Assignment 2

AI1110: Probability and Random Variables
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12.13.2.13 Question: Two balls are drawn at random with replacement from a box containing 10 black balls and 8 red balls. Find the probability that

(i) both balls are red
(ii) first ball is black and second is red
(iii) one of them is black and other is red

Solution:

Since we are drawing balls with replacement, the probability p of drawing red ball on any draw is independent of other draws

$$p = \frac{8}{18} = \frac{4}{9} \tag{1}$$

If we consider drawing a red ball as success then, Each draw is a Bernoulli trial with probability of success begin p

When two balls are drawn, this is a Binomial distribution with 2 independent Bernoulli trials

Let X be the random variable denoting the number of successes in a Binomial distribution

The probability of r successes in a Binomial distribution with n independent Bernoulli trials where probability of success in each trial is p is

$$P_X(r) = {}^nC_r p^r (1-p)^{n-r} \tag{2}$$

Here, $n = 2$ and $p = \frac{4}{9}$

$$X \in \{0, 1, 2\} \tag{3}$$

$$X \sim Bin\left(2, \frac{4}{9}\right) \tag{4}$$

From (1) and (2),

$$\therefore \; P_X(r) = {}^2C_r \left(\frac{4}{9}\right)^r \left(\frac{5}{9}\right)^{2-r} \tag{5}$$

(i) Since both balls are red, the required probability is $P_X(2)$

$$P_X(2) = {}^2C_2 \left(\frac{4}{9}\right)^2 \left(\frac{5}{9}\right)^0 \tag{6}$$

$$= \frac{16}{81} \tag{7}$$

$$\therefore \; P_X(2) = \frac{16}{81} \tag{8}$$

(ii) Let E be the event that first ball is black and other is red. The required probability $\Pr(E)$ is

$$\Pr(E) = (1-p) \times p \tag{9}$$

$$= \frac{1}{2} \times P_X(1) \tag{10}$$

$$= \frac{1}{2} \times {}^2C_1 \left(\frac{4}{9}\right)^1 \left(\frac{5}{9}\right)^1 \tag{11}$$

$$= \frac{1}{2} \times 2 \times \frac{4}{9} \times \frac{5}{9} \tag{12}$$

$$= \frac{20}{81} \tag{13}$$

$$\therefore \; \Pr(E) = \frac{20}{81} \tag{14}$$

(iii) Since only one of the two balls has to be red, The required probability is $P_X(1)$

$$P_X(1) = {}^2C_1 \left(\frac{4}{9}\right)^1 \left(\frac{5}{9}\right)^1 \tag{15}$$

$$= 2 \times \frac{4}{9} \times \frac{5}{9} \tag{16}$$

$$= \frac{40}{81} \tag{17}$$

$$\therefore \; P_X(1) = \frac{40}{81} \tag{18}$$