Cheat sheet

This is a rather incomplete and loose collection of equations and constants

Use with caution and check / think critical if equation is applicable

Electric charge & field

$$k \approx 9.0 \times 10^9 \mathrm{N\,m^2/C^2}$$

$$\epsilon_0 = \frac{1}{4\pi k} = 8.854 \times 10^{-12} \frac{\mathrm{C^2}}{\mathrm{N\,m^2}}$$

$$F = k\frac{Q_1 Q_2}{r^2} \quad \text{in [N]}$$

$$\vec{\mathbf{F}}_{12} = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r_{21}^2}\hat{\mathbf{r}}_{21}$$

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q}$$

$$E = \frac{F}{q} = \frac{ma}{q} \quad \text{in [N/C]}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}$$

$$\vec{\mathbf{E}} = \int d\vec{\mathbf{E}}$$

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \quad \text{e.g. for parallel, oppositely charged plates}$$

$$\text{Note on } [E] = \frac{\mathrm{N}}{\mathrm{C}} = \frac{\mathrm{N\,m}}{\mathrm{C\,m}} = \frac{\mathrm{J}}{\mathrm{C\,m}} = \frac{\mathrm{V}}{\mathrm{m}} \quad \text{with } \mathrm{J} = \mathrm{Nm} \text{ \\& } \mathrm{V} = \mathrm{J/C}$$

Electric flux & electric potential

$$\Phi_E = \vec{\mathbf{E}}\vec{\mathbf{A}} = EA\cos\theta = E_\perp A = EA_\perp$$

$$\Phi_E = \oint \vec{\mathbf{E}} d\vec{\mathbf{A}} = \frac{Q_{enc}}{\epsilon_0}$$

$$W = Fd = qEd = -\Delta U \quad \text{for a homogenous E field}$$

$$V_{BA} = V_B - V_A = \frac{U_B - U_A}{q} = \frac{\Delta U_{BA}}{q} = -\frac{W_{BA}}{q} \quad \text{in [J/C] =[V]}$$

$$\Delta U_{BA} = -\int_A^B \vec{\mathbf{F}} d\vec{\mathbf{l}}$$

$$V_{BA} = -\int_A^B \vec{\mathbf{E}} d\vec{\mathbf{l}}$$

$$V_{BA} = -Ed \quad \text{for uniform E-field}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} \quad \text{for a point charge with } V = 0 \text{ at } r = \infty$$

Capacitance, batteries, & resistance

$$Q = CV$$

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C V^2 = \frac{1}{2} QV$$

$$\epsilon = K\epsilon_0$$

$$C = \frac{Q}{V} = K\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d} \quad \text{for a parallel-plate capacitor}$$

$$I = \frac{dQ}{dt}$$

$$V = IR$$

$$R = \rho \frac{l}{A}$$

$$P = VI = I^2 R = \frac{V^2}{R}$$

$$I = \int \vec{\mathbf{j}} d\vec{\mathbf{A}}$$

DC circuits & Kirchhoff' laws

$$R_{eq} = \sum R_i \quad \text{for resistances in series}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_i} \quad \text{for resistances in parallel}$$

$$\frac{1}{C_{eq}} = \sum \frac{1}{C_i} \quad \text{for capacities in series}$$

$$C_{eq} = \sum C_i \quad \text{for capacities in parallel}$$

$$\sum_i I_i = 0 \quad \leftrightarrow \quad \sum_j I_{in,j} = \sum_k I_{out,k} \quad \text{junction rule}$$

$$\sum_i V_i = 0 \quad \text{loop rule}$$

$$V_C(t) = V_0(1 - e^{-\frac{t}{RC}}) \quad \text{RC circuit; charging}$$

$$I(t) = \frac{V_0}{R} e^{-\frac{t}{RC}} \quad \text{RC circuit; charging}$$

$$V_C(t) = V_0 e^{-\frac{t}{RC}} \quad \text{RC circuit; discharging}$$

$$I(t) = I_0 e^{-\frac{t}{RC}} \quad \text{RC circuit; discharging}$$

Magnetism & magnetic field

$$F = IlB\sin\theta \quad \text{within an uniform magnetic field}$$

$$d\vec{\mathbf{F}} = I d\vec{\mathbf{l}} \times \vec{\mathbf{B}} \quad \text{in general}$$

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} \quad \text{single moving particle}$$

$$\frac{e}{m_e} = \frac{v}{Br} = \frac{E}{B^2 r} \quad \text{within uniform fields}$$

$$qvB = m\frac{v^2}{r} \quad \text{within uniform magnetic field}$$

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \quad \text{Lorentz Equation}$$

$$U_H = E_H d = v_d B d \quad \text{Hall voltage}$$

$$B = \frac{\mu_0}{2\pi}\frac{I}{r} \quad \text{magnetic field of a straight wire}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{enc} \quad \text{Ampère's law}$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2} \quad \text{Biot-Savart law}$$

Electromagnets, induction & inductance

$$B = \frac{\mu_0 N I}{l} \quad \text{for solenoids}$$

$$B = \frac{\mu_0 N I}{2\pi r} \quad \text{for toroids}$$

$$\Phi_B = B_\perp A = BA\cos\theta = \vec{\mathbf{B}}\vec{\mathbf{A}} \quad \text{for uniform magnetic field}$$

$$\Phi_B = \int \vec{\mathbf{B}} d\vec{\mathbf{A}}$$

$$V_{ind} = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

$$V_1 = -M\frac{dI_2}{dt}$$

$$V_{ind} = Blv \quad \text{for a conductor moving perp. in a uni. mag. field}$$

$$\oint \vec{\mathbf{E}} d\vec{\mathbf{l}} = -\frac{d\Phi_B}{dt} \quad \text{general form of Faraday's law}$$

$$U = \frac{1}{2}LI^2$$

Alternating current (AC)

$$V(t) = V_0\sin(2\pi f t) = V_0\sin(\omega t)$$

$$I(t) = \frac{V(t)}{R} = \frac{V_0}{R}\sin(2\pi f t) = I_0\sin(\omega t)$$

$$I_{rms} = \sqrt{\bar{I}^2} = \sqrt{\frac{1}{2}I_0^2} = \frac{I_0}{\sqrt{2}}$$

$$V_{rms} = \sqrt{\bar{V}^2} = \sqrt{\frac{1}{2}V_0^2} = \frac{V_0}{\sqrt{2}}$$

$$\bar{P} = \frac{1}{2}I_0^2 R = \frac{1}{2}\frac{V_0^2}{R}$$

$$\bar{P} = I_{RMS}V_{RMS} = I_{RMS}^2 R = \frac{V_{RMS}^2}{R}$$

AC circuits & Electromagnetic oscillations

LR-circuit

$$\tau = \frac{L}{R}$$

$$I(t) = \frac{V_0}{R}(1 - e^{-\frac{t}{\tau}}) \quad \text{switching DC on}$$

$$I(t) = I_0 e^{-\frac{t}{\tau}} \quad \text{switching DC off}$$

LC-circuit

$$I(t) = -\frac{dQ}{dt} = \omega Q_0 \sin(\omega t + \phi) = I_0 \sin(\omega t + \phi)$$

$$\omega = \sqrt{\frac{1}{LC}}$$

$$U = U_B + U_E = \frac{Q_0^2}{2C}\sin^2(\omega t + \phi) + \frac{Q_0^2}{2C}\cos^2(\omega t + \phi) = \frac{Q_0^2}{2C}$$

R, L & C in AC circuits

Component	Voltage-Current Relationship	Phase Difference	Reactance
Resistor (R)	$V = IR$	In phase (0°)	R
Inductor (L)	$V = L\frac{dI}{dt}$	Voltage leads current by 90°	$X_L = \omega L$
Capacitor (C)	$I = C\frac{dV}{dt}$	Current leads voltage by 90°	$X_C = \frac{1}{\omega C}$

$$Z = R + jX,$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

LRC in AC mode

$$V_0 = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2} = I_0\sqrt{R^2 + (X_L - X_C)^2} = I_0 Z$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan\phi = \frac{V_{L0} - V_{C0}}{V_{R0}} = \frac{I_0(X_L - X_C)}{I_0 R} = \frac{X_L - X_C}{R}$$

$$\cos\phi = fracV_{R0}V_0 = fracI_0RI_0Z = fracRZ$$

$$P = I_{rms}^2 R = I_{rms}^2 Z \cos\phi = V_{rms} I_{rms} \cos\phi$$

Maxwell's equations & Electromagnetic waves

Equation Name	Integral Form	Differential Form
Gauss's law for electricity	$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q}{\epsilon_0}$	$\vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$
Gauss's law for magnetism	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$	$\vec{\nabla} \cdot \vec{\mathbf{B}} = 0$
Faraday's law of induction	$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d\Phi_B}{dt}$	$\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$
Ampère's law with Maxwell's correction	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$	$\vec{\nabla} \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{j}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$

$$E = E_y = E_0 \sin(kx - \omega t)$$

$$B = B_z = B_0 \sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi f$$

$$\nu = f\lambda = \frac{\omega}{k}$$

$$\nu = \frac{E_0}{B_0} = \frac{\omega}{k}$$

$$\nu = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = c$$

$$c = \frac{E}{B}$$

$$u = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

$$u = \epsilon_0 E^2 = \epsilon_0 c^2 B^2 = \frac{B^2}{\mu_0} = \epsilon_0 c E B = \sqrt{\frac{\epsilon_0}{\mu_0}} E B$$

$$\overline{S} = \frac{1}{2}\epsilon_0 c E_0^2 = \frac{1}{2}\frac{c}{\mu_0} B_0^2 = \frac{E_0 B_0}{2\mu_0} = \frac{E_{rms} B_{rms}}{\mu_0}$$

$$P = \frac{\overline{S}}{c} \quad \text{radiation fully absorbed}$$

$$P = \frac{\overline{2S}}{c} \quad \text{radiation fully reflected}$$

$$c = \frac{E}{B} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 300.000.000 \text{ m/s} = 300 \times 10^6 \text{ m/s}$$

$$c = \lambda f$$

Geometrical optics: Refection & refraction

Mirror equation:

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$$

The angle of incidence and the angle of reflection are equal $\theta_i = \theta_r$.

Snell's law of refraction: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Lateral magnification (mirror): $m = \frac{h_i}{h_o} = -\frac{d_i}{d_0}$

Geometrical optics: Optical instruments

Lensmaker's equation: $\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2})$

Thin lens equation for convex/converging lenses: $\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$

Thin lens equation for concave/diverging lenses: $\frac{1}{d_0} - \frac{1}{d_i} = \frac{1}{f}$

Lateral magnification (lens): $m = \frac{h_i}{h_o} = -\frac{d_i}{d_0}$

Wave optics: Interference & polarization

Malus's law: $I = I_0 \cos^2 \theta$

Brewster's law: $\tan \theta_p = \frac{n_2}{n_1}$

Newtons rings:

- **Dark rings (destructive interference):** Occur when this condition is met: $2t + \frac{\lambda}{2} = \frac{\lambda}{2}(2m+1) = (m+\frac{1}{2})\lambda$ with $m = 0, 1, 2, ...$ as the order of the dark rings. Rearranging this, we get $2t = \lambda m$.
- **Bright rings (constructive interference):** Occur when this condition is met: $2t + \frac{\lambda}{2} = m\lambda$, with $m = 1, 2, ...$ as the order of the bright rings. Rearranging this, we get $2t = (m - \frac{1}{2})\lambda$.

Double slit:

- **Constructive interference**, resulting in a bright fringe on the screen, happens when the path difference is an integer multiple of the wavelength (λ) of the monochromatic light:

$$d \sin \theta = m\lambda, \quad m = 0, 1, 2, ...$$

- **Destructive interference**, resulting in a dark fringe on the screen, occurs when the path difference is a half-integer multiple of the wavelength:

$$d \sin \theta = (m + \frac{1}{2})\lambda, \quad m = 0, 1, 2, ...$$

Wave optics: Diffraction

Diffraction at single-slit:

- $\Delta = D \sin \theta \leftrightarrow D \sin \theta = m\lambda$
- **Central maximum:** Rays passing straight through the slit are in phase, creating a central bright region at an angle $\theta = 0$.
- **Minima:** Minima occur at angles θ where the path difference between rays from the top and bottom of the slit is an integer multiple of the wavelength λ:

$$D \sin \theta = m\lambda, \quad m = \pm 1, \pm 2, ...$$

- **Higher-order maxima:** Between the minima, weaker maxima appear approximately where the path difference is a half-integer multiple of the wavelength:

$$D \sin \theta \approx (m + \frac{1}{2})\lambda, \quad m \approx . \pm \frac{3}{2}, \pm \frac{5}{2}, ...$$

Intensity of diffraction patterns:

- **single-slit** $I_\theta = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2 = I_0 \left(\frac{\sin\left(\frac{\pi D \sin \theta}{\lambda}\right)}{\frac{\pi D \sin \theta}{\lambda}} \right)^2$

- **double-slit** $I_\theta = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2 \cos^2 \left(\frac{\delta}{2} \right)$