The black hole entropy function in the presence of a gauge Chern-Simons term in five dimensions

Hendrik Jennen

Supervisor: Dr. Jan Perz

June 24, 2010

Outline

Equations of motion: Einstein's equations

Action

$$\mathcal{S} = \int R \star 1 - F \wedge \star F$$

- Einstein's equations (metric eqs. of motion)

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = 0$$

Equations of motion: Einstein's equations

Action

$$\mathcal{S} = \int R \star 1 - F \wedge \star F$$

- Einstein's equations (metric eqs. of motion)

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = T_{\mu\nu}$$

Equations of motion: Einstein's equations

Action

$$\mathcal{S} = \int R \star 1 - F \wedge \star F$$

- Einstein's equations (metric eqs. of motion)

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = T_{\mu\nu}$$

$$\text{geometry} \leftrightarrow \text{energy}$$

Equations of motion: Einstein's equations

Action

$$\mathcal{S} = \int R \star 1 - F \wedge \star F$$

- Einstein's equations (metric eqs. of motion)

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = T_{\mu\nu}$$

$$\text{geometry} \leftrightarrow \text{energy}$$

- Finding a generic solution is highly non-trivial
  - Anticipate solutions with symmetry: Ansätze

Black hole solutions

- Charged (q) non-rotating mass (M) at “origin”
  - Static spherically symmetric Ansatz

Black hole solutions

- Charged (q) non-rotating mass (M) at “origin”
  - Static spherically symmetric Ansatz

Space-time metric (Reissner-Nordström)

$$ds^2 = -\Delta(r)dt^2 + \Delta^{-1}(r)dr^2 + r^2 d\Omega_2$$

- $\Delta(r)$ has two coordinate singularities $r_\pm = M \pm \sqrt{M^2 - q^2}$
  - Horizons—black hole
- If $M = q$: black hole is extremal
  - Maximal charge for given mass

Black hole solutions

- Charged (q) non-rotating mass (M) at "origin"
  - Static spherically symmetric Ansatz

Space-time metric (Reissner-Nordström)

$$ds^2 = -\Delta(r)dt^2 + \Delta^{-1}(r)dr^2 + r^2 d\Omega_2$$

- $\Delta(r)$ has two coordinate singularities $r_\pm = M \pm \sqrt{M^2 - q^2}$
  - Horizons—black hole
- If $M = q$: black hole is extremal
  - Maximal charge for given mass

Black hole solutions

- Charged (q) non-rotating mass (M) at "origin"
  - Static spherically symmetric Ansatz

Space-time metric (Reissner-Nordström)

$$ds^2 = -\Delta(r)dt^2 + \Delta^{-1}(r)dr^2 + r^2 d\Omega_2$$

- $\Delta(r)$ has two coordinate singularities $r_\pm = M \pm \sqrt{M^2 - q^2}$
  - Horizons—black hole
- If $M = q$: black hole is extremal
  - Maximal charge for given mass

Black hole thermodynamics

- Classically black holes cannot radiate
  - Zero temperature
- Laws of black hole mechanics
  - Reminiscent of laws of thermodynamics
  - E.g. black hole horizon area never decreases
  - Macroscopic entropy $\sim$ area? [Bekenstein]
- Quantum field theory on classical curved background
  - Thermodynamical spectrum of black body [Hawking]

$$S = \frac{A}{4\hbar}$$

Black hole thermodynamics

- Classically black holes cannot radiate
  - Zero temperature
- Laws of black hole mechanics
  - Reminiscent of laws of thermodynamics
  - E.g. black hole horizon area never decreases
  - Macroscopic entropy $\sim$ area? [Bekenstein]
- Quantum field theory on classical curved background
  - Thermodynamical spectrum of black body [Hawking]

$$S = \frac{A}{4\hbar}$$

Black hole thermodynamics

- Classically black holes cannot radiate
  - Zero temperature
- Laws of black hole mechanics
  - Reminiscent of laws of thermodynamics
  - E.g. black hole horizon area never decreases
  - Macroscopic entropy $\sim$ area? [Bekenstein]
- Quantum field theory on classical curved background
  - Thermodynamical spectrum of black body [Hawking]

$$S = \frac{A}{4\hbar}$$

Black hole thermodynamics

- Classically black holes cannot radiate
  - Zero temperature
- Laws of black hole mechanics
  - Reminiscent of laws of thermodynamics
  - E.g. black hole horizon area never decreases
  - Macroscopic entropy $\sim$ area? [Bekenstein]
- Quantum field theory on classical curved background
  - Thermodynamical spectrum of black body [Hawking]

$$S = \frac{A}{4\hbar}$$

Black hole thermodynamics

- Classically black holes cannot radiate
  - Zero temperature
- Laws of black hole mechanics
  - Reminiscent of laws of thermodynamics
  - E.g. black hole horizon area never decreases
  - Macroscopic entropy $\sim$ area? [Bekenstein]
- Quantum field theory on classical curved background
  - Thermodynamical spectrum of black body [Hawking]

$$S = \frac{A}{4\hbar}$$

Generalized Einstein-Maxwell theory

action

$$\mathcal{S} = \int R \star 1 - g_{rs}(\varphi) d\varphi^r \wedge \star d\varphi^s - h_{ij}(\varphi) F^i \wedge \star F^j$$

- Gravitation coupled to a set of scalar and vector fields
- Coupling matrices g_{rs} and h_{ij} depend on scalars
- No potential terms
- Static spherically symmetric extremal black hole solutions
- Einstein-Maxwell: special case

- Structure suggested by supergravity (bosonic sector)

Generalized Einstein-Maxwell theory

action

$$\mathcal{S} = \int R \star 1 - g_{rs}(\varphi) d\varphi^r \wedge \star d\varphi^s - h_{ij}(\varphi) F^i \wedge \star F^j$$

- Gravitation coupled to a set of scalar and vector fields
- Coupling matrices g_{rs} and h_{ij} depend on scalars
- No potential terms
- Static spherically symmetric extremal black hole solutions
- Einstein-Maxwell: special case

- Structure suggested by supergravity (bosonic sector)

Generalized Einstein-Maxwell theory

action

$$\mathcal{S} = \int R \star 1 - g_{rs}(\varphi) d\varphi^r \wedge \star d\varphi^s - h_{ij}(\varphi) F^i \wedge \star F^j$$

- Gravitation coupled to a set of scalar and vector fields
- Coupling matrices g_{rs} and h_{ij} depend on scalars
- No potential terms
- Static spherically symmetric extremal black hole solutions
- Einstein-Maxwell: special case

- Structure suggested by supergravity (bosonic sector)

The attractor mechanism

- Non-trivial dynamics only in radial direction
  - Static spherically symmetric extremal black holes
- Integrate out the fields along other directions

$$\mathcal{S}_{\text{eff}} = \int dr\, g_{ij}\dot{\varphi}^i\dot{\varphi}^j(r) + V_{\text{BH}}(\phi^i, q) + [\ldots]$$

- Effective potential for the scalars
  - Scalars at horizon do not depend on asymptotic values: $\varphi^i_{\text{H}}(q)$
  - Attractor mechanism
- Macroscopic entropy depends on charges only

$$S \sim \text{Area}(\varphi^i_{\text{H}}(q), q)$$

The attractor mechanism

- Non-trivial dynamics only in radial direction
  - Static spherically symmetric extremal black holes
- Integrate out the fields along other directions

$$\mathcal{S}_{\text{eff}} = \int dr\, g_{ij}\dot{\varphi}^i\dot{\varphi}^j(r) + V_{\text{BH}}(\phi^i, q) + [\ldots]$$

- Effective potential for the scalars
  - Scalars at horizon do not depend on asymptotic values: $\varphi^i_{\text{H}}(q)$
  - Attractor mechanism
- Macroscopic entropy depends on charges only

$$S \sim \text{Area}(\varphi^i_{\text{H}}(q), q)$$

The attractor mechanism

- Non-trivial dynamics only in radial direction
  - Static spherically symmetric extremal black holes
- Integrate out the fields along other directions

$$\mathcal{S}_{\mathrm{eff}} = \int dr\, g_{ij}\dot{\varphi}^i \dot{\varphi}^j(r) + V_{\mathrm{BH}}(\phi^i, q) + [\ldots]$$

- Effective potential for the scalars
  - Scalars at horizon do not depend on asymptotic values: $\varphi^i_{\mathrm{H}}(q)$
  - Attractor mechanism
- Macroscopic entropy depends on charges only

$$S \sim \mathrm{Area}(\varphi^i_{\mathrm{H}}(q), q)$$

The attractor mechanism

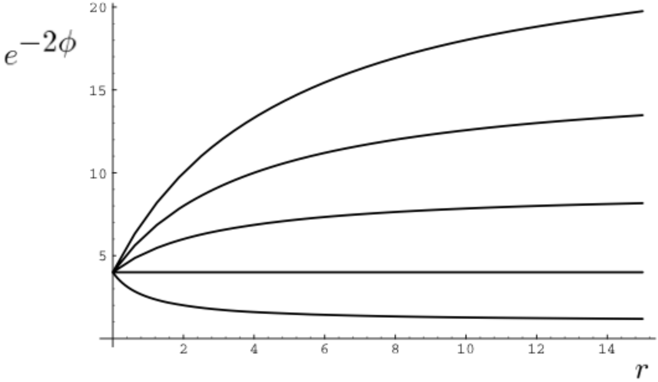


Figure: Adapted from arXiv:0805.2498v2 [hep-th]

The attractor mechanism

- Non-trivial dynamics only in radial direction
  - Static spherically symmetric extremal black holes
- Integrate out the fields along other directions

$$\mathcal{S}_{\text{eff}} = \int dr\, g_{ij}\dot{\varphi}^i\dot{\varphi}^j(r) + V_{\text{BH}}(\phi^i, q) + [\ldots]$$

- Effective potential for the scalars
  - Scalars at horizon do not depend on asymptotic values: $\varphi^i_{\text{H}}(q)$
  - Attractor mechanism
- Macroscopic entropy depends on charges only

$$S \sim \text{Area}(\varphi^i_{\text{H}}(q), q)$$

$\rightarrow$ Importance of attractor mechanism

The entropy function

- Goal: function with two practical properties
  - On-shell fields at horizon—variational principle
  - Entropy at extremum

The entropy function

- Goal: function with two practical properties
  - On-shell fields at horizon—variational principle
  - Entropy at extremum
- No explicit dependence on gauge potentials
- Assume symmetry
- How? Legendre transform w.r.t. e-fields

Entropy function [Sen]

$$\mathcal{E} \equiv q_i e^i - \int_{\mathrm{H}} d\theta \, d\phi \, \mathcal{L}(g, \varphi^s, F^i)$$

The entropy function

- Goal: function with two practical properties
  - On-shell fields at horizon—variational principle
  - Entropy at extremum
- No explicit dependence on gauge potentials
- Assume symmetry
- How? Legendre transform w.r.t. e-fields

Entropy function [Sen]

$$\mathcal{E} \equiv q_i e^i - \int_{\mathrm{H}} d\theta \, d\phi \, \mathcal{L}(g, \varphi^s, F^i)$$

The entropy function

- Goal: function with two practical properties
  - On-shell fields at horizon—variational principle
  - Entropy at extremum
- No explicit dependence on gauge potentials
- Assume symmetry
- How? Legendre transform w.r.t. e-fields

Entropy function [Sen]

$$\mathcal{E} \equiv q_i e^i - \int_{\mathrm{H}} d\theta \, d\phi \, \mathcal{L}(g, \varphi^s, F^i)$$

The entropy function

- Goal: function with two practical properties
  - On-shell fields at horizon—variational principle
  - Entropy at extremum
- No explicit dependence on gauge potentials
- Assume symmetry
- How? Legendre transform w.r.t. e-fields

Entropy function [Sen]

$$\mathcal{E} \equiv q_i e^i - \int_{\mathrm{H}} d\theta\, d\phi\, \mathcal{L}(g, \varphi^s, F^i)$$

- Attractor mechanism

Chern-Simons term and Page charge

Action - 5D

$$\mathcal{S} = \int F \wedge \star F + A \wedge F \wedge F + [\ldots]$$

$$d \star F = -\frac{3}{2} F \wedge F \quad \Longleftrightarrow \quad d(\star F + \frac{3}{2} A \wedge F) = 0$$

Maxwell charge
- Conserved
- Not localized

$$Q \equiv \int d(\ldots)$$

Page charge
- Conserved
- Localized

Chern-Simons term and Page charge

Action - 5D

$$\mathcal{S} = \int F \wedge \star F + A \wedge F \wedge F + [\ldots]$$

$$d \star F = -\frac{3}{2} F \wedge F \qquad \Longleftrightarrow \qquad d(\star F + \frac{3}{2} A \wedge F) = 0$$

Maxwell charge

- Conserved
- Not localized

$$Q = \int d(\ldots)$$

Page charge

- Conserved
- Localized

Chern-Simons term and Page charge

Action - 5D

$$\mathcal{S} = \int F \wedge \star F + A \wedge F \wedge F + [\dots]$$

$$d \star F = -\frac{3}{2} F \wedge F \qquad \Longleftrightarrow \qquad d(\star F + \frac{3}{2} A \wedge F) = 0$$

Maxwell charge

- Conserved
- Not localized

$$Q \equiv \int d(\dots)$$

Page charge

- Conserved
- Localized

Chern-Simons term and Page charge

Action - 5D

$$\mathcal{S} = \int F \wedge \star F + A \wedge F \wedge F + [\dots]$$

$$d \star F = -\frac{3}{2} F \wedge F \qquad \Longleftrightarrow \qquad d(\star F + \frac{3}{2} A \wedge F) = 0$$

Maxwell charge

- Conserved
- Not localized

$$Q \equiv \int d(\dots)$$

Page charge

- Conserved
- Localized

Chern-Simons term and Page charge

Action - 5D

$$\mathcal{S} = \int F \wedge \star F + A \wedge F \wedge F + [\ldots]$$

$$d \star F = -\frac{3}{2} F \wedge F \quad \Longleftrightarrow \quad d(\star F + \frac{3}{2} A \wedge F) = 0$$

Maxwell charge

- Conserved
- Not localized

$$Q \equiv \int d(\ldots)$$

Page charge

- Conserved
- Localized

Dimensional reduction [Cardoso, Oberreuter, Perz; Goldstein, Jena]

- Establish connection between 5D and 4D black holes

Entropy function with CS-terms (1)

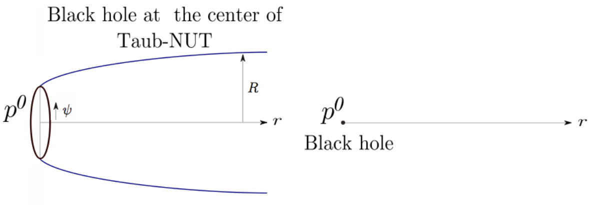
entropy function 5D black hole

=

entropy function associated 4D black hole

Dimensional reduction [Cardoso, Oberreuter, Perz; Goldstein, Jena]

- Establish connection between 5D and 4D black holes



Entropy function with CS-terms (1)

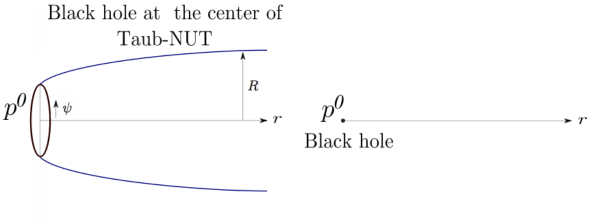
entropy function 5D black hole

$=$

entropy function associated 4D black hole

Dimensional reduction [Cardoso, Oberreuter, Perz; Goldstein, Jena]

- Establish connection between 5D and 4D black holes



Entropy function with CS-terms (1)

entropy function 5D black hole

$$\equiv$$

entropy function associated 4D black hole

Direct 5D procedure [Arsiwalla]

- Entropy function defined analogously as in 4D
- Integrate Lagrangian along horizon of black hole
- Charges conjugate to e-fields
- Entropy function as Legendre transform

4D

- $\mathcal{F}_4$
- $q = \partial\mathcal{F}_4/\partial e$
- $\mathcal{E}_4 \equiv qe - \mathcal{F}_4$

$\longrightarrow$

5D

- $\mathcal{F}_5$
- $Q \equiv \partial\mathcal{F}_5/\partial e$
- $\mathcal{E}_5 \equiv Qe - \mathcal{F}_5$

Direct 5D procedure [Arsiwalla]

- Entropy function defined analogously as in 4D
- Integrate Lagrangian along horizon of black hole
- Charges conjugate to e-fields
- Entropy function as Legendre transform

4D

- $\mathcal{F}_4$
- $q = \partial\mathcal{F}_4/\partial e$
- $\mathcal{E}_4 \equiv qe - \mathcal{F}_4$

$\longrightarrow$

5D

- $\mathcal{F}_5$
- $Q = \partial\mathcal{F}_5/\partial e$
- $\mathcal{E}_5 \equiv Qe - \mathcal{F}_5$

Direct 5D procedure [Arsiwalla]

- Entropy function defined analogously as in 4D
- Integrate Lagrangian along horizon of black hole
- Charges conjugate to e-fields
- Entropy function as Legendre transform

4D

- $\mathcal{F}_4$
- $q = \partial\mathcal{F}_4/\partial e$
- $\mathcal{E}_4 = qe - \mathcal{F}_4$

$\longrightarrow$

5D

- $\mathcal{F}_5$
- $Q \equiv \partial\mathcal{F}_5/\partial e$
- $\mathcal{E}_5 = Qe - \mathcal{F}_5$

Direct 5D procedure [Arsiwalla]

- Entropy function defined analogously as in 4D
- Integrate Lagrangian along horizon of black hole
- Charges conjugate to e-fields
- Entropy function as Legendre transform

4D

- $\mathcal{F}_4$
- $q = \partial\mathcal{F}_4/\partial e$
- $\mathcal{E}_4 \equiv qe - \mathcal{F}_4$

$\longrightarrow$

5D

- $\mathcal{F}_5$
- $Q \equiv \partial\mathcal{F}_5/\partial e$
- $\mathcal{E}_5 \equiv Qe - \mathcal{F}_5$

Direct 5D procedure [Arsiwalla]

- Entropy function defined analogously as in 4D
- Integrate Lagrangian along horizon of black hole
- Charges conjugate to e-fields
- Entropy function as Legendre transform

4D

- $\mathcal{F}_4$
- $q = \partial \mathcal{F}_4 / \partial e$
- $\mathcal{E}_4 \equiv qe - \mathcal{F}_4$

$\longrightarrow$

5D

- $\mathcal{F}_5$
- $Q \equiv \partial \mathcal{F}_5 / \partial e \quad (?)$
- $\mathcal{E}_5 \equiv Qe - \mathcal{F}_5$

Direct 5D procedure - discussion

- Q the Page charge?
  - Mismatch in relative factors due to CS-terms
  - Explicit appearance of gauge potentials

When action depends explicitly on gauge potentials the conjugate charges are not conserved quantities.

- EF as Legendre transform w.r.t. e-fields not possible

Direct 5D procedure - discussion

- Q the Page charge? No.
  - Mismatch in relative factors due to CS-terms
  - Explicit appearance of gauge potentials

When action depends explicitly on gauge potentials the conjugate charges are not conserved quantities.

- EF as Legendre transform w.r.t. e-fields not possible

Direct 5D procedure - discussion

- Q the Page charge? No.
  - Mismatch in relative factors due to CS-terms
  - Explicit appearance of gauge potentials

When action depends explicitly on gauge potentials the conjugate charges are not conserved quantities.

- EF as Legendre transform w.r.t. e-fields not possible

Direct 5D procedure - discussion

- Q the Page charge? No.
  - Mismatch in relative factors due to CS-terms
  - Explicit appearance of gauge potentials

When action depends explicitly on gauge potentials the conjugate charges are not conserved quantities.

- EF as Legendre transform w.r.t. e-fields not possible

Direct 5D procedure - discussion

- Q the Page charge? No.
  - Mismatch in relative factors due to CS-terms
  - Explicit appearance of gauge potentials

When action depends explicitly on gauge potentials the conjugate charges are not conserved quantities.

- EF as Legendre transform w.r.t. e-fields not possible
- However: Page charge seems right quantity for EF
  - Consistency with attractor mechanism

Conclusion

- We compared two proposals to extend the entropy function to BH solutions in the presence of CS-terms (5D).
- Page charge is the natural quantity to use in a 5D entropy function as it is measurable at the horizon.
- Entropy function defined as a Legendre transform w.r.t. the electric fields seems not possible due to explicit gauge potentials in the action.

- Outlook
  - Dimensional reduction of black holes with other asymptotic geometries
  - Entropy function not as Legendre transform in five dimensions