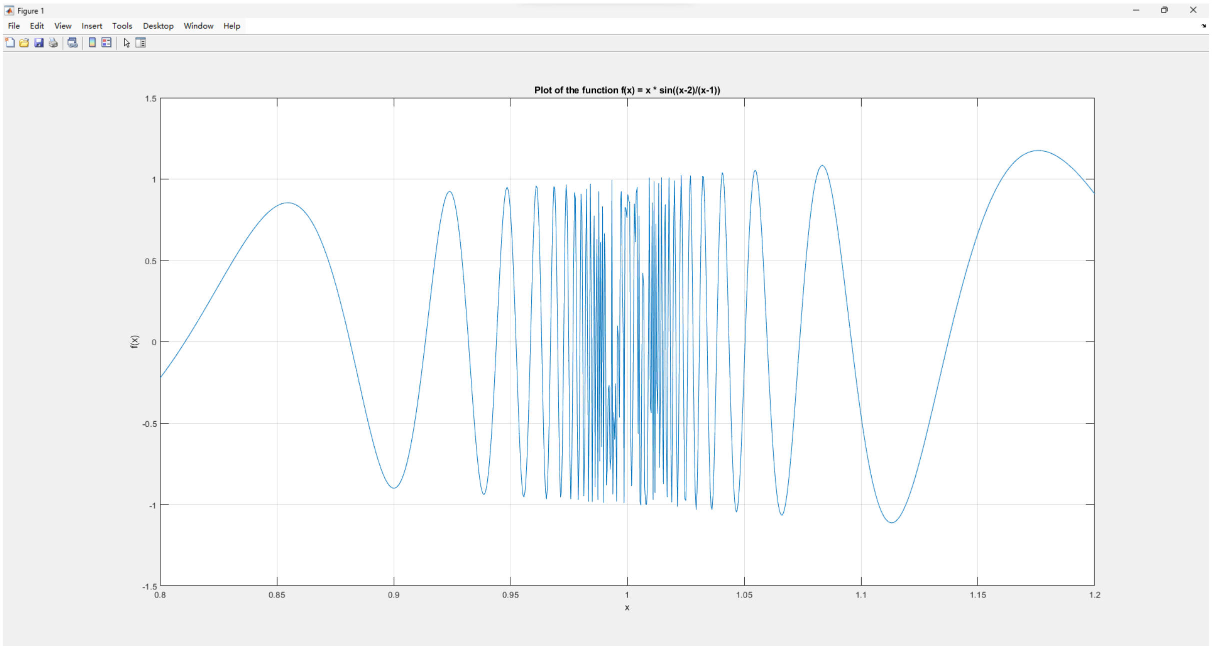
Assignment 1

學號：111550129　姓名：林彥亨

1. (20%) The function $f(x) = x * \sin\frac{(x-2)}{(x-1)}$ has many zeros, especially near $x = 1$ where the function is discontinuous. Find the four zeros nearest to $x = 0.95$ by bisection, correct to five significant figures. How can you find good starting intervals?

- This is the picture we get through the matlab.



So we choose four zeros near x = 0.95 with bisection,

when the range from 0.95 to 0.953, we get the result is x = 0.95236

```
>> problem1
there is 8 interation(s).
Root near x = 0.95 is x = 0.95236
```

when the range from 0.94 to 0.95, we get the result is x = 0.94397

```
>> problem1
there is 9 interation(s).
Root near x = 0.95 is x = 0.94397
```

when the range from 0.955 to 0.96, we get the result is x = 0.95856

```
>> problem1
there is 8 interation(s).
Root near x = 0.95 is x = 0.95856
```

when the range from 0.96 to 0.965, we get the result is x = 0.96333

```
>> problem1
there is 8 interation(s).
Root near x = 0.95 is x = 0.96333
```

---

2. (20%) Repeat Problem 1 but use the secant method. How many fewer iterations are required?

	bisection	secant method
0.95236	8	3
0.94397	9	6
0.95856	8	4
0.96333	8	5

when the range from 0.95 to 0.953

```
>> problem2
Root found: 0.952361 after 3 iterations
```

when the range from 0.94 to 0.95

```
>> problem2
Root found: 0.943976 after 6 iterations
```

when the range from 0.955 to 0.96

```
>> problem2
Root found: 0.958563 after 4 iterations
```

when the range from 0.96 to 0.965

```
>> problem2
Root found: 0.963335 after 5 iterations
```

---

3. (20%) This polynomial obviously has roots at $x = 2$ and at $x = 4$; one is a double root, the other is a triple root:

$$\begin{aligned} p(x) &= (x-2)^3(x-4)^2 \\ &= x^5 - 14x^4 + 76x^3 - 200x^2 + 256x - 128 \end{aligned}$$

(a) Which root can you get with bisection? Which root can't you get?

(b) Repeat part (a) with the secant method.

(c) If you begin with the interval [1, 5], which root will you get with (1) bisection, (2) the secant method, (3) false position?

(a)

Due to the bisection, we get the root of $x = 2$; the root of $x = 4$ cannot get.

(b)

Due to the scant method, we get the root of $x = 2$; the root of $x = 4$ cannot get.

(c)

(1) bisection: get the root $x = 2$.

Given that the triple root at *x* = 2 causes a sign change while the double root at *x* = 4 does not, the bisection method should find the triple root at *x* = 2.

(2) secant method: get the root $x = 2$

In the secant method, it is possible to find 2 or 4. Due to the result of the function, we only get the answer of $x = 2$.

- Here is the result.

```
>> problem3_b
Secant method converged to x = 2.000000
```

(3) false position: get the root $x = 2$.

The false position method requires a sign change over the interval. It will, therefore, also likely converge to the triple root at *x*=2. Similar to bisection, it relies on the Intermediate Value Theorem, which guarantees a root within an interval where the function changes sign.

---

4. (20%) Use Muller's method to find roots of these equations.

   (a) $4x^3 - 3x^2 + 2x - 1 = 0$, root near $x = 0.6$.

   (b) $x^2 + e^x = 5$, roots near $x = 1, x = -2$.

(a)

the root is 0.6058.

```
>> problem4_a
Root found: 0.605830.
```

(b)

the root near x = 1 is 1.2411; the root near x = -2 is -2.2114.

```
>> problem4_b
For root near x = 1:
Root found: 1.241143.

For root near x = -2:
Root found: -2.211438.
```

---

5. (20%) Most functions can be rearranged in several ways to give $x = g(x)$ with which to begin the fixed-point method. For $f(x) = e^x - 2x^2$, one $g(x)$ is

$$x = \pm\sqrt{\frac{e^x}{2}}$$

1

(a) Show that this converges to the root near 1.5 if the positive value is used and to the root near -0.5 if the negative is used.

(b) There is a third root 2.6. Show that we do not converge to this root even though values near to the root are used to begin the iterations. Where does it converge if $x_0 = 2.5$? If $x_0 = 2.7$?

(c) Find another rearrangement that does converge correctly to the third root.

(a)

Here is the result.

```
>> problem5_a
For the positive root:
Fixed-point iteration converged to x = 1.487987

For the negative root:
Fixed-point iteration converged to x = -0.539834
```

(b)

the x_0= 2.5 can get the coverge to x =1.5, but $x_0 = 2.7$ cannot converged.

```
>> problem5_b
Starting from x0 = 2.500000, fixed-point iteration converged to x = 1.487989
Starting from x0 = 2.700000, fixed-point iteration did not converge, last x = Inf
```

(c)

we take each side with nature log, so we get the $g(x) = ln(2x^2)$, with $x_0 = 2.5$ and we get the result is about x= 2.6.

```
>> problem5_c
Converged to x = 2.617836
```

---

6. (20%) Solve the following system of nonlinear equations using Newton's method.

$$y = cos^2(x)$$
$$x^2 + y^2 - x = 2$$

The system of equations provided is:

$$y = cos^2(x)$$

$$x^2 + y^2 - x = 2$$

To apply Newton's method, we need to express this system as*F*(*x*,*y*)=0, where *F* is a vector-valued function. For this system, we can write:

$$F_1(x, y) = y - cos^2(x)$$

$$F_2(x, y) = x^2 + y^2 - x - 2$$

The Jacobian matrix *J* for this system is:

$$J(x, y) = \begin{bmatrix} \partial x/\partial F1 & \partial F1/\partial y \\ \partial x/\partial F2 & \partial y/\partial F2 \end{bmatrix} = \begin{bmatrix} 2sin(x)cos(x) & 1 \\ 2x - 1 & 2y \end{bmatrix}$$

Newton's method for systems uses the following iteration formula:

$$\begin{bmatrix} x_n + 1 \\ y_n + 1 \end{bmatrix} = \begin{bmatrix} x_n \\ y_n \end{bmatrix} - J(x_n, y_n)^-1 \cdot F(x_n, y_n)$$

- Here is the result we may get two answers using Newton's method.

$$x_0 = 2, y_0 = 0.5$$

```
>> problem6
Solution converged to x = 1.990759, y = 0.166241
```

$$x_0 = 1, y_0 = 1$$

```
>> problem6
Solution converged to x = -0.964417, y = 0.324782
```