Statistics - 5

① 1. $H_0: \mu = 25$, $H_1: \mu \neq 25$ ✓ correctly stated

2. $H_0: \sigma > 10$, $H_1: \sigma = 10$ ✗ not correct

3. $H_0: \bar{x} = 50$, $H_1: \bar{x} \neq 50$ ✓ correctly stated

4. $H_0: P = 0.1$, $H_1: P = 0.5$ ✗ not correct

5. $H_0: S = 30$, $H_1: S > 30$ ✗ not correct

1 is correct because μ (population mean) is specified with $=$ and $\neq$

2 is not correct because σ should be in alternate hypothesis

3 is again correct because $\bar{x}$ (simple mean) is mentioned on 50 with $=$ & $\neq$

4 is not correct because P values (Probability) are different in H_0 and H_1

5 is not correct because we don't specify hypothesis testing on sample size

② $\mu = 52$ Avg cost of text book

$\sigma = 4.5$ Std deviation

$n = 100$ Sample size

$\bar{x} = 52.8$

5% significance

$$Z_{\alpha} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

2 tail test 5%

$$\left(\bar{X} - 1.96\frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96\frac{\sigma}{\sqrt{n}}\right)$$

Hypothesis testing

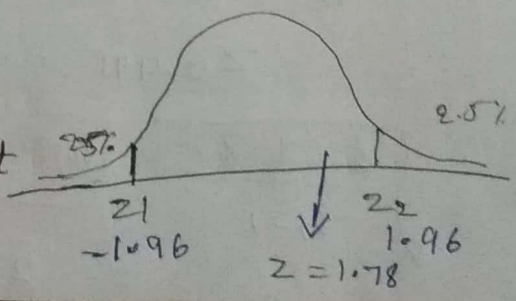
Null Hypothesis: $H_0: \mu = 52$

Alternate Hypothesis: $H_1: \mu \neq 52$

5% significance

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{52.8 - 52.0}{\frac{4.5}{\sqrt{100}}} = \frac{0.8}{0.45} = 1.78$$

Since $z = 1.78$ falls in range of -1.96 to 1.96 i.e in the acceptance region, we accept null hypothesis, i.e Avg cost of text book is $52



(3) chemical pollutant mean $\mu = 34$ ppm

Standard deviation $\sigma = 8$ ppm

level of significance = 1%

$H_0: \mu = 34$

$H_1: \mu \neq 34$

Sample size $n = 50$

Sample mean $\bar{x} = 32.5$ ppm

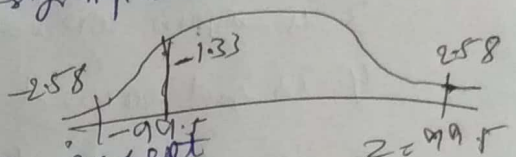
$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{32.5 - 34}{\frac{8}{\sqrt{50}}} = \frac{-1.5}{\frac{8}{5\times\sqrt{2}}} = \frac{-1.5}{1.13} = -1.33$$

(1.414)

critical value of z with 1% significance is

$-2.58 < z < 2.58$

-1.33



since z falls in the given range, we accept null hypothesis.

Hence company's claim of lowering pollution is true.

(4) Avg dental expenditure $\mu = 1135$ $

$n = 22$ no of families of size 4

22 families expenditure $\alpha = 0.5$

1008, 812, 1117, 1323, 1308, 1415, 831, 1021, 1287, 851, 930, 730, 699, 872, 913, 944, 954, 987, 1695, 995, 1003, 994

$H_0: \mu = 1135$

$H_1: \mu \neq 1135$

Average of 22 families with respondents $= \frac{22,869}{22}$

$= 1037.32$

$z_{0.995} = 2.58$

$$S = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n-1}} = \sqrt{\frac{(1008 - 1031.32)^2 + (812 - 1037.2)^2 + \cdots + (994 - 1037.2)^2}{22-1}}$$

$= 240.37$

Avg warehouse cost per sft $\mu = \$32.28$

$n = 19$

$\bar{X} = \$31.67$

$\sigma = \$1.29$

5% significance

$$-z_{97.5} < z < z_{97.5}$$

$$-1.96 < z < 1.96$$

$H_0 : \mu = \$32.28$

$H_1 : \mu \neq \$32.28$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{31.67 - 32.28}{1.29/\sqrt{19}} = \frac{-0.61}{0.29} = -2.1$$

$-1.96 \not< -2.1 < 1.96$

z is not in acceptance region, i.e falls in rejection region, Hence we reject null hypothesis. Hence the avg cost per sft is not $32.28 and it is changed now

$-2.58 < z < z_{99.5}$

$z_{-99.5} < z < z_{99.5}$

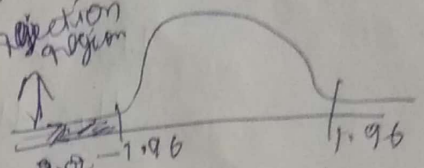
~~z falls in acceptance~~

$\alpha = 0.5, \quad \frac{\alpha}{2} = 0.025$

$z_{.025} = 1.96$

$-1.96 \neq z \neq 1.96$

-2.02



since z does not lie in acceptance region, we reject null hypothesis.

Hence average dental expense of $1135 is not correct

(5) Avg family income $\mu = \$48,432$

$n = 400$

$\bar{x} = \$48,574$

$\sigma = \$2000$

null hypothesis $H_0: \mu = \$48,432$

$H_1: \mu \neq \$48,432$

significance level of 10%.

$\alpha = 10, \quad \alpha/2 = 5\%. \quad z_{95\%} = 1.645$

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{48,574 - 48,432}{2000/\sqrt{400}} = \frac{142}{100} = 1.42$$

$z_{critical} = 1.645$

$z_{-95\%}$ [-1.645] $< z <$ $z_{95\%}$ [1.645]

[1.42]

$z = 1.42$ falls in acceptance region, Hence accept null hypothesis. Hence the given report is Valid

(6) Avg warehouse cost per sft $\mu = \$32.28$

$n = 19$

$\bar{x} = \$31.67$

$\sigma = \$1.29$

5% significance

$$z_{-97.5} < z < z_{+97.5}$$

$$-1.96 < z < 1.96$$

$H_0 : \mu = \$32.28$

$H_1 : \mu \neq \$32.28$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{31.67 - 32.28}{1.29/\sqrt{19}} = \frac{-0.61}{0.29} = -2.1$$

$-1.96 \not< -2.1 < 1.96$

z is not in acceptance region, i.e falls in rejection region, Hence we reject null hypothesis. Hence the avg cost per sft is not \$32.28 and it is changed now

(8) $n = 16,\ \mu = 10,\ \bar{x} = 12,\ S = 1.5$

$$t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{12 - 10}{\frac{1.5}{\sqrt{16}}} = 5.33$$

(9) degree of freedom $= n - 1$
$= 16 - 1 = 15$

at 5% significance on two tailed test

$$t_{15}\ P > 0.05 = 2.131$$

at 99% significance of one tailed test

$$t_{15}\ P \geq 0.01 = 2.602$$

(10) $n = 25,\ \mu = 60,\ S = 4,$

$df = 25 - 1 = 24$

$t_{95\%}$

$$t = \frac{\bar{x} - 60}{\frac{4}{\sqrt{25}}} = \frac{\bar{x} - 60}{\frac{4}{5}} = \frac{(\bar{x} - 60) \times 5}{4} \simeq 95?$$

95

$1 - (0.1 + 0.05) = 0.985$

$\bar{x} - 60 \times \frac{5}{4} = 2.064$

$\bar{x} = 61.652$

95% 25 df

degree $= 24$

$t_{0.05} < t < t_{0.10}$

$s = 0.981$

degree $= 24$

$1.711 < t < 2.064$

$t > 0.255$

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