Weekly Meeting

Topic: validate $k = 6$; find property (β) and (γ)

Presenter: Heng-Tse Chou @ NTHU STAT

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A grouping for $k = 4$

α	β	$\alpha \cdot \beta$	$\alpha \cdot \beta^2$
14	23	1234	$12^2 34^2$
$1^2 4$	$2^2 3$	$1^2 2^2 34$	$1^2 234^2$
24	$1^2 3$	$1^2 234$	1234^2
$2^2 4$	13	$12^2 34$	$1^2 2^2 34^2$
123	$12^2 4$	$1^2 34$	$2^2 34^2$
$1^2 2^2 3$	$1^2 24$	134	234^2
$12^2 3$	$1^2 2^2 4$	234	$1^2 34^2$
$1^2 23$	124	$2^2 34$	134^2

$k = 4 \to k = 6$

Assume we have A_k, B_k, B'_k, B''_k.

$$
\begin{aligned}
A_{k+2} = (&A_k, A_k e_{k+1}, A_k e_{k+1}^2, A_k e_{k+2}, A_k e_{k+2}^2,\\
&A_k e_{k+1} e_{k+2}, A_k e_{k+1}^2 e_{k+2}^2, A_k e_{k+1} e_{k+2}^2, A_k e_{k+1}^2 e_{k+2})\\
B_{k+2} = (&B_k, B_k e_{k+2}, B_k e_{k+2}^2, B_k e_{k+1} e_{k+2}, B_k e_{k+1}^2 e_{k+2}^2,\\
&B_k e_{k+1} e_{k+2}^2, B_k e_{k+1}^2 e_{k+2}, B_k e_{k+1}, B_k e_{k+1}^2)\\
B'_{k+2} = (&B'_k, B'_k e_{k+1} e_{k+2}, B'_k e_{k+1}^2 e_{k+2}^2, B'_k e_{k+1} e_{k+2}^2, B'_k e_{k+1}^2 e_{k+2},\\
&B'_k e_{k+1}, B'_k e_{k+1}^2, B'_k e_{k+2}, B'_k e_{k+2}^2)\\
B''_{k+2} = (&B''_k, B''_k e_{k+1} e_{k+2}^2, B''_k e_{k+1}^2 e_{k+2}, B''_k e_{k+1}, B''_k e_{k+1}^2,\\
&B''_k e_{k+2}, B''_k e_{k+2}^2, B''_k e_{k+1} e_{k+2}, B''_k e_{k+1}^2 e_{k+2}^2)
\end{aligned}
$$

Property (β) for $s = 2$

(β): stratifications on $s^2 \times s \times s$, $s \times s^2 \times s$ and $s \times s \times s^2$ grids.

Thm: D has property β iff

1. A is of resolution IV or higher.
2. $(B, B') \subseteq \bar{A}$.
3. (B, B') does not contain any interaction column involving two factors from A.

Property (β) for $s = 2$

The thm is based on (a_i, a_j, a_u, b_u) being $\mathrm{OA}(n, 4, s, 4)$.

1. (a_i, a_j, a_u) do not form a word of length 3: A is resolution IV.
2. (a_i, a_j, b_u) do not form a word of length 3: B does not contain any 2fi from A.
3. (a_i, a_u, b_u) do not form a word of length 3: $(B, B') \subseteq \bar{A} = S \setminus A$.
4. (a_i, a_j, a_u, b_u) do not form a word of length 3: B' does not contain any 2fi from A.

Construction of (β) for $s = 2$

Let P_0 consists of $e_3, \ldots, e_k$ and all their interactions.

Let $P = (I, P_0)$.

Then, we have $S = (P_0, e_1P, e_2P, e_1e_2P) \to A = e_1P$ and $B = e_2P$.

For $s = 3$

Let P_0 consists of $e_3, \ldots, e_k$ and all their interactions.

Let $P = (I, P_0)$.

Then, we have

$$S = (P, e_1P, e_1^2P, e_2P, e_2^2P, e_1e_2P, e_1^2e_2^2P, e_1e_2^2P, e_1^2e_2P)$$

After Meeting

1. The grouping for (α) should use the correct permutation.
2. Now `s22` passed but `s111` still need to figure out.
3. (β) looks promising. Try verify it.