REVISITING THE LOAD FOLLOWING ADDER

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1. Introduction and Review

The load following adder (LFA) is one of the most nuanced and mathematical concepts involved in pricing a deal on the Full Requirements (FR) desk. It is sometimes interpreted as something we charge to cover the mismatch between the flat hedges we procure and the realized load, but I believe that is an incorrect interpretation. I have come across an interpretation where the LFA is equivalent to the value of an option to follow the load, but still struggle to understand that. However, I do recognize that the formula we're going to come up with is reminiscent of the Black-Scholes formula for pricing a vanilla European call/put option. After having spent some time reviewing the approved methodology paper by risk, I am rewriting the derivation in more detail, in the hope that analysts who are mathematically inclined and curious about LFA, benefit from it.

1.1. Review.

Before delving into the derivation, basic concepts from probability, statistics, linear algebra and stochastic calculus need to be revisited including the following:

(1) Normal Distribution

(2) Lognormal Distribution

(3) Wiener Processes and Geometric Brownian Motion

(4) Ito's Lemma

(5) Cholesky Decomposition

2. Main Derivation

2.1. Risk-neutral valuation.

The Risk Analytics (RA) side of the FR desk prices deals using a risk-neutral valuation approach where the expected value of the future payoff of the derivative is zero. For the sake of simplicity, we can assume that the deal is energy-only with no capacity, ancillary services or RECs. We can also start off by assuming that this is a "1-hour" deal, and then generalize this to a real-world case towards the end of the derivation.

Let's define the following variables

(1) L - Load

(2) P - Floating Price
(3) CP - Fixed Contract Price

In a typical FR deal, we sell L MWhs of power at the fixed contract price CP while we buy power at the floating market P price. The payoff of this "1-hour" deal is $(CP - P)L$

Let's say we want to find the fair contract price CP such that $\mathbb{E}[(CP - P)L]$ is 0. Expanding the term inside the expectation, we get $\mathbb{E}[(CP.L) - (P.L)] = 0$. Note that while P and L are random variables, CP is deterministic.

Hence we get $CP = \frac{\mathbb{E}[\mathbb{P}.\mathbb{L}]}{E[L]}$. It has been statistically observed by the team that P and L tend to be correlated random variables in practice. Hence, by using the definition of covariance, we get

$$CP = \mathbb{E}[P] + \frac{cov(P, L)}{\mathbb{E}[L]} \approx SEP + LFA \tag{2.1}$$

When we extrapolate this oversimplified "1-hour" deal to a more general case, the first term is known as the shaped-energy price (SEP), which simply put, is the load-weighted average price of energy. The second term, is the LFA, which is the main topic of discussion for this paper. Calculating the covariance of price and load would require us to make assumptions on the nature of the these variables which we shall do in the subsequent section.

2.2. **Geometric Brownian Motion.** The risk-approved methodology to calculate the LFA assumes that price and load follow a correlated geometric Brownian motion. While this is a fairly standard model for stock prices, applying this to electricity prices and load is something worth researching further. Nevertheless, it is something the team has gotten comfortable with over the years, and will form the basis for this paper as well.

Using the definitions of geometric Brownian motion, we get

$$\frac{dP}{P} = \mu_P dt + \sigma_P dz_1 \tag{2.2}$$

$$\frac{dL}{L} = \mu_L dt + \sigma_L dz_2 \tag{2.3}$$

where μ_L and μ_P are the means of the return on load and price, while σ_L and σ_P are the standard deviations of the return on load and price respectively. dz_1 and dz_2 are Wiener processes with a correlation of ρ. It is important to note that load and price volatility are, by definition, the standard deviations of the return on load and price, and not load and price itself. The discretized versions of (2.2) and (2.3) give us

$$\frac{\delta P}{P} = \mu_P \delta t + \sigma_P \sqrt{\delta t} \phi_1 \tag{2.4}$$

$$\frac{\delta L}{L} = \mu_L \delta t + \sigma_L \sqrt{\delta t} \phi_2 \tag{2.5}$$

where ϕ_1 and ϕ_2 are standard normal variables with a correlation of ρ. Note that over a period δt, the mean of the return on (say) load is $\mu_L \delta t$ while the variance of the return on load is $\sigma_L^2 \delta t$. This arises as a consequence of the Markov property of geometric Brownian motion, where the future state of the process is only dependent on the current state and independent of history. Since the variance per unit time is σ_L^2, the variance over a period δt is $\sigma_L^2 \delta t$ (since the variance of the sum of independent random normal variables is the sum of the variances of the variables) and consequently, the standard deviation over a period δt is $\sigma_L \sqrt{\delta t}$.

Applying Ito's Lemma to (2.2) and (2.3) leads us to the lognormal property of load and prices where

$$\log \frac{P_t}{P_o} \sim N\left((\mu_P - \frac{\sigma_P^2}{2})t, \sigma_P \sqrt{t} \right) \tag{2.6}$$

$$\log \frac{L_t}{P_o} \sim N\left((\mu_L - \frac{\sigma_L^2}{2})t, \sigma_L \sqrt{t} \right) \tag{2.7}$$

Although a complete review of the theoretical underpinnings of geometric Brownian motion is beyond the scope of this document, the above equations should be sufficient to help us derive the equation for the LFA. For more depth in understanding, I highly recommend reviewing the chapters on Wiener processes and stock price modeling in [1] .

2.3. **Cholesky Decomposition.** The Cholesky decomposition method allows us to generate correlated random variables. Given a vector of independent random variables $X = [\epsilon_1, \epsilon_2]^T$, Cholesky decomposition creates a new vector Z of correlated random variables as follows,

$$Z = LX \tag{2.8}$$

and

$$L = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{bmatrix} \tag{2.9}$$

which gives us

$$Z = \begin{bmatrix} \epsilon_1 \\ \rho\epsilon_1 + \sqrt{1-\rho^2}\epsilon_2 \end{bmatrix} \tag{2.10}$$

2.4. **Calculating the covariance.** Now that we've established the foundation for the derivation, we can go ahead and express P and L as functions of time, and calculate their covariance and ultimately the LFA. Expressing load and price as a function of time using (2.6), (2.7) and applying Cholesky decomposition using (2.10), we get

$$P_t = P_0 \exp\left((\mu_p - \frac{-\sigma_P^2}{2})t + \sigma_P\sqrt{t}\epsilon_1\right) \tag{2.11}$$

and

$$L_t = L_0 \exp\left((\mu_L - \frac{-\sigma_L^2}{2})t + \sigma_L\sqrt{t}(\rho\epsilon_1 + \sqrt{1-\rho^2}\epsilon_2)\right) \tag{2.12}$$

Note that

$$E[P_t] = P_0 \exp(\mu_P t) \tag{2.13}$$

and

$$E[L_t] = L_0 \exp(\mu_L t) \tag{2.14}$$

The covariance of P and L is now calculated, by definition, as

$$cov(P_t, L_t) = E[P_tL_t] - E[P_t]E[L_t] \tag{2.15}$$

Since P and L are both functions of time, the covariance will also be a function of time. Substituting (2.11) and (2.12) into $E[P_tL_t]$ in (2.15), by making use of the fact that ϵ_1 and ϵ_2 are independent random variables and grouping terms, we get

$$\begin{aligned} E[P_tL_t] = P_0L_0 \exp((\mu_p + \mu_L)t) \exp\left(\frac{-\sigma_P^2 t - \sigma_L^2 t}{2}\right) E\left[\exp((\sigma_P\sqrt{t} + \rho\sigma_L\sqrt{t})\epsilon_1)\right] \\ E\left[\exp(\sqrt{1-\rho^2}\sigma_L\sqrt{t}\epsilon_2)\right] \end{aligned} \tag{2.16}$$

Since ϵ_1 and ϵ_2 are standard normal random variables, the terms inside the exponentials in (2.16) are normal random variables, which also implies that the exponentials of these are lognormal variables, by definition. Recalling that the expectation of a lognormal distribution with parameters μ and σ is $\exp(\mu + \frac{\sigma^2}{2})$, we get

$$E[P_tL_t] = P_0L_0 \exp\left((\mu_p+\mu_L)t\right) \exp\left(\left(\frac{-\sigma_P^2}{2}+\frac{-\sigma_L^2}{2}\right)t\right) \exp\left(\frac{(\sigma_P\sqrt{t}+\rho\sigma_L\sqrt{t})^2}{2}\right) \exp\left(\frac{(\sqrt{1-\rho^2}\sigma_L\sqrt{t})^2}{2}\right) \tag{2.17}$$

Expanding and simplifying (2.17), we get

$$E[P_tL_t] = P_0L_0 \exp\left((\mu_p+\mu_L)t\right) \exp\left(\rho\sigma_P\sigma_L t\right) \tag{2.18}$$

Substituting (2.18), (2.13) and (2.14) in (2.15), we get

$$cov(P_t,L_t) = P_0L_0 \exp\left((\mu_p+\mu_L)t\right)\left(\exp\left(\rho\sigma_P\sigma_L t\right)-1\right) = E[P_t]E[L_t]\left(\exp\left(\rho\sigma_P\sigma_L t\right)-1\right) \tag{2.19}$$

Substituting (2.19) in (2.1), we get

$$LFA = \frac{cov(P_t,L_t)}{E[L_t]} = \frac{E[P_t]E[L_t]\left(\exp\left(\rho\sigma_P\sigma_L t\right)-1\right)}{E[L_t]} = E[P_t]\left(\exp\left(\rho\sigma_P\sigma_L t\right)-1\right) \tag{2.20}$$

Note that the correlation and the load volatility is something we calculate internally, while we get the expected value of the future price from the futures/forwards desk and the price volatility from the options desk. I presume that the option traders are able to calculate the implied volatilities based on what the options are trading at. They are probably not using the standard Black-Scholes equation, but some variant of it adapted to the more exotic electricity options that the FR desk trades in.

2.5. **Load Volatility.** On this desk, we calculate load volatility by the approach described below:

We first calculate the variance of load, $var(L_t)$, and the expected load based on our forecast, $E[L_t]$, and then use these variables to calculate the load volatility, which as mentioned earlier, is the standard deviation of the return on load. Using (2.7) and the properties of a lognormal distribution, we get

$$var[L_t] = (E[L_t])^2\left(\exp\left(\sigma_L^2 t\right)-1\right) \tag{2.21}$$

Solving for σ_L in (2.21), we get

$$\sigma_L = \sqrt{\frac{1}{t}\log\left(\frac{var[L_t]}{(E[L_t])^2}+1\right)} \tag{2.22}$$

This is the equation used to compute the load volatility from the standard deviation and expectation of load. Note that in practice, $\frac{var[L_t]}{(E[L_t])^2}$ tends to be lesser than 1. If we use the Taylor series expansion about 0 of $log(1+x) \approx x$ by ignoring the higher order terms, we can simplify (2.22) as

$$\sigma_L \approx \frac{1}{\sqrt{t}}\frac{stdev(L_t)}{E[L_t]} \tag{2.23}$$

Note that I am explicitly stating the standard deviation of L_t and not using the symbol σ_L as the latter is the standard deviation of the return on load and not load itself. With all of these variables calculated, one could plug them into (2.1) and calculate the contract price that we should charge, such that on average, we make no money on the deal. The term that the team uses for this is the P-50 price. Strictly speaking, the P-50 value of a distribution is its median, but the team uses the term interchangeably with the mean.

2.6. **Extrapolating to a real deal.** Congratulations for following along thus far! We have already covered the bulk of the derivation, and all that remains is to extrapolate this to a real-world deal.

Let us say we first want to find the fair contract price CP for a deal that runs for M months and that we split each month into B blocks (say WD1, WD2, WD3, WD4, WE1... for example). The payoff from this deal is $\sum_{m=1}^{M}\sum_{b=1}^{B}(CP - P_{b,m})L_{b,m}$. By the principle of risk-neutral valuation, setting the expected payoff to 0, we get

$$E\left[\sum_{m=1}^{M}\sum_{b=1}^{B}(CP - P_{b,m})L_{b,m}\right] = 0 \tag{2.24}$$

Rearranging the terms and by applying linearity of expectation to (2.24), we can solve for the contract price CP as follows:

$$CP = \frac{\sum_{m=1}^{M}\sum_{b=1}^{B} E\left[P_{b,m}L_{b,m}\right]}{\sum_{m=1}^{M}\sum_{b=1}^{B} E\left[L_{b,m}\right]} \tag{2.25}$$

Using the definition of covariance, (2.25) can be rewritten as:

$$CP = \frac{\sum_{m=1}^{M}\sum_{b=1}^{B} E[P_{b,m}]E[L_{b,m}]}{\sum_{m=1}^{M}\sum_{b=1}^{B} E\left[L_{b,m}\right]} + \frac{\sum_{m=1}^{M}\sum_{b=1}^{B} cov(P_{b,m}, L_{b,m})}{\sum_{m=1}^{M}\sum_{b=1}^{B} E\left[L_{b,m}\right]} = SEP + LFA \tag{2.26}$$

As before, the first term is the SEP and the second term is the LFA. Note that this time though, to calculate the SEP, we would need to shape the forward price for each month into each block and forecast the load for each block of each month. To calculate the LFA, we can substitute (2.19) into (2.26) as follows:

$$LFA = \frac{\sum_{m=1}^{M}\sum_{b=1}^{B} cov(P_{b,m}, L_{b,m})}{\sum_{m=1}^{M}\sum_{b=1}^{B} E\left[L_{b,m}\right]} = \frac{\sum_{m=1}^{M}\sum_{b=1}^{B} E[P_{b,m}]E[L_{b,m}]((exp(\rho_{b,m}(\sigma_P)_{b,m}(\sigma_L)_{b,m}t) - 1)}{\sum_{m=1}^{M}\sum_{b=1}^{B} E\left[L_{b,m}\right]} \tag{2.27}$$

We're almost there! The only piece of information we have glossed over is the time value of money. Strictly speaking, the present value of the expected payoff in (2.24) should be set to 0, by discounting the cash flows at the appropriate rates. This leads to a minor modification to (2.27) and the final formula for LFA as follows:

$$LFA = \frac{\sum_{m=1}^{M}\sum_{b=1}^{B} D_m E[P_{b,m}]E[L_{b,m}]((exp(\rho_{b,m}(\sigma_P)_{b,m}(\sigma_L)_{b,m}t) - 1)}{\sum_{m=1}^{M}\sum_{b=1}^{B} D_m E\left[L_{b,m}\right]} \tag{2.28}$$

where D_m is the appropriate discounting factor. On our desk, we assume continuous compounding which leads to a form $\exp(-rt)$, where r is the risk-free interest rate plus a spread based on our credit rating. This concludes the derivation of the formula for the LFA. It is important to note that we only get ON-peak hub volatilities from the options desk, and use price volatility multipliers to transform these into monthly zonal volatilities for the different time blocks.

2.7. **Price-Load Correlation.** The term ρ in (2.9) is the correlation between ϕ_1 and ϕ_2 in (2.4) and (2.5). However, we need to come up with a way to express the correlation as a function of load and price in order to be able to calculate it statistically. Since ϕ_1 and ϕ_2 are standard normal variables, their correlation equals their covariance. Using this fact, rearranging and solving for ϕ_1 and ϕ_2 in (2.4) and (2.5), we get

$$\rho = cov(\phi_1, \phi_2) = cov\left(\frac{\log\frac{P_t}{P_0} - \mu_P t + \frac{\sigma_P^2}{2}t}{\sigma_P\sqrt{t}}, \frac{\log\frac{L_t}{L_0} - \mu_L t + \frac{\sigma_L^2}{2}t}{\sigma_L\sqrt{t}}\right) \tag{2.29}$$

Simplifying this, we observe that the constants drop off and we end up with $\rho = correl(\log P_t, \log L_t)$. which explains why the team currently estimates correlations between the natural logarithm of load and price.

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References

1. Hull, John, 1946-. Options, Futures, and Other Derivatives. Boston: Prentice Hall, 2012.