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Assignment 2

AI1110: Probability and Random Variables Indian Institute of Technology Hyderabad

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12.13.1.13 Question: An instructor has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

Solution: Let *X* and *Y* be two random variables in which X describes the difficulty level of question and Y describes the type of question according to the table I:

Variable	Event
X = 0	Easy question
X = 1	Difficult question
Y = 0	True/False question
Y = 1	Multiple choice question

TABLE I

From the law of total probability,

$$p_X(0) + p_X(1) = 1 (1)$$

$$p_Y(0) + p_Y(1) = 1$$

$$p_X(0) = p_{X,Y}(0,0) + p_{X,Y}(0,1)$$
 (3)

$$p_Y(0) = p_{X,Y}(0,0) + p_{X,Y}(1,0)$$
 (4)

$$p_Y(0) = p_{X,Y}(0,0) + p_{X,Y}(1,0)$$

$$p_X(0) = \frac{300 + 500}{300 + 200 + 500 + 400}$$

$$= \frac{4}{7}$$

$$p_Y(0) = \frac{300 + 200}{300 + 200 + 500 + 400}$$

$$= \frac{5}{14}$$

$$p_Y(0) = \frac{300 + 200}{300 + 200 + 500 + 400}$$

$$=\frac{5}{14}$$

From (1), (2), (6) and (8),

$$p_X(1) = 1 - p_X(0) (9)$$

$$=\frac{3}{7}\tag{10}$$

$$p_Y(1) = 1 - p_Y(0) \tag{11}$$

$$=\frac{9}{14}\tag{12}$$

$$p_{X,Y}(0,0) = \frac{300}{1400} \tag{13}$$

$$=\frac{3}{14}\tag{14}$$

$$p_{X,Y}(0,0) = \frac{1}{1400}$$

$$= \frac{3}{14}$$

$$p_{X,Y}(1,1) = \frac{400}{1400}$$

$$= \frac{2}{7}$$
(16)
$$(16)$$

$$(17)$$

$$=\frac{2}{7}\tag{16}$$

From (3), (4), (6), (8) and (14),

$$p_{X,Y}(0,1) = p_X(0) - p_{X,Y}(0,0)$$
 (17)

$$=\frac{5}{14}\tag{18}$$

$$p_{X,Y}(1,0) = p_Y(0) - p_{X,Y}(0,0)$$

$$= \frac{1}{7}$$
(20)

$$=\frac{1}{7}\tag{20}$$

The required probability is $p_{X|Y}(0|1)$ (2)

From the definition of conditional probability, We know that,

$$p_{X|Y}(x|y) = \frac{p_{X,Y}(x,y)}{p_Y(y)}$$
 (21)

From (12) and (18),

(5)

(6)

(7)

(8)

$$p_{X|Y}(0|1) = \frac{p_{X,Y}(0,1)}{p_Y(1)}$$
 (22)

$$=\frac{\frac{5}{14}}{\frac{9}{14}}\tag{23}$$

$$=\frac{5}{9}\tag{24}$$

$$=\frac{\frac{5}{14}}{\frac{9}{14}}$$

$$=\frac{5}{9}$$

$$\therefore p_{X|Y}(0|1) = \frac{5}{9}$$
(23)
$$(24)$$