1.8. Fluid dynamics

In this chapter we introduce the physics of **fluids in motion**, where velocity fields, pressure variations, and flow patterns evolve in space and time. We begin by characterizing different types of flow and using **streamlines** to visualize motion. We then develop the **continuity equation**, expressing mass conservation in moving fluids, and derive **Bernoulli's equation** as a key relation between pressure, speed, and height in ideal flows. Finally, we examine real-world effects such as **viscosity**, **drag**, and the **Magnus effect**, which arise when internal friction and rotation influence fluid motion.

1.8.1 Fluids in Motion

To describe fluids in motion, we treat the velocity of the fluid as a **vector field**. With one temporal and three spatial dimensions, we can write:

$$\vec{v}(x, y, z, t) = v_x(x, y, z, t)\,\hat{i} + v_y(x, y, z, t)\,\hat{j} + v_z(x, y, z, t)\,\hat{k}.$$

At each point in space and time, the fluid has a velocity vector. This lets us visualize and analyze flow patterns.

Types of Fluid Flow

Fluid motion is commonly classified along two independent axes:

1. **Steady vs. Unsteady Flow**
   - **Steady (Stationary) Flow:** The velocity field does not change with time at any point, i.e. all properties are time-independent.
   - **Unsteady Flow:** The velocity field evolves in time.
2. **Laminar vs. Turbulent Flow**
   - **Laminar Flow:** Smooth, ordered motion; neighboring fluid layers slide past one another without mixing.
   - **Turbulent Flow:** Chaotic motion with eddies, rapid fluctuations in velocity and pressure.
   - **Transitional Flow:** Between laminar and turbulent, occurring near a critical Reynolds number.

Streamlines

A **streamline** is a curve everywhere tangent to the instantaneous velocity field. **Key points** for stream line are:

- Streamlines never intersect.
- Closer spacing means faster flow.
- In **steady** flow, streamlines coincide with the actual particle paths.
- Streamlines exist for any flow, even unsteady or turbulent; they simply change over time.

1.8.2 Continuity Equation

When a fluid flows through a pipe, it transports both **volume** and **mass**. In **steady flow**, whatever enters a section per unit time must also leave it — a direct consequence of **mass conservation**.

Fluid Compressibility

- **Incompressible fluids:** density remains constant $\rightarrow$ both volume and mass flow rate stay constant along the pipe.
- **Compressible fluids:** density may change $\rightarrow$ mass conservation still holds, but not volume conservation.

Volume Flow Rate

Lets define the **volume flow rate** as

$$Q = \frac{dV}{dt} = Av,$$

with A as the cross-sectional area and v as the average speed.

Mass Flow Rate

The **mass flow rate** is defined as

$$\frac{dm}{dt} = \rho\,\frac{dV}{dt} = \rho Av.$$

We can write this more compactly as

$$\dot{m} = \rho Av = \rho Q.$$

For incompressible fluids ($\rho = \text{constant}$), both Q and $\dot{m}$ remain constant along the pipe.

Continuity Equation

Conservation of mass in steady flow requires

$$\dot{m} = \rho Av = \text{constant}.$$

Thus, between two cross-sections:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2.$$

For incompressible fluids ($\rho_1 = \rho_2$):

$$A_1 v_1 = A_2 v_2 = Q = \text{constant}.$$

Narrow sections (small A) increase velocity; wider sections decrease it.

Conceptual Notes

- Q measures **volume per unit time**; $\dot{m} = dm/dt$ measures **mass per unit time**.
- In steady flow, **mass flow rate is constant** along a streamline.
- The continuity equation is simply **mass conservation** applied to moving fluids.
- For incompressible fluids, $Av = \text{constant}$ always holds.

1.8.3 Bernoulli's Equation

Bernoulli's equation is an energy conservation law for **fluids in steady motion**.
Along a streamline, the sum of **pressure energy**, **kinetic energy**, and **gravitational potential energy per unit volume** remains constant.

We assume:

- **steady (stationary) flow** (no time dependence),
- an **incompressible** fluid (density ρ constant),
- **non-viscous** flow (no frictional losses),
- motion **along a streamline**.

Under these ideal conditions, a small fluid volume element carries its mechanical energy unchanged as it moves. Thus, the total mechanical energy per unit volume satisfies

$$E_{\text{pressure}} + E_{\text{kinetic}} + E_{\text{potential}} = \text{constant along a streamline}.$$

Derivation

Consider a small fluid element of volume dV and mass $m = \rho\, dV$. Its total energy can be decomposed in the following components:

- **Kinetic energy:** $K = \frac{1}{2}mv^2 = \frac{1}{2}\rho v^2\, dV$
- **Potential energy:** $U = mgh = \rho gh\, dV$
- **Pressure energy** (work done on the element): Pressure does work $P\, dV$ on the volume element as it moves (important, sign is due to the fact that the work done **on** not **by** the volume element). Thus, $E_{\text{pressure}} = P\, dV$

Since the sum of these energies remains constant along a streamline,

$$P\, dV + \frac{1}{2}\rho v^2\, dV + \rho gh\, dV = \text{constant}.$$

For an incompressible fluid, dV is the same everywhere and nonzero, so we divide by dV:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant along a streamline.}$$

This is the **Bernoulli equation**.

Each term represents **energy per unit volume** (J/m³):

- P — pressure energy density,
- $\frac{1}{2}\rho v^2$ — kinetic energy density,
- $\rho g h$ — gravitational potential energy density.

If the fluid speeds up (larger v), the constant total must be preserved by either a decrease in pressure P or an increase in height h. Conversely, slower regions tend to have higher pressure. This inverse relation between speed and pressure is the essence of **Bernoulli's principle** and explains many effects such as narrowed water jets and spray bottles.

Venturi Effect (Horizontal Flow)

If the fluid flows horizontally so that the height does not change ($h_1 = h_2$), Bernoulli's equation simplifies to

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2,$$

showing that regions of higher flow speed correspond to lower pressure.

In a pipe with a constriction, the continuity equation requires $A_1 v_1 = A_2 v_2$, so the velocity increases where the cross-sectional area decreases. Inserting this into Bernoulli's equation gives

$$\Delta P = P_1 - P_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2).$$

Thus, the pressure drops in the narrower section of the pipe. This relationship forms the basis of Venturi meters, carburetors, and many flow-measurement devices.

Torricelli's Theorem (Pressure–Height Effects)

If the fluid velocity is the same or negligible at two points, Bernoulli's equation reduces to

$$P_1 - P_2 = \rho g(h_2 - h_1),$$

which is the familiar hydrostatic pressure law.

To determine the efflux speed from a small hole in a tank, we apply Bernoulli's equation between two points on the same streamline:
(1) the liquid surface at height h above the hole, where $v_1 \approx 0$ and $P_1 = P_{\text{atm}}$, and
(2) a point just outside the hole at height 0, where $v_2 \neq 0$, and the pressure is also atmospheric $P_2 = P_{\text{atm}}$.

$$P_{\text{atm}} + \rho g h = P_{\text{atm}} + \tfrac{1}{2}\rho v^2.$$

The atmospheric pressures cancel, yielding

$$\rho g h = \tfrac{1}{2}\rho v^2, \qquad \Rightarrow \qquad v = \sqrt{2gh}.$$

This is **Torricelli’s theorem**: the speed of the outflowing jet equals the speed a body would acquire when falling freely through the height h. Note that the height h is the height of liquid above the hole, not the distance of the hole to the ground.

Aside: Why Bernoulli Alone Cannot Explain Airplane Lift

Bernoulli's equation can describe *pressure differences* once the velocity field is known, but it does **not** explain why air moves faster above an airfoil.

Real lift requires:

- viscosity and formation of a boundary layer
- circulation around the wing (Kutta condition)
- asymmetric flow pattern determined by wing shape and angle of attack

Thus, the common “faster flow → lower pressure → lift” explanation is incomplete.

Conceptual Notes

- Bernoulli’s equation expresses **conservation of mechanical energy per unit volume** in an ideal fluid.
- It applies **along a streamline** for **steady, incompressible, non-viscous** flow.
- Faster flow corresponds to **lower pressure** when the height is constant (basis of the Venturi effect).
- Pressure increases with **depth** in a fluid at rest, and Bernoulli reproduces the hydrostatic law.
- Torricelli’s theorem is a direct consequence: a fluid exiting a hole at depth h has speed $v = \sqrt{2gh}$.
- Bernoulli’s equation connects changes in **pressure**, **velocity**, and **height** in a unified framework.

1.8.4. Viscosity

Real fluids exhibit **internal friction** that resists relative motion between adjacent layers. This property, called **viscosity**, quantifies how strongly a fluid opposes shear. It plays a major role in the flow of liquids and gases, lubrication, blood flow, and many industrial processes.

Shear Flow and Definition of Viscosity

Consider a fluid confined between two parallel plates separated by distance d. The lower plate is stationary, while the upper plate moves at speed v under a tangential force $F_\parallel$, thus, $v \parallel F_\parallel$. The fluid layers slide past one another, creating a **velocity gradient**

$$\frac{dv}{dy} = \frac{v}{d},$$

where y measures distance from the stationary plate. This expression simply states that the fluid's velocity increases linearly from 0 to v across the gap of thickness d. The **tangential force per area**, which is the shear stress τ (not confused with torque), required to keep the top plate moving is proportional to this velocity gradient:

$$\frac{F_\parallel}{A} = \eta \frac{dv}{dy},$$

or equivalently,

$$F_\parallel = \eta A \, \frac{v}{d}.$$

The proportionality constant η is the **dynamic viscosity**, which characterizes a fluid's resistance to shear: Large η means strong internal friction. The tangential force you must apply to keep the plate moving **equals the viscous resisting force** exerted by the fluid; **viscosity is the internal friction** opposing the motion of adjacent layers. Note that shear stress $\frac{F_\parallel}{A}$ has the same units as normal stress $\frac{F_\perp}{A}$, but refers to a force **parallel**, not perpendicular, to the area.

The SI unit of viscosity is

$$[\eta] = \mathrm{Pa} \cdot \mathrm{s} = \frac{\mathrm{N} \cdot \mathrm{s}}{\mathrm{m}^2},$$

with common alternatives being **Poise (P):** $1\,\mathrm{P} = 0.1\,\mathrm{Pa} \cdot \mathrm{s}$ and **Centipoise (cP):** $1\,\mathrm{cP} = 10^{-3}\,\mathrm{Pa} \cdot \mathrm{s}$.

Newtonian and Non-Newtonian Fluids

A fluid is **Newtonian** if its viscosity is constant and the shear stress is proportional to the shear rate. Examples include water, air, and many simple liquids. In **Non-Newtonian** fluids, viscosity depends on shear rate or time. Examples include blood, ketchup, toothpaste, polymer melts, cornstarch mixtures.

Parabolic Laminar Flow

The parallel-plate setup from before produced a **linear** velocity profile, because each layer moves at a constant rate relative to the next. The idea extends to **flow in a cylindrical tube** even though there is no physical "moving plate" inside. The tube wall is stationary and enforces the **no-slip condition**: fluid in contact with the wall has zero velocity. Fluid in the center moves fastest. Because viscosity couples neighboring layers, each ring-shaped fluid layer "drags along" the next one. The outer layers move slowly, the inner layers more quickly.

This interaction creates a smooth distribution of velocities that must satisfy both the viscous shear relation and the cylindrical geometry. As a result, the velocity profile becomes **parabolic** rather than linear:

$$v(r) = v_{\max}\left(1 - \frac{r^2}{R^2}\right),$$

with r the distance from the centerline. The curvature of this profile is a **direct consequence** of viscosity and the stationary wall. Thus, laminar tube flow (Poiseuille flow) requires no moving plate inside the tube; the **combination of viscosity and fixed boundaries** automatically generates a shear field in which velocity changes smoothly from zero at the wall to a maximum at the center.

Reynolds Number and Flow Regimes

The **Reynolds number**

$$R = \frac{\rho v D}{\eta}$$

characterizes the relative importance of inertial to viscous forces.

- $R < 2300$: laminar
- $2300 < R < 4000$: transitional
- $R > 4000$: turbulent

Low viscosity increases R and makes turbulence more likely.

Energy Dissipation

Viscosity converts mechanical energy into heat. The rate of viscous dissipation per unit volume is

$$\dot{W} = \eta\left(\frac{dv}{dy}\right)^2.$$

This explains why fluids warm up when stirred or forced through narrow channels.

Temperature Dependence

- **Liquids:** η decreases with increasing temperature; molecules slide more easily

$$\eta = \eta_0 e^{B/T}.$$

- **Gases:** η increases with temperature due to enhanced momentum transfer.

Viscous Drag: Stokes' Law

A small sphere of radius r moving slowly through a viscous fluid experiences a drag force

$$F_D = 6\pi\eta r v,$$

valid for very small Reynolds numbers ($R < 1$). This "creeping flow" regime is dominated entirely by viscous forces, i.e. the viscous forces are overwhelmingly dominant compared to the inertial forces.

The Magnus Effect

A spinning object drags nearby fluid through viscous interaction in the boundary layer, creating asymmetric flow speeds.
The side with faster flow has lower pressure; the side with slower flow has higher pressure.
This pressure difference produces a sideways force — the **Magnus effect** — responsible for curved trajectories of spinning balls.
The effect relies fundamentally on **viscosity**; without viscous adhesion, no lift or side force would develop.

Conceptual Notes

- Viscosity measures internal friction and resistance to shear.
- Newtonian fluids obey $\frac{F_\parallel}{A} \propto dv/dy$; non-Newtonian fluids do not.
- Flow profiles are linear (shear flow) or parabolic (pipe flow).
- Reynolds number determines laminar vs. turbulent flow.
- Viscosity causes energy loss and heating.
- Stokes drag and Magnus force arise from viscous interactions with boundaries.

1.8.5 Poiseuille's Equation

When a viscous fluid flows steadily through a cylindrical tube, internal friction causes energy loss and a pressure drop along the tube. For slow, smooth, **laminar flow**, the relationship between pressure difference, flow rate, and tube dimensions is described by **Poiseuille's equation**, a result of central importance in fluid mechanics and physiology.

As established in the previous section on viscosity, laminar flow in a cylindrical tube develops a **parabolic velocity profile**: Velocity rises smoothly from zero at the wall (no-slip) to a maximum at the center. To relate this profile to pressure and viscosity, we analyze the forces acting on a thin cylindrical fluid shell of radius r and thickness dr under the assumption of steady, incompressible, Newtonian flow.

In a real viscous flow, mechanical energy is continually **dissipated** by internal friction. To maintain steady flow, an upstream–downstream **pressure drop** must therefore be imposed; otherwise the fluid would slow down and eventually stop. This required pressure drop is **not** predicted by Bernoulli's equation because Bernoulli applies only to **ideal, inviscid** flows with no energy loss. In viscous flow, pressure must decrease along the tube to supply the energy lost to friction.

Derivation of Poiseuille's Law

Consider a short segment of the tube of length dx. The pressure at x and $x + dx$ acts on the same cross-sectional area $A_\perp = \pi r^2$. The **differential net pressure force** on the fluid slice is therefore

$$dF_P = -\,dP\,\pi r^2,$$

or per unit length,

$$\frac{dF_P}{dx} = -\frac{dP}{dx}\,\pi r^2.$$

where $-\frac{dP}{dx} > 0$ because pressure decreases downstream.

Opposing this is the **viscous shear force** exerted by neighboring fluid layers on the cylindrical surface $dA_\parallel = 2\pi r dx$ of the shell. The differential viscous force on the shell is

$$dF_v = \eta\, dA_\parallel \frac{dv}{dr} = 2\pi r\, dx\, \eta\, \frac{dv}{dr},$$

so

$$\frac{dF_v}{dx} = 2\pi r\, \eta\, \frac{dv}{dr}.$$

Balancing pressure and viscous forces and simplifying gives us:

$$-\frac{dP}{dx}\,\pi r^2 = 2\pi r\, \eta\, \frac{dv}{dr},$$

$$dv = -\frac{1}{2\eta}\,\frac{dP}{dx}\, r\, dr.$$

Integrating,

$$\int_{v(r)}^{0} dv = -\frac{1}{2\eta}\,\frac{dP}{dx}\int_{r}^{R} r\, dr,$$

which gives

$$v(r) = \frac{1}{4\eta}\,\frac{dP}{dx}\,(R^2 - r^2).$$

Over the full tube length L the constant pressure gradient is

$$\frac{dP}{dx} = -\frac{\Delta P}{L},$$

with $\Delta P = P_1 - P_2$. Inserting this gives

$$v(r) = \frac{\Delta P}{4\eta L}(R^2 - r^2),$$

a parabolic profile with maximum speed at the center and zero at the wall.

Therefore, the total **volume flow rate** (Poiseuille's equation) is obtained by summing the contributions of all concentric cylindrical shells of area element each with $dA = 2\pi r\, dr$:

$$Q = \int_0^R v(r)\,2\pi r\,dr = \frac{\pi R^4 \Delta P}{8\eta L}.$$

This is **Poiseuille's equation**, valid for laminar flow of incompressible, Newtonian fluids in cylindrical tubes. Note for area of a concentric shell $dA = \pi r_o^2 - \pi r_i^2 = \pi((r_i + dr)^2 - r_i^2)$ gives for small dr and with $r_i = r$ we obtain $dA = 2\pi r_i\,dr$.

Key Dependencies

$$Q \propto \frac{R^4}{L}\frac{\Delta P}{\eta}.$$

- Flow rate is **proportional** to pressure difference.
- Flow rate is **inversely proportional** to viscosity and tube length.
- Flow rate depends on the **fourth power of radius**, so even small changes in radius cause large changes in Q.

Further, **pressure decreases linearly** along the tube.

Average Velocity and Pressure Drop

The volume flow rate through the cross-section must satisfy

$$Q = \pi R^2 v_{\text{avg}}.$$

From Poiseuille's equation,

$$Q = \frac{\pi R^4 \Delta P}{8\eta L}.$$

Equating both expressions gives

$$\pi R^2 v_{\text{avg}} = \frac{\pi R^4 \Delta P}{8\eta L},$$

so

$$v_{\text{avg}} = \frac{R^2 \Delta P}{8\eta L}.$$

Because the maximum velocity occurs at the center of the tube,

$$v(r=0) = \frac{\Delta P}{4\eta L}(R^2 - 0) = \frac{R^2 \Delta P}{4\eta L},$$

and therefore

$$v_{\max} = 2\,v_{\text{avg}}.$$

Validity and Reynolds Number

Poiseuille's equation holds when:

- the flow is **laminar**,
- the fluid is **Newtonian**,
- the flow is **steady**,
- the tube is **rigid and circular**.

The Reynolds number is

$$R = \frac{\rho \, v_{\text{avg}}(2R)}{\eta},$$

and must be below about **2000** for laminar flow to remain stable.

Application: Blood Flow and Deviations from Ideal Behavior

In large arteries and veins, blood behaves approximately as a Newtonian fluid, so Poiseuille's law provides a good first description of flow. Flow is extremely sensitive to vessel radius ($Q \propto R^4$), meaning vasoconstriction strongly reduces flow while vasodilation increases it. In smaller vessels, however, deviations appear: Red blood cells alter the effective viscosity, blood becomes **shear-thinning** (its viscosity decreases with shear rate), and pulsatile pressure from the heart introduces unsteady flow. Although blood is not perfectly Newtonian—especially in capillaries where cell deformation dominates—Poiseuille's law remains an excellent first approximation for much of the systemic circulation.

Conceptual Notes

- Poiseuille's equation links **pressure**, **radius**, **viscosity**, and **flow rate** in laminar flow.
- Flow depends very strongly on tube radius $\propto R^4$.
- Velocity profile is parabolic; $v_{\max} = 2v_{\text{avg}}$.
- Viscosity causes linear pressure drop along the tube.
- Blood flow regulation relies strongly on vessel diameter.
- The law breaks down for turbulent or non-Newtonian flow.

1.8.6 The Navier–Stokes Equation: A Crowd Analogy

Physics does not stop with steady, laminar flow but there are more general equation describing non-steady, turbulent conditions. One equation is the incompressible Navier–Stokes equation (incompressibility condition $\nabla \cdot \vec{v} = 0$):

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \vec{v} + \vec{g},$$

To build intuition for the terms in the Navier–Stokes equation, we follow a thought experiment inspired by Prof. René Matzdorf's excellent explanation.

Imagine you are a **person inside a dense moving crowd**. You represent a small **fluid volume element** whose motion is influenced by its surroundings. Each term of the Navier–Stokes equation corresponds to a physical effect you experience in this crowd.

Local acceleration

Your own speed can change simply because **you decide to walk faster or slower**. This corresponds to the local time derivative

$$\frac{\partial \vec{v}}{\partial t},$$

the immediate change of velocity at your location.

Convection (advection)

Even if you do not change your own pace, your velocity changes if you are **pushed into a region where the crowd moves faster or slower**.A single sideways step may place you into a stream of faster moving people. This effect is the *convective acceleration*.

To write this mathematically, we introduce the symbol ∇ (nabla), which is a vector of partial derivatives:

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right).$$

For a scalar field $f(x, y, z)$,

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right),$$

which measures how f changes in each spatial direction.

For a vector field $\vec{v} = (v_x, v_y, v_z)$,
the object $\nabla \vec{v}$ (also called the velocity gradient tensor) is the matrix of all spatial derivatives,

$$\nabla \vec{v} = \begin{pmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{pmatrix}.$$

The combination $(\vec{v} \cdot \nabla)\vec{v}$ means: take the directional derivative of the velocity field **in the direction of the flow itself**.

In the crowd analogy, this corresponds to being carried into a region where the surrounding crowd already moves differently—your velocity changes simply because you entered a new flow region.

Gravity

If the entire concert meadow lies on a slope, the whole crowd is **pulled downhill**. Gravity accelerates every person in the same direction, giving the body force term

$$\vec{g}.$$

Pressure gradient

If the left side of the crowd is packed and pushing while the right side is relatively empty, you feel a **net shove toward the low-pressure region**. This corresponds to a spatial change in pressure,

$$-\frac{1}{\rho}\nabla p,$$

where ∇p is the same spatial-derivative operator introduced in the convection term. A pressure gradient therefore accelerates fluid from regions of high pressure toward regions of low pressure.

Viscous forces

You constantly **bump shoulders** with the people around you. If those beside you move faster, they **pull you along**; if they move slower, they **hold you back**. This interpersonal "friction" corresponds to viscous forces described by

$$\nu\nabla^2\vec{v},$$

which smooth out velocity differences between neighboring fluid layers. Here $\nu = \eta/\rho$ is the **kinematic viscosity**.

To understand this term, we introduce the **Laplacian** operator.
For a scalar field $f(x, y, z)$, the Laplacian is

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2},$$

a measure of how much the value of f differs from its surroundings. It is the mathematical operator that also appears in the heat equation and describes diffusion.

For a vector field $\vec{v} = (v_x, v_y, v_z)$, the Laplacian acts on each component:

$$\nabla^2\vec{v} = \left(\nabla^2 v_x,\ \nabla^2 v_y,\ \nabla^2 v_z\right).$$

Thus the term

$$\nu\nabla^2\vec{v}$$

describes **momentum diffusion**: regions moving faster than their neighbors slow down, while slower regions are dragged along.

This diffusion of velocity corresponds exactly to how shoulder-to-shoulder interactions in a dense crowd tend to even out differences in motion.

The incompressible Navier–Stokes equation

Combining all contributions, the motion of an incompressible viscous fluid is governed by

$$\boxed{\underbrace{\frac{\partial \vec{v}}{\partial t}}_{\text{local acceleration}} + \underbrace{(\vec{v}\cdot\nabla)\vec{v}}_{\text{convective acceleration}} = \underbrace{-\frac{1}{\rho}\nabla p}_{\text{pressure force}} + \underbrace{\nu\nabla^2\vec{v}}_{\text{viscous diffusion}} + \underbrace{\vec{g}}_{\text{gravity}}}$$

together with the incompressibility condition

$$\nabla \cdot \vec{v} = 0.$$

In the crowd analogy, incompressibility means the local density of people stays constant: they do not pile up or vanish, so flow into any small region is balanced by flow out.

Interpretation through the crowd analogy

- **Local acceleration:** you change your own speed.
- **Convective acceleration:** you are carried along by faster or slower parts of the crowd.
- **Pressure force:** you move from high-pressure crowded regions to low-pressure emptier ones.
- **Viscous diffusion:** shoulder–to–shoulder interactions pull or brake you depending on how neighbors move.
- **Gravity:** the whole crowd is pulled downhill if the terrain slopes.

Hopefully this analogy by Prof. Rene Matzdorf provides a memorable physical picture for one of the most important and complex equations in fluid dynamics.