Weekly Meeting

Topic: Issues regarding grouping and permutations

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Issues

1. Check if $m > 8$ is possible, by trying different mutiplication to the permutation.
2. Dig into the grouping algorithm when $s = 2$, and think about if it can be extended to $s = 3$.

Extendable grouping for $s = 2$

From Shi and Tang, 2020.

We next present a recursive construction of designs A, B and B' needed in Theorem 1. Recall that $B' = (b'_1, \ldots, b'_m)$ is determined by $A = (a_1, \ldots, a_m)$ and $B = (b_1, \ldots, b_m)$ via $b'_j = a_j b_j$. Let A_k, B_k and B'_k, based on k independent factors $e_1, \ldots, e_k$, satisfy the condition in Theorem 1 that A_k is of resolution IV or higher and (A_k, B_k, B'_k) is of resolution III or higher. Then A_{k+2}, B_{k+2} and B'_{k+2}, based on $k+2$ independent factors $e_1, \ldots, e_{k+2}$, can be constructed to satisfy the requirement in Theorem 1. This is done by defining

$$\begin{aligned} A_{k+2} &= (A_k, e_{k+1}A_k, e_{k+2}A_k, e_{k+1}e_{k+2}A_k), \\ B_{k+2} &= (B_k, e_{k+2}B_k, e_{k+1}e_{k+2}B_k, e_{k+1}B_k). \end{aligned} \tag{2}$$

Then $B'_{k+2} = (B'_k, e_{k+1}e_{k+2}B'_k, e_{k+1}B'_k, e_{k+2}B'_k)$. It is straightforward to verify that A_{k+2} has resolution IV or higher and $(A_{k+2}, B_{k+2}, B'_{k+2})$ has resolution III or higher.

Extendable grouping for $s = 2$

Theorem 1. *If an* SOA$(n, m, 8, 3)$ *is to be constructed using regular* A, B *and* C *with their columns selected from a saturated design* S, *then it has property* α *if and only if* A *is of resolution IV or higher and* (A, B, B') *has resolution III or higher, where* $B' = (b'_1, \ldots, b'_m)$ *with* $b'_j = a_j b_j$.

Extendable grouping for $s = 2$

Table 2. Maximum numbers of factors SOAs of strength three and four

		Strength three			
k	$n = 2^k$	Family 1	Family 2	Family 3	Strength four
4	16	5	4	3	2
5	32	9	8	7	3
6	64	20	16	15	4
7	128	40	32	31	5
8	256	80	64	63	8

$k = 4 \to k = 6$

Assume we have A_k, B_k, B'_k, B''_k.

$$
\begin{aligned}
A_{k+2} = (&A_k, A_k e_{k+1}, A_k e_{k+1}^2, A_k e_{k+2}, A_k e_{k+2}^2,\\
&A_k e_{k+1} e_{k+2}, A_k e_{k+1}^2 e_{k+2}^2, A_k e_{k+1} e_{k+2}^2, A_k e_{k+1}^2 e_{k+2})\\
B_{k+2} = (&B_k, B_k e_{k+2}, B_k e_{k+2}^2, B_k e_{k+1} e_{k+2}, B_k e_{k+1}^2 e_{k+2}^2,\\
&B_k e_{k+1} e_{k+2}^2, B_k e_{k+1}^2 e_{k+2}, B_k e_{k+1}, B_k e_{k+1}^2)\\
B'_{k+2} = (&B'_k, B'_k e_{k+1} e_{k+2}, B'_k e_{k+1}^2 e_{k+2}^2, B'_k e_{k+1} e_{k+2}^2, B'_k e_{k+1}^2 e_{k+2},\\
&B'_k e_{k+1}, B'_k e_{k+1}^2, B'_k e_{k+2}, B'_k e_{k+2}^2)\\
B''_{k+2} = (&B''_k, B''_k e_{k+1} e_{k+2}^2, B''_k e_{k+1}^2 e_{k+2}, B''_k e_{k+1}, B''_k e_{k+1}^2,\\
&B''_k e_{k+2}, B''_k e_{k+2}^2, B''_k e_{k+1} e_{k+2}, B''_k e_{k+1}^2 e_{k+2}^2)
\end{aligned}
$$

$k = 4 \to k = 6$

- Now we have $m = 8$ for $k = 4, s = 3$.
- 32 effects in total (full factorial: 40 effects).
- By the proposed method, we have $m = 8 \times 9 = 72$ $k = 6, s = 3$.
- 288 effects in total (full factorial: 364 effects).

A grouping for $k = 4$

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
14	23	1234	$12^2 34^2$
$1^2 4$	$2^2 3$	$1^2 2^2 34$	$1^2 234^2$
24	$1^2 3$	$1^2 234$	1234^2
$2^2 4$	13	$12^2 34$	$1^2 2^2 34^2$
123	$12^2 4$	$1^2 34$	$2^2 34^2$
$1^2 2^2 3$	$1^2 24$	134	234^2
$12^2 3$	$1^2 2^2 4$	234	$1^2 34^2$
$1^2 23$	124	$2^2 34$	134^2

Why 34 cannot be put in α or β

Take the first row for example.

Instead of multiply by $(3, 4)$, we multiply it by $(3, 34)$.

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
3	34	$3^2 4$	4^2

Why 34 cannot be put in α or β

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
13	234	123^24	12^24^2
1^24	2^23	1^22^234	1^2234^2
24	1^23	1^2234	1234^2
2^24	13	12^234	$1^22^234^2$
123	12^24	1^234	2^234^2
1^22^23	1^224	134	234^2
12^23	1^22^24	234	1^234^2
1^223	124	2^234	134^2

Why 34 cannot be put in α or β

- (1, 2) and (7, 3) are duplicated.
- (1, 3) and (4, 4) are duplicated.