# 2.8. Maxwell's equations & electromagnetic waves

**Why does holding a car key to the head increase the range to open the car?**
**[YouTube - Top Gear]**

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- key fob **transmits** electromagnetic (EM) waves
- $\rightarrow$ we need to **understand EM waves**

# Maxwell & electromagnetism

- Initially, **understanding of electricity and magnetism** (Oersted, Ampère, and Faraday) was **fragmented**, with phenomena being studied in isolation
- **James Clerk Maxwell** (1831–1879):
    - **unified electricity and magnetism** through four fundamental equations $\rightarrow$ Maxwell’s equations
    - used **concept of fields** (introduced by Faraday)
    - predicted the existence of **electromagnetic waves**, which were later experimentally confirmed by *Heinrich Hertz* (1857-1894)

## Maxwell’s correction of Ampère’s law: Recap what we know

We already know that:

1. time-constant electric currents generate time-constant magnetic fields (first observed by Ørsted & Ampère's law)

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{\text{enc}}$$

2. changing magnetic fields generate electric fields (Faraday's law)

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d\Phi_B}{dt}$$

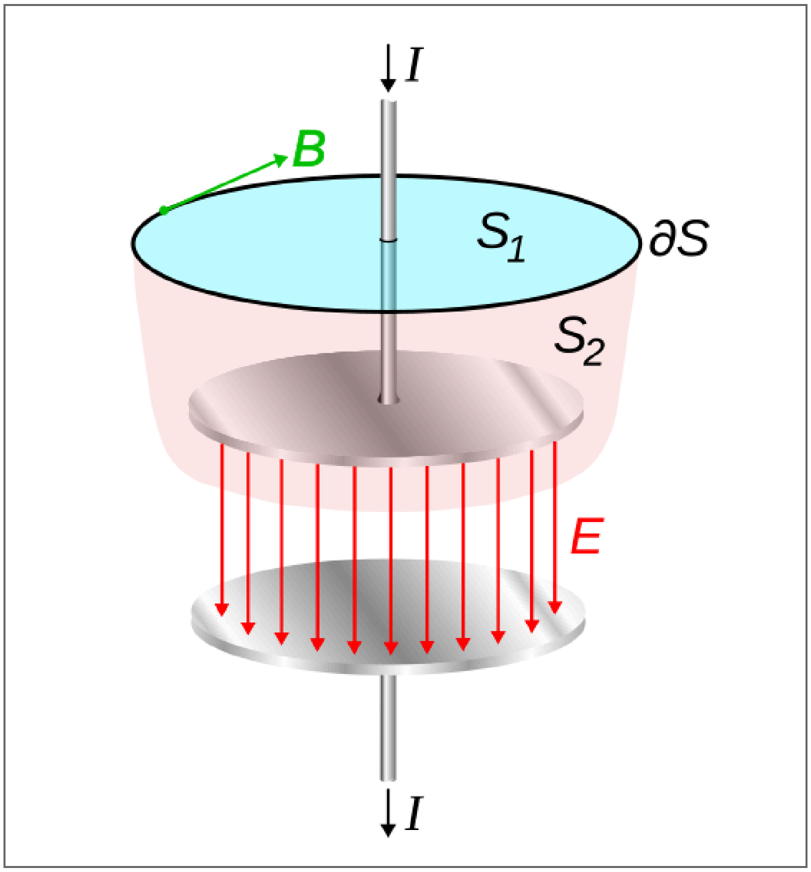
$\rightarrow$ Maxwell's contribution/idea: **changing electric field produces a magnetic field**, leading to the concept of **displacement current**

# Maxwell’s correction of Ampère’s law: The need for displacement current

- Ampère's law: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{\text{enc}}$
  - $\rightarrow$ **magnetic field $\vec{\mathrm{B}}$ around a closed loop is proportional to enclosed current $I_{\text{enc}}$**
- Maxwell realized there are **scenarios where Ampère's law fails**:
  - imaging charing capacitor:
    - during charging, current in wire generates magnetic field
    - no charges pass gap between plates

# Maxwell's correction of Ampère's law: The need for displacement current (cont')

- **important** $\oint$ is a line integral over the closed path, but $I_{\text{enc}}$ considers the current through the surface bound by the closed path
- for surface $S_1$: **everything works**, enclosed current generates magnetic field $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{\text{enc}} \to B = \frac{\mu_0 I_{\text{enc}}}{2\pi r}$
- for surface $S_2$: **fails because no current through** $S_2$: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} \neq 0$ but $\mu_0 I_{\text{enc}} = 0$

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# Maxwell’s correction of Ampère’s law:
# Introducing the displacement current

- Maxwell resolved issue by introducing **displacement current** $I_D$ with $Q = CV = \epsilon_0 \frac{A}{d} Ed = \epsilon_0 AE$:

$$I_D = \frac{dQ}{dt} = \epsilon_0 \frac{AdE}{dt} = \epsilon_0 \frac{d\Phi_e}{dt}$$

- displacement current is **not a real flow of charge**, but it represents the effect of a time-varying electric field to generate the observed magnetic field
- **Ampère's law with displacement current:**

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{\text{enc}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

# Physics & symmetry (obsession): Gauss's law for magnetism

`sim - Gausss' laws`

- **we already know**, the electric flux through a closed surface is

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

- **since no magnetic monopoles exist**, the magnetic flux through a close surface is

$$\Phi_B = \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

# Putting everything together: Maxwell's equations

- **4 equations to unify electricity and magnetism into electromagnetism**
    - Gauss's law for electricity:

      $$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0}$$

    - Gauss's law for magnetism:

      $$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

    - Faraday's law of induction:

      $$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d\Phi_B}{dt}$$

    - Ampère's law with Maxwell correction:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

# Maxwell's Equations: (1) Gauss's law for electricity

$$\oint \vec{\mathbf{E}} d\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0}$$

This states that **electric charges produce electric fields**, and the **total electric flux** of $\mathbf{E}$ **through a closed surface** is **proportional to the charge enclosed**.

```
Note: We take the integral over a closed surface.
```

# Maxwell's Equations: (2) Gauss's law for magnetism

$$\oint \vec{\mathbf{B}} d\vec{\mathbf{A}} = 0$$

This implies that magnetic field lines have no beginning or end, meaning **no magnetic monopoles exist**.

`Note: We take the integral over a closed surface.`

# Maxwell's Equations: (3) Faraday's law of induction:

$$\oint \vec{\mathbf{E}} d\vec{\mathbf{l}} = -\frac{d\Phi_B}{dt}$$

This describes how a **changing magnetic field induces an electric field**, which is the principle behind electrical generators.

```
Note: We take the integral around a closed path.
```

# Maxwell’s Equations: (4) Ampère’s law with Maxwell correction:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{\text{enc}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

This states that an **electric current and/or a changing electric field generates a magnetic field**.

```
Note: We take the integral around a closed path.
```

## Significance of Maxwell’s equations

1. **Unification of electricity and magnetism:** Electricity and magnetism are intrinsically linked
2. **Prediction of electromagnetic waves and light:** Predicted existence of electromagnetic waves that propagate at a speed calculated from $\epsilon_0$ and $\mu_0$ (speed of light). Light itself is an electromagnetic wave, i.e. unifying optics with electromagnetism.
3. **Foundation of Classical Electromagnetism:** Maxwell's equations form the complete and consistent foundation of classical electromagnetism and describe all classical electromagnetic phenomena.
4. **Basis for Special Relativity:** While formulated within classical physics, Maxwell's equations played crucial role in development of Special Relativity, i.e. the fact that speed of light is constant for all observers.

# Electromagnetic waves:

`ew11 + ew23 - transmit / receive EM waves`

- let's start with an experiment and observe
- **antenna length and orientation** seems to play a role, **why?**

## Electromagnetic waves: Propagation without matter

- Maxwell equations for empty space
- no charges $\rightarrow$ **closed E-field lines** & **only displacement current**

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

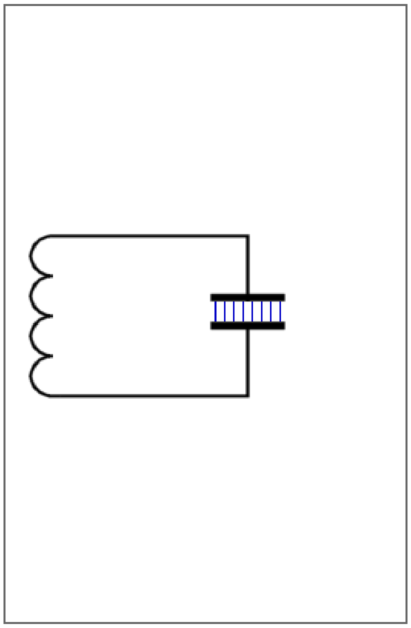
$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d\Phi_B}{dt}$$

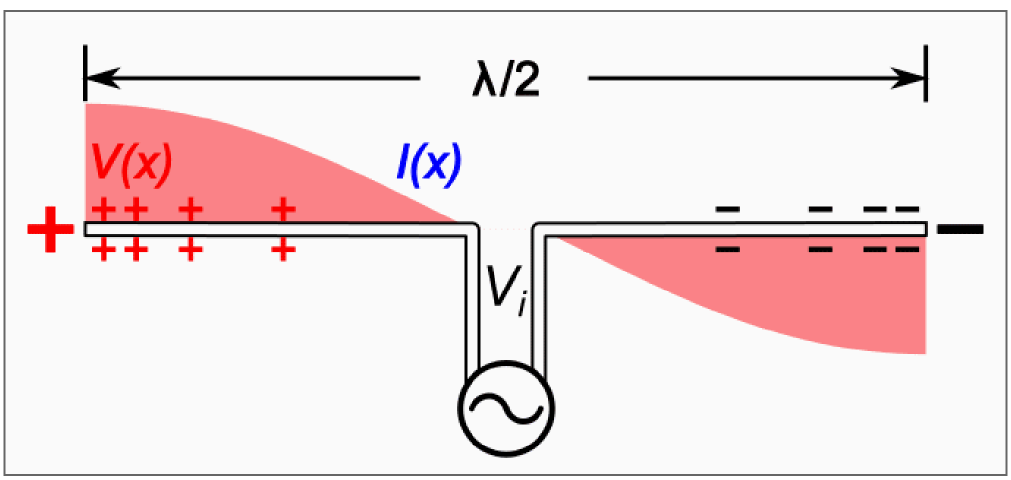
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

**changing electric field produces a changing magnetic field and vice versa** $\rightarrow$ **wave propagation**

# Electromagnetic waves: Transmission by dipole antenna

- in essence LC-circuit with AC supply





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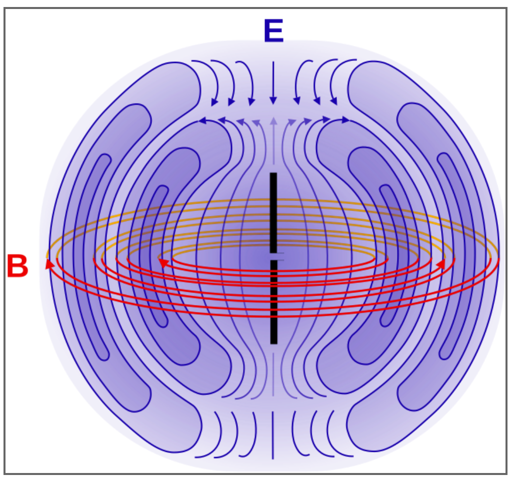
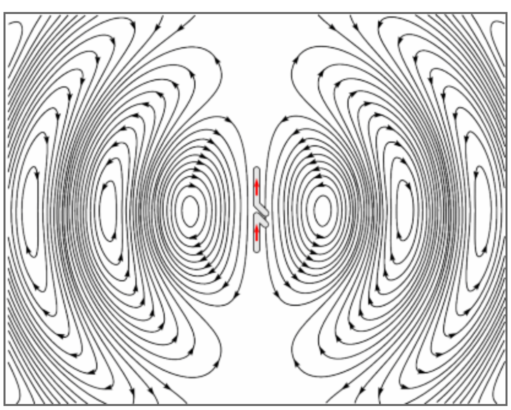
# Electromagnetic waves: Transmission by dipole antenna (cont')

- while complex to describe in the near-field, EM-waves can be are reasonably flat in the far field
- $\rightarrow$ EM-waves are **plane waves** in the radiation/far field



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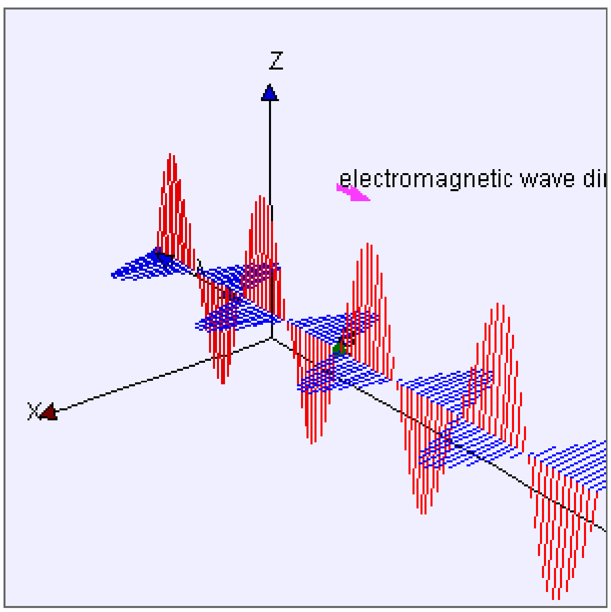
# Electromagnetic waves: Transversal waves

- by definition $\vec{\mathbf{E}}$ perpendicular to $\vec{\mathbf{B}}$ $\rightarrow$ **wave travels perpendicular to E- and B-field** ( $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$)
- $\rightarrow$ EM waves are **transversal waves**
- further, B and E are **in phase**

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# Electromagnetic waves: Traveling, sinusoidal waves

- EM waves with wavelength $\lambda$ generated by sinusoidal oscillations at antenna
- in general, traveling wave with velocity $\nu = f\lambda = \frac{\omega}{k}$, angular frequency $\omega = 2\pi f$, and wave number $k = \frac{2\pi}{\lambda}$:

$$D(x,t) = A\sin\Big(\frac{2\pi}{\lambda}(x - \nu t)\Big) = A\sin(kx - \omega t)$$

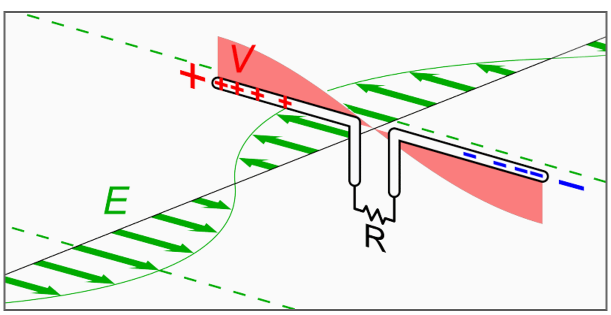
- sinusoidal description of plane EM waves:
  - $E = E_0 \sin(kx - \omega t)$ with the E-field oscillating in $y$ direction
  - $B = B_0 \sin(kx - \omega t)$ with the B-field oscillating in $z$ direction

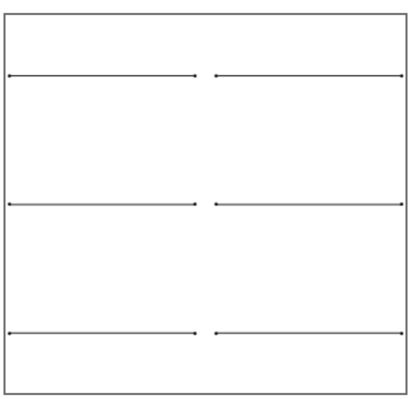
*Note: waves moving to the right, i.e.* $(kx - \omega t)$*, waves moving to the left, i.e.* $(kx + \omega t)$

# Electromagnetic waves: Receiving & wavelength

`recap ew11 + ew23`

- EM wave needs to **induce voltage** in antenna to be "received"
- alternating E-field of EM-waves causes electric oscillations in antenna
- dipole antenna **resonant** if is length is approx. half EM wavelength
- at resonance, induced currents **add constructively** (remember standing waves)



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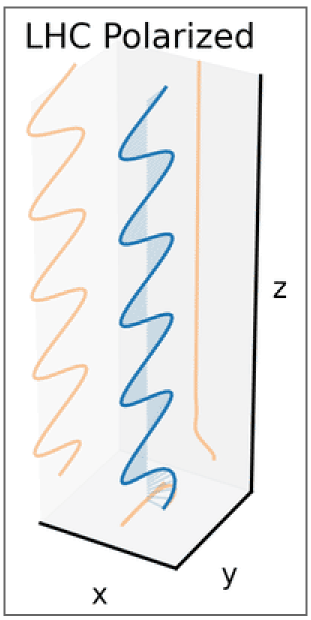
# Electromagnetic waves: Receiving & polarization

`recap ew11 + ew23`

- orientation of E-field to antenna needs to match to induce voltage
- most common polarizations: linear and circular

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# Electromagnetic waves: Velocity & speed of light

`ow28`

- wave velocity defined as:

$$\nu = \frac{\omega}{k} = f\lambda$$

- **is the speed of light constant?**

## Electromagnetic waves: Velocity & speed of light (cont')

- for EM wave in vacuum & since E and B in phase:

$$\nu = \frac{E}{B} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = c$$

- thus, **speed of EM waves in vacuum is the speed of light** and $c = \lambda f = \mathrm{const}$
- for EM wave in matter, things are different:
    - $\nu \leq c$:
    - frequency remains constant
    - wavelength changes $\lambda_n = \frac{v}{f} = \frac{c}{nf} = \frac{\lambda}{n}$

`ew22 - H20 and decimeter waves`

# Revisiting "Why does holding a car key to the head increase the range to open the car?"

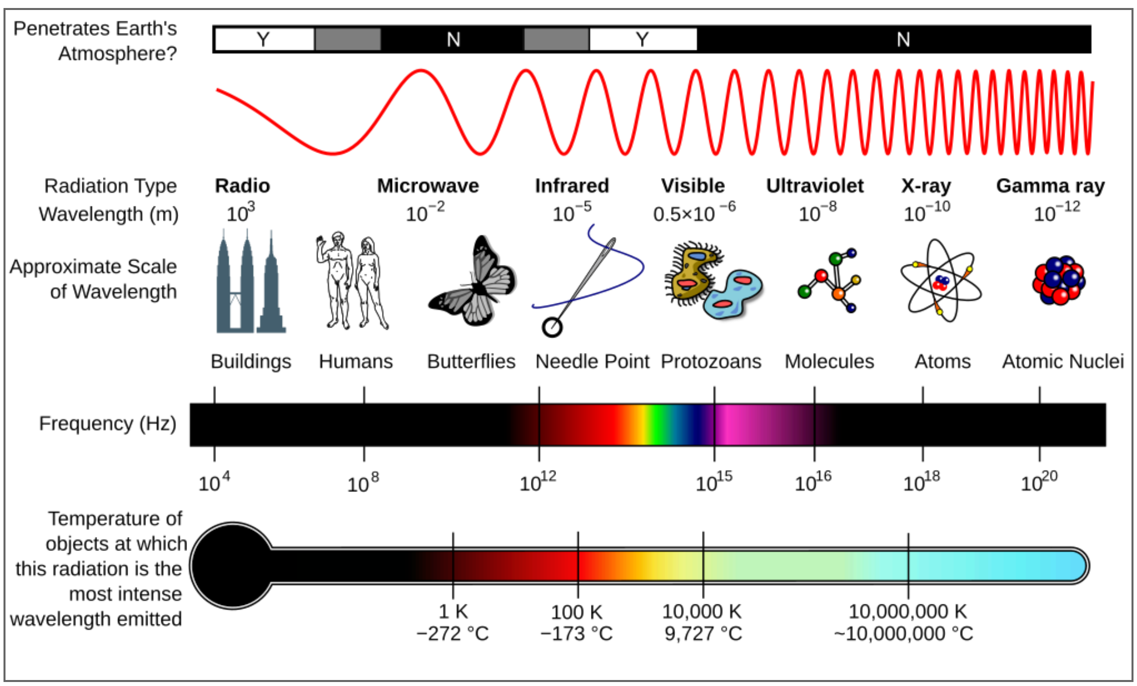
`ew22 - H20 and decimeter waves`

*from **wikipedia** under **CC0 1.0 Universal***

- key fob transmits EM-waves at 300 MHz - 433 MHz
- in air: wave length $\approx 1$ m (@ 300 MHz)
- in water/brain: wave length $\approx 10 - 12$ cm (@ 300 MHz)
- head acts as resonator (constructive interference) to amplify signal/boost range

# Electromagnetic spectrum & light

- all electromagnetic waves propagate at $c = \lambda f = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 300 \times 10^6\,\mathrm{m/s}$
- visible light spans frequencies from $(4.0 - 7.5) \times 10^{14}\,\mathrm{Hz}$ (tera-hertz) and wavelengths between 400 nm and 750 nm

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