1.1. Kinematics in one dimension

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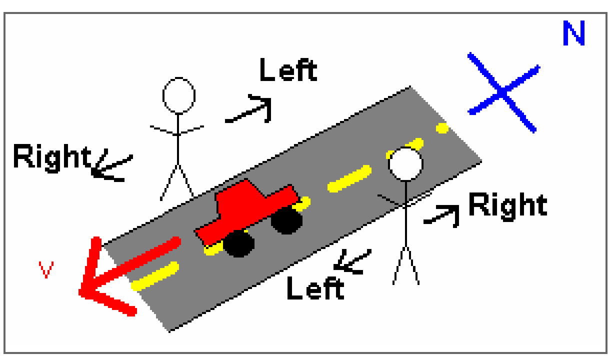
What are kinematics?

- **kinematics** describes *how* objects move — not *why* they move
- focuses on the **geometry of motion** in space and time
- in **one dimension**, motion occurs along a single straight line (e.g. horizontal or vertical)

Frame of reference

- **reference frame** defines the stage on which motion is observed
- usually includes an **origin**, **axes**, a **clock**, and an **observer**
- motion is always **relative**, e.g. a passenger can be at rest in the train but moving relative to the ground
- in everyday (Galilean) mechanics, switching frames means simply **adding or subtracting velocities**
- $\rightarrow$ **laws of physics stay the same**; only the numerical values and signs may differ.



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Velocity as a vector (in 1D)

- **velocity** is a **vector** $\rightarrow$ has magnitude and direction
- in 1D, direction is given by the **sign**:
    - positive $\rightarrow$ motion along $+\hat{\imath}$ (or $+\hat{\jmath}$)
    - negative $\rightarrow$ motion in the opposite direction
- using $\vec{v} = v\,\hat{\imath}$ or $\vec{v} = v\,\hat{\jmath}$ keeps direction explicit.
- changing the **coordinate convention** (origin or positive direction) affects all signs equally, but **does not change the underlying physics**

Average velocity

`sim avg. vs inst. Velocity`

- describes motion over a **finite time interval**:

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

  - based on **displacement** from initial position, not total distance traveled
  - on an $x(t)$ graph, it is the **slope of the secant line** between two points

Instantaneous velocity

`sim avg. vs inst. Velocity`

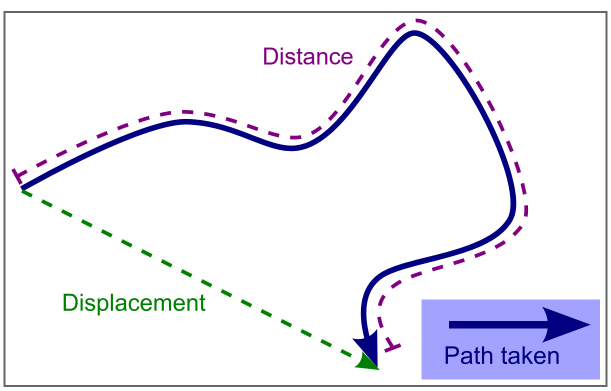
- captures motion **at a specific instant**:

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

  - it is the **slope of the tangent** to the $x(t)$ curve
  - gives the **rate and direction** of change of position

Distance vs. displacement & speed vs. velocity

- **displacement** measures the net change in position
- **distance** is the **total path length** traveled
- **speed** is the **magnitude** of velocity, computed from the (total) distance traveled, and is always positive
- **velocity** includes direction (can be positive or negative) and is computed from the (net) displacement



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Acceleration

- describes how velocity changes with time:

$$a(t) = \frac{dv}{dt}$$

    - is the **slope of the tangent** to the $v(t)$ curve
    - indicates the **rate of change of velocity**
- **sign of** a determines how v changes:
    - $a > 0 \rightarrow$ velocity increases (speeds up in the positive direction)
    - $a < 0 \rightarrow$ velocity decreases (slows down or speeds up in the negative direction)
- *deceleration* simply means acceleration opposite to motion

Uniform motion

`mb06`

- **velocity remains constant** $(v = \mathrm{const})$
- equal displacements occur in equal time intervals:

$$x(t) = x_0 + vt$$

- zero acceleration ($a = 0$)
- **average velocity = instantaneous velocity**
- **graphical view:**
  - $x(t)$ → straight line (slope = v)
  - $v(t)$ → horizontal line (constant value)

Uniformly accelerated motion

`mb06`

- constant acceleration ($a = \mathrm{const}$)
- velocity changes **linearly** with time

$$v(t) = v_0 + at$$

$$x(t) = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

- **graphical view:**
  - $v(t)$ is a **straight line** (slope = a)
  - $x(t)$ is a **parabola** (slope = v)
- applies to:
  - motion under steady thrust or uniform braking
  - free fall near Earth’s surface (neglecting air resistance)

Relations between Position, Velocity, & Acceleration

`sim x, v, a relation`

- **Position, velocity, and acceleration** are linked by **derivatives and integrals**:

$$v(t) = \frac{dx}{dt}, \quad \& \quad a(t) = \frac{dv}{dt}$$

$$x(t) = \int v(t)\,dt, \quad \& \quad v(t) = \int a(t)\,dt$$

- **slopes** and **areas** connect them graphically:
    - slope of $x(t) \to$ velocity
    - slope of $v(t) \to$ acceleration
    - area under $v(t) \to$ displacement
    - area under $a(t) \to$ change in velocity

Free Fall and Its Equations

`mb04`

- **free fall** is a case of **uniformly accelerated motion** under gravity:

$$a = \pm g, \quad g \approx 9.81\,\mathrm{m/s}^2$$

- choose sign convention, e.g. “up” positive $\rightarrow$ $a = -g$ or “down” positive $\rightarrow a = +g$
- equations of motion:

$$v(t) = v_0 \pm gt, \qquad y(t) = y_0 + v_0 t \pm \tfrac{1}{2} g t^2$$

- if the object starts from rest and in origin ($v_0 = 0$ & $y_0 = 0$):

$$v(t) = \pm gt, \qquad y(t) = \pm \tfrac{1}{2} g t^2$$

Universality of free fall

- near Earth’s surface, **all bodies accelerate equally**, regardless of mass
- in **vacuum**, light and heavy objects fall at the same rate $\rightarrow$ **Otto von Guericke’s vacuum experiments**
- real conditions include **air resistance**, which grows with speed and cross-section
- at high speeds, air resistance balances gravity, leading to **terminal velocity**, where acceleration effectively becomes zero