Universidad Industrial de Santander, Colombia
Numerical Analysis, 2021-1
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Lab 0. Preliminary

1 Instructions

- Make a **pdf** report including the solution to each point of the practice with name *Lab0_name_lastname.pdf*.
- Send the report and all created files in a rar or zip file with name *Lab0_name_lastname.rar* in the Moodle.
- You are allowed to use internet, notes, and .m files that you have created before.

2 Purposes

- To understand the rounding arithmetic.
- To apply the floating point format and rounding arithmetic.
- To implement the floating point format and rounding arithmetic in Matlab.

3 Practice

3.1 Understanding

Answer with your own words the following questions:

- (0.2 points) What is the absolute and relative error?

- (0.2 points) How to calculate the significant digits of a number?

- (0.2 points) What properties does the order approximation have?

3.2 Applying

- (0.5 points) Use three-digit rounding arithmetic to compute the following sums (sum in the given order):

  a) $\sum_{k=1}^{6} \frac{1}{3^k}$

  b) $\sum_{k=1}^{6} \frac{1}{3^{7-k}}$

- (1.0 points) *Improving the Quadratic Formula.* Assume that $a \neq 0$ and $b^2 - 4ac > 0$ and consider the equation $ax^2 + bx + c = 0$. The roots can be computed with the quadratic formulas

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \; x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}. \tag{1}$$

  Show that these roots can be calculated with the equivalent formulas

$$x_1 = \frac{-2c}{b + \sqrt{b^2 - 4ac}}, \; x_2 = \frac{-2c}{b - \sqrt{b^2 - 4ac}}. \tag{2}$$

  Hint. Rationalize the numerators in (1). Remark. In the cases when $|b| \approx \sqrt{b^2 - 4ac}$, one must proceed with caution to avoid loss of precision due to a catastrophic cancellation. If $b > 0$, then x_1 should be computed with formula (2), and x_2 should be computed using (1). However, if $b < 0$, then x_1 should be computed using (1), and x_2 should be computed using (2).

3.3 Implementing

- (1.0 point) Create a Matlab function called *floating_point_function_name_lastname()* to determine the floating point format to stored decimal numbers in a computer with 16 bits of precision distributed as shown below.

Sign	Exponent							Mantissa							

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% P: Floating-point representation
% N: Decimal number
P=floating_point_function_name_lastname(N);
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- (0.5 points) Use the following decimal numbers to test the created function.

  a) 1612.078125_{10}

  b) 6317.9136_{10}

  c) -962.0153_{10}

- (0.5 points) Calculate the relative and absolute error between the given decimal numbers above and the stored decimal numbers in a computer with 16 bits of precision.

- (1.0 point) Use the results obtained in *Improving the Quadratic Formula* to construct a Matlab program that will accurately compute the roots of a quadratic equation in all situations, including the troublesome ones when $|b| \approx \sqrt{b^2 - 4ac}$.

- (0.5 points) Find the roots of the following quadratic equation from the created function:

  a) $x^2 - 1,000.001x + 1 = 0$

  b) $x^2 - 10,000.0001x + 1 = 0$

  c) $x^2 - 100,000.00001x + 1 = 0$

  d) $x^2 - 1,000,000.000001x + 1 = 0$