

GATE EE 2018 PAPER

Question 18

Let f be a real-valued function of a real variable defined as

$$f(x) = x - \lfloor x \rfloor,$$

where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x .

Compute

$$\int_{0.25}^{1.25} f(x) dx \text{ (up to 2 decimal places).}$$

Solution

Lets splite the integral

$$\int_{0.25}^{1.25} (x - \lfloor x \rfloor) dx = \int_{0.25}^1 x dx + \int_1^{1.25} (x - 1) dx$$

Lets compute 1st part of intergal

$$\int_{0.25}^1 x dx = \left. \frac{x^2}{2} \right|_{0.25}^1 = \frac{1}{2} - \frac{0.25^2}{2} = 0.5 - 0.03125 = 0.46875$$

Lets compute 2nd part of integral

$$\int_1^{1.25} (x - 1) dx = \left. \frac{x^2}{2} \right|_1^{1.25} - x \Big|_1^{1.25} = (0.78125 - 0.5) - 0.25 = 0.03125$$

Add the individually computed values.

$$\int_{0.25}^{1.25} f(x) dx = 0.46875 + 0.03125 = 0.5$$

The final answer is 0.50