Lecture tutorial 2E

LC parallel circuit

`ew06`

Voltage and current characterized by parallel circuit design

- $V(t) = V_0 \cos(\omega t) = V_0 \cos(2\pi f t)$
- $V_{L\|C} = V_L = V_C$
- $I = I_C + I_L$ (Kirchhoff's node rule)

Reactance & impedance

In general:

- $Z_C = \frac{1}{j\omega C}$
- $Z_L = j\omega L$

The impedance of the circuit is therefore (remember rule for resistance/impedance in parallel):

$$\frac{1}{Z} = \frac{1}{Z_C} + \frac{1}{Z_L}$$

$$\frac{1}{Z} = \frac{1}{\frac{1}{j\omega C}} + \frac{1}{j\omega L} = j\omega C + \frac{1}{j\omega L}$$

- we perform at trick based on $j^2 = -1$:

$$\frac{1}{j} = \frac{1 \cdot j}{j \cdot j} = \frac{j}{-1} = -j$$

- thus, we can rearrange the inductive part:

$$\frac{1}{Z} = j\omega C - \frac{j}{\omega L} = j(\omega C - \frac{1}{\omega L})$$

- take the reciprocal

$$Z = \frac{1}{j(\omega C - \frac{1}{\omega L})} = \frac{-j}{(\omega C - \frac{1}{\omega L})}$$

Phasor diagram

- parallel circuit, so sensible to **use $V_{L\|C}$ as reference**
- at **capacitor**: current I_C **lags 90° behind $V_{L\|C}$**
- at **inductor**: current I_L **lead 90° before $V_{L\|C}$**
- **$I_{L\|C}$: vector product of I_C and I_L**

Frequency behavior & (anti-)resonance

$$Z = \frac{1}{j(\omega C - \frac{1}{\omega L})}$$

- **if** $\omega C = -\frac{1}{\omega L}$ the denominate goes to zero and therefore, the **impedance to infinity** $\rightarrow$ so-called, **anti-resonance**
- the resonance frequency therefore is:

$$\omega C - \frac{1}{\omega L} = 0$$

$$\omega C = \frac{1}{\omega L}$$

$$\omega^2 C = \frac{1}{L}$$

$$\omega^2 = \frac{1}{LC}$$

$$f = \sqrt{\frac{1}{2\pi LC}}$$

Lissajous curve

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Lissajous Curve

- A **Lissajous curve** is formed by the **superposition of two perpendicular harmonic oscillations**, typically represented as:

$$x(t) = A \sin(\omega_x t + \delta), \quad y(t) = B \sin(\omega_y t)$$

  - A, B as the amplitudes
  - ω_x, ω_y as the angular frequencies,
  - δ is the phase difference between the oscillations

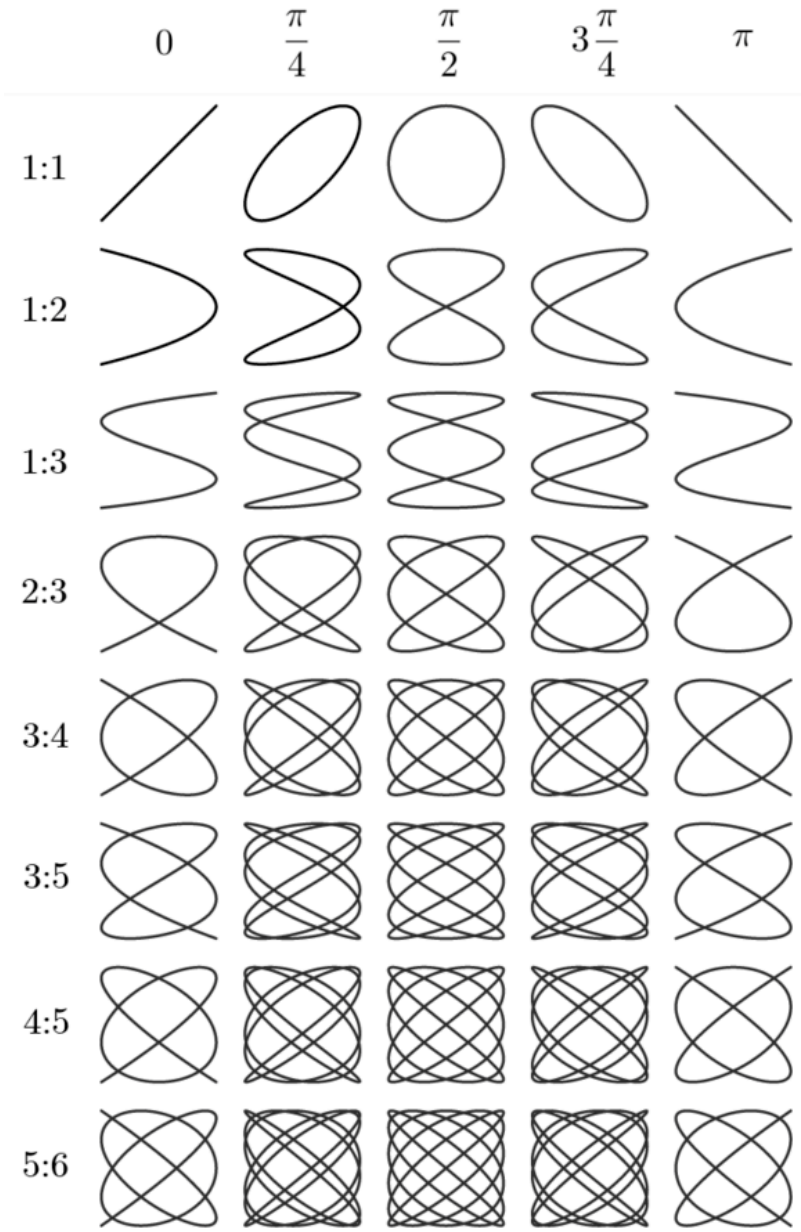
- The **ratio of the frequencies** $\frac{\omega_x}{\omega_y}$ determines the **shape and number of loops** (or "knots") in the figure.

  - If the ratio is rational (e.g., 1:1, 2:3), the pattern is **closed** and periodic.
  - If irrational, the figure never exactly closes and densely fills a region.

- The **relative phase** δ affects the **orientation and symmetry** of the figure:

  - $\delta = 0$ or π: the curve is symmetric and aligned with the axes
  - $\delta = \frac{\pi}{2}$: the figure is often a ellipse (or circle if amplitudes are equal).
  - Varying δ smoothly rotates or skews the figure.

- Lissajous figures are often visualized on oscilloscopes using **XY mode**, where one channel drives horizontal deflection and the other vertical deflection.



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Microwaves & waveguides

`ew13+ew14+ew15`

- Microwaves propagate inside a waveguide via reflections and interference
- This forms TE (Transverse Electric) and TM (Transverse Magnetic) modes, depending on the field configuration (waveguides do not support TEM (Transverse Electromagnetic) modes).
- Only waves above the cutoff frequency f_c can travel: $f_c = \frac{c}{2a}$ with
  - c - speed of light
  - a - wider dimension of the waveguide cross-section
- Waveguide confines and efficiently directs the energy with minimal loss.