Assignment 3

AI1110: Probability and Random Variables
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12.13.6.3 Question: Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability that this person being male? Assume that there are equal number of males and females.

Solution: Let X and Y be two random variables which describe the gender and color of hair of the person respectively according to table I :

Variable	Event
$X = 0$	Men
$X = 1$	Women
$Y = 0$	Non-grey hair
$Y = 1$	grey hair

TABLE I

It is given that,

$$p_X(0) = p_X(1) = \frac{1}{2} \tag{1}$$

$$p_{Y|X}(1|0) = \frac{5}{100} \tag{2}$$

$$= \frac{1}{20} \tag{3}$$

$$p_{Y|X}(1|1) = \frac{0.25}{100} \tag{4}$$

$$= \frac{1}{400} \tag{5}$$

From the law of total probability,

$$p_Y(1) = p_{Y|X}(1|0) \times p_X(0) + p_{Y|X}(1|1) \times p_X(1) \tag{6}$$

$$= \frac{1}{20} \times \frac{1}{2} + \frac{1}{400} \times \frac{1}{2} \tag{7}$$

$$= \frac{21}{800} \tag{8}$$

$$\therefore \; p_Y(1) = \frac{21}{800} \tag{9}$$

The required probability is $p_{X|Y}(0|1)$.

From the law of conditional probability,

$$p_{X|Y}(x|y) = \frac{p_{X,Y}(x,y)}{p_Y(y)} \tag{10}$$

$$p_{X,Y}(x,y) = p_{Y|X}(y|x) \times p_X(x) \tag{11}$$

From (1), (3) and (11)

$$p_{X,Y}(0,1) = p_{Y|X}(1|0) \times p_X(0) \tag{12}$$

$$= \frac{1}{20} \times \frac{1}{2} \tag{13}$$

$$= \frac{1}{40} \tag{14}$$

$$\therefore \; p_{X,Y}(0,1) = \frac{1}{40} \tag{15}$$

From (9), (10) and (15)

$$p_{X|Y}(0|1) = \frac{p_{X,Y}(0,1)}{p_Y(1)} \tag{16}$$

$$= \frac{\frac{1}{40}}{\frac{21}{800}} \tag{17}$$

$$= \frac{20}{21} \tag{18}$$

$$\therefore \; p_{X|Y}(0|1) = \frac{20}{21} \tag{19}$$