Indian Institute of Technology, Kharagpur
Department of Physics
Statistical Mechanics-I (PH41023)

Home work set - 5

To be submitted on Moodle by: Saturday 20^{th} November, 2021

1 Problem 1

Consider the one-dimensional Ising model in a Canonical ensemeble, we start with the Canonical partition function that was derived in Tutorial Class 9.

(a) Find the thermodynamic quantities of internal energy U, Helmoholtz free energy A and the entropy S.

(b) Plot the magnetization per spin $m = M/N = -\frac{\partial A}{\partial B}$ for a few values of βJ as a function of βB.

(c) Find the critical exponents of the one dimensional Ising model at the zero temperature. The exponents to be found are $\alpha, \beta, \delta, \gamma$. They are defined as,

$$m \sim (T_c - T)^{\beta}, \tag{1}$$

$$\Xi \sim \frac{1}{|T - T_c|^{\gamma}}, \tag{2}$$

$$c_v \sim \frac{1}{|T - T_c|^{\alpha}}, \tag{3}$$

$$m \sim |h|^{1/\delta}. \tag{4}$$

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2 Problem 2

Consider the modified Van der-Waal equation for a real gas given by,

$$(P + a/v^n)(v - b) = RT, \quad n > 1. \tag{5}$$

Find the critical constants P_c, V_c, T_c and the critical exponents $\alpha, \beta, \gamma, \delta$.

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3 Problem 3

Computer simulation: Use either your computer or a free online coding portal like Google Collab.

Consider a brownian particle in a two-dimensional plane that obeys the following Langevin equation,

$$\frac{dv_x}{dt} = -v_x + \zeta_x(t), \tag{6}$$

$$\frac{dv_y}{dt} = -v_y + \zeta_y(t), \tag{7}$$

$$\frac{dx}{dt} = v_x, \tag{8}$$

$$\frac{dy}{dt} = v_y. \tag{9}$$

where $\zeta(t)$ is a random Gaussian white noise that satisfies the following statistics, $\langle\zeta(t)\rangle = 0, \langle\zeta(t)\zeta(t')\rangle = 2D\delta(t - t')$ with the angular brackets denoting ensemble average.

We will try to solve this equation numerically now, let us discretize the time axis. Start with some initial non-zero velocity (v_x, v_y) withe the particle at the origin $(x, y) = (0, 0)$. Choose a value of D and numerically integrate the above equations to find $x(t), y(t), v_x(t), v_y(t)$ upto some time $t > 1$ time unit. Trace the trajectory of such a particle in the two-dimensional plane. Add the code and the snapshot of the trajectory in your homework assignment.
