THE PUBLIC IS MORE FAMILIAR WITH BAD DESIGN THAN GOOD DESIGN. IT IS, IN EFFECT, CONDITIONED TO PREFER BAD DESIGN, BECAUSE THAT IS WHAT IT LIVES WITH. THE NEW BECOMES THREATENING, THE OLD REASSURING.

PAUL RAND

A DESIGNER KNOWS THAT HE HAS ACHIEVED PERFECTION NOT WHEN THERE IS NOTHING LEFT TO ADD, BUT WHEN THERE IS NOTHING LEFT TO TAKE AWAY.

ANTOINE DE SAINT-EXUPÉRY

. . . THE DESIGNER OF A NEW SYSTEM MUST NOT ONLY BE THE IMPLEMENTOR AND THE FIRST LARGE-SCALE USER; THE DESIGNER SHOULD ALSO WRITE THE FIRST USER MANUAL. . . IF I HAD NOT PARTICIPATED FULLY IN ALL THESE ACTIVITIES, LITERALLY HUNDREDS OF IMPROVEMENTS WOULD NEVER HAVE BEEN MADE, BECAUSE I WOULD NEVER HAVE THOUGHT OF THEM OR PERCEIVED WHY THEY WERE IMPORTANT.

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Dedicated to those who appreciate LaTeX

and the work of Edward R. Tufte and Donald E. Knuth.

Introduction

This is a collection of psuedocode for classical algorithms.

Basic Iterative and Recursive Algorithms

Algorithm 1 Horner rule for polynomial evaluation.

procedure Horner($A[0 \ldots n], x$) $\triangleright$ $A : \{a_0 \ldots a_n\}$
    $p \leftarrow A[n]$
    for $i \leftarrow n-1$ **downto** 0 **do**
        $p \leftarrow px + A[i]$
    return p

Algorithm 2 Integer Multiplication.

procedure Int-Mult(y, z) $\triangleright$ $y, z \geq 0;\ y, z \in \mathbb{Z}$
    if $z = 0$ **then**
        return 0
    return Int-Mult($cy, \lfloor \frac{z}{c} \rfloor$) $+ y(z \bmod c)$

Sorting

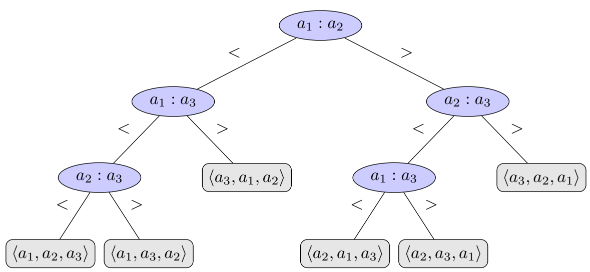

Figure 1: 3-element sorting algorithm represented by a decision tree.

Algorithm 3 Bubblesort (Bubble the smallest element each time.).

procedure BUBBLESORT($A[1 \cdots n]$)
    for $i \leftarrow 1$ **to** $n-1$ **do**
        for $j \leftarrow n$ **downto** $i+1$ **do**
            if $A[j] < A[j-1]$ **then**
                SWAP($A[j], A[j-1]$)

Algorithm 4 Bubblesort (Bubble the largest element each time.).

procedure BUBBLESORT($A[1 \cdots n]$)
    for $i \leftarrow n$ **downto** 2 **do**
        for $j \leftarrow 1$ **to** $i-1$ **do**
            if $A[j] > A[j+1]$ **then**
                SWAP($A[j], A[j+1]$)

Algorithm 5 Bubblesort (An improved version; from wiki).

```
procedure BUBBLESORT(A[1···n])
    repeat
        newn ← 0
        for i ← 1 to n − 1 do
            if A[i − 1] > A[i] then
                SWAP(A[i − 1], A[i])
                newn ← i
        n ← newn
    until n = 0
```

Quicksort

HOARE-PARTITION (alg4)

Algorithm 6 Hoare partition.

```
procedure HOARE-PARTITION(A, l, r)
    x ← A[l]
    i ← l
    j ← r + 1

    while TRUE do
        while A[+ + i] < x do
            if i = r then
                break
        while A[− − j] > x do
            if j = l then
                break

        if i < j then
            SWAP(A, i, j)

    SWAP(A, l, j)
```

Linear Time Sorting

Algorithm 7 Selection Sort.

```
procedure SELECTION-SORT(A, n)
    for i ← 1 to n − 1 do
        for j ← i + 1 to n do
            if A[j] < A[i] then
                SWAP(A[j], A[i])
```

Algorithm 8 COUNTING-SORT in place in $O(n + k)$ time.

```
procedure COUNTING-SORT(A, k)
    TODO
```

Selection

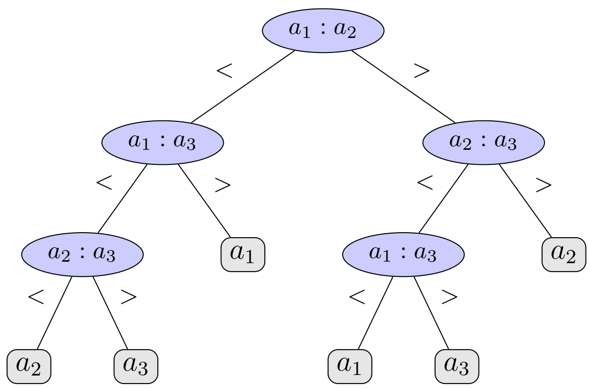

Figure 2: 3-element median selection algorithm represented by a decision tree.

Searching

Algorithm 9 (see Problem $5-2$ of [1]) searches for a value x in an unsorted array A consisting of n elements by checking $A[1], A[2], \cdots, A[n]$ in order until either it finds $A[i] = x$ or it reaches the end of the array.

[1] Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein. *Introduction to Algorithms, Third Edition*. The MIT Press, 3rd edition, 2009. ISBN 0262033844, 9780262033848

Algorithm 9 Deterministic search.

```
procedure DETERMINISTIC-SEARCH(A[1···n], x)
    i ← 1
    while i ≤ n do
        if A[i] = x then
            return true
        i ← i + 1
    return false
```

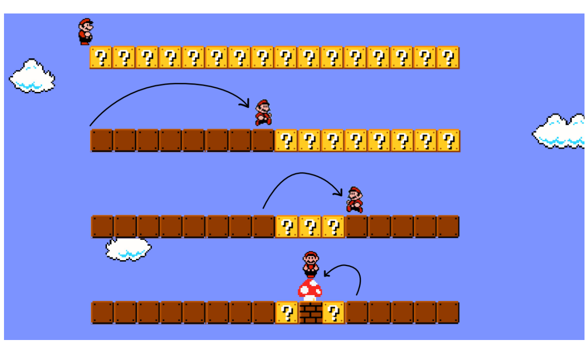

Figure 3: Binary stride.

Algorithm 10 Iterative binary search.

procedure BINARYSEARCH($A[0 \cdots n-1], x$)
    $L \leftarrow 0$
    $R \leftarrow n - 1$
    while $L \leq R$ **do**
        $m \leftarrow L + (R - L)/2$
        if $A[m] < x$ **then**
            $L \leftarrow m + 1$
        else if $A[m] > x$ **then**
            $R \leftarrow m - 1$
        else
            return m
    return -1

Algorithm 11 Recursive binary search.

procedure BINARYSEARCH(A, L, R, x)
    if $R < L$ **then**
        return -1
    $m \leftarrow L + (R - L)/2$
    if $A[m] = x$ **then**
        return m
    else if $A[m] > x$ **then**
        return BINARYSEARCH($A, L, m - 1, x$)
    else
        return BINARYSEARCH($A, m + 1, R, x$)

Randomized Algorithms

Algorithm 12 One in three.

```
procedure OneInThree
    if FairBit = 0 then
        return 0
    else
        return 1 − OneInThree
```

Dynamic Programming

Algorithm 13 Computing $\binom{n}{k}$ recursively.

procedure $\textsc{Binom}(n, k)$ ▷ Required: $n \geq k \geq 0$
    if $k = 0 \vee n = k$ **then**
        return 1
    return $\textsc{Binom}(n-1, k) + \textsc{Binom}(n-1, k-1)$

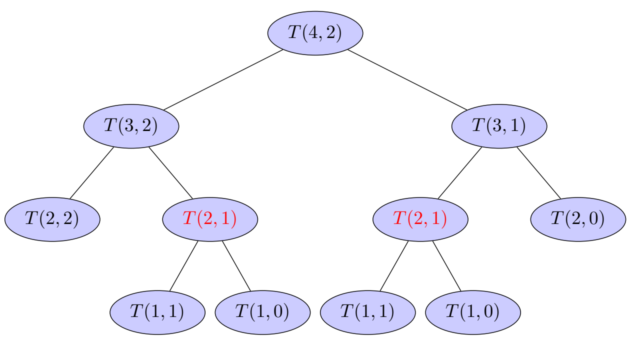

Figure 4: Calculate $\binom{4}{2}$ recursively.

$$
\begin{array}{ccccccccccc}
 & & & & & \binom{0}{0} & & & & & \\
 & & & & \binom{1}{0} & & \binom{1}{1} & & & & \\
 & & & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & & & \\
 & & \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} & & \\
 & \binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4} & \\
\binom{5}{0} & & \binom{5}{1} & & \binom{5}{2} & & \binom{5}{3} & & \binom{5}{4} & & \binom{5}{5}
\end{array}
$$

Figure 5: Pascal triangle for binomial coefficients.

Algorithm 14 Computing $\binom{n}{k}$ by dynamic programming.

```
procedure BINOM(n, k)                              ▷ Required: n ≥ k ≥ 0
    for i ← 0 to n − k do
        B[i][0] ← 1
    for i ← 1 to k do
        B[i][i] ← 1
    for j ← 1 to k do
        for d ← 1 to n − k do
            i ← j + d
            B[i][j] ← B[i − 1][j] + B[i − 1][j − 1]
    return B[n][k]
```

$$(n-k+1)+(k)+k(n-k)=nk-k^2+n+1$$

Algorithm 15 Max-sum subarray.

```
procedure MSS(A[1 ··· n])
    MSS[0] ← 0
    for i ← 1 to n do
        MSS[i] ← max {MSS[i − 1] + A[i], 0}
    return max_{1≤i≤n} MSS[i]
```

Algorithm 16 Max-sum subarray (Implementation Simplified).

```
procedure MSS(A[1 ··· n])
    mss ← 0
    MSS ← 0
    for i ← 1 to n do
        MSS ← max {MSS + A[i], 0}
        mss ← max {mss, MSS}
    return mss
```

Traversal on Trees

DFS on Trees

Algorithm 17 Recursive DFS pre-order traversal on binary tree.

```
procedure RECURSIVE-DFS(t)
    print t.key

    if t.left ≠ NIL then
        RECURSIVE-DFS(t.left)
    if t.right ≠ NIL then
        RECURSIVE-DFS(t.right)

RECURSIVE-DFS(T.root)
```

Algorithm 18 Iterative DFS pre-order traversal on binary tree.

```
procedure ITERATIVE-DFS(t)
    S.PUSH(t)                                   ▷ S : stack

    while S ≠ ∅ do
        v ← S.POP()
        print v.key

        if v.right ≠ NIL then
            S.PUSH(v.right)
        if v.left ≠ NIL then
            S.PUSH(v.left)

ITERATIVE-DFS(T.root)
```

Algorithm 19 Recursive DFS traversal on a rooted tree stored using the "left-child, right-sibling" representation.

```
procedure RECURSIVE-DFS(t)
    print t.key

    if t.left-child ≠ NIL then
        RECURSIVE-DFS(t.left-child)
    if t.right-sibling ≠ NIL then
        RECURSIVE-DFS(t.right-sibling)

RECURSIVE-DFS(T.root)
```

Algorithm 20 Calculate the sum of depths of all nodes of a tree T.

```
procedure SUM-OF-DEPTHS()
    return SUM-OF-DEPTHS(T, 0)

procedure SUM-OF-DEPTHS(r, depth)                ▷ r: root of a tree
    if r is a leaf then
        return depth
    sum ← depth
    for all child vertex v of r do
        sum ← sum + SUM-OF-DEPTHS(v, depth + 1)
    return sum
```

Algorithm 21 Count the number of nodes in T at depth K.

```
procedure NODES-AT-DEPTH()
    return NODES-AT-DEPTH(T, K)

procedure NODES-AT-DEPTH(r, k)                   ▷ r: root of a tree
    if k = 0 then
        return 1
    if r is a leaf then
        return 0
    num ← 0
    for all child vertex v of r do
        num ← num + NODES-AT-DEPTH(v, k − 1)
    return num
```

Algorithm 22 Check whether a tree T has any leaf at an even depth.

```
procedure LEAF-AT-EVEN-DEPTH()
    return LEAF-AT-DEPTH(T, even = 0)

procedure LEAF-AT-DEPTH(r, parity)                ▷ r: root of a tree
    if r is a leaf then
        return 1 − parity
    result ← 0
    for all child vertex v of r do
        result ← result ∨ LEAF-AT-DEPTH(v, 1 − parity)
    return result
```

BFS on Trees

Algorithm 23 Calculate the sum of contents of nodes of a tree T at each depth.

```
procedure SUM-AT-DEPTH(r)                ▷ r: root of the tree T
    r.depth ← 0
    Q ← ∅
    ENQUEUE(Q, r)
    while Q ≠ ∅ do
        u ← DEQUEUE(Q)
        sumAtDepth[u.depth] += u.content
        for all child vertex v of u do
            v.depth ← u.depth + 1
            ENQUEUE(Q, v)
```

Algorithm 24 Count the number of nodes of a tree T at each depth.

```
procedure NODES-AT-DEPTH(r)                    ▷ r: root of the tree T
    r.depth ← 0
    Q ← ∅
    ENQUEUE(Q, r)
    while Q ≠ ∅ do
        u ← DEQUEUE(Q)
        nodesAtDepth[u.depth] += 1
        for all child vertex v of u do
            v.depth ← u.depth + 1
            ENQUEUE(Q, v)
    return argmax_K nodesAtDepth[k]
```

Stack and Queue

Stack

Queue

Algorithm 25 Circular queue.

```
procedure ENQUEUE(Q, x)
    if Q.head = Q.tail + 1 then
        return "OVERFLOW"

    Q[Q.tail] = x

    if Q.tail = Q.length then
        Q.tail = 1
    else
        Q.tail = Q.tail + 1

procedure DEQUEUE(Q)
    if Q.head = Q.tail then
        return "UNDERFLOW"

    x = Q[Q.head]

    if Q.head = Q.length then
        Q.head = 1
    else
        Q.head = Q.head + 1

    return x
```

Stack and Queue

Algorithm 26 Simulating a queue using two stacks S_1, S_2.

```
procedure Enq(x)
    Push(S_1, x)

procedure Deq()
    if S_2 = ∅ then
        while S_1 ≠ ∅ do
            Push(S_2, Pop(S_1))

    Pop(S_2)
```

DAG

Paths in Graphs

Single-Source Shortest Paths (SSSP)

Algorithm 27 Dijkstra's algorithm for SSSP

```
for all v ∈ V do
    dist[v] ← ∞
dist[s] ← 0

Q ← MinPQ(V)
while Q ≠ ∅ do
    u ← DeleteMin(Q)
    for all (u, v) ∈ E ∧ v ∉ Q do
        if dist[v] > dist[u] + l(u, v) then
            dist[v] ← dist[u] + l(u, v)
            DecreaseKey(Q, v)
```

All-Pairs Shortest Paths (APSP)

Algorithm 28 Floyd-Warshall algorithm for APSP

```
for all (i, j) do
    if (i, j) ∈ E then
        dist(i, j, 0) ← l(i, j)
    else
        dist(i, j, 0) ← ∞

for k ← 1 to n do
    for i ← 1 to n do
        for j ← 1 to n do
            dist(i, j, k) = min(dist(i, j, k − 1), dist(i, k, k − 1) + dist(k, j, k − 1))
```

Eulerian Path and Eulerian Circuit

Algorithm 29 Hierholzer's algorithm for finding an Eulerian circuit.

$VE \leftarrow \emptyset$
$C \leftarrow \emptyset$

while $VE \neq E$ **do**
  $u \leftarrow \textsc{Choose}(u : (u \rightarrow v) \notin VE)$
  $C' \leftarrow \textsc{Circuit}(u, E \setminus VE)$
  $VE \leftarrow VE \cup C'$
  $C \leftarrow C \cup C'$

Bibliography

Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein. *Introduction to Algorithms, Third Edition*. The MIT Press, 3rd edition, 2009. ISBN 0262033844, 9780262033848.