Transmission Lines

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UNIT-V

Transmission Lines

A transmission line basically consists of two or more parallel conductors used to connect a source to a load. The source may be a generator, a transmitter, or an oscillator; the load may be a factory, an antenna, or an oscilloscope, respectively.

→ Transmission lines are commonly used in power distribution, communications, electrical laboratories and transmission lines such as the twisted-pair and coaxial cables are used in computer networks such as the ethernet and internet.

→ Transmission line problems are usually solved using EM field theory and electric circuit theory, the two major theories on which electrical engineering is based.

Types of Transmission lines:

a) Two-wire line
b) Coaxial line
c) Planar line
d) Wire above conducting plane
e) Micro strip line
f) Waveguides
g) Optical cable

a) Two-wire line: This transmission line consists of a pair of parallel conducting wires separated by a uniform distance .These are used in power systems or telephones lines.

Fig. Two-wire line.

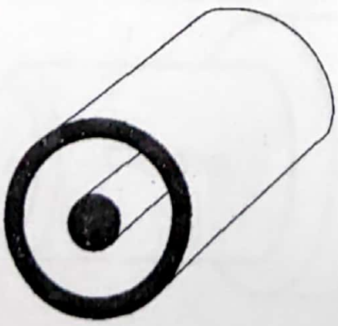
Merits:

1. The cost of two wire transmission line is very low as compared to other types of lines.
2. To design the open two line transmission line is quite simple and easy too.
3. Open two wire lines are capable of handling high power.

Demerits:

1. The external interference of the signal in open two wire lines is more.
2. Due to external interference the output at the load end of two wire transmission line will be noisy.
3. To use the two wire transmission lines in the twisty paths is quite difficult.
4. It cannot be used on very high frequencies because it will generate skin effect.

b) Coaxial line: The co-axial cable can be constructed by placing a solid conductor inside a hallow cylindrical coaxially and these two are separated by a dielectric. They are used as TV cables, telephones cables and power cables.



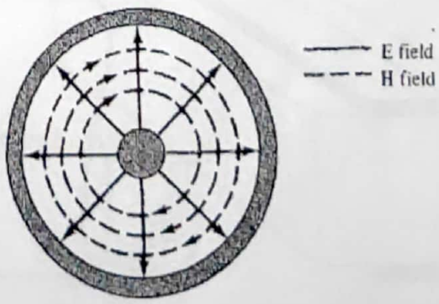


Fig.(a) Coaxial cable (b).E and H fields in coaxial cable

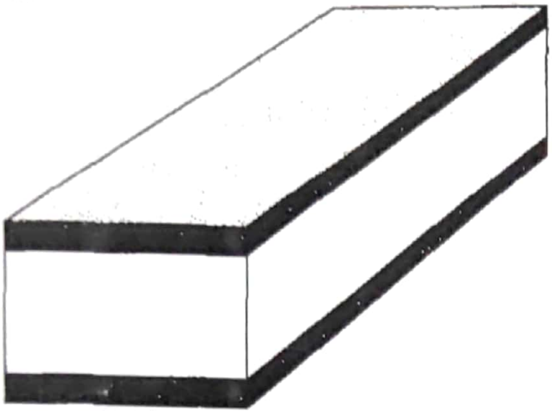
Merits:

1. As the outer conductor (braded wire) is grounded, therefore the possibility of external interference is minimized. The output of the load end will be less noised.
2. The coaxial cable is used for high frequencies transmission.
3. This type of transmission cables can be easily used if the path of energy from source to load is twisty or complicated.
4. Coaxial cable occupies less space as compared to two wire lines.
5. The conductor which carries the energy from source to load is protected from dust, rust etc. due to proper insulation.

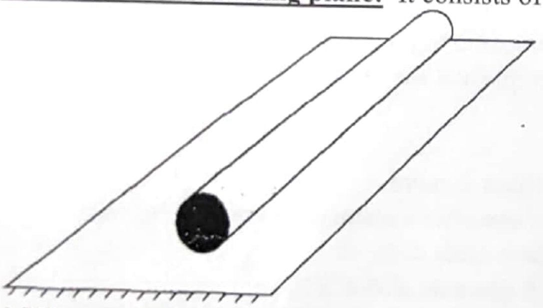
Demerits:

1. This type of transmission line is costly with respect to two wire lines.
2. Designing of coaxial cable is difficult as compared to two wire lines.
3. This type of transmission lines handles low power transmissions.

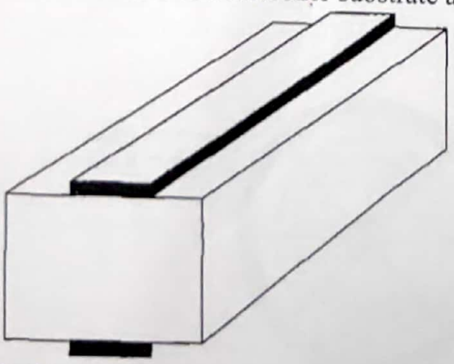
(c) Planar line: It consists of two parallel conductors separated by dielectric medium as shown in fig. It supports TE and TM waves.



d) Wire above conducting plane: It consists of a conducting wire above the ground plane as shown in fig.



c) Micro strip line: The Micro strip line is transmission line geometry with a single conductor trace on one side of a dielectric substrate and a single ground plane on the other side.



Merits:

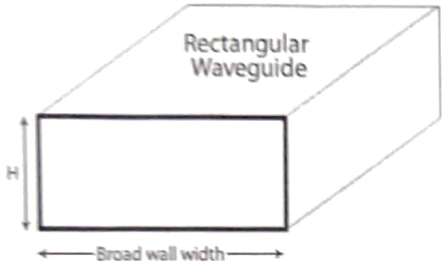
1. Very high frequency.

2. Small size
3. Low weight.
4. Losses are minimum.
5. This type of transmission line is used for very high frequency.
6. Micro strip lines are used in integrated circuits where distance between load and source is very short.
7. As the path of energy is made of very good conductor like gold, therefore the losses of energy are minimum possible.
8. The weight of micro strip line is low.

Demerits:

1. The cost of micro strip is very high as compared to coaxial and two wire line.
2. The micro strip line cannot be used as a transmission line when the distance between source and load is long.
3. This type of transmission line cannot be used in twisty paths between source and load.

d) Wave guides: The wave guides are hallowed or dielectric filled conductor used to transmit the electromagnetic energy at micro wave frequency ranges.
The wave propagates in TE ,TM,TEM modes.



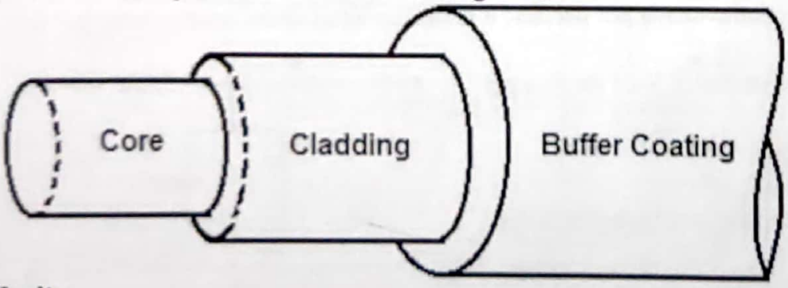
Merits:

1. The large surface area of waveguides greatly reduces copper (I^2R) losses.
2. Dielectric losses are also lower in wave guides than in two-wire and coaxial transmission line

Demerits:

1. Physical size is the primary lower-frequency limitation of waveguides. The width of a waveguide must be approximately a half wavelength at the frequency of the wave to be transported
2. Waveguides are difficult to install because of their rigid, hollow-pipe shape. Special couplings at the joints are required to assure proper operation.
3. The inside surfaces of waveguides are often plated with silver or gold to reduce skin effect losses. These requirements increase the costs and decrease the practicality of waveguide systems at any other than microwave frequencies.

e) Optical fibers transmission line: It consists of core and cladding. Information passes through the core in the form of totally internal reflected TEM light waves.



Merits:

1. The fiber optics offers the high bandwidth.
2. Fiber immune to electromagnetic interference, Fiber has a very low rate of bit error (10 in

10-13), Fiber-optic transmission is virtually noise free.
3. Fiber provides an extremely secure transmission medium.
4. When high freq signal are propagated through the optical fiber the loss is very low.
5. Because of very small size and light in weight and large flexibility, it is easy to install and compatibility with digital technology.
6. As optical fiber has no electrical conductivity, therefore grounding and protection are not necessary.
7. Lack of electrical signals in the fiber, so it cannot shock or other hazards. This makes optical fibers suitable for work in explosive atmospheres.

Demerits:

1. Installing fiber optic cabling is still relatively costly.
2. Equipment used in the fiber optics is expensive, specialized optical test equipment is needed in testing of optical fiber.
3. Fiber is a small and compact cable, and it is highly susceptible to becoming cut or damaged during installation or construction activities.
4. Damage to Fiber Optic Cables from birds, ants, Sharks etc
5. Even though the raw material for making optical fibers, sand, is cheap, optical fibers are still more expensive per meter than copper.
6. The glass can be affected by various chemicals including hydrogen gas (a problem in underwater cables).
7. Optical fiber cannot be joined together as a easily as copper cable and requires additional training of personnel and expensive precision splicing and measurement equipment.
8. As optical fibers have no electrical conductivity, therefore additional copper cable is not used with optical fiber to provide power supply to the repeaters.

Transmission Line Parameters:

The transmission line is described in terms of its line parameters, which are its resistance per unit length R, inductance per unit length L, conductance per unit length G, and capacitance per unit length C. These are also called primary parameters. These are independent of frequency.
These parameters are not lumped but distributed that means these are uniformly distributed along the entire length of the line.
Resistance(R) : A series resistance is due to the internal resistance of the conductors of a transmission line. It depends on the conductivity and cross-sectional area of the conductors. But at high frequencies, it depends on skin depth. It is measured as loop resistance per unit length of the line. Its units are Ω/m.
Inductance (L):A series inductance is due to the magnetic flux produced around the conductors of a transmission line. The flux linkage per unit current gives the inductance of the line. It is measured as loop inductance per unit length of the line. Its units are H/m.
Capacitance(C): Two conductors of a transmission line separated by a dielectric form a capacitor. Thus a shunt capacitance is formed due to the electric field between the conductors. It is measured as shunt capacitance per unit length of the line. Its units are F/m.
Conductance (G): A shunt conductance is due to the leakage current between the conductors of a line since the dielectric medium between the conductors is not perfect. It is measured as shunt conductance per unit length of the line. Its units are ℧/m.
The series impedance Z and shunt admittance Y of the line per unit length can be expressed as

$$Z = R + j\omega L$$
$$Y = G + j\omega C$$

The equivalent circuit of the transmission line is shown in fig

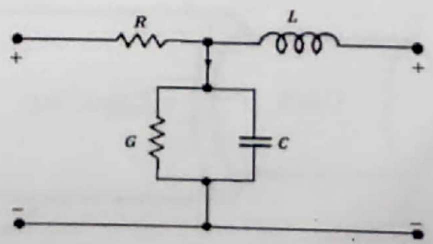


Fig. Equivalent circuit of transmission line

→ * → Characterstic Impedence of Tx line Z_0 :-

→ * → The ratio of the amplitude of voltage & current of a single wave propagation along the line,

i.e a wave travelling in one direction in the absence of reflection in the other direction.

$Z_0 = \frac{V}{I}$

Tx line

$I^+(z)$ → direction

$V^+(z)$

forward direction wave → +ve z

$$\left.\begin{aligned} V^+(z) &= V_0^+ e^{-\gamma z} \\ I^+ &= I_0^+ e^{-\gamma z} \end{aligned}\right\}$$ time representation of Forward wave eq.

$$Z_0 = \frac{V^+(z)}{I^+(z)} = \frac{V_0^+ \cancel{e^{-\gamma z}}}{I_0^+ \cancel{e^{-\gamma z}}}$$

$$\boxed{Z_0 = \frac{V_0^+}{I_0^+}} \longrightarrow (1)$$

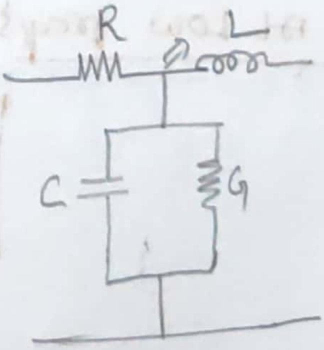
taking

P — q

←dz→

$$-\frac{dV}{dz} = (R+j\omega L) I$$

$\frac{dV}{dz} = (R+j\omega L)I$



$$-\frac{dV^+(z)}{dz} = (R+j\omega L) I_0^+ e^{-\gamma z}$$

$$-\frac{d\, V_0^+ e^{-\gamma z}}{dz} = (R+j\omega L) I_0^+ e^{-\gamma z}$$

$$-\left[-\gamma\, V_0^+ \cancel{e^{-\gamma z}}\right] = (R+j\omega L) I_0 \cancel{e^{-\gamma z}}$$

$$\frac{V_0^+}{I_0^+} = \frac{R+j\omega L}{\gamma}$$

$$\frac{V_0^+}{I_0^+} = \frac{R+j\omega L}{\gamma} = \frac{Z}{\gamma}$$

$$= \frac{R+j\omega L}{\sqrt{(R+j\omega L)(G+j\omega L)}}$$

$\gamma = \sqrt{ZY}$ - adimittance

$$\frac{V_0^+}{I_0^+} = \sqrt{\frac{R+j\omega L}{G+j\omega L}}$$

$$\boxed{Z_0 = \frac{V_0^+}{I_0^+} \sqrt{\frac{R+j\omega L}{G+j\omega L}}}$$

$$Z_0 = \frac{Z}{\sqrt{ZY}} = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega L}}$$

(or)

*) At High freq's

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

*) At Low freq's

$$\boxed{Z_0 = \sqrt{\frac{R}{G}}}$$

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Transmission Line Equations: consider a transmission line with two parallel conductors. Let R, L, C and G be the primary parameters. Consider a point P on the line at a distance x from the source as shown in fig.

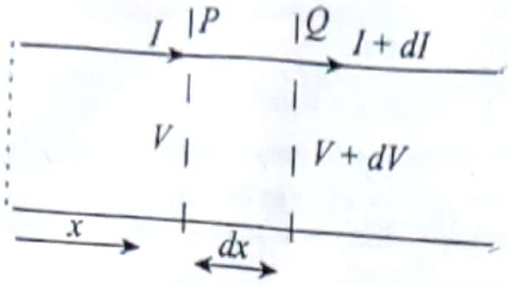

Fig. Voltages and currents on the transmission line

Let Q be the another point at distance dx from the P.

Let V and I be the voltage and current at point P respectively. Let V+dV and I+dI be the voltage and current at point Q respectively.

For a small length dx of the line, the series impedance is $(R + j\omega L)dx$ and shunt admittance is $(G + j\omega C)dx$.

The potential difference between P and Q is

$$V - (V + dV) = I(R + j\omega L)dx$$

$$-\frac{dV}{dx} = (R + j\omega L)I \quad ----(1)$$

The current difference between P and Q is

$$I - (I + dI) = V(G + j\omega C)dx$$

$$-\frac{dI}{dx} = (G + j\omega C)V \quad ----(2)$$

Taking differentiation of eq.(1)

$$-\frac{d^2V}{dx^2} = (R + j\omega L)\frac{dI}{dx} \quad -----(3)$$

Substitute the eq.(2) in eq.(3)

$$\frac{d^2V}{dx^2} = (R + j\omega L)(G + j\omega C)V$$

Similarly differentiate eq.(2) and substitute eq.(1) we get

$$\frac{d^2I}{dx^2} = (R + j\omega L)(G + j\omega C)I$$

The propagation constant

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Where α=attenuation constant and β=phase shift constant

Therefore

$$\frac{d^2V}{dx^2} = \gamma^2 V \quad ----(4)$$

$$\frac{d^2I}{dx^2} = \gamma^2 I \quad ----(5)$$

The equations (4) and (5) are the second order differential equations in terms of voltage and current whose solutions are given by

$$V = ae^{\gamma x} + be^{-\gamma x} \quad ---(6)$$
$$I = ce^{\gamma x} + de^{-\gamma x} \quad ----(7)$$

Where a, b, c and d are the constants.

In terms of hyperbolic functions the above equations become

$$e^{\gamma x} = \cosh \gamma x + \sinh \gamma x$$
$$e^{-\gamma x} = \cosh \gamma x - \sinh \gamma x$$

$$V = A\cosh\gamma x + B\sinh\gamma x \quad --(8)$$
$$I = C\cosh\gamma x + D\sinh\gamma x \quad --(9)$$

Instead of four constants A,B,C and D the above equations can simplified to only two constants, by substituting the value of V from eq.(8) in eq.(1)

$$-\frac{d}{dx}(A\cosh\gamma x + B\sinh\gamma x) = (R + j\omega L)I$$
$$-(\gamma A\sinh\gamma x + B\gamma\cosh\gamma x) = (R + j\omega L)I$$
$$I = \frac{-\gamma}{(R + j\omega L)}(A\sinh\gamma x + B\cosh\gamma x)$$
$$I = -\sqrt{\frac{G + j\omega C}{R + j\omega L}}(A\sinh\gamma x + B\cosh\gamma x)$$
$$I = -\frac{1}{Z_0}(A\sinh\gamma x + B\cosh\gamma x)$$

Where

$$\text{Characteristic impedance } \boxed{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}}$$

$$V = A\cosh\gamma x + B\sinh\gamma x$$
$$I = -\frac{1}{Z_0}(A\sinh\gamma x + B\cosh\gamma x)$$

The constants A and B can be obtained by using initial conditions.
Let V_s and I_s be the source voltage and current respectively. At source end, x=0, the voltage V= V_s and current I= I_s
Then

$$V_s = A\cosh\gamma(0) + B\sinh\gamma(0)$$

Therefore $$V_s = A$$

$$I_s = -\frac{1}{Z_0}(A\sinh\gamma(0) + B\cosh\gamma(0))$$

Therefore $$B = -I_s Z_0$$

Substituting the constants A and B in above equations

$$\boxed{\begin{aligned} V &= V_s\cosh\gamma x - I_s Z_0\sinh\gamma x \\ I &= I_s\cosh\gamma x - \frac{V_s}{Z_0}\sinh\gamma x \end{aligned}}$$

These are called transmission line equations. They give voltage and current at a point of distance x from the sending end in terms of source voltage and current.

Infinite line: A line is said to be infinite if all the input signals are consumed by the line and there is no reflected signal.

$$I = ce^{\gamma x} + de^{-\gamma x} \quad ---(1)$$

When x=0, the current at the sending end is $I = I_{Si}$
Substitute x=0 in Eq.(1)

$$I_{Si} = c + d$$

When x=∞, the current at receiving end is I=0
Substituting x=∞ in Eq.(1)

$$0 = ce^{\gamma\infty} + de^{-\gamma\infty}$$
$$0 = c\infty$$
$$c = 0$$

Therefore

$$d = I_{Si},$$

The current at any point on the infinite line is given by

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$$I = I_{SI}\, e^{-\gamma x}$$

Similarly the voltage at any point the infinite line is given by

$$V = V_{SI}\, e^{-\gamma x}$$

Secondary Constants: The propagation constant γ and the characteristic impedance Z_0 are referred as secondary constants.

1.Propagation Constant(γ) : The propagation constant γ is a complex quantity.

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

1 neper = 8.6dB

Where α is the attenuation constant (in nepers per meter or decibels per meter), and β is the phase constant (in radians per meter).

2.Characteristic impedance(Z_0) : The characteristic impedance Z_0 of the line is the ratio of positively traveling voltage wave to current wave at any point on the line. The characteristic impedance is also defined as the input impedance of an infinite line.

We know that

$$-\frac{dV}{dx} = (R + j\omega L)I$$

For infinite line

$$V = V_S e^{-\gamma x}$$
$$I = I_S e^{-\gamma x}$$
$$-\frac{d}{dx}(V_S e^{-\gamma x}) = (R + j\omega L) I_S e^{-\gamma x}$$
$$\gamma V_S e^{-\gamma x} = (R + j\omega L) I_S e^{-\gamma x}$$
$$\gamma V_S = (R + j\omega L) I_S$$
$$\frac{V_S}{I_S} = \frac{R + j\omega L}{\gamma}$$

The input impedance of infinite line is given by

$$Z_0 = \frac{V_S}{I_S}$$

$$\text{Characteristic impedance } Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

The propagation constant γ and the characteristic impedance Zo are important properties of the line because they both depend on the line parameters *R*, *L*, *G*, and *C* and the frequency of operation.

Attenuation and phase constants:

The propagation constant γ is a complex quantity.

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Squaring the magnitude of γ we get

$$\alpha^2 + \beta^2 = \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)} \quad ---(1)$$

Squaring γ on the both sides

$$(\alpha + j\beta)^2 = (R + j\omega L)(G + j\omega C)$$
$$\alpha^2 - \beta^2 + j2\alpha\beta = RG - \omega^2 LC + j(\omega LG + \omega RC)$$

Equating the real parts we get

$$\alpha^2 - \beta^2 = RG - \omega^2 LC \quad ----(2)$$

Adding Eq.(1) and Eq.(2)

$$2\alpha^2 = \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)} + RG - \omega^2 LC$$

The real part of γ is called attenuation constant. It determines the reduction or attenuation in voltage and current along the line. Its unit is neper per km. 1neper=8.686dB.

$$\alpha = \sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + (RG-\omega^2LC)\right]} \quad NP/km$$

Subtracting Eq.(2) from Eq.(1)

$$\beta = \sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} - (RG-\omega^2LC)\right]} \quad rad/km$$

The imaginary part of γ is called the phase constant. It determines the variation in phase position of voltage and current along the line. Its unit is radians per km.

a) **Lossless Line** ***(R = 0 = G):*** A transmission line is said to be loss less if the conductors of the line are perfect($\sigma_c \approx \infty$) and the dielectric medium separating them is lossless ($\sigma_d \approx 0$).
Condition for losses less line is, $R = 0 = G$.
The high frequency lines are termed as lossless lines because the $\omega = 2\pi f$ in the series impedance($R + j\omega L$) and shunt admittance ($G + j\omega C$) becomes very large due to the high frequency.
Therefore the real part of the series impedance can be neglected.
The propagation constant and characteristic impedance for losses line are given by

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

Substitute $R = 0 = G$

Therefore

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}$$

$$\alpha = 0 \text{ and } \beta = j\omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

b) **Distortion less Line** ***(R/L = G/C):*** A transmission line is said to be distortion less if the attenuation constant 'α' is frequency independent while the phase constant 'β' is linearly dependent on frequency.
The condition for the distortion less line is $R/L = G/C$.
The propagation constant for distortion less line is given by

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{RG\left(1+\frac{j\omega L}{R}\right)\left(1+\frac{j\omega C}{G}\right)}$$

For the distortion less line

Therefore

$$\frac{R}{L} = \frac{G}{C}$$

$$\gamma = \sqrt{RG}\left(1+\frac{j\omega C}{G}\right)$$

$$\alpha + j\beta = \sqrt{RG} + j\omega LC$$

$$\alpha = \sqrt{RG} \text{ and } \beta = \omega LC$$

The characteristic impedance for distortion less line is given by

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{R(1+j\omega L/R)}{G(1+j\omega C/G)}} = \sqrt{\frac{R}{G}}$$

$$R_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} \quad \text{and } X_0 = 0$$

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The phase velocity is

$$v_0 = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

Note:

1. The phase velocity is independent of frequency.
2. The phase velocity and characteristic impedance are same for both lossless and distortionless lines.
3. A lossless line is also a distortionless line, but a distortionless line is not necessarily lossless. Although lossless lines are desirable in power transmission, telephone lines are required to be distortionless.

Input Impedance:

A transmission line terminated with any load impedance Z_R at x=L is shown in fig.

The V and I equations at a point of distance x from the sending end in terms of source voltage and current are given by.

$$V = V_s \cosh \gamma x - I_s Z_0 \sinh \gamma x$$

$$I = I_s \cosh \gamma x - \frac{V_s}{Z_0} \sinh \gamma x$$

$$V_R = V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l$$

$$I_R = I_s \cosh \gamma l - \frac{V_s}{Z_0} \sinh \gamma l$$

The load impedance Z_R is given by

$$Z_R = \frac{V_R}{I_R}$$

$$Z_R = \frac{V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l}{I_s \cosh \gamma l - \frac{V_s}{Z_0} \sinh \gamma l}$$

$$Z_R \left(I_s \cosh \gamma l - \frac{V_s}{Z_0} \sinh \gamma l \right) = V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l$$

$$Z_R I_s \cosh \gamma l - Z_R \frac{V_s}{Z_0} \sinh \gamma l = V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l$$

$$Z_R I_s \cosh \gamma l + I_s Z_0 \sinh \gamma l = V_s \cosh \gamma l + Z_R \frac{V_s}{Z_0} \sinh \gamma l$$

$$I_s (Z_R \cosh \gamma l + Z_0 \sinh \gamma l) = V_s \left(\cosh \gamma l + \frac{Z_R}{Z_0} \sinh \gamma l \right)$$

$$\frac{V_s}{I_s} = \frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{\cosh \gamma l + \frac{Z_R}{Z_0} \sinh \gamma l}$$

The input impedance of the line is given by

$$Z_{in} = \frac{V_s}{I_s}$$

$$Z_{in} = Z_0 \left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \right]$$

$$Z_{in} = Z_0 \left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l} \right]$$

For lossless line γ=jβ

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan \beta l}{Z_0 + jZ_R \tan \beta l} \right]}$$

Voltage and Current at any point on the transmission line:

Consider the transmission line of length l terminating with an impedance Z_R. Let V_R and I_R be the voltage and current at the load Z_R.

The voltage and current at a point of distance x from the sending end in terms of source voltage and current are given by

$$V = A\cosh\gamma x + B\sinh\gamma x \quad ---(1)$$

$$I = -\frac{1}{Z_0}(A\sinh\gamma x + B\cosh\gamma x) \quad ---(2)$$

Where A and B are constants.

At $x=l$ $\quad V=V_R$ and $I=I_R$

Substituting these values in eq.(1) and eq.(2)

$$V_R = A\cosh\gamma l + B\sinh\gamma l \quad ----(3)$$

$$I_R = -\frac{1}{Z_0}(A\sinh\gamma l + B\cosh\gamma l) \quad ---(4)$$

Multiplying Eq.(3) with $\cosh\gamma l$ and Eq.(4) with $\sinh\gamma l$

$$V_R\cosh\gamma l = A\cosh^2\gamma l + B\sinh\gamma l\cosh\gamma l \quad ----(5)$$

$$Z_0 I_R\sinh\gamma l = -A\sinh^2\gamma l - B\cosh\gamma l\sinh\gamma l \quad ---(6)$$

Adding Eq.(5) and Eq.(6)

$$V_R\cosh\gamma l + Z_0 I_R\sinh\gamma l = A(\cosh^2\gamma l - \sinh^2\gamma l)$$

Therefore

$$A = V_R\cosh\gamma l + Z_0 I_R\sinh\gamma l$$

Multiplying Eq.(3) with $\sinh\gamma l$ and Eq.(4) with $\cosh\gamma l$

$$V_R\sinh\gamma l = A\cosh\gamma l\sinh\gamma l + B\sinh^2\gamma l \quad ----(7)$$

$$Z_0 I_R\cosh\gamma l = -A\sinh\gamma l\cosh\gamma l - B\cosh^2\gamma l \quad ---(8)$$

Adding Eq.(7) and Eq.(8)

$$V_R\sinh\gamma l + Z_0 I_R\cosh\gamma l = -(\cosh^2\gamma l - \sinh^2\gamma l)$$

$$B = -(V_R\sinh\gamma l + Z_0 I_R\cosh\gamma l)$$

Substituting the values of A and B in Eq.(1)

$$V = (V_R\cosh\gamma l + Z_0 I_R\sinh\gamma l)\cosh\gamma x - (V_R\sinh\gamma l + Z_0 I_R\cosh\gamma l)\sinh\gamma x$$

$$V = (V_R\cosh\gamma l\cosh\gamma x + Z_0 I_R\sinh\gamma l\cosh\gamma x) - (V_R\sinh\gamma l\sinh\gamma x + Z_0 I_R\cosh\gamma l\sinh\gamma x)$$

$$V = V_R(\cosh\gamma l\cosh\gamma x - \sinh\gamma l\sinh\gamma x) + Z_0 I_R(\sinh\gamma l\cosh\gamma x - \cosh\gamma l\sinh\gamma x)$$

$$V = V_R\cosh\gamma(l-x) + Z_0 I_R\sinh\gamma(l-x)$$

Similarly Substituting the values of A and B in Eq.(2)

$$I = I_R\cosh\gamma(l-x) + +\frac{V_R}{Z_0}\sinh\gamma(l-x)$$

If $y=l-x$ then

$$\boxed{\begin{aligned} V &= V_R\cosh\gamma y + Z_0 I_R\sinh\gamma y \\ I &= I_R\cosh\gamma y + +\frac{V_R}{Z_0}\sinh\gamma y \end{aligned}}$$

These are the voltage and current equations at a point of distance y from the load end in terms of terminal voltage and current.

Transmission line terminated with characteristic impedance:

Consider a line of length l which is terminated with characteristic impedance Z_0 as shown in fig.

The input impedance of the line is given by

$$Z_{in} = Z_0\left[\frac{Z_R + Z_0\tanh\gamma l}{Z_0 + Z_R\tanh\gamma l}\right]$$

Here $Z_R=Z_0$

$$\boxed{Z_{in} = Z_0\left[\frac{Z_0 + Z_0 \tanh\gamma l}{Z_0 + Z_0 \tanh\gamma l}\right]}$$

$$Z_{in} = Z_0$$

Hence the input impedance of a finite line terminated with characteristic impedance Z_0 is equal to characteristic impedance Z_0.

Therefore a finite line terminated with characteristic impedance Z_0 is equal to infinite line.

A line terminated with characteristic impedance Z_0 is called matched line.

Standing Waves In Transmission Lines: Signal energy is transmitted through a transmission line from the source to the load in the form of voltage and current waves. When the terminated load impedance is different from the characteristic impedance of the line, then the some part of the transmitted signal is returns back and there exists a reflected wave .These incident and reflected travelling waves create a standing waves.

The voltage and current equations along the transmission line equation are given by

$$V = V_i e^{-\gamma x} + V_r e^{\gamma x}$$
$$I = I_i e^{-\gamma x} + I_r e^{\gamma x}$$

In the above equations 1st term is called incident wave and 2nd term is called reflected wave.

Reflection Coefficient:

The reflection coefficient is defined as the ratio of reflected voltage to incident voltage. The reflection coefficient is also defined as the ratio of reflected current to incident current. It is denoted by K.

Let V_i and V_r are the incident and reflected voltages respectively.

$$K = \frac{V_r}{V_i}$$

Let I_i and I_r are the incident and reflected currents respectively.

$$K = -\frac{I_r}{I_i}$$

The voltage at the load Z_R is given by

$$V_R = V_i + V_r \quad ----(1)$$
$$I_R = I_i + I_r \quad ----(2)$$

But characteristic impedance Z_0 is $\quad Z_0 = \frac{V_i}{I_i} = -\frac{V_r}{I_i} \quad ----(3)$

Substitute the Eq.(3) in Eq.(2), we get

$$I_R = \frac{V_i}{Z_0} - \frac{V_r}{Z_0}$$
$$Z_0 I_R = V_i - V_r \quad ---(4)$$

Divide the Eq.(1) by Eq.(4), we get

$$\frac{V_R}{Z_0 I_R} = \frac{V_i + V_r}{V_i - V_r}$$

We know that

$$Z_R = \frac{V_R}{I_R}$$

$$\frac{Z_R}{Z_0} = \frac{1 + \frac{V_r}{V_i}}{1 - \frac{V_r}{V_i}}$$

We know that the reflection coefficient K is

$$K = \frac{V_r}{V_i}$$

$$\frac{Z_R}{Z_0} = \frac{1+K}{1-K}$$

$$Z_R(1-K) = Z_0(1+K)$$
$$(Z_R - Z_RK) = (Z_0 + Z_0K)$$
$$Z_R - Z_0 = Z_RK + Z_0K$$
$$K(Z_R + Z_0) = Z_R - Z_0$$

The reflection coefficient K is

$$\boxed{K = \frac{Z_R - Z_0}{Z_R + Z_0}}$$

The range of reflection coefficient K is $-1 \le |K| \le 1$

Voltage standing wave ratio (VSWR): The VSWR is defined as the ratio of the maximum voltage to the minimum voltage on the line having standing waves.

$$VSWR = S = \frac{V_{max}}{V_{min}} = \frac{V_i + V_r}{V_i - V_r} = \frac{1+|K|}{1-|K|}$$

$$\boxed{S = \frac{1+|K|}{1-|K|}}$$

Similarly the current standing wave ratio can be defined as the ratio between maximum current to the minimum current.

The range of VSWR is $1 \le S \le \infty$

$$(Z_{in})_{max} = \frac{V_{max}}{I_{min}} = SZ_o$$
$$(Z_{in})_{min} = \frac{V_{min}}{I_{max}} = \frac{Z_o}{S}$$

a **Short circuited line ($Z_R = 0$):** If transmission line is terminated with short circuit, then the line is called short circuited line.

The input impedance of the short circuited line can be obtained by substituting the $Z_R = 0$ in the input impedance of the transmission line.

$$Z_{in} = Z_0\left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l}\right]$$

If Z_R=0 then

$$Z_{sc} = Z_0 \tanh \gamma l$$

The input impedance of the short circuited lossless transmission line is given by

$$Z_{sc} = jZ_0 \tan \beta l$$

The reflection coefficient of the short circuited transmission line is K= -1.

The VSWR of the short circuited transmission line is S=∞.

The variation of input impedance of SC line is shown in fig.

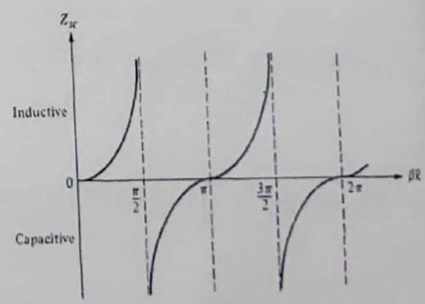
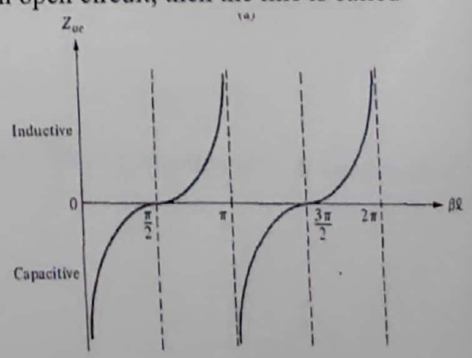


Fig . Input impedance of SC

b **Open circuited line ($Z_R = \infty$):** If transmission line is terminated with open circuit, then the line is called open circuited line.

The input impedance of the open circuited line can be obtained by substituting the Z_R=∞ in the input impedance of the transmission line.



$$Z_{in} = Z_0\left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l}\right]$$

If Z_R=∞ then

$$Z_{oc} = Z_0 \coth \gamma l$$

The input impedance of the open circuited line lossless transmission line is given by

$$Z_{oc} = -jZ_0 \cot \beta l$$

Fig. Input impedance of OC

The reflection coefficient of the open circuited line transmission line is K= 1.
The VSWR of the open circuited line transmission line is S=∞.
The variation of input impedance of OC line is shown in fig.

c) **λ/8- line:** If length of the line is λ/8 then it is called λ/8 line or eight-wave line.
The input impedance of the lossless transmission line is given by

$$Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan \beta l}{Z_0 + jZ_R \tan \beta l}\right]$$

$$\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{8} = \frac{\pi}{4}$$

$$Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan\frac{\pi}{4}}{Z_0 + jZ_R \tan\frac{\pi}{4}}\right]$$

$$Z_{in} = Z_0 \left[\frac{Z_R + jZ_0}{Z_0 + jZ_R}\right]$$

$$|Z_{in}| = Z_0$$

The λ/8 line is used to transform any load impedance Z_R to an input impedance Z_{in} whose magnitude is equal to magnitude of Z_0.

d) **Quarter-wave line:** If length of the line is λ/4 then it is called λ/4 line or quarter-wave line.
The input impedance of the lossless transmission line is given by

$$Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan \beta l}{Z_0 + jZ_R \tan \beta l}\right]$$

$$\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan\frac{\pi}{2}}{Z_0 + jZ_R \tan\frac{\pi}{2}}\right]$$

$$Z_{in} = Z_0 \left[\frac{\frac{Z_R}{\tan\frac{\pi}{2}} + jZ_0}{\frac{Z_0}{\tan\frac{\pi}{2}} + jZ_R}\right]$$

$$Z_{in} = \frac{{Z_0}^2}{Z_R}$$

The λ/4 line can transform a low impedance into a high impedance and vice versa, thus it can be considered as an impedance inverter.
The quarter wave line may be used as a transformer for impedance matching of load Z_R with input impedance Z_{in}. For impedance matching Z_R and Z_{in} , the line characteristic impedance Z_o may be selected as $Z_o = \sqrt{Z_R Z_{in}}$

e) **λ/2- line(half-wave line):** If length of the line is λ/2 then it is called λ/2 line or half-wave line.
The input impedance of the lossless transmission line is given by

$$Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan \beta l}{Z_0 + jZ_R \tan \beta l}\right]$$

$$\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$$

$$Z_{in} = Z_0\left[\frac{Z_R + jZ_0 \tan\pi}{Z_0 + jZ_R \tan\pi}\right]$$

$$Z_{in} = Z_R$$

The input impedance Z_{in} of the $\lambda/2$ line is equal to load impedance Z_R.

Transmission lines of various lengths can be used as circuit elements: Different lengths of transmission lines can be used as circuit elements as discussed below.

The input impedance of the short circuited lossless transmission line is given by

$$Z_{sc} = jZ_0 \tan\beta l \quad --- (1)$$

The input impedance of the open circuited lossless transmission line is given by

$$Z_{oc} = -j\,Z_0 \cot\beta l \quad --- (2)$$

The Eq.s(1) and (2) shows that input impedance of an open and short circuited lossless line is a pure reactance. Desired value of the reactance is obtained by varying the electrical length βl of the stubs. If length of the short circuited line is less than λ/4, It will act as inductance. If length of the short circuited line is greater than λ/4 and less then λ/2, It will act as capacitance. If length of the short circuited line is equal to λ/4, It will act as parallel resonance circuit with high impedance. If length of the short circuited line is equal to λ/2, It will act as series resonance circuit with low impedance.

(If length of the open circuited line is less than λ/4, It will act as capacitance. If length of the open circuited line is greater than λ/4 and less then λ/2, It will act as inductance. If length of the open circuited line is equal to λ/4, It will act as series resonance circuit with low impedance. If length of the open circuited line is equal to λ/2, It will act as parallel resonance circuit with high impedance.

****The Smith Chart:**

The Smith chart is the most commonly used graphical techniques in solving transmission problems in simple way. It is basically a graphical indication of the impedance and VSWR of a transmission line as one moves along the line.

The construction of the chart is based on the reflection coefficient

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

Instead of having separate Smith charts for transmission lines with different characteristic impedances such as Z_o = 60,100, and 120. A normalized chart can be used for any line in which all impedances are normalized with respect to the characteristic impedance Z_o of the particular line under consideration.

$$K = \frac{\frac{Z_R}{Z_0} - 1}{\frac{Z_R}{Z_0} + 1} = \frac{z_r - 1}{z_r + 1}$$

Where z_r is the normalized impedance

$$z_r = \frac{1 + K}{1 - K}$$

Since z_r and K both are complex quantities, we have

$$R + jX = \frac{1 + K_r + jK_x}{1 - (K_r + jK_x)}$$

$$R + jX = \frac{(1+K_r) + jK_x}{(1-K_r) - jK_x}$$

Rationalizing on the right hand side, we get

$$R + jX = \frac{(1+K_r) + jK_x}{(1-K_r) - jK_x} \times \frac{(1-K_r) + jK_x}{(1-K_r) + jK_x}$$

$$R + jX = \frac{1 - K_r^2 - K_x^2 + 2jK_x}{(1-K_r)^2 + K_x^2}$$

Equating real and imaginary parts on both sides, we get

$$R = \frac{1 - K_r^2 - K_x^2}{(1-K_r)^2 + K_x^2} \quad ---(1)$$

$$X = \frac{2K_x}{(1-K_r)^2 + K_x^2} \quad ---(2)$$

Eq.s(1) and (2) will yield two set of orthogonal circles when solved separately. Eq.(1) will results in family of circle called R-circle while Eq.(2) will results in family of circle called X-circle.

(i) The constant R-circle:

$$R\{1 + K_r^2 - 2K_r + K_x^2\} = 1 - K_r^2 - K_x^2$$

$$R + RK_r^2 - 2RK_r + RK_x^2 = 1 - K_r^2 - K_x^2$$

$$K_r^2(R+1) + K_x^2(R+1) - 2RK_r = 1 - R$$

$$K_r^2 + K_x^2 - \frac{2R}{1+R}K_r = \frac{1-R}{1+R}$$

Adding $\frac{R^2}{(1+R)^2}$ on both sides, we get

$$K_r^2 - \frac{2R}{1+R}K_r + \frac{R^2}{(1+R)^2} + K_x^2 = \frac{1-R}{1+R} + \frac{R^2}{(1+R)^2}$$

$$\left(K_r - \frac{R}{1+R}\right)^2 + K_x^2 = \left(\frac{1}{1+R}\right)^2$$

This equation represents a family of circles on the reflection co-efficient plane. These circles are called constant –R circles with center $\left(\frac{R}{1+R}, 0\right)$ and radius $\frac{1}{1+R}$.

(i) The constant X-circle:

$$X = \frac{2K_x}{(1-K_r)^2 + K_x^2}$$

$$(1-K_r)^2 + K_x^2 = \frac{2K_x}{X}$$

$$(1-K_r)^2 + K_x^2 - \frac{2K_x}{X} = 0$$

Adding $\left(\frac{1}{X}\right)^2$ on both sides, we get

$$(1-K_r)^2 + K_x^2 - \frac{2K_x}{X} + \left(\frac{1}{X}\right)^2 = \left(\frac{1}{X}\right)^2$$

$$(K_r - 1)^2 + \left(K_x - \frac{1}{X}\right)^2 = \left(\frac{1}{X}\right)^2$$

This equation represents a family of circles on the reflection co-efficient plane. These circles are called constant –X circles with center $\left(1, \frac{1}{X}\right)$ and radius $\frac{1}{X}$.

Smith chart

Smith chart is a simple graphical tool which consists of locus of constant resistance value as circles on real axis & locus of constant resistance value as circles on real axis & locus of reactance values as circles on imaginary axis

Application of Smith chart :—

(a) Smith chart is used as admittance diagram

(b) It is used for converting a impedence into admittance

(c) It is used to determine the load Impedence.

(d) It is used to determine the input impedance & admittance of short circuted line & open ~~line~~ circuit line

10

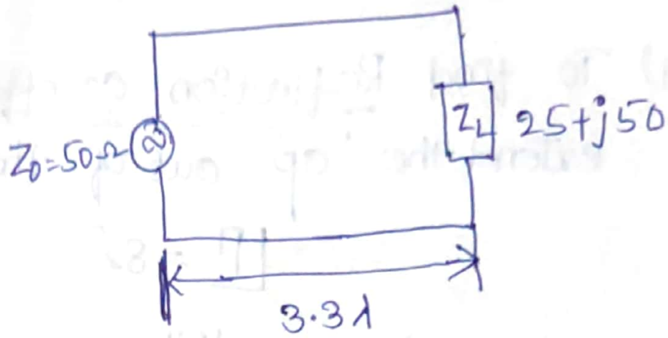
Q*→ A 50Ω Tx line is terminated to load of 25+j50 the length of the Tx line is 3.3λ.

Find $|\Gamma| \angle \Gamma$

VSWR

Zin

Sol. using Smith chart



Given data

$l = 3.3\lambda$

$Z_0 = 50\Omega$

$Z_L = 25+j50$

① Find we have to find normalized Load Impedence

$$Z_{LN} = \frac{Z_L}{Z_0} = \frac{25+j50}{50} = 0.5+j1$$

② Note Z_{LN} on smith chart.

Resistance ○le – 0.5 (R)

Reactance ○le – 1 (X)

mark a point on smith chard wher R & X point intersect at mark it as p.

③ Take center of the graph (1,0) at mark point as 'O'. & take a radius of OP &draw the ○le from the center (1,0)

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where that o^{le} touches the resistance circle is the VSWR value

i.e $\boxed{VSWR = 4.8}$

(4) To find Reflection co-efficient magnitude & angle?

extend the 'op' out of the rim's (o^{le})

$\angle \Gamma = 82^\circ$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$4.8 = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$4.8 - 4.8\Gamma = 1+|\Gamma|$$

$$4.8+1 = [4.8+1]|\Gamma|$$

Theortical value $|\Gamma| = 1$.

(or)

magnitute

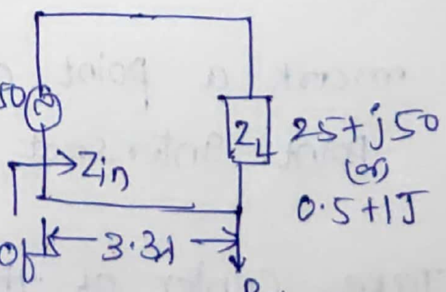
$$|\Gamma| = \frac{dd}{d.} = \frac{5.3}{8.4} = 0.64$$

$\therefore \Gamma = 0.64 \angle 82^\circ$

excat value

(5) Input Impedence :-

To find Z_{in} take refrecence point from Z_L at moves towards source (Generator) to the length of 3.31



→ * In Smith chart one VSWR circle ~~covers~~ moves 1/2 distance. So we have to take multiples of 1/2 i.e 3.3λ can be written as

$3\lambda + 0.3\lambda$

$6\frac{\lambda}{2} + 0.3\lambda \rightarrow$ this distance is travelled from Z_L to Z_{in}

it moves 6 VSWR circles to stand on Z_{iN}

$= 0.135\lambda + 0.3\lambda$

$= 0.435\lambda$

i.e $0.135\lambda \rightarrow$ we get is take wavelength toward generator the point that is 'op' is extended to wavelength graph on smith chart.

So, now we have to move 0.435λ to get Z_{in} in Smith chart ~~tow~~ toward the generator wavelength.

normalised value $Z_{inN} = 0.24 - j0.5$

Generalised value $\rightarrow Z_{in} = \underbrace{50}_{Z_0}(0.24 - j0.5)$

$$\boxed{Z_{in} = 12 - j25}$$

⑥ Input admittance :—

Extend the Z_{inN} line to the other end of the VSWR O^{le}.

$Y_N = 1.1 + j\,1.65$

=

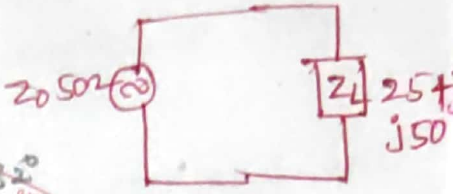
** A 50Ω Tx line is terminated to load of the length of Tx line is 3.3λ.

Find

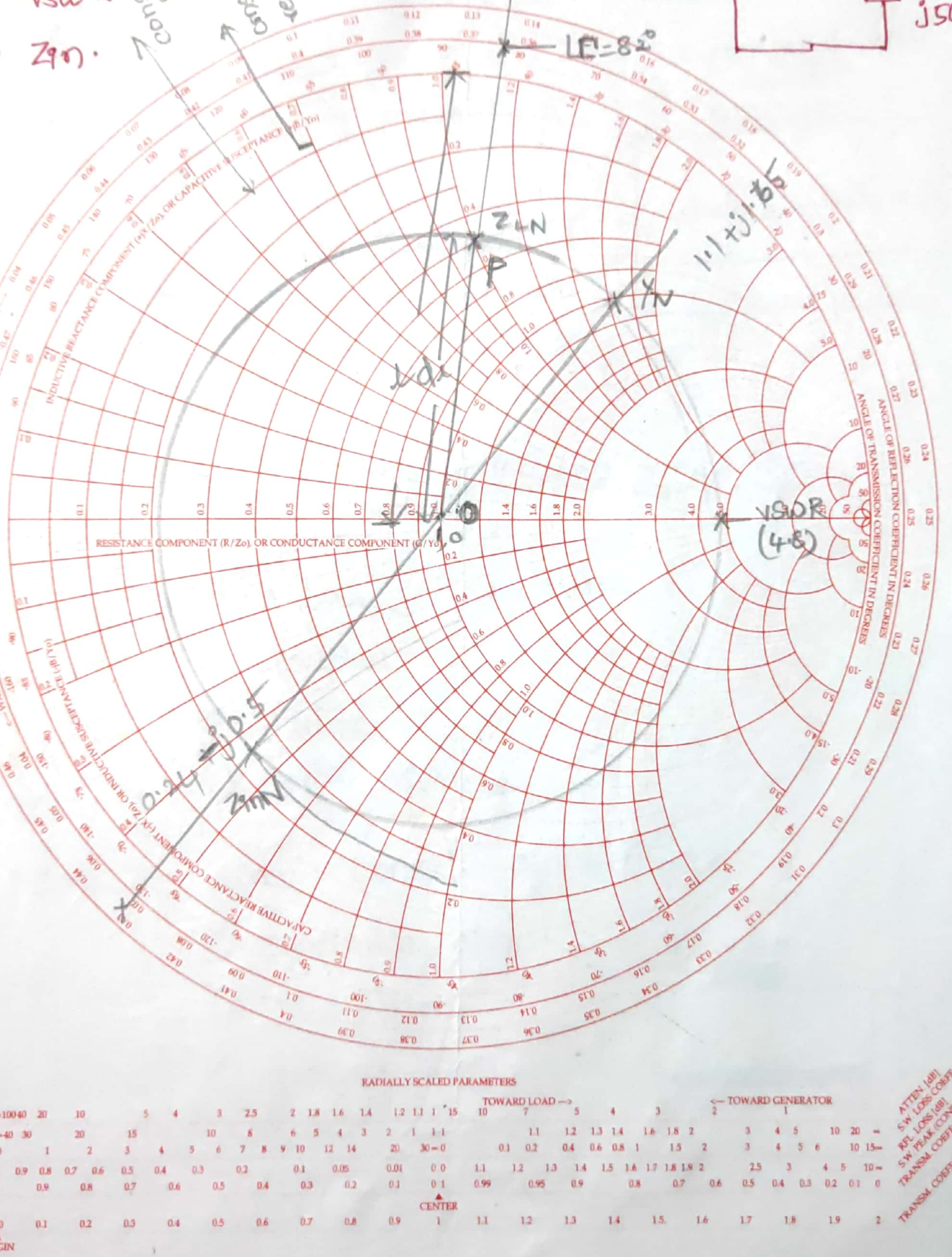
(a) Γ

(b) VSWR

(c) Zin.



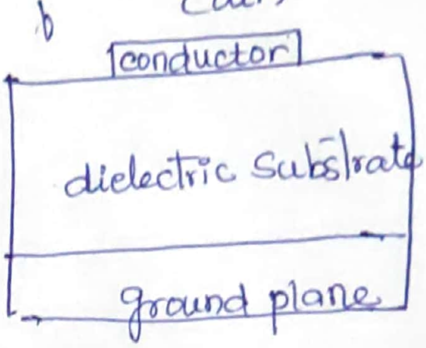
Smith impedance (or Amittance) chart



→ Microstrip transmission line :— carries EM waves (or) microwave freq sig

→ Microstrip is a type of electrical Tx line which can be fabricated using printed circuit board technology.

→ cross-section of micro-strip geometry



Conductor is separated from ground plane by dielectric substrate. upper dielectric is typically air.

→ It is used to convey microwave freq sigs.

→ Microstrips can form

(a) Antennas
(b) Couplers
(c) filters
(d) power dividers.
(e) MMIC

→ Microstrip lines consists of 3 layers

(a) conducting strip
(b) Dielectric
(c) Ground plane.