1.7. Hydrostatics

In this chapter we introduce the physics of **fluids at rest**, focusing on how pressure, density, and gravity shape fluid behavior. We discuss the relevant **phases of matter**, define **pressure** and **buoyancy**, and show how these lead to **Pascal's principle** and **Archimedes' principle**. We also touch on surface effects such as **surface tension** and **capillarity**, which become important at fluid interfaces and small length scales.

1.7.1. Phases of Matter

All substances in the universe exist in one or more of three familiar states or **phases**: solid, liquid, and gas. These phases differ mainly in the **spacing**, **motion**, and **intermolecular forces** between their constituent particles. Understanding these distinctions is essential for fluid mechanics, since both liquids and gases—collectively called **fluids**—are able to flow and continuously deform under applied forces.

Solids

In a solid, particles are tightly packed and held in place by strong intermolecular forces. They can vibrate about their equilibrium positions but cannot move freely. As a result, solids possess a definite shape and volume, exhibit high density, and are essentially incompressible. Although they can deform under applied stress, the deformation is finite and solids return to equilibrium once the stress is removed.

Examples include metals, ice, and crystalline salts.

Liquids

In a liquid, particles remain close together but are no longer fixed in a rigid structure. Intermolecular forces are weaker than in solids, allowing particles to move and rearrange. Liquids therefore have a definite volume but no fixed shape; they take the shape of their container. They are only slightly compressible and can flow, meaning they continue to deform as long as shear stress is applied.

Examples include water, oil, and mercury.

Gases

In a gas, particles are far apart and move rapidly in all directions. Intermolecular forces are negligible except during collisions. Gases have neither definite shape nor definite volume and expand to fill any container. They are highly compressible and considerably less dense than liquids or solids.

Examples include air, oxygen, and carbon dioxide.

The Role of Temperature and Pressure

Temperature measures the **average kinetic energy** of the particles in a substance. As temperature increases, molecular vibrations grow stronger and the average separation between particles tends to increase. If the thermal energy becomes large enough to overcome the attractive forces between molecules, the substance may undergo a **phase change**. At sufficiently high temperature, intermolecular attraction becomes negligible and the material exists as a gas.

Pressure influences phase in the opposite direction. Increasing pressure forces particles closer together, strengthening the effect of intermolecular forces and favoring condensed phases such as liquids or solids. Conversely, lowering pressure reduces the constraint on particle spacing, making it easier for a substance to vaporize. This interplay between thermal motion and external pressure determines whether a material is solid, liquid, or gas under given conditions.

Phase Transitions

A **phase transition** occurs when a substance changes between solid, liquid, and gaseous states. During such transitions, energy is absorbed or released in the form of **latent heat**, which is used to rearrange molecular binding rather than to increase temperature. This is why a phase change at constant pressure occurs at **constant temperature**: the added or removed energy modifies molecular configuration without altering the average kinetic energy.

Common transitions include melting and freezing between the solid and liquid phases, vaporization and condensation between liquid and gas, and sublimation or deposition between solid and gas. In each case, an endothermic transition absorbs heat to weaken molecular binding, whereas an exothermic transition releases heat as molecular attraction strengthens.

Phase Diagrams

The dependence of phase on **temperature** and **pressure** is conveniently summarized in a **phase diagram**. The diagram shows the regions where a substance exists as solid, liquid, or gas, separated by **phase boundaries** along which transitions occur. Two special points on such diagrams are particularly important:

- The **triple point**, where solid, liquid, and gas coexist in equilibrium.
- The **critical point**, which marks the end of the liquid–gas boundary. Beyond this point, the distinction between liquid and gas disappears and the substance becomes a **supercritical fluid**.

Plasma: The Fourth State

At very high temperatures, atoms become ionized. Ionized means the electrons are separated from the nuclei, creating a mixture of free electrons and charged ions called **plasma**.

Key properties:

- Electrically conductive,
- Strongly influenced by electric and magnetic fields,
- Found in stars, lightning, and fluorescent lamps.

Key Points

Property	Solid	Liquid	Gas
Molecular spacing	Very close	Close	Far apart
Forces between molecules	Strong	Moderate	Weak
Shape	Fixed	Variable	Variable
Volume	Fixed	Fixed	Variable
Compressibility	Very small	Small	Large
Flow	No	Yes	Yes

- Near the **critical point**, the distinction between liquid and gas disappears, forming a **supercritical fluid**.
- Phase changes depend on **temperature** and **pressure**, visualized in a **phase diagram**.
- **Shear stress** leads to *finite* deformation in solids (they reach equilibrium) but *continuous* deformation in fluids (they keep deforming as long as shear is applied).
- Liquids and gases are both **fluids** because they flow under applied forces.

1.7.2. Density

The **density** of a substance describes how much mass is contained in a given volume. It is defined as the mass per unit volume,

$$\rho = \frac{m}{V},$$

with mass m measured in kilograms and volume V in cubic meters. The SI unit is therefore $\mathrm{kg/m^3}$, though densities of solids and liquids are often expressed in $\mathrm{g/cm^3}$, where $1 \mathrm{\ g/cm^3} = 1000 \mathrm{\ kg/m^3}$.

Dependence on Temperature & Pressure

Density can vary with temperature and pressure. Most substances expand when heated, causing their density to decrease. Solids show only minimal thermal expansion, while liquids exhibit a modest but still relatively small temperature dependence. Gases, however, are highly sensitive to both temperature and pressure, since their volume adjusts strongly when either quantity changes. We can model this dependency as:

$$\rho(T) = \frac{\rho_0}{1 + \beta(T - T_0)},$$

where β is the coefficient of volume expansion.

A notable example is water, which reaches its maximum density at about $4°C$. This anomaly ensures that colder water remains near the surface while denser water sinks, a behavior vital for aquatic ecosystems.

1.7.3. Pressure in Fluids

When a fluid is in contact with a surface, it exerts a force that is always directed **perpendicular** to that surface. To describe this direction mathematically, we introduce the **surface normal** $\hat{n}$, which points outward from the surface. The component of the force acting along this normal is the **normal force**, and its intensity per unit area defines the **pressure** of the fluid.

Pressure is therefore

$$P = \frac{F_{\perp}}{A},$$

where $F_{\perp}$ is the normal component of the force acting on an area A. Pressure is a scalar quantity, but the force it produces has a well-defined direction: it always pushes **against** the surface along $-\hat{n}$.

For a small surface element we introduce the area vector $d\vec{A} = \hat{n}\,dA$, whose magnitude equals the area dA and whose direction is given by the outward surface normal $\hat{n}$. The force exerted by the fluid on this element can then be written as

$$d\vec{F} = -\,p\,d\vec{A}.$$

This compact form encodes that the force acts perpendicular to the surface and is uniformly distributed over the area element. The minus sign is due to the convention that force is considered towards the surface element, while the normal vector points outward.

The SI unit of pressure is the Pascal,

$$1\,\mathrm{Pa} = 1\,\mathrm{N/m}^2.$$

Other frequently used units include:

- $1\,\mathrm{atm} \approx 1.013 \times 10^5\,\mathrm{Pa}$
- $1\,\mathrm{bar} = 10^5\,\mathrm{Pa}$
- $1\,\mathrm{mmHg} = 133\,\mathrm{Pa}$
- $1\,\mathrm{psi} = 6895\,\mathrm{Pa}$

Pressure at a Point in a Fluid

In a fluid at rest, pressure at any point is **the same in all directions**. Otherwise, the fluid would experience net force and start to flow.

This **isotropy** of pressure means that in static fluids, the pressure at a point **does not** depend on orientation or shape of the container.

Pressure & Depth

Consider a fluid of density ρ at rest. Take a thin horizontal layer of thickness $\mathrm{d}y$ at depth y. The pressure below this layer must support not only the pressure from above but also the **weight** of the fluid in the layer itself.

The mass of this layer is $\rho\,A\,\mathrm{d}y$, so its weight is $\rho g\,A\,\mathrm{d}y$. The net upward force from pressure must balance this weight:

- upward force: $(P + \mathrm{d}P)A$
- downward force: PA
- layer's weight: $\rho g\,A\,\mathrm{d}y$

Force balance gives

$$(P + \mathrm{d}P)A - PA = \rho g\,A\,\mathrm{d}y,$$

which simplifies to

$$\mathrm{d}P = \rho g\,\mathrm{d}y, \qquad \frac{\mathrm{d}P}{\mathrm{d}y} = \rho g.$$

To determine the absolute pressure, we integrate from the surface ($y = 0$), where the pressure is P_0. This surface pressure is usually the atmospheric pressure (see later section) acting on the fluid.

Integrating to a depth h:

$$P = P_0 + \rho g h.$$

Thus, in a fluid of uniform density, **pressure increases linearly with depth**, starting from the external pressure applied at the surface.

Pressure Differences and the Hydrostatic Paradox

From the hydrostatic relation $P = P_0 + \rho g h$, the pressure difference between two depths h_1 and h_2 in a fluid of uniform density is

$$\Delta P = \rho g(h_2 - h_1).$$

This result shows that the pressure depends only on the **vertical distance** between the points. It does not depend on the total volume of fluid, the width of the container, or how the container is shaped.

This leads to the **hydrostatic paradox**: Containers of very different shape can exert the **same pressure at the bottom** when filled to the same height, even though they may hold very different amounts of fluid.

The reason is that pressure in a static fluid is determined solely by the **weight of the fluid column directly above the point**, not by the overall amount of fluid in the container.

Pressure in a Gas

For gases, the same relationship applies if the density is constant. However, for the atmosphere, density decreases with altitude, so pressure variation is not exactly linear. For small height differences, $\Delta P = -\rho g \Delta h$ still holds approximately.

Key Points

- Pressure in a fluid increases linearly with depth when density is constant.
- Pressure acts **perpendicular** to surfaces and is the same in all directions at a point.
- The shape or volume of the container does not affect the pressure at a given depth (hydrostatic paradox).
- Pressure is a **scalar** quantity, even though it produces directional forces.

1.7.4. Atmospheric Pressure and Gauge Pressure

The air surrounding the Earth exerts a force on every surface it contacts. This **atmospheric pressure** is substantial and arises from the weight of the air column above us. Understanding how to measure and compare this pressure is essential for hydrostatics, weather, and engineering applications.

Origin of Atmospheric Pressure

Air has mass and therefore weight. At any point in the atmosphere, the pressure is determined by the **weight of the air above**:

$$P = \int_{h}^{\infty} \rho g \, \mathrm{d}h.$$

At sea level, the average atmospheric pressure is

$$P_{\mathrm{atm}} = 1.013 \times 10^5 \ \mathrm{Pa} \approx 1 \ \mathrm{atm}.$$

Equivalent units:

- 101.3 kPa
- 760 mmHg
- 14.7 psi
- 1.013 bar

This value corresponds to supporting a **10.3 m column of water** per square meter.

Variation of Atmospheric Pressure with Altitude

As we move higher in the atmosphere, the pressure drops because there is **less air above** to contribute weight. For small changes in height, we can treat the air density ρ as approximately constant. The hydrostatic relation then becomes

$$\frac{\mathrm{d}P}{\mathrm{d}h} = -\rho g,$$

showing that pressure decreases with increasing altitude.

Integrating from sea level ($h = 0$, $P = P_0$) to height h gives the linear approximation

$$P = P_0 - \rho g h.$$

Over larger altitude ranges, however, the density of air is no longer constant. Assuming the atmosphere behaves approximately as an ideal gas with slowly varying temperature T, we can write the density as

$$\rho = \frac{MP}{RT},$$

where M is the molar mass of air and R is the gas constant.
Substituting this into the hydrostatic relation yields

$$\frac{\mathrm{d}P}{\mathrm{d}h} = -\frac{Mg}{RT}P.$$

For roughly constant temperature, the factor in front of P is constant. Separating variables:

$$\frac{1}{P}\,\mathrm{d}P = -\frac{Mg}{RT}\,\mathrm{d}h.$$

Integrating from ($h = 0$, $P = P_0$) to (h, P):

$$\int_{P_0}^{P} \frac{1}{P'}\,\mathrm{d}P' = -\frac{Mg}{RT}\int_{0}^{h}\mathrm{d}h',$$

which gives

$$\ln\left(\frac{P}{P_0}\right) = -\frac{Mg}{RT}h.$$

Apply exponential,

$$P = P_0\,\mathrm{e}^{-h/h_0},$$

with the **scale height**

$$h_0 = \frac{RT}{Mg}$$

is the characteristic height over which atmospheric pressure decreases by a factor of e. In other words, h_0 is the characteristic decay length. For typical conditions near 20°C, $h_0 \approx 8.4\ \mathrm{km}$.

Absolute, Gauge, Differential Pressure, & Vacuum

Pressure measurements require a reference:

1. **Absolute Pressure (P_{abs})**
   Pressure measured relative to **vacuum** (zero pressure).
   Vacuum is simply the condition where the absolute pressure approaches zero.

2. **Atmospheric Pressure (P_{atm})**
   The surrounding air pressure; the natural reference for many instruments.

3. **Gauge Pressure (P_g)**
   Pressure measured relative to atmospheric pressure:

$$P_g = P_{\text{abs}} - P_{\text{atm}}.$$

   - $P_g > 0$: pressure above atmospheric
   - $P_g < 0$: pressure **below** atmospheric → **partial vacuum**

   Negative gauge pressure therefore indicates that the system pressure is lower than the surrounding air.

4. **Differential Pressure**
   The difference between two arbitrary pressures:

$$\Delta P = P_2 - P_1.$$

Measuring Pressure with Fluid Columns (Manometers)

A **U-tube manometer** compares a system pressure with atmospheric pressure. At equilibrium, the height difference h satisfies

$$P_{\text{system}} = P_{\text{atm}} + \rho g h.$$

If the system pressure is **below** atmospheric, the fluid rises on that side:

$$P_{\text{system}} = P_{\text{atm}} - \rho g h,$$

demonstrating **negative gauge pressure** (partial vacuum).

If both legs are connected to different systems, the instrument measures their pressure difference:

$$\Delta P = \rho g (h_2 - h_1).$$

Used for small pressure differences in gas lines or across flow elements.

Mercury Barometer

A mercury barometer consists of a long, closed tube completely filled with mercury and inverted into a mercury reservoir. When the mercury column settles, a small **vacuum** (the Torricellian vacuum) forms at the top of the tube. Atmospheric pressure acting on the reservoir supports the mercury column:

$$P_{\text{atm}} = \rho_{\text{Hg}}gh.$$

At sea level this gives

$$h = \frac{P_{\text{atm}}}{\rho_{\text{Hg}}g} \approx 0.760 \text{ m},$$

so standard atmospheric pressure corresponds to a **760 mm Hg** column.

Atmospheric Pressure and the Human Body

Although atmospheric pressure is large, we are not crushed because the pressures inside our body—blood, lungs, and tissues—match the external pressure, maintaining **pressure equilibrium**. Problems arise only when pressure changes too quickly, such as during diving, flying, or rapid altitude changes, where the body must adjust gradually to avoid discomfort or injury.

A familiar example of pressure differences in the body is **blood pressure measurement**. In the standard auscultatory method, an inflatable cuff temporarily blocks blood flow in an artery. As the cuff pressure is slowly reduced, blood begins to flow again in pulses, producing characteristic **Korotkoff sounds** heard with a stethoscope. These sounds identify:

- the **systolic pressure** (first sound): blood just starts to push through the compressed artery
- the **diastolic pressure** (last sound): the artery is fully open and flow becomes smooth again

This technique effectively compares **arterial pressure** to the known pressure inside the cuff, demonstrating how pressure differences govern fluid flow inside the human body.

Key Points

- Atmospheric pressure arises from the **weight of the air column**.
- Pressure decreases **exponentially** with altitude.
- **Absolute pressure** is referenced to vacuum; **gauge pressure** to the atmosphere.
- **Negative gauge pressure** corresponds to a **partial vacuum**.
- Fluid column instruments (manometers, barometers) visualize pressure differences directly.

1.7.5. Pascal's Principle

Pascal's principle describes how pressure behaves in a confined, incompressible fluid.

> **Pascal's Principle:**
> A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

This means that an applied pressure change

$$\Delta P = \frac{\Delta F}{A}$$

has the same value everywhere in the fluid. All parts of the fluid and all surfaces in contact with it experience the **same pressure increase**.

The idea can be visualized with a sealed container filled with an incompressible fluid. When a small piston pushes on the fluid, the fluid cannot be compressed, so the applied pressure spreads uniformly throughout the system. This allows hydraulic devices—such as presses, lifts, and brakes—to multiply force: a small input force on a small area produces a much larger output force on a larger area, while energy is conserved.

The Hydraulic Press

A hydraulic press consists of two connected cylinders filled with an incompressible fluid. Pushing down on a small piston of area A_1 with force F_1 produces a pressure

$$P = \frac{F_1}{A_1}.$$

By Pascal's principle, this pressure is transmitted undiminished to the larger piston of area A_2, which therefore experiences the force

$$F_2 = PA_2 = \frac{A_2}{A_1}F_1.$$

The press thus provides a **mechanical advantage**

$$\frac{F_2}{F_1} = \frac{A_2}{A_1},$$

allowing a small input force to generate a much larger output force when the output piston has a larger area.

Conservation of Energy

Although the force is amplified, **energy is conserved**.
The smaller piston must move through a proportionally greater distance.

If the small piston moves downward by distance d_1, the large piston rises by distance d_2 such that:

$$W_1 = F_1 d_1, \qquad W_2 = F_2 d_2.$$

Use $F_1 = PA_1$ and $F_2 = PA_2$:

$$W_1 = PA_1 d_1, \qquad W_2 = PA_2 d_2.$$

With $A_1 d_1 = A_2 d_2$ from volume conservation:

$$W_1 = P A_1 d_1 = P A_2 d_2 = W_2.$$

Thus,

$$F_1 d_1 = F_2 d_2,$$

showing that the **work input equals the work output**: the press multiplies force, not energy.

Pressure and Depth Combined

If the large piston holds a car at a height difference h relative to the small piston, the transmitted pressure must overcome both the car's weight and the hydrostatic pressure difference in the fluid:

$$P_2 = P_1 + \rho g h.$$

In most hydraulic systems this height difference is small and the correction is negligible, but in tall installations (e.g., hydraulic elevators) it must be taken into account.

Real Hydraulic Systems

Real systems are not ideal. Pressure losses arise from fluid friction, minor leaks, trapped air, and imperfect seals.
Air bubbles are especially problematic because they compress easily, reducing stiffness and efficiency; therefore, hydraulic lines must be **bled** to remove air.

Key Points

- Pressure applied to a confined fluid is transmitted equally in all directions.
- Force multiplication results from constant pressure acting on different areas.
- No energy is gained: larger force corresponds to smaller displacement.
- Pascal's principle applies only to **incompressible fluids** in **static equilibrium**.
- Hydraulic systems can generate large forces but require proper sealing and air-free operation.

1.7.6. Archimedes' Principle & Buoyancy

When an object is placed in a fluid, it experiences an **upward force** called the **buoyant force**, caused by pressure increasing with depth. This phenomenon determines whether an object floats, sinks, or remains suspended. The governing law, discovered over 2000 years ago, is **Archimedes' Principle**.

Origin of the Buoyant Force

In a fluid at rest, pressure increases with depth:

$$P = P_0 + \rho g h.$$

For a submerged object, the pressure on its bottom surface (deeper) is greater than on its top surface. If A is the cross-sectional area and h_t and h_b are the depths of the top and bottom surfaces,

$$F_{\text{top}} = \rho g h_t A, \qquad F_{\text{bottom}} = \rho g h_b A.$$

The net upward force is

$$F_B = F_{\text{bottom}} - F_{\text{top}} = \rho g (h_b - h_t) A = \rho g V,$$

where V is the displaced fluid volume. This is the **buoyant force**.

Archimedes' Principle

> **Archimedes' Principle:**
> Any object wholly or partially immersed in a fluid experiences an upward buoyant force equal to the **weight of the fluid displaced**.

Thus,

$$F_B = \rho_{\text{fluid}} g V_{\text{disp}}.$$

Net Force and Apparent Weight

The apparent weight of a submerged object is reduced by the buoyant force:

$$W_{\text{app}} = W - F_B.$$

- If $F_B < W$: the object sinks
- If $F_B = W$: it remains suspended
- If $F_B > W$: it rises and floats

Floating Objects

An object floats when its weight equals the buoyant force on the submerged part:

$$\rho_{\text{obj}} g V_{\text{obj}} = \rho_{\text{fluid}} g V_{\text{sub}}.$$

Cancelling g gives

$$\frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{\rho_{\text{obj}}}{\rho_{\text{fluid}}},$$

which determines the fraction submerged.

Buoyancy in Gases

Archimedes' principle applies equally to gases. An object of volume V in air experiences

$$F_B = \rho_{\text{air}} g V,$$

small for everyday objects but essential for balloons, blimps, and airships.

Key Points

- The buoyant force equals the **weight of displaced fluid**, not the object's weight.
- Apparent weight in a fluid is reduced by the buoyant force.
- Archimedes' principle applies to both **liquids** and **gases**.

1.7.8. Surface Tension & Capillarity

Liquids display remarkable behavior at their surfaces due to two types of molecular interactions: **cohesion** (attraction between molecules of the same substance) and **adhesion** (attraction between molecules of different substances).

Molecular Origin

The origin of surface tension lies in how molecular binding differs between the bulk and the surface. A molecule deep inside the liquid is surrounded by neighbors on all sides and forms many cohesive bonds. These attractive interactions place it in a **low-energy, well-bound** state.

A molecule at the surface, however, has neighbors only to the sides and below. It forms **fewer cohesive bonds**, which means it is **less well bound** and therefore in a **higher-energy** state. To move a molecule from the interior to the surface, some binding interactions must be broken, and this requires **work**.

Because physical systems tend to minimize energy, a liquid naturally acts to **reduce its surface area**, decreasing the number of high-energy surface molecules. The inward pull caused by these unbalanced cohesive forces is what we observe macroscopically as **surface tension** γ.

Definition of Specific Surface Energy & Surface Tension

To enlarge a liquid surface by an amount ΔA, work ΔW must be done.
The **specific surface energy** is

$$\varepsilon = \frac{\Delta W}{\Delta A}.$$

A surface also pulls tangentially along any line in the interface.
The **surface tension** is the force per unit length:

$$\gamma = \frac{F}{L}.$$

In a static situation (equilibrium/fluid at rest/no net force), both describe the same property:

$$\gamma = \varepsilon.$$

Units: Surface tension and specific surface energy have the same unit, **N/m = J/m²** because J/m² = Nm/m² = N/m.

Spherical Droplet (one surface)

Consider a spherical droplet of radius r. We increase its radius by a small amount Δr. The work needed to create this additional surface must equal the increase in surface energy.

1. Work done by the pressure

The pressure difference $p = \Delta P = P_{\text{in}} - P_{\text{out}}$ acts on the surface and expands the droplet.

A small radial expansion Δr changes the volume by

$$\Delta V = 4\pi r^2 \,\Delta r.$$

The work done by pressure is ($\Delta W = F\Delta s = pA\Delta s = pV$)

$$\Delta W = p\,\Delta V = p\,(4\pi r^2 \Delta r).$$

2. Increase in surface energy

The surface energy of a droplet is

$$E = \varepsilon A = \varepsilon(4\pi r^2),$$

with ε the specific surface energy.

Increasing the radius from r to $r + \Delta r$ increases the surface energy $\Delta E = E(r + \Delta r) - E(r)$ by

$$\Delta E = \varepsilon\left[4\pi(r + \Delta r)^2 - 4\pi r^2\right] = 4\pi\varepsilon\left[(r + \Delta r)^2 - r^2\right] = 4\pi\varepsilon\left[r^2 + 2r\Delta r + (\Delta r)^2 - r^2\right]$$
$$= 4\pi\varepsilon\left[2r\Delta r + (\Delta r)^2\right].$$

Since $(\Delta r)^2$ is very small,

$$\Delta E \approx 4\pi\varepsilon(2r\,\Delta r) = 8\pi r\varepsilon\,\Delta r.$$

3. Energetic balance

The work done must equal the increase in surface energy:

$$\Delta W = \Delta E.$$

Thus,

$$p\,(4\pi r^2 \Delta r) = 8\pi r\varepsilon\,\Delta r.$$

Cancel $4\pi r\Delta r$ on both sides:

$$pr = 2\varepsilon.$$

So the pressure difference inside the droplet is

$$p = \frac{2\varepsilon}{r}.$$

Since $\gamma = \varepsilon$ in equilibrium, this gives the familiar result:

$$p = \Delta P = \frac{2\gamma}{r}.$$

Soap Bubble (two surfaces)

A soap bubble has **two** liquid–air interfaces (inner and outer).
The surface energy is therefore doubled:

$$\Delta E_{\text{bubble}} = 2 \times (8\pi r\varepsilon\Delta r) = 16\pi r\varepsilon\Delta r.$$

Repeating the same balance leads to

$$p = \Delta P = \frac{4\varepsilon}{r} = \frac{4\gamma}{r}.$$

Thus a soap bubble has **twice** the excess pressure of a liquid droplet of the same radius.

Capillary Action

When a narrow tube (capillary) is placed in a liquid, the liquid level inside can rise or fall relative to the surrounding surface. This occurs because of surface tension and the balance between cohesive forces within the liquid and adhesive forces between the liquid and the tube wall.

The height h of the rise (or depression) is

$$h = \frac{2\gamma \cos\theta}{\rho g r},$$

where γ is the surface tension, θ the contact angle, ρ the fluid density, and r the tube radius.

If adhesion exceeds cohesion → the liquid wets the surface and rises in the tube ($\theta < 90°$).
If cohesion exceeds adhesion → the liquid does not wet the surface and the level is depressed ($\theta > 90°$).

Examples:

- Water rises in glass tubes (wetting)
- Mercury is depressed in glass (non-wetting)

Key Points

- Surface tension originates from cohesive molecular forces and acts tangentially along the interface.
- It produces pressure differences across curved surfaces and drives capillary rise.
- Capillary height is **inversely proportional** to tube radius.
- Capillarity plays essential roles in cleaning, lubrication, and biological transport.