Assignment 6 - EE2703 Applied Programming Lab

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1 Overview

In this assignment we, use the *scipy.signal* module to solve LTI systems. We solve the damping oscillator problem, coupled springs problem and the RLC circuit finally.

2 Code and Generated Outputs

Importing required libraries.

```
In [1]: from pylab import *
        import scipy.signal as sp
```

2.1 Damped Oscillator

We are given :

$$f(t) = cos(1.5t) * exp(-0.5t) * u_0(t),$$

So,

$$F(s) = \frac{s+0.5}{(s+0.5)^2 + 2.25}$$

We have our damping equation as :

$$\ddot{x} + 2.25x = f(t)$$

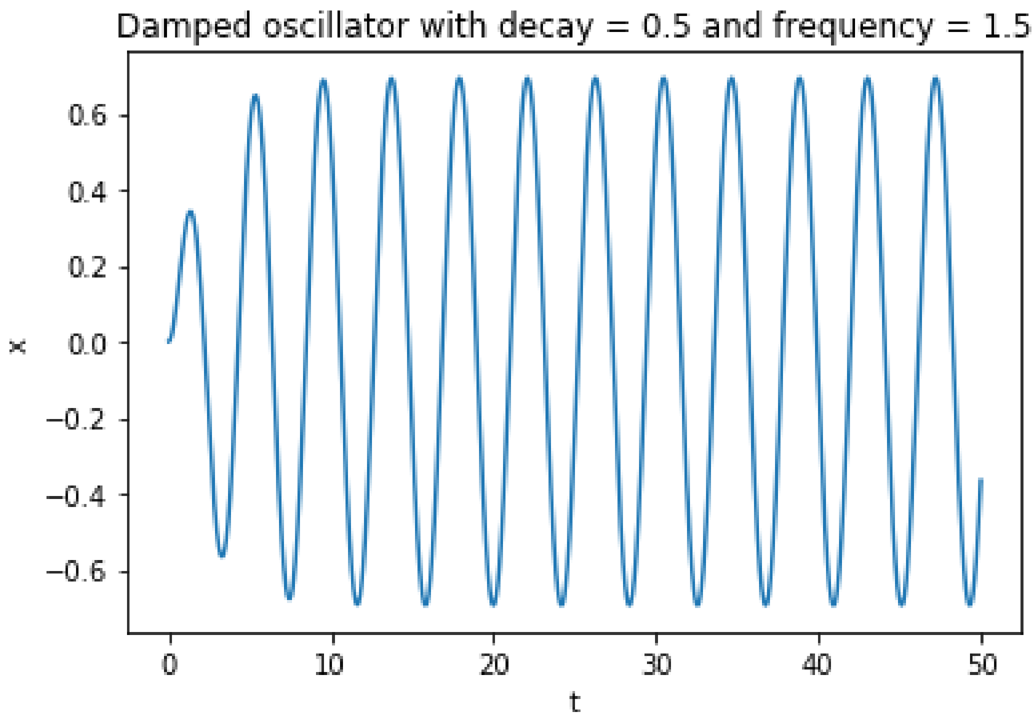
So,

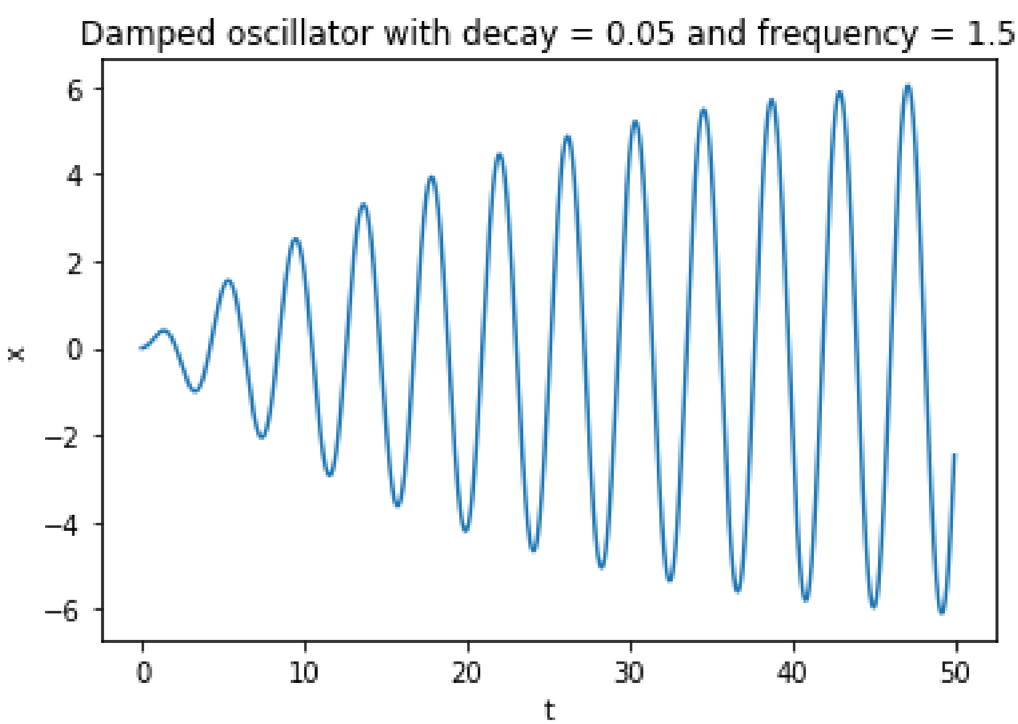
$$X(s) = \frac{F(s)}{s^2 + 2.25}$$

The function *springResponse* takes decay constant and frequency as parameters and plots x versus t for $0 < t < 50$. $x(t)$ is computed using the *scipy.impulse* function from $X(s)$. We plot the graphs for decay constants 0.5 and 0.05.

```
In [2]: def springResponse(f,d,t):
            X = sp.lti([1,d],polymul([1,0,2.25],[1,2*d,d**2+f**2]))
            t,x = sp.impulse(X,None,linspace(0,t,5001))
            title("Damped oscillator with decay = "+str(d)+" and frequency = "+str(f))
            xlabel('t')
            ylabel('x')
            plot(t,x)
```

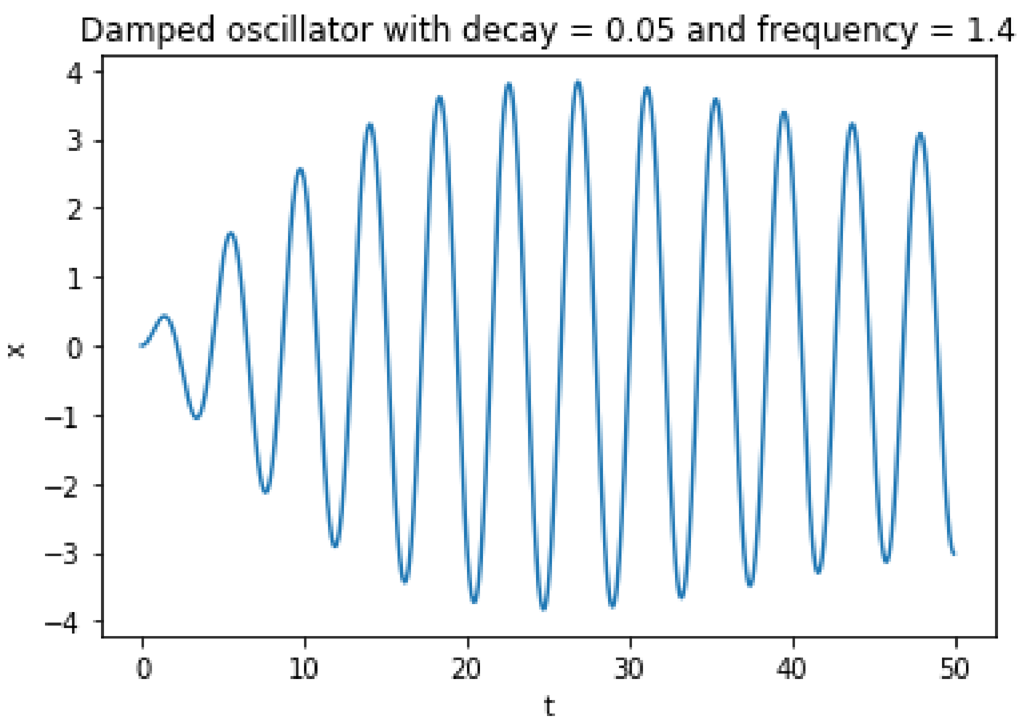
```
    show()
springResponse(1.5,0.5,50)
springResponse(1.5,0.05,50)
```

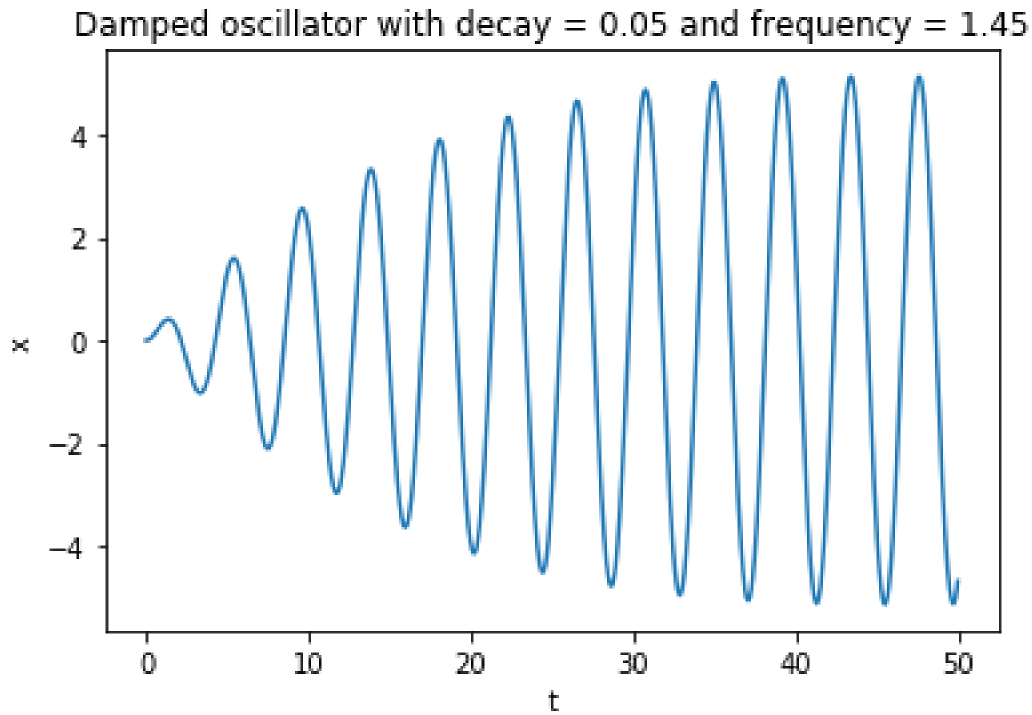


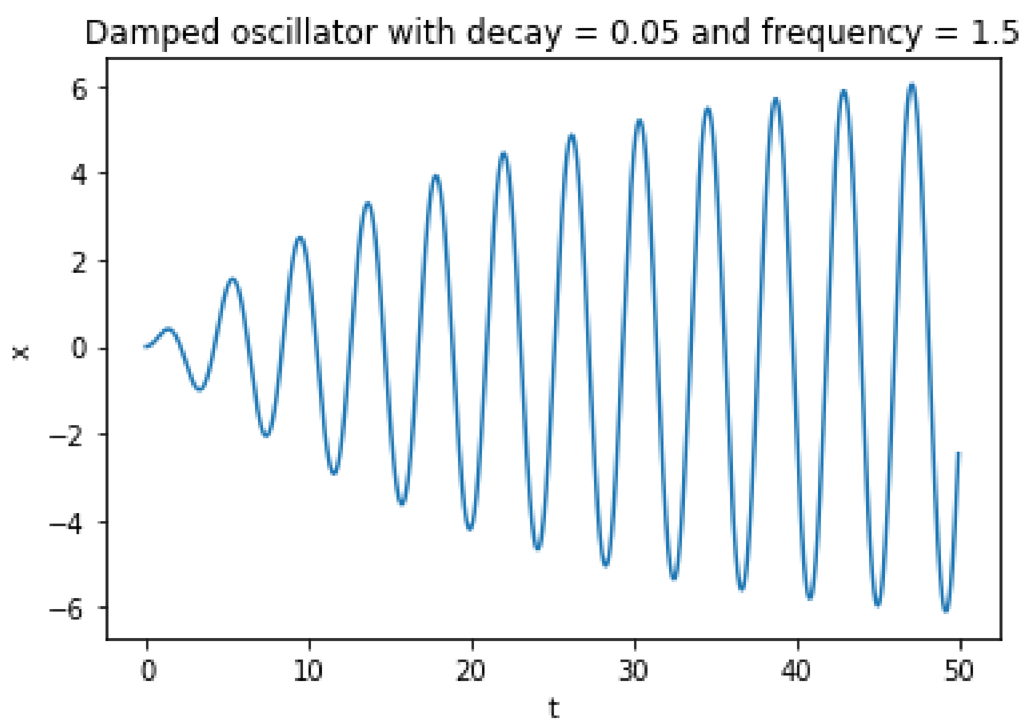


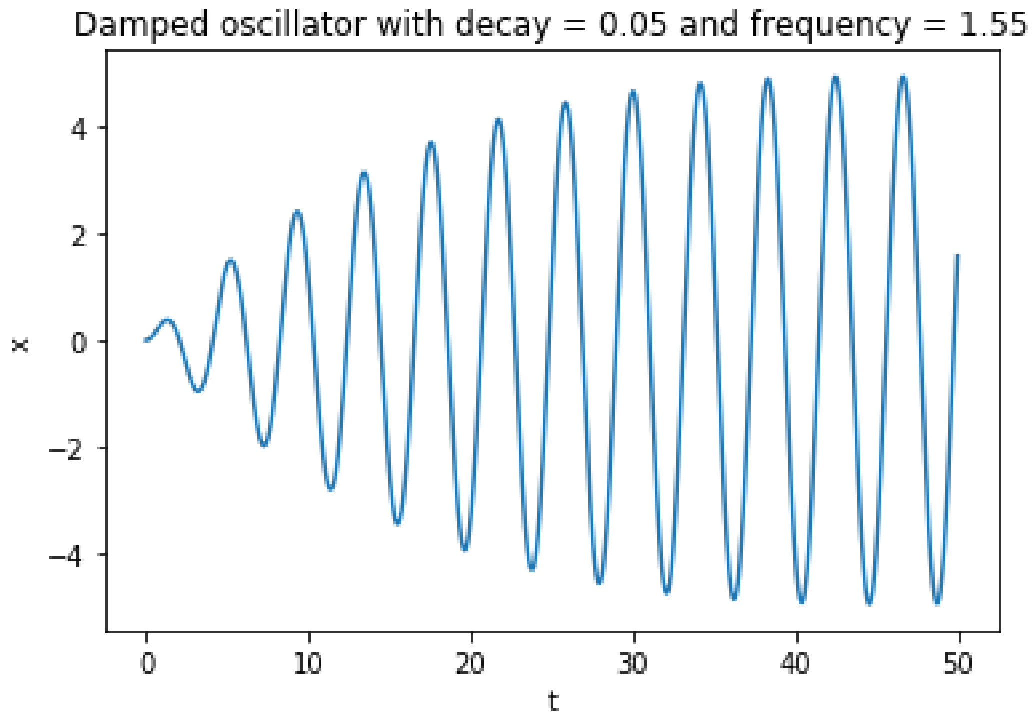
Now, for the same decay constant, we vary the frequency from 1.4 to 1.6 and see the variation in the plots.

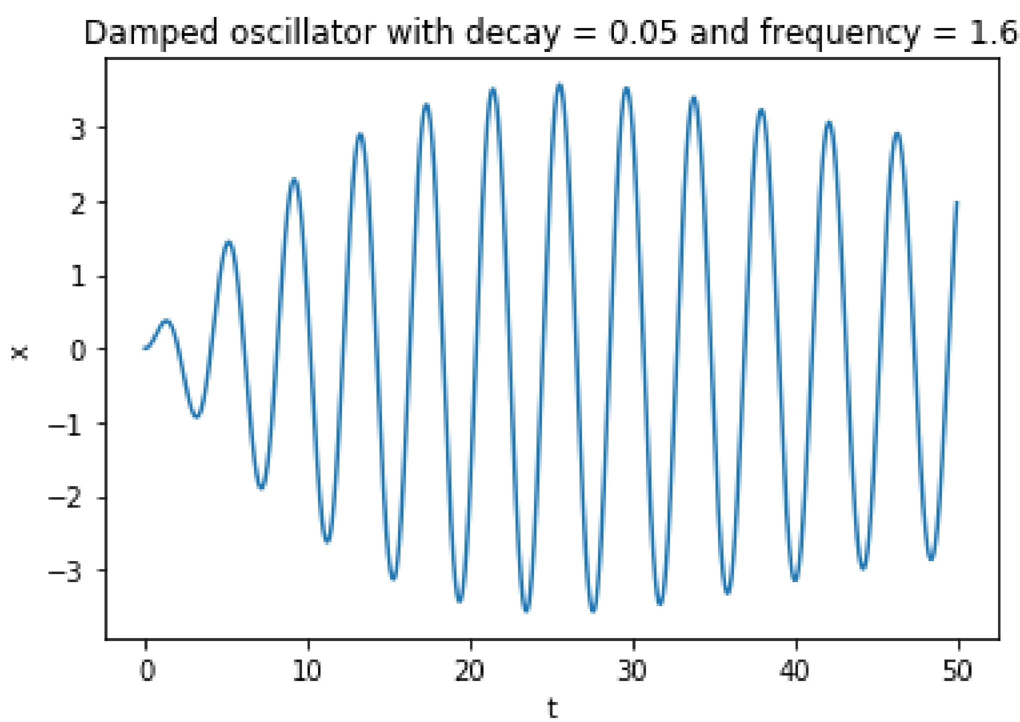
```
In [3]: for f in arange(1.4,1.61,0.05):
            springResponse(f,0.05,50)

        def springResponse(f,d,t):
            X = sp.lti([1,d],polymul([1,0,2.25],[1,2*d,d**2+f**2]))
            t,x = sp.impulse(X,None,linspace(0,t,5001))
            title("Damped oscillator with decay = "+str(d)+" and frequency = "+str(f))
            xlabel('t')
            ylabel('x')
            plot(t,x)

        for f in arange(1.4,1.61,0.05):
            springResponse(f,0.05,50)
        legend(['1.4','1.45','1.5','1.55','1.6'],title='w =')
        show()
```

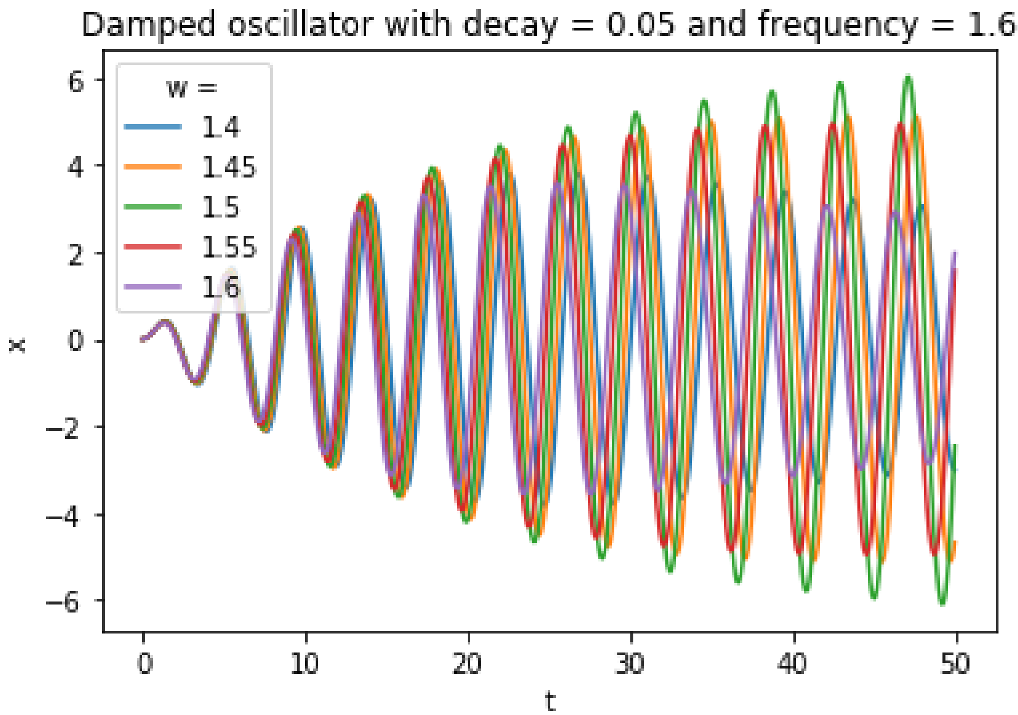



Damped oscillator with decay = 0.05 and frequency = 1.45
x
4
2
0
−2
−4
0
10
20
30
40
50
t


Damped oscillator with decay = 0.05 and frequency = 1.5
x
6
4
2
0
−2
−4
−6
0
10
20
30
40
50
t


Damped oscillator with decay = 0.05 and frequency = 1.55
x
t


Damped oscillator with decay = 0.05 and frequency = 1.6
x
t



Since the resonant frequency is 1.5, we observe the maximum amplitude to be occuring at at that frequency.

2.2 Coupled spring problem

We are given :

$$\ddot{x} + x - y = 0$$

$$\ddot{y} + 2(y - x) = 0$$

$$x(0) = 1$$

and other initial conditions to be zero. Solving it, we get :

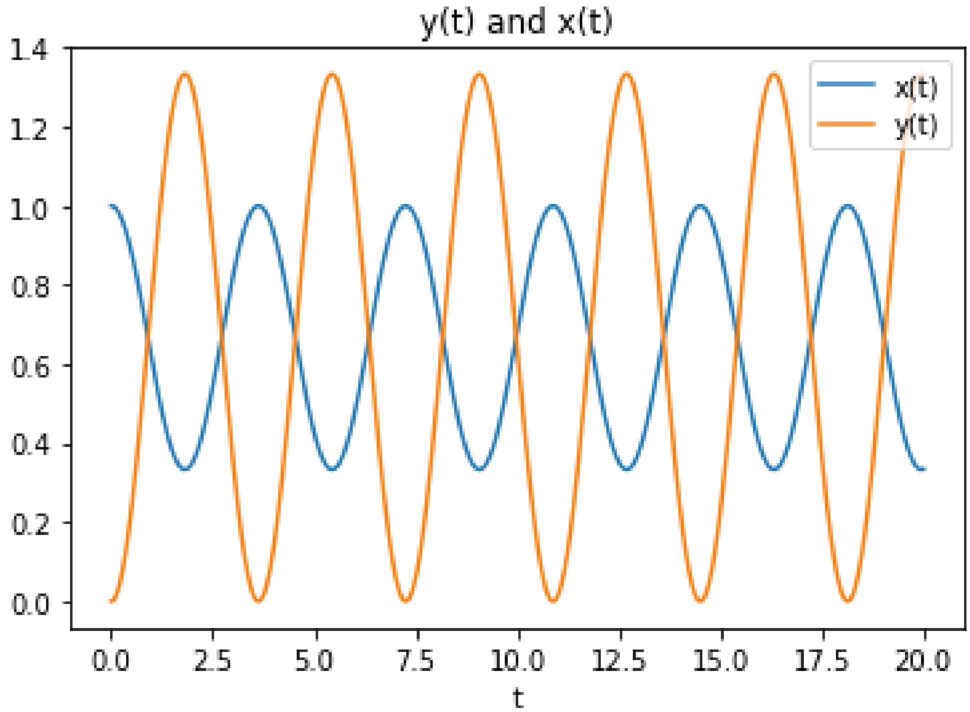
$$X(s) = \frac{s^2 + 2}{s^3 + 3s}$$

$$Y(s) = \frac{2}{s^3 + 3s}$$

Using the above results, we use those values in *scipy.lti* and plot them.

```
In [4]: X = sp.lti([1,0,2],[1,0,3,0])
        Y = sp.lti([2],[1,0,3,0])
        t,x = sp.impulse(X,None,linspace(0,20,501))
        plot(t,x)
        t,x = sp.impulse(Y,None,linspace(0,20,501))
        title("y(t) and x(t)")
```

```
xlabel('t')
plot(t,x)
legend(['x(t)','y(t)'],loc = 'upper right')
show()
```



2.3 RLC Circuit

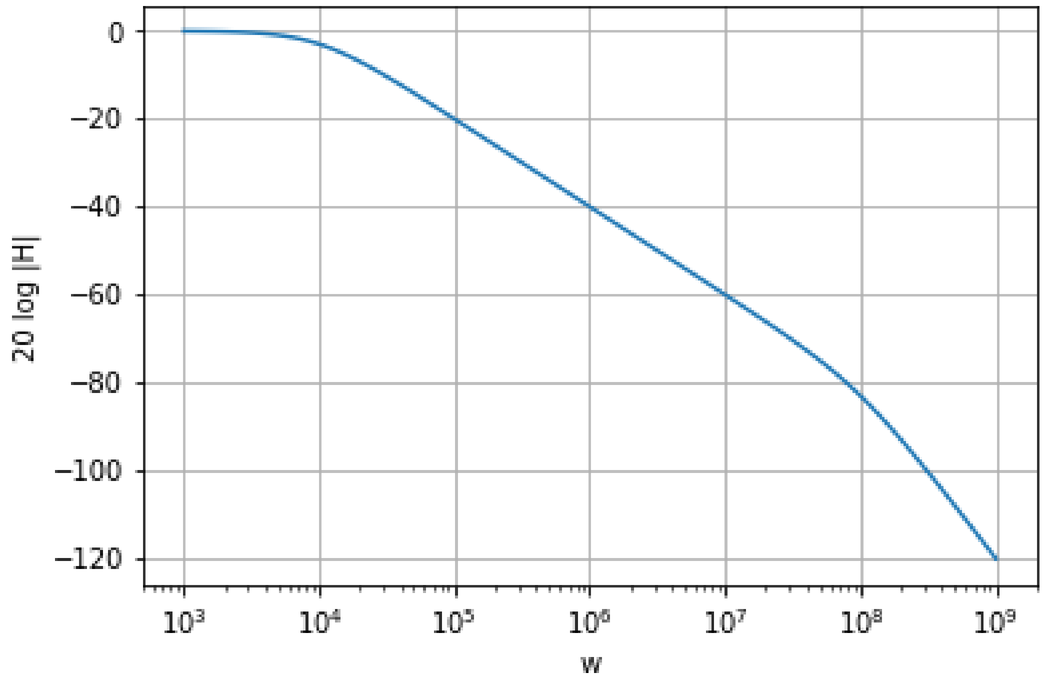
For the given circuit :

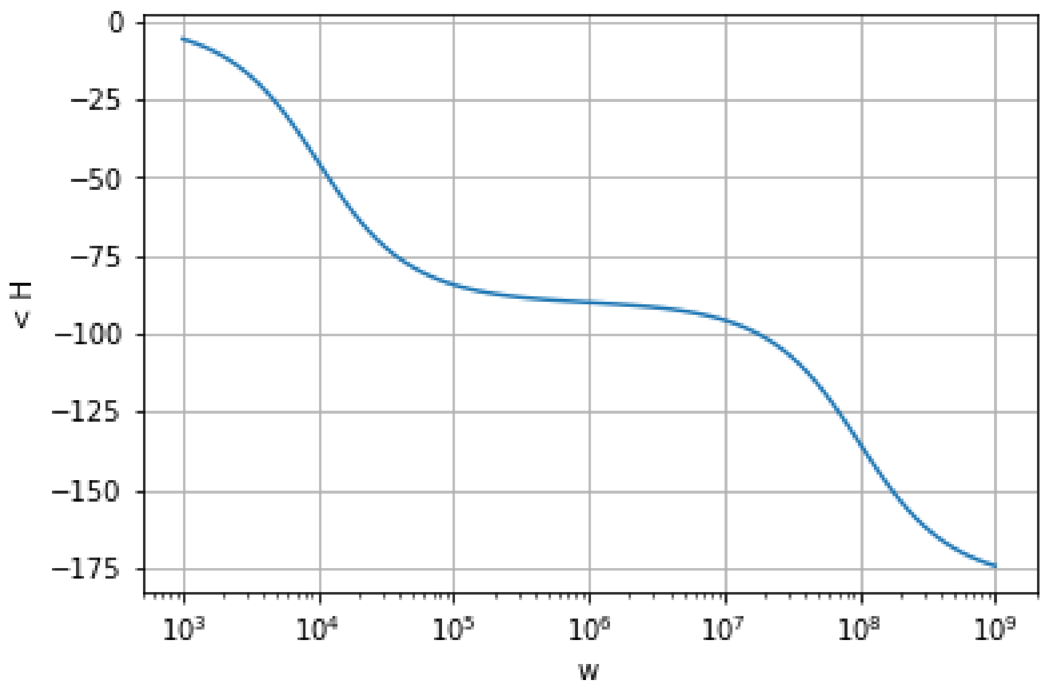
$$\frac{V_o(s)}{V_i(s)} = \frac{1}{LCs^2 + RCs + 1}$$

We use the values given, obtain bode plots using *signal.bode* and plot them.

```
In [5]: R = 100
        L = 10**-6
        C = 10**-6
        H = sp.lti([1],[L*C,R*C,1])
        w,S,phi = H.bode()
        ylabel('20 log |H|')
        semilogx(w,S)
        grid()
        xlabel('w')
        show()
        ylabel('< H')
```

```
semilogx(w,phi)
xlabel('w')
grid()
show()
```



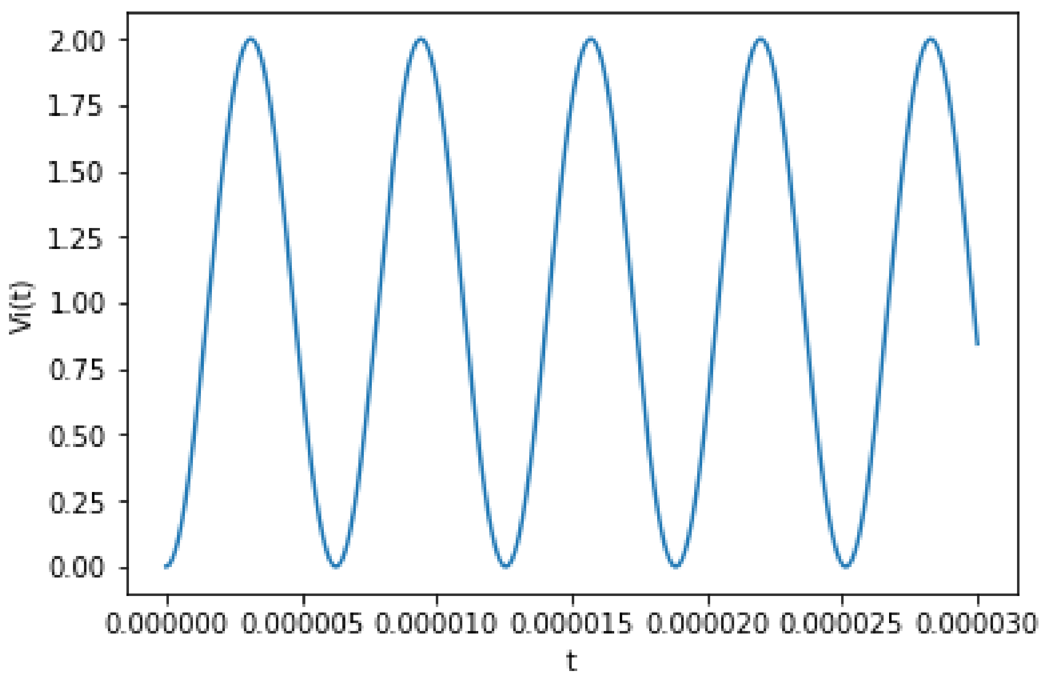


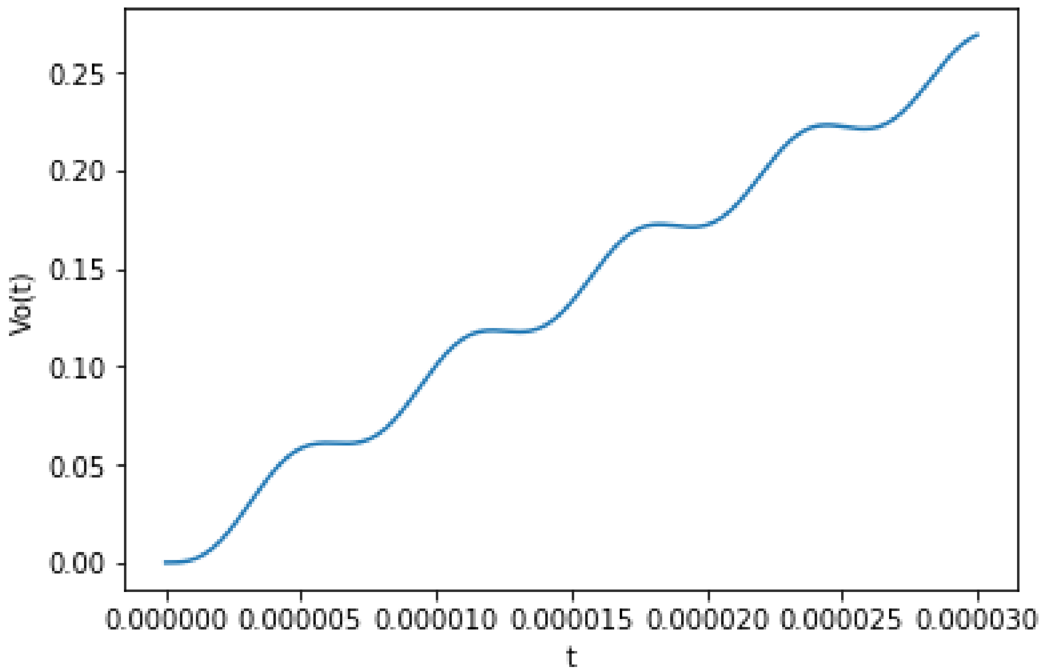
Now that we have the transfer function, we are given an input :

$$v_i(t) = (cos(10^3 t) - cos(10^6 t)) * u(t)$$

Using the transfer function calculated above and the *signal.lsim*, the response is calculated and plotted for two case.
The first case is for $0 < t < 30\mu s$.

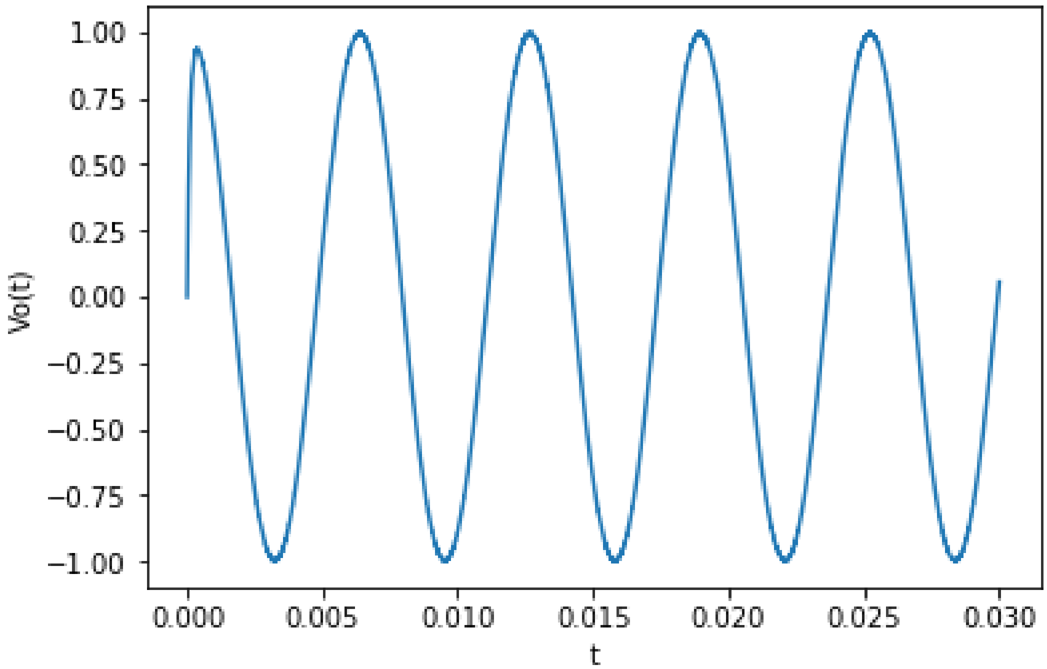
```
In [6]: t = np.linspace(0,30*(10**-6),10000)
        vi = cos(1000*t)-cos(10**6*t)
        t,vo,_ = sp.lsim(H,vi,t)
        plot(t,vi)
        xlabel('t')
        ylabel('Vi(t)')
        show()
        plot(t,vo)
        xlabel('t')
        ylabel('Vo(t)')
        show()
```





Next, we plot the long term response, i.e for $0 < t < 30ms$.

```
In [7]: t = np.linspace(0,30*(10**-3),10000)
        vi = cos(1000*t)-cos(10**6*t)
        t,vo,_ = sp.lsim(H,vi,t)
        plot(t,vo)
        xlabel('t')
        ylabel('Vo(t)')
        show()
```



3 Conclusion

In this assignment, we used *scipy*'s signal processing library to solve some LTI systems.