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1 |----- MODULE Consensus -----|
2 | EXTENDS Naturals, FiniteSets |
3 |-----|
4 | CONSTANT Value  the set of all values that can be chosen |
5 |-----|
6 | VARIABLE chosen the set of all values that have been chosen |
7 |-----|
8 | TypeOK  $\triangleq$ 
9 |    $\wedge$  IsFiniteSet(chosen)
10 |    $\wedge$  chosen  $\subseteq$  Value
11 |-----|
12 | Init  $\triangleq$  chosen = {}
13 |-----|
14 | Next  $\triangleq$ 
15 |    $\wedge$  chosen = {}
16 |    $\wedge \exists v \in \text{Value} : \text{chosen}' = \{v\}$ 
17 |-----|
18 | Spec  $\triangleq$  Init  $\wedge \Box[\text{Next}]_{\text{chosen}}$ 
19 |-----|
20 | Inv  $\triangleq$ 
21 |    $\wedge$  TypeOK
22 |    $\wedge \text{Cardinality}(\text{chosen}) \leq 1$  Safety: at most one value is chosen
23 |-----|
24 | THEOREM Invariance  $\triangleq$  Spec  $\Rightarrow \Box$ Inv
25 | <1>1. Init  $\Rightarrow$  Inv
26 | <1>2. Inv  $\wedge [\text{Next}]_{\text{chosen}} \Rightarrow \text{Inv}'$ 
27 | <1>3. QED
28 | <2>1. Inv  $\wedge \Box[\text{Next}]_{\text{chosen}} \Rightarrow \Box$ Inv
29 |   BY <1>2 and a TLA proof rule
30 | <2>2. QED
31 |   BY <1>1, <2>1 and simple logic
32 |-----|
33 | Success  $\triangleq \Diamond(\text{chosen} \neq \{\})$  Liveness: a value is eventually chosen
34 | LiveSpec  $\triangleq$  Spec  $\wedge \text{WF}_{\text{chosen}}(\text{Next})$ 
35 |-----|
36 | THEOREM LivenessTheorem  $\triangleq$  LiveSpec  $\Rightarrow$  Success
37 |-----|

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