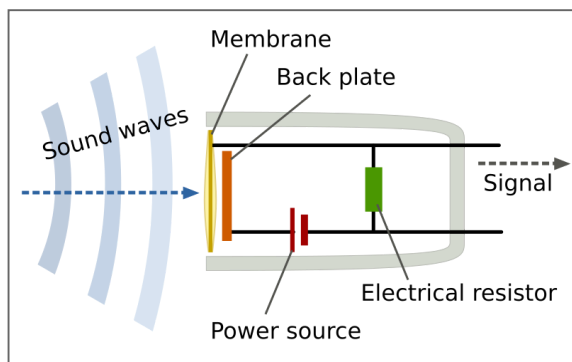
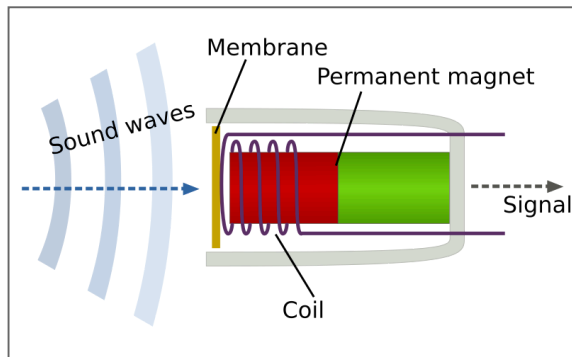


## 2.6. Electromagnets, induction & inductance



### How does a microphone work?



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## Interaction of coil & magnet

- previous lecture we used **constant** magnetic field to understand:
  1. electric current produces magnetic field
  2. magnetic field exerts a force on electric currents/moving electric particles

### **What happens when we change the magnetic field over time?**

em18 - magnet drop

- Joseph Henry (1797-1878) & Michael Faraday (1791-1867) described independently that **changing magnetic field induces an electric field**
- **concepts to understand:**
  - alternating voltage/current
  - electromagnetic induction
  - inductance & electromagnets

What influences the induced voltage ? (1/2)

emf - flux

**Impact of geometry and orientation**

## The flux strikes back: Magnetic flux

simulation mag. flux

- magnetic flux  $\Phi_B$  defined as:

$$\Phi_B = \int \vec{\mathbf{B}} d\vec{\mathbf{A}}$$

- here  $\vec{\mathbf{A}}$  is the vector normal to the area  $A$
- in contrast to the electric flux  $\Phi_E$ , "regular" integral instead of closed surface integral
- for a **uniform field** simplifies to:

$$\Phi_B = B_{\perp} A = BA \cos \theta = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$$

- measured in *weber* [Wb] = [T m<sup>2</sup>]

What influences the induced voltage ? (2/2)

em12 - flux

### **Impact of change over time and number of loops**

- **only changing magnetic fields induce a voltage**
- rate of change scales with induced voltage, i.e.  $\frac{d\Phi_B}{dt}$
- for same  $\frac{d\Phi_B}{dt}$ , coil with more loops shows higher induced voltage

## Faraday's law of induction

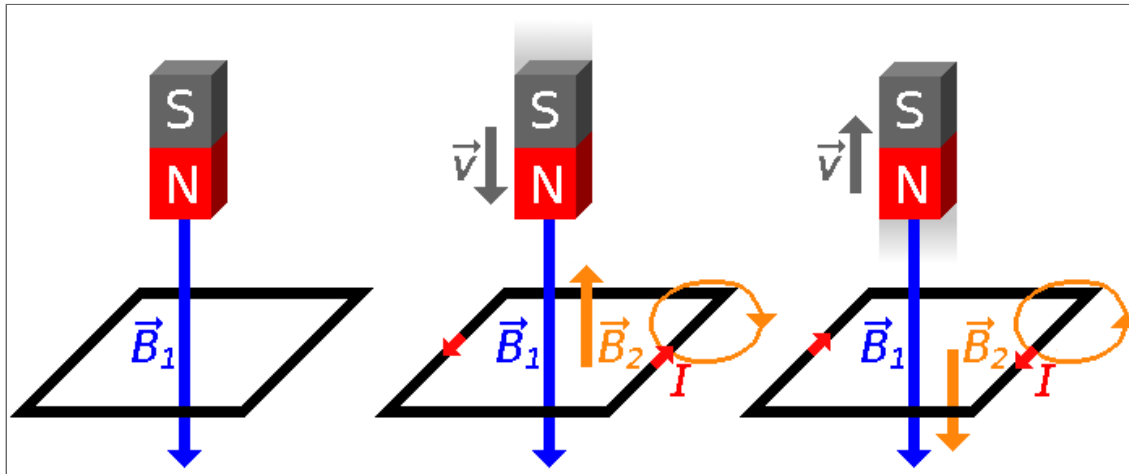
- $V_{ind} = -N \frac{d\Phi_B}{dt}$
- with:
  - N - number of loops in coil
  - $\frac{d\Phi_B}{dt}$  - change in magnetic flux

**Why is there a minus?**

## Lenz's law

em14 - Rings

**A change in the magnetic flux through a conducting loop induces a voltage, so that the resulting current generates a magnetic field that opposes the change in magnetic flux.**



from [wikipedia](#) under [CC0 1.0 Universal](#); image was edited



## E-field from electrostatics vs. electromagnetism

- remember electrostatics: **static charges generate electric field**:
  - $V_{AB} = \int_a^b \vec{\mathbf{E}} d\vec{\mathbf{l}}$
  - thus, for closed path,  $V = \oint \vec{\mathbf{E}} d\vec{\mathbf{l}} = 0 \rightarrow$  work is zero
  - **conservative**  $\rightarrow$  path independence & can define potential energy
- **changing magnetic flux induce voltage that generate an electric field**:
  - voltage induced by changing magnetic flux is:  $V_{ind} = -\frac{d\Phi_B}{dt}$
  - work per unit charge due to induced voltage:  $V_{ind} = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$
  - **general form of Faraday's law**  $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d\Phi_B}{dt}$
  - work for closed path not necessarily zero  $\rightarrow$  **non-conservative**
  - changing magnetic field induces **electric field with closed field lines**

## Eddy currents

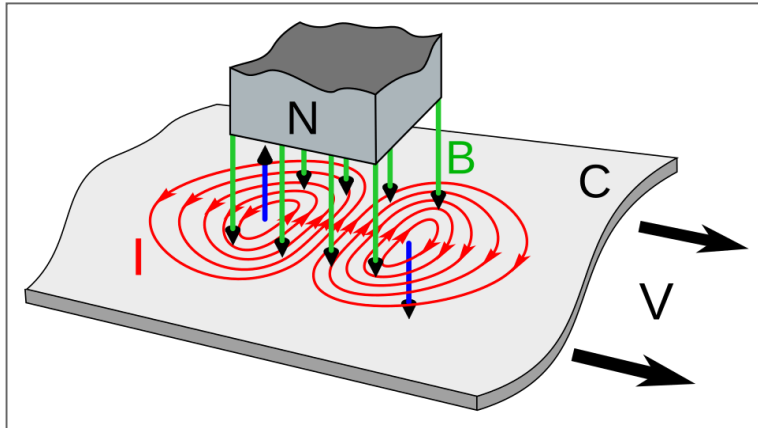
### em25 - Waltenhofen's pendulum

- conductor (pendulum) moves through magnetic field generated by an electromagnet
- → changing magnetic flux in conductor → voltage induced
- part leaving the electromagnetic field:
  - decrease in magnetic flux (number of field lines passing through)
  - generates a field that repels this motion (Lenz's law), i.e. field generated that **attracts** towards the electromagnet
- part entering the electromagnetic field:
  - increase in magnetic flux (number of field lines passing through)

- generates a field that repels this motion (Lenz's law), i.e. field generated that **repels** towards the electromagnet

## Eddy currents (cont')

em25 - Waltenhofen's pendulum



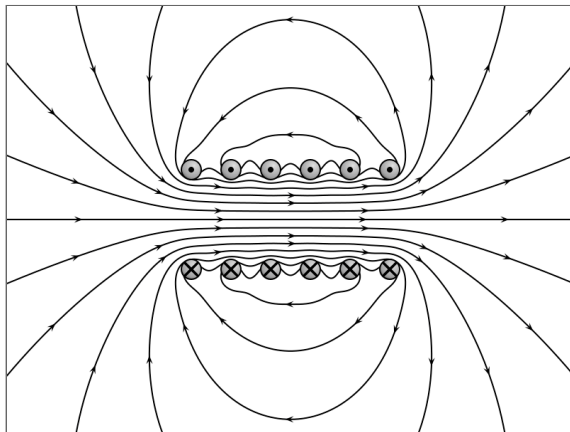
from [wikipedia](#) under [CC0 1.0 Universal](#)

- direction of induced current via RHR-1'

## Electromagnets - solenoids

### em10 + simulation B-field solenoid

- tightly wound helical coils of wire
- found in many devices: generators, loudspeakers, MRI scanners, and many more
- produce an almost uniform magnetic field inside the coil and weak field outside
- with loop density  $n$ , the field is:  $B = \mu_0 \frac{N}{l} I = \mu_0 n I$



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## AC electric generator

em38

- convert mechanical energy into electrical energy via a **rotating armature** in a **static magnetic field**
- assuming a uniform magnetic field, **induced voltage in rotating coil** is:

$$V_{ind} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \left( \int \vec{\mathbf{B}} d\vec{\mathbf{A}} \right) = -\frac{d}{dt} (BA \cos \theta)$$

- **only angle  $\theta$  is time dependent**
- assuming initial angle  $\theta_0 = 0$  and constant rotation velocity  $\omega = \frac{d\theta}{dt}$ :

$$V_{ind} = -BA \frac{d \cos(\omega t)}{dt} = BA\omega \sin(\omega t)$$

- for  $N$  loops and maximally induced voltage  $V_{ind,0} = NBA\omega$ :

$$V_{ind} = NBA\omega \sin(\omega t) = V_{ind,0} \sin(\omega t)$$

- principle can be **reversed** to work as **electric motors**



## Primer on alternating current (AC)

- AC == time varying voltage & current
- for sinusoidal behavior, with  $\omega = 2\pi f$ , voltage alternates between  $+V_0$  and  $-V_0$  described as:

$$V(t) = V_0 \sin(2\pi ft) = V_0 \sin(\omega t)$$

- current is given by:

$$I(t) = \frac{V(t)}{R} = I_0 \sin(\omega t)$$

## Using Root-Mean-Squared (RMS) values

- often RMS values are used in AC:

$$V_{\text{RMS}} = \sqrt{V(t)^2} = \sqrt{V_0^2 \sin^2(\omega t)}$$

- with  $\sin^2(\omega t) = \frac{1}{2}$

$$V_{\text{RMS}} = \frac{V_0}{\sqrt{2}}$$

$$I_{\text{RMS}} = \frac{I_0}{\sqrt{2}}$$

- for example, a 220V outlet has a peak voltage  $V_0 \approx 311 \text{ V}$

## Power in AC circuits

- power is  $P(t) = I(t)V(t)$
- for sinusoidal current/voltage, average power over a period is **non-zero**:

$$\bar{P} = I_0 \sin(\omega t) V_0 \sin(\omega t) = I_0 V_0 \sin^2(\omega t)$$

- for one period  $\sin^2(\omega t) = \frac{1}{2}$ ,  $I_{RMS} = \frac{I_0}{\sqrt{2}}$  and  $V_{RMS} = \frac{V_0}{\sqrt{2}}$

$$\bar{P} = \sqrt{2}I_{RMS}\sqrt{2}V_{RMS}\frac{1}{2} = I_{RMS}V_{RMS}$$

# Transformers

em46

- transformer consists of a *primary* and *secondary* coil
- two coils **do not** form a closed circuit, but only their proximity is essential for the transformer to work

**What influences the voltage in the secondary coil?**

## Transformers - What influences the voltage in the secondary coil?

- iron core (geometry, layered, etc.)
- ratio of loops:
  - without energy loss, the induced voltages in both coils are:

$$V_2 = N_2 \frac{d\Phi_B}{dt} \quad \& \quad V_1 = N_1 \frac{d\Phi_B}{dt}$$

- taking the voltage ratio & using conservation of power,  $P_1 = P_2$ , we get:

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

- depending on the ratio of the loop numbers:
  - step-up transformer: For  $N_2 > N_1$  the voltage  $V_2$  will be larger than  $V_1$

- step-down transformer: For  $N_2 < N_1$  the voltage  $V_2$  will be smaller than  $V_1$

## Mutual inductance

- relates the induced voltage in one coil to the rate of change of current in a neighboring coil
- defined as proportionality constant:

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1}$$

- the induced voltage in one coil can directly be related to change in current in the other coil:

$$V_2 = -N_2 \frac{d\Phi_{21}}{dt} = -M_{21} \frac{dI_1}{dt} \quad \& \quad V_1 = -M_{12} \frac{dI_2}{dt}$$

- mutual inductance only depends on geometric variables, i.e. the spacing between the coils and their individual shape, size, and number of loops:

$$M = M_{21} = M_{12}$$

- **inductance** is expressed in **units of henry**  $[H] = [V \text{ s} / A] = [\Omega \text{ s}]$



## Self-inductance & inductors

### YouTube - James Bond

em13

- even for a **single coil**, we find the phenomenon of inductance, a.k.a **self-inductance**
- **changing current** passes through the coil **induces voltage in coil itself** with **opposed direction**
- defined as proportionality constant (depends on coil geometry):

$$L = \frac{N\Phi_B}{I}$$

$$V_{ind} = -N \frac{d\Phi_B}{dt} = -L \frac{dI}{dt}$$

- often referred simply to as inductance and measured in henry [H].

- **inductors are fundamental circuit elements alongside resistors and capacitors**

## Energy stored in the magnetic field

em17 - Thomson

- power supplied to an inductor is:

$$P = I V_{\text{ind}} = I L \frac{dI}{dt}$$

- the work done to build up the current is:

$$dW = P dt = I L \frac{dI}{dt} dt = I L dI$$

$$W = \int_0^I I L dI = \frac{1}{2} L I^2$$

- energy stored in the magnetic field is:

$$U = \frac{1}{2} L I^2$$

- analogous to the energy stored in a capacitor:

$$U = \frac{1}{2}CV^2$$