Assignment 1

AI1110: Probability and Random Variables
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10.15.1.3 Question: Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?

Solution:

Sample space of tossing a coin is

$$S = \{H, T\}, |S| = 2$$

Number of heads in sample space $n(H) = 1$

Let us assign Head(H) to one team and Tail(T) to the other team.

Probability of an event E, $\Pr(E)$ is defined as

$$\Pr(E) = \frac{\text{Number of outcomes favourable to event } E}{\text{Total number of outcomes}} \tag{1}$$

Let X be a random variable which takes the values 0 and 1.

$$X = \begin{cases} 1, & \text{if coin toss results in Head} \\ 0, & \text{if coin toss results in Tail} \end{cases} \tag{2}$$

From law of total probability,

$$\Pr(X = 0) + \Pr(X = 1) = 1 \tag{3}$$

From (1) and (2),

$$\Pr(X = 1) = \Pr(H) \tag{4}$$

$$= \frac{n(H)}{|S|} \tag{5}$$

$$\therefore \quad \Pr(X = 1) = \frac{1}{2} \tag{6}$$

From (3),

$$\Pr(X = 0) = 1 - \Pr(X = 1) \tag{7}$$

$$\therefore \quad \Pr(X = 0) = \frac{1}{2} \tag{8}$$

From (6) and (8), Probability of a coin toss resulting in Head and Tail is equal i.e, both teams have equal probability of winning the coin toss. Hence, This is a fair way of deciding which team should get the ball at the beginning of a football game.