Weekly Meeting

Topic: Exhausive search for 9×9 ($m = 11$)

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Date: Sept. 18, 2024

Property for finding 9×9

If $D = (d_1, \ldots, d_m)$ is constructed via $D = sA + B$,and D is $\mathrm{SOA}(2+)$.

For the case where $s = 3$, these statements are equivalent, given $i \neq j$, $i < j$:

1. (d_i, d_j) achieve stratification over $s^2 \times s^2$ grids.
2. (a_i, b_i, a_j, b_j) is $\mathrm{OA}(n, 4, 3, 4)$.
3. $(b_i, b_j, a_i b_i, a_i b_i^2, a_j b_j, a_j b_j^2)$ are different factors from S.

Why (3) ?

$$
\begin{aligned}
a_i \times b_i \times b_j \neq I &\rightarrow a_i \times b_i \times b_j \neq I \rightarrow b_j \neq a_i b_i \\
&\rightarrow a_i \times b_i \times b_j^2 \neq I \rightarrow b_j^2 \neq a_i b_i \rightarrow b_j \neq a_i b_i \\
&\rightarrow a_i \times b_i^2 \times b_j \neq I \rightarrow b_j \neq a_i b_i^2 \\
&\rightarrow a_i \times b_i^2 \times b_j^2 \neq I \rightarrow b_j^2 \neq a_i b_i^2 \rightarrow b_j \neq a_i b_i^2
\end{aligned}
$$

$$
\begin{aligned}
a_j \times b_i \times b_j \neq I &\rightarrow a_j \times b_i \times b_j \neq I \rightarrow b_i \neq a_j b_j \\
&\rightarrow a_j \times b_i \times b_j^2 \neq I \rightarrow b_i \neq a_j b_j^2 \\
&\rightarrow a_j \times b_i^2 \times b_j \neq I \rightarrow b_i^2 \neq a_j b_j \rightarrow b_i \neq a_j b_j \\
&\rightarrow a_j \times b_i^2 \times b_j^2 \neq I \rightarrow b_i^2 \neq a_j b_j^2 \rightarrow b_i \neq a_j b_j^2
\end{aligned}
$$

$$
\begin{aligned}
b_i \times b_j \neq I &\rightarrow b_i \times b_j \neq I \rightarrow b_i \neq b_j \\
&\rightarrow b_i \times b_j^2 \neq I \rightarrow b_i \neq b_j^2 = b_j
\end{aligned}
$$

Why (3) ?

$a_i \times b_i \times a_j \times b_j \neq I$

$\rightarrow a_i \times a_j \times b_i \times b_j \neq I \rightarrow a_i b_i \neq a_j b_j$

$\rightarrow a_i \times a_j \times b_i \times b_j^2 \neq I \rightarrow a_i b_i \neq a_j b_j^2$

$\rightarrow a_i \times a_j \times b_i^2 \times b_j \neq I \rightarrow a_i b_i^2 \neq a_j b_j$

$\rightarrow a_i \times a_j \times b_i^2 \times b_j^2 \neq I \rightarrow a_i b_i^2 \neq a_j b_j^2$

$\rightarrow a_i \times a_j^2 \times b_i \times b_j \neq I \rightarrow a_i b_i \neq a_j^2 b_j \rightarrow a_i b_i \neq a_j b_j^2$

$\rightarrow a_i \times a_j^2 \times b_i \times b_j^2 \neq I \rightarrow a_i b_i \neq a_j^2 b_j^2 \rightarrow a_i b_i \neq a_j b_j$

$\rightarrow a_i \times a_j^2 \times b_i^2 \times b_j \neq I \rightarrow a_i b_i^2 \neq a_j^2 b_j \rightarrow a_i b_i^2 \neq a_j b_j^2$

$\rightarrow a_i \times a_j^2 \times b_i^2 \times b_j^2 \neq I \rightarrow a_i b_i^2 \neq a_j^2 b_j^2 \rightarrow a_i b_i^2 \neq a_j b_j$