Dark energy as a kinematic effect

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We present a generalization of teleparallel gravity that is consistent with local spacetime kinematics regulated by the de Sitter group $SO(1,4)$. The mathematical structure of teleparallel gravity is shown to be given by a nonlinear Riemann–Cartan geometry without curvature, which inspires us to build the generalization on top of a de Sitter–Cartan geometry with a cosmological function. The cosmological function is given its own dynamics and naturally emerges nonminimally coupled to the gravitational field in a manner akin to teleparallel dark energy models or scalar-tensor theories in general relativity. New in the theory here presented, the cosmological function gives rise to a kinematic contribution in the deviation equation for the world lines of adjacent free-falling particles. While having its own dynamics, dark energy manifests itself in the local kinematics of spacetime.

I. INTRODUCTION

Physically equivalent to general relativity in its description of the gravitational interaction, teleparallel gravity is mathematically and conceptually rather different from Einstein's *opus magnum.* Although the precise implementation of general relativity differs from the one of teleparallel gravity, their geometric structures are related by switching between certain subclasses of Riemann–Cartan spacetimes. On the one hand, general relativity being the standard model for classical gravity, it is naturally well known that the fundamental field is the vierbein, which is accompanied by the Levi-Civita spin connection. The resulting spacetime is thus characterized by a Riemann–Cartan geometry without torsion. On the other hand, teleparallel gravity takes a different route in order to generalize the geometry of Minkowski space to incorporate the dynamics of the gravitational field, for it is torsion instead of curvature that is turned on by gravitating sources [1]. Since the curvature remains zero, the spin connection retains its role of representing inertial effects only. Therefore, the description can be extended to account for a breakdown of the weak equivalence principle [2].

The rationale behind Riemann–Cartan geometry underlying both theories is closely related to the assumption that kinematics is locally governed by the Poincaré group $ISO(1,3)$. Be that as it may, there is significant evidence that our universe momentarily undergoes accelerated expansion [3, 4], which indicates that the large-scale kinematics of spacetime is approximated better by the de Sitter group $SO(1,4)$ [5]. We shall take this evidence to heart and conjecture that local kinematics is regulated by the de Sitter group. Looked at from a mathematical standpoint, this amounts to have the Riemann–Cartan geometry replaced by a Cartan geometry modeled on de Sitter space [6]. The corresponding spacetime is everywhere approximated by de Sitter spaces, whose combined set of cosmological constants in general varies from event to event, hence resulting in the cosmological function [7].

In the present article we propose an extended theory of gravity as we generalize teleparallel gravity for such a de Sitter–Cartan geometry. Quite similar to the cosmological constant in teleparallel gravity or general relativity, we model the dark energy driving the accelerated expansion by a cosmological function Λ of dimension one over length squared. Fundamentally different, however, the cosmological function alters the kinematics governing physics around any point, such that spacetime is approximated locally by a de Sitter space of cosmological constant Λ. To be exact, a congruence of particles freely falling in an external gravitational field exhibits a relative acceleration, not only due to the nonhomogeneity of the gravitational field, but also because of the local kinematic properties of spacetime that are determined by the cosmological function.

The organization of the article is as follows. In Sec. II the basic tools for de Sitter–Cartan geometry that are used in subsequent sections are reviewed briefly. Afterwards, we show in Sec. III that the mathematical structure underlying teleparallel gravity is that of a nonlinear Riemann–Cartan geometry, which is consistent with the standard interpretation of teleparallel gravity as a gauge theory for the Poincaré translations. This is an important observation, since we leave the gauge picture for what it is upon generalizing the theory for a cosmological function in de Sitter teleparallel gravity. Section IV is devoted to the main results of this article, in which we discuss in sequence the fundamentals of de Sitter teleparallel gravity, the phenomenology of the kinematic effects, and the dynamics of the gravitational field and the cosmological function. We conclude in Sec. V.

II. DE SITTER–CARTAN GEOMETRY WITH A COSMOLOGICAL FUNCTION

The Cartan geometry modeled on $(\mathfrak{so}(1,4), SO(1,3))$ consists of a principal Lorentz bundle over spacetime together with a $\mathfrak{so}(1,4)$-valued one-form, which is called

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the Cartan connection [6, 8–10]. The Lie algebra $\mathfrak{so}(1,4)$ that generates the de Sitter group is subject to the commutation relations

$$\begin{aligned} -i[M_{ab}, M_{cd}] &= \eta_{ac}M_{bd} - \eta_{ad}M_{bc} + \eta_{bd}M_{ac} - \eta_{bc}M_{ad}, \\ -i[M_{ab}, P_c] &= \eta_{ac}P_b - \eta_{bc}P_a, \\ -i[P_a, P_b] &= -l^{-2}M_{ab}, \end{aligned} \tag{1}$$

where $\eta_{ab} = (+,-,-,-)$ and we parametrize elements of $\mathfrak{so}(1,4)$ by $\frac{i}{2}\lambda^{ab}M_{ab} + i\lambda^a P_a$, so that the M_{ab} span the Lorentz subalgebra, while the $P_a \equiv M_{a4}/l$ are a basis for the de Sitter transvections. The length scale l is related to the cosmological constant of the corresponding de Sitter space $SO(1,4)/SO(1,3)$, namely, [6]

$$\Lambda = \frac{3}{l^2}. \tag{2}$$

Because the Cartan connection is valued pointwise in copies of (1), the set of length scales $l(x)$ may form an arbitrary function of spacetime [7, 11]. Concomitantly, the cosmological constants (2) of the local de Sitter spaces constitute a nonconstant cosmological function Λ.

Corresponding to the reductive nature of the relations (1), the Cartan connection is decomposed in a spin connection $\omega^a{}_{b\mu}$ and vierbein $e^a{}_\mu$, from which it follows that the vierbein has dimension of length. Furthermore, the curvature and torsion of the geometry are given by [7]

$$R^a{}_{b\mu\nu} = B^a{}_{b\mu\nu} + \frac{1}{l^2}(e^a{}_\mu e_{b\nu} - e^a{}_\nu e_{b\mu}) \tag{3a}$$

and

$$T^a{}_{\mu\nu} = G^a{}_{\mu\nu} - \partial_\mu \ln l \, e^a{}_\nu + \partial_\nu \ln l \, e^a{}_\mu, \tag{3b}$$

where the two-forms

$$B^a{}_{b\mu\nu} = \partial_\mu\omega^a{}_{b\nu} - \partial_\nu\omega^a{}_{b\mu} + \omega^a{}_{c\mu}\omega^c{}_{b\nu} - \omega^a{}_{c\nu}\omega^c{}_{b\mu} \tag{4a}$$

and

$$G^a{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu + \omega^a{}_{b\mu}e^b{}_\nu - \omega^a{}_{b\nu}e^b{}_\mu \tag{4b}$$

are the exterior covariant derivatives of the spin connection, respectively, vierbein. When we solve (4b) for the spin connection, the Ricci theorem is recovered:

$$\omega^a{}_{b\mu} = \mathring{\omega}^a{}_{b\mu} + K^a{}_{b\mu}, \tag{5}$$

where $\mathring{\omega}^a{}_{b\mu} = \frac{1}{2}e^c{}_\mu(\Omega_{bc}{}^a + \Omega_b{}^a{}_c + \Omega_c{}^a{}_b)$ is the Levi-Civita spin connection, with $\Omega_{abc} = e_a{}^\mu e_b{}^\nu(\partial_\mu e_{c\nu} - \partial_\nu e_{c\mu})$ the coefficients of anholonomy, and where

$$K^a{}_{b\mu} = \frac{1}{2}(G^a{}_{\mu b} + G_\mu{}^a{}_b + G_b{}^a{}_\mu) \tag{6}$$

is the contortion of $\omega^a{}_{b\mu}$.

Subsequently, it is straightforward to define algebraic covariant differentiation by $D_\mu = \partial_\mu + \omega_\mu$, whereas $\nabla_\mu = \partial_\mu + \Gamma_\mu$ stands for the corresponding spacetime covariant derivative, i.e., $\Gamma^\rho{}_{\nu\mu} = e_a{}^\rho D_\mu e^a{}_\nu$. A metric structure is readily constructed as well by defining $g_{\mu\nu} = e^a{}_\mu e_{a\nu}$, a symmetric tensor that is covariantly constant.

To conclude we remark that the pointwise decomposition of the Cartan connection in a spin connection and vierbein is manifestly local Lorentz invariant. The decomposition, albeit arbitrary, is not unique, for the possibilities are in a bijective correspondence with sections of the fibre bundle $Q[dS]$ of de Sitter spaces, associated with a principal $SO(1,4)$ bundle Q over spacetime [12]. The hidden symmetries, which are generic local de Sitter transformations that do not belong to the Lorentz subgroup, are manifestly restored when the geometric objects are constructed such that they have an explicit dependence on the section chosen. This can be achieved through a nonlinear realization of the Cartan connection and it has been shown in [7] that such a nonlinear de Sitter–Cartan geometry with a cosmological function has a structure identical to the one outlined above. Therefore, we may assume the geometry of this section to be $SO(1,4)$ invariant.

III. THE CARTAN GEOMETRIC STRUCTURE OF TELEPARALLEL GRAVITY

In order to make manifest that teleparallel gravity is constructed over a nonlinear Riemann–Cartan geometry, we briefly review its fundamentals [1]. These fundamentals were originally devised to form a gauge theory for the Poincaré translations. By pinning down its Cartan geometric structure, it will be easy to understand how to allow for a different kind of kinematics in Sec. IV.

Because kinematics is governed by the Poincaré group, spacetime $\mathcal{M}$ is the base manifold of a principal $ISO(1,3)$ bundle Q, while the local Minkowski spaces are the fibres $\{M_x \,|\, x \in \mathcal{M}\}$ of the associated bundle $Q \times_{ISO(1,3)} M$. The $\mathfrak{iso}(1,3)$-valued connection $A_\mu dx^\mu$ determines how adjacent fibres compare under an infinitesimal displacement dx^μ.

In the absence of gravity the connection is flat, and spacetime can be identified with every local Minkowski space as follows. We adopt spacetime coordinates $x^\mu = \delta^\mu_a \xi^a$ for the Cartesian system ξ^a, while the section $\xi^a(x) = \delta^a_\mu x^\mu$ of $Q[M]$ defines the points of tangency between the local Minkowski spaces and spacetime. It follows that the vierbein is given trivially by $e^a{}_\mu = \delta^a_\mu$. After a general coordinate transformation $\delta^\mu_a \xi^a \mapsto x^\mu(\xi)$ the vierbein assumes the form $e^a{}_\mu = \partial_\mu \xi^a$. There yet remains the freedom to consider local $ISO(1,3)$ transformations on the tangent Minkowski spaces. Since elements of M_x transform as vectors under local Lorentz transformations $\xi^a \mapsto \Lambda^a{}_b \xi^b$, we provide the vierbein with a corresponding connection, i.e., $e^a{}_\mu = \partial_\mu \xi^a + A^a{}_{b\mu}\xi^b$. The connection is purely inertial, i.e., it takes the form $\Lambda^a{}_c \partial_\mu \Lambda_b{}^c$, so that no gravitational degrees of freedom are attributed to it. Lastly, the components of Lorentz vectors are invariant under local Poincaré translations $\xi^a \mapsto \xi^a + \epsilon^a$. Therefore, one includes a term in its definition

$$e^a{}_\mu = \partial_\mu \xi^a + A^a{}_{b\mu}\xi^b + A^a{}_\mu, \tag{7}$$

which transforms as $A^a{}_\mu \mapsto A^a{}_\mu - \partial_\mu \epsilon^a - A^a{}_{b\mu}\epsilon^b$. Note that the connection $A^a{}_{b\mu}$ is invariant under local Poincaré translations, hence

$$\omega^a{}_{b\mu} = A^a{}_{b\mu} \tag{8}$$

is a well-defined spin connection, being independent of the gauge ξ^a chosen.

The curvature and torsion then take the form

$$B^a{}_{b\mu\nu} = F^a{}_{b\mu\nu} \tag{9}$$

and

$$G^a{}_{\mu\nu} = \xi^b F^a{}_{b\mu\nu} + F^a{}_{\mu\nu}\,, \tag{10}$$

where the two-forms $F^a{}_{b\mu\nu}$ and $F^a{}_{\mu\nu}$ are the exterior covariant derivatives of $A^a{}_{b\nu}$ and $A^a{}_\nu$, respectively, with respect to $A^a{}_{b\mu}$. Because the spin connection (8) is purely inertial, the curvature (9) vanishes, so that the torsion is determined entirely by $F^a{}_{\mu\nu}$, which itself is nonzero only if $A^a{}_\mu \neq \partial_\mu \epsilon^a + A^a{}_{b\mu}\epsilon^b$ for every choice of ϵ^a, namely, it cannot be set to zero everywhere by a local Poincaré translation. The form $A^a{}_\mu$ is therefore generally given the role of gravitational gauge potential, whereas the torsion is said to be its field strength.

The geometry defined by the spin connection (8) and vierbein (7), together with their curvature (9) and torsion (10), is a nonlinear Riemann–Cartan geometry [7, 13]. This signifies that Poincaré transformations acting on these objects are realized nonlinearly by elements of its Lorentz subgroup. Local Poincaré translations, for example, are realized trivially by the identity matrix. The implied invariance of the vierbein and torsion under such translations is a crucial ingredient to allow for the interpretation of teleparallel gravity as a gauge theory. Indeed, as they play the role of covariant derivative and field strength, respectively, they must transform with the adjoint representation of the gauge group, which is the trivial representation due to the latter's abelian nature.

Because the set of de Sitter translations do not constitute a group, it does not appear feasible to construct teleparallel gravity with local kinematics regulated by $SO(1,4)$ through the gauge paradigm. By observing that the structure underlying teleparallel gravity is a nonlinear Riemann–Cartan geometry, it is natural to incorporate local de Sitter kinematics by generalizing for a nonlinear de Sitter–Cartan geometry.

IV. DE SITTER TELEPARALLEL GRAVITY

A. Fundamentals of de Sitter teleparallel gravity

Whilst the geometry summarized in Sec. II is the mathematical framework we shall employ to model teleparallel gravity with a cosmological function Λ, there remains to be specified how precisely it intends to accommodate the kinematics due to Λ and the dynamical degrees of freedom of the gravitational field and the cosmological function.

To begin with, it is postulated that a gravitational field is present if and only if the exterior covariant derivative (4b) of the vierbein has a value not equal to zero. This characterization to indicate whether or not there are gravitational degrees of freedom is formally the same as in teleparallel gravity, which may be argued to be natural. *A priori*, we would like to generalize for a different kinematics only and not alter the geometrical representation of the dynamics of the gravitational field. Nevertheless, this does not mean that the dynamics itself remains unaltered. On the contrary, we shall see in Sec. IV C how the presence of a cosmological function modifies the gravitational field equations.

In further similarity with teleparallel gravity, the spin connection does not bear any gravitational degrees of freedom. Being a connection for local Lorentz transformations, it naturally continues to represent fictitious forces existing in a certain class of frames. The final and most important issue that must be settled in specifying for the geometry is then— *How are the local kinematics, whose defining group in the presence of the cosmological function is $SO(1,4)$, accounted for?*

The question is given an answer by postulating that the curvature (3a) vanishes at every spacetime event, i.e.,

$$B^a{}_{b\mu\nu} = -\frac{\Lambda}{3}(e^a{}_\mu e_{b\nu} - e^a{}_\nu e_{b\mu}). \tag{11}$$

The curvature of the spin connection hence equals the curvature of the Levi-Civita connection on a de Sitter space with cosmological constant given by Λ, which varies from point to point. If the cosmological function goes to zero over the whole of spacetime, the geometric structure of teleparallel gravity is recovered. The prescription (11) to implement the kinematics in the geometric framework is of great importance, for the kinematic effects will be observable as fictitious forces between adjacent free-falling particles, something which will be clarified in Sec. IV B.

B. Particle mechanics and kinematic effects

The motion of a particle of nonzero rest mass m in the presence of a gravitational field and cosmological function is determined by the action ($c = 1$)

$$\mathcal{S} = -m \int u_a e^a, \tag{12}$$

where $u^a = e^a{}_\mu dx^\mu/d\tau$ is the four-velocity of the particle. Hence, as usual (12) is proportional to the particle's proper time τ. The equations of motion are given by

$$u^\rho D_\rho u^a = K^a{}_{b\rho} u^b u^\rho, \tag{13}$$

which are identical in form to the ones governing particle mechanics in teleparallel gravity [14], i.e., when the cosmological function vanishes. In particular, (13) complies with the weak equivalence principle, yet a breakdown of the latter most likely could be coped with along the lines it is done in teleparallel gravity [2]. Despite the fact it is not immediately obvious from (13), a nonzero cosmological function has an impact on the motion of particles. The first change is rather indirect and stems from a modification in the gravitational field equations, thus altering the value of the contortion for a given distribution of energy-momentum that sources gravity.

The second change reflects the alteration in kinematics, now regulated by the de Sitter group. In order to clarify this, we consider a one-parameter family of solutions $x_\sigma(\tau)$ of (13), parametrized by σ. These solutions form a two-dimensional surface, to which the vector fields $u = d/d\tau$ and $v = d/d\sigma$ are tangent. For every σ, u is the four-velocity of the particle with world line x_σ. The field v is tangent to constant τ slices, connecting the world lines of neighboring particles during their motion through spacetime. The vector field [15]

$$\mathfrak{a}^a = u^\mu D_\mu (u^\nu D_\nu v^a)$$

is therefore the relative acceleration between the world lines, measured by a free-falling observer. Because $[u, v]$ equals zero and

$$u^\mu D_\mu v^a - v^\mu D_\mu u^a = u^\mu v^\nu G^a{}_{\mu\nu},$$

while u^a satisfies the equations (13), the relative acceleration may be written as

$$\begin{aligned}\mathfrak{a}^a = {} & u^\mu v^\nu u^b B^a{}_{b\mu\nu} \\ & + v^\mu D_\mu (K^a{}_{b\nu} u^b u^\nu) + u^\mu D_\mu (u^\lambda v^\nu G^a{}_{\lambda\nu}),\end{aligned} \tag{14}$$

where $B^a{}_{b\mu\nu}$ is the curvature (11) of the local de Sitter spaces. The first term is therefore only present when the cosmological function is nonzero.

Equation (14) is a chief result of de Sitter teleparallel gravity, for it describes what the phenomenology is of the local de Sitter kinematics. The last two terms are dynamical in nature and come from a nonhomogeneous gravitational field. The first term originates in the cosmological function Λ, as can be seen from (11), and is caused by the kinematics. This contribution manifests itself in that two particles separated by the infinitesimal v^a deviate as if they were moving in a de Sitter space with cosmological constant Λ. Hence, two neighboring free-falling particles have world lines that deviate, not only because they move in a nonhomogeneous gravitational field, but also because of the kinematics that is determined by the cosmological function. According to this approach, dark energy has its origins in the cosmological function and reveals itself as a kinematic effect.

C. Dynamics of the gravitational field and the cosmological function

Having specified the particle mechanics caused by a gravitational field and cosmological constant, we now prescribe the dynamics of the latter two themselves. We shall define the gravitational action as a reasonable generalization of the action for the gravitational field in teleparallel gravity, for which the Lagrangian is given by [16]

$$\mathcal{L}_{\rm tg} = \frac{1}{4} G^\rho{}_{\mu\nu} G_\rho{}^{\mu\nu} + \frac{1}{2} G^\rho{}_{\mu\nu} G^{\nu\mu}{}_\rho - G^\nu{}_{\mu\nu} G^{\rho\mu}{}_\rho.$$

In teleparallel gravity, the two-form $G^a{}_{\mu\nu}$ is the torsion of the underlying Riemann–Cartan geometry. Because we have generalized for a de Sitter–Cartan geometry, for which the torsion is given by (3b), it appears a natural proposition to define the Lagrangian for de Sitter teleparallel gravity as

$$\mathcal{L}_{\rm dStg} = \frac{1}{4} T^\rho{}_{\mu\nu} T_\rho{}^{\mu\nu} + \frac{1}{2} T^\rho{}_{\mu\nu} T^{\nu\mu}{}_\rho - T^\nu{}_{\mu\nu} T^{\rho\mu}{}_\rho.$$

Substituting for (3b) restates this function as a Lagrangian for the gravitational field and cosmological function, namely,

$$\mathcal{L}_{\rm dStg} = \mathcal{L}_{\rm tg} - \frac{3}{2} \partial_\mu \ln\Lambda\, \partial^\mu \ln\Lambda - 2 G^{\mu\nu}{}_\mu \partial_\nu \ln\Lambda. \tag{15}$$

The action for de Sitter teleparallel gravity is thus given by ($c = \hbar = 1$)

$$\mathcal{S}_{\rm dStg} = \frac{1}{2\kappa} \int d^4x\, e\, \mathcal{L}_{\rm dStg}, \tag{16}$$

where $\kappa = 8\pi G_N$ and $e = \det e^a{}_\mu$.

The action (16) reminds, on the one hand, of the scheme in which scalar-tensor theories modify gravity in the framework of general relativity [17–21], or, on the other hand, of teleparallel dark energy, where a scalar field is coupled nonminimally to teleparallel gravity [22–24]. To be precise, it specifies for a gravitational sector modeled by teleparallel gravity— for a spin connection with curvature (11)— that interacts with the cosmological function due to a nonminimal coupling between the trace of the exterior covariant derivative of the vierbein and the logarithmic derivative of Λ. A theory quite similar in structure was discussed in [25]. Despite the similarity, there is a crucial discrepancy it has in common with any of the other modifications of general relativity or teleparallel gravity that introduce nonminimal couplings to scalar fields. Usually, these fields are added to the theory in a manner rather reminiscent of *ad hoc* hypotheses, and are not an essential feature of the spacetime geometry. In de Sitter teleparallel gravity by contrast, the scalar field is the cosmological function, which forms an integral part of the geometric structure and, moreover, quantifies the kinematics locally governed by the de Sitter group in the sense of Sec. IV B.

Note that the cosmological function appears in the action only through its logarithmic derivative, which is a direct consequence of (3b). Factors of $\partial_\mu \ln \Lambda$ are naturally dimensionless, which renders a correct overall dimension for the action. Because spacetime coordinates are numbers and the dimension of the vierbein components is that of length, i.e., $[e^a{}_\mu] = L$, while $[\kappa] = L^2$, $[e] = L^4$, $[g^{\mu\nu}] = L^{-2}$, and $[G^{\rho\mu}{}_\rho] = L^{-2}$, the action (16) indeed has dimension of $\hbar = 1$.

The field equations for the vierbein are

$$\begin{aligned} &D_\rho(e\, W_a{}^{\rho\mu}) + e\, t_a{}^\mu - 2e\, G^{\rho\mu}{}_\rho e_a{}^\nu \partial_\nu \ln \Lambda \\ &- 3e\, e_a{}^\rho \partial_\rho \ln \Lambda\, \partial^\mu \ln \Lambda + \frac{3}{2} e\, e_a{}^\mu \partial_\rho \ln \Lambda\, \partial^\rho \ln \Lambda \\ &\qquad - 2e\, e_a{}^\mu \Box \ln \Lambda + 2e\, e_a{}^\rho \nabla_\rho \partial^\mu \ln \Lambda = 0, \end{aligned} \tag{17}$$

where $\Box = g^{\mu\nu} \nabla_\mu \partial_\nu$ is the d'Alembertian, while

$$\begin{aligned} W_a{}^{\mu\nu} \equiv G_a{}^{\mu\nu} + G^{\nu\mu}{}_a - G^{\mu\nu}{}_a \\ - 2e_a{}^\nu G^{\lambda\mu}{}_\lambda + 2e_a{}^\mu G^{\lambda\nu}{}_\lambda, \end{aligned}$$

and

$$t_a{}^\mu = G^b{}_{\rho a} W_b{}^{\rho\mu} - e_a{}^\mu \mathcal{L}_{\rm tg}.$$

are the superpotential, respectively, the gravitational energy-momentum current [16]. The gravitational field equations (17) solve for the components of the vierbein, but do not determine the cosmological function. In de Sitter teleparallel gravity Λ is given its own dynamics, which is dictated by the field equation

$$\Box \ln \Lambda + G^{\mu\rho}{}_\mu \partial_\rho \ln \Lambda = -\frac{2}{3}(\nabla_\mu G^{\rho\mu}{}_\rho + G^\mu{}_{\rho\mu} G^{\nu\rho}{}_\nu). \tag{18}$$

The coupling of matter fields to the gravitational sector is carried out by taking the sum of the matter action

$$\mathcal{S}_{\rm m} = \int d^4x\, e\, \mathcal{L}_{\rm m}$$

and the action (16) for the gravitational field and cosmological function. The energy-momentum current $\delta\mathcal{L}_{\rm m}/\delta e^a{}_\mu$ of matter is a source for the gravitational field equations (17), but does not appear in the equation of motion (18) for the cosmological function. According to this scheme, energy-momentum generates gravity, which in turn sources the cosmological function.

V. CONCLUSIONS

In the present work we formulated a theory of gravity consistent with local spacetime kinematics regulated by the de Sitter group, namely, de Sitter teleparallel gravity. It was made plain first that teleparallel gravity, a theory physically equivalent to general relativity, has the mathematical structure of a nonlinear Riemann–Cartan geometry. This inspired us to generalize for de Sitter kinematics by considering de Sitter–Cartan geometry in the presence of a nonconstant cosmological function Λ.

The theory has the structure of a gravitational sector described by teleparallel gravity that couples nonminimally to the cosmological function. Dynamical degrees of freedom of the gravitational field are present if and only if the exterior covariant derivative of the vierbein does not vanish. Further, the cosmological function has its own dynamics, sourced by the trace of the exterior covariant derivative of the vierbein, but not directly by the matter energy-momentum current. It is thence similar in form to teleparallel dark energy, or scalar-tensor theories in the framework of general relativity.

A crucial difference between these models and the theory here proposed is that the cosmological function modifies the local kinematics of spacetime. Indeed, at every spacetime point we put forward that the curvature of the spin connection is equal to the curvature of the Levi-Civita connection of a de Sitter space with cosmological constant given by the value of the cosmological function. We saw that such a choice gives rise to a kinematic contribution in the deviation equation for the world lines of adjacent free-falling particles, that is, they undergo a relative acceleration that is kinematic in origin. This result is arguably the one of most importance of this article, for it specifies in exactly what manner the kinematics due to the cosmological function are to be observed. Hence, dark energy may be interpreted as a kinematic effect or, alternatively, as the cosmological function causing this effect.

It is interesting to note that there exists a link between the dynamics and kinematics of the theory, in the sense that the value of the cosmological function is determined dynamically by its interaction with the gravitational field, while the resulting value determines the local spacetime kinematics, which in its turn affects the motion of matter. The theory thus gives a precise model that prescribes how the kinematics of high energy physics may be modified locally and becomes spacetime-dependent [26]. Although there is a connection between them, dynamics and kinematics remain logically separated in the geometric representation of de Sitter teleparallel gravity. Nontrivial dynamics gives way to the torsion of the de Sitter–Cartan geometry being nonzero, whereas the value of the curvature of the spin connection encodes the inertial effects of a given frame and the local de Sitter kinematics. This is a natural generalization of the geometric representation of teleparallel gravity, which is recovered when the cosmological function vanishes.

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