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Chern-Simons terms and the Three Notions of Charge

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Abstract: In theories with Chern-Simons terms or modified Bianchi identities, it is useful to define three notions of either electric or magnetic charge associated with a given gauge field. A language for discussing these charges is introduced and the properties of each charge are described. 'Brane source charge' is gauge invariant and localized but not conserved or quantized, 'Maxwell charge' is gauge invariant and conserved but not localized or quantized, while 'Page charge' conserved, localized, and quantized but not gauge invariant. This provides a further perspective on the issue of charge quantization recently raised by Bachas, Douglas, and Schweigert. For the Proceedings of the E.S. Fradkin Memorial Conference.

Keywords: Supergravity, p–branes, D-branes.

Contents

1. Introduction

One of the intriguing properties of supergravity theories is the presence of Abelian Chern-Simons terms and their duals, the modified Bianchi identities, in the dynamics of the gauge fields. Such cases have the unusual feature that the equations of motion for the gauge field are non-linear in the gauge fields even though the associated gauge groups are Abelian. For example, massless type IIA supergravity contains a relation of the form

$$d\tilde{F}_4 + F_2 \wedge H_3 = 0, \tag{1.1}$$

where $\tilde{F}_4, F_2, H_3$ are gauge invariant field strengths of rank $4, 2, 3$ respectively.

Such relations complicate our usual understanding of charge in a gauge theory. On the one hand, the fields F_2 and H_3 are invariant under the gauge transformations naively associated with $\tilde{F}_4$ so that one would not consider them to carry charge. On the other, these fields are clearly sources of $\tilde{F}_4$. Thus, one may ask what is the proper definition of charge in a theory with Chern-Simons terms. This question is central to the issue raised by Bachas, Douglas, and Schweigert [1] and continued by several authors [2, 3, 4] as to in just what sense D0-brane charge should be quantized.

The approach adopted here is not to argue for a particular notion of charge, but instead to discuss the fact that there are at least three natural notions of charge in a theory with Chern-Simons terms or a modified Bianchi identity. A closely related discussion in which multiple notions of charge were of use can be found in [5]. Which notion of charge is most useful depends on the goal that one has in mind. One of the main purposes of this work is to provide a language for the proper discussion of these ideas. The notions of charge discussed below are referred to as 'brane source charge,' 'Maxwell charge,' and 'Page charge.'

Brane source charge is a notion of charge most directly associated with external objects coupled to the theory. As implied by the name, this charge is localized. That is to say that it is not carried by the gauge fields but is instead associated directly with external sources (or topological non-trivialities of the spacetime manifold) which take the shape of various branes. This charge is gauge invariant, but not conserved. However, the non-conservation rules take a precise form which can be directly related to the Hanany-Witten effect [6]. The relationship is a generalization of the argument for the case of D0/D8-branes presented in [7, 8]. In general, brane source charge is not quantized. It is in fact this charge that was directly computed by Bachas, Douglas, and Schweigert [1] and found not to be quantized in a particularly interesting example. This is also the notion of charge used to identify the branes in the supergravity solution of [9].

Another notion, 'Maxwell charge,' is conserved and gauge invariant but not localized. Instead, it is carried by the gauge fields themselves and so is diffused throughout a classical solution. As a result, the Dirac quantization argument does not require its integral over an arbitrary volume to be quantized and, in general, it will be quantized only when integrated to infinity with appropriate fall-off conditions on the fields. It is this charge that was recently discussed by Taylor [2].

The third type of charge is "Page charge." Here we follow tradition (e.g., [10]) by naming this charge after the author of the paper in which it first appeared [11]. This charge is again localized and not carried by the gauge fields. It is also conserved and under appropriate conditions it is invariant under small gauge transformations. However, it does transform under large gauge transformations. By looking at how Chern-Simons terms and modified Bianchi identities originate in Kaluza-Klein reduction, one can argue that the Page charge is quantized. The Page charge quantization conditions were matched with the Dirac quantization conditions of the higher dimensional theory in [12]. From the perspective of the theory on the D2-brane, this is the charge that was conjectured to be quantized in [1] and, although it was not discussed in these terms there, it also matches the notion of charge discussed by Alekseev, Mironov, and Morozov in [4].

These types of charge are not new, as they have all appeared in the literature. However, as is clear from the recent discussion of D0-brane charge in [1, 2, 4], a coherent discussion of these charges will prove useful and a proper language for discussing these charges is needed.

We discuss in turn the brane source, Maxwell, and Page notions of charge in sections II-IV. Due to limitations of space, we discuss the details only in the particularly illustrative case of D4-brane charge in type IIA supergravity. In each case, we make a number of observations about that particular notion of charge and the relation to D0-brane charge in the setting of Bachas, Douglas, and Schweigert. A few closing comments are contained in section V.

2. Brane Source Charge and Brane-ending effects

Let us recall that that type IIA supergravity contains a three-form Ramond-Ramond gauge field A_3 for which D4-branes carry magnetic charge. One class of gauge transformations act on this field as $A_3 \to A_3 + d\Lambda_2$ for an arbitrary smooth two-form Λ_2. Throughout this work, we find it convenient to indicate the rank of each form with a subscript. An unusual property of this field, however, is that it also transforms under the gauge transformations normally associated with the Ramond-Ramond potential A_1:

$$(A_1, A_3) \to (A_1 + d\Lambda_0, A_3 - B_2 \wedge d\Lambda_0), \tag{2.1}$$

where B_2 is the Neveu-Schwarz two-form (i.e., the Kalb-Ramond field). This means that the field strength $F_4 = dA_3$ is not gauge invariant, but instead transforms as $F_4 \to F_4 - H_3 \wedge d\Lambda_0$. Here, $H_3 = dB_2$ is the gauge invariant Neveu-Schwarz field strength. As a result, it is convenient to introduce the gauge invariant 'improved field strength' $\tilde{F}_4 = dA_3 - A_1 \wedge H_3$ and to write the Bianchi identity in the form of equation (1.1). Such a relation is known as a modified Bianchi identity. Similar equations appear involving the dual field $*\tilde{F}_4$ (associated with D2-brane charge) in the equations of motion due to Chern-Simons terms of the form $A_i \wedge F_j \wedge F_k$ for various i, j, k in the type IIA action. One can often exchange a modified Bianchi identity for a Chern-Simons term by performing an electromagnetic duality transformation. Due to their similar forms, our discussion in all cases below applies equally well to the effects of modified Bianchi identities and those of Chern-Simons terms.

We wish to discuss the various notions of charge in terms of a language of currents associated with external sources. This language, however, is sufficiently general so as to be useful for what one might call 'solitonic charge' associated with topological nontrivialities (such as black holes, any singularities that one might deem to allow, and so on). Suppose for example that we are given a spacetime containing a wormhole that is threaded by some electric flux. Then we may choose to consider a related spacetime in which the neck of the wormhole has been rounded off by hand. The new spacetime will of course not satisfy the supergravity equations of motion in the region that has been modified. We can describe this departure from pure supergravity by saying that some external source is present in this region. Using such a language will allow us to suppose that we work on the manifold R^n and that the spacetime is smooth.

We begin with what, from the standpoint of the modified Bianchi identity, is perhaps the most natural parameterization of this external source. We simply define the nonvanishing of the modified Bianchi identity to be the dual $*j^{bs}_{D4}$ of some current, which will in some way be associated with D4-branes. Thus, we have

$$d\tilde{F}_4 + F_2 \wedge H_3 = *j^{bs}_{D4}. \tag{2.2}$$

We repeat that this is nothing other than a definition of $*j^{bs}_{D4}$, now providing a parameterization of the external sources. In general, we would write each modified Bianchi identity and equation of motion for the gauge fields as a polynomial in the gauge invariant improved field strengths, their hodge duals, and exterior derivatives of these and then let the right hand side be some $*j$. Each such current will be associated with some brane, either a D-brane, NS5-brane, or a fundamental string. Similar sources for the metric are associated with energy and momentum, while sources for the dilaton are associated with NS instantons and NS7-branes.

Let us make a few simple observations about the current defined in (2.2). Examining the left-hand side, we see that our current is gauge invariant. It is also 'localized' in the sense that it vanishes wherever the spacetime is described by pure supergravity. In this sense, it is naturally associated with *external* brane sources that are coupled to supergravity. For this reason, we refer to this notion of charge as 'brane source charge.'

We note that this notion of charge coincides with many familiar conventions. For example, suppose that we rewrite type IIA supergravity in terms of the magnetic field strength A_5 dual to A_3. Then the modified Bianchi identity for A_3 becomes an equation of motion for A_5. In this case, the brane source current is just what results from additional terms of the form $-\int A_5 \wedge *j^{bs}_{D4}$ that one would add to the action to represent external sources. A similar discussion for the case of D0-brane charge on a D2-brane coupled to supergravity shows that since brane source charge arises from varying the brane action with respect to the gauge field, it is this notion of charge which raised the puzzle in [1], as they found this charge not to be quantized.

In fact, supergravity considerations also lead one to expect this charge not to be quantized. This follows from the fact that it is not conserved, and that its non-conservation takes a special form. Let us simply take the exterior derivative of (2.2), allowing also sources $*j^{bs}_{D6} = dF_2$ and $*j^{bs}_{NS5} = dH_3$ for the other relevant gauge fields. We find:

$$d * j^{bs}_{D4} = F_2 \wedge *j^{bs}_{NS5} + *j^{bs}_{D6} \wedge H_3, \tag{2.3}$$

so that both NS5-branes and D6-branes can be sources of our charge in the proper backgrounds. What is particularly interesting about this result is that, due to the ranks of the forms involved, it has components in which all indices take spatial values. This means that such components have no time derivatives and instead constitute a *constraint*, telling us how D4-brane charge must change in spatial directions. In particular, integrating this result over some six-dimensional volume V_6 tells us that the net number of D4-branes (as counted by brane-source charge) ending inside V_6 is controlled by the fluxes of gauge fields captured by NS5-branes and D6-branes inside V_6:

$$\int_{V_6} *j_{D4} = \int_{V_6 \cap NS5} F_2 + \int_{V_6 \cap D6} H_3. \tag{2.4}$$

Note that the intersection of V_6 with the worldvolume of an NS5-brane is generically of dimension 2, and that the intersection with the worldvolume of a D6-brane is generically of dimension 3. The normalization is such that if a single NS5-brane captures all of the F_2 flux emerging from a D6-brane, then this constraint states that exactly one D4-brane worth of charge will begin (or end, depending on the sign) on the NS5-brane. This constraint tells us that D4-brane source charge must be created continuously over the world volume of NS5- and D6-branes. Since constraints are typically not significantly modified by quantization, it would be quite surprising if such a charge were quantized. This point was also made in [1] working from the perspective of the worldvolume theory on a brane.

Such constraints connect Chern-Simons terms and modified Bianchi identities with the same types of branes ending on branes as in Townsend's 'Brane Surgery' argument [13]. These arguments are not equivalent, however, as [13] considers that case where brane source charge (say, for a D4-brane) is not created or destroyed, but instead flows away through the worldvolume of the other (D5- or D6-) brane.

Finally, we note (see also [9]) that such constraints provide yet another derivation of the Hanany-Witten effect [6]. The argument is a generalization of the argument of that of [7, 8] for the D0/D8 case. Suppose that an NS5-brane lies on one side of a D6-brane in such a way that there is no D4-brane charge in the vicinity. Typically, the constraints can still be satisfied if other NS5- and D6-branes are nearby. When the NS5-brane is moved past the D6-brane, the flux captured by each of these branes changes by one unit. The NS5-brane must then be a source of one D4-brane, while the D6-brane must be a sink. If the branes are moved quickly, causality considerations show that we must now have a D4-brane stretching from the NS5-brane to the D6-brane. Whether or not one wishes to use brane source charge to count 'real D4-branes,' one finds that some sort of brane must be stretched between the NS5- and D6-branes. A corresponding argument from the perspective of the worldvolume theory was presented in [14, 15] but it is nice to arrive at this result via such a short argument in supergravity. Other complimentary derivations of this effect can be found in [16, 17, 18, 19, 20, 21]. Some of these derivations use an 'anomaly inflow' argument, and we refer the reader to [5] to connect such a perspective directly with the present discussion, closing the circle of ideas.

3. Maxwell Charge and Asymptotic Conditions

Our next notion of charge follows from the idea that any source of the gauge field should be considered to constitute a charge. Consider again the relation $d\tilde{F}_4 + F_2 \wedge H_3 = 0$ which holds in the absence of external sources. Clearly, $F_2 \wedge H_3$ is a source for the field strength $\tilde{F}_4$, so that we might count it as carrying charge. To this end, let us define the Maxwell charge current to be the exterior derivative of the gauge invariant field strength:

$$d\tilde{F}_4 = *j_{D4}^{Maxwell}. \tag{3.1}$$

Such a relation describes the familiar currents of Yang-Mills theories, in which the gauge fields also carry charge. A similar idea allowing gravitational fields to contribute to energy and momentum is captured by the ADM mass for gravity. A study of [2] shows that this is in fact the notion of charge used by Taylor in that reference.

This current has many useful properties. It is manifestly gauge invariant and conserved. However, it is not localized, as it is carried by the bulk fields. This means that the conservation law for Maxwell charge is somewhat less useful that one might hope. Consider for a moment integrating $\tilde{F}_4$ over some surface ∂V to obtain the total charge associated with some region V. The charge measured in this way is unchanged when we deform the surface ∂V so long as this surface does not pass through any charge. Since Maxwell charge is carried by the bulk fields, such charge-preserving deformations may not exist at all.

This of course is the case in Yang-Mills theory or gravity, where one solves the problem by using Gauss' law for surfaces at infinity where the bulk charge density vanishes under appropriate fall-off conditions. This works well for charges carried by pointlike objects, but is somewhat less satisfactory for the present case in which the sources are branes. The point is that one might like the charge measured to remain unchanged when the Gauss' law surface is deformed in space as well as when translated in time. A charge associated with p-branes is measured by a Gauss' law surface of co-dimension $p+2$, so that interesting deformations of the Gauss' law surface in space are indeed possible for $p > 0$.

Consider in particular the D4-brane case. Note that the Maxwell and brane source currents are related by $*j_{D4}^{Maxwell} = *j_{D4}^{bs} - F_2 \wedge H_3$. Suppose that we have some region

V with $\partial V = S_1 - S_2$. Then $\int_{S_1} \tilde{F}_4 = \int_{S_2} \tilde{F}_4$ if and only if $\int_V * j_{D4}^{Maxwell} = 0$. In a region of infinity in which the supergravity equations of motion hold (and thus there are no external sources), we have $\int_V * j_{D4}^{Maxwell} = - \int_V F_2 \wedge H_3$. Note that this will not in general vanish (so long as V spans a finite fraction of infinity) as $\int F_2$ measures the D6-brane charge while $\int H_3$ measures the NS5-brane charge. The asymptotically flat version of [9] or, analogously [22] for D3-branes and test D5-branes, are examples in which this can be seen. Note that one does not need the complete supergravity solution to obtain this result.

Thus, even at infinity the Maxwell charge is not localized. In fact, unless F_2 and H_3 flux is confined, the Maxwell D4 charge in a region V must change continuously with V even at infinity. This means that Maxwell charge associated with generic surfaces at infinity cannot be quantized. Note, however, that in the case of D0-brane charge studied in [2] there is a unique sphere at infinity at which Gauss' law can be applied and the issue does not arise.

4. Page Charge and Kaluza-Klein reduction

The final notion of charge that we will consider is one first introduced by Page in [11]. The idea is first to write the modified Bianchi identity (or equation of motion with a Chern-Simons term) as the exterior derivative of some differential form, which in general will not be gauge invariant. In the presence of an external source, it is this exterior derivative that is identified with a current or charge. Thus, for our case of D4-branes we would write

$$d(\tilde{F}_4 + A_1 \wedge H_3) = * j_{D4}^{Page}. \tag{4.1}$$

There is some ambiguity in this process as the second term could also have been taken to be of the form $F_2 \wedge B_2$. This ambiguity will be discussed further below.

We see immediately that the Page current is conserved and localized, in the sense that it vanishes when the pure supergravity equations of motion hold. However, it is also clear that this current is gauge dependent as it transforms nontrivially under gauge transformations of A_1. This problem is to some extent alleviated by integrating the current over some five-volume V_5 to form a charge:

$$Q_{D4,V}^{Page} = \int_{V_5} * j_{D4}^{Page} = \int_{\partial V} (\tilde{F}_4 + A_1 \wedge H_3). \tag{4.2}$$

If A_1 is a well-defined 1-form on ∂V and $dH_3 = 0$ on ∂V, then an integration by parts shows that the Page charge is invariant under small gauge transformations $A_1 \to A_1 + d\Lambda_0$. However, in general it will still transform under large gauge transformations.

The qualification that A_1 be a well-defined 1-form means that there can be no 'Dirac strings' of A_1 passing through ∂V in the chosen gauge. A similar integration by parts shows that, when ∂V does not intersect any NS5 or D6 branes or the associated Dirac strings, the same page charge would be obtained from $\tilde{F}_4 + F_2 \wedge B_2$.

We note that the Page charge differs from the Maxwell charge only by the boundary term discussed in the last section. That is, we have $Q^{Page}_{D4,V_5} = Q^{Maxwell}_{D4,V_5} + \int_{\partial V} A_1 \wedge H_3$. A similar expression holds for D0-brane charge. For the case studied by Taylor in [2], the corresponding boundary term was explicitly assumed to vanish when ∂V was the sphere at infinity. Thus, although [2] began with the idea of Maxwell charge, in that case a discussion in terms of Page charge would be equivalent. Similarly, when one works out the D0-brane Page charge for the case of [1] one finds $*j^{Page}_{D0} = *j^{bs}_{D0} - \int B \wedge *j^{bs}_{D2}$. It was exactly a term of the form $\int B \wedge *j^{bs}_{D2}$ that created the puzzle in [1], and we see that it is explicitly cancelled in the Page charge. Computing the Page charge for other examples agrees with [4], although it was discussed there in a somewhat different language.

We would now like to argue that it is the Page charge which is naturally quantized. The argument that we will give is essentially contained in [12] and perhaps earlier works as well. However, let us first embark on a small tangent which is in fact not a convincing argument for quantization. We note that D2-branes couple electrically to $\tilde{F}_4$ and that the D2-brane action contains a term $\int_{D2} A_3$. In order for $e^{iS_{D2}}$ to be insensitive to Dirac strings, $\int_\Sigma A_3$ should be quantized for any 3-surface Σ wrapping tightly around a Dirac string. But $\int_{\partial_V} (\tilde{F}_4 + A_1 \wedge H_3) = \int_{\partial_V} (dA_3) = \int_\Sigma A_3$ where Σ wraps tightly around all Dirac strings of A_3 passing through ∂V. Thus, requiring $e^{iS_{D2}}$ to be well-defined in the presence of Dirac strings would force quantization of the Page charge. We agree with [1], however, that this is not by itself a convincing argument for quantization of Page charge as it assumes that the effective action of the D2-brane is known a priori. In fact, the Chern-Simons terms of such an effective action are typically deduced from properties of the bulk fields. Nevertheless, it is reassuring that Page charge quantization is consistent with the usual D2-brane action.

Now, for a more convincing argument. Recall that many of the Chern-Simons terms and modified Bianchi identities of type IIA supergravity arise from the Kaluza-Klein reduction of 11-dimensional supergravity. Of course, 11-dimensional supergravity has its own Chern-Simons terms as required by supersymmetry. Nevertheless, our discussion of D4-brane charge would be the same if, instead of type IIA supergravity, we considered the reduction to ten dimensions of an 11-dimensional Einstein Maxwell theory given by

$$S_{11} = \int \sqrt{g} R + \frac{1}{2} F^M_4 \wedge * F^M_4, \tag{4.3}$$

and in particular having no Chern-Simons term. We have labelled the 4-form field strength of this pseudo M-theory F_4^M in order to distinguish it from the F_4 of the ten dimensional theory.

In such a simple Einstein-Maxwell theory, charge quantization is believed to be well understood with $\int_{\partial V} F_4^M$ and $\int_{\partial V} *F_4^M$ being quantized. In Kaluza-Klein reduction along x_{10}, the relation between 10- and 11-dimensional fields is just

$$F_4^M = F_4 + H_3 \wedge dx_{10} = (\tilde{F}_4 + A_1 \wedge H_3) + H_3 \wedge dx_{10}. \tag{4.4}$$

As a result, if $Q_{D4}^{Page}(S_4) = \int_{S_4}(\tilde{F}_4 + A_1 \wedge H_3)$ is the Page charge associated with the surface S_4, we see that this is identical to the M5-brane charge $Q_{M5}(S_4, x_{10} = const)$ defined by integrating F_4^M over the surface at constant x_{10} that projects to S_4 in the ten-dimensional spacetime. This observation was used in [12] to match the ten- and eleven-dimensional Dirac quantization conditions. Thus, it is the Page charge that lifts to the familiar notion of charge in 11-dimensions. Quantization of the usual charge in 11-dimensional Einstein-Maxwell theory directly implies quantization of D4-brane Page charge in ten-dimensions. It is for this reason that we have chosen to use D4-brane charge as our example system. Quantization of the Page charge for other branes then follows from T-duality. T-duality directly implies Page charge quantization in systems with sufficient translational symmetry, and one can use homotopy invariance of the Page charge to complete the argument. Quantization of the Page charge in 2+1 dimensional theories with $A \wedge F$ Chern-Simons terms was derived in [23].

Note that under the Kaluza-Klein assumption of translation invariance in x_{10} the precise value of x_{10} is unimportant. Furthermore, under a change of gauge $A_1 \to A_1 + d\Lambda_0$ in the 10-dimensional spacetime, we have $x_{10} \to x_{10} - \Lambda$. This means that a change of gauge in ten dimensions corresponds to a change of *surface* in 11-dimensions. This provides a clear physical meaning to the change in the Page charge under a large gauge transformations: in the 11-dimensional theory, we have replaced the M5-brane charge contained in one surface with the M5-brane charge contained in a homotopically inequivalent surface.

5. Discussion

We have seen that three notions of charge can be useful in theories with Chern-Simons terms. Brane source charge is gauge invariant and localized, but not conserved or quantized. Its non-conservation, however, summarizes consistency conditions that must be satisfied by external sources coupled to the theory and leads directly to the Hanany-Witten brane creation effect.

In contrast, Maxwell charge is carried by the bulk fields and so is not localized. It is quite similar to the ADM mass, energy, and momentum of gravitating systems, which is in fact one of the reasons for its use in [2]. This charge is both gauge invariant and conserved. However, in certain interesting cases involving p-branes with $p > 0$, the fall-off conditions at infinity are too weak for this conservation law to be as useful as one might like.

Finally, while it transforms nontrivially under large gauge transformations, the Page charge is localized and conserved. When the Chern-Simons term or modified Bianchi identity arises from Kaluza-Klein reduction, this charge is naturally associated with charge in the higher dimensional theory. As a result, it is this charge that is naturally taken to be quantized. The gauge dependence of the Page charge is nothing other than the ambiguity associated with choosing a surface in the higher dimensional theory that projects onto the chosen surface in the lower dimensional spacetime. Note that, due to its relation to the higher dimensional fields, it is also the Page charge which is naturally associated with supersymmetry.

It is interesting to consider Page charge in the context of branes created in the Hanany-Witten effect. In many cases involving D0- and D8-branes, the created string clearly has a Page charge of zero as the associated Gauss' law surface can be slipped over the end of the D0-brane and contracted to a point. However, a non-zero Page charge can arise for higher branes. A number of examples are under investigation.

Such considerations apply not only to supergravity, but also for example to the D2-brane theory directly investigated by Bachas, Douglas, and Schweigert. They argued that a certain charge $\int F$ should be quantized, where $F = dA$ is a gauge field on the D2-brane that is in fact not gauge invariant. One can check that this is also a Page charge of the D2-brane theory. Again, Kaluza-Klein reduction provides a useful perspective. If one investigates the relation between the D2-brane theory and the theory of an M2-brane, one finds that $\int F$ is exactly the canonical momentum of the M2-brane in the compact x_{10} direction, and so is again naturally quantized.

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