Class Exercise 1

Fit the parameters r, a, s, b of the predator-prey system of ODE below to the data provided in ODE2_data.mat using fminsearch optimizer. (The data provided is for the state N_p).

$$\frac{dN_p}{dt} = rN_p - aN_pN_h$$
$$\frac{dN_h}{dt} = -sN_h + bN_pN_h$$

using the initial conditions for prey and predator given as $N_p(0)$=40, $N_h(0)$=9.
Use initial guess of parameters as param_init = [0.2 0.025 0.02 0.01];
Compute the approximative covariance matrix of the parameters using the Jacobian of the model numerically.

Class Exercise 2

Consider the predator-prey model above, where the measurement noise $\varepsilon \sim \mathbf{N}(0, 1)$. Assume that the true parameter values are r, a, s, b = [0.0972 0.0047 0.0193 0.001]. Generate synthetic measurements by adding noise to the model solution with true parameter values (use sigma=0.5), using the time data t=linspace(0,100). Estimate the parameters with the simulated measurements. Repeat the data generation and estimation 1000 times (or more) and collect the obtained samples for the parameters. Compute the covariance matrix from the samples. Make 1D (individual chain plots) and 2D (pairwise plot) visualizations of the results. Visualize the true curve and also the curves from the estimations.

Class Exercise 3

Find the LSQ fit of the parameters θ_1, θ_2 of the model $y = \frac{1}{1+\exp(-(\theta_1+\theta_2 x))}$ with the data given in data_1.mat. Study the uncertainty of the solution by bootstrap. Plot the data, fit and the curves from bootstrap.
Hint: function `lsqcurvefit` might be easier to use in this case.