UNIT-2

COLUMNS AND STRUTS

COLUMN

A column is a long vertical slender bar or vertical member, subjected to an axial compressive load and fixed rigidly at both ends.

For example

Vertical pillar between the Roof and Floor.

STRUT

A strut is a slender bar or member in any position other than vertical subjected to a compressive load and fixed rigidly or Hinged or pin joints at one or both the ends.

For example

Piston rods, Connecting rods, side link in forging machine etc.

The failure of such member will occurs

* By pure compression
* By buckling
* By combination of pure compression and buckling, depending upon a slenderness ratio.

TYPES OF COLUMNS

Depending upon the slenderness ratio or length to diameter ratio, columns can be divided into three

* Short Columns
* Medium size Columns
* Long columns

Short Columns

* It is that column in which the ratio of effective length to the least lateral dimension is less than 12

$$\boxed{\frac{L_e}{D} < 12}$$

∴ (L_e – Effective length)

* When short columns are subjected to compressive load, their buckling is generally negligible and as such the buckling stresses are very small as compared with direct compressive.

* It is assumed that short columns are always subjected to direct compressive stresses only.

Medium Size Columns

The columns which have their length varying from 8 times their diameter to 30 times their respective diameters or their slenderness ratio lying between 32 and 120 are called medium size column (or) Intermediate columns.

* In these columns, both the buckling as well as direct stresses are of significant values.

* Therefore, in the design of intermediate columns, both these stresses are taken into account.

Long columns

* It is that column in which the effective length to least lateral dimension ratio is greater than 12

$$\frac{Le}{D} > 12$$

* They are usually subjected to buckling stress only
* Direct compressive stress is very small as compared with buckling stress, and hence it is neglected.

EULER'S THEORY FOR LONG COLUMNS

In 1757, Swiss mathematician Leonhard Euler gave the formula for stability of long column

* Euler consider only bending of column as the direct compression was negligible compared to bending.

* Euler neglected the direct compression of column.

* Hence Euler's is only applicable of long column.

ASSUMPTION MADE IN EULER'S COLUMN THEORY

* The column is initially straight and of uniform lateral dimension.
* The compressive load is exactly axial and it passes through the centroid of the column section.
* The material of the column is perfectly homogeneous and isotropic
* The material of the column is Elastic & obey's Hooke's Law (σ vs e)
* The length of the column is very large compared to the other dimension.
* Self weight of the column was negligible
* limit of proportionality is not exceed.

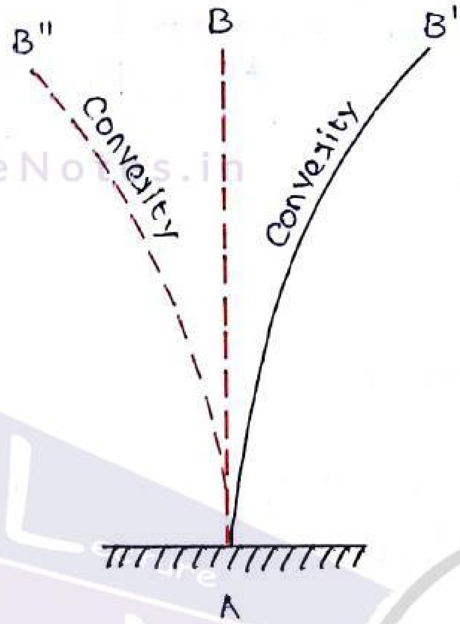
END CONDITION FOR LONG COLUMNS

The following four types of end conditions of the columns are important.

* Both the ends of the column are hinged (or pinned)
* one end is fixed and the other end is free
* Both the end of the column are fixed.
* one end is fixed and the other is pinned.

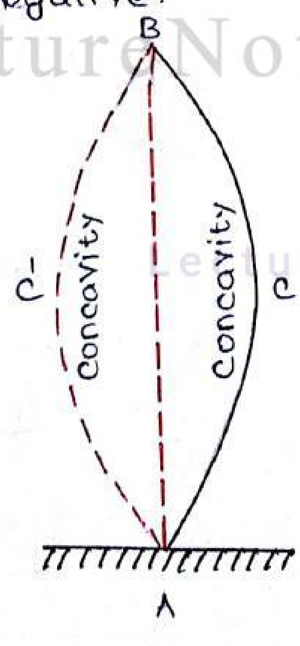
Sign Convention for Bending Moments

A Bending moment which bends the column so as to present convexity towards the initial centre line of the member will be regarded as positive.



Positive

A bending moment which bends the column as to present concavity towards the initial centre line of the member will be regarded as negative.



Negative

EULER'S FORMULA

Euler's formula is used for calculating the critical load for a column or strut.

$$\boxed{P_{Euler} = \frac{\pi^2 EI}{Le^2}}$$

Where,

P - Critical load

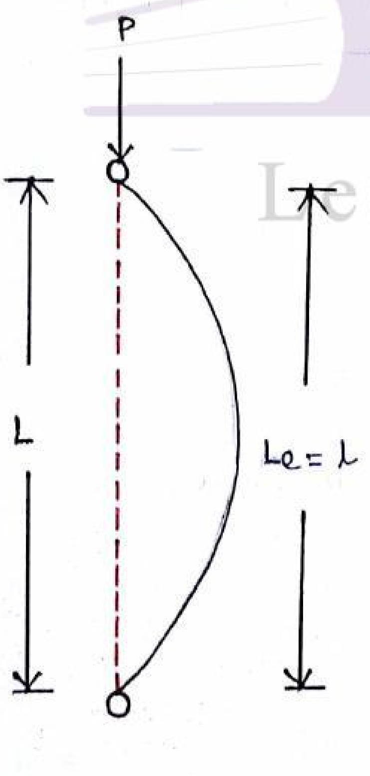
E - Modulus of elasticity

I - least moment of inertia of section of the column

Le - Equivalent length of the strut.

END CONDITIONS TAKEN IN THE EULER'S ANALYSIS

BOTH END HINGED



P - Crippling load

L - Actual length

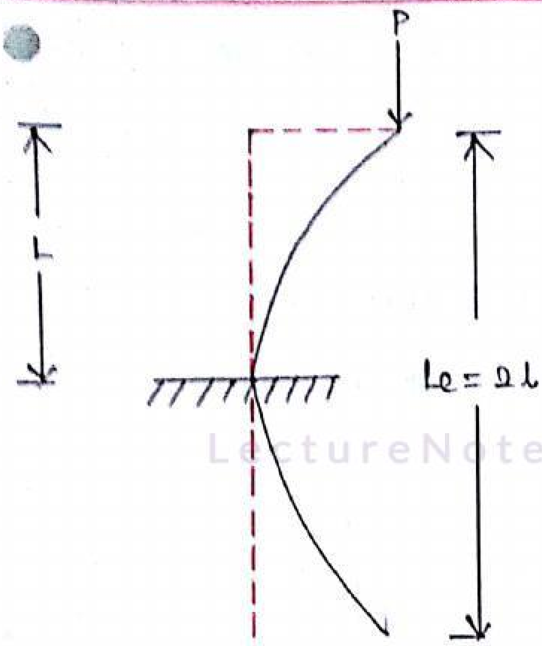
Le - Equivalent Length

Euler's formula

$$P = \frac{\pi^2 EI}{Le^2}$$

$$\boxed{P = \frac{\pi^2 EI}{L}}$$

ONE END FIXED OTHER END FREE



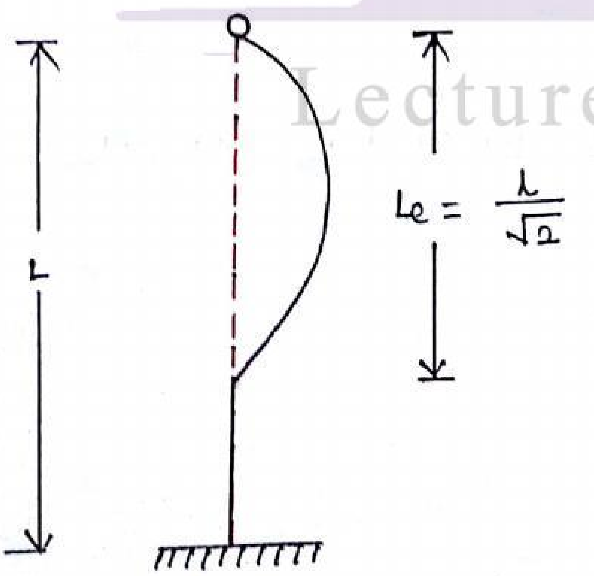
Euler's formula

$$P = \frac{\pi^2 EI}{L_e^2}$$

$$= \frac{\pi^2 EI}{(2L)^2}$$

$$\boxed{P = \frac{\pi^2 EI}{4l^2}}$$

ONE END FIXED OTHER END HINGED



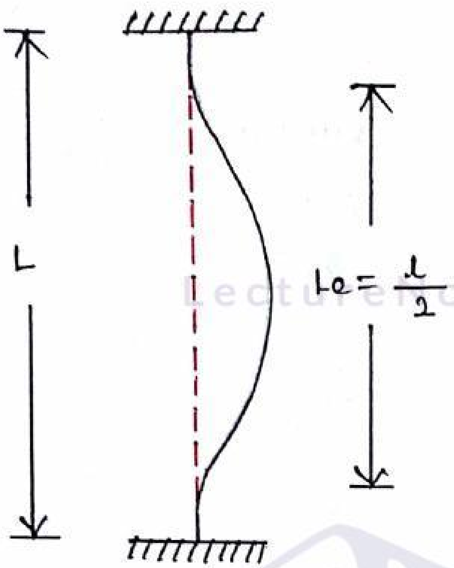
Euler's formula

$$P = \frac{\pi^2 EI}{L_e^2}$$

$$= \frac{\pi^2 EI}{\left(\frac{l}{\sqrt{2}}\right)^2}$$

$$\boxed{P = \frac{2\pi^2 EI}{l^2}}$$

Both End Fixed



Euler's formula

$$P = \frac{\pi^2 EI}{L_e^2}$$

$$= \frac{\pi^2 EI}{\left(\frac{l}{2}\right)^2}$$

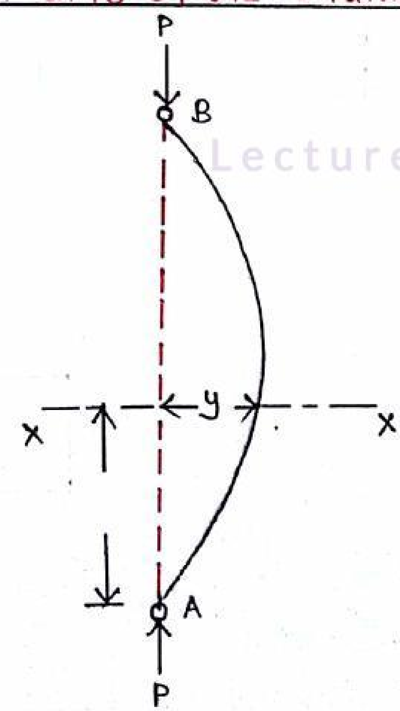
$$= \frac{\pi^2 EI}{l^2/4}$$

$$\boxed{P = \frac{4\pi^2 EI}{l^2}}$$

Derivation of Euler's Formula for Differend End Condition

Case-1

When both ends of the column are hinged or pinned



We know that

Bending equation

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\frac{M}{EI} = \frac{1}{R} \rightarrow (1)$$

R – Radius of curvature

$\frac{1}{R}$ – curvature

y – Deflection

$$\frac{dy}{dx} = \text{Slope} = \theta$$

$$\left.\frac{d^2y}{dx^2} = \text{Moment} = M \ \text{(or)} \ \text{Curvature}\right\} \rightarrow (2)$$

Sub. (1) and (2) equation

$$\frac{M}{EI} = \frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$EI\frac{d^2y}{dx^2} = Mx-x$$

$$EI\frac{d^2y}{dx^2} = -P \times y$$

$$\frac{d^2y}{dx^2} = \frac{-Py}{EI}$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = 0$$

Second degree differential equation

y = Complementary Solution + Particular Integral

$$y = C_1 \cos\left(x \times \sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(x \times \sqrt{\frac{P}{EI}}\right) \rightarrow (1)$$

Boundary Condition

1) At 'B' $x = 0$; $y = 0$

2) At 'A' $x = L$; $y = 0$

Apply boundary condition

Ist Boundary Condition

$x = 0$; $y = 0$

Sub. these values in equation (1)

$$0 = C_1 \cos\left(0 \times \sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(0 \times \sqrt{\frac{P}{EI}}\right)$$

$$= C_1 (1) + C_2 (0)$$

$$\boxed{C_1 = 0}$$

IInd Boundary Condition

$x = L$; $y = 0$

Sub. these values in equation (1)

$$0 = (0) \cos\left(L \times \sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(L \times \sqrt{\frac{P}{EI}}\right)$$

$$= 0 + C_2 \sin\left(L \times \sqrt{\frac{P}{EI}}\right)$$

$$C_2 \sin\left(l \times \sqrt{\frac{P}{EI}}\right) = 0$$

$$\sin\left(l \times \sqrt{\frac{P}{EI}}\right) = 0$$

$\therefore$ (Here C_2 cannot be zero)

$$= \sin 0 \text{ (or) } \sin\pi \text{ (or) } \sin 2\pi \text{ (or) } \sin 3\pi$$

$$l \times \sqrt{\frac{P}{EI}} = 0 \text{ (or) } \pi \text{ (or) } 2\pi \text{ (or) } 3\pi$$

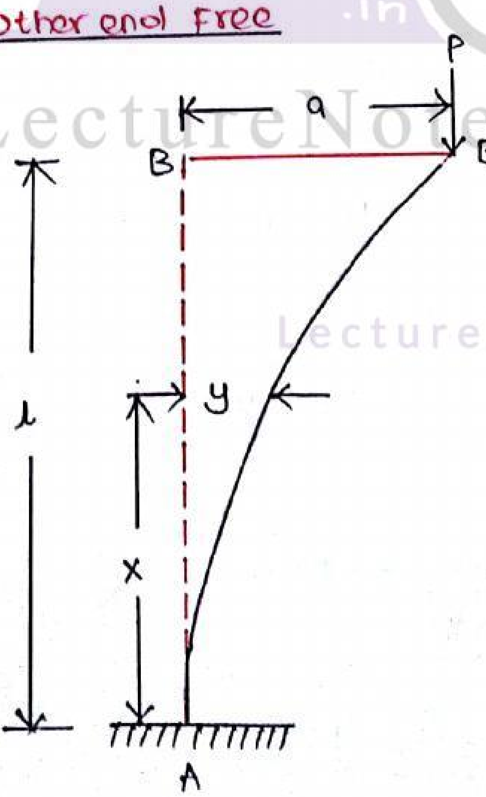
Taking the least practical value

$$l \times \sqrt{\frac{P}{EI}} = \pi$$

$$\boxed{P = \frac{\pi^2 EI}{l^2}}$$

Case-2

One End Fixed, other end Free



We know that

Bending equation

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\frac{M}{EI} = \frac{1}{R} \rightarrow (1)$$

y = Deflection

$$\frac{dy}{dx} = \text{slope} = \theta$$

$$\left.\frac{d^2y}{dx^2} = \text{Moment} = M \text{ (or) Curvature}\right\} \rightarrow (2)$$

Sub equation (1) and (2)

$$\frac{M}{EI} = \frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$EI\frac{d^2y}{dx^2} = M x - x$$

$$EI\frac{d^2y}{dx^2} = P(a-y)$$

$$EI\frac{d^2y}{dx^2} = P \times a - P \times y$$

$$EI\frac{d^2y}{dx^2} + P \times y = P \times a$$

$$\frac{d^2y}{dx^2} + \frac{P \times y}{EI} = \frac{P \times a}{EI}$$

Second degree differential equation

$$y = C_1 \cdot \cos\left(x \times \sqrt{\frac{P}{EI}}\right) + C_2 \cdot \sin\left(x \times \sqrt{\frac{P}{EI}}\right) + a \rightarrow (1)$$

Boundary Condition

* At point 'A'

For a Fixed end, the deflection as well as slope is zero

At end 'A' (which is fixed), Deflection $(y) = 0$

Slope $\left(\frac{dy}{dx}\right) = 0$

$x = 0, \; y = 0 \Rightarrow$ (Deflection)

$x = 0, \; \frac{dy}{dx} = 0 \Rightarrow$ (Slope)

* At point 'B' (Free end)

$x = l, \; y = a$

Apply Boundary Condition

Ist Boundary condition

$x = 0, \; y = 0$

Sub. these values in equtn (1)

$$0 = C_1 \cdot \cos\left(0 \times \sqrt{\frac{P}{EI}}\right) + C_2 \cdot \sin\left(0 \times \sqrt{\frac{P}{EI}}\right) + a$$

$$= C_1 \times 1 + C_2 \times 0 + a$$

$$= C_1 + a$$

$$\boxed{C_1 = -a} \rightarrow (2)$$

$x = 0, \ \frac{dy}{dx} = 0$

Differentiating equation (1) w.r.t 'x' we get

$$\frac{dy}{dx} = -C_1 \sin\left(x \times \sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}} + C_2 \cos\left(x \times \sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}} + 0$$

$$= -C_1 \sqrt{\frac{P}{EI}} \cdot \sin\left(x \times \sqrt{\frac{P}{EI}}\right) + C_2 \sqrt{\frac{P}{EI}} \cdot \cos\left(x \times \sqrt{\frac{P}{EI}}\right)$$

$$0 = -C_1 \sqrt{\frac{P}{EI}} \cdot \sin\left(0 \times \sqrt{\frac{P}{EI}}\right) + C_2 \sqrt{\frac{P}{EI}} \cdot \cos\left(0 \times \sqrt{\frac{P}{EI}}\right)$$

$$= -C_1 \sqrt{\frac{P}{EI}} \cdot \sin(0) + C_2 \sqrt{\frac{P}{EI}} \cdot \cos(0)$$

$$= -C_1 \cdot \sqrt{\frac{P}{EI}} \times 0 + C_2 \cdot \sqrt{\frac{P}{EI}} \times 1$$

$$= C_2 \sqrt{\frac{P}{EI}}$$

$$\boxed{C_2 = 0} \rightarrow (3) \qquad \therefore \left(\sqrt{\frac{P}{EI}} \text{ cannot be equal to zero}\right)$$

Sub. the value of $C_1 = -a$ and $C_2 = 0$ in equtn (1)

$$y = -a \cos\left(x \times \sqrt{\frac{P}{EI}}\right) + 0 + a \rightarrow (4)$$

II nd Boundary Condition

$x = l \; ; \; y = a$

Sub. these values in equtn (4)

$$a = -a\cos\left(l \times \sqrt{\frac{P}{EI}}\right) + a$$

$$a - a = -a\cos\left(l \times \sqrt{\frac{P}{EI}}\right)$$

$$0 = -a\cos\left(l \times \sqrt{\frac{P}{EI}}\right)$$

$\therefore$ ('a' cannot be equal zero)

$$\cos\left(l \times \sqrt{\frac{P}{EI}}\right) = 0$$

$$= \cos \pi/2 \text{ or } \cos 3\pi/2 \text{ or } \cos 5\pi/2$$

$$l \times \sqrt{\frac{P}{EI}} = \pi/2 \text{ or } 3\pi/2 \text{ or } 5\pi/2$$

Taking the least practical values

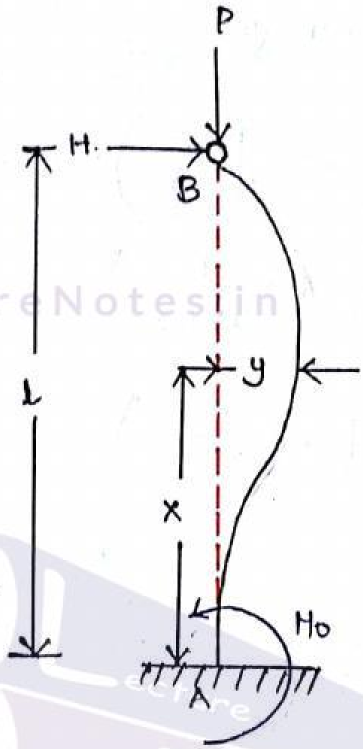
$$l \times \sqrt{\frac{P}{EI}} = \frac{\pi}{2}$$

$$\sqrt{\frac{P}{EI}} = \frac{\pi}{2l}$$

$$\boxed{P = \frac{\pi^2 EI}{4l^2}}$$

Case - 3

One End Fixed, Other End Pin Jointed



We know that

Bending Equation

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\frac{M}{EI} = \frac{1}{R} \rightarrow (1)$$

y = Deflection

$\frac{dy}{dx}$ = Slope $\Rightarrow (\theta)$

$\frac{d^2y}{dx^2}$ = Moment $\Rightarrow (M)$ (or) Curvature $\rightarrow (2)$

Sub equation (1) & (2)

$$\frac{M}{EI} = \frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$EI\frac{d^2y}{dx^2} = M x - x$$

$$EI\frac{d^2y}{dx^2} = -P \times y + H(l-x)$$

$$EI\frac{d^2y}{dx^2} + P \times y = H(l-x).$$

$$\frac{d^2y}{dx^2} + \frac{P \times y}{EI} = \frac{H(l-x)}{EI}$$

Second degree differential equation

$y =$ Complementary solution + Particular Integral

$$y = C_1 \cos\left(x \times \sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(x \times \sqrt{\frac{P}{EI}}\right) + \frac{H}{P}(l-x) \rightarrow (1)$$

Boundary Condition

* At the Fixed end 'A'

$x = 0,\ y = 0$ and also $\frac{dy}{dx} = 0$

* At the Hinged end 'B'

$x = l$ and $y = 0$

Apply Boundary condition

I^{st} Boundary Condition

$x = 0;\ y = 0$

sub these values in equtn ①

$$0 = C_1 \cdot \cos\left(0 \times \sqrt{\frac{P}{EI}}\right) + C_2 \cdot \sin\left(0 \times \sqrt{\frac{P}{EI}}\right) + \frac{H}{P}(l-0)$$

$$= C_1 \cos(0) + C_2 \sin(0) + \frac{H}{P}(l-0)$$

$$= C_1(1) + C_2(0) + \frac{H}{P}(l-0)$$

$$\boxed{C_1 = -\frac{H}{P} \times l}$$

$x = 0 ; \quad \frac{dy}{dx} = 0$

Differentiating equation ① w.r.t 'x' we get

$$\frac{dy}{dx} = -C_1 \sin\left(x \times \sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}} + C_2 \cdot \cos\left(x \times \sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}} - \frac{H}{P}$$

$$= -C_1 \sqrt{\frac{P}{EI}} \cdot \sin\left(x \times \sqrt{\frac{P}{EI}}\right) + C_2 \sqrt{\frac{P}{EI}} \cdot \cos\left(x \times \sqrt{\frac{P}{EI}}\right) - \frac{H}{P}$$

$$0 = -\left(-\frac{H}{P} \times l\right) \sqrt{\frac{P}{EI}} \cdot \sin\left(0 \times \sqrt{\frac{P}{EI}}\right) + C_2 \sqrt{\frac{P}{EI}} \cdot \cos\left(0 \times \sqrt{\frac{P}{EI}}\right) - \frac{H}{P}$$

$$= -\left(-\frac{H}{P} \times l\right) \sqrt{\frac{P}{EI}}\,(0) + C_2 \sqrt{\frac{P}{EI}}\,(1) - \frac{H}{P}$$

$$C_2 \sqrt{\frac{P}{EI}} = \frac{H}{P}$$

$$\boxed{C_2 = \frac{H}{P} \sqrt{\frac{P}{EI}}}$$

IInd Boundary Condition

- At the end 'B'

$x = l \; ; \; y = 0$

$$0 = -\frac{H}{P} l \cdot \cos\left(l \times \sqrt{\frac{P}{EI}}\right) + \frac{H}{P}\sqrt{\frac{EI}{P}} \sin\left(l \times \sqrt{\frac{P}{EI}}\right) + \frac{H}{P}(l - l)$$

$$= -\frac{H}{P} l \cdot \cos\left(l \times \sqrt{\frac{P}{EI}}\right) + \frac{H}{P}\sqrt{\frac{EI}{P}} \cdot \sin\left(l \times \sqrt{\frac{P}{EI}}\right)$$

$$\frac{H}{P}\sqrt{\frac{EI}{P}} \cdot \sin\left(l \times \sqrt{\frac{P}{EI}}\right) = \frac{H}{P} l \cdot \cos\left(l \times \sqrt{\frac{P}{EI}}\right)$$

$$\sin\left(l \times \sqrt{\frac{P}{EI}}\right) = \frac{H}{P} \cdot l \times \frac{P}{H} \times \sqrt{\frac{EI}{P}} \cdot \cos\left(l \times \sqrt{\frac{P}{EI}}\right)$$

$$= l \cdot \sqrt{\frac{P}{EI}} \cdot \cos\left(l \cdot \sqrt{\frac{P}{EI}}\right)$$

$$\tan\left(l \times \sqrt{\frac{P}{EI}}\right) = l \cdot \sqrt{\frac{P}{EI}}$$

$$l \times \sqrt{\frac{P}{EI}} = 4.5 \text{ radians}$$

Squaring both sides

$$l^2 \times \frac{P}{EI} = 4.5^2$$

$$P = 20.25 \frac{EI}{l^2}$$

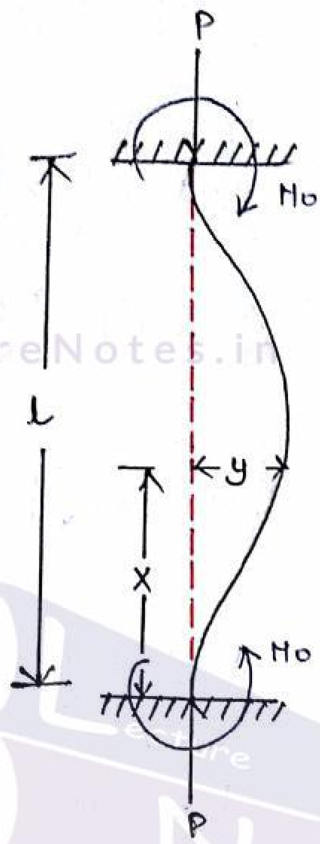
$$\boxed{P = \frac{2\pi^2 EI}{l^2}}$$

But approximately

$20.25 = 2\pi^2$

Case - 4

Both End Fixed



We know that

Bending Equation

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\frac{M}{EI} = \frac{1}{R} \rightarrow (1)$$

y = Deflection

$$\frac{dy}{dx} = \text{Slope}$$

$$\frac{d^2y}{dx^2} = \text{Moment (or) curvature} \rightarrow (2)$$

Sub equation (1) and (2)

$$\frac{M}{EI} = \frac{1}{R} = \frac{d^2y}{dx^2} \Rightarrow \frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$EI\frac{d^2y}{dx^2} = Mx - x$$

$$EI \frac{d^2y}{dx^2} = M_0 - P \times y$$

$$EI \frac{d^2y}{dx^2} + P \times y = M_0$$

$$\frac{d^2y}{dx^2} + \frac{P \times y}{EI} = \frac{M_0}{EI}$$

Second degree differential equation

y = Complementry solution + particular Integral

$$y = C_1 \cdot \cos\left(x \times \sqrt{\frac{P}{EI}}\right) + C_2 \cdot \sin\left(x \times \sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P} \rightarrow (1)$$

Boundary Condition

* At 'A' $x = 0$, $y = 0$ and also $\frac{dy}{dx} = 0$ as A is a fixed end.

* At 'B' $x = L$, $y = 0$ and also $\frac{dy}{dx} = 0$ as B is also a fixed end.

Apply Boundary Condition

I[st] Boundary Condition

$$x = 0, \ y = 0$$

Sub. those values in equtn (1)

$$0 = C_1 \cdot \cos\left(0 \times \sqrt{\frac{P}{EI}}\right) + C_2 \cdot \sin\left(0 \times \sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P}$$

$$0 = C_1(1) + C_2(0) + \frac{M_0}{P}$$

$$\boxed{C_1 = -\frac{M_0}{P}}$$

$x=0$; $\frac{dy}{dx}=0$

Differentiating equtn ① w.r.t 'x' we get

$$\frac{dy}{dx} = -C_1 \cdot \sin\left(x \times \sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}} + C_2 \cdot \cos\left(x \times \sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}} + 0$$

$$\frac{dy}{dx} = -C_1 \sqrt{\frac{P}{EI}} \cdot \sin\left(x \times \sqrt{\frac{P}{EI}}\right) + C_2 \sqrt{\frac{P}{EI}} \cdot \cos\left(x \times \sqrt{\frac{P}{EI}}\right) + 0$$

$$0 = -C_1 \sqrt{\frac{P}{EI}} \cdot \sin\left(0 \times \sqrt{\frac{P}{EI}}\right) + C_2 \sqrt{\frac{P}{EI}} \cdot \cos\left(0 \times \sqrt{\frac{P}{EI}}\right) + 0$$

$$= -C_1 \sqrt{\frac{P}{EI}}\,(0) + C_2 \sqrt{\frac{P}{EI}}\,(1)$$

$$\boxed{C_2 = 0}$$

$\therefore$ (The value of $\sqrt{\frac{P}{EI}}$ cannot be equal to zero)

Sub. these values of $C_1 = -\frac{M_0}{P}$ and $C_2 = 0$ in equtn ① we get

$$y = -\frac{M_0}{P} \cdot \cos\left(x \times \sqrt{\frac{P}{EI}}\right) + 0 \cdot \sin\left(x \times \sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P}$$

$$y = -\frac{M_0}{P} \cdot \cos\left(x \times \sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P} \rightarrow ②$$

IInd Boundary Condition

$x = l$; $y = 0$

Sub. these values in equtn ②

$$0 = -\frac{M_0}{P} \cdot \cos\left(l \times \sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P}$$

$$\frac{M_0}{P} \cos\left(l \cdot \sqrt{\frac{P}{EI}}\right) = \frac{M_0}{P}$$

$$\cos\left(l \cdot \sqrt{\frac{P}{EI}}\right) = \frac{M_0}{P} \times \frac{P}{M_0}$$

$$\cos\left(l \times \sqrt{\frac{P}{EI}}\right) = \cos 0,\ \cos 2\pi,\ \cos 4\pi,\ \cos 6\pi$$

$$l \times \sqrt{\frac{P}{EI}} = 0,\ 2\pi,\ 4\pi,\ 6\pi$$

Taking the least practical value

$$l \times \sqrt{\frac{P}{EI}} = 2\pi$$

$$\boxed{P = \frac{4\pi^2 EI}{l^2}}$$

S.No	END CONDITION OF COLUMN	CRIPPLING LOAD IN TERMS OF ACTUAL LENGTH	CRIPPLING LOAD IN TERMS OF EFFECTIVE LENGTH	RELATION B/W EFFECTIVE LENGTH AND ACTUAL LENGTH
1	Both end hinged	$\frac{\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{Le^2}$	$Le = l$
2	One end is fixed and other is free	$\frac{\pi^2 EI}{4l^2}$	$\frac{\pi^2 EI}{Le^2}$	$Le = 2l$
3	Both end fixed	$\frac{4\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{Le^2}$	$Le = \frac{l}{2}$
4	One end Fixed and other is Hinged	$\frac{2\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{Le^2}$	$Le = \frac{l}{\sqrt{2}}$

EFFECTIVE LENGTH OR EQUIVALENT LENGTH OF A COLUMN

The Effective length of a given column with given end conditions is the length of an equivalent column of the same material and cross section with hinged ends, and having the value of the crippling load equal to that of the given column. Effective length is also called Equivalent length.

Let,

L_e - Effective length of a column

l - Actual length of the column

P - Crippling load for the column

Crippling load for any type of end condition

$$\boxed{P = \frac{\pi^2 EI}{L_e^2}}$$

SLENDERNESS RATIO

The ratio of the actual length of a column to the least radius of gyration of the column is known as slenderness ratio

$$\text{Slenderness ratio} = \frac{\text{Actual length}}{\text{Least radius of gyration}}$$

$$\boxed{\text{Slenderness ratio} = \frac{l}{K}}$$

EULER'S CRITICAL STRESS

Crippling stress in terms of Effective length and Radius of Gyration

The Moment of Inertia (I) can be expressed in terms of Radius of Gyration (K)

Radius of Gyration

$$K = \sqrt{\frac{I}{A}}$$

$$I = AK^2$$

Where,

A - Area of cross section

I - Least value of Moment of Inertia

K - least Radius of Gyration of the column Section

Now,

Crippling load 'P' in terms of Effective length

$$P = \frac{\pi^2 EI}{Le^2}$$

$$= \frac{\pi^2 E \times AK^2}{Le^2}$$

$$= \frac{\pi^2 E \times A}{Le^2/K^2}$$

$$\boxed{P = \frac{\pi^2 E \times A}{\left(\frac{Le}{K}\right)^2}}$$

Stress corresponding to crippling Load

$$\text{Crippling stress} = \frac{\text{Crippling Load}}{\text{Area}}$$

$$= \frac{P}{A}$$

(Sub. the value of 'P')

$$= \frac{\pi^2 E \times \cancel{A}}{\cancel{A} \cdot \left(\frac{Le}{K}\right)^2}$$

$$\boxed{\text{Crippling stress} = \frac{\pi^2 E}{\left(\frac{Le}{K}\right)^2}}$$

LIMITATION OF EULER'S FORMULA

$$\text{Crippling stress} = \frac{\pi^2 E}{\left(\frac{Le}{K}\right)^2}$$

For a column with both end hinged

$$Le = l$$

$$\text{Crippling stress} = \frac{\pi^2 E}{\left(\frac{l}{K}\right)^2}$$

Where,

$\frac{l}{K}$ - Slenderness ratio

* If the slenderness ratio $\left(\frac{l}{k}\right)$ is small, the crippling stress or the stress at failure will be high

* But for the column material, the crippling stress cannot be greater than the crushing stress.

* When the slenderness ratio is less than a certain limit, Euler's formula gives a value of crippling stress greater than the crushing stress.

* If the limiting case, we can find out the value of $\left(\frac{l}{k}\right)$ for which crippling stress is equal to crushing stress

For example

For a Mild steel column with both ends hinged

crushing stress $= 330\ N/mm^2$

young's modulus $= 2.1\times10^5\ N/mm^2$

Equating the crippling stress to the crushing stress corresponding to the minimum value of slenderness ratio

crippling stress = crushing stress

$$\frac{\pi^2 E}{\left(\frac{l}{k}\right)^2} = 330$$

$$\frac{\pi^2\times 2.1\times10^5}{\left(\frac{l}{k}\right)^2} = 330$$

$$\left(\frac{l}{k}\right)^2 = \frac{\pi^2\times 2.1\times10^5}{330}$$

$$= 6281$$

$$\frac{l}{k} = \sqrt{6282}$$

$$\frac{l}{k} = 79.27 \text{ say } 80$$

If the slenderness ratio is less than 80 for mild steel column with both end hinged, then Euler's formula will not be valid

Prblm. no: 1

A Solid round bar 3m long and 5cm in diameter is used as a strut with both ends hinged. Determine the crippling (or collapsing) load. Take $E = 2.0 \times 10^5$ N/mm^2

Given data

Length of bar $(L) = 3m \Rightarrow 3 \times 1000 m \Rightarrow 3000 mm$

Diameter of bar $(d) = 5cm \Rightarrow 5 \times 10 \Rightarrow 50 mm$

Young's modulus $(E) = 2.0 \times 10^5$ N/mm^2

Moment of Inertia $(I) = \frac{\pi}{64} \times (d)^4$

$$= \frac{\pi}{64} \times (5)^4$$

$$I = 30.68 \text{ cm}^4 \Rightarrow 30.68 \times 10^4 \text{ mm}^4$$

let,

P - crippling load

Solution

Both the ends of the bar are hinged, Hence the crippling load

$$P = \frac{\pi^2 EI}{l^2}$$

$$= \frac{\pi^2 \times (2\times 10^5) \times 20.68 \times 10^4}{(2000)^2}$$

$= 67288\ N$

$1\ N \Rightarrow \frac{1}{1000}\ N$

$P = 67.288\ kN$

$67288\ N \Rightarrow \frac{67288}{1000}$

$\Rightarrow 67.288\ kN$

Prblm.no:2

From the problem.no:1 Determine the crippling load, when the given strut is used with the following conditions

(i) One end of the strut is fixed and the outer end is free

(ii) Both the end of strut are fixed

(iii) One end is fixed and other is Hinged

Given data

$L = 2000\ mm$

$d = 50\ mm$

$E = 2\times 10^5\ N/mm^2$

$I = 20.68 \times 10^4\ mm^4$

To, P - Crippling load

Solution

(i) Crippling load when one end is fixed and other is free

$$P = \frac{\pi^2 EI}{4L^2}$$

$$= \frac{\pi^2 \times 2\times 10^5 \times 20.68 \times 10^4}{4 \times 2000^2}$$

$$\boxed{P = 16822\ N}$$

Alternate Method

The crippling load for any type of end condition

$$P = \frac{\pi^2 EI}{l_e^2} \rightarrow (1)$$

$\therefore$ (l_e - Effective length)

When one end is fixed and other end is free

$$l_e = 2L$$

$$= 2 \times 3000$$

$$l_e = 6000 \text{ mm}$$

Sub. the value of L in equation (1)

$$P = \frac{\pi^2 \times (2 \times 10^5) \times 30.68 \times 10^4}{6000^2}$$

$$\boxed{P = 16822 \text{ N}}$$

(ii) Crippling load when both the ends are fixed

$$P = \frac{4\pi^2 EI}{l^2}$$

$$= \frac{4\pi^2 \times (2 \times 10^5) \times 30.68 \times 10^4}{3000^2}$$

$$= 269152 \text{ N}$$

$$\boxed{P = 269.152 \text{ kN}}$$

Alternate Method

$$P = \frac{\pi^2 EI}{le^2}$$

$$le = \frac{L}{2} = \frac{3000}{2}$$ $\therefore$ (Le - Effective length)

$$Le = 1500mm$$

$$P = \frac{\pi^2 \times 2 \times 10^5 \times 30.68 \times 10^4}{1500^2}$$

$$\boxed{P = 269152\ N}$$

(iii) Crippling load when one end is fixed and the other is hinged

$$P = \frac{2\pi^2 EI}{l^2}$$

$$= \frac{2 \times \pi^2 \times 2 \times 10^5 \times 30.68 \times 10^4}{3000^2}$$

$$\boxed{P = 134576\ N}$$

Alternate Method

$$P = \frac{\pi^2 EI}{le^2}$$

le = Effective length

$$le = \frac{l}{\sqrt{2}}$$

$$le = \frac{3000}{\sqrt{2}}$$

$$= \frac{\pi^2 \times (2 \times 10^5) \times 30.68 \times 10^4}{\left(\frac{3000}{\sqrt{2}}\right)^2}$$

$$\boxed{P = 134576\ N}$$

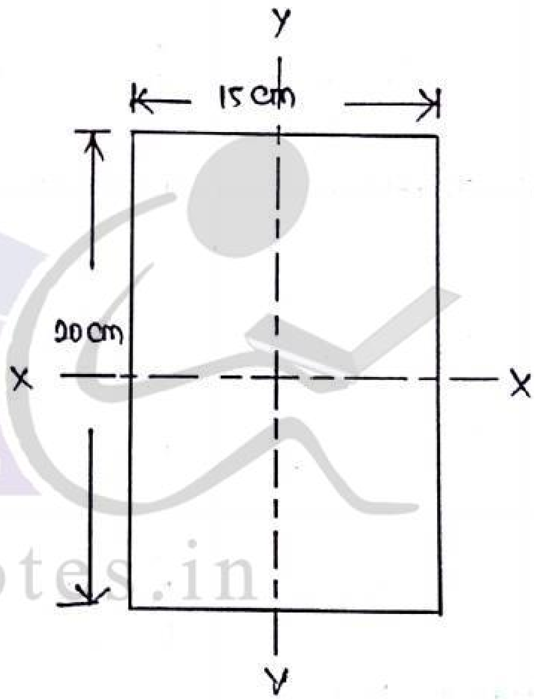
Prblm.no: 2

A Column of Timber section 15cm x 20cm is 6 metre long both ends being Fixed. If the young's modulus for Timber = 17.5 kN/m², Determine

(i) Crippling load

(ii) Safe Load for the Column if Factor of Safety = 3

Given data



Dimension of section = 15cm x 20 cm

Actual length (l) = 6m ⇒ 6000mm

young's modulus (E) = 17.5 kN/m²

Let,

P - Crippling load

Solution

$$P = \frac{\pi^2 EI}{Le^2}$$

$$Le = \frac{l}{2}$$ ∴ (Le - Effective length) When both the ends are Fixed

$$= \frac{600}{2}$$

$$\boxed{le = 3000\ mm}$$

I - least value of Moment of Inertia

Moment of Inertia of the section about $x-x$ axis

$$I_{xx} = \frac{bd^3}{12}$$

$$= \frac{15 \times 20^3}{12}$$

$$= 10000 \text{ cm}^4$$

$$\boxed{I_{xx} = 10000 \times 10^4 \text{ mm}^4}$$

Moment of Inertia of the section about $y-y$ axis

$$I_{yy} = \frac{db^3}{12}$$

$$= \frac{20 \times 15^3}{12}$$

$$= 5625 \text{ cm}^4$$

$$\boxed{I_{yy} = 5625 \times 10^4 \text{ mm}^4}$$

$\therefore$ I_{yy} is less than I_{xx}

- Therefore the column will tend to buckle in $y-y$ direction

- The value of 'I' will be least value of the two Moment of Inertia

$$I = 5625 \times 10^4 \text{ mm}^4$$

Sub. the values of $I = 5625 \times 10^4$ mm^4 and $L = 2000$ mm in equtn ①

$$P = \frac{\pi^2 \times 17.5 \times 5625 \times 10^4}{2000}$$

$$\boxed{P = 1079.48 \text{ kN}}$$

(ii) Safe Load for the column

Factor of safety = 3.0

$$\text{Safe Load} = \frac{\text{crippling load}}{\text{Factor of Safety}}$$

$$= \frac{1079.48}{3}$$

$$= 359.8$$

$$\boxed{\text{Safe Load} = 360\ kN}$$

Problm. no: 4

A Hollow mild steel tube 6m long 4cm internal diameter and 5mm thick is used as a strut with both ends hinged. Find the crippling load and safe load taking factor of safety as 3. Taking $E = 2\times10^5\ N/mm^2$

Given data

Length of Tube $(l) = 6m \Rightarrow 600\ cm$

Internal dia $(d) = 4\ cm$

Thickness $(t) = 5mm \Rightarrow 0.5cm$

External dia $(D) = d + 2t$

$= 4 + 2\times 0.5$

$= 4+1$

$$\boxed{D = 5cm}$$

Young's Modulus $(E) = 2\times10^5\ N/mm^2$

Factor of safety = 3

Moment of Inertia of section $= \frac{\pi}{64}(D^4 - d^4)$

$= \frac{\pi}{64}[5^4 - 4^4]$

$= \frac{\pi}{64}[625 - 256]$

$= 18.11 \text{ cm}^4$

$= 18.11 \times 10^4 \text{ mm}^4$

Both Ends of the strut are Hinged

Effective length $(le) = l$

$= 600 \text{ cm}$

$le = 6000 \text{ mm}$

Let,

P - Crippling Load

Solution

$$P = \frac{\pi^2 E I}{le^2}$$

$$= \frac{\pi^2 \times 2 \times 10^5 \times 18.11 \times 10^4}{6000^2}$$

$$= 9929.9$$

$$\boxed{P = 9930 \text{ N}}$$

$$\text{Safe Load} = \frac{\text{Crippling Load}}{\text{Factor of Safety}}$$

$$= \frac{9930}{3}$$

$$\boxed{\text{Safe Load} = 3310\ N}$$

Prblm.no:5

A Simply Supported beam of length 4 metre is subjected to a Uniformly distributed load of 30 kN/m over the whole span and deflects 15mm at the centre. Determine the crippling loads when this beam is used as a column with the following conditions

(i) One end fixed and other end hinged

(ii) Both the end pin jointed

Given data.

Length $(l) = 4m \Rightarrow 4000mm$

Uniformly distributed Load $(\omega) = 30\ kN/m \Rightarrow 30{,}000\ N/m$

$$= \frac{30{,}000}{1000}\ N/mm$$

$$\omega = 30\ N/mm$$

Deflection at centre $\delta = 15mm$

Solution

Simply supported beam, carrying U.D.L over the whole span the deflection at the centre

$$\delta = \frac{5}{384} \times \frac{\omega \times L^4}{EI}$$

$$15 = \frac{5}{384} \times \frac{3 \times 4000^4}{EI}$$

$$EI = \frac{5}{384} \times \frac{30 \times 4000^4}{15}$$

$$= \frac{5}{384} \times \frac{3 \times 256}{15} \times 10^{13}$$

$$EI = \frac{2}{3} \times 10^{13} \text{ N mm}^2$$

(i) Crippling load when the beam is used as a column with one end fixed and other end hinged.

The crippling load 'P' for this case in terms of Actual length

$$P = \frac{2\pi^2 \times EI}{l^2} \quad \therefore (l - \text{Actual length})$$

$$= \frac{2 \times \pi^2 \times \frac{2}{3} \times 10^{13}}{4000^2}$$

$$\boxed{P = 8224.5 \text{ KN}}$$

(ii) Crippling load when both the end are pin-jointed

$$P = \frac{\pi^2 \times EI}{l^2}$$

$$= \frac{\pi^2 \times \frac{2}{3} \times 10^{13}}{4000^2}$$

$$\boxed{P = 4112.25 \text{ KN}}$$

Prblm.no:6

A Solid round bar 4m long and 5cm in diameter was found to extend 4·6mm under a Tensile load of 50 KN. This bar is used as a strut with both ends hinged. Determine the buckling load for the bar and also the safe load taking factor of safety as 4·0

Given data

Actual length of bar $(L) = 4m \Rightarrow 4000\,mm$

Dia. of bar $(d) = 5cm$

Area of bar $(A) = \frac{\pi d^2}{4} \Rightarrow \frac{\pi \times (5)^2}{4}$

$A = 19.62\,cm^2 \Rightarrow 19.62 \times 10^2$

$A = 1962\,mm^2$

Extension of bar $(\delta L) = 4.6mm$

Tensile Load $(W) = 50\,KN \Rightarrow 50 \times 1000$

$W = 50{,}000\,N$

Solution

The value of young's modulus (E) is not given.

$$\text{Young's modulus } (E) = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

$$= \frac{\text{Tensile load / Area}}{\text{Extension of bar / Length of bar}}$$

$$= \frac{W/A}{\delta L/L}$$

$\therefore$ Stress $= \frac{\text{Load}}{\text{Area}}$

$\therefore$ Strain $= \frac{\delta L}{L}$

$$= \frac{W}{A} \times \frac{L}{\delta L}$$

$$= \frac{50000}{1963} \times \frac{4000}{4.6}$$

$$\boxed{E = 2.2148 \times 10^4 \ N/mm^2}$$

Strut is hinged at its both ends

Effective length (l_e) = Actual length (l)

$l_e = 4000$ mm

Buckling load

$$P = \frac{\pi^2 EI}{l_e^2}$$

$$= \frac{\pi^2 \times 2.214 \times 10^4 \times \frac{\pi}{64} \times 5^4 \times 10^4}{4000 \times 4000}$$

$$= 4189.99$$

$$\boxed{P = 4190 \ N}$$

Safe Load

$$\text{Safe Load} = \frac{\text{Crippling Load}}{\text{Factor of Safety}}$$

$$= \frac{4190}{4}$$

$$\boxed{\text{Safe load} = 1047.5\ N}$$

Prblm. no: 7

A Hollow alloy tube 5m long with external and internal diameter 40mm and 25mm respectively was found to extend 6.4mm under a tensile load of 60 kN. Find the buckling load for the tube when used as a column with end both pinned. Also find the safe load for the tube, taking a factor of safety = 4

Given data

Actual length $(L) = 5m \Rightarrow 5000\ mm$

External. dia $(D) = 40mm$

Internal. dia $(d) = 25mm$

Extension $(\delta L) = 6.4\ mm$

Tensile load $(W) = 60\ kN \Rightarrow 60 \times 1000\ N$

$W = 60,000\ N$

Safety factor $= 4$

Solution

Area $(A) = \frac{\pi}{4}(D^2 - d^2)$

$= \frac{\pi}{4}(40^2 - 25^2)$

$A = 766\ mm^2$

Moment of Inertia $(I) = \frac{\pi}{64}(D^4 - d^4)$

$= \frac{\pi}{64}(40^4 - 25^4)$

$$= \frac{\pi}{64}(2560000 - 390625)$$

$$= \frac{\pi}{64} \times 2169375$$

$$I = 106500 \text{ mm}^4$$

Solution

Young's Modulus

$$E = \frac{\text{Tensile Stress}}{\text{Tensile Strain}}$$

$$= \frac{\left(\frac{W}{A}\right)}{\left(\frac{\delta L}{L}\right)}$$

$$= \frac{\left(\frac{60{,}000}{766}\right)}{\left(\frac{6.4}{5000}\right)}$$

$$= \frac{60{,}000}{766} \times \frac{5000}{6.4}$$

$$\boxed{E = 6.11945 \times 10^4 \text{ N/mm}^2}$$

Column is pinned at both the Ends

$$\text{Effective length } (Le) = \text{Actual length } (L)$$

$$Le = L$$

$$\boxed{Le = 5000 \text{ mm}}$$

Buckling load

$$P = \frac{\pi^2 E I}{Le^2}$$

$$= \frac{\pi^2 \times 6.11945 \times 10^4 \times 106500}{5000^2}$$

$$\boxed{P = 2573\ N}$$

Safe Load

$$\text{Safe Load} = \frac{\text{Buckling Load}}{\text{Factor of safety}}$$

$$= \frac{2573}{4}$$

$$\boxed{\text{Safe Load} = 643.2\ N}$$

Prblm-no: 8

Calculate the safe compressive load on a hollow cast iron column (one end rigidly fixed and other hinged) of 15cm external diameter, 10 cm internal diameter and 10m in length. Use Euler's formula with a Factor of safety of 5 and E = 95 kN/mm²

Given data

External dia (D) = 15 cm

Internal dia (d) = 10 cm

Actual length of column (l) = 10 m ⇒ 10000 mm

Factor of safety = 5

Young's modulus (E) = 95 kN/mm² ⇒ 95000 N/mm²

Moment of Inertia of hollow column

$$(I) = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi^2 \times 6.11945 \times 10^4 \times 106500}{5000^2}$$

$$\boxed{P = 2573 \text{ N}}$$

Safe Load

$$\text{Safe Load} = \frac{\text{Buckling Load}}{\text{Factor of safety}}$$

$$= \frac{2573}{4}$$

$$\boxed{\text{Safe Load} = 643.2 \text{ N}}$$

Prblm.no: 8

Calculate the safe compressive load on a hollow cast iron column (one end rigidly fixed and other hinged) of 15cm external diameter, 10 cm internal diameter and 10m in length. Use Euler's formula with a Factor of safety of 5 and $E = 95 \text{ kN/mm}^2$

Given data

External dia $(D) = 15$ cm

Internal dia $(d) = 10$ cm

Actual length of column $(l) = 10$ m $\Rightarrow 10000$ mm

Factor of safety $= 5$

Young's modulus $(E) = 95 \text{ kN/mm}^2 \Rightarrow 95000 \text{ N/mm}^2$

Moment of Inertia of hollow column

$$(I) = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}(15^4 - 10^4)$$

$$= \frac{\pi}{64}(50625 - 10000)$$

$$= 1994.175 \text{ cm}^4$$

$$I = 1994.175 \times 10^4 \text{ mm}^4$$

One end of the column is fixed and other end is hinged

$$\text{Effective length } (l_e) = \frac{l}{\sqrt{2}}$$

$$= \frac{10000}{\sqrt{2}}$$

Let,

P - Crippling load

Solution

$$P = \frac{\pi^2 EI}{l_e^2}$$

$$= \frac{\pi^2 \times 95000 \times 1994.175 \times 10^4}{\left(\frac{1000}{\sqrt{2}}\right)^2}$$

$$= \frac{\pi^2 \times 95000 \times 1994.175 \times 10^4}{10000 \times 10000}$$

$$= 372950 \text{ N}$$

$$\boxed{P = 372.95 \text{ kN}}$$

$$\text{Safe load} = \frac{\text{Crippling load}}{\text{F.O.S}}$$

$$= \frac{372.95}{5}$$

$$= 74.79\ \text{KN}$$

Problem no: 9

Determine Euler's crippling load for an I - section joist 40 cm × 20 cm × 1 cm and 5 m long which is used as a strut with both end fixed. Take young's modulus for the joist as $2.1 \times 10^5\ N/mm^2$.

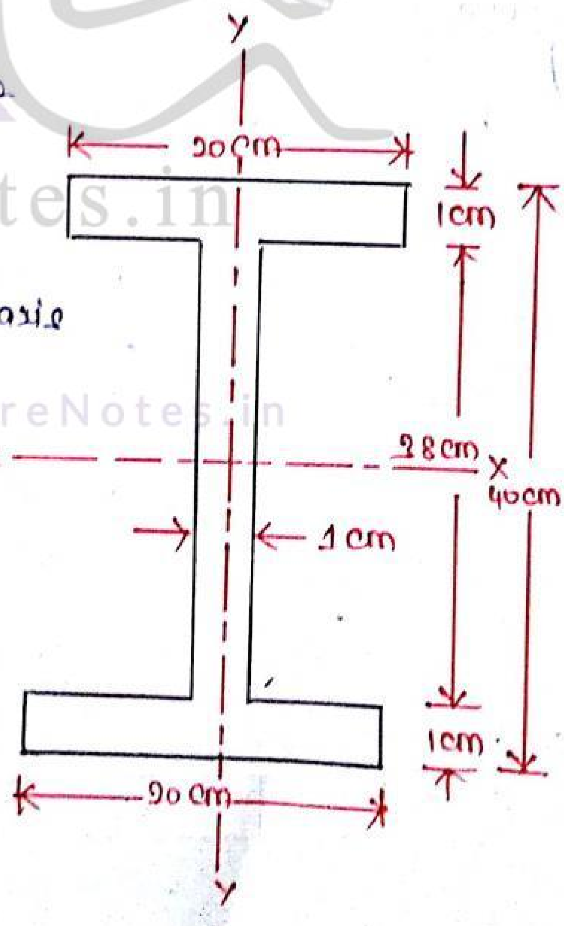
Given data

Dimension of I - section = 40 cm × 20 cm × 1 cm

Actual length (l) = 5 m ⇒ 5000 mm

young's modulus $(E) = 2.1 \times 10^5\ N/mm^2$

Solution



Moment of Inertia of the section about X - X axis

$$I_{xx} = \frac{20 \times 40^3}{12} - \frac{(20-1) \times (40-2\times1)^3}{12}$$

$$= 19786\ cm^4$$

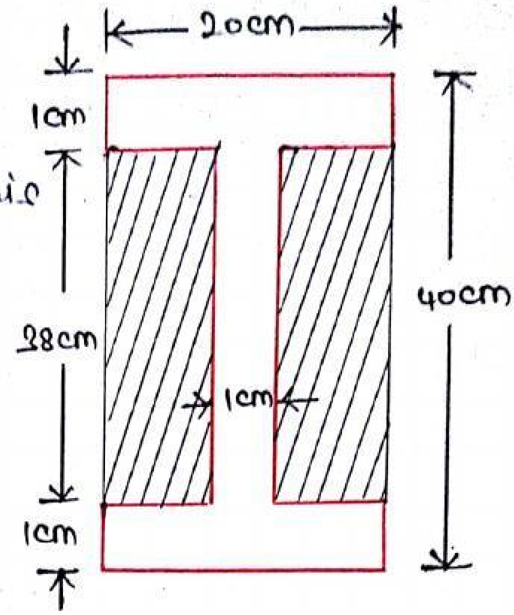
$$\boxed{I_{xx} = 19786 \times 10^4\ mm^4}$$

$|||^{ly}$ Moment of Inertia of the section about Y - Y axis

$$I_{yy} = \frac{38 \times (1)^3}{12} + \left(2 \times \frac{1 \times 20^3}{12}\right)$$

$= 3.166 + 1333.33$

$$\boxed{I_{yy} = 1336.5\ cm^4}$$



least value of the MOI is about y-y axis

$$\boxed{I = 1336.5 \times 10^4\ mm^4}$$

Both the end of the strut are fixed

Effective length $(le) = \frac{l}{2}$

$= \frac{5000}{2}$

$$\boxed{le = 2500mm}$$

Crippling Load

$$P = \frac{\pi^2 EI}{le^2}$$

$$= \frac{\pi^2 \times 2.1 \times 10^5 \times 1336.5 \times 10^4}{2500^2}$$

$= 4432080\ N$

$$\boxed{P = 4.432\ MN}$$

Prblm. no: 10

Determine the crippling load for a T-section of dimensions 10cm x 10cm x 2cm and of length 5m when it is used as strut with both of its ends hinged. Take Young's modulus $E = 2 \times 10^5\ N/mm^2$

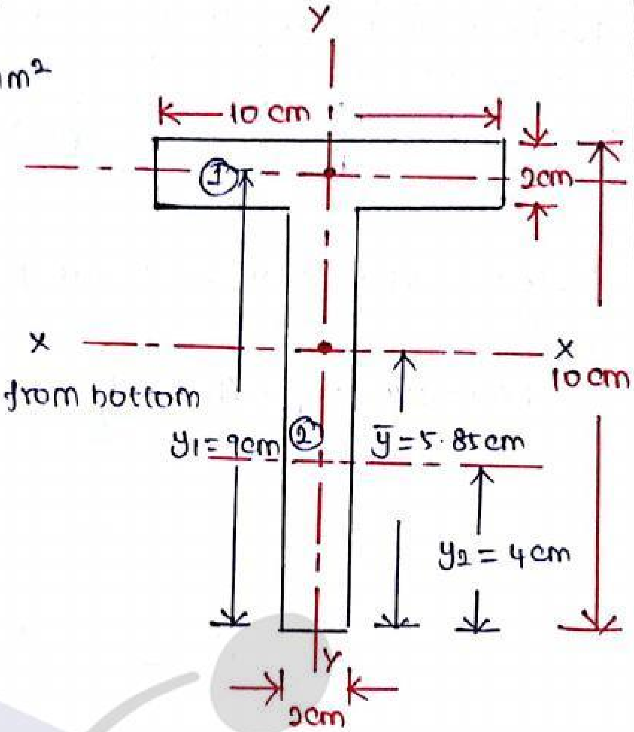
Given data

Dimension of T - section = 10 cm × 10 cm × 2 cm

Actual length (l) = 5 m ⇒ 5000 mm

Young's modulus (E) = 2.0×10^5 N/mm^2

Solution



Symmetrical about the axis Y-Y

let $\bar{y}$ = Distance of C.G of the section from bottom end.

Flange

Area of Flange (a_1) = 10×2

$$\boxed{a_1 = 20 \text{ cm}^2}$$

y_1 - Distance of C.G of area a_1 from the bottom

$$= 8 + \frac{2}{2}$$

$$= 8 + 1$$

$$\boxed{y_1 = 9 \text{ cm}}$$

Web

Area of Web (a_2) = 7×1

$$\boxed{a_2 = 7 \text{ cm}^2}$$

y_2 - Distance of C.G of area a_2 from the bottom end

$$y_2 = \frac{8}{2}$$

$$\boxed{y_2 = 4 \text{ cm}}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{(20 \times 9) + (16 \times 4)}{20 + 16}$$

$$= \frac{180 + 64}{36}$$

$$\boxed{\bar{y} = 6.777\,\text{cm}}$$

Moment of Inertia about the axis X – X

$$I_{xx} = \left[\frac{bd^3}{12} + a_1 \times (y_1 - \bar{y})^2 + \frac{bd^3}{12} + a_2 \times (\bar{y} - y_2)^2 \right]$$

$$= \left[\frac{10 \times 8^3}{12} + 20 \times (9 - 6.777)^2 \right] + \left[\frac{2 \times 8^3}{12} + 16 \times (6.777 - 4)^2 \right]$$

$$= (6.667 + 98.834) + (85.333 + 123.387)$$

$$\boxed{I_{xx} = 314.221\,\text{cm}^4}$$

Moment of Inertia about the axis Y - Y

$$I_{yy} = \left[\frac{db^3}{12} + a_1 \times (x_1 - \bar{x})^2 \right] + \left[\frac{db^3}{12} + a_2 \times (x_2 - \bar{x})^2 \right]$$

$$= \frac{2 \times 10^3}{12} + 20\,(0 - 0)^2 + \frac{8 \times 2^3}{12} + 16\,(0 - 0)^2$$

$$= \frac{2 \times 10^3}{12} + \frac{8 \times 2^3}{12}$$

$$= 166.67 + 5.33$$

$$\boxed{I_{yy} = 172\ cm^4}$$

Least value of Moment of Inertia is about y-axis

$$\boxed{I = I_{yy} = 839160\ mm^4}$$

Strut is fixed at both ends

Effective length $(l_e) = l$

$$\boxed{l_e = 5000\ mm}$$

Crippling Load

$$P = \frac{\pi^2 EI}{l_e^2}$$

$$= \frac{\pi^2 \times (2\times10^5) \times (172\times10^4)}{5000^2}$$

$$\boxed{P = 135805.4\ N}$$

Prblm.no: 11

Using Euler's formula, calculate the critical stresses for a series of struts having slenderness ratio of 40, 80, 120, 160 and 200 under the following conditions

(i) Both end Hinged

(ii) Both end Fixed

Given data

Solution

(i) Critical stresses when both ends hinged

Slenderness ratio $\left(\frac{l}{k}\right) = 40, 80, 120, 160$ and 200

The Critical stress or crippling stress

$$\text{Crippling Stress} = \frac{\pi^2 \times E}{\left(\frac{le}{K}\right)^2}$$

(le - Effective length)

Both End Hinged

$le = l$ ∴ (l - Actual length)

$$\text{Critical stress} = \frac{\pi^2 \times E}{\left(\frac{l}{K}\right)^2}$$

When $\frac{l}{K} = 40$, the critical stress

$$= \frac{\pi^2 \times E}{40^2}$$

$$= \frac{\pi^2 \times 2.05 \times 10^5}{1600}$$

$$= 1264.54 \text{ N/mm}^2$$

When $\frac{l}{K} = 80$, the critical stress

$$= \frac{\pi^2 \times E}{80^2}$$

$$= \frac{\pi^2 \times 2.05 \times 10^5}{6400}$$

$$= 316.135 \ N/mm^2$$

When $\frac{l}{K} = 120$, the critical stress becomes

$$= \frac{\pi^2 \times E}{120^2}$$

$$= \frac{\pi^2 \times 2.05 \times 10^5}{14400}$$

$$= 140.5 \ N/mm^2$$

When $\frac{l}{K} = 160$, the critical stress

$$= \frac{\pi^2 \times E}{160}$$

$$= \frac{\pi^2 \times 2.05 \times 10^5}{25600}$$

$$= 79.02 \ N/mm^2$$

When $\frac{l}{K} = 200$, the critical stress

$$= \frac{\pi^2 \times E}{200^2}$$

$$= \frac{\pi^2 \times 2.05 \times 10^5}{40000}$$

$$= 50.58 \ N/mm^2$$

(ii) Critical stress when both ends fixed

$$\text{crippling or critical stress} = \frac{\pi^2 \times E}{\left(\frac{le}{K}\right)^2}$$

When both ends fixed

$$le = \frac{l}{2}$$

$$\text{critical stress} = \frac{\pi^2 \times E}{\left(\frac{l}{2K}\right)^2} = \frac{4\pi^2 \times E}{\left(\frac{l}{K}\right)^2}$$

When $\frac{l}{K} = 40$, the critical stress

$$= \frac{4\pi^2 \times E}{\left(\frac{l}{K}\right)^2}$$

$$= \frac{4 \times \pi^2 \times 2.05 \times 10^5}{40^2}$$

$$= 5058.16 \text{ N/mm}^2$$

When $\frac{l}{K} = 80$, the critical stress

$$= \frac{4\pi^2 \times E}{\left(\frac{l}{K}\right)^2}$$

$$= \frac{4 \times \pi^2 \times 2.05 \times 10^5}{80^2}$$

$$= 1264.54 \text{ N/mm}^2$$

When $\frac{l}{K} = 120$, the critical stress

$$= \frac{4\pi^2 \times E}{120^2}$$

$$= \frac{4 \times \pi^2 \times 2.05 \times 10^5}{120^2}$$

$$= 562.02 \text{ N/mm}^2$$

When $\frac{l}{K} = 160$, the critical stress becomes

$$= \frac{4\pi^2 \times E}{160^2}$$

$$= \frac{4\pi^2 \times 2.05 \times 10^5}{160^2}$$

$$= 316.125 \text{ N/mm}^2$$

When $\frac{l}{K} = 200$, the critical stress

$$= \frac{4\pi^2 \times E}{200^2}$$

$$= \frac{4\pi^2 \times 2.05 \times 10^5}{200^2}$$

$$= 202.32 \text{ N/mm}^2$$

RANKINE'S FORMULA

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E} \rightarrow (1)$$

Where,

P - Crippling load by Rankine's formula

P_c - crushing Load $\Rightarrow (\sigma_c \times A)$

σ_c - Ultimate crushing stress

A - Area of cross section

P_E - Crippling load by Euler's formula.

$$P_E = \frac{\pi^2 EI}{l_e^2}$$

$\therefore$ (l_e - Effective length)

For a given column material

- Crushing stress σ_c is a constant
- crushing load $(P_c) = \sigma_c \times A$ will also be constant

In equation (1) P_c is constant

- The value of P depends upon the value of P_E
- The value of P_E depends upon the Effective length.

* If the column is short, which means the value of l_e is small, then the value of P_E will be large.

* Hence the value of $\frac{1}{P_E}$ will be small enough and is negligible as compared to the value of $\frac{1}{P_c}$

Neglecting the value of $\frac{1}{P_E}$ in equtn (1)

$$\frac{1}{P} \to \frac{1}{P_c}$$

(or)

$$P \to P_c$$

Hence the crippling load by Rankine's formula for a short column is approximately equal to crushing load.

* If the column is long, which means the values of Le is large.

Then the value of P_E will be small

* Hence the Rankine's formula $\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E}$ Large enough compared with $\frac{1}{P_c}$

Neglecting the value of $\frac{1}{P_c}$ in Rankine's formula equating

$$\frac{1}{P} \to \frac{1}{P_E}$$

(or)

$$P \to P_E$$

Hence the crippling load by Rankine's formula for long columns is approximately equal to to crippling load given by Euler's formula.

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E}$$

$$\frac{P_E + P_c}{P_c \times P_E}$$

Taking Reciprocal both sides

$$P = \frac{P_c \cdot P_E}{P_E + P_c}$$

$$= \frac{P_c}{1 + \frac{P_c}{P_E}}$$

$$= \frac{\sigma_c \times A}{1 + \frac{\sigma_c \cdot A}{\left(\frac{\pi^2 \cdot EI}{Le^2}\right)}}$$

$\therefore (P_c = \sigma_c \times A)$

$\therefore \left(P_E = \frac{\pi^2 \cdot EI}{Le^2}\right)$

$I = AK^2$

K - least radius of Gyration

The above equation become as

$$P = \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c \cdot A \cdot Le^2}{\pi^2 \cdot E \cdot AK^2}}$$

$$= \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c}{\pi^2 \cdot E} \cdot \left(\frac{Le}{K}\right)^2}$$

$$P = \frac{\sigma_c \cdot A}{1 + a\left(\frac{Le}{K}\right)^2} \rightarrow (2)$$

Where,

$a = \frac{\sigma_c}{\pi^2 E}$ and is known as Rankine's constant.

Equtn (2) gives crippling load by Rankine's Formula

* The value of 'a' is taken from the results of the experiments and is not calculated from the values of σ_c and E.

* The value of σ_c and a for different columns materials

S.NO	MATERIAL	σ_c in N/mm^2	a
1	Wrought Iron	250	$\frac{1}{9000}$
2	Cast Iron	550	$\frac{1}{1600}$
3	Mild Steel	320	$\frac{1}{7500}$
4	Timber	50	$\frac{1}{750}$

Prblm.no:12

The External and Internal diameter of a hollow cast iron column are 5cm and 4cm respectively. If the length of this column is 3 m and both of its ends are fixed, Determine the crippling load using Rankine's formula. Take the values of $\sigma_c = 550\ N/mm^2$ and $a = \frac{1}{1600}$ in Rankine's formula.

Given data

External dia(D) = 5 cm

Internal dia (d) = 4 cm

$$\text{Area}(A) = \frac{\pi}{4}(D^2 - d^2)$$

$$= \frac{\pi}{4}(5^2 - 4^2)$$

$$= 7.068\ cm^2 \Rightarrow 7.068 \times 10^2$$

$$A = 706.8\ mm^2$$

$$\text{Moment of Inertia}(I) = \frac{\pi}{64}[D^4 - d^4]$$

$$= \frac{\pi}{64}[5^4 - 4^4]$$

$$= 18.1132\ cm^4 \Rightarrow 18.1132 \times 10^4$$

$$I = 188132\ mm^4$$

Least Radius of Gyration

$$K = \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{181132}{706.8}}$$

$$K = 16.008\ mm$$

Length of column $(l) = 2m \Rightarrow 2000\ mm$

Let,

P - crippling load by Rankine's Formula

crushing stress $(\sigma_c) = 550\ N/mm^2$

Rankine's constant $(a) = \frac{1}{1600}$

Solution

Both the ends are fixed

Effective length $l_e = \frac{l}{2}$

$= \frac{3000}{2}$

$$\boxed{l_e = 1500mm}$$

Crippling Load by Rankine's Formula

$$P = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l_e}{k}\right)^2}$$

$$= \frac{550 \times 706.8}{1 + \left(\frac{1}{1600}\right) \times \left(\frac{1500}{16.009}\right)^2}$$

$$= \frac{353400}{6.4876}$$

$$\boxed{P = 54473.14 \text{ N}}$$

Prblm. no: 12

A Hollow cylindrical cast Iron column is 4m long with both end fixed. Determine the minimum diameter of the column if it has to carry a safe load of 250 kN with a factor of safety of 5. Take the Internal diameter as 0.8 times the external diameter. Take $\sigma_c = 550 N/mm$ and $a = \frac{1}{1600}$ in Rankine's formula.

Given data

length of column $(l) = 4m \Rightarrow 4000mm$

End conditions = Both end Fixed

Effective length $(le) = \frac{l}{2}$

$= \frac{4000}{2}$

$le = 2000mm$

Safe load = 250 KN

Factor of safety = 5

crushing stress $(\sigma_c) = 550 \ N/mm^2$

$a = \frac{1}{1600}$ in Rankine's formula

Let

External dia = D

Internal dia (d) = $0.8 \times D$

Area of column (A) = $\frac{\pi}{4}[D^2 - d^2]$

$$= \frac{\pi}{4}\left[D^2 - (0.8D)^2\right]$$

$d = 0.8 \times D$

$$= \frac{\pi}{4}\left[D^2 - 0.64 D^2\right]$$

$$= \frac{\pi}{4} \times 0.36 D^2$$

$$\boxed{A = 0.28 D^2}$$

Moment of Inertia $\Big\}$ $I = \frac{\pi}{64}\left[D^4 - d^4\right]$

$$= \frac{\pi}{64}\left[D^4 - (0.8D)^4\right]$$

$$= \frac{\pi}{64}\left[D^4 - 0.84 \times D^4\right]$$

$$= \frac{\pi}{64}\left[D^4 - 0.4096 D^4\right]$$

$$= \frac{\pi}{64} \times 0.5904 D^4$$

$$\boxed{I = 0.0289 D^4}$$

$I = AK^2$

Where,

K - Radius of Gyration

$$K = \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{0.0287 D^4}{0.28 D^2}}$$

$$\boxed{K = 0.32 D}$$

Solution

$$P = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l_e}{K}\right)^2}$$

$$1250000 = \frac{550 \times 0.28 D^2}{1 + \frac{1}{1600} \times \left(\frac{2000}{0.32 D}\right)^2}$$

$$\frac{1250000}{550 \times 0.28} = \frac{D^2}{1 + \frac{24414}{D^2}}$$

$$8116.88 D^2 + 8116.88 \times 24414 = D^4$$

$$D^4 - 8116.88 D^2 - 8116.88 \times 24414 = 0$$

$$D^4 - 8116.88 D^2 - 198165508.3 = 0$$

The above equation is a Quadratic equation in D^2

$$\therefore \left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

$$D^2 = \frac{8116.88 \pm \sqrt{8116.88 + 4 \times 1 \times 198165508.3}}{2 \times 1}$$

$$= \frac{8116.88 \pm 28154.39}{2}$$

$$= \frac{8116.88 + 28154.39}{2}$$

$$= 18135.44 \text{ mm}^2$$

$$= \sqrt{18135.44}$$

$$\boxed{D = 134.66 \text{ mm}}$$

External dia (D) = 134.66 mm

Internal dia (d) = 0.8 × 134.66

$$\boxed{d = 107.7 \text{ mm}}$$

Prblm.no : 14

A 1.5 m long column has a circular cross section of 5 cm diameter. One of the ends of the column is fixed in direction and position and other end is free. Taking Factor of safety as 3, Calculate the safe load using.

(a) Rankine's formula, taking yield stress $\sigma_c = 560$ N/mm^2 and $a = \frac{1}{1600}$ for pinned ends.

(b) Euler's formula, young's modulus for C.I $= 1.2 \times 10^5$ N/mm^2

Given data

Length $(l) = 1.5\,m \Rightarrow 1500\,mm$

Diamoter $(d) = 5\,cm$

Area $(A) = \frac{\pi}{4} \times d^2 \Rightarrow \frac{\pi}{4} \times (5)^2$

$= 19.635\,cm^2$

$A = 19.635 \times 10^2\,mm^2$

Moment of Inertia $(I) = \frac{\pi}{64} \times D^4$

$= \frac{\pi}{64} \times 5^4$

$= 30.7\,cm^4$

$= 30.7 \times 10^4\,mm^4$

least radius of Gyration

$$K = \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{30.7 \times 10^4}{19.635 \times 10^2}}$$

$$K = 12.5\,mm$$

End condition $\rightarrow$ one end is fixed and other end is free

Effective length $l_e = 2l$

$= 2 \times 1500$

$l_e = 3000\,mm$

Factor of Safety = 3

Solution

$$P = \frac{\pi^2 EI}{l_e^2} \rightarrow \text{wrong}$$

(i) Safe Load by Rankine's Formula

$$P = \frac{\sigma_c \times A}{1 + a\left(\frac{l_e}{k}\right)^2}$$

$$= \frac{560 \times 1963.5}{1 + \frac{1}{1600} \times \left(\frac{3000}{12.5}\right)^2}$$

$$\boxed{P = 29708.1 \text{ N}}$$

(ii) Safe Load by Euler's formula

$$P = \frac{\pi^2 EI}{l_e^2}$$

$$= \frac{\pi^2 \times 1.2 \times 10^5 \times (30.7 \times 10^4)}{(3000)^2}$$

$$\boxed{P = 40200 \text{ N}}$$

$$\text{Safe load} = \frac{\text{crippling load}}{\text{Factor of Safety}}$$

$$= \frac{40200}{3}$$

$$\boxed{\text{Safe Load} = 13400 \text{ N}}$$

Prblm.no: 15

A Short length of tube, 4 cm Internal diameter and 5 cm external diameter, failed by compression at a load of 240 kN. When a 2 metre length of the same tube was tested as a Strut with fixed ends, the load at failure was 158 kN. Assuming that σ_c in Rankine's formula is given by the first test, find the value of the constant a in the same formula. What will be the crippling load of this tube if it is used as a strut 3 m long with one end fixed and the other hinged?

Given data

External diameter $(D) = 5\,cm$

Internal diameter $(d) = 4\,cm$

$$\text{Area } (A) = \frac{\pi}{4}(D^2 - d^2)$$

$$= \frac{\pi}{4}(5^2 - 4^2)$$

$$= 7.068\ cm^2$$

$$A = 7.068 \times 10^2\ mm^2$$

$$\text{Moment of Inertia } (I) = \frac{\pi}{64}[D^4 - d^4]$$

$$= \frac{\pi}{64}[5^4 - 4^4]$$

$$= \frac{\pi}{64}[625 - 256]$$

$$I = 18.113\ cm^4$$

$= 18.113 \times 10^4 \ mm^4$

$I = 181130 \ mm^4$

Least radius of Gyration

$$K = \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{181130}{706.8}}$$

$$K = 16mm$$

Crushing Load (P) = 240 KN

The value of σ_c in Rankine's Formula by the crushing load of 240 KN

$$\sigma_c = \frac{\text{crushing load}}{\text{Area}}$$

$$= \frac{240 \cancel{\times 10^3}}{706.8}$$

$$\sigma_c = 0.3395 \ KN/mm^2$$

Length of the strut (l) = 2m ⇒ 2000mm

End conditions - Both the ends are Fixed.

Effective length (Le) = $\frac{l}{2}$

$$= \frac{2000}{2}$$

$$Le = 1000 \ mm$$

Crushing load of strut (P) = 158 kN

Solution

(i) Value of constant 'a'

$$P = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l_e}{k}\right)^2}$$

$$158 = \frac{0.32953 \times 706.8}{1 + a\left(\frac{1000}{16}\right)^2}$$

$$= \frac{232.92}{1 + 3906.25 \times a}$$

$$1 + 3906.25 \times a = \frac{232.92}{158}$$

$$= 1.5189$$

$$a = \frac{1.5189 - 1}{3906.25}$$

$$= 0.0001328$$

$$\boxed{a = \frac{1}{7530}}$$

(ii) Crippling load for the strut of length 3 m when one end is fixed and other is hinged

Actual length (l) = 3 m ⇒ 3000 mm

End conditions = one end fixed and other is hinged

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Effective length $(l_e) = \frac{l}{\sqrt{2}}$

$$= \frac{3000}{\sqrt{2}}$$

Crippling Load

$$P = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l_e}{k}\right)^2}$$

$$= \frac{0.33953 \times 706.8}{1 + \frac{1}{7530} \times \left(\frac{3000}{\sqrt{2} \times 16}\right)^2}$$

$$= \frac{0.33953 \times 706.8}{1 + 2.3344}$$

$$\boxed{P = 71.97 \text{ kN}}$$

Prblm.no:16

Find the Euler crushing load for a hollow cylindrical cast iron column 20 cm external diameter and 25 mm thick if it is 6 m long and is hinged at both ends. Take $E = 1.2 \times 10^5 \text{ N/mm}^2$

Compare the load with the crushing load as given by the Rankine's formula, taking $\sigma_c = 500 \text{ N/mm}^2$ and $a = \frac{1}{1600}$.

for what length of the column would these two formulae give the same crushing load ?

Given data

External diameter $(D) = 20\,cm$

Thickness $(t) = 25\,mm \Rightarrow 2.5\,cm$

Internal diameter $(d) = (D - 2 \times t)$

$= 20 - 2 \times 2.5$

$d = 15\,cm$

Area $(A) = \frac{\pi}{4}(D^2 - d^2)$

$= \frac{\pi}{4}(20^2 - 15^2)$

$= 137.44\,cm^2 \Rightarrow 137.44 \times 10^2\,mm^2$

$A = 13744\,mm^2$

Moment of Inertia $(I) = \frac{\pi}{64}[D^4 - d^4]$

$= \frac{\pi}{64}(20^4 - 15^4)$

$= \frac{\pi}{64}(160000 - 50625)$

$= 5668.93\,cm^4 \Rightarrow 5368.92 \times 10^4$

$I = 53689200\,mm^4$

Least radius of Gyration

$$K = \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{53689200}{13744}}$$

$k = 62.5 mm$

Length of column $(l) = 6m \Rightarrow 6000\ mm$

End condition $\rightarrow$ Both end are hinged

Effective length $(Le) = L$

$$Le = 6000mm$$

$$E = 1.2 \times 10^5\ N/mm^2$$

Solution

Crushing Load by Euler's formula

$$P = \frac{\pi^2 EI}{Le^2}$$

$$= \frac{\pi^2 \times 1.2 \times 10^5 \times 53689300}{(6000)^2}$$

$$\boxed{P = 1766307\ N}$$

Crushing Load by Rankine's formula

$$\sigma_c = 550\ N/mm^2$$

$$a = \frac{1}{1600}$$

$$P = \frac{\sigma_c \cdot A}{1 + a\left(\frac{Le}{K}\right)^2}$$

$$= \frac{550 \times 13744}{1 + \frac{1}{1600} \times \left(\frac{6000}{62.5}\right)^2}$$

$$\boxed{P = 118224.8\ N}$$

The length of the column for which the above two formulae gives the same crushing load.

let,

L - length of the column

Crushing Load by Euler's formula

$$= \frac{\pi^2 EI}{L^2}$$

Crushing load by Rankine's formula

$$= \frac{\sigma_c \cdot A}{1+a\left(\frac{L}{K}\right)^2}$$

Equating the Crushing Load

$$\frac{\pi^2 EI}{L^2} = \frac{\sigma_c \cdot A}{1+a\left(\frac{L}{K}\right)^2}$$

$$\frac{\pi^2 \times 1.2\times 10^5 \times 52689200}{L^2} = \frac{550 \times 12744}{1+\frac{1}{1600}\times\left(\frac{L}{62.5}\right)^2}$$

$$\frac{\pi^2 \times 1.2\times 10^5 \times 52689200}{550\times 12744} = \frac{L^2}{1+\frac{L^2}{6250000}}$$

$$8411800 = \frac{L^2}{1+\frac{L^2}{6250000}}$$

$$8411800\left(1+\frac{L^2}{6250000}\right) = L^2$$

$$8411800 + \frac{8411800 L^2}{6250000} = L^2$$

$$8411800 + 1.346 L^2 = L^2$$

$$1.346 L^2 - L^2 = -8411800$$

$$0.346 L^2 = -8411800$$

$$L = \sqrt{\frac{-8411800}{0.346}}$$

The above equation give the Imaginary Value of Length

∴.. Hence it is not possible to have the same length of columns, which have the same crushing load for the two given Formula.

Prblm.no: 17

A Hollow cast Iron column 200mm outside diameter and 150 mm inside diameter 8m Long has both End Fixed. It is subjected to an axial compressive load. Taking Factor of safety as 6, $\sigma_c = 560\ N/mm^2$, $a = \frac{1}{1600}$. Determine the safe Rankine load.

Given data

External dia (D) = 200mm

Internal dia (d) = 150mm

Length (l) = 8m ⇒ 8000mm

End conditions = Both the ends are Fixed

Crushing stress $\sigma_c = 560\ N/mm^2$

Rankine's constant $(a) = \frac{1}{1600}$

Safety Factor $= 6$

Area of cross section $(A) = \frac{\pi}{4}(D^2 - d^2)$

$= \frac{\pi}{4}(200^2 - 150^2)$

$= \frac{\pi}{4}(40000 - 22500)$

$A = 13744\ mm^2$

Moment of Inertia $(I) = \frac{\pi}{64}(D^4 - d^4)$

$= \frac{\pi}{64}(200^4 - 150^4)$

$= \frac{\pi}{64}(1600000000 - 506250000)$

$I = 53689000\ mm^4$

Least Radius of Gyration $(K) = \sqrt{\frac{I}{A}}$

$= \sqrt{\frac{53689000}{13744}}$

$K = 62.5mm$

Let,

P- crippling load by Rankine's formula.

Solution

$$P = \frac{\sigma_c \times A}{1 + a\left(\frac{L_e}{K}\right)^2}$$

$$L_e = \frac{L}{2}$$

$$= \frac{8000}{2}$$

$$= 4000mm$$

$$= \frac{560 \times 13744}{1 + \frac{1}{1600} \times \left(\frac{4000}{62.5}\right)^2}$$

$$= \frac{7696640}{1 + 2.56}$$

$$= \frac{7696640}{3.56}$$

$$= 2161977 N$$

$$\boxed{P = 2161.977 kN}$$

$$\text{Safe load} = \frac{\text{Crippling load}}{\text{Factor of safety}}$$

$$= \frac{2161.977}{6}$$

$$\boxed{\text{Safe load} = 360.3295 kN}$$

Columns with Eccentric Load

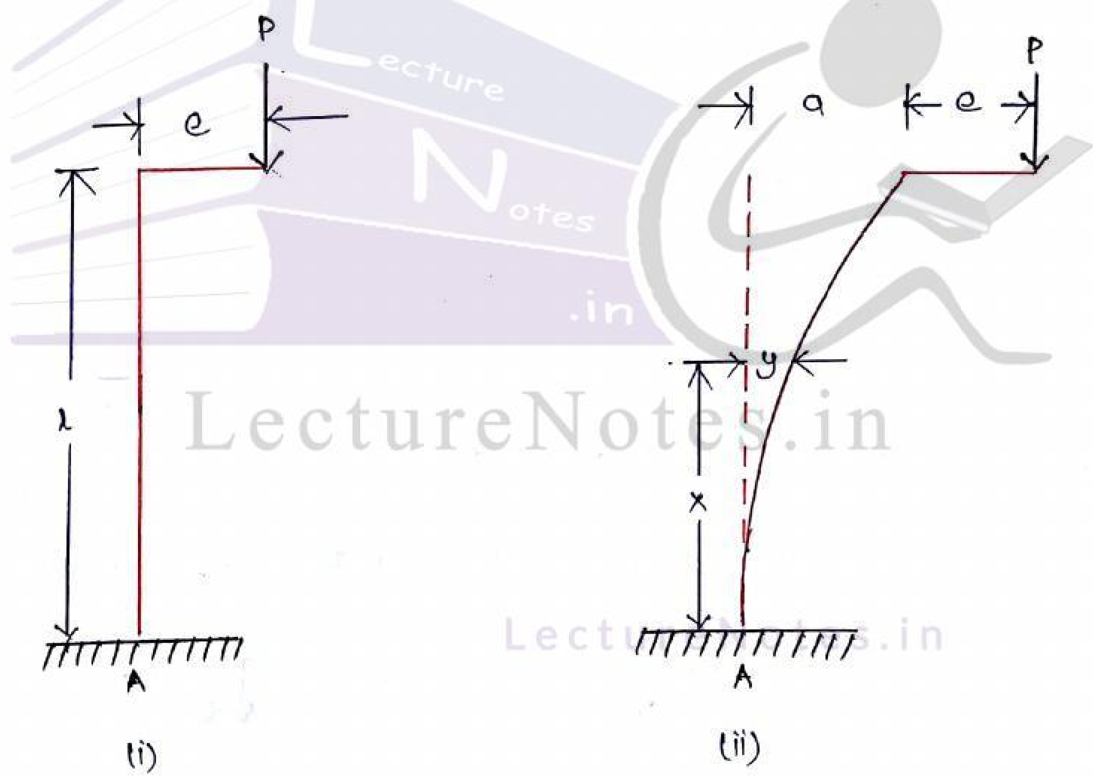
A column AB of length 'l' fixed at end 'A' and free at end 'B'. The column is subjected to a load 'P' which is eccentric by an amount of 'e'. The free end will sway side sway by an amount 'a' and the column will deflect.

a - Deflection at free end B

e - Eccentricity

A - Area of cross section of column.

Consider any section at a distance 'x' from the fixed end 'A'



We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\frac{M}{EI} = \frac{1}{R} \rightarrow (1)$$

y = Deflection

$$\frac{dy}{dx} = \text{Slope}$$

$$\frac{d^2y}{dx^2} = \text{Moment or Curvature} \rightarrow (2)$$

Sub ① & ②

$$\frac{M}{EI} = \frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$EI\frac{d^2y}{dx^2} = Mx - x$$ $\therefore$ (Moment = Load × Distance)

$$EI\frac{d^2y}{dx^2} = P \times (a+e-y)$$

$$EI\frac{d^2y}{dx^2} = P(a+e) - P \times y$$

$$EI\frac{d^2y}{dx^2} + P \times y = P(a+e)$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} \times y = \frac{P}{EI}(a+e)$$

Second degree differential equation

y = Complementary Solution + particular integral

$$y = C_1 \cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(x\sqrt{\frac{P}{EI}}\right) + a + e \rightarrow ③$$

<u>Boundary Condition</u>

* At point 'A'

$x = 0$, $y = 0$ and also $\frac{dy}{dx} = 0$

* At point 'B'

$x = l$, $y = 0$

Apply Boundary Condition

I^{st} Boundary Condition

From equtn (2)

$$0 = C_1 \cos\left(0 \times \sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(0 \times \sqrt{\frac{P}{EI}}\right) + a + e$$

$$0 = C_1(1) + C_2(0) + a + e$$

$$0 = C_1 + a + e$$

$$\boxed{C_1 = -(a+e)}$$

Differential equtn (2) with respect 'x' we get

$$\frac{dy}{dx} = -C_1\left[\sin\left(x \times \sqrt{\frac{P}{EI}}\right)\right] \times \sqrt{\frac{P}{EI}} + C_2\left[\cos\left(x \times \sqrt{\frac{P}{EI}}\right)\right] \times \sqrt{\frac{P}{EI}} + 0 \rightarrow (4)$$

$$0 = -C_1\left[\sin\left(0 \times \sqrt{\frac{P}{EI}}\right)\right] \times \sqrt{\frac{P}{EI}} + C_2\left[\cos\left(0 \times \sqrt{\frac{P}{EI}}\right)\right] \times \sqrt{\frac{P}{EI}}$$

$$= 0 + C_2(1) \times \sqrt{\frac{P}{EI}}$$

$$= C_2 \times \sqrt{\frac{P}{EI}}$$

$$\boxed{C_2 = 0}$$

$\therefore \left(\sqrt{\frac{P}{EI}} \rightarrow \text{cannot be zero}\right)$

Substituting the values of C_1 and C_2 in equtn (2)

$$y = -(a+e)\cos\left(x \times \sqrt{\frac{P}{EI}}\right) + 0 + a + e \rightarrow (5)$$

IInd Boundary Condition

$x = l, \; y = a$

From equn (5)

$$a = -(a+e)\cdot\cos\left(l \times \sqrt{\frac{P}{EI}}\right) + a + e$$

$$(a+e)\cdot\cos\left(l\times\sqrt{\frac{P}{EI}}\right) = \cancel{a} + e - \cancel{a}$$

$$(a+e)\cdot\cos\left(l\times\sqrt{\frac{P}{EI}}\right) = e$$

$$a+e = \frac{e}{\cos\left(l\times\sqrt{\frac{P}{EI}}\right)}$$

$$a+e = e\sec\left(l\times\sqrt{\frac{P}{EI}}\right) \rightarrow (6)$$

Maximum stress

Let,

Find the maximum compressive stress for the column section.

Due to eccentricity there will be bending stress and also direct stress.

Maximum Compressive Stress $\}$ = Direct stress + Bending stress

$$\sigma_{max} = \sigma_0 + \sigma_b$$

$$\therefore \left(\sigma_0 = \frac{P}{A}\right)$$

The maximum bending stress will be at the section where bending moment is maximum.

Bending moment is maximum at the fixed end 'A'

$$\text{Max. BM at A } (M) = P\times(a+e)$$

$$= P \times e \sec\left(l \times \sqrt{\frac{P}{EI}}\right)$$

Using Bending Equation

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\sigma_b = \frac{M}{I} \times y$$

$$= \frac{M}{\left(\frac{I}{y}\right)}$$

Section modulus

$$\therefore \left(Z = \frac{I}{y}\right)$$

$$= \frac{M}{Z}$$

$$\sigma_b = \frac{P \times e \cdot \sec\left(l \times \sqrt{\frac{P}{EI}}\right)}{Z}$$

Hence Maximum compressive stress

$$\sigma_{max} = \sigma_0 + \sigma_b$$

$$\boxed{\sigma_{max} = \frac{P}{A} + \frac{P \times e \cdot \sec\left(l \times \sqrt{\frac{P}{EI}}\right)}{Z}} \rightarrow (7)$$

* Equation (7) is used for One End fixed, Other end is free and load is eccentric to the column.

* In this equation l is the actual length of the column.

* The relation between actual length and Effective length for a column whose end is fixed and other end is free.

$$l_e = 2l$$

$$l = \frac{l_e}{2}$$

Sub. the value of 'l' in equ^n no: 7

$$\boxed{\sigma_{max} = \frac{P}{A} + \frac{P \times e \times \sec\left(\frac{l_e}{2} \times \sqrt{\frac{P}{EI}}\right)}{Z}}$$

Prblm. no: 18

A column of circular section is subjected to a load of 120 KN. The load is parallel to the axis but eccentric by an amount of 2.5mm. The external and internal diameters of columns are 60mm and 50mm respectively. If both the ends of the column are hinged and column is 2.1m long, then determine the maximum stress in the column. Take $E = 200\ GN/m^2$

Given data

Load (P) = 120 KN ⇒ 120×10^3 N

Eccentricity (e) = 2.5 mm ⇒ 2.5×10^{-3} m

D = 60 mm ⇒ 0.06m

d = 50 mm ⇒ 0.05m

l = 2.1m

Both ends are hinged

$$l_e = l$$

$L_e = 2.1 m$

Value of $E = 200 \, GN/m^2 \Rightarrow 200 \times 10^9 \, N/m^2$

Solution

Maximum stress

$$\sigma_{max} = \frac{P}{A} + \frac{P \times e \times \sec\left(\frac{L_e}{2} \times \sqrt{\frac{P}{EI}}\right)}{Z} \rightarrow (1)$$

Area of section (A)

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$= \frac{\pi}{4}(0.06^2 - 0.05^2)$$

$$= \frac{\pi}{4} \times 0.0011$$

$$A = 8.639 \times 10^{-4} \, m^2$$

Moment of Inertia (I)

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}(0.06^4 - 0.05^4)$$

$$= \frac{\pi}{64}(1.296 \times 10^{-5} - 0.625 \times 10^{-5})$$

$$I = 0.0329 \times 10^{-5}$$

Section Modulus (Z)

$$Z = \frac{I}{y}$$

$$= \frac{0.0329 \times 10^{-5}}{0.03}$$

$$Z = 1.0975 \times 10^{-5} \, m^3$$

$$y = \frac{D}{2}$$

$$= \frac{0.06}{2}$$

$$y = 0.03$$

$$\sec\left(\frac{10}{2}\times\sqrt{\frac{P}{EI}}\right) = \sec\left(\frac{2.1}{2}\times\sqrt{\frac{120\times10^{3}}{(200\times10^{9})\times0.0229\times10^{-5}}}\right)$$

$$= \sec(1.4179 \text{ radians})$$

$$= 1.4179\times\frac{180}{\pi}$$

$$= 81.229$$

$$= \sec(81.229)$$

$$= 6.566$$

Sub. these value in equn ①

$$\sigma_{max} = \frac{120\times10^{3}}{8.629\times10^{-4}} + \frac{(120\times10^{3})\times(2.5\times10^{-2})\times6.566}{1.0975\times10^{-5}}$$

$$= 138.9\times10^{6} + 179.48\times10^{6}$$

$$= 318.38\times10^{6}\ N/m^{2}$$

$$\boxed{\sigma_{max} = 318.38\ N/mm^{2}}$$

Prblm. no: 19

- If the given column of problem: 18 is subjected to an eccentric load of 100 kN and maximum permissible stress is limited to 320 MN/m², then determine the maximum eccentricity of the load.

Given data

Data from prblm. no: 18

$D = 60mm \Rightarrow 0.06m$

$d = 50mm \Rightarrow 0.05m$

$l = 2.1m \;;\; l_e = l = 2.1m$

$E = 200\ GN/m^2 \Rightarrow 200\times10^9\ N/m^2$

$I = 0.0329\times10^{-5}\ m^4$

$Z = 1.0975\times10^{-5}\ m^3$

Area $(A) = 8.639\times10^{-4}\ m^2$

$P = 100\ kN \Rightarrow 100\times10^3\ N$

$\sigma_{max} = 320\ MN/m^2 \Rightarrow 320\times10^6\ N/m^2$

let,

e - Eccentricity

Solution

Maximum stress

$$\sigma_{max} = \frac{P}{A} + \frac{P \times e \times \sec\left(\frac{l_e}{2}\times\sqrt{\frac{P}{EI}}\right)}{Z} \rightarrow (1)$$

$$\sec\left(\frac{l_e}{2} \times \sqrt{\frac{P}{EI}}\right) = \sec\left[\frac{2.1}{2} \times \sqrt{\frac{100 \times 10^3}{200 \times 10^9 \times 0.0329 \times 10^{-5}}}\right]$$

$$= \sec(1.294 \text{ radians})$$

$$= \sec\left(1.294 \times \frac{180^\circ}{\pi}\right)$$

$$= \sec(74.16^\circ)$$

$$= 3.665$$

Sub. the known values in equtn ①

$$320 \times 10^6 = \frac{100 \times 10^3}{8.639 \times 10^{-4}} + \frac{(100 \times 10^3) \times e \times 3.665}{1.0975 \times 10^{-5}}$$

$$= 115.754 \times 10^6 + 33394.e \times 10^6$$

$$320 = 115.754 + 33394 \times e$$

$$e = \frac{320 - 115.754}{33394} \text{ m}$$

$$\boxed{e = 6.116 \times 10^{-3} \text{ m}}$$

STRAIGHT LINE FORMULA

The Euler's formula and Rankine's formula give only the approximate values of crippling load due to the following reasons.

* The pin Joints are not practically frictionless
* The End fixation is never perfectly rigid.
* In case of Euler's formula, the effect of direct compression has been neglected.
* The Load is not exactly applied as desired
* The Members are never perfectly straight and uniform in section
* The Materials of the member is not homogenous and isotropic

The Empirical straight line formula are commonly used in practical designing

$$P = \sigma_c \times A - n\left(\frac{L_e}{K}\right) \times A$$

Where,

P - crippling load on the column

σ_c - compressive yield stress

A - Area of cross section of the column

$$\frac{L_e}{K} = \text{slenderness ratio}$$

n - A constant whose value depends upon the material of the column.

P - Plotted against $\left(\frac{L_e}{K}\right)$, we will get a straight line and hence the represent a straight line formula.

$$\frac{P}{A} = \sigma_c - n \cdot \left(\frac{l_e}{k}\right)$$

Where,

$\frac{P}{A}$ = represent the corresponding to load 'A'

PROF. PERRY'S FORMULA

In cases where we have to determine the safe load that can be applied on a column at a given eccentricity prof. perry's formula prove quite useful.

let,

σ_d - stress due to direct load $\Rightarrow (P/A)$

σ_{max} - Maximum permissible stress

l_e - Effective length of the column

σ_b - Maximum compressive stress due to bending moment

$$= \frac{M}{Z} = \frac{M \times y_c}{AK^2}$$

y_c - Distance from the neutral axis of the extreme layer in compression

$$= \frac{P \cdot e \sec \frac{l_e}{2}\sqrt{\frac{P}{EI}}}{AK^2} \cdot y_c$$

$$= \frac{P \cdot e \cdot y_c}{AK^2} \sec \frac{\pi}{2}\sqrt{\frac{P}{P_{Euler}}}$$

Where

$$P_{Euler} = \frac{\pi^2 EI}{l_e^2}$$

$$\sigma_{max} = \frac{P}{A} + \frac{P \cdot e \cdot y_c}{A k^2} \sec \cdot \frac{\pi}{2}\sqrt{\frac{P}{P_{Euler}}}$$

$$\sigma_d = \frac{P}{A}$$

$$\sigma_{max} = \sigma_d \left[1 + \frac{e y_c}{k^2} \sec \cdot \frac{\pi}{2}\sqrt{\frac{P}{P_{Euler}}}\right]$$

According to prof. perry

$$\sec \cdot \frac{\pi}{2}\sqrt{\frac{P}{P_{Euler}}} = \frac{1.2\, P_{Euler}}{P_{Euler} - P}$$

Let,

$$\sigma_{Euler} = \frac{P_{Euler}}{A}$$

$$\sec \frac{\pi}{2}\sqrt{\frac{P}{P_{Euler}}} = \frac{1.2\, P_{Euler}}{P_{Euler} - P}$$

$$= \frac{1.2\, \sigma_{Euler}}{\sigma_{Euler} - \sigma_d}$$

$$\sigma_{max} = \sigma_d \left[1 + \frac{e y_c}{k^2} \cdot \frac{1.2\, P_{Euler}}{P_{Eular} - \sigma_d}\right]$$

$$\sigma_{max} = \sigma_d \left[1 + \frac{e y_c}{k^2} \cdot \frac{1.2\, \sigma_{Euler}}{\sigma_{Euler} - \sigma_d}\right]$$

$$\left[\frac{\sigma_{max}}{\sigma_d} - 1\right] = \frac{e y_c}{k^2} \cdot \frac{1.2\, \sigma_{Euler}}{\sigma_{Euler} - \sigma_d}$$

$$\left(\frac{\sigma_{max}}{\sigma_d} - 1\right)\left(\frac{\sigma_{Euler} - \sigma_d}{\sigma_{Euler}}\right) = \frac{1.2\, e \cdot y_c}{k^2}$$

$$\boxed{\left(\frac{\sigma_{max}}{\sigma_d}-1\right)\left(1-\frac{\sigma_d}{\sigma_{Euler}}\right)=\frac{1.2\,e\cdot y_c}{K^2}}$$

$\hookrightarrow$ Prof. perry's Formula

BEAM COLUMNS

DEFINITION

A structural member that is subjected to axial compression and tranverse bending at the same time. A beam column differs from a column only by the presence of the eccentricity of the load application, and moment or Transverse load.

STRUT WITH LATERAL LOAD (OR BEAM COLUMNS)

Columns carry axial compressive loads. If the columns are also subjected to transverse loads, then they are known as beam columns. The transverse load is generally uniformly distributed. But let us consider two cases

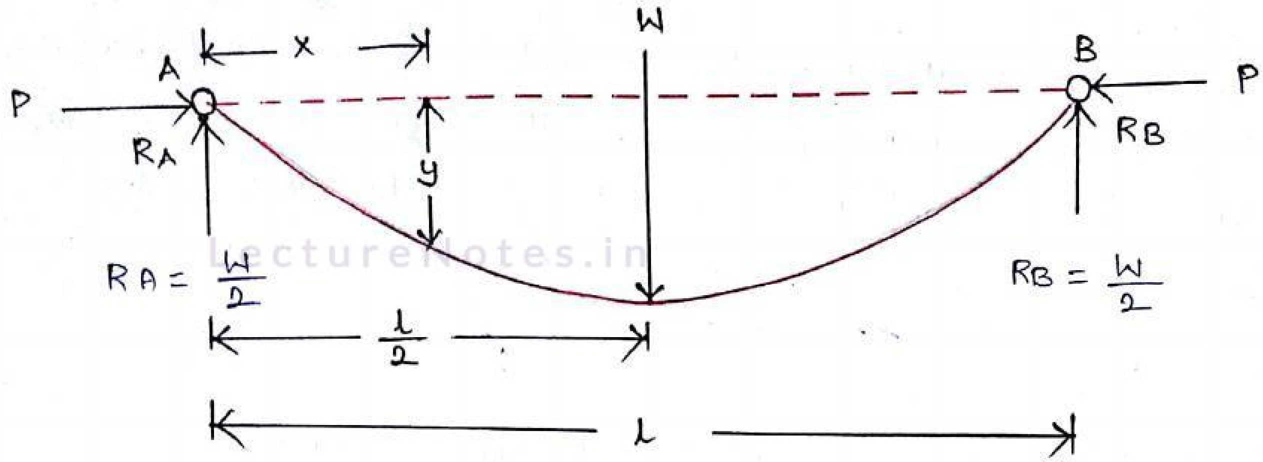
* Transverse load is a point load and acts at the centre
* transverse load is uniformly distributed

STRUT SUBJECTED TO COMPRESSIVE AXIAL LOAD OR AXIAL THRUST AND A TRANSVERSE POINT LOAD AT THE CENTRE

BOTH ENDS ARE PINNED

A Strut AB of length 'l' subjected to compressive axial load P and a transverse point load W at the centre. The strut is pinned at both of its ends. Consider any section at a distance x from the end A. Let 'y' is the deflection at this section.

The Bending Moment at the Section



P - Axial Compressive load

W - Transverse point load at centre

y - Deflection

We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\frac{M}{EI} = \frac{1}{R} \rightarrow (1)$$

y - Deflection

$\frac{dy}{dx}$ - slope

$\frac{d^2y}{dx^2}$ - Moment (or) curvature $\rightarrow (2)$

sub (1) & (2)

$$\frac{M}{EI} = \frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$EI \frac{d^2y}{dx^2} = Mx - x$$

$$EI \frac{d^2y}{dx^2} = -Py - \frac{W}{2}x$$

$$\frac{d^2y}{dx^2} = \frac{-P}{EI} \times y - \frac{W}{2EI} \times x$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} \times y = -\frac{W}{2EI} \times x$$

Second degree differential equation

y = Complementary solution + particular Integral

$$y = C_1 . \cos\left(x . \sqrt{\frac{P}{EI}}\right) + C_2 . \sin\left(x \times \sqrt{\frac{P}{EI}}\right) - \frac{W \times x}{2P} \rightarrow (3)$$

Boundary Conditions

Ist Boundary Condition

At $x = 0$, $y = 0$

At $x = \frac{L}{2}$, $\frac{dy}{dx} = 0$

Apply Boundary Conditions

From equtn no: (3)

$x = 0$, $y = 0$

$$0 = C_1 . \cos\left(0 \times \sqrt{\frac{P}{EI}}\right) + C_2 . \sin\left(0 \times \sqrt{\frac{P}{EI}}\right) - \frac{W \times 0}{2P}$$

$$= C_1 . (1) + C_2 (0) - \frac{W \times 0}{2P}$$

$$\boxed{C_1 = 0}$$

Differential equtn (3) with respect 'x' we get

$$\frac{dy}{dx} = -C_1\left[\sin\left(x \times \sqrt{\frac{P}{EI}}\right)\right] \times \sqrt{\frac{P}{EI}} + C_2\left[\cos\left(x \times \sqrt{\frac{P}{EI}}\right)\right] \times \sqrt{\frac{P}{EI}} - \frac{W}{2P}$$

$$0 = -C_1\left[\sin\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)\right] \times \sqrt{\frac{P}{EI}} + C_2\left[\cos\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)\right] \times \sqrt{\frac{P}{EI}} - \frac{W}{2P}$$

$$= 0 \cdot \left[\sin\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)\right] \times \sqrt{\frac{P}{EI}} + C_2\left[\cos\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)\right] \times \sqrt{\frac{P}{EI}} - \frac{W}{2P}$$

$$= 0 + C_2\left[\cos\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)\right] \times \sqrt{\frac{P}{EI}} - \frac{W}{2P}$$

$$= \frac{W}{2P} \times \sqrt{\frac{EI}{P}} \times \frac{1}{\cos\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)}$$

$$C_2 = \frac{W}{2P} \times \sqrt{\frac{EI}{P}} \times \sec\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)$$

Sub. the value of C_1 and C_2 in equtn (3)

$$y = 0 + \frac{W}{2P} + \sqrt{\frac{EI}{P}} \times \sec\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) \cdot \sin\left(x \times \sqrt{\frac{P}{EI}}\right) - \frac{W \times x}{2P}$$

$$y = \frac{W}{2P} \times \sqrt{\frac{EI}{P}} \times \sec\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) \cdot \sin\left(x \times \sqrt{\frac{P}{EI}}\right) - \frac{W \times x}{2P} \rightarrow (4)$$

Let,

To Find

Maximum deflection

Maximum Bending moment

Maximum stress

Maximum deflection (y_{max})

The deflection is maximum at the centre $x = \frac{l}{2}$

Substituting $x = \frac{l}{2}$ in equtn (4)

$$y_{max} = \frac{W}{2P} \times \sqrt{\frac{EI}{P}} \times \sec\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) \sin\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) - \frac{W}{2P} \times \frac{l}{2}$$

$$y_{max} = \frac{W}{2P} \times \sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) - \frac{W \times l}{4P} \rightarrow (5)$$

Maximum Bending Moment (M_{max})

The B.M is given by equation

$$M = -\left(P \times y + \frac{W}{2} \times x\right)$$

The bending B.M will be maximum at the centre

Where,

$$y = y_{max}$$

$$x = \frac{l}{2}$$

$$M_{max} = -\left(P \times y_{max} + \frac{W}{2} \times \frac{l}{2} \right)$$

$$= -\left[P \times \left\{ \frac{W}{2P} \times \sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}} \right) - \frac{W \times l}{4P} \right\} + \frac{W}{2} \times \frac{l}{2} \right]$$

Sub. y_{max} here from equ^n

$$= -\frac{W\cancel{P}}{2\cancel{P}} \times \sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}} \right) - \frac{W\cancel{P} \times l}{4\cancel{P}} + \frac{W}{2} \times \frac{l}{2}$$

$$= -\left[\frac{W}{2} \times \sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}} \right) - \cancel{\frac{W \times l}{4}} + \cancel{\frac{W \times l}{4}} \right]$$

$$= -\left[\frac{W}{2} \times \sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}} \right) \right]$$

-ve sign is due to sign convention.

Hence the magnitude of maximum B.M is given by

$$M_{max}\ (\text{Magnitude}) = \frac{W}{2} \times \sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}} \right)$$

Maximum stress (σ_{max})

Maximum stress induced is due to direct axial compressive load and due to maximum bending stress.

$$\sigma_{max} = \sigma_0 + \sigma_b$$

Where,

- σ_o - Stress due to Direct axial compressive load.

  σ_b - Stress due to bending.

We know that,

Bending Equation

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

The stress due to bending of strut

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{M \times y}{I}$$

$$\therefore (I = AK^2)$$

$$\boxed{\sigma_b = \frac{M \times y_c}{AK^2}}$$

Where,

y_c - Distance of the external layer in Compression from Neutral axis

k - Radius of Gyration

M - M_{max}

$$\text{Maximum bending stress} = \frac{M_{max} \times y_c}{AK^2}$$

$$= \left[\frac{\frac{W}{2} \times \sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)}{AK^2} \right] \times y_c$$

$$\boxed{\sigma_{max} = \frac{P}{A} + \left[\frac{W}{2} \times \sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)\right] \times \frac{y_c}{Ak^2}}$$

Prblm.no: 20

Determine the maximum stress induced in a cylindrical steel strut of length 1.2m and diameter 30mm. The strut is hinged at both its ends and subjected to an axial thrust of 20 kN at its ends and a transverse point load of 1.8 kN at the centre. Take E = 208 GN/m^2.

Given data

$l = 1.2m$

$d = 30\ mm \Rightarrow 0.03m$

Axial thrust (P) $= 20\ kN \Rightarrow 20 \times 10^3\ N$

Transverse point load (W) $= 1.8\ kN \Rightarrow 1.8 \times 10^3\ N$

$E = 208\ GN/m^2 \Rightarrow 208 \times 10^9\ N/m^2$

Area (A) $= \frac{\pi}{4} d^2$

$= \frac{\pi}{4} \times (0.03)^2$

$A = 7.068 \times 10^{-4}\ m^2$

Moment of Inertia (I) $= \frac{\pi}{64} d^4$

$= \frac{\pi}{64} \times 0.03^4$

$I = 3.976 \times 10^{-8}\ m^4$

Solution:

Direct stress is due to axial thrust

$$\sigma_0 = \frac{P}{A}$$

$$= \frac{20 \times 10^3}{7.068 \times 10^{-4}}$$

$$\sigma_0 = 28.29 \times 10^6 \ N/m^2$$

Maximum bending stress

$$\sigma_b = \frac{M_{max} \times y_c}{I}$$

Max. Bending moment

$$M_{max} = \frac{W}{2} \times \sqrt{\frac{EI}{P}} \times \tan\left(\frac{L}{2} \times \sqrt{\frac{P}{EI}}\right)$$

Let us find $\sqrt{\frac{P}{EI}}$

$$\sqrt{\frac{P}{EI}} = \sqrt{\frac{20 \times 10^3}{(208 \times 10^9) \times (3.976 \times 10^{-8})}}$$

$$= \sqrt{2.4183}$$

$$= 1.555$$

$$\sqrt{\frac{EI}{P}} = \frac{1}{1.555}$$

$$= 0.643$$

$$\frac{l}{2} \times \sqrt{\frac{P}{EI}} = \frac{1.2}{2} \times 1.555$$

$$= 0.933 \times \text{rad}$$

$$= 0.933 \times \frac{180^\circ}{\pi}$$

$$= 53.45^\circ$$

$$\tan\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) = \tan(0.933 \text{ rad})$$

$$= \tan(53.45^\circ)$$

$$= 1.349$$

Sub. the above equtn ①

$$\sigma_b = \frac{780.66 \times y_c}{I}$$

$$y_c = \frac{d}{2} = \frac{30}{2}$$

$$= \frac{780.66 \times 0.015}{3.976 \times 10^{-8}}$$

$y = 15 mm \Rightarrow 0.015 m$

$$\sigma_b = 294.51 \times 10^6 \ N/m^2$$

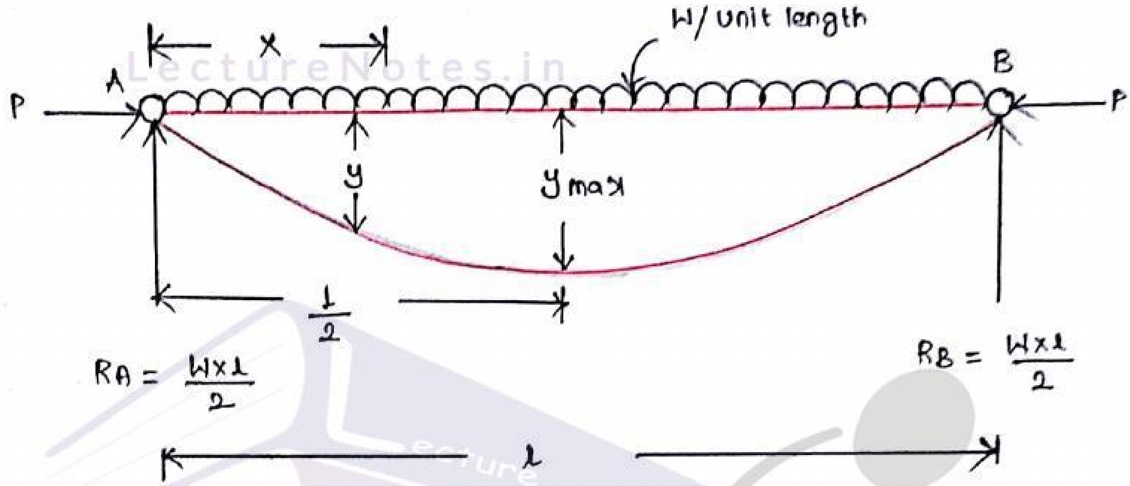
Maximum stress induced

$$\sigma_{max} = \sigma_0 + \sigma_b$$

$$= 28.29 \ MN/m^2 + 294.51 \ MN/m^2$$

$$\boxed{\sigma_{max} = 322.8 \ MN/m^2}$$

STRUT SUBJECTED TO COMPRESSIVE AXIAL LOAD OR AXIAL THRUST AND A TRANSVERSE UNIFORMLY DISTRIBUTED LOAD OF INTENSITY w PER UNIT LENGTH



$$M = -P \times y + (w \times x) \times \frac{x}{2} - \frac{w \times l}{2} \times x$$

$$= -P xy + \frac{wx^2}{2} - \frac{w \times l \times x}{2} \rightarrow (1)$$

We know that

Bending equation

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\frac{M}{EI} = \frac{1}{R} \rightarrow (2)$$

y = Deflection

$\frac{dy}{dx}$ = Slope

$\frac{d^2y}{dx^2}$ = Moment (or) Curvature $\rightarrow (3)$

Sub (2) & (3)

$$\frac{M}{EI} = \frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$EI \frac{d^2y}{dx^2} = M$$

Bending Moment is also given by

$$M = EI \frac{d^2y}{dx^2} \rightarrow (4)$$

Differentiating the equtn (1) w.r.t 'x'

$$\frac{dM}{dx} = -P\frac{dy}{dx} + \frac{\omega \times 2x}{2} - \frac{\omega \times l}{2}$$

$$= -P\frac{dy}{dx} + \omega \times x - \frac{\omega \times l}{2}$$

Differentiating the above equation again

$$\frac{d^2M}{dx^2} = -P\frac{d^2y}{dx^2} + \omega \rightarrow (5)$$

From equtn (4) sub this value in equtn (5)

$$\frac{d^2M}{dx^2} = -P \times \frac{M}{EI} + \omega$$

$$\frac{d^2M}{dx^2} + \frac{P}{EI} \times M = \omega$$

Second degree differential equation

$y =$ Complementary Solution + particular Integral.

$$M = C_1 \cos\left(x \cdot \sqrt{\frac{P}{EI}}\right) + C_2 \cdot \sin\left(x \cdot \sqrt{\frac{P}{EI}}\right) + \frac{\omega \times EI}{P} \rightarrow (6)$$

The equation is a differential equation in 'M' and is more useful as the maximum bending moment can be obtained directly from this.

Boundary Conditions

-

* At $x=0$, $M=0$

* At $x=\frac{L}{2}$, $\frac{dM}{dx}=0$

Apply Boundary Conditions

From equⁿ ⑥

y= complementary solution + particular integral

$$M = C_1 \cdot \cos\left(0 \times \sqrt{\frac{P}{EI}}\right) + C_2 \cdot \sin\left(0 \times \sqrt{\frac{P}{EI}}\right) + \frac{\omega \times EI}{P}$$

$$= C_1(1) + 0 + \frac{\omega \times EI}{P}$$

$$\boxed{C_1 = -\frac{\omega \times EI}{P}}$$

Differentiating equation ⑥ with respect 'x' we get

$$\frac{dM}{dx} = -C_1 \times \sqrt{\frac{P}{EI}} \cdot \sin\left(x \times \sqrt{\frac{P}{EI}}\right) + C_2 \times \sqrt{\frac{P}{EI}} \cdot \cos\left(x \times \sqrt{\frac{P}{EI}}\right) + 0$$

$$0 = -C_1 \sqrt{\frac{P}{EI}} \cdot \sin\left(\frac{L}{2} \times \sqrt{\frac{P}{EI}}\right) + C_2 \times \sqrt{\frac{P}{EI}} \cdot \cos\left(\frac{L}{2} \times \sqrt{\frac{P}{EI}}\right)$$

$$= -\left(-\frac{\omega \times EI}{P}\right) \times \sqrt{\frac{P}{EI}} \cdot \sin\left(\frac{L}{2} \times \sqrt{\frac{P}{EI}}\right) + C_2 \cdot \sqrt{\frac{P}{EI}} \cos\left(\frac{L}{2} \times \sqrt{\frac{P}{EI}}\right)$$

$$0 = \frac{\omega \times EI}{P} \times \sin\left(\frac{L}{2} \times \sqrt{\frac{P}{EI}}\right) + C_2 \cdot \cos\left(\frac{L}{2} \times \sqrt{\frac{P}{EI}}\right)$$

$$= -\frac{\omega \times EI}{P} \times \frac{\sin \frac{L}{2} \times \sqrt{\frac{P}{EI}}}{\cos \frac{L}{2} \times \sqrt{\frac{P}{EI}}}$$

$$= -\frac{\omega \times EI}{P} \cdot \tan\left(\frac{L}{2} \times \sqrt{\frac{P}{EI}}\right)$$

Sub. the value of C_1 and C_2 in equtn ⑥

$$M = \left(-\frac{w \times EI}{P}\right) \times \cos\left(x \times \sqrt{\frac{P}{EI}}\right) + \left[-\frac{wEI}{P}\tan\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)\right]$$

$$\sin\left(x \times \sqrt{\frac{P}{EI}}\right) + \frac{w \times EI}{P}$$

$$= \left(\frac{-w \times EI}{P}\right)\left[\cos\left(x \times \sqrt{\frac{P}{EI}}\right) + \tan\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)\sin\left(x \times \sqrt{\frac{P}{EI}}\right) - 1\right]$$

Let us find

Maximum bending moment

Maximum deflection

Maximum stress

Maximum Bending Moment

The bending moment is maximum at $x = \frac{l}{2}$

∴ Hence above equation becomes as

$$M_{max} = \left(-\frac{w \times EI}{P}\right)\left[\cos\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) + \tan\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) \times \sin\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) - 1\right]$$

$$= -\frac{w \times EI}{P}\left[\cos\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) + \frac{\sin\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)}{\cos\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)} \times \sin\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) - 1\right]$$

$$= -\frac{w \times EI}{P}\left[\frac{\cos^2\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) + \sin^2\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)}{\cos\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)} - 1\right]$$

$$= -\frac{w \times EI}{P}\left[\frac{1}{\cos\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right)} - 1\right]$$

$$\boxed{M_{max} = -\frac{w \times EI}{P}\left[\sec\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) - 1\right]} \rightarrow (7)$$

Maximum deflection

$M = M_{ax}$, $y = y_{max}$ and $x = \frac{l}{2}$

From equtn (1)

$$M_{max} = -P \times y_{max} + \frac{w}{2} \times \left(\frac{l}{2}\right)^2 - \frac{wl}{2} \times \frac{l}{2}$$

$$= -P \times y_{max} + \frac{w \times l^2}{8} - \frac{w \times l^2}{4}$$

$$= -P \times y_{max} - \frac{w \times l^2}{8}$$

$$= -\left(P \times y_{max} + \frac{wl^2}{8}\right)$$

From equtn (7)

$$M_{max} = -\frac{w \times EI}{P}\left[\sec\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) - 1\right]$$

Equating the two values of M_{max}

$$-\left(P \times y_{max} + \frac{wl^2}{8}\right) = -\frac{w \times EI}{P}\left[\sec\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) - 1\right]$$

$$P \times y_{max} = \frac{w \times EI}{P}\left[\sec\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) - 1\right] - \frac{wl^2}{8}$$

$$\boxed{y_{max} = \frac{w \times EI}{P^2}\left[\sec\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) - 1\right] - \frac{wl^2}{8}} \rightarrow (8)$$

Maximum stress

$$\sigma_{max} = \sigma_0 + \sigma_b$$

$$= \frac{P}{A} + \frac{M_{max}}{Z}$$

Where,

$$\frac{M_{max}}{Z} = \frac{M_{max}}{\left(\frac{I}{y_c}\right)}$$

$$= \frac{M_{max} \times y_c}{I}$$

$$\boxed{\sigma_{max} = \frac{P}{A} + \frac{M_{max} \times y_c}{I}}$$

y_c - Distance of extreme layer in compression from N.A

Prblm no: 21

Determine the maximum stress induced in a horizontal strut of length 2.5m and of rectangular cross section 40mm wide and 80 mm deep when it carries an axial thrust of 100 kN and a vertical load of 6 kN/m length. The strut is having pin joints at its ends. Take E = 208 GN/m²

Given data

$l = 2.5 m$

$b = 40 mm \Rightarrow 0.04 m$

$d = 80 mm \Rightarrow 0.08 m$

Axial thrust $(P) = 100$ kN $\Rightarrow 100 \times 10^3$ N

Uniformly distributed load $(w) = 6\ kN/m \Rightarrow 6 \times 10^3\ N/m$

$$E = 208\ GN/m^2 \Rightarrow 208 \times 10^6\ N/m^2$$

$$Area\ (A) = b \times d$$

$$= 0.04 \times 0.08$$

$$A = 0.0032\ m^2$$

$$I = \frac{bd^3}{12}$$

$$= \frac{0.04 \times 0.08^3}{12}$$

$$I = 1.7066 \times 10^{-6}\ m^4$$

let,

Maximum bending moment

Solution

$$M_{max} = \frac{w \times EI}{P}\left[\sec\left(\frac{1}{2} \times \sqrt{\frac{P}{EI}}\right) - 1\right]$$

magnitude (-ve sign)

$$\frac{P}{EI} = \frac{100 \times 1000}{208 \times 10^9 \times 1.7066 \times 10^{-6}}$$

$$= 0.2817$$

$$\frac{EI}{P} = \frac{1}{0.2817}$$

$$= 3.55$$

$$\sqrt{\frac{P}{EI}} = \sqrt{0.2817}$$

$$= 0.5307$$

$$\frac{l}{2} \times \sqrt{\frac{P}{EI}} = \frac{2.5}{2} \times 0.5307$$

$$= 0.6634 \text{ radians}$$

$$= \frac{0.6634 \times 180^\circ}{\pi}$$

$$= 38^\circ$$

$$\sec\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) = \sec(38^\circ)$$

$$= 1.269$$

$$M_{max} = w \times \frac{EI}{P}\left[\sec\left(\frac{l}{2} \times \sqrt{\frac{P}{EI}}\right) - 1\right]$$

$$= (6 \times 10^3) \times 3.55\,[1.269 - 1]$$

$$M_{max} = 5729.70 \text{ Nm}$$

Stress due to direct load

$$\sigma_0 = \frac{P}{A}$$

$$= \frac{100 \times 1000}{0.0032}$$

$$\boxed{\sigma_0 = 31.25 \times 10^6 \text{ N/m}^2}$$

Max. bending stress (σ_b)

$$\sigma_b = \frac{M_{max} \times y_c}{I}$$

y_c = Distance of extreme layer in compression from neutral axis

$$y_c = \frac{d}{2}$$

$$= \frac{80}{2}$$

$$y_c = 40\,mm \Rightarrow 0.04\,m$$

$$\sigma_b = \frac{5729.7 \times 0.04}{1.4066 \times 10^{-6}}$$

$$\boxed{\sigma_b = 134.3 \times 10^{6}\ N/m^2}$$

Maximum stress

$$\sigma_{max} = \sigma_o + \sigma_b$$

$$= 31.25 \times 10^{6} + 134.3 \times 10^{6}$$

$$= 165.55 \times 10^{6}\ N/m^2$$

$$\boxed{\sigma_{max} = 165.55\ MN/m^2}$$