

The Probability of Sudoku: The Bounds of the Cardinality of Minimal Fair Sudoku Puzzles

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Background

- A **Sudoku puzzle** is an incomplete **Sudoku board**, which is a 9x9 matrix of numbers 1-9 placed so that no number appears twice in any row, column, or one of the nine 3x3 blocks.
- The numbers in the puzzle are known as the **clues** or **givens**.
- A **determining set**, also known as a fair puzzle, is a Sudoku puzzle with exactly one unique solution.
- Critical sets** are the determining sets where none of the current givens in the puzzle can be removed without making the puzzle unfair [Cooper&Kirkpatrick].
- Cardinality** is the numbers of givens in a puzzle.
- Sudoku boards can also be created that are not 9x9 as long as they are in the form $(n^2) \times (n^2)$, such as 4x4, 16x16, and 100x100.
- Equivalence classes** are groups of unique Sudoku puzzles with the same cardinality that have different solutions, but their only difference is in two squares.
- It has been proven that there are 288 equivalence classes of 4x4 Sudoku boards [Oddson].
- We also know that there are 5,472,730,538 equivalence classes of 9x9 Sudoku boards [Russell&Jarvis].

Example of a Shidoku (4x4) Board

1	2	3	4
3	4	1	2
2	3	4	1
4	1	2	3

Research

- What are the different possible cardinalities (i.e., the "spectrum") of the critical sets?
- What are the minimum minimum cardinalities of critical sets?
- What are the maximum minimum cardinalities of critical sets?
- What are the minimum maximum cardinalities of critical sets?
- What are the maximum maximum cardinalities of critical sets?

Method

- We converted our spectrum problem into a series of satisfiability problems.
- Satisfiability problems** consist of finding truth assignments to variables that together satisfy a given Boolean formula.
- We ran our satisfiability problems in a free world-class satisfiability solver, or SAT solver, which tells us if the puzzles that we feed it are solvable and if they have unique solutions.

Methods

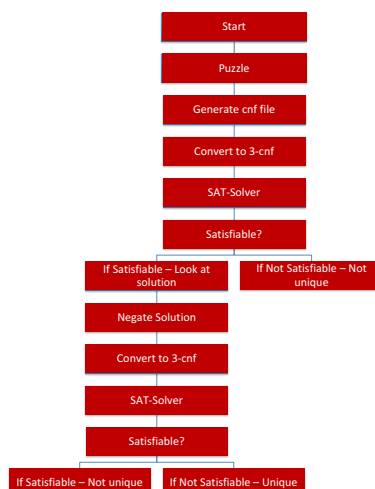
Here is an example of the Shidoku board that is now unique: (use the Shidoku board on the left)

1	1	1	1
1	1	-1	-1
1	-1	-1	1
1	-1	-1	1

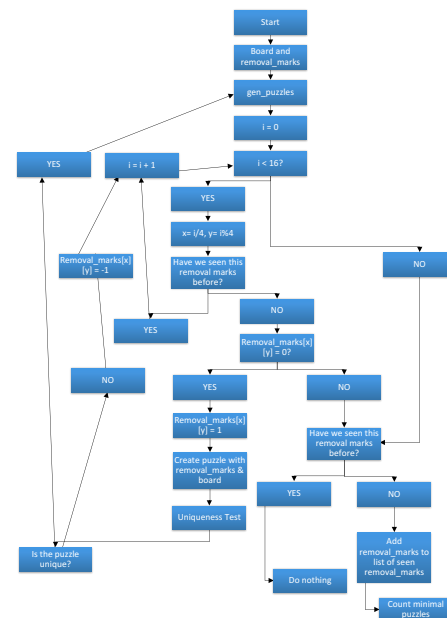
Here is the solution of the unique Shidoku board:

		1	2
	3	4	
	1	2	

Here is a flow chart for our uniqueness checker:



Results



Citations

http://www.afjarvis.staff.shef.ac.uk/sudoku/felgenhauer_jarvis_spec1.pdf

http://sudopedia.enjoysudoku.com/Shi_Doku.html

<http://people.math.sc.edu/cooper/criticalsets.pdf>

<http://www.sudoku-dragon.com/sudokuhistory.htm>

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