

Weekly Meeting

Topic: Property α for SOA of strength 3 with 3 levels

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Goal

- Make SOA of strength 3 with 3 levels having property α , that is, **stratification on $s^2 \times s^2$ grids in all two dimensions.**

Recap

Lemma 1. (from Shi and Tang 2020)

$$D : SOA(n, m, s^3, 3)$$

$$A = (a_1, \dots, a_m)$$

$$B = (b_1, \dots, b_m)$$

$$C = (c_1, \dots, c_m)$$

D exists if and only if A , B and C exist such that (a_i, a_j, a_u) , (a_i, a_j, b_j) and (a_i, b_i, c_i) are $OA(n, 3, s, 3)$ s for all $i \neq j$, $i \neq u$ and $j \neq u$.

They are linked through $D = s^2 A + sB + C$.

Recap

Proposition 1. (i) (from Shi and Tang, 2020)

An $\text{SOA}(n, m, s^3, 3)$ as characterized in Lemma 1 through A , B and C has property α if and only if (a_i, b_i, a_j, b_j) is an $\text{OA}(n, 4, s, 4)$ for all $i \neq j$.

Recap

Theorem 1. (from Shi and Tang, 2020)

If an $\text{SOA}(n, m, s^3, 3)$ for $s = 2$ is to be constructed using regular A , B , and C with their columns selected from a saturated design S , then it has property α if and only if:

1. A is of resolution IV or higher
2. (A, B, B') has resolution III or higher, that is, no repeated columns, where $B' = (b'_1, \dots, b'_m)$ with $b'_j = a_j b_j$

Breaking down

We first focus on the first two conditions of Lemma 1 and Proposition 1 (i):

1. A is of resolution $IV \iff (a_i, a_j, a_u)$ is $OA(n, 3, s, 3)$
2. (a_i, b_i, a_j, b_j) being $OA(n, 4, s, 4) \implies (a_i, a_j, b_j)$ being $OA(n, 3, s, 3)$

Breaking down

For $s = 2$, (a_i, b_i, a_j, b_j) having strength 4

→ Four columns are independent, orthogonal

→ No defining words among them

→ $a_i b_i a_j b_j \neq I$

→ $a_i b_i \neq a_j b_j$

→ (A, B, B') having no repeated columns can assure this

Breaking down

Finally, to choose c_j :

- Take c_j to be any column other than a_j , b_j and $a_j b_j$
- (a_j, b_j, c_j) is $\text{OA}(n, 3, s, 3)$
- All requirements from Lemma 1 and Proposition 1 (i) are satisfied

Same idea goes for $s = 3$

- A still need to be of resolution IV or higher for (a_i, a_j, a_u) being $OA(n, 3, s, 3)$
- (a_i, b_i, a_j, b_j) is strength 4 $\longrightarrow I \neq a_i b_i a_j b_j$ and $a_i b_i a_j b_j^2$
- It means $a_i b_i \neq a_j b_j$ and $a_i b_i \neq a_j b_j^2$
- (A, B, B', B'') having no repeated columns can assure this, where $b'_j = a_j b_j$ and $b''_j = a_j b_j^2$

To sum up

If an $\text{SOA}(n, m, s^3, 3)$ for $s = 3$ is to be constructed using regular A , B , and C with their columns selected from a saturated design S , then it has property α if and only if:

1. A is of resolution IV or higher
2. (A, B, B', B'') has resolution III or higher where $B' = (b'_1, \dots, b'_m)$ with $b'_j = a_j b_j$, and $B'' = (b''_1, \dots, b''_m)$ with $b''_j = a_j b_j^2$.