

# 1.1. Kinematics in one dimension



Kinematics describes how objects move without explaining why they move. In one dimension, motion unfolds along a single straight line—usually the horizontal  $x$ -axis or vertical  $y$ -axis—and is expressed through quantities like position, displacement, velocity, and acceleration. To describe motion unambiguously, we must define a reference frame, specifying an origin, a time scale, and which direction counts as positive.

## 1.1.1. Frame of reference

A reference frame is the stage on which motion takes place. It consists of an observer, an origin, coordinate axes, and a clock. **All measurements of position, velocity, and acceleration are made relative to that frame.** For instance, a passenger walking inside a moving train may be nearly at rest relative to the train but moving rapidly relative to the ground—both descriptions are correct within their respective frames. In everyday (Galilean) mechanics, transforming between frames involves simply adding or subtracting velocities depending on direction. **Although numerical values may differ, all observers agree on the physical events themselves**, such as when the passenger reaches the door. Stating the reference frame and axis convention at the outset prevents sign errors and ensures consistent interpretation of motion.

## 1.1.2. Velocity as a vector (in 1D)

Velocity  $v$  is fundamentally a **vector** quantity. In one dimension, its direction reduces to a sign, but it can still be useful to write  $\vec{v} = v \hat{i}$  (horizontal) or  $\vec{v} = v \hat{j}$  (vertical). This notation keeps direction explicit and makes relationships like “ $\vec{a}$  is opposite to  $\vec{v}$ ” precise.

It also reminds us that coordinate conventions (e.g. moving the origin or flipping the positive direction) must be applied consistently to position, velocity, and acceleration together. **The physics does not change when coordinates do, only the algebraic signs change.**

## 1.1.3. Average velocity

Average velocity summarizes motion over a finite time interval as:

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

It uses **displacement**, the signed change in position, not the total path length. Because of this, direction reversals can make  $\bar{v}$  small or even zero, even when the traveled distance is large. On a position–time graph,  $\bar{v}$  equals the slope of the **secant line** connecting  $(t_0, x_0)$  and  $(t, x)$ . A straight line indicates constant velocity, while curvature means that velocity changes over time.

## 1.1.4. Instantaneous velocity

Instantaneous velocity describes motion at a specific moment by shrinking the time interval to zero:

$$v(t) = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

It is the **derivative of the position function** and represents the **slope of the tangent** to the  $x(t)$  curve. In other words,  $v(t)$  gives the rate at which position changes with time. The **sign** of  $v$  indicates direction along the chosen axis—positive for motion in the positive direction and negative for the opposite.

```
interactive(children=Dropdown(description='Function:', options=['x(t) = t^2', 'x(t) = t^3', 'x(t) = 0.5 + 0.5...'])
```

## 1.1.5 Speed vs. velocity; distance vs. displacement

**Velocity** is a signed quantity that carries direction information, while **speed** is its magnitude and always nonnegative. Likewise, **displacement** is a signed change in position, whereas **distance traveled** is the total path length accumulated over time. These pairs answer different questions: displacement compares beginning and end only, while distance records all the wandering in between.

Because of direction reversals, the magnitude of the average velocity generally differs from the average speed unless motion stays in one direction. When reading a problem, decide early whether it asks for a magnitude (speed, distance) or a signed quantity (velocity, displacement). That single choice avoids most ambiguities and sign errors.

## 1.1.6. Acceleration

Acceleration describes how velocity changes with time. Over a finite interval, the average acceleration is  $\bar{a} = \frac{\Delta v}{\Delta t}$  and at a specific instant:

$$a(t) = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

It is the **derivative of the velocity function** and represents the **slope of the tangent** to the  $v(t)$  curve. In other words,  $a(t)$  gives the rate at which velocity changes with time.

The **sign of  $a$**  shows how velocity changes along the chosen axis:  $a > 0$  makes  $v$  more positive,  $a < 0$  makes it more negative. When  $a$  and  $v$  share the same sign, the object speeds up; when they have

opposite signs, it slows down. The latter is often called deceleration, though in physics it simply means acceleration opposite to the motion.

## 1.1.7 Motion types

A **uniform motion** occurs when the velocity remains constant ( $v = \text{const}$ ). The object covers equal displacements in equal time intervals, and its position changes linearly with time:

$$x(t) = x_0 + vt$$

In this case, there is **no acceleration** ( $a = 0$ ), the average and instantaneous velocity are equal, and the position–time graph is a straight line.

A **uniformly accelerated motion** occurs when the velocity changes linearly with time, that is, when acceleration is constant ( $a = \text{const}$ ). The key relations are:

$$v(t) = v_0 + at, \quad \& \quad x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

These formulas connect position, velocity, time, and acceleration and are valid only under the constant- $a$  assumption.

The first equation shows that  $v(t)$  is a straight line whose slope equals  $a$ . The second equation indicates that  $x(t)$  includes a linear term from the initial velocity and a quadratic term from constant acceleration.

Constant acceleration is an idealization but an excellent one for motion under steady thrust or uniform braking, and for vertical motion near Earth's surface when air resistance is negligible.

## 1.1.8 Relations between position, velocity, & acceleration

Position, velocity, and acceleration form a **hierarchy of motion** linked by derivatives and integrals.

**Velocity is the time derivative of position,**

$$v(t) = \frac{dx}{dt},$$

and **acceleration is the time derivative of velocity**,

$$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}.$$

**Conversely, integrating reverses these relationships:**

$$v(t) = \int a(t) dt, \quad \& \quad x(t) = \int v(t) dt.$$

Graphically, these operations correspond to **slopes** and **areas**.

- On a position–time graph, the **slope** of the tangent gives the instantaneous velocity.

- On a velocity–time graph, the **slope** gives acceleration, while the **area under the curve** represents the change in position (displacement).
- On an acceleration–time graph, the **area under the curve** represents the change in velocity.

This structure means that motion can be understood geometrically: the steeper the  $x(t)$  curve, the faster the object moves; the steeper the  $v(t)$  curve, the stronger the acceleration; and the area under each curve connects the quantity below it in the hierarchy. These relationships hold for all one-dimensional motions, whether the quantities vary smoothly or in piecewise-constant segments.

```
interactive(children=(Dropdown(description='Function:', options=('x(t) = t^2', 'x(t) = t^3', 'x(t) = 0.5 + 0.5...')),
```

## 1.1.9 Freely falling objects

Near Earth’s surface and neglecting air resistance, all objects accelerate downward with the same magnitude  $g \approx 9.81 \text{ m/s}^2$ . You may choose “up” as positive so that  $a = -g$ , or “down” as positive so that  $a = +g$ —either convention is valid if applied consistently. This **universality of free fall** is remarkable: in a vacuum, light and heavy bodies share the same acceleration. Historically, this principle became visible through early vacuum experiments, such as Otto von Guericke’s demonstrations showing the dramatic effects of removing air.

In our one-dimensional framework, free fall is simply a case of **uniformly accelerated motion** with  $a = \pm g$ , where we use  $y$  instead of  $x$  to represent vertical position. The familiar equations apply:

$$v(t) = v_0 \pm gt, \quad y(t) = y_0 + v_0 t \pm \frac{1}{2}gt^2.$$

Assuming  $v_0 = 0 \text{ m/s}$ , the equations further simplify to:

$$v(t) = \pm gt, \quad y(t) = \pm \frac{1}{2}gt^2.$$

Two practical notes refine this idealization without breaking it. First,  $g$  varies slightly with latitude and altitude, but the standard value is sufficient for most calculations. Second, **air resistance** increases with speed and cross-section; over long drops or for light objects it eventually balances gravity, leading to a **terminal velocity** where acceleration effectively vanishes.