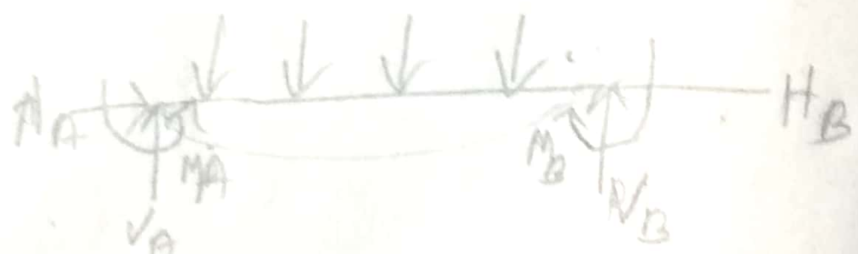
Date
11-02-20

Unit - III

Fixed Beams and Continous Beams.

Fixed Beam

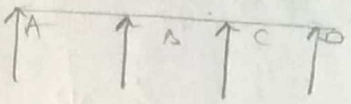
A beam is two fixed end is called as two fixed Beam.



Continous Beam

A beam with more than two supports is consider as continous Beam

+ hogie
− sage

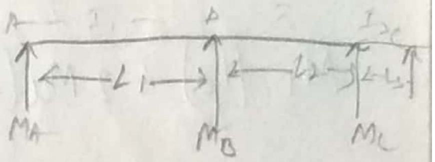


clapeyron's three moment therom (3 Moment eqⁿ.

$$M_a\left(\frac{L_1}{I_1}\right) + 2M_b\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_c\left(\frac{L_2}{I_2}\right) = \frac{6a_1\bar{x}_1}{L_1 I_1} + \frac{6a_2\bar{x}_2}{L_2 I_2}$$

If cross section area are different

If AB and BC cross section's are similar then



$$I_1 = I_2$$

$$\frac{M_a L_1}{I_1} + 2M_b\left(\frac{L_1+L_2}{I_1}\right) + M_c\left(\frac{L_2}{I_1}\right) = \frac{6a\bar{x}_1}{L_1 I_1} + \frac{6a_2\bar{x}_2}{L_2 I_1}$$

$$M_a L_1 + 2M_b(L_1+L_2) + M_c L_2 = \frac{6a\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}$$

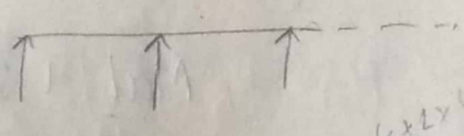
Note

$$M_B\left(\frac{L_2}{I_2}\right) + 2M_c\left(\frac{L_2}{I_2}+\frac{L_3}{I_3}\right) + M_D\left(\frac{L_3}{I_3}\right) = \frac{6a_2\bar{x}_2}{L_2 I_2} + \frac{6a_3\bar{x}_3}{L_3 I_3}$$

Case-1

Beam with fixed end

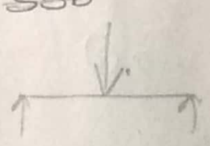
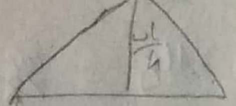
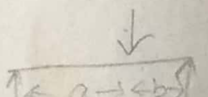
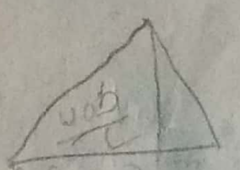
Just replace the fixed end with an imagine beam try to apply the clapeyron's three moment therom



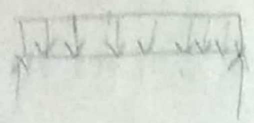
$\frac{1}{2}\times L\times w$

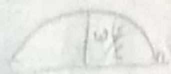
$\frac{wL^2}{4}$

$\frac{wL^2}{8}$

SSB	Bending Moment	Area
		$\frac{wL^2}{8}$
		$\frac{wab}{2}$

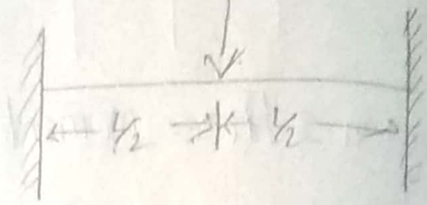
$\frac{1}{2}\times b\times h$

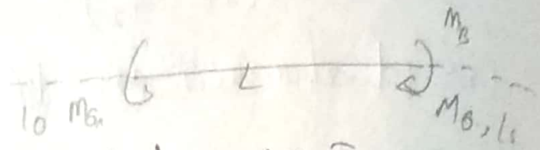




$\frac{wl^3}{12}$ $(\frac{2}{3} \times bm)$

→ A fixed beam with central point load





$$M_{0_1}(L_0) + 2M_a(L_0+L_1) + M_b(L_1) = \frac{6a_0\bar{x}_0}{L_0} + \frac{6a_1\bar{x}_1}{L_1}$$

$$2M_aL_1 + M_bL = \frac{6a_1x_1}{L}$$

$$(2M_a + M_b)L = \frac{6\left(\frac{wL^2}{8}\right)\left(\frac{L}{2}\right)}{L}$$

$$2M_a + M_b = \frac{3wL}{8} \to (1)$$

ABO

$$M_a(L) + 2M_b(L+L_0) + M_{0_2}(L_0) = \frac{6a_1\bar{x}_1}{L} + \frac{6a_0\bar{x}_0}{L}$$

$$M_aL + 2M_bL = \frac{6\left(\frac{wL^2}{8}\right)\left(\frac{L}{2}\right)}{L}$$

$$\cancel{M_aL+}$$

$$(M_a + 2M_b)L = \frac{3}{8}wL^2$$

$$M_a + 2M_b = \frac{3}{8}wL \to (2)$$

From (1) (2)

$$2M_a + M_b = \frac{3wL}{8}$$

$$M_a + 2M_b = \frac{3}{8}wL$$

$-M_a + M_b = 0$

$M_a = M_b$

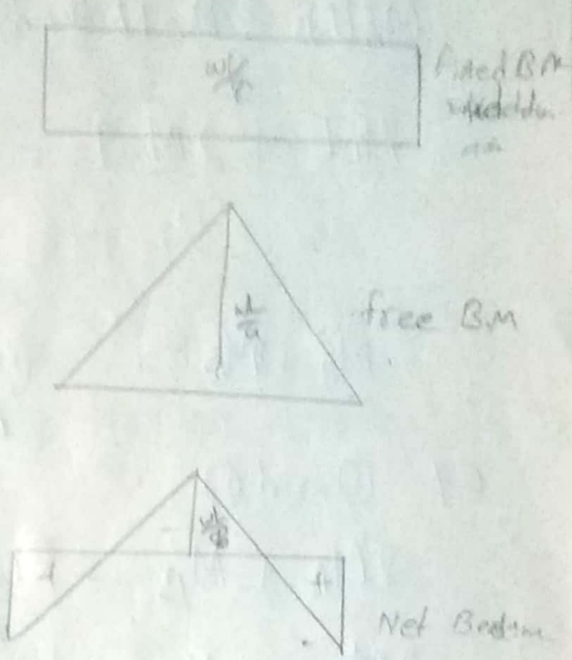
sub eq (1) in $M_a = M_b$

$2M_a + M_b = \frac{3WL}{8}$

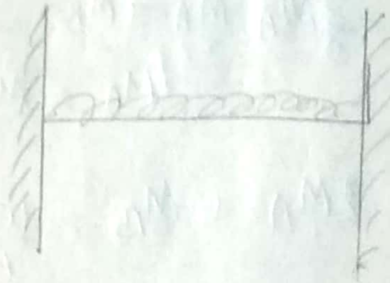
$3M_a = \frac{3WL}{8}$

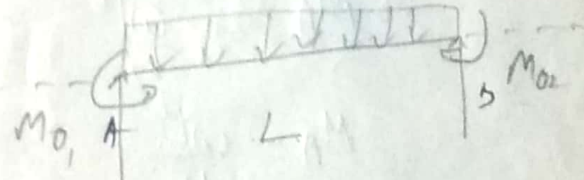
$M_a = \frac{WL}{8}$

$\therefore M_a = M_b = \frac{WL}{8}$



→ A fixed beam with udl load.





By applying ~~Can~~ clapeyron's therom. $M_{0_1} A B$

$$M_0(L_0) + 2M_A(L_0+L) + M_B(L) = \frac{6a_0\bar{x}_0}{L_0} + \frac{6a_1\bar{x}_1}{L_1}$$

$$+2M_A L + M_B L = \frac{6\left(\frac{wL^3}{12}\right)\left(\frac{L}{2}\right)}{L}$$

$$(2M_A + M_B)L = \frac{\frac{3wL^4}{12}}{L}$$

$$2M_A + M_B = \frac{wL^2}{4} \rightarrow (1)$$

on AB M_{0_2}

$$M_A(L) + 2M_B(L+L_0) + M_{0_2}(L_0) = \frac{6a_2\bar{x}_2}{L} + \frac{6a_0}{L_0}$$

$$M_A L + 2M_B L = \frac{6\left(\frac{wL^3}{12}\right)\left(\frac{L}{2}\right)}{L} + 0$$

$$M_A + 2M_B = \frac{wL^2}{4} \rightarrow ②$$

Eq ① and ②

$$2M_A + M_B = \frac{wL^2}{4}$$

$$M_A + 2M_B = \frac{wL^2}{4}$$

$$\cancel{3M_A} \quad M_A - M_B = 0$$

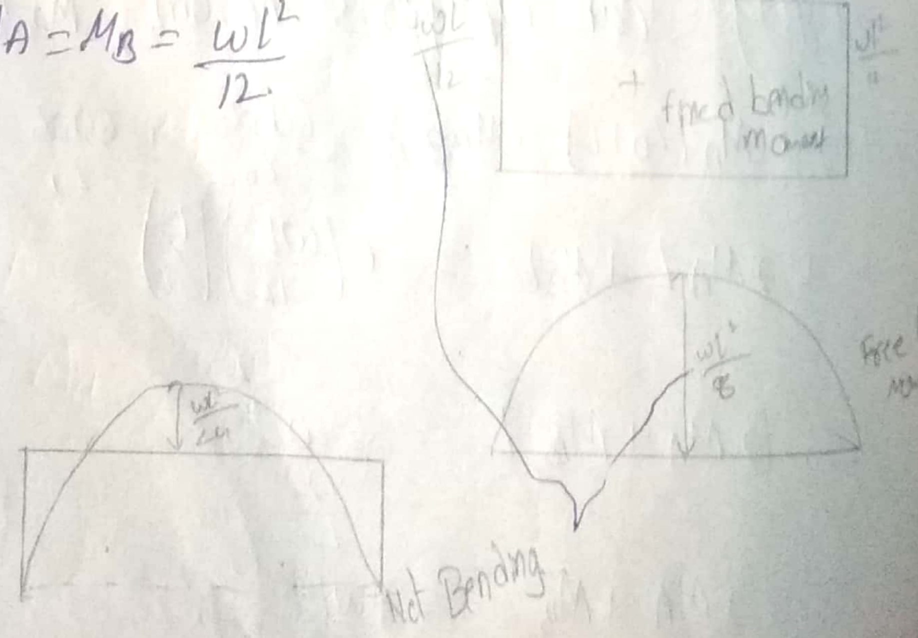
$$M_A = M_B$$

$$2M_A + M_B = \frac{wL^2}{4}$$

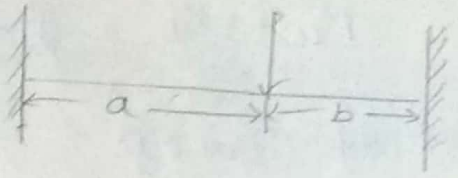
$$3M_A = \frac{wL^2}{4}$$

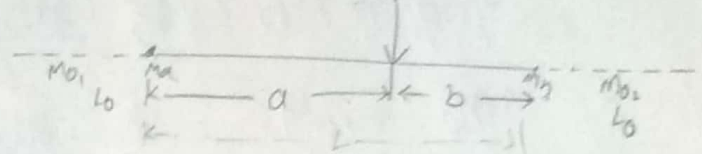
$$M_A = \frac{wL^2}{12}$$

$$M_A = M_B = \frac{wL^2}{12}$$



$\rightarrow$





Applying the clapeyron's three moment equation.

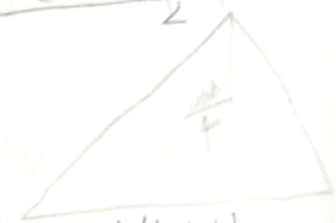
for M_{o_1} A B

$$\rightarrow M_0(L_0) + 2M_a(L_0+L) + M_b(L) = \frac{6a_0\bar{x}_0}{L_0} + \frac{6a_1\bar{x}_1}{L_1}$$

$$2M_aL + M_bL = \frac{6a\bar{x}}{L}$$

$$2M_aL + M_bL = \frac{6\left(\frac{wab}{2}\right)\left(\frac{2}{3}a\right)\left(\frac{L+b}{3}\right)}{L}$$

$\bar{x} = \frac{L+b}{2}$



$$\rightarrow 2M_aL + M_bL = \frac{3}{3}\frac{wab(L+b)}{L}$$

$$2M_a + M_b = \frac{wab(L+a)}{L^2} \rightarrow (1)$$

$\rightarrow$ For ABM_{o_2}

$$M_a(L) + 2M_b(L+L_0) + M_{o_2}(L_0) = \frac{6a\bar{x}}{L} + \frac{6a_0\bar{x}_0}{L}$$

$$M_a(L) + 2M_bL = \frac{6a\bar{x}}{L}$$

$$M_a(L) + 2M_bL = \frac{6\left(\frac{wab}{2}\right)\left(\frac{L+a}{2}\right)}{L}$$

$$M_aL + 2M_bL = \frac{3}{3}\frac{wab(L+a)}{L}$$

$\cancel{M_a(\ ) + 2M_b(\ )}$

$$M_a + 2M_b = \frac{wab(L+a)}{L^2} \rightarrow (2)$$

solving (1) and (2).

$$2M_a + M_b = \frac{wab(L+b)}{L^2}$$

$$2M_a + 4M_b = 2\frac{wab(L+a)}{L^2}$$

$$3M_b = 2\frac{wab}{L^2}(L+a) - \frac{wab}{L^2}(L+b)$$

$$3M_b = \frac{wab}{L^2}\left[2(L+a) - (L+b)\right]$$

$$3M_b = \frac{wab}{L^2}\left[2L+2a-L-b\right]$$

$$3M_b = \frac{wab}{L^2}\left[L+2a-b\right]$$

$$3M_b = \frac{wab}{L^2}\left[(a+b)+2a-b\right]$$

$$\cancel{3}M_b = \frac{wab}{L^2}\left[\cancel{3}a\right]$$

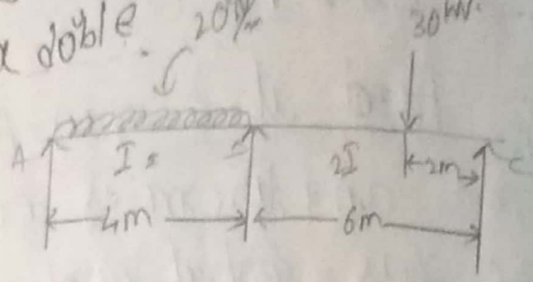
$$\boxed{M_b = \frac{wa^2b}{L^2}}$$

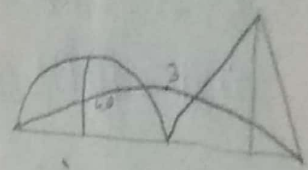
∴ IIly

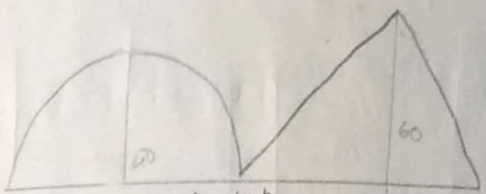
$$M_a = \frac{wab^2}{L^2}$$

→ There is a beam ABC which is continous beam AB of span AB of span 4m . Moment of Inertia it carries a UDL of 20 kN/m all over the span and beam BC ~~span~~ carries a point load 30 kN/m at 2m from the

B' . BC has a doble moment of enertia BC = 6M







At Beam end Moment is zero?

$\frac{2}{3} \times b \times h$

Area of Parabola $= \frac{2}{3} \times L \times \left(\frac{wL^2}{8}\right)$

$\frac{wL^3}{12}$

$= \frac{20 \times 4^3}{12}$

$= 106.66$

centriod $= \bar{x} = \frac{L}{2} = \frac{4}{2}$

$= 2$

Area of trangle $= \frac{1}{2} \times b \times h$

$\frac{1}{2} \times b \times \frac{wab}{L}$

$\frac{1}{2} \times 6 \times \frac{(30 \times 4 \times 2)}{6}$

$= 120$

centriod

$\bar{x} = \frac{L+b}{3}$

$= \frac{6+2}{3}$

$= 2.66$

∴ Apply in clapyron's three moment ABC

$$M_A\left(\frac{4}{I}\right) + 2M_B\left(\frac{4}{I} + \frac{6}{2I}\right) + M_C\left(\frac{6}{2I}\right) = \frac{6(106.6)2}{4I} + \frac{6(120)\left(\frac{8}{3}\right)}{6(2I)}$$

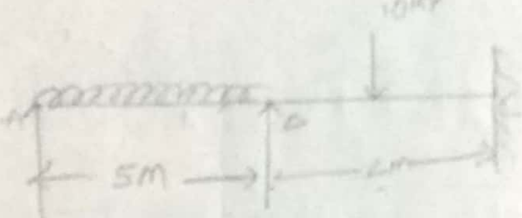
$$0 + 2M_B\left(\frac{4}{I} + \frac{6}{2I}\right) + 0 = \frac{319.8}{I} + \frac{160}{I}$$

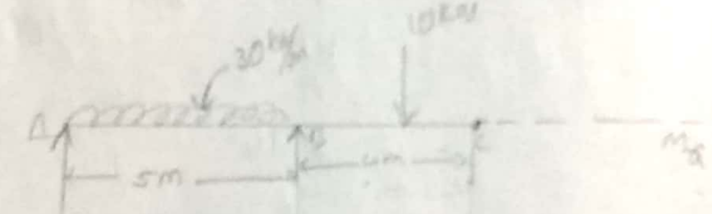
$$2M_b \frac{(4+3)}{\cancel{I}} = \frac{479.8}{\cancel{I}}$$

$$14 M_b = 479.8$$

$$M_b = 34.27 \text{ kN-m}.$$

→ A beam ABC fixed at C span AB = 5m span BC is 4m span and AB carries a UDL of 30 kN/m and span BC has central point of 10 kN

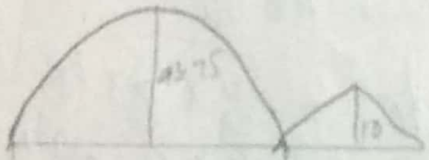


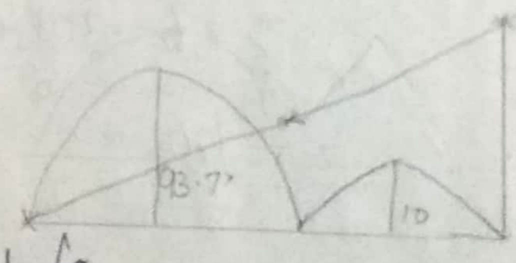


$\frac{wL^2}{8}$, $\frac{wL}{4}$

$\frac{30(5)^2}{8}$, $\frac{(10)(4)}{4}$

93.75 $10





Area of Parabola $= \frac{wL^3}{12}$, centriod $\bar{x} = \frac{L}{2}$

$= \frac{30(5)^3}{12}$ $= \frac{5}{2}$

$= 312.5$ $= 2.5$

Area of $\triangle = \frac{2}{3} \times \frac{WL^2}{8}$ | centriod $= \bar{x} = \frac{L+a}{3}$

$= \frac{10 \times 4^2}{8}$ | $= \frac{4+2}{3}$

$= 20$ | $= 2$

$\frac{4+3}{2} = 2$

Apply the clapeyron's therom. ABC

$$M_a(5) + 2M_b(5+4) + M_c(4) = \frac{6(13.12.5)(2.5)}{5} + \frac{6(20)(2)}{4}$$

$$5M_a + 18M_b + 4M_c = 937.5 + 60$$

At the endof continous beam the moment is not there so it is zero

$$18M_b + 4M_c = 997.5 \rightarrow (1)$$

Apply the clapeyron's therom BCD

$$M_b(4) + 2M_c(4+L_0) + M_0(L_0) = \frac{6(20)(2)}{4} + \frac{6A_0\bar{x}_0}{L_0}$$

$$4M_b + 2M_c(4+0) + M_0(0) = 60 + 0$$

$$4M_b + 8M_c = 60 \rightarrow (2)$$

Equating (1) and (2) ; Multiply ing eq 2 with 2

$$18M_b + 4M_c = 997.5$$
$$8M_b + 16M_c = 120$$

$$10M_b = 877.5$$

Scanned with CamScanner

$M_b = \frac{877.5}{10}$

$\boxed{M_b = 87.75\ KN\text{-}m}$

sub M_b value in eq ①

$4(87.75) + 2(M_c) = 60$

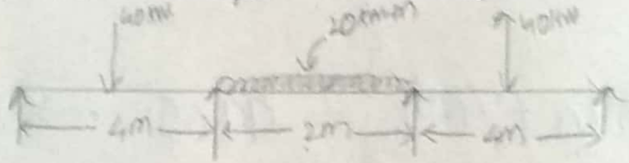
$351 + 2M_c = 60$

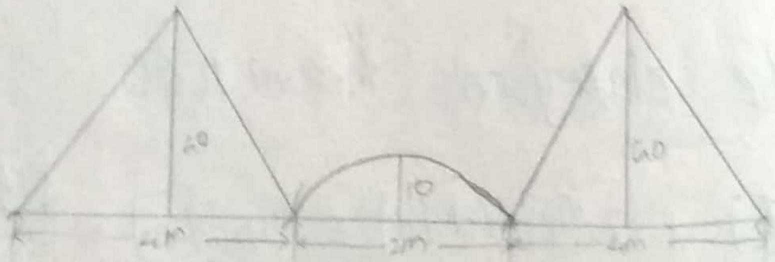
$2M_c = 60 - 351$

$2M_c = -291$

$\boxed{M_c = -145.5\ KN\text{-}m}$

→ ABCD is a continous beam carring span AB 4m, span BC = 2m CD = 4m AB carries a points load 40KN at center and span BC carries UDL 20KN-m span CD carries point load 40KN





Area

$\frac{WL^2}{8} \Rightarrow \frac{40(4)^2}{8}$; $\frac{WL^3}{12} = \frac{20(2)^3}{12}$; $\frac{WL^2}{8} = \frac{40(4)^2}{8}$

80 ; $= 13.3$; $= 80$

centroid $\frac{L}{2} = \frac{\cancel{8}4}{2}$; $\frac{L}{2} = \frac{\cancel{8}2}{2} = 1$; $\frac{L}{2} = \frac{4}{2}$

$= \cancel{4}2$; ; $= 2$

Apply the clapeyron's three moment theoam

$M_A(L_1) + 2M_B(L_1 + L_2) + M_C(L_2) = \frac{6a_1\bar{x}_1}{L_1 I_1} + \frac{6a_2\bar{x}_2}{L_2 I_2}$

$$M_A(4) + 2M_B(4+2) + M_C(2) = \frac{6(80)(12)}{4} + \frac{6(13.33)(1)}{2}$$

$$2M_A + 12M_B + 2M_C = 240 + 39.90$$

$$4M_A + 12M_B + 2M_C = 279.99 \rightarrow (1)$$

again applying on BCD

$$M_B(2) + 2M_C(2+4) + M_D(6) = \frac{6(13.33)(1)}{2} + \frac{6(80)(12)}{4}$$

$$2M_B + 12M_C + 6M_D = 39.99 + 240$$

$$2M_B + 2M_C + 6M_D = 279.99 \rightarrow (2).$$

Net bending eq^n

$$4M_A + 12M_B + 2M_C = 279.99$$

$$+ 2M_B + 12M_C + 6M_D = 279.99$$

At the simple supports beam at ends A and D are no moment so that '0' places in M_A and M_D

$$\therefore \quad 12M_B + 2M_C = 279.99 \rightarrow (3)$$

$$2M_B + 12M_C = 279.99 \rightarrow (4)$$

By Multplying with 6 to eq (4)

$$12M_B + 72M_C = 1679.94 \rightarrow (5)$$

solving eq (3) and (5)

$$12M_B + 2M_C = 279.99$$

$$12M_B + 72M_C = 1679.94$$

$$70M_C = 1399.95$$

$$M_C = 19.99 \text{ N-m}.$$

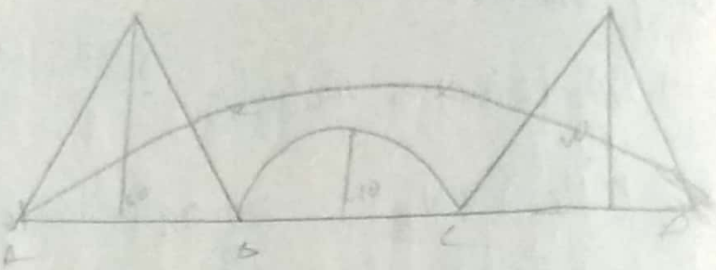
sub M_C in value in (3)

$$12M_B + 2(19.99) = 279.99$$

$12 M_B + 39.98 = 279.99$

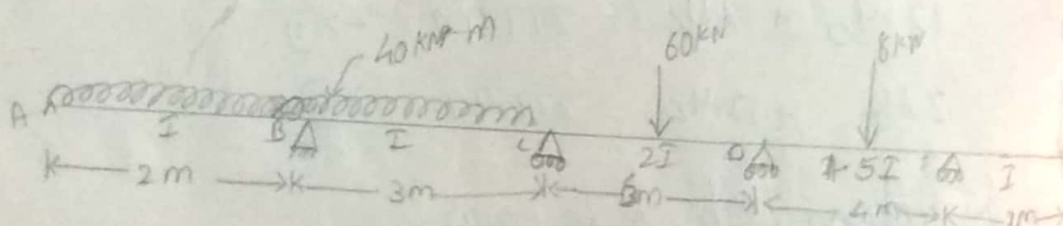
$12 M_B = 249.99$

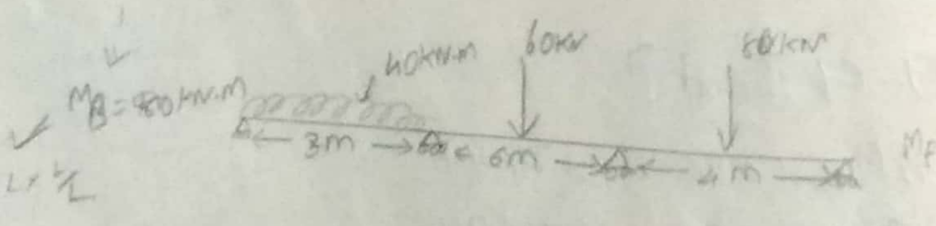
$M_B = 19.99$ N-m

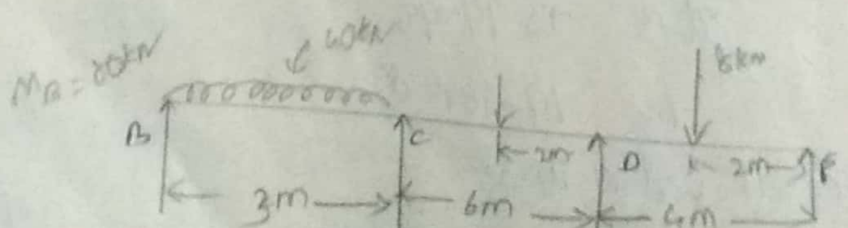


15-03

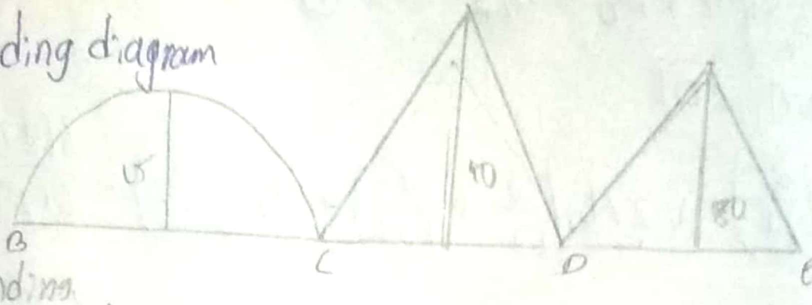
A beam AB,CD,EF, hinged at B rollers at C,D,E, span AB 2m, BC 3m, CD 6m, DE 4m EF 2m moment of inertia of AB,B EF = I, Moment of inertia of CD = 2I, DE = 1.5I AB and BC carri UDL of intecity 40kN-m all over the span. CD carres point load 60 kN at the center and span DE carries 80 kN at the center and they is point of 40kN at F







Free Bending diagram



maximum bending

Parabola $\frac{wl^2}{8}$

$= \frac{40(3)^2}{8}$

$= 45\,M$

point $\frac{wL}{4}$

$\frac{60(6)}{4}$

$= 90\,m$

$\frac{wL}{4}$

$\frac{80(4)}{4}$

$80\,m$

Area of Bending

UDL ~~Parabola~~ $= \frac{wl^3}{12}$

$\frac{40(3)^2}{12}$

$a_1 = 90\,m^2$

Point load

$= \frac{wl^2}{8}$

$\frac{60(6)^2}{8}$

$a_2 = 270\,m^2$

point load

$\frac{wl^2}{8}$

$= \frac{80(4)^2}{8}$

$a_3 = 160\,m^2$

centriod

$l/2 = \frac{3}{2}$

$\bar{x}_1 = 1.5\,m$

Point load

$l/2 = 6/2$

$\bar{x}_2 = 3\,m$

Point load

$l/2 = \frac{4}{2}$

$\bar{x}_3 = 2\,m$

Applying clapeyron's three bending theorom

From Beam BCD

$$M_B\left(\frac{3}{I}\right) + 2M_C\left(\frac{3}{I} + \frac{6}{2I}\right) + M_D\left(\frac{6}{2I}\right) = \frac{6(90)(1.5)}{3I} + \frac{6(270)(3)}{6\times 2I}$$

$$80\left(\frac{3}{I}\right) + 2M_C\left(\frac{3}{I} + \frac{6}{2I}\right) + M_D\left(\frac{6}{2I}\right) = \frac{6(90)(1.5)}{3I} + \frac{6(270)(3)}{6\times 2I}$$

$$\frac{3}{I}\left[80 + 4M_C + 2M_D\right] = \frac{270}{I} + \frac{405}{I}$$

$$\frac{3}{I}\left[80 + 4M_C + 2M_D\right] = \frac{675}{I}$$

$$80 + 4M_C + 2M_D = 225$$

$4M_C + M_D = 145 \rightarrow ①$

For Beam CDE

$$M_C\left(\frac{6}{2I}\right) + 2M_D\left(\frac{6}{2I} + \frac{4}{1.5I}\right) + M_E\left(\frac{4}{1.5I}\right) = \frac{6(270)3}{6\times 2I} + \frac{6(160)2}{4\times 1.5I}$$

$$M_C\left(\frac{6}{2I}\right) + 2M_D\left[\frac{6}{2I} + \frac{4}{1.5I}\right] + 80\left(\frac{4}{1.5I}\right) = \frac{405}{I} + \frac{320}{I}$$

Multiply with I on B.S

$$M_C\left(\frac{6}{2}\right) + 2M_D\left[\frac{6}{2} + \frac{4}{1.5}\right] + 80\left(\frac{4}{1.5}\right) = 725$$

$$3M_C + \frac{34}{3}M_D + \frac{640}{3} = 725$$

$$3M_C + \frac{34}{3}M_D = \frac{1535}{3}$$

Multiply by 3

$9M_C + 34M_D = 1535 \rightarrow ②$

By solving the eq ① and ②

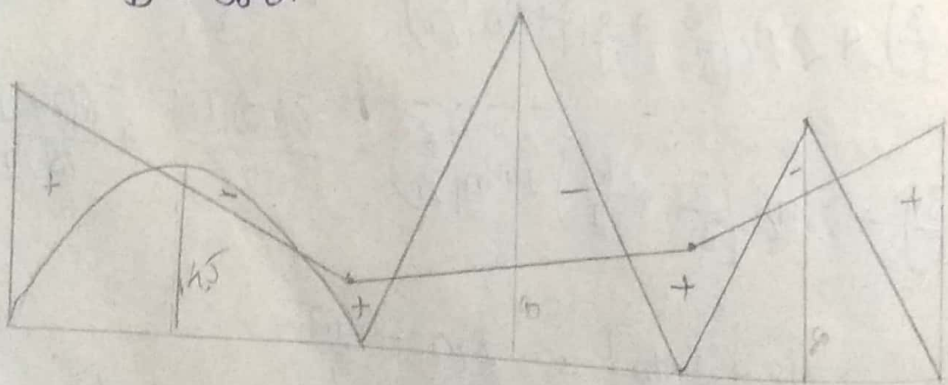
$4M_C + M_D = 145$

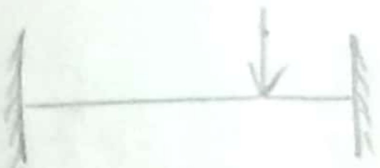
$9M_C + 34M_D = 1535$ (By eq^n mode)

$M_C = \cancel{28}0$

26.73

$M_D = 38.07$



		$M_{\cancel{ab}\,AB}$	M_{BA}
①		$-\frac{wl^2}{12}$	$+\frac{wl^2}{12}$
			$\frac{wl^2}{20}$
②		$-\frac{wL}{8}$	$+\frac{wL}{8}$
③		$-\frac{wab^2}{L^2}$	$+\frac{wa^2b}{L^2}$