Assignment 7 - EE2703 Applied Programming Lab

EE18B132 Hemanth Ram

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1 Overview

In this week, we analyze the low pass filter and the high pass filter using Laplace transforms. For doing this, we use the *sympy* library of python for solving the nodal equations in the s-domain. Then, we use the *Poly* class of *sympy* to get coefficients and finally use *scipy.signal* for arriving at results and plotting.

2 Code and Generated Outputs

Importing required libraries and defining the symbol s.

```
In [1]: from __future__ import division
        from sympy import *
        import scipy.signal as sp
        import pylab as p
        init_printing()
        s = symbols('s')
```

Funciton to get the coefficients of numerator polynomial and denominator polynomial given an expression, so that it can be used to construct the transfer funciton using *sp.lti*. It first simplifies the expressions using $sympy.simplify$ and then separates into *num* (numerator) and *den* (denominator) using $sympy.fraction$. Then, we use the *Poly* class to get the coefficients.

```
In [2]: def st(y):
            Y = (simplify(y))
            n,d = fraction(Y)
            n,d = Poly(n,s), Poly(d,s)
            num,den = n.all_coeffs(), d.all_coeffs()
            num,den = [float(f) for f in num], [float(f) for f in den]
            return num,den
```

Function to return the solution of the nodal equations for the low-pass filter given the parameters. Giving V_i as 1 would yield us the transfer function at V_o which is the expression in $V[3]$. We then pass it to $st()$ to get the coefficients of the polynomials in the fraction.

```
In [3]: def lowpass(R1,R2,C1,C2,G,Vi):
            A = Matrix([[0,0,1,-1/G],[-1/(1+s*R2*C2),1,0,0],[0,-G,G,1],[-1/R1-1/R2-s*C1,1/R2,0,s
            b = Matrix([0,0,0,-Vi/R1])
            V = A.inv()*b
            return (A,b,V)

        A,b,V = lowpass(10000,10000,1e-9,1e-9,1.586,1)
        n,d = st(V[3])
        print("The transfer function for the low-pass filter")
        simplify(V[3])
```

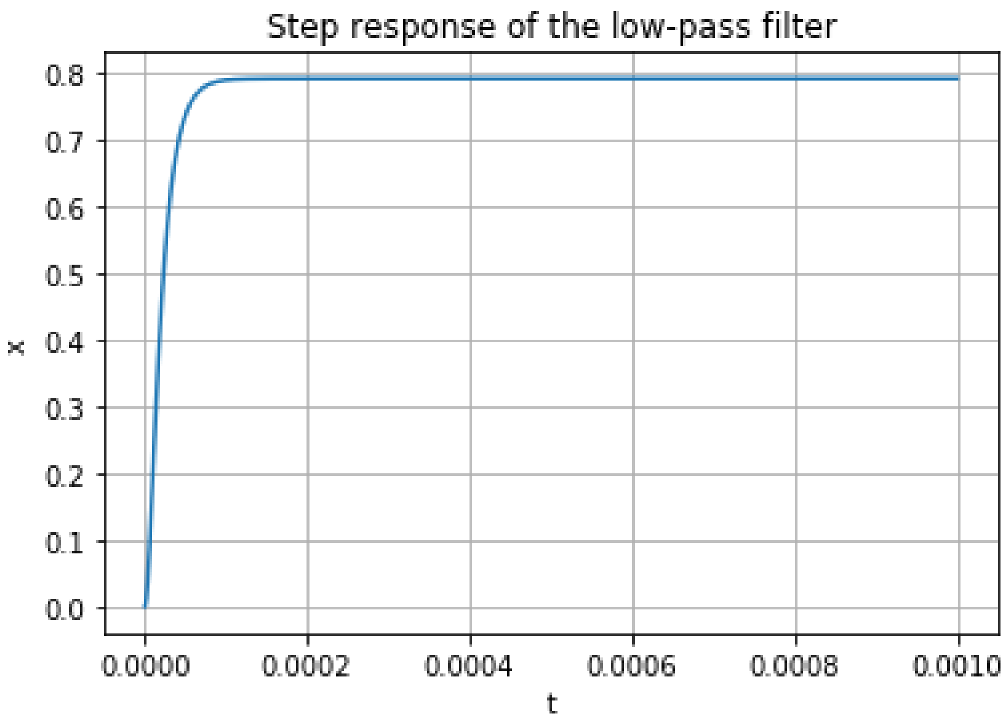
```
The transfer function for the low-pass filter
```

```
Out[3]:
```

$$\frac{0.0001586}{2.0 \cdot 10^{-14} s^2 + 4.414 \cdot 10^{-9} s + 0.0002}$$

Using the values of the coefficients obtained above, we get the step response just by adding an extra 0 to the denominator coefficients which is equivalent to multiplying $H(s)$ with $1/s$. We then obtain the plot using *sp.impulse*.

```
In [4]: d.append(0)
        X = sp.lti(n,d)
        d.pop()
        t,x = sp.impulse(X,None,p.linspace(0,0.001,5001))
        p.xlabel('t')
        p.ylabel('x')
        p.title("Step response of the low-pass filter")
        p.plot(t,x)
        p.grid()
        p.show()
```

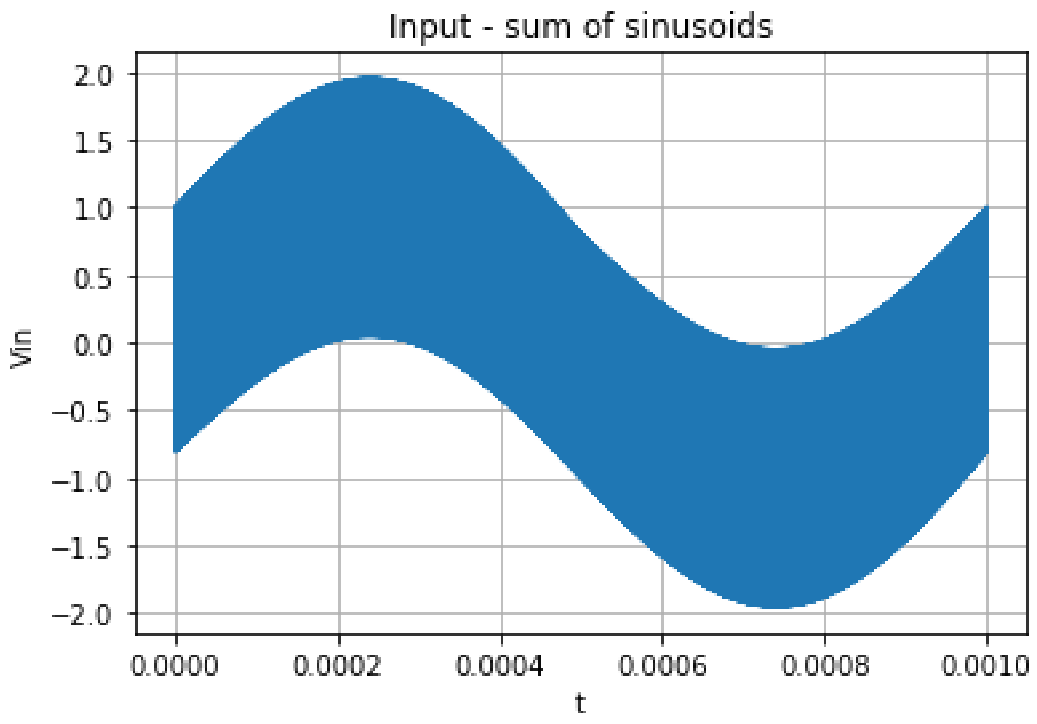


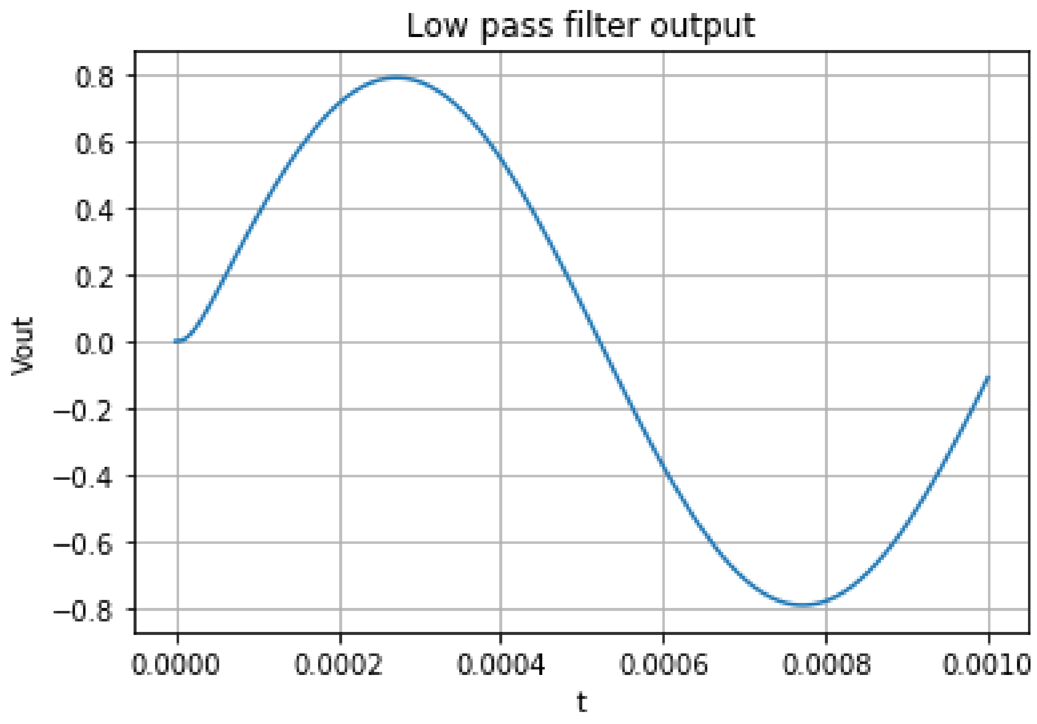
Next we get the response of the low-pass filter for a sum of sinusoids :

$$V_i(t) = (sin(2000\pi t) + cos(2.10^6\pi t)) * u_o(t)$$

We first make and input and use *signal.lsim* to obtain the response.

```
In [5]: t = p.linspace(0,0.001,5000)
        Vin = (p.sin(2000*t*p.pi)+p.cos(2*(10**6)*p.pi*t))
        p.title("Input - sum of sinusoids")
        p.plot(t,Vin)
        p.xlabel('t')
        p.ylabel('Vin')
        p.grid()
        p.show()
        X = sp.lti(n,d)
        t,y,s_v = sp.lsim(X,Vin,t)
        p.title("Low pass filter output")
        p.xlabel('t')
        p.ylabel('Vout')
        p.plot(t,y)
        p.grid()
        p.show()
```





Function to get the transfer function of high-pass filter just like we did for the low-pass filter.

```
In [6]: def highpass(R1,R3,C1,C2,G,Vi) :
            s = symbols ('s')
            A = Matrix([[0,-1,0,1/G],[s*C2*R3/(s*C2*R3+1),0,-1,0],[0,G,-G,1],[-s*C2-1/R1-s*C1,0,
            b = Matrix([0,0,0,-Vi*s*C1])
            V = A.inv()*b
            return (A,b,V)

        A,b,V = highpass(1e4,1e4,1e-9,1e-9,1.586,1)
        n,d = st(V[3])
        print("The transfer function for the high-pass filter")
        simplify(V[3])
```

```
The transfer function for the high-pass filter
```

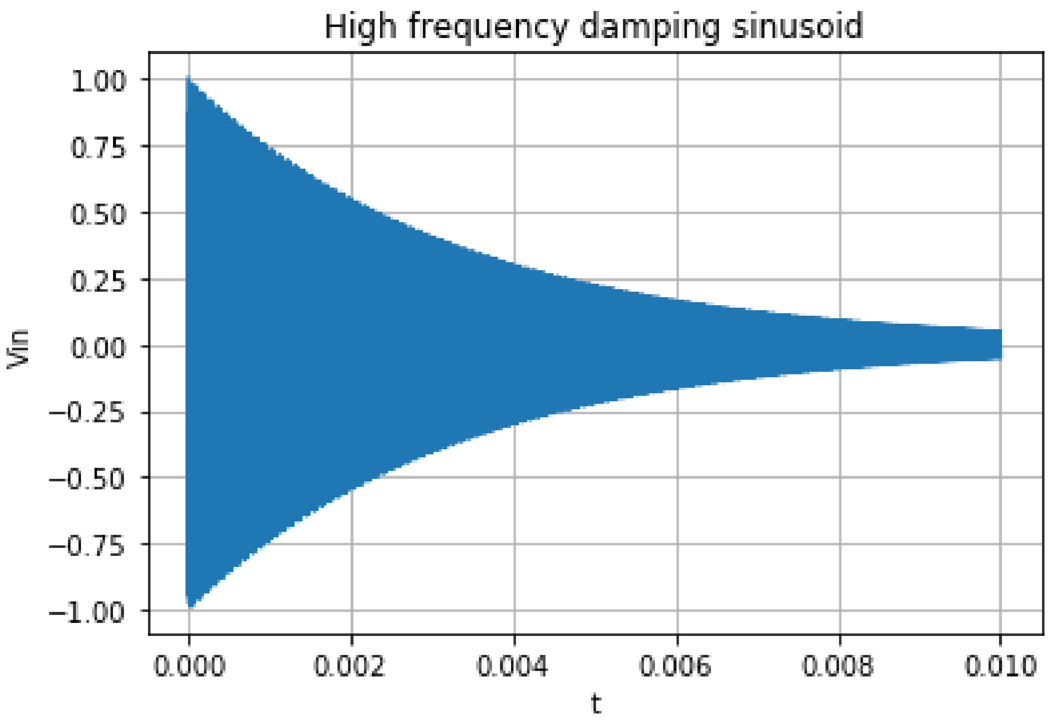
```
Out[6]:
```

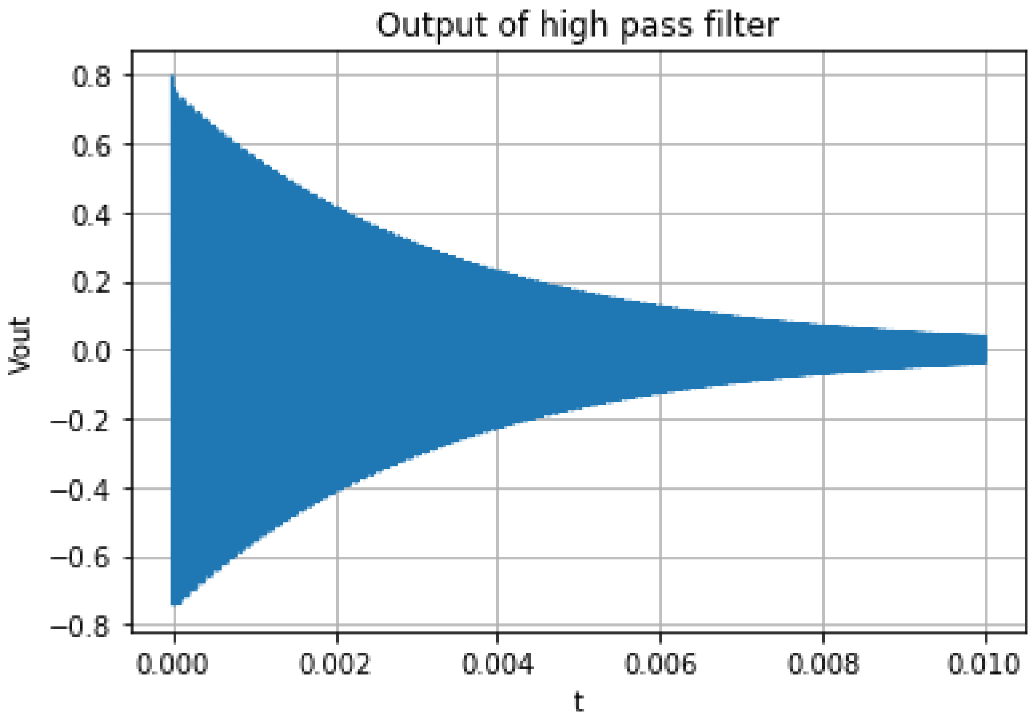
$$\frac{1.586 \cdot 10^{-14} s^2}{2.0 \cdot 10^{-14} s^2 + 4.414 \cdot 10^{-9} s + 0.0002}$$

Now we find the response for a damping sinusoid of high frequency for a high-pass filter.

$$V_i(t) = cos(10^7 t).e^{-300t}$$

```
In [7]: t = p.linspace(0,0.01,5000)
        Vin = (p.cos(1e7*t))*p.exp(-300*t)
        p.plot(t,Vin)
        p.xlabel('t')
        p.ylabel('Vin')
        p.title('High frequency damping sinusoid')
        p.grid()
        p.show()
        X = sp.lti(n,d)
        t,y,s_v = sp.lsim(X,Vin,t)
        p.title('Output of high pass filter')
        p.xlabel('t')
        p.ylabel('Vout')
        p.plot(t,y)
        p.grid()
        p.show()
```

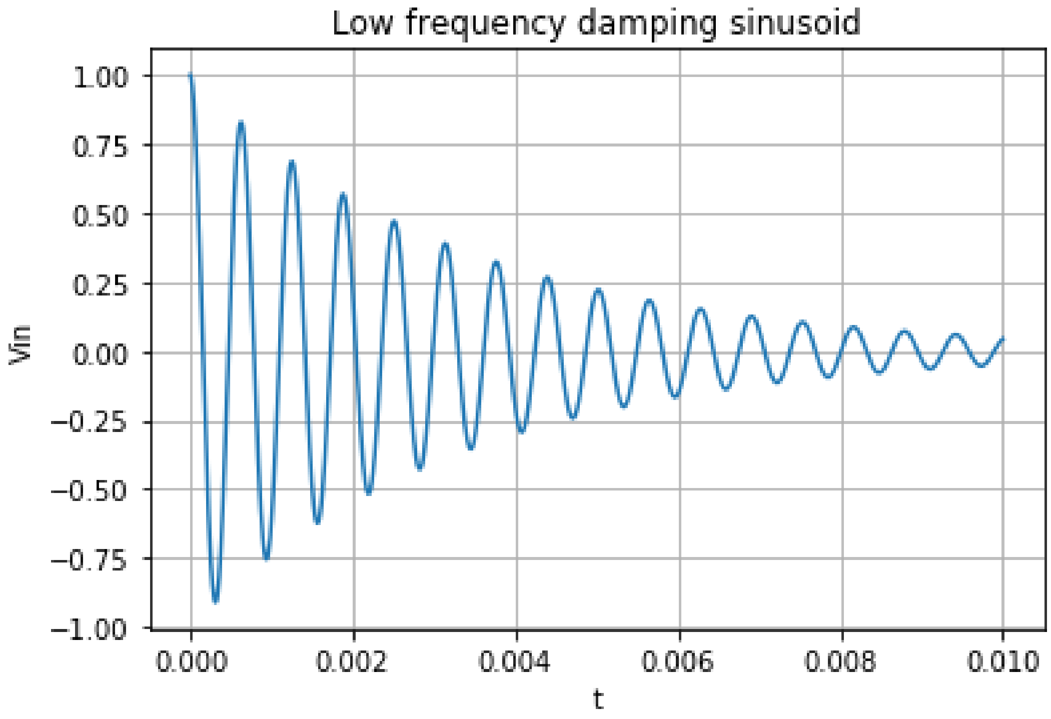

High frequency damping sinusoid
Vin
t
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.000
0.002
0.004
0.006
0.008
0.010

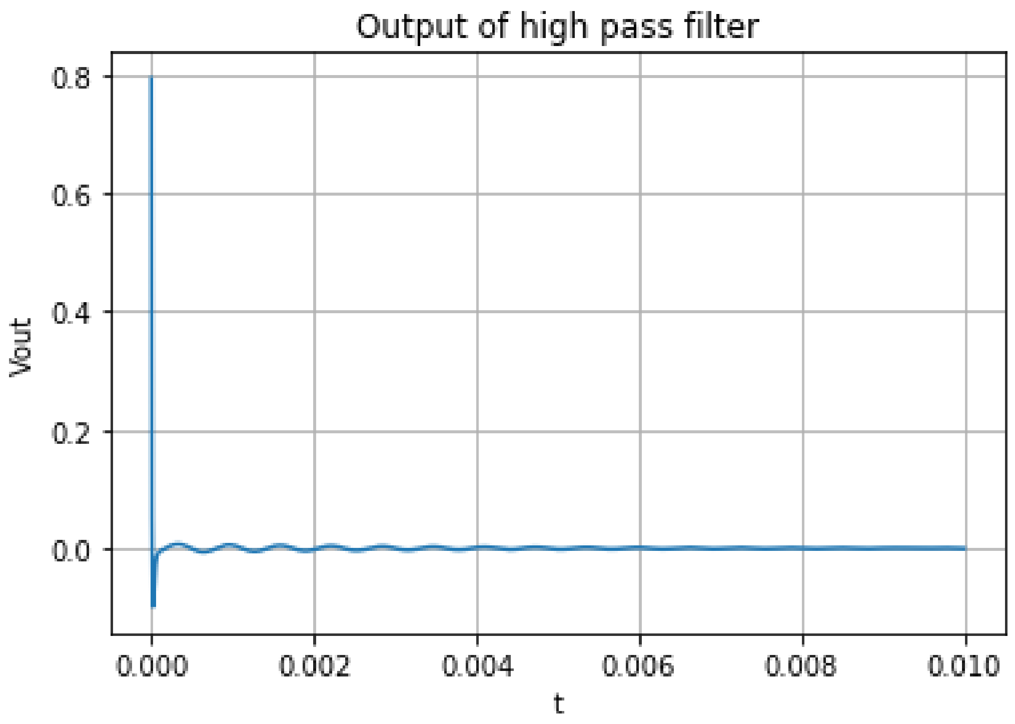

Output of high pass filter
Vout
t
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
0.000
0.002
0.004
0.006
0.008
0.010

Next we find the response for a damping sinusoid of high frequency for a high-pass filter.

$$V_i(t) = cos(10^4 t).e^{-300t}$$

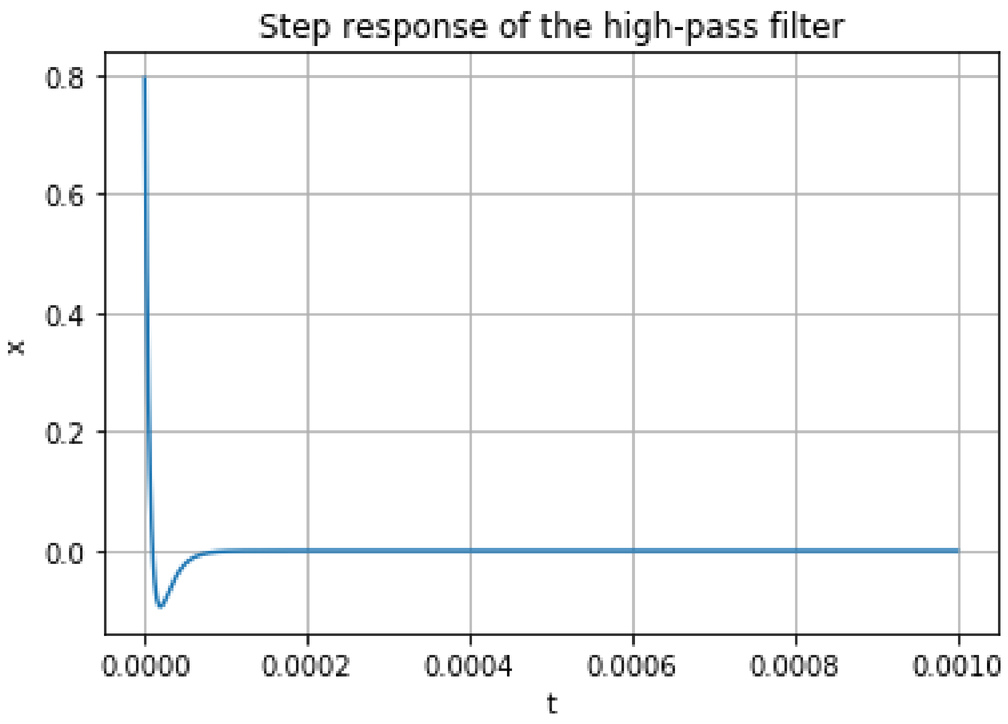
```
In [8]: t = p.linspace(0,0.01,5000)
        Vin = (p.cos(1e4*t))*p.exp(-300*t)
        p.plot(t,Vin)
        p.xlabel('t')
        p.ylabel('Vin')
        p.title('Low frequency damping sinusoid')
        p.grid()
        p.show()
        X = sp.lti(n,d)
        t,y,s_v = sp.lsim(X,Vin,t)
        p.title('Output of high pass filter')
        p.xlabel('t')
        p.ylabel('Vout')
        p.plot(t,y)
        p.grid()
        p.show()
```





Finally, we find the step response of the high-pass filter just like we did for the low-pass filter.

```
In [9]: d.append(0)
        X = sp.lti(n,d)
        d.pop()
        t,x = sp.impulse(X,None,p.linspace(0,0.001,5001))
        p.title("Step response of the high-pass filter")
        p.xlabel('t')
        p.ylabel('x')
        p.plot(t,x)
        p.grid()
        p.show()
```



3 Conclusion

The *sympy* module helped us in analytically solving the equations in the s-domain and then we interpreted these equations using the signals toolbox. We got the step responses of both the filters and saw the responses of the systems for low frequency and high frequency inputs.