

Lecture tutorial 2B

Millikan / Oil drop experiment

es29

Set-up:

- two parallel plates generating an approximately uniform electric field E
- voltage V and plate distance d can be used to estimate E
- gap in between plates viewed through e.g. microscope → **up-down flip/mirror-inverted image**
- vaporizer/atomizer/sprayer used to inset oil droplets
- due to friction, droplets charged → motion can be observed due to:
 - electrical field
 - gravitation
 - buoyancy

Task L2B.1.

We have an oil droplet with an idealized spherical shape. The density of oil and air are $\rho_o = 900\text{kg/m}^3$ and $\rho_a = 1.29\text{kg/m}^3$, respectively. What fraction of the gravitational force is the buoyancy force?

Given

- $\rho_o = 900\text{kg/m}^3$
- $\rho_a = 1.29\text{kg/m}^3$

Find

$$F_b/F_g$$

Solutions

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$$F_g = m_o g = \rho_o V g = \rho_o \frac{4}{3}\pi r^3 g$$

$$F_b = m_a g = \rho_a V g = \rho_a \frac{4}{3}\pi r^3 g$$

$$\frac{F_b}{F_g} = \frac{\rho_a \frac{4}{3}\pi r^3 g}{\rho_o \frac{4}{3}\pi r^3 g}$$

$$\frac{F_b}{F_g} = \frac{\rho_a}{\rho_o} = 0.00143$$

The strength of the buoyancy is approximately 1% of the gravitational force.

Task L2B.2.

Imagine a spherical oil droplet ($\rho_o = 900\text{kg/m}^3$, $r = 1\mu\text{m}$) inside an uniform electric field generated by two charged plates (plate distance $d = 3\text{mm}$). The applied voltage of 981V causes the droplet to hover/levitate. What is the charge of the droplet if we neglect buoyancy?

Given

- $d = 3\text{mm}$
- $V = 981\text{V}$
- $\rho_o = 900\text{kg/m}^3$
- $r = 1\mu\text{m}$

Find

- q

Solution

$$F_g = F_e$$

$$mg = qE$$

$$\rho_o \frac{4}{3} \pi r^3 g = q \frac{V}{d}$$

$$q = \frac{4\pi r^3 \rho_o g d}{3V} = 1.131 \times 10^{-19} \text{C}$$

Task L2B.3

Derive the electric field very close to any conducting surface if the charge density $\sigma = \frac{Q}{A}$.

Solution:

- a conducting material will only have an electric field outside, thus **charges accumulate only on the surface**
- if we decompose any shape of conductor, we can approximate the field near to the surface to be uniform
- we use a Gaussian surface as a small cylindrical box that covers the charges on the surface entirely (surface can extend beyond the surface)
- according to Gauss's law, we get

$$\oint \vec{E} d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

- as we assume the field to be **uniform near and perpendicular to the surface** and the area of the box to be simply A :

$$EA = \frac{Q_{enc}}{\epsilon_0}$$

- the charge density is $\sigma = \frac{Q}{A} \leftrightarrow Q = \sigma A$

$$EA = \frac{\sigma A}{\epsilon_0}$$

- thus, we obtain our final equation:

$$E = \frac{\sigma}{\epsilon_0}$$

Since this result is true for any conduction surface, we can also use it to describe the **field of a conducting plate** i.e. capacity if we assume that the plate are large compared to their distance and the field lines are perpendicular to the plates.

Task L2B.4

es28

Starting from the force equilibrium for es28, derive an equation for the voltage which would be present.

Solution

- the electrostatic force pulls the scale up, but only one plate is charge/one moves which gives us $F_e = \frac{1}{2}QE$
- gravitation force is $F_g = mg$

$$mg = \frac{1}{2}QE$$

- using the concept about the electric field from the previous task, $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$, we can state:

$$mg = \frac{1}{2}Q^2 \frac{1}{\epsilon_0 A}$$

- the charge of a capacitor is $Q = CV = \frac{\epsilon_0 A}{d} V$

$$mg = \frac{1}{2} \frac{\epsilon_0^2 A^2}{d^2} V^2 \frac{1}{\epsilon_0 A}$$

$$mg = \frac{1}{2} \frac{\epsilon_0 A}{d^2} V^2$$

$$V = \sqrt{\frac{2d^2 mg}{\epsilon_0 A}}$$