1.4. Work, Energy, & Power

In this chapter, we link **force and motion** to the broader concept of **energy** — the ability to do work.
We define **work** as the transfer of energy by a force, derive expressions for **kinetic** and **potential energy**, and show how these combine into the conserved quantity of **mechanical energy** when only conservative forces act.

We then extend the analysis to **nonconservative** and **dissipative forces**, which transform mechanical energy into other forms such as heat, and introduce **power** as the rate of energy transfer or conversion.

1.4.1. Definition of Work

In everyday language, the word **work** describes any physical or mental effort. In physics, however, the term has a precise and quantitative definition. Work is closely linked to **energy** which will be introduced later in this script.

In physics, **work is defined as** the energy transferred to or from an object when a force acts on it and causes a displacement in the direction of that force.

1.4.2. Work done by a Constant Force

When a **constant force** $\vec{F}$ acts on an object that undergoes a displacement $\vec{d}$, the work done by the force is defined as:

$$W = \vec{F} \cdot \vec{d}.$$

This dot product implies that **work is a scalar quantity** (it has magnitude but no direction), even though it results from two vectors.

Work is measured in **joule (J)** which is defined as:

$$1\,\mathrm{J} = 1\,\mathrm{N} \cdot \mathrm{m} = 1\,\mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2.$$

If we know the angle θ between $\vec{F}$ and $\vec{d}$, we can rewrite the equation into:

$$W = Fd\cos\theta.$$

The angle θ determines the **sign** of the work:

- If $0^\circ \leq \theta < 90^\circ$, $\cos\theta > 0$ and $W > 0$.
  The force has a component in the direction of motion, and **positive work** is done.

- If $\theta = 90^\circ$, $\cos\theta = 0$ and $W = 0$.
  The force is perpendicular to motion, thus the work is zero.

- If $90^\circ < \theta \le 180^\circ$, $\cos\theta < 0$ and $W < 0$.
  The force opposes motion and **negative work** is done.

Note that the dot product $\vec{F} \cdot \vec{d} = Fd\cos\theta$ measures the projection of $\vec{F}$ onto $\vec{d}$, or in other words, it measures the component of $\vec{F}$ parallel to $\vec{d}$, i.e. $F_{\parallel}$

$$W = F_{\parallel} d.$$

We can conclude:

- Only the component of a force **along the direction of displacement** contributes to work.
- A force that is applied without causing motion — for example, pushing against a wall that does not move — does **no work** in the physical sense.

Examples for Conceptual Understanding

- **Pushing against a wall:**
  Even if you apply a large force, if the wall does not move, the displacement is zero. Therefore **no work** is done.

- **Lifting an object vertically:**
  When you lift a book upward, your applied force and the displacement are in the same direction. You do **positive work** on the book.

- **Lowering an object slowly:**
  When lowering the same book carefully, your force is upward but the motion is downward. You do **negative work**.

- **Carrying an object at constant height:**
  The force from your arm acts vertically, but the displacement is horizontal. Since the angle between them is 90°, **no work** is done on the book.

- **Pulling a box at an angle:**
  Only the **horizontal component** of your pulling force contributes to the work. The vertical component does no work because it is perpendicular to the displacement.

- **Sliding a box with friction:**
  When you push a box across the floor, your applied force does **positive work**, while friction does **negative work** of equal or smaller magnitude.
  The **net work** is the sum of all individual works:

  $$W_{\text{net}} = \sum_i W_i$$

or, equivalently,

$$W_{\text{net}} = \vec{F}_{\text{net}}\,\vec{d}\,.$$

- **Circular motion:**
  The centripetal force is always perpendicular to the velocity of the object, so it does **no work**. It only changes the **direction**, not the **speed**, of motion.

These examples illustrate that only the **component of a force parallel to motion** transfers energy to or from an object, while perpendicular forces do no work at all. Further, the **net work** equals the **sum of all individual works** or, equivalently, the **net force times the displacement**.

1.4.3. Work Done by a Varying Force

In the previous sections, we restricted our discussion to constant forces, where both the magnitude and direction of the force remain unchanged throughout the displacement.
However, in many real-world situations forces vary either in magnitude, in direction, or both (e.g. the gravitational force acting on a spacecraft as it moves far from Earth). In such cases, the simple equation $W = Fd\cos\theta$ is no longer valid, since F and θ may change along the path.
We must instead consider infinitesimal contributions to work and then **integrate** over the entire displacement.

If the force on an object varies with position, we can still define the infinitesimal work done over a small displacement $d\vec{l}$ as

$$dW = \vec{F} \cdot d\vec{l}\,.$$

In essence, over the infinitesimal displacement $d\vec{l}$, the force is constant and, therefore, we re-use our previous equation.

To find the total work done by the force as the object moves from point A to point B along any path, we **integrate**:

$$W = \int_A^B \vec{F} \cdot d\vec{l}\,.$$

This is known as a **line integral** of the force along the path of motion.

If $\vec{F}$ and $d\vec{l}$ are not parallel, the dot product ensures that only the component of the force in the direction of motion contributes to the work.

In one-dimensional motion, where the force acts along the x-axis, this simplifies to

$$W = \int_{x_A}^{x_B} F(x)\,dx.$$

Graphically, this integral corresponds to the **signed area under the force–position curve** $F(x)$. Positive areas represent positive work (force aiding the motion), while negative areas represent negative work (force opposing the motion).

If a force has components in more than one direction, the total work can be expressed as the sum of the work done by each component:

$$W = \int_{x_1}^{x_2} F_x \, dx + \int_{y_1}^{y_2} F_y \, dy + \int_{z_1}^{z_2} F_z \, dz.$$

This general form is valid for any path in three-dimensional space and and can be extended to higher dimensions if needed.

1.4.4. Springs, Work, and Hooke's Law

A particularly important example of a **varying force** is the restoring force of an ideal spring, which obeys **Hooke's law**:

$$F_S = -kx,$$

where

- k is the **spring constant** (a measure of stiffness), and
- x is the **displacement from the equilibrium position** ($x = 0$).

The **negative sign** indicates that the spring **force always points opposite to the displacement**, acting to restore the system to its equilibrium position.

Work Done by the Spring

When the spring moves through a small displacement dx, the infinitesimal work done **by the spring** is

$$dW_S = F_S \, dx = (-kx) \, dx.$$

To find the total work done as the spring changes from position x_1 to x_2, we integrate:

$$W_S = \int_{x_1}^{x_2} (-kx) \, dx = -\tfrac{1}{2} k (x_2^2 - x_1^2).$$

The negative sign shows that the **spring does positive work** when it pushes the mass back toward equilibrium (reducing $|x|$), and **negative work** when it is being stretched or compressed further.

Work Done on the Spring

If an external agent slowly stretches the spring, the applied force must exactly balance the spring force:

$$F_{\text{ext}} = +kx.$$

The work done **on the spring** by this external force is

$$W_{\text{ext}} = \int_{x_1}^{x_2} F_{\text{ext}}\, dx = \int_{x_1}^{x_2} kx\, dx = \tfrac{1}{2}k(x_2^2 - x_1^2).$$

For a stretch starting from the relaxed position ($x_1 = 0$):

$$W_{\text{ext}} = \tfrac{1}{2}kx^2.$$

This quantity represents the **work required to stretch or compress the spring**. As we will see in the next section, this work is stored in the system as energy and can later reappear as motion when the spring returns toward equilibrium.

Key points:

- The spring force increases **linearly** with displacement ($F \propto x$).
- The work done equals the **area under the F–x curve**, which forms a triangle.
- Because the work equals $W = \frac{1}{2}kx^2 = \frac{1}{2}F_{\text{max}}x$, we can see that only half of the maximum force acts on average during the stretch. For a linearly increasing force, this corresponds exactly to the average force $\frac{1}{2}F_{\text{max}}$ acting over the displacement.

1.4.5. The Work–Energy Principle & Kinetic Energy

We have defined **work** as the transfer of energy by a force acting through a displacement.
We now examine how this concept connects directly to the **motion of an object**, leading to the definition of **translational kinetic energy** and the **work–energy principle**. In the following, we will simply refer to kinetic energy to mean translational kinetic energy. A second form — rotational kinetic energy — will be introduced later when we study rotational motion.

From Work to Kinetic Energy

Consider an object of mass m moving under a **net force** $\vec{F}_{\text{net}}$.
From Newton's second law,

$$\vec{F}_{\text{net}} = m\vec{a}.$$

The infinitesimal work done by this force over a small displacement $d\vec{l}$ is

$$dW = \vec{F}_{\text{net}} \cdot d\vec{l}.$$

Substituting $\vec{F}_{\text{net}} = m\vec{a} = m\frac{d\vec{v}}{dt}$ and using $d\vec{l} = \vec{v}\,dt$ gives

$$dW = m\frac{d\vec{v}}{dt} \cdot \vec{v}\,dt = m\vec{v} \cdot d\vec{v}.$$

Integrating between two states with velocities $\vec{v}_1$ and $\vec{v}_2$ gives

$$W_{\text{net}} = \int_{\vec{v}_1}^{\vec{v}_2} m\vec{v} \cdot d\vec{v} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2.$$

The **net work done on the object** therefore equals the **change in** the quantity $\frac{1}{2}mv^2$:

$$W_{\text{net}} = \Delta K = K_2 - K_1, \qquad K = \tfrac{1}{2}mv^2.$$

Thus, **work done on an object changes its kinetic energy** — it modifies the object's motion.

Consequently, like work, **energy is a scalar quantity** and **is measured in **joules (J)**.

The Work–Energy Principle for Kinetic Energy

The relationship we have just derived,

$$W_{\text{net}} = \Delta K = K_2 - K_1,$$

is known as the **work–energy theorem**:

> The net work done on an object by all the forces acting on it equals the change in its *kinetic energy*.

This theorem holds **for any motion**, whether the forces are constant or variable, and regardless of the path.
Later, we will extend this idea to include other forms of energy, such as potential energy, leading to the principle of **mechanical energy conservation**.

Physical Interpretation

- **Positive net work** ($W_{\text{net}} > 0$): the object gains kinetic energy and speeds up.
- **Negative net work** ($W_{\text{net}} < 0$): the object loses kinetic energy and slows down.
- **Zero net work** ($W_{\text{net}} = 0$): the kinetic energy remains constant, and the motion is uniform.

In other words, **work changes motion** — it converts force and displacement into energy of motion.

The work–energy principle establishes the bridge between **forces** and **energy**. In the next section, we will extend this idea to include **stored energy** — work associated with position rather than motion — introducing the concept of **potential energy** and the **conservation of mechanical energy**.

1.4.6. Potential Energy

We derived **kinetic energy** from the work done by a net force, finding that $W_{\text{net}} = \Delta K$. We now turn to another form of energy — **potential energy** — which arises from forces that depend on position.

While kinetic energy describes **motion**, potential energy represents the **capacity to do work** due to an object's **position or configuration**.

Potential energy is particularly useful for describing situations where energy is stored and later released, such as in gravity or elastic systems.

From Work to Potential Energy

From the definition of work,

$$dW = \vec{F} \cdot d\vec{l}\,.$$

If the force $\vec{F}$ depends on position (for example, gravity or a spring), the total work done by this force between points A and B is

$$W_{AB} = \int_A^B \vec{F} \cdot d\vec{l}\,.$$

In many systems, the **work done** by a force **depends only on the initial and final positions** of the object and not on the path taken between them. Such forces are called **conservative forces** (see next section). Examples include the gravitational force and the elastic spring force described by Hooke's law. For such cases, it is convenient to define a scalar function — the **potential energy** — whose change equals the **negative of the work done by the force**:

$$\Delta U = U_B - U_A = -W_{AB} = -\int_A^B \vec{F} \cdot d\vec{l}\,.$$

This definition ensures that when the force does **positive work**, the potential energy decreases, and when work is done **against** the force, the potential energy increases.

Gravitational Potential Energy

Consider a mass m in a uniform gravitational field near Earth's surface.
The gravitational force acting on it is

$$\vec{F}_G = -mg\,\hat{y},$$

where $\hat{y}$ is the unit vector in the upward vertical direction.

The work done by gravity as the object moves vertically from y_1 to y_2 is

$$W_G = \int_{y_1}^{y_2} \vec{F}_G \cdot d\vec{l} = \int_{y_1}^{y_2} (-mg)\,dy = -mg(y_2 - y_1).$$

We define the change in gravitational potential energy as the negative of the work done by gravity.
Applying this definition gives

$$\Delta U = -W_G = mg(y_2 - y_1).$$

Choosing $U = 0$ at $y = 0$ gives

$$U_{\text{grav}} = mgy.$$

The **gravitational potential energy is a property of the Earth–object system, not of the object alone**. It **depends** on their **relative positions**: lifting the object increases the system's stored energy, while letting it fall allows that energy to convert into kinetic energy. In a reference frame moving together with the object and the observer, where their relative positions remain constant, the potential energy would not change.

Elastic Potential Energy (Spring Force)

The spring force obeys **Hooke's law**:

$$F_S = -kx.$$

The work done by the spring as it moves from x_1 to x_2 is

$$W_S = \int_{x_1}^{x_2} (-kx)\, dx = -\tfrac{1}{2}k(x_2^2 - x_1^2).$$

Using $\Delta U = -W_S$, the change in potential energy is

$$\Delta U = \tfrac{1}{2}k(x_2^2 - x_1^2).$$

If we choose $U(0) = 0$, the **elastic potential energy** stored in the spring becomes

$$U_{\text{elastic}} = \tfrac{1}{2}kx^2.$$

This energy represents the work required to stretch or compress the spring and can later be released as motion when the spring returns to equilibrium.

The Reference Position and Zero of Potential Energy

In both gravitational and elastic systems, the **zero point** of potential energy is **arbitrary**.
Only **changes** in potential energy (ΔU) have physical meaning, since forces depend on spatial **differences** in U, not its absolute value.

For example, in the spring system, we chose $U(0) = 0$ at the **equilibrium position**, leading to

$$U_{\text{elastic}} = \tfrac{1}{2}kx^2.$$

However, we could equally choose $U = 0$ at any other position x_0:

$$U(x) = \tfrac{1}{2}k(x^2 - x_0^2),$$

and all observable results, such as forces and energy differences, would remain unchanged.

The same principle applies to **gravitational potential energy**, where one might set $U = 0$ at ground level, at the table surface, or even at infinity.
What matters physically is not where $U = 0$, but how U **changes** between two points.

Hence, the **reference position** affects only the numerical value of U, not the dynamics or the measurable energy transformations of the system.

```
VBox(children=(HBox(children=(FloatSlider(value=5.0, description='x1', max=12.566370614359172, step=0.01), Flo…
Output()
```

Force–Potential Energy Relationship

For a one-dimensional motion where $\vec{F} = F(x)\hat{i}$,

$$U(x) = -\int F(x)\,dx + C,$$

where C is an arbitrary constant defining the zero of potential energy.

Differentiating gives the inverse relation:

$$F(x) = -\frac{dU(x)}{dx}.$$

Thus, the force always points in the direction of **decreasing potential energy**.

This concept can be extended to higher dimensions. In three dimensions, the concept generalizes using partial derivatives:

$$\vec{F} = -\nabla U = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right).$$

The **gradient** of U points toward the direction of greatest increase of potential energy, while the force acts oppositely, i.e. toward lower potential energy.

Key Insights

- Only **changes** in potential energy are physically meaningful; the zero level can be chosen arbitrarily.
- Potential energy is a property of the **system**, not the object alone (e.g., the Earth–mass system).
- When the force does positive work, the potential energy decreases by the same amount: $W = -\Delta U$.

The definition above applies to forces for which the work depends only on the endpoints of motion. In the next section, we will define such forces formally as **conservative forces** and contrast them with **nonconservative** ones.

1.4.7. Conservative and Nonconservative Forces

In the previous section, we defined potential energy for cases where the work done by a force depends only on the initial and final positions.

For such forces, it is possible to recover mechanical energy completely when the system returns to its original configuration and we call these forces **conservative forces**

Conservative Forces

A **conservative force** is one for which the work done moving an object between two points is **independent of the path** taken.
Equivalently, the work done by a conservative force around any closed path (indicated by $\oint$) is zero:

$$W_{\text{closed}} = \oint \vec{F} \cdot d\vec{l} = 0.$$

This property means that energy stored in the system can be **recovered** without loss, since all work done in one direction can be undone by returning along the same path.

Typical examples include:

- Gravity
- Elastic (spring) force
- Electrostatic force

For all these, the force can be expressed as the **negative gradient** of a potential energy function:

$$\vec{F} = -\nabla U.$$

Example: Rollercoaster and Path Independence

Consider a simplified **rollercoaster track** where a cart of mass m moves under the influence of gravity, neglecting friction and air resistance.
The work done by gravity as the cart moves from point A to point B depends **only** on the change in height, not on the path or shape of the track:

$$W_{AB} = -mg(y_B - y_A).$$

If the rollercoaster returns to its starting point ($y_B = y_A$), the **net work done by gravity** over the entire loop is zero:

$$W_{\text{loop}} = 0.$$

This remains true whether the cart moves **clockwise** or **counterclockwise** around the loop, so that

$$W_{AB} = -W_{BA}.$$

Hence, the gravitational force is **conservative**: the potential energy of the system depends only on the vertical position of the cart, and completing a full ride restores the system to its initial energy state.

Nonconservative Forces

In contrast, **nonconservative forces** are those for which the work done depends on the path taken. When such forces act, some mechanical energy is transformed into other forms (such as heat or sound) and **cannot** be fully recovered.

Common examples include:

- Friction
- Air resistance
- Viscous drag

The work done by friction over a distance d is, for instance,

$$W_{\text{fr}} = -F_{\text{fr}} d,$$

and if the path is longer, more energy is dissipated:

$$W'_{\text{fr}} = -F_{\text{fr}} d' \quad \text{with} \quad d' > d.$$

Energy Conservation and System Classification

Type of Force/System	Path Dependence	Recoverable Energy?	Example
Conservative	No (depends only on endpoints)	Yes	Gravity, springs
Nonconservative	Yes (depends on path)	No (energy dissipated)	Friction, drag

The distinction between conservative and nonconservative forces forms the foundation of **energy conservation** in mechanics, developed in the next section.

1.4.8. Mechanical Energy and Its Conservation

We now bring together the concepts of **kinetic** and **potential energy** to describe the total energy of a system. When only **conservative forces** act, energy may transform between kinetic and potential forms, but the total **mechanical energy** remains constant.

From Work to Energy Conservation

From the work–energy theorem,

$$W_{\text{net}} = \Delta K.$$

If we separate the total work into contributions from **conservative** and **nonconservative** forces,

$$W_{\text{net}} = W_{\text{cons}} + W_{\text{noncons}}.$$

For conservative forces we defined

$$W_{\text{cons}} = -\Delta U.$$

Substituting gives

$$\Delta K = -\Delta U + W_{\text{noncons}}.$$

Rearranging,

$$\Delta K + \Delta U = W_{\text{noncons}}.$$

Conservation of Mechanical Energy

If no nonconservative forces act ($W_{\text{noncons}} = 0$), then

$$\Delta K + \Delta U = 0 \quad \Rightarrow \quad K_1 + U_1 = K_2 + U_2.$$

We define the **mechanical energy**

$$E = K + U,$$

so that in a conservative system

$$E = \text{constant}.$$

> **Principle of Conservation of Mechanical Energy:**
> In an isolated system acted on only by conservative forces, the total mechanical energy remains constant.

Physical Meaning

When potential energy decreases, kinetic energy increases by the same amount, and vice versa.
A falling object, a swinging pendulum, or a mass on a spring all illustrate this continuous exchange between K and U while E remains constant.

In many systems, **several forms of potential energy** act simultaneously.
For example, in a **vertical spring–mass system**, both **gravitational** and **elastic** potential energy contribute:

$$E = K + U_{\text{grav}} + U_{\text{elastic}} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 = \text{constant}.$$

Here, gravity and the spring force continually exchange energy with the moving mass, but the total mechanical energy of the system stays the same.

This principle provides a powerful shortcut for analyzing motion **without directly applying Newton's laws**.
Knowing

$$E = \frac{1}{2}mv^2 + U(x),$$

one can determine the speed or position of an object anywhere along its path using **energy conservation alone**.

Extension to Three Dimensions

For three-dimensional motion, the same relation holds:

$$E = K + U = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) + U(x, y, z) = \text{constant}.$$

The corresponding force components follow from the potential:

$$F_x = -\frac{\partial U}{\partial x}, \qquad F_y = -\frac{\partial U}{\partial y}, \qquad F_z = -\frac{\partial U}{\partial z}.$$

Key Points

- **Mechanical energy:** $E = K + U$
- **Conservation:** $\Delta E = 0$ if only conservative forces act
- **Nonconservative work:** $\Delta E = W_{\text{noncons}}$
- **Interpretation:** changes in potential and kinetic energy exactly balance in conservative systems

1.4.9. The Law of Conservation of Energy

We have seen that when only **conservative forces** act, the total **mechanical energy** $E = K + U$ remains constant. In real systems, however, additional forces such as friction, air resistance, or deformation are often present. These forces transform mechanical energy into other forms — typically **thermal energy** — but the **total energy** of the system remains unchanged.

This leads to one of the most fundamental principles in all of physics: The **law of conservation of energy**.

The Universal Conservation of Energy

Energy exists in many forms — mechanical, thermal, chemical, electrical, nuclear, or radiant.
It can be **transformed** between these forms or **transferred** between systems, but the total energy of an **isolated system** never changes.

$$\text{Total Energy} = \text{constant}.$$

Thus, the law of conservation of energy extends the work–energy principle beyond mechanics:
it states that although the *form* of energy may change, the *total amount* remains constant in every physical process — from a falling object to a chemical reaction or a nuclear fusion event.

Mechanical Energy with Nonconservative Forces

When nonconservative forces such as **friction** or **air resistance** act, they do **negative work** on the system, converting part of the mechanical energy into **thermal energy**.

The total mechanical energy therefore decreases, but the total energy (mechanical + thermal) remains constant:

$$\Delta(K + U) + \Delta E_{\text{thermal}} = 0.$$

For example, as a roller-coaster car moves along a track with friction, it loses mechanical energy but gains internal (heat) energy.
It will not reach its original height because some of the gravitational potential energy has been transformed into heat.

Dissipative Forces and Energy Transformation

Friction and drag are **dissipative forces**, a class of **nonconservative forces** that remove mechanical energy from a system.
They convert the macroscopic (bulk) motion of an object into disordered molecular motion, perceived as **heat**.
From the microscopic point of view, thermal energy is the sum of the kinetic and potential energies of atoms and molecules.

Thus, when an object slows due to friction,

$$\Delta K_{\text{object}} + \Delta U_{\text{thermal}} = 0.$$

No energy is lost — it merely changes form from mechanical to internal energy.

The Law of Conservation of Energy (General Form)

> **Law of Conservation of Energy:**
> The total energy of an isolated system remains constant.
> Energy can be transformed from one form to another or transferred between objects, but it cannot be created or destroyed.

In symbolic form:

$$\Delta K + \Delta U + \Delta E_{\text{other}} = 0,$$

where E_{other} may include heat, sound, chemical, electrical, or nuclear energy.

Universality and Scope

The conservation of energy is one of the most universal and experimentally verified principles in physics.
It applies to systems ranging from galaxies to subatomic particles and holds even in quantum and relativistic contexts.
In relativity, energy and mass are related by $E = mc^2$, so the total **energy–mass** of an isolated system is constant.

For mechanical systems, Newton's laws and the energy principle are equivalent descriptions, but the **energy formulation** is often more powerful because it unifies all physical processes — mechanical, thermal, electrical, and beyond.

Conceptual Summary

- Energy can be **transferred** or **transformed**, but never created or destroyed.
- In the absence of nonconservative forces, **mechanical energy** ($K + U$) is constant.
- When nonconservative forces act, mechanical energy changes, but the **total energy** (including heat and other forms) remains constant.
- The conservation of energy is a universal law — the total energy of the universe is constant.

1.4.10. Power and Efficiency

After defining work and energy, we now consider **how fast** energy is transferred or transformed. This rate is described by the physical quantity **power**.

Definition of Power

Power measures the rate of doing work or transferring energy. It is measured in **watt (W)** $1 \text{ W} = 1 \text{ J/s}$.

Average power:

$$P_{\text{avg}} = \frac{W}{t}.$$

Instantaneous power:

$$P = \frac{dW}{dt} = \frac{dE}{dt}.$$

From $dW = \vec{F} \cdot d\vec{l}$, we obtain

$$P = \frac{dW}{dt} = \frac{\vec{F} d\vec{l}}{dt} = \vec{F} \cdot \vec{v}.$$

Power is positive when the force **adds** energy to a system and negative when it **removes** energy.

Efficiency

No real process converts energy perfectly from one form to another.
The **efficiency** η of a machine is the ratio of useful output power to total input power:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}}.$$

Typical efficiencies:

- Gasoline engine: 15–25 %
- Diesel engine: 30–40 %
- Electric motor: 80–95 %

If a car engine operates at 20 % efficiency, only one-fifth of the fuel's chemical energy becomes mechanical energy; the rest appears as heat.

Conceptual Summary

- **Power** quantifies how quickly energy is converted or transferred.
- Two machines performing the same work differ in power if one completes it faster.
- Power can be **positive** (energy supplied) or **negative** (energy extracted).
- **Efficiency** compares useful output to total input and is always less than 1.