Fourier Transforms

⇒ Both Fourier Series & Fourier Transform are the mathematical tools for representing signal in frequency domain.

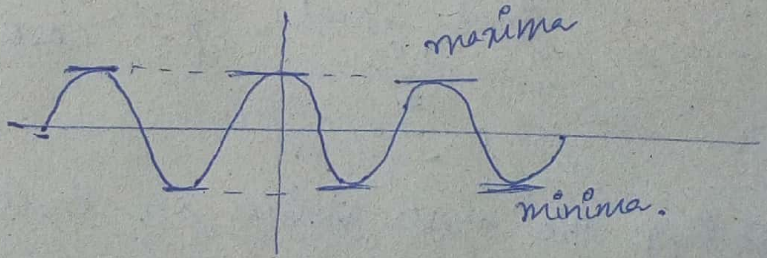
⇒ Fourier series is mainly used for periodic signals & Fourier transform is used for aperiodic signals

condition for Existence: (Dirichlet's condition).

(i) $f(t)$ should be absolutely integrable.

$$\text{i.e } \int_{-\infty}^{\infty} |f(t)|\, dt < \infty.$$

(ii) The function must have Finite number of maxima & minima.



(iii) The function must have finite number of discontinuties in any interval of time.

$$x(t) \xrightarrow{F.T.} X(j\omega)$$

$$X(j\omega) \longrightarrow \int_{-\infty}^{\infty} x(t).\, e^{-j\omega t}.\, dt.$$

The general F.S. is given by.

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

where a_0, a_n, b_n are the co-efficients of Fourier ~~series~~ analysis.

$$a_0 = \frac{2}{T}\int_0^T x(t)\,dt \rightarrow \text{average Value}$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cdot \cos n\omega t\cdot dt \qquad T = \text{time Period.}$$

$$b_n = \frac{2}{T}\int_0^T x(t)\,\sin n\omega t\cdot dt.$$

⇒ Trignometric Form & Exponential form of Fourier Series

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos n\omega t + b_n \sin n\omega t\right) \rightarrow ①$$

$$\left(\omega = \frac{2\pi}{T}\right)$$

but

$$\sin n\omega t = \frac{e^{jn\omega t} - e^{-jn\omega t}}{2j} \qquad \left(\because \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}\right)$$

$$\cos n\omega t = \frac{e^{jn\omega t} + e^{-jn\omega t}}{2} \qquad \left(\because \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}\right)$$

Substituting the above values in Eq ①, we get

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left\{a_n\cdot\left(\frac{e^{jn\omega t} + e^{-jn\omega t}}{2}\right) + b_n\left(\frac{e^{jn\omega t} - e^{-jn\omega t}}{2j}\right)\right\}$$

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(\frac{a_n - jb_n}{2}\right)e^{jn\omega t} + \sum_{n=1}^{\infty}\left(\frac{a_n + jb_n}{2}\right)e^{-jn\omega t} \rightarrow ②$$

Let $c_0 = \frac{a_0}{2}$

$$C_n = \frac{a_n - jb_n}{2}$$

$$c_{-n} = \frac{a_n + jb_n}{2}.$$

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$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n e^{jn\omega t} + \sum_{n=1}^{\infty} C_{-n} \cdot e^{+jn\omega t}.$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t} \longrightarrow (3)$$

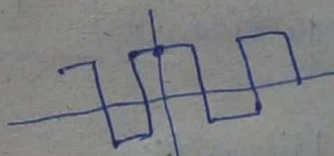
where

$$C_n = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega t} \cdot dt.$$

⇒ Condition of Symmetry

In Fourier Analysis of there are three types of symmetries. they are

(1) Even symmetry
(2) Odd symmetry
(3) Half wave symmetry.



① Even Symmetry: (mirror symmetry) A function is said to be even symmetry if it satisfies the condition.

$$\boxed{x(t) = x(-t)}.$$

Even
$\cos(-t) = \cos t$
$\sin(-t) = -\sin t$.
odd.

Fourier Series expansion of an even signal does not contain "sine" terms. ($\because b_n = 0$)
only a_0 & a_n Values.

② Odd Symmetry (or) (rotation symmetry) A function is said to be odd symmetry if it satisfies the condition

$$\boxed{x(-t) = -x(t)}$$

Fourier Series of an odd signal contains only sine values means $(a_0 = 0,\ a_n = 0\ \ b_n \neq 0)$.

(3) Half wave symmetry:

The function is said to be a half wave symmetry if it satisfies the condition

$$\boxed{x(t) = -x\left(t \pm \frac{T}{2}\right)}$$

Fourier Series expansion of HWS signal contains only odd harmonics.

$a_0 = 0$, a_n in only odd values of n

b_n in only odd values of n.

⇒ Properties of F.T

(1) Linearity

if $F[f_1(t)] = F_1(j\omega)$

& $F[f_2(t)] = F_2(j\omega)$

then $F[a_1 f_1(t) + a_2 f_2(t)] = a_1 F_1(j\omega) + a_2 F_2(j\omega)$

a_1 & a_2 are constants.

(2) Time shifting: $F[f(t)] — F(j\omega)$

then $F[f(t-t_0)] = e^{-j\omega t_0} \cdot F(j\omega)$.

(3) Time reversal: if $F[f(t)] — F(j\omega)$

then $F[f(-t)] = F(-j\omega)$

④ frequency shifting:

if $F[f(t)] = F(j\omega)$

then $F[f(t) \cdot e^{j\omega_0 t}] = F[j(\omega - \omega_0)]$

⑤ Time Scaling: if $F[f(t)] = F(j\omega)$

then $F[f(at)] = \frac{1}{|a|} F\left(\frac{j\omega}{a}\right)$.

⑥ Differention in time:

if $F[f(t)] = F(j\omega)$

then $F\left[\frac{d}{dt} f(t)\right] = j\omega \cdot F(j\omega)$.

⑦ Differentiation in frequency: if $F[f(t)] - F(j\omega)$

then $F[t \cdot f(t)] - j \frac{d}{d\omega} F(j\omega)$.

⑧ Conjugation: if $F[f(t)] = F(j\omega)$

then $F[f^*(t)] = F^*(-j\omega)$.

⑨ Duality: if $F(f(t)] = F(j\omega)$

then $F[F(t)] = 2\pi f(-j\omega)$.

⑩ Time convolution: if $F[f_1(t)] = F_1(j\omega)$

& $F[f_2(t)] = F_2(j\omega)$

then

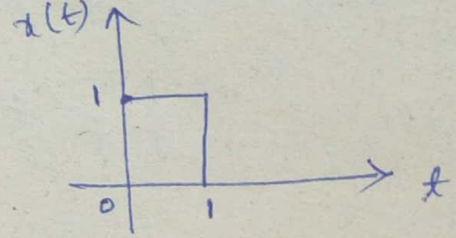
$F[f_1(t) \cdot f_2(t)] = \frac{1}{2\pi} F_1(j\omega) * F_2(j\omega)$.

in case multiplication

$F[f_1(t) \cdot f_2(t)] = \frac{1}{2\pi}[F_1(j\omega) * F_2(j\omega)]$.

//

→ find Fourier Transform of the Signal.



$x(t) = 1.$

Sol: F.T.

$$x(\omega) = \int_0^1 x(t) . e^{-j\omega t} . dt$$

$$= \int_0^1 1 . e^{-j\omega t} dt$$

$$= \int_0^1 e^{-j\omega t} dt \qquad \left(\because \int e^{xt} . dt = \frac{e^{xt}}{x} \right)$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega} \right]_0^1$$

$$= \left[\frac{e^{-j\omega (1)} - e^{-j\omega . (0)}}{-j\omega} \right]$$

$$= \left[\frac{e^{-j\omega} - 1}{-j\omega} \right]$$

$$X(\omega) = \left(\frac{1 - e^{-j\omega}}{j\omega} \right) // \qquad \left(e^{j\theta} = \cos\theta + j\sin\theta \right)$$

$$= \frac{1}{j\omega} \left\{ 1 - (\cos\omega - j\sin\omega) \right\}$$

$$= \frac{1}{j\omega} \left[1 - \cos\omega + j\sin\omega \right]$$

$$= \frac{1}{j\omega} (1 - \cos\omega) + \cancel{j} \frac{\sin\omega}{\cancel{j}\omega}$$

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$$x(\omega) = \frac{\sin\omega}{\omega} - j\frac{(1-\cos\omega)}{\omega}$$ //

— x —

⇒ $x(t) = e^{-at}u(t)$ find F.T ?

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty} \qquad \left(\because e^{-\infty} = 0,\ e^{0} = 1\right)$$

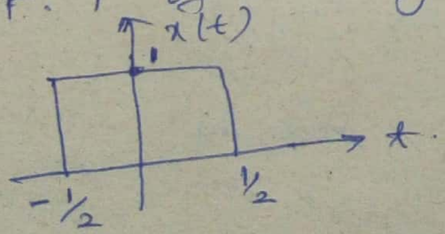
$$= \frac{-1}{(a+j\omega)}\left[e^{-(a+j\omega)\infty} - e^{-(a+j\omega)0}\right]$$

$$= \frac{-1}{(a+j\omega)}\left[0 - 1\right]$$

$$X(\omega) = \frac{1}{a+j\omega}$$ //.

— x —

⇒ Find F.T of the signal is rectangle.



Sol: F.T $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$.

$$x(\omega) = \int_{-\frac{1}{2}}^{\frac{1}{2}} 1 \cdot e^{-j\omega t} \cdot dt$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j\omega t} \cdot dt$$

$$= \frac{\left[e^{-j\omega t}\right]_{-1/2}^{1/2}}{-j\omega}$$

$$= \frac{-1}{j\omega}\left[e^{-j\frac{\omega}{2}} - e^{j\frac{\omega}{2}}\right]$$

$$= \frac{\left(-e^{-j\frac{\omega}{2}} + e^{j\frac{\omega}{2}}\right)}{j} \times \frac{2}{2\omega}$$

$$= \frac{2}{\omega}\left[\frac{e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}}{2j}\right]$$

$$X(\omega) = \frac{2}{\omega} \sin\frac{\omega}{2}$$

—— x ——

$\Rightarrow$ $x(t) = e^{-2(t-1)} u(t-1)$ find F.T ?

Sol: $e^{-2t} u(t) \longrightarrow \frac{1}{2+j\omega}$

~~Here~~

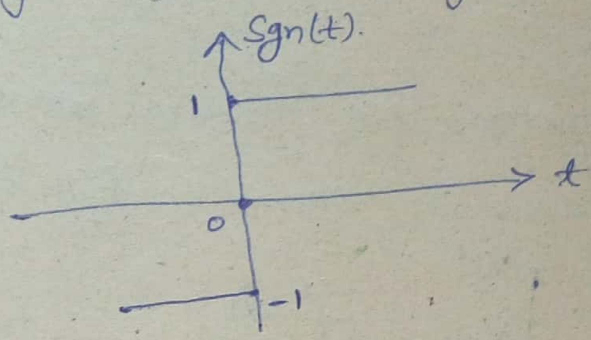
$e^{-2(t-1)} \cdot u(t-1) \rightarrow$

Here applying time shifting Property

$$\rightarrow \frac{e^{-j\omega}}{2+j\omega}$$

⇒ find Fourier Transform of the Signum signal.
(or)
Signum function.

Sol: The Signum function is given as,



$$sgn(t) = -1 \text{ for } t<0$$
$$= 0 \text{ for } t=0$$
$$= 1 \text{ for } t>0.$$

F.T. $$X(\omega) = \int_{-\infty}^{\infty} x(t).\, e^{-j\omega t}.\, dt$$

$$= \int_{-\infty}^{\infty} sgn(t).\, e^{-j\omega t}.\, dt.$$

$$= \int_{-\infty}^{0} sgn(t).\, e^{-j\omega t}.\, dt + \int_{0}^{\infty} sgn(t).\, e^{-j\omega t}.\, dt$$

$$= \int_{-\infty}^{0} (-1).\, e^{-j\omega t}.\, dt + \int_{0}^{\infty} 1.\, e^{-j\omega t}.\, dt.$$

$$= -\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-\infty}^{0} + \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{0}^{\infty}$$

$$= \frac{+1}{j\omega}\left[e^{0} - e^{-\infty}\right] - \frac{1}{j\omega}\left[e^{-\infty} - e^{0}\right]$$

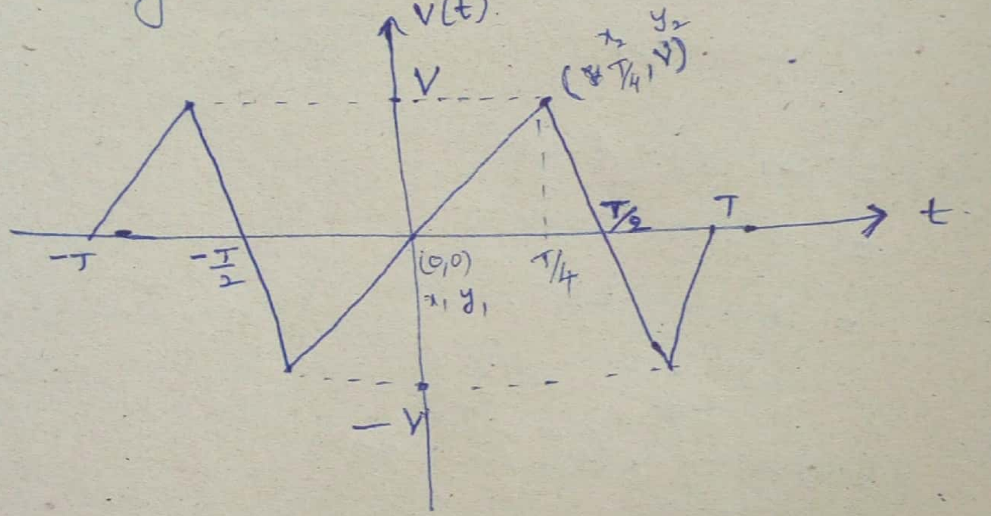
$$= \frac{1}{j\omega}\left[1-0\right] - \frac{1}{j\omega}\left[0-1\right]$$

$$X(\omega) = \frac{1}{j\omega} + \frac{1}{j\omega}$$

$$X(\omega) = \frac{2}{j\omega}$$ //.

— x —

⇒ Find the trigonometric Fourier Series of the traingular wave form shown in fig. below.



sol: The given wave from it looks like an odd function as it can be observed from the waveform

$$v(-t) = -v(t).$$

Hence the Fourier Series expansion contains only Sine terms. i.e $a_0 = 0$, & $a_n = 0$. $b_n \neq 0$.

So we find coefficient of b_n.

$$b_n = \frac{2}{T}\int_0^T v(t) \cdot \sin n\omega t \, dt.$$

The v(t) function is given as

The quarter wave of the given waveform can be represented by equation of a straight line

$y = mx.$

where $y = v(t).$ $x = t.$

$m = \text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$v(t) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) t \qquad \int uv = uv' - \int v' u'$$

$$= \left\{\frac{(V - 0)}{\left(\frac{T}{4} - 0\right)}\right\} t$$

$$v(t) = \frac{V}{\left(\frac{T}{4}\right)} \times t = \frac{4V}{T} t. \qquad 0 \le t \le T/4$$

$(T = T/4)$

$\omega = \frac{2\pi}{T}.$

$$\therefore b_n = \frac{2}{(T/4)} \int_0^{T/4} \frac{4V}{T} t . \sin n\left(\frac{2\pi}{T}\right) t . dt$$

$$= \frac{8}{T} \cdot \frac{4V}{T} \int_0^{T/4} t . \sin\left(\frac{2\pi n}{T}\right) t . dt$$

$$b_n = \frac{32V}{T^2} \left\{ \left. \frac{-t \cos\left(\frac{2\pi n}{T}\right) t}{\frac{2n\pi}{T}} \right|_0^{T/4} + \int_0^{T/4} 1 . \frac{\cos\left(\frac{2\pi n}{T}\right) t}{\frac{2\pi n}{T}} dt \right\}_0^{T/4}$$

$$= \frac{32V}{T^2} \times \frac{T}{2\pi n} \left\{ \left[-t . \cos\left(\frac{2\pi n}{T}\right) t\right]_0^{T/4} + \left[\frac{\sin\left(\frac{2\pi n}{T}\right) t}{\frac{2\pi n}{T}}\right]_0^{T/4} \right\}$$

$$= \frac{16V}{Tn\pi} \left\{ -\frac{T}{4} \cos\left(\frac{2\pi n}{T}\right) \frac{T}{4} \Big/ 0 + \left[\frac{\sin\left(\frac{2\pi n}{T}\right) \frac{T}{4}}{\left(\frac{2\pi n}{T}\right)}\right]_0 \right\}$$

$$b_n = \frac{\cancel{16}^{\,8} V}{\cancel{\pi}\, n\pi} \cdot \frac{\cancel{\pi}}{\cancel{2}\pi n} \sin\left[\frac{n\pi}{2}\right]$$

$$= \frac{8V}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right).$$

$$= \frac{8V}{n^2\pi^2} \quad \text{for } n = 1, 5, 9 \ldots$$

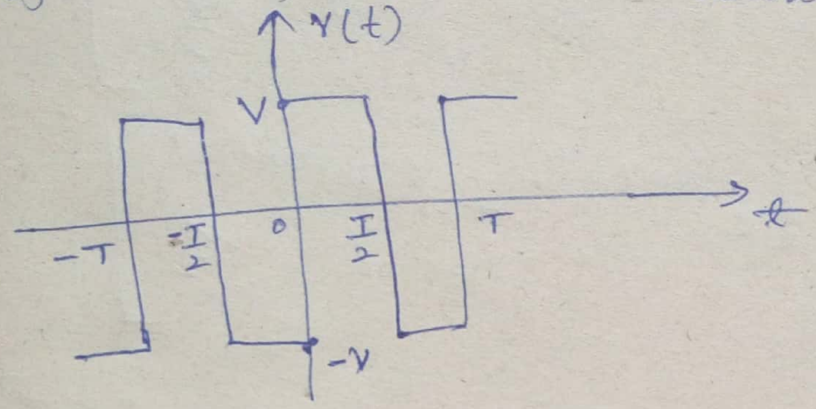
$$= -\frac{8V}{n^2\pi^2} \quad \text{for } n = 3, 7, 11 \ldots$$

$$= 0 \quad \text{for } n = 2, 4, 6 \ldots$$

Hence we have

$$V(t) = \frac{8V}{\pi^2}\left[\sin\omega t - \frac{1}{3^2}\sin 3\omega t + \frac{1}{5^2}\sin 5\omega t + \ldots\right]$$

→ Determine the Fourier Series in exponential form for the function shown below in figure.



Sol: The given waveform can be expressed as,

$$v(t) = V \quad \text{for} \quad 0 \le t \le \frac{T}{2}$$

$$= -V \quad \text{for} \quad \frac{T}{2} \le t \le T.$$

The exponential Fourier Series is given by,

$$v(t) = \sum_{n=-\infty}^{\infty} C_n \, e^{jn\omega t}$$

where $C_n = \frac{1}{T} \int_0^T v(t) . e^{-jn\omega t} . dt$

$$C_n = \frac{1}{T} \left[\int_0^{T/2} v(t) . e^{-jn\omega t} . dt + \int_{T/2}^{T} v(t) . e^{-jn\omega t} . dt \right]$$

$$C_n = \frac{1}{T} \left[\int_0^{T/2} V . e^{-jn\omega t} . dt + \int_{T/2}^{T} (-V) . e^{-jn\omega t} . dt \right]$$

$$= \frac{V}{T} \left[\int_0^{T/2} e^{-jn\omega t} . dt - \int_{T/2}^{T} e^{-jn\omega t} . dt \right]$$

$$C_n = \frac{V}{T}\left[\left(\frac{e^{-jn\omega t}}{-j\omega n}\right)_0^{\frac{T}{2}} - \left(\frac{e^{-jn\omega t}}{-j\omega n}\right)_{\frac{T}{2}}^{T}\right]$$

$$= \frac{-V}{j\omega T n}\left[\left(e^{-jn\omega t}\right)_0^{T/2} - \left(e^{-jn\omega t}\right)_{T/2}^{T}\right]$$

$$= \frac{-V}{j\omega T n}\left[e^{-j\omega n\frac{T}{2}} - e^{-0} - e^{-jn\omega T} + e^{-j\omega n\frac{T}{2}}\right]$$

$$= \frac{-V}{j\omega n T}\left[2e^{-j\omega n\frac{T}{2}} - 1 - e^{-j\omega n T}\right] \quad \left(\omega = \frac{2\pi}{T}\right)$$

$$= \frac{-V}{j\omega n T}\left[2 \cdot e^{-jn\frac{T}{2}\left(\frac{2\pi}{T}\right)} - 1 - e^{-jnT\left(\frac{2\pi}{T}\right)}\right]$$

$$= \frac{-V}{j\omega n T}\left[2e^{-jn\pi} - 1 - e^{-j2\pi n}\right]$$

$$= \frac{-V}{j\omega n T}\left[2(\cos n\pi - j\sin(n\pi)) - 1 - (\cos 2\pi n - j\sin 2\pi n)\right]$$

$$\left[\because \cos n\pi = (-1)^n,\ \sin n\pi = 0 \quad \cos 2\pi n = 1,\ \sin 2\pi n = 0\right]$$

$$= \frac{-V}{j\omega n T}\left[2(-1)^n - 1 - 1\right]$$

if n is even

$$C_n = \frac{-V}{j\omega nT}(2-1-1) = 0.$$

if n is odd

$$C_n = \frac{-V}{j\omega nT}(-2-1-1) = \frac{4V}{j\left(\frac{2\pi}{\cancel{T}}\right)n\cancel{T}}$$

$$C_n = \frac{\overset{2}{\cancel{4}}V}{j\cancel{2}\pi n} = \frac{2V}{j\pi n} = \frac{-j2V}{\pi n}.$$

∴ The exponential fourier Series is

$$V(t) = \sum_{n=-\infty}^{\infty} C_n e^{+jn\omega t}.$$

$$= \sum_{n=1,3,5}^{\infty} \frac{-j2V}{\pi n} e^{jn\omega t}.$$

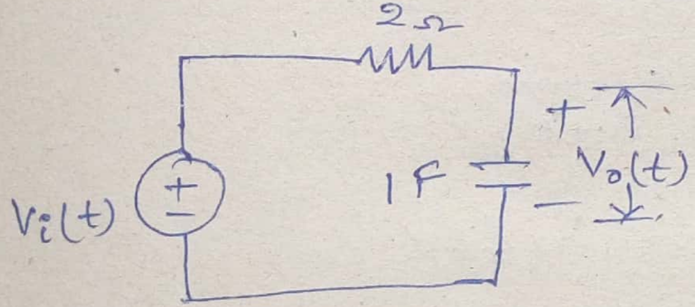
$$\boxed{V(t) = \frac{-j2V}{\pi} \sum_{n=1,3,5}^{\infty} \frac{e^{jn\omega t}}{n}}$$

Application of Fourier Transform for Circuit Analysis

t	s (L.T)	f (F.T)
$R \Rightarrow$	$R \longrightarrow$	R
$L \Rightarrow$	$Ls \longrightarrow$	$j\omega L$
$C \Rightarrow$	$\frac{1}{Cs} \longrightarrow$	$\frac{1}{j\omega C}$

$(\because s = j\omega)$

⇒ Find $V_o(t)$ in the circuit for $V_i(t) = 2e^{-3t}u(t)$.

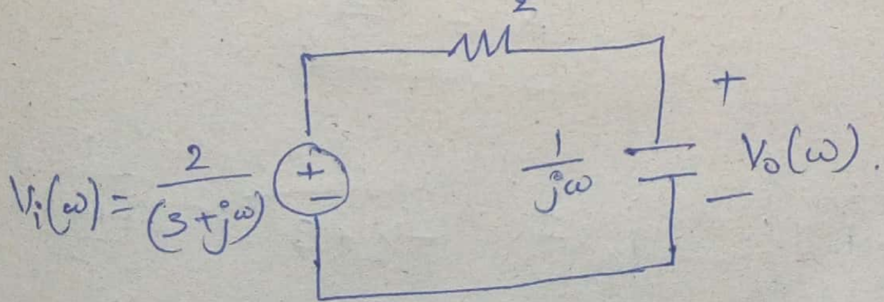


Sol. $C = 1F \rightarrow \frac{1}{j\omega C} = 1/j\omega$.

$V_i(t) = 2e^{-3t}u(t)$.

$V_i(\omega) = \frac{2}{(3+j\omega)}$

Redraw the circuit in frequency domain.



using voltage division rule

$$V_o(\omega) = \frac{V_i(\omega) \cdot 1/j\omega}{2 + 1/j\omega}$$

$$V_o(\omega) = \frac{\frac{2}{(3+j\omega)} \cdot \cancel{\frac{1}{j\omega}}}{\frac{(j2\omega+1)}{\cancel{j\omega}}}$$

$$= \frac{2}{(3+j\omega)(1+j2\omega)}$$

$$= \frac{\cancel{2}}{(3+j\omega)\cancel{2}(j\omega+\frac{1}{2})}$$

$$V_o(\omega) = \frac{1}{(3+j\omega)(j\omega+\frac{1}{2})}$$

Applying Partial fractions

$$V_o(\omega) = \frac{A}{(j\omega+3)} + \frac{B}{(j\omega+\frac{1}{2})}$$

$$A = \left.\frac{1}{(j\omega+\frac{1}{2})}\right|_{j\omega=-3} = \frac{1}{-3+\frac{1}{2}}$$

$$= \frac{1}{\left(\frac{-6+1}{2}\right)}$$

$$\boxed{A = \frac{-2}{5}}$$

$$B = \left.\frac{1}{(j\omega+3)}\right|_{j\omega=-\frac{1}{2}} = \frac{1}{-\frac{1}{2}+3} = \frac{1}{\left(\frac{-1+6}{2}\right)}$$

$$\boxed{B = \frac{2}{5}}$$

$$V_o(\omega) = \frac{\frac{-2}{5}}{(j\omega+3)} + \frac{\frac{2}{5}}{(j\omega+\frac{1}{2})}$$

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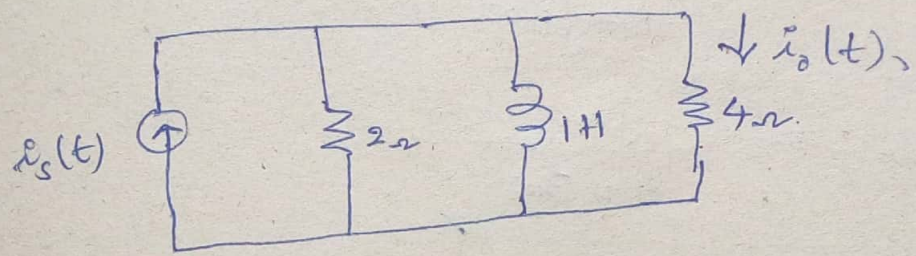
Apply Inverse fourier Transform.

$$\boxed{v_o(t) = \left[-\frac{2}{5} e^{-3t} + \frac{2}{5} e^{-\frac{1}{2}t}\right] u(t).\ V}$$

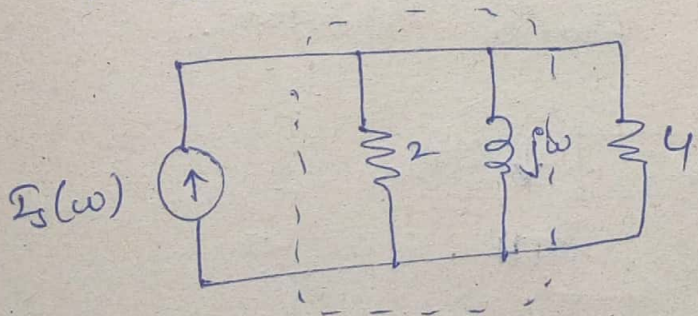
⇒ Find Transfer function $\frac{I_o(\omega)}{I_s(\omega)}$ for the ckt



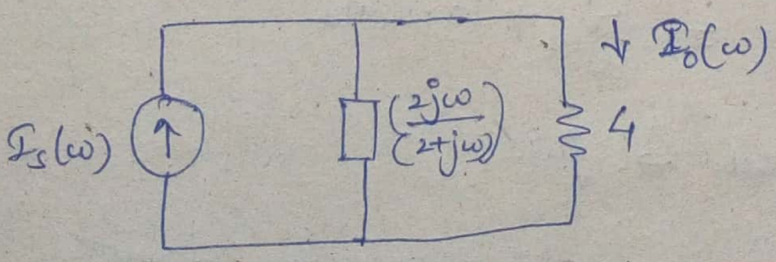
Sol:

1H — $j\omega L \rightarrow j\omega$.

redraw the ckt in frequency domain.



$$2 \| j\omega = \frac{2 \times j\omega}{2 + j\omega} = \frac{j2\omega}{(2+j\omega)}$$



using Current division rule.

$$I_o(\omega) = \frac{I_s(\omega) \cdot \frac{2j\omega}{(2+j\omega)}}{4 + j2\omega/2+j\omega)}$$

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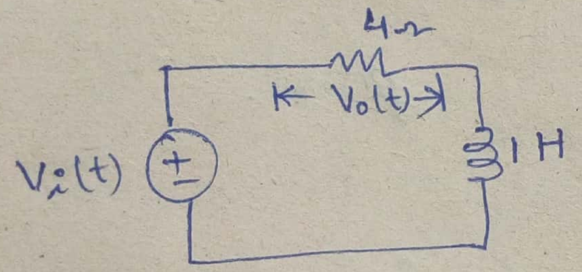
$$H(\omega) = \frac{I_o(\omega)}{I_s(\omega)} = \frac{\left(\dfrac{j2\omega}{\cancel{2+j\omega}}\right)}{\left(\dfrac{8+j4\omega+j2\omega}{\cancel{2+j\omega}}\right)}$$

$$= \frac{j2\omega}{8+j6\omega}.$$

$$\boxed{H(\omega) = \frac{j\omega}{4+j3\omega}}$$

⇒ Determine $V_o(t)$ if $V_i(t) = 2\,sgn(t) = -2 + 4u(t)$



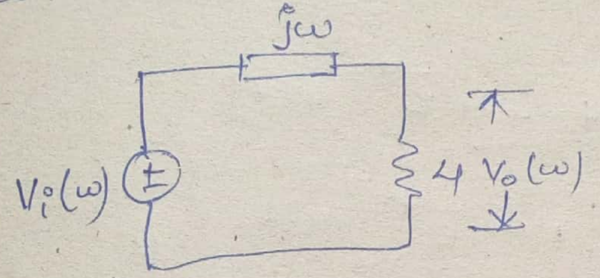
Sol:

$$V_i(t) = 2\,Sgn(t).$$

$$V_i(\omega) = 2\left(\frac{2}{j\omega}\right) = \frac{4}{j\omega}.$$

$$1H \rightarrow j\omega L \rightarrow j\omega.$$

redraw the Ckt in frequency domain



using voltage division rule.

$$V_o(\omega) = \frac{V_i(\omega) \cdot 4}{4 + j\omega}$$

$$= \frac{\frac{4}{j\omega} \cdot 4}{4 + j\omega}$$

$$V_o(\omega) = \frac{16}{j\omega(4 + j\omega)}$$

Apply partial fraction.

$$V_o(\omega) = \frac{A}{j\omega} + \frac{B}{(4 + j\omega)}$$

$$A = \left. \frac{16}{(j\omega + 4)} \right|_{j\omega = 0} = \frac{16}{4} = 4$$

$$\boxed{A = 4}$$

$$B = \left. \frac{16}{j\omega} \right|_{j\omega = -4} = \frac{16}{-4} = -4.$$

$$\boxed{B = -4}$$

$$V_o(\omega) = \frac{4}{j\omega} + \frac{(-4)}{(4 + j\omega)}$$

$$= 2\left(\frac{2}{j\omega}\right) + \frac{(-4)}{(4 + j\omega)}$$

Apply inverse Fourier Transform.

$$V_o(t) = 2 \cdot sgn(t) - 4 e^{-4t} \cdot u(t).$$

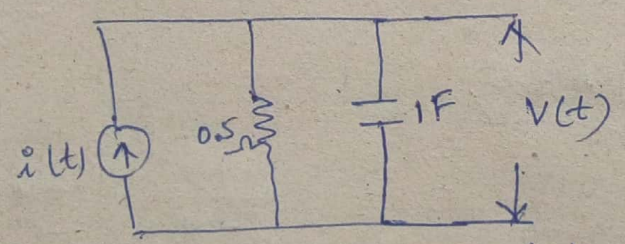
$$= -2 + 4u(t) - 4e^{-4t} u(t)$$

$$\boxed{V_o(t) = -2 + 4\left[1 - e^{-4t}\right]u(t).} \; V.$$

⇒ An input voltage $V_i(t) = 5e^{-3t}u(t)$ is applied to a Series RL Circuit with $R = 4\Omega$ & $L = 2H$. Find the voltage $V_o(t)$ across the inductor using frequency domain analysis (FT method).

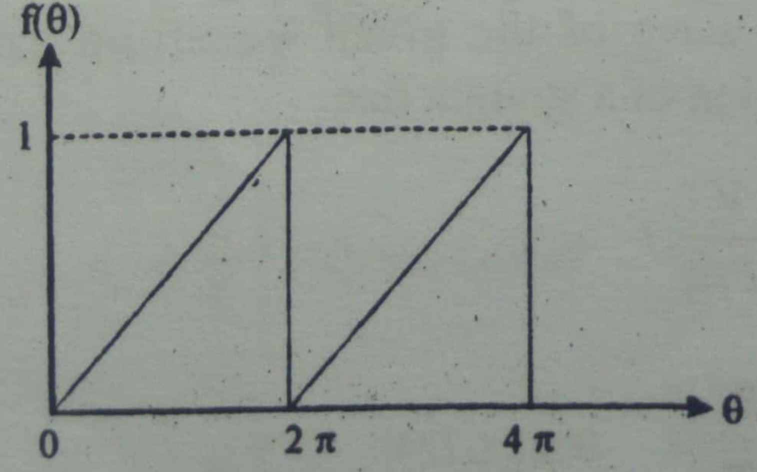
A: $5\,u(t)\left[3e^{-3t} - 2e^{-2t}\right]$ V.

⇒ Determine the output voltage response $v(t)$ across the capacitor. the current source excitation $i(t) = e^{-t}u(t)$. as shown in fig.



A: $V(t) = \left(e^{-t} - e^{-2t}\right)u(t)$. V

Q16. **Determine the fourier series for the saw tooth function shown in figure.**



Figure

Ans:

$$= \frac{16V}{Tn\pi} \frac{T}{2n\pi} . \sin\left[\frac{2n\pi}{T}\left(\frac{T}{4}\right)\right]$$

$$= \frac{8V}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right)$$

$$= \frac{8V}{n^2\pi^2} \quad \text{for } n = 1, 5, 9, ...$$

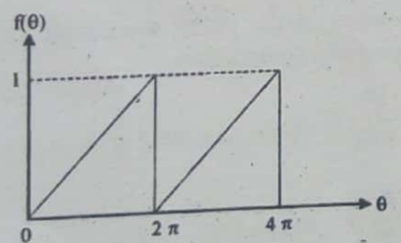
$$= -\frac{8V}{n^2\pi^2} \quad \text{for } n = 3, 7, 11, ...$$

$$= 0 \text{ for } n = 2, 4, 6, ...$$

Hence, we have,

$$v(t) = \frac{8V}{\pi^2}\left(\sin \omega t - \frac{1}{3^2}\sin 3\omega t + \frac{1}{5^2}\sin 5\omega t + ...\right)$$

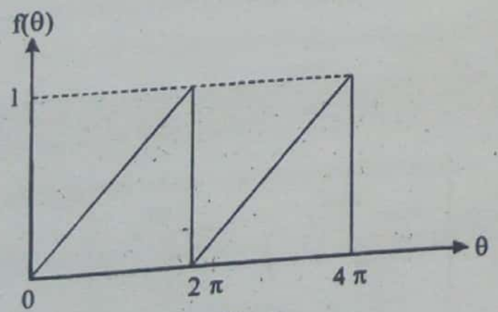
Q16. Determine the fourier series for the saw tooth function shown in figure.



Figure

Ans:

Given waveform is,



Figure

The function $f(\theta)$ is given by,

$$f(\theta) = \frac{\theta}{2\pi} \qquad (0 < \theta < 2\pi)$$

By fourier series expansion, we have,

$$f(\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\theta + b_n \sin n\theta) \qquad ... (1)$$

Now,

The fourier coefficient a_0 is given as,

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(\theta)d\theta$$

$$= \frac{1}{\pi}\int_0^{2\pi}\frac{\theta}{2\pi}d\theta = \frac{1}{2\pi^2}\left[\frac{\theta^2}{2}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi^2}\left[\frac{(2\pi)^2}{2} - 0\right] = \frac{1}{2\pi^2} \times \frac{4\pi^2}{2}$$

$$a_0 = 1$$

The fourier coefficient a_n is given as,

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\cos n\theta\, d\theta$$

$$= \frac{1}{\pi}\int_0^{2\pi}\frac{\theta}{2\pi}\cos n\theta\, d\theta$$

$$= \frac{1}{2\pi^2}\left\{\theta\frac{\sin n\theta}{n} - \int_0^{2\pi} 1.\frac{\sin n\theta}{n}d\theta\right\}$$

$$= \frac{1}{2\pi^2}\left\{\theta\frac{\sin n\theta}{n} - \left(\frac{-\cos n\theta}{n^2}\right)\right\}_0^{2\pi}$$

$$= \frac{1}{2\pi^2}\left\{\frac{\theta}{n}\sin n\theta + \frac{1}{n^2}\cos n\theta\right\}_0^{2\pi}$$

$$= \frac{1}{2\pi^2}\left[\frac{2\pi}{n}\sin(2\pi)n + \frac{1}{n^2}\cos(2\pi)n - 0 - \frac{1}{n^2}\cos(0)\right]$$

$$= \frac{1}{2\pi^2}\left[0 + \frac{1}{n^2}(1) - \frac{1}{n^2}(1)\right]$$

$$a_n = 0$$

Similarly, the fourier coefficient b_n is given as,

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\sin n\theta\, d\theta$$

$$= \frac{1}{\pi}\int_0^{2\pi}\frac{\theta}{2\pi}\sin n\theta\, d\theta$$

$$= \frac{1}{2\pi^2}\left\{\theta\left(\frac{-\cos n\theta}{n}\right) - \int_0^{2\pi} 1\frac{-\cos n\theta}{n}d\theta\right\}$$

$$= \frac{1}{2\pi^2}\left[\frac{-\theta}{n}\cos n\theta + \frac{\sin n\theta}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi^2}\left[\frac{-2\pi}{n}\cos(2\pi)n + \frac{\sin(2\pi)n}{n^2} + 0 - \frac{\sin(0)}{n^2}\right]$$

$$= \frac{1}{2\pi^2}\left[\frac{-2\pi}{n}(1)\right]$$

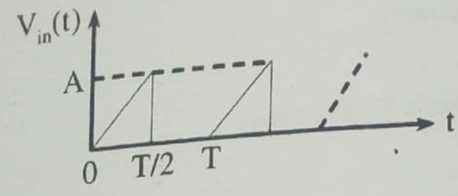
$$b_n = \frac{-1}{n\pi}$$

Substituting the coefficients in equation (1), we get,

$$f(\theta) = \frac{1}{2} + \sum_{n=1}^{\infty}\left(0 - \frac{1}{n\pi}\sin n\theta\right)$$

$$f(\theta) = \frac{1}{2} - \frac{1}{\pi}\sin\theta - \frac{1}{2\pi}\sin 2\theta - \frac{1}{3\pi}\sin 3\theta...$$

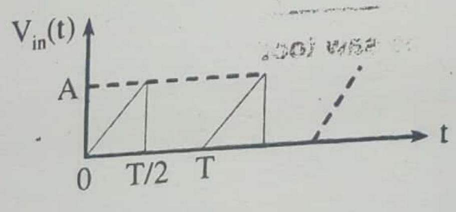
Q17. Obtain the trigonometric Fourier series for the waveform shown in figure.



Figure

Ans:

Given waveform is as shown in figure,



Figure

Required to determine,

Trigonometric Fourier series = ?

By inspection, it is observed that the given waveform is neither odd nor even. Thus the series will contain both sine cosine terms.

We also observe that,

$$V_{in}(t) = \frac{A}{T}\,t\,;\ \text{for } 0 < t < \frac{T}{2}$$

$$= 0\,;\ \text{for } \frac{T}{2} < t < T$$

And also,

$$\omega = \frac{2\pi}{T}$$

Now,

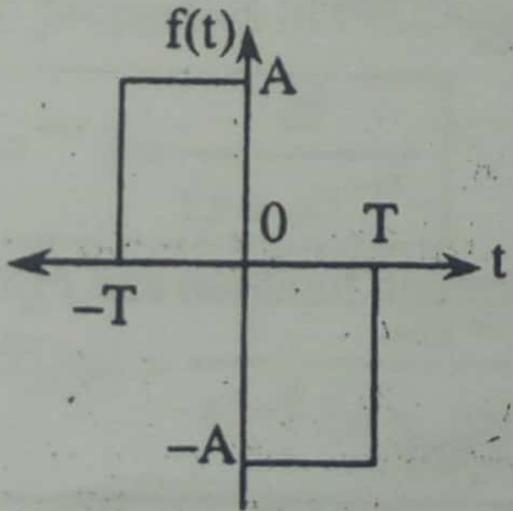
$$a_0 = \frac{1}{T}\int_0^T V_{in}(t)\,dt$$

$$= \frac{1}{T}\left[\int_0^{T/2}\left(\frac{A}{T}t\right)dt + \int_{T/2}^{T}(0)\,dt\right] = \frac{1}{T}\left[\frac{A}{T}\int_0^{T/2} t\,dt + 0\right]$$

$$= \frac{1}{T}\left\{\frac{A}{T}\left[\frac{t^2}{2}\right]_0^{T/2}\right\} = \frac{1}{T}\cdot\frac{A}{2T}\left[\left(\frac{T}{2}\right)^2 - 0\right]$$

$$= \frac{A}{8}$$

Q46. Determine the Fourier transform of the function shown in figure.



Figure

Ans:

Applying fourier transform on both sides, we get,

$$F[f''(t)] = \frac{A}{b-a} F[\delta(t+b) - \delta(t+a) - \delta(t-a) + \delta(t-b)]$$

By the differentiation property, we have,

$$\frac{d^n f(t)}{dt^n} = (j\omega)^n F(j\omega)$$

And by time shifting theorems,

$$F[\delta(t-t_0)] = e^{j\omega t_0}$$

$$\therefore\ (j\omega)^2 F(j\omega) = \frac{A}{b-a}[e^{j\omega b} - e^{j\omega a} - e^{-j\omega a} + e^{-j\omega b}]$$

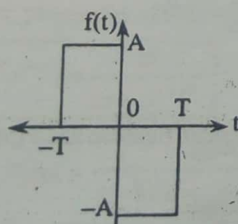
$$-\omega^2 F(j\omega) = \frac{A}{b-a}[(e^{j\omega b} + e^{-j\omega b}) - (e^{j\omega a} + e^{-j\omega a})]$$

$$F(j\omega) = \frac{-A}{(b-a)\omega^2}[2\cos\omega b - 2\cos\omega a]$$

$$= \frac{-2A}{(b-a)\omega^2}(\cos\omega b - \cos\omega a)$$

$$= \frac{2A}{(b-a)\omega^2}(\cos\omega a - \cos\omega b)$$

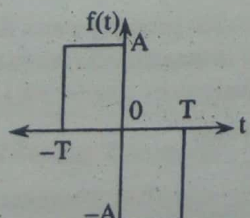
Q46. Determine the Fourier transform of the function shown in figure.



Figure

Ans:

Given function is,



Figure

From figure,

$$f(t) = A \text{ for } -T < t < 0$$

$$= -A \text{ for } 0 < t < T$$

Now, the Fourier transform is given by,

$$f(j\omega) = F[f(t)]$$

$$= \int_{-\infty}^{\infty} f(t).e^{-j\omega t}\,dt$$

$$= \int_{-T}^{T} f(t)e^{-j\omega t}\,dt$$

$$= \int_{-T}^{0} f(t)e^{-j\omega t}\,dt + \int_{0}^{T} f(t).e^{-j\omega t}\,dt$$

$$= \int_{-T}^{0} A.e^{-j\omega t}\,dt + \int_{0}^{T} -A.e^{-j\omega t}\,dt$$

$$= A\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-T}^{0} - A\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{0}^{T}$$

$$= \frac{-A}{j\omega}\left[e^{-j\omega t}\right]_{-T}^{0} + \frac{+A}{j\omega}\left[e^{-j\omega t}\right]_{0}^{T}$$

$$= \frac{-A}{j\omega}\left[e^{0} - e^{-j\omega(-T)}\right] + \frac{A}{j\omega}\left[e^{-j\omega t} - e^{0}\right]$$

$$= \frac{-A}{j\omega}\left[1 - e^{j\omega t}\right] + \frac{A}{j\omega}\left[e^{-j\omega t} - 1\right]$$

$$= \frac{-A}{j\omega} + \frac{A.e^{j\omega t}}{j\omega} + \frac{Ae^{-j\omega t}}{j\omega} - \frac{A}{j\omega}$$

$$= \frac{-2A}{j\omega} + \frac{A}{j\omega}\left(e^{j\omega t} + e^{-j\omega t}\right)$$

$$= \frac{-2A}{j\omega} + \frac{A}{j\omega}(\cos\omega T + j\sin\omega T$$

$$+ \cos(-\omega T) - j\sin\omega T)$$

$$= \frac{-2A}{j\omega} + \frac{A}{j\omega}(2\cos(\omega T)$$

$$= \frac{-2A}{j\omega} + \frac{2A}{j\omega}\cos(\omega T)$$

$$\therefore\quad F(j\omega) = \frac{2A}{j\omega}[\cos\omega T - 1]$$

3.7 PROPERTIES OF FOURIER TRANSFORMS

Q47. Write the properties of Fourier transform and explain.

OR

State and explain the properties of Fourier transform.

Ans:

Model Paper-II, Q6(b)

The properties of Fourier transforms are given as follows,

$$I(j\omega) = \frac{2A}{(1-\omega^2)\left(\sqrt{1+\omega^2}\right)} \cos\frac{\omega\pi}{2} \angle \frac{-\omega\pi}{2} - \tan^{-1}\omega$$

Magnitude,

$$|I(j\omega)| = \frac{2A}{(1-\omega^2)\left(\sqrt{1+\omega^2}\right)} \cos\frac{\omega\pi}{2}$$

Phase,

$$\theta(j\omega) = \frac{-\omega\pi}{2} - \tan^{-1}\omega$$

The magnitude and phase spectra of $I(j\omega)$ are shown in figure (4) and figure (5).

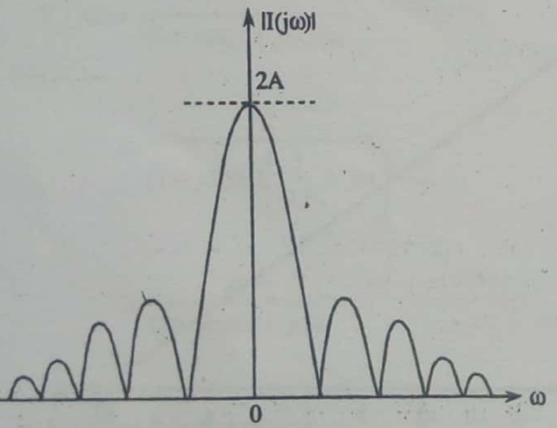


Figure (4)

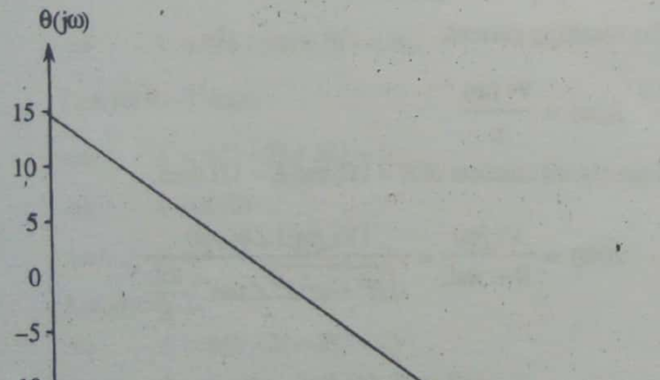


Figure (5)

Q58. An input voltage $V_i(t) = 5\,e^{-3t}\,u(t)$ is applied to a series RL circuit with R = 4 Ω and L = 2 H. Find the output voltage $V_o(t)$ across the inductor using frequency domain analysis.

Ans:

Given that,

$V_i(t) = 5\,e^{-3t}\,u(t)$

$R = 4\,\Omega$ and $L = 2$ H

$V_o(t) = ?$

The series RL circuit is as shown in the figure (1),

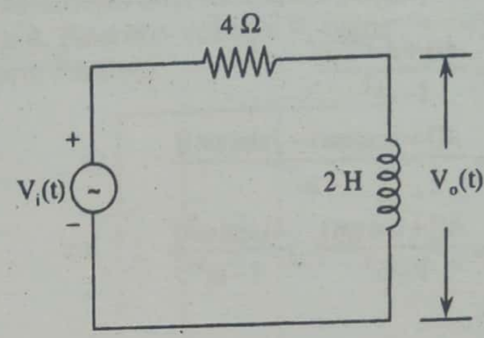


Figure (1)

Fourier transformed network for the above circuit is as follows,

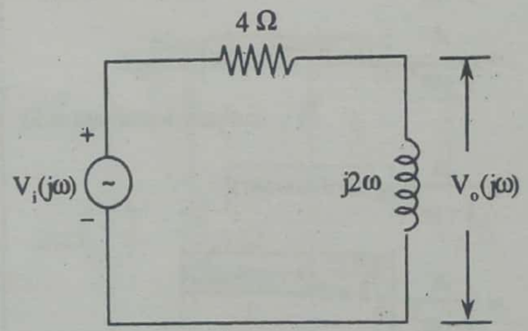


Figure (2)

Fourier transform of input function is given by,

$$V_i(j\omega) = FT\{5e^{-3t}\,u(t)\}$$

$$= 5.\,FT\,\{e^{-3t}\,u(t)\}$$

$$= \frac{5}{j\omega+3} \quad \text{... (1)}$$

$$\text{System function, } H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} \quad \text{... (2)}$$

$$= \frac{j2\omega}{j2\omega+4}$$

Fourier transform of output function is given by,

$$V_o(j\omega) = H(j\omega).\,V_i(j\omega)$$

$$= \frac{j2\omega}{j2\omega+4}.\frac{5}{j\omega+3}$$

$$= \frac{j\omega}{j\omega+2}.\frac{5}{j\omega+3} \quad \text{... (3)}$$

Applying partial fraction to the above equation (3), we get,

$$V_o(j\omega) = \frac{j\omega}{j\omega+2}.\frac{5}{j\omega+3} = \frac{A}{j\omega+3} + \frac{B}{j\omega+2} \quad \text{... (4)}$$

$$\Rightarrow \quad \frac{5\,j\omega}{(j\omega+2)(j\omega+3)} = \frac{A(j\omega+2)+B(j\omega+3)}{(j\omega+2)(j\omega+3)}$$

$$\Rightarrow \quad 5j\omega = A(j\omega+2)+B(j\omega+3) \quad \text{... (5)}$$

Put, $j\omega = -3$ in equation (5), we get,

$$5(-3) = A(-3+2) + B(-3+3)$$

$$-15 = -A$$

$$\boxed{\therefore\ A = 15}$$

Put $j\omega = -2$ in equation (5), we get,

$$5(-2) = A(-2+2) + B(-2+3)$$

$$-10 = B$$

$$\boxed{\therefore\ B = -10}$$

Substitute A and B values in equation (4), we get,

$$V_o(j\omega) = \frac{15}{j\omega+3} + \frac{(-10)}{j\omega+2}$$

$$V_o(j\omega) = \frac{15}{j\omega+3} - \frac{10}{j\omega+2} \qquad \text{... (6)}$$

Applying inverse Fourier transform to equation (6), we get,

$$FT^{-1}\{V_o(j\omega)\} = FT^{-1}\left\{\frac{15}{j\omega+3} - \frac{10}{j\omega+2}\right\}$$

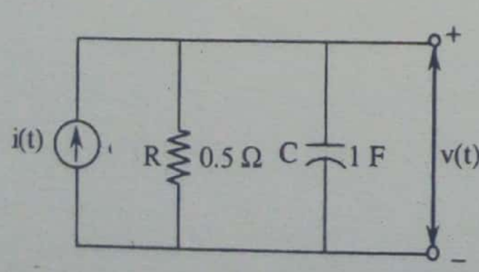
$$V_o(t) = FT^{-1}\left\{\frac{15}{j\omega+3}\right\} - FT^{-1}\left\{\frac{10}{j\omega+2}\right\}$$

$$= 15\,FT^{-1}\left\{\frac{1}{j\omega+3}\right\} - 10\,FT^{-1}\left\{\frac{1}{j\omega+2}\right\}$$

$$= 15.e^{-3t}\,u(t) - 10.e^{-2t}.u(t)$$

$$\therefore\quad V_o(t) = 5\,u(t)\,[3e^{-3t} - 2e^{-2t}]$$

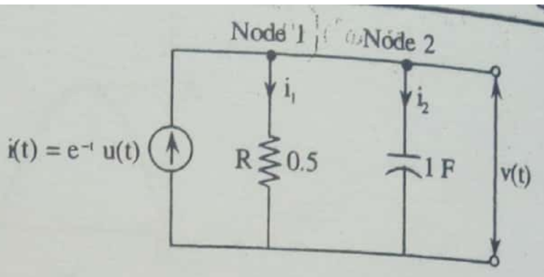
Q59. Determine the output voltage response v(t) across the capacitor to a current source excitation i(t) = e^{-t} u(t) as shown in figure.



Figure

Ans:

The given circuit is



Figure

Applying KCL at node 1, we have,

$$i(t) = i_1 + i_2$$

$$\Rightarrow\quad e^{-t}u(t) = \frac{v(t)}{R} + j\omega C\, v(t)$$

$$\Rightarrow\quad e^{-t}u(t) = v(t)\left[\frac{1}{R} + j\omega C\right]$$

From figure,

$$R = 0.5\,\Omega$$

$$C = 1\,\text{F}$$

$$\Rightarrow\quad e^{-t}u(t) = v(t)\left[\frac{1}{0.5} + j\omega(1)\right]$$

$$\Rightarrow\quad e^{-t}u(t) = v(t)\left[\frac{1}{0.5} + j\omega\right]$$

$$\Rightarrow\quad e^{-t}u(t) = v(t)\,[2 + j\omega] \qquad \text{...(1)}$$

Let,

$$F[v(t)] = V(j\omega)$$

Applying Fourier transform on both sides of equation (1), we get,

$$F[e^{-t}u(t)] = F[v(t)\,(2+j\omega)]$$

$$\Rightarrow\quad \frac{1}{j\omega+1} = V(j\omega)\,(2+j\omega)$$

$$\Rightarrow\quad V(j\omega) = \frac{1}{(1+j\omega)(2+j\omega)}$$

$$= \frac{A}{1+j\omega} + \frac{B}{2+j\omega} \qquad \text{...(2)}$$

$$\Rightarrow\quad 1 = A(2+j\omega) + B(1+j\omega)$$

Let, $j\omega = -1$ then,

$$\Rightarrow\quad 1 = A(2-1) + B(1-1)$$

$$\Rightarrow\quad A = 1$$

Let, $j\omega = -2$ then,

$\Rightarrow \quad 1 = A(2-2) + B(1-2)$

$\Rightarrow \quad 1 = B(-1)$

$\Rightarrow \quad B = -1$

Substituting A, B values in equation (2), we get,

$$V(j\omega) = \frac{A}{1+j\omega} + \frac{B}{2+j\omega}$$

$$= \frac{1}{1+j\omega} - \frac{1}{2+j\omega}$$

Taking inverse Fourier transform, we get,

$$F^{-1}[V(j\omega)] = F^{-1}\left[\frac{1}{1+j\omega} - \frac{1}{2+j\omega}\right]$$

$$\Rightarrow \quad v(t) = F^{-1}\left[\frac{1}{1+j\omega}\right] - F^{-1}\left[\frac{1}{2+j\omega}\right]$$

$$= e^{-t}\,u(t) - e^{-2t}\,u(t)$$

$$= (e^{-t} - e^{-2t})\,u(t)$$

$\therefore$ The voltage response across the capacitor is given by,

$$v(t) = (e^{-t} - e^{-2t})\,u(t)$$

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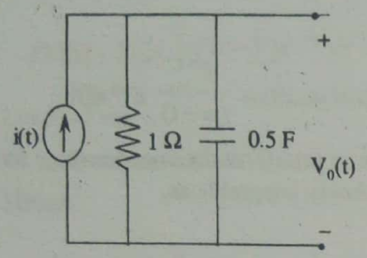
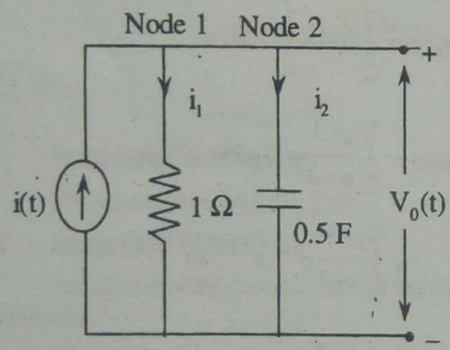
Q56. The current source in the figure, is i(t) = 4e^{-t} for t ≥ 0. Find the voltage V$_0$ using Fourier transform method.



Figure

Ans:

The given circuit is,



Applying KCL at node 1, we have,

$$i(t) = i_1 + i_2$$

Given

$$i(t) = 4\,e^{-t} \text{ for } t \geq 0$$

$$\therefore \quad 4\,e^{-t} = \frac{V_0(t)}{R} + j\omega C\, V_0(t)$$

$$4\,e^{-t} = V_0(t)\left[\frac{1}{R} + j\omega C\right]$$

From figure,

$$R = 1\,\Omega$$

$$C = 0.5\,\text{F}$$

$$\Rightarrow \quad 4\,e^{-t} = V_0(t)\left[\frac{1}{1} + j\omega(0.5)\right]$$

$$\Rightarrow \quad 4\,e^{-t} = V_0(t)\,[1 + 0.5\,j\omega]$$

$$\Rightarrow \quad 4\,e^{-t} = V_0(t)\,0.5\left(\frac{1}{0.5} + j\omega\right)$$

$$\Rightarrow \quad \frac{4}{0.5}\,e^{-t} = V_0(t)\,(2 + j\,\omega)$$

$$8\,e^{-t} = V_0(t)\,(2 + j\,\omega) \qquad \text{...(1)}$$

Let,

$$F[V_0(t)] = V_0(j\omega)$$

Applying Fourier transform on both sides of equation (1), we ge

$$\Rightarrow \quad \frac{8}{1 + j\omega} = V_0(j\,\omega)\,(2 + j\omega)$$

$$\Rightarrow \quad V_0\,(j\,\omega) = \frac{8}{(1 + j\omega)(2 + j\omega)}$$

$$\Rightarrow \quad V_0(j\omega) = \frac{A}{1 + j\omega} + \frac{B}{2 + j\omega}$$

$$\Rightarrow \quad 8 = A\,(2 + j\,\omega) + B\,(1 + j\,\omega)$$

Let, $j\,\omega = -1$ then,

$$\Rightarrow \quad 8 = A\,(2 - 1) + B\,(1 - 1)$$

$$\therefore \quad A = 8$$

Let, $j\,\omega = -2$

$$\Rightarrow \quad 8 = A\,(2 - 2) + B\,(1 - 2)$$

$$\Rightarrow \quad 8 = -B$$

$$\therefore \quad B = -8$$

Substituting A, B values in equation (2), we get,

$$V_0\,(j\,\omega) = \frac{A}{1 + j\omega} + \frac{B}{2 + j\omega}$$

$$= \frac{8}{1 + j\omega} - \frac{8}{2 + j\omega}$$

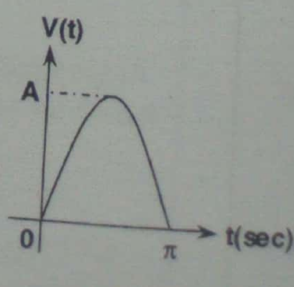
Taking inverse Fourier transform, we get,

$$F^{-1}\,[(V_0)j\,\omega)] = F^{-1}\left[\frac{8}{1 + j\omega} - \frac{8}{2 + j\omega}\right]$$

$$\Rightarrow \quad V_0(t) = F^{-1}\left[\frac{8}{1 + j\omega}\right] - F^{-1}\left[\frac{8}{2 + j\omega}\right]$$

$$\therefore \quad V_0(t) = 8e^{-t} - 8\,e^{-2t} \text{ for } t \geq 0$$

Q57. Find the Fourier transform of the sine pulse shown in fi spectra. If this voltage is applied to a series RL circuit amplitude and phase spectra for the resulting current i(t).

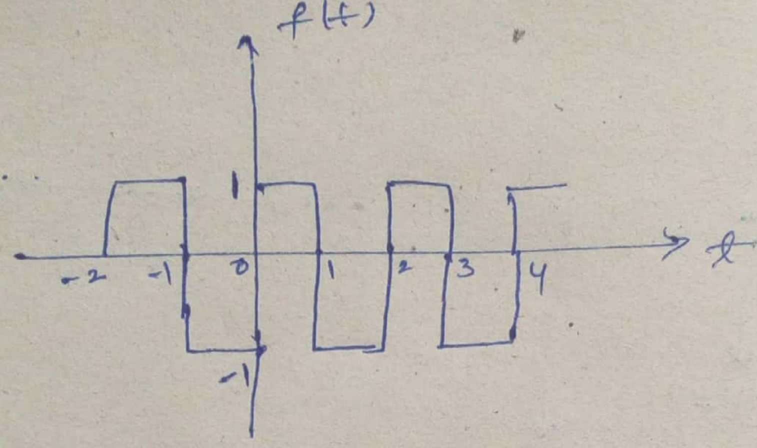


Figure

Ans:

The given waveform is shown i...

⇒ Find F.S.



Sol:

$T = 2$ & $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$.

F.S

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\pi t + b_n \sin n\omega t\right)$$

Here the f(t) is odd symmetry. The f(t) value only in b_n sin terms ($a_0 = a_n = 0, b_n \neq 0$)

$$b_n = \frac{2}{T} \int_0^T f(t) \sin n\omega t \, dt.$$

$$= \frac{2}{2} \int_0^2 f(t) \sin n\omega t \, dt$$

$$= \frac{2}{2} \left[\int_0^1 f(t) \sin n\omega t \, dt + \int_1^2 f(t) \sin n\omega t \, dt \right]$$

$$= \frac{2}{2} \left[\int_0^1 (1) \sin n\omega t \, dt + \int_1^2 (-1) \sin n\omega t \, dt \right]$$

$(\omega = \pi)$

$$= \frac{2}{2} \left[-\left(\frac{\cos n\pi t}{n\pi}\right)_0^1 + \left(\frac{\cos n\pi t}{n\pi}\right)_1^2 \right]$$

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$$b_n = \frac{+1}{n\pi}\left(-\cos n\pi + \cos 0\right) + \frac{1}{n\pi}\left(\cos(2n\pi) - \cos n\pi\right)$$

$$\left(\cos(n\pi) = (-1)^n,\ \cos(2n\pi) = 1\right)$$

$n = \text{odd}$

$$= \frac{1}{n\pi}(1+1) + \frac{1}{n\pi}(1+1)$$

$$= \frac{2}{n\pi} + \frac{2}{n\pi}$$

$$\boxed{b_n = \frac{4}{n\pi}}$$

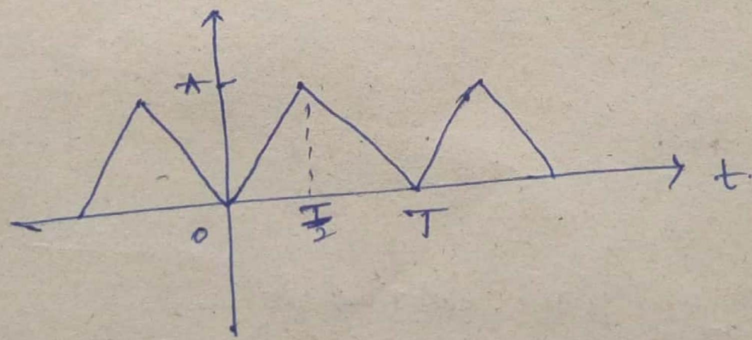
The F.S is

$$f(t) = \sum_{n=1}^{\infty} b_n \sin n\omega t.$$

$$= \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{4}{n\pi} \sin n\omega t$$

$$\boxed{f(t) = \frac{4}{\pi}\sin \omega t + \frac{4}{3\pi}\sin 3\omega t + \frac{4}{5\pi}\sin 5\omega t.}$$

⇒ Find F.S $f(t)$.



Sol. It is an even function. $(b_n = 0,\ a_0 \neq a_n \neq 0)$

$$f(t) = \begin{cases} \frac{2A}{T}t & \text{for } 0 < t < \frac{T}{2} \\ 2A - \frac{2A}{T}t & \text{for } \frac{T}{2} < t < T. \end{cases}$$

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FS is

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t.$$

$$a_0 = \frac{2}{T}\int_0^{T/2} f(t).\ dt$$

$$= \frac{2}{T}\left[\int_0^{T/2} f(t)\,dt + \cancel{\int_{T/2}^{T} f(t)\,dt}\right]$$

$$= \frac{2}{T}\left[\int_0^{T/2} \frac{2A}{T} t\ dt\right]$$

$$= \frac{2}{T}.\frac{2A}{T}\int_0^{T/2} t.\,dt$$

$$= \frac{4A}{T^2}\left[\frac{t^2}{2}\right]_0^{T/2}$$

$$= \frac{4A}{2\cancel{T^2}}.\frac{\cancel{T^2}}{4}$$

$$\boxed{a_0 = \frac{A}{2}}$$

$$a_n = \frac{2}{T}\int_0^{T} f(t).\ \cos n\omega t.\,dt.$$

$$= \frac{2}{(T/2)}\int_0^{T/2} \frac{2A}{T} t.\ \cos n\omega t.\ dt.$$

$$= \frac{4}{T}.\frac{2A}{T}\underbrace{\int_0^{T/2} t.\ \cos n\omega t.\ dt}_{\int uv.}$$

$$a_n = \frac{8A}{T^2}\left\{\left[\frac{t}{n\omega}\sin n\omega t\right]_0^{T/2} - \int_0^{T/2} \frac{1}{n\omega}\sin n\omega t.\,dt\right\}$$

$$= \frac{8A}{T^2}\left[\frac{\frac{T}{2}}{n\left(\frac{2\pi}{T}\right)}\cdot\cancel{\sin\left(n\cdot\frac{2\pi}{T}\cdot\frac{T}{2}\right)} + \frac{1}{n^2\omega^2}\left[\cos n\frac{2\pi}{T}t\right]_0^{T/2}\right]$$

0.

$$= \frac{8A}{T^2\omega^2 n^2}\left[\cos n\cdot\frac{2\pi}{T}\cdot\frac{T}{2} - \cos 0\right]$$

$$= \frac{8A}{T^2 n^2 \omega^2}\left[-1-1\right]$$

$\cos n\pi = (-1)^n.$

$n = \text{odd}.$

$$= \frac{8A}{n^2 T^2\left(\frac{2\pi}{T}\right)^2}\cdot\left[-1-1\right]$$

$$= \frac{-2\cdot\overset{2}{\cancel{8}}A\,\cancel{T^2}}{n^2\cancel{T^2}\,\cancel{4}\pi^2}$$

$$a_n = \frac{-4A}{n^2\pi^2}.$$

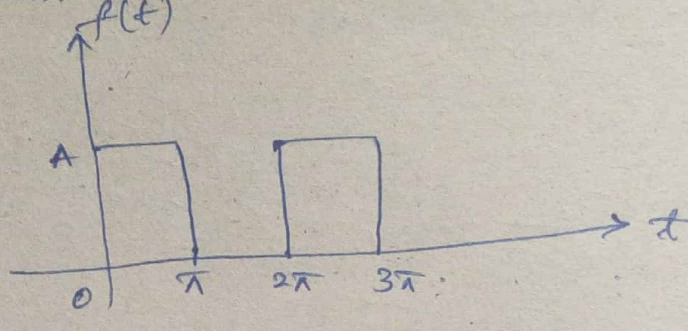
$\therefore$ The F.S is

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(\frac{-4A}{n^2\pi^2}\right)\cos n\omega t.\,dt$$

$$= \frac{A}{2} - \frac{4A}{\pi^2}\sum_{\substack{n=1\\ n=\text{odd}}}^{\infty}\frac{\cos n\omega t}{n^2}.\,dt$$

$$\boxed{f(t) = \frac{A}{2} - \frac{4A}{\pi^2}\cos\omega t - \frac{4A}{3^2\pi^2}\cos 3\omega t - \frac{4A}{5^2\pi^2}\cos 5\omega t - \ldots}$$

⇒ find the trigonometric fourier Series of the wave form.



Sol:

Time period $T = 2\pi$.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1.$$

$$f(t) = \begin{cases} A & \text{for } 0 < t < \pi \\ 0 & \text{for } \pi < t < 2\pi. \end{cases}$$

$$a_0 = \frac{1}{T}\int_0^T f(t).dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} f(t).dt$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} f(t).dt + \underbrace{\int_{\pi}^{2\pi} f(t).dt}_{0}\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} A\,dt$$

$$= \frac{A}{2\pi}\int_0^{\pi} dt = \frac{A}{2\pi}\left[t\right]_0^{\pi}$$

$$= \frac{A\cancel{\pi}}{2\cancel{\pi}}$$

$$\boxed{a_0 = \frac{A}{2}}$$

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$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\, dt$$ put $\omega = 1$.

$$= \frac{2}{2\pi}\int_0^{2\pi} f(t)\cos nt\, dt$$

$$= \frac{2}{2\pi}\int_0^{\pi} f(t)\cdot\cos nt\, dt$$

$$= \frac{1}{\pi}\int_0^{\pi} A\cdot\cos nt\, dt$$

$$= \frac{A}{\pi}\int_0^{\pi} \cos nt\, dt$$

$$= \frac{A}{\pi}\left[\frac{\sin nt}{n}\right]_0^{\pi}$$

$$= \frac{A}{\pi n}\left[\sin n\pi - \sin n(0)\right]$$

$$\boxed{a_n = 0.}$$

$$b_n = \frac{2}{T}\int_0^T f(t)\cdot\sin n\omega t\, dt$$ put $\omega = 1$.

$$= \frac{2}{2\pi}\int_0^{2\pi} f(t)\cdot\sin nt\, dt$$

$$= \frac{1}{\pi}\int_0^{\pi} A\cdot\sin nt\, dt$$

$$= \frac{A}{\pi}\left[\frac{-\cos nt}{n}\right]_0^{\pi}$$

$$= \frac{A}{\pi n}\left[-\cos n\pi + \cos n(0)\right]$$

$$= \frac{A}{\pi n}\left[-(-1)^n + 1\right]$$

$$b_n = \begin{cases} \dfrac{2A}{n\pi} & \text{for } n = \text{odd value} \\ 0 & \text{for } n = \text{even value.} \end{cases}$$

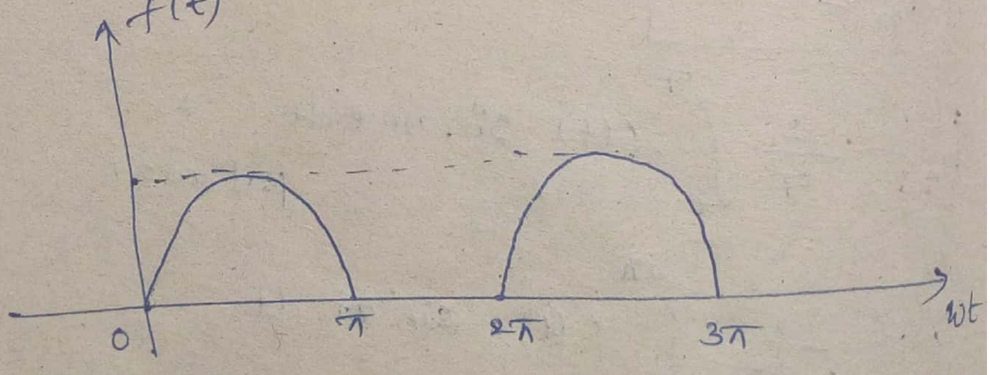
$\therefore$ The fourier Series is

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$= \frac{A}{2} + \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{2A}{n\pi} \sin n\omega t.$$

$$\boxed{f(t) = \frac{A}{2} + \frac{2A}{\pi} \sin\omega t + \frac{2A}{3\pi} \sin 3\omega t + \text{-----}}$$

$\Rightarrow$ Find the exponential fourier Series of the wave form.



Sol:

$$T = 2\pi, \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1.$$

$$f(t) = A \sin\omega t, \quad 0 < \omega t < \pi$$

$$= 0, \quad \pi < \omega t < 2\pi.$$

the exponential F. S is given by

$$f(t) = \sum_{n=-\infty}^{\infty} C_n . e^{jn\omega t}.$$

$$C_n = \frac{1}{T}\int_0^T f(t). e^{-jn\omega t}.dt$$

$$= \frac{1}{2\pi}\int_0^{\cancel{2}\pi} A \sin\omega t . e^{-jn\omega t}.dt$$

$$\int e^{ax} \sin bx . dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

$a = -jn$, $b = 1$, $x = \omega t$.

$$C_n = \frac{A}{2\pi}\left[\frac{e^{-jn\omega t}}{1-n^2}(-jn\sin\omega t - 1.\cos\omega t)\right]_0^\pi$$

$$= \frac{A}{2\pi}\left[\frac{e^{-jn\pi}}{1-n^2}(1) + \frac{1}{1-n^2}\right]$$

$$C_n = \frac{A}{2\pi(1-n^2)}\left[1 + e^{-jn\pi}\right]$$ //

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{A}{2\pi(1-n^2)}(1+e^{-jn\pi}).e^{jn\omega t}.$$

put $\omega = 1$.

$$\boxed{f(t) = \frac{A}{2\pi}\sum_{n=-\infty}^{\infty}\frac{(1+e^{-jn\pi})}{(1-n^2)}e^{jnt}}$$

Let

$$V = V_0 + V_1 \sin(\omega t + \theta_1) + V_2 \sin(\omega t + \theta_2) + \cdots$$

$$I = I_0 + I_1 \sin(\omega t + \psi_1) + I_2 \sin(\omega t + \psi_2) + \cdots$$

I_0 & V_0 - is DC compoment.

$$V_{rms} \text{ (or) } V_{eff} = \sqrt{V_0^2 + \left(\frac{V_1}{\sqrt{2}}\right)^2 + \left(\frac{V_2}{\sqrt{2}}\right)^2}$$

$$= \sqrt{V_0^2 + \frac{V_1^2}{2} + \frac{V_2^2}{2}}.$$

$$I_{rms} \text{ (or) } I_{eff} = \sqrt{I_0^2 + \frac{I_1^2}{2} + \frac{I_2^2}{2}}.$$

$$P_{avg} = V_0 I_0 + \frac{V_1 I_1}{2}\cos(\theta_1 - \psi_1) + \frac{V_2 I_2}{2}\cos(\theta_2 - \psi_2).$$

(or)
True power

apparent power $= V_{rms} I_{rms}$

$$\text{power factor} = \frac{\text{true power}}{\text{apparent power}}$$

//

⇒ The input voltage in volts to a Series RL is

$e(t) = 180 \sin(314t + 10^\circ) + 56 \sin(942t + 35^\circ) + 18$.

The values of R and L are 18Ω & $0.0413H$;

Determine

(i) The expression of current

(ii) The rms value of voltage & current.

(iii) The power factor of the circuit.

<u>Sol</u>: input voltage

$$e(t) = 18 + 180\sin(314t + 10^\circ) + 56 \sin(942t + 35^\circ)$$

$$e(t) = e_0(t) +$$

$$e(t) = e_0(t) + e_1(t) + e_3(t).$$

where $e_0(t) = 18$

$e_1(t) = 180 \sin(314t + 10^\circ)$

$e_3(t) = 56 \sin(942t + 35^\circ)$

Here $\omega = 314$, $R = 18$, $L = 0.0413H$.

For 1st harmonic. $\boxed{X_{Ln} = n\omega L}$ → n^{th} harmonic

$$X_{L_1} = \omega L$$

$$= 314 \times 0.0413$$

$$X_{L_1} = 12.9682\ \Omega$$

$$\therefore Z_1 = R_1 + jX_{L_1} = 18 + j12.9682$$

$$= 22.185 \angle 35.77^\circ\ \Omega$$

$$i_1(t) = \frac{e_1(t)}{Z_1} = \frac{18 \sin(314t + 10^\circ)}{22.185 \angle 35.77}$$

$$= 8.113 \sin(314t + 10^\circ - 35.77)$$

$i_1(t) = 8.113 \sin(314t - 25.77°)$ A.

For 3^{rd} harmonic

$$X_{L_3} = 3\omega L = 3 \times 314 \times 0.0413 = 38.9046\ \Omega$$

$$Z_3 = R + jX_{L_3}$$

$$= 18 + j\,38.9046$$

$$= 42.867 \angle 65.17°\ \Omega$$

$$\therefore i_3(t) = \frac{e_3(t)}{Z_3} = \frac{56 \sin(942t + 35°)}{42.867 \angle 65.17°}$$

$$= 1.306 \sin(942t + 35° - 65.17)$$

$$= 1.306 \sin(942t - 30.17)\ \text{A}.$$

$$i_0(t) = \frac{e_0(t)}{(R + j0)} = \frac{e_0(t)}{R} = \frac{18}{18} = 1.$$

(i) Expression for Current

$$i(t) = i_0(t) + i_1(t) + i_3(t)$$

$$= 1 + 8.113 \sin(314t - 25.77°)$$
$$+ 1.306 \sin(942t - 30.17°).$$

(iii) $$V_{rms} = \sqrt{e_0(t)^2 + \frac{e_1(t)^2}{2} + \frac{e_3(t)^2}{2}}.$$

$$= \sqrt{18^2 + \frac{(180)^2}{2} + \frac{(56)^2}{2}}.$$

$$V_{rms} = 134.506\ \text{V}$$

$$I_{rms} = \sqrt{i_0(t)^2 + \frac{i_1(t)^2}{2} + \frac{i_3(t)^2}{2}}$$

$$= \sqrt{1 + \frac{(8.113)^2}{2} + \frac{(1.306)^2}{2}}$$

$$= 5.896 A$$

(iii) $\text{power factor} = \frac{\text{true power (or) avg. power}}{\text{Apparant power}}$

$$\text{true power} = e_0 i_0 + \frac{e_1 i_1}{2}\cos(\theta_1 - \psi_1) + \frac{e_3 i_3}{2}\cos(\theta_3 - \psi_3)$$

$$P_{avg} = 18 \times 1 + \frac{180 \times 8.113}{2}\cos[10 - (-25.77)]$$
$$+ \frac{56 \times 1.306}{2}\cos[35 - (-30.17)]$$

$$= 18 + 730.17\cos(35.77) + 36.57\cos(65.17)$$

$$P_{avg} = 625.79 W$$

$$\text{Appalant power} = V_{rms} \times I_{rms}$$

$$= 134.506 \times 5.896$$

$$= 793.047 \text{ VA}.$$

$$\therefore \text{power factor} = \frac{\text{true power}}{\text{Apparant power}}$$

$$= \frac{625.79}{793.047}$$

$$= 0.789 \text{ Legging}.$$

⇒ A periodic current source is given by $i(t) = 5 + 3\cos(100t + 45^\circ) + 2\cos(200t - 10^\circ)$. is applied to series RC circuit and 0.5Ω & $0.02F$. calculate the response $V(t)$ and average power.

Sol:

$$i(t) = \underbrace{5}_{I_0(t)} + \underbrace{3\cos(100t + 45^\circ)}_{I_1(t)} + \underbrace{2\cos(200t - 10)}_{I_2(t)}$$

$$= 5 + 3\cos(100t + 45^\circ) + 2\cos(2 \times 100t - 10)$$

$$\omega = 100.$$

$R = 0.5\Omega$ & $C = 0.02F$.

To determine the $V(t)$ is

$$V(t) = V_0(t) + V_1(t) + V_2(t).$$

where $V_0(t) = I_0(t).Z_0.$

$$V_1(t) = I_1(t).Z_1$$

$$V_2(t) = I_2(t)Z_2.$$

For 1st harmonic

$$X_{c_1} = \frac{1}{\omega C} = \frac{1}{100 \times 0.02} = \frac{1}{2} = 0.5$$

$$\therefore Z_1 = R - jX_{c_1}$$

$$= 0.5 - j0.5$$

$$= 0.707 \angle -45^\circ.$$

$$V_1(t) = I_1(t).Z_1$$

$$= 3\cos(100t + 45)\ 0.707 \angle -45^\circ$$

$$= 2.121\cos(100t + 45 - 45)$$

$$= 2.121\cos(100t)$$

For 2^{nd} harmonic

$$X_{C_2} = \frac{1}{2\omega C} = \frac{1}{2\times 100\times 0.02} = \frac{1}{4} = 0.25$$

$$\therefore Z_2 = R - jX_{C_2}$$
$$= 0.5 - j0.25 = 0.559\angle -26.56^\circ$$

$$V_2(t) = I_2(t) . Z_2$$
$$= 2\cos(200t - 10) \times 0.559\angle -26.56$$
$$= 1.118 \cos(200t - 10 - 26.56)$$
$$= 1.118 \cos(200t - 36.56^\circ)$$

For 0 harmonic

$$X_{C_0} = 0.$$
$$Z_0 = R - jX_{C_0} = R = 0.5.$$

$$V_0(t) = I_0(t). Z_0$$
$$= 5\times 0.5 = 2.5.$$

$\therefore$ The response of $V(t)$ is.

$$V(t) = V_0(t) + V_1(t) + V_2(t)$$
$$= 2.5 + 2.121\cos(100t) + 1.118\cos(200t - 36.56).$$

Now average power is given by,

$$P_{avg} = V_0 I_0 + \frac{V_1 I_1}{2}\cos(\theta_1 - \varphi_1) + \frac{V_2 I_2}{2}\cos(\theta_2 - \varphi_2)$$
$$= V_0 I_0 + \frac{V_1 I_1}{2}\cos(0 - 45) + \frac{V_2 I_2}{2}\cos(-36.56 + 10)$$
$$= V_0 I_0 + \frac{V_1 I_1}{2}\cos(45) + \frac{V_2 I_2}{2}\cos(26.56)$$

$$P_{avg} = 2.5 \times 5 + \frac{2.121 \times 3}{2} \times 0.707 + \frac{1.118 \times 2}{2} \times 0.894$$

$$= 12.5 + 2.249 + 0.999$$

$$\boxed{P_{avg} = 15.748 \text{ Watt.}}$$

Q30. The current in an RL circuit with $R = 12\ \Omega$, $L = 6$ H and $i(t) = 12 \sin(900t) + 7 \sin(2700t) + \sin(4500t)$. Determine the effective applied voltage and average power.

Ans:

Q30. The current in an RL circuit with R = 12 Ω, L = 6 H and i(t) = 12 sin (900t) + 7 sin (2700t) + sin (4500t). Determine the effective applied voltage and average power.

Ans:

Given that,

An *RL* circuit,

$R = 12\,\Omega$

$L = 6\,\text{H}$

$i(t) = 12 \sin(900\,t) + 7 \sin(2700\,t) + \sin(4500\,t)$

Required to determine,

Effective applied voltage, $V_{eff} = ?$

Average power, $P_{avg} = ?$

From the given expression of current,

$i(t) = 12 \sin 900\,t + 7 \sin 2700\,t + \sin 4500\,t$

We have, $\omega = 900$

It can also be observed that there are three harmonics i.e., fundamental, third, fifth harmonics.

For 1st Harmonic

$R_1 = 12\,\Omega$

$L = 6\,\text{H} \quad \Rightarrow X_{L1} = \omega L = 900 \times 6 = 5400\,\Omega$

$Z_1 = R_1 + j\,8X_{L1}$

$= (12 + j\,5400)\,\Omega = 5400.013\,\angle 89.873\,\Omega$

$\therefore V_1(t) = Z_1\, i_1(t)$

$= (5400.013\angle 89.873)\,[12 \sin 900\,t]$

$= 64800.156 \sin(900\,t + 89.873°)$

For 3rd Harmonic

$$R_3 = 12\,\Omega$$

$$X_{L_3} = 3\,\omega L = 3 \times 900 \times 6 = 16200\,\Omega$$

$$Z_3 = R_3 + j\,X_{L_3} = (12 + j\,16200)\,\Omega$$

$$= (16200.00\,\angle 89.957)\,\Omega$$

$$\therefore\ V_3(t) = Z_3\, i_3(t)$$

$$= (16200.004\,\angle 89.957)\,(7 \sin 2700\,t)$$

$$= 113400.028 \sin (2700\,t + 89.957)$$

For 5th Harmonic

$$R_5 = 12\,\Omega$$

$$X_{L_5} = 5\,\omega L = 5 \times 900 \times 6$$

$$= 27000\,\Omega$$

$$Z_5 = R_5 + jX_{L5}$$

$$= (12 + j27000)\,\Omega$$

$$= (27000.003\,\angle 89.974)\,\Omega$$

$$\therefore\ V_5(t) = Z_5\, i_5(t)$$

$$= (27000.003\,\angle 89.974)\,[\sin 4500\,t]$$

$$= 27000.003 \sin (4500\,t + 89.974)$$

$$\therefore\ V(t) = V_1(t) + V_3(t) + V_5(t)$$

$$= 64800.156 \sin (900\,t + 89.873) + 113400.028 \sin (2700\,t + 89.957) + 27000.003 \sin (4500\,t + 89.974)$$

$\therefore$ Effective value of $V(t)$,

$$V_{eff} = \sqrt{\frac{(64800.156)^2 + (113400.028)^2 + (108000.012)^2}{2}}$$

$$= 50282.4851\ \text{V}$$

$\therefore$ Average power, P_{avg}

$$= \frac{V_{m_1} i_{m_1}}{2} \cos(\theta_1 - \phi_1) + \frac{V_{m_3} i_{m_3}}{2} \cos(\theta_3 - \phi_3) + \frac{V_{m_5} i_{m_5}}{2} \cos(\theta_5 - \theta_5)$$

$$= \frac{64800.156 \times 12}{2} \cos (89.873 - 0) + \frac{113400.028 \times 7}{2} \cos (89.957 - 0) + \frac{27000.003}{2} \cos (89.974 - 0)$$

$$= 388800.936 \cos 89.873 + 396900.098 \cos 89.957 + 27000.003 \cos 89.974$$

$$= 861.803 + 297.870 + 12.252$$

$$= 1171.925\ \text{W}$$

$\therefore$ Effective applied voltage = 119838.702 V

Average power = 1171.925 W