2.8. TITLE

Why does holding a car key to the head increase the range to open the car?

- ANSWER FRAGMENT

-> answer requires addition to the script
\lambda_n = \lambda / n

Ovgu_fnw_logo

- university logo for ovgu fnw

Maxwell's equations & electromagnetic waves

- maxwell unified electricity and magnetism with four fundamental equations
- electromagnetic waves were predicted and later confirmed experimentally
- these concepts form the basis for modern communication and special relativity

revisiting ampère's law: changing electric fields produce magnetic fields

- electric currents generate magnetic fields and, via faraday's law, magnetic fields generate electric fields
- a changing electric field produces a magnetic field; this is captured by the concept of displacement current

- original ampère's law in integral form is given by

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{\text{enc}}$$

- displacement current is defined as

$$I_D = \epsilon_0 \frac{d\Phi_E}{dt}$$

- with the correction, ampère's law becomes

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{\text{enc}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

- the magnetic field outside a capacitor is

$$B = \frac{\mu_0 I}{2\pi r}$$

gauss's law for magnetism

- the electric flux through a closed surface is

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

- since no magnetic monopoles exist, the magnetic flux is

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

maxwell’s equations

- the four equations in integral form are:
    - gauss's law for electricity:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q}{\epsilon_0}$$

    - gauss's law for magnetism:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

    - faraday's law of induction:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d\Phi_B}{dt}$$

    - ampère's law with maxwell correction:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

- in differential form the equations are:
  - $$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$$
  - $$\nabla \cdot \vec{\mathbf{B}} = 0$$
  - $$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$
  - $$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$$
- these equations not only unify electricity and magnetism but also predict electromagnetic waves

electromagnetic waves

- in empty space (no charges or currents) the integral equations simplify to

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0 \quad , \quad \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

- a changing electric field produces a changing magnetic field and vice versa, enabling wave propagation

- sinusoidal solutions for plane waves are:
  - $$E = E_0 \sin(kx - \omega t)$$

    with the electric field oscillating in the (y) direction
  - $$B = B_0 \sin(kx - \omega t)$$

    with the magnetic field oscillating in the (z) direction
- the wave parameters are defined by:
  - wave number:

    $$k = \frac{2\pi}{\lambda}$$
  - angular frequency:

    $$\omega = 2\pi f$$
  - wave velocity:

    $$\nu = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = c$$

- the poynting vector, representing energy flux, is given by

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

- the energy density of the electromagnetic field is

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

- radiation pressure is determined by the poynting vector:
    - for fully absorbed radiation:

$$P = \frac{S}{c}$$

    - for fully reflected radiation:

$$P = \frac{2S}{c}$$

electromagnetic spectrum & and light

- all electromagnetic waves propagate at

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8 \, \mathrm{m/s}$$

- visible light spans frequencies from (4.0 \times 10^{14}) to (7.5 \times 10^{14},\text{Hz}) and wavelengths between 400 nm and 750 nm
- the relation between frequency and wavelength is

$$c = \lambda f$$

- beyond visible light, the spectrum includes infrared, ultraviolet, radio waves, microwaves, x-rays, and gamma rays

summary

- maxwell’s equations unify electricity and magnetism and predict electromagnetic waves
- the introduction of the displacement current resolves inconsistencies in ampère's law
- electromagnetic waves are pure field oscillations that carry energy and momentum, as described by the poynting vector
- these principles are fundamental to modern communication technologies and underpin

special relativity