Based on their total scores, 200 candidates of civil service examination are divided into two groups, the upper 30% and the remaining 70%. The first question of the examination is considered as a sample. Among the first group, 40 had the correct answer, whereas among the second group, 80 had the correct answer. Based on these results, can one conclude that the first question is not good at discriminating the ability of the students being examined here? (Take $\alpha = 0.05$)

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Q. 1. Let, $p1$ = proportion of correct answer in the upper 30% group
$p2$ = proportion of correct answer in the remaining 70% group.

Given, $n = 200$
so $n_1 = 60$, $n_2 = 140$

1. Formulate Hypothesis

Null hypothesis $H_0 : p1 = p2$ (No difference in discrimination ability)

Alterative Hypothesis $H_1 : p1 \neq p2$ (Difference in discrimination ability)

2. Significance level at 5%

$\alpha = 0.05$ $\left[Z_{\frac{\alpha}{2}} = Z_{0.002} = \pm 1.96 \right]$

3. Standard Normal distribution

4. Test Statistic:

proportions:

$$p1 = \frac{x_1}{n_1} = \frac{40}{60} = 0.6667$$

$$p2 = \frac{x_2}{n_2} = \frac{80}{140} = 0.5714$$

Pooled proportion $p = \frac{40+80}{60+140} = \frac{120}{200}$

$p = 0.6$
$q = 0.4$

Standard Error $(P_1 - P_2)$:

$$SE(P_1 - P_2) = \sqrt{\frac{pq}{n_1} + \frac{pq}{n_2}}$$

$$= \sqrt{\frac{0.6 \times 0.4}{60} + \frac{0.6 \times 0.4}{140}}$$

$$= \sqrt{0.004 + 0.001714}$$

$$= \sqrt{0.005714}$$

$$SE(P_1 - P_2) \simeq 0.0755$$

Z - test statistic

$$Z_{obs} = \frac{(P_1 - P_2)}{SE}$$

$$= \frac{0.6667 - 0.5714}{0.0755}$$

$$= \frac{0.0953}{0.0755}$$

$$Z_{obs} = 1.2596$$

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5. Rejection Criteria:

Since $H_0 : p_1 = p_2$ vs $H_1 : p_1 \neq p_2$ is a two tailed hypothesis,

Reject H_0 if $Z_{obs} < Z_{\alpha/2}$

otherwise Accept H_0

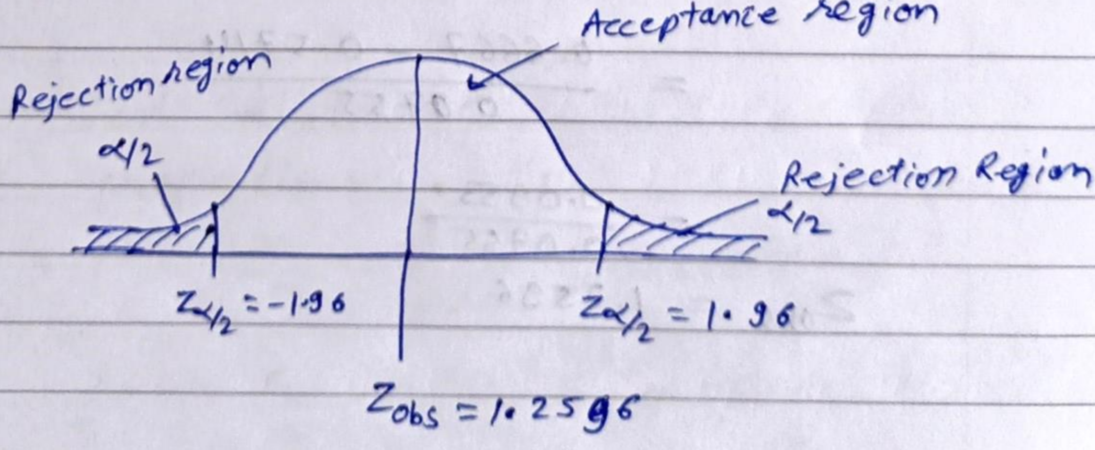
whenever $-Z_{\alpha/2} \leq Z_{obs} \leq Z_{\alpha/2}$

accept H_0 beacause

$Z_{obs} = 1.2596 < Z_{\alpha/2} = 1.96$

Conclusion:

we cannot reject the null hypothesis at the 0.05 level of significance. Therefore, we do not have enough evidence to conclued that the first question is not good at discriminating the ability of the student being examined here.



An experiment was performed to compare the abrasive wear of two different laminated materials. Twelve pieces of material 1 were tested by exposing each piece to a machine measuring wear. Ten pieces of material 2 were similarly tested. In each case, the depth of wear was observed. The samples of material 1 gave an average (coded) wear of 85 units with a sample standard deviation of 4, while the samples of material 2 gave an average of 81 with a sample standard deviation of 5. Can we conclude at 0.05 level of significance that the abrasive wear of material 1 exceeds that of material 2 by more than 2 units? Assume
the populations to be approximately normal with equal variances.

Q.2. Given data

for M1 : $n_1 = 12$, $\bar{x}_1 = 85$, $s_1 = 4$

M2 : $n_2 = 10$, $\bar{x}_2 = 81$, $s_2 = 5$

1. Formulation Hypothesis

null Hypothesis $H_0: \mu_1 - \mu_2 \leq 2$

vs Alternative hypothesis $H_1: \mu_1 - \mu_2 > 2$

H_0: The mean abrasive wear of material 1 is equal to or less than mean abrasive of material 2 by no more than 2 units

H_1: The mean abrasive wear of material 1 exceeds the mean abrasive wear of material 2 by more than 2 units

2. Significance level : $\alpha = 0.05$

3. Student's t-distribution : Unpaired T test for the diff. in means.

4. Test-Statistic :

$$t_{obs} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{(\alpha, n_1 + n_2 - 2)}$$

$$S_p^2 = \frac{(n_1 - 1) s_1^2 + (n_2 - 1) s_2^2}{(n_1 + n_2 - 2)}$$

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$$S_p^2 = \frac{(12-1)\times 4^2 + (10-1)\times 5^2}{12+10-2}$$

$$= \frac{(11\times 16) + (9\times 25)}{20}$$

$$= \frac{176+225}{20}$$

$$S_p^2 = 20.05$$

$$S_p = 4.48$$

$$t_{obs} = \frac{(x_1 - x_2) - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$= \frac{85-81-2}{4.48\sqrt{\frac{1}{12}+\frac{1}{10}}} = \frac{2}{4.48\times\sqrt{0.1833}}$$

$$t_{obs} = 1.0427$$

For $\alpha = 0.05$, $t_{(\alpha,20)} = t_{(0.05,20)} = 1.7247$

Degree of freedom(df) $= n_1 + n_2 - 2$
$= 12+10-2 = 20$

5. Rejection Criteria:

Since $H_0: \mu_1 - \mu_2 \le 2$ vs $H_1: \mu_1 - \mu_2 > 2$ is a one tailed test hypothesis,

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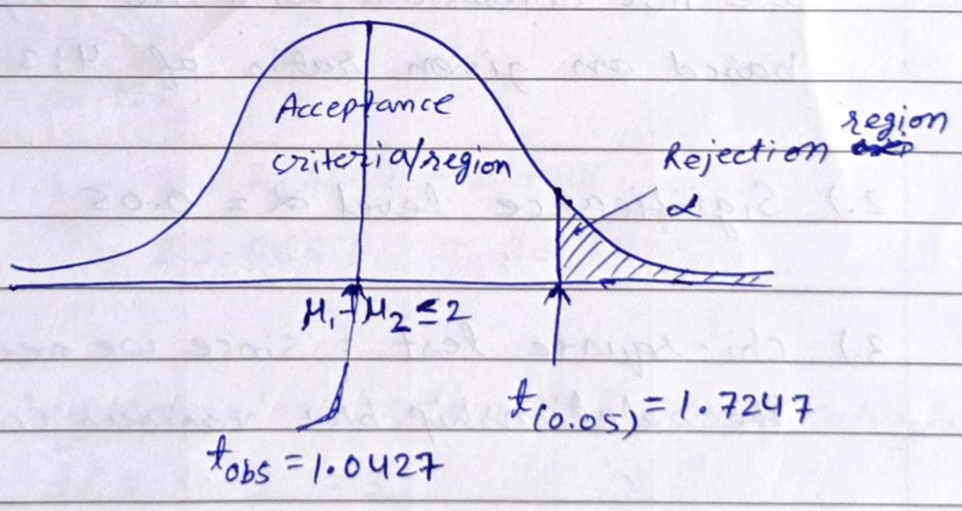
Reject H_0 if $t_{obs} > t_{(0.05, 20)}$
otherwise Accept H_0 if $t_{obs} \leq t_{(0.05, 20)}$

6. Conclusion:

Since $t_{obs} = 1.0427 < t_{(0.05, 20)} = 1.7247$
we fail to reject the null hypothesis H_0.

There is insufficient evidence to conclude that the abrasive wear of material 1 exceeds that of material 2 by more than 2 units at the 0.05 level of significance.



A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured a third class, 90 were placed in second class and 20 got a first class. Do these figures commensurate with the general examination result which is in ratio of 4:3:2:1 for the various categories respectively? Test at 5% Level of Significance.

Q.3

Given data: $n = 500$

Obervations, failed $= 220 = O_1$

3^{rd} class $= 170 = O_2$

2^{nd} class $= 90 = O_3$

1^{st} class $= 20 = O_4$

$\alpha = 0.05$

1.) Null Hypothesis Ho : The oberved frequencies are consistent with the expected frequencies based on given ratio of 4:3:2:1

Alternative Hypothesis H_1 : The observed freq. are not consistent with the expected freq. based on given ratio of 4:3:2:1

2.) Significance level $\alpha = 0.05$

3.) Chi-square test : Since we need to measure the ralationship b/w various categories.

4.) Calculation of expected frequencies:

$$E_1 = \frac{4}{10} \times 500 = 200$$

$$E_2 = \frac{3}{10} \times 500 = 150$$

$$E_3 = \frac{2}{10} \times 500 = 100$$

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$$E_4 = \frac{1}{10} \times 500 = 50$$

5.) Calculation of chi-square statistic:

class	O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
Failed	220	200	20	400	2
3rd	170	150	20	400	2.6667
2nd	90	100	-10	100	1
1st	20	50	-30	900	18
Total	500	500			$\chi^2 = 23.6667$

$$\chi^2 = \sum_{i=1}^{4} \frac{(O-E)^2}{E}$$

$$= 2 + 2.6667 + 1 + 18$$

$$= 23.6667 \simeq 23.67$$

6.) Degree of freedom df: no. of categories - 1

$df = 4 - 1 = 3$

7) Critical value at $\alpha = 0.05$ and $df = 3$

$$\chi^2_{(0.05, 3)} = 7.81$$

$$\chi^2 = 23.67 > \chi^2_{(0.05, 3)} = 7.81$$

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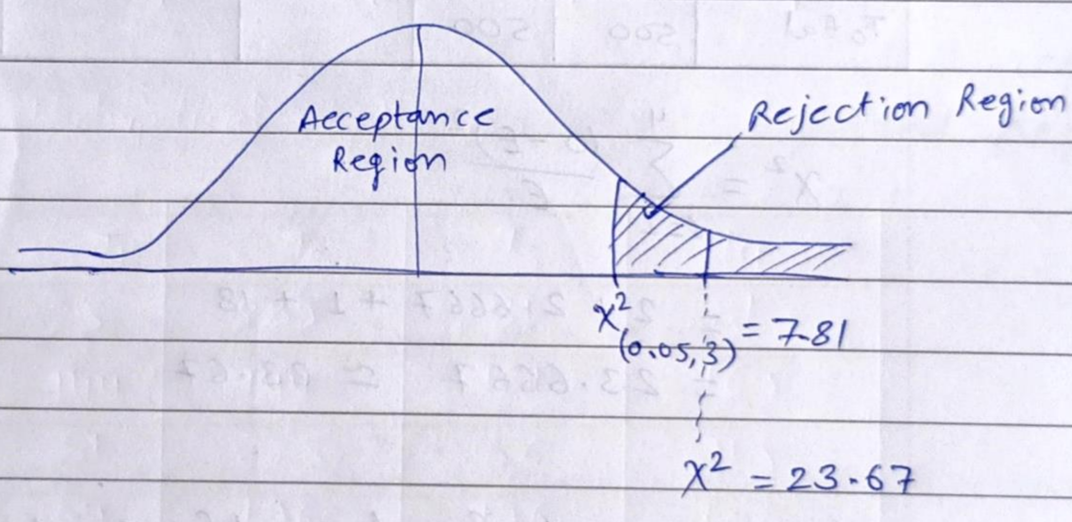
Inference:

Since the chi-square statistic $\chi^2 = 23.67$ is greater than critical value $\chi^2_{(0.05,3)} = 7.81$, we reject the null hypothesis.

This means that the observed frequencies are not consistent with the expected frequencies based on given ratio of 4:3:2:1.



4. The following are the numbers of mistakes made in 5 successive days for 4 technicians working for a photographic laboratory:

Technician I	Technician II	Technician III	Technician IV
5	17	9	9
12	12	11	13
9	15	6	7
8	14	14	10
11	17	10	11

Test at the level of significance $\alpha = 0.01$ whether the differences among the 4-sample means can be attributed to chance.

Q.4. Step 1: Formulation of Hypothesis

Null Hypothesis $H_0 = \mu_1 = \mu_2 = \mu_3 = \mu_4$

That is, there is no difference among 4-sample means can be attributed to chance

Alternative Hypothesis $H_1 : \mu_i \neq \mu_j$ for atleast one pair (i, j); $i \neq j$

$i, j = 1, 2, 3, 4$

That is, there is difference ~~among~~ in mean at least one pair of technicians.

Step 2: Data

T1	T2	T3	T4
5	17	9	9
12	12	11	13
9	15	6	7
8	14	14	10
11	17	10	11

Step 3: Level of Significance $\alpha = 0.01$

Step 4: Test statistic

$$F_{obs} = \frac{MST}{MSE}$$

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Step 5: Calculation of Test Statistic

	T_1	T_2	T_3	T_4	
	5	17	9	9	
	12	12	11	13	
	9	15	6	7	
	8	14	14	10	
	11	17	10	11	
	$x_{1\cdot} = 45$	$x_{2\cdot} = 75$	$x_{3\cdot} = 50$	$x_{4\cdot} = 50$	$\Sigma x_{i\cdot} = 220 = G$
Square	2025	5625	2500	2500	12650

Individual Squares

	T_1	T_2	T_3	T_4	
	25	289	81	81	
	144	144	121	169	
	81	225	36	49	
	64	196	196	100	
	121	289	100	121	
Total	435	1143	534	520	2632

$$G = 45 + 75 + 50 + 50 = 220$$

$$\overline{x}_G = \frac{220}{20} = 11$$

$$\Sigma\Sigma x_{ij}^2 = 2632$$

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1. Correction Factor :

$$CF = \frac{G^2}{n} = \frac{(220)^2}{20} = \frac{48400}{20}$$

$$CF = 2420$$

2. Total Sum of Squares :

$$TSS = \sum\sum x_{ij}^2 - CF$$

$$= 2632 - 2420 = 212$$

3. Sum of Squares b/w Treatments :

$$SST = \frac{\sum x_i^2}{n_i} - CF$$

$$= \frac{12650}{5} - 2420$$

$$= 2530 - 2420 = 110$$

4. Sum of Squares due to Error :

$$SSE = TSS - SST$$

$$= 212 - 110 = 102$$

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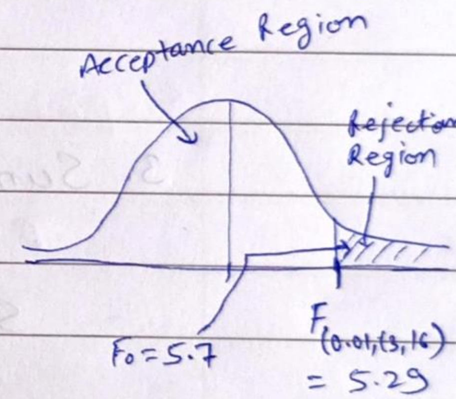
ANOVA Table (one way)

Source of variation	Sum of squares	Degrees of freedom	Mean Sum of squres	F-ratio
Between treatments	SST = 110	$k-1$ $4-1=3$	$MST = \frac{SST}{k-1}$ $\frac{110}{3} = 36.67$	$F_{obs} = \frac{MST}{MSE}$
Error	SSE = 102	$n-k$ $20-4=16$	$MSE = \frac{SSE}{n-k}$ $\frac{102}{16} = 6.375$	$= \frac{36.67}{6.375}$
Total	TSS = 212	$n-1$ $20-1=19$		$= 5.7521$

$F_{obs} = 5.7521$



Step 6: Critical Value

$$F_{(0.01,(3,16))} = 5.29$$

Step 7: Decision

Reject H_0 if $F_{obs} > F_{(0.01,(3,16))}$
otherwise Accept H_0

As $F_{obs} = 5.7521 > F_{(0.01,(3,16))} = 5.29$
the null Hypothesis H_0 is rejected. That means ~~the~~ there is difference in mean atleast one pair of technicians.