

1.4. Work, Energy, & Power

md28 - Pendel



What is the min. height required for the object to do a looping?

⇒ Solution at the end

New concepts required:

- Introduce Hooke's law, work, & power
- Derive kinetic energy & potential energy from work
- Conservative vs. nonconservative force
- Fundamental concept of **conservation of energy**

Definition of Work

- In physics, work has a strict quantitative meaning (not “effort” as in everyday language)
- **Work describes energy transfer caused by a force acting on an object along the direction of displacement**



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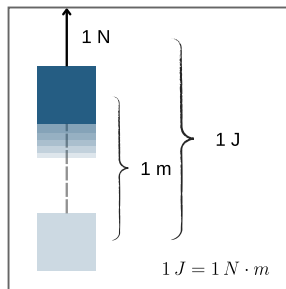
Work for Straight-Line Motion under a Constant Force

- Work is defined by the dot product of \vec{F} and the displacement \vec{d} :

$$W = \vec{F} \cdot \vec{d}$$

- Dot product \rightarrow work is a **scalar**
- Inputs: force vector + displacement vector
- Units:

$$1 \text{ J} = 1 \text{ N} \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$$



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Work for Straight-Line Motion under a Constant Force (cont')

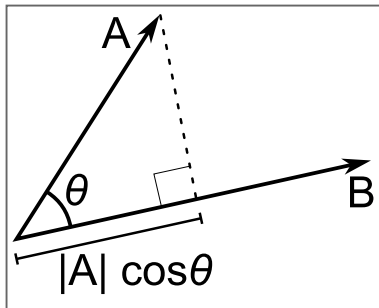
- With θ as the angle between \vec{F} & \vec{d}

$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$$

- Projection interpretation:

$$\vec{F} \cdot \vec{d} = F_{\parallel} d$$

→ Only component of force parallel to displacement does work.



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Work and constant Force: Examples

Example	Situation / Key Idea	Work Result
Pushing against a wall		
Lifting an object		
Lowering an object slowly		
Carrying an object		
Pulling a box at an angle		
Sliding a box with friction		
Circular motion		

Work and constant Force: Solutions

Example	Situation / Key Idea	Work Result
Pushing against a wall	Force applied, but displacement = 0	no work
Lifting an object up	Force \uparrow , displacement \uparrow	positive work
Lowering an object slowly	Force \uparrow , displacement \downarrow	negative work
Carrying an object	Force \uparrow , displacement \rightarrow (horizontal), angle = 90°	no work
Pulling a box at an angle	Only horizontal component F_{\parallel} contributes	positive work
Sliding a box with friction	Applied force \rightarrow positive; friction \rightarrow negative	net work = sum of works

Example	Situation / Key Idea	Work Result
Circular motion	Centripetal force \perp velocity	no work

Work and constant Force: Summary

- Work from constant force:

$$W = \vec{F} \vec{d} = Fd \cos \theta$$

- Sign from angle:
 - $0^\circ \leq \theta < 90^\circ \rightarrow$ **positive work**
 - $\theta = 90^\circ \rightarrow$ **zero work**
 - $90^\circ < \theta \leq 180^\circ \rightarrow$ **negative work**
- Only the **parallel component** of force transfers energy
- Net work: $W_{\text{net}} = \sum_i W_i = \sum_i \vec{F}_i \vec{d} = \vec{F}_{\text{net}} \vec{d}$

Work Done by a Variable Force

- Force may vary in magnitude and/or direction
- Split path into infinitesimal segments along the position vector \vec{r} :

$$dW = \vec{F}(\vec{r}) \cdot d\vec{r}$$

- Total work = sum of all infinitesimal contributions:

$$W = \int dW = \int \vec{F}(\vec{r}) \cdot d\vec{r}$$

- Dot product ensures only the **parallel component** contributes

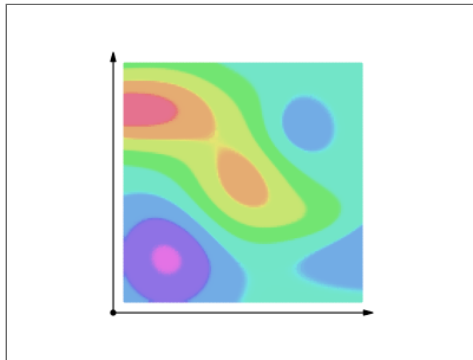
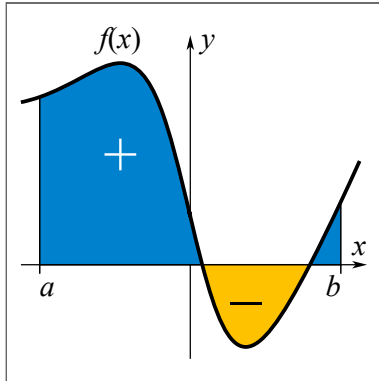
Work as a Line Integral

- General form between \vec{r}_1 and \vec{r}_2 : $W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r}) \cdot d\vec{r} \Rightarrow$ **signed area under the $F(\vec{r})$ curve**
- 1D case:

$$W = \int_{x_1}^{x_2} F(x) dx$$

- 3D case:

$$W = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z dz$$



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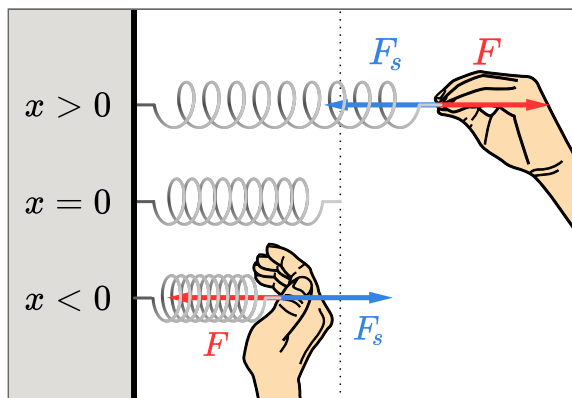
Hooke's Law: Spring Force

mi17 - Feder

- Ideal spring force with spring constant (stiffness) k & displacement from equilibrium x :

$$F_s = -kx$$

- Negative sign \rightarrow restoring force (points opposite to x)
- Force increases **linearly** with displacement
 \rightarrow straight line in the F - x diagram



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Work Done by a Spring

- Work done **by the spring** (from x_1 to x_2):

$$W_S = \int_{x_1}^{x_2} F_S \, dx = \int_{x_1}^{x_2} (-kx) \, dx = -\frac{1}{2}k(x_2^2 - x_1^2)$$

- Work done **on the spring** by external force to stretch from 0 to x :

$$F_{\text{ext}} = +kx$$

$$W_{\text{ext}} = \int_0^x (+kx) \, dx = \frac{1}{2}kx^2$$

- \rightarrow equal to **area under the F - x graph** (triangle)

Deriving Kinetic Energy from Work

md36 - Schuss

- Infinitesimal work:

$$dW = \vec{F}_{\text{net}} \cdot d\vec{l} = m\vec{a} \cdot d\vec{l}$$

- With $\vec{a} = \frac{d\vec{v}}{dt}$ and $d\vec{l} = \vec{v} dt$:

$$dW = m \frac{d\vec{v}}{dt} \cdot \vec{v} dt = m \vec{v} \cdot d\vec{v}$$

- Integrate from \vec{v}_1 to \vec{v}_2 :

$$W_{\text{net}} = \int_{\vec{v}_1}^{\vec{v}_2} m \vec{v} \cdot d\vec{v} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

- Define work–energy relation & kinetic energy:

$$W_{\text{net}} = \Delta K, \quad K = \frac{1}{2}mv^2$$

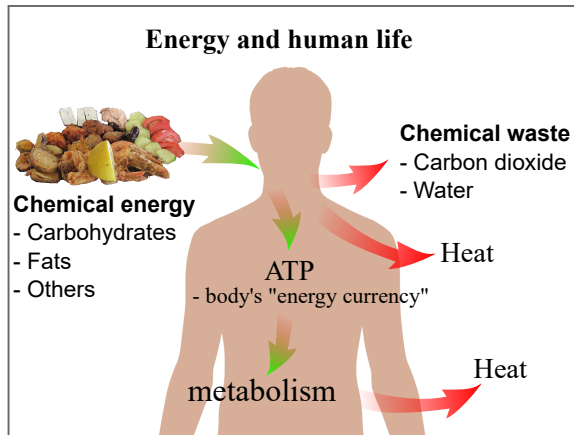
Energy

Energy is a scalar quantity that measures the ability of a system to perform work or to produce changes in the system or its environment.

→ Energy also measure in joule [J]

→ Several forms of energy, e.g. kinetic energy describing motion, or potential energy representing capacity to do work due to position/configuration





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Deriving Potential Energy from Work

- Total work from A to B:

$$W_{AB} = \int \vec{F}(\vec{r}) \cdot d\vec{l}$$

- For **conservative forces** (gravity, spring), **work depends only on start/end points**
- Define potential energy change:

$$\Delta U = U_B - U_A = -W_{AB} = - \int \vec{F}(\vec{r}) \cdot d\vec{l}$$

Gravitational Potential Energy

md25 - Anheben Kette

- Gravitational force (uniform field) with \hat{y} pointing upwards:

$$\vec{F}_G = -mg \hat{y}$$

- Work done by gravity on the object:

$$W_G = \int_{y_1}^{y_2} (-mg) dy = -mg(y_2 - y_1)$$

- Define potential energy with $U = 0$ at $y = 0$:

$$U_{\text{grav}} = -W_G = mgy$$

Elastic Potential Energy (Spring)

- Spring force obeying Hooke's law:

$$F_S = -kx$$

- Work done by the spring on the object:

$$W_S = \int_{x_1}^{x_2} (-kx) \, dx = -\frac{1}{2}k(x_2^2 - x_1^2)$$

- Define potential energy with $U(x = 0) = 0$:

$$U_{\text{elastic}} = -W_S = \frac{1}{2}kx^2$$

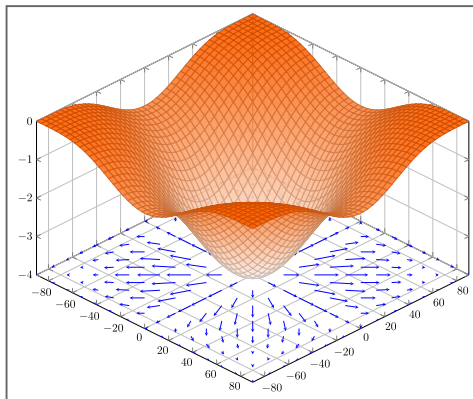
Reference Point & Force–Potential Relation

sim - reference height

- Zero of potential energy is **arbitrary**
→ only **changes** in U matter
- Force–potential connection (1D):

$$U(x) = - \int F(x) \, dx + C \quad \Leftrightarrow \quad F(x) = - \frac{dU}{dx}$$

- Generalization: $\vec{F} = -\nabla U$
- → Force points toward **lower** potential energy (work by force: $W = -\Delta U$)



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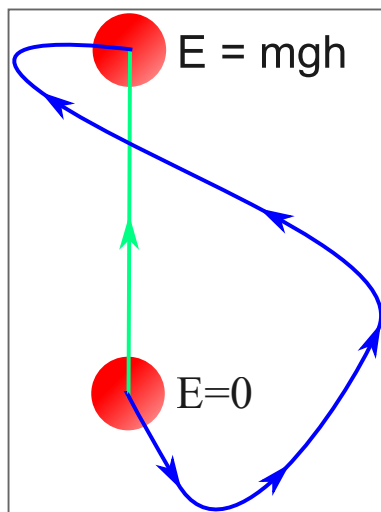
Conservative vs. Nonconservative Forces

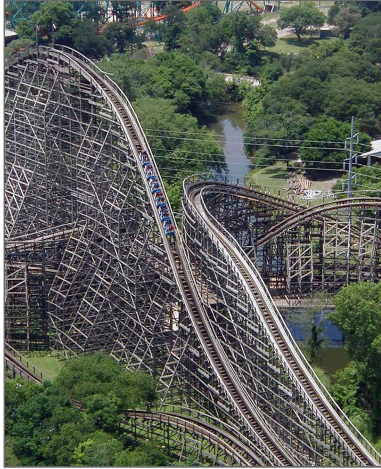
- **Conservative forces:**

- **Work independent of path**, e.g. gravity, springs, electrostatic force
- Closed-path work: $\oint \vec{F} \cdot d\vec{l} = 0$

- **Nonconservative forces:**

- **Work depends on path**, e.g. friction, air resistance, drag
- Mechanical energy not fully recoverable





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Conservation of Energy

md35 - Vase

md29 - Looping

Law of Conservation of Energy:

The total energy of an isolated system remains constant.

Energy may be transferred or transformed, but cannot be created or destroyed.

- Total energy includes all forms:
mechanical, thermal, chemical, electrical, nuclear, radiant
- Dissipative forces (friction, drag):
convert mechanical energy → thermal energy

- Energy balance (isolated system):

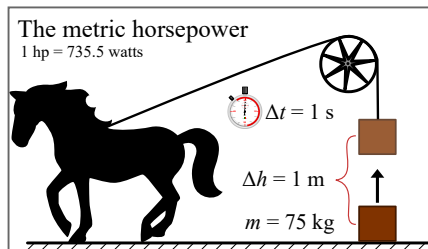
$$K + U + E_{\text{other}} = \text{const} \Leftrightarrow \Delta(K + U + E_{\text{other}}) = 0$$

Power & Efficiency

- **Power** is the **rate of doing work**:

$$P_{\text{avg}} = \frac{W}{\Delta t}, \quad \& \quad P = \frac{dW}{dt} = \frac{\vec{F} d\vec{l}}{dt} = \vec{F} \cdot \vec{v}$$

- **Units**: watt ($W = \text{J/s}$)
- **Efficiency** (useful output vs. input): $\eta = \frac{E_{\text{useful}}}{E_{\text{input}}}$
- Express as decimal or percentage;
ideal $\eta = 1$ not achievable due to losses



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Revisit first experiment

md28

What height required for the object to do a looping?



Bonus I

md30

Which track has the most energy?
Which track is the fastest?



Bonus II

mi17

At what height does the object reach its maximum velocity?

