# VOLUME AS A FUNCTION OF PRICE - AN ALTERNATE APPROACH

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## 1. Introduction

The energy component of Full Requirements (FR) deals is usually a short position in a variable-volume swap, where the volumetric component is typically valued using an actuarial approach. This paper explores an alternative approach where the volume can be approximated as a function of price to come up with a semi-analytical formulation for the arbitrage-free price and Greeks for hedging the swap. This valuation approach allows the exotic swap to be booked as a standard derivative since its value depends only on traded parameters.

## 2. Valuation

Let's define the following variables:

(1) $L$ - Load (Volume)
(2) $P$ - Floating Price
(3) $k$ - Fixed Contract Price
(4) $t$ - Time

In a typical FR deal, we sell $L$ MWhs of power at the fixed contract price $k$ while we buy power at the floating market $P$ price. Setting the expected value of the payoff of this deal to 0, we get

$$E\left[\sum_{m=1}^{M}(k - P_m)L_m\right] = 0 \tag{2.1}$$

where the deal runs for M months. Solving for the fair contract price $k$, we get

$$k = \frac{\sum_{m=1}^{M} E\left[P_m L_m\right]}{\sum_{m=1}^{M} E\left[L_m\right]} = \frac{\sum_{m=1}^{M} E[P_m]E[L_m]exp(\rho_m(\sigma_P)_m(\sigma_L)_m t_m)}{\sum_{m=1}^{M} E\left[L_m\right]} \tag{2.2}$$

Note that this approach requires the computation of the expected load and the load volatility, which is done actuarially. This paper explores an alternative approach where the volume can be approximated as a function of price using a call spread function, which allows us to price the swap and compute its Greeks while eliminating the load parameter. Also note that (2.2) accounts for seasonality by effectively evaluating a $k_m$ for each month and converting that to a single $k$ for


FIGURE 1. Fitting a call spread functional form to all historical load and price data

the term.

2.1. **Functional form of load.** The BGE RES load from the MD-SOS deal has been chosen as an example for this study, given the importance of this load to the FR desk and its highly weather-responsive nature. As has been consistently observed by the team, the load and price are positively correlated, but the correlation tends to be suppressed at extremely low and high prices. A call spread function of the following form was hence chosen to model load as a function of price.

$$L(P) = \begin{cases} L_H, & \text{if } P \geq k_H, \\ L_L + \frac{L_H - L_L}{k_H - k_L}(P - k_L), & \text{if } k_L < P < k_H, \\ L_L, & \text{if } P \leq k_L \end{cases} \tag{2.3}$$

(2.3) can be combined into a single equation as follows:

$$L(P) = N_L + lev(max(P - k_L, 0) - max(P - k_H, 0)) \tag{2.4}$$

where $lev = \frac{L_H - L_L}{k_H - k_L}$. The arbitrage-free strike $k_m$ for a given month can then be solved for as

$$k_m = \frac{E\left[P_m L_m\right]}{E\left[L_m\right]} \tag{2.5}$$

Note that we calculate the $k_m$ for a given month, given that electricity forwards and options trade for a given month. $k_m$ can then be solved as

$$k_m = F_m \frac{(L_L)_m + lev_m CS(\widetilde{F_m})}{(L_L)_m + lev_m CS(F_m)} \tag{2.6}$$

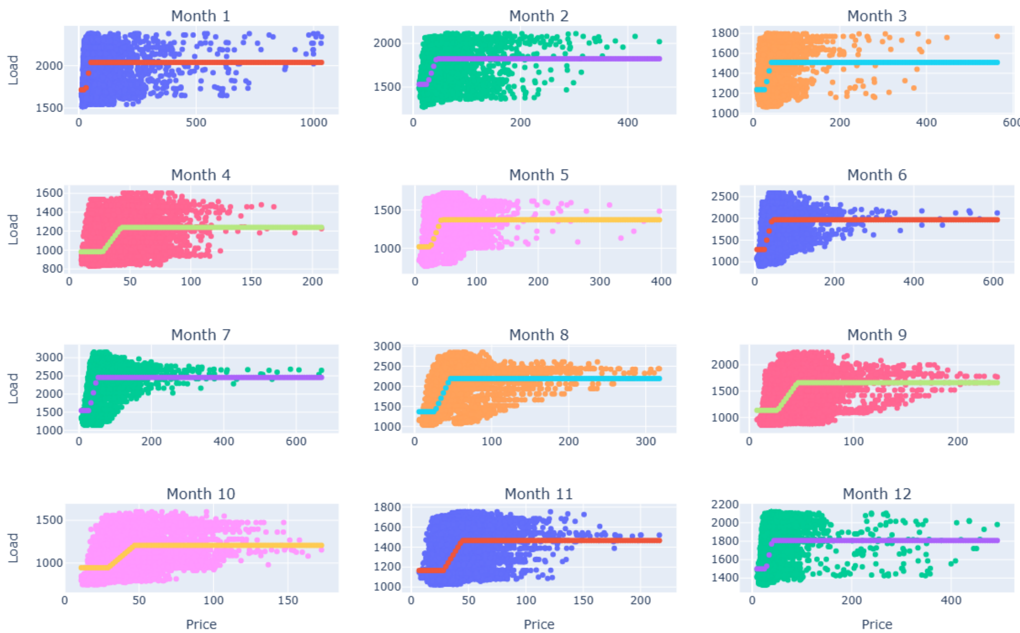


FIGURE 2. Fitting a call spread to data monthly to capture seasonality

where $F_m$ is the forward for month $m$ and $\widetilde{F_m} = F_m exp(\sigma^2 t_m)$, where $\sigma_m$ is the implied volatility for month $m$ and $t_m$ is the time from the evaluation date to the middle of the month in question. $CS$ is the call spread which can be evaluated using the Black-Scholes equation.

The call spreads in 1 and 2 were fitted using the SQSLP (Sequential Least SQuares Programming) optimizer method within the scipy.optimize module, to calculate the optimal parameters $k_L, k_H, L_L, L_H$ for each month, which are then plugged into (2.6) to calculate the $k_m$ for each month. These are then compared to that obtained from FR's MM (MasterModel) as seen in 3. As observed, the results are reasonably similar, with differences averaging to approximately 8%.

## 3. Greeks and Hedging

The $\Delta_m$ of the swap for a given month can be calculated as

$$\Delta_m = k_m.lev_m.\Delta(CS(F_m)) - \Big((L_L)_m + lev_m.CS(F_m) + F_m.lev_m.\Delta(CS(\widetilde{F_m})).exp(\sigma_m^2 t_m)\Big) \tag{3.1}$$

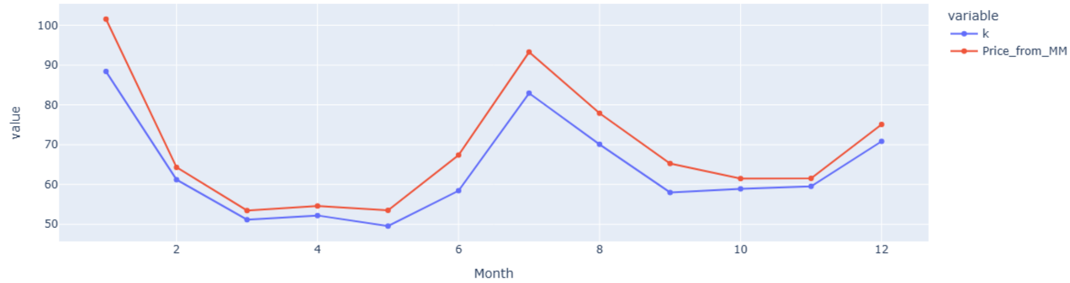


FIGURE 3. Comparison of calculated $k_m$ with MM results

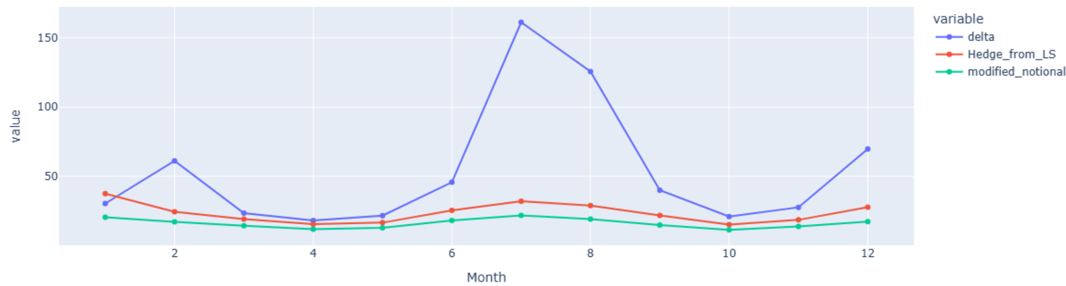


FIGURE 4. Comparison to hedge volumes from LS

The $\Delta$ neutral hedge volumes calculated using (3.1), compared to the expected volumes (labeled as modified notional) and FR's hedge volumes from the Load Summary (LS) can be seen in 4. It is evident that the FR hedge volumes are calculated using actuarial overhedge percentages applied to the expected volume. The $\Delta$ calculated using (3.1) changes dynamically depending on the forward and the implied volatility and would require rebalancing of the portfolio.

3.1. **Conclusion.** Coming up with a fixed price in a variable volume swap (volume can be anything from 0 to $\infty$) involves using historical data. One can also solve this problem using the principles of financial derivatives by describing the volume as a piecewise linear function of the price, defined by parameters. Optimizing these parameters will result in a "mean" functional form. AI can help come up with a more (potentially) accurate and predictive form using current data, such as demand/supply and weather forecasts.

## REFERENCES

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