CS7646 Project 1 Martingale

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1 INTRODUCTION

The assignment aims to study a Monte Carlo simulation of the Martingale betting strategy. The American wheel of Roulette has 36 holes with 2 greens, compared to the European 1 (Small and Tse, 2012).

The odds of winning with an outside bet of black or red can be calculated as:

$$P_{win} = NumberofColour/TotalNumbers = 18/38 = 0.4737 \quad (1)$$

Using the odds the Monte Carlo simulation can be carried out by comparing the result to a random number generator and simulating wins and loses per spin. With each loss, the bet will be doubled, and with each win the bet resets back to the initial $1 bet until the target is reached. In the second experiment, a bankroll is added, limiting the possible losses.

2 QUESTION 1

From 1000 episodes using the Martingale strategy with unlimited resources, each episode resulted in meeting the desired $80 target by 1000 sequential bets as calculated from the output array proportion of winnings at $80 to all episodes at the end of 1000 spins (Figure 1, Figure 2).

The win probability is 0.4737. While each event is independent and believing otherwise is the Gambler's fallacy, for a fair game, the random simulated walks should approach the win probability within some error (Kovic and Kristiansen, 2019). The target of $80 following the Martingale means the gambler requires 79 wins as each win will earn the gambler a new dollar as the initial bet is set to $1. For 1000 spins, the win ratio required is 7.9%. Compare this to the odds of 47.4% and it is very likely, but not guaranteed, that the gambler should meet their target given the Martingale strategy. It so happened in this simulation that the gambler won all simulated episodes, giving 100% probability of success with unlimited resources.

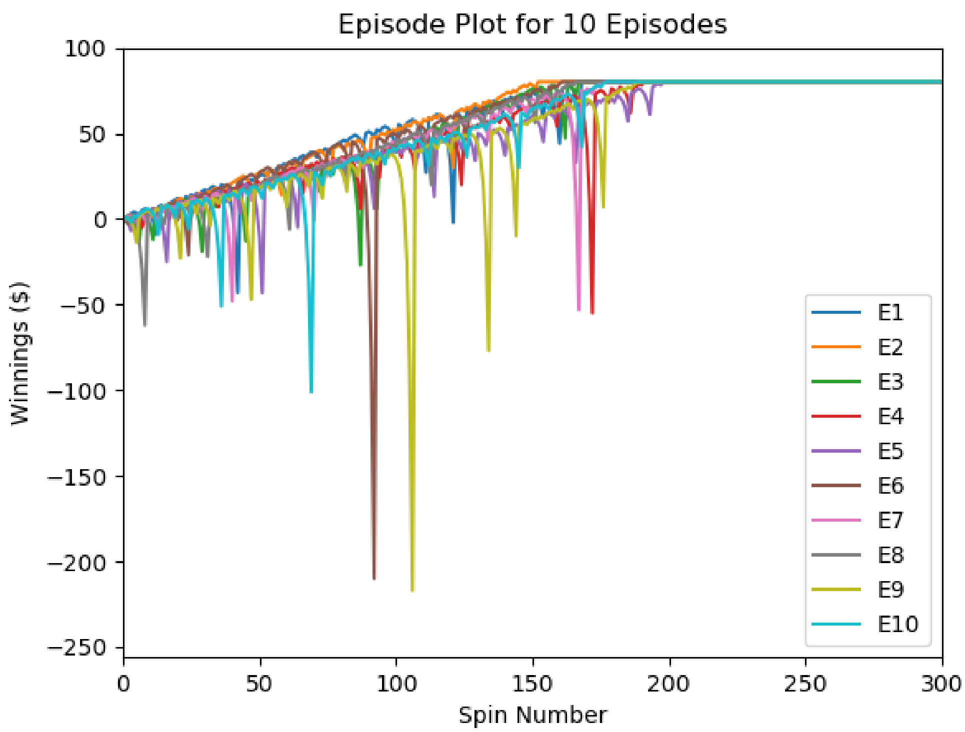

Figure 1—10 Episode Results of Martingale

3 QUESTION 2

Over 1000 episodes, the Monte Carlo simulation demonstrates that the expected value of winnings after 1000 sequential bets is $80.

Given the expected value of a random variable (Grinstead and Laurie Snell, 2020):

$$E[X] = \sum_{i=1}^{k} x_i * p_i \tag{2}$$

Where x is the random variable at i and p is the probability weighting. The simulation results were based on a pseudo-random number generated which was compared to the win probability.

The simulated realization are equiprobable, so the expected value reduces to the mean of the spins between episodes. By 1000 sequential bets, following the Martingale strategy with unlimited resources, all iterations converged to the target of $80 (Figure 2).

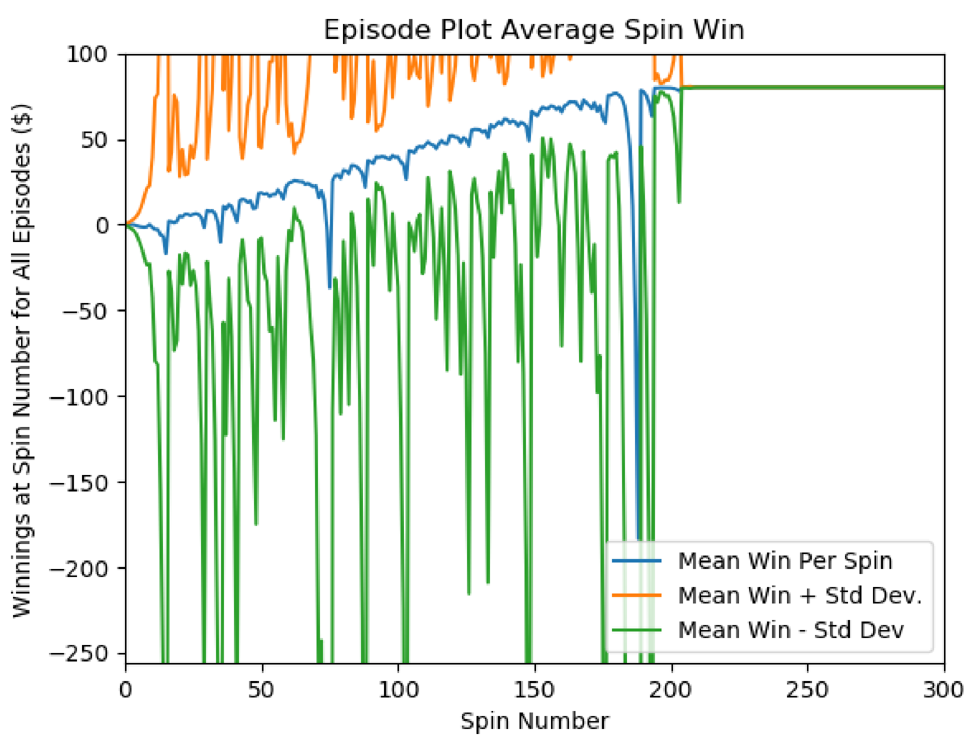

Figure 2—Average Episode Results Per Spin of Martingale

There is variation between spins causing the average to fluctuate. For an idea of the central value of the data, the median can be used and shows the trend without sensitivities seen in the average (Figure 3).

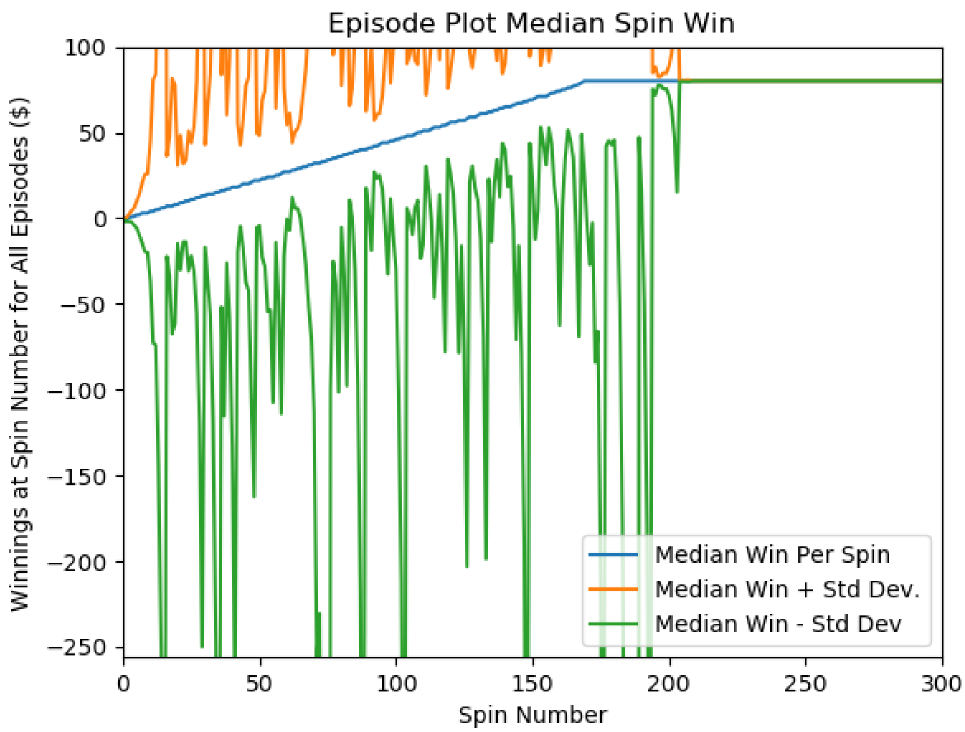

Figure 3—Median Episode Results Per Spin of Martingale

4 QUESTION 3

From Figure 2, it can be seen that the standard deviations begin to stabilize before converging to the mean of $80 when the winnings reach the target $80. There is a maximum reached before oscillating towards convergence. The reason is that the episodes each reach the target at different times, but because there is a set upper limit that is carried forwards, the odds of not converging became increasingly unlikely given the win odds of 47.4% as the spins continue given unlimited resources.

5 QUESTION 4

Unlike in the previous example where resources were unlimited, now the bet is limited to a bankroll of $256. Due to this, the odds of reaching the target are limited.

From the simulation, the probability at the end of 1000 spins for all episodes to meet the target was only 10.6 % as calculated from the simulation results (number of episodes where the target was reached over the total episodes).

6 QUESTION 5

The expected value of winnings was calculated to be -$54.50 from the simulation, the average winnings at the end of the simulated episodes. The explanation of this is given in Question 2, as the expected value is the mean between the episodes at the end of 1000 sequential bets.

7 QUESTION 6

Unlike previously where the standard deviation and average oscillated, there seems to be a steady decrease to the mean winnings per spin. The standard deviation bounds appear to stabilize with the slope of the mean as can be seen by the average and median plots (Figures 4-5). It is likely that this is due to two bounds being introduced, $80 like before and now -$256. As the probability of achieving the target was 10.6% in this simulation, then the standard deviation cannot converge as there is variation between realizations.

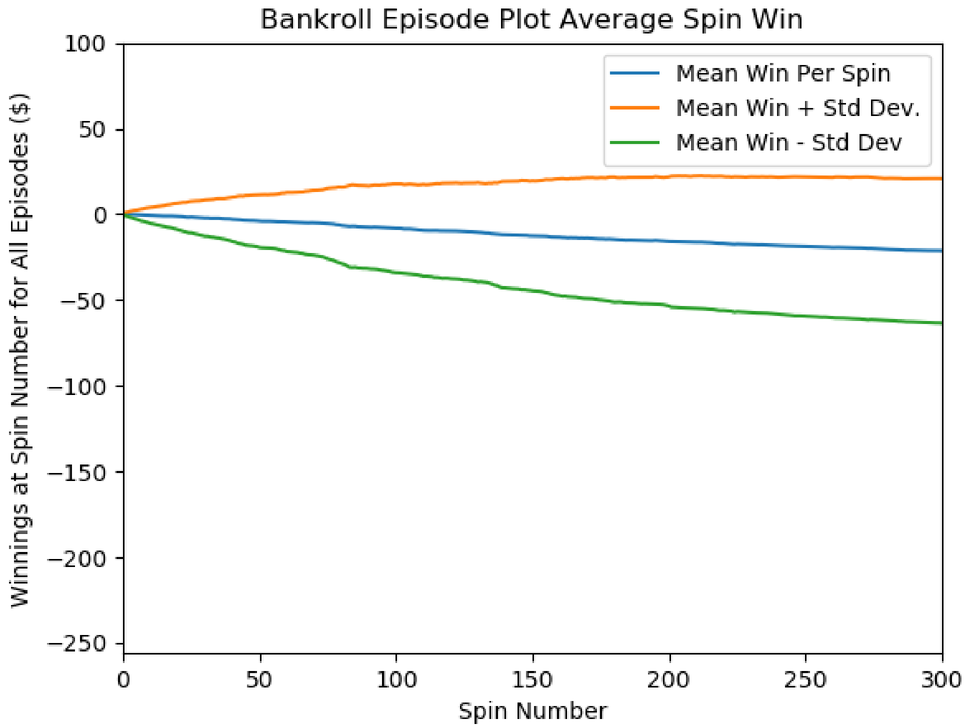

Figure 4—Bank Rolled Average Episode Results Per Spin of Martingale

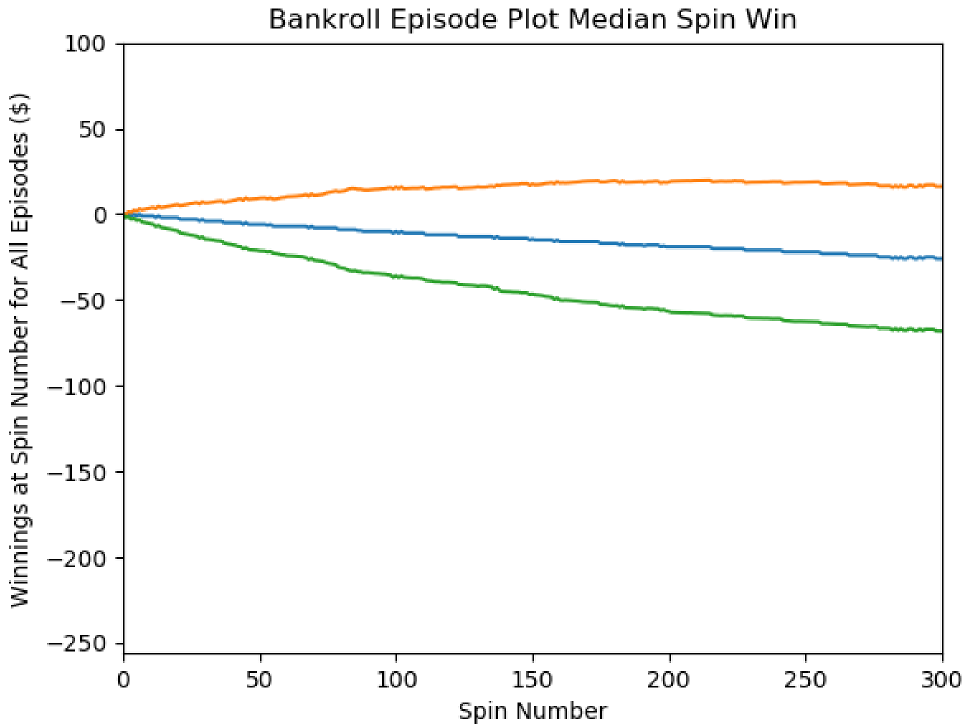

Figure 5—Bank Rolled Median Episode Results Per Spin of Martingale

8 QUESTION 7

The benefits of using expected values, and Monte Carlo simulation in general, is to get an idea of various realizations of probabilistic scenarios. This is important to understand risk and levels of uncertainty in certain decisions. From a probability standpoint, the Martingale strategy is expected to lose. Through a Monte Carlo simulation, one can see an example of how that realization occurs, and under different constraints, such as the bankroll of $256 in the second experiment. Through using the expected values, a depiction of the most likely return on the action can be estimated across realizations.

9 CONCLUSIONS

The project explores the use of Monte Carlo simulation for applying the Martingale betting strategy. It was found that given truly no constraint on betting amounts and bankroll, the strategy will allow the gambler to reach their goal.

However, if there is a limit on bankroll, the gambler will quickly find that they will lose very frequently, observed as 10.6% of the time in the simulation of 1000 sequential bets over 1000 realizations. The Monte Carlo simulation was shown as an effective way to explore how stochastic situations apply to set probabilities and to better quantify risk.