

# Cheat sheet



This is a rather incomplete and loose collection of equations and constants

**Use with caution** and check / think critical if equation is applicable

## Electric charge & field

$$k \approx 9.0 \times 10^9 \text{ N m}^2 / \text{C}^2$$

$$\epsilon_0 = \frac{1}{4\pi k} = 8.854 \times 10^{-12} \frac{\text{C}^2}{\text{N m}^2}$$

$$F = k \frac{Q_1 Q_2}{r^2} \quad \text{in [N]}$$

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r_{21}^2} \hat{r}_{21}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$E = \frac{F}{q} = \frac{ma}{q} \quad \text{in [N/C]}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dQ}{r^2}$$

$$\vec{E} = \int d\vec{E}$$

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \quad \text{e.g. for parallel, oppositely charged plates}$$

$$\text{Note on [E]} = \frac{\text{N}}{\text{C}} = \frac{\text{N m}}{\text{C m}} = \frac{\text{J}}{\text{C m}} = \frac{\text{V}}{\text{m}} \quad \text{with J = Nm \& V = J/C}$$

## Electric flux & electric potential

$$\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \theta = E_{\perp} A = EA_{\perp}$$

$$\Phi_E = \oint \vec{\mathbf{E}} d\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0}$$

$$W = Fd = qEd = -\Delta U \quad \text{for a homogenous E field}$$

$$V_{BA} = V_B - V_A = \frac{U_B - U_A}{q} = \frac{\Delta U_{BA}}{q} = -\frac{W_{BA}}{q} \quad \text{in [J/C] = [V]}$$

$$\Delta U_{BA} = - \int_A^B \vec{\mathbf{F}} d\vec{\mathbf{l}}$$

$$V_{BA} = - \int_A^B \vec{\mathbf{E}} d\vec{\mathbf{l}}$$

$$V_{BA} = -Ed \quad \text{for uniform E-field}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} \quad \text{for a point charge with } V = 0 \text{ at } r = \infty$$

## Capacitance, batteries, & resistance

$$Q = CV$$

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

$$\epsilon = K\epsilon_0$$

$$C = \frac{Q}{V} = K\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d} \quad \text{for a parallel-plate capacitor}$$

$$I = \frac{dQ}{dt}$$

$$V = IR$$

$$R = \rho \frac{l}{A}$$

$$P = VI = I^2 R = \frac{V^2}{R}$$

$$I = \int \vec{\mathbf{j}} d\vec{\mathbf{A}}$$

## DC circuits & Kirchhoff' laws

$$R_{eq} = \sum R_i \quad \text{for resistances in series}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_i} \quad \text{for resistances in parallel}$$

$$\frac{1}{C_{eq}} = \sum \frac{1}{C_i} \quad \text{for capacities in series}$$

$$C_{eq} = \sum C_i \quad \text{for capacities in parallel}$$

$$\sum_i I_i = 0 \quad \leftrightarrow \quad \sum_j I_{in,j} = \sum_k I_{out,k} \quad \text{junction rule}$$

$$\sum_i V_i = 0 \quad \text{loop rule}$$

$$V_C(t) = V_0(1 - e^{-\frac{t}{RC}}) \quad \text{RC circuit; charging}$$

$$I(t) = \frac{V_0}{R} e^{-\frac{t}{RC}} \quad \text{RC circuit; charging}$$

$$V_C(t) = V_0 e^{-\frac{t}{RC}} \quad \text{RC circuit; discharging}$$

$$I(t) = I_0 e^{-\frac{t}{RC}} \quad \text{RC circuit; discharging}$$

## Magnetism & magnetic field

$$F = IlB \sin \theta \quad \text{within an uniform magnetic field}$$

$$d\vec{\mathbf{F}} = Id\vec{\mathbf{l}} \times \vec{\mathbf{B}} \quad \text{in general}$$

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} \quad \text{single moving particle}$$

$$\frac{e}{m_e} = \frac{v}{Br} = \frac{E}{B^2 r} \quad \text{within uniform fields}$$

$$qvB = m \frac{v^2}{r} \quad \text{within uniform magnetic field}$$

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \quad \text{Lorentz Equation}$$

$$U_H = E_H d = v_d B d \quad \text{Hall voltage}$$

$$B = \frac{\mu_0}{2\pi} \frac{I}{r} \quad \text{magnetic field of a straight wire}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{enc} \quad \text{Ampère's law}$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2} \quad \text{Biot-Savart law}$$

# Electromagnets, induction & inductance

$$B = \frac{\mu_0 N I}{l} \quad \text{for solenoids}$$

$$B = \frac{\mu_0 N I}{2\pi r} \quad \text{for toroids}$$

$$\Phi_B = B_{\perp} A = B A \cos \theta = \vec{\mathbf{B}} \vec{\mathbf{A}} \quad \text{for uniform magnetic field}$$

$$\Phi_B = \int \vec{\mathbf{B}} d\vec{\mathbf{A}}$$

$$V_{ind} = -N \frac{d\Phi_B}{dt} = -L \frac{dI}{dt}$$

$$V_1 = -M \frac{dI_2}{dt}$$

$$V_{ind} = Blv \quad \text{for a conductor moving perp. in a uni. mag. field}$$

$$\oint \vec{\mathbf{E}} d\vec{\mathbf{l}} = -\frac{d\Phi_B}{dt} \quad \text{general form of Faraday's law}$$

$$U = \frac{1}{2} L I^2$$

## Alternating current (AC)

$$V(t) = V_0 \sin(2\pi ft) = V_0 \sin(\omega t)$$

$$I(t) = \frac{V(t)}{R} = \frac{V_0}{R} \sin(2\pi ft) = I_0 \sin(\omega t)$$

$$I_{rms} = \sqrt{\bar{I}^2} = \sqrt{\frac{1}{2} I_0^2} = \frac{I_0}{\sqrt{2}}$$

$$V_{rms} = \sqrt{\bar{V}^2} = \sqrt{\frac{1}{2} V_0^2} = \frac{V_0}{\sqrt{2}}$$

$$\bar{P} = \frac{1}{2} I_0^2 R = \frac{1}{2} \frac{V_0^2}{R}$$

$$\bar{P} = I_{RMS} V_{RMS} = I_{RMS}^2 R = \frac{V_{RMS}^2}{R}$$

## AC circuits & Electromagnetic oscillations

# LR-circuit

$$\tau = \frac{L}{R}$$

$$I(t) = \frac{V_0}{R} (1 - e^{-\frac{t}{\tau}}) \quad \text{switching DC on}$$

$$I(t) = I_0 e^{-\frac{t}{\tau}} \quad \text{switching DC off}$$

# LC-circuit

$$I(t) = -\frac{dQ}{dt} = \omega Q_0 \sin(\omega t + \phi) = I_0 \sin(\omega t + \phi)$$

$$\omega = \sqrt{\frac{1}{LC}}$$

$$U = U_B + U_E = \frac{Q_0^2}{2C} \sin^2(\omega t + \phi) + \frac{Q_0^2}{2C} \cos^2(\omega t + \phi) = \frac{Q_0^2}{2C}$$

# R, L & C in AC circuits

Component	Voltage-Current Relationship	Phase Difference	Reactance
Resistor ( $R$ )	$V = IR$	In phase ( $0^\circ$ )	$R$
Inductor ( $L$ )	$V = L \frac{dI}{dt}$	Voltage leads current by $90^\circ$	$X_L = \omega L$
Capacitor ( $C$ )	$I = C \frac{dV}{dt}$	Current leads voltage by $90^\circ$	$X_C = \frac{1}{\omega C}$

$$Z = R + jX,$$

$$\tan \phi = \frac{X_L - X_C}{R}$$

# LRC in AC mode

$$V_0 = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2} = I_0 \sqrt{R^2 + (X_L - X_C)^2} = I_0 Z$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan \phi = \frac{V_{L0} - V_{C0}}{V_{R0}} = \frac{I_0(X_L - X_C)}{I_0 R} = \frac{X_L - X_C}{R}$$

$$\cos \phi = \frac{V_{R0}}{V_0} = \frac{I_0 R}{I_0 Z} = \frac{R}{Z}$$

$$P = I_{rms}^2 R = I_{rms}^2 Z \cos \phi = V_{rms} I_{rms} \cos \phi$$

## Maxwell's equations & Electromagnetic waves

Equation Name	Integral Form	Differential Form
Gauss's law for electricity	$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q}{\epsilon_0}$	$\vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$
Gauss's law for magnetism	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$	$\vec{\nabla} \cdot \vec{\mathbf{B}} = 0$
Faraday's law of induction	$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d\Phi_B}{dt}$	$\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$
Ampère's law with Maxwell's correction	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$	$\vec{\nabla} \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$

$$E = E_y = E_0 \sin(kx - \omega t)$$

$$B = B_z = B_0 \sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi f$$

$$\nu = f\lambda = \frac{\omega}{k}$$

$$\nu = \frac{E_0}{B_0} = \frac{\omega}{k}$$

$$\nu = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = c$$

$$c = \frac{E}{B}$$

$$u = u_E + u_B = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{B^2}{\mu_0}$$

$$u = \epsilon_0 E^2 = \epsilon_0 c^2 B^2 = \frac{B^2}{\mu_0} = \epsilon_0 c E B = \sqrt{\frac{\epsilon_0}{\mu_0}} E B$$

$$\bar{S} = \frac{1}{2} \epsilon_0 c E_0^2 = \frac{1}{2} \frac{c}{\mu_0} B_0^2 = \frac{E_0 B_0}{2 \mu_0} = \frac{E_{rms} B_{rms}}{\mu_0}$$

$$P = \frac{\bar{S}}{c} \quad \text{radiation fully absorbed}$$

$$P = \frac{2\bar{S}}{c} \quad \text{radiation fully reflected}$$

$$c = \frac{E}{B} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 300.000.000 \text{ m/s} = 300 \times 10^6 \text{ m/s}$$

$$c = \lambda f$$

## Geometrical optics: Refection & refraction

**Mirror equation:**

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$$

**The angle of incidence and the angle of reflection are equal**  $\theta_i = \theta_r$ .

**Snell's law of refraction:**  $n_1 \sin \theta_1 = n_2 \sin \theta_2$

**Lateral magnification (mirror):**  $m = \frac{h_i}{h_o} = -\frac{d_i}{d_0}$

## Geometrical optics: Optical instruments

**Lensmaker's equation:**  $\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$

**Thin lens equation for convex/converging lenses:**  $\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$

**Thin lens equation for concave/diverging lenses:**  $\frac{1}{d_0} - \frac{1}{d_i} = \frac{1}{f}$

**Lateral magnification (lens):**  $m = \frac{h_i}{h_o} = -\frac{d_i}{d_0}$

## Wave optics: Interference & polarization

**Malus's law:**  $I = I_0 \cos^2 \theta$

**Brewster's law:**  $\tan \theta_p = \frac{n_2}{n_1}$

**Newtons rings:**

- **Dark rings (destructive interference):** Occur when this condition is met:  
 $2t + \frac{\lambda}{2} = \frac{\lambda}{2}(2m + 1) = (m + \frac{1}{2})\lambda$  with  $m = 0, 1, 2, \dots$  as the order of the dark rings.  
Rearranging this, we get  $2t = \lambda m$ .
- **Bright rings (constructive interference):** Occur when this condition is met:  $2t + \frac{\lambda}{2} = m\lambda$ , with  $m = 1, 2, \dots$  as the order of the bright rings. Rearranging this, we get  $2t = (m - \frac{1}{2})\lambda$ .

**Double slit:**

- **Constructive interference**, resulting in a bright fringe on the screen, happens when the path difference is an integer multiple of the wavelength ( $\lambda$ ) of the monochromatic light:

$$d \sin \theta = m\lambda, \quad m = 0, 1, 2, \dots$$

- **Destructive interference**, resulting in a dark fringe on the screen, occurs when the path difference is a half-integer multiple of the wavelength:

$$d \sin \theta = (m + \frac{1}{2})\lambda, \quad m = 0, 1, 2, \dots$$

## Wave optics: Diffraction

**Diffraction at single-slit:**

- $\Delta = D \sin \theta \leftrightarrow D \sin \theta = m\lambda$
- **Central maximum:** Rays passing straight through the slit are in phase, creating a central bright region at an angle  $\theta = 0$ .
- **Minima:** Minima occur at angles  $\theta$  where the path difference between rays from the top and bottom of the slit is an integer multiple of the wavelength  $\lambda$ :

$$D \sin \theta = m\lambda, \quad m = \pm 1, \pm 2, \dots$$

- **Higher-order maxima:** Between the minima, weaker maxima appear approximately where the path difference is a half-integer multiple of the wavelength:

$$D \sin \theta \approx (m + \frac{1}{2})\lambda, \quad m \approx \pm \frac{3}{2}, \pm \frac{5}{2}, \dots$$

**Intensity of diffraction patterns:**

- **single-slit**  $I_\theta = I_0 \left( \frac{\sin(\beta/2)}{\beta/2} \right)^2 = I_0 \left( \frac{\sin\left(\frac{\pi D \sin \theta}{\lambda}\right)}{\frac{\pi D \sin \theta}{\lambda}} \right)^2$



- **double-slit**  $I_{\theta} = I_0 \left( \frac{\sin(\beta/2)}{\beta/2} \right)^2 \cos^2 \left( \frac{\delta}{2} \right)$