CS161 hw 5

1.

P	Q	$P \Rightarrow Q$ ($\neg P \vee Q$)	$\neg Q \Rightarrow \neg P$ ($\neg P \vee Q$)
false	false	true	true
false	true	true	true
true	false	false	false
true	true	true	true

$P \Rightarrow Q = \neg P \vee Q$

$\neg Q \Rightarrow \neg P = Q \vee \neg P = \neg P \vee Q$

$\therefore P \Rightarrow Q = \neg Q \Rightarrow \neg P$.

Thus, $P \Rightarrow Q$ and $\neg Q \Rightarrow \neg P$ are equivalent.

P	Q	$P \Leftrightarrow \neg Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
false	false	false	false
false	true	true	true
true	false	true	true
true	true	false	false

$P \Leftrightarrow \neg Q = (P \Rightarrow \neg Q) \wedge (\neg Q \Rightarrow P)$
$= (\neg P \vee \neg Q) \wedge (Q \vee P)$

Thus, $P \Leftrightarrow \neg Q$ and $((P \wedge \neg Q) \vee (\neg P \wedge Q))$ are equivalent.

2.

Smoke	Fire	$(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$ ($Smoke \vee \neg Fire$)
false	false	true
false	true	false
true	false	true
true	true	true

Thus, $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$ is satisfiable.

Smoke	Fire	Heat	$(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \vee Heat) \Rightarrow Fire)$	$Smoke \Rightarrow Fire$	$(Smoke \vee Heat) \Rightarrow Fire$
false	false	false	true	true	true
false	false	true	false	true	false
false	true	false	true	true	true
false	true	true	true	true	true
true	false	false	true	false	false
true	false	true	true	false	false
true	true	false	true	true	true
true	true	true	true	true	true

Thus, $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \vee Heat) \Rightarrow Fire)$ is satisfiable.

Smoke	Heat	Fire	$(Smoke \land Heat) \Rightarrow Fire$	$Smoke \Rightarrow Fire$	$Heat \Rightarrow Fire$	$((Smoke \land Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \lor (Heat \Rightarrow Fire))$
false	false	false	true	true	true	true
false	false	true	true	true	true	true
false	true	false	true	true	false	true
false	true	true	true	true	true	true
true	false	false	true	false	true	true
true	false	true	true	true	true	true
true	true	false	false	false	false	true
true	true	true	true	true	true	true

Thus, $((Smoke \land Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \lor (Heat \Rightarrow Fire))$ is <u>valid</u>

3. (a)

KB: (knowledge base)

$Mythical \Rightarrow Immortal$

$\neg Mythical \Rightarrow (\neg Immortal \land Mammal)$

$(Immortal \lor Mammal) \Rightarrow Horned$

$Horned \Rightarrow Magical$

Mythical: the unicorn is mythical.
Immortal: the unicorn is immortal.
Mammal: the unicorn is mammal.
Horned: the unicorn is horned.
Magical: the unicorn is magical.

(b) Clauses of knowledge base (KB):

1. $\neg Mythical \lor Immortal$
2. $Mythical \lor \neg Immortal$
3. $Mythical \lor Mammal$
4. $\neg Immortal \lor Horned$
5. $\neg Mammal \lor Horned$
6. $\neg Horned \lor Magical$

$\neg Mythical \Rightarrow (\neg Immortal \land Mammal)$
$= Mythical \lor (\neg Immortal \land Mammal)$
$= (Mythical \lor \neg Immortal) \land (Mythical \lor Mammal)$

$(Immortal \lor Mammal) \Rightarrow Horned$
$= \neg(Immortal \lor Mammal) \lor Horned$
$= (\neg Immortal \land \neg Mammal) \lor Horned$
$= (\neg Immortal \lor Horned) \land (\neg Mammal \lor Horned)$

CNF of knowledge base:

$(\neg Mythical \lor Immortal) \land (Mythical \lor \neg Immortal) \land (Mythical \lor Mammal) \land$
$(\neg Immortal \lor Horned) \land (\neg Mammal \lor Horned) \land (\neg Horned \lor Magical)$.

1° clause of $\neg\alpha$: (α: Mythical)

(c) 7. $\neg$Mythical

Applying the resolution rule to the clauses, we get

8. $\neg$Immortal (from 2,7)
9. Mammal (from 3,7)
10. Horned (from 5,9)
11. Magical (from 6,10) $\therefore$ There's no contradiction anywhere.

$\therefore$ We cannot use the knowledge base to prove that the unicorn is mythical.

2° clause of $\neg\beta$ (β: Magical)

12. $\neg$Magical

Applying the resolution rule to the clauses, we get

13. $\neg$Horned (from 6,12)
14. $\neg$Mammal (from 5,13)
15. $\neg$Immortal (from 4,13)
16. $\neg$Mythical (from 1,15)
17. Mythical (from 3,14)
18. Empty clause (from 16,17) $\therefore KB \models \beta$

Therefore, we can use the knowledge base to prove that the unicorn is magical.

3° clause of $\neg\gamma$: (γ: Horned)

19. $\neg$Horned

Applying the resolution rule to the clauses, we get

20. $\neg$Mammal (from 5,19)
21. $\neg$Immortal (from 4,19)
22. $\neg$Mythical (from 1,21)
23. Mythical (from 3,20)
24. Empty clause (from 22,23) $\therefore KB \models \gamma$

Therefore, we can use the knowledge base to prove that the unicorn is horned.

4. Figure 1:

Figure 1 is decomposable because the subcircuits feeding into an and-gate share no variables for all and-gates in Figure 1.

Figure 1 is not smooth because we can look at the second or-gate from left to right on the lower level; its left input contains variable c, but its right input contains variable c and d. Since this or-gate's input variables are not the same, Figure 1 is not smooth.

Figure 1 is not deterministic because a counterexample would be we assign A=true, B=false, C=true, D=false, and in this case both inputs for the or-gate at the root node would be high.

Figure 2 is decomposable because the subcircuits feeding into an and-gate share no variables for all and-gates in Figure 2.
Figure 2 is smooth because the subcircuits feeding into an or-gate contain the same variables for all or-gates in Figure 2.
Figure 2 is not deterministic because a counterexample would be the third or-gate from left to right on the lower level; both of this or-gate's inputs are $\neg A \wedge B$, if we set A=false and B=true, then $(\neg A \wedge B)$ will evaluate to true, and this or-gate will have two high inputs.

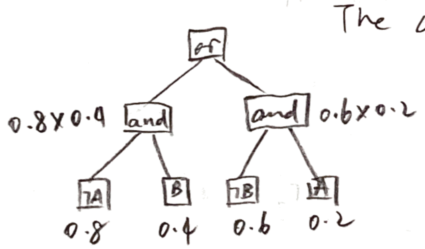
5. (a)

A	B	$(\neg A \wedge B)$	$(\neg B \wedge A)$	$(\neg A \wedge B) \vee (\neg B \wedge A)$	
false	false	false	false	false	
false	true	true	false	true	✓
true	false	false	true	true	✓
true	true	false	false	false	

$W(\neg A, B) = W(\neg A)W(B) = 0.8 \times 0.4 = 0.32$
$W(A, \neg B) = W(A)W(\neg B) = 0.2 \times 0.6 = 0.12$
$W(\neg A, B) + W(A, \neg B) = 0.32 + 0.12 = \underline{0.44}$

(b). For the decomposable, deterministic and smooth NNF circuit in Figure 3, the count on the root is the same as the Weighted Model Count for the formula.

The count on the root is $0.8 \times 0.4 + 0.6 \times 0.2 = \underline{0.44} = W(\neg A, B) + W(A, \neg B) = 0.44$



(c) $(W(\neg A, B) + W(\neg B, A)) \times (W(C, D) + W(\neg D, \neg C)) + (W(\neg A, \neg B) + W(B, A)) \times (W(C, \neg D) + W(D, \neg C))$
$= (0.8 \times 0.4 + 0.6 \times 0.2) \times (0.6 \times 08 + 0.2 \times 0.4) + (0.8 \times 0.6 + 0.4 \times 0.2) \times (0.6 \times 0.2 + 0.8 \times 0.4)$
$= 0.44 \times 0.56 + 0.56 \times 0.44$
$= \underline{0.4928}$