Searching & Sorting in Java – Binary Search

If the data you wish to search is already in order, a sequential search will still, on average, take the same amount of time to find the item you want. It is possible, however, to greatly improve the speed of a search on sorted data.

The *binary search* algorithm is one such way to improve performance using sorted data. This algorithm is an example of a *divide and conquer* algorithm, of which there are many other examples. This type of algorithm solves a problem by quickly reducing its size. For the binary search, at each stage of the problem we cut the size of the problem roughly in half.

To illustrate, consider the following list, and suppose we are searching for the value 47.

16	19	22	24	27	29	37	40	43	44	47	52	56	60	64
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

To start the process, we initially examine the item in the middle of the array. The middle item, 40, is not the one we want, but it is less than the value we are looking for. Since the list is sorted, we use this information to eliminate all of the items in the lower half of the list. Our search now only looks at the remaining (upper) half of the list.

16	19	22	24	27	29	37	40	43	44	47	52	56	60	64
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

We repeat our strategy on these items. The middle value is now 52, which is too high, so we eliminate the upper half of the remaining list.

16	19	22	24	27	29	37	40	43	44	47	52	56	60	64
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

The middle value is now 44, which is too small. Eliminating everything below this value leaves us with only a single item that hasn't been eliminated, which is the location of our target value.

16	19	22	24	27	29	37	40	43	44	47	52	56	60	64
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

To implement this algorithm in Java, we will search for `item` in an array called `list`. Through the process of elimination, the upper and lower bounds of the array that we need to search will change, so we will track them with `int` variables called `end` and `front`. Similarly, we need to track the `middle` value, also an `int`.

For each iteration, we can find the value of middle by taking the average of the end and front. If the value at middle is equal to item, then obviously our search is done. If our middle value is too low, the front becomes middle + 1. If our middle value is too high, the end becomes middle - 1.

If our search value is not in the list, this process will continue until front and end and middle are all equal to each other (i.e., we are looking at a single element of the array). On the next step, end or front will change such that end < front, which signals the end of our search, at which point we return a value to indicate a failure (-1).

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```
public static int binSearch ( double[] list, double item) {
   int front = 0;                  // lower bound of searching
   int end = list.length - 1;      // upper bound of searching
   int middle;                     // current search candidate
   boolean found = false;
   int location = -1;              // location of item, -1 for failure
   while (front <= end && !found)
   {
      middle = (front + end)/2; // integer division, auto-truncate
      if (list[middle] == item) {
         location = middle;         // success!
         found = true;
      }
      else if (list[middle] < item)
      {
         front = middle + 1;    // look only in end half
      }
      else
      {
         end = middle - 1;      // look only in front half
      }
   }
   return location;
}
```

Suppose we want to perform a binary search for the value 75 on the following data.

12	34	47	62	75	87	90

Initially, we need to search the entire array, so front and end are set to 0 and 6, while middle is set to 3.

front	middle	end
0	3	6

12	34	47	62	75	87	90
0	1	2	3	4	5	6

Since 62 < 75, the item we are seeking cannot be in the left half of the array. We discard this half by setting `front` to `middle + 1 = 4`. The middle of the remaining interval is `(4 + 6)/2 = 5`.

front	middle	end
4	5	6

12	34	47	62	75	87	90
0	1	2	3	4	5	6

Since `87 > 75`, the value 75 cannot be in the upper half of the sublist, so we discard it by setting `end` to `middle – 1 = 4`. The new value of middle will be `(4 + 4)/2 = 4`.

front	middle	end
4	4	4

12	34	47	62	75	87	90
0	1	2	3	4	5	6

Once the value has been found at middle, the search ends successfully.

Now let us consider a failed search, were the final element was not equal to our search value.

front	middle	end
4	4	4

12	34	47	62	80	87	90
0	1	2	3	4	5	6

Since `80 > 75`, the value 75 cannot be in the upper half of the sublist, so we discard it by setting `end` to `middle – 1 = 3`. Now we have the situation where `end < front`, so our searching ends without a successful result.

Note: The Java libraries include methods for sorting arrays of any primitive type (int, long, float, double, char), or even objects (e.g., String). These methods are *overloaded*, which means they can be called using the same method name, `sort`. Since they are part of the `Arrays` class, the call will be:

```
Arrays.sort(<array name>);
```

For example, consider the following arrays (integers and strings) which are sorted using `sort`.

```
int[] numbers = {4, 3, 5, 6, 7, 4, 8, 3, 4, 1};
String[] names = {"Ed", "Bob", "Alice", "Rob", "Gayle"};
Array.sort(numbers);
Array.sort(names);
```

In order to use methods from the `Arrays` class, we must *import* the library into our current program. A full program, including the required import statement, is shown below. Notice that the import must come before the class declaration.

Also included is a method, toString, which allows the array to be easily displayed on a single line (if it is short enough).

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```
import java.util.Arrays;

public class ArraySortJavaLib {

    public static void main(String[] args)     {

         //... 1. Sort strings - or any other Comparable objects.
        String[] names = {"Zoe", "Alison", "David"};
        Arrays.sort(names);
        System.out.println(Arrays.toString(names));

        //... 2. Sort doubles or other primitives.
        double[] lengths = {120.0, 0.5, 0.0, 999.0, 77.3};
        Arrays.sort(lengths);
        System.out.println(Arrays.toString(lengths));
    }
}
```

Comparing Sequential and Binary Search

We judge the effectiveness of a search by looking at the worst case scenario on the number of comparisons performed to find your element.

<u>Sequential Search</u> – Worst case is your element is at the end of the array.

# of Comparisons = n where n is the size of the array

ie find 71:

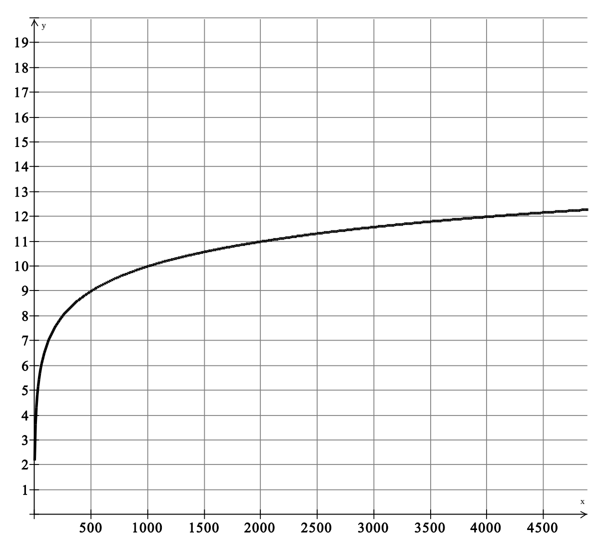
16 19 24 27 29 37 40 43 44 47 52 56 60 64 71

There would have to be 15 comparisons until the element is found.

<u>Binary Search</u> – Worst case is you have divided your array in such a way that First, Last and Middle all point to the same element in the array.

# of Comparisons = $\frac{\log(n)}{\log(2)}$ (rounded up) where n is the size of the array.

# of comparisons



# of elements

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Exercises

1. Suppose that an array contains the following elements.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
23	27	30	34	41	49	51	55	57	60	67	72	78	83	96

   Trace the execution of the method `binSearch` shown in this section as it searches for the following values of `item`. In each trace, show the progress of the search by using diagrams similar to those in previous examples.

   (a) 72 (b) 41 (c) 62

2. What changes would have to be made to `binSearch` so that it will search an array in *descending* order?

3. Rewrite binSearch so that, if a search is unsuccessful, the method will return the index of the value *nearest* to item, instead of returning -1. If there is a tie, return the smaller index.

4. What is the maximum number of comparisons that might be necessary to perform a binary search on a list containing seven items?

5. Repeat the previous question for lists with indicated sizes.

(a) 3	(e) 31
(b) 15	(f) 63
(c) 1000	(g) 100
(d) 10000	(h) 500