Weekly Meeting

Topic: fix `s111` stratification for property α with $k = 6$; find property β

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Issues

1. `s111` stratification for property α with $k = 6$.
2. Construct property β.

Breaking down property β

(a_i, a_j, a_u, b_u) being OA with strength 4 implies

1. $a_i a_j a_u \neq I \rightarrow A$ is resolution IV.
2. $a_i a_j b_u \neq I \rightarrow B$ contains no 2fi from A.
3. $a_i a_u b_u \neq I \rightarrow (B', B'') \subseteq \bar{A}$
4. $a_i a_j a_u b_u \neq I \rightarrow B'$ and B'' contains no 2fi from A.

Breaking down property β

Therefore, $\mathrm{SOA}(n, m, 27, 3)$ has property β iff:

1. A is resolution IV.
2. $(B, B', B'') \subseteq \bar{A}$.
3. (B, B', B'') contains no 2fi from A.

Construction for property β

Take $k = 4$ for example.

$P_0 = (e_3, e_4, e_3e_4, e_3e_4^2)$
$P = (I, P_0, P_0^2)$

$A = e_1P$
$B = e_2P$
$B' = e_1e_2P$
$B'' = e_1e_2^2P \rightarrow S = (P_0, A, B, B', B'')$

Construction for property β

This construction passed `s11` and `s21` but not passing `s111` and `s211`.

Reason: Duplicated P_0 in A.

This is similar to why our construction for property α with $k = 6$ is not passign `s111`.

Setup for property α with $k = 6$

$A_6 = (A_4, e_5 A_4, e_5^2 A_4, e_6 A_4, e_6^2 A_4, e_5 e_6 A_4, e_5^2 e_6^2 A_4, e_5 e_6^2 A_4, e_5^2 e_6 A_4)$

Possible workaround: similar to how we construct grouping with $k = 4$.

$\rightarrow$ Divide into 2 groups that any 2 columns from group 1 won't form I with any 1 column from group 2.

Grouping with $k = 4$

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
14	23	1234	$12^2 3^2 4$
$1^2 4$	$2^2 3$	$1^2 2^2 34$	$1^2 23^2 4$
24	$1^2 3$	$1^2 234$	$123^2 4$
$2^2 4$	13	$12^2 34$	$1^2 2^2 3^2 4$
123	$12^2 4$	$1^2 34$	$2^2 34^2$
$1^2 2^2 3$	$1^2 24$	134	234^2
$12^2 3$	$1^2 2^2 4$	234	$1^2 34^2$
$1^2 23$	124	$2^2 34$	134^2

Grouping with $k = 6$

α	$5\cdot?$	$5^2\cdot?$	$6\cdot?$	$6^2\cdot?$	$56\cdot?$	$5^2 6^2\cdot?$	$56^2\cdot?$	$5^2 6\cdot?$
β	$6\cdot?$	$6^2\cdot?$	$5^2\cdot?$	$5\cdot?$	$56^2\cdot?$	$5^6 2\cdot?$	$5^2 6^2\cdot?$	$56\cdot?$
$\alpha\cdot\beta$	$56\cdot?$	$5^2 6^2\cdot?$	$5^2 6\cdot?$	$56^2\cdot?$	$5^2\cdot?$	$5\cdot?$	$6\cdot?$	$6^2\cdot?$
$\alpha\cdot\beta^2$	$56^2\cdot?$	$5^2 6\cdot?$	$56\cdot?$	$5^2 6^2\cdot?$	$6^2\cdot?$	$6\cdot?$	$5^2\cdot?$	$5\cdot?$

Building blocks: A_4, A_4^*, B_4, B_4^*

$$A_4 B_4 = B'$$
$$A_4 B_4^* = A_4 B_4^2 = B''$$
$$A_4^* B_4 = A_4^2 B_4 = A_4 B_4^2 = B''$$
$$A_4^* B_4^* = A_4^2 B_4^2 = A_4 B_4 = B'$$

After Meating

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
$5 \cdot A$	$6 \cdot B$	$56 \cdot AB$	$56^2 \cdot AB^2$
$5 \cdot A^2$	$6 \cdot B^2$	$56 \cdot A^2B^2$	$56^2 \cdot A^2B$
$5 \cdot B$	$6 \cdot A^2$	$56 \cdot A^2B$	$56^2 \cdot AB$
$5 \cdot B^2$	$6 \cdot A$	$56 \cdot AB^2$	$56^2 \cdot A^2B^2$
$6 \cdot AB$	$5 \cdot AB^2$	$56 \cdot A^2$	$5^26 \cdot B^2$
$6 \cdot A^2B^2$	$5 \cdot A^2B$	$56 \cdot A$	$5^26 \cdot B$
$6 \cdot AB^2$	$5 \cdot A^2B^2$	$56 \cdot B$	$5^26 \cdot A^2$
$6 \cdot A^2B$	$5 \cdot AB$	$56 \cdot B^2$	$5^26 \cdot A$

After Meeting

- A, A^*, B, B^* idea is essentially re-ordering columns, so not working.
- Permutate by A and B seems good but still not working.

To-do

1. Verify A, B permutations (not working).
2. Use permutation idea to solve β property.
3. Write clearly about β property's construction.