Assignment 2

AI1110: Probability and Random Variables
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12.13.2.13 Question: Two balls are drawn at random with replacement from a box containing 10 black balls and 8 red balls. Find the probability that

(i) both balls are red
(ii) first ball is black and second is red
(iii) one of them is black and other is red

Solution:

Let X and Y be two random variables which describe the color of first and second ball drawn from the box respectively according to table I:

Variable	Event
$X = 0$	1^{st} ball is red
$X = 1$	1^{st} ball is black
$Y = 0$	2^{nd} ball is red
$Y = 1$	2^{nd} ball is black

TABLE I

Since we are drawing balls with replacement, X and Y are independent of each other.

Probability	Value
$\Pr(X = 0)$	$\frac{8}{18} = \frac{4}{9}$
$\Pr(X = 1)$	$\frac{10}{18} = \frac{5}{9}$
$\Pr(Y = 0)$	$\frac{4}{9}$
$\Pr(Y = 1)$	$\frac{5}{9}$

TABLE II

(i) Since both balls are red, the required probability is $\Pr(X = 0, Y = 0)$

$$\Pr(X = 0, Y = 0) = \Pr(X = 0) \times \Pr(Y = 0) \tag{1}$$

$$= \frac{4}{9} \times \frac{4}{9} \tag{2}$$

$$= \frac{16}{81} \tag{3}$$

$$\therefore \quad \Pr(X = 0, Y = 0) = \frac{16}{81} \tag{4}$$

(ii) The required probability is $\Pr(X = 1, Y = 0)$

$$\Pr(X = 1, Y = 0) = \Pr(X = 1) \times \Pr(Y = 0) \tag{5}$$

$$= \frac{5}{9} \times \frac{4}{9} \tag{6}$$

$$= \frac{20}{81} \tag{7}$$

$$\therefore \quad \Pr(X = 1, Y = 0) = \frac{20}{81} \tag{8}$$

(iii) Let event E be one of them is black and other is red.
The required probability, $\Pr(E)$ is

$$\Pr(E) = \Pr(X = 1, Y = 0) + \Pr(X = 0, Y = 1) \tag{9}$$

From (8),

$$\Pr(X = 1, Y = 0) = \frac{20}{81} \tag{10}$$

$$\Pr(X = 0, Y = 1) = \Pr(X = 0) \times \Pr(Y = 1) \tag{11}$$

$$= \frac{4}{9} \times \frac{5}{9} \tag{12}$$

$$= \frac{20}{81} \tag{13}$$

From (10) and (13),

$$\Pr(E) = \frac{20}{81} + \frac{20}{81} \tag{14}$$

$$= \frac{40}{81} \tag{15}$$

$$\therefore \quad \Pr(E) = \frac{40}{81} \tag{16}$$