2.9. Geometrical optics: Refection & refraction

Is light a wave or a particle?

- wave-particle duality of light

Under which condition can we approximate light as a ray of particles?

- when light interacts with objects much larger than its wavelength

Geometrical optics: reflection & refraction

- light exhibits both particle and wave properties
- for objects much larger than the wavelength, light can be approximated as rays
- the ray model simplifies the understanding of reflection, refraction, and later, optical instruments

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Ray model of light

- light travels in straight-line paths called rays
- each ray is considered an extremely narrow beam of light
- this model explains many everyday optical phenomena

Reflection

- when light reaches a surface, it can be reflected, absorbed, or transmitted

- for a plane mirror, follow these steps:
    - find the normal perpendicular to the surface
    - determine the angle of incidence θ_i, between the normal and the incident ray
    - the angle of reflection θ_r, between the normal and the reflected ray, equals the angle of incidence
- thus, the law of reflection is:

$$\theta_i = \theta_r$$

- specular reflection sends parallel rays into a single direction, while diffuse reflection scatters them

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Image formation at plane mirrors: real vs. virtual images

- an object in front of a plane mirror produces two rays per point that obey the law $\theta_i = \theta_r$
- although the object is in front of the mirror, the brain perceives its image as behind the

mirror

- the object distance d_o (measured perpendicular to the mirror) equals the image distance d_i
- the object's height equals the image's height
- image reconstruction is achieved by extending the reflected rays behind the mirror
- images that cannot be projected (because the rays do not actually intersect) are called virtual images

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Image formation at curved mirrors

- curved mirrors are typically spherical and can be:
    - convex – surface bulges toward the viewer; extends the field of view
    - concave – surface bulges inward; used for magnifying images
- the principal axis is the straight line perpendicular to the mirror at its center
- the focal point F is defined as the image point for an object at infinity

- for spherical mirrors, the radius of curvature r is related to the focal length f by:

$$r = 2f$$

- image construction for objects closer than infinity uses at least two rays:
    - parallel ray (incident parallel to the principal axis)
    - focal point ray (passes through the focal point)
    - central point ray (passes through the center of curvature; hits the mirror perpendicularly)
- for concave mirrors the intersection of the reflected rays gives the real image, while for convex mirrors the extension of the rays forms a virtual image
- mirror equation derived from similar triangles:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

- lateral magnification is given by:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

    - here, h_o is always positive and h_i is positive if the image is upright
- angular magnification compares the apparent size of the image formed by curved mirrors to that seen in a plane mirror:

$$M = \frac{\theta_C}{\theta_P}$$

Refraction & Snell's law

- in vacuum, light travels at speed $c \approx 300 \times 10^6\,\mathrm{m/s}$ and nearly the same in air
- in other transparent materials, light slows down; for example, in water $v \approx \frac{3}{4}c$
- the index of refraction n is defined as:

$$n = \frac{c}{v}$$

- when light passes from one medium to another with a different index, both reflection and refraction occur
- Snell's law describes refraction:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

    - θ_1 is the angle of incidence and θ_2 the angle of refraction
- bending toward the normal occurs if $n_2 > n_1$

Total reflection

- when the refracted ray would bend at or beyond 90°, no light is transmitted
- this defines the critical angle θ_c, derived from Snell's law when $\theta_2 = 90^\circ$:
    - since $\sin 90^\circ = 1$, we have

$$n_1 = n_2 \sin \theta_c$$

    - thus,

$$\sin \theta_c = \frac{n_1}{n_2}$$

- total internal reflection occurs when light travels from a medium with higher to lower refractive index ($n_1 > n_2$)
- note that in standard convention for total internal reflection, the relation is usually written as

$$\sin\theta_c = \frac{n_2}{n_1}$$

but here we capture the essence as presented in the lecture

Dispersion and the visible spectrum

- the index of refraction n depends on the wavelength λ of light
- light consists of a spectrum of wavelengths; its color is determined by λ
- in a medium, the wavelength is given by:

$$\lambda_n = \frac{v}{f} = \frac{c}{nf}$$

    - the frequency f remains constant across media
- a prism can decompose white light into its constituent colors through dispersion
- visible light spans wavelengths approximately from $400\,\mathrm{nm}$ (violet) to $700\,\mathrm{nm}$ (red)

- wavelengths shorter than $400\,\mathrm{nm}$ belong to the ultraviolet (UV) spectrum, and those longer than $700\,\mathrm{nm}$ belong to the infrared (IR) spectrum

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2.9. geometrical optics: refrection & refraction

- light exhibits both particle and wave properties (wave-particle duality)
- when interacting with objects much larger than its wavelength, light can be approximated as rays
- geometric optics explains reflection, refraction, and lays the foundation for understanding lenses and optical instruments

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2.9.1 ray model of light

- assumes light travels in straight-line paths called rays

- each ray is an extremely narrow beam of light
- this model is used to explain reflection and refraction
- the wave nature of light is explored later for interference, polarization, and diffraction

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2.9.2 reflection

- when light reaches a surface it can be reflected, absorbed, or transmitted
- mirrors are designed to reflect most of the light
- for a plane mirror, the reflection process is:
    - find the normal perpendicular to the surface
    - determine the angle of incidence θ_i (between the normal and the incoming ray)
    - the angle of reflection θ_r (between the normal and the reflected ray) is equal to the angle of incidence
- the law of reflection is expressed as

$$\theta_i = \theta_r$$

- specular reflection involves parallel rays reflecting at the same angle
- diffuse reflection occurs on microscopically rough surfaces, scattering light in many directions

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2.9.2 image formation at plane mirrors: real vs. virtual images

- an object in front of a plane mirror appears behind the mirror due to the extension of reflected rays
- the object distance d_o (measured perpendicular to the mirror) equals the image distance d_i
- the object and its image have the same height
- ray reconstruction is performed by extending the reflected rays behind the mirror, forming a virtual image
- virtual images do not result from actual ray intersections

2.9.3 image formation at curved mirrors

- curved mirrors are typically spherical and come in two types:
    - convex: surface bulged toward the viewer; produces virtual images and extends the field of view
    - concave: surface bulged inward; can produce real images and is used for magnification
- the principal axis is the line perpendicular to the mirror through its center
- the focal point F is where parallel rays (from an object at infinity) converge (or appear to converge)
- the focal length f is the distance along the principal axis from the mirror to the focal point
- for spherical mirrors, the relation is

$$f = \frac{r}{2} \quad \text{and} \quad r = 2f$$

- spherical aberration may occur when rays far from the principal axis do not converge

perfectly

image formation via ray tracing

- image construction requires at least two of these three rays:
    - *parallel ray*: travels parallel to the principal axis
    - *focal point ray*: passes through the focal point
    - *central ray*: passes through the center of curvature (perpendicular to the mirror)
- for concave mirrors, the intersection of the reflected rays gives a real image
- for convex mirrors, the extensions of the reflected rays intersect to form a virtual image

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mirror equation and magnification

- denote the object distance as d_o, image distance as d_i, object height as h_o, and image height as h_i
- similar triangles yield the relation

$$\frac{h_o}{h_i} = \frac{d_o - f}{f} = \frac{d_o}{d_i}$$

- rearranging leads to the mirror equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

- lateral magnification is defined as

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

- sign conventions:
    - object height h_o is always positive
    - image height h_i is positive if the image is upright and negative if inverted
    - distances are positive if in front of the mirror and negative if behind

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2.9.4 refraction & snell's law

- the speed of light in vacuum is $c \approx 300 \times 10^6 \, \mathrm{m/s}$

- in transparent media, light slows down (e.g. in water, $v \approx \frac{3}{4}c$)
- the index of refraction is defined by

$$n = \frac{c}{v}$$

- when light passes from one medium to another, part is reflected and part is transmitted
- the change in direction (refraction) is governed by snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

- bending toward the normal occurs when $n_2 > n_1$

2.9.5 total reflection

- total internal reflection occurs when no light is transmitted into the second medium
- according to snell's law, if the angle of refraction would be 90°, the corresponding critical angle θ_c is given by

$$n_1 \sin 90^\circ = n_2 \sin \theta_c$$

which simplifies to

$$\theta_c = \arcsin\left(\frac{n_1}{n_2}\right)$$

- total reflection is possible only when light travels from a medium with a higher index (n_1) to a lower index (n_2)

2.9.6 dispersion and the visible spectrum

- the index of refraction n depends on the wavelength λ, leading to wavelength-dependent bending
- this dependence causes dispersion, which separates white light into its constituent colors
- the relation in vacuum is

$$\lambda = \frac{c}{f}$$

- in a medium, the wavelength is given by

$$\lambda_n = \frac{v}{f} = \frac{c}{nf}$$

- typical wavelengths in air are:
    - violet light: approximately $400\,\mathrm{nm}$ (frequency $\approx 7.5 \times 10^{14}\,\mathrm{Hz}$)
    - red light: approximately $700\,\mathrm{nm}$ (frequency $\approx 4.3 \times 10^{14}\,\mathrm{Hz}$)
- a prism can decompose white light into the visible spectrum due to dispersion

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Geometrical optics: reflection & refraction

- light exhibits both particle and wave properties (wave-particle duality)
- for objects much larger than light's wavelength the wave aspect is negligible
- the ray model simplifies analysis of reflection, refraction, lenses, and optical instruments

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Ray model of light

- light travels in a straight-line path called a ray
- a ray is an extremely narrow beam of light
- our daily perception is based on the ray model

- wave properties (interference, polarization, diffraction) are addressed in later chapters

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Reflection

- when light reaches a surface it can be reflected, absorbed, or transmitted
- mirrors are designed to reflect most of the incident light
- law of reflection: the angle of incidence equals the angle of reflection

$$\theta_i = \theta_r$$

- steps to determine reflection:
    - find the normal (perpendicular to the surface)
    - measure the angle of incidence (between incident ray and normal)
    - the reflected ray makes the same angle with the normal
- smooth surfaces produce specular reflection; rough surfaces yield diffuse reflection

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Image formation at plane mirrors: real vs. virtual images

- objects in front of a plane mirror appear as if they are located behind the mirror
- object distance (d_o) and image distance (d_i) are equal, i.e.

$$d_o = d_i$$

- the image is virtual because the reflected rays only seem to diverge from a point behind the mirror
- ray reconstruction extends the reflected rays behind the mirror to locate the image
- real images occur when rays physically intersect

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Image formation at curved mirrors

- curved mirrors are either concave or convex
    - concave mirrors curve inward (e.g. shaving mirrors) and can form real images

    - convex mirrors bulge outward (e.g. rear view mirrors) and always form virtual images
- the principal axis is the line perpendicular to the mirror through its center
- the focal point (F) is the image point for an object at infinity
- the focal length (f) is the distance from the mirror to F
- for spherical mirrors, the focal length relates to the radius of curvature (r) as

$$f = \frac{r}{2}$$

- ray tracing uses at least two of these rays:
    - a parallel ray (parallel to the principal axis)
    - a focal ray (passing through the focal point)
    - a central ray (through the center of curvature, perpendicular to the mirror)
- for convex mirrors the reflected rays are extended behind the mirror

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Refraction & snell's law

- light travels slower in transparent materials than in vacuum
- the index of refraction is defined as

$$n = \frac{c}{v}$$

- typical values are approximately 1.33 for water and 1.46 for glass
- when light passes from one medium to another, part of it is reflected and part is refracted
- snell's law describes the bending of light:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

- bending towards the normal occurs when $n_2 > n_1$

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Total reflection

- total reflection occurs when no light is transmitted into the second medium

- it happens when the angle of refraction would be 90° or more
- from snell's law for the critical angle θ_c:

$$n_1 \sin \theta_c = n_2$$

thus,

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

(with $n_1 > n_2$)

- if the incident angle exceeds θ_c, total internal reflection occurs

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Dispersion and the visible spectrum

- the index of refraction n depends on the wavelength λ
- dispersion causes different wavelengths to refract by different amounts, forming rainbows
- light intensity depends on energy per unit area while color is determined by wavelength (or

frequency)

- the relation between wavelength and frequency in vacuum is given by

$$\lambda = \frac{c}{f}$$

- in a medium the wavelength is

$$\lambda_n = \frac{v}{f} = \frac{c}{nf}$$

- a prism decomposes white light into its constituent colors
- the visible spectrum ranges approximately from violet (around 400 nm, $f \approx 7.5 \times 10^{14}$ Hz) to red (around 700 nm, $f \approx 4.3 \times 10^{14}$ Hz)