

## 2.2. Electric Flux & Electric Potential

**Why do birds sitting on high voltage power lines not get electrocuted?**

es03



- need to understand:
  - concept of electric potential and voltage
  - relation to electric potential energy
  - deeper understanding of electric fields, i.e. electric flux & Gauss's law
- start by revisiting electric field experimentally es05

## Electric flux for uniform electric field

script simulation: rectangle in field

- For a uniform electric field, the electric flux is defined as

$$\Phi_E = \vec{\mathbf{E}} \vec{\mathbf{A}}$$

where  $\vec{\mathbf{A}}$  is the vector perpendicular to the surface with magnitude  $A$

- Using  $\cos \theta$  as the angle between  $\vec{\mathbf{E}}$  and  $\vec{\mathbf{A}}$ , the equation can be rewritten as

$$\Phi_E = EA \cos \theta = E_{\perp} A = EA_{\perp}$$

- The number of field lines  $N$  passing through an area perpendicular to the field is proportional to the electric flux

$$N \propto E_{\perp} A = \Phi_E$$

## Electric flux for non-uniform electric field

- arbitrary surface can be decomposed into infinitesimal areas  $d\vec{A}$
- for each  $d\vec{A}$ , associated field is uniform
- integral over closed surface gives the **total flux through closed surface**:

$$\Phi_E = \oint \vec{E} d\vec{A}$$

- **by conventions**
  - $d\vec{A}$  points outwards from the surface of the enclosed volume
  - flux leaving the surface is positive
  - flux entering the surface is negative
- **consequences on net flux:**
  - if  $\Phi_E$  is positive, there is a net flux out of the volume
  - if  $\Phi_E$  is negative, there is a net flux into the volume
  - if  $\Phi_E = 0$ , there is no net flux

script simulation circle in field

## Gauss's law

- named after Karl Friedrich Gauss (1777-1855)
- **Gauss's law: Electric flux through a closed surface is equal to the net enclosed charge divided by the permittivity of free space:**

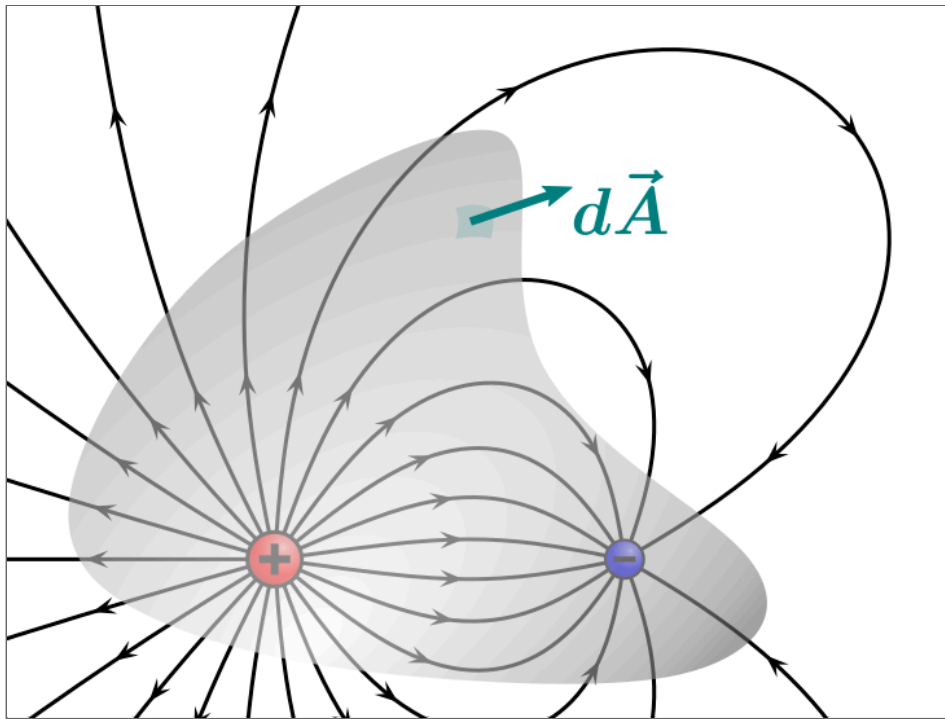
$$\Phi_E = \oint \vec{\mathbf{E}} d\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0}$$

- flux through surface is **independent** of:
  - distribution of the enclosed charge within the volume
  - charges outside the surface that may affect the position but not the number of field lines
- **principle of superposition applicable:**

$$\oint \vec{\mathbf{E}} \, d\vec{\mathbf{A}} = \oint \left( \sum_i \vec{\mathbf{E}}_i \right) d\vec{\mathbf{A}} = \sum_i \frac{Q_i}{\epsilon_0} = \frac{Q_{enc}}{\epsilon_0}$$

## Examples for Gauss's law

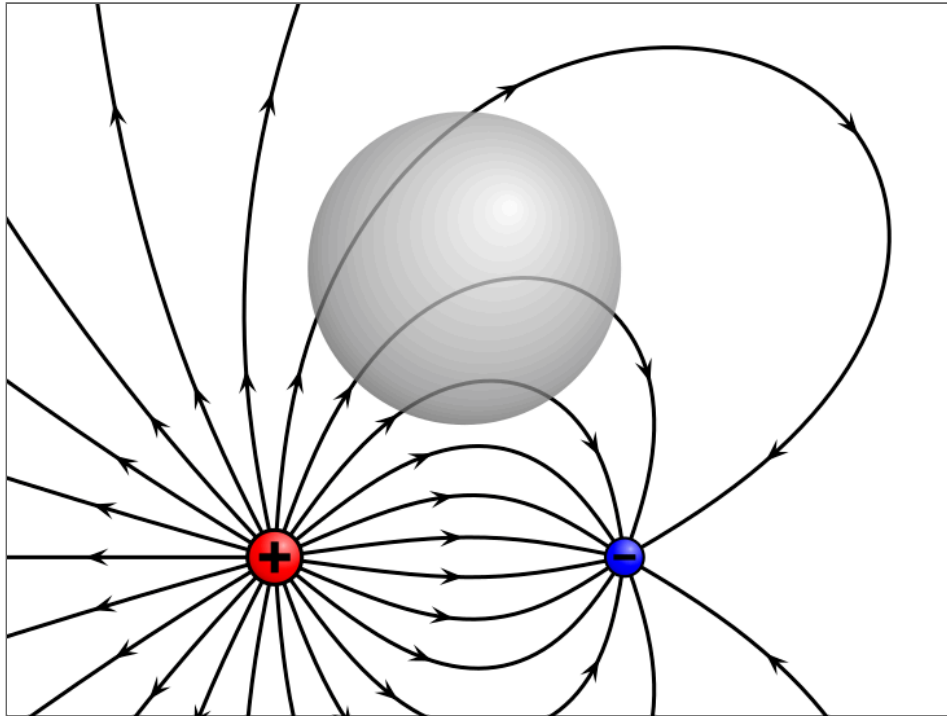
$$\Phi_E = \oint \vec{\mathbf{E}} d\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0} > 0$$



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$$\Phi_E = \oint \vec{\mathbf{E}} d\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0} = 0$$



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## Gauss's law: Relation to Coulomb's law

### thought experiment

- consider a spherical surface (radius  $r$ ) with a single enclosed charge  $Q_{enc}$  at the center
- the  $\vec{\mathbf{E}}$  field is oriented radially with the same magnitude everywhere on the surface
- field lines penetrate the spherical surface perpendicular to  $d\vec{\mathbf{A}}$
- surface area of the sphere is  $\oint dA = 4\pi r^2$
- This leads to Coulomb's law in electric field form:

$$\frac{Q_{enc}}{\epsilon_0} = \oint \vec{\mathbf{E}} d\vec{\mathbf{A}} = E \oint dA = 4\pi r^2 E$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \quad \text{with} \quad Q_{enc} = Q$$

## Complementary nature of Coulomb's and Gauss's law

	Coulomb's Law	Gauss's Law
<b>Equation</b>	$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \leftrightarrow F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$	$\oint \vec{E} d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$
<b>Focus</b>	Field/force & (point) charges	Flux through surface & enclosed net charge
<b>Applicability</b>	Point charges, any distribution (complex)	Mostly used for symmetrical distributions, although applicable to any case
<b>Relevant Charge Sources</b>	All charges present contribute	Only enclosed charges contribute to flux calculation

## Electric potential energy

- concept of **conservation of energy** in electricity is analogous to mechanics
- **electric potential energy is defined for conservative forces**, e.g. the electrostatic force  $\vec{F} = \vec{E}q$  (conservative force  $\rightarrow$  path independence)
- **in uniform electric field**, the **work** to move a test charge  $q$  over a distance  $d$  is:

$$W = Fd = qEd$$

- change in **potential energy**  $\Delta U$  from point  $A$  to  $B$  is:

$$\Delta U = U_B = -W$$

- note the similarity to mechanics, i.e.  $mgh$ , but as we have two types of charge:

- → **the closer a charge is to a charge of the same polarity, the higher the electrostatic force, thus, the higher the electric potential energy**

Voltage: The difference in electric potential

- **electric potential**  $V$  is defined as the electric potential energy per unit charge

$$V = \frac{U}{q}$$

- **voltage is the difference in potential** between points  $A$  and  $B$ :

$$V_{BA} = V_B - V_A \quad \text{in [J/C] = [V]}$$

- acknowledging Alessandro Volta, the unit is called **volt** [V]
- **reference point** (typically ground or infinity) is chosen where  $V = 0$

Putting it all together

- **electric potential** is defined at a **single point** in space
- only potential **differences**, i.e. voltage, are **measurable**
- **reference point** with zero potential is **defined arbitrarily**
- combining the definitions gives

$$V_{BA} = V_B - V_A = \frac{U_B}{q} - \frac{U_A}{q} = \frac{\Delta U_{BA}}{q} = -\frac{W_{BA}}{q}$$

- **work** performed on a charge is given by

$$W_{BA} = Fd = -qV_{BA}$$

- $\rightarrow$  **voltage is a measure of the work a charge can perform**

## The electron volt

- electron volt (eV) is the energy gained by an electron when moving through a potential difference of 1 V

$$1 \text{ eV} \approx 1.60 \times 10^{-19} \text{ J}$$

- handy when dealing with small energies, i.e. charged particles

## Relating electric potential & electric field

- analogously to mechanics, change in potential energy  $\Delta U_{BA}$  from  $A$  to  $B$  along the path  $\vec{l}$  is defined as:

$$\Delta U_{BA} = - \int_A^B \vec{F} d\vec{l}$$

- we know  $V = \frac{U}{q}$  and  $\vec{E} = \frac{\vec{F}}{q} \leftrightarrow \vec{F} = \vec{E}q$ , therefore:

$$V_{BA} = -\frac{1}{q}q \int_A^B \vec{E} d\vec{l}$$

$$V_{BA} = - \int_A^B \vec{E} d\vec{l}$$

## Relating electric potential & electric field (cont'd)

$$V_{BA} = - \int_A^B \vec{\mathbf{E}} d\vec{\mathbf{l}}$$

- in **uniform electric field**, the integral simplifies to

$$V_{BA} = -Ed \quad \text{with distance } d$$

- for an infinitesimal change, the relation is

$$dV = -\vec{\mathbf{E}} d\vec{\mathbf{l}}$$

- therefore, electric field is the gradient of the electric potential:

$$\vec{\mathbf{E}} = -\text{grad}V = -\nabla V$$

## Equipotential lines & surfaces

script simulation: equipotential

- equipotential lines (2D) and surfaces (3D) represent regions where the **electric potential is constant**
- **moving perpendicular** to the electric field ( $\vec{E}_{\perp} d\vec{l}$ ) does not change the potential

Unconventional ways to make light

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