2.2. Electric Flux & Electric Potential

Why do birds sitting on high voltage power lines not get electrocuted?
`es03`

- need to understand:
    - concept of electric potential and voltage
    - relation to electric potential energy
    - deeper understanding of electric fields, i.e. electric flux & Gauss's law
- start by revisiting electric field experimentally `es05`

Electric flux for uniform electric field

`script simulation: rectangle in field`

- For a uniform electric field, the electric flux is defined as

$$\Phi_E = \vec{\mathbf{E}}\vec{\mathbf{A}}$$

  where $\vec{\mathbf{A}}$ is the vector perpendicular to the surface with magnitude A
- Using $\cos\theta$ as the angle between $\vec{\mathbf{E}}$ and $\vec{\mathbf{A}}$, the equation can be rewritten as

$$\Phi_E = EA\cos\theta = E_\perp A = EA_\perp$$

- The number of field lines N passing through an area perpendicular to the field is proportional to the electric flux

$$N \propto E_{\perp} A = \Phi_E$$

Electric flux for non-uniform electric field

- arbitrary surface can be decomposed into infinitesimal areas $d\vec{\mathbf{A}}$
- for each $d\vec{\mathbf{A}}$, associated field is uniform
- integral over closed surface gives the **total flux though closed surface**:

$$\Phi_E = \oint \vec{\mathbf{E}}\, d\vec{\mathbf{A}}$$

- **by conventions**
  - $d\vec{\mathbf{A}}$ points outwards from the surface of the enclosed volume
  - flux leaving the surface is positive
  - flux entering the surface is negative
- **consequences on net flux**:
  - if Φ_E is positive, there is a net flux out of the volume
  - if Φ_E is negative, there is a net flux into the volume
  - if $\Phi_E = 0$, there is no net flux

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script simulation circle in field
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Gauss's law

- named after Karl Friedrich Gauss (1777-1855)
- **Gauss's law**: **Electric flux through a closed surface is equal to the net enclosed charge divided by the permittivity of free space**:

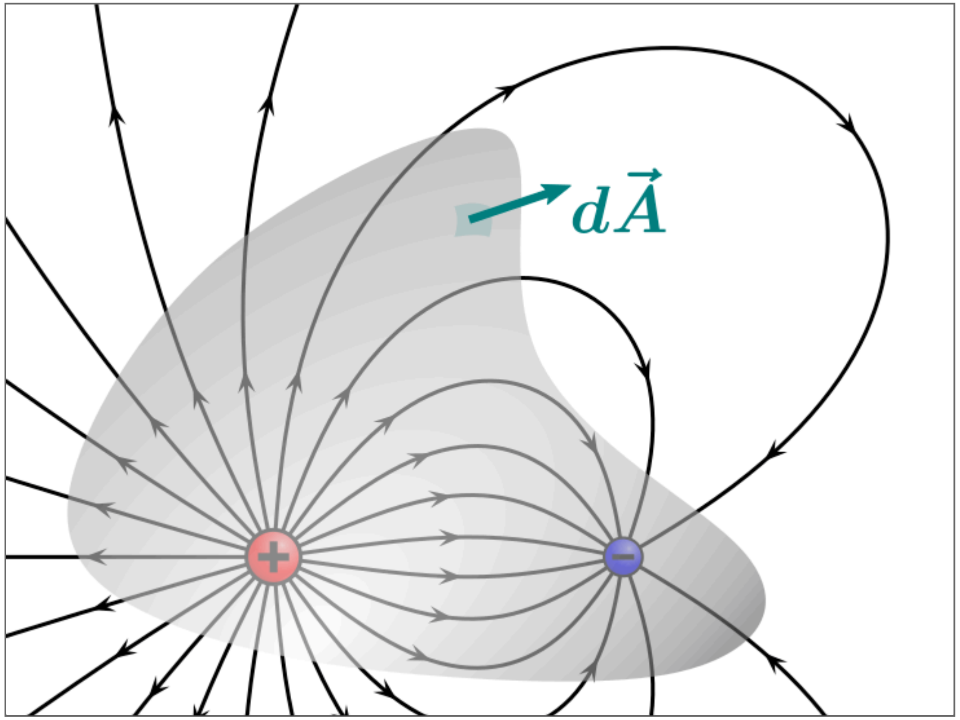
$$\Phi_E = \oint \vec{\mathbf{E}}\, d\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0}$$

- flux through surface is **independent** of:
  - distribution of the enclosed charge within the volume
  - charges outside the surface that may affect the position but not the number of field lines
- **principle of superposition applicable**:

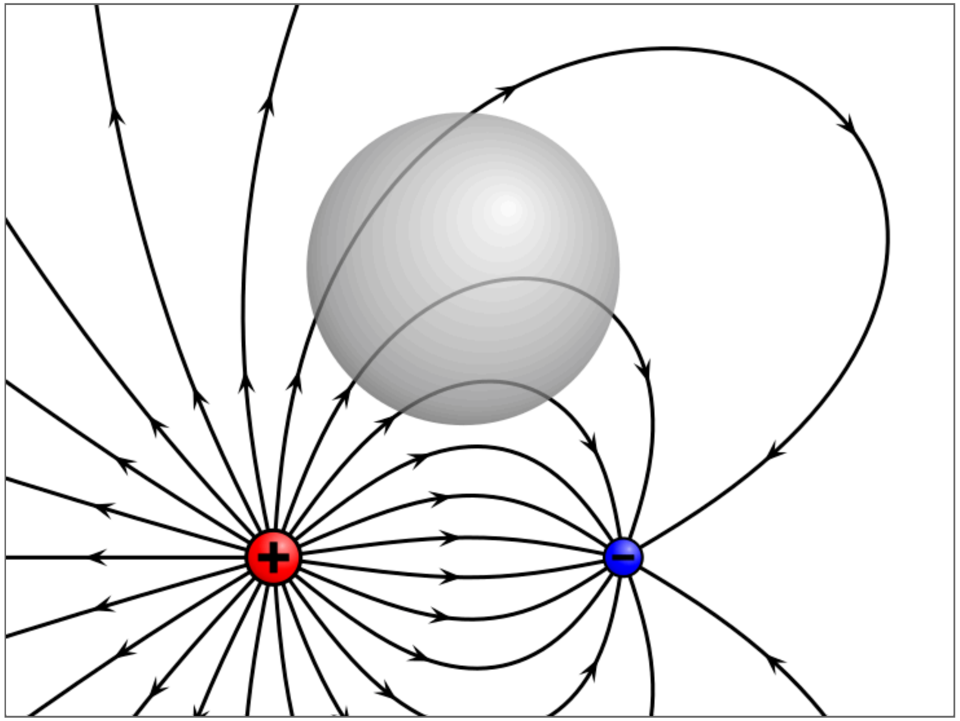
$$\oint \vec{\mathbf{E}}\, d\vec{\mathbf{A}} = \oint \left(\sum_i \vec{\mathbf{E}}_i \right) d\vec{\mathbf{A}} = \sum_i \frac{Q_i}{\epsilon_0} = \frac{Q_{enc}}{\epsilon_0}$$

Examples for Gauss's law

$$\Phi_E = \oint \vec{\mathbf{E}}\, d\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0} > 0$$



$$\Phi_E = \oint \vec{\mathbf{E}}\, d\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0} = 0$$



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Gauss's law: Relation to Coulomb's law

`thought experiment`

- consider a spherical surface (radius r) with a single enclosed charge Q_{enc} at the center
- the $\vec{\mathbf{E}}$ field is oriented radially with the same magnitude everywhere on the surface
- field lines penetrate the spherical surface perpendicular to $d\vec{\mathbf{A}}$
- surface area of the sphere is $\oint dA = 4\pi r^2$
- This leads to Coulomb's law in electric field form:

$$\frac{Q_{enc}}{\epsilon_0} = \oint \vec{\mathbf{E}} d\vec{\mathbf{A}} = E \oint dA = 4\pi r^2 E$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \quad \text{with} \quad Q_{enc} = Q$$

Complementary nature of Coulomb's and Gauss's law

	Coulomb's Law	**Gauss's Law**
Equation	$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \leftrightarrow F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}$	$\oint \vec{\mathbf{E}}\, d\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0}$
Focus	Field/force & (point) charges	Flux through surface & enclosed net charge
Applicability	Point charges, any distribution (complex)	Mostly used for symmetrical distributions, although applicable to any case
Relevant Charge Sources	All charges present contribute	Only enclosed charges contribute to flux calculation

Electric potential energy

- concept of **conservation of energy** in electricity is analogous to mechanics
- **electric potential energy is defined for conservative forces**, e.g. the electrostatic force $\vec{\mathbf{F}} = \vec{\mathbf{E}}q$ (conservative force $\rightarrow$ path independence)
- **in uniform electric field**, the **work** to move a test charge q over a distance d is:

$$W = Fd = qEd$$

- change in **potential energy** ΔU from point A to B is:

$$\Delta U = U_B = -W$$

- note the similarity to mechanics, i.e. mgh, but as we have two types of charge:

- **$\rightarrow$ the closer a charge is to a charge of the same polarity, the higher the electrostatic force, thus, the higher the electric potential energy**

Voltage: The difference in electric potential

- **electric potential** V is defined as the electric potential energy per unit charge

$$V = \frac{U}{q}$$

- **voltage is the difference in potential** between points A and B:

$$V_{BA} = V_B - V_A \quad \text{in } [\mathrm{J/C}] = [\mathrm{V}]$$

- acknowledging Alessandro Volta, the unit is called **volt** [V]
- **reference point** (typically ground or infinity) is chosen where $V = 0$

Putting it all together

- **electric potential** is defined at a **single point** in space
- only potential **differences**, i.e. voltage, are **measurable**
- **reference point** with zero potential is **defined arbitrarily**
- combining the definitions gives

$$V_{BA} = V_B - V_A = \frac{U_B}{q} - \frac{U_A}{q} = \frac{\Delta U_{BA}}{q} = -\frac{W_{BA}}{q}$$

- **work** performed on a charge is given by

$$W_{BA} = Fd = -qV_{BA}$$

- $\rightarrow$ **voltage is a measure of the work a charge can perform**

The electron volt

- electron volt (eV) is the energy gained by an electron when moving through a potential difference of 1 V

$$1\,\mathrm{eV} \approx 1.60 \times 10^{-19}\,\mathrm{J}$$

- handy when dealing with small energies, i.e. charged particles

Relating electric potential & electric field

- analogously to mechanics, change in potential energy ΔU_{BA} from A to be B along the path $\vec{\mathbf{l}}$ is defined as:

$$\Delta U_{BA} = -\int_A^B \vec{\mathbf{F}} d\vec{\mathbf{l}}$$

- we know $V = \frac{U}{q}$ and $\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} \leftrightarrow \vec{\mathbf{F}} = \vec{\mathbf{E}}q$, therefore:

$$V_{BA} = -\frac{1}{q} q \int_A^B \vec{\mathbf{E}} d\vec{\mathbf{l}}$$

$$V_{BA} = -\int_A^B \vec{\mathbf{E}} d\vec{\mathbf{l}}$$

Relating electric potential & electric field (cont'd)

$$V_{BA} = -\int_A^B \vec{\mathbf{E}} d\vec{\mathbf{l}}$$

- in **uniform electric field**, the integral simplifies to

$$V_{BA} = -Ed \quad \text{with distance}\, d$$

- for an infinitesimal change, the relation is

$$dV = -\vec{\mathbf{E}}\, d\vec{\mathbf{l}}$$

- therefore, electric field is the gradient of the electric potential:

$$\vec{\mathbf{E}} = -\mathrm{grad} V = -\nabla V$$

Equipotential lines & surfaces

`script simulation: equipotential`

- equipotential lines (2D) and surfaces (3D) represent regions where the **electric potential is constant**
- **moving perpendicular** to the electric field ($\vec{\mathbf{E}}\perp d\vec{\mathbf{l}}$) does not change the potential

Unconventional ways to make light

`es31`