2.5. Magnetism & magnetic field

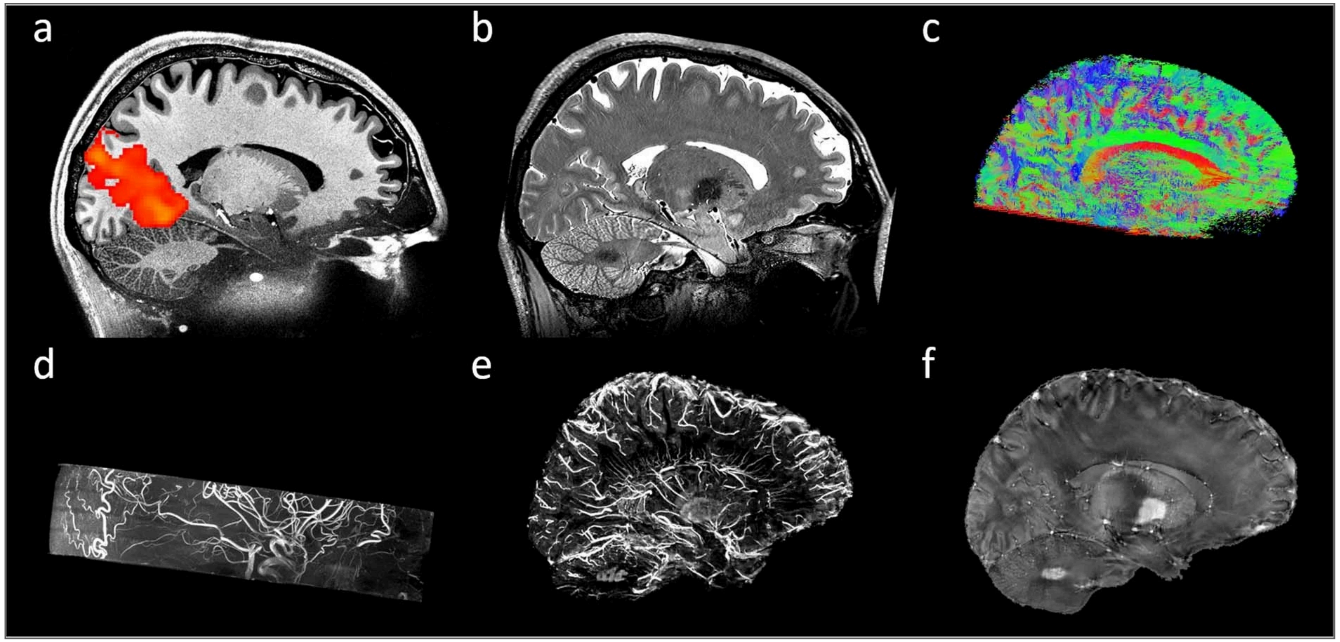
`em11`

What happens if you cut a magnet into two pieces?

- every magnet has a north and south pole
- opposite poles attract while like poles repel
- cutting a magnet always yields smaller magnets with both poles
- all **magnets are dipoles**, **no magnetic monopoles** have been observed

Primer on magnetism

- magnets and their fields are found in **daily life** e.g. loudspeakers, generators, HDD, and magnetic resonance imaging
- magnetic ore was discovered in the **greek region of magnesia**, giving the phenomenon its name
- the **interdependency of magnetism and electricity** was discovered in the 19th century



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Disclaimer

- there are two entities to describe the magnetic field
  - $\vec{\mathbf{B}}$: magnetic flux density and measured in tesla [T]
  - $\vec{\mathbf{H}}$: actual magnetic field strength and measured in amperes per meter [A/m]
  - in vacuum: $\vec{\mathbf{H}} = \frac{1}{\mu_0}\vec{\mathbf{B}} - \vec{\mathbf{M}}$ with μ_0 and $\vec{\mathbf{M}}$ being the permeability of free space and magnetization
- will use $\vec{\mathbf{B}}$ / B-field to describe the magnetic field instead of $\vec{\mathbf{H}}$ / H-field

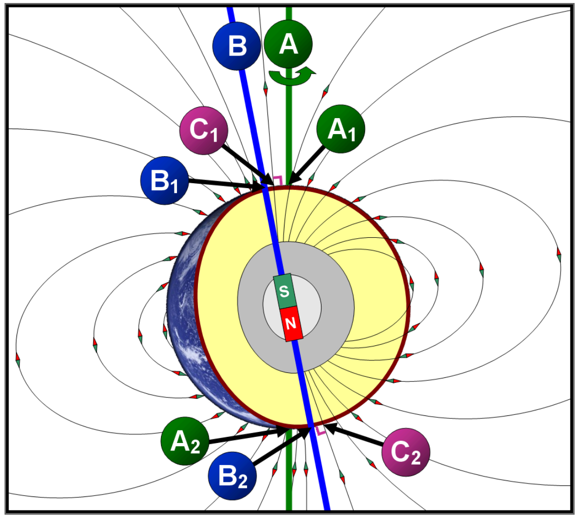
Magnetic field

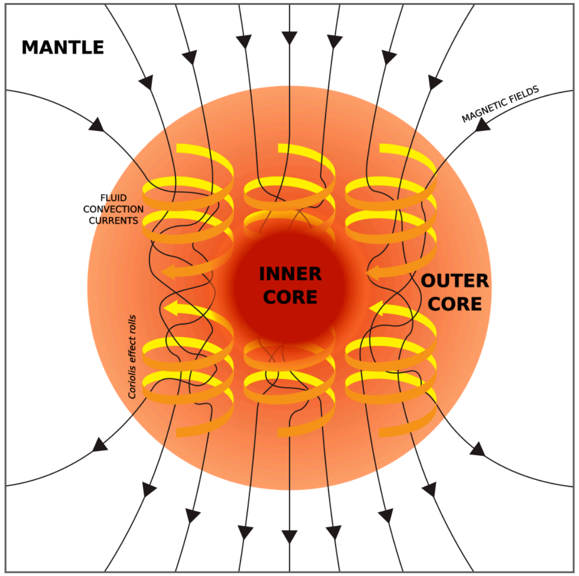
`simulation two bar magnets`

- field line density is **proportional** to the strength of the magnetic field
- the magnetic field direction is **tangent** to the field lines at any point
- **difference to electric field lines** / consequence of no magnetic monopoles:
    - field lines run from north to south and **always form closed loops**
    - field lines continue through the magnet itself

Earth’s magnetic field

- earth's magnetic field is believed to be produced by motion of conductive fluid in the earth's core, the **geodynamo**
- the earth’s magnetic field can be **approximated as a large bar magnet** (position changes and **not** aligned with geographic poles)
- the compass pole pointing toward geographic north actually aligns with the earth’s magnetic south





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Electric currents produce magnetic fields

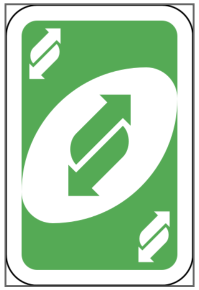
`em01 + simulation B-field of wire`

- **static** electric charge and magnet show no interaction
- Hans Christian Ørsted (1777-1851) found that compass needle deflect if put near a wire running current
- **current running through a wire generates a magnetic field**
- magnetic field **not uniform** in direction and magnitude but **forms circular lines around the wire**
- experimentally we can see field magnitude B:
    - increases with the current I
    - decreases with distance r
    - $B = \propto \frac{I}{r}$
- in general, the magnitude of magnetic field B produced by a long, straight current-running wire is:
    - $B = \frac{\mu_0}{2\pi} \frac{I}{r}$

- with *permeability of free space* $\mu_0 = 4\pi \times 10^-7$ T m /A.

Magnetic fields exert a force on currents

`em02 + simulation straight wires in B-field`



- Hans Christian Ørsted (1777-1851) found that a **current-carrying wire in a magnetic field experiences a force**
- observations for straight wire in homogenous magnetic field (approximated by horseshoe magnet):
    - force F perpendicular to B
    - F scales with l, I, and B
- in vector form:

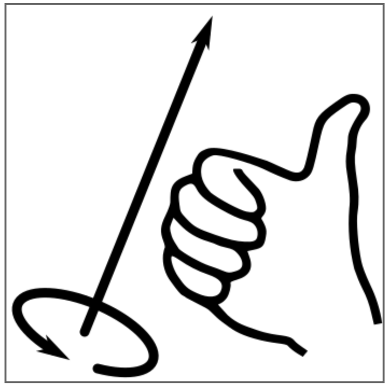
$$\vec{F} = I\vec{l} \times \vec{\mathbf{B}}$$

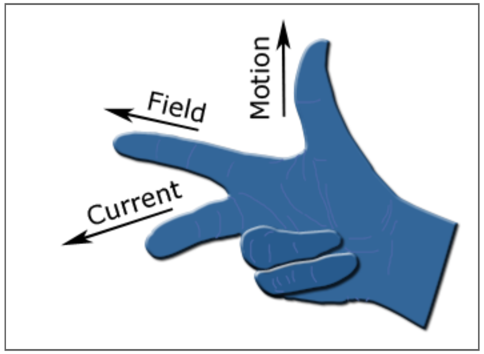
- with θ as angle between B and l:

$$F = IlB\sin\theta$$

Right-hand rules

- convention: magnetic field perpendicularly coming out of the paper shown as $\times$ and going into the paper as $\odot$
- *Right-Hand Rule 1* (RHR-1):
  - thumb pointing along the direction of the current I in a wire
  - wrapped fingers "around"
  - fingers curl in the direction the magnetic field $\vec{\mathbf{B}}$
- *Right-Hand Rule 2* (RHR-2): *
  - index finger pointing along the direction of the current I
  - middle finger points perpendicular to your thumb along the direction of the magnetic field $\vec{\mathbf{B}}$
  - thumb perpendicular to index & middle finger, points in the direction of the force
- disclaimer: use the conventional current, i.e. running from positive to negative, and not the true physical direction of freely moving electrons





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Ampère's law

- So far, know only B-field for a long straight, current-running wire: $B = \frac{\mu_0}{2\pi}\frac{I}{r}$
- **how to generalize to arbitrary configurations?**
- solution by André Marie Ampère (1775-1836)
  - considered a closed path around a current I_{enc}
  - decompose path into many, (infinitesimally) short, straight segments
  - consider only magnetic field component parallel to the path
  - taking the integral yields Amère's law: $\oint \vec{\mathbf{B}} \cdot d\vec{l} = \mu_0 I_{enc}$

Example for Ampère’s law

- consider long straight wire running the current I
- closed path integral for a circle centered around wires, thus, $I_{enc} = I$:

$$\oint \vec{\mathbf{B}} \cdot d\vec{l} = \mu_0 I$$

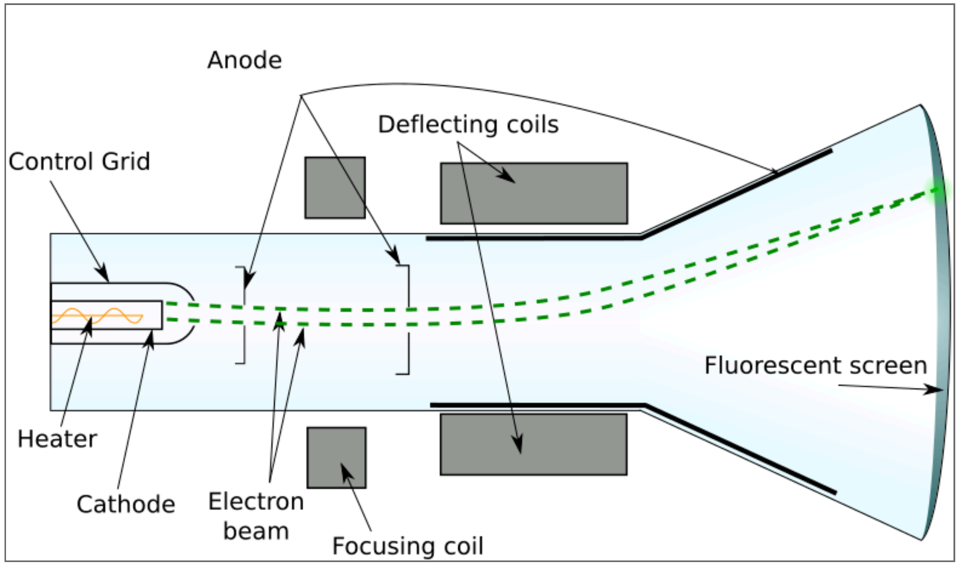
- $\vec{\mathbf{B}}$ is tangent to closed path (const. magnitude) and circumference of circle is $2\pi r$

$$\oint \vec{\mathbf{B}} \cdot d\vec{l} = B \oint d\vec{l} = B 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

Individual charges moving through magnetic fields

`em08`



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Force on a moving charge due to a magnetic field

- for a current-running wire we know $\vec{\mathbf{F}} = I\vec{\mathbf{l}} \times \vec{\mathbf{B}}$
- current is charge by unit time $I = \frac{Q}{t} = \frac{Nq}{t}$
- a single charged particles travel the distance l depending on their speed: $\vec{\mathbf{l}} = t\vec{\mathbf{v}}$
- thus, we obtain:

$$\vec{\mathbf{F}} = I\vec{\mathbf{l}} \times \vec{\mathbf{B}} = \frac{Nq}{t} t\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

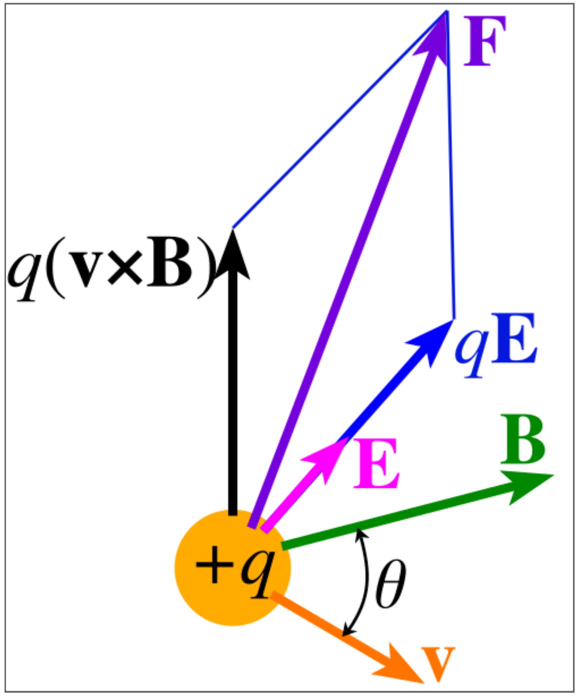
$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

- if $\vec{\mathbf{B}}$ is uniform, the equation can be simplified to $F = qvB\sin\theta$ with θ as the angle between the magnetic field and the direction the charge in moving

Lorentz equation

- the total force on a charged particle due to electric and magnetic fields is given by

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$$



Hall effect

`em05/em48 + simulation Hall effect`

- current-running wire, a.k.a. **confined** space, in a magnetic field (assuming perpendicular, uniform)
- $\vec{\mathbf{F}}_B = e\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}$ with $\vec{\mathbf{v}}_d$ being the drift velocity of the electron
- **Hall field**: electron will be **deflected** towards one side of the conducting wire **creating an electric field** $\vec{\mathbf{E}}_H$
- Hall field itself causes a force with the same magnitude but opposite direction to the magnetic force:

$$eE_H = ev_dB$$

$$E_H = v_dB$$

- **Hall voltage** V_H in the presence of uniform, perpendicular fields and thin, but long conducting wire is:

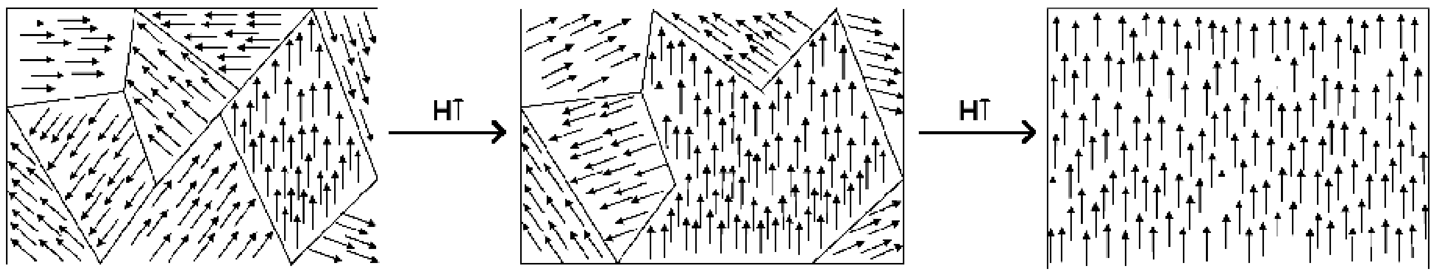
$$V_H = E_H d = v_d B d$$

- with **Hall effect** we can differentiate between positive and negative charges

Microscopic view of ferromagnetism

`em21`

- ferromagnetic materials are divided into domains that act like tiny bar magnets
- in an unmagnetized state, the domains are randomly oriented so their fields cancel
- applying an external magnetic field aligns the domains and magnetizes the material
- heating or mechanical shock can randomize the domains again



Magnetic permeability & susceptibility

- a material's permeability μ relates to the free space permeability μ_0
- the relative permeability is defined as

$$K_m = \frac{\mu}{\mu_0}$$

- magnetic susceptibility is given by

$$\chi = K_m - 1$$

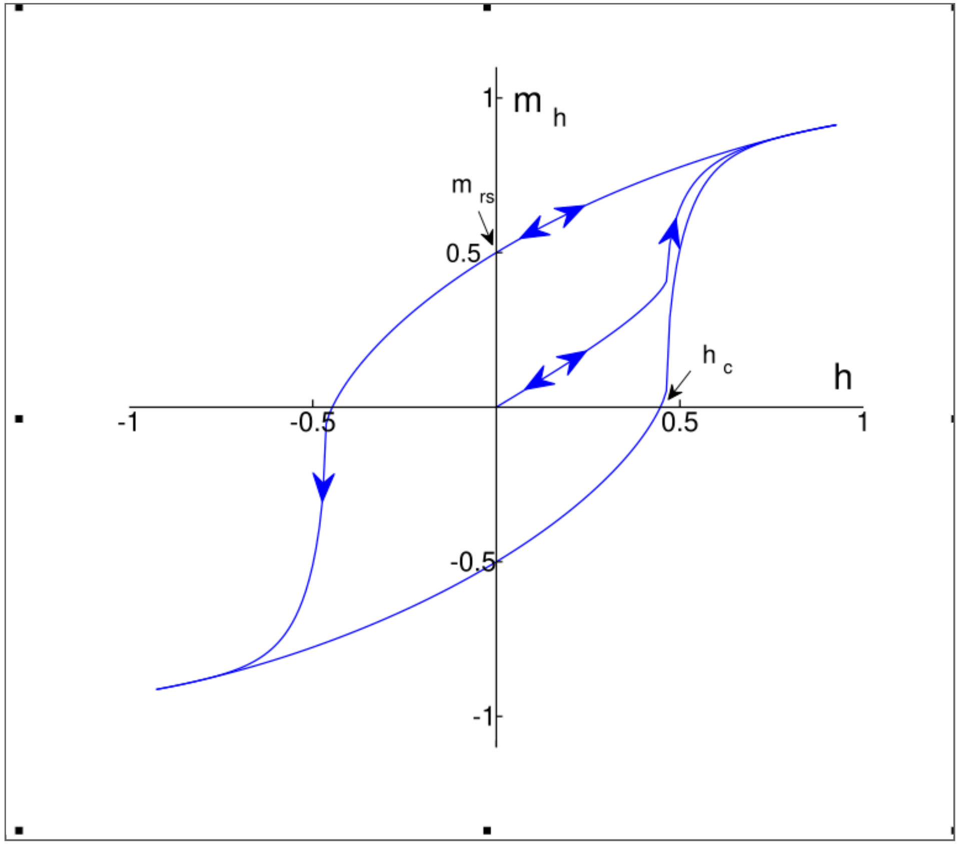
Magnetic materials

`em32`

- **diamagnetic materials** $\chi < 0$:
    - push the magnetic field out
    - examples: gold, silver, water, oxygenated blood
- **paramagnetic materials** $\chi > 0$:
    - pull the magnetic field in
    - examples: lithium, aluminium, deoxygenated blood
- **ferromagnetic materials** $\chi \gg 1$:
    - have a strong pulling effect
    - examples: iron, nickel, cobalt

Hysteresis

- hysteresis describes the lag in a material’s magnetic response to an external field
- as the external field is increased, the material’s magnetic field increases until saturation
- when the external field is reduced, the material retains some magnetization
- completely removing the magnetization requires applying a reverse external field
- permanent magnets exhibit broad hysteresis loops while electromagnets show shallower curves



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