2.4. DC Circuits & Kirchhoff's Laws

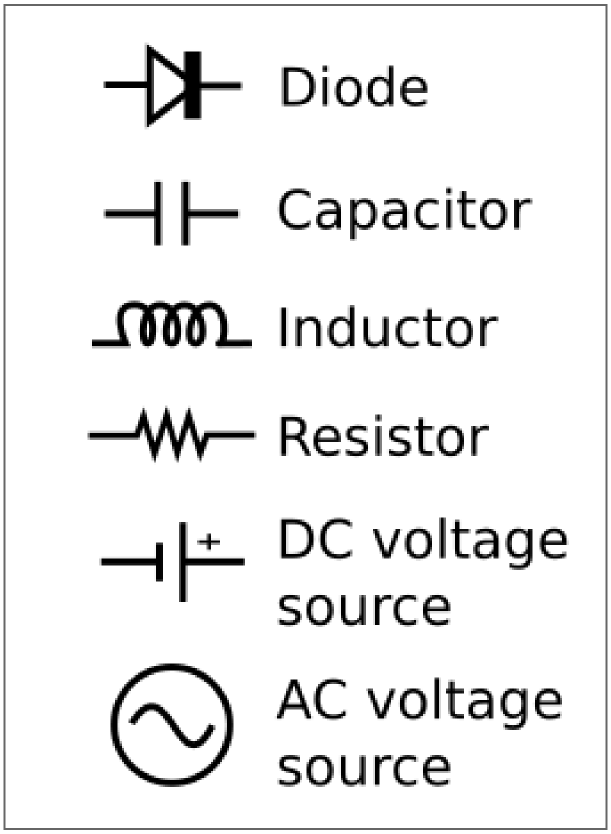
Imagine you're designing a camera flash. What components would you need to quickly store and release a large amount of electrical energy, and how would you control the timing?

- need to design a circuit with at least
  - voltage source
  - capacitor
  - resistor
  - switch

Circuits 101

- **Electric circuit:** Elements (batteries, resistors, capacitors) connected by wires
- **Circuit states:**
  - Open: No current flow
  - Closed: Current flow
  - Short: Excessive current (hazard)
- **Voltage source (V):** Provides potential difference, drives current (I)
- **Ground:** Reference potential (e.g., Earth)
- **Voltage drop:** Potential difference across an element
- **Element connections:** Series or parallel
- **Equivalent circuits:** Simplification by combining elements into e.g. an effective resistance

Circuits 101 (cont'd)



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Open-circuit voltage of a DC source

- **Source:** Energy conversion to electrical potential
- **Open-circuit voltage** V_0**:** Voltage without current flow
- **Internal resistance** R_{int}**:** Real sources have internal resistance
- **Voltage with current:**

$$V = V_0 - IR_{int}$$

- **Non-constant** R_{int}**:** Leads to non-linear current-voltage characteristics

Measuring Voltages and Currents

`el36 & el28`

- **Ammeter:** Measures current (series connection)
- **Voltmeter:** Measures voltage (parallel connection)
- **Ohmmeter:** Measures resistance
- **Multimeter:** Combines ammeter, voltmeter, ohmmeter
- **Internal resistance:**
    - Voltmeter: High (limits current)
    - Ammeter: Low (limits voltage drop)
- **Simultaneous measurement:** Configuration choice depends on resistor and device internal resistances

Resistors in series & parallel

`el41`

- **Series resistors:**
  - equivalent resistance: $R_{eq} = \sum R_i$
  - voltage drops add up
  - **interpretation for equivalent resistor:** increase effective length
- **Parallel Resistors:**
  - equivalent resistance: $\frac{1}{R_{eq}} = \sum \frac{1}{R_i}$
  - current splits
  - net resistance is smaller than the smallest individual resistance
  - **interpretation for equivalent resistor:** increase effective cross-sectional area

Capacitors in series & parallel

`el23`

- **Parallel Capacitors:**
  - equivalent capacitance: $C_{eq} = \sum C_i$
  - voltage is the same
  - total charge adds up
  - **interpretation for equivalent capacitor:** increase effective plate area
- **Series Capacitors:**
  - equivalent capacitance: $\frac{1}{C_{eq}} = \sum \frac{1}{C_i}$
  - charge is the same
  - voltage drops add up
  - net capacitance is smaller than the smallest individual capacitance
  - **interpretation for equivalent capacitor:** increase effective plate distance

Kirchhoff's Rules

- **Junction rule**

$$\sum_{i} I_i = 0 \quad \leftrightarrow \quad \sum_{j} I_{in,j} = \sum_{k} I_{out,k}$$

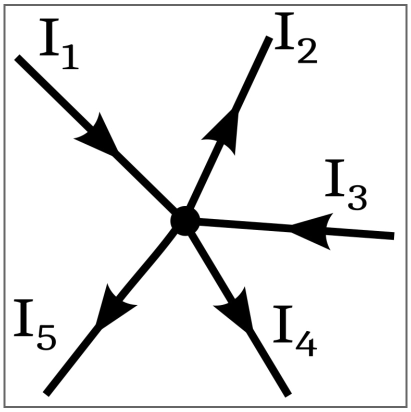
  - → **current conservation**

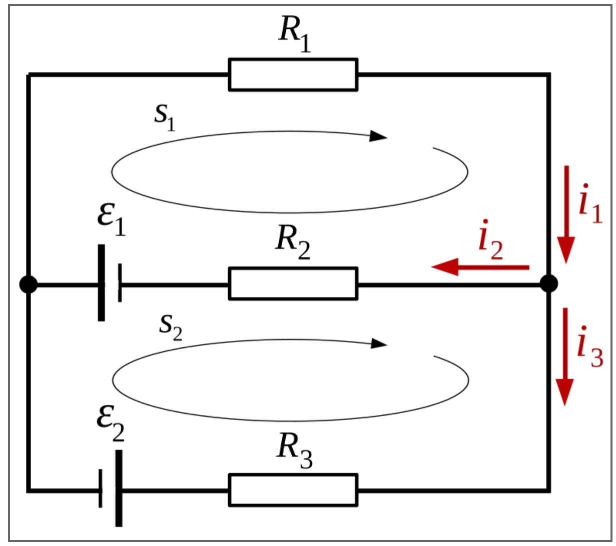
- **Loop rule**

$$\sum_{i} V_i = 0$$

  - → **energy conservation**

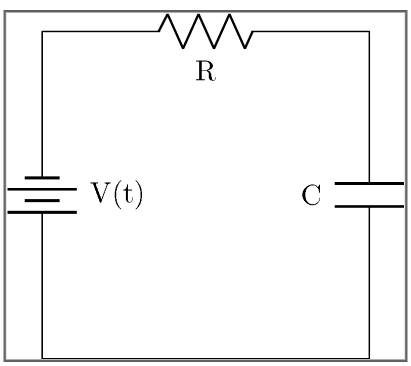
- **Circuit analysis steps:**
  1. Label currents
  2. Identify unknowns
  3. Apply junction rule
  4. Apply loop rule
  5. Solve equations
  6. Vector/matrix form of equation


I1
I2
I3
I4
I5


R1
s1
ε1
R2
i1
i2
s2
i3
ε2
R3

RC-circuit

- fundamental circuit for:
    - creating precise time delays → camera flash (charging of a capacitor determines the flash duration)
    - audio filtering → smooths out unwanted fluctuations
    - simple oscillators → periodic waveforms



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RC-circuit: Charging

- **Circuit:** Resistor, capacitor, voltage source in series
- **Kirchhoff's loop rule:**

$$V_0 = IR + \frac{Q}{C}$$

- **Differential equation:** $(I = \frac{dQ}{dt})$

$$V_0 = R\frac{dQ}{dt} + \frac{Q}{C}$$

$$\frac{dt}{RC} = \frac{dQ}{V_0C - Q}$$

$$\frac{1}{RC}\int_0^t dt = \int_0^Q \frac{dQ}{V_0C - Q}$$

RC-circuit: Charging (cont'd)

- **Differential equation:** $(I = \frac{dQ}{dt})$

$$\frac{1}{RC}\int_0^t dt = \int_0^Q \frac{dQ}{V_0C - Q}$$

- **Solution:**

$$Q(t) = Q_0(1 - e^{-t/RC})$$

$$V_C(t) = V_0(1 - e^{-t/RC})$$

$$I(t) = \frac{V_0}{R}e^{-t/RC}$$

- → **Decaying current** and **increasing charge & voltage** over time
- **Time constant ($\tau = RC$):**
    - Time for V_C to reach 63% of V_0
    - Time for I to decay to 37% of initial value

- Charging is considered complete after $5\ \tau$

RC-circuit: Discharging

- **Circuit:** Capacitor and resistor in series; **no voltage source**
- **Kirchhoff's Loop Rule:**

$$IR = \frac{Q}{C}$$

- **Differential Equation:**

$$-\frac{dQ}{dt}R = \frac{Q}{C}$$

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

$$\int_{Q_0}^{Q} \frac{1}{Q} dQ = -\frac{1}{RC} \int_{0}^{t} dt$$

RC-circuit: Discharging (cont'd)

- **Differential Equation:**

$$\int_{Q_0}^{Q} \frac{1}{Q} dQ = -\frac{1}{RC} \int_{0}^{t} dt$$

- **Solution:**

$$Q(t) = Q_0 e^{-t/RC}$$

$$V_C(t) = V_0 e^{-t/RC}$$

$$I(t) = I_0 e^{-t/RC}$$

- **→ Decaying voltage and current over time**

RC-circuit: Periodically switching

`Script simulation RC-circuit`
`el11`

- **Sawtooth Voltage:** Created by periodic charging and discharging
- **Practical Implementation:** Voltage source, capacitor, two resistors, and a tube
- **Charging Loop:** Capacitor charges through one resistor
- **Discharging Loop:** Tube conducts at breakdown voltage, discharging capacitor through the other resistor
- **Repeat Cycle:** Sawtooth waveform generated
- **Application:** Heart pacemaker, (music) , etc.