MATHEMATICS

SECTION A

January 4, 2024

1 Real Numbers

1. Prove that $2 + 5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.
2. Using Euclid's Algorithm, find the HCF of 2048 and 960.
3. If HCF $(336, 54) = 6$, find LCM $(336, 54)$.
4. Write the smallest number which is divisible by both 306 and 657.

2 Algebra

5. For what value of k, is the polynomial $f(x) = 3x^4 - 9x^3 + x^2 + 15x + k$ completely divisible by $3x^2 - 5$?
6. The total cost of a certain length of a piece of cloth is ₹200. If the piece was $5m$ longer and each metre of cloth costs ₹2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre ?
7. Sumit is 3 times as old as his son. Five years later, he shall be two and a half times as old as his son. How old is Sumit at present ?
8. In a class test, the sum of Arun's marks in Hindi and English is 0. Had he got 2 marks more in Hindi and 3 marks less in English, the product of the marks would have been 210. Find his marks in the two subjects.
9. Find the value(s) of k so that the pair of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has a unique solution.
10. For what value of k, will the following pair of equations have infinitely many solutions :

$$2x + 3y = 7 \text{ and } (k + 2)\,x - 3\,(1 - k)\,y = 5k + 1$$

11. Find the nature of the roots of the quadratic equation

$$4x^2 + 4\sqrt{3}x + 3 = 0$$

12. Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained.
13. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.
14. Solve for x :

$$x^2 + 5x - \left(a^2 + a - 6\right) = 0$$

15. Find the nature of roots of the quadratic equation $2x^2 - 4x - 3 = 0$.

16. Find the 21^{st} term of the A.P. $-4\frac{1}{2}, -3, -1\frac{1}{2}, ...$

17. Find the common difference of the Arithmetic Progression (A.P.)

$$\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, ..(a \neq 0)$$

18. Which term of the Arithmetic Progression $-7, -12, -17, -22, ...$ will be -82 ? Is -100 any term of the A.P. ? Give reason for your answer.

19. How many terms of the Arithmetic Progression $45, 39, 33, ...$must be taken so that their sum is 180 ? Explain the double answer.

3 Coordinate Geometry

20. Write the coordinates of a point P on x-axis which is equidistant from the points $A(-2, 0)$ and $B(6, 0)$.

21. Find a relation between x and y if the points $A(x, y)$, $B(-4, 6)$ and $C(-2, 3)$ are collinear.

22. Point **A** lies on the line segment **XY** joining $X(6, -6)$ and $Y(-4, -1)$ in such a way that $\frac{XA}{XY} = \frac{2}{5}$. If point A also lies on the line $3x + k(y + 1) = 0$, find the value of k.

23. Find the ratio in which the y-axis divides the line segment joining the points $(-1, -4)$ and $(5, -6)$. Also find the coordinates of the point of intersection.

24. Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points $(-2, -5)$ and $(6, 3)$. Find the coordinates of the point of intersection.

4 Geometry

25. Find A and B if $\sin(A + 2B) = \frac{\sqrt{3}}{2}$ and $\cos(A + 4B) = 0$, where A and B are acute angles.

26. In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite the first side is a right angle.

27. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides.

28. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

29. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that $AP \times PC = BP \times DP$.

30. Find the area of a triangle whose vertices are given as $(1, -1)$, $(-4, 6)$ and $(-3, -5)$.

31. In Figure 1, ABC is an isosceles triangle right angled at C with $AC = 4cm$. Find the length of AB.

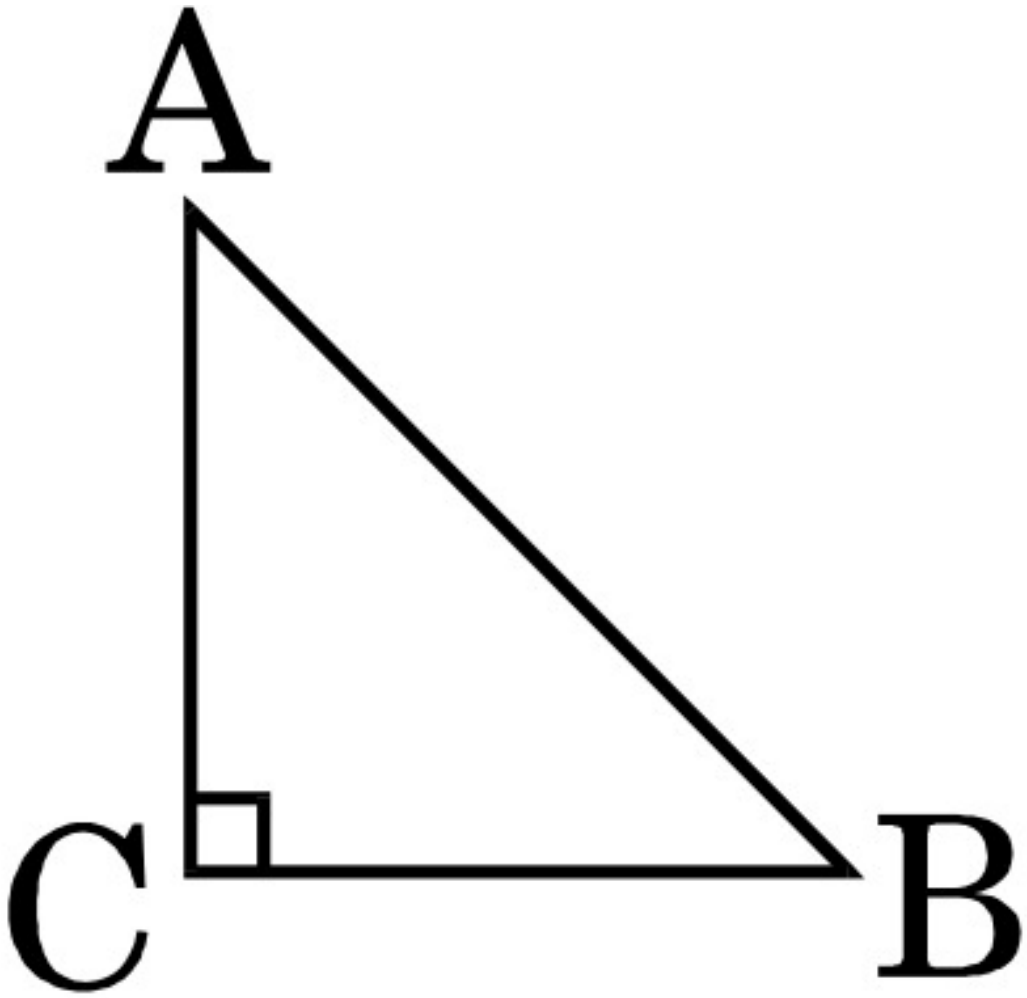

Figure 1: Triangle ABC

32. In Figure 2, $DE \parallel BC$. Find the length of side AD, given that $AE = 1.8cm$, $BD = 7.2cm$ and $CE = 5.4cm$.

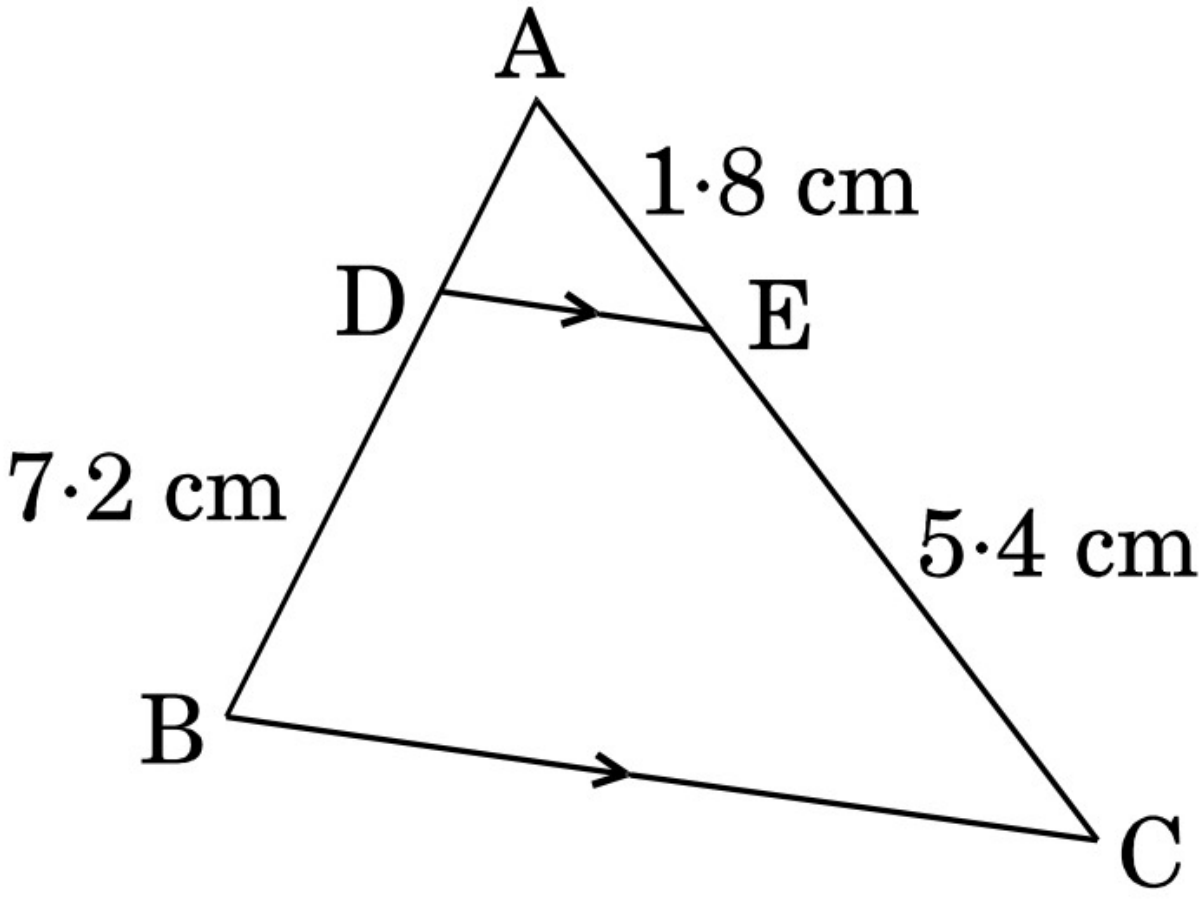

Figure 2: Triangle ABC

33. In Figure 3, PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting PQ at A and RS at B. Prove that $\angle AOB = 90°$

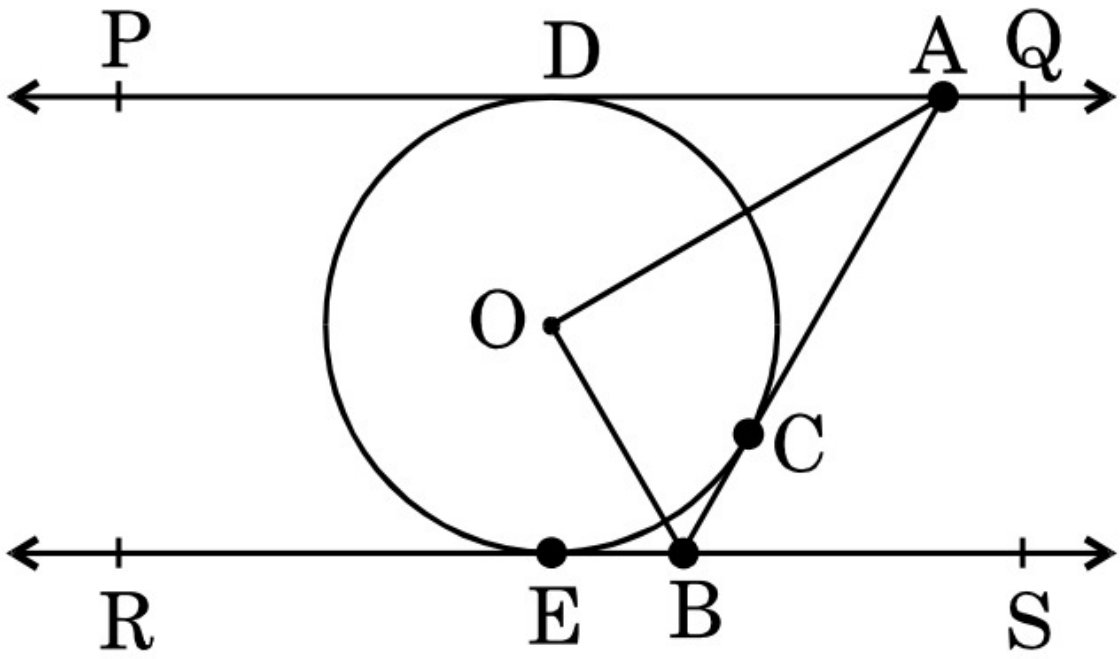

Figure 3: Tangent and Circle

34. In Figure 4, a square $OABC$ is inscribed in a quadrant $OPBQ$. If $OA = 15cm$, find the area of the shaded region. ($Use\ \ \pi = 3.14$)

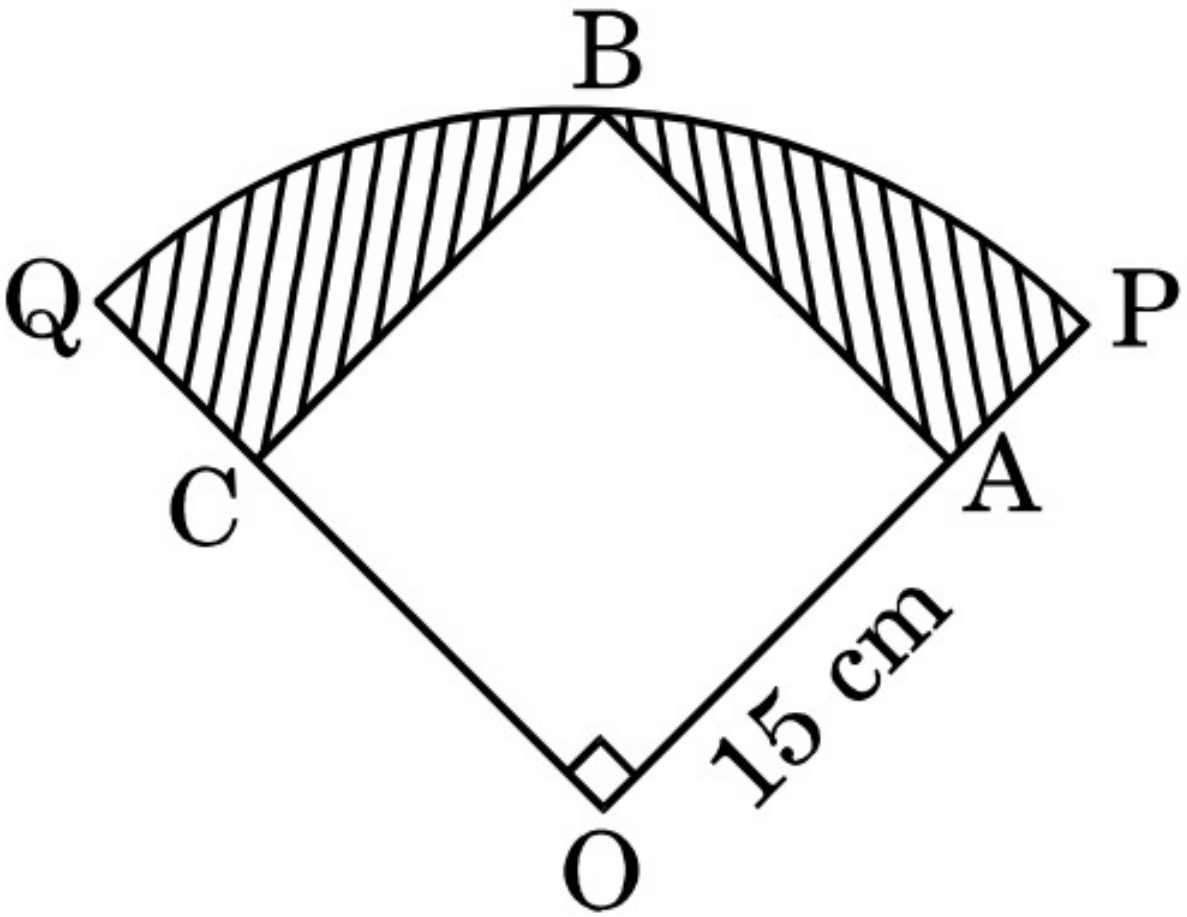

Figure 4: Square $OABC$

35. In Figure 5, $ABCD$ is a square with side $2\sqrt{2}cm$ and inscribed in a circle. Find the area of the shaded region. ($Use\ \ \pi = 3.14$).

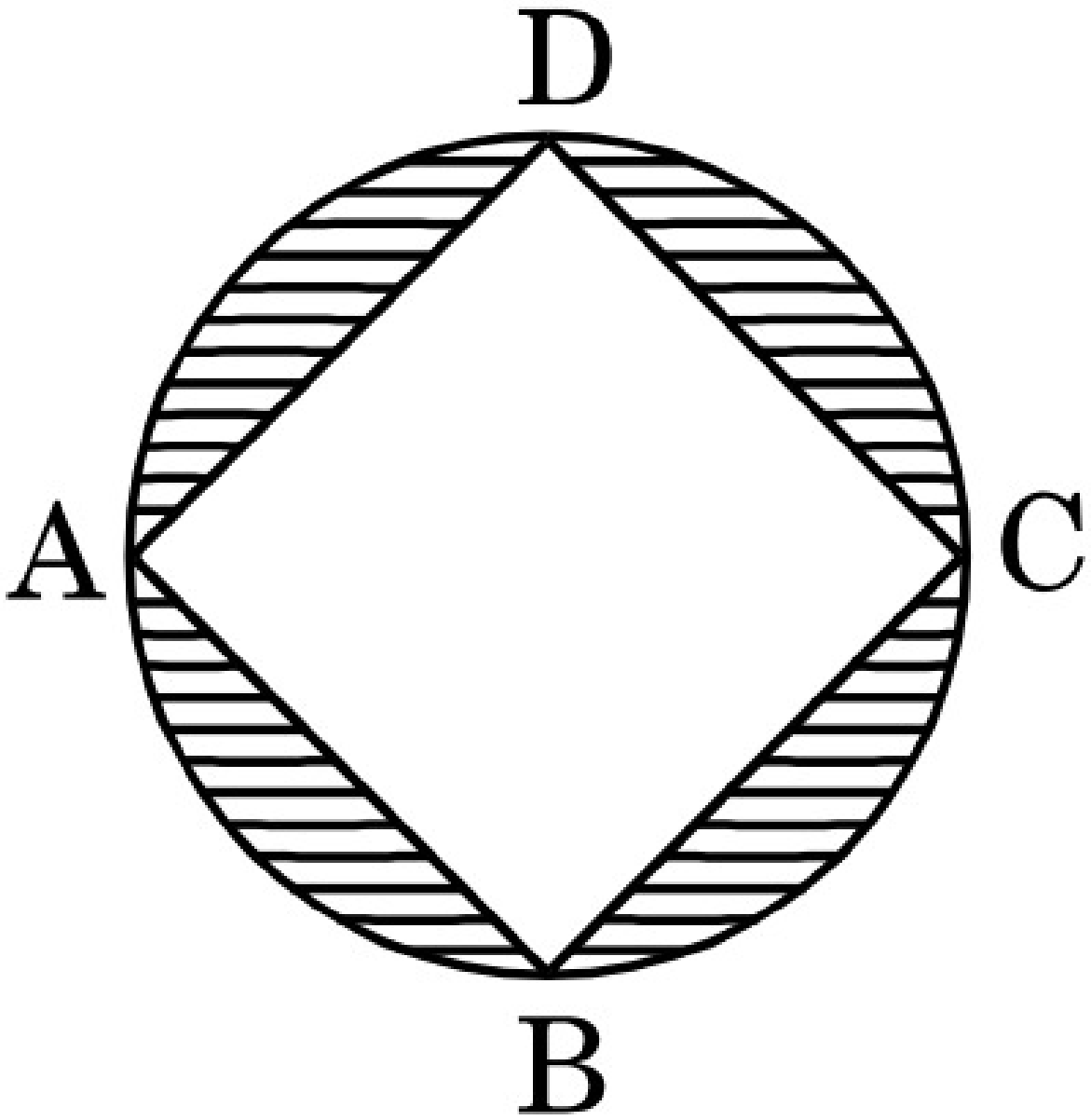

Figure 5: Square $ABCD$

36. Draw two concentric circles of radii $2cm$ and $5cm$. Take a point P on the outer circle and construct a pair of tangents PA and PB to the smaller circle. Measure PA.

37. Construct an equilateral $\triangle ABC$ with each side $5cm$. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of $\triangle ABC$.

38. Diagonals of a trapezium $PQRS$ intersect each other at the point O, $PQ \parallel RS$ and $PQ = 3RS$. Find the ratio of the areas of triangles POQ and ROS.

5 Trignometry

39. The larger of two supplementary angles exceeds the smaller by $18°$. Find the angles.

40. If $\sin A = \frac{3}{4}$, calculate $\sec A$.

41. Two poles of equal heights are standing opposite to each other on either side of the road which is $80m$ wide. From a point P between them on the road, the angle of elevation of the top of a pole is $60°$ and the angle of depression from the top of the other pole of point P is $30°$. Find the heights of the poles and the distance of the point P from the poles.

42. Amit, standing on a horizontal plane, finds a bird flying at a distance of $200m$ from him at an elevation of $30°$. Deepak standing on the roof of a $50m$ high building, finds the angle of elevation of the same bird to be $45°$. Amit and Deepak are on opposite sides of the bird. Find the distance of the bird from Deepak.

43. From a point P on the ground, the angle of elevation of the top of a tower is $30°$ and that of the top of the flag-staff fixed on the top of the tower is $\sqrt{5}$. If the length of the flag-staff is $5m$, find the height of the tower. ($Use\ \sqrt{3} = 1.732$).

44. Evaluate :

$$\sin^2 60° + 2\tan 45° - \cos^2 30°.$$

45. Evaluate :

$$\left(\frac{3\tan 41°}{\cot 90°}\right)^2 - \left(\frac{\sin 3° \sec 55°}{\tan 10° \tan 20° \tan 60° \tan 70° \tan 80°}\right)^2$$

46. Prove that :

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\csc\theta$$

47. Prove that :

$$\frac{\sin\theta}{\cot\theta + \csc\theta} = 2 + \frac{\sin\theta}{\cot\theta - \csc\theta}$$

48. Evaluate :

$$\left(\frac{3\sin 43°}{\cos 47°}\right)^2 - \frac{\cos 37° \csc 53°}{\tan 5° \tan 25° \tan 45° \tan 65° \tan 85°}$$

6 Mensuration

49. A solid iron pole consists of a cylinder of height $220cm$ and base diameter $24cm$, which is surmounted by another cylinder of height $60cm$ and radius $8cm$. Find the mass of the pole, given that $1cm^3$ of iron has approximately $8gm$ mass. ($use\ \ \pi = 3.14$)

50. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is $20cm$ and the diameter of the cylinder is $7cm$. Find the total volume of the solid. ($use\ \ \pi = \frac{22}{7}$).

51. Two spheres of same metal weigh $1kg$and $7kg$. The radius of the smaller sphere is $3cm$. The two spheres are melted to form a single big sphere. Find the diameter of the new sphere.

52. A right cylindrical container of radius $6cm$ and height $15cm$ is full of ice-cream, which has to be distributed to 10 children in equal cones having hemispherical shape on the top. If the height of the conical portion is four times its base radius, find the radius of the ice-cream cone.

7 Probability

53. A die is thrown twice. Find the probability that

(a) 5 will come up at least once .

(b) 5 will not come up either time .

54. The probability of selecting a blue marble at random from a jar that contains only blue, black and green marbles is $\frac{1}{5}$. The probability of selecting a black marble at random from the same jar is $\frac{1}{4}$. If the jar contains 11 green marbles, find the total number of marbles in the jar.