# QUEEN'S UNIVERSITY

KINGSTON, ONTARIO

## Faculty of Engineering and Applied Science

## APSC221 - Economics and Business Practices in Engineering – Fall 2022

Final Examination – December 17, 2022

INSTRUCTOR – Jan Sneep

This examination consists of 10 questions and is out of a total of **102.5** marks.

Time available **3** hours.

**INSTRUCTIONS:**

1. Answers to ALL questions must be written in the answer booklet that is provided.
    a. Put your name, student number on the front page(s) of all answer booklet(s).
    b. Do not remove any pages from the booklet that is provided.
    c. If there is insufficient space, ask the proctor for another answer booklet.
2. Do not use RED COLOUR for your presentation in any form, written or pictorial; this colour is reserved for marking your submission.
3. **ONLY** the use of the Queen's University-approved Casio 991 calculator will be allowed.
4. The Formula sheet and Interest tables are provided at the back of this exam.
5. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer a clear statement of any assumptions made. **Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.**
6. Marks will be deducted if appropriate interest factors, their designations and appropriate values of interest rate (i) and period (n) are not indicated. You are strongly encouraged to provide cash flow diagrams where appropriate.
7. You must hand in all answer booklets, questions sheets and reference sheets on completion of the test.

There will be penalties if any of the above instructions are not followed.

GOOD LUCK

# Engineering Economics
# Final Exam - Fall 2022

**DO NOT ANSWER ON THIS TEST PAPER, All work and answers must be shown in your examination booklet.**

## Question 1: Multiple Choice (½ mark for each = 10 marks)

Please write the answers in a single column, one answer per line, on a page of its own.

Select the one alternative that best completes the statement or answers the question.

A. A typical fixed cost would be _____.
   1. insurance
   2. general management salaries
   3. license fees
   4. interest costs on borrowed capital
   5. all of the above

B. There may be uncertainty in estimating project parameter values as a result of _____.
   1. technological change
   2. change in sales volume or market share
   3. inflation
   4. general economic activity levels within an industry
   5. all of the above

C. Parametric cost estimating is the use of historical cost data and statistical techniques to predict _____ costs.
   1. sunk
   2. variable
   3. depreciation
   4. future
   5. all of the above

D. A Cost Estimating Relationship (CER) is a mathematical model that describes the cost of an engineering project as a function of one or more _____ variables.
   1. cost
   2. design
   3. depreciation
   4. fixed
   5. all of the above

E. Market equivalence is a consequence of the ability to _____ one cash flow for another at zero cost.
   1. understand
   2. predict impact of
   3. exchange
   4. analyse
   5. all of the above

F. For present worth calculations when N->∞ you can use _____.
   1. (P/A, i, n)
   2. P/i
   3. A/i
   4. F/i
   5. All of the above

G. The simplest relation between projects occurs when they are _____.
   1. equivalent annuities
   2. mathematically equivalent
   3. mutually exclusive
   4. independent
   5. all of the above

H. Projects are _____ if, in the process of choosing one, all other alternatives are excluded.
   1. equivalent annuities
   2. mathematically equivalent
   3. mutually exclusive
   4. independent
   5. all of the above

I. The first step in the process of comparing several mutually exclusive alternatives using IRR is to _____.
   1. calculate the present worth of all the alternatives
   2. calculate the annual worth of all the alternatives
   3. order the alternatives from largest to smallest first cost
   4. order the alternatives from smallest to largest first cost
   5. order the alternatives from largest to smallest present worth

J. The external rate of return (ERR) is the rate of return on a project where any cash flows _____.
   1. has to be turned into a present worth
   2. not invested in the project are assumed to earn the MARR
   3. has to be turned into a future worth
   4. has to first be ordered from smallest to largest
   5. any of the above will work

K. Within the CCA system, a firm's assets are grouped by _____.
   1. first cost
   2. current salvage value
   3. method of depreciation
   4. ½-year rule
   5. asset class

L. Installing a new piece of equipment sometimes involves substantial up-front costs, called installation costs. Which of the following is an example of such a cost?
   1. Disruption of production
   2. Training of workers
   3. Reorganization of processes
   4. Upgrading of electrical wiring
   5. all of the above

M. Which of the following methods of incorporating inflation into project evaluations is not correct because it will bias the evaluation in favour of the investment?
   1. Real MARR and real cash flows
   2. Real MARR and actual cash flows
   3. Actual MARR and actual cash flows
   4. Convert Actual MARR to Real MARR then use real cash flows
   5. Convert Real MARR to Actual MARR then use actual cash flows

N. The severity of negative consequences in a project's life span is _____.
   1. unknown until detailed modeling is concluded
   2. greatest early in the project's life
   3. bounded or limited in scope
   4. minimal early in the project's life
   5. the period of greatest worry

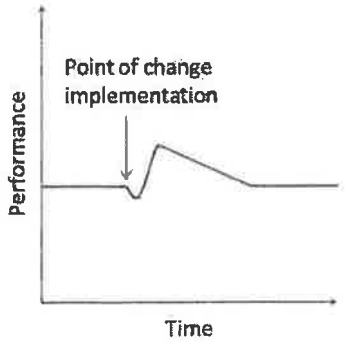
O. An organization can choose to _____ risk.
   1. accept
   2. minimize
   3. share
   4. transfer
   5. all of the above

P. Contingency reserves in several forms, including financial and managerial, are among the most common methods to _____ project risk.
   1. account for
   2. determine
   3. evaluate
   4. mitigate
   5. all of the above

Q. In dealing with risk and uncertainty, "Rolling Back", has nothing to Walmart and prices, it is _____.
   1. a procedure involving backward induction and assessing the best expected values
   2. a process to evaluate activity on node diagrams to determine LF and LS values
   3. the points of intersection on a price elasticity curve were revenue equals total costs
   4. how we calculate the payback period by deducting each year of savings from the first cost
   5. none of the above

R. What are the two dimensions of change management?
   1. organisational and strategic
   2. human and organisational
   3. tactical and strategic
   4. stories and legends
   5. business and competitive

S. Looking at the picture below match the phrase, from the given list, that best describes the change outcome.
   1. Ideal
   2. Alterations in customer demand
   3. Partial Success
   4. Lost Investment
   5. Mergers and acquisitions



T. The duration and structure of change initiatives vary significantly, but all have a degree of complexity and _____.

1. involve cost accounting and management accounting
2. are outside the normal daily workload
3. monitor resources, costs, quality and budget
4. vary the structure and duration
5. require equity sources of funding

## Question 2: Cost Estimation (9 marks)

You work for an injection moulding company and one of your biggest customers has just asked you to help out with a backorder problem they have. They want you to take on a one-time order to produce 300,000 plastic parts.

Because your company does not have the machinery required to produce the part, and it is just a one-time contract, it has been decided to rent a machine. You check local suppliers and for the best machine available, the following information is provided.

| | |
|---|---|
| Fee for delivery and setup | $ 2,000.00 |
| Rental Cost / month* | $ 8,000.00 |
| Heating Fuel Cost / Hour | $ 14.00 |
| Parts / Hour | 100 |
| Defect Rate | 4% |

*Note: Machines can only be rented by the full month not by the day.

You will require an operator to run the machine at $25 / hour and you will have the machine in operation for two shifts per day for five days a week. Each shift is 8 hours long, however with lunch and coffee breaks the machine will only be producing parts for 7 hours per shift. Each part weights 250 grams and the required plastic costs $0.82 per kg. Defective parts will have to be shipped to a recycling centre. The hauling company charges $80 per trip and can take a maximum of 1000 kg per trip.

Your boss has asked you to put together the quotation for the customer using a 30% profit markup.

a) How many parts in total will you have to make so you can ship 300,000 parts?
b) How many hours will you need to operate the machine for?
c) How many months will it take you to finish the job (assume exactly 52 weeks per year)?
d) What will be the total cost to rent the machine?
e) What will be the total cost of the material?
f) What will be the total cost of labour?
g) What will be the total operating cost, including both labour and heating fuel?
h) What will be the total recycling cost, assuming defective parts use the same amount of material as good parts?
i) What is the grand total you will bill the customer for the job?

Show all your work, including units, for full marks.

## Question 3: Time Value of Money (6.5 marks)

a) Victory Visa, Magnificent Master Card, and Amazing Express are credit card companies that charge different interest on overdue accounts. Victory Visa charges 27.8% compounded daily, Magnificent Master Card charges 28% compounded weekly, and Amazing Express charges 28.2% compounded monthly. Assuming that there are exactly 365 days in a year, 52 weeks in a year, and on the basis of interest rate, which credit card has the best deal?

b) Today, an investment you made three years ago has matured and is now worth $3,000. It was a locked-in three-year deposit at a locked-in interest rate of 10% per year. You knew at the time that you were taking a risk in making such an investment because interest rates vary over time. Looking back over the past three years, interest rates for similar one-year investments did indeed vary. The interest rates were 8% the first year, 10% the second, and 14% in the third. Did you lose out or gain by locking into a 10% rate for the three years? If so, by how much?

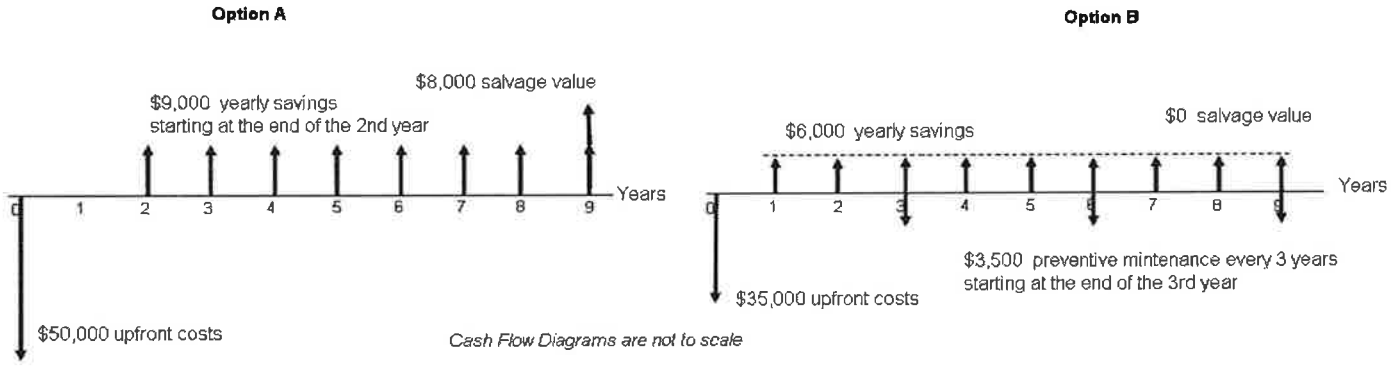
## Question 4: Comparison Methods (9 marks)

Two mutually exclusive projects are under consideration. The cash flow diagrams below depict the costs and savings. Option A is a 9-year project with an upfront cost of $50,000, yearly savings of $9,000 starting at the end of the second year, and a salvage value of $8,000. Option B is a 9-year project with an upfront cost of $35,000, yearly savings of $6,000 starting at the end of the first year, and no salvage value. Option B also has a preventive maintenance cost of $3,500 every 3 years. Assume a company MARR of 10% and using the present worth method:

a) What is the PW of each option?

b) Which is the better option and why?

c) Which project, if any, should be selected? Fully justify your answer.

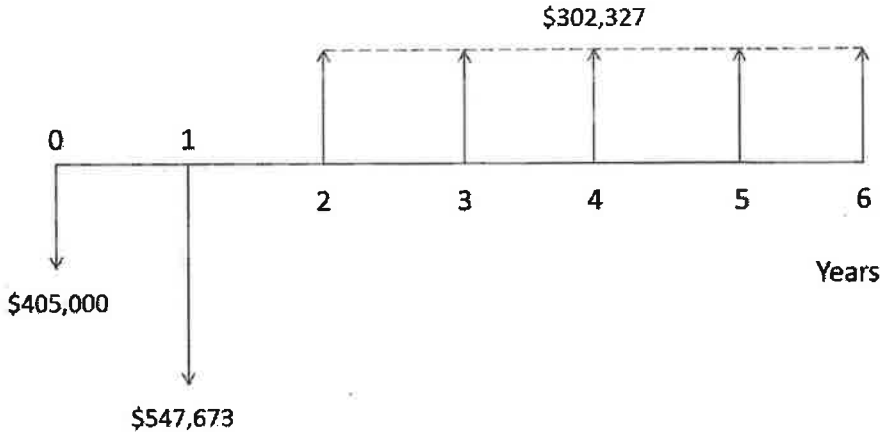



## Question 5: Equivalent Annual Cost – Replacement Decisions (16 marks)

Novelis, a subsidiary of Alcan, produces aluminium foil for household use. They purchased a set of working rollers, for thinning the processed aluminium, 10 years ago for $43,502 dollars. The Operations Project Manager has decided to conduct an analysis to determine when the rollers should be replaced. He has predicted the O&M costs for the next 5 years will $3,100 this coming year and then increase by $280 per year after that. Novelis uses a MARR of 12% for this type of project and a declining-balance depreciation model with a depreciation rate of 20% to calculate salvage values. What is the economic life for the existing rollers and the EAC*.

## Question 6: Rates of Return (8 marks)

Determine the precise rate of return for the project depicted by the cash flow diagram below.




## Question 7: Inflation (8 marks)

The Napanee Town Council is currently investigating giving their downtown area a facelift and is looking at a proposal brought to them by a developer. The deal is essentially that for an initial cost to the town of $2.4 million the developer will contract to keep actual annual maintenance costs to $125,000 per year and provide actual revenues from parking totaling $525,000 per year. The Town's Chief Financial Officer identifies that the real dollar MARR is 8% and that she is anticipating inflation to average 1.852% per year for the entire 30 year study period. The town would have to pay for major upgrades to the system in year 10 and year 20. The cost for such an upgrade today would be $50,000. Calculate the present worth (PW) to determine if the Town Council should invest in this project. Fully justify your answer.

## Question 8: Taxes (8 marks)

Churchill Metal Product (CMP) opened for business in 2006. Over the following years, their transactions for CCA Class 8 assets are listed in the table below. Class 8 assets have a declining balance depreciation rate of 20%.

| Date | Item | Activity | Amount |
|---|---|---|---|
| March 11, 2006 | Machine 1 | Purchase | $50,000 |
| April 24, 2006 | Machine 2 | Purchase | $150,000 |
| November 3, 2008 | Machine 3 | Purchase | $250,000 |
| November 22, 2008 | Machine 1 | Sale | $10,000 |
| July 2, 2009 | Machine 3 | Sale | $45,000 |

Compute the Undepreciated Capital Costs (UCC) for the period 2006 – 2009. Create a table with the headings, as a minimum, shown below. You can have more columns if you like.

| Year | CCA | $UCC_{End}$ |
|---|---|---|

## Question 9: Dealing with Uncertainty and Risk (8 marks)

You are considering the purchase of a new $70,000 dump truck. The truck is expected to generate average net revenues of between $12,000 and $20,000 per year over the five-year service life of the project. At the end of the project the salvage value of the truck is estimated to be $20,000. Your MARR for this project is 12.32% compounded monthly.

a) At what average annual revenue does this project breakeven?

b) After speaking with the accounting department about similar projects in the past you estimate the following probabilities about what revenues will average over the next five years. Based on expected revenue should you invest in this project?

| Average annual revenue | Probability |
|---|---|
| $12,000 | 30% |
| $14,000 | 25% |
| $16,000 | ? |
| $18,000 | 15% |
| $20,000 | 10% |

## Question 10: Wonder Workshop - Business Questions (20 marks)

Business Idea:

Create a pair of robots, named Dash and Dot, that teach kids how to code.

Pitch:

All parents want to help their kids develop skills that will prepare them for future success. Meet Dash and Dot – which are fun, easy-to-use, interactive robots that teach kids how to code while they play. Dash is the bigger of the two – he has wheels that allow him to scoot around on the floor. Dot is equally interactive, but does not move about. Both are pretty sophisticated. They have sensors, accelerometers, microphones, and speakers, all of which give plenty of personality and capabilities. Kids learn to code while they teach Dash and Dot to make sounds, blink their lights, move about, change directions, play games, go on adventures, and much more. Take a minute to see what Dash and Dot look like by visiting Wonder Workshop's website.

Here's how it works. Dash and Dot are accompanied by five free apps that can be controlled by smartphones or tablets (more apps are in the works). The apps – named Go, Path, Xylo, Blockly, and Wonder – are aimed at different age groups, which allows Dash and Dot to grow with the families that own them. The Go app is appropriate for children as young as five. This app teaches the target children how to use basic commands to bring Dash and Dot to life. By using Go, kids can instruct Dash to move forward or backward, move his head from side-to-side, make sounds (the elephant sounds is a favorite), change color, record their voices, and many other things. The Blockly app, which is suitable for kids as young as eight, teaches block programming to instruct Dask and Dot to do fun and creative things. In computer programming, a block or code block is a section of code that is grouped together. There are several challenges embedded within the Blockly app, such as Driving School (which teaches Dash how to drive), which require children to actually write code to play the game, For example, a snippet of the code an eight-year old might write to help teach Dash to drive might say:

```
When Start
If Dash See Dot
Backward 50 feet normal
Turn right 90
Say Dash Hi
All lights red
Turn to voice
Stop wheels
```

The commands are displayed in the app in the same format as most basic coding programs. Don't know what the block of code shown above means? An eight-year old following-along in the Blockly app will not only learn what it means, but will be capable of writing code that makes Dash and Dot engage in many activities. The Wonder app teaches even more advanced programming to older children.

Das and Dot can do many other things, which makes them not only educational but fun. The overarching goal is to stretch children's imagination while at the same time preparing them for the future by teaching the basics of coding.

**Wonder Workshop Pro Forma Balance Sheet - Projected $ (000's) for year ending**

| | 2023 | 2024 | 2025 |
|---|---|---|---|
| **Assets** | | | |
| Current Assets | | | |
| Cash | 75 | 72 | 70 |
| Accounts receivable | 125 | 150 | 180 |
| Inventories | 250 | 263 | 276 |
| Total Current Assets | 450 | 485 | 525 |
| | | | |
| Fixed Assets | | | |
| Distribution Centres | 1,000 | 1,015 | 1,030 |
| Manufacturing Equipment | 1,386 | 1,455 | 1,528 |
| Other Assets | 3 | 30 | 30 |
| **Total Assets** | 2,839 | 2,985 | 3,114 |
| | | | |
| **Liabilities and Owners' Equity** | | | |
| Current Liabilities | | | |
| Due to Bank | 130 | 137 | 143 |
| Accounts Payable | 644 | 676 | 710 |
| Wages Payable | 213 | 245 | 282 |
| Income Tax Payable | 63 | 93 | 124 |
| Total Current Liabilities | 1,050 | 1,151 | 1,259 |
| | | | |
| Long-Term Liabilities | | | |
| Long-term Debt | 1,000 | 975 | 951 |
| Total Long-Term Liabilities | 1,000 | 975 | 951 |
| | | | |
| Owner's Equity | | | |
| Capital Stock | 700 | 770 | 847 |
| Retained earnings | 89 | 89 | 57 |
| Total Owners' Equity | 789 | 859 | 904 |
| | | | |
| **Total Liabilities and Owners' Equity** | 2,839 | 2,985 | 3,114 |

**Wonder Workshop Pro Forma Income Statement - Projected $ (000's) for year ending**

| | | | 2023 | 2024 | 2025 |
|---|---|---|---|---|---|
| Revenues | | | | | |
| | Total Revenue | | 9,355 | 9,682 | 10,021 |
| | Less: Costs | | 8,281 | 8,405 | 8,531 |
| | | Net revenue | 1,074 | 1,277 | 1,490 |
| | | | | | |
| Expenses | | | | | |
| | Marketing | | 600 | 660 | 726 |
| | Depreciation | | 147 | 169 | 194 |
| | Interest | | 76 | 75 | 74 |
| | Income Taxes | | 126 | 187 | 248 |
| | | Total expenses | 948 | 1,091 | 1,242 |
| | | | | | |
| Net Income | | | 126 | 187 | 248 |

| Financial Ratios - Projected | 2023 | 2024 | 2025 | Closest Industry Norm |
|---|---|---|---|---|
| Current ratio | 0.43 | 0.42 | 0.42 | 1.8 |
| Acid-test ratio | 0.19 | 0.19 | 0.20 | 0.95 |
| Inventory Turnover | 37 | 37 | 36 | 35 |
| Return-on-equity ratio | 15.9% | 21.7% | 27.4% | 18% |
| Profit Margin | 11% | 13% | 15% | 15% |
| Debt to Equity Ratio | 2.60 | 2.47 | 2.44 | 2.56 |

**See formula sheet for the definitions of the above ratios.**

Based ONLY on the information provided above answer the following questions. We are looking for short sentences and / or bullet points, **NOT** long-winded rambling mini essays.

First Screen (6 Marks)

a) Using a Low (-1), Moderate (0), High (1) potential, score the following and you **must** provide justification from the case for each in order to receive full marks. (5 marks)

Your answer in the answer booklet should look like:

9. a) 1. {score} – {Justification}
2. {score} – {Justification}
Etc.

| Strength of the Business Idea | Low ( -1 ) | Moderate ( 0 ) | High ( 1 ) |
|---|---|---|---|
| 1. Extent to which the idea: Takes advantage of an environmental trend, and/or solves a problem, and/or addresses an unfilled gap in the marketplace. | Weak | Moderate | Strong |
| 2. Timeliness of entry to market. | Not timely | Moderately timely | Very timely |
| 3. Extent to which the idea "adds value" for its buyer or end user. | Low | Medium | High |
| 4. Extent to which the customer is satisfied by competing products that are already available. | Very satisfied | Moderately satisfied | Not very satisfied or ambivalent |
| 5. Degree to which the idea requires customers to change their basic practices or behaviors. | Substantial changes required | Moderate changes required | Small to no changes required |

b) Total your scores from part a) and provide a suggestion for improving the potential. (1 mark)

Your answer in the answer booklet should look like:

9. b) 1. {Total score} – {Suggestion}

Feasibility Analysis (4 Marks)

a) Comment on what you think about the Industry Attractiveness. One plus and one negative. (2 mark)
b) What do you think the two markets are that should be studied to make an informed decision about the Target Market? (2 marks)

Financial Strength (6 Marks)

You have been asked to evaluate Wonder Workshop's projected financial strength for a potential investor based on the Pro Forma Balance Sheet, Income Statement and Financial Ratios provided above for the next three years. Provide a concise comment on the implications of each of the ratios show above, not describing what the ratio is (definitions provided on the formula sheet), but what it implies and / or a potential reason why it would be what it is.

Business Plans: Marketing and Operations (4 Marks)

Assuming that you are fully in charge of Wonder Workshop and will make all of the decisions;

a) Marketing Strategy: Describe what your overall approach for marketing Wonder Workshop's products and services would be. (2 marks)
   i. How would you position it?
   ii. How would you differentiate it from competitors?
b) Operations Plan: Articulate what your general approach to operations will be. (2 marks)
   Frame your discussion in terms of;
   i. "back stage" – the behind the scenes activities
   ii. "front stage" – what the customer will see and experience

# APSC 221 – Examination Formula Sheet

## Interest Factors:

$(F/P, i, n) = (1 + i)^n$

$(P/F, i, n) = \frac{1}{(1+i)^n}$

$P = \lim_{n\to\infty} A\left(\frac{P}{A}, i, n\right) = \frac{A}{i}$

To calculate effective interest rates:

$$i_e = (1 + i_s)^m - 1$$

$$i_e = (1 + \frac{r}{m})^m - 1$$

## Cost Estimation Techniques:

$C_n = C_k\left(\frac{I_n}{I_k}\right)$ Estimate year n, based on year k

$Zu = K[u^{\left(\frac{\log S}{\log 2}\right)}]$ u=output unit #

K=resources for 1st unit

S=learning curve parameter

$\frac{C_A}{C_B} = (\frac{S_A}{S_B})^X$ C=Cost in $, same point in time

S=Size, same physical units

X=cost-capacity factor

## Risk:

Expected Value, $EV(X) = \sum_{i=1}^{m} x_i p(x_i)$

Project Risk Factor = $P_f + C_f - (P_f)(C_f)$

## Taxes:

$$CTF = 1 - \frac{td\left(1+\frac{i}{2}\right)}{(i+d)(1+i)}$$

$$CSF = 1 - \frac{td}{(i+d)}$$

## Equivalent Annual Costs:

$EAC_{Total} = EAC_{Capital} + EAC_{O\&M}$

$EAC_{Capital} = [P + I - S(n)](A/P, i, n) + S(n)i$

## Depreciation:

$BV_{SL}(n) = P - n(\frac{P-S}{N})$, $D_{SL}(n) = \frac{P-S}{N}$

$BV_{DB}(n) = P(1-d)^n$, $D_{DB}(n) = BV_{DB}(n-1)\, d$

Where; $d = 1 - \sqrt[n]{\frac{S}{P}}$

## Inflation:

$$i_{Real} = \frac{1 + i_{Actual}}{1 + f} - 1$$

$i_{Actual} = (1 + i_{Real})(1 + f) - 1$

Actual$ = Real$ $(1 + f)^n$

## Financial Ratios:

Current Ratio = Current Assets / Current Liabilities
Quick or Acid Test Ratio = Quick Assets / Current Liabilities
Equity Ratio = Total Equity / Total Assets
Inventory Turnover = Sales / Inventories
Return on Assets (ROA) = Net Income / Average Total Assets
Return on Equity (ROE) = Net Income / Average Shareholder Equity
Profit Margin = Net Income / Net Sales
Debt Ratio = Total Debt / Total Assets
Debt to Equity Ratio = Total Liabilities / Total Equity

**i = 8%**

| n | [F/P, i, n] | [P/F, i, n] | [F/A, i, n] | [A/F, i, n] | [P/A, i, n] | [A/P, i, n] | [A/G, i, n] | [P/G, i, n] | n |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.0800 | 0.9259 | 1.0000 | 1.0000 | 0.9259 | 1.0800 | 0.0000 | 0.0000 | 1 |
| 2 | 1.1664 | 0.8573 | 2.0800 | 0.4808 | 1.7833 | 0.5608 | 0.4808 | 0.8573 | 2 |
| 3 | 1.2597 | 0.7938 | 3.2464 | 0.3080 | 2.5771 | 0.3880 | 0.9487 | 2.4450 | 3 |
| 4 | 1.3605 | 0.7350 | 4.5061 | 0.2219 | 3.3121 | 0.3019 | 1.4040 | 4.6501 | 4 |
| 5 | 1.4693 | 0.6806 | 5.8666 | 0.1705 | 3.9927 | 0.2505 | 1.8465 | 7.3724 | 5 |
| 6 | 1.5869 | 0.6302 | 7.3359 | 0.1363 | 4.6229 | 0.2163 | 2.2763 | 10.5233 | 6 |
| 7 | 1.7138 | 0.5835 | 8.9228 | 0.1121 | 5.2064 | 0.1921 | 2.6937 | 14.0242 | 7 |
| 8 | 1.8509 | 0.5403 | 10.6366 | 0.0940 | 5.7466 | 0.1740 | 3.0985 | 17.8061 | 8 |
| 9 | 1.9990 | 0.5002 | 12.4876 | 0.0801 | 6.2469 | 0.1601 | 3.4910 | 21.8081 | 9 |
| 10 | 2.1589 | 0.4632 | 14.4866 | 0.0690 | 6.7101 | 0.1490 | 3.8713 | 25.9768 | 10 |
| 11 | 2.3316 | 0.4289 | 16.6455 | 0.0601 | 7.1390 | 0.1401 | 4.2395 | 30.2657 | 11 |
| 12 | 2.5182 | 0.3971 | 18.9771 | 0.0527 | 7.5361 | 0.1327 | 4.5957 | 34.6339 | 12 |
| 13 | 2.7196 | 0.3677 | 21.4953 | 0.0465 | 7.9038 | 0.1265 | 4.9402 | 39.0463 | 13 |
| 14 | 2.9372 | 0.3405 | 24.2149 | 0.0413 | 8.2442 | 0.1213 | 5.2731 | 43.4723 | 14 |
| 15 | 3.1722 | 0.3152 | 27.1521 | 0.0368 | 8.5595 | 0.1168 | 5.5945 | 47.8857 | 15 |
| 16 | 3.4259 | 0.2919 | 30.3243 | 0.0330 | 8.8514 | 0.1130 | 5.9046 | 52.2640 | 16 |
| 17 | 3.7000 | 0.2703 | 33.7502 | 0.0296 | 9.1216 | 0.1096 | 6.2037 | 56.5883 | 17 |
| 18 | 3.9960 | 0.2502 | 37.4502 | 0.0267 | 9.3719 | 0.1067 | 6.4920 | 60.8426 | 18 |
| 19 | 4.3157 | 0.2317 | 41.4463 | 0.0241 | 9.6036 | 0.1041 | 6.7697 | 65.0134 | 19 |
| 20 | 4.6610 | 0.2145 | 45.7620 | 0.0219 | 9.8181 | 0.1019 | 7.0369 | 69.0898 | 20 |
| 25 | 6.8485 | 0.1460 | 73.1059 | 0.0137 | 10.6748 | 0.0937 | 8.2254 | 87.8041 | 25 |
| 30 | 10.0627 | 0.0994 | 113.2832 | 0.0088 | 11.2578 | 0.0888 | 9.1897 | 103.4558 | 30 |
| 35 | 14.7853 | 0.0676 | 172.3168 | 0.0058 | 11.6546 | 0.0858 | 9.9611 | 116.0920 | 35 |
| 40 | 21.7245 | 0.0460 | 259.0565 | 0.0039 | 11.9246 | 0.0839 | 10.5699 | 126.0422 | 40 |
| 50 | 46.9016 | 0.0213 | 573.7702 | 0.0017 | 12.2335 | 0.0817 | 11.4107 | 139.5928 | 50 |
| 60 | 101.2571 | 0.0099 | 1253.2133 | 0.0008 | 12.3766 | 0.0808 | 11.9015 | 147.3000 | 60 |

**i = 10%**

| n | [F/P, i, n] | [P/F, i, n] | [F/A, i, n] | [A/F, i, n] | [P/A, i, n] | [A/P, i, n] | [A/G, i, n] | [P/G, i, n] | n |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.1000 | 0.9091 | 1.0000 | 1.0000 | 0.9091 | 1.1000 | 0.0000 | 0.0000 | 1 |
| 2 | 1.2100 | 0.8264 | 2.1000 | 0.4762 | 1.7355 | 0.5762 | 0.4762 | 0.8264 | 2 |
| 3 | 1.3310 | 0.7513 | 3.3100 | 0.3021 | 2.4869 | 0.4021 | 0.9366 | 2.3291 | 3 |
| 4 | 1.4641 | 0.6830 | 4.6410 | 0.2155 | 3.1699 | 0.3155 | 1.3812 | 4.3781 | 4 |
| 5 | 1.6105 | 0.6209 | 6.1051 | 0.1638 | 3.7908 | 0.2638 | 1.8101 | 6.8618 | 5 |
| 6 | 1.7716 | 0.5645 | 7.7156 | 0.1296 | 4.3553 | 0.2296 | 2.2236 | 9.6842 | 6 |
| 7 | 1.9487 | 0.5132 | 9.4872 | 0.1054 | 4.8684 | 0.2054 | 2.6216 | 12.7631 | 7 |
| 8 | 2.1436 | 0.4665 | 11.4359 | 0.0874 | 5.3349 | 0.1874 | 3.0045 | 16.0287 | 8 |
| 9 | 2.3579 | 0.4241 | 13.5795 | 0.0736 | 5.7590 | 0.1736 | 3.3724 | 19.4215 | 9 |
| 10 | 2.5937 | 0.3855 | 15.9374 | 0.0627 | 6.1446 | 0.1627 | 3.7255 | 22.8913 | 10 |
| 11 | 2.8531 | 0.3505 | 18.5312 | 0.0540 | 6.4951 | 0.1540 | 4.0641 | 26.3963 | 11 |
| 12 | 3.1384 | 0.3186 | 21.3843 | 0.0468 | 6.8137 | 0.1468 | 4.3884 | 29.9012 | 12 |
| 13 | 3.4523 | 0.2897 | 24.5227 | 0.0408 | 7.1034 | 0.1408 | 4.6988 | 33.3772 | 13 |
| 14 | 3.7975 | 0.2633 | 27.9750 | 0.0357 | 7.3667 | 0.1357 | 4.9955 | 36.8005 | 14 |
| 15 | 4.1772 | 0.2394 | 31.7725 | 0.0315 | 7.6061 | 0.1315 | 5.2789 | 40.1520 | 15 |
| 16 | 4.5950 | 0.2176 | 35.9497 | 0.0278 | 7.8237 | 0.1278 | 5.5493 | 43.4164 | 16 |
| 17 | 5.0545 | 0.1978 | 40.5447 | 0.0247 | 8.0216 | 0.1247 | 5.8071 | 46.5819 | 17 |
| 18 | 5.5599 | 0.1799 | 45.5992 | 0.0219 | 8.2014 | 0.1219 | 6.0526 | 49.6395 | 18 |
| 19 | 6.1159 | 0.1635 | 51.1591 | 0.0195 | 8.3649 | 0.1195 | 6.2861 | 52.5827 | 19 |
| 20 | 6.7275 | 0.1486 | 57.2750 | 0.0175 | 8.5136 | 0.1175 | 6.5081 | 55.4069 | 20 |
| 25 | 10.8347 | 0.0923 | 98.3471 | 0.0102 | 9.0770 | 0.1102 | 7.4580 | 67.6964 | 25 |
| 30 | 17.4494 | 0.0573 | 164.4940 | 0.0061 | 9.4269 | 0.1061 | 8.1762 | 77.0766 | 30 |
| 35 | 28.1024 | 0.0356 | 271.0244 | 0.0037 | 9.6442 | 0.1037 | 8.7086 | 83.9872 | 35 |
| 40 | 45.2593 | 0.0221 | 442.5926 | 0.0023 | 9.7791 | 0.1023 | 9.0962 | 88.9525 | 40 |
| 50 | 117.3909 | 0.0085 | 1163.9085 | 0.0009 | 9.9148 | 0.1009 | 9.5704 | 94.8889 | 50 |
| 60 | 304.4816 | 0.0033 | 3034.8164 | 0.0003 | 9.9672 | 0.1003 | 9.8023 | 97.7010 | 60 |

**i = 12%**

| n | [F/P, i, n] | [P/F, i, n] | [F/A, i, n] | [A/F, i, n] | [P/A, i, n] | [A/P, i, n] | [A/G, i, n] | [P/G, i, n] | n |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.1200 | 0.8929 | 1.0000 | 1.0000 | 0.8929 | 1.1200 | 0.0000 | 0.0000 | 1 |
| 2 | 1.2544 | 0.7972 | 2.1200 | 0.4717 | 1.6901 | 0.5917 | 0.4717 | 0.7972 | 2 |
| 3 | 1.4049 | 0.7118 | 3.3744 | 0.2963 | 2.4018 | 0.4163 | 0.9246 | 2.2208 | 3 |
| 4 | 1.5735 | 0.6355 | 4.7793 | 0.2092 | 3.0373 | 0.3292 | 1.3589 | 4.1273 | 4 |
| 5 | 1.7623 | 0.5674 | 6.3528 | 0.1574 | 3.6048 | 0.2774 | 1.7746 | 6.3970 | 5 |
| 6 | 1.9738 | 0.5066 | 8.1152 | 0.1232 | 4.1114 | 0.2432 | 2.1720 | 8.9302 | 6 |
| 7 | 2.2107 | 0.4523 | 10.0890 | 0.0991 | 4.5638 | 0.2191 | 2.5515 | 11.6443 | 7 |
| 8 | 2.4760 | 0.4039 | 12.2997 | 0.0813 | 4.9676 | 0.2013 | 2.9131 | 14.4714 | 8 |
| 9 | 2.7731 | 0.3606 | 14.7757 | 0.0677 | 5.3282 | 0.1877 | 3.2574 | 17.3563 | 9 |
| 10 | 3.1058 | 0.3220 | 17.5487 | 0.0570 | 5.6502 | 0.1770 | 3.5847 | 20.2541 | 10 |

**i = 13%**

| n | [F/P, i, n] | [P/F, i, n] | [F/A, i, n] | [A/F, i, n] | [P/A, i, n] | [A/P, i, n] | [A/G, i, n] | [P/G, i, n] | n |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.1300 | 0.8850 | 1.0000 | 1.0000 | 0.8850 | 1.1300 | 0.0000 | 0.0000 | 1 |
| 2 | 1.2769 | 0.7831 | 2.1300 | 0.4695 | 1.6681 | 0.5995 | 0.4695 | 0.7831 | 2 |
| 3 | 1.4429 | 0.6931 | 3.4069 | 0.2935 | 2.3612 | 0.4235 | 0.9187 | 2.1692 | 3 |
| 4 | 1.6305 | 0.6133 | 4.8498 | 0.2062 | 2.9745 | 0.3362 | 1.3479 | 4.0092 | 4 |
| 5 | 1.8424 | 0.5428 | 6.4803 | 0.1543 | 3.5172 | 0.2843 | 1.7571 | 6.1802 | 5 |
| 6 | 2.0820 | 0.4803 | 8.3227 | 0.1202 | 3.9975 | 0.2502 | 2.1468 | 8.5818 | 6 |
| 7 | 2.3526 | 0.4251 | 10.4047 | 0.0961 | 4.4226 | 0.2261 | 2.5171 | 11.1322 | 7 |
| 8 | 2.6584 | 0.3762 | 12.7573 | 0.0784 | 4.7988 | 0.2084 | 2.8685 | 13.7653 | 8 |
| 9 | 3.0040 | 0.3329 | 15.4157 | 0.0649 | 5.1317 | 0.1949 | 3.2014 | 16.4284 | 9 |
| 10 | 3.3946 | 0.2946 | 18.4197 | 0.0543 | 5.4262 | 0.1843 | 3.5162 | 19.0797 | 10 |

**i = 14%**

| n | [F/P, i, n] | [P/F, i, n] | [F/A, i, n] | [A/F, i, n] | [P/A, i, n] | [A/P, i, n] | [A/G, i, n] | [P/G, i, n] | n |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.1400 | 0.8772 | 1.0000 | 1.0000 | 0.8772 | 1.1400 | 0.0000 | 0.0000 | 1 |
| 2 | 1.2996 | 0.7695 | 2.1400 | 0.4673 | 1.6467 | 0.6073 | 0.4673 | 0.7695 | 2 |
| 3 | 1.4815 | 0.6750 | 3.4396 | 0.2907 | 2.3216 | 0.4307 | 0.9129 | 2.1194 | 3 |
| 4 | 1.6890 | 0.5921 | 4.9211 | 0.2032 | 2.9137 | 0.3432 | 1.3370 | 3.8957 | 4 |
| 5 | 1.9254 | 0.5194 | 6.6101 | 0.1513 | 3.4331 | 0.2913 | 1.7399 | 5.9731 | 5 |
| 6 | 2.1950 | 0.4556 | 8.5355 | 0.1172 | 3.8887 | 0.2572 | 2.1218 | 8.2511 | 6 |
| 7 | 2.5023 | 0.3996 | 10.7305 | 0.0932 | 4.2883 | 0.2332 | 2.4832 | 10.6489 | 7 |
| 8 | 2.8526 | 0.3506 | 13.2328 | 0.0756 | 4.6389 | 0.2156 | 2.8246 | 13.1028 | 8 |
| 9 | 3.2519 | 0.3075 | 16.0853 | 0.0622 | 4.9464 | 0.2022 | 3.1463 | 15.5629 | 9 |
| 10 | 3.7072 | 0.2697 | 19.3373 | 0.0517 | 5.2161 | 0.1917 | 3.4490 | 17.9906 | 10 |

**i = 15%**

| n | [F/P, i, n] | [P/F, i, n] | [F/A, i, n] | [A/F, i, n] | [P/A, i, n] | [A/P, i, n] | [A/G, i, n] | [P/G, i, n] | n |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.1500 | 0.8696 | 1.0000 | 1.0000 | 0.8696 | 1.1500 | 0.0000 | 0.0000 | 1 |
| 2 | 1.3225 | 0.7561 | 2.1500 | 0.4651 | 1.6257 | 0.6151 | 0.4651 | 0.7561 | 2 |
| 3 | 1.5209 | 0.6575 | 3.4725 | 0.2880 | 2.2832 | 0.4380 | 0.9071 | 2.0712 | 3 |
| 4 | 1.7490 | 0.5718 | 4.9934 | 0.2003 | 2.8550 | 0.3503 | 1.3263 | 3.7864 | 4 |
| 5 | 2.0114 | 0.4972 | 6.7424 | 0.1483 | 3.3522 | 0.2983 | 1.7228 | 5.7751 | 5 |
| 6 | 2.3131 | 0.4323 | 8.7537 | 0.1142 | 3.7845 | 0.2642 | 2.0972 | 7.9368 | 6 |
| 7 | 2.6600 | 0.3759 | 11.0668 | 0.0904 | 4.1604 | 0.2404 | 2.4498 | 10.1924 | 7 |
| 8 | 3.0590 | 0.3269 | 13.7268 | 0.0729 | 4.4873 | 0.2229 | 2.7813 | 12.4807 | 8 |
| 9 | 3.5179 | 0.2843 | 16.7858 | 0.0596 | 4.7716 | 0.2096 | 3.0922 | 14.7548 | 9 |
| 10 | 4.0456 | 0.2472 | 20.3037 | 0.0493 | 5.0188 | 0.1993 | 3.3832 | 16.9795 | 10 |