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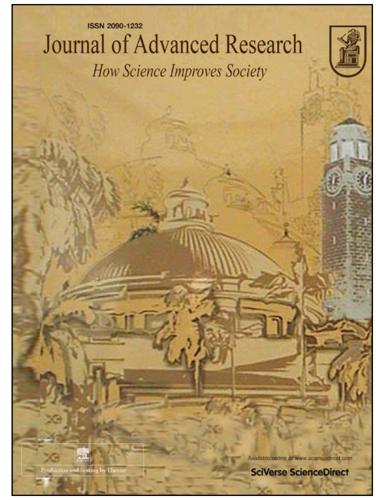

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PII: S2090-1232(21)00197-1
DOI: https://doi.org/10.1016/j.jare.2021.09.015
Reference: JARE 1075

To appear in: *Journal of Advanced Research*

Received Date: 21 April 2021
Accepted Date: 16 September 2021

Please cite this article as: Zhang, M., Xie, X., Manafian, J., Ilhan, O.A., Singh, G., Characteristics of the new multiple rogue wave solutions to the fractional generalized CBS-BK equation, *Journal of Advanced Research* (2021), doi: https://doi.org/10.1016/j.jare.2021.09.015

Characteristics of the new multiple rogue wave solutions to the fractional generalized CBS-BK equation

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12 Sep 2021

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Abstract

Introduction: The multiple Exp-function scheme is employed for searching the multiple soliton solutions for the fractional generalized Calogero-Bogoyavlenskii-Schiff-Bogoyavlensky- Konopelchenko equation.
Objectives: Moreover, the Hirota bilinear technique is utilized to detecting the lump and interaction with two stripe soliton solutions.
Methods: The multiple Exp-function scheme and also, the semi-inverse variational principle will be used for the considered equation.
Results: We have obtained more than twelve sets of solutions including a combination of two positive functions as polynomial and two exponential functions. The graphs for various fractional-order α are designed to contain three dimensional, density, and y-curves plots. Then, the classes of rogue waves-type solutions to the fractional generalized Calogero-Bogoyavlenskii-Schiff-Bogoyavlensky- Konopelchenko equation within the frame of the bilinear equation, is found.
Conclusion: Finally, a direct method which is called the semi-inverse variational principle method was used to obtain solitary waves of this considered model. These results can help us better understand interesting physical phenomena and mechanism. The dynamical structures of these gained lump and its interaction soliton solutions are analyzed and indicated in graphs by choosing suitable amounts. The existence conditions are employed to discuss the available got solutions.

Keywords: Multiple Exp-function method; Hirota bilinear technique; Lump solitons; Semi-inverse variational principle

Introduction

It is known that the integrability of mathematics physics area has been better investigated in recent years. There are diverse definitions of integrability of nonlinear evolution differential equations. Among them, there are existing some indicators such as Wronskian, Casoratian, Lie symmetry analysis, homotopy perturbation method and bilinear Bäcklund transformations have been associated with rational function solutions [1]-[6], etc. Therefore, in order to study the integrability of nonlinear evolution differential equations, we need to determine these indicators. To the best of our knowledge, the multi exp-function approach ([7]-[11]) is an effective tool to construct the bilinear equation. Lump wave is localized in space and will not disappear due to changes in time. In 2015, Prof. Ma suggested a

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n ... Ma provided theoretical support for this technique and proved it, making the lump solution has been greatly developed [13]. Later, this technique was widely used by researchers, and plenty lump solutions and interaction solutions of nonlinear partial differential equations were acquired ([14]-[21]).

Here, we mainly consider the below dynamical model, which can be used to explain some interesting (2+1)-dimensional waves of physics, namely, the (2+1)-dimensional Bogoyavlenski equation [22]. That is

$$\begin{aligned} &4\Psi_t + \Psi_{xxy} - 4\Psi^2\Psi_y - 4\Psi_x\Phi = 0, \\ &\Psi\Psi_y = \Phi_x, \end{aligned} \tag{0.1}$$

in which the plenty of researchers have been worked on it at Refs. ([23]-[26]). Abadi and Naja [27] presented a modified form of Eq. (0.1) as below frame

$$4\Psi_{xt} + 8\Psi_x\Psi_{xy} + 4\Psi_y\Psi_{xx} + \Psi_{xxxy} = 0, \tag{0.2}$$

which called the breaking soliton equation. Moreover, Eq. (0.2) is considered as Bogoyavlensky-Konopelchenko (BK) equation [28, 29] by below form

$$a\Psi_{xt} + b\Psi_{xxxx} + c\Psi_{xxy} + d\Psi_x\Psi_{xx} + e\Psi_x\Psi_{xy} + k\Psi_{xx}\Psi_y = 0. \tag{0.3}$$

The generalized BK equation [30] is given as below

$$\Psi_t + \alpha(6\Psi\Psi_x + \Psi_{xxx}) + \beta(\Psi_{xxy} + 3\Psi\Psi_y + 3\Psi_x\Phi_y) + \gamma_1\Psi_x + \gamma_2\Psi_y + \gamma_3\Phi_{yy} = 0, \tag{0.4}$$

in which $\Phi_x = \Psi$, and $\alpha, \beta, \gamma_1, \gamma_2$, and γ_3 are specified values. Eq. (0.4) can be written as

$$\Phi_{xt} + \alpha(6\Phi_x\Phi_{xx} + \Phi_{xxxx}) + \beta(\Phi_{xxxy} + 3\Phi_x\Phi_{xy} + 3\Phi_{xx}\Phi_{xy}) + \gamma_1\Phi_{xx} + \gamma_2\Phi_{xy} + \gamma_3\Phi_{yy} = 0. \tag{0.5}$$

In this paper, we will investigate the following the (2+1)-dimensional the fractional generalized Calogero-Bogoyavlenskii-Schiff-Bogoyavlensky- Konopelchenko (gCBS-BK) equation

$$\begin{aligned} &D_t^{\alpha}\Psi + \Psi_{xxy} + 3\Psi_x\Psi_y + \delta_1\Psi_y + \delta_2\Phi_{yy} + \delta_3\Psi_x + \delta_4(3\Psi_x^2 + \Psi_{xxx}) + \delta_5(3\Phi_{yy}^2 + \Psi_{yyyy}) + \\ &\delta_6(3\Psi_y\Phi_{yy} + \Psi_{yyy}) = 0, \quad \Psi_x = \Phi, \quad 0 < \alpha \leq 1. \end{aligned} \tag{0.6}$$

The below fractional transformation

$$\tau = \frac{t^{\alpha}}{\Gamma(\alpha+1)}, \tag{0.7}$$

will change Eq. (0.6) as follows

$$\begin{aligned} &\Psi_\tau + \Psi_{xxy} + 3\Psi_x\Psi_y + \delta_1\Psi_y + \delta_2\Phi_{yy} + \delta_3\Psi_x + \delta_4(3\Psi_x^2 + \Psi_{xxx}) + \delta_5(3\Phi_{yy}^2 + \Psi_{yyyy}) + \\ &\delta_6(3\Psi_y\Phi_{yy} + \Psi_{yyy}) = 0, \quad \Psi_x = \Phi, \quad 0 < \alpha \leq 1. \end{aligned} \tag{0.8}$$

For $\alpha = 1$, by getting $\delta_3 = \delta_4 = \delta_5 = \delta_6 = 0$, Eq. (0.8) transforms to generalized CBS equation [31, 32]. While the arbitrary constants supposed $\delta_5 = \delta_6 = 0$, then Eq. (0.8) transforms to generalized BK equation [32, 33, 34].

The fractional generalized CBS-BK equation is

$$\begin{aligned} \mathbb{P}_{gCBS-BK}(\Upsilon) := {} &\Psi_\tau + \Psi_{xxy} + 3\Psi_x\Psi_y + \delta_1\Psi_y + \delta_2\Psi_{xyy} + \delta_3\Psi_x + \delta_4(3\Psi_x^2 + \Psi_{xxx}) + \delta_5(3\Psi_{xyy}^2 + \Psi_{yyyy}) + \\ &\delta_6(3\Psi_y\Psi_{xyy} + \Psi_{yyy}) = 0. \end{aligned} \tag{0.9}$$

The Hirota derivatives [35] is considered as

$$\prod_{i=1}^{3} D_{\sigma_i}^{\omega_i} f.\zeta = \prod_{i=1}^{3}\left(\frac{\partial}{\partial\sigma_i} - \frac{\partial}{\partial\sigma_i'}\right)^{\omega_i} f(\sigma)\zeta(\sigma')\Bigg|_{\sigma'=\sigma}, \tag{0.10}$$

where the vectors $\sigma = (\sigma_1, \sigma_2, \sigma_3) = (x, y, t)$, $\sigma' = (\sigma_1', \sigma_2', \sigma_3') = (x', y', t')$ and $\omega_1, \omega_2, \omega_3$ are the free amounts. The bilinear form of the generalized KDKK equation is as:

$$\mathbb{B}_{gCBS-BK}(\mathfrak{f}) := \left(\delta_4 D_x^4 + D_x^3 D_y + \delta_3 D_x^2 + D_x D_\tau + \delta_2 D_y^2 + \delta_1 D_x D_y + \delta_5 D_y^4 + \delta_6 D_y^3 D_x\right)\mathfrak{f}.\mathfrak{f} \tag{0.11}$$

$$= 2\left[\delta_4(\mathfrak{f}\mathfrak{f}_{xxxx} - 4\mathfrak{f}_x\mathfrak{f}_{xxx} + 3\mathfrak{f}_{xx}^2) + (\mathfrak{f}\mathfrak{f}_{xxxy} - \mathfrak{f}_y\mathfrak{f}_{xxx} - 3\mathfrak{f}_x\mathfrak{f}_{xxy} + 3\mathfrak{f}_{xx}\mathfrak{f}_{xy}) + \delta_3(\mathfrak{f}\mathfrak{f}_{xx} - \mathfrak{f}_x^2) + (\mathfrak{f}\mathfrak{f}_{x\tau} - \mathfrak{f}_x\mathfrak{f}_\tau) + \right.$$
$$\left. \delta_2(\mathfrak{f}\mathfrak{f}_{yy} - \mathfrak{f}_y^2) + \delta_1(\mathfrak{f}\mathfrak{f}_{xy} - \mathfrak{f}_x\mathfrak{f}_y) + \delta_5(\mathfrak{f}\mathfrak{f}_{yyyy} - 4\mathfrak{f}_y\mathfrak{f}_{yyy} + 3\mathfrak{f}_{yy}^2) + \delta_6(\mathfrak{f}\mathfrak{f}_{yyyx} - \mathfrak{f}_x\mathfrak{f}_{yyy} - 3\mathfrak{f}_y\mathfrak{f}_{yyx} + 3\mathfrak{f}_{yy}\mathfrak{f}_{yx})\right].$$

Employ the below bilinear frame

$$\Psi = \Psi_0 + 2(\ln \mathfrak{f})_x, \quad \Phi = 2(\ln \mathfrak{f})_{xx}. \tag{0.12}$$

The Bell polynomial will be as

$$\mathbb{P}_{gCBS-BK}(\Psi) = \left[\frac{\mathbb{B}_{TOE}(\mathfrak{f})}{\mathfrak{f}}\right]_x. \tag{0.13}$$

Use the modified Riemann-Liouville derivative of order α [36] as fom

$$D_t^\alpha u(t) = \begin{cases} \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\vartheta)^{-\alpha} (u(\vartheta) - u(0))\, d\vartheta, & if \quad 0 < \alpha \leq 1, \\ \left[u^{(n)}(t)\right]^{(\alpha-n)}, & if \quad n \leq \alpha < n+1, n \geq 1, \end{cases} \tag{0.14}$$

with the below relations [37, 38, 39]

$$\begin{cases} D_J^\alpha\big(f(x)g(x)\big) = \sum_{j=0}^{+\infty} \begin{pmatrix} \alpha \\ j \end{pmatrix} f^{(j)}(x) g_{R-L}^{(\alpha-j)}(x) - \frac{f(0)g(0)}{x^\alpha \Gamma(1-\alpha)}, \\ D_J^\alpha\big(f(g(x))\big) = \sum_{j=0}^{+\infty} \begin{pmatrix} \alpha \\ j \end{pmatrix} \frac{x^{j-\alpha} j!}{\Gamma(j-\alpha+1)} \sum_{m=1}^{j} f^{(m)}(g) \sum \prod_{k=1}^{j} \frac{1}{P_k!} \left(\frac{g^{(k)}}{k!}\right)^{P_k} + \frac{f(g(x)) - f(g(0))}{x^\alpha \Gamma(1-\alpha)}, \\ D_t^\alpha t^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(1+\alpha-\gamma)} t^{\gamma-\alpha}, \ \gamma > 0, \end{cases} \tag{0.15}$$

where Γ denotes the Gamma function. Khader et al [40] investigated the spectral collocation method with help of Chebyshev polynomials to the space fractional Korteweg-de Vries (KdV) and KdV-Burgers equations based on the Caputo-Fabrizio fractional derivative. Authors of [41] studied the joint effect for the presence of two fractional derivative parameters by considering a novel analytical solution scheme for the fractional initial value problems. In [42], the local fractional Poisson equation was considered by employing q-homotopy analysis transform method. The generalized Calogero-Bogoyavlenskii-Schiff equation to extract new complex solutions has been investigated by using the Bernoulli sub-equation function method and Modified exponential function method in Ref. [43]. Odibat and Baleanu used a generalized Caputo-type fractional derivative and also presented an adaptive predictor corrector method for the numerical solution of generalized Caputo-type initial value problem [44].
The outline of this paper organized as follows. In section 2, the multiple Exp-function method (MEFM) are investigated. In section 3, multiple soliton solutions (MSS) with its corresponding method will be obtained and will obtain different solutions and their corresponding three dimensional, density, and two dimensional plots which can illustrate their dynamic structure. In addition, the discussion of the lump and interaction with two stripe soliton solutions have been offered. The semi-inverse variational method is given in section 5. A few of conclusions and outlook will be given in the final section.

MEFM

In this section, we briefly give certain basic knowledge of MEFM which are required in below with steps:
Step 1. Consider the NLPDE

$$\mathcal{N}(x, y, t, \Psi, \Psi_x, \Psi_y, \Psi_t, \Psi_{xx}, \Psi_{tt}, ...) = 0. \tag{0.16}$$

Commence the transformations $\xi_i = \xi_i(x, y, t), 1 \leq i \leq n$, by below form

$$\xi_{i,x} = \alpha_i \xi_i, \quad \xi_{i,y} = \beta_i \xi_i, \quad \xi_{i,t} = \lambda_i \xi_i, \quad 1 \leq i \leq n, \tag{0.17}$$

where $\alpha_i, \beta_i, 1 \leq i \leq n$, and occur the below function solutions,

$$\xi_i = \varpi_i e^{\theta_i}, \quad \theta_i = \alpha_i x + \beta_i y - \lambda_i t, \quad 1 \leq i \leq n, \tag{0.18}$$

w

Step 2. Let the solution of the Eq. (0.16) to be of the below form in term of $\xi_i, 1 \le i \le n$:

$$\Psi = \frac{\Delta(\xi_1, \xi_2, ..., \xi_n)}{\Omega(\xi_1, \xi_2, ..., \xi_n)}, \quad \Delta = \sum_{p,q=1}^{n} \sum_{d,e=0}^{M} \Delta_{pq,de} \xi_p^d \xi_q^e, \quad \Omega = \sum_{p,q=1}^{n} \sum_{d,e=0}^{N} \Omega_{pq,de} \xi_p^d \xi_q^e, \tag{0.19}$$

in which $\Delta_{pq,de}$ and $\Omega_{pq,de}$ are amounts to be settled. Then, we have

$$\Psi(x, y, t) = \frac{\Delta(\varpi_1 e^{\alpha_1 x+\beta_1 y-\lambda_1 t}, ..., \varpi_n e^{\alpha_n x+\beta_n y-\lambda_n t})}{\Omega(\varpi_1 e^{\alpha_1 x+\beta_1 y-\lambda_1 t}, ..., \varpi_n e^{\alpha_n x+\beta_n y-\lambda_n t})}, \tag{0.20}$$

and also we have

$$\Delta_t = \sum_{d=1}^{n} \Delta_{\xi_d} \xi_{d,t}, \quad \Omega_t = \sum_{d=1}^{n} \Omega_{\xi_d} \xi_{d,t}, \quad \Delta_x = \sum_{d=1}^{n} \Delta_{\xi_d} \xi_{d,x}, \quad \Omega_x = \sum_{d=1}^{n} \Omega_{\xi_d} \xi_{d,x}, \quad \Delta_y = \sum_{d=1}^{n} \Delta_{\xi_d} \xi_{d,y}, \quad \Omega_y = \sum_{d=1}^{n} \Omega_{\xi_d} \xi_{d,y}, \tag{0.21}$$

$$\Psi_t = \frac{\Omega \sum_{d=1}^{n} \Delta_{\xi_i} \xi_{d,t} - \Delta \sum_{d=1}^{n} \Omega_{\xi_d} \xi_{d,t}}{\Omega^2}, \quad \Psi_x = \frac{\Omega \sum_{d=1}^{n} \Delta_{\xi_i} \xi_{d,x} - \Delta \sum_{d=1}^{n} \Omega_{\xi_d} \xi_{d,x}}{\Omega^2},$$

$$\Psi_y = \frac{\Omega \sum_{d=1}^{n} \Delta_{\xi_i} \xi_{d,y} - \Delta \sum_{d=1}^{n} \Omega_{\xi_d} \xi_{d,y}}{\Omega^2}.$$

MSS for the fractional gCBS-BK equation

Set I: One-wave solution

Suppose that solution of Eq. (0.5) be as below form

$$\Psi(x, y, t) = \Psi_0 + \frac{2\Delta_1}{\Omega_1}, \quad \Omega_1 = 1 + \rho_1 e^{\alpha_1 x+\beta_1 y-\lambda_1 \frac{t^\alpha}{\Gamma(\alpha+1)}}, \quad \Delta_1 = \rho_1 e^{\alpha_1 x+\beta_1 y-\lambda_1 \frac{t^\alpha}{\Gamma(\alpha+1)}}, \tag{0.22}$$

where σ_1 and σ_2 are unspecified values. Appending (0.22) into Eq. (0.5), one get

$$\rho_1 = \rho_1, \quad \alpha_1 = \Omega\beta_1, \quad \beta_1 = \beta_1, \quad \lambda_1 = \frac{\beta_1 \left(\Omega^2 \delta_3 + \Omega\, \delta_1 + \delta_2\right)}{\Omega}, \quad \rho_1 = \rho_1, \tag{0.23}$$

where Ω solves $\Omega^4 \delta_4 + \Omega^3 + \Omega\, \delta_6 + \delta_5 = 0$, therefore, the final solution will be as

$$\Psi(x, y, t) = \Psi_0 + \frac{2\rho_1 e^{\Omega\beta_1 x+\beta_1 y-\frac{\beta_1\left(\Omega^2\delta_3+\Omega\,\delta_1+\delta_2\right)}{\Omega}\frac{t^\alpha}{\Gamma(\alpha+1)}}}{1 + \rho_1 e^{\Omega\beta_1 x+\beta_1 y-\frac{\beta_1(\Omega^2\delta_3+\Omega\,\delta_1+\delta_2)}{\Omega}\frac{t^\alpha}{\Gamma(\alpha+1)}}}. \tag{0.24}$$

Set II: 2-wave solutions

Suppose that solution of Eq. (0.5) be as below form

$$\Psi(x, y, t) = \Psi_0 + \frac{2\Delta_2}{\Omega_2}, \tag{0.25}$$

$$\Omega_2 = 1 + \rho_1 e^{\alpha_1 x+\beta_1 y-\lambda_1 \frac{t^\alpha}{\Gamma(\alpha+1)}} + \rho_2 e^{\alpha_2 x+\beta_2 y-\lambda_2 \frac{t^\alpha}{\Gamma(\alpha+1)}} + \rho_1\rho_2\rho_{12} e^{(\alpha_1+\alpha_2)x+(\beta_1+\beta_2)y-(\lambda_1+\lambda_2)\frac{t^\alpha}{\Gamma(\alpha+1)}}, \tag{0.26}$$

$$\Delta_2 = \alpha_1\rho_1 e^{\alpha_1 x+\beta_1 y-\lambda_1 \frac{t^\alpha}{\Gamma(\alpha+1)}} + \alpha_2\rho_2 e^{\alpha_2 x+\beta_2 y-\lambda_2 \frac{t^\alpha}{\Gamma(\alpha+1)}} + (\alpha_1+\alpha_2)\rho_1\rho_2\rho_{12} e^{(\alpha_1+\alpha_2)x+(\beta_1+\beta_2)y-(\lambda_1+\lambda_2)\frac{t^\alpha}{\Gamma(\alpha+1)}}.$$

Plugging (0.25) along with (0.26) into Eq. (0.5), become

$$\rho_1 = \rho_1, \quad \rho_2 = \rho_2, \quad \rho_{12} = \rho_{12}, \quad \alpha_1 = \alpha_1, \quad \alpha_2 = \Omega\,\beta_2, \quad \beta_1 = \frac{\alpha_1}{\Omega}, \quad \beta_2 = \beta_2, \tag{0.27}$$

$$\lambda_1 = \frac{\alpha_1 \left(\Omega^2 \delta_3 + \Omega\, \delta_1 + \delta_2\right)}{\Omega^2}, \quad \lambda_2 = \frac{\beta_2 \left(\Omega^2 \delta_3 + \Omega\, \delta_1 + \delta_2\right)}{\Omega},$$

where Ω solves $\Omega^4 \delta_4 + \Omega^3 + \Omega\, \delta_6 + \delta_5 = 0$, then, the final solution will be as

$$\Psi_2(x, y, t) = \Psi_0 + 2 \left(\alpha_1\rho_1 e^{\alpha_1 x+\frac{\alpha_1}{\Omega} y-\frac{\alpha_1\left(\Omega^2\delta_3+\Omega\,\delta_1+\delta_2\right)}{\Omega^2}\frac{t^\alpha}{\Gamma(\alpha+1)}} + \Omega\,\beta_2\rho_2 e^{\Omega\,\beta_2 x+\beta_2 y-\lambda_2\frac{t^\alpha}{\Gamma(\alpha+1)}} + \right. \tag{0.28}$$

$$(\alpha_1+\Omega\,\beta_2)\rho_1\rho_2\rho_{12}e^{(\alpha_1+\Omega\,\beta_2)x+(\frac{\alpha_1}{\Omega}+\beta_2)y-\frac{\left(\Omega^2\delta_3+\Omega\,\delta_1+\delta_2\right)(\Omega\,\beta_2+\alpha_1)}{\Omega^2}\frac{t^\alpha}{\Gamma(\alpha+1)}}\Bigg)\Bigg/\Bigg(1+\rho_1e^{\alpha_1x+\frac{\alpha_1}{\Omega}y-\frac{\alpha_1\left(\Omega^2\delta_3+\Omega\,\delta_1+\delta_2\right)}{\Omega^2}\frac{t^\alpha}{\Gamma(\alpha+1)}}+$$

$$\rho_2e^{\Omega\,\beta_2x+\beta_2y-\frac{\beta_2\left(\Omega^2\delta_3+\Omega\,\delta_1+\delta_2\right)}{\Omega}\frac{t^\alpha}{\Gamma(\alpha+1)}}+\rho_1\rho_2\rho_{12}e^{(\alpha_1+\Omega\,\beta_2)x+(\frac{\alpha_1}{\Omega}+\beta_2)y-\frac{\left(\Omega^2\delta_3+\Omega\,\delta_1+\delta_2\right)(\Omega\,\beta_2+\alpha_1)}{\Omega^2}\frac{t^\alpha}{\Gamma(\alpha+1)}}\Bigg).$$

Lump and two stripe solitons

Here, we will further investigate the interaction of lump and 2-kink solutions. First of all, consider the below relations

$$\mathfrak{f}(x,y,\tau)=\left(\sum_{i=1}^{4}a_ix_i\right)^2+\left(\sum_{i=5}^{8}a_ix_i\right)^2+\exp\left(\sum_{i=9}^{12}a_ix_i\right)+\exp\left(\sum_{i=13}^{16}a_ix_i\right)+a_{17},\qquad \tau=\frac{t^\alpha}{\Gamma(\alpha+1)},\tag{0.29}$$

$$\frac{d\mathfrak{f}}{dx}(x,y,\tau)=2a_1\left(\sum_{i=1}^{4}a_ix_i\right)+2a_5\left(\sum_{i=5}^{8}a_ix_i\right)+a_9\exp\left(\sum_{i=9}^{12}a_ix_i\right)+a_{13}\exp\left(\sum_{i=13}^{16}a_ix_i\right),$$

$$(x_1,x_2,x_3,x_4)=(x_5,x_6,x_7,x_8)=(x_9,x_{10},x_{11},x_{12})=(x_{13},x_{14},x_{15},x_{16})=(x,y,\tau,1),$$

where $a_i, i=1,...,13$ are unspecified values. Putting (0.29) into Eq. (0.9), get the below options:

Option I:

$$a_1=a_1,\ \ a_2=\frac{a_1a_{14}}{a_{13}},\ \ a_3=-\frac{{a_1}^2{a_{13}}^2\delta_3+{a_1}^2a_{13}a_{14}\delta_1+{a_1}^2{a_{14}}^2\delta_2-{a_6}^2{a_{13}}^2\delta_2}{a_1{a_{13}}^2},\ \ a_4=a_4,\ \ a_5=0,\ \ a_6=a_6,\tag{0.30}$$

$$a_7=-\frac{a_6\left(a_{13}\delta_1+2\,a_{14}\delta_2\right)}{a_{13}},\ \ a_8=a_8,\ \ a_9=a_{13},\ \ a_{10}=a_{14},\ \ a_{11}=-\frac{3\,{a_1}^2{a_{13}}^2\delta_3+3\,{a_1}^2a_{13}a_{14}\delta_1+3\,{a_1}^2{a_{14}}^2\delta_2-{a_6}^2{a_{13}}^2\delta_2}{3{a_1}^2a_{13}},$$

$$a_{12}=a_{12},\ \ \ a_{13}=a_{13},\ \ \ a_{14}=a_{14},\ \ \ a_{15}=-\frac{3\,{a_1}^2{a_{13}}^2\delta_3+3\,{a_1}^2a_{13}a_{14}\delta_1+3\,{a_1}^2{a_{14}}^2\delta_2-{a_6}^2{a_{13}}^2\delta_2}{3{a_1}^2a_{13}},\ \ \ a_{16}=a_{16},$$

$$a_{17}=\frac{-3\,{a_6}^2{a_{13}}^5+2\,{a_1}^2{a_{14}}^3\delta_2}{2{a_{13}}^2{a_{14}}^3\delta_2},\ \ \ \delta_4=-\frac{1}{6}\frac{3\,{a_1}^2a_{13}a_{14}+2\,{a_6}^2\delta_2}{{a_{13}}^2{a_1}^2},\ \ \ \delta_5=\frac{1}{2}\frac{{a_{13}}^3}{{a_{14}}^3},\ \ \ \delta_6=-\frac{{a_{13}}^2}{{a_{14}}^2},$$

by utilizing the function (0.30), the exact solution will be as

$$\Psi_1=\Psi_0+\frac{2d\mathfrak{f}_1(x,y,\tau)/dx}{\mathfrak{f}_1(x,y,\tau)}.\tag{0.31}$$

Option II:

$$a_1=a_1,\ \ \ a_2=\frac{a_5a_{14}\left(\Omega+2\right)}{a_{13}},\ \ \ a_3=-\frac{\Omega\,{a_1}^2a_{14}\delta_1+\Omega\,{a_5}^2a_{14}\delta_1+2\,{a_1}^2a_{14}\delta_1+2\,{a_5}^2a_{14}\delta_1-a_1a_7a_{13}}{a_{13}a_5},\tag{0.32}$$

$$a_4=a_4,\ \ \ a_5=a_5, a_6=-\frac{a_1a_{14}(\Omega+2)}{a_{13}},\ \ \ a_7=a_7,\ \ \ a_8=a_8,\ \ \ a_9=\Omega\,a_{13},\ \ \ a_{10}=a_{14},$$

$$a_{11}=\frac{-15\Omega a_5{a_{13}}^2a_{14}+3\Omega a_1a_{14}\delta_1-10a_5{a_{13}}^2a_{14}+3\Omega a_7a_{13}+3a_1a_{14}\delta_1-3a_5a_{14}\delta_1}{3a_5},\ \ a_{12}=a_{12},\ \ a_{13}=a_{13},\ \ a_{14}=a_{14},$$

$$a_{15}=-\frac{-15\,\Omega\,a_5{a_{13}}^2a_{14}+3\,\Omega\,a_1a_{14}\delta_1-10\,a_5{a_{13}}^2a_{14}+6\,a_1a_{14}\delta_1+3\,a_5a_{14}\delta_1-3\,a_7a_{13}}{3a_5},\ \ \ a_{16}=a_{16},$$

$$a_{17}=\frac{\left({a_1}^2+{a_5}^2\right)\left(\Omega+3\right)}{{a_{13}}^2},\ \ \delta_2=\frac{5}{2}\frac{{a_{13}}^3\Omega}{a_{14}\left(\Omega+2\right)},\ \ \delta_3=\frac{-5\,\Omega\,a_5{a_{13}}^2a_{14}+2\,\Omega\,a_1a_{14}\delta_1-5\,a_5{a_{13}}^2a_{14}+4\,a_1a_{14}\delta_1-2\,a_7a_{13}}{2a_{13}a_5},$$

$$\delta_4=-\frac{a_{14}}{6a_{13}\left(\Omega+1\right)},\ \ \ \delta_5=-\frac{{a_{13}}^3\Omega}{\left(\Omega+2\right){a_{14}}^3},\ \ \ \delta_6=-\frac{2}{3}\frac{{a_{13}}^2\left(\Omega+1\right)}{\left(\Omega+2\right){a_{14}}^2},\ \ \ \Omega=-\frac{3}{2}\pm\frac{\sqrt{5}}{2},$$

by utilizing the function (0.30), the exact solution will be as

$$\Psi_2=\Psi_0+\frac{2d\mathfrak{f}_2(x,y,\tau)/dx}{\mathfrak{f}_2(x,y,\tau)}.\tag{0.33}$$

$$a_1 = a_1,\ \ a_2 = \frac{1}{2}\frac{a_1 {a_{10}}^2\delta_6}{\delta_2},\ \ a_3 = -\frac{1}{4}\frac{a_1\left({a_{10}}^4{\delta_6}^2 + 2\,{a_{10}}^2\delta_1\delta_6 + 4\,\delta_2\delta_3\right)}{\delta_2},\ \ a_4 = a_4,\ \ a_5 = a_5,\ \ a_6 = \frac{1}{2}\frac{\delta_6 {a_{10}}^2 a_5}{\delta_2},\ (0.34)$$

$$a_7 = -\frac{1}{4}\frac{a_5\left({a_{10}}^4{\delta_6}^2 + 2\,{a_{10}}^2\delta_1\delta_6 + 4\,\delta_2\delta_3\right)}{\delta_2},\ \ a_8 = a_8,\ \ a_9 = 0,\ \ a_{10} = \frac{\sqrt{-\delta_5\delta_2}}{\delta_5},\ \ a_{11} = -a_{10}\delta_1,\ \ a_{12} = a_{12},\ \ a_{13} = 0,$$

$$a_{14} = a_{10},\ \ a_{15} = -a_{10}\delta_1,\ \ a_{16} = a_{16},\ \ a_{17} = a_{17},\ \ \delta_4 = \frac{1}{16}\frac{\delta_6\left({\delta_6}^3 + 8\,{\delta_5}^2\right)}{{\delta_5}^3},$$

by utilizing the function (0.30), the exact solution will be as

$$\Psi_3 = \Psi_0 + \frac{2d\mathfrak{f}_3(x,y,\tau)/dx}{\mathfrak{f}_3(x,y,\tau)}. \tag{0.35}$$

Option IV:

$$a_1 = a_1,\ \ a_2 = \frac{a_1 a_{10}}{a_9},\ \ a_3 = \frac{1}{2}\frac{a_1\left({a_9}^2{a_{14}}^2 - 2\,a_9 a_{10}\delta_3 - 2\,{a_{10}}^2\delta_1\right)}{a_9 a_{10}},\ \ a_4 = a_4,\ \ a_5 = a_5,\ \ a_6 = \frac{a_{10}a_5}{a_9}, \tag{0.36}$$

$$a_7 = \frac{1}{2}\frac{a_5\left({a_9}^2{a_{14}}^2 - 2\,a_9 a_{10}\delta_3 - 2\,{a_{10}}^2\delta_1\right)}{a_9 a_{10}},\ \ a_8 = a_8,\ \ a_9 = a_9,\ \ a_{10} = a_{10},\ \ a_{11} = \frac{1}{2}\frac{{a_9}^2{a_{14}}^2 - 2\,a_9 a_{10}\delta_3 - 2\,{a_{10}}^2\delta_1}{a_{10}},$$

$$a_{12} = a_{12},\ \ a_{13} = 0,\ \ a_{14} = 4\,\sqrt{\delta_2\delta_4}\,\delta_4,\ \ a_{15} = -a_{14}\delta_1,\ \ a_{16} = a_{16},\ \ a_{17} = a_{17},\ \ \delta_5 = -\frac{1}{16}{\delta_4}^{-3},\ \ \delta_6 = -\frac{1}{4}{\delta_4}^{-2},$$

by utilizing the function (0.30), the exact solution will be as

$$\Psi_4 = \Psi_0 + \frac{2d\mathfrak{f}_4(x,y,\tau)/dx}{\mathfrak{f}_4(x,y,\tau)}. \tag{0.37}$$

Option V:

$$a_1 = a_1,\ a_2 = -\frac{1}{2}\frac{a_1\delta_6}{\delta_5},\ a_3 = 0,\ a_4 = a_4,\ a_5 = a_5,\ a_6 = -\frac{1}{2}\frac{a_5\delta_6}{\delta_5},\ a_7 = 0,\ a_8 = a_8,\ a_9 = -\frac{2\delta_5\left(a_{10} - a_{14}\right)}{\delta_6}, \tag{0.38}$$

$$a_{10} = a_{10},\ a_{11} = -a_{14}\delta_1,\ a_{12} = a_{12},\ a_{13} = 0,\ a_{14} = \frac{\sqrt{\delta_6\left(-\delta_1 + \sqrt{{\delta_1}^2 - 4\,\delta_2\delta_3}\right)}}{\delta_6},\ a_{15} = -a_{14}\delta_1,\ a_{16} = a_{16},\ a_{17} = a_{17},$$

by employing the function (0.30), the exact solution will be as

$$\Psi_5 = \Psi_0 + \frac{2d\mathfrak{f}_5(x,y,\tau)/dx}{\mathfrak{f}_5(x,y,\tau)}. \tag{0.39}$$

Option VI:

$$a_1 = a_1,\ a_2 = \frac{\left(a_{10} - a_{14}\right)a_1}{a_9},\ a_3 = \frac{1}{2}\frac{a_1\left({a_9}^2{a_{14}}^2 - 2\,a_9 a_{10}\delta_3 + 2\,a_9 a_{14}\delta_3 - 2\,{a_{10}}^2\delta_1 + 4\,a_{10}a_{14}\delta_1 - 2\,{a_{14}}^2\delta_1\right)}{a_9\left(a_{10} - a_{14}\right)}, \tag{0.40}$$

$$a_4 = a_4,\ a_5 = a_5,\ a_6 = \frac{\left(a_{10} - a_{14}\right)a_5}{a_9},\ a_7 = \frac{1}{2}\frac{a_5\left({a_9}^2{a_{14}}^2 - 2\,a_9 a_{10}\delta_3 + 2\,a_9 a_{14}\delta_3 - 2\,{a_{10}}^2\delta_1 + 4\,a_{10}a_{14}\delta_1 - 2\,{a_{14}}^2\delta_1\right)}{a_9\left(a_{10} - a_{14}\right)},$$

$$a_8 = a_8,\ a_9 = a_9, a_{10} = 2\,\delta_4\left(2\,\sqrt{\delta_2\delta_4} - a_9\right),\ a_{11} = \frac{1}{2}\frac{{a_9}^2{a_{14}}^2 - 2\,a_9 a_{10}\delta_3 + 2\,a_9 a_{14}\delta_3 - 2\,{a_{10}}^2\delta_1 + 2\,a_{10}a_{14}\delta_1}{a_{10} - a_{14}},$$

$$a_{12} = a_{12},\ \ a_{13} = 0,\ a_{14} = 4\,\sqrt{\delta_2\delta_4}\,\delta_4,\ a_{15} = -a_{14}\delta_1,\ a_{16} = a_{16},\ a_{17} = a_{17},\ \ \delta_5 = -\frac{1}{16}{\delta_4}^{-3},\ \ \delta_6 = -\frac{1}{4}{\delta_4}^{-2},$$

by utilizing the function (0.30), the exact solution will be as

$$\Psi_6 = \Psi_0 + \frac{2d\mathfrak{f}_6(x,y,\tau)/dx}{\mathfrak{f}_6(x,y,\tau)}. \tag{0.41}$$

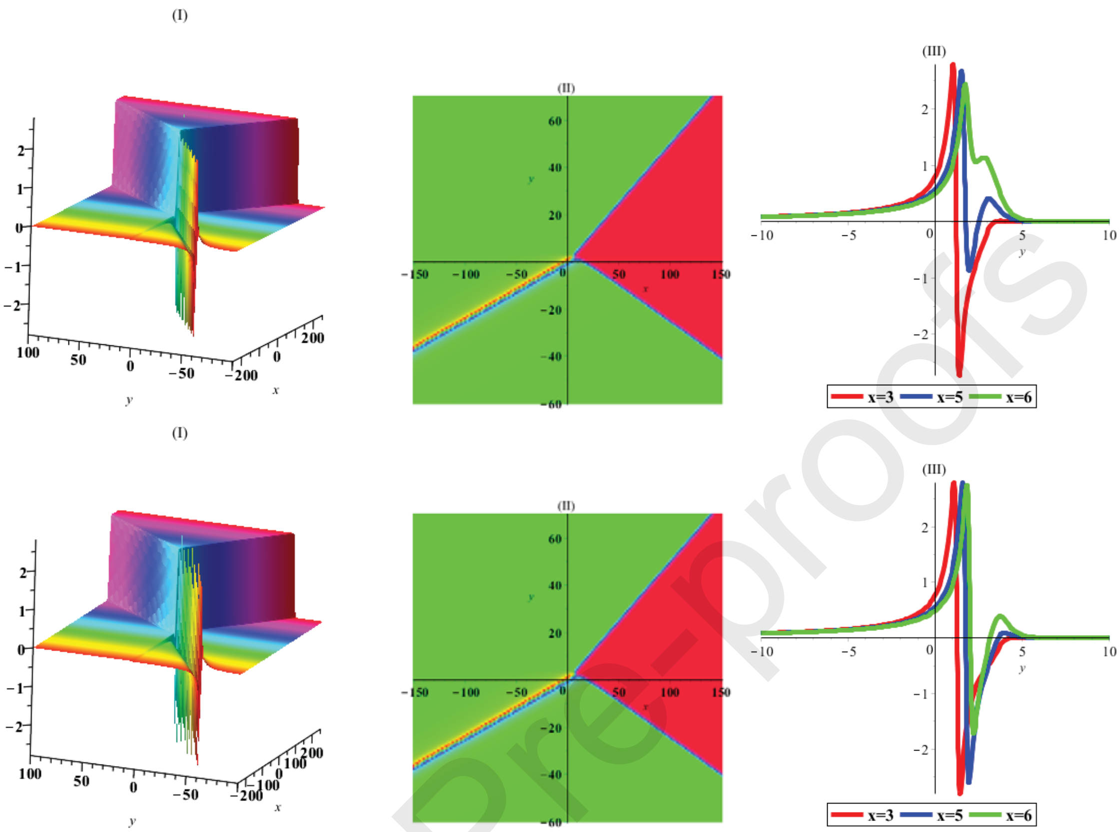

Figure 1: The graph of lump with two stripe solitons (0.41) with FO $\alpha = 0.5$ for the first line and $\alpha = 0.95$ for the second line.

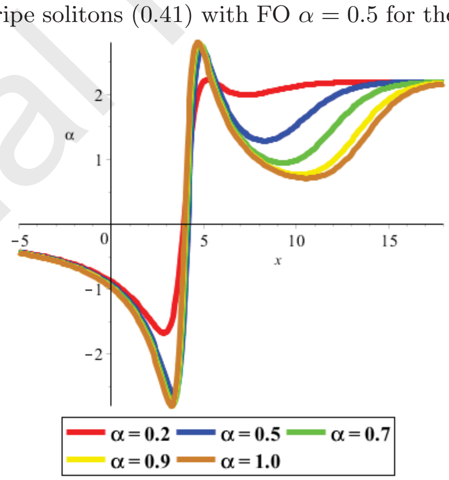

Figure 2: The 2D graph of lump with two stripe solitons (0.41) with the different fractional orders.

M s. 1 including three dimensional, density plot, and y-curves plots. By taking the novel parameters such as $a_1 = 1.2, a_4 = 2, a_5 = 1.5, a_9 = 1.1, a_8 = 1, a_{10} = 1.2, a_{12} = 1.5, a_{13} = 1.3, \delta_1 = 1.5, \delta_2 = 2, \delta_3 = 1.6, \delta_4 = 1, \delta_6 = 1.5, a_{16} = 2, a_{17} = 1, \Psi_0 = 0, t = 2$, the acquired lump with two stripe solutions in Option VI are shown with two diverse fractional orders. Also, 2D graph of the lump soliton by selecting amounts of the various fractional order (FO) α is designed in Fig. 2.

Option VII:

$$a_1 = a_1,\ a_2 = -2\,a_1\delta_4,\ a_3 = -\frac{a_1\left(16\,{\delta_2}^2{\delta_4}^2 - 8\,\delta_1\delta_2\delta_4 + 4\,\delta_2\delta_3\right)}{\delta_2},\ \ a_4 = a_4,\ a_5 = a_5,\ \ a_6 = -2\,\delta_4 a_5, \tag{0.42}$$

$$a_7 = -\frac{1}{4}\frac{a_5\left(16\,{\delta_2}^2{\delta_4}^2 - 8\,\delta_1\delta_2\delta_4 + 4\,\delta_2\delta_3\right)}{\delta_2},\ \ a_8 = a_8,\ a_9 = a_9,\ a_{10} = 0,\ a_{11} = -\frac{1}{2}\frac{\sqrt{\delta_2\delta_4}\left(16\,{\delta_2}^2{\delta_4}^2 + 4\,\delta_2\delta_3\right)}{\delta_2},$$

$$a_{12} = a_{12},\ \ a_{13} = 0,\ a_{14} = 4\,\frac{(\delta_2\delta_4)^{3/2}}{\delta_2},\ \ a_{15} = -4\,\frac{(\delta_2\delta_4)^{3/2}\,\delta_1}{\delta_2},\ \ a_{16} = a_{16},\ a_{17} = a_{17},\ \ \delta_5 = -\frac{1}{16}{\delta_4}^{-3},\ \ \delta_6 = -\frac{1}{4}{\delta_4}^{-2},$$

by utilizing the function (0.30), the final solution will be as

$$\Psi_7 = \Psi_0 + \frac{2d\mathfrak{f}_7(x, y, \tau)/dx}{\mathfrak{f}_7(x, y, \tau)}. \tag{0.43}$$

Moreover, by choosing suitable amounts, the dynamical structures of periodic wave solutions are presented in Figs. 3 including three dimensional, density, and y-curves plots. By taking the novel parameters containing $a_1 = 1.2, a_4 = 2, a_5 = 1.5, a_9 = 1.1, a_8 = 1, a_{10} = 1.2, a_{12} = 1.5, a_{13} = 1.3, \delta_1 = 1.5, \delta_2 = 2, \delta_3 = 1.6, \delta_4 = 1, \delta_6 = 1.5, a_{16} = 2, a_{17} = 1, \Psi_0 = 0, t = 2$, the acquired lump with two stripe solutions in Option VII are shown with two different FOs. Also, 2D plot of the lump soliton by selecting amounts of the various FO α is designed in Fig. 4.

Option VIII:

$$a_1 = a_1,\ a_2 = -2\,a_1\delta_4,\ a_3 = -\frac{4a_1\left(16\,{\delta_2}^2{\delta_4}^2 - 8\,\delta_1\delta_2\delta_4 + 4\,\delta_2\delta_3\right)}{\delta_2},\ \ a_4 = a_4,\ a_5 = a_5,\ \ a_6 = -2\,\delta_4 a_5, \tag{0.44}$$

$$a_7 = -\frac{1}{4}\frac{a_5\left(16\,{\delta_2}^2{\delta_4}^2 - 8\,\delta_1\delta_2\delta_4 + 4\,\delta_2\delta_3\right)}{\delta_2},\ \ a_8 = a_8,\ a_9 = a_9,\ a_{10} = 2\,\frac{{a_9}^3}{\delta_2},\ a_{11} = -\frac{\sqrt{\delta_2\delta_4}\left(4\,{\delta_2}^2{\delta_4}^2 + 2\,\delta_1\delta_2\delta_4 + \delta_2\delta_3\right)}{\delta_2},$$

$$a_{12} = a_{12},\ \ a_{13} = 0,\ a_{14} = 4\,\frac{(\delta_2\delta_4)^{3/2}}{\delta_2},\ \ a_{15} = -4\,\frac{(\delta_2\delta_4)^{3/2}\,\delta_1}{\delta_2},\ \ a_{16} = a_{16},\ a_{17} = a_{17},\ \ \delta_5 = -\frac{1}{16}{\delta_4}^{-3},\ \ \delta_6 = -\frac{1}{4}{\delta_4}^{-2},$$

by utilizing the function (0.30), the final solution will be as

$$\Psi_8 = \Psi_0 + \frac{2d\mathfrak{f}_8(x, y, \tau)/dx}{\mathfrak{f}_8(x, y, \tau)}. \tag{0.45}$$

Moreover, by choosing suitable amounts, the dynamical structures of periodic wave solutions are presented in Figs. 5 including three dimensional, density, and y-curves plots. By taking the novel parameters including $a_1 = 1.2, a_4 = 2, a_5 = 1.5, a_9 = 1.1, a_8 = 1, a_{10} = 1.2, a_{12} = 1.5, a_{13} = 1.3, \delta_1 = 1.5, \delta_2 = 2, \delta_3 = 1.6, \delta_4 = 1, \delta_6 = 1.5, a_{16} = 2, a_{17} = 1, \Psi_0 = 0, t = 2$, the acquired lump with two stripe solutions in Option VIII are offered with two various FOs. Also, 2D graph of the lump stripe soliton by selecting amounts of the different fractional order α is designed in Fig. 6.

Option IX:

$$a_1 = a_2\Omega,\ a_2 = a_2,\ a_3 = -\frac{{a_1}^2\delta_3 + a_1a_2\delta_1 + {a_2}^2\delta_2}{a_1},\ \ a_4 = a_4,\ a_5 = a_5,\ \ a_6 = \frac{a_2a_5}{a_1}, \tag{0.46}$$

$$a_7 = -\frac{a_5\left({a_1}^2\delta_3 + a_1a_2\delta_1 + {a_2}^2\delta_2\right)}{{a_1}^2},\ \ a_8 = a_8,\ a_9 = a_9,\ a_{10} = \frac{a_2a_9}{a_1},\ a_{11} = -\frac{a_9\left({a_1}^2\delta_3 + a_1a_2\delta_1 + {a_2}^2\delta_2\right)}{{a_1}^2},$$

$$a_{12} = a_{12},\ \ a_{13} = a_{13},\ a_{14} = \frac{a_2a_{13}}{a_1},\ \ a_{15} = -\frac{a_{13}\left({a_1}^2\delta_3 + a_1a_2\delta_1 + {a_2}^2\delta_2\right)}{{a_1}^2},\ \ a_{16} = a_{16},\ a_{17} = a_{17},$$

in which Ω, solves $\delta_4\Omega^4 + \Omega^3 + \delta_6\Omega + \delta_5 = 0$, and by utilizing the function (0.30), the exact solution will be as

$$\Psi_9 = \Psi_0 + \frac{2d\mathfrak{f}_9(x, y, \tau)/dx}{\mathfrak{f}_9(x, y, \tau)}. \tag{0.47}$$

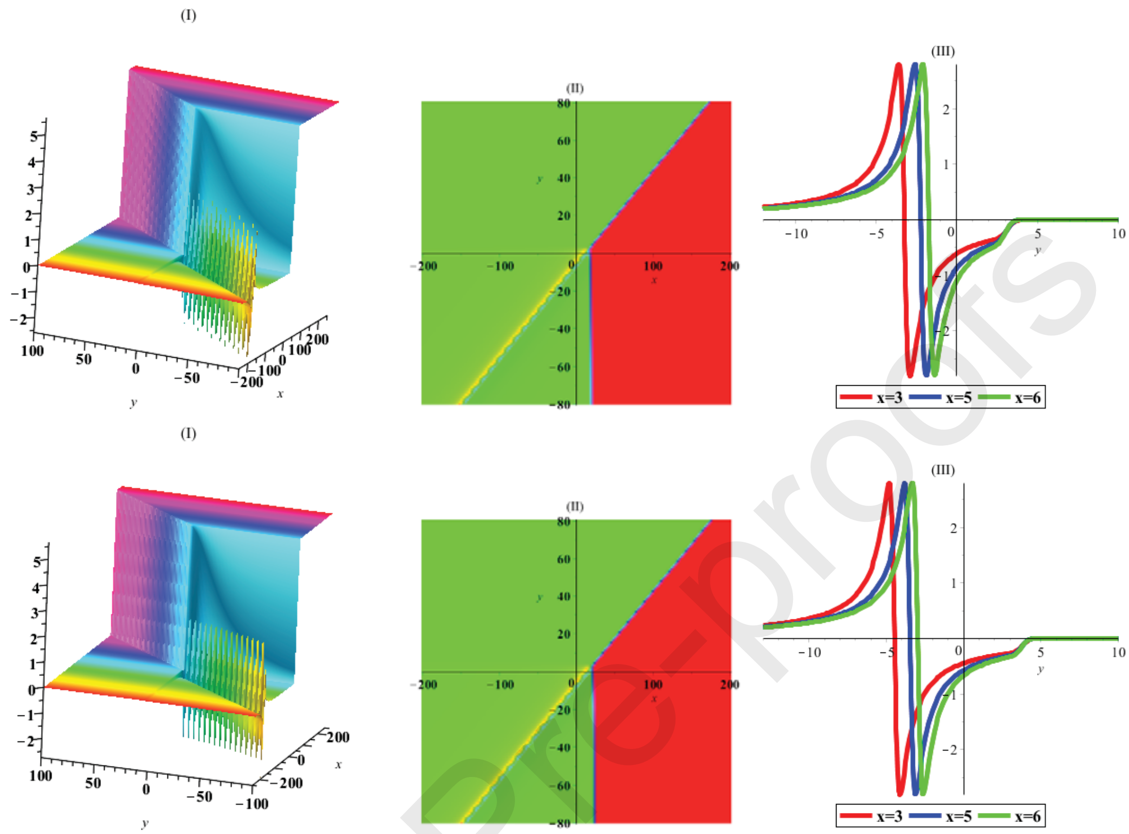

Figure 3: The graph of lump with two stripe solitons (0.43) with FO $\alpha = 0.5$ for the first line and $\alpha = 0.95$ for the second line.

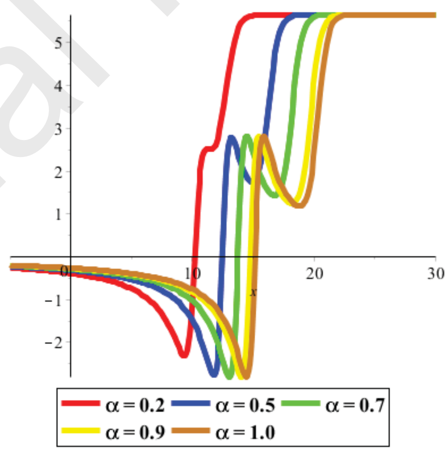

Figure 4: The 2D graph of lump with two stripe solitons (0.43) with the various fractional orders.

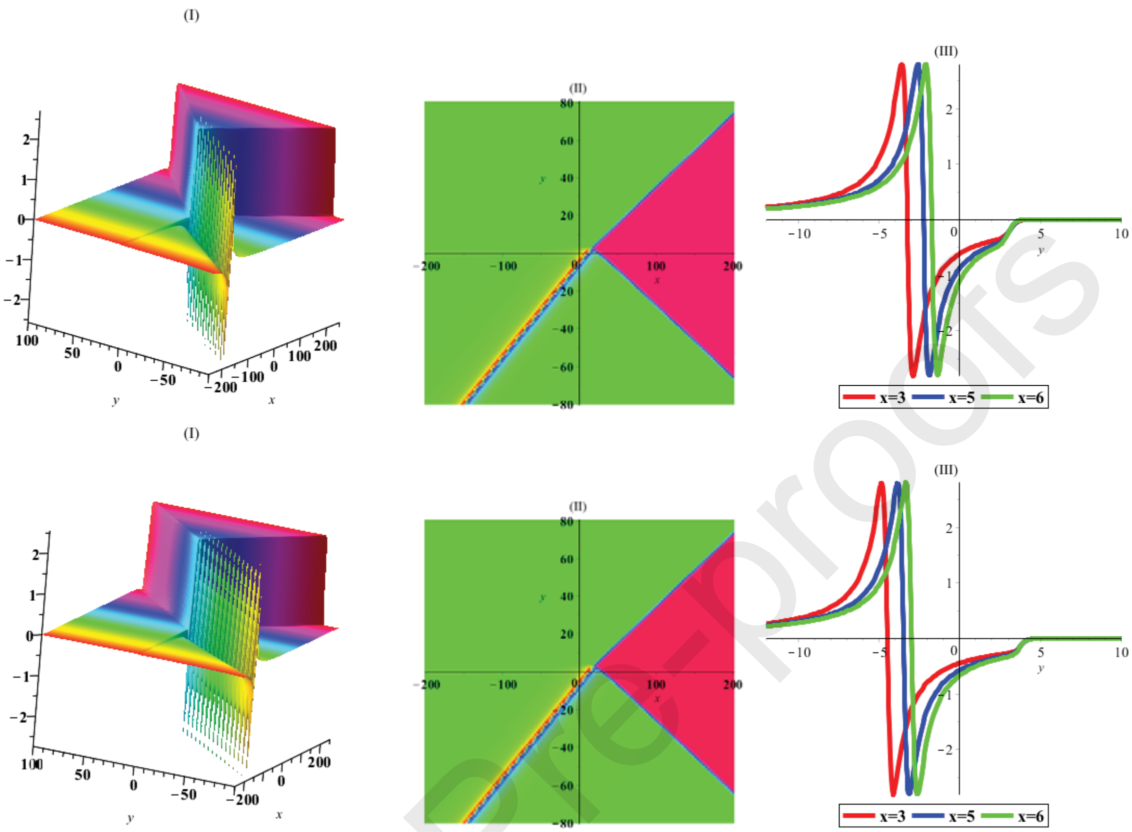

Figure 5: The graph of lump with two stripe solitons (0.45) with FO $\alpha = 0.5$ for the first line and $\alpha = 0.95$ for the second line.

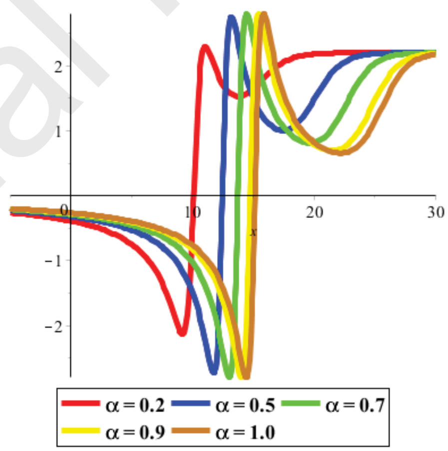

Figure 6: The 2D graph of lump with two stripe solitons (0.45) with the different fractional orders.

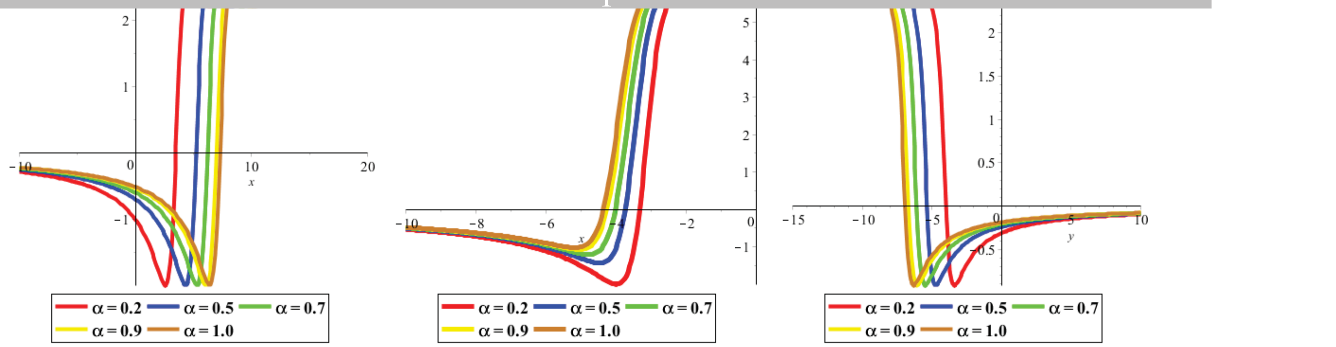

Figure 7: The 2D graph of lump with two stripe solitons (0.47) (Left), (0.49) (Center), and (0.51) (Right) with the different fractional orders.

Via choosing suitable amounts, including the novel parameters $a_1 = 1.2, a_4 = 2, a_5 = 1.5, a_9 = 1.1, a_8 = 1, a_{10} = 1.2, a_{12} = 1.5, a_{13} = 1.3, \delta_1 = 1.5, \delta_2 = 2, \delta_3 = 1.6, \delta_4 = 1, \delta_5 = 1, \delta_6 = 1, a_{16} = 2, a_{17} = 1, \Psi_0 = 0, y = 1.5, t = 2$, the 2D graph of the lump stripe soliton by selecting amounts of the various FO α is designed in Fig. 7 (Left).

Option X:

$$a_1 = a_1,\ a_2 = a_2,\ a_3 = \frac{1}{2}\frac{{a_1}^2{a_{10}}^2 - 2\,a_1a_2a_9a_{10} + {a_2}^2{a_9}^2 - 2\,a_1a_2\delta_3 - 2\,{a_2}^2\delta_1}{a_2},\ a_4 = a_4,\ a_5 = a_5,\ a_6 = \frac{a_2a_5}{a_1},\tag{0.48}$$

$$a_7 = \frac{1}{2}\frac{a_5\left({a_1}^2{a_{10}}^2 - 2\,a_1a_2a_9a_{10} + {a_2}^2{a_9}^2 - 2\,a_1a_2\delta_3 - 2\,{a_2}^2\delta_1\right)}{a_1a_2},\quad a_8 = a_8,\ a_9 = \frac{1}{2}\frac{-a_{10} + 4\sqrt{\delta_2\delta_4}\delta_4}{\delta_4},\ a_{10} = a_{10},$$

$$a_{11} = \frac{1}{2}\frac{{a_1}^2a_9{a_{10}}^2 - 2\,a_1a_2{a_9}^2a_{10} + {a_2}^2{a_9}^3 - 2\,a_1a_2a_9\delta_3 - 2\,a_1a_2a_{10}\delta_1}{a_1a_2},\quad a_{12} = a_{12},\quad a_{13} = a_{13},\ a_{14} = \frac{a_2a_{13}}{a_1},$$

$$a_{15} = 1/2\,\frac{a_{13}\left({a_1}^2{a_{10}}^2 - 2\,a_1a_2a_9a_{10} + {a_2}^2{a_9}^2 - 2\,a_1a_2\delta_3 - 2\,{a_2}^2\delta_1\right)}{a_1a_2},\ a_{16} = a_{16},\ a_{17} = a_{17},\ \delta_5 = -\frac{1}{16}{\delta_4}^{-3},\ \delta_6 = -\frac{1}{4}{\delta_4}^{-2},$$

in which Ω, solves $\delta_4\Omega^4 + \Omega^3 + \delta_6\Omega + \delta_5 = 0$, and by employing the function (0.30), the final solution will be as

$$\Psi_{10} = \Psi_0 + \frac{2d\mathfrak{f}_{10}(x, y, \tau)/dx}{\mathfrak{f}_{10}(x, y, \tau)}.\tag{0.49}$$

Furthermore, by choosing suitable amounts, including the novel parameters $a_1 = 1.2, a_2 = 2, a_4 = 2, a_5 = 1.2, a_8 = 1.5, a_{10} = 1.3, a_{12} = 1.1, a_{13} = 1.3, \delta_1 = 1.5, \delta_2 = 2, \delta_3 = 1.6, \delta_4 = 1, a_{16} = 2, a_{17} = 1, \Psi_0 = 0, y = 1.5, t = 2$, the 2D graph of the lump stripe soliton by selecting amounts of various FO α is designed in Fig. 7 (Center).

Option XI:

$$a_1 = -\frac{1}{2}\frac{a_2\delta_2}{{a_9}^2},\ a_2 = a_2,\ a_3 = \frac{1}{2}\frac{\left(4\,{a_9}^4 - 2\,{a_9}^2\delta_1 + \delta_2\delta_3\right)a_2}{{a_9}^2},\quad a_4 = a_4,\ a_5 = a_5,\quad a_6 = -2\,\frac{{a_9}^2a_5}{\delta_2},\tag{0.50}$$

$$a_7 = -\frac{a_5\left(4\,{a_9}^4 - 2\,{a_9}^2\delta_1 + \delta_2\delta_3\right)}{\delta_2},\quad a_8 = a_8,\ a_9 = \sqrt{\delta_2\delta},\ a_{10} = 2\,\frac{{a_9}^3}{\delta_2},$$

$$a_{11} = \frac{1}{2}\frac{{a_1}^2a_9{a_{10}}^2 - 2\,a_1a_2{a_9}^2a_{10} + {a_2}^2{a_9}^3 - 2\,a_1a_2a_9\delta_3 - 2\,a_1a_2a_{10}\delta_1}{a_1a_2},\quad a_{12} = a_{12},\quad a_{13} = a_{13},\ a_{14} = -2\,\frac{{a_9}^2a_{13}}{\delta_2},$$

$$a_{15} = -\frac{a_{13}\left(4\,{a_9}^4 - 2\,{a_9}^2\delta_1 + \delta_2\delta_3\right)}{\delta_2},\quad a_{16} = a_{16},\ a_{17} = a_{17},\quad \delta_5 = -\frac{1}{16}{\delta_4}^{-3},\quad \delta_6 = -\frac{1}{4}{\delta_4}^{-2},$$

in which Ω, solves $\delta_4\Omega^4 + \Omega^3 + \delta_6\Omega + \delta_5 = 0$, and by utilizing the function (0.30), the exact solution will be as

$$\Psi_{11} = \Psi_0 + \frac{2d\mathfrak{f}_{11}(x, y, \tau)/dx}{\mathfrak{f}_{11}(x, y, \tau)}.\tag{0.51}$$

F $_2 = 1.1, a_{13} = 1.3, \delta_1 = 1.5, \delta_2 = 2, \delta_3 = 1.6, \delta_4 = 1, a_{16} = 2, a_{17} = 1, \Psi_0 = 0, x = 1.5, t = 2$, the 2D graph of the lump stripe soliton by selecting amounts of various FO α is designed in Fig. 7 (Right).

Option XII:

$$a_1 = a_1,\ a_2 = a_2,\ a_3 = -\frac{{a_1}^2\delta_3 + a_1a_2\delta_1 + {a_2}^2\delta_2}{a_1},\quad a_4 = a_4,\ a_5 = a_5,\quad a_6 = \frac{a_2a_5}{a_1}, \tag{0.52}$$

$$a_7 = -\frac{a_5\left({a_1}^2\delta_3 + a_1a_2\delta_1 + {a_2}^2\delta_2\right)}{{a_1}^2},\quad a_8 = a_8,\ a_9 = -a_{13},\quad a_{10} = -\frac{a_2a_{13}\left(a_1{a_{13}}^2 - a_2\delta_2\right)}{a_1\left(a_1{a_{13}}^2 + a_2\delta_2\right)},$$

$$a_{11} = \frac{a_{13}\left({a_1}^4{a_{13}}^4\delta_3 + {a_1}^3a_2{a_{13}}^4\delta_1 + {a_1}^2{a_2}^2{a_{13}}^4\delta_2 + 2{a_1}^3a_2{a_{13}}^2\delta_2\delta_3 - 2\,a_1{a_2}^3{a_{13}}^2{\delta_2}^2 + {a_1}^2{a_2}^2{\delta_2}^2\delta_3 - a_1{a_2}^3\delta_1{\delta_2}^2 + {a_2}^4{\delta_2}^3\right)}{{a_1}^2\left(a_1{a_{13}}^2 + a_2\delta_2\right)^2},$$

$$a_{12} = a_{12},\quad a_{13} = a_{13},\ a_{14} = \frac{a_2a_{13}\left(a_1{a_{13}}^2 - a_2\delta_2\right)}{a_1\left(a_1{a_{13}}^2 + a_2\delta_2\right)},$$

$$a_{15} = -\frac{a_{13}\left({a_1}^4{a_{13}}^4\delta_3 + {a_1}^3a_2{a_{13}}^4\delta_1 + {a_1}^2{a_2}^2{a_{13}}^4\delta_2 + 2{a_1}^3a_2{a_{13}}^2\delta_2\delta_3 - 2a_1{a_2}^3{a_{13}}^2{\delta_2}^2 + {a_1}^2{a_2}^2{\delta_2}^2\delta_3 - a_1{a_2}^3\delta_1{\delta_2}^2 + {a_2}^4{\delta_2}^3\right)}{{a_1}^2\left(a_1{a_{13}}^2 + a_2\delta_2\right)^2},$$

$$a_{16} = a_{16},\ a_{17} = a_{17},\quad \delta_4 = -\frac{1}{2}\frac{\left(a_1{a_{13}}^2 - a_2\delta_2\right)a_2}{{a_1}^2{a_{13}}^2},\quad \delta_5 = \frac{1}{2}\frac{{a_1}^2\left(a_1{a_{13}}^2 + a_2\delta_2\right)^2}{{a_{13}}^2\left(a_1{a_{13}}^2 - a_2\delta_2\right){a_2}^3},\quad \delta_6 = -\frac{{a_1}^2\left(a_1{a_{13}}^2 + a_2\delta_2\right)}{{a_2}^2\left(a_1{a_{13}}^2 - a_2\delta_2\right)},$$

in which Ω, solves $\delta_4\Omega^4 + \Omega^3 + \delta_6\Omega + \delta_5 = 0$, and by utilizing the function (0.30), the exact solution will be as

$$\Psi_{12} = \Psi_0 + \frac{2d\mathfrak{f}_{12}(x, y, \tau)/dx}{\mathfrak{f}_{12}(x, y, \tau)}. \tag{0.53}$$

Furthermore, by choosing suitable amounts, the dynamical structures of periodic wave solutions are presented in Figs. 8 including three dimensional, density, and y-curves plots. By taking novel parameters including $a_1 = 1.5, a_2 = 2, a_4 = 2, a_5 = 1.5, a_9 = 1.1, a_8 = 1, a_{10} = 1.2, a_{12} = 1.5, a_{13} = 1.3, \delta_1 = 1.5, \delta_2 = 2, \delta_3 = 1.6, \delta_4 = 1, \delta_6 = 1.5, a_{16} = 2, a_{17} = 1, \Psi_0 = 0, t = 2$, the acquired lump with two stripe solutions in Option XII are shown with two various FOs. Also, 2D graph of the lump stripe soliton by choosing values of various FO α is designed in Fig. 9.

Solitary solutions for Eq. (0.6)

Via $\xi = k\left(x + ay - \frac{c}{\Gamma(\alpha+1)}t^\alpha\right)$ in Eq. (0.6), one gets

$$F(\Psi) := k\left(a\delta_1 - \frac{c}{\Gamma(\alpha+1)} + \delta_3\right)\Psi' + 3k^2(a+\delta_4)\Psi'^2 + k^3\left(a^3\delta_6 + a^2\delta_2 + a + \delta_4\right)\Psi''' + \tag{0.54}$$

$$3k^6a^4\delta_5\Psi'''^2 + 3k^4a^3\delta_6\Psi'\Psi''' + k^5a^4\Psi''''' = 0.$$

Based on Refs. [45, 46, 47], we have

$$\Lambda(\Psi) = \int\left(F(\Psi)\frac{d\Psi}{d\xi}\right)d\xi, \tag{0.55}$$

$$J = \int_{-\infty}^{\infty}\Lambda(\Psi)d\xi. \tag{0.56}$$

Case I:

Consider the below function

$$u(\xi) = A\ sech(B\xi). \tag{0.57}$$

By using

$$J = \frac{1}{3360}kA^2\left(\frac{1}{2}BA\pi\,k\left(427\,k^4a^4B^4\delta_5 - 912\,k^2B^2a^3\delta_6 + 924\,a + 924\,\delta_4\right) - \right. \tag{0.58}$$

$$\left. 2520\,k^2B^2\delta_4 + 21000\,k^4a^4B^4 - 2520\,k^2B^2a - 2520\,k^2B^2a^2\delta_2 - 2520\,k^2B^2a^3\delta_6\right),$$

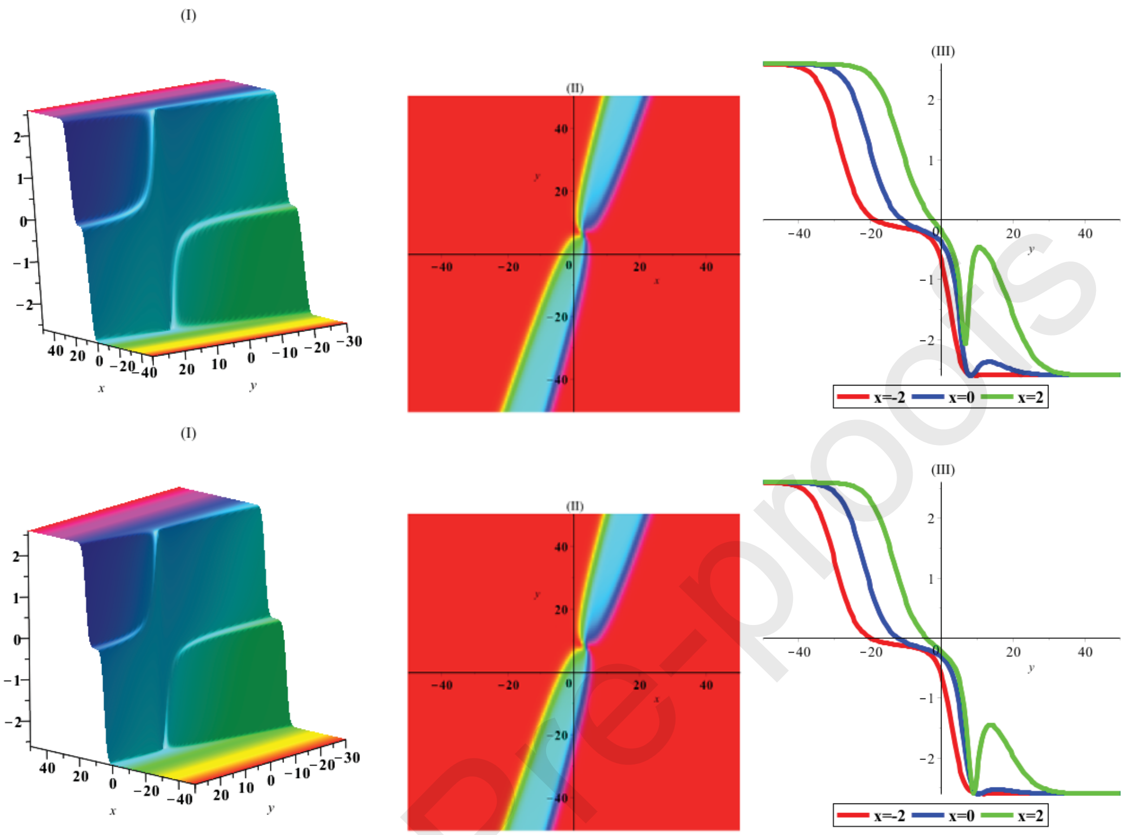

Figure 8: The graph of lump with two stripe solitons (0.53) with FO $\alpha = 0.5$ for the first line and $\alpha = 0.95$ for the second line.

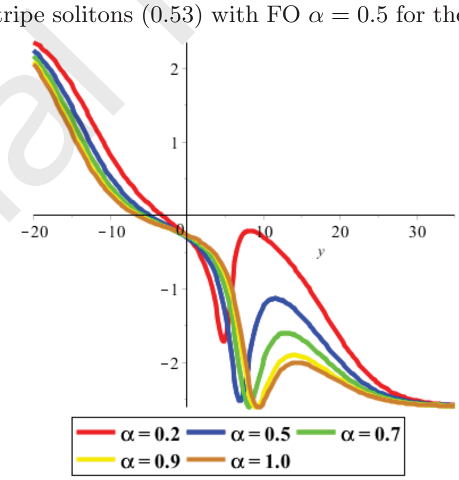

Figure 9: The 2D graph of lump with two stripe solitons (0.53) with the different fractional orders.

w

$$c=\Gamma(\alpha+1)\left[\frac{31\,k^4a^4B^4}{7}-\frac{7}{5}k^2B^2\left(a^3\delta_6+a^2\delta_2+a+\delta_4\right)+a\delta_1+\delta_3\right],$$

and employing the below relations

$$\frac{\partial J}{\partial A}=0 \tag{0.59}$$

and

$$\frac{\partial J}{\partial B}=0. \tag{0.60}$$

The nonlinear algebraic system will be concluded as

$$\frac{k^2A^2B\pi\,\left(427\,k^4a^4B^4\delta_5-912\,k^2B^2a^3\delta_6+924\,a+924\,\delta_4\right)}{3360}+\frac{k^2A^2B\pi\,\left(427\,k^4a^4B^4\delta_5-912\,k^2B^2a^3\delta_6+924\,a+924\,\delta_4\right)}{6720}$$
$$+\frac{kA\left(21000\,k^4a^4B^4-2520\,k^2B^2a^3\delta_6-2520\,k^2B^2a^2\delta_2-2520\,k^2B^2a-2520\,k^2B^2\delta_4\right)}{1680}=0, \tag{0.61}$$

$$\frac{1}{3360}kA^2\left(\frac{1}{2}A\pi\,k\left(427\,k^4a^4B^4\delta_5-912\,k^2B^2a^3\delta_6+924\,a+924\,\delta_4\right)+\frac{1}{2}BA\pi\,k\left(1708\,B^3a^4k^4\delta_5-1824\,Ba^3k^2\delta_6\right)-\right.$$
$$\left.5040\,k^2B\delta_4+84000\,B^3a^4k^4-5040\,k^2Ba-5040\,k^2Ba^2\delta_2-5040\,Ba^3k^2\delta_6\right)=0. \tag{0.62}$$

By solving (0.61) and (0.62), receive the solutions

$$A=-\frac{1120\Omega\,\left(25\,a^2\Omega^2-3\,a^3\delta_6-3\,a^2\delta_2-3\,a-3\,\delta_4\right)}{a\pi\,\left(427\,\Omega^4\delta_5-912\,\Omega^2a\delta_6+924\,a+924\,\delta_4\right)}, \tag{0.63}$$

$$B=\frac{\Omega}{ka}, \tag{0.64}$$

in which Ω, solves equation $\lambda_6\Omega^6+\lambda_4\Omega^4+\lambda_2\Omega^2+\lambda_0=0$ and the coefficients are as

$$\lambda_0=-5544\,a^4\delta_6-5544\,a^3\delta_4\delta_6-5544\,a^3\delta_2-5544\,a^2\delta_2\delta_4-5544\,a^2-11088\,a\delta_4-5544\,{\delta_4}^2, \tag{0.65}$$
$$\lambda_2=115500a^3+115500a^2\delta_4,$$
$$\lambda_4=2562\,a^3\delta_5\delta_6-68400\,a^3\delta_6+2562\,a^2\delta_2\delta_5+2562\,a\delta_5+2562\,\delta_4\delta_5,$$
$$\lambda_6=10675a^2\delta_5.$$

The conditions are

$$k\neq 0,\quad 427\,\Omega^4\delta_5-912\,\Omega^2a\delta_6+924(a+\delta_4)\neq 0,\quad 27{\lambda_0}^2{\lambda_6}^2-18\lambda_0\lambda_2\lambda_4\lambda_6+4\lambda_0{\lambda_4}^3+4{\lambda_2}^3\lambda_6-{\lambda_2}^2{\lambda_4}^2>0. \tag{0.66}$$

The final solutions is as

$$u(x,y,t)=-\frac{1120\Omega\left(25a^2\Omega^2-3a^3\delta_6-3a^2\delta_2-3a-3\delta_4\right)}{a\pi\left(427\Omega^4\delta_5-912\Omega^2a\delta_6+924a+924\delta_4\right)} \tag{0.67}$$
$$\times\mathrm{sech}\left[\frac{\Omega}{a}\left(x+ay-\left(\frac{31}{4}\Omega^4-\frac{7\Omega^2}{5a^2}\left(a^3\delta_6+a^2\delta_2+a+\delta_4\right)+a\delta_1+\delta_3\right)t^\alpha\right)\right].$$

Results and discussion

For investigating the fractional gCBS-BK equation, the interaction of lump and 2-kink solutions are analyzed in Figs. (1)-(9) demonstrate the transfer of the lump solution from one soliton to the another soliton. The dynamical structure of periodic wave solutions are presented in Figs. 1 including three dimensional, density plot, and y-curves plots. By taking the novel parameters such as $a_1=1.2, a_4=2, a_5=1.5, a_9=1.1, a_8=1, a_{10}=1.2, a_{12}=1.5, a_{13}=1.3, \delta_1=1.5, \delta_2=2, \delta_3=1.6, \delta_4=1, \delta_6=1.5, a_{16}=2, a_{17}=1, \Psi_0=0, t=2$. Also, 2D graph of the lump soliton by selecting amounts of the various fractional order (FO) α is designed in Fig. 2. The novel parameters containing $a_1=1.2, a_4=2, a_5=1.5, a_9=1.1, a_8=1, a_{10}=1.2, a_{12}=1.5, a_{13}=1.3, \delta_1=1.5, \delta_2=2, \delta_3=1.6, \delta_4=1, \delta_6=1.5, a_{16}=2, a_{17}=1, \Psi_0=0, t=2$ for Figs. 3 and 4 the acquired lump with two stripe solutions. And for Figs.

5 … $\delta_1 = 1.5, \delta_2 = 2, \delta_3 = 1.6, \delta_4 = 1, \delta_6 = 1.5, a_{16} = 2, a_{17} = 1, \Psi_0 = 0, t = 2$ are considered. Moreover, the parameters $a_1 = 1.2, a_4 = 2, a_5 = 1.5, a_9 = 1.1, a_8 = 1, a_{10} = 1.2, a_{12} = 1.5, a_{13} = 1.3, \delta_1 = 1.5, \delta_2 = 2, \delta_3 = 1.6, \delta_4 = 1, \delta_5 = 1, \delta_6 = 1, a_{16} = 2, a_{17} = 1, \Psi_0 = 0, y = 1.5, t = 2$ for Fig. 7 (Left) and the parameters $a_1 = 1.2, a_2 = 2, a_4 = 2, a_5 = 1.2, a_8 = 1.5, a_{10} = 1.3, a_{12} = 1.1, a_{13} = 1.3, \delta_1 = 1.5, \delta_2 = 2, \delta_3 = 1.6, \delta_4 = 1, a_{16} = 2, a_{17} = 1, \Psi_0 = 0, y = 1.5, t = 2$ for Fig. 7 (Center), and the parameters $a_2 = 2, a_4 = 2, a_5 = 1.2, a_8 = 2, a_{12} = 1.1, a_{13} = 1.3, \delta_1 = 1.5, \delta_2 = 2, \delta_3 = 1.6, \delta_4 = 1, a_{16} = 2, a_{17} = 1, \Psi_0 = 0, x = 1.5, t = 2$ for Fig. 7 (Right) are considered. Finally, the parameters including $a_1 = 1.5, a_2 = 2, a_4 = 2, a_5 = 1.5, a_9 = 1.1, a_8 = 1, a_{10} = 1.2, a_{12} = 1.5, a_{13} = 1.3, \delta_1 = 1.5, \delta_2 = 2, \delta_3 = 1.6, \delta_4 = 1, \delta_6 = 1.5, a_{16} = 2, a_{17} = 1, \Psi_0 = 0, t = 2$ are considered the acquired lump with two stripe solutions in in Figs. 8 and 9.

Conclusion

In this paper, the multiple solitons, lump solitons and interaction with two stripe soliton solution, and solitary wave solution for the fractional gCBS-BK equation are investigated. The MEFM was utilized and obtained one-soliton and two-soliton for the fractional gCBS-BK equation. The Hirota bilinear method is utilized which contains the lump and interaction between a lump and two stripe solitons. Moreover, we studied the solitary, bright and dark soliton wave solutions of the fractional gCBS-BK equation by the help of SIVP in the previous section. The graphs for various fractional-order α were plotted containing three dimensional, density, and y-curves plots. In particular, the interaction of lump and 2-kink solutions are analyzed in Figs. (1)-(9) demonstrate the transfer of the lump solution from one soliton to the another soliton. Finally, new solitary waves of this study problem can be discovered by using a test function of the SIVP. These results can assist us in better understanding interesting physical phenomena and mechanism. For other cases of the rogue and solitary wave solutions and also interaction lump with periodic wave solutions to appear in this problem, we will further to discuss in future work.

Compliance with Ethics Requirements:

This article does not contain any studies with human or animal subjects.

Declaration of Competing Interest:

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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