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GATE EE 2018 PAPER

Question

Let f be a real-valued function defined by

$$f(x) = x - \lfloor x \rfloor,$$

where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. Compute

$$\int_{0.25}^{1.25} f(x)\,dx$$

(give the answer up to two decimal places).

Solution

The function $f(x) = x - \lfloor x \rfloor$ is the fractional part of x. Split the integration interval at the integer $x = 1$:

$$\int_{0.25}^{1.25} f(x)\,dx = \int_{0.25}^{1} f(x)\,dx + \int_{1}^{1.25} f(x)\,dx.$$

For $0.25 \le x < 1$, $\lfloor x \rfloor = 0$ so $f(x) = x$. For $1 \le x \le 1.25$, $\lfloor x \rfloor = 1$ so $f(x) = x - 1$.
Thus

$$\int_{0.25}^{1} f(x)\,dx = \int_{0.25}^{1} x\,dx = \left.\frac{x^2}{2}\right|_{0.25}^{1} = \frac{1^2}{2} - \frac{0.25^2}{2} = \frac{1}{2} - \frac{0.0625}{2} = 0.5 - 0.03125 = 0.46875,$$

$$\int_{1}^{1.25} f(x)\,dx = \int_{1}^{1.25} (x-1)\,dx = \left(\left.\frac{x^2}{2}\right|_{1}^{1.25}\right) - (1.25 - 1)$$
$$= \left(\frac{1.5625}{2} - \frac{1}{2}\right) - 0.25 = (0.78125 - 0.5) - 0.25 = 0.28125 - 0.25 = 0.03125.$$

Adding both parts:

$$\int_{0.25}^{1.25} f(x)\,dx = 0.46875 + 0.03125 = 0.5.$$

Therefore, up to two decimal places, the value is $\boxed{0.50}$.