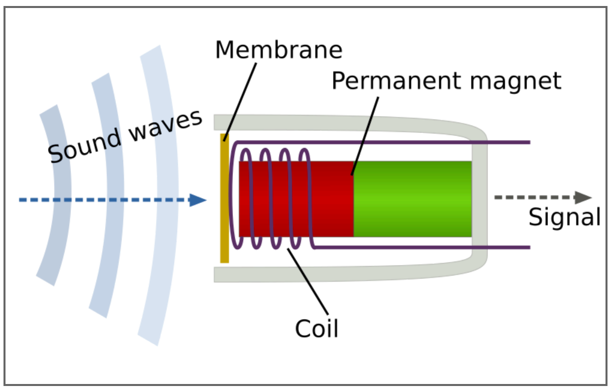
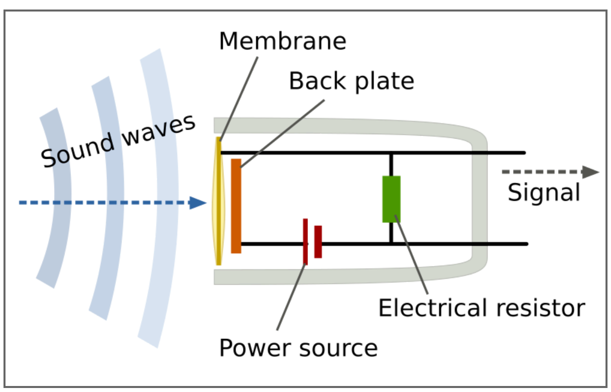
2.6. Electromagnets, induction & inductance

How does a microphone work?





Interaction of coil & magnet

- previous lecture we used **constant** magnetic field to understand:
    1. electric current produces magnetic field
    2. magnetic field exerts a force on electric currents/moving electric particles

What happens when we change the magnetic field over time?

`em18 - magnet drop`

- Joseph Henry (1797-1878) & Michael Faraday (1791-1867) described independently that **changing magnetic field induces an electric field**
- **concepts to understand**:
    - alternating voltage/current
    - electromagnetic induction

- inductance & electromagnets

What influences the induced voltage ? (1/2)

`em12 - flux`

Impact of geometry and orientation

The flux strikes back: Magnetic flux

`simulation mag. flux`

- magnetic flux Φ_B defined as:

$$\Phi_B = \int \vec{B} \, d\vec{A}$$

- here $\vec{A}$ is the vector normal to the area A
- in contrast to the electric flux Φ_E, "regular" integral instead of closed surface integral
- for a **uniform field** simplifies to:

$$\Phi_B = B_{\perp} A = BA \cos\theta = \vec{B} \cdot \vec{A}$$

- measured in *weber* [Wb] = [T m²]

What influences the induced voltage ? (2/2)

`em12 - flux`

Impact of change over time and number of loops

- **only changing magnetic fields induce a voltage**
- rate of change scales with induced voltage, i.e. $\frac{d\Phi_B}{dt}$
- for same $\frac{d\Phi_B}{dt}$, coil with more loops shows higher induced voltage

Faraday's law of induction

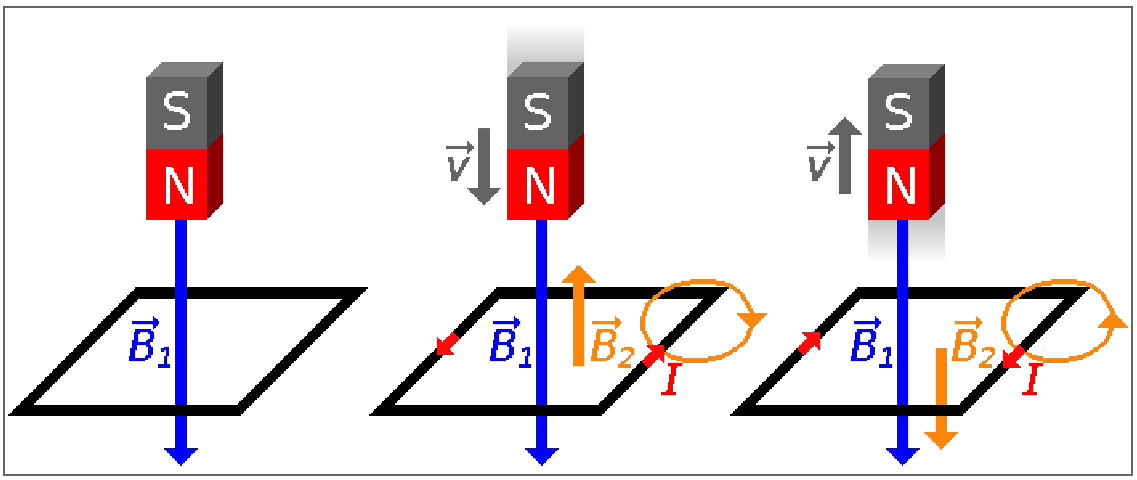
- $V_{ind} = -N \frac{d\Phi_B}{dt}$
- with:
    - N - number of loops in coil
    - $\frac{d\Phi_B}{dt}$ - change in magnetic flux

Why is there a minus?

Lenz’s law

`em14 - Rings`

A change in the magnetic flux through a conducting loop induces a voltage, so that the resulting current generates a magnetic field that opposes the change in magnetic flux.



*from **wikipedia** under **CC0 1.0 Universal**; image was edited*

E-field from electrostatics vs. electromagnetism

- remember electrostatics: **static charges generate electric field**:
  - $V_{AB} = \int_a^b \vec{E} d\vec{l}$
  - thus, for closed path, $V = \oint \vec{E} d\vec{l} = 0$ → work is zero
  - **conservative** → path independence & can define potential energy
- **changing magnetic flux induce voltage that generate an electric field**:
  - voltage induced by changing magnetic flux is: $V_{ind} = -\frac{d\Phi_B}{dt}$
  - work per unit charge due to induced voltage: $V_{ind} = \oint \vec{E} \cdot d\vec{l}$
  - **general form of Faraday's law** $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$
  - work for closed path not necessarily zero → **non-conservative**

- changing magnetic field induces
  electric field with closed field lines

Eddy currents

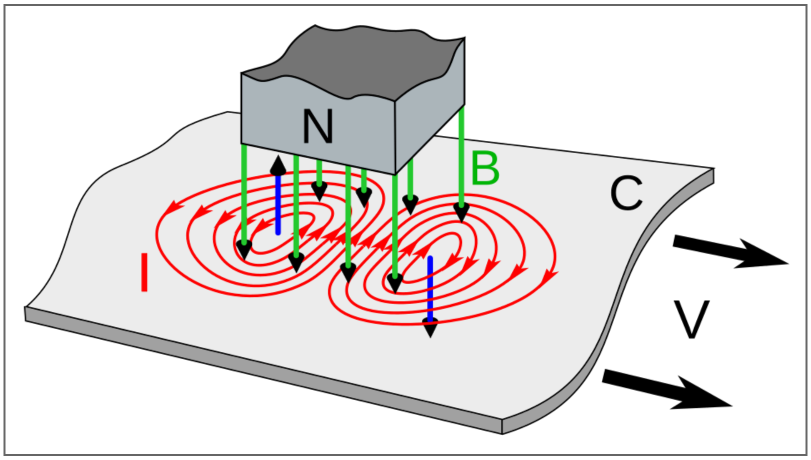
`em25 - Waltenhofen's pendulum`

- conductor (pendulum) moves through magnetic field generated by an electromagnet
- → changing magnetic flux in conductor → voltage induced
- part leaving the electromagnetic field:
    - decrease in magnetic flux (number of field lines passing through)
    - generates a field that repels this motion (Lenz's law), i.e. field generated that **attracts** towards the electromagnet
- part entering the electromagnetic field:
    - increase in magnetic flux (number of field lines passing through)

- generates a field that repels this motion (Lenz's law), i.e. field generated that **repels** towards the electromagnet

Eddy currents (cont')

`em25 - Waltenhofen's pendulum`



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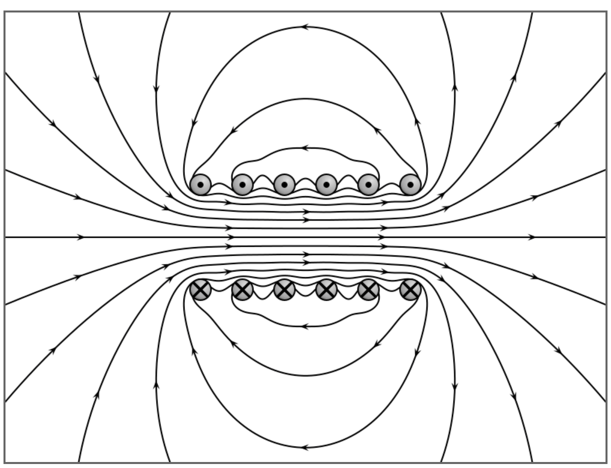
- **direction of induced current via RHR-1'**

Electromagnets - solenoids

`em10 + simulation B-field solenoid`

- tightly wound helical coils of wire
- found in many devices: generators, loudspeakers, MRI scanners, and many more

- produce an almost uniform magnetic field inside the coil and weak field outside
- with loop density n, the field is: $B = \mu_0 \frac{N}{l} I = \mu_0 n I$



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AC electric generator

`em38`

- convert mechanical energy into electrical energy via a **rotating armature** in a **static magnetic field**
- assuming a uniform magnetic field, **induced voltage in rotating coil** is:

$$V_{ind} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\int \vec{B}\, d\vec{A}\right) = -\frac{d}{dt}(BA\cos\theta)$$

- **only angle θ is time dependent**
- assuming initial angle $\theta_0 = 0$ and constant rotation velocity $\omega = \frac{d\theta}{dt}$:

$$V_{ind} = -BA\frac{d\cos(\omega t)}{dt} = BA\omega\sin(\omega t)$$

- for N loops and maximally induced voltage $V_{ind,0} = NBA\omega$:

$$V_{ind} = NBA\omega\sin(\omega t) = V_{ind,0}\sin(\omega t)$$

- principle can be **reversed** to work as **electric motors**

Primer on alternating current (AC)

- AC == time varying voltage & current
- for sinusoidal behavior, with $\omega = 2\pi f$, voltage alternates between $+V_0$ and $-V_0$ described as:

$$V(t) = V_0 \sin(2\pi f t) = V_0 \sin(\omega t)$$

- current is given by:

$$I(t) = \frac{V(t)}{R} = I_0 \sin(\omega t)$$

Using Root-Mean-Squared (RMS) values

- often RMS values are used in AC:

$$V_{RMS} = \sqrt{\overline{V(t)^2}} = \sqrt{\overline{V_0^2 \sin^2(\omega t)}}$$

- with $\overline{\sin^2(\omega t)} = \frac{1}{2}$

$$V_{RMS} = \frac{V_0}{\sqrt{2}}$$

$$I_{RMS} = \frac{I_0}{\sqrt{2}}$$

- for example, a 220V outlet has a peak voltage $V_0 \approx 311$ V

Power in AC circuits

- power is $P(t) = I(t)V(t)$
- for sinusoidal current/voltage, average power over a period is **non-zero**:

$$\overline{P} = I_0 \sin(\omega t) V_0 \sin(\omega t) = I_0 V_0 \sin^2(\omega t)$$

- for one period $\sin^2(\omega t) = \frac{1}{2}$, $I_{RMS} = \frac{I_0}{\sqrt{2}}$ and $V_{RMS} = \frac{V_0}{\sqrt{2}}$

$$\overline{P} = \sqrt{2} I_{RMS} \sqrt{2} V_{RMS} \frac{1}{2} = I_{RMS} V_{RMS}$$

Transformers

`em46`

- transformer consists of a *primary* and *secondary* coil
- two coils **do not** form a closed circuit, but only their proximity is essential for the transformer to work

What influences the voltage in the secondary coil?

Transformers - What influences the voltage in the secondary coil?

- iron core (geometry, layered, etc.)
- ratio of loops:
  - without energy loss, the induced voltages in both coils are:

$$V_2 = N_2 \frac{d\Phi_B}{dt} \quad \& \quad V_1 = N_1 \frac{d\Phi_B}{dt}$$

  - taking the voltage ratio & using conservation of power, $P_1 = P_2$, we get:

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

- depending on the ratio of the loop numbers:
  - step-up transformer: For $N_2 > N_1$ the voltage V_2 will be larger than V_1
  - step-down transformer: For $N_2 < N_1$ the voltage V_2 will be smaller than V_1

Mutual inductance

- relates the induced voltage in one coil to the rate of change of current in a neighboring coil
- defined as proportionality constant:

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1}$$

- the induced voltage in one coil can directly be related to change in current in the other coil:

$$V_2 = -N_2 \frac{d\Phi_{21}}{dt} = -M_{21} \frac{dI_1}{dt} \quad \& \quad V_1 = -M_{12} \frac{dI_2}{dt}$$

- mutual inductance only depends on geometric variables, i.e. the spacing between the coils and their individual shape, size, and number of loops:

$$M = M_{21} = M_{12}$$

- **inductance** is expressed in **units of henry** [H] = [V s / A] = [Ω s]

Self-inductance & inductors

YouTube - James Bond

`em13`

- even for a **single coil**, we find the phenomenon of inductance, a.k.a **self-inductance**
- **changing current** passes through the coil **induces voltage in coil itself** with **opposed direction**
- defined as proportionality constant (depends on coil geometry):

$$L = \frac{N\Phi_B}{I}$$

$$V_{ind} = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

- often referred simply to as inductance and measured in henry [H].
- **inductors are fundamental circuit elements alongside resistors and capacitors**

Energy stored in the magnetic field

`em17 - Thomson`

- power supplied to an inductor is:

$$P = I\,V_{ind} = I\,L\frac{dI}{dt}$$

- the work done to build up the current is:

$$dW = P\,dt = IL\frac{dI}{dt}dt = ILdI$$

$$W = \int_0^I I\,L\,dI = \frac{1}{2}LI^2$$

- energy stored in the magnetic field is:

$$U = \frac{1}{2}LI^2$$

- analogous to the energy stored in a capacitor:

$$U = \frac{1}{2}CV^2$$