Lecture tutorial 2B

Millikan / Oil drop experiment

`es29`

Set-up:

- two parallel plates generating an approximately uniform electric field E
- voltage V and plate distance d can be used to estimate E
- gap in between plates viewed through e.g. microscope $\rightarrow$ **up-down flip/mirror-inverted image**
- vaporizer/atomizer/sprayer used to inset oil droplets
- due to friction, droplets charged $\rightarrow$ motion can be observed due to:
  - electrical field
  - gravitation
  - buoyancy

Task L2B.1.

We have an oil droplet with an idealized spherical shape. The density of oil and air are $\rho_o = 900\mathrm{kg/m^3}$ and $\rho_a = 1.29\mathrm{kg/m^3}$, respectively. What fraction of the gravitational force is the buoyancy force?

Given

- $\rho_o = 900\mathrm{kg/m^3}$
- $\rho_a = 1.29\mathrm{kg/m^3}$

Find

$$F_b/F_g$$

Solutions

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$$F_g = m_o g = \rho_o V g = \rho_o \frac{4}{3}\pi r^3 g$$

$$F_b = m_a g = \rho_a V g = \rho_a \frac{4}{3}\pi r^3 g$$

$$\frac{F_b}{F_g} = \frac{\rho_a \frac{4}{3}\pi r^3 g}{\rho_o \frac{4}{3}\pi r^3 g}$$

$$\frac{F_b}{F_g} = \frac{\rho_a}{\rho_o} = 0.00143$$

The strength of the buoyancy is approximately 1% of the gravitational force.

Task L2B.2.

Imagine a spherical oil droplet ($\rho_o = 900\text{kg/m}^3$, $r = 1\mu\text{m}$) inside an uniform electric field generated by two charged plates (plate distance $d = 3\text{mm}$). The applied voltage of $981V$ causes the droplet to hover/levitate. What is the charge of the droplet if we neglect buoyancy?

Given

- $d = 3\text{mm}$
- $V = 981\text{V}$
- $\rho_o = 900\text{kg/m}^3$
- $r = 1\mu\text{m}$

Find

- q

Solution

$$F_g = F_e$$

$$mg = qE$$

$$\rho_o \frac{4}{3}\pi r^3 g = q\frac{V}{d}$$

$$q = \frac{4\pi r^3, \rho_o g d}{3V} = 1.131 \times 10^{-19}\text{C}$$

Task L2B.3

Derive the electric field very close to any conducting surface if the charge density $\sigma = \frac{Q}{A}$.

Solution:

- a conducting material will only have an electric field outside, thus **charges accumulate only on the surface**
- if we decompose any shape of conductor, we can approximate the field near to the surface to be uniform
- we use a Gaussian surface as a small cylindrical box that covers the charges on the surface entirely (surface can extend beyond the surface)
- according to Gauss's law, we get

$$\oint \vec{\mathbf{E}} d\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0}$$

- as we assume the field to be **uniform near and perpendicular to the surface** and the area of the box to be simply A:

$$EA = \frac{Q_{enc}}{\epsilon_0}$$

- the charge density is $\sigma = \frac{Q}{A} \leftrightarrow Q = \sigma A$

$$EA = \frac{\sigma A}{\epsilon_0}$$

- thus, we obtain our final equation:

$$E = \frac{\sigma}{\epsilon_0}$$

Since this result is true for any conduction surface, we can also use it do describe the **field of a conducting plate** i.e. capacity if we assume that the plate are large compared to their distance and the field lines are perpendicular to the plates.

Task L2B.4

`es28`

Starting from the force equilibrium for `es28`, derive an equation for the voltage which would be present.

Solution

- the electrostatic force pulls the scale up, but only one plate is charge/one moves which gives us $F_e = \frac{1}{2}QE$
- gravitation force is $F_g = mg$

$$mg = \frac{1}{2}QE$$

- using the concept about the electric field from the previous task, $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$, we can state:

$$mg = \frac{1}{2}Q^2\frac{1}{\epsilon_0 A}$$

- the charge of a capacitor is $Q = CV = \frac{\epsilon_0 A}{d}V$

$$mg = \frac{1}{2}\frac{\epsilon_0^2 A^2}{d^2}V^2\frac{1}{\epsilon_0 A}$$

$$mg = \frac{1}{2}\frac{\epsilon_0 A}{d^2}V^2$$

$$V = \sqrt{\frac{2d^2 mg}{\epsilon_0 A}}$$