Maths MCQ and FB questions from the New Revised Edition of Boscoss II PU Text Book to be released shortly

Multiple Choice Questions

Ch. 1 : Relations & Functions

Choose the correct answer

1. Let R be the relation in the set **N** given by $R = \{(a, b) : a = b - 2, b > 6\}$.

A) $(2, 4) \in R$ B) $(3, 8) \in R$ C) $(6, 8) \in R$ D) $(8, 7) \in R$ **(NCERT)**

2. Let R be the relation in the set (1, 2, 3, 4} given by

$R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$. Choose the correct answer.

A) R is reflexive and symmetric but not transitive.

B) R is reflexive and transitive but not symmetric.

C) R is symmetric and transitive but not reflexive.

D) R is equivalence relation. **(NCERT)**

3. The relation $R = \{(a, b) : \gcd (a, b) = 1, 2a \neq b, a, b \in Z\}$ is _____

A) transitive but not reflexive B) symmetric but not transitive

C) reflexive but not symmetric D) neither symmetric nor transitive **(JEE-M 23)**

4. A set A has 4 elements. Then the number of relations on A is

A) 2^4 B) 2^{16} C) 16^2 D) 2^8

5. Let R be a relation on N, the set of all natural numbers given by $R = \{(a, b) : a \leq b\}$. Then,

A) R is reflexive and symmetric B) R is symmetric and transitive

C) R is reflexive and transitive but not symmetric D) R is an equivalence relation.

6. Divisibility relation on Z is

A) reflexive B) symmetric C) transitive D) equivalence relation

7. Let L denote the set of all straight lines in a plane. Let R be the relation on L defined by $R = \{(l, m) : l \text{ is perpendicular to } m\}$. Then R is

A) reflexive B) symmetric C) transitive D) equivalence relation

8. In the set of all integers z, which of the following relations is not an equivalence relation ?

A) $\{(x, y) : x \leq y\}$ B) $\{(x, y) : x = y\}$ C) $\{(x, y) : x - y \text{ is even integer}\}$ D) $Z \times Z$

9. The relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is

A) reflexive B) symmetric C) transitive D) equivalence **(23-M, MQP)**

10. The function $f : Z \to Z$ given by $f(x) = x^2$ is

A) bijective B) one-one but not onto

C) onto but not 1-1 D) neither 1-1 nor onto

11. The greatest integer function $f : \mathbf{R} \to \mathbf{R}$, given by f(x) = [x], is

A) bijective B) one-one but not onto

C) onto but not 1-1 D) neither 1-1 nor onto

12. The modulus function $f : \mathbf{R} \to \mathbf{R}$, given by f(x) = |x|, is

A) bijective B) one-one but not onto

C) onto but not 1-1 D) neither 1-1 nor onto

13. The Signum function $f : \mathbf{R} \to \mathbf{R}$, given by $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$ is.

A) bijective B) one-one but not onto

C) onto but not 1-1 D) neither 1-1 nor onto

14. Let $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = x^4$. Choose the correct answer.

A) f is one-one onto B) f is many-one onto

C) f is one-one but not onto D) f is neither one-one nor onto. **(NCERT, MQP)**

15. Let $f : \mathbf{R} \to \mathbf{R}$ defined by f(x) = 3x. Choose the correct answer.

A) f is one-one onto B) f is many-one onto

C) f is one-one but not onto D) f is neither one-one nor onto. **(NCERT)**

16. If f = {(5, 2), (6, 3)}, then which of the following is true?

A) Domain of f = N B) Domain of f is {2, 3, 5, 6}

C) Range of f = {2, 3} D) Range of f is {5, 6}

17. A set A has 3 elements and the set B has 4 elements. Then the number of injective functions that can be defined from A to B is

A) 4! B) 3! C) 12! D) 64!

18. The number of all one – one functions from the set A = {a, b, c} to itself is.

A) 3 B) 6 C) 27 D) 1

19. If A contains 3 elements and B contains 2 elements, then the number of one – one functions from A to B is___

A) 3 B) 0 C) 3^2 D) 3!

20. Relation R in the set A = {1, 2, 3, ..., 13, 14} defined as R = {(x, y) : 3x – y = 0}

Then R is

A) reflexive B) symmetric C) transitive D) none of these

21. If $f: R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$ is onto, then the interval S is

A) [–1, 3] B) [–1, 1] C) [0, 1] D) [0, 3] **(AIEEE 04)**

22. $f : R \to R$ given by $f(x) = x + \sqrt{x^2}$ is

A) one-one B) onto C) bijective D) many one-into

23. $f : R \to R$ given by $f(x) = 5x + |\cos x|$ is

A) one-one and onto B) one-one and into

C) many one and into D) many one and onto

24. In the set Z of all integers, which of the following relation R is not an equivalence relation ?

A) $x R y$ if $x \leq y$ B) $x R y$ if $x = y$

C) $x R y$ if $x - y$ is an even integer D) $x R y$ if $|x| = |y|$

25. If A = {x, y, z}, then the relation R = {(x, y), (y, x), (x, x)} on A is

A) reflexive B) symmetric and transitive

C) symmetric only D) transitive only

26. For $x, y \in R$, define a relation R by $x R y$ if and only if $x - y + \sqrt{2}$ is an irrational number. Then R is

A) an equivalence relation B) R is symmetric

C) R is reflexive D) R is transitive

27. Let A = {1, 2, 3} and consider the relation

R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}. Then R is

A) reflexive but not symmetric B) reflexive but not transitive

C) symmetric and transitive D) neither symmetric nor transitive

28. Let $f : R \to R$ be defined by $f(x) = e^x - e^{-|x|}$. Then

A) the range of f is $(-\infty, 0]$ B) f is 1 – 1

C) the range of f is $[0, \infty)$ D) f is onto

29. A is a set having 6 distinct elements. The number of distinct functions from A to A which are not bijections is **(CET 18)**

A) $6! - 6$ B) $6^6 - 6$ C) $6^6 - 6!$ D) $6!$

30. If $A = \{x \mid x \in \mathbb{N}, x \le 5\}$ $B = \{x \mid x \in \mathbb{Z}, x^2 - 5x + 6 = 0\}$, then the number of onto functions from A to B is (CET 19)

A) 30 B) 2 C) 32 D) 23

31. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set $A \times B$. Then : (JEE-M 21)

A) $y = 273x$ B) $2y = 91x$ C) $y = 91x$ D) $2y = 273x$

32. If $f : R \to R$ defined by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then $(f \circ f)(x) =$

A) $3 - x^3$ B) x C) x^3 D) $-x$

33. If $f : R \to R$ given by $f(x) = 7x + 8$ and $f^{-1}(12) = \frac{k}{7}$, then the value of k is

A) 7 B) 1 C) 4 D) 8

34. If $f(x) = \frac{3x+2}{5x-3}$, $x \in R - \left\{\frac{3}{5}\right\}$, then

A) $f^{-1}(x) = f(x)$ B) $f(f(x)) = -x$ C) $f^{-1}(x) = -f(x)$ D) Inverse does not exist

35. If a set A has m elements and set B has 7 elements and the number of injections from A to B is 2520, then the value of m is

A) 2 B) 7 C) 6 D) 5

36. For any two real numbers θ and ϕ, θ R ϕ iff $\sec^2\theta - \tan^2\phi = 1$. Then the relation R is

A) reflexive but not transitive B) symmetric but not reflexive

C) an equivalence relation D) both reflexive and symmetric but not transitive.

37. A function $f : [0, \infty) \to [0, \infty)$ defined by $f(x) = \frac{x}{1+x}$ is

A) one-one and onto B) one-one but not onto

C) onto but not one-one D) neither one-one nor onto

38. Let a function $f : N \to N$ be defined by $f(n) = \begin{cases} 2n, & n = 2, 4, 6, 8, \ldots\ldots\ldots \\ n-1, & n = 3, 7, 11, 15, \ldots\ldots \\ \frac{n+1}{2}, & n = 1, 5, 9, 13, \ldots\ldots \end{cases}$, then f is

A) one-one but not onto B) onto but not one-one

C) neither one-one nor onto D) one-one and onto

39. Let $f : R \to R$ be defined by $f(x) = \frac{1}{x}, \forall x \in R$, then f is …………. (CET 15)

A) one – one B) onto C) bijective D) f is not defined

40. Let A = {x : x ∈ R; x is not a positive integer} Define $f : A \rightarrow R$ as $f(x) = \frac{2x}{x-1}$, then f is **(CET 21)**

A) Injective but not surjective B) surjective but not injective

B) bijective D) neither injective nor surjective

41. The function $f(x) = \sqrt{3}\sin 2x - \cos 2x + 4$ is one-one in the interval **(CET 21)**

A) $\left[\frac{-\pi}{6}, \frac{\pi}{3}\right]$ B) $\left(\frac{\pi}{6}, \frac{-\pi}{3}\right]$ C) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ D) $\left[\frac{-\pi}{6}, \frac{-\pi}{3}\right)$

42. Let $f: R \rightarrow R$ be defined by $f(x) = 2x + 6$ which is a bijective mapping then $f^{-1}(x)$ is given by **(CET 16)**

A) $\frac{x}{2} - 3$ B) $2x + 6$ C) $x - 3$ D) $6x + 2$

43. If $f : R \rightarrow R$ is defined by $f(x) = 2x + 3$, then $f^{-1}(x)$ **(CET 12)**

A) is given by $\frac{x-3}{2}$ B) is given by $\frac{1}{2x+3}$

C) does not exist because 'f ' is not injective

D) does not exist because 'f ' is not surjective

44. The number of bijective functions from the set A to itself, if A contains 108 elements is **(COMEDK 15)**

A) 108 B) (108)! C) $(108)^2$ D) 2^{108}

45. The set A has 4 elements and the set B has 5 elements then the number of injective mappings that can be defined from A to B is **(CET 16)**

A) 144 B) 72 C) 60 D) 120

46. If the set x contains 7 elements and set y contains 8 elements, then the number of bijections from x to y is **(CET 22)**

A) 0 B) 7 ! C) $8\ P_7$ D) 8 !

47. If $f(x) = e^x$ and $g(x) = \log e^x$, then which of the following is TRUE ?

A) $f\{g(x)\} \neq g\{f(x)\}$ B) $f\{g(x)\} = g\{f(x)\}$

C) $f\{g(x)\} + g\{f(x)\} = 0$ D) $f\{g(x)\} - g\{f(x)\} = 1$

48. $f : R \rightarrow R$ and $g : [0, \infty) \rightarrow R$ are defined by $f(x) = x^2$ and $g(x) = \sqrt{x}$. Which one of the following is not true ? **(CET 19, 23)**

A) (fog) (2) = 2 B) (gof) (4) = 4 C) (gof) (– 2) = 2 D) (fog) (– 4) = 4

49. Let $f : R \to R$ be defined by $f(x) = 3x^2 - 5$ and $g : R \to R$ by $g(x) = \dfrac{x}{x^2+1}$, then gof is **(CET 23)**

A) $\dfrac{3x^2}{x^4+2x^2-4}$ B) $\dfrac{3x^2-5}{9x^4-30x^2+26}$ C) $\dfrac{3x^2}{9x^4+30x^2-2}$ D) $\dfrac{3x^2-5}{9x^4-6x^2+26}$

50. Let $f(x) = \sin 2x + \cos 2x$ and $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain **(CET 23)**

A) $x \in \left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$ B) $x \in \left[\dfrac{-\pi}{4}, \dfrac{\pi}{4}\right]$ C) $x \in \left[0, \dfrac{\pi}{4}\right]$ D) $x \in \left[\dfrac{-\pi}{8}, \dfrac{\pi}{8}\right]$

Answers

1. **(C)**

$2 = 4 - 2$ but $4 < 6$ $\therefore (2, 4) \notin R$; $(3, 8) \notin R$ $[\because 3 \neq 8 - 2]$. $(6, 8) \in R$ because $8 > 6$ and $a = 8 - 2$.

2. **(B)**

$(a, a) \in R, \ \forall a \in \{1, 2, 3, 4\}$; $(1, 2) \in R$ but $(2, 1) \notin R$

$\therefore$ R is not symmetric ; it is trivially transitive.

3. **(D)**

4. **(B)**

No. of relations $= 2^{n(A\times A)} = 2^{n(A).n(A)} = 2^{16}$

5. **(C)**

$a \leq a \ \forall a$; $2 \leq 3$ but $3 \not\leq 2$; $a \leq b$ and $b \leq c \Rightarrow a \leq c$

6. **(C)**

$0 \nmid 0$; $4 | 2$ but $2 \nmid 4$; $a | b$ and $b | c \Rightarrow a | c$

7. **(B)**

8. **(A)**

9. **(B)**

10. **(D)**

$f(2) = f(-2) = 4 \Rightarrow$ f is not 1 – 1

range of $f = W \neq Z \Rightarrow$ not on to

11. **(D)**

$f(1 . 2) = f(1 . 9) = 1 \Rightarrow$ f is not 1 – 1

range of $f = Z \neq R \Rightarrow$ not onto

12. **(D)**

$f(2) = f(-2) = 2 \Rightarrow$ f is not 1 – 1

range of $f = [0, \infty) \neq R \Rightarrow$ not onto

13. (D)

$f(1) = f(2) = 1 \Rightarrow$ not 1 – 1

range of f = $\{-1, 0, 1\} \neq R$

14. (D)

$f(1) = f(-1)$; Range = $\mathbf{R}_* \neq \mathbf{R}$

15. (A)

$f(a) = f(b) \Rightarrow a = b;\ f\left(\frac{b}{3}\right) = b\ ;\ \frac{b}{3} \in \mathbf{R}$ when $b \in \mathbf{R}$.

16. (C)

Domain = {5, 6}, Range = {2, 3}

17. (A)

Required = ${}^4P_3 = 4!$

18. (B)

Required = ${}^3P_3 = 3!$

19. (B)

If n(A) > n(B) then no one-one functions.

20. (D)

$R = \{(x, y) : 3x - y = 0\}$ i.e. $R = \{(x, y) : 3x = y\}$

If R is to be reflexive, $(x, x) \in R,\ \forall\ x \in A$.

Now, $(x, x) \in R$ if $3x = x$, which is true only for $x = 0$.

In other words, $(1, 1) \notin R$ because $3.1 \neq 1\ \therefore$ R is not reflexive.

If R is to be symmetric, then $(x, y) \in R \Rightarrow (y, x) \in R$.

Now, $(x, y) \in R \Rightarrow 3x = y \Rightarrow x = \frac{1}{3}y$ i.e. $3y \neq x \Rightarrow (y, x) \notin R$.

For example, $(1, 3) \in R$ but $(3, 1) \notin R$. $\therefore$ R is not symmetric

Let (x, y) and $(y, z) \in R$. Then $3x = y$ and $3y = z$

Then $3x = \frac{1}{3}z \Rightarrow 3x \neq z$ i.e. $(x, z) \notin R$.

For example, $(1, 3)$ and $(3, 9) \in R$ but $(1, 9) \notin R\ \therefore$ R is not transitive.

21. (A)

Max. f = 1 + 2; Min. f = 1 – 2

22. (D)

$$f(x) = x + \sqrt{x^2} = x + |x| = \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

23. (A)

$f(x) = 5x \Rightarrow f'(x) = 5, \quad \forall x \in R$

$\Rightarrow$ f(x) is strictly increasing function

$\therefore$ $f(x) = 5x + |\cos x|$ is also strictly increasing function $\Rightarrow$ it is both 1 – 1 and onto

24. (A)

25. (C)

26. (C)

Since $x - x + \sqrt{2} = \sqrt{2}$ which is an irrational number, so xRx, $\forall\ x \in R$ is possible. $\therefore$ R is reflexive.

But R is not symmetric, for, $\left(\sqrt{2},\ 1\right) \in R$ but $\left(1,\ \sqrt{2}\right) \notin R$

Also R is not transitive, for, $\left(\sqrt{2},\ 1\right) \in R$ and $\left(1,\ 2\sqrt{2}\right) \in R$

but $\left(\sqrt{2},\ 2\sqrt{2}\right) \notin R$

27. (1)

Clearly, R is reflexive, for, $(1, 1) \in R$, $(2, 2) \in R$, $(3, 3) \in R$

But R is not symmetric, for, $(2, 3) \in R$ but $(3, 2) \notin R$

28. (C)

When $x \geq 0$, $f(x) = e^{x} - e^{-x}$

When $x < 0$, $f(x) = e^{x} - e^{-(-x)} = 0$

Clearly f is not 1 – 1

When $x > 0$, $e^{x} > e^{-x}$ $\quad \therefore f(x) > 0, \forall x > 0 \quad \therefore$ The range is $[0, \infty) \neq R$

$\therefore$ f is not onto.

29. (C)

Required = No. of functions – number of bijective functions = $6^6 - 6!$

30. (A)

$A = \{1, 2, 3, 4, 5\}$ & $B = \{2, 3\}$

Use : If $n(A) = n\ (n \geq 2)$ & $n(B) = 2$, then the number of onto functions from A to B is

$2^n - 2 = 2^5 - 2 = 30.$

31. (B)

$x = {}^{5}P_{3} = 5 . 4 . 3$

$y = {}^{15}P_{3} = 15 . 14 . 13 \qquad \therefore \frac{y}{x} = \frac{15.14.13}{5.4.3} = \frac{91}{2} \Rightarrow 2y = 91x$

32. (B)

33. (C)

$$f^{-1}(x) = \frac{x-8}{7};\ f^{-1}(12) = \frac{4}{7} \Rightarrow k = 4$$

34. (A)

$$f^{-1}(x)=\frac{-3x-2}{-5x+3}=\frac{3x+2}{5x-3}=f(x)$$

35. (D)

$2520 = {}^{7}P_{m} \Rightarrow m = 5$

36. (C)

37. (B)

$$f(x)=\frac{x+1-1}{x+1}=1-\frac{1}{x+1}$$

$$f'(x)=\frac{1}{(x+1)^2}>0 \Rightarrow f \text{ is } 1-1$$

Range of f = [0, 1) $\neq$ codomain $\Rightarrow$ f is not onto

38. (D)

39. (D)

$f(0) = \frac{1}{0}$, which is meaningless !

$\therefore$ f is not a well defined function.

40. (A)

Domain = R – N; $f(x) = \dfrac{2}{1-\frac{1}{x}}$

$a, b \in A \Rightarrow f(a) = f(b) \Rightarrow \dfrac{2}{1-\frac{1}{a}} = \dfrac{2}{1-\frac{1}{b}} \Rightarrow a = b \quad \therefore$ f is 1 - 1

Let y $\in$ R be such that y = f(x)

Then $y=\dfrac{2}{1-\frac{1}{x}} \Rightarrow 1-\dfrac{1}{x}=\dfrac{2}{y}$

$\dfrac{1}{x}=1-\dfrac{2}{y}=\dfrac{y-2}{y} \quad \therefore x=\dfrac{y}{y-2} \notin Domain\ if\ \ y=2$

OR

$f(x)\neq 2 \ \because \dfrac{2x}{x-1}\neq 2\ and\ 2\in R$

$\therefore$ Range $\neq$ R (codomain) $\quad \therefore$ It is not onto $\therefore$ (A) is the correct option

41. (A)

$$f(x) = \sqrt{3}\,\sin 2x - \cos 2x + 4 = 2\left(\frac{\sqrt{3}}{2}\sin 2x - \frac{1}{2}\cos 2x\right) + 4$$

$$= 2\left(\cos\frac{\pi}{6}\,.\,\sin 2x - \sin\frac{\pi}{6}\,.\,\cos 2x\right) + 4 = 2.\sin\left(2x - \frac{\pi}{6}\right) + 4$$

$$\sin\left(2x - \frac{\pi}{6}\right) \textit{ is } 1-1 \text{ in the interval: } \left(2x - \frac{\pi}{6}\right) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\therefore\ 2x \in \left[-\frac{\pi}{3}, \frac{2\pi}{3}\right] \quad \therefore\ x \in \left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$$

Aliter: The options in (B) and (D) are not intervals!!

$\therefore$ (A) or (C) is the correct answer

Take (C) : $f\left(\frac{\pi}{2}\right) = 5 = f\left(-\frac{\pi}{2}\right)$ $\quad\therefore$ f is not 1 – 1

$\therefore$ (A) should be the correct answer

42. (A)

We have, f(x) = 2x + 6. f is a bijective function $\Rightarrow f^{-1}$ exists.

Let $x \in R$ then there exists $y \in R$ such that $f(x) = y \Rightarrow 2x + 6 = y$

$\Rightarrow x = \frac{y-6}{2} \Rightarrow f^{-1}(y) = \frac{y-6}{2} \quad \therefore\ f^{-1}(x) = \frac{x-6}{2}$ for all $x \in R$

$x \neq 3$ because $[x]^2 - [x] - 6 = 0$ when $x = 3$ $\quad\therefore$ (A) is the correct answer

43. (A)

Let $y = 2x + 3 \ \Rightarrow\ 2x = y - 3 \ \Rightarrow\ x = \frac{1}{2}(y - 3)\ \ \therefore f^{-1}(x) = \frac{1}{2}(x - 3)$

44. (B)

From memory !

45. (D)

Set A has 4 elements and set B has 5 elements, hence the number of injective mappings from A to B = ${}^5P_4 = 120$

46. (A)

An $n(X) < n(Y)$, no onto function is possible and hence bijective function from $X \to Y$ is not possible.

47. (B)

$f(g(x)) = e^{g(x)} = f(x)$; $f(x) = x$; $g(f(x)) = f(x) = f(g(x))$

48. (D)

$(fog)(-4) = f(g(-4))$; but $g(-4) = \sqrt{-4}$ doesn't exist

49. (B)

$$(gof)(x) = g(f(x)) = \frac{f}{f^2+1} = \frac{3x^2-5}{(3x^2-5)^2+1} = \frac{3x^2-5}{9x^4-30x^2+26}$$

50. (D)

$g(f(x)) = (\sin 2x + \cos 2x)^2 - 1 = 2 \sin 2x \, . \cos 2x = \sin 4x$;

It is invertible if $4x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ i.e., $x \in \left[-\frac{\pi}{8}, \frac{\pi}{8}\right]$

Remark: sin x is bijective and hence invertible in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Ch. 2: Inverse Trigonometric Functions

Choose the correct answer

1. The principal value branch of $\sin^{-1} x$ is

A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ B) $(0, \pi)$ C) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ D) $[0, \pi]$

2. The domain of $f(x) = \sin^{-1} x$ is

A) $[-1, 1]$ B) $(-1, 1)$ C) $(-\infty, \infty)$ D) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

3. The principal value branch of $f(x) = \cos^{-1} x$ is

A) $[-1, 1]$ B) $(0, \pi)$ C) $[0, \pi]$ D) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

4. The domain of $f(x) = \cos^{-1} x$ is

A) $(-1, 1)$ B) $[-1, 1]$ C) $[0, \pi]$ D) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

5. The principal value branch of $f(x) = \tan^{-1} x$ is

A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ C) $(0, \pi)$ D) $[0, \pi]$ **(23-M, 23-J)**

6. The domain of $f(x) = \tan^{-1} x$ is

A) $R - (-1, 1)$ B) $(0, \pi)$ C) $(-\infty, \infty)$ D) $[-1, 1]$

7. The domain of $f(x) = \cot^{-1} x$ is

A) $(-\infty, \infty)$ B) $R - (-1, 1)$ C) $[-1, 1]$ D) $R - [-1, 1]$

8. The principal value branch of $\cot^{-1} x$ is **(MQP)**

A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ C) $(0, \pi)$ D) $[0, \pi]$

9. The domain of $\sec^{-1} x$ is

A) $(-\infty, \infty)$ B) $R-(-1, 1)$ C) $[-1, 1]$ D) $R-[-1, 1]$

10. The principal value branch of $\sec^{-1} x$ is

A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)-\{0\}$ B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]-\{0\}$ C) $(0, \pi)-\left\{\frac{\pi}{2}\right\}$ D) $[0, \pi]-\left\{\frac{\pi}{2}\right\}$

11. The domain of $\operatorname{cosec}^{-1} x$ is

A) $(-\infty, \infty)$ B) $R-(-1, 1)$ C) $[-1, 1]$ D) $R-[-1, 1]$

12. The principal value branch of $\operatorname{cosec}^{-1} x$ is

A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)-\{0\}$ B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]-\{0\}$ C) $(0, \pi)-\left\{\frac{\pi}{2}\right\}$ D) $[0, \pi]-\left\{\frac{\pi}{2}\right\}$

13. The value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is

A) $-\frac{\pi}{6}$ B) $\frac{\pi}{6}$ C) $\frac{5\pi}{6}$ D) $-\frac{\pi}{3}$

14. Value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is

A) $\frac{\pi}{3}$ B) $-\frac{\pi}{3}$ C) $\frac{2\pi}{3}$ D) $\frac{5\pi}{6}$

15. The value of $\tan^{-1}(-1)$ is

A) $\frac{\pi}{4}$ B) $-\frac{\pi}{4}$ C) $\frac{3\pi}{4}$ D) $-\frac{3\pi}{4}$

16. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=$

A) $\frac{2\pi}{3}$ B) $-\frac{\pi}{3}$ C) $\frac{\pi}{3}$ D) $-\frac{2\pi}{3}$

17. $\sin(\tan^{-1} x)=$

A) $\frac{x}{\sqrt{1-x^2}}$ B) $\frac{x}{\sqrt{x^2-1}}$ C) $\frac{x}{\sqrt{1+x^2}}$ D) $\pm\frac{x}{\sqrt{1+x^2}}$

18. $\tan^{-1}(-\sqrt{3}) =$

A) $-\frac{\pi}{3}$ B) $-\frac{\pi}{6}$ C) $\frac{5\pi}{6}$ D) $\frac{2\pi}{3}$

19. If $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, then the value of x is

A) $0, \frac{1}{2}$ B) $1, \frac{1}{2}$ C) 0 D) $\frac{1}{2}$

20. If $\sin^{-1}x = y$, then

A) $0 \le y \le \pi$ B) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ C) $0 < y < \pi$ D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

21. The solution set of the equation $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$ is

A) $\{0, 1\}$ B) $\{-1, 1\}$ C) $\left\{0, \frac{1}{2}\right\}$ D) $\left\{-1, \frac{1}{2}\right\}$

22. The value of $\sin^{-1}\left[\cos\left\{\sin^{-1}\left(\frac{1}{2}\tan\frac{\pi}{3}\right)\right\}\right]$ is

A) $2\tan^{-1}\left(2-\sqrt{3}\right)$ B) $2\tan^{-1}\left(\sqrt{2}-1\right)$ C) $2\tan^{-1}\left(\sqrt{2}+1\right)$ D) $2\tan^{-1}\left(2+\sqrt{3}\right)$

23. The value of the expression $2\sec^{-1}2 + \operatorname{cosec}^{-1}2$ is equal to

A) $\frac{\pi}{6}$ B) $\frac{5\pi}{6}$ C) $\frac{7\pi}{6}$ D) 2π

24. The principal value of $\sin^{-1}\left[\cos\left(\sin^{-1}\frac{1}{2}\right)\right]$ is

A) $\frac{\pi}{4}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{2}$

25. $\cos^{-1}\left(\cos\frac{14\pi}{3}\right)$ is

A) $\frac{\pi}{3}$ B) $\frac{2\pi}{3}$ C) $\frac{\pi}{6}$ D) $\frac{5\pi}{6}$

Fill in the blanks

1. $\tan^{-1}\sqrt{3} + \cot^{-1}\left(-\sqrt{3}\right) =$ _______

2. $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) =$ _______

3. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) =$ ______

4. $\operatorname{cosec}^{-1}(-\sqrt{2}) = \_\_\_\_\_$

5. $\sec^{-1}\left(\dfrac{2}{\sqrt{3}}\right) = \_\_\_\_\_\_$

6. The value of $\cos(2\cos^{-1}(0.8))$ is ____

7. $\cos^{-1}\left(\cos\dfrac{8\pi}{3}\right) = \_\_\_\_\_$

8. $\sin^2\left(\tan^{-1}\dfrac{3}{4}\right) + \sin^2\left(\cos^{-1}\dfrac{3}{5}\right) =$

9. $\cos^{-1}\left(\cot\dfrac{\pi}{2}\right) + \cos^{-1}\left(\sin\dfrac{2\pi}{3}\right) =$

10. $\dfrac{\tan^{-1}\sqrt{3} - \sec^{-1}(-2)}{\operatorname{cosec}^{-1}(-\sqrt{2}) + \cos^{-1}\left(-\dfrac{1}{2}\right)} =$

11. $\sin^{-1}(\sin(-600°)) = \_\_\_\_$

12. If the inverse trigonometric functions take principal values, then

$$\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\frac{4}{3}\right) + \frac{2}{5}\sin\left(\tan^{-1}\frac{4}{3}\right)\right) = \_\_\_$$

13. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then $\alpha(\beta+\gamma) + \beta(\gamma+\alpha) + \gamma(+\beta) = \_\_$

14. $\tan^{-1}\left(\dfrac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\dfrac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right) = \_\_\_\_\_$

15. If $\dfrac{x^2+1}{x^2-1} = A + \dfrac{B}{x^2-1}$, then $\tan^{-1} A + \operatorname{cosec}^{-1} B = \_\_\_\_$

Answer

1. **(C)** **2.** **(A)** **3.** **(C)** **4.** **(C)** **5.** **(A)** **6.** **(C)** **7.** **(A)** **8.** **(C)**

9. **(B)** **10.** **(C)** **11.** **(B)** **12.** **(B)** **13.** **(A)** **14.** **(C)** **15.** **(B)** **16.** **(C)**

17. **(C)**

$$\sin(\tan^{-1}x) = \sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right) = \frac{x}{\sqrt{1+x^2}}$$

18. **(A)** **19.** **(C)** **20.** **(B)**

21. (C)

Hint: Go from the alternatives

$x \neq -1 \quad \because \sin^{-1}(1-x)$ would become $\sin^{-1} 2$.

When $x = 1$, LHS $= \dfrac{\pi}{2} \neq 0 =$ RHS

When $x = \dfrac{1}{2}$, LHS $= \sin^{-1}\dfrac{1}{2} + \sin^{-1}\dfrac{1}{2} = \cos^{-1}\dfrac{1}{2}$

22. (A)

$$\sin^{-1}\left[\cos\left\{\sin^{-1}\frac{\sqrt{3}}{2}\right\}\right] = \sin^{-1}\left[\cos\frac{\pi}{3}\right] = \sin^{-1}\frac{1}{2} = \frac{\pi}{6} = 2\tan^{-1}(2-\sqrt{3})$$

23. (B)

$$\text{G.E.} = 2\,.\,\frac{\pi}{3} + \frac{\pi}{6} = \frac{5\pi}{6}$$

24. (C)

$$\text{G.E.} = \sin^{-1}\left[\cos\frac{\pi}{6}\right] = \frac{\pi}{3}$$

25. (B)

$$\cos\frac{14\pi}{3} = \cos\frac{2\pi}{3} \qquad \therefore \cos^{-1}\cos\frac{2\pi}{3} = \frac{2\pi}{3}$$

Fill in the blanks

1. $\left(\dfrac{5\pi}{6}\right)$ **2. (1)** **3.** $\dfrac{5\pi}{6}$ **4.** $\left(-\dfrac{\pi}{4}\right)$ **5.** $\left(\dfrac{\pi}{6}\right)$

6. $$\cos\left(2\cos^{-1}\frac{8}{10}\right) = \cos\left(2\cos^{-1}\frac{4}{5}\right) = \cos 2\alpha = 2\cos^2\alpha - 1$$
$$= 2\left(\frac{4}{5}\right)^2 - 1\left(\alpha = \cos^{-1}\frac{4}{5}\right) = \frac{7}{25} = 0.28$$

7. $$\frac{8\pi}{3} = 2\pi + \frac{2\pi}{3} \Rightarrow \text{Required} = \cos^{-1}\cos\left(2\pi + \frac{2\pi}{3}\right) = \frac{2\pi}{3}$$

8. $$\tan^{-1}\frac{3}{4} = \sin^{-1}\frac{3}{5},\ \cos^{-1}\frac{3}{5} = \sin^{-1}\frac{4}{5} \qquad \therefore \text{Required} = \frac{9}{25} + \frac{16}{25} = 1$$

9. $$\text{G.E.} = \cos^{-1}0 + \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$$

10. $\text{G.E.} = \dfrac{\frac{\pi}{3} - \left(\pi - \frac{\pi}{3}\right)}{-\frac{\pi}{4} + \pi - \frac{\pi}{3}} = \dfrac{\frac{-\pi}{3}}{\frac{5\pi}{12}} = -\dfrac{4}{5}$

11. $\dfrac{\pi}{3}$

12. $\tan^{-1}\dfrac{4}{3} = \sin^{-1}\dfrac{4}{5} = \cos^{-1}\dfrac{3}{5}$ $\quad \therefore$ Required $= \cos^{-1}\left(\dfrac{3}{10}\cdot\dfrac{3}{5} + \dfrac{2}{5}\cdot\dfrac{4}{5}\right) = \cos^{-1}\dfrac{1}{2} = \dfrac{\pi}{3}$

13. Max. value of $\cos^{-1} x = \pi \Rightarrow \alpha = \beta = \gamma = -1$

$\therefore$ Required = 2 + 2 + 2 = 6

14. $\dfrac{1+\sqrt{3}}{3+\sqrt{3}} = \dfrac{1}{\sqrt{3}}$; $\dfrac{8+4\sqrt{3}}{6+3\sqrt{3}} = \dfrac{4}{3}$

$\therefore$ Required $= \tan^{-1}\dfrac{1}{\sqrt{3}} + \sec^{-1}\dfrac{2}{\sqrt{3}} = \dfrac{\pi}{6} + \dfrac{\pi}{6} = \dfrac{\pi}{3}$

15. $\dfrac{x^2+1}{x^2-1} = \dfrac{x^2-1+2}{x^2-1} = 1 + \dfrac{2}{x^2-1} \Rightarrow A = 1,\ B = 2$

$\therefore$ Required $= \dfrac{\pi}{4} + \dfrac{\pi}{6} = \dfrac{5\pi}{12}$

Ch.3: Matrices

Choose the correct answer

1. If O (A) = 4 × 3 , O (B) = 4 × 5 , then $O\left((A'\,B)'\right)$ is

A) 3 × 5 B) 3 × 4 C) 4 × 3 D) 5 × 3

2. If a matrix has 8 elements, then the total number of possible different matrices is

A) 8 B) 6 C) 4 D) 2

3. For any square matrix $A = [a_{ij}]$, with $a_{ij} = 0$ when $i \neq j$, then A is a

A) Unit matrix B) Scalar matrix C) Diagonal matrix D) Zero matrix

4. The number of all possible different matrices of order 3 × 3 with each entry 0 or 1 is

A) 27 B) 18 C) 81 D) 512 **(MQP)**

5. If $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 5 & y-2 \\ 8 & 4 \end{bmatrix}$, then x and y are respectively

A) $-\dfrac{2}{3}, 7$ B) $\dfrac{2}{3}, \dfrac{7}{3}$ C) $-\dfrac{2}{3}, -7$ D) $-\dfrac{1}{3}, 7$

6. If $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$, $X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$, then the matrix X is

A) $\begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$ B) $\begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$ C) $\begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$ D) $\begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$

7. Let X be a matrix of order $2 \times n$ and Z is matrix of order $2 \times p$. If $n = p$, then the order of the matrix is

A) $p \times 2$ B) $2 \times n$ C) $n \times 3$ D) $p \times n$

8. If A and B are matrices of order $m \times n$ and $n \times n$ respectively, then which of the following are defined?

A) Both AB and BA B) AB, A^2 C) A^2, B^2 D) AB, B^2

9. If $A = \begin{bmatrix} 3 & x \\ y & 0 \end{bmatrix}$ and $A = A'$, then

A) $x = 0, y = 3$ B) $x + y = 3$ C) $x = y$ D) $x = -y$

10. If A and B are symmetric matrices of same order then AB – BA is

A) Symmetric matrix B) Skew symmetric matrix
C) Unit matrix D) Null matrix

11. If $A = \begin{bmatrix} 2 & x-3 & x-2 \\ 3 & -2 & -1 \\ 4 & -1 & -5 \end{bmatrix}$ is a symmetric matrix, then $x =$

A) 0 B) 3 C) 6 D) 8

12. If A is a square matrix then $A - A'$ is

A) Unit matrix B) Null matrix
C) Symmetric matrix D) Skew symmetric matrix

13. If $A = \begin{bmatrix} 1 & -2 \\ -4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$, then $3A - 2B =$

A) $\begin{bmatrix} -5 & -8 \\ -6 & 5 \end{bmatrix}$ B) $\begin{bmatrix} -5 & -8 \\ 6 & -5 \end{bmatrix}$ C) $\begin{bmatrix} 5 & -8 \\ -6 & 5 \end{bmatrix}$ D) $\begin{bmatrix} 5 & 8 \\ 6 & 5 \end{bmatrix}$

14. If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, then the value of x and y are

A) $x = 3, y = 3$ B) $x = -3, y = 3$ C) $x = 3, y = -3$ D) $x = -3, y = -3$

15. If $\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+2 \end{bmatrix}$ is a symmetric matrix, then $x =$

A) 2 B) 5 C) –1 D) 3

16. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then the value of α for which $A + A' = I$ is

A) $\frac{\pi}{6}$ B) $\frac{\pi}{3}$ C) π D) $\frac{3\pi}{2}$

17. Let $O(X) = 2 \times n$, $O(Y) = 3 \times k$, $O(Z) = 2 \times p$, $O(W) = n \times 3$, $O(P) = p \times k$ respectively. Then the restriction on n, k, p so that $PY + WY$ will be defined are

A) $k = 3$, $p = n$ B) k is arbitrary, $p = 2$

C) p is arbitrary D) $k = 2$, $p = 3$

18. Values of x and y for which $\begin{bmatrix} x-4 & 3 & 5 \\ -3 & 0 & 2-y \\ -5 & 2y-3 & 0 \end{bmatrix}$ is a skew symmetric matrix is

A) $x = 0$, $y = 1$ B) $x = 4$, $y = 2$ C) $x = -4$, $y = 2$ D) $x = 4$, $y = 1$

19. If $\begin{bmatrix} 3+x & 0 \\ y-1 & 2 \end{bmatrix}$ is a scalar matrix, then the values of x and y are respectively

A) $-1, 1$ B) $-1, -1$ C) $-3, 1$ D) $-3, -1$

20. If $\begin{bmatrix} 5 & 0 & y-7 \\ 0 & 4-x & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the values of x and y are respectively

A) $5, -7$ B) $-1, 7$ C) $-1, -7$ D) $1, 7$

21. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

A) A B) $I - A$ C) I D) 3A

22. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

A) $1 + \alpha^2 + \beta\gamma = 0$ B) $1 - \alpha^2 + \beta\gamma = 0$

C) $1 - \alpha^2 - \beta\gamma = 0$ D) $1 + \alpha^2 - \beta\gamma = 0$

23. Which of the given values of x and y make the following pair of matrices equal $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}$, $\begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

A) $x = -\frac{1}{3}, y = 7$ B) Not possible to find

C) $y = 7, x = -\frac{2}{3}$ D) $x = -\frac{1}{3}, y = -\frac{2}{3}$

24. If $\begin{bmatrix} x-2 & y-3 \\ 0 & 4 \end{bmatrix}$ is a scalar matrix, then

A) x = 2, y = 3 B) x = 6, y = 3 C) x = 4, y = – 3 D) x = 6, y $\in$ R

25. If $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then a + b + c + d =

A) 0 B) 3 C) 2 D) 4

26. The values of x and y if $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$ are given by

A) x = 2, y = 9 B) x = 2, y = 8 C) x = 3, y = 6 D) x = 4, y = 7

27. If $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$, then x + y + z + t =

A) 20 B) 18 C) 21 D) 24

28. If $\begin{bmatrix} 2x+y & 3y \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 6 & 4 \end{bmatrix}$, then x + y =

A) 3 B) 4 C) 0 D) 5

29. The matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is

A) a symmetric matrix B) scalar matrix

C) skew symmetric matrix D) a non-square matrix

30. The matrix $\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is

A) a skew symmetric matrix B) symmetric matrix

C) a non-singular matrix D) both symmetric and skew symmetric matrix

31. If $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$, then AB =

A) $\begin{bmatrix} 2 & 0 \\ 3 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 0 & 2 \\ 0 & 3 \end{bmatrix}$ C) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

32. If $\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is a symmetric matrix, then x =

A) 3 B) 5 C) 4 D) 2

Answer

1. (D) **2.** (C) **3.** (C) **4.** (D) **5.** (A) **6.** (D) **7.** (B) **8.** (D) **9.** (C) **10.** (B)
11. (C) **12.** (D) **13.** (A) **14.** (A) **15.** (B) **16.** (B) **17.** (A) **18.** (D) **19.** (A) **20.** (B)
21. (C) **22.** (C) **23.** (B) **24.** (B) **25.** (C) **26.** (A) **27.** (D) **28.** (A) **29.** (A) **30.** (A)
31. (D) **32.** (A)

Ch. 4: Determinants

Choose the correct answer

1. If $A = kB$, where A and B are square matrices of order n, then $|A| =$

A) $k\,|B|$ B) $k^n\,|B|$ C) $k^{n+1}\,|B|$ D) $nk\,|B|$

2. If $\begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix} = \begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix}$ then the value of x is

A) $\sqrt{3}$ B) $\pm\sqrt{6}$ C) $\pm\sqrt{3}$ D) $\sqrt{6}$

3. If A is a matrix of order 3×3, then $|kA|$ is equal to

A) $k|A|$ B) $k^2|A|$ C) $k^3|A|$ D) $3k|A|$

4. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then $|2A| =$

A) $2|A|$ B) $3|A|$ C) $4|A|$ D) $|A|$

5. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then $|3A|$ is equal to

A) 27 B) 4 C) 54 D) 108

6. If $A_n = \begin{bmatrix} 1-n & n \\ n & 1-n \end{bmatrix}$, then $|A_1| + |A_2| + \ldots\ldots\ldots + |A_{2021}| =$

A) – 2021 B) $(2021)^2$ C) $-(2021)^2$ D) 4042 **(CET 22)**

7. If there are two values of ‘a’ which makes determinant $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, then the sum of these numbers is

A) – 4 B) 4 C) 9 D) 5 **(CET 22)**

8. If $A = \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ *and* $\det\left(A^2 - \frac{1}{2}I\right) = 0$, then a possible value of α is

A) $\frac{\pi}{2}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{6}$ **(JEE-M 21)**

9. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is

A) 4 B) 1 C) 6 D) 12 **(JEE-M 21)**

10. If $f(x) = \begin{vmatrix} x^3 - x & a + x & b + x \\ x - a & x^2 - x & c + x \\ x - b & x - c & 0 \end{vmatrix}$, then

A) f(2) = 0 B) f(0) = 0 C) f(–1) = 0 D) f(1) = 0 **(CET 2020)**

11. Let $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, $a \in R$ be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If det(Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to :

A) 36 B) 24 C) 45 D) 18 **(JEE-M 21)**

12. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, then adj A is

A) $\begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix}$ B) $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ C) $\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$ D) $\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$

13. If A is a non-singular matrix of order 3, then $|\text{adj } A|$ is equal to

A) $|A|$ B) $|A|^2$ C) $|A|^3$ D) $3|A|$

14. If A is an invertible matrix of order 3, then $\det(A^{-1})$ is

A) det A B) $\frac{1}{\det A}$ C) $\frac{1}{(\det A)^2}$ D) $(\det A)^2$

15. If A is a square matrix of order 3 with $|\text{adj } A| = 25$, then $|A| =$

A) $\frac{1}{25}$ B) 25^2 C) ± 5 D) $\pm\frac{1}{5}$ **(MQP)**

16. The inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$ is

A) $\begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$ B) $\begin{bmatrix} 6 & -3 \\ -4 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ D) does not exist

17. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then det (adj A) is

A) a^{27} B) a^9 C) a^6 D) a^2

18. If A is a matrix of order 3, such that $A\,(\text{adj } A) = 10I$, then $|\text{adj } A| =$

A) 1 B) 10 C) 100 D) 5

19. If $A = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$, then A (adj A) is eqaul to

A) $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ B) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ C) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ D) $\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

20. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A^{-1} =$

A) $\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ B) $\frac{-1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ D) $\begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$

21. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the matrix A is

A) $\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$ B) $\begin{bmatrix} -2 & 1 \\ 3 & -2 \end{bmatrix}$ C) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ D) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

22. If $A = [a_{ij}]$ is a 3 x 3 matrix and A_{ij} is the cofactor of element a_{ij} then,

$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} =$

A) $|A|$ B) $-|A|$ C) 0 D) 1

23. If x, y, z are non-zero real numbers, then the inverse of the matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is

A) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$ B) $xyz\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$ C) $\frac{1}{xyz}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ D) $\frac{1}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

24. The system of equations $x + y + z = 5,\ x + 2y + 3z = 9,\ x + 3y + 4z = 2$ has

A) Unique solution B) No solution

C) Infinitely many solutions D) Exactly two solutions

25. If A is a 3 × 3 square matrix and $|A| = 6$, then $|2 \text{ adj } A|$ is equal to

A) 48 B) 8 C) 288 D) 12

Fill in the blanks

1. If $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ then x ______ **(23-J)**

2. $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix} =$ __________

3. If A(adj A) = 5I, where I is the identity matrix of order 3, then |adj A| = ___

4. If $A = \left[a_{ij}\right]$ is a 3×3 matrix and A_{ij} is the cofactor of element a_{ij} then,

$a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} =$ _______

5. The co-factor of the element 6 in $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ is __________

6. A square matrix A is singular if |A| = ____ **(MQP)**

7. If A is a matrix of order 3 and $B = |A|A^{-1}$. If |A| = – 5, then |B| is equal to ____

8. If $B = \begin{bmatrix} p & 3 \\ 3 & p \end{bmatrix}$ and $|B^7| = 128$, then p = _____

9. If the system of equations x + ky – z = 0, 3x – ky – z = 0 and x – 3y + z = 0, has non-zero solution, then k is equal to _____

10. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then |A| |adj. A| is equal to

11. If A is a 3 × 3 non-singular matrix and if |A| = 3, then $|(5A)^{-1}|$ = _____

12. If $A = \begin{bmatrix} x & 1 & -x \\ 0 & 1 & -1 \\ x & 0 & 7 \end{bmatrix}$ and det (A) = $\begin{vmatrix} 3 & 0 & 1 \\ 2 & -1 & 2 \\ 0 & 0 & 3 \end{vmatrix}$, then the value of x is ____

13. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, then the value of the determinant $|A^{2020} - 5A^{2019}|$ is _____

14. If $A = \begin{bmatrix} 2 & 3 \\ 4 & -2 \end{bmatrix}$, $B = \begin{bmatrix} -4 & 1 \\ 0 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 8 & -5 \\ -7 & 3 \end{bmatrix}$, then |BAC| = ____

15. For any 2 × 2 matrix A, if A. adj.(A) = $\begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$, then |A| is equal to ___

Answer

1. **(B)** **2.** **(C)** **3.** **(B)** **4.** **(C)** **5.** **(D)**

6. **(C)**

$|A_n| = (1-n)^2 - n^2 = -(2n-1)$

$\Sigma|An| = -\Sigma(2n-1) = -n^2 = -(2021)^2$

7. **(A)**

$1(2a^2+4) + 2(4a-0) + 5(8) = 86 \Rightarrow 2a^2 + 8a + (\ ) = 0 \Rightarrow a_1 + a_2 = -\dfrac{8}{2} = -4$

8. **(C)**

$$A^2 = \sin^2\alpha \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (\sin^2\alpha)\, I \Rightarrow A^2 - (\sin^2\alpha)\, I = O \Rightarrow |A^2 - \sin^2\alpha I| = 0$$

$\therefore$ Comparing with $\left|A^2 - \dfrac{1}{2}I\right| = 0$, we get $\sin^2\alpha = \dfrac{1}{2}$ $\quad \therefore \alpha = \dfrac{\pi}{4} \in \left[0, \dfrac{\pi}{2}\right]$

9. **(A)**

$$A^2 = \begin{pmatrix} a & b \\ b & c \end{pmatrix}\begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} a^2+b^2 & b(a+c) \\ b(a+c) & b^2+c^2 \end{pmatrix}$$

The sum of the diagonal entries of A^2 is $a^2 + 2b^2 + c^2$

As $a^2 + 2b^2 + c^2 = 1$, $a, b, c \in Z$

$\Rightarrow b = 0$ and $a^2 + c^2 = 1$ $\quad \therefore$ $a = \pm 1$ and $c = 0$ or $a = 0$ and $c = \pm 1$

Number of matrices = 2 + 2 = 4

10. **(B)**

Clearly $f(0) = \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = 0$, being a determinant of a skew symmetric matrix of order 3.

11. **(A)**

$$P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{pmatrix} 4 & 3+a \\ a+3 & 0 \end{pmatrix}$$

$$Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{pmatrix} 0 & 3-a \\ a-3 & 0 \end{pmatrix}$$

Now, $|Q| = \dfrac{1}{4}(a-3)^2 = 9 \Rightarrow (a-3)^2 = 36$ $\quad \therefore a = 9$ or -3

Now, $|P| = -\dfrac{1}{4}(a+3)^2$

When a = – 3, |P| = 0

When a = 9, $|P| = -\frac{1}{4}.144 = -36$

Required = |0 + (– 36)| = 36

12. (D) 13. (B) 14. (B) 15. (C) 16. (D) 17. (C) 18. (C) 19. (A) 20. (B) 21. (A) 22. (A)
23. (A) 24. (A)
25. (C)

$$|2\ \text{adj.}A| = 2^3|\text{adj}A| = 8|A|^2 = 8(36) = 288$$

Fill in the blanks

1. $\left(\pm\ 2\sqrt{2}\right)$ **2. (0)** **3. (25)** **4. (0)** **5. (4)** **6. (0)**

7. $B = |A|A^{-1} \Rightarrow |B| = \left||A|A^{-1}\right| = |A|^n\left|A^{-1}\right| = |A|^n\frac{1}{|A|} = |A|^{n-1} = (-5)^2 = 25$

8. $|B^7| = 128 \Rightarrow |B^7| = 2^7 \Rightarrow |B| = 2 \Rightarrow p^2 - 9 = 2 \Rightarrow p = \pm\sqrt{11}$

9. The system has non-zero solution, if

$$\begin{vmatrix} 1 & k & -1 \\ 3 & -k & -1 \\ 1 & -3 & 1 \end{vmatrix} = 0 \Rightarrow (-k-3) - k(4) - 1(-9+k) = 0\ ;\ -6k = -6 \Rightarrow k = 1$$

10. $|A||\text{adj}A| = a^3|A|^2 = a^3\left(a^3\right)^2 = a^9 \quad \left(\because |\text{adj}A| = |A|^{n-1}\right)$

11. $|(5A)^{-1}| = \frac{1}{|5A|} = \frac{1}{5^3|A|} = \frac{1}{125\times 3} = \frac{1}{375}$

12. $|A| = \begin{vmatrix} x & 1 & -x \\ 0 & 1 & -1 \\ x & 0 & 7 \end{vmatrix} = 7x - 1(x) - x(-x) = x^2 + 6x \Rightarrow \det(A) = \begin{vmatrix} 3 & 0 & 1 \\ 2 & -1 & 2 \\ 0 & 0 & 3 \end{vmatrix} = -9$

$\therefore x^2 + 6x = -9 \Rightarrow x^2 + 6x + 9 = 0 \Rightarrow x^2 + 3x + 3x + 9 = 0$

$\Rightarrow x(x+3) + 3(x+3) = 0 \Rightarrow x = -3$

13. $|A^{2020} - 5A^{2019}| = |A^{2019}(A - 5I)| = |A^{2019}||A - 5I|$ $\qquad A - 5I = \begin{bmatrix} -4 & 2 \\ 3 & 0 \end{bmatrix}$

$= |A|^{2019}|A - 5I| = (-1)^{2019}(-6) = 6$

14. $|BAC| = |B|\,|A|\,|C| = \begin{vmatrix} -4 & 1 \\ 0 & 6 \end{vmatrix}\begin{vmatrix} 2 & 3 \\ 4 & -2 \end{vmatrix}\begin{vmatrix} 8 & -5 \\ -7 & 3 \end{vmatrix} = (-14)(-16)(-11) = -4224$

15. $A.\text{adj}(A) = \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$

$$\Rightarrow |A|I = \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}\left(\because A.\text{adj}(A) = |A|I\right) \therefore |A|\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow |A| = 7$$

Ch. 5: Differentiation

Choose the correct answer

1. Number of points in (0, 2) in which f(x) = [x], where [,] denotes the greatest integer functions is not differentiable is

A) 0 B) 1 C) 2 D) 3

2. The function f(x) = [x] where [,] denotes the greatest integer function is continuous at

A) 1 B) 1.5 C) 2 D) 3

3. The function f(x) = [x] is

A) continuous at x = 0 B) differentiable at x = 0

C) both continuous and differentiable D) discontinuous at x = 0

4. The derivative of f(x) = |x| at x = 0 is

A) 0 B) 1 C) –1 D) not existing

5. Let f(x) = [x]. Then f(x) is

A) continuous for all $x \in R$ B) both continuous and differentiable for all $x \in R$

C) differentiable for all $x \in R$ D) differentiable for all non integral points.

6. Left hand derivative of f(x) = |x| at x = 0 is

A) 1 B) –1 C) 0 D) not existing

7. If y = log (log x), then y′ =

A) $\dfrac{1}{\log x}$ B) $\dfrac{1}{x \log x}$ C) $\dfrac{\log x}{x}$ D) $\dfrac{1}{x \log x} \cdot \dfrac{1}{x}$

8. If $y = e^{\log x}$, then $\dfrac{d^2y}{dx^2} =$

A) 1 B) 0 C) e log x D) $\dfrac{\log x}{x}$

9. $\dfrac{d}{dx}(\cos^{-1}(\sin x))$, $0 \le x \le \dfrac{\pi}{2}$, is equal to

A) tan x B) 1 C) –1 D) – tan x

10. If $y = \frac{e^{\log x}}{x}$, then $y' =$

A) x B) $\frac{1}{x}$ C) 0 D) 1

11. If y = log x, then $\frac{d^2y}{dx^2} =$

A) $\frac{1}{x}$ B) $\frac{1}{x^2}$ C) $-\frac{1}{x^2}$ D) x^2

12. Derivative of e^{x^3} w.r.t. x is

A) $3e^{x^3}$ B) $3x^2e^{x^3}$ C) e^{3x^2} D) e^{x^3}

13. If $y = \cos\sqrt{x}$, then $y' =$ **(CET)**

A) $-\sin\sqrt{x}$ B) $\frac{-\sin\sqrt{x}}{\sqrt{x}}$ C) $\frac{-\sin\sqrt{x}}{2\sqrt{x}}$ D) $\frac{\sin\sqrt{x}}{2\sqrt{x}}$

14. Derivative of cos (1 – x) w.r.t. x is

A) sin (1 – x) B) – sin (1 – x) C) sin (1 – x) (1 – x) D) 2 sin x(1 – x)

15. The derivative of $\sin^{-1} x$ exists in the interval **(23-M)**

A) [–1, 1] B) (–1, 1) C) R D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

16. The derivative of f given by $f(x) = \cos^{-1} x$ exists, if $x \in$

A) (–1, 1) B) [–1, 1] C) $(-\infty, \infty)$ D) R – (1, 1)

17. If $y = \sin(\cos x)$, then $\frac{dy}{dx} =$

A) cos (cos x) B) cos (cos x)sin x

C) – cos cos x) sin x D) – cos (cos x) cos x

18. If $y = e^x + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}$, then $\frac{dy}{dx}$ value at $x = 1$

A) 5e B) 15e C) 15 D) 5

19. The number of points in R in which the function f(x) = |x| + |x + 1|is not differentiable, is

A) 0 B)10 C)2 D) infinite

20. If $y = A\sin x + B\cos x$, then $\frac{d^2y}{dx^2} =$

A) y B) $-y$ C) x D) y^2

21. If $f(x) = \sec^{-1} x,\ f'(-2) =$

A) $\frac{1}{2\sqrt{3}}$ B) $-\frac{1}{2\sqrt{3}}$ C) $\frac{1}{6}$ D) doesn't exist

22. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is

A) $\frac{1}{2}$ B) $\frac{1}{\sqrt{2}}$ C) 1 D) $\sqrt{2}$

23. Derivative of $5^{\log_5 \sin x}$ w.r.t. x is

A) sin x B) cos x C) $5^{\sin x}$ D) $5^{\cos x}$

24. If $f(x) = \begin{cases} [x]+[-x], x \neq 2 \\ K, x = 2 \end{cases}$ then f(x) is continuous at x = 2, provided K is equal to

A) 2 B) 1 C) –1 D) 0

25. $f(x) = \begin{cases} 3x-8 & \text{if } x \leq 5 \\ 2k & \text{if } x > 5 \end{cases}$ is continuous, find k **(CET 15)**

A) $\frac{2}{7}$ B) $\frac{3}{7}$ C) $\frac{4}{7}$ D) $\frac{7}{2}$

26. If $f(x) = \begin{cases} Kx^2 & \text{if } x \leq 2 \\ 3 & \text{if } x > 2 \end{cases}$ is continuous at x = 2, then the value of K is

(CET 17)

A) 4 B) 3 C) $\frac{4}{3}$ D) $\frac{3}{4}$

27. The function f(x) = cot x is discontinuous on every point of the set **(CET 23)**

A) $\left\{x = (2n+1)\frac{\pi}{2};\ n \in Z\right\}$ B) $\{x = n\pi;\ n \in Z\}$

C) $\left\{x = \frac{n\pi}{2};\ n \in Z\right\}$ D) {x = 2np ; n ∈ Z}

28. If the function is $f(x) = \frac{1}{x+2}$, then the point of discontinuity of the composite function y = f(f(x)) is **(CET 23)**

A) $\frac{2}{5}$ B) $\frac{-5}{2}$ C) $\frac{1}{2}$ D) $\frac{5}{2}$

29. If $y = f(x^2 + 2)$ and $f'(3) = 5$, then $\frac{dy}{dx}$ at $x = 1$ is .... **(CET 13, 15)**

A) 5 B) 25 C) 15 D) 10

30. If $f(x) = |\cos x - \sin x|$, then $f'\left(\frac{\pi}{6}\right)$ equal to **(CET 18)**

A) $-\frac{1}{2}(1+\sqrt{3})$ B) $\frac{1}{2}(1+\sqrt{3})$ C) $-\frac{1}{2}(1-\sqrt{3})$ D) $\frac{1}{2}(1-\sqrt{3})$

31. If [x] represents the greatest integer function and $f(x) = x - [x] - \cos x$ then $f'\left(\frac{\pi}{2}\right) =$

A) 2 B) 0 C) does not exist D) 1 **(CET 19)**

32. If $y = (\cos x^2)^2$, then $\frac{dy}{dx}$ is equal to **(CET 21)**

A) $-4x \sin 2x^2$ B) $-x \sin x^2$ C) $-2x \sin 2x^2$ D) $-x \cos 2x^2$

33. If $f(1) = 1$, $f'(1) = 3$ then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is **(CET 22)**

(A) 10 (B) 35 (C) 33 (D) 12

34. If $y = e^{\log_e[1+x+x^2+.....]}$, then $\frac{dy}{dx} =$ **(CET 12)**

A) $\frac{1}{(1+x)^2}$ B) $\frac{1}{(1-x)^2}$ C) $\frac{-1}{(1+x)^2}$ D) $\frac{-1}{(1-x)^2}$

35. $\frac{d}{dx}\left[\cos^2\left(\cot^{-1}\sqrt{\frac{2+x}{2-x}}\right)\right]$ is …….. **(CET 11)**

A) $\frac{-3}{4}$ B) $\frac{-1}{2}$ C) $\frac{1}{2}$ D) $\frac{1}{4}$

36. If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $f'(\sqrt{3})$ is **(CET 2020)**

A) $\frac{1}{2}$ B) $\frac{1}{\sqrt{3}}$ C) $-\frac{1}{\sqrt{3}}$ D) $-\frac{1}{2}$

37. If $\sec\left(\frac{x+y}{x-y}\right) = c$, then $\frac{dy}{dx} =$

A) $x - y$ B) $\frac{y}{x}$ C) $y - x$ D) $\frac{x}{y}$

38. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ is **(CET 2020)**

A) -2^{y-x} B) 2^{x-y} C) $\frac{2^y - 1}{2^x - 1}$ D) 2^{y-x}

39. If $x = ct$ and $y = \frac{c}{t}$, find $\frac{dy}{dx}$ at $t = 2$ **(CET 15)**

A) $\frac{1}{4}$ B) 4 C) $\frac{-1}{4}$ D) 0

40. The derivative of $\tan^{-1}\left[\frac{\sin x}{1 + \cos x}\right]$ with respect to $\tan^{-1}\left[\frac{\cos x}{1 + \sin x}\right]$ is **(CET 11)**

1) -2 2) 0 3) -1 4) 2

41. If $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $\frac{du}{dv}$ is **(CET 23)**

A) $\frac{1-x^2}{1+x^2}$ B) $\frac{1}{2}$ C) 1 D) 2

42. If $y = x^{\sin x} + (\sin x)^x$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ is **(CET 22)**

A) $\frac{4}{\pi}$ B) 1 C) $\pi \log \frac{\pi}{2}$ D) $\frac{\pi^2}{2}$

43. If the three functions f(x), g(x) and h(x) are such that h(x) = f(x). g(x) and $f'(x) \cdot g'(x) = c$, where c is a constant, then $\frac{f''(x)}{f(x)} + \frac{g''(x)}{g(x)} + \frac{2c}{f(x)g(x)}$ is equal to **(CET10)**

A) $h'(x) \cdot h(x)$ B) $\frac{h(x)}{h''(x)}$ C) $\frac{h''(x)}{h(x)}$ D) $\frac{h(x)}{h'(x)}$

44. If $f(x) = be^{ax} + ae^{bx}$, then $f''(0) =$ **(CET 08)**

A) 2ab B) 0 C) ab D) ab (a + b)

45. If $f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \ldots\ldots + x^n$, then $f''(1) =$ **(CET 09, 22)**

A) $n(n-1)2^{n-1}$ B) $(n-1)2^{n-1}$ C) $n(n-1)2^{n-2}$ D) $n(n-1)2^n$

46. If $y = 2x^{n+1} + \frac{3}{x^n}$, then $x^2 \frac{d^2y}{dx^2}$ is **(CET 20)**

A) $n(n+1)\,y$ B) $x\frac{dy}{dx} + y$ C) y D) $6n\,(n+1)y$

47. If $y = \log(\log x)$ then $\frac{d^2y}{dx^2}$ is equal to **(CET 17)**

A) $\frac{(1+\log x)}{(x\log x)^2}$ B) $\frac{(1+\log x)}{x^2 \log x}$ C) $\frac{-(1+\log x)}{x^2 \log x}$ D) $\frac{-(1+\log x)}{(x\log x)^2}$

48. If $y = e^{\sqrt{x\sqrt{x\sqrt{x}}\ldots\ldots}}, x > 1$ then $\frac{d^2y}{dx^2}$ at $x = \log_e 3$ is **(CET 22)**

A) 3 B) 0 C) 5 D) 1

49. If $x + y = \tan^{-1} y$ and $\frac{d^2y}{dx^2} = f(y)\frac{dy}{dx}$, then $f(y) =$ _____ **(CET 13)**

A) $\frac{-2}{y^3}$ B) $\frac{2}{y^3}$ C) $\frac{1}{7}$ D) $\frac{-1}{7}$

50. If $x = a\sec^2\theta$, $y = a\tan^2\theta$ then $\frac{d^2y}{dx^2} =$ **(CET 19)**

A) 0 B) 2a C) 4 D) 1

Answer

1. **(B)** **2.** **(B)** **3.** **(D)** **4.** **(D)** **5.** **(D)** **6.** **(B)** **7.** **(B)** **8.** **(B)** **9.** **(C)** **10.** **(C)**

11. **(C)** **12.** **(B)** **13.** **(C)** **14.** **(A)** **15.** **(B)** **16.** **(C)** **17.** **(C)** **18.** **(C)** **19.** **(C)** **20.** **(B)**

21. **(A)**

$$f'(x) = \frac{1}{|x|\sqrt{x^2-1}} = \frac{1}{2\sqrt{4-1}} = \frac{1}{2\sqrt{3}}$$

22. **(B)** **23.** **(B)**

24. **(C)**

$[x] + [-x] = 0$ if $x \in Z$ and $[x] + [-x] = -1$ if $x \notin Z$

Now $f(2) = k$; when x is close to 2, $f(x) = -1$ $\therefore L = -1 = k$

25. **(D)**

LHL = RHL = f (5) $\Rightarrow$ 2k = 15 – 8 $\therefore$ k = $\frac{7}{2}$

26. **(D)**

Continuous at x = 2 $\Rightarrow$ LHL = f(2) = RHL $\therefore$ $k.2^2 = 3$ $\Rightarrow$ $k = \frac{3}{4}$

27. **(B), (D)**

cot x is continuous at every multiple of π

(B) is the correct answer

But the set in **(D)** is a subset of the set in **(B)**

i.e. **(D)** is also a correct answer

28. **(B)**

$$f(f(x)) = \frac{1}{\frac{1}{x+2}+2} = \frac{x+2}{2x+5} \text{ is discontinuous at } x = \frac{-5}{2}$$

29. **(D)**

$$\frac{dy}{dx} = f'(x^2+2).2x = f'(3).2 = 5.2 = 10$$

30. **(A)**

Around $x = \frac{\pi}{6}$, f(x) = cos x – sin x because cos x > sin x

$$f'(x) = -\sin x - \cos x \quad \therefore f\left(\frac{\pi}{6}\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2} = -\frac{1}{2}(1+\sqrt{3})$$

31. **(A)**

Around $x = \frac{\pi}{2}$, $[x] = 1$ $\therefore$ f(x) = x – 1 – cos x

$$f'(x) = 1 + \sin x \quad \therefore \quad f'\left(\frac{\pi}{2}\right) = 2$$

32. **(C)**

$$y = \cos^2 x^2 \quad \therefore \frac{dy}{dx} = 2\cos(x^2)(-\sin(x^2)).2x = -2x\sin(2x^2)$$

33. **(C)**

Let F(x) be the given function

$$F'(x) = f'(f(f(x))).f'(f(x)).f'(x) + 2f(x).f'(x)$$

$$F'(1) = f'(f(f(1))).f'f(1)).f'(1) + 2f(1).f'(1)$$

$$= f'(1).f'(1).f'(1) + 2f'(1) = 27 + 6 = 33$$

34. **(B)**
$y = 1 + x + x^2 + \ldots\ldots$
This infinite sum of the Geometric Series will exist if $|x| < 1$.

Then $y = \dfrac{"a"}{1-r} = \dfrac{1}{1-x} \Rightarrow \dfrac{dy}{dx} = -\dfrac{1}{(1-x)^2}.(-1) = \dfrac{1}{(1-x)^2}$

Remark: As $|x| < 1$ was not mentioned in the question, it was considered as a faulty question.

35. **(D)**

Put $x = 2\cos\theta$. Then $\sqrt{\dfrac{2+x}{2-x}} = \cot\dfrac{\theta}{2}$ $\quad\therefore \cot^{-1}\sqrt{\dfrac{2+x}{2-x}} = \dfrac{\theta}{2}$

$\therefore$ Required $= \dfrac{d}{dx}\left[\cos^2\dfrac{\theta}{2}\right] = \dfrac{d}{dx}\,.\left[\dfrac{1}{2}(1+\cos\theta)\right] = \dfrac{d}{dx}\left[\dfrac{1}{2}\left(1+\dfrac{x}{2}\right)\right] = \dfrac{1}{4}$

36. **(D)**

Recall: $\sin^{-1}\dfrac{2x}{1+x^2} = \begin{cases} -\pi - 2\tan^{-1}x,\ x < -1 \\ 2\tan^{-1}x,\ -1 \le x \le 1 \\ \pi - 2\tan^{-1}x,\ x > 1 \end{cases}$

Here $x = \sqrt{3} > 1$ $\quad\therefore\ f(x) = \sin^{-1}\dfrac{2x}{1+x^2} = \pi - 2\tan^{-1}x \quad \therefore f'(x) = -2.\dfrac{1}{1+x^2}$

$f'\left(\sqrt{3}\right) = -2\,.\dfrac{1}{1+3} = -\dfrac{1}{2}$

37. **(B)**

$\sec\left(\dfrac{x+y}{x-y}\right) = c \Rightarrow \sec\left(\dfrac{1+\dfrac{y}{x}}{1-\dfrac{y}{x}}\right) = c$

$\therefore\ f\left(\dfrac{x}{y}\right) = c \Rightarrow \dfrac{d}{dx}\left(\dfrac{x}{y}\right) = 0 \Rightarrow \dfrac{y.1 - x.\dfrac{dy}{dx}}{y^2} = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x}$

38. **(A)**
$2^x + 2^y = 2^{x+y} \Rightarrow 2^{-y} + 2^{-x} = 1$

$\therefore \dfrac{dy}{dx} = -\dfrac{f_x}{f_y} = -\dfrac{2^{-x}\log 2(-1)}{2^{-y}\log 2\,(-1)} = -2^{y-x}$

39. **(C)**

$\dfrac{dy}{dx} = \dfrac{-\dfrac{c}{t^2}}{c} = -\dfrac{1}{4}$ at $t = 2$.

40. **(C)**

Use $\dfrac{\sin x}{1+\cos x} = \tan\dfrac{x}{2}$ and $\dfrac{\cos x}{1+\sin x} = \tan\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right)$

$\therefore$ Required = Derivative of $\dfrac{x}{2}$ w.r.t. $\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right)$ which is –1

41. **(C)**

Required is derivative of $2\tan^{-1}x$ w.r.t. $2\tan^{-1}x$ and hence it is 1

42. **(B)**

$$\frac{dy}{dx} = x^{\sin x}.\frac{d}{dx}(\sin x.\log x) + (\sin x)^x.\frac{d}{dx}(x\log(\sin x))$$

$$= x^{\sin x}[\sin x.\frac{1}{x} + \cos x.\log x] + (\sin x)^x[1.\log\sin x + x\cot x]$$

$$\text{At } x = \frac{\pi}{2},\ \frac{dy}{dx} = \frac{\pi}{2}\left[\frac{2}{\pi}+0\right] + 1[0+0] = 1$$

43. **(C)**

$h = f.g \Rightarrow h' = f'.g + f.g' \Rightarrow h'' = f''.g + 2f'.g' + f.g''$

$$\text{G.E.} = \frac{f''g + fg'' + 2c}{fg} = \frac{f''g + fg'' + 2f'g'}{fg} = \frac{h''}{h}$$

44. **(D)**

$f'(x) = b.ae^{ax} + a.be^{bx}$; $f''(x) = ab.ae^{ax} + ab.be^{bx}$

$\therefore f''(0) = ab.a + ab.b = ab(a+b)$

45. **(C)**

$f(x) = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + ... + {}^nC_n x^n = (1+x)^n$

$\therefore f'(x) = n(1+x)^{n-1}$; $f''(x) = n(n-1)(1+x)^{n-2}$ $\quad\therefore f''(1) = n(n-1)(2)^{n-2}$

Aliter : Put $n = 2$. Then $f(x) = 1 + 2x + x^2$

$\therefore f'(x) = 2 + 2x$ and $f(x) = 2$ $\quad\therefore f''(1) = 2$

Now go to the alternatives and put $n = 2$ and detect (3) as the correct answer

46. **(A)**

$$\frac{dy}{dx} = 2.(n+1)x^n + 3\left(-\frac{n}{x^{n+1}}\right)$$

$$\frac{d^2y}{dx^2} = 2(n+1).nx^{n-1} + 3n.\frac{(n+1)}{x^{n+2}}$$

$$\therefore\ \frac{x^2d^2y}{dx^2} = 2(n+1)n.x^n + \frac{3n(n+1)}{x^n} = n(n+1)\left(2x^n + \frac{3}{x^n}\right) = n(n+1)y$$

47. **(D)**

$$\frac{dy}{dx}=\frac{1}{\log x}\cdot\frac{1}{x}=\frac{1}{x\log x}\;;\;\frac{d^2y}{dx^2}=-\frac{1}{(x\ \log x)^2}\cdot\frac{d}{dx}(x\log x)$$

i.e., $\frac{d^2y}{dx^2}=-\frac{1}{(x\log x)^2}\cdot\left(1.\log x+x.\frac{1}{x}\right)=-\frac{1}{(x\log x)^2}(\log x+1)$

48. **(A)**

Let $z=\sqrt{x\sqrt{x\sqrt{x........}}}\Rightarrow z=\sqrt{xz}$

$\therefore z^2=xz\Rightarrow z=x$

$\therefore y=e^x\quad\therefore\frac{d^2y}{dx^2}=e^x=e^{\log_e 3}=3(\text{at } x=\log_e 3)$

49. **(B)**

$$\frac{dx}{dy}+1=\frac{1}{1+y^2}\Rightarrow\frac{dx}{dy}=\frac{1}{1+y^2}-1=-\frac{y^2}{1+y^2}$$

$$\therefore\frac{dy}{dx}=-\frac{1+y^2}{y^2}=-\frac{1}{y^2}-1\quad\therefore\frac{d^2y}{dx^2}=-\left(-\frac{2}{y^3}\right)\cdot\frac{dy}{dx}\quad\therefore f(y)=\frac{2}{y^3}$$

50. **(A)**

$x-y=a(\sec^2\theta-\tan^2\theta)=a^2\quad\therefore\ 1-y'=0\Rightarrow y''=0$

Ch. 6: Application of Derivatives

Choose the correct answer

1. The rate of change of the area of a circle per second w.r.t. its radius r when r = 5cm is……cm^2/s.

A) 10π B) 12π C) 8π D) 11π

2. The total revenue Rupees received from the sale of x units of a product is given by $R(x)=13x^2+26x-15$, then the marginal revenue when $x=1$ in Rupees is

A) 26 B) 13 C) 52 D) 104

3. The radius of an air bubble is increasing at the rate of 1cm/s, then the volume of the bubble increasing when the radius is 1cm is …….. cc/sec

A) 4π B) 2 C) 2π D) 8π

4. The function $f(x)=3x+17$ is strictly increasing on

A) $(-\infty,\infty)$ B) $(0,\infty)$ C) $(-\infty,0)$ D) $(0,3)$

5. If a function f is such that $f'(c)=0$ and $f'(c)<0$ for some 'c' on an interval 'I', then at c the function f attains its

A) absolute maximum value B) absolute minimum value

C) local maximum value D) local minimum value

6. Minimum value of the function f given by $f(x) = |x|, x \in (0, 1)$ is

A) 1 B) 2 C) 0 D) does not exist

7. Maximum value of the function f given by $f(x) = \text{sgn}(x), x \in (0, 1)$ is

A) 0 B) 1 C) 2 D) does not exist

8. Which of the following function is decreasing on $\left(0, \frac{\pi}{2}\right)$

A) sin x B) tan x C) cos x D) cos 3x

9. The critical point of the function $f(x) = 3 + |x|$ is

A) (0, 0) B) (0, 3) C) (1, 4) D) not existing

10. Point of inflection of x^3 is

A) (0, 0) B) (1,1) C) (0, 1) D) not existing

11. Point of inflection of sin x is

A) $x = n\pi$ B) $x = (2n+1)\frac{\pi}{2}$ C) $\frac{\pi}{2}$ D) not existing

12. If a ball is thrown vertically upwards and the height 's' reached in time 't' is given by $s = 22t - 11t^2$, then the total distance travelled by the ball is **(CET 12)**

A) 44 units B) 33 units C) 11 units D) 22 units

13. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/sec. At that instant, when the radius of circular wave is 8 cm, how fast is the enclosed area increasing ? **(CET 14)**

A) 6π cm^2/s B) 8π cm^2/s C) $\frac{8}{3}$ cm^2/s D) 80π cm^2/s

14. The point on the circle $x^2 + y^2 = 2$ at which the abscissa and ordinate increase at the same rate is

A) (–1, –1) B) (1, –1) C) (1, 1) D) (–1, 4)

15. A particle starts from rest and its angular displacement (in radians) is given by $\theta = \frac{t^2}{20} + \frac{t}{5}$. If the angular velocity at the end of t = 4 is k, then the value of 5k is **(CET 21)**

A) 0.6 B) 5 C) 5k D) 3

16. A circular plate of radius 5 cm is heated. Due to expansion, its radius increases at the 0.05 cm/sec. The rate at which its area is increasing when radius is 5.2 cm is **(CET 23)**

A) 5.05 π cm^2/sec B) 5.2 π cm^2/sec C) 0.52 π cm^2/sec D) 24.4 π cm^2/sec

17. The distance 's' in meters travelled by a particle in 't' seconds is given by $s = \frac{2t^3}{3} - 18t + \frac{5}{3}$. The acceleration when the particle comes to rest is **(CET 23)**

A) 12 m^2/sec B) 3 m^2 /sec C) 18 m^2 / sec D) 10 m^2 /sec

18. A particle moves along the curve $\frac{x^2}{16}+\frac{y^2}{4}=1$. When the rate of change of abscissa is 4 times that of its ordinate, then the quadrant in which the particle lies is **(CET 23)**

A) III or IV B) I or III C) II or III D) II or IV

19. The function $f(x) = x^2 - 2x$ is strictly decreasing in the interval **(CET 21)**

A) $(-\infty, 1)$ B) $(1, \infty)$ C) R D) $(-\infty, \infty)$

20. The function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly **(CET 22)**

A) decreasing in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ B) increasing in $\left[\pi, \frac{3\pi}{2}\right]$

C) decreasing in $\left[0, \frac{\pi}{2}\right]$ D) decreasing in $\left[\frac{\pi}{2}, \pi\right]$

21. If x is real, then the minimum value of $x^2 - 8x + 17$ is ….. **(CET 15)**

A) 1 B) 2 C) 3 D) 4

22. The maximum value of xe^{-x} is **(CET 12)**

A) e B) $\frac{1}{e}$ C) $-e$ D) $-\frac{1}{e}$

23. The maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$ is **(CET 21)**

A) 1 B) 23 C) 5 D) -23

24. The maximum value of $\left(\frac{1}{x}\right)^x$ is **(CET 16, 18)**

A) e B) e^e C) $e^{\frac{1}{e}}$ D) $\left(\frac{1}{e}\right)^e$

25. An enemy fighter jet is flying along the curve given by $y = x^2 + 2$. A soldier is placed at (3, 2) wants to shoot down the jet when it is nearest to him. Then the nearest distance is **(CET 23)**

A) 2 units B) $\sqrt{3}$ units C) $\sqrt{5}$ units D) $\sqrt{6}$ units

Answer

1. (A) 2. (C) 3. (A) 4. (A) 5. (C) 6. (D) 7. (B) 8. (C) 9. (B) 10. (A)

11. (A)

12. (D)

$\frac{ds}{dt} = 22 - 22t \Rightarrow 0 \Rightarrow t = 1$

$\therefore$ Greatest height attained = s (at t = 1) = 22 – 11 = 11 units

$\therefore$ Total distance travelled by the ball = 2s = 22 units

13. (D)

$$A = \pi a^2 \Rightarrow \frac{dA}{dt} = 2\pi a \frac{da}{dt} = 2\pi.5.8 = 80\pi \ \text{cm}^2/\text{s}$$

14. (B)

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0\,;\ \frac{dy}{dt} = \frac{dx}{dt} \Rightarrow x + y = 0 \quad \therefore \text{(2) is the correct answer}$$

15. (D)

$$\frac{d\theta}{dt}\ at\ t = 4\ is\ k \Rightarrow \frac{2t}{20} + \frac{1}{5} = k\ \ at\ t = 4$$

$$i.e.,\ \frac{2}{5} + \frac{1}{5} = k \Rightarrow k = \frac{3}{5} \Rightarrow 5k = 3$$

16. (C)

$A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt} \qquad \therefore\ \left.\frac{dA}{dt}\right|_{r=5.2} = 2\pi \times 5.2 \times 0.05 = 0.52\,\pi$$

17. (A)

$$\frac{ds}{dt} = 2t^2 - 18\,;\ \frac{ds}{dt} = 0 \Rightarrow t = 3$$

$$\frac{d^2s}{dt^2} = 4t \quad \therefore\ \text{Required} = 12\ \text{m}^2/\text{s}$$

18. (D)

$$\frac{2x\frac{dx}{dt}}{16} = -\frac{2y\frac{dy}{dt}}{4}\ ;\ \text{Given}\ \frac{dx}{dt} = 4\frac{dy}{dt} \Rightarrow \frac{x\,.\,\frac{4\,dy}{dt}}{16} = \frac{-y\frac{dy}{dt}}{4} \Rightarrow x = -y$$

As x and y are of opposite signs, (D) is the correct answer.

19. (A)

20. (D)

$f'(x) = (12 \sin^2 x - 12 \sin x + 12) \cos x$

$12 \sin^2 x - 12 \sin x + 12 > 0,\ \forall x$

The sign of $f'(x)$ depends on the sign of cos x

$\therefore\ f'(x) < 0$ when $x \in \left(\frac{\pi}{2}, \pi\right)$

21. (1)

$x^2 - 8x + 17 = (x-4)^2 + 1 \geq 1$

22. (2)

$f(x) = xe^{-x} \Rightarrow f'(x) = 1.e^{-x} + x(-e^{-x}) = 0 \Rightarrow 1 - x = 0 \Rightarrow x = 1$

$\therefore$ Max. value $= 1.e^{-1} = \frac{1}{e}$. Is it a risky solution ?!

23. (C)

$$\frac{dy}{dx} = -3x^2 + 6x + 2 = m = slope$$

$$\frac{dm}{dx} = -6x + 6; \quad \frac{d^2m}{dx^2} = -6 < 0$$

$$\frac{dm}{dx} = 0 \Rightarrow x = 1$$

$\therefore$ m is maximum when x = 1 $\quad\therefore$ max. value of m = – 3 + 6 + 2 = 5

24. (3)

Hint : The maximum value of $x^{-x} = \dfrac{1}{\text{The minimum value of } x^x} = e^{\frac{1}{e}}$

Aliter : $y = x^{-x} \Rightarrow \log_e y = -x \log_e x$

Let $z = -x \log_e x$

$$\Rightarrow \frac{dz}{dx} = -x\frac{1}{x} + \log_e x(-1) = -1 - \log_e x \Rightarrow \frac{dz}{dx} = 0 \Rightarrow -(1 + \log_e x) = 0 \Rightarrow x = e^{-1} = \frac{1}{e}$$

Now $\dfrac{d^2z}{dx^2} = -\dfrac{1}{x} \Rightarrow \dfrac{d^2z}{dx^2} < 0$ at $x = \dfrac{1}{e}$ $\therefore$ Maximum value of $\dfrac{1}{x^x}$ is $e^{\frac{1}{e}}$

25. (C)

z = Distance between (3, 2) and $(x, x^2 + 2) = \sqrt{(x-3)^2 + x^4}$

z is min $\Leftrightarrow z^2$ is min : let $f(x) = z^2 = (x-3)^2 + x^4$

$f'(x) = 2(x-3) + 4x^3 \Rightarrow f'(1) = 0$

$\therefore$ z is min at x = 1 and min $z = \sqrt{4+1} = \sqrt{5}$

Ch.7 : Integration

Choose the correct answer

1. $\displaystyle\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}\, dx =$

A) tan x – cot x + c B) tan x + cot x + c
C) tan x – sec x + c D) sec x – tan x + c

2. $\int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx =$

A) $\frac{2}{3}\times x^{\frac{3}{2}} + 2\sqrt{x} + c$ B) $\frac{3}{2}\times x^{\frac{2}{3}} + 2\sqrt{x} + c$

C) $\frac{3}{2}\times x^{\frac{3}{2}} - 2\sqrt{x} + c$ D) $\frac{3}{2}\times x^{\frac{3}{2}} + 2\sqrt{x} + c$

3. $\int \cos 2x\, dx =$

A) $\frac{\sin 2x}{2} + c$ B) $\frac{\cos 2x}{2} + c$ C) $\frac{-\sin 2x}{2} + c$ D) $\frac{-\cos 2x}{2} + c$

4. $\int \sin^{-1}(\cos x)\, dx,\ 0 < x < \frac{\pi}{2}$, is equal to

A) $\frac{\pi}{2}x - \frac{x^2}{2} + c$ B) $\frac{\pi}{2} + \frac{x^2}{2} + c$ C) $\frac{\pi}{2}x + \frac{x^2}{2} + c$ D) $\frac{\pi}{2}x + \frac{\pi}{2} + c$

5. $\int \sqrt{ax+b}\, dx =$

A) $\frac{1}{a}\times\frac{2}{3}(ax+b)^{\frac{3}{2}} + c$ B) $\frac{1}{b}\times\frac{2}{3}(ax+b)^{\frac{3}{2}} + c$

C) $\frac{1}{b}\times\frac{3}{2}(ax+b)^{\frac{5}{2}} + c$ D) $\frac{1}{a}\times\frac{3}{2}(ax+b)^{\frac{2}{3}} + c$

6. $\int e^x\left(\tan^{-1} x + \frac{1}{1+x^2}\right) dx =$

A) $e^x \tan^{-1} x + c$ B) $-e^x \tan^{-1} x + c$ C) $e^x \tan^{-1}(1 + x^2) + c$ D) $-e^x \cot^{-1} x + c$

7. $\int e^x\left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}}\right) dx =$

A) $e^x \sin^{-1} x + c$ B) $e^x \cos^{-1} x + c$ C) $e^x(-\sin^{-1} x) + c$ D) 0

8. $\int e^x \sec x(1+\tan x)\, dx =$ **(23-M)**

A) $e^x \sec x + c$ B) $e^x \tan x + c$ C) $e^x \sec^2 x + c$ D) $e^x \sin x + c$

9. $\int e^{2x} dx =$

A) $e^{2x} + c$ B) $2e^{2x} + c$ C) $\frac{e^{2x}}{2} + c$ D) $\frac{e^x}{2} + c$

10. $\int_0^{\pi}\left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right) dx =$

A) π B) $\frac{-\pi}{2}$ C) 1 D) 0

11. $\int_{-1}^{1} e^x \, dx =$

A) $e - \frac{1}{e}$ B) $e + \frac{1}{e}$ C) $-e + \frac{1}{e}$ D) $e + \frac{2}{e}$

12. $\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

A) $\frac{\pi}{3}$ B) $\frac{2\pi}{3}$ C) $\frac{\pi}{6}$ D) $\frac{\pi}{12}$

13. $\int_{2}^{3} x^2 dx =$

A) 2 B) 19 C) $\frac{19}{2}$ D) $\frac{19}{3}$

14. $\int x^2 \left(1 - \frac{1}{x^2}\right) dx =$

A) $\frac{x^3}{3} - 1 + c$ B) $2x + c$ C) $\frac{x^3}{3} - x + c$ D) $\frac{x^3}{3} + c$

15. $\int_{-1}^{1} \sin^{-1} x \, dx =$

A) $\frac{\pi}{2} - 1$ B) π C) $\frac{\pi}{2}$ D) 0

16. The anti-derivative of $\frac{x^3 - 1}{x^2}$ w.r.t. x is

A) $\frac{x^2}{2} - \frac{1}{x} + c$ B) $\frac{x^2}{2} + \frac{1}{x} + c$ C) $\frac{x^2}{2} + \frac{2}{x} + c$ D) $\frac{x^2}{2} - \frac{2}{x} + c$

17. $\int \left(x^{\frac{3}{2}} + 2e^x - \frac{1}{x} \right) dx =$

A) $\frac{2x^{\frac{5}{2}}}{5} + 2e^x + \log|x| + c$ B) $\frac{2x^{\frac{5}{2}}}{5} + 2e^x - \log|x| + c$

C) $\frac{5x^{\frac{5}{2}}}{2} + 2e^x - \log|x| + c$ D) $\frac{5x^{\frac{5}{2}}}{2} + 2e^x + \log|x| + c$

18. $\int\left(x^{\frac{2}{3}}+1\right)dx =$

A) $\dfrac{3x^{\frac{5}{3}}}{5}+c$ B $\dfrac{3x^{\frac{5}{3}}}{5}+x+c$ C) $\dfrac{5x^{\frac{5}{2}}}{3}+x+c$ D) $\dfrac{5x^{\frac{5}{2}}}{3}+c$

19. $\int(\sin x+\cos x)\,dx$

A) $\sin x-\cos x+c$ B) $\sin x+\cos x+c$

C) $\cos x-\sin x+c$ D) $-(\sin x+\cos x)+c$

20. $\int \operatorname{cosec} x\,(\operatorname{cosec} x+\cot x)\,dx =$

A) $-\cot x-\operatorname{cosec} x+c$ B) $\cot x-\operatorname{cosec} x+c$

C) $-\cot x+\operatorname{cosec} x+c$ D) $\cot x+\operatorname{cosec} x+c$

21. $\int \tan^2 2x\,dx$ is

A) $\sec 2x-x+c$ B) $\sec 2x+x+c$ C) $\dfrac{\tan 2x}{2}-x+c$ D) $\dfrac{\tan 2x}{2}+x+c$

22. $\int\dfrac{1-x}{\sqrt{x}}\,dx$ is

A) $2\sqrt{x}+\dfrac{3x^{\frac{3}{2}}}{2}+c$ B) $2\sqrt{x}-\dfrac{2x^{\frac{3}{2}}}{3}+c$

C) $2\sqrt{x}+\dfrac{3x^{\frac{3}{2}}}{2}+c$ D) $2\sqrt{x}-\dfrac{3x^{\frac{3}{2}}}{2}+c$

23. $\int\dfrac{(1-\sin x)\,dx}{\cos^2 x} =$

A) $\sec x-\tan x+c$ B) $\sec x+\tan x+c$

C) $\tan x-\sec x+c$ D) $-(\sec x+\tan x)+c$

24. $\int\dfrac{dx}{e^x+e^{-x}} =$

A) $\tan^{-1}\left(e^x\right)+c$ B) $\tan^{-1}\left(e^{-x}\right)+c$

C) $\log\left|e^x+e^{-x}\right|+c$ D) $\log\left|e^x-e^{-x}\right|+c$

25. $\int\dfrac{\sec^2 x}{\operatorname{cosec}^2 x}\,dx =$

A) $\sec x-x+c$ B) $\tan x+x+c$ C) $\sec x+x+c$ D) $\tan x-x+c$

26. $\int\dfrac{dx}{1+\cos x}\,dx =$

A) $\dfrac{1}{2}\tan\dfrac{x}{2}+c$ B) $\tan\dfrac{x}{2}+c$ C) $\cot\dfrac{x}{2}+c$ D) $\sec\dfrac{x}{2}+c$

27. $\int \frac{\cos^2 x}{1+\sin x}\, dx =$

A) $x - \sin x + c$ B) $x + \sin x + c$ C) $x + \cos x + c$ D) $x - \cos x + c$

28. $\int \frac{1}{\sin^2 x \cos^2 x}\, dx =$

A) $\tan x - \cot x + c$ B) $\tan x + \cot x + c$ C) $\cot x - \tan x + c$ D) $-\cot x - \tan x + c$

29. $\int \frac{2\sin x - 3\cos x}{2\cos x + 3\sin x}\, dx =$

A) $\log|2\cos x + 3\sin x| + c$ B) $\log|2\cos x - 3\sin x| + c$

C) $-\log|2\cos x + 3\sin x| + c$ D) $-\log|2\sin x - 3\cos x| + c$

30. $\int \frac{(\log x)^2}{x}\, dx =$

A) $\frac{(\log x)^3}{3} + c$ B) $\frac{(\log x)^3}{3x} + \log x + c$

C) $\frac{(\log x)^2}{2} + c$ D) $2(\log x)^3 + \frac{1}{x} + c$

31. $\int x^x (1 + \log x)\, dx =$

A) $x^x + c$ B) $x^{-x} + c$ C) $x \log x + c$ D) $\log x + c$

32. The value of $\int \frac{e^{6\log x} - e^{5\log x}}{e^{4\log x} - e^{3\log x}}\, dx$ is equal to **(CET 16)**

A) 0 B) $\frac{x^3}{3}$ C) $\frac{3}{x^3}$ D) $\frac{1}{x}$

33. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta}\, dx$ is equal to **(CET 17, 22)**

A) $2(\sin x + x\cos\theta) + c$ B) $2(\sin x - x\cos\theta) + c$

C) $2(\sin x + 2x\cos\theta) + c$ D) $2(\sin x - 2x\cos\theta) + c$

34. $\int \frac{\sin^2 x}{1+\cos x}\, dx =$ **(CET 15)**

A) $x + \sin x + c$ B) $x - \sin x + c$ C) $\sin x + c$ D) $\cos x + c$

35. $\int \frac{\sin 2x}{\sin^2 x + 2\cos^2 x}\, dx =$ **(CET 14)**

A) $-\log(1 + \sin^2 x) + c$ B) $\log(1 + \cos^2 x) + c$

C) $-\log(1 + \cos^2 x) + c$ D) $\log(1 + \tan^2 x) + c$

36. $\int_0^{\pi/2} \sqrt{\sin \theta} \cos^3 \theta \, d\theta$ is equal to **(CET 22)**

A) $\frac{8}{23}$ B) $\frac{8}{21}$ C) $\frac{7}{23}$ D) $\frac{7}{21}$

37. $\int_0^{\pi/2} \frac{\cos x \sin x}{1+\sin x} dx$ is equal to **(CET 22)**

A) log 2 – 1 B) – log 2 C) log 2 D) 1 – log 2

38. $\int \frac{x^3 \sin(\tan^{-1}(x^4))}{1+x^8} dx$ *is equal to* **(CET 21)**

A) $\frac{-\cos(\tan^{-1}(x^4))}{4} + c$ B) $\frac{\cos(\tan^{-1}(x^4))}{4} + c$

C) $\frac{-\cos(\tan^{-1}(x^3))}{3} + c$ D) $\frac{\sin(\tan^{-1}(x^4))}{4} + c$

39. *The value of* $\int \frac{x^2 dx}{\sqrt{x^6+a^6}}$ *is equal to* **(CET 21)**

A) $\log |x^3 + \sqrt{x^6+a^6}| + c$ B) $\log |x^3 - \sqrt{x^6+a^6}| + c$

C) $\frac{1}{3}\log |x^3 + \sqrt{x^6+a^6}| + c$ D) $\frac{1}{3}\log |x^3 - \sqrt{x^6+a^6}| + c$

40. $\int \frac{1}{1+3\sin^2 x + 8\cos^2 x} dx =$ **(CET 23)**

A) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + c$ B) $\frac{1}{6}\tan^{-1}(2\tan x) + c$

C) $6\tan^{-1}\left(\frac{2\tan x}{3}\right) + c$ D) $\tan^{-1}\left(\frac{2\tan x}{3}\right) + c$

41. $\int \frac{1}{\sqrt{3-6x-9x^2}} dx$ is equal to **(CET 18)**

A) $\sin^{-1}\left(\frac{3x+1}{2}\right)+c$ B) $\sin^{-1}\left(\frac{3x+1}{6}\right)+c$

C) $\frac{1}{3}\sin^{-1}\left(\frac{3x+1}{2}\right)+c$ D) $\sin^{-1}\left(\frac{2x+1}{3}\right)+c$

42. $\int_1^e \log x \, dx =$

A) 1 B) e – 1 C) e + 1 D) 0

43. The value of $\int_0^4 |x-1|\, dx$ is …. **(CET 11)**

A) 1 B) 4 C) 5 D) $\frac{5}{2}$

44. $\int_{0.2}^{3.5} [x]\, dx$ is equal to **(CET 17)**

A) 4.5 B) 3.5 C) 4 D) 3

45. The value of the integral $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \log(\sec\theta - \tan\theta)\, d\theta$ is **(CET 14)**

A) 0 B) $\frac{\pi}{4}$ C) π D) $\frac{\pi}{2}$

46. The value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^{103} x . \cos^{101} x\, dx$ is **(CET 16)**

A) $\left(\frac{\pi}{4}\right)^{101}$ B) 0 C) $\left(\frac{\pi}{4}\right)^{103}$ D) 2

47. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1+\cos 2x}$ is equal to **(CET 15)**

A) 2 B) 1 C) 4 D) 0

48. The value of $\int_{-1/2}^{1/2} \cos^{-1} x\, dx$ is **(CET 20)**

A) $\frac{\pi}{2}$ B) 1 C) $\frac{\pi^2}{2}$ D) π

49. $\int_{-2}^{0} (x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1))dx =$ **(CET 23)**

A) 4 B) 0 C) 1 D) 3

50. $\int_2^8 \frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}} + 5^{\sqrt{10-x}}}\, dx =$ **(CET 23)**

A) 4 B) 5 C) 3 D) 6

Answer

1. (B)	**2. (A)**	**3. (A)**	**4. (A)**	**5. (A)**	**6. (A)**	**7. (A)**	**8. (A)**	**9. (C)**	**10. (D)**
11. (A)	**12. (D)**	**13. (D)**	**14. (C)**	**15. (D)**	**16. (B)**	**17. (B)**	**18. (A)**	**19. (A)**	**20. (A)**
21. (C)	**22. (B)**	**23. (C)**	**24. (A)**	**25. (A)**	**26. (B)**	**27. (C)**	**28. (A)**	**29. (C)**	**30. (A)**

31. (B)

$$\frac{d}{dx}[x^x] = x^x(1+\log x) \Rightarrow \int x^x(1+\log x)\, dx = x^x + c$$

32. (B)

$$I=\int\frac{e^{\log(x^6)}-e^{\log(x^5)}}{e^{\log(x^4)}-e^{\log(x^3)}}dx=\int\frac{x^6-x^5}{x^4-x^3}dx=\int\frac{x^5\ (x-1)}{x^3\ (x-1)}dx=\int x^2\ dx=\frac{x^3}{3}+c$$

33. (A)

$$\cos 2x-\cos 2\theta=(2\cos^2 x-1)-(2\cos^2\theta-1)=2(\cos^2 x-\cos^2\theta)$$

$$\therefore\ \frac{\cos 2x-\cos 2\theta}{\cos x-\cos\theta}=\frac{2(\cos^2 x-\cos^2\theta)}{\cos x-\cos\theta}=2(\cos x+\cos\theta)$$

$$\therefore\ I=2\int(\cos x+\cos\theta)\,dx=2(\sin x+(\cos\theta)x)+c$$

34. (B)

$$I=\int(1-\cos x)\ dx=x-\sin x+C$$

35. (C)

$$\frac{d}{dx}[\sin^2 x+2\cos^2 x]=\sin 2x+2\,.\,(-\sin 2x)=-\sin 2x$$

$$\therefore\ I=-\int\frac{f'}{f}dx=-\log f=-\log(\sin^2 x+2\cos^2 x)=-\log(1+\cos^2 x)+c$$

36. (B)

$$I=\int_0^{\pi/2}(\sin\ \theta)^{1/2}.\ (1-\sin^2\theta)\ d(\sin\ \theta)=\int_0^1(t^{1/2}-t^{5/2})dt=\frac{2}{3}-\frac{2}{7}=\frac{8}{21}$$

37. (D)

$$I=\int_0^1\frac{t}{1+t}dt\ \text{ where } t=\sin\ x\ ;\ \frac{t}{1+t}=1-\frac{1}{1+t}$$

$$=[t-\log(1+t)]\Big|_0^1=1-\log 2$$

38. (A)

$$I=\frac{1}{4}\int\sin(\tan^{-1}(x^4))\ .\ d\,(\tan^{-1}(x^4))=-\frac{1}{4}\cos\ (\tan^{-1}x^4)+c$$

39. (C)

$$I=\frac{1}{3}\int\frac{d(x^3)}{\sqrt{(x^3)^2+a^6}}=\frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right|+c$$

40. (A)

$$\int\frac{\sec^2 x}{\sec^2 x+3\tan^2 x+8}dx=\int\frac{\sec^2 x}{3^2+(2\tan x)^2}dx\ \left|\begin{array}{l}2\tan x=t\\ \sec^2 x\ dx=\frac{1}{2}dt\end{array}\right.$$

$$=\int\frac{\frac{1}{2}dt}{3^2+t^2}=\frac{1}{2}\times\frac{1}{3}\tan^{-1}\frac{t}{3}=\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$$

41. (C)

$3 - 6x - 9x^2 = -[9x^2 + 6x - 3] = -[(3x+1)^2 - 4] = 4 - (3x+1)^2$

$$\therefore\ I = \frac{1}{3}\int \frac{d(3x+1)}{\sqrt{4-(3x+1)^2}} = \frac{1}{3}\sin^{-1}\frac{3x+1}{2} + c$$

42. (A)

$I = (x \log x - x)|_1^e = 1$

43. (C)

$$I = \int_0^4 |x-1|\, d(x-1) = \frac{1}{2}(x-1)\,|x-1|\Big|_0^4 = \frac{1}{2}\quad (9+1) = 5$$

Aliter : $I = \int_0^1 -(x-1)\,dx + \int_1^4 (x-1)\,dx$ etc.

44. (A)

$$I = \int_{0.2}^{1} 0\,dx + \int_1^2 1\,dx + \int_2^3 2\,dx + \int_3^{3.5} 3\,dx = 0 + 1 + 2 + 3\,(0.5) = 3 + 1.5 = 4.5$$

45. (A)

$$f(-\theta) = \log[\sec(-\theta) - \tan(-\theta)] = \log(\sec\theta + \tan\theta) = \log(\sec\theta - \tan\theta)^{-1}$$
$$= -\log(\sec\theta - \tan\theta) = -f(\theta)$$

$\therefore$ $f(\theta)$ is odd $\quad \therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} f(\theta)\,d\theta = 0$

46. (B)

From memory !

$\sin^{103}x \,.\, \cos^{101}x$ is an odd function $\quad \therefore I = 0$

47. (B)

$$I = 2\int_0^{\frac{\pi}{4}} \frac{1}{1+\cos 2x}\,dx = 2\int_0^{\frac{\pi}{4}} \frac{1}{2\cos^2 x}\,dx = \int_0^{\frac{\pi}{4}} \sec^2 x\,dx = \tan x\Big|_0^{\frac{\pi}{4}} = 1$$

48. (A)

$$\int_{-1/2}^{1/2} \cos^{-1}x\,dx = \int_{-1/2}^{1/2}\left(\frac{\pi}{2} - \sin^{-1}x\right)dx = \frac{\pi}{2}x\Big|_{-1/2}^{1/2} - 0 \quad [\because \sin^{-1}x \text{ is odd}]$$
$$= \frac{\pi}{4} - \left[-\frac{\pi}{4}\right] = \frac{\pi}{2}$$

Aliter: $\int_{-a}^{a} f(x)\,dx = \int_0^a [f(x) + f(-x)]\,dx$

$$\int_{-1/2}^{1/2}\cos^{-1}x\,dx = \int_0^{1/2}[\cos^{-1}x + \cos^{-1}(-x)]dx = \int_0^{1/2}[\cos^{-1}x + \pi - \cos^{-1}x]dx = \int_0^{1/2}\pi\,dx = \pi x\Big|_0^{1/2} = \frac{\pi}{2}$$

Third Method :

$$I = x\,.\cos^{-1}x\Big|_{-1/2}^{1/2} - \int_{-1/2}^{1/2} -\frac{x}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\cos^{-1}\frac{1}{2} + \frac{1}{2}\cos^{-1}\left(-\frac{1}{2}\right) + 0 \quad \because \frac{x}{\sqrt{1-x^2}} \text{ is odd}$$
$$= \frac{1}{2}\left(\frac{\pi}{3} + \frac{2\pi}{3}\right) = \frac{\pi}{2}$$

49. (A)

$$\int_{-2}^{0}((x+1)^3+2)\,dx+\int_{-2}^{0}(x+1)\cos(x+1)\,dx$$

$$=\left[\frac{(x+1)^4}{4}+2x\right]_{-2}^{0} + \int_{-1}^{1} t\cos t\,dt = \left(\frac{1}{4}+0\right) - \left(\frac{1}{4}-4\right) + 0 = 4$$

Aliter: Put $t = x + 1$

$$\therefore\ I=\int_{-1}^{1}(t^3+2+t\cos t)=\int_{-1}^{1}2\,dt=4$$

50. (C)

$I=\frac{1}{2}(8-2)=3$. From memory !

Remark: $\int_a^b \frac{f(a+b-x)}{f(x)+f(a+b-x)}\,dx = \frac{1}{2}(b-a)$

Ch. 9: Diffential Equation

Fill in the blanks by choosing the appropriate answer from given in brackets.

$\left(2, \frac{1}{4}, 3, \frac{1}{4}, 0, -1, \frac{1}{2}, 6, 4, \frac{15}{4}, 1, -2, 4, 7, -\frac{1}{2}, \frac{1}{\sqrt{2}}, \sqrt{2}\right)$

1. The number of independent arbitrary constabts in a differential equation of order 4 and degree 2 is ______

2. The number of arbitrary constants in the particular solution of a differential equation of fourth order is______

3. The number of arbitrary constants in the general solution of a differential equation of fourth order is_____

4. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)+\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}=0$ is ___ **(CET)**

5. The degree of the differential equation $\frac{d^2y}{dx^2}=\sqrt[3]{1+\left(\frac{dy}{dx}\right)^2}$ is _____ **(CET 18)**

6. If m and n are degree and order of $(1+y_1^2)^{\frac{2}{3}}=y_2$ then the value of $\frac{m+n}{m-n}$ is ___ **(CET 11)**

7. The sum of the degree and order of the differential equation $(1+y_1^2)^{2/3}=y_2$ is_____ **(CET 22)**

8. The product of the degree and order of the D.E. $\left(\frac{d^2y}{dx^2}\right)^2-\left(\frac{dy}{dx}\right)^3=y^3$ is ____

9. The order of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)\right]^{\frac{3}{4}}=\frac{d^2y}{dx^2}$ is___ **(CET 16)**

10. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^2=\frac{d^2y}{dx^2}$ is ___ **(CET 17)**

11. The degree of the differential equation $1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2=\sqrt[3]{\frac{d^2y}{dx^2}+1}$ is___ **(CET 23)**

12. The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0$ is ____

13. If the integrating factor of the D.E. $\frac{dy}{dx}+\frac{y}{x}=x^2$ is f(x), then $f(\sqrt{2})$ is ___

14. The general solution of $\frac{dx}{dy}=xy$ is $f(x)=\frac{y^2}{2}+c$, then f(1) is _____

15. The sum of the order and degree of the differential equation $\left(\frac{dy}{dx}\right)^2+x=0$ is ____

16. The number of independent arbitrary constants in the solution of $y'+y=e^x$ is ___

17. If the integrating factor of the differential equation $x\frac{dy}{dx}-y=2x^2$ is f(x), then f(2) ___

18. The integrating factor of the differential equation $\frac{dy}{dx}-y=\cos x$ is e^{kx}, then k is ___

19. The integrating factor of the differential equation $x\frac{dy}{dx}+2y=x^2$ is x^k, then k is ___

20. The integrating factor of the differential equation $\frac{dy}{dx}=x+xy$ is e^{kx^2}, then k is _____

21. If $\frac{dy}{dx}=y+3>0$ and y(0) = 2, then y (log 2) is equal to ___ **(AIEEE 11)**

22. If $y'=y+1$ and y(0) = 1, then find y (log 2) ___ **(IIT 09)**

23. If f(x) is the I.F. of the differential equation $(x\log x)\frac{dy}{dx}+y=2x\log x$, $(x\geq 1)$. Then $f(e^2)=$ ____

24. If $\frac{dy}{dx}+\frac{y}{x}=x^2$, then $2y(2)-y(1)=$ ____ **(CET 22)**

25. If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$ and y (0) = 1, then $y\left(\frac{\pi}{2}\right)$ is equal to:______ **(JEE-M 17)**

Answer

1. **(4)** **2.(0)** **3.** **(4)** **4.** **(2)** **5.** **(3)** **6.** **(3)** **7.** **(5)** **8.** **(4)** **9.** **(2)** **10.** **(1)**

11. **(6)** **12.** **(2)** **13.** **($\sqrt{2}$)** **14.** **(0)** **15.** **(3)** **16.** **(1)** **17.** **($\frac{1}{2}$)**

18. **(–1)** **19.** **(2)** **20.** **(–1)**

21. **(7)**

Hint: $\frac{dy}{y+3} = dx \Rightarrow \log(y+3) = x + c \text{ or } y+3 = Ae^x$

$y(0) = 2 \Rightarrow 5 = A \quad \therefore \text{ When } x = \log 2,\ y = 10 - 3 = 7$

22. **(3)**

23. **(2)**

$$f(x) = e^{\int \frac{1}{x \log x} dx} = e^{\int \frac{\frac{1}{x}}{\log x} dx} = e^{\log(\log x)} = \log x \quad \therefore f(e^2) = \log e^2 = 2$$

24. **($\frac{15}{4}$)**

$x\,dy + y\,dx = x^3\,dx \Rightarrow xy = \frac{x^4}{4} + c$

Put $x = 2, x = 1$: $2.\,y(2) - 1\,.\,y(1) = \frac{2^4}{4} - \frac{1^4}{4} \Rightarrow 2\,y(2) - y(1) = 4 - \frac{1}{4} = \frac{15}{4}$

25. **($\frac{1}{4}$)**

$(2 + \sin x)\,dy + (y + 1)\cos x\,dx = 0$ i.e., $(2 + \sin x)\,d(y + 1) + (y + 1)\,d(\sin x) = 0$

$\Rightarrow (2 + \sin x)(y + 1) = c$; $y(0) = 1 \Rightarrow 2(2) = c$

$\therefore (2 + \sin x)(y + 1) = 4$; when $x = \frac{\pi}{2}$, $3(y+1) = 4$