## UNIT-1

## Control systems

**system:-** When a number of elements or components are connected in a sequence to perform a specific function, the group thus formed is called a "system."

**Control system:-**

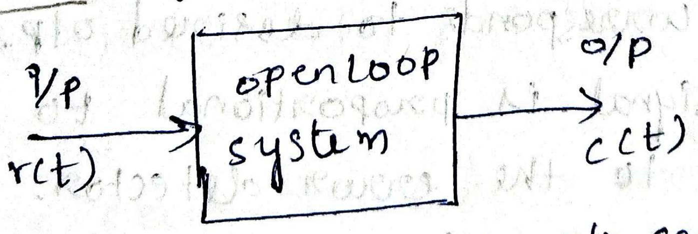
In a system when the o/p quantity is controlled by varying the i/p quantity is called "Control system"

→ The o/p quantity is called controlled variable or response and i/p quantity is called command signal or excitation

**open loop systems:-**

→ Control system in which the o/p quantity has no effect upon the i/p quantity are called "open-loop control system"

→ This means that the o/p is not fed back to the i/p for correction.




→ In open loop system the o/p can be varied by varying the i/p. But due to external disturbance the system o/p may change

→ In open loop system the changes in o/p are corrected by changing the i/p manually.

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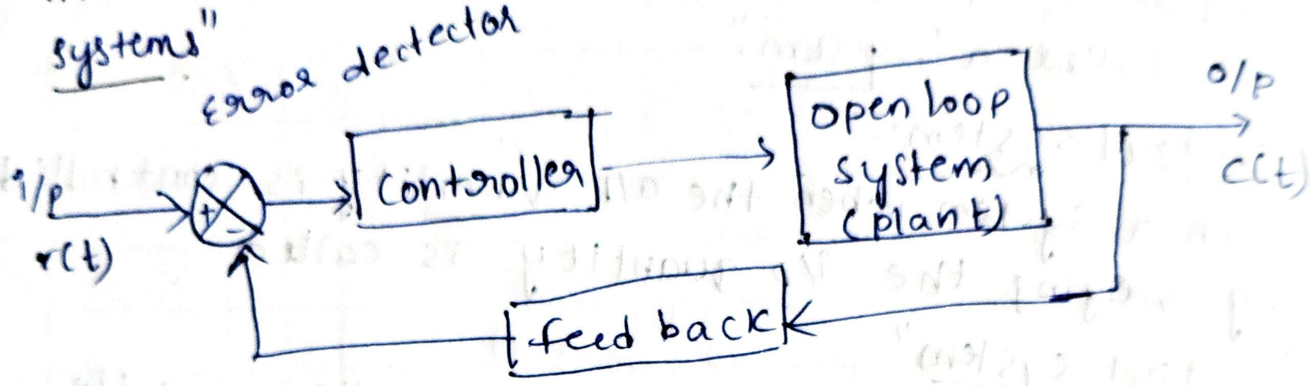
## Closed loop system

Control systems in which the o/p quantity has an effect upon i/p quantity in order to maintain the desired o/p value are called "closed loop systems"




→ The open loop system can be modified as closed loop system by providing feed back.

→ The provision of feedback automatically correct the changes in o/p due to disturbances. Hence the closed loop system is also called automatic control system.

→ It consist of an error detector, a controller, plant (openloop system) and feedback path elements.

→ The i/p signal corresponds to desired o/p.

→ The feedback signal is proportional to o/p signal and it is fed to the error detector.

→ The error signal generated by the error detector is the difference b/w reference s/l and feedback signal.

→ The controller modifies and amplifies the error signal to produce better control action.

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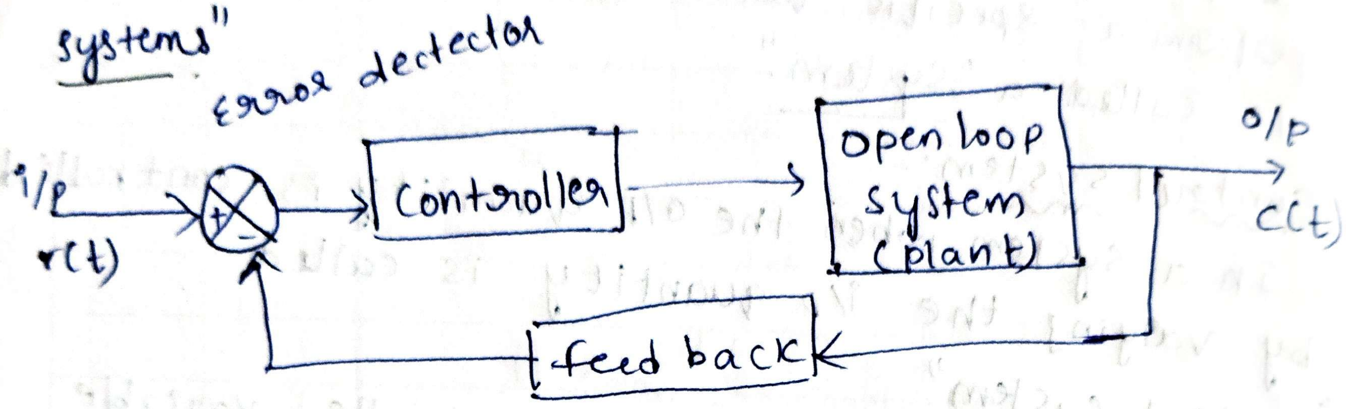


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## Advantages of open loop systems

1) The open loop systems are simple and economical.
2) The open loop systems are easier to construct
3) Generally the open loop systems are stable

## Disadvantages of open loop systems

1) The open loop systems are inaccurate & unreliable
2) The changes in the o/p due to external disturbances are not corrected automatically.

## Advantages of closed loop systems

1) The closed loop systems are accurate
2) The closed loop systems are less affected by noise
3) The sensitivity of the system may be made small to make the system more stable.

## Disadvantages of closed loop systems.

1) The closed loop systems are complex & costly
2) The feedback reduces the overall gain of the system.
3) Stability is a major problem in closed loop system and more care is needed to design a stable closed loop system.

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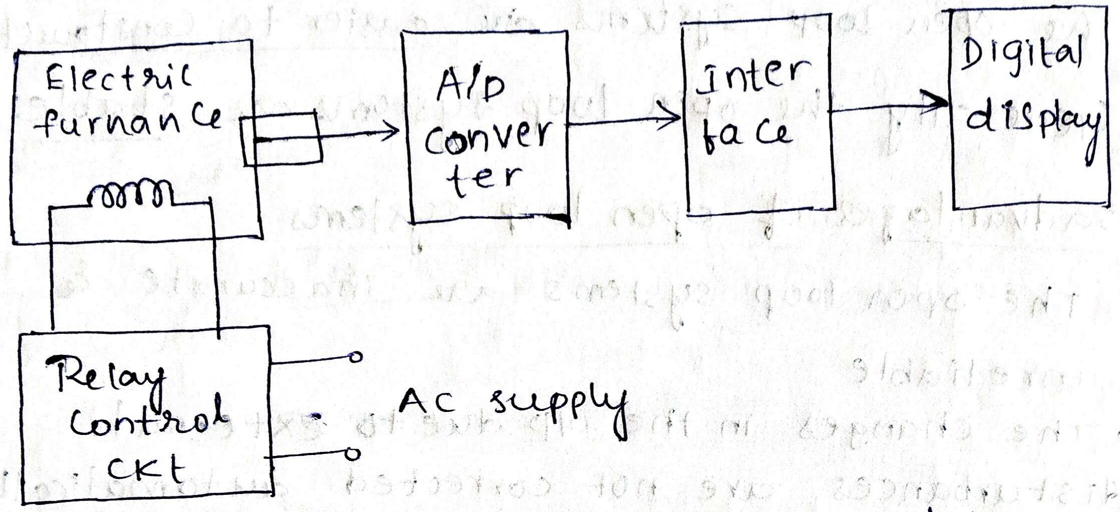
Examples of control system:

Temparature Control system

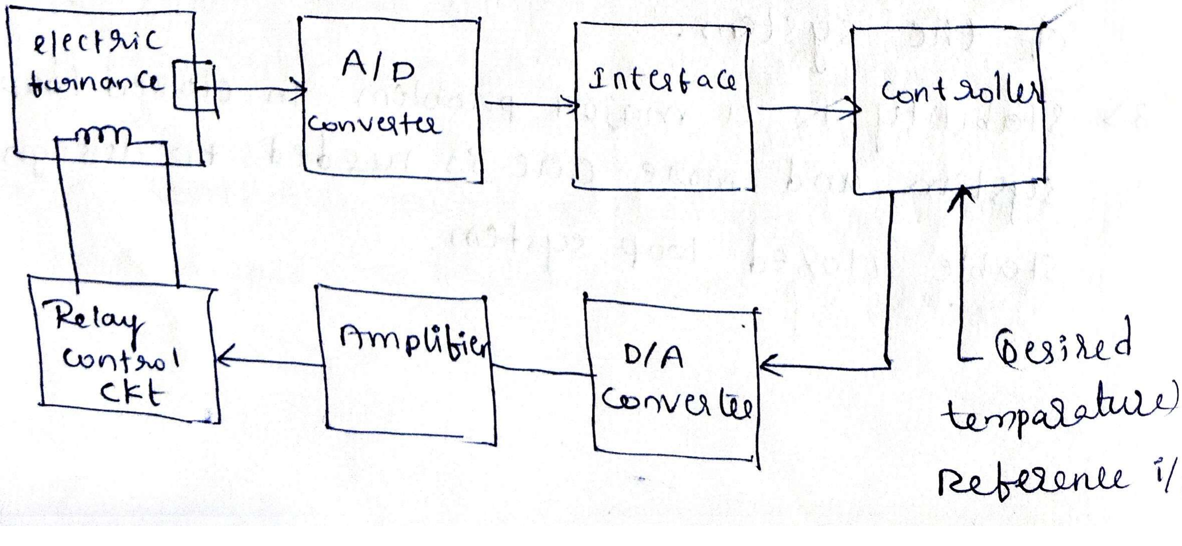
open loop system




The o/p in the system is the desired temparature and it depends on the time during which the supply to the heater remains ON.

The ON & OFF of the supply is governed by the time setting of the relay. the temparature is measured by sensor and is converted to digital signal by analogy to digital converter. the digital signal is given to digital display to display the temparature in this system if there is any change in the o/p temparature then the time setting of the delay is not alternated automatically

closed loop system.



The o/p of the system is the desired temparature and it depends on the time during which supply to the heater remains ON. 5

The switching ON & off of the relay is controlled by a controller which is a digital system or a computer. The actual temparature is sensed by a sensor and converted to digital signal by analog to digital converter.

The computer reads the actual temparature and compares with the desired temparature if it finds any difference then it send signals to switch ON. or OFF the relay through the analog to digital converter and amplifies. does the system automatically corrects any changes in the output

Traffic Control system.

→ Traffic control by means of a traffic signals operated on a time basis constitutes as open loop system the sequence of control signals are based on a time slot given for each signal

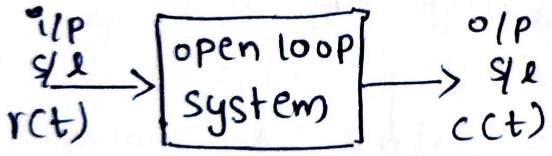
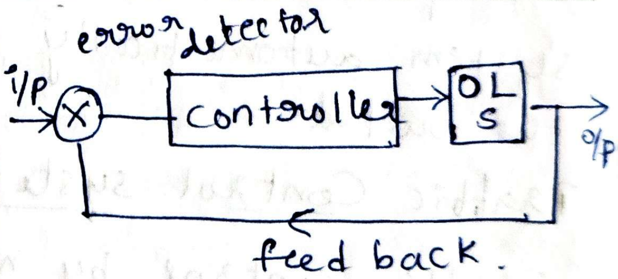
→ The system will not measure the density of the topic before giving the signals

→ since the time slot doesnot change according to the traffic density that system is a open loop system.

Closed loop system

Traffic control system can be made as a closed loop system if the time slots of the signals are decided based on the density of the traffic in closed loop traffic control system, the density of the traffic is measured by a computer on all the sides and the information is fed to a computer. the timings of the control signals are decided by the computer based on the density of traffic. Since the closed loop system dynamically changes the timings, the flow of vehicles will be better than open loop system

Differences b/w open loop system and closed loop system

| open loop system | closed loop system. |
|---|---|
|   |   |
| → No feed back is used | → feed back is used |
| → These are simple to construct and less in cost | → These are complicate to construct and costly |
| → These are generally stable | → These are unstable during certain conditions. |
| → The open loop systems are simple | → The closed loop systems are complex. |

# Mathematical models of control systems



→ A control system is a collection of physical objects connected together to serve an objective

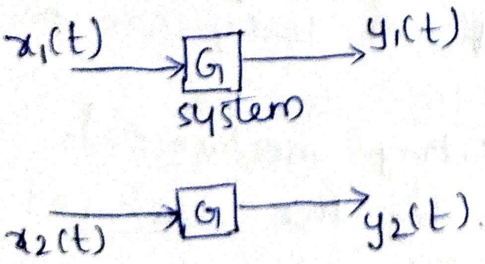
→ The i/p o/p relations of various physical components of a system are governed by differential equations.

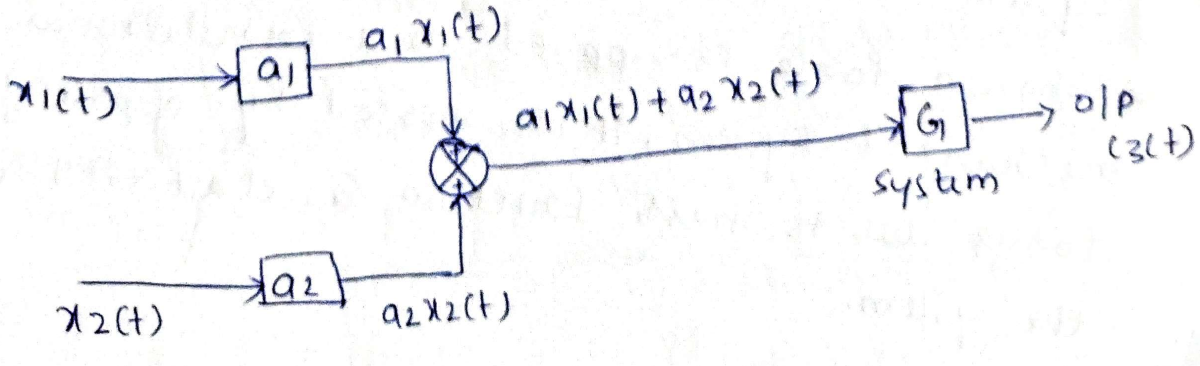
→ The mathematical model of a control system constitutes a set of differential equations.

→ The response or o/p of the system can be studied by solving the differential eqn's for various i/p conditions.

→ The mathematical model of a system is linear if it obeys the principle of superposition and homogenity

→ This principle implies that if a system model has responses $y_1(t)$ & $y_2(t)$ to any i/p's $x_1(t)$ & $x_2(t)$ respectively, then the system response to the linear combination of these i/p s $\boxed{a_1x_1(t) + a_2x_2(t)}$ is given by linear combinations of the individual o/p's $\boxed{a_1y_1(t) + a_2y_2(t)}$ where $a_1$ & $a_2$ are constants.



A mathematical model will be linear, if the differential eqn's describing the system has constant coefficients.

→ If the cofficients of the differential eq'n describing the system are constants then the model is linear time invarient.

→ If the coff t's of the differential eq'n governing the system are functions of time then the model is linear time varying

→ The transfer function of a system is defined as the ratio of laplace transform of o/p to the L.T of i/p with zero conditions.

$$\boxed{\text{Transfer functn} = \frac{\text{L.T of o/p}}{\text{L.T of i/p}}}$$ with zero initial conditions.

## Mechanical Translational systems.

→ The body of M.T.S can be obtained by using three basic elements mass, spring, & dash pot

These '3' elements represents '3' essential phenomena which occur in various ways in mechanical systems.

→ The weight of the mechanical system is represented by the element Mass and it is assumed to be concentrated at the center of the body.

→ The elastic deformation of the body can be represented by a spring.

→ The friction existing in rotating mechanical system can be represented by the dash-pot

→ when a force is applied to a translational mechanical system, it is opposed by opposing forces due to mass, friction & elasticity of the system.

9 The force acting on a mechanical body are governed by Newton's second law of motion. for translational system it states that the sum of forces acting on a body is zero.

[Newton's 2nd law state's that the sum of applied forces is equal to the sum of opposing forces on a body]

List of symbols used in mechanical Translational system.

$x$ = Displacement, m.

$v = \frac{dx}{dt}$ = velocity, m/sec

$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ = Acceleration m/sec²

$f$ = applied force, N (Newtons).

$f_m$ = opposing force offered by mass of the body in N.

$f_k$ = " " " " elasticity of the body (spring) in N

$f_b$ = " " " " friction of the body (dash-pot), N

$M$ = mass, Kg.

$K$ = stiffness of spring, N/m

$B$ = viscous friction coefficient N-sec/m.

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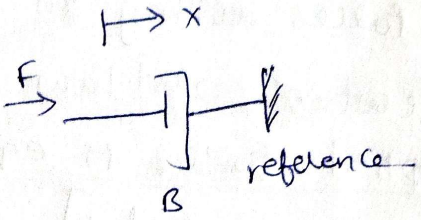
Consider an idel mass element which has negligible friction & elasticity. let a force be applied on it. The mass will offers opposing force which is proportional to acceleration of the body.

$f$ = Applied force

$f_m$ = opposing force due to mass


$f_m \propto \frac{d^2x}{dt^2} \Rightarrow \boxed{f_m = M\frac{d^2x}{dt^2} = f}$ (By Newton's 2nd law)

Consider an idel frictional element dashpot which has negligible mass & elasticity. let Forece be applied on it. The dashpot will offers an opposing force which is proportional to velocity of the Body.




Let, $f$ = Applied force

$f_b$ = opposing force due to friction.

$f_b \propto \frac{dx}{dt} \Rightarrow \boxed{f_b = B\frac{dx}{dt} = f}$

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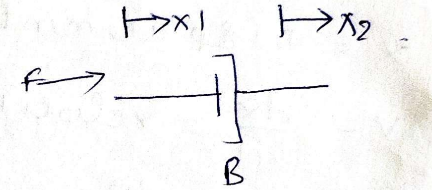
when the dashpot has displacement at both ends the opposing force is proportional to differential velocity




$f_b \propto \frac{d}{dt}(x_1 - x_2)$

$\boxed{f_b = B\frac{d}{dt}(x_1 - x_2) = f}$

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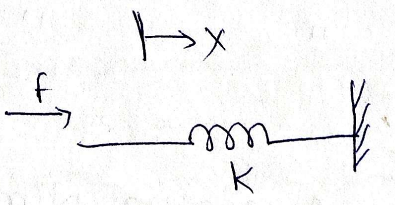
Consider an ideal elastic element spring which has negligible mass and friction. let a force is applied on it. The spring will offers an opposing force which is proportional to displacement of the body.

$f$ = Applied force

$f_K$ = opposing force due to elasticity.




$f_K \propto x \Rightarrow \boxed{f_K = Kx = f}$

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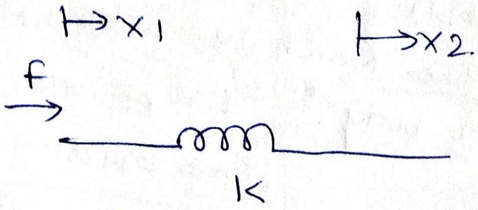
when the spring has displacement at both ends the opposing force is proportional to differential displacement.

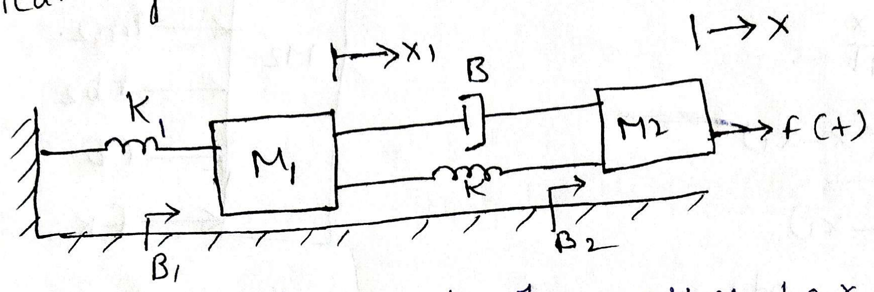
$f_K \propto (x_1 - x_2)$

$\boxed{f_K = K(x_1 - x_2) = f}$



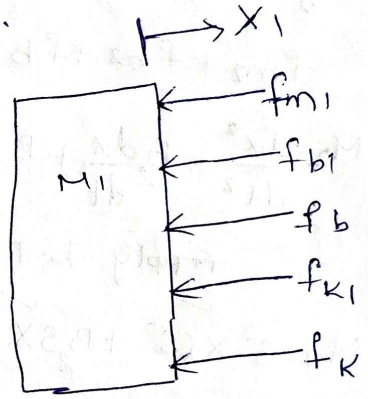
example ① 11

write the differential eqn's governing the mechanical system and determine the transfer functn




$\frac{X(S)}{F(S)}$

let the displacement of mass $M_1$ be $x_1$ the free body diagram of mass $M_1$.




$$f_{m1} = M_1 \frac{d^2x_1}{dt^2}$$

$$f_{b1} = B_1 \frac{dx_1}{dt}$$

$$f_b = B \frac{d}{dt}(x_1 - x)$$

$$f_{K1} = K_1 x_1$$

$$f_K = K(x_1 - x).$$

By Newton's second law.

$$f_{m1} + f_{b1} + f_b + f_{K1} + f_K = 0.$$

$$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B \frac{d}{dt}(x_1 - x) + K_1 x_1 + K(x_1 - x) = 0.$$

Apply L.T o.b.s

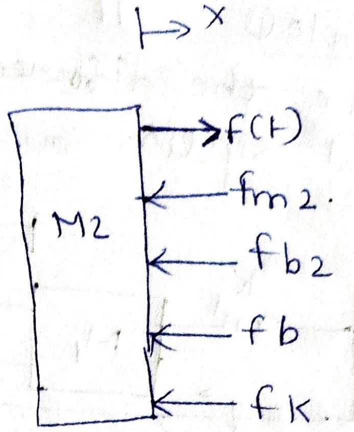
$$M_1 s^2 X_1(S) + B_1 S X_1(S) + B\, S[X_1(S) - X(S)] + K_1 X_1(S) + K[X_1(S) - X(S)] = 0.$$

$$M_1 S^2 X_1(S) + B_1 S X_1(S) + BS\, X_1(S) - BS\, X(S) + K_1 X_1(S) + K X_1(S) - K X(S) = 0.$$

$$X_1(S)[M_1 S^2 + B_1 S + BS + K_1 + K] - X(S)[BS + K] = 0.$$

$$X_1(S)[M_1 S^2 + (B_1 + B) S + (K_1 + K)] = X(S)[BS + K]$$

$$\therefore X_1(S) = \frac{X(S)[BS + K]}{M_1 s^2 + (B_1 + B) S + (K_1 + K)}$$




$f_{m2} = M_2 \frac{d^2x}{dt^2}$

$f_{b2} = B_2 \frac{dx}{dt}$

$f_b = B \frac{d}{dt}(x - x_1)$

$f_k = K(x - x_1)$

By Newton's 2nd law,

$f_{m2} + f_{b2} + f_b + f_k = f(t)$

$$M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + B \frac{d}{dt}(x - x_1) + K(x - x_1) = f(t)$$

~~Apply~~ L.T o.b.s

$$M_2 s^2 X(s) + B_2 s X(s) + Bs X(s) - Bs X_1(s) + K X(s) - K X_1(s) = F(s)$$

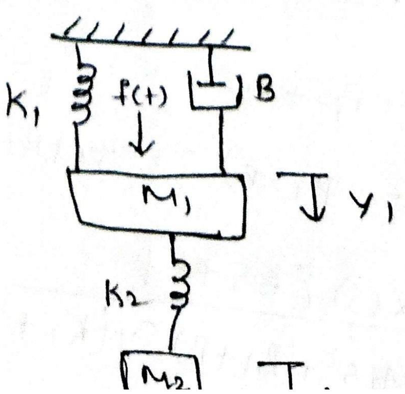
$$X(s)[M_2 s^2 + B_2 s + Bs + K] - X_1(s)[Bs + K] = F(s)$$

$$X(s)[M_2 s^2 + (B_2 + B)s + K] \; - X(s) \frac{[Bs+K]^2}{M_1 s^2 + (B_1+B)s + (K_1+K)} = F(s)$$

$$X(s)\left[\frac{[M_1 s^2 + (B_1+B)s + (K_1+K)][M_2 s^2 + (B_2+B)s + K] - (Bs+K)^2}{M_1 s^2 + (B_1+B)s + (K_1+K)}\right] = F(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{M_1 s^2 + (B_1+B)s + (K_1+K)}{[M_1 s^2 + (B_1+B)s + (K_1+K)][M_2 s^2 + (B_2+B)s + K] - (Bs+K)^2}}$$

② find T.F $\frac{Y_2(s)}{F(s)}$,



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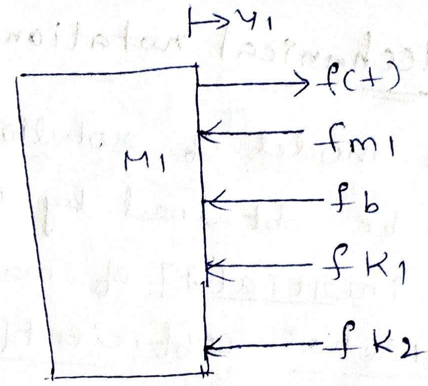



$$f_{m1} = M_1 \frac{d^2 y_1}{dt^2}$$

$$f_b = B \frac{dy_1}{dt}$$

$$f_{K1} = K_1 y_1$$

$$f_{K2} = K_2 (y_1 - y_2)$$

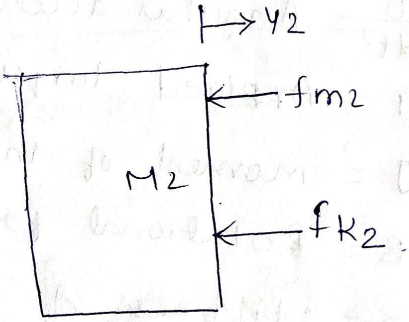
$$f_{m1} + f_b + f_{K1} + f_{K2} = f(t).$$

$$M_1 \frac{d^2 y_1}{dt^2} + B\frac{dy_1}{dt} + K_1 y_1 + K_2 (y_1 - y_2) = f(t).$$

Apply L.T o.b.s

$$M_1 s^2 Y_1(S) + BS\, Y_1(S) + K_1 Y_1(S) + K_2 Y_1(S) - K_2 Y_2(S) = F(S).$$

$$Y_1(S)[M_1 s^2 + BS + (K_1 + K_2)] - K_2 Y_2(S) = F(S) \rightarrow ①$$




$$f_{m2} = M_2 \frac{d^2 y_2}{dt^2}$$

$$f_{K2} = K_2 (y_2 - y_1)$$

$$f_{m2} + f_{K2} = 0.$$

$$M_2 \frac{d^2 y_2}{dt^2} + K_2 (y_2 - y_1) = 0.$$

L.T o.b.s

$$M_2 s^2 Y_2(S) + K_2 Y_2(S) - K_2 Y_1(S) = 0.$$

$$Y_2 [M_2 s^2 + K_2] - K_2 Y_1(S) = 0.$$

$$K_2 Y_1(S) = Y_2(S)[M_2 s^2 + K_2]$$

$$Y_1(S) = \frac{Y_2(S)[M_2 s^2 + K_2]}{K_2} \rightarrow ②$$ , ② in ①, we get

$$Y_2(S) \frac{[M_2 s^2 + K_2]}{K_2} [M_1 s^2 + Bs + (K_1 + K_2)] - K_2 Y_2(S) = F(S)$$

$$Y_2(S)\left[\frac{[M_2 s^2 + K_2][M_1 s^2 + BS + K_1 + K_2] - K_2^2}{K_2}\right] = F(S).$$

$$\boxed{\frac{Y_2(S)}{F(S)} = \frac{K_2}{[M_1 s^2 + BS + (K_1 + K_2)][M_2 s^2 + K_2] - K_2^2}}$$

# Mechanical rotational systems

The model of rotational mechanical systems can be obtained by using '3' elements, Moment of inertia[J] of mass, dash-pot with rotational frictional coefficient[B] and torsional spring with stiffness [K]

## List of symbols used in Mechanical Rotational system.

$\theta$ = Angular displacement, rad.

$\frac{d\theta}{dt}$ = Angular velocity, rad/sec

$\frac{d^2\theta}{dt^2}$ = Angular accelaration, rad/sec2

T = Applied torque, N-m

J = moment of inertia, Kg-m2/rad

B = Rotational frictional coefficients, N-m

K = stiffness of the spring, N-m/rad.

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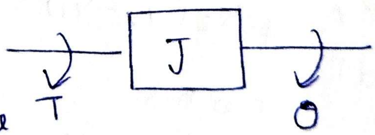
friction, elasticity is negligible

* the opposing Torque is due to moment of inertia is proportional to the angular accelaration




Ideal rotational mass element.

$T$ = Applied torque

$T_j$ = opposing torque

$T_j \propto \frac{d^2\theta}{dt^2} \Rightarrow \boxed{T_j = J\frac{d^2\theta}{dt^2} = T}$

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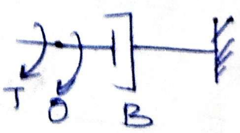
inertia & elasticity is negligible

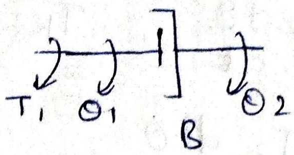
$T_b \propto \frac{d\theta}{dt}$

$$\boxed{T_b = B\frac{d\theta}{dt} = T}$$




friction

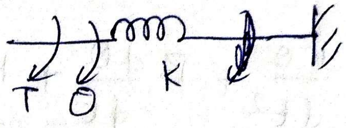
$$T_b \propto \frac{d}{dt}(\theta_1 - \theta_2)$$




$$\boxed{T_b = B\frac{d}{dt}(\theta_1 - \theta_2) = T}$$

T = Applied torque.

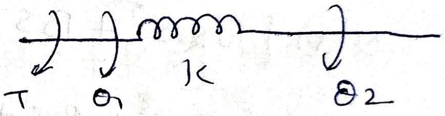
TK = opposing torque.




$T_K \propto \theta$.

$$\boxed{T_K = K\theta = T}$$

$T_K \propto (\theta_1 - \theta_2)$.




$$\boxed{T_K = K(\theta_1 - \theta_2) = T}$$

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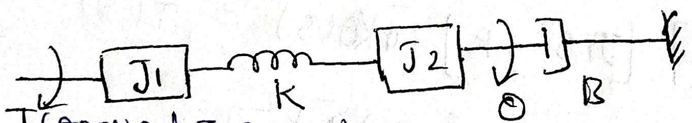
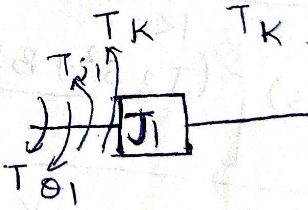
**Ex①**

write the differential eqn's governing the mechanical rotational system




$\left.\begin{matrix} T_{J_1} \\ T_K \end{matrix}\right\}$.

$$T_{J_1} = \frac{J_1 d^2\theta_1}{dt^2} \; ; \; T_K = K(\theta_1 - \theta).$$




$$T_{J_1} + T_K = T$$

$$J_1\frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) = T.$$

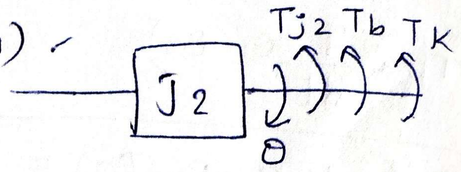
$$J_1\frac{d^2\theta_1}{dt^2} + K\theta_1 - K\theta = T.$$

Apply L.T on b.s

$$J_1 s^2\theta_1(s) + K\theta_1(s) - K\theta(s) = T(s).$$

$$\theta_1(s)\left[J_1 s^2 + K\right] - K\theta(s) = T(s) \longrightarrow ①$$

$T_b = B\frac{d\theta}{dt}$ ; $T_K = K(\theta - \theta_1)$




$T_{j2} = J_2 \frac{d^2\theta}{dt^2}$

$T_{j2} + T_b + T_K = 0$

$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0$

Apply L.T on b.s

$J_2 s^2\theta(s) + Bs\theta(s) + K\theta(s) - K\theta_1(s) = 0$

$\theta(s)[J_2 s^2 + Bs + K] - K\theta_1(s) = 0$

$\theta(s)[J_2 s^2 + Bs + K] = K\theta_1(s)$

$\theta_1(s) = \frac{\theta(s)[J_2 s^2 + Bs + K]}{K} \rightarrow ②$

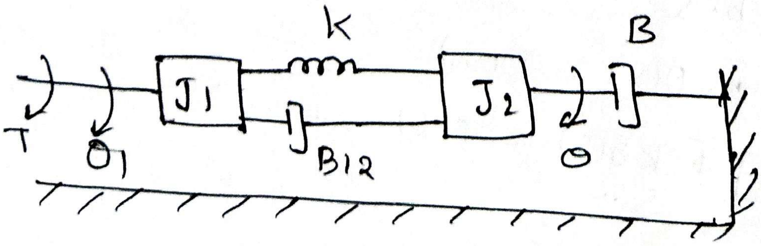
Sub ② in ①, we get

$\frac{\theta(s)[J_2 s^2 + Bs + K]}{K}[J_1 s^2 + K] - K\theta(s) = T(s)$

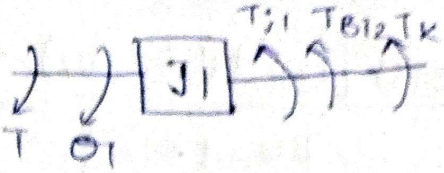
$\theta(s)\left[\frac{(J_2 s^2 + Bs + K)(J_1 s^2 + K) - K^2}{K}\right] = T(s)$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{K}{(J_2 s^2 + Bs + K)(J_1 s^2 + K) - K^2}}$$

② find $\theta(s)/T(s)$.



$T_{J_1} = J_1 \frac{d^2\theta_1}{dt^2}$




$T_{B_{12}} = B_{12} \frac{d}{dt}(\theta_1 - \theta)$

$T_K = K(\theta_1 - \theta)$

w.k.T

$T_{j1} + T_{B12} + T_K = T$.

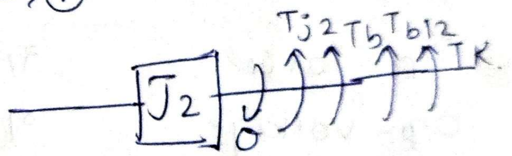
$J_1 \frac{d^2\theta_1}{dt^2} + B_{12}\frac{d}{dt}(\theta_1 - \theta) + K(\theta_1 - \theta) = T$.

Apply L.T o.b.s

$J_1 s^2 \theta_1(S) + B_{12}\, S\theta_1(S) - B_{12} S\theta(S) + K\,\theta_1(S) - K\,\theta(S) = T(S)$

$\theta_1(S)[J_1 s^2 + B_{12} S + K] - \theta(S)[B_{12}S + K] = T(S)$ $\rightarrow$ ①

$T_{J2} = J_2 \frac{d^2\theta}{dt^2}$




$T_b = B \frac{d\theta}{dt}$

$T_{B12} = B_{12}\frac{d}{dt}(\theta - \theta_1)$

$T_K = K(\theta - \theta_1)$

$T_{j2} + T_b + T_{b12} + T_K = 0$.

$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + B_{12}\frac{d}{dt}(\theta - \theta_1) + K(\theta - \theta_1) = 0$.

Apply L.T o.b.s

$J_2 s^2\theta(S) + BS\theta(S) + B_{12} S\theta(S) - B_{12} S\theta_1(S) + K\theta(S) - K\theta_1(S) = 0$.

$\theta(S)[J_2 S^2 + BS + B_{12}S + K] - \theta_1(S)[B_{12}S + K] = 0$.

$\theta(S)[J_2 S^2 + BS + B_{12}S + K] = \theta_1(S)[B_{12}S + K]$

$\theta_1(S) = \frac{\theta(S)[J_2 S^2 + BS + B_{12}S + K]}{B_{12}S + K}$ $\rightarrow$ ②

Sub ② in ①, we get.

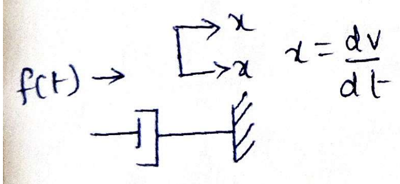
$$\theta(s)\,\frac{\left[J_2 s^2+(B + B_{12})s+K\right]\left[J_1 s^2+B_{12}s+K\right]}{B_{12}s+K} - \left[B_{12}s+K\right]\theta(s) = T(s)$$

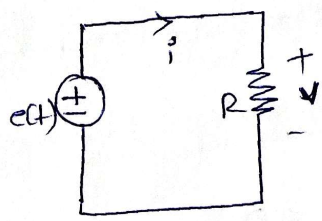
$$\theta(s)\left[\frac{\left[J_2 s^2+(B+B_{12})s+K\right]\left[J_1 s^2+B_{12}s+K\right]-\left[B_{12}s+K\right]^2}{B_{12}s+K}\right] = T(s)$$

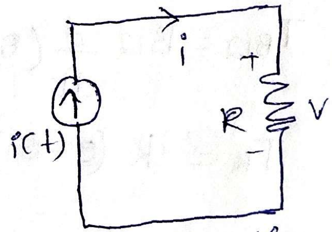
$$\boxed{\frac{\theta(s)}{T(s)} = \frac{B_{12}s+K}{\left[J_2 s^2+(B+B_{12})s+K\right]\left[J_1 s^2+B_{12}s+K\right]-\left[B_{12}s+K\right]^2}}$$

| Mechanical system | Electrical system | Electrical system. |
|---|---|---|
| i/p – force | i/p: vtg source | i/p: current source |
| o/p – voltage across the element | o/p: current element | o/p: voltage across the element. |

$x = \frac{dv}{dt}$



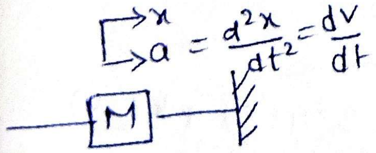



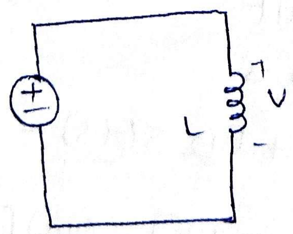



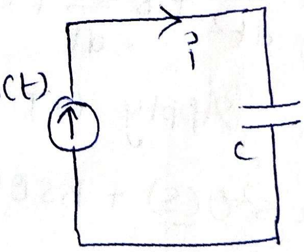

$f = B\frac{dv}{dt}$ $\qquad V = iR$ $\qquad i(t) = \frac{V}{R}$

$e(t) = iR$

$a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$



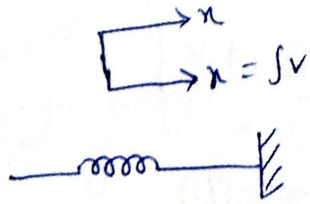



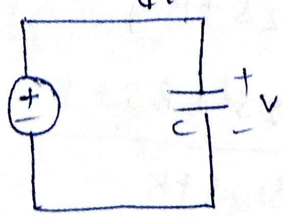



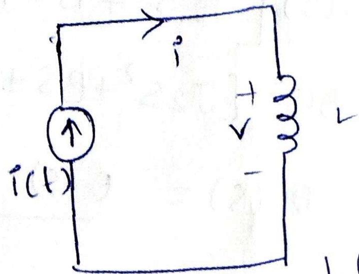

$i = C\frac{dv}{dt}$

$V = L\frac{di}{dt} = e(t)$

$x = \int v\,dt$




$f(t) = Kx$

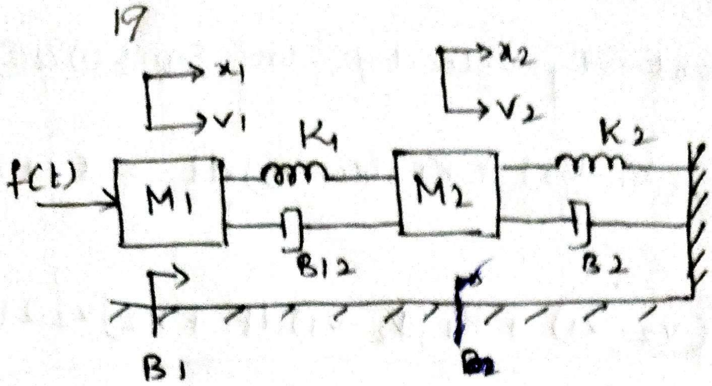
$= K\int v\,dt$

$v = \frac{1}{C}\int i\,dt = e(t)$ $\qquad i(t) = \frac{1}{L}\int v\,dt$

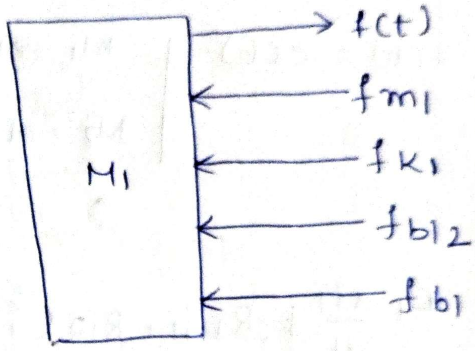
|← force vtg analogy →|

|← force current analogy →|

Ex ①.




free body diagram of Mass 'M1'.




$f_{m1} = M_1 \frac{d^2x_1}{dt^2}$.

$f_{K1} = K_1(x_1 - x_2)$

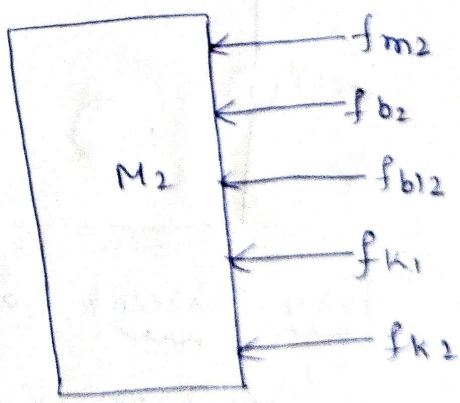
$f_{b12} = B_{12} \frac{d}{dt}(x_1 - x_2)$

$f_{b1} = B_1 \frac{dx_1}{dt}$

$f_{m1} + f_{b1} + f_{b12} + f_{K1} = f(t)$

$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_{12} \frac{d}{dt}(x_1 - x_2) + K_1(x_1 - x_2) = f(t).$ → ①.

free body diagram of Mass $M_2$




$f_{m2} = M_2 \frac{d^2x_2}{dt^2}$

$f_{b2} = B_2 \frac{dx_2}{dt}$

$f_{b12} = B_{12} \frac{d}{dt}(x_2 - x_1)$.

$f_{K1} = K_1(x_2 - x_1)$

$f_{K2} = K_2 x_2$.

$f_{m2} + f_{b2} + f_{b12} + f_{K1} + f_{K2} = 0$.

$M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B_{12} \frac{d}{dt}(x_2 - x_1) + K_1(x_2 - x_1) + K_2 x_2 = 0$ → ②

$\boxed{v = \frac{dx}{dt} \Rightarrow x = \int v\, dt}$

Replace displacement by velocity in Eqn's ① & ②

$$M_1 \frac{dV_1}{dt} + B_1 V_1 + B_{12}(V_1 - V_2) + K_1 \int (V_1 - V_2)\,dt = f(t)$$

$$M_2 \frac{dV_2}{dt} + B_2 V_2 + B_{12}(V_2 - V_1) + K_1 \int (V_2 - V_1)\,dt + K_2 \int V_2\,dt = 0$$

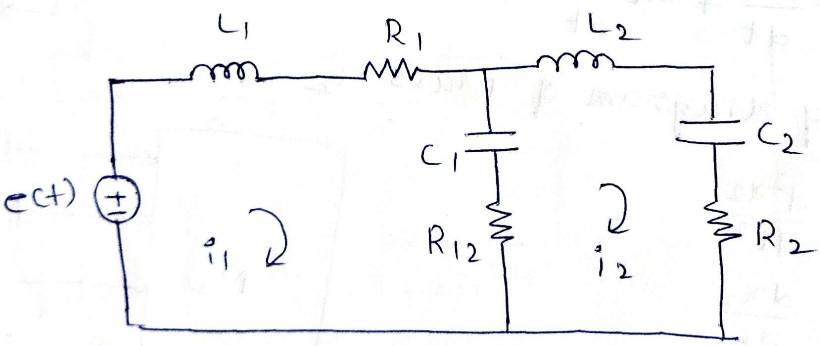
"force-voltage analogeous ckt"

| | | | | |
|---|---|---|---|---|
| $f(t) = e(t)$ | $M_1 \to L_1$ | $B_1 \to R_1$ | $k_1 \to 1/C_1$ | $V_1 \to i_1$ |
| | $M_2 \to L_2$ | $B_2 \to R_2$ | $k_2 \to 1/C_2$ | $V_2 \to i_2$ |
| | | $B_{12} \to R_{12}$ | | |

$$L_1 \frac{di_1}{dt} + R_1 i_1 + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\,dt = e(t) \rightarrow ③$$

$$L_2 \frac{di_2}{dt} + R_2 i_2 + R_{12}(i_2 - i_1) + \frac{1}{C_1}\int (i_2 - i_1)\,dt + \frac{1}{C_2}\int i_2\,dt = 0$$

$\hookrightarrow ④$

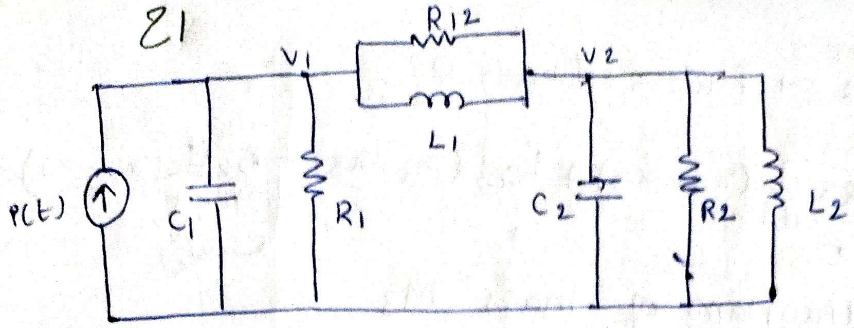



"force-current analogous ckt"

| | | | | |
|---|---|---|---|---|
| $i(t) = f(t)$ | $M_1 \to C_1$ | $B_1 \to 1/R_1$ | $K_1 \to 1/L_1$ | $V_1 \to V_1$ |
| | $M_2 \to C_2$ | $B_2 \to 1/R_2$ | $K_2 \to 1/L_2$ | $V_2 \to V_2$ |
| | | $B_{12} \to 1/R_{12}$ | | |

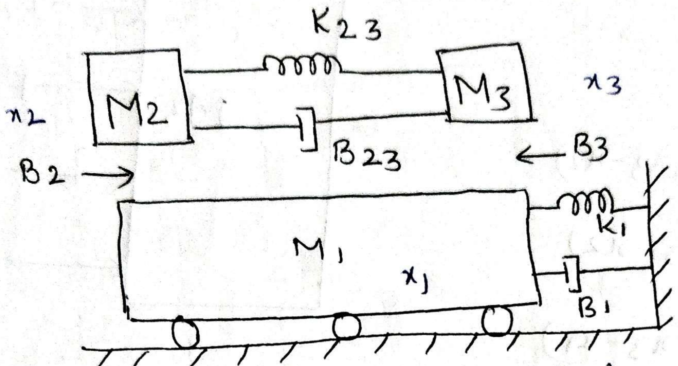
$$C_1 \frac{dV_1}{dt} + \frac{1}{R_1} V_1 + \frac{1}{R_{12}}(V_1 - V_2) + \frac{1}{L_1}\int (V_1 - V_2)\,dt = i(t)$$

$$C_2 \frac{dV_2}{dt} + \frac{1}{R_2} V_2 + \frac{1}{R_{12}}(V_2 - V_1) + \frac{1}{L_1}\int (V_2 - V_1)\,dt + \frac{1}{L_2}\int V_2\,dt = 0$$

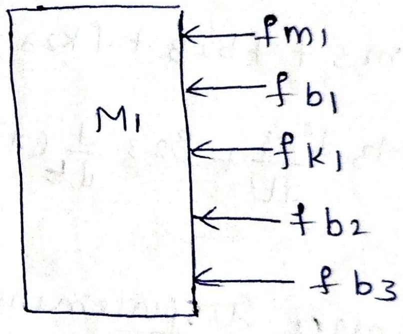



Ex

1.35




Sol. free body diagram of Mass 'M1'




$$f_{m1} = M \frac{d^2 x_1}{dt^2}$$

$$f_{b1} = B_1 \frac{dx}{dt}$$

$$f_{k1} = K_1 x_1$$

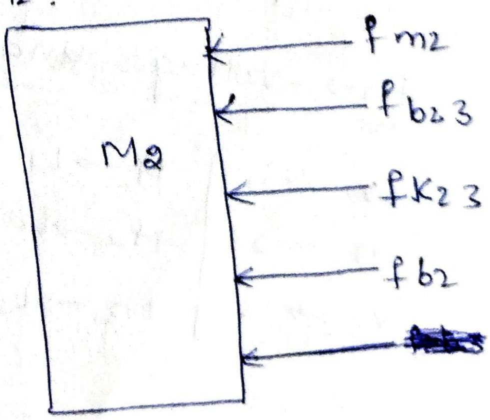
$$f_{b2} = B_2 \frac{d}{dt}(x_1 - x_2)$$

$$f_{b3} = B_3 \frac{d}{dt}(x_1 - x_3)$$

$$f_{m1} + f_{b1} + f_{K1} + f_{b2} + f_{b3} = 0.$$

$$M \frac{d^2 x_1}{dt^2} + B_1 \frac{dx}{dt} + K_1 x_1 + B_2 \frac{d}{dt}(x_1 - x_2) + B_3 \frac{d}{dt}(x_1 - x_3) = 0 \quad \text{(1)}$$

free Body diagram of Mass M2.




$$f_{m2} = M_2 \frac{d^2 x_2}{dt^2}$$

$$f_{b23} = B_{23} \frac{d}{dt}(x_2 - x_3)$$

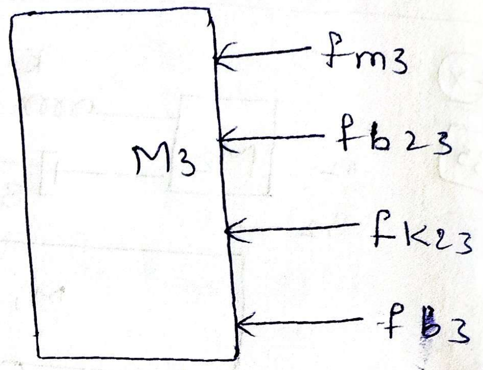
$$f_{K23} = K_{23}(x_2 - x_3)$$

$$f_{b2} = B_2 \frac{d}{dt}(x_2 - x_1).$$

$$f_{m2} + f_{b23} + f_{k23} + f_{b2} = 0$$

$$M_2 \frac{d^2 x_2}{dt^2} + B_{23} \frac{d}{dt}(x_2 - x_3) + K_{23}(x_2 - x_3) + B_2 \frac{d}{dt}(x_2 - x_1) = 0 \quad \rightarrow (2)$$

Free body diagram of mass $M_3$




$$f_{m3} = M_3 \frac{d^2 x_3}{dt^2}$$

$$f_{b23} = B_{23} \frac{d}{dx}(x_3 - x_2)$$

$$f_{k23} = K_{23}(x_3 - x_2)$$

$$f_{b3} = B_3 \frac{d}{dx}(x_3 - x_1)$$

$$f_{m3} + f_{b23} + f_{k23} + f_{b3} = 0.$$

$$M_3 \frac{d^2 x_3}{dt^2} + B_{23} \frac{d}{dt}(x_3 - x_2) + K_{23}(x_3 - x_2) + B_3 \frac{d}{dt}(x_3 - x_1) = 0 \quad \rightarrow (3).$$

Replace displacement by velocity in (1) (2) & (3)

$$M_1 \frac{dv_1}{dt} + B_1 v_1 + K_1 \int v_1 + B_2 (v_1 - v_2) + B_3 (v_1 - v_3) = 0$$

$$M_2 \frac{dv_2}{dt} + B_{23}(v_2 - v_3) + K_{23} \int (v_2 - v_3)\,dt + B_2 (v_2 - v_1) = 0$$

$$M_3 \frac{dv_3}{dt} + B_{23}(v_3 - v_2) + K_{23} \int (v_3 - v_2)\,dt + B_3 (v_3 - v_1) = 0.$$

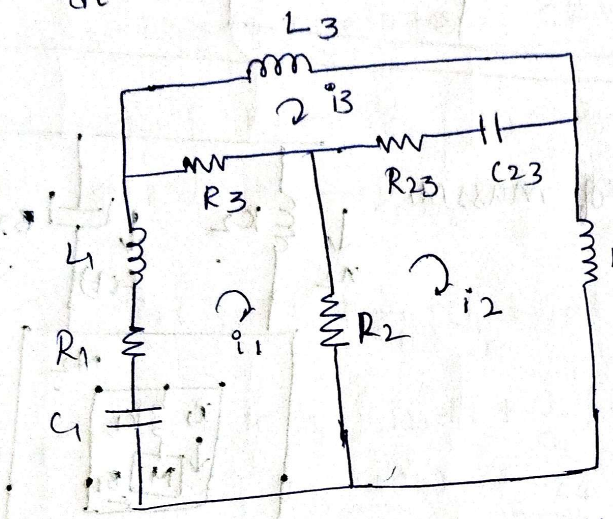
Force-voltage analogous ckt

| | | | |
|---|---|---|---|
| $v_1 \rightarrow i_1$ | $M_1 \rightarrow L_1$ | $B_1 \rightarrow R_1$ | $K_1 \rightarrow 1/C_1$ |
| $v_2 \rightarrow i_2$ | $M_2 \rightarrow L_2$ | $B_2 \rightarrow R_2$ | $K_{23} \rightarrow 1/C_{23}$. |
| $v_3 \rightarrow i_3$ | $M_3 \rightarrow L_3$ | $B_3 \rightarrow R_3$ | |
| | | $B_4 \rightarrow R_4$ | |

$$L_1 \frac{d^2 i_1}{dt^2} + R_1 i_1 + \frac{1}{C_1}\int i_1 dt + R_2(i_1 - i_2) + R_3(i_1 - i_3) = 0$$

$$L_2 \frac{d^2 i_2}{dt^2} + R_{23}(i_2 - i_3) + \frac{1}{C_{23}}\int (i_2 - i_3)dt + R_2(i_2 - i_1) = 0.$$

$$L_3 \frac{di_3}{dt} + R_{23}(i_3 - i_2) + \frac{1}{C_{23}}\int (i_3 - i_2)dt + R_3(i_3 - i_1) = 0.$$




force - current analogous ckt

| | | | |
|---|---|---|---|
| $V_1 \to V_1$ | $M_1 \to C_1$ | $K_1 \to 1/L_1$ | $B_1 \to 1/R_1$ |
| $V_2 \to V_2$ | $M_2 \to C_2$ | $K_{23} \to 1/L_{23}$ | $B_2 \to 1/R_2$ |
| $V_3 \to V_3$ | $M_3 \to C_3$ | | $B_3 \to 1/R_3$ |
| | | | $B_{23} \to 1/R_{23}$ |

$$M_1 \frac{dV_1}{dt} + B_1 V_1 + K_1\int V_1 dt + B_2(V_1 - V_2) + B_3(V_1 - V_3) = 0$$

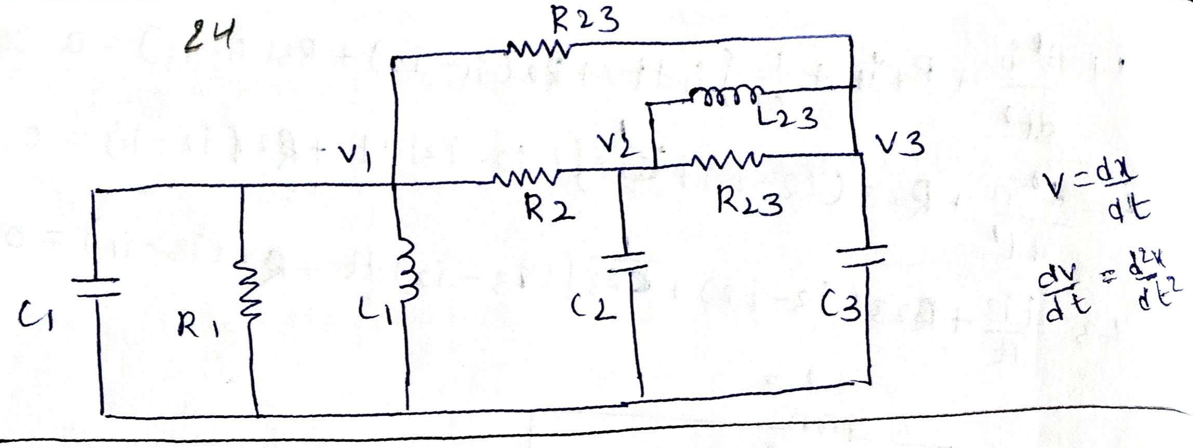
$$M_2 \frac{dV_2}{dt} + B_{23}(V_2 - V_3) + K_{23}\int (V_2 - V_3)dt + B_2(V_2 - V_1) = 0$$

$$M_3 \frac{dV_3}{dt} + B_{23}(V_3 - V_2) + K_{23}\int (V_3 - V_2)dt + B_3(V_3 - V_1) = 0$$

$$C_1 \frac{dV_1}{dt} + \frac{1}{R_1}V_1 + \frac{1}{L_1}\int V_1 dt + \frac{1}{R_2}(V_1 - V_2) + \frac{1}{R_3}(V_1 - V_3) = 0$$

$$C_2 \frac{dV_2}{dt} + \frac{1}{R_{23}}(V_2 - V_3) + \frac{1}{L_{23}}\int (V_2 - V_3)dt + \frac{1}{R_2}(V_2 - V_1) = 0$$

$$C_3 \frac{dV_3}{dt} + \frac{1}{R_{23}}(V_3 - V_2) + \frac{1}{L_{23}}\int (V_3 - V_2)dt + \frac{1}{R_3}(V_3 - V_1) = 0.$$

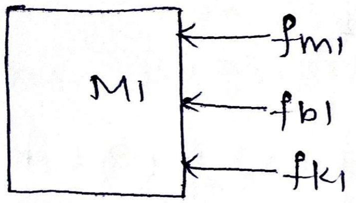


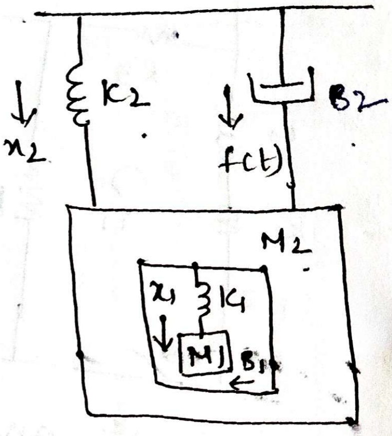

$V = \frac{dx}{dt}$

$\frac{dv}{dt} = \frac{d^2x}{dt^2}$

Ex.

free body diagram of mass $M_1$




$f_{m_1} = M_1 \frac{d^2x_1}{dt^2}$

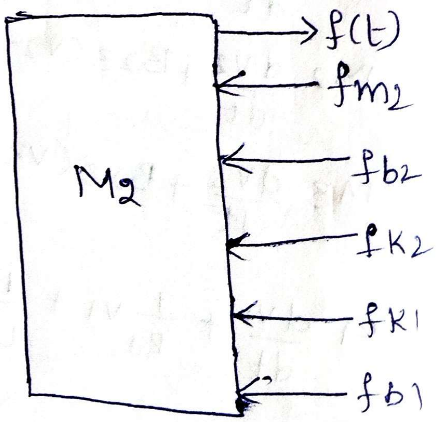
$f_{b_1} = B_1 \frac{d}{dt}(x_1 - x_2)$

$f_{K_1} = K_1(x_1 - x_2)$

$f_{m_1} + f_{b_1} + f_{k_1} = 0.$

$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{d}{dt}(x_1 - x_2) + K_1(x_1 - x_2) = 0 \longrightarrow (1)$

Free body diagram of Mass $M_2$




$f_{m_2} = M_2 \frac{d^2x_2}{dt^2}$ ; $f_{b_2} = B_2 \frac{dx_2}{dt}$

$f_{K_2} = K_2 x_2$ ; $f_{K_1} = K_1(x_2 - x_1)$

$f_{b_1} = B_1 \frac{d}{dt}(x_2 - x_1)$

$f_{m_2} + f_{b_2} + f_{K_2} + f_{K_1} + f_{b_1} = f(t).$

$M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + K_2 x_2 + K_1(x_2 - x_1) + B_1 \frac{d}{dt}(x_1 - x_2)$

$= f(t) \longrightarrow (2)$

Replace displacement by velocity.

$$M_1 \frac{dv_1}{dt} + B_1 (v_1 - v_2) + K_1 \int (v_1 - v_2)\, dt = 0.$$

$$M_2 \frac{dv_2}{dt} + B_2 v_2 + K_2 \int v_2 \, dt + K_1 \int (v_2 - v_1)\, dt + B_1 (v_1 - v_2) = f(t)$$

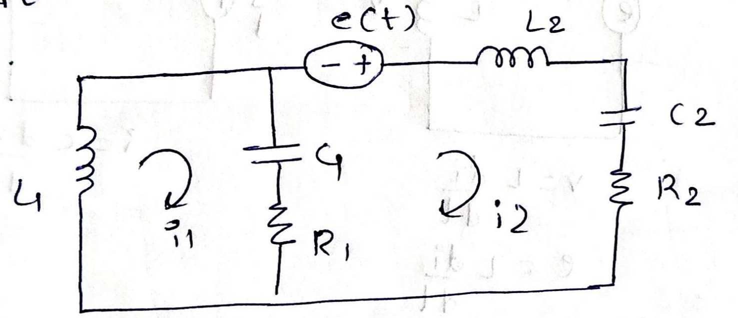
**Force voltage analogous ckt**

$M_1 \to L_1$ | $K_1 \to 1/C_1$ | $B_1 \to R_1$ | $f(t) = e(t)$ | $v_1 \to i_1$

$M_2 \to L_2$ | $K_2 \to 1/C_2$ | $B_2 \to R_2$ | | $v_2 \to i_2$

$$L_1 \frac{di_1}{dt} + R_1 (i_1 - i_2) + \frac{1}{C_1} \int (i_1 - i_2)\, dt = 0$$

$$L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2} \int i_2 \, dt + \frac{1}{C_1} \int (i_2 - i_1)\, dt + R_1 (i_1 - i_2) = e(t)$$



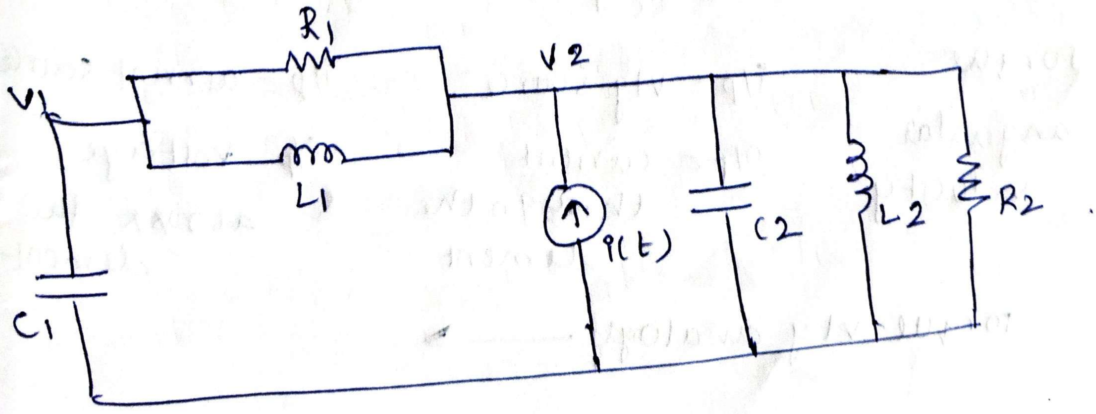

**Force current analogy ckt**

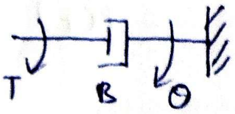
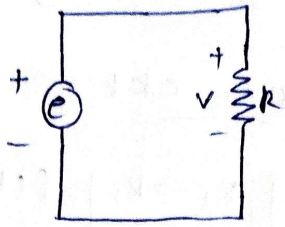
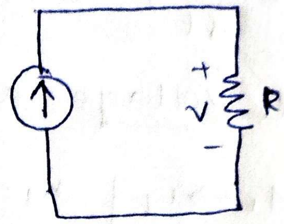
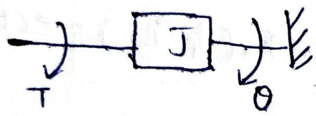
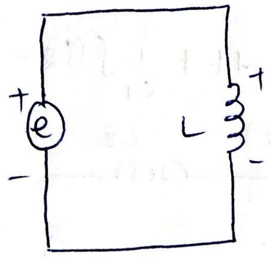
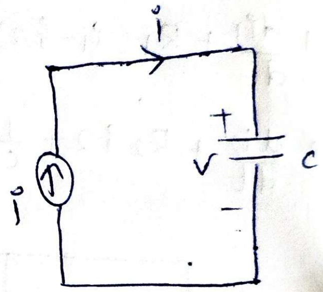
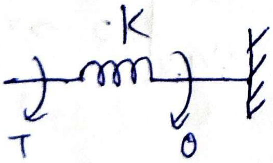
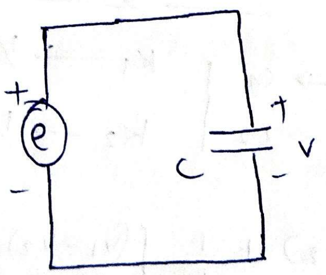
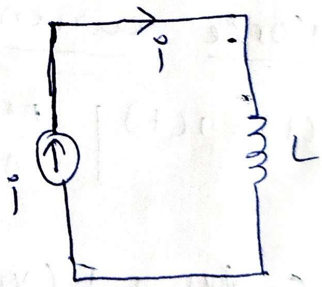
$f(t) = i(t)$ | $M_1 \to C_1$ | $K_1 \to 1/L_1$ | $B_1 \to 1/R_1$ | $v_1 \to v_1$

| $M_2 \to C_2$ | $K_2 \to 1/L_2$ | $B_2 \to 1/R_2$ | $v_2 \to v_2$.

$$C_1 \frac{dv_1}{dt} + \frac{1}{R_1} (v_1 - v_2) + \frac{1}{L_1} \int (v_1 - v_2)\, dt = 0.$$

$$C_2 \frac{dv_2}{dt} + \frac{1}{R_2} v_2 + \frac{1}{L_2} \int v_2 \, dt + \frac{1}{L_1} \int (v_2 - v_1)\, dt + \frac{1}{R_1} (v_1 - v_2) = i(t)$$

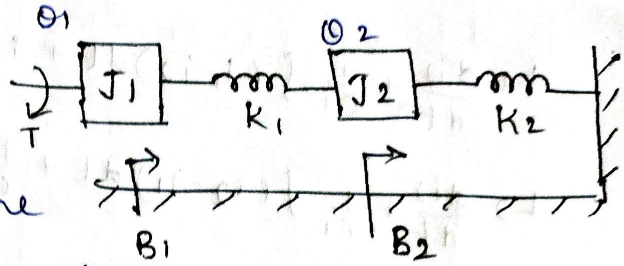


| Mechanical Rotational system | Electrical system | Electrical system. |
|---|---|---|
|  |  |  |
| $T = B\frac{d\theta}{dt} = B\omega$ | $v = iR$<br>$e = iR$ | $i = v/R$ |
|  |  |  |
| $T = J\frac{d^2\theta}{dt^2}$<br>$= J\frac{d\omega}{dt}$ | $v = L\frac{di}{dt}$<br>$e = L\frac{di}{dt}$ | $i = C\frac{dv}{dt}$ |
|  |  |  |
| $T = K\theta$<br>$= K\int \omega dt$ | $v = \frac{1}{C}\int i dt$<br>$e = \frac{1}{C}\int i dt$ | $i = \frac{1}{L}\int v dt$ |
| i/p = Torque<br>o/p = angualar velocity | i/p = vtg source<br>o/p = current through the element | i/p = current sou...<br>o/p = voltage across the elem... |

← Torque vtg analogy →

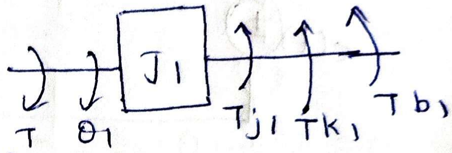
← Torque current analogy →

Ex




Let angular velocity $J_1$ & $J_2$ be $\theta_1$ & $\theta_2$. The corresponding angular velocities be $\omega_1$ & $\omega_2$.

Free body diagram of $J_1$




$$T_{J1} = J_1 \frac{d^2\theta_1}{dt^2}$$

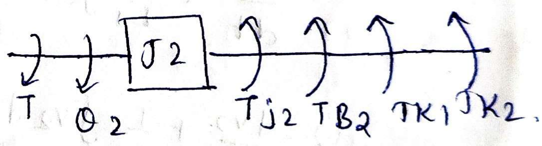
$$T_{b1} = B_1 \frac{d\theta_1}{dt} \; ; \; T_{K1} = K_1(\theta_1 - \theta_2)$$

$$T_{j1} + T_{b1} + T_{K1} = T$$

$$J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2) = T \rightarrow ①$$

$$\omega = \frac{d\theta}{dt}$$
$$\theta = \int \omega\, dt$$

Free body diagram of $J_2$




$$T_{J2} = J_2 \frac{d^2\theta_2}{dt^2} \; ;$$

$$T_{b2} = B_2 \frac{d\theta_2}{dt} \; ;$$

$$T_{K1} = K_1(\theta_2 - \theta_1) \; ; \; T_{K2} = K_2\theta_2$$

$$T_{j2} + T_{b2} + T_{K1} + T_{K2} = 0$$

$$J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + K_1(\theta_2 - \theta_1) + K_2\theta_2 = 0 \rightarrow ②$$

Replace angular displacement by angular velocity.

$$J_1 \frac{d\omega_1}{dt} + B_1\omega_1 + K_1 \int(\omega_1 - \omega_2)\,dt = T$$

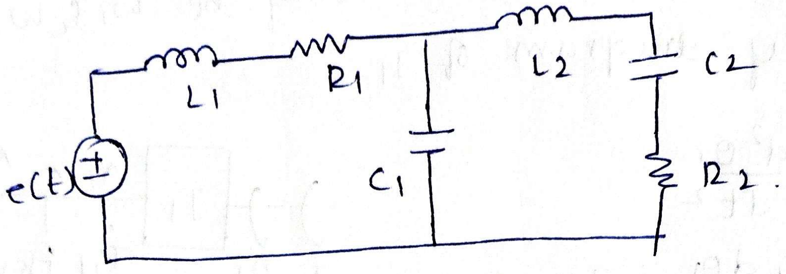
$$J_2 \frac{d\omega_2}{dt} + B_2 d\omega_2 + K_1 \int(\omega_2 - \omega_1)\,dt + K_2 \int \omega_2\, dt = 0$$

Torque-voltage analogy.

| | | | |
|---|---|---|---|
| $\omega_1 \rightarrow i_1$ | $J_1 \rightarrow L_1$ | $K_1 \rightarrow 1/C_1$ | $B_1 \rightarrow R_1$ |
| $\omega_2 \rightarrow i_2$ | $J_2 \rightarrow L_2$ | $K_2 \rightarrow 1/C_2$ | $B_2 \rightarrow R_2$ |

$$L_1\frac{di_1}{dt}+\frac{1}{C_1}\int(i_1-i_2)dt+i_1R_1=e(t)$$

$$L_2\frac{di_2}{dt}+\frac{1}{C_1}\int(i_2-i_1)dt+\frac{1}{C_2}\int i_2dt+R_2i_2=0$$

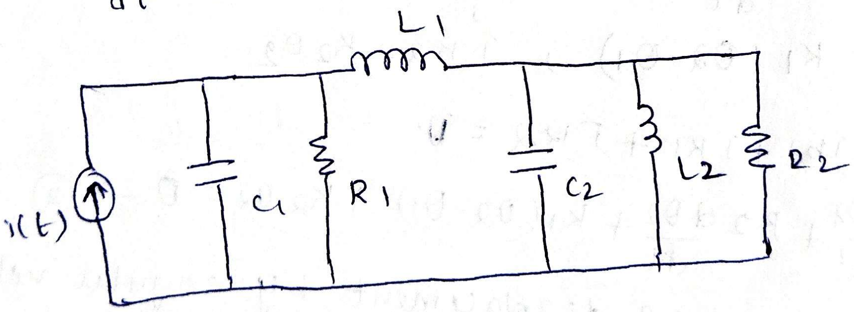



Torque current analogy

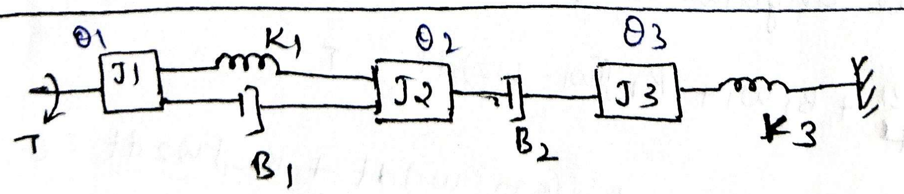
| | | | |
|---|---|---|---|
| $\omega_1 \rightarrow v_1$ | $J_1 \rightarrow C_1$ | $B_1 \rightarrow 1/R_1$ | $K_1 \rightarrow 1/L_1$ |
| $\omega_2 \rightarrow v_2$ | $J_2 \rightarrow C_2$ | $B_2 \rightarrow 1/R_2$ | $K_2 \rightarrow 1/L_2$ |

$$C_1\frac{dv_1}{dt}+\frac{1}{L_1}\int(v_2-v_1)dt+\frac{1}{R_1}v_1=i(t)$$

$$C_2\frac{dv_2}{dt}+\frac{1}{L_2}\int v_2dt+\frac{1}{L_2}\int(v_1-v_2)dt+\frac{1}{R_2}v_2=0.$$

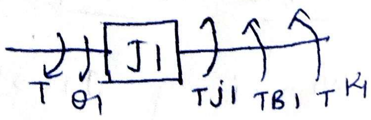



Ex




free body diagram of $J_1$

$$T_{J_1}=J_1\frac{d^2\theta_1}{dt^2}$$




$$T_{B_1}=B_1\frac{d}{dt}(\theta_1-\theta_2)$$

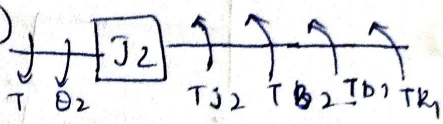
$$T_{K_1}=K_1(\theta_1-\theta_2)$$

$T_{J1} + T_{b1} + T_{K1} = T$

$$J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d}{dt}(\theta_1 - \theta_2) + K_1(\theta_1 - \theta_2) = T \rightarrow (1)$$

free body diagram of $J_2$

$T_{j2} = J_2 \frac{d^2\theta_2}{dt^2}$ ; $T_{b2} = B_2 \frac{d}{dt}(\theta_2 - \theta_3)$



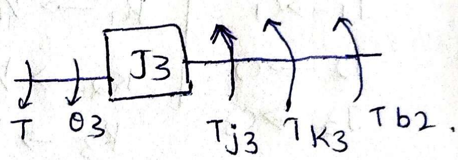

$T_{b1} = B_1 \frac{d}{dt}(\theta_2 - \theta_1)$ ; $T_{K1} = K_1(\theta_2 - \theta_1)$.

$T_{j2} + T_{b2} + T_{b1} + T_{K1} = 0$

$$J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d}{dt}(\theta_2 - \theta_3) + B_1 \frac{d}{dt}(\theta_2 - \theta_1) + K_1(\theta_2 - \theta_1) = 0 \rightarrow (2)$$

free body diagram of $J_3$

$T_{j3} = J_3 \frac{d^2\theta_3}{dt^2}$




$T_{K3} = K_3(\theta_3)$

$T_{b2} = B_2(\theta_3 - \theta_2)$.

$T_{j3} + T_{K3} + T_{b2} = 0$.

$$\left[ \omega = \frac{d\theta}{dt} \quad \theta = \int \omega \, dt \right]$$

$$J_3 \frac{d^2\theta_3}{dt^2} + K_3\theta_3 + B_2 \frac{d}{dt}(\theta_3 - \theta_2) = 0. \rightarrow (3).$$

replace angular displacement angular velocity.

$$J_1 \frac{d\omega_1}{dt} + B_1(\omega_1 - \omega_2) + K_1 \int (\omega_1 - \omega_2)\, dt = T.$$

$$J_2 \frac{d\omega_2}{dt} + B_2(\omega_2 - \omega_3) + B_1(\omega_2 - \omega_1) + K_1 \int (\omega_2 - \omega_1)\, dt = 0.$$

$$J_3 \frac{d\omega_3}{dt} + K_3 \int \omega_3 \, dt + B_2(\omega_3 - \omega_2) = 0$$

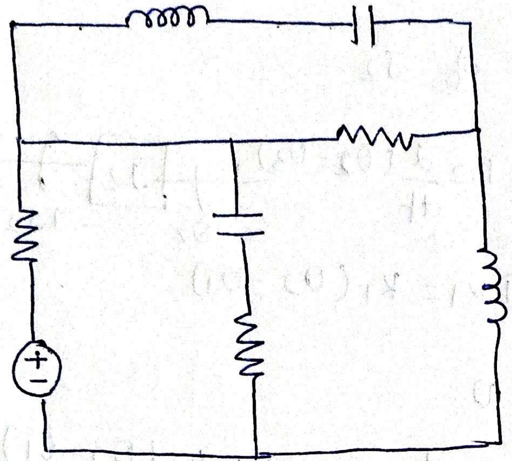
Torque voltage analogy.

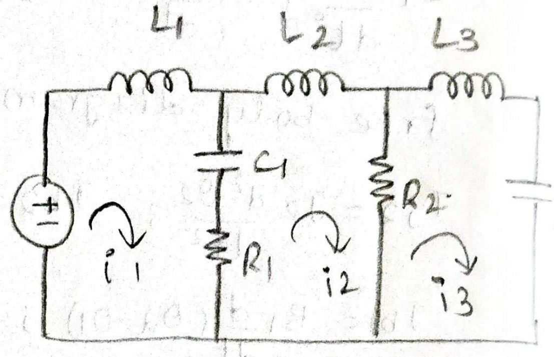
| | | | |
|---|---|---|---|
| $\omega_1 \rightarrow i_1$ | $J_1 \rightarrow L_1$ | $K_1 \rightarrow 1/C_1$ | $B_1 \rightarrow R_1$ |
| $\omega_2 \rightarrow i_2$ | $J_2 \rightarrow L_2$ | $K_2 \rightarrow 1/C_2$ | $B_2 \rightarrow R_2$. |
| $\omega_3 \rightarrow i_3$ | | $K_3 \rightarrow 1/C_3$ | |

$$J_1 \frac{di_1}{dt} + R_2(i_1 - i_2) + \frac{1}{C_1} \int (i_1 - i_2)\, dt = e(t)$$

$$J_2 \frac{di_2}{dt} + R_2(i_2 - i_3) + R_1(i_2 - i_1) + \frac{1}{C_1} \int (i_2 - i_1)\, dt = 0.$$

$$J_3 \frac{di_3}{dt} + \frac{1}{C_3}\int i_3\,dt + R_2(i_3 - i_2) = 0$$





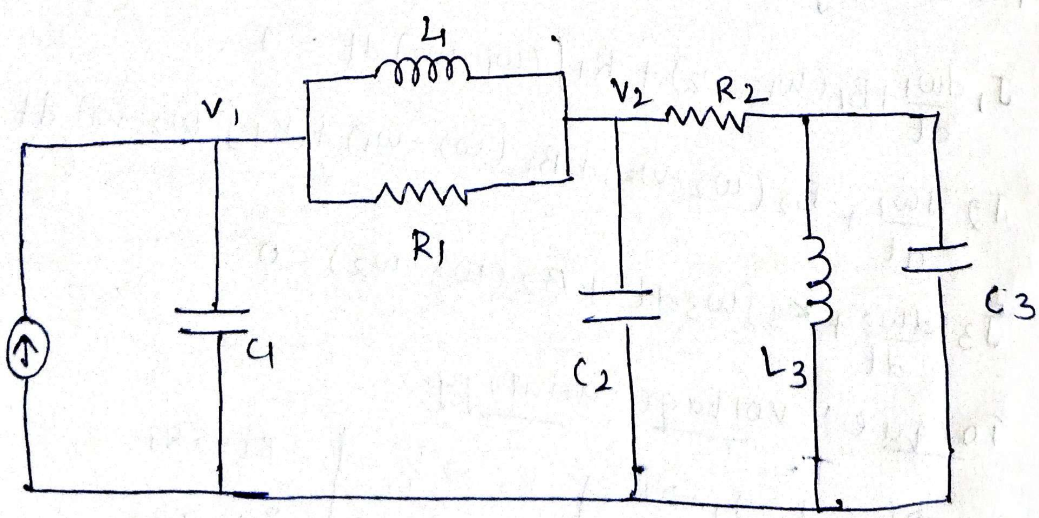

Torque current analogy.

| | | | | |
|---|---|---|---|---|
| $J_1 \to C_1$ | $K_1 \to 1/L_1$ | $B_1 \to 1/R_1$ | $T = i(t)$ | $\omega_1 \to v_1$ |
| $J_2 \to C_2$ | $K_2 \to 1/L_2$ | $B_2 \to 1/R_2$ | | $\omega_2 \to v_2$ |
| $J_3 \to C_3$ | $K_3 \to 1/L_3$ | | | |

$$C_1 \frac{dv_1}{dt} + \frac{1}{L_1}\int (v_1 - v_2)\,dt + \frac{1}{R_1}(v_1 - v_2) = i(t)$$

$$C_2 \frac{dv_2}{dt} + \frac{1}{L_2}\int (v_2 - v_1)\,dt + \frac{1}{R_1}(v_2 - v_1) + \frac{1}{R_2}(v_3 - v_2) = 0$$

$$C_3 \frac{dv_3}{dt} + \frac{1}{L_3}\int v_3\,dt + \frac{1}{R_2}(v_3 - v_2) = 0$$

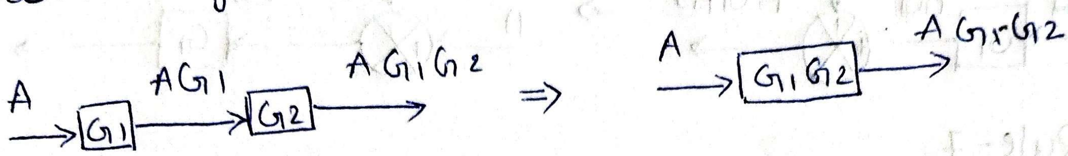


# Block diagram Reduction

## Rules of Block diagram Algebra.

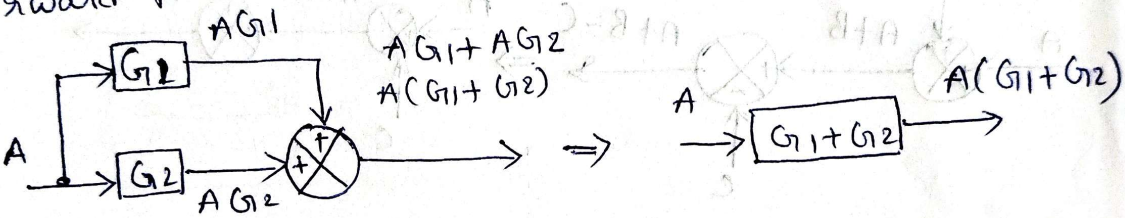
### Rule-1.

Combining the blocks in cascade



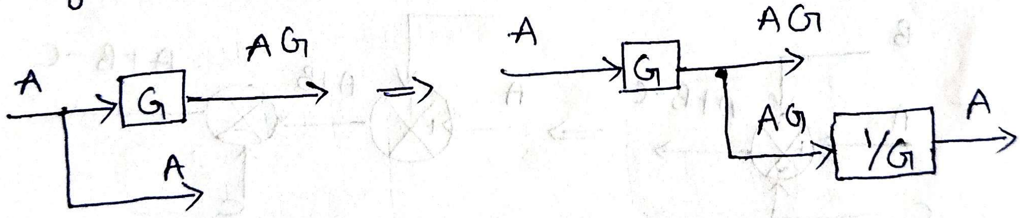

### Rule-2.

Combining ||el blocks (or) combining feed forward paths.



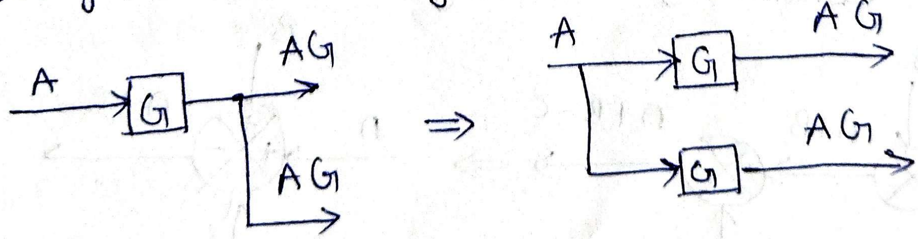

### Rule-3.

Moving the Branch point ahead of the block.



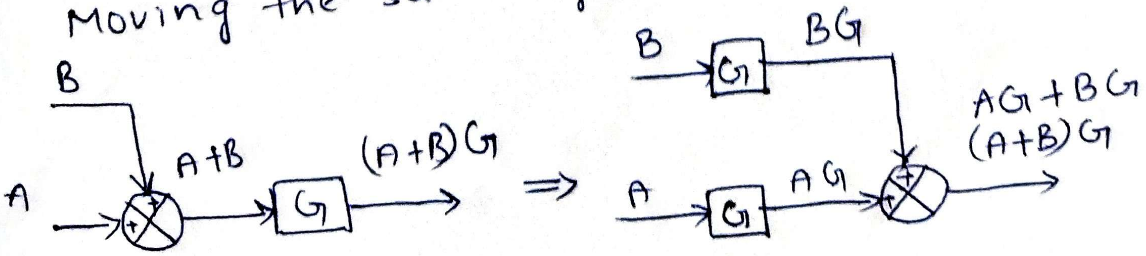

### Rule-4.

Moving the Summing point before the block.




### Rule-5

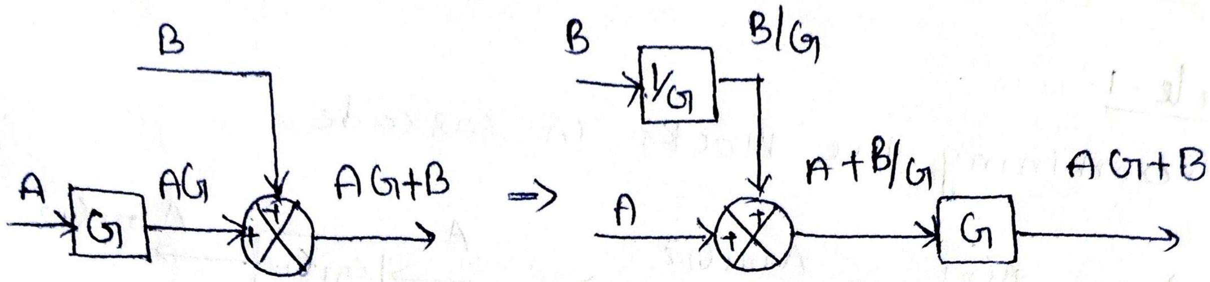
Moving the summing point ahead of the block.




Scanned with ACE Scanner

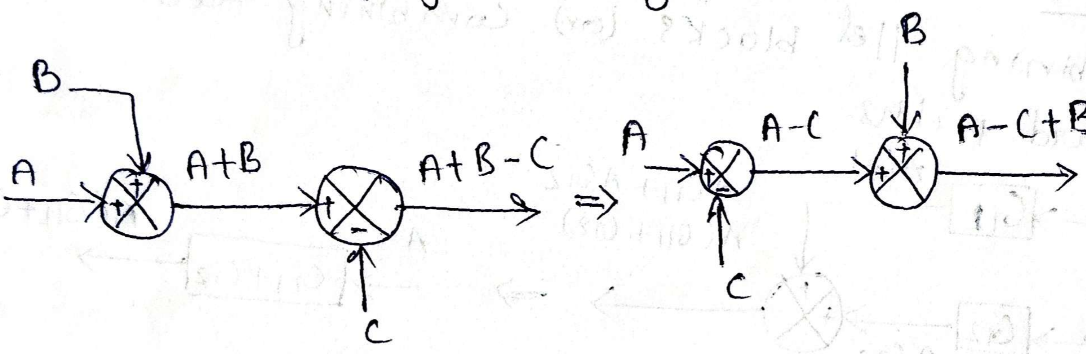
## Rule-6

Moving the summing point before the block.



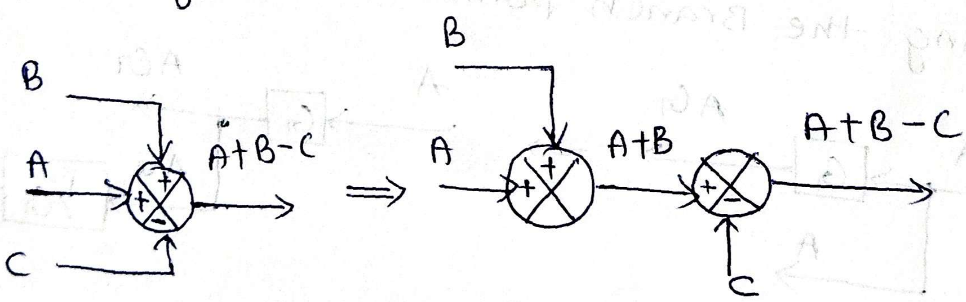

## Rule-7

Interchanging summing point



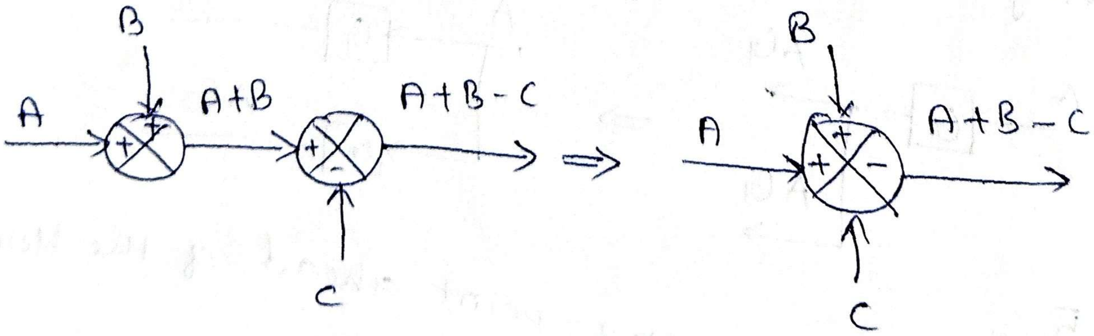

## Rule-8

Splitting summing points

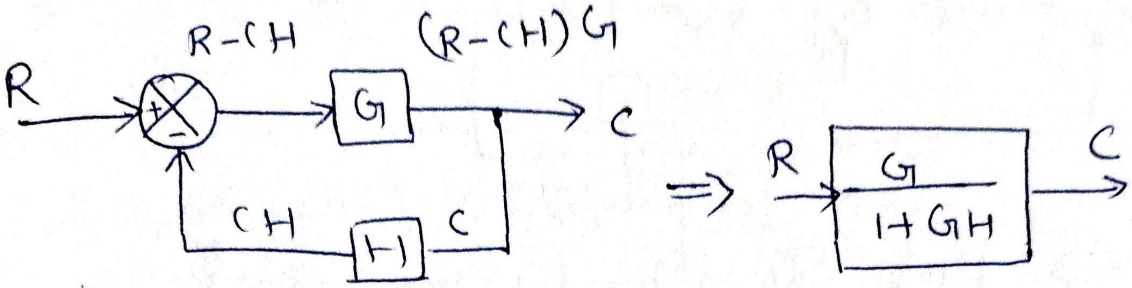



## Rule-9

Combining summing points



**Rule-10** Elimination of (−ve) feedback loop.




**proof.**

$$C=(R-CH)G$$

$$C=RG-CHG$$

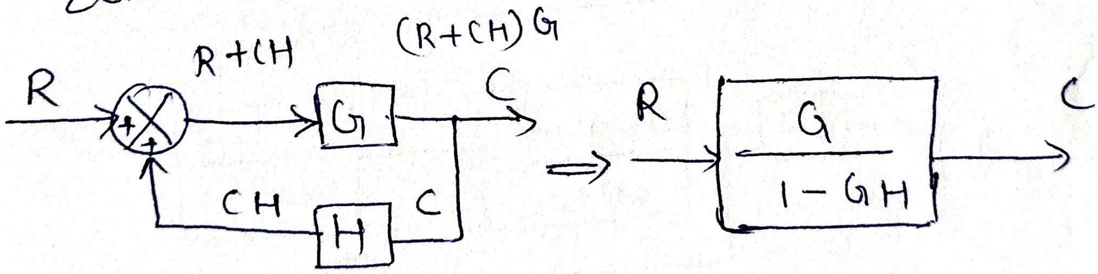
$$C+CHG=RG$$

$$C(1+HG)=RG$$

$$\boxed{\frac{C}{R}=\frac{G}{1+HG}}$$

**Rule-11**

Eliminate of (+ve) feed back loop.



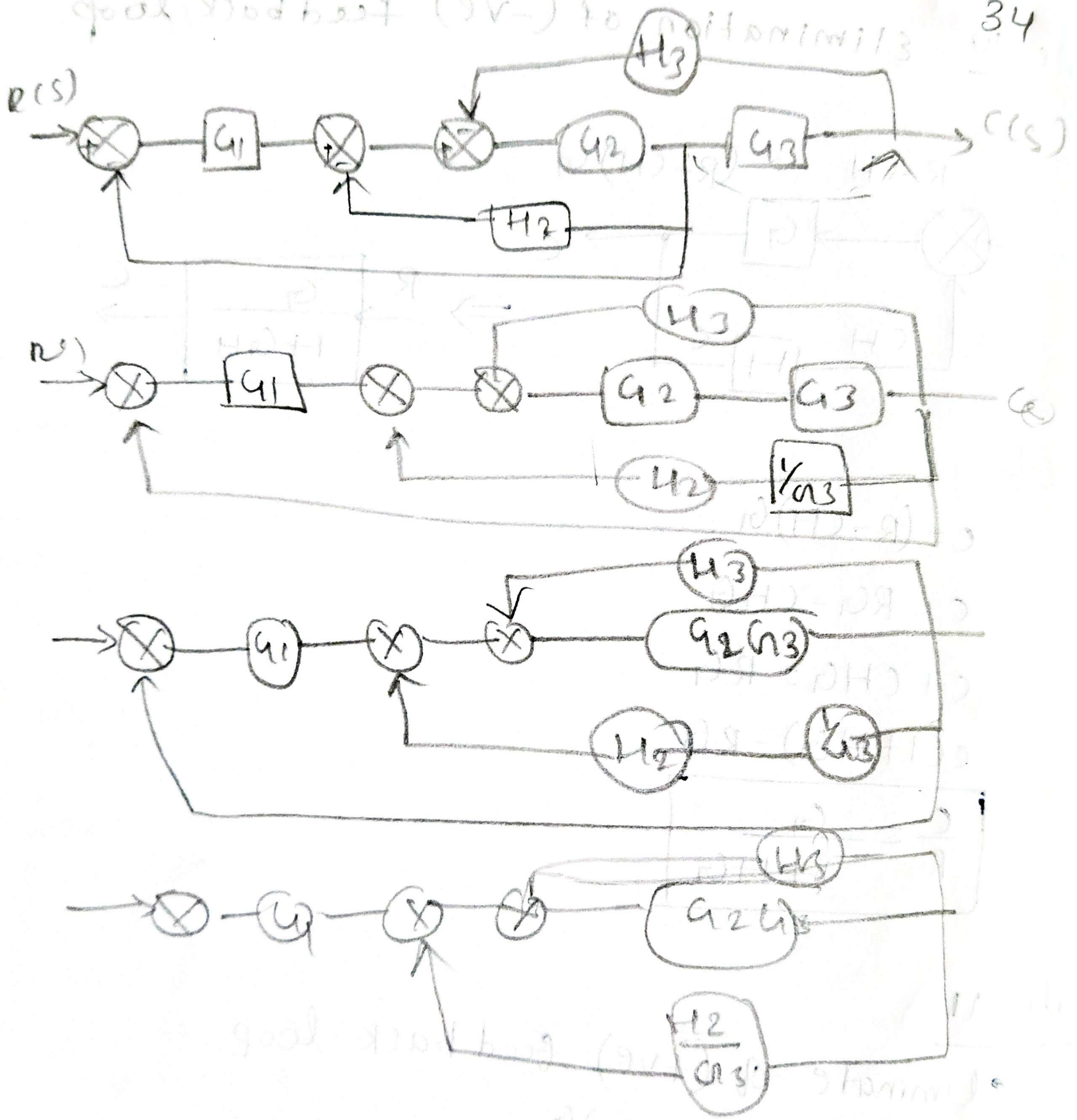

$$C=(R+CH)G$$

$$C=RG+CHG$$

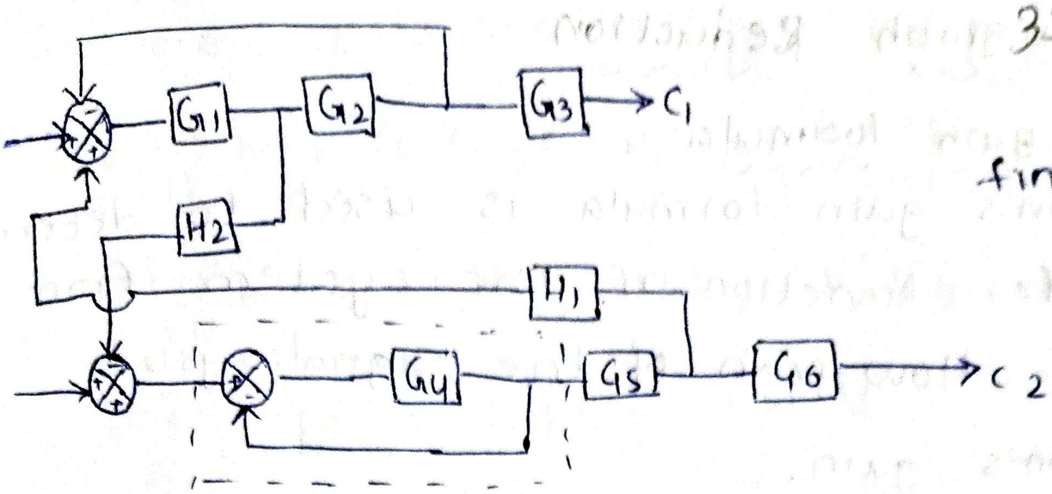
$$C-CHG=RG$$

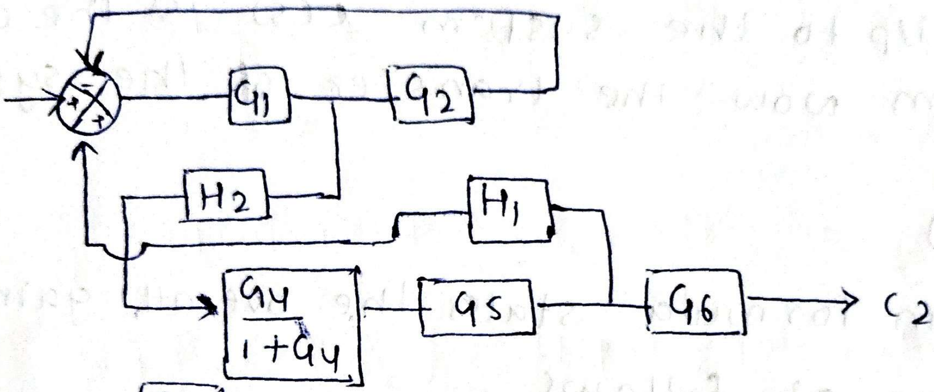
$$C(1-HG)=RG$$

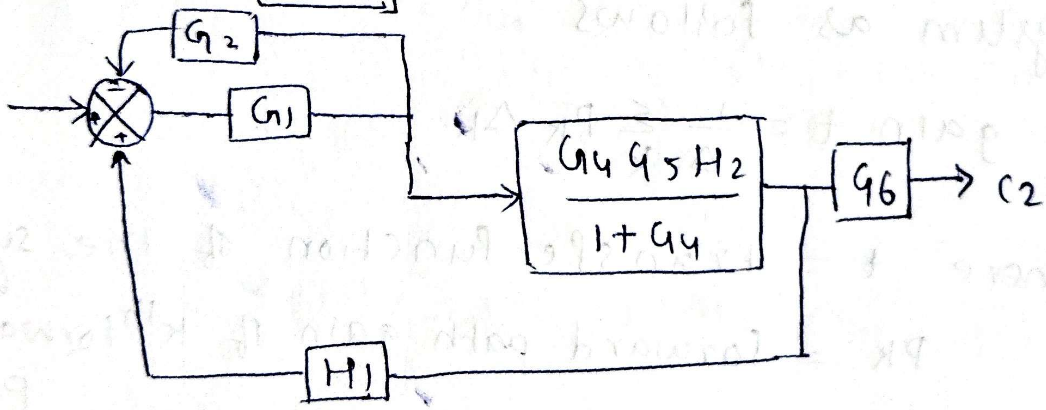
$$\boxed{\frac{C}{R}=\frac{G}{1-HG}}$$

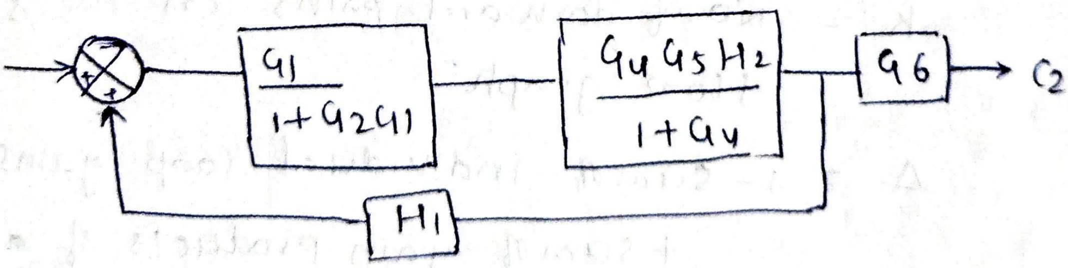

R(s)
G1
G2
G3
H3
H2
C(s)
G1
G2
G3
H3
H2
1/G3
G1
G2G3
H3
H2
1/G3
G1
G2G3
H3
H2/G3

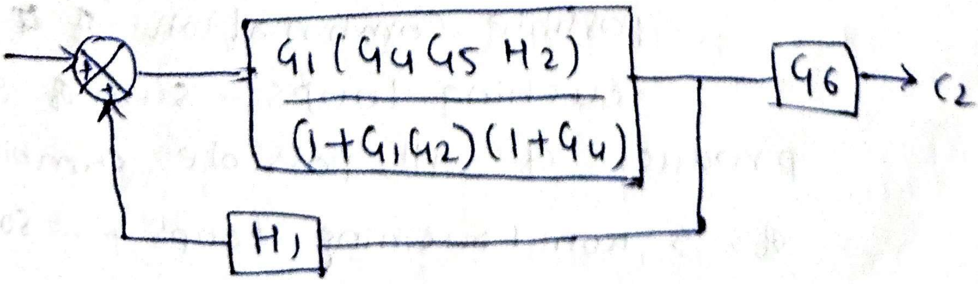
find $\frac{C_2}{R_1}$




$G_1$

# Signal flow graph Reduction

## Mason's gain formula

The mason's gain formula is used to determin the transfer function of the system from the signal flow grah of the ~~Signal~~ system

~~The mason's gain~~.

Let $r(s)$ is i/p to the system $c(s)$ is the o/p of the system. Now the transfer of the system

$$t(s) = \frac{c(s)}{r(s)}$$

Mason's gain formula states the overall gain of the system as follows

$$\text{overall gain } t = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$$

where $t$ = transfer function of the system

$P_K$ = forward path gain of $K^{th}$ forward path

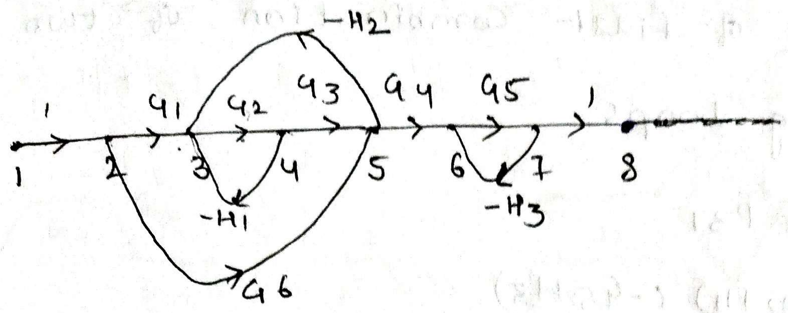
$K$ = No. of forward paths in the signal flow graph

$\Delta$ = 1 - sum of individual loop gains + sum of gain products of all possible combinations of 2 non touching loops - sum of gain products of all possible combinations of 3 non touching loops + ... so on...

$\Delta K$ = $\Delta$ for that part of the graph which is non touching $K^{th}$ forward path.

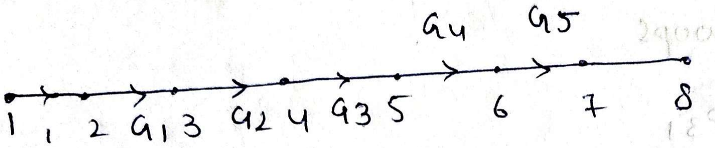
37

a)

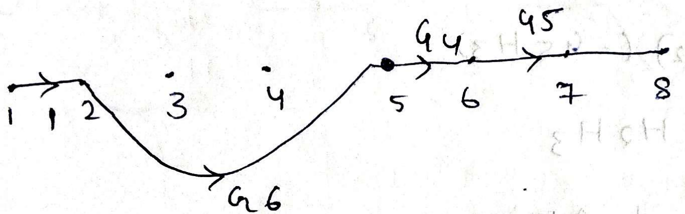



**Forward path gain:-**

There are two forward paths $\therefore K=2$



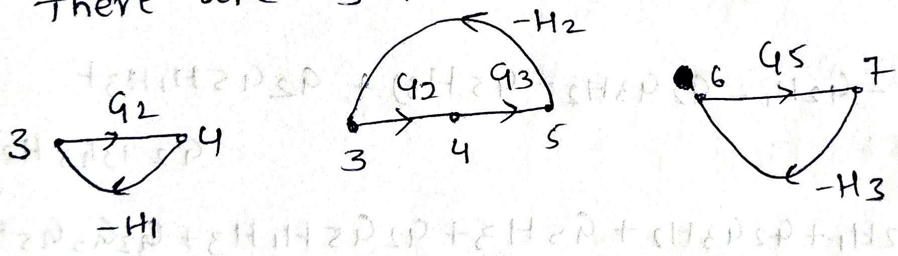

Gain of forward path 1
$P_1 = G_1G_2G_3G_4G_5$




Gain of forward path 2
$P_2 = G_6G_4G_5$

**Individual loop gains**

There are '3' individual loops




Loop gain of individual loop-1 $P_{11} = -G_2H_1$

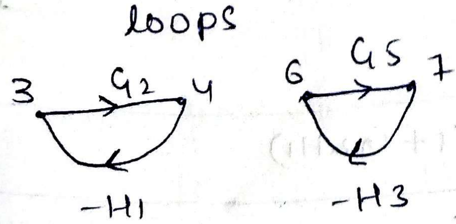
Loop gain of individual loop-2 $P_{21} = -G_2G_3H_2$

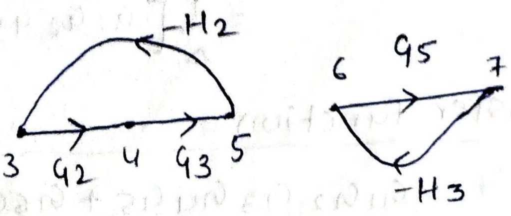
Loop gain of individual loop-3 $P_{31} = -G_5H_3$

Loop gain of

**Gain product of two non-touching loops**

There are two combinations of two non-touching loops



Gain product of first combination of two non-touching loops.

$$P_{12} = P_{11}\, P_{31}$$
$$= (-G_2H_1)(-G_5H_3)$$
$$= G_2G_5H_1H_3$$

Gain product of second combination of two non-touching loops

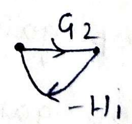
$$P_{22} = P_{21} \cdot P_{31}$$
$$= (-G_2G_3H_2)(-G_5H_3)$$
$$= G_2G_3G_5H_2H_3$$

Calculation of $\Delta$ and $\Delta_K$ :-

$$\Delta = 1-(P_{11}+P_{21}+P_{31})+(P_{12}+P_{22})$$
$$=1-(-G_2H_1 - G_2G_3H_2 - G_5H_3) + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3$$

$$\Delta = 1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3$$

$$\Delta_1 = 1, \quad \Delta_2 = 1-[-G_2H_1] = 1+G_2H_1$$




Transfer function :-

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$$

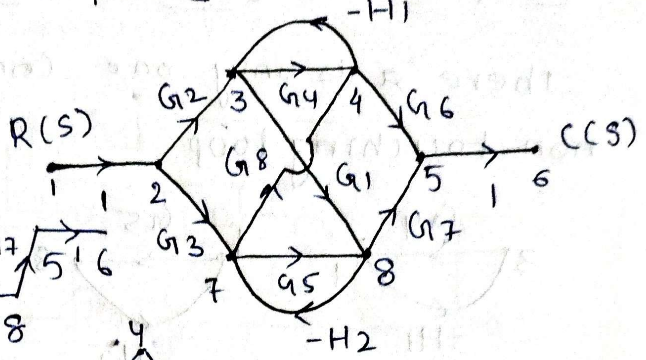
$$= \frac{1}{\Delta}\left[G_1G_2G_3G_4G_5(1) + G_6G_4G_5(1+G_2H_1)\right]$$

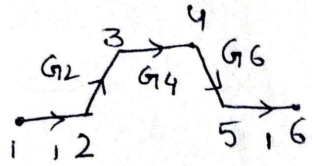
Transfer function

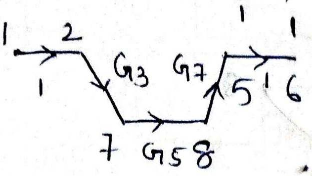
$$\boxed{T = \frac{G_1G_2G_3G_4G_5 + G_6G_4G_5(1+G_2H_1)}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}}$$

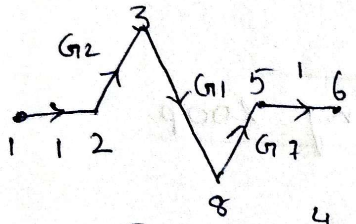
Q) There are 6 forward paths

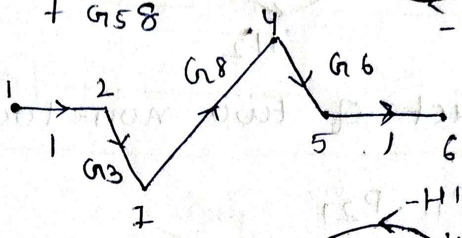
$\therefore K = 6$

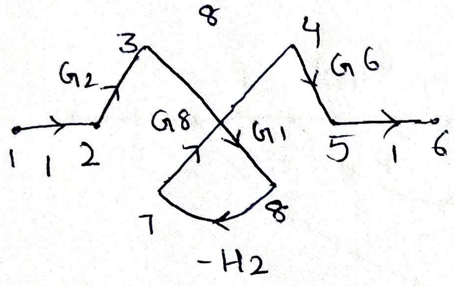


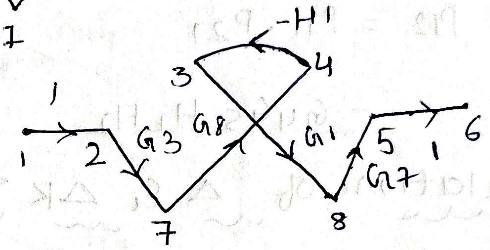




Gain of forward path - 1 $P_1 = G_2 G_4 G_6$

" " 2 $P_2 = G_3 G_5 G_7$

" " 3 $P_3 = G_2 G_1 G_7$

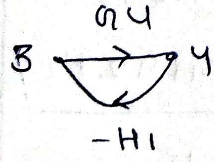
" " 4 $P_4 = G_3 G_8 G_6$

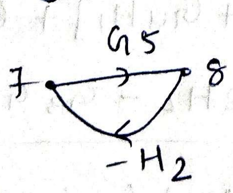
" " 5 $P_5 = -G_2 G_1 G_8 G_6 H_2$

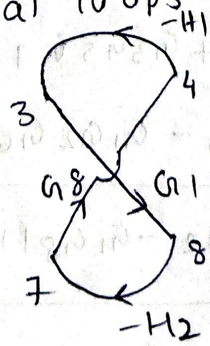
" " 6 $P_6 = -G_3 G_8 G_1 G_7 H_1$

Individual loop gains

There are '3' individual loops



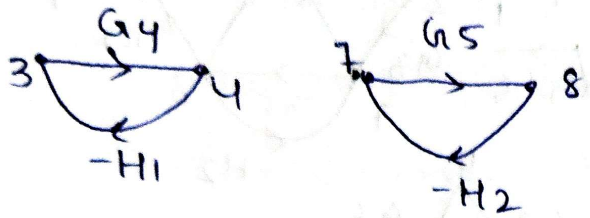

loop gain of individual loop - 1 $P_{11} = -G_4 H_1$

" " " 2 $P_{21} = -G_5 H_2$

" " " 3 $P_{31} = G_1 G_8 H_1 H_2$

Gain product of two non-touching loops :-

There a is only one combination of two non-touching loop




Gain product of two non-touching loop

$$P_{12} = P_{11} \cdot P_{21}$$

$$= G_4 G_5 H_1 H_2$$

Calculation of $\Delta$ & $\Delta K$ :-

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + P_{12}$$

$$= 1 - (-G_4H_1 - G_5H_2 + G_1 G_8 H_1 H_2) + G_4G_5 H_1 H_2$$

$$= 1 + G_4 H_1 + G_5 H_2 - G_1 G_8 H_1 H_2 + G_4 G_5 H_1 H_2$$

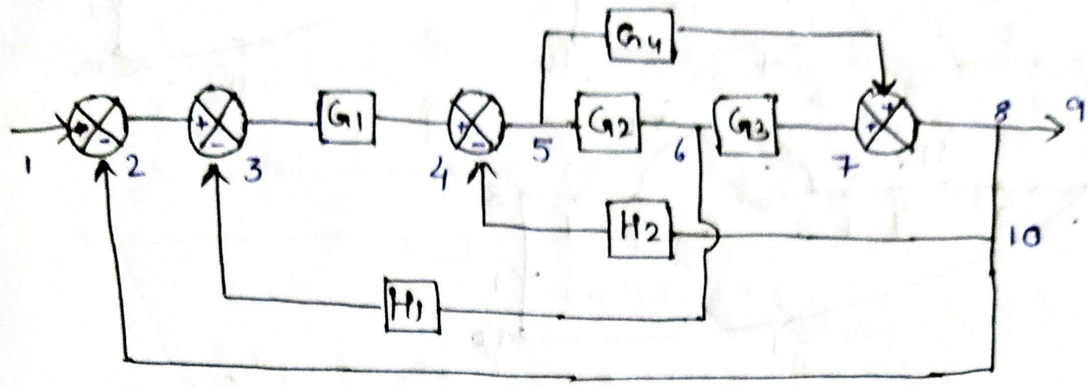
$\Delta_1 = 1 + G_5 H_2$ $\quad \Delta_2 = 1 + G_4 H_1$ $\quad \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$

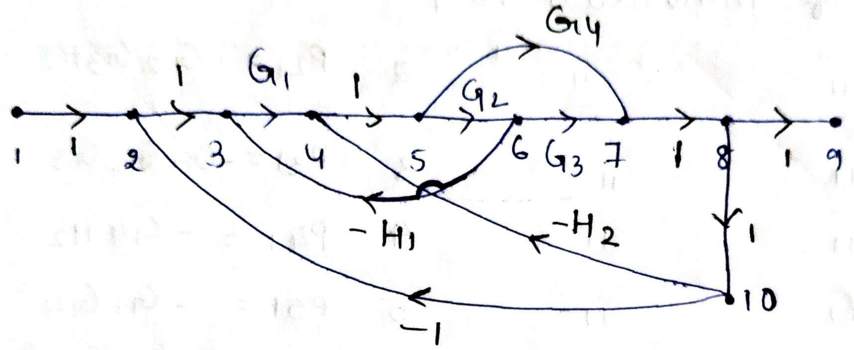
Transfer function T

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K = \frac{1}{\Delta} [P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6]$$

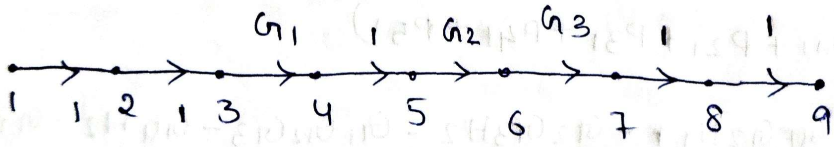
$$\boxed{T = \frac{G_2G_4G_6[1+G_5H_2] + G_3G_5G_7[1+G_4H_1] + G_1G_2G_7 + G_3G_8G_6 - G_1G_2G_6G_8H_2 - G_3G_8G_1G_2H_1}{1+G_4H_1 + G_5H_2 - G_1G_8H_1H_2 + G_4G_5H_1H_2}}$$

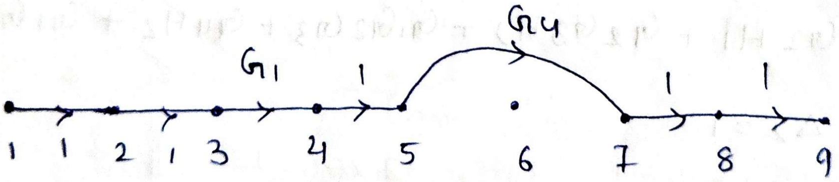
41) Convert the block diagram to signal flow graph & determine C/R




Forward path gains



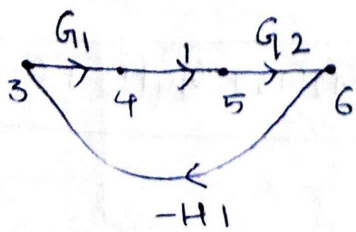



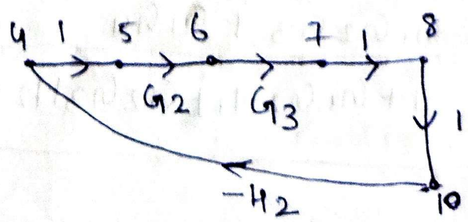

Forward path gain - 1 $P_1 = G_1 G_2 G_3$

" " 2 $P_2 = G_1 G_4$

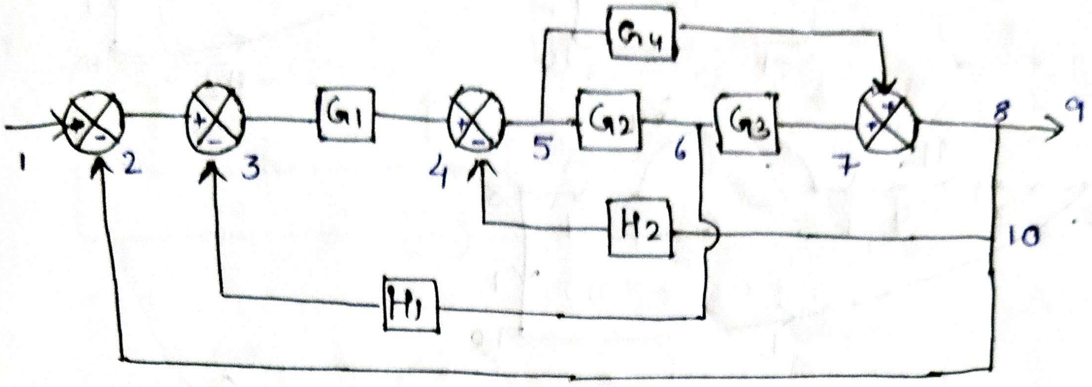
Individual loop gains

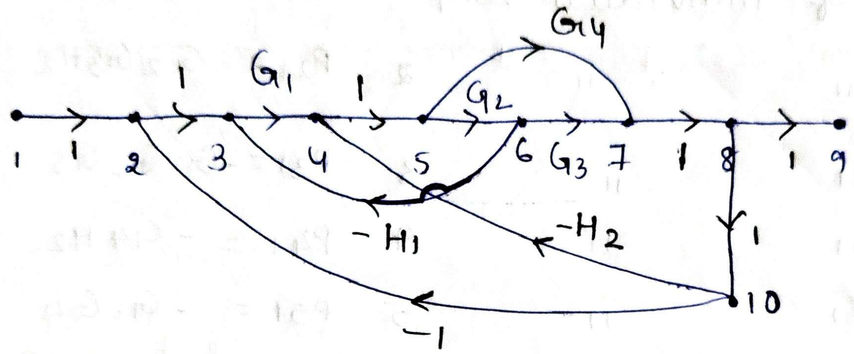
There are 5 individual loops



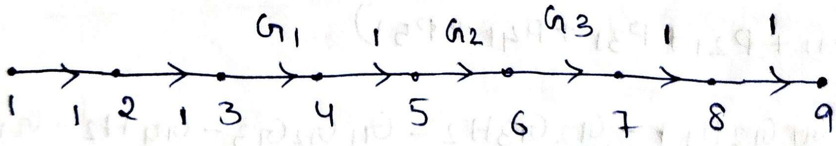



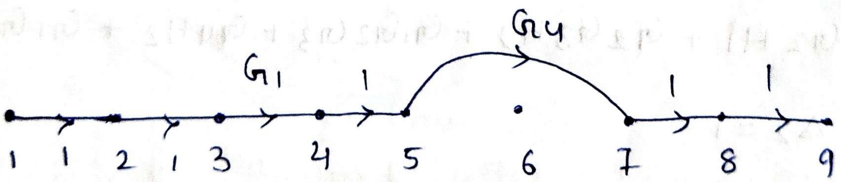
4) convert the block diagram to signal flow graph & determine C/R




Forward path gains



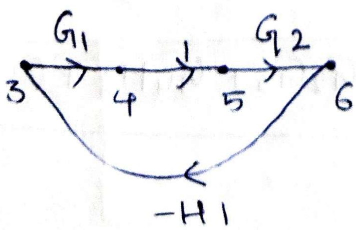



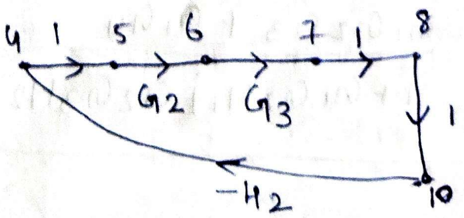

Forward path gain - 1 $P_1 = G_1 G_2 G_3$

" " 2 $P_2 = G_1 G_4$

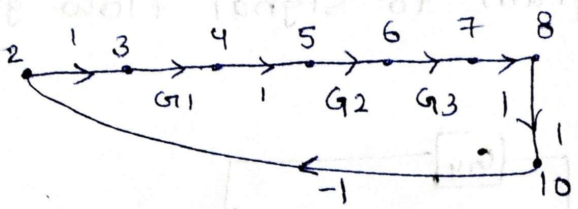
Individual loop gains

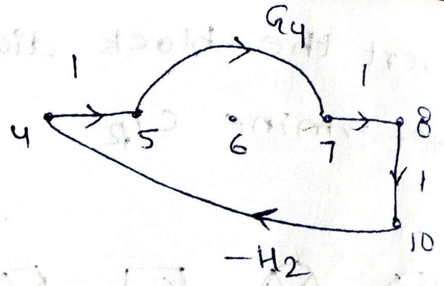
There are 5 individual loops

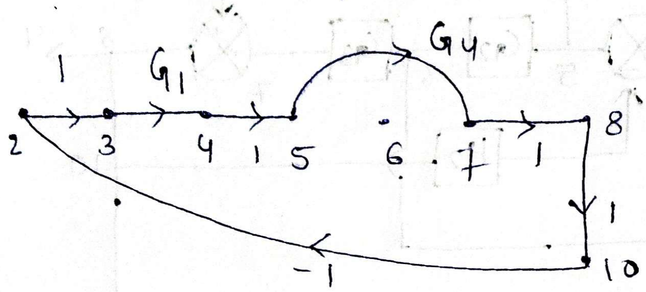



Scanned with ACE Scanner




loop gain of individual loop-1 $P_{11} = -G_1G_2H_1$

" " " 2 $P_{21} = -G_2G_3H_2$

" " " 3 $P_{31} = -G_1G_2G_3$

" " " 4 $P_{41} = -G_4H_2$

" " " 5 $P_{51} = -G_1G_4$

Calculation of $\Delta$ & $\Delta K$ :-

$$\Delta = 1-(P_{11}+P_{21}+P_{31}+P_{41}+P_{51})$$

$$= 1-(-G_1G_2H_1 - G_2G_3H_2 - G_1G_2G_3 - G_4H_2 - G_1G_4)$$

$$\Delta = 1+G_1G_2H_1+G_2G_3H_2+G_1G_2G_3+G_4H_2+G_1G_4$$

$\Delta_1 = 1$ , $\Delta_2 = 1$

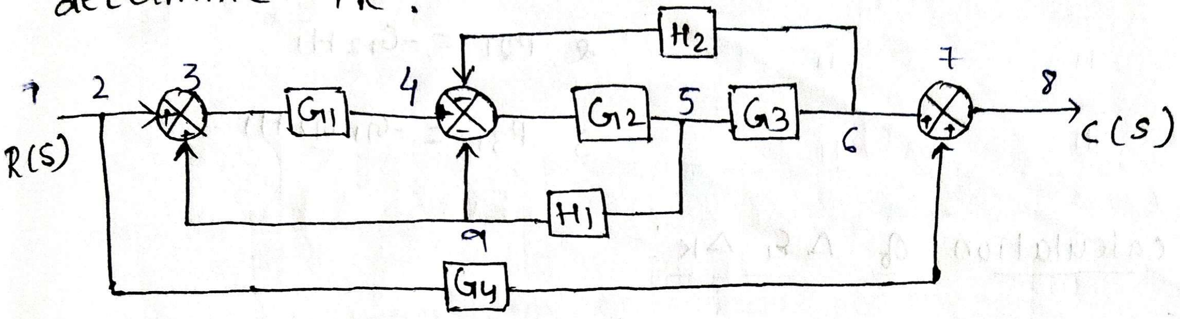
Transfer function :-

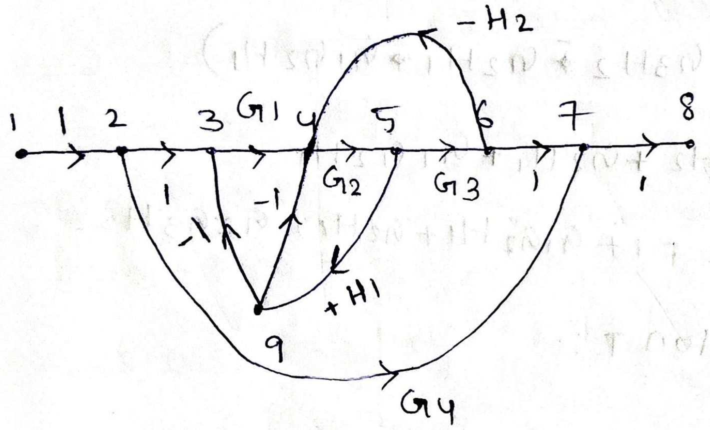
$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K$$

$$= \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$$

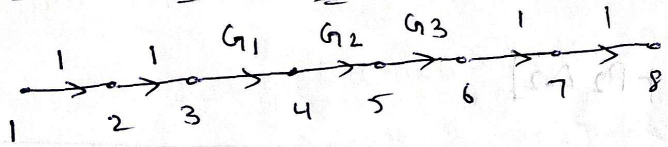
$$\boxed{T = \frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_1G_2G_3+G_4H_2+G_1G_4}}$$

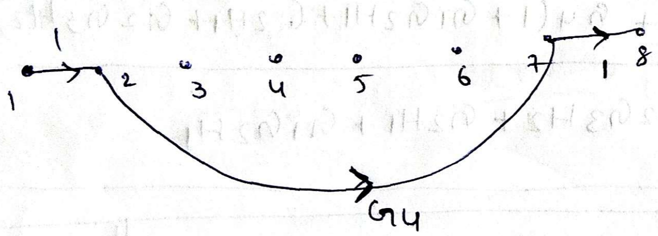
convert the block diagram to signal flow graph & determine C/R ?




forward path gains



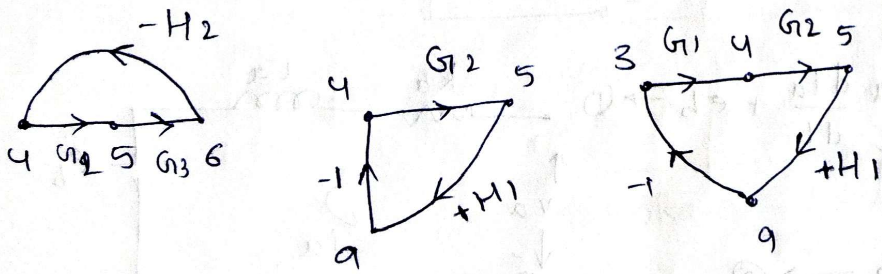




Gain of forward path - 1 $P_1 = G_1G_2G_3$

" " 2 $P_2 = G_4$

individual loop gains :-

There are 3 individual loops.



gain of individual loop -1 $P_{11} = -G_2 G_3 H_2$

" " " 2 $P_{21} = -G_2 H_1$

" " " 3 $P_{31} = -G_1 G_2 H_1$

calculation of $\Delta$ & $\Delta_K$ :-

$$\Delta = 1 - (P_{11} + P_{21} + P_{31})$$

$$= 1 - (-G_2 G_3 H_2 - G_2 H_1 - G_1 G_2 H_1)$$

$$\Delta = 1 + G_2 G_3 H_2 + G_2 H_1 + G_1 G_2 H_1$$

$\Delta_1 = 1$ $\quad\quad$ $\Delta_2 = 1 + G_1 G_2 H_1 + G_2 H_1 + G_2 G_3 H_2.$

Transfer function T :-

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K$$

$$T = \frac{1}{\Delta} [P_1 \Delta_1 + P_2 \Delta_2]$$

$$\boxed{T = \frac{G_1 G_2 G_3 + G_4 (1 + G_1 G_2 H_1 + G_2 H_1 + G_2 G_3 H_2)}{1 + G_2 G_3 H_2 + G_2 H_1 + G_1 G_2 H_1}}$$