Weekly Meeting

Topic: Issues regarding grouping and permutations

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Date: May 1, 2024

Issues

1. Check if $m > 8$ is possible, by trying different mutiplication to the permutation.
2. Dig into the grouping algorithm when $s = 2$, and think about if it can be extended to $s = 3$.

Extendable grouping for $s = 2$

$k = 4 \to k = 6$

Assume we have A_k, B_k, B'_k, B''_k.

$$
\begin{aligned}
A_{k+2} &= (A_k, A_k e_{k+1}, A_k e_{k+1}^2, A_k e_{k+2}, A_k e_{k+2}^2, \\
&\qquad A_k e_{k+1} e_{k+2}, A_k e_{k+1}^2 e_{k+2}^2, A_k e_{k+1} e_{k+2}^2, A_k e_{k+1}^2 e_{k+2}) \\
B_{k+2} &= (B_k, B_k e_{k+2}, B_k e_{k+2}^2, B_k e_{k+1} e_{k+2}, B_k e_{k+1}^2 e_{k+2}^2, \\
&\qquad B_k e_{k+1} e_{k+2}^2, B_k e_{k+1}^2 e_{k+2}, B_k e_{k+1}, B_k e_{k+1}^2) \\
B'_{k+2} &= (B'_k, B'_k e_{k+1} e_{k+2}, B'_k e_{k+1}^2 e_{k+2}^2, B'_k e_{k+1} e_{k+2}^2, B'_k e_{k+1}^2 e_{k+2}, \\
&\qquad B'_k e_{k+1}, B'_k e_{k+1}^2, B'_k e_{k+2}, B'_k e_{k+2}^2) \\
B''_{k+2} &= (B''_k, B''_k e_{k+1} e_{k+2}^2, B''_k e_{k+1}^2 e_{k+2}, B''_k e_{k+1}, B''_k e_{k+1}^2, \\
&\qquad B''_k e_{k+2}, B''_k e_{k+2}^2, B''_k e_{k+1} e_{k+2}, B''_k e_{k+1}^2 e_{k+2}^2)
\end{aligned}
$$

$k = 4 \rightarrow k = 6$

- Now we have $m = 8$ for $k = 4, s = 3$.
- 32 effects in total (full factorial: 40 effects).
- By the proposed method, we have $m = 8 \times 9 = 72$ $k = 6, s = 3$.
- 288 effects in total (full factorial: 364 effects).

A grouping for $k = 4$

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
14	23	1234	$12^2 34^2$
$1^2 4$	$2^2 3$	$1^2 2^2 34$	$1^2 234^2$
24	$1^2 3$	$1^2 234$	1234^2
$2^2 4$	13	$12^2 34$	$1^2 2^2 34^2$
123	$12^2 4$	$1^2 34$	$2^2 34^2$
$1^2 2^2 3$	$1^2 24$	134	234^2
$12^2 3$	$1^2 2^2 4$	234	$1^2 34^2$
$1^2 23$	124	$2^2 34$	134^2

Why 34 cannot be put in α or β

Take the first row for example.

Instead of multiply by $(3, 4)$, we multiply it by $(3, 34)$.

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
3	34	$3^2 4$	4^2

Why 34 cannot be put in α or β

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
13	234	123^24	12^24^2
1^24	2^23	1^22^234	1^2234^2
24	1^23	1^2234	1234^2
2^24	13	12^234	$1^22^234^2$
123	12^24	1^234	2^234^2
1^22^23	1^224	134	234^2
12^23	1^22^24	234	1^234^2
1^223	124	2^234	134^2

Why 34 cannot be put in α or β

- (1, 2) and (7, 3) are duplicated.
- (1, 3) and (4, 4) are duplicated.