixed or moving, is placed in the path of the jet, a force is exerted by the jet on the plate. This force is obtained by Newton's second law of motion or from Impulse – Momentum equation.

$$F = ma$$

Thus the impact of jet means, the force exerted by the jet on a plate, which may be stationary or moving.

The following cases of impact of jet i.e. the force exerted by the jet on a plate will be considered.

1) Force exerted by the jet on a stationary plate, when

- a) Plate is vertical to the jet.
- b) Plate is inclined to the jet and
- c) Plate is curved

2) Force exerted by the jet on a moving plate, when

- a) Plate is vertical to the jet.
- b) Plate is inclined to the jet.
- c) Plate is curved.

### **a) FORCE EXERTED BY THE JET ON A STATIONARY VERTICAL PLATE:**

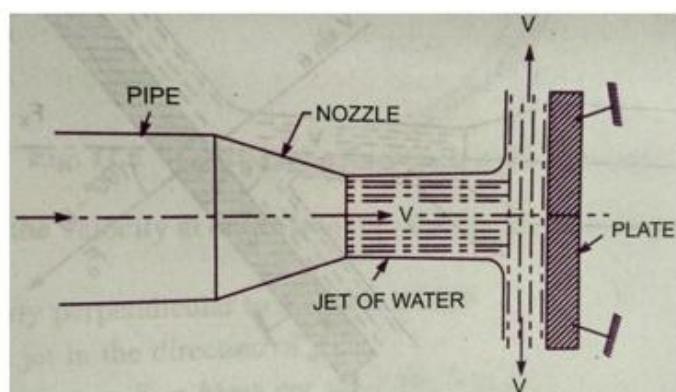
Consider a jet of water coming out from the nozzle, strikes a flat vertical plate.

Let  $V$  = Velocity of jet.

$d$  = Diameter of jet.

$$a = \text{Area of cross-section of jet.} = \frac{\pi}{4} d^2$$

The jet of water after striking the plate will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking will be deflected through  $90^\circ$ .



Hence the component of the velocity of the jet, in the direction of jet, after striking will be zero.

The force exerted by the jet on the plate in the direction of jet,

$F_x = \text{Rate of change of momentum in the direction of force.}$

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{Final velocity}}{\text{Time}}$$

For deriving the above equation, we have taken initial velocity minus final velocity and not final velocity minus initial velocity. If the force exerted on the jet is to be calculated, then final velocity minus initial velocity is to be taken. But if the force exerted by the jet on the plate is to be calculated, then initial velocity minus final velocity is to be taken.

### b) Force exerted by a jet on a stationary inclined Flat plate:

Let a jet of water coming out from the nozzle, strikes an inclined flat plate.

$V$  = Velocity of jet in the direction of X

$a$  = Area of cross-section of jet.

Mass of water per second striking the plate =  $\square a v$

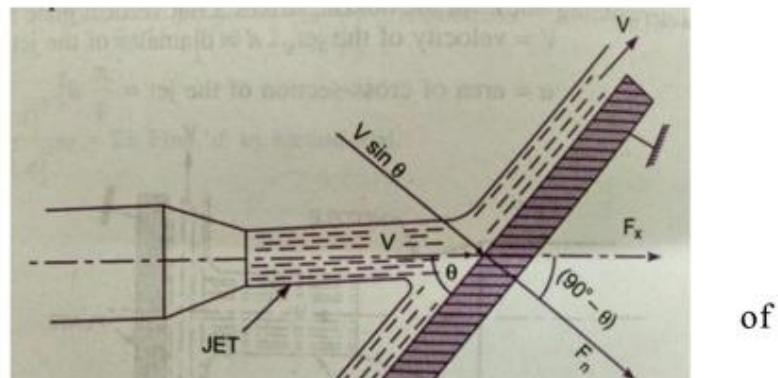
If the plate is smooth and there is no loss energy due to impact of the jet, the jet will move over the plate after striking with a velocity equal to initial velocity.

with a velocity  $V$ . Let us find the force exerted by the jet on the plate ~~in the direction normal to the plate~~. Let this force is represented by  $F_n$ .

Then  $F_n = \text{Mass of jet striking per second}$

$\times (\text{Initial velocity of jet before striking in the direction of } n)$

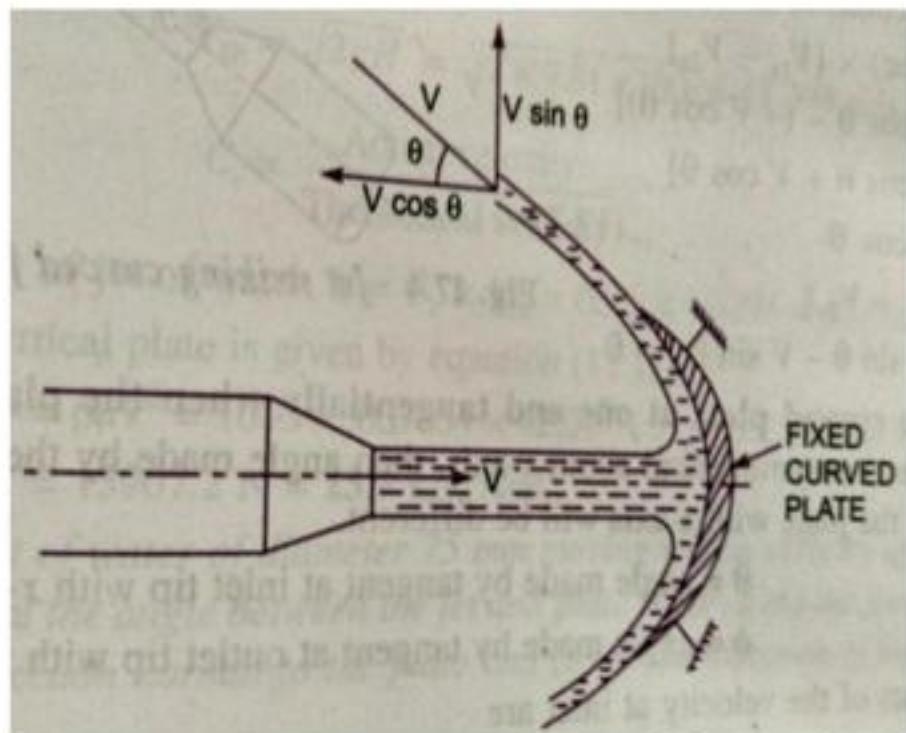
$- (\text{Final velocity of jet after striking in the direction of } n)$



### c) Force exerted by a jet on a stationary Curved plate:

#### i) Jet strikes the curved plate at the centre:

The jet after striking the plate comes out with same velocity, if the plate is smooth and there is no loss of energy due to impact of the jet, in the tangential direction of the curved plate. The velocity at the outlet of the plate can be resolved into two components, one in the direction of the jet and other perpendicular to the direction of jet.



Component of velocity in the direction of jet

(-ve sign is taken as the velocity at outlet is in the opposite direction of the jet of water coming out from nozzle.)

Component of velocity perpendicular to the jet

Force exerted by the jet in the direction of the jet

$$F_x = \text{Mass per sec} (V_{1x} - V_{2x})$$

Where  $V_{1x}$  = Initial velocity in the direction of jet =  $V$

$V_{2x}$  = Final velocity in the direction of jet

Similarly  $F_y = \text{Mass per second} (V_{1y} - V_{2y})$

Where  $V_{1y}$  = Initial velocity in the direction of  $y = 0$

$V_{2y}$  = Final velocity in the direction of  $y$

-ve sign means the force is acting in the downward direction.

In this case the angle of deflection of jet =  $180^\circ - \theta$

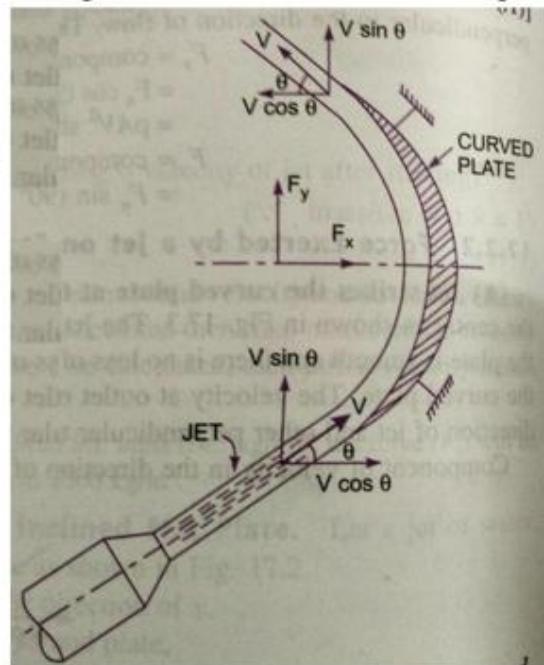
## ii) Jet strikes the curved plate at one end tangentially when the plate is symmetrical:

Let the jet strikes the curved fixed plate at one end tangentially. Let the curved plate is symmetrical about  $x$ -axis. Then the angle made by the tangents at the two ends of the plate will be same.

Let  $V$  = Velocity of jet of water.

Angle made by the jet with  $x$ -axis at the inlet tip of the curved plate.

If the plate is smooth and loss of energy due to impact is zero, then the velocity of water at the outlet tip of the curved plate will be equal to  $V$ . The force exerted by the jet of water in the direction of  $x$  and  $y$  are



## iii) Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical:

When the curved plate is unsymmetrical about  $x$ - axis, then the angles made by tangents drawn at inlet and outlet tips of the plate with  $x$ - axis will be different.

Let Angle made by tangent at the inlet tip with  $x$ -axis.

Angle made by tangent at the outlet tip with  $x$ -axis

The two components of velocity at inlet are

# FORCE EXERTED BY A JET ON MOVING PLATES

The following cases of the moving plates will be considered:

- Flat vertical plate moving in the direction of jet and away from the jet.
- Inclined plate moving in the direction of jet and
- Curved plate moving in the direction of jet or in the horizontal direction.

## a) Force on flat vertical plate moving in the direction of jet:

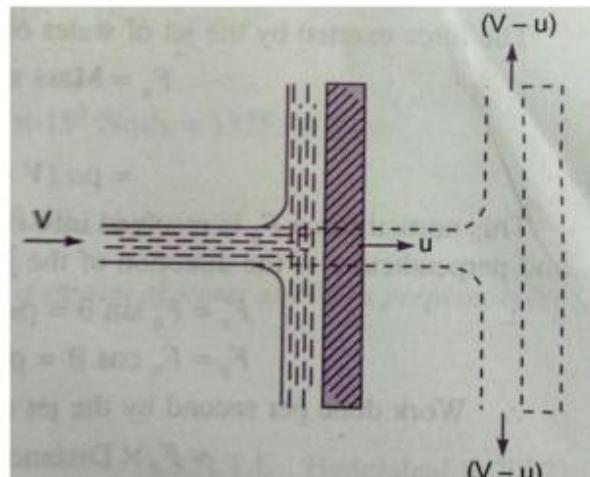
Let a jet of water striking a flat vertical plate moving with a uniform velocity away from the jet.

Let  $V$  = Velocity of jet.

$a$  = Area of cross-section of jet.

$u$  = Velocity of flat plate.

In this case, the jet does not strike the plate with a velocity  $v$ , but it strikes with a relative velocity, which is equal to the absolute velocity of jet of water minus velocity of the plate.



Hence relative velocity of the jet with respect to plate =  $V - u$

Mass of water striking the plate per second

∴ Force exerted by the jet on the moving in the direction of the plate

$F_x = \text{mass of water striking per second} \times (\text{Initial velocity with which water strikes} - \text{Final velocity})$

Since final velocity in the direction of jet is zero.

In this, case the work will be done by the jet on the plate, as the plate is moving. For stationary plates, the work done is zero.

∴ The work done per second by the jet on the plate

Time

$$= F_x \times u = \square a(v - u)^2 \times u \quad \dots \dots \dots (2)$$

In the above equation (2), if the value of  $\square$  for water is taken in S.I units (i.e.  $1000 \text{ kg/m}^3$ ) the work done will be in  $\text{N m/s}$ . The term  $\frac{\text{Nm}}{\text{s}}$  is equal to Watt (W).

## b) Force on inclined plate moving in the direction of jet:

Let a jet of water strikes an inclined plate, which is moving with a uniform velocity in the direction of jet.

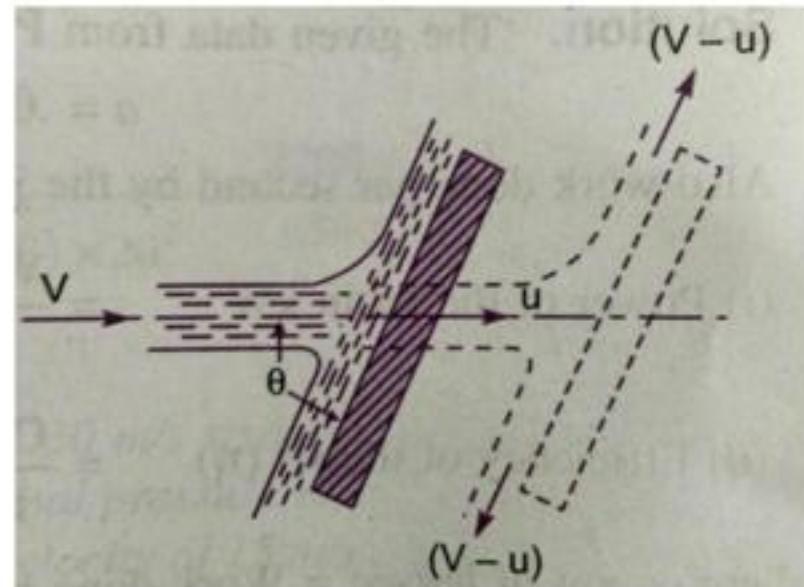
Let  $V$  = Absolute velocity of water.

$u$  = Velocity of plate in the direction of jet.

$a$  = Cross-sectional area of jet

Angle between jet and plate.

Relative velocity of jet of water



The velocity with which jet strikes

Mass of water striking per second =  $a(V - u)$

If the plate is smooth and loss of energy due to impact of the jet is assumed zero, the jet of water will leave the inclined plate with a velocity equal

The force exerted by the jet of water on the plate in the direction normal to the plate is given as

$F_n$  = Mass striking per sec  $\times$  (Initial velocity in the normal direction with which jet strikes – final velocity)

Work done per second by the jet on the plate

$$= F_x \times \text{Distance per second in the direction of } x$$

### c) Force on the curved plate when the plate is moving in the direction of jet:

Let a jet of water strikes a curved plate at the centre of the plate, which is moving with a uniform velocity in the direction of jet.

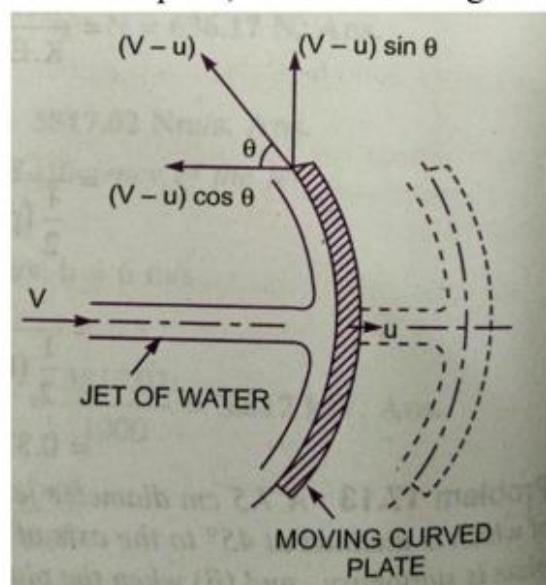
Let  $V$  = absolute velocity of jet.

$a$  = area of jet.

$u$  = Velocity of plate in the direction of jet.

Relative velocity of jet of water or the velocity with which jet strikes the curved plate =  $V - u$

If the plate is smooth and the loss of energy due to impact of jet is zero, then the velocity with which the jet will be leaving the curved vane =  $(V - u)$



This velocity can be resolved into two components, one in the direction of jet and the other perpendicular to the direction of jet.

Component of the velocity in the direction of jet =  $-(V - u) \cos\theta$

(-ve sign is taken as at the outlet, the component is in the opposite direction of the jet).

Component of velocity in the direction perpendicular to the direction of jet =  $(V - u) \sin\theta$

∴ Mass of water striking the plate = velocity with which jet strikes the plate.

$$= (V - u)$$

∴ Force exerted by the jet of water on the curved plate in the direction of jet  $F_x$

$F_x$  = Mass striking per sec [Initial velocity with which jet strikes the plate in the direction of jet - Final velocity]

Work done by the jet on the plate per second

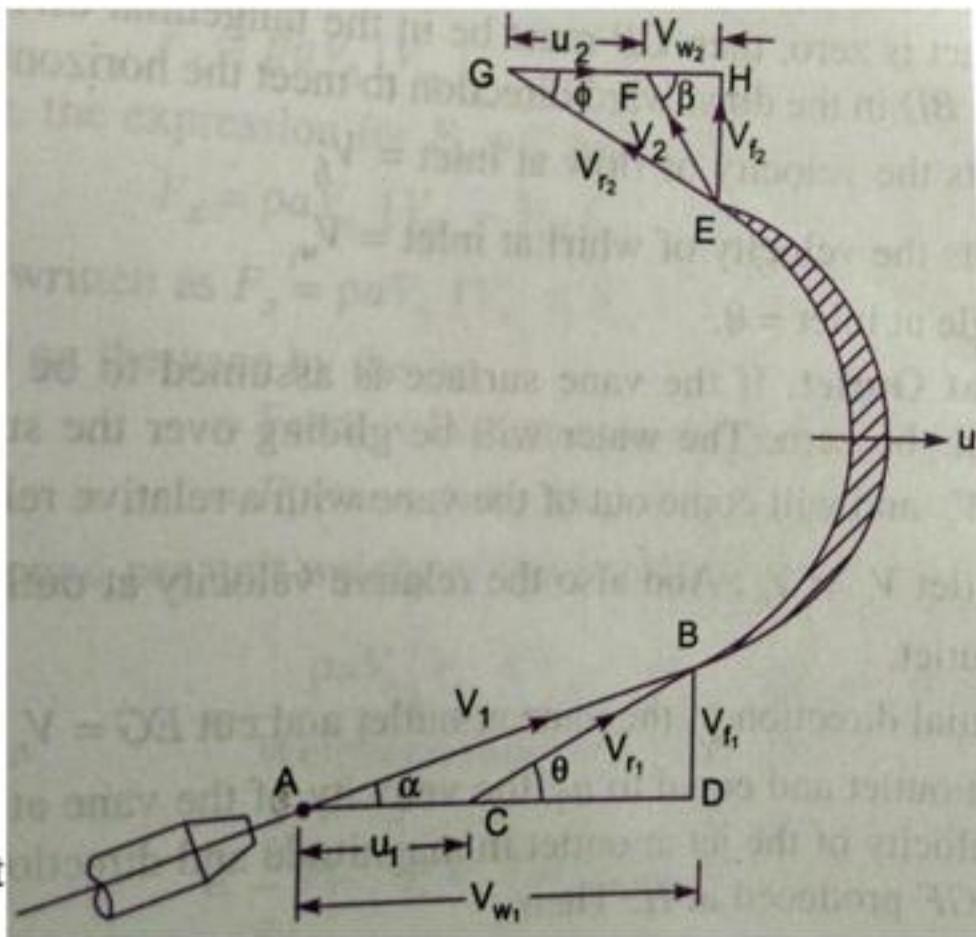
$$= F_x \times \text{Distance travelled per second in the direction of } x$$

#### d) Force exerted by a jet of water on an un-symmetrical moving Curved plate when Jet strikes tangentially at one of the tips:

Let a jet of water striking a moving curved plate tangentially, at one of its tips.

As the jet strikes tangentially, the loss of energy due to impact of the jet will be zero. In this case as plate is moving, the velocity with which the jet of water strikes is equal to the relative velocity of jet with respect to the plate. Also as the plate is moving in different direction of the jet, the relative velocity at inlet will be equal to the vector difference of velocity of jet and velocity of the plate at inlet.

The triangles ABD and EGH are called velocit



**Velocity triangle at inlet:** Take any point A and draw a line AB =  $V_1$  in magnitude and direction which means line AB makes an angle  $\alpha$  with the horizontal line AD. Next draw a line AC =  $u_1$  in magnitude. Join C to B. Then CB represents the relative velocity of the jet at inlet. If the loss of energy at inlet due to impact is zero, then CB must be in the tangential direction to the vane at inlet. From B draw a vertical line BD in the downward direction to meet the horizontal line AC produced at D.

Then  $BD = \text{Represents the velocity of flow at inlet} =$

$\square u_1$   $AD = \text{Represents the velocity of whirl at inlet} =$

$\square u_1$

$\angle BCD = \text{Vane angle at inlet} = \alpha$

**Velocity triangle at outlet:** If the vane surface is assumed to be very smooth, the loss of energy due to friction will be zero. The water will be gliding over the surface of the vane with a relative velocity  $u_1$  and will come out of the vane with a relative velocity  $u_2$ . This means that the relative

velocity at outlet  $u_2 = u_1$ . The relative velocity at outlet should be in tangential direction to the vane at outlet.

Draw EG in the tangential direction of the vane at outlet and cut EG =  $u_2$ . From G, draw a line GF in the direction of vane at outlet and equal to  $u_2$ , the velocity of vane at outlet. Join EF. Then EF represents the absolute velocity of the jet at outlet in magnitude and direction. From E draw a vertical line EH to meet the line GF produced at H.

Then,  $EH = \text{Velocity of flow at outlet.} = \square u_2$ .

$FH = \text{Velocity of whirl at outlet} = \square u_2$

$\phi = \text{Angle of vane at outlet}$

$\alpha = \text{Angle made by } V_2 \text{ with the direction of motion of vane at outlet.}$

If vane is smooth and is having velocity in the direction of motion at inlet and outlet equal, then we have

$u_1 = u_2 = \square = \text{velocity of vane in the direction of motion and}$

$$u_1 = u_2$$

Now mass of water striking the vane per second  $\square \square u_1$  ..... (1)

Where  $a = \text{area of jet of water}$ ,  $u_1 = \text{Relative velocity at inlet.}$

$\therefore$  Force exerted by the jet in the direction of motion

$F_x = \text{Mass of water striking per second} \times [\text{Initial velocity with which jet strikes in the}]$

direction of motion – Final velocity of jet in the direction of motion] ----- (2)

## HYDRAULIC TURBINES

Turbines are defined as the hydraulic machines which converts hydraulic energy in to mechanical energy. This mechanical energy is used in running an electric generator which is directly coupled to the shaft of Turbine. Thus mechanical energy is converted in to electrical energy. The electric power which is obtained from the hydraulic energy is known as the Hydro-electric power.

**Efficiency of a Turbine:** The following are the important efficiencies of Turbine.

- a) Hydraulic Efficiency,  $\eta_h$
- b) Mechanical Efficiency,  $\eta_m$
- c) Volumetric Efficiency,  $\eta_v$
- d) Overall Efficiency,  $\eta_o$

**a) Hydraulic Efficiency ( $\eta_h$ ):** it is defined as the ratio of power given by the water to the runner of a turbine (runner is a rotating part of a turbine and on the runner vanes are fixed) to the power supplied by the water at the inlet of the turbine. The power at the inlet of the turbine is more and this power goes on decreasing as the water flows over the vanes of the turbine due to hydraulic losses as the vanes are not smooth. Hence power delivered to the runner of the turbine will be less than the power available at the inlet of the turbine.

$$\eta_h = \frac{\text{Power delivered to the runner}}{\text{Power supplied at inlet}} = \frac{\text{R.P}}{\text{W.P}}$$

$$\text{R.P} = \text{Power delivered to the runner} = \frac{W}{g} \frac{[V_{w_1} + V_{w_2}] \times u}{1000} \quad \text{kW} \quad \text{for Pelton Turbine}$$

$$= \frac{W}{g} \frac{[V_{w_1} u_1 + V_{w_2} u_2] \times u}{1000} \quad \text{kW} \quad \text{Radial flow Turbine.}$$

$$\text{W.P} = \text{power supplied at inlet of turbine} = \frac{W \times H}{1000} \quad \text{kW}$$

Where  $W$  = weight of water striking the vanes of the turbine per second =  $\rho g Q$

$Q$  = Volume of water per second

$V_{w_1}$  = Velocity of whirl at inlet.

$V_{w_2}$  = Velocity of whirl at outlet

$u$  = Tangential velocity of vane

$u_1$  = Tangential velocity of vane at inlet of radial vane.

$u_2$  = Tangential velocity of vane at outlet of radial vane.

$H$  = Net head on the Turbine.

Power supplied at the inlet of the turbine in S I Units is known as Water Power.

$$W.P = \frac{\rho \times g \times Q \times H}{1000} \text{ K.W} \quad (\text{For water } \rho = 1000 \text{Kg/m}^3)$$

$$= \frac{1000 \times g \times Q \times H}{1000} = g \times Q \times H \text{ kW}$$

**b) Mechanical Efficiency ( $\eta_m$ ):** The power delivered by the water to the runner of a turbine is transmitted to the shaft of the turbine. Due to mechanical losses, the power available at the shaft of the turbine is less than the power delivered to the runner of the turbine. The ratio of power available at the shaft of the turbine (Known as S.P or B.P) to the power delivered to the runner is defined as Mechanical efficiency.

$$\eta_m = \frac{\text{Power at the shaft of the turbine}}{\text{Power delivered by the water to the runner}} = \frac{S.P}{R.P}$$

**c) Volumetric Efficiency ( $\eta_v$ ):** The volume of the water striking the runner of the turbine is slightly less than the volume of water supplied to the turbine. Some of the volume of the water is discharged to the tailrace without striking the runner of the turbine. Thus the ratio of the volume of the water supplied to the turbine is defined as Volumetric Efficiency.

$$\eta_v = \frac{\text{Volume of water actually striking the Runner}}{\text{Volume of water supplied to the Turbine}}$$

**d) Overall Efficiency ( $\eta_0$ )** It is defined as the ratio of power available at the shaft of the turbine to the power supplied by the water at the inlet of the turbine.

$$\begin{aligned} \eta_0 &= \frac{\text{Power available at the shaft of the turbine}}{\text{Power supplied at the inlet of the turbine}} = \frac{\text{Shaft power}}{\text{Water power}} \\ &= \frac{S.P}{W.P} = \frac{S.P}{W.P} \times \frac{R.P}{R.P} \\ &= \frac{S.P}{R.P} \times \frac{R.P}{W.P} \end{aligned}$$

$$\eta_0 = \eta_m \times \eta_v$$

If shaft power (S.P) is taken in kW, Then water power should also be taken in kW. Shaft power is represented by P.

$$\text{Water power in } kW = \frac{\rho \times g \times Q \times H}{1000} \quad \text{Where } \rho = 1000 \text{Kg/m}^3$$

$$\eta_0 = \frac{\text{Shaft Power in kW}}{\text{Water Power in kW}} = \frac{P}{\frac{\rho \times g \times Q \times H}{1000}} \quad \text{Where } P = \text{Shaft Power}$$

## **CLASSIFICATION OF HYDRAULIC TURBINES:**

The Hydraulic turbines are classified according to the type of energy available at the inlet of the turbine, direction of flow through the vanes, head at the inlet of the turbine and specific speed of the turbine. The following are the important classification of the turbines.

1. According to the type of energy at inlet:
  - (a) Impulse turbine and
  - (b) Reaction turbine
2. According to the direction of flow through the runner:
  - (a) Tangential flow turbine
  - (b) Radial flow turbine.
  - (c) Axial flow turbine
  - (d) Mixed flow turbine.
3. According to the head at inlet of the turbine:
  - (a) High head turbine
  - (b) Medium head turbine and
  - (c) Low head turbines.
4. According to the specific speed of the turbine:
  - (a) Low specific speed turbine
  - (b) Medium specific speed turbine
  - (c) High specific speed turbine.

If at the inlet of turbine, the energy available is only kinetic energy, the turbine is known as **Impulse turbine**. As the water flows over the vanes, the pressure is atmospheric from inlet to outlet of the turbine. If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy, the turbine is known as **Reaction turbine**. As the water flows through runner, the water is under pressure and the pressure energy goes on changing into kinetic energy. The runner is completely enclosed in an air-tight casing and the runner and casing is completely full of water.

If the water flows along the tangent of runner, the turbine is known as **Tangential flow turbine**. If the water flows in the radial direction through the runner, the turbine is called **Radial flow turbine**. If the water flows from outward to inwards radially, the turbine is known as **Inward** radial flow turbine, on the other hand, if the water flows radially from inward to outwards, the turbine is known as **outward** radial flow turbine. If the water flows through the runner along the direction parallel to the axis of rotation of the runner, the turbine is called **axial flow** turbine. If the water flows through the runner in the radial direction but leaves in the direction parallel to the axis of rotation of the runner, the turbine is called **mixed flow** turbine.

## **PELTON WHEEL (Turbine)**

It is a tangential flow impulse turbine. The water strikes the bucket along the tangent of the runner. The energy available at the inlet of the turbine is only kinetic energy. The pressure at the inlet and outlet of turbine is atmospheric. This turbine is used for high heads and is named after L.A.Pelton an American engineer.

The water from the reservoir flows through the penstocks at the outlet of which a nozzle is fitted. The nozzle increases the kinetic energy of the water flowing through the penstock. At the outlet of the nozzle, the water comes out in the form of a jet and strikes the buckets (vanes) of the runner. The main parts of the Pelton turbine are:

1. Nozzle and flow regulating arrangement (spear)
2. Runner and Buckets.
3. Casing and
4. Breaking jet

**1. Nozzle and flow regulating arrangement:** The amount of water striking the buckets (vanes) of the runner is controlled by providing a spear in the nozzle. The spear is a conical needle which is operated either by hand wheel or automatically in an axial direction depending upon the size of the unit. When the spear is pushed forward in to the nozzle, the amount of water striking the runner is reduced. On the other hand, if the spear is pushed back, the amount of water striking the runner increases.

**2. Runner with buckets:** It consists of a circular disc on the periphery of which a number of buckets evenly spaced are fixed. The shape of the buckets is of a double hemispherical cup or bowl. Each bucket is divided in to two symmetrical parts by a dividing wall, which is known as splitter.

The jet of water strikes on the splitter. The splitter divides the jet in to two equal parts and the jet comes out at the outer edge of the bucket. The buckets are shaped in such a way that the jet gets deflected through an angle of  $160^{\circ}$  or  $170^{\circ}$ . The buckets are made of cast Iron, cast steel, Bronze or stainless steel depending upon the head at the inlet of the turbine.

**3. Casing:** The function of casing is to prevent the splashing of the water and to discharge the water to tailrace. It also acts as safeguard against accidents. It is made of Cast Iron or fabricated steel plates. The casing of the Pelton wheel does not perform any hydraulic function.

**4. Breaking jet:** When the nozzle is completely closed by moving the spear in the forward direction, the amount of water striking the runner reduces to zero. But the runner due to inertia goes on revolving for a long time. To stop the runner in a short time, a small nozzle is provided, which directs the jet of water on the back of the vanes. This jet of water is called Breaking jet.

## Velocity triangles and work done for Pelton wheel:

The jet of water from the nozzle strikes the bucket at the splitter, which splits up the jet into two parts. These parts of the jet, glides over the inner surfaces and comes out at the outer edge. The splitter is the inlet tip and outer edge of the bucket is the outlet tip of the bucket. The inlet velocity triangle is drawn at the splitter and outer velocity triangle is drawn at the outer edge of the bucket.

Let

$H$  = Net head acting on the Pelton Wheel

$$= H_g - h_f$$

Where  $H_g$  = Gross Head

$$h_f = \frac{4fLV^2}{D^4 \times 2g}$$

Where  $D^*$  = diameter of penstock,

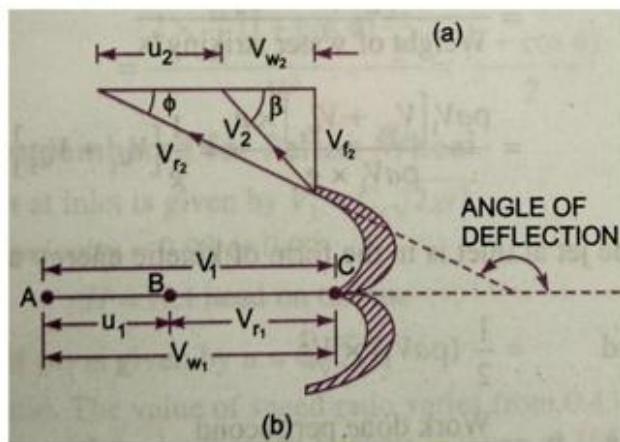
$D$  = Diameter of wheel,

$d$  = Diameter of Jet,

$N$  = Speed of the wheel in r.p.m

Then  $V_1$  = Velocity of jet at inlet

$$= \sqrt{2gH} \quad u = u_1 = u_2 = \frac{\pi DN}{60}$$



The Velocity Triangle at inlet will be a straight line where

$$V_{r_1} = V_1 - u_1 = V_1 - u$$

$$V_{w_1} = V_1 \quad \alpha = 0^\circ \quad \text{and} \quad \theta = 0^\circ$$

From the velocity triangle at outlet, we have

$$V_{r_2} = V_{r_1} \quad \text{and} \quad V_{w_2} = V_{r_2} \cos \theta - u_2$$

The force exerted by the Jet of water in the direction of motion is

$$F_x = \rho a V_1 [V_{w_1} + V_{w_2}] \quad (1)$$

As the angle  $\beta$  is an acute angle, +ve sign should be taken. Also this is the case of series of vanes, the mass of water striking is  $\rho a V_1$  and not  $\rho a V_{r_2}$ . In equation (1), 'a' is the area of the jet =  $\frac{\pi}{4} d^2$

Now work done by the jet on the runner per second

$$= F_x \times u = \rho a V_1 [V_{w_1} + V_{w_2}] \times u \quad \text{Nm/s}$$

$$\text{Power given to the runner by the jet} = \frac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{1000} \text{ kW}$$

$$\begin{aligned} \text{Work done/s per unit weight of water striking/s} &= \frac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{\text{Weight of water } \frac{\text{striking}}{s}} \\ &= \frac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{\rho a V_1 \times g} = \frac{1}{g} [V_{w_1} + V_{w_2}] \times u \quad (3) \end{aligned}$$

The energy supplied to the jet at inlet is in the form of kinetic energy

$$\therefore \text{K.E. of jet per second} = \frac{1}{2} m V^2 = \frac{1}{2} (\rho a V_1) \times V_1^2$$

$$\begin{aligned} \therefore \text{Hydraulic efficiency, } \eta_h &= \frac{\text{Work done per second}}{\text{K.E. of jet per second}} \\ &= \frac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{\frac{1}{2} (\rho a V_1) \times V_1^2} \\ &= \frac{2 [V_{w_1} + V_{w_2}] \times u}{V_1^2} \quad (4) \end{aligned}$$

$$\text{Now } V_{w_1} = V_1 \text{ and } V_{r_1} = V_1 - u_1 = (V_1 - u)$$

$$\begin{aligned} \therefore V_{r_2} &= (V_1 - u) \\ \text{And } V_{w_2} &= V_{r_2} \cos \phi - u_2 \end{aligned}$$

$$= \frac{V_r \cos \phi - u}{(V_1 - u) \cos \phi - u}$$

Substituting the values of  $V_{w_1}$  and  $V_{w_2}$  in equation (4)

$$\begin{aligned} \eta_h &= \frac{2 [V_1 + (V_1 - u) \cos \phi - u] \times u}{V_1^2} = \frac{2 [V_1 - u + (V_1 - u) \cos \phi] \times u}{V_1^2} \\ &= \frac{2(V_1 - u)[1 + \cos \phi]u}{V_1^2} \quad (5) \end{aligned}$$

The efficiency will be maximum for a given value of  $V_1$  when

$$\frac{d}{du} (\eta_h) = 0 \text{ or } \frac{d}{du} \left[ \frac{2u(V_1 - u)[1 + \cos \phi]}{V_1^2} \right] = 0$$

$$\text{Or } \frac{(1 + \cos \phi)}{V_1^2} \frac{d}{du} (2uV_1 - 2u^2) = 0$$

$$\text{Or } \frac{d}{du} [2uV_1 - 2u^2] = 0 \quad \left( \because \frac{1 + \cos \phi}{V_1^2} \neq 0 \right)$$

$$\text{Or } 2V_1 - 4u = 0 \quad \text{Or} \quad u = \frac{V_1}{2} \quad (6)$$

Equation (6) states that hydraulic efficiency of a Pelton wheel will be maximum when the velocity of the wheel is half the velocity of the jet water at inlet. The expression for maximum efficiency will be obtained by substituting the value of  $u = \frac{V_1}{2}$  in equation (5)

$$\text{Max. } \eta_h = \frac{2 \left( V_1 - \frac{V_1}{2} \right) (1 + \cos \phi) \times \frac{V_1}{2}}{V_1^2}$$

$$= \frac{2 \times \frac{V_1}{2} (1 + \cos \phi) \frac{V_1}{2}}{V_1^2} = \frac{(1 + \cos \phi)}{2} \quad \text{--- (7)}$$

## RADIAL FLOW REACTION TURBINE:

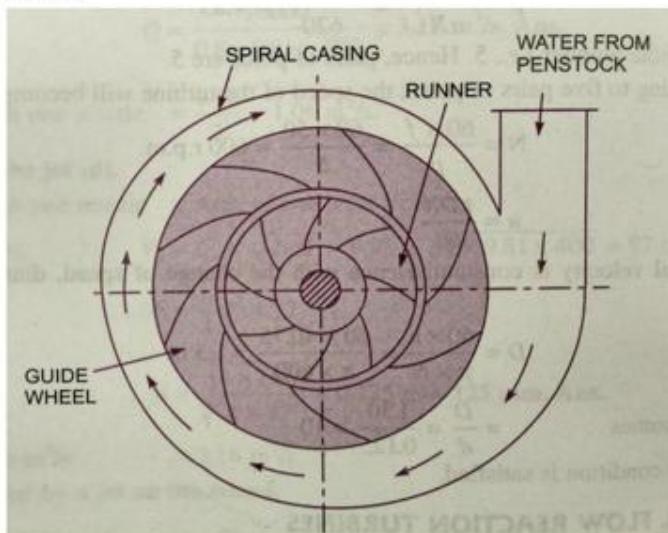
In the Radial flow turbines water flows in the radial direction. The water may flow radially from outwards to inwards (i.e. towards the axis of rotation) or from inwards to outwards. If the water flows from outwards to inwards through the runner, the turbine is known as **inwards radial flow turbine**. And if the water flows from inwards to outwards, the turbine is known as **outward radial flow turbine**.

Reaction turbine means that the water at the inlet of the turbine possesses kinetic energy as well as pressure energy. As the water flows through the runner, a part of pressure energy goes on changing into kinetic energy. Thus the water through the runner is under pressure. The runner is completely enclosed in an air-tight casing and the runner is always full of water.

### Main parts of a Radial flow Reaction turbine:

1. Casing
2. Guide mechanism
3. Runner and
4. Draft tube.

**1. Casing:** in case of reaction turbine, casing and runner are always full of water. The water from the penstocks enters the casing which is of spiral shape in which area of cross-section one of the casing goes on decreasing gradually. The casing completely surrounds the runner of the turbine. The water enters



the runner at constant velocity throughout the circumference of the runner.

**2. Guide Mechanism:** It consists of a stationary circular wheel all around the runner of the turbine. The stationary guide vanes are fixed on the guide mechanism. The guide vanes allow the water to strike the vanes fixed on the runner without shock at inlet. Also by suitable arrangement, the width between two adjacent vanes of guide mechanism can be altered so that the amount of water striking the runner can be varied.

**3. Runner:** It is a circular wheel on which a series of radial curved vanes are fixed. The surfaces of the vanes are made very smooth. The radial curved vanes are so shaped that the water enters and leaves the runner without shock. The runners are made of cast steel, cast iron or stain less steel. They are keyed to the shaft.

**4. Draft - Tube:** The pressure at the exit of the runner of a reaction turbine is generally less than atmospheric pressure. The water at exit can't be directly discharged to the tail race. A tube or pipe of gradually increasing area is used for discharging the water from the exit of the turbine to the tail race. This tube of increasing area is called draft-tube.

**Inward Radial Flow Turbine:** In the inward radial flow turbine, in which case the water from the casing enters the stationary guiding wheel. The guiding wheel consists of guide vanes which direct the water to enter the runner which consists of moving vanes. The water flows over the moving vanes in the inward radial direction and is discharged at the inner

diameter of the runner. The outer diameter of the runner is the inlet and the inner diameter is the outlet.

### Velocity triangles and work done by water on runner:

Work done per second on the runner by water

$$= \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]$$

$$= \rho Q [V_{w_1} u_1 \pm V_{w_2} u_2] \quad \dots \quad (1) \quad (\because aV_1 = Q)$$

The equation represents the energy transfer per second to the runner.

Where  $V_{w_1}$  = Velocity of whirl at inlet

$V_{w_2}$  = Velocity of whirl at outlet

$u_1$  = Tangential velocity at inlet

$$= \frac{\pi D_2 \times N}{60}, \quad \text{Where } D_1 = \text{Outer dia. Of runner,}$$

$u_2$  = Tangential velocity at outlet

$$= \frac{\pi D_2 \times N}{60}$$

Where  $D_1$  = Inner dia. Of runner,

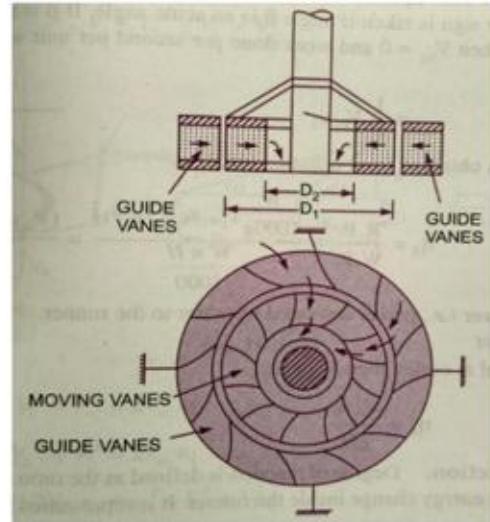
$N$  = Speed of the turbine in r.p.m.

The work done per second per unit weight of water per second

$$= \frac{\text{work done per second}}{\text{weight of water striking per second.}}$$

$$= \frac{\rho Q [V_{w_1} u_1 \pm V_{w_2} u_2]}{\rho Q \times g}$$

$$= \frac{1}{g} [V_{w_1} u_1 \pm V_{w_2} u_2] \quad \dots \quad (2)$$



Equation (2) represents the energy transfer per unit weight/s to the runner. This equation is known by **Euler's equation**.

In equation +ve sign is taken if  $\beta$  is an acute angle,

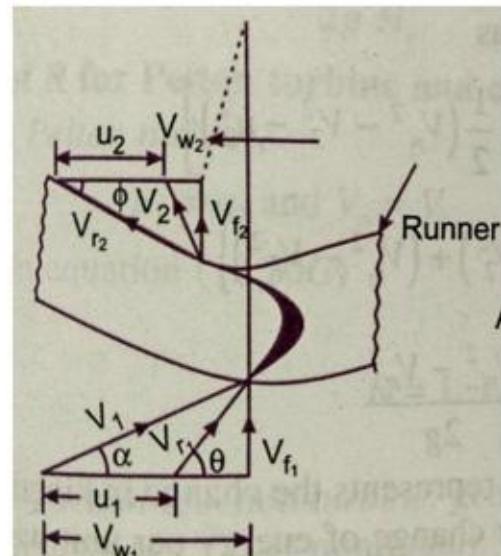
-ve sign is taken if  $\beta$  is an obtuse angle.

If  $\beta = 90^\circ$  then  $V_{w_2} = 0$  and work done per second per unit weight of water striking/s

$$\text{Work done} = \frac{1}{g} V_{w_1} u_1$$

$$\text{Hydraulic efficiency } \eta_h = \frac{\text{R.P.}}{\text{W.P.}} = \frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}}$$

$$= \frac{\frac{W}{1000 \times g} [V_{w_1} u_1 \pm V_{w_2} u_2]}{\frac{W \times H}{1000}} = \frac{(V_{w_1} u_1 \pm V_{w_2} u_2)}{gH} \quad \dots \quad (3)$$



Where R.P. = Runner Power i.e. power delivered by water to the runner  
W.P. = Water Power

If the discharge is radial at outlet, then  $V_{w_2} = 0$

$$\eta_h = \frac{V_{w_1} u_1}{gH}$$

### Definitions:

The following terms are generally used in case of reaction radial flow turbines which are defined as:

**1. Speed Ratio:** The speed ratio is defined as  $= \frac{u_1}{\sqrt{2gH}}$

Where  $u_1$  = tangential velocity of wheel at inlet

**2. Flow Ratio:** The ratio of velocity of flow at inlet ( $V_{f_1}$ ) to the velocity given  $\sqrt{2gH}$  is known as the flow ratio.

$$= \frac{V_{f_1}}{\sqrt{2gH}} \quad \text{Where } H = \text{Head on turbine}$$

**3. Discharge of the turbine:** The discharge through a reaction radial flow turbine is

$$Q = \pi D_1 B_1 \times V_{f_1} = \pi D_2 B_2 \times V_{f_2}$$

Where  $D_1$  = Dia of runner at inlet

$D_2$  = Dia of runner at outlet

$B_1$  = Width of the runner at inlet

$B_2$  = Width of runner at outlet

$V_{f_1}$  = Velocity of flow at inlet

$V_{f_2}$  = Velocity of flow at outlet

If the thickness of the vanes are taken into consideration then the area through which flow takes place is given by

$$= \pi D_1 - n \times t \quad \text{Where } n = \text{Number of vanes and}$$

$t$  = Thickness of each vane

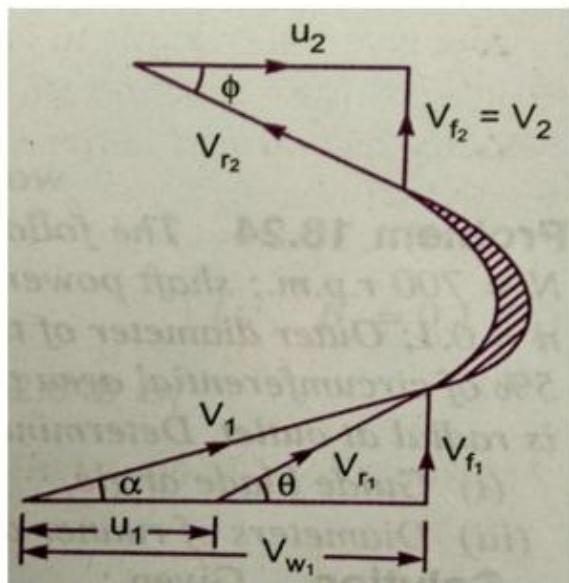
**4. Head:** The (H) on the turbine is given by

$$H = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} \quad \text{Where } P_1 = \text{Pressure at inlet}$$

**5. Radial Discharge:** This means the angle made by absolute velocity with the tangent on the wheel is  $90^\circ$  and the component of whirl velocity is zero. The radial discharge at outlet means  $\beta = 90^\circ$  and  $V_{w_2} = 0$  while radial discharge at inlet means  $\alpha = 90^\circ$  and  $V_{w_1} = 0$

6. If there is no loss of energy when the water flows through the vanes then we have

$$H - \frac{V_2^2}{2g} = \frac{1}{g} [V_{w_1} u_1 \pm V_{w_2} u_2]$$



### FRANCIS TURBINE:

The inward flow reaction turbine having radial discharge at outlet is known as Francis Turbine. The water enters the runner of the turbine in the radial direction at outlet and leaves in the axial direction at the inlet of the runner. Thus the Francis turbine is a mixed flow type turbine.

The velocity triangle at inlet and outlet of the Francis turbine are drawn in the same way as in case of inward flow reaction turbine. As in case of inward radial flow turbine. The discharge of Francis turbine is radial at outlet; the velocity of whirl at outlet ( $V_{w_2}$ ) will be zero. Hence the work done by water on the runner per second will be

$$= \rho Q [V_{w_1} u_1]$$

The work done per second per unit weight of water striking/sec =  $\frac{1}{g} [V_{w_1} u_1]$

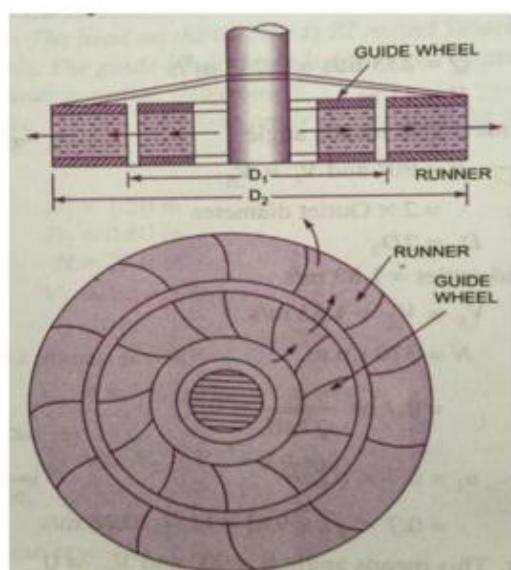
$$\text{Hydraulic efficiency } \eta_h = \frac{V_{w_1} u_1}{g H}$$

### Important relations for Francis turbines:

1. The ratio of width of the wheel to its diameter is given as  $n = \frac{B_1}{D_1}$ . The value of n varies from 0.10 to 0.40
2. The flow ratio is given as

$$\text{Flow ratio} = \frac{V_{f_1}}{\sqrt{2gH}} \text{ and varies from 0.15 to 0.30}$$

3. The speed ratio =  $\frac{u_1}{\sqrt{2gH}}$  varies from 0.6 to 0.9



## Outward radial flow Reaction Turbine:

In the outward radial flow reaction turbine water from the casing enters the stationary guide wheel. The guide wheel consists of guide vanes which direct the water to enter the runner which is around the stationary guide wheel. The water flows through the vanes of the runner in the outward radial direction and is discharged at the outer diameter of the runner. The inner diameter of the runner is inlet and outer diameter is the outlet.

The velocity triangles at inlet and outlet will be drawn by the same procedure as adopted for inward flow turbine. The work done by the water on the runner per second, the horse power developed and hydraulic efficiency will be obtained from the velocity triangles. In this case as the inlet of the runner is at the inner diameter of the runner, the tangential velocity at inlet will be less than that of an outlet. i.e.

$$u_1 < u_2 \quad \text{As } D_1 < D_2 \text{ All}$$

the working conditions flow through the runner blades without shock. As such eddy losses which are inevitable in Francis and propeller turbines are almost completely eliminated in a Kaplan turbine.

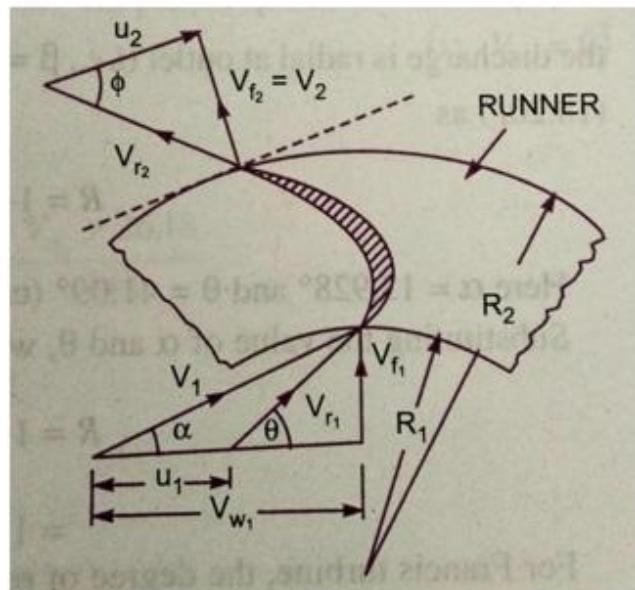
The discharge through the runner is obtained as

$$Q = \frac{\pi}{4} (D_o^2 - D_b^2) \times V_{f_1}$$

Where  $D_o$  = outer diameter of the runner

$D_b$  = Diameter of the hub

$V_{f_1}$  = Velocity of flow at inlet



### Important points for Kaplan turbine:

1. The peripheral velocity at inlet and outlet are equal.

$$u_1 = u_2 = \frac{\pi D_o N}{60}$$

Where  $D_o$  = Outer diameter of runner.

2. Velocity of flow at inlet and outlet are equal.

$$V_{f_1} = V_{f_2}$$

3. Area of flow at inlet = Area of flow at outlet

$$= \frac{\pi}{4} (D_o^2 - D_b^2)$$

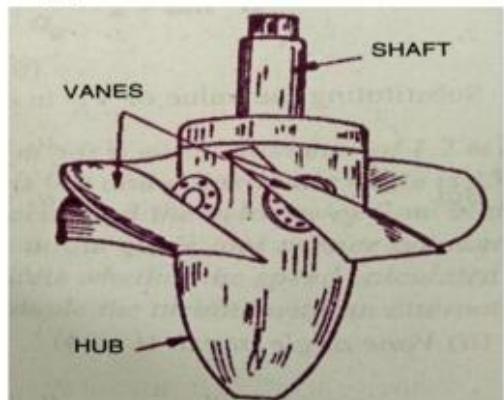
## AXIAL FLOW REACTION TURBINE:

If the water flows parallel to the axis of the rotation shaft the turbine is known as axial flow turbine. If the head at inlet of the turbine is the sum of pressure energy and kinetic energy and during the flow of the water through the runner a part of pressure energy is converted into kinetic energy, the turbine is known as reaction turbine.

For axial flow reaction turbine, the shaft of the turbine is vertical. The lower end of the shaft is made longer known as "hub" or "boss". The vanes are fixed on the hub and acts as a runner for the axial flow reaction turbine. The important types of axial flow reaction turbines are:

1. Propeller Turbine
2. Kaplan Turbine

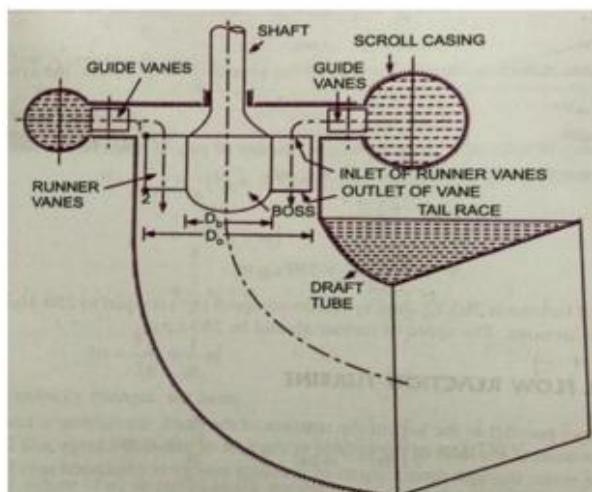
When the vanes are fixed to the hub and they are not adjustable the turbine is known as propeller turbine. But if the vanes on the hub are adjustable, the turbine is known as Kaplan turbine. This turbine is suitable, where large quantity of water at low heads is available.



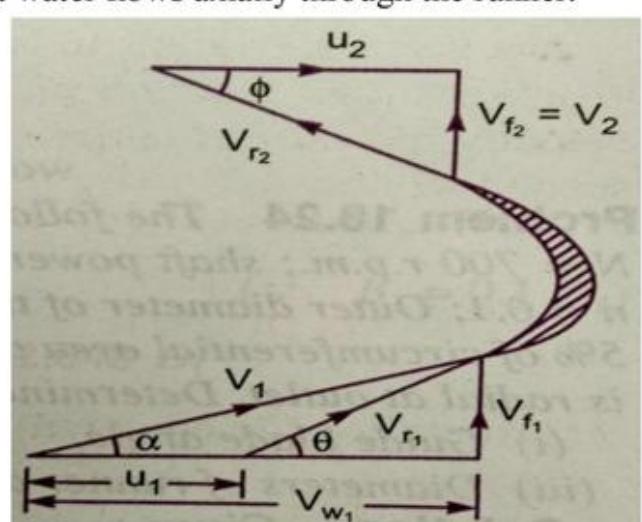
The main parts of the Kaplan turbine are:

1. Scroll casing
2. Guide vanes mechanism
3. Hub with vanes or runner of the turbine
4. Draft tube

Between the guide vanes and the runner the water in the Kaplan turbine turns through a right angle into the axial angle direction and then passes through the runner. The runner of the Kaplan turbine has four or six or eight in some cases blades and it closely resembles a ship's propeller. The blades (vanes) attached to a hub or bosses are so shaped that water flows axially through the runner.



The runner blades of a propeller turbine are fixed but the Kaplan turbine runner heads can be turned about their own axis, so that their angle of inclination may be adjusted while the turbine is in motion. The adjustment of the runner blades is usually carried out automatically by means of a servomotor operating inside the hollow coupling of turbine and generator shaft. When both guide vane angle and runner blade angle may thus be varied a



high efficiency can be maintained over a wide range of operating conditions. i.e. even at part load, when a lower discharge is flowing through the runner a high efficiency can be attained in case of Kaplan turbine. The flow through turbine runner does not affect the shape of velocity triangles as blade angles are simultaneously adjusted, the water under all the working conditions flows through the runner blades without shock. The eddy losses which are inevitable in Francis and propeller turbines are completely eliminated in a Kaplan Turbine.

## **Working Proportions of Kaplan Turbine:**

The main dimensions of Kaplan Turbine runners are similar to Francis turbine runner. However the following are main deviations,

- i. Choose an appropriate value of the ratio  $n = \frac{d}{D}$ , where d in hub or boss diameter and D is runner outside diameter. The value of n varies from 0.35 to 0.6
- ii. The discharge Q flowing through the runner is given by

$$Q = \frac{\pi}{4} (D^2 - d^2) V_f = \frac{\pi}{4} (D^2 - d^2) \psi \sqrt{2gH}$$

The value of flow ratio  $\psi$  for a Kaplan turbine is 0.7

- iii. The runner blades of the Kaplan turbine are twisted, the blade angle being greater at the outer tip than at the hub. This is because the peripheral velocity of the blades being directly proportional to radius. It will vary from section to section along the blade, and hence in order to have shock free entry and exit of water over the blades with angles varying from section to section will have to be designed.

## **DRAFT TUBE:**

The draft tube is a pipe of gradually increasing area, which connects the outlet of the runner to the tail race. It is used for discharging water from the exit of the turbine to the tail race. This pipe of gradually increasing area is called a draft tube. One end of the draft tube is connected to the outlet of the runner and the other end is submerged below the level of water in the tail race. The draft tube in addition to save a passage for water discharge has the following two purposes also:

1. It permits a negative head to be established at the outlet of the runner and thereby increase the net head on the turbine. The turbine may be placed above the tail race without any loss of net head and hence turbine may be inspected properly.
2. It converts a large portion of the kinetic energy  $\left(\frac{V_2^2}{2g}\right)$  rejected at the outlet of the turbine into useful energy. Without the draft tube the kinetic energy rejected at the turbine will go waste to the tail race.

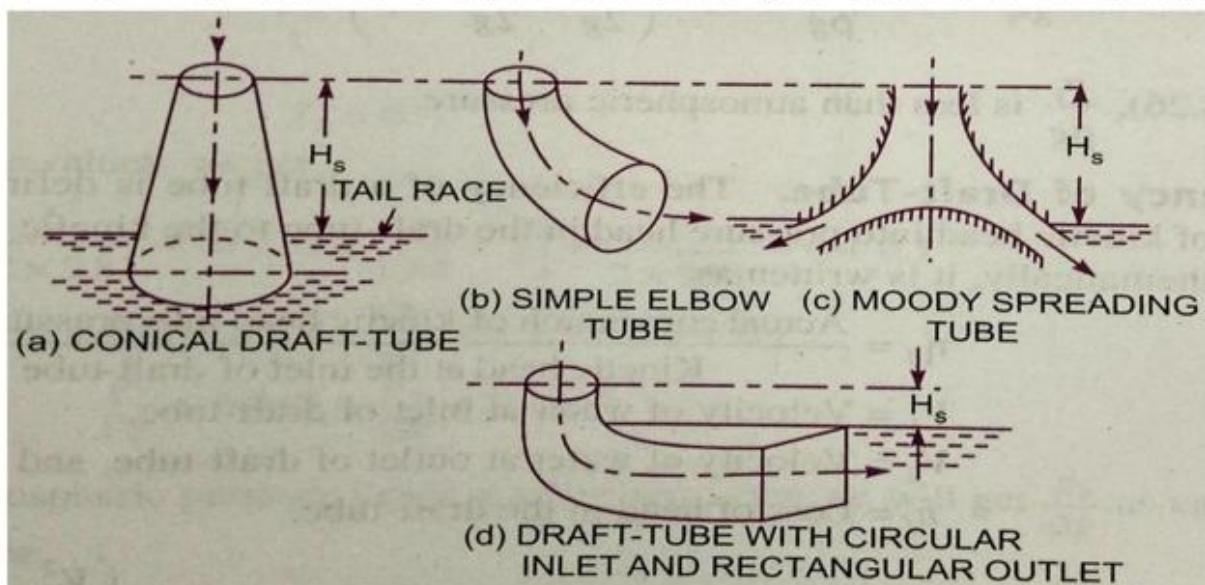
Hence by using the draft tube, the net head on turbine increases. The turbine develops more power and also the efficiency of the turbine increase.

If a reaction turbine is not fitted with a draft tube, the pressure at the outlet of the runner will be equal to atmospheric pressure. The water from the outlet of the runner will discharge freely into the tail race. The net head on the turbine will be less than that of

a reaction turbine fitted with a draft tube. Also without draft tube the kinetic energy ( $\frac{V_2^2}{2g}$ ) rejected at the outlet of the will go water to the tail race.

### Types of Draft Tube:

1. Conical Draft Tube
2. Simple Elbow Tubes
3. Moody Spreading tubes
4. Elbow Draft Tubes with Circular inlet and rectangular outlet



and moody spreading draft tubes are most efficient while simple elbow draft tube and elbow draft tubes with circular inlet and rectangular outlet require less space as compared to other draft tubes.

**Draft tube theory:** Consider a conical draft tube

$H_s$  = Vertical height of draft tube above tail race

$y$  = Distance of bottom of draft tube from tail race.

Applying Bernoulli's equation to inlet section 1-1 and outlet section 2-2 of the draft tube and taking section 2-2 a datum, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + (H_s + y) = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + 0 + h_f \quad (1)$$

Where  $h_f$  = loss of energy between section 1-1 and 2-2.

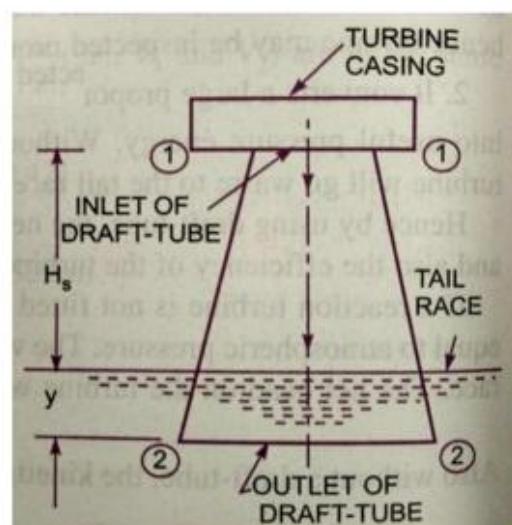
But  $\frac{p_2}{\rho g} = \text{Atmospheric Pressure} + y = \frac{p_a}{\rho g} + y$

Substituting this value of  $\frac{p_2}{\rho g}$  in equation (1) we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + (H_s + y) = \frac{p_a}{\rho g} + y + \frac{V_2^2}{2g} + h_f$$

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + H_s = \frac{p_a}{\rho g} + \frac{V_2^2}{2g} + h_f$$

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} + \frac{V_2^2}{2g} + h_f - \frac{V_1^2}{2g} - H_s$$



\_\_\_\_\_ (2)

In equation (2) is less than atmospheric pressure.

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - H_s - \left[ \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_f \right]$$

**Efficiency of Draft Tube:** the efficiency of a draft tube is defined as the ratio of actual conversion of kinetic head in to pressure in the draft tube to the kinetic head at the inlet of the draft tube.

$$\eta_d = \frac{\text{Actual conversion of Kinetic head in to Pressure head}}{\text{Kinetic head at the inlet of draft tube}}$$

Let  $V_1$  = Velocity of water at inlet of draft tube

$V_2$  = Velocity of water at outlet of draft tube

$h_f$  = Loss of head in the draft tube

Theoretical conversion of Kinetic head into Pressure head in

$$\text{Draft tube} = \left[ \frac{V_1^2}{2g} - \frac{V_2^2}{2g} \right]$$

$$\text{Actual conversion of Kinetic head into pressure head} = \left[ \frac{V_1^2}{2g} - \frac{V_2^2}{2g} \right] - h_f$$

Now Efficiency of draft tube

$$\eta_d = \frac{\left[ \frac{V_1^2}{2g} - \frac{V_2^2}{2g} \right] - h_f}{\frac{V_1^2}{2g}}$$

## PROBLEMS

1. A pelton wheel has a mean bucket speed of 10m/s with a jet of water flowing at the rate of 700lts/sec under a head of 30 m. the buckets deflect the jet through an angle of  $160^\circ$  calculate the power given by the water to the runner and hydraulic efficiency of the turbine? Assume co-efficient of velocity=0.98

**Given:**

Speed of bucket  $u = u_1 = u_2 = 10 \text{ m/s}$

Discharge  $Q = 700 \text{ lts/sec} = 0.7 \text{ m}^3/\text{s}$

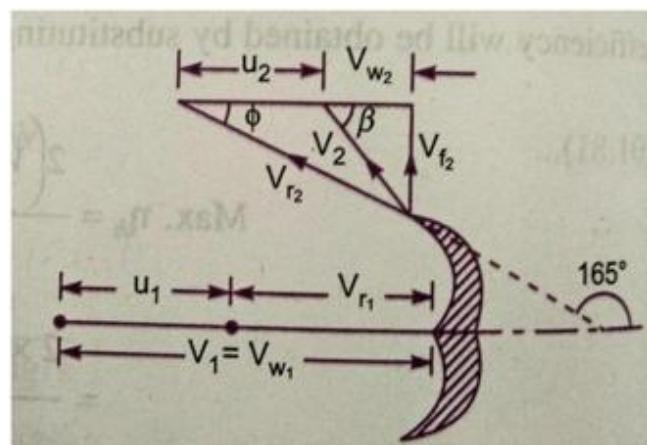
Head of water  $H = 30 \text{ m}$

Angle deflection =  $160^\circ$

$\therefore \text{Angle } \phi = 180 - 160 = 20^\circ$

Co-efficient of velocity  $C_v = 0.98$

The velocity of jet  $V_1 = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 9.81 \times 30} = 23.77 \text{ m/s}$



$$V_{r_2} = V_1 - u_1 = 23.77 - 10 = 13.77 \text{ m/s}$$

$$V_{w_2} = V_1 = 23.77 \text{ m/s}$$

From the outlet velocity triangle

$$V_{r_2} = V_{r_2} = \frac{13.77}{\sin \theta}$$

$$V_{w_2} = V_{r_2} \cos \theta - u_2$$

$$= 13.77 \cos 20^\circ - 10 = 2.94 \text{ m/s}$$

Work done by the jet/sec on the runner is given by equation

$$\begin{aligned} &= \rho a V_1 [V_{w_1} + V_{w_2}] \times u \\ &= 1000 \times 0.7 [23.77 + 2.94] \times 10 \\ &= 186970 \text{ Nm/s} \end{aligned}$$

$$\text{Power given to the turbine} = \frac{186970}{1000} = 186.97 \text{ kW}$$

The hydraulic efficiency of the turbine is given by equation

$$\eta_h = \frac{2[V_{w_1} + V_{w_2}] \times u}{V_1^2} = \frac{2[23.77 + 2.94] \times 10}{23.77 \times 23.77} = 0.9454$$

Or

$$= 94.54\%$$

2. A reaction turbine works at 450rpm under a head of 120m. its diameter at inlet is 120cm and flow area is  $0.4 \text{ m}^2$ . The angles made by absolute and relative velocities at inlet are  $20^\circ$  and  $60^\circ$  respectively, with the tangential velocity. Determine

- i) Volume flow rate
- ii) the power developed
- iii) The hydraulic efficiency. Assume whirl at outlet is zero.

**Given:** Speed of turbine  $N = 450 \text{ rpm}$

$$\text{Head} \quad H = 120 \text{ m}$$

$$\text{Diameter of inlet } D_1 = 120 \text{ cm} = 1.2 \text{ m}$$

$$\text{Flow area } \pi D_1 \times B_1 = 0.4 \text{ m}^2$$

$$\text{Angle made by absolute velocity } \alpha = 20^\circ$$

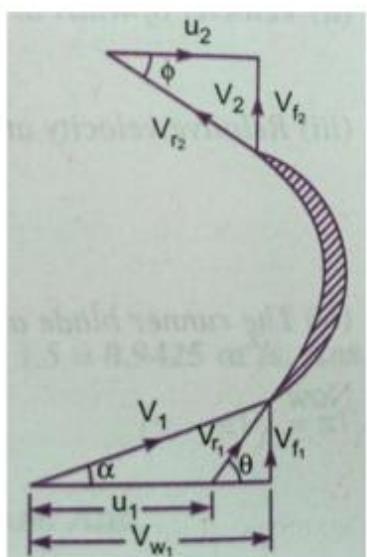
$$\text{Angle made by relative velocity } \theta = 60^\circ$$

$$\text{Whirl at outlet} \quad V_{w_2} = 0$$

$$\text{Tangential velocity of the turbine at inlet}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 450}{60} = 28.27 \text{ m/s}$$

$$\text{From inlet triangle} \quad \tan \alpha = \frac{V_{f_1}}{V_{w_1}}$$



$$\tan 20^\circ = \frac{V_{f_1}}{V_{w_1}} = 0.364,$$

$$V_{f_1} = 0.364 V_{w_1} \quad \text{(1)}$$

Also  $\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{0.364 V_{w_1}}{V_{w_1} - 28.27} \quad (\because \tan \theta = \tan 60^\circ = 1.732)$

$$1.732 = \frac{0.364 V_{w_1}}{V_{w_1} - 28.27}$$

$$0.364 V_{w_1} = 1.732 (V_{w_1} - 28.27)$$

$$0.364 V_{w_1} = 1.732 V_{w_1} - 28.27 \times 1.732$$

$$V_{w_1} (1.732 - 0.364) = 48.96$$

$$V_{w_1} = \frac{48.96}{1.732 - 0.364} = 35.79 \text{ m/s}$$

From equation (1)  $V_{f_1} = 0.364 V_{w_1} = 0.364 \times 35.79 = 13.027 \text{ m/s}$

i) Volume flow rate is given by equation as  $Q = \pi D_1 B_1 V_{f_1}$

$$Q = 0.4 \times 13.027 = 5.211 \text{ m}^3/\text{sec} \quad (\because \pi D_1 B_1 = 0.4 \text{ m}^2)$$

ii) Work done per second on the turbine is given by equation

$$= \rho Q [V_{w_1} \times u_1]$$

$$= 1000 \times 5.211 [35.79 \times 28.27] = 5272.402 \text{ Nm/s}$$

Power developed in  $kW = \frac{\text{work done per sec}}{1000} = \frac{5272.402}{1000} = 5272.402 \text{ kW}$

iii) The hydraulic efficiency is given by equation

$$\eta_h = \frac{V_{w_1} \times u_1}{g \times H} = \frac{35.79 \times 28.27}{9.81 \times 120} = 0.8595$$

$$= 85.95\%$$

3. The internal and external diameters of an outward flow reaction turbine are 2m and 2.75m respectively. The turbine is running at 250rpm and the rate of flow of water through the turbine is  $5 \text{ m}^3/\text{s}$ . the width of the runner is constant at inlet and outlet is equal to 250mm. the head on the turbine is 150m. Neglecting the thickness of the vanes and taking discharge radial at outlet, determine:

i) Vane angle at inlet and outlet

ii) velocity of flow inlet and outlet

**Given:** Internal diameter  $D_1 = 2 \text{ m}$

External diameter  $D_2 = 2.75 \text{ m}$

Speed of turbine  $N = 250 \text{ rpm}$

Discharge  $Q = 5 \text{ m}^3/\text{s}$

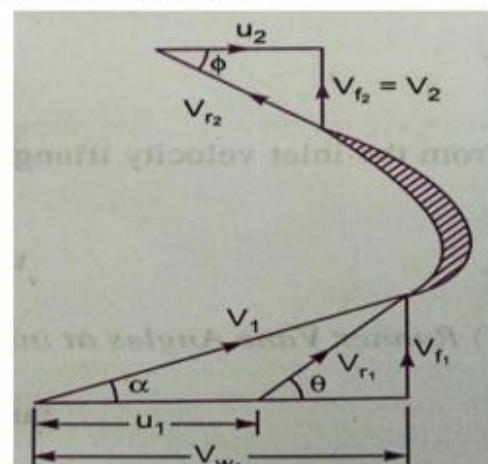
Width at inlet and outlet  $B_1 = B_2 = 250 \text{ mm} = 0.25 \text{ m}$

Head  $H = 150 \text{ m}$

Discharge at outlet = radial

$$\therefore V_{w_2} = 0 \text{ and } V_{f_2} = V_2$$

The tangential velocity of turbine at inlet and outlet



$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 2 \times 250}{60} = 26.18 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 2.75 \times 250}{60} = 36 \text{ m/s}$$

The discharge through the turbine is given by

$$V_{f_2} = \frac{Q}{\pi D_1 B_1} = \frac{5}{\pi \times 2.75 \times 0.25} = 2.315 \text{ m/s}$$

$$\text{Using equation } H - \frac{V_2^2}{2g} = \frac{V_{w_1} u_1}{g} \quad (\because V_{w_2} = 0)$$

But for radial discharge  $V_2 = V_{f_2} = 2.315 \text{ m/s}$

$$150 - \frac{(2.315)^2}{2 \times 9.81} = \frac{V_{w_1} \times 26.18}{9.81} \quad \text{Or} \quad 149.73 = \frac{V_{w_1} \times 26.18}{9.81}$$

$$V_{w_1} = \frac{149.73 \times 9.81}{26.18} = 56.1 \text{ m/s}$$

i) Vane angle at inlet and outlet

$$\text{From the inlet velocity triangle } \tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{3.183}{56.1 - 26.18} = 0.1064$$

$$\therefore \theta = \tan^{-1} 0.1064 = 6^{\circ}072' \quad \text{or} \quad 6^{\circ}4.32'$$

$$\text{From outlet velocity triangle } \tan \theta = \frac{V_{f_2}}{u_2} = \frac{2.315}{36} = 0.0643$$

ii) Velocity of flow at inlet and outlet

$$\therefore \theta = \tan^{-1}(0.0643) = 3.68' \quad \text{or} \quad 3^{\circ}40.8'$$

$$V_{f_1} = 3.183 \text{ m/s} \quad \text{and} \quad V_{f_2} = 2.315 \text{ m/s}$$

4. A Francis turbine with an overall efficiency of 75% is required to produce 148.25kW power. It is working under a head of 7.62m. The peripheral velocity =  $0.26 \sqrt{2gh}$  and the radial velocity of flow at inlet is  $0.96 \sqrt{2gh}$ . The wheel runs at 150rpm and the hydraulic losses in the turbine are 22% of the available energy. Assuming radial discharge determine

i) The guide blade angle.                            ii) The wheel vane angle at inlet

iii) The diameter of the wheel at inlet, and iv) Width of the wheel at inlet.

**Given:** Overall efficiency  $\eta_0 = 75\% = 0.75$

Head H=7.62rpm

Power Produced S.P. = 148.25kW

Speed N= 150rpm

Hydraulic loses = 22% of energy

$$\text{Peripheral velocity } u_1 = 0.26 \sqrt{2gh} = 0.26 \sqrt{2 \times 9.81 \times 7.62} = 3.179 \text{ m/s}$$

Discharge at outlet = Radial

$$V_{w_2} = 0 \quad V_{f_2} = V_2$$

The hydraulic efficiency

But ,

i) The guide blade angle i.e. From inlet velocity triangle

ii) The wheel angle at inlet

iii) The diameter of wheel at inlet ( $D_1$ )

Using relation

$$u_1 = \frac{\pi D_1 N}{60}$$

$$D_1 = \frac{60 \times u_1}{\pi \times N} = \frac{60 \times 3.179}{\pi \times 150} = 0.4047m$$

iv) Width of the wheel at inlet ( $B_1$ )

$$\eta_0 = \frac{SP}{WP} = \frac{148.25}{WP}$$

But

$$WP = \frac{W \times H}{1000} = \frac{\rho \times g \times Q \times H}{1000} = \frac{1000 \times 9.81 \times Q \times 7.62}{1000}$$

$$\eta_0 = \frac{148.25 \times 1000}{1000 \times 9.81 \times Q \times 7.62}$$

$$Q = \frac{148.25 \times 1000}{1000 \times 9.81 \times 7.62 \times \eta_0} = \frac{148.25 \times 1000}{1000 \times 9.81 \times 7.62 \times 0.75} = 2.644 m^3/s \quad (\because \eta_0 = 75\%)$$

Using equation

$$Q = \pi D_1 B_1 \times V_{f_1}$$

$$2.644 = \pi \times 0.4047 \times B_1 \times 11.738$$

$$B_1 = \frac{2.644}{\pi \times 0.4047 \times 11.738} = 0.177m$$

5. A Kaplan turbine runner is to be designed to develop 7357.5kW shaft power. The net available head is 5.50m. Assume that the speed ratio is 2.09 and flow ratio is 0.68 and the overall efficiency is 60%. The diameter of boss is  $\frac{1}{3}$  of the diameter of runner. Find the diameter of the runner, its speed and specific speed.

**Given:** Shaft power P = 7357.5kW

Head H = 5.5m

Speed ratio  $= \frac{u_1}{\sqrt{2gH}} = 2.09$

$$\therefore u_1 = 2.09 \times \sqrt{2 \times 9.81 \times 5.5} = 21.71 m/s$$

Flow ratio  $= \frac{V_{f_1}}{\sqrt{2gH}} = 0.68$

$$\therefore V_{f_1} = 0.68 \times \sqrt{2 \times 9.81 \times 5.5} = 7.064 \text{ m/s}$$

Overall Efficiency  $\eta_0 = 60\% = 0.60$

Diameter of boss  $D_b = \frac{1}{3} \times D_0$

Using the relation  $\eta_0 = \frac{\text{Shaft power}}{\text{water power}} = \frac{7357.5}{\rho g Q H}$

$$0.60 = \frac{7357.5 \times 1000}{1000 \times 9.81 \times Q \times 5.5}$$

Discharge  $Q = \frac{7357.5 \times 1000}{1000 \times 9.81 \times 5.5 \times 0.60} = 227.27 \text{ m}^3/\text{s}$

Using equation for discharge

$$Q = \frac{\pi}{4} [D_0^2 - D_b^2] \times V_{f_1}$$

$$227.27 = \frac{\pi}{4} \left[ D_0^2 - \left( \frac{D_0}{3} \right)^2 \right] \times V_{f_1}$$

$$227.27 = \frac{\pi}{4} \times \frac{8}{9} D_0^2 \times 7.064$$

$$D_0^2 = 227.27 \times \frac{4}{\pi} \times \frac{9}{8} \times \frac{1}{7.064}$$

$$D_0 = 6.788 \text{ m}$$

$$D_b = \frac{1}{3} D_0 = \frac{6.788}{3} = 2.262 \text{ m}$$

Using the relation  $u_2 = \frac{\pi D_0 N}{60}$

$$N = \frac{60 \times u_1}{\pi D_0} = \frac{60 \times 21.71}{\pi \times 6.788} = 61.08 \text{ rpm}$$

( $\because u_1 = u_2$ ) The specific speed

$$N_s = \frac{N \sqrt{P}}{H^{5/4}} = \frac{61.08 \times \sqrt{7357.5}}{(5.5)^{5/4}} = 622 \text{ rpm}$$

## Geometric similarity

The geometric similarity must exist between the model and its prototype. the ratio of all corresponding linear dimensions in the model and its prototype are equal.

Let  $L_m$  = length of model

$b_m$  = Breadth of model

$D_m$  = Diameter of model

$A_m$  = Area of model

$V_m$  = Volume of model

And  $L_p, b_p, D_p, A_p, V_p$  = Corresponding values of the prototype.

For geometrical similarity between model and prototype, we must have the relation,

$$\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r$$

Where  $L_r$  is called scale ratio.

For area's ratio and volume's ratio the relation should be,

$$\frac{A_p}{A_m} = \frac{L_p \times b_p}{L_m \times b_m} = L_r \times L_r = L_r^2$$

$$\frac{V_p}{V_m} = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{b_p}{b_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3 = L_r^3$$

## Performance of Hydraulic Turbines

In order to predict the behavior of a turbine working under varying conditions of head, speed, output and gate opening , the results are expressed in terms of quantities which may be obtained when the head on the turbine is reduced to unity. The conditions of the turbine under unit head are such that the efficiency of the turbine remains unaffected. The three important unit quantities are:

1. Unit speed,
2. Unit discharge, and
3. Unit power

**1. Unit Speed:** it is defined as the speed of a turbine working under a unit head. It is denoted by ' $N_u$ '. The expression of unit speed ( $N_u$ ) is obtained as:

Let  $N$  = Speed of the turbine under a head  $H$

$H$  = Head under which a turbine is working

$u$  = Tangential velocity.

The tangential velocity, absolute velocity of water and head on turbine are related as:

$$u \propto V \quad \text{Where } V \propto \sqrt{H}$$

$$\propto \sqrt{H} \quad \text{_____ (1)}$$

Also tangential velocity ( $u$ ) is given by

$$u = \frac{\pi D N}{60} \quad \text{Where } D = \text{Diameter of turbine.}$$

For a given turbine, the diameter ( $D$ ) is constant

$$u \propto N \quad \text{Or } N \propto u \quad \text{Or } N \propto \sqrt{H} \quad (\because \text{From (1), } u \propto \sqrt{H})$$

$$\therefore N = K_1 \sqrt{H} \quad \text{_____ (2) Where } K_1 \text{ is constant of proportionality.}$$

If head on the turbine becomes unity, the speed becomes unit speed or

When  $H = 1, N = N_u$

Substituting these values in equation (2), we get

$$N_u = K_1 \sqrt{1.0} = K_1$$

Substituting the value of  $K_1$  in equation (2)

$$N = N_u \sqrt{H} \quad \text{or} \quad N_u = \frac{N}{\sqrt{H}} \quad \text{_____ (I)}$$

**2. Unit Discharge:** It is defined as the discharge passing through a turbine, which is working under a unit head (i.e. 1 m). It is denoted by ' $Q_u$ ' the expression for unit discharge is given as:

Let  $H$  = head of water on the turbine

$Q$  = Discharge passing through turbine when head is  $H$  on the turbine.

$a$  = Area of flow of water

The discharge passing through a given turbine under a head ' $H$ ' is given by,

$$Q = \text{Area of flow} \times \text{Velocity}$$

But for a turbine, area of flow is constant and velocity is proportional to  $\sqrt{H}$

$$Q \propto \text{velocity} \propto \sqrt{H}$$

Or 
$$Q = K_2 \sqrt{H} \quad \dots \quad (3)$$

Where  $K_2$  is constant of proportionality

If  $H = 1, Q = Q_u$  (By definition)

Substituting these values in equation (3) we get

$$Q_u = K_2 \sqrt{1.0} = K_2$$

Substituting the value of  $K_2$  in equation (3) we get

$$\begin{aligned} Q &= Q_u \sqrt{H} \\ Q_u &= \frac{Q}{\sqrt{H}} \quad \dots \quad (\text{II}) \end{aligned}$$

**3. Unit Power:** It is defined as the power developed by a turbine working under a unit head (i.e. under a head of 1m). It is denoted by ' $P_u$ '. The expression for unit power is obtained as:

Let  $H$ = Head of water on the turbine

$P$ = Power developed by the turbine under a head of  $H$

$Q$ = Discharge through turbine under a head  $H$

The overall efficiency ( $\eta_0$ ) is given as

$$\eta_0 = \frac{\text{Power developed}}{\text{Water power}} = \frac{P}{\frac{pgQH}{1000}}$$

$$\begin{aligned} P &= \eta_0 \times \frac{pgQh}{1000} \\ &\propto Q \times H \propto \sqrt{H} \times H \quad (\because Q \propto \sqrt{H}) \\ &\propto H^{3/2} \end{aligned}$$

$$P = K_3 H^{3/2} \quad \dots \quad (4) \text{ Where } K_3 \text{ is a constant of proportionality}$$

When  $H=1 \text{ m}, P = P_u$

$$\therefore P_u = K_3 (1)^{3/2} = K_3$$

Substituting the value of  $K_3$  in equation (4) we get

$$\begin{aligned} P &= P_u H^{3/2} \\ P_u &= \frac{P}{H^{3/2}} \quad \dots \quad (\text{III}) \end{aligned}$$

**Use of Unit Quantities ( $N_u, Q_u, P_u$ ):**

If a turbine is working under different heads, the behaviour of the turbine can be easily known from the values of the unit quantities i.e. from the value of unit speed, unit discharge and unit power.

Let  $H_1, H_2, H_3, \dots$  are the heads under which a turbine works,

$N_1, N_2, N_3, \dots$  are the corresponding speeds,

$Q_1, Q_2, Q_3, \dots$  are the discharge and

$P_1, P_2, P_3, \dots$  are the power developed by the turbine.

Using equation I, II, III respectively,

$$\left. \begin{aligned} N_u &= \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}} = \frac{N_3}{\sqrt{H_3}} \\ Q_u &= \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}} = \frac{Q_3}{\sqrt{H_3}} \\ P_u &= \frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}} = \frac{P_3}{H_3^{3/2}} \end{aligned} \right\} \text{(IV)}$$

Hence, if the speed, discharge and power developed by a turbine under a head are known, then by using equation (IV) the speed, discharge, power developed by the same turbine at different head can be obtained easily.

### CHARACTERISTIC CURVES OF HYDRAULIC TURBINES:

Characteristic curves of a hydraulic turbine are the curves, with the help of which the exact behaviour and performance of the turbine under different working conditions can be known. These curves are plotted from the results of the tests performed on the turbine under different working conditions.

The important parameters which are varied during a test on turbine are:

- |              |  |                  |
|--------------|--|------------------|
| 1) Speed (N) | 2) Head (H)                            | 3) Discharge (Q) |
| 4) Power (P) | 5) Overall Efficiency ( $\eta_0$ ) and | 6) Gate opening. |

Out of the above six parameters, three parameters namely speed (N), Head (H) and discharge (Q) are independent parameters.

Out of the three independent parameters, (N, H, Q) one of the parameter is kept constant (say H) and the variation of other two parameters with respect to any one of the remaining two independent variables (say N and Q) are plotted and various curves are obtained. These curves are called characteristic curves. The following are the important characteristic curves of a turbine.

1. Main Characteristic Curves or Constant Head Curves.
2. Operating Characteristic Curves or Constant Speed Curves.
3. Muschel Curves or Constant Efficiency Curves.

#### 1. Main Characteristic Curves or Constant Head Curves:

These curves are obtained by maintaining a constant head and a constant gate opening (G.O.) on the turbine. The speed of the turbine is varied by changing the load on the turbine. For each value of the speed, the corresponding values of the power (P) and discharge (Q) are obtained. Then the overall efficiency ( $\eta_0$ ) for each value of the speed is calculated. From these readings the values of unit speed ( $N_u$ ), unit power ( $P_u$ ) and unit discharge ( $Q_u$ ) are determined. Taking  $N_u$  as abscissa, the values of  $Q_u$ ,  $P_u$  and  $\eta_0$  are plotted. By changing the gate opening, the values of  $Q_u$  and  $N_u$  are determined and taking  $N_u$  as abscissa, the values of  $Q_u$ ,  $P_u$  and  $\eta_0$  are plotted.

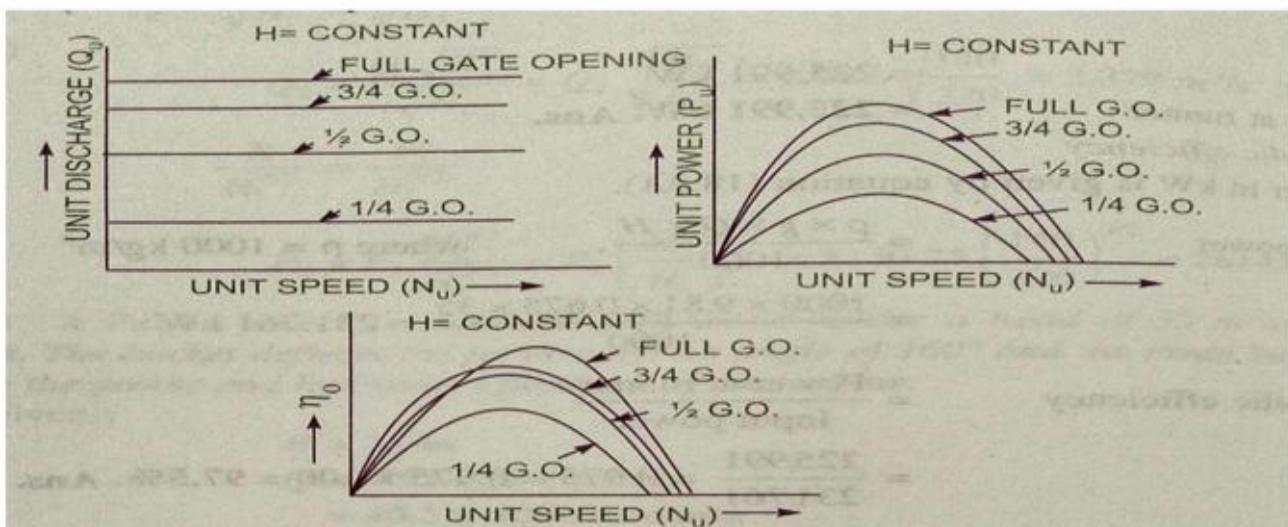
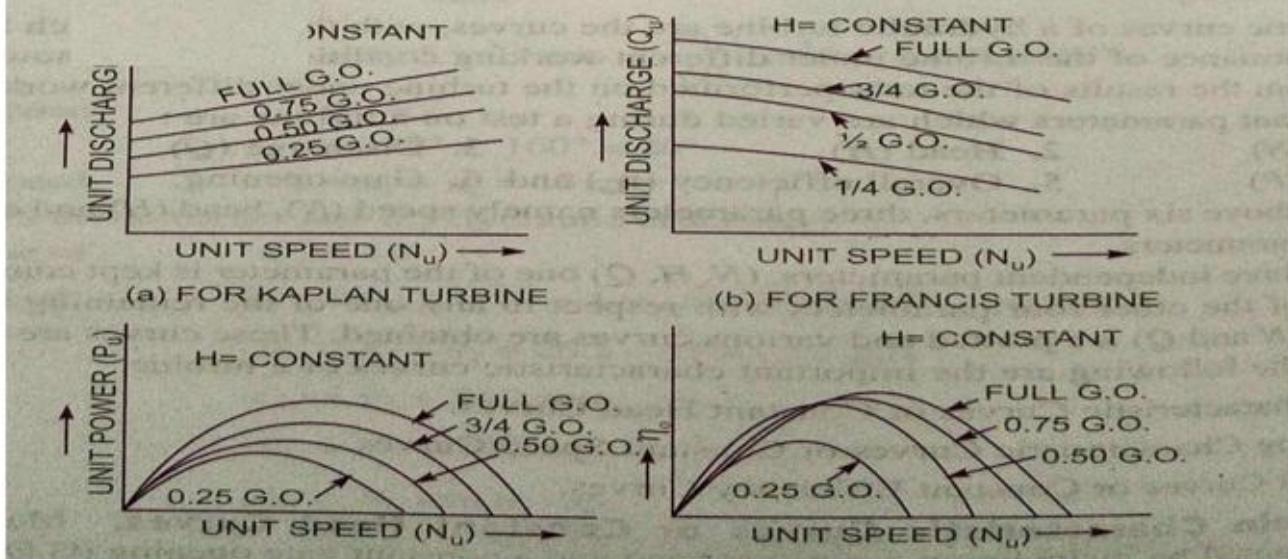
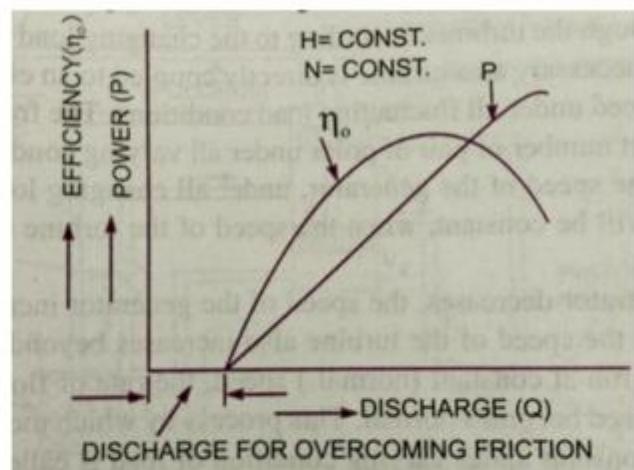


Fig. 18.35 Main characteristic curves for a Pelton wheel.



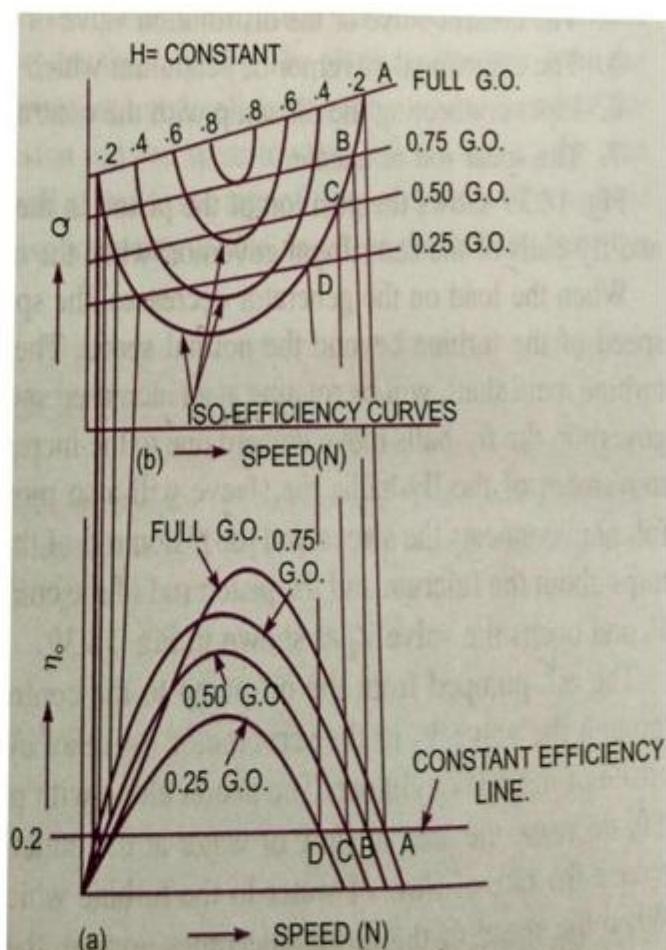
## 2. Operating Characteristic Curves or Constant Speed Curves:

These curves are plotted when the speed on the turbine is constant. In case turbines, the head is generally constant. As already discussed there are three independent parameters namely  $N$ ,  $H$  and  $Q$ . For operating characteristics  $N$  and  $H$  are constant and hence the variation of power and the efficiency with respect to discharge  $Q$  are plotted. The power curve for the turbine shall not pass through the origin, because certain amount of discharge is needed to produce power to overcome initial friction. Hence the power and efficiency curves will be slightly away from the origin on the x-axis, as to overcome initial friction certain amount of discharge will be required.



## 3. Constant Efficiency Curves or Muschel Curves or Iso - Efficiency Curves:

These curves are obtained from the speed vs. efficiency and speed vs. discharge curves for different gate openings. For a given efficiency from the  $N_u$  vs.  $\eta_0$  curves there are two speeds. From the  $N_u$  vs.  $Q_u$  curves, corresponding to two values of speeds there are two values of discharge. Hence for a given efficiency there are two values of discharge for a particular gate opening. This means for a given efficiency there are two values of speeds and two values of discharge for a given gate opening. If the efficiency is maximum there is only one value. These two values of speed and two values of discharge. Corresponding to a particular gate opening are plotted. The procedure is repeated for different gate openings and the curves  $Q$  vs.  $N$  are plotted. The points having the same efficiencies are joined. The curves having the same efficiency



are called Iso-efficiency curves. These curves are helpful for determining the zone of constant efficiency and for predicated the performance of the turbine at various efficiencies.

For plotting the Iso-efficiency curves, horizontal lines representing the same efficiency are drawn on the  $\eta_0 \sim$  speed curves. The points at which these lines cut the efficiency curves at various gate opening are transferred to the corresponding  $Q \sim$  speed curves. The points having the same efficiency are then joined by smooth curves. These smooth curves represent the Iso-efficiency curves.

**Cavitation :** Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid in a region, where the pressure of the liquid falls below its vapour pressure and the sudden collapsing of these vapour bubbles in a region of higher pressure. When the vapour bubbles collapse, a very high pressure is created. The metallic surfaces, above which these vapour bubbles collapse, is subjected to these high pressures, which cause pitting action on the surface. Thus cavities are formed on the metallic surface and also considerable noise and vibrations are produced.

Cavitation includes formation of vapour bubbles of the flowing liquid and collapsing of the vapour bubbles. Formation of vapour bubbles of the flowing liquid take place only whenever the pressure in any region falls below vapour pressure. When the pressure of the flowing liquid is less than its vapour pressure, the liquid starts boiling and the vapour bubbles are formed. These vapour bubbles are carried along with the flowing liquid to higher pressure zones, where these vapour condense and the bubbles collapse. Due to sudden collapsing of the bubbles on the metallic surface, high pressure is produced and metallic surfaces are subjected to high local stress. Thus the surfaces are damaged.

**Precaution against Cavitation:** The following are the Precaution against cavitation

- i. The pressure of the flowing liquid in any part of the hydraulic system should not be allowed to fall below its vapour pressure. If the flowing liquid is water, then the absolute pressure head should not be below 2.5m of water.
- ii. The special materials or coatings such as Aluminum-bronze and stainless steel, which are cavitation resistant materials, should be used.

**Effects of Cavitation:** the following are the effects of cavitation.

- i. The metallic surfaces are damaged and cavities are formed on the surfaces.
- ii. Due to sudden collapse of vapour bubbles, considerable noise and vibrations are produced.
- iii. The efficiency of a turbine decreases due to cavitation. Due to pitting action, the surface of the turbine blades becomes rough and the force exerted by the water on the turbine blades decreases. Hence, the work done by water or output horse power becomes less and efficiency decreases.

**Hydraulic Machines Subjected to Cavitation:** The hydraulic machines subjected to Cavitation are reaction turbine and centrifugal pumps.

**Cavitation in Turbines:** in turbines, only reaction turbines are subjected to cavitation. In reaction turbines the cavitation may occur at the outlet of the runner or at the inlet of the draft tube where the pressure is considerably reduced. (i.e. which may be below vapour pressure of the liquid flowing through the turbine) Due to cavitation, the metal of the runner vanes and draft tube is gradually eaten away, which results in lowering the efficiency of the turbine. Hence the cavitation in a reaction turbine can be noted by a sudden drop in efficiency. In order to determine whether cavitation will occur in any portion of a reaction turbine, the critical value of Thoma's cavitation factors  $\sigma$  is calculated.

$$\sigma = \frac{H_b - H_s}{H} = \frac{(H_{atm} - H_v) - H_s}{H},$$

Where  $H_b$  =Barometric pressure head in m of water,

$H_{atm}$  =Atmospheric pressure head in m of water,

$H_v$  = Vapour pressure head in m of water,

Suction pressure at the outlet of reaction turbine in m of water or height of turbine runner above the tail water surface,

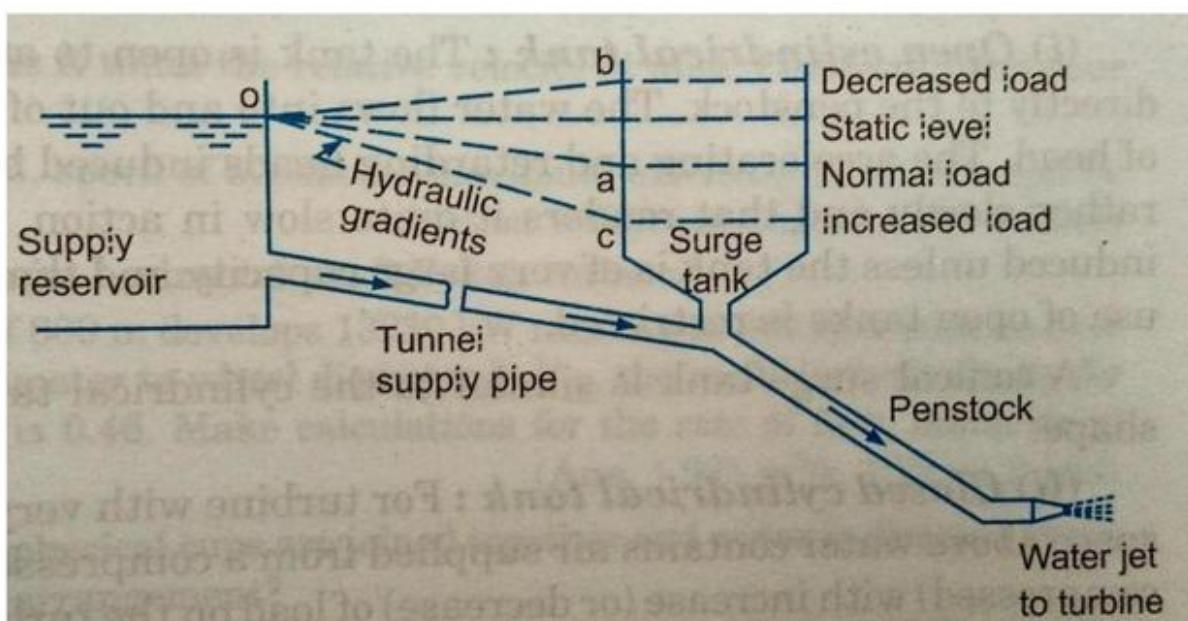
H = Net head on the turbine in m.

## Surge Tank:

When the load on the generator decreases, the governor reduces the rate of flow of water striking the runner to main constant speed for the runner. The sudden reaction of rate of flow in the penstock may lead to water hammer in pipe due to which the pipe may burst. When the load on the generator increases the turbine requires more water. Surge tanks are employed in case of high head and medium head hydro power plants where the penstock is very long and fore bays are suitable for medium and low head hydro power plants where the length of penstock is short.

An ordinary sugar tank is a cylindrical open topped storage reservoir, which is connected to the penstock at a point as close as possible to the turbine. The upper lip of the tank is kept well above the maximum water level in the supply reservoir. When the load on the turbine is steady and normal and there are no velocities variations in the pipe line there will be normal pressure gradient  $oaa_1$ . The water surface in the surge tank will be lower than the reservoir surface by an amount equal to friction head loss in the pipe connecting reservoir and sugar tank. When the load on the generator is reduced, the turbine gates are closed and the water moving towards the turbine has to move back ward. The rejected water is then stored in the surge tank, raising the pressure gradient. The retarding head so built up in the surge tank reduces the velocity of flow in the pipe line corresponding to the reduced discharge required by the turbine.

When the load on the generator increases the governor opens the turbine gates to increase the rate of flow entering the runner. The increased demand of water by the turbine is partly met by the water stored in the surge tank. As such the water level in the surge tank falls and falling pressure gradient is developed. In other words, the surge tank develops an accelerating head which increases the velocity of flow in the pipe line to a valve corresponding to the increased discharge required by the turbine.



## Water Hammer:

Consider a long pipe AB, connected at one end to a tank containing water at a height of H from the centre of the pipe. At the other end of the pipe, a valve to regulate the flow of water is provided. When the valve is completely open, the water is flowing with velocity, V in the pipe. If now the valve is suddenly closed, the momentum of the flowing water will be destroyed and consequently a wave of high pressure will be set up. This wave of high pressure will be transmitted along the pipe with a velocity equal to the velocity of sound wave and may create noise called knocking. Also this wave of high pressure has the effect of hammering action on the walls of the pipe and hence it is known as water hammer.

The pressure rise due to water hammer depends upon:

1. Velocity of flow of water in pipe.
2. The length of pipe.
3. Time taken to close the valve.
4. Elastic properties of the material of the pipe.

The following cases of water hammer in pipes will be considered.

1. Gradual closure of valve
2. Sudden closure of valve considering pipe in rigid
3. Sudden closure of valve considering pipe elastic.

### 1. Gradual Closure of Valve:

$$\text{Mass of water in pipe} = \rho \times \text{volume of water} = \rho \times A \times L$$

Where A = Area of cross-section of the pipe

L = Length of pipe

The valve is closed gradually in time „T“ seconds and hence the water is brought from initial velocity V to zero velocity in time seconds.

$$\therefore \text{Retardation of water} = \frac{\text{change of velocity}}{\text{Time}} = \frac{V-0}{T} = \frac{V}{T}$$

$$\therefore \text{Retarding force} = \text{Mass} \times \text{Retardation} = \rho AL \times \frac{V}{T} \quad (1)$$

If p is the intensity of pressure wave produced due to closure of the valve, the force due to pressure wave

$$= p \times \text{Area of pipe} = p \times A \quad (2)$$

Equating the two forces given by equation (1) & (2)

$$\rho AL \times \frac{V}{T} = p \times A$$

$$p = \frac{\rho LV}{T}$$

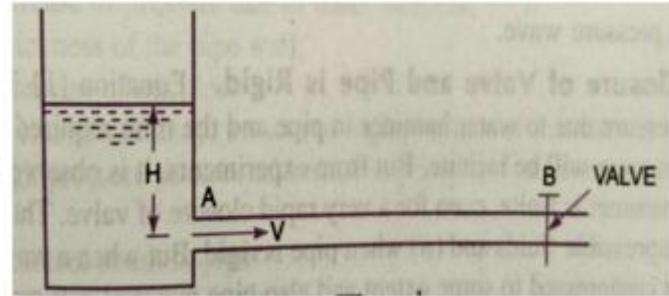
$$\text{Head of pressure} \quad H = \frac{p}{\rho g} = \frac{\rho LV}{\rho g \times T} = \frac{LV}{gT}$$

i) The valve closure is said to be gradual if  $T > \frac{2L}{C}$

Where  $T$  = Time in sec,  $C$  = Velocity of Pressure wave

ii) The valve closure is said to be sudden if  $T < \frac{2L}{C}$

Where  $C$  = Velocity of Pressure Wave



## 2) Sudden Closure of Valve and Pipe is Rigid:

In sudden closure of valve T=0, the increase in pressure will be infinite when wave of high pressure is created the liquid gets compressed to some extent and pipe material gets stretched. For a sudden closure of valve, the value of T is very small and hence a wave of high pressure is created.

When the valve is closed suddenly, the kinetic energy of flowing water is converted in to strain energy of water if the effect of friction is neglected and pipe wall is assumed to be rigid.

$$\therefore \text{loss of kinetic energy} = \frac{1}{2} \times \text{mass of water in pipe} \times V^2 \\ = \frac{1}{2} \times \rho A L \times V^2$$

$$\text{Gain of strain energy} = \frac{1}{2} \left[ \frac{p^2}{K} \right] \times \text{volum} = \frac{1}{2} \frac{p^2}{K} \times AL$$

Equating loss of Kinetic energy to gain of strain energy

$$\frac{1}{2} \times \rho A L \times V^2 = \frac{1}{2} \frac{p^2}{K} \times AL$$

$$p^2 = \frac{1}{2} \times \rho A L \times V^2 \times \frac{2K}{AL} = \rho K V^2$$

$$p = \sqrt{\rho K V^2} = V \sqrt{K \rho} = V \sqrt{\frac{K \rho^2}{\rho}}$$

$$p = \rho V \sqrt{\frac{K}{\rho}} \quad \left( \because \sqrt{\frac{K}{\rho}} = C \right)$$

$$p = \rho V \times C$$

Where C = velocity of pressure wave.

## CENTRIFUGAL PUMPS

The hydraulic machines which convert the mechanical energy in to hydraulic energy are called pumps. The hydraulic energy is in the form of pressure energy. If the mechanical energy is converted in to pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called centrifugal pump.

The centrifugal pump acts as a reversed of an inward radial flow reaction turbine. This means that the flow in centrifugal pumps is in the radial outward directions. The centrifugal pump works on the principle of forced vertex flow which means that when a certain mass of liquid is rotated by an external torque, the rise in pressure head of the rotating liquid takes place. The rise in pressure head at any point of the rotating liquid is proportional to the square of tangential velocity of the liquid at that point. (i.e. rise in pressure head =  $\frac{\omega^2}{2}$  or  $\frac{r^2 \omega^2}{2}$ ). Thus the outlet of the impeller, where radius is more, the rise in pressure head will be more and the liquid will be discharged at the outlet with a high pressure head. Due to this high pressure head, the liquid can be lifted to a high level.

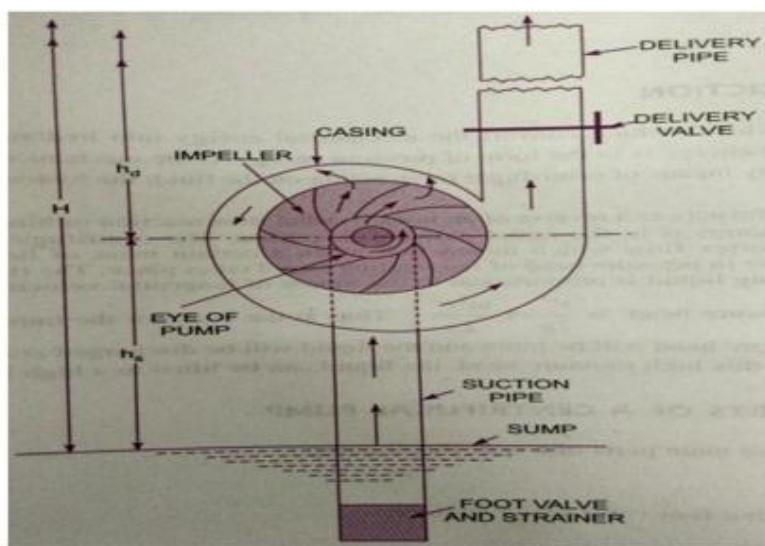
The following are the main parts of a centrifugal pump.

- 1) Impeller. 2) Casing. 3) Suction pipe with foot valve and a strainer 4) Delivery pipe.

**1. Impeller:** The rotating part of a centrifugal pump is called impeller. It consists of a series of backward curved vanes. The impeller is mounted on a shaft which is connected to the shaft of an electric motor.

**2. Casing:** the casing of a centrifugal pump is similar to the casing of a reaction turbine. It is an air tight passage surrounding the impeller and is designed in such a way that the kinetic energy of the water discharged at the outlet of the impeller is converted in to pressure energy before the water leaves the casing and enters the delivery pipe. The following three types of the casing are commonly adopted.

- a) Volute
- b) Vortex
- c) Casing with guide blades



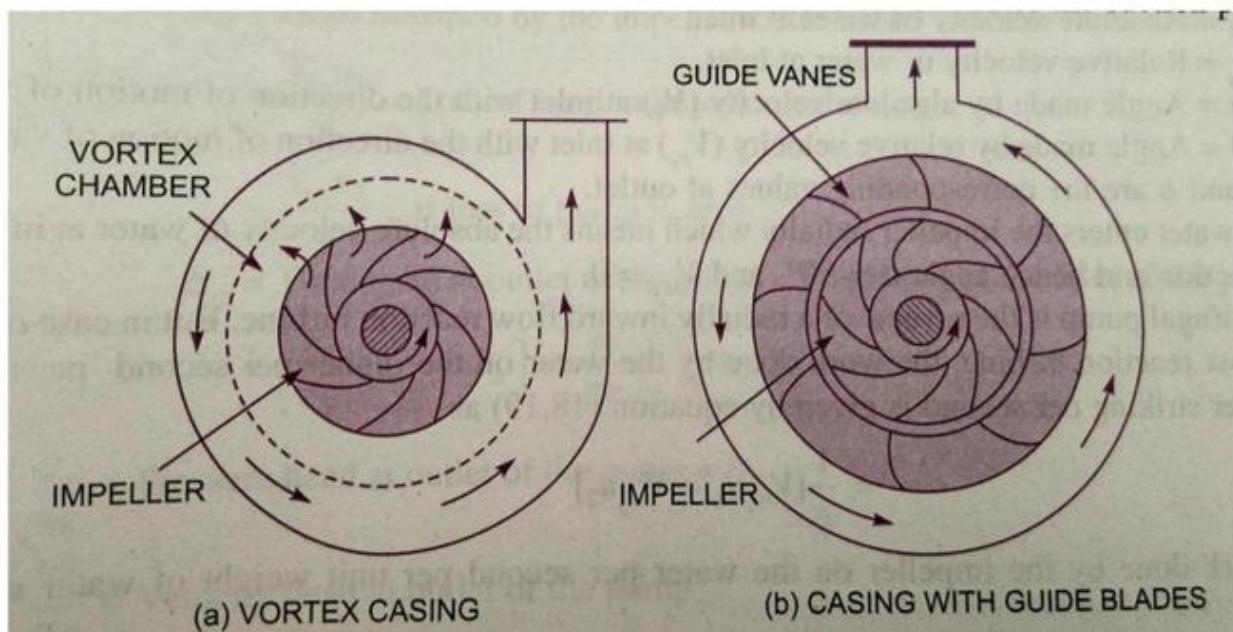
**a) Volute Casing:** It is the casing surrounding the impeller. It is of a spiral type, in which area of flow increases gradually. The increase in area of flow decreases the velocity of flow. The decrease in velocity increases the pressure of the water flowing through the casing. It has been observed that in case of volute casing, the efficiency of the pump increase slightly as a large amount of energy is lost due to the formation of eddies in this type of casing.

**b) Vortex Casing:** If a circular chamber is introduced between the casing and the impeller, the casing is known as vortex casing. By introducing the circular chamber, the loss of energy due to the formation of eddies is reduced to a considerable extent. Thus the efficiency of the pump is more than the efficiency when only volute casing is provided.

**c) Casing with guide blades:** in this type of casing the impeller is surrounded by a series of guide blades mounted on a ring known as diffuser. The guide vanes are designed in such a way that the water from the impeller enters the guide vanes without shock.

Also the area of guide vanes increases thus reducing the velocity of flow through guide vanes and consequently increasing the pressure of the water. The water from the guide vanes then pass through the surrounding casing, which is in most of the cases concentric with the impeller.

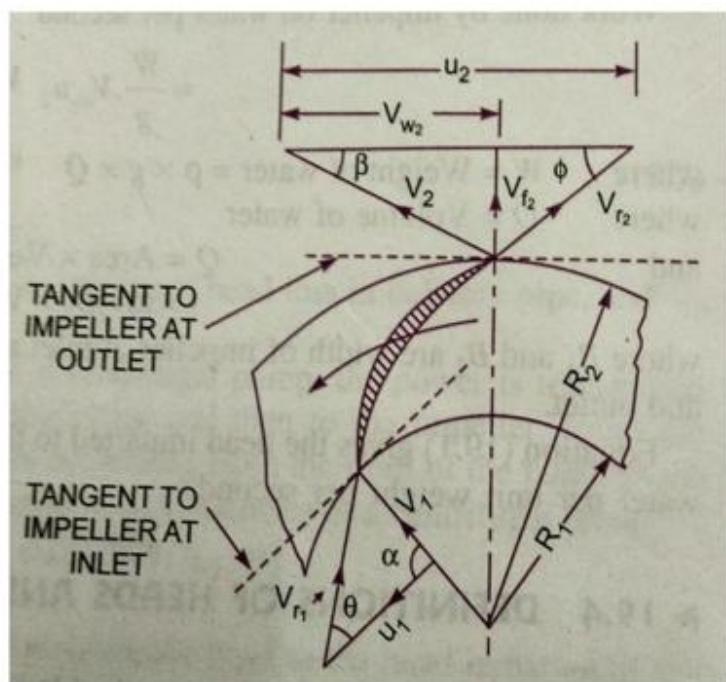
**3. Suction pipe with a foot valve and a strainer:** A pipe whose one end is connected to the inlet of the pump and other end dips in to water in a sump is known as suction pipe. A foot valve which is a non-return valve or one-way type of valve is fitted at the lower end of the suction pipe. The foot valve opens only in the upward direction. A strainer is also fitted at the lower end of the suction pipe.



**4. Delivery pipe:** A pipe whose one end is connected to the outlet of the pump and the other end delivers the water at the required height is known as delivery pipe.

## Work done by the centrifugal pump on water:

In the centrifugal pump, work is done by the impeller on the water. The expression for the work done by the impeller on the water is obtained by drawing velocity triangles at inlet and outlet of the impeller in the same way as for a turbine. The water enters the impeller radially at inlet for best efficiency of the pump, which means the absolute velocity of water at inlet makes an angle of  $90^\circ$  with the direction of motion of the impeller at inlet. Hence angle  $\alpha = 90^\circ$  and  $\beta_1 = 0$  for drawing the velocity triangles the same notations are used as that for turbines.



Let  $N$  = Speed of the impeller in r.p.m.

$D_1$  = Diameter of impeller at inlet

$$D_1 = \text{Tangential velocity of impeller at inlet} = \frac{\pi D_1 N}{60}$$

$D_2$  = Diameter of impeller at outlet

$$D_2 = \text{Tangential velocity of impeller at outlet} = \frac{\pi D_2 N}{60}$$

$V_1$  = Absolute velocity of water at inlet.

$V_{r1}$  = Relative velocity of water at inlet

$\alpha$  = Angle made by absolute velocity  $V_1$  at inlet with the direction of motion of vane

$\beta$  = Angle made by relative velocity ( $V_{r1}$ ) at inlet with the direction of motion of vane And  $D_2$ ,  $D_1$ ,  $\alpha$ ,  $\beta$ ,  $\phi$  are the corresponding values at outlet.

As the water enters the impeller radially which means the absolute velocity of water at inlet is in the radial direction and hence angle  $\alpha = 90^\circ$  and  $\beta_1 = 0$ .

A centrifugal pump is the reverse of a radially inward flow reaction turbine. But in case of a radially inward flow reaction turbine, the work done by the water on the runner per second per unit weight of the water striking per second is given by equation.

$$= - \frac{1}{D_1} D_1 - D_2 D_2$$

$\therefore$  Work done by the impeller on the water per second per unit weight of water striking/second

$$= - \frac{1}{D_1} D_1 - D_2 D_2$$

$$\begin{aligned}
 &= -\frac{1}{\rho} (\rho_1 u_1^2 - \rho_2 u_2^2) \\
 &= -\frac{1}{\rho} \rho_2 u_2^2 - \frac{1}{\rho} \rho_1 u_1^2 \\
 &= \frac{1}{\rho} \rho_2 u_2^2 \quad \dots \quad (1) \qquad \because u = 0
 \end{aligned}$$

Work done by the impeller on water per second

$$= \frac{W}{Q} \times Q \quad \text{Where } W = \text{Weight of water} = \rho \times g \times h$$

$Q = \text{Volume of water}$

$Q = \text{Area} \times \text{Velocity of flow}$

$$= \rho_1 A_1 \times u_1$$

$$= \rho A_2 u_2 \times u_2$$

Where  $A_1$  and  $A_2$  are width of impeller at inlet and outlet and

$u_1$  And  $u_2$  are velocities of flow at inlet and outlet

**Head imparted to the water by the impeller or energy given by impeller to water per unit weight per second**

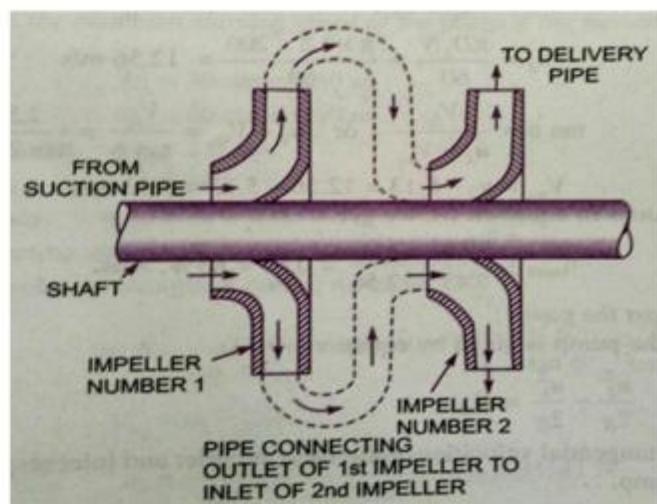
$$= \frac{H}{g}$$

### MULTI-STAGE CENTRIFUGAL PUMPS:

If centrifugal pump consists of two or more impellers, the pump is called a multi-stage centrifugal pump. The impeller may be mounted on the same shaft or on different shafts. A multi-stage pump is having the following two important functions:

- 1) To produce a high head and 2) To discharge a large quantity of liquid.

If a high head is to be developed, the impellers are connected in series (or on the same shaft) while for discharging large quantity of liquid, the impellers (or pumps) are connected in parallel.



**Multi-Stage Centrifugal Pumps for High Heads:** For developing a high head, a number of impellers are mounted in series on the same shaft.

The water from suction pipe enters the 1<sup>st</sup> impeller at inlet and is discharged at outlet with increased pressure. The water with increased pressure from the outlet of the 1<sup>st</sup> impeller is taken to the inlet of the 2<sup>nd</sup> impeller with the help of a connecting pipe. At the outlet of the 2<sup>nd</sup> impeller the pressure of the water will be more than the water at the outlet of the 1<sup>st</sup> impeller. Thus if more impellers are mounted on the same shaft, the pressure at the outlet will be increased further.

Let  $n$  = Number of identical impellers mounted on the same shaft,

$\square \square$  = Head developed by each impeller.

Then total Head developed =  $\square \times \square \square$

The discharge passing through each impeller is same.

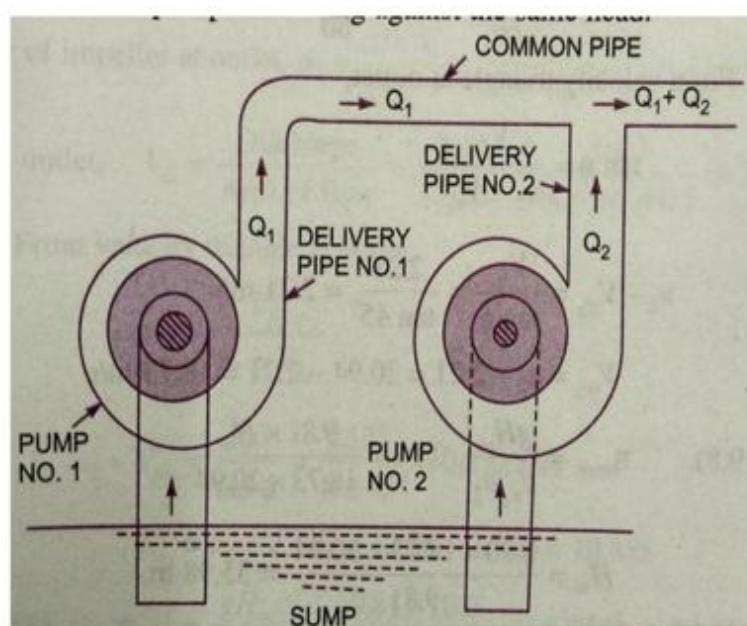
### **Multi-Stage Centrifugal Pumps for High Discharge:**

For obtaining high discharge, the pumps should be connected in parallel. Each of the pumps lifts the water from a common sump and discharges water to a common pipe to which the delivery pipes of each pump is connected. Each of the pumps is working against the same head.

Let  $n$  = Number of identical pumps  
arranged in parallel.

$Q$  = Discharge from one pump.

$\therefore$  Total Discharge =  $\square \times \square$

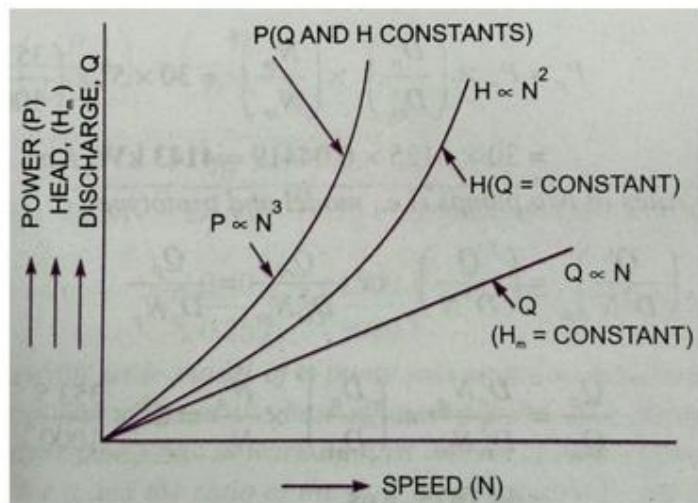


## Performance Characteristic Curves of centrifugal pumps

The characteristic curves of a centrifugal pump are defined as those curves which are plotted from the results of a number of tests on the centrifugal pump. These curves are necessary to predict the behavior and performance of the pump, when the pump is working under different flow rate, head and speed. The following are the important characteristic curves for the pumps:

1. Main characteristic curves.
2. Operating characteristic curves and
3. Constant efficiency or Muschel curves.

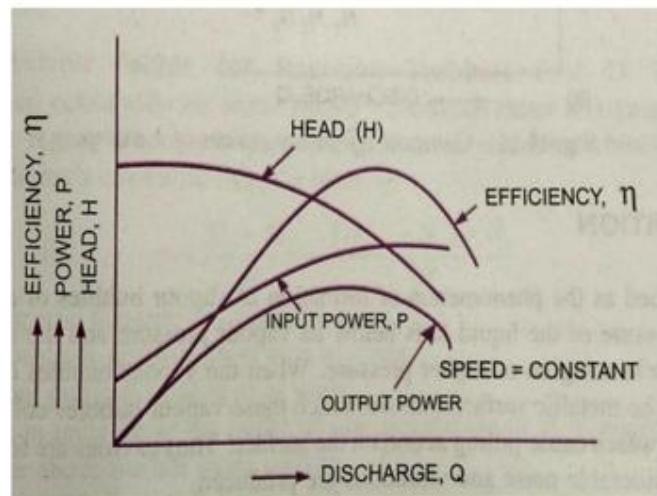
**1. Main Characteristic Curves:** the main characteristic curves of a centrifugal pump consists of a head (Manometric head  $\square$ ) power and discharge with respect to speed. For plotting curves of Manometric head versus speed, discharge is kept constant. For plotting curves of discharge versus speed, Manometric head ( $\square$ ) is kept constant. For plotting curves power versus speed, Manometric head and discharge are kept constant.



### 2 Operating Characteristic Curves:

If the speed is kept constant, the variation of Manometric head, power and efficiency with respect to discharge gives the operating characteristics of the pump.

The input power curve for pumps shall not pass through the origin. It will be slightly away from the origin on the y-axis, as even at zero discharge some power is needed to overcome mechanical losses.



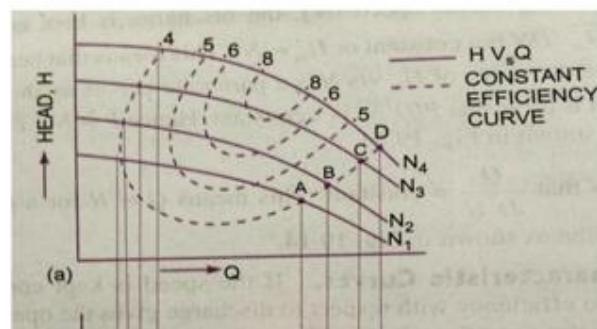
The head curve will have maximum value of head when the discharge is zero.

The output power curve will start from origin as at  $Q=0$ , output power  $\square \square \square \square$  will be zero. The efficiency curve will start from origin as at  $Q=0$ ,  $\eta = 0$ .  $\therefore \eta = \frac{\text{Output Power}}{\text{Input Power}}$

### 3 Constant Efficiency Curves:

For obtaining constant efficiency curves for a pump, the head versus discharge curves and efficiency v/s discharge curves for different speeds are used. By combining these curves

$\sim \square \square \square \square \square \square \square \square \sim \square \square \square \square \square \square \square \square$  constant



efficiency curves are obtained.

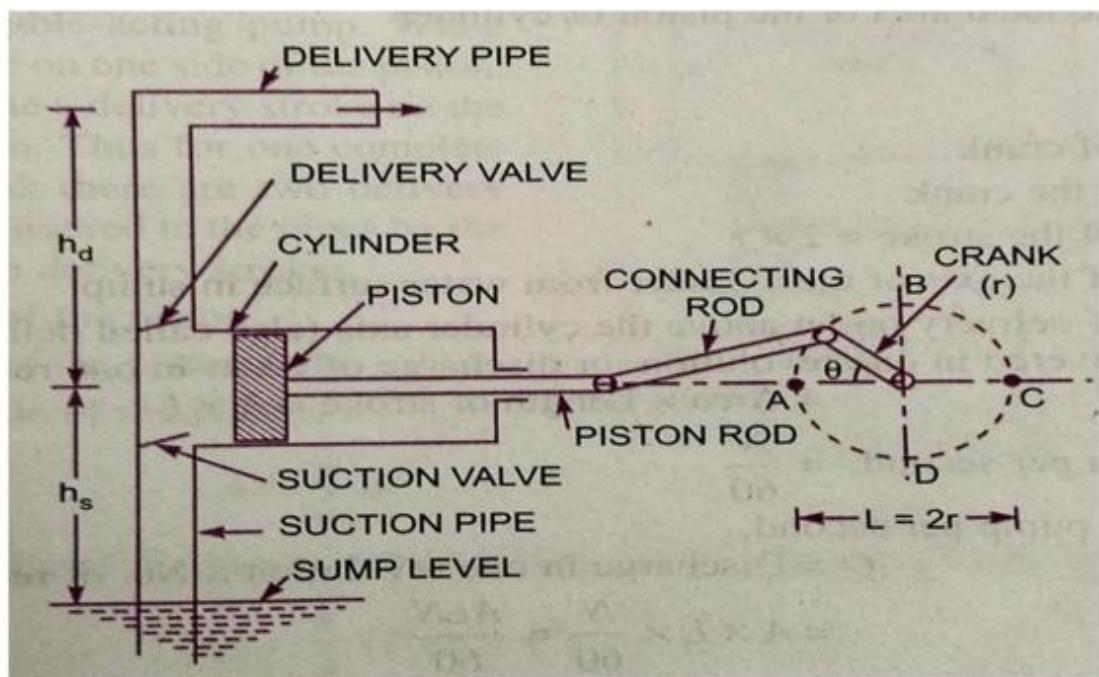
For plotting the constant efficiency curves (Iso- efficiency curves), horizontal lines representing constant efficiencies are drawn on the  $\square \sim \square$  curves. The points, at which these lines cut the efficiency curves at various speeds, are transferred to the corresponding  $\square \sim \square$  curves. The points having the same efficiency are then joined by smooth curves. These smooth curves represent the iso - efficiency curves.

For any pump installation, a distinction is made between the required NPSH and the available NPSH. The value of required NPSH is given by the pump manufacturer. This value can also be determined experimentally. For determining its value the pump is tested and minimum value of  $\square \square$  is obtained at which the pump gives maximum efficiency without any noise. (i.e. cavitation free). The required NPSH varies with the pump design, speed of the pump and capacity of the pump.

When the pump is installed, the available NPSH is calculated from the above equation (2). In order to have cavitation free operation of centrifugal pump, the available NPSH should be greater than the required NPSH.

## RECIPROCATING PUMPS

The mechanical energy is converted in to hydraulic energy (pressure energy) by sucking the liquid in to a cylinder in which a piston is reciprocating, which exerts the thrust on the liquid and increases its hydraulic energy (pressure energy) the pump is known as reciprocating pump.



A single acting reciprocating pump consists of a piston, which moves forwards and backwards in a close fitting cylinder. The movement of the piston is obtained by connecting the piston rod to crank by means of a connecting rod. The crank is rotated by means of an electric motor. Suction and delivery pipes with suction valve and delivery valve are connected to the cylinder. The suction and delivery valves are one way valves or non-return



valves, which allow the water to flow in one direction only. Suction valve allows water from suction pipe to the cylinder which delivery valve allows water from cylinder to delivery pipe only.