

面向分布式系统的复制数据类型理论研究概述

(CCF 2018 第九届优博论坛)

魏恒峰

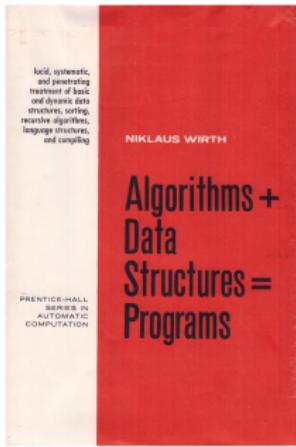
南京大学软件所

2018 年 08 月 08 日



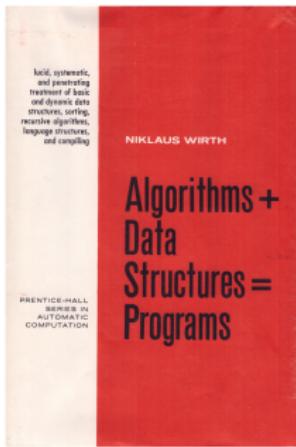
Abstract Data Types (ADT) [Liskov and Zilles, 1974]

(单线程; 顺序语义)



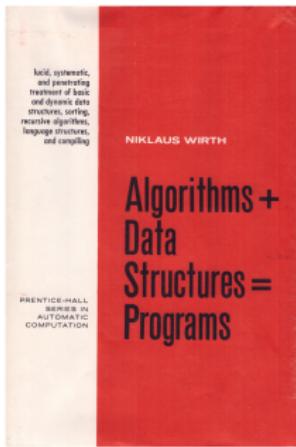
Abstract Data Types (ADT) [Liskov and Zilles, 1974]

(单线程; 顺序语义)



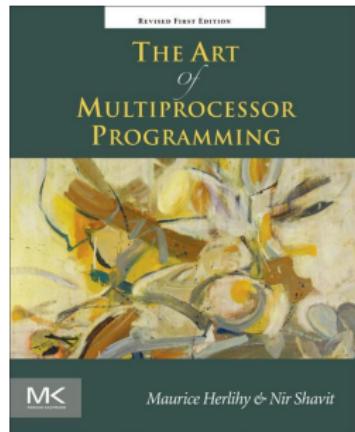
Abstract Data Types (ADT) [Liskov and Zilles, 1974]

(单线程; 顺序语义)



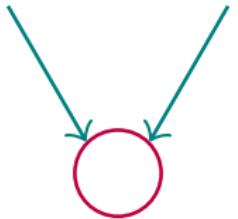
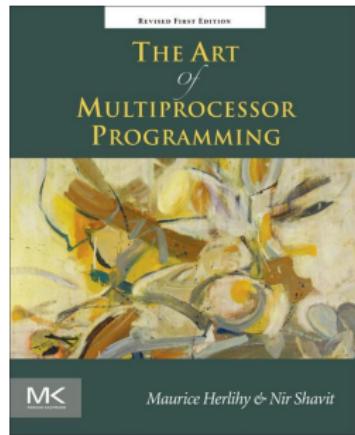
Concurrent Data Types [Herlihy and Wing, 1990]

(多线程; 并发语义)



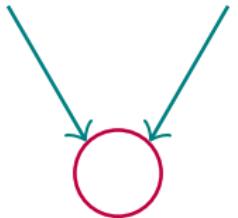
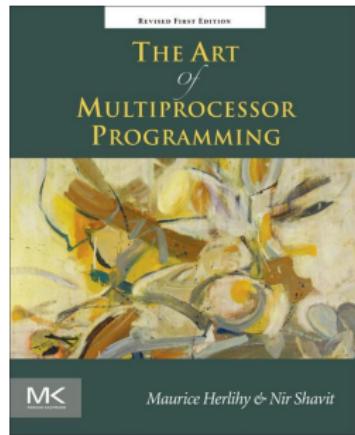
Concurrent Data Types [Herlihy and Wing, 1990]

(多线程; 并发语义)



Concurrent Data Types [Herlihy and Wing, 1990]

(多线程; 并发语义)

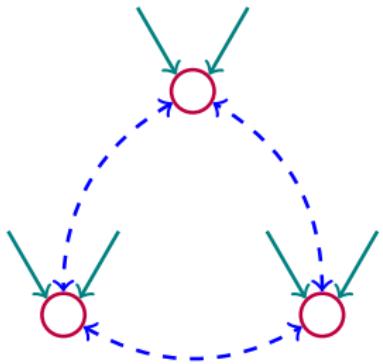


Replicated Data Types (RDT; \approx 2010 年) [Burckhardt et al., 2014]

(多副本; 复制语义)

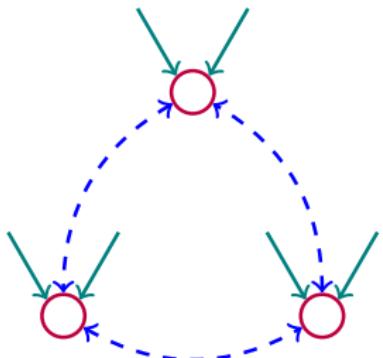
Replicated Data Types (RDT; \approx 2010 年) [Burckhardt et al., 2014]

(多副本; 复制语义)



Replicated Data Types (RDT; \approx 2010 年) [Burckhardt et al., 2014]

(多副本; 复制语义)







新平台

大规模分布式系统



新浪微博社交应用¹:

- ▶ 日均用户近一亿名
- ▶ 日均消息近一亿条

¹2015 第三季度; 数据来自 China Internet Watch.

大规模分布式系统



新浪微博社交应用¹:

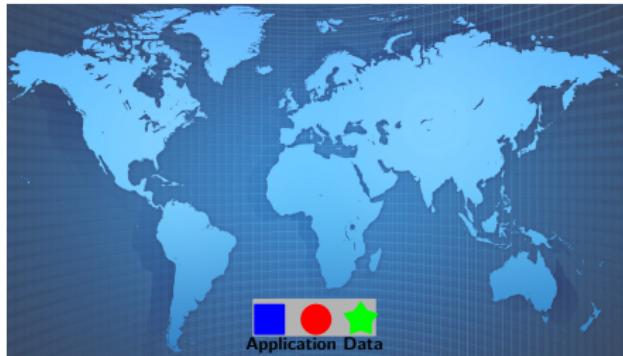
- ▶ 日均用户近一亿名
- ▶ 日均消息近一亿条

特性需求:

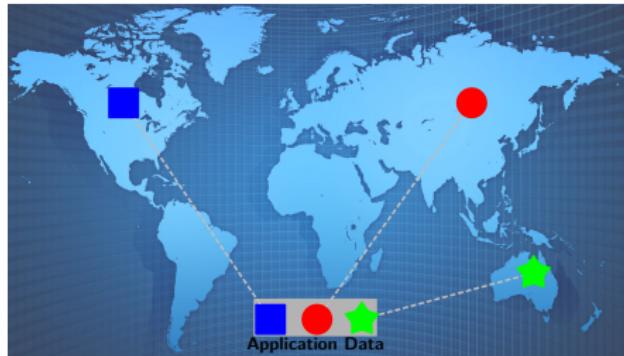
- ▶ 低延迟, 高可用性 (4 个 9²)
- ▶ 高容错性, 高可扩展性

¹2015 第三季度; 数据来自 China Internet Watch.

²数据来自 InfoQ.

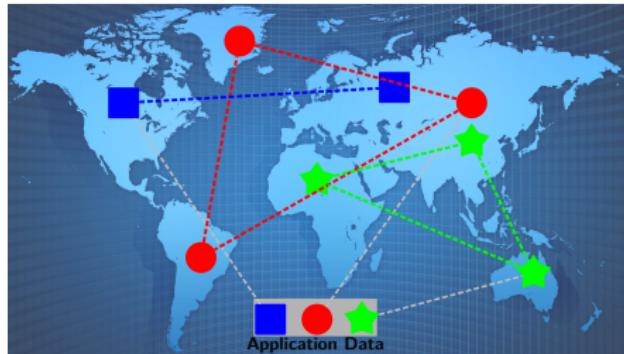


分布数据 (distributed data):



分布数据 (distributed data):

1. 分区 (partition): 水平扩展



分布数据 (distributed data):

1. 分区 (partition): 水平扩展
2. 副本 (replication) : 就近访问, 容灾备份

复制数据类型 [Shapiro et al., 2011a]

- ▶ Read/Write Register
- ▶ Counter
- ▶ Set
- ▶ List
- ▶ HashMap
- ▶ Disjoint Set
- ▶ Graph
- ▶ ...

What's
new?

新问题, 新挑战

What's
new?

新问题, 新挑战

Replicated Data Types: Specification, Verification, Optimality

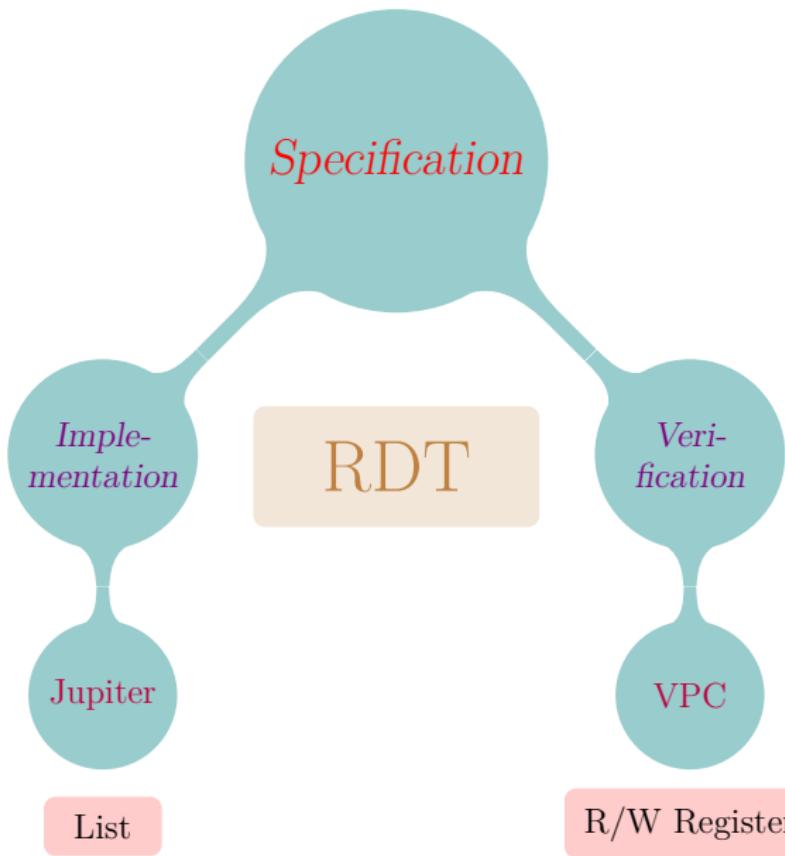
Sebastian Burckhardt

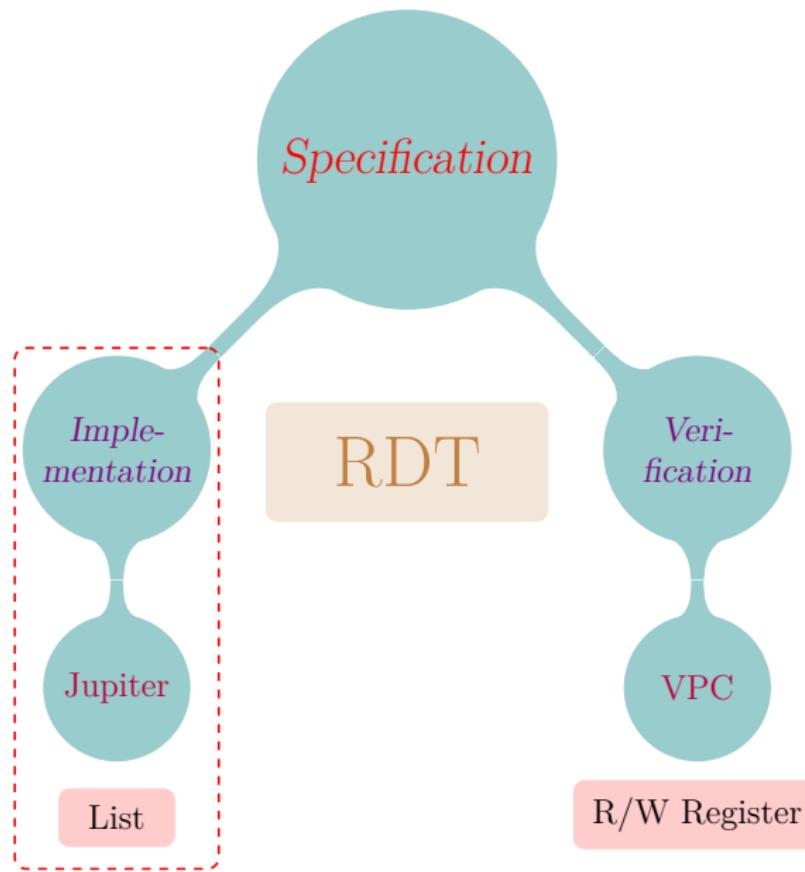
Alexey Gotsman

Hongseok Yang

Marek Zawirski

[Burckhardt et al., 2014]





Brief Announcement @ PODC'2018 ³

实现复制列表的 Jupiter 协议 [Nichols et al., 1995]^a 满足
weak list specification [Attiya et al., 2016]^b.

^aDavid A. Nichols et al. (1995). "High-latency, Low-bandwidth Windowing in the Jupiter Collaboration System". In: *Proceedings of the 8th Annual ACM Symposium on User Interface and Software Technology*. UIST '95. ACM, pp. 111–120.

^bHagit Attiya et al. (2016). "Specification and complexity of collaborative text editing". In: *Proceedings of the 2016 ACM Symposium on Principles of Distributed Computing*. PODC '16. ACM, pp. 259–268.

Brief Announcement @ PODC'2018 ³

实现复制列表的 Jupiter 协议 [Nichols et al., 1995]^a 满足
weak list specification [Attiya et al., 2016]^b.

^aDavid A. Nichols et al. (1995). "High-latency, Low-bandwidth Windowing in the Jupiter Collaboration System". In: *Proceedings of the 8th Annual ACM Symposium on User Interface and Software Technology*. UIST '95. ACM, pp. 111–120.

^bHagit Attiya et al. (2016). "Specification and complexity of collaborative text editing". In: *Proceedings of the 2016 ACM Symposium on Principles of Distributed Computing*. PODC '16. ACM, pp. 259–268.

Weak List Specification

基于副本的协同文本编辑系统



(a) Google Docs



(b) Apache Wave



(c) Wikipedia



(d) LATEX Editor

复制列表对象: 建模编辑系统的核心功能

$\text{INS}(a, p)$: 在 p 位置插入元素 a

$\text{DEL}(p)$: 删除 p 位置上的元素

READ : 返回该列表

定义 (最终收敛性 (Eventual Convergence) [Ellis and Gibbs, 1989])

当用户不再提交更新操作时, 每个 *replica* 上的列表是相同的。

定义 (最终收敛性 (Eventual Consistency) [Ellis and Gibbs, 1989])

当用户不再提交更新操作时, 每个 *replica* 上的列表是相同的。

定义 (强最终一致性 (Strong Eventual Consistency) [Shapiro et al., 2011b])

如果两个 *replica* 处理了同一组用户操作, 那么这两个 *replica* 上对列表是相同的。

定义 (最终收敛性 (Eventual Consistency) [Ellis and Gibbs, 1989])

当用户不再提交更新操作时, 每个 *replica* 上的列表是相同的。

定义 (强最终一致性 (Strong Eventual Consistency) [Shapiro et al., 2011b])

如果两个 *replica* 处理了同一组用户操作, 那么这两个 *replica* 上对列表是相同的。

对系统的中间状态缺少足够的约束

Specification and Complexity of Collaborative Text Editing

Hagit Attiya
Technion

Sebastian Burckhardt
Microsoft Research

Alexey Gotsman
IMDEA Software Institute

Adam Morrison
Technion

Hongseok Yang
University of Oxford

Marek Zawirski^{*}
Inria & Sorbonne Universités,
UPMC Univ Paris 06, LIP6

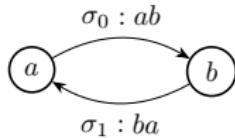
定义 (Weak List Specification $\mathcal{A}_{\text{weak}}$ [Attiya et al., 2016])

Informally, $\mathcal{A}_{\text{weak}}$ requires the ordering between elements that are not deleted to be consistent across the system.

定义在系统所有列表状态上的全局性质

定义 (状态对兼容性 (Pairwise State Compatibility Property))

任给两个列表状态 σ_0 、 σ_1 , 若它们含有两个共同元素 a 、 b ,
则 a 、 b 在 σ_0 与 σ_1 中的相对顺序保持一致。

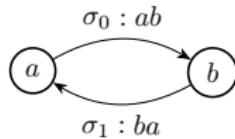
$$\boxed{\sigma_0 : ab}$$
$$\boxed{\sigma_1 : ba}$$


定义 (状态对兼容性 (Pairwise State Compatibility Property))

任给两个列表状态 σ_0, σ_1 , 若它们含有两个共同元素 a, b ,
则 a, b 在 σ_0 与 σ_1 中的相对顺序保持一致。

$$\sigma_0 : ab$$

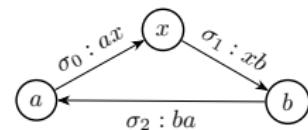
$$\sigma_1 : ba$$



$$\sigma_0 : ax$$

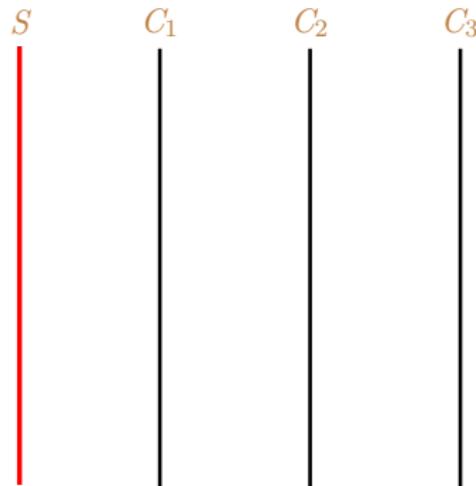
$$\sigma_1 : xb$$

$$\sigma_2 : ba$$

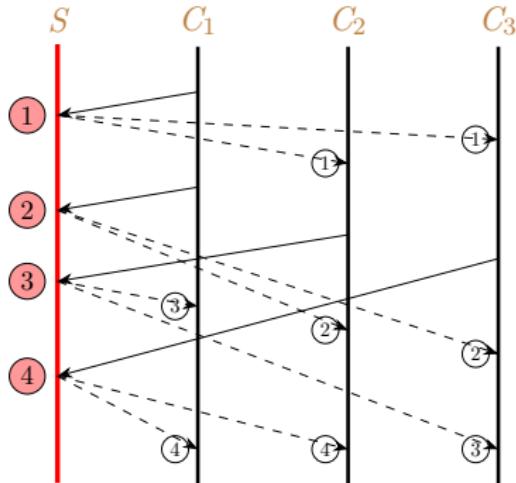


Jupiter

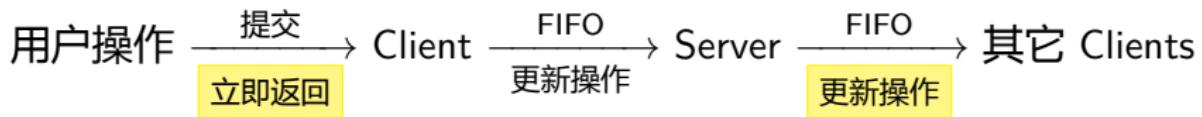
$(n + 1)$ replica \triangleq (n) Client + (1) Server [Nichols et al., 1995]



$$(n+1) \text{ replica} \triangleq (n) \text{ Client} + (1) \text{ Server} \quad [\text{Nichols et al., 1995}]$$

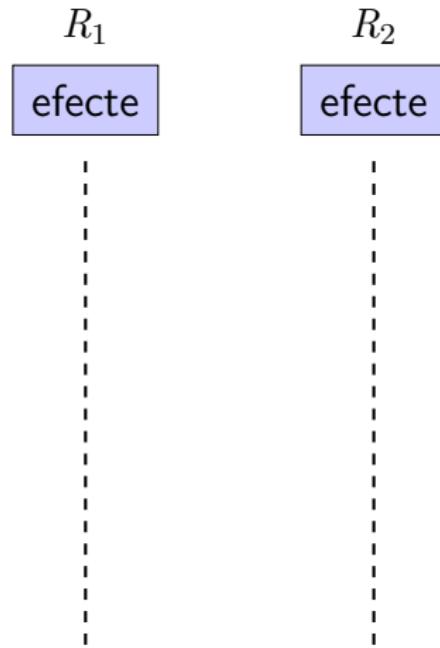


Server 负责将所有操作序列化

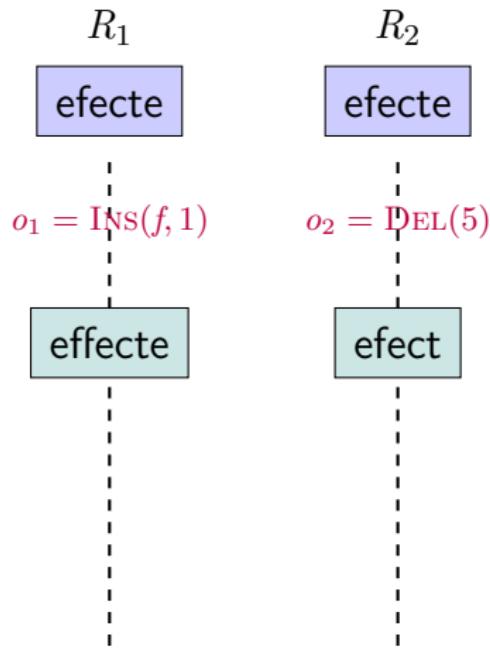


操作转换 (Operational Transformation; OT) [Ellis and Gibbs, 1989] 技术

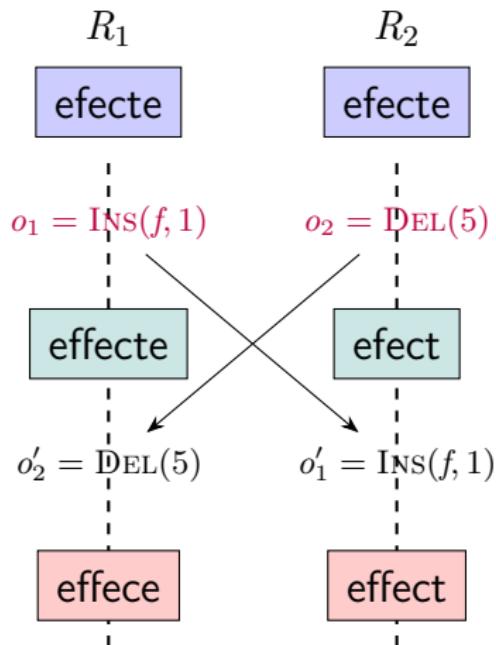
操作转换 (Operational Transformation; OT) [Ellis and Gibbs, 1989] 技术



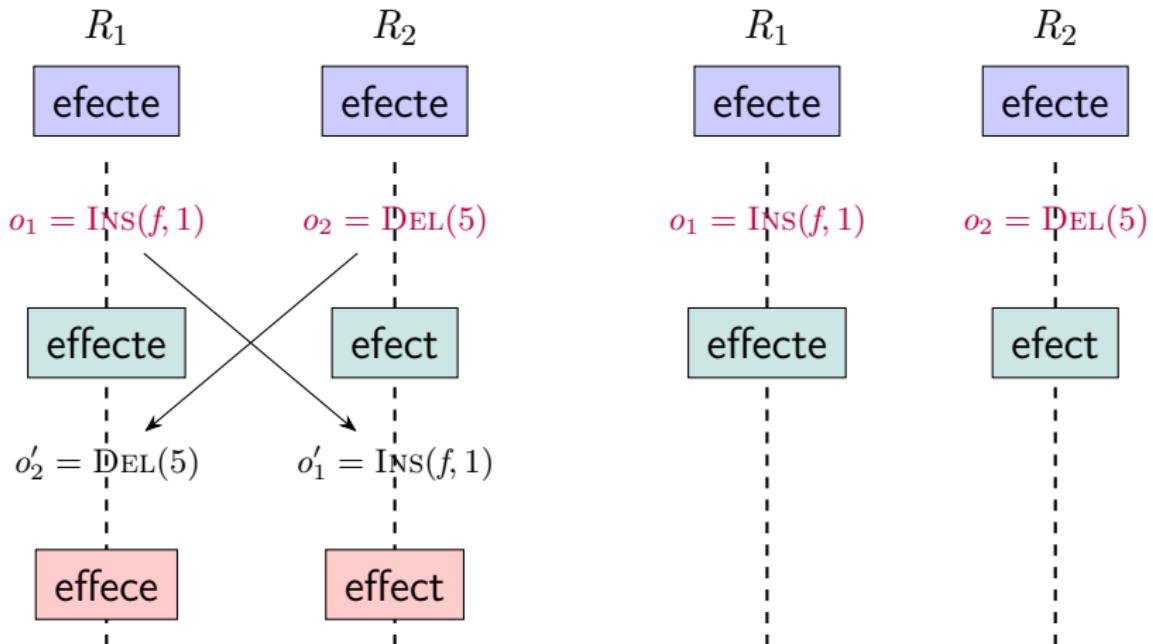
操作转换 (Operational Transformation; OT) [Ellis and Gibbs, 1989] 技术



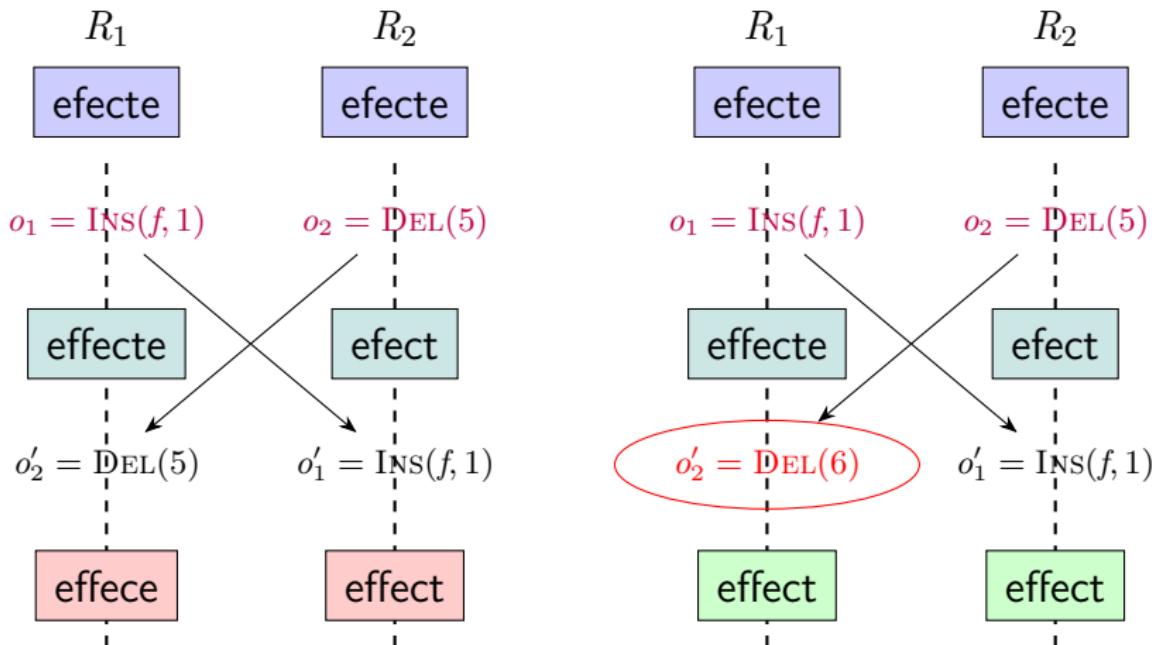
操作转换 (Operational Transformation; OT) [Ellis and Gibbs, 1989] 技术

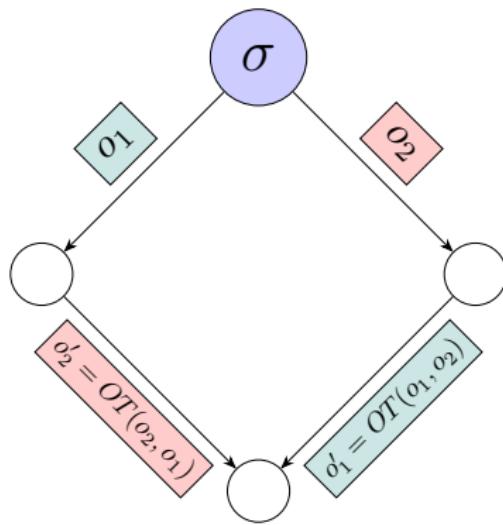


操作转换 (Operational Transformation; OT) [Ellis and Gibbs, 1989] 技术



操作转换 (Operational Transformation; OT) [Ellis and Gibbs, 1989] 技术





交换律 $\sigma; o_1; o'_2 \equiv \sigma; o_2; o'_1$

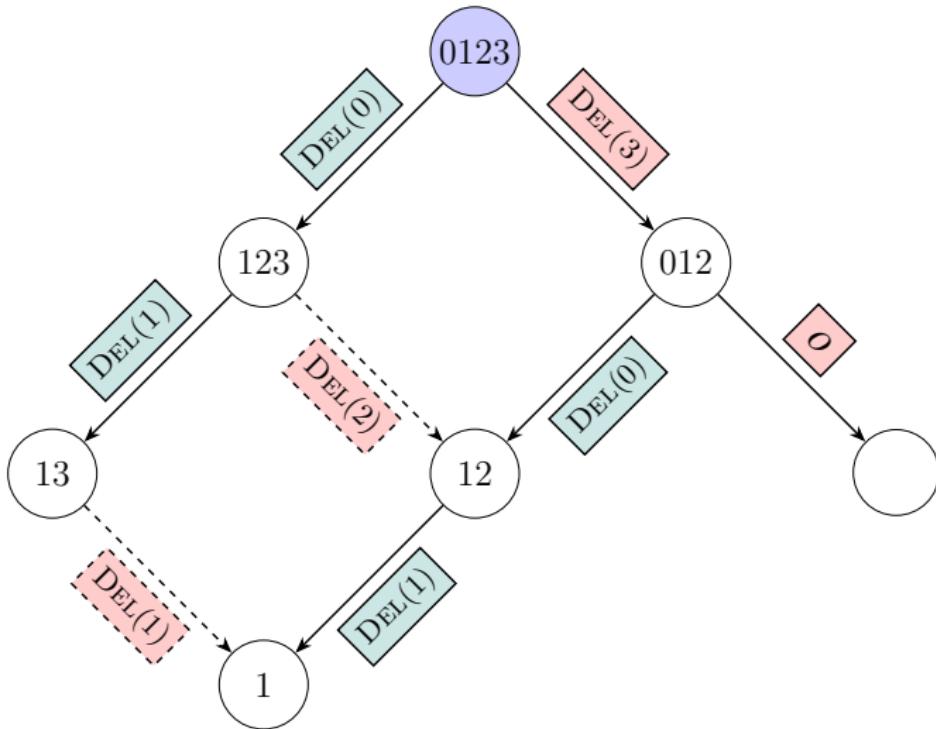
针对列表的操作转换函数 [Ellis and Gibbs, 1989]

$$OT\left(\text{INS}(a_1, p_1, pr_1), \text{INS}(a_2, p_2, pr_2) \right) = \begin{cases} \text{INS}(a_1, p_1, pr_1) & p_1 < p_2 \\ \text{INS}(a_1, p_1 + 1, pr_1) & p_1 > p_2 \\ \text{NOP} & p_1 = p_2 \wedge a_1 = a_2 \\ \text{INS}(a_1, p_1 + 1, pr_1) & p_1 = p_2 \wedge a_1 \neq a_2 \wedge pr_1 > pr_2 \\ \text{INS}(a_1, p_1, pr_1) & p_1 = p_2 \wedge a_1 \neq a_2 \wedge pr_1 \leq pr_2 \end{cases}$$

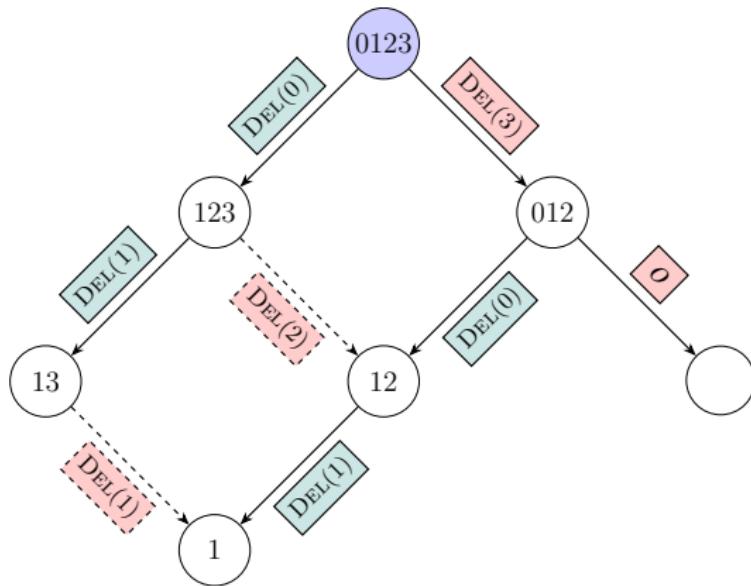
$$OT\left(\text{INS}(a_1, p_1, pr_1), \text{DEL}(_, p_2, pr_2) \right) = \begin{cases} \text{INS}(a_1, p_1, pr_1) & p_1 \leq p_2 \\ \text{INS}(a_1, p_1 - 1, pr_1) & p_1 > p_2 \end{cases}$$

$$OT\left(\text{DEL}(_, p_1, pr_1), \text{INS}(a_2, p_2, pr_2) \right) = \begin{cases} \text{DEL}(_, p_1, pr_1) & p_1 < p_2 \\ \text{DEL}(_, p_1 + 1, pr_1) & p_1 \geq p_2 \end{cases}$$

$$OT\left(\text{DEL}(_, p_1, pr_1), \text{DEL}(_, p_2, pr_2) \right) = \begin{cases} \text{DEL}(_, p_1, pr_1) & p_1 < p_2 \\ \text{DEL}(_, p_1 - 1, pr_1) & p_1 > p_2 \\ \text{NOP} & p_1 = p_2 \end{cases}$$

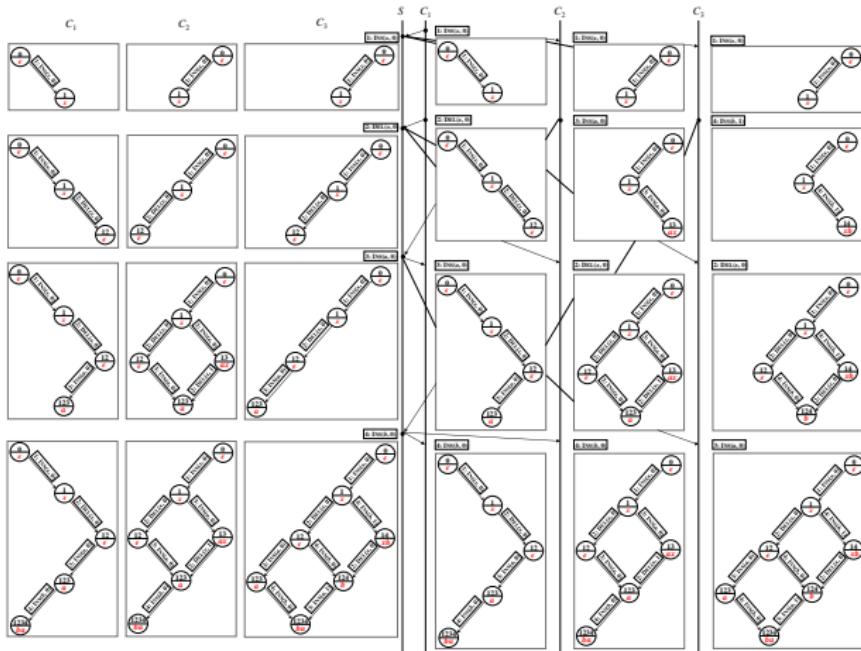


利用数据结构 2D 状态空间 [Xu, Sun, and Li, 2014] 控制何时以及如何执行“操作转换”



2D: LOCAL vs. GLOBAL

每个 Client 维护一个 2D 状态空间



Server 维护 n 个 2D 状态空间, 与 n 个 Clients 对应

$\mathcal{A}_{\text{weak}}$ 所规定的全局性质



Jupiter 协议中, 每个 replica 所维护的局部视图

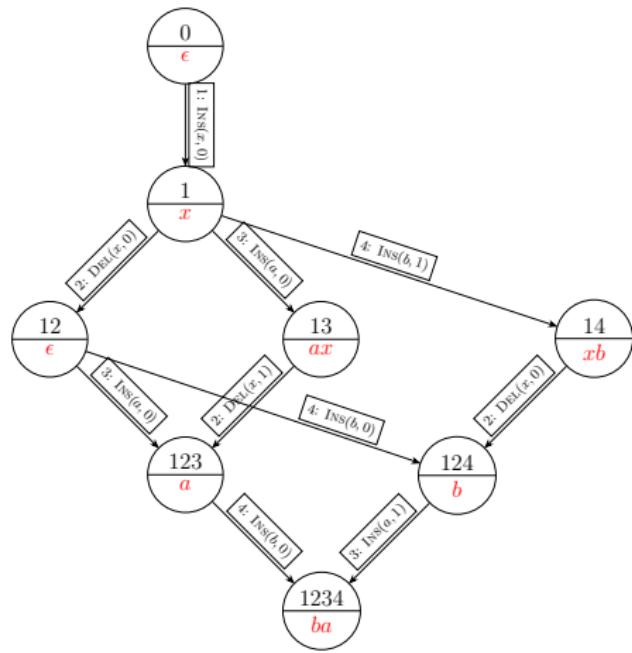
CJupiter (Compact Jupiter)

CJupiter (Compact Jupiter)

Theorem (等价性)

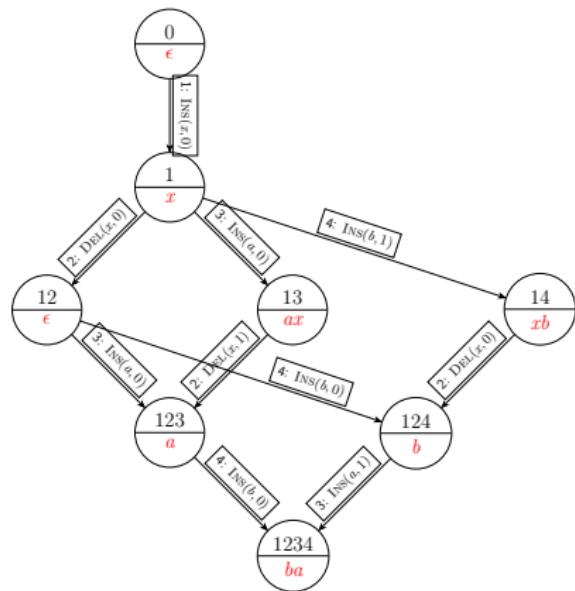
在相同的操作调度下, *CJupiter* 与 *Jupiter* 中的对应 *replica* 的行为 (状态序列) 是相同的。

CJupiter 为每个 replica 维护一个 n -ary 有序状态空间



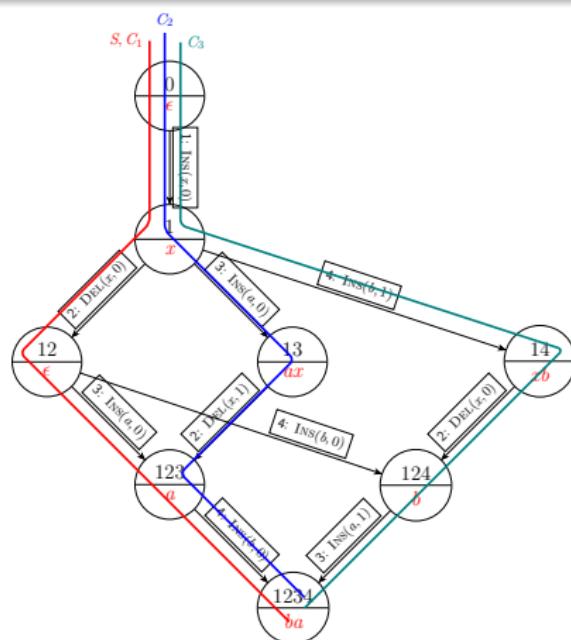
命题 (Compactness of CJupiter)

CJupiter 所维护的 $(n + 1)$ 个 n -ary 有序状态空间是相同的。



命题 (Compactness of CJupiter)

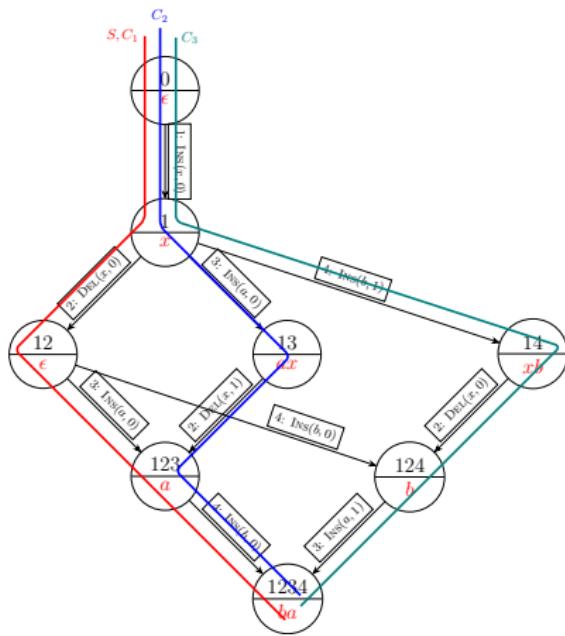
CJupiter 所维护的 $(n + 1)$ 个 n -ary 有序状态空间是相同的。



每个 replica 的行为对应于该状态空间中的一条路径

CJupiter 满足 Weak List Specification

关注某个 n -ary 有序状态空间, 三步骤证明状态对兼容性

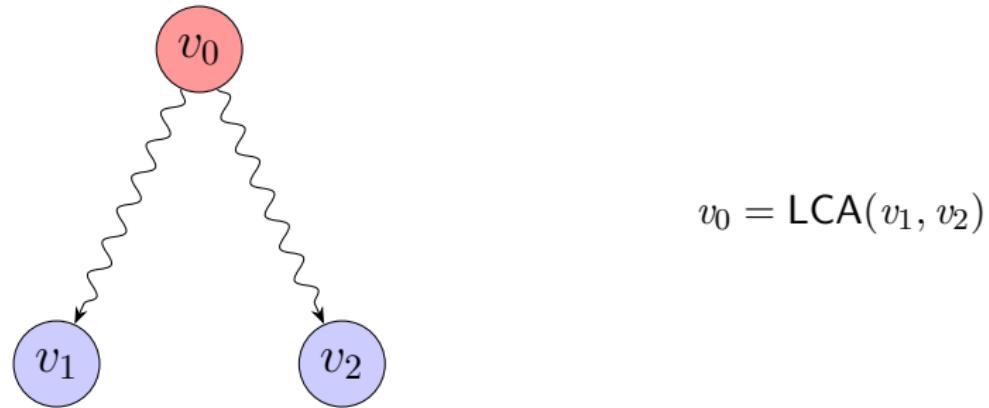


1

任取两个状态节点 v_1 和 v_2

引理 (LCA (Lowest Common Ancestor))

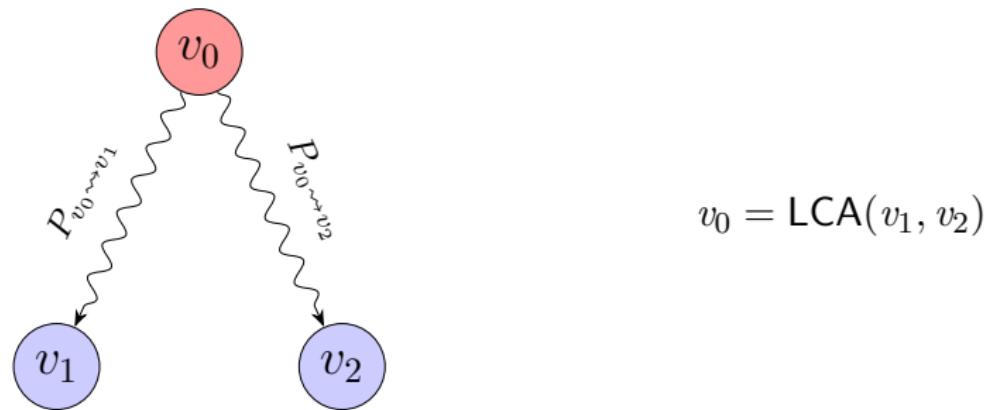
n -ary 有序状态空间中的任意一对状态节点都有唯一的最近公共祖先。



2 考虑从 $v_0 = \text{LCA}(v_1, v_2)$ 到 v_1 和 v_2 的两条路径

引理 (Disjoint Paths)

路径 $P_{v_0 \rightsquigarrow v_1}$ 上包含的操作集 $O_{v_0 \rightsquigarrow v_1}$ 与路径 $P_{v_0 \rightsquigarrow v_2}$ 上包含的操作集 $O_{v_0 \rightsquigarrow v_2}$ 不相交。

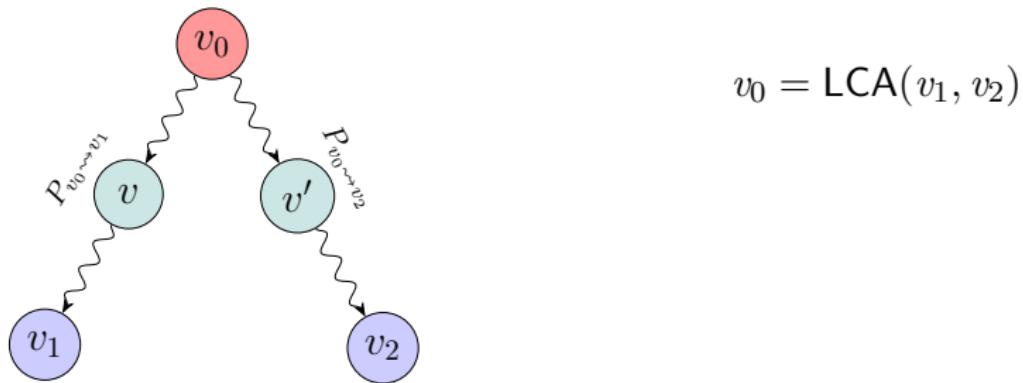


3

考虑两条路径上的状态

引理 (Compatible Paths)

$P_{v_0 \rightsquigarrow v_1}$ 上的任一状态 v 与 $P_{v_0 \rightsquigarrow v_2}$ 上的任一状态 v' 是兼容的。

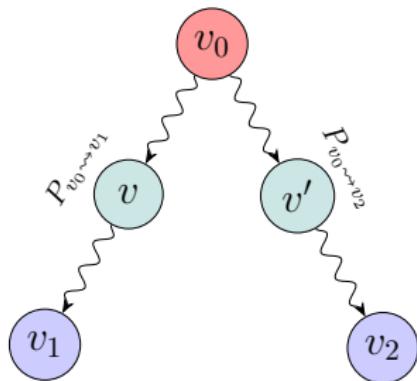


3

考虑两条路径上的状态

引理 (Compatible Paths)

$P_{v_0 \rightsquigarrow v_1}$ 上的任一状态 v 与 $P_{v_0 \rightsquigarrow v_2}$ 上的任一状态 v' 是兼容的。



$$v_0 = \text{LCA}(v_1, v_2)$$

$\therefore v_1$ 和 v_2 是兼容的

个人体会: 基于 OT 思想的协议晦涩难懂



个人体会: 基于 OT 思想的协议晦涩难懂



- ▶ 协议多种多样
- ▶ 经常不加证明
- ▶ 证明是错误的

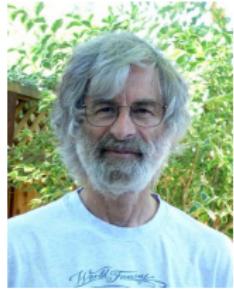
个人体会: 基于 OT 思想的协议晦涩难懂



- ▶ 协议多种多样
- ▶ 经常不加证明
- ▶ 证明是错误的
- ▶ **勘误也是错的**

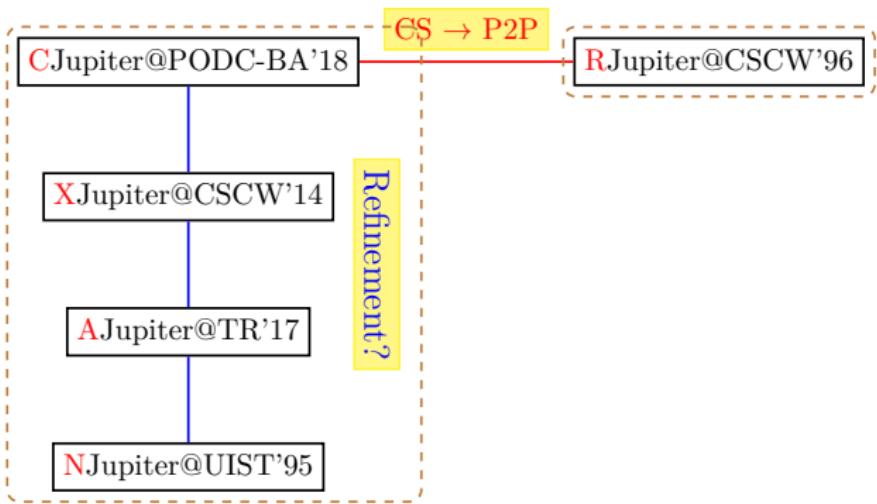
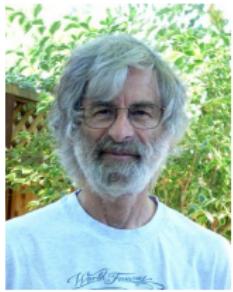
Model Checking: 使用 TLA+

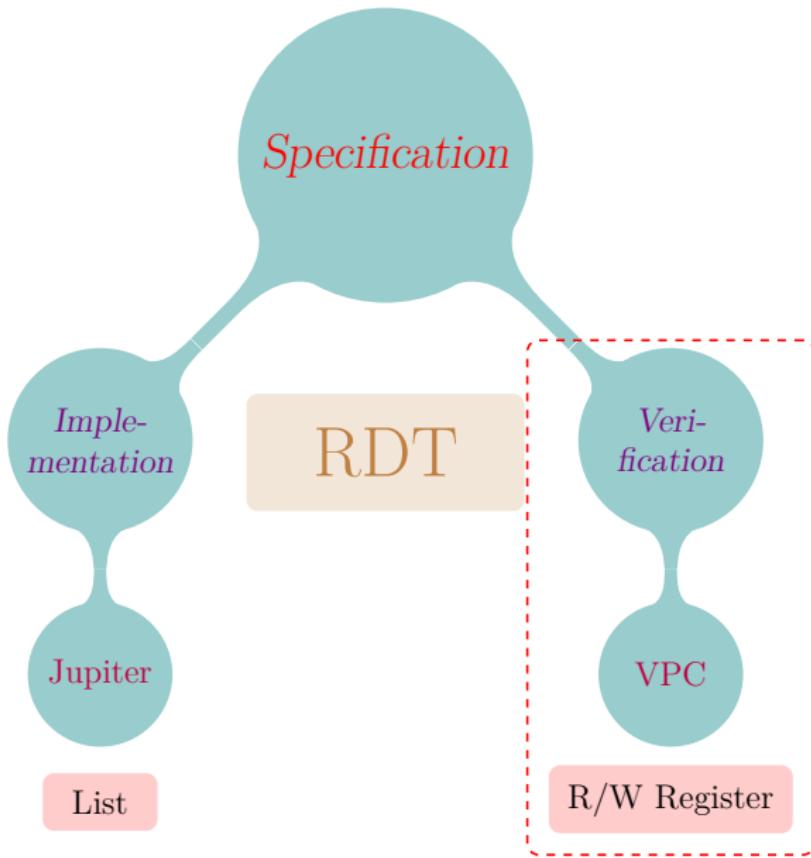
jupiter-tlaplus@github



Model Checking: 使用 TLA+

jupiter-tlaplus@github





NOW
IS THE
TIME





协议验证 (Verification of a Protocol)

执行验证 (Verification of an Execution)

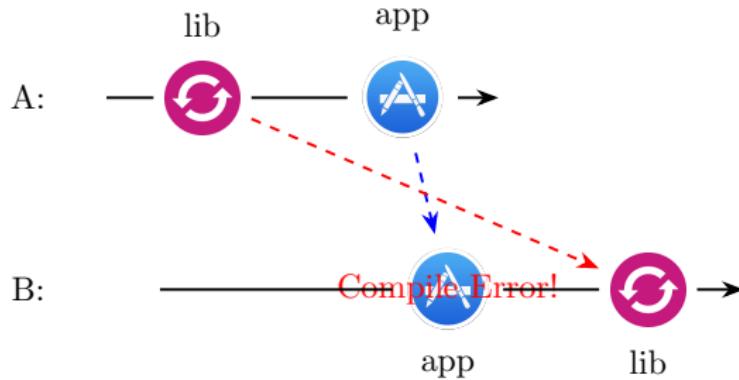
执行验证 (Verification of an Execution)



黑盒测试/确认系统是否提供了其所声称的数据一致性

[DeCandia et al., 2007] [Golab, Li, and Shah, 2011]

PRAM: 包含存储系统常提供的最基本的“会话”(session)一致性
[Terry et al., 1994] [Brzezinski, Sobaniec, and Wawrzyniak, 2004]



PRAM 保证“单调写”性质

定义 (VPC: Verifying PRAM Consistency)

VPC 判定问题:

实例: 系统执行 (*execution e*)

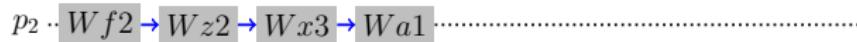
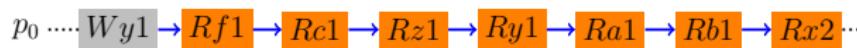
问题: 该执行 e 是否满足 PRAM 一致性模型 (\mathcal{C})?

$$e \in \mathcal{C} \Rightarrow \{0, 1\}?$$

定义 (系统执行)

系统执行 $e \triangleq \{h_p \mid h_p : \text{进程 } p \text{ 上的读写操作序列}\}$

规模 n : 系统执行中读写操作的总数

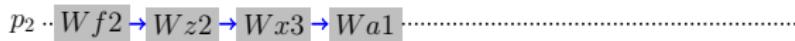
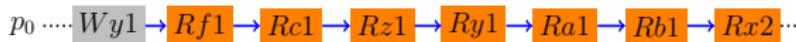


定义 (PRAM 一致性模型)

系统执行 e 满足 PRAM 一致性



$\forall p : p$ 上所有操作与其它进程上所有写操作存在合法调度

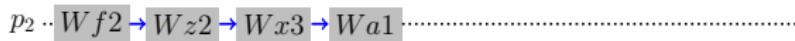
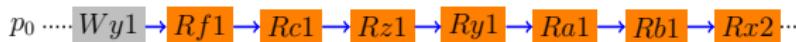


定义 (PRAM 一致性模型)

系统执行 e 满足 PRAM 一致性



$\forall p : p$ 上所有操作与其它进程上所有写操作存在合法调度



$p_0 : W f 2 \ W f 1 \ W z 2 \ W z 1 \ W y 2 \ W y 1 \ R f 1 \ W x 5 \ W x 3 \ W x 2 \ W c 1 \ R c 1$
 $R z 1 \ R y 1 \ W a 1 \ R a 1 \ W b 1 \ R b 1 \ R x 2$

表: VPC 问题的四种变体 (按“执行”的类型) 及复杂度
([*] : 本文工作)

	<i>(S)ingle variable</i>	<i>(M)ultiple variables</i>
<i>write (D)uplicate values</i>	VPC-SD	VPC-MD
<i>write (U)nique value</i>	VPC-SU	VPC-MU

表: VPC 问题的四种变体 (按“执行”的类型) 及复杂度
([*] : 本文工作)

	<i>(S)ingle variable</i>	<i>(M)ultiple variables</i>
<i>write (D)uplicate values</i>	VPC-SD <i>(NP-complete)</i> [*]	VPC-MD <i>(NP-complete)</i> [*]
<i>write (U)nique value</i>	VPC-SU (P) [Golab, Li, and Shah, 2011]	VPC-MU (P) [*]

表: VPC 问题的四种变体 (按“执行”的类型) 及复杂度
([*] : 本文工作)

	<i>(S)ingle variable</i>	<i>(M)ultiple variables</i>
<i>write (D)uplicate values</i>	VPC-SD <i>(NP-complete)</i> [*]	VPC-MD <i>(NP-complete)</i> [*]
<i>write (U)nique value</i>	VPC-SU (P) [Golab, Li, and Shah, 2011]	VPC-MU (P) [*]

Read-mapping [Gibbons and Korach, 1997]: $\forall r, \exists! w, f(r) = w.$

VPC-SD (VPC-MD) 是 NP-complete 问题

多项式规约: 从 UNARY 3-PARTITION 到 VPC-SD.

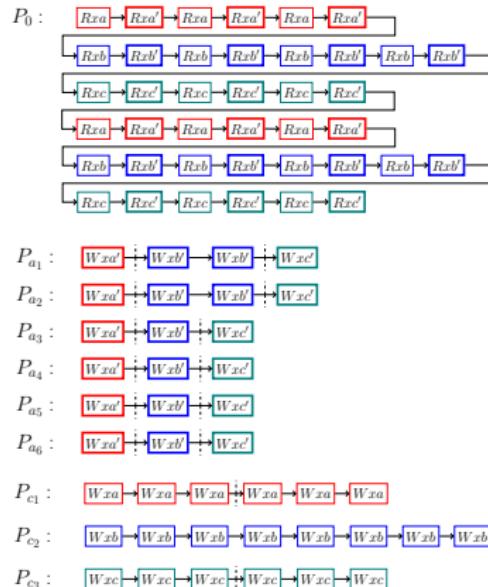


图: UNARY 3-PARTITION 实例 $A = \{2, 2, 1, 1, 1, 1\}$, $m = 2, B = 4$ 对应的 VPC-SD 执行

VPC-MU 的多项式算法 RW-CLOSURE

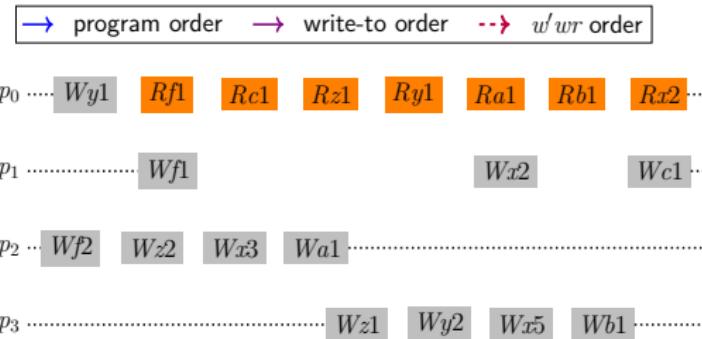


图: RW-CLOSURE 算法示例: 在传递闭包之上迭代应用 $w'wr$ 规则

VPC-MU 的多项式算法 RW-CLOSURE

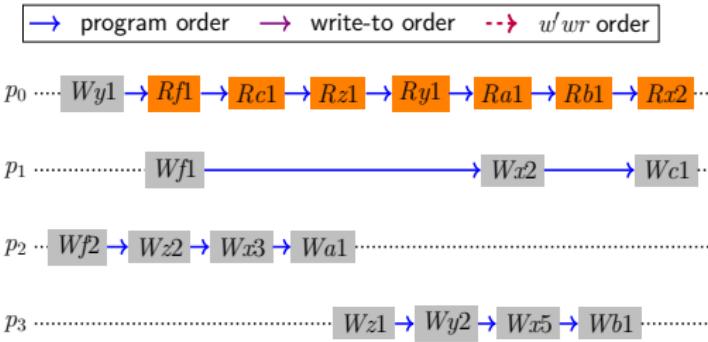


图: RW-CLOSURE 算法示例: 在传递闭包之上迭代应用 $w'wr$ 规则

VPC-MU 的多项式算法 RW-CLOSURE

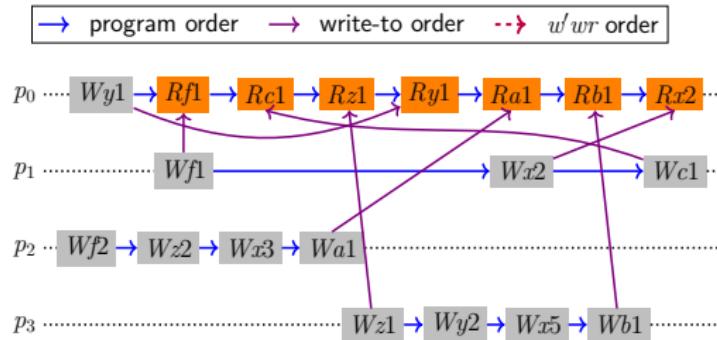


图: RW-CLOSURE 算法示例: 在传递闭包之上迭代应用 $w'wr$ 规则

VPC-MU 的多项式算法 RW-CLOSURE

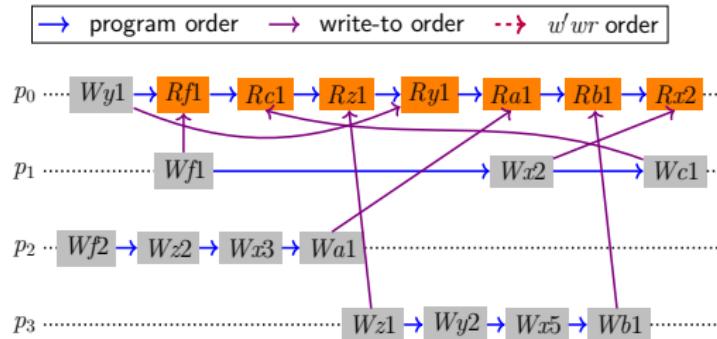


图: RW-CLOSURE 算法示例: 在传递闭包之上迭代应用 $w'wr$ 规则

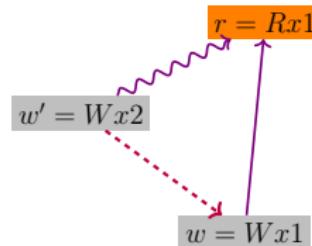


图: $w'wr$ 规则

VPC-MU 的多项式算法 RW-CLOSURE

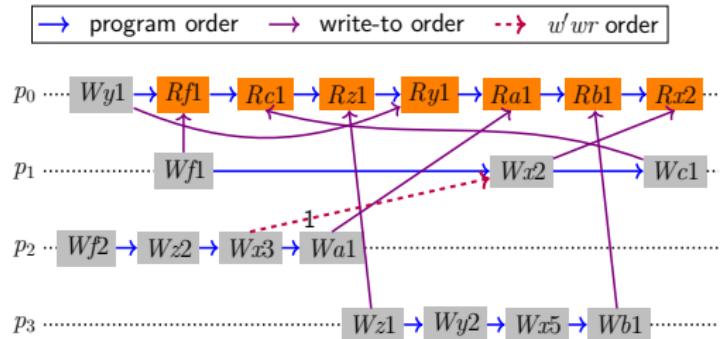


图: RW-CLOSURE 算法示例: 在传递闭包之上迭代应用 $w' wr$ 规则

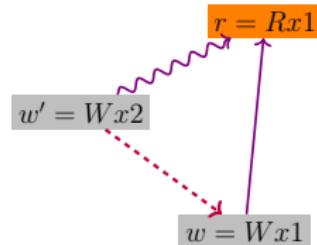


图: $w' wr$ 规则

VPC-MU 的多项式算法 RW-CLOSURE

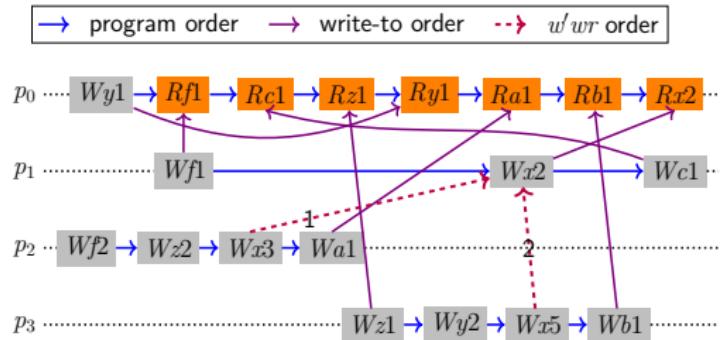


图: RW-CLOSURE 算法示例: 在传递闭包之上迭代应用 $w'wr$ 规则

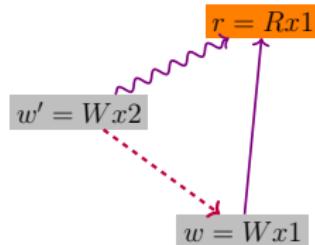
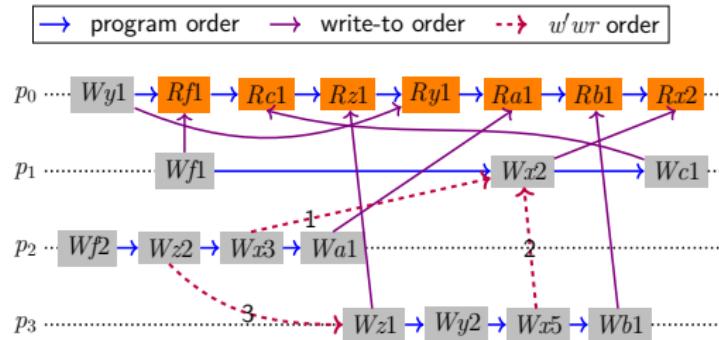
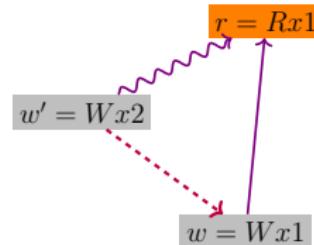


图: $w'wr$ 规则

VPC-MU 的多项式算法 RW-CLOSURE



图：RW-CLOSURE 算法示例：在传递闭包之上迭代应用 $w' wr$ 规则



图： $w' wr$ 规则

VPC-MU 的多项式算法 RW-CLOSURE

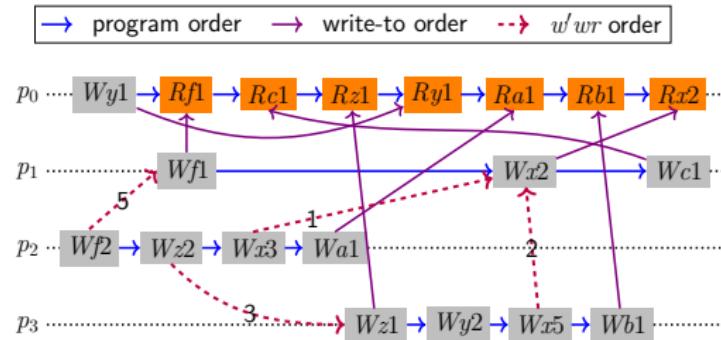


图: RW-CLOSURE 算法示例: 在传递闭包之上迭代应用 $w' wr$ 规则

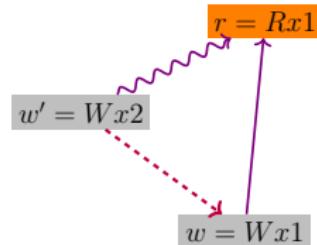


图: $w' wr$ 规则

VPC-MU 的多项式算法 RW-CLOSURE

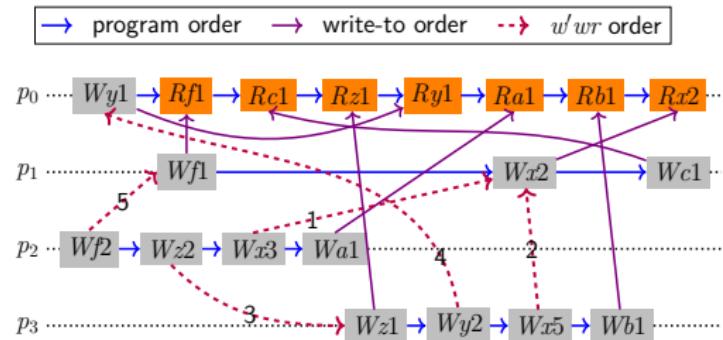


图: RW-CLOSURE 算法示例: 在传递闭包之上迭代应用 $w' wr$ 规则

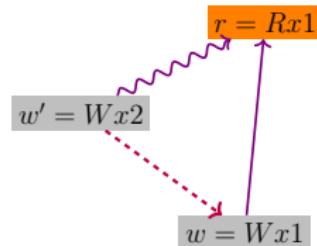


图: $w' wr$ 规则

定理 (RW-CLOSURE 算法正确性)

VPC-MU 实例满足 PRAM 一致性



RW-CLOSURE 算法所得图是 DAG 图

定理 (RW-CLOSURE 算法正确性)

VPC-MU 实例满足 PRAM 一致性



RW-CLOSURE 算法所得图是 DAG 图

证明

“ \implies ” 反证法

“ \impliedby ” 难点: DAG 图蕴含着多个全序

技巧: 对读操作作数学归纳, 构造合法调度

定理 (RW-CLOSURE 算法正确性)

VPC-MU 实例满足 PRAM 一致性



RW-CLOSURE 算法所得图是 DAG 图

证明

“ \implies ” 反证法

“ \iff ” 难点: DAG 图蕴含着多个全序

技巧: 对读操作作数学归纳, 构造合法调度

RW-CLOSURE 算法复杂度:

$$\underbrace{O(n^2)}_{\# \text{loops}} \cdot \underbrace{O(n^3)}_{\text{transitive closure}} = O(n^5)$$

RW-CLOSURE 算法的缺点:

- ▶ 在全图上应用 $w'wr$ 规则
- ▶ 应用 $w'wr$ 规则无特定顺序

RW-CLOSURE 算法的缺点:

- ▶ 在全图上应用 $w'wr$ 规则
- ▶ 应用 $w'wr$ 规则无特定顺序

VPC-MU 的多项式算法 READ-CENTRIC 要点:

- ▶ 增量式调度每个读操作
- ▶ 在读操作诱导的局部子图上按逆拓扑序应用 $w'wr$ 规则

定理 (READ-CENTRIC 算法正确性)

$VPC-MU$ 实例满足 $PRAM$ 一致性



READ-CENTRIC 算法所得图是 DAG 图

定理 (READ-CENTRIC 算法正确性)

VPC-MU 实例满足 PRAM 一致性



READ-CENTRIC 算法所得图是 DAG 图

证明

READ-CENTRIC $\xrightleftharpoons{\text{Reachability}}$ RW-CLOSURE

难点: $\#w'wr_{\text{READ-CENTRIC}} \leq \#w'wr_{\text{RW-CLOSURE}}$

READ-CENTRIC 算法复杂度:

$$\underbrace{O(n)}_{\text{iterations}} \cdot \underbrace{O(n \cdot n^2)}_{\text{TOPO-SCHEDULE}} = O(n^4)$$

引理 (TOPO-SCHEDULE 的非迭代性)

设 TOPO-SCHEDULE 正在处理读操作 r ,
则局部子图中的每个写操作最多只有一次机会
在满足规则 $w'wr$ 的三元组中扮演 “ w' 角色”。

实验评估

实验目的¹：

1. 考察 READ-CENTRIC 算法的实际效率 (*vs.* 渐近时间复杂度)
2. 对比 READ-CENTRIC 算法与 RW-CLOSURE 算法的效率

¹机器配置: Intel Core i7 3.40GHZ, 4GB RAM.

实验评估

实验目的¹：

1. 考察 READ-CENTRIC 算法的实际效率 (*vs.* 渐近时间复杂度)
2. 对比 READ-CENTRIC 算法与 RW-CLOSURE 算法的效率

两类负载：

1. 随机生成的系统执行
2. 满足 PRAM 一致性的系统执行 (\approx 最坏情况输入)

¹机器配置: Intel Core i7 3.40GHZ, 4GB RAM.

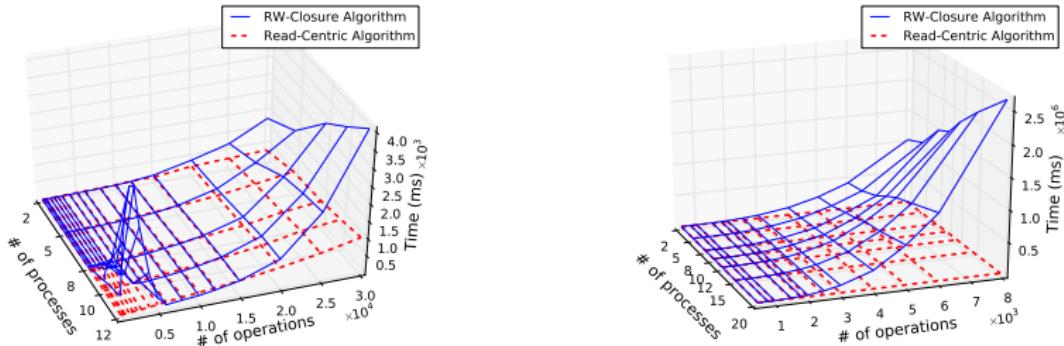


图: RW-CLOSURE 算法与 READ-CENTRIC 算法在 (左) 随机生成的执行及 (右) 满足 PRAM 一致性的执行上的运行时间。

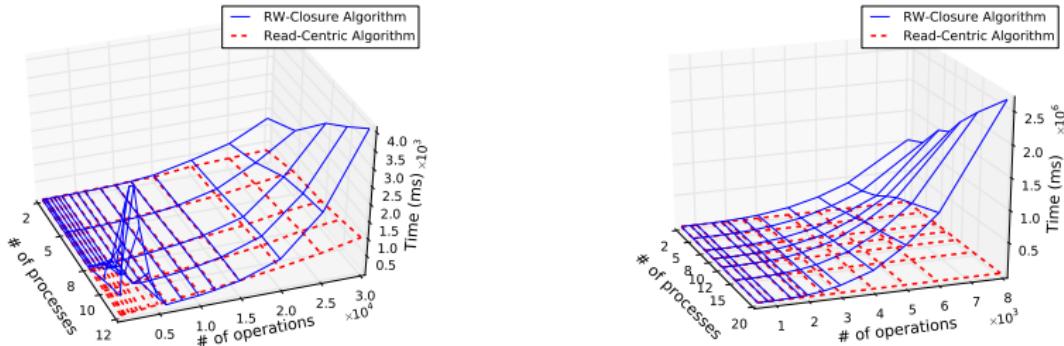


图: RW-CLOSURE 算法与 READ-CENTRIC 算法在 (左) 随机生成的执行及 (右) 满足 PRAM 一致性的执行上的运行时间。

(右) 20 个进程、8,000 个操作:
READ-CENTRIC 可获得 694 倍加速.

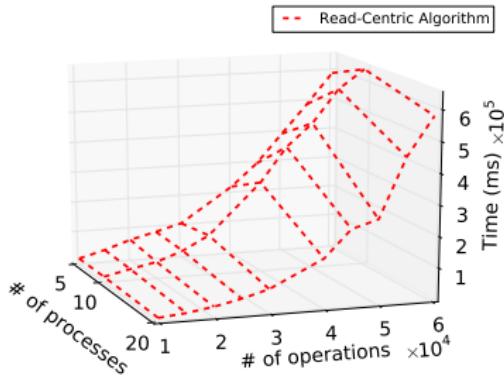


图: READ-CENTRIC 算法在满足 PRAM 一致性的执行上的运行时间

READ-CENTRIC: 20 个进程、60,000 个操作 < 600s ¹

RW-CLOSURE: 20 个进程、8,000 个操作 > 3,000s

¹ 用于测试，规模可用

VPC 在相关工作中的意义

较早关注 (分布式系统领域) “弱一致性模型验证”问题 (2013~):

强一致性: [Gibbons and Korach, 1997] [Cantin, Lipasti, and Smith, 2005]
[Golab, Li, and Shah, 2011]

弱一致性: [Furbach et al., 2014] [Bouajjani et al., 2017]
[Emmi and Enea, 2018]

**表: VSC (Verifying Sequential Consistency) 与 VL (Verifying Linearizability)
问题的复杂度 [Gibbons and Korach, 1997]**

Variants	VSC	VL
General	NP-complete	NP-complete
2 Operations/Process	NP-complete	w.l.o.g. ⁴
2 Variables	NP-complete	w.l.o.g.
3 Processes	NP-complete	$O(n \log n)$
Read-mapping	NP-complete	$O(n \log n)$
Write-order	NP-complete	$O(n \log n)$
read&write only	NP-complete	NP-complete
Conflict-order	$O(n \log n)$	$O(n \log n)$

⁴The complexity is not affected by the given restriction.

表: VMC (Verifying Memory Coherence) 问题的复杂度 [Cantin, Lipasti, and Smith, 2005]

Variants	Read/Write	Read-Modify-Write
1 Operation/Process	$O(n \lg n)$	$O(n^2)$
2 Operations/Process	?	NP-complete
3+ Operations/Process	NP-complete	NP-complete
Constant k processes	$O(n^k)$	$O(n^k)$
1 Write/Value (Read-mapping)	$O(n)$	$O(n \lg n)$
2 Writes/Value	NP-complete	?
3+ Writes/Value	NP-complete	NP-complete
Write-order	$O(n^2)$	$O(n)$

表: Atomicity⁵ 相关一致性模型验证问题复杂度 (假设: 不允许写重复值)

	Safety	Regularity	Atomicity	Sequential
Offline [Anderson et al., 2010]	$O(n^2)$	$O(n^2)$	$O(n^3)$	<i>not studied</i>
Online ⁶ [Golab, Li, and Shah, 2011]	$O(n)$	$O(n)$	$O(n \log n)$	$\text{Poly}(n)$

⁵也称 Linearizability

⁶包含其它假设

表: k -AV (k -Atomicity Verification) 问题复杂度

Problems	Variants	Results	Work
1-AV	General	NP-complete	[Gibbons and Korach, 1997]
1-AV	Write unique value	$O(n \log n)$	[Gibbons and Korach, 1997]
2-AV	Write unique value	$O(n \log n)$	[Golab, Hurwitz, and Li, 2013]
k -AV	Write unique value	$O(n^2)$	[Golab et al., 2015]
	Bounded concurrency		[Golab et al., 2018]
k -AV	Write unique value		

表: k -AV (k -Atomicity Verification) 问题复杂度

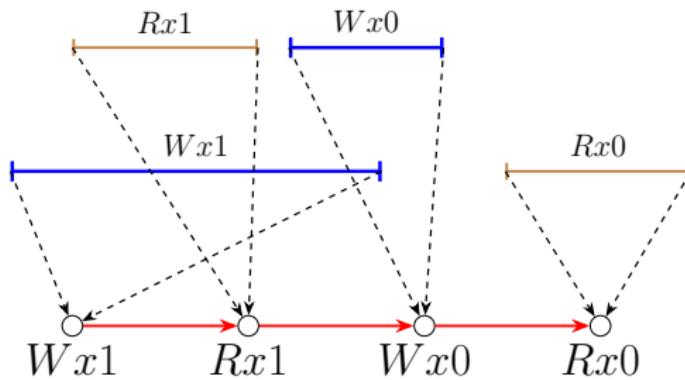
Problems	Variants	Results	Work
1-AV	General	NP-complete	[Gibbons and Korach, 1997]
1-AV	Write unique value	$O(n \log n)$	[Gibbons and Korach, 1997]
2-AV	Write unique value	$O(n \log n)$	[Golab, Hurwitz, and Li, 2013]
k -AV	Write unique value	$O(n^2)$	[Golab et al., 2015]
	Bounded concurrency		[Golab et al., 2018]
k -AV	Write unique value		

表: k -AV (k -Atomicity Verification) 问题复杂度

Problems	Variants	Results	Work
1-AV	General	NP-complete	[Gibbons and Korach, 1997]
1-AV	Write unique value	$O(n \log n)$	[Gibbons and Korach, 1997]
2-AV	Write unique value	$O(n \log n)$	[Golab, Hurwitz, and Li, 2013]
k -AV	Write unique value Bounded concurrency	$O(n^2)$	[Golab et al., 2015] [Golab et al., 2018]
k -AV	Write unique value		

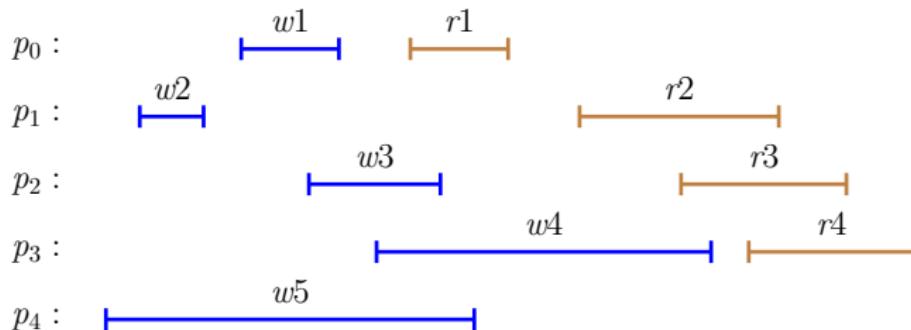
Atomicity = 实时序 + 读写语义

[Lamport, 1986]



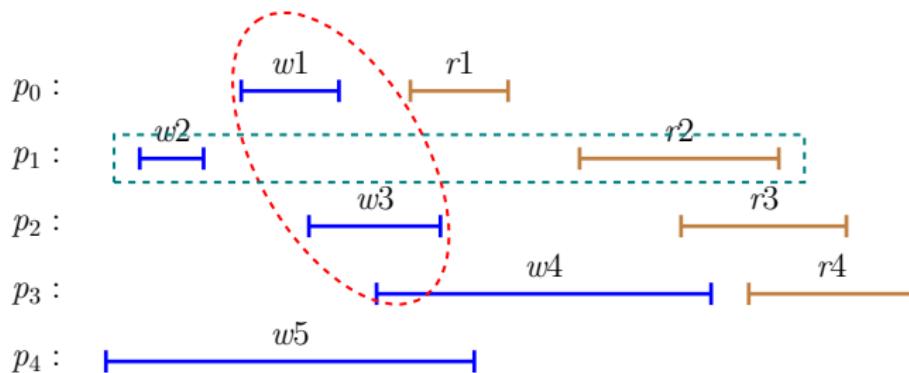
k -Atomicity = 实时序 + k -读写语义

[Aiyer, Alvisi, and Bazzi, 2005] [Taubenfeld, 2013]



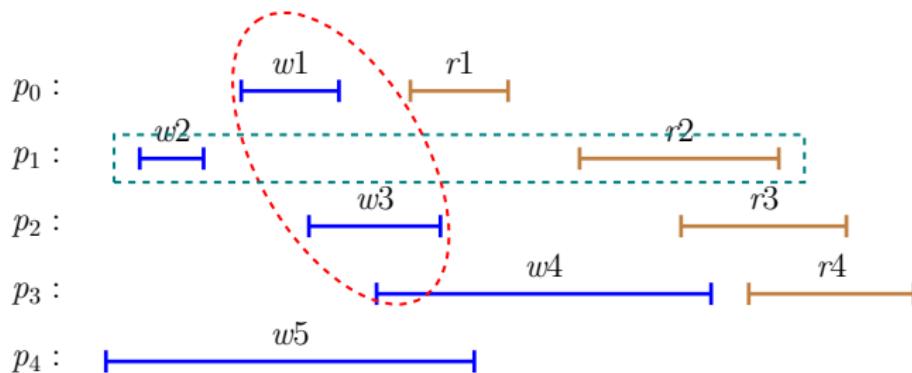
k -Atomicity = 实时序 + k -读写语义

[Aiyer, Alvisi, and Bazzi, 2005] [Taubenfeld, 2013]



k -Atomicity = 实时序 + k -读写语义

[Aiyer, Alvisi, and Bazzi, 2005] [Taubenfeld, 2013]



3-Atomicity : $w5 \quad w2 \quad w1 \quad r1 \quad w3 \quad w4 \quad r2 \quad r3 \quad r4$

定义 (k -AV (k -Atomicity Verification) 判定问题)

实例: 系统执行 e (不允许写重复值)、参数 k

问题: 该执行 e 是否满足 k -Atomicity?

\mathcal{I} -Atomicity = i -实时序 + 读写语义

\mathcal{I} : Inversions

\mathcal{I} -Atomicity = i -实时序 + 读写语义

\mathcal{I} : Inversions

$$f(\{\text{inversions}\}) \leq i$$

\mathcal{I} -Atomicity = i -实时序 + 读写语义

\mathcal{I} : Inversions

$$f(\{\text{inversions}\}) \leq i$$

定义框架





协议量化分析

[Lee and Welch, 2005] [Bailis et al., 2012] [Chatterjee and Golab, 2017]



协议量化分析

[Lee and Welch, 2005] [Bailis et al., 2012] [Chatterjee and Golab, 2017]

PA2AM: Probabilistically-Atomic 2-Atomicity

-  Aiyer, Amitanand, Lorenzo Alvisi, and Rida A. Bazzi (2005). "On the Availability of Non-strict Quorum Systems". In: *Proceedings of the 19th International Conference on Distributed Computing*. DISC '05. Springer-Verlag, pp. 48–62.
-  Anderson, Eric et al. (2010). "What Consistency Does Your Key-value Store Actually Provide?" In: *Proceedings of the Sixth International Conference on Hot Topics in System Dependability*. HotDep'10. Vancouver, BC, Canada: USENIX Association, pp. 1–16. URL: <http://dl.acm.org/citation.cfm?id=1924908.1924919>.
-  Attiya, Hagit et al. (2016). "Specification and complexity of collaborative text editing". In: *Proceedings of the 2016 ACM Symposium on Principles of Distributed Computing*. PODC '16. ACM, pp. 259–268.
-  Bailis, Peter et al. (2012). "Probabilistically Bounded Staleness for Practical Partial Quorums". In: *Proc. VLDB Endow.* 5.8, pp. 776–787.
-  Bouajjani, Ahmed et al. (2017). "On Verifying Causal Consistency". In: *Proceedings of the 44th ACM SIGPLAN Symposium on Principles of Programming Languages*. POPL 2017. Paris, France: ACM, pp. 626–638. ISBN: 978-1-4503-4660-3. DOI: 10.1145/3009837.3009888. URL: <http://doi.acm.org/10.1145/3009837.3009888>.

-  Brzezinski, J, C Sobaniec, and D Wawrzyniak (2004). "From session causality to causal consistency". In: *Proceedings of the 12th Euromicro Conference on Parallel, Distributed and Network-Based Processing*, pp. 152–158.
-  Burckhardt, Sebastian et al. (2014). "Replicated Data Types: Specification, Verification, Optimality". In: *Proceedings of the 41st ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*. POPL '14. San Diego, California, USA: ACM, pp. 271–284. ISBN: 978-1-4503-2544-8. DOI: 10.1145/2535838.2535848. URL: <http://doi.acm.org/10.1145/2535838.2535848>.
-  Cantin, Jason F., Mikko H. Lipasti, and James E. Smith (2005). "The complexity of verifying memory coherence and consistency". In: *IEEE Transactions on Parallel and Distributed Systems* 16.7, pp. 663–671.
-  Chatterjee, Shankha and Wojciech Golab (2017). "Brief Announcement: A Probabilistic Performance Model and Tuning Framework for Eventually Consistent Distributed Storage Systems". In: *Proceedings of the ACM Symposium on Principles of Distributed Computing*. PODC '17. Washington, DC, USA: ACM, pp. 259–261. ISBN: 978-1-4503-4992-5. DOI: 10.1145/3087801.3087850. URL: <http://doi.acm.org/10.1145/3087801.3087850>.

-  DeCandia, Giuseppe et al. (2007). "Dynamo: Amazon's Highly Available Key-value Store". In: *Proceedings of 21st ACM SIGOPS Symposium on Operating Systems Principles*. SOSP '07. ACM, pp. 205–220.
-  Ellis, C. A. and S. J. Gibbs (1989). "Concurrency Control in Groupware Systems". In: *Proceedings of the 1989 ACM SIGMOD International Conference on Management of Data*. SIGMOD '89. ACM, pp. 399–407.
-  Emmi, Michael and Constantin Enea (2018). "Monitoring Weak Consistency". In: *Computer Aided Verification*. Ed. by Hana Chockler and Georg Weissenbacher. Cham: Springer International Publishing, pp. 487–506. ISBN: 978-3-319-96145-3.
-  Furbach, F. et al. (2014). "Memory Model-Aware Testing - A Unified Complexity Analysis". In: *2014 14th International Conference on Application of Concurrency to System Design*, pp. 92–101.
-  Gibbons, Phillip B. and Ephraim Korach (1997). "Testing shared memories". In: *SIAM J. Comput.* 26.4, pp. 1208–1244.
-  Golab, Wojciech, Jeremy Hurwitz, and Xiaozhou (Steve) Li (2013). "On the k-Atomicity-Verification Problem". In: *Proceedings of the 33rd IEEE International Conference on Distributed Computing Systems*. ICDCS '13. IEEE Computer Society, pp. 591–600.

-  Golab, Wojciech, Xiaozhou Li, and Mehul A. Shah (2011). "Analyzing Consistency Properties for Fun and Profit". In: *Proceedings of the 30th Annual ACM SIGACT-SIGOPS Symposium on Principles of Distributed Computing*. PODC '11. ACM, pp. 197–206.
-  Golab, Wojciech et al. (2015). "Computing Weak Consistency in Polynomial Time: [Extended Abstract]". In: *Proceedings of the 2015 ACM Symposium on Principles of Distributed Computing*. PODC '15. ACM, pp. 395–404.
-  Golab, Wojciech et al. (2018). "Computing k -Atomicity in Polynomial Time". In: 47, pp. 420–455.
-  Herlihy, Maurice P. and Jeannette M. Wing (1990). "Linearizability: A Correctness Condition for Concurrent Objects". In: *ACM Trans. Program. Lang. Syst.* 12.3, pp. 463–492. ISSN: 0164-0925. DOI: 10.1145/78969.78972. URL: <http://doi.acm.org/10.1145/78969.78972>.
-  Lamport, Leslie (1986). "On interprocess communication (Part II: algorithms)". In: *Distrib. Comput.* 1.2, pp. 86–101.
-  Lee, Hyunyoung and Jennifer L. Welch (2005). "Randomized Registers and Iterative Algorithms". In: *Distrib. Comput.* 17.3, pp. 209–221.

-  Liskov, Barbara and Stephen Zilles (1974). "Programming with Abstract Data Types". In: *Proceedings of the ACM SIGPLAN Symposium on Very High Level Languages*. Santa Monica, California, USA: ACM, pp. 50–59. DOI: 10.1145/800233.807045. URL: <http://doi.acm.org/10.1145/800233.807045>.
-  Nichols, David A. et al. (1995). "High-latency, Low-bandwidth Windowing in the Jupiter Collaboration System". In: *Proceedings of the 8th Annual ACM Symposium on User Interface and Software Technology*. UIST '95. ACM, pp. 111–120.
-  Shapiro, Marc et al. (2011a). *A comprehensive study of Convergent and Commutative Replicated Data Types*. Research Report RR-7506. Inria – Centre Paris-Rocquencourt ; INRIA, p. 50. URL: <https://hal.inria.fr/inria-00555588>.
-  Shapiro, Marc et al. (2011b). "Conflict-free Replicated Data Types". In: *Proceedings of the 13th International Conference on Stabilization, Safety, and Security of Distributed Systems*. SSS'11. Springer-Verlag, pp. 386–400.
-  Taubenfeld, Gadi (2013). "Weak Read/Write Registers". In: *Distributed Computing and Networking*. Ed. by Davide Frey et al. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 423–427. ISBN: 978-3-642-35668-1.
-  Terry, Douglas B. et al. (1994). "Session Guarantees for Weakly Consistent Replicated Data". In: *Proceedings of the 3rd International Conference on Parallel and Distributed Information Systems*. PDIS '94, pp. 140–149.



Xu, Yi, Chengzheng Sun, and Mo Li (2014). "Achieving Convergence in Operational Transformation: Conditions, Mechanisms and Systems". In: *Proceedings of the 17th ACM Conference on Computer Supported Cooperative Work*. CSCW '14. ACM, pp. 505–518.

Thank
You!

