2.7. TITLE

TEASER?
`ew10 - Teslatransformator`
RECAP LENZ? `em17 - Thomson`

- ANSWER FRAGMENT

Ac circuits & electromagnetic oscillations

- in this chapter we combine resistance, capacity and inductance in ac circuits
- focus on electromagnetic oscillations and the exchange of energy between electric and magnetic fields

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The lr-circuit

Switching on the dc supply

- consider a simple dc circuit with an ideal voltage source V_0, a resistor R and an inductor L in series
- applying kirchhoff's loop rule gives

$$V_0 = L\frac{dI}{dt} + IR$$

- solving the differential equation yields

$$I(t) = \frac{V_0}{R}\left(1 - e^{-t/\tau}\right)$$

with the time constant

$$\tau = \frac{L}{R}$$

- after about 5τ the current nearly reaches its steady state value

Switching off the dc supply

- when the dc supply is switched off ($V_0 = 0$) the inductor resists the change in current
- kirchhoff's loop rule becomes

$$L\frac{dI}{dt} + IR = 0$$

- the solution for the decaying current is

$$I(t) = I_0 e^{-t/\tau}$$

with

$$\tau = \frac{L}{R}$$

Electromagnetic oscillations

Lc-circuit

- the lc-circuit consists only of an inductor and a capacitor in series
- initially the capacitor is charged to Q_0, with voltage

$$V = \frac{Q}{C}$$

- applying kirchhoff's rule leads to

$$-L\frac{dI}{dt} + \frac{Q}{C} = 0$$

  and using $I = -\frac{dQ}{dt}$, we obtain the differential equation

$$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

- its general solution is

$$Q(t) = Q_0 \cos\Big(\omega t + \phi\Big)$$

with the angular frequency

$$\omega = \sqrt{\frac{1}{LC}}$$

- the energy oscillates between the capacitor (electric field) and the inductor (magnetic field)

Lrc-circuit

- adding a resistor introduces damping into the oscillations
- the governing equation becomes

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

- the system can be underdamped, overdamped or critically damped depending on the relation between R^2 and $4LC$

Ac circuits

Resistance vs. reactance

- resistance R is independent of frequency and dissipates energy as heat
- reactance (inductive X_L and capacitive X_C) depends on frequency and temporarily stores energy
- the phase difference between voltage and current arises due to reactance

Resistor in an ac circuit

- follows ohm's law:

$$V = IR$$

- voltage and current are in phase
- the average power is given by

$$\bar{P} = I_{rms}^2 R$$

Inductor in an ac circuit

- the voltage across an inductor is

$$V = L\frac{dI}{dt}$$

- for a sinusoidal current $I(t) = I_0 \cos(\omega t)$ the voltage is

$$V(t) = \omega L I_0 \cos\left(\omega t + \frac{\pi}{2}\right)$$

- the inductive reactance is

$$X_L = \omega L$$

- the voltage leads the current by 90°

Capacitor in an ac circuit

- the voltage across a capacitor is determined by

$$V = \frac{Q}{C}$$

- for a sinusoidal current the resulting voltage is

$$V(t) = \frac{I_0}{\omega C} \cos\left(\omega t - \frac{\pi}{2}\right)$$

- the capacitive reactance is

$$X_C = \frac{1}{\omega C}$$

- the voltage lags the current by 90°

Summary of ac circuit components

- resistor: no phase shift; dissipates energy as heat
- inductor: voltage leads current; reactance $X_L = \omega L$
- capacitor: voltage lags current; reactance $X_C = \frac{1}{\omega C}$

Filters

- high-pass filters allow high-frequency signals to pass while attenuating low-frequency ones
- low-pass filters allow low-frequency signals to pass while attenuating high-frequency ones
- these filters are common in signal processing and audio electronics

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Impedance

- impedance combines resistance and reactance into a complex quantity:

$$Z = R + j\left(X_L - X_C\right)$$

- its magnitude is

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

- the phase angle is given by

$$\tan\phi = \frac{X_L - X_C}{R}$$

- impedance determines the relationship between voltage and current in ac circuits

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Ac lrc-circuit & phasor diagrams

- in a series lrc-circuit the sum of the voltage drops equals the source voltage:

$$V = V_R + V_L + V_C$$

- in the phasor diagram:
    - V_R is drawn along the positive x-axis (in phase with the current)
    - V_L is drawn 90° ahead of V_R
    - V_C is drawn 90° behind V_R

- the resultant voltage is found by vector addition:

$$V_0 = I_0 Z$$

with

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

- similarly, for rms values:

$$V_{rms} = I_{rms} Z$$

Resonance in ac circuits

- resonance occurs when the inductive and capacitive reactances cancel ($X_L = X_C$)
- the resonant angular frequency is

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

and the resonant frequency is

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

- at resonance:
    - the impedance is purely resistive ($Z = R$)
    - voltage and current are in phase ($\phi = 0$)
    - the current is maximized, with

$$I_{max} = \frac{V_{rms}}{R}$$

- the sharpness of the resonance depends on the value of R

Impedance matching

- maximum power transfer occurs when the source impedance matches the load impedance:

$$Z_1 = Z_2$$

- for purely resistive circuits, maximum power is delivered when $R_1 = R_2$

- mismatched impedances lead to reduced power transfer efficiency and potential signal distortion

Wah-Wah inc. schematics + hendrix video