STATS790 A2

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1 Question 1(a)

Derive/show how to compute linear regression coefficients (for general choice of y and X) using the following four methods: naive linear algebra; QR decomposition; SVD; and Cholesky decomposition.

Consider residual sum-of-squares of the linear regression model:

$$\mathbf{RSS}(\beta) = (y - \mathbf{X}\beta)^T (y - \mathbf{X}\beta) \tag{1}$$

We want to minimize it by differentiating with respect to β:

$$\frac{\partial \mathbf{RSS}}{\partial \beta} = -2\mathbf{X}^T (y - \mathbf{X}\beta) \tag{2}$$

So set it to 0 to obtain the normal equation:

$$\mathbf{X}^T \mathbf{X}\beta = \mathbf{X}^T y \tag{3}$$

We can derive the linear regression coefficients by using the following 4 methods.

- Naive linear algebra

$$\begin{aligned} \mathbf{X}^T \mathbf{X}\beta &= \mathbf{X}^T y \\ \hat{\beta} &= (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T y \end{aligned} \tag{4}$$

- QR decomposition
  By the definition of QR decomposition. For a real $n \times p$ matrix A, where $n \geq p$, matrix A can be decomposed into two matrices, A and Q:

$$A = QR \tag{5}$$

  where $Q^T Q = I_p$ and R is an $p \times p$ upper triangular matrix. Hence, we can decompose the matrix X into QR to get:

$$
\begin{aligned}
\mathbf{X}^T\mathbf{X}\beta &= \mathbf{X}^T y \\
(QR)^T(QR)\beta &= (QR)^T y \\
R^T Q^T Q R\beta &= R^T Q^T y \\
R^T R\beta &= R^T Q^T y \\
R\beta &= Q^T y \\
\beta &= R^{-1} Q^T y
\end{aligned} \tag{6}
$$

- SVD decomposition
  By definition of SVD decomposition. A $n \times p$ matrix A can be decomposed into the following form:

$$A = U\Sigma V^T \tag{7}$$

  where U and V are $n \times p$ and $p \times p$ orthogonal matrices, and D is a $p \times p$ diagonal matrix contain singular value of A. And $U^T U = I$ and $VV^T = I$. So we can decompose X to obtain:

$$
\begin{aligned}
\mathbf{X}^T\mathbf{X}\beta &= \mathbf{X}^T y \\
(U\Sigma V^T)^T(U\Sigma V^T)\beta &= (U\Sigma V^T)^T y \\
V\Sigma^T U^T U\Sigma V^T\beta &= (U\Sigma V^T)^T y \\
V\Sigma^T\Sigma V^T\beta &= V\Sigma^T U^T y \\
\Sigma V^T\beta &= U^T y \\
V^T\beta &= \Sigma^{-1} U^T y \\
VV^T\beta &= V\Sigma^{-1} U^T y \\
\beta &= V\Sigma^{-1} U^T y
\end{aligned} \tag{8}
$$

  where $V\Sigma^{-1}U^T$ is the pseudo-inverse of X.

- Cholesky decomposition
  By definition of Cholesky decomposition. If the matrix A is positive definite and symmetric, then there exists a lower triangular matrix, L, such that

$$A = LL^T \tag{9}$$

  We know $X^T X$ is symmetric. If it is also positive definite(full rank), then we can decompose it as $\mathbf{X}^T\mathbf{X} = LL^T$ to obtain:

$$
\begin{aligned}
\mathbf{X}^T\mathbf{X}\beta &= \mathbf{X}^T y \\
LL^T\beta &= X^T y \\
L(L^T\beta) &= X^T y
\end{aligned} \tag{10}
$$

Let $c = L^T\beta$, we first solve $Lc = X^Ty$ to get:

$$c = L^{-1}X^Ty \tag{11}$$

Then solve $L^T\beta = c$ to get the regression coefficient:

$$\beta = (L^T)^{-1}c \tag{12}$$

2 Question 2

Suppose augmenting X and y into $\mathbf{B}$ and $\mathbf{y}^*$:

$$\mathbf{B} = \begin{bmatrix} \mathbf{X} \\ \sqrt{\lambda}\mathbf{I} \end{bmatrix} \qquad \mathbf{y}^* = \begin{bmatrix} \mathbf{y} \\ 0 \end{bmatrix} \tag{13}$$

We can solve it using standard linear regression by QR decomposition from Q1. Let:

$$\mathbf{B} = \mathbf{QR} \tag{14}$$

The normal equation will be:

$$\begin{aligned} \mathbf{B}^T\mathbf{B}\beta &= \mathbf{B}^T \begin{bmatrix} \mathbf{y} \\ 0 \end{bmatrix} \\ \mathbf{B}^T\mathbf{B}\beta &= \mathbf{B}^T y^* \\ (\mathbf{QR})^T(\mathbf{QR})\beta &= (\mathbf{QR})^T y^* \end{aligned} \tag{15}$$

Thus, β will be:

$$\beta^{ridge} = \mathbf{R}^{-1}\mathbf{Q}^T y^* \tag{16}$$

3 ESL 3.6

Considering the posterior distribution of β^{ridge}:

$$\mathbf{P}(\beta|\mathbf{y}, \mathbf{X}) = \frac{\mathbf{P}(\mathbf{y}|\beta, \mathbf{X})\mathbf{P}(\beta)}{\mathbf{P}(y|\mathbf{X})} \propto \mathbf{P}(\mathbf{y}|\beta, \mathbf{X})\mathbf{P}(\beta) \tag{17}$$

If we choose a Gaussian prior $\beta \sim \mathcal{N}(0, \tau\mathbf{I})$ and $y \sim \mathcal{N}(\mathbf{X}\beta, \sigma^2\mathbf{I})$, we see that:

$$\begin{aligned} \mathbf{P}(\beta|\mathbf{y}, \mathbf{X}) &\propto \mathbf{P}(\mathbf{y}|\beta, \mathbf{X})\mathbf{P}(\beta) \\ &\propto \frac{1}{(2\pi)^{\frac{p}{2}}|\sigma|} \exp\left(-\frac{(y - \mathbf{X}\beta)^T(y - \mathbf{X}\beta)}{2\sigma^2}\right) \frac{1}{(2\pi)^{\frac{p}{2}}|\tau|^{\frac{1}{2}}} \exp\left(-\frac{\beta^T\beta}{2\tau}\right) \\ &\propto \exp\left(-\frac{(y - \mathbf{X}\beta)^T(y - \mathbf{X}\beta)}{2\sigma^2}\right) \exp\left(-\frac{\beta^T\beta}{2\tau}\right) \end{aligned} \tag{18}$$

Since the prior is Gaussian prior is conjugate prior for Gaussian distribution. By matching the pattern, the covariance matrix and the mean of Gaussian distribution will be:

$$\Sigma = \frac{1}{\sigma^2} X^T X + \frac{1}{\tau} I \tag{19}$$

$$\begin{aligned} \mu &= \frac{1}{\sigma^2} \Sigma^{-1} X^T y \\ &= (X^T X + \frac{\sigma^2}{\tau} I)^{-1} X^T y \end{aligned} \tag{20}$$

We can take the log to **MAP** and ignore the constant:

$$\begin{aligned} \arg\max_\beta (\log \mathbf{P}(\beta | \mathbf{y}, \mathbf{X})) &= \arg\max_\beta \left(\log(\exp\left(-\frac{(y - \mathbf{X}\beta)^T (y - \mathbf{X}\beta)}{2\sigma^2} \right)) + \log\exp\left(-\frac{\beta^T \beta}{2\tau} \right) \right) \\ &= \arg\max_\beta \left(-\frac{(y - \mathbf{X}\beta)^T (y - \mathbf{X}\beta)}{2\sigma^2} - \frac{\beta^T \beta}{2\tau} \right) \\ &= \arg\max_\beta \left(-\frac{1}{\sigma^2} \left((y - \mathbf{X}\beta)^T (y - \mathbf{X}\beta) + \frac{\sigma^2}{\tau} \beta^T \beta \right) \right) \\ &= \arg\min_\beta \left((y - \mathbf{X}\beta)^T (y - \mathbf{X}\beta) + \frac{\sigma^2}{\tau} \beta^T \beta \right) \end{aligned} \tag{21}$$

which is the same for the ridge regression in matrix form. Here $\lambda = \frac{\sigma^2}{\tau}$, we can also derive the β. And clearly, it is equivalent to the mean:

$$\beta^{ridge} = (\mathbf{X}^T \mathbf{X} + \frac{\sigma^2}{\tau} \mathbf{I})^{-1} \mathbf{X}^T y \tag{22}$$

If we have a smaller variance τ for the prior, λ will increase.

4 ESL 3.19

By using SVD decomposition to solve ridge regression, we can obtain:

$$\begin{aligned} \beta^{ridge} &= (X^T X + \lambda I)^{-1} X^T y \\ &= ((V \Sigma^T U^T)(U \Sigma V^T) + \lambda V V^T)^{-1} V \Sigma U^T y \\ &= (V \Sigma^2 V^T + \lambda V V^T)^{-1} V \Sigma U^T y \\ &= ((V \Sigma^2 + \lambda V) V^T)^{-1} V \Sigma U^T y \\ &= (V^T)^{-1} (\Sigma^2 + \lambda I)^{-1} V^{-1} V \Sigma U^T y \\ &= V (\Sigma^2 + \lambda I)^{-1} \Sigma U^T y \end{aligned} \tag{23}$$

Then, consider the L2 norm of β^{ridge}:

$$
\begin{aligned}
\|\beta^{ridge}\| &= \sqrt{\beta^{ridge T}\beta^{ridge}} \\
&= \sqrt{y^T U\Sigma(\Sigma^2+\lambda I)^{-1}V^T V(\Sigma^2+\lambda I)^{-1}\Sigma U^T y} \\
&= \sqrt{y^T U\Sigma(\Sigma^2+\lambda I)^{-2}\Sigma U^T y} \\
&= \sqrt{(U^T y)^T \Sigma(\Sigma^2+\lambda I)^{-2}\Sigma U^T y} \\
&= \sqrt{(U^T y)^T \frac{\Sigma^2}{(\Sigma^2+\lambda I)^2} U^T y} \\
&= \sqrt{\sum_{j=1}^{p} (U^T y)_j^2 \frac{d_j^2}{(d_j^2+\lambda)^2}}
\end{aligned}
\tag{24}
$$

Since $\lambda \geq 0$, it's easy to see $0 \leq \frac{d_j^2}{(d_j^2+\lambda)^2} \leq 1$. As $\lambda \to 0$, the fraction $\frac{d_j^2}{(d_j^2+\lambda)^2} \to 1$. And $\|\beta^{ridge}\|$ also increases. This also holds for lasso regression. Consider the constraint optimization of lasso regression:

$$
\begin{aligned}
\beta^{lasso} &= \arg\min_{\beta} \left(\sum_{i=1}^{N} \left(y_i - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 \right) \\
s.t &\sum_{j}^{p} |\beta_j| \leq t
\end{aligned}
\tag{25}
$$

When λ decrease, t will increase. The feasible region for $\sum_j^p |\beta_j|$ will also increase. So smaller λ will allow the model to select a larger β. Another way to explain this is to consider the other format, which is equivalent to the above minimization problem of the lasso:

$$
\beta^{lasso} = \arg\min_{\beta} \left(\sum_{i=1}^{N} \left(y_i - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^{p} |\beta_j| \right) \tag{26}
$$

If we have a larger complexity parameter λ, the shrinkage will be larger. This will shrink the coefficient toward 0 and vice versa. Hence, we can conclude $\|\beta\|$ increases as its tuning parameter approach 0.

5 ESL 3.28

Suppose t is fixed. The original lasso regression can be written as:

$$
\beta^{lasso} = \arg\min_{\beta} ||\mathbf{y} - \mathbf{X}^{ori}\beta^{ori}||_2^2 + \lambda||\beta^{ori}||_1 \tag{27}
$$

If we add $X_j^* = X_j$, we will obtain

$$\beta^{new} = \arg\min_{\beta} ||\mathbf{y} - \mathbf{X}^{new}\beta^{new}||_2^2 + \lambda \sum_{k=0}^{p} |\beta_k^{ori}| + \lambda|\beta_j^*| \tag{28}$$

Let $\beta_j^c = \beta_j + \beta_j^*$, we can rewrite the above equation in the following way:

$$\beta^{lasso} = \arg\min_{\beta} ||\mathbf{y} - \mathbf{X}^{new}\beta^{new}||_2^2 + \lambda \sum_{k\neq j}^{p} |\beta_k| + \lambda|\beta_j^c| + (\lambda|\beta_j| + \lambda|\beta_j^*| - \lambda|\beta_j^c|) \tag{29}$$

By triangle inequality:

$$|\beta_j| + |\beta_j^*| \geq |\beta_j + \beta_j^*| = |\beta_j^c| \tag{30}$$

The later term in equation(27) $\lambda|\beta_j| + \lambda|\beta_j^*| - \lambda|\beta_j^c|$ has to be positive. Also, $\beta_j = \beta_j^*$. Given $\beta_j^{lasso} = a$, the term $\lambda|\beta_j| + \lambda|\beta_j^*| - \lambda|\beta_j^c|$ becomes 0. This implies $\beta_j = \beta_j^* = \frac{a}{2}$ give the optimal solution.

6 ESL 3.30

We want to form a lasso problem in the space of augmented space. Assume we are augmenting X and y into $\mathbf{B}$ and $\mathbf{y}^*$ in the general form.

$$\mathbf{B} = \begin{bmatrix} \mathbf{X} \\ \eta\mathbf{I} \end{bmatrix} \qquad \mathbf{y}^* = \begin{bmatrix} \mathbf{y} \\ 0 \end{bmatrix} \tag{31}$$

We can form the minimization problem as:

$$\begin{aligned} \beta &= \arg\min_{\beta} ||\mathbf{y}^* - \mathbf{B}\beta||_2^2 \\ &= \arg\min_{\beta} \left|\left| \begin{bmatrix} \mathbf{y} - \mathbf{X}\beta \\ 0 - \eta\beta \end{bmatrix} \right|\right|_2^2 \\ &= \arg\min_{\beta} ||\mathbf{y} - \mathbf{X}\beta||_2^2 + ||0 - \eta\beta||_2^2 \\ &= \arg\min_{\beta} ||\mathbf{y} - \mathbf{X}\beta||_2^2 + \eta^2||\beta||_2^2 \end{aligned} \tag{32}$$

Clearly, this is the ridge regression(L2 norm). We can also form a lasso regression in the augmented space. And replace the minimization with the above ridge regression since $||\mathbf{y}^* - \mathbf{B}\beta||_2^2 = ||\mathbf{y} - \mathbf{X}\beta||_2^2 + \eta^2||\beta||_2^2$.

$$\begin{aligned} \beta^{lasso} &= \arg\min_{\beta} ||\mathbf{y}^* - \mathbf{B}\beta||_2^2 + \delta||\beta||_1 \\ &= \arg\min_{\beta} ||\mathbf{y} - \mathbf{X}\beta||_2^2 + \eta^2||\beta||_2^2 + \delta||\beta||_1 \end{aligned} \tag{33}$$

Consider the elastic net:

$$
\begin{aligned}
&\arg\min_{\beta} ||y - X\beta||_2^2 + \lambda[\alpha||\beta||_2^2 + (1-a)||\beta||] \\
&= \arg\min_{\beta} ||y - X\beta||_2^2 + \lambda\alpha||\beta||_2^2 + \lambda(1-a)||\beta||
\end{aligned}
\tag{34}
$$

Let $\eta = \sqrt{\lambda\alpha}$ and $\delta = \lambda(1-\alpha)$. Then we successfully convert the elastic net to a lasso problem. We can solve the lasso by:

$$
\beta^{lasso} = \arg\min_{\beta} ||\mathbf{y}^* - \mathbf{B}\beta||_2^2 + \lambda(1-\alpha)||\beta||_1 \tag{35}
$$

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Question 1(b)

```
#Naive Linear Algebra

beta.la <- function(X,y){
  beta <- solve(t(X) %*% X) %*%t(X) %*% y

  return(beta)
}


#QR decomposition

beta.qr <- function(X,y){
  QR <- qr(X)
  Q <- qr.Q(QR)
  R <- qr.R(QR)

  beta <- solve(R) %*% t(Q) %*% y

  return(beta)
}


#SVD decomposition

beta.svd <- function(X,y){
  svd <- svd(X)
  u <- svd$u
  d <- diag(svd$d)
  v <- svd$v

  beta <- v %*% solve(d) %*% t(u) %*% y

  return(beta)

}
```

```
library(microbenchmark)
#my_lm <- function(X,y) rnorm(ncol(X)) ## trivial: for testing
simfun <- function(n, p) {
    y <- rnorm(n)
    X <- matrix(rnorm(p*n), ncol = p)
```

```r
    X <- as.matrix(cbind(rep(1,n),X))
    list(X = X, y = y)
}
set.seed(101)
s <- simfun(100, 10)


beta.qr(s$X, s$y)
```

```
##              [,1]
##  [1,] -0.05491747
##  [2,]  0.14166868
##  [3,] -0.02815091
##  [4,] -0.13411563
##  [5,] -0.01479834
##  [6,]  0.12601089
##  [7,]  0.06444634
##  [8,] -0.03591869
##  [9,]  0.19412958
## [10,]  0.04473605
## [11,] -0.02575748
```

```r
beta.la(s$X, s$y)
```

```
##              [,1]
##  [1,] -0.05491747
##  [2,]  0.14166868
##  [3,] -0.02815091
##  [4,] -0.13411563
##  [5,] -0.01479834
##  [6,]  0.12601089
##  [7,]  0.06444634
##  [8,] -0.03591869
##  [9,]  0.19412958
## [10,]  0.04473605
## [11,] -0.02575748
```

```r
beta.svd(s$X, s$y)
```

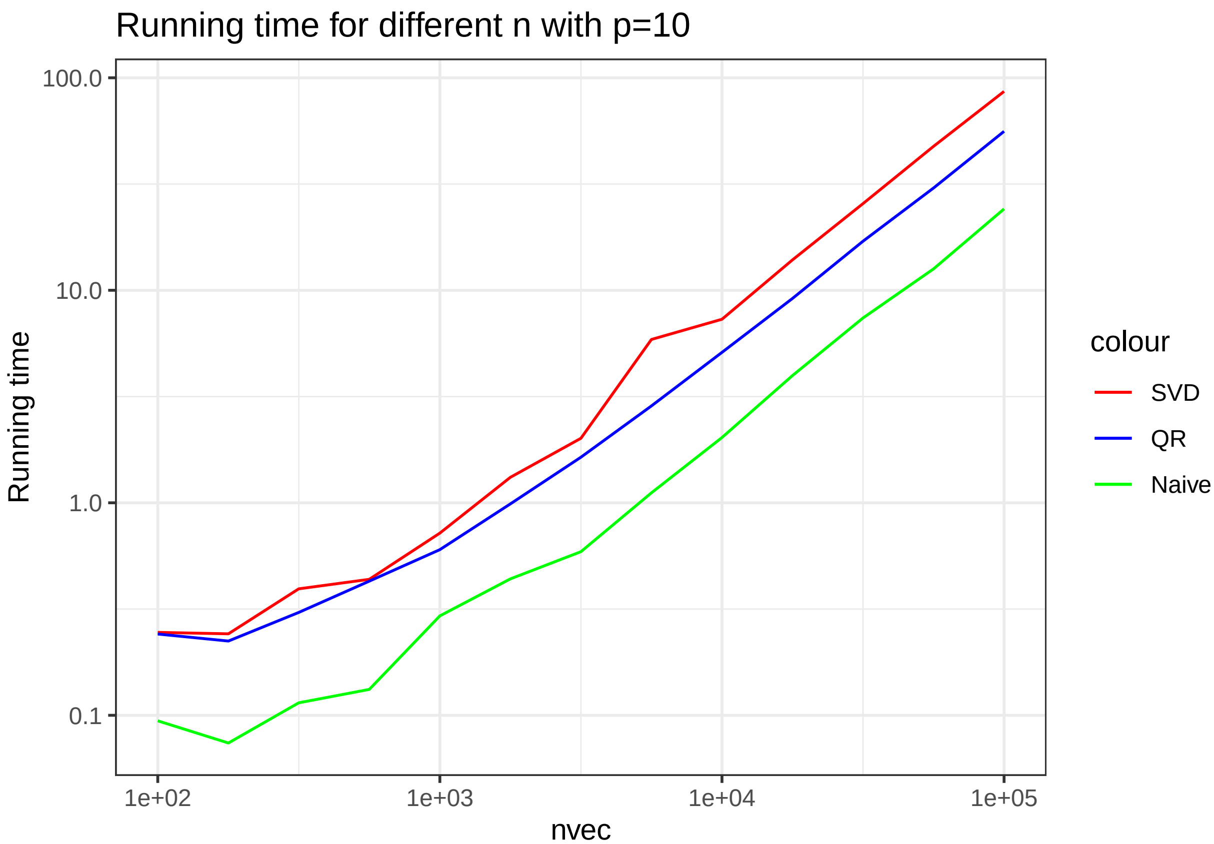
```
##              [,1]
##  [1,] -0.05491747
##  [2,]  0.14166868
##  [3,] -0.02815091
##  [4,] -0.13411563
##  [5,] -0.01479834
##  [6,]  0.12601089
##  [7,]  0.06444634
##  [8,] -0.03591869
##  [9,]  0.19412958
## [10,]  0.04473605
## [11,] -0.02575748
```

```r
nvec <- round(10^seq(2, 5, by = 0.25))

#store the result:
```

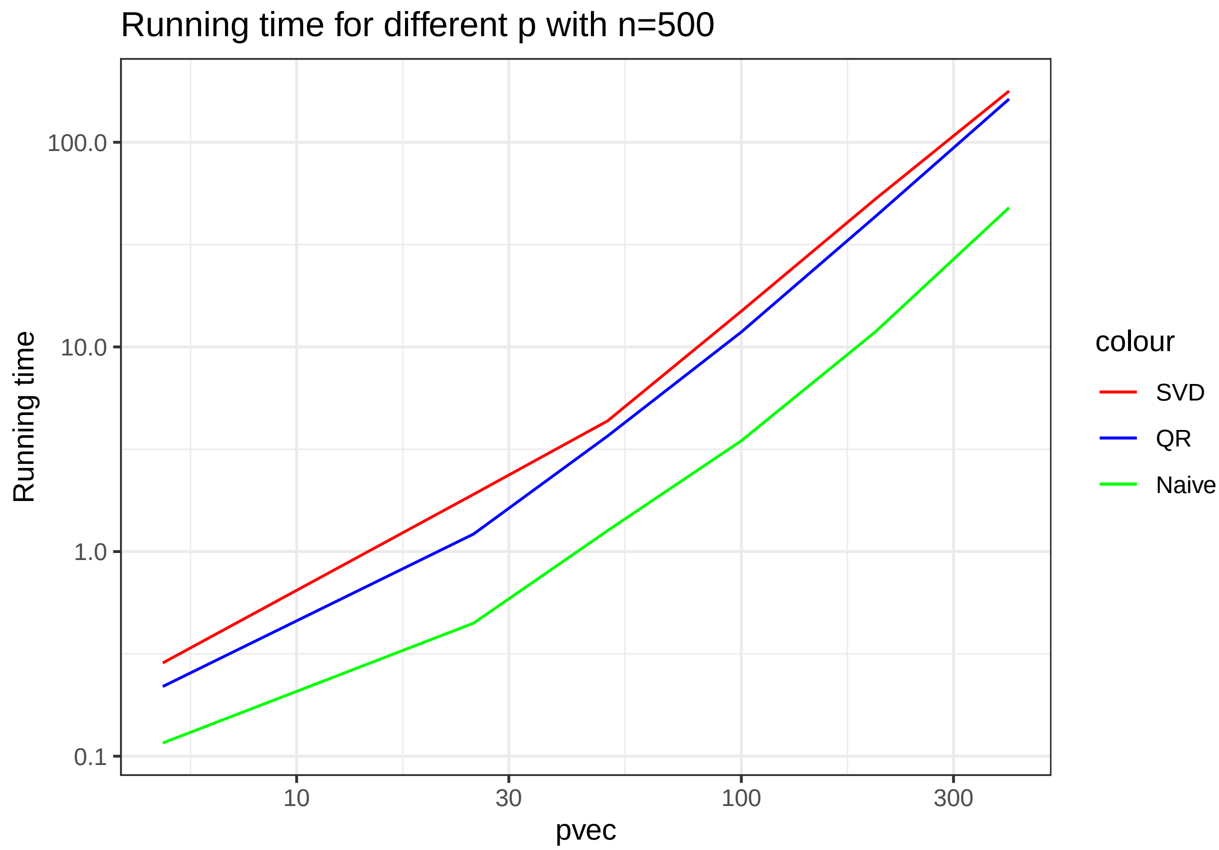
```r
ren.la <- c()
ren.qr <- c()
ren.svd <- c()

#looping over values of n
for (i in 1:length(nvec)) {

    s <- simfun(nvec[i], p = 10)
    m <- microbenchmark(
        beta.la(s$X, s$y),
        beta.qr(s$X, s$y),
        beta.svd(s$X, s$y))
    re <- summary(m, unit ='ms')
    ren.la <- append(ren.la, re$mean[1])
    ren.qr <- append(ren.qr, re$mean[2])
    ren.svd <- append(ren.svd, re$mean[3])

}

ren.df <- data.frame(Naive = ren.la, QR = ren.qr, SVD = ren.svd)





#pvec <- round(10^seq(1, 2, by = 0.25))

pvec <- c(5,25,50,100,200,400)
rep.la <- c()
rep.qr <- c()
rep.svd <- c()


#looping over values of p


for (i in 1:length(pvec)) {
    s <- simfun(500, p = pvec[i])
    m <- microbenchmark(
        beta.la(s$X, s$y),
        beta.qr(s$X, s$y),
        beta.svd(s$X, s$y))
    re <- summary(m,unit = 'ms')
    rep.la <- c(rep.la, re$mean[1])
    rep.qr <- c(rep.qr, re$mean[2])
    rep.svd <- c(rep.svd, re$mean[3])
}

rep.df <- data.frame(Naive = rep.la, QR = rep.qr, SVD = rep.svd)
```

```r
library(ggplot2)

#plot Running time for different n with p=10
```

```r
(ggplot()
  + geom_line(aes(x= nvec,y = ren.la,colour = "red"))
  + geom_line(aes(x= nvec,y = ren.qr,colour = "blue"))
  + geom_line(aes(x= nvec,y = ren.svd,colour = "green"))
  + scale_color_manual(labels = c("SVD", "QR","Naive"), values = c("red", "blue","green"))
  + scale_x_continuous(trans='log10')
  + scale_y_continuous(trans='log10')
  + ggtitle("Running time for different n with p=10")
  + ylab("Running time")
  + theme_bw())
```




```r
#plot Running time for different n with p=10
(ggplot()

  + geom_line(aes(x= pvec,y = rep.svd,colour = "green"))
    + geom_line(aes(x= pvec,y = rep.la,colour = "red"))

  + geom_line(aes(x= pvec,y = rep.qr,colour = "blue"))

  + scale_color_manual(labels = c("SVD", "QR","Naive"), values = c("red", "blue","green"))
  + scale_x_continuous(trans='log10')
  + scale_y_continuous(trans='log10')
  + ggtitle("Running time for different p with n=500")
  + ylab("Running time")
  + theme_bw())
```




```
#fit linear model for n time

#Naive linear algebra
naive.n.lm <- lm(
  log(ren.la)~ log(nvec)
)
naive.n.lm
```

```
##
## Call:
## lm(formula = log(ren.la) ~ log(nvec))
##
## Coefficients:
## (Intercept)    log(nvec)
##     -7.1132       0.8653
```

```
#QR
qr.n.lm <- lm(
  log(ren.qr)~ log(nvec)
)
qr.n.lm
```

```
##
## Call:
## lm(formula = log(ren.qr) ~ log(nvec))
##
## Coefficients:
## (Intercept)    log(nvec)
```

```
##     -6.2134       0.9047
```

```r
#SVD
svd.n.lm <- lm(
  log(ren.svd)~ log(nvec)
)
svd.n.lm
```

```
##
## Call:
## lm(formula = log(ren.svd) ~ log(nvec))
##
## Coefficients:
## (Intercept)    log(nvec)
##     -5.9740       0.8385
```

```r
#fit linear model for p predictor

#Naive linear algebra
naive.p.lm <- lm(
  log(rep.la)~ log(pvec)
)
naive.p.lm
```

```
##
## Call:
## lm(formula = log(rep.la) ~ log(pvec))
##
## Coefficients:
## (Intercept)    log(pvec)
##      -4.843        1.377
```

```r
#QR
qr.p.lm <- lm(
  log(rep.qr)~ log(pvec)
)
qr.p.lm
```

```
##
## Call:
## lm(formula = log(rep.qr) ~ log(pvec))
##
## Coefficients:
## (Intercept)    log(pvec)
##      -3.939        1.475
```

```r
#SVD
svd.p.lm <- lm(
  log(rep.svd)~ log(pvec)
)
svd.p.lm
```

```
##
## Call:
## lm(formula = log(rep.svd) ~ log(pvec))
##
## Coefficients:
```

```
## (Intercept)    log(pvec)
##      -4.363        1.522
```

Question 2

```
#Ridge regression by data augmentation(QR decompsition)

lmridge <- function(X,y,lambda){

  X <- unname(X)
  n <- dim(X)[1]
  p <- dim(X)[2]

  #augmenting X
  B <- as.matrix(rbind(as.matrix(X),sqrt(lambda)*diag(p)))

  B <- as.matrix(cbind(c(rep(1,n),rep(0,p)),B))

  #augmentin y

  y <- as.matrix(c(y,rep(0,p)))


  #QR decomposition
  QR <- qr(B)
  Q <- qr.Q(QR)
  R <- qr.R(QR)


  beta <- solve(R) %*% t(Q) %*% as.matrix(y)



  return(beta)
}

# read data
df <- read.table("https://hastie.su.domains/ElemStatLearn/datasets/prostate.data")
head(df)
```

```
##       lcavol  lweight age      lbph svi       lcp gleason pgg45       lpsa
## 1 -0.5798185 2.769459  50 -1.386294   0 -1.386294       6     0 -0.4307829
## 2 -0.9942523 3.319626  58 -1.386294   0 -1.386294       6     0 -0.1625189
## 3 -0.5108256 2.691243  74 -1.386294   0 -1.386294       7    20 -0.1625189
## 4 -1.2039728 3.282789  58 -1.386294   0 -1.386294       6     0 -0.1625189
## 5  0.7514161 3.432373  62 -1.386294   0 -1.386294       6     0  0.3715636
## 6 -1.0498221 3.228826  50 -1.386294   0 -1.386294       6     0  0.7654678
##   train
## 1  TRUE
## 2  TRUE
## 3  TRUE
## 4  TRUE
## 5  TRUE
## 6  TRUE
```

```
train <- df[df$train==TRUE,]
train <- scale(train,TRUE,TRUE)
train_x <- train[,1:8]
```

```r
train_y <- train[,9]

test <- df[df$train==FALSE,]
test <- scale(test,TRUE,TRUE)
test_x <- test[,1:8]
test_y <- test[,9]
```

```r
#ridge regression by data augmenting(QR)

beta.ridge <- lmridge(train_x,train_y,2)

pred.ridge <- as.matrix(cbind(rep(1,nrow(test_x)),test_x)) %*% beta.ridge

#RSS error
sum((test_y - pred.ridge)^2)
```

```
## [1] 13.9301
```

```r
#ridge regression by glmnet
fit <- glmnet::glmnet(
  x = train_x,
  y = train_y,
  family = "gaussian",
  alpha = 0,  ## ridge penalty
  lambda = 2,
  standardize = FALSE,
)

fit_pred <- predict(
  fit,
  s = 2,
  newx = test_x
)

#RSS Error
sum((test_y - fit_pred)^2)
```

```
## [1] 16.38894
```

```r
m1 <- microbenchmark(
    lmridge(train_x,train_y,2),
    fit <- glmnet::glmnet(
    x = train_x,
    y = train_y,
    family = "gaussian",
    alpha = 0,  ## ridge penalty
    lambda = 2,
    standardize = FALSE,
    intercept = FALSE,
    )
    )

results <- summary(m1)

#time comparsion
```

```r
rownames(results) <- c("lmridge", "glmnet")
results[,-1]
```

```
##              min        lq      mean   median       uq      max neval
## lmridge 146.440  169.8665  212.1053  212.617  235.971  332.804   100
## glmnet  989.267 1020.2280 1114.6138 1048.236 1230.810 1410.080   100
```