

Lady Tasting Tea

A woman with long blonde hair, wearing a dark top, is smiling and pointing her right index finger towards the camera. She is in a library or study room with bookshelves filled with books in the background.

“Original” hypothesis test

Exact p-value calculation

Accept H_A , retain H_0

More test intuition

1920's

Refined
social
gathering



College dons



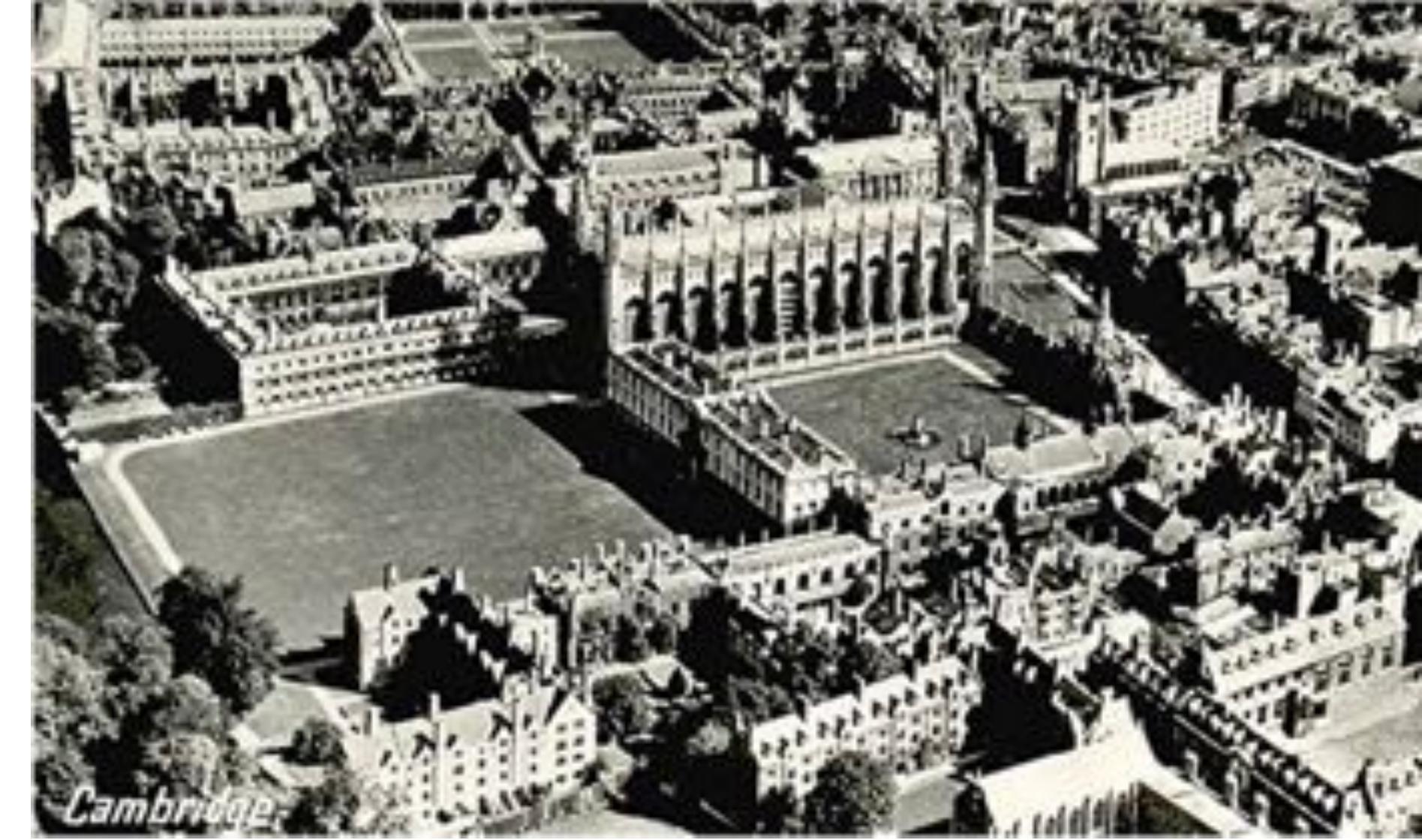
UNIVERSITY OF
CAMBRIDGE

Soon...

Devolves into...

Rowdy brawl

Topic of
contention?



Proper Preparation of

In preparing
milk tea



Should
tea



or
milk



be added
first?

Most
say

Doesn't matter

Can't tell the difference!

Algologist Lady
Muriel Bristol



Adamant

Tea first or it's foul!

Is she right?

Is she wrong?

Can she even tell???

Luckily...

There is a Dr. in the house...

Statistics Dr.

H_0 Lady cannot tell which of tea or milk was poured first

H_A Lady can discern order with probability > 0.5

Fisher's Test

8 cups of tea

4 Tea first



End up



4 Milk first



Mix them up

Ask Lady Bristol



Which 4 are which?



Ronald Fisher

Decision & Interpretation

Lady knows

4



4



Asked for

4



4



Random guesses

4



4



If she can discern

> 4



ideally

8



Calculate probabilities

Ever Even

Always guess same number of tea-first and milk-first correctly



T M T T M M M

Guess 3 tea-first correctly as tea-first

Guess 1 tea-first as milk-first

Guess 3 milk-first as milk-first correctly

Always Guess same # correctly

x tea-first → x milk-first

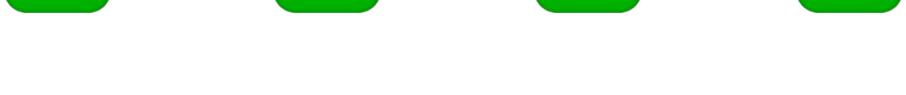
7 correct



3 tea-first ✓



3 milk-first ✓



Always guess even
number correctly

Probability

H_0 Lady randomly chooses 4 tea-first cups

Start with 6

$P_{H_0}(x \text{ cups correct}) = ?$

Lady chooses four tea-first cups



ways to make such choices

$$\binom{8}{4} = \frac{8!}{4! \cdot 4!} = \frac{\cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 70$$

6 correct

\leftrightarrow 3 tea-first & 3 milk-first correct



\leftrightarrow Of lady's 4 tea-first choices, 3 are tea-first and 1 is milk-first

ways to make such choices

$$\binom{4}{3} \cdot \binom{4}{1} = 16$$

$P(6 \text{ correct}) = \frac{16}{70} \approx 0.229$

p Values

H_0

Lady guesses randomly

H_A

Correct with prob. > 0.5

c specific

C random

correct guesses

p value of c

$P_{H_0}(c \text{ or more correct guesses})$

$P_{H_0}(C \geq c)$



c

0



2



4



6



8



	$P_{H_0}(C = c)$	$P_{H_0}(C \geq c)$
0	$\binom{4}{0} \binom{4}{4} / 70 = 1/70$	1
2	$\binom{4}{1} \binom{4}{3} / 70 = 16/70$	$69/70 \approx 0.986$
4	$\binom{4}{2} \binom{4}{2} / 70 = 36/70$	$53/70 \approx 0.757$
6	$\binom{4}{3} \binom{4}{1} / 70 = 16/70$	$17/70 \approx 0.243$
8	$\binom{4}{4} \binom{4}{0} / 70 = 1/70$	$1/70 \approx 0.014$

p Value → Test

H_0 Lady guesses randomly

H_A Correct w.p. > 0.5

5% significance level

Prob. type-I error $\leq 5\%$

$P_{H_0}(\text{Accept } H_A) \leq 5\%$

p value
of c

H_0 probability of
c or more correct

$> 5\%$
 $\leq 5\%$

Outcome likely under H_0
Unlikely under H_0

Retain H_0
Accept H_A

c	p value	%	v. 5%	Hyp
0	1	100	>	H_0
2	69/70	98.6	>	H_0
4	53/70	75.7	>	H_0
6	17/70	24.3	>	H_0
8	1/70	1.4	\leq	H_A

Need all 8 cups
correct to accept
 H_A with 5%
significance level

And Lady Bristol?

Of 8 cups

How many did she guess correctly?

8



Accept H_A with 5% significance level

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Test intuition



Z and T Tests

