CS 161 hw7

1. Base cases:

$n=1$ $Pr(\alpha_1|\beta) = Pr(\alpha_1|\beta)$ ✓

$n=2$ $Pr(\alpha_1,\alpha_2|\beta) = \frac{P(\alpha_1,\alpha_2,\beta)}{P(\beta)} = \frac{P(\alpha_1|\alpha_2,\beta)P(\alpha_2,\beta)}{P(\beta)} = P(\alpha_1|\alpha_2,\beta)P(\alpha_2|\beta)$ ✓

Assume for $n=k$ the equation holds, so

$Pr(\alpha_1,\dots,\alpha_k|\beta) = Pr(\alpha_1|\alpha_2,\dots,\alpha_k,\beta)Pr(\alpha_2|\alpha_3,\dots,\alpha_k,\beta)\dots Pr(\alpha_k|\beta)$

For $n=k+1$,

$$Pr(\alpha_1,\dots,\alpha_k,\alpha_{k+1}|\beta) = \frac{Pr(\alpha_1,\dots,\alpha_k,\alpha_{k+1},\beta)}{Pr(\beta)} = \frac{Pr(\alpha_1,\dots\alpha_k|\alpha_{k+1},\beta)}{P(\beta)}$$

$$= Pr(\alpha_1,\dots\alpha_k|\alpha_{k+1},\beta)Pr(\alpha_{k+1}|\beta)$$

$$= Pr(\alpha_1|\alpha_2,\dots,\alpha_k,\alpha_{k+1},\beta)Pr(\alpha_2|\alpha_3,\dots,\alpha_k,\alpha_{k+1},\beta)\dots Pr(\alpha_k|\alpha_{k+1},\beta)Pr(\alpha_{k+1}|\beta)$$

$$= Pr(\alpha_1|\alpha_2,\dots\alpha_{k+1},\beta)Pr(\alpha_2|\alpha_3,\dots,\alpha_{k+1},\beta)\dots Pr(\alpha_{k+1}|\beta)$$ ✓

Thus, we have proven by induction that:

$Pr(\alpha_1,\dots\alpha_n|\beta) = Pr(\alpha_1|\alpha_2,\dots,\alpha_n,\beta)Pr(\alpha_2|\alpha_3,\dots,\alpha_n,\beta)\dots Pr(\alpha_n|\beta)$.

2. $Pr(oil) = 0.5$

oil: oil is present

$Pr(gas) = 0.2$

gas: gas is present

$Pr(neither) = 0.3$

neither: neither oil nor gas is present

$Pr(T|oil) = 0.9$

T: Test comes out positive

$Pr(T|gas) = 0.3$

$Pr(T|neither) = 0.1$

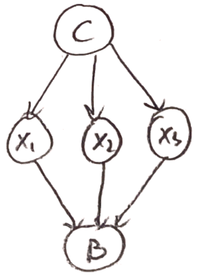
$$Pr(oil|T) = \frac{Pr(T|oil)Pr(oil)}{Pr(T)} = \frac{0.9\times0.5}{0.54} = \boxed{0.8333}$$

$Pr(T) = Pr(T|oil)Pr(oil) + Pr(T|gas)Pr(gas) + Pr(T|neither)Pr(neither)$

$= 0.9\times0.5 + 0.3\times0.2 + 0.1\times0.3$

$= 0.54$

3.



C: coin drawn

X_1, X_2, X_3: outcomes of coin flip

B: bell ring status

C	Pr(C)
a	1/3
b	1/3
c	1/3

C	X_1	$Pr(X_1 \mid C)$
a	heads	0.2
a	tails	0.8
b	heads	0.4
b	tails	0.6
c	heads	0.8
c	tails	0.2

C	X_2	$Pr(X_2 \mid C)$
a	heads	0.2
a	tails	0.8
b	heads	0.4
b	tails	0.6
c	heads	0.8
c	tails	0.2

C	X_3	$Pr(X_3 \mid C)$
a	heads	0.2
a	tails	0.8
b	heads	0.4
b	tails	0.6
c	heads	0.8
c	tails	0.2

X_1	X_2	X_3	B	$Pr(B \mid X_1, X_2, X_3)$
heads	heads	heads	on	1
heads	heads	tails	on	0
heads	tails	heads	on	0
heads	tails	tails	on	0
tails	heads	heads	on	0
tails	heads	tails	on	0
tails	tails	heads	on	0
tails	tails	tails	on	1
heads	heads	heads	off	0
heads	heads	tails	off	1
heads	tails	heads	off	1
heads	tails	tails	off	1
tails	heads	heads	off	1
tails	heads	tails	off	1
tails	tails	heads	off	1
tails	tails	tails	off	0

4\. 1(a)

$I(A, \emptyset, BE)$

$I(B, \emptyset, AC)$

$I(C, A, BDE)$

$I(D, AB, CE)$

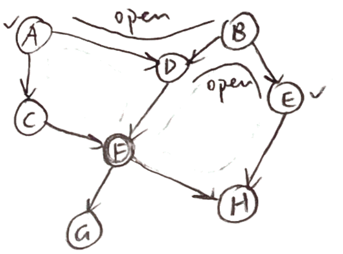
$I(E, B, ACDFG)$

$I(F, CD, ABE)$

$I(G, F, ABCDEH)$

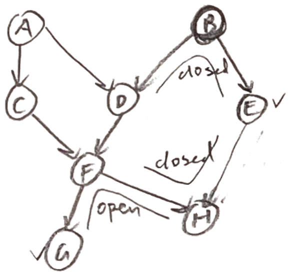
$I(H, EF, ABCDG)$

(b)



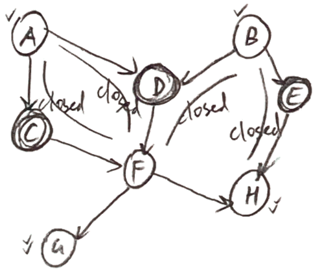
There's a path between A and E that is not blocked by F.

So, d-separation (A, F, E) is $\boxed{\text{false}}$



Every path between G and E is blocked by B.

So, d-separation (G, B, E) is $\boxed{\text{true}}$.



Every path between a node in $\{A, B\}$ and a node in $\{G, H\}$ is blocked by $\{C, D, E\}$.

So, d-separation (C, D, E) is $\boxed{\text{true}}$.

1(c). $Pr(a, b, c, d, e, f, g, h) = Pr(a) \cdot Pr(b) \cdot Pr(c|a) Pr(d|a,b) Pr(e, b) Pr(f|c,d) Pr(g|f) \cdot Pr(h|e,f)$

A is independent of B

(d). $Pr(A=1, B=1) = Pr(A=1) \cdot Pr(B=1) = .2 \times .7 = \boxed{0.14}$

A is independent of E

$Pr(E=0 | A=0) = Pr(E=0) = Pr(E=0|B=0) Pr(B=0) + Pr(E=0|B=1) Pr(B=1)$

$= .1 \times .3 + .9 \times .7$

$= \boxed{0.66}$

5. (a) $\alpha: A \Rightarrow B$

$\alpha = \neg A \vee B$

models of α: $\boxed{w_0, w_2, w_3}$

(b) $Pr(\alpha) = 0.3 + 0.1 + 0.4 = \boxed{0.8}$

(c)

	A	B	$Pr(A, B \mid \alpha)$
w_0	T	T	$0.3/0.8 = 0.375$
w_1	T	F	0
w_2	F	T	$0.1/0.8 = 0.125$
w_3	F	F	$0.4/0.8 = 0.5$

(d) $A \Rightarrow \neg B = \neg A \vee \neg B = \neg(A \wedge B)$

$Pr(A \Rightarrow \neg B \mid \alpha) = 0.125 + 0.5 = \boxed{0.625}$