2.11. Wave optics: Interference & polarization

What happens when we shin light through a hole?

`ow05 - adjustable`

- wave-particle duality of light
- ray model breaks down if **object size and wavelength are on same order of magnitude**
- **agenda for today**:
    - **new model** to understand wave nature of light
    - revisit **polarization** & **interference** with application to light

Similarities of water waves & EM-waves

`sim - ray vs wave`

`ms17 - wave machine`

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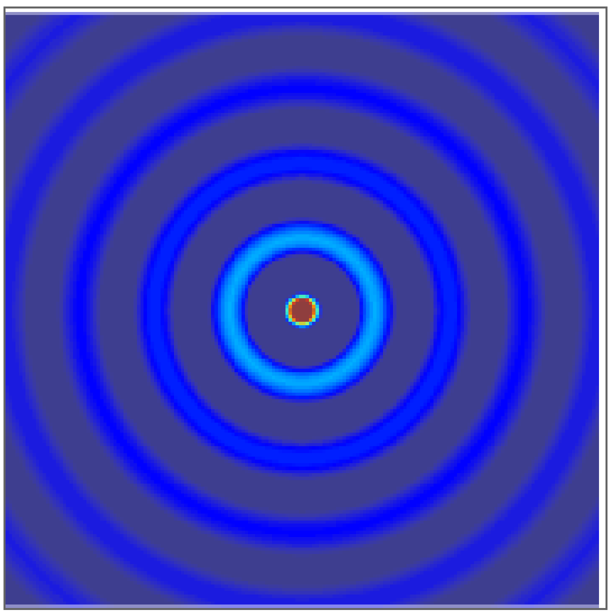
Huygens' principle: Framework

- **Christian Huygens** (1629-1695) proposed a principle to analyze wave propagation
- imagine **every point** of a wavefront as a **source of secondary spherical wavelets**



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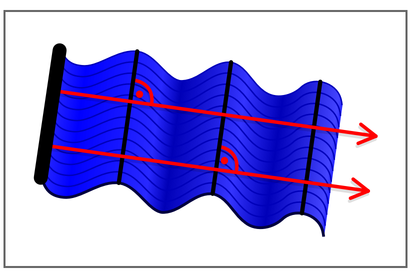
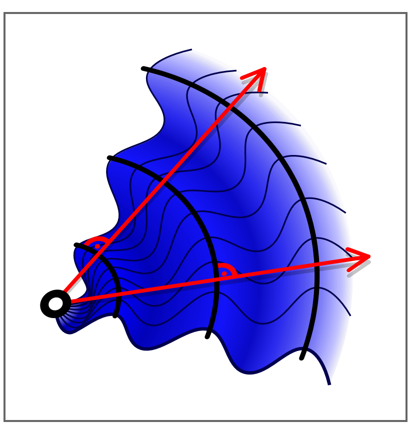
Huygens' principle: Framework (cont')

- **Christian Huygens** (1629-1695) proposed a principle to analyze wave propagation
- imagine **every point** of a wavefront as a **source of secondary spherical wavelets**
- these **wavelets propagate outwards** in all directions at the speed of the wave
- the **new wavefront** at a later time is the **tangent surface to all of these secondary wavelets**





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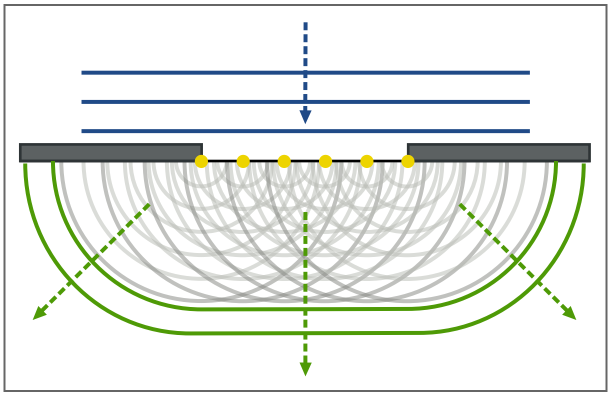
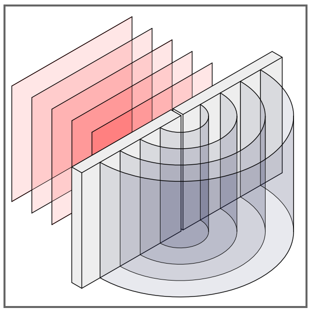
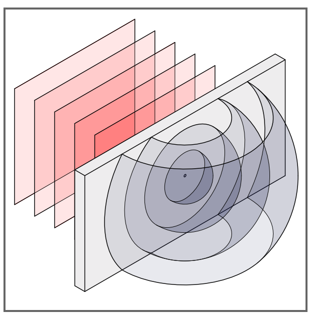
Understanding wavefronts, wavelets, & propagation

- **wavefront**: a surface connecting points of a wave that are in phase
    - example: ripples on a pond
    - for light: connects points with oscillating electric and magnetic fields in the same way
- think of a wavefront as a line (in 2D) or a surface (in 3D) of many tiny sources
- each tiny source emits a **wavelet**, spreading out like tiny ripples
- **superposition of wavelets generates the new wavefront**
    - individual wavelets interfere with each other
    - their combined effect determines the shape and position of the wavefront at the next instant (tangent to all wavelets)







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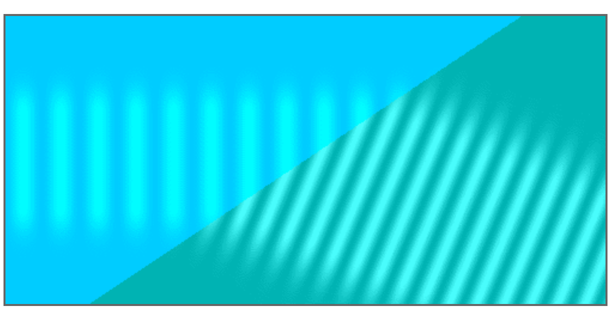
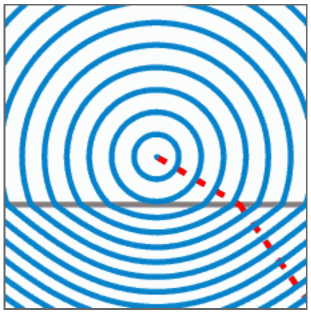
Refraction & wave speed

- light travels from medium 1 to medium 2
- mediums are isotropic (same wave speed v in all directions) but have different refractive indices n_1 and n_2
- distance traveled by a wave in time t: $r = vt$
- speed of light in a vacuum: c
- speed of light in a medium with refractive index n: $v = \frac{c}{n}$
- therefore, in our two mediums: $v_1 = \frac{c}{n_1}$ and $v_2 = \frac{c}{n_2}$
- if $n_1 \neq n_2$, distance traveled in the same time t will be different: $r_1 = v_1 t \neq r_2 = v_2 t$





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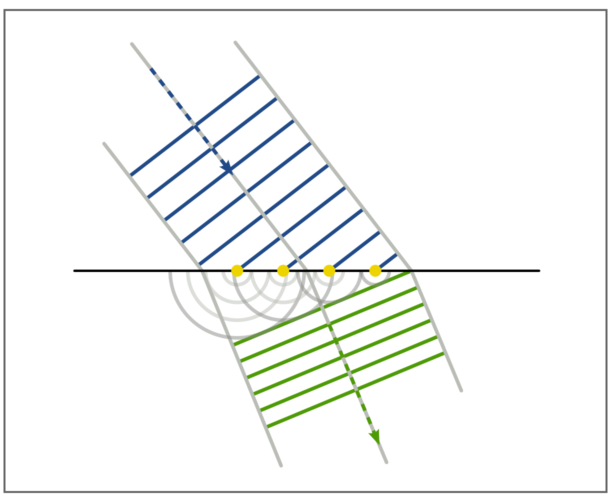
Deriving Snell's law with waves

- consider a wavefront approaching the interface at angle θ_1 $(\theta_1 \neq 0)$
- at point A, wavefront hits medium 2 first
- at point B, another part of wavefront hits after time t
- during time t, the part at A travels $v_2 t$, while the part at B travels $v_1 t$
- AB is the length of the wavefront segment about to cross the interface



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Deriving Snell's law with waves (cont')

- from the geometry:

  $$\sin\theta_1 = \frac{v_1 t}{AB} \quad \& \quad \sin\theta_2 = \frac{v_2 t}{AB}$$

- dividing the first by the second:

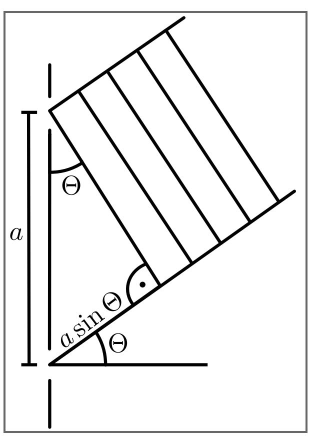
  $$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1 t/AB}{v_2 t/AB} = \frac{v_1}{v_2}$$

- substituting $v_1 = \frac{c}{n_1}$ and $v_2 = \frac{c}{n_2}$:

  $$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

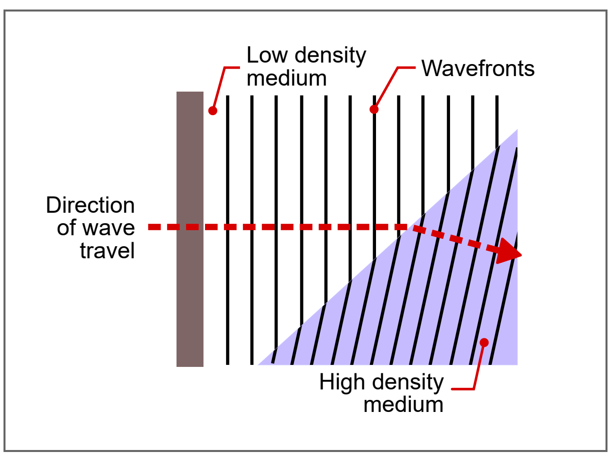
- rearranging gives Snell's Law:

  $$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



Frequency & wavelength changes

- **frequency of the wave does not change** during refraction (determined by the source)
- **wavelength** λ **does change** because speed v changes ($v = \lambda f \leftrightarrow \lambda = \frac{v}{f}$)
- ratio of wavelengths in the two mediums: $\frac{\lambda_1}{\lambda_2} = \frac{v_1/f}{v_2/f} = \frac{v_1}{v_2}$
- since $\frac{v_1}{v_2} = \frac{n_2}{n_1}$, we have: $\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$
- **wavelength is shorter in a medium with a higher refractive index**



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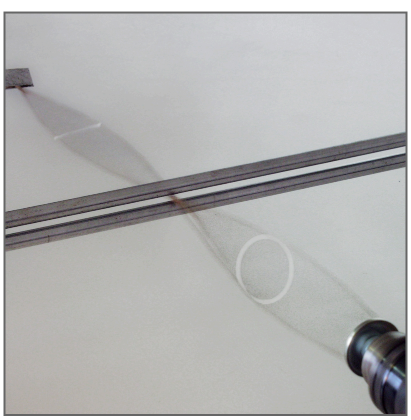
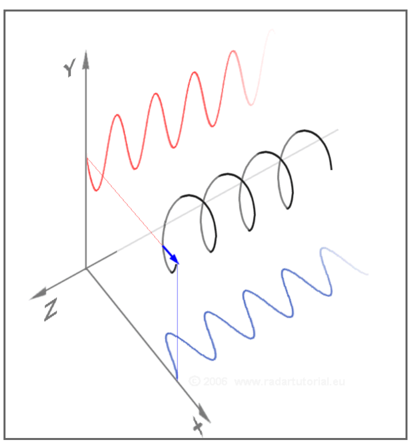
Polarization: Transverse nature of light

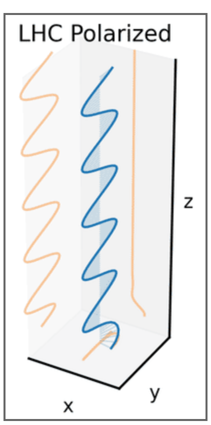
- electromagnetic (EM) waves are transversal waves and can be polarized
- **unpolarized light**: electric field oscillates randomly in all directions perpendicular to propagation
- **linearly polarized** (plane-polarized waves): oscillate in a single plane
    - imagine a rope swung horizontally through a vertical slit: only horizontal oscillations pass



Polarization: Transverse nature of light (cont')

- electromagnetic (EM) waves are transversal waves and can be polarized
- **unpolarized light**: electric field oscillates randomly in all directions perpendicular to propagation
- **linearly polarized** (plane-polarized waves): oscillate in a single plane
- **circular polarized**: oscillation vector rotates

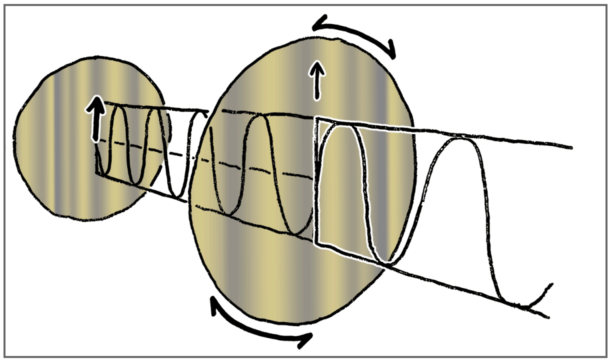




Polarization by absorption (polaroids)

`os27 - cross-polarization`

- polaroid sheets contain **long molecules aligned parallel to each other** (act like parallel slits)
- component of **electric field parallel to molecules causes electron movement** and energy transfer (absorbed)
- light oscillating **perpendicular to molecules is transmitted**
- **transmission axis** (polarization axis): direction of the transmitted electric field (perpendicular to molecules)
- a polaroid acts as a **polarizer**, creating linear/plane-polarized light from unpolarized light



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Polarization by absorption: Malus's law

- intensity of polarized light transmitted through a polaroid:

$$I = I_0 \cos^2 \theta$$

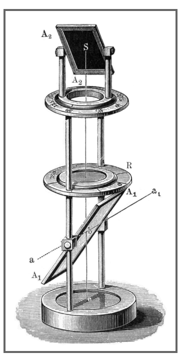
  - I_0: incident intensity
  - θ: angle between incident polarization and transmission axis
- for unpolarized light, the transmitted intensity is halved:

$$I = \frac{1}{2} I_0$$

Polarization by reflection

`ow02 - Nörrenberg`

- unpolarized light can be **polarized upon reflection** from a non-metallic surface at non-perpendicular angles
- reflected light is preferentially **polarized parallel to the reflecting surface**
- **polaroid sunglasses** reduce glare from horizontal surfaces



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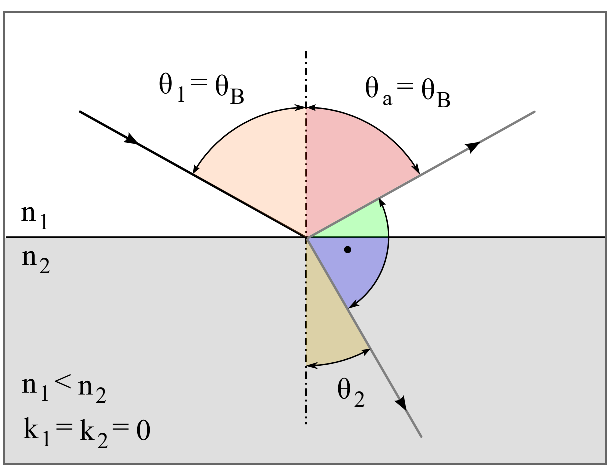
Polarization by reflection: Brewster's angle

- degree of polarization in reflected beam varies with the angle of incidence
- 100% polarization at the **polarizing angle** or **Brewster's angle** (θ_B)
- Brewster's Law:

$$\tan\theta_B = \frac{n_2}{n_1}$$

- at Brewster's angle, reflected and refracted rays are perpendicular ($\theta_B + \theta_2 = 90°$)



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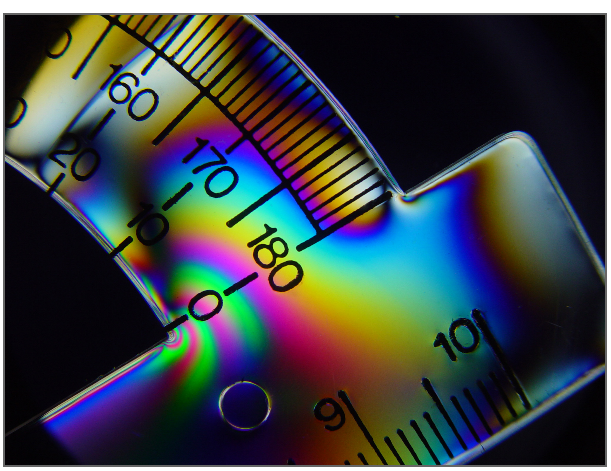
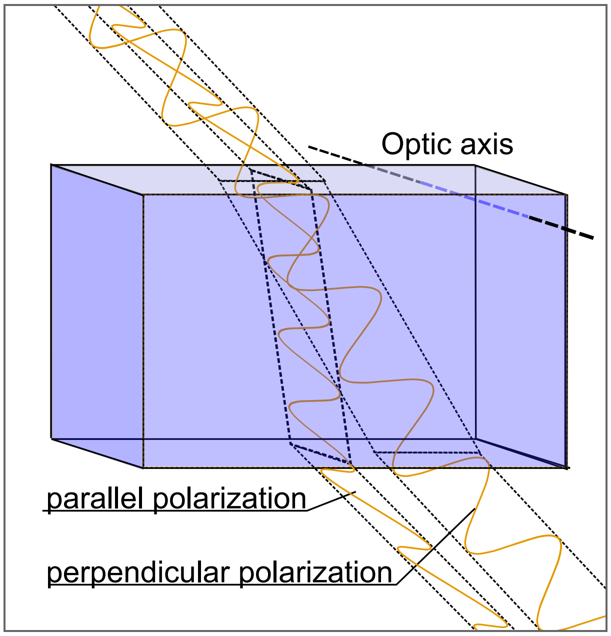
Birefringence & stress patterns

`os27 - polarization and mechanical stress`

- **birefringence**: optical property where material has different refractive indices for different polarization directions
- occurs in **anisotropic crystals** or **isotropic materials under stress**
- incoming light splits into two beams
- the two rays experience different phase velocities ⇒ **phase shift**
- plastic under mechanical stress becomes birefringent





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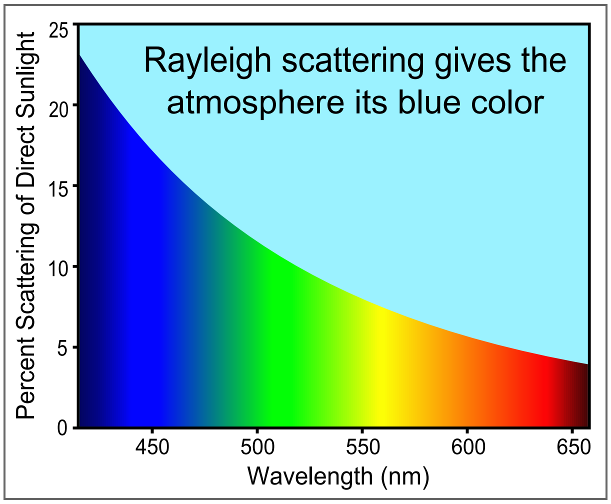
Rayleigh scattering in the atmosphere or why is the sky blue?

`ow34 - milky sunset`

- **light scattered by atmospheric molecules** e.g. nitrogen, oxygen
- sunlight causes molecules' electrons to oscillate, re-emitting light in all directions
- amount of scattering is **inversely proportional to** the fourth power of **wavelength** ($1/\lambda^4$)
- **shorter wavelengths** (blue, violet) **scatter much more** effectively than longer wavelengths (red, orange)
- **blue light** is preferentially **scattered in all directions**, making the sky appear blue
- **at sunset**, sunlight travels through a **much longer path** in the atmosphere
- light reaching our eyes directly from the setting sun (or reflected off clouds) is depleted in blue and appears reddish



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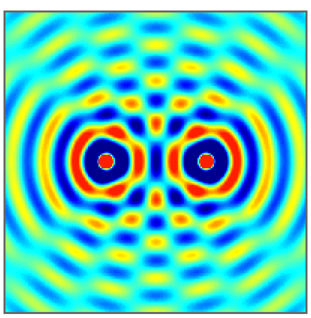
Interference & light

- **wave nature for light can result in interference patterns**
- let's start easy by using **monochromatic light**: single wavelength or narrow range of wavelengths (e.g., lasers, specific filters)



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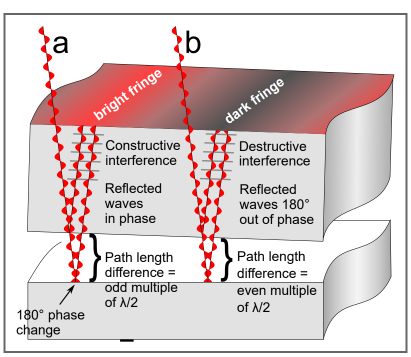
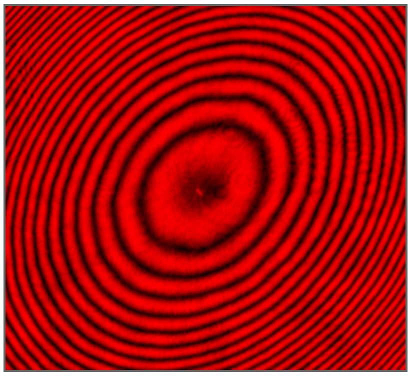
Newton’s rings with monochromatic light

`ow04 - Newton's rings in mono`

- plano-convex lens on a flat glass surface ⇒ thin, wedge-shaped air gap
- **Newton's rings**: interferences of monochromatic light creates concentric bright and dark circular rings
- **interference pattern** due to light reflected from two surfaces:
    - bottom surface of the lens (glass-air interface)
    - top surface of the flat glass plate (air-glass interface)
    - ⇒ **path difference**





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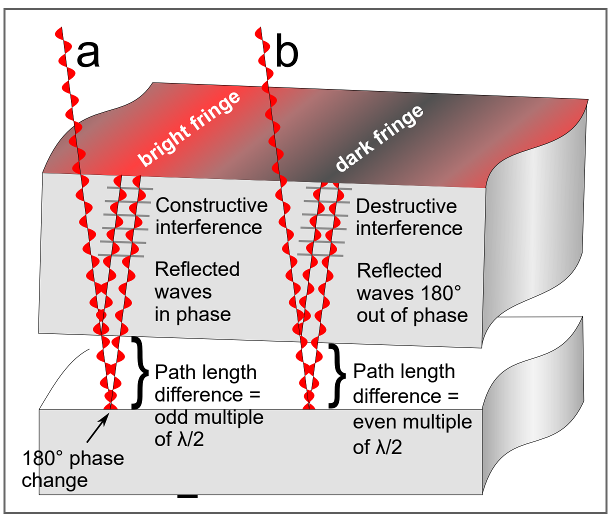
The math for Newton's rings

- second wave travels an extra distance of approximately twice the air gap thickness $(2t)$
- additionally 180° phase shift at air-glass interface (occurs only from low to high refractive index interface)
- $\Rightarrow \frac{\lambda}{2}$ phase shift
- **total path difference is**: $\Delta = 2t + \frac{\lambda}{2}$



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Conditions for bright and dark rings

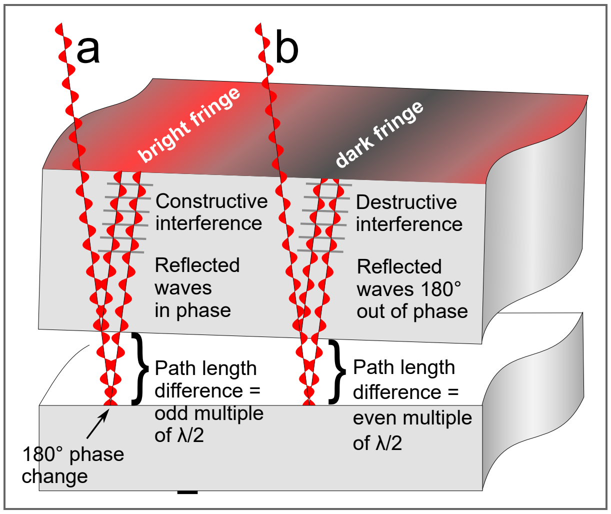
- path difference ($2t$) combined with $\lambda/2$ phase shift determines interference:
- **dark rings (destructive interference)**: total path difference is half-integer multiple of wavelength

$$2t + \frac{\lambda}{2} = m\lambda \implies 2t = (m - \frac{1}{2})\lambda, \quad m = 0, 1, 2, \ldots$$

  - $m = 0$ corresponds to the dark center
- **bright rings (constructive interference)**: total path difference is integer multiple of wavelength

$$2t + \frac{\lambda}{2} = (m + \frac{1}{2})\lambda \implies 2t = m\lambda, \quad m = 0, 1, 2, \ldots$$

  - $m = 1$ is the first bright ring



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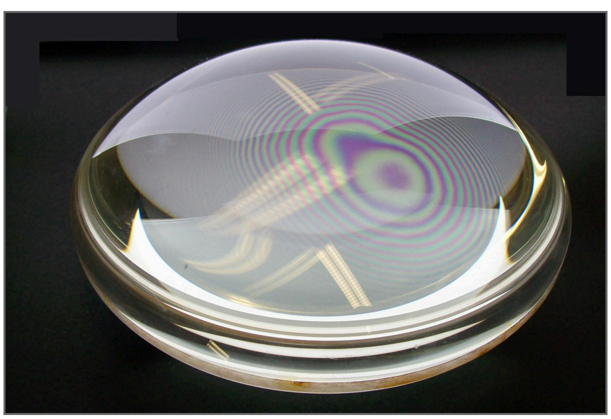
Newton's rings with white light

`ow04 - Newton's rings in color`

- using white light (spectrum of wavelengths) results in colored concentric rings
- constructive interference occurs for different wavelengths at different air gap thicknesses
- rings closest to the center are vibrant (rainbow sequence)
- further from the center, rings overlap more, color contrast decreases, eventually becoming nearly white or blurry



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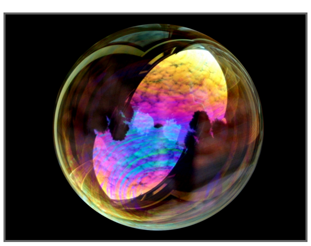
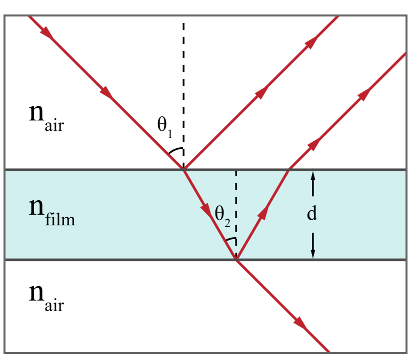
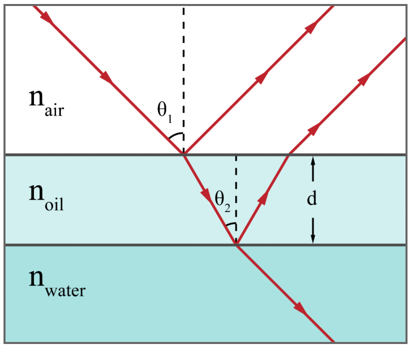
Interference in thin films

`ow04 - soap bubble`

- interference also occurs in other thin films, e.g. soap bubbles or oil films on water
- colors are due to interference of light reflected from the two interfaces
- for near-normal incidence, path difference is approximately $2nd$ (n: refractive index of film, d: thickness)
- phase shifts depend on refractive indices at each interface
- if reflection at both interfaces is lower to higher (or vice versa), phase shifts cancel







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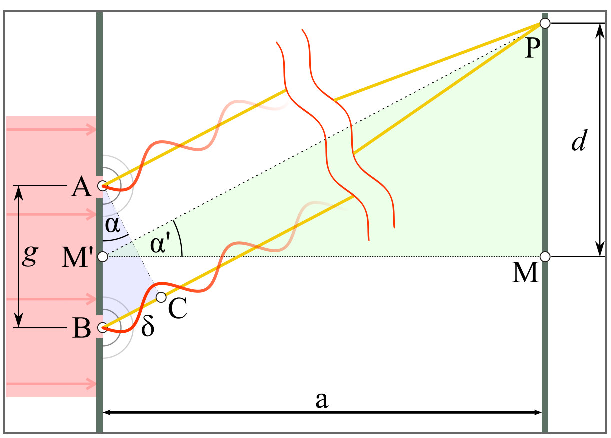
Interference: Young's double-slit experiment

`ow05 - double slit`

- Thomas Young (1773-1829) provided strong evidence for the wave nature of light
- illuminated a screen with two closely spaced, narrow slits S_1 and S_2 (separation d)
- if d is on the order of the wavelength, interference occurs
- **diffraction** also occurs (next chapter), conceptually similar to interference at this level



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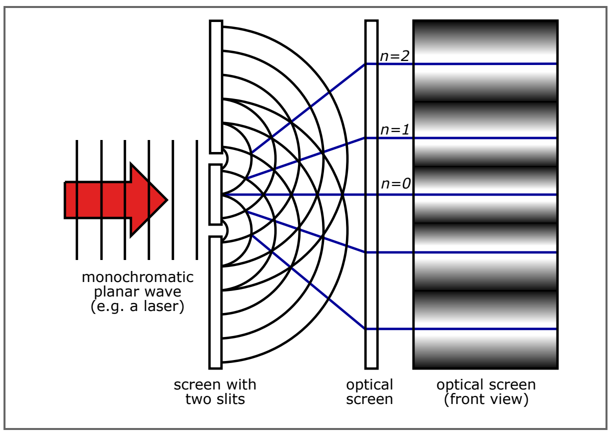
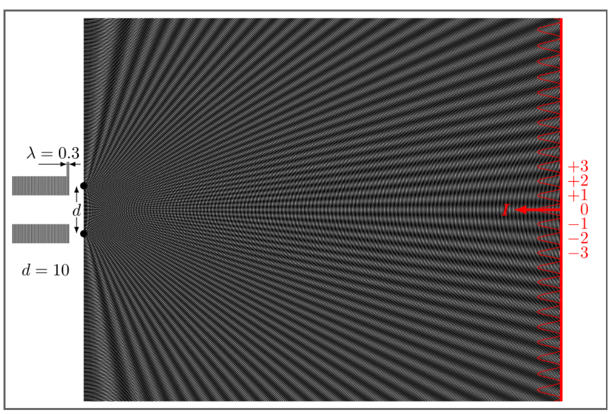
Double-Slit with monochromatic light

`sim - double slit`

- two slits act as **coherent sources** (same wavelength, constant phase relationship)
- waves diffract and overlap behind the screen





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Interference at the double-slit

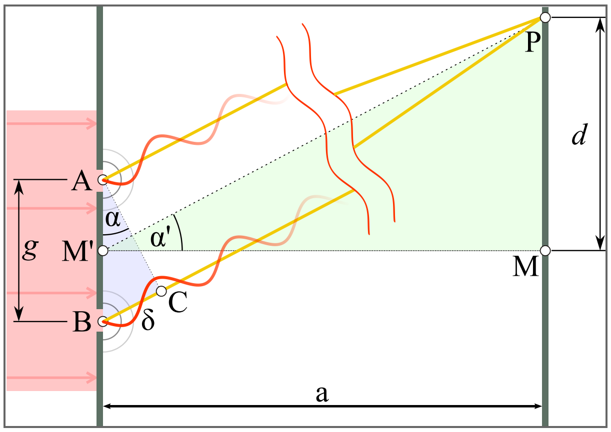
- **path difference** (Δ) between waves from S_1 and S_2 is approximately $\Delta = g \sin \alpha$ (g distance slits)
- **constructive interference** occurs when path difference is an integer multiple of the wavelength λ:

$$g \sin \alpha = m\lambda, \quad m = 0, 1, 2, \ldots$$

  - m is the order of the bright fringe
  - **central bright fringe** ($m = 0$) at $\alpha = 0$ (zero path difference, waves in phase)
  - first-order bright fringes ($m = 1$) at angles where path difference is one wavelength
  - distance between bright fringes is approximately constant for small angles ($\sin \alpha \approx \alpha$)
- **destructive interference** occurs when the path difference is a half-integer multiple of the

wavelength λ:

$$g \sin \alpha = (m + \frac{1}{2})\lambda, \quad m = 0, 1, 2, \ldots$$



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Take-home messages:

- transverse nature of light results in polarization
- superposition of light waves leads to interference patterns