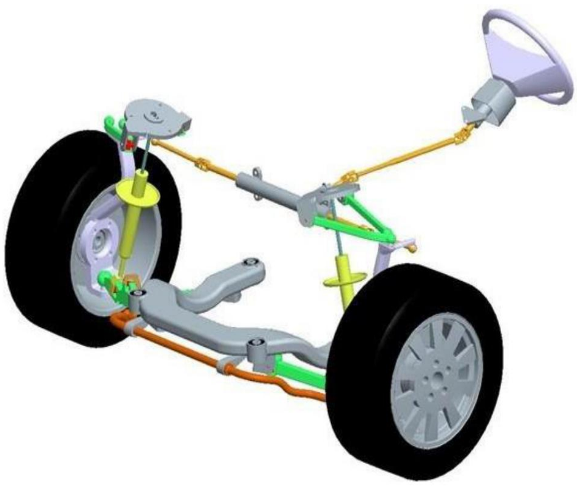
2

Special Mechanisms



Course Contents

2.1 Straight Line Mechanisms

- It permits only relative motion of an oscillatory nature along a straight line. The mechanisms used for this purpose are called ***straight line mechanisms***.
  1. In which only turning pairs are used
  2. In which one sliding pair is used.
- These two types of mechanisms may produce exact straight line motion or approximate straight line motion.
- **Need of Straight Line:**
  1. Sewing Machine converts rotary motion to up/down motion.
  2. Want to constrain pistons to move only in a straight line.
  3. How do you create the first straight edge in the world? (Compass is easy)
  4. Windshield wipers, some flexible lamps made of solid pieces connected by flexible joints.

2.2 Exact Straight Line Motion Mechanisms Made Up Of Turning Pairs

- The principle adopted for a mathematically correct or exact straight line motion is described in Fig.4.1
- Let O be a point on the circumference of a circle of diameter OP. Let OA be any chord and B is a point on OA produced, such that

$$OA \times OB = \text{constant}$$

- The triangles OAP and OBQ are similar.

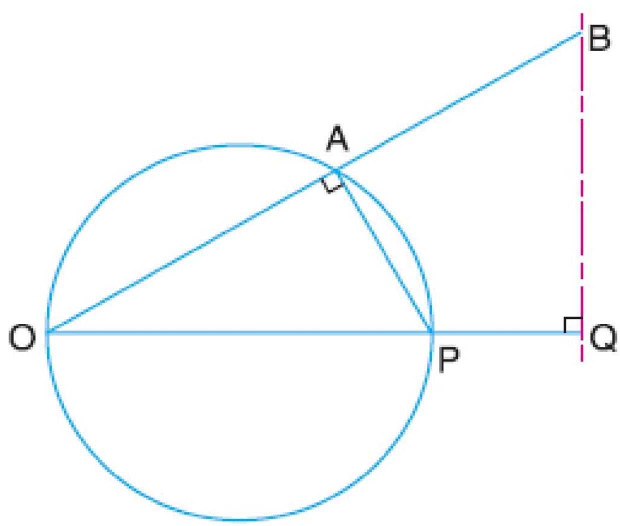


Fig. 4.1 Exact straight line motion mechanism

$$\frac{OA}{OP} = \frac{OQ}{OB}$$

$$OP \times OQ = OA \times OB$$

$$OQ = \frac{OA \times OB}{OP}$$

- But *OP* is constant as it is the diameter of a circle; therefore, if *OA* × *OB* is constant, then *OQ* will be constant.
- Hence

$$OA \times OB = constant$$

- So point B moves along the straight line.

2.3 Peaucellier Mechanism (Exact Straight Line)

- It consists of a fixed link OO1 and the other straight links O1A, OC, OD, AD, DB, BC and CA are connected by turning pairs at their intersections, as shown in Fig. 4.2
- The pin at A is constrained to move along the circumference of a circle with the fixed diameter OP, by means of the link O1A. In Fig. 4.2
- AC = CB = BD = DA
- OC = OD
- $OO_1 = O_1A$

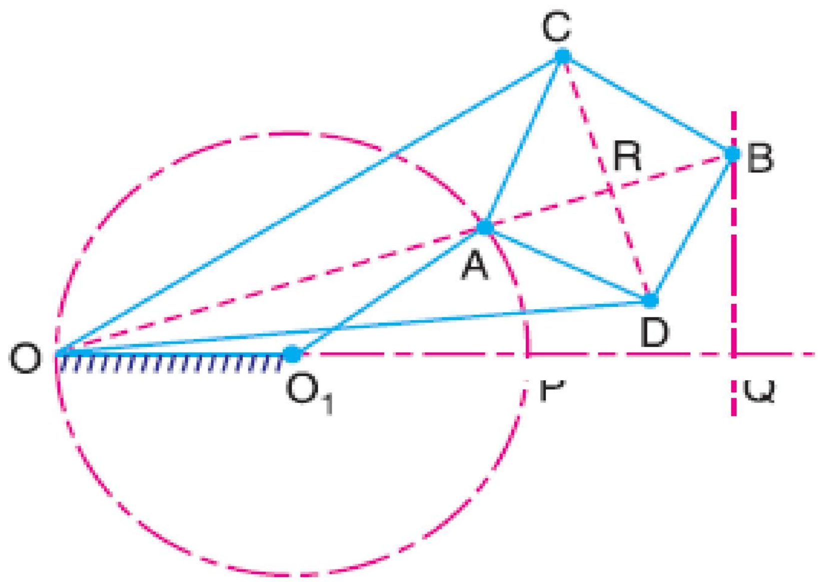


Fig. 4.2 Peaucellier Mechanism

- From right angled triangles *ORC* and *BRC*, we have

$$OC^2 = OR^2 + RC^2 \qquad \text{(I)}$$

$$BC^2 = RB^2 + RC^2 \qquad (ii)$$

- From (i) and (ii)

$$OC^2 - BC^2 = OR^2 - RB^2$$

$$= (OR - RB)(OR + RB)$$

$$= OB \times OA$$

- Since OC and BC are of constant length, therefore the product $OB \times OA$ remains constant.

Hart's Mechanism

- This mechanism requires only six links as compared with the eight links required by the Peaucellier mechanism.
- It consists of a fixed link OO1 and other straight links O1A, FC, CD, DE and EF are connected by turning pairs at their points of intersection, as shown in Fig. 4.3.
- The links FC and DE are equal in length and the lengths of the links CD and EF are also equal. The points O, A and B divide the links FC, CD and EF in the same ratio. A little consideration will show that BOCE is a trapezium and OA and OB are respectively parallel to FD and CE.

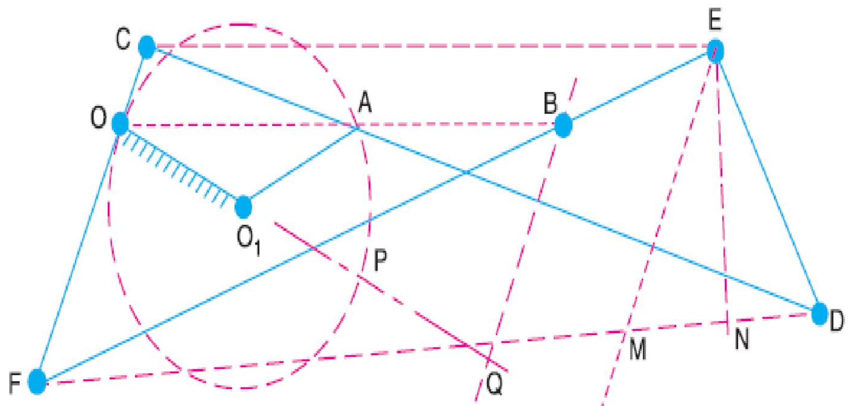


Fig. 4.3 Hart's Mechanism

- Here, FC = DE & CD = EF
- The point O, A and B divide the links FC, CD and EF in the same ratio.
- From similar triangles CFE and OFB,

$$\frac{CE}{FC} = \frac{OB}{OF} \text{ or CB} = \frac{CE \times OF}{FC} \ldots \ldots \text{(i)}$$

- From similar triangle FCD and OCA

$$\frac{FD}{FC} = \frac{OA}{OC} \quad or\ OA = \frac{FD \times OC}{FC} \quad \ldots . (ii)$$

- From above equations,

$$OA \times OB = \frac{FD \times OC}{FC} \times \frac{CE \times OF}{FC}$$

$$= FD \times CE \times \frac{OC \times OF}{FC^2}$$

- Since the lengths of OC, OF and FC are fixed, therefore

$$OA \times OB = FD \times CE \times cons. \ldots (iii)$$

- From point E, draw EM parallel to CF and EN perpendicular to FD.

$$FD \times CE = FD \times FM \quad (CE = FM)$$
$$= (FN + ND)(FN - MN)$$
$$= FN^2 - ND^2 \; (\text{MN} = \text{ND})$$
$$= (FE^2 - NE^2) - (ED^2 - NE^2) \text{ (From right}$$

angle triangles FEN and EDN)
$$= E^2 - ED^2 = constant \quad (iv)$$

– From equation (iii) and (iv),

$$OA \times OB = constant$$

Exact Straight Line Motion consisting of one sliding pair-Scott Russell's Mechanism

– A is the middle point of PQ and OA = AP = AQ. The instantaneous center for the link PAQ lies at I in OA produced and is such that IP is perpendicular to OP.

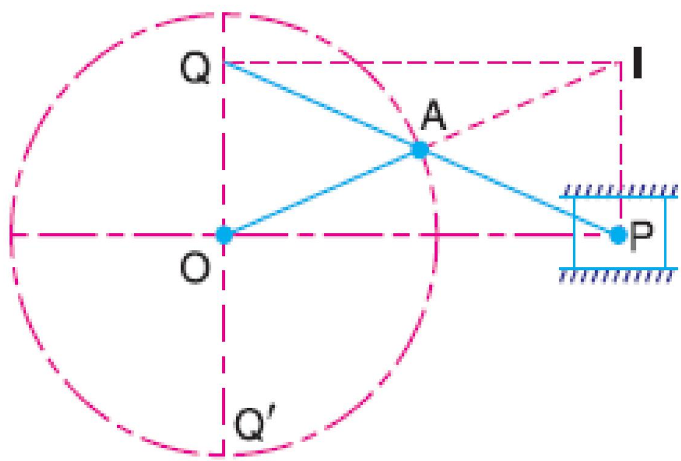


Fig. 4.4 Scott Russell's Mechanism

– Join IQ. Then Q moves along the perpendicular to IQ. Since OPIQ is a rectangle and IQ is perpendicular to OQ, therefore Q moves along the vertical line OQ for all positions of QP. Hence Q traces the straight line OQ′.

– If OA makes one complete revolution, then P will oscillate along the line OP through a distance 2 OA on each side of O and Q will oscillate along OQ′ through the same distance 2 OA above and below O. Thus, the locus of Q is a copy of the locus of P.

Approximate straight line motion mechanisms

Watt's Mechanism

- It has four links as shown in fig. OB, O1A, AB and OO1.

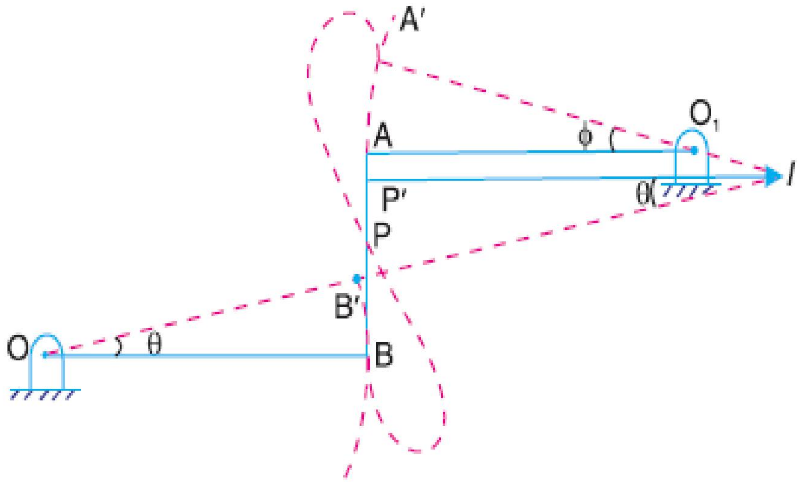


Fig. 4.5 watt's mechanism

- OB and O1A oscillates about centers O and O1 respectively. P is a point on AB such that,

$$\frac{O_1}{OB} = \frac{PB}{PA}$$

- As OB oscillates the point P will describe an approximate straight line.

Modified Scott-Russel Mechanism

- This is similar to Scott-Russel mechanism but in this case AP is not equal to AQ and the points P and Q are constrained to move in the horizontal and vertical directions.

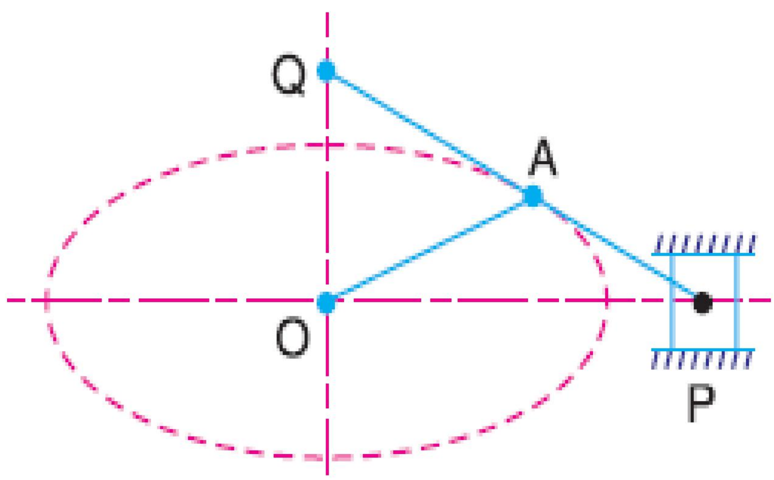


Fig. 4.6 Modified Scott-Russel Mechanisms

- A little consideration will show that it forms an elliptical trammel, so that any point *A* on *PQ* traces an ellipse with semi-major axis *AQ* and semi minor axis *AP*.
- If the point *A* moves in a circle, then for point *Q* to move along an approximate straight line, the length *OA* must be equal (*AP*)2 / *AQ*. This is limited to only small displacement of *P*.

Grasshopper Mechanism

- In this mechanism, the centers O and O1 are fixed. The link OA oscillates about O through an angle AOA1 which causes the pin P to move along a circular arc with O1 as center and O1P as radius.

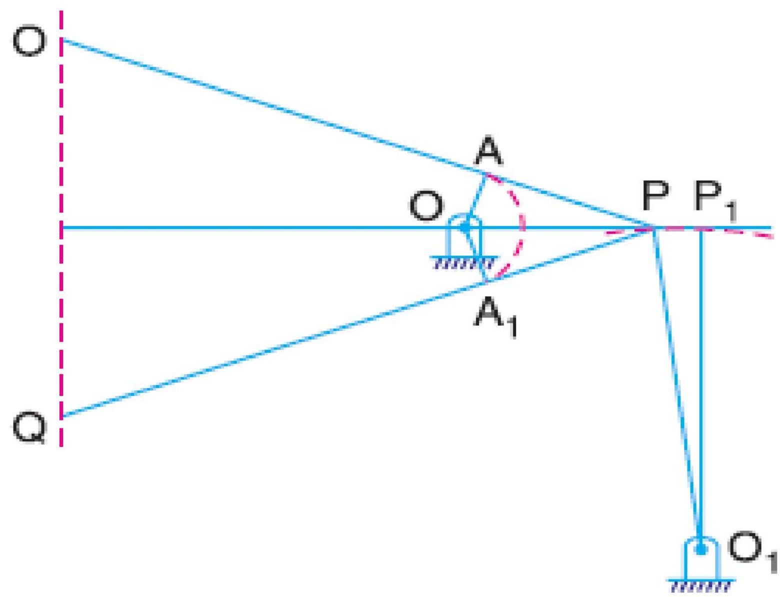


Fig. 4.7 Grasshopper Mechanism

- For small angular displacements of OP on each side of the horizontal, the point Q on the extension of the link PA traces out an approximately a straight path QQ′. if the lengths are such that

$$OA = \frac{AP^2}{AQ}$$

Tchebicheff's Mechanism

- It is a four bar mechanism in which the crossed links OA and O1B are of equal length, as shown in Fig. 4.8.
- The point P, which is the mid-point of AB, traces out an approximately straight line parallel to OO1.

– The proportions of the links are, usually, such that point P is exactly above O or O1 in the extreme positions of the mechanism i.e. when BA lies along OA or when BA lies along BO_1.

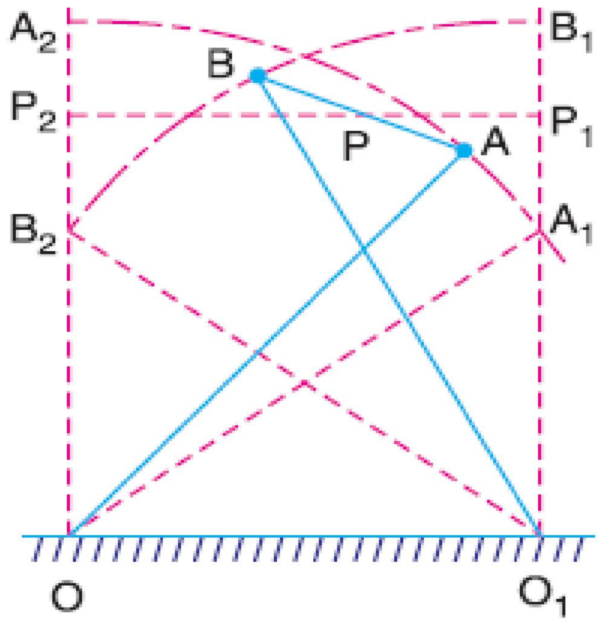


Fig. 4.8 Tchebicheff's mechanism

– It may be noted that the point *P* will lie on a straight line parallel to OO_1, in the two extreme positions and in the mid position, if the lengths of the links are in proportions

$$AB : OO_1 : OA = 1 : 2 : 4.5.$$

Roberts Mechanism

– It is also a four bar chain mechanism, which, in its mean position, has the form of a trapezium. The links OA and O1 B are of equal length and OO1 is fixed. A bar PQ is rigidly attached to the link AB at its middle point P.

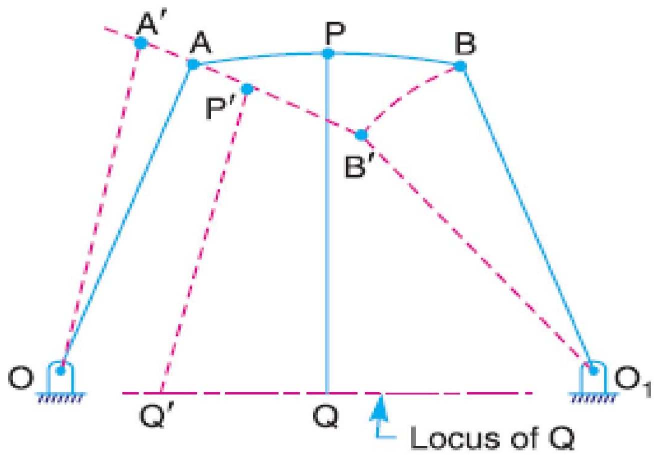


Fig. 4.9 Robert's Mechanism

– A little consideration will show that if the mechanism is displaced as shown by the dotted lines in Fig. the point *Q* will trace out an approximately straight line.

Steering gear mechanism

– The steering gear mechanism is used for changing the direction of two or more of the wheel axles with reference to the chassis, so as to move the automobile in any desired path.

– Usually the two back wheels have a common axis, which is fixed in direction with reference to the chassis and the steering is done by means of the front wheels.

– In automobiles, the front wheels are placed over the front axles, which are pivoted at the points A and B, as shown in Fig. 4.10.

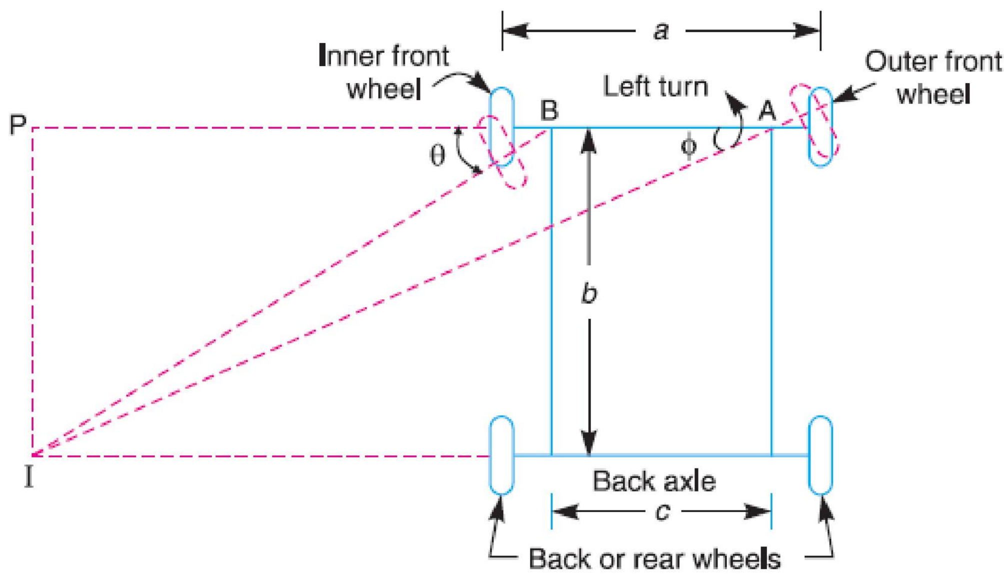


Fig. 4.10 steering gear mechanism

– These points are fixed to the chassis. The back wheels are placed over the back axle, at the two ends of the differential tube. When the vehicle takes a turn, the front wheels along with the respective axles turn about the respective pivoted points. The back wheels remain straight and do not turn. Therefore, the steering is done by means of front wheels only.

– In order to avoid skidding (i.e. slipping of the wheels sideways), the two front wheels must turn about the same instantaneous centre I which lies on the axis of the back wheels. If the instantaneous centre of the two front wheels do not coincide with the instantaneous Centre of the back wheels, the skidding on the front or back wheels will definitely take place, which will cause more wear and tear of the tyres.

- Thus, the condition for correct steering is that all the four wheels must turn about the same instantaneous centre. The axis of the inner wheel makes a larger turning angle θ than the angle φ subtended by the axis of outer wheel.
- Let, a = wheel track
  b = wheel base
  c = Distance between the pivots A and B of the front axle.
- Now from triangle IBP,

$$\cot\theta = \frac{BP}{IP}$$

- And from triangle IAP,

$$\cot \emptyset = \frac{AP}{IP} = \frac{AB + BP}{IP} = \frac{c}{b} + \cot \theta$$

$$\boldsymbol{\cot \emptyset - \cot \theta = \frac{c}{b}}$$

- This is the fundamental equation for correct steering.

Devis Steering Mechanism

- The Davis steering gear is shown in Fig. 9.16. It is an exact steering gear mechanism. The slotted links AM and BH are attached to the front wheel axle, which turn on pivots A and B respectively.
- The rod CD is constrained to move in the direction of its length, by the sliding members at P and Q. These constraints are connected to the slotted link AM and BH by a sliding and a turning pair at each end. The steering is affected by moving CD to the right or left of its normal position. C ′D′ shows the position of CD for turning to the left.

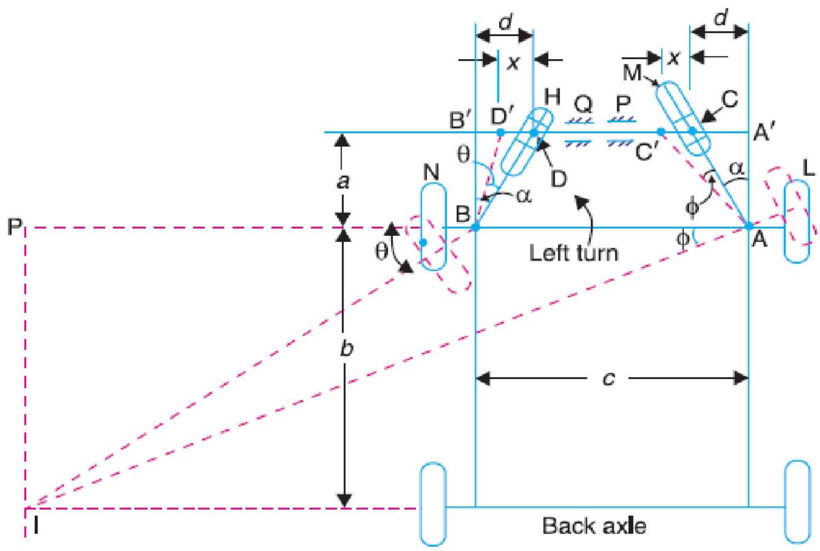


Fig. 4.11 Devis steering gear mechanism

– Let,

a = Vertical distance between AB and CD,
b = Wheel base,
d = Horizontal distance between AC and BD,
c = Distance between the pivots A and B of the front axle.
x = Distance moved by AC to AC ′ = CC ′ = DD′, and
α = Angle of inclination of the links AC and BD, to the vertical.

– From triangle AA’C ′

$$\tan(\alpha + \emptyset) = \frac{A'C'}{A'A'} = \frac{d + x}{a} \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

– From triangle AA’C

$$\tan\alpha = \frac{A'C}{A'A'} = \frac{d}{a} \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

– From triangle BB’D’

$$\tan(\alpha - \theta) = \frac{B'D'}{BB'} = \frac{d - x}{a} \ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

– We know that,

$$\tan(\alpha + \emptyset) = \frac{\tan\alpha + \tan\emptyset}{1 - \tan\alpha \times \tan\emptyset}$$

$$\frac{d + x}{a} = \frac{{}^{d}/_{a} + \tan\emptyset}{1 - /_{a} \times \tan\emptyset} = \frac{d + (\ \times \tan\emptyset)}{a - (\ \times \tan\emptyset)}$$

$$d \cdot x \times (a - d \times \tan\emptyset) = a \times (d + a \times \tan\emptyset)$$

$$a \cdot d - d^2 \times \tan\emptyset + a \cdot x - d \times x \times \tan\emptyset = a \cdot d + {}^2 \times \tan\emptyset$$

$$\tan\emptyset \times (a^2 + d^2 + d \cdot x) = a \cdot x$$

$$\tan\emptyset = \frac{a \cdot x}{(a^2 + d^2 + d \cdot x)} \ldots\ldots\ldots\ldots\ldots (iv)$$

– Similarly from $\tan(\alpha - \theta) = \frac{d-x}{a}$, we get

$$\tan\theta = \frac{a \cdot x}{(a^2 + d^2 - d \cdot x)} \ldots\ldots\ldots\ldots\ldots (v)$$

– We know that for correct steering,

$$\cot\emptyset - \cot\theta = \frac{c}{b}$$

$$\frac{(a^2 + d^2 + d \cdot x)}{a \cdot x} - \frac{(a^2 + {}^2 - d \cdot x)}{a \cdot x} = \frac{c}{b}$$

$$\frac{2\,d}{a} = \frac{c}{b}$$

$$2\ \tan\alpha = \frac{c}{b}$$

$$\tan\alpha = \frac{c}{2\,b}$$

Ackerman steering Gear

- The Ackerman steering gear mechanism is much simpler than Davis gear. The difference between the Ackerman and Davis steering gears are :
  1 The whole mechanism of the Ackerman steering gear is on back of the front wheels;
  whereas in Davis steering gear, it is in front of the wheels.
  2 The Ackerman steering gear consists of turning pairs, whereas Davis steering gear consists of sliding members.

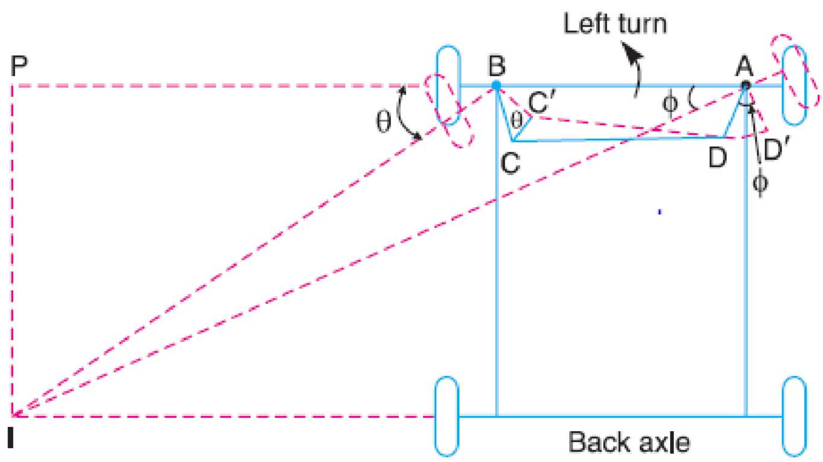


Fig. 4.12 Ackerman steering mechanism

- In Ackerman steering gear, the mechanism ABCD is a four bar crank chain, as shown in Fig. 4.12. The shorter links BC and A D are of equal length and are connected by hinge joints with front wheel axles. The longer links A B and CD are of unequal length.
- The following are the only three positions for correct steering.
  1 When the vehicle moves along a straight path, the longer links A B and CD are parallel and the shorter links BC and AD are equally inclined to the longitudinal axis of the vehicle, as shown by firm lines in Fig. 4.12.
  2 When the vehicle is steering to the left, the position of the gear is shown by dotted lines in Fig. 4.12. In this position, the lines of the front wheel axle intersect on the back wheel axle at I, for correct steering.
  3 When the vehicle is steering to the right, the similar position may be obtained.

Universal or Hooke's Joint

- A Hooke's joint is used to connect two shafts, which are intersecting at a small angle, as shown in Fig.4.10. The end of each shaft is forked to U-type and each fork provides two bearings for the arms of a cross.
- The arms of the cross are perpendicular to each other. The motion is transmitted from the driving shaft to driven shaft through a cross.

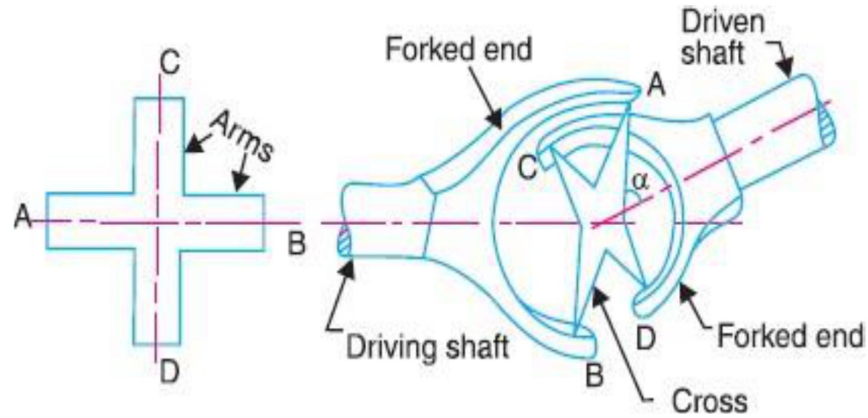


Fig. 4.13 Hooke's Joint

- The main application of the Universal or Hooke's joint is found in the transmission from the gear box to the differential or back axle of the automobiles. It is also used for transmission of power to different spindles of multiple drilling machines.

Ratio of shaft velocities

- The top and front views connecting the two shafts by a universal joint are shown in Fig. 4.11. Let the initial position of the cross be such that both arms lie in the plane of the paper in front view, while the arm AB attached to the driving shaft lies in the plane containing the axes of the two shafts.
- Let the driving shaft rotates through an angle θ, so that the arm AB moves in a circle to a new position A1 B1 as shown in front view.
- A little consideration will show that the arm CD will also move in a circle of the same size. This circle when projected in the plane of paper appears to be an ellipse. Therefore the arm CD takes new position C1D1 on the ellipse, at an angle θ. But the true angle must be on the circular path.
- To find the true angle, project the point C1 horizontally to intersect the circle at C2. Therefore the angle COC2 (equal to φ) is the true angle turned by the driven shaft.

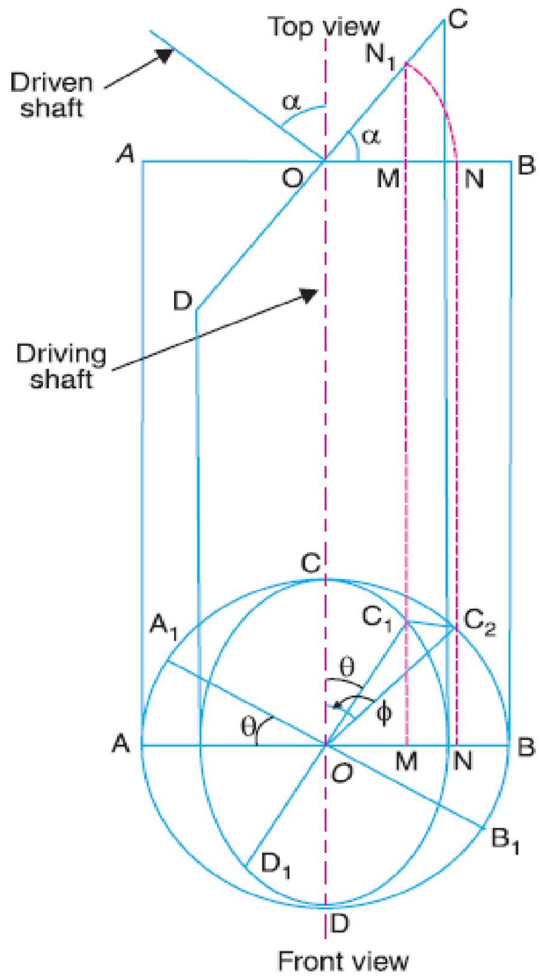


Fig. 4.14 ration of shaft velocities

– In triangle OC_1M, angle $OC_1M = \Theta$

$$\tan\theta = \frac{OM}{MC_1} \ldots\ldots\ldots\ldots (i)$$

– In triangle OC_2N, angle $OC_2N = \emptyset$

$$\tan\emptyset = \frac{ON}{NC_2} = \frac{ON}{MC_1} \ldots\ldots (ii) \quad (NC_2 = MC_1)$$

– Dividing eq. (i) by (ii)

$$\tan\emptyset = \frac{ON}{NC_2} = \frac{ON}{MC_1}$$

– But

$OM = N_1 \cos\alpha = ON\cos\alpha$ (α = angle of inclination of driving and driven shaft)

$$\frac{\tan\theta}{\tan\emptyset} = \frac{ON\cos\alpha}{ON} = \cos\alpha$$

$$\tan\theta = \tan\emptyset \times \cos\alpha \ldots\ldots\ldots\ldots (iii)$$

– Let,

$\omega =$ angular velocity of driving shaft $= \frac{d\theta}{dt}$

$\omega_1 =$ angular velocity of driven shaft $= \frac{d\emptyset}{dt}$

- Differentiating both side of eq. (iii)

$$\sec^2\theta \times \frac{d\theta}{dt} = \cos\alpha \times \sec^2\emptyset \times \frac{\emptyset}{dt}$$

$$\sec^2\theta \times = \cos\alpha \times \sec^2\emptyset \times \omega_1$$

$$\frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\cos\alpha \times \sec^2\emptyset}$$

$$= \frac{1}{\cos^2\theta \times \cos\alpha \times \sec^2\emptyset} \ldots\ldots.(iv)$$

- We know that,

$$\sec^2\emptyset = 1 + \tan^2\emptyset = 1 + \frac{\tan^2\theta}{\cos^2\alpha}$$

$$= 1 + \frac{\sin^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

$$= \frac{\cos^2\theta \times \cos^2\alpha + \sin^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

$$= \frac{\cos^2\theta \times (1 - \sin^2\alpha) + \sin^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

$$= \frac{\cos^2\theta - \sin^2\alpha \times \cos^2\theta + \sin^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

$$= \frac{1 - \sin^2\alpha \times \cos^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

- Substituting this value in eq. (iv)

$$\frac{\omega_1}{\omega} = \frac{1}{\cos^2\theta \times \cos\alpha} \times \frac{\cos^2\theta \times \cos^2\alpha}{1 - \sin^2\alpha \times \cos^2\theta}$$

Maximum and Minimum speed of Driven Shaft

$$\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1 - \sin^2\alpha \times \cos^2\theta}$$

$$\omega_1 = \frac{\omega \times \cos\alpha}{1 - \sin^2\alpha \times \cos^2\theta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

- The value of ω_1 will be minimum for a given value of α, if the denominator of eq. (I) is minimum.

$\cos^2\theta = 1$, i.e. $\Theta = 0^o, 180^o, 360^o$ etc.

- Maximum speed of the driven shaft,

$$\omega_{1(\max)} = \frac{\omega\cos\alpha}{1 - \sin^2\alpha} = \frac{\omega \times \cos\alpha}{\cos^2\alpha} = \frac{\omega}{\cos\alpha}$$

$$N_{1(\max)} = \frac{N}{\cos}$$

– Similarly, the value of ω_1 is minimum , if the denominator of eq. (i) is maximum, this will happen, when $(\sin^2\alpha \times \cos^2\theta)$ is maximum, or

$$\cos^2\theta = 0, \text{ i.e. } \Theta = 90^o, 270^o \text{ etc.}$$

Polar diagram – salient features of driven shaft speed

– For one complete revolution of the driven shaft, there are two points i.e. at 0° and 180° as shown by points 1 and 2 in Fig. Where the speed of the driven shaft is maximum and there are two points i.e. at 90° and 270° as shown by point 3 and 4 where the speed of the driven shaft is minimum.

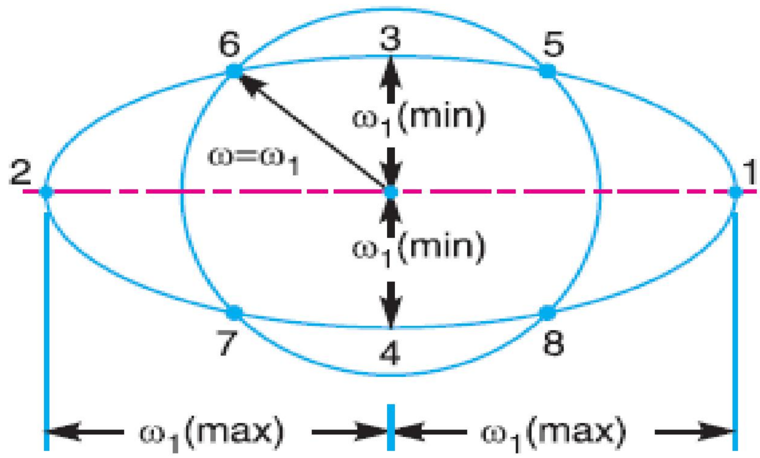


Fig. 4.15 polar diagram

– Since there are two maximum and two minimum speeds of the driven shaft, therefore there are four points when the speeds of the driven and driver shaft are same. This is shown by points, 5, 6, 7 and 8 in Fig.

– Since the angular velocity of the driving shaft is usually constant, therefore it is represented by a circle of radius ω. The driven shaft has a variation in angular velocity, the maximum value being ω/cos α and minimum value is ω cos α. Thus it is represented by an ellipse of semi-major axis ω/cos α and semi-minor axis ω cos α, as shown in Fig.4.15.

Double Hooke's Joint

– The velocity of the driven shaft is not constant, but varies from maximum to minimum values. In order to have a constant velocity ratio of the driving and driven shafts, an intermediate shaft with a Hooke's joint at each end as shown in Fig. , is used. This type of joint is known as double Hooke's joint.

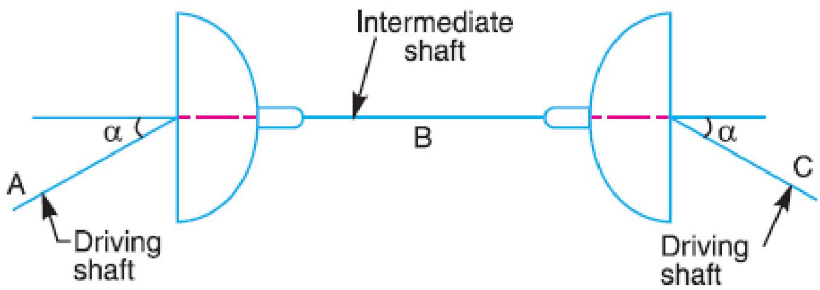


Fig. 4.16 double Hooke's joint

- For shaft A and B,

$$\tan\theta = \tan\emptyset \times \cos\alpha$$

- For shaft B and C,

$$\tan\gamma = \tan\emptyset \times \cos\alpha$$

- This shows that the speed of the driving and driven shaft is constant. In other words, this joint gives a velocity ratio equal to unity, if
  1. The axes of the driving and driven shafts are in the same plane, and
  2. The driving and driven shafts make equal angles with the intermediate shaft.

Examples:

1. In a Davis steering gear, the distance between the pivots of the front axle is 1.2 metres and the wheel base is 4.7 metres. Find the inclination of the track arm to the longitudinal axis of the car, when it is moving along a straight path .

– **Given:** c = 1.2 m ; b = 4.7 m

- Let, α = Inclination of the track arm to the longitudinal axis.
- We know that

$$\tan\alpha = \frac{C}{2\,b} = \frac{1.2}{2\times 4.7} = 0.222$$

$$= 14.5°$$

2. Two shafts with an included angle of 160° are connected by a Hooke's joint. The driving shaft runs at a uniform speed of 1500 r.p.m. The driven shaft carries a flywheel of mass 12 kg and 100 mm radius of gyration. Find the maximum angular acceleration of the driven shaft and the maximum torque required .

- **Given:** N = 1500 rpm ; m = 12 kg; k = 100 mm ; α = 20^o
- We know that angular speed of driving shaft,

$$\omega = 2\,\pi\,\frac{1500}{60} = 157\,\frac{rad}{s}$$

- The mass moment of inertia of the driven shaft,

$$I = m \times K^2 = 12 \times 0.1^2 = 0.12\ kg.m$$

Max. angular acceleration of driven shaft,

$$\cos 2\theta = \frac{\sin^2 \alpha \times 2}{2 - \sin^2 \alpha} = \frac{\sin^2 20 \times 2}{2 - \sin^2 20} = 0.124$$

$$= 41.45°$$

$$\frac{d\omega_1}{dt} = \frac{\omega^2 \times \cos \alpha \times \sin 2\theta \times \sin^2 \alpha}{(1 - \sin^2 \alpha \times \cos^2 \theta)^2}$$

$$= \frac{157^2 \times \cos 20 \times \sin 84.9 \times \sin^2 20}{(1 - \sin^2 20 \times \cos^2 44.45)^2} = 3090 \frac{rad}{s^2}$$

– Max torque req.

$$= I \times \frac{d\omega_1}{dt} = 0.12 \times 3090 = 371\ N.m$$

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2. Theory of Machines by R.S. Khurmi & J.K.Gupta,S.Chand publication.