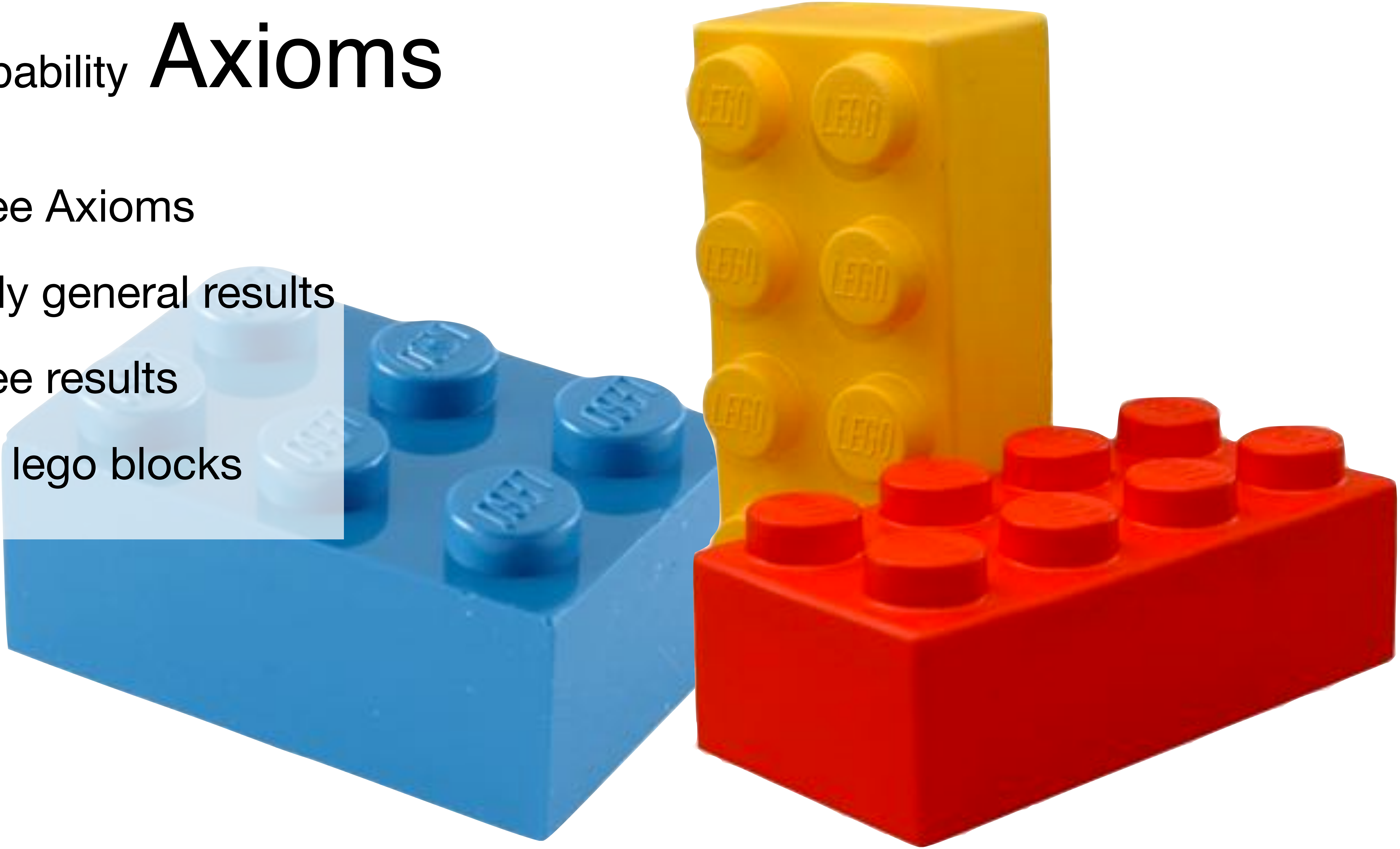
Probability Axioms

Three Axioms

Imply general results

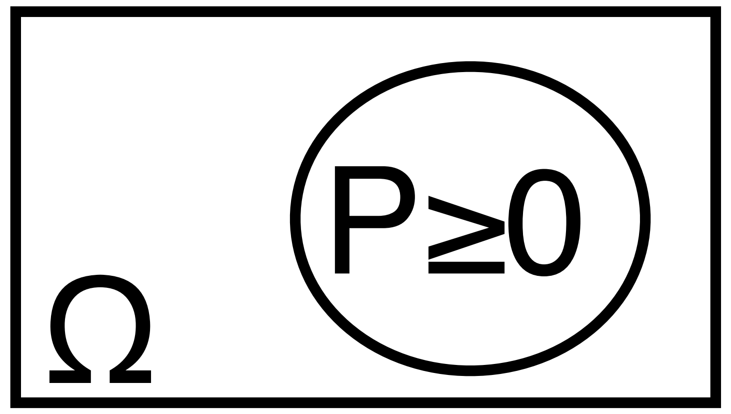
Three results

Use lego blocks



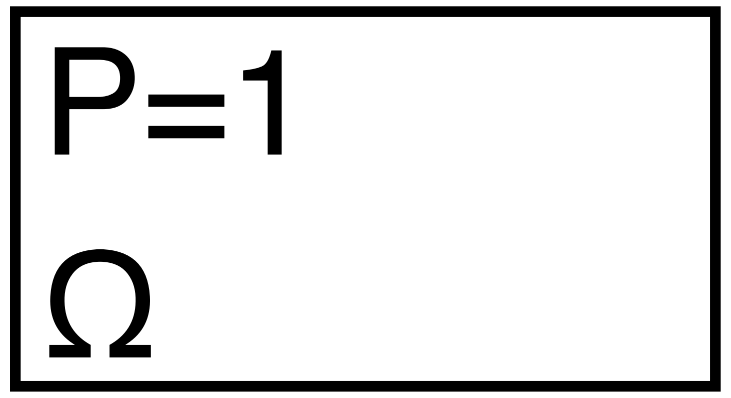
Three Axioms

Non-negativity $P(A) \geq 0$

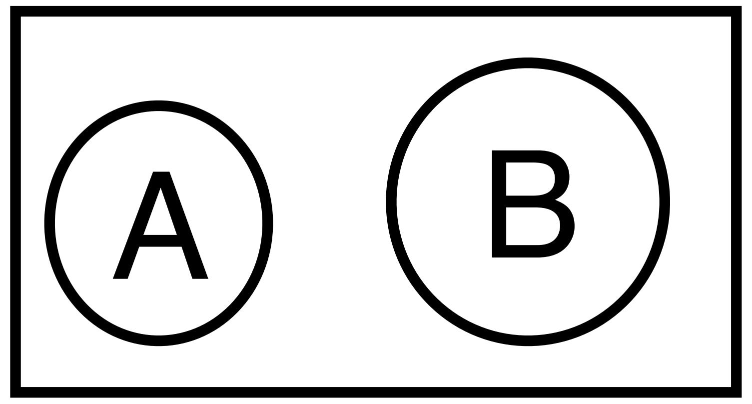


Short for $\forall A \;\; P(A) \geq 0$

Unitarity $P(\Omega) = 1$



Addition rule A, B disjoint $\rightarrow$ $P(A \cup B) = P(A) + P(B)$



$A_1, A_2, \ldots$ disjoint $\rightarrow$ $P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots$

Countable unions only

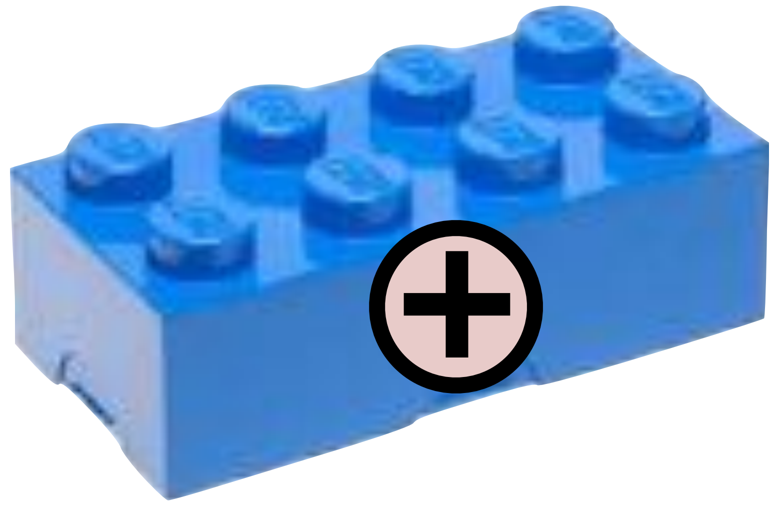
Uncountable later

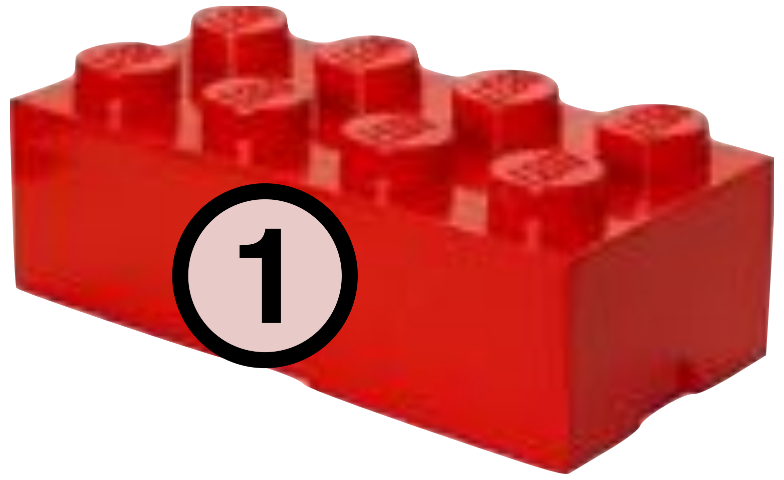
“Building” Results

Use lego blocks

construct three simple results

Complement rule

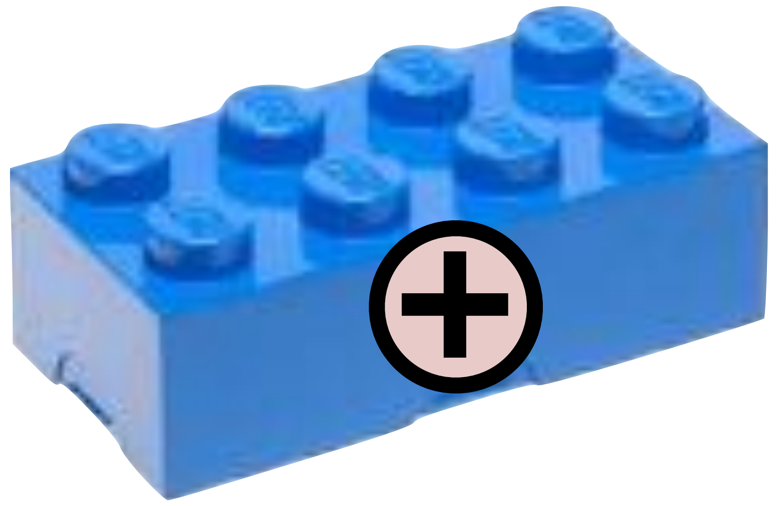




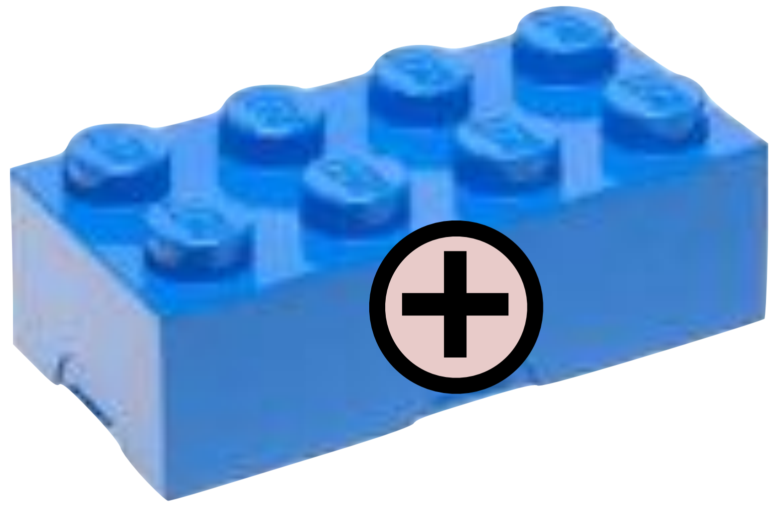
Subtraction

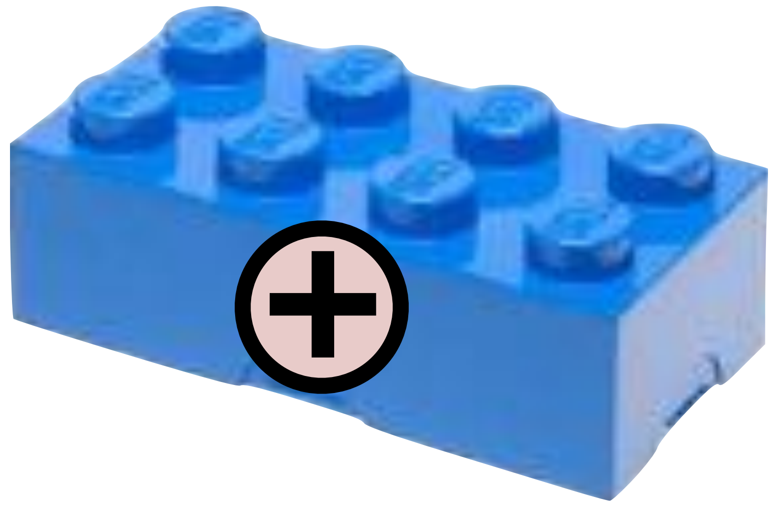
nested sets

general sets



Inclusion-exclusion





Complement Rule

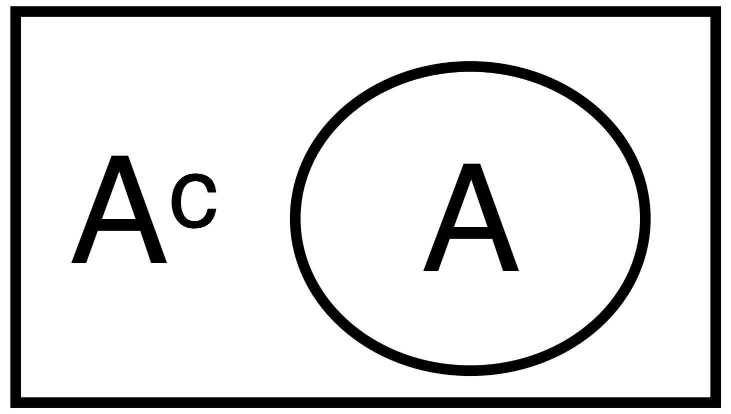
Complement rule for counting

$|A^c| = |\Omega| - |A|$

Same for 

$A \cup A^c = \Omega$

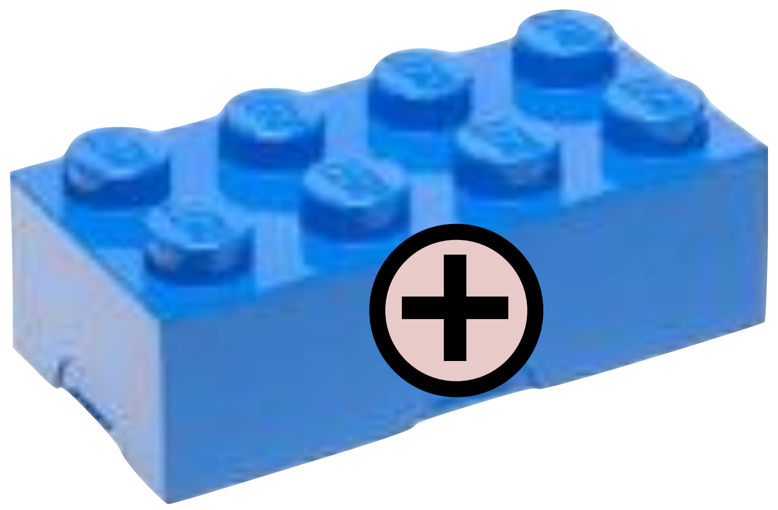
Ω

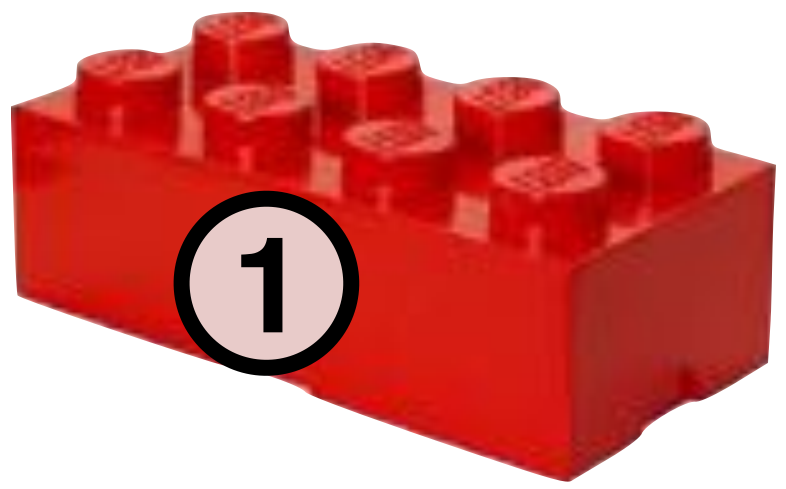


$P(A) + P(A^c) = P(A \cup A^c) = P(\Omega) = 1$

$P(A^c) = 1 - P(A)$

Complement rule for probability

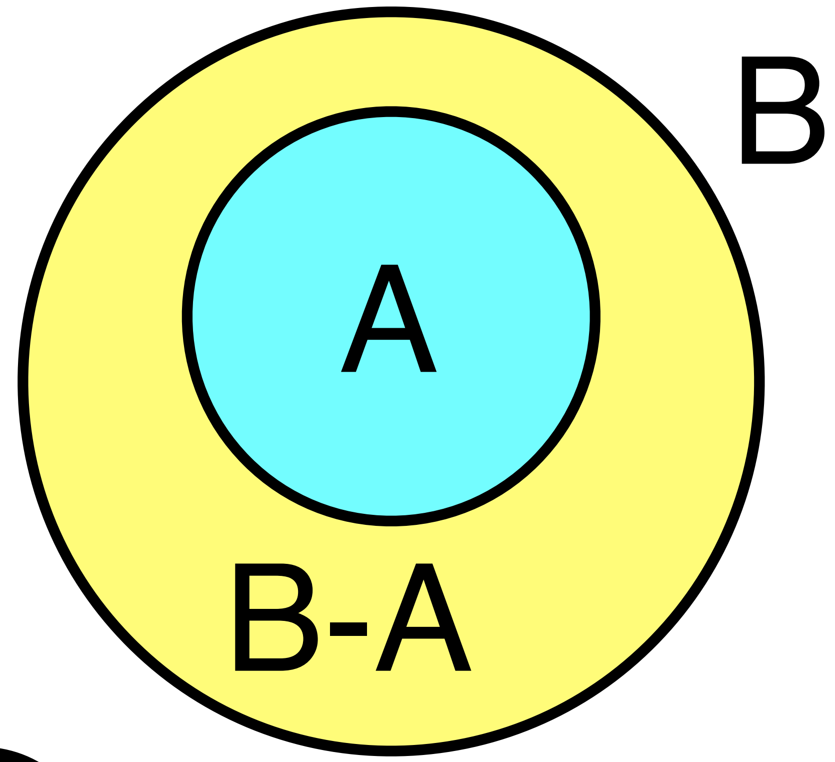


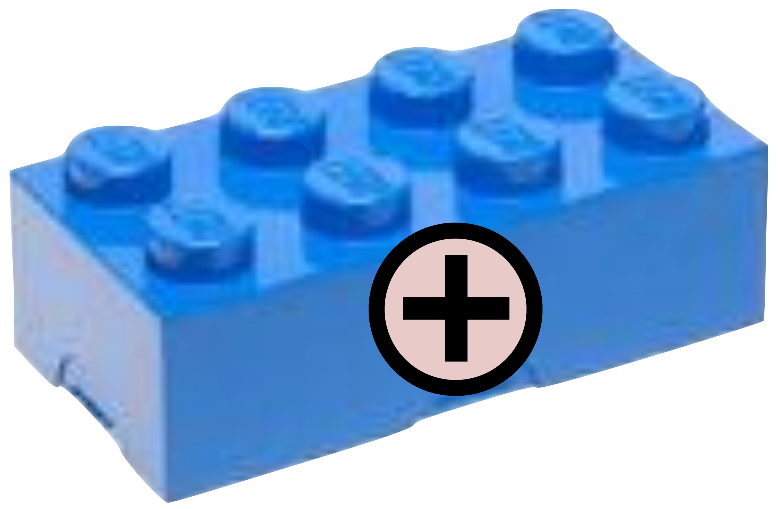


Subtraction Rule - Nested Sets

Complement rule $A \subseteq \Omega$ $P(A^c) = 1 - P(A)$ $P(\Omega - A) = P(\Omega) - P(A)$

Generalize $A \subseteq B$ $\rightarrow$ $P(B-A) = P(B) - P(A)$





$A \subseteq B$ $\rightarrow$ $B = A \cup (B-A)$

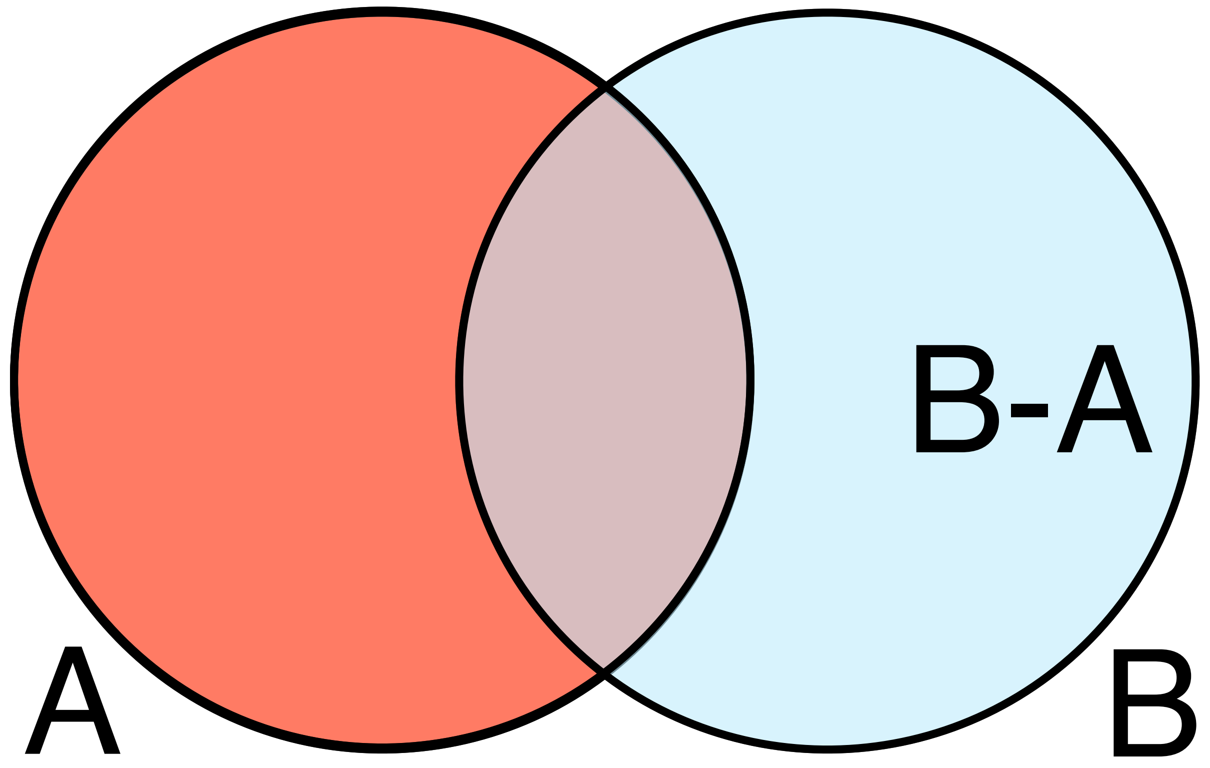
$P(B) = P(\,A \cup (B-A)\,) = P(A) + P(B-A)$

$P(B-A) = P(B) - P(A)$ **Subtraction rule for nested sets**

Subtraction Rule - General Sets

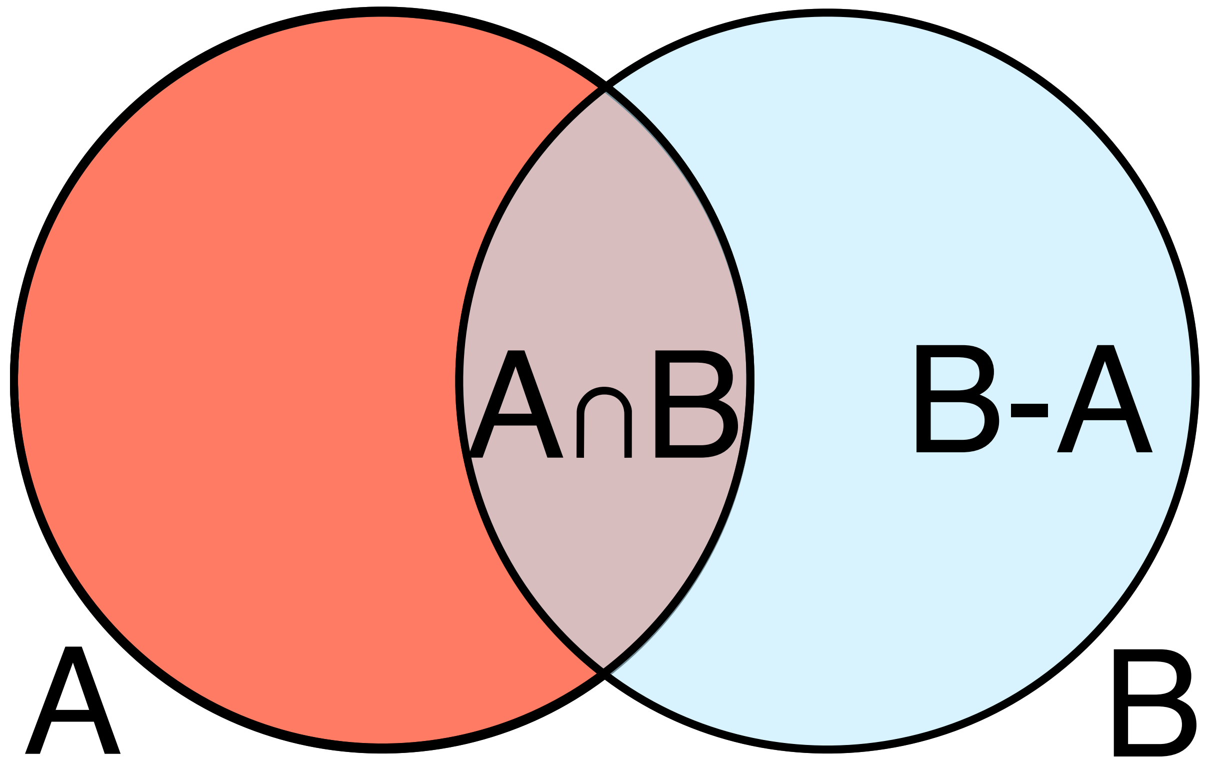
Nested $A \subseteq B$ $P(B-A) = P(B) - P(A)$

General $\forall A, B$ $P(B-A) = P(B) - P(A \cap B)$

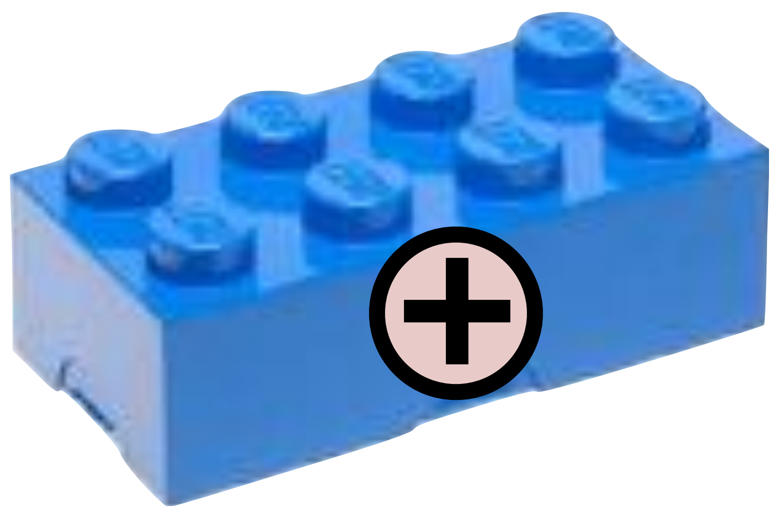


$B - A = B - (A \cap B)$

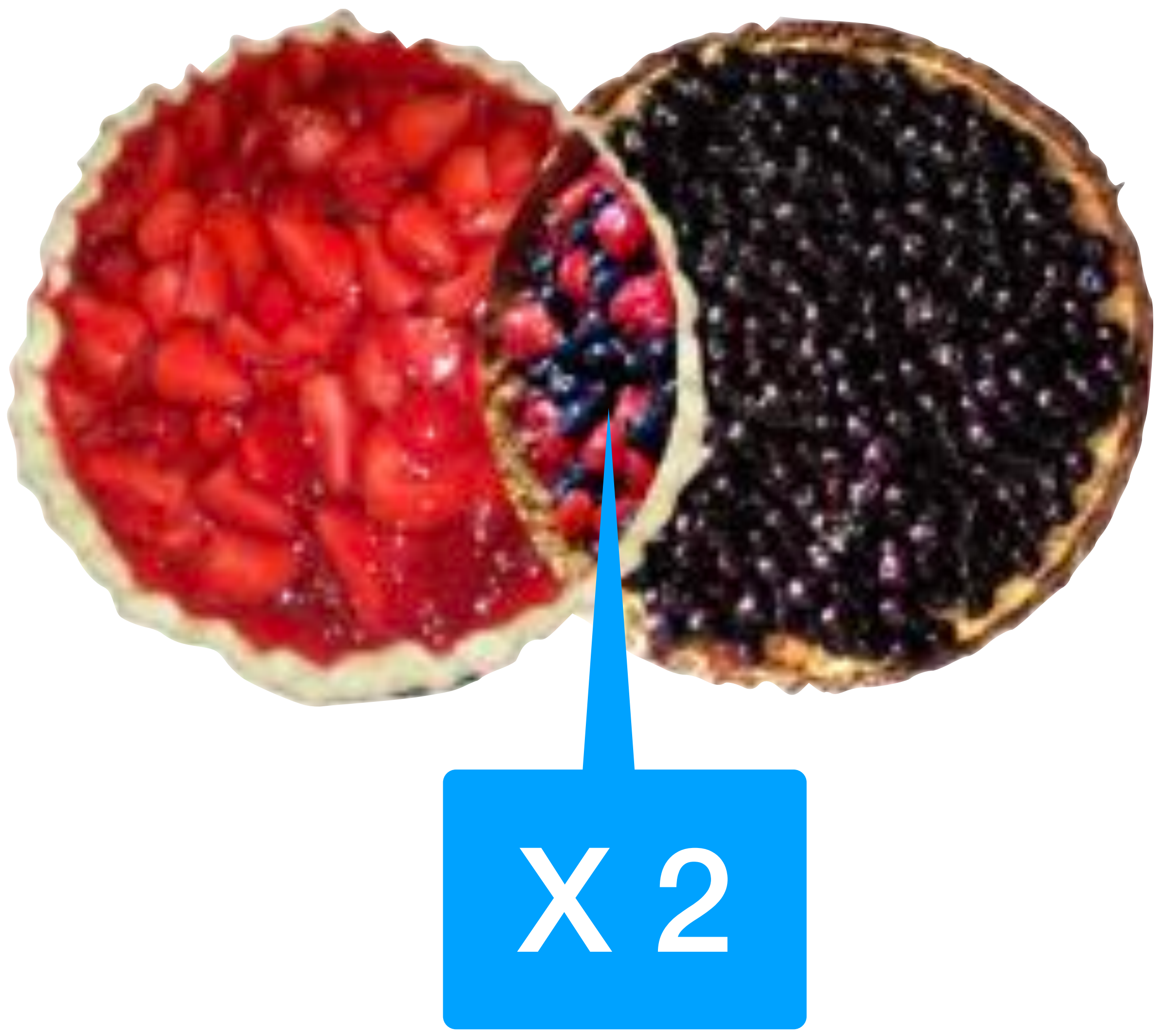
$A \cap B \subseteq B$



$P(B - A) = P(\, B - (A \cap B)\,) = P(B) - P(A \cap B)$

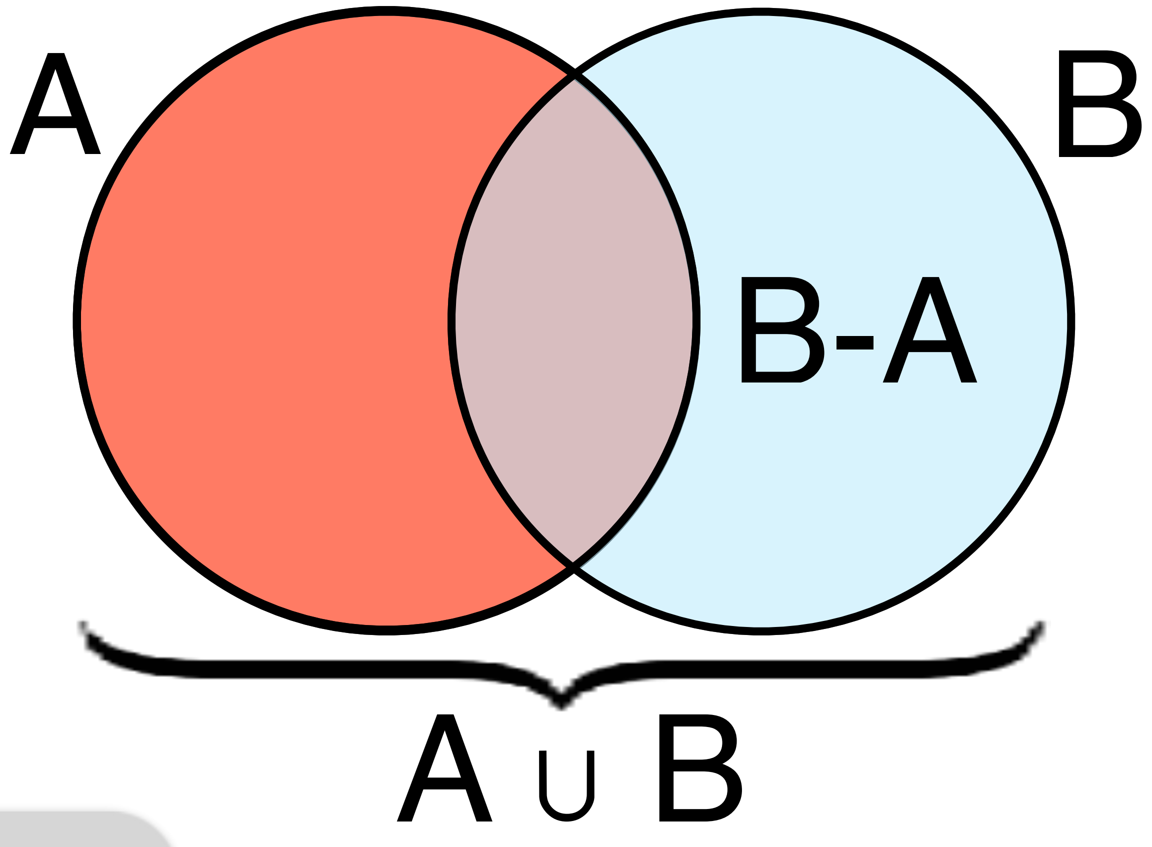


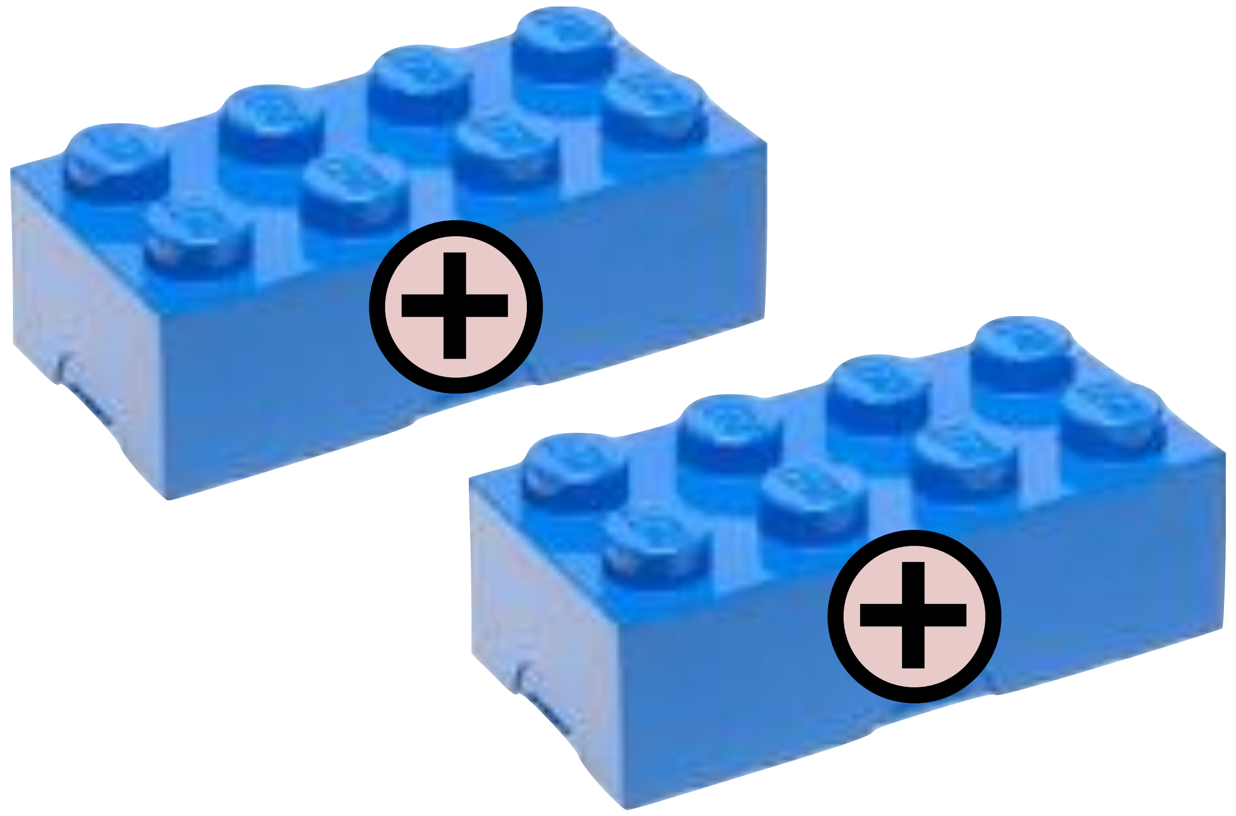
Inclusion Exclusion



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$A \cup B = A \cup B\text{-}A$$





$$P(A \cup B) = P(A \cup B\text{-}A)$$

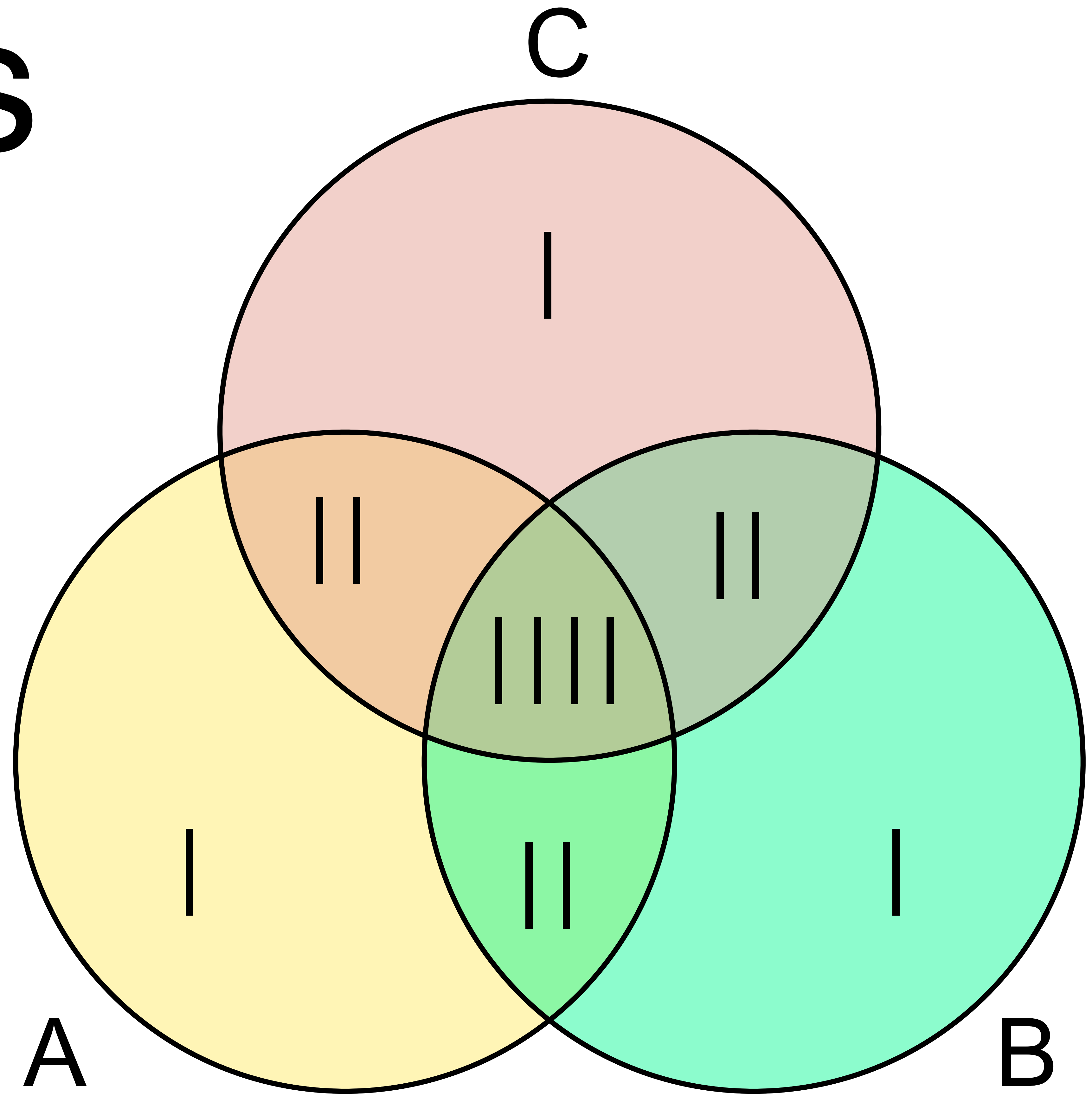
$$= P(A) + P(B\text{-}A)$$

General subtraction

$$P(B\text{-}A) = P(B) - P(A \cap B)$$

$$= P(A) + P(B) - P(A \cap B)$$

More Sets



Two sets

P(A∪B) = P(A) + P(B) - P(A∩B)

Three sets

P(A∪B∪C) = P(A) + P(B) + P(C)
- P(A ∩ B) - P(A ∩ C) - P(B ∩ C)
+ P(A ∩ B ∩ C)

n sets

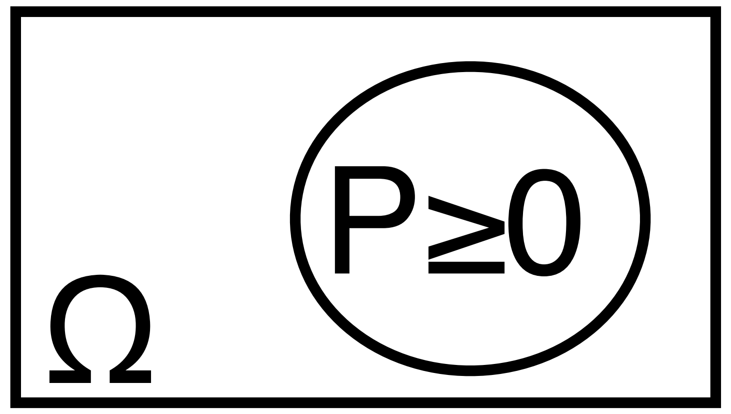
$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{1\leq i\leq n} P(A_i) - \sum_{1\leq i<j\leq n} P(A_i \cap A_j) + \ldots + (-1)^{n-1} P\left(\bigcap_{i=1}^{n} A_i\right)$$

Probability **Inequalities**

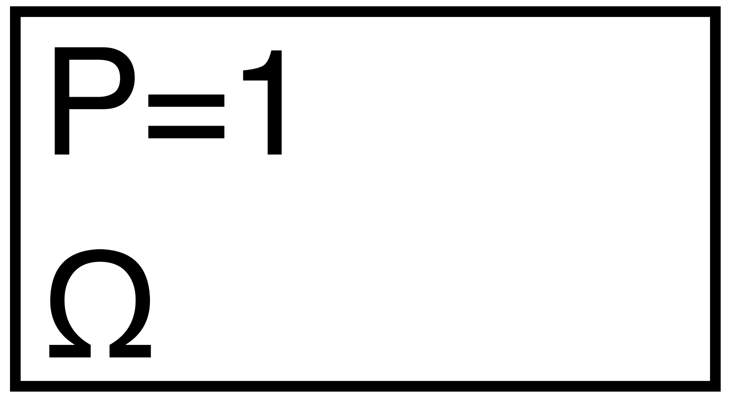
- Three Axioms
- Inequalities
- Union bound
- $1.4M question

Three Axioms

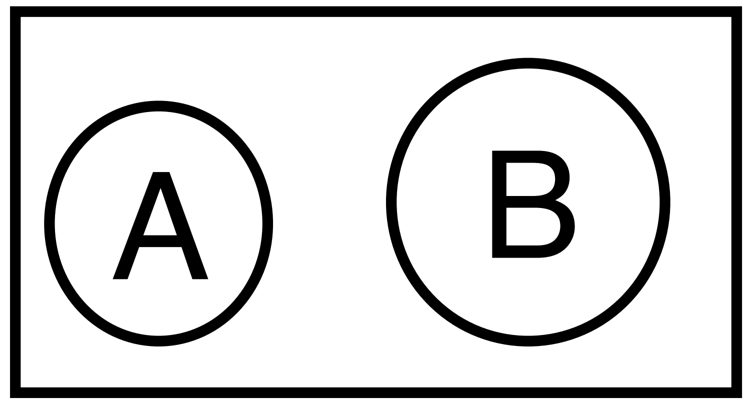
Non-negativity | $P(A) \geq 0$



Unitarity | $P(\Omega) = 1$



Addition rule | A, B disjoint $\rightarrow$ $P(A \cup B) = P(A) + P(B)$



$A_1, A_2, \ldots$ disjoint $\rightarrow$ $P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots$

Equalities $\rightarrow$ Inequalities

Probability of Null Event

① $P(\Omega) = 1$

What about $P(\varnothing)$?

Show $P(\varnothing) = 0$

$\Omega = \varnothing \cup \Omega$

$$\cancel{P(\Omega)} = P(\varnothing \cup \Omega) \stackrel{+}{=} P(\varnothing) + \cancel{P(\Omega)}$$

$P(\varnothing) = 0$

The null event has nil probability

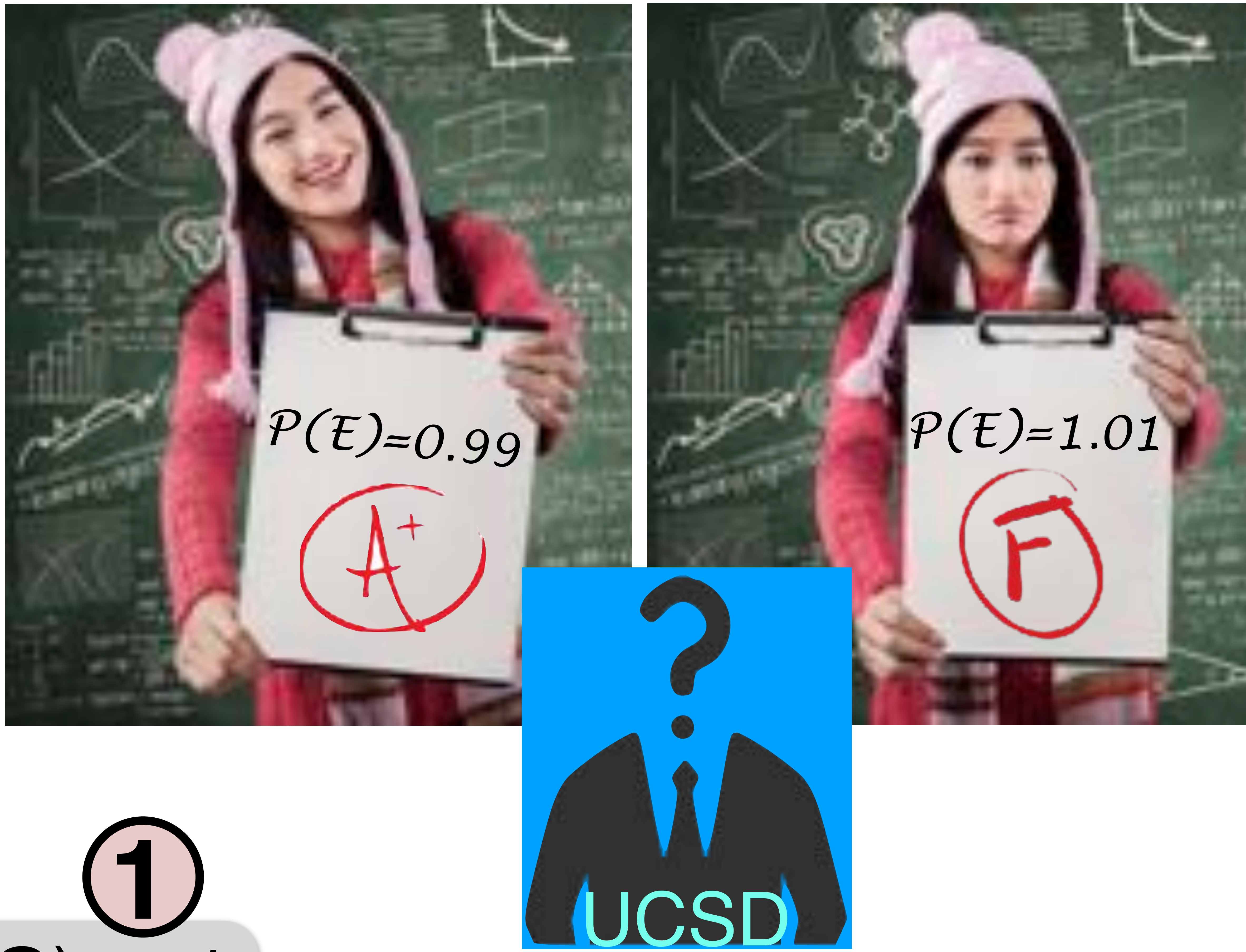
$$0 \le P(A) \le 1$$

$\geq$ $P(A) \geq 0$ Show $P(A) \leq 1$

$A \cup A^c = \Omega$

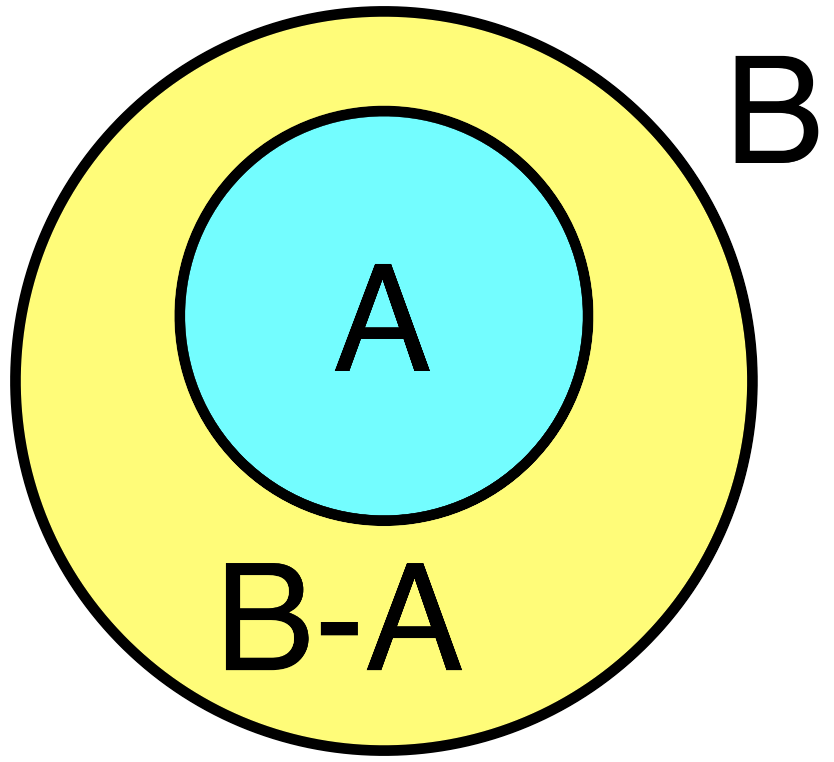
$$P(A) \overset{\geq}{\leq} P(A) + P(A^c) \overset{+}{=} P(A \cup A^c) = P(\Omega) \overset{1}{=} 1$$

$P(A) \leq 1$ Probability always between 0 and 1



Subset

$A \subseteq B$ $P(A) \leq P(B)$



$A \subseteq B$ $B = A \cup (B-A)$

$$P(B) = P(\,A \cup (B-A)\,) = P(\,A) + P(B-A) \geq P(A)$$

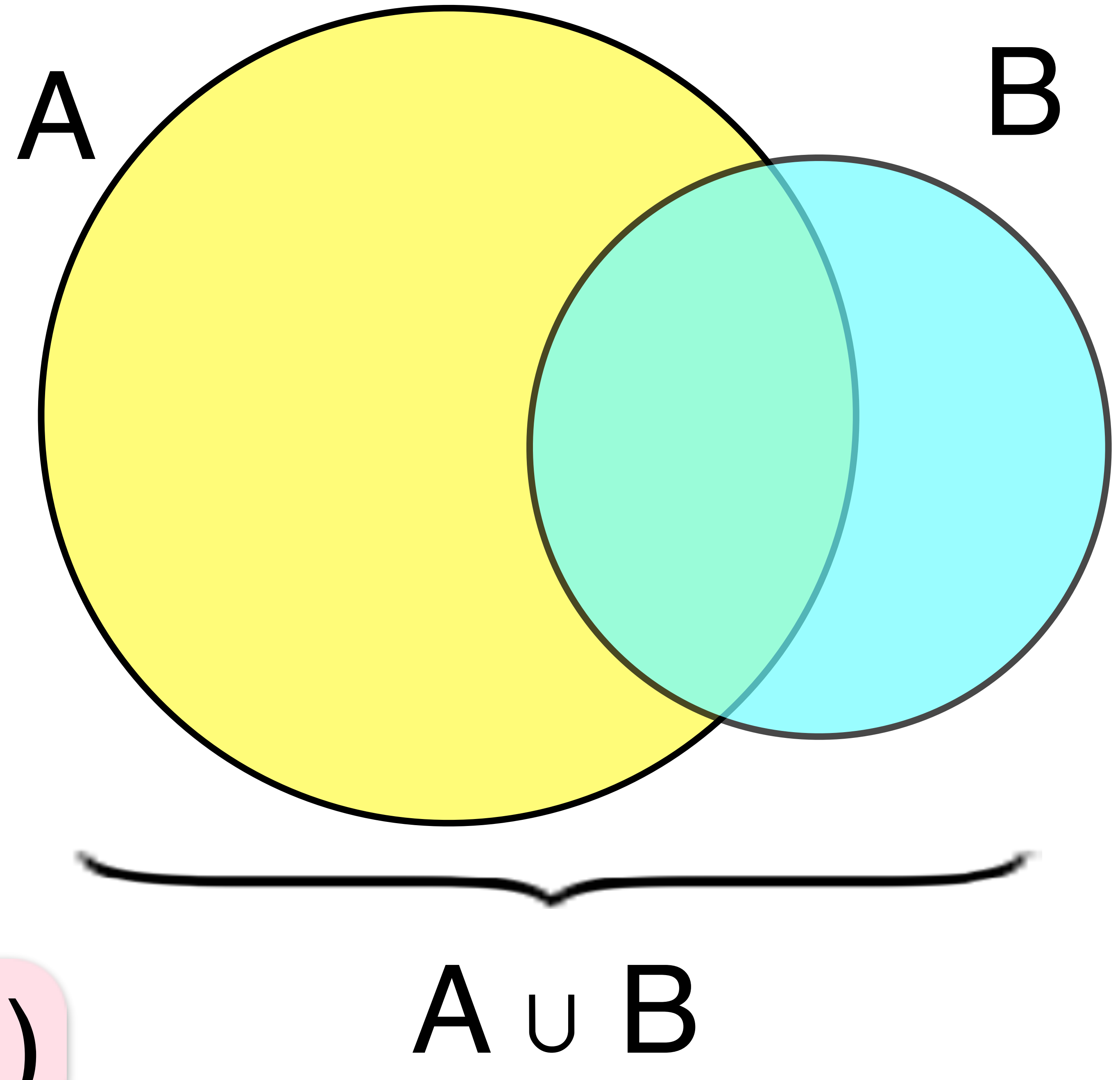
Obvious?

Union

$\max(P(A),P(B)) \leq P(A \cup B) \leq P(A) + P(B)$

Left $\leq$ $A, B \subseteq A \cup B$ $P(A), P(B) \leq P(A \cup B)$

Right $\leq$  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



$P(A \cup B) \leq P(A) + P(B)$ Union bound Very useful

$1,400,000 Question

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

1. Linda is a bank teller

2. Linda is a bank teller and is active in the feminist movement

Please answer in the poll below

The Linda Problem

Psychological Review

VOLUME 90 NUMBER 4 OCTOBER 1983

Extensional Versus Intuitive Reasoning:
The Conjunction Fallacy in Probability Judgment

Amos Tversky
Stanford University

Daniel Kahneman
University of British Columbia, Vancouver,
British Columbia, Canada

88 UBC Students

85% Bank teller **&** activist more likely

B - Bank teller

A - Active in feminist movement

$P(B)$ vs. $P(B \cap A)$

$B \supseteq B \cap A$

$P(B) \geq P(B \cap A)$

Irrational concept of probability

Related Questions

Several conjunction fallacy problems

Bjorn Borg Preeminent tennis player of late ’70’s

6 French opens Wimbledon ’76-‘80

1980 Suppose Borg reaches the ’81 Wimbledon Finals.

More likely?

A Borg will lose first set

B Borg will lose first set but win match

72% chose B Again $B \subseteq A$ $P(B) \leq P(A)$

1981 Reached final Won first set Lost match to 

1983 Retired Age 26

The Ultimate Reward

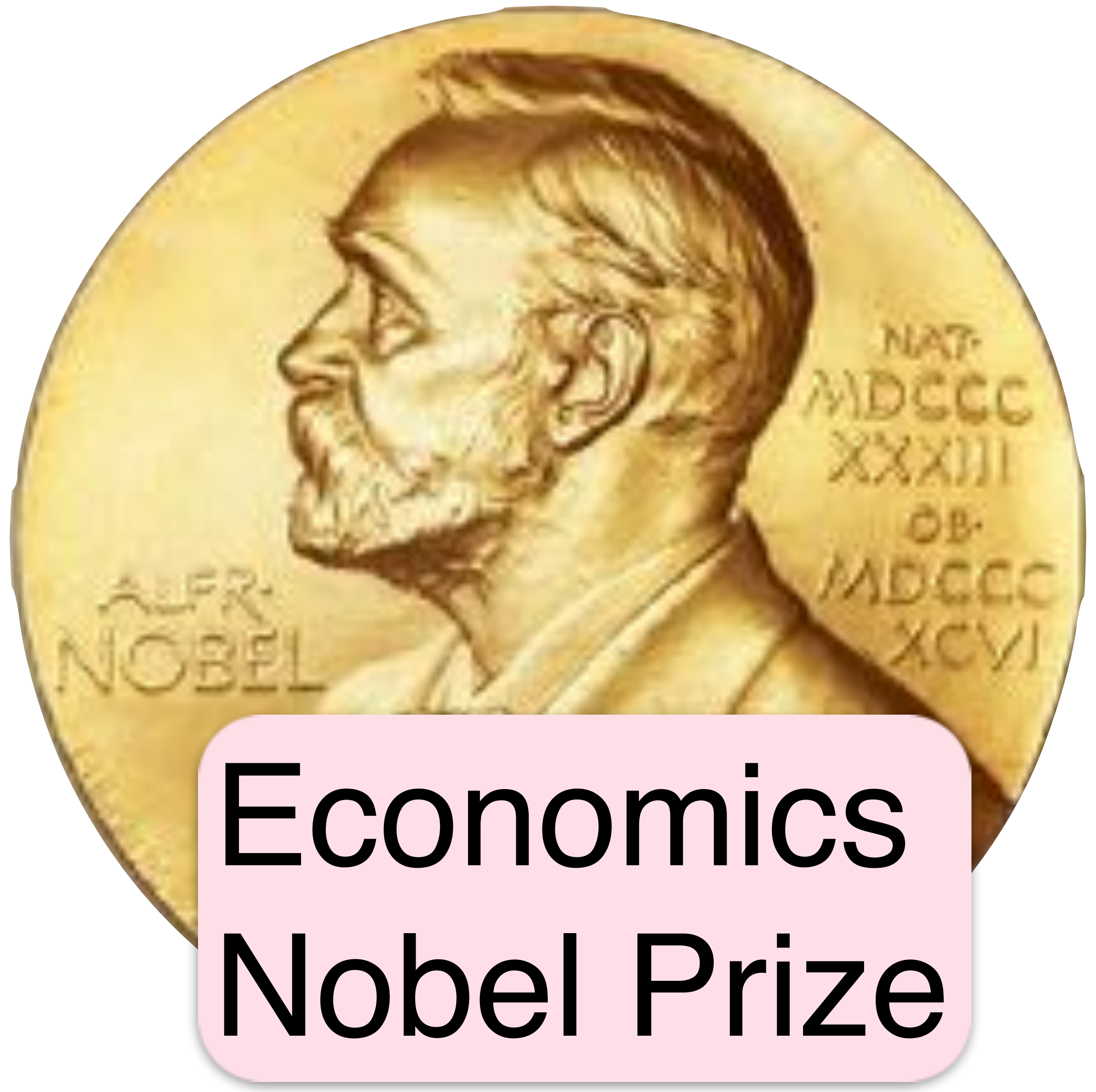
T & K — Many probability-perception studies

1996 — Tversky passed away

2002 — Kahneman

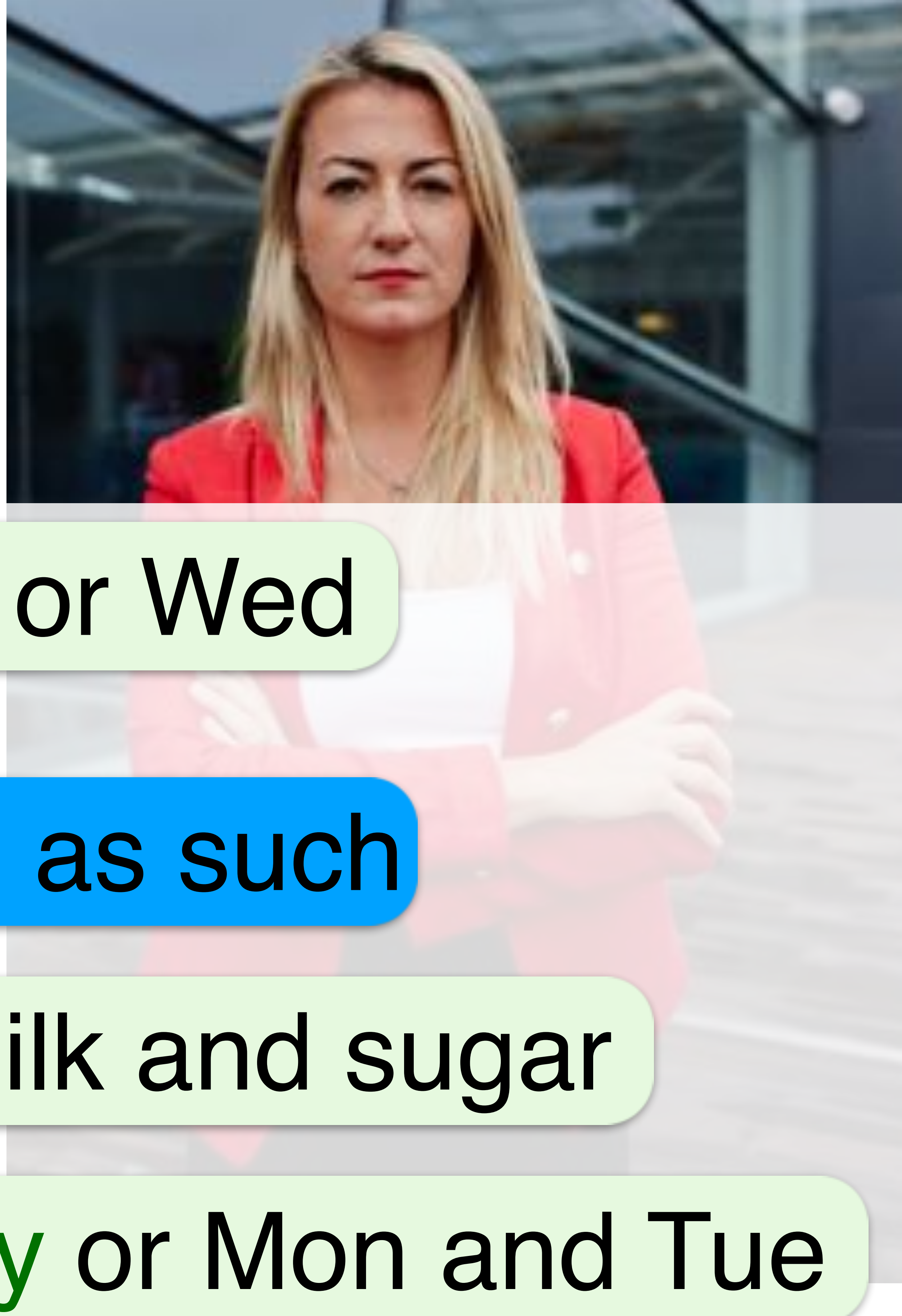
best known

"for integrating insights from psychological research into economic science, especially regarding human judgment and decision-making under uncertainty"



Economics
Nobel Prize

Criticism



Multiple choice and ranking questions often disjoint

Coffee or Tea | Tea hot or cold | Rank Mon, Tue, or Wed

Often, when not explicitly disjoint, we still interpret them as such

With milk or with milk and sugar → Milk only or Milk and sugar

Meet Monday or Monday and Tuesday → Mon only or Mon and Tue

Students may have similarly interpreted the Linda question

Teller or Teller and activist → Teller only or Teller and activist

Simply thought Linda more likely active in movement than not

Bottom Line

Humans only moderately good at estimating probability

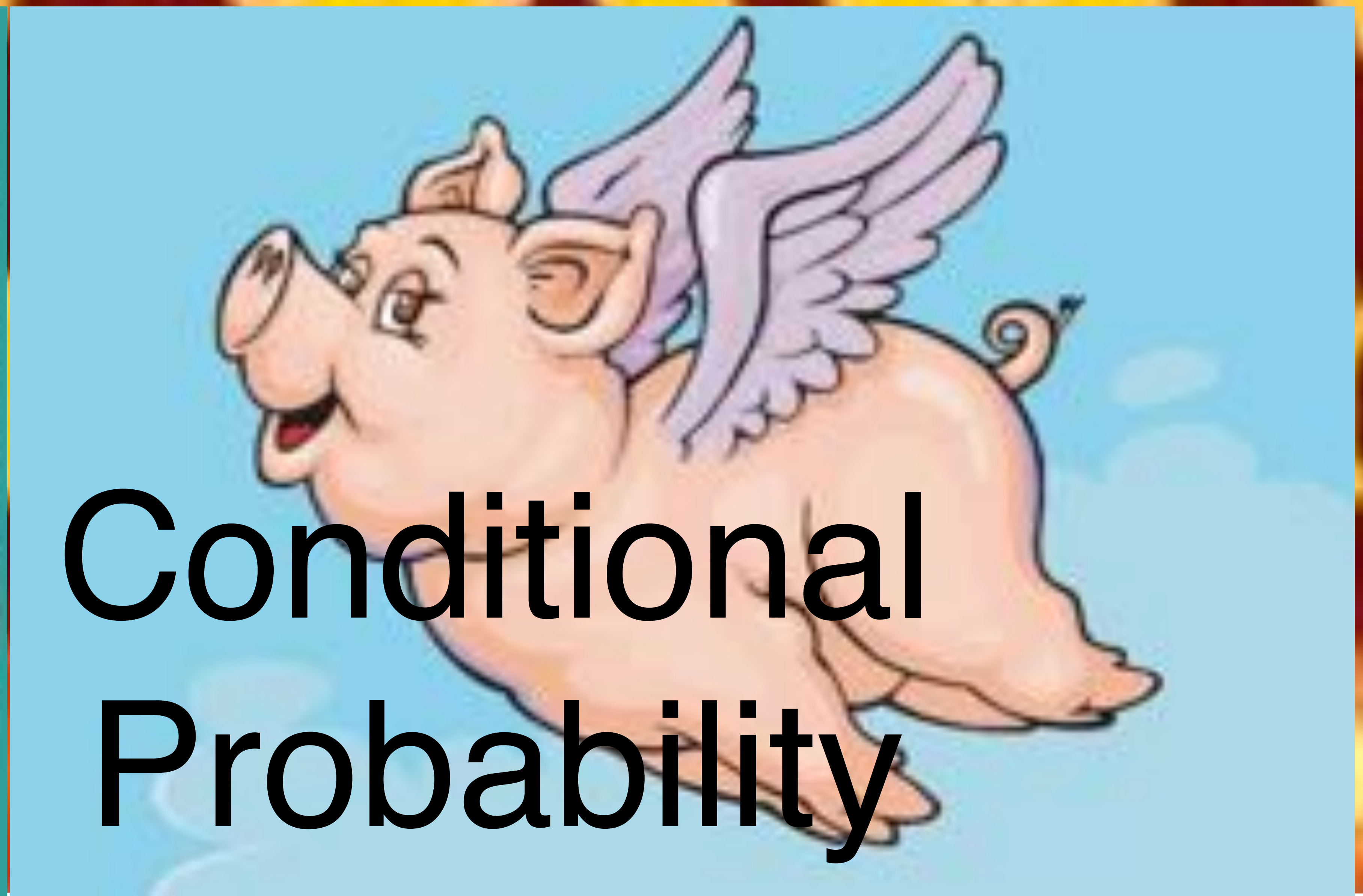
Probability **Inequalities**

Three Axioms

Inequalities

Union bound

$1.4M question



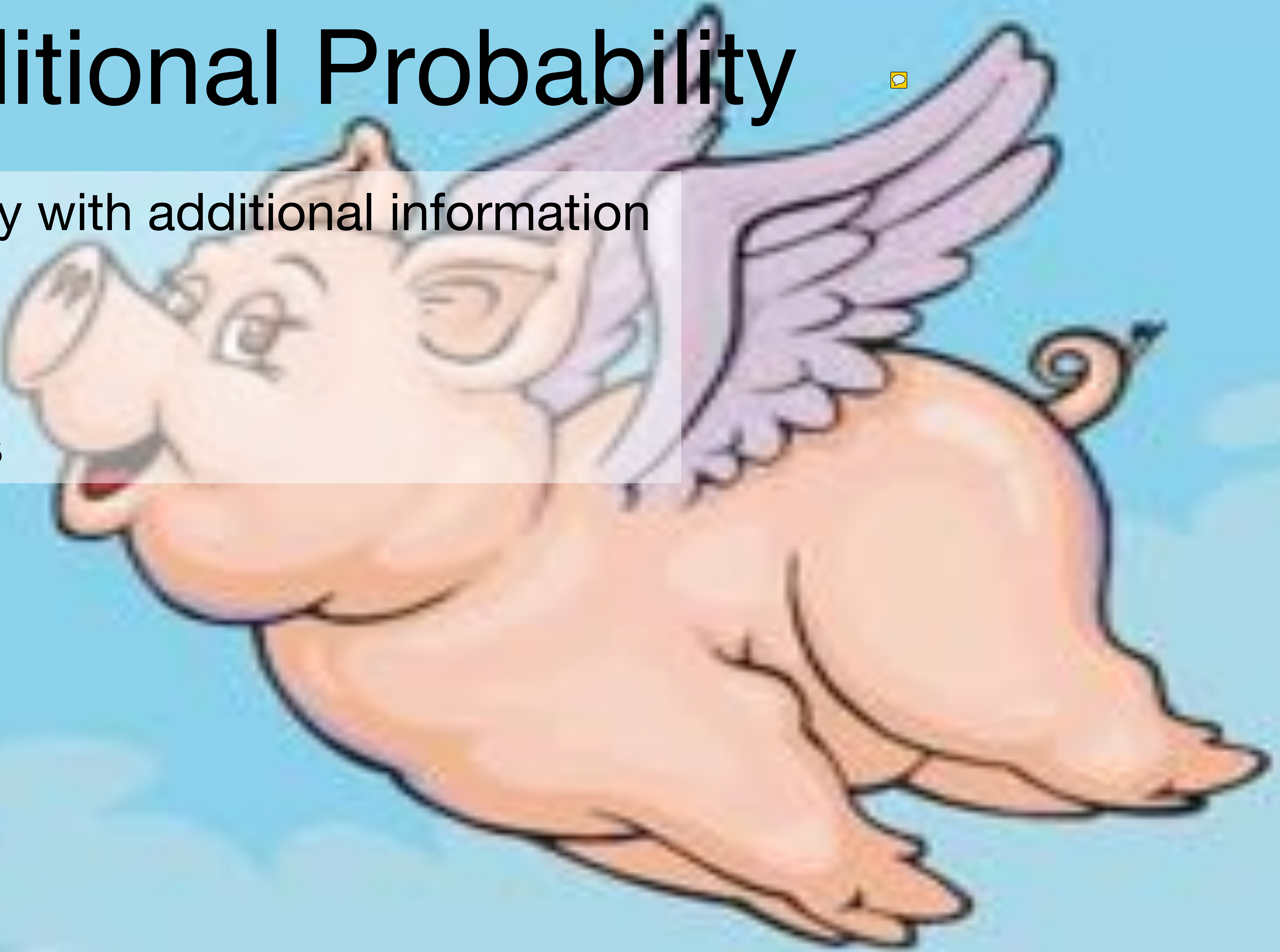
Conditional Probability

Conditional Probability

Probability with additional information

Definition

Examples



Why Condition

- Often have partial information about the world
- Modifies event probabilities
    - Unemployment numbers - stock prices
    - LeBron James injured - Cavaliers game result
    - Sunny weekend - beach traffic
- Can help
    - Improve estimates
    - Determine original unconditional probabilities

Back to Basics

Empirical frequency interpretation of probability

The probability P(E) of an event E is the fraction of experiments where E occurs when the number of experiments grows to infinity

To estimate P(E) repeat the experiment many times, calculate the fraction of experiments where E occurs

Fair Die

$$P(2) = \frac{2}{12} = \frac{1}{6}$$

Estimate

$$\underbrace{\boxed{2}\ 1\ 3\ 6\ 4\ \boxed{2}\ 5\ 4\ 3\ 6\ 5\ 1}_{12}$$

Conditional Probability

Let E and F be events. The conditional probability P(F I E) of F given E is the fraction of times F occurs in experiments where E occurs

To estimate P(F I E) take many samples, consider only experiments where E occurs, and calculate the fraction therein where F occurs too

$$P(2 \mid \text{Even}) = \frac{2}{6} = \frac{1}{3}$$

Even = {2, 4, 6}

$\boxed{2}$ 1 3 6 4 $\boxed{2}$ 5 4 3 6 5 1

Die

$P(\{2\}) = P(2) = 1/6$

$P(2 \mid \text{Odd}) = P(2 \mid \{1,3,5\}) = \frac{0}{6} = 0$

2 1 3 6 4 2 5 4 3 6 5 1

P(2) = ?

$P(\leq 2) = P(\{1,2\}) = 1/3$

P(≤ 2) = ?

$P(\leq 2 \mid \geq 2) = P(\{1,2\} \mid \{2,3,4,5,6\}) = \frac{2}{10} = \frac{1}{5}$

$\boxed{2}$ 1 3 6 4 $\boxed{2}$ 5 4 3 6 5 1

General Events - Uniform Spaces

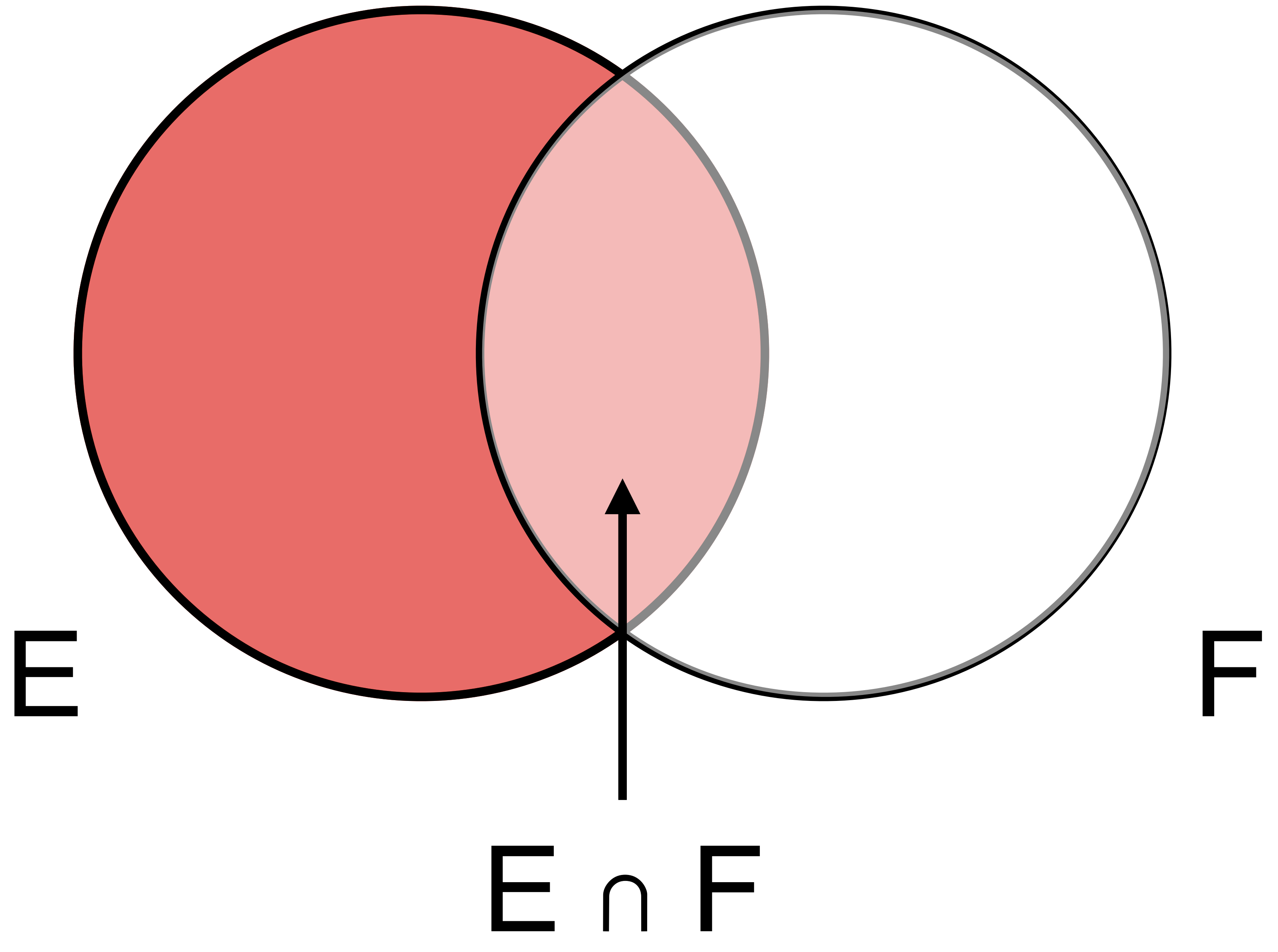
$$P(F \mid E) = P(X \in F \mid X \in E)$$

$$= P(X \in E \text{ and } X \in F \mid X \in E)$$

$$= P(X \in E \cap F \mid X \in E)$$

$$= P(E \cap F \mid E)$$

$$= \frac{|E \cap F|}{|E|}$$



Fair Die Again

$$P(\text{Prime} \mid \text{Odd}) = P(\{2,3,5\} \mid \{1,3,5\})$$

$$= \frac{|\{2,3,5\} \cap \{1,3,5\}|}{|\{1,3,5\}|} = \frac{|\{3,5\}|}{|\{1,3,5\}|} = \frac{2}{3}$$

$$P(\{4\} \mid \text{Prime}) = P(\{4\} \mid \{2,3,5\})$$

$$= \frac{|\{4\} \cap \{2,3,5\}|}{|\{2,3,5\}|} = \frac{|\emptyset|}{|\{2,3,5\}|} = 0$$

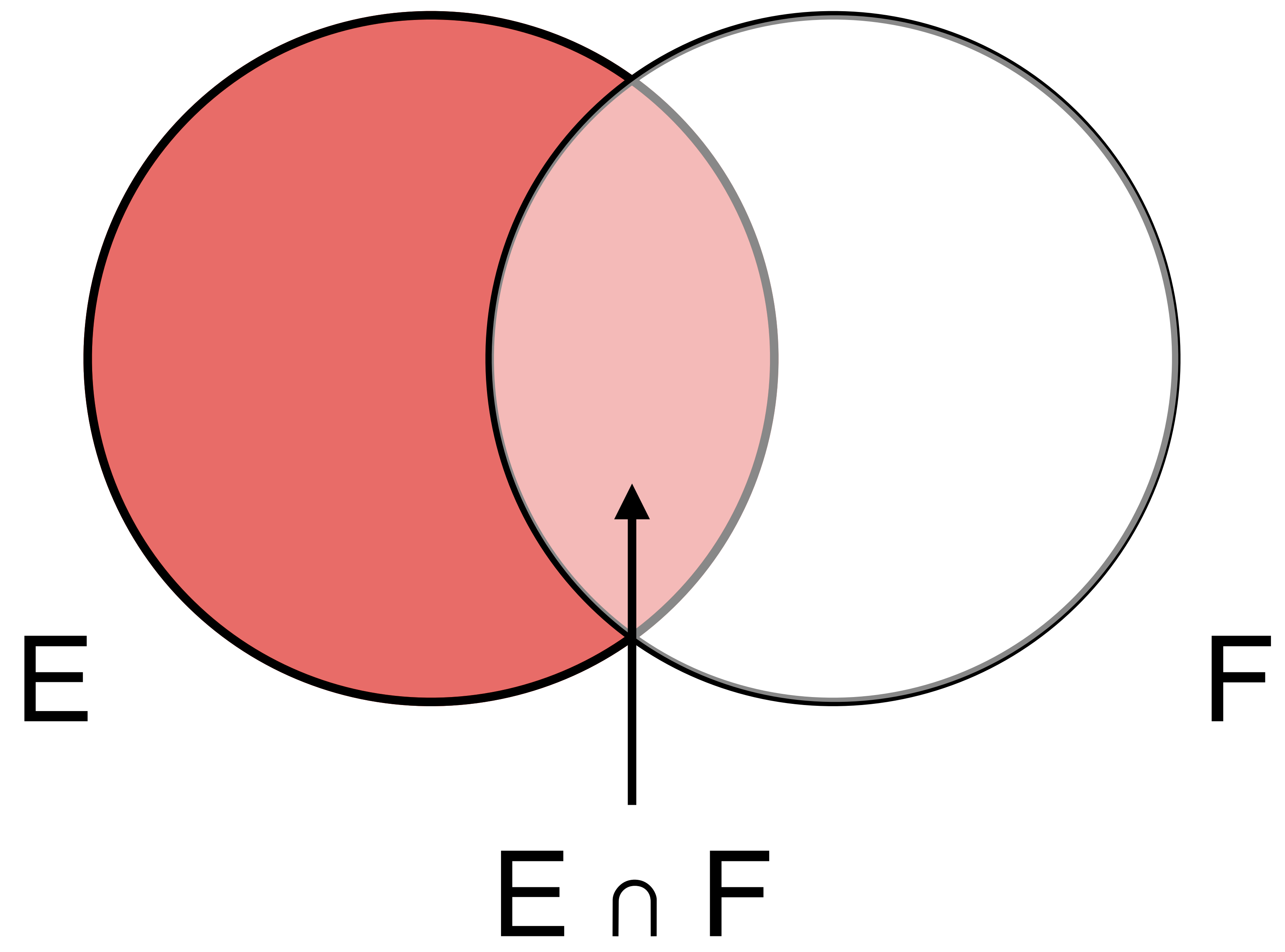
General Spaces

$$P(F \mid E) = P(X \in F \mid X \in E)$$

$$= P[X \in E \cap X \in F \mid X \in E]$$

$$= P[X \in E \cap F \mid X \in E]$$

$$= \frac{P(E \cap F)}{P(E)}$$



4-Sided Die

Face	1	2	3	4
Probability	.1	.2	.3	.4

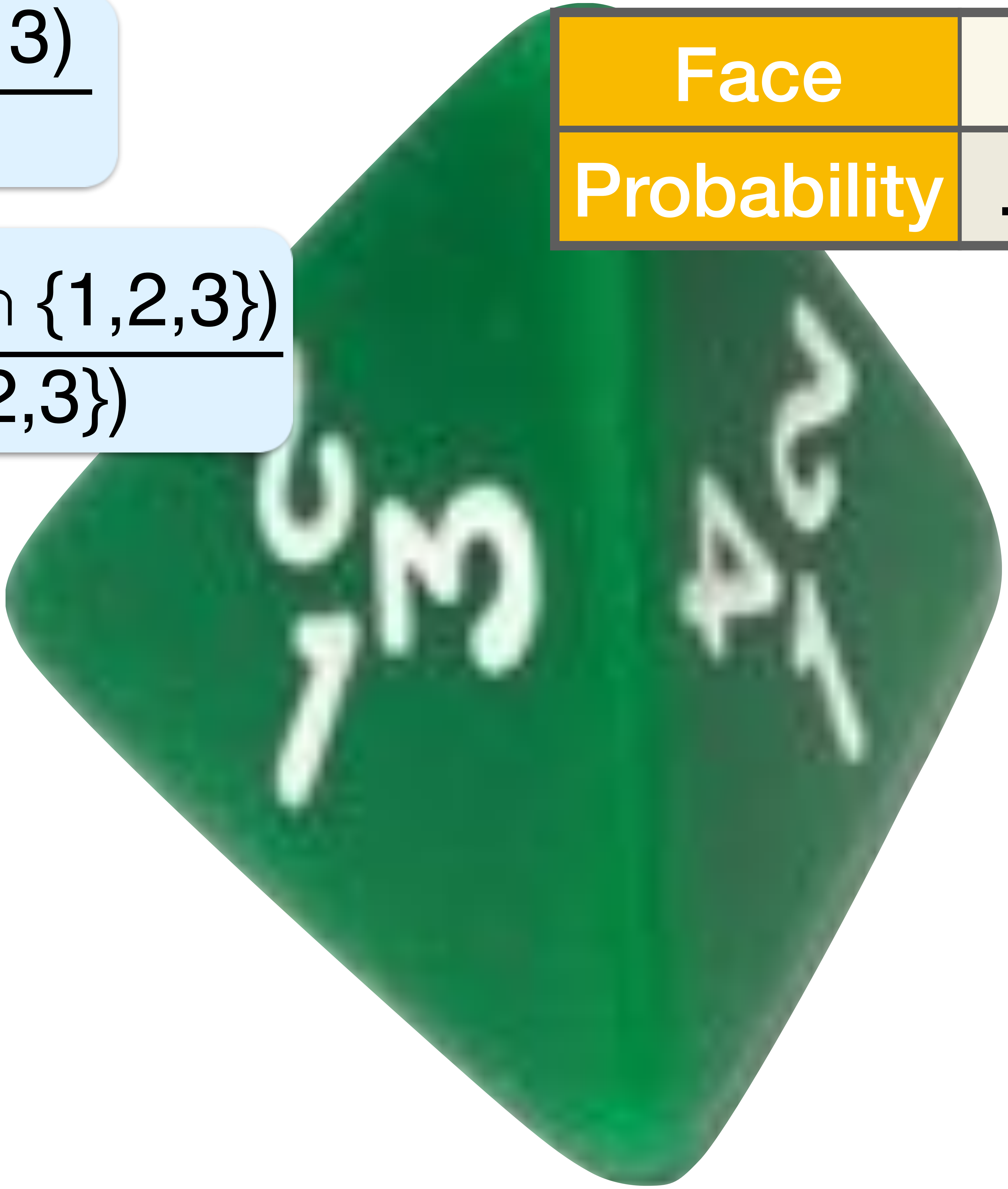
$P(\geq 2 \mid \leq 3)$

$$= \frac{P(\geq 2 \cap \leq 3)}{P(\leq 3)}$$

$$= \frac{P(\{2,3,4\} \cap \{1,2,3\})}{P(\{1,2,3\})}$$

$$= \frac{P(\{2,3\})}{P(\{1,2,3\})}$$

$$= \frac{.5}{.6} = \frac{5}{6}$$




KIDS
ARE PEOPLE TOO

Conditionals are Probabilities Too

$P(x \mid a) \geq 0$

$\sum_{x \in \Omega} P(x \mid a) = 1$

$P(x \mid a) \geq 0$

Conditional Probability

Probability with additional information

Definition

Examples

Independence

Independence

Independence

Informal and formal definition

Examples

Motivation

$P(F \mid E) > P(F)$

$P(2 \mid \text{Even}) = \frac{1}{3} > \frac{1}{6} = P(2)$

E ↗ probability of F

$P(F \mid E) < P(F)$

$P(2 \mid \text{Odd}) = 0 < \frac{1}{6} = P(2)$

E ↘ probability of F

$P(F \mid E) = P(F)$

$P(\text{Even} \mid \leq 4) = \frac{1}{2} = P(\text{Even})$

E neither ↗ nor ↘ probability of F

Whether or not E occurs, does not change P(F)

motivation → intuitive definition → formal

Independence - Intuitive

Events E and F are independent, denoted $E \perp\!\!\!\perp F$, if the occurrence of one does not affect the other's probability

$P(F \mid E) = P(F)$

Visually

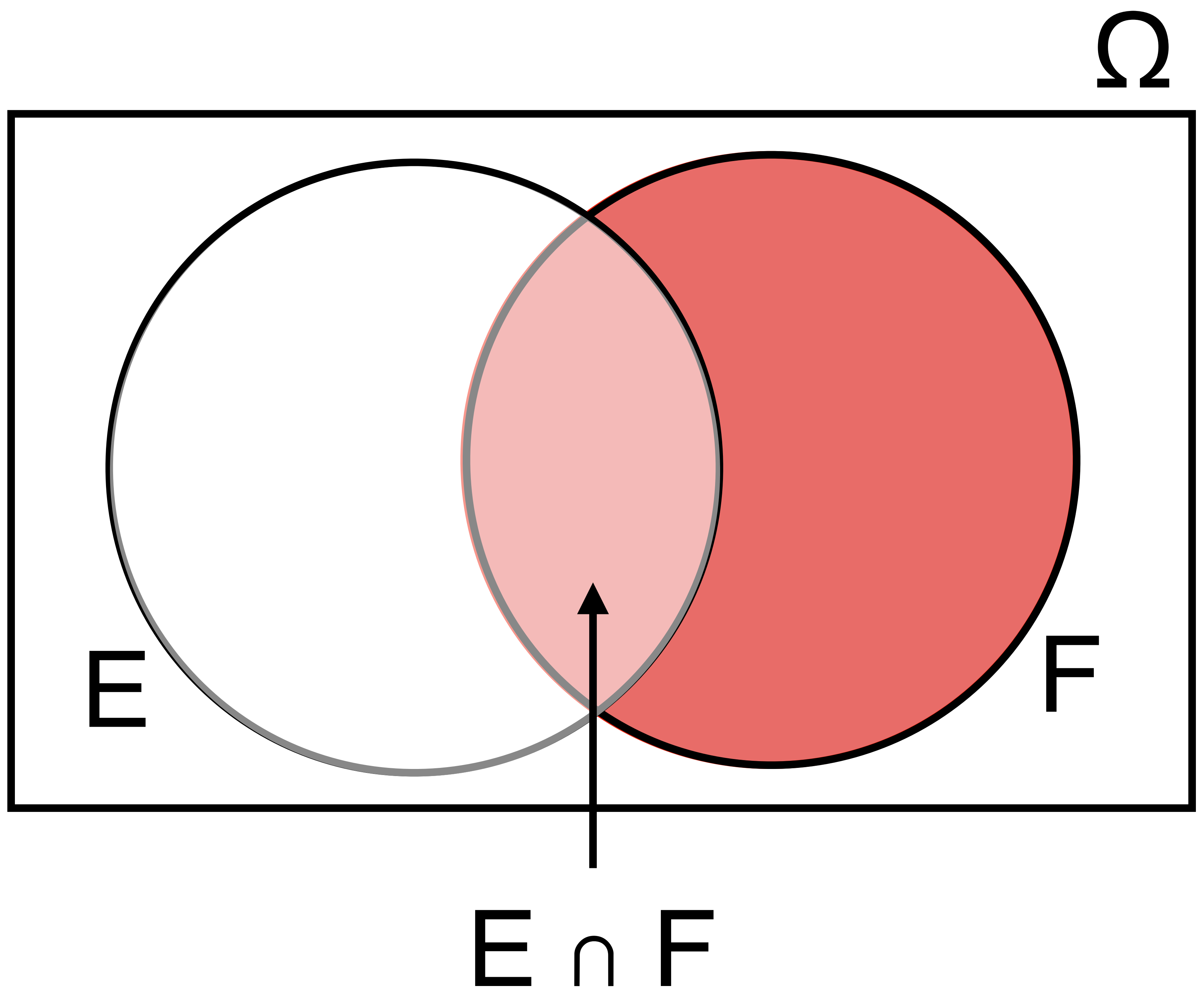
$$P(F) = \frac{P(F)}{P(\Omega)}$$

F as a fraction of Ω

=

$$P(F \mid E) \triangleq \frac{P(E \cap F)}{P(E)}$$

$E \cap F$ as a fraction of E



Two issues

Asymmetric

Undefined if $P(E)=0$

Independence - Formal

Informally

$P(F) = P(F \mid E) \triangleq \frac{P(E \cap F)}{P(E)}$

Asymmetric

Undefined if $P(E)=0$

Formally

E and F are independent if $P(E \cap F) = P(E) \cdot P(F)$

Otherwise, dependent

Symmetric ✅

Applies when $P = 0$ ✅

Implies

intuitive def. $P(F|E) = P(F)$

$P(E|F) = P(E)$

$P(F \mid \overline{E}) = P(F)$

$P(E \mid \overline{F}) = P(E)$

Non-Surprising Independence

Two coins

H_1 First coin heads $P(H_1)=\frac{1}{2}$

H_2 Second coin heads $P(H_2)=\frac{1}{2}$

$H_1 \cap H_2$ Both coins heads $P(H_1 \cap H_2)=\frac{1}{4}$

$P(H_1 \cap H_2) = \frac{1}{4} = P(H_1) \cdot P(H_2)$ $\quad H_1 \perp\!\!\!\perp H_2$

Not surprising as two separate coins

Can have $\perp\!\!\!\perp$ even for one experiment

Single Die

Three events

Event	Set	Probability
Prime	{ 2, 3, 5 }	½
Odd	{ 1, 3, 5 }	½
Square	{ 1, 4 }	⅓

Which pairs are $\perp\!\!\!\perp$ and $\not\perp\!\!\!\perp$

Intersection	Set	Prob	Product	=?	Independence
Prime ∩ Odd	{ 3, 5 }	⅓	½ · ½ = ¼	≠	dependent
Prime ∩ Square	∅	0	½ · ⅓ = ⅙	≠	dependent
Odd ∩ Square	{ 1 }	⅙	½ · ⅓ = ⅙	=	independent

Three Coins

Three events

Event	Description	Set	Probability
H_1	first coin heads	$\{h{*}{*}\}$	½
H_2	second coin heads	$\{{*}h{*}\}$	½
HH	exactly 2 heads in a row	{hht, thh}	¼

Which pairs are $\perp\!\!\!\perp$ and $\not\perp\!\!\!\perp$

Intersection	Set	Prob	=?	Product	Independence
$H_1 \cap H_2$	$\{hh{*}\}$	¼	=	½ · ½ = ¼	independent
$H_2 \cap HH$	{hht, thh}	¼	≠	½ · ¼ = ⅛	dependent
$H_1 \cap HH$	{hht}	⅛	=	½ · ¼ = ⅛	independent

Independence of Ω and $\varnothing$

$\forall A$ $\quad P(\Omega \cap A) = P(A) = P(\Omega) \cdot P(A)$ $\quad \Omega \perp\!\!\!\perp$ of any event

A occurring doesn't modify likelihood of Ω

$\forall A$ $\quad P(\varnothing \cap A) = P(\varnothing) = P(\varnothing) \cdot P(A)$ $\quad \varnothing \perp\!\!\!\perp$ of any event

A occurring doesn't modify likelihood of $\varnothing$

Independence

- Informal and formal definitions
- Examples

Total Probability

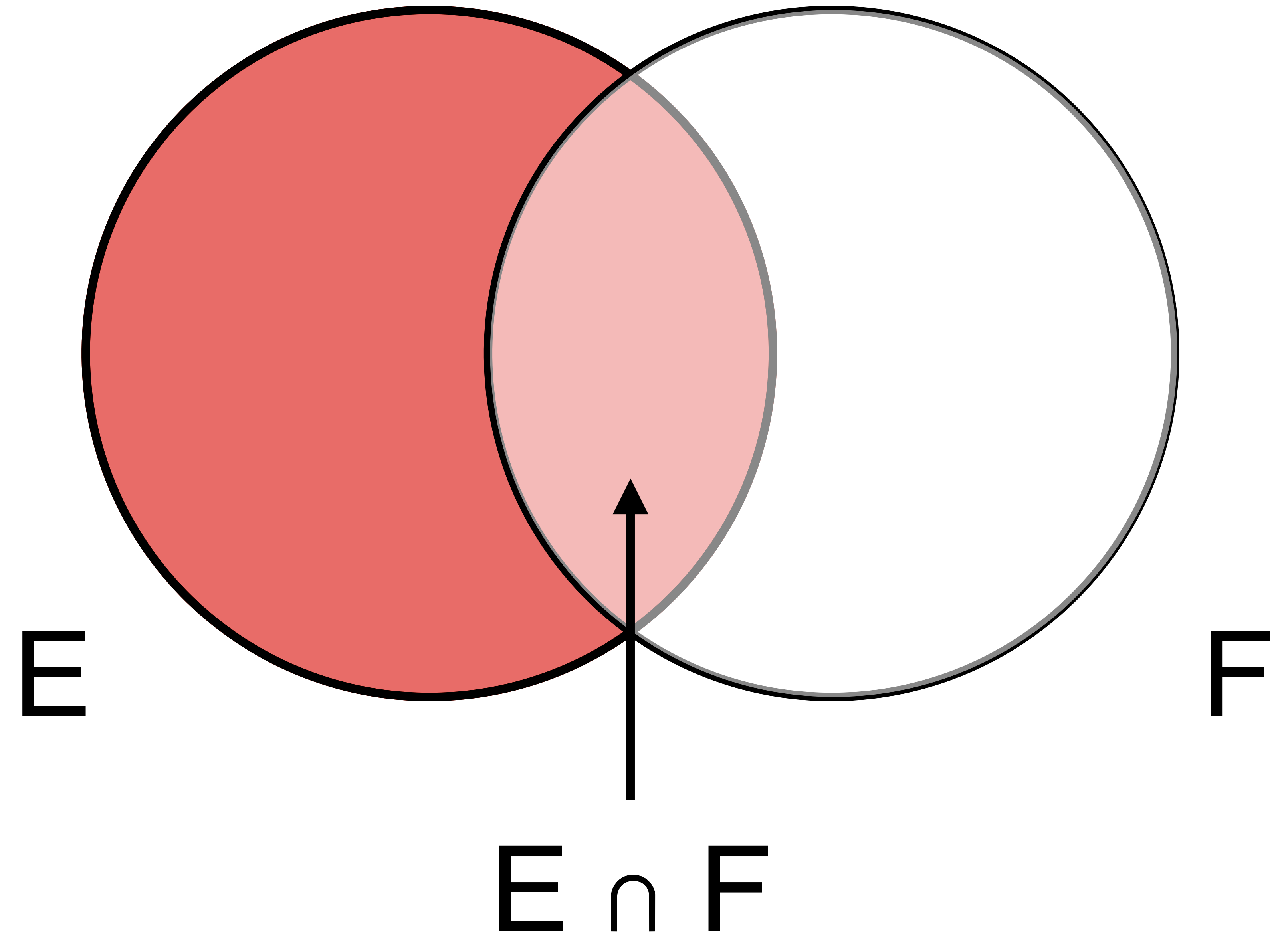
Sequential Probability

Product Rule

Recall conditional probability

$$P(F \mid E) = \frac{P(E \cap F)}{P(E)}$$

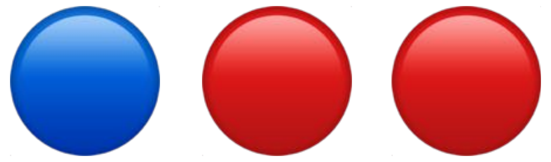
$$P(E \cap F) = P(E) \cdot P(F \mid E)$$



Helps calculate regular (not conditional) probabilities

Sequential Selection

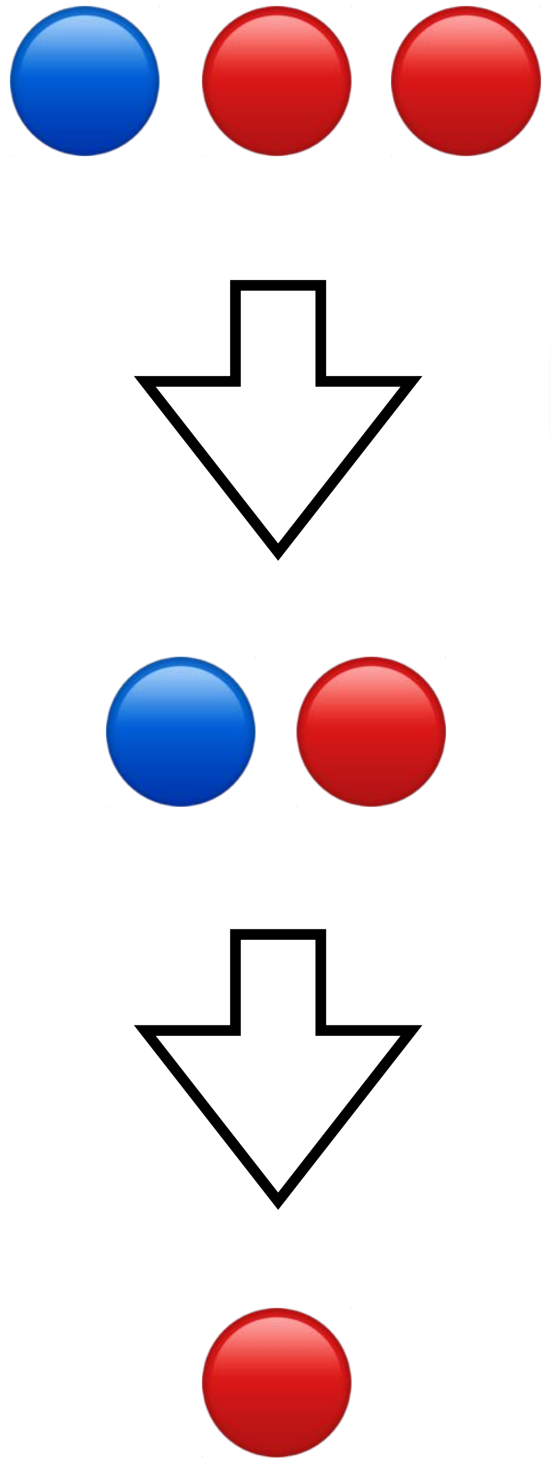
1 blue, 2 red balls



Draw 2 balls without replacement

P(both red) = ?

R_i - i'th ball is red



$P(R_1) = 2/3$

$P(R_2 \mid R_1) = 1/2$

P(both red)

$= P(R_1, R_2)$

$= P(R_1) \cdot P(R_2 \mid R_1)$

$= 2/3 \cdot 1/2$

$= 1/3$

General Product Rule

For 3 events

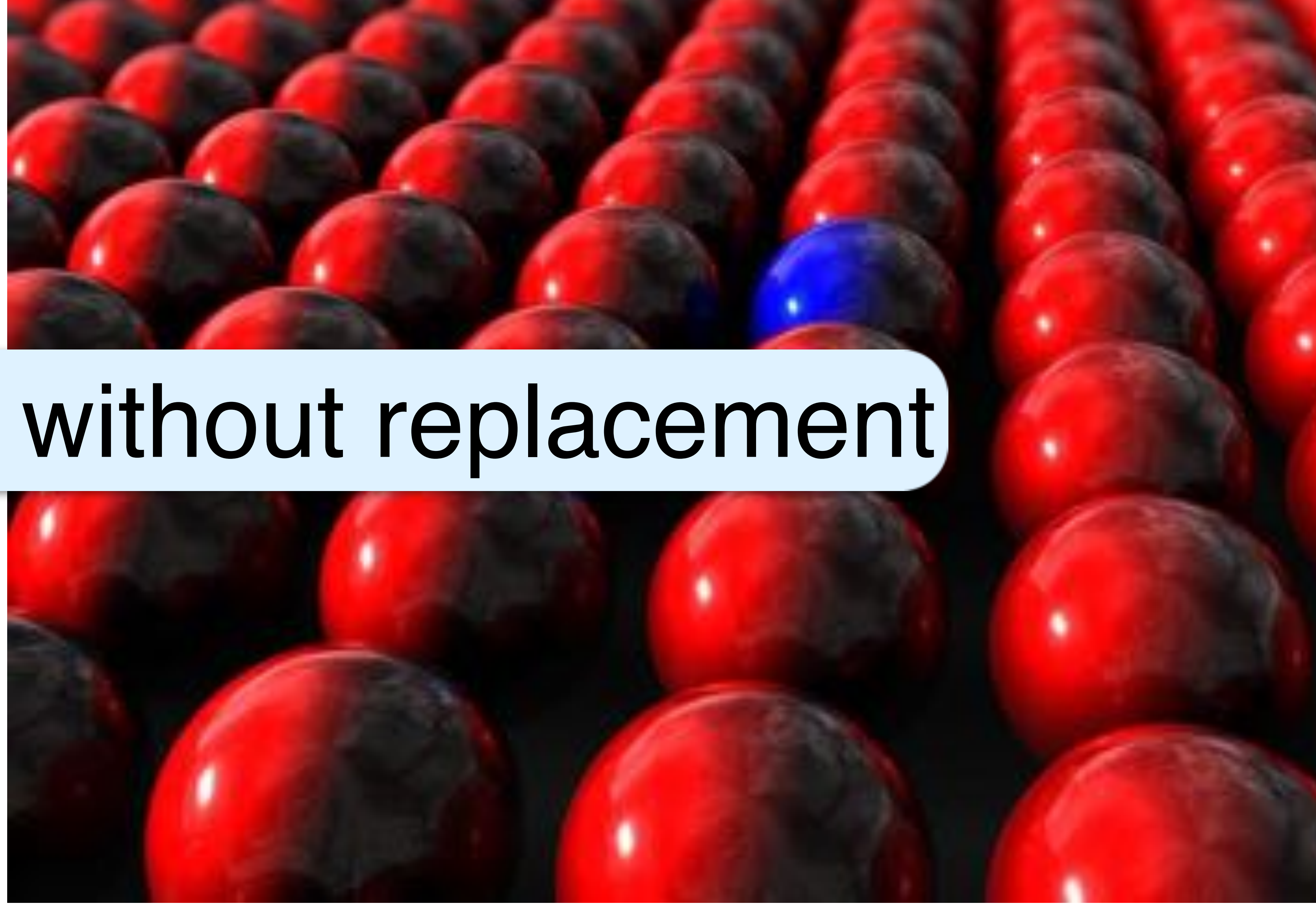
$P(E \cap F \cap G)$ $= P((E \cap F) \cap G)$

$= P(E \cap F) \cdot P(G \mid E \cap F)$

$= P(E) \cdot P(F \mid E) \cdot P(G \mid E \cap F)$

Similarly for more events

Odd Ball



n-1 red balls and one blue ball

Pick n balls without replacement

P(last ball is blue) = ?

R_i - ith ball is red

R^i - $R_1, R_2, \ldots, R_i$

$$P(\text{last ball blue}) = P(R_1)P(R_2|R_1)P(R_3|R^2)\ldots P(R_{n-1}|R^{n-2})$$

$$= \frac{n-1}{n}\,\frac{n-2}{n-1}\,\frac{n-3}{n-2}\cdots\frac{2}{3}\,\frac{1}{2} = \frac{1}{n}$$

Or.. Arrange in row, probability last ball is blue = 1/n

The Birthday Paradox

How many people does it take so that two will likely share a birthday?

- Assume that every year has 365 days
- Everyone is equally likely to be born on any given day

Probabilistically

- Choose n random integers, each $\in \{1, \dots, 365\}$, with replacement
- $B(n)$ - probability that two (or more) are the same
- For which n does $B(n)$ exceed, say, $1/2$?

Some first think it $n \approx 365$, but in fact much smaller

First Attempt

Consider the n people in order, say alphabetically

List their birthdays 2, 10, 365, 180, 10, …

Selection with replacement

Set of all possible birthdays sequences $\Omega = \{1,2,\ldots, 365\}^n$ $|\Omega| = 365^n$

Individual birthday uniform → Ω uniform

B_n {sequences with repetition}

P(repetition) = $|B_n| / |\Omega|$ Evaluating $|B_n|$ involved

Complement

B_n n people have birthday repetition

B_n^c n people, no two share a birthday

Evaluate sequentially

Person i different b/day from all previous

Calculation

$$1 - x \le e^{-x}$$

Among n people

P(no two people share a birthday)

$$= \frac{364}{365} \cdot \frac{363}{365} \cdot \ldots \cdot \frac{365-n+1}{365}$$

$$= \prod_{i=1}^{n-1} \left(1 - \frac{i}{365}\right)$$

$$\le \prod_{i=1}^{n} e^{-\frac{i}{365}}$$

$$= \exp\left(-\frac{1}{365} \cdot \sum_{i=1}^{n-1} i\right)$$

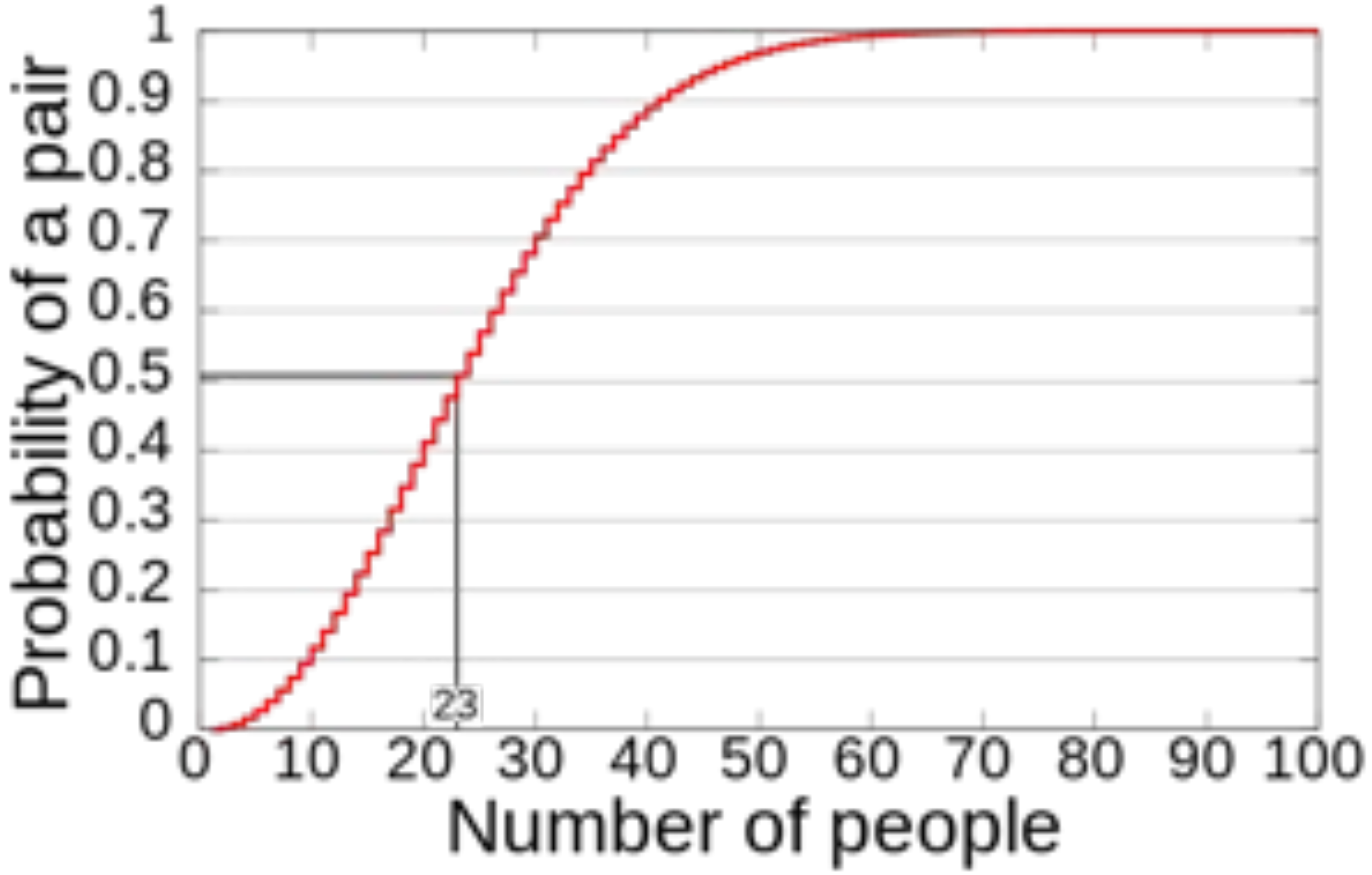
$$= \exp\left(-\frac{n(n-1)}{2 \cdot 365}\right)$$

$$\approx \exp\left(-\frac{n^2}{2 \cdot 365}\right) = 0.5$$

When the probability is 0.5

$$-\frac{n^2}{2 \cdot 365} = \ln 0.5 = -\ln 2$$

$$n \approx \sqrt{-2 \cdot 365 \cdot \ln 0.5} = 22.494$$



This lecture: Sequential Probability

Next: Total Probability

Total Probability

Divide and Conquer

When evaluating probability of an event

Sometimes easier to split event into different parts

Calculate probability of each part

Add probabilities

Law of Total Probability

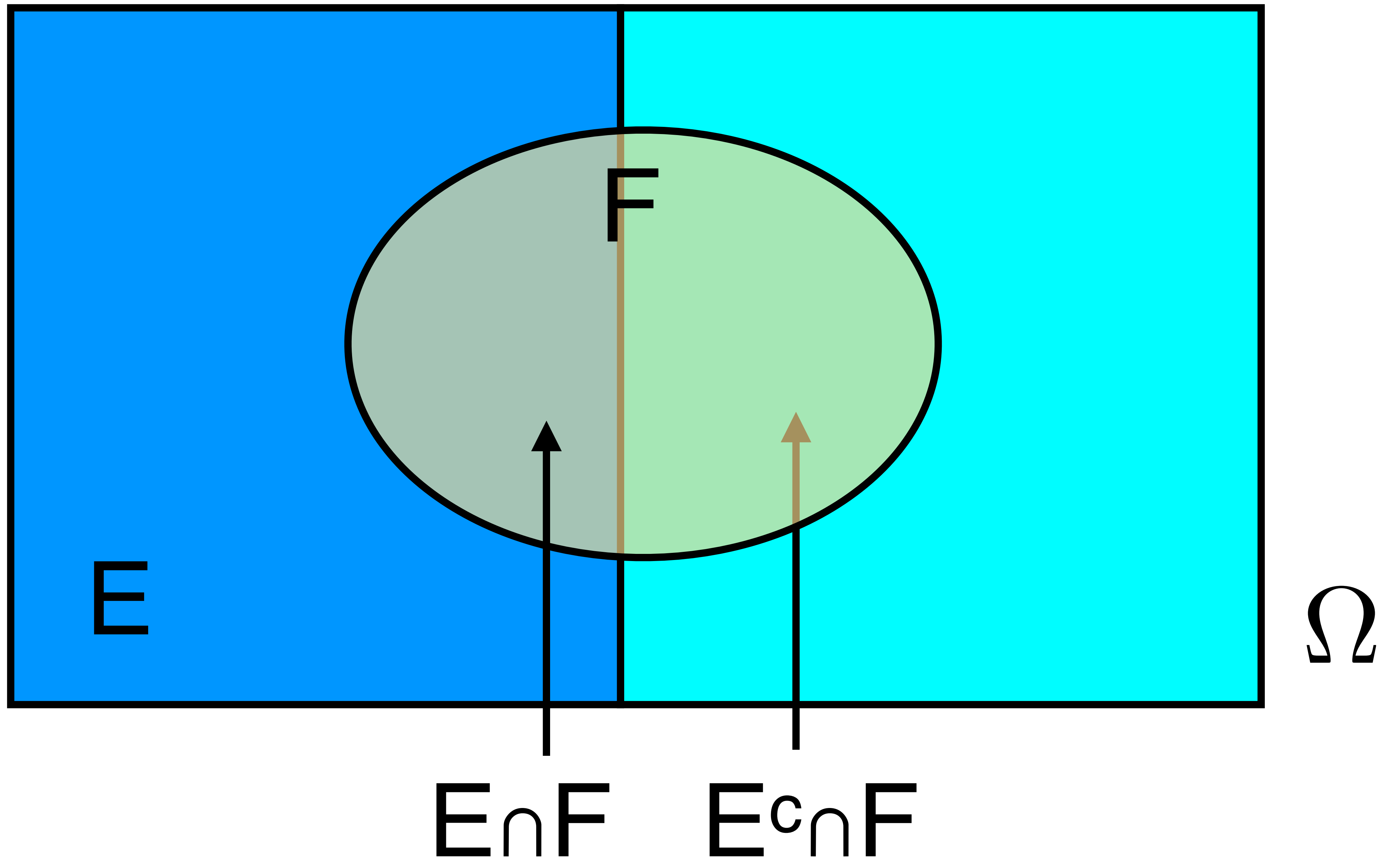
E,F events, P(F)=?

$F = E \cap F \cup E^c \cap F$

$P(F) = P(E \cap F) + P(E^c \cap F)$

$= P(E) \cdot P(F \mid E) + P(E^c) \cdot P(F \mid E^c)$

Product rule



2 Fair Coins

H_i - coin i is h

$\exists H$ - at least one h

$P(\exists H)$?

$$P(\exists H) = \frac{|\exists H|}{|\Omega|} = \frac{3}{4}$$

h h
h t
t h
t t

$$P(\exists H) = P(H_1 \cap \exists H) + P(H_1^C \cap \exists H)$$

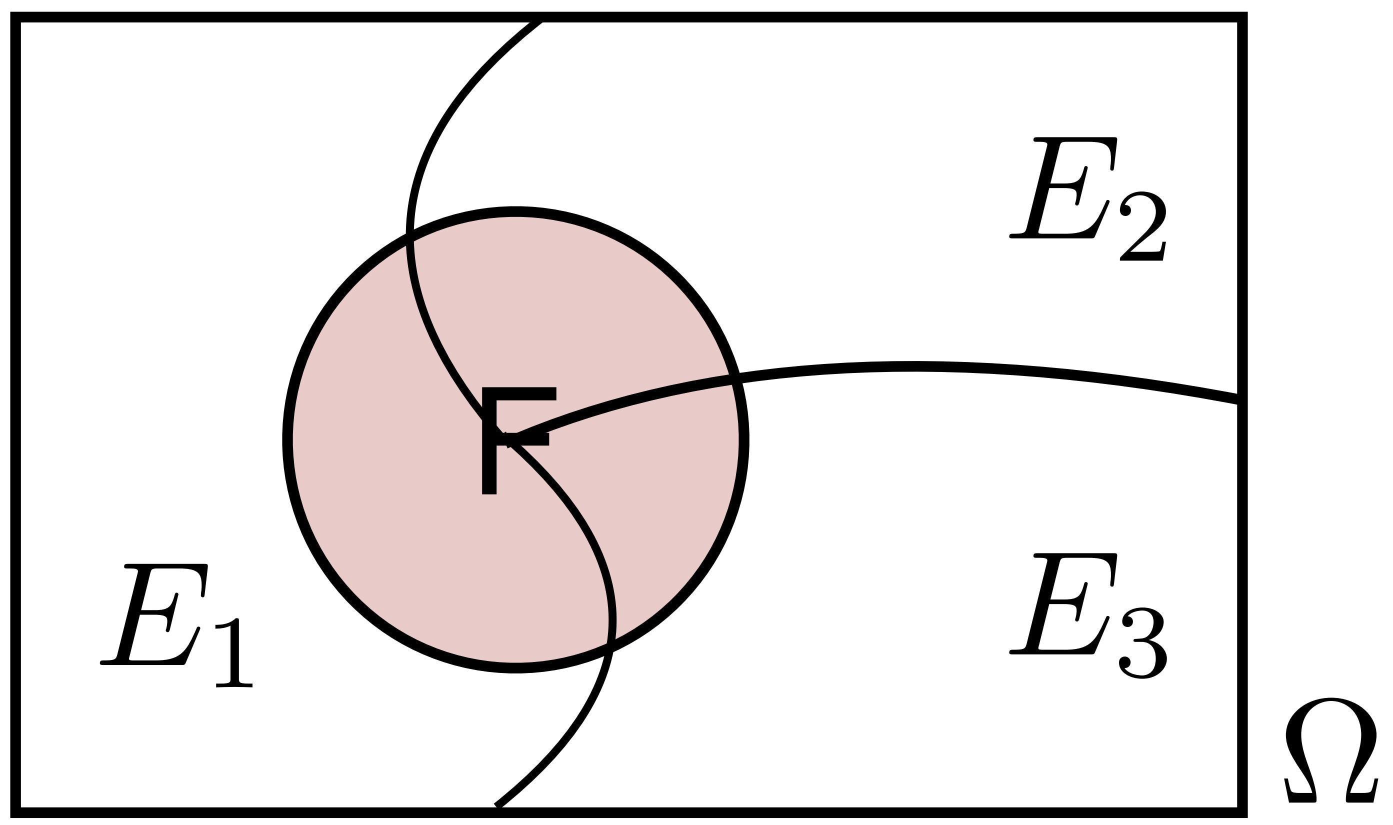
$$= P(H_1) \cdot P(\exists H \mid H_1) + P(H_1^C) \cdot P(\exists H \mid H_1^C)$$

$$= \tfrac{1}{2} \cdot 1 + \tfrac{1}{2} \cdot \tfrac{1}{2} = \tfrac{3}{4}$$

Total Probability - n Conditions

Let $E_1, E_2, \ldots E_n$ partition Ω

$$F = \bigcup_{i=1}^{n} (E_i \bigcap F)$$



$$P(F) = \sum_{i=1}^{n} P(E_i \cap F) \quad = \sum_{i=1}^{n} P(E_i) \cdot P(F|E_i)$$

2 Dice

D_i - outcome of die i

$S = D_1 + D_2$ sum of 2 dice

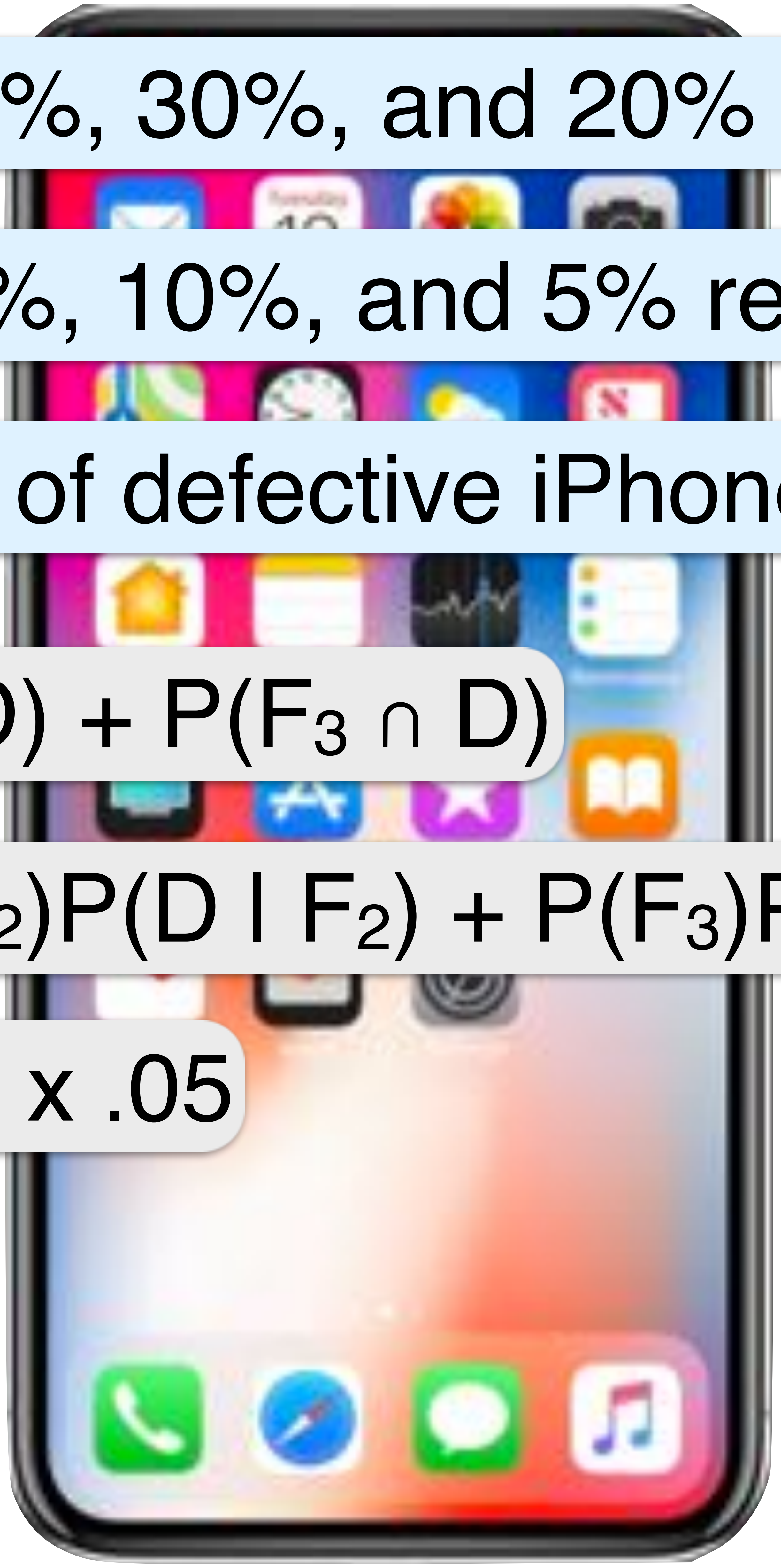
$P(S = 5) = ?$

$$P(S = 5) = \sum_{i=1}^{4} P(D_1 = i) \cdot P(D_2 = 5 - i \mid D_1 = i)$$

$$= \sum_{i=1}^{4} P(D_1 = i) \cdot P(D_2 = 5 - i)$$

$$= 4 \cdot \frac{1}{36} = 1/9$$

iPhone X



Three factories produce 50%, 30%, and 20% of iPhones

Their defective rates are 4%, 10%, and 5% respectively

What is the overall fraction of defective iPhones?

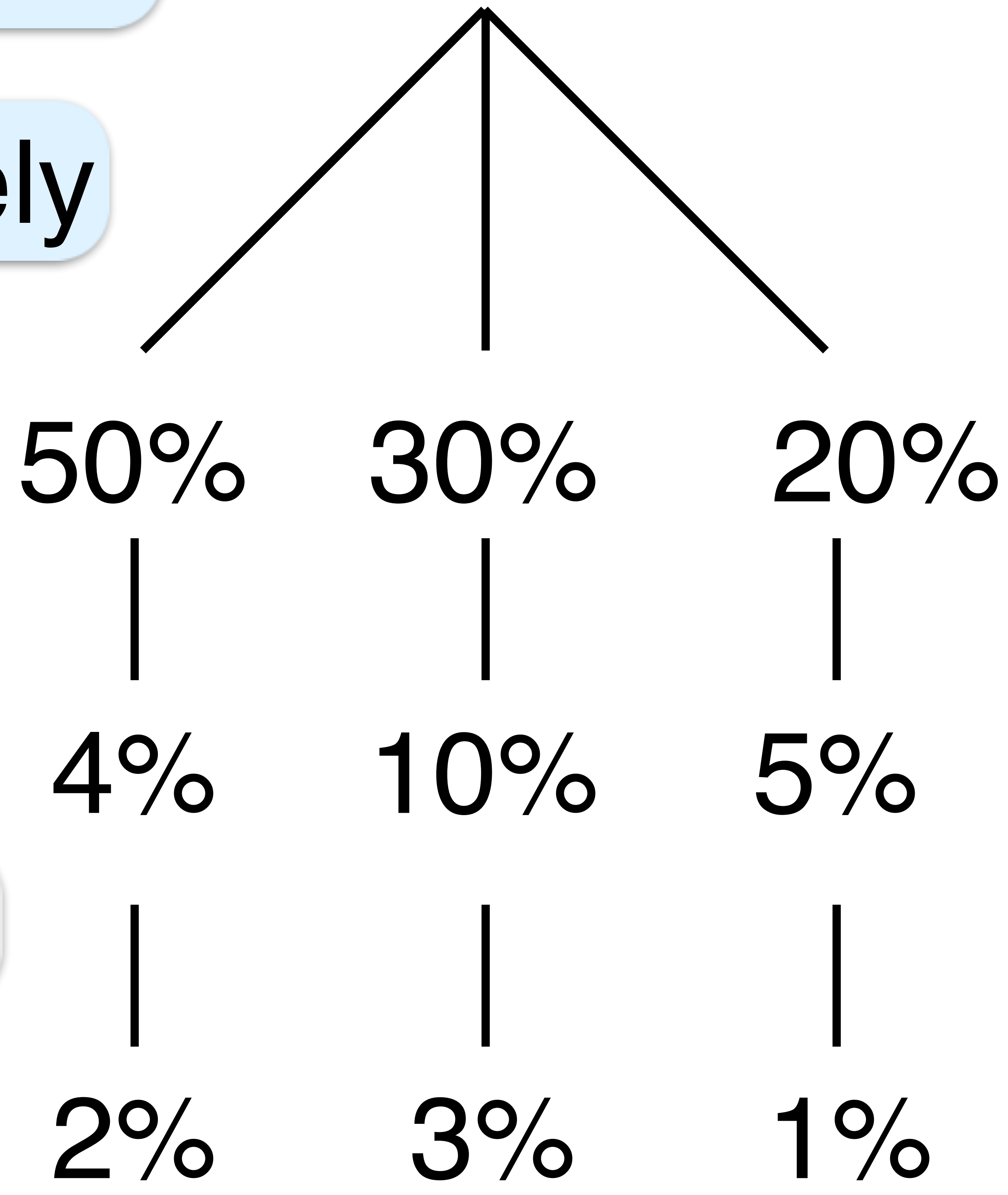
$P(D) = P(F_1 \cap D) + P(F_2 \cap D) + P(F_3 \cap D)$

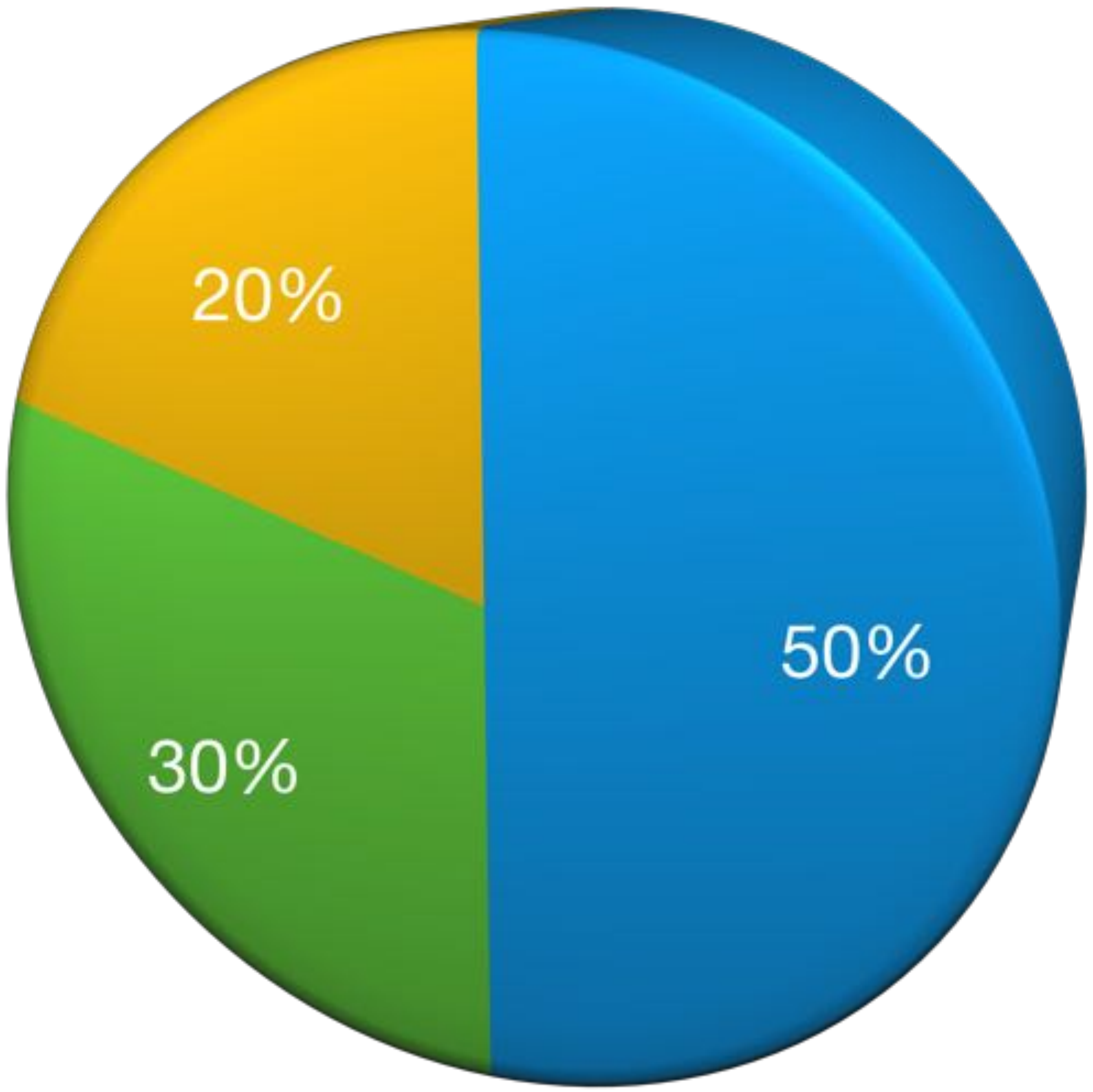
$= P(F_1)P(D \mid F_1) + P(F_2)P(D \mid F_2) + P(F_3)P(D \mid F_3)$

$= .5 \times .04 + .3 \times .1 + .2 \times .05$

$= .02 + .03 + .01$

$= .06$




20%
50%
30%

This lecture: Total Probability

Next: Bayes' Rule

Bayes' Rule

Asymmetry

Bought Google at IPO → Wealthy ←?

Alive today → Born after 1800 ←?

Forward - Backward

At times | $P(F \mid E)$ - easy | $P(E \mid F)$ - hard

2 coins | H_i - coin i is h | $\exists H$ - at least one h

$P(\exists H \mid H_1) = 1$ | $P(H_1 \mid \exists H)$?

2 dice | D_i - face of die i | $S = D_1+D_2$ sum of 2 faces

$P(S=5 \mid D_1=2) = P(D_2=3) = 1/6$ | $P(D_1=2 \mid S=5)$?

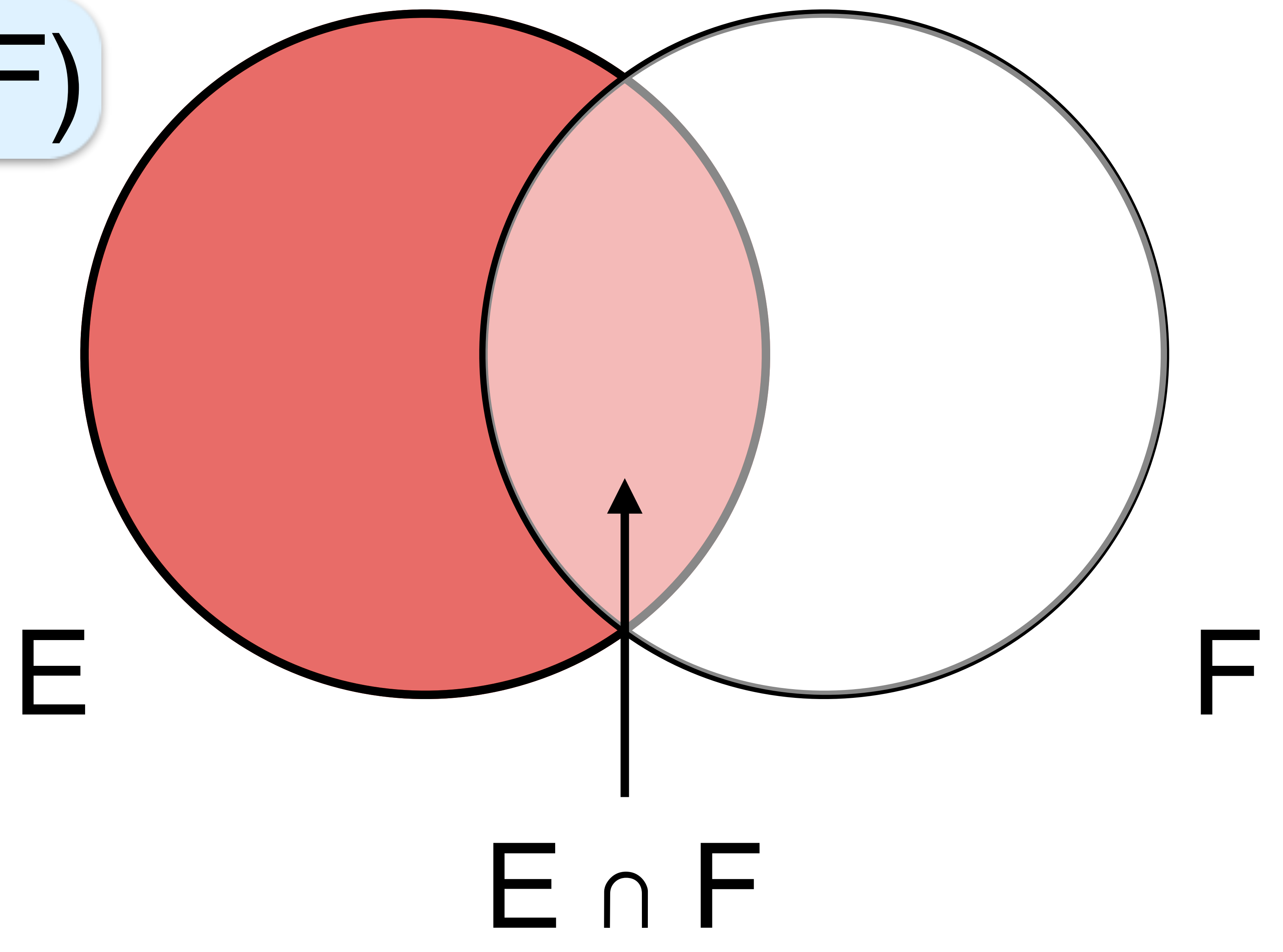
Bayes' Rule | Method for converting $P(F \mid E)$ to $P(E \mid F)$

Bayes' Rule

P(E), P(F)

Given P(F | E) (and a bit more) determine P(E | F)

$$\boxed{P(E \mid F) = \frac{P(E) \cdot P(F \mid E)}{P(F)}}$$



μ-proof

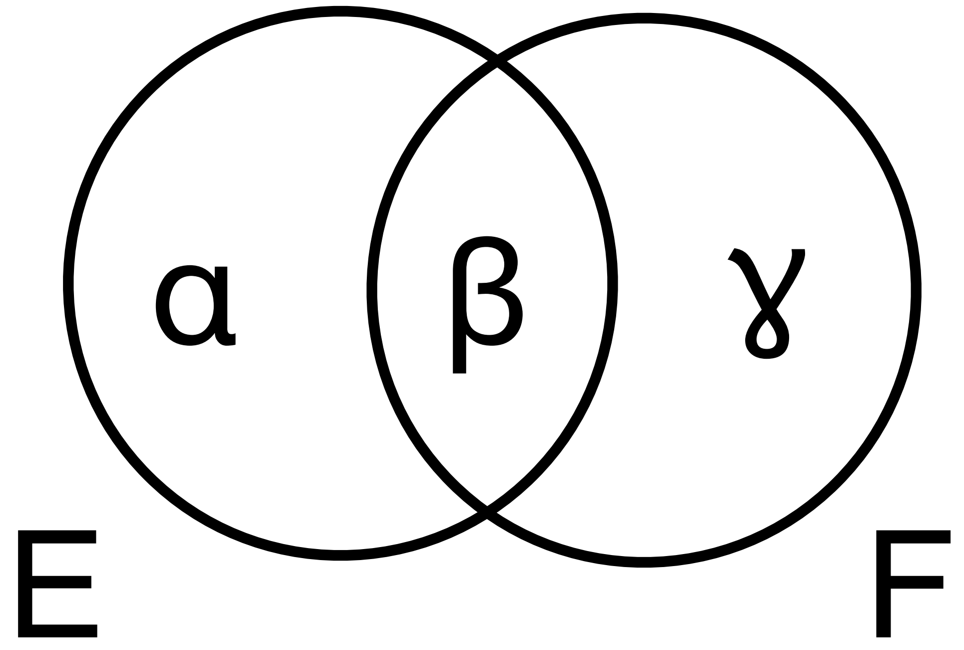
$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{P(E) \cdot P(F \mid E)}{P(F)}$$

Two More Views

$$\boxed{P(E \mid F) = \frac{P(E) \cdot P(F \mid E)}{P(F)}}$$

$$P(F) \cdot P(E \mid F) = P(E \cap F) = P(E) \cdot P(F \mid E)$$

$$P(F \mid E) = \frac{\beta}{\alpha + \beta}$$



$$P(E \mid F) = \frac{\beta}{\beta + \gamma}$$

$$P(E \mid F) = \frac{\beta}{\beta + \gamma} = \frac{\beta}{\alpha + \beta} \cdot \frac{\alpha + \beta}{\beta + \gamma} = \frac{P(F \mid E) \cdot P(E)}{P(F)}$$

Two Fair Coins

H_i - coin i is h

$\exists H$ - at least one h

$$P(H_1 \mid \exists H) = \frac{|H_1 \cap \exists H|}{|\exists H|}$$

$$\exists H \left\{ \begin{array}{c} \left.\begin{array}{c} h\ h \\ h\ t \end{array}\right\} H_1 \\ t\ h \end{array} \right.$$

$$t\ t$$

$P(H_1 \mid \exists H) = ?$

$$P(H_1 \mid \exists H) = P(\exists H \mid H_1) \cdot \frac{P(H_1)}{P(\exists H)} = 1 \cdot \frac{1/2}{3/4} = 2/3$$

$P(\exists H \mid H_1) = 1$

$P(H_1) = 1/2$

$P(\exists H) = 3/4$ Last video

Two Fair Dice

D_i - outcome of die i $\quad$ $S = D_1 + D_2$ Sum of 2 dice

$P(D_1 = 2 \mid S = 5)$?

$$P(D_1 = 2 \mid S = 5) = \frac{P(S = 5 \mid D_1 = 2) \cdot P(D_1 = 2)}{P(S = 5)} = \frac{\frac{1}{6} \cdot \frac{1}{6}}{\frac{1}{9}} = \frac{1}{4}$$

$$P(S = 5 \mid D_1 = 2) = P(D_2 = 3 \mid D_1 = 2) = P(D_2 = 3) = \frac{1}{6}$$

$$P(D_1 = 2) = \frac{1}{6}$$

$P(S = 5) = \frac{1}{9}$ $\quad$ Last video

$$S = 5 \begin{cases} 1\ 4 \\ 2\ 3 \} \ D_1 = 2 \\ 3\ 2 \\ 4\ 1 \end{cases}$$

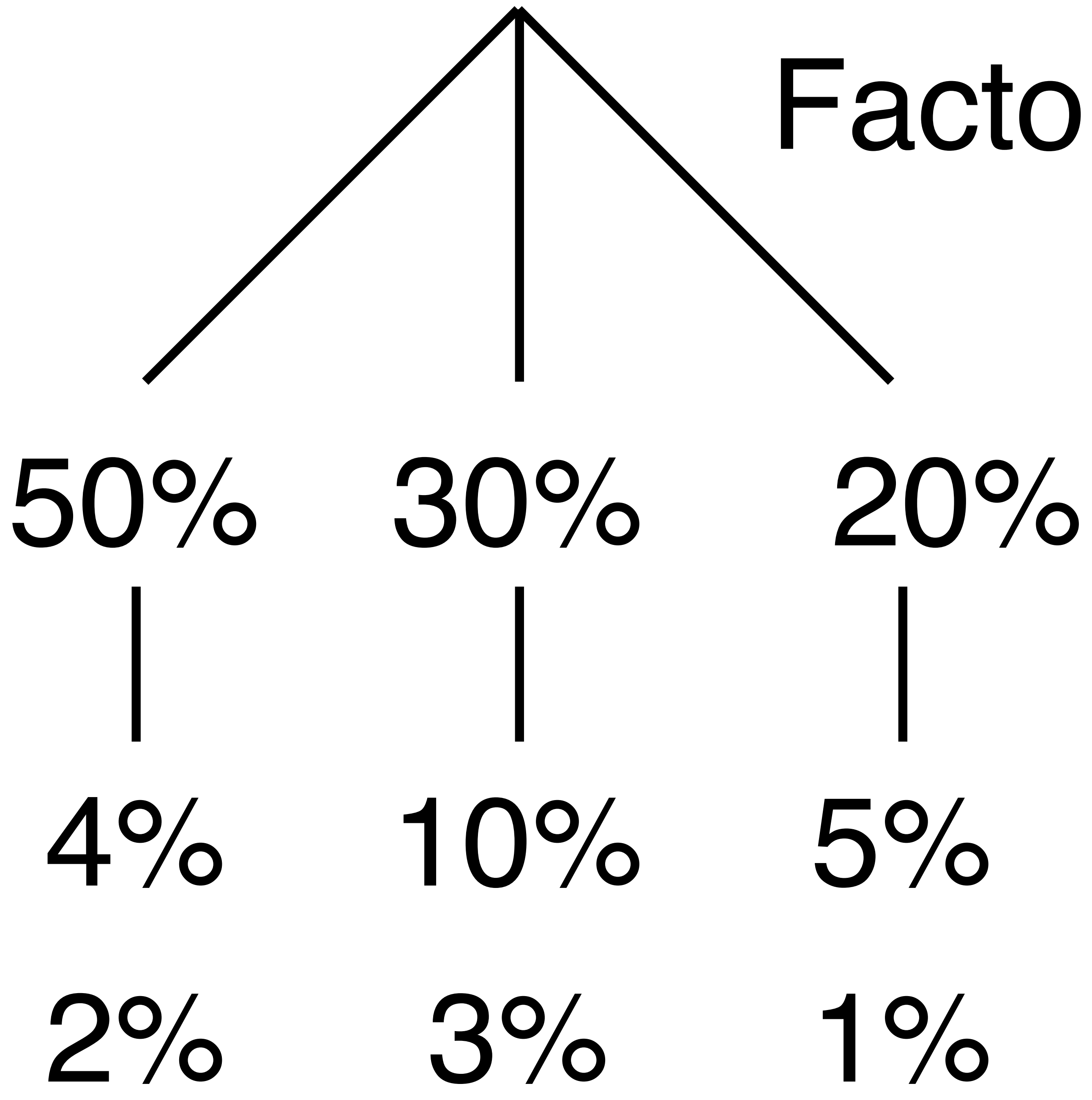
$$P(D_1 = 2 \mid S = 5) = \frac{|D_1 = 2 \cap S = 5|}{|S = 5|}$$

Foxconn

Foxconn has 3 factories producing 50%, 30%, and 20% of its iPhones

Factory defective fractions 4%, 10%, and 5% respectively

Overall fraction of defective phones?



$$\begin{aligned} P(D) &= P(D \cap F_1) + P(D \cap F_2) + P(D \cap F_3) \\ &= P(F_1)P(D \mid F_1) + P(F_2)P(D \mid F_2) + P(F_3)P(D \mid F_3) \\ &= .5 \times .04 + .3 \times .1 + .2 \times .05 \\ &= .02 + .03 + .01 \\ &= .06 \end{aligned}$$

Culprit?

$$P(F_1 \mid D) = \frac{P(D \mid F_1) \cdot P(F_1)}{P(D)} = \frac{.04 \cdot .5}{.06} = \frac{.02}{.06} = ⅓$$

$P(D \mid F_1) = .04$ $\quad$ $P(F_1) = .5$ $\quad$ $P(D) = .06$

$$P(F_2 \mid D) = \frac{.1 \cdot .3}{.06} = \frac{.03}{.06} = ½$$

$$P(F_3 \mid D) = \frac{.05 \cdot .2}{.06} = \frac{.01}{.06} = ⅙$$

Conditional probabilities add to 1

Conditional order determined by both $P(F_i)$ and $P(D \mid F_i)$

Taxi

This Lecture: Bayes' Rule

Next: Random Variables