

## 2.3. Capacitance, resistance & current

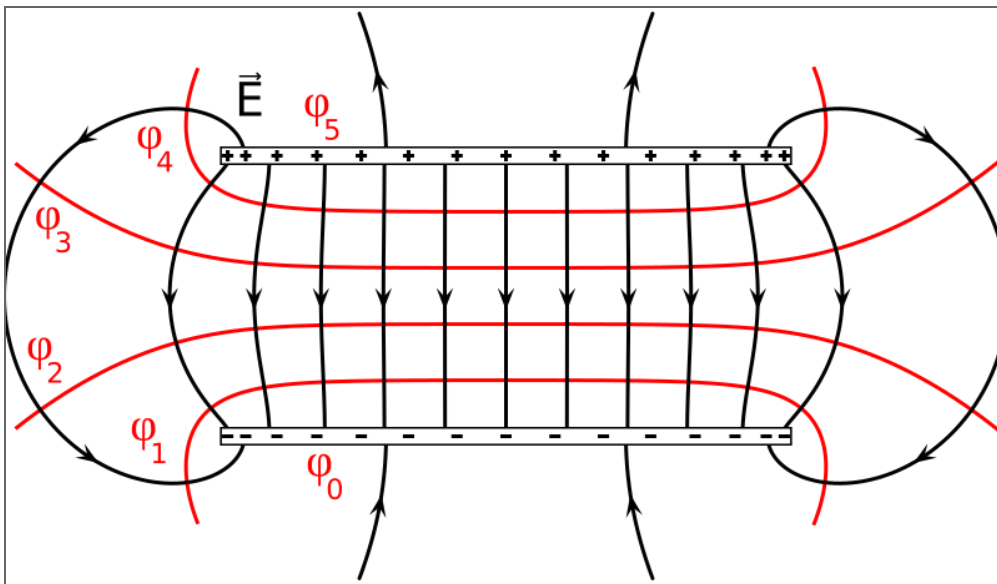


### How can we store electric energy?

- using circuit elements such as:
  - capacitors (separated charges)
  - electric batteries (electrochemical voltage source)
- transition from **static electricity to flow of charges**
- associated physical concepts: dielectrics, electric power, current, resistance, & Ohm's law
- **disclaimer:** simplify notation for voltage to
$$V = V_{BA} = V_B - V_A$$

## Capacitors - Basic concept

- two plates with opposite charge produce an electric field



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## Capacitors - Relation of $V$ and $d$

es27

- simple configuration:
  - two parallel plates of area  $A$  separated by distance  $d$
  - voltage source connected
  - $\rightarrow$  plates accumulate charge  $Q$  with equal magnitude but opposite sign

**What happens if we change the distance?** (if  $Q = \text{const}$ )

- **voltage increases with distance** (unit breakdown voltage reached)

$$V = V_B - V_A = - \int_A^B \vec{\mathbf{E}} d\vec{\mathbf{l}} = +E \int_A^B l = E d$$

**Notes:**

- electric field for two parallel planes:  $E = \frac{Q}{\epsilon_0 A}$

- angle between  $\vec{\mathbf{E}}$  and  $d\vec{\mathbf{l}}$  is  $180^\circ$   
 $\rightarrow El \cos(180^\circ) = -El.$
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## Capacitance $c$

- **fundamental relation:**  $Q = CV$
- **capacitance:**
  - proportionality constant  $C = \frac{Q}{V}$
  - unit: **farad** [F] = [C/V], typically capacitors in picofarad to microfarad range
  - capacitance determined by **geometry**: size, shape, relative position of plates
- determining capacitance **analytically** for uniform  $\vec{\mathbf{E}}$ :

$$V = Ed = \frac{Qd}{\epsilon_0 A}$$

$$\frac{V}{Q} = \frac{d}{\epsilon_0 A}$$

$$C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}$$

## Storing electric energy

- **conservation of energy:** the work  $W$  required to charge the capacitor is equal to the electric energy stored in the capacitor  $U$
- work required to move small amount of charge in presence of potential difference:

$$dW = -Vdq$$

- integrating over the entire charge  $Q$  and with  $V = \frac{q}{C}$ , we get:

$$W = - \int_0^Q V dq = - \frac{1}{C} \int_0^Q q dq = - \frac{1}{2} \frac{Q^2}{C}$$

- $\rightarrow$  energy  $U$  "stored" is:

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

script simulation: C, Q, E, U

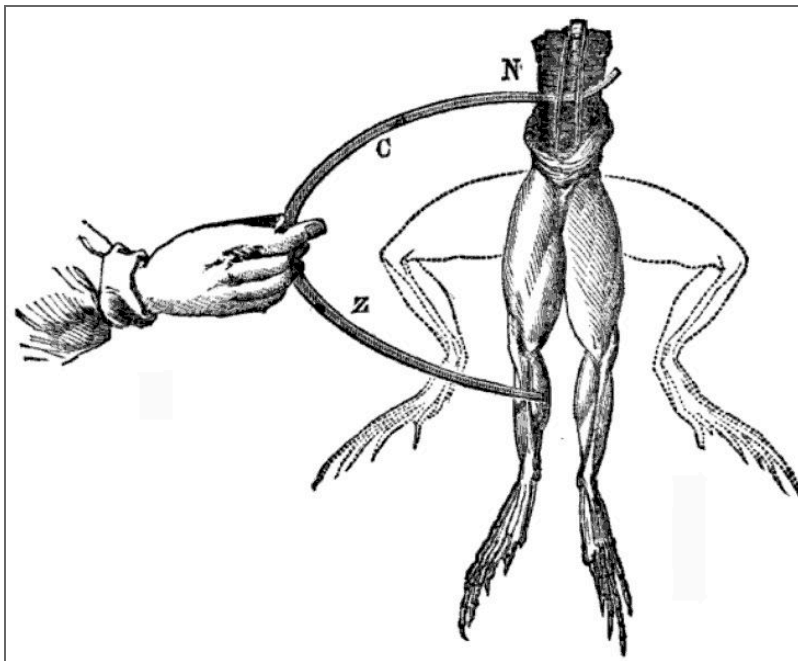
# Dielectrics

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- dielectrics are **insulating material**
- dielectric increases capacitance by a factor  $K$ :  
$$C = K C_0$$
- material permittivity is defined as  $\epsilon = K \epsilon_0$
- for a parallel-plate capacitor with a dielectric:  
$$C = \frac{\epsilon A}{d}$$
- inserting a dielectric increases the breakdown voltage and allows smaller gaps between plates

## History of electric battery: Galvani vs. Volta

- Luigi Galvani (1737-1798) connected a copper and iron wire to a frog leg and saw muscle contraction
- → interpreted as *life-force*
- Alessandro Volta (1745-1827) disagreed and realized the potential (pun intended) of dissimilar metal
- → combined cells of zinc & silver soaked in salt solution to form a *battery* of cells





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# Concept of electric battery

e123 + e125

- battery consists of:
  - **electrodes** - metal rods connected to terminals:
    - **cathode** - negative electrode  
e.g. Pb (lead)
    - **anode** - positive electrode e.g.  
PbO<sub>2</sub> (lead dioxide)
  - **electrolyte** e.g. sulfuric acid, apple, or frog leg
- oversimplified reaction:  $\text{Pb} + \text{acid} \rightarrow$  net effect of accumulation of electrons in cathode
- if connected, battery provides a voltage that drives current in a circuit
- **no charges are generated, merely separated, obeying the laws of conservation**

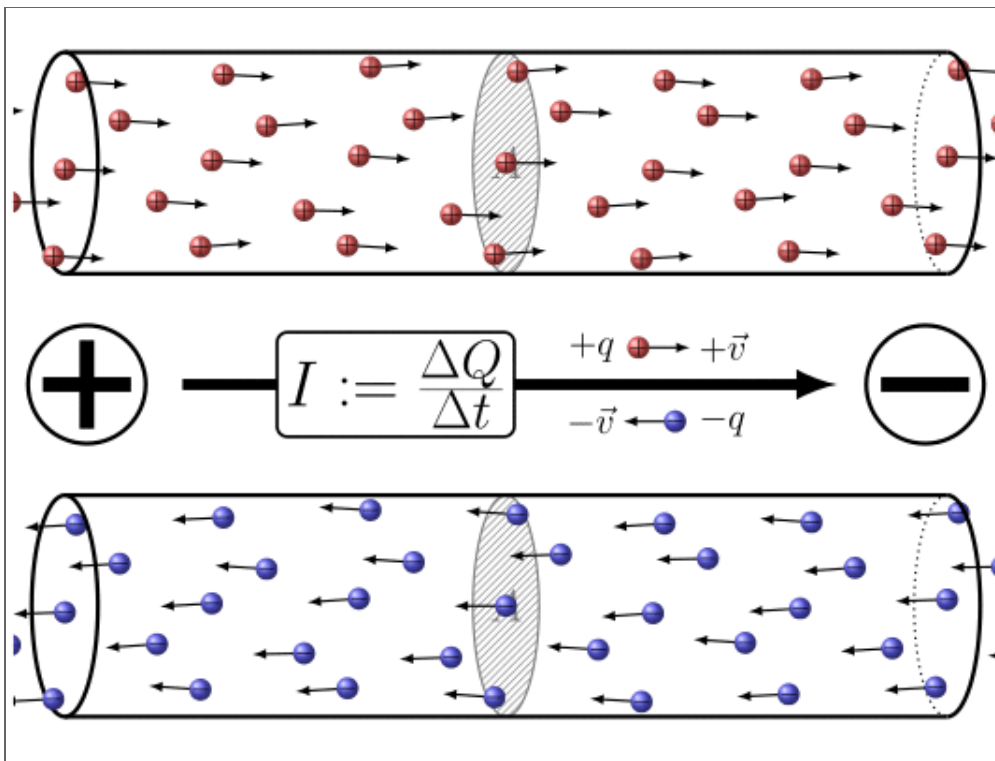
Start moving: electric current, resistance &  
ohm's law

es15

**Time to leave electrostatics, i.e. resting  
charges, and consider moving charges**

## Electric current

- average current defined as  $\bar{I} = \frac{\Delta Q}{\Delta t}$
- instantaneous current defined as  $I = \frac{dQ}{dt}$
- **unit** *ampere* [A]=[C/s] in recognition of André Ampère (1775-1836)
- **conservation of charge**: current is constant throughout a continuous conductor
- **flow direction**:
  - conventional current flows from positive to negative (Franklin), while electrons move from negative to positive (physics)
  - directionality (usually) non-critical/yield equal results (exception e.g. Hall effect)



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## Ohm's law & resistors

- **Ohm's law** relates voltage, current, and resistance:  $V = I R \leftrightarrow I = \frac{V}{R}$
- **resistance:**
  - proportionality constant which quantifies the **hindrance of electron flow**
  - unit of resistance: **Ohm** ( $\Omega = \frac{V}{A}$ ) in recognition of Georg Simon Ohm (1787-1854)
- Ohmic resistors have a constant  $R$ , while non-ohmic resistors change with conditions such as temperature

## Resistivity & conductivity

e132 + e104

- **resistance** in a uniform wire **depends on**:
  - cross-sectional area  $A$
  - wire length  $l$ ,
  - material used, i.e. resistivity  $\rho$  in [ $\Omega$  m]:

$$R = \rho \frac{l}{A}$$

- alternative material property: conductivity:  
 $\sigma = \frac{1}{\rho}$

## Temperature dependency of resistivity

e103

- **temperature dependence of resistivity** can be approximated as

$$\rho(T) = \rho_0 (1 + \alpha (T - T_0))$$

- $\alpha$  being the material-specific **temperature coefficient** of resistivity
- **negative temperature coefficients (NTC)**, i.e. lower resistance when heated, such as semiconductor which have more free electrons available at higher temperatures
- **positive temperature coefficients (PTC)**, i.e. higher resistance when heated, such as many metals as the higher temperature increases the likelihood of atom-electron collision



## Electric power

- **electric circuits transmit electric energy**  $\rightarrow$   
amount of electric power  $P$  delivered is  
therefore of interest (at the very least to you  
energy provider)

- **electric power is the energy per unit time:**

$$P = \frac{dU}{dt}$$

- unit: **Watt** [W]=[J/s]

- using  $V = \frac{U}{q} \rightarrow dU = Vdq$  as well as  $I = \frac{dq}{dt}$

$$P = \frac{dU}{dt} = \frac{dq}{dt} V$$

$$P = VI$$

- applying Ohm's law  $V = RI$ , we can extend  
this to:

$$P = VI = I^2 R = \frac{V^2}{R}$$

## Current density

e107

- switch from macro- to microscopic perspective
- **current density**  $j$ : current per unit cross-sectional area:  $j = \frac{I}{A} \leftrightarrow I = jA$
- for non-uniform current:  $I = \int \vec{j} \cdot d\vec{A}$
- $I$  is a macroscopic quantity, defined for e.g. the entire cross-section of a wire, while  $\vec{j}$  is a microscopic quantity, defined for each point.

## Drift speed

- macroscopic perspective: electricity moves with the speed of light
- microscopic perspective:
  - electrons collide with lattice
  - electrons move with an **average drift speed**  $v_d \approx 0.05 \text{ mm/s}$
- relate drift speed to (macroscopic) current via number of free electron per unit volume

$$n = \frac{N}{Al}:$$

$$I = \frac{\Delta Q}{\Delta t} = \frac{-eN}{\Delta t} = \frac{-enAl}{\Delta t} = \frac{-enAv_d\Delta t}{\Delta t} = -enA$$

$$j = \frac{I}{A} = -env_d$$

- minus sign indicates that electrons drift in opposite direction to macroscopic current
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## Microscopic view on Ohm's law

- relate current density and electric field in idealized conditions:
  - resistances is related to the conductors geometry and resistivity:  $R = \rho \frac{l}{A}$
  - in a uniform field:  $V = \int E dl = El$
  - current  $I$  is:  $I = jA$
  - Ohm's law states:  $V = RI$

$$V = IR$$

$$El = jA\rho \frac{l}{A}$$

$$El = j\rho l$$

$$E = j\rho$$

- **generalization to microscopic view of Ohm's law**
  - electric field as proxy for  $V$
  - current density as proxy for  $I$
  - resistivity as proxy for  $R$

$$\vec{\mathbf{E}} = \rho \vec{\mathbf{j}} \quad \Leftrightarrow \quad \vec{\mathbf{j}} = \frac{1}{\rho} \vec{\mathbf{E}}$$