2.2. Electric Flux & Electric Potential

**Why do birds sitting
on high voltage power lines not get
electrocuted?**

`es03`

- need to understand:
    - concept of electric potential and voltage
    - relation to electric potential energy
    - deeper understanding of electric fields, i.e. electric flux & Gauss's law
- start by revisiting electric field experimentally `es05`

Electric flux for uniform electric field

`script simulation: rectangle in field`

- For a uniform electric field, the electric flux is defined as

$$\Phi_E = \vec{\mathbf{E}}\vec{\mathbf{A}}$$

  where $\vec{\mathbf{A}}$ is the vector perpendicular to the surface with magnitude A
- Using $\cos\theta$ as the angle between $\vec{\mathbf{E}}$ and $\vec{\mathbf{A}}$, the equation can be rewritten as

$$\Phi_E = EA\cos\theta = E_{\perp}A = EA_{\perp}$$

- The number of field lines N passing through an area perpendicular to the field is proportional to the electric flux

$$N \propto E_{\perp}A = \Phi_E$$

Electric flux for non-uniform electric field

- arbitrary surface can be decomposed into infinitesimal areas $d\vec{\mathbf{A}}$
- for each $d\vec{\mathbf{A}}$, associated field is uniform
- integral over closed surface gives the **total flux though closed surface**:

$$\Phi_E = \oint \vec{\mathbf{E}}\, d\vec{\mathbf{A}}$$

- **by conventions**
  - $d\vec{\mathbf{A}}$ points outwards from the surface of the enclosed volume
  - flux leaving the surface is positive
  - flux entering the surface is negative
- **consequences on net flux**:
  - if Φ_E is positive, there is a net flux out of the volume
  - if Φ_E is negative, there is a net flux into the volume
  - if $\Phi_E = 0$, there is no net flux

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script simulation circle in field
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Gauss’s law

- named after Karl Friedrich Gauss (1777-1855)
- **Gauss's law**: **Electric flux through a closed surface is equal to the net enclosed charge divided by the permittivity of free space**:

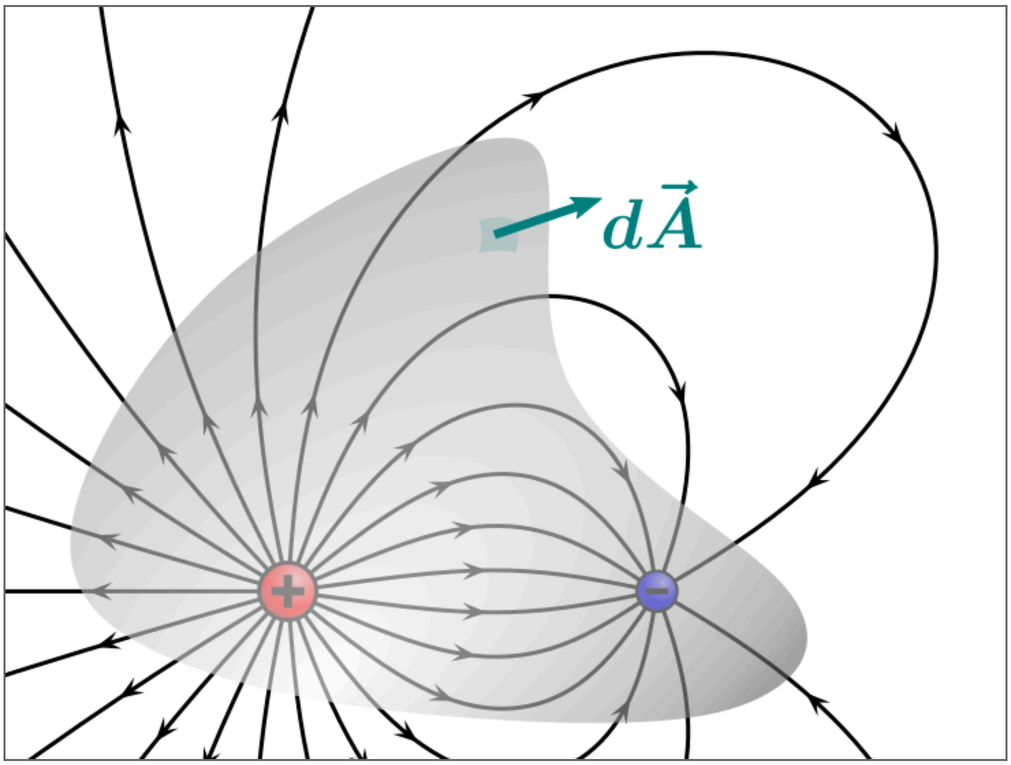
$$\Phi_E = \oint \vec{\mathbf{E}}\, d\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0}$$

- flux through surface is **independent** of:
    - distribution of the enclosed charge within the volume
    - charges outside the surface that may affect the position but not the number of field lines
- **principle of superposition applicable**:

$$\oint \vec{\mathbf{E}}\, d\vec{\mathbf{A}} = \oint (\sum_i \vec{\mathbf{E}}_i)\, d\vec{\mathbf{A}} = \sum_i \frac{Q_i}{\epsilon_0} = \frac{Q_{enc}}{\epsilon_0}$$

Examples for Gauss's law

$$\Phi_E = \oint \vec{\mathbf{E}}\, d\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0} > 0$$



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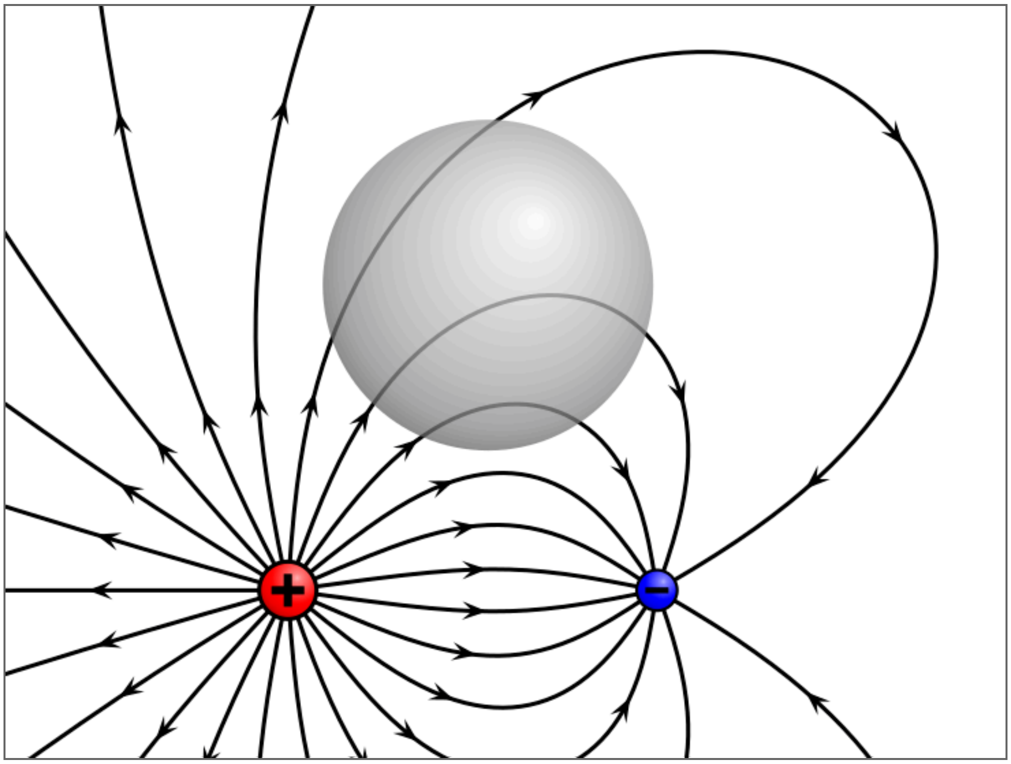
unedited

$$\Phi_E = \oint \vec{\mathbf{E}}\, d\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0} = 0$$



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Gauss's law: Relation to Coulomb's law

`thought experiment`

- consider a spherical surface (radius r) with a single enclosed charge Q_{enc} at the center
- the $\vec{\mathbf{E}}$ field is oriented radially with the same magnitude everywhere on the surface
- field lines penetrate the spherical surface perpendicular to $d\vec{\mathbf{A}}$
- surface area of the sphere is $\oint dA = 4\pi r^2$
- This leads to Coulomb's law in electric field form:

$$\frac{Q_{enc}}{\epsilon_0} = \oint \vec{\mathbf{E}} d\vec{\mathbf{A}} = E \oint dA = 4\pi r^2 E$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \quad \text{with} \quad Q_{enc} = Q$$

Complementary nature of Coulomb’s and Gauss’s law

	Coulomb's Law	Gauss's
Equation	$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \leftrightarrow F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}$	$\oint \vec{\mathbf{E}}\, d$
Focus	Field/force & (point) charges	Flux t surfac enclo charg
Applicability	Point charges, any distribution (complex)	Most for symm distri altho appli any c

	Coulomb's Law	Gauss's
Relevant Charge Sources	All charges present contribute	Only charg contr flux calcu

Electric potential energy

- concept of **conservation of energy** in electricity is analogous to mechanics
- **electric potential energy is defined for conservative forces**, e.g. the electrostatic force $\vec{\mathbf{F}} = \vec{\mathbf{E}}d$ (conservative force $\rightarrow$ path independence)
- **in uniform electric field**, the **work** to move a test charge q over a distance d is:

$$W = Fd = qEd$$

- change in **potential energy** ΔU from point A to B is:

$$\Delta U = U_B = -W$$

- note the similarity to mechanics, i.e. mgh, but as we have two types of charge:
    - $\rightarrow$ **the closer a charge is to a charge of the same polarity, the higher the**

electrostatic force, thus, the higher the electric potential energy

Voltage: The difference in electric potential

- **electric potential** V is defined as the electric potential energy per unit charge

$$V = \frac{U}{q}$$

- **voltage is the difference in potential** between points A and B:

$$V_{BA} = V_B - V_A \quad \text{in } [\text{J}/\text{C}] = [\text{V}]$$

- acknowledging Alessandro Volta, the unit is called **volt** [V]
- **reference point** (typically ground or infinity) is chosen where $V = 0$

Putting it all together

- **electric potential** is defined at a **single point** in space
- only potential **differences**, i.e. voltage, are **measurable**
- **reference point** with zero potential is **defined arbitrarily**
- combining the definitions gives

$$V_{BA} = V_B - V_A = \frac{U_B}{q} - \frac{U_A}{q} = \frac{\Delta U_{BA}}{q} = -\frac{W_{BA}}{q}$$

- **work** performed on a charge is given by

$$W_{BA} = Fd = -qV_{BA}$$

- $\rightarrow$ **voltage is a measure of the work a charge can perform**

The electron volt

- electron volt (eV) is the energy gained by an electron when moving through a potential difference of 1 V

$$1\,\mathrm{eV} \approx 1.60 \times 10^{-19}\,\mathrm{J}$$

- handy when dealing with small energies, i.e. charged particles

Relating electric potential & electric field

- analogously to mechanics, change in potential energy ΔU_{BA} from A to be B along the path $\vec{\mathbf{l}}$ is defined as:

$$\Delta U_{BA} = -\int_{A}^{B} \vec{\mathbf{F}} d\vec{\mathbf{l}}$$

- we know $V = \frac{U}{q}$ and $\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} \leftrightarrow \vec{\mathbf{F}} = \vec{\mathbf{E}}q$, therefore:

$$V_{BA} = -\frac{1}{q} q \int_{A}^{B} \vec{\mathbf{E}} d\vec{\mathbf{l}}$$

$$V_{BA} = -\int_{A}^{B} \vec{\mathbf{E}} d\vec{\mathbf{l}}$$

Relating electric potential & electric field (cont'd)

$$V_{BA} = -\int_A^B \vec{\mathbf{E}} d\vec{\mathbf{l}}$$

- in **uniform electric field**, the integral simplifies to

$$V_{BA} = -Ed \quad \text{with distance}\, d$$

- for an infinitesimal change, the relation is

$$dV = -\vec{\mathbf{E}}\, d\vec{\mathbf{l}}$$

- therefore, electric field is the gradient of the electric potential:

$$\vec{\mathbf{E}} = -\mathrm{grad} V = -\nabla V$$

Equipotential lines & surfaces

`script simulation: equipotential`

- equipotential lines (2D) and surfaces (3D) represent regions where the **electric potential is constant**
- **moving perpendicular** to the electric field ($\vec{\mathbf{E}} \perp d\vec{\mathbf{l}}$) does not change the potential

Unconventional ways to make light

`es31`