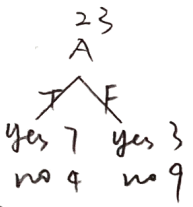
CS161 hw9

1.



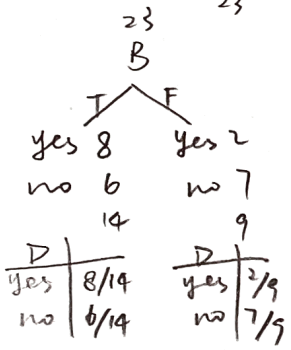
A=T	
D	
yes	7/11
no	4/11

A=F	
D	
yes	3/12
no	9/12

$ENT(D|A=T) = -\frac{7}{11}\log_2\frac{7}{11} - \frac{4}{11}\log_2\frac{4}{11} \approx 0.946$

$ENT(D|A=F) = -\frac{3}{12}\log_2\frac{3}{12} - \frac{9}{12}\log_2\frac{9}{12} \approx 0.811$

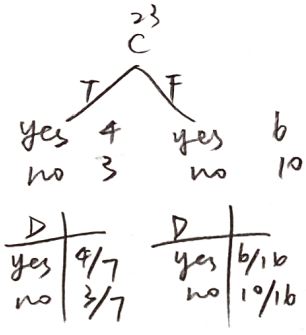
$ENT(D|A) = \frac{11}{23} \times 0.946 + \frac{12}{23} \times 0.811 \approx \underline{0.876}$



$ENT(D|B=T) = -\frac{8}{14}\log_2\frac{8}{14} - \frac{6}{14}\log_2\frac{6}{14} = 0.985$

$ENT(D|B=F) = -\frac{2}{9}\log_2\frac{2}{9} - \frac{7}{9}\log_2\frac{7}{9} = 0.764$

$ENT(D|B) = \frac{14}{23} \times 0.985 + \frac{9}{23} \times 0.764 = \underline{0.899}$

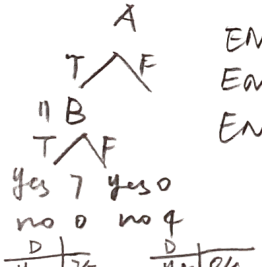


$ENT(D|C=T) = -\frac{4}{7}\log_2\frac{4}{7} - \frac{3}{7}\log_2\frac{3}{7} = 0.985$

$ENT(D|C=F) = -\frac{6}{16}\log_2\frac{6}{16} - \frac{10}{16}\log_2\frac{10}{16} = 0.954$

$ENT(D|C) = \frac{7}{23} \times 0.985 + \frac{16}{23} \times 0.954 = \underline{0.963}$

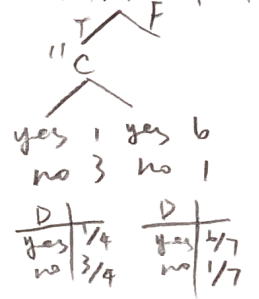
At the first step, we choose A which has the smallest entropy.



$ENT(D|A=T, B=T) = 0$

$ENT(D|A=T, B=F) = 0$

$ENT(D|A=T, B) = 0$

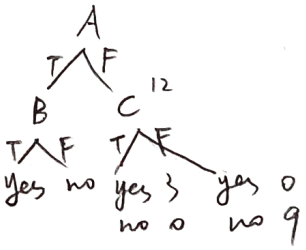


$ENT(D|A=T, C=T) = -\frac{1}{4}\log_2\frac{1}{4} - \frac{3}{4}\log_2\frac{3}{4} = 0.811$

$ENT(D|A=T, C=F) = -\frac{6}{7}\log_2\frac{6}{7} - \frac{1}{7}\log_2\frac{1}{7} = 0.592$

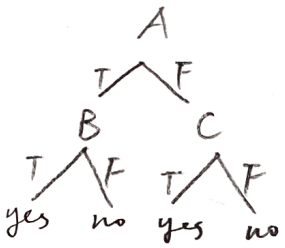
$ENT(D|A=T, C) = \frac{4}{11} \times 0.811 + \frac{7}{11} \times 0.592 = 0.672$

At the second step, we choose B which has the smallest entropy.



$ENT(D|A=F, c) = 0$

Decision tree:

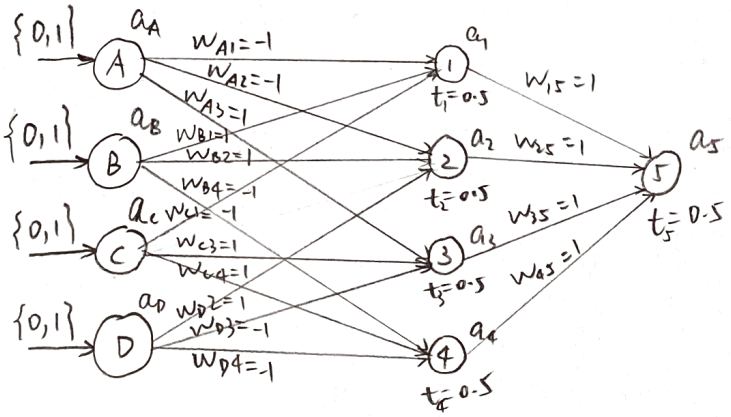


2. $(A \vee \neg B) \text{ XOR } (\neg C \vee D)$

$= ((\overline{(A \vee \neg B)} \wedge (\neg C \vee D)) \vee ((A \vee \neg B) \wedge \overline{(\neg C \vee D)})$

$= ((\neg A \wedge B \wedge (\neg C \vee D)) \vee ((A \vee \neg B) \wedge (C \wedge \neg D))$

$= (\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge B \wedge D) \vee (A \wedge C \wedge \neg D) \vee (\neg B \wedge C \wedge \neg D)$



$w_{A1} = w_{C1} = w_{A2} = w_{D3} = w_{B4} = w_{D4} = -1$

$w_{B1} = w_{B2} = w_{D2} = w_{A3} = w_{C3} = w_{C4} = 1$

$w_{15} = w_{25} = w_{35} = w_{45} = 1$

activation function g, $g(x) = \begin{cases} 1 & \text{if } x \geq 0.5 \\ 0 & \text{if } x < 0.5 \end{cases}$

$a_1 = g(w_{A1}a_A + w_{B1}a_B + w_{C1}a_C) = g(-a_A + a_B - a_C)$

$a_2 = g(w_{A2}a_A + w_{B2}a_B + w_{D2}a_D) = g(-a_A + a_B + a_D)$

$a_3 = g(w_{A3}a_A + w_{C3}a_C + w_{D3}a_D) = g(a_A + a_C - a_D)$

$a_4 = g(w_{B4}a_B + w_{C4}a_C + w_{D4}a_D) = g(-a_B + a_C - a_D)$

$a_5 = g(w_{15}a_1 + w_{25}a_2 + w_{35}a_3 + w_{45}a_4) = g(a_1 + a_2 + a_3 + a_4)$

$= g(g(-a_A + a_B - a_C) + g(-a_A + a_B + a_D) + g(a_A + a_C - a_D) + g(-a_B + a_C - a_D))$