

Assignment 2

AI1110: Probability and Random Variables

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12.13.1.13 Question: An instructor has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

Solution: Let X and Y be two random variables in which X describes the difficulty level of question and Y describes the type of question according to the table I :

Variable	Event
$X = 0$	Easy question
$X = 1$	Difficult question
$Y = 0$	True/False question
$Y = 1$	Multiple choice question

TABLE I

The probabilities of these random variables are according the below table II

Probability	Value
$p_X(0)$	$\frac{4}{7}$
$p_X(1)$	$\frac{3}{7}$
$p_Y(0)$	$\frac{5}{14}$
$p_Y(1)$	$\frac{9}{14}$
$p_{X,Y}(0, 0)$	$\frac{3}{14}$
$p_{X,Y}(0, 1)$	$\frac{5}{14}$
$p_{X,Y}(1, 0)$	$\frac{1}{7}$
$p_{X,Y}(1, 1)$	$\frac{2}{7}$

TABLE II

From the definition of conditional probability, We know that,

$$p_{X|Y}(x|y) = \frac{p_{X,Y}(x, y)}{p_Y(y)} \quad (1)$$

From (1) and table II,

$$p_{X|Y}(0|1) = \frac{p_{X,Y}(0, 1)}{p_Y(1)} \quad (2)$$

$$= \frac{\frac{5}{14}}{\frac{9}{14}} \quad (3)$$

$$= \frac{5}{9} \quad (4)$$

$$\therefore p_{X|Y}(0|1) = \frac{5}{9} \quad (5)$$

The required probability is $p_{X|Y}(0|1)$