

2.1. Electric charge & electric field

What comes to your mind when you hear electricity?



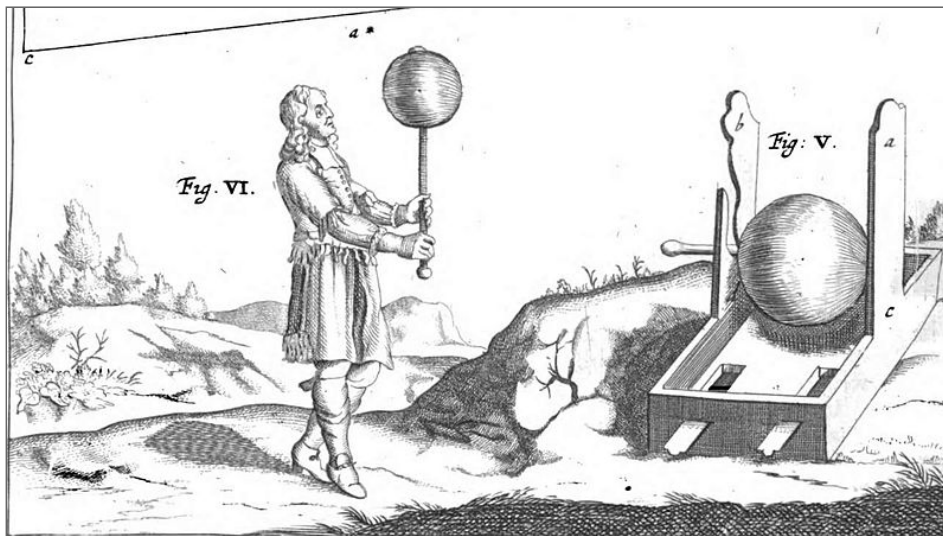
- modern technology, i.e. EVs, computers, smartphones, etc.
- human body relies on electrical signals i.e. heart & brain
- electric forces
 - involved in holding atoms & molecules together
 - are the origin of forces such as friction or normal force can at the atomic level
- gravitational forces remains a separate entity

Short history of studying electricity

- since ancient Greece, the effect of static electricity known
 - "elektron" is Greek for "amber"
 - rub amber with cloth → amber attracts small pieces
- scientific investigation of electricity around late 1700:
 - Charles Augustin de Coulomb (1736 - 1806)
 - Benjamin Franklin (1706–1790)
 - Michael Faraday (1791 - 1867)
 - Karl Friedrich Gauss (1777-1855)
 - Alessandro Volta's (1745-1827)
 - ... and many more ...

Otto von Guericke & his sulfur globe

- 1672: reported that after rubbing sulfur globe, globe repelled/attracted light objects
- → there is **contactless** force beyond gravitation → es07



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Static electricity

- are there different types of charge?
- when do they attract and when do they repel each other?
- → es13 + es04



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Static electricity

- two types of charge: **positive** and **negative**
- arbitrary convention by **Benjamin Franklin** (1706–1790):
 - rubbing amber/plastic → **negative charge**
 - rubbing glass rod → **positive charge**
- charges of **same type repel** each other
- charges of **opposite type attract** each other

Charge origin

- **simplified atomic model:**
 - tiny nucleus with **positively charged protons** and neutral neutrons
 - **negatively charged electrons** orbit nucleus
- **neutral atom:** equal number of protons & electrons
- **ion:** atom that gained or lost electrons
- **polar molecules:** charge **distribution is non-uniform**, e.g. water molecule with positive side (H) and negative side (O)
- **elementary charge**
 - smallest observed charge: **electron or proton**
 - $e = 1.602 \times 10^{-19} \text{C}$

Conductors vs. Insulators

- **free electrons:**
 - electrons that can "detach from their parent atoms" and move through the material/lattice
- **conductors** (e.g. metals): high abundance of free electrons
- **insulators** (e.g. wood): low abundance of free electrons
- **semiconductors** (e.g. silicon): intermediate abundance of free electrons

Conservation of charge

- net electric charge **cannot be created or destroyed**
- → **charges can only be separated** but their sum remains constant
- **example:** rubbing a glass rod with a cloth:
 - glass rod gains positive charge by emitting free electrons
 - cloth gains equal negative charge by receiving free electrons
 - **total charge remains zero**
- **grounding:**
 - connection to Earth
 - Earth acts as a **charge reservoir**

Inducing charge by conduction

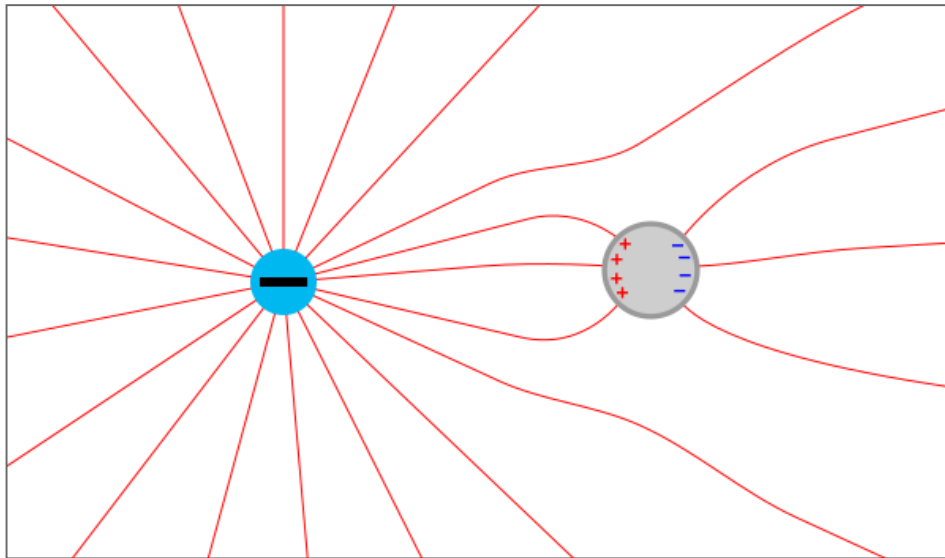
es09 + es10

- **contact required**
- free electrons move from one conductor to another
- after contact, both objects have the **same charge**

Inducing charge by (electrostatic) induction

es09 + es10

- **no contact required**



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Inducing charge by (electrostatic) induction

es09 + es10

- **no contact required**
- in conductor/influence:
 - **redistribution of free electrons** in nearby conductor which causes charge separation
- in insulators/polarization:
 - electrons cannot move freely
 - **molecules change their orientation** which causes charge separation
- → induced charge separation **does not change total charge**

Electrostatic force & Coulomb's law

es08 + script simulation

- how to quantify/compute electrostatic force?
- investigated by **Charles Augustin de Coulomb** (1736–1806)
- honoring his contribution, charge Q is measured in the SI unit **coulomb**
[C]

Electrostatic force & Coulomb's law

$$F = k \frac{Q_1 Q_2}{r^2}$$

- with:
 - $k \approx 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$ (Coulomb's proportionality constant)
 - r is the distance between charges
 - Q_1 and Q_2 are the charges
- **force direction**
 - force acts **along the line connecting both charges**
 - **like charges repel, opposite charges attract**

Vector Form of Coulomb's Law

$$\vec{\mathbf{F}}_{12} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2} \hat{r}_{21}$$

- with:
 - $\vec{\mathbf{F}}_{12}$: force Q_2 exerts on Q_1
 - fundamental constant **permittivity of free space** ϵ_0
 - $k = \frac{1}{4\pi\epsilon_0}$
 - \hat{r}_{21} is **unit vector** pointing from Q_2 to Q_1 .
- **superposition principle**: for multiple charges, net force is the vector sum of all forces

Concept of the Electric Field

- observations so far:
 - charges q exert electrostatic force $\vec{\mathbf{F}}$
 - electrostatic forces act at a distance & contactless
- **electric field $\vec{\mathbf{E}}$** connects q and $\vec{\mathbf{F}}$:
 - $\vec{\mathbf{E}}$ is a **vector field**
 - concept by Michael Faraday (1791–1867)
 - analogy with gravitational field: $\vec{\mathbf{F}} = q\vec{\mathbf{E}} \leftrightarrow \vec{\mathbf{F}} = m\vec{\mathbf{g}}$

Electric field form of Coulomb's law

- **idea:** measure field created by charge Q by placing a small *test charge* q (q has negligible effect of Q)
- using Coulomb's law, $\vec{\mathbf{E}}$ is then:

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2} \hat{r}_{21} \frac{1}{q} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}_{21}$$

- **superposition principle:** net field is the sum of individual fields:

$$\vec{\mathbf{E}}_{net} = \vec{\mathbf{E}}_A + \vec{\mathbf{E}}_B + \vec{\mathbf{E}}_C + \dots$$

- **integral form:** integrate over distribution of **infinitesimal small charges**
 dQ

$$d\vec{\mathbf{E}} = \frac{1}{4\pi\epsilon_0} \frac{dQ}{r^2} \hat{r}$$

$$\vec{\mathbf{E}} = \int d\vec{\mathbf{E}}$$

Field lines of electric field

script simulation

- motivation:
 - \vec{E} is a vector field, i.e. each point in space is represented by a vector
 - vector length == magnitude of field
 - vector orientation == direction of field
 - as plot, **often perceived as cluttered**
- **concept of field lines:**
 - arrows indicate **direction & magnitude** of \vec{E}
 - **density of lines** represents field's magnitude
 - direction of \vec{E} **tangent** to field lines
 - field lines **start on positive, end on negative** charges

- **field lines do not cross**

Motion of charged particles inside electric field

- a charge q in an **electric field** \vec{E} experiences force:

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

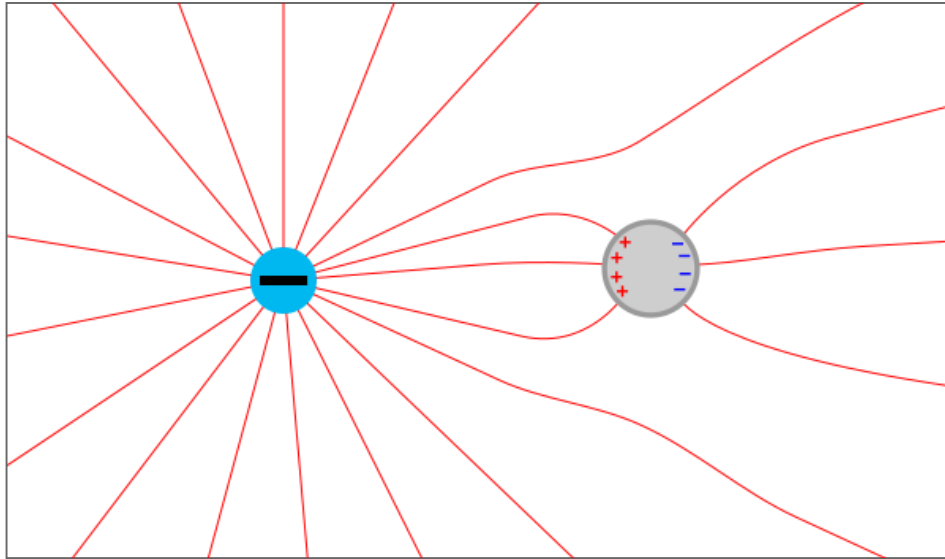
- for an **electron in uniform** E , acceleration is:

$$a = \frac{-eE}{m_e}$$

see lecture tutorial

Conductors in an electric field

- reminder: **electrostatics, i.e. charges at rest**
- for conductors in static condition:
 - **electric field inside conductor is zero:** $\vec{E} = 0$
 - *if this would not be the case, there would be a force acting on free electrons causing them to move until they reach a position in which the net force is zero, i.e. the field is zero and they are at rest ($\vec{F} = q\vec{E}$)*
 - **net charge distributes on surface**
 - electric field **perpendicular** to surface

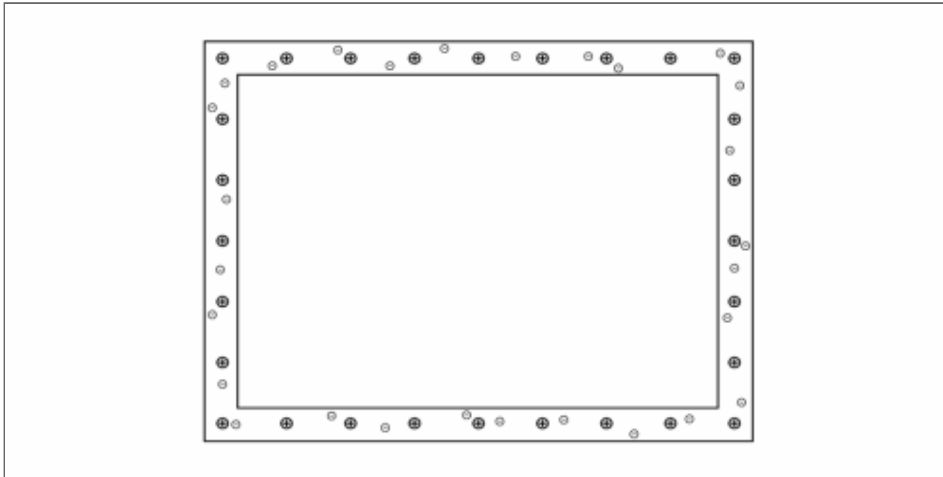


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Faraday cage

es16

- conducting, closed surface with $\vec{E} = 0$ inside



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