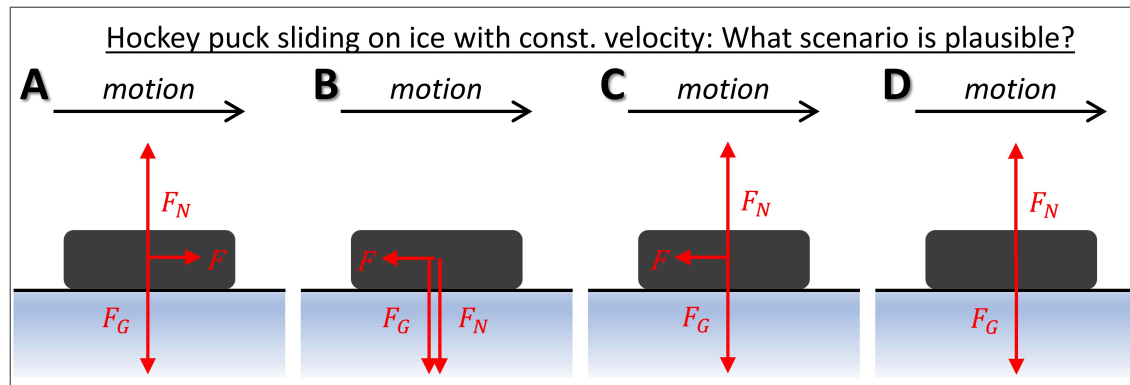


## 1.3. Dynamics: Newton's Laws of Motion



- need to introduce force, Newton's laws and their applications
- solution at the end of the lecture

# Force

- **Action** that can:
  - cause an object to change its velocity, its shape, or to resist other forces
  - cause changes of pressure in a fluid
- Acts via **contact** (friction, tension, normal) or **at a distance** (gravity, electromagnetism)
- Can be **measured** with a spring scale (stretch  $\propto$  force)
- Unit: Newtons  $[F] = \text{N}$
- **Vector quantity**  $\rightarrow$  combine by **vector addition**

## Galileo's idea of inertia

md09 – Luftkissenbahn, stays in motion

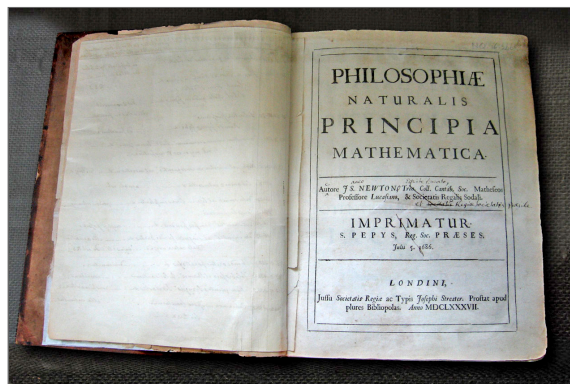
- **Aristotle**: motion requires continuous force; rest is the natural state
- **Galileo**:
  - Imagined motion on a **frictionless surface**
  - In the **ideal case (no friction)** → no force needed to maintain constant velocity
  - → Objects **retain their motion** unless acted upon by a force
  - → **Friction** is a real force that resists motion, not proof that motion needs effort

# Newton's first law of motion — the law of inertia

## mb05 - Trägheit verschiedener Körper

- **Inertia:** tendency to resist changes in motion
- Motion remains **constant** (speed and direction) if **net force = 0**
- Building on Galileo's work, Newton formulated his **first law of motion:**

*Every object continues in its state of rest or of uniform motion in a straight line unless acted upon by a net external force.*



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## Inertial reference frames & when the first law holds

- Any frame moving with **constant velocity** relative to an inertial frame is itself inertial
- Newton's first law **fails in accelerating frames**
- **Non-inertial frames**: accelerating or rotating; show **apparent (fictitious) forces**
  - Example: a smartphone on the dashboard **slides back** as a car accelerates forward
- **Earth**  $\approx$  inertial for most lab situations

# Mass, weight & inertia

## md23 - Änderung der Kraft bei Beschleunigung

- **Mass:**

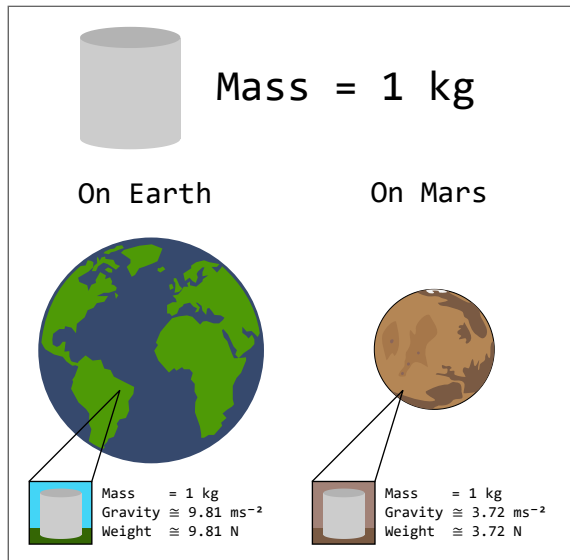
- Measures **inertia**, the resistance to acceleration or change in motion
- Larger mass  $\rightarrow$  **more force** needed for same acceleration
- **Intrinsic property** of matter; independent of location
- **Unit:** kilogram (kg)

- **Weight:**

- A **force** due to gravity  $\rightarrow \vec{W} = m\vec{g}$
- Direction: toward Earth's center
- Magnitude:  $|\vec{g}| \approx 9.81 \text{ m/s}^2$

## Mass, weight & inertia: Example:

→ **Mass** stays constant; **weight** depends on local gravity



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## Newton's second law of motion

md09 – Luftkissenbahn,  $a$  in Abhängigkeit von  $m$

- **First law:** motion stays constant if **net force = 0**
- **Second law:** describes motion when a **net force acts** → object **accelerates**

*At any instant of time, the net force on a body is equal to the body's acceleration multiplied by its mass or, equivalently, the rate at which the body's momentum is changing with time*

$$\vec{F}_{\text{net}} = m\vec{a}$$

- **Unit:** Newton (N) →  $1 \text{ N} = 1 \frac{\text{kg m}}{\text{s}^2}$
- $\vec{a} \propto \vec{F}_{\text{net}}, \vec{a} \propto \frac{1}{m}$

- Direction of  $\vec{a}$  is the same as  $\vec{F}_{\text{net}}$

## Newton's third law of motion

### md11 - Wechselwirkungsgesetz

- **Forces arise from interactions** between two objects
- In every interaction, **each object exerts a force on the other**
- Newton's **third law of motion**, also known as **action–reaction principle**, states:

*If two bodies exert forces on each other, these forces have the same magnitude but opposite directions.*

## Newton's third law of motion (cont')

- **Symbolic form:**

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

where  $\vec{F}_{AB}$  acts on B by A, and  $\vec{F}_{BA}$  acts on A by B

- The two forces are
  - **Equal in magnitude**
  - **Opposite in direction**
  - **Act on different bodies** → never cancel within one free-body diagram
- Key idea: every force is part of an **interaction pair**

## Newton's third law of motion: Ice skater & wall

- Skater **pushes backward** on the wall → force **on the wall**
- Wall **pushes forward** on the skater → equal and opposite **reaction force**
- With minimal ice friction, this reaction **accelerates** the skater backward
- Skater moves due to the **force exerted by the wall on her**, not her own push

## Newton's third law of motion: Walking & propulsion

- **Walking:**

- Foot pushes **backward** on ground → ground pushes **forward** on foot
- The **forward reaction force** propels you ahead
- Without **friction** (e.g., on ice), no effective push → no motion

- **Rocket propulsion:**

- Engine pushes **gases backward** → gases push **rocket forward**
- Rocket accelerates due to the **force on it** from escaping gases, not from pushing on air or ground
- The forces are equal, but their effects differ:
  - gas has small mass, thus, gains large acceleration

- rocket has large mass, thus, gains smaller acceleration in the opposite direction

## Newton's third law of motion: Clarifying action & reaction

- Always note **which object exerts** and **which object receives** the force
- Example: person  $P$  pulls sled  $S$ 
  - $P$  exerts forward force on  $S$ :  $\vec{F}_{PS}$
  - $S$  exerts backward force on  $P$ :  $\vec{F}_{SP} = -\vec{F}_{PS}$
- Forces act on **different objects**  $\rightarrow$  cannot cancel each other
- **Sled moves** if pull  $>$  friction
- **Person moves** if ground's forward reaction  $>$  sled's backward pull



## Newton's third law of motion: Key points

- **Forces always occur in pairs.** A single, isolated force does not exist.
- **Action–reaction forces act on different bodies.** Only the forces acting *on a single body* enter its  $\sum \vec{F} = m\vec{a}$  equation.
- **The origin of forces is mutual interaction.** Both objects experience forces of equal strength at the same instant.
- **Equal forces can produce different accelerations,** depending on each object's mass.

## Solving problems with Newton's laws: Free-body diagrams (FBD)

- **FBD** isolates one object and shows all **external forces** acting on it
- **Internal forces** or forces the object exerts on others are **not included**

### Steps:

1. **Select the object** and sketch all external forces as vectors
2. **Label each force** with source and direction (e.g.,  $\vec{F}_N$ ,  $\vec{F}_g$ ,  $\vec{F}_T$ )
3. **Choose coordinates** that simplify the motion (e.g., along incline or direction of motion)
4. **Resolve forces** into components and apply Newton's second law:

$$\sum F_x = ma_x, \quad \sum F_y = ma_y, \quad \text{etc.}$$

5. Keep **signs and units consistent**; opposite directions carry opposite signs.

## Net force composition

md08 – Kräfteparallelogramm

sim – net force

- When multiple forces act on a point, the **net (resultant) force** is their **vector sum**
- To solve, use **parallelogram rule** or **tail-to-tip rule**

## Normal force & inclined planes

- **Normal force**  $\vec{N}$ : surface's **perpendicular reaction** preventing interpenetration
- **Horizontal surface:**  $N = mg$
- **Incline (angle  $\theta$ ):**  $N = mg \cos \theta$
- Weight  $\vec{W} = m\vec{g}$  resolves into
  - **Perpendicular:**  $W_y = -mg \cos \theta$
  - **Parallel:**  $W_x = -mg \sin \theta$
- The **downslope component**  $mg \sin \theta$  drives motion (if frictionless)

## Tension

- **Tension:** pulling force transmitted by a taut cord/rope/cable
- **Massless, inextensible cord**  $\rightarrow$  **same tension** everywhere
- Cords **pull** but do **not push**; tension acts **along the cord** away from the object

### Example (two boxes on frictionless table):

Boxes  $A$  and  $B$  (masses  $m_A, m_B$ ) connected by a cord; external pull  $F_P$  on  $A$ .

Box  $A$ :  $F_P - T = m_A a$

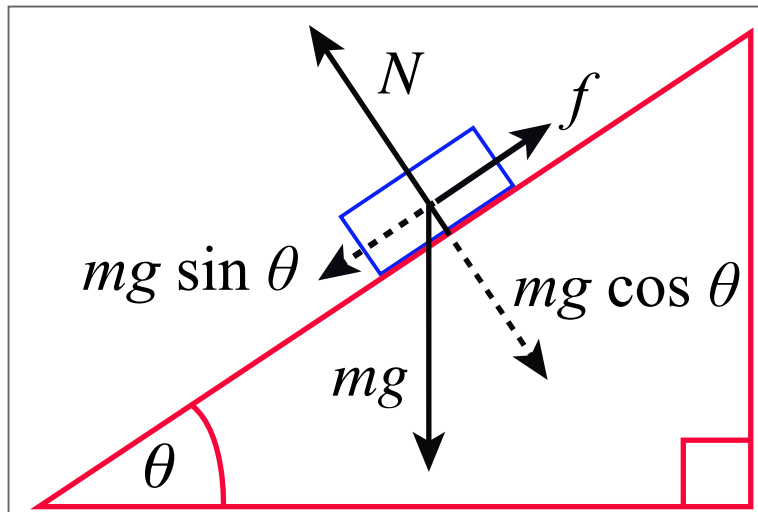
Box  $B$ :  $T = m_B a$

$$\Rightarrow a = \frac{F_P}{m_A + m_B}, \quad T = \frac{m_B}{m_A + m_B} F_P$$

## Using Newton's laws with friction

- **Friction:** surface interaction that **opposes relative motion**
- Origin: microscopic **asperities and bonding** at contact points
- Acts **parallel** to surface, magnitude  $\propto$  **normal force**  $N$
- **Static friction:**
  - up to  $F_{\text{fr}} \leq \mu_s N$
  - Motion starts when required force  $> \mu_s N$
- **Kinetic friction** (sliding):
  - constant magnitude:  $F_{\text{fr}} = \mu_k N$
- **Rolling resistance:** much smaller, from surface deformation
- Coefficients  $\mu_s, \mu_k$  depend on **materials** and **surface conditions**, nearly independent of speed or area
- Usually  $\mu_s > \mu_k \rightarrow$  starting motion requires more force than maintaining it

Interim summary: Linear dynamics



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## Dynamics of uniform circular motion

### md42 - Demonstration der Zentrifugalkraft mit einem Ball

- Object moving in a circle at constant speed → continuously **accelerates toward the center**
- Requires a **net inward (centripetal) force**:

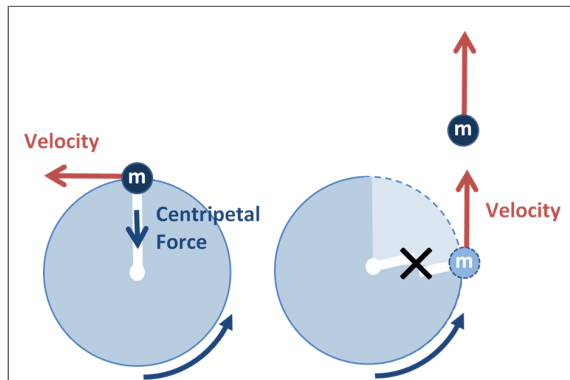
$$\sum F_R = ma_R = m \frac{v^2}{r} = m\omega^2 r$$

- **Centripetal force** acts **toward the center**, maintaining circular motion



## Dynamics of uniform circular motion: No centrifugal force

- There is **no real outward force**
- → **centrifugal force** is seen only in **non-inertial frames**
- Examples:
  - Turning car: Your **inertia** gives the **illusion** of an outward pull
  - Ball on a string: When released, it moves **tangentially**, not radially outward



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## Vertical circles

- In vertical motion, **tension** (or **normal force**) changes with position as **weight** adds or subtracts from the inward (centripetal) force

$$\sum \vec{F} = m\vec{a}_r$$

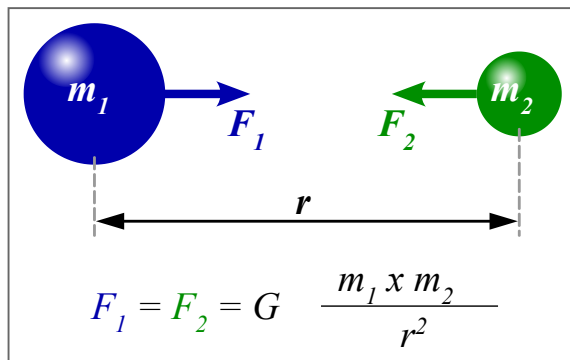
- **At the top:**  $T_{\text{top}} + mg = m\frac{v_{\text{top}}^2}{r}$
- **At the bottom:**  $T_{\text{bottom}} - mg = m\frac{v_{\text{bottom}}^2}{r}$
- **Minimum speed** for a taut string (tension = 0 at top):  $v_{\text{min}} = \sqrt{gr}$

## Gravitational forces & planetary motion

- Newton's **law of universal gravitation**: every mass **attracts** every other mass along the line joining their centers

$$\vec{F}_G = G \frac{m_1 m_2}{\vec{r}_{12}^2}$$

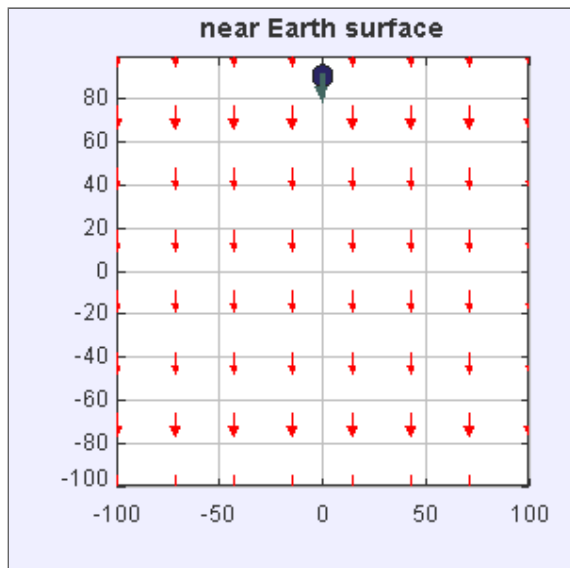
- **Gravitational constant**:  $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$
- Force is **mutual** (acts on both bodies equally and oppositely)
- Obeys an **inverse-square law**  $\rightarrow$  decreases rapidly with distance



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## Gravity near Earth's surface

- Near Earth, gravitational force simplifies to **weight**:  $F_G = mg$
- $g = G \frac{m_E}{r_E^2} \approx 9.81 \text{ m/s}^2$  where  $m_E$  and  $r_E$  are Earth's mass and radius
- $g$  is slightly **smaller** at high altitude or near the equator



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## Satellites & orbits

- For a body of mass  $m$  orbiting a planet of mass  $M$ :

$$\frac{GMm}{r^2} = m\frac{v^2}{r}$$

- **Orbital speed** and **period**:

$$v = \sqrt{\frac{GM}{r}}, \quad T = 2\pi\sqrt{\frac{r^3}{GM}}$$

- All bodies at the same orbital radius  $\rightarrow$  same **speed** and **period**, independent of their mass
- **Gravity provides** the centripetal acceleration keeping objects in orbit
- **Weightlessness** arises because satellite and occupants **fall together**  $\rightarrow$  no contact force  $\rightarrow$  **continuous free fall**

Revisit first question:

