

Introduction :-

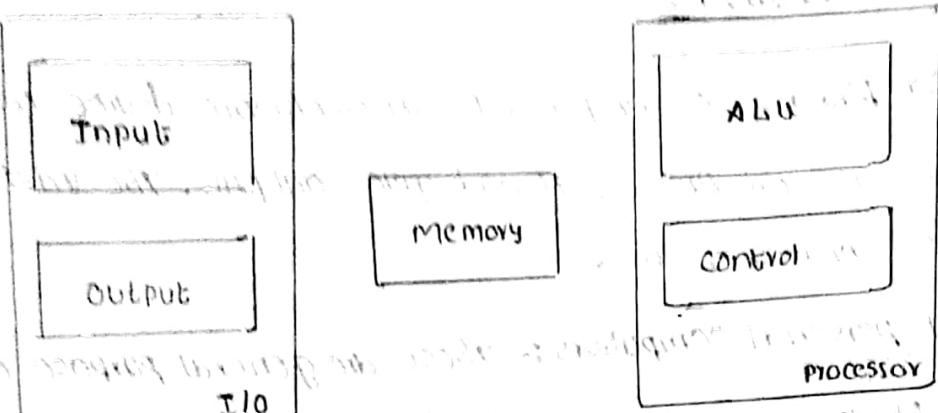
Computer :- A computer is an electronic device which takes input and process it and gives output. The various types of computers are :-

1. Personal Computers :- These are general purpose computers that are widely used in homes, schools, etc.
2. Notebook Computers (laptop) :- These are the compact version of personal computers. These are portable.
3. Work Stations :- These computers give more performance than PC's. These are used for scientific purpose.
4. Main Frames :- These computers have more storage and faster than work stations. These are used in business data processing.
5. Super Computers :- These are fastest computers that are used for large scale numerical calculations.
6. Hand held Computers :- It is also called as PDA (personal Digital Assistant). It can fits into pocket and runs with batteries.

Functional Units :-

A Computer consists of 5 main parts as shown are:-

1. Input unit
2. Memory unit
3. Arithmetic logic unit
4. Output unit
5. Control unit



1. Input unit :- Computer accepts input through input units.

Eg. - Keyboard, Mouse.

2. Memory unit :-

→ The function of memory unit is to store programs

and data.

→ These are two classes of storage, primary storage

and secondary storage. They are:-

(i) primary memory

(ii) secondary memory

→ primary Memory :-

* The Primary memory is the fastest memory.

* Memory in which any location can be reached in short and fixed amount of time after specifying its address is called Random Access Memory (RAM).

→ secondary Memory :-

* It is used when large amount of data needs to be stored.

Eg. - CD's, Pendrives, ... etc.

3. Arithmetic logic unit :-

It performs arithmetic and logical operations such as addition, subtraction, multiplication, AND, OR etc.

4. Output unit :-

Its function is to send processed result to outside world.

5. Control unit :-

Controls all the activities in a computer by passing the control signals.

Software :-

- Software is a collection of programs.
- To run application software, the system must already contains some system software in the memory.
- Software of system performs the following functions:
 - * Receiving and interpreting user commands.
 - * Entering and editing application programs and storing them as files in secondary storage.
- Application programs are usually written in high-level programming language, such as C, C++, Java, -- etc.
- A system software program called a compiler translates high-level language program to a suitable machine language program.

Languages :-

→ Computer programming languages are divided into three categories. They are :

(i) High-level languages

(ii) Assembly languages

(iii) Machine languages

High-level languages :-

→ Languages with the highest level of abstraction are called high-level language.

→ The same program code can be converted and run the program.

Eg: C++, JAVA, etc.

Assembly language :-

→ Languages are at much lower level of abstraction.

→ A program written in the assembly language of one microprocessor cannot be run on computer.

→ Assembly languages are platform dependent.

Machine language :-

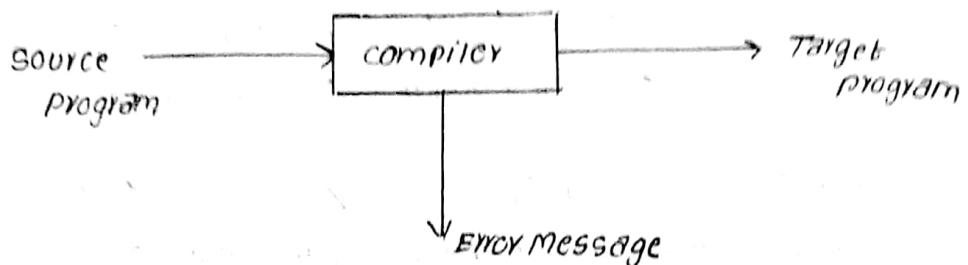
→ These language is a low-level language, which has binary values.

→ Programs never written in machine language, it will be translated into machine language.

→ Machine languages are also platform dependent; each microprocessor has its own machine language.

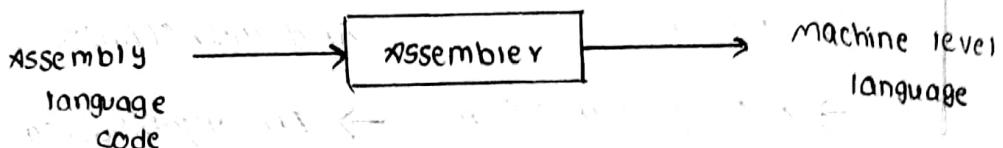
Compiler :-

→ It is a translator which is used to convert programs in a high-level language to low-level language. It translates the entire programs and also report the errors in source code.



Assembler :-

Assembler is a translator which is used to translate the assembly language code into machine language code.



Loader :-

A loader is a part of computer operating system that is responsible for loading programs and libraries. It is one of the essential stages in process of starting program.

Linker :-

After compilation (or) Assembling process - object code is generated. Linker links the object code with other required Object code .

Operating System :-

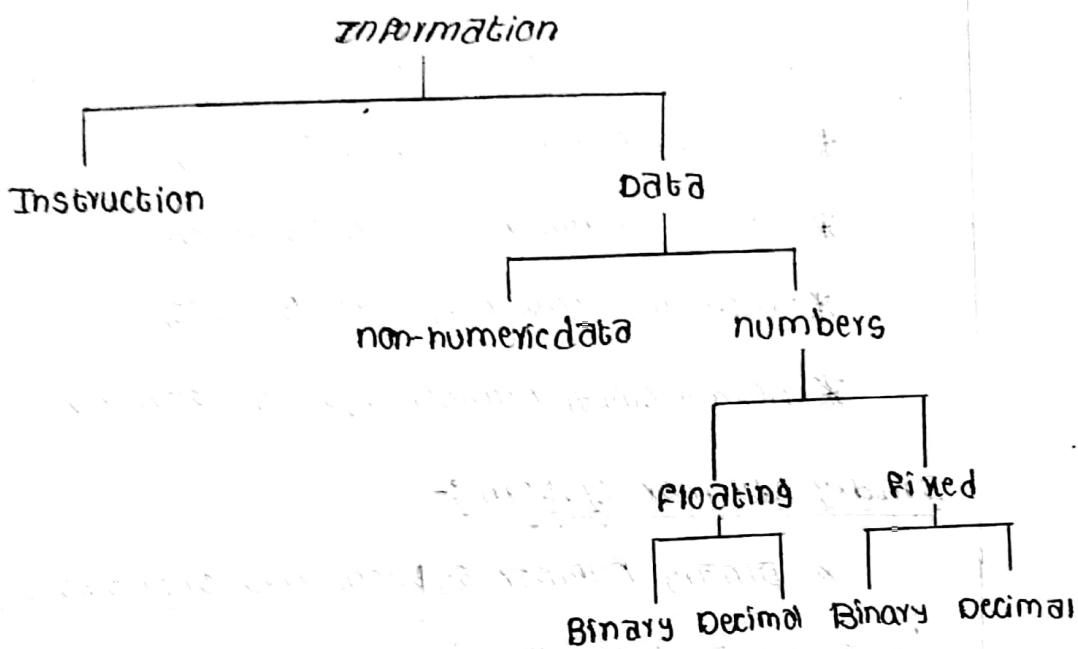
OS is a large program, or actually a collection of routines, that is used to control the sharing of and interaction among various computer units.

Computer Architecture	Computer organisation.
→ It is concerned with the way hardware components are connected together to form a computer system.	→ It is concerned with the structure and behaviour of a computer system as seen by the user.
→ It acts as the interface between hardware and software.	→ It deals with the components of a connection in a system.
→ It helps us to understand the functionalities of system.	→ It tells us how exactly all the units are arranged and interconnected.
→ A programmer can view architecture in terms of instructions.	→ In organisation express the realization of the architecture.
→ It involves high level design issues.	→ It involves low level design issues.

1. DATA TYPES:-

* DATA :- DATA is set of lines (or) statements, which has some information about something homogeneous issues.

→ The basic form of information handled by a computer are instructions and data.



* Data types:-

→ Binary information in digital computers is stored in memory (or) processor registers. Registers contain either data (or) control information.

→ Data are numbers and binary coded information that are operated on to achieve required.

→ Data types found in registers of digital computers may be classified as being one of following.

* numbers used in arithmetic computations.

* letters or the alphabets used in data processing.

* other discrete symbols used for specific purpose.

Number System :-

- Number systems are the technique to represent numbers in the computer system architecture, every value that you are saving or getting into / from the computer memory has a defined number system.
- Computer architecture supports following number systems.
- * Binary number system (Base 2).
 - * Octal number system (Base 8).
 - * Decimal number system (Base 10).
 - * Hexadecimal number system (Base 16).

Binary Number System :-

A Binary number system has only two digits, which are 0 and 1. Every number is represented with 0 and 1 in number system. The base of binary numbers is 2.

$$\text{Eq :- } (101101)_2 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ = 32 + 0 + 8 + 4 + 0 + 1 \\ = (45)_{10}$$

Octal Number System :-

Octal number system has eight digits i.e (0 to 7), Every number is represented with 0, 1, 2, 3, 4, 5, 6 and 7 in number system. The base of octal number is 8.

$$\text{Eq :- } (736.4)_8 = 7 \times 8^2 + 3 \times 8^1 + 6 \times 8^0 + 4 \times 8^{-1}, \\ = 7 \times 64 + 3 \times 8 + 6 \times 1 + 4/8 \\ = (478.5)_{10}$$

Decimal number system :-

Decimal number system has 10 digits i.e (0 to 9). Every number is represented with 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. In this decimal system the base is 10.

$$\text{Eg:- } (724.5)_{10} = 7 \times 10^2 + 2 \times 10^1 + 4 \times 10^0 + 5 \times 10^{-1} \\ = 7 \times 100 + 2 \times 10 + 4 \times 1 + 5/10 \\ = (724.5)_{10}$$

Hexa-decimal number system :-

Hexa decimal number system has 16 alpha numeric values from 0 to 9 and A to F. Every number is represented with 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E and F. In this system, the base is 16. Here A=10; B=11; C=12; D=13; E=14; F=15.

$$\text{Eg:- } (FB)_{16} = F \times 16^1 + B \times 16^0 \\ = 15 \times 16 + 3 \\ = (243)_{10}$$

Conversions :-

The process of converting the numbers of one base to the another base is known as conversion. Basically there are three types of conversions. They are:

1. Decimal number system to other base

$$\text{i.e } ()_{10} = ()_2 / ()_8 / ()_{16}$$

2. Other system (base) to decimal number system.

$$\text{i.e } ()_2 / ()_8 / ()_{16} = ()_{10}$$

3. Other base to other Base

$$()_2 / ()_8 / ()_{16} = ()_2 / ()_8 / ()_{16}$$

1- Decimal number to other Base System :-

Step 1 :- The separation of number into integers and fractions.

Step 2 :- Divide the number part integer and multiply the fraction successively by R.

→ Decimal to Binary :-

* The process of continuous division of a number by 2 until zero as result.

Eg:- convert 13 into binary

$$\begin{array}{r} 2 \Big| 13 \\ 2 \Big| 6-1 \\ 2 \Big| 3-0 \\ 2 \Big| 1-1 \\ 0-1 \end{array} \quad \text{Answer} = 1101$$

$$\therefore (13)_{10} = (1101)_2$$

TYPE-2 :- Convert 10.25 into binary

integer part

$$\begin{array}{r} 2 \Big| 10 \\ 2 \Big| 5-0 \\ 2 \Big| 2-1 \\ 1-0 \end{array}$$

$$0.25 \times 2 = 0.50$$

$$0.50 \times 2 = 1.00$$

$$(0.25)_{10} = (0.01)_2$$

$$(10)_{10} = (1010)_2$$

$$\therefore (10.25)_{10} = (1010.01)_2$$

→ Decimal to octal :-

* The process of continuous division of a number by 8 until zero as result.

Eg:- convert 440 into octal

$$\begin{array}{r} 8 | 440 \\ 8 | 55 - 0 \\ 8 | 6 - 7 \\ \quad\quad\quad 0 - 6 \\ \quad\quad\quad \rightarrow \end{array}$$

Answer = 0670

$$\therefore (440)_{10} = (670)_8$$

→ Decimal to hexa decimal :-

* This process is same as that of decimal to binary except the base taken 16 instead of 2.

Eg:- convert $(765.245)_{10}$ to hexa decimal

integer part :-

$$\begin{array}{r} 16 | 765 \\ 16 | 47 - 13 \\ 16 | 2 - 15 \\ \quad\quad\quad 0 - 2 \end{array}$$

Fractional part :-

$$\begin{array}{r} 0.245 \times 16 \\ \hline 3.920 \times 16 \\ \hline 14.720 \times 16 \\ \downarrow \qquad\qquad\qquad 11.520 \end{array}$$

$$\therefore (765.245)_{10} = (2F0.3EB)_{16}$$

Q. Other number System into decimal number system:-

* Multiply each digit by their weights and sum up the values.

→ Binary to Decimal number :-

In this process, we have to multiply each digit by their weights and sum up them.

Eg:- Convert $(101101.11)_2$ into decimal.

A) Given number = $(101101.11)_2$

$$= 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$$

$$= (1 \times 32) + 0 + (1 \times 8) + (1 \times 4) + 0 + (1 \times 1) + (1 \times \frac{1}{2}) + (1 \times \frac{1}{4})$$

$$= 32 + 0 + 8 + 4 + 1 + 0.5 + 0.25 = (45.75)_{10}$$

→ Octal to Decimal number :-

In this process, we have to follow above process.

Eg:- Convert $(53)_8$ into decimal.

A) Given number = $5 \times 8^1 + 3 \times 8^0$

$$= 5 \times 8 + 3 \times 1 = 40 + 3$$

$$= (43)_{10}$$

→ Hexadecimal to Decimal number :-

In this process, we have to follow above process.

Eg:- Convert $(3F6)_{16}$ into decimal

A) Given number = $3 \times 16^2 + 15 \times 16^1 + 6 \times 16^0$

$$= 3 \times 256 + 15 \times 16 + 6 \times 1$$

$$= 768 + 240 + 6$$

$$= (1014)_{10}$$

3. Other Base to other Base :-

- * There are mainly two methods are there. They are
 - (i) Direct method
 - (ii) Indirect method

→ Direct Method :-

→ Converting Binary to Octal by grouping binary bits.

Eg:- Convert 101011100 into octal

A) Given number = $(101011100)_2$
To convert binary to octal, we have to make 3-bit groups and convert them directly.

$$\begin{array}{cccc} 001 & 010 & 111 & 100 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 7 & 4 \end{array} \Rightarrow (1274)_8$$

→ Indirect Method :-

→ Convert the Binary number to decimal and then decimal to octal.

Eg:- Convert $(10110)_2$ into octal

A) Given number = $(10110)_2$

Step 1 :- Convert binary to decimal

$$= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$$= 16 + 0 + 4 + 2 + 0$$

$$= (22)_{10}$$

Step 2 :- Convert decimal to octal

$$= (22)_{10}$$

$$(22)_{10} = (1026)_8$$

8	22
8	2 - 6
8	16 - 6
0 - 2	

Q. Complements :-

- Compliments are used in digital computers for simplifying the subtraction operation and for logical manipulation.
- There are mainly two types of complements for each base system. They are

(i) r's - complement

(ii) $(r-1)$'s - complement.

- When the base value r is substituted in the name, the two types are 2's and 10's complement for the binary and 10's and 9's complement for decimal's.

r's complement :-

The r's complement of a n-digit number N in base r is defined as $r-N$ for $N \neq 0$ and 0 for $N=0$.

Eg:- The 10's Complement of 2389 is

$$\begin{array}{r} 9999 \\ -2389 \\ \hline 7610 +1 = 7611 \end{array}$$

The 2's complement of 101100 is

$$101100 \text{ 's } c = 010011$$

$$2's c = 1's c + 1$$

$$\begin{array}{r} \Rightarrow 010011 \\ +100011 \\ \hline 010100 \end{array}$$

$(r-1)$'s complement :-

Given a number N having base r and having n digits, the $(r-1)$'s complement of N is defined as $(r-1)-N$.

Eg:- For decimal numbers $r=10$

$$r-1 = 9 \Rightarrow r = 9+1$$

So 9's complement is $(10^n - 1) - N$.

Then, 10's complement \Rightarrow 10's complement - 1

$$\begin{aligned} \text{9's C of } 546700 &\Rightarrow (10^6 - 1) - N \\ &= (1000000 - 1) - N \\ &= \underline{\quad\quad\quad\quad\quad\quad\quad\quad} \\ &\quad - 546700 \\ &\quad \underline{453299} \end{aligned}$$

Subtraction with complements :-

\rightarrow The subtraction of two n -digit unsigned numbers $M-N$ in base r can be done as

1. Add the minuend M to r 's complement of subtrahend N .

No.

2. If $M \geq N$, the sum will produce and end carry r^n , which can be discarded, what is left is result.

3. If $M < N$, the sum will not produce and end carry and is equal to r^n . To get answer, take the r 's complement of sum and place a negative sign in front.

Eg:- using 10's complement subtract $3250 - 72532$.

A) Here $M = 03250$ and $N = 72532$. Here $N > M$

$$M = 03250$$

$$\begin{array}{r} \text{10's Compliment of } N = \\ \hline 27468 \\ \hline 30718 \end{array}$$

\therefore The answer is $-(10\text{'s C of } 30718) = -69282$.

→ Subtraction of unsigned numbers can also be done by means of the $(r-1)$'s complement.

→ Remember that $(r-1)$'s complement is one less than r 's complement.

Ex: Given two binary numbers $X = 1010100$ and $Y = 1000011$.

Perform the subtraction (a) $X-Y$ and (b) $Y-X$ using 2's complement.

(a) Given binary numbers are $X = 1010100$

$$\begin{array}{r} \text{1010100} \\ - 1000011 \\ \hline \end{array}$$

$$Y = 1000011$$

$$\begin{array}{r} \text{1010100} \\ - 1000011 \\ \hline \end{array}$$

$$1\text{'s complement of } Y = 0111100$$

$$\begin{array}{r} X = 1010100 \\ + 0111100 \\ \hline 10010000 \end{array}$$

$$\begin{array}{r} \text{Here Carry is added} \\ + 1 \\ \hline 0010001 \end{array}$$

$$(b) Y-X = 1000011 - 1010100$$

$$Y = 1000011$$

$$1\text{'s complement of } X = 0101011$$

$$\begin{array}{r} 0101011 \\ + 1101110 \\ \hline 1110001 \end{array}$$

Here there is no end carry, Hence the answer is

$$Y-X = -(1\text{'s complement of } 1101110) = -0010001$$

Same result as

Subtraction of unsigned numbers via

2's complement

3. Fixed point numbers :-

- In binary numbers system, a number can be represented as an integer (or) a fraction.
- In integer numbers, radix point is fixed and assumed to be to the right most digit.
- As radix point is fixed, the number system is referred to as fixed point number system.

Fixed point Representation :-

- The fixed-point method assumes that the binary point is always fixed in one position. The two positions most widely used are
 - * A binary point in the extreme left of the register to make the stored number a fraction.
 - * A binary point in the extreme right of the register to make the stored number an integer.
- * In either case, the binary number point is not actually present, but its presence is assumed from the fact the number stored in register.

Integer Representation :-

- When an integer binary number is positive, the sign is represented by 0 and the magnitude by a positive binary number.
- When the number is negative, the sign is represented by 1 but the rest of numbers will be represented in three possible ways, they are
 1. Signed magnitude representation.
 2. Signed 1's complement representation.

3. Signed - 2's Complement Representation.

Ex:- Consider an 8-bit register and number +14.

→ The way to represent is 00001110

Consider an 8-bit register and number -14

→ In signed-magnitude represented as 1-0001110

In signed - 1's complement as 1 1110001

In signed - 2's complement as 11110010

Arithmetic Addition :-

→ The addition of two numbers in the signed-magnitude system follows the rules of ordinary arithmetic.

→ Addition of two signed-magnitude numbers follow normal rules.

* If have same sign, add two numbers, use same sign.

* If they have different sign means follow the highest number sign and assign same sign.

* Must compare the signs and magnitudes and then either add (or) subtract.

→ Addition of two signed 2's complement number, it doesn't require a comparison. (or) subtraction, only addition and complementation.

* Add two numbers including their sign bit.

* Discard any carry out or sign bit position.

* All Negative numbers must be in 2's complement.

* If the sum obtained is negative, then it is in

2's complement form.

$$\text{Eg:-(i) } +6 \quad \begin{array}{r} 0000\ 0110 \\ +13 \\ \hline \underline{0001\ 0011} \end{array} \quad \text{(ii) } -6 \quad \begin{array}{r} 1\ 111\ 010 \\ +13 \\ \hline \underline{0000\ 1101} \\ +7 \\ \hline \underline{0000\ 1111} \end{array}$$

$$\text{(iii) } +6 \quad \begin{array}{r} 0000\ 0110 \\ -13 \\ \hline \underline{1\ 111\ 0011} \\ -7 \\ \hline \underline{1\ 111\ 1001} \end{array} \quad \text{(iv) } -6 \quad \begin{array}{r} 1\ 111\ 010 \\ -13 \\ \hline \underline{1\ 111\ 0011} \\ -19 \\ \hline \underline{1\ 110\ 1101} \end{array}$$

→ In each of four cases, the operation performed is always addition, including the sign bit, any carry out the sign bit is discarded, and negative results are in 2's complement form.

Arithmetic subtraction :-

→ The subtraction of the two signed binary numbers when negative numbers are in 2's complement form is very simple and can be stated as follows: Take the 2's complement of the subtrahend and add it to the minuend. A carry out of sign bit position is discarded.

→ A carry out of sign bit position is discarded.

→ This procedure stems from the fact that a subtraction operation can be changed into an other addition operation, if the sign of subtrahend is changed. These are demonstrated by following variations.

$$(\pm A) - (+B) = (\pm A) + (-B)$$

$$\text{and also } (\pm A) - (-B) = (\pm A) + (+B)$$

$$\text{and also } (+A) - (+B) = (+A) + (-B)$$

$$\text{and also } (+A) - (-B) = (+A) + (+B)$$

Eg:- consider the subtraction of

$$(-6) - (-13) = +7$$

$$-6 = 100001010$$

$$-13 = 10001101 \quad [\text{where } 13 \text{ will be } 2's \text{ comp of } 13]$$

$$\begin{array}{r} & \underline{11} \\ + & \underline{100010111} \end{array}$$

[here 1 is carry]

$$= 00010111$$

[remove carry]

$$= (+7)$$

Decimal Fixed-point Representation:-

- the representation of decimal numbers in registers is a function of binary code used to represent a decimal digit.
- this takes much more space than the equivalent binary representation and the circuits required to perform decimal arithmetic are more complex.
- Representation of signed decimal numbers in BCD is similar to the representation of signed numbers in binary.
- To obtain the 10's complement of a BCD number, first take the 9's complement and then add one to least significant digit.

Eg:- $(+375) + (-240) = (+135)$

$$0375 = (0000 \ 0011 \ 0111 \ 0101) \text{ BCD}$$

$$9760 = (1001 \ 0111 \ 0110 \ 0000) \text{ BCD}$$

$$\underline{0135 = (0000 \ 0001 \ 0010 \ 0101) \text{ BCD}}$$

Floating-point Representation :-

- The floating-point representation of a number has two parts. They are :- (i) Mantissa
(ii) Exponent
- The first part represents a fixed point number, it is called mantissa. The second part designates the position of decimal point and is called exponent.

Eg:- Fraction Exponent
+ 0.6132789 +04

\uparrow + 6132.789 (represented) above.

- +04 represents is an actual decimal position of the decimal point here the 4 position to right that is indicated decimal point in the fraction as

$$0.6132789 \times 10^4 = 6132.789$$

- The floating point is always represent as

$m \times r^e$
here m = mantissa, e = exponent, r = radix

- For a binary number +1001.11 is represented with an 8-bit fraction and 6-bit exponent as

Fraction Exponent
01001110 000100

- For Binary sign doesn't shown. The exponent has the equivalent binary number +4. The floating point is $m \times 2^e = +(.1001110)_2 \times 2^{+4}$

Normalization :-

- A floating-point number is said to be normalized if the most significant digit of mantissa is non-zero.
- The number is normalized only if its leftmost digit is non-zero.

Eg:- Let us take 8-bit binary number 00011010 is not normalized because of three leading 0's. The number can be normalized by shifting it three positions towards left and discarding leading 0's.

so, the result will be 11010000. ←

- Two main standard forms of floating-point numbers are from the following organisations that decide standards are :

(i) ANSI (American National Standards Institute)

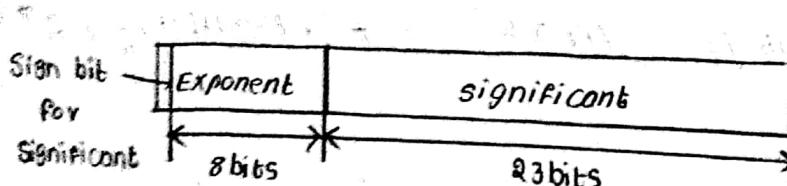
(ii) IEEE (Institute of Electrical and Electronic Engineers, London = 1975 Engineering).

- 32 bit floating point format. ←

- Leftmost bit = sign bit.

- Exponent in the next 8 bits. Use a biased way of representation.

- Final portion of word is significant.



4. Other Binary Codes :-

Other binary codes for decimal numbers and the alphanumeric characters are sometimes used. Digital computers are also employ other binary codes. They are:-

1. Gray Code :-

The reflected binary code (or) Gray code is an ordering of two binary number system, such that two successive value differ by one bit only.

Gray codes are also useful in normal sequence of binary numbers generated by hardware.

→ Gray code is also known as reflected binary code.

The Gray code is not suitable for arithmetic operations.

Decimal	Binary	Gray Code
0	000	000
1	001	001
2	010	011
3	011	010
4	100	110
5	101	111
6	110	101
7	111	100

→ Convert $(010)_2$ to Gray code.

A) $\begin{array}{r} 0 \ 1 \ 0 \\ \downarrow \ \uparrow \ \downarrow \\ 0 \ 1 \ 1 \end{array}$ Hence $(010)_2 = (011)_{GC}$

2. BCD (or) Binary Coded Decimal :-

→ In this method, each decimal digit is represented by a 4-bit binary number.

→ BCD is the way to express each decimal number with binary number.

Decimal	0	1	2	3	4	5	6	7	8	9
BCD	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001

Eg :- Convert $(11101)_2$ to BCD.

a) First convert it into decimal.

$$\Rightarrow 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 1 \times 2^4$$

$$\Rightarrow 1 + 0 + 4 + 8 + 16$$

$$= (29)_{10}$$

$$(11101)_2 = (00101001)_{BCD}.$$

3. Excess-3 Code (or) X33 Code :-

→ In this method, we have to add 3 to the decimal number and then we convert into BCD code.

Eg :- Convert $(426)_{10}$ to excess-3 code.

a) Each digit is added by 3 and it is converted to BCD.

$$\begin{array}{r}
 426 \\
 + 333 \\
 \hline
 759
 \end{array}
 \Rightarrow (0111\ 0101\ 1001)_{X33}.$$

4. ASC-II Code :-

It is abbreviated as American Standard Code for information interchange, is a character encoding standard for electronic communication.

ASCII codes represents text in computers and tele-communications and other devices.

5. Error Detection code :-

- whenever a message is transmitted, it may get Scrambled by noise (or) data may corrupted.
- To avoid this, we use error detecting codes, which are additional data added to a given message.
- A simple example for error detection code is parity check.

Error Correcting Code :-

- Along with error-detecting code, we can also pass some data to figure out original message from the received message.
- This type of code is called Error-correcting code.

Hamming Code :-

- Hamming Code is a block code that is capable of detecting up to two simultaneous bit errors and the correcting single-bit errors.
- It was developed by R.W Hamming for the error correction, in this coding method, the source encodes that message by inserting redundant bits within the message.

Register Transfer language :-

→ micro operations :- A micro operation is an elementary operation performed on data stored in one or more registers.

Eg :- shift, count, clear and load.

→ The internal hardware organisation of a digital computers is best defined by specifying :

1. The set of registers it contains and their function.

2. The sequence of micro operations performed on data stored in registers.

3. The control that initiates the sequence of the micro operations.

→ Register Transfer language :-

→ Micro operations can be explained in terms of words but if it is a lengthy procedure, then we use the symbols to represent information transfer between them.

→ The symbolic notation used to describe the micro operation transfer register is called as "Register Transfer language".

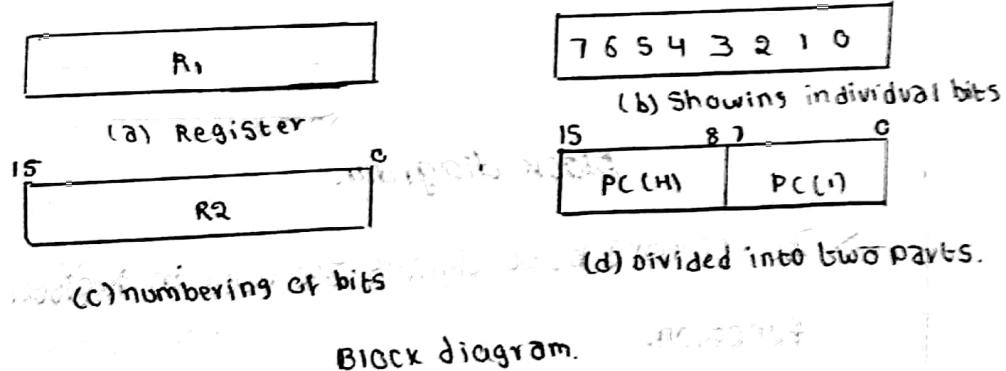
→ The term "register transfer" implies that the availability of hardware logic circuits that can perform a stated micro operation.

- The term "language" implies programming language that specifies the writing of symbols to specify a given computational process.
- Register Transfer language is an appropriate tool for designing the internal organisation of digital computers.

Register TRANSFER :-

Registers :-

- Computer Registers are designated by capital letters to denote function of Register.
- Eg:- MAR (Memory Address Register), PC (Program Counter), IR (Instruction Register), RT (Process Register).



Register Transfer :-

- Information transfers from one register to another register. It is designated in symbolic form by means of replacement operator.

$$R_2 \leftarrow R_1$$

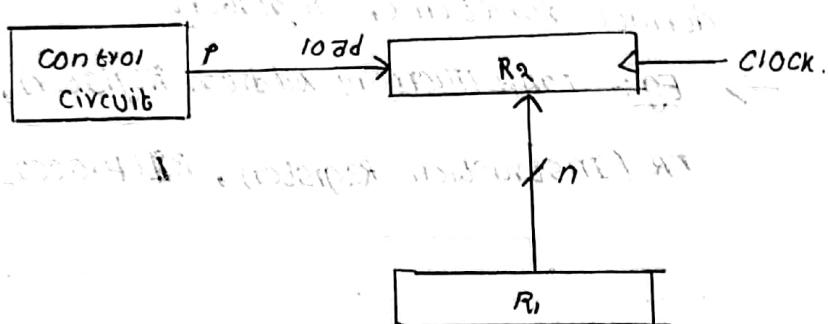
- If we want the transfer to occur under a predefine condition, this can be shown by if-then statement.

\downarrow
if ($p=1$) then ($R_2 \leftarrow R_1$)
 \downarrow
Control Signal.

→ This statement can also be written by using a control function as follows. A control statement is a Boolean variable that is equal to 1 (or) 0.

$$P : R_2 \leftarrow R_1$$

→ The block diagram for the register transfer from R_1 to R_2 is shown below. The n outputs of register R_1 is connected to n inputs of register R_2 . Here the letter n is used to indicate number of bits for the Register.



block diagram.

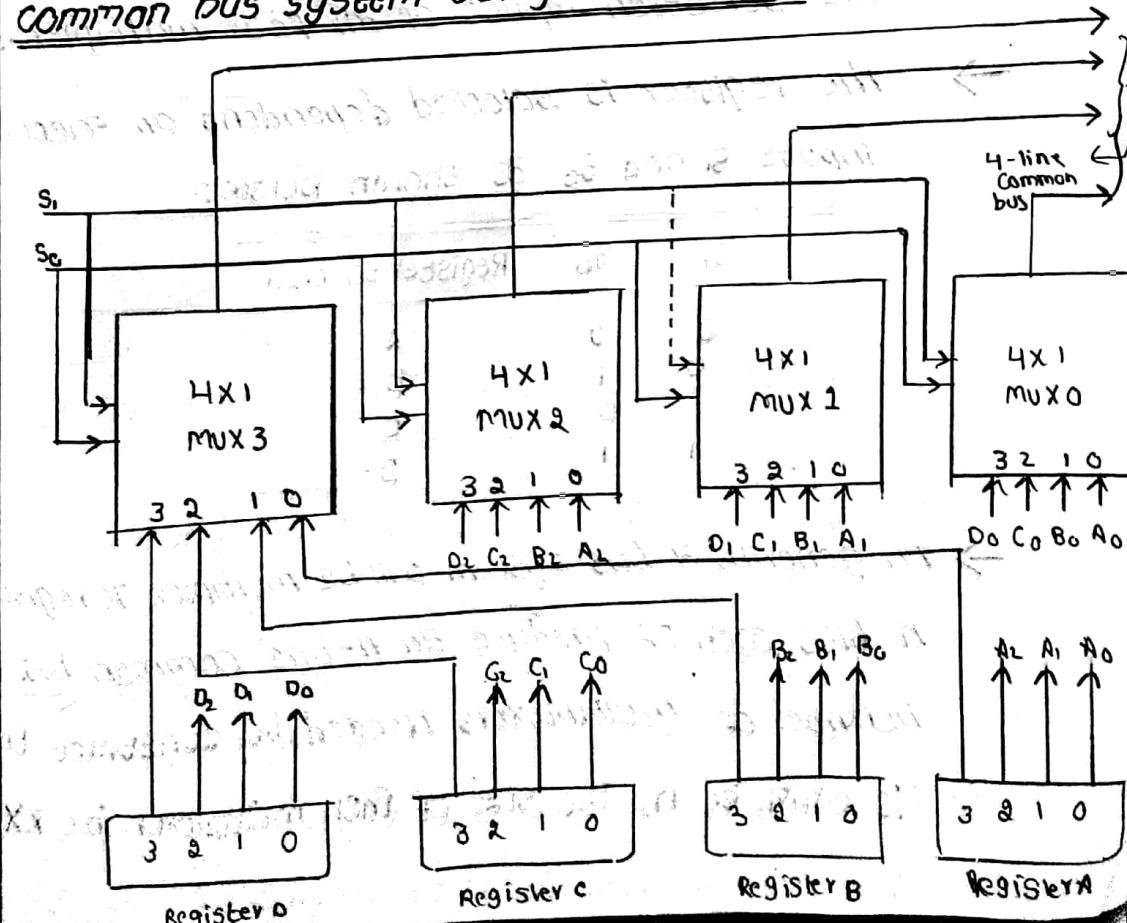
→ The following basic signals are used in Register transfer function.

Symbol	Description	Examples
Registers and Numerals	Denotes a Register	MAR, R_2
Parantheses ()	Denote a part of register	$R_2(0-7)$, $R_2(L)$.
Arrow \leftarrow	Denotes transfer of information	$R_2 \leftarrow R_1$
Comma (,)	Separation	$R_2 \leftarrow R_1, R_2(L)$.

BUS AND MEMORY :-

- A computer has many registers and paths must be provided to transfer information from one register to another. The number of wires will be more if separate lines are used between each register and all other registers.
- A more efficient scheme for transferring information between registers is a common bus system. A bus has set of common lines, one for each bit of register.
- Control signals determine which registers is selected by bus.
- The common bus system is implemented in 2 ways:
 - 1) using multiplexers
 - 2) using three-state bus buffers.

common bus system using multiplexers :-



- The construction of bus system for four registers is as shown in figure above.
- Each register has four bits, numbered 0 through 3.
- The bus consists of four 4×1 multiplexers each having four data inputs, 0 through 3, and two selection input S_1 and S_0 .
- All the 0 bits of the registers (i.e. A_0, B_0, C_0 and D_0) are connected to the MUX 0. All the 1 bits of the registers are connected to MUX 1.
- MUX 0 multiplexes the four 0 bits of the registers, MUX 1 multiplexes the four 1 bits of the registers, and similarly the other two.
- The two selection lines S_1 and S_0 are connected to the selection inputs of all four multiplexers.
- The register is selected dependent on selection inputs S_1 and S_0 as shown below:

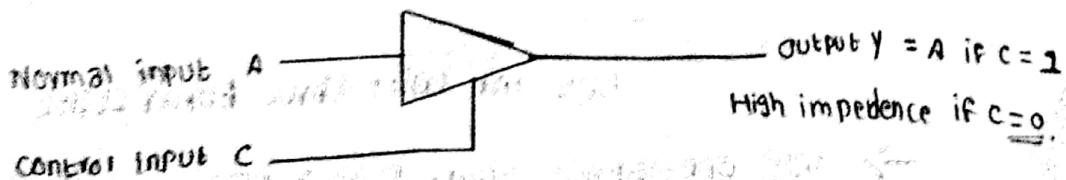
S_1	S_0	Register Selected
0	0	A
0	1	B
1	0	C
1	1	D

- In general, a bus system will be multiplex K registers of n bits each or produce an n -line common bus. The number of multiplexers needed to construct the bus is equal to n . The size of each multiplexer be $K \times 1$.

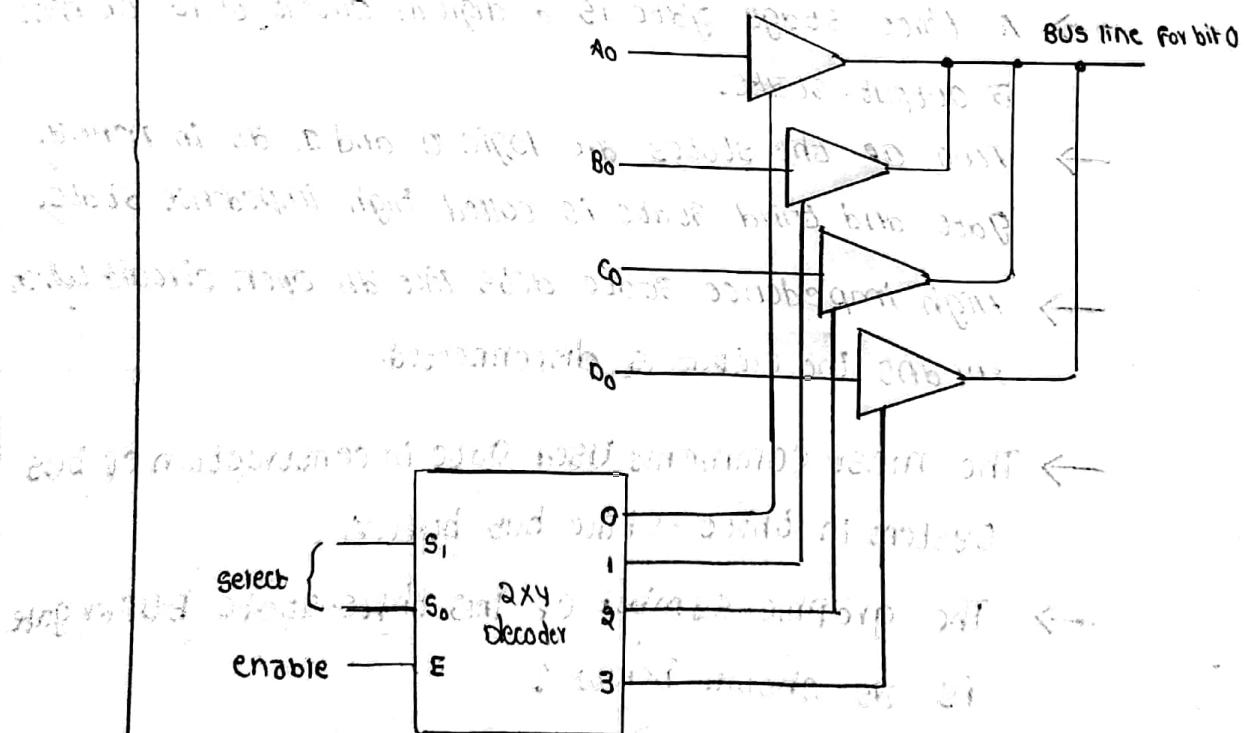
- the data present in the bus can be transferred to one (or) more destination registers by connecting the inputs of the register to bus line and enabling the load control of the destination Register.
- The symbolic statement for bus transfer is shown below : $BUS \leftarrow C$, $R_1 \leftarrow BUS$
- The contents of register C are placed on the bus, and the content of bus is loaded into register R_1 by activating load control input.
 $R_1 \leftarrow C$.

Three State Bus Buffers :-

- A bus system can be constructed with three-state gates instead of multiplexers.
- A three-stage gate is a digital circuit that can have 3 output state.
- Two of the states are logic 0 and 1 as in normal gate and third state is called high impedance state.
- High impedance state acts like an open circuit which means the output is disconnected.
- The most commonly used gate in construction of bus system is three-state bus buffer.
- The graphic symbol of this three-state Buffer gate is as shown below :



- It has both normal input and control input.
- The control input determines the output state.
- When the control input is equal to 1, the output is enabled and gate behaves as normal buffers.
- When the control input is 0, then, the output is disabled and gate goes to a high impedance state.
- The construction of a bus system with three-state buffers is demonstrated in following figures.
- The output of four buffers are connected together to form a single bus line.
- No more than one buffer may be in active state at a given time.



Bus line with three Buffer State

- Only one three-state buffer has access to the bus line while all other buffers are maintained in high-impedance

State.

- One way to make no more than one control input is active at any given time is to use a decoder, as shown in figure.
- When the enable input of decoder is 0, all buffers are disabled.
- When the enable input is active, one of three-state buffers will be active, depending on the binary value of the selection inputs of decoder (S_1 , S_0).

Memory transfer :-

- The transfer of information from a memory word to the outside environment is called a **read operation**.
- The transfer of new information to be stored in the memory is called **write operation**.
- A memory word is indicated by letter M . It is necessary to specify the address of M while doing the memory transfer operation.

* Read operation :-

Read : $DR \leftarrow M[AR]$

- This causes a transfer of information into DR (data register) from the memory word M , selected by the address in address in AR.

* Write operation :-

Write : $M[AR] \leftarrow R$

- This causes a transfer of information from R , in to the memory word selected by address in AR.

Arithmetic micro operations :-

→ A micro operation is an elementary operation, that is performed with data stored in registers. The micro operations in computers are classified as

1. Register transfer micro operations :-

It transfers binary information from one register to another.

2. Arithmetic micro operations :-

It performs arithmetic operations on data stored in registers.

3. logic micro operations :-

It performs bit manipulation operations.

4. Shift micro operations :-

It performs shift operation on the data stored in the registers.

→ The arithmetic micro operations is defined as

add micro operation :-

It states that contents of Register R₁ are added

to contents of Register R₂. sum transferred to Register R₃.

$$R_3 \leftarrow R_1 + R_2$$

Subtraction of micro operations:-

→ subtraction is most often implemented through the complementation and addition. Instead of writing the minus operation, we can specify it as

$$R_3 \leftarrow R_1 + \overline{R_2} + 1$$

- In above statements $\overline{R_2}$ is symbol for 2's complement of R_2 . Adding 1 to the 1's complement gives us a 2's complement. Hence it is equal to $R_1 - R_2$.
- The following table shows arithmetic micro operations.

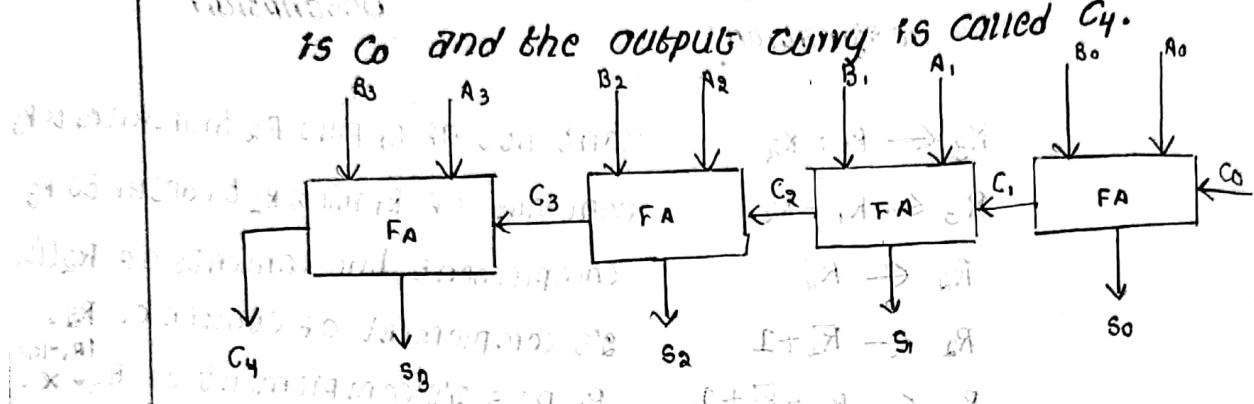
Symbolic designation	Description
$R_3 \leftarrow R_1 + R_2$	Contents of R_1 plus R_2 transferred to R_3 .
$R_3 \leftarrow R_1 - R_2$	Contents of R_1 minus R_2 transfer to R_3 .
$R_2 \leftarrow \overline{R_2}$	Complement the contents of R_2 (i.e. 2's complement).
$R_2 \leftarrow \overline{R_2} + 1$	1's complement of contents of R_2 .
$R_3 \leftarrow R_1 + \overline{R_2} + 1$	R_1 plus 2's complement of $R_2 - 1$.
$R_1 \leftarrow R_1 + 1$	Increment the R_1 contents by 1.
$R_1 \leftarrow R_1 - 1$	Decrement the R_1 contents by 1.

Binary Adder :-

- To implement add micro operation with hardware, we need the registers that holds the data and the digital component that performs the arithmetic addition.
- The digital circuit that forms the arithmetic sum of the two bits and a previous carry is called a full-adder.
- The digit circuit that generates the arithmetic sum of the binary numbers of any length is called a binary adder.

→ The binary adder is constructed with full-adder circuits connected to the input carry of the next full adder.

→ The following figure shows interconnection of four full-adders (FA) to provide a 4-bit binary adder. The input carry to the binary adder is C_0 and the output carry is called C_4 .

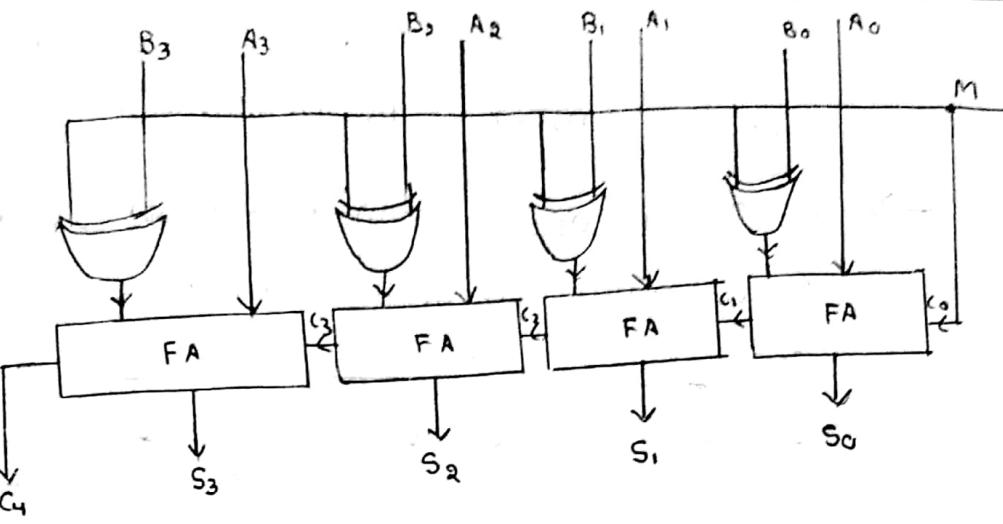


→ An n -bit binary adder requires n full-adders. The output carry from each full-adder is connected to the input carry of the next-higher order full adder.

Binary Adder-Subtractor

→ The subtraction of binary numbers can be done using complements. Remember subtraction $A-B$ can be done by taking 2's complement of B and adding it to A .

→ The addition and subtraction operation can be combined into one common circuit by including an ex-or gate with each full adder. A 4-bit adder-subtractor circuit is as shown in figure.



4-bit adder-subtractor.

- The mode input controls the operation, when $M=0$, the circuit is an adder and when $M=1$, the circuit will becomes subtractor.
- Each EX-OR gate receives input M and one of them is B .
- When $M=0$, we have $B \text{ (XOR) } 0 = B$. The full adders receive the value of B the input carry is 0, then the circuit performs $A+B$.
- When $M=1$, we have $B \text{ (XOR) } 1 = \bar{B}$. The $C_0 = 1$, and B inputs are complemented and 1 is added through input carry. The circuit performs the 2's complement of B and then added to A . The result is $A-B$.

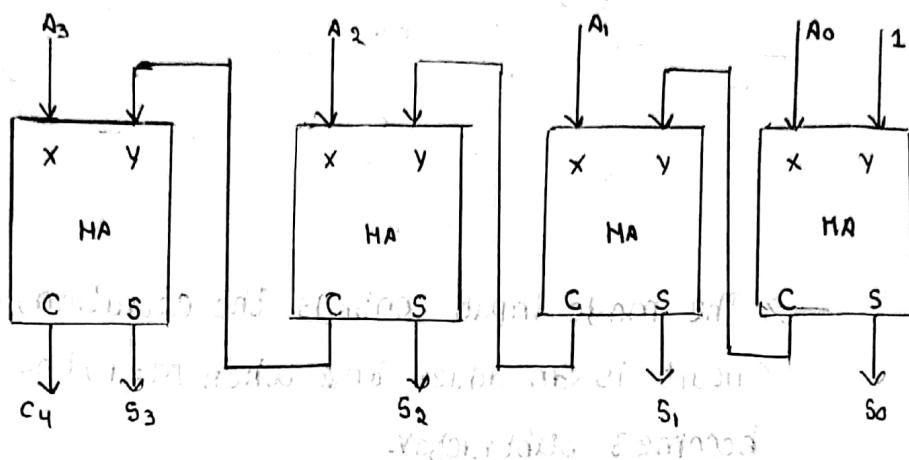
Binary Incrementer :-

- The increment operation adds one to a number in a register.

Eg:- Let a Binary number $\Rightarrow 0110$
After increment $\Rightarrow 0111$.

- The diagram of a 4-bit combinational circuit incrementer is shown below.

- One of the inputs to the least significant bit of the one number to be incremented.
- The output carry from one half-adder is connected to one of inputs to the next-higher order half-adder.



- The circuit can be extended to an n -bit binary incrementer by extending the diagram by adding n -half adders.

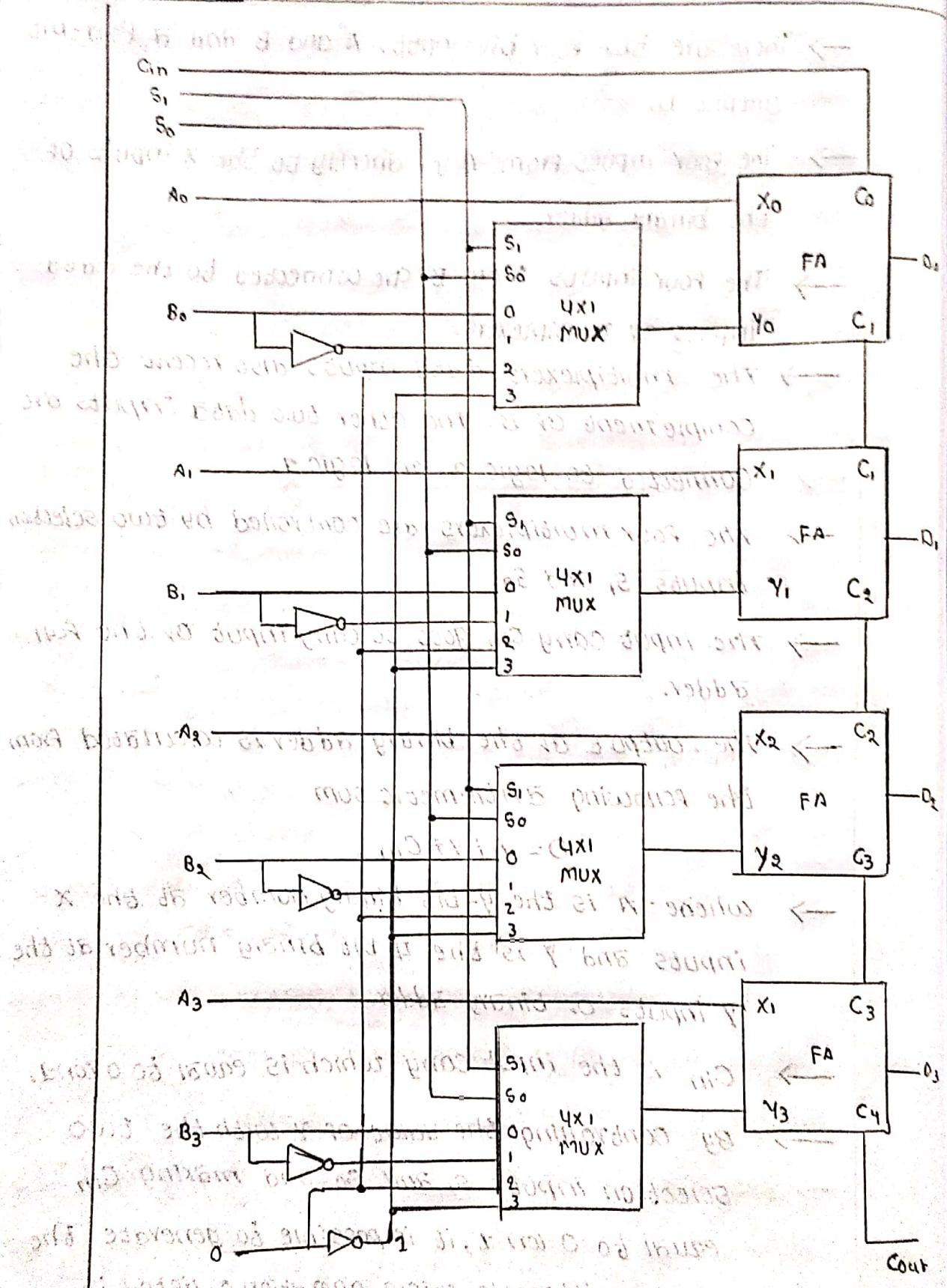
Arithmetic Circuit :-

- The arithmetic operations listed in above table (addition, subtraction, increment and decrement) can be implemented in one composite arithmetic circuit.

- The basic component of the arithmetic circuit is the parallel adder. By controlling the data inputs to the adder.

- The diagram for 4-bit arithmetic circuit is as shown in figure below. It has 4 full adder circuits that has 4-bit adder.

- There are two four bit inputs A and B and a four-bit output D.
 - The four inputs from A go directly to the X inputs of the binary adder.
 - The four inputs from B are connected to the data inputs of multiplexers.
 - The multiplexer's data inputs also receive the complements of B. The other two data inputs are connected to logic 0 (or) logic 1.
 - The four multiplexers are controlled by two selection inputs s_1 and s_0 .
 - The input carry C_{in} goes to carry input of the full adder.
 - The output of the binary adder is calculated from the following arithmetic sum
- $$D = A + Y + C_{in} .$$
- where A is the 4-bit binary number at the X inputs and Y is the 4-bit binary number at the Y inputs of binary adder.
 - C_{in} is the input carry which is equal to 0 (or) 1.
 - By controlling the value of Y with the two selection inputs s_1 and s_0 and making C_{in} equal to 0 (or) 1, it is possible to generate the eight arithmetic micro operations listed in below table.



4-bit arithmetic circuit.

Select S_1	S_0	C_{in}	Input Y	Output $D = A + Y + C_{in}$	Microoperation
0	0	0	B	$D = A + B$	Add
0	0	1	B	$D = A + B + 1$	Add with carry
0	1	0	\bar{B}	$D = A + \bar{B}$	Subtract with borrow
0	1	1	\bar{B}	$D = A + \bar{B} + 1$	Subtract
1	0	0	0	$D = A$	Transfer A
1	0	1	0	$D = A + 1$	Increment A
1	1	0	1	$D = A - 1$	Decrement A
1	1	1	1	$D = A$	Transfer A

→ Addition :- When $S_1, S_0 = 00$, the value of B is applied to the Y inputs of the adder. If $C_{in} = 0$, the output $D = A + B$. If $C_{in} = 1$, output $D = A + B + 1$. Both cases perform add operations.

→ Subtraction :- when $S_1, S_0 = 01$, the complement of B is applied to the Y inputs of the adder. If $C_{in} = 1$, the output $D = A + B' + 1$. This produces the $A + 2'C$ of B. which is equal to $A - B$. When $C_{in} = 0$, then $D = A + B'$.

→ Increment :- when $S_1, S_0 = 10$, the inputs from B are neglected and instead, all 0's are inserted into Y inputs. The outputs becomes $D = A + 0 + C_{in}$. This gives $D = A$ when $C_{in} = 0$ and $D = A + 1$ when $C_{in} = 1$. In this first case we have direct transfer of input A to output D.

→ Decrement :- When $S_1 S_0 = 11$, all 1's are inserted in to the Y inputs to the adder to produce the decrement operation $D = A - 1$, when $C_{in} = 0$. This is because a number (with all 1's equal to 2's complement of 1.

Logic micro Operations :-

→ logic micro operations specify micro binary operations for strings of bits stored in registers. These operations are considered each bit of the register separately and treated them as binary.

→ For Eg :- The exclusive-OR micro operation with the contents of two register R_1 and R_2 is symbolized by statements

$$P: R_1 \leftarrow R_1 \oplus R_2$$

→ It specifies a logic micro operation to be executed on the individual bits of registers. As numeric example, assume each register has 4 bits.

→ Special Symbols :-

The symbol \vee is used to denote an OR operation and the symbol \wedge is used to denote an AND operation. The compliment micro operation is same as its compliment and uses a bar on top of the register name.

List of logic micro operations :-

→ There are 16 different operations that can be performed with two binary variables. They can be determined from all possibilities truth table obtained with two binary variables as F_0 to F_{15} .

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	0	0	0	0	0	0	↑	1	1	1	1	1	1	1	1
0	1	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	1
1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Truth table for 16 functions of 2 variables.

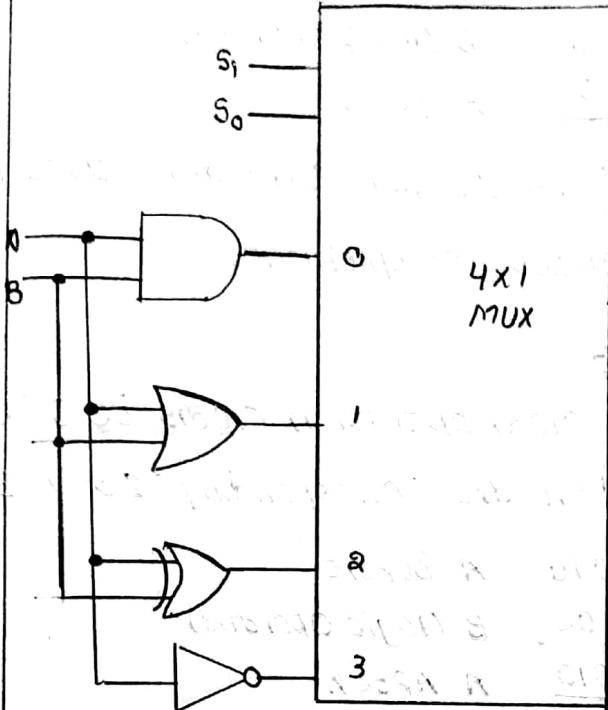
→ The 16 Boolean functions of two variables x and y are expressed in the algebraic form in the first column of table below, The relationship between two binary contents of two registers A and B. Name of the micro operation is in the third column.

Boolean function	Microoperation	Name
$F_0 = 0$	$F \leftarrow 0$	Clear
$F_1 = XY$	$F \leftarrow A \wedge B$	AND
$F_2 = X\bar{Y}$	$F \leftarrow A \wedge \bar{B}$	TRANSFER A
$F_3 = \bar{X}$	$F \leftarrow \bar{A}$	
$F_4 = X'Y'$	$F \leftarrow \bar{A} \wedge \bar{B}$	TRANSFER B
$F_5 = Y$	$F \leftarrow B$	
$F_6 = X \oplus Y$	$F \leftarrow A \oplus B$	EXCLUSIVE-OR
$F_7 = X + Y$	$F \leftarrow A \vee B$	OR
$F_8 = (X+Y)'$	$F \leftarrow \bar{A} \vee \bar{B}$	NOR
$F_9 = (X \oplus Y)'$	$F \leftarrow \bar{A} \oplus \bar{B}$	EXCLUSIVE-NOR
$F_{10} = Y'$	$F \leftarrow \bar{B}$	Complement

$F_{11} = x + y$	$F \leftarrow A \vee B$	
$F_{12} = x'$	$F \leftarrow \bar{A}$	complement
$F_{13} = x' + y$	$F \leftarrow \bar{A} \vee B$	
$F_{14} = (xy)'$	$F \leftarrow \bar{A} \wedge \bar{B}$	NAND
$F_{15} = 1$	$F \leftarrow \text{all } 1's$	Set to all 1's.

Hardware Implementation :-

- The hardware implementation of logic microoperations requires that logic gates be inserted for each bit (or) pair of bits in registers.
- Although there are 16 logic microoperations, most computers use only NOT, AND, OR, XOR and complement from which all others can be derived.
- Figure shows one stage of circuit that generates the four basic logic microoperations, it has four gates and a multiplexer.
- Each of the four logic operations is generated through a gate that performs required logic.
- The outputs of the gates are applied to the data inputs of multiplexers.
- The two selection inputs choose one of the data inputs of the multiplexer and direct its value to the output.



E_i	S_1	S_0	OUTPUT	Operation
	0	0	$E = A \wedge B$	AND
	0	1	$E = A \vee B$	OR
	1	0	$E = A \oplus B$	XOR
	1	1	$E = \bar{A}$	Complement

Function table

→ (a) logic diagram

Some Applications :-

1. Selective - Set :

→ The selective set operation sets the bits in the register A to 1, where there are corresponding 1's in the register B. It does not effect bit position that have 0's in B.

Eg:- $\begin{array}{r} 1010 \\ \times 1100 \\ \hline 1110 \end{array}$ A before

 B (logic operand)

A after.

→ "OR" microoperation can be used for performing selective-set.

2. Selective Complement :-

→ The selective complement operation complements bits in A where there are corresponding 1's in B. It does not effect bit positions that have 0's in B.

$$\begin{array}{r} \text{Eg:-} \quad 1010 \quad A \text{ before} \\ \quad \quad \quad 1100 \quad B \text{ (logic operand)} \\ \hline \quad \quad \quad 0110 \quad A \text{ after.} \end{array}$$

→ "EXCLUSIVE OR" microoperation can be used for performing Selective Complement.

3. Selective Clear :-

→ The selective clear operation clears to 0 the bits in A where there are corresponding 1's in B.

$$\begin{array}{r} \text{Eg:-} \quad 1010 \quad A \text{ before} \\ \quad \quad \quad 1100 \quad B \text{ (logic operand)} \\ \hline \quad \quad \quad 0010 \quad A \text{ after.} \end{array}$$

→ The corresponding microoperations is $A \leftarrow A \wedge \bar{B}$.

4. MASK :-

→ The mask operation is similar to selective-clear operation except that the bits of A are cleared only where there are corresponding 0's in B.

$$\begin{array}{r} \text{Eg:-} \quad 1010 \quad A \text{ before} \\ \quad \quad \quad 1100 \quad B \text{ (logic operand)} \\ \hline \quad \quad \quad 0000 \quad A \text{ after-masking} \end{array}$$

→ The mask operation is "AND" micro operation.

5. Clear :-

→ The clear operation compares the word in A and B and produces an all 0's result if the two numbers are equal. This is "EXCLUSIVE OR" micro operation.

$$\begin{array}{r} \text{Eg:-} \quad 1010 \quad A \\ \quad \quad \quad 1010 \quad B \\ \hline \quad \quad \quad 0000 \quad A \oplus B \end{array}$$

Shift Micro Operations

- Shift micro operations are used to shift the contents of register to the left (or) right shift microoperations. They are used for serial transfer of data.
- There are three types of shifts :

1. logical Shift
2. CIRCULAR SHIFT
3. ARITHMETIC SHIFT.

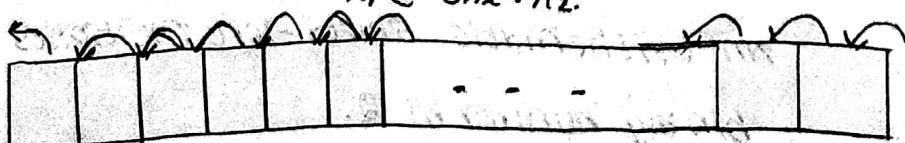
- The symbolic notation for shift micro operations are :-

Symbolic designation	Description
$R \leftarrow Shl R$	Shift-left register R.
$R \leftarrow Shr R$	Shift-right register R.
$R \leftarrow Cls R$	Circular left-shift register R
$R \leftarrow Csr R$	Circular right-shift Register R.
$R \leftarrow Ashl R$	Arithmetic shift-left R
$R \leftarrow Ashr R$	Arithmetic shift-right R

logical shift :-

- A logical shift is one that transfer 0 through the serial input.
- Logical shift(shl) :- Specify one bit shift to the left of content of register R_1 .

$$R_1 \leftarrow Shl \cdot R_1$$



→ logical shift right (Shr) :-

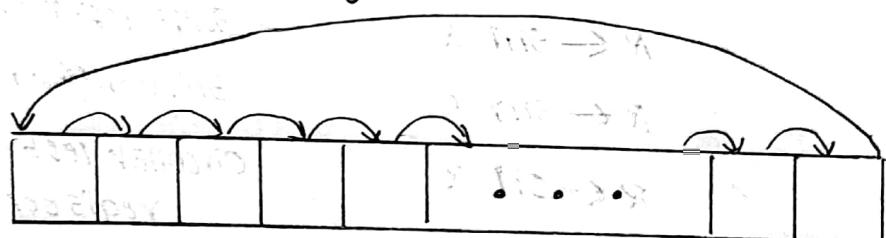
→ Specify one bit shift to the right of content of register R_a .

$$R_a \leftarrow \text{shr } R_a$$

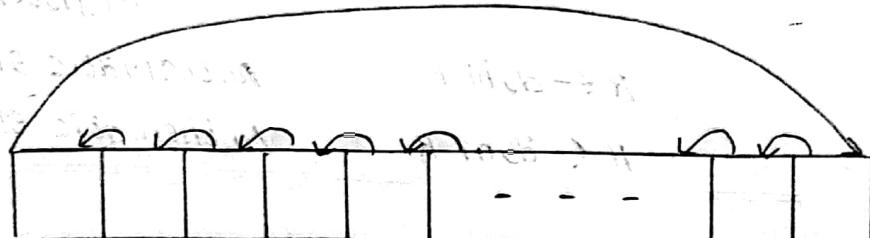


Circular shift :-

→ The circular shift circulates the bits of the register around the two ends without loss of information.
we will use the symbol Crl and Cir for circular left shift and right shift.



Circular right shift.



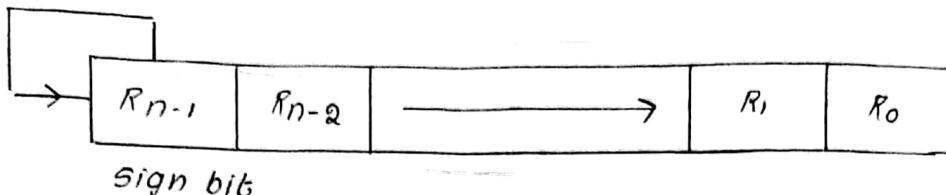
circular left shift.

Arithmetic shift :-

→ An Arithmetic shift is a micro operation that shifts a signed binary number to the left (or right).

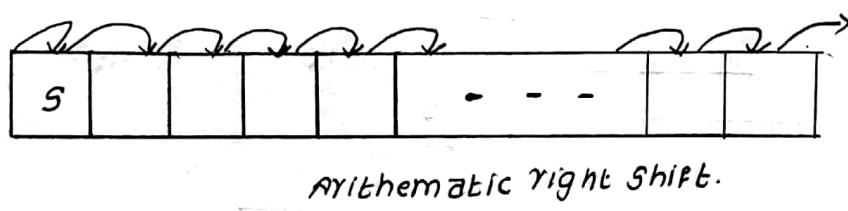
An arithmetic shift-left multiplies a signed binary number by 2.

- Arithmetic shifts must leave the sign bit unchanged.
- The left most bit in the register holds the sign bit and the remaining bits hold the number.



Arithmetic Shift right:-

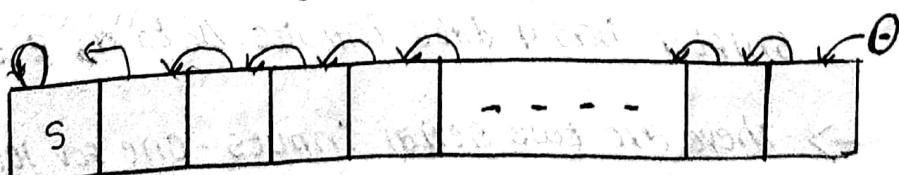
- The arithmetic shift-right leaves the sign bit as unchanged . Thus R_{n-1} remains same as R_{n-2} receives the bit from R_{n-1} . The bit R_0 is lost



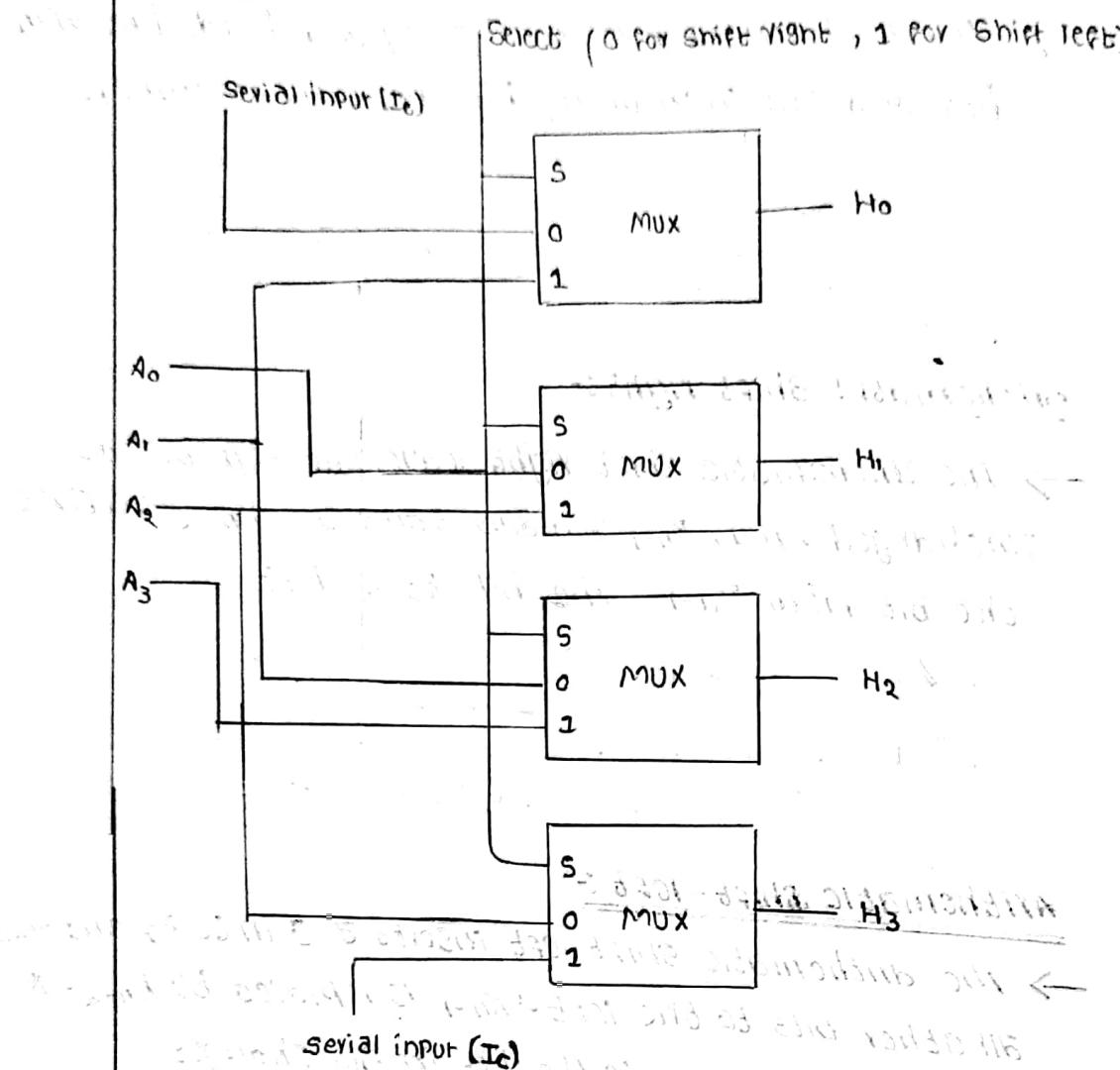
Arithmetic Shift-left :-

- The arithmetic shift-left inserts a 0 in to R_0 and shift all other bits to the left. R_{n-1} is replaced by R_{n-2} . A sign reversal occur if the bit in R_{n-1} changes.
- An overflow occurs if the bit in sign bit position (R_{n-1}) after performing shift operation doesn't match with previous value.
- An overflow flipflop us can be used to detect the arithmetic shift-left overflow.

$$V_S = R_{n-1} \oplus R_{n-2}$$



Hardware implementation :-



Function table

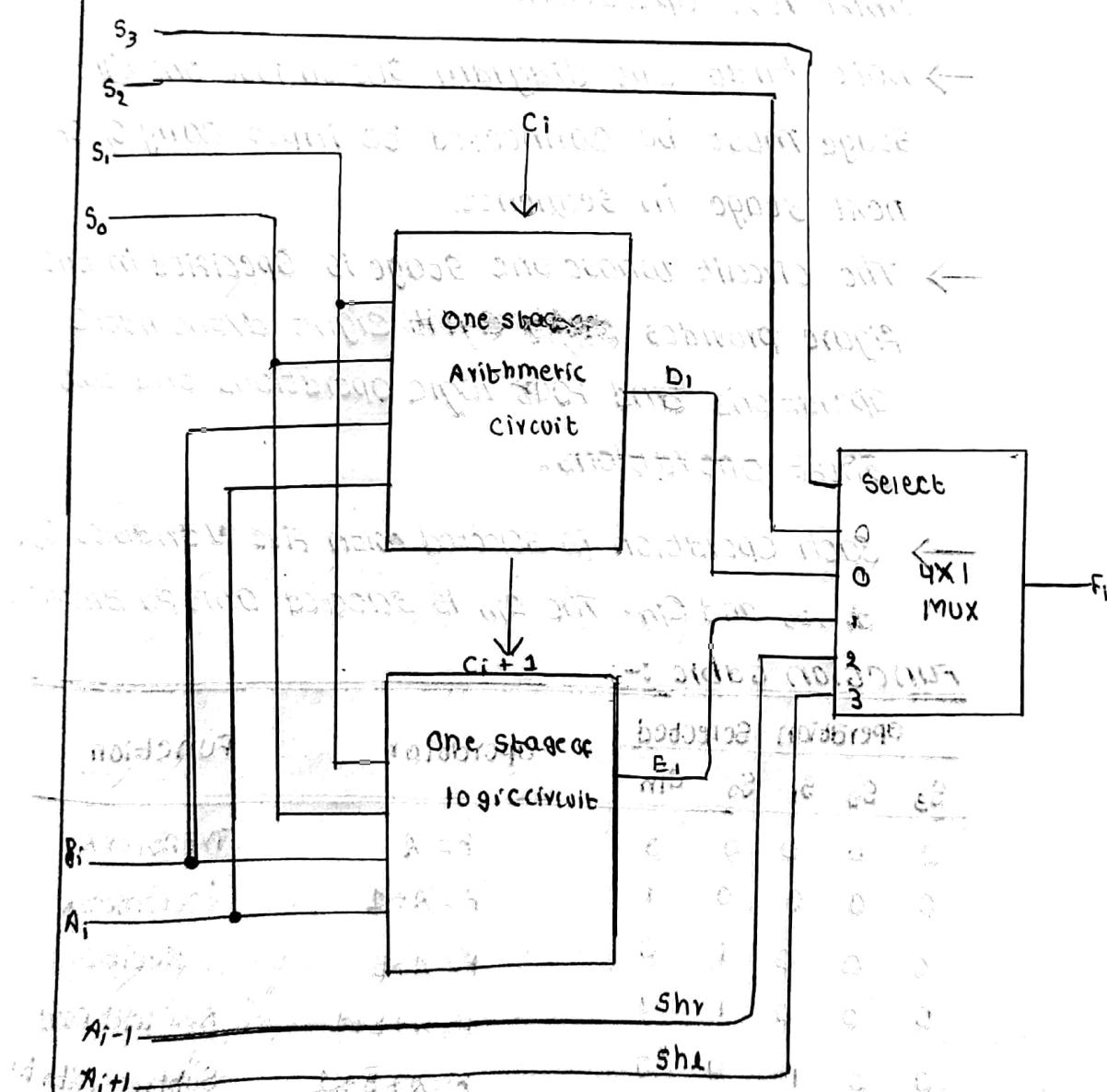
Select	Output
0 (IR)	$H_0 \ A_0 \ A_1 \ A_2$
1 (IL)	$A_1 \ A_2 \ A_3 \ I_0$

- A Combinational circuit shifter can be constructed with multiplexers as shown in figure. The 4-bit shifter has 4-data inputs A_0 to A_3 & outputs H_0 to H_3 .
- There are two serial inputs. One for left shift and another for right shift.

→ when the Selection input $s=0$, the data are shifted right. when $s=1$, the data are shifted left.

Arithmetic logic shift unit :-

→ the arithmetic logic and shift units can be combined in to one ALU with common selection variables.
one stage of ALU is as shown in below.



→ Inputs A_i and B_i are applied to both the arithmetic and logic units. A particular micro operation is selected with input s_3 and s_0 .

→ A 4x1 multiplexer at the output chooses between

an arithmetic output in E , and logic output in H .
The data in multiplexer are selected with input S_3 ,
and S_2 .

- The other two data inputs to the multiplexer receive inputs A_{i-1} for shift right operation and A_{i+1} for shift left operation.
- Note that the diagram shown just one typical stage must be connected to input carry C_i of next stage in sequence.
- The circuit whose one stage is specified in the figure provides ~~only~~ eight arithmetic operations and four logic operations and two shift operations.
- Each operation is selected with five variables S_3, S_4, S_5, S_6 and Cin . The Cin is selected only for arithmetic.

Function table :-

Operation Selected					Operation	Function
S_3	S_2	S_1	S_0	Cin		
0	0	0	0	0	$F = A$	TRANSFER A
0	0	0	0	1	$F = A + 1$	increment A
0	0	0	1	0	$F = A + B$	Addition
0	0	0	1	1	$F = A + B + 1$	Add with Carry
0	0	1	0	0	$F = A + \bar{B}$	Subtract with borrow
0	0	1	0	1	$F = A + \bar{B} + 1$	Subtraction
0	0	1	1	0	$F = A - 1$	Decrement A
0	0	1	1	1	$F = A$	TRANSFER A
0	1	0	0	X	$F = A \wedge B$	AND
0	1	0	1	X	$F = A \vee B$	OR

0	1	1	0	X	$F = A \oplus B$	XOR
0	1	1	1	X	$F = \bar{A}$	complement A
1	0	X	X	X	$F = \text{shv } A$	shift right A into F
1	1	X	X	X	$F = \text{shl } A$	shift left A into F

Example 1.8.1 Consider the memory system of a computer storing the following data :

Address in Hex	Data stored (binary)
2000	00111000
2001	00110100
2002	00110010
2003	00111001

Interpret the storage as numbers in the manner indicated below and find their decimal values in each case.

- i) Big-endian storage of 2 hex words of 4-digits each.
- ii) Big-endian storage of 2 BCD words of 4 -digits each.
- iii) Little-endian storage, in ASCII, of a 4-digit signed hex word.
- iv) Little endian storage, in ASCII, of a 4-digit BCD word.

Solution : We know that, big endian storage uses lower byte address for more-significant bytes and little-endian storage uses lower byte address for less-significant bytes. Therefore, in big-endian storage given two words are : 3834H, 3239H and in little-endian storage given two words are : 3932H, 3438H.

- i) Big-endian storage of 2 hex words of 4-digit each are : 3834H, 3239H

$$3834H = 3 \times 16^3 + 8 \times 16^2 + 3 \times 16^1 + 4 \times 16^0 = 14388 \text{ decimal}$$

$$3239H = 3 \times 16^3 + 2 \times 16^2 + 3 \times 16^1 + 9 \times 16^0 = 12857 \text{ decimal}$$

- ii) Big-endian storage of 2 BCD words of 4-digit each are : 3834, 3239.

$$3834 \text{ (BCD)} = 3834 \text{ decimal}$$

$$3239 \text{ (BCD)} = 3239 \text{ decimal}$$

- iii) Little-endian storage in ASCII, of 4-digit signed hex word is : 9248H. **Note :** We subtract 30H from ASCII code to get the digit value in hex.

$$9248H = 9 \times 16^3 + 2 \times 16^2 + 4 \times 16^1 + 8 \times 16^0 = 37448 \text{ decimal}$$

- iv) Little-endian storage, in ASCII, of a 4-digit BCD word is : 9248

$$9248 \text{ BCD} = 9248 \text{ decimal}$$

Example 1.8.2 Convert the following binary number to decimal number.

a) 110101_2 b) 111100_2

May-09, Marks 2

Solution :

$$\begin{aligned}(110101)_2 &= 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 32 + 16 + 0 + 4 + 0 + 1 = (53)_{10}\end{aligned}$$

$$\begin{aligned}(111100)_2 &= 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 \\ &= 32 + 16 + 8 + 4 + 0 + 0 = (60)_{10}\end{aligned}$$

Example 1.9.1 Find 1's complement of $(11010100)_2$.

Solution :

1	1	0	1	0	1	0	0
▽	▽	▽	▽	▽	▽	▽	▽
0	0	1	0	1	0	1	1

Number

NOT operation

1's complement of number

Example 1.9.2 Find 2's complement of $(11000100)_2$.

Solution : Let us see how -6 is represented in these three formats. Consider the number 6 represented in binary with eight bits.

Signed-magnitude representation : 10000110

1	1	0	0	0	1	0	0	Number
					1	1		Carry
0	0	1	1	1	0	1	1	1's complement of number
+							1	Add 1
0	0	1	1	1	1	1	0	2's complement of number

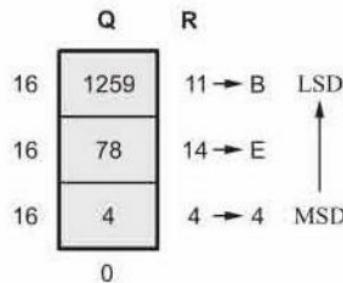
Signed-1's complement representation : 11111001

Signed-2's complement representation : 11111010

Example 1.10.1 Represent 1259.125_{10} in single precision and double precision formats

Solution : Step 1 : Convert decimal number in binary format

Integer part :

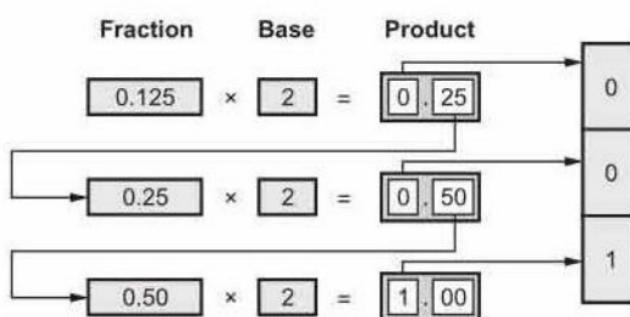


$$\therefore (1259)_{10} = (4EB)_{16}$$

4	E	B	Hex number
0	1	0	Binary number
1	1	1	0 1 0 0 1 1 1 0 1 0 1 1

$$\therefore (1259)_{10} = (100\ 1110\ 1011)_2$$

Fractional part :



$$(0.125)_{10} = (0.001)_2$$

$$\begin{aligned} \therefore \text{Binary number} &= 1\ 0\ 0\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ .\ 0\ 0\ 1 \\ &= 1\ 0\ 0\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ .\ 0\ 0\ 1 \end{aligned}$$

Step 2 : Normalize the number

$$1\ 0\ 0\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ .\ 0\ 0\ 1 = 1.0\ 0\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 1 \times 2^{10}$$

Now we will see the representation of the numbers in single precision and double precision formats.

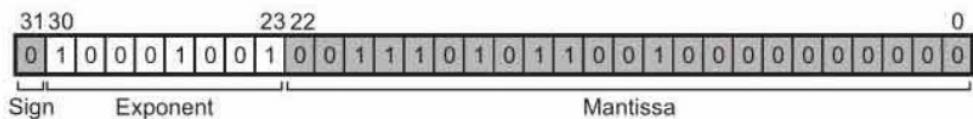
Step 3 : Single precision representation

For a given number S = 0, E = 10 and M = 0 0 1 1 1 0 1 0 1 1 0 0 1

Bias for single precision format is = 127

$$\therefore E' = E + 127 = 10 + 127 = 137_{10} = 10001001_2$$

\therefore Number in single precision format is given as



Step 4 : Double precision representation

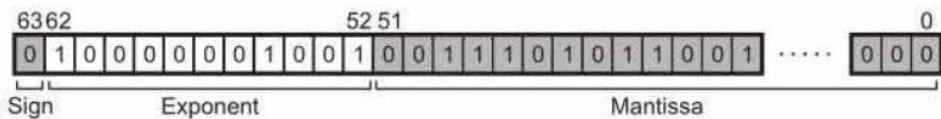
For a given number

S = 0, E = 10, and M = 0 0 1 1 1 0 1 0 1 1 0 0 1

Bias for double precision format is = 1023

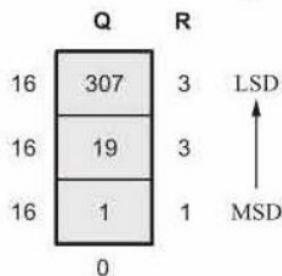
$$\therefore E' = E + 1023 = 10 + 1023 = 1033_{10} = 100000001001_2$$

\therefore Number in double precision format is given as



Example 1.10.2 Represent -307.1875_{10} in single precision and double precision formats.

Solution : Step 1 : Convert decimal number in binary format integer part

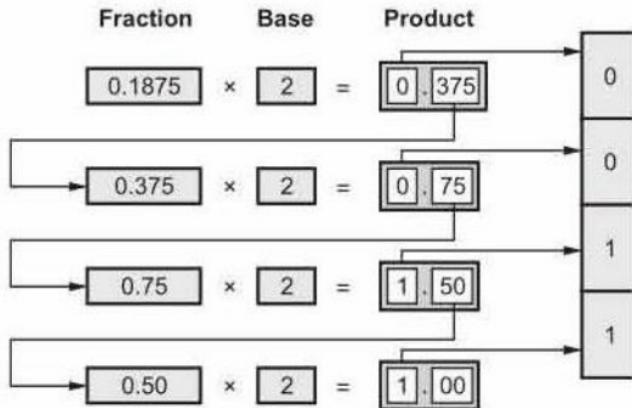


Integer part :

1	3	3	Hex number
0 0 0 1	0 0 1 1	0 0 1 1	Binary number

$$\therefore (307)_{10} = (133)_{16}$$

$$\therefore (307)_{10} = (100110011)_2$$



Fractional part :

$$\therefore (0.1875)_{10} = (0.0011)_2$$

$$\begin{aligned}\therefore \text{Binary number} &= -100110011 + .0011 \\ &= -100110011 .0011\end{aligned}$$

Step 2 : Normalize the number

$$-100110011 .0011 = -1.001100110011 \times 2^8$$

Now we will see the representation of the numbers in single precision and double precision formats.

Step 3 : Single precision representation

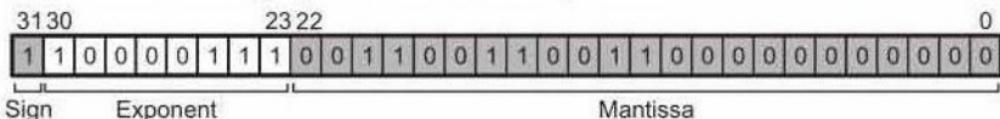
For a given number

$$S = 1, E = 8, \text{ and } M = 0011\ 0011\ 0011$$

Bias for single precision format is = 127

$$\therefore E' = E + 127 = 8 + 127 = 135_{10} = 10000111_2$$

\therefore Number in single precision format is given as



Step 4 : Double precision representation

For a given number

$$S = 1, E = 8, \text{ and } M = 0011\ 0011\ 0011$$

Bias for double precision format is = 1023

$$\therefore E' = E + 1023 = 8 + 1023 = 1031_{10} = 10000000111_2$$

\therefore Number in double precision format is given as

Sign $\underbrace{100\ 0000\ 0111}_{\text{Exponent}} \quad \underbrace{0011001100110.....0}_{\text{Mantissa (52- bits)}}$

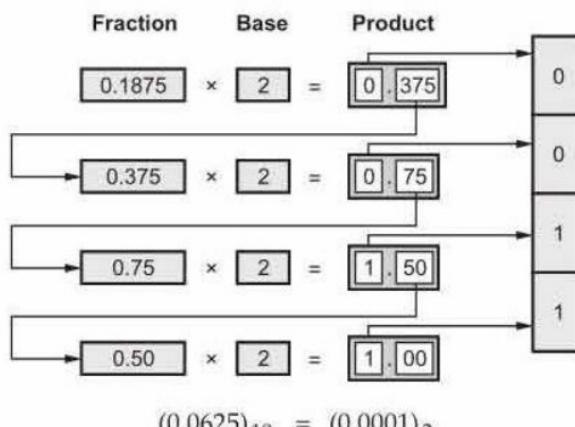
$\begin{matrix} 6362 & & 52.51 & & 0 \\ \boxed{1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\} & \boxed{0\ 0\ 0} \end{matrix}$
 Sign Exponent Mantissa

Example 1.10.3 Represent 0.0625_{10} in single precision and double precision formats.

Solution : Step 1 : Convert decimal number in binary format

Integer part : 0

Fractional part :



Step 2 : Normalize the number

$$0.0001 = 1.0 \times 2^{-4}$$

Now we will see the representation of the numbers in single precision and double precision formats

Step 3 : Single precision representation

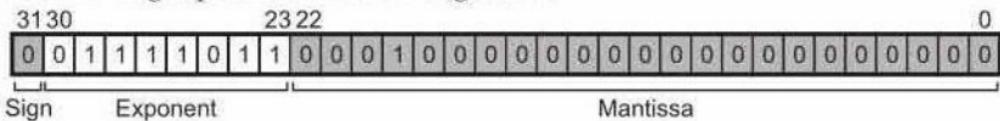
For a given number

$$S = 0, \quad E = -4, \quad \text{and } M = 0001$$

Bias for single precision format is = 127

$$\therefore E' = E + 127 = -4 + 127 = 123_{10} = 01111011_2$$

\therefore Number in single precision format is given as



Step 4 : Double precision representation

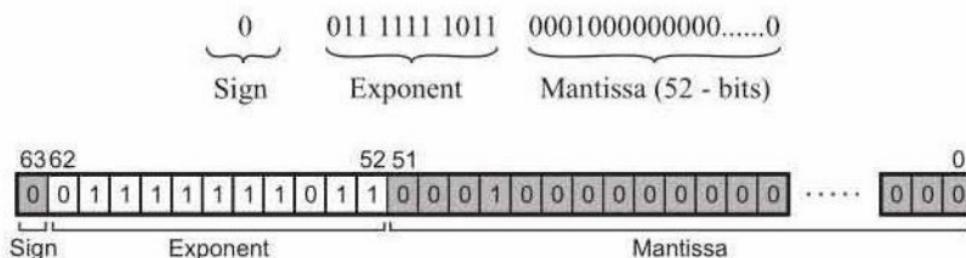
For a given number

$$S = 0, \quad E = -4, \quad \text{and } M = 0001$$

Bias for double precision format is = 1023

$$\therefore E' = E + 1023 = -4 + 1023 = 1019 = 01111111011_2$$

\therefore Number in double precision format is given as



Example 1.13.1 Registers R1 and R2 of a computer contain the decimal values 1200 and 2400 respectively. What is the effective address of the memory operand in each of the following instructions?

- 1) Load 20 (R1), R5
- 2) Add - (R2), R5
- 3) Move # 3000, R5
- 4) Sub (R1) +, R5

May-06, Marks 6

Solution : R1 1200 R2 2400

Sr. No.	Instruction	Effective address of memory
1	Load 20(R1), R5	$1200 + 20 = 1220$
2	Add - (R2), R5	$2400 - 1 = 2399$
3	Move #3000, R5	3000
4	Sub (R1)+, R5	1200

Example 1.13.2 Registers R1 and R2 of a computer contain the decimal values 1200 and 4600. What is the effective address of the memory operand in each of the following instructions ?

- a) Load 20 (R1), R5
- b) Add- (R2), R5.

May-08, Marks 2

Solution : a) EA : $1200 + 20$ b) EA : 4599.

Example 1.13.3 What must the address field of an indexed addressing mode instruction be to make it the same as a register indirect mode instruction?

Solution : The effective address in the indexed mode is given by $EA = \text{Offset} + R$ and the effective address in the register indirect mode is given by $EA = R$. Thus, when offset is 0, the indexed addressing mode instruction is same as that of register indirect mode instruction.

Example 1.13.4 At memory address 200, two word instruction, load to AC is stored with a mode bit as a most significant bit. At location 201 the address stored is 500. At location 202 next instruction is stored. The following numbers are stored at different memory locations as shown ahead.

Memory location (Address)	Memory contents
399	450
400	700
500	800
600	900
702	325
800	300

If the content of PC is 200, while the contents of register R1 is 400. XR register is 100. If all the numbers and addresses are in decimal number, find out contents of AC and effective address for the following addressing modes.

- i) Direct address ii) Indirect address
- iii) Relative address iv) Indexed address
- v) Register indirect addressing mode.

Solution : The Fig. 1.13.3 shows the given information for our convenience.

- i) Direct address : $EA = 500 \therefore AC = 800$
- ii) Indirect address : $EA = [500] = 800 \therefore AC = 300$
- iii) Relative address : $EA = PC + \text{Address part of instruction}$
 $= 202 + 500 = 702$
 $\therefore AC = 325$
- iv) Indexed address : $EA = XR + \text{Address part of instruction}$
 $= 100 + 500 = 600$
 $\therefore AC = 900$
- v) Register indirect address : $EA = R1 = 400$
 $\therefore AC = 700$

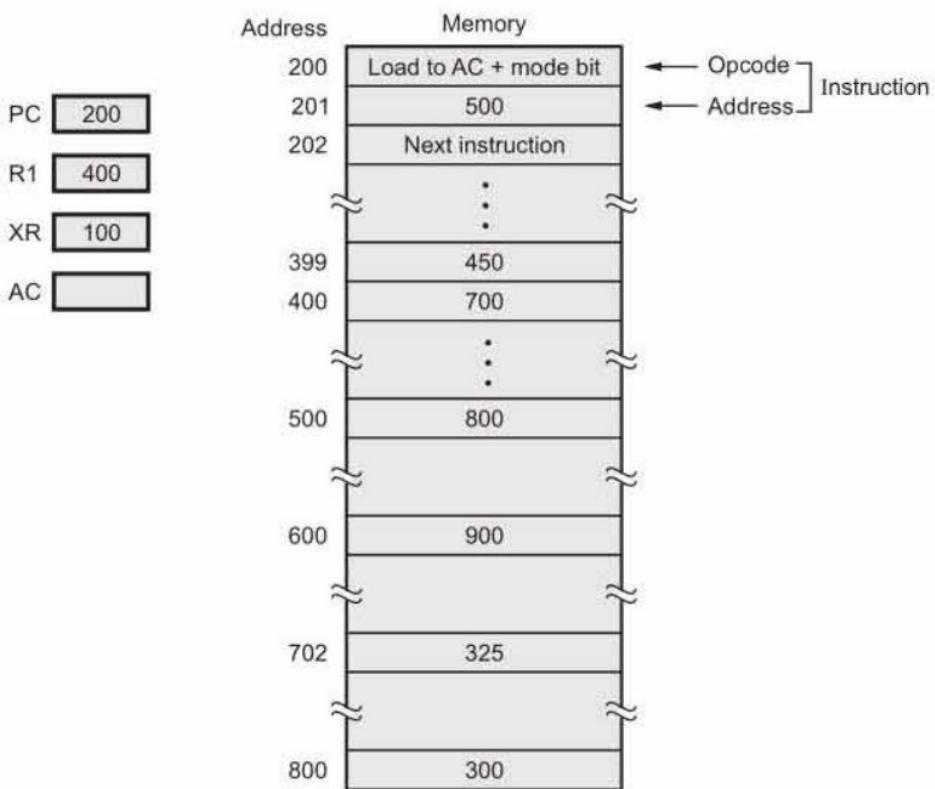


Fig. 1.13.3

Example 1.13.5 An instruction is stored at location 300 with its address field at location 301. The address field has the value 400. A processor register R1 contains the number 200. Evaluate the effective address if the addressing mode of the instruction is :

- Immediate
- Direct
- Register indirect
- Relative
- Index with R1 as the index register.

Solution :



i) **Immediate mode** : In immediate addressing mode, the address bits of an instruction code specify the actual operand. Hence for this addressing mode the address of the operand i.e. effective address is 301.

ii) **Direct mode** : Effective Address (EA) : 400

iii) **Register indirect mode** : Effective Address (EA) : 200

iv) **Relative mode** : Effective Address (EA) : PC + Address part of instruction = $302 + 400 = 702$

v) **Index with R1 as the index register** : Effective address (EA)

$$= R1 + \text{Address part of the instruction} = 200 + 400$$

$$= 600$$

Example 2.2.3 Perform arithmetic operation with binary numbers with negative numbers in signed 2's complementary form $(-35) + (-40)$.

Dec.-03, Marks 2

Solution :

<table border="1" style="border-collapse: collapse; width: 100px;"> <tr><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>1</td></tr> </table>	0	1	0	0	0	1	1	1	0	1	1	1	0	0	1	0	1	1	1	0	1	35 in binary - 35 in 1's complement form - 35 in 2's complement form			
0	1	0	0	0	1	1																			
1	0	1	1	1	0	0																			
1	0	1	1	1	0	1																			
<table border="1" style="border-collapse: collapse; width: 100px;"> <tr><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>1</td><td>0</td><td>1</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td></tr> </table>	0	1	0	1	0	0	0	1	0	1	0	1	1	1	1	0	1	1	0	0	0	40 in binary - 40 in 1's complement form - 40 in 2's complement form			
0	1	0	1	0	0	0																			
1	0	1	0	1	1	1																			
1	0	1	1	0	0	0																			
$\begin{array}{r} \therefore (-35) \\ + (-40) \end{array}$	2's complement of - 35 2's complement of - 40 2's complement of 75, i.e. (-75)																								
Verification : $\begin{array}{r} 0 \\ + \end{array}$ <table border="1" style="border-collapse: collapse; width: 100px;"> <tr><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>1</td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td></tr> </table>	1	0	0	0	1	0	1	1								1	0	1	0	0	1	0	1	0	(+ 75)
1	0	0	0	1	0	1	1																		
							1																		
0	1	0	0	1	0	1	0																		

Example 2.2.1 Perform addition of $(11001100)_2$ and $(11011010)_2$.

Solution :

1	1	1		1				Carry
		1	1	0	0	1	1	0
+		1	1	0	1	1	0	
		1	1	0	1	0	0	
		1	1	0	1	0	1	0

In the above example, we have seen the addition of two unsigned binary numbers. In case of the signed numbers, we have to consider the sign of the number and we may require sign extension to avoid possible overflow.

Example 2.2.2 Add $(28)_{10}$ and $(15)_{10}$ by converting them into binary.

Solution : Using decimal to binary conversion technique we have,

$$(28)_{10} = (011100)_2 \text{ and } (15)_{10} = (01111)_2$$

Sign Extension → + → Sign →	<table border="1" style="border-collapse: collapse; width: 100px;"> <tr><td></td><td style="text-align: right;">1</td><td style="text-align: right;">1</td><td style="text-align: right;">1</td><td></td><td></td><td></td><td></td></tr> <tr><td style="text-align: right;">0</td><td style="text-align: right;">0</td><td style="text-align: right;">1</td><td style="text-align: right;">1</td><td style="text-align: right;">1</td><td style="text-align: right;">0</td><td style="text-align: right;">0</td><td></td></tr> <tr><td style="text-align: right;">0</td><td style="text-align: right;">0</td><td style="text-align: right;">0</td><td style="text-align: right;">1</td><td style="text-align: right;">1</td><td style="text-align: right;">1</td><td style="text-align: right;">1</td><td></td></tr> <tr><td style="text-align: right;">0</td><td style="text-align: right;">1</td><td style="text-align: right;">0</td><td style="text-align: right;">1</td><td style="text-align: right;">0</td><td style="text-align: right;">1</td><td style="text-align: right;">1</td><td></td></tr> </table>		1	1	1					0	0	1	1	1	0	0		0	0	0	1	1	1	1		0	1	0	1	0	1	1		Carry Binary equivalent of $(28)_{10}$ Binary equivalent of $(15)_{10}$ Result : Binary equivalent of $(43)_{10}$	$(28)_{10}$ $+ (15)_{10}$ <hr style="width: 100px; margin-left: 0;"/> $(43)_{10}$
	1	1	1																																
0	0	1	1	1	0	0																													
0	0	0	1	1	1	1																													
0	1	0	1	0	1	1																													

Example 2.2.4 Perform $1010100 - 1000100$ using 1's and 2's complement.

May-08, CSE, Marks 2

Solution : Subtraction using 1's complement

1	0	0	0	1	0	0
0	1	1	1	0	1	1

Subtrahend

1's complement of subtrahend

1	1	1					
	1	0	1	0	1	0	0
+	0	1	1	1	0	1	1
1	0	0	0	1	1	1	1
			1	1	1	1	
1	0	0	0	1	1	1	1
+							1
	0	0	1	0	0	0	0

Carry

Minuend

1's complement of subtrahend

Result

Carry

Result

Add end-around carry

Final result

Subtraction using 2's complement

1	0	0	0	1	0	0
0	1	1	1	0	1	1
+						1
0	1	1	1	1	0	0

Subtrahend

1's complement of subtrahend

2's complement of subtrahend

1	1	1	1	1			
	1	0	1	0	1	0	0
+	0	1	1	1	1	0	0
X	0	0	1	0	0	0	0

Carry

Minuend

2's complement of subtrahend

Result

Example 2.2.5 Using 2's complement method perform $(1010_2 - 1111_2)$.

Dec.-08, CSE/IT, Marks 2

Solution :

	0	0	0	0
+				1
	0	0	0	1

1's complement of 1111

Add 1

2's complement of 1111

	1	0	1	0
+	0	0	0	1
	1	0	1	1

2's complement of 1111

**No carry, answer is negative and
in 2's complement form**

Two Marks Questions with Answers

Q.1 Define computer architecture.

Ans. : Computer architecture is defined as the functional operation of the individual H/W unit in a computer system and the flow of information among the control of those units.

Q.2 Define computer H/W.

Ans. : Computer H/W is the electronic circuit and electro mechanical equipment that constitutes the computer.

Q.3 Name the functional units of a computer.

Dec.-08

Ans. : The functional units of a computer are :

1. Input unit
2. Output unit
3. Control unit
4. Memory unit
5. Arithmetic and logical unit

Q.4 What is meant by Central Processing Unit (CPU) ?

Dec.-09

Ans. : The arithmetic and logic unit in conjunction with control unit is commonly called Central Processing Unit (CPU).

Q.5 What is the function of input unit ?

Ans. : A computer accepts a digitally coded information through input unit using input devices such as keyboard and mouse.

Q.6 What are the functions of control unit ?

Ans. : The control unit co-ordinates and controls the activities amongst the functional units. The basic function of control unit is to fetch the instructions stored in the main memory, identify the operations and the devices involved in it, and accordingly generate control signals to execute the desired operations.

Q.7 What is the function of arithmetic and logic unit ?

Ans. : The arithmetic and logic unit (ALU) is responsible for performing arithmetic operations such as add, subtract, division and multiplication, and logical operations such as ANDing, ORing, Inverting etc.

Q.8 What is meant by the stored program concept ? Discuss.

May-07

Ans. : According to stored-program concept memory can contain the program (source code), the corresponding compiled machine code, editor program and even the compiler that generated the machine code.

Q.39 Convert the following binary number to decimal number.

a) 110101_2 b) 111100_2

May-09

Ans. :

$$(110101)_2 = 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= 32 + 16 + 0 + 4 + 0 + 1 = (53)_{10}$$

$$(111100)_2 = 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$$

$$= 32 + 16 + 8 + 4 + 0 + 0 = (60)_{10}$$

Q.40 How do you relate addition and subtraction ?

Ans. : We can relate addition and subtraction operations of numbers by the following relationship :

$$(\pm A) - (+B) = (\pm A) + (-B) \text{ and}$$

$$(\pm A) - (-B) = (\pm A) + (+B)$$

Q.41 What is one's complement of numbers ?

Ans. : The 1's complement of a binary number is the number that results when we change all 1's to zeros and the zeros to ones.

Q.42 What is 2's complement of numbers ?

Ans. : The 2's complement is the binary number that results when we add 1 to the 1's complement. It is given as 2's complement = 1's complement + 1.

Q.43 Define IEEE floating point single and double precision standard.

Dec.-06, 07

Ans. : The 32-bit standard representation (**single-precision representation**) occupies a single 32-bit word. The 32-bits are divided into three fields as shown below :

(field 1) Sign \leftarrow 1 - bit

(field 2) Exponent \leftarrow 8 - bits

(field 3) Mantissa \leftarrow 23 - bits

The 64-bit standard representation (**double-precision representation**) occupies two 32-bit words. The 64-bits are divided into three fields as shown below :

(field 1) Sign \leftarrow 1 - bit

(field 2) Exponent \leftarrow 1 - bit

(field 3) Mantissa \leftarrow 52 - bits

Q.1 Define half adder and full adder.

Ans. :

- **Half adder :** The logic circuit which performs the arithmetic sum of two bits is called a half adder.
- **Full adder :** The logic circuit which performs the arithmetic sum of 3 bits (bit 1 : input 1, bit 2 : input 2, bit 3 : carry from the previous addition) is called a full adder.

Q.2 Define half subtractor and full subtractor.

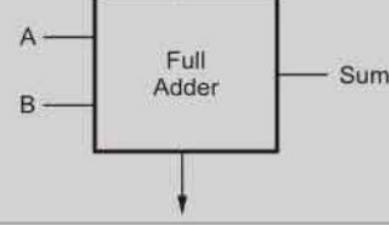
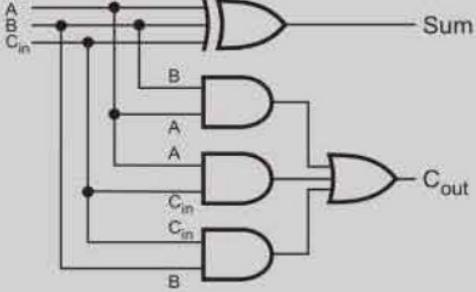
Ans. :

- **Half subtractor :** It is a combinational circuit that subtracts two bits and produces their difference and borrow.
- **Full subtractor :** It is a combinational circuit that performs a subtraction between 2 bits. It also takes into account borrow of the lower significant stage.

Q.3 What is the difference between half adder and full adder?

Dec.-07

Ans. :

Sr. No.	Half adder	Full adder
1.	Half-adder takes two binary-inputs i.e. augend and addend bits and gives out two binary outputs as sum and carry.	Full-adder, alongwith augend and addend takes third additional bit Cin as input. Cin represents the carry from the previous lower significant position.
2.	Half-adder is not used in practice	Full adder is used in practice
3.	Block diagram : 	Block diagram : 
4.	Logic diagram : 	Logic diagram : 

generator circuit. As shown in Fig. 3-3, this circuit consists of one exclusive-OR and one exclusive-NOR gate. Since $P(\text{even})$ is the exclusive-OR of x , y , z , and $P(\text{odd})$ is the complement of $P(\text{even})$, it is necessary to employ an exclusive-NOR gate for the needed complementation. The message and the odd-parity bit are transmitted to their destination where they are applied to a parity checker. An error has occurred during transmission if the parity of the four bits received is even, since the binary information transmitted was originally odd. The output of the parity checker would be 1 when an error occurs, that is, when the number of 1's in the four inputs is even. Since the exclusive-OR function of the four inputs is an odd function, we again need to complement the output by using an exclusive-NOR gate.

It is worth noting that the parity generator can use the same circuit as the parity checker if the fourth input is permanently held at a logic-0 value. The advantage of this is that the same circuit can be used for both parity generation and parity checking.

It is evident from the example above that even-parity generators and checkers can be implemented with exclusive-OR functions. Odd-parity networks need an exclusive-NOR at the output to complement the function.

PROBLEMS

- 3-1. Convert the following binary numbers to decimal: 101110; 1110101; and 110110100.
- 3-2. Convert the following numbers with the indicated bases to decimal: $(12121)_3$; $(4310)_5$; $(50)_7$; and $(198)_{12}$.
- 3-3. Convert the following decimal numbers to binary: 1231; 673; and 1998.
- 3-4. Convert the following decimal numbers to the bases indicated.
 - a. 7562 to octal
 - b. 1938 to hexadecimal
 - c. 175 to binary
- 3-5. Convert the hexadecimal number F3A7C2 to binary and octal.
- 3-6. What is the radix of the numbers if the solution to the quadratic equation $x^2 - 10x + 31 = 0$ is $x = 5$ and $x = 8$?
- 3-7. Show the value of all bits of a 12-bit register that hold the number equivalent to decimal 215 in (a) binary; (b) binary-coded octal; (c) binary-coded hexadecimal; (d) binary-coded decimal (BCD).
- 3-8. Show the bit configuration of a 24-bit register when its content represents the decimal equivalent of 295: (a) in binary; (b) in BCD; (c) in ASCII using eight bits with even parity.
- 3-9. Write your name in ASCII using an 8-bit code with the leftmost bit always 0. Include a space between names and a period after a middle initial.

- 3-10. Decode the following ASCII code:

1001010 1001111 1001000 1001110 0100000 1000100 1001111 1000101

- 3-11. Obtain the 9's complement of the following eight-digit decimal numbers: 12349876; 00980100; 90009951; and 00000000.
- 3-12. Obtain the 10's complement of the following six-digit decimal numbers: 123900; 090657; 100000; and 000000.
- 3-13. Obtain the 1's and 2's complements of the following eight-digit binary numbers: 10101110; 10000001; 10000000; 00000001; and 00000000.
- 3-14. Perform the subtraction with the following unsigned decimal numbers by taking the 10's complement of the subtrahend.
- a. $5250 - 1321$ b. $1753 - 8640$
c. $20 - 100$ d. $1200 - 250$
- 3-15. Perform the subtraction with the following unsigned binary numbers by taking the 2's complement of the subtrahend.
- a. $11010 - 10000$ b. $11010 - 1101$
c. $100 - 110000$ d. $1010100 - 1010100$
- 3-16. Perform the arithmetic operations $(+42) + (-13)$ and $(-42) - (-13)$ in binary using signed-2's complement representation for negative numbers.
- 3-17. Perform the arithmetic operations $(+70) + (+80)$ and $(-70) + (-80)$ with binary numbers in signed-2's complement representation. Use eight bits to accommodate each number together with its sign. Show that overflow occurs in both cases, that the last two carries are unequal, and that there is a sign reversal.
- 3-18. Perform the following arithmetic operations with the decimal numbers using signed-10's complement representation for negative numbers.
- a. $(-638) + (+785)$
b. $(-638) - (+185)$
- 3-19. A 36-bit floating-point binary number has eight bits plus sign for the exponent and 26 bits plus sign for the mantissa. The mantissa is a normalized fraction. Numbers in the mantissa and exponent are in signed-magnitude representation. What are the largest and smallest positive quantities that can be represented, excluding zero?
- 3-20. Represent the number $(+46.5)_{10}$ as a floating-point binary number with 24 bits. The normalized fraction mantissa has 16 bits and the exponent has 8 bits.
- 3-21. The Gray code is sometimes called a reflected code because the bit values are reflected on both sides of any 2^n value. For example, as shown in Table 3-5, the values of the three low-order bits are reflected over a line drawn between 7 and 8. Using this property of the Gray code, obtain:
- a. The Gray code numbers for 16 through 31 as a continuation of Table 3-5.
b. The excess-3 Gray code for decimals 10 to 19 as a continuation of the list in Table 3-6.
- 3-22. Represent decimal number 8620 in (a) BCD; (b) excess-3 code; (c) 2421 code; (d) as a binary number.

- 3-23. List the 10 BCD digits with an even parity in the leftmost position (total of five bits per digit). Repeat with an odd-parity bit.
- 3-24. Represent decimal 3984 in the 2421 code of Table 3-6. Complement all bits of the coded number and show that the result is the 9's complement of 3984 in the 2421 code.
- 3-25. Show that the exclusive-OR function $x = A \oplus B \oplus C \oplus D$ is an odd function. One way to show this is to obtain the truth table for $y = A \oplus B$ and for $z = C \oplus D$ and then formulate the truth table for $x = y \oplus z$. Show that $x = 1$ only when the total number of 1's in A , B , C , and D is odd.
- 3-26. Derive the circuits for a 3-bit parity generator and 4-bit parity checker using an even-parity bit. (The circuits of Fig. 3-3 use odd parity.)

CHAPTER 3

3-1

$$(101110)_2 = 32 + 8 + 4 + 2 = 46$$

$$(1110101)_2 = 64 + 32 + 16 + 4 + 1 = 117$$

$$(110110100)_2 = 256 + 128 + 32 + 16 + 4 = 436$$

3-2

$$(12121)_3 = 3^4 + 2 \times 3^3 + 3^2 + 2 \times 3 + 1 = 81 + 54 + 9 + 6 + 1 = 151$$

$$(4310)_5 = 4 \times 5^3 + 3 \times 5^2 + 5 = 500 + 75 + 5 = 580$$

$$(50)_7 = 5 \times 7 = 35$$

$$(98)_{12} = 12^2 + 9 \times 12 + 8 = 144 + 108 + 8 = 260$$

3-3

$$(1231)_{10} = 1024 + 128 + 64 + 15 = 2^{10} + 2^7 + 2^6 + 2^3 + 2^2 + 2 + 1 = (10011001111)_2$$

$$(673)_{10} = 512 + 128 + 32 + 1 = 2^9 + 2^7 + 2^5 + 1 = (1010100001)_2$$

$$\begin{aligned} (1998)_{10} &= 1024 + 512 + 256 + 128 + 64 + 8 + 4 + 2 \\ &= 2^{10} + 2^9 + 2^8 + 2^7 + 2^6 + 2^3 + 2^2 + 2^1 = (1111001110)_2 \end{aligned}$$

3-4

$$(7562)_{10} = (16612)_8$$

$$(1938)_{10} = (792)_{16}$$

$$(175)_{10} = (10101111)_2$$

3-5

$$(F3A7C2)_{16} = (1111\ 0011\ 1010\ 0111\ 1100\ 0010)_2$$

$$= (74723702)_8$$

3-6

$$(x^2 - 10x + 31)_r = [(x-5)(x-8)]_{10}$$

$$= x^2 - (5+8)_{10} x + (40)_{10} x$$

$$\text{Therefore : } (10)_r = (13)_{10} \quad \underline{\underline{r=13}}$$

$$\text{Also } (31)_r = 3 \times 13 + 1 = (40)_{10} \quad (r=13)$$

$$3-7 \quad (215)_{10} = 128 + 64 + 16 + 7 = (11010111)_2$$

- (a) 0000 11010111 Binary
 (b) 000 011 010 111 Binary coded octal
 (c) 0000 1101 0111 Binary coded hexadecimal
 (d) 0010 0001 0101 Binary coded decimal

$$\begin{array}{r} 0 \\ 2 \\ \hline 1 \\ \end{array} \quad \begin{array}{r} 3 \\ 2 \\ \hline 7 \\ \end{array} \quad \begin{array}{r} 0 \\ \downarrow \\ 7 \\ \end{array} \quad \begin{array}{r} 2 \\ 1 \\ \hline 5 \\ \end{array}$$

$$3-8 \quad (295)_{10} = 256 + 32 + 7 = (100100111)_2$$

- (a) 0000 0000 0000 0001 0010 0111
 (b) 0000 0000 0000 0010 1001 0101
 (c) 10110010 00111001 00110101

3-10 JOHN DOE

3-11 87650123; 99019899; 09990048; 999999.

3-12 876100; 909343; 900000; 000000

3-13 01010001; 0111110; 0111111; 1111110; 1111111
 01010010; 0111111; 10000000; 1111111; 00000000

3-14

(a) 5250	(b) 1753	(c) 020	(d) 1200
+ 8679	+ 1360	+ 900	+ 9150
1) 3929	0) 3113	0) 920	1) 0950
\downarrow 10's complement			
- 6887		- 080	

3-15

(a)	(b)	(c)	(d)
11010	11010	000100	1010100
+ 10000	10011	010000	0101100
1) 01010	1) 01101	0) 010100	1) 0000000
(26-16=10)		(84-84=0)	
		- 101100 (4-48=-44)	

3-16

$$+42 = 0101010$$

$$-42 = 1010110$$

$$(+42) \quad 0101010$$

$$(-13) \quad 1110011$$

$$(+29) \quad 0011101$$

$$+13 = 0001101$$

$$-13 = 1110011$$

$$(-42) \quad 1010110$$

$$(+13) \quad 0001101$$

$$(-29) \quad 1100011$$

3-17

01 ← last two carries → 1 0

$$+70 \quad 01000110$$

$$+80 \quad 01010000$$

$$+150 \quad 10010110$$

greater than +127
negative

$$-70 \quad 10111010$$

$$-80 \quad 10110000$$

$$-150 \quad 01101010$$

less than -128
positive

3-18

$$(a) (-638) \quad 9362 \\ (+785) \quad +0785 \\ (+147) \quad 0147$$

$$(b) (-638) \quad 9362 \\ (-185) \quad +9815 \\ (-823) \quad 9177$$

3-19

Mantissa

[+]	2.6 bits
-----	----------

Largest: +0.1111...1

$$1 - 2^{-26}$$

Exponent

[S]	8 bits
-----	--------

36 bits

$$+11111111$$

$$+255$$

$$(1 - 2^{-26}) \times 2^{+255}$$

Smallest: +0.1000...0

(normalized) 2^{-1}

$$-11111111$$

$$-255$$

$$2^{-256}$$

3-20

$$46.5 = 32 + 8 + 4 + 2 + 0.5 = (101110, 1)_2$$

Sign

$$\begin{array}{r} 0 \quad 1011101000000000 \\ \hline \text{24-bit mantissa} \end{array}$$

$$\begin{array}{r} 00000110 \\ \hline \text{8-bit exponent (+6)} \end{array}$$

3-21 (2)

<u>Decimal</u>	<u>Gray code</u>
16	11000
17	11001
18	11011
19	11010
20	11110
21	11111
22	11101
23	11100
24	10100
25	10101
26	10111
27	10110
28	10010
29	10011
30	10001
31	10000

(b)

<u>Decimal</u>	<u>Excess-3 Gray</u>
9	0010 1010
10	0110 1010
11	0110 1110
12	0110 1111
13	0110 1101
14	0110 1100
15	0110 0100
16	0110 0101
17	0110 0111
18	0110 0110
19	0110 0010
20	0111 0010

3-22 8620

- (a) BCD 1000 0110 0010 0000
 (b) XS-3 1011 1001 0101 0011
 (c) 2421 1110 1100 0010 0000
 (d) Binary 10000110101100 (8192+256+128+32+8+4)

3-23

<u>Decimal</u>	<u>BCD with even parity</u>	<u>BCD with odd parity</u>
0	0 0000	1 0000
1	1 0001	0000·1
2	1 0010	00010
3	0 0011	1 0011
4	1 0100	0 0100
5	0 0101	1 0101
6	0 0110	1 0110
7	1 0111	0 0111
8	1 1000	0 1000
9	0 1001	1 1001

3-24

$$3984 = 0011 \quad 1111 \quad 1110 \quad 0100 \\ 1100 \quad 0000 \quad 0001 \quad 1011 = 6015$$

3-25

A	B	$y = A \oplus B$	C	D	$z = C \oplus D$
0	0	0	0	0	0
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	1	1	0

y	z	$x = y \oplus z$	<u>ABCD</u>
0	0	0	0001, 0010, 1101, 1110
0	1	1	$\left[\begin{array}{l} AB=00 \text{ or } 11 \\ CD=01 \text{ or } 10 \end{array} \right]$
1	0	1	$\left[\begin{array}{l} AB=01 \text{ or } 10 \\ CD=00 \text{ or } 11 \end{array} \right]$
1	1	0	0100, 0111, 1000, 1011 Always odd number of 1's

3-26

Same as in Fig. 3-3 but without the complemented circles in the outputs of the gates.

$$P = x \oplus y \oplus z \\ \text{Error} = x \oplus y \oplus z \oplus P$$

PROBLEMS

- 4-1. Show the block diagram of the hardware (similar to Fig. 4-2a) that implements the following register transfer statement:

$$yT_2: R2 \leftarrow R1, R1 \leftarrow R2$$

- 4-2. The outputs of four registers, $R0$, $R1$, $R2$, and $R3$, are connected through 4-to-1-line multiplexers to the inputs of a fifth register, $R5$. Each register is eight bits long. The required transfers are dictated by four timing variables T_0 through T_3 as follows:

$$\begin{aligned} T_0: & R5 \leftarrow R0 \\ T_1: & R5 \leftarrow R1 \\ T_2: & R5 \leftarrow R2 \\ T_3: & R5 \leftarrow R3 \end{aligned}$$

The timing variables are mutually exclusive, which means that only one variable is equal to 1 at any given time, while the other three are equal to 0. Draw a block diagram showing the hardware implementation of the register transfers. Include the connections necessary from the four timing variables to the selection inputs of the multiplexers and to the load input of register $R5$.

- 4-3. Represent the following conditional control statement by two register transfer statements with control functions.

$$\text{If } (P = 1) \text{ then } (R1 \leftarrow R2) \text{ else if } (Q = 1) \text{ then } (R1 \leftarrow R3)$$

- 4-4. What has to be done to the bus system of Fig. 4-3 to be able to transfer information from any register to any other register? Specifically, show the connections that must be included to provide a path from the outputs of register C to the inputs of register A .

- 4-5. Draw a diagram of a bus system similar to the one shown in Fig. 4-3, but use three-state buffers and a decoder instead of the multiplexers.

- 4-6. A digital computer has a common bus system for 16 registers of 32 bits each. The bus is constructed with multiplexers.

- a. How many selection inputs are there in each multiplexer?
- b. What size of multiplexers are needed?
- c. How many multiplexers are there in the bus?

- 4-7. The following transfer statements specify a memory. Explain the memory operation in each case.

- a. $R2 \leftarrow M[AR]$
- b. $M[AR] \leftarrow R3$
- c. $R5 \leftarrow M[R5]$

- 4-8. Draw the block diagram for the hardware that implements the following statements:

$$x + yz: AR \leftarrow AR + BR$$

where AR and BR are two n -bit registers and x , y , and z are control variables. Include the logic gates for the control function. (Remember that the symbol $+$ designates an OR operation in a control or Boolean function but that it represents an arithmetic plus in a microoperation.)

- 4-9. Show the hardware that implements the following statement. Include the logic gates for the control function and a block diagram for the binary counter with a count enable input.

$$xyT_0 + T_1 + y'T_2: AR \leftarrow AR + 1$$

- 4-10. Consider the following register transfer statements for two 4-bit registers $R1$ and $R2$.

$$\begin{aligned} xT: R1 &\leftarrow R1 + R2 \\ x'T: R1 &\leftarrow R2 \end{aligned}$$

Every time that variable $T = 1$, either the content of $R2$ is added to the content of $R1$ if $x = 1$, or the content of $R2$ is transferred to $R1$ if $x = 0$. Draw a diagram showing the hardware implementation of the two statements. Use block diagrams for the two 4-bit registers, a 4-bit adder, and a quadruple 2-to-1-line multiplexer that selects the inputs to $R1$. In the diagram, show how the control variables x and T select the inputs of the multiplexer and the load input of register $R1$.

- 4-11. Using a 4-bit counter with parallel load as in Fig. 2-11 and a 4-bit adder as in Fig. 4-6, draw a block diagram that shows how to implement the following statements:

$$\begin{array}{ll} x: R1 \leftarrow R1 + R2 & \text{Add } R2 \text{ to } R1 \\ x'y: R1 \leftarrow R1 + 1 & \text{Increment } R1 \end{array}$$

where $R1$ is a counter with parallel load and $R2$ is a 4-bit register.

- 4-12. The adder-subtractor circuit of Fig. 4-7 has the following values for input mode M and data inputs A and B . In each case, determine the values of the outputs: S_3 , S_2 , S_1 , S_0 , and C_4 .

	M	A	B
a.	0	0111	0110
b.	0	1000	1001
c.	1	1100	1000
d.	1	0101	1010
e.	1	0000	0001

- 4-13. Design a 4-bit combinational circuit decrementer using four full-adder circuits.
- 4-14. Assume that the 4-bit arithmetic circuit of Fig. 4-9 is enclosed in one IC package. Show the connections among two such ICs to form an 8-bit arithmetic circuit.
- 4-15. Design an arithmetic circuit with one selection variable S and two n -bit data inputs A and B . The circuit generates the following four arithmetic operations in conjunction with the input carry C_{in} . Draw the logic diagram for the first two stages.

S	$C_{in} = 0$	$C_{in} = 1$
0	$D = A + B$ (add)	$D = A + 1$ (increment)
1	$D = A - 1$ (decrement)	$D = A + \bar{B} + 1$ (subtract)

- 4-16. Derive a combinational circuit that selects and generates any of the 16 logic functions listed in Table 4-5.
- 4-17. Design a digital circuit that performs the four logic operations of exclusive-OR, exclusive-NOR, NOR, and NAND. Use two selection variables. Show the logic diagram of one typical stage.
- 4-18. Register A holds the 8-bit binary 11011001. Determine the B operand and the logic microoperation to be performed in order to change the value in A to:
 a. 01101101
 b. 11111101
- 4-19. The 8-bit registers AR , BR , CR , and DR initially have the following values:

$$\begin{aligned} AR &= 11110010 \\ BR &= 11111111 \\ CR &= 10111001 \\ DR &= 11101010 \end{aligned}$$

Determine the 8-bit values in each register after the execution of the following sequence of microoperations.

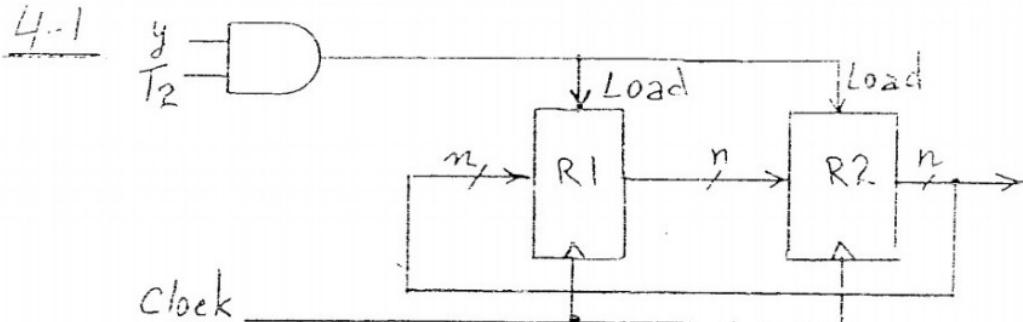
$AR \leftarrow AR + BR$	Add BR to AR
$CR \leftarrow CR \wedge DR$, $BR \leftarrow BR + 1$	AND DR to CR , increment BR
$AR \leftarrow AR - CR$	Subtract CR from AR

- 4-20. An 8-bit register contains the binary value 10011100. What is the register value after an arithmetic shift right? Starting from the initial number 10011100, determine the register value after an arithmetic shift left, and state whether there is an overflow.
- 4-21. Starting from an initial value of $R = 11011101$, determine the sequence of binary values in R after a logical shift-left, followed by a circular shift-right, followed by a logical shift-right and a circular shift-left.

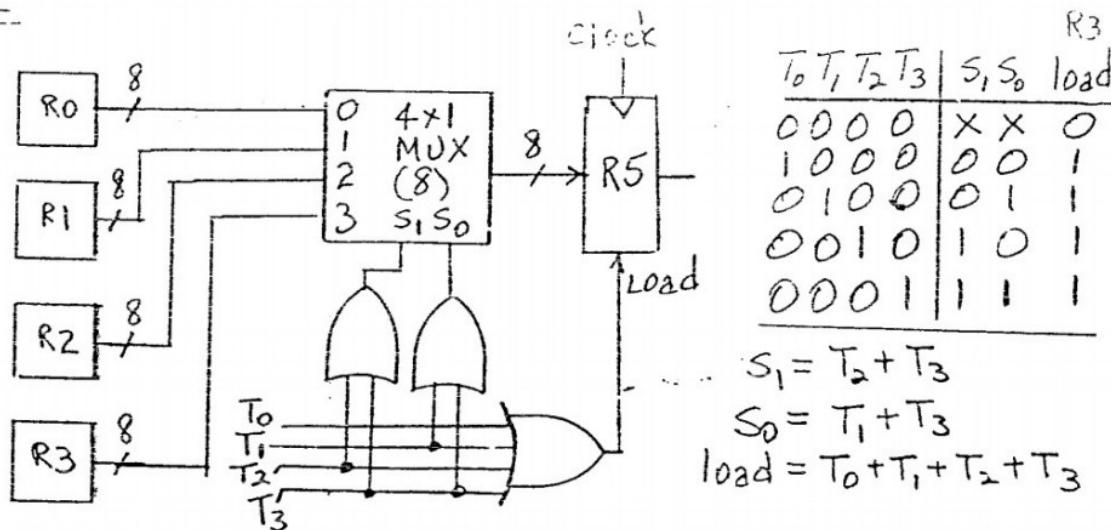
- 4-22. What is the value of output H in Fig. 4-12 if input A is 1001, $S = 1$, $I_R = 1$, and $I_L = 0$?
- 4-23. What is wrong with the following register transfer statements?
- $xT: AR \leftarrow \overline{AR}, AR \leftarrow 0$
 - $yT: R1 \leftarrow R2, R1 \leftarrow R3$
 - $zT: PC \leftarrow AR, PC \leftarrow PC + 1$

CHAPTER 4

4-1



4-2



4-3

P: $R1 \leftarrow R2$

$P'Q$: $R1 \leftarrow R3$

4-4

Connect the 4-line common bus. to the four inputs of each register.

Provide a "load" control input in each register.

Provide a clock input for each register.

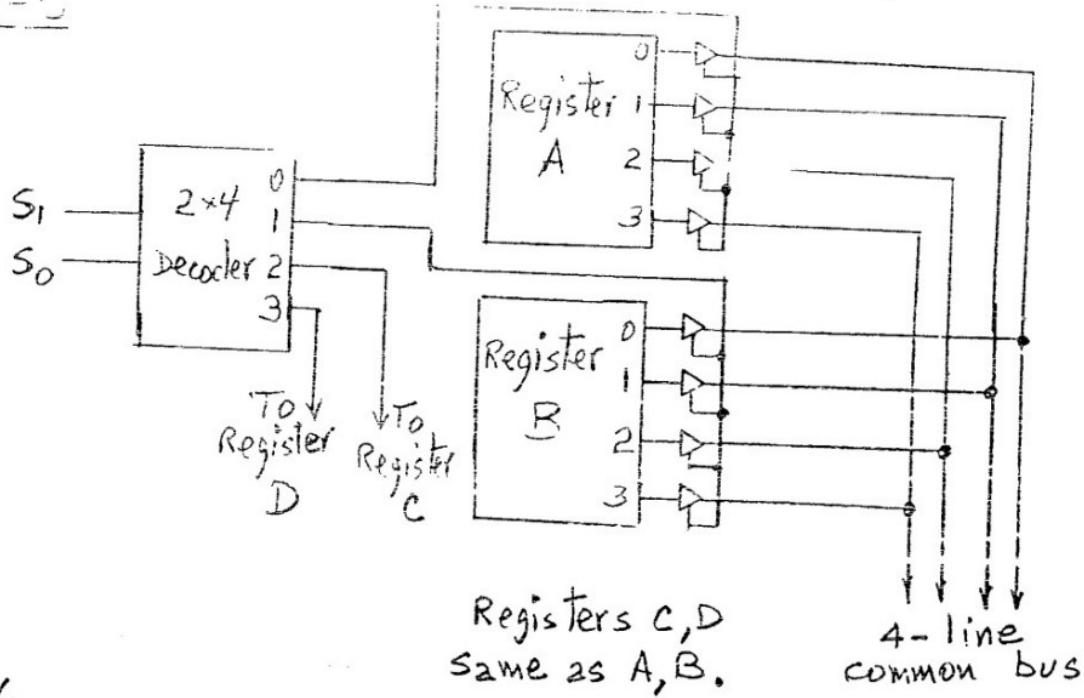
To transfer from register C to register A:

Apply $S_1, S_0 = 10$ (to select C for the bus.)

Enable the load input of A

Apply a clock pulse.

4-5



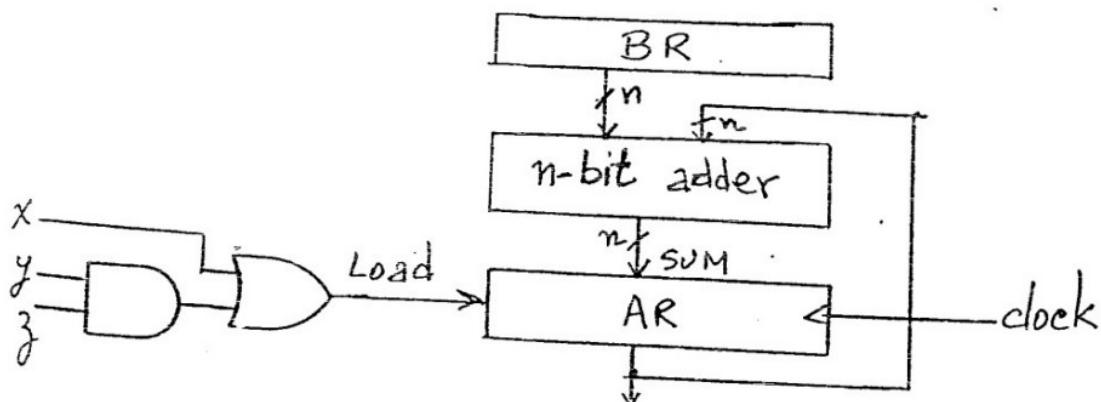
4-6

- (a) 4 selection lines to select one of 16 registers.
- (b) 16×1 multiplexers
- (c) 32 multiplexers, one for each bit of the registers.

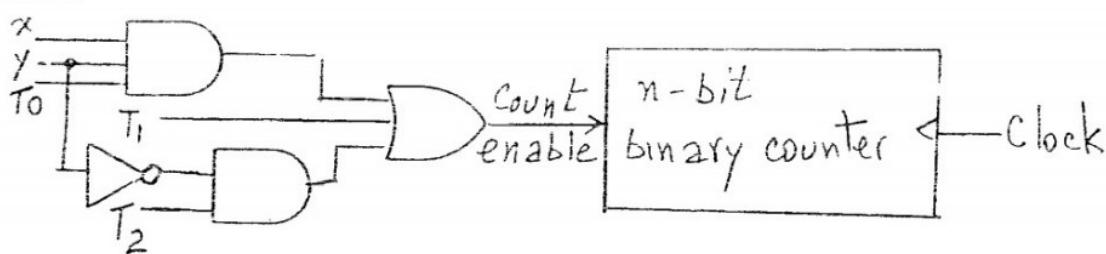
4-7

- (a) Read memory word specified by the address in AR into register R2.
- (b) Write content of register R3 into the memory word specified by the address in AR.
- (c) Read memory word specified by the address in R5 and transfer content to R5 (destroys previous value)

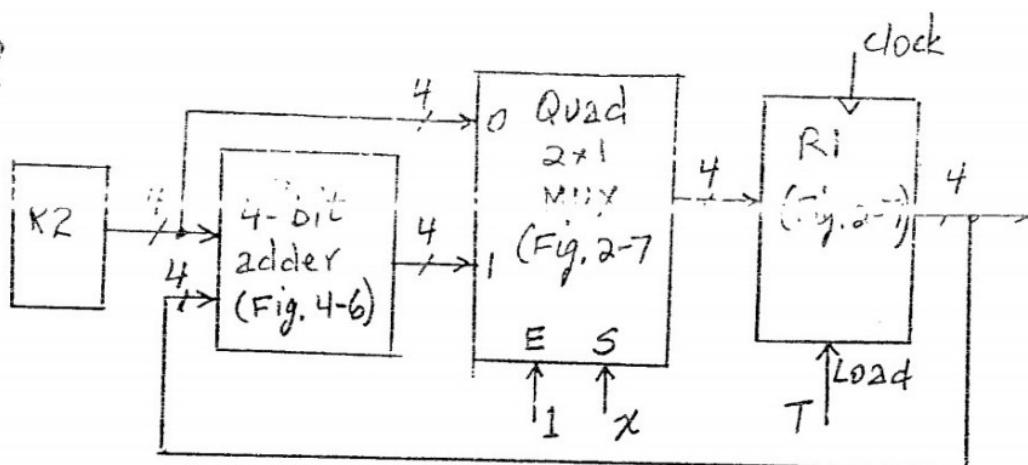
4-8



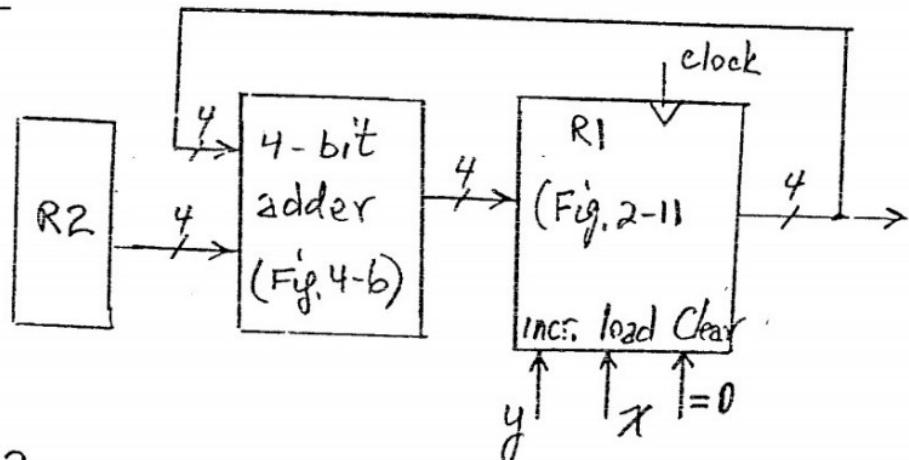
4-9



4-10



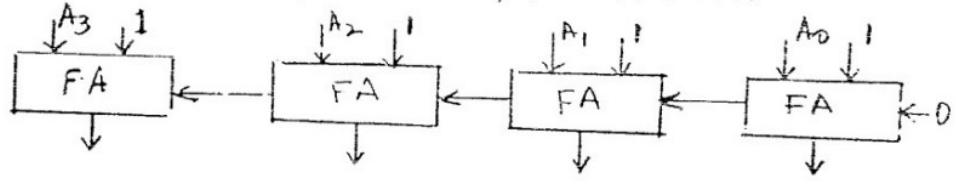
4-11



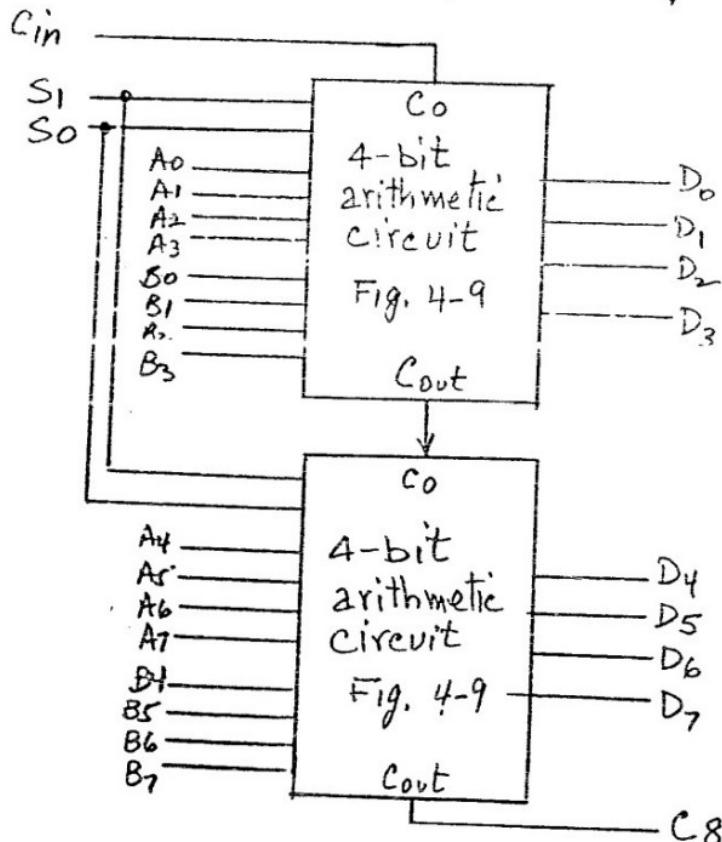
4-12

M	A	B	Sum	C ₄	
0	0111 + 0110		1101	0	7+6 = 13
0	1000 + 1001		0001	1	8+9 = 16+1
1	1100 - 1000		0100	1	12-8 = 4
1	0101 - 1010		1011	0	5-10 = -5 (in 2's comp.)
1	0000 - 0001		1111	0	0-1 = -1 (in 2's comp.)

$$4-13 \quad A-1 = A + 2's \text{ complement of } 1 = A + 1111$$

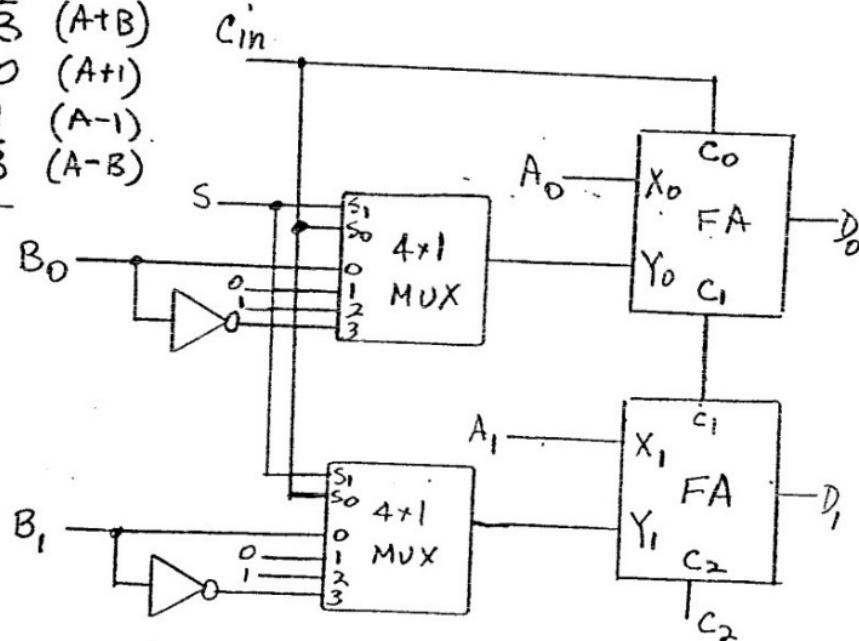


4-14

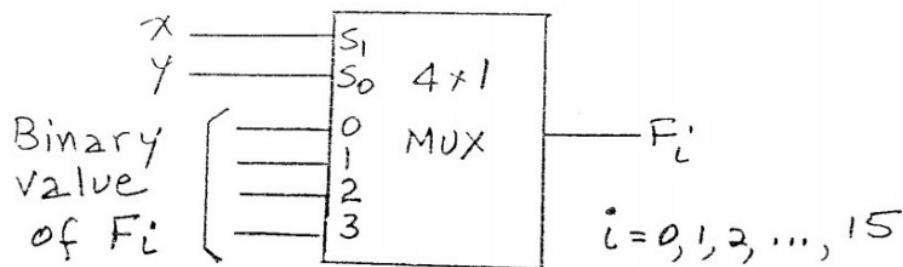


4-15

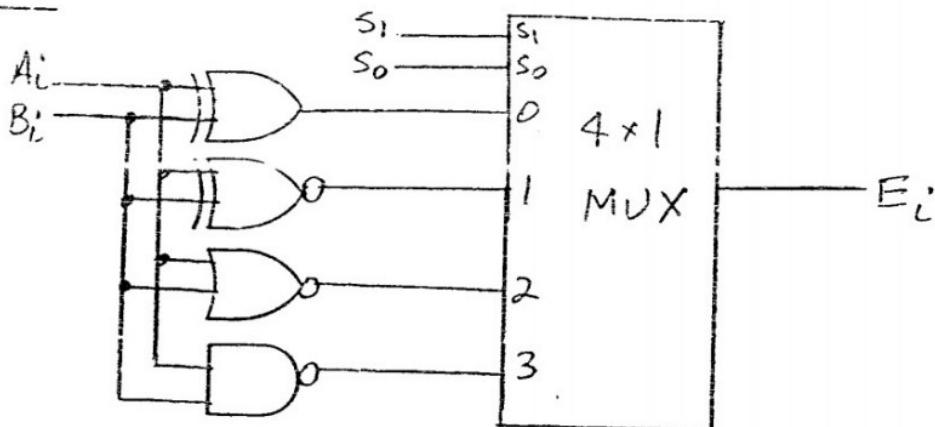
S	Cin	X	Y	
0	0	A	B	$(A+B)$
0	1	A	0	$(A+1)$
1	0	A	1	$(A-1)$
1	1	A	\bar{B}	$(A-B)$



4-16



4-17



4-18

$$(a) \begin{array}{l} A = 11011001 \\ B = 10110100 \\ A \leftarrow A \oplus B \end{array} \quad \frac{B}{01101101}$$

$$\begin{array}{l} A = 11011001 \\ B = 11111101 \text{ (OR)} \\ \hline \end{array} \quad A \leftarrow A \vee B$$

4-19

$$(2) AR = 11110010 \quad BR = 11111111 (+)$$

$$AR = \overline{11110001} \quad BR = 11111111 \quad CR = 10111001 \quad DR = 11101010$$

$$(b) \begin{array}{l} CR = 10111001 \\ DR = 11101010 \quad (\text{AND}) \end{array} \quad BR = 11111111 + 1$$

$$CR = \overline{10101000} \quad BR = \overline{00000000} \quad AR = 11110001 \quad DR = 11101010$$

$$(c) \begin{array}{l} AR = 11110001 (-) \\ CR = 10101000 \end{array}$$

$$AR = \overline{01001001}; \quad BR = 00000000; \quad CR = 10101000; \quad DR = 11101010$$

4-20

$$R = 10011100$$

Arithmetic shift right : 11001110

Arithmetic shift left : 00111000 overflow because a negative number changed to positive

4-21

logical shift left : $\overset{R=11011101}{10111010} \leftarrow$

circular shift right : $\overset{\leftarrow}{01011101}$

logical shift right : $\overset{\rightarrow}{00101110}$

circular shift left : $\overset{\rightarrow}{01011100}$

4-22

$S=1$ Shift left

$A_0 A_1 A_2 A_3 I_L$

$H = \overset{1}{\leftarrow} \overset{0}{\leftarrow} \overset{0}{\leftarrow} \overset{1}{\leftarrow} \overset{0}{\leftarrow}$ shift left ..

4-23

- (a) Cannot complement and increment the same register at the same time,
- (b) Cannot transfer two different values (R_2 and R_3) to the same register ($R1$) at the same time,
- (c) Cannot transfer a new value into a register (PC) and increment the original value by one at the same time,