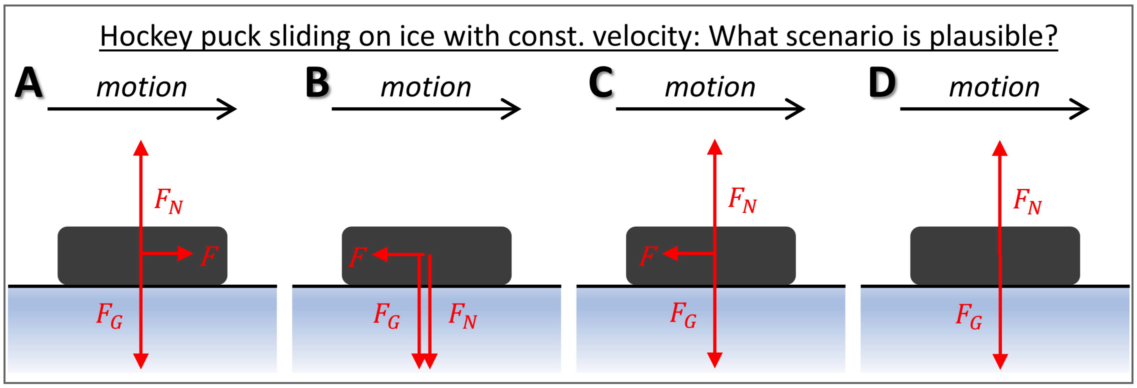
1.3. Dynamics: Newton’s Laws of Motion



- need to introduce force, Newton's laws and their applications
- solution at the end of the lecture

Force

- **Action** that can:
    - cause an object to change its velocity, its shape, or to resist other forces
    - cause changes of pressure in a fluid
- Acts via **contact** (friction, tension, normal) or **at a distance** (gravity, electromagnetism)
- Can be **measured** with a spring scale (stretch $\propto$ force)
- Unit: Newtons $[F] = \mathrm{N}$
- **Vector quantity** $\rightarrow$ combine by **vector addition**

Galileo's idea of inertia

`md09 - Luftkissenbahn, stays in motion`

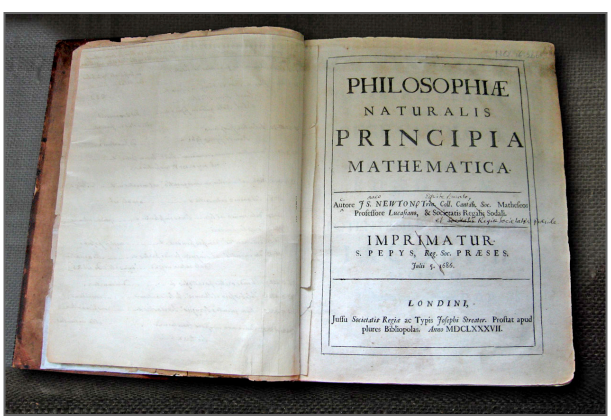
- **Aristotle**: motion requires continuous force; rest is the natural state
- **Galileo**:
    - Imagined motion on a **frictionless surface**
    - In the **ideal case (no friction)** $\rightarrow$ no force needed to maintain constant velocity
    - $\rightarrow$ Objects **retain their motion** unless acted upon by a force
    - $\rightarrow$ **Friction** is a real force that resists motion, not proof that motion needs effort

Newton’s first law of motion — the law of inertia

`mb05 – Trägheit verschiedener Körper`

- **Inertia:** tendency to resist changes in motion
- Motion remains **constant** (speed and direction) if **net force = 0**
- Building on Galileo’s work, Newton formulated his **first law of motion**:

> *Every object continues in its state of rest or of uniform motion in a straight line unless acted upon by a net external force.*



Inertial reference frames & when the first law holds

- Any frame moving with **constant velocity** relative to an inertial frame is itself inertial
- Newton’s first law **fails in accelerating frames**
- **Non-inertial frames**: accelerating or rotating; show **apparent (fictitious) forces**
    - Example: a smartphone on the dashboard **slides back** as a car accelerates forward
- **Earth** $\approx$ inertial for most lab situations

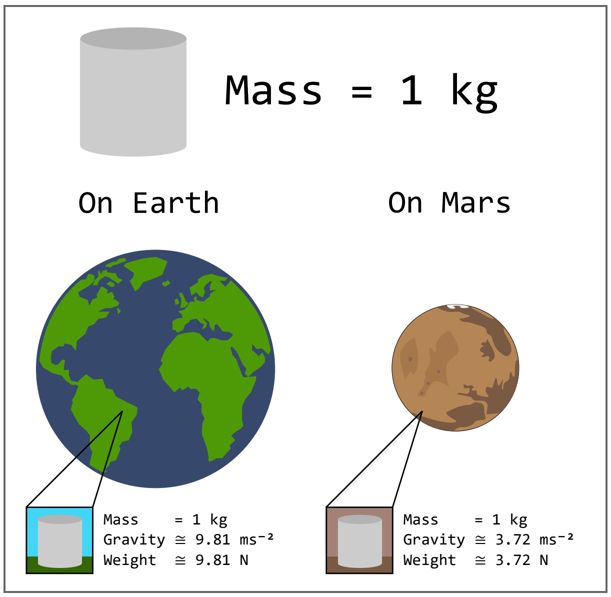
Mass, weight & inertia

`md23 - Änderung der Kraft bei Beschleunigung`

- **Mass**:
  - Measures **inertia**, the resistance to acceleration or change in motion
  - Larger mass $\rightarrow$ **more force** needed for same acceleration
  - **Intrinsic property** of matter; independent of location
  - **Unit:** kilogram (kg)
- **Weight**:
  - A **force** due to gravity $\rightarrow \vec{W} = m\vec{g}$
  - Direction: toward Earth's center
  - Magnitude: $|\vec{g}| \approx 9.81\,\mathrm{m/s}^2$

Mass, weight & inertia: Example:

$\rightarrow$ **Mass** stays constant; **weight** depends on local gravity



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Newton’s second law of motion

`md09 – Luftkissenbahn, a in Abhängigkeit von m`

- **First law**: motion stays constant if **net force = 0**
- **Second law**: describes motion when a **net force acts** $\rightarrow$ object **accelerates**

> *At any instant of time, the net force on a body is equal to the body's acceleration multiplied by its mass or, equivalently, the rate at which the body's momentum is changing with time*

$$\vec{F}_{\mathrm{net}} = m\vec{a}$$

- **Unit:** Newton (N) $\rightarrow 1\,\mathrm{N} = 1\,\frac{\mathrm{kg\,m}}{\mathrm{s}^2}$
- $\vec{a} \propto \vec{F}_{\mathrm{net}}, \ \ \vec{a} \propto \frac{1}{m}$
- Direction of $\vec{a}$ is the same as $\vec{F}_{\mathrm{net}}$

Newton’s third law of motion

`md11 - Wechselwirkungsgesetz`

- **Forces arise from interactions** between two objects
- In every interaction, **each object exerts a force on the other**
- Newton’s **third law of motion**, also know as **action–reaction principle**, states:

> *If two bodies exert forces on each other, these forces have the same magnitude but opposite directions.*

Newton's third law of motion (cont')

- **Symbolic form:**

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

  where $\vec{F}_{AB}$ acts on B by A, and $\vec{F}_{BA}$ acts on A by B
- The two forces are
  - **Equal in magnitude**
  - **Opposite in direction**
  - **Act on different bodies** → never cancel within one free-body diagram
- Key idea: every force is part of an **interaction pair**

Newton’s third law of motion: Ice skater & wall

- Skater **pushes backward** on the wall → force **on the wall**
- Wall **pushes forward** on the skater → equal and opposite **reaction force**
- With minimal ice friction, this reaction **accelerates** the skater backward
- Skater moves due to the **force exerted by the wall on her**, not her own push

Newton's third law of motion: Walking & propulsion

- **Walking:**
    - Foot pushes **backward** on ground $\rightarrow$ ground pushes **forward** on foot
    - The **forward reaction force** propels you ahead
    - Without **friction** (e.g., on ice), no effective push $\rightarrow$ no motion
- **Rocket propulsion:**
    - Engine pushes **gases backward** $\rightarrow$ gases push **rocket forward**
    - Rocket accelerates due to the **force on it** from escaping gases, not from pushing on air or ground
    - The forces are equal, but their effects differ:
        - gas has small mass, thus, gains large acceleration

- rocket has large mass, thus, gains smaller acceleration in the opposite direction

Newton's third law of motion: Clarifying action & reaction

- Always note **which object exerts** and **which object receives** the force
- Example: person P pulls sled S
  - P exerts forward force on S: $\vec{F}_{PS}$
  - S exerts backward force on P: $\vec{F}_{SP} = -\vec{F}_{PS}$
- Forces act on **different objects** $\rightarrow$ cannot cancel each other
- **Sled moves** if pull $>$ friction
- **Person moves** if ground's forward reaction $>$ sled's backward pull

Newton's third law of motion: Key points

- **Forces always occur in pairs.** A single, isolated force does not exist.
- **Action–reaction forces act on different bodies.** Only the forces acting *on a single body* enter its $\sum \vec{F} = m\vec{a}$ equation.
- **The origin of forces is mutual interaction.** Both objects experience forces of equal strength at the same instant.
- **Equal forces can produce different accelerations**, depending on each object's mass.

Solving problems with Newton's laws: Free-body diagrams (FBD)

- **FBD** isolates one object and shows all **external forces** acting on it
- **Internal forces** or forces the object exerts on others are **not included**

Steps:

1. **Select the object** and sketch all external forces as vectors
2. **Label each force** with source and direction (e.g., $\vec{F}_N$, $\vec{F}_g$, $\vec{F}_T$)
3. **Choose coordinates** that simplify the motion (e.g., along incline or direction of motion)
4. **Resolve forces** into components and apply Newton's second law:

$$\sum F_x = ma_x, \qquad \sum F_y = ma_y, \quad \text{etc.}$$

5. Keep **signs and units consistent**; opposite directions carry opposite signs.

Net force composition

`md08 – Kräfteparallelogramm`
`sim – net force`

- When multiple forces act on a point, the **net (resultant) force** is their **vector sum**
- To solve, use **parallelogram rule** or **tail-to-tip rule**

Normal force & inclined planes

- **Normal force** $\vec{N}$: surface's **perpendicular reaction** preventing interpenetration
- **Horizontal surface:** $N = mg$
- **Incline (angle θ):** $N = mg\cos\theta$
- Weight $\vec{W} = m\vec{g}$ resolves into
  - **Perpendicular:** $W_y = -mg\cos\theta$
  - **Parallel:** $W_x = -mg\sin\theta$
- The **downslope component** $mg\sin\theta$ drives motion (if frictionless)

Tension

- **Tension**: pulling force transmitted by a taut cord/rope/cable
- **Massless, inextensible cord** $\rightarrow$ **same tension** everywhere
- Cords **pull** but do **not push**; tension acts **along the cord** away from the object

Example (two boxes on frictionless table):
Boxes A and B (masses m_A, m_B) connected by a cord; external pull F_P on A.
Box A: $F_P - T = m_A a$
Box B: $T = m_B a$

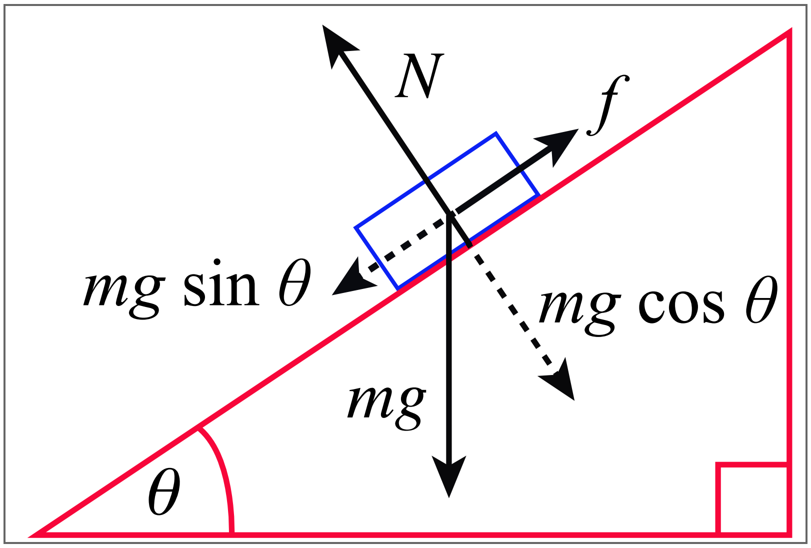
$$\Rightarrow\ a = \frac{F_P}{m_A + m_B}, \quad T = \frac{m_B}{m_A + m_B}\,F_P$$

Using Newton's laws with friction

- **Friction**: surface interaction that **opposes relative motion**
- Origin: microscopic **asperities and bonding** at contact points
- Acts **parallel** to surface, magnitude $\propto$ **normal force** N
- **Static friction**:
  - up to $F_{\mathrm{fr}} \leq \mu_s N$
  - Motion starts when required force $> \mu_s N$
- **Kinetic friction** (sliding):
  - constant magnitude: $F_{\mathrm{fr}} = \mu_k N$
- **Rolling resistance**: much smaller, from surface deformation
- Coefficients μ_s, μ_k depend on **materials** and **surface conditions**, nearly independent of speed or area

- Usually $\mu_s > \mu_k \rightarrow$ starting motion requires more force than maintaining it

Interim summary: Linear dynamics



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Dynamics of uniform circular motion

`md42 - Demonstration der Zentrifugalkraft mit einem Ball`

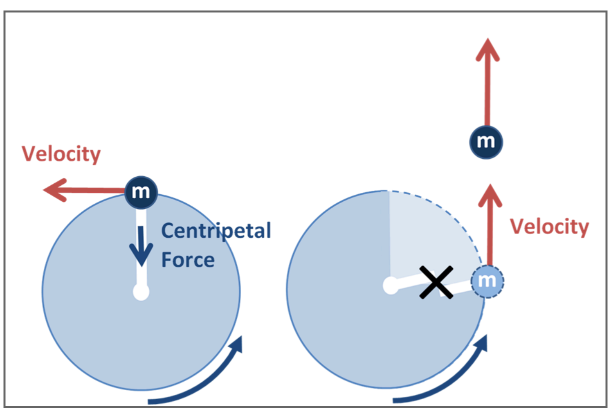
- Object moving in a circle at constant speed $\rightarrow$ continuously **accelerates toward the center**
- Requires a **net inward (centripetal) force**:

$$\sum F_R = m a_R = m\frac{v^2}{r} = m\omega^2 r$$

- **Centripetal force** acts **toward the center**, maintaining circular motion

Dynamics of uniform circular motion: No centrifugal force

- There is **no real outward force**
- $\rightarrow$ **centrifugal force** is seen only in **non-inertial frames**
- Examples:
  - Turning car: Your **inertia** gives the **illusion** of an outward pull
  - Ball on a string: When released, it moves **tangentially**, not radially outward



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Vertical circles

- In vertical motion, **tension** (or **normal force**) changes with position as **weight** adds or subtracts from the inward (centripetal) force

$$\sum \vec{F} = m\vec{a}_r$$

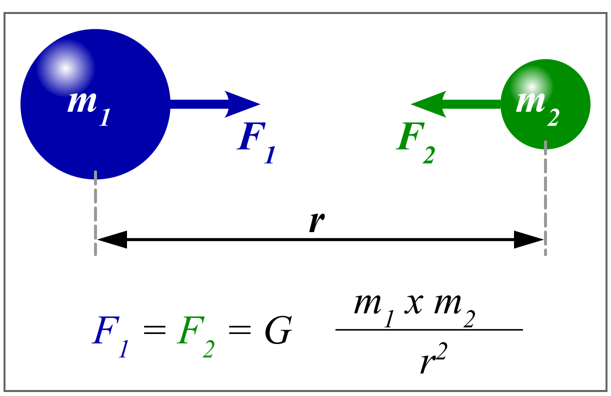
- **At the top:** $T_{\text{top}} + mg = m\dfrac{v_{\text{top}}^2}{r}$
- **At the bottom:** $T_{\text{bottom}} - mg = m\dfrac{v_{\text{bottom}}^2}{r}$
- **Minimum speed** for a taut string (tension $= 0$ at top): $v_{\min} = \sqrt{gr}$

Gravitational forces & planetary motion

- Newton's **law of universal gravitation**: every mass **attracts** every other mass along the line joining their centers

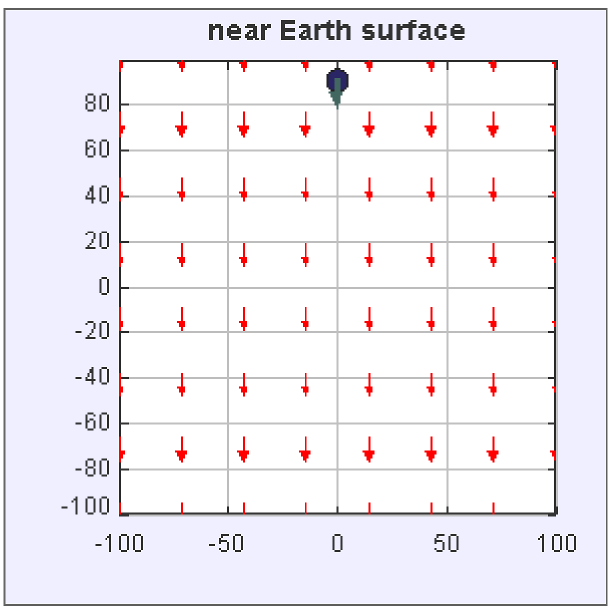
$$\vec{F}_G = G\frac{m_1 m_2}{\vec{r}_{12}^{\,2}}$$

- **Gravitational constant:** $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$
- Force is **mutual** (acts on both bodies equally and oppositely)
- Obeys an **inverse-square law** $\rightarrow$ decreases rapidly with distance



Gravity near Earth's surface

- Near Earth, gravitational force simplifies to **weight**: $F_G = mg$
- $g = G\frac{m_E}{r_E^2} \approx 9.81\,\mathrm{m/s}^2$ where m_E and r_E are Earth's mass and radius
- g is slightly **smaller** at high altitude or near the equator



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Satellites & orbits

- For a body of mass m orbiting a planet of mass M:

$$\frac{GMm}{r^2} = m\frac{v^2}{r}$$

- **Orbital speed** and **period:**

$$v = \sqrt{\frac{GM}{r}}, \qquad T = 2\pi\sqrt{\frac{r^3}{GM}}$$

- All bodies at the same orbital radius $\rightarrow$ same **speed** and **period**, independent of their mass
- **Gravity provides** the centripetal acceleration keeping objects in orbit
- **Weightlessness** arises because satellite and occupants **fall together** $\rightarrow$ no contact force $\rightarrow$ **continuous free fall**

Revisit first question: