

# Lecture Tutorial 1C:

## Atwood, Weightlessness, & Gravitation



**How strong is gravity acting on a person on  
the ISS?**

# What is an Atwood machine?

md04

An (ideal) Atwood machine consists of:

- Two masses  $m_1$  and  $m_2$
- Connected by a massless, inextensible cable
- Over a frictionless pulley

Key assumptions:

1. Cable mass = 0 → tension is the same on both sides
2. Pulley friction  $\approx 0$  → tension does not change direction
3. Cable length fixed → both masses accelerate with the same magnitude  $|a|$ , in opposite directions

This is mathematically identical to an elevator with a counterweight.

# Physics of the Atwood machine

Let  $m_1$  be heavier (moves down) and  $m_2$  lighter (moves up).

For  $m_1$  (downward positive):  $m_1g - T = m_1a$

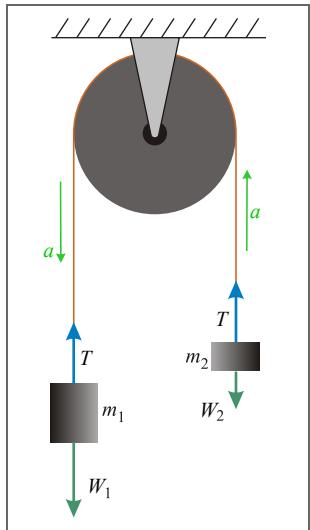
For  $m_2$  (upward positive):  $T - m_2g = m_2a$

Add both equations to eliminate  $T$ :

$$(m_1 - m_2)g = (m_1 + m_2)a$$

Thus the acceleration is

$$a = \frac{m_1 - m_2}{m_1 + m_2} g.$$



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## Atwood machine: Special cases and application

- Balanced masses ( $m_1 = m_2$ ):  $a = 0 \rightarrow$  system is in equilibrium
- One mass zero ( $m_2 = 0$ ):  $a = g \rightarrow$  heavier mass free-falls
- For an elevator  $m_E$  with counterweight  $m_C$  ( $m_E \approx m_C$ ):
  - Acceleration is very small
  - System is nearly balanced
  - Motor only overcomes friction and minor imbalances  $\rightarrow$  minimal energy use

# Niederfinow Boat Lift: History & Key Facts

- **Niederfinow Boat Lift** (opened 1934) on the Oder–Havel Canal
- Overcomes a **36 m height difference** between canal levels
- Trough: length ~ **85 m**, width ~ **12 m**, water depth ~ **2.5 m**
- Total moving mass (trough + water) ~ **4,290 t**
- Suspended by **256 cables** and balanced by **192 counterweights**



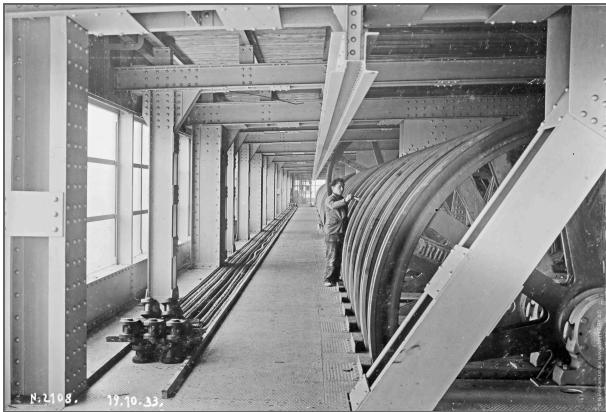


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# Niederfinow Boat Lift: A Giant Atwood Machine

- The entire lift works as a **scaled-up Atwood machine**
- Trough (with water + ship) on one side
- Counterweights of equal mass on the other side
- Cables run over guide wheels: forces mirror an Atwood system
- Motors only apply small forces → friction, acceleration, synchronization
- Core idea: **balance the large mass so movement requires little effort**



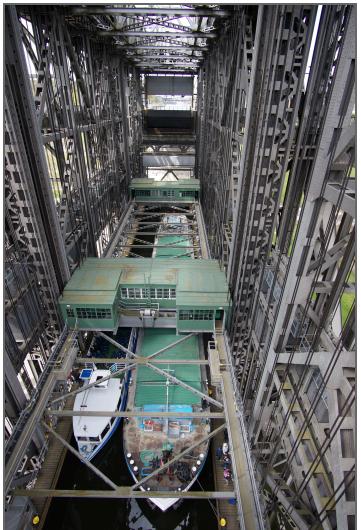


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# Niederfinow Boat Lift: Ship Displacement

- When a ship enters the water-filled caisson, it **displaces its own weight** in water
- Archimedes principle → **ship weight = weight of water pushed out before the gate closes**
- Therefore:
  - Before ship: trough + water
  - After ship: trough + (less water) + ship
  - **Total mass is unchanged**
- Result: counterweights can be tuned once and work for all ships





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# Niederfinow Boat Lift: Why the Lift Is Energy Efficient

- Counterweights match the mass of the caisson  
→ nearly perfect balance
- Motors only overcome:
  - **rolling and guide friction**
  - **small water-level differences**
  - **starting and stopping inertia**
- No need to pump large volumes of water  
(unlike lock systems)  
  
Energy use is comparable to a **well-balanced elevator**
- Near Magdeburg:
  - boat lift uses Archimedes' principle
  - the submerged float bodies provide buoyancy that balances the weight of the water-filled trough
- Examples of **engineering using fundamental physics** for efficiency



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# Atwood Machine & Cable-Break: A Thought Experiment

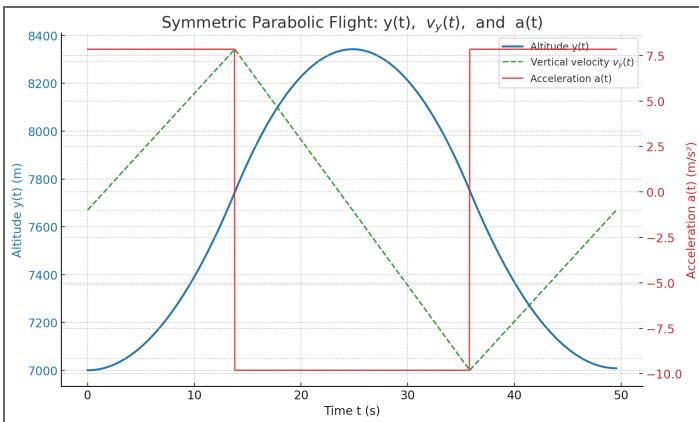
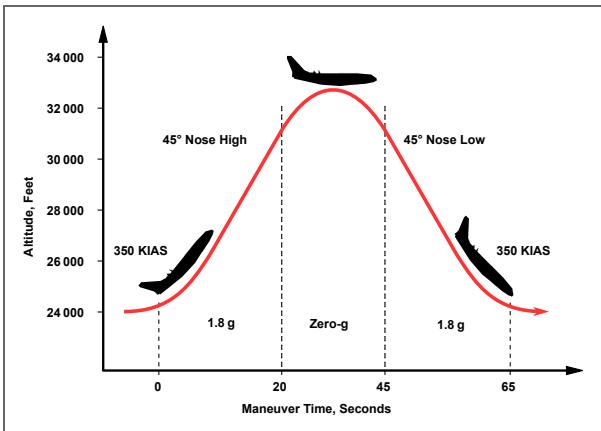
- Atwood machine: two masses connected by a rope over a pulley; motion determined by mass difference.
- If the rope breaks, a mass experiences **free fall**.
- In free fall the acceleration is

$$a = -g$$

- A falling elevator and a person inside accelerate downward **together**, both at  $-g$ .
- With no supporting force from the floor, the normal force becomes zero → the person feels **weightless**.
- Key idea: **free fall = no normal force = apparent weightlessness**.

## Parabolic Flights (Zero-G Flights)

- Pilots fly a special “parabola” so the plane follows the same free-fall path as an object thrown upward.
- During the top part of this parabola, both plane and passengers accelerate downward at  $a = -g$ .
- Since they fall together, passengers float — **apparent weightlessness**.
- Duration of weightlessness: typically 20–25 seconds per parabola.
- Before and after the zero-g phase, the aircraft performs **hyper-g pull-up and pull-out** maneuvers, giving **apparent acceleration**  $a > g$ .
- Fun fact: NASA’s KC-135 was nicknamed the **“Vomit Comet”**



[left] from [wikipedia](#), **Attribution-Share Alike 4.0 International**; [right]  $y(t)$ ,  $v(t)$ , and  $a(t)$  diagram

# Apparent Weightlessness: Dropping Your Phone

## phyphox - Beschleunigung mit g

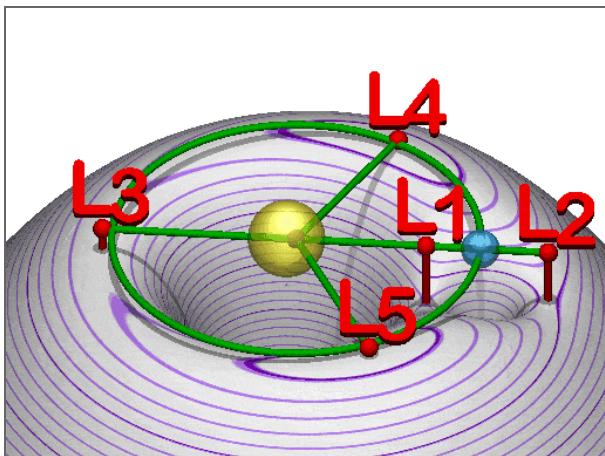
- A phone's accelerometer does **not** measure gravity — it measures the **proper acceleration** caused by the **normal force** acting on the sensor.
- When resting in your hand or on a table, the phone feels an upward normal force → accelerometer reads  $\approx 1 \text{ g}$ .
- As soon as you **drop** the phone (or during a jump when it leaves your hand), it enters **free fall**:
  - gravity acts, but **no surface pushes on the sensor**
  - normal force  $N = 0$
  - accelerometer reading →  $\approx 0 \text{ g}$

- The phone is still pulled downward by gravity, but it *feels* weightless because the **supporting normal force disappears.**

# Apparent vs. Gravitational Weightlessness

- **Gravitational weightlessness (true microgravity):**
  - Occurs where gravitational forces from two (or more) bodies nearly **cancel**, e.g. at the **L1 Lagrange point**.
  - Net gravity is extremely small → objects experience **very low acceleration** even without being in free fall.
  - This is a *hypothetical pure case* of microgravity created by gravitational balance.
- **Apparent weightlessness:**
  - Gravity is fully present, but there is **no supporting force** (normal force) acting on the body.
  - Happens in parabolic flights, falling elevators, roller-coaster airtime, or when jumping.

- With  $N = 0$ , you *feel* weightless even though gravity acts normally.



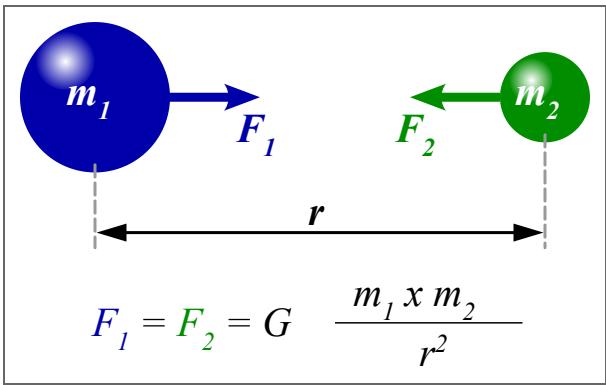
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# Newton's Law of Gravitation

- **Every mass attracts every other mass with a force along the line connecting them.**
- Two point masses  $m_1$  and  $m_2$  at positions  $\vec{r}_1$  and  $\vec{r}_2$ :
  - Displacement from  $m_1$  to  $m_2$ :
$$\vec{r} = \vec{r}_2 - \vec{r}_1, \quad r = |\vec{r}|$$
  - Using the unit vector  $\hat{r}_{12} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$ :
- Newton's law of gravitation in vector form

$$\vec{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{r}_{12} = +G \frac{m_1 m_2}{r^2} \hat{r}_{21}$$

- Note: The minus sign ensures the gravitational force is **attractive**, i.e. it points in the **opposite direction** of the unit vector  $\hat{r}_{12}$  (which points from  $m_1$  to  $m_2$ ).



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## Example: Two Masses on a Line

- Masses  $m_1$  and  $m_2$  on the  $x$ -axis at positions  $x_1$  and  $x_2$ :

- $\vec{r}_1 = (x_1, 0, 0)$
- $\vec{r}_2 = (x_2, 0, 0)$

- Displacement:

$$\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1, 0, 0), \quad r = |x_2 - x_1|$$

- Force on  $m_2$  by  $m_1$ :

$$\vec{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{x} \quad \text{if } x_2 > x_1$$

- Interpretation:

- $\vec{F}_{21} = -\vec{F}_{12}$  due to Newton's third law
- Force points along the line joining the masses.

- The same formula works in any dimension, just with the appropriate vector  $\vec{r}$ .

## Gravity Near Earth's Surface: Deriving $g$

- Earth mass  $M_E$ , radius  $R_E$ .
- Force on mass  $m$  at the surface with the distance  $r = R_E$  from Earth's center:

$$F = G \frac{M_E m}{R_E^2}$$

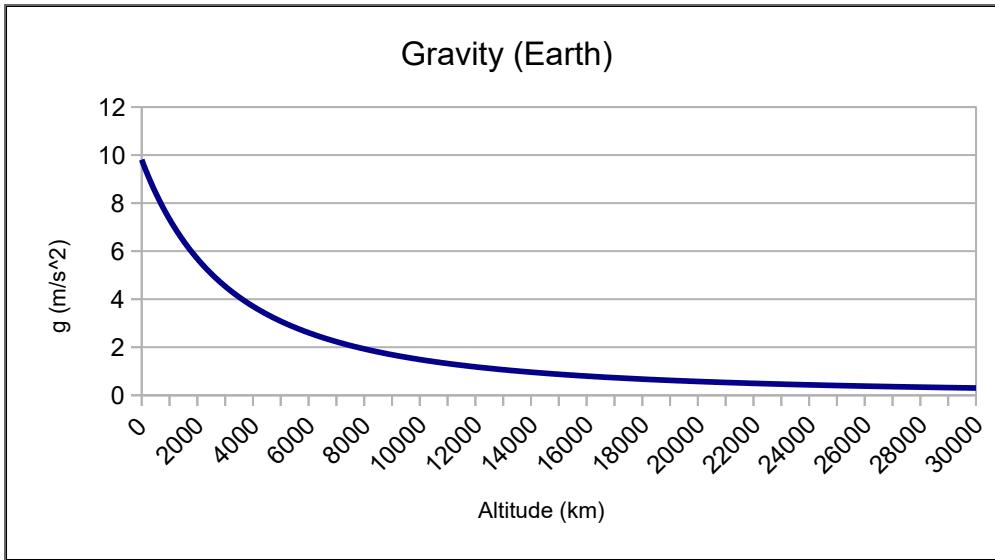
- By definition, weight is  $W = mg$ .
- Equating:

$$mg = G \frac{M_E m}{R_E^2} \quad \Rightarrow \quad g = G \frac{M_E}{R_E^2}$$

- $g$  is **not a universal constant**; it depends on  $M_E$  and  $R_E$ .

# Gravity vs. Altitude

⇒ **inverse square law**  $g \propto \frac{1}{r^2}$



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# Satellites: Circular Orbit as Free Fall

- Satellite of mass  $m$  in circular orbit at radius  $r$  (from Earth's center).
- Gravity provides centripetal force:

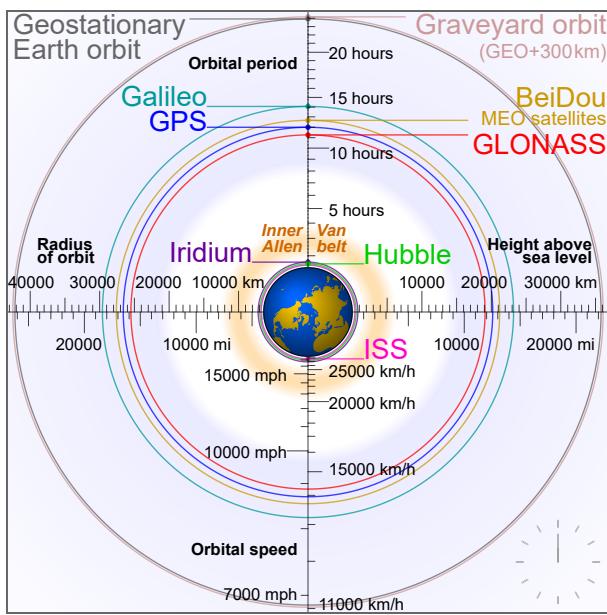
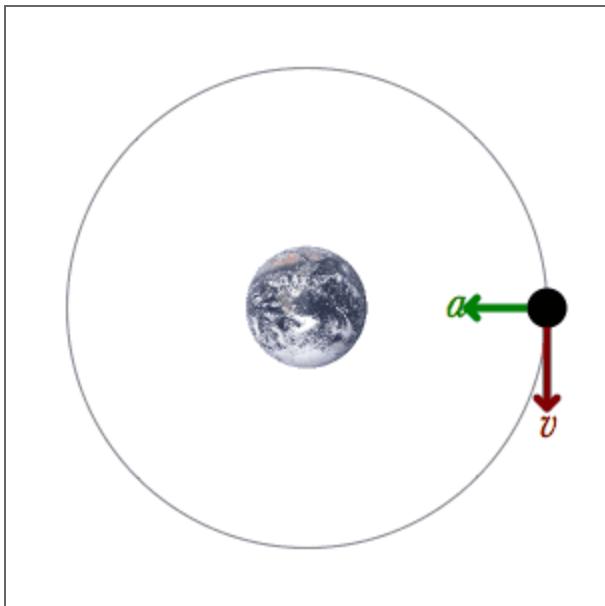
$$G \frac{M_E m}{r^2} = m \frac{v^2}{r}$$

$$v = \sqrt{\frac{GM_E}{r}}$$

- Interpretation:
  - Velocity **independent** of satellites mass
  - Satellite is in continuous **free fall** toward Earth.
  - **Tangential speed**  $v$  is just right so it keeps "missing" Earth.
  - **Orbit = free fall** whose path curves around the planet.

# Orbital Period and Free-Fall Picture

- Orbital period:  $T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM_E}}$

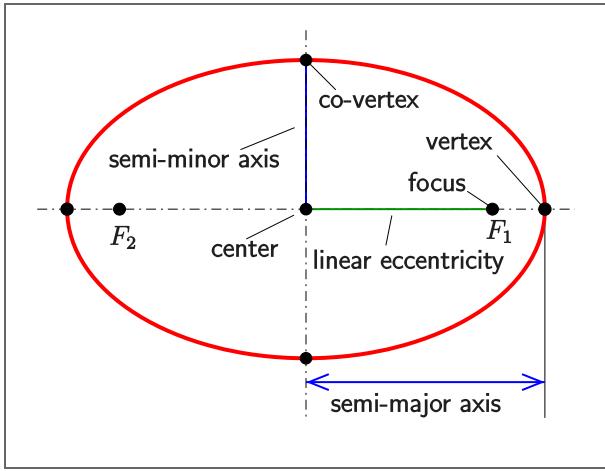


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## Kepler's Laws: Overview

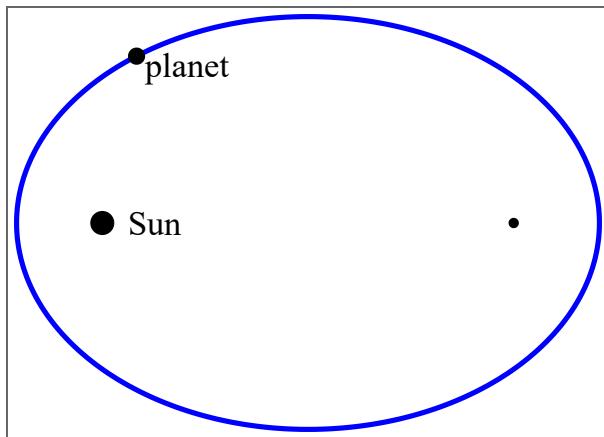
- Before Newton, Kepler summarized planetary motion in three empirical laws:
  1. Planets move in **ellipses** with the Sun at one focus.
  2. Planets sweep out **equal areas in equal times**.
  3. For each planet,  $T^2 \propto a^3$ , where  $T$  is **orbital period** and  $a$  **the semi-major axis**.
- Newton later showed:
  - Kepler's laws follow from universal gravitation plus Newton's laws of motion.
  - Celestial mechanics obey the same physics as falling apples.

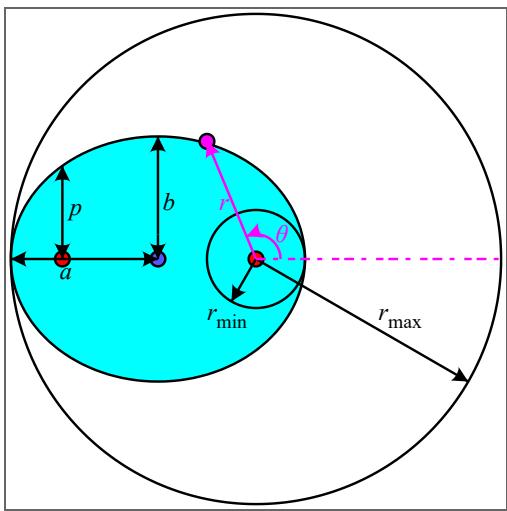


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# Kepler's First Law: Elliptical Orbits

- Planets move in **elliptical orbits** with the Sun at one focus.
- Ellipse parameters:
  - Semi-major axis  $a$  (sets the “size” of the orbit)
  - Eccentricity  $e$  (shape):  $0 \leq e < 1$
- Special cases:
  - $e = 0 \rightarrow$  circle
  - Small  $e \rightarrow$  nearly circular (like Earth)



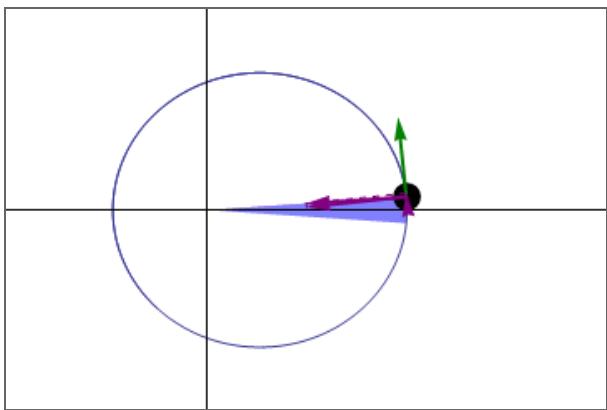


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## Kepler's Second Law: Equal Areas in Equal Times

- Statement: A line from the Sun to a planet sweeps out **equal areas in equal times**.
- Observations:
  - Planet moves **faster** near perihelion (closest approach).
  - **Slower** near aphelion (furthest point).
- Newton's explanation (intuition):
  - Gravity is a **central force**, so torque is zero → angular momentum  $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$  is conserved.
  - Smaller distance  $r$ , so to keep  $L$  constant, speed  $v$  must increase.
  - Result: **same area in the same time** → variable orbital speed.



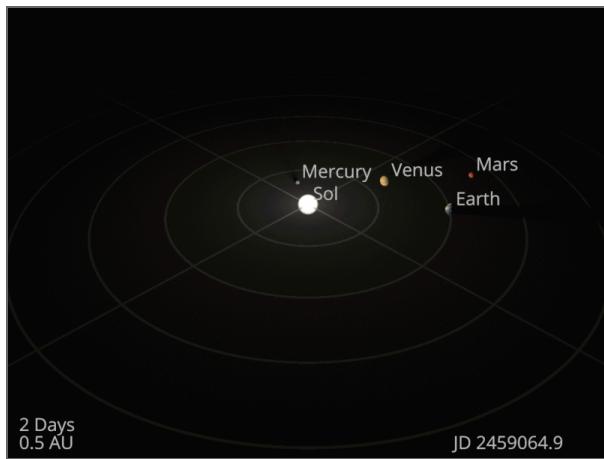
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## Kepler's Third Law: $T^2 \propto a^3$

- Statement: For all planets orbiting the **same central mass**,

$$\frac{T^2}{a^3} = \text{constant}$$

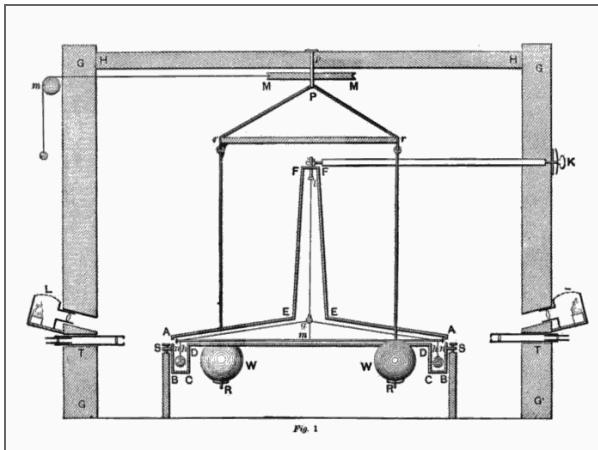
- Intuition: Orbits further out have a much larger radius and move more slowly → strongly increased period.



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# Cavendish Experiment: Measuring $G$

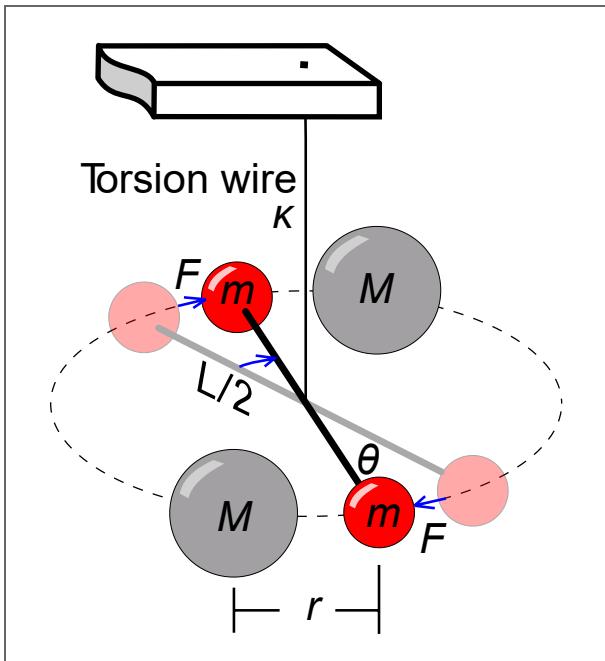
- Goal:
  - Measure the tiny gravitational attraction between known laboratory masses.
  - From this, determine  $G$  and then the mass (and density) of Earth.
- Key idea:
  - Use a very sensitive torsion balance: gravitational attraction twists a thin wire.
  - Measure the tiny angular deflection to infer the force.



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# Cavendish: Experimental Setup

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## Cavendish: Physics and Determining $G$

- Gravitational force between one large and one small sphere:

$$F = G \frac{mM}{r^2}$$

- This force creates a torque on the rod and attached mirror causes deflection of laser beam.
- $G$  can be determined using the oscillatory motion and/or stationary end point (equilibrium).
- Literature values  $G = 6.67430 \times 10^{-11} \frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$

## Cavendish: Interpretation and Error Sources

- Once  $G$  is known, Earth's mass follows from:

$$g = \frac{GM_E}{R_E^2} \quad \Rightarrow \quad M_E = \frac{gR_E^2}{G}$$

- Cavendish effectively "weighed the Earth."
- Main challenges and error sources:
  - Extremely small forces (on the order of  $10^{-10}$ – $10^{-11}$  N).
  - Air currents, temperature gradients, and mechanical vibrations.
  - Imperfect knowledge of mass distribution and distances.