Assignment 4

AI1110: Probability and Random Variables
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12.13.6.3 Question: In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.

Solution: Let X and Y be two random variables which describes the event of getting six in a die roll and number of die rolls at which he quits

Variable	Event
$X = 1$	Getting a six
$X = 0$	Not getting a six

TABLE I

$Y \in \{1, 2, 3\}$

$$p_X(1) = \frac{1}{6} \tag{1}$$

$$p_X(0) = \frac{5}{6} \tag{2}$$

Let $P(n)$ denote the amount he wins if he throws the die n times.

$$P(n) = \begin{cases} 2 - n, & \text{if he get a six in } n^{th} \text{ die roll} \\ -3, & \text{if } n = 3 \text{ and he doesn't get six anytime} \end{cases} \tag{3}$$

Let $E(P(n))$ be the expected value of amount he wins in this game

$$\begin{aligned} E(P(n)) &= \sum_{n=1}^{n=3} p_Y(n) \times P(n) \\ &= p_X(1) \times (2-1) + p_X(0) \times p_X(1) \times (2-2) + \\ &\quad p_X(0)^2 \times p_X(1) \times (2-3) + p_X(0)^3 \times -3 \\ &= \frac{1}{6} + \frac{0}{36} - \frac{25}{216} - \frac{375}{216} \\ &= -\frac{364}{216} \\ &\approx -1.685 \end{aligned}$$

$\therefore$ The expected amount he wins is ≈ -1.685