Weekly Meeting

Topic: Property α for SOA of strength 3 with $s = 3$

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Date: Mar 27, 2024

Goal

- Find similar grouping scheme and permutations to Wu, C.-F.J. (1989) for $s = 3$.

Permutations

This illustrate the idea under $s = 2$.

- B_k: all effects for a full factorial of k independent factors plus I.
- All the elements in B_k can be expressed as $w_i = w_{\pi(i)} \cdot w_{r(i)}$.
- Then, the exclusive sets $(w_{\pi(i)} \cdot (k+1))$, $(w_{r(i)} \cdot (k+2))$, $(w_i \cdot (k+1)(k+2))$, each of which is of the form $(\alpha, \beta, \alpha \cdot \beta)$ for $k = 4$.

Permutations

Example.

$$I \cdot I = I$$
$$1 \cdot 12 = 2$$
$$2 \cdot 1 = 12$$
$$12 \cdot 2 = 1$$

$\longrightarrow$

$$3 \cdot 4 = 34$$
$$13 \cdot 124 = 234$$
$$23 \cdot 14 = 1234$$
$$123 \cdot 24 = 134$$

$$1 \cdot 12 = 2$$

For $s = 3$

We want to find the grouping scheme of $(\alpha, \beta, \alpha \cdot \beta. \alpha \cdot \beta^2)$ for $s^2 \times s^2$ stratification property, which requires the design A has resolution IV and (A, B, B', B'') has resolution III.

The major difference is that each effect in a 3 level design **contains 2 components**.

Therefore, instead the permutations of *all effects*, we want to find the permutations of *all components*.

For $s = 3$

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
1	2	12	12^2
1^2	2^2	$1^2 2^2$	1^2
2	1^2	$1^2 2$	12
2^2	1	12^2	$1^2 2^2$
12	12^2	1^2	2^2
$1^2 2^2$	$1^2 2$	1	2
12^2	$1^2 2^2$	2	1^2
$1^2 2$	12	2^2	1

For $s = 3$

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
13	24	1234	$12^2 34^2$
$1^2 3$	$2^2 4$	$1^2 2^2 34$	$1^2 34^2$
23	$1^2 4$	$1^2 234$	1234^2
$2^2 3$	14	$12^2 34$	$1^2 2^2 34^2$
123	$12^2 4$	$1^2 34$	$2^2 34^2$
$1^2 2^2 3$	$1^2 24$	134	234^2
$12^2 3$	$1^2 2^2 4$	234	$1^2 34^2$
$1^2 23$	124	$2^2 34$	134^2

For $s = 3$

... adding up another group $(1, 2, 12, 12^2)$, this gives us the grouping scheme (A, B, B', B'') we desired.

Note that $(3, 4, 34, 34^2)$ cannot be included, otherwise it will not properly form a design A of resolution IV.

Side note: res. IV = cannot form I with any 3 elements.

Next ...

1. Generate $D = 9A + 3B + C$.
2. Check if D is SOA of strength 3.
3. Check if D has α property.
4. Find the permutations for $k = 3$.