# 2.10. Geometrical optics: Optical instruments

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## TEASER?

- ANSWER FRAGMENT

... dummy ...

# geometrical optics: optical instruments

- using reflection and refraction to study lenses and optical instruments
- covers simple devices like magnifying glasses to complex ones like telescopes and the human eye
- introduces key concepts such as image formation, aberrations, and optical limitations

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## primer on lenses & the lensmaker’s equation

- lenses have two surfaces which can be planar, convex or concave
- convex lenses are converging while concave lenses are diverging
- when a ray enters air-to-lens and then lens-to-air, it is refracted according to snell's law
- focal length is the same on both sides (assuming a thin lens)

## lensmaker's equation

$$frac1f = (n-1)$$
$$left($$
$$frac1R_1 +$$
$$frac1R_2$$
$$right)$$

- $n$ is the index of refraction
- $R_1$ and $R_2$ are the radii of curvature (positive for convex, negative for concave)

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## thin lenses model

- approximates the lens as having negligible thickness
- assumes paraxial rays (small angle approximations:
  $sin($
  $theta)$
  $approx$
  $theta$
  ,

$tan($

$theta)$

$approx$

$theta$

,

$cos($

$theta)$

$approx 1$

)

- uses a single principal plane instead of two separate refraction planes
- distinguishes between real and virtual focal points

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## image formation at thin lenses via ray tracing

- three principal rays determine the image location:
    - parallel ray: travels parallel to the principal axis
    - focal point ray: passes through the focal point
    - central (optical center) ray: goes through the lens center with little

deviation

- for convex lenses:
    - parallel ray refracts through the focal point on the opposite side
- for concave lenses:
    - refracted rays appear to diverge from the focal point on the same side as the object

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## thin lens equation & lateral magnification

- object distance $d_o$ and image distance $d_i$ are measured along the principal axis
- for convex lenses, the thin lens equation is:

$$frac1d_o + frac1d_i = frac1f$$

- for concave lenses, the equation becomes:

$$frac1d_o - frac1d_i = frac1f$$

- lateral magnification is defined as:

$$m = frach_i h_o = - fracd_i d_o$$

- sign conventions:
  - $h_o$ always positive
  - $h_i$ positive if upright, negative if inverted
  - $d_o$ positive for real objects
  - $d_i$ positive for real images and negative for virtual images
  - $f$ positive for convex and negative for concave lenses

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## combining lenses

- analyze the first lens using the thin lens equation:
  - calculate $d_{i1}$ and magnification $m_1 = - fracd_{i1} d_{o1}$
- treat the image from the first lens as the object for the second lens

- determine the second object distance $d_{o2}$ based on the lens separation and image location

- apply the thin lens equation for the second lens:
    - calculate $d_{i2}$ and magnification $m_2 = -$
      $fracd_{i2}d_{o2}$
- total magnification is:

$$m_{eff} = m_1$$
$$timesm_2$$

- careful use of sign conventions is crucial

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## pinhole cameras

- a lensless camera where a tiny pinhole focuses light onto a light-sensitive surface
- image formation is based on similar triangles:

$$frach_i d_i =$$
$$frach_o d_o$$

- optimal pinhole diameter is given by:

$$d_{pin}$$
$$approx$$

$sqrt2.44$

$lambdaf$

- $lambda$
  is the wavelength of light
- $f$ is the distance from the pinhole to the image plane
- advantages:
  - simple, inexpensive, infinite depth of field
- disadvantages:
  - low light sensitivity and sharpness limited by diffraction

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## the human eye

- functions similarly to a camera with additional biological sophistication
- key components:
  - cornea ( $n$ $approx1.376$ ): primary site of light bending

    - iris and pupil: control light entry
    - lens ( $n$ $approx 1.385$ $textto 1.405$ ): adjusts shape (accommodation) to focus light
    - retina and fovea: convert light into neural signals; fovea is the sharpest point
    - aqueous humor ( $n$ $approx 1.336$ ) and vitreous humor ( $n$ $approx 1.337$ ): transparent media filling the eye
- the eye has a fixed image distance
- corrective lenses are used to adjust the focus for nearsightedness, farsightedness, and astigmatism

corrective lenses

- nearsightedness (myopia):
    - image forms in front of the retina

    - corrected using diverging (negative) lenses
- farsightedness (hyperopia):
    - image forms behind the retina
    - corrected using converging (positive) lenses
- astigmatism:
    - irregular cornea or lens shape causes blurred images
    - corrected with cylindrical lenses

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## magnifying glass

- a simple magnifier that uses a single converging lens to enlarge the apparent size of objects
- the apparent size is based on the angular magnification
  $M =$
  $fractheta'$
- with a relaxed eye (image at infinity):
    - object is placed at the focal point $f$
    - angular magnification is:

$$M =$$
$$frac N f$$

- where
  $N = 25$

  ,
  $textcm$
  is the near point
- when the eye is focused at the near point:
  - the virtual image is formed at
    $d_i = -N$
  - using the thin lens equation:

$$frac1f =$$
$$frac1d_o -$$
$$frac1N$$

- the object distance becomes:

$$d_o =$$
$$fracNfN + f$$

- angular magnification is then:

$$M = 1 +$$
$$fracNf$$

- a larger subtended angle leads to a larger apparent size

## Telescopes

### introduction

- telescopes are optical instruments designed to magnify distant objects
- they allow observation of celestial bodies and faraway phenomena in greater detail

### components

- objective lens/mirror collects light from the object and forms a real image
- eyepiece lens magnifies the image formed by the objective

### angular magnification

- the total angular magnification $M$ of a telescope is defined as the ratio of the angle subtended by the image at the eye to the angle subtended by the object as seen by the unaided eye

- for an objective with focal length $f_o$ and an eyepiece with focal length $f_e$, the magnification is given by

$$M = \frac{\theta'}{\theta} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}$$

- with the small angle approximation $\theta' \approx h/f_e$ and $\theta \approx h/f_o$

image formation

- the image of the first lens (objective) becomes the object for the second lens (eyepiece)
- objective lens forms a real, inverted image near its focal point
- eyepiece lens acts as a magnifying glass producing a virtual, inverted final image

types of telescopes

REFRACTING TELESCOPES (KEPLERIAN)

- design uses two converging lenses
- objective lens has a long focal length to collect light and form a real, inverted, diminished image at its focal plane

- eyepiece lens has a short focal length and magnifies the real image
- advantages include simple design and good image quality for small apertures
- disadvantages include heavy, expensive objective lenses and chromatic aberration

### REFLECTING TELESCOPES (NEWTONIAN)

- design uses a concave mirror as the objective to collect and focus light
- light reflects off the primary mirror forming a real, inverted image at the focal point
- a secondary flat mirror reflects the image to the side for viewing through an eyepiece
- advantages include absence of chromatic aberration and easier manufacture of large mirrors
- disadvantages include diffraction effects from the secondary mirror and the need for periodic realignment

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## Aberration

## introduction

- aberration refers to imperfections in image formation due to the breakdown of the small angle approximation
- real lenses have thickness and imperfections that can lead to blurring or distortion

## types of aberrations

- monochromatic aberrations occur with a single wavelength and include spherical aberration, coma, astigmatism, curvature of field, and distortion
- chromatic aberration arises from dispersion where different wavelengths are refracted by different amounts

## spherical aberration

- occurs when rays far from the optical axis focus at different points than rays close to the axis
- causes a blurred image or a circle of least confusion

- correction methods include aspherical lenses, aperture stops, or compound lenses

coma

- causes off-axis points to be imaged as asymmetrical, comet-shaped blurs
- results from off-axis rays being refracted by different amounts
- can be minimized using aspherical or carefully designed compound lenses

astigmatism

- happens when a lens or mirror has different focal lengths in two perpendicular planes
- a point object is imaged as two short perpendicular line segments
- correction involves using a combination of lenses or cylindrical lenses

curvature of field

- the image of a flat object is formed on a curved surface instead of a flat plane
- results in a sharp center with blurred edges or vice versa

- field-flattening lenses or specific lens designs can correct this aberration

distortion

- causes the shape of the image to differ from the shape of the object
- barrel distortion makes straight lines bow outwards and pincushion distortion makes them bow inwards
- symmetric compound lenses or software correction can reduce distortion

chromatic aberration

- arises because the refractive index of a lens varies with the wavelength of light
- results in colored fringes especially at high contrast edges
- achromatic doublets or apochromatic lenses are used for correction

aberrations in mirrors

- spherical aberration in mirrors occurs for the same geometric reasons as in lenses

- parabolic mirrors can correct spherical aberration for objects at infinity
- mirrors do not exhibit chromatic aberration because reflection is independent of wavelength

summary of aberrations

- spherical aberration: caused by the spherical shape of the lens/mirror; corrected by aspherical lenses, aperture stops, or compound lenses
- coma: off-axis rays cause a comet-shaped blur; minimized with aspherical or compound lenses
- astigmatism: different focal lengths in perpendicular planes; corrected by lens combinations or cylindrical lenses
- curvature of field: focal points on a curved surface lead to uneven focus; corrected with field-flattening lenses
- distortion: non-uniform magnification leads to curved lines; minimized with symmetric compound lenses or digital correction

- chromatic aberration: dispersion causes colored fringes; corrected with achromatic doublets or apochromatic lenses