Transient Response Analysis

- Generally the transient response state in represent the changing state. Practically all the electrical equipment are designed to operate in steady state but a designer is more concerned about parametric changes in transient then the steady state.
- Transients occur in any s/m becoz of memory elements in (L & C). electrical circuit analysis

  An inductor can store energy in the form of Electromagnetic field ($\frac{1}{2}LI^2$)

  An Capacitor " " " " " Electro static field. ($\frac{1}{2}CV^2$)
- When a sudden changes in the circuit occurs these L & C are not allow the voltage & current changes suddenly and circuit goes into oscillations which is transient behaviour.
- The total response of a circuit (or) Network is the sum of response are obtained from initial stored energy in the circuit and also from external source applied to the circuit. (or) Network.
- The response obtained only from the internal stored energy is called as source free response (or) natural response (or) transient response.

  This response depends upon the circuit elements only and independent to the source.
- The response are obtained purely from input source is called as force response (or) steady state response.

$\therefore$ $\boxed{\text{Total Response} = \text{natural response} + \text{force response}}$

if the initial conditions of the ckt are zero

$\therefore$ $\boxed{\text{Total Response} = \text{force response}}$

⇒ Behaviour of N/w elements during transient Operate.

element	DC steady state	AC Steady state	Transient State.
R	R	R in phase element	R
L	Short ckt (S.C)	lagging (I lags V)	O.C. (open circuit)
C	O.C	Leading (I leads V)	S.C.

⇒ initial conditions

initial conditions are those conditions that exist in the circuit immediatly after switching operation to know the values of current & voltag

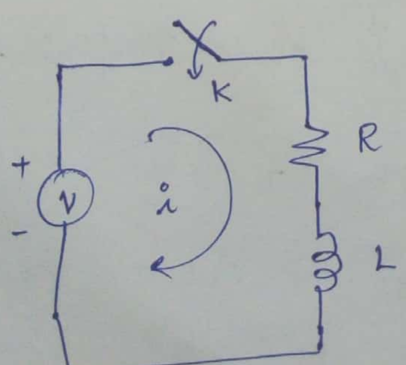
these instants of time are indicated as

$t = 0^-$ ⇒ just before switch operated.

$t = 0$ ⇒ Exact instant of switch operation.

$t = 0^+$ ⇒ just after switch operation.

⇒ Transient Response of Series RL circuit having DC Excitation (first Order circuit).



Let a d.c. Voltage V be applied suddenly (i.e at $t=0$) by closing a switch K in a Series RL circuit as shown in fig above.

applying KVL to the Circuit

$$Ri + L\frac{di}{dt} = V$$

$$\frac{di}{dt} + i\frac{R}{L} = \frac{V}{L}$$

(∵ where $p = \frac{d}{dt}$)

$$pi + i\frac{R}{L} = \frac{V}{L}$$

$$i\left(p + \frac{R}{L}\right) = \frac{V}{L} \rightarrow ①$$

from Eq ① is a Non-homogeneous differential equation and the forced response is obtained from its solution, the solution is given by $i(t) = i_c + i_p$

i_c = complementary function that always goes to zero value in a relatively short time (transient solution)

$$i_c = c\,e^{-(R/L)t}$$

where c is constant.

i_p is the particular solution of i that provides the steady state response.

$$i_p = e^{(-R/L)t}\int e^{(R/L)t}\left(\frac{V}{L}\right)dt = \frac{V}{R}$$

$$\therefore\ i(t) = i_c + i_p$$

$$\boxed{i(t) = c\,e^{-(R/L)t} + \frac{V}{R}} \rightarrow ②$$

Since, the inductor in the ckt Now current through it just before switching is same to the current just after the switching, This is represented as

$$i(0^-) = i(0^+)$$

However before switching, there was no current through the inductor and hence at time $t = 0^+$ the current through the inductor will also be zero. i.e $i(0^+) = 0$. $(\because t = 0^+)$

with initial condition,

from Eq ② ⇒ $0 = c\, e^{-(R/L).0} + \frac{V}{R}$

$$C = -\frac{V}{R}$$

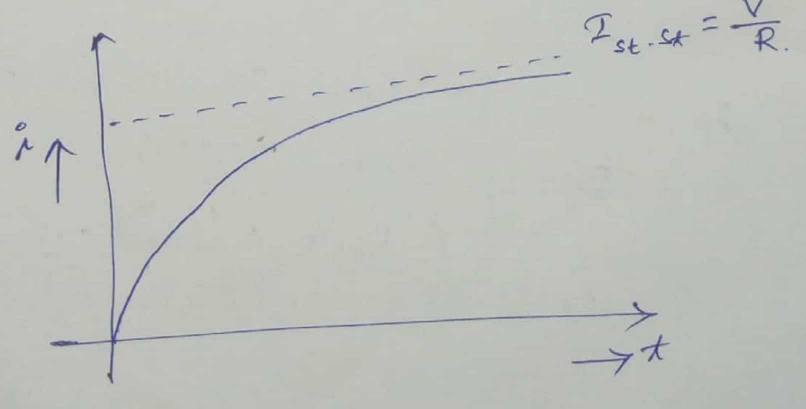
the above value substitute in Eq ②, we get

$$i(t) = \frac{-V}{R} e^{-(R/L)t} + \frac{V}{R}$$

$$\boxed{i(t) = \frac{V}{R} - \frac{V}{R} e^{-(R/L)t} \ A} \rightarrow$$

$$i(t) = \frac{V}{R}\left(1 - e^{-(R/L)t}\right). A. \rightarrow ③$$

from Eq ③ clearly shows the exponential rise of current i charging the inductor.



where in R–L circuit the time constant $(\tau) = \frac{L}{R}$.

and damping ratio is inverse of time constant.

Voltage across the resistance $V_R(t) = iR = \frac{V}{\cancel{R}}\left(1 - e^{-(R/L)t}\right).\cancel{R}$

$$V_R(t) = V\left(1 - e^{-(R/L)t}\right). \rightarrow ④$$

Voltage across the inductor $V_L = L\frac{di}{dt} = L\frac{d}{dt}\left[\frac{V}{R}\left(1 - e^{-(R/L)t}\right)\right]$

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$$V_L(t) = V e^{-(R/L)t} \rightarrow (5)$$

The instantaneous powers in Resistor & inductor are given as

$$P_R = V_R\, i = V\left(1 - e^{-(R/L)t}\right) \cdot \frac{V}{R}\left(1 - e^{-(R/L)t}\right) \text{ Watt}$$

$$P_L = V_L \cdot i = V \cdot e^{-(R/L)t} \cdot \frac{V}{R}\left(1 - e^{-(R/L)t}\right) \text{ Watt.}$$

from Eq (4) & Eq (5), Eq (3) substitute the values of $t = 0$, $t = \infty$, $t = \tau = \frac{L}{R}$.

(i) at $t = 0$,

$$i(t) = \frac{V}{R}\left(1 - e^{-(R/L)0}\right)$$

$$= \frac{V}{R}\left(1 - e^{0}\right) \qquad (\because e^{0} = 1)$$

$$= \frac{V}{R}(1-1)$$

$$i(t) = 0.$$

$$V_R(t) = V\left(1 - e^{-(R/L)0}\right)$$

$$= V(1-1)$$

$$V_R(t) = 0,$$

$$V_L(t) = V e^{-(R/L) \cdot 0} = V \cdot e^{0} = V.$$

$$V_L(t) = V \qquad (\because L \text{ acts as open circuit at } t = 0)$$

(ii) at $t = \infty$.

$$i(t) = \frac{V}{R}\left(1 - e^{-(R/L)\infty}\right) \qquad (\because e^{-\infty} = 0)$$

$$= \frac{V}{R}(1 - 0) = \frac{V}{R}.$$

$$V_R(t) = V\left(1 - e^{-(R/L)\infty}\right)$$

$$V_R(t) = V.$$

$$\therefore \quad V_L(t) = V e^{-(R/L)\infty}$$

$$V_L(t) = 0. \qquad (\because \text{ L acts as short circuit at } t=\infty).$$

(iii) at $t = \frac{L}{R} = T$.

$$i(t) = \frac{V}{R}\left(1 - e^{-(R/L)\frac{L}{R}}\right)$$

$$= \frac{V}{R}\left(1 - e^{-1}\right) = \frac{V}{R}\left(1 - 0.368\right). \qquad (\because e^{-1} = 0.368)$$

$$= 0.632\,\frac{V}{R}.$$

$$V_R(t) = V\left(1 - e^{-(R/L)\frac{L}{R}}\right)$$

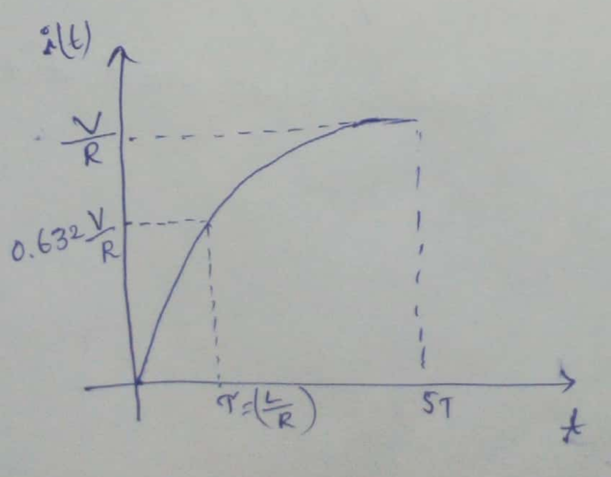
$$= V\left(1 - e^{-1}\right)$$

$$= V\left(1 - 0.368\right) = 0.632\,V.$$

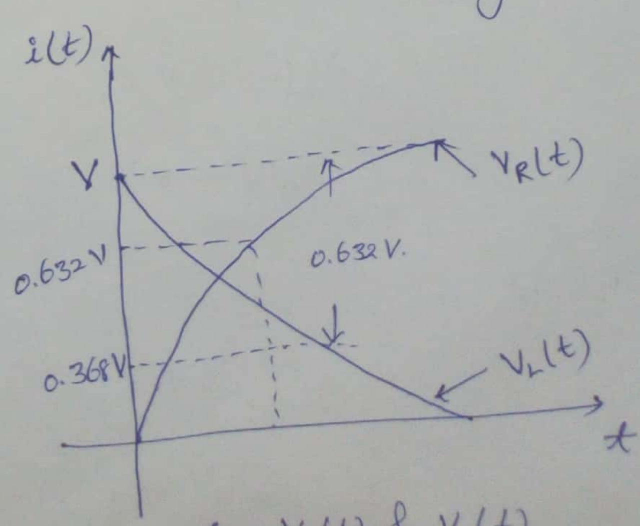
$$V_L(t) = V e^{-(R/L)L/R} = V.e^{-1}$$

$$= 0.368.\,V.$$

The values of $i(t)$, $V_R(t)$, $V_L(t)$ are shown in figures below



for $i(t)$.



for $V_R(t)$ & $V_L(t)$.

EXAMPLE 8.2 *Find the current in a series R-L circuit having $R = 2\,\Omega$ and $L = 10$ H while a d.c. voltage of 100 V is applied. What is the value of this current after 5 sec. of switching on ?*

SOLUTION. Time constant $= \frac{L}{R} = \frac{10}{2} = 5$ sec.

Charging constant is given by

$$i = I(1 - e^{-t/T});$$

I being the steady state current.

Here, $$i = \frac{100}{2}(1 - e^{-t/5}) = 50\,(1 - e^{-t/5})$$

$$\left[\because I = \frac{100\ \text{V}}{2\,\Omega}\right]$$

$\therefore \quad i_{\text{steady state}} = 50 \text{ A}; \quad i_{\text{transient}} = -50\, e^{-t/5} \text{ A}$

After $\quad t = 5 \text{ sec}, \quad i_{\text{transient}} = -50\, e^{-t/5}$

$$= -50 \times \frac{1}{e} = -18.518 \text{ A}$$

Thus i (after 5 sec.)

$$= i_{\text{st. state}} + i_{\text{transient}}$$

$$= 50 - 18.518 = \mathbf{31.482 \text{ A.}}$$

EXAMPLE 8.3 *A series R-L circuit has $R = 25$ ohm and $L = 5$ Henry. A d.c. voltage of 100 V is applied at $t = 0$. Find (a) the equations for charging current, voltage across R and L and (b) the current in the circuit 0.5 second later and (c) the time at which the drops across R and L are same.*

SOLUTION. (*a*) $\dfrac{L}{R}$ = time constant $(T) = \dfrac{5}{25} = \dfrac{1}{5}$ sec.

The charging current is given by

$$i = I\,(1 - e^{-t/T});$$

T being the final steady state current.

Here, $\quad i = \dfrac{100}{25}(1 - e^{-t/(1/5)})$ or $\mathbf{4\,(1 - e^{-5t})\ A}$

Voltage drop across R is

$$v_R = iR = 4 \times 25\,(1 - e^{-5t})$$

$$= \mathbf{100\,(1 - e^{-5t})\ V}$$

Voltage drop across L is,

$$v_L = L\frac{di}{dt}$$

or $\quad v_L = 5 \times \dfrac{d}{dt}[4\,(1 - e^{-5t})] = \mathbf{100\, e^{-5t}\ V}$

(*b*) At $t = 0.5$ sec.,

$$i = 4\,(1 - e^{-5(0.5)}) = 4\,(1 - e^{-2.5}) = \mathbf{3.67\ A.}$$

(*c*) To satisfy the condition of $v_R \equiv v_L$, $v_R = v_L = 50$ V, since applied voltage is 100 V.

$\therefore \quad 50 = L\dfrac{di}{dt} = 100\, e^{-5t}$

or, $\quad 0.5 = e^{-5t}$ or $t = \mathbf{0.139\ sec.}$

EXAMPLE 8.4 *A d.c. voltage of 100 V is applied to a coil having $R = 100\,\Omega$ and $L = 10$ H. What is the value of the current 0.1 sec later the switching on ? What is the time taken by the current to reach half of its final value ?*

SOLUTION. Final current $I = \dfrac{V}{R} = \dfrac{100}{10} = 10$ A.

$$T \text{ (time constant)} = \frac{L}{R} = \frac{10}{10} = 1 \text{ sec.}$$

The charging current is given by

$$i = I\,(1 - e^{-t/T}) \quad \text{or} \quad i = 10\,(1 - e^{-t})$$

The value of current 0.1 sec. later is

$$i = 10\,(1 - e^{-0.1}) = \mathbf{0.95\ A.}$$

Again, when the current will be half of final value,

$$5 = 10\,(1 - e^{-t})$$

or, $\quad 0.5 = (1 - e^{-t}) \quad \therefore\ t = 0.69$ sec.

Hence, after **0.69 sec.** of switching, the current will be just half the final value.

EXAMPLE 8.5 *A coil having resistance of 10 Ω and inductance of 1 H is switched on to a direct voltage of 100 V. Calculate the rate of change of the current (a) at the instant of closing the switch and (b) when $t = L/R$ (c) Also find the steady state value of the current.*

SOLUTION. The charging current,

$$i = \frac{V}{R}(1 - e^{-t/T})$$

where$T$ is the time constant $= \dfrac{1}{10} = 0.1$ sec.

$\therefore \quad i = \dfrac{100}{10}(1 - e^{-t/0.1})$ or $i = 10\,(1 - e^{-10t})$

$\therefore \quad \dfrac{di}{dt} = 100\, e^{-10t}$

Thus the rate of change of current at the instant of closing the switch ($t = 0$) is

$$\frac{di}{dt} = 100\, e^{-10 \times 0} = \mathbf{100\ A/sec.}$$

(*b*) When $t = \dfrac{L}{R} = 0.1$ sec.,

$$\frac{di}{dt} = 100\, e^{-10 \times 0.1} = \mathbf{36.8\ A/sec.}$$

(*c*) The final steady state current

$$i_{\text{st.}} = \frac{100 \text{ V}}{10\,\Omega} = \mathbf{10\ A.}$$

EXAMPLE 8.6 *Determine the voltage at the terminals of a coil having $R = 10\,\Omega$ and $L = 15$ H at the instant when the current is 10 A and increasing @ 5 A/sec. Also find the stored energy in the inductor. Compute the same exercise for the case when the current decreases @ 5 A/sec.*

SOLUTION. In the L-R circuit,

$$E = iR + L\frac{di}{dt}$$

[i = current, E = voltage at coil terminals]

$$= 10i + 15\frac{di}{dt} = 10 \times 10 + 15 \times 5 = \mathbf{175\ V.}$$

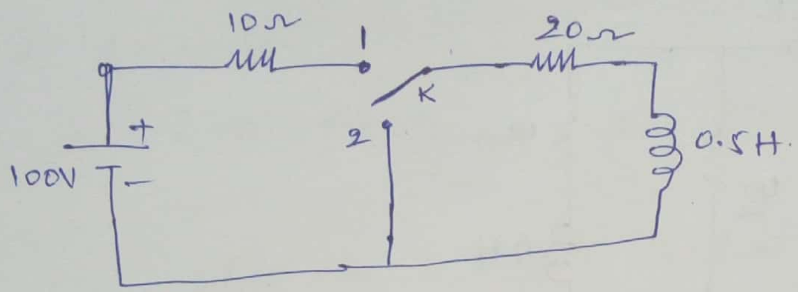
Energy stored $= \frac{1}{2} LI^2 = \frac{1}{2} \times 15 \times 10^2 = \mathbf{750\ J}$

When the current is decaying,

$$E = iR - L\frac{di}{dt} = 10 \times 10 - 15 \times 5 = \mathbf{25\ V.}$$

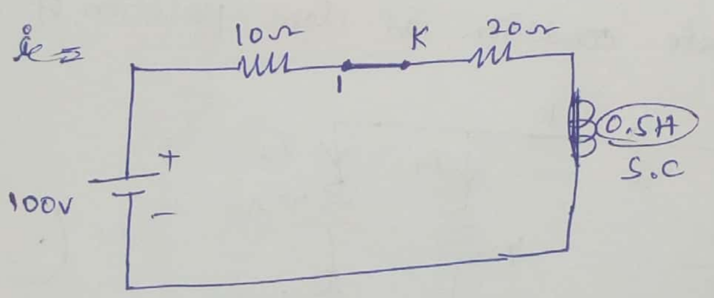
Energy stored $= \frac{1}{2} LI^2 = \frac{1}{2} 15 \times 10^2 = \mathbf{750\ J.}$

⇒ In below figure the switch K is kept first at position 1. and steady state condition is reached. At $t = 0$. switch is moved to position 2. Find the current in both the cases.



sol: when the switch is at position 1, the steady state current becomes,

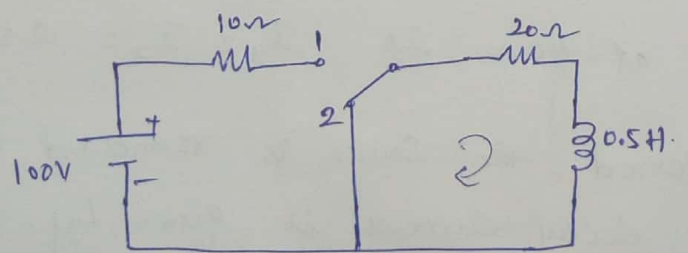
$i =$



(∵ L becomes S.C)

$$I = \frac{100}{(10+20)} = \frac{100}{30} = 3.33\ A.$$

As soon as the switch is moved to position 2 at $t=0$, the RL circuit starts decaying and the decay current is given by



time constant $\tau = \frac{L}{R} = \frac{0.5}{20} = 0.025$ sec.

$$i = I e^{-t/\tau}$$

$$= 3.33\ e^{-t/0.025}$$

$$i = 3.33\ e^{-40t}.$$

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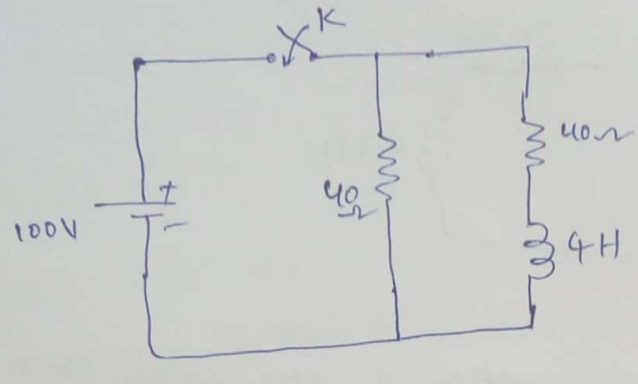
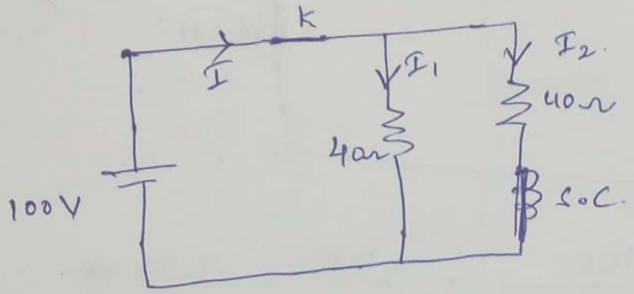
⇒ In figure below steady state condition is reached with 100V dc Source. At $t=0$, switch K is suddenly opened. Find the expression of Current through the inductor after $t = \frac{1}{2}$ sec.



Sol: At steady state condition of close position of switch K.

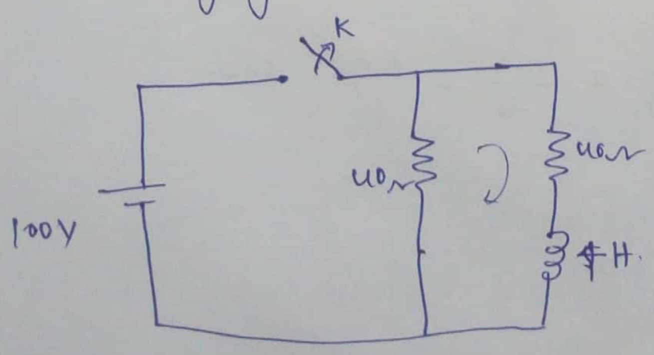


(∵ L is short Circuit)

$$I = \frac{100}{(40 \| 40)} = \frac{100}{20} = 5A.$$

Here I is total current to the circuit. The Currents I_1 & I_2 in the Resistances of 40Ω is $I_1 = I_2 = 2.5A$.

As soon as K is opened, the source is removed and LR ckt starts discharging. The decay current is given by



$$i = I e^{-t/T} \quad (\because I = 2.5A)$$

time constant $T = \frac{L}{R}$

$$= \frac{4}{(40+40)}$$

$$= \frac{4}{80}$$

$$T = 0.05 \text{ sec.}$$

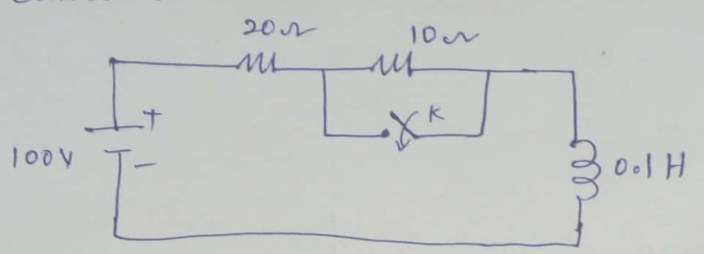
$$i = 2.5\, e^{-t/0.05}$$

$$= 2.5\, e^{-20t}$$

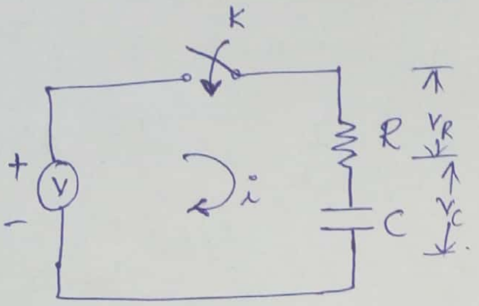
at $t = \frac{1}{2}$ sec

$$i = 2.5\, e^{-20 \times \frac{1}{2}} = 2.5 \times e^{-10} = 1.14 \times 10^{-4}\ A$$

⇒ A dc Voltage of 100 V is applied in the circuit (fig below) and the switch K is open. The switch K is closed at $t=0$. Find the complete expression for the current.



⇒ Transient Response of Series RC Circuit having DC Excitation (first order circuit)



Let a dc voltage V be applied (at $t=0$) by closing a switch K in a Series RC circuit.

$$\left(\because V_C = \frac{1}{C}\int i\,dt \right.$$
$$\left. V_R = iR \right)$$

apply KVL to the loop.

$$V_R + V_C = V$$

$$Ri + \frac{1}{C}\int i\,dt = V$$

Differentiation of above expression w.r.t ~~with~~ t.

$$R\frac{di}{dt} + \frac{i}{C} = 0 \longrightarrow (1)$$

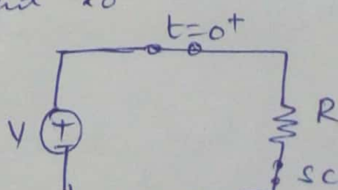
Eq (1) is a homogeneous differential equation whose solution will contain only complementary function. their is no particular solution.

$$i = i_c = K e^{-t/RC} \quad — (2)$$

where K is a constant.

with application of voltage and assuming no initial charge across the capacitor, the capacitor will not produce any voltage across it but acts as a short circuit causing the current to the circuit to be $\left(\frac{V}{R}\right)$.

i.e at $t = 0^+$, $i(0^+) = \frac{V}{R}$.



from Eq (2)

$$\frac{V}{R} = K e^{-\left(\frac{1}{RC}\right)(0)}$$

$$\frac{V}{R} = K e^{-0} \qquad (\because e^0 = 1)$$

$$K = \frac{V}{R}.$$

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Finally we obtain that $\boxed{i = \frac{V}{R} e^{-t/RC}}$ A — (3)

Voltage drop across the resistor $V_R = i R$

$$= \frac{V}{\cancel{R}} \cdot e^{-t/RC} \cdot \cancel{R}$$

$$\boxed{V_R = V e^{-t/RC}} \rightarrow (4)$$

- Voltage drop across the Capacitor

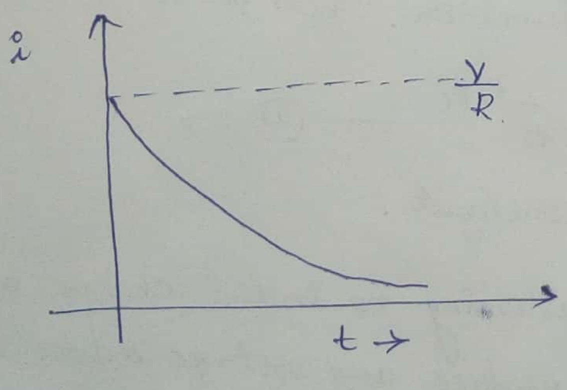
$$V_C = \frac{1}{C} \int i\,dt$$

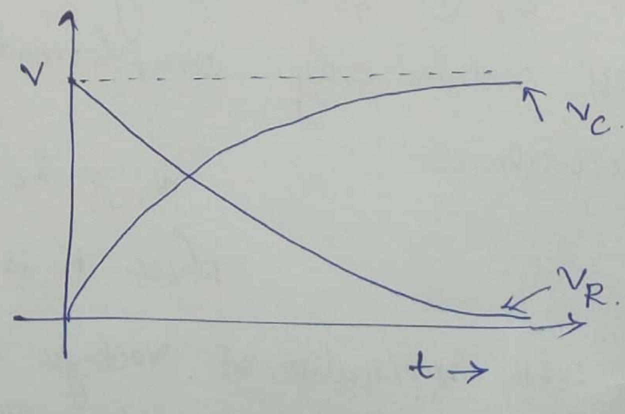
$$= \frac{1}{C} \int \frac{V}{R} e^{-t/RC} \cdot dt$$

$$= \frac{V}{RC} \int e^{-t/RC} \cdot dt$$

$$\boxed{V_C = V (1 - e^{-t/RC})} \longrightarrow (5)$$

Observing Eq's (3), (4) & Eq (5) as shown in figures below.





The time constant of RC Circuit $\boxed{\tau = RC}$ .. Sec.

The instantaneous powers are given by

$$P_R = i V_R = \left(\frac{V}{R} \cdot e^{-t/RC}\right)\left(V e^{-t/RC}\right) = \frac{V^2}{R} e^{-2\frac{t}{RC}} \text{ Watt.}$$

and

$$P_C = i V_C = \left(\frac{V}{R} e^{-t/RC}\right)\left[V(1 - e^{-t/RC})\right]$$

$$= \frac{V^2}{R}\left(e^{-t/RC} - e^{-2t/RC}\right) \text{ Watt}$$

the capacitor is charging. The charge stored in capacitor is

$$q = CV_C = CV(1 - e^{-t/RC})$$

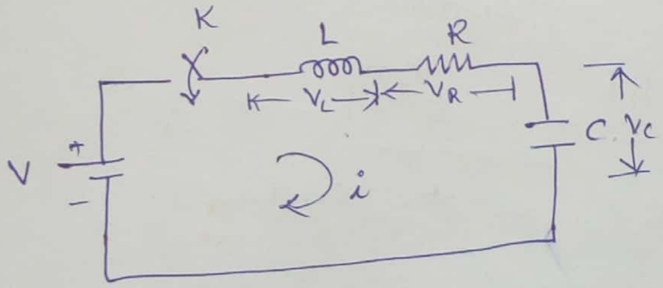
$$\boxed{q = Q(1 - e^{-t/RC})} \text{ columbs } (\because Q = CV).$$

While that capacitor is discharging. The charge stored in capacitor

$$q = CV_C = C.V e^{-t/RC}$$

$$\boxed{q = Q e^{-t/RC}} \text{ columbs. } (\because Q = CV).$$

⇒ Transient Response of series RLC Circuit with DC Excitation (Second order circuit).



in the Series RLC Circuit $t=0^+$ after the switch K is closed,

Apply KVL to the loop.

$$P = \frac{d}{dt}$$

$$V = V_R + V_L + V_C$$

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

By differentiation

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0.$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0. \rightarrow (\times)$$

$$i\left(P^2 + \frac{R}{L}P + \frac{1}{LC}\right) = 0.$$

$$P^2 + \frac{R}{L}P + \frac{1}{LC} = 0 \rightarrow (1)$$

from Eq (1) shows like is a Second order, lineal homogeneous differential equation.

The roots of the above Equation is,

$$P_1, P_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4/LC}}{2}$$

~~Let~~ $\cancel{\alpha =}$

$$= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{4}{4LC}}$$

$$= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - 1/LC}$$

Let $\alpha = \frac{-R}{2L}$, $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - 1/LC}$.

Hence $P_1 = \alpha + \beta$, $P_2 = \alpha - \beta$.

Also, the solution of differential equation (1) becomes

$$\boxed{i = C_1 e^{P_1 t} + C_2 e^{P_2 t}} \longrightarrow (2)$$

where C_1 & C_2 are the constants.

Case 1: when $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$.

This time β is positive real quantity. Hence, the roots are P_1 & P_2 are real but not equal. ($P_1 \neq P_2$). give the over damping response.

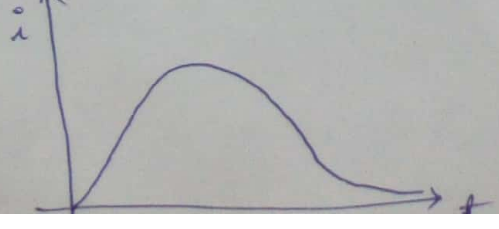
$P_1 = \alpha + \beta$, $P_2 = \alpha - \beta$.

$$i = C_1 e^{(\alpha+\beta)t} + C_2 e^{(\alpha-\beta)t}$$

$$= C_1\left(e^{\alpha t} * e^{\beta t}\right) + C_2\left(e^{\alpha t} * e^{-\beta t}\right)$$

$$= \cancel{e^{\alpha t}(C_1 + C_2)}\ e^{\alpha t}\left(C_1 e^{\beta t} + C_2 e^{-\beta t}\right).$$

the Current response is overdamped



Case 2: when $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$.

This time β is imaginary and then the roots P_1 & P_2 are Complex conjugates.

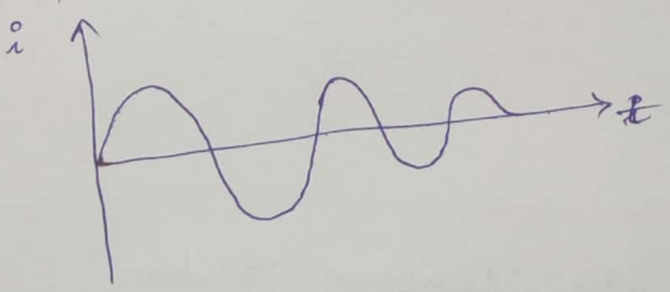
$$P_1 = \alpha + j\beta, \quad P_2 = \alpha - j\beta.$$

$$i = C_1 e^{(\alpha+j\beta)t} + C_2 e^{(\alpha-j\beta)t}$$

$$= e^{\alpha t}\left(C_1 e^{j\beta t} + C_2 e^{-j\beta t}\right).$$

$$= e^{\alpha t}\left[C_1 \cos(\beta t) + C_2 \sin(\beta t)\right]$$

The Current response is undamped.



Case 3: when $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$.

This time β is Zero

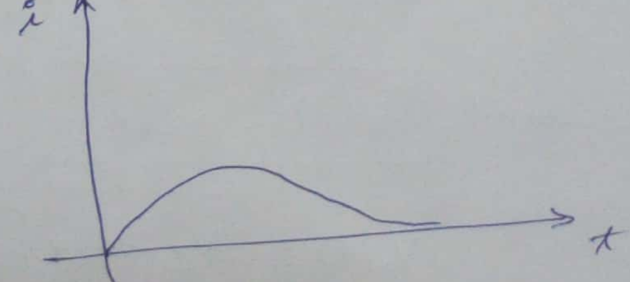
Hence roots P_1 & P_2 are real and equal

$$P_1 = P_2 = \alpha.$$

$$i = C_1 e^{\alpha t} + C_2 e^{\alpha t} . t$$

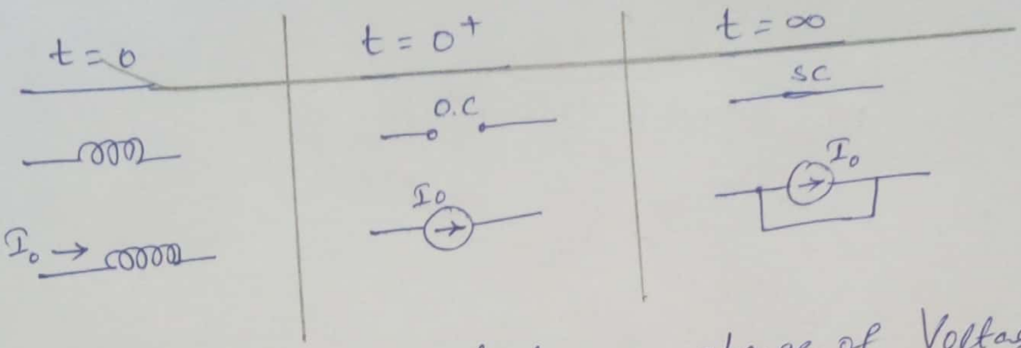
$$i = e^{\alpha t}\left(C_1 + t C_2\right).$$

The Current response is Critically damped

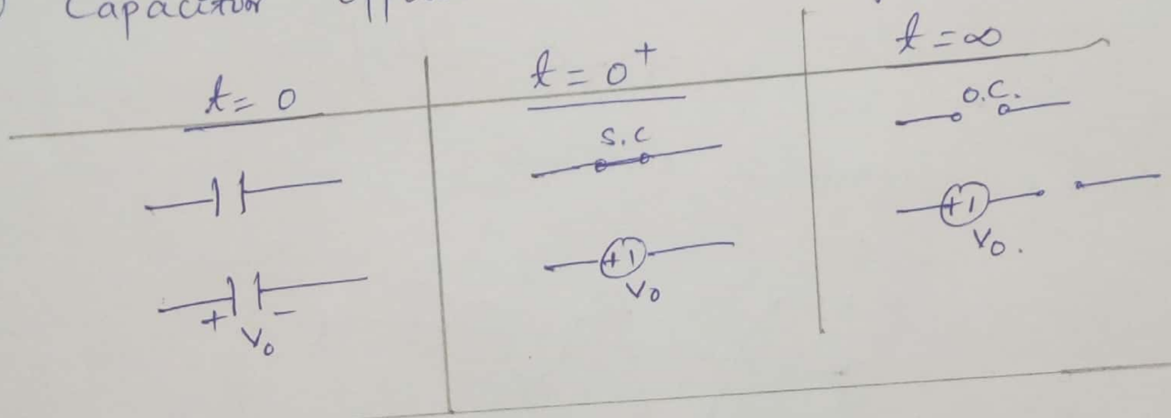


⇒ Behaviour of Circuit elements :

① inductor opposes instantaneous change of current.



② Capacitor opposes instantaneous change of Voltage



Steps to solve Transient problems

① Draw the circuit at time $t = 0^+$

② find initial conditions : $V_c(0^+)$ (or) $I_L(0^+)$.

③ Find time constant of the circuit $(t \to \infty)$

$$\underbrace{\tau = \frac{L_{eq}}{R_{eq}}}_{RL \text{ circuit}} \quad (\text{or}) \quad \underbrace{\tau = R_{eq} * C_{eq}}_{RC \text{ circuit}}$$

④ Find Final values of V_c (or) I_L (at $t \to \infty$)

⑤ total solution = Final Value + (initial value − Final Value) $e^{-t/\tau}$

inductor Current $\boxed{I_L(t) = I_L(\infty) + \left[I_L(0) - I_L(\infty)\right] e^{-t/\tau}}$

Capacitor voltage. $\boxed{V_c(t) = V_c(\infty) + \left[V_c(0) - V_c(\infty)\right] e^{-t/\tau}}$

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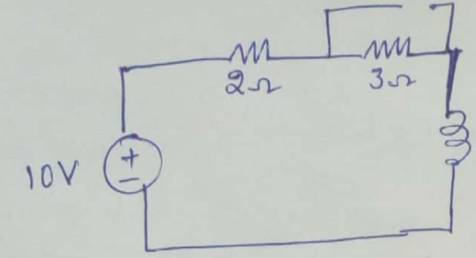
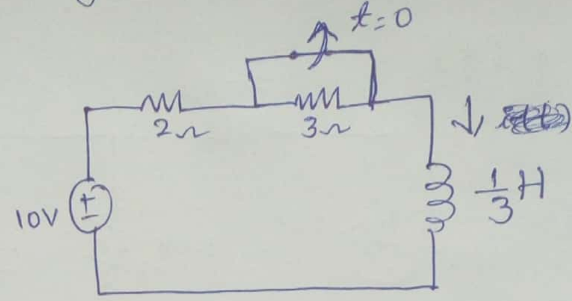
⇒ find the complete expression for $i(t)$.



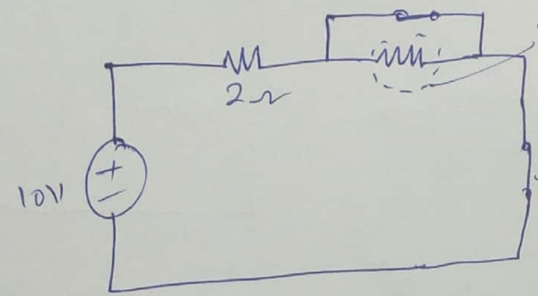


sol: the complete expression

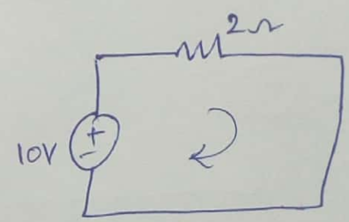
$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/T}$$

at $t=0$: Switch is closed. & L is acts as a short ckt.

neglected the 3Ω Resistor.

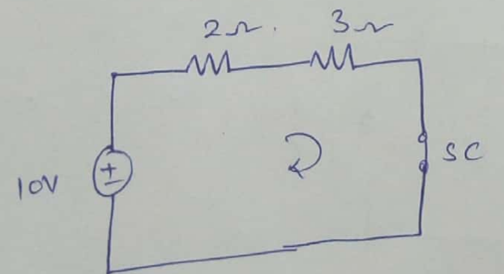


⇒



$$i(0) = \frac{10}{2} = 5A.$$

at $t=\infty$: Switch is opened. & L as a S.C.



$$\Rightarrow i(\infty) = \frac{10}{(2+3)} = \frac{10}{5} = 2A.$$

$t=\infty$. after switch open the time constant $T = \frac{L}{R} = \frac{(1/3)}{(2+3)} = \frac{1}{5\times3} = \frac{1}{15}$ sec

The complete expression

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/T}$$

$$= 2 + \left[5 - 2\right] e^{-t/(1/15)}$$

$$\boxed{i(t) = 2 + 3e^{-15t} \text{ A.}}$$

Q18. **A constant voltage of 100 V is applied at t = 0 to a series R-C circuit having R = 5 MΩ and C = 20 μF. Assuming no initial charge to the capacitor, find the expression for i, voltage across R and C.**

Ans:

According to given data, the circuit obtained is shown in figure (1).

Let $i(t)$ be the current through the circuit after the switch is closed at $t = 0$.

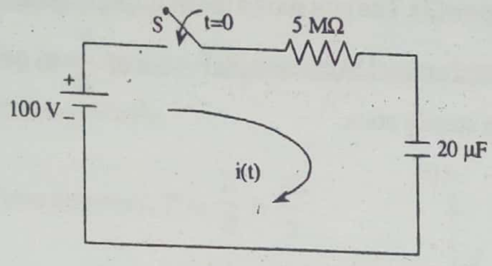


Figure (1)

(i) Expression for Current

Applying KVL to the circuit, we get,

$$100 = 5\times 10^{6} i(t) + \frac{1}{20\times 10^{-6}}\int i(t)dt \quad \ldots(1)$$

Differentiating equation (1), with respect to t, we get,

$$0 = 5\times 10^{6}\frac{di}{dt} + \frac{i}{20\times 10^{-6}}$$

$$\frac{di}{dt} + 0.01i = 0$$

$$(D+0.01)i = 0 \quad \ldots(2)$$

The solution for equation (2) is given by,

$$i(t) = Ce^{-0.01t} \quad \ldots(3)$$

At $t = 0^+$, i.e., immediately after closing the switch, capacitor behaves as short circuit as shown in figure (2).

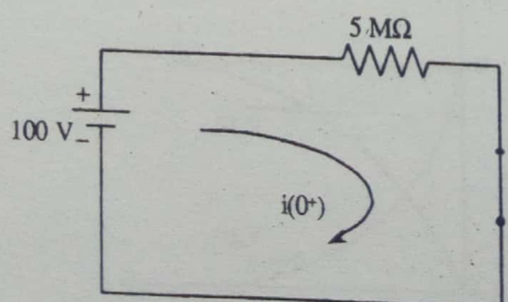


Figure (2)

The current through the circuit is now given as,

$$i(0^+) = \frac{100}{5\times 10^{6}}$$

$$= 20\times 10^{-6}\text{ A}$$

Substituting the value of $i(0^+)$ in equation (3), we get,

At $t = 0^+$,

$$i(0^+) = 20 \times 10^{-6}$$

Now,

$$i(0^+) = C e^0$$

$$C = i(0^+)$$

$$C = 20 \times 10^{-6}$$

Substituting the value of C in equation (3), we get,

$$i(t) = 20 \times 10^{-6} e^{-0.01t}$$

(ii) Voltage Across R

$$V_R = i(t)R$$

$$= 20 \times 10^{-6} e^{-0.01t} \times 5 \times 10^6$$

$$= 100 e^{-0.01t}$$

(iii) Voltage Across C

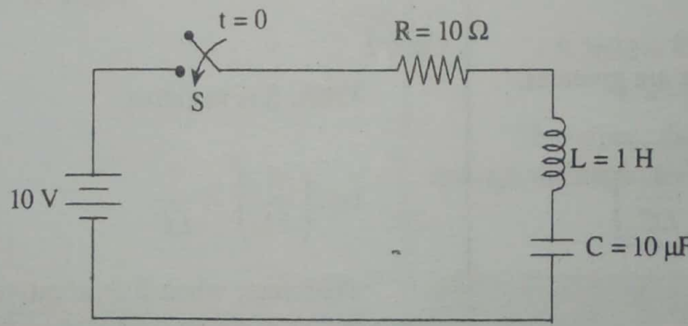
$$V_C = V - V_R$$

$$= 100 - 100e^{-0.01t}$$

$$= 100(1 - e^{-0.01t})$$

...work shown in figure the switch in posi-

Q24. **For the circuit shown in figure, determine $i(0^+)$, $\frac{di}{dt}(0^+)$ and $\frac{d^2i}{dt^2}(0^+)$.**



Ans:

The given circuit is shown in figure (1).

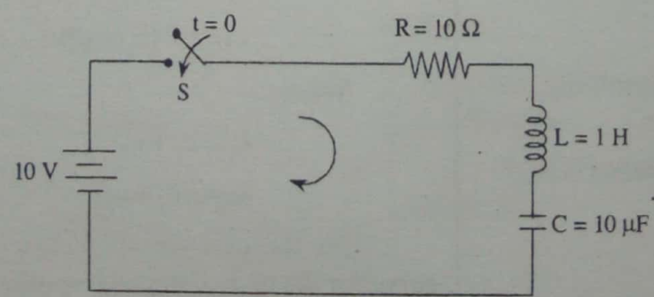


Figure (i)

To determine,

$i(0^+) = ?$

$\frac{di}{dt}(0^+) = ?$

$\frac{d^2i}{dt^2}(0^+) = ?$

Applying KVL to the loop, we get,

$$iR + \frac{Ldi}{dt} + \frac{1}{C}\int_{-\infty}^{t} idt = V \qquad \text{... (1)}$$

While finding the initial conditions, $I(0^-)$ and $V_c(0^-) = 0$. Therefore, the inductor is replaced by open circuit and capacitor by short circuit as shown in figure (ii).

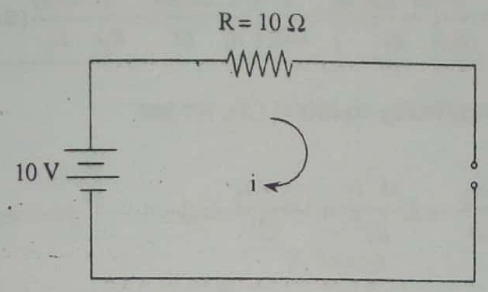


Figure (ii)

$\therefore \quad I(0^+) = 0$

At $t = 0^+$ the voltage across the capacitor = 0

$\therefore$ Equation (1) becomes,

$$iR + \frac{Ldi}{di} = V$$

$$\frac{di}{dt} = \frac{V - iR}{L}$$

$$\left.\frac{di}{dt}\right|_{t=0^+} = \frac{V}{L} \qquad [\because I = 0 \text{ at } t = 0^+]$$

$$\frac{di}{dt} = \frac{10}{1} = 10 \text{ Amp/sec}$$

To determine the second derivative, differentiate the equation (1),

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{c} = \frac{dv}{dt}$$

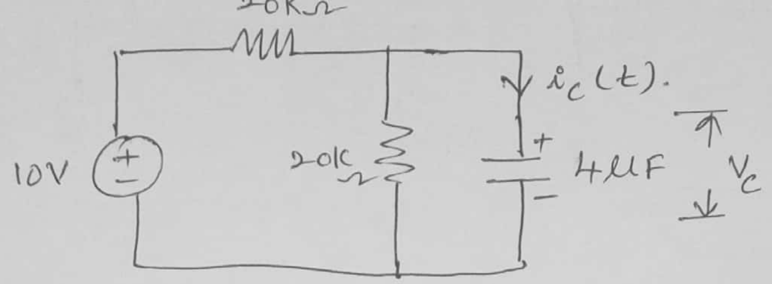
$$L\frac{d^2i}{dt^2} = -R\frac{di}{dt} - \frac{i}{c}$$

$$\frac{d^2i}{dt} = \frac{l}{L}\left(-R\frac{di}{dt} - \frac{i}{c}\right)$$

$$\frac{d^2i}{dt}(0^+) = \frac{1}{1}(-10 \times 10 - 0)$$

$$= -100 \text{ Amp/sec}^2$$

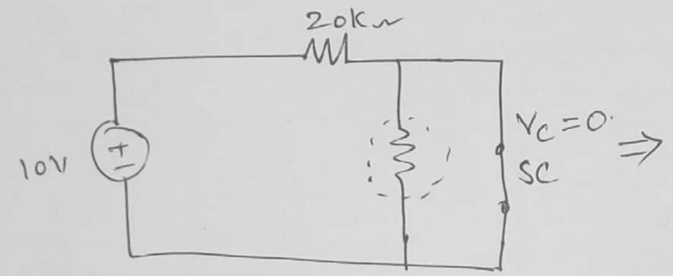
⇒ V_c is zero at time $t = 0$ sec, for $t > 0$, find the capacitor current $i_c(t)$.



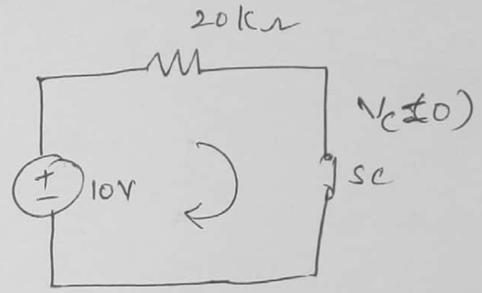
at $t = 0$ is DC steady state condition
$L \rightarrow SC$
$C \rightarrow OC$.
~~at t=0 → SC~~

sol:

at $t = 0^+$. the capacitor is short ckt.

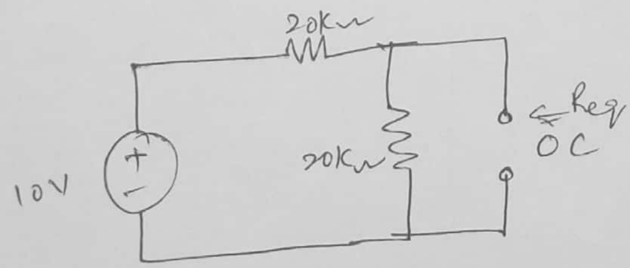


⇒



$$i_c(0^+) = \frac{10}{20K} = 0.5mA.$$

at $t = \infty$ The capacitor is open circuit.



$i_c(\infty) = 0.$

time constant $T = R_{eq}.C = (20||20) . 4 \times 10^{-6}$
$= 10 \times 4 \times 10^{-6} = 40 m sec.$

Capacitor current

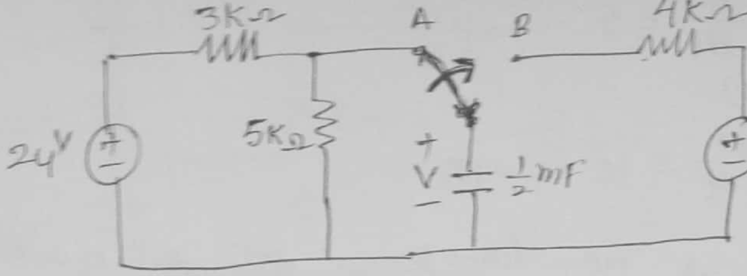
$$i_c(t) = i_c(\infty) + [i_c(0) - i_c(\infty)] e^{-t/T}$$

$$= 0 + [0.5 - 0] e^{-t/T}$$

$$= 0.5 e^{-t/T} = 0.5 e^{-t/40msec}$$

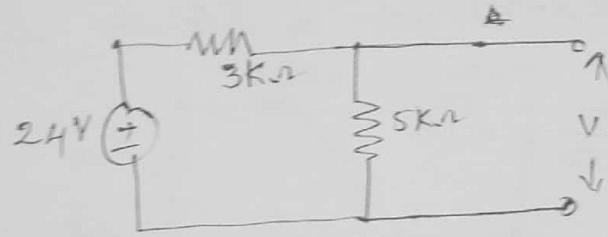
$$\boxed{i_c(t) = 0.5 e^{-25t} A.} \quad t > 0.$$

⇒ find $V(t)$ at $t = 4\,sec$.



Sol: The solution of $V(t) = V(\infty) + [V(0) - V(\infty)]\, e^{-t/T}$.

at $t=0$: The switch at position A. & Capacitor acts as O.C.

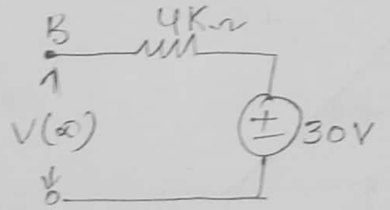


$$I(0) = \frac{24}{8}\ A. = 3\,mA.$$

Voltage at terminals $V(0) = I(0) \times 5K\Omega$

$$= 3 \times 10^{-3} \times 5 \times 10^{3} = 15\ V$$

at $t = \infty$: The switch at position B & capacitor acts as O.C.



$$V(\infty) = 30\,V.$$

Time Constant $T = R \times C = 4 \times 0.5 = 2\,sec.$

$$\therefore\ V(t) = V(\infty) + [V(0) - V(\infty)]\, e^{-t/T}$$

$$= 30 + [15 - 30]\, e^{-t/2}$$

$$= 30 - 15\, e^{-0.5t}$$

at $t = 4\,sec$.

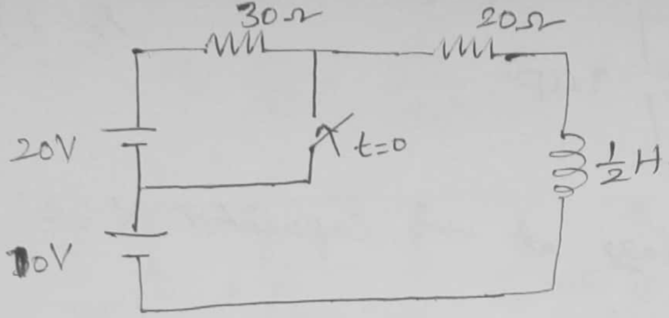
$$V(t) = 30 - 15 \cdot e^{-0.5 \times 4}$$ $(\because e^{-2} = 0.135)$

$$= 30 - 15\, e^{-2}$$

$$= 30 - 15 \times (0.135)$$

$$\boxed{V(t)\Big|_{t=4sec} = 27.97\,V}$$

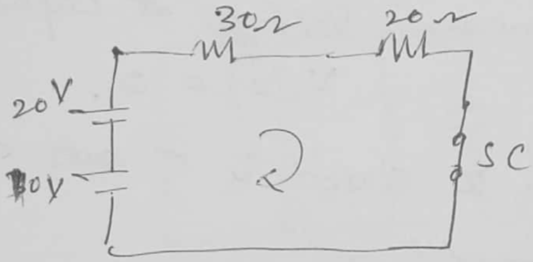
⇒ find the current $i(t)$.



Sol: The general solution of

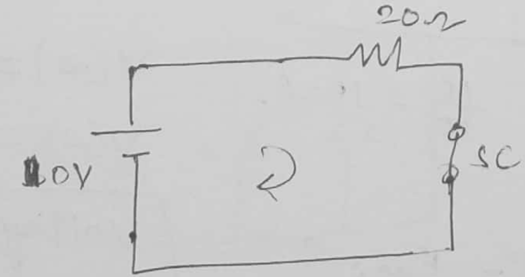
$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/T}$$

at $t=0$ switch is open & L acts as a short circuit.



$$i(0) = \frac{30}{50} = \frac{3}{5} \text{ A},$$

at $t=\infty$ switch is closed & L acts as a short circuit.



$$i(\infty) = \frac{10}{20} = \frac{1}{2} \text{ A.}$$

time constant $T = \frac{L}{R} = \frac{0.5}{20} = \frac{1}{40}$ sec.

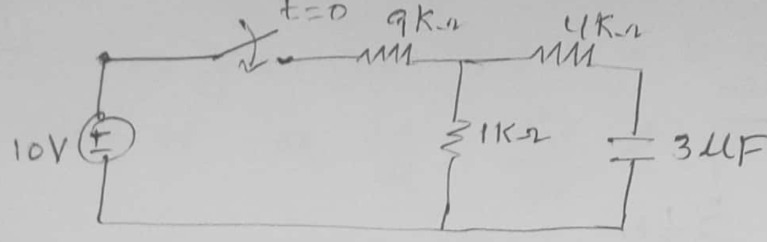
∴ The solution of $i(t)$

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/T}$$

$$= \frac{1}{2} + \left[\frac{3}{5} - \frac{1}{2}\right] e^{-t/(1/40)}$$

$$\boxed{i(t) = \frac{1}{2} + \left[\frac{3}{5} - \frac{1}{2}\right] e^{-40t}} \text{ A.}$$

$\Rightarrow$



find current in Capacitor $i_c(t)$.

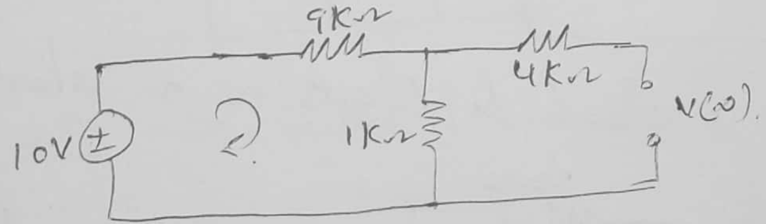
Sol: first find the voltage at the capacitor $V(t)$ and then $i_c = C\frac{dv(t)}{dt}$.

$$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/T}$$

at $t=0$ The switch is open and capacitor is in initial no charge means voltage at capacitor is zero.

$$V(0) = 0.$$

at $t=\infty$: The switch is closed & C acts as O.C.



$V(\infty) = I \times 1K\Omega$

$$I = \frac{10}{(9+1)} = \frac{10}{10} = 1mA.$$

$V(\infty) = 1m \times 1K$

$V(\infty) = 1V$

Voltage Source is S.C
current Source is O.C

time constant $T = R_{eq}\, C$

$= \cancel{\{(9+1)\|4\} \times 3\mu}$

$= \{(9\|1) + 4\}\, 3\mu F$

$= \left(\frac{9\times1}{9+1} + 4\right) 3\mu F = \left(\frac{49}{10}K\right) 3\mu$

$= 14.7\, m\,sec$

$V(t) = \left(1 - e^{-t/T}\right) V$

Current in Capacitor $i_c = C\frac{dv(t)}{dt}$

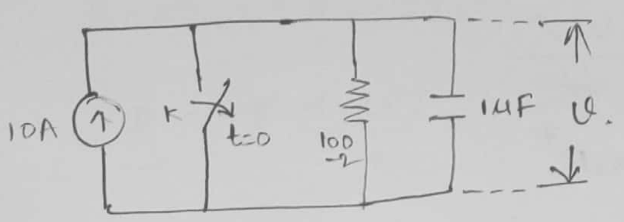
$\because \frac{d}{dt}e^{xt} = x\, e^{xt}$

$= 3\times10^{-6}\frac{d\left(1 - e^{-t/14.7m}\right)}{dt}$

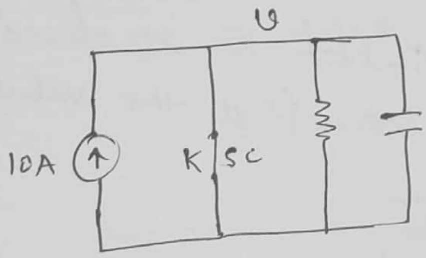
$= -3\times10^{-6} \times \frac{1}{14.7\,m} \times e^{-68t}$ A.

$=$

⇒ In the given CKt switch K is opened at $t=0$. find the Value of v, $\frac{dv}{dt}$, $\frac{d^2v}{dt^2}$ at $t=0^+$.



sol: at $t=0^-$. the switch K is closed

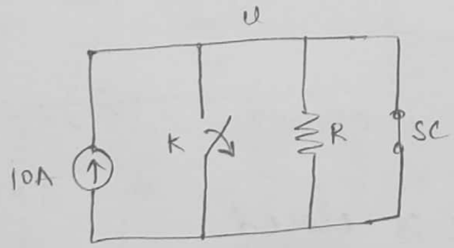


$t=0^-$

$V_c(0^-)=0$

$V(0^-)=0.$

at $t=0^+$ the switch is opened.



$V_c(0^+)=0.$

$v(0^+)=0.$

Nodal analysis apply to the main CKt $t>0$.

$$I = \frac{V}{R} + C\frac{dv}{dt} \quad \text{—(1)}$$

$t=0^+ \rightarrow V = V(0^+)$

$$I = \frac{V(0^+)}{R} + C\frac{d\,V(0^+)}{dt}$$

$$\frac{d\,V(0^+)}{dt} = \frac{I}{C} = \frac{10A}{1\times10^{-6}} = 10^7 \text{ V/sec.}$$

Eq (1) is differentiation,

$$0 = \frac{dv}{dt}\times\frac{1}{R} + C\frac{d^2v}{dt^2}$$

$t=0^+ \rightarrow V = v(0^+)$

$$0 = \frac{1}{R}\cdot\frac{d\,v(0^+)}{dt} + C\,\frac{d^2 v(0^+)}{dt^2}$$

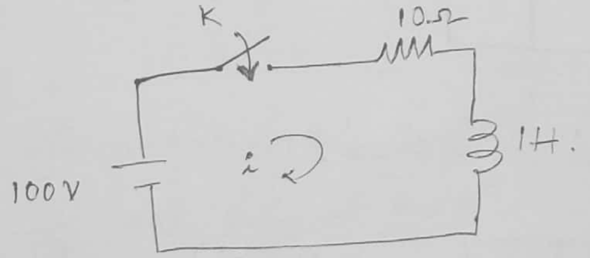
$$\frac{d^2 v(0^+)}{dt^2} = -\frac{1}{RC}\,\frac{d\,v(0^+)}{dt}$$

$$= \frac{-1}{100\times 1\times 10^{-6}} \times 10^{7}$$

$$= -10^{11}\ V/sec^2.$$

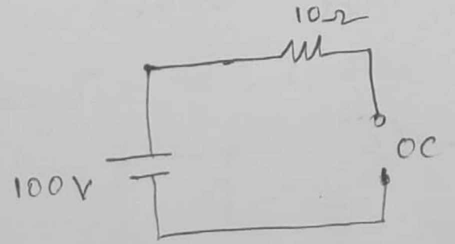
— x —

⇒ In the given network, switch K is closed at $t=0$. with Zero current in the inductor. find the value i, $\frac{di}{dt}$ & $\frac{d^2 i}{dt^2}$ at $t=0^+$.



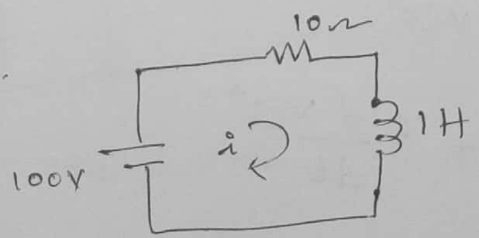
Sol. at $t=0^-$ The switch K is opened. $i(0^-)=0$.

at $t=0^+$ The switch K is closed.



$i(0^+) = 0.$

at $t>0.$



KVL apply to the ckt. $V = iR + L\frac{di}{dt} \rightarrow ①$

$t=0^+ \rightarrow i = i(0^+)$

$$V = R\,i(0^+) + L\,\frac{d\,i(0^+)}{dt}$$

$$\frac{d\,i(0^+)}{dt} = \frac{V}{L} = \frac{100}{1} = 100\ A/sec.$$

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Eq ① is differentiation

$$0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2}$$

$$t = 0^+ \rightarrow i = i(0^+)$$

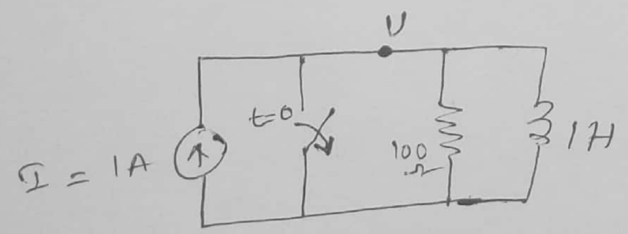
$$0 = R\frac{d\,i(0^+)}{dt} + L\frac{d^2 i(0^+)}{dt^2}$$

$$\frac{d^2 i(0^+)}{dt^2} = \frac{-R}{L}\frac{d\,i(0^+)}{dt}$$

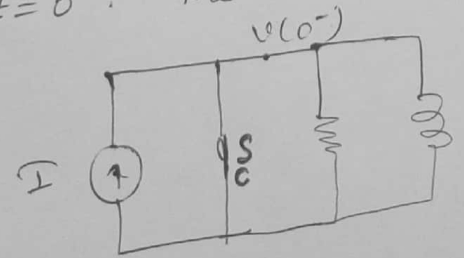
$$= -\frac{10}{1} \times 100$$

$$= -1000 \text{ A/sec}^2 \text{ //.}$$

⇒ In the given network, switch K is opened at $t=0$. solve for V, $\frac{dv}{dt}$ and $\frac{d^2v}{dt^2}$ at $t=0^+$.

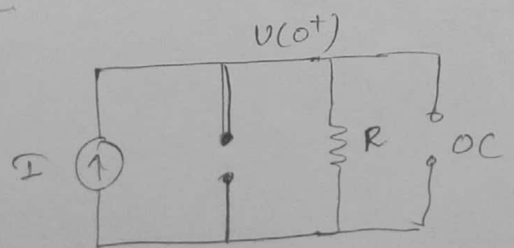


Sol: at $t=0^-$. The switch K is closed.



$v(0^-) = 0.$

at $t=0^+$ The switch K is opened.



$$V(0^+) = I \times R$$

$$= 1 \times 100$$

$$= 100 \text{ V.}$$

Nodal analysis to the main ckt.

$$I = \frac{V}{R} + \frac{1}{L}\int_{-\infty}^{t} V\,dt \rightarrow (*)$$

Differentiation

$$0 = \frac{1}{R}\cdot\frac{dv}{dt} + \frac{v}{L} \rightarrow (1)$$

$t = 0^+ \rightarrow \quad v = v(0^+)$

$$0 = \frac{1}{R}\frac{d\,v(0^+)}{dt} + \frac{v(0^+)}{L}.$$

$$\frac{d\,v(0^+)}{dt} = \frac{-v(0^+)}{L} \times R = \frac{-100 \times 100}{1}$$

$$= -10^4 \ \text{V/sec.}$$

Eq (1) again diffentiation

$$0 = \frac{1}{R}\frac{d^2v}{dt^2} + \frac{1}{L}\frac{dv}{dt}$$

$t = 0^+ \rightarrow \quad v = v(0^+)$

$$0 = \frac{1}{R}\frac{d^2 v(0^+)}{dt^2} + \frac{1}{L}\frac{d\,v(0^+)}{dt}$$

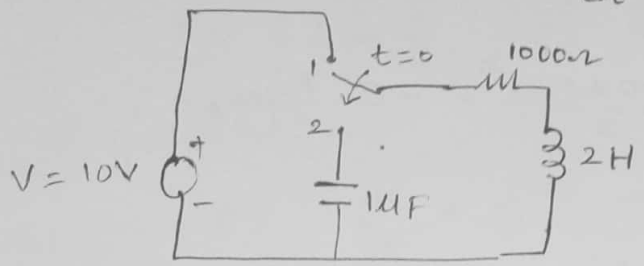
$$-\frac{1}{L}\frac{dv(0^+)}{dt} = \frac{1}{R}\frac{d^2 v(0^+)}{dt^2}$$

$$\Rightarrow \frac{d^2 v(0^+)}{dt^2} = -\frac{R}{L}\frac{d\,v(0^+)}{dt}$$

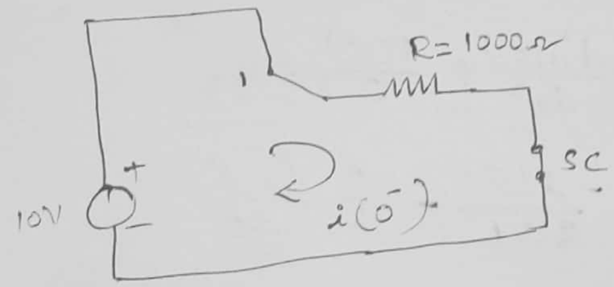
$$= -\frac{100}{1} \times -10^4$$

$$= 10^6 \ \text{V/sec}^2$$

⇒ The switch in below figure is changed from position 1 to 2 at $t=0$. Solve for i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0^+$



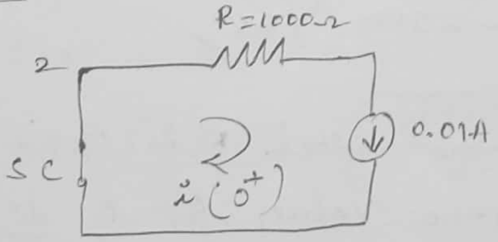
Sol: at $t=0^-$ switch is at position 1.



(∵ L becomes short ckt)

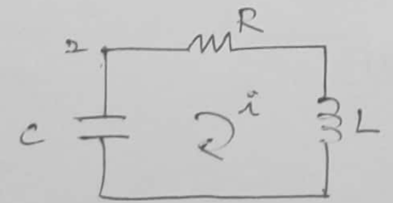
$$i(0^-) = \frac{V}{R} = \frac{10}{1000} = 0.01\,A.$$

at $t=0^+$ The switch is at position 2.



$$i(0^+) = 0.01\,A.$$

at $t>0$. the switch is at position 2.



KVL apply to the ckt

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i\,dt = 0.$$

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{0} i\,dt + \frac{1}{C}\int_{0}^{t} i\,dt = 0$$

$$0.$$

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{0}^{t} i\,dt = 0. \rightarrow ①$$

$$t=0^+ \rightarrow i = i(0^+)$$

$$R\,i(0^+) + L\frac{d\,i(0^+)}{dt} + \frac{1}{C}\int_{0}^{0^+} i(0^+)\,dt = 0$$

$$R\,i(0^+) + L\frac{d}{dt}i(0^+) = 0.$$

$$\frac{d\,i(0^+)}{dt} = -\frac{R}{L}\,i(0^+)$$

$$= -\frac{1000 \times 0.01}{2} = -5 \text{ A/sec}$$

Eq ① again differentiation

$$R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{i}{C} = 0$$

$t = 0^+ \rightarrow i = i(0^+)$

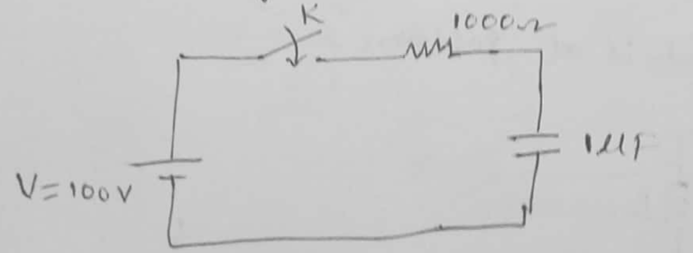
$$R\frac{d\,i(0^+)}{dt} + L\frac{d^2 i(0^+)}{dt^2} + \frac{i(0^+)}{C} = 0.$$

$$\frac{d^2 i(0^+)}{dt^2} = \frac{-i(0^+)}{RCL} - \frac{R}{L}\frac{d\,i(0^+)}{dt}$$

$$= \frac{-0.01}{2 \times 1 \times 10^{-6}} - \frac{1000}{2} \times (-5).$$

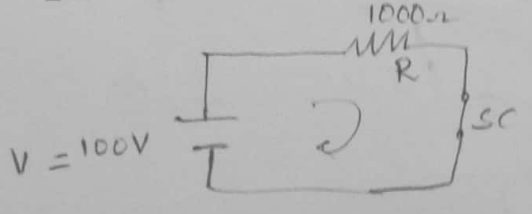
$$= -2500 \text{ A/sec}^2$$

⇒ In the given network, the switch K is closed at $t=0$ with capacitor uncharged. Find the values for i, $\frac{di}{dt}$ & $\frac{d^2 i}{dt^2}$ at $t = 0^+$



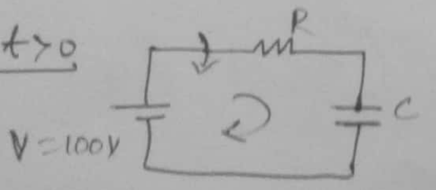
sol: $v_c(0^-) = 0.$

$t = 0^+$ The switch K is closed.



$$i(0^+) = \frac{V}{R} = \frac{100}{1000} = 0.1\text{ A}.$$

at $t > 0$



Apply KVL to the loop

$$V = iR + \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

$$V = iR + \underbrace{\frac{1}{C}\int_{-\infty}^{0} i\,dt}_{0} + \frac{1}{C}\int_{0}^{t} i\,dt.$$

$$V = Ri + \frac{1}{C}\int_{0}^{t} i\,dt$$

Differentiation

$$0 = R\frac{di}{dt} + \frac{1}{C}\times i \rightarrow (1)$$

$$t = 0^+ \rightarrow i = i(0^+)$$

$$R\frac{d\,i(0^+)}{dt} + \frac{i(0^+)}{C} = 0$$

$$\frac{d\,i(0^+)}{dt} = \frac{-i(0^+)}{RC} = \frac{-0.1}{1000\times 1\times 10^{-6}} = -100\ \text{A/sec}.$$

Eq (1) differentiation

$$0 = R\frac{d^2 i}{dt^2} + \frac{di}{dt}\times\frac{1}{C}$$

$$t = 0^+ \rightarrow i = i(0^+)$$

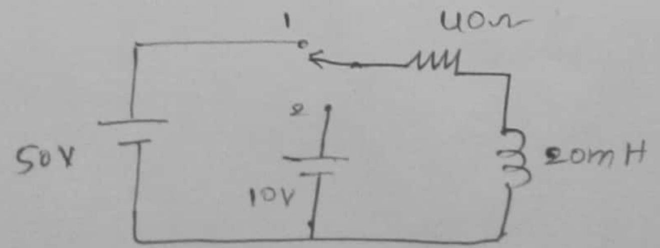
$$R\frac{d^2 i(0^+)}{dt^2} + \frac{1}{C}\frac{d\,i(0^+)}{dt} = 0$$

$$\frac{d^2 i(0^+)}{dt^2} = \frac{-1}{RC}\frac{d\,i(0^+)}{dt}$$

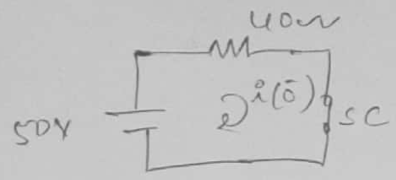
$$= \frac{-1}{1000\times 1\times 10^{-6}}(-100)$$

$$= 10^5\ \text{A/sec}^2$$

⇒ The Network is under steady state with switch at position 1 at $t=0$ switch is moved to position 2. find $i(t)$.



Sol: at $t=0^-$ N/w has attained steady state condition

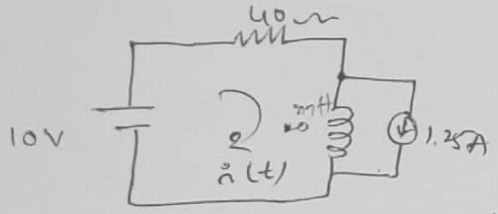


$$i(0^-) = \frac{V}{R} = \frac{50}{40} = 1.25A$$

Current through inductor canot change instanteneously $\therefore i(0^-) = i(0^+) = 1.25A$

at $t>0$

Switch at position 2.



apply KVL to the loop.

$$-10 + 40i + 20\times10^{-3}\frac{di}{dt} = 0.$$

$$\boxed{\frac{di}{dt} + Pi = Q.}$$

$$\frac{di}{dt} + \underbrace{2\times10^{3}}_{P}\, i = \underbrace{500}_{Q.}$$

$$i(t) = \frac{Q}{P} + Ke^{-Pt}.$$

$$= \frac{500}{2\times10^3} + K.e^{-2\times10^3 t}.$$

$$i(t) = 0.25 + Ke^{-2000t}.$$

at $t=0$.

$$i(0) = 0.25 + K.e^{-0}$$

$$i(0) = 0.25 + K \Rightarrow K = i(0) - 0.25 = 1.25 - 0.25$$

$$K = 1$$

$$i(t) = 0.25 + (1)\cdot e^{-2000t}$$

$$\boxed{i(t) = 0.25 + e^{-2000t}}\ A \quad \text{for } t>0.$$

Transients Using Laplace Transforms:

Laplace Transforms of some useful functions.

$L[u(t)] = \frac{1}{s}$, $L[e^{-at}] = \frac{1}{(s+a)}$

$L[\sin\omega t] = \frac{\omega}{(s^2+\omega^2)}$, $L[e^{-at} \cdot t^n] = \frac{n!}{(s+a)^{n+1}}$

$L[\cos\omega t] = \frac{s}{(s^2+\omega^2)}$, $L[e^{-at} \cdot \sin\omega t] = \frac{\omega}{(s+a)^2+\omega^2}$

$L[t^n] = \frac{n!}{s^{n+1}}$, $L[e^{-at} \cdot \cos\omega t] = \frac{(s+a)}{(s+a)^2+\omega^2}$

$L[\delta(t)] = 1$, $L[e^{at}] = \frac{1}{(s-a)}$

$$L\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0^+)$$

$$L\left[\int f(t)\right] = \frac{1}{s}\int f(t)dt\Big|_{0^+} + \frac{1}{s}F(s)$$

voltage source $V \rightarrow \frac{V}{s}$, $R \rightarrow R$

$L[v(t)] \longrightarrow V(s)$, $L \rightarrow sL$

current source $(I) \rightarrow \frac{I}{s}$, $C \rightarrow \frac{1}{Cs}$.

$L[i(t)] \longrightarrow I(s)$. ,

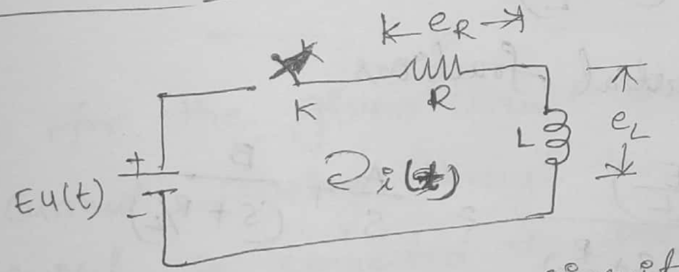
$$v(t) = L\frac{di(t)}{dt} \xrightarrow{L.S} V(s) = L\left[sI(s) - i(0^+)\right]$$

$$v(t) = V_0 + \frac{1}{C}\int i(t)dt \xrightarrow{L.S} V(s) = \frac{V_0}{s} + \frac{1}{C}\cdot\frac{I(s)}{s}.$$

Applications of Laplace transform to Electric Circuit:

1. it is a very powerful method solving linear differential equation
2. Transient behaviour of electric circuit can be easily determine.
3. This method automatically take care of initial condition.
4. It also provide solution of non-homogeneous differential equation.

Series R-L Circuit:



Consider a series R L circuit with R Ω & L H which is supplied by DC voltage with a unit step.

Voltage across resistor $e_R = iR$.

Voltage across inductor $e_L = L\frac{di}{dt}$.

By applying KVL for the given circuit switch is closed.

$$Eu(t) = e_R + e_L$$

$$Eu(t) = i(t)R + L\frac{di(t)}{dt} \quad - (1)$$

apply Laplace transform to Eq (1).

$$\frac{E}{s} = R\,I(s) + L\left[s\,I(s) - i(0)\right]$$

Here $i(0)$ indicates the initial value of the current which is zero. $(\because i(0) = 0)$

$$\frac{E}{S} = I(s).R + L[sI(s) - 0]$$

$$= I(s)R + Ls\,I(s).$$

$$\frac{E}{s} = I(s)[R + Ls]$$

$$I(s) = \frac{E/s}{R + Ls.}$$

$$= \frac{E}{s\left(s + \frac{R}{L}\right)L.}$$

$$I(s) = \frac{E/L}{s\left(s + \frac{R}{L}\right)}$$

by using partial fractions

$$I(s) = \frac{\left(\frac{E}{L}\right)}{s\left(s + \frac{R}{L}\right)} = \frac{A}{S} + \frac{B}{(S + R/L)}$$

$$A = \left.\frac{\left(\frac{E}{L}\right)}{(s + R/L)}\right|_{s=0} = \frac{E/L}{0 + \frac{R}{L}} = \frac{E}{R}.$$

$$B = \left.\frac{E/L}{s}\right|_{s=-R/L} = \frac{E/L}{(-R/L)} = \frac{-E}{R}.$$

Thus

$$I(s) = \frac{\left(\frac{E}{R}\right)}{s} - \frac{(E/R)}{(s + R/L)}$$

taking inverse Laplace transform.

$$i(t) = \left(\frac{E}{R} - \frac{E}{R}e^{-\frac{R}{L}t}\right) \quad A_{//}$$

Voltage across Resistor

$$e_R = i(t) \cdot R$$

$$= \frac{E}{\cancel{R}}\left(1 - e^{-R/L\, t}\right) \cdot \cancel{R}$$

$$= E\left(1 - e^{-R/L\, t}\right) V$$

Voltage across inductor

$$e_L = L \frac{di(t)}{dt}$$

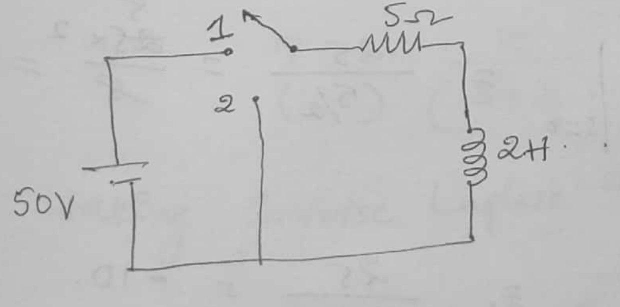
$$= L \frac{d}{dt} \frac{E}{R}\left(1 - e^{-R/L \cdot t}\right)$$

$$= \cancel{L} \cdot \frac{E}{\cancel{R}} \cdot \frac{\cancel{R}}{\cancel{L}} \cdot e^{-R/L\, t}$$

$$e_L = E \cdot e^{-R/L\, t}\ V.$$

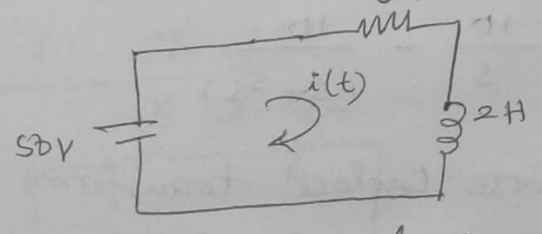
---

① For the given circuit shown in fig. calculate current flowing through the circuit when the switch is connected to position 1 & position 2.



Sol:

Case ①: when switch is connected to position 1.



apply KVL to the loop.

$$50 = 5\, i(t) + 2 \frac{d\, i(t)}{dt}$$

taking Laplace transform on both sides

$$L[50] = L\left[i(t).R + 2\frac{d\,i(t)}{dt} \right]$$

$$50\, L[1] = R\, L[i(t)] + 2\, L\left[\frac{d}{dt} i(t)\right]$$

$$50 \times \frac{1}{S} = R\, I(S) + 2\left[S\, I(S) - i(0) \right] \quad (\because i(0)=0)$$

$$\frac{50}{S} = R\, I(S) + 2S\, I(S)$$

$$= I(S)\left[5 + 2S\right]$$

$$I(s) = \frac{50/s}{(5+2s)} = \frac{\cancel{50}}{s\left(s+\frac{5}{2}\right)\cancel{2}}$$

$$I(s) = \frac{25}{s\left(s+\frac{5}{2}\right)}$$

By using Partial fractions

$$I(s) = \frac{25}{s\left(s+\frac{5}{2}\right)} = \frac{A}{S} + \frac{B}{(S+5/2)}$$

$$A = \left.\frac{25}{\left(S+\frac{5}{2}\right)}\right|_{S=0} = \frac{25}{(5/2)} = \frac{\cancel{25}^{5} \times 2}{\cancel{5}} = 10.$$

$$B = \left.\frac{25}{S}\right|_{S=\frac{-5}{2}} = \frac{25}{\left(-\frac{5}{2}\right)} = -10.$$

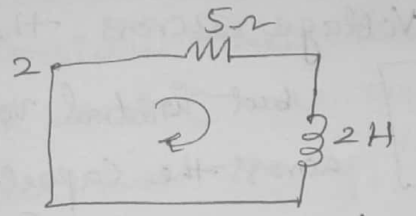
$$I(s) = \frac{10}{S} - \frac{10}{(S+5/2)}$$

taking inverse Laplace transform

$$i(t) = 10 - 10.e^{-\frac{5}{2}t}$$

$$= 10\left(1 - e^{-\frac{5}{2}t}\right) A$$

Case ②: when switch is connected to position 2



$$i(0) = \frac{E}{R} = \frac{50}{5} = 10.$$

apply KVL to the loop.

$$5\,i(t) + 2\frac{d}{dt}i(t) = 0$$

apply Laplace transform on both sides

$$5\,I(s) + 2\left[sI(s) - i(0)\right] = 0$$

$$5\,I(s) + 2s\,I(s) - 2\times 10 = 0$$

$$5\,I(s) + 2s.I(s) = 20$$

$$I(s)\left[5 + 2s\right] = 20$$

$$I(s) = \frac{20}{(5+2s)}$$

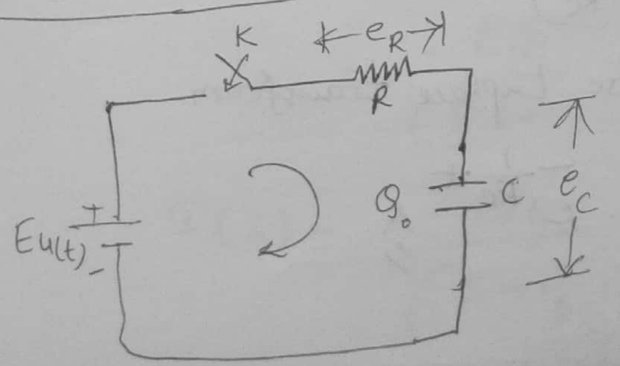
$$= \frac{20}{(s+\frac{5}{2})^2}$$

$$I(s) = \frac{10}{(s+\frac{5}{2})}$$

taking inverse Laplace transform.

$$i(t) = 10.\,e^{-\frac{5}{2}t}\ A$$

⇒ Series RC Circuit:



Q_0 = initial charge on Capacitor.

Let the Capacitance has an initial charge of Q_0 columbs. then initial Voltage across the capacitor V_0 is given by $\boxed{V_0 = \frac{Q_0}{C}}$. but initial voltage across the capacitor is zero. $(\because V_0 = 0)$.

By applying KVL for the loop.

$$E u(t) = e_R + e_C$$

$$= i(t) R + \frac{1}{C}\int i(t) dt + V_0 \quad (\because V_0 = 0)$$

$$E u(t) = i(t) R + \frac{1}{C}\int i(t).dt$$

taking Laplace transform on both sides

$$\frac{E}{s} = R\, I(s) + \frac{1}{C}\left[\frac{I(s)}{s}\right]$$

$$\frac{E}{s} = I(s)\left[R + \frac{1}{Cs}\right]$$

$$I(s) = \frac{E/s}{\left(R + \frac{1}{Cs}\right)} = \frac{E/\cancel{s}}{(RCs+1)\, C\cancel{s}}$$

$$I(s) = \frac{E/C}{(1 + RCs)}$$

$$I(s) = \frac{E/\cancel{C}}{\left(s + \frac{1}{RC}\right) R\cancel{C}} = \frac{E/R}{\left(s + \frac{1}{RC}\right)}$$

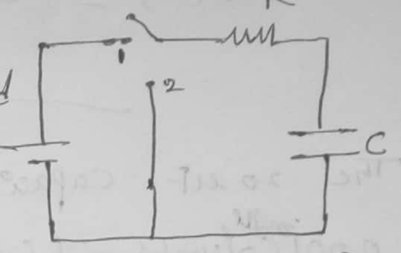
$$\therefore I(s) = \frac{E/R}{\left(s + \frac{1}{RC}\right)}$$

taking inverse Laplace transform

$$i(t) = \frac{E}{R} . e^{-\frac{1}{RC}t} \quad A.$$

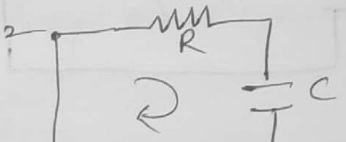
RC decay transients:

consider the circuit with R and C are connected in Series and the applied dc voltage is E. intially



the Switch has been in position 1. for sufficient time. ~~when~~ before switch is move to position 2 the capacitor gets charged to the voltage E.

when Switch is move to position 2.



By applying KVL

$$i(t)R + \frac{1}{C}\int i(t)dt + E = 0.$$

$$i(t)R + \frac{1}{C}\int i(t)dt = -E.$$

apply laplace transfrom on both sides

$$I(s).R + \frac{1}{C}\left(\frac{I(s)}{s}\right) = \frac{-E}{s}$$

$$I(s)\left[R + \frac{1}{Cs}\right] = -E/s.$$

$$I(s) = \frac{-E/s}{\left(R + \frac{1}{Cs}\right)}$$

$$= \frac{-E/\cancel{s}}{(RCs+1).C\cancel{s}}$$

$$= \frac{-E/C}{(RCs+1)} = \frac{-E/\cancel{C}}{\left(s+\frac{1}{RC}\right)R\cancel{C}}$$

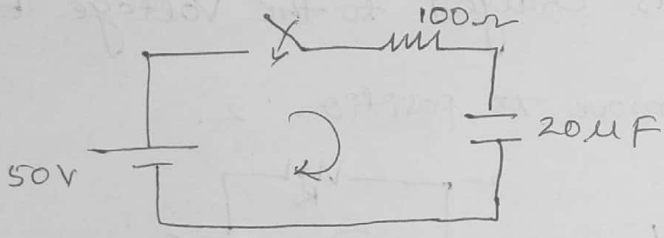
$$I(s) = \frac{-E/R}{\left(s+\frac{1}{RC}\right)}.$$

taking inverse Laplace transform.

$$i(t) = \frac{-E}{R} e^{-\frac{1}{RC}t} \text{ A}$$

---- x ----

$\Rightarrow$ The 20 μF capacitor has initial charge of $q_0 =$ 0.001 milli Columbs which is connected to in series to 100 Ω. The total circuit is supplied by 50V DC. find the Transient Current $i(t)$.



$$V_0 = \frac{q_0}{C} = \frac{0.001 \times 10^{-3}}{20 \times 10^{-6}}$$

$$\boxed{V_0 = 5}$$

Sol: By applying KVL to the loop.

$$R\,i(t) + \frac{1}{C}\int i(t)dt + V_0 = V$$

$$100\,i(t) + \frac{1}{20 \times 10^{-6}} \int i(t).dt + 5 = 50$$

$$100\,i(t) + \frac{10^6}{20} \int i(t)dt = 45$$

apply Laplace transform on both sides

$$100\,I(s) + \frac{10^6}{20}\left(\frac{I(s)}{s}\right) = \frac{45}{s}$$

$$100\,I(s) + 5 \times 10^4 \frac{I(s)}{s} = \frac{45}{s}$$

$$I(s)\left[100 + \frac{5 \times 10^4}{s}\right] = \frac{45}{s}$$

$$I(s) = \frac{(45/s)}{\left(100 + \frac{5 \times 10^4}{s}\right)}$$

$$= \frac{45/s}{(100s + 5 \times 10^4)/s}$$

$$I(s) = \frac{45}{(100s + 5 \times 10^4)}$$

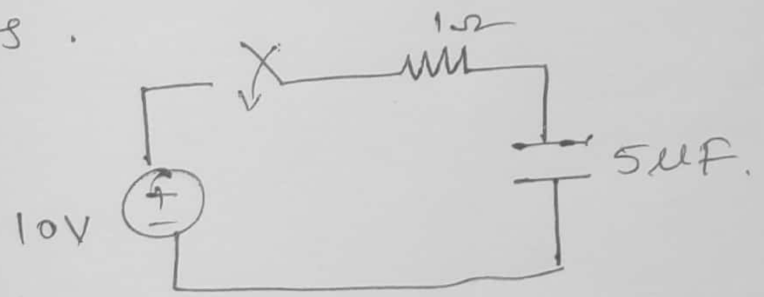
$$I(s) = \frac{45}{100\left(s + \frac{5\times10^4}{100}\right)}$$

$$I(s) = \frac{0.45}{(s+500)}$$

taking inverse Laplace transform

$$i(t) = 0.45 \cdot e^{-500t} \ A$$

⇒ Find $i(t)$ in fig. below following switching at $t=0$. Assume initial charge on capacitor 250 μC colombs.



1.1.3.3 R-L-C Circuits

Q33. Derive the expression for transient response of RLC series circuit with unit step input.

Ans: **Model Paper-I, Q3**

Consider a series RLC circuit shown in figure (1).

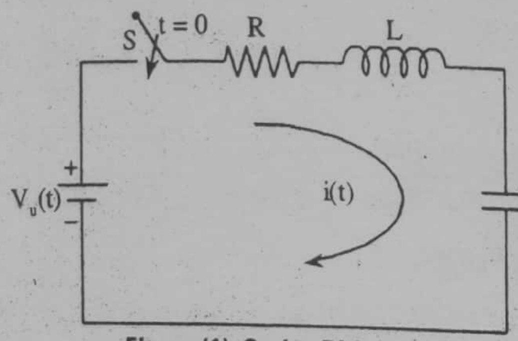


Figure (1): Series RLC Circuit

$V_u(t)$ is a unit step voltage applied to the series RLC circuit.

Let $i(t)$ be the current flowing through the circuit. At $t=0$, the switch is closed. Let us assume that the initial current through inductor and initial voltage across capacitor is zero i.e.,

$$i(0^+) = i(0^-) = 0 \text{ and } V_c(0^+) = V_c(0^-) = 0$$

Applying KVL to the circuit, we get,

$$V_u(t) = R_i(t) + L\frac{di(t)}{dt} + \frac{1}{c}\int i(t)\,dt \quad \text{... (1)}$$

Applying Laplace transform to the equation (1), we get,

$$\frac{V}{S} = RI(S) + L[SI(S) - i(0)] + \frac{1}{C}\left[\frac{I(S)}{S} + \frac{q_0}{S}\right]$$

$$\frac{V}{S} = I(S)\left[R + SL + \frac{1}{CS}\right]$$

($\because$ Initial conditions are zero)

$$\frac{V}{S} = I(S)\left[\frac{RCS + S^2LC + 1}{CS}\right]$$

$$I(S) = \frac{V_c C}{S^2LC + RCS + 1}$$

$$= \frac{V/L}{S^2 + \frac{R}{L}S + \frac{1}{LC}} \quad \text{... (2)}$$

The roots of the above equation are,

$$S_1, S_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$S_1, S_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Based on the relation between $\left(\frac{R}{2L}\right)^2$ and $\frac{1}{LC}$ we get three types of roots and hence, three types of response.

Case (i): $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

When $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$, the roots will be real and unequal and the system is said to be over-damped.

The response of the system can be determined as,

$$I(S) = \frac{V/L}{(S+S_1)(S+S_2)} = \frac{A}{S+S_1} + \frac{B}{S+S_2}$$

$$I(S) = \frac{V/L(S_2-S_1)}{S+S_1} + \frac{V/L(S_1-S_2)}{S+S_2}$$

$$i(t) = \frac{V}{L(S_2-S_1)}[e^{-S_1t} - e^{-S_2t}]$$

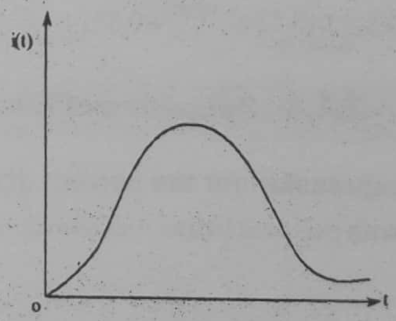


Figure (2)

Figure (2) shows the time response when the system is over-damped.

Case (ii): $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

When $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$, the roots of the system will be real and equal and the system is said to be critically damped.

$$I(S) = \frac{V/L}{(S+S_1)^2}$$

$$i(t) = \frac{V}{L}te^{S_1t}$$

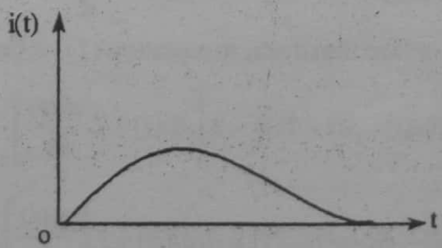


Figure (3)

Figure (3) shows the time response of the critically damped system.

Case (iii): $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

When $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$, the roots of the system will be complex conjugate and the system is said to be under-damped system.

The response of the system can be determined as,

Let the two roots be,

$$S_1 = -A + jB \text{ and } S_2 = -A - jB$$

Where,

$$A = \frac{R}{2L} \text{ and } B = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\therefore \quad I(S) = \frac{V/L}{(S+A-jB)(S+A+jB)}$$

$$I(S) = \frac{V/j2BL}{S+A-jB} - \frac{V/j2BL}{S+A+jB}$$

$$i(t) = \frac{V}{j2BL}e^{-At}e^{jBt} - \frac{V}{j2BL}e^{-At}e^{-jBt}$$

$$= \frac{Ve^{-At}}{BL}\left[\frac{e^{jBt} - e^{-jBt}}{2j}\right]$$

$$i(t) = \frac{Ve^{-At}}{BL}\sin Bt$$

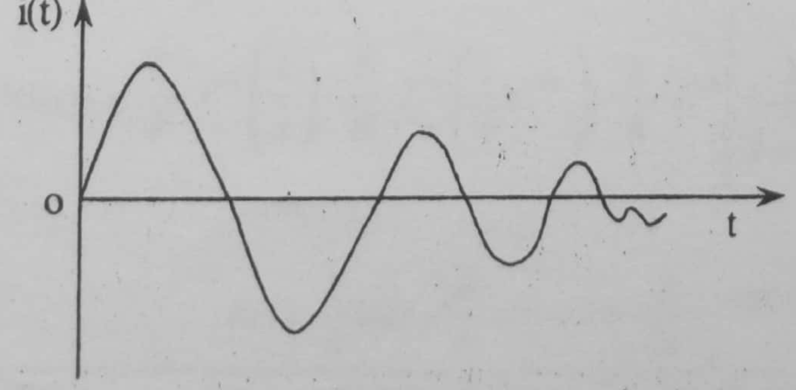


Figure (4)

The response of the under-damped system is shown in figure (4).

⇒ A Series RLC Circuit has $R = 10\Omega$ & $L = 2H$. what is the value of capacitance will make the circuit critically damped.

sol: The circuit is ~~A~~ critically damped at

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

$$\left(\frac{10}{2\times 2}\right)^2 = \frac{1}{2\times C}$$

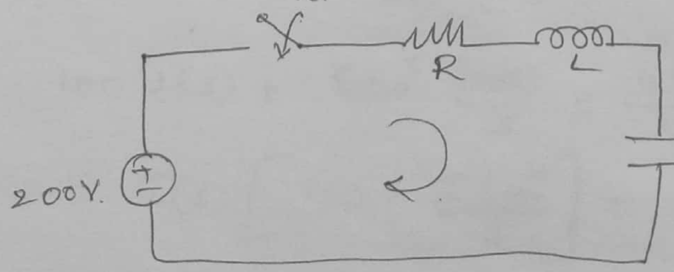
$$\Rightarrow \quad 2C = \left(\frac{4}{10}\right)^2 = \frac{16}{100}$$

$$C = \frac{16^{8}}{100\times \cancel{2}} = \frac{8}{100}$$

$$\boxed{C = 0.08\ F}$$

⇒ A Series RLC Circuit with $R = 100\Omega$ & $L = 0.1H$ and $C = 100\mu F$ has a dc voltage of 200V applied to each at $t = 0$ through a switch find the expression for transient current by assuming initially relaxed circuit condition.

~~sol~~:



initially relaxed means $\boxed{I (or)\ (or)\ V(0) = 0}$

by applying KVL to the loop

$$V = Ri + L\,di/dt + \frac{1}{C}\int i\,dt$$

~~200~~ =

~~(t). 100 +~~

taking Laplace transform on both sides

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$$\frac{V}{S} = RI(s) + LsI(s) + \frac{I(s)}{Cs}$$

$$\frac{V}{S} = I(s)\left[R + Ls + \frac{1}{Cs}\right]$$

$$I(s) = \frac{V/s}{R + Ls + \frac{1}{Cs}} = \frac{V/\cancel{s}}{\cancel{s}\left[Rs + Ls^2 + \frac{1}{C}\right]}$$

$$I(s) = \frac{V}{\left(Rs + Ls^2 + \frac{1}{C}\right)}$$

Substitute ~~d~~ V, R, L, & C values in the above expression

$$I(s) = \frac{200}{\left(100s + 0.1s^2 + \frac{1}{100 \times 10^{-6}}\right)}$$

$$I(s) = \frac{200}{\left(0.1s^2 + 100s + 10^4\right)} \rightarrow (1)$$

$0.1s^2 + 100s + 10^4 = 0$ is 2^{nd} order equation

the roots are

$$s_1, s_2 = \frac{-100 \pm \sqrt{100^2 - 4 \times 0.1 \times 10^4}}{2 \times 0.1}$$

$$= \frac{-100 \pm \sqrt{10{,}000 - 4000}}{0.2}$$

$$= \frac{-100 \pm \sqrt{6000}}{0.2} = \frac{-100 \pm 20\sqrt{15}}{0.2}$$

$$= \cancel{-100 \pm}$$

$$s_1 = -112.70 \text{ \& } s_2 = -887.29.$$

Eq① ⇒ $I(s) = \dfrac{200}{(s+112.70)(s+887.29)}$

by partial fractions

$$\frac{200}{(s+112.70)(s+887.29)} = \frac{A}{(s+112.70)} + \frac{B}{(s+887.29)}$$

$$A = \left.\frac{200}{(s+887.29)}\right|_{s=-112.70}$$

$$= \frac{200}{(-112.70+887.29)}$$

$$\boxed{A = 0.258}$$

$$B = \left.\frac{200}{(s+112.70)}\right|_{s=-887.29}$$

$$= \frac{200}{-887.29+112.70}$$

$$\boxed{B = -0.258}$$

$$I(s) = \frac{0.258}{(s+112.70)} - \frac{0.258}{(s+887.29)}$$

taking inverse Laplace transform.

$$i(t) = \left(0.258\, e^{-112.70t} - 0.258\, e^{-887.29}\right) A$$

⇒ . A Series RLC Circuit with $R = 300\Omega$, $L = 1H$, and $C = 100\mu F$ has voltage $V = 50V$. find the current $i(t)$ by assuming zero initial condition.

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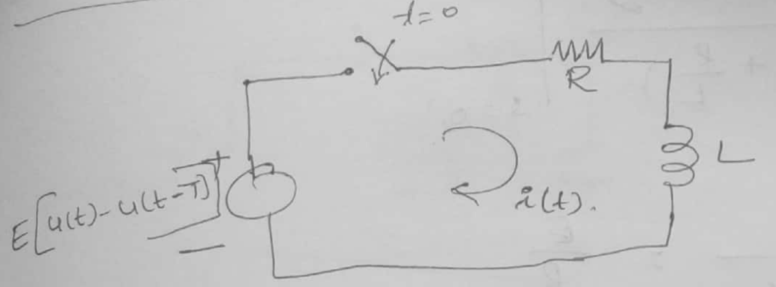
Response of RL Circuit with Pulse Excitation



$i(0^-)=0.$

$L[u(t)] = \frac{1}{s}$

$L[u(t-T)] = \frac{e^{-sT}}{s}.$

apply KVL to the loop.

$$E[u(t)-u(t-T)] = R\,i(t) + L\frac{d}{dt}i(t)$$

apply Laplace transform on both sides.

$$\frac{E}{s} - \frac{E e^{-sT}}{s} = R\,I(s) + L[sI(s) - I(0)]$$

$$= R\,I(s) + Ls\,I(s)$$

$$\frac{E}{s}(1-e^{-sT}) = I(s)[R+Ls]$$

$$I(s) = \frac{\frac{E}{s}(1-e^{-sT})}{R+Ls}.$$

$$= \frac{\frac{E}{s}\cdot(1-e^{-sT})}{\left(s+\frac{R}{L}\right)\cdot L}$$

$$I(s) = \frac{\frac{E}{L}(1-e^{-sT})}{s\left(s+\frac{R}{L}\right)}$$

by using partial fractions

$$\frac{\frac{E}{L}\cancel{(1-e^{-sT})}}{s\left(s+\frac{R}{L}\right)} = \frac{A}{s} + \frac{B}{(s+R/L)}$$

$$A = \left.\frac{\frac{E}{L}\left(1-e^{-sT}\right)}{\left(s+\frac{R}{L}\right)}\right|_{s=0}$$

$$= \frac{E/L}{R/L} = \frac{E}{R}$$

$$\boxed{A = \frac{E}{R}}$$

$$B = \left.\frac{E/L}{s}\right|_{s=-R/L} = \frac{E/L}{-R/L}$$

$$\boxed{B = -\frac{E}{R}}$$

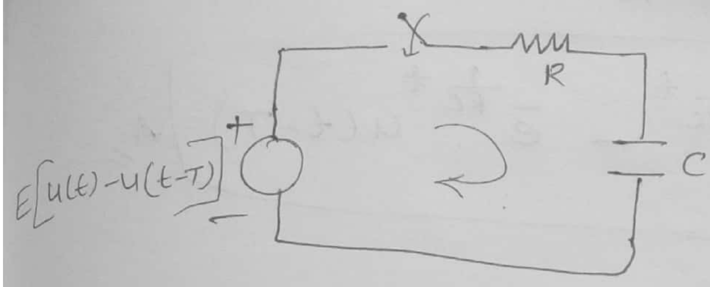
$$I(s) = \left(1-e^{-sT}\right)\left[\frac{\frac{E}{R}}{s} - \frac{E/R}{\left(s+R/L\right)}\right]$$

$$= \frac{E/R}{s} - \frac{E/R}{\left(s+R/L\right)} - \frac{\frac{E}{R}\cdot e^{-sT}}{s} + \frac{\frac{E}{R}\cdot e^{-sT}}{\left(s+R/L\right)}$$

taking Inverse Laplace transform

$$\boxed{i(t) = \frac{E}{R}u(t) - \frac{E}{R}e^{-R/L\,t} - \frac{E}{R}u(t-T) + \frac{E}{R}e^{-R/L\,t}u(t-T)}$$

⇒ Response of RC Circuit with pulse Excitation



No initial charge

$\therefore V_0 = 0.$

apply KVL to the loop.

$$E[u(t)-u(t-T)] = R\,i(t) + \frac{1}{C}\int i(t)\,dt.$$

apply Laplace transform on both sides.

$$\frac{E}{s} - \frac{E e^{-sT}}{s} = R I(s) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{V_0}{s}\right]$$

$$= R I(s) + \frac{I(s)}{Cs}.$$

$$\frac{E}{s}\left(1 - e^{-sT}\right) = I(s)\left[R + \frac{1}{Cs}\right]$$

$$I(s) = \frac{E/s\cdot\left(1-e^{-sT}\right)}{\left(R+\frac{1}{Cs}\right)}$$

$$= \frac{\frac{E}{\cancel{s}}\left(1-e^{-sT}\right)}{\left(RCs+1\right)C\cancel{s}}$$

$$= \frac{\frac{E}{\cancel{C}}\cdot\left(1-e^{-sT}\right)}{\left(s+\frac{1}{RC}\right)R\cancel{C}}$$

$$I(s). = \frac{\frac{E}{R}\left(1-e^{-sT}\right)}{\left(s+\frac{1}{RC}\right)}$$

$$I(s) = \frac{E}{R}\cdot\frac{1}{\left(s+\frac{1}{RC}\right)} - \frac{E}{R}\cdot\frac{e^{-sT}}{\left(s+\frac{1}{RC}\right)}$$

taking Inverse Laplace transform.

$$i(t) = \frac{E}{R} e^{-\frac{1}{RC}t} - \frac{E}{R} e^{-\frac{1}{RC}t} u(t-T)$$

$$\boxed{i(t) = \frac{E}{R}\left(e^{-\frac{1}{RC}t} - e^{-\frac{1}{RC}t} u(t-T)\right) A}$$

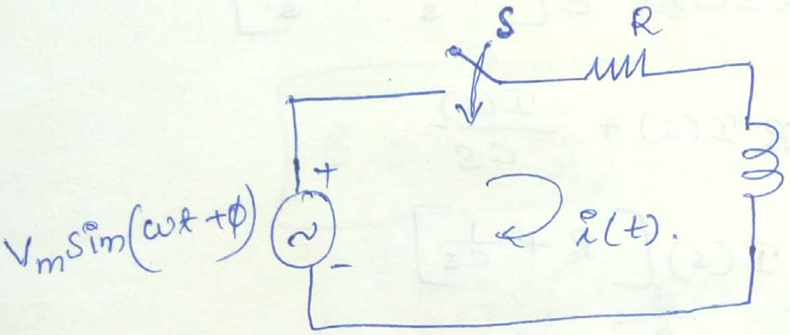
$$\boxed{i(t) = \frac{E}{R}\left(e^{-\frac{1}{RC}t} - e^{-\frac{1}{RC}t}\,u(t-T)\right) A}$$

— x —

AC Transient Analysis

→ **Series R-L Circuit :**

consider the R-L circuit as shown in figure.



when switch is closed, applying KVL to the ckt.

$$R\,i(t) + L\frac{di(t)}{dt} = V_m \sin(\omega t + \phi) \rightarrow (1)$$

For the above differential equation the particular solution is given by

$$i_p = A\left[\cos(\omega t + \phi)\right] + B\sin(\omega t + \phi) \quad -(2)$$

Eq (2) is differentiation,

$$\frac{di_p}{dt} = i_p' = -A\omega\sin(\omega t + \phi) + B\omega\cos(\omega t + \phi).$$

$$Eq\,(1) \Rightarrow R\left[A\cos(\omega t + \phi) + B\sin(\omega t + \phi)\right]$$
$$+ L\left[-A\omega\sin(\omega t + \phi) + B\omega\cos(\omega t + \phi)\right]$$
$$= V_m\sin(\omega t + \phi).$$

$$\sin(\omega t+\phi)\left[BR - LA\omega\right] + \cos(\omega t+\phi)\left[AR + B\omega L\right] = V_m \sin(\omega t+\phi)$$

Equating the co-efficients of like terms, we get

$$BR - A\omega L = V_m \rightarrow (3)$$

$$AR + B\omega L = 0 \rightarrow (4)$$

solving Eq (3) & Eq (4), we get the values of A & B.

$$\text{Eq}(3)\times R \Rightarrow -A\omega L.R + BR^2 = V_m R \rightarrow (5)$$

$$\text{Eq}(4)\times \omega L \Rightarrow \omega LAR + B\omega^2L^2 = 0. \rightarrow (6)$$

$$\text{from Eq}(6) \Rightarrow A\omega LR = -B\omega^2L^2$$

the above value substitute in Eq (5), we get

$$B\omega^2L^2 + BR^2 = V_m.R$$

$$B(R^2+\omega^2L^2) = V_m.R$$

$$B = \frac{V_m R}{(R^2+\omega^2L^2)}.$$

The above value B substitute in Eq (4).

$$AR + \frac{V_m.R.\omega L}{(R^2+\omega^2L^2)} = 0.$$

$$A\cancel{R} = \frac{-V_m.\cancel{R}.\omega L}{(R^2+\omega^2L^2)}$$

$$A = \frac{-V_m.\omega L}{(R^2+\omega^2L^2)}$$

Substitute the values A & B in Eq (2), we get

$$i_p = \frac{-V_m\,\omega L}{(R^2+\omega^2L^2)}\cos(\omega t+\phi) + \frac{V_m R}{(R^2+\omega^2L^2)}\times\sin(\omega t+\phi).$$

$$i_p = \frac{V_m}{(R^2+\omega^2L^2)}\left[R\sin(\omega t+\phi) - \omega L\cos(\omega t+\phi)\right]$$

Let $R = K\cos\theta$

$\omega L = K\sin\theta$

thus

$$i_p = \frac{V_m}{(R^2+\omega^2L^2)}\left[K\cos\theta \cdot \sin(\omega t+\phi) - K\sin\theta \cdot \cos(\omega t+\phi)\right]$$

$$= \frac{V_m K}{(R^2+\omega^2L^2)}\left[\cos\theta \cdot \sin(\omega t+\phi) - \sin\theta \cdot \cos(\omega t+\phi)\right]$$

$$i_p = \frac{V_m K}{(R^2+\omega^2L^2)} \sin(\omega t+\phi-\theta)$$

where

$R^2+\omega^2L^2 = K^2\cos^2\theta + K^2\sin^2\theta$

$= K^2(\sin^2\theta+\cos^2\theta)$

$(R^2+\omega^2L^2) = K^2(1)$

$\Rightarrow K = \sqrt{(R^2+\omega^2L^2)}$

$\frac{\omega L}{R} = \frac{K\sin\theta}{K\cos\theta}$

$\tan\theta = \frac{\omega L}{R}$

$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$i_p = \frac{V_m \cdot \sqrt{R^2+\omega^2L^2}}{(R^2+\omega^2L^2)} \sin(\omega t+\phi-\theta)$$

$$i_p = \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \sin(\omega t+\phi-\theta)$$

the complementary solution of the Eq (1)

$i_c = C e^{-R/L\, t}$

$\therefore$ The total solution of Eq (1)

$i(t) = i_c + i_p$

$$i(t) = c\,e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \sin\left(\omega t+\phi - \tan^{-1}\frac{\omega L}{R}\right) \rightarrow (7)$$

no current in the circuit before the switch was closed hence $i(0) = 0$. then at $t = 0$.

$$i(0) = c\,e^{-\frac{R}{L}\cdot 0} + \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \sin\left(\omega\cdot 0+\phi - \tan^{-1}\frac{\omega L}{R}\right).$$

$$0 = c + \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \sin\left(\phi - \tan^{-1}\frac{\omega L}{R}\right).$$

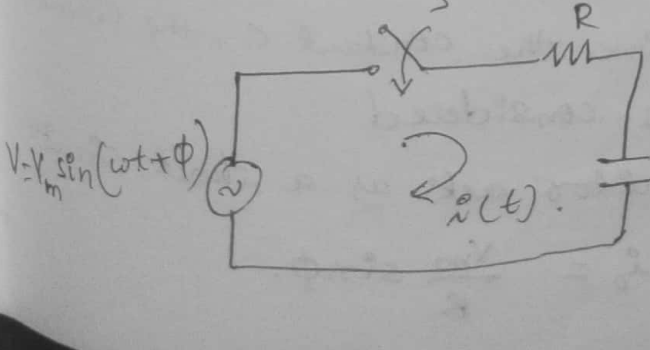
$$c = \frac{-V_m}{\sqrt{R^2+\omega^2L^2}} \sin\left(\phi - \tan^{-1}\frac{\omega L}{R}\right).$$

The above c value substitute in Eq (7) we get

$$i(t) = e^{-\frac{R}{L}t}\left[-\frac{V_m}{\sqrt{R^2+\omega^2L^2}} \sin\left(\phi - \tan^{-1}\frac{\omega L}{R}\right)\right] + \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \sin\left(\omega t+\phi - \tan^{-1}\frac{\omega L}{R}\right).$$

⇒ Series RC Circuit :

Consider a series RC circuit excited by a sinusoidal source as shown in fig. below.



when switch 'S' is closed at $t=0$, we applying KVL to the circuit, we get.

$$R\,i(t) + \frac{1}{C}\int i(t)\,dt = V_m \sin(\omega t + \phi)$$

by using diffentiation to above expression.

$$R\frac{di(t)}{dt} + \frac{i(t)}{C} = V_m \cdot \omega \cdot \cos(\omega t + \phi)$$

$$\frac{di(t)}{dt} + \frac{i(t)}{RC} = \frac{V_m \omega}{R}\cos(\omega t + \phi) \rightarrow (1)$$

The general sollution of Eq (1) is

$$i(t) = i_c + i_p.$$

where $i_c = C e^{-\frac{1}{RC}t}$

$$i_p = A\cos(\omega t + \phi) + B\sin(\omega t + \phi)$$

$$\frac{di_p}{dt} = i_p' = -A\omega \sin(\omega t + \phi) + B\omega \cdot \cos(\omega t + \phi)$$

Substitute the values i_p & $\frac{di_p}{dt}$ in Eq (1) we get

The particular funtion i_p has been obtained in a similar way as been done for the case with R-L circuit under sinusoidal response.

$$i(t) = C e^{-\frac{t}{RC}} + \frac{V_m}{\sqrt{R^2 + (1/\omega C)^2}} \sin\left(\omega t + \phi + \tan^{-1} 1/\omega CR\right) \rightarrow (2)$$

in order to determine the constant C, the circuit condition at $t=0$ is considered

at $t=0$ the capacitor acts as a short circuit

initial current $i_0 = \frac{V_m}{R}\sin\phi.$

Hence setting $t=0$, the complete solution becomes

$$\frac{V_m}{R}\sin\phi = C + \frac{V_m}{\sqrt{R^2+(1/\omega C)^2}}\sin\left(\phi+\tan^{-1}1/\omega CR\right).$$

$$C = \frac{V_m}{R}\sin\phi - \frac{V_m}{\sqrt{R^2+(1/\omega C)^2}}\sin\left[\phi+\tan^{-1}(1/\omega CR)\right]$$

the above c value substitute in Eq ② we get.

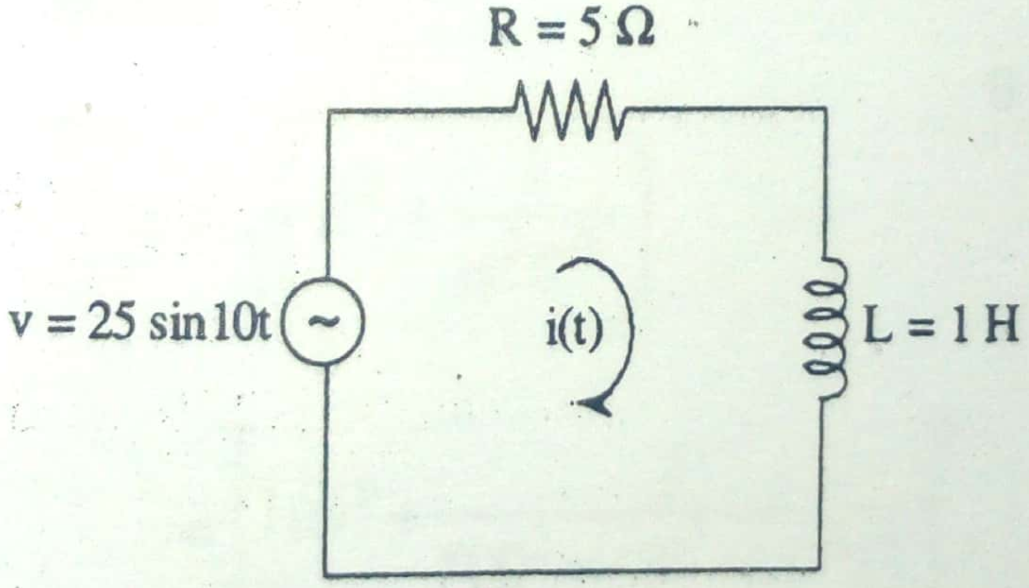
$$i(t) = e^{-\frac{t}{RC}}\left(\frac{V_m}{R}\sin\phi - \frac{V_m}{\sqrt{R^2+(1/\omega C)^2}}\sin\left(\phi+\tan^{-1}1/\omega CR\right)\right)$$

$$+ \frac{V_m}{\sqrt{R^2+(1/\omega C)^2}}\sin\left(\omega t+\phi+\tan^{-1}1/\omega CR\right)$$

//.

A sinusoidal voltage 25 sin 10t is applied at time t = 0 to series R-L circuit having R = 5 Ω and L = 1 H. Find i(t) and sketch it. Assume zero current through the inductor before applying the voltage.

The R-L circuit is shown in figure.



Figure

From figure, we get,

$$R = 5\,\Omega,\ L = 1\text{ H},\ \theta = 10,\ V = 25\text{ V}$$

By applying KVL to the circuit, we get,

$$5i + \frac{di}{dt} = 25\sin 10t$$

$$\frac{di}{dt} + 5i = 25\sin 10t \quad \text{...(1)}$$

$$(D+5)i = 25\sin 10t \quad \text{...(2)}$$

The general solution of equation (2) consists of two parts, i.e., complementary function and particular integral.

The complementary function of the solution is,

$$i_c = ce^{-(5)t} \qquad \text{... (3)}$$

By assuming particular integral as, $i_p = A \sin 10t + B \cos 10t$, we get,

$$i_p = \frac{V}{\sqrt{R^2 + (\omega L)^2}} \sin\left[\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right] \qquad \text{... (4)}$$

Substituting the value of L, ω, R and V in the above equation, we get,

$$i_p = \frac{25}{\sqrt{5^2 + (10)^2}} \sin\left(10t - \tan^{-1}\left(\frac{10}{5}\right)\right)$$

$$i_p = \sqrt{5} \sin(10t - 63.43^\circ) \qquad \text{... (5)}$$

The complete solution is,

$$i(t) = i_c + i_P$$

$$\therefore \quad i(t) = ce^{-5t} + \sqrt{5} \sin(10t - 63.43^\circ) \qquad \text{... (6)}$$

At $t = 0$,

The current flowing through circuit is zero, i.e., $i(0) = 0$,

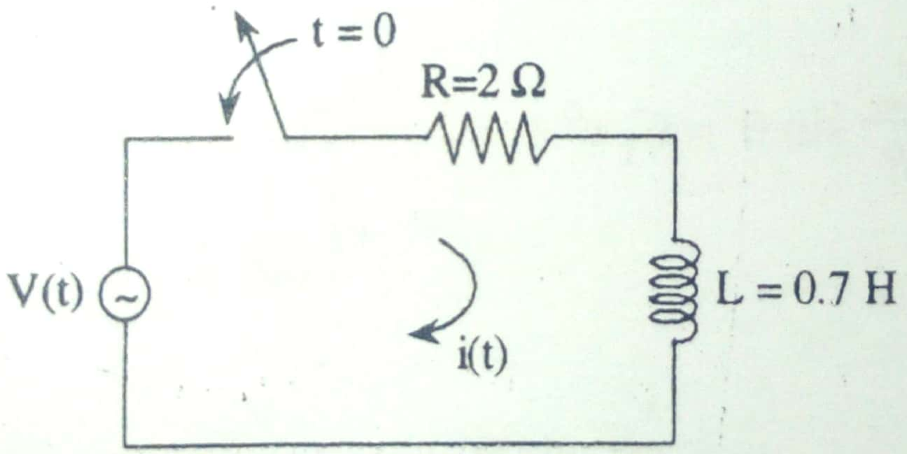
Using this condition in equation (6), we get,

$$0 = c + \sqrt{5} \sin(-63.43^\circ)$$

$$\therefore \; c = -2$$

Hence, $i(t) = 2e^{-5t} + \sqrt{5} \sin(10t - 63.43^\circ)$

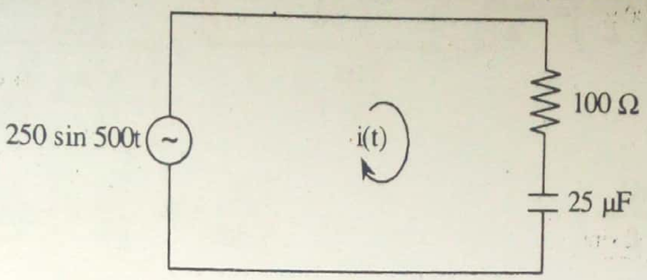
Q39. For the circuit given below in figure, the applied voltage is V(t) =10 sin(200 t + 60°). Find the current through the circuit for $t \geq 0$. Assume zero initial condition. Use time domain approach.



Q41. A series R-C circuit with R = 100 Ω and C = 25 μF has a sinusoidal excitation V(t) = 250 sin 500t. Find the total current assuming that the capacitor is initially uncharged.

OR

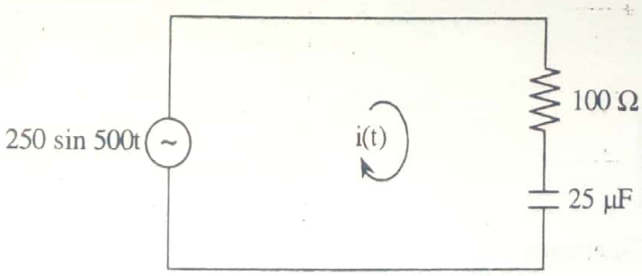
For the circuit shown find the total current assuming that the capacitor is initially uncharged.



Figure

Ans:

Given circuit is shown in figure.



Figure

$v = 250 \sin 500t$

$R = 100\,\Omega$

$C = 25\,\mu\text{F} = 25 \times 10^{-6}\,\text{F}$

$\omega = 500$ rad/sec

The total current solution for the circuit is given as,

$$i(t) = c_1 e^{-t/RC} + \left(\frac{V_m}{z}\right)\sin\left(\omega t + \tan^{-1}\left(\frac{1}{\omega CR}\right)\right) \quad \text{...(1)}$$

Where,

$$z = \sqrt{\left(R^2 + \frac{1}{\omega^2 C^2}\right)}$$

$$= \left[100^2 + \frac{1}{500^2 \times (25 \times 10^{-6})^2}\right]^{1/2}$$

$$= 128.062\,\Omega$$

Now substituting all corresponding values in equation (1), we get,

$$i(t) = c_1 e^{-t/100 \times 25 \times 10^{-6}} + \frac{250}{128.062}\sin\left(500t + \tan^{-1}\left(\frac{1}{500 \times 25 \times 10^{-6} \times 100}\right)\right)$$

$$= c_1 e^{-400t} + 1.952 \sin[500\,t + \tan^{-1}(0.8)]$$

$$= c_1 e^{-400t} + 1.952 \sin(500\,t + 38.66°) \quad \text{...(2)}$$

Applying initial conditions,

At, $t = 0^+, V_c(0^+) = 0$ and $i(0^+) = 0$

Therefore, equation (2) becomes,

$$0 = c_1 \times e^0 + 1.952 \sin(500 \times 0 + 38.66°)$$

$$= c_1 + 1.952 \sin(38.66°)$$

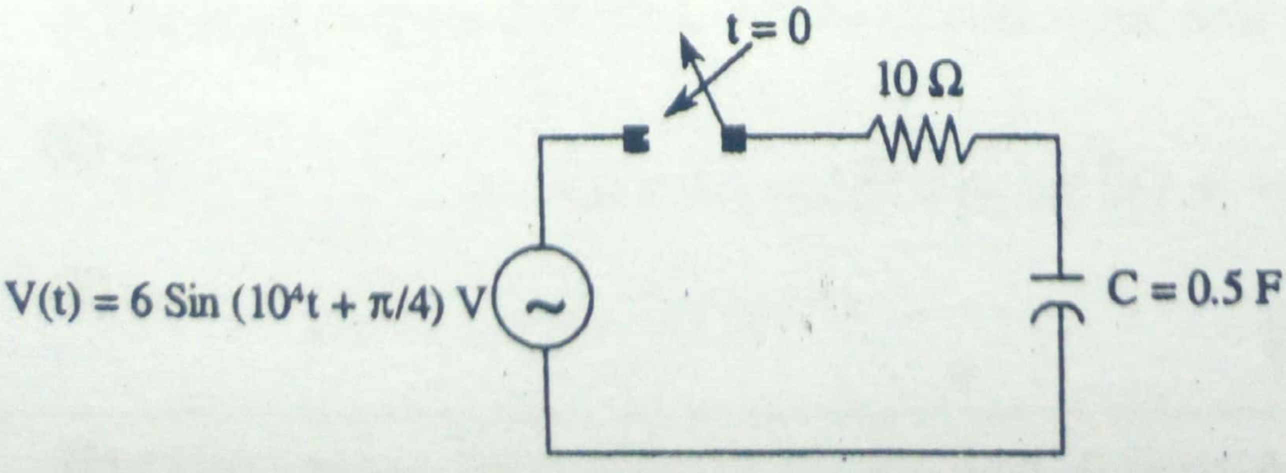
$$c_1 = -1.952 \sin(38.66°)$$

$$= -1.221$$

$\therefore$ The total current is,

$$I(t) = -1.221\, e^{-400\,t} + 1.952 \sin(500t + 38.66°)$$

Q42. For the circuit shown in figure, determine the particular solution for i(t) through the circuit. Assume zero initial conditions.



Figure