2.12. Wave optics: Diffraction

TEASER?

- ANSWER FRAGMENT

michelson interferometer

`os26`

- splits a light beam into two paths using a beam splitter
- each path reflects off a mirror and recombines at the detector
- interference depends on **optical path difference (OPD)** between arms
- if one mirror moves by distance Δx, the OPD changes by $2\Delta x$
- interference fringes shift, allowing precise distance or wavelength measurements

$$\mathrm{OPD} = 2\Delta x \quad \Rightarrow \quad m\lambda = 2\Delta x$$

- used in metrology, gravitational wave detection (LIGO), and coherence experiments

introduction to diffraction

- building on interference and polarization (wave nature of light).

- **diffraction**: bending of waves as they pass through an aperture or around an obstacle.
- direct consequence of light's wave nature.
- noticeable when aperture/obstacle size is comparable to wavelength.

2.12.1 revisiting the double-slit experiment: diffraction

`os05 - double slit w\ red and green`

- previously, double-slit: interference of two discrete waves.
- waves **bend** (diffract) at interfaces comparable to wavelength.
- interference pattern: bright (constructive) and dark (destructive) fringes.

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the catch: intensity modulation

- simple interference predicts equally bright fringes.
- **reality**: intensity of fringes is modulated.
    - central ones brightest.

        - intensity decreases away from center.
- each slit acts as a source of waves (Huygens' principle) that interfere in a more complex way.
- this phenomenon, responsible for intensity variations, is **diffraction**.

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interference vs. diffraction

- **fundamentally the same phenomenon**: superposition of coherent waves.
- **distinction often lies in conceptualization/source arrangement**:
    - **interference**: superposition from a few discrete sources (e.g., two rays).
    - **diffraction**: superposition from a continuous distribution of sources or many closely spaced sources.
- diffraction: interference of a wave with itself.
- each point on a wavefront acts as a source of secondary wavelets (Huygens' principle).

2.12.2 diffraction at a single-slit

`sim - single slit`

`os05 - single slit`

- easier setup before double-slit intensity.
- monochromatic light from coherent source (λ, phase).
- single narrow slit of width D ($D \approx \lambda$).
- results in a diffraction pattern on a distant screen:
    - central bright maximum.
    - flanked by minima and weaker secondary maxima.

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single-slit pattern formation

- arises from interference of waves originating from different parts of the slit.
- path difference: $\Delta = D \sin\theta$ (screen far from slit).
- relation to wavelength λ:

$$D \sin\theta = m\lambda$$

- m is the order.

- *note*: for minima, m is an integer. for higher-order maxima, m is approximately a half-integer.

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single-slit maxima and minima summary

- **central maximum**:
    - rays passing straight through slit ($\theta = 0$) are in phase.
    - creates a central bright region.
- **minima**:
    - occur at angles θ where path difference between rays from top and bottom of slit is an integer multiple of λ:

$$D \sin\theta = m\lambda, \quad m = \pm 1, \pm 2, \ldots$$

- **higher-order maxima**:
    - weaker maxima appear between minima.
    - approximately where path difference is a half-integer multiple of λ:

$$D\sin\theta \approx (m+\frac{1}{2})\lambda, \quad m \approx \pm\frac{3}{2}, \pm\frac{5}{2}, \ldots$$

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intensity in single-slit diffraction pattern

- consider a slit split into N thin strips, each thickness Δy.
- each strip emits coherent wavelets (Huygens' principle).
- consider parallel rays at angle θ.
- path difference: $\Delta = \Delta y \sin\theta$.
- phase difference: $\Delta\beta = \frac{2\pi}{\lambda}\Delta y \sin\theta$.

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phasor technique for intensity

- each strip has electric field ΔE_0 (amplitude).
- phase differs between strips. electric field is a vector (magnitude & phase).
- intensity on screen: vector sum of all strips.
- total phase difference β across all slits (width $D = N\Delta y$):

$$\beta = N\Delta\beta = \frac{2\pi}{\lambda}N\Delta y \sin\theta = \frac{2\pi}{\lambda}D\sin\theta$$

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phasor summation

- if $\beta = 2\pi$, all vectors cancel (first minimum).
- minima for $\beta = \pm 2\pi, \pm 4\pi, \ldots$.
- higher-order maxima for $\beta = \pm 3\pi, \pm 5\pi, \ldots$
  - portion of vectors cancel, reducing intensity.

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deriving the intensity formula

- central maximum ($\theta = 0$): phasors in phase, resultant $E_0 = N\Delta E_0$.
- general angle θ: phasors form circular arc.
- arc length $E_0 = r\beta$.
- resultant field E_θ is chord of arc:

$$\sin\left(\frac{\beta}{2}\right) = \frac{E_\theta/2}{r}$$

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single-slit intensity formula

- dividing relations:

$$\frac{E_\theta}{E_0} = \frac{\sin(\beta/2)}{\beta/2}$$

- intensity is proportional to square of electric field amplitude ($I \propto E^2$):

$$\frac{I_\theta}{I_0} = \left(\frac{E_\theta}{E_0}\right)^2 = \left(\frac{\sin(\beta/2)}{\beta/2}\right)^2$$

- substituting β:

$$I_\theta = I_0 \left(\frac{\sin\left(\frac{\pi D \sin\theta}{\lambda}\right)}{\frac{\pi D \sin\theta}{\lambda}}\right)^2$$

2.12.3 diffraction at a double-slit

- previous interference analysis determines maxima/minima positions.
- reality: finite number of peaks, brightest at center, lower intensity surrounding.
- this is due to **diffraction** from each slit.

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combining diffraction and interference

- double-slit: each slit has width D, separation d.

- each slit contributes an electric field modulated by its own diffraction:

$$E_{single} = E_{0,single}\frac{\sin(\beta/2)}{\beta/2}$$

- where $\frac{\beta}{2} = \frac{\pi D \sin \theta}{\lambda}$

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phase difference between slits

- path difference between light from two slits: $\Delta = d \sin \theta$.
- phase difference δ:

$$\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda} d \sin \theta$$

- electric fields from the two slits:

$$E_1 = E_{single} e^{i\delta/2}, \quad E_2 = E_{single} e^{-i\delta/2}$$

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total electric field for double slit

- superposition:

$$E_{total} = E_1 + E_2 = E_{single}(e^{i\delta/2} + e^{-i\delta/2})$$

- using
e^{ix}
$+ e^{-ix}$
$= 2\cos$
x
:

$$E_{total} = 2E_{single}\cos\left(\frac{\delta}{2}\right)$$

- substituting E_{single}:

$$E_{total} = 2E_{0,single}\frac{\sin(\beta/2)}{\beta/2}\cos\left(\frac{\delta}{2}\right)$$

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double-slit intensity formula

- intensity $I_\theta \propto E_{total}^2$
- let I_0 be the intensity of the central maximum:

$$I_\theta = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2\cos^2\left(\frac{\delta}{2}\right)$$

interpretation of double-slit intensity

`sim - intensity`

- **diffraction factor (envelope)**: $\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2$ from each slit (D)
- **interference factor**: $\cos^2\left(\frac{\delta}{2}\right)$ from path difference between slits (d)
- diffraction envelope modulates finer interference fringes
- zeros of diffraction pattern cause disappearance of interference fringes

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2.12.4 limits of resolution & circular apertures

- lenses (circular apertures of diameter D) cannot image a point perfectly (diffraction & aberration)
- light from a point source forms an **Airy disk**
- angular half-width θ of Airy disk:

$$\theta \approx 1.22\frac{\lambda}{D}$$

resolution limit of a pinhole camera

`os07 - resolution limit`

- the resolution is limited mainly by two effects: diffraction and geometric blur
- diffraction occurs because light waves spread out when passing through the pinhole, causing image blur
- geometric blur happens if the pinhole is too large, letting rays from one point spread on the image plane
- there is an optimal pinhole diameter balancing diffraction and geometric blur for the sharpest image

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diffraction limit and optimal pinhole size

- diffraction-limited angular resolution θ for a circular aperture of diameter d and wavelength λ is approximately

$$\theta \approx 1.22 \frac{\lambda}{d}$$

- the optimal pinhole diameter to minimize blur is approximately

$$d_{\text{opt}} \approx 1.9\sqrt{\lambda f}$$

  where f is the distance from the pinhole to the image plane (focal length)
- this optimal diameter balances diffraction and geometric blur for best image sharpness

practical considerations

- visible light wavelength λ is about 550 nm (green light)
- for typical pinhole cameras with f on the order of centimeters, optimal pinhole diameter is tens to hundreds of micrometers
- resolution limit corresponds roughly to the size of the diffraction spot on the image plane, often on the order of tens of micrometers
- this translates to an angular resolution of a few arcminutes in typical setups

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Rayleigh criterion

- resolution limit: ability to distinguish two closely spaced objects
- **Rayleigh criterion**: just resolvable when one center overlaps other's first minimum
- minimum angular separation:

$$\theta_{min} = 1.22\frac{\lambda}{D}$$

- smaller θ_{min} → better resolution
- applies to telescopes and mirrors (D = objective diameter)
- **ultimate limit of resolution**: $RP \approx \frac{\lambda}{2}$

`ow13 - spect grating` `ow10 + ow15 - spect lamps` ow13 Spektren am Reflexionsgitter ow10 Spektrum einer Quecksilberdampflampe ow15 Spektrum einer Natriumdampflampe

2.12.5 diffraction grating & spectroscopy

- **diffraction grating**: many equally spaced slits (spacing d)
- thousands of lines per cm/mm
- used for precise wavelength measurements
- maxima occur at angles:

$$\sin\theta = \frac{m\lambda}{d}, \quad m = 0, \pm 1, \pm 2, \ldots$$

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diffraction grating features

- central (zero-order) maximum is brightest
- **sharper higher-order maxima** than double-slit
- even slight angle change → destructive interference across many slits
- **transmission grating**: light passes through
- **reflection grating**: lines ruled on mirror → light reflected

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white light with diffraction grating

- white light instead of monochromatic
- **central white peak** ($m = 0$): all wavelengths overlap constructively
- for $m \neq 0$: different λ diffract at different angles:

$$\sin\theta = \frac{m\lambda}{d}$$

- result: spectrum of colors per order
- key principle behind **spectroscopy**

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spectrometer / spectroscope

- measures light wavelength with high precision
- uses diffraction grating or prism
- **components**:
    1. light enters slit in **collimator**
    2. slit at focal point of lens → parallel beam
    3. beam sent to grating/prism
    4. movable telescope focuses dispersed light
