QUEEN'S UNIVERSITY

KINGSTON, ONTARIO

Faculty of Engineering and Applied Science

APSC221 - Economics and Business Practices in Engineering - Winter 2013

SECTION 100

Final Examination – April 24, 2013

INSTRUCTOR – Jan Sneep

This examination constitutes 60% of your final course mark.

Time available **3** hours. The test is out of a total of **72** marks. (Average of 2.5 minutes per mark)

INSTRUCTIONS:

1. Answers to ALL questions must be written in the answer booklet that is provided.
    a. Put your name and student number on the front and all pages of all answer booklets.
    b. Do not remove any pages from the booklet that is provided.
    c. If there is insufficient space, ask the proctor for another answer booklet.
2. Do not use RED COLOUR for your presentation in any form, written or pictorial; this colour is reserved for marking your submission.
3. **ONLY** the use of a University-approved, non-programmable, non-communicating calculator will be allowed.
    - Blue sticker, Gold sticker or, the approved model, Casio 991.
4. Formula sheet, Interest tables, and Standard Normal Distribution table are provided at the back of the exam.
5. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer a clear statement of any assumptions made. **Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.**
6. Marks will be deducted if appropriate interest factors, their designations and appropriate values of interest rate (i) and period (n) are not indicated. You are strongly encouraged to provide cash flow diagrams where appropriate.
7. You must hand in **all** answer booklets, questions sheets and reference sheets on completion of the test.

There will be penalties if any of the above instructions are not followed.

GOOD LUCK.

APSC 221 - Engineering Economics
Winter 2013

Final Examination – Wednesday April 24, 2013

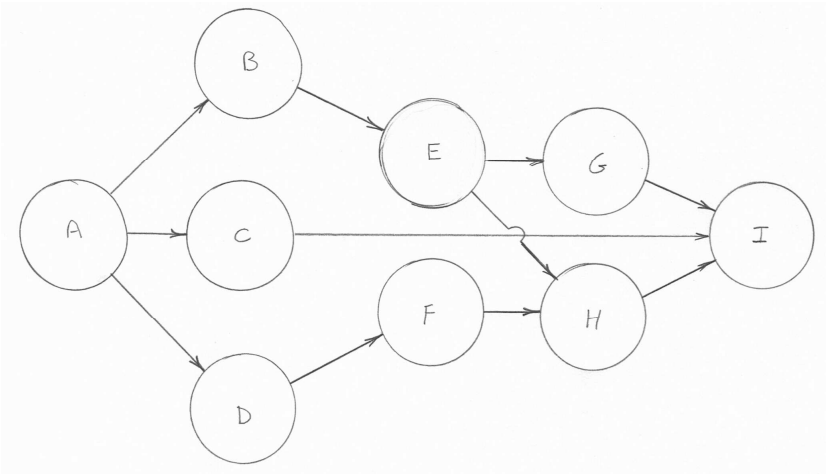
DO NOT ANSWER ON THIS TEST PAPER, All work and answers must be shown in your examination booklet(s).

Question 1: Scheduling (10 marks – 14%)

a) Use the information in the table below to calculate the Expected Time and Variance for each activity using the PERT method. Record your answers using a table similar to the one below in your answer booklet. (2.5 marks)

Activity	Optimistic (weeks)	Most Likely (weeks)	Pessimistic (weeks)	Expected Time	Variance
A	3	5	7		
B	4.5	5	5.5		
C	9	11	19		
D	4	6	8		
E	2	3	10		
F	1	2	3	2	0.11
G	4	5.5	10	6	1.00
H	5	6.5	11	7	1.00
I	1.4	2.4	7	3	0.87

b) Use the expected values above and the activity-on-node diagram below to determine the critical path activities and the expected number of weeks to complete the project. Hint: The expected time is the same as the activity duration. Use the forward pass and backward pass of the CPM method to calculate the earliest start, earliest finish, latest finish and latest start times to determine which activities are on the critical path. (3.5 marks)



c) Determine the probability that the project completes in less than or equal to 23 weeks. (2 marks)

d) It is company tradition to hold a celebration party after a project completes. Assume that crews only work on regular business days, i.e. Monday through Friday and there are no holidays during the project so every week has exactly five days. If you want to be 98% confident, what week and day of the week do you tell your co-workers to book in their calendars for the party? For example your answer should be something like "Thursday of week 23". (2 marks)

Question 2: Time Value of Money (10 marks – 14%)

a) What is the amount of interest earned on $800 for 5 years at 10.5% simple interest? (½ mark)
b) What is the amount of interest earned on $800 for 5 years at 10.5% compounded monthly? (½ mark)
c) How much would you have to deposit every month to have the $1,350 in the bank if the term, interest rate and compounding are the same as in part b) above? (½ mark)
d) How much interest have you earned in c) ? (½ mark)
e) Find the 7 uniform annual deposits that can provide a single withdrawal of $48,500.00, 2 years after the last deposit is made at an interest rate of 8% per year. (1 mark)
f) $152,000 was deposited in a savings account 7 years ago, and the account earned interest at the rate of 8% per year. What is the amount of equal annual withdrawals that can be made to completely deplete the fund 7 years from now if the first withdrawal will be made now? (1 mark)
g) What value of G makes the two series of cash flows described below equivalent to each other at an interest rate of 7.77% per year, compounded every 3 months? (4 marks)
   Cash Flow A: 22 annual deposits in the amount of $200
   Cash Flow B: 11 annual deposits, starting with $200 in the first year, and increasing by $G each year.
h) A small electronics manufacturer expects total revenues to decrease over the next 4 years due to the recession. The company forecasts that the revenue will be $19 million in the first year, but will decline by $2 million every year for the next 3 years. What is the present worth of total revenue at an interest rate of 11.387% per year, compounded monthly? (2 marks)

Question 3: Cost Concepts (5 marks – 7%)

You are deciding between three types of space heaters for a winter construction project. The associated costs are shown below. The cost of operation for propane and oil heaters is estimated by (41,045/Efficiency Factor) x Fuel Cost per Btu per day, and the cost to operate the electric heaters is estimated by (12.03/Efficiency Factor) x Electricity Cost per kWh per day. The selected heater will be used December 1 to March 31 (31 + 31 + 28 + 31 = 121 days) then sold at the market value. Which alternative should be selected?

Alternative	Electric	Gas	Oil
Price of space heater	$28,000	$23,500	$25,000
Efficiency Factor	2.0	0.57	0.75
Fuel Cost	$0.16/kWh	$0.00001/Btu	$0.00001/Btu
Maintenance costs	$300	$220	$250
Market value	$25,200	$20,700	$22,500

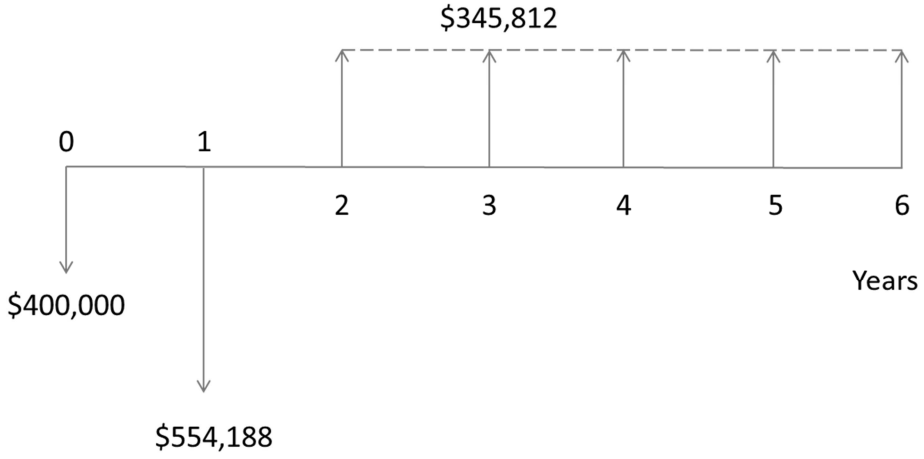
Question 4: Comparison Methods (5 marks – 7%)

Three mutually exclusive projects are under consideration for an irrigation system. The first is a concrete reservoir with a steel pipe system and the second is an earthen dam with a wooden aqueduct. Both projects have essentially infinite lives, provided adequate maintenance is performed. Or you could buy a truck and haul water from a nearby lake. Calculate the present worths of the alternatives using an interest rate of 8%. What assumption do you need to make to be able to perform the comparison? Which alternative should be chosen?

	Concrete Reservoir	Earthen Dam	Truck
First Cost	$500,000	$200,000	$50,000
Annual costs	$2,000	$12,000	$25,000
Replace wooden aqueduct every 15 years.	N/A	$100,000	N/A
Salvage at 10 years	N/A	N/A	$2,000

Question 5: Internal Rate of Return (5 marks – 7%)

Determine the internal rate of return for the project depicted by the cash flow diagram below.



Question 6: Inflation (3 marks – 4%)

Wexler Mining Systems wants to determine when it should buy a second new front-end loader. They can buy a new one for $200,000 today or sign a contract to pay $300,000 for a new one three years from now. Inflation is currently 1.786% and is expected to rise to 2.679% a year from now. From an economic perspective should the company buy now or buy later if their real MARR is 12%?

Question 7: Taxes (4 marks – 6%)

A new energy efficient boiler costs $55,000 and would save $17,000 per year compared to the existing one. Salvage value is estimated to be $1,000 at the end of its 15 year life. The CCA rate is 20% and the corporate tax rate is 38%. The after-tax MARR is 12%. What is the after-tax present worth of the new boiler?

Question 8: Financial Accounting and Capital Budgeting (8 marks – 11%)

Investing in equity is more risky than investing in debt. Equity owners are paid only if the company first meets its contractual obligations to lenders. This higher risk means that equity owners have an expectation of a greater return on average than the interest rate paid to debt holders. Consider a simple case in which a company has three possible performance outcomes - weak results, normal results, and strong results. Investors do not know which performance outcome will actually occur. Each outcome is equally probable. Assume that **all** after-tax income is paid to equity holders as dividends. The available data is shown in the following Table.

	Weak Results	Average Results	Strong Results
Net Operating income ($/year)[1]	130,000	200,000	270,000
Interest payments ($/year)	21,000	21,000	21,000
Income before tax ($/year)	109,000	179,000	249,000
Tax at 40% ($/year)	43,600	71,600	99,600
After-tax Net Income = Dividends ($/year)	65,400	107,400	149,400
Debt ($)	150,000	150,000	150,000
Equity = Book value of shares ($)	537,000	537,000	537,000

[1] Net operating income per year is revenue per year minus costs (other than interest) per year.

a) What is the return the lenders are being paid? (½ mark)
b) What are the three possible returns the owners can receive, i.e the three Return on Equity (ROE) ratios? (1.5 marks)
c) What is the weighted average cost of capital (WACC) for this company? (3 marks)
d) If the market returned 8% this year and Canadian Government bonds paid 2%, using the CAPM model what would the three beta (weak, average, strong) values be? (1.5 marks)
e) If Current Liabilities were $45,000 calculate the Equity ratio and the after-tax Return on Assets ratio for the average results? (1.5 marks)

Question 9: Dealing with Uncertainty and Risk (12 marks – 17%)

You are trying to decide whether to buy the patent for a product designed by another company. The decision to buy will mean an investment of around $6 million, however the demand for the product is not known. If demand is light, you expect a return of $1.3 million each year for **three years**. If demand is moderate, you expect the return will be $2.5 million each year for **four years**. If demand is high, then estimates are that you can expect revenues of $4.0 million a year for **five years**. Based on previous investment history you have determined that the probability of light demand is 0.2 and that there is a 0.3 probability of high demand. The corporate MARR is 12%.

a) Calculate the expected **present worth** of the patent. On this basis, should you recommend making the investment? (7 marks)
b) How sensitive is the analysis to changes in the initial cost if your negotiating skills prove to be poor; by what percent does the expected present worth of the patent change for a +10% increase in initial cost? (2.5 marks)
c) Your boss comes to you at the last minute and tells you he has found another mutually exclusive investment with an expected present worth of $2.5 million. You figure that your negotiating skills are only 50/50, meaning that 50 percent of the time you will get the patent deal for $6 million and the other 50 percent of the time you will pay 10% more for the deal. Construct a well labeled decision tree to represent this scenario and make a recommendation to your boss about which deal the company should pursue. (2.5 marks)

Question 10: All Chapters (½ mark for each = 10 marks – 14%)

Write your answers in your exam booklet **NOT** on this page.
Please write the answers in a single column, one answer per line, on a page of its own.
Select the one alternative that best completes the statement or answers the question.

A. Individuals who both recognize and seize opportunities are referred to as
   1) independents.
   2) entrepreneurs.
   3) intrapreneurs.
   4) franchisees.
   5) speculators.

B. Low start-up costs and simplicity of operations are among the main advantages of _____.
   1) a co-operative
   2) a sole proprietorship
   3) selling shares
   4) incorporating
   5) none of these

C. A decrease over time in average prices of goods and services is called _____.
   1) project variance
   2) monetary easing
   3) sunk costs
   4) payback
   5) deflation

D. Project managers face ethical decisions on a daily basis. Which is an example of such?
   1) Offers of gifts from contractors
   2) Pressure to alter status reports
   3) False reports for charges of time
   4) Pressures to compromise quality to meet penalty schedules
   5) all of these

E. When alternatives for accomplishing a specific task are being compared over one year or less they are referred to as Present Economic Studies and _____.
   1) the before-tax MARR is used
   2) only the Present Worth is calculated
   3) the time value of money can be ignored
   4) the present worth of costs is set equal to the present worth of revenues
   5) none of these

F. One significant cost that is often forgotten is the effect on the time of everyone affected by change. Many people in the organisation will need to give up time to _____.
   1) explain how things work to those involved in designing changes
   2) be consulted on the effectiveness and realism of planned change
   3) provide ideas for improvements
   4) be trained and have changes explained to them
   5) all of the above

G. The effective interest rate is the actual but not usually stated interest rate when the compounding period is not calculated on an annual basis. Which of the following statement is true?
   1) Effective will always be above nominal, regardless of the number of compounding periods
   2) Effective will always be below nominal, regardless of the number of compounding periods
   3) Effective will always be above nominal, when there is more than one compound period in the year
   4) Effective will always be below nominal, when there is more than one compound period in the year
   5) none of these

H. The formulas we use for arithmetic gradients always convert the gradients into either a Present or Annual worth cash flow _____ time periods before the first non-zero gradient amount.
   1) one
   2) two
   3) three
   4) four
   6) none of these

I. Which of the following is **NOT** one of the risk management steps?
   1) analysis of probability and consequences
   2) mitigation strategies
   3) planning and scheduling
   4) risk identification
   5) control and documentation

J. If the annual savings are not constant, we can calculate the payback period by _____.
   1) setting the present worth of all costs equal to the present worth of all savings
   2) the ratio of first cost over the first year sum of all savings
   3) accounting for the need to recover capital quickly
   4) deducting each year of savings from the first cost until the first cost is recovered
   5) none of these

K. The text states that a disadvantage of using the internal rate of return method is that _____.
   1) we need to calculate the after-tax MARR
   2) the method used is only an approximation
   3) there may be more than one internal rate of return
   4) interest tables are not provided in fractional amounts
   5) linear interpolation between interest rates is not 100% accurate

L. The CAPM asserts that the return on any stock depends upon its risk relative to the _____.
   1) weighted average cost of capital
   2) the portion of debt financing
   3) market
   4) risk free interest rate
   5) weighted average cost of equity

M. Sensitivity analysis is an approach to project evaluation used to gain a better understanding of how _____.
   1) management decisions effect the likelihood of change management success or failure
   2) uncertainty affects the evaluation by examining changes to project parameters
   3) the probability distribution function changes over time
   4) to assess the consequences, or effect, of its occurrence
   5) to mitigate the severity of negative consequences

N. Companies are taxed on _____.
   1) revenues
   2) net profits
   3) expenses
   4) assets
   5) equity

O. Once an asset has been installed and has been operating for some time, the costs of installation and other expenses incurred up to that time are _____.
   1) are depreciated over the estimated life of the asset after taking into account the half-year rule
   2) called sunk costs and are no longer relevant to any decision to replace the current asset
   3) are referred to as real dollars and the cash flow needs to be adjusted to make them actual
   4) are required to be reported to the Canada Revenue agency so an accurate salvage value can be provided
   5) all of the above

P. Inflation _____.
   1) describes the increase in monetary exchange rates as determined by market forces
   2) is the increase over time in average prices of goods and services
   3) is how the probability of tax rate changes due to environmental forces
   4) assesses the consequences of expanding market share
   5) mitigates the severity of negative consequences

Q. One particularly useful summary statistic is the expected value, which is the _____ of a random variable.
   1) variance
   2) mode
   3) mean
   4) standard deviation
   5) range

R. A project organization can choose to _____ risk.
   1) accept
   2) minimize
   3) share
   4) transfer
   5) all of the above

S. What are the two dimensions of change management?
   1) organisational and strategic
   2) human and organisational
   3) tactical and strategic
   4) stories and legends
   5) business and competitive

T. When conducting a replacement analysis, where the defender and challenger are identical, what condition needs to be true?
   1) Equivalent annual cost of capital must be increasing over time.
   2) Total annual costs can be described as a simple parabola ($Y=aX^2+bX+c$).
   3) Revenues are the same for both alternatives.
   4) Sunk costs are known and can be incorporated in the analysis.
   5) Equivalent annual cost of maintenance expenses must be decreasing.

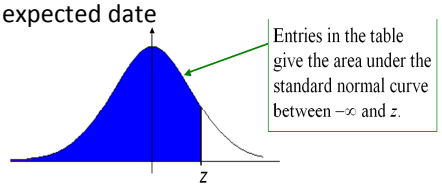
$$z = \frac{x - \mu}{\sigma}$$

X = due date or target date

μ = mean or expected date

σ = Project Standard Deviation

Standard Normal Distribution Table



z	0.0000	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700	0.0800	0.0900
-3.0000	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9000	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8000	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7000	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6000	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5000	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4000	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3000	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2000	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1000	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0000	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9000	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8000	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7000	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6000	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5000	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4000	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3000	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2000	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1000	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0000	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9000	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8000	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7000	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6000	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5000	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4000	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3000	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2000	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1000	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0000	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0000	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1000	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2000	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3000	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4000	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5000	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6000	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7000	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8000	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9000	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0000	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1000	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2000	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3000	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4000	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5000	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6000	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7000	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8000	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9000	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0000	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1000	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2000	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3000	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4000	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5000	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6000	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7000	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8000	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9000	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0000	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

i = **0.875%**

n	[F/P, i, n]	[P/F, i, n]	[F/A, i, n]	[A/F, i, n]	[P/A, i, n]	[A/P, i, n]	[A/G, i, n]	[P/G, i, n]	n
1	1.00875	0.99133	1.00000	1.00000	0.99133	1.00875	0.00000	0.00000	1
2	1.01758	0.98273	2.00875	0.49782	1.97405	0.50657	0.49782	0.98273	2
3	1.02648	0.97420	3.02633	0.33043	2.94826	0.33918	0.99419	2.93113	3
4	1.03546	0.96575	4.05281	0.24674	3.91401	0.25549	1.48911	5.82839	4
5	1.04452	0.95738	5.08827	0.19653	4.87138	0.20528	1.98258	9.65789	5
6	1.05366	0.94907	6.13279	0.16306	5.82045	0.17181	2.47459	14.40325	6
7	1.06288	0.94084	7.18645	0.13915	6.76129	0.14790	2.96515	20.04828	7
8	1.07218	0.93268	8.24933	0.12122	7.69397	0.12997	3.45427	26.57702	8
9	1.08156	0.92459	9.32152	0.10728	8.61856	0.11603	3.94193	33.97372	9
10	1.09103	0.91657	10.40308	0.09613	9.53513	0.10488	4.42814	42.22283	10
11	1.10057	0.90862	11.49411	0.08700	10.44374	0.09575	4.91289	51.30901	11
12	1.11020	0.90074	12.59468	0.07940	11.34448	0.08815	5.39620	61.21710	12
13	1.11992	0.89292	13.70488	0.07297	12.23740	0.08172	5.87806	71.93217	13
14	1.12972	0.88518	14.82480	0.06745	13.12258	0.07620	6.35847	83.43948	14
15	1.13960	0.87750	15.95452	0.06268	14.00008	0.07143	6.83742	95.72447	15
16	1.14957	0.86989	17.09412	0.05850	14.86997	0.06725	7.31493	108.77279	16
17	1.15963	0.86234	18.24369	0.05481	15.73231	0.06356	7.79099	122.57026	17
18	1.16978	0.85486	19.40333	0.05154	16.58717	0.06029	8.26560	137.10292	18
19	1.18001	0.84745	20.57311	0.04861	17.43462	0.05736	8.73876	152.35697	19
20	1.19034	0.84010	21.75312	0.04597	18.27471	0.05472	9.21047	168.31880	20
21	1.20076	0.83281	22.94346	0.04359	19.10752	0.05234	9.68074	184.97498	21
22	1.21126	0.82559	24.14421	0.04142	19.93311	0.05017	10.14956	202.31227	22
23	1.22186	0.81842	25.35548	0.03944	20.75153	0.04819	10.61693	220.31760	23
24	1.23255	0.81132	26.57734	0.03763	21.56286	0.04638	11.08286	238.97808	24
25	1.24334	0.80429	27.80989	0.03596	22.36715	0.04471	11.54734	258.28098	25
30	1.29869	0.77001	34.13632	0.02929	26.28514	0.03804	13.84807	363.99852	30
35	1.35651	0.73718	40.74442	0.02454	30.03613	0.03329	16.11276	483.96484	35
40	1.41691	0.70576	47.64672	0.02099	33.62723	0.02974	18.34148	616.77317	40
50	1.54589	0.64688	62.38694	0.01603	40.35676	0.02478	22.69145	915.75351	50
60	1.68660	0.59291	78.46891	0.01274	46.52483	0.02149	26.89897	1251.46988	60

i = **8.00%**

n	[F/P, i, n]	[P/F, i, n]	[F/A, i, n]	[A/F, i, n]	[P/A, i, n]	[A/P, i, n]	[A/G, i, n]	[P/G, i, n]	n
1	1.08000	0.92593	1.00000	1.00000	0.92593	1.08000	0.00000	0.00000	1
2	1.16640	0.85734	2.08000	0.48077	1.78326	0.56077	0.48077	0.85734	2
3	1.25971	0.79383	3.24640	0.30803	2.57710	0.38803	0.94874	2.44500	3
4	1.36049	0.73503	4.50611	0.22192	3.31213	0.30192	1.40396	4.65009	4
5	1.46933	0.68058	5.86660	0.17046	3.99271	0.25046	1.84647	7.37243	5
6	1.58687	0.63017	7.33593	0.13632	4.62288	0.21632	2.27635	10.52327	6
7	1.71382	0.58349	8.92280	0.11207	5.20637	0.19207	2.69366	14.02422	7
8	1.85093	0.54027	10.63663	0.09401	5.74664	0.17401	3.09852	17.80610	8
9	1.99900	0.50025	12.48756	0.08008	6.24689	0.16008	3.49103	21.80809	9
10	2.15892	0.46319	14.48656	0.06903	6.71008	0.14903	3.87131	25.97683	10
11	2.33164	0.42888	16.64549	0.06008	7.13896	0.14008	4.23950	30.26566	11
12	2.51817	0.39711	18.97713	0.05270	7.53608	0.13270	4.59575	34.63391	12
13	2.71962	0.36770	21.49530	0.04652	7.90378	0.12652	4.94021	39.04629	13
14	2.93719	0.34046	24.21492	0.04130	8.24424	0.12130	5.27305	43.47228	14
15	3.17217	0.31524	27.15211	0.03683	8.55948	0.11683	5.59446	47.88566	15
16	3.42594	0.29189	30.32428	0.03298	8.85137	0.11298	5.90463	52.26402	16
17	3.70002	0.27027	33.75023	0.02963	9.12164	0.10963	6.20375	56.58832	17
18	3.99602	0.25025	37.45024	0.02670	9.37189	0.10670	6.49203	60.84256	18
19	4.31570	0.23171	41.44626	0.02413	9.60360	0.10413	6.76969	65.01337	19
20	4.66096	0.21455	45.76196	0.02185	9.81815	0.10185	7.03695	69.08979	20
21	5.03383	0.19866	50.42292	0.01983	10.01680	0.09983	7.29403	73.06291	21
22	5.43654	0.18394	55.45676	0.01803	10.20074	0.09803	7.54118	76.92566	22
23	5.87146	0.17032	60.89330	0.01642	10.37106	0.09642	7.77863	80.67259	23
24	6.34118	0.15770	66.76476	0.01498	10.52876	0.09498	8.00661	84.29968	24
25	6.84848	0.14602	73.10594	0.01368	10.67478	0.09368	8.22538	87.80411	25
30	10.06266	0.09938	113.28321	0.00883	11.25778	0.08883	9.18971	103.45579	30
35	14.78534	0.06763	172.31680	0.00580	11.65457	0.08580	9.96107	116.09199	35
40	21.72452	0.04603	259.05652	0.00386	11.92461	0.08386	10.56992	126.04220	40
50	46.90161	0.02132	573.77016	0.00174	12.23348	0.08174	11.41071	139.59279	50
60	101.25706	0.00988	1253.21330	0.00080	12.37655	0.08080	11.90154	147.30001	60

i = **12.00%**

n	[F/P, i, n]	[P/F, i, n]	[F/A, i, n]	[A/F, i, n]	[P/A, i, n]	[A/P, i, n]	[A/G, i, n]	[P/G, i, n]	n
1	1.12000	0.89286	1.00000	1.00000	0.89286	1.12000	0.00000	0.00000	1
2	1.25440	0.79719	2.12000	0.47170	1.69005	0.59170	0.47170	0.79719	2
3	1.40493	0.71178	3.37440	0.29635	2.40183	0.41635	0.92461	2.22075	3
4	1.57352	0.63552	4.77933	0.20923	3.03735	0.32923	1.35885	4.12731	4
5	1.76234	0.56743	6.35285	0.15741	3.60478	0.27741	1.77459	6.39702	5
6	1.97382	0.50663	8.11519	0.12323	4.11141	0.24323	2.17205	8.93017	6
7	2.21068	0.45235	10.08901	0.09912	4.56376	0.21912	2.55147	11.64427	7
8	2.47596	0.40388	12.29969	0.08130	4.96764	0.20130	2.91314	14.47145	8
9	2.77308	0.36061	14.77566	0.06768	5.32825	0.18768	3.25742	17.35633	9
10	3.10585	0.32197	17.54874	0.05698	5.65022	0.17698	3.58465	20.25409	10
11	3.47855	0.28748	20.65458	0.04842	5.93770	0.16842	3.89525	23.12885	11
12	3.89598	0.25668	24.13313	0.04144	6.19437	0.16144	4.18965	25.95228	12
13	4.36349	0.22917	28.02911	0.03568	6.42355	0.15568	4.46830	28.70237	13
14	4.88711	0.20462	32.39260	0.03087	6.62817	0.15087	4.73169	31.36242	14
15	5.47357	0.18270	37.27971	0.02682	6.81086	0.14682	4.98030	33.92017	15
16	6.13039	0.16312	42.75328	0.02339	6.97399	0.14339	5.21466	36.36700	16
17	6.86604	0.14564	48.88367	0.02046	7.11963	0.14046	5.43530	38.69731	17
18	7.68997	0.13004	55.74971	0.01794	7.24967	0.13794	5.64274	40.90798	18
19	8.61276	0.11611	63.43968	0.01576	7.36578	0.13576	5.83752	42.99790	19
20	9.64629	0.10367	72.05244	0.01388	7.46944	0.13388	6.02020	44.96757	20
21	10.80385	0.09256	81.69874	0.01224	7.56200	0.13224	6.19132	46.81876	21
22	12.10031	0.08264	92.50258	0.01081	7.64465	0.13081	6.35141	48.55425	22
23	13.55235	0.07379	104.60289	0.00956	7.71843	0.12956	6.50101	50.17759	23
24	15.17863	0.06588	118.15524	0.00846	7.78432	0.12846	6.64064	51.69288	24
25	17.00006	0.05882	133.33387	0.00750	7.84314	0.12750	6.77084	53.10464	25
30	29.95992	0.03338	241.33268	0.00414	8.05518	0.12414	7.29742	58.78205	30
35	52.79962	0.01894	431.66350	0.00232	8.17550	0.12232	7.65765	62.60517	35
40	93.05097	0.01075	767.09142	0.00130	8.24378	0.12130	7.89879	65.11587	40
50	289.00219	0.00346	2400.01825	0.00042	8.30450	0.12042	8.15972	67.76241	50
60	897.59693	0.00111	7471.64111	0.00013	8.32405	0.12013	8.26641	68.81003	60

i = **14.00%**

n	[F/P, i, n]	[P/F, i, n]	[F/A, i, n]	[A/F, i, n]	[P/A, i, n]	[A/P, i, n]	[A/G, i, n]	[P/G, i, n]	n
1	1.14000	0.87719	1.00000	1.00000	0.87719	1.14000	0.00000	0.00000	1
2	1.29960	0.76947	2.14000	0.46729	1.64666	0.60729	0.46729	0.76947	2
3	1.48154	0.67497	3.43960	0.29073	2.32163	0.43073	0.91290	2.11941	3
4	1.68896	0.59208	4.92114	0.20320	2.91371	0.34320	1.33701	3.89565	4
5	1.92541	0.51937	6.61010	0.15128	3.43308	0.29128	1.73987	5.97313	5
6	2.19497	0.45559	8.53552	0.11716	3.88867	0.25716	2.12182	8.25106	6
7	2.50227	0.39964	10.73049	0.09319	4.28830	0.23319	2.48324	10.64888	7
8	2.85259	0.35056	13.23276	0.07557	4.63886	0.21557	2.82457	13.10280	8
9	3.25195	0.30751	16.08535	0.06217	4.94637	0.20217	3.14632	15.56286	9
10	3.70722	0.26974	19.33730	0.05171	5.21612	0.19171	3.44903	17.99055	10
11	4.22623	0.23662	23.04452	0.04339	5.45273	0.18339	3.73331	20.35673	11
12	4.81790	0.20756	27.27075	0.03667	5.66029	0.17667	3.99977	22.63988	12
13	5.49241	0.18207	32.08865	0.03116	5.84236	0.17116	4.24909	24.82471	13
14	6.26135	0.15971	37.58107	0.02661	6.00207	0.16661	4.48194	26.90094	14
15	7.13794	0.14010	43.84241	0.02281	6.14217	0.16281	4.69904	28.86229	15
16	8.13725	0.12289	50.98035	0.01962	6.26506	0.15962	4.90110	30.70567	16
17	9.27646	0.10780	59.11760	0.01692	6.37286	0.15692	5.08884	32.43046	17
18	10.57517	0.09456	68.39407	0.01462	6.46742	0.15462	5.26299	34.03800	18
19	12.05569	0.08295	78.96923	0.01266	6.55037	0.15266	5.42429	35.53107	19
20	13.74349	0.07276	91.02493	0.01099	6.62313	0.15099	5.57343	36.91354	20
21	15.66758	0.06383	104.76842	0.00954	6.68696	0.14954	5.71113	38.19006	21
22	17.86104	0.05599	120.43600	0.00830	6.74294	0.14830	5.83807	39.36581	22
23	20.36158	0.04911	138.29704	0.00723	6.79206	0.14723	5.95494	40.44627	23
24	23.21221	0.04308	158.65862	0.00630	6.83514	0.14630	6.06237	41.43713	24
25	26.46192	0.03779	181.87083	0.00550	6.87293	0.14550	6.16100	42.34410	25
30	50.95016	0.01963	356.78685	0.00280	7.00266	0.14280	6.54226	45.81324	30
35	98.10018	0.01019	693.57270	0.00144	7.07005	0.14144	6.78240	47.95191	35
40	188.88351	0.00529	1342.02510	0.00075	7.10504	0.14075	6.92996	49.23764	40
50	700.23299	0.00143	4994.52135	0.00020	7.13266	0.14020	7.07135	50.43751	50
60	2595.91866	0.00039	18535.13328	0.00005	7.14011	0.14005	7.11974	50.83566	60

i = **15.00%**

n	[F/P, i, n]	[P/F, i, n]	[F/A, i, n]	[A/F, i, n]	[P/A, i, n]	[A/P, i, n]	[A/G, i, n]	[P/G, i, n]	n
1	1.15000	0.86957	1.00000	1.00000	0.86957	1.15000	0.00000	0.00000	1
2	1.32250	0.75614	2.15000	0.46512	1.62571	0.61512	0.46512	0.75614	2
3	1.52088	0.65752	3.47250	0.28798	2.28323	0.43798	0.90713	2.07118	3
4	1.74901	0.57175	4.99338	0.20027	2.85498	0.35027	1.32626	3.78644	4
5	2.01136	0.49718	6.74238	0.14832	3.35216	0.29832	1.72281	5.77514	5
6	2.31306	0.43233	8.75374	0.11424	3.78448	0.26424	2.09719	7.93678	6
7	2.66002	0.37594	11.06680	0.09036	4.16042	0.24036	2.44985	10.19240	7
8	3.05902	0.32690	13.72682	0.07285	4.48732	0.22285	2.78133	12.48072	8
9	3.51788	0.28426	16.78584	0.05957	4.77158	0.20957	3.09223	14.75481	9
10	4.04556	0.24718	20.30372	0.04925	5.01877	0.19925	3.38320	16.97948	10
11	4.65239	0.21494	24.34928	0.04107	5.23371	0.19107	3.65494	19.12891	11
12	5.35025	0.18691	29.00167	0.03448	5.42062	0.18448	3.90820	21.18489	12
13	6.15279	0.16253	34.35192	0.02911	5.58315	0.17911	4.14376	23.13522	13
14	7.07571	0.14133	40.50471	0.02469	5.72448	0.17469	4.36241	24.97250	14
15	8.13706	0.12289	47.58041	0.02102	5.84737	0.17102	4.56496	26.69302	15
16	9.35762	0.10686	55.71747	0.01795	5.95423	0.16795	4.75225	28.29599	16
17	10.76126	0.09293	65.07509	0.01537	6.04716	0.16537	4.92509	29.78280	17
18	12.37545	0.08081	75.83636	0.01319	6.12797	0.16319	5.08431	31.15649	18
19	14.23177	0.07027	88.21181	0.01134	6.19823	0.16134	5.23073	32.42127	19
20	16.36654	0.06110	102.44358	0.00976	6.25933	0.15976	5.36514	33.58217	20
21	18.82152	0.05313	118.81012	0.00842	6.31246	0.15842	5.48832	34.64479	21
22	21.64475	0.04620	137.63164	0.00727	6.35866	0.15727	5.60102	35.61500	22
23	24.89146	0.04017	159.27638	0.00628	6.39884	0.15628	5.70398	36.49884	23
24	28.62518	0.03493	184.16784	0.00543	6.43377	0.15543	5.79789	37.30232	24
25	32.91895	0.03038	212.79302	0.00470	6.46415	0.15470	5.88343	38.03139	25
30	66.21177	0.01510	434.74515	0.00230	6.56598	0.15230	6.20663	40.75259	30
35	133.17552	0.00751	881.17016	0.00113	6.61661	0.15113	6.40187	42.35864	35
40	267.86355	0.00373	1779.09031	0.00056	6.64178	0.15056	6.51678	43.28299	40
50	1083.65744	0.00092	7217.71628	0.00014	6.66051	0.15014	6.62048	44.09583	50
60	4383.99875	0.00023	29219.99164	0.00003	6.66515	0.15003	6.65298	44.34307	60

i = **20.00%**

n	[F/P, i, n]	[P/F, i, n]	[F/A, i, n]	[A/F, i, n]	[P/A, i, n]	[A/P, i, n]	[A/G, i, n]	[P/G, i, n]	n
1	1.20000	0.83333	1.00000	1.00000	0.83333	1.20000	0.00000	0.00000	1
2	1.44000	0.69444	2.20000	0.45455	1.52778	0.65455	0.45455	0.69444	2
3	1.72800	0.57870	3.64000	0.27473	2.10648	0.47473	0.87912	1.85185	3
4	2.07360	0.48225	5.36800	0.18629	2.58873	0.38629	1.27422	3.29861	4
5	2.48832	0.40188	7.44160	0.13438	2.99061	0.33438	1.64051	4.90612	5
6	2.98598	0.33490	9.92992	0.10071	3.32551	0.30071	1.97883	6.58061	6
7	3.58318	0.27908	12.91590	0.07742	3.60459	0.27742	2.29016	8.25510	7
8	4.29982	0.23257	16.49908	0.06061	3.83716	0.26061	2.57562	9.88308	8
9	5.15978	0.19381	20.79890	0.04808	4.03097	0.24808	2.83642	11.43353	9
10	6.19174	0.16151	25.95868	0.03852	4.19247	0.23852	3.07386	12.88708	10
11	7.43008	0.13459	32.15042	0.03110	4.32706	0.23110	3.28929	14.23296	11
12	8.91610	0.11216	39.58050	0.02526	4.43922	0.22526	3.48410	15.46668	12
13	10.69932	0.09346	48.49660	0.02062	4.53268	0.22062	3.65970	16.58825	13
14	12.83918	0.07789	59.19592	0.01689	4.61057	0.21689	3.81749	17.60078	14
15	15.40702	0.06491	72.03511	0.01388	4.67547	0.21388	3.95884	18.50945	15
16	18.48843	0.05409	87.44213	0.01144	4.72956	0.21144	4.08511	19.32077	16
17	22.18611	0.04507	105.93056	0.00944	4.77463	0.20944	4.19759	20.04194	17
18	26.62333	0.03756	128.11667	0.00781	4.81219	0.20781	4.29752	20.68048	18
19	31.94800	0.03130	154.74000	0.00646	4.84350	0.20646	4.38607	21.24390	19
20	38.33760	0.02608	186.68800	0.00536	4.86958	0.20536	4.46435	21.73949	20
21	46.00512	0.02174	225.02560	0.00444	4.89132	0.20444	4.53339	22.17423	21
22	55.20614	0.01811	271.03072	0.00369	4.90943	0.20369	4.59414	22.55462	22
23	66.24737	0.01509	326.23686	0.00307	4.92453	0.20307	4.64750	22.88671	23
24	79.49685	0.01258	392.48424	0.00255	4.93710	0.20255	4.69426	23.17603	24
25	95.39622	0.01048	471.98108	0.00212	4.94759	0.20212	4.73516	23.42761	25
30	237.37631	0.00421	1181.88157	0.00085	4.97894	0.20085	4.87308	24.26277	30
35	590.66823	0.00169	2948.34115	0.00034	4.99154	0.20034	4.94064	24.66140	35
40	1469.77157	0.00068	7343.85784	0.00014	4.99660	0.20014	4.97277	24.84691	40
50	9100.43815	0.00011	45497.19075	0.00002	4.99945	0.20002	4.99451	24.96978	50
60	56347.51435	0.00002	281732.57177	0.00000	4.99991	0.20000	4.99894	24.99423	60

APSC221 – Examination Formula Sheet

Interest Factors:

$(F/P, i, n) = (1 + i)^n$

$(P/F, i, n) = \frac{1}{(1+i)^n}$

$(F/A, i, n) = \frac{(1+i)^n-1}{i}$

$(A/F, i, n) = \frac{i}{(1+i)^n-1}$

$(P/A, i, n) = \frac{(1+i)^n-1}{i(1+i)^n}$

$(A/P, i, n) = \frac{i(1+i)^n}{(1+i)^n-1}$

$(A/G, i, n) = \frac{1}{i} - \frac{n}{(1+i)^n-1}$

$(P/G, i, n) = \frac{(1+i)^n-in-1}{i^2(1+i)^n}$

$P = \lim_{n\to\infty} A\left(\frac{P}{A}, i, n\right) = \frac{A}{i}$

To calculate effective interest rates:

$$i_e = (1 + i_s)^m - 1$$

$$i_e = (1 + \frac{r}{m})^m - 1$$

$$i_e = \lim_{m\to\infty} (1 + \frac{r}{m})^m - 1 = e^r - 1$$

Cost Estimation Techniques:

$$C_n = C_k(\frac{\bar{I}_n}{\bar{I}_k})$$

Project Scheduling:

$t = (a + 4m + b)/6$

$\sigma^2 = [(b - a)/6]^2$

$Z = (X - \mu)/ \sigma$

Crash Cost per period =

$$\frac{(Crash\ Cost - Normal\ Cost)}{(Normal\ Time - Crash\ Time)}$$

Risk:

Expected Value, $EV(X) = \sum_{i=1}^{m} x_i p(x_i)$

Project Risk Factor = $P_f + C_f - (P_f)(C_f)$

Taxes:

$$CTF = 1 - \frac{td\left(1 + \frac{i}{2}\right)}{(i + d)(1 + i)}$$

$$CSF = 1 - \frac{td}{(i + d)}$$

Equivalent Annual Costs:

$EAC_{Total} = EAC_{Capital} + EAC_{O\&M}$

$EAC_{Capital} = (P- S)(A/P, i, n) +Si$

Depreciation:

$BV_{SL}(n) = P - n(\frac{P-S}{N})$, $D_{SL}(n) = \frac{P-S}{N}$

$BV_{DB}(n) = P(1 – d)^n$, $D_{DB}(n) = BV_{DB}(n-1)\ d$

Where; $d = 1 - \sqrt[N]{\frac{S}{P}}$

Inflation:

$$i_{Real} = \frac{1 + i_{Actual}}{1 + f} - 1$$

$$i_{Actual} = (1 + i_{Real})(1 + f) - 1$$

$Actual\$ = Real\$\ (1 + f)^n$

Financial Ratios:

Acid Test Ratio = Quick Assets / Current Liabilities
Current Ratio = Current Assets / Current Liabilities
Equity Ratio = Total Equity / (Total Equity + Total Liabilities)
Inventory Turnover = Sales / Inventory
Working Capital = Current Assets – Current Liabilities
Return on Assets (ROA) = Net Income / Total Assets
Return on Equity (ROE) = Net Income / Total Equity

Return on Capital Employed (ROCE) = $\frac{EBIT(1-t)}{Debt + Equity}$

Debt to Capital Employed (λ) = Debt / (Debt + Equity)

Weighted average Cost of Capital:

$$WACC = \lambda(1\text{-}t)i_B + (1\text{-}\lambda)e_a$$

Capital Asset Pricing Model (CAPM)

$$e_a = R_S = R_F + \beta_S(R_M – R_F)$$

Where;

- EBIT is the earnings before interest and tax
- Debt includes all interest bearing liabilities
- λ is the ratio of total debt (short and long-term) to total debt plus equity.
- t is the effective tax rate and can be computed as t = tax paid / earnings before tax
- i_B is the cost of debt financing
- e_a is the cost of equity financing
- R_S is the return on a stock
- R_M is the market return
- R_F is the risk free return
- β_S is the level of risk associated with the stock