problems on discrete random variable :-

1. A Sample of 4 items is selected at a random from box containing 12 items of which 5 are defective find expected Number of defective items?

Let 'x' be the number of defective items among 4 items drawn from 12 items.

clearly x will takes the values 0, 1, 2, 3 or 4

Given that:

No. of items = 12

No. of defective items = 5

No. of good items = 7

i. The probability of getting no defective items is given

$$P(x=0) = P(\text{no defective})$$

$$= \frac{7C_4}{12C_4}$$

$$= \frac{35}{495} = \frac{7}{99}$$

ii. The probability of getting one defective item and three good items

$$P(x=1) = \frac{5C_1 \times 7C_3}{12C_4} = \frac{175}{495} = \frac{35}{99}$$

iii. The probability of getting two defective item and two good items

$$P(x=2) = \frac{5C_2 \times 7C_2}{12C_4} = \frac{14}{33}$$

iv. The probability of getting 3 defective items and good item

$$P(x=3 = \frac{5C_3 \times 7C_1}{12C_4} = \frac{14}{99}$$

v. The probability of getting 4 defective items

The probability distribution of x

x:	0	1	2	3	4
$p(x)$	$\frac{35}{495}$	$\frac{35}{99}$	$\frac{14}{33}$	$\frac{14}{99}$	$\frac{1}{99}$

The expected value of x is

$$E(x) = \sum_{i=0}^{4} x_i p_i$$

$$= 0 + \frac{35}{99} + 2\left(\frac{14}{33}\right) + 3\left(\frac{14}{99}\right) + 4\left(\frac{1}{99}\right)$$

$$= \frac{5}{3}$$

2. Two cards are drawn successfully with replacement from a well shuffled pack of cards. Find the probability distribution of the number of kings that can be drawn.

Let 'x' be the random variable No. of kings

Here x takes the values 0, 1, 2.

No. of cards = 52

No. of kings = 4

The probability of drawing a king is given by

$$\frac{4C_1}{52C_{12}} = \frac{1}{13}$$

The probability of not getting a king $= 1 - \frac{1}{13}$

$$= \frac{12}{13}$$

i. $p(x=0) = \frac{12}{13} \cdot \frac{12}{13} = \frac{144}{169}$

ii. $p(x=1) = \frac{1}{13} \cdot \frac{12}{13} + \frac{12}{13} \cdot \frac{1}{13} = \frac{12}{169} + \frac{12}{169} = \frac{24}{169}$

iii. $p(x=2) = \frac{1}{13} \cdot \frac{1}{13} = \frac{1}{169}$

The probability distribution is

x : 0 1 2

3. find the probability distribution of the No. of balls drawn. when 3 balls are drawn without replac from a bag containing 4 white and 6 red balls.

1. $p(a \le x \le b) = \int_a^b f(x)\,dx.$

2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

3. $\int_a^b f(x)\,dx = F(b) - F(a)$

4. mean $= \mu = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx.$

(or)

$E(\phi(x)) = \int_{-\infty}^{\infty} \phi(x) f(x)\,dx$

5. median :- If m is median, then

$$\int_a^m f(x)\,dx = \int_m^b f(x)\,dx = \frac{1}{2}$$

6. mode :- to find mode ; $f'(x) = 0$ & $f''(x) < 0$

7. variance $v(x) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

(or)

$$E((x-\mu)^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \left[\int_{-\infty}^{\infty} x f(x)\,dx\right]^2$$

1. If x is a continuous random variable & $y = ax+b$ prove that $E(y) = aE(x)+b$ and Variance$(y) = a^2 var(x)$

Sol :- By definition, we have

i. $E(y) = E(ax+b) = \int_{-\infty}^{\infty} (ax+b) f(x)\,dx$

$$= a\int_{-\infty}^{\infty} x f(x)\,dx + b\int_{-\infty}^{\infty} f(x)\,dx$$

$= aE(x) + b(1)$

$E(y) = aE(x) + b$

ii. By ① we have $E(y) = aE(x) + b \rightarrow ①$

Now, we have $y = ax + b \rightarrow ②$

② − ① gives $y - E(y) = a(x - E(x))$

Squaring on b.s we get

$$(y - E(y))^2 = a^2 (x - E(x))^2$$

Taking expectations on both sides, we get

$$E(y - E(y))^2 = a^2 E(x - E(x))^2$$

$$V(Y) = a^2 V(x)$$

2. If x is a continuous random variable & k is a constant then prove that

i. $var(x+k) = var(x)$

ii. $var(kx) = k^2 var(x)$

Sol :- By the definition

$$var(x+k) = \int_{-\infty}^{\infty} (x+k)^2 f(x)dx - \left[\int_{-\infty}^{\infty} (x+k) f(x) dx\right]^2$$

$$= \int_{-\infty}^{\infty} (x^2 + k^2 + 2xk) f(x) dx - \left[\int_{-\infty}^{\infty} x f(x) dx + k \int_{-\infty}^{\infty} f(x) dx\right]^2$$

$$= \int_{-\infty}^{\infty} x^2 f(x) dx + 2k \int_{-\infty}^{\infty} x f(x) dx + k^2 \int_{-\infty}^{\infty} f(x) dx - \left(\int_{-\infty}^{\infty} x f(x) dx + k \int_{-\infty}^{\infty} f(x) dx\right)^2$$

$$= E(x^2) + 2kE(x) + k^2(1) - (E(x) + k)^2$$

$$= E(x^2) + 2kE(x) + k^2 - (E(x)^2 + k^2 + 2kE(x))$$

$$= E(x^2) + \cancel{2kE(x)} + \cancel{k^2} - E(x)^2 - \cancel{k^2} - \cancel{2kE(x)}$$

$$= E(x^2) - E^2(x)$$

ii. $var(kx) = \int_{-\infty} (kx)f(x)\,dx - \left[\int_{-\infty} (kx)f(x)\,dx\right]$

$$= k^2\int_{-\infty}^{\infty} x^2 f(x)\,dx - k^2\left[\int_{-\infty}^{\infty} x f(x)\,dx\right]^2$$

$$= k^2E(x^2) - k^2(E(x))^2$$

$$= k^2(E(x^2) - E(x)^2)$$

$$= k^2 var(x)$$

3. for the continuous probability function $f(x) = kx^2e^{-x}$ when $x \geq 0$ find i. k ii. mean 3. variance

a. We know that $\int_{-\infty}^{\infty} f(x)\,dx = 1$ since $x \geq 0$

$$\int_0^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_0^{\infty} kx^2e^{-x}\,dx = 1$$

$\because \int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \dots$

where $v_1 = \int v\,dx$, $v_2 = \int v_1\,dx$ $v_3 = \int v_2\,dx$

$$= k\int_0^{\infty} \underset{u}{x^2}\,\underset{v}{e^{-x}}\,dx = 1$$

$$= k\left[x^2(-e^{-x}) - 2x(e^{-x}) + (2)(-e^{-x})\right]_0^{\infty} = 1$$

$$= k\left[e^{-x}(-x^2 - 2x - 2)\right]_0^{\infty} = 1$$

$$= k(0 - (-2)) = 1$$

$$= 2k = 1$$

$$= \boxed{k = 1/2}$$

ii. mean $= \mu = E(x) = \int_0^{\infty} x(kx^2e^{-x})\,dx$

$$= k\int_0^{\infty} x^3e^{-x}\,dx$$

$$+6x(-e \ldots$$

$$= \frac{1}{2}\left[x^3(-e^{-x}) - 3x^2(e^{-x}) + 6x(-e \ldots\right]_0^\infty$$

$$= \frac{1}{2}\left[e^{-x}(-x^3 - 3x^2 - 6x - 6)\right]_0^\infty$$

$$= \frac{1}{2}(0 - (-6))$$

$$= \frac{\cancel{6}\,3}{\cancel{2}}$$

$$= 3$$

iii. Variance $V(x) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

$$= \int_0^\infty x^2(k x^2 e^{-x})\,dx - \mu^2$$

$$= k\int_0^\infty x^4 e^{-x}\,dx - \mu^2$$

$$= \frac{1}{2}\left[x^4(-e^{-x}) - 4x^3(e^{-x}) + 12x^2(-e^{-x}) - 24x(e^{-x}) \ldots 24(-e^{-x})\right]_0^\infty - \mu^2$$

$$= \frac{1}{2}\left[e^{-x}\left[-x^4 - 4x^3 - 12x^2 - 24x - 24\right]\right]_0^\infty - \mu^2$$

$$= \frac{1}{2}\left[0 - (-24)\right] - 9$$

$$= \cancel{\frac{24}{2}}\ \cancel{\ldots} = 7.5$$

$$= \frac{24}{2} = 12 - 9$$

$$- 2$$

... function of a random variable x is

$f(x) = \begin{cases} \frac{1}{2}\sin x, & \text{for } 0 \le x \le \pi \\ 0 & \text{otherwise} \end{cases}$ find the mean, mode, median of the distribution & also find the probability b/w 0 & $\pi/2$

Sol :- i. mean :- By definition mean $= \mu = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$$= \int_{-\infty}^{0} x f(x)\,dx + \int_{0}^{\pi} x f(x)\,dx + \int_{\pi}^{\infty} x f(x)\,dx.$$

$$= 0 + \int_{0}^{\pi} x\left(\tfrac{1}{2}\sin x\right)dx + 0$$

$$= \frac{1}{2}\int_{0}^{\pi} \underset{u}{x}\ \underset{v}{\sin x}\,dx$$

$$= \frac{1}{2}\Big[x(-\cos x) - (1)(-\sin x)\Big]_{0}^{\pi}$$

$\cos\pi = -1$

$\cos 0 = 1$

$\sin\pi = 0$

$\sin 0 = 0$

$$= \frac{1}{2}\Big[\pi(-\cos\pi) - (1)(-\sin\pi) + (0 + \sin 0)\Big]$$

$$= \frac{1}{2}[\pi]$$

$$= \frac{\pi}{2}$$

$$\boxed{\mu = \frac{\pi}{2}}$$

ii. mode :- Let $f'(x) = \frac{1}{2}(\cos x),\ 0 \le x \le \pi$

Let $f'(x) = 0 \Rightarrow \cos x = 0 \Rightarrow x = \pi/2$

$f''(x) = \frac{1}{2}(-\sin x),\ \frac{-\sin x}{2}$

$f''(\pi/2) = \frac{-\sin\pi/2}{2} = -\frac{1}{2} < 0$

iii. median :- Let m be the median of r.v. x

$$\int_a^m f(x)\,dx = \int_m^b f(x)\,dx = 1/2$$

$$\Rightarrow \int_0^m \frac{1}{2}\sin x\,dx = \int_m^{\pi} \frac{1}{2}\sin x\,dx = 1/2$$

consider

$$\int_0^m \frac{1}{2}\sin x\,dx = 1/2$$

$$\frac{1}{2}\int_0^m \sin x\,dx = \frac{1}{2}$$

$$\int_0^m \sin x\,dx = 1$$

$$\left[-\cos x\right]_0^m = 1$$

$$\left[-\cos m + \cos 0\right] = 1$$

$$1 - \cos m = 1$$

$$\cos m = 0$$

$$\boxed{m = \pi/2}$$

$\therefore$ mean = mode = median = $\pi/2$

iv. $P(0 < x < \pi/2) = \int_0^{\pi/2} f(x)\,dx$

$$= \frac{1}{2}\int^{\pi/2} \sin x\,dx$$

$= \frac{1}{2}\left[-\cos x\right]_0^{\pi/2}$

$= \frac{1}{2}\left[-\cos \pi/2 + \cos 0\right]$

$= \frac{1}{2}(0+1)$

$= 1/2$

5. A continuous random variable x has the distribution function $F(x) = \begin{cases} 0, & \text{if } x \le 1 \\ k(x-1)^4, & \text{if } 1 \le x \le 3 \\ 1, & \text{if } x > 3 \end{cases}$ determine

i. $f(x)$

ii. k

iii. mean

Sol We know that $f(x) = \frac{d}{dx}(F(x))$

$\therefore f(x) = \begin{cases} 0, & \text{if } x \le 1 \\ 4k(x-1)^3, & \text{if } 1 \le x \le 3 \\ 0, & \text{if } x > 3 \end{cases}$

ii. find k

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{-\infty}^{1} f(x)\,dx \int_{1}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx = 1.$$

$$\Rightarrow \int_{1}^{3} 4k(x-1)^3\,dx = 1$$

$$= 4k\int_{1}^{3}(x-1)^3\,dx = 1$$

$$\Rightarrow 4k\left[\frac{(x-1)^4}{}\right]^3 = 1$$

$\Rightarrow 4k\left[\frac{(3-1)^4}{4} - \frac{(1-1)^4}{4}\right] = 1$

$\Rightarrow \cancel{4}k\left[\frac{16}{\cancel{4}}\right] = 1$

$\therefore 16k = 1$

$k = 1/16$

iii. mean $= \mu = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_{-\infty}^{1} x f(x)\,dx + \int_{1}^{3} x f(x)\,dx + \int_{3}^{\infty} x f(x)\,dx$

$\Rightarrow \int_{1}^{3} x(4k(x-1)^3)\,dx$

$= 4k \int_{1}^{3} x(x-1)^3\,dx$

put $x - 1 = t$
$x = 1 + t$
$dx = dt$
when $x = 1 \Rightarrow t = 0$
$x = 3 \Rightarrow t = 2$

$= 4k \int_{0}^{2} (1+t)\,t^3\,dt$

$= 4k \int_{0}^{2} t^3 + t^4\,dt$

$= 4k\left[\frac{t^4}{4} + \frac{t^5}{5}\right]_0^2$

$= 4k\left[\left[\frac{2^4}{4} + \frac{2^5}{5}\right] - \left[\frac{t^0}{4} + \frac{t^0}{5}\right]\right]$

$= 4K\left[\frac{52}{5}\right]$

$= \frac{4}{16}\left[\frac{52}{5}\right]$

$\mu = \frac{13}{5}$

6. The trouble shooting capacity of Ic chip in a circuit is a RV x whose distribution function is given by

$$P(x) = \begin{cases} 0, & \text{for } x \leq 3 \\ 1 - \frac{9}{x^2}, & \text{for } x > 3 \end{cases}$$

where x denotes the no. of years

find the probability that the Ic chip will work properly

i. less than 8 years

ii. beyond 8 years

iii. anywhere from 5 to 7 yrs

iv. " " 2 to 5 yrs

Sol :- We know that $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt$

and $f(x) = \int_a^b f(x)\,dx = F(b) - F(a)$

$\therefore f(x) = \int_a^b f(x)\,dx = F(b) - F(a)$

i. $P(X \leq 8) = \int_0^8 \left(1 - \frac{9}{x^2}\right)dx = F(8) - F(0)$

$= 1 - \frac{9}{64}$

$= 55/64$

ii. $P(X > 8) = 1 - P(x \leq 8)$

$1 - \frac{55}{}$

iii. $P(5 \le x \le 7) = \int_{5} f(x)\,dx = f(7) - f(5)$

$$= 1 - \frac{9}{49} - 1 - \frac{9}{25}$$

$$= \frac{40}{49} - \frac{16}{25}$$

$$= \frac{216}{1225} = \cancel{0.01959}\ 0.176$$

iv. $P(2 \le x \le 5) = \int_{2}^{5} f(x)\,dx = f(5) - f(2)$

$$= 1 - \frac{9}{25} - 1 - \frac{9}{4}$$

$$= \frac{16}{25} + \frac{5}{4}$$

$$= \frac{189}{100} = 1.89$$

Discrete uniform distribution :- A random variable x has a discrete uniform distribution iff it's probability distribution is given by $P(x) = \frac{1}{n}$, for $x = x_1, x_2 ---- x_n$
The random variable x is that called discrete uniform random variable.

discreete = mean = Σ
Contin = " = $\int$.

Eg:-

x	0	1
$P(x)$	$1/2$	$1/2$

Bernoulli's distribution :- A random variable x which takes two values 0 and 1 with probability q and p respectively i.e; $P(x=0) = q$ and $P(x=1) = p$ is called, Bernoulli's discrete random variable and is said to have a bernoulli's distribution.

The probability function of Bernoulli's distribution is
$P(x) = p^x q^{1-x} = p^x (1-p)^{1-x}$ $x = 0, 1$

mean of discrete uniform random variable :-

We know that the mean of the random variable x is given by $E(x) = \Sigma x f(x) \Rightarrow E(x) = \Sigma x \frac{1}{n} = \frac{1}{n} \sum_{x=1}^{n} x$

$$= \frac{1}{n}[1 + 2 + ---- + n]$$

$$= \frac{1}{\cancel{n}} \frac{\cancel{n}(n-1)}{2}$$

$$= \frac{(n-1)}{2}$$

Variance :- We know that the variance of random variable is $V(x) = E(x^2) - (E(x))^2$

$$= \sum_{x=1}^{n} x^2 \cancel{f(x)} - \left(\frac{(n-1)}{2}\right)^2$$

$$= \sum_{x=1}^{n} x^2 \frac{1}{n} - \frac{(n-1)^2}{4}$$

$$= \frac{1}{n} \frac{n(n+1)(2n+1)}{3!} - \frac{(n-1)^2}{4}$$

$$= \frac{(n+1)(2n+1)}{3!} - \frac{(n-1)^2}{4}$$

$$\Rightarrow \frac{(n+1)(n-1)}{12}$$

Mean of Bernoulli's random variable :- We know that the mean of the random variable x is given by

$E(x) = \Sigma x f(x)$

$\Rightarrow E(x) = \sum_{x=0}^{1} x_i f(x_i)$

$= \sum_{x=0}^{1} x \, p^{x}(1-p)^{1-x}$

when $x=0$
$= 0$

when $x = 1 = p$

$= 0 + p = p$

$E(x) = p$

Variance :- We know that the variance of random variable is $V(x) = E(x^2) - (E(x))^2$

$= \sum_{x=0}^{1} x^2 f(x) - p^2$

$= \sum_{x=0}^{1} x^2 p^{x}(1-p)^{1-x} - p^2$

$= 0 + p - p^2$

$= p(1-p)$

$V(x) = pq$

independent trails such that for each trail, p is the probability of a success and q that of a failure. then the probability of getting exactly r success in n trails is given by $^nC_r p^r q^{n-r}$ when r takes any integer value b/w 0 and n.
The probability of the no. of success so obtained is called the binomial distribution, bcz the probabilities are the successive terms of the expension of binomial $(q+p)^n$.

Definitions :-

A random variable x has a binomial distribution if it assumes only non-negative values and it's probability distribution is given by;

$$p(x=r) = p(r) = \begin{cases} {}^nC_r\, p^r q^{n-r}, & \text{for } r=0,1,2 \cdots n \\ 0, & \text{otherwise} \end{cases}$$

(or)

$$p(x=x) = p(x) = \begin{cases} {}^nC_x\, p^x q^{n-x}, & \text{for } x=0,1,2,\cdots n \\ 0, & \text{otherwise.} \end{cases}$$

Constants of binomial distribution [2M]

Mean of a binomial distribution :- The binomial distribution is given by $f(x) = p(x) = {}^nC_x\, p^x q^{n-x}$, $x=0,1,\cdots$ and $q = 1-p$

We know that the mean of RV is

$$\mu = E(x) = \sum_{x=0}^{n} x f(x)$$

$$= \sum_{x=0}^{n} x\, {}^nC_x\, p^x q^{n-x}$$

$$= 0\, q^n + 1 \cdot {}^nC_1\, p^1 q^{n-1} + 2\, {}^nC_2\, p^2 q^{n-2} + \cdots + n p^n$$

$$= npq^{n-1} + 2\frac{n(n-1)}{2} p^2 q^{n-2} + \cdots + np^n$$

$$= np\,[q^{n-1} \cdots$$

$\boxed{\mu = np}$

Variance of the binomial distribution :- The b.d is given by $f(x) = p(x) = nc_x p^x q^{n-x}$, $x = 0, 1, 2 \dots n$ and $q = 1-p$

We know that variance of RV is

$$v(x) = E(x^2) - (E(x))^2 = E(x^2) - \mu^2$$

$$\left.\begin{aligned} &= \sum_{x=0}^{n} x^2 f(x) - \mu^2 \\ &= \sum_{x=0}^{n} x^2 nc_x p^x q^{n-x} - n^2p^2 \\ &= -n^2p^2 + 1nc_1 p^1 q^{n-1} - n^2p^2 \end{aligned}\right\} \text{where}$$

$$E(x^2) = \sum_{x=0}^{n} x^2 f(x)$$

$$= \sum_{x=0}^{n} x^2 nc_x p^x q^{n-x}$$

$$= \sum_{x=0}^{n} (x^2 \cancel{-x} + x) nc_x p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) + x \; nc_x p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) nc_x p^x q^{n-x} + \sum_{n=0}^{n} x \, nc_x p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) nc_x p^x q^{n-x} + \mu$$

$$= 2nc_2 p^2 q^{n-2} + 3 \cdot 2 nc_3 p^3 q^{n-3} + \dots + n(n-1)p^n) + \mu$$

$$= \cancel{2}\frac{n(n-1)}{\cancel{2}} p^2 q^{n-2} + \cancel{6} \cdot \frac{n(n-1)(n-2)}{\cancel{6}} nc_3 p^3 q^{n-3} + \dots$$

$$+ n(n-1)p^n) + \mu$$

$= n(n-1)p^2 (q+p)^{n-2} + \mu$

$= n(n-1)p^2 + \mu$

$V(x) = n(n-1)p^2 + \mu - \mu^2$

$= n^2p^2 - np^2 + np - n^2p^2$

$= np(1-p)$

$V(x) = npq$

poisson distribution :- [Manu]

The poisson distribution can be derived as a limiting case of the binomial distribution under the conditions that

i. p is very small

ii. n is very large

iii. $np = \lambda$

Definition :- A random variable x is said to follow a poisson distribution if it assumes only non-negative integers and its probability mass function is given by

$$p(x, \lambda) = P(X = x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, & \text{for } x = 0,1,2,\dots, \lambda > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Here λ is called the parameter of the distribution. the notation $X \sim P(\lambda)$ denote that x is a poisson variable with poisson distribution.

Note :- i. $\sum_{x=0}^{\infty} P(X = x) = \sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!}$

$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!}$

$= e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} \dots\right]$

$\left[e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \dots \right.$

$= e^{-\lambda} \cdot e^{\lambda}$

The poisson distribution function is $f(x) = P(x = x) =$

$$\sum_{r=0}^{\infty} p(r) = e^{-\lambda} \sum_{r=0}^{\infty} \frac{\lambda^r}{r!}$$

mean of poisson distribution :- The mean of P.D is given by $\mu = E(x) = \sum_x x\, p(x)$

we have that

$$p(x) = \frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2, ---$$

$$\therefore E(x) = \sum_{x=0}^{\infty} x \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{\cancel{x} \lambda^x}{\cancel{x}(x-1)!}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!}$$

put $x - 1 = y$

$x = y + 1$

$x = 1 \quad = y = 0$

$x = \infty, \quad y = \infty$

$$= e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^{y+1}}{y!}$$

$$= \lambda e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^y}{y!} \qquad \left[\sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = e^{\lambda} \right]$$

$$= \lambda e^{-\lambda} e^{\lambda}$$

$$= \lambda e^{0} = \lambda$$

$$\boxed{E(x) = \lambda}$$

variance of poisson distribution :- The variance of poisson distribution is given by $V(x) = E(x^2) - (E(x))$

$$E(x^2) = \sum_{x=0} x^2 p(x)$$

$$= \sum_{x=0}^{\infty} x^2 \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{x^2 \lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{x^{\cancel{2}} \lambda^x}{\cancel{x}(x-1)!}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{x \lambda^x}{(x-1)!}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{[(x-1)+1] \lambda^x}{(x-1)!}$$

$$= e^{-\lambda} \left\{ \sum_{x=1}^{\infty} \frac{(x-1) \lambda^x}{(x-1)!} + \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!} \right\}$$

$$= e^{-\lambda} \left\{ \sum_{x=2}^{\infty} \frac{\lambda^x}{(x-2)!} + \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!} \right\} \qquad \left\{ \frac{(x-1)!}{(x-1)} = (x-2)! \right\}$$

$$= e^{-\lambda} \left\{ \sum_{y=0}^{\infty} \frac{\lambda^{y+2}}{y!} + \sum_{z=0}^{\infty} \frac{\lambda^{z+1}}{z!} \right\}$$

put $x-2 = y$

$x-1 = z$

then $x = y+2$

$x = z+1$

$$= e^{-\lambda} \left\{ \lambda^2 \sum_{y=0}^{\infty} \frac{\lambda^y}{y!} + \lambda \sum_{z=0}^{\infty} \frac{\lambda^z}{z!} \right\}$$

$$= e^{-\lambda} \left\{ \lambda^2 e^{\lambda} + \lambda e^{\lambda} \right\}$$

$$E(x^2) = \lambda^2 + \lambda$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= \lambda^2 + \lambda - \lambda^2$$

Mode of the poisson distribution :- Mode is the value of x for which the probability $p(x)$ is maximum. The mode of poisson distribution lies between

$p(x) \geq p(x+1)$ and $p(x) \geq p(x-1)$

$$\frac{e^{-\lambda}\lambda^{x}}{x!} \geq \frac{e^{-\lambda}\lambda^{(x+1)}}{(x+1)!}$$

$$1 \geq \frac{\lambda}{(x+1)}$$

$\Rightarrow \lambda \leq x+1 \Rightarrow x \geq \lambda - 1$

Ily $p(x) \geq p(x-1) \Rightarrow x \leq \lambda$

$\therefore$ The mode of poisson distribution lies between $\lambda - 1$ and λ

Recurrence relation :-

$$p(x) = \frac{\lambda}{x} p(x-1)$$

Movement Generating function of poisson distribution

The movement Generating function of poisson of the distribution is given by $M_x(t) = E(e^{tx})$

$$\Rightarrow M_x(t) = \sum_x x\, p(x)$$

$$= \sum_{x=0} e^{tx} \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^{t})^{x}}{x!}$$

$$= e^{-\lambda} e^{\lambda e^{t}}$$

$$= e^{\lambda(e^{t}-1)}$$

Problems :-

1. 20% of the bolts produced in a factory found to be defective. find the probability that in a sample of 10 bolts choosen at random, exactly 2 will be defective by using i. binomial distribution ii. poisson approximation to binomial distribution.

Sol :- The probability of defective bolts $= p = \frac{20}{100} = 0.2$

$q = 1 - p = 1 - 0.2 = 0.8$

i. By binomial distribution we have.

$p(x) = {}^{n}c_{x} p^{x} q^{n-x}$

$n = 10$ and $x = 2$

$p(x) = 10c_2 p^2 q^{10-2}$

$= 10c_2 (0.2)^2 (0.8)^8$

$= 45 \times 0.04 \times 0.1677$

$p(x=2) = 0.3019$

ii. we know that $\lambda = np$

$= 10 \times 0.2$

$= 2$

By poisson distribution we have

$$p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-2}\,2^{2}}{2!}$$

$$= \frac{0.135 \times 4}{2} = \frac{0.54}{2} = 0.27$$

2. If a probability of a defective fuse from a manufacturing unit is 2%, in a box of 200 fuses, find the probability i. more than 3 ... are defective

Sol :- $p = \frac{2}{100} = 0.02$

$q = 1 - p$

$= 1 - 0.02$

$= 0.98$

i. $P(x=4) = \frac{e^{-\lambda} \lambda^x}{x!}$

$n = 200 \quad x = 4$

when $\lambda = np$

$\lambda = 200 \times 0.02 = 4$

$P(x=4) = \frac{e^{-4}\, 4^4}{4!} = \frac{0.0183 \times 256}{24} = \frac{4.6848}{24}$

$P(x=4) = 0.1952$

ii. $P(x \geq 3) = 1 - [P(x=0) + P(x=1) + P(x=2)] + P(x=3)$

$= 1 - \left[\frac{e^{-\lambda} \lambda^0}{0!} + \frac{e^{-\lambda} \lambda^1}{1!} + \frac{e^{-\lambda} \lambda^2}{2!} + \frac{e^{-\lambda} \lambda^3}{3!}\right.$

$= 1 - \frac{e^{-4} (4)^0}{0!} + \frac{e^{-4} (4)^1}{1!} + \frac{e^{-4} (4)^2}{2!} + \frac{e^{-4} (4)^3}{3!}$

$= 1 - \frac{0.0183 \times 1}{1} + \frac{0.0183 \times 4}{1} + \frac{0.0183 \times 16}{2} + \frac{0.0183 \times 64}{6}$

$= 1.39683$

3. In a given city, 6% of all drivers get at least one parking ticket per year. Use the poisson approximation with binomial distribution to determine the probability that among 80 drivers, i. 4 will get atleast one parking ticket in any given year ii. atleast 3 will get atleast 1 parking ticket in any given year. iii. Anywhere from 3 to 6, inclusive, will get atleast 1 parking ticket in any given year

Sol) :- $p = 6\% = \frac{6}{100} = 0.06$

$q = 1-p = 1-0.06 = 0.94$, $n = 80$

i. $p(x=4) =$

$\lambda = np = 0.06 \times 80 = 4.8$

~~i. probabilit~~

i. $p(x=4) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-4.8}(4.8)^4}{4!}$

$p(x=4) = 0.1820$

ii. $p(x \geq 3) = p(x=0) + p(x=1) + p(x=2) + p(x=3)$

$= e^{-4.8}$

ii. $p(x \geq 3) = 1 - [p(x=0) + p(x=1) + p(x=2)]$

$= 1 - \left[\frac{e^{-4.8}(4.8)^0}{0!} + \frac{e^{-4.8}(4.8)^1}{1!} + \frac{e^{-4.8}(4.8)^2}{2!}\right]$

$p(x \geq 3) = \cancel{0.8860}\ 0.8574$

iii. $p(3 \leq x \leq 6) = p(x=3) + p(x=4) + p(x=5) + p(x=6)$

$= e^{-4.8}(4.8)^3, e^{-4.8}(4.8)^4, e^{-4.8}(4.8)^5, e^{-4.8}(4.8)^6$

$= 0.6482$

4. If x is a poisson variable such that $p(x=2) = 9p(x=4)$ $+ 90p(x=6)$ find the mean & variance [Manu 5M]

Sol :- Let x be a poisson variable with parameter λ.

then by poisson distribution; we have

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad \text{for } x = 0, 1, 2 - - - -, \lambda > 0$$

Given $p(x=2) = 9p(x=4) + 90p(x=6)$

$$\frac{e^{-\lambda}\lambda^{2}}{2!} = 9\frac{e^{-\lambda}\lambda^{4}}{4!} + 90\frac{e^{-\lambda}\lambda^{6}}{6!}$$

$$\frac{e^{-\lambda}\lambda^{2}}{2!} = e^{-\lambda}\left[\frac{9\lambda^{4}}{4!} + \frac{90\lambda^{6}}{6!}\right]$$

$$\frac{\cancel{e^{-\lambda}}\cancel{\lambda^{2}}}{2!} = \cancel{e^{-\lambda}}\cancel{\lambda^{2}}\left[\frac{9\lambda^{2}}{4!} + \frac{90\lambda^{4}}{6!}\right]$$

$$\frac{1}{2} = \frac{9\lambda^{2}}{24} + \frac{90\lambda^{4}}{720}$$

$$\frac{1}{2} = \frac{3\lambda^{2}}{8} + \frac{\lambda^{4}}{8}$$

$$1 = \frac{3\lambda^{2}}{4} + \frac{\lambda^{4}}{4}$$

$$\lambda^{4} + 3\lambda^{2} = 4$$

$\lambda^4 + 4\lambda^2 - \lambda^2 - 4 = 0$

$\lambda^2(\lambda^2+4) - (\lambda^2+4) = 0$

$(\lambda^2-1)(\lambda^2+4) = 0$

$\lambda^2 = 1, \quad \lambda^2 = -4$

$\lambda = 1$ [$\because$ λ must be +ve & real number]

for poisson distribution mean = variance = $\lambda = 1$

5. Assuming that one in 80 births is a case of twins, calculate the probability of two or more sets of twins on a day when 30 births occur. Compare the results obtained by using i. binomial distribution ii. poisson distribution.

Sol :- Here $n = 30$

$p = \frac{1}{80} = 0.0125$

$q = 1 - p = 1 - 0.0125 = 0.987$

i. binomial distribution :-

$p(x \geq 2) = {}^nC_x \, p^x q^{n-x}$

$= 1 - p(x<2)$

$= 1 - [p(x=0) + p(x=1)]$

$= 1 - [{}^{30}C_0 (0.0125)^0 (0.987)^{30} + {}^{30}C_1 (0.0125)^1 (0.987)^{29}]$

$= 1 - [0.695 + 30 \times 0.0125 \times 0.684]$

$= 1 - [0.695 + 0.2565]$

$= 1 - [0.9515]$

$= 0.0485$

$\lambda = np$

$\lambda = 30 \times 0.0125 = 0.375$

ii. poisson distribution :-

$p(x \geq 2) = 1 - p(x < 2)$

$= 1 - p(x=0) + p(x=1)$

$= 1 - \frac{e^{-\lambda}\lambda^x}{x!} + \frac{e^{-\lambda}\lambda^x}{x!}$

$= 1 - \frac{e^{-(0.375)}(0.375)^0}{0!} + \frac{e^{-(0.375)}(0.375)^1}{1!}$

$= 1 - 0.6872 + 0.2577$

$= 1 - 0.9449$

$= 0.0551$

Normal distribution for continuous RV :-

A RV x is said to have a normal distribution, if it's probability density function is given by

$f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \; -\infty < x < \infty,$

$-\infty < \mu < \infty$ and $\sigma > 0$ where μ is the mean and σ is standard deviation are the two parameters of normal distribution.

The RV x is then said to be a normal RV (or) normal variate

Note :- $x \sim N(\mu, \sigma^2)$

ii. if $x \sim N(\mu, \sigma^2)$ then $z = \frac{x-\mu}{\sigma}$ is a standard normal variate with $\mu = 0$, $V(z) = 1$

iii. The probability density function is given by and the corresponding density function is given by

$\phi(z) = P(Z \le z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{u^2}{2}} du$

iv. $\phi(-z) = 1 - \phi(z)$

$P(a \le x \le b) = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$

$P(z \le a) = P(z \ge a)$

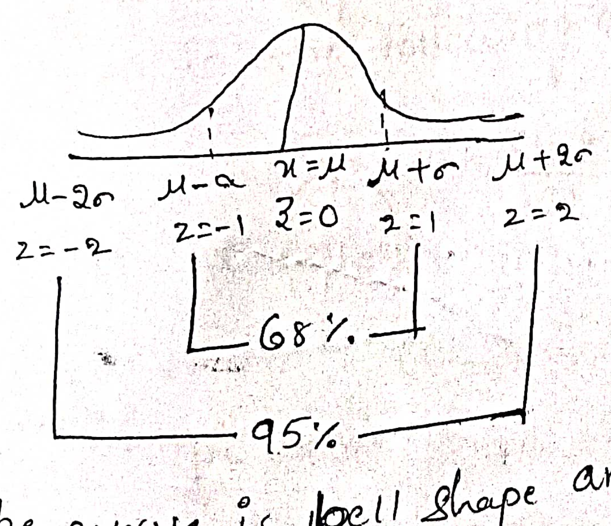
for normal distribution we have,

mean = median = mode

Area under the normal curve :-

By taking $z = \frac{x-\mu}{\sigma}$, the standard normal curve is formed. The total area under this curve is '1'. The area under the curve is divided into two equal parts by $z=0$. left hand side area and the right hand side area to $z=0$ is 0.5.

Normal curve :- [2M]



The curve is bell shape and symmetrical about the x-axis.

Problem

X is normally distributed and the mean of x is ... standard deviation is 4. a. find the probability

b. Find x_1 when $p(x \geq x_1) = 0.25$, $p(x_1 < x < x_2) = 0.50$ and $p(x > x_2) = 0.025$

$p(x_1 < x < x_2) = 0.50$ and $p(x > x_2) = 0.025$

Sol :- Given $\mu = 12$ & $\sigma = 4$

we have x is normally distributed

$X \sim N(\mu, \sigma^2)$ and $z = \dfrac{x-\mu}{\sigma}$

a.

i. $p(x \geq 20)$ when $x = 20$, $z = \dfrac{20-\mu}{\sigma} = \dfrac{20-12}{4} = 2$

$\therefore p(x \geq 20) = p(z \geq 2)$

$= 0.5 - p(0 \leq z \leq 2)$

$= 0.5 -$ Area from 0 to 2

$= 0.5 - 0.4772$

$= 0.0228$

ii. $p(x \leq 20) = z = \dfrac{20-\mu}{\sigma} = 2$

$= 1 - p(x \geq 20)$

$= 1 - 0.0228$

$= 0.9772$

iii. $p(0 \leq x \leq 12) =$

We have $z = \dfrac{x-\mu}{\sigma} = \dfrac{x-12}{4}$

when $x = 0$, $z = -3$, when $x = 12$, $z = 0$

$p(0 \leq x \leq 12) = p(-3 \leq x \leq 0)$

$= p(0 \leq x \leq 3)$ from Symmetry

$=$ Area from $z = 0$ to $z = 3$

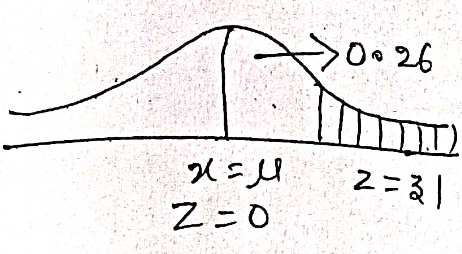
$= 0.49865$

$p(x > x_1) = 0.24$ $\quad \frac{x_1 \ldots}{4} = z_1$ (say)

$= P(z > z_1) = 0.24$

$= P(0 < z < z_1) = 0.26$

$z_1 = 0.65$ (approx) from Normal table.



Hence $\frac{x_1 - 12}{4} = 0.65$

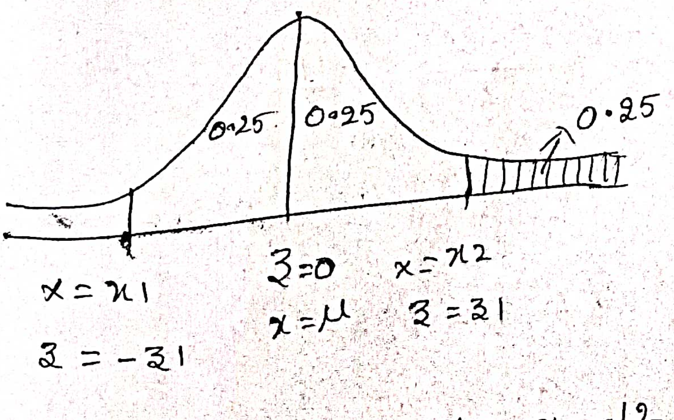
$x_1 = 12 + (0.65)4$

$= 14.6$

c. when $x = x_1$, $x = x_2$, $z = \frac{x - \mu}{\sigma} = \frac{x_1 - 12}{4} = z_1$ (say)

$p(x_1 < x < x_2) = 0.50$
$p(x > x_2) = 0.25$ $\Big\} \rightarrow ①$

from ①, clearly the points x_1 and x_2 are located as shown in the following curve



when $x = x_2 \Rightarrow z_1 = \frac{x_2 - \mu}{\sigma} = \frac{x_2 - 12}{4} \rightarrow$ (say) ②

Given $P(x > x_2) = P(z > z_2) = 0.25$ by curve

$= p(0 \le z \le z_1) = 0.25$

when $x = x_1 \Rightarrow z_2 = \frac{x_1 - \mu}{\sigma} = -z_1$ say

$$= \frac{x_1 - 12}{4} = -0.68$$

$$x_1 = -0.68 \times 4 + 12$$

$$= 12 - 2.72$$

$$x_1 = 9.28$$

from (2), we get

$$\frac{x_2 - 12}{4} = 0.68$$

$$x_2 = 0.68 \times 4 + 12$$

$$x_2 = 14.72$$

2. If x is a normal variate with $\mu = 30$ & $\sigma = 5$

find i. $P(26 \le x \le 40)$ ii. $P(x \ge 45)$

a. Given that

mean $\mu = 30$

S.D $\sigma = 5$

we have $x \sim N(\mu, \sigma^2)$ and $z = \frac{x - \mu}{\sigma} = \frac{x - 30}{5}$

i. $P(26 \le x \le 40)$:-

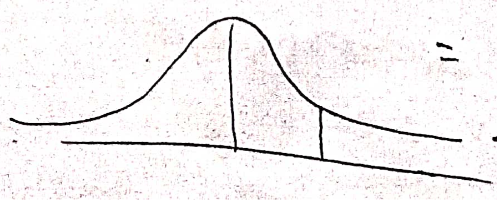
$x = 26 \quad z = \frac{26 - 30}{5} = -\frac{4}{5} = -0.8$

$x = 40 \quad z = \frac{40 - 30}{5} = \frac{10}{5} = 2$

$$P(-0.8 \le z \le 2) = P(2 \le z \le 0.8)$$

= Area from $z = 0$ to -0.8 +

" " $z = 0$ to 2



$= 0.2881 + 0.4772$

$= 0.7653$

ii. $p(x \geq 45)$ =

$x = 45 \quad z = \frac{x-\mu}{\sigma} = \frac{45-30}{5} = \frac{15}{5} = 3$

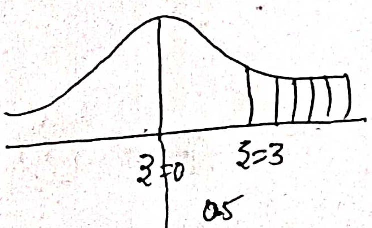
$p(x \geq 45) = p(z \geq 3)$

$= 0.5 - p(0 \leq z \leq 3)$

$= 0.5 -$ Area from 0 to 3

$= 0.5 - 0.4987$

$= 0.0013$



8. In a normal distribution 31% of items are under 45 and 8% are over 64 find the mean and s.D of the distribution [Manu 5M]

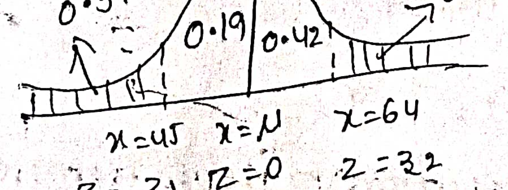
Sol:- Let x denotes the normal variate

Given that $P(X<45) = \frac{31}{100} = 0.31$ & $P(x>64) = 0.08$

if x has a normal distribution with μ & σ, then the standard variables corresponding to $x = 45$ & 64 are given below.

we have $z = \frac{x-\mu}{\sigma} \Rightarrow \frac{45-\mu}{\sigma} = -z_1$ (say) → (1)

$z = \frac{x-\mu}{\sigma} = \frac{64-\mu}{\sigma} = z_2$ (say) → (2)



from the fig, we have $P(0<z<z_2) = 0.42$

$z_2 = 1.41$ from table → (3)

$P(-z_1<z<0) = 0.19$

$$\frac{45-\mu}{\sigma} = -0.50 \Rightarrow \begin{array}{l} 45-\mu = -0.50\sigma \\ \mu = 45+(0.50)\sigma \rightarrow (5) \end{array}$$

$$\frac{64-\mu}{\sigma} = 1.41 \Rightarrow \begin{array}{l} 64-\mu = 1.41\sigma \\ \mu = 1.41 \\ \mu = 64-1.41\sigma \rightarrow (6) \end{array}$$

Solve (5) & (6)

$$\mu = 45+(0.50)\sigma$$
$$\mu = 64-(1.41)\sigma$$
$$\underline{- \quad\quad - \quad\quad +}$$
$$= -19+1.91\sigma$$
$$+19 = +1.91\sigma$$
$$\frac{19}{1.91} = \sigma$$
$$\sigma = 9.947 \quad \text{sub in (5)}$$

$$\mu = 45+(0.50)9.947$$
$$= 45+4.9735$$
$$= 49.9735$$

$\therefore$ mean $= \mu = 49.9735$

S.D $= \sigma = 9.947$

4. In a normal distribution 7% of items are under 35 and 89% are under 63. determine the mean & variance of distribution

Sol: Let x be the normal variate

$p(x \leq 63) = 0.89$

$p(x \leq 63) + P(x > 63) = 1$

$p(x > 63) = 1 - p(x \leq 63)$

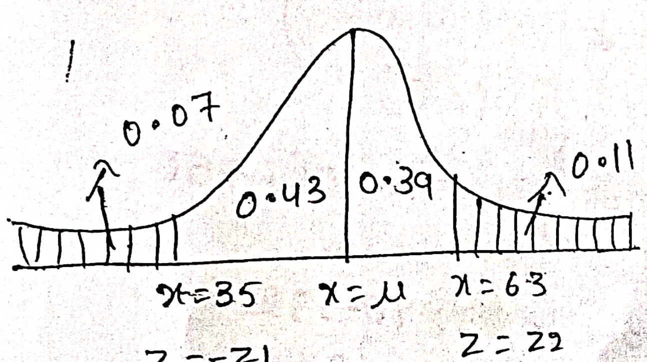
$= 1 - 0.89$

$p(x > 63) = 0.11$

$p(x < 35) = 7\% = 0.07$

$p(x < 35) = 0.07$

The points $x = 35$ & $x = 63$ shown in the following figure.



from the fig, we have $p(0 < z < z_2) = 0.39$

$z_2 = 1.23$ from table

$p(-z_1 < z < 0) = 0.43$

$p(0 < z < z_1) = 0.43$

$z_1 = 1.048$ from table.

$z = \frac{x - \mu}{\sigma} = -z_1 \Rightarrow \frac{35 - \mu}{\sigma} = -1.48$

$35 - \mu = -1.48\sigma$

$\mu = 35 + (1.48)\sigma \rightarrow (1)$

$z = \frac{x - \mu}{\sigma} = z_2 \Rightarrow \frac{63 - \mu}{\sigma} = 1.23$

$$63 - \mu = 1.23\sigma$$

$$\mu = 63 - 1.23\sigma \rightarrow (2)$$

from (1) & (2)

$$\mu = 35 + (1.48)\sigma$$
$$\mu = 63 - (1.23)\sigma$$
$$- \quad +$$

$$= -28 + 2.71\sigma$$

$$= \cancel{-}28 = \cancel{-}2.71\sigma$$

$$\frac{28}{2.71} = \sigma$$

sub in (2)

$$10.332 = \sigma \Rightarrow \sigma^2 = 106.750$$

mean : $\mu = 63 - 1.23 \times 10.332$

$$\mu = 50.29164$$

5. Given that the mean heights of students in a class is 158 cm with S.D of 20 cm find how many students heights lie b/w 150 cm & 170 cm if there are 100 students in the class.

Exponential distribution :- [Manu 2M]

A random variable x is said to have an exponential distribution with parameter θ with 0 if its probability density function is given by

$$f(x,\theta) = \begin{cases} \theta e^{-\theta x}, & x \geq 0, \text{othe} \\ 0, & \text{otherwise.} \end{cases}$$

The cummulative distribution function $F(x)$ is given by

$$F(x) = \int_0^x f(x)\,dx = \theta \int_0^x e^{-\theta x}\,dx$$

$$= \theta \left[\frac{e^{-\theta x}}{-\theta}\right]_{x=0}^{x} = \left[-e^{-\theta x}\right]_{x=0}^{x}$$

$$= -e^{-\theta x} + 1 = 1 - e^{-\theta x}.$$

$$\therefore F(x) = \begin{cases} 1 - e^{-\theta x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Note :- The mean of exponential distribution $= \frac{1}{\theta}$ and

variance $= \frac{1}{\theta^2}$

problem :-

~~The pro~~ Show that if x has an e.d, then for every constant $a \geq 0$, $P(Y \leq x \,/\, x \geq a) = P(X \leq x)$, for all x where $Y = X - a$

a. The probability density function of e.d with parameter θ is $f(x) = \theta e^{-\theta x}$, $x \geq 0$

We have $P(Y \leq x \,/\, x \geq a) = P(X - a \leq x \,/\, X \geq a)$

$= P(X \leq x + a \,/\, X \geq a)$

an

$P(Y$

ponential
proba-

$$= \int_a^{x+4} f(x)\,dx$$

$$= \int_a^{x+a} \theta e^{-\theta x}\,dx$$

$$= \theta \int_a^{x+a} e^{-\theta x}\,dx$$

iven by

$$= \cancel{\theta} \cdot \left[\frac{e^{-\theta x}}{-\cancel{\theta}}\right]_a^{x+a}$$

$$= \left(-e^{-\theta x}\right]_a^{x+a}$$

$$= -e^{-\theta(x+a)} + e^{-\theta a}$$

$$= -e^{a\theta}\left(1 - e^{-\theta x}\right)$$

- and

and $p(x \geq a) = p(a \leq x \leq \infty)$

$$= \theta \int_a^{\infty} e^{-\theta x}\,dx$$

$$= \cancel{\theta}\left[\frac{e^{-\theta x}}{-\cancel{\theta}}\right]_a^{\infty}$$

stant

$$= -\left[e^{-\theta x}\right]_a^{\infty}$$

ie $y = x - a$

with

$$= -\cancel{e^{-\theta a}} + \cancel{e^{-\theta \infty}}$$

$$= e^{-\infty} + e^{-a\theta}$$

$$= e^{-a\theta}$$

$$p(y \leq x / x \geq a) = \frac{p(y \leq x \wedge x \geq a)}{p(x \geq a)}$$

Also, $P(X \le x) = P(0 \le X \le x)$

$$= \theta \int_0^x e^{-\theta x} \, dx$$

upper - lower

$$= \theta \left[\frac{e^{-\theta x}}{-\theta} \right]_0^x$$

$$= -e^{-\theta x} + e^{-\theta 0}$$

$$= 1 - e^{-\theta x}$$

Geometric distribution :- If p is the probability of success and k be the no. of failures preceeding the 1st success than distribution is given by $P(k) = q^k p$,

$k = 0, 1, 2, \ldots$, $q = 1 - p$

clearly. $\sum_{k=0}^{\infty} P(k) = 1$

1. A die is cast untill 6 appears. what is the probability that it must be cast more than 5 times.

Sol :- Here the probability of getting 6 is $p = \frac{1}{6}$.

$$q = 1 - p$$
$$= 1 - \frac{1}{6} = \frac{5}{6}$$

if X is the no. of tosses required for the 1st success, then $P(X = x) = q^{x-1} p$

required probability $= P(x > 5)$

$$= 1 - P(x \le 5)$$

$$= 1 - \sum_{x=0}^{5} q^{x-1} p$$

$$= 1 - \left[\left(\cancel{\left(\frac{5}{6}\right)^{-1} \left(\frac{1}{6}\right)} \right) + \left(\frac{5}{6}\right)^0 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right) \ldots \right.$$

ility
e 1^{st}
kp,

obability

$= \frac{1}{6}$.

uccess,

$\left(\frac{5}{6}\right)^1$

$= \left(\frac{5}{6}\right)^4 \left(\frac{1}{6}\right)$

$$= 1 - \frac{1}{6}\left[1 + \frac{5}{6} + \frac{25}{36} + \frac{125}{216} + \frac{625}{1296}\right]$$

$= 0.4018$

Uniform distribution or rectangular distribution. [2M]

A random variable x is said to be uniformly distributed over the interval $-\infty < a < b < \infty$ if it's density function is given by

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

The distribution given by 1 is called uniform distribution in this distribution x takes the values with the same probability.

$$\text{mean} = \mu = \int_a^b x\, f(x)\,dx = \int_a^b x \cdot \frac{1}{b-a}\,dx$$

$$= \frac{1}{b-a}\int_a^b x\,dx$$

$$= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{1}{b-a}\left[\left[\frac{b^2}{2} - \frac{a^2}{2}\right]\right]$$

$$= \frac{b^2 - a^2}{2(b-a)} = \frac{(b+a)\cancel{(b-a)}}{2\cancel{(b-a)}}$$

$$= \frac{b+a}{2}$$

$$\int_a^b x^2 f(x)\,dx - \mu^2$$

$$\int_a^b x^2 \frac{1}{b-a}\,dx - \mu^2$$

$$\frac{1}{b-a}\int_a^b x^2\,dx - \mu^2$$

$$\frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b - \frac{(a+b)^2}{4}$$

$$\frac{1}{b-a}\left[\frac{b^3}{3}-\frac{a^3}{3}\right] - \frac{(a+b)^2}{4}$$

$$= \frac{b^3-a^3}{3(b-a)} - \frac{(a+b)^2}{4}$$

$$= \frac{\cancel{(b-a)}(b^2+ab+a^2)}{3\cancel{(b-a)}} - \frac{(a+b)^2}{4}$$

$$= \cancel{(a+b)}\ \frac{(b-a)^2}{12}$$