Lecture Tutorial 1C: Atwood, Weightlessness, & Gravitation

OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG

How strong is gravity acting on a person on the ISS?

What is an Atwood machine?

`md04`

An (idal) Atwood machine consists of:

- Two masses m_1 and m_2
- Connected by a massless, inextensible cable
- Over a frictionless pulley

Key assumptions:

1. Cable mass = 0 → tension is the same on both sides
2. Pulley friction ≈ 0 → tension does not change direction
3. Cable length fixed → both masses accelerate with the same magnitude $|a|$, in opposite directions

This is mathematically identical to an elevator with a counterweight.

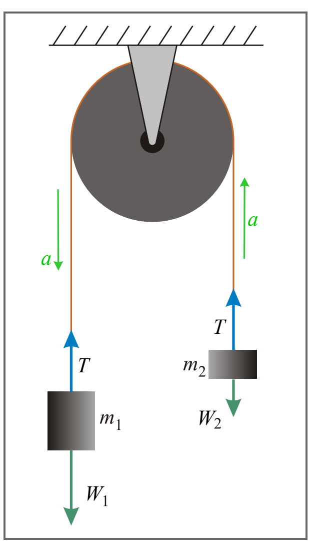
Physics of the Atwood machine

Let m_1 be heavier (moves down) and m_2 lighter (moves up).
For m_1 (downward positive): $m_1 g - T = m_1 a$
For m_2 (upward positive): $T - m_2 g = m_2 a$
Add both equations to eliminate T:

$$(m_1 - m_2)g = (m_1 + m_2)a$$

Thus the acceleration is

$$a = \frac{m_1 - m_2}{m_1 + m_2}\, g.$$



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Atwood machine: Special cases and application

- Balanced masses ($m_1 = m_2$): $a = 0 \rightarrow$ system is in equilibrium
- One mass zero ($m_2 = 0$): $a = g \rightarrow$ heavier mass free-falls
- For an elevator m_E with counterweight m_C ($m_E \approx m_C$):
  - Acceleration is very small
  - System is nearly balanced
  - Motor only overcomes friction and minor imbalances $\rightarrow$ minimal energy use

Niederfinow Boat Lift: History & Key Facts

- **Niederfinow Boat Lift** (opened 1934) on the Oder–Havel Canal
- Overcomes a **36 m height difference** between canal levels
- Trough: length ~ **85 m**, width ~ **12 m**, water depth ~ **2.5 m**
- Total moving mass (trough + water) ~ **4,290 t**
- Suspended by **256 cables** and balanced by **192 counterweights**

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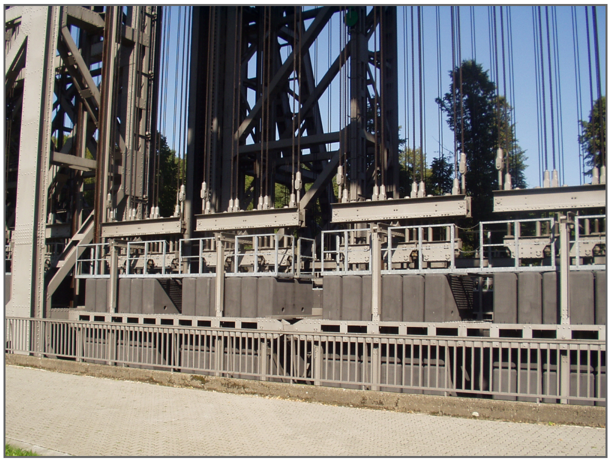
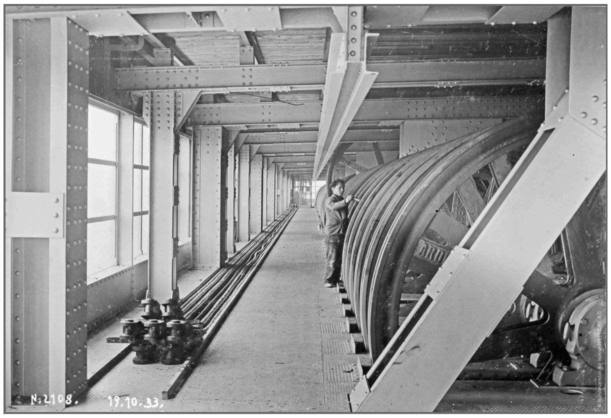
Niederfinow Boat Lift: A Giant Atwood Machine

- The entire lift works as a **scaled-up Atwood machine**
- Trough (with water + ship) on one side
- Counterweights of equal mass on the other side
- Cables run over guide wheels: forces mirror an Atwood system
- Motors only apply small forces → friction, acceleration, synchronization
- Core idea: **balance the large mass so movement requires little effort**





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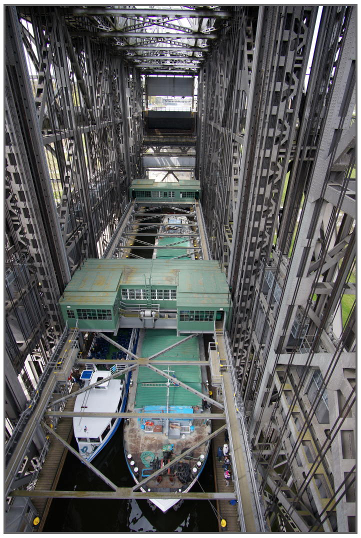
Niederfinow Boat Lift: Ship Displacement

- When a ship enters the water-filled caisson, it **displaces its own weight** in water
- Archimedes principle → **ship weight = weight of water pushed out before the gate closes**
- Therefore:
    - Before ship: trough + water
    - After ship: trough + (less water) + ship
    - **Total mass is unchanged**
- Result: counterweights can be tuned once and work for all ships



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Niederfinow Boat Lift: Why the Lift Is Energy Efficient

- Counterweights match the mass of the caisson → nearly perfect balance
- Motors only overcome:
  - **rolling and guide friction**
  - **small water-level differences**
  - **starting and stopping inertia**
- No need to pump large volumes of water (unlike lock systems)

  Energy use is comparable to a **well-balanced elevator**
- Near Magdeburg:
  - boat lift uses Archimedes’ principle
  - the submerged float bodies provide buoyancy that balances the weight of the water-filled trough
- Examples of **engineering using fundamental physics** for efficiency

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Atwood Machine & Cable-Break: A Thought Experiment

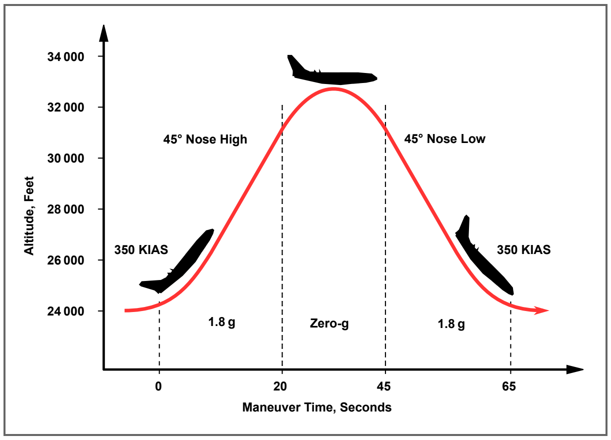
- Atwood machine: two masses connected by a rope over a pulley; motion determined by mass difference.
- If the rope breaks, a mass experiences **free fall**.
- In free fall the acceleration is

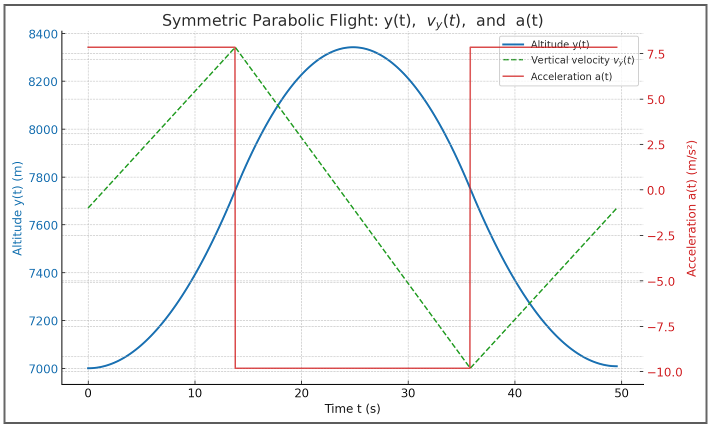
$$a = -g$$

- A falling elevator and a person inside accelerate downward **together**, both at $-g$.
- With no supporting force from the floor, the normal force becomes zero → the person feels **weightless**.
- Key idea: **free fall = no normal force = apparent weightlessness**.

Parabolic Flights (Zero-G Flights)

- Pilots fly a special “parabola” so the plane follows the same free-fall path as an object thrown upward.
- During the top part of this parabola, both plane and passengers accelerate downward at $a = -g$.
- Since they fall together, passengers float — **apparent weightlessness**.
- Duration of weightlessness: typically 20–25 seconds per parabola.
- Before and after the zero-g phase, the aircraft performs **hyper-g pull-up and pull-out** maneuvers, giving **apparent acceleration** $a > g$.
- Fun fact: NASA’s KC-135 was nicknamed the **“Vomit Comet”**





*[left] from **wikipedia**, **Attribution-Share Alike 4.0 International**; [right] $y(t)$, $v(t)$, and $a(t)$ diagram*

Apparent Weightlessness: Dropping Your Phone

`phyphox - Beschleunigung mit g`

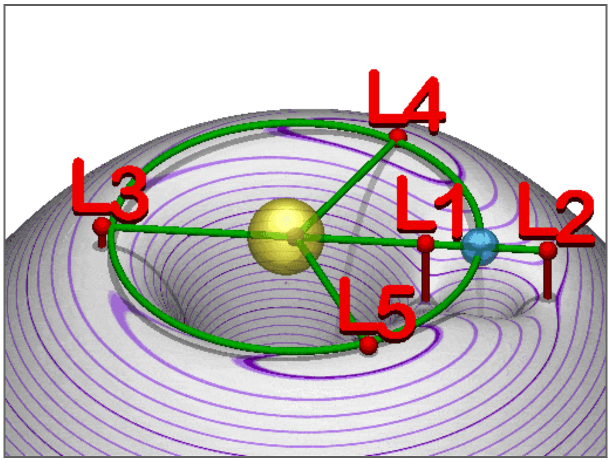
- A phone's accelerometer does **not** measure gravity — it measures the **proper acceleration** caused by the **normal force** acting on the sensor.
- When resting in your hand or on a table, the phone feels an upward normal force → accelerometer reads **≈ 1 g**.
- As soon as you **drop** the phone (or during a jump when it leaves your hand), it enters **free fall**:
    - gravity acts, but **no surface pushes on the sensor**
    - normal force $N = 0$
    - accelerometer reading → **≈ 0 g**

- The phone is still pulled downward by gravity, but it *feels* weightless because the **supporting normal force disappears**.

Apparent vs. Gravitational Weightlessness

- **Gravitational weightlessness (true microgravity):**
    - Occurs where gravitational forces from two (or more) bodies nearly **cancel**, e.g. at the **L1 Lagrange point**.
    - Net gravity is extremely small → objects experience **very low acceleration** even without being in free fall.
    - This is a *hypothetical pure case* of microgravity created by gravitational balance.
- **Apparent weightlessness:**
    - Gravity is fully present, but there is **no supporting force** (normal force) acting on the body.
    - Happens in parabolic flights, falling elevators, roller-coaster airtime, or when jumping.

- With $N = 0$, you *feel* weightless even though gravity acts normally.



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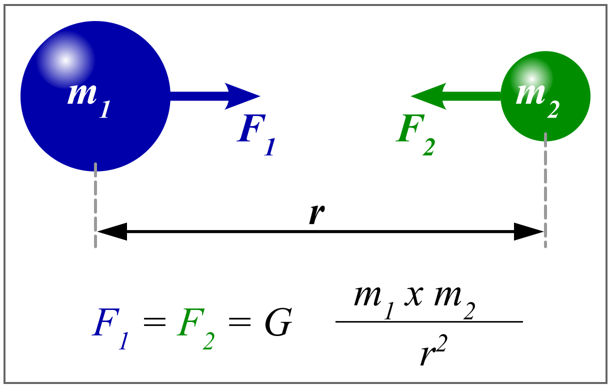
Newton's Law of Gravitation

- **Every mass attracts every other mass with a force along the line connecting them.**
- Two point masses m_1 and m_2 at positions $\vec{r}_1$ and $\vec{r}_2$:
    - Displacement from m_1 to m_2:
      $\vec{r} = \vec{r}_2 - \vec{r}_1, \quad r = |\vec{r}|$
    - Using the unit vector $\hat{r}_{12} = \dfrac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$:
- Newton's law of gravitation in vector form

$$\vec{F}_{12} = -G\frac{m_1 m_2}{r^2}\,\hat{r}_{12} = +G\frac{m_1 m_2}{r^2}\,\hat{r}_{21}$$

- Note: The minus sign ensures the gravitational force is **attractive**, i.e. it points in the **opposite direction** of the unit vector $\hat{r}_{12}$ (which points from m_1 to m_2).



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Example: Two Masses on a Line

- Masses m_1 and m_2 on the x-axis at positions x_1 and x_2:
    - $\vec{r}_1 = (x_1, 0, 0)$
    - $\vec{r}_2 = (x_2, 0, 0)$
- Displacement:

$$\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1, 0, 0), \quad r = |x_2 - x_1|$$

- Force on m_2 by m_1:

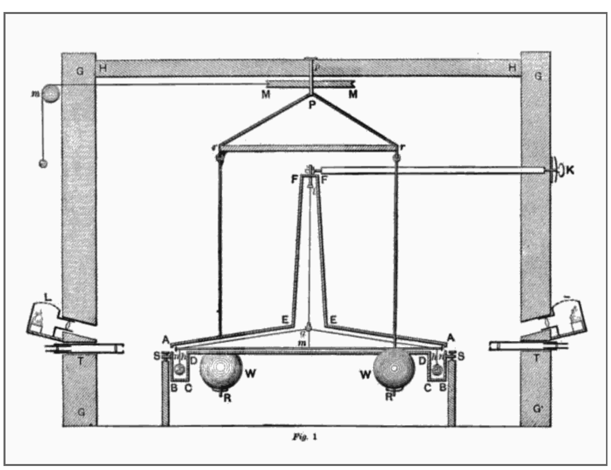
$$\vec{F}_{12} = -G\frac{m_1 m_2}{r^2}\,\hat{x} \quad \text{if } x_2 > x_1$$

- Interpretation:
    - $\vec{F}_{21} = -\vec{F}_{12}$ due to Newton's third law
    - Force points along the line joining the masses.

- The same formula works in any dimension, just with the appropriate vector $\vec{r}$.

Cavendish Experiment: Measuring G

- Goal:
    - Measure the tiny gravitational attraction between known laboratory masses.
    - From this, determine G and then the mass (and density) of Earth.
- Key idea:
    - Use a very sensitive torsion balance: gravitational attraction twists a thin wire.
    - Measure the tiny angular deflection to infer the force.



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Cavendish: Experimental Setup

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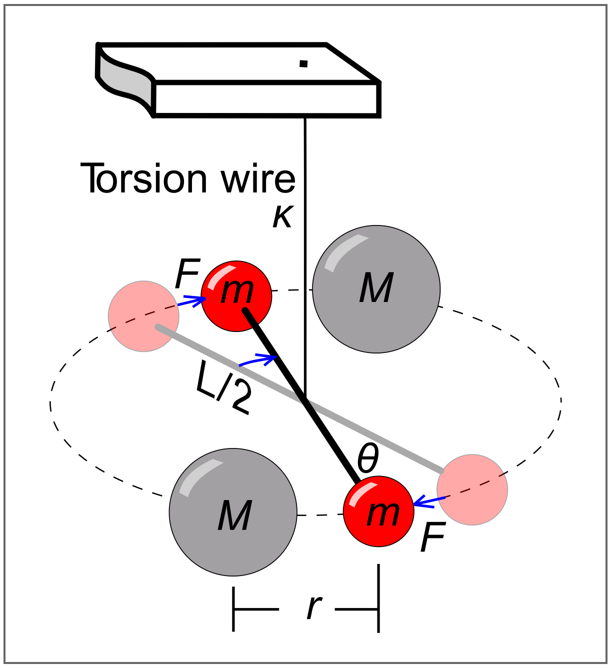

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Gravity Near Earth's Surface: Deriving g

- Earth mass M_E, radius R_E.
- Force on mass m at the surface with the distance $r = R_E$ from Earth's center:

$$F = G\frac{M_E m}{R_E^2}$$

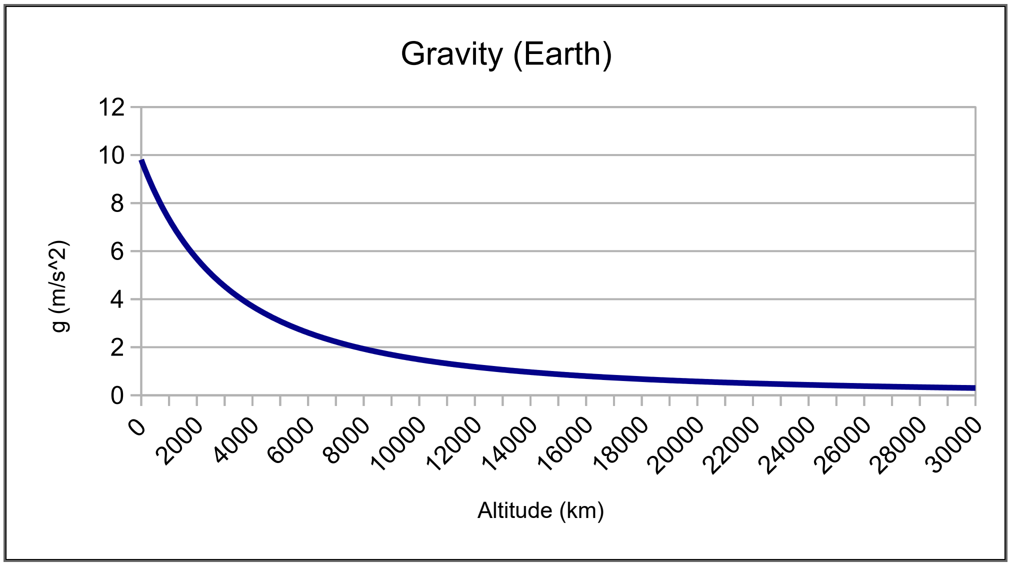
- By definition, weight is $W = mg$.
- Equating:

$$mg = G\frac{M_E m}{R_E^2} \quad \Rightarrow \quad g = G\frac{M_E}{R_E^2}$$

- g is **not a universal constant**; it depends on M_E and R_E.

Gravity vs. Altitude

$\Rightarrow$ **inverse square law** $g \propto \frac{1}{r^2}$



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Satellites: Circular Orbit as Free Fall

- Satellite of mass m in circular orbit at radius r (from Earth's center).
- Gravity provides centripetal force:

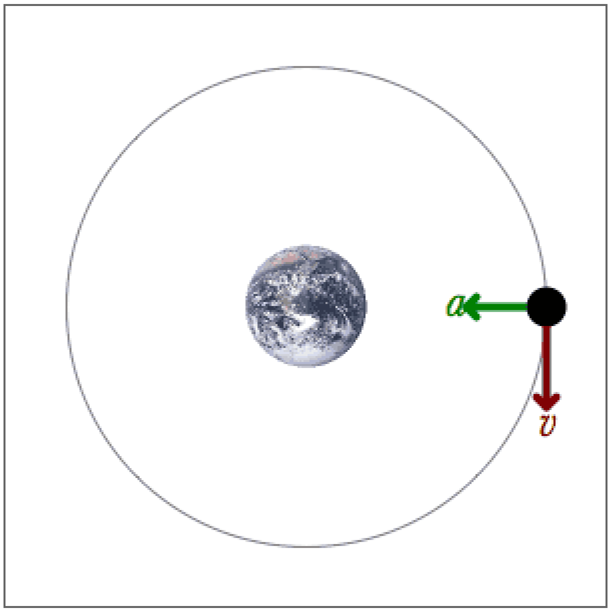
$$G\frac{M_E m}{r^2} = m\frac{v^2}{r}$$

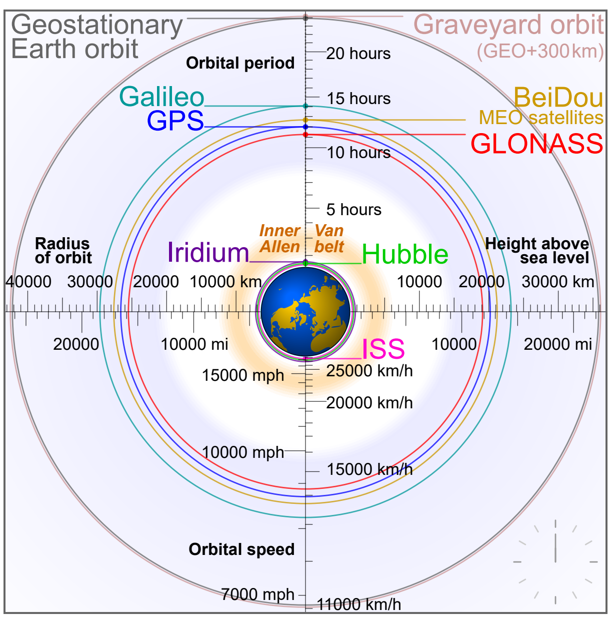
$$v = \sqrt{\frac{GM_E}{r}}$$

- Interpretation:
    - Velocity **independent** of satellites mass
    - Satellite is in continuous **free fall** toward Earth.
    - **Tangential speed** v is just right so it keeps "missing" Earth.
    - **Orbit = free fall** whose path curves around the planet.

Orbital Period and Free-Fall Picture

- Orbital period: $T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM_E}}$





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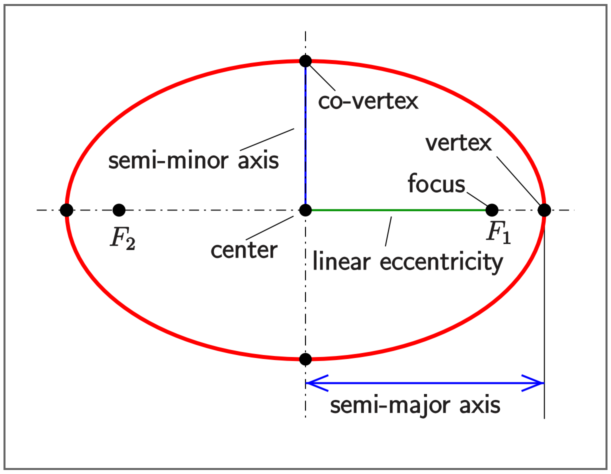
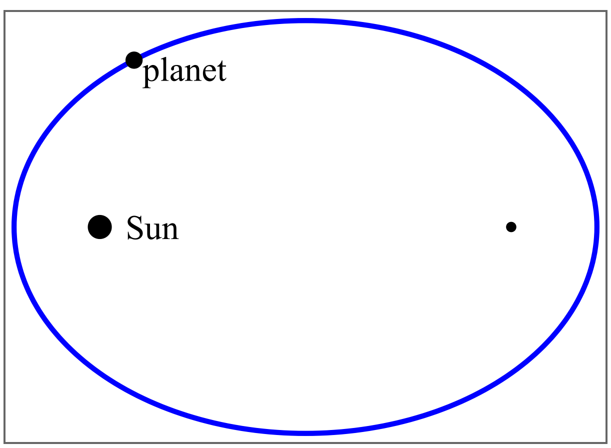
Kepler's Laws: Overview

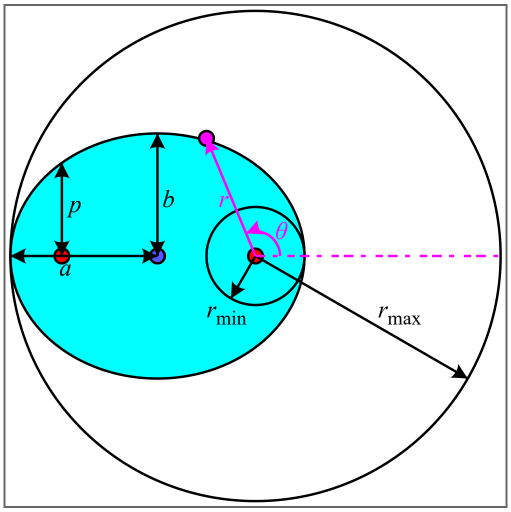
- Before Newton, Kepler summarized planetary motion in three empirical laws:
    1. Planets move in **ellipses** with the Sun at one focus.
    2. Planets sweep out **equal areas in equal times**.
    3. For each planet, $T^2 \propto a^3$, where T **is orbital period** and a **the semi-major axis**.
- Newton later showed:
    - Kepler's laws follow from universal gravitation plus Newton's laws of motion.
    - Celestial mechanics obey the same physics as falling apples.


co-vertex
vertex
semi-minor axis
focus
F_2
center
F_1
linear eccentricity
semi-major axis

Kepler's First Law: Elliptical Orbits

- Planets move in **elliptical orbits** with the Sun at one focus.
- Ellipse parameters:
  - Semi-major axis a (sets the "size" of the orbit)
  - Eccentricity e (shape): $0 \leq e < 1$
- Special cases:
  - $e = 0$ → circle
  - Small e → nearly circular (like Earth)





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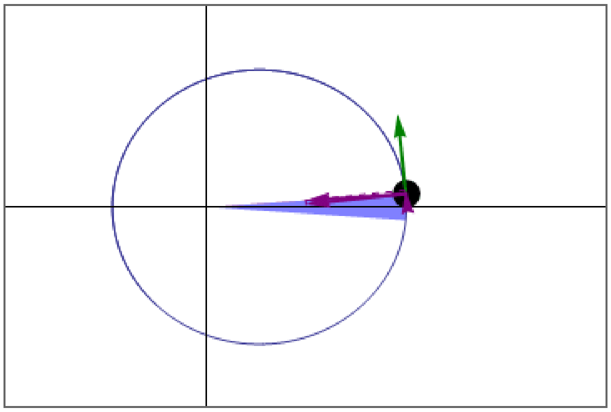
Kepler's Second Law: Equal Areas in Equal Times

- Statement: A line from the Sun to a planet sweeps out **equal areas in equal times**.
- Observations:
  - Planet moves **faster** near perihelion (closest approach).
  - **Slower** near aphelion (furthest point).
- Newton's explanation (intuition):
  - Gravity is a **central force**, so torque is zero → angular momentum $\vec{L} = \vec{r} \times m\vec{v}$ is conserved.
  - Smaller distance r, so to keep L constant, speed v must increase.
  - Result: **same area in the same time** → variable orbital speed.



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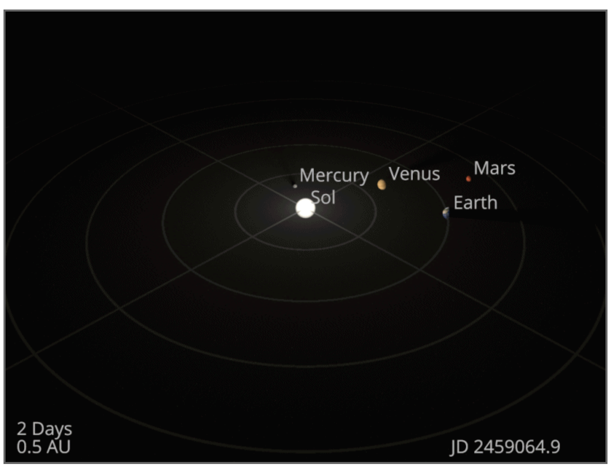
Kepler's Third Law: $T^2 \propto a^3$

- Statement: For all planets orbiting the **same central mass**,

$$\frac{T^2}{a^3} = \text{constant}$$

- Intuition: Orbits further out have a much larger radius and move more slowly → strongly increased period.



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Cavendish: Physics and Determining G

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- Gravitational force between one large and one small sphere:

$$F = G\frac{mM}{r^2}$$

- This force creates a torque on the rod and attached mirror causes deflection of laser beam.
- G can be determined using the oscillatory motion and/or stationary end point (equilibrium).
- Literature values $G = 6.67430 \times 10 - 11\frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}$

Cavendish: Interpretation and Error Sources

- Once G is known, Earth's mass follows from:

$$g = \frac{GM_E}{R_E^2} \quad \Rightarrow \quad M_E = \frac{gR_E^2}{G}$$

- Cavendish effectively "weighed the Earth."
- Main challenges and error sources:
  - Extremely small forces (on the order of 10^{-10}–10^{-11} N).
  - Air currents, temperature gradients, and mechanical vibrations.
  - Imperfect knowledge of mass distribution and distances.