16/04/21 Friday

UNIT-I

1. Solution of Algebraic & Transcendental Equations

I. Bisection method :-

P1: find a positive root of $x^3-x-1=0$ correct of to two decimal places by bisection method.

IM 4 times

Sol: given that,

$$x^3-x-1=0$$

let $f(x) = x^3-x-1=0$

put $x=0$

$f(0) = 0^3-0-1 = 0-0-1 = -1$

$f(0) = -1 < 0$

put $x=1$

$f(1) = 1^3-1-1 = \cancel{1}-\cancel{1}-1 = -1$

$f(1) = -1 < 0$

put $x=2$

$f(2) = 2^3-2-1 = 8-2-1 = 5$

$f(2) = 5 > 0$

The root of equation lies b/w 1 & 2

here $x_0 = 1$ & $x_1 = 2$

$x_0 = 1 = -$

$x_1 = 2 = +$

$x_2 = 1.5 = +$

$x_3 = 1.25 = -$

$x_4 = 1.38 = +$

$x_5 = 1.32 = -$

$x_6 = 1.35 = +$

$x_7 = 1.34 = +$

$x_8 = 1.33 = +$

$x_9 = 1.33 = +$

According to bisection method

$$x_2 = \frac{x_1 + x_0}{2} \Rightarrow \frac{1+2}{2} = \frac{3}{2} = 1.5$$

$$\boxed{x_2 = 1.5}$$

$f(x) = x^3 - x - 1$

$f(x_2) = x_2^3 - x_2 - 1$

$f(1.5) = (1.5)^3 - (1.5) - 1$

$= (1.5)^3 - 1.5 - 1$

$f(1.5) = 0.875 > 0$

The root of equation lies b/w 1 & 1.5

According to bisection method

$$x_3 = \frac{x_0 + x_2}{2} \Rightarrow \frac{1+1.5}{2} = \frac{2.5}{2} = 1.25$$

$$\boxed{x_3 = 1.25}$$

$f(x) = x^3 - x - 1$

$f(x_3) = x_3^3 - x - 1$

$f(1.25) = 1.25^3 - 1.25 - 1 = -0.29687$

$f(1.25) = -0.29687 < 0$

The root of equation lies b/w 1.5 & 1.25

according to bisection method

$$x_4 = \frac{x_2 + x_3}{2} = \frac{1.5 + 1.25}{2} = 1.375$$

$$\boxed{x_4 = 1.375}$$

by problem he mentioned 2 decimal places.
so that

$\boxed{x_4 = 1.38}$

$f(x) = x^3 - x - 1$

$f(x_4) = x_4^3 - x_4 - 1$

$f(1.38) = (1.38)^3 - 1.38 - 1 = 0.248072$

$f(1.38) = 0.24807 > 0$

The root of equation lies b/w 1.25 & 1.38
according to bisection method,

$x_5 = \frac{x_3 + x_4}{2} = \frac{1.25 + 1.38}{2} = 1.315$

$\boxed{x_5 = 1.32}$

$f(x) = x^3 - x - 1$

$f(x_5) = x_5^3 - x_5 - 1$

$f(1.32) = (1.32)^3 - 1.32 - 1 = -0.020032$

$f(1.32) = -0.020032 > 0$

The root of the equation lies b/w 1.38 & 1.32
according to bisection method (x_6) =

$x_6 = \frac{x_4 + x_5}{2} = \frac{1.38 + 1.32}{2} = 1.35$

$\boxed{x_6 = 1.35}$

$f(x) = x^3 - x - 1$

$f(x_6) = x_6^3 - x_6 - 1$

$f(1.35) = (1.35)^3 - (1.35) - 1 = 0.110375$

$f(1.35) = 0.110375 > 0$

The root of the equation lies b/w 1.35 & 1.32 according to bisection method

$$x_7 = \frac{x_5 + x_6}{2} = \frac{1.32 + 1.35}{2} = 1.335$$

$\boxed{x_7 = 1.34}$

$f(x) = x^3 - x - 1$

$f(x_7) = x_7^3 - x_7 - 1$

$f(1.34) = (1.34)^3 - (1.34) - 1 = 0.06604$

$f(1.34) = 0.066104 > 0$

The root of the equation lies b/w 1.34 & 1.32 according to bisection method

$$x_8 = \frac{x_5 + x_7}{2} = \frac{1.34 + 1.32}{2} = 1.33$$

$\boxed{x_8 = 1.33}$

$f(x) = x^3 - x - 1$

$f(x_8) = x_8^3 - x_8 - 1$

$f(1.33) = 1.33^3 - 1.33 - 1 = 0.022637$

$f(1.33) = 0.022637 > 0$

The root of the equation lies b/w 1.33 & 1.32 according to bisection method

$$x_9 = \frac{x_5 + x_8}{2} = \frac{1.33 + 1.32}{2} = 1.325$$

$\boxed{x_9 = 1.33}$

$f(x) = x^3 - x - 1$

$f(x_9) = (1.33)^3 - 1.33 - 1$

$f(1.33) = (1.33)^3 - 1.33 - 1 = 0.022637$

$f(1.33) = 0.022637 > 0$

$\therefore$ The positive real root is 1.33 //

17/04/2021
saturday

2. find out the square root of 25 given $x_0 = 2.0$ & $x_1 = 7.0$ using bisection method.

A given that,
the square root of 25
Let $x^2 = 25$
$x^2 - 25 = 0$
let $f(x) = x^2 - 25$
given $x_0 = 2.0 = 2$

$f(x) = x^2 - 25$
put $x = 2$
$f(2) = 2^2 - 25$
$f(2) = 4 - 25$
$f(2) = -21$

put $x = 7$
$f(7) = 7^2 - 25$
$f(7) = 49 - 25$
$f(7) = 24$

The root of equation lies b/w 2 & 7
here $x_0 = 2$ & $x_1 = 7$
according to bisection method

$x_0 = 2 = -$
$x_1 = 1 = +$
$x_2 = 4.5 = -$
$x_3 = 5.75 = +$
$x_4 = 5.13 = +$
$x_5 = 4.82 = -$
$x_6 = 4.98 = -$
$x_7 = 5.06 = +$
$x_8 = 5.02 = +$
$x_9 = 5 = +$
$x_{10} = 4.99 = -$
$x_{11} = 4.99 = -$
$x_{12} = 4.99 = -$

7. $x_2 = \frac{x_0+x_1}{2} \Rightarrow \frac{2+7}{2} \Rightarrow \frac{9}{2} \Rightarrow 4.5$

$\boxed{x_2 = 4.5}$

$f(x) = x^2 - 25$

$f(x_2) = 4.5^2 - 25 = -4.75$

$f(x_2) = -4.75$

The root of the equation lies b/w 7 & 4.5 according to bisection method

$x_3 = \frac{x_2+x_1}{2} \Rightarrow \frac{4.5+7}{2} \Rightarrow 5.75$

$\boxed{x_3 = 5.75}$

$f(x) = x^2 - 25$

$f(x_3) = (5.75)^2 - 25 = 8.06$

$f(x_3) = 8.06$

The root of the equation lies b/w 5.75 & 4.5 according to bisection method

$x_4 = \frac{x_3+x_2}{2} \Rightarrow \frac{5.75+4.5}{2} = 5.125$

$\boxed{x_4 = 5.13}$

$f(x) = x^2 - 25$

$f(x_4) = (5.13)^2 - 25 = 1.31$

$f(x_4) = 1.31$

The root of the equation lies b/w 5.13 & 4.5 according to bisection method

$x_5 = \frac{x_4+x_2}{2} \Rightarrow \frac{5.13+4.5}{2} = 4.815$

$\boxed{x_5 = 4.82}$

$f(x) = x^2 - 25$

$f(x_5) = (4.82)^2 - 25$

$f(x_5) = -1.76$

The root of equation lies b/w the 4.82 & 5.13 according to bisection method

$$x_6 = \frac{x_4 + x_5}{2} = \frac{5.13 + 4.82}{2} = 4.975$$

$$\boxed{x_6 = 4.98}$$

$f(x) = x^2 - 25$

$f(x_6) = 4.98 - 25 = -0.1996$

$f(x_6) = -0.1996$

The root of equation lies b/w the 4.98 & 5.13 according to bisection method

$$x_7 = \frac{x_4 + x_6}{2} = \frac{4.98 + 5.13}{2} = 5.055$$

$$\boxed{x_7 = 5.06}$$

$f(x) = x^2 - 25$

$f(x_7) = 5.06^2 - 25$

$f(x_7) = 0.60$

The root of equation lies b/w the 5.06 & 4.98 according to bisection method

$$x_8 = \frac{x_6 + x_7}{2} = \frac{5.06 + 4.98}{2} = 5.02$$

$$\boxed{x_8 = 5.02}$$

$f(x) = x^2 - 25$

$f(x_8) = 5.02^2 - 25$

$f(x_8) = 0.2004$

The root of equation lies b/w the 5.02 & 4.98 according to bisection method

$$x_9 = \frac{x_8 + x_6}{2} = \frac{5.02 + 4.98}{2} = 5$$

$$\boxed{x_9 = 5}$$

9

$f(x) = x^2 - 25$

$f(x_9) = 5^2 - 25$

$f(x_9) = 0$

The root of equation lies b/w the 5 & 4.98 according to bisection method

$x_{10} = \frac{x_9 + x_6}{2} = \frac{5 + 4.98}{2} = 4.99$

$\boxed{x_{10} = 4.99} \approx 5$

$f(x) = x^2 - 25$

$f(x_{10}) = 4.99^2 - 25$

$f(x_{10}) = -0.0999$

The root of equation lies b/w the 4.99 & 5 according to bisection method

$x_{11} = \frac{x_{10} + x_9}{2} = \frac{4.99 + 5}{2} = 4.995$

$\boxed{x_{11} = 4.99} \approx 5$

$f(x) = x^2 - 25$

$f(x_{11}) = 4.99^2 - 25$

$f(x_{11}) = -0.0999$

The root of equation lies b/w the 4.99 & 5 according to bisection method

$x_{12} = \frac{x_{11} + x_9}{2} = \frac{4.99 + 5}{2} = 4.995$

$\boxed{x_{12} = 4.99} \approx 5$

$f(x_{12}) = 4.99^2 = 25$

$f(x_{12}) = -0.0999$

$\therefore$ The square root of 25 is 5

P_3: find a real root of equation $x \cdot \log_{10}^{x} = 1.2$ which lies b/w 2 & 3 by using bisection method?

P_4:- find real root of equation $3x = e^x$ by bisection method?

① Rcl + M + shift + M + AC
STO AC
② shift + c

P_5:- find a real root of the equation $3x = \cos x + 1$ by bisection method

P_6:- using bisection method, find root of $f(x) = x - \cos x = 0$ find an approximate

P_7:- By using bisection method, find an approximate root of equation $\sin x = \frac{1}{x}$ that lies b/w $x = 1$ & $x = 1.5$ (measure in radians, carry out computation upto 7^{th} stage)

shift + mode + 4 + AC ⇒ for conversion into radians
shift + mode + 3 + AC ⇒ for conversion into degrees

[∴ Note, first convert it into radians)

A given that,

$\sin x = \frac{1}{x}$

$\sin x = 1$

$\sin x - 1 = 0$ (by cross multiplication)

Let $f(x) = x.\sin x - 1$

put $x \cdot x_0 = 1$

$f(1) = 1.\sin 1 - 1 = 0.1585$

$f(1) = -0.1585 < 0$

put $x = x_1 = 1.5$

$f(1.5) = 1.5 \sin 1.5 - 1 = 0.49624$

$f(1.5) = 0.49624 > 0$

$x_0 = 1 = -$
$x_1 = 1.5 = +$
$x_2 = 1.25 = +$
$x_3 = 1.125 = +$
$x_4 = 1.0625 = -$
$x_5 = 1.09375 = -$
$x_6 = 1.109375 = -$
$x_7 = 1.1171875 = -$
$x_8 = 1.12109375 = +$

the root of equation lies b/w 1 & 1.5

here $x_0 = 1$ & $x_1 = 1.5$

according to bisection method

$x_2 = \frac{x_0 + x_1}{2} = \frac{1 + 1.5}{2} = 1.25$

$\boxed{x_2 = 1.25}$

$f(1.25) = 1.25 \times (\sin 1.25) - 1 = 0.1862370$

the root of the equation lies b/w 1.25 & 1

according to bisection method.

$x_3 = \frac{x_0 + x_2}{2} = \frac{1.25 + 1}{2} = 1.125$

$\boxed{x_3 = 1.125}$

$f(x) = x \cdot \sin x - 1$

$f(x_3) = 1.125 \times \sin(1.125) - 1$

$f(x_3) = 0.01505$

the root of the equation lies b/w 1.125 & 1

according to bisection method

$x_4 = \frac{x_0 + x_3}{2} = \frac{1.125 + 1}{2} = 1.0625$

$\boxed{x_4 = 1.0625}$

$f(x_4) = 1.0625 \times \sin(1.0625) - 1$

$f(x_4) = -0.0718263138$

the root of the equation lies b/w 1.0625 & 1.125

according to bisection method

$x_5 = \frac{x_3 + x_4}{2} = \frac{1.0625 + 1.125}{2} = 1.09375$

$\boxed{x_5 = 1.09375}$

$f(x_5) = 1.09375 \times \sin(1.09375) - 1$

$x_5 = -0.0273697226$

The root of the equation lies b/w 1.09375 & 1.125
according to bisection method

$$x_6 = \frac{x_5 + x_3}{2} = \frac{1.125 + 1.09375}{2} = 1.109375$$

$\boxed{x_6 = 1.109375}$

$f(x_6) = 1.109375 \times \sin(1.109375) - 1$

$f(x_6) = -6.64277486 \times 10^{-3}$

$f(x_6) = -0.0066428$

The root of the equation lies b/w 1.109375 & 1.125
according to bisection method

$$x_7 = \frac{x_3 + x_6}{2} \Rightarrow \frac{1.125 + 1.109375}{2}$$

$\boxed{x_7 = 1.1171875}$

$f(x_7) = 1.1171875 \times \sin(1.1171875) - 1$

$f(x_7) = 4.2080340 11 \times 10^{-3}$

$f(x_7) = -0.004208034011$

The root of equation lies b/w 1.1171875 & 1.125
according to bisection method

$$x_8 = \frac{x_3 + x_7}{2} = \frac{1.1171875 + 1.125}{2}$$

$\boxed{x_8 = 1.12109375}$

$f(x_8) = 1.12109375$

$f(x_8) = 1.12109375 \times \sin(1.12109375) - 1$

$f(x_8) = 9.63060860 3 \times 10^{-3}$

$f(x_8) = 0.009630608603$

P3 Sol

given that

$x \cdot \log x = 1.2$

$x \cdot \log x - 1.2 = 0$

$f(x) = x \cdot \log x - 1.2$

$f(2) = 2 \cdot \log 2 - 1.2$

$= 2(0.30102) - 1.2$

$= 0.60204 - 1.2$

$= -0.59796$

$f(3) = x \cdot \log x - 1.2$

$f(3) = 3 \cdot \log 3 - 1.2$

$f(3) = 3 \cdot (0.47712) - 1.2$

$f(3) = 1.431363 - 1.2$

$f(3) = 0.2313637642$

$x_0 = 2 = -$

$x_1 = 3 = +$

$x_2 = 2.5 = -$

$x_3 = 2.75 = +$

$x_4 = 2.625 = -$

$x_5 = 2.6875 = -$

$x_6 = 2.71875 = -$

$x_7 = 2.734375 = -$

$x_8 = 2.7421875 = +$

$x_9 = 2.73828125 = -$

$x_{10} = 2.740234375 =$

The root of equation lies b/w 2 & 3

here $x_0 = 2$ & $x_1 = 3$

according to bisection method

$$x_2 = \frac{x_0 + x_1}{2} = \frac{2+3}{2} = \frac{5}{2} = 2.5$$

$\boxed{x_2 = 2.5}$

$f(x_2) = x \cdot \log x - 1.2$

$f(2.5) = 2.5 \log 2.5 - 1.2$

$f(2.5) = -0.2051499783$

the root of equation lies b/w 3 & 2.5

according to bisection method

$$x_3 = \frac{x_1 + x_2}{2} = \frac{3+2.5}{2} = 2.75$$

$\boxed{x_3 = 2.75}$

$f(x_3) = x \log x - 1.2$

$f(2.75) = 2.75 \log 2.75 - 1.2$

$f(2.75) = 8.164908033 \times 10^{-3}$

$f(2.75) = 0.008164908033$

The root of equation lies b/w 2.75 & 2.5

according to bisection method

$x_4 = \frac{x_3 + x_2}{2} = \frac{2.75 + 2.5}{2} = 2.625$ $\boxed{x_4 = 2.625}$

$f(x_4) = x \log x - 1.2$

$f(2.625) = 2.625 \log 2.625 - 1.2$

$f(2.625) = -0.0997855 6718$

The root of equation lies b/w 2.625 & 2.75

according to bisection method

$x_5 = \frac{x_4 + x_3}{2} = \frac{2.625 + 2.75}{2} = 2.6875$ $\boxed{x_5 = 2.6875}$

$f(x_5) = x \log x - 1.2$

$f(2.6875) = 2.6875 \log (2.6875) - 1.2$

$f(2.6875) = -0.04612597802$

The root of equation lies b/w 2.6875 & 2.75

according to bisection method

$x_6 = \frac{x_5 + x_3}{2} = \frac{2.75 + 2.6875}{2} = 2.71875$ $\boxed{x_6 = 2.71875}$

$f(x_6) = 2.71 \log 2.71 - 1.2$

$f(x_6) = 2.71875 \log 2.71875 - 1.2$

$f(x_6) = -0.019058535 5$

The root of equation lies b/w 2.71875 & 2.75

according to bisection method

$x_7 = \frac{x_3 + x_6}{2} = \frac{2.71875 + 2.75}{2} = 2.734375$ $\boxed{x_7 = 2.734375}$

$f(x) = x \cdot \log x - 1.2$

$f(x_7) = x_7 \log x_7 - 1.2$

$f(2.734375) = 2.734375 \log 2.734375 - 1.2$

$f(2.734375) = -5.466201986 \times 10^{-3}$

$= -0.005466201986$

The root of equation lies b/w 2.734375 & 2.75

according to bisection method

$x_8 = \frac{x_3 + x_7}{2} = \frac{2.734375 + 2.75}{2} = 2.7421875$

$\boxed{x_8 = 2.7421875}$

$f(x_8) = 2.7421875 \times \log(2.7421875) - 1.2$

$= -1.344519779 \times 10^{-3}$

$= +0.001344519779$

The root of the equation lies b/w 2.7421875 &

2.734375

according to bisection method

$x_9 = \frac{x_7 + x_8}{2} = \frac{2.734375 + 2.7421875}{2}$

$\boxed{x_9 = 2.73828125}$

$f(x_9) = 2.73828125 \cdot \log(2.73828125) - 1.2$

$f(x_9) = -2.062051129 \times 10^{-3}$

$f(x_9) = -0.002062051129$

The root of equation lies b/w = 2.7382812 & 2.7421875

$x_{10} = \frac{x_8 + x_9}{2} = \frac{2.7421875 + 2.73828125}{2}$

$\boxed{x_{10} = 2.740234375} \simeq 2.74$

$f(x_{10}) = 2.740234375 \times \log(2.740234375) - 1.2$

$f(x_{10}) = -3.590679615 \times 10^{-4}$

15 $f(x_{10}) = -0.0003590679615$

The root of equation lies b/w 2.740234375 & 2.7421875

$$x_{11} = \frac{2.740234375+2.7421875}{2}$$

$\boxed{x_{11} = 2.741210938} \approx 2.74$

$f(x_{11}) = 2.741210938 \log(2.741210938) - 1.2$

$f(x_{11}) = 4.926505852 \times 10^{-4}$

$f(x_{11}) = 0.0004926505852$

∴ The Real root is 2.74

P4
Sol: $f(x) = 3x - e^x$

$\boxed{\text{Put } x=0}$

$f(0) = 3(0) - e^0$

$f(0) = -1$

$\boxed{\text{Put } x=1}$

$f(1) = 3(1) - e^1$

$f(1) = 0.2817181715$

The root of equation lies b/w 0 & 1

according to bisection method

$x_2 = \frac{0+1}{2} = 0.5$

$\boxed{x_2 = 0.5}$

$f(x_2) = 3(0.5) - e^{0.5}$

$f(x_2) = -0.1487212707$

$x_0 = 0 = -$

$x_1 = 1 = +$

$x_2 = 0.5 = -$

$x_3 = 0.75 = +$

$x_4 = 0.625 = +$

$x_5 = 0.5625 = -$

$x_6 = 0.59375 = -$

$x_7 = 0.609375 = -$

$x_8 = 0.6171875 = -$

$x_9 = 0.62109375 = +$

$x_{10} = 0.61914025 = +$

The root of the equation lies b/w 0.5 & 1

according to bisection method

$x_3 = \frac{0.5+1}{2} = 0.75$ $\boxed{x_3 = 0.75}$

$f(x_3) = 3(0.75) - e^{0.75}$

$f(x_3) = 0.1329999834$

The root of equation lies b/w 0.75 & 0.5

$x_4 = \frac{0.75+0.5}{2} = 0.625$ $\boxed{x_4 = 0.625}$

$f(x_4) = 3(0.625) - e^{0.625}$

$f(x_4) = 6.754042568 \times 10^{-3}$

$f(x_4) = 0.006754042568$

The root of equation lies b/w 0.625 & 0.5

$x_5 = \frac{x_2+x_4}{2} = \frac{0.5+0.625}{2} = 0.5625$ $\boxed{x_5 = 0.5625}$

$f(x_5) = 3(0.5625) - e^{0.5625}$

$f(x_5) = -0.06755465696$

The root of equation lies b/w 0.5625 & 0.625
according to bisection method

$x_6 = \frac{0.5625+0.625}{2} = 0.59375$ $\boxed{x_6 = 0.59375}$

$f(x_6) = 3(0.59375) - e^{0.59375}$

$f(x_6) = -0.02951607212$

The root of equation lies b/w 0.59375 & 0.625
according to bisection method

$x_7 = \frac{0.59375+0.625}{2} =$

$\boxed{x_7 = 0.609375}$

$f(x_7) = 3(0.609375) - e^{0.609375}$

$f(x_7) = -0.01115648854$

The root of the equation lies b/w 0.609375 & 0.625
according to bisection method

$x_8 = \frac{x_4+x_7}{2} = \frac{0.625+0.609375}{2}$

17. $\boxed{x_8 = 0.6171875}$

$f(x_8) = 3(0.6171875) - e^{0.6171875}$

$f(x_8) = -2.144652046 \times 10^{-3}$

$f(x_8) = -0.00214465$

The root of equation lies b/w 0.6171875 & 0.625

$x_9 = \frac{x_8 + x_4}{2} = \frac{0.6171875 + 0.625}{2}$

$\boxed{x_9 = 0.62109375}$

$f(x_9) = 3(0.62109375) - e^{0.62109375}$

$f(x_9) = 2.31893295 \times 10^{-3}$

$f(x_9) = 0.0023188932 75$

The root of eq. lies b/w 0.62109375 & 0.6171875

$x_{10} = \frac{0.62109375 + 0.6171875}{2}$

$\boxed{x_{10} = 0.619140625}$

$f(x_{10}) = 3(0.619140625) - e^{0.619140625}$

$f(x_{10}) = 9.066320344 \times 10^{-5}$

$f(x_{10}) = 0.0000906632$

The root of the eq. lies b/w 0.6171875 & 0.6191402 5
according to bisection method

$x_{11} = \frac{x_{10} + x_8}{2} = \frac{0.6171875 + 0.61914025}{2}$

$\boxed{x_{11} = 0.618163875}$

$f(x_{11}) = 3(0.618163875) - e^{0.618163875}$

$f(x_{11}) = -1.026324232 \times 10^{-3}$

$\therefore$ The real root is "0.62"

8 given

$f(x) = x - \cos x = 0$

$\boxed{\text{Put } x = 0}$

$f(0) = 0 - \cos 0$

$f(0) = -1$

$\boxed{\text{Put } x = 1}$

$f(1) = 1 - \cos(1)$

$f(1) = 0.459697$

$x_0 = 0 = -$

$x_1 = 1 = +$

$x_2 = 0.5 = -$

$x_3 = 0.75 = +$

$x_4 = 0.625 = -$

$x_5 = 0.6875 = -$

$x_6 = 0.71875 = -$

$x_7 = 0.734375 = -$

$x_8 = 0.7421875 = +$

$x_9 = 0.7328175 = -$

$x_{10} = 0.740234625 = +$

The root of eq. lies b/w 0 & 0.45967

according to bisection method

$$x_2 = \frac{x_0 + x_1}{2} = \frac{0+1}{2} = 0.5$$

$\boxed{x_2 = 0.5}$

$f(x_2) = 0.5 - \cos(0.5)$

$f(x_2) = -0.37758$

The root of eq. lies b/w 0.5 & 1

according to bisection method

$$x_3 = \frac{x_1 + x_2}{2} = \frac{1+0.5}{2} = 0.75 \quad \boxed{x_3 = 0.75}$$

$f(x_3) = 0.75 - \cos(0.75)$

$f(x_3) = 0.0183111$

The root of eq. lies b/w 0.75 & 0.5

according to bisection method

$$x_4 = \frac{x_3 + x_2}{2} = \frac{0.5+0.75}{2} = 0.625 \quad \boxed{x_4 = 0.625}$$

$f(x_4) = 0.625 - \cos(0.625)$

$f(x_4) = -0.185963$

The root of eq. lies b/w 0.625 & 0.75

according to bisection method

$$x_5 = \frac{x_3 + x_4}{2} = \frac{0.625+0.75}{2} \quad \boxed{x_5 = 0.6875}$$

$f(x_5) = 0.6875 - \cos(0.6875)$

19. $f(x_5) = -0.0853349$

The root of eq. lies b/w 0.6875 & 0.75 according to bisection method.

$$x_6 = \frac{x_5 + x_3}{2} = \frac{0.75 + 0.6875}{2} = 0.71875$$

$\boxed{x_6 = 0.71875}$

$f(x_6) = 0.71875 - \cos(0.71875)$

$f(x_6) = -0.033879$

The root of eq. lies b/w 0.71875 & 0.75 according to bisection method

$$x_7 = \frac{x_3 + x_6}{2} = \frac{0.75 + 0.71875}{2}$$

$\boxed{x_7 = 0.734375}$

$f(x_7) = 0.734375 - \cos(0.734375)$

$f(x_7) = -7.874725 \times 10^{-3}$

$f(x_7) = -0.007874725$

The root of eq. lies b/w 0.734375 & 0.75 according to bisection method

$$x_8 = \frac{x_7 + x_3}{2} = \frac{0.734375 - 0.75}{2}$$

$\boxed{x_8 = 0.7421875}$

$f(x_8) = 0.7421875 - \cos(0.7421875)$

$f(x_8) = 5.195711744 \times 10^{-3}$

$f(x_8) = 0.005195711744$

The root of eq. lies b/w 0.7421875 & 0.73475 according to bisection method

$$x_9 = \frac{x_7 + x_8}{2} = \frac{0.7421875 + 0.734375}{2}$$

$\boxed{x_9 = 0.73828175}$

$f(x_9) = 0.73828175 - \cos 0.73828175$

$f(x_9) = -1.344813243 \times 10^{-3}$

$f(x_9) = -0.001344813243$

The root of equation lies b/w 0.73828175 & 0.7421875

according to bisection method

$$x_{10} = \frac{x_8+x_9}{2} = \frac{0.73828175+0.7421875}{2}$$

$$\boxed{x_{10} = 0.740234625}$$

$f(x_{10}) = 0.740234625 - \cos(0.740234625)$

$f(x_{10}) = 1.924291396 \times 10^{-3}$

$f(x_{10}) = 0.001924291396$

The root of eq. lies b/w 0.740234625 & 0.73828175

according to bisection method

$$x_{11} = \frac{x_9+x_{10}}{2} = \frac{0.740234625+0.73828175}{2}$$

$$\boxed{x_{11} = 0.7392581875}$$

$f(x_{11}) = 0.7392581875 - \cos(0.7392581875)$

$f(x_{11}) = 2.896367992 \times 10^{-4}$

$f(x_{11}) = 0.0002896367992$

$\therefore$ The real root is "0.74"

P5 Sol/n

given

$3x = \cos x + 1$

$f(x) = 3x - \cos x - 1 = 0$

put $x=0$

$f(0) = 3(0) - \cos(0) - 1$

$f(0) = -2$

put $x=1$

$f(1) = 3(1) - \cos(1) - 1$

$f(1) = 1.459697694$

$x_0 = 0 = -$

$x_1 = 1 = +$

$x_2 = 0.5 = +$

$x_3 = 0.25 = -$

$x_4 = 0.375 = -$

$x_5 = 0.4375 = -$

$x_6 = 0.46875 = -$

$x_7 = 0.484375 = -$

$x_8 = 0.4921875 = -$

$x_9 = 0.49609375 = -$

$x_{10} = 0.498046875 = -$

$x_{11} = 0.4990234375 = -$

The root of eq. lies b/w 0 & 1

Taken as 0.5

according to bisection method

$$x_2 = \frac{x_1+x_0}{2} = \frac{0+1}{2} = 0.5$$

$\boxed{x_2 = 0.5}$

$f(x_2) = (0.5) - \cos(0.5) - 1$

$f(x_2) = 0.3163738428$

The root of eq lies b/w 0 & 0.5

according to bisection method

$x_3 = \frac{x_0 + x_2}{2} = \frac{0+0.5}{2}$

$\boxed{x_3 = 0.25}$

$f(x_3) = \frac{x_0 + x_2}{2} = \frac{0+0.5}{2} = 0.25$

$f(x_3) = 3(0.25) - \cos(0.25) - 1$

$f(x_3) = -0.2733156837$

The root of eq lies b/w 0.25 & 0.5

$x_4 = \frac{0.25+0.5}{2} = 0.375$

$\boxed{x_4 = 0.375}$

$f(x_4) = 3(0.375) - \cos(0.375) - 1$

$f(x_4) = -0.8055076219$

The root of eq lies b/w 0.375 & 0.5

$x_5 = \frac{x_4 + x_2}{2} = \frac{0.375+0.5}{2}$

$\boxed{x_5 = 0.4375}$

$f(x_5) = 3(0.4375) - \cos(0.4375) - 1$

$f(x_5) = -0.5433136834$

The root of eq lies b/w 0.4375 & 0.5

$x_6 = \frac{x_5 + x_2}{2} = \frac{0.4375+0.5}{2}$

$\boxed{x_6 = 0.46875}$

$f(x_6) = 3(0.46875) - \cos(0.46875) - 1$

$f(x_6) = -0.4995636$

The root of eq lies b/w 0.46875 & 0.5

$x_7 = \frac{x_2 + x_6}{2} = \frac{0.46875 + 0.5}{2}$ $\boxed{x_7 = 0.484375}$

$f(x_7) = 3(0.484375) - \cos(0.484375) - 1$

$f(x_7) = -0.6718411565$

The root of eq. lies b/w 0.484375 & 0.5

$\boxed{x_8 = 0.4921875}$

$x_8 = \frac{0.484375 + 0.5}{2} = 0.4921875$

$f(x_8) = 3(0.4921875) - \cos(0.4921875) - 1$

$f(x_8) = -0.40473875$

The root of eq. lies b/w 0.4921875 & 0.5

$\boxed{x_9 = 0.49609375}$

$x_9 = \frac{0.4921875 + 0.5}{2} = 0.49609375$

$f(x_9) = 3(0.49609375) - \cos(0.49609375) - 1$

$f(x_9) = -0.3913173677$

The root of eq. lies b/w 0.49609375 & 0.5

$\boxed{x_{10} = 0.498046875}$

$x_{10} = \frac{0.49609375 + 0.5}{2} = 0.498046875$

$f(x_{10}) = -0.3843766404$

The root of eq. lies b/w 0.498046875 & 0.5

The root of eq. according to bisection method

$\boxed{x_{11} = 0.4990234375}$

$x_{11} = \frac{0.498046875 + 0.5}{2} = 0.4990234375$

$f(x_{11}) = 3(0.4990234375) - \cos(0.4990234375) - 1$

$f(x_{11}) = -0.3809800199$

∴ The real root is "0.5"

False position method (or) Regula-falsi method

Find the roots of equation $x^3 - x - 4 = 0$ using false position method.

$a = 1$ = −
$b = 2$ +
$x_1 = 1.66667$ = −
$x_2 = 1.78049$ = −
$x_3 = 1.79447$ = −
$x_4 = 1.79610$ = −
$x_5 = 1.79629$ = −

Given that,

$x^3 - x - 4 = 0$

let $f(x) = x^3 - x - 4$

put $x = 0$

$f(x) = x^3 - x - 4$

$f(0) = 0^3 - 0 - 4$

$f(0) = -4 < 0$

put $x = 1$

$f(x) = x^3 - x - 4$

$f(1) = 1^3 - 1 - 4$

$f(1) = 1 - 1 - 4$

$f(1) = -4 < 0$

put $x = 2$

$f(2) = 2^3 - 2 - 4$

$f(2) = 8 - 2 - 4$

$f(2) = 2 > 0$

f(2) is positive & f(1) is positive

f(1) & f(2) are opposite.

The roots of eq. lies b/w 1 & 2

here $a = 1$ & $b = 2$

The first approximation to the root is

$$x_1 = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

here $a = 1$ $b = 2$

$f(a) = f(1) = -4$

$f(b) = f(2) = 2$

$$x_1 = \frac{1 \cdot 2 - 2(-4)}{2 - (-4)}$$

$x_1 = \frac{2+4}{6} = \frac{10}{6} = 1.666667$

$\boxed{x_1 = 1.66667}$

$f(x) = x^3 - x - 4$

$f(x_1) = x_1^3 - x_1 - 4$

$f(x_1) = 1.66667^3 - 1.66667 - 4$

$f(x_1) = -1.0304037$

$f(x_1) = -1.03704 < 0$

f(1.66667) is negative & f(2) is positive

f(1.66667) & f(2) are opposite direction

The root of the eq. lies b/w 1.66667 & 2

here $x_1 = 1.66667$ & $b = 2$

Second approximation to root is

here $x_1 = 1.66667$

$f(x_1) = -1.03704$ $b = 2$

$$x_2 = \frac{(x_1) f(b) - b f(x_1)}{f(b) - f(x_1)}$$

$$x_2 = \frac{(1.66667)(2) - 2(-1.03704)}{2 - (-1.03704)}$$

$\boxed{x_2 = 1.78049}$

$f(x_2) = x^3 - x - 4$

$f(1.78049) = (1.78049)^3 - (1.78049) - 4$

$f(1.78049) = -0.13607$

$\boxed{f(x_2) = -0.13607}$

$f(1.78049)$ is negative & $f(2)$ is positive

$f(1.78049)$ & $f(2)$ are in opposite direction

The root of equation lies b/w 1.78049 & 2

here $x_2 = 1.78049$ & $b = 2$

$$x_3 = \frac{x_2 f(b) - b f(x_2)}{f(b) - f(x_2)}$$

$$x_3 = \frac{1.78049(2) - 2(-0.13608)}{2 - (-0.13608)}$$

$\boxed{x_3 = 1.79447}$

$f(1.79447) = x^3 - x - 4$

$f(1.79447) = 1.79447^3 - 1.79447 - 4$

$\boxed{f(1.79447) = -0.016056}$

$f(1.79447)$ is negative & $f(2)$ is positive

$f(1.79447)$ & $f(2)$ are in opposite direction

The root of eq. lies b/w 1.79447 & 2

$x_3 = 1.79447$ & $b = 2$

$f(x_3) = -0.016056$

$$x_4 = \frac{x_3 f(b) - b f(x_3)}{f(b) - f(x_3)}$$

$$x_4 = \frac{1.79447(2) - 2(-0.016056)}{2 - (-0.016056)}$$

$\boxed{x_4 = 1.79610}$

$f(x) = x^3 - x - 4$

$f(x_4) = x_4^3 - x_4 - 4$

$f(1.79610) = 1.79610^3 - 1.79610 - 4$

$f(1.79610) = -1.925925 \times 10^{-3}$

$f(1.79610) = -0.0019259$

$f(1.79610)$ is negative & $f(2)$ is positive

$f(1.79610)$ & $f(2)$ are in opposite direction

The root of eq. lies b/w 1.79610 & 2

$x_4 = 1.79610$ & $b = 2$

$f(x_4) = -0.001925$

$$x_5 = \frac{x_4 f(b) - b f(x_4)}{f(b) - f(x_4)}$$

$$x_5 = \frac{1.79610(2) - 2(-0.001925)}{2 - (-0.001925)}$$

$\boxed{x_5 = 1.79629}$

$f(x) = x^3 - x - 4$

$f(1.79629) = 1.79629^3 - 1.79629 - 4$

$f(1.79629) = -2.79624 \times 10^{-4}$

$= -0.0002769$

∴ The root of the equation is "1.79"

Q2:- find the root of the equation $x \log_{10}^{x} = 1.2$ by using false position method.

A. given that

$x \log_{10}^{x} = 1.2$

$x \log_{10} x - 1.2 = 0$

let $f(x) = x \log_{10} x - 1.2$

put $x = 0$

$f(0) = 0 \log_{10}^{0} - 1.2$

$f(0) = -1.2$

put $x = 1$

$f(1) = 1 \log_{10}(1) - 1.2$

$f(1) = -1.2$

put $x = 2$

$f(2) = 2 \log_{10}(2) - 1.2$

$f(2) = -0.59794$

put $x = 3$

$f(3) = 3 \log_{10}(3) - 1.2$

$f(3) = 0.23136$

$f(2)$ is negative & $f(3)$ is positive

$f(2)$ & $f(3)$ are opposite direction

The root of the equation b/w 2 & 3

here $a = 2$, $b = 3$

The first approximation to the root is

$a = 2 = -$

$b = 3 = +$

$x_1 = 2.72102 = -$

$x_2 = 2.74019 = +$

$x_3 = 2.74062 = -$

$$x_1 = \frac{a \cdot f(b) - b\, f(a)}{f(b) - f(a)}$$

here $a = 2$, $b = 3$

$f(a) = f(2) = -0.59794$

$f(b) = f(3) = 0.23136$

$$x_1 = \frac{2(0.23136) - 3(-0.59794)}{0.23136 - (-0.59794)}$$

$$\boxed{x_1 = 2.72102}$$

$f(x) = x \log_{10}^{x_1} - 1.2$

$f(x_1) = x_1 \log_{10}^{x_1} - 1.2$

$f(2.72102) = 2.72102 \log 2.72102 - 1.2$

$f(2.72102) = -0.01708$

$f(2.72102)$ is negative & $f(3)$ is positive

$f(2.72102)$ & $f(3)$ are opposite direction.

The root of eq. lies b/w 2.72102 & 3

here $x_1 = 2.72102$ & $b = 3$

The second approximation to the root is

$$x_2 = \frac{x_1 \cdot f(b) - b(f x_1)}{f(b) - f(x_1)}$$

$$x_2 = \frac{2.72102 \cdot (0.23136) - 0.3(-0.01708)}{0.23136 - (-0.01708)}$$

$$\boxed{x_2 = 2.74019}$$

$f(x_2) = 2.74019 \log 2.74019 - 1.2$

$f(x_2) = 3.8716 \times 10^{-4} = -0.00038716$ (x)

$= -3.977664 \times 10^{-4} = -0.00039776$

$f(2.74109)$ & $f(3)$ are opposite direction

The root of eq lies b/w 2.74109 & 3

here $x_2 = 2.74019$ & $b = 3$

The third approximation to the root is

$x_2 = 2.74019 \quad b = 3$

$f(x_2) = -0.0003974$

$$x_3 = \frac{x_2 f(b) - b f(x_2)}{f(b) - f(x_2)} = \frac{2.74019(0.23136) - 3(-0.000399)}{0.23136 - (-0.0003974)}$$

$\boxed{x_3 = 2.74062}$

$f(x) = x \cdot \log x - 1.2$

$f(x_3) = 2.74062 \log(2.74062) - 1.2$

$f(x_3) = -2.27594 \times 10^{-5}$

$f(x_3) = -0.00002275$

$\therefore$ The root of equation is "2.74"

3- find the root of the equation $2x - \log_{10} x = 7$ which lies b/w 3.5 & 4 by regula-falsi method.

A. given that

$2x - \log_{10} x = 7$

$2x - \log_{10} x - 7 = 0$

let $f(x) = 2x - \log_{10} x - 7 = 0$

put $x = 0$

$f(0) = 2(0) - \log 0 - 7$

$f(0) = -7$

put $x = 1$

$f(1) = 2(1) - \log(1) - 7$

28 $f(1) = -5$

put $x = 2$

$f(2) = 2(2) - \log(2) - 7$

A. put $x_1 = 3.5$

$f(x_1) = 2(3.5) - \log 3.5 - 7$

$f(x_1) = -0.54407$

here

$x_1 = a$ $\quad a = 3.5 = -$

$x_2 = b$ $\quad b = 4 = +$

$x_1 = 3.78878 = -$

$x_2 = 3.78928 = -$

put $x_1 = 4$

$f(x_2) = 2(4) - \log(4) - 7$

$f(x_2) = 0.39794$

f (a) is negative & f(b) is positive

3.5 is negative & 4 is positive

f(3.5) & f(4) are opposite direction

The root of equation lies b/w 3.5 & 4

here $a = 3.5$ $\quad b = 4$ $\quad f(a) = -0.54407$ $\quad f(b) = 0.39794$

The first approximation to root is

$$x_1 = \frac{a.f(b) - b\,f(a)}{f(b) - f(a)} = \frac{3.5(0.39794) - 4(-0.54407)}{0.39794 - (-0.54407)}$$

$x_1 = 3.78878$

$f(x) = 2x - \log x - 7$

$f(3.78878) = 2(3.78878) - \log(3.78878) - 7$

$f(x_1) = -9.39388 \times 10^{-4} = -0.00093938$

f(4) is positive & $f(x_1)$ = negative

4 & 3.78878 are opposite direction

The root of eq. lies b/w 4 & 3.78878

here $x_1 = 3.78878$ $\quad b = 4$ $\quad f(x_1) = -0.00093938$ $\quad f(b) = 0.39794$

$$x_2 = \frac{x_1 f(b) - b f(x_1)}{f(b) - f(x_1)} = \frac{3.78878(0.39794) - 4(-0.00093938)}{0.39794 - (-0.00093938)}$$

$x_2 = 3.78927$

$f(x_2) = 2(3.78927) - \log(3.78927) - 7 = -1.55515 \times 10^{-5}$

$f(x_2) = -0.0000155$

$\therefore$ The positive root is "3.789."

Q:- Find the square root of equation $\log x = \cos x$ by using regula-falsi method. (in radians)

A: given that

$$\log x = \cos x$$

$$\log x - \cos x = 0$$

let $f(x) = \log x - \cos x$

$\left[\because \log 1 = 0 \quad \because \log 0 = 1 \right]$

How

$x \cdot e^x = 3$

$x e^x = 2$

$e^x \cdot \sin x = 1$ (degree)

$x \cdot e^x = \cos x$

put $x = 0$

$f(0) = \log(0) - \cos(0)$

$f(0) = 0$

put $x = 1$

$f(1) = \log(1) - \cos(1)$

$f(1) = -0.99984 < 0$

$f(0) = 0$

$f(1) = -0.54030$

put $x = 2$

$f(2) = \log(2) - \cos(2)$ (x)

$f(2) = -0.69836 < 0$

put $x = 3$

$f(3) = \log(3) - \cos(3)$

$f(3) = -0.52150$

put $x = 4$

$f(4) = \log(4) - \cos(4)$

$f(4) = -0.39551$

put $x = 5$

$f(5) = \log(5) - \cos(5)$

$f(5) = -0.29723$

put $x = 6$

$f(6) = \log(6) - \cos(6)$ (X)

$f(6) = -0.21637$

Q. given that

$\log x = \cos x$

$\log x - \cos x = 0$

let $f(x) = \log x - \cos x$

put $x = 0$

$f(0) = \log(0) - \cos(0)$

$f(0) = 0$

$x = 0 =$

$a = 1 = -$

$b = 2 = +$

$x_1 = 1.42967 = +$

$x_2 = 1.41838 = -$

$x_3 = 1.41870 = +$

put $x = 1$

$f(1) = \log(1) - \cos(1)$

$f(1) = -0.54031$

put $x = 2$

$f(2) = \log(2) - \cos(2)$

$f(2) = 0.71717$

f(1) is negative & f(2) is positive

f(1) & f(2) are opposite direction.

The root of the equation lies b/w 1 & 2

here $a = 1$ $b = 2$ $f(a) = -0.54031$ $f(b) = 0.71717$

The first approximation to root is

$$x_1 = \frac{a \cdot f(b) - b f(a)}{f(b) - f(a)} = \frac{1(0.71717) - 2(-0.54031)}{0.71717 - (-0.54031)}$$

$x_1 = 1.42967$

$f(x) = \log x - \cos x$

$f(x_1) = \log(1.42967) - \cos(1.42967)$

$\boxed{f(x_1) = 0.01457}$

$f(1)$ is negative & $f(x_1)$ is positive

$f(1)$ & $f(x_1)$ are in opposite direction

The root of eq. lies b/w 1 & 1.42967

here $a = 1$ $x_1 = 1.42967$

$f(a) = -0.54031$ $f(x_1) = 0.01457$

The second approximation to root is

$$x_2 = \frac{a \cdot f(x_1) - x_1 f(a)}{f(x_1) - f(a)} = \frac{1(0.01457) - 1.42967(-0.54031)}{0.01457 - (-0.54031)}$$

$\boxed{x_2 = 1.41838}$

$f(x_2) = \log(1.41838) - \cos(1.41838)$

$f(x_2) = -3.42891 \times 10^{-5}$

$f(x_2) = -0.0000342891$

$f(x_2)$ is negative & $f(x_1)$ is positive

$f(x_2)$ & $f(x_1)$ are in opposite direction

The root of eq. lies b/w 1.42967 & 1.41838

here $x_1 = 1.42967$ $x_2 = 1.41838$

$f(x_1) = 0.01457$ $f(x_2) = (-0.0000342891)$

The third approximation to root is

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} = 1.42967($$

$$= \frac{1.42967(-0.0000342891) - 1.41838(0.01457)}{(-0.0000342891) - (0.01457)}$$

$$\boxed{x_3 = 1.41840}$$

$f(x_3) = \log(1.41840) - \cos(1.41870)$

$f(x_3) = 3.799788 \times 10^{-4}$

$= 0.0003799$

$f(x_2)$ is negative & $f(x_3)$ is positive

$f(x_2)$ & $f(x_3)$ are in opposite direction

The root of eq. lies b/w 1.41838 & 1.41840

here $x_2 = 1.42967$ $x_3 = 1.41840$

$f(x_2) = -0.00003$ $f(x_3) = 0.00037$

The fourth approximation to root is

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{1.42967(0.00037) - 1.41840(-0.00003)}{0.00037 - (-0.00003)}$$

$$\boxed{x_4 = 1.41882}$$

∴ The root of eq is root is "1.41"

P5. find the root of the equation $x.e^x = 3$ by using regula-falsi method.

A. given that,

$a = 1 \quad = -$

$b = 2 \quad = +$

$x_1 = 1.02335 = -$

$x_2 = 1.03583 = -$

$x_3 = 1.04246 = -$

$x_4 = 1.04597 = -$

$x.e^x = 3$

$x.e^x - 3 = 0$

$f(x) = x.e^x - 3$

put $x = 0$

$f(0) = 0.e^0 - 3 = 0$

$f(0) = -3 < 0$

put $x = 1$

$f(1) = 1.e^1 - 3$

$f(1) = -0.28171$

put $x = 2$

$f(2) = 2.e^2 - 3$

$f(2) = 11.77812$

f(a) is negative & f(b) is positive

f(a) & f(b) are in opposite direction

The equation of root lies b/w 1 & 2

here $a = 1 \quad b = 2 \quad f(a) = -0.28171 \quad f(b) = 11.77812$

The first approximation to root is

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{1(11.77812) - 2(-0.28191)}{11.77812 - (-0.28171)}$$

$\boxed{x_1 = 1.02335}$

$f(x) = x.e^x - 3$

$f(1.02335) = 1.02335 . e^{1.02335} - 3$

$f(1.02335) = -0.15252$

34 $f(x_1)$ is negative & f(b) is positive

$f(x_1)$ & $f(b)$ are in opposite direction

The equation of root lies b/w 1.02335 & 2

$x_1 = 1.02335 \quad b = 2 \quad f(x_1) = -0.15252 \quad f(b) = 11.77812$

$$x_2 = \frac{x_1 f(b) - b f(x_1)}{f(b) - f(x_1)} = \frac{1.02335(11.77812) - 2(-0.15252)}{11.77812 - (-0.15252)}$$

$\boxed{x_2 = 1.03583}$

$f(x_2) = x_2 - e^{x_2} - 3$

$f(1.03583) = 1.03583 - e^{1.03583} - 3$

$f(1.03583) = -0.08160$

$f(x_2)$ & $f(b)$ are in opposite direction

$f(x_2)$ is negative & $f(b)$ is positive

The eq. of root lies b/w 1.03583 & 2

$x_2 = 1.03583 \quad b = 2 \quad f(x_2) = -0.08160 \quad f(b) = 11.77812$

$$x_3 = \frac{x_2 f(b) - b f(x_2)}{f(b) - f(x_2)} = \frac{1.03583(11.77812) - 2(-0.08160)}{11.77812 - (-0.08160)}$$

$\boxed{x_3 = 1.04246}$

$f(1.04246) = 1.04246 - e^{1.04246} - 3$

$f(1.04246) = -0.04339$

$f(x_3)$ & $f(b)$ are in opposite direction

$f(x_3)$ is negative & $f(b)$ are positive

The eq of root lies b/w 1.04246 & 2

$x_3 = 1.04246 \quad b = 2 \quad f(x_3) = -0.04339 \quad f(b) = 11.77812$

$$x_4 = \frac{x_3 f(b) - b f(x_3)}{f(b) - f(x_3)} = \frac{1.04246(11.77812) - 2(-0.04339)}{11.77812 - (-0.04339)}$$

$\boxed{x_4 = 1.04597}$

$f(1.04597) = 1.04597 - e^{1.04597} - 3$

35 $f(1.04597) = -0.02300$

∴ The real root is "1.04"

Q:- Find the root of the equation $x \cdot e^x = 2$ by using regula-falsi method.

A: Given data,

$a = 0 = -$

$b = 1 = +$

$x \cdot e^x = 2$

$x_1 = 0.73575 = -$

$x \cdot e^x - 2 = 0$

$x_2 = 0.83951 = -$

$f(x) = x \cdot e^x - 2 = 0$

$x_3 = 0.85119 = -$

put $x = 0$

$f(0) = 0 \cdot e^0 - 2 = 0$

$f(0) = -2 < 0$

put $x = 1$

$f(1) = 1 \cdot e^1 - 2 = 0$

$f(1) = 0.71829 > 0$

$f(0)$ is negative & $f(1)$ is positive

$f(0)$ & $f(1)$ are in opposite direction

The root of the equation lies b/w 0 & 1

here $a = 0$ $b = 1$, $f(a) = -2$ $f(b) = 0.71829$

The first approximation to root is

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{0(0.71829) - 1(-2)}{0.71829 - (-2)}$$

$\boxed{x_1 = 0.73575}$ $\Leftrightarrow$ $f(0.73575) = 0.73575 \cdot e^{0.73575} - 2$

$= -0.46445$

$f(x_1)$ is negative & $f(b)$ is positive

$f(a)$ & $f(b)$ are in opposite direction

The root of the equation lies b/w 1 & 0.73575

The second approximation to the root is

$b = 1 \quad x_1 = 0.73575 \quad f(b) = 0.71829 \quad f(x_1) = -0.46445$

$$x_2 = \frac{x_1 f(b) - b f(x_1)}{f(b) - f(x_1)} = \frac{0.73575(+0.71829) - 1(-0.46445)}{0.71829 - (-0.46445)}$$

$\boxed{x_2 = 0.83951}$

$f(x_2) = 0.83951 \cdot e^{0.83951} - 2$

$f(x_2) = -0.05633$

$f(x_2)$ is negative & $f(b)$ is positive

$f(x_2)$ & $f(b)$ are in opposite direction

The root of the eq. lies b/w 0.83951 & 1

The third approximation to root is

$b = 1 \quad x_2 = 0.83951 \quad f(b) = 0.71829 \quad f(x_2) = -0.05633$

$$x_3 = \frac{x_2 f(b) - b f(x_2)}{f(b) - f(x_2)} = \frac{0.83951(0.71829) - 1(-0.05633)}{0.71829 - (-0.05633)}$$

$\boxed{x_3 = 0.85118}$

$f(x_3) = 0.85118 \cdot e^{0.85118} - 2$

$f(x_3) = -6.18808 \times 10^{-3}$

$= -0.006188$

$f(x_3)$ is negitive & $f(b)$ is positive

$f(x_3)$ & $f(b)$ are in opposite direction

The root of eq. lies b/w 0.85118 & 1

The fourth approximation to root is

$b = 1 \quad x_3 = 0.85118 \quad f(b) = 0.71829 \quad f(x_3) = -0.00618$

$$x_4 = \frac{x_3 f(b) - b\, f(x_3)}{f(b) - f(x_3)} = \frac{0.85118(0.71829) - 1(-0.00618)}{0.71829 - (-0.00618)}$$

$\boxed{x_4 = 0.85244}$

$f(x_4) = 0.85244 \cdot e^{0.85244} - 2$

$= -7.1913 \times 10^{-4}$

$= 0.00071913$

$\therefore$ The real root is "0.85"

P7:- find the root of equation $e^x \cdot \sin x = 1$ by using regula-falsi method. (degrees)

A. Given that

$e^x \cdot \sin x = 1$

$e^x \cdot \sin x = 1$

$f(x) = e^x \sin x - 1$

put $x = 0$

$f(0) = e^0 \sin 0 - 1$

$f(0) = -1$

put $x = 1$

$f(1) = e^1 \cdot \sin 1 - 1$

$f(1) = -0.95255$

put $x = 2$

$f(2) = e^2 \cdot \sin 2 - 1$

$f(2) = -0.74218$

put $x = 3$

$f(3) = e^3 \cdot \sin 3 - 1$

$f(3) = 0.05119$

$a = 2 = -$

$b = 3 = +$

$x_1 = 2.93547 = -$

$x_2 = 2.96196 = -$

$x_3 = 2.92954 = -$

f(a) is negative & f(b) is positive

f(a) & f(b) are in opposite direction

The root of eq. lies b/w 2 & 3

The first approximate to root is

$a = 2 \quad b = 3 \quad f(a) = -0.74212 \quad f(b) = 0.05119$

$$x_1 = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)} = \frac{2(0.05119) - 3(-0.74212)}{0.05119 - (-0.74212)}$$

$$\boxed{x_1 = 2.93547}$$

$f(x_1) = e^{x_1} \cdot \sin x_1 - 1$

$f(2.93547) = e^{2.93547} \cdot \sin(2.93547) - 1$

$f(2.93547) = -0.03567$

f(x_1) is negative & f(b) is positive

f(x_1) & f(b) are in opposite direction

The root of eq. lies b/w 2.93547 & 3

The second approximate to root is

$x_1 = 2.93547 \quad b = 3 \quad f(x_1) = -0.03567 \quad f(b) = 0.05119$

$$x_2 = \frac{x_1\,f(b) - b\,f(x_1)}{f(b) - f(x_1)} = \frac{2.93547(0.05119) - 3(-0.03567)}{0.05119 - (-0.03567)}$$

$$\boxed{x_2 = 2.96196}$$

$f(x_2) = e^{x_2} \cdot \sin x_2 - 1$

$f(2.96196) = e^{2.96196} \cdot \sin 2.96196 - 1$

$f(2.96196) = -8.60834 \times 10^{-4}$

$= -0.0008608$

$f(x_2)$ is negative & $f(b)$ is positive

$f(x_2)$ & $f(b)$ are in Opposite direction

The roots of equation lies b/w 2.96196 & 3

The third approximate root is

$$x_3 = \frac{x_2 f(b) - b f(x_2)}{f(b) - f(x_2)} = \frac{2.96196(0.05119) - 3(-0.00086)}{0.05119 - (-0.00086)}$$

$x_2 = 2.96196$ $b = 3$ $f(x_2) = -0.00086$ $f(b) = 0.05119$

$$\boxed{x_3 = 2.92954}$$

$f(x_3) = e^{2.92954} \cdot \sin(2.92954) - 1$

$f(x_3) = -0.04331$

$f(x_3)$ is negative & $f(b)$ is positive

$f(x_3)$ & $f(b)$ are in opposite direction

The roots of eq. lies b/w 2.92954 & 3

The fourth approximation to root is

$x_3 = 2.92954$ $b = 3$ $f(x_3) = -0.04331$ $f(b) = 0.05119$

$$x_4 = \frac{x_3 f(b) - b f(x_3)}{f(b) - f(x_3)} = \frac{2.92954(0.05119) - 3(-0.04331)}{0.05119 - (-0.04331)}$$

$$\boxed{x_4 = 2.96183}$$

$f(x_4) = e^{2.96183} \cdot \sin(2.96183) - 1$

$= -1.034 \times 10^{-3}$

$= -0.00103$

$f(x_4)$ is negative & $f(b)$ is positive

$f(x_4)$ & $f(b)$ are in opposite direction

The roots of eq lies b/w 2.96183 & 3

The fifth approximation to root is

$x_4 = 2.96183 \quad b = 3 \quad f(x_4) = -0.00103 \quad f(b) = 0.05119$

$$x_5 = \frac{x_4 f(b) - b f(x_4)}{f(b) - f(x_4)} = \frac{2.96183(0.05119) - 3(-0.00103)}{0.05119 - (-0.00103)}$$

$$\boxed{x_5 = 2.96258}$$

$f(x_5) = e^{x_5} \cdot \sin x_5 - 1$

$= e^{2.96258} \cdot \sin(2.96258) - 1$

$f(x_5) = -3.20923 \times 10^{-5}$

$f(x_5) = 0.0000320$

$\therefore$ The root of eq is "2.96"

P8:- Find the root of eq. $x \cdot e^x = \cos x$ by using regula falsi method.

A. given data,

$x \cdot e^x = \cos x$

$x \cdot e^x - \cos x = 0$

$f(x) = x \cdot e^x - \cos x$

put $x = 0$

$f(0) = 0 \cdot e^0 - \cos(0)$

$f(0) = -1$

put $x = 1$

$f(1) = 1 \cdot e^1 - \cos(1)$

$f(1) = 1.71843$

$a = 0 = -$

$b = 1 = +$

$x_1 = 0.36785 = -$

$x_2 = 0.50328 = -$

$x_3 = 0.54738 = -$

$x_4 = 0.56109 = -$

$x_5 = 0.56528 = -$

$f(a)$ is negative & $f(b)$ is positive

$f(a)$ & $f(b)$ are in opposite direction

The root of eq. lies b/w 0 & 1

The first approximation to root is

$a = 0 \quad b = 1 \quad f(a) = -1 \quad f(b) = 1.71843$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{0(1.71843) - 1(-1)}{1.71843 - (-1)}$$

$$\boxed{x_1 = 0.36785}$$

$f(x_1) = x_1 \cdot e^{x_1} - \cos(x_1) = 0.36785 \times e^{0.36785} - \cos(0.36785)$

$f(x_1) = -0.46857$

$f(x_1)$ is negative & $f(b)$ is positive

$f(x_1)$ & $f(b)$ are in opposite direction

The root of eq. lies b/w 0.36785 & 1

The second approximation to root is

$x_1 = 0.36785 \quad b = 1 \quad f(x_1) = -0.46857 \quad f(b) = 1.71843$

$$x_2 = \frac{x_1 f(b) - b f(x_1)}{f(b) - f(x_1)} = \frac{0.36785(1.71843) - 1(-0.46857)}{1.71843 - (-0.46857)}$$

$$\boxed{x_2 = 0.50328}$$

$f(x_2) = 0.50328 \cdot e^{0.50328} - \cos(0.50328)$

$f(x_2) = -0.16746$

$f(x_2)$ is negative & $f(b)$ is positive

$f(x_2)$ & $f(b)$ are in opposite direction

The root of eq. lies b/w 0.50328 & 1

The third approximation to root is

$x_2 = 0.50328 \quad b = 1 \quad f(x_2) = -0.16746 \quad f(b) = 1.71843$

$$x_3 = \frac{x_2 f(b) - b f(x_2)}{f(b) - f(x_2)} = \frac{0.50328(1.71843) - 1(-0.16746)}{1.71843 - (-0.16746)}$$

$$\boxed{x_3 = 0.54738}$$

$f(x_3) = 0.54738 \cdot e^{0.54738} - \cos(0.54738)$

$f(x_3) = -0.05368$

$f(x_3)$ is negative & $f(b)$ is positive

$f(x_3)$ & $f(b)$ are in opposite direction

The root of eq. lies b/w 0.54738 & 1

The fourth approximation to root is

$x_3 = 0.54738 \quad b = 1 \quad f(x_3) = -0.05368 \quad f(b) = 1.71843$

$$x_4 = \frac{x_3 f(b) - b f(x_3)}{f(b) - f(x_3)} = \frac{0.54738(1.71843) - 1(-0.05368)}{1.71843 - (-0.05368)}$$

$$\boxed{x_4 = 0.56109}$$

$f(x_4) = 0.56109 \cdot e^{0.56109} - \cos(0.56109)$

$f(x_4) = -0.01659$

$f(x_4)$ is negative & $f(b)$ is positive

$f(x_4)$ & $f(b)$ are in opposite direction

The root of eq. lies b/w 0.56109 & 1

The fifth approximation to root is

$x_4 = 0.56109 \quad b = 1 \quad f(x_4) = -0.01659 \quad f(b) = 1.71843$

$$x_5 = \frac{x_4 f(b) - b f(x_4)}{f(b) - f(x_4)} = \frac{0.56109(1.71843) - 1(-0.01659)}{1.71843 - (-0.01659)}$$

$$\boxed{x_5 = 0.56528}$$

$$f(x_5) = e^{0.56528} \cdot 0.56528 - \cos(0.56528)$$

$$f(x_5) = -5.09216 \times 10^{-3}$$

$$= -0.005092$$

$\therefore$ The real root of equation is "0.56"

which ...

The Iteration method (or) Method of Successive approximations

Let the given equation be

$f(x) = 0 \longrightarrow (1)$

The equation (1) can be written in the form

$x = \phi(x) \longrightarrow (2)$

let $x = x_0$ be an approximate value of the required root

The first approximation is given by $x = \phi(x_0)$

The successive approximation are given by

$x_2 = \phi(x_1)$

$x_3 = \phi(x_2)$

$x_4 = \phi(x_3)$

$\vdots$

$x_n = \phi(x_{n-1})$

The sequence $(x_0, x_1, x_2, x_3 \ldots x_n)$ is called the sequence of successive approximations

III. Iterative Method

P:- find a positive root of eq. $x^3+x^2-1=0$ by iteration method.

Sol. given equation is $x^3+x^2-1=0$

let $f(x) = x^3+x^2-1$

put $x=0$

$f(0) = 0^3+0^2-1$

$f(0) = -1 < 0$

put $x=1$

$f(1) = 1^3+1^2-1$

$f(1) = 1$

$f(1) = 1 > 0$

f(0) is negitive & f(1) is positive

the root of the equation lies b/w 0 & 1

This equation can be written in the form.

$$x = \phi(x)$$

we get $x^3+x^2-1=0$

$$x^3+x^2=1$$

$$x^2(x+1)=1$$

$$x^2 = \frac{1}{x+1}$$

$$x^2 = \sqrt{\frac{1}{x+1}}$$

$$x = \frac{1}{\sqrt{x+1}} \quad (\text{or}) \quad x = \frac{1}{\sqrt{1+x}}$$

$$\phi(x) = \frac{1}{\sqrt{1+x}}$$

$$\phi(x) = \frac{1}{(1+x)^{1/2}} = (1+x)^{-1/2}$$

$(\because x^n = n \cdot x^{n-1})$

$$\phi(x) = (1+x)^{-1/2}$$

$$\phi'(x) = -\frac{1}{2}(1+x)^{-\frac{1}{2}-1}$$

$$\phi'(x) = -\frac{1}{2}(1+x)^{\frac{-1-2}{2}} = -\frac{1}{2}(1+x)^{-3/2}$$

$$|\phi'(x)| = \left|-\frac{1}{2} \cdot \frac{1}{(1+x)^{3/2}}\right|$$

put $x = 0$

$$|\phi'(0)| = \left|-\frac{1}{2} \cdot \frac{1}{(1+0)^{3/2}}\right| = \left|-\frac{1}{2}(1)\right| = \left|-\frac{1}{2}\right| = \frac{1}{2}$$

$$|\phi'(0)| = \frac{1}{2} = 0.5 < 1$$

put $x = 1$

$$|\phi'(1)| = \left|-\frac{1}{2} \cdot \frac{1}{(1+1)^{3/2}}\right| = |-0.176776|$$

$$|\phi'(1)| = 0.176776$$

So the iteration method can be applied

$$x_0 = \frac{0+1}{2} = 0.5$$

$$x = \frac{1}{\sqrt{1+x}}$$

$x_1 = \frac{1}{\sqrt{1+0.176}} = 0.92183$

$x_2 = \frac{1}{\sqrt{1+x_1}} = \frac{1}{\sqrt{1+0.8164}} = 0.741983$

$x_3 = \frac{1}{\sqrt{1+x_2}} = \frac{1}{\sqrt{1+0.741983}} = 0.757666$

$x_4 = \frac{1}{\sqrt{1+x_3}} = \frac{1}{\sqrt{1+0.75427}} = 0.754278$

$x_4 = \frac{1}{\sqrt{1+x_4}} = \frac{1}{\sqrt{1+0.754278}} = 0.755066$

$x_5 = \frac{1}{\sqrt{1+x_5}} = \frac{1}{\sqrt{1+755066}} = 0.754850$

$x_6 = \frac{1}{\sqrt{1+x_6}} = \frac{1}{\sqrt{1+0.754850}} = 0.754883$

$x_7 = \frac{1}{\sqrt{1+x_7}} = \frac{1}{\sqrt{1+0.754883}} = 0.754876$

$x_8 = \frac{1}{\sqrt{1+x_8}} = \frac{1}{\sqrt{1+0.754876}} = 0.754878$

$\therefore$ The positive root of eq is 0.75488

P2:- solve $x^3 = 2x+5$ for a positive root by iteration method.

A. Given eq $x^3 = 2x+5$

$x^3 - 2x - 5 = 0$

Let $f(x) = x^3 - 2x - 5 = 0$

put $x = 0$

$f(0) = 0^3 - 2(0) - 5$

$f(0) = -5 < 0$

put $x = 1$

$f(1) = 1^3 - 2(1) - 5$

$f(1) = -6 < 0$

put $x = 2$

$f(2) = 2^3 - 2(2) - 5$

$f(2) = -1 < 0$

put $x = 3$

$f(3) = 3^3 - 2(3) - 5$

$f(3) = 16 > 0$

$f(2)$ is negitive & $f(3)$ is positive

The root of eq. lies b/w 2 & 3

The eq. can be written in the form

$x = \phi(x)$

We get $x^3 = 2x + 5$

$x = (2x)^{1/3}$

i.e $\phi(x) = (2x+5)^{1/3}$

$\phi'(x) = \frac{1}{3}(2x+5)^{1/3 - 1} \cdot (2)$

$= \frac{2}{3}(2x+5)^{-2/3}$

$\phi'(x) = \frac{2}{3(2x+5)^{2/3}}$

put $x = 2$

$\phi'(2) = \frac{2}{3(2(2)+5)^{2/3}}$

$\phi'(2) = 0.154080 < 1$

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put $x = 3$

$$\phi'(3) = \frac{2}{3(2(3)+5)^{2/3}}$$

$\phi'(3) = 0.1347866 < 0$

so the iteration is applied

$x_0 = \frac{2+3}{2} = \frac{5}{2} = 2.5$ $\boxed{x_0 = 2.5}$

$\boxed{x_1 = (2x+5)^{1/3}}$

$x_1 = (2(2.5)+5)^{1/3} = 2.154434$

$\boxed{x_1 = 2.154434}$

$x_2 = (2(2.154434)+5)^{1/3} = 2.103612$

$\boxed{x_2 = 2.103612}$

$x_3 = (2(2.103612)+5)^{1/3} = 2.095927$

$\boxed{x_3 = 2.095927}$

$x_4 = (2(2.095927)+5)^{1/3} = 2.094760$

$\boxed{x_4 = 2.094760}$

$x_5 = (2(2.094760)+5)^{1/3} = 2.09458316$

$\boxed{x_5 = 2.09458316}$

$$x_6 = \left(2(2.094503)+5\right)^{1/3} = 2.094544$$

$$x_6 = 2.094544$$

$$x_7 = \left(2(2.094544)+5\right)^{1/3} = 2.094550$$

$$x_7 = 2.094550$$

$\therefore$ The real root of the equation is "2.09455"

$$x_1 = (2(2.5)+5)^{1/3} = 2.154434$$

$$x_2 = (2(2.154434)+5)^{1/3} = 2.103612$$

$$x_3 = (2(2.103612)+5)^{1/3} = 2.095927$$

$$x_4 = (2(2.095927)+5)^{1/3} = 2.094760$$

$$x_5 = (2(2.094760)+5)^{1/3} = 2.0945\overset{31}{86}$$

$$x_6 = (2(2.094503)+5)^{1/3} = 2.094544$$

$$x_7 = (2(2.094544)+5)^{1/3} = 2.094550$$

$\therefore$ The real root of the equation is 2.0945" i3 find a real root of $2x - \log x = 7$ by iterative method.

A given that

$$2x - \log x = 7$$

$$2x - \log x - 7 = 0$$

let $f(x) = 2x - \log x - 7$

put $x = 0$

$f(0) = 2(0) - \log(0) - 7$

$f(0) = -7$

put $x = 1$

$f(1) = 2(1) - \log(1) - 7$

$f(1) = -5$

put $x = 2$

$f(2) = 2(2) - \log(2) - 7$

$f(2) = -3.3010$

put $x = 3$

$f(3) = 2(3) - \log(3) - 7$

$f(3) = -1.477121$

put $x = 4$

$f(4) = 2(4) - \log(4) - 7$

$f(4) = 0.397940 < 0$

$f(3)$ is negitive & $f(4)$ is positive

The root of eq. lies b/w 3 & 4

The eq. can be written in the form

$$x = \phi(x)$$

$2x - \log x = 7$

$2x = \log x + 7$

$x = \dfrac{\log x + 7}{2}$

i.e $\phi(x) = \dfrac{\log x + 7}{2}$

$\phi'(x) = \dfrac{\frac{1}{x}}{2}$

put $x = 3$

$|\phi'(3)| = \dfrac{\frac{1}{3}}{2} = 0.166666 < 1$

put $x = 4$

$|\phi'(4)| = \dfrac{\frac{1}{4}}{2} = 0.125 < 1$

So iteration method can be applied

$x_0 = \dfrac{3+4}{2} = \dfrac{7}{2} = 3.5$

$\boxed{x_0 = 3.5}$

$\boxed{x = \dfrac{\log x + 7}{2}}$

$x_1 = \dfrac{\log(x_0) + 7}{2} = \boxed{3.772034 = x_1}$

$x_2 = \dfrac{\log(x_1) + 7}{2} = \boxed{3.788287 = x_2}$

$x_3 = \dfrac{\log(x_2) + 7}{2} = \boxed{3.789221 = x_3}$

$x_4 = \dfrac{\log(x_3) + 7}{2} = \boxed{3.78927 = x_4}$

$x_5 = \dfrac{\log(x_4) + 7}{2} = \boxed{3.789278 = x_5}$

$x_6 = \frac{\log(x_5)+7}{2} = \boxed{3.789278 = x_6}$

$x_7 = \frac{\log(x_6)+7}{2} = \boxed{3.789278 = x_7}$

$\therefore$ The real root is "3.789278"

Q4. Find a real root of equation $x = \frac{1}{2} + \sin x$ using iterative method.

Given that, $x = \frac{1}{2} + \sin x$

$x - \frac{1}{2} - \sin x = 0.$

let $f(x) = x - \frac{1}{2} - \sin x$

put $x = 0$

$f(0) = 0 - \frac{1}{2} - \sin(0)$

$f(0) = -0.5.$

put $x = 1$

$f(1) = 1 - \frac{1}{2} - \sin(1)$

$f(1) = 0.482544$

$\cdot$ $f(0)$ is negitive & $f(1)$ is positive

The root of eq lies b/w 0 & 1

The eq. can be written in the form

$x = \phi(x)$

we get, $x = \frac{1}{2} + \sin x$

i,e $\phi(x) = \frac{1}{2} + \sin x$

$\phi'(x) = \cos x$

put $x = 0$

$|\phi'(0)| = \cos 0 = 1$

$|\phi'(1)| = \cos 1 = 0.9998476$

$x_0 = 0 + \frac{1}{2} = \frac{0+1}{2} = 0.5$

$x_0 = 0.5$

$x = \frac{1}{2} + \sin x$

$x_1 = \frac{1}{2} + \sin(x_0) = \frac{1}{2} + \sin(0.5) = 0.508726$

$\boxed{x_1 = 0.508726}$

$x_2 = \frac{1}{2} + \sin(x_1)$

$x_2 = 0.508878$

$x_3 = \frac{1}{2} + \sin(x_3)$

$x_3 = 0.508881$

$\therefore$ The real root is "0.508881"

$x_4 = \frac{1}{2} + \sin(x_4)$

$x_4 = 0.508881$

$x_5 = \frac{1}{2} + \sin(x_5)$

$x_5 = 0.508881$

$x_6 = \frac{1}{2} + \sin(x_6)$

$x_6 = 0.508881$

$x_7 = \frac{1}{2} + \sin(x_7)$

$x_7 = 0.508881$

P5. using the iterative method, find a real root of equation $\cos x = 3x - 1$ correct the 3 decimal places (here $x = 0$ degrees & $x = 90^\circ$ degrees)

A. given that $\cos x = 3x - 1$

$$\cos x - 3x + 1 = 0$$

put $x = 0$

$f(0) \cos(0) - 3(0) + 1 = 0$

$f(0) = 2$

$f(90) = \cos(90) - 3(90) + 1 = 0$

$f(1) = -269$

$f(0)$ is positive, $f(1)$ is negitive

The root of eq lies b/w 0 & 90

The eq. can be written in the form

$$x = \phi(x)$$

we get, $\cos x = 3x - 1$ $(\phi x) = \frac{\cos x + 1}{3}$

i,e $\phi'(x) = \frac{-\sin x}{3}$

put $x = 0$

$\phi'(0) = \left| \frac{-\sin 0}{3} \right|$

$\phi'(0) = 0$

Put $x = 90^\circ$

$\phi(90^\circ) = \left| -\frac{\sin 90}{3} \right| = \left| -0.3333 \right|$

So iteration method can be multiplie

$\boxed{x_0 = 45^\circ} = \frac{0+90}{2}$

$$x = \frac{1+\cos x}{3}$$

$$x_1 = \frac{1+\cos x_0}{3} = \frac{1+\cos(45)}{3} = 0.5690$$

$$x_2 = \frac{1+\cos x_1}{3} = 0.666650$$

$$x_3 = \frac{1+\cos x_2}{3} = 0.666644$$

$$x_4 = \frac{1+\cos x_3}{3} = 0.666644$$

$\therefore$ The real root is "0.666644"

P6 find a real root of the equation $x \cdot \sin x = 1$

Given that $x \cdot \sin x = 1$

$$x \cdot \sin x - 1 = 0$$

$$f(x) = x \cdot \sin x - 1$$

put $x = 0$

$$f(0) = 0 \cdot \sin(0) - 1$$

$$f(0) = -1 < 0$$

put $x = 0.5$

$$f(0.5) = 0.5 \cdot \sin(0.5) - 1$$

$$f(0.5) = -0.995636$$

put $x = \frac{\pi}{2} = 90^\circ$

$$f(90^\circ) = 90^\circ \cdot \sin(90^\circ) - 1$$

$$f(90^\circ) = 89$$

$f(90) = 89 \geq 0$

$f(0.5)$ is negative & $f(90°)$ is positive

The root of eq. lies b/w 0.5 & 90°

The eq. can be written in the form of

$$x = \phi(x)$$

we get $x = \sin x = 1$

$$x = \frac{1}{\sin x}$$

$$\phi(x) = \frac{1}{\sin x}$$

$$\phi'(x) = -\cot x \cdot \operatorname{cosec} x$$

put $x = 0.5$

$$|\phi(0)| = |-\cot(0.5) \cdot \operatorname{cosec}(0.5)$$

$$|\phi(0)| = |-\frac{1}{\tan}(0.5) \cdot \frac{1}{\sin}(0.5)$$

$$\phi(0) = 0 < 1$$

put $x = 1$

30/04/21 Friday

IV. Newton Rasphson Method:-

Q. using Newton rasphson method. find a real root of $x^3-x-2=0$

A. given that $x^3-x-2=0$

let $f(x) = x^3-x-2$

put $x=0$

$f(0) = 0^3-0-2$

$f(0) = -2 < 0$

put $x=1$

$f(1) = 1^3-1-2$

$f(1) = -2 < 0$

put $x=2$

$f(2) = 2^3-2-2$

$f(2) = 4 > 0$

f(1) is negitive & f(2) is positive

∴ the root of equation lies b/w 1 & 2

we get, $f(x) = x^3-x-2$

$f'(x) = 3x^2-1-0$

$f'(x) = 3x^2-1$

$x_0 = \frac{1+2}{2} =$

$\boxed{x_0 = 1.5}$

according to Newton raphson's method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

where $\boxed{x_0 = 1.5}$

$f(x_0) = x_0^3 - x_0 - 2$

$f(1.5) = 1.5^3 - 1.5 - 2$

$\boxed{f(1.5) = -0.125}$

$f'(x_0) = 3x_0^2 - 1$

$f'(x_0) = 3(1.5)^2 - 1$

$\boxed{f'(x_0) = 5.75}$

$$x_1 = 1.5 - \frac{(-0.125)}{5.75}$$

$$x_1 = 1.5 + \frac{0.125}{5.75}$$

$x_1 = 1.5 + 0.021739$

$\boxed{x_1 = 1.521739}$

by Newton's raphson's method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$x_1 = 1.521739$

$f(x_1) = 1.521739^3 - 1.521739 - 2$

$f(x_1) = 2.136152 \times 10^{-3}$

$\boxed{f(x_1) = 0.0021361}$

$f'(x_1) = 3(1.521739)^2 - 1$

$\boxed{f'(x_1) = 5.947068}$

$$x_2 = 1.521739 - \frac{-(0.0021361)}{5.947068}$$

$$\boxed{x_2 = 1.521379}$$

by newton's raphson's method

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$x_3 = 1.521379$

$f(x_2) = 1.521379^3 - 1.521379 - 2$

$f(x_2) = -4.201094 \times 10^{-6}$

$f(x_2) = -0.000004201$

$f'(x_2) = 3(1.521379)^2 - 1$

$f'(x_2) = 5.943782$

$$x_3 = 1.521379 - \frac{-(0.0000042)}{5.943782}$$

$$\boxed{x_3 = 1.521379}$$

$\therefore$ The real root of the equation given

is "1.521379"

P_2 · Using newton-raphson method, find a positive root of $x^4 - x - 9 = 0$.

P_3 · find a real root of $x + \log_{10} x - 2 = 0$ using newton raphson method.

A. given that $x + \log_{10} x - 2 = 0$

let $f(x) = x + \log_{10} x - 2$

put $x = 0$

$f(0) = 0 + \log 0 - 2$

$f(0) = -2 < 0$

put $x = 1$

$f(1) = 1 + \log 1 - 2$

$f(1) = -1 < 0$

put $x = 2$

$f(2) = 2 + \log 2 - 2$

$f(2) = 0.301029 > 0$

$f(1)$ is negitive & $f(2)$ is positive

$\therefore$ The root of equation lies b/w 1 & 2

we get, $f(x) = x + \log_{10} x - 2$

$$f'(x) = 1 + \frac{1}{x} - 0$$

$$f'(x) = 1 + \frac{1}{x}$$

$$x_0 = \frac{1+2}{2}$$

$$\boxed{x_0 = 1.5}$$

according to Newton raphson's method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

where $x_0 = 1.5$

$$f(x_0) = x_0 + \log_{10} x_0 - 2$$

$$f(1.5) = 1.5 + \log_{10} 1.5 - 2$$

$$f(1.5) = -0.323908$$

$$f'(x_0) = 1 + \frac{1}{x_0}$$

$$f'(1.5) = 1 + \frac{1}{1.5}$$

$$f'(1.5) = 1.66666$$

$$x_1 = 1.5 - \frac{-(0.323908)}{1.666666}$$

$$\boxed{x_1 = 1.694344}$$

by Newton's raphson's method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$x_1 = 1.694344$

$$f(x_1) = 1.694344 + \log 1.694344 - 2$$

68 $f(x_1) = -0.076654$

$$f'(x_1) = 1 + \frac{1}{x_1}$$

$$f'(x_1) = 1 + \frac{1}{1.694344}$$

$$f'(x_1) = 1.590198$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1.694344 - \frac{-(0.076654)}{1.590198}$$

$$\boxed{x_2 = 1.742548}$$

by Newton raphson's method

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$x_2 = 1.742548$

$f(x_2) = 1.742548 + \log(1.742548) - 2$

$f(x_2) = -0.0162672$

$f'(x_2) = 1 + \frac{1}{x_2}$

$f'(x_2) = 1.5738722$

$$x_3 = 1.742548 - \frac{-0.016267}{1.5738722}$$

$$\boxed{x_3 = 1.752883}$$

by Newton raphson method

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$x_3 = 1.752883$

$f(x_2) = 1.752883 + \log(1.752883) - 2$

$f(x_2) = -3.36407 \times 10^{-3}$

$f(x_2) = -0.003364$

$f'(x_2) = 1 + \frac{1}{x_3}$

$f'(x_2) = 1 + \frac{1}{1.752883}$

$f'(x_2) = 1.570488$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$x_4 = 1.752883 - \frac{(-0.003364)}{1.570488}$

$\boxed{x_4 = 1.755025}$

∴ The real root of eq. is "1.75"

P5 P4 • find a real root of $x \cdot \log_{10} x = 1.2$ by using Newton raphson method.

Hint (:- $f(x) = x \cdot \log_{10} x - 1.2$

$u \cdot v$

$f'(x) = x \cdot \frac{1}{x} + \log x (1) - 0$

$= 1 + \log_{10} x$

P5 find a real root for $x \cdot \tan x + 1 = 0$ using Newton raphson method

(Q)

find the root of eq. $x \cdot \sin x + \cos x = 0$

P6. find a root of $e^{x} \cdot \sin x = 1$ (near) using newton raphson method

P7 find a real root of the equation $x \cdot e^{x} - \cos x = 0$ using newton raphson method

(or)

by using newton raphson method find a positive root of $\cos x, x \cdot e^{x} = 0$

P8 find a real root of the equation $\cos x - x^2 - x = 0$ using Newton-raphson method.

P9. find a real root of the equation $3x - \cos x - 1 = 0$ using newton-raphson method.

P10 find a real root of $f(x) = x^3 - 19$ correct upto 3 decimal places using Newton-raphson method.

for exam point of view

$x = e^{-x}$

$x - e^{-x} = 0$

$x - \frac{1}{e^{x}} = 0$

$\frac{x, e^{x} - 1 = 0}{e^{x}}$

by cross mutiplyns

$f(x) \cdot x \cdot e^{x} - 1 = 0$

$f(x) = x \cdot e^{x} - 1 = 0$

$f'(x) = x \cdot e^{x} + e^{x}(1) - 0 = 0$

$= x \cdot e^{x} + e^{x}$

P2:- Ans given that $x^4-x-9=0$

let $f(x)=x^4-x-9$

put $x=0$

$f(0)=0^4-0-9$

$f(0)=-9<0$

put $x=1$

$f(1)=1^4-1-9$

$f(1)=-9<0$

put $x=2$

$f(2)=2^4-2-9$

$f(2)=5>0$

f(1) is negitive & f(2) is positive

the root of equation lies b/w 1 & 2

we get, $f(x)=x^4-x-9$

$f'(x)=4x^3-1-0$

$f'(x)=4x^3-1$

$x_0=\frac{1+2}{2}=$

$\boxed{x_0=1.5}$

according to newton raphson method

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$$

72

$x_0 = 1.5$

$f(x_0) = x_0^4 - x_0 - 9$

$f(1.5) = 1.5^4 - 1.5 - 9$

$f(1.5) = -5.4375$

$f'(x_0) = 4x_0^3 - 1$

$f'(x_0) = 4(1.5)^3 - 1$

$f'(x_0) = 12.5$

$$x_1 = 1.5 - \frac{(-5.4375)}{(12.5)}$$

$$\boxed{x_1 = 1.935}$$

by newton raphson method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$x_1 = 1.935$

$f(x_1) = x_0^4 - x_0 - 9$

$f(x_1) = 3.0842$

$f'(x_1) = 4x_1^3 - 1$

$f'(x_1) = 4(1.935)^3 - 1$

$f'(x_1) = 27.980301$

$$x_2 = 1.935 - \frac{3.0345}{27.980301}$$

13 $\boxed{x_2 = 1.826548}$

by newton raphson method

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$x_2 = 1.826548$

$f(x_2) = 1.826548^4 - 1.826548 - 9$

$f(x_2) = 0.304200$

$f'(x_2) = 4(1.826548)^3 - 1$

$f'(x_2) = 23.375484$

$$x_3 = 1.826548 - \frac{0.304200}{23.375484}$$

$\boxed{x_3 = 1.813534}$

by newton raphson method

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$x_3 = 1.813534$

$f(x_3) = 1.813534^4 - 1.813534 - 9$

$f(x_3) = 3.365842 \times 10^{-3}$

$f(x_3) = 0.003365$

$f'(x_3) = 4(1.813534)^3 - 1$

$f'(x_3) = 22.858168$

$$x_4 = 1.813534 - \frac{0.003365}{22.858168}$$

$\boxed{x_4 = 1.813386}$

$\therefore$ The real root of equation is "1.813"

P_4:- given that,

$$x \cdot \log_{10} x = 1.2$$

$$f(x) = x \cdot \log_{10} x - 1.2 = 0$$

Put $x = 0$

$$f(0) = 0 \cdot \log_{10} 0 - 1.2$$

$$f(0) = -1.2 < 0$$

put $x = 1$

$$f(1) = 1 \cdot \log_{10} 1 - 1.2$$

$$f(1) = -1.2 < 0$$

put $x = 2$

$$f(2) = 2 \cdot \log 2 - 1.2$$

$$f(2) = -0.597940$$

put $x = 3$

$$f(3) = 3 \cdot \log 3 - 1.2$$

$$f(3) = 0.231363$$

$f(2)$ negitive & $f(3)$ is positive

The root of the eq. lies b/w 2 & 3

we get, $x \cdot \log_{10} x = 1.2$

$$f(x) = x \cdot \log_{10} x - 1.2$$

$$f'(x) = x \cdot \frac{1}{x} + \log x (1)$$

$$= 1 + \log_{10} x$$

$$x_0 = \frac{2+3}{2} = 2.5$$

75 $\boxed{x_0 = 2.5}$

according to newton raphson method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$x_0 = 2.5$

$f(x_0) = 2.5 . \log 2.5 - 1.2$

$f(x_0) = -0.205149$

$f'(x_0) = 1 + \log x$

$f'(x_0) = 1.397940$

$$x_1 = 2.5 - \frac{-0.205149}{1.397940}$$

$\boxed{x_1 = 2.646750}$

by newton raphson method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$x_1 = 2.646750$

$f(x_0) = 2.646750 \cdot \log(2.646750) - 1.2$

$f(x_0) = -0.081184$

$f'(x_0) = 1 + \log x$

$f'(x_0) = 1 + \log(2.646750)$

$f'(x_0) = 1.422712$

$$x_2 = 2.646750 - \frac{-(0.081184)}{1.422712}$$

$\boxed{x_2 = 2.703812}$

by newton raphson method

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$x_2 = 2.703812$

$f(x_2) = 2.703812 . \log(2.703812) - 1.2$

$f(x_2) = -0.032016$

$f'(x_2) = 1 + \log(2.709812)$

$f'(x_2) = 1.432939$

$$x_3 = 2.703812 - \frac{-0.032016}{1.432939}$$

$\boxed{x_3 = 2.726154}$

by newton raphson method

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$x_3 = 2.726154$

$f(x_3) = 2.726154 . \log(2.726154) - 1.2$

$f(x_3) = -0.012622$

$f'(x_3) = 1 + \log(2.726154)$

$f'(x_3) = 1.435550$

$$x_4 = 2.726154 - \frac{-(0.012622)}{1.435550}$$

$\boxed{x_4 = 2.734946}$

by newton raphson method

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$$

$x_4 = 2.734946$

$f(x_4) = 2.734946 . \log(2.734946) - 1.2$

$f(x_4) = -4.96855 \times 10^{-3} = -0.000496$

$f'(x_4) = 1 + \log(2.734946)$

$f'(x_4) = 1.436948$

$$x_5 = 2.734946 - \frac{-(0.000496)}{1.436948}$$

$\boxed{x_5 = 2.735291}$

$\therefore$ The real root of the equation is "2.73"

P5. Given that,

$x . \tan x + 1 = 0$ ($\because$ change casio mode Degrees $\longrightarrow$ radians)

$f(x) = x . \tan x + 1$

put $x = 0$

$f(0) = 0 . \tan 0 + 1$

$f(0) = 1$

put $x = 1$

$f(1) = 1 . \tan 1 + 1$

$f(1) = 2.557407$

put $x = 2$

$f(2) = 2 . \tan 2 + 1$

$f(2) = -3.370079$

$f(2)$ is nigitive & $f(1)$ is positive

The roots of given eq. lies b/w 2 & 1

we get, $x.\tan x+1=0$

$f(x) = x.\tan x+1$

$f'(x) = x.\sec^2 x + \tan x(1)$ $\left[\because 1+\tan^2\theta = \sec^2\theta\right.$

$f'(x) = x.\sec^2 x+\tan x$

$x_0 = \frac{1+2}{2} =$

$\boxed{x_0 = 1.5}$

according to newton rasphson method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$\boxed{x_0 = 1.5}$

$f(x_0) = x_0 \tan x_0+1$

$f(x_0) = 1.5.\tan(1.5)+1$

$f(x_0) = 22.152129$

$f'(x_0) = x.\sec^2 x+\tan x$

$f'(x_0) = x.(1+\tan^2 x)+\tan x$

$f'(x_0) = x_0 \cdot (1+\tan^2 x_0)+\tan x_0$

$f'(x_0) = 1.5\left(1+\tan^2(1.5)\right)+\tan(1.5)$

$f'(x_0) = 313.876486$

$$x_1 = 1.5 - \frac{22.152129}{313.876486}$$

$\boxed{x_1 = 1.429424}$

according to newton rasphson method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$\boxed{x_1 = 1.429424}$

$f(x_1) = x_1 . \tan x_1 + 1$

$f(x_1) = 1.429424 . \tan(1.429424) + 1$

$f(x_1) = 11.043609$

$f'(x_1) = 1.429424(1 + \tan^2(1.429424)) + \tan(1.429424)$

$f'(x_1) = 79.025500$

$$x_2 = 1.429429 - \frac{11.043609}{79.025500}$$

$\boxed{x_2 = 1.289681}$

according to newton rasphson method

$f(x_2) = x_2 . \tan x_2 + 1$

$f(x_2) = 1.289681 . \tan 1.289681 + 1$

$f(x_2) = 5.466238$

$f'(x_2) = 1.289681(1 + \tan^2(1.289681)) + \tan(1.289681)$

$f'(x_2) = 20.219571$

$$x_3 = 1.289681 - \frac{5.466238}{20.219571}$$

$\boxed{x_3 = 1.019337}$

according to newton rasphson method

$f(x_3) = x_3 . \tan x_3 + 1$

$f(x_3) = 1.020364 . \tan(1.020364) + 1$

$f(x_3) = 2.657148$

$f'(x_3) = x_3 \cdot (1+\tan^2 x_3) + \tan x_3$

$f'(x_3) = 1.020364(1+\tan^2(1.020364)) + \tan(1.020364)$

$f'(x_3) = 5.339097$

$$x_4 = 1.020364 - \frac{2.662642}{5.359033}$$

$x_4 = 0.523512$

01/08/21
saturday

P6 given that, $e^x \cdot \sin x = 1$

$f(x) = e^x \cdot \sin x - 1 = 0$

put $x = 0$

$f(0) = e^0 \cdot \sin(0) - 1$

$f(0) = -1 < 0$

put $x = 1$

$f(1) = e^1 \cdot \sin(1) - 1$

$f(1) = -0.952559 < 0$

put $x = 2$

$f(2) = -0.742125 < 0$

put $x = 3$

$f(3) = e^3 \cdot \sin(3) - 1$

$f(3) = 0.051195 > 0$

$f(2)$ is negitive & $f(3)$ is positive

the roots of eq lies b/w 2 & 3

we get $e^x \cdot \sin x = 1$

$f(x) = e^x \cdot \sin x - 1$

$f'(x) = e^x \cdot \cos x + \sin x \cdot e^x$

$x_0 = \frac{2+3}{2} =$

$\boxed{x_0 = 2.5}$

according to newton rasphson method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$x_0 = 2.5$

$f(x) = e^{2.5} \cdot \sin 2.5 - 1 = -0.468607$

$f'(x) = e^{2.5} \cdot \cos 2.5 + \sin 2.5 \cdot e^{2.5} = 12.702291$

$$x_1 = 2.5 - \frac{-0.468607}{12.702291}$$

$\boxed{x_1 = 2.503689}$

according to newton rasphson method

$$x_2 = x_1 - \frac{f(x_0)}{f'(x_0)}$$

$x_1 = 2.503689$

$f(x) = e^{2.503689} \cdot \sin 2.503689 - 1 = -0.465856$

$f'(x) = e^{2.503689} \cdot \cos(2.503689) + \sin(2.503689) \cdot e^{2.503689}$

$= 12.654030$

$$x_2 = 2.503689 - \frac{-0.465856}{12.654030}$$

$\boxed{x_2 = 2.540503}$

82 according to Newton rasphson method

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$x_2 = 2.540503$

$f(x) = e^{2.540503} \cdot \sin(2.540503) - 1 = -0.437683$

$f'(x) = e^{2.540503} \cdot \cos(2.540503) + \sin(2.540503) \cdot e^{2.540503}$

$= 13.235898$

$$x_3 = 2.540503 - \frac{-(0.437683)}{13.235898}$$

$\boxed{x_3 = 2.573570}$

$\therefore$ The real root of equation is "2.5"

P_7: given that $x \cdot e^x - \cos x = 0$

$f(x) = x \cdot e^x - \cos x$

put $x = 0$

$f(0) = 0 \cdot e^0 - \cos(0)$

$f(0) = -1$

put $x = 1$

$f(1) = 1 \cdot e^1 - \cos(1)$

$f(1) = 1.718434$

$f(0)$ is negative & $f(1)$ positive

The root of the eq. lies b/w 0 & 1

we get, $f(x) = x \cdot e^x - \cos x$

$f'(x) = x \cdot e^x + e^x(1) - \cos x$

$f'(x) = x \cdot e^x + e^x - \cos x$

$$x_0 = \frac{0+1}{2} =$$

$$\boxed{x_0 = 0.5}$$

according to newton raphson's method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$x_0 = 0.5$

$$f(x) = (0.5).\,e^{0.5} - \cos(0.5) = -0.175601$$

$$f'(x) = (0.5).\,e^{0.5} + e^{0.5} - \cos(0.5) = 1.473119$$

$$x_1 = 0.5 - \frac{-(0.175601)}{1.473119}$$

$$\boxed{x_1 = 0.619203}$$

according to newton raphson's method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$x_1 = 0.619203$

$$f(x) = 0.619203.\,e^{0.619203} - \cos(0.619203) = 0.150195$$

$$f'(x) = 0.619203.\,e^{0.619203} + e^{0.619203} - \cos(0.619203) = 2.007642$$

$$x_2 = 0.619203 - \frac{0.150195}{2.007642}$$

$$\boxed{x_2 = 0.544391}$$

according to newton raphson's method

$$x_3 = x_2 - \frac{f(x_2)}{f'(x^2)}$$

84 $x_2 = 0.544391$

$$f(x) = 0.544391.\,e^{0.544391} - \cos(0.544391) = -0.06166$$

$f'(x_2) = 0.544391 \times e^{0.544391} + e^{0.544391} - \cos(0.54439)$

$= 1.661893$

$$x_3 = 0.544391 - \frac{-(0.06166)}{1.661893}$$

$\boxed{x_3 = 0.581493}$

according to newton raphson method

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$x_3 = 0.581493$

$f(x_3) = 0.581493 \times e^{0.581493} - \cos(0.581493) = 0.040172$

$f'(x_3) = 0.581493 \times e^{0.581493} + e^{0.581493} - \cos(0.581493) = 1.828879$

$$x_4 = 0.581493 - \frac{0.040172}{1.828879}$$

$\boxed{x_4 = 0.559527}$

$\therefore$ The real root of equation is "0.5"

P8: given that, $\cos x - x^2 - x = 0$

$f(x) = \cos x - x^2 - x$

put $x = 0$

$f(0) = \cos(0) - 0^2 - 0$

$f(0) = 1 > 0$

put $x = 1$

85 $f(1) = \cos(1) - 1^2 - 1$

$= -1.000152 < 0$

$f(0)$ is positive & $f(1)$ is negitive

The root of eq. lies b/w 0 & 1

we get, $f(x) = \cos x - x^2 - x$

$f'(x) = -\sin x - 2x - 1$

$f'(x) = -\sin x - 2x - 1$

$\therefore x_0 = \frac{0+1}{2} = 0.5$

$\boxed{x_0 = 0.5}$

according to newton raphson's method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$x_0 = 0.5$

$f(x) = \cos(0.5) - 0.5^2 - 0.5 = 0.249961$

$f'(x) = -\sin(0.5) - 2(0.5) - 1 = -2.008726$

$$x_1 = 0.5 - \frac{0.249961}{-2.008726}$$

$\boxed{x_1 = 0.624437}$

according to newton raphson's method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$x_2 = 0.624437$

$f(x_2) = \cos(0.624437) - 0.624437^2 - 0.624437 = -0.014417$

$f'(x_2) = -\sin(0.624437) - 2(0.624437) - 1 = -2.259772$

$$x_2 = 0.624437 - \frac{-(0.014417)}{-(2.259772)}$$

$\boxed{x_2 = 0.618057}$

according to newton raphson method

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$x_2 = 0.618057$

$f(x_2) = \cos(0.618057) - (0.618057)^2 - (0.618057) = -1.0963$...

$= -0.000109$

$f'(x_2) = -\sin(0.618057) - 2(0.618057) - 1 = -2.246900$

$$x_3 = 0.618057 - \frac{-0.000109}{-2.246900}$$

$\boxed{x_3 = 0.618008}$

$\therefore$ The real root of equation is "0.61"

Pq. Given that, $3x - \cos x - 1 = 0$

$f(x) = 3x - \cos x - 1$

put $x = 0$

$f(0) = 3(0) - \cos(0) - 1$

$f(0) = -2 < 0$

put $x = 1$

$f(1) = 3(1) - \cos(1) - 1$

$f(1) = 1.000152 > 0$

$f(0)$ is negitive & $f(1)$ is positive

The root of eq. lies b/w 0 & 1

we get, $f(x) = 3x - \cos x - 1$

$f'(x) = 3 - (-\sin x) - 0$

$f'(x) = 3 + \sin x$

$x_0 = \frac{0+1}{2}$

$\boxed{x_0 = 0.5}$

according to Newton raphson method

$x_1 = x_0 - \frac{f(x_0)}{f(x_0)}$

$x_0 = 0.5$

$f(x_0) = 3(x_0) - \cos(x_0) - 1 = 3(0.5) - \cos(0.5) - 1 = -0.499961$

$f'(x_0) = 3 + \sin(x_0) = 3 + \sin(0.5) = 3.008726$

$x_1 = 0.5 - \frac{-0.499961}{3.008726}$

$\boxed{x_1 = 0.666170}$

according to Newton raphson method

$x_2 = x_1 - \frac{f(x_0)}{f(x_1)}$

$x_1 = 0.666170$

$f(x_1) = 3(0.666170) - \cos(0.666170) - 1 = -1.42240 \times 10^{-3}$
$= -0.001422$

$f'(x_1) = 3 + \sin(0.666170) = 3.011626$

$x_2 = 0.666170 - \frac{-(0.001422)}{3.011626}$

$\boxed{x_2 = 0.6666421}$

$\therefore$ The real root of equation is "0.666"

P10

Given that,

$f(x) = x^3 - 19 =$

$f(x) = x^3 - 19 = 0$

put $x = 0$

$f(0) = 0^3 - 19 =$

$f(0) = -19 < 0$

put $x = 1$

$f(1) = 1^3 - 19$

$f(1) = -18$

put $x = 2$

$f(2) = 2^3 - 19$

$f(2) = -11$

put $x = 3$

$f(3) = 3^3 - 19$

$f(3) = 8$

$f(2)$ is negitive & $f(3)$ is positive

The roots of eq. lies b/w 2 & 3.

we get, $f(x) = x^3 - 19$

$f'(x) = 3x^2 - 0$

$f'(x) = 3x^2$

$x_0 = \frac{2+3}{2} = 2.5$

$f(x_0) = x_0^3 - 19 = 2.5^3 - 19 = -3.375$

$f'(x_0) = 3x_0^2 = 3(2.5)^2 = 18.75$

$x_1 = 2.5 - \frac{-3.375}{18.75}$

$\boxed{x_1 = 2.68}$

according to Newton raphson's method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$x_1 = 2.68$

$f(x_1) = 2.68^3 - 19 = 0.248832$

$f'(x_1) = 3(2.68)^2 = 21.5472$

$$x_2 = 2.68 - \frac{0.248832}{21.5472}$$

$$\boxed{x_2 = 2.668451}$$

according to Newton raphson's method

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$x_2 = 2.668451$

$f(x_2) = 2.668451^3 - 19 = -1.05421 \times 10^{-3} = -0.001054$

$f'(x_2) = 3(2.668451)^2 = 21.361892$

$$x_3 = 2.668451 - \frac{-(0.001054)}{21.361892}$$

$$\boxed{x_3 = 2.668500}$$

$\therefore$ The root of equation is "2.668"

Q. Using Newton - Raphson method find square root of a number "N".

A. Let $\sqrt{N} = x$

is $x = \sqrt{N}$

Squaring on both sides

$x^2 = N$

$x^2 - N = 0$

$f(x) = x^2 - N$

$f'(x) = 2x$

By Newton - Raphson method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$= x_i - \frac{(x_i^2 - N)}{2x_i} = \frac{2x_i \cdot x_i - x_i^2 + N}{2x_i}$$

$$= \frac{2x_i^2 - x_i^2 + N}{2x_i} = \frac{x_i^2 + N}{2x_i} = \frac{1}{2}\left(\frac{x_i^2 + N}{x_i}\right)$$

$$x_{i+1} = \frac{1}{2}\left[\frac{x_i^2}{x_i} + \frac{N}{x_i}\right] = \frac{1}{2}\left[x_i + \frac{N}{x_i}\right]$$

$$x_{i+1} = \frac{1}{2}\left(x_i + \frac{N}{x_i}\right)$$

for example we will find the sequence root of $N = 24$

A) Here $N = 24 \longrightarrow B,$

let the initial approximation be $x_0 = 4.8$

$x_1 = \frac{1}{2}\left[x_0 + \frac{N}{x_0}\right] = \frac{1}{2}\left[4.8 + \frac{24}{4.8}\right] = 4.9$

$\boxed{x_1 = 4.9}$

$x_2 = \frac{1}{2}\left(x_1 + \frac{N}{x_1}\right) = \frac{1}{2}\left(4.9 + \frac{24}{4.9}\right) = 4.899$

$\boxed{x_2 = 4.899}$

$x_3 = \frac{1}{2}\left[x_2 + \frac{N}{x_2}\right] = \frac{1}{2}\left[4.899 + \frac{24}{4.899}\right] = 4.899$

since $x_2 = x_3 = 4.899$

$\therefore$ The solution to $f(x) = x^2 - 24$ is 4.899.

$\therefore$ the square root of 24 is "4.899"

Q. using Newton raphson method. find Reciprocal of a number.

A. Let $\frac{1}{N} = x$

Let N = number

Reciprocal of $N = \frac{1}{N}$

by cross multiplication

$1 = Nx$

$\frac{1}{x} = N$

$\frac{1}{x} - N = 0$

let $f(x) = \frac{1}{x} - N$

$f'(x) = x^{-1} = -1 \cdot x^{-1-1} = -1 x^{-2} = -1 \times \frac{1}{x^2} = \frac{-1}{x^2}$

$\therefore f'(x) = -\frac{1}{x^2}$

According to Newton-raphson method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$x_{i+1} = x_i - \frac{\left(\frac{1}{x_i} - N\right)}{-\frac{1}{x_i^2}}$$

$$x_{i+1} = x_i - \left(\frac{\frac{1 - x_i N}{x_i}}{-\frac{1}{x_i^2}}\right)$$

$$x_{i+1} = x_i - \left\{\frac{1 - x_i \cdot N}{x_i} \times -\frac{x_i^2}{1}\right\}$$

$$x_{i+1} = x_i - \left[(1 - x_i N)(-x_i)\right]$$

$$x_{i+1} = x_i - \left[(1 - x_i N)(-x_i)\right]$$

$$x_{i+1} = x_i - \left[-x_i + x_i^2 N\right]$$

$$= x_i + x_i - x_i^2 N$$

$$= 2x_i - x_i^2 \cdot N$$

$$= x_i \left[2 - N \cdot x_i\right]$$

$$x_{i+1} = x_i \left[2 - x_i \cdot N\right]$$

for example, the calculation of reciprocal of 22 is as follows

$N = 22$

Assume the initial approximation

be $x_0 = 0.045$

$x_1 = x_0 (2 - N.x_0)$

$x_1 = 0.045 (2 - (22)(0.045))$

$\boxed{x_1 = 0.0455}$

$x_2 = x_1 (2 - Nx_1)$

$x_2 = 0.0455 (2 - 22 \times 0.0455)$

$\boxed{x_2 = 0.0455}$

$x_3 = x_2 (2 - Nx_2)$

$x_3 = 0.0455 (2 - (22)(0.0455))$

$\boxed{x_3 = 0.0455}$

$x_4 = x_3 (2 - Nx_3)$

$x_4 = 0.0455 (2 - 22 \times 0.0455)$

$\boxed{x_4 = 0.0455}$

$\therefore$ The reciprocal of 22 is 0.0455

Q. Derive a formulae to find the cube root of N using N-R Method. Hence find the cube root of 15.

Q. Using Newton - raphson Method. find cube root.

A. Let $\sqrt[3]{N} = x$

$x = \sqrt[3]{N}$

$x = (N)^{1/3}$ Taking power 3 on both sides

$x^3 = (N^{1/3})^3$

$x^3 = N$

$x^3 - N = 0$

$f(x) = x^3 - N$

$f'(x) = 3x^2$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$x_{i+1} = x_i - \frac{(x_i^3 - N)}{3x_i^2}$$

$$x_{i+1} = \frac{3x_i - x_i^3 + N}{3x_i^2} = \frac{2x_i^3 + N}{3x_i^2}$$

$$x_{i+1} = \frac{1}{3}\left\{2x_i + \frac{N}{x_i^2}\right\} \longrightarrow (1)$$

using the above iteration method formula root of any number can be found out.

Q. To find the cube root of 15.

A. Let $N = 15$

let the initial approximation be $x_0 = 2.44$

According Newton - Raphson method

$$x_{i+1} = \frac{1}{3}\left[2x_i + \frac{N}{x_i^2}\right] \longrightarrow (1)$$

$$x_1 = \frac{1}{3}\left(2x_0 + \frac{N}{x_0^2}\right) = \frac{1}{3}\left[2(2.44) + \frac{15}{(2.44)^2}\right]$$

$$x_1 = \frac{1}{3}\left[4.88 + 2.519484\right) = 2.466$$

$$\boxed{x_1 = 2.466}$$

$$x_2 = \frac{1}{3}\left[2 \times 2.466 + \frac{15}{(2.466)^2}\right]$$

$$= 4.932 + 2.46664/3$$

$$\boxed{x_2 = 2.466}$$

$$\boxed{x_3 = 2.466}$$

$$\boxed{x_4 = 2.466}$$

we take $\sqrt[3]{15} = 2.466$

Q. Evaluate $\sqrt{28}$ to four decimal places by Newton's iterative method.

A. Given that $\sqrt{28}$

let $x = \sqrt{28}$

Squaring on both sides

$x^2 = 28$

$x^2 - 28 = 0$

$f(x) = x^2 - 28$

put $x = 0$

$f(0) = -28 < 0$

put $x = 1$

$f(1) = -27 < 0$

put $x = 2$

$f(2) = -24 < 0$

put $x = 3$

$f(3) = -19 < 0$

put $x = 5$

$f(5) = -3 < 0$

put $x = 4$

$f(4) = -12 < 0$

put $x = 6$

$f(6) = 8 > 0$

f(5) is negative & f(6) is positive

The root lies b/w 5 & 6

$x_0 = \frac{5+6}{2} = \frac{11}{2} = 5.5$

$\boxed{x_0 = 5.5}$

$f(x_0) = f(5.5) = (5.5)^2 - 28 = 2.25$

$f'(x_0) = 2x = 2(5.5) = 11$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 5.5 - \frac{2.25}{11}$

$x_1 = 5.5 - 0.204545$

$\boxed{x_1 = 5.295455}$

$f'(x_1) = f(5.295455) = 0.041844$

$f'(x_1) = (2 \times 5.295455) = 10.5909$

$x_2 = x_1 - \frac{f(x)}{f'(x_1)}$

$x_2 = 5.295455 - \frac{0.041844}{10.58091}$

$\boxed{x_2 = 5.291504}$

$f(x_2) = f(5.291504) = +0.0000145 82$

$f'(x_2) = 2 \times 5.291504 = 10.583008$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 5.291504 - \frac{+(0.000014582)}{10.583008}$$

$$x_3 = 5.291504 - 0.0000013779$$

$$\boxed{x_3 = 5.291503}$$

Since $x_2 = x_3$ up to four decimal places

we have $\sqrt{28} = 5.2915$

V. Guass - Jordon procedure

working procedure to solve the Supline of equation :

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

using guass - Jordon Method

1) construct the matrix with the coefficients of x, y, z & the constants in the right side Such a matrix is called Augmented matrix of the given equation

$\therefore$ The Augmented matrix of the given matrix of equation is

$$\begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{pmatrix}$$

2) Reduce the above Augmented matrix to form

$$\begin{pmatrix} 1 & 0 & 0 & \alpha \\ 0 & 1 & 0 & \beta \\ 0 & 0 & 1 & \gamma \end{pmatrix}$$

by applying suitable elementary row operation only.

3) The solution of given supline of equations is

$$x = \alpha, \quad y = \beta, \quad z = \gamma.$$

7/05/21
Tuesday
5:17am

Solve $x-y+z=2$ using guass-Jordon method.

A. given that

$2x-y+3z=9 \rightarrow (1)$

$x+y+z=6 \rightarrow (2)$

$x-y+z=2 \rightarrow (3)$

Augmentied matrix of the given system of equation is

$$[A/B] = \begin{array}{c} \begin{bmatrix} 2 & -1 & 3 & 9 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \end{bmatrix} \end{array} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$R_1 \longleftrightarrow R_3$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 9 \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$R_3 \longrightarrow R_3 - 2R_1$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 5 \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$R_3 \longrightarrow R_3 - 2R_1$

$$\begin{array}{cccc} 2 & -1 & 3 & 9 \\ 2 & -2 & 2 & 4 \\ \hline 0 & 1 & 1 & 5 \end{array}$$

$R_2 \rightarrow R_2 - R_1$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$\begin{array}{ccccc} & 1 & 1 & 1 & 6 \\ (-) & 1 & -1 & 1 & 2 \\ \hline & 0 & 2 & 0 & 4 \end{array}$$

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$R_2 \longrightarrow R_2/2$

$$\begin{array}{c c} \begin{pmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 5 \end{pmatrix} & \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \end{array}$$

$R_1 \longrightarrow R_1 + R_2$

$$\begin{array}{c c} \begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 5 \end{pmatrix} & \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \end{array} \Longrightarrow \begin{array}{r r r r r} & 1 & -1 & 1 & 2 \\ (+) & 0 & 1 & 0 & 2 \\ \hline & 1 & 0 & 1 & 4 \end{array}$$

$R_3 \longrightarrow R_3 - R_2$

$$\begin{array}{c c} \begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix} & \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \end{array} \Longrightarrow \begin{array}{r r r r r} & 0 & 1 & 1 & 5 \\ (-) & 0 & 1 & 0 & 2 \\ \hline & 0 & 0 & 1 & 3 \end{array}$$

$R_1 \longrightarrow R_1 - R_3$

$$\begin{array}{c c} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix} & \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \end{array} \Longrightarrow \begin{array}{r r r r} 1 & 0 & 1 & 4 \\ 0 & 0 & 1 & 3 \\ \hline 1 & 0 & 0 & 1 \end{array}$$

The solution of given equations is

$x = 1, \quad y = 2, \quad z = 3$

2. Solve the equation $10x+y+z=12$, $2x+10y+z$ & $x+y+5z=7$ by guass Zordon method.

A. given that,

$$10x+y+z=12 \longrightarrow ①$$

$$2x+10y+z=13 \longrightarrow ②$$

$$x+y+5z=7 \longrightarrow ③$$

Augmented matrix of given System of equation

$$[A/B] = \begin{array}{c} \begin{bmatrix} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{bmatrix} \end{array} \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

$R_1 \rightarrow R_1 - 9R_3$

$$\begin{bmatrix} 1 & -8 & -44 & -51 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{bmatrix} \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \Rightarrow \begin{array}{rrrr} 10 & 1 & 1 & 12 \\ (-)\ 9 & 9 & 45 & 63 \\ \hline 1 & -8 & -44 & -51 \end{array}$$

$R_2 \longrightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & -8 & -44 & -51 \\ 0 & 26 & 89 & 115 \\ 1 & 1 & 5 & 7 \end{bmatrix} \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \Rightarrow \begin{array}{rrrr} 2 & 10 & 1 & 13 \\ (-)\ 2 & -16 & -88 & -102 \\ \hline 0 & 26 & 89 & 115 \end{array}$$

$R_3 \longrightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & -8 & -44 & -51 \\ 0 & 26 & 89 & 115 \\ 0 & 9 & 49 & 58 \end{bmatrix} \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \Rightarrow \begin{array}{rrrr} 1 & 1 & 5 & 7 \\ (-)\ 1 & -8 & -44 & -51 \\ \hline 0 & 9 & 49 & 58 \end{array}$$

$R_2 \longrightarrow -R_2 + 3R_3$

$$\begin{pmatrix} 1 & -8 & -44 & -51 \\ 0 & 1 & 58 & 59 \\ 0 & 9 & 49 & 58 \end{pmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \Rightarrow \begin{array}{ccc} -26 & -89 & 115 \\ 27 & 147 & 174 \\ \hline 1 & 58 & 59 \end{array}$$

$R_1 \longrightarrow R_1 + 8R_2$

$$\begin{pmatrix} 1 & 0 & 420 & 421 \\ 0 & 1 & 58 & 59 \\ 0 & 9 & 49 & 58 \end{pmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \Rightarrow \begin{array}{cccc} 1 & -8 & -44 & -51 \\ 0 & 8 & 464 & 472 \\ \hline 1 & 0 & 420 & 421 \end{array}$$

$R_3 \rightarrow R_3 - 9R_2$

$$\begin{pmatrix} 1 & 0 & 420 & 421 \\ 0 & 1 & 58 & 59 \\ 0 & 0 & -473 & -473 \end{pmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \Rightarrow \begin{array}{ccc} 9 & 49 & 58 \\ 9 & 522 & 531 \\ \hline 0 & -473 & -473 \end{array}$$

$R_3 \rightarrow R_3 / -473$

$$\begin{pmatrix} 1 & 0 & 420 & 421 \\ 0 & 1 & 58 & 59 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \qquad \begin{array}{cccc} 1 & 0 & 420 & 421 \\ 0 & 0 & 420 & 420 \\ \hline 1 & 0 & 0 & 1 \end{array}$$

$R_1 \rightarrow R_1 - 420R_3$

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 58 & 59 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \qquad \begin{array}{cccc} 0 & 1 & 58 & 59 \\ 0 & 0 & 58 & 58 \\ \hline 0 & 1 & 0 & 1 \end{array}$$

$R_2 \rightarrow R_2 - 58R_3$

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

$\therefore$ The solution of equation is

$x = 1,$

$y = 1,$

$z = 1,$

Home work

using guage- Jordon method, solve the simplin

1) $2x+y+z=10$; $3x+2y+3z=18$; $x+4y+9z=16$

$\boxed{\text{Ans } x=7, \ y=-9, \ z=5}$

2) $10x_1+x_2+x_3=12$; $x_1+10x_2-x_3=10$; $x_1-2x_2+10x_3=9$

$\boxed{\text{Ans } x_1=1, \ x_2=1, \ x_3=1}$

3) $3x+4y+5z=18$; $2x-y+8z=13$; $5x-2y+7z=20$

$\boxed{\text{Ans } x=3, \ y=1, \ z=1}$

Q. Solve the system of equation $2x+3y+z=11$; $x+2y+z=8$; $3x-y-2z=5$ using guass Jordon method.

A. Given that

$$2x+3y+z=11 \rightarrow (1)$$
$$x+2y+z=8 \rightarrow (2)$$
$$3x-y-2z=5 \rightarrow (3)$$

The Augmented matrix of the given system of equation is

$$(A|B) = \begin{array}{c} \begin{pmatrix} 2 & 3 & 1 & 11 \\ 1 & 2 & 1 & 8 \\ 3 & -1 & -2 & 5 \end{pmatrix} \end{array} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$R_1 \longleftrightarrow R_2$$

$$\begin{pmatrix} 1 & 2 & 1 & 8 \\ 2 & 3 & 1 & 11 \\ 3 & -1 & -2 & 5 \end{pmatrix} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$R_3 \rightarrow R_3 - 3R_1$

$$\begin{pmatrix} 1 & 2 & 1 & 8 \\ 2 & 3 & 1 & 11 \\ 0 & -7 & -5 & -29 \end{pmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

$$\begin{array}{cccc} 3 & -1 & -2 & 5 \\ 3 & 6 & 3 & 24 \\ \hline 0 & -7 & -5 & -29 \end{array}$$

$R_3 \rightarrow R_3/-1$

$$\begin{pmatrix} 1 & 2 & 1 & 8 \\ 2 & 3 & 1 & 11 \\ 0 & 7 & 5 & 29 \end{pmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$$\begin{pmatrix} 1 & 2 & 1 & 8 \\ 0 & -1 & -1 & -5 \\ 0 & 7 & 5 & 29 \end{pmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

$$\begin{array}{cccc} 2 & 3 & 1 & 11 \\ 2 & 4 & 2 & 16 \\ \hline 0 & -1 & -1 & -5 \end{array}$$

$R_2 \rightarrow R_2/-1$

$$\begin{pmatrix} 1 & 2 & 1 & 8 \\ 0 & 1 & 1 & 5 \\ 0 & 7 & 5 & 29 \end{pmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

$R_1 \rightarrow R_1 - 2R_2$

$$\begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 5 \\ 0 & 7 & 5 & 29 \end{pmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

$$\begin{array}{cccc} 1 & 2 & 1 & 8 \\ 0 & 2 & 2 & 10 \\ \hline 1 & 0 & -1 & -2 \end{array}$$

$R_3 \rightarrow R_3 - 7R_2$

$$\begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & -2 & -6 \end{pmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

$$\begin{array}{ccccc} 7 & 5 & 29 & 9 \\ 7 & 7 & 35 & \\ \hline 0 & -2 & -6 & \end{array}$$

$R_3 \rightarrow R_3/-2$

$$\begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{pmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

$R_1 \rightarrow R_1 + R_3$

$$\begin{array}{c} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{pmatrix} \end{array} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array} \qquad \begin{array}{cccc} 1 & 0 & -1 & -2 \\ 0 & 0 & 1 & 3 \\ \hline 1 & 0 & 0 & 1 \end{array}$$

$R_2 \rightarrow R_2 - R_3$

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix} \qquad \begin{array}{cccc} 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \\ \hline 0 & 1 & 0 & 2 \end{array}$$

hence the solution of given equation is

$x = 1, \; y = 2, \; z = 3$

04/05/2021
Tuesday
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VI. Guass - Seidel Iteration method:

The System of (evaluation) equation

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}\right\} \rightarrow (1)$$

The System of equation (1) can be written as

$$\left.\begin{aligned} x_1 &= \frac{1}{a_{11}}\left(b_1 - a_{12}x_2 - a_{13}x_3\right) \\ x_2 &= \frac{1}{a_{22}}\left(b_2 - a_{21}x_1 - a_{23}x_3\right) \\ x_3 &= \frac{1}{a_{33}}\left(b_3 - a_{31}x_1 - a_{32}x_2\right) \end{aligned}\right\} \rightarrow (2)$$

let the initial approximate solution be x_1^0, x_2^0, x_3^0, substituting x_2^0, x_3^0 in the first equation of (2)

we get

$$x_1^1 = \frac{1}{a_{11}}\left(b_1 - a_{12}x_2^0 - a_{13}x_3^0\right) \longrightarrow (3)$$

This is taken as the first approximation of x_1

Substituting x_1^1 for x_1 & x_3^0 for x_3^0 in the second equation of (2) we get

$$x_2^1 = \frac{1}{a_{22}}\left(b_2 - a_{21}x_1^1 - a_{23}x_3^0\right) \longrightarrow (4)$$

This is taken as the first approximation of x_2

Next Substituting x_1^1 for x_1 & x_2^1 for x_2 in the last equation of (2) we get

$$x_3^1 = \frac{1}{a_{33}}\left(b_3 - a_{31}x_1^1 - a_{32}x_2^1\right) \longrightarrow (5)$$

This is taken as the first approximation of x_3

The values obtained in 3,4,5 constitute the first iteration of the solution

proceeding in the same way we get successive iterates

The $(n+1)^{th}$ iterates are given by

$$x_1^{n+1} = \frac{1}{a_{11}}\left[b_1 - a_{12}x_2^{n} - a_{13}x_3^{n}\right]$$

$$x_2^{n+1} = \frac{1}{a_{22}}\left[b_2 - a_{21}x_1^{(n+1)} - a_{23}x_3^{n}\right]$$

$$x_3^{n+1} = \frac{1}{a_{33}}\left[b_3 - a_{31}x_1^{(n+1)} - a_{32}x_2^{(k+1)}\right]$$

the iteration process is stopped when the desired order of approximatiom is reached to or two successive iteration are nearly the same.

2/05/21 Tuesday

here 2nd & 3rd iterations are approximately equal & required solution is $x=1, y=1, z=1$.

2. Solve $10x_1 + \cancel{2}x_2 + x_3 = 6$

$10x_1 + \cancel{10}x_2 + x_3 = 6$

$x_1 + x_2 + 10x_3 = 6$

A. Given that,

$10x_1 + x_2 + x_3 = 6$

$x_1 + 10x_2 + x_3 = 6$

$x_1 + x_2 + 10x_3 = 6$

Let $x_1 = \frac{1}{10}\left[6 - x_2 - x_3\right]$

$x_2 = \frac{1}{10}\left[6 - x_1 - x_3\right]$

$x_3 = \frac{1}{10}\left[6 - x_1 - x_2\right]$

Let the initial solution of system is

$(x_1)_0 = (x_2)_0 = (x_3)_0 = 0$

first iteration:

$(x_1)_1 = \frac{1}{10}(6-(x_2)_0-(x_3)_0)$

$(x_1)_1 = \frac{1}{10}(6-0-0)$

$(x_1) = 0.6$

$(x_2)_1 = \frac{1}{10}(6-x_1-(x_3)_0)$

$(x_2)_1 = \frac{1}{10}(6-0.6-0)$

$= 0.54$

$(x_3)_1 = \frac{1}{10}(6-x_1-x_2)$

$= \frac{1}{10}(6-0.6-0.54)$

$= 0.486$

second iteration:

$(x_1)_2 = \frac{1}{10}(6-0.54-0.486)$

$= \frac{1}{10}(6-0.54-0.486)$

$= 0.4974$

$(x_2)_2 = \frac{1}{10}(6-0.4974-0.486)$

$= 0.5016$

$(x_3)_2 = \frac{1}{10}(6-0.4974-0.5016)$

$= 0.5001$

Third iteration

$$(x_1)_3 = \frac{1}{10}\left(6 - 0.5016 - 0.5001\right)$$
$$= 0.4998$$

$$(x_2)_3 = \frac{1}{10}\left(6 - 0.4998 - 0.5001\right)$$
$$= 0.5001$$

$$(x_3)_3 = \frac{1}{10}\left(0.4998 - 0.5001\right)$$
$$= 0.5001$$

here 2nd & 3rd iteration the approximation solution is equal & required iteration

$x = 0.5$ $y = 0.5$, $z = 0.5$

P 3. $10x + 2y + z = 9$

$2x + 20y - 2z = -44$

$-2x + 3y + 10z = 22$

A. Given

$$10x + 2y + z = 9$$
$$x + 10y - z = -22$$
$$-2x + 3y + 10z = 22$$

$$x = \frac{1}{10}\left(9 - 2y - z\right)$$
$$y = \frac{1}{10}\left(-22 - x + z\right)$$
$$z = \frac{1}{10}\left(22 + 2x - 3y\right)$$ initial $x_0 = y_0 = z_0 = 0$

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first iteration:

$x_1 = \frac{1}{10}(9-(2(0))-0)$

$x_1 = 0.9$

$y_1 = \frac{1}{10}(-22-0.9+0)$

$= -2.29$

$z_1 = \frac{1}{10}(22+2(0.9)-3(-2.29))$

$= 3.067$

second iteration:-

$x_2 = \frac{1}{10}(9-2(-2.29)-3.067)$

$= 1.0513$

$y_2 = \frac{1}{10}(-22-1.0513+3.067)$

$= -1.9984$

$z_2 = \frac{1}{10}(22+2(1.0513)-3(-1.9984))$

$= 3.00978$

Third iteration:-

$x_3 = \frac{1}{10}(9-2(-1.9984)-3.00978)$

$= 0.9987$

$y_3 = \frac{1}{10}(-22-0.9987+3.00978)$

$= -1.9988$

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$z_3 = \frac{1}{10}\left[22 + 2(0.9987) - 3(-1.9988)\right]$

$= 2.99938$

here 2^{nd} & 3^{rd} iterations are approximately equal & required solution is

$x = 1, \quad y = -2, \quad z = 3.$

{ Or }

Given data

$10x + 2y + z = 9 \longrightarrow ①$

$2x + 20y - 2z = -44 \longrightarrow ②$

$-2x + 3y + 10z = 22 \longrightarrow ③$

from equation ①

$10x + 2y + z = 9$

$10x = 9 - 2y - z$

$\boxed{x = \frac{1}{10}(9 - 2y - z)}$

from equation ②

$2x + 20y - 2z = -44$

$20x + 20y = -44 + 2z$

$20y = -44 + 2z - 2x$

$20y = -44 + 2z - 2x$

$\boxed{y = \frac{1}{20}(-44 + 2z - 2x)}$

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from equation ③

$$-2x + 3y + 10z = 22$$

$$10z = +22 + 2x - 3y$$

$$\boxed{z = \frac{1}{10}[22 + 2x - 3y]}$$

Let the initial solution of system is

$$x_0 = 0, \quad y_0 = 0, \quad z_0 = 0$$

first solution iteration :-

$$x = \frac{1}{10}(9 - 2y - z_0)$$

$$x_1 = \frac{1}{10}(9 - 2y_0 - z_0)$$

$$x_1 = \frac{1}{10}(9 - 2(0) - 0) = \frac{1}{10}(9)$$

$$\boxed{x_1 = 0.9}$$

$$y = \frac{1}{20}(-44 + 2z - 2x)$$

$$y_1 = \frac{1}{20}(-44 + 2z_0 - 2(x_1))$$

$$y_1 = \frac{1}{20}(-44 + 2(0) - 2(0.9))$$

$$\boxed{y_1 = -2.29}$$

$$z = \frac{1}{10}[22 + 2x - 3y]$$

$$z_1 = \frac{1}{10}[22 + 2(x_1) - 3(y_1)]$$

$$= \frac{1}{10}\{22 + 2(0.9) - 3(-2.29)\}$$

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$$\boxed{z_1 = 3.067}$$

Second iteration :-

$$x = \frac{1}{10}(9-2y-z)$$

$$x_2 = \frac{1}{10}(9-2y_1-z_1)$$

$$= \frac{1}{10}(9-2(-2.29)-3.067)$$

$$\boxed{x_2 = 1.0513}$$

$$y = \frac{1}{20}(44+2z-2x)$$

$$y_2 = \frac{1}{20}(-44+2z_1-2(x_2))$$

$$y_2 = \frac{1}{20}(-44+2(3.067)-2(1.0513))$$

$$\boxed{y_2 = -1.99843}$$

$$z = \frac{1}{10}(22+2x-3y)$$

$$z_2 = \frac{1}{10}(22+2(x_2)-3(y_2))$$

$$z_2 = \frac{1}{10}(22+2(1.0513)-3(-1.99843))$$

$$\boxed{z_2 = 3.009789}$$

Third iteration :-

$$x = \frac{1}{10}(9-2y-z)$$

$$x_3 = \frac{1}{10}(9-2y_2-z_2)$$

$$= \frac{1}{10}(9-2(-1.99843)-3.009789)$$

115 $\boxed{x_3 = 0.998707}$

$y = \frac{1}{20}(-44 + 2z - 2x)$

$y_3 = \frac{1}{20}(-44 + 2z_2 - 2x_3)$

$= \frac{1}{20}(-44 + 2(3.009789) - 2(0.998707))$

$\boxed{y_3 = -1.998891}$

$z = \frac{1}{10}(22 + 2x - 3y)$

$z_3 = \frac{1}{10}(22 + 2x_3 - 3y_3)$

$= \frac{1}{10}(22 + 2(0.998707) - 3(-1.998891))$

$\boxed{z_3 = 2.999408}$

fourth iteration:-

$x = \frac{1}{10}(9 - 2y - z)$

$x_4 = \frac{1}{10}(9 - 2y_3 - z_3)$

$= \frac{1}{10}(9 - 2(-1.998891) - 2.999408)$

$\boxed{x_4 = 0.999837}$

$y = \frac{1}{20}(-44 + 2z - 2x)$

$y_4 = \frac{1}{20}(-44 + 2z_3 - 2x_4)$

$= \frac{1}{20}(-44 + 2(2.999408) - 2(0.999837))$

$\boxed{y_4 = -2.000042}$

$$z = \frac{1}{10}(22 + 2x - 3y)$$

$$z_4 = \frac{1}{10}(22 + 2x_4 - 3y_4)$$

$$= \frac{1}{10}(22 + 2(0.999837) - 3(-2.000042))$$

$$\boxed{z_4 = 2.99998}$$

fifth iteration :-

$$x = \frac{1}{10}(9 - 2y - z)$$

$$x_5 = \frac{1}{10}(9 - 2y_4 - z_4)$$

$$= \frac{1}{10}(9 - 2(-2.000042) - 2.99998)$$

$$\boxed{x_5 = 1.0000104}$$

$$y = \frac{1}{20}(-44 + 2z - 2x)$$

$$y_5 = \frac{1}{20}(-44 + 2z_4 - 2x_5)$$

$$= \frac{1}{20}(-44 + 2(2.99998) - 2(1.0000104))$$

$$\boxed{y_5 = -2.000003}$$

$$z = \frac{1}{10}(22 + 2x - 3y)$$

$$z_5 = \frac{1}{10}(22 + 2x_5 - 3y_5)$$

$$= \frac{1}{10}(22 + 2(1.0000104) - 3(-2.00003))$$

$$\boxed{x_5 = 3.00000298}$$

x	y	z
$x_1 = 0.9$	$y_1 = -2.29$	$z_1 = 3.067$
$x_2 = 1.0513$	$y_2 = -1.98943$	$z_2 = 3.009789$
$x_3 = 0.998707$	$y_3 = -1.998891$	$z_3 = 2.999408$
$x_4 = 0.99837$	$y_4 = -2.000042$	$z_4 = 2.99998$
$x_5 = 1.0000104$	$y_5 = -2.000003$	$z_5 = 3.0000298$

P:- $10x + y + z = 12$

$2x + 10y + z = 13$

3. $2x + 2y + 10z = 14$

A. Given data,

$10x + y + z = 12$

$2x + 10y + z = 13$

$2x + 2y + 10z = 14$

for eq → 1 $\quad 10x + y + z = 12$

$10x = 12 - y - z$

$$\boxed{x = \frac{1}{10}(12 - y - z)}$$

for eq → 2 $\quad 2x + 10y + z = 13$

$2x + 10y = 13 - z$

$10y = 13 - z - 2x$

$$\boxed{y = \frac{1}{10}(13 - z - 2x)}$$

for eq → 3 $\quad 2x + 2y + 10z = 14$

$10z = 14 - 2x - 2y$

$$\boxed{z = \frac{1}{10}(14 - 2x - 2y)}$$

let initial solution of system is

$x_0 = 0 , y_0 = 0 , z_0 = 0$

first iteration :-

$x = \frac{1}{10}(12 - y - z)$

$x_1 = \frac{1}{10}(12 + 0 - 0)$

$\boxed{x_1 = 1.2}$

$y = \frac{1}{10}(13 - z - 2x_1)$

$y = \frac{1}{10}(13 - 0 - 2(1.2))$

$\boxed{y_1 = 1.06}$

$z = \frac{1}{10}(14 - 2x_1 - 2y_1)$

$z_1 = \frac{1}{10}(14 - 2(1.2) - 2(1.06))$

$\boxed{z_1 = 0.948}$

Second iteration :-

$x = \frac{1}{10}(12 - y_1 - z_1)$

$x_2 = \frac{1}{10}(12 - 1.06 - 0.948)$

$\boxed{x_2 = 0.9992}$

$y = \frac{1}{10}(13 - z_1 - 2x_2)$

$y_2 = \frac{1}{10}(13 - 0.948 - 2(0.9992))$

$\boxed{y_2 = 1.00536}$

$$z = \frac{1}{10}(14 - 2x_2 - 2y_2)$$

$$z_2 = \frac{1}{10}(14 - 2(0.9992) + 2(1.00536))$$

$$\boxed{z_2 = 0.999088}$$

Third iteration:-

$$x = \frac{1}{10}(12 - y_2 - z_2)$$

$$x_3 = \frac{1}{10}(12 - 1.00536 - 0.999088)$$

$$\boxed{x_3 = 0.9995552}$$

$$y = \frac{1}{10}(13 - z_2 - 2x_3)$$

$$y_3 = \frac{1}{10}(13 - 0.99088 - 2(0.999555))$$

$$\boxed{y_3 = 1.001001}$$

$$z = \frac{1}{10}(14 - 2x_3 - 2y_3)$$

$$z_3 = \frac{1}{10}(14 - 2(0.999555) - 2(1.001001))$$

$$\boxed{z_3 = 0.9998888}$$

fourth iteration:-

$$x = \frac{1}{10}(12 - y_3 - z_3)$$

$$x_4 = \frac{1}{10}(12 - 1.001001 - 0.9998888)$$

$$\boxed{x_4 = 0.99991102}$$

$y = \frac{1}{10}(13 - z_3 - 2x_4)$

$y_4 = \frac{1}{10}(13 - 0.9998888 - 2(0.99991102))$

$\boxed{y_4 = 1.0000289}$

$z = \frac{1}{10}(14 - 2x - 2y)$

$z_4 = \frac{1}{10}(14 - 2(0.99991102) - 2(1.0000289))$

$\boxed{z_4 = 1.00000120}$

fifth iteration:-

$x = \frac{1}{10}(12 - y_4 - z_4)$

$x_5 = \frac{1}{10}(12 - 1.0000289 - 1.0000126)$

$\boxed{x_5 = 0.99996}$

$y = \frac{1}{10}(13 - z_4 - 2x_5)$

$y_5 = \frac{1}{10}(13 - 1.0000120 - 2(0.9996))$

$\boxed{y_5 = 1.000007}$

$z = \frac{1}{10}(14 - 2x_5 - 2y_5)$

$z = \frac{1}{10}(14 - 2(0.99996) - 2(1.00007))$

$\boxed{z_5 = 1.00006}$

x	y	z
$x_1 = 1.2$	$y_1 = 1.06$	$z_1 = 0.948$
$x_2 = 0.9992$	$y_2 = 1.00536$	$z_2 = 0.999088$
$x_3 = 0.999555$	$y_3 = 1.0001001$	$z_3 = 0.999888$
$x_4 = 0.999111$	$y_4 = 1.0000289$	$z_4 = 1.00001$
$x_5 = 0.999996$	$y_5 = 1.000007$	$z_5 = 1.00000$

P_2.

$$8x - 3x_2 + 2x_3 = 20$$
$$4x_1 + 11x_2 + x_3 = 33$$
$$6x_1 + 3x_2 + 12x_3 = 36$$

A. Given that

$$8x_1 - 3x_2 + 2x_3 = 20 \longrightarrow (1)$$
$$4x_1 + 11x_2 - x_3 = 33 \longrightarrow (2)$$
$$6x_1 + 3x_2 + 12x_3 = 36 \longrightarrow (3)$$

for first equation:-

$$8x_1 - 3x_2 + 2x_3 = 20$$
$$8x_1 = 20 + 3x_2 - 2x_3$$
$$\boxed{x_1 = \frac{1}{8}(20 + 3x_2 - 2x_3)}$$

for second equation:

$$4x_1 + 11x_2 - x_3 = 33$$

$$11x_2 = 33 + x_3 - 4x_1$$

$$\boxed{x_2 = \frac{1}{11}(33 + x_3 - 4x_1)}$$

for third equation

$$6x_1 + 3x_2 + 12x_3 = 36$$

$$12x_3 = 36 - 6x_1 - 3x_2$$

$$\boxed{x_3 = \frac{1}{12}(36 - 6x_1 - 3x_2)}$$

first iteration :-

$$x_1 = \frac{1}{8}(20 + 3(0) - 2(0))$$

$$\boxed{x_1 = 2.5}$$

$$x_2 = \frac{1}{11}(33 + 0 - 4(2.5))$$

$$\boxed{x_2 = 2.09090}$$

$$x_3 = \frac{1}{12}(36 - 6(2.5) - 3(2.09090))$$

$$\boxed{x_3 = 1.227275}$$

second iteration :-

$$x_1' = \frac{1}{8}(20 + 3(2.09090) - 2(1.227275))$$

$$\boxed{x_1' = 2.77268}$$

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$x_2' = \frac{1}{11}(30+1.227275-4(2.977268)$

$\boxed{x_2' = 2.0289275}$

$x_3' = \frac{1}{12}(36-6(2.977668)-3(2.0289725)$

$\boxed{x_3' = 1.0004134}$

Third iteration :-

$x_1'' = \frac{1}{8}(20+3(2.0289275)-2(1.004134)$

$\boxed{x_1'' = 3.009314}$

$x_2'' = \frac{1}{11}(33+1.004134-4(3.009814))$

$\boxed{x_2'' = 1.996807}$

$x_3'' = \frac{1}{12}(36-6(3.009814)-3(1.996807)$

$\boxed{x_3'' = 0.995891}$

fourth iteration :-

$x_1''' = \frac{1}{8}(20+3(1.996807)-2(0.985891)$

$\boxed{x_1''' = 2.999829}$

$x_2''' = \frac{1}{11}(33+0.995891-4(2.999829)$

$\boxed{x_2''' = 1.999688}$

$$x_3''' = \frac{1}{12}\left[36 - 6(2.999929) - 3(1.999688)\right)$$

$$\boxed{x_3''' = 1.000163}$$

fifth iteration :-

$$x_1^{iv} = \frac{1}{8}\left[20 + 3(1.999688) - 2(1.000163)\right)$$

$$\boxed{x_1^{iv} = 2.999842}$$

$$x_2^{iv} = \frac{1}{11}\left(33 + 1.000163 - 4(2.994842)\right)$$

$$\boxed{x_2^{iv} = 2.000072}$$

$$x_3^{iv} = \frac{1}{12}\left(36 - 6(2.999842) - 3(2.000074)\right)$$

$$\boxed{x_3^{iv} = 1.000061}$$

x_1	x_2	x_3
$x_1 = 2.5$	$x_2 = 2.09090$	$x_3 = 1.227275$
$x_1' = 2.9177268$	$x_2' = 2.0289275$	$x_3' = 1.004134$
$x_1'' = 3.009314$	$x_2'' = 1.996807$	$x_3'' = 0.995891$
$x_1''' = 2.999829$	$x_3''' = 1.999688$	$x_3''' = 1.000163$
$x_1^{iv} = 2.999842$	$x_2^{iv} = 2.000072$	$x_3^{iv} = 1.000061$

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