Weekly Meeting

Topic: Need A to be resolution IV

Presenter: Heng-Tse Chou @ NTHU STAT

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Issues

1. Property α with design A having resolution IV.
2. Property β with design A having resolution IV.

Property α

$\mathrm{SOA}(n, m, 27, 3)$ has property α iff:

1. A is resolution IV.
2. (B, B', B'') is resolution III, i.e., no repeated columns.

Property β

$\mathrm{SOA}(n, m, 27, 3)$ has property β iff:

1. A is resolution IV.
2. $(B, B', B'') \subseteq \bar{A}$.
3. (B, B', B'') contains no 2 factor interaction from A.

Property α for $k = 4$

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
14	23	1234	$12^2 3^2 4$
$1^2 4$	$2^2 3$	$1^2 2^2 34$	$1^2 23^2 4$
24	$1^2 3$	$1^2 234$	$123^2 4$
$2^2 4$	13	$12^2 34$	$1^2 2^2 3^2 4$
123	$12^2 4$	$1^2 34$	$2^2 34^2$
$1^2 2^2 3$	$1^2 24$	134	234^2
$12^2 3$	$1^2 2^2 4$	234	$1^2 34^2$
$1^2 23$	124	$2^2 34$	134^2

Property α with $k = 4$

This construction provides D with 8 factors (8/10).

Property α with $k = 6$

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
$5 \cdot A_{(1)}$	$6 \cdot B_{(1)}$	$56 \cdot A_{(1)}B_{(1)}$	$56^2 \cdot A_{(1)}B^2_{(1)}$
$5^2 \cdot A_{(1)}$	$6^2 \cdot B_{(1)}$	$5^2 6^2 \cdot A_{(1)}B_{(1)}$	$5^2 6 \cdot A_{(1)}B^2_{(1)}$
$6 \cdot A_{(1)}$	$5^2 \cdot B_{(1)}$	$5^2 6 \cdot A_{(1)}B_{(1)}$	$56 \cdot A_{(1)}B^2_{(1)}$
$6^2 \cdot A_{(1)}$	$5 \cdot B_{(1)}$	$56^2 \cdot A_{(1)}B_{(1)}$	$5^2 6^2 \cdot A_{(1)}B^2_{(1)}$
$56 \cdot A_{(2)}$	$56^2 \cdot B_{(2)}$	$5^2 \cdot A_{(2)}B_{(2)}$	$6^2 \cdot A_{(2)}B^2_{(2)}$
$5^2 6^2 \cdot A_{(2)}$	$5^2 6 \cdot B_{(2)}$	$5 \cdot A_{(2)}B_{(2)}$	$6 \cdot A_{(2)}B^2_{(2)}$
$56^2 \cdot A_{(2)}$	$5^2 6^2 \cdot B_{(2)}$	$6 \cdot A_{(2)}B_{(2)}$	$5^2 \cdot A_{(2)}B^2_{(2)}$
$5^2 6 \cdot A_{(2)}$	$56 \cdot B_{(2)}$	$6^2 \cdot A_{(2)}B_{(2)}$	$5 \cdot A_{(2)}B^2_{(2)}$

Property α with $k = 6$

Where

$$A_{(1)} = (14, 1^2 4, 24, 2^2 4)$$

$$A_{(2)} = (123, 1^2 2^2 3, 12^2 3, 1^2 23)$$

$$B_{(1)} = (23, 2^2 3, 1^2 3, 13)$$

$$B_{(2)} = (12^2 4, 1^2 24, 1^2 2^2 4, 124)$$

This construction provides D with 32 factors (32/91).

Grouping with A not having res. IV

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
$5 \cdot A$	$6 \cdot B$	$56 \cdot AB$	$56^2 \cdot AB^2$
$5^2 \cdot A$	$6^2 \cdot B$	$5^2 6^2 \cdot AB$	$5^2 6 \cdot AB^2$
$6 \cdot A$	$5^2 \cdot B$	$5^2 6 \cdot AB$	$56 \cdot AB^2$
$6^2 \cdot A$	$5 \cdot B$	$56^2 \cdot AB$	$5^2 6^2 \cdot AB^2$
$56 \cdot A$	$56^2 \cdot B$	$5^2 \cdot AB$	$6^2 \cdot AB^2$
$5^2 6^2 \cdot A$	$5^2 6 \cdot B$	$5 \cdot AB$	$6 \cdot AB^2$
$56^2 \cdot A$	$5^2 6^2 \cdot B$	$6 \cdot AB$	$5^2 \cdot AB^2$
$5^2 6 \cdot A$	$56 \cdot B$	$6^2 \cdot AB$	$5 \cdot AB^2$

Grouping with A not having res. IV

This construction provides D with 64 factors (64/91).

Property β for $s = 2$

$P_0 =$ all combinations of $e_3, \ldots, e_k$.
$P = (I, P_0)$

$A = e_1 P$
$B = e_2 P$
$B' = e_1 e_2 P \to S = (P_0, A, B, B')$

`s11`, `s21`, `s111` and `s211` are all satisfied.

Property β for $s = 3$

$P_0 =$ all combinations of $e_3, \ldots, e_k$.
$P = (I, P_0, P_0^2)$

$A = e_1 P$
$B = e_2 P$
$B' = e_1 e_2 P$
$B'' = e_1 e_2^2 P \to S = (P_0, A, B, B', B'')$

However, A does not have res. IV. `s111` and `s211` are not satisfied.

Post-Meeting Notes

New stuffs:

- Find a crirerion to quantify how close a design to α property
- Simulated Annealing to generate a good design (See MaxPro)
- Minimum moment aberration as a criterion to start with.

Minimum moment aberration

For an (N, s^n)-design $D = [r_{ij}]_{N\times n}$ and a positive integer t, define the tth power moment to be $K_t(D) = [N(N-1)/2]^{-1} \sum_{1\le i<j\le N} [\delta_{ij}(D)]^t$, where

$$\delta_{ij}(D) = \sum_{k=1}^{n} \delta(r_{ik}, r_{jk}) \tag{2}$$

is the number of coincidences between the ith and jth rows and $\delta(x, y)$ is the Kronecker delta function, equal to 1 if $x = y$ and 0 otherwise. It is important to note that $n - \delta_{ij}(D)$ is known as the *Hamming distance* between the ith and jth rows in algebraic coding theory.