2.10. Geometrical optics: Optical instruments

`os15`

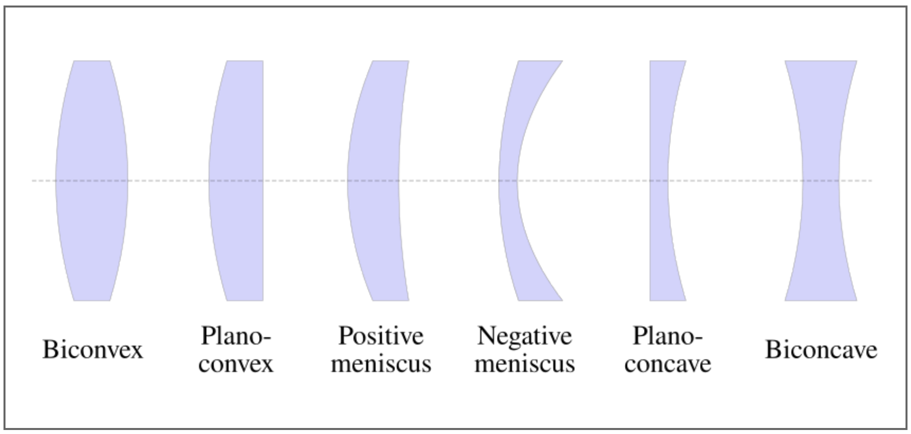
Explain this magic!

Agenda:

- use reflection and refraction to study lenses and optical instruments
- introduces key concepts such as image formation, aberrations, and optical limitations

Lens shapes

- lenses have two surfaces which can be planar, convex or concave
- convex lenses are converging
- concave lenses are diverging



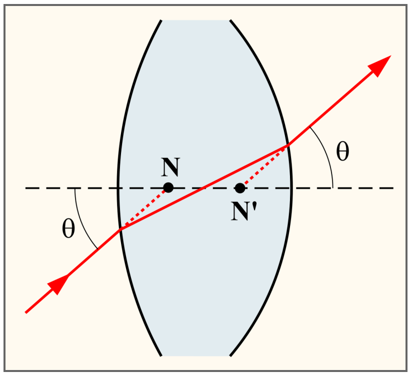
Ray tracing at a (thick) lens

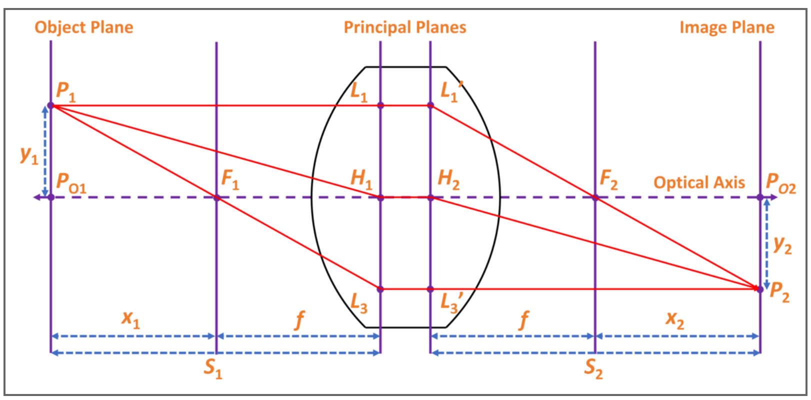
`sim - ray tracing thick lens`

- when a **ray** enters air-to-lens and then lens-to-air, it is **refracted according to Snell's law**

Ray tracing at a (thick) lens (cont')

`os15`





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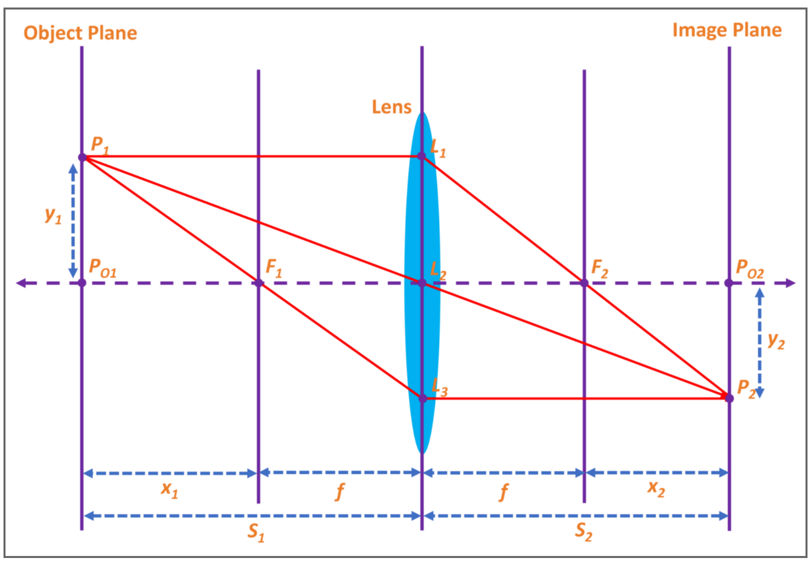
Thin Lens Model: Key assumptions

- **negligible lens thickness & single principal plane**:
  - both refractions are occurring at a single plane at the center of the lens
- **paraxial ray approximation**
  - only rays close to and nearly parallel with the optical axis are considered
  - small-angle approximations applies: $\sin(\theta) \approx \theta$, $\tan(\theta) \approx \theta$, $\cos(\theta) \approx 1$

Thin Lens Model: Advantages

- simplifies ray tracing and widely used in geometric optics
- focal length is the same on both sides in the thin lens approximation



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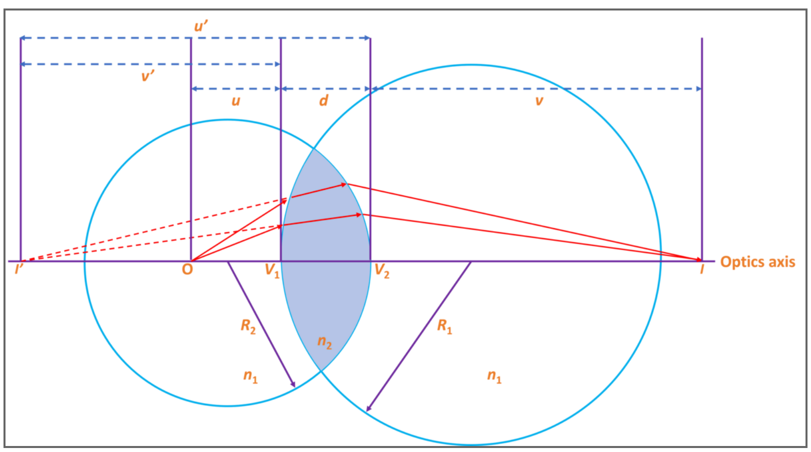
Lensmaker's equation

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

- n is the index of refraction
- R_1 and R_2 are the radii of curvature (positive for convex and negative for concave)
- **only valid for thin lenses**



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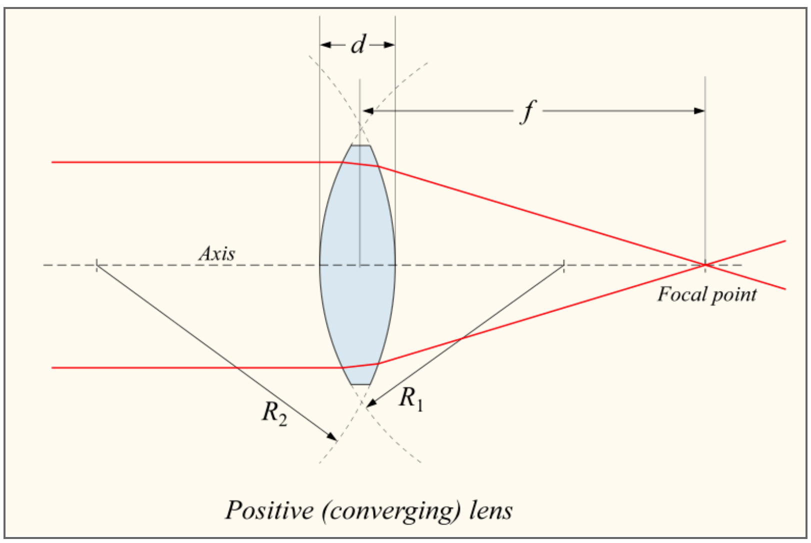
Thin lens model: Converging / convex lens

`os01`

- parallel rays converge at **focal point**, i.e image point for an object a infinite distance
- **focal plane**: perpendicular to axis, through focal point



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Thin lens model: Diverging / concave lens

`os01`

- parallel rays appear to diverge from a **virtual** focal point

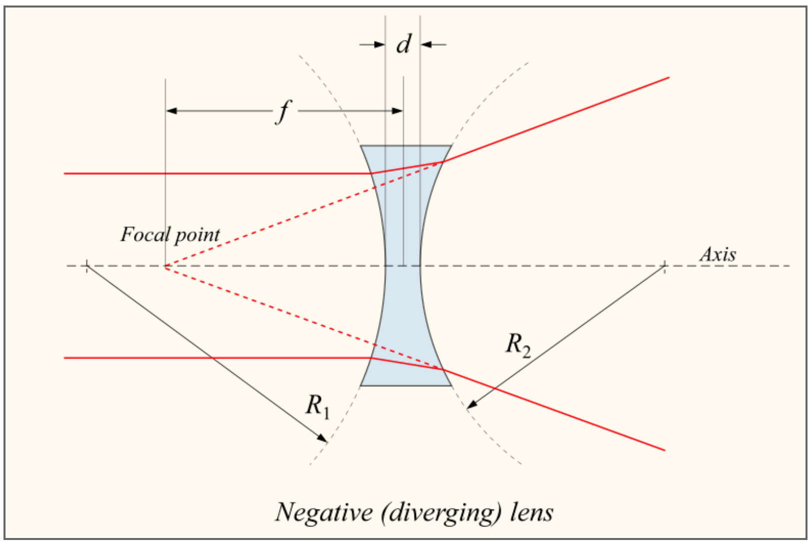

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Image formation at thin lenses via ray tracing

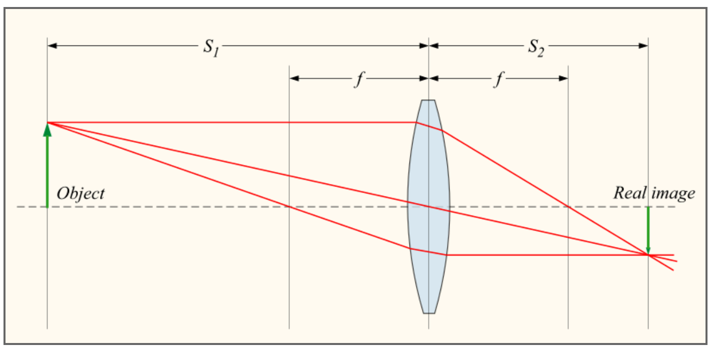
`sim - image formation thin lens`

- to locate image, trace these three **principal rays**:
    - **parallel ray**: travels parallel to the principal axis; becomes focal point ray after refraction
    - **focal point ray**: passes through the focal point; becomes parallel ray after refraction
    - **central (optical center) ray**: goes through the lens center; remains central ray

Image formation at converging / convex lens

`os08`

- parallel ray refracts through the focal point on the opposite side
- real for $d_0 > f$ and virtual image for $d_o < f$



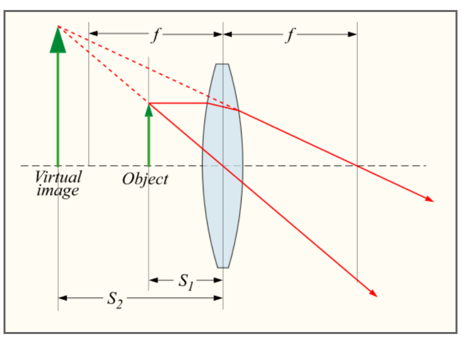


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Image formation at diverging / concave lens

`os08`

- refracted rays appear to diverge from the focal point on the same side as the object
- virtual image

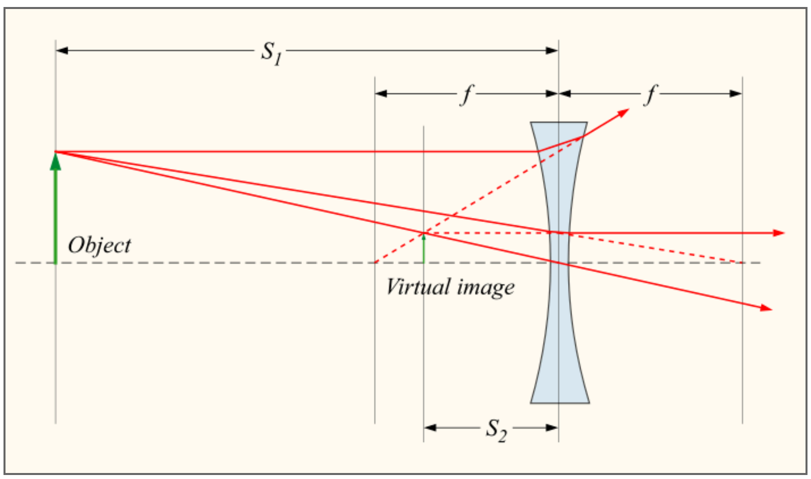

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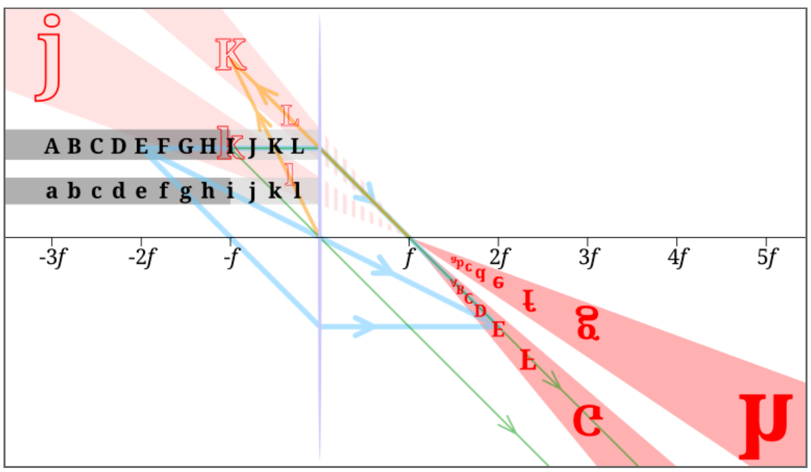
Image formation summary

	Real Image	Virtual Image
Image enlarged	Convex lens: $f < d_o < 2f$	Convex lens: $d_o < f$
Image diminished	Convex lens: $d_o > 2f$	Concave lens: always



Thin lens equation: Definitions

we define:

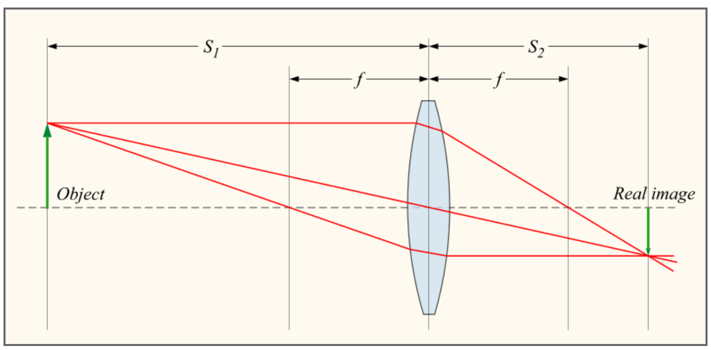
- object distance: d_o (from lens to object)
- image distance: d_i (from lens to image)
- object height: h_o
- image height: h_i

→ these are all measured along the **principal axis**

Thin lens equation: Derivation

- consider a ray through the **optical center** (undeviated, straight line).
- using similar triangles:

$$\frac{h_i}{h_o} = \frac{d_i}{d_o}$$



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Thin lens equation: Derivation (cont')

- now analyze the geometry of rays through the **focal point**.
- for a convex/converging lens, similar triangles give:

$$\frac{h_i}{h_o} = \frac{d_i - f}{f}$$

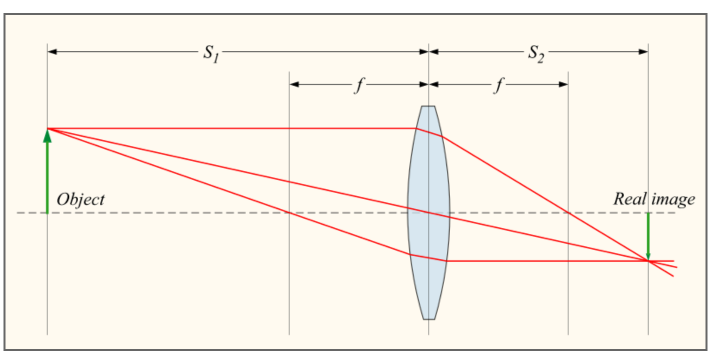
- using both equations:

$$\frac{d_i - f}{f} = \frac{d_i}{d_o}$$

- solving for f:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

- virtually the same as for mirrors



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Sign conventions for thin lenses

- $h_o > 0$ (always)
- $h_i > 0$ if image is **upright**, $h_i < 0$ if **inverted**
- $d_o > 0$ for **real objects** (light comes from that side)
- $d_i > 0$ for **real images** (on opposite side of lens)
- $d_i < 0$ for **virtual images** (same side as object)
- $f > 0$ for **converging** lenses
- $f < 0$ for **diverging** lenses

Lateral magnification

defined as:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

- $|m| > 1$: image is **enlarged**
- $|m| < 1$: image is **diminished**
- $m > 0$: **upright**
- $m < 0$: **inverted**

FYI: same formula as for mirrors

Combining lenses

to analyze a system of two lenses:

- step 1: First lens
    - use thin lens equation → find d_{i1}
    - compute magnification:

      $m_1 = -\frac{d_{i1}}{d_{o1}}$
- step 2: Second lens
    - use image from lens 1 as object
    - $d_{o2} = \text{separation} - d_{i1}$
    - use thin lens equation → find d_{i2}
    - $m_2 = -\frac{d_{i2}}{d_{o2}}$

Total effect of two lenses

- effective magnification:

$$m_{eff} = m_1 \times m_2$$

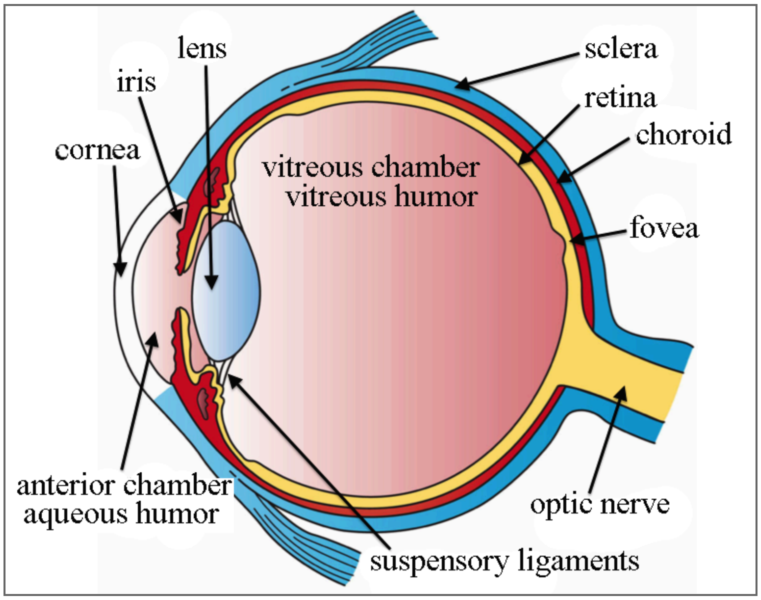
- **important:** watch out for **sign conventions**
    - negative d_i: virtual image
    - positive d_i: real image
    - same applies for object distances and focal lengths

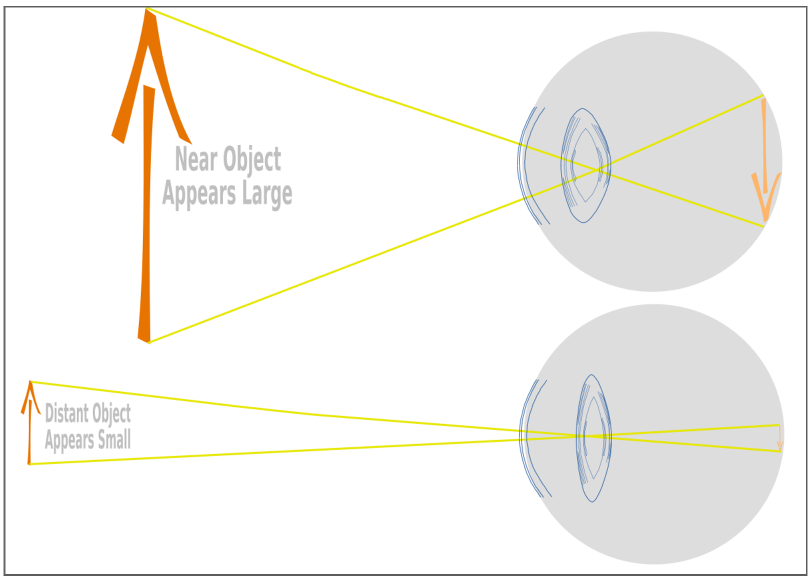
→ consider a ray diagram to double-check your reasoning!

The Human eye

`os19 - the eye`

- eye works like a **biological camera**, with fixed image distance and dynamic focusing
- eye accommodates by adjusting the **lens curvature**
- key components:
    - **Cornea** ($n \approx 1.376$): primary refraction
    - **Aqueous humor** ($n \approx 1.336$): fluid behind cornea
    - **Lens** ($n \approx 1.385–1.405$): changes shape to focus
    - **Vitreous humor** ($n \approx 1.337$): gel-like interior
    - **Iris and pupil**: control light entry
    - **Retina and fovea**: light detection; fovea = sharp vision





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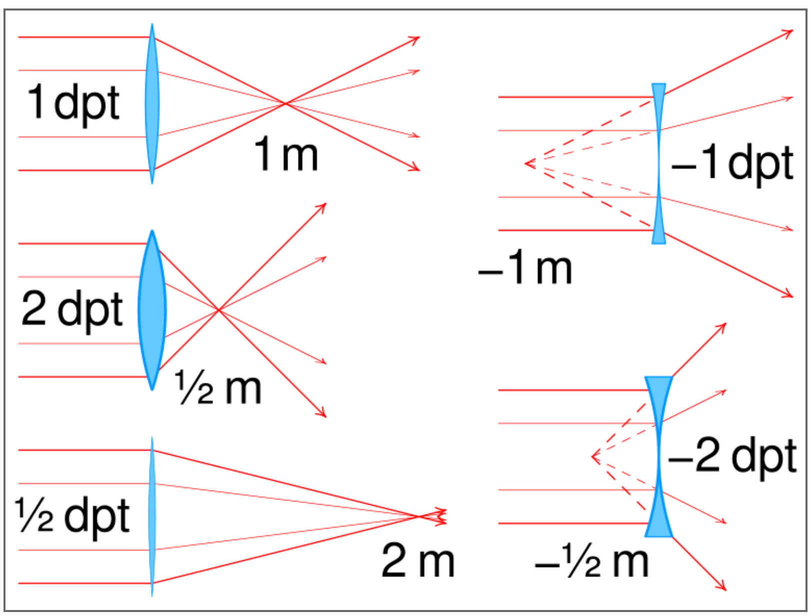
Vision errors & correction

`os01 - Haftoptik with Auge`

- **nearsightedness (Myopia)**
    - image forms **in front of the retina**
    - corrected with **diverging (–)** lenses
- **farsightedness (Hyperopia)**
    - image forms **behind the retina**
    - corrected with **converging (+)** lenses



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Introduction to aberration

`os10 - Linsenfehler`

- **abberration** = image imperfections caused by deviations from ideal lens behavior
- arises when the **small-angle (paraxial) approximation** breaks down
- real lenses:
    - have **non-negligible thickness**
    - exhibit **material and surface imperfections**
- results in **blurring**, **distortion**, or **color fringing**

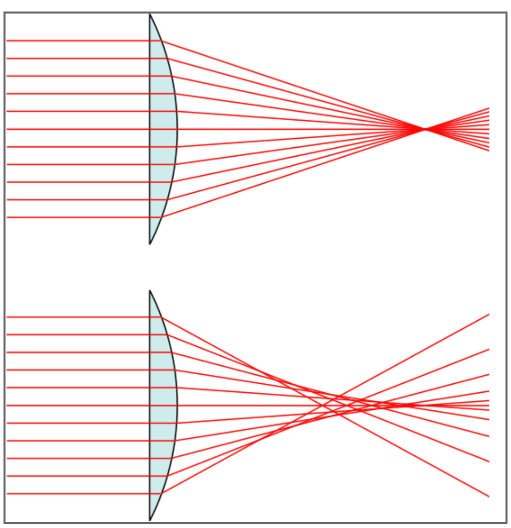
Types of aberrations

- **monochromatic aberrations** (single wavelength):
  - caused by geometry, not color
  - include:
  - **spherical aberration**: rays far from axis focus incorrectly
  - **coma**: off-axis points appear comet-shaped
  - **astigmatism**: radial and tangential rays focus differently
  - **field curvature**: image forms on a curved surface
  - **distortion**: magnification varies with position (barrel/pincushion)
- **chromatic aberration**:
  - caused by **dispersion**: refractive index varies with wavelength
  - different colors focus at different points → results in **color fringing**,

especially at edges

Spherical aberration

- rays farther from the optical axis focus **closer** to the lens than paraxial rays
- leads to image blur or a **circle of least confusion** instead of a sharp point
- common in **spherical lenses and mirrors**
- reduced by:
    - using **aspherical surfaces**
    - limiting aperture size (e.g. with a stop)
    - combining lenses in a **compound system**



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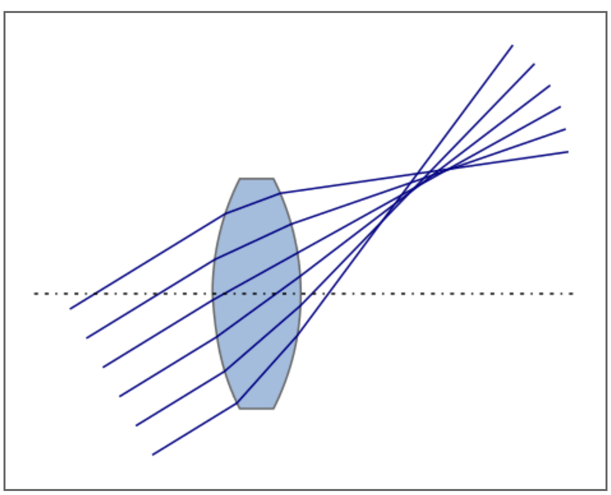
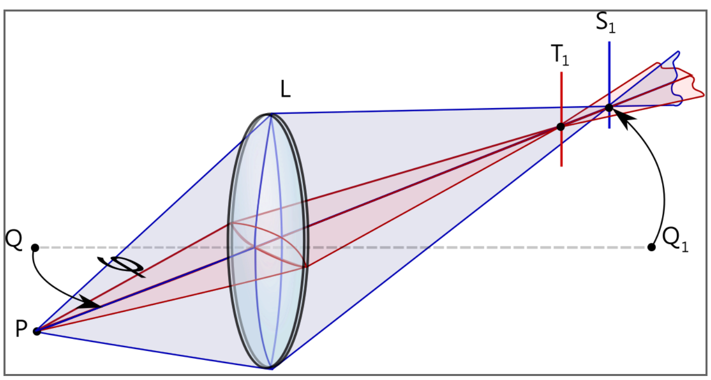
Coma

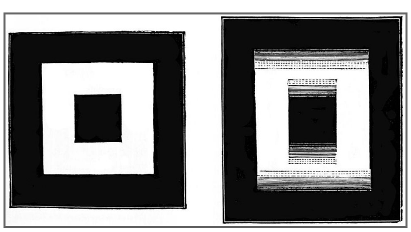
- off-axis points appear as **asymmetrical, comet-shaped blurs**
- caused by **off-axis rays** being refracted differently depending on their height in the lens
- image blur increases with distance from the optical axis
- reduced by:
    - using **aspherical lenses**
    - optimizing lens shape in **compound systems**



Astigmatism

- occurs when a lens or mirror has **different focal lengths in two perpendicular planes**
- a point object appears as two short, perpendicular line segments (radial and tangential foci)
- corrected using cylindrical lenses or specially designed compound lenses





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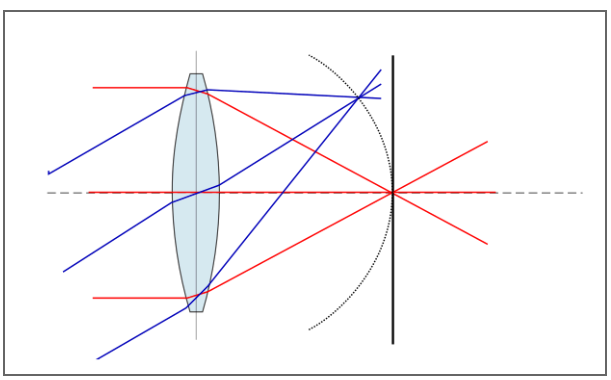
Curvature of field

- **cause**: geometry of lenses; even if the lens focuses rays properly, the image forms on a curved surface
- the image of a flat object forms on a **curved surface** instead of a flat image plane
- leads to a **sharp center** with blurred edges, or vice versa
- corrected using **field-flattening lenses** or optimized multi-element designs



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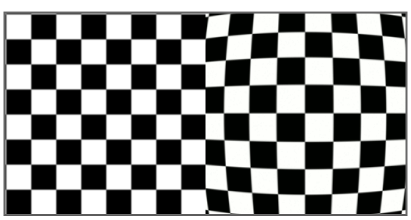
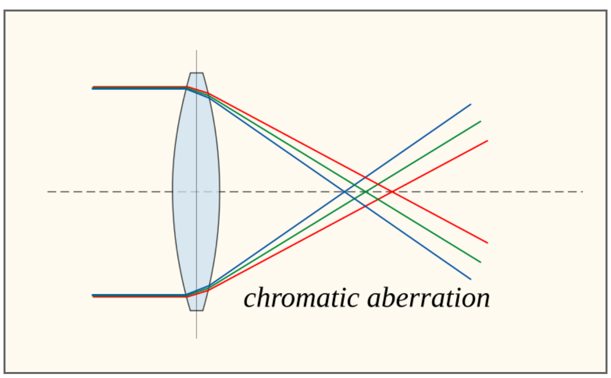
Distortion

- **cause**: different magnification at different parts of the image field, often due to lens design
- **shape of the image to differ from the shape of the object**
- barrel distortion: straight lines bow outward
- pincushion distortion: straight lines bow inward
- reduced using symmetric compound lenses or software correction



Chromatic aberration

- caused by **dispersion** — refractive index depends on wavelength
- different colors focus at different points along the optical axis
- leads to **color fringing**, especially at high-contrast edges
- reduced using **achromatic doublets**, low-dispersion glass, or digital correction



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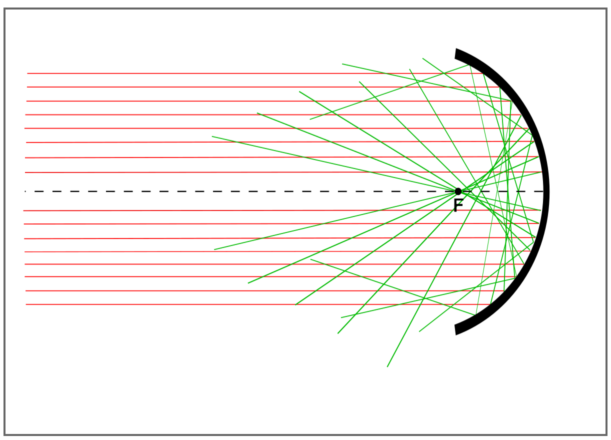
Aberrations in mirrors

- spherical aberration in mirrors occurs for the same geometric reasons as in lenses
- **parabolic mirrors can correct spherical aberration** for objects at infinity
- mirrors do **not exhibit chromatic aberration** because reflection is independent of wavelength



Summary of aberrations

Aberration	Cause	Effect	Correcti Method
Spherical Aberration	Spherical shape of lens/mirror causing varying focal points.	Blurred image, circle of least confusion.	Asphe lenses apertu stops, comp lenses
Coma	Off-axis rays focusing at different points.	Comet-shaped blur for off-axis points.	Asphe lenses comp lenses satisfy Abbe condit
Astigmatism	Different focal lengths in perpendicular planes for off-axis points.	Point objects imaged as lines.	Comb of lens cylind lenses

Aberration	**Cause**	**Effect**	**Correcti Method**
Curvature of Field	Focal points lie on a curved surface.	Image of a flat object is curved, edges may be blurred.	Field- flatter lenses specif design
Distortion	Non-uniform magnification across the field of view.	Straight lines appear curved (barrel or pincushion).	Symm comp lenses softwa correc
Chromatic Aberration	Dispersion (variation of refractive index with wavelength).	Colored fringes in the image.	Achro doubl apoch lenses