2.1. Electric charge & electric field

In this chapter we are going to introduce charges, how they interact with each other, and what force come into play. Further, we will introduce the concept of the electric field.

2.1.1. Static electricity & conservation of electric charge

- object can become charged due to e.g. friction and possess *net electric charge*
- *two types* of electric charge:
  - *positive* & *negative*
  - rubbed amber (or plastic) is by convention as negative charge, hence, rubbed glass rod is associated with positive charge
  - arbitrary choice by Benjamin Franklin (1706–1790)
- *charges of same type repel, charges of opposite type attracts each other*

Law of conservation of electric charge:

- **no net electric charge can be created or destroyed**
- charges can only be **separated** (see section "Inducing charge")
- i.e. when rubbing a glass rod with a cloth, the cloth acquires the equal amount of negative charges as the glass rod acquires positive charges, ergo *algebraically*, net change in both is zero

2.1.2. Charge origin, insulators, & conductors

- use simplified model of an atom
  - tiny but massive nucleus with *positively charged protons* and neutrons
  - nucleus surrounded by *negatively charged electrons*
  - same number of protons and electrons with same magnitude but opposite direction of electric charge
  - therefore, *atom has no net charge*
  - if atom loses or gains electrons, it possess a charge and is called an *ion*
- explanation charging by friction:
  - electrons can move quite freely
  - rubbing causes them to "transfer" from one object to another
- *polar* molecules:
  - while overall molecule is neutral, its charge can be *distributed non-uniformly*
  - example: *water molecule* with opposite charges on different ends, i.e. positively at H and O

    - water molecules in air are responsible for electrostatically charged objects to return to neutral state
    - free electron "leak off" or "hope off" via the water molecules in the air
    - therefore, static electricity more noticeable on dry days
- *conductors* of electricity (e.g. metals) have very loosely bound electrons which can move freely and react to charged objects
- *semiconductors* (e.g. silicon & germanium) have fewer and *insulators/nonconductors* (e.g. wood & plastic) have almost no free electrons

2.1.3. Inducing charge

Charging by conduction

- if two conductors are in physical contact, electrons can exchange from one to the other depending on initial charge of objects
- if a positive conductor "touches" a neutral one, electrons from the neutral object "flow" to the positively charged one and, therefore, the neutral object becomes positively charges as well
- results: both conductors have *same sign of the charge* after being charged by conduction

Charging by (electrostatic) induction

- without direct contact a charged conductor can alter the charge distribution in another conductor in its proximity
- if a positive conductor is close to a neutral one, but does not "touch" it, the electrons in the neutral conductor will be attracted towards the positive conductor
- results: charges in the initially neutral conductor will be separated while the net charge remains zero
- notes on insulators/nonconductor:
    - while electrons in insulators cannot move freely within the object, they can move within heir own atoms/molecule
    - charged object can alter alignment of the atoms/molecules in the insulators
    - hence, the insulators as a whole can still experience charge separation by induction although it remains net neutral

Ground

- object connect to *ground*, i.e. earth
- earth acts as reservoir of charge as it is large and can conduct
- hence, earth accepts or provides electrons from/to connected object

Electroscope

- device to detect charges
- two conducting, thin leafs attached to a rod

- if charged, leafs will repeal each other
- can be used to illustrate charging by conduction and induction
- inherently cannot differentiate between positive or negative charge as only the amount of charge is indicated by the leafs

2.1.4. Coulomb's law

- in the 1780s, Charles Augustin de Coulomb (1736 - 1806) investigated what factors affect the force of repulsion and attraction between charged objects
- using different ratios of charges and a suspended rod, so-called torsion balance, he could investigate the *electrostatic force* between charged objects
- honoring his contribution, charge Q is measured in the SI unit **coulomb** [C]
- *magnitude* of the *electrostatic force*:
  - the force is proportional to the product of the magnitude of the charges, i.e. Q_1 & Q_2
  - the force is inversely proportional to the square root of the distance r between the charges
  - there is a proportionality constant k
- this yields that the *magnitude* of the *electrostatic force* that either charge exerts on the other is:

$$F = k\frac{Q_1 Q_2}{r^2}$$

- *direction* of the electrostatic force:
  - force is acting along the line connecting both charges
  - two charges with same sign, i.e. repelling each other, the force on either charge is directed away from the other
  - two charges with opposite sign, i.e. attract each other, the force on either charge is directed towards the other
- the proportionality constant $k \approx 9.0 \times 10^9 \mathrm{N\ m^2/C^2}$ is related to the fundamental constant *permittivity of free space* ϵ_0 by the following relation:

$$\epsilon_0 = \frac{1}{4\pi k} = 8.854 \times 10^{-12} \frac{\mathrm{C^2}}{\mathrm{N\ m^2}}$$

Notes on Coulomb's law:

- assumes point charges at rest, i.e. electrostatic force
- describes the force Q_1 exerts on Q_2
- reminder Newton's third law: **the force charge A exerts on the charge B is equal but opposite to that exerted by charge B on charge A**
- strong similarity to gravitational forces, although gravitation always an attractive force

Vectorization of Coulomb's law:

- electrostatic force can be vectorized, i.e. it possess a magnitude and direction
- in vector form we add two subscript, e.g. $\vec{\mathbf{F}}_{12}$, to indicate the force acts on charge 1 and charge 2 exerts the force
- therefore, $\vec{\mathbf{F}}_{12}$ will be the vector force on charge Q_1, due to Q_2:

$$\vec{\mathbf{F}}_{12} = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r_{21}^2}\hat{\mathbf{r}}_{21}$$

- with $\hat{\mathbf{r}}_{21}$ as the unit vector pointing from the "source" charge towards the charge "acted upon", i.e. from Q_2 to Q_1; note that unit vector can be computed from the vector from Q_2 to Q_1 and its length $\hat{\mathbf{r}}_{21} = \frac{\vec{\mathbf{r}}_{21}}{\|\vec{\mathbf{r}}_{21}\|}$
- Q_1 and Q_2 and be positive or negative
- if both charges have the same sign, i.e. $Q_1 Q_2 > 0$, then the force $\vec{\mathbf{F}}_{12}$ is repulsive and points away from Q_2
- if both charges have opposite sign, i.e. $Q_1 Q_2 < 0$, then the force $\vec{\mathbf{F}}_{12}$ is attractive and points towards Q_2

Superposition of Coulomb's law:

- *principle of superposition* can be applied if more than two charges exert forces on each other
- net force will be vector sum of the individual forces due to all charge interactions
- for example, for five charges, the net force on charge A is $\vec{\mathbf{F}}_{A,net} = \vec{\mathbf{F}}_B + \vec{\mathbf{F}}_C + \vec{\mathbf{F}}_D + \vec{\mathbf{F}}_E$

Elementary charge e

- e is the smallest charge observed in nature, i.e. charge of an electron or proton
- $e = 1.602 \times 10^{-19}\mathrm{C} \approx 1.6 \times 10^{-19}\mathrm{C}$
- proton has charge of e, electron of $-e$
- while *quantized*, i.e. no fraction of an electron exist and we only observe multiples of e, $2e$, $3e$ ... , $1\mu\mathrm{C} \approx 10^{13}$ electrons, ergo, appeals continuous on macroscopic level

```
interactive(children=(FloatSlider(value=1.0, description='q1', max=1.0), FloatSlider(value=0.
5, description='q…
interactive(children=(FloatSlider(value=0.1, description='r', max=1.0, min=0.1), IntSlider(va
lue=5, descriptio…
```

```
<function __main__.plot_coulomb_r(r, n_trace, show_analytical, transform_y_axis)>
```

2.1.5. Concept of the electric field

- electric forces do not act by contact but rather *contact less & over a distance*
- similar to the gravitation, we need the concept of an **electric field** (first developed by Michael Faraday (1791 - 1867))

Thought experiment

- place small positive *test charge* q near the charge we want to study Q
- test charge q is so small that its effect on Q can be neglected
- if we measure the force (magnitude and direction) Q exerts on q at every point in space we have a surrogate for the electric field $\vec{\mathbf{E}}$

Vector field $\vec{\mathbf{E}}$

- **electric field** $\vec{\mathbf{E}}$ is defined as the force $\vec{\mathbf{F}}$ exerted in on tiny positive test charge q divided by the magnitude of test charge which approaches zero ($q \to 0$):

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q}$$

- using Coulomb's law and considering the magnitude of the $\vec{\mathbf{E}}$ field, the test charge q cancels out and give use the so-called *electric field form* of Coulomb's law

$$E = \frac{F}{q} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\frac{1}{q} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

- therefore, E depends on the charge we study Q and the distance to it r

- if $\vec{\mathbf{E}}$ is known, $\vec{\mathbf{F}}$ on a test charge q can therefore be described as

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

- if q is negative, $\vec{\mathbf{E}}$ & $\vec{\mathbf{F}}$ point in opposite direction

- if q is positive, $\vec{\mathbf{E}}$ & $\vec{\mathbf{F}}$ point in same direction

Superposition & electric fields

- the *principle of superposition* for electric fields is supported by experimental evidence
- therefore, the net field $\vec{\mathbf{E}}_{net}$ can be described by the super position of the electric fields of charge Q_A, Q_B, Q_C, and so on:

$$\vec{\mathbf{E}}_{net} = \vec{\mathbf{E}}_A + \vec{\mathbf{E}}_B + \vec{\mathbf{E}}_C + \ldots$$

Integral form of the electric field

- for charge distributions, we can assume the distribution to be continuous on macroscopic scale (e caus charge to be actually quantized)

- dividing the charge into infinitesimal small charges dQ yields the field portion dE at a distance r

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}$$

- by integration we get the vector field

$$\vec{\mathbf{E}} = \int d\vec{\mathbf{E}}$$

- often it is useful to define:
  - the *charge per unit length* λ (in units of (C/m))
  - the *charge per unit area* σ (in units of (C/m^2))
- for an infinite plane with uniform surface charge density σ:

$$E = \frac{\sigma}{2\epsilon_0}$$

- for two parallel, oppositely charged plates at close distance:

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Field lines

- $\vec{\mathbf{E}}$ vector field can be represented by arrow with its length and orientation indicating the magnitude and direction of the electric field
- however, arrows for each point in space would be cluttered and cumbersome to interpret
- instead, use *field lines*:
  - indicate the *direction* of force due to the $\vec{\mathbf{E}}$ field has on a positive test charge
  - indicate the *magnitude* of the force by the *density* of the field lines
  - point *radially* inwards for negative and outwards for positive charges
  - *begin at positive and end at negative* charges
  - direction of $\vec{\mathbf{E}}$ tangent to field lines
- therefore,
  - number of field lines beginning/ending at a particular charge is proportional to the charge itself
  - magnitude of $\vec{\mathbf{E}}$ is proportional to number of field lines crossing a perpendicular unit area
  - field lines do not cross as the underlying, tangent (net) electric field cannot have two directions at the same time

```
interactive(children=(FloatSlider(value=1.0, description='q1_mag', max=1.0, min=-1.0), FloatS
lider(value=-1.0,…
```

```
<function __main__.plot_e_field(q1_mag, q2_mag, plot_q2, q2_x, q2_y, vector_field)>
```

Notes on the simulation:

- we apply the principle of superposition, i.e. compute the electric field per charge and sum over all fields
- we compute use the vector for of Coulomb's law,
- we implicitly compute the unit vector with $\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{\|\vec{\mathbf{r}}\|} = \frac{\vec{\mathbf{r}}}{r}$

- therefore, the nominator will involve the multiplication of Q and the respective vector components of $\vec{\mathbf{r}}$
- further, the denominator is r^3 instead of r^2, as the length of the vector (used to normalize the vector) needs to be factored in

$$\vec{\mathbf{E}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} = \frac{1}{4\pi\epsilon_0}\frac{Q * \vec{\mathbf{r}}}{r^3}$$

```
interactive(children=(FloatSlider(value=1.0, description='distance', max=4.0, min=0.5), Outpu
t()), _dom_classe…
```

```
<function __main__.plot_e_field(distance)>
```

2.1.6. Effect of electric fields

Conductors

There are some key properties for conductors inside electric fields:

- in the static condition, i.e. charges are at rest, the *electric field is zero inside the conductor*
  - if not the case, the freely moving electrons would be subject to a force ($\vec{\mathbf{F}} = q\vec{\mathbf{E}}$) which would move them until they reach a position where the net force experienced by them is zero, i.e. the electric field is zero
- for charged conductors, any net charge on a conductor distributes itself on the surface of the conductor as the field inside the conductor is zero
- the electric field is always perpendicular to the surface outside of a conductor
  - otherwise, there would be a component parallel to the surface which would exert a force on the free electrons at the surface causing them to move

These key properties are the reason for *Faraday cage*.

Charged particle inside an electric field

If a charge q is inside an electric field if will experience a force due to $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$.

For the trivial example of an electron (with charge -e) inside an homogenous electric field (e.g. generated by two parallel and charged plates), we can compute the acceleration a of the electron due to the field as:

$$E = \frac{F}{q} = \frac{ma}{q}$$

$$a = \frac{-eE}{m_e}$$

Electric dipoles

Two equal charges with opposite in sign, i.e. $+Q$ & $-Q$, and separated by a distance l, are referred to as an *electric dipole*. Electric dipoles have a *dipole moment* $\vec{\mathbf{p}}$ with a magnitude of $p = Ql$