Weekly Meeting

Topic: Issues regarding grouping and permutations

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Issues

1. Check if $m > 8$ is possible, by trying different mutiplication to the permutation.
2. Dig into the grouping algorithm when $s = 2$, and think about if it can be extended to $s = 3$.

Extendable grouping for $s = 2$

From Shi and Tang, 2020.

We next present a recursive construction of designs A, B and B' needed in Theorem 1. Recall that $B' = (b'_1, \ldots, b'_m)$ is determined by $A = (a_1, \ldots, a_m)$ and $B = (b_1, \ldots, b_m)$ via $b'_j = a_j b_j$. Let A_k, B_k and B'_k, based on k independent factors $e_1, \ldots, e_k$, satisfy the condition in Theorem 1 that A_k is of resolution IV or higher and (A_k, B_k, B'_k) is of resolution III or higher. Then A_{k+2}, B_{k+2} and B'_{k+2}, based on $k+2$ independent factors $e_1, \ldots, e_{k+2}$, can be constructed to satisfy the requirement in Theorem 1. This is done by defining

$$\begin{aligned} A_{k+2} &= (A_k, e_{k+1}A_k, e_{k+2}A_k, e_{k+1}e_{k+2}A_k), \\ B_{k+2} &= (B_k, e_{k+2}B_k, e_{k+1}e_{k+2}B_k, e_{k+1}B_k). \end{aligned} \tag{2}$$

Then $B'_{k+2} = (B'_k, e_{k+1}e_{k+2}B'_k, e_{k+1}B'_k, e_{k+2}B'_k)$. It is straightforward to verify that A_{k+2} has resolution IV or higher and $(A_{k+2}, B_{k+2}, B'_{k+2})$ has resolution III or higher.

Extendable grouping for $s = 2$

Theorem 1. *If an* SOA$(n, m, 8, 3)$ *is to be constructed using regular* A, B *and* C *with their columns selected from a saturated design* S, *then it has property* α *if and only if* A *is of resolution IV or higher and* (A, B, B') *has resolution III or higher, where* $B' = (b'_1, \ldots, b'_m)$ *with* $b'_j = a_j b_j$.

Extendable grouping for $s = 2$

Table 2. Maximum numbers of factors SOAs of strength three and four

		Strength three			
k	$n = 2^k$	Family 1	Family 2	Family 3	Strength four
4	16	5	4	3	2
5	32	9	8	7	3
6	64	20	16	15	4
7	128	40	32	31	5
8	256	80	64	63	8

$k = 4 \to k = 6$

Assume we have A_k, B_k, B'_k, B''_k.

$$
\begin{aligned}
A_{k+2} = (&A_k, A_k e_{k+1}, A_k e_{k+1}^2, A_k e_{k+2}, A_k e_{k+2}^2, \\
&A_k e_{k+1} e_{k+2}, A_k e_{k+1}^2 e_{k+2}^2, A_k e_{k+1} e_{k+2}^2, A_k e_{k+1}^2 e_{k+2}) \\
B_{k+2} = (&B_k, B_k e_{k+2}, B_k e_{k+2}^2, B_k e_{k+1} e_{k+2}, B_k e_{k+1}^2 e_{k+2}^2, \\
&B_k e_{k+1} e_{k+2}^2, B_k e_{k+1}^2 e_{k+2}, B_k e_{k+1}, B_k e_{k+1}^2) \\
B'_{k+2} = (&B'_k, B'_k e_{k+1} e_{k+2}, B'_k e_{k+1}^2 e_{k+2}^2, B'_k e_{k+1} e_{k+2}^2, B'_k e_{k+1}^2 e_{k+2}, \\
&B'_k e_{k+1}, B'_k e_{k+1}^2, B'_k e_{k+2}, B'_k e_{k+2}^2) \\
B''_{k+2} = (&B''_k, B''_k e_{k+1} e_{k+2}^2, B''_k e_{k+1}^2 e_{k+2}, B''_k e_{k+1}, B''_k e_{k+1}^2, \\
&B''_k e_{k+2}, B''_k e_{k+2}^2, B''_k e_{k+1} e_{k+2}, B''_k e_{k+1}^2 e_{k+2}^2)
\end{aligned}
$$

$k = 4 \rightarrow k = 6$

- Now we have $m = 8$ for $k = 4, s = 3$.
- 32 effects in total (full factorial: 40 effects).
- By the proposed method, we have $m = 8 \times 9 = 72$ $k = 6, s = 3$.
- 288 effects in total (full factorial: 364 effects).

A grouping for $k = 4$

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
14	23	1234	$12^2 34^2$
$1^2 4$	$2^2 3$	$1^2 2^2 34$	$1^2 234^2$
24	$1^2 3$	$1^2 234$	1234^2
$2^2 4$	13	$12^2 34$	$1^2 2^2 34^2$
123	$12^2 4$	$1^2 34$	$2^2 34^2$
$1^2 2^2 3$	$1^2 24$	134	234^2
$12^2 3$	$1^2 2^2 4$	234	$1^2 34^2$
$1^2 23$	124	$2^2 34$	134^2

Why 34 cannot be put in α or β

Take the first row for example.

Instead of multiply by $(3, 4)$, we multiply it by $(3, 34)$.

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
3	34	$3^2 4$	4^2

Why 34 cannot be put in α or β

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
13	234	123^24	12^24^2
1^24	2^23	1^22^234	1^2234^2
24	1^23	1^2234	1234^2
2^24	13	12^234	$1^22^234^2$
123	12^24	1^234	2^234^2
1^22^23	1^224	134	234^2
12^23	1^22^24	234	1^234^2
1^223	124	2^234	134^2

Why 34 cannot be put in α or β

- (1, 2) and (7, 3) are duplicated.
- (1, 3) and (4, 4) are duplicated.

After meeting

1. Hold the validation of $m > 8$ and the grouping for $k = 3$.
2. Some issues with `strength.py` since the A with $m = 8$ does not have enough degrees of freedom to have strength 5.
3. Look into property β. It should be quick because it resembles the construction of property α.
4. If there is still time, look into property γ.