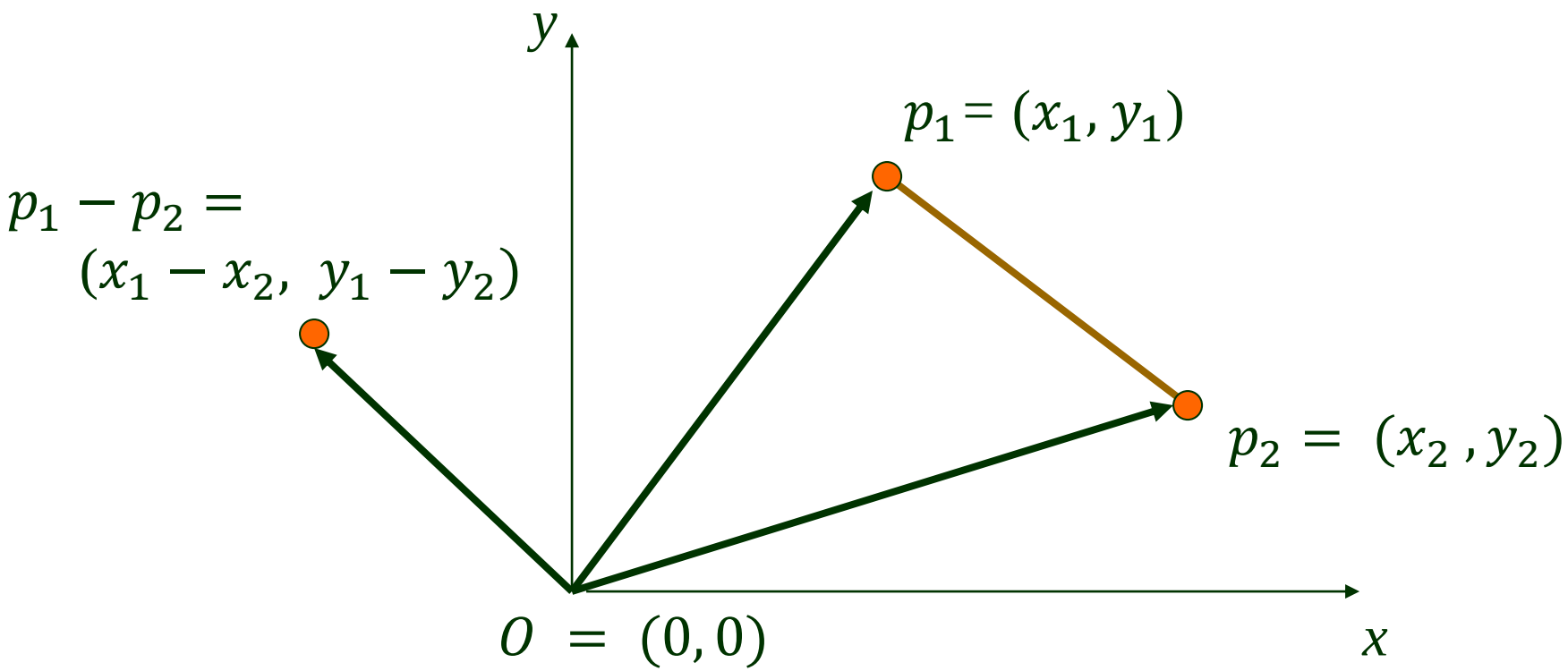
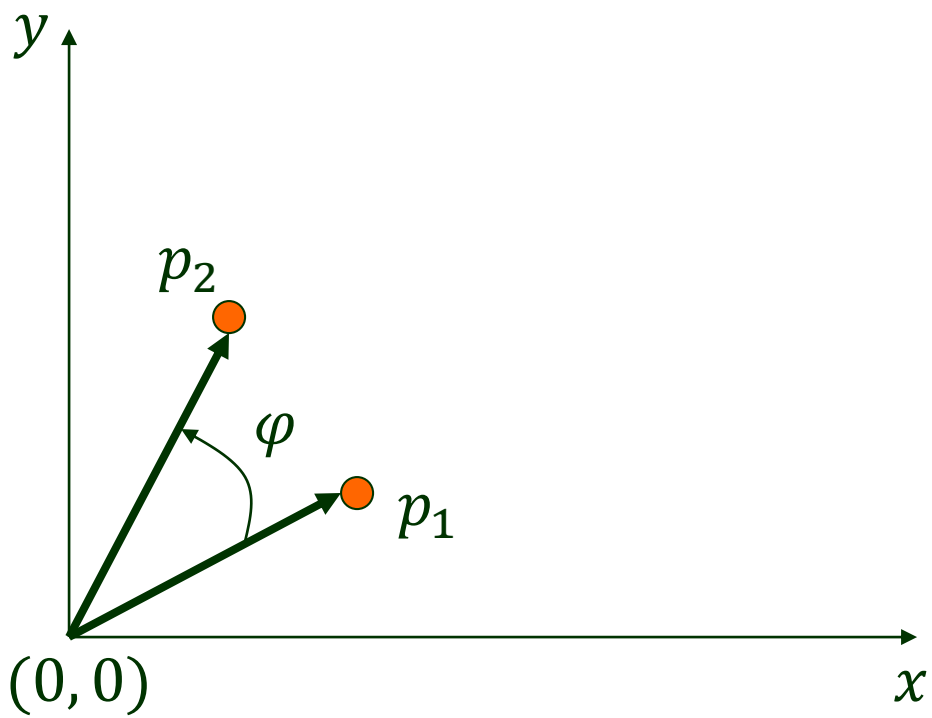
Line Segments & Vectors



Points (vectors): $p_1, p_2, p_1 - p_2 = \overrightarrow{p_2 p_1}$

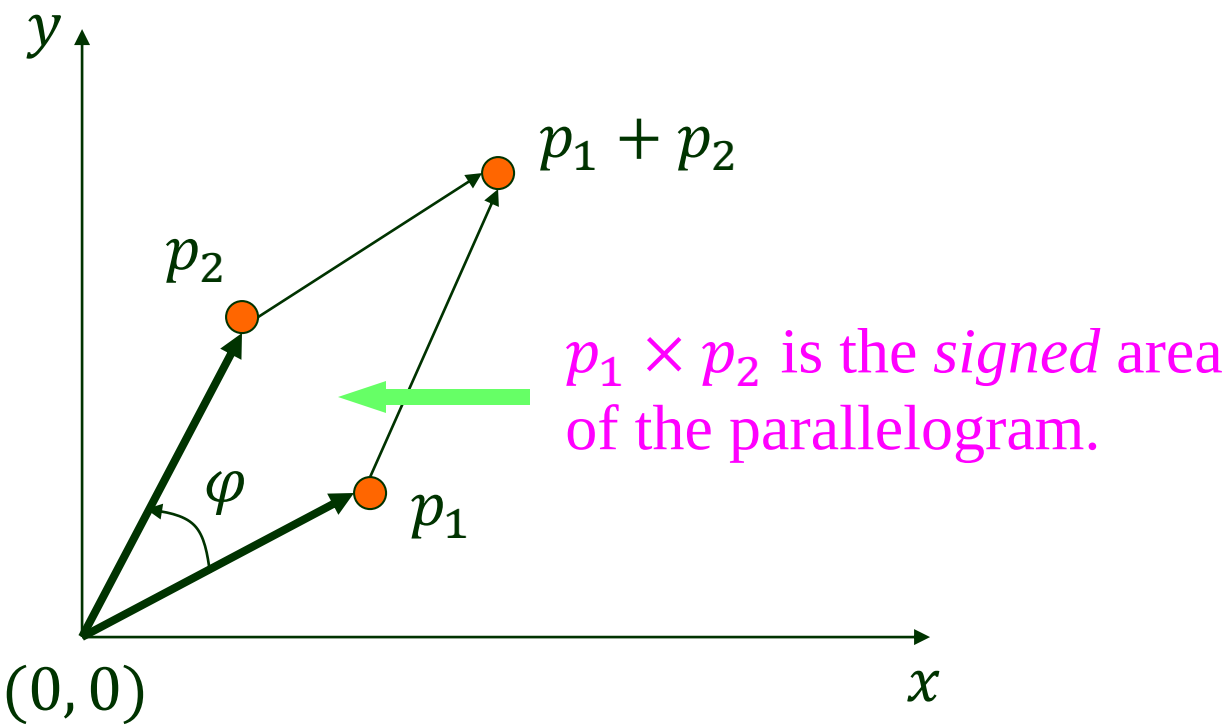
Line segment: $\overline{p_2 p_1} = \overline{p_1 p_2}$

Dot (Inner) Product



$$p_1 \cdot p_2 = x_1 x_2 + y_1 y_2 = p_2 \cdot p_1 = |p_1|\,|p_2|\,\cos\varphi$$

Cross (Vector) Product

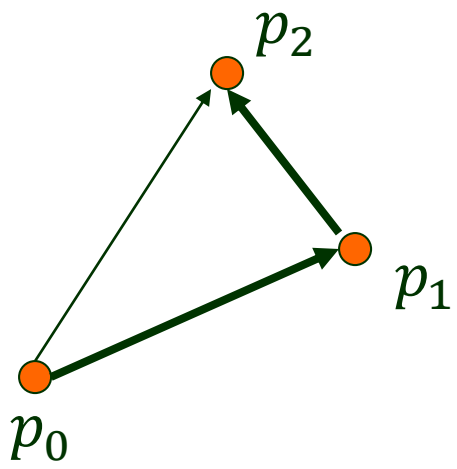


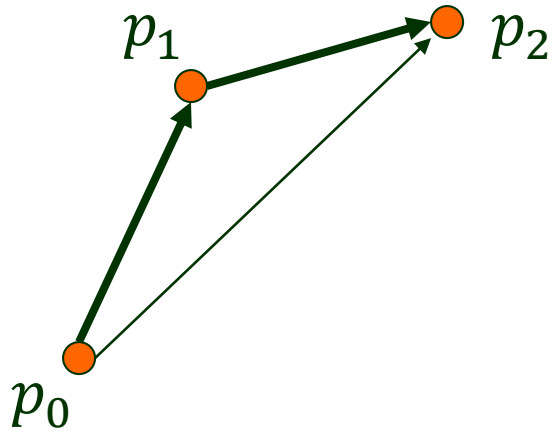
$$p_1 \times p_2 = x_1 y_2 - x_2 y_1 = -p_2 \times p_1 = |p_1|\,|p_2|\,\sin\varphi$$

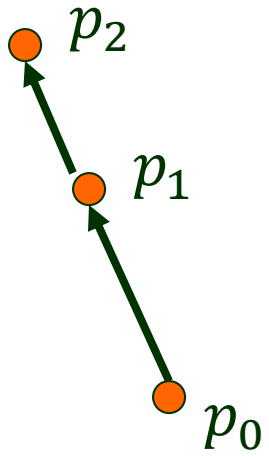
p_1 and p_2 are *collinear* with the origin iff $p_1 \times p_2 = 0$.

Turning of Consecutive Segments

Segments $\overline{p_0p_1}$ and $\overline{p_1p_2}$. Move from p_0 to p_1 then to p_2.







Counterclockwise

$\overrightarrow{p_0p_1} \times \overrightarrow{p_1p_2} > 0$

Clockwise

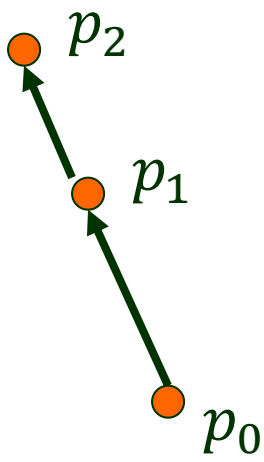
$\overrightarrow{p_0p_1} \times \overrightarrow{p_1p_2} < 0$

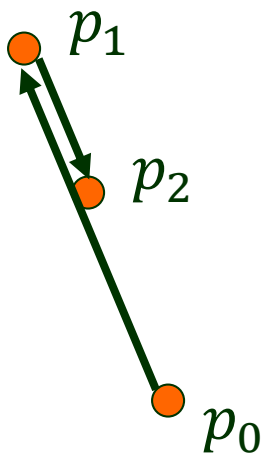
Turn of 0 or π

$\overrightarrow{p_0p_1} \times \overrightarrow{p_1p_2} = 0$

Collinear Points

$\overrightarrow{p_0p_1} \times \overrightarrow{p_1p_2} = 0 \Longrightarrow p_0, p_1, p_2$ are collinear.





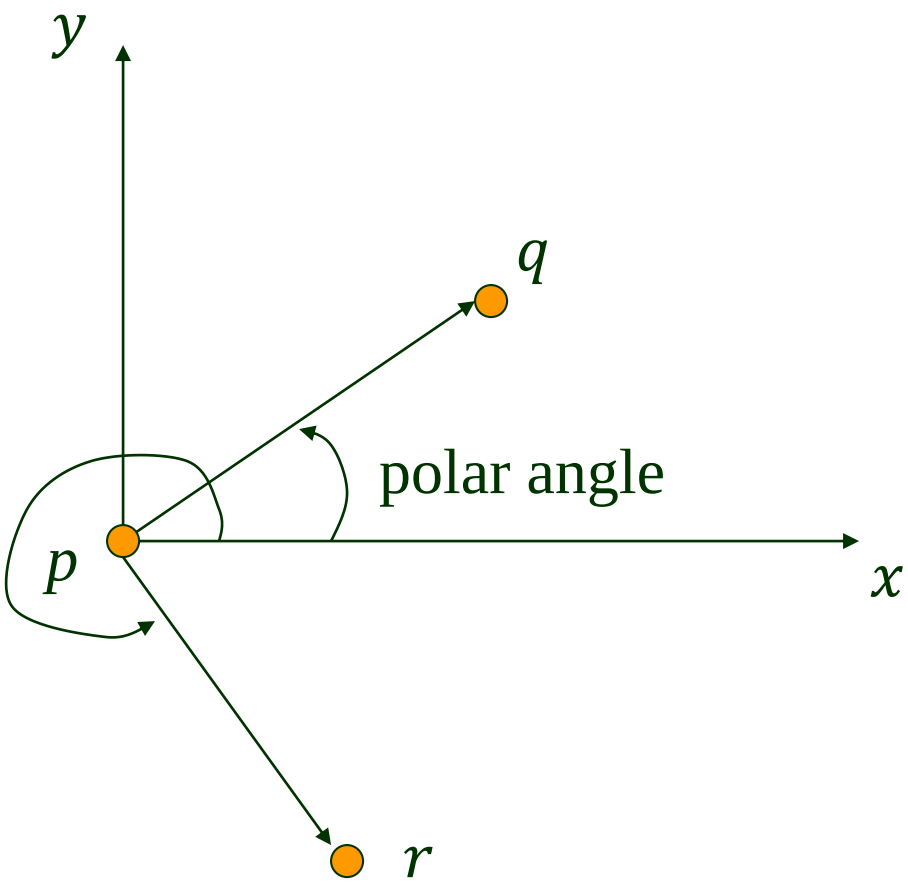
No change of direction

Direction reversal

$\overrightarrow{p_0p_1} \cdot \overrightarrow{p_1p_2} > 0$

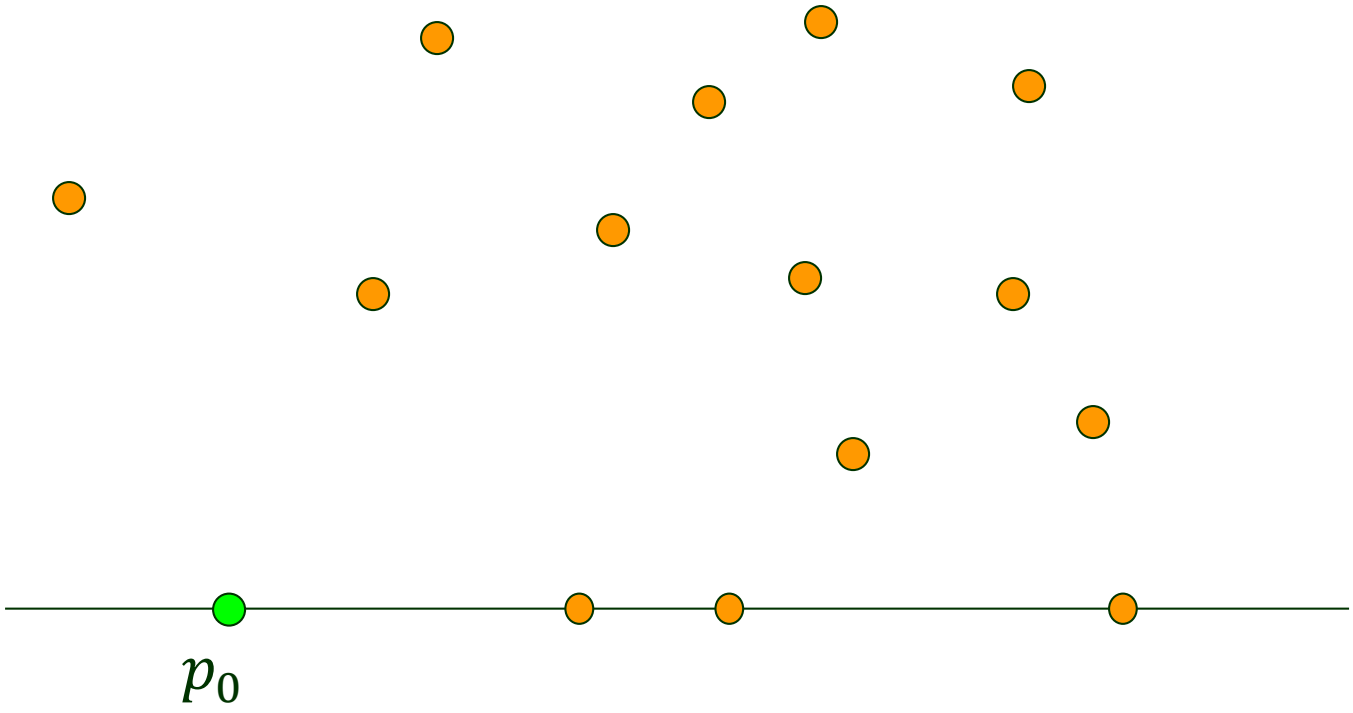
$\overrightarrow{p_0p_1} \cdot \overrightarrow{p_1p_2} < 0$

Polar Angle



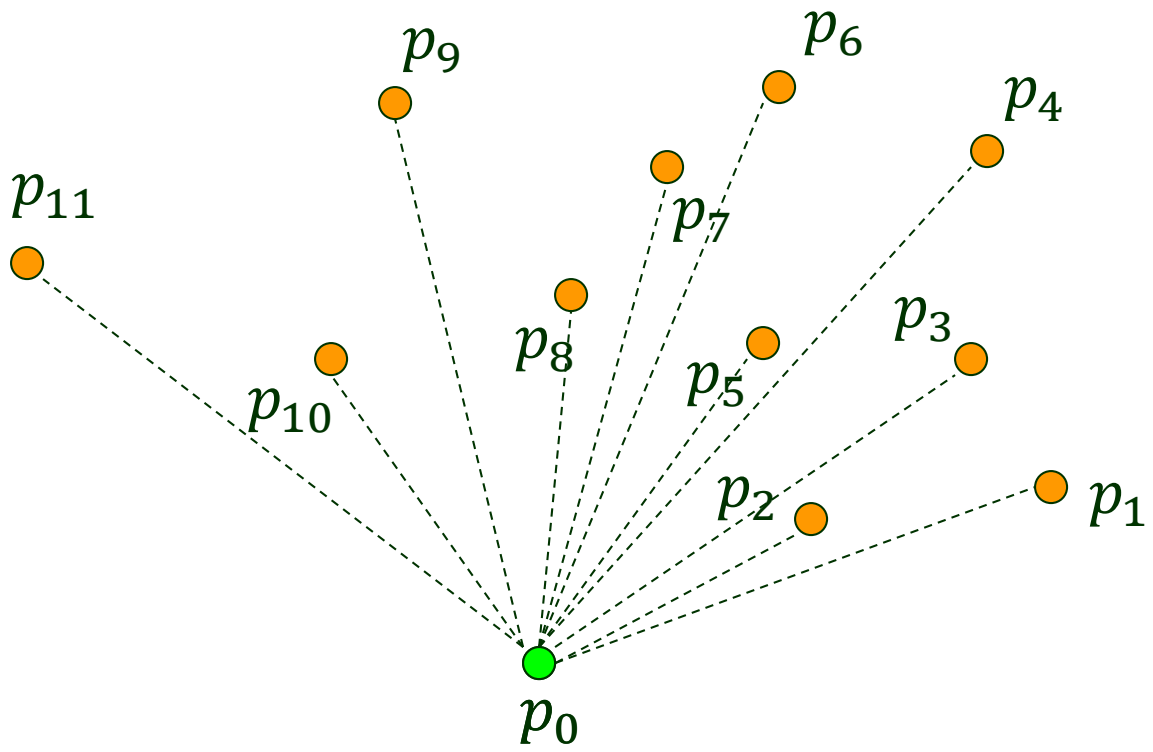
Tie Breaking (1)

When more than one point has the smallest y coordinate, pick the *leftmost* one.



Sorting by Polar Angle

2) Sort by polar angle with respect to p_0.



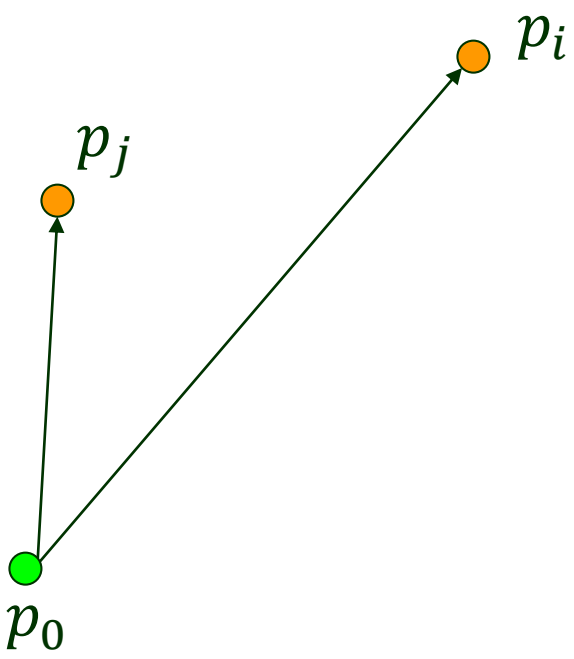
Labels are in the polar angle order.

No Polar Angle Evaluation

p_0 is the lowest (and leftmost) ⟹ all polar angles $\in [0, \pi)$.

Use cross product!

$$p_i < p_j \text{ if } \overrightarrow{p_0 p_i} \times \overrightarrow{p_0 p_j} > 0$$



Tie Breaking (2)

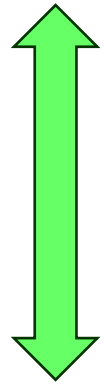
What if p_0, p_i, p_j are on the same line?

Order them by distance from p_0.

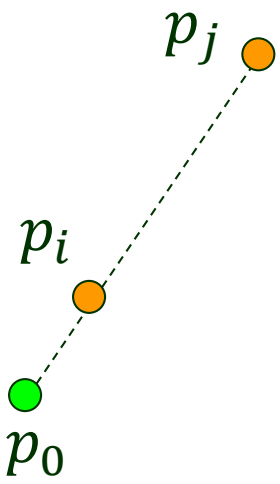
$p_i < p_j$ if $\overrightarrow{p_0p_i} \times \overrightarrow{p_0p_j} = 0$ and

$\boxed{|\overrightarrow{p_0p_i}| < |\overrightarrow{p_0p_j}|}$

No square roots.
Use dot product!

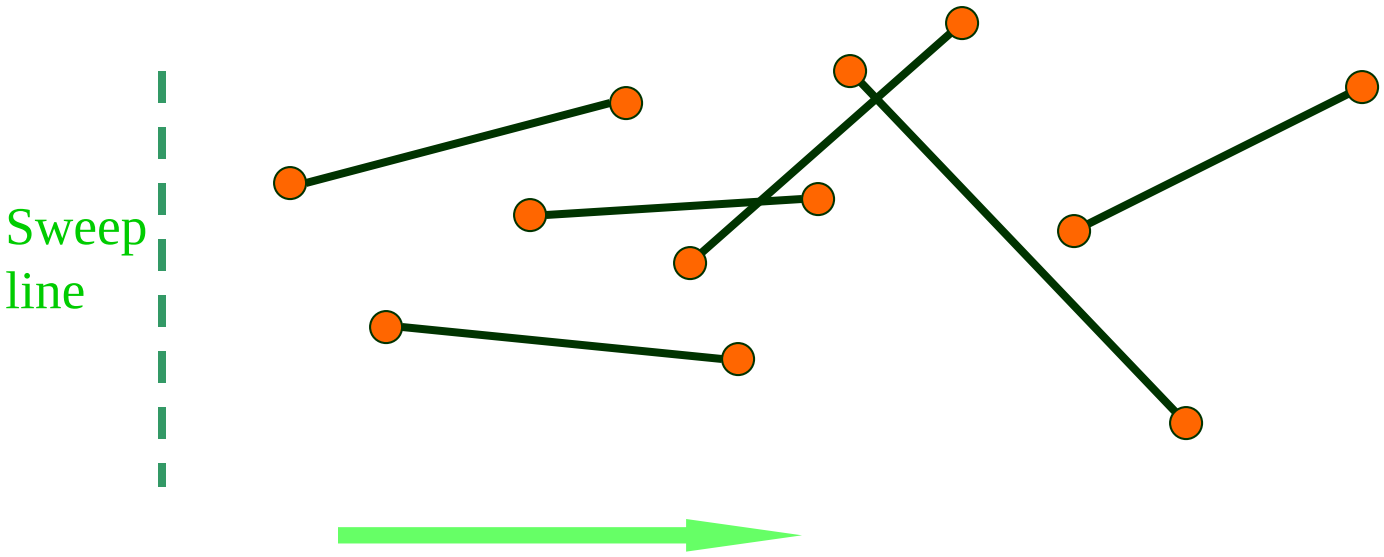


$$\overrightarrow{p_0p_i} \cdot \overrightarrow{p_0p_i} < \overrightarrow{p_0p_j} \cdot \overrightarrow{p_0p_j}$$



Sorting by x-coordinate

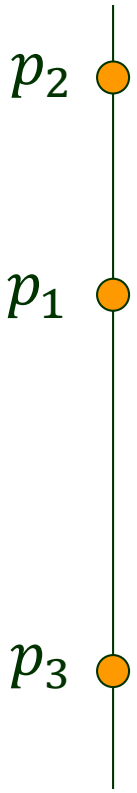
Imagine a vertical line sweeping across the plane from left to right. It hits the endpoints of these line segments one by one.



Tie Breaking (3)

What if two or endpoints are on the same vertical line?

Order them by y-coordinate.



$p_3 < p_1 < p_2$