Dark energy as a kinematic effect

Merging de Sitter special relativity with teleparallel gravity

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Outline

Cosmological principle

- Homogeneous and isotropic universe at large scales
  - $\sim$ 300 million light-years

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Friedman-Lemaître-Robertson-Walker

$$ds^2 = dt^2 - a(t)^2\big[d\chi^2 + f_k(\chi)^2\, d\Omega_2\big]$$

- Comoving coordinates
- Symmetric universe / cosmological principle
- Space with constant curvature
  - Spherical for $k = 1$ $[f_k = \sin\chi]$
  - Flat for $k = 0$ $[f_k = \chi]$
  - Hyperbolic for $k = -1$ $[f_k = \sinh\chi]$
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Present-day accelerated expansion

- Determine evolution of scale factor today (t_*)
  - No a-priori assumptions on dynamical model
- Luminosity distance d_L vs. redshift z for SNe Ia
  - $d_L^2 \sim L_{\text{absolute}}/F_{\text{observed}}$
  - $z = a_*/a - 1$

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Hubble law

$$d_L(z) = \frac{z}{H_*} + [1 - q_*]\frac{z^2}{2H_*} - \left[1 - q_* - 3q_*^2 + j_* + \frac{k}{a_*^2 H_*^2}\right]\frac{z^3}{6H_*}$$

- $H = \dot{a}/a, \quad q = -\ddot{a}a/\dot{a}^2, \quad j = \dddot{a}a^2/\dot{a}^3, \quad \ldots$
- Inflation $\Longrightarrow$ negligible curvature term
- Evidence for accelerated expansion $\longleftrightarrow$ $\dot{a}_*, \ddot{a}_* > 0$

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Dark energy

Einstein's equations

$$R_{\mu\nu} - g_{\mu\nu} R + g_{\mu\nu} \Lambda = T_{\mu\nu}$$

- No known component of repulsive gravitation
- Expansion driven by dark energy
- Cosmological constant
- Vacuum solution given by de Sitter space
- Change of large-scale kinematic group
  - $ISO(1,3) \longrightarrow SO(1,4)$
  - Degree of deformation depends on Λ
  - de Sitter special relativity

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Working objective

Kinematic group at every point is $SO(1,4)$

- Pointwise deformation $ISO(1,3) \xrightarrow{\Lambda} SO(1,4)$
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- How to implement mathematically?
  - Cartan geometry modeled on de Sitter space
- How to implement physically?
  - Generalize teleparallel gravity
  - Deform curvature of spin connection

Klein geometry

Klein geometry

- Geometry of homogeneous spaces
  - Minkowski space, de Sitter space, ...
- Symmetry Lie group $G \longrightarrow$ transitivity
  - $ISO(1,3)$, $SO(1,4)$, ...
- Subgroup $H \subset G$ defines isotropy around any point
  - $SO(1,3)$, $SO(1,3)$, ...
- Homogeneous space $\longleftrightarrow G/H$

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Minkowski space $ISO(1,3)$ $\xrightarrow{\Lambda}$ de Sitter space $SO(1,4)$

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Cartan geometry

- Let G/H be homogeneous model space
- Give up homogeneity, retain symmetry locally
- Endow manifold M with Cartan connection A
  - 1-form valued in $\mathfrak{g}$
  - $\dim M = \dim G/H$

Cartan connection

$$A : TM \mapsto \mathfrak{g} = \mathfrak{h} \oplus \mathfrak{g}/\mathfrak{h}$$

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- $G = SO(3)$, $H = SO(2)$
- For each direction, A returns an element of $\mathfrak{so}(3)$

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- $A_{\mathfrak{h}}$ is H-connection
- $A_{\mathfrak{p}}$ is vielbein $\longrightarrow$ soldering

Cartan curvature

$$F = dA + \frac{1}{2}[A, A]$$

- $F_{\mathfrak{h}}$ is curvature
- $F_{\mathfrak{p}}$ is torsion
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- Generic spacetime with locally $SO(1,4)$ kinematics
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$$\mathfrak{so}(1,4) = \mathfrak{so}(1,3) \oplus \mathfrak{p}$$

- de Sitter translations $\quad [\mathfrak{p} = \mathfrak{so}(1,4)/\mathfrak{so}(1,3)]$

Commutation relations

$$\begin{aligned} [\mathfrak{so}(1,3), \mathfrak{so}(1,3)] &\subset \mathfrak{so}(1,3) \\ [\mathfrak{so}(1,3), \mathfrak{p}] &\subset \mathfrak{p} \\ [\mathfrak{p}, \mathfrak{p}] &\subset \mathfrak{so}(1,3) \end{aligned}$$

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  - $\Lambda \to 0$ gives Minkowski space
  - $\mathfrak{p} \longrightarrow$ Poincaré translations

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Spin connection & vierbein

$$\begin{tikzcd} & \mathfrak{so}(1,3) \arrow[r] & \omega^a{}_{b\mu} \\ A \arrow[ur, dash, bend left] \arrow[dr, dash, bend right] & & \\ & \mathfrak{p} \arrow[r] & e^a{}_\mu \end{tikzcd}$$

Curvature & torsion

$$\begin{tikzcd} & \mathfrak{so}(1,3) \arrow[r] & R^a{}_{b\mu\nu} \\ F \arrow[ur, dash, bend left] \arrow[dr, dash, bend right] & & \\ & \mathfrak{p} \arrow[r] & T^a{}_{\mu\nu} \end{tikzcd}$$

- Decompose according to $\mathfrak{so}(1,4) = \mathfrak{so}(1,3) \oplus \mathfrak{p}$
- Local $SO(1,3)$ invariance
- $F = dA + \frac{1}{2}[A, A]$

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Introduction to teleparallel gravity

- Description for classical gravity
- Predictions equivalent to general relativity
- Mathematical structure is related but different
  - Riemann–Cartan spacetime without curvature
  - Gravitational degrees of freedom encoded in torsion
- Conceptually quite unlike general relativity
  - No geometrization of gravitational interaction
  - Restores the concept of force
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- Gauge theory for the Poincaré translations
  - Nonlinear realization of Riemann–Cartan connection
  - Not further considered

Geometric objects

- Kinematics ruled by Poincaré group
  - Riemann–Cartan geometry

$$B^a{}_{b\mu\nu} = \partial_\mu \omega^a{}_{b\nu} - \partial_\nu \omega^a{}_{b\mu} + \omega^a{}_{c\mu}\omega^c{}_{b\nu} - \omega^a{}_{c\nu}\omega^c{}_{b\mu}$$
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Ricci theorem

$$\omega^a{}_{b\mu} = \mathring{\omega}^a{}_{b\mu} + K^a{}_{b\mu}$$

- Levi-Civita connection
- Contortion

- General relativity $\longrightarrow G^a{}_{\mu\nu} \equiv 0$
  - $K^a{}_{b\mu} = 0$
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  - Both inertial and gravitational effects in spin connection

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- Teleparallel gravity $\longrightarrow B^a{}_{b\mu\nu} \equiv 0$
- $\omega^a{}_{b\mu} = \Lambda^a{}_c \partial_\mu \Lambda_b{}^c$
  - Spin connection encodes inertial effects only
- Torsion represents gravitational degrees of freedom
- 10 off-shell degrees of freedom
  - Same as in Riemannian spacetime
- Dictionary general relativity – teleparallel gravity

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Particle mechanics

Action

$$-m\int d\tau = -m\int u_a e^a$$

- Mass m in gravitational field
- Nonzero torsion

Equations of motion

$$u^\rho D_\rho u^a = K^a{}_{b\rho} u^b u^\rho$$

- Inertial vs. gravitational effects
- Force equation

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- Equivalence with geodesic equation

$$u^\rho \partial_\rho u^a + [\underbrace{\omega^a{}_{b\rho} - K^a{}_{b\rho}}_{\mathring{\omega}^a{}_{b\rho}}]u^b u^\rho = 0$$

Gravitional field equations

Lagrangian teleparallel gravity

$$\mathcal{L}_{\text{tg}} = \frac{1}{4} G^{\rho}{}_{\mu\nu} \, G_{\rho}{}^{\mu\nu} + \frac{1}{2} G^{\rho}{}_{\mu\nu} \, G^{\nu\mu}{}_{\rho} - G^{\nu}{}_{\mu\nu} \, G^{\rho\mu}{}_{\rho}$$

Field equations

$$D_{\rho}(e \, W_{a}{}^{\rho\mu}) + e \, t_{a}{}^{\mu} = 0$$

- Superpotential $W_{a}{}^{\rho\mu}$
- Gravitational energy-momentum current $t_{a}{}^{\mu}$
  - Conserved charges $q_a = \int t_{a}{}^{0} \longrightarrow \dot{q}_a = 0$
- Equivalence with general relativity
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Lagrangian teleparallel gravity

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- Deform local kinematics $ISO(1,3) \longrightarrow SO(1,4)$
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- de Sitter teleparallel gravity
- Deform the curvature of spin connection
  - Minkowski space $\longrightarrow$ de Sitter space
  - Deformation specified by Λ
  - Kinematic contribution to deviation equation
- Gravity relates to torsion of vierbein
- Cosmological function acquires own dynamics

Fundamentals

- Kinematics ruled by de Sitter group
  - de Sitter–Cartan geometry

$$R^a{}_{b\mu\nu} = B^a{}_{b\mu\nu} + \frac{\Lambda}{3}(e^a{}_\mu e_{b\nu} - e^a{}_\nu e_{b\mu})$$

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Particle mechanics & kinematic effects

Action

$$-m \int u_a e^a$$

Equations of motion

$$u^\rho D_\rho u^a = K^a{}_{b\rho}\, u^b u^\rho$$

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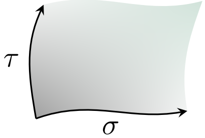

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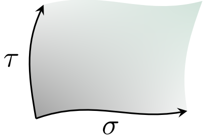

Relative acceleration free-falling particles

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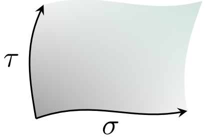

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Dynamics of de Sitter teleparallel gravity

Lagrangian de Sitter teleparallel gravity

$$\mathcal{L}_{\mathsf{dStg}} = \mathcal{L}_{\mathsf{tg}} - \frac{3}{2}\partial_\mu \ln\Lambda\, \partial^\mu \ln\Lambda - 2\, G^{\mu\nu}{}_{\mu}\, \partial_\nu \ln\Lambda$$

- Gravitational sector modeled by teleparallel gravity
- Kinetic term for the cosmological function
- Interaction through nonminimal coupling
- Similarities with theories of modified gravity
  - Teleparallel dark energy
  - Scalar-tensor theories
- Important differences
  - Λ deforms local kinematics
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Field equations

Gravitational field

$$e^{-1} D_\rho(e\, W_a{}^{\rho\mu}) + t_a{}^\mu - 2G^{\rho\mu}{}_\rho\, e_a{}^\nu \partial_\nu \ln\Lambda - 2e_a{}^\mu \Box \ln\Lambda$$
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Cosmological function

$$\Box \ln\Lambda + G^{\mu\rho}{}_\mu\, \partial_\rho \ln\Lambda = -\frac{2}{3}(\nabla_\mu G^{\rho\mu}{}_\rho + G^\mu{}_{\rho\mu}\, G^{\nu\rho}{}_\nu)$$

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  - Point-dependent length scale in de Sitter translations
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