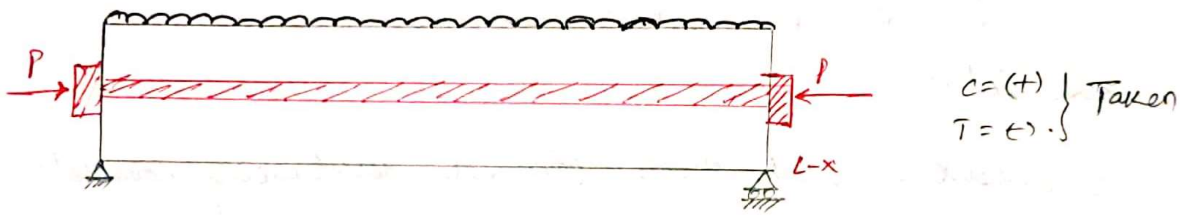
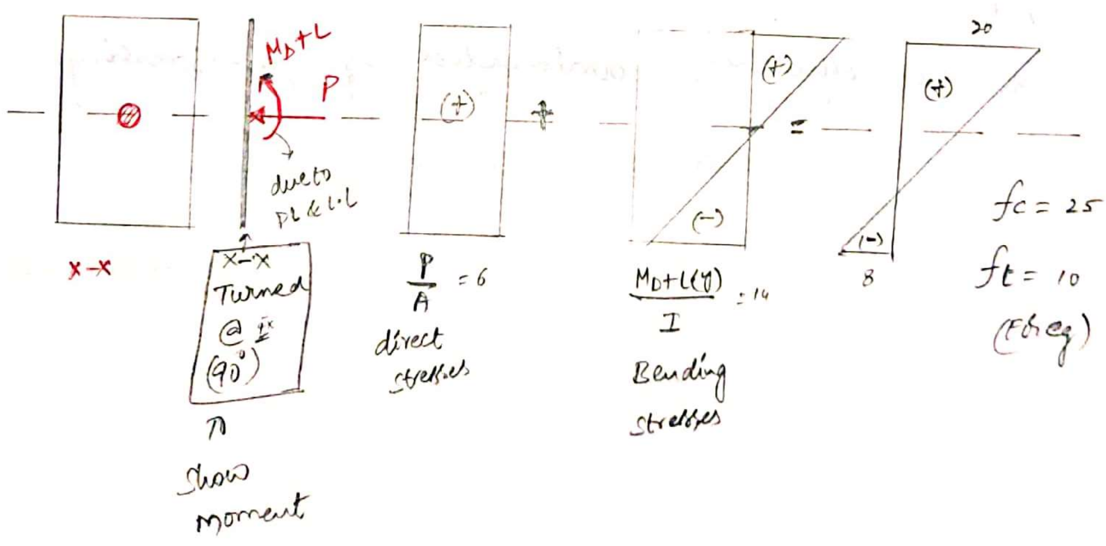
LEC-01

"ANALYSIS OF PRE-STRESSED CONCRETE MEMBER" (PSC)

- An psc is basically concrete in which internal stresses of suitable magnitude and distribution are introduced so that stress resulting from external loads are counteracted to a desired degree.

- A psc is different from conventional RCC due to application of an initial load on it prior to its use





- In final stress diagram, stress at top and bottom of concrete are within permissible limits.
- If same moment is applied on section without pre-stressing force, the stress at bottom fibre exceeds the permissable limit.
- It means, member is made safe by pre-stressing.

NOTE:

- For concrete internal stresses are induced due to the following reasons.

(i) Tensile strength of concrete is less and hence cracks may develop at early stages of loading, flexural members such as beam and slabs.

(ii) It enhances bond, shear, torsional resistance of concrete

(iii) In pipes and liquid storage tanks the hoop tensile stress is can be effectively counteracted by pre-stressing.

TYPES OF PRE-STRESSING :

- pre-stressing can be classified on the basis of following.

(I) SOURCE OF PRE-STRESSING FORCE! (PS)

(a) HYDRAULIC PS:-

- This is the simplest type of pre-stressing producing large pre-stressing force
- The hydraulic jack is used for tensioning of tendons, comprises of calibrated pressure guages which indicate the magnitude of PS force [Ex pile load test - reaction truss method in SM].

(b) MECHANICAL PS:

- In this type of pre-stressing, the devices include weight, with (or) without lever transmission, gears, pulley blocks, screwjack etc
- It is used for mass scale production.

(c) ELECTRICAL PS:

- In this type of PS, the steel wires are electrically heated and anchored before, placing concrete in the moulds.

- This is also termed as "Thermo-electric pre-stressing."

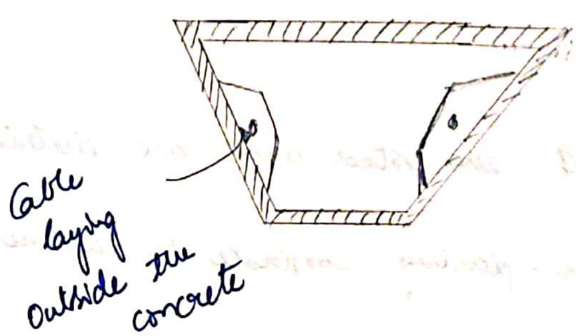
ⓓ CHEMICAL PS:

- In this case, Expanding cement is used which is composed of 75% postland cement, 15% high alumina cement & 10% Gypsum.
- In this case since expansion of concrete is restrained by high tension wire, compressive stresses are induced in it.
- This system is also known as "SELF PRE STRESSING."

Ⓘ APPLICATION OF PRE-STRESSING FORCE:

ⓐ EXTERNAL PRE-STRESSING:

- When the pre-stressing is achieved by elements located outside the concrete it is termed as external pre-stressing
- The tendons can lie outside the member (or) inside the hollow space. of the member in this case



Ex Bridges

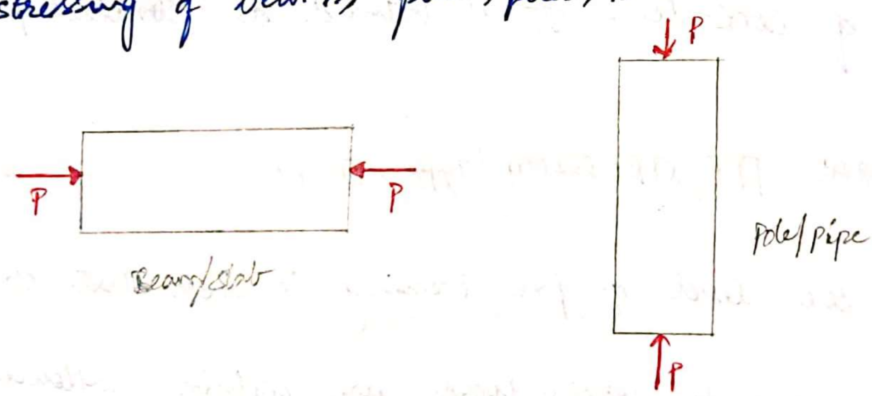
ⓑ INTERNAL PRE-STRESSING:

- when the pre-stressing is achieved by the element located inside the concrete member, by embedded tendons it is termed as INTERNAL PRESTRESSING.
- Most of the applications of pre-stressing are internal pre-stressing

ⓔ ⅲ DIRECTION OF PRE-STRESSING FORCE:

ⓐ LINEAR PRESTRESSING:

- when the pre-stressed members are straight (or) flat in the direction of pre-stressing, the pre-stressing is called linear ps
- Pre-stressing of beams, piles, poles, slabs



ⓑ CIRCULAR PRE-STRESSING:

- when the pre-stressed members are curved in the direction of prestressing it is called circular p.s.

- For Example pipes, Tanks, domes, silos etc

(VI) EXTENT OF PRE-STRESSING:

(A) FULL PRESTRESSING / TYPE-I PS [Ex Bridges]

- when the level of ps is such that no tensile stress is allowed in concrete under Service load it is called full pre-stressing (or) type I pre-stressing.

(B) LIMITED PRESTRESSING / TYPE-II PS

- when the level of pre-stressing such that the tensile stress under service load is within the cracking stress of concrete. It is termed as limited p.s / Type-II ps

(C) PARTIAL PRE STRESSING / TYPE-III ps. [light loaded structures].

- when the level of pre stressing is such that the tensile stresses due to service load are within allowable limit it is termed as Partial ps / Type-III ps.

(V) AXIS OF PRE-STRESSING!

(a) UNI AXIAL PS : ~~Bars~~ Tendons are parallel to one-axis

(b) BI-AXIAL PS ! Tendons are parallel to TWO-axis.

(c) MULTI AXIAL PS: Tendons are parallel to more than Three axis. [Ex-dome].

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(VI) TIME OF PRE STRESSING!

(a) PRE TENSIONING:

- The tension is applied to the tendons before casting of the concrete.
- The pre compression is transimitted from steel to concrete in this case through "BOND" b/w steel & concrete over the transmission length near the end
- Moulds are kept on casting bed and wires are inserted through moulds.
- wires are fixed at position and tensioned from the ends.

Scanned with CamScanner

- concrete is casted in the moulds and the wires are kept in direct contact with concrete.

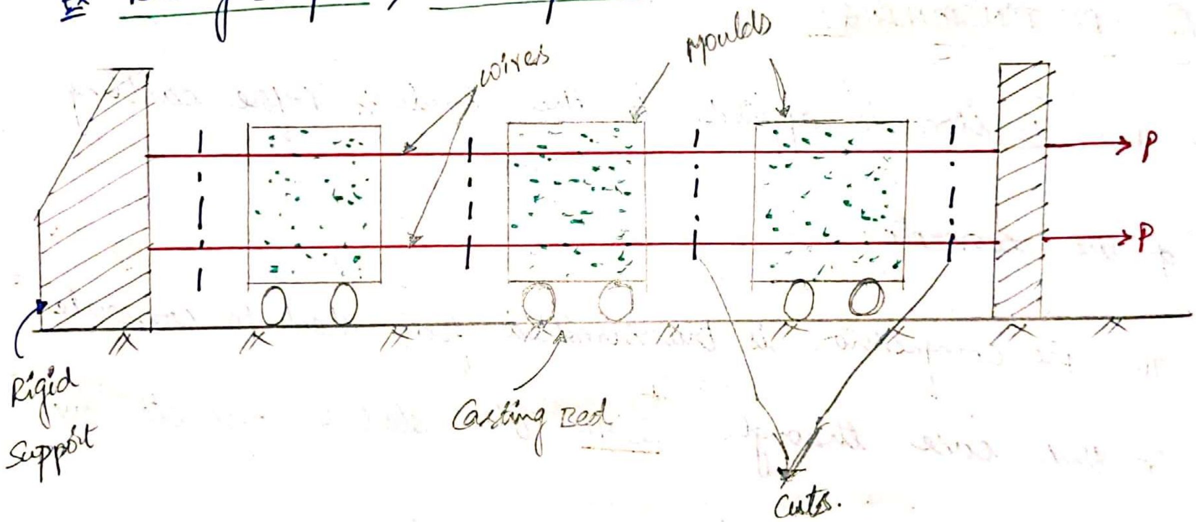
- concrete is allowed to get sufficient strength.

- now wires are cut at ends of each mould & pre-stressing force is transferred to concrete. by bond action.

- This method is termed as "LONG LINE/HOYER METHOD".

- It is generally used for small repetative type of work

Ex Railway sleepers., electric poles. etc



- various stages involved in pre tensioning

(i) Anchoring of tendons against the end supports.

(ii) Placing of jack

(iii) applying tension to the tendons

(iv) Casting of concrete

(v) cutting of the tendons.

- Here no anchoring devices are required but pre-stress bed is required; there is a waiting period in this case for pre-stress bed to be reused.

- This method requires following devices

(i) pre-stressing bed

(ii) END ABUTMENT

(iii) shuttering/MOULD

(iv) ANCHORING device

(v) jack

(vi) Harping device
[Bent]

(b) POST TENSIONING:

- The tension is applied to the tendons after the hardening of the concrete

- The pre compressive force in this case is transmitted by ~~bearing action~~ "BEARING ACTION".

Scanned with CamScanner

- concrete is casted in desired shape & size with the duct inside the member at desired location.
- Duct may be pvc (or) steel as it is flexible, so it can be provided in any shape.
- concrete is allowed to get sufficient strength.
- now cables are provided in duct & tensioned from end

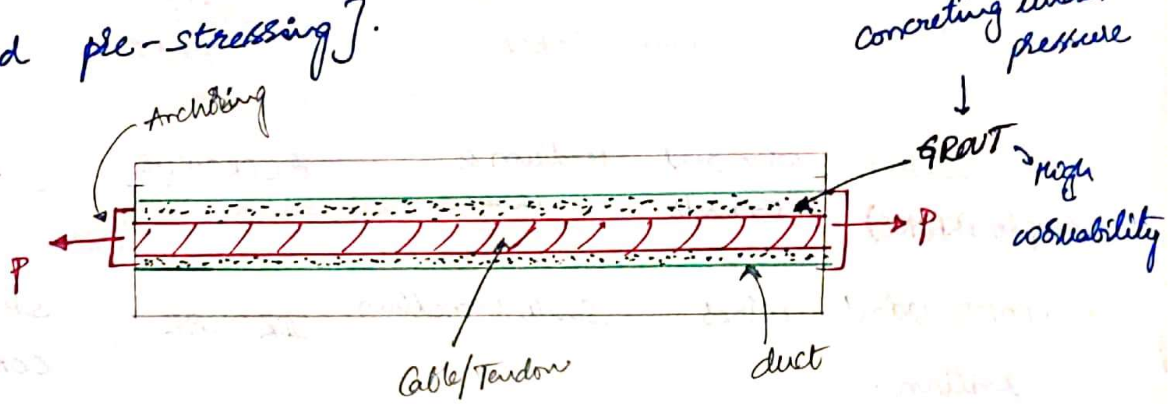
[NOTE:
Cable can be provide either before & after the concreting.]

- Tensioning is done either from one end (or) from both ends
- After tensioning cables are Anchored by any suitable mechanism

(i) Freyssinet system
(ii) Lee Macall system
(iii) Magnel Blaton system
(iv) Gifford udall system
(v)

- After Anchoring, cables are cut and pre-stressing force is transferred on the member

- Remaining space of duct can be filled with mortar of very high workability. [In this case it is termed as Bonded pre-stressing].



- It is suitable for heavy cast insitu members &

  Ex Bridge girders.
- The waiting period in this case is comparetively less
- Here transfer of pre stress is independent of transmission length.
- It is comparetively complex & costly than pre-tensioning as it requries anchorage, grouting device.

- Various systems of post-Tensioning are as follows.

POST TENSIONING SYSTEM.	TYPE OF TENDON	RANGE OF FORCE	CABLE DUCT	TYPE OF ANCHORAGE
FREYSSINET (devp in FRANCE)	wire and strands	Medium & large	~~A~~ CIRCULAR	CONICAL
GIFFORD Udall Britian.	wires	Small & medium	CIRCULAR	SPLIT CONICAL
Lee - Mc Call Britian	Bars [@ ends it is Threaded]	Small to large	CIRCULAR	NUT & BOLT action
Magnel Blaton Belgium	wires	Small to large	RECTANGULAR	WEDGE

- For pre stressing high strength steel and concrete is required as

(i) In pre-stress, there are losses on account of short term effect [Ex elastic shortening, friction, Anchorage ~~[illegible]~~ slip & also long term effect like ~~of~~ Creep, Shrinkage, relaxation of steel]

(ii) Due to all these effects the total loss of strain in steel is approx 8×10^{-4}, that would result in loss of stress in

$\Rightarrow$ loss of stress = $8 \times 10^{-4} \times 2 \times 10^5 = 160\ N/mm^2$

modulus of elasticity (MOE)

(iii) Hence if conventional steel is used [Fe_{250}, Fe_{415}, Fe_{500}].

maximum stresses would be lost

(iv) Hence, in this case pre-stress of around 1200 – 2000 N/mm^2 is required, in which "losses are of order" of 200 N/mm^2.

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- High strength concrete offers high resistance to tension, shear, bond & bearing.
- In case of pre-Tensioned members, tensile stress in steel of very high magnitude should be transferred to concrete as pre-stress through (i) Bonding (ii) ~~Bearing~~.
- In case of post tension members, it is transferred by bearing
- High strength concrete is less liable to shrink and high modulus of elasticity & smaller creep strain as

a result loss of pre stress in steel is reduced

- use of high strength concrete results in reduction of cross-sectioned dimension of prestressed concrete & hence also reduces weight of the structure.

- As per IS 1343 - 2012

min. Grade of concrete

Pre Tensioning	M40
Post Tensioning	M30

- ADVANTAGES OF PRE-STRESSING:

(a) section remains uncracked at service load hence durability of steel increases.

(b) Full section of concrete is utilized

(c) shear resistance increases

(d) Improved performance under dynamic & fatigue loading

cyclic [EQ, Rain]

(e) High span to depth ratio is possible in this case

~~XX~~ [L/D = 40-45]

— DISADVANTAGE OF PRE-STRESSING:

(a) Pre-stressing need skilled technology.

(b) Use of high strength material is coastly

(c) There is additional cost of equipments

(d) strict quality control & inspection is required

ANALYSIS OF PRE-STRESS:

NOTE! - Analysis of prestress is done by W.S.M

- For design we can either use W.S.M (or) L.S.M

- Assumptions in the analysis of pre-stress.

(a) Concrete is assumed to be homogenous and elastic

(b) For both steel and concrete hook's law is valid in working stress

(c) plane section before bending remains plane after bending.

(d) Stress in pre-stressing cables does not change along the length of the cable

(e) ~~vasit~~ Variation of stress in pre-stressing cable due to external loading is neglected

UNIT-3

Flexure & Shear.

Part -I - "Flexure strength of psc members"

- when psc members are subjected to bending loads of different types of flexural failures are possible at critical sections depending upon following parameters

1. percentage of reinforcements in the section
2. Degree of bond between tendon and concrete
3. Compressive strength of concrete
4. Ultimate tensile strength of concrete

- various types of structural failures encountered in psc members are

(A) Fracture of steel in tension

(B) Failure of under reinforced sections.

(C) Failure of over reinforced sections.

(D) Shear failures

→ P.T.O

(E) Failure of bond between steel and concrete

(F) Anchorage failures.

(A) Fracture of steel in tension:

- sudden failure of pre stressed member without any warning is generally due to the fracture of steel in tension zone. This type of failure takes place due to the provision of lowering the percentage of steel in the section, because of less amount of steel, the concrete in tension zone develops cracks and steel cannot bear any additional tensile stresses transferred to it by the cracked concrete and the psc member fails due to the fracture of steel.

IS 1343 -1980, prescribes a minimum longitudinal reinforcement of 0.2% of the c/s area in all cases

→ P.T.O

Except in the case of pre-tensioned ~~and~~ of smaller sections

To prevent this type of failure, when HYSD bars are used, the minimum steel % is reduced to 0.15%.

(B) <u>Failure of UNDER REINFORCED SECTIONS:</u>

- When the amount of steel provided in the section is less than that required for balanced section, failure takes place by an excessive elongation of steel followed by the crushing of the concrete. As bending loads are increased, excessive elongation of steel takes place which shifts the neutral axis towards the compression side

The member approaches failure due to the gradual reduction of the compressive zone exhibiting large deflection and cracks which developed at soffit and progressed towards the compression plate.

→ P.T.O

- When the area of the concrete in compression zone is insufficient to resist the resultant compressive force, the ultimate flexure failure of member takes place through crushing of the concrete. Large deflection and wide cracks are the characteristics features of under Reinforced section.

This type of behaviour generally since there is considerable warning before the failure

(c) FAILURE OF OVER REINFORCED SECTIONS (ORS):

- Over reinforced sections are the sections in which amount of steel is provided is more than that required for balanced section. Generally ORS members fail due to sudden crushing of concrete by small deflection and narrow cracks are the characteristic features of ORS. The area of steel being comparitively large, the stresses developed in steel at the failure

→ P.T.O

of the members may not reach tensile strength of steel

In structural concrete member, it is undesirable to have sudden failure without any warning in the form of excessive deflection and wide spread cracks. So, generally, the use of ORS are discouraged

(iv) **SHEAR FAILURE:** If the sections are not adequately designed for shear, they may fail by shear force before attaining their full flexural strength.

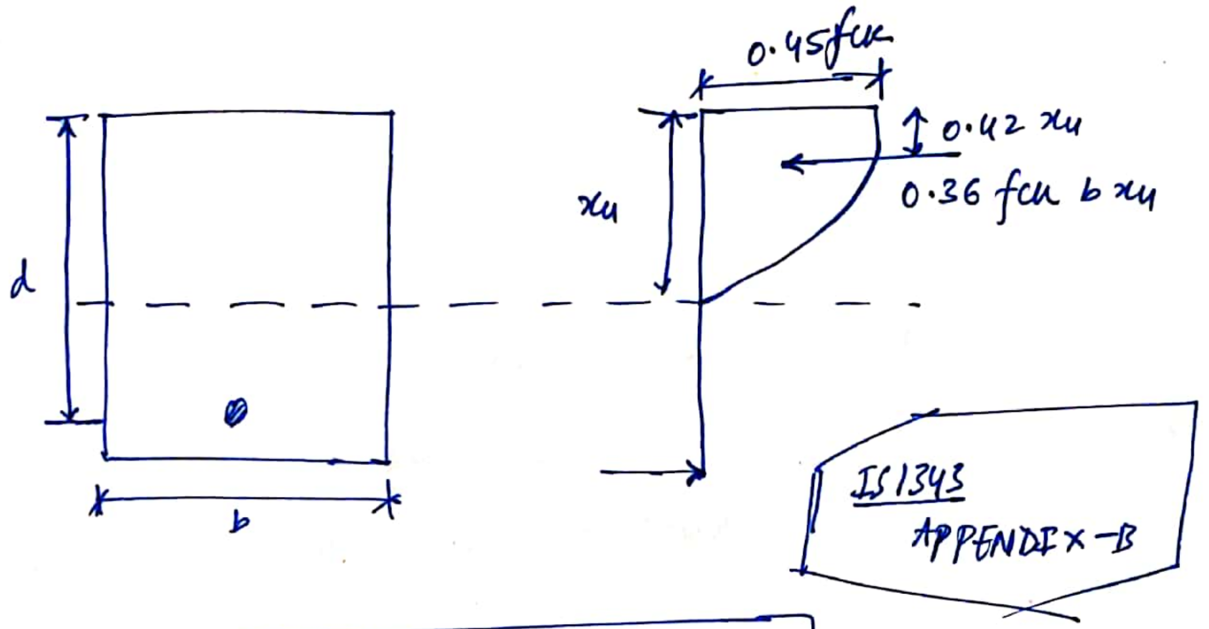
(v) **Failure of bond between steel and concrete:**

This type of failure ~~takes~~ takes place due to inadequate transmission lengths at the ends of the member

(vi) **Anchorage failures:**

These failures may takes place in end blocks, if not properly designed in post tensioned members to resist transverse tensile forces

MOMENT OF RESISTENCE OF RECTANGULAR SECTION (IS1343)



$$\boxed{MR_U = f_{pu}\ A_p\ (d - 0.42\, x_u)}$$

where; MR_U - Ultimate moment of resistence of the section

b - width of the member

d - effective depth

f_{pu} - Tensile stress developed in tendons @ the failure stage of the beam.

f_p - characteristic tensile strength of prestressing steel

f_{pe} - effective prestress in tendons after losses.

A_p - Area of prestressing tendons

x_u - neutral-Axis depth.

→ The value of 'f_{pu}' depends upon the effective reinforcement ratio

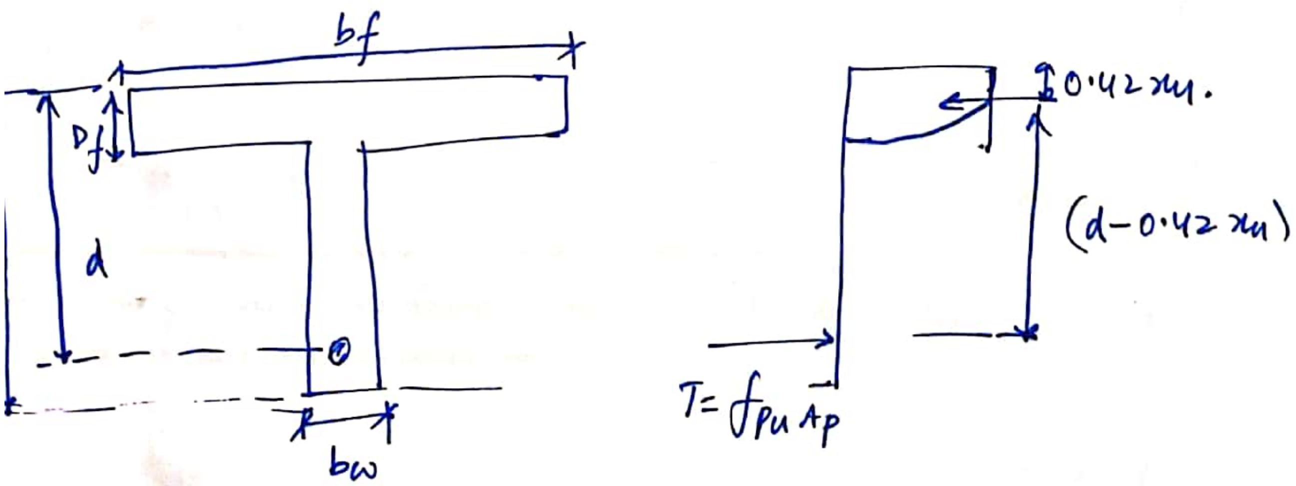
$$\left(\frac{A_p f_p}{bd\, f_{ck}}\right).$$

Table (11) – Conditions at the ultimate L.S for rectangular Beams with pre tensioned Tendons (or) with post-tensioned tendons having effective Bond.

Table (12) – Conditions at the ultimate L.S for post tensioned Rectangular Beams having unbonded Tendons.

MOR of T (or) I sections.

(a) when N.A lies in flange:



$$\boxed{MOR = f_{pu} A_p (d - 0.42 x_u)}$$

(i) MOR of pre tensioned and post tensioned T (or) I sections.

- Find effective reinforcement ratio $\frac{A_p f_p}{b d f_{ck}}$

- For the above effective reinforcement ratio,

  Find $\frac{f_{pu}}{0.87 f_p}$ and x_u/d from Table (11) of IS 1343.

  The find x_u and f_{pu}.

If $x_u < D_f$

Find the MOR of rectangular / T & I section using formula

$$\boxed{MOR = f_{pu} A_p (d - 0.42 x_u)}$$

(ii) MOR of unbonded T/I section (post tensioned)

- Find the effective reinforcement ratio $\frac{A_p f_{pe}}{b d f_{cu}}$ & $\frac{l}{d}$ ratio

- For above effective reinforcement ratio, and l/d ratio

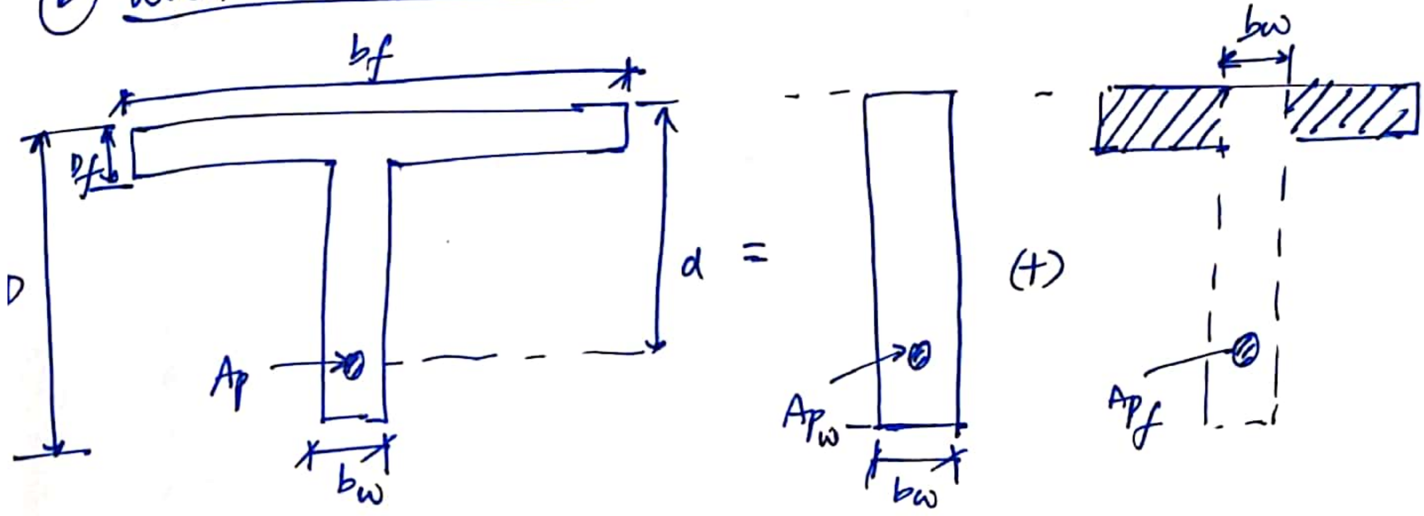
  find $\frac{f_{pu}}{f_{pe}}$ and x_u/d from Table (12) of IS 1343.

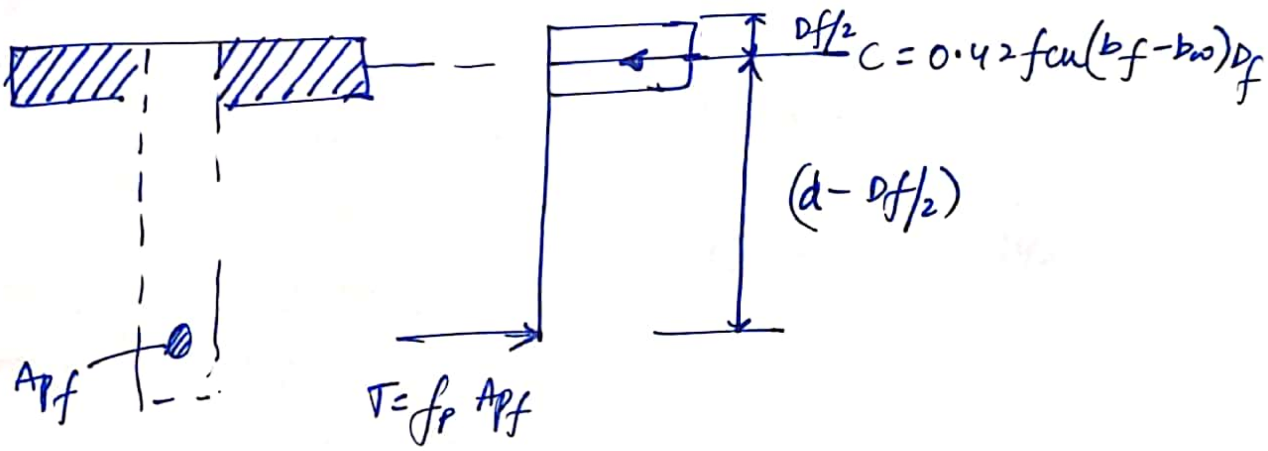
  and find x_u and f_{pu}

→ P.T.O

If $x_u < D_f$; $\boxed{MOR = f_{pu} A_p (d - 0.42 x_u)}$

(b) when N.A lies in web:





$$\boxed{A_p = A_{pf} + A_{pw}}$$

Compression = Tension

$$0.45 f_{cu} (b_f - b_w) D_f = f_p \cdot A_{pf}.$$

→ P.T.O

$$\boxed{M_{Total} = \underbrace{f_{pu} A_{pw} (d - 0.42 x_u)}_{M_2} + \underbrace{A_{pf} f_p (d - D_f/2)}_{M_1}}$$

$\Downarrow$

$$\boxed{\begin{array}{l} M_{flange} = M_1 = f_p A_{pf} (d - D_f/2) \\ \hline M_{web} = M_2 = f_{pu} A_{pw} (d - 0.42 x_u) \end{array}}$$

(1) MOR of pre tensioned and post tensioned bonded flanged sections.

- Find effective reinforcement ratio $\frac{A_p f_p}{b_f d f_{ck}}$

- For the above effective reinforcement ratio, find $\frac{f_{pu}}{0.87 f_p}$ and $\frac{x_u}{d}$ from Table (11) of IS 1343.

  and find f_{pu} and x_u.

- If $x_u > D_f$, find A_{pf} and A_{pw} using the following expressions.

$$\boxed{A_{pf} = 0.45 f_{ck} (b - b_w) D_f / f_p}$$

$$\boxed{A_{pw} = A_p - A_{pf}}$$

- Again find new effective reinforcement ratio $\frac{A_{pw} f_p}{b_w d f_{ck}}$

- For the new effective reinforcement ratio, from table (11) of IS 1343, find $\frac{f_{pu}}{0.87 f_p}$ and x_u/d.

  Then find f_{pu} and x_u.

- Obtain the MOR of the flanged section using the formula.

$$\boxed{MOR = f_{pu} \; A_{pw} \left(d - 0.42 x_u\right) + A_{pf} f_p \left(d - D_f/2\right)}$$

i) MOR of post tensioned unbonded flanged section

- Find the effective reinforcement ratio $\frac{A_p f_{pe}}{b d f_{ck}}$ and l/d

- For the above effective reinforcement ratio and l/d ratio, find $\frac{f_{pu}}{f_{pe}}$ and $\frac{x_u}{d}$ from table (12) of IS 1343 and

  Then find f_{pu} and x_u

→ P.T.O

- If $x_u > D_f$: find A_{pf} and A_{pw} using the following equations.

$$\boxed{A_{pf} = 0.45\, f_{cu}\,(b - b_w)\, D_f / f_p}$$

$$\boxed{A_{pw} = A_p - A_{pf}}$$

- Again find new effective reinforcement ratio,

$\dfrac{A_{pw}\, f_{pe}}{b_w\, d\, f_{ck}}$ and l/d ratio.

- For the new effective reinforcement ratio and l/d ratio from Table (12) IS 1343.

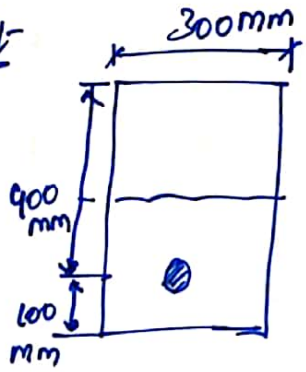
Find $\dfrac{f_{pu}}{f_{pe}}$ and $\dfrac{x_u}{d}$ and then find f_{pu} and x_u

- obtain MOR of flange section using the following relation

$$\boxed{MOR = f_{pu}\, A_{pw}\, (d - 0.42\, x_u) + f_p\, A_{pf}\,(d - D_f/2)}$$

Q. Calculate the ultimate moment carrying capacity of a pretensioned section of size (300×500) mm having an effective cover of 100mm. Take $f_{cu} = 42\ N/mm^2$; $f_p = 1900\ N/mm^2$, $A_p = 600\ mm^2$?

A-



$$\frac{A_p f_p}{b d f_{cu}} = \frac{600\times1900}{300\times400\times42} = 0.22$$

From Table (11).

$\frac{A_p f_p}{b d f_{cu}} \Rightarrow$

0.20 → 1
0.22 → 1 — By interpolation
0.25 → 1.

So, $\frac{f_{pu}}{0.87 f_p} = 1 \Rightarrow f_{pu} = 0.87\times1900\times1 = 1653\ N/mm^2$

$\frac{x_u}{d} \Rightarrow$

0.20 — 0.435
0.22 — 0.477.
0.25 — 0.542

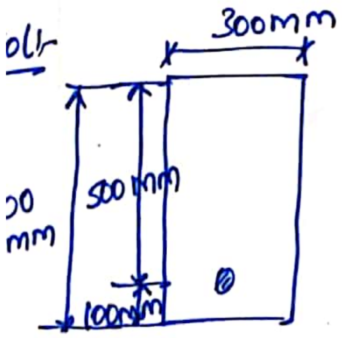
So, $\frac{x_u}{d} = 0.477 \Rightarrow x_u = 400\times0.477 = 190.8\,mm$

So, $MOR_U = f_{pu} A_p (d - 0.42 x_u) = 1653\times600 (d - 0.42\,190.8)$

$$\boxed{MOR_U = 317.2\ KN.m}$$

$\underline{P_2}$ A psc rectangular beam (300×600) mm has a span of 10m. It is prestressed with 800 mm^2 steel wires and CG of steel is 100 mm above bottom edge. Take $f_p = 1700 \text{ N/mm}^2$, $f_{ck} = 40 \text{ N/mm}^2$. Find the ultimate moment carrying capacity of section and the maximum U.D.L of the beam can carry?

Sol:-



$f_p = 1700 \text{ N/mm}^2$; $A_p = 800 \text{ mm}^2$

$f_{ck} = 40 \text{ N/mm}^2$; $d = 500 \text{ mm}$

$$\frac{A_p f_p}{b d f_{ck}} = \frac{800 \times 1700}{300 \times 500 \times 40} = 0.226$$

From Table (11) —— Because f_{pe} is not given so it is pretensioned.

0.2 — 1

$\boxed{0.226 — 1}$

0.25 — 1

So, $\frac{f_{pu}}{0.87 f_p} = 1$

$f_{pu} = 1 \times 0.87 \times 1700 = 1479$ Mpa

$\|ly$ 0.2 — 0.435

$\boxed{0.226 — 0.49}$

0.25 — 0.542

$\frac{x_u}{d} = 0.49 \Rightarrow x_u = 0.49 \times 500$

$\boxed{x_u = 245 \text{mm}}$

$MOR_U = f_{pu}\, A_p\,(d - 0.42x_u)$.

$MOR_U = 1479 \times 800\,[500 - 0.42\,(243)]$.

$\boxed{MOR_U = 469.85 \text{ kN-m}}$

$\therefore$ B.M created due to external loads = MOR_U.

For SSB carrying UDL $\rightarrow$ So, $\frac{wl^2}{8} = 469.85$.

$w = \frac{469.85 \times 8}{10^2}$

$w = 37.59$ kN/m

$DL = \gamma_c \cdot A_c$
$= 24\,(0.3 \times 0.6)$
$= 4.32$ kN/m.

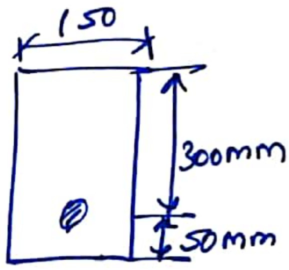
$W = DL + LL \Rightarrow LL = W - DL = 37.59 - 4.32$

$\boxed{L.L = 33.29 \text{ kN/m}}$

$\rightarrow$ P.T.O

Q3 A post tensioned beam having a rectangular section (150×350) mm having a cover of 50mm. If $A_p = 461\ mm^2$, $f_{cu} = 40$ Mpa, $f_p = 1600$ Mpa. estimate the ultimate moment carrying capacity of the section assuming the ratio of $\frac{\text{eff span}}{D}$ as 20; Effective stress in tendon after all losses is 800 Mpa?

Sol:- $f_p = 1600$ Mpa ; $A_p = 461\ mm^2$



$f_{cu} = 40$ Mpa ; $d = 300$ mm

$b = 150$ mm ; $\frac{L}{D} = 20$

$f_{pe} = 800\ N/mm^2$ —— f_{pe} is given means Unbounded Tendon

From Table (12)

$$\frac{A_p f_{pe}}{b\,d\,f_{cu}} = \frac{461 \times 800}{150 \times 300 \times 40} = 0.205$$

$$\frac{f_{pu}}{f_{pe}} = 1.16 \Rightarrow f_{pu} = 1.16 \times 800 = 928\ N/mm^2$$

$$\frac{x_u}{d} = 0.58 \Rightarrow x_u = 0.58 \times 300 = 174\ mm$$

$$MOR_U = f_{pu} A_p (d - 0.42 x_u)$$

$$= 928 \times 461 (300 - 0.42(174))$$

$$\boxed{MOR_U = 97.08 \text{ KN-m}}$$

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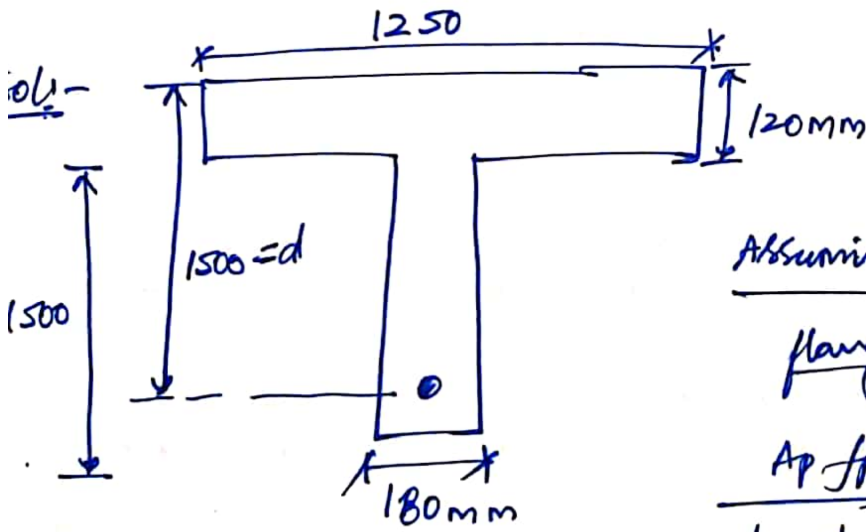
4 A pre Tensioned T-beam has a flange of 1250mm width 120mm thickness. The depth and thickness of the web are 1500 and 180mm respectively. The high Tensile Steel wires of area 4700 mm^2 are located at an effective depth of 1500mm. M40 grade Concrete is used Take $f_p = 1600 \, N/mm^2$. Calculate the ultimate flexure Strength of T-beam

Sol:-



Assuming NA lying within the flange

$$\frac{A_p f_p}{b_f \, d \, f_{cu}} = \frac{4700 \times 1500}{1250 \times 1500 \times 40} = 0.1$$

$$\frac{x_u}{d} = 0.217 \Rightarrow x_u = 0.217 \times 1500 = 325.5\,mm > D_f.$$

$\therefore$ $\boxed{\text{NA lies inside the web.}}$

$$A_p = A_{pf} + A_{pw}$$

$$\Rightarrow 0.45 f_{ck} [b_f - b_w] D_f = f_p A_{pf}.$$

$$0.45(40) [1250 - 180] 120 = 1600 \times A_{pf}.$$

so, $\boxed{A_{pf} = 1444.5\,mm^2}$

$$A_{pw} = A_p - A_{pf} = 4700 - 1444.5 = 3255.5\,mm^2$$

So, $$\frac{A_{pw} f_p}{b_w\, d\, f_{ck}} = \frac{3255.5 \times 1600}{180 \times 1500 \times 40} = 0.48.$$

$$\frac{f_{pu}}{0.87 f_p} = 0.9 \Rightarrow f_{pu} = 0.9 \times 0.87 \times 1600$$

$$f_{pu} = 1252.8\ N/mm^2$$

$\longrightarrow$ P.T.O

$$\frac{x_u}{d} = 0.783 \Rightarrow \text{ \& } x_u = 0.783 \times 1500 = 1174.5\,mm.$$

$$MORU = f_{pu}\, A_{pw}\,(d - 0.42 x_u) + f_p\, A_{pf}\left(d - \frac{D_f}{2}\right).$$

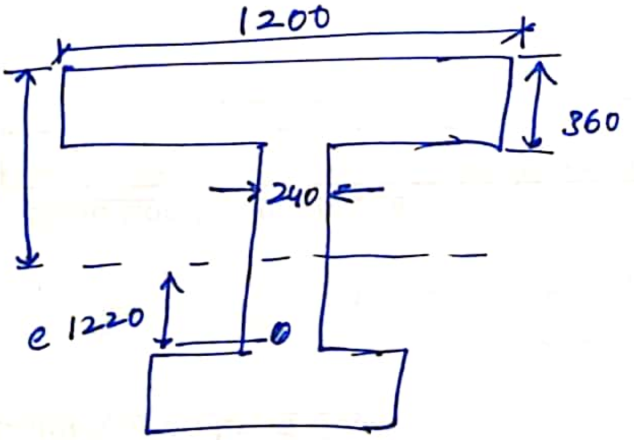
$$= 1252.8 \times 3255.5\,(1500 - 0.42(1174.5))$$

$$+ 1600 \times 1444.5\left(1500 - \frac{120}{2}\right).$$

$$\boxed{MORU = 7433.98\ kN\text{-}m}$$

Q5 An unsymmetrical I-section bridge girdles has the following sectional properties: width and thickness of flange is 1200 and 360mm web portion is 240mm thick. Centroid of the section is located at 580mm from the top. The girdle is used over a span of 40m and the tendons are unbonded with a c/s area 7000 mm^2 are parabolic with an eccentricity of 1220mm at the center of the span f_{cu} = 45 Mpa, f_{pe} = 1200 Mpa, estimate the ultimate flexural strength of the ~~Concrete~~ section?

Sol-



$f_{cu} = 45$ Mpa

$f_{pe} = 1200$ Mpa.

$A_p = 7000 mm^2$

$d = 580 + 1220 = 1800mm$

$$\frac{\text{Span}}{\text{depth}} = \frac{40\times10^3}{1800} = 22.22.$$

NA is lies ~~with~~ In the flange

$$\frac{A_p f_{pe}}{b\, d\, f_{cu}} = \frac{7000\times1200}{1200\times1800\times45} = 0.086$$

	30	~~22~~ 22	20
0.05 —	0.16	0.16	0.16
$\boxed{0.086}$ —		$\boxed{0.271}$	
0.10 —		0.315	

$$\frac{x_u}{d} = 0.271 \Rightarrow x_u = 0.271 \times 1800 = 487.8\,mm > D_f.$$

So, assumption is wrong

N.A is ~~wo~~ in the web

$$A_p = A_{pf} + A_{pw}.$$

when f_p is given put f_p here

$$0.45\, f_{cu}\,(b_f - b_w)\,D_f = f_{pe}\; A_{pf}$$

$$\Rightarrow 0.45 \times 45\,(1200 - 240)\,360 = 1200\; A_{pf}$$

$$\boxed{A_{pf} = 5832\,mm^2}$$

$$\text{So, } A_{pw} = A_p - A_{pf} = 7000 - 5832 = \cancel{1158}\ 1168\ mm^2$$

$$\frac{A_{pw}\,f_p}{b_w\,d\,f_{cu}} = \frac{1168 \times 1200}{240 \times 1800 \times 45} = 0.072$$

	30	22022	20
0.05	1.21	1.295	
0.072		1.272	
0.10	1.18	1.242	

$$\frac{f_{pu}}{0.87 f_p} = 1.272 \Rightarrow f_{pu} = 1.272 \times 1200 = 1526.4$$

$$\frac{x_u}{d} = 0.228$$

	30	22.22	20
0.05	0.16	0.16	0.16
0.072		0.228	
0.10	0.3	0.315	0.32

$$x_u = 0.228 \times 1800 = 410.4 \text{mm}$$

$$MOR_v = f_{pu} \; A_{pw} (d - 0.42 x_u) + f_{pe} \; A_{pf} (d - D_f/2)$$

$$MOR_v = 1526.4 \times 1168 \, (1800 - 0.42 (410.4)) + 1200 \times 5832 \, (1800 - 360/2)$$

$$\boxed{MOR = 14239.2 \text{ KN-m}}$$

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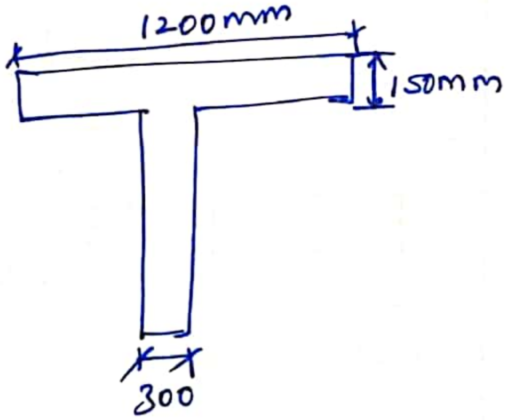
Q6 A post tensioned unbonded prestressed concrete beam of T-section having a $b_f = 1200\,mm$ and $D_f = 150\,mm$, $b_w = 300\,mm$ is prestressed by $4700\,mm^2$ of high tensile steel located at an effective depth of 1600 mm. $f_{cu} = 40\,Mpa$, $f_p = 1600\,Mpa$. The span to depth ratio is 20 and effective prestress after all losses is 1000 Mpa. Estimate the ultimate flexural strength of unbonded section?

Sol:-



$A_p = 4700\,mm^2$

$f_{cu} = 40\,Mpa$

$f_p = 1600\,Mpa$

$f_{pe} = 1000\,Mpa$

$d = 1600\,mm$

$\frac{l}{d} = 20.$

NA lies in flange

$$\frac{A_p f_{pe}}{b_f d f_{cu}} = \frac{4700 \times 1000}{1200 \times 1600 \times 40} = 0.06.$$

0.05	0.16
$\boxed{0.06}$	$\boxed{0.192}$
0.10	0.32

$$\frac{x_u}{d} = 0.192 \Rightarrow x_u = 0.192 \times 1600 = 307.2\,mm > D_f.$$

so, assumption is wrong

NA lies in web

$$A_p = A_{pf} + A_{pw}.$$

$$\Rightarrow 0.45\, f_{ck} (b_f - b_w)\, D_f = f_p\, A_{pf}$$

$$0.45 \times 40\,(1200 - 300)\, 150 = 1600 \times A_{pf}$$

$$\boxed{A_{pf} = 1518.75\,mm^2}.$$

$$A_{pw} = A_p - A_{pf} = 4700 - 1518.75 = 3181.25\,mm^2$$

$$\frac{A_{pw}\, f_{pe}}{b_w\, d\, f_{ck}} = \frac{3181.25 \times 1000}{300 \times 1600 \times 40} = 0.165.$$

0.15	1.2
$\boxed{0.165}$	$\boxed{1.188}$
0.20	1.16

$$\frac{f_{pu}}{f_{pe}} = 1.188 \Rightarrow f_{pu} = 1.188 \times 1000 = 1188 \text{ Mpa.}$$

$$0.15 - 0.46$$

$$\boxed{0.165 - 0.496}$$

$$0.2 - 0.58$$

$$\frac{x_u}{d} = 0.496 \Rightarrow x_u = 0.496 \times 1600 = 793.6 \text{ mm.}$$

$$MOR_U = f_{pu} A_{pw} (d - 0.42 x_u) + f_p A_{pf} (d - D_f/2)$$

$$= 1188 \times 3181.25 (1600 - 0.42(793.6)) + 1600 \times \cancel{1}1518.75 (1600 - 150/2)$$

$$\boxed{MOR_U = 8.493 \times 10^3 \text{ kN-m}}$$

~~Q2 A post tensioned bridge grides with unbonded tendons is of box section of overall dimension (1200×1800mm) mm with a wall thickness of 150mm. The high tensile steel has an area of 4000 mm² is located at an effective~~ ~~depth prestress in steel after~~

→ P.T.O

Q7 A pre tensioned beam of rectangular section (300x700) mm is stressed by 800 mm² of high tensile steel located at an effective depth of 600 mm. The beam is also reinforced with supplementary reinforcement consisting of 4 bars of 25 mm ϕ of fe415 grade HYSD steel located 100 mm from soffit. Estimate the flexural strength of the section. Assume characteristic strength tendon as 1600 Mpa and fcu = 40 Mpa?

Sol:

$$A_s = 4 \times \left(\pi/4 \times 25^2\right) = 1963.5 \text{ mm}^2$$

$f_{cu} = 40$ Mpa

$f_p = 1600$ Mpa

$A_p = 800$ mm²

$b = 300$ mm

$d = 600$ mm

$f_y = 415$ Mpa.

The untensioned supplementary reinforcement is replaced by an equivalent area of prestressing steel by using formula.

$$\frac{A_s f_y}{f_p} = \frac{1963.5 \times 415}{1600} = 509.28 \text{ mm}^2$$

∴ Total area of prestressing steel, $A_p = A_p' + 509.28$

$$A_p = 800 + 509.8 = 1309.28 \text{ mm}^2$$

$$\frac{A_p f_p}{b\,d\,f_{cu}} = \frac{1309.28 \times 1600}{300 \times 600 \times 40} = 0.29.$$

$$\frac{f_{pu}}{0.87 f_p} \Rightarrow 2$$

0.25	0.592
0.29	0.632
0.30	0.655

$$\frac{x_u}{d} = 0.632 \Rightarrow x_u = 0.632 \times 600 = 379.2\,mm.$$

$$M_{ORU} = f_{pu} \cdot A_p \, (d - 0.42 x_u)$$

$$= 1392 \times 1809.28 \left(600 - 0.42\,(379.2)\right)$$

$$\boxed{M_{ORU} = 803.25 \text{ kN-m}}$$

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UNIT-4

ANALYSIS FOR SHEAR

The shear stress distribution of a member is the function of shear force and the properties of cross section of the member

$$\tau_v = \frac{V(A\bar{y})}{Ib}.$$

where, τ_v = Shear stress in N/mm^2

$A\bar{y}$ = first moment of area [statical moment].

V = shear force in N

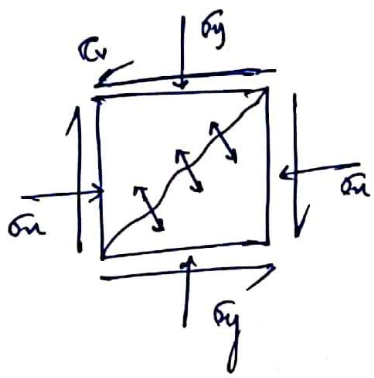
I = moment of inertia about its centroidal axis in mm^4

b = bredth of section in mm

[Incase of T & I section $b = b_w$]

- In a psc member, the shear stress is generally accompanied by direct stress in axial direction of a member and if transverse vertical prestressing is adopted, compressive stress perpendicular to member axis will be present in addition to the axial prestress.

- The most general case of an element subjected to a 2D stress system is shown in figure



$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x + \sigma_y)^2 + 4\tau_v^2}$$

analysis of shear

$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_v^2}$$
(C)

$$\sigma_{min} = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_v^2}$$
(T)

σ_{max} = major principle stress.

σ_{min} = minor principle stress.

- In a psc member, the direct stress σ_x and σ_y being compressive, the magnitude of principle tensile stress is considerably reduced and in some cases even eliminated so that under working load, both major and minor principle stresses are compressive, thereby eleminating the risk of diagonal tension cracks in concrete.

- Minor principle stress (σ_{min}) develops tensile stress and have diagonal crack will occur minor principle stress is also called as diagonal tension.

- In general there are 3 ways of improving the shear resistence of structural concrete member by pre-stressing techniques.

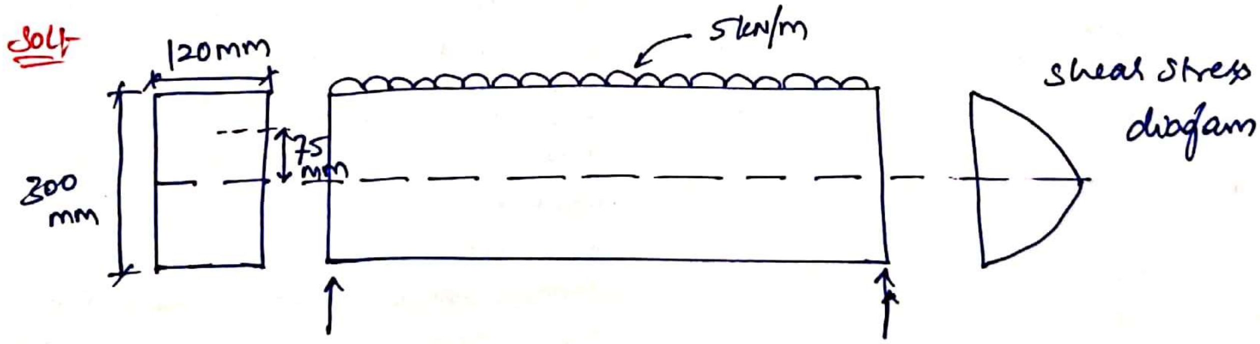
① Horizantal (or) axial prestressing

② verticle (or) transverse prestressing

③ prestressing by inclined (or) sloping cables.

Q1 A psc beam of span 10m is of rectangular section 120 mm wide and 300mm deep is axially prestressed by a cable carrying an effective force of 180 kN. The beam supports a total UDL of 5kN/m which includes self weight of the member. Compare the magnitude of principle tension developed in the beam with & without the axial prestress?

Sol:-



R = 25kN

$$R = \frac{wl}{2}$$

$$= \frac{5\times10}{2} = 25kN$$

$$A = 120 \times 300 = 36{,}000\ mm^2$$

$$I = \frac{120 \times 300^3}{12} = 270 \times 10^6\ mm^4.$$

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$$\tau_v = \frac{V(A\bar{y})}{Ib} = \frac{25\times10^2\,(120\times150)\,75}{(270\times10^6)\,(120)} = 1.041\ N/mm^2$$

ⓐ without axial prestressing

$\sigma_x = 0$; $\sigma_y = 0$; $\tau_v = 1.041\ N/mm^2$

$$\sigma_{max} = \frac{\sigma_x+\sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau_v^2}$$

$$= 0 + \frac{1}{2}\sqrt{0^2+4(1.04)^2} = 1.041\ N/mm^2$$

$$\sigma_{min} = \frac{\sigma_x+\sigma_y}{2} - \frac{1}{2}\sqrt{(0)^2+4(1.04)^2} = (-)\ 1.041\ N/mm^2$$

ⓑ with axial prestress:

$$\sigma_x = \frac{P\cos\theta}{A} = \frac{(80\times10^3)\cos 0}{120\times300} = 5\ N/mm^2$$

$\sigma_y = 0$

$\tau_v = 1.041\ N/mm^2$

$$\sigma_{max} = \frac{5+0}{2} + \frac{1}{2}\sqrt{(5-0)^2+4(1.04)^2} = 5.208\ N/mm^2$$

$$\sigma_{min} = \frac{5+0}{2} - \frac{1}{2}\sqrt{(5-0)^2+4(1.04)^2} = (-)\ 0.208\ N/mm^2$$

$\therefore$ The percentage reduction in principle tension

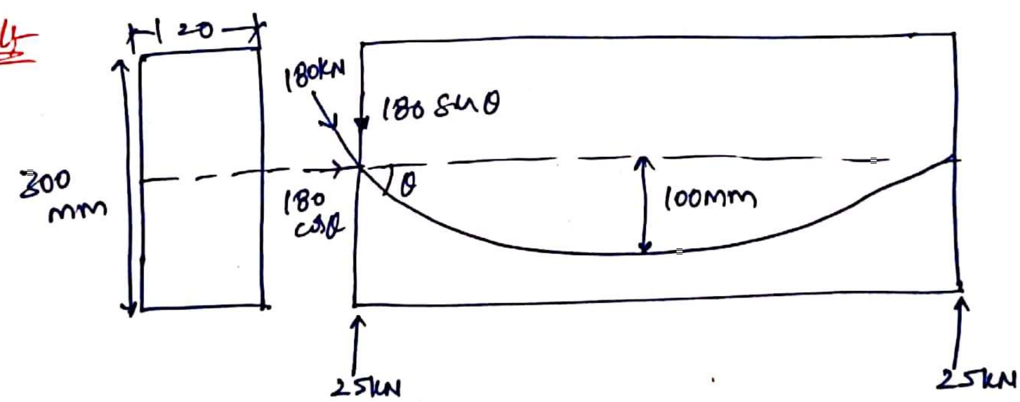
$$= \left(\frac{1.041 - 0.208}{1.041}\right) \times 100$$

$$= 80.02\ \%$$

Q2 Continued ---→

From the above problem, consider a curved cable having an eccentricity of 100mm at the center of span and reducing to zero at the supports is used. Estimate the percentage principal tension reduction in comparison with the case of axial prestressing?

Sol:



$$\theta = \tan^{-1}\left(\frac{4e}{l}\right) = \tan^{-1}\left(\frac{4 \times 0.1}{10}\right) = 2.29^\circ$$

$$V = wl/2 - P\sin\theta$$

$= 25 - (180 \times \sin 2.29)$

$$\boxed{V = 17.81\ kN}$$

$$\tau_v = \frac{V(A\bar{y})}{Ib} = \frac{(17.81)\times 10^3\,(120\times 150)75}{\frac{120\times 300^3}{12}(120)} = 0.74\ N/mm^2$$

$$\sigma_x = \frac{P\cos\theta}{A} = \frac{(180\times 10^3)\cos 2.29^\circ}{120\times 300} = 4.99\ N/mm^2$$

$$\sigma_y = 0$$

$$\sigma_{max} = \frac{4.99+0}{2} + \frac{1}{2}\sqrt{(4.99-0)^2 + 4(0.74)^2} = 5.09\ N/mm^2$$

$$\sigma_{min} = \frac{4.99+0}{2} - \frac{1}{2}\sqrt{(4.99-0)^2 + 4(0.74)^2} = (-)\,0.107\ N/mm^2$$

$\therefore$ Percentage reduction in principle tension with axial prestressing $\Big\} = \left(\frac{0.208 - 0.107}{0.208}\right) \times 100 = 48.56\%$

Q3 Continued:-

If the beam is additionally prestressed by verticle cables imparting a stress of 2.5 N/mm² in the direction of depth of beam, estimate the nature of principle stress?

Sol:-

$\sigma_x = 4.99\ N/mm^2$

$\sigma_y = 2.5\ N/mm^2$

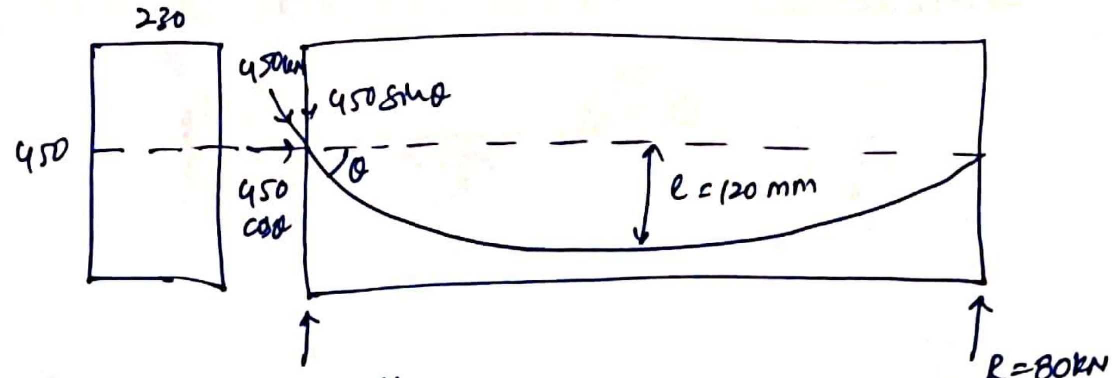
$\tau_v = 0.74\ N/mm^2$

$$\sigma_{max} = \left(\frac{4.99+2.5}{2}\right) + \frac{1}{2}\sqrt{(4.99-2.5)^2+4(0.74)^2} = 5.193\ N/mm^2$$

$$\sigma_{min} = \left(\frac{4.99-2.5}{2}\right) - \frac{1}{2}\sqrt{(4.99-2.5)^2+4(0.74)^2} = (+)\ 2.296\ N/mm^2$$

Q4 A rectangular beam (230×450) mm is simply supported over a span of 8m. it carries an UDL of 20kN/m. inclusive of self weight. It is prestressed with a parabolic cable with an eccentricity of 120mm at the center and reduces to zero at support. prestress in the cable is 450 kN. Determine the principle tensions. Also determine the verticle prestress required to reduce the principle tensions to zero?

Sol:-



$R = wl/2$

$= \frac{20\times 8}{2} = 80kN$

8 m.

$\theta = \tan^{-1}(4e/l) = \tan^{-1}\left(\frac{4(0.12)}{8}\right) = 3.43^\circ$

$A = 230 \times 450 = 103500\ mm^2$

$I = \frac{230 \times 450^3}{12} = 1.746 \times 10^9\ mm^4$

$V = wl/2 - 450 \sin 3.43 = 80 - 450 \sin 3.43 = 53.07\ kN$

$\sigma_x = \frac{P\cos\theta}{A} = \frac{450\times 10^3 \times \cos 3.43}{103500} = 4.34\ N/mm^2$

$\sigma_y = 0$

Area above NA $= 230 \times 450/2 = 51750\ mm^2$

$\tau_v = \frac{V(A\bar{y})}{Ib} = \frac{53.07\times 10^3 \times 51750 \times 112.5}{(1.746\times 10^9)(230)} = 0.77\ N/mm^2$

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$$\Rightarrow \sigma_{max} = \frac{4.34+0}{2} + \frac{1}{2}\sqrt{(4.34-0)^2 + 4(0.77)^2} = 4.47 \text{ N/mm}^2$$

$$\sigma_{min} = \frac{4.34+0}{2} - \frac{1}{2}\sqrt{(4.34-0)^2 + 4(0.77)^2} = (-)0.13 \text{ N/mm}^2$$

Now,

$$\sigma_{min} = \frac{\sigma_x+\sigma_y}{2} - \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2 + 4(\tau_v)^2}$$

$$0 = \frac{4.34+\sigma_y}{2} - \frac{1}{2}\sqrt{(4.34-\sigma_y)^2 + 4(0.77)^2}$$

$$\left(\frac{4.34+\sigma_y}{\cancel{2}}\right)^2 = \left(\cancel{\tfrac{1}{2}}\right)^2 (4.34-\sigma_y)^2 + 4(0.77)^2$$

$$(4.34)^2 + \sigma_y^2 + 8.68\,\sigma_y = (4.34)^2 + \sigma_y^2 - 8.68\,\sigma_y + 2.3716$$

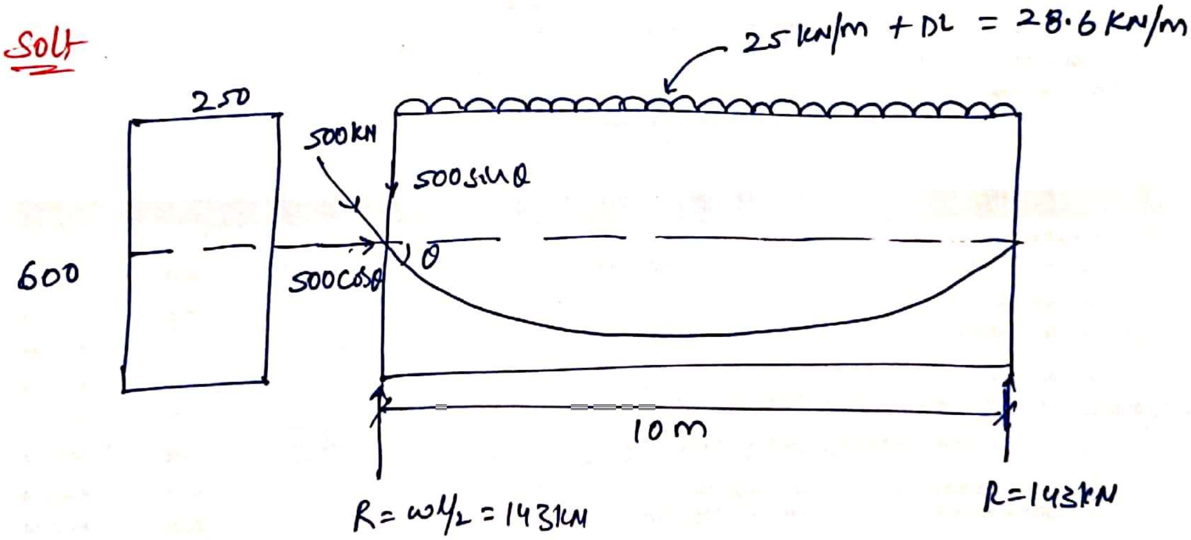
$$17.36\,\sigma_y = 2.3716$$

$$\boxed{\sigma_y = 0.136 \text{ N/mm}^2}$$

Q5 A beam (250×600) mm in a section supported over a span of 10m. It carries a UDL of 25 kN/m in addition to its own weight. It is prestressed by a parabolic cable with maximum eccentricity of 240mm at center and zero at support. Effective prestress in cable is 500kN.

Determine the principle tension at the support at the centroidal axis. Is the beam safe, if M45 grade concrete is used?

Sol:



$$\theta = \tan^{-1}\frac{4e}{l} = \tan^{-1}\left(\frac{4\times 0.24}{10}\right) = 5.48^\circ$$

$$DL = \gamma_c \times A_c = 24\times(0.25\times 0.6) = 3.6 \text{ kN/m.}$$

$$\text{Total Load} = 25+3.6 = 28.6 \text{ kN/m}$$

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$$V = \left[\frac{wl}{2} - p\sin\theta\right]$$

$$= 143 - 500 \sin 5.48 = 95.25\,kN$$

$$\sigma_x = \frac{p\cos\theta}{A} = \frac{500\times10^3 \times \cos 5.48}{250\times 600} = 3.32\ N/mm^2$$

$$\sigma_y = 0$$

ⓐ @ support ; Centroidal axis

$$\tau_v = \frac{V(A\bar{y})}{Ib} = \frac{95.25\times10^3\,(250\times300\times150)}{\frac{250\times600^3}{12}\,(250)}$$

$$\boxed{\tau_v = 0.952\ N/mm^2}$$

$$\sigma_{max} = \frac{3.32+0}{2} + \frac{1}{2}\sqrt{(3.32-0)^2 + 4\,(0.952)^2} = 3.57\ N/mm^2$$

$$\sigma_{min} = \frac{3.32+0}{2} - \frac{1}{2}\sqrt{(3.32-0)^2 + 4\,(0.952)^2} = (-)\,0.254\ N/mm^2$$

(b) @ support ; 60 mm above centroidal axis.

$$\tau_v = \frac{V(A\bar{y})}{Ib} = \frac{95.25\times10^3\,(250\times240)\,180}{\frac{250\times600^3}{12}\times 250} = 0.914\ N/mm^2$$

$$\sigma_{max} = \frac{3.32+0}{2} + \frac{1}{2}\sqrt{(3.32-0)^2 + 4(0.914)^2} = 3.55\ N/mm^2$$

$$\sigma_{min} = \frac{3.32+0}{2} - \frac{1}{2}\sqrt{(3.32-0)^2 + 4(0.914)^2} = (-)\,0.235\ N/mm^2$$

From page 46

permissable principle Tension $= f_t = 0.24\sqrt{f_{ck}}$

$= 0.24\,(45) = 1.61\ N/mm^2$

$> (-)0.235\ N/mm^2$

As the values are within permisable limits

Hence beam is safe

Q6 An I-Section has following sectional details.

TOP flange = (600 × 200) mm

web = (150 × 800) mm

Bottom flange = (300 × 200) mm

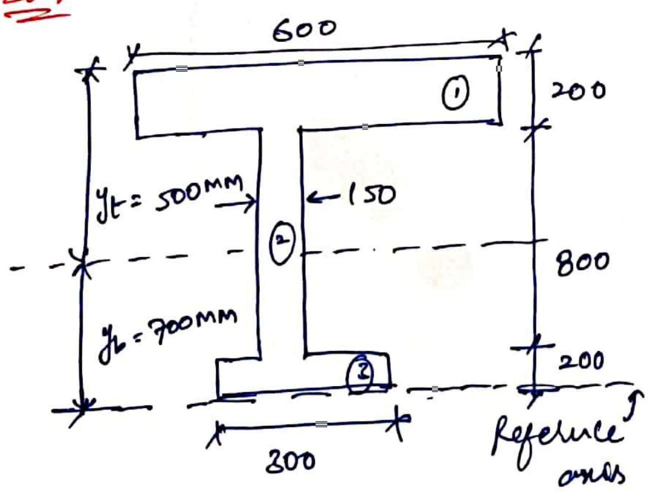
Shear force across section = 250 KN

Effective pre stress in Cable = 1500 KN

Inclination of Tendon @ given section = $\sin^{-1}(1/20)$

Fibre stress distribution across section varies linearly from 11 N/mm² compression at top and 1 N/mm² compression @ soffit. Find the principal stress @ the centroidal axis and at the junction of web and lower flange?

Sol:-



$A_1 = 200 \times 600 = 120000\ mm^2$

$A_2 = 150 \times 800 = 120000\ mm^2$

$A_3 = 300 \times 200 = 60000\ mm^2$

$y_1 = 200 + 800 + \frac{200}{2} = 1100\ mm$

$y_2 = 200 + 800/2 = 600\ mm$

$y_3 = \frac{200}{2} = 100\ mm$

$$\bar{y} = y_b = \frac{(120000 \times 1100) + (120000 \times 600) + (60000 \times 100)}{12000 + 12000 + 60000}$$

$y_b = 700$ mm ; $y_t = 500$ mm

Applying parallel axis theorem

$$I_{①} = I_{x_1 x_1} + A_1 (\bar{y} - y_1)^2$$

$$= \frac{600 \times 200^3}{12} + 120000\,(700 - 1100)^2 = 1.96 \times 10^{10}\ \text{mm}^4$$

$$I_{②} = I_{x_2 x_2} + A_2 (\bar{y} - y_2)^2$$

$$= \frac{150 \times 800^3}{12} + 120000\,(700 - 600)^2 = 7.6 \times 10^9\ \text{mm}^4$$

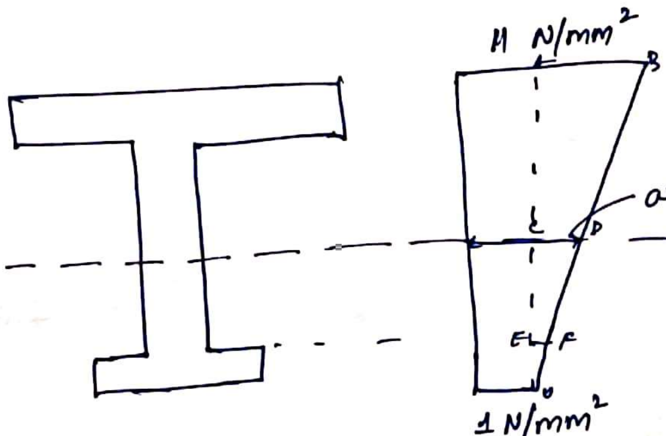
$$I_{③} = I_{x_3 x_3} + A_3 (\bar{y} - y_3)^2$$

$$= \frac{300 \times 200^3}{12} + 60000\,(700 - 100)^2 = 2.18 \times 10^{10}\ \text{mm}^4$$

$$I = I_1 + I_2 + I_3 = (1.96 + 0.76 + 2.18) \times 10^{10}$$

$$\boxed{I = 4.9 \times 10^{10}\ \text{mm}^4}$$

To find the principal stresses @ the centroidal axis.



axial prestress @ centroidal axis. (σ_x)

From similar Δles.

ΔAB = OCD

$$\frac{10}{1200} = \frac{y}{700}$$

$y = 5.83$ N/mm²

$\sigma_x = 1 + 5.83 = 6.83$ N/mm²

$\sigma_y = 0$ (NO PS in verticle direction)

$\theta = \sin^{-1}(1/20) = 2.86°$

Shear force $V = 250 - P\sin\theta = 250 - 1500 \sin(2.86°) = 175$ KN

$$\tau_v = \frac{V(A\bar{y})}{Ib}$$

$\bar{y} = y/2$ @ above NA

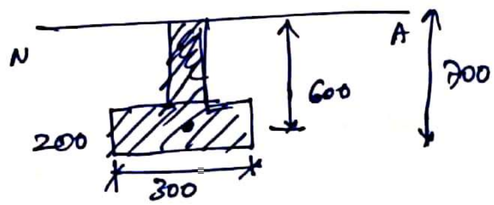
$$= \frac{175 \times 10^3 \left[(600\times200)\,\overset{(300+100)}{400} + (150\times300)\,150\right]}{4.9\times10^{10}\times150}$$

$= 1.304$ N/mm²

$$\sigma_{max} = \frac{6.83+0}{2} + \frac{1}{2}\sqrt{(6.83)^2 + 4(1.304)^2} = 7.07 \text{ N/mm}^2$$

$$\sigma_{min} = \frac{6.83+0}{2} - \frac{1}{2}\sqrt{(6.83)^2 + 4(1.304)^2} = (-)0.24 \text{ N/mm}^2$$

To find the principal tension at the junction of web and lower flange:



From similar tri

OAB = OEF

$$\frac{10}{1200} = \frac{y'}{200}$$

$y' = 1.67\ N/mm^2$.

$\sigma_x = 1 + 1.67 = 2.67\ N/mm^2$

$\sigma_y = 0$. (No ps in verticle direction)

$$\tau_v = \frac{V \cdot (A\bar{y})}{Ib}$$

$$\tau_v = \frac{\cancel{0.}75 \times 10^3 \left((300 \times 200 \times \overset{500+100}{600}) \right)}{4.9 \times 10^{10} \times 150} = 0.857\ N/mm^2$$

$$\sigma_{max} = \frac{2.67 + 0}{2} + \frac{1}{2}\sqrt{(2.67)^2 + 4(0.85)^2} = 2.92\ N/mm^2$$

$$\sigma_{min} = \frac{2.67 + 0}{2} - \frac{1}{2}\sqrt{(2.67)^2 + 4(0.85)^2} = (-)0.251\ N/mm^2$$

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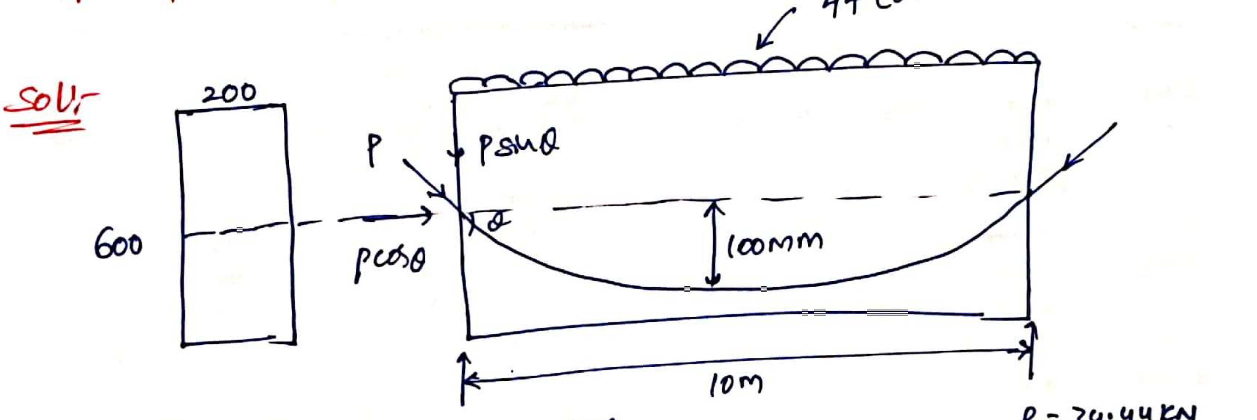
Q7 A concrete beam of rectangular section (200x600mm) is prestressed by a parabolic cable located at an eccentricity of 100mm at mid span and zero at supports. If the beam has a span of 10m and carries an UDL of 4kN/m. Find the effective force necessary in the cable for zero shear stresses at the support section. For this condition. Calculate the principle stresses Take $\gamma_c = 24\,kN/m^3$.

Sol:-



$R = \frac{wl}{2} = \frac{6.88\times}{2}$

$R = 34.4\,kN$

$$\theta = \tan^{-1}\left(\frac{4e}{l}\right) = \tan^{-1}\left(\frac{4(0.1)}{10}\right) = 2.29^\circ$$

$$V = \frac{wl}{2} - P\sin\theta.$$

$$0 = 34.4 - P\sin 2.29.$$

$$\boxed{P = 860.92\,kN}$$

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$$\sigma_x = \frac{P\cos\theta}{A} = \frac{860.92 \times 10^3 \times \cos 2.29}{200 \times 600} = 7.17\ N/mm^2$$

$\sigma_y = 0$

$\tau_v = 0$

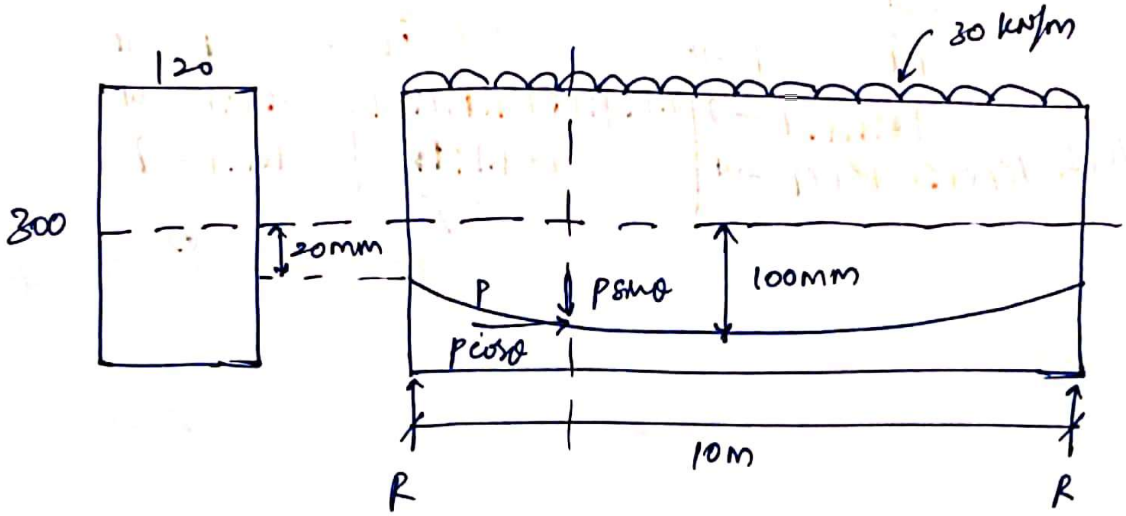
$$\sigma_{max} = \frac{7.17+0}{2} - \frac{1}{2}\sqrt{(7.17)^2 + 0^2} = 7.17\ N/mm^2$$

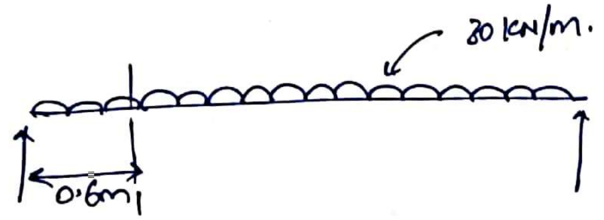
$$\sigma_{min} = \frac{7.17+0}{2} - \frac{1}{2}\sqrt{(7.17)^2 + 0^2} = 0\ N/mm^2.$$

Q8 A simply supported beam is 150mm wide and 300mm deep has a span of 10m is prestressed with a parabolic cable which has a maximum eccentricity of 100mm at mid span and minimum eccentricity of 200mm at supports. Effective prestressing force in cable is 300 KN. The beam carries an UDL of 30kN/m inclusive of DL.

Determine the principal tension at 0.6m from left support and 20mm above the centroidal axis?

Soln → PTO





R=150 kN

Shear force @ 0.6m $= 150 - (30 \times 0.6) = 132\ kN$

$$V = 132 - P\sin\theta$$

$$y = \frac{4ex}{l^2}(l-x)$$

$$y = \frac{4ex}{l} - \frac{4ex^2}{l^2}$$

$$\frac{dy}{dx} = \frac{4e}{l} - \frac{8ex}{l^2}$$

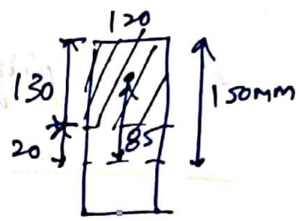
$$\tan\theta = \frac{4(80)}{10{,}000} - \frac{8(80 \times 600)}{(10{,}000)^2}$$

$\theta = \tan^{-1}(0.02816)$

$\boxed{\theta = 1.61^\circ}$

Slope @ 0.6m from left support $= 1.61^\circ$

$V = 132 - P\sin\theta = 132 - 300 \sin 1.61 = 123.56$ kN



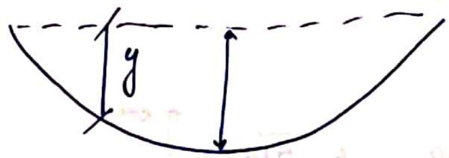
net shear force @ 0.6m from left support $\Big\} = 123.56$ kN

$$\tau_v = \frac{V(A\bar{y})}{Ib} = \frac{1.23\times 56 \times 10^3\,(130\times 120\times 85)}{\frac{120\times 300^3}{12}\times 120}$$

$\boxed{\tau_v = 5.05\ N/mm^2}$

NOTE: To find σ @ support, it is just prestressing force [Horizontal component / Area of concrete], but σ at intermediate span of beam, consider BM due to prestress @ eccentricity and BM due to DL and LL

$\longrightarrow$ P.T.O

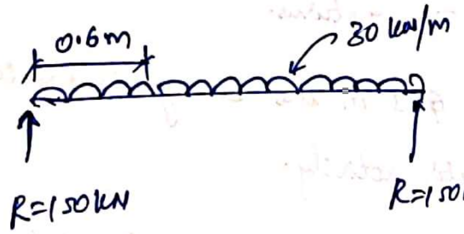


$$y = \frac{4rx}{L} - \frac{4rx^2}{L^2}.$$

$$y = \frac{4\times 80\times 600}{10{,}000^2}\,(10{,}000 - 600)$$

$$y = 18.05\ \text{mm}.$$

$$e = 20 + y = 20 + 18.05 = 38.05\ \text{mm}.$$



$$M_{xx} = (150\times 0.6) - \left(30\times \frac{0.6^2}{2}\right)$$

$$\boxed{M_{xx} = 84.6\ \text{kN-m}}$$

$$\sigma_x = \frac{P\cos\theta}{A} - \frac{P\cos\theta\, e}{I}\,y + \frac{My}{I}.$$

$$= \left[\frac{300\times 10^3 \cos 1.61}{120\times 300}\right] - \left[\frac{300\times 10^3 \times \cos 1.61 \times 38.5}{\frac{120\times 300^3}{12}}\times 20\right]$$

$$+ \left[\frac{84.6\times 10^6}{\frac{120\times 300^3}{12}}\right]\times 20$$

$$\sigma_x = 13.75\ \text{N/mm}^2\ ;\ \sigma_y = 0$$

σ_{max}

$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$= \frac{13.75+0}{2} + \frac{1}{2}\sqrt{(13.75-0)^2 + 4(5.05)^2}$$

$$\boxed{\sigma_{max} = 15.405\ N/mm^2}$$

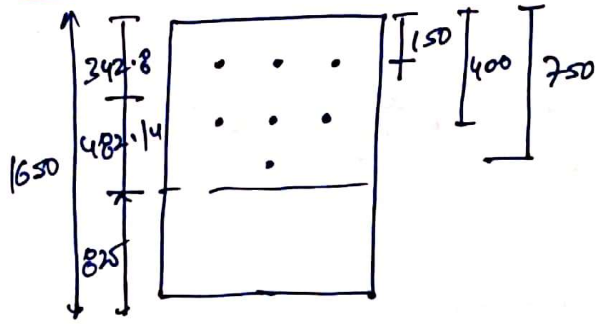
$$\sigma_{min} = \frac{13.75+0}{2} - \frac{1}{2}\sqrt{(13.75-0)^2 + 4(5.05)^2}$$

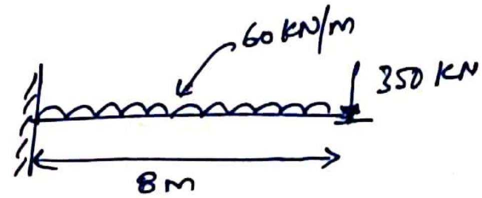
$$\boxed{\sigma_{min} = (-)\,1.65\ N/mm^2}$$

Q9 A cantilever portion of a prestressed concrete bridge of a rectangular c/s 600mm wide and 1650mm deep is 8m long and carries a reaction of 350 kN from the suspended span at the free end together with an UDL of 60kN/m inclusive of its own weight. The beam is prestressed by 7 cables carrying a force of 1000 kN (each cable) of which '3' are located @ 150mm, 3 @ 400mm and 1 @ 750mm from the top edge. Calculate the magnitude of the principle

Stresses @ 550 mm from top of Cantilever at Support section?

Sol:-



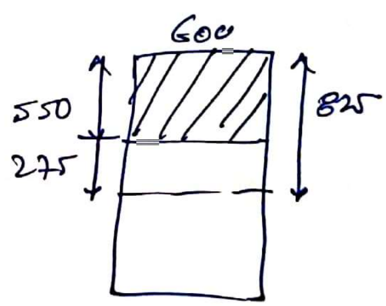


$$\bar{y} = \frac{(3\times 150) + (3\times 400) + (1\times 750)}{3+3+1}$$

$$\boxed{\bar{y} = 342.8 \text{ mm}}$$

$$e = 825 - \bar{y} = 825 - 342.8$$

$$\boxed{e = 482.14 \text{ mm}}$$



$$V = (60\times 8) + 350$$

$$\boxed{V = 830 \text{ kN}}$$

$$\tau_v = \frac{V(A\bar{y})}{Ib}$$

$$= \frac{(830\times 10^3)(550\times 600)(275 + 550/2)}{\frac{600\times 1650^3}{12}\times 600}$$

$$\tau_v = 1.12 \text{ N/mm}^2$$

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$$BM = -(350 \times 8) - (60 \times 8 \times 8/2)$$

(-ve) sign show hogging

$$= (-)4720 \text{ KNM}$$

$$\sigma_x = \frac{P\cos\theta}{A} + \frac{P\cos\theta\, e\, y}{I} - \frac{My}{Z}.$$

$$= \left(\frac{7 \times 1000 \times 10^3 \times \cos 0}{600 \times 1650}\right) + \left(\frac{7 \times 1000 \times 10^3 \times \cos 0 \times 482.14 \times 275}{\frac{600 \times 1650^3}{12}}\right)$$

$$- \left(\frac{4720 \times 10^6}{\frac{600 \times 1650^3}{12}}\right) \times 275.$$

$\boxed{\sigma_x = 5.42 \text{ N/mm}^2}$; $\boxed{\sigma_y = 0}$

$$\sigma_{max} = \frac{5.42 + 0}{2} + \frac{1}{2}\sqrt{(5.42)^2 + 4(1.12)^2} = 5.64 \text{ N/mm}^2$$

$$\sigma_{min} = \frac{5.42 + 0}{2} - \frac{1}{2}\sqrt{(5.42)^2 + 4(1.12)^2} = (-)0.22 \text{ N/mm}^2$$

SHEAR DESIGN OF PSC BEAMS.

Design steps!

(1) For External load, calculate ultimate shear force V_u,

Take load factor 1.5

(2) Calculate the Capacity of the section (or) ultimate

shear resistance

$$V_{co} = 0.67\, bD \sqrt{f_t^2 + 0.87\, f_{cp} f_t}$$

[Clause 22.4.1 page 46]

where, b = bredth of the member

D = overall depth of the member

f_t = maximum principle tensile stress.

$= 0.24 \sqrt{f_{ck}}$

f_{cp} = Compress stress @ centroidal axis due to prestress

(3) Comparing V_{co} and V_u.

→ PTO

If $V_{co} > V_u$; provide minimum shear reinforcement

$$\frac{A_{sv}}{b S_v} = \frac{0.4}{0.87 f_y} \quad \text{[clause 22.4.3.1, Page 48]}$$

where A_{sv} = area of stirrups

S_v = ~~sti~~ stirrups spacing

f_y = characteristic strength of the stirrups. ($\ngtr$ 415 N/mm²)

If $V_u > V_{co}$;

$$\frac{A_{sv}}{S_v} = \frac{V_u - V_{co}}{0.87 f_y d_t} \quad \text{[clause 22.4.3.2, P-48]}$$

d_t = depth from the extreme compression fibre either to the longitudinal bars (or) to the centroid of the tendons whichever is greater

∴ Maximum spacing of stirrups $\ngtr 0.75 d_t$ (or).

$\ngtr 4b$ [If $V_u > 1.8 V_{co}$ spacing $\ngtr 0.5 d_t$]

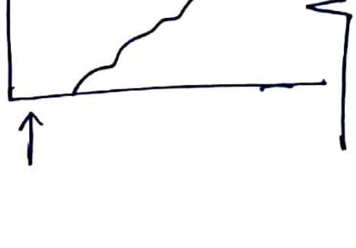
modes of failure due to shear

Failure due to shear is sudden as compared to failure due to flexure. Following ⑤ modes of failure due to shear are identified.

1. Diagonal Tension failure
2. Shear compression failure
3. Shear tension failure
4. web crushing failure
5. Arched rib failure.

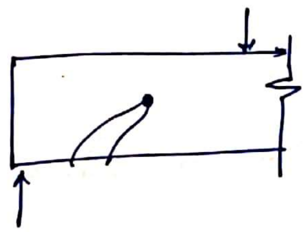
The occurance of mode of failure depends on span to depth ratio, loading, cross-section, amount of anchorage reinforcement

① Diagonal Tension failure:



In this mode an inclined crack propogates rapidly due to inadequate shear reinforcement

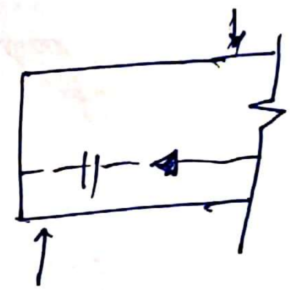
② Shear compression failure:



There is crushing of concrete near the compression flange above the tip of the inclined crack

③ Shear Tension failure:

Due to in adequate anchorage of longitudinal bars, the diagonal cracks propogate horizantally along the bar.



④ Web crushing failure:

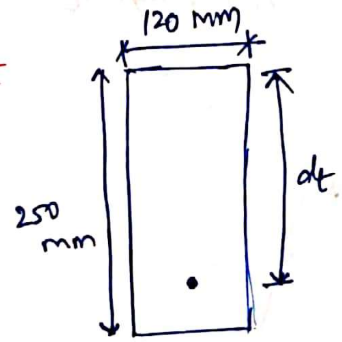
Concrete in the web crushes due to inadequate web thickness.

⑤ Arched rib failure:

For deep beams, the web may buckle and subsequently crush, There can be failure due to bearing stress.

Q The support of an psc beam 120mm wide and 250mm deep is required to support an ultimate shear force of 60KN A compressive prestress at centroidal axis is $5 N/mm^2$. Characteristic cube strength of concrete is $40 N/mm^2$. The cover to the tension reinforcement is 50mm. The characteristic strength of steel is $250 N/mm^2$. Design suitable reinforcement using IS 1343 provision?

Sol:



$d_t = 250 - 50 = 200mm$

$V_u = 60 KN$

$\sigma_x = f_{cp} = 5 N/mm^2$

$b = 120 mm, \ D = 250 mm$

$f_{cu} = 40 N/mm^2$

$f_y = 250 N/mm^2$

So, $f_t = 0.24\sqrt{f_{cu}} = 0.24\sqrt{40} = 1.52 \ N/mm^2$

$V_{co} = 0.67\, b D \sqrt{f_t^2 + 0.8 . f_{cp} f_t}$

$= 0.67 (120)(250) \sqrt{(1.52)^2 + 0.8(5)(1.52)}$

$$\boxed{V_{co} = 58.22 \text{ KN}}$$

$\rightarrow$ $V_u > V_{co}$

So, $$\frac{A_{sv}}{S_v} = \frac{V_u - V_{co}}{0.87 f_y d_t}.$$

providing 2LVS, 6 mmϕ bars. (LVS – legged vertical stirrups)

$$A_{sv} = 2 \times \frac{\pi}{4}(6)^2 = 56.55 \text{mm}^2$$

$$\Rightarrow \frac{56.55}{S_v} = \frac{(60 - 58.22)}{0.8\,(250)(200)}.$$

$$\boxed{S_v = 1381.98 \text{ mm}}$$

Maximum:

$S_v \not> 0.75 d_t$ (or) $\not> 4b$.

(i) $S_v \not> 0.75(200) = 150$ mm

(ii) $S_v \not> 4(120) = 480$ mm

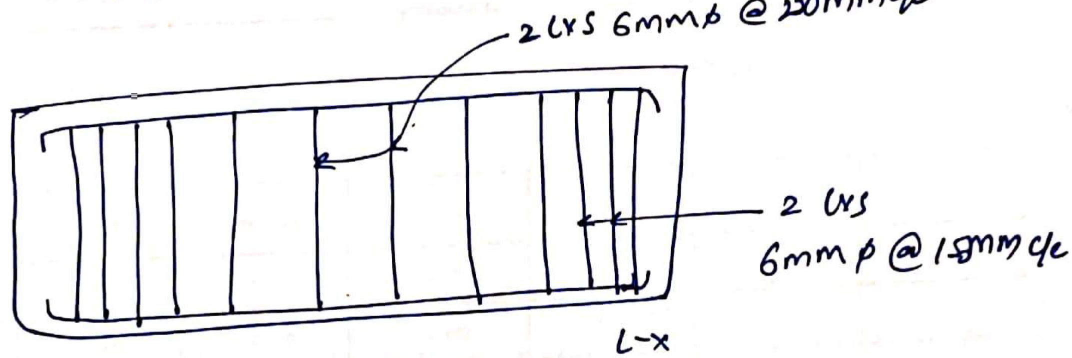
minimum

$$\frac{A_{sv}}{b S_v} = \frac{0.4}{0.87 f_y} \Rightarrow \frac{56.55}{120\,(S_v)} = \frac{0.4}{0.87\,(250)}$$

$$\boxed{S_v = 256.24 \text{ mm}}$$

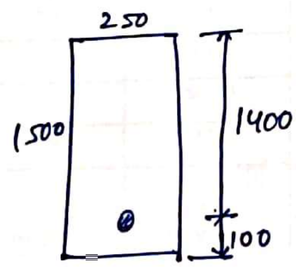
∴ provide 2-Lys 6mmφ @ 150 mm @ supports

and 2 Lys 6mmφ @ 250 mm @ center span



Q2 A psc beam 250 mm wide and 1500mm deep is subjected to a shear force of 900kN. The fibre stress under working loads is 4 N/mm². If the effective prestress is 1000 N/mm² and area of the cable is 1500mm², design shear reinforcement. The cables are inclined @ an angle of $\sin^{-1}(1/6)$ effective cover is 100mm, $f_{ck} = 40\ N/mm^2$

Sol:



$\theta = \sin^{-1}(1/6) = 9.59^\circ$

$V = 900\ kN$

$V_u = 1.5\ V = 1.5\ (900) = 1350\ kN$

$f_{cp} = 4\ N/mm^2;\ f_{ck} = 40\ N/mm^2;\ P = 1000\ N/mm^2$

$A_{st} = 1500\ mm^2$

$f_t = 0.24\sqrt{f_{ck}} = 0.24\sqrt{40} = 1.52\ N/mm^2$

NOTE! $P\sin\theta$ is considered if the cable profile is parabolic, circular (or) inclined

$\Rightarrow$ $P = A_{st}\, p = 1500 \times 1000 = 1.5 \times 10^6$ N

$$V_{co} = 0.67\, bD \sqrt{f_t^2 + 0.8 f_{cp} f_t} + P\sin\theta$$

$$= 0.67 (250)(1500) \sqrt{1.52^2 + 0.8 (4)(1.52)} + (1.5 \times 10^6) \sin(9.58)$$

$$\boxed{V_{co} = 922.97 \text{ kN}}$$

$\Rightarrow$ $V_u > V_{co}$.

So, $$\frac{A_{sv}}{S_v} = \frac{V_u - V_{co}}{0.87 f_y d_t}$$

providing 2L's, 8mm ϕ bars.

$$A_{sv} = \pi/4 \, (8)^2 \times 2 = 100.53 \text{ mm}^2$$

$$\frac{100.53}{S_v} = \frac{(1350 - 922.27)\, 10^3}{0.87 (415)(1400)}$$

$$\boxed{S_v = 118.95 \text{ mm}}$$

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Minimum

$$\frac{A_{sv}}{b S_v} = \frac{0.4}{0.87 f_y}.$$

$$\frac{100.53}{250 \times S_v} = \frac{0.4}{0.87 (415)}.$$

$$\boxed{S_v = 362.96 \text{ mm}}$$

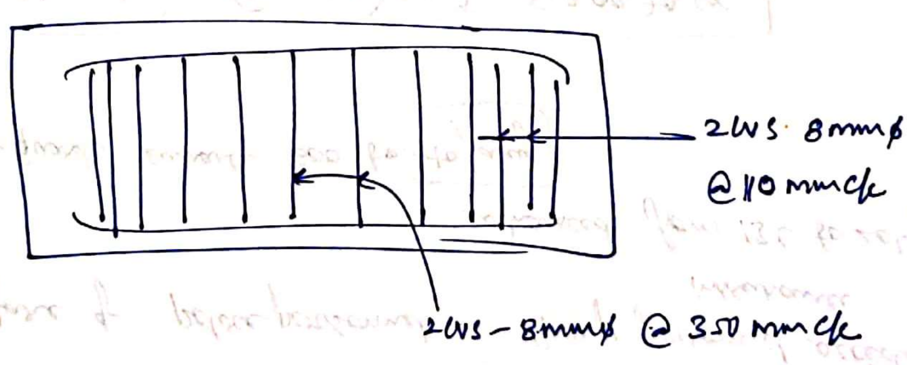
Maximum.

(1) $0.75 d_t = 0.75 (1400) = 1050 \text{mm}$

(2) $4b = 4 \times 250 = 1000 \text{mm}.$

∴ provide 2 LVS 8mm ϕ @ 110 mm c/c @ support

and 2 LVS 8mm ϕ @ 350 mm c/c @ mid span.



Q A PSC beam of Symmetrical I Sections has overall depth of 2m. Thickness of web is 200mm Effective Span is 40m. The beam is prestressed by Cables which are Concentric @ Supports and have an eccentricity of 750mm @ Center Span.

Force in Cable is 1200kN @ transfer stage, $f_{cu} = 60 N/mm^2$ estimate the ultimate Shear resistance at Support Section

If ultimate shear resistance @ support due to loads is 2834 kN and loss ratio is 0.8, design the suitable shear reinforcement using Fe 415 Steel. Area of Section is 0.88×10^6 mm^2

Sol:

$D = 2000 mm.$

$P_i = 1200 \, kN$

$f_{cu} = 60 N/mm^2$

$V_u = 2834 \, kN$

$\eta = 0.8$

$f_y = 415 \, N/mm^2$

$A = 0.88 \times 10^6 \, mm^2$

$b_w = 200 mm$

$$f_t = 0.24 \sqrt{f_{cu}} = 0.24 \sqrt{60} = 1.86 \, N/mm^2$$

$$\theta = \tan^{-1}(4e/L) = \tan^{-1}\left(\frac{4(750)}{40}\right) = 4.29^\circ.$$

$P_f = 0.8 P_i = 0.8 \times 1200$

P_i = prestress initial

P_f = prestress final

$f_{cp} = S_x = \frac{P_f \cos\theta}{A}$.

$$= \frac{0.8 (1200) \times 10^3 \times \cos 4.29}{0.88 \times 10^6} = 1.09 \text{ N/mm}^2$$

$$V_{co} = 0.67 \, bD \sqrt{f_t^2 + 0.8 \, f_{cp} \, f_t} + P_f \sin\theta$$

$$= 0.67 (200)(2000) \sqrt{1.86^2 + 0.8 (1.09)(1.86)} + [0.8 (1200) \times 10^3 \times \sin 4.29]$$

$$\boxed{V_{co} = 675.94 \text{ KN}}$$

$\Rightarrow V_u > V_{co}$.

$$\frac{A_{sv}}{S_v} = \frac{V_u - V_{co}}{0.87 \, f_y \, d_t}$$

Providing 4-LVS. of 8mm ϕ bars.

$A_{sv} = \pi/4 \, (8^2)(4) = 201.06 \text{ mm}^2$

$$\frac{201.06}{S_v} = \frac{(2834 - 675.94)}{(0.87)(415)(1900)} \Rightarrow \boxed{S_v = 63.91 \text{mm}}$$

Minimum

$$\frac{A_{sv}}{b\, S_v} = \frac{0.4}{0.87 f_y}$$

$$\frac{201.06}{200\,(S_v)} = \frac{0.4}{0.87\,(415)}$$

$$\boxed{S_v = 907.41\ mm}$$

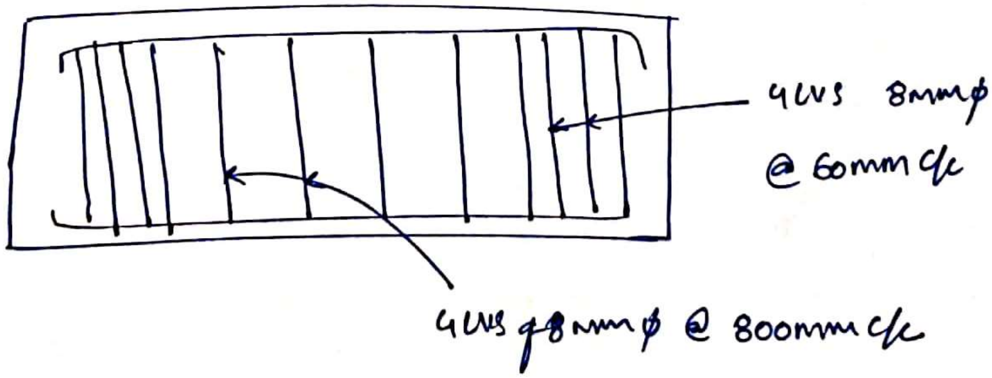
Maximum:

(i) $0.75 d_t = 0.75 \times 1900 = 1425\ mm$

(ii) $4b = 4(200) = 800\ mm$

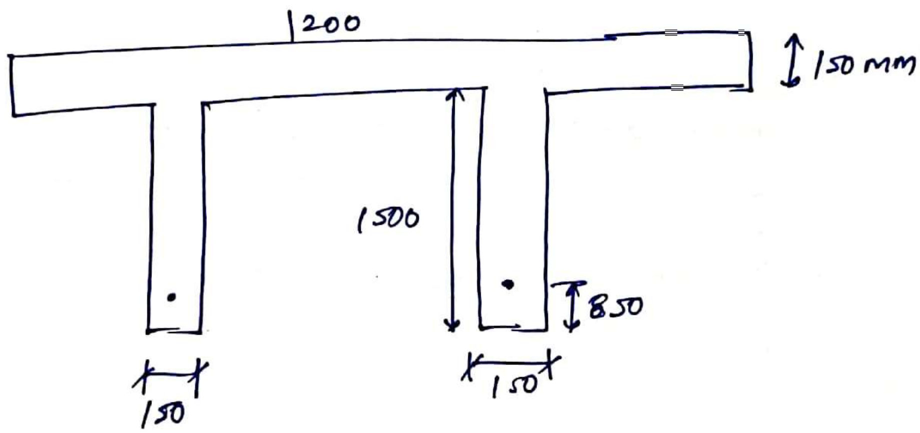
provide 4 LVS of 8mm ϕ @ 60mm c/c @ supports.

and 4 LVS of 8mm ϕ @ 800mm c/c @ mid span



Q A double T-section used as a bridge girder is made up of flange 1200mm wide and 150mm thick. The ribs are 150mm wide and 1500mm deep. The section is prestressed by high tensile steel wires of area 4700 mm² located @ 850mm from soffit of girder. Initial stress in wires is 1200 N/mm². f_{cu} = 40 N/mm². Estimate the ultimate shear strength of support section @ girder?

Soln



D = 1650 mm.

P = 1200 N/mm²

f_{cu} = 40 N/mm²

f_{p} =

A_{st} = 4700 N/mm²

d_t = 1650 − 850
= 800mm

$f_t = 0.24\sqrt{f_{cu}} = 1.52$ N/mm²

$P = P \times A_{st} = 5.64 \times 10^6 N = 5640$ kN

$$f_{cp} = \sigma_x = P/A = \frac{5.64 \times 10^6}{[(1200 \times 150) + 2(1500 \times 150)]} = 8.25 \text{ N/mm}^2$$

$$V_{co} = 0.67\ bD \sqrt{f_t^2 + 0.8 f_{cp} f_t}$$

$$V_{co} = 0.67\ (300)\ (1650) \sqrt{1.52^2 + (0.8 \times 8.95 \times 1.52)}$$

$$\boxed{V_{co} = 1204.65\ \text{kN}}$$

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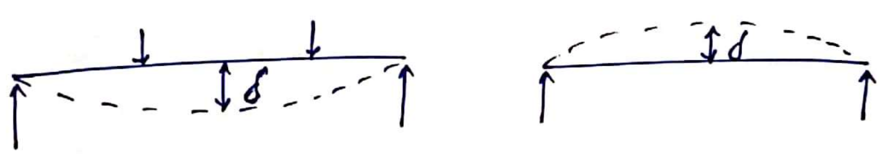
UNIT-4

Deflection of prestressed concrete beams.

- The verticle distance between straight longitudinal axis and bentup longitudinal axis is called deflection and is denoted by "δ" and measured in 'mm'

- The deflection produced due to prestress and eccentricity, dead load and liveload is called Short term (δ_i) instantaneous deflection. The deflection due to creep and shrinkage is called longterm deflection.



- upward deflection is taken as negtive and downward deflection is taken as positive.

Fasd

→ P.T.O

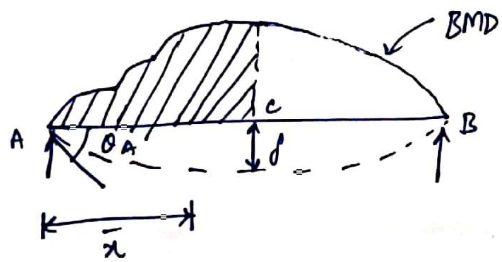
Factors affecting deflection:

1. magnitude of imposed loads and self weight
2. Magnitude of pre-stressing force
3. span of the member
4. c/s area of the member
5. modulus of elasticity of the material
6. moment of inertia of c/s.
7. Eccentricity of the cable
8. Shape of the cable
9. Finity conditions.
10. Creep, shrinkage of concrete
11. Relaxation of steel stress
12. Grade of concrete and grade of steel.
13. Humidity, temperature, w/c ratio etc...

Instantaneous deflection:

It can be calculated by moment area method. Short term deflection of a prestress member are governed by moment and flexural rigidity of the member.

- Mohr's moment area table is readily available for the estimation of deflections due to prestressing force (or) self weight (or) imposed loads.



Slope at A = area of BMD between A and C divided by EI

$$\theta_A = \frac{\text{BMD Area between A and C}}{EI}$$

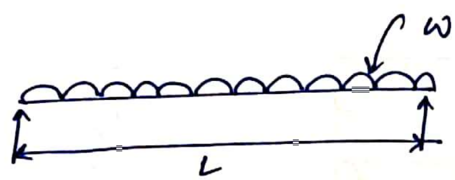
Deflection at C = $\dfrac{\text{moment of area of BMD between A \& C and moment taken about A}}{EI.}$

i.e., $\theta_A = \dfrac{A}{EI}$; $\delta = \dfrac{A\bar{x}}{EI}$

Short term deflections:

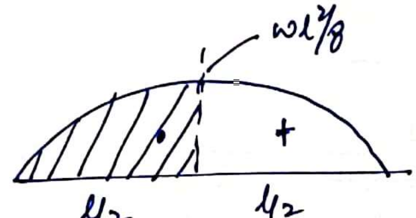
(a) Deflection due to L.L:

(i) Simply supported beam loaded with an UDL



From mohr's theorem.

$$\Delta = \left[\frac{A\bar{x}}{EI}\right] = \left[\frac{\frac{2}{3}\left(\overset{b}{\tfrac{l}{2}}\right)\left(\overset{h}{\tfrac{wl^2}{8}}\right)\left(\tfrac{5}{8}\times\tfrac{l}{2}\right)}{EI}\right]$$

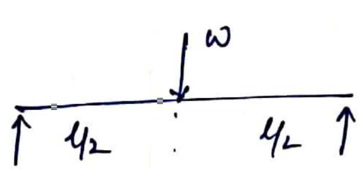


$$\boxed{\Delta = \frac{5}{384}\,\frac{wl^4}{EI}}\ (\downarrow)\ (+ve)$$

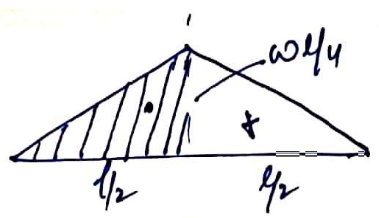
Sagging BM

$\bar{x}$ = centroidal distance of the shaded area from the support

(ii) Simply supported beam loaded with a point load:

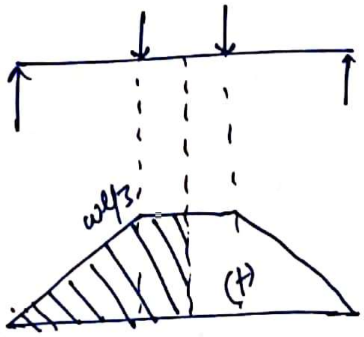


$$\Delta = \frac{\left[\frac{1}{2}\times\frac{l}{2}\times\frac{wl}{4}\right]\left[\frac{2}{3}\times\frac{l}{2}\right]}{EI}$$



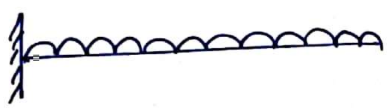
$$\boxed{\Delta = \frac{1}{48}\,\frac{wl^3}{EI}}\ (\downarrow)$$

(iii) Simply supported beam loaded with a (2) point loads!



$$\boxed{\Delta = \frac{23}{648}\frac{wl^3}{EI}}\ (\downarrow).$$

(iv) Cantilever beam loaded with UDL!



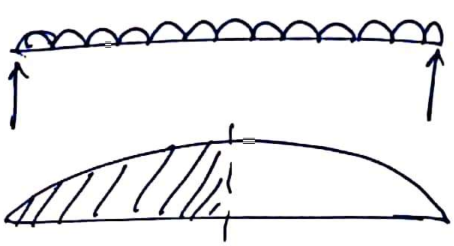
$$\boxed{\Delta = wl^4/8EI}\ (\downarrow).$$

(v) Cantilever beam loaded with point load!

$$\boxed{\Delta = wl^3/3EI}\ (\downarrow).$$

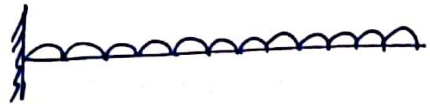
(b) Deflection due to DLs!

(i) Simply supported beam loaded with UDL



$$\boxed{\Delta = \frac{5}{384}\frac{wl^4}{EI}}\ (\downarrow)$$

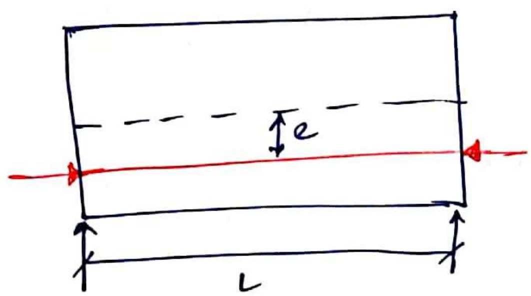
(iii) Cantilever beam loaded with UDL



$$\boxed{\Delta = \frac{wL^4}{8EI}} \quad (\downarrow)$$

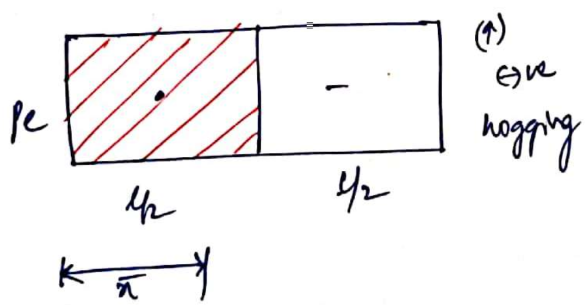
→ Deflection due to tendon profile [prestressing force].

(i) Simply supported beam carrying a cable which has constant eccentricity throughout the span.



$$\Delta = \frac{A\bar{x}}{EI}$$

$$= \frac{(Pe\,L/2)\,L/4}{EI}$$

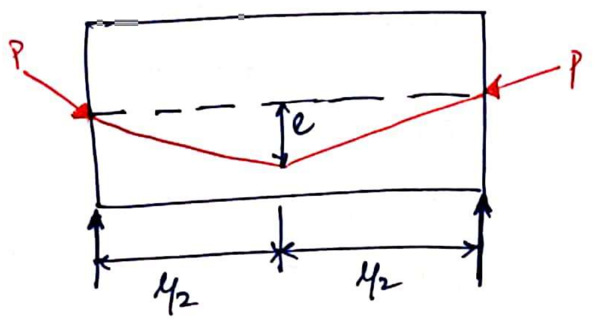


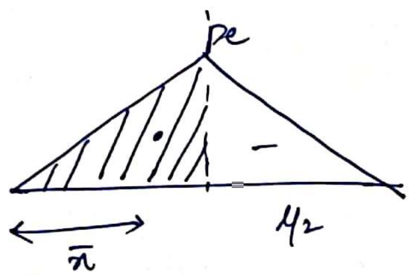
$$\boxed{\Delta = \frac{Pe\,L^2}{8EI}} \quad (\uparrow)\ (-ve)$$

hogging

→ P.T.O

② Simply supported beam carrying a cable which is linearly bent, having maximum eccentricity 'e' at mid-span and zero eccentricity at supports.





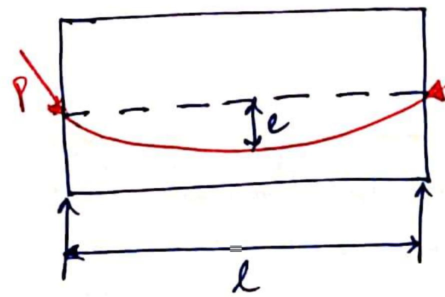
$$\Delta = \frac{A\bar{x}}{EI}.$$

$$= \frac{\left[\left(l/2 \times 1/2 \times Pe\right)\left(2/3 \times l/2\right)\right]}{EI}$$

$$\boxed{\Delta = 1/12 \left(\frac{Pel^2}{EI}\right)} \ (\uparrow)$$

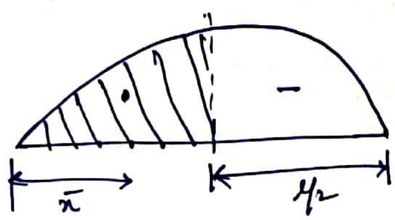
③ Simply supported beam having parabolic cable profile which has eccentricity 'e' at mid span and zero at supports. [concentric at supports].

⟶ P.T.O



$$\Delta = \frac{A\bar{x}}{EI}.$$

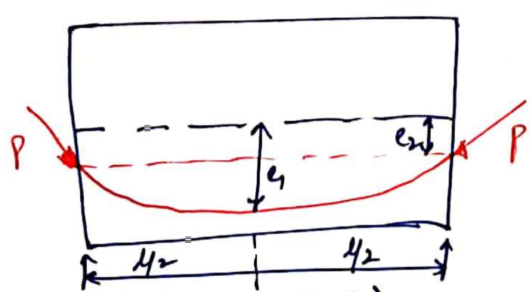
$$= \frac{\left[\left(\frac{2}{3}\,\frac{l}{2}\,Pe\right)\left(\frac{5}{8}\times\frac{l}{2}\right)\right]}{EI}$$



$$\boxed{\Delta = \frac{5}{48}\,\frac{Pel^2}{EI}}\ (\uparrow)$$

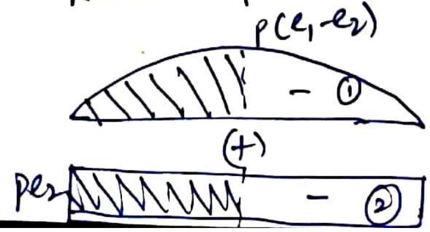
$\bar{x} = \frac{5}{8}\,\frac{l}{2}$

(4) Simply supported beam having parabolic cable, having maximum eccentricity 'e_1' at mid span and minimum eccentricity 'e_2' at supports below the centroidal axis / towards the soffit of the beam.



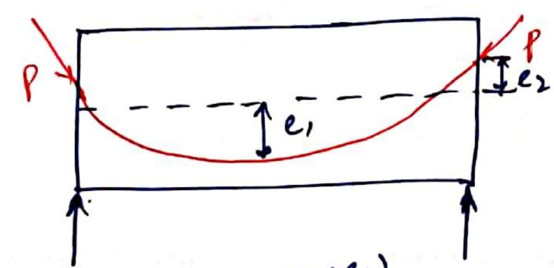
$$\Delta = \frac{A\bar{x}}{EI}.$$

$$= \frac{\left[\left(\frac{2}{3}\,\frac{l}{2}\,P(e_1-e_2)\right)\left(\frac{5}{8}\times\frac{l}{2}\right) + \left(Pe_2\,\frac{l}{2}\,\frac{l}{4}\right)\right]}{EI}$$

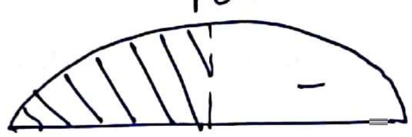


$$\boxed{\Delta = \frac{Pl^2}{48EI}\,(e_2+5e_1)}\ (\uparrow)$$

(5) Simply supported beam having parabolic cable, having eccentricity 'e_1' towards soffit at mid span and eccentricity 'e_2' at support above the centroidal axis.



$$\Delta = \frac{A\bar{x}}{EI}.$$



$$= \frac{(-)\left\{ \frac{2}{3} \times \frac{l}{2} \times P(e_1+e_2)\left(\frac{5}{8} \cdot \frac{l}{2}\right) \right\} \quad (+)\left\{ Pe_2 \times \frac{l}{2} \times \frac{l}{4} \right\}.}{EI}$$

$$= \frac{-\left\{ \frac{1}{3} P(e_1+e_2)\left(\frac{5l}{16}\right) \right\} + \frac{Pe_2 l^2}{8}}{EI.}$$

$$= (-)\frac{5Pl^2e_1}{48EI} - \frac{5Pl^2e_2}{48EI} + \frac{Pl^2e_2}{8EI}.$$

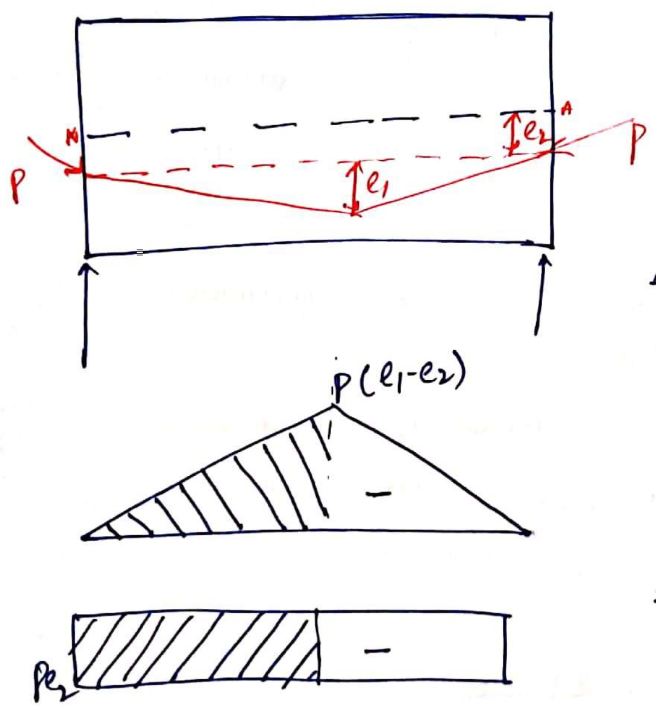
$$= \frac{(-)5Pl^2e_1}{48EI} - \frac{5Pl^2e_2 + 6Pl^2e_2}{48EI.}$$

$$= (-)\frac{5l^2Pe_1}{48EI} + \frac{Pl^2e_2}{48EI}$$

$$\Delta = \frac{Pl^2}{48EI}\left(e_2 - 5e_1\right)$$

$$\therefore \boxed{\Delta = \frac{Pl^2}{48EI}\left(e_2 - 5e_1\right)} \quad (7).$$

(6) Simply supported beam having linearly bent cable having eccentricity of e_1 (maximum) at mid span and eccentricity 'e_2' at supports below the centroidal axis (8) towards the soffit of beam.



$$\Delta = \frac{A\bar{x}}{EI}.$$

$$\Delta = \frac{\left[\left(\frac{1}{2} \times \frac{l}{2} \times P(e_1-e_2)\left(\frac{2}{3} \times \frac{l}{2}\right)\right) + \left[Pe_2 \frac{l}{2}\left(\frac{l}{4}\right)\right]\right]}{EI}$$

$$= \frac{Pl^2 e_1}{12} - \frac{Pl^2 e_2}{12} + \frac{Pe_2 l^2}{8}$$

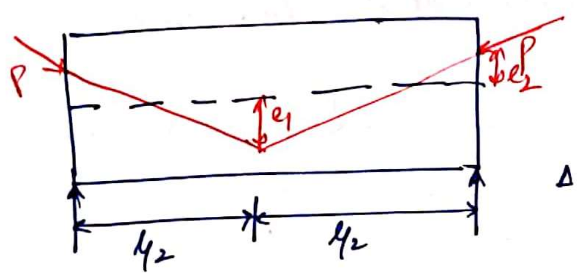
$$= \frac{Pl^2 e_1}{12} - \frac{2Pl^2 e_2 + 3Pe_2 l^2}{24}$$

Scanned with CamScanner

$$\Delta = \frac{Pl^2 e_1}{12} + \frac{Pe_2 l^2}{24}.$$

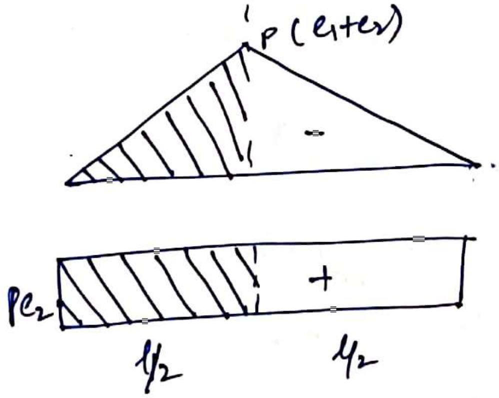
$$\boxed{\Delta = \frac{Pl^2}{24EI}\left(e_2 + 2e_1\right)} \quad (\uparrow).$$

(7) Simply supported beam having linearly bent cable having eccentricity (e_1) below NA at mid span and eccentricity (e_2) at supports above the centroidal axis.



$$\Delta = \frac{A\bar{x}}{EI}.$$

$$\Delta = \frac{\left[(-)\left(\frac{1}{2}\times\frac{l}{2}\times P(e_1+e_2)\left(\frac{2}{3}\frac{l}{2}\right)\right) + \left[Pe_2 \frac{l}{2}\frac{l}{4}\right)\right]}{EI}.$$



$$= (-)\frac{Pe_1 l^2}{12EI} - \frac{Pe_2 l^2}{12EI} + \frac{Pe_2 l}{8EI}.$$

$$= (-)\frac{Pe_1 l^2}{12EI} - \frac{2Pe_2 l^2 + 3Pe_2 l^2}{24EI}$$

$$= (-)\frac{Pe_1 l^2}{12EI} + \frac{Pel^2}{24EI}.$$

$$\boxed{\Delta = \frac{Pl^2}{24EI}\left[-2e_1 + e_2\right]} \quad (\uparrow).$$

→ Long term deflections:

Deflection of PSC member may vary due to time dependent such as creep and shrinkage. Deflection produced due to L.L and D.L and prestress including all time dependent loss there will be decrease in prestress and corresponding decrease in deflection

Lin has suggested a simplified approximate method for determining the long term deflection:

$$\boxed{\Delta = \left[\Delta_{DL} + \Delta_{LL} + \Delta_P\left(\frac{P_{eff}}{P_i}\right)\right]\left[1+\phi\right]}$$

where, Δ_{LL} = deflection due to live load

Δ_{DL} = deflection due to dead load

Δ_P = deflection due to prestress and eccentricity.

P_{eff} = effective prestressing force | $\frac{P_{eff}}{P_i} = \eta$ = loss ratio

P_i = initial prestressing force

ϕ = creep co-efficient

Final downward deflection $\ngtr$ span/250

Final upward deflection $\ngtr$ span/300

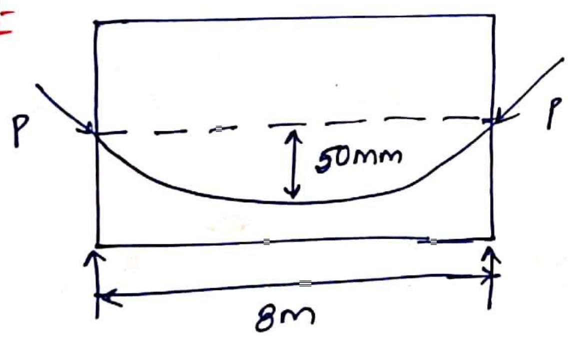
Q1 A Concrete beam with a c/s area of 32×10^3 mm^2 and a radius of gyration 72 mm is prestressed by a parabolic cable carrying an effective stress of 1000 N/mm². Span of the beam is 8m. The Cable is composed of 6 - 7mm φ wires and has an eccentricity of 50mm at the center and zero at the supports. Neglecting all losses, find the central deflection at the beam as follows.

(a) Pre stress (+) self weight

(b) Prestress (+) self weight (+) L.L of 2kN/m

Take M40 and $\gamma_c = 24$ kN/m³

Sol:-



$$r = \sqrt{\frac{I}{A}}.$$

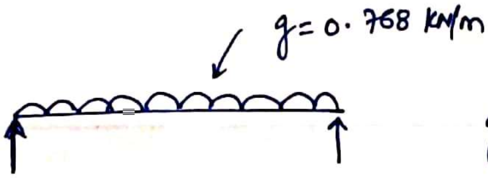
$$72 = \sqrt{\frac{I}{32 \times 10^3}}.$$

$$\boxed{I = 165.88 \times 10^6 \text{ mm}^4}$$

Prestressing force $= 1000 \times \pi/4 \times 7^2 \times 6 = 230.9$ kN

→ P.T.O

(a)



intensity of DL.

$$q = \gamma_c \times A_c$$

$$= 24 \times 32 \times 10^3 \times 10^{-6}$$

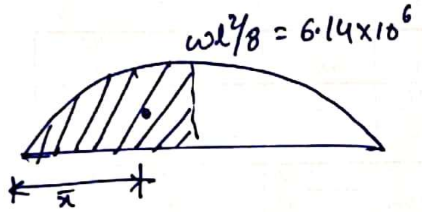
$$q = 0.768\ \text{KN/m} = 0.768\ \text{N/mm}$$

$$\Delta = \frac{5}{384} \frac{wl^4}{EI}.$$

$$E_c = 5700\sqrt{f_{ck}} = 36.05 \times 10^3\ \text{N/mm}^2$$

$$\Delta x = \frac{5}{384} \times \frac{0.768 \times (8 \times 10^3)^4}{36.05 \times 10^3 \times 165.88 \times 10^6} = 6.85\ \text{mm} (\downarrow).$$

(b) moment area method



$$BM = \frac{0.768 \times 8^2}{8} = 6.14\ \text{KN-m}$$

$$BM = 6.14 \times 10^6\ \text{N-mm}$$

$$\Delta = \frac{\left[-\frac{2}{3} \times 4000 \times 6.14 \times 10^6\right]\left[\frac{5}{8} \times 4000\right]}{36.05 \times 10^3 \times 165.88 \times 10^6}$$

$$\Delta = 6.85\ \text{mm}.$$

$\longrightarrow$ P.T.O

(b) $u = 2 kN/m = 2 N/mm$ $\frac{10^3}{10^3}$

$$\Delta u = \frac{5}{384} \times \frac{2 \times (8000)^4}{36.05 \times 10^3 \times 165.88 \times 10^6} = 17.84 \text{ mm} (\downarrow).$$

(c) P and e

$$\Delta = \frac{5 Pe L^2}{48 EI}$$

$$= \frac{5 \times 230.9 \times 10^3 \times 50 \times (8000)^2}{48 \times 36.05 \times 10^3 \times 165.88 \times 10^6}$$

$\Delta = (-) 12.87 mm (\uparrow).$

(8) moment area method

$Pe = 230.9 \times 10^3 \times 50 = 11.69 \times 10^6$ N-mm

$$\Delta = \frac{\left[\frac{2}{3} (4000)(11.54) \times 10^6 \right] \left[\frac{5}{8} \times 4000 \right]}{(36.05 \times 10^3)(165.88 \times 10^6)}.$$

$\Delta = (-) 12.87 (mm) (\uparrow).$

$\longrightarrow$ P.T.O

(A) Prestress and self weight.

$PS + SW = (-)12.87 + 6.85 = (-)6.02\,mm \not> \frac{L}{300}$.

$L/300 = \frac{8000}{300} = 26.67\,mm$

So, $(-)6.02 \not> 26.67\,mm$ Hence Safe

(B) PS + SW + LL $= (-)12.87 + 6.85 + 17.84 = 11.22\,mm$

$\Rightarrow L/250 = \frac{8000}{250} = 30\,mm$

So, $11.22 \not> 30\,mm$, Hence Safe

Q2 A concrete beam having a rectangular section (150 x 300) mm is prestressed by a parabolic cable at an eccentricity of 75mm at center and towards bottom soffit and eccentricity of 25mm towards top. Effective prestressing force is 350 KN. The beam supports a concentrated load of 20KN at center of span in addition to ~~its~~ self wt. Span is 8m

→ PTO

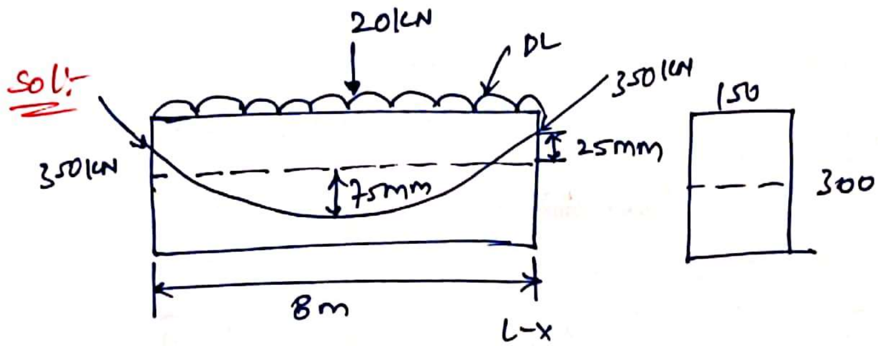
Find.

(i) short term deflection at center of span at PS+SW+LL

(ii) Long term deflection, if the loss ratio is 0.8.

Take Ec as 38 N/mm^2 and γ_{conc} as 24 kN/m^3, Creep co-efficient = 1.6

Sol:-



(i) DL

$$\Delta = \frac{5}{384} \frac{wl^4}{EI}$$

$$I = \frac{150 \times 300^3}{12} = 337.5 \times 10^6 \ mm^4$$

$$g = \gamma_c \times A_c = 24 \times 150 \times 300 = 1.08 \ kN/m = 1.08 \ N/mm$$

$$\Delta = \frac{5}{384} \times \frac{1.08 \times 8000^4}{38 \times 10^3 \times 337.5 \times 10^6} = 4.49 \ mm \ (\downarrow)$$

→ P.T.O

② LL

$$\Delta = \frac{wl^3}{48EI} = \frac{20\times10^3 \times 8000^3}{48\times 38\times10^3 \times 337.5\times10^6} = 16.63\text{mm} (\downarrow)$$

③ P and e

$$\Delta = \frac{Pl^2}{48EI}(e_2 - 5e_1)$$

$$\Delta = \frac{350\times10^3\times8000^2}{48\times337.5\times10^6 \times 38\times10^3}(25 - 5(75))$$

$$\boxed{\Delta = (-)\,12.74\text{ mm}}\ (\uparrow).$$

ⓐ Short term deflection

$$\Delta = \Delta_{DL} + \Delta_{LL} + \Delta_{ps} = 4.49 + 16.68 - 12.74$$

$$\boxed{\Delta = 8.38\text{ mm } (\downarrow)}$$

ⓑ Long term deflection

$$\Delta = \left[\Delta_{LL} + \Delta_{DL} + \Delta_{ps}\left(\frac{P_{eff}}{P_i}\right)\right][1+\phi]$$

$$\Delta = \left[4.49 + 16.63 - 12.74\,(0.8)\right](1+1.6)$$

$$\boxed{\Delta = 28.4\text{ mm}}\ (\downarrow)$$

$$l/250 = \frac{8000}{250} = 30mm$$

So, $28.49 \not> 30mm$

Hence Safe

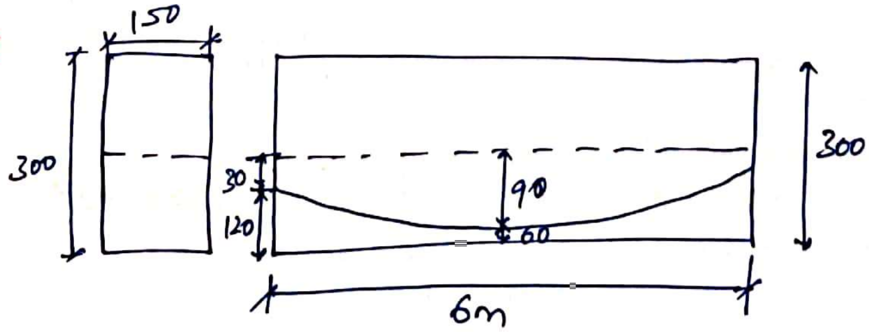
Q3 A rectangular concrete beam of c/s 150mm × 300mm is simply supported over a span of 6m. It is prestressed by a symmetric parabolic cable at a distance of 120mm from bottom fibre at support at a distance of 60mm from bottom fibre at mid span. Initial prestress in cable is 300KN determine.

(a) Max deflection of beam at center

(b) Central concentrated force to be applied to nullify the above deflection

Take unit weight of concrete $= 25 kN/m^3$, $E_c = 3.84 \times 10^4 N/mm^2$

Sol:



$e_1 = 90mm$

$e_2 = 30mm$

(a) (1) DL

$$\Delta = \frac{5}{384} \frac{wl^4}{EI}.$$

$$I = \frac{150 \times 300^3}{12} = 337.5 \times 10^6 \text{ mm}^4$$

$W = \gamma_c \cdot A_c = 25 \times 0.15 \times 0.8 = 1.125$ kN/m $= 1.125$ N/mm

(1) Rawdx

$$\Delta = \frac{5}{384} \times \frac{1.125 \times (6000)^4}{3.8 \times 10^4 \times 3375 \times 10^6} = 1.48 \text{mm} (\downarrow).$$

(2) Pande

$$\Delta = \frac{Pl^2}{48EI} \left[e_2 + 5e_1\right]$$

$$= \frac{300 \times 10^3 \times 6000^2}{48 \times (337.5) \times 10^6 \times 9.8 \times 10^4} \left[30 + 5(90)\right]$$

$\boxed{\Delta = (-) 8.42 \text{mm}}$ $(\uparrow)$.

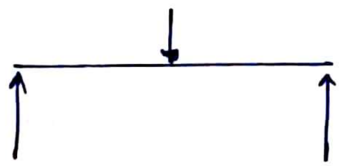
Net $\Delta = (-)8.42 + 1.48$

$= 6.94$ mm.

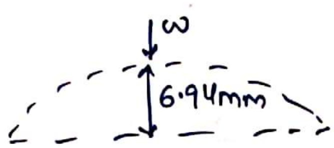
(3) Live load

$\longrightarrow$ P.T.O

(b) LL



central concentrated load required to cause 6.94 mm deflection.



$$\Delta = \frac{1}{48} \frac{wl^3}{EI}.$$

$$\Rightarrow \quad 6.94 = \frac{1}{48} \times \frac{w\,(6000)^3}{3.8\times10^4 \times 337.5\times10^6}$$

$$\Rightarrow \quad \boxed{w = 19.8\ \text{KN}}$$

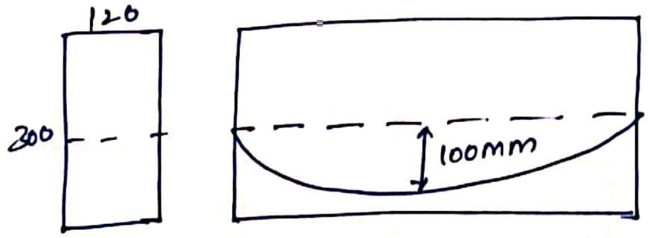
Q4 A rectangular beam (120x 300)mm is simply supported over a span of 6m. It is prestressed with a parabolic cable having a max eccentricity of 100mm at mid-span and zero at supports. Effective prestress in cable is 300 KN. Determine

(a) max. deflection due to Prestress and self weight

(b) central concentrated load required to cause max deflection $\frac{L}{300}$ in downward direction.

Take $E_c = 38.2$ GPa, $\gamma_c = 24\ \text{KN/m}^3$.

Sol:-



(a) ① DL

$E_c = 38.2 \times 10^9\ N/m^2 = 38.2 \times 10^3\ N/mm^2$

$$I = \frac{120 \times 300^3}{12} = 270 \times 10^6\ mm^4$$

$\omega = A_c \times \gamma_c = 24 \times 0.12 \times 0.3 = 0.864\ kN/m = 0.864\ N/mm$

$$\Delta = \frac{5}{384}\ \frac{\omega l^4}{EI}.$$

$$= \frac{5}{384} \times \frac{0.864 \times 6000^4}{38.2 \times 10^3 \times 270 \times 10^6}$$

$\boxed{\Delta = 1.41\ mm}$ (↓)

② P and e

$$\Delta = \frac{5}{384}\ \frac{Pe\,L^2}{EI}$$

$$= \frac{5}{384} \times \frac{300 \times 10^3 \times 100 \times 6000^2}{38.2 \times 10^3 \times 270 \times 10^6}$$

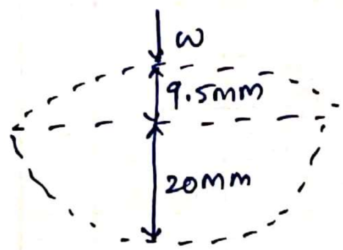
$\boxed{\Delta = (-)\,10.91\,mm}$ (↑)

(b) LL

$\rightarrow \frac{L}{300} = \frac{6000}{300} = 20MM.$

After prestress the beam will hog. So, deflection is in upward direction

net deflection, $\Delta = (+)1.41 (-) 10.91 = (-) 9.5mm (\uparrow)$.



Here, we require a deflection of $\frac{L}{300}$ i.e., 20mm in downward direction. First the load need to Straighten the beam. Then Cause a deflection of 20mm downwards. So the deflection is additive

$\Delta = 9.5 + 20 = 29.5 mm.$

$$\Delta = \frac{wl^3}{EI} \times \frac{1}{48}$$

$$29.5 = \frac{1}{48} \times \frac{w \times 6000^3}{38.2 \times 10^3 \times 270 \times 10^6}.$$

$$\boxed{w = 67.614 \ KN}$$

$\longrightarrow$ P.T.O

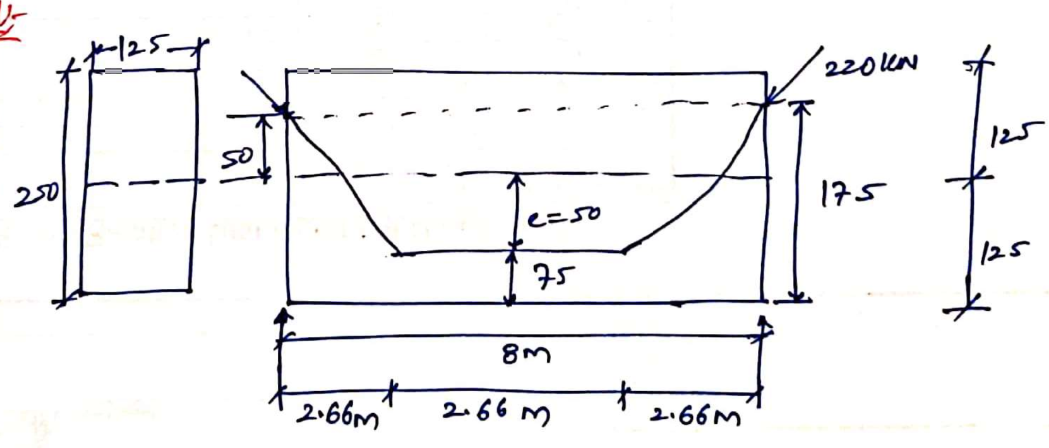
Q5 A simply supported concrete beam of span 8m is of rectangular c/s 125 mm wide and 250 mm deep is prestressed by a single cable in which total tensile force is 220kN. The central line of cable is parallel to the axis of the beam and 75 mm above the soffit over the middle third of the span and curved upwards in a parabolic manner over the outer third of the span to a distance of 17.5mm above the soffit at the support. If the E_c is 35 kN/m^2 and γ_c = 24 kN/m^3 Calculate.

(a) upward deflection due to only prestress.

(b) deflection due to prestress and self weight

(c) magnitude of concentrated load placed at 1/3rd points of the span which would result in short term deflection of $\frac{\text{span}}{500}$?

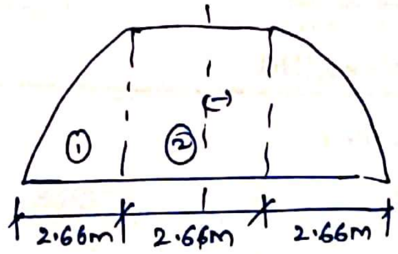
Sol:-

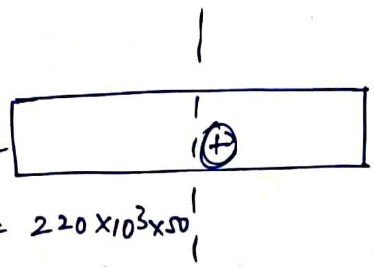


→ P.T.O

(a) ① <u>Deflection due to prestress and eccentricity (Pande)</u>

$$P(e_1+e_2) = 220\times10^3\,(50+50) = 22\times10^6 \text{ N-mm}$$





Pe_2

$= 220\times10^3\times50$

$= 11\times10^6$ N-mm

$$\Delta = \frac{(-)\left[\left(\tfrac{2}{3}\times 22\times10^6\times 2.66\times10^3\right)\left(\tfrac{5}{2}\times 2666\right)\right] (-)\left[\left(2.22\times10^6\times \tfrac{2666}{2}\right)\left(2666+\tfrac{1333}{2}\right)\right] + \left[\left(11\times10^6\times\tfrac{8000}{2}\right)\left(\tfrac{4000}{2}\right)\right]}{35\times10^3\times162.76\times10^6}$$

$$I = \frac{125\times250^3}{12}$$

$$= 162.76\times10^6 \text{ mm}^4$$

$$\boxed{\Delta = (-)\,13.15\text{ mm}}\ (\uparrow)$$

② <u>DL</u>

$$\Delta = \frac{5}{384}\,\frac{wl^4}{EI}.$$

$$w = 24\times0.125\times0.25 = 0.75 \text{ kN/m} = 0.75 \text{ N/mm}$$

$$\Delta = \frac{5}{384}\times\frac{0.75\times8000^4}{35\times10^3\times162.75\times10^6} = 7.02\text{ mm } (\downarrow).$$

(b) $(-)\,13.15 + 7.02 = (-)6.13$ mm.

(P.S + SW)

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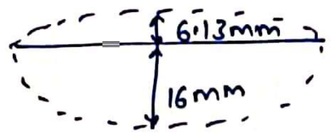
(c) $\frac{span}{500} = \frac{8000}{500} = 16mm$

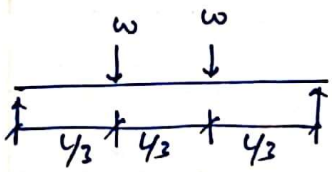


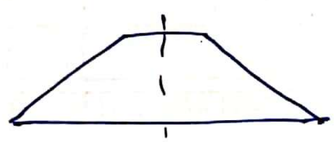
$\Delta = 6.13 + 16 = 22.13$ mm

$\Delta = \frac{23}{648} \cdot \frac{wl^3}{EI}$.

$22.13 = \frac{23}{648} \times \frac{w(8000)^3}{35 \times 10^3 \times 162.76 \times 10^6}$

$\boxed{w = 6.937 kN}$





Q6 A prestressed concrete beam of rectangular section (150×400)mm is stressed by (4) cables each carry an effective force of 250 KN, span of beam is 12m. The 1st cable is parabolic with an eccentricity of 50mm below the centroidal axis at mid span, 60mm above centroidal axis at support.

2nd cable is also parabolic with zero eccentricity at support and with an eccentricity of 50 mm at midspan

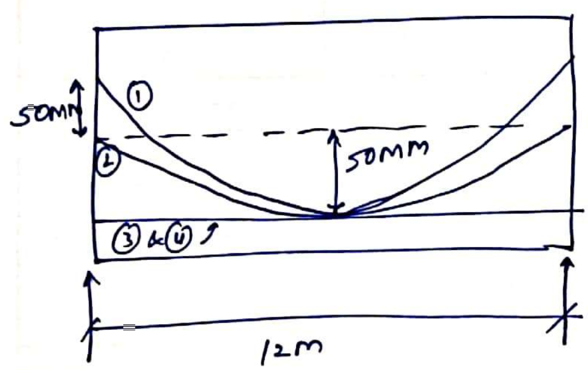
3rd and 4th cable are straight with constant eccentricity of 50 mm below centriodal axis. If the beam supports an UDL of 10kN/m, Ec = 40 kN/m³ and γ_c = 25 kN/m³. Estimate the instantaneous deflection at following stages?

(a) Ps + Sw

(b) Ps + Sw + LL

(c) Deflection taking into account the effect of creep. Take ϕ as 1.6 and loss ratio as 0.8?

Sol:-



$$\Delta_1 = \frac{Pl^2}{48EI}(e_2 - 5e_1)$$

$$I = \frac{150 \times 400^3}{12}$$

$$I = 800 \times 10^6 \text{ mm}^4$$

$$= \frac{250 \times 10^3 \times (12000)^2}{48 \times 40 \times 10^3 \times 800 \times 10^6} (50 - 5(50))$$

$$\boxed{\Delta_1 = (-)\, 4.694 \text{ mm}} \quad (\uparrow).$$

→ P.T.O

$$\Delta_2 = \frac{5}{48} \frac{Pe l^2}{EI}.$$

$$= \frac{5}{48} \times \frac{250 \times 10^3 \times 50 \times 12000^2}{40 \times 10^3 \times 800 \times 10^6}$$

$$\boxed{\Delta_2 = (-) 5.86 \text{ mm}} \ (\uparrow)$$

$$\Delta_3 = \Delta_4 = \frac{Pe l^2}{8EI}$$

$$= 1/8 \times \frac{250 \times 10^3 \times 50 \times 12000^2}{40 \times 10^3 \times 800 \times 10^6}.$$

$$\boxed{\Delta_3 = \Delta_4 = (-) 7.03 \text{ mm}} \uparrow$$

<u>DL</u>

$$\Delta = \frac{5}{384} \frac{wl^4}{EI}.$$

$$= \frac{5}{384} \frac{1.5 \times 12000^4}{40 \times 10^3 \times 800 \times 10^6}$$

$w = A_c \times \gamma_c$

$= 24 \times 0.15 \times 0.4$

$= 1.5$ kN/m $= 1.5$ N/mm

$$\boxed{\Delta = 12.65 \text{ mm}} \ (\downarrow)$$

<u>LL</u> $\Delta = \frac{5}{384} \frac{wl^4}{EI}. = \frac{5}{384} \times \frac{10 \times 12000^4}{40 \times 10^3 \times 800 \times 10^6}.$

$$\boxed{\Delta = 84.88 \text{ mm}} \ (\downarrow)$$

$\rightarrow$ P.T.O

ⓐ Short term deflection

~~$\Delta = (-)6.$~~

$$\Delta = (-)4.69 (-)7.03 (-)7.03 (-)5.86 + 12.65 + 84.38$$

$$\boxed{\Delta = 72.42 \text{ mm}} \ (\downarrow)$$

ⓑ Long term deflection

$$\Delta = \left[\Delta_{DL} + \Delta_{LL} + \Delta_{Pande} \left(\cancel{\frac{P_{eff}}{P_i}} \right)^{\eta} \right] (1+\phi)$$

$$\Delta = \left[12.65 + 84.88 - (4.69 + 5.86 + 7.03 + 7.03)0.8\right]\left[1+1.6\right]$$

$$\boxed{\Delta = 201.09 \text{ mm}} \ (\downarrow).$$

Q7 A rectangular beam of span 8m, 150mm wide and 350mm deep. The beam is prestressed by a parabolic cable having an eccentricity of 75mm below centroidal axis at midspan and 25mm above the centroidal axis at support. The initial force in cable is 350kN. The beam supports 3 concentrated loads of 10kN each at an interval of 2m. $E_c = 38 \text{ N/mm}^2$

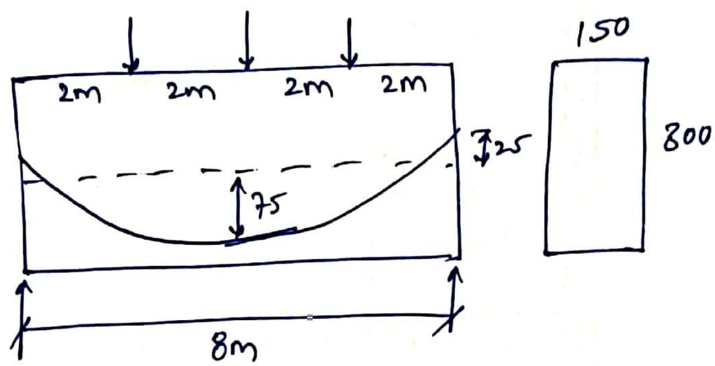
→ P.T.O

$\gamma_c = 24\,kN/m^3$, neglecting losses of pre stress, estimate

(a) The short term deflection due to SW + PS + LL

(b) allowing for 20% loss of PS, estimate the long term deflection assume creep co-efficient as 1.8?

Sol:-



$e_1 = 75mm$

$e_2 = 25mm$

$$I = \frac{150 \times 300^3}{12} = 337.5 \times 10^6\, mm^4$$

(1) DL

$$\Delta = \frac{5}{384}\,\frac{wl^4}{EI}.$$

$W = A_c \times \gamma_c = 24 \times 0.15 \times 0.3$

$w = 1.08\, kN/m = 1.08\, N/mm$

$$= \frac{5}{384} \times \frac{1.08 \times 8000^4}{337.5 \times 10^6 \times 38 \times 10^3}$$

$\boxed{\Delta = 4.49\,m}$ ($\downarrow$).

(2) Parde

$$\Delta = \frac{Pl^2}{48EI}\,(e_2 - 5e_1)$$

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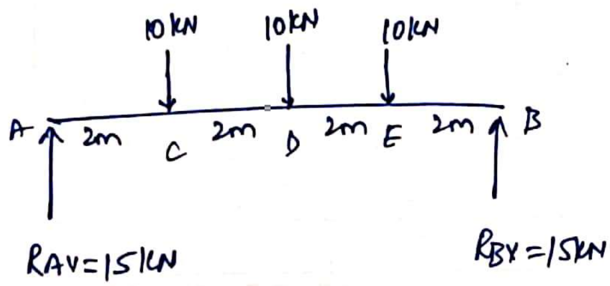
$$\Delta = \frac{350\times10^3 \times 8000^2}{48\,(337.5\times10^6)\,(38\times10^3)} \cdot \left(25 - 5(75)\right)$$

$$\boxed{\Delta = (-)\,12.74\ \text{mm}}\ (\uparrow)$$

(3) a



$\Sigma V = 0$

$-10 - 10 - 10 + R_{AV} + R_{BY} = 0$

$R_{AV} + R_{BV} = 30$

$\Sigma M_A = 0$

$10(2) + 10(4) + 10(6)$

$- R_B(8) = 0$

$R_B = 15\ \text{kN}$

$R_A = 15\ \text{kN}$

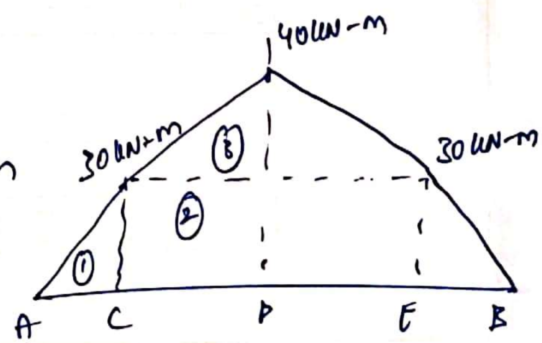
$BM_{@A} = 0$

$BM@C = 15(2) = 30\ \text{kN-m}$

$BM@D = 15(4) - 10(2) = 40\ \text{kN-m}$

$BM@E = 15(2) = 30\ \text{kN-m}$

$BM@B = 0$



$\longrightarrow$ P.T.O

$$\Delta = \frac{\left[\left(\frac{1}{2} \times 2\times10^3 \times 30\times10^6\right)\left(\frac{2}{3} \times 2\times10^3\right)\right] + \left[\left(2\times10^3 \times 30\times10^6\right)\left(2 + \frac{1}{2}\times10^3\right)\right] + \left[\left(\frac{1}{2}\times10^3 \times 60\times10^6\right)\left(2 + \frac{2}{3}(2)\times10^3\right)\right]}{38\times10^3 \times 337.5\times10^6}$$

$$\Delta = \frac{253.33\times10^{12}}{38\times10^3 \times 337.5\times10^6} = 19.75 \text{ mm} \downarrow$$

(a) Short term deflection:

$$\Delta = 4.49 - 12.74 + 19.75$$

$$\Delta = 11.50 \text{ mm}$$

$$\eta = \frac{P_{eff}}{P_i} = 100 - 20 = 80\%$$
$$= 0.8$$

(b) Long term deflection:

$$\Delta = \left[\Delta_{DL} + \Delta_{LL} + \Delta_{pe}\left(\frac{P_{eff}}{P_i}\right)\right](1+\phi)$$

$$\Delta = \left[4.49 + 19.75 - 12.74(0.8)\right]\left[1+1.8\right].$$

$$\boxed{\Delta = 39.33 \text{ mm}} (\downarrow).$$

UNIT-5

Composite Beams

- A composite beam is a structural element made up of two structural elements of different materials having different moduli which resists the applied as a single homogenous unit.

- The composite construction is mainly used in highway bridges. A prestressed precast concrete unit with cast insitu slab from the composite beam. The composite sections are formed by using standard I-sections, double T-sections and trough shaped sections. The precast griders are spanning between the piers. The cast insitu slab mechanically bonded to the precast griders by shear connectors. The composite construction is used in buildings too. Also, in case of multi beam bridge construction, the standard box sections

→ P.T.O

with shear connectors are placed side by side and the concrete slab is cast at top.

- In the composite construction, the prestressed concrete precast element is first cast and the composite cross-Section is completed by adding the cast insitu slab. In simple words, the composite construction involves Two Stages of operation. unlike the single operation carried out in casting simple pre-stressed concrete beams.

- The behaviour of composite beam can be understood with a note that the precast unit and insitu concrete slab have different elastic moduli, different quality of concrete having different strengths, shrinkage and creep characteristics.

→ P.T.O

ADVATAGES:

1) considerable saving in cost is possible in composite construction than conventional reinforced concrete construction & in fully - prestressed concrete and construction.

2) The composite construction does not require formwork and scaffolding and it facilitates a continuous work without any interruption of regular traffic beneath.

3) The precast concrete unit supports the forms of the cast insitu slab, and hence, there is reduction in the false work and shoring cost

4) In practice, the insitu slab is of grade M20 and the precast unit is between M30 to M60. As the insitu slab is lighter in weight when compared to the precast prestressed concrete unit, the overall cost of construction leads to economy

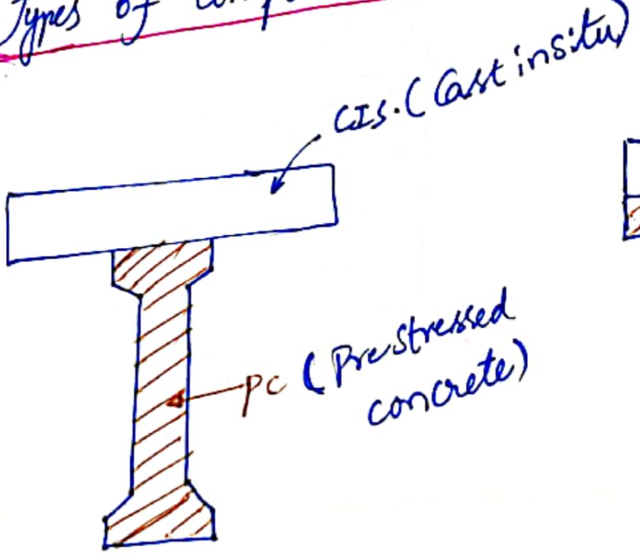
⑤ The construction time gets reduced with the use of precast elements

⑥ The cast insitu slab provides continuity at the ends of the precast elements over adjacent spans.

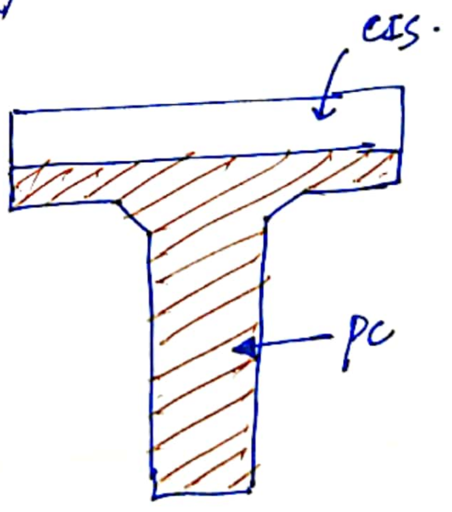
⑦ The cast insitu slab provides to the stability to the girders

⑧ As the composite construction includes the precast unit and it will be economical, if more precast units are manufactured in large numbers.

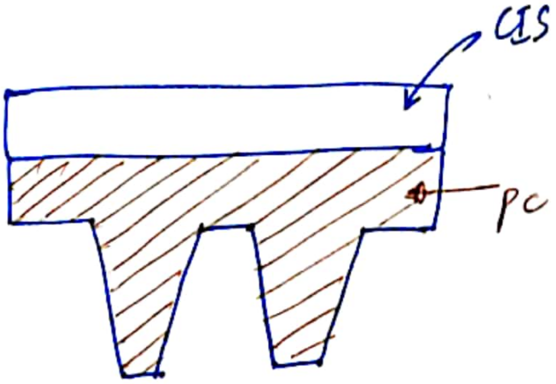
Types of Composite beams:



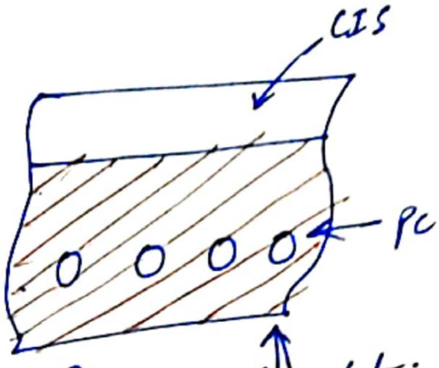
ⓐ slab and girder



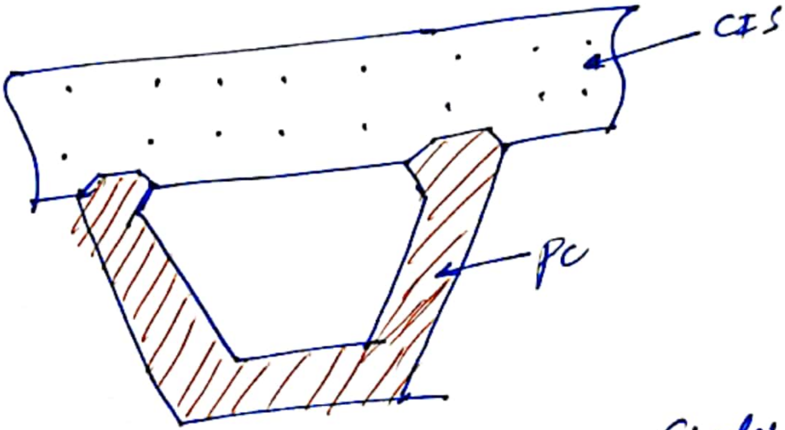
ⓑ single T-girder



ⓒ double T-Grides



ⓓ Hollow core slab.
[most economical]



ⓔ Trough shaped precast Girder
[Trapezoidal].

→ P.T.O

Composite Beams – propped & unpropped:

- In composite construction, the precast prestressed units are made in the factory with good quality control. The cast insitu slab is made at site and placed at the top of the precast unit. The interface between these two structural units i.e., the precast prestressed beam and the cast insitu slab are either roughened and the shear connectors are provided to enable composite action.

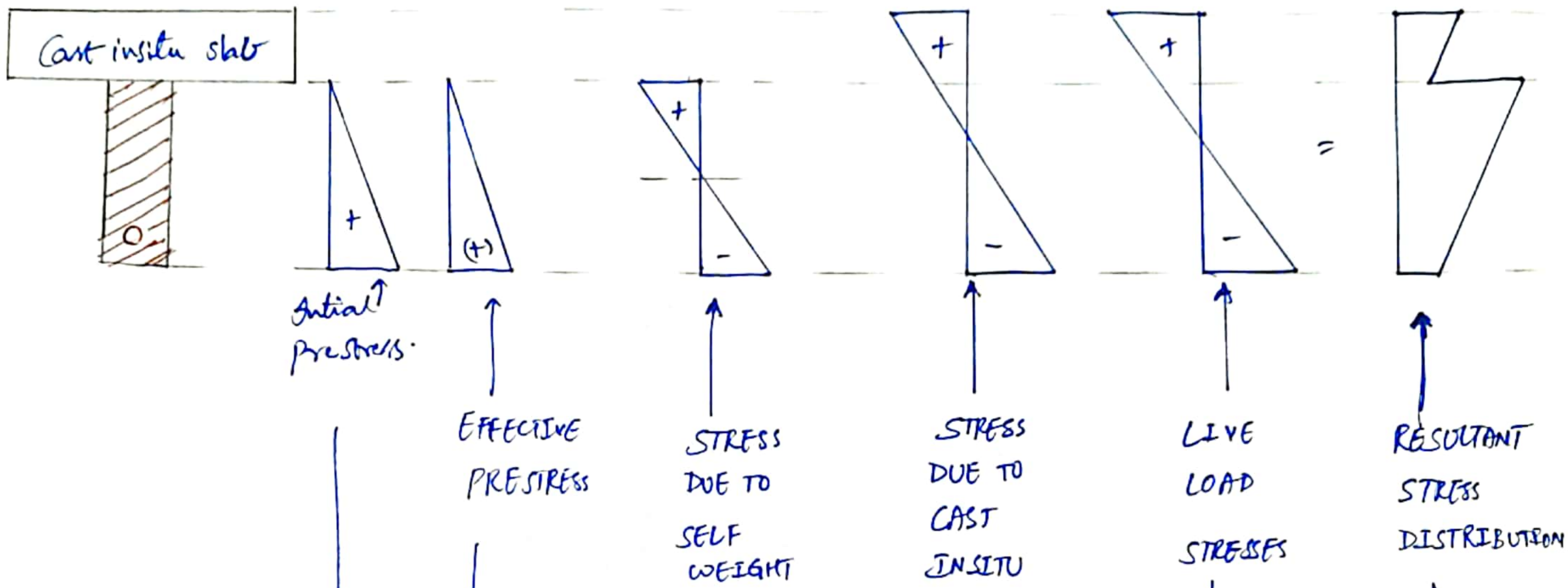
- In case of propped construction i.e., when the precast prestressed units are propped, the self weight of the cast insitu slab is acting when the props are removed. The flexural stresses are calculated using the sectional properties of composition section.

- In the case of unpropped construction i.e., if the precast unit is unpropped when the cast insitu slab concrete is placed, the self weight of the

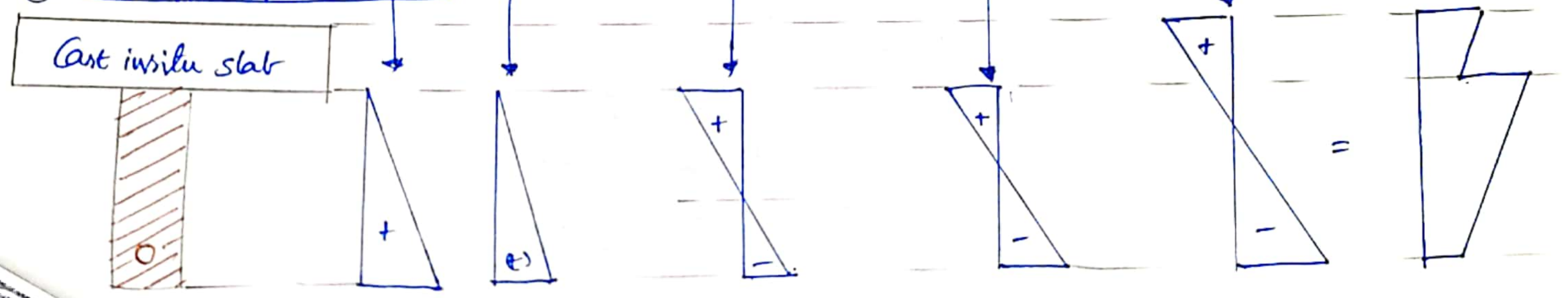
Cast insitu slab acts as a dead load on the precast unit. The flexural stresses are calculated using the sectional properties of the precast units only.

→ P.T.O

(a) PROPPED PRECAST UNIT



(b) UNPROPPED PRECAST UNIT:



Example ① A precast pretensioned beam of span 6m is of size 125 X 250 mm. The prestressing force is applied through bottom kern point. An initial pre-stressing force of 250 KN was applied with a loss of prestress 15%. The cast insitu slab is of 450 X 40 mm. The composite beam supports a line load of 5 KN/m. Calculate the resultant stresses developed in the precast unit and cast insitu slab assuming the pretensioned unit as (i) unpropped and (ii) propped during casting of slab. Assume the modulus of elasticity of concrete for the precast unit as 1.25 times the modulus of elasticity of the cast insitu slab.?

Sol:- Stresses due to precast pretensioned beam

Area of cross-section $= 125 \times 250$

$= 31250 \ mm^2$

Section modulus $= \frac{bD^2}{6} = \frac{125 \times 250^2}{6}$

$= 1.3 \times 10^6 \ mm^3$

Initial pre-stressing force, $P_0 = 250$ KN

As the pre-stressing force is applied at the bottom kern point, therefore

$$f = \frac{P}{A} + \frac{M}{Z}.$$

$$f = \frac{P}{A} + \frac{Pe}{Z}. \qquad [\because e = D/6].$$

$$f = \frac{P}{A} + \frac{PD}{6 \times \frac{bD^2}{6}} = \frac{P}{A} + \frac{P}{A}.$$

$$f = \frac{2P}{A} = \frac{2 \times 250 \times 10^3}{125 \times 250} = 16 \text{ N/mm}^2$$

Final pre-stressing force $= 0.85 P_0 = 0.85 \times 250$

$= 212.5$ KN

Self weight of the precast beam $= 0.125 \times 0.25 \times 24 \times 10^3$

$= 0.75$ KN/m

$= 750$ N/m.

Moment due to Self weight $= 750 \times 6^2/8 = 3375$ Nm.

Stress at the top and bottom $= \frac{M}{Z} = \frac{3375 \times 10^3}{1.3 \times 10^6} = 2.6\ N/mm^2$

So, Stress at top $= 2.6\ N/mm^2$ (compression)

Stress at bottom $= 2.6\ N/mm^2$ [Tension].

Self weight of the cast insitu slab $= \frac{40}{1000} \times \frac{450}{1000} \times 1 \times 24$

$= 0.432$ KN/m.

moment due to self weight $= 0.432 \times 6^2/8 = 1.94$ KNm.

(compression)

Stress at the top of the precast unit $= \frac{1.94 \times 10^6}{1.3 \times 10^6}$

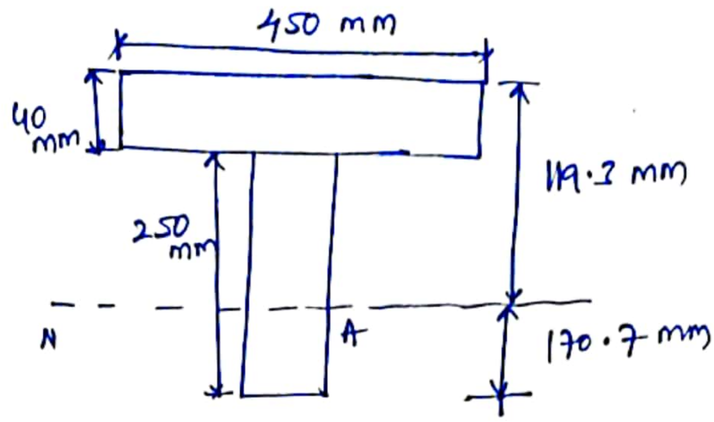
$= 1.49\ N/mm^2$ [Tension].

→ Sectional properties of the composite section

Taking the bottom as the reference

$$\bar{y} = \frac{450 \times 40 \times 270 + 1.25 \times (125 \times 250 \times 125)}{1.25 \times 125 \times 250 + 450 \times 40}$$

$$\boxed{\bar{y} = 170.1\,mm}$$



$$I_{xx} = \frac{450 \times 40^3}{12} + 450 \times 40 \times (119.3 - 20)^2$$

$$+ 1.25 \left[\frac{125 \times 250^3}{12} + 125 \times 250 \times (125 - 170.7)^2 \right].$$

$$I_{xx} = 464.93 \times 10^6\ mm^4$$

Maximum line load moment $= 5 \times 6^2/8 = 22.5\,kNm.$

$$\because Z = \frac{I}{y_{max}}$$

$$Z_f = \frac{464.93 \times 10^6}{119.3} = 3.9 \times 10^6\ mm^3$$

$$Z_b = \frac{464.93 \times 10^6}{170.7} = 2.7 \times 10^6\ mm^3$$

⟶ P.T.O

Live load stresses in the Composite ~~beam~~ Section.

$$\text{Stress at top} = \frac{M_L}{Z_t} = \frac{22.5\times10^6}{3.9\times10^6} = 5.77 \text{ Mpa [Comp]}.$$

$$\text{Stress at bottom} = \frac{M_L}{Z_b} = \frac{22.5\times10^6}{2.7\times10^6} = 8.33 \text{ Mpa [Tension]}.$$

If the prestressed beam is propped, the self weight of the slab acts on the composite section

moment due to slab weight = 1940 Nm.

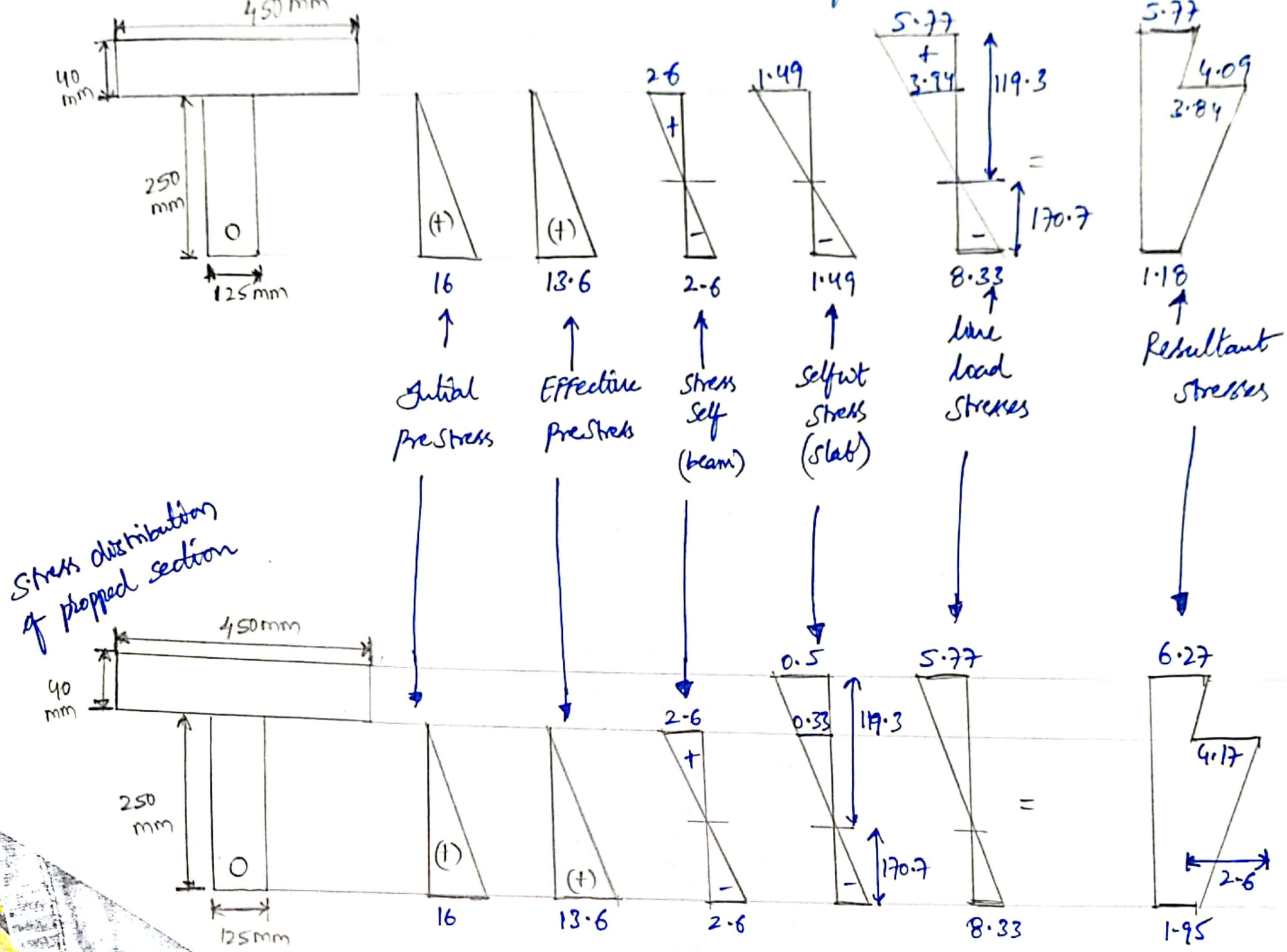
Stress due to this moment in the composite section

$$\text{At top} = \frac{1940\times10^3}{3.9\times10^6} = 0.5 \text{ N/mm}^2 \text{ [Comp]}.$$

$$\text{At bottom} = \frac{1940\times10^3}{2.7\times10^6} = 0.72 \text{ N/mm}^2 \text{ [Tension]}$$

STRESS distribution of unpropped section

Stress distribution of propped section



mple ②

A Composite beam is made by casting 300 X 900 mm precast prestressed with cast insitu slab of 1500 X 150 mm. The effective prestress of 12 Mpa at soffit and zero at the top. Calculate the uniformly distributed for the composite action on a Simply Supported span of 16 m for the following conditions, concrete weights 25 KN/m³

(i) Unpropped construction

(ii) Propped construction

Sol:- Self weight of the beam $= 0.3 \times 0.9 \times 1 \times 25 = 6.75$ KN/m.

Bending moment due to precast beam $= \dfrac{6.75 \times 16^2}{8} = 216$ KNm

Second moment area of section $= \dfrac{300 \times 900^3}{12} = 1.82 \times 10^{10}\ mm^4$

Flexural stress due to precast beam $= \dfrac{M}{Z}$

$$= \frac{216 \times 10^6}{\left(300 \times \frac{900^2}{6}\right)} = 5.33\ Mpa$$

⟶ P.T.O

Thin Cylinders ... shells- Wire wound thin cylinders

insitu slab weight per meter $= 1.5 \times 0.15 \times 1 \times 25 = 5.63$ KN/m

Bending moment due to self weight of cast insitu slab

$$= 5.63 \times 16^2/8 = 180.2 \text{ KN m}.$$

modulus of section $= Z = 300 \times 900^2/6$

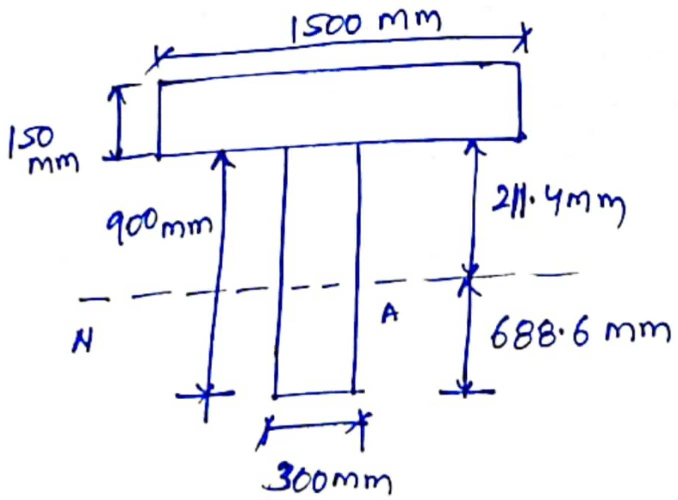
$$= 40.5 \times 10^6 \text{ mm}^3.$$

Flexural stresses developed $= \frac{M}{Z}$

$$= \frac{180.2 \times 10^6}{40.5 \times 10^6} = 4.45 \text{ N/mm}^2$$

Bending moment due to live load $= wl^2/8$

$$= \frac{w(16)^2}{8} = 32w \text{ KNm}$$



$\rightarrow$ P.T.O

Taking the soffit of the beam as reference

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}.$$

$$\bar{y} = \frac{(300 \times 900) \times 450 + (1500 \times 150) \times 975}{(300 \times 900) + (1500 \times 150)}$$

$$\bar{y} = 688.6 \text{ mm}.$$

$$I_{xx} = \frac{300 \times (688.6)^3}{3} + \frac{300 \times (211.4)^3}{3} + \frac{1500\,(361.4)^3}{3} - \frac{1200 \times (211.4)^3}{3}.$$

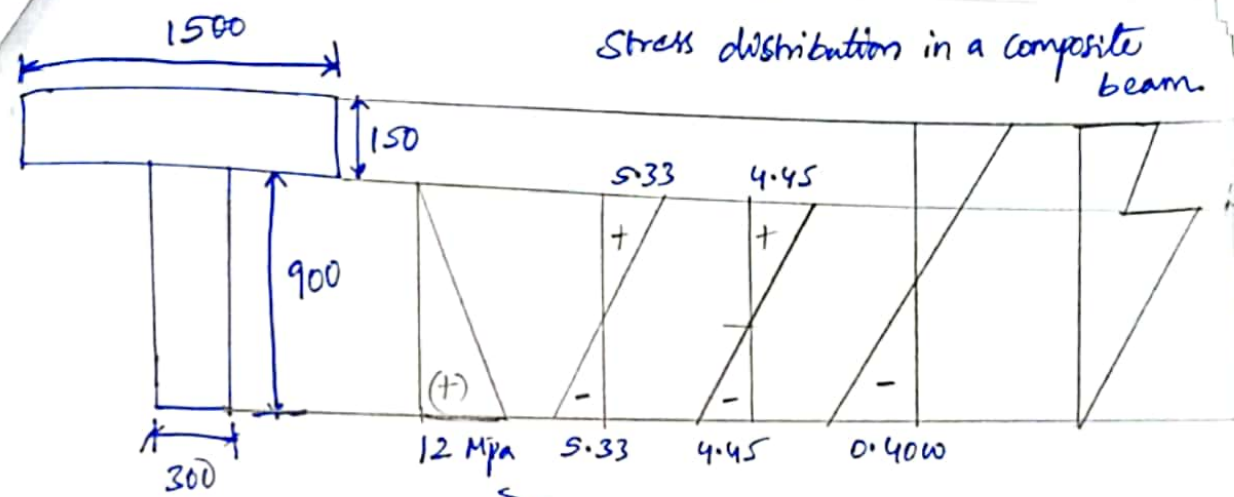
$$I_{xx} = 5.68 \times 10^{10} \text{ mm}^4$$

$$Z_t = \frac{I_{xx}}{y_t} = \frac{5.68 \times 10^{10}}{150 + 211.4} = 157.17 \times 10^6 \text{ mm}^3$$

$$Z_b = \frac{I_{xx}}{y_b} = \frac{5.68 \times 10^{10}}{688.6} = 82.5 \times 10^6 \text{ mm}^3$$

Stress due to live load at the soffit of the beam

$$= \frac{32\,w \times 10^6}{82.5 \times 10^6} = 0.45w.$$



Refering to the above figure.

$$12 - 5.33 - 4.45 - 0.42w = 0$$

$$w = 5.29 \text{ KN/m}$$

Load intensity per $KN/m^2 = \frac{5.29}{1.50} = 3.53 \text{ KN/m}^2$

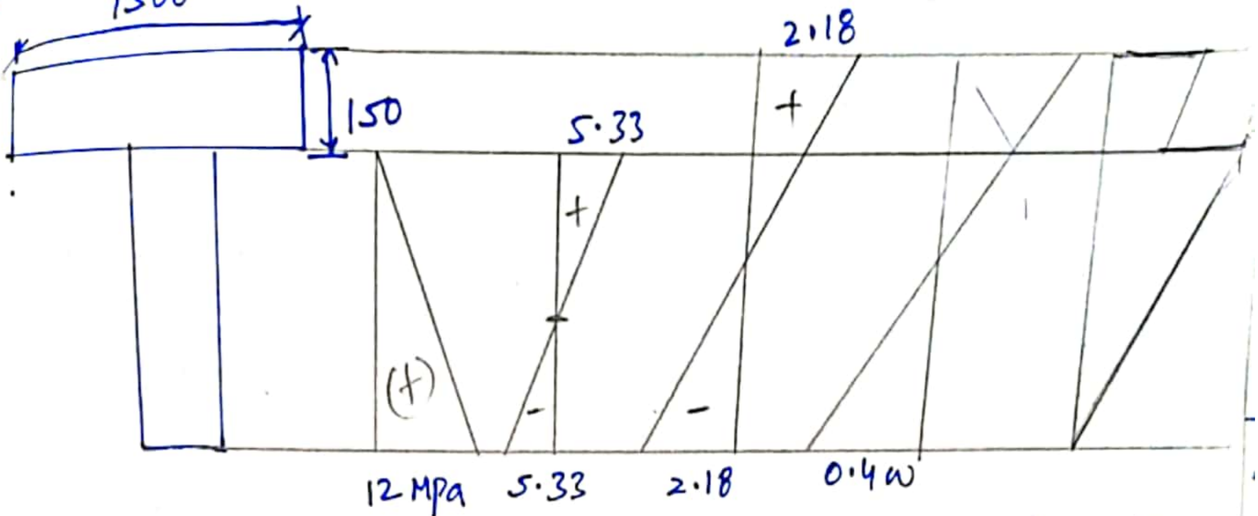
Propped construction

Stress due to cast insitu slab $= \frac{M_s}{Z_b}$

$$= \frac{180.2 \times 10^6}{82.5 \times 10^6} = 2.18 \text{ N/mm}^2$$

→ P.T.O

Stress distribution in a composite beam.



stress at the soffit is zero and hence refering the above fig

$$12 - 5.33 - 2.18 - 0.4w = 0$$

$$w = 11.22 \text{ KN/m.}$$

$$\text{Live load intensity} = \frac{11.22}{1.50} = 7.48 \text{ KN/m}^2$$

Differential shrinkage in Composite beams:

In composite construction, the precast prestressed beams are resisting the applied loads along with cast insitu slab. As the precast elements were placed very much earlier, creep and shrinkage would have already taken place. The wet concrete of slab is laid over the precast unit, and the shrinkage and creep continues. The stresses are computed as follows.

Let ε_{cs}, E_c and A_i be the shrinkage strain, modulus of elasticity of insitu concrete and the cross-sectional area of the cast insitu slabs. The magnitude of the tensile force (N_{sh}) is computed from the following equation

$$\boxed{N_{sh} = \varepsilon_{cs} \cdot E_c \cdot A_i}$$

This tensile force is balanced by a compressive force applied at the centroid of equal magnitude. The force applied at the cast insitu slab causes a direct force acting at the centroid of the composite section together with a bending moment

This creates a direct force and a bending moment causes direct and bending stresses. The estimation of shrinkage stress can be explained briefly as follows

(i) If the top of the cast insitu slab has restrained against contraction, a uniform tensile stress would dnelope due to differential shrinkage

(ii) This imaginary ~~resistant~~ restraint force acts at the mid depth of slab

(iii) As in self equilibrating system; a balancing compressive force will act in the composite section. This results in a compressive force and a moment in the composite section.

(iv) The axial and bending stresses can be calculated for the above effects and added to the uniform tensile stress to obtain the stresses due to differential shrinkage

The formula to compute the stresses (fig below) due to differential shrinkage are as follows.

- Stress at the top fibre of the slab $= \frac{P_s}{A_c} + \frac{M}{Z_t} - f$

- Stress at the bottom fibre of the slab $= \frac{P_s}{A_c} + \frac{M}{Z_b} - f$.

$\longrightarrow$ P.T.O

. stress at the top fibre of the beam $= \frac{P_s}{A_c} + \frac{M}{Z_t}$.

. stress at the bottom fibre of the beam $= \frac{P_s}{A_c} + \frac{M}{Z_b}$.

where,

f = uniform tensile stress at center of the slab

P_s and M = direct compressive force and the bending moment

A_c = area of the composite section

Z_t, Z_f, Z_b = section moduli at top, junction and bottom of the precast beam respectively.

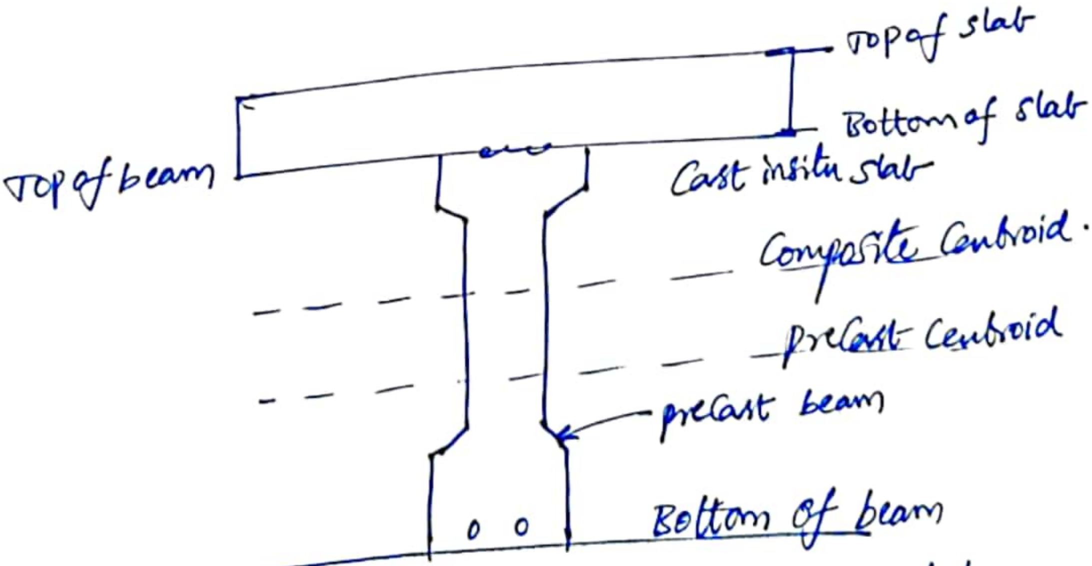


fig: details of the compound beam.