UNIT-5

INTRODUCTION :

In rolling contact bearings, the contact between the bearing surfaces is rolling instead of sliding as in sliding contact bearings. We have already discussed that the ordinary sliding bearing starts from rest with practically metal-to-metal contact and has a high coefficient of friction. It is an outstanding advantage of a rolling contact bearing over a sliding bearing that it has a low starting friction.

TYPES OF ROLLING CONTACT BEARINGS

Following are the two types of rolling contact bearings:

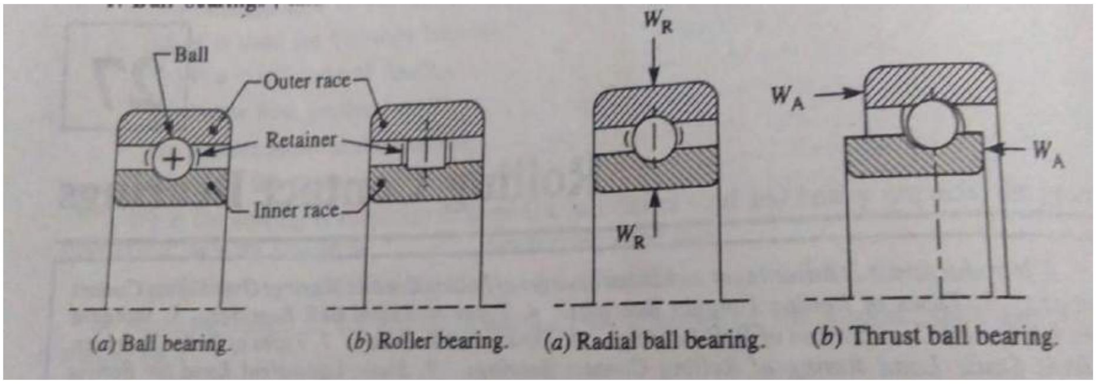
1. Ball bearings:
2. Roller bearings.

The ball and roller bearings consist of an inner race which is mounted on the shaft or journal and an outer race which is carried by the housing or casing. In between the inner and outer race, number of balls or rollers are used and these are held at proper distances by retainers so that they do not touch each other. The retainers are thin strips and used for the balls have been properly spaced.

The ball bearings are used for light loads and the roller bearings are used for heavier loads.

The rolling contact bearings, depending upon the load to be carried, are classified as :

(a) Radial bearings, and (b) Thrust bearings.



ADVANTAGES AND DISADVANTAGES OF ROLLING CONTACT BEARINGS OVER

SLIDING CONTACT BEARINGS:

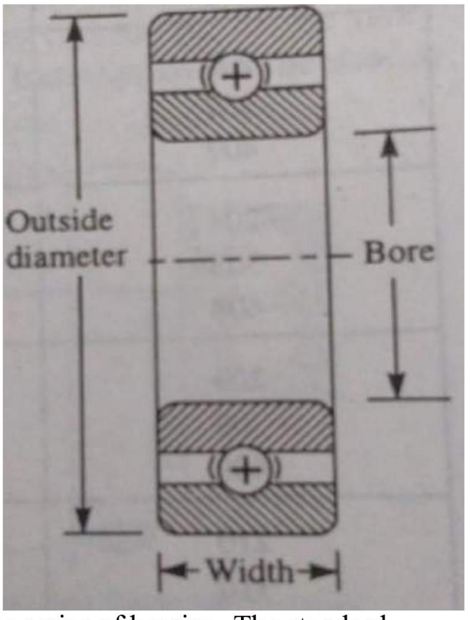
ADVANTAGES

1. Low starting and running friction except at very high speeds.
2. Ability to withstand momentary shock loads.
3. Accuracy of shaft alignment.
4. Low cost of maintenance, as no lubrication is required while in service.
5. Small overall dimensions.
6. Reliability of service.
7. Easy to mount and erect.
8. Cleanliness.

DISADVANTAGES

1. More noisy at very high speeds.
2. Low resistance to shock loading.
3. More initial cost.
4. Design of bearing housing complicated.

STANDARD DIMENSIONS AND DESIGNATIONS OF BALL BEARINGS:



The dimensions that have been standardised on an international basis are shown in Fig. These dimensions are a function of the bearing bore and the series of bearing. The standard dimensions are given in millimetres. There is no standard for the size and number of steel balls. The bearings are designated by a number. In general, the number consists of atleast three digits. Additional digits or letters are used to indicate special features e.g. deep groove, filling notch etc.

The last three digits give the series and the bore of the bearing. The last two digits from 04 onwards, when multiplied by 5, give the bore diameter in millimetres. The third from the last digit designates the series of the bearing.

The most common ball bearings are available in four series as follows :

1. Extra light (100), 2. Light (200), 3. Medium (300), 4. Heavy (400)

BASIC STATIC LOAD RATING OF ROLLING CONTACT BEARINGS

The load carried by a non-rotating bearing is called a static load. The basic static load rating is defined as the static radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which corresponds to a total permanent deformation of the ball (or roller) and race, at the most heavily stressed contact, equal to 0.0001 times the ball (or roller) diameter. In single row angular contact ball bearings, the basic static load relates to the radial component of the load, which causes a purely radial displacement of the bearing rings in relation to each other.According to IS : 3823–1984, the basic static load rating (C0) in newtons for ball and roller bearings may be obtained as discussed below :

1. **For radial ball bearings, the basic static radial load rating (C0) is given by**

$$C_0 = f_0.i.Z.D^2 \cos \alpha$$

Where,

i = Number of rows of balls in any one bearing,

Z = Number of ball per row,

D = Diameter of balls, in mm,

α = Nominal angle of contact i.e. the nominal angle between the line of action of the ball load and a plane perpendicular to the axis of bearing, and f_0 = A factor depending upon the type of bearing. The value of factor (f_0) for bearings made of hardened steel are taken as follows :

f_0 = 3.33, for self-aligning ball bearings

= 12.3, for radial contact and angular contact groove ball bearings.

2. For radial roller bearings, the basic static radial load rating is given by

$$C0 = f_0.i.Z.le .D \cos \alpha$$

Where,

i = Number of rows of rollers in the bearing,

Z = Number of rollers per row,

le = Effective length of contact between one roller and that ring

D = Diameter of roller in mm. It is the mean diameter in case of tapered rollers,

α = Nominal angle of contact. It is the angle between the line of action of the roller resultant load and a plane perpendicular to the axis of the bearing, and

f_0 = 21.6, for bearings made of hardened steel.

3 For thrust ball bearings,

The basic static axial load rating is given by

$$\mathbf{C0 = f_0.Z.D^2} \sin \alpha$$

where

Z = Number of balls carrying thrust in one direction, and

f_0 = 49, for bearings made of hardened steel.

4 For thrust roller bearings,

The basic static axial load rating is given by

$$\mathbf{C0 = f_0.Z.l_e.D.\sin \alpha}$$

where Z = Number of rollers carrying thrust in one direction, and

f_0 = 98.1, for bearings made of hardened steel.

STATIC EQUIVALENT LOAD FOR ROLLING CONTACT BEARINGS

The static equivalent load may be defined as the static radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which, if applied, would cause the same total permanent deformation at the most heavily stressed ball (or roller) and race contact as that which occurs under the actual conditions of loading. The static equivalent radial load (W_{0R}) for radial or roller bearings under combined radial and axial or thrust loads is given by the greater magnitude of those obtained by the following two equations, i.e.

$$\mathbf{W_{0R} = X_0.W_R + Y_0.W_A\ ;}$$

X_0 = Radial load factor, and

Y_0 = Axial or thrust load factor.

According to IS : 3824 – 1984, the values of X0 and Y0 for different bearings are given in the **databook table**

S. No.	Type of bearing	Single row bearing		Double row bearing	
		X_0	Y_0	X_0	Y_0
1.	Radial contact groove ball bearings.	0.60	0.50	0.60	0.50
2.	Self aligning ball or roller bearings and tapered roller bearing.	0.50	0.22 cot θ	1	0.44 cot θ
3.	Angular contact groove bearings :				
	$\alpha = 15°$	0.50	0.46	1	0.92
	$\alpha = 20°$	0.50	0.42	1	0.84
	$\alpha = 25°$	0.50	0.38	1	0.76
	$\alpha = 30°$	0.50	0.33	1	0.66
	$\alpha = 35°$	0.50	0.29	1	0.58
	$\alpha = 40°$	0.50	0.26	1	0.52
	$\alpha = 45°$	0.50	0.22	1	0.44

LIFE OF A BEARING

The life of an individual ball (or roller) bearing may be defined as the number of revolutions (or hours at some given constant speed) which the bearing runs before the first evidence of fatigue develops in the material of one of the rings or any of the rolling elements. The rating life of a group of apparently identical ball or roller bearings is defined as the number of revolutions (or hours at some given constant speed) that 90 per cent of a group of bearings will complete or exceed before the first evidence of fatigue develops.

The term minimum life is also used to denote the rating life. It has been found that the life which 50 per cent of a group of bearings will complete or exceed is approximately 5times the life which 90 per cent of the bearings will complete or exceed. In other words, we maysay that the average life of a bearing is 5 times the rating life (or minimum life).

It may be noted that the longest life of a single bearing is seldom longer than the4 times the average life and the maximum life of a single bearing is about 30 to 50 times the minimum life.

DYNAMIC LOAD RATING OF ROLLING CONTACT BEARINGS

The basic dynamic load rating is defined as the constant stationary radial load (in case of radial ball or roller bearings) or constant axial load (in case of thrust ball or roller bearings) which a group of apparently identical bearings with stationary outer ring can endure for a rating life of one million revolutions with only 10 per cent failure. The basic dynamic load rating (C) in newtons for ball and roller bearings may be obtained as discussed below :

1. According to IS: 3824 (Part 1)– 1983, the basic dynamic radial load rating for radial and angular contact ball bearings, except the filling slot type, with balls not larger than 25.4 mm in diameter, is given by

$$C = f_c\,(i \cos \alpha)^{0.7}\, Z^{2/3} .\, ^{D1.8}$$

and for balls larger than 25.4 mm in diameter,

$$C = 3.647\, f_c\,(i \cos \alpha)^{0.7}\, Z^{2/3} .\, D^{1.4}$$

DYNAMIC EQUIVALENT LOAD FOR ROLLING CONTACT BEARINGS

The dynamic equivalent load may be defined as the constant stationary radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which, if applied to a bearing with rotating inner ring and stationary outer ring, would give the same life as that which the bearing will attain under the actual conditions of load and rotation.

The dynamic equivalent radial load (W) for radial and angular contact bearings, except the filling slot types, under combined constant radial load (W_R) and constant axial or thrust load (W_A) is given by

$$\mathbf{W = X \cdot V \cdot W_R + Y \cdot W_A}$$

where V = A rotation factor, = 1, for all types of bearings when the inner race is rotating,
= 1, for self-aligning bearings when inner race is stationary,
= 1.2, for all types of bearings except self-aligning, when inner race is stationary.

The values of radial load factor (X) and axial or thrust load factor (Y) for the dynamically loaded bearings may be taken from **Data Table:**

DYNAMIC LOAD RATING FOR ROLLING CONTACT BEARINGS UNDER VARIABLE LOADS

The approximate rating (or service) life of ball or roller bearings is based on the fundamental equation,

$$\mathbf{L = (C / W)^k \times 10^6 \quad revolutions}$$

Or $$\mathbf{C = W (L/10^6)^{1/k}}$$

Where

L = Rating life,

C = Basic dynamic load rating,

W = Equivalent dynamic load, and

k = 3, for ball bearings, = 10/3, for roller bearings.

The relationship between the life in revolutions (L) and the life in working hours (LH) is given by

$$\mathbf{L = 60\, N \cdot L_H} \quad \text{revolutions}$$

MATERIALS AND MANUFACTURE OF BALL AND ROLLER BEARINGS

Since the rolling elements and the races are subjected to high local stresses of varying magnitude with each revolution of the bearing, therefore the material of the rolling element (i.e. steel) should be of high quality.

The balls are generally made of high carbon chromium steel. The material of both the balls and races are heat treated to give extra hardness and toughness. The balls are manufactured by hot forging on hammers from steel rods. They are then heattreated, ground and polished. The races are also formed by forging and then heat-treated, ground and polished.

LUBRICATION OF BALL AND ROLLER BEARINGS

The ball and roller bearings are lubricated for the following purposes :

1. To reduce friction and wear between the sliding parts of the bearing,
2. To prevent rusting or corrosion of the bearing surfaces,
3. To protect the bearing surfaces from water, dirt etc., and
4. To dissipate the heat. In general, oil or light grease is used for lubricating ball and roller bearings. Only pure mineral oil or calcium-base grease should be used. If there is a possibility of moisture contact, then potassium or sodium-base greases may be used.

SOLVE QUESTIONS

1. A single row angular contact ball bearing number 310 is used for an axial flow compressor. The bearing is to carry a radial load of 2500 N and an axial or thrust load of 1500 N. Assuming light shock load, determine the rating life of the bearing. Solution.

Given : $W_R = 2500$ N ; $W_A = 1500$ N

From data Table we find that for single row angular contact ball bearing, the values of radial factor (X) and thrust factor (Y)

For $WA / WR = 1500 / 2500 = 0.6$ are $X = 1$ and $Y = 0$

Since the rotational factor (V) for most of the bearings is 1,

therefore dynamic equivalent load,

$$W = X.V.W_R + Y.W_A$$

$$= 1 \times 1 \times 2500 + 0 \times 1500$$

$= 2500$ N 1016

we find that for light shock load, the service factor (KS) is 1.5.

Therefore the design dynamic equivalent load should be taken as

$W = 2500 \times 1.5 = 3750$ N From Table

we find that for a single row angular contact ball bearing number 310, the basic dynamic capacity, C = 53 kN = 53 000 N

We know that rating life of the bearing in revolutions,

$L = (C/W)^k \times 10^6$ revolutions

5. $= 2823 \times 10^6$ rev Ans. ...

When the angle of contact of the bearing with the journal is 120°, then the bearing is said to be partial journal bearing. This type of bearing has less friction than full journal bearing, but it can be used only where the load is always in one direction. The most common application of the partial journal bearings is found in rail road car axles. The full and partial journal bearings may be called as clearance bearings because the diameter of the journal is less than that of bearing.

When a partial journal bearing has no clearance i.e. the diameters of the journal and bearing are equal, then the bearing is called a fitted bearing, The sliding contact bearings, according to the thickness of layer of the lubricant between the bearing and the journal, may also be classified as follows :

1. Thick film bearings. The thick film bearings are those in which the working surfaces are completely separated from each other by the lubricant. Such type of bearings are also called as hydrodynamic lubricated bearings.

2. Thin film bearings. The thin film bearings are those in which, although lubricant is present, the working surfaces partially contact each other atleast part of the time. Such type of bearings are also called boundary lubricated bearings.

3. Zero film bearings. The zero film bearings are those which operate without any lubricant present. 4. Hydrostatic or externally pressurized lubricated bearings.

The hydrostatic bearings are those which can support steady loads without any relative motion between the journal and the bearing. This is achieved by forcing externally pressurized lubricant between the members.

HYDRODYNAMIC LUBRICATED BEARINGS

In hydrodynamic lubricated bearings, there is a thick film of lubricant between the journal and the bearing. When the bearing is supplied with sufficient lubricant, a pressure is build up in the clearance space when the journal is rotating about an axis that is eccentric with the bearing axis. The load can be supported by this fluid pressure without any actual contact between the journal and bearing. The load carrying ability of a hydrodynamic bearing arises simply because a viscous fluid resists being pushed around. Under the proper conditions, this resistance to motion will develop a pressure distribution in the lubricant film that can support a useful load. The load supporting pressure in hydrodynamic bearings arises from either

1. the flow of a viscous fluid in a converging channel (known as wedge film lubrication), or
2. the resistance of a viscous fluid to being squeezed out from between approaching surfaces (known as squeeze film lubrication).

PROPERTIES OF SLIDING CONTACT BEARING MATERIALS

When the journal and the bearings are having proper lubrication i.e. there is a film of clean, non-corrosive lubricant in between, separating the two surfaces in contact, the only requirement of the bearing material is that they should have sufficient strength and rigidity. However, the conditions under which bearings must operate in service are generally far from ideal and thus the other properties as discussed below must be considered in selecting the best material.

1. **Compressive strength**. The maximum bearing pressure is considerably greater than the average pressure obtained by dividing the load to the projected area. Therefore the bearing material should have high compressive strength to withstand this maximum pressure so as to prevent extrusion or other permanent deformation of the bearing.
2. **Fatigue strength**. The bearing material should have sufficient fatigue strength so that it can withstand repeated loads without developing surface fatigue cracks. It is of major importance in aircraft and automotive engines.
3. **Conformability**. It is the ability of the bearing material to accommodate shaft deflections and bearing inaccuracies by plastic deformation (or creep) without excessive wear and heating.
4. **Embedability**. It is the ability of bearing material to accommodate (or embed) small particles of dust, grit etc., without scoring the material of the journal.
5. **Bandability**. Many high capacity bearings are made by bonding one or more thin layers of a bearing material to a high strength steel shell. Thus, the strength of the bond i.e. bondability is an important consideration in selecting bearing material.
6. **Corrosion resistance.** The bearing material should not corrode away under the action of lubricating oil. This property is of particular importance in internal combustion engines where the same oil is used to lubricate the cylinder walls and bearings. In the cylinder, the lubricating oil comes into contact with hot cylinder walls and may oxidise and collect carbon deposits from the walls.
7. **Thermal conductivity**. The bearing material should be of high thermal conductivity so as to permit the rapid removal of the heat generated by friction.
8. **Thermal expansion**. The bearing material should be of low coefficient of thermal expansion,

so that when the bearing operates over a wide range of temperature, there is no undue change in the clearance. All these properties as discussed above are, however, difficult to find in any particular bearing material. The various materials are used in practice, depending upon the requirement of the actual service conditions.

ASSUMPTIONS IN HYDRODYNAMIC LUBRICATED BEARINGS

The following are the basic assumptions used in the theory of hydrodynamic lubricated bearings:

1. The lubricant obeys Newton's law of viscous flow.
2. The pressure is assumed to be constant throughout the film thickness.
3. The lubricant is assumed to be incompressible.
4. The viscosity is assumed to be constant throughout the film.
5. The flow is one dimensional, i.e. the side leakage is neglected.

MATERIALS USED FOR SLIDING CONTACT BEARINGS

The materials commonly used for sliding contact bearings are discussed below :

1. Babbit metal.

The tin base and lead base babbits are widely used as a bearing material, because they satisfy most requirements for general applications. The babbits are recommended where the maximum bearing pressure (on projected area) is not over 7 to 14 N/mm^2 ; When applied in Marine bearings, automobiles, the babbit is generally used as a thin layer, 0.05 mm to 0.15 mm thick, bonded to an insert or steel shell.

The composition of the babbit metals is as follows :

Tin base babbits: Tin 90%; Copper 4.5%; Antimony 5%; Lead 0.5%. Lead base babbits: Lead 84%; Tin 6%; Antimony 9.5%; Copper 0.5%.

2. Bronzes.

The bronzes (alloys of copper, tin and zinc) are generally used in the form of machined bushes pressed into the shell. The bush may be in one or two pieces. The bronzes commonly used for bearing material are gun metal and phosphor bronzes.

The gun metal (Copper 88% ; Tin 10% ; Zinc 2%) is used for high grade bearings subjected to high pressures (not more than 10 N/mm2 of projected area) and high speeds.

The phosphor bronze (Copper 80% ; Tin 10% ; Lead 9% ; Phosphorus 1%) is used for bearings subjected to very high pressures (not more than 14 N/mm^2 of projected area) and speeds.

3. Cast iron.

The cast iron bearings are usually used with steel journals. Such type of bearings are fairly successful where lubrication is adequate and the pressure is limited to 3.5 N/mm2 and speed to 40 metres per minute.

4. Silver.

The silver and silver lead bearings are mostly used in aircraft engines where the fatigue strength is the most important consideration.

5. Non-metallic bearings.

The various non-metallic bearings are made of carbon-graphite, rubber, wood and plastics. The carbon-graphite bearings are self lubricating, dimensionally stable over a wide range of operating conditions, chemically inert and can operate at higher temperatures than other bearings.

PROPERTIES OF LUBRICANTS

1. Viscosity. It is the measure of degree of fluidity of a liquid. It is a physical property by virtue of which an oil is able to form, retain and offer resistance to shearing a buffer film-under heat and pressure. The greater the heat and pressure, the greater viscosity is required of a lubricant to prevent thinning and squeezing out of the film. The motion is accompanied by a linear slip or shear between the particles throughout the entire height (h) of the film thickness. If A is the area of the plate in contact with the lubricant, then the unit shear stress is given by $\tau = P/A$

2 **Oiliness.** It is a joint property of the lubricant and the bearing surfaces in contact. It is a measure of the lubricating qualities under boundary conditions where base metal to metal is prevented only by absorbed film. There is no absolute measure of oiliness.

3. **Density.** This property has no relation to lubricating value but is useful in changing the kinematic viscosity to absolute viscosity. Mathematically Absolute viscosity = $\rho \times$ Kinematic viscosity (in m2/s) where ρ = Density of the lubricating oil. The density of most of the oils at 15.5°C varies from 860 to 950 kg / m3 (the average value may be taken as 900 kg / m3). The density at any other temperature (t) may be obtained from the following relation, i.e. $\rho t = \rho 15.5 - 0.000\,657\, t$ where $\rho 15.5$ = Density of oil at 15.5° C.

4. **Viscosity index.** The term viscosity index is used to denote the degree of variation of viscosity with temperature.

5. **Flash point**. It is the lowest temperature at which an oil gives off sufficient vapour to support a momentary flash without actually setting fire to the oil when a flame is brought within 6 mm at the surface of the oil.

6. **Fire point**. It is the temperature at which an oil gives off sufficient vapour to burn it continuously when ignited.

7. **Pour point or freezing point.** It is the temperature at which an oil will cease to flow when cooled.

TERMS USED IN HYDRODYNAMIC JOURNAL BEARING A hydrodynamic journal bearing is shown in Fig. in which O is the centre of the journal and O′ is the centre of the bearing.

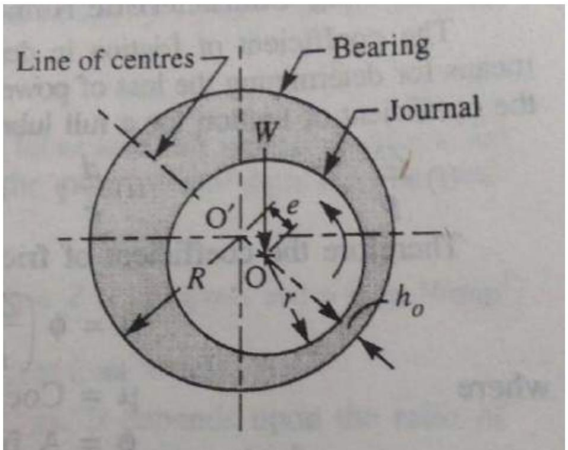


Fig. 26.7. Hydrodynamic journal bearing.

Let, D = Diameter of the bearing,

d = Diameter of the journal, and

l = Length of the bearing.

The following terms used in hydrodynamic journal bearing are important from the subject point of view :

1. Diametral clearance. It the difference between the diameters of the bearing and the journal. Mathematically, diametral clearance,

$$c = D - d$$

Note : The diametral clearance (c) in a bearing should be small enough to produce the necessary velocity gradient, so that the pressure built up will support the load. Also the small clearance has the advantage of decreasing side leakage. A commonly used clearance in industrial machines is 0.025 mm per cm of journal diameter.

2. **Radial clearance**. It is the difference between the radii of the bearing and the journal. Mathematically, radial clearance,

$$c_1 = (D-d)/2 = R - r = c/2$$

3. **Diametral clearance ratio**. It is the ratio of the diametral clearance to the diameter of the journal. Mathematically, diametral clearance ratio

$$= c/d = (D- d)/d$$

4. **Eccentricity**. It is the radial distance between the centre (O) of the bearing and the displaced centre (O′) of the bearing under load. It is denoted by e.

5. **Minimum oil film thickness**. It is the minimum distance between the bearing and the journal, under complete lubrication condition. It is denoted by h_0 and occurs at the line of centres as shown in fig. Its value may be assumed as c / 4.

6. **Attitude or eccentricity ratio**. It is the ratio of the eccentricity to the radial clearance. Mathematically, attitude or eccentricity ratio, $\varepsilon = e/c_1 = 1-h_o/c_1$

7. **Short and long bearing.** If the ratio of the length to the diameter of the journal (i.e. l / d) is less than 1, then the bearing is said to be short bearing.

On the other hand, if l/d is greater than 1, then the bearing is known as long bearing

BEARING CHARACTERISTIC NUMBER AND BEARING MODULUS FOR JOURNAL BEARINGS

The coefficient of friction in design of bearings is of great importance, because it affords a means for determining the loss of power due to bearing friction. It has been shown by experiments that the coefficient of friction for a full lubricated journal bearing is a function of three variables, i.e. (i) ; ZN /p (ii) d/ c and (iii) l /d

Therefore the coefficient of friction may be expressed as

$$\mu = \varphi \left[ZN/p,\ d/c\ ,\ l/d \right]$$

where

μ = Coefficient of friction,

φ = A functional relationship,

Z = Absolute viscosity of the lubricant, in kg / m-s,

N = Speed of the journal in r.p.m.,

p = Bearing pressure on the projected bearing area in N/mm^2,

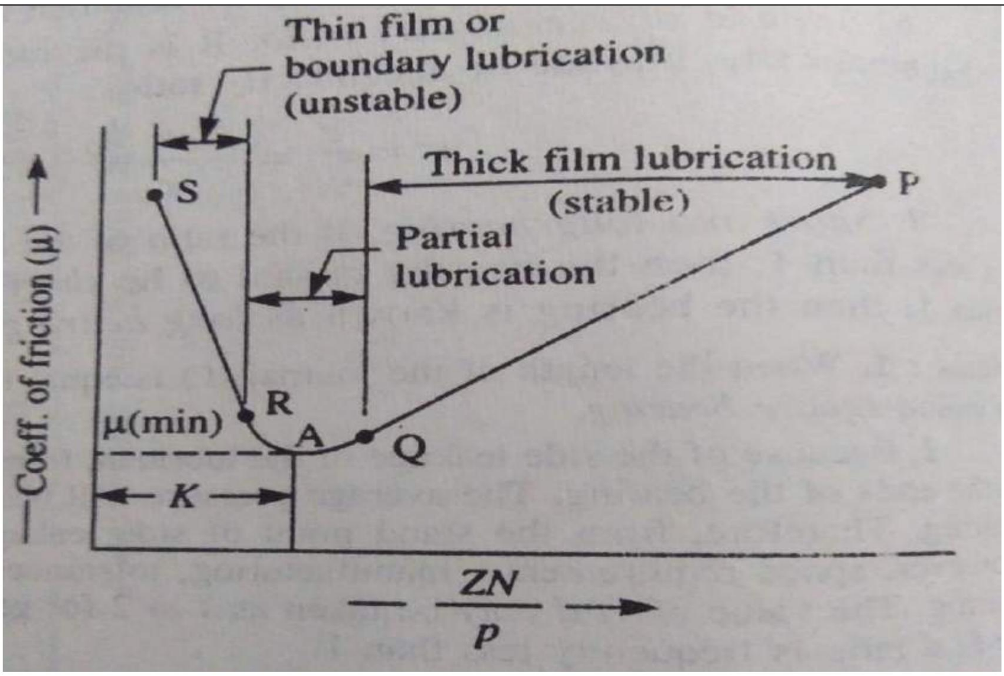
= Load on the journal $\div l \times d$

d = Diameter of the journal,

l = Length of the bearing, and

c = Diametral clearance.

The factor ZN / p is termed as bearing characteristic number and is a dimensionless number. The variation of coefficient of friction with the operating values of bearing characteristic number (ZN / p) as obtained by McKee brothers (S.A. McKee and T.R. McKee) in an actual test of friction is shown in Fig. The factor ZN/p helps to predict the performance of a bearing.



COEFFICIENT OF FRICTION FOR JOURNAL BEARINGS

In order to determine the coefficient of friction for well lubricated full journal bearings, the following empirical relation established by McKee based on the experimental data, may be used. Coefficient of friction,

$\mu = (33/10^{8})\,(ZN/p)\,(d/c) + k$ (when Z is in kg / m-s and p is in N / mm^2)

where Z, N, p, d and c have usual meanings as discussed in previous article, and

k = Factor to correct for end leakage. It depends upon the ratio of length to the diameter of the bearing (i.e. l/d).

= 0.002 for l/d ratios of 0.75 to 2.8.

The operating values of ZN / p should be compared with values given in Table of design values of journal bearing to ensure safe margin between operating conditions and the point of film breakdown.

CRITICAL PRESSURE OF THE JOURNAL BEARING

The pressure at which the oil film breaks down so that metal to metal contact begins, is known as critical pressure or the minimum operating pressure of the bearing. It may be obtained by the following empirical relation, i.e.

Critical pressure or minimum operating pressure,

$p = (ZN/P)(d/c)^2$

SOMMERFELD NUMBER

The Sommerfeld number is also a dimensionless parameter used extensively in the design of journal bearings. Mathematically,

Sommerfeld number = $(ZN/4.75 \times 10^6)(d/c)^2$

For design purposes, its value is taken as follows : 14.3×10^6

(when Z is in kg / m-s and p is in N / mm2)

HEAT GENERATED IN A JOURNAL BEARING

The heat generated in a bearing is due to the fluid friction and friction of the parts having relative motion. Mathematically, heat generated in a bearing,

$Q_g = \mu.W.V$ N-m/s or J/s or watts ...(i)

where μ = Coefficient of friction,

W = Load on the bearing in N,

DESIGN PROCEDURE FOR JOURNAL BEARING

The following procedure may be adopted in designing journal bearings, when the bearing load, the diameter and the speed of the shaft are known.

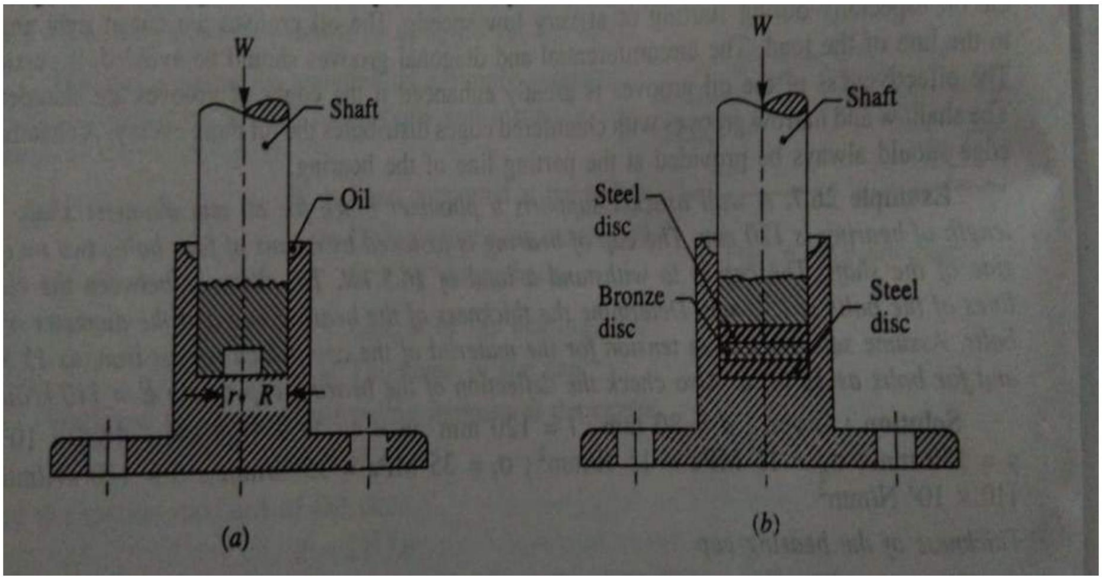
1. Determine the bearing length by choosing a ratio of l/d from Table
2. Check the bearing pressure, p = W / l.d from Table for probable satisfactory value.
3. Assume a lubricant from Table 26.2 and its operating temperature (t 0). This temperature should be between 26.5°C and 60°C with 82°C as a maximum for high temperature installations such as steam turbines.
4. Determine the operating value of ZN / p for the assumed bearing temperature and check this value with corresponding values in Table, to determine the possibility of maintaining fluid film operation.
5. Assume a clearance ratio c/d from Table
6. Determine the coefficient of friction (μ) by using the relation
7. Determine the heat generated by using the relation as discussed
8. Determine the heat dissipated by using the relation as discussed
9. Determine the thermal equilibrium to see that the heat dissipated becomes atleast equal to the heat generated. In case the heat generated is more than the heat dissipated then either the bearing is redesigned or it is artificially cooled by water

THRUST BEARINGS

A thrust bearing is used to guide or support the shaft which is subjected to a load along the axis of the shaft. Such types of bearings are mainly used in turbines and propeller shafts. The thrust bearings are of the following two types :

1. Foot step or pivot bearings, and
2. Collar bearings.

In a foot step or pivot bearing, the loaded shaft is vertical and the end of the shaft rests within the bearing. In case of collar bearing, the shaft continues through the bearing. The shaft may be vertical or horizontal with single collar or many collars.



Footstep or Pivot Bearings

A simple type of footstep bearing, suitable for a slow running and lightly loaded shaft, is shown in Fig. If the shaft is not of steel, its end must be fitted with a steel face. The shaft is guided in a gunmetal bush, pressed into the pedestal and prevented from turning by means of a pin. Since the wear is proportional to the velocity of the rubbing surface, which increases with the distance from the axis (i.e. radius) of the bearing, therefore the wear will be different at different radii. It may be noted that the wear is maximum at the outer radius and zero at the Centre. In order to compensate for end wear, the following two methods are employed.

1. The shaft is counter-bored at the end, as shown in Fig.

2. The shaft is supported on a pile of discs. It is usual practice to provide alternate discs of different materials such as steel and bronze, as shown in Fig. (b), so that the next disc comes into play, if one disc seizes due to improper lubrication. It may be noted that a footstep bearing is difficult to lubricate as the oil is being thrown outwards from the centre by centrifugal force. In designing,

it is assumed that the pressure is uniformly distributed throughout the bearing surface.

Let W = Load transmitted over the bearing surface,

R = Radius of the bearing surface (or shaft),

A = Cross-sectional area of the bearing surface,

p = Bearing pressure per unit area of the bearing surface between rubbing surfaces,

μ = Coefficient of friction, and

N = Speed of the shaft in r.p.m.

When the pressure in uniformly distributed over the bearing area, then

$$p = W/A = W/\pi R^2$$

and the total frictional torque,

$$T = (2/3)\mu W R$$

∴ Power lost in friction, $P = (2 \pi N T)/60$ ... (T being in N-m)

DESIGN OF SPUR GEARS

Introduction

We have discussed earlier that the slipping of a belt or rope is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slipping is to reduce the velocity ratio of the system. In precision machines, in which a definite velocity ratio is of importance (as in watch mechanism), the only positive drive is by *gears* or *toothed wheels*. A gear drive is also provided, when the distance between the driver and the follower is very small.

Friction Wheels

The motion and power transmitted by gears is kinematically equivalent to that transmitted by frictional wheels or discs. In order to understand how the motion can be transmitted by two toothed wheels, consider two plain circular wheels *A* and *B* mounted on shafts. The wheels have sufficient rough surfaces and press against each other as shown in Fig

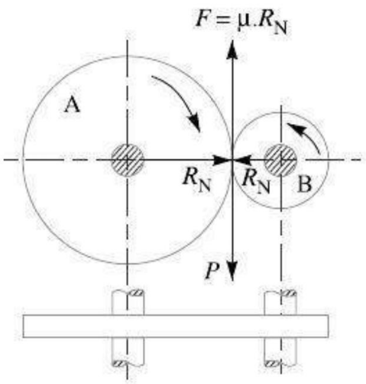


Fig. Friction wheels.

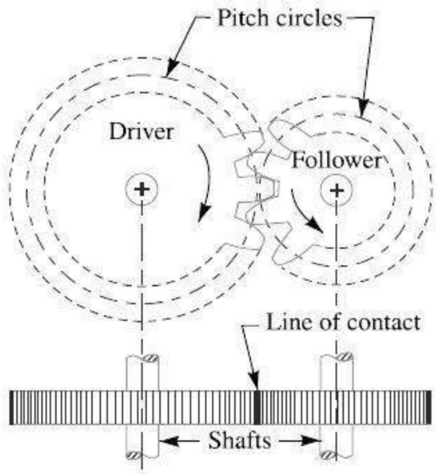


Fig. Gear or toothed wheel.

Let the wheel *A* is keyed to the rotating shaft and the wheel *B* to the shaft to be rotated. A little consideration will show that when the wheel *A* is rotated by a rotating shaft, it will rotate the wheel *B* in the opposite direction as shown in Fig.. The wheel *B* will be rotated by the wheel *A* so long as the tangential force exerted by the wheel *A* does not exceed the maximum frictional resistance between the two wheels. But when the tangential force (*P*) exceeds the *frictional resistance (*F*), slipping will take place between the two wheels.

In order to avoid the slipping, a number of projections (called teeth) as shown in Fig. are provided on the periphery of the wheel *A* which will fit into the corresponding recesses on the periphery of the wheel *B*. A friction wheel with the teeth cut on it is known as *gear* or *toothed wheel*. The usual connection to show the toothed wheels is by their pitch circles.

Advantages and Disadvantages of Gear Drives

The following are the advantages and disadvantages of the gear drive as compared to other drives, *i.e*. belt, rope and chain drives :

Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It may be used for small centre distances of shafts.
4. It has high efficiency.
5. It has reliable service.
6. It has compact layout.

1. Since the manufacture of gears require special tools and equipment, therefore it is costlier than other drives.
2. The error in cutting teeth may cause vibrations and noise during operation.
3. It requires suitable lubricant and reliable method of applying it, for the proper operation of gear drives.

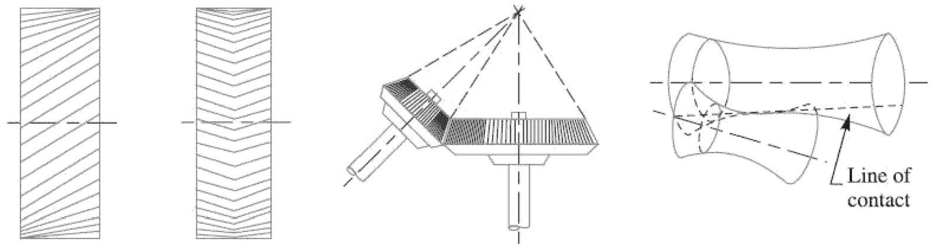
Classification of Gears

The gears or toothed wheels may be classified as follows :

1. *According to the position of axes of the shafts*. The axes of the two shafts between which the motion is to be transmitted, may be
(a) Parallel, (b) Intersecting, and (c) Non-intersecting and non-parallel.

The two parallel and co-planar shafts connected by the gears is shown in Fig. These gears are called spur gears and the arrangement is known as spur gearing. These gears have teeth parallel to the axis of the wheel as shown in Fig. Another name given to the spur gearing is helical gearing, in which the teeth are inclined to the axis. The single and double helical gears connecting parallel shafts are shown in Fig. (a) and (b) respectively. The object of the double helical gear is to balance out the end thrusts that are induced in single helical gears when transmitting load. The double helical gears are known as herringbone gears. A pair of spur gears are kinematically equivalent to a pair of cylindrical discs, keyed to a parallel shaft having line contact.

The two non-parallel or intersecting, but coplanar shafts connected by gears is shown in Fig.(c). These gears are called bevel gears and the arrangement is known as bevel gearing. The bevel gears, like spur gears may also have their teeth inclined to the face of the bevel, in which case they are known as helical bevel gears.



(*a*) Single helical gear. (*b*) Double helical gear. (*c*) Bevel gear. (*d*) Spiral gear.

The two non-intersecting and non-parallel i.e. non-coplanar shafts connected by gears is shown in Fig.(d). These gears are called skew bevel gears or spiral gears and the arrangement is known as skew bevel gearing or spiral gearing. This type of gearing also have a line contact, the rotation of which about the axes generates the two pitch surfaces known as hyperboloids.

Notes: (*i*) When equal bevel gears (having equal teeth) connect two shafts whose axes are mutually perpendicular, then the bevel gears are known as *mitres*.

(*ii*) A hyperboloid is the solid formed by revolving a straight line about an axis (not in the same plane), such that every point on the line remains at a constant distance from the axis.

(*iii*) The worm gearing is essentially a form of spiral gearing in which the shafts are usually at right angles.

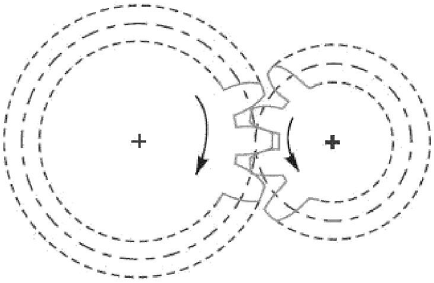
2. *According to the peripheral velocity of the gears*. The gears, according to the peripheral velocity of the gears, may be classified as:

(a)Low velocity, (b) Medium velocity, and (c) High velocity.

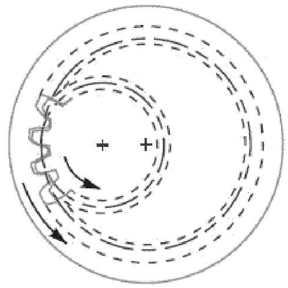
The gears having velocity less than 3 m/s are termed as low velocity gears and gears having velocity between 3 and 15 m / s are known as medium velocity gears. If the velocity of gears is more than 15 m / s, then these are called high speed gears.

3. *According to the type of gearing*. The gears, according to the type of gearing, may be classified as :

(*a*) External gearing, (*b*) Internal gearing, and (*c*) Rack and pinion.



(*a*) External gearing.



(*b*) Internal gearing.

In external gearing, the gears of the two shafts mesh externally with each other as shown in Fig. (a). The larger of these two wheels is called spur wheel or gear and the smaller wheel is called pinion. In an external gearing, the motion of the two wheels is always unlike, as shown in Fig. (a).

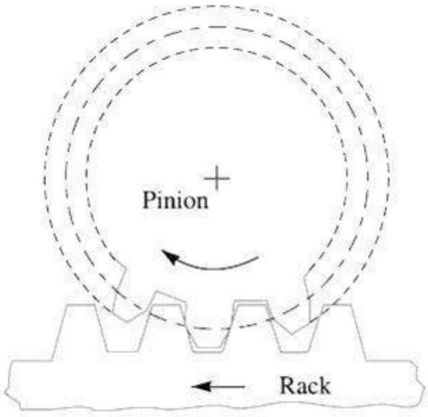


Fig. Rack and pinion.

In internal gearing, the gears of the two shafts mesh internally with each other as shown in Fig. (b). The larger of these two wheels is called annular wheel and the smaller wheel is called pinion. In an internal gearing, the motion of the wheels is always like as shown in Fig.(b). Sometimes, the gear of a shaft meshes externally and internally with the gears in a straight line, as shown in Fig. Such a type of gear is called rack and pinion. The straight line gear is called rack and the circular wheel is called pinion. A little consideration will show that with the help of a rack and pinion, we can convert linear motion into rotary motion and vice-versa as shown in Fig.

4. *According to the position of teeth on the gear surface*. The teeth on the gear surface may be (*a*) Straight, (*b*) Inclined, and (*c*) Curved. We have discussed earlier that the spur gears have straight teeth whereas helical gears have their teeth inclined to the wheel rim. In case of spiral gears, the teeth are curved over the rim surface.

Terms used in Gears

The following terms, which will be mostly used in this chapter, should be clearly understood at this stage. These terms are illustrated in Fig.

1. *Pitch circle*. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.
2. *Pitch circle diameter.* It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also called as *pitch diameter*.
3. *Pitch point*. It is a common point of contact between two pitch circles.
4. *Pitch surface*. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.
5. *Pressure angle or angle of obliquity*. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by φ. The standard pressure angles are 14 /2° and 20°.
6. *Addendum*. It is the radial distance of a tooth from the pitch circle to the top of the tooth.
7. *Dedendum*. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.
8. *Addendum circle*. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.
9. *Dedendum circle*. It is the circle drawn through the bottom of the teeth. It is also called *root circle*.

Note: Root circle diameter = Pitch circle diameter × cos φ, where φ is the pressure angle.

10. *Circular pitch*. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c.

Mathematically,

Circular pitch, $p_c = \pi D/T$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

Note : If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively; then for them to mesh correctly,

$$p_c \quad \frac{D_1}{T_1} \quad \frac{D_2}{T_2} \text{ or } \frac{D_1}{D_2} \quad \frac{T_1}{T_2}$$

11. *Diametral pitch.* It is the ratio of number of teeth to the pitch circle diameter in millimetres. It denoted by P_d.

Mathematically,

$$\text{Diametral pitch, } p_d \quad \frac{T}{D} \quad \frac{}{p_c}$$

Where T = Number of teeth, and

D = Pitch circle diameter.

12. *Module*. It is the ratio of the pitch circle diameter in millimetres to the number of teeth. It is usually denoted by m.

Mathematically,

$$\text{Module, } m = D / T$$

Note : The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40 and 50.

The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5,5.5, 7, 9, 11, 14, 18, 22, 28, 36 and 45 are of second choice.

13. *Clearance.* It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as *clearance circle*.

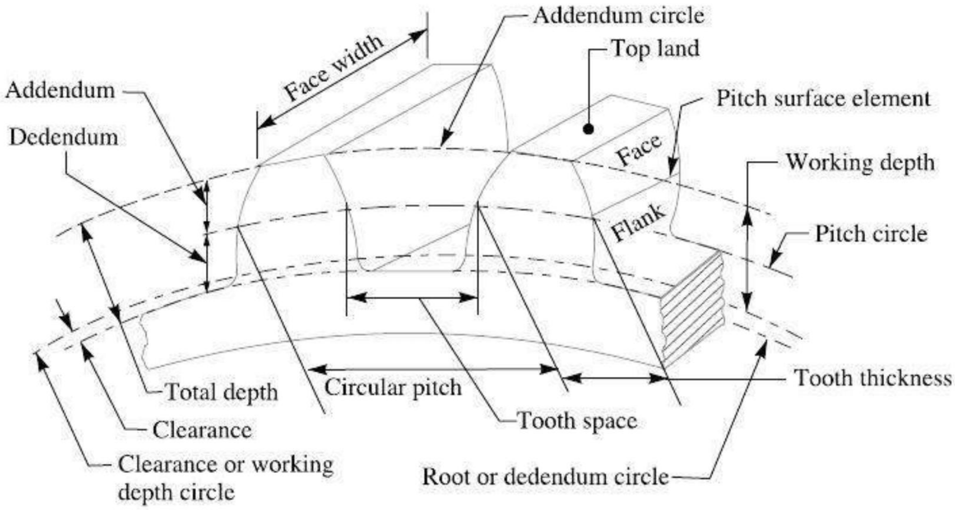


Fig. Terms used in gears.

14. *Total depth*. It is the radial distance between the addendum and the dedendum circle of a gear. It is equal to the sum of the addendum and dedendum.

15. *Working depth*. It is radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. *Tooth thickness*. It is the width of the tooth measured along the pitch circle.

17. *Tooth space*. It is the width of space between the two adjacent teeth measured along the pitch circle.

18. *Backlash*. It is the difference between the tooth space and the tooth thickness, as measured on the pitch circle.

19. *Face of the tooth*. It is surface of the tooth above the pitch surface.

20. *Top land*. It is the surface of the top of the tooth.

21. *Flank of the tooth*. It is the surface of the tooth below the pitch surface.

22. *Face width*. It is the width of the gear tooth measured parallel to its axis.

23. *Profile*. It is the curve formed by the face and flank of the tooth.

24. *Fillet radius*. It is the radius that connects the root circle to the profile of the tooth.

25. *Path of contact*. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

26. *Length of the path of contact*. It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

27. *Arc of contact*. It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, *i.e.*

(*a*) *Arc of approach*. It is the portion of the path of contact from the beginning of the engagement to the pitch point.

(*b*) *Arc of recess*. It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Note: The ratio of the length of arc of contact to the circular pitch is known as *contact ratio i.e*. number of pairs of teeth in contact.

<u>Systems of Gear Teeth</u>

The following four systems of gear teeth are commonly used in practice.14 /2° Composite system, 2. 14 /2° Full depth involute system, 3. 20° Full depth involute system, and 4. 20° Stub involute system.

The 14 /2° composite system is used for general purpose gears. It is stronger but has no interchangeability. The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion. The teeth are produced by formed milling cutters or hobs. The tooth profile of the 14 /2° full depth involute system was developed for use with gear hobs for spur and helical gears.

The tooth profile of the 20° full depth involute system may be cut by hobs. The increase of the pressure angle from 14 /2° to 20° results in a stronger tooth, because the tooth acting as a beam is wider at the base. The 20° stub involute system has a strong tooth to take heavy loads.

<u>Minimum Number of Teeth on the Pinion in Order to Avoid Interference</u>

We have seen in the previous article that the interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth. The minimum number of teeth on the pinion which will mesh with any gear (also rack) without interference are given in the following table.

Table. Minimum number of teeth on the pinion in order to avoid interference.

S. No.	Systems of gear teeth	Minimum number of teeth on the pinion
1.	$14\frac{1}{2}°$ Composite	12
2.	$14\frac{1}{2}°$ Full depth involute	32
3.	20° Full depth involute	18
4.	20° Stub involute	14

The number of teeth on the pinion (T_P) in order to avoid interference may be obtained from the following relation :

$$T_P = \frac{2A_W}{G\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1\right]}$$

where A_W = Fraction by which the standard addendum for the wheel should be multiplied,

G = Gear ratio or velocity ratio = $T_G / T_P = D_G / D_P$,

ϕ = Pressure angle or angle of obliquity.

Gear Materials

The material used for the manufacture of gears depends upon the strength and service conditions like wear, noise etc. The gears may be manufactured from metallic or non-metallic materials. The metallic gears with cut teeth are commercially obtainable in cast iron, steel and bronze. The nonmetallic materials like wood, rawhide, compressed paper and synthetic resins like nylon are used for gears, especially for reducing noise.

The cast iron is widely used for the manufacture of gears due to its good wearing properties, excellent machinability and ease of producing complicated shapes by casting method. The cast iron gears with cut teeth may be employed, where smooth action is not important.

The steel is used for high strength gears and steel may be plain carbon steel or alloy steel. The steel gears are usually heat treated in order to combine properly the toughness and tooth hardness.

Design Considerations for a Gear Drive

In the design of a gear drive, the following data is usually given:

1. The power to be transmitted.

2. The speed of the driving gear,
3. The speed of the driven gear or the velocity ratio, and
4. The centre distance.

The following requirements must be met in the design of a gear drive:

(*a*) The gear teeth should have sufficient strength so that they will not fail under static loading or dynamic loading during normal running conditions.

(*b*) The gear teeth should have wear characteristics so that their life is satisfactory.

(*c*) The use of space and material should be economical.

(*d*) The alignment of the gears and deflections of the shafts must be considered because they effect on the performance of the gears. (*e*) The lubrication of the gears must be satisfactory.

Beam Strength of Gear Teeth – Lewis Equation

The beam strength of gear teeth is determined from an equation (known as Lewis equation) and the load carrying ability of the toothed gears as determined by this equation gives satisfactory results. In the investigation, Lewis assumed that as the load is being transmitted from one gear to another, it is all given and taken by one tooth, because it is not always safe to assume that the load is distributed among several teeth. When contact begins, the load is assumed to be at the end of the driven teeth and as contact ceases, it is at the

end of the driving teeth. This may not be true when the number of teeth in a pair of mating gears is large, because the load may be distributed among several teeth. But it is almost certain that at some time during the contact of teeth, the proper distribution of load does not exist and that one tooth must transmit the full load. In any pair of gears having unlike number of teeth, the gear which have the fewer teeth (*i.e.* pinion) will be the weaker, because the tendency toward undercutting of the teeth becomes more pronounced in gears as the number of teeth becomes smaller.

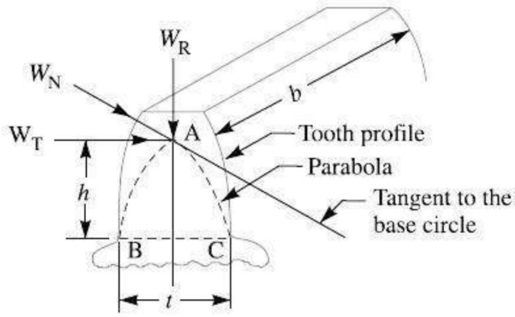


Fig. Tooth of a gear.

Consider each tooth as a cantilever beam loaded by a normal load (W_N) as shown in Fig. It is resolved into two components *i.e.* tangential component (W_T) and radial component (W_R) acting perpendicular and parallel to the centre line of the tooth respectively. The tangential component (W_T) induces a bending stress which tends to break the tooth. The radial component (WR) induces a compressive stress of relatively small magnitude, therefore its effect on the tooth may be neglected. Hence, the bending stress is used as the basis for design calculations. The critical section or the section of maximum bending stress may be obtained by drawing a parabola through A and tangential to the tooth curves at B and C. This parabola, as shown dotted in Fig., outlines a beam of uniform strength, *i.e.* if the teeth are shaped like a parabola, it will have the same stress at all the sections. But the tooth is larger than the parabola at every section except BC. We therefore, conclude that the section BC is the section of maximum stress or the critical section. The maximum value of the bending stress (or the permissible working stress), at the section BC is given by

$$\sigma_w = M.y / I \qquad ...(i)$$

where M = Maximum bending moment at the critical section $BC = W_T \times h$,

W_T = Tangential load acting at the tooth,

h = Length of the tooth,

y = Half the thickness of the tooth (t) at critical section $BC = t/2$,

I = Moment of inertia about the centre line of the tooth = $b.t^3/12$,

b = Width of gear face.

Substituting the values for M, y and I in equation (i), we get

$$\sigma_w = \frac{(W_T \times h)\ t/2}{b.t^3/12} = \frac{(W_T \times h) \times 6}{b.t^2}$$

or

$$W_T = \sigma_w \times b \times t^2 / 6\ h$$

In this expression, t and h are variables depending upon the size of the tooth (*i.e.* the circular pitch) and its profile.

Let $t = x \times p_c$, and $h = k \times p_c$; where x and k are constants.

$$\therefore \quad W_T = \sigma_w \times b \times \frac{x^2 . p_c^2}{6k.p_c} = \sigma_w \times b \times p_c \times \frac{x^2}{6k}$$

Substituting $x^2 / 6k = y$, another constant, we have

$$W_T = \sigma_w . b . p_c . y = \sigma_w . b . \pi m . y \qquad ...(\because p_c = \pi m)$$

The quantity y is known as ***Lewis form factor*** or ***tooth form factor*** and W_T (which is the tangential load acting at the tooth) is called the ***beam strength of the tooth.***

Since $y = \frac{x^2}{6k} = \frac{t^2}{(p_c)^2} \times \frac{p_c}{6h} = \frac{t^2}{6h.p_c}$, therefore in order to find the value of y, the quantities t, h and p_c may be determined analytically or measured from the drawing similar to Fig. 28.12. It may be noted that if the gear is enlarged, the distances t, h and p_c will each increase proportionately. Therefore the value of y will remain unchanged. A little consideration will show that the value of y is independent of the size of the tooth and depends only on the number of teeth on a gear and the system of teeth. The value of y in terms of the number of teeth may be expressed as follows :

$$y = 0.124 - \frac{0.684}{T}, \text{ for } 14\tfrac{1}{2}^\circ \text{ composite and full depth involute system.}$$

$$= 0.154 - \frac{0.912}{T}, \text{ for } 20^\circ \text{ full depth involute system.}$$

$$= 0.175 - \frac{0.841}{T}, \text{ for } 20^\circ \text{ stub system.}$$

Permissible Working Stress for Gear Teeth in the Lewis Equation

The permissible working stress (σ_w) in the Lewis equation depends upon the material for which an allowable static stress (σ_a) may be determined. The *allowable static stress* is the stress at the elastic limit of the material. It is also called the *basic stress*. In order to account for the dynamic effects which become more severe as the pitch line velocity increases, the value of σ_W is reduced. According to the Barth formula, the permissible working stress,

$$\sigma_w = \sigma_o \times C_v$$

where σ_o = Allowable static stress, and

C_v = Velocity factor.

The values of the velocity factor (C_v) are given as follows :

$$C_v = \frac{3}{3+v}, \text{ for ordinary cut gears operating at velocities upto 12.5 m / s.}$$

$$= \frac{4.5}{4.5+v}, \text{ for carefully cut gears operating at velocities upto 12.5 m/s.}$$

$$= \frac{6}{6+v}, \text{ for very accurately cut and ground metallic gears operating at velocities upto 20 m / s.}$$

$$= \frac{0.75}{0.75+\sqrt{v}}, \text{ for precision gears cut with high accuracy and operating at velocities upto 20 m / s.}$$

$$= \left(\frac{0.75}{1+v}\right) + 0.25, \text{ for non-metallic gears.}$$

In the above expressions, v is the pitch line velocity in metres per second.

The following table shows the values of allowable static stresses for the different gear materials.

Dynamic Tooth Load

In previous article, the velocity factor was used to make approximate allowance for the effect of dynamic loading. The dynamic loads are due to the following reasons:

1. Inaccuracies of tooth spacing,
2. irregularities in tooth profiles, and
3. Deflection of teeth under load.

Closer approximations to the actual conditions may be made by the use of equations based on extensive series of tests, as follows:

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1. Inaccuracies of tooth spacing,
2. Irregularities in tooth profiles, and
3. Deflections of teeth under load.

A closer approximation to the actual conditions may be made by the use of equations based on extensive series of tests, as follows :

$$W_D = W_T + W_I$$

where W_D = Total dynamic load,

W_T = Steady load due to transmitted torque, and

W_I = Increment load due to dynamic action.

The increment load (W_I) depends upon the pitch line velocity, the face width, material of the gears, the accuracy of cut and the tangential load. For average conditions, the dynamic load is determined by using the following Buckingham equation, *i.e.*

$$W_D = W_T + W_I = W_T + \frac{21\,v\,(b.C + W_T)}{21\,v + \sqrt{b.C + W_T}} \quad ...(i)$$

where W_D = Total dynamic load in newtons,

W_T = Steady transmitted load in newtons,

v = Pitch line velocity in m/s,

b = Face width of gears in mm, and

C = A deformation or dynamic factor in N/mm.

A deformation factor (C) depends upon the error in action between teeth, the class of cut of the gears, the tooth form and the material of the gears. The following table shows the values of deformation factor (C) for checking the dynamic load on gears.

The increment load W_I depends upon the pitch line velocity, the face width, material of the gears, the accuracy of cut and tangential load. For average conditions, the dynamic load is determined by using the following Buckingham equation i.e.

Where W_D = Total dynamic load in newtons,

W_T = steady transmitted load in newtons,

v= Pitch line velocity in m/s

b= face width of gears in mm and

C = a deformation or dynamic factor in N/mm.

A deformation factor (C) depends upon the error in action between teeth, the class of cut of the gears, the tooth form and the material of the gears.

The value of C in N/mm may be determined by using the following relation:

where K = A factor depending upon the form of the teeth.

= 0.107, for $14\frac{1}{2}°$ full depth involute system.

= 0.111, for 20° full depth involute system.

= 0.115 for 20° stub system.

E_P = Young's modulus for the material of the pinion in N/mm^2.

Static Tooth Load

The ***static tooth load*** (also called ***beam strength*** or ***endurance strength*** of the tooth) is obtained by Lewis formula by substituting flexural endurance limit or elastic limit stress (σ_e) in place of permissible working stress (σ_w).

∴ Static tooth load or beam strength of the tooth,

$$W_S = \sigma_e . b . p_c . y = \sigma_e . b . \pi\, m . y$$

For safety, against tooth breakage, the static tooth load (W_S) should be greater than the dynamic load (W_D). Buckingham suggests the following relationship between W_S and W_D.

For steady loads, $W_S \geq 1.25\, W_D$

For pulsating loads, $W_S \geq 1.35\, W_D$

For shock loads, $W_S \geq 1.5\, W_D$

Note : For steel, the flexural endurance limit (σ_e) may be obtained by using the following relation :

$$\sigma_e = 1.75 \times \text{B.H.N. (in MPa)}$$

Wear Tooth Load

The maximum load that gear teeth can carry, without premature wear, depends upon the radii of curvature of the tooth profiles and on the elasticity and surface fatigue limits of the materials. The maximum or the limiting load for satisfactory wear of gear teeth, is obtained by using the following Buckingham equation, *i.e.*

$$W_w = D_P . b . Q . K$$

where W_w = Maximum or limiting load for wear in newtons,

D_P = Pitch circle diameter of the pinion in mm,

b = Face width of the pinion in mm,

Q = Ratio factor

$$= \frac{2 \times V.R.}{V.R. + 1} = \frac{2T_G}{T_G + T_P}, \text{ for external gears}$$

$$= \frac{2 \times V.R.}{V.R. - 1} = \frac{2T_G}{T_G - T_P}, \text{ for internal gears.}$$

$V.R.$ = Velocity ratio = T_G / T_P,

K = Load-stress factor (also known as material combination factor) in N/mm^2.

The load stress factor depends upon the maximum fatigue limit of compressive stress, the pressure angle and the modulus of elasticity of the materials of the gears. According to Buckingham, the load stress factor is given by the following relation :

$$K = \frac{(\sigma_{es})^2 \sin\phi}{1.4}\left(\frac{1}{E_P} + \frac{1}{E_G}\right)$$

where σ_{es} = Surface endurance limit in MPa or N/mm^2,

ϕ = Pressure angle,

E_P = Young's modulus for the material of the pinion in N/mm^2, and

E_G = Young's modulus for the material of the gear in N/mm^2.

Notes : 1. The surface endurance limit for steel may be obtained from the following equation :

$$\sigma_{es} = (2.8 \times \text{B.H.N.} - 70) \text{ N/mm}^2$$

2. The maximum limiting wear load (W_w) must be greater than the dynamic load (W_D).

Causes of Gear Tooth Failure

The different modes of failure of gear teeth and their possible remedies to avoid the failure, are as follows:

1. *Bending failure*. Every gear tooth acts as a cantilever. If the total repetitive dynamic load acting on the gear tooth is greater than the beam strength of the gear tooth, then the gear tooth with fail in bending, *i.e*. the gear tooth with break. In order to avoid such failure, the module and face width of the gear is adjusted so that the beam strength is greater than the dynamic load.

2. *Pitting*. It is the surface fatigue failure which occurs due to many repetitions of Hertz contact stresses. The failure occurs when the surface contact stresses are higher than the endurance limit of the material. The failure starts with the formation of pits which continue to grow resulting in the rupture of the tooth surface. In order to avoid the pitting, the dynamic load between the gear tooth should be less than the wear strength of the gear tooth.

3. *Scoring*. The excessive heat is generated when there is an excessive surface pressure, high speed or supply of lubricant fails. It is a stick-slip phenomenon in which alternate shearing and welding takes place rapidly at high spots. This type of failure can be avoided by properly designing the parameters such as speed, pressure and proper flow of the lubricant, so that the temperature at the rubbing faces is within the permissible limits.

4. *Abrasive wear*. The foreign particles in the lubricants such as dirt, dust or burr enter between the tooth and damage the form of tooth. This type of failure can be avoided by providing filters for the lubricating oil or by using high viscosity lubricant oil which enables the formation of thicker oil film and hence permits easy passage of such particles without damaging the gear surface.

5. *Corrosive wear.* The corrosion of the tooth surfaces is mainly caused due to the presence of corrosive elements such as additives present in the lubricating oils. In order to avoid this type of wear, proper anti-corrosive additives should be used.

Design Procedure for Spur Gears

In order to design spur gears, the following procedure may be followed :

1. First of all, the design tangential tooth load is obtained from the power transmitted and the pitch line velocity by using the following relation :

$$W_T = \frac{P}{v} \times C_S \qquad \text{...(i)}$$

where W_T = Permissible tangential tooth load in newtons,

P = Power transmitted in watts,

*v = Pitch line velocity in m / s $= \dfrac{\pi D N}{60}$,

D = Pitch circle diameter in metres,

N = Speed in r.p.m., and

C_S = Service factor.

Note : **1.** The above values for service factor are for enclosed well lubricated gears. In case of non-enclosed and grease lubricated gears, the values given in the above table should be divided by 0.65.

2. Apply the Lewis equation as follows :

$$W_T = \sigma_w.b.p_c.y = \sigma_w.b.\pi\, m.y$$

$$= (\sigma_o.C_v)\, b.\pi\, m.y \qquad ...(\because \sigma_w = \sigma_o.C_v)$$

Notes : (*i*) The Lewis equation is applied only to the weaker of the two wheels (*i.e.* pinion or gear).

(*ii*) When both the pinion and the gear are made of the same material, then pinion is the weaker.

(*iii*) When the pinion and the gear are made of different materials, then the product of $(\sigma_w \times y)$ or $(\sigma_o \times y)$ is the *deciding factor. The Lewis equation is used to that wheel for which $(\sigma_w \times y)$ or $(\sigma_o \times y)$ is less.

(*iv*) The product $(\sigma_w \times y)$ is called ***strength factor*** of the gear.

(*v*) The face width (b) may be taken as $3\,p_c$ to $4\,p_c$ (or $9.5\,m$ to $12.5\,m$) for cut teeth and $2\,p_c$ to $3\,p_c$ (or $6.5\,m$ to $9.5\,m$) for cast teeth.

3. Calculate the dynamic load (W_D) on the tooth by using Buckingham equation, *i.e.*

$$W_D = W_T + W_I$$

$$= W_T + \frac{21v\,(b.C + W_T)}{21v + \sqrt{b.C + W_T}}$$

In calculating the dynamic load (W_D), the value of tangential load (W_T) may be calculated by neglecting the service factor (C_S) *i.e.*

$$W_T = P / v, \text{ where } P \text{ is in watts and } v \text{ in m / s.}$$

4. Find the static tooth load (*i.e.* beam strength or the endurance strength of the tooth) by using the relation,

$$W_S = \sigma_e.b.p_c.y = \sigma_e.b.\pi\, m.y$$

For safety against breakage, W_S should be greater than W_D.

5. Finally, find the wear tooth load by using the relation,

$$W_w = D_P.b.Q.K$$

The wear load (W_w) should not be less than the dynamic load (W_D).

Problem-1:

The following particulars of a single reduction spur gear are given:

Gear ratio = 10: 1; Distance between centers = 660 mm approximately; Pinion transmits 500kW at 1800 r.p.m.; Involute teeth of standard proportions (addendum = with pressure angle of 22.5°; Permissible normal pressure between teeth = 175 N per mm of width. Find :1. The nearest standard module if no interference is to occur;

2. The number of teeth on each wheel;

3. The necessary width of the pinion; and

4. The load on the bearings of the wheels due to power transmitted.

Solution : Given : $G = T_G / T_P = D_G / D_P = 10$; $L = 660$ mm ; $P = 500$ kW $= 500 \times 10^3$ W ; $N_P = 1800$ r.p.m. ; $\phi = 22.5°$; $W_N = 175$ N/mm width

1. *Nearest standard module if no interference is to occur*

Let m = Required module,

T_P = Number of teeth on the pinion,

T_G = Number of teeth on the gear,

D_P = Pitch circle diameter of the pinion, and

D_G = Pitch circle diameter of the gear.

We know that minimum number of teeth on the pinion in order to avoid interference,

$$T_P = \frac{2 A_W}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1\right]}$$

$$= \frac{2 \times 1}{10\left[\sqrt{1 + \frac{1}{10}\left(\frac{1}{10} + 2\right)\sin^2 22.5°} - 1\right]} = \frac{2}{0.15} = 13.3 \text{ say } 14$$

$$\dots (\because A_W = 1 \text{ module})$$

$$\therefore \quad T_G = G \times T_P = 10 \times 14 = 140 \qquad \dots(\because T_G / T_P = 10)$$

Problem-2:

A bronze spur pinion rotating at 600 r.p.m. drives a cast iron spur gear at a transmission ratio of 4: 1. The allowable static stresses for the bronze pinion and cast iron gear are 84 MPa and 105 MPa respectively. The pinion has 16 standard 20° full depth involute teeth of module 8 mm. The face width of both the gears is 90 mm. Find the power that can be transmitted from the standpoint of strength.

Solution. Given : $N_P = 600$ r.p.m. ; $V.R. = T_G / T_P = 4$; $\sigma_{OP} = 84$ MPa $= 84$ N / mm^2 ; $\sigma_{OG} = 105$ MPa $= 105$ N/mm^2 ; $T_P = 16$; $m = 8$ mm ; $b = 90$ mm

We know that pitch circle diameter of the pinion,

$$D_P = m.T_P = 8 \times 16 = 128 \text{ mm} = 0.128 \text{ m}$$

$\therefore$ Pitch line velocity,

$$v = \frac{\pi D_P . N_P}{60} = \frac{\pi \times 0.128 \times 600}{60} = 4.02 \text{ m/s}$$

Since the pitch line velocity (v) is less than 12.5 m/s, therefore velocity factor,

$$C_v = \frac{3}{3+v} = \frac{3}{3+4.02} = 0.427$$

We know that for 20° full depth involute teeth, tooth form factor for the pinion,

$$y_P = 0.154 - \frac{0.912}{T_P} = 0.154 - \frac{0.912}{16} = 0.097$$

and tooth form factor for the gear,

$$y_G = 0.154 - \frac{0.912}{T_G} = 0.154 - \frac{0.912}{4 \times 16} = 0.14 \qquad \ldots (\because T_G / T_P = 4)$$

$$\therefore \qquad \sigma_{OP} \times y_P = 84 \times 0.097 = 8.148$$

$$\text{and} \qquad \sigma_{OG} \times y_G = 105 \times 0.14 = 14.7$$

Since ($\sigma_{OP} \times y_P$) is less than ($\sigma_{OG} \times y_G$), therefore the pinion is weaker. Now using the Lewis equation for the pinion, we have tangential load on the tooth (or beam strength of the tooth),

$$W_T = \sigma_{wP}.b.\pi\, m.y_P = (\sigma_{OP} \times C_v)\, b.\pi\, m.y_P \qquad (\because \sigma_{WP} = \sigma_{OP}.C_v)$$

$$= 84 \times 0.427 \times 90 \times \pi \times 8 \times 0.097 = 7870 \text{ N}$$

$\therefore$ Power that can be transmitted

$$= W_T \times v = 7870 \times 4.02 = 31\,640 \text{ W} = 31.64 \text{ kW}$$ **Ans.**

Problem-3

A pair of straight teeth spur gears is to transmit 20 kW when the pinion rotates at 300 r.p.m. The velocity ratio is 1 : 3. The allowable static stresses for the pinion and gear materials are 120 MPa and 100 MPa respectively.

The pinion has 15 teeth and its face width is 14 times the module. Determine: 1. module; 2. face width; and 3. pitch circle diameters of the pinion and the gear from the standpoint of strength only, taking into consideration the effect of the dynamic loading. The tooth form factor y can be taken as

0.912

y 0.154 ~~No.of teeth~~

and the velocity factor C_V as

$C_V 3 \frac{3}{\quad} v$,where v is expressed in m/s

Solution. Given : P = 20 kW = 20 × 10^3 W ; N_P = 300 r.p.m. ; $V.R.$ = T_G / T_P =3 ; σ_{OP} = 120 MPa = 120 N/mm² ; σ_{OG} = 100 MPa = 100 N/mm² ; T_P = 15 ; b = 14 module = 14 m

1. ***Module***

Let m = Module in mm, and

D_P = Pitch circle diameter of the pinion in mm.

We know that pitch line velocity,

$$v = \frac{\pi D_P N_P}{60} = \frac{\pi m . T_P . N_P}{60} \qquad \ldots (\because D_P = m.T_P)$$

$$= \frac{\pi m \times 15 \times 300}{60} = 236\ m \text{ mm/s} = 0.236\ m \text{ m/s}$$

Assuming steady load conditions and 8-10 hours of service per day, the service factor (C_S) from Table 28.10 is given by

$$C_S = 1$$

We know that design tangential tooth load,

$$W_T = \frac{P}{v} \times C_S = \frac{20 \times 10^3}{0.236\ m} \times 1 = \frac{84\ 746}{m} \text{ N}$$

and velocity factor,
$$C_v = \frac{3}{3 + v} = \frac{3}{3 + 0.236\ m}$$

We know that tooth form factor for the pinion,

$$y_P = 0.154 - \frac{0.912}{T_P} = 0.154 - \frac{0.912}{15}$$
$$= 0.154 - 0.0608 = 0.0932$$

and tooth form factor for the gear,

$$y_G = 0.154 - \frac{0.912}{T_G} = 0.154 - \frac{0.912}{3 \times 15}$$
$$= 0.154 - 0.203 = 0.1337 \quad \ldots (\because T_G = 3T_P)$$

$\therefore \quad \sigma_{OP} \times y_P = 120 \times 0.0932 = 11.184$

and $\quad \sigma_{OG} \times y_G = 100 \times 0.1337 = 13.37$

Since $(\sigma_{OP} \times y_P)$ is less than $(\sigma_{OG} \times y_G)$, therefore the pinion is weaker. Now using the Lewis equation to the pinion, we have

$$W_T = \sigma_{wP}.b.\pi\, m.y_P = (\sigma_{OP} \times C_v)\, b.\pi\, m\,.\, y_P$$

$$\therefore \quad \frac{84\,746}{m} = 120\left(\frac{3}{3 + 0.236\, m}\right) 14\, m \times \pi\, m \times 0.0932 = \frac{1476\, m^2}{3 + 0.236\, m}$$

or $\quad 3 + 0.236\, m = 0.0174\, m^3$

Solving this equation by hit and trial method, we find that

$$m = 6.4 \text{ mm}$$

The standard module is 8 mm. Therefore let us take

$$m = 8 \text{ mm } \textbf{Ans.}$$

2. *Face width*

We know that the face width,

$$b = 14\, m = 14 \times 8 = 112 \text{ mm } \textbf{Ans.}$$

3. *Pitch circle diameter of the pinion and gear*

We know that pitch circle diameter of the pinion,

$$D_P = m.T_P = 8 \times 15 = 120 \text{ mm } \textbf{Ans.}$$

and pitch circle diameter of the gear,

$$D_G = m.T_G = 8 \times 45 = 360 \text{ mm } \textbf{Ans.} \quad \ldots (\because T_G = 3\, T_P)$$

Problem-4

A gear drive is required to transmit a maximum power of 22.5 kW. The velocity ratio is 1:2 and r.p.m. of the pinion is 200. The approximate centre distance between the shafts may be taken as 600 mm. The teeth have 20° stub involute profiles. The static stress for the gear material (which is cast iron) may be taken as 60 MPa and face width as 10 times the module. Find the module, face width and number of teeth on each gear. Check the design for dynamic and wear loads. The deformation or dynamic factor in the Buckingham equation may be taken as 80 and the material combination factor for the wear as 1.4.

Solution. Given : $P = 22.5$ kW = 22 500 W ; $V.R. = D_G/D_P = 2$; $N_P = 200$ r.p.m. ; $L = 600$ mm ; $\sigma_{OP} = \sigma_{OG} = 60$ MPa = 60 N/mm^2 ; $b = 10$ m ; $C = 80$; $K = 1.4$

Module

Let m = Module in mm,

D_P = Pitch circle diameter of the pinion, and

D_G = Pitch circle diameter of the gear.

We know that centre distance between the shafts (L),

$$600 = \frac{D_P}{2} + \frac{D_G}{2} = \frac{D_P}{2} + \frac{2\,D_P}{2} = 1.5\ D_P \qquad \text{... } (\because D_G = V.R. \times D_P)$$

$\therefore$ $D_P = 600 / 1.5 = 400$ mm = 0.4 m

and $D_G = 2\ D_P = 2 \times 400 = 800$ mm = 0.8 m

Since both the gears are made of the same material, therefore pinion is the weaker. Thus the design will be based upon the pinion.

We know that pitch line velocity of the pinion,

$$v = \frac{\pi D_P \,.\, N_P}{60} = \frac{\pi \times 0.4 \times 200}{60} = 4.2 \text{ m / s}$$

Since v is less than 12 m / s, therefore velocity factor,

$$C_v = \frac{3}{3 + v} = \frac{3}{3 + 4.2} = 0.417$$

We know that number of teeth on the pinion,

$$T_P = D_P / m = 400 / m$$

$\therefore$ Tooth form factor for the pinion,

$$y_P = 0.175 - \frac{0.841}{T_P} = 0.175 - \frac{0.841 \times m}{400} \qquad \text{... (For 20° stub system)}$$

$$= 0.175 - 0.0021\ m \qquad \text{...}(i)$$

Assuming steady load conditions and 8–10 hours of service per day, the service factor (C_S) from Table 28.10 is given by

$$C_S = 1$$

We know that design tangential tooth load,

$$W_T = \frac{P}{v} \times C_S = \frac{22\ 500}{4.2} \times 1 = 5357 \text{ N}$$

We also know that tangential tooth load (W_T),

$$5357 = \sigma_{wP} \,.\, b.\, \pi\, m.y_P = (\sigma_{OP} \times C_v)\, b.\pi\, m.y_P$$
$$= (60 \times 0.417)\ 10\ m \times \pi\, m\,(0.175 - 0.0021\ m)$$
$$= 137.6\ m^2 - 1.65\ m^3$$

Solving this equation by hit and trial method, we find that

$$m = 0.65 \text{ say } 8 \text{ mm } \textbf{Ans.}$$

Face width

We know that face width,

$$b = 10\ m = 10 \times 8 = 80 \text{ mm } \textbf{Ans.}$$

Number of teeth on the gears

We know that number of teeth on the pinion,

$$T_P = D_P / m = 400 / 8 = 50 \textbf{ Ans.}$$

and number of teeth on the gear,

$$T_G = D_G / m = 800 / 8 = 100 \textbf{ Ans.}$$

Checking the gears for dynamic and wear load

We know that the dynamic load,

$$W_D = W_T + \frac{21v\,(b.C + W_T)}{21v + \sqrt{b.C + W_T}}$$
$$= 5357 + \frac{21 \times 4.2\ (80 \times 80 + 5357)}{21 \times 4.2 + \sqrt{80 \times 80 + 5357}}$$

$$= 5357 + \frac{1.037 \times 10^6}{196.63} = 5357 + 5273 = 10\ 630 \text{ N}$$

From equation (*i*), we find that tooth form factor for the pinion,

$$y_P = 0.175 - 0.0021\ m = 0.175 - 0.0021 \times 8 = 0.1582$$

From Table 28.8, we find that flexural endurance limit (σ_e) for cast iron is 84 MPa or 84 N/mm^2.

∴ Static tooth load or endurance strength of the tooth,

$$W_S = \sigma_e \,.\, b.\, \pi\, m.\, y_P = 84 \times 80 \times \pi \times 8 \times 0.1582 = 26\ 722 \text{ N}$$

We know that ratio factor,

$$Q = \frac{2 \times V.R.}{V.R. + 1} = \frac{2 \times 2}{2 + 1} = 1.33$$

∴ Maximum or limiting load for wear,

$$W_w = D_P \,.\, b.\, Q \,.\, K = 400 \times 80 \times 1.33 \times 1.4 = 59\ 584 \text{ N}$$

Since both W_S and W_w are greater than W_D, therefore the design is safe.