Coq, Chapar, and Coq Again

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ICS, NJU

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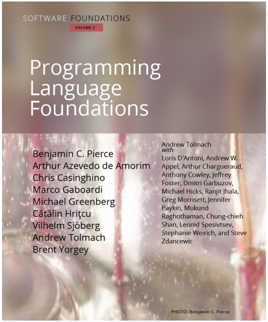
The Coq Proof Assistant



"Software Foundations"

Chapar: Certified Causally Consistent Distributed Key-Value Stores

Mohsen Lesani Christian J. Bell Adam Chlipala
Massachusetts Institute of Technology, USA
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Chapar@POPL'2016

Chapar: Certified Causally Consistent Distributed Key-Value Stores

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"A framework for modular verification of causal consistency for replicated key-value store implementations and their client programs."

$\mathbb{P}$: Client Program

$\mathbb{I}$: KV Store Implementation

$\mathbb{P}$: Client Program

Causally Content

$\mathbb{I}$: KV Store Implementation

Causal Consistency

$\mathbb{P}$: Client Program

Causally Content

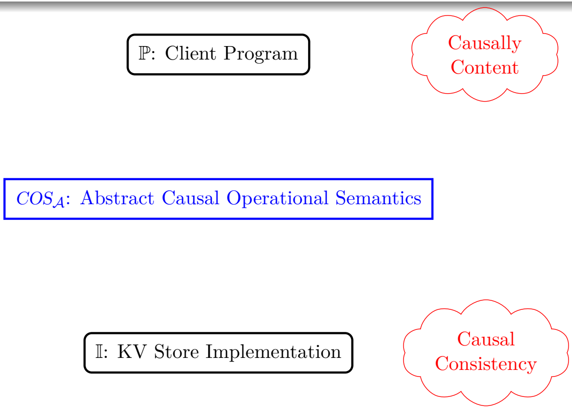
$COS_{\mathcal{A}}$: Abstract Causal Operational Semantics

$\mathbb{I}$: KV Store Implementation

Causal Consistency

Definition (Causally Content)

A client program is causally content if it avoids assertion failures when executed with $COS_{\mathcal{A}}$.



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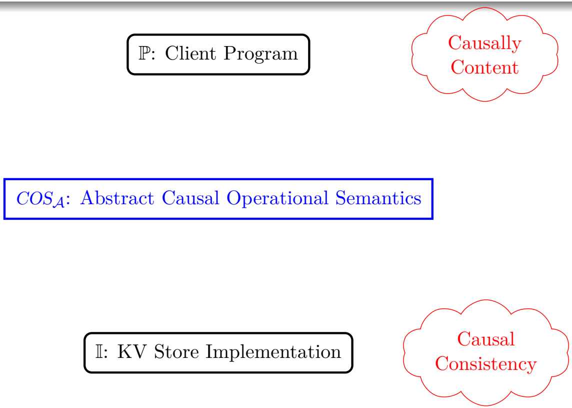

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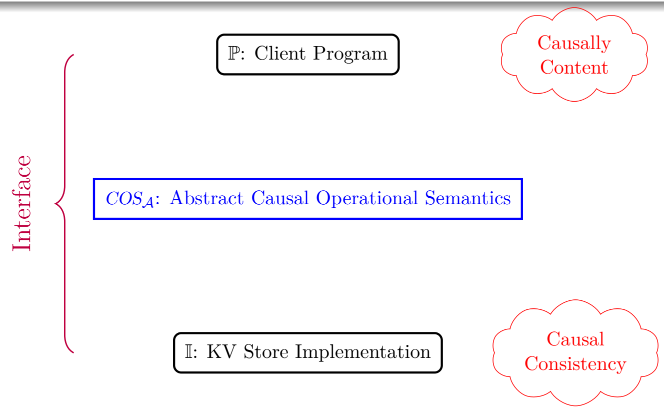

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Interface

$\mathbb{P}$: Client Program

$COS_{\mathcal{A}}$: Abstract Causal Operational Semantics

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Interface

$\mathbb{P}$: Client Program

$COS_{\mathcal{A}}$: Abstract Causal Operational Semantics

$\mathbb{I}$: KV Store Implementation

$\mathbb{P}$: get, put

$COS_{\mathcal{A}}$: get, put, update

$\mathbb{I}$: get, put, guard, update

$\mathbb{P}$: Client Program

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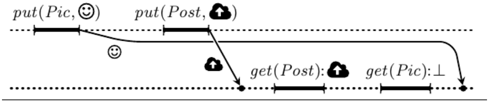

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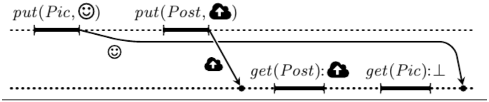

Program 1 (p_1): Uploading a photo and posting a status

$0 \rightarrow$	**Alice**
$put(Pic, ☺)$;	▷ uploads a new photo
$put(Post, ☁)$	▷ announces it to her friends
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$post \leftarrow get(Post)$;	▷ checks Alice's post
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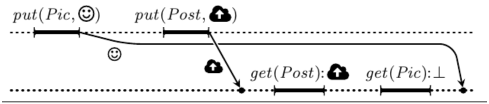

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$$assert(post \neq \perp \implies photo \neq \perp)$$

$\boxed{\mathbb{P}\text{: Client Program}}$

$\boxed{\mathit{COS}_{\mathcal{A}}\text{: Abstract Causal Operational Semantics}}$

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Abstract:

without referring to the details of specific implementations;

do not involving message passing.

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labelled transition system

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without referring to the details of specific implementations;
do not involving message passing.

Operational:

labelled transition system
executable (like TLA+)

Causal:

explicitly track happens-before dependencies

$$W_{\mathcal{A}} \ : \ N \rightarrow (S \times D \times U \times A \times M)$$

$$W_{\mathcal{A}} \ : \ N \to (S \times D \times U \times A \times M)$$

$c \ : C$ Clock

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$d \;: D = \mathcal{P}(N \times C)$ — Dependencies

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$u \;:\; U = (K \times V \times D)^*$	Updates
$a \;:\; A = N \to C$	Applied
$m : M = K \to (V \times N \times C \times D)$	Store

PUT

$$\frac{\begin{array}{cc} u' = u +\!\!+ [(k,v,d)] & a' = a[n \mapsto a(n)+1] \\ m' = m[k \mapsto (v, n, |u'|, \emptyset)] & d' = d \cup \{(n, |u'|)\} \end{array}}{\begin{array}{c} W_{\mathcal{A}}[n \mapsto (put(k,v); s, d, u, a, m)] \\ \xrightarrow{n,\ |u'|\ \triangleright\ put(k,v)}_{\mathcal{A}} \\ W_{\mathcal{A}}[n \mapsto (s, d', u', a', m')] \end{array}}$$

GET

$$\frac{\begin{array}{c} m(k) = (v, n'', c'', d'') \\ d' = \begin{cases} d \cup \{(n'', c'')\} \cup d'' & \text{if } n'' \neq n_0 \\ d & \text{otherwise} \end{cases} \end{array}}{\begin{array}{c} W_{\mathcal{A}}[n \mapsto (x \leftarrow get(k); s, d, u, a, m)] \\ \xrightarrow{n'',\ c'',\ n\ \triangleright\ get(k):\ v}_{\mathcal{A}} \\ W_{\mathcal{A}}[n \mapsto (s[x := v], d', u, a, m)] \end{array}}$$

UPDATE

$$\frac{\begin{array}{cc} a_1(n_2) < |u_2| & u_2[\![a_1(n_2)]\!] = (k, v, d) \\ \bigwedge_{(n,c)\in d} c \leq a_1(n) & a_1' = a_1[n_2 \mapsto a_1(n_2)+1] \\ \multicolumn{2}{c}{m_1' = m_1[k \mapsto (v, n_2, a_1'(n_2), d)]} \end{array}}{\begin{array}{c} W_{\mathcal{A}}[n_1 \mapsto (s_1, d_1, u_1, a_1, m_1)][n_2 \mapsto (s_2, d_2, u_2, a_2, m_2)] \\ \xrightarrow{n_2,\ a_1'(n_2),\ n_1\ \triangleright\ update(k,v)}_{\mathcal{A}} \\ W_{\mathcal{A}}[n_1 \mapsto (s_1, d_1, u_1, a_1', m_1')][n_2 \mapsto (s_2, d_2, u_2, a_2, m_2)] \end{array}}$$

ASSERTFAIL

$$\frac{}{C[n \mapsto (assertfail, d, u, a, m)] \xrightarrow{assertfail}_{\mathcal{A}} C[n \mapsto (skip, d, u, a, m)]}$$

GET

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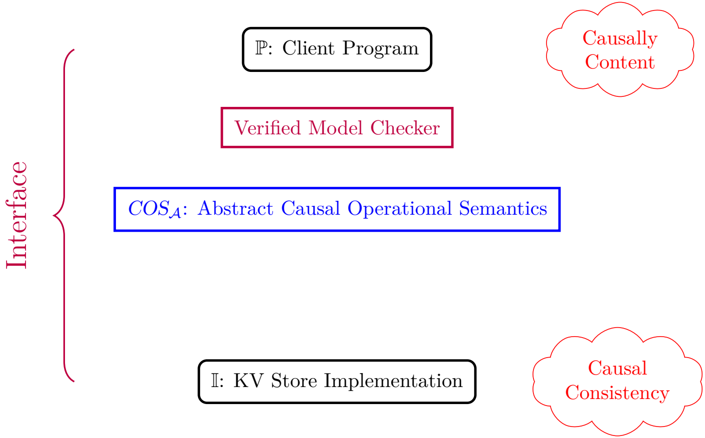
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Instrumented Concrete Operational Semantics

Concrete Operational Semantics

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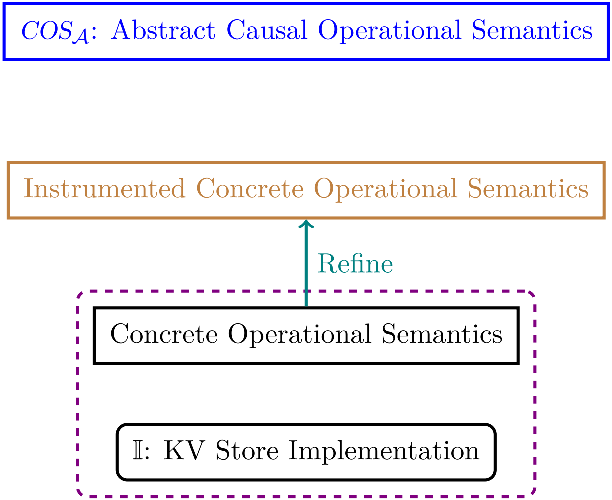
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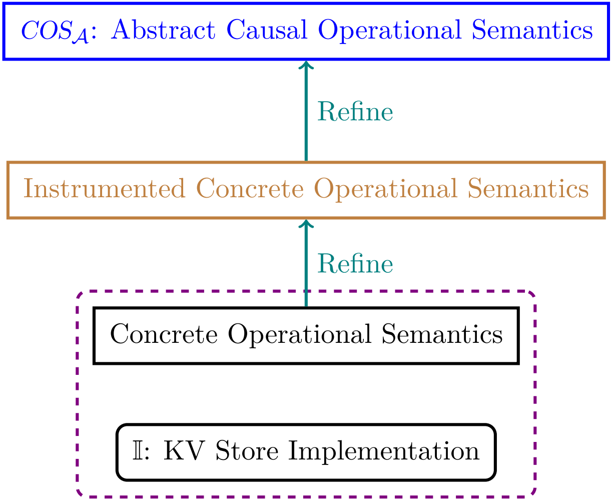
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$$W_{\mathcal{C}} \;:\; H \times T$$

Concrete Operational Semantics

$\mathbb{I}$: KV Store Implementation

$$W_{\mathcal{C}} \; : \; H \times T$$

$$h \; : H = N \to (S \times \text{State}(V)) \qquad \text{Hosts}$$

$$t \; : T = \mathcal{P}(M) \qquad \text{Transit}$$

Concrete Operational Semantics

$\mathbb{I}$: KV Store Implementation

$$W_{\mathcal{C}} \; : \; H \times T$$

h	$: H = N \to (S \times \mathrm{State}(V))$	Hosts
t	$: T = \mathcal{P}(M)$	Transit
m	$: M = N \times K \times V \times \mathrm{Update}(V)$	Message
σ	$: \mathrm{State}(V)$	Alg State
u	$: \mathrm{Update}(V)$	Alg Update

PUT

$$\frac{\mathsf{put}(V, n, \sigma, k, v) \rightsquigarrow^* (\sigma', u) \qquad t' = t \cup \{(n', k, v, u) \mid n' \in N \setminus \{n\}\}}{(h[n \mapsto (put(k,v); s, \sigma)], t) \xrightarrow{n \rhd\ put(k,v)}_{\mathcal{C}(\mathbb{I})} (h[n \mapsto (s, \sigma')], t')}$$

GET

$$\frac{\mathsf{get}(V, n, \sigma, k) \rightsquigarrow^* (v, \sigma')}{(h[n \mapsto (x \leftarrow get(k); s, \sigma)], t) \xrightarrow{n \rhd\ get(k):\ v}_{\mathcal{C}(\mathbb{I})} (h[n \mapsto (s[x := v], \sigma')], t)}$$

UPDATE

$$\frac{\mathsf{guard}(V, n, \sigma, k, v, u) \rightsquigarrow^* true \qquad \mathsf{update}(V, n, \sigma, k, v, u) \rightsquigarrow^* \sigma'}{(h[n \mapsto (s, \sigma)], t \cup \{(n, k, v, u)\}) \xrightarrow{n \rhd\ update(k,v)}_{\mathcal{C}(\mathbb{I})} (h[n \mapsto (s, \sigma')], t)}$$

ASSERTFAIL

$$\frac{}{(h[n \mapsto (assertfail, \sigma)], t) \xrightarrow{assertfail}_{\mathcal{C}(\mathbb{I})} (h[n \mapsto (skip, \sigma)], t)}$$

GET

$$\frac{\mathsf{get}(V, n, \sigma, k) \rightsquigarrow^* (v, \sigma')}{\begin{array}{c}(h[n \mapsto (x \leftarrow get(k); s, \sigma)], t) \\ \xrightarrow{n \triangleright\ get(k):\ v}_{\mathcal{C}(\mathbb{I})} \\ (h[n \mapsto (s[x := v], \sigma')], t)\end{array}}$$

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Parametric on the implementation $\mathbb{I}$

$$\textsc{Get}\quad \frac{\mathsf{get}(V, n, \sigma, k) \rightsquigarrow^* (v, \sigma')}{(h[n \mapsto (x \leftarrow get(k); s, \sigma)], t) \xrightarrow{n \triangleright\, get(k):\, v}_{\mathcal{C}(\mathbb{I})} (h[n \mapsto (s[x := v], \sigma')], t)}$$

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Operational: Model the executions of the implementation $\mathbb{I}$

$COS_{\mathcal{A}}$: Abstract Causal Operational Semantics

Instrumented Concrete Operational Semantics

Concrete Operational Semantics

$\boxed{COS_{\mathcal{A}}\text{: Abstract Causal Operational Semantics}}$

$\boxed{\text{Instrumented Concrete Operational Semantics}}$

$\boxed{\text{Concrete Operational Semantics}}$

Parametric on the implementation $\mathbb{I}$

$COS_{\mathcal{A}}$: Abstract Causal Operational Semantics

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Parametric on the implementation $\mathbb{I}$

$\rightarrow_{\mathcal{I}(\mathbb{I})}$ is similar to non-instrumented $\rightarrow_{\mathcal{C}(\mathbb{I})}$

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Parametric on the implementation $\mathbb{I}$

$\rightarrow_{\mathcal{I}(\mathbb{I})}$ is similar to non-instrumented $\rightarrow_{\mathcal{C}(\mathbb{I})}$

"Uniquely identify put *operations to track causal dependencies between them."*

$\boxed{\color{blue}{COS_{\mathcal{A}}}\text{: Abstract Causal Operational Semantics}}$

$\uparrow$ Refine

$\boxed{\text{Instrumented Concrete Operational Semantics}}$

$\boxed{\text{Concrete Operational Semantics}}$

$COS_{\mathcal{A}}$: Abstract Causal Operational Semantics

Well-Reception $\uparrow$ Refine

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$COS_{\mathcal{A}}$: Abstract Causal Operational Semantics

Well-Reception | Refine

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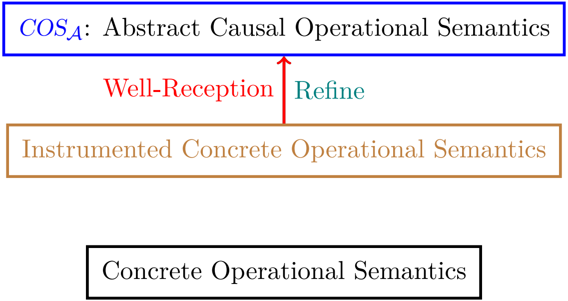
Concrete Operational Semantics

Theorem (Sufficiency of Well-Reception)

Every well-receptive implementation is causally consistent.



Definition (Well-Reception)

An implementation is *well-receptive* iff there exists a *function* Rec for the implementation such that th four conditions InitCond, StepCond, CauseCond, and SeqCond are satisfied.

$$c \leftarrow Rec(\sigma, n)$$

$$
\begin{array}{l}
\mathsf{WellRec}(\mathbb{I}) \;\triangleq \\
\quad let\ (\mathsf{State}, \_, \_, \_, \_, \_, \_) = \mathbb{I}\ in \\
\quad \exists \mathsf{Rec} \colon (\mathsf{State}(IV), N) \to C \colon \\
\quad let\ \mathsf{Rec}'(W, n', n) = \\
\qquad let\ (H[n' \mapsto (\_, \sigma, \_)], \_) = W\ in \\
\qquad \mathsf{Rec}(\sigma, n)\ in \\
\quad \mathsf{InitCond}(\mathbb{I}, \mathsf{Rec}') \wedge \mathsf{StepCond}(\mathbb{I}, \mathsf{Rec}') \wedge \mathsf{CauseCond}(\mathbb{I}, \mathsf{Rec}') \\
\quad \wedge\ \mathsf{SeqCond}(\mathbb{I})
\end{array}
$$

$$
\begin{array}{l}
\mathsf{InitCond}(\mathbb{I}, \mathsf{Rec}') \;\triangleq\; \forall p, n, n' \colon \\
\quad \mathsf{Rec}'(W_{\mathcal{I}0}(p), n, n') = 0
\end{array}
$$

$$
\begin{array}{l}
\mathsf{StepCond}(\mathbb{I}, \mathsf{Rec}') \;\triangleq\; \forall p, h_{\mathcal{I}}, W_{\mathcal{I}}, l_{\mathcal{I}}, W'_{\mathcal{I}} \colon \\
\quad (W_{\mathcal{I}0}(p) \xrightarrow{h_{\mathcal{I}}}{}^{*}_{\mathcal{I}(\mathbb{I})} W_{\mathcal{I}} \;\wedge\; W_{\mathcal{I}} \xrightarrow{l_{\mathcal{I}}}_{\mathcal{I}(\mathbb{I})} W'_{\mathcal{I}}) \Rightarrow \\
\quad \left\{
\begin{array}{l}
\textsc{Case}\ l_{\mathcal{I}} = n, \_ \triangleright put(\_, \_, \_) \colon \_, \_ \\
\quad \mathsf{Rec}'(W'_{\mathcal{I}}, n, n) = \mathsf{Rec}'(W_{\mathcal{I}}, n, n) + 1 \wedge \\
\quad \forall n' \colon n' \neq n \Rightarrow \mathsf{Rec}'(W'_{\mathcal{I}}, n, n') = \mathsf{Rec}'(W_{\mathcal{I}}, n, n') \\
\textsc{Case}\ l_{\mathcal{I}} = n, \_ \triangleright get(\_, \_) \colon \_, \_ \\
\quad \forall n' \colon \mathsf{Rec}'(W'_{\mathcal{I}}, n, n') = \mathsf{Rec}'(W_{\mathcal{I}}, n, n') \\
\textsc{Case}\ l_{\mathcal{I}} = n', c', n \triangleright update(\_, \_, \_, \_) \colon \_ \\
\quad \mathsf{Rec}'(W_{\mathcal{I}}, n, n') + 1 = c' \wedge \\
\quad \mathsf{Rec}'(W'_{\mathcal{I}}, n, n') = \mathsf{Rec}'(W_{\mathcal{I}}, n, n') + 1 \wedge \\
\quad \forall n'' \colon n'' \neq n' \Rightarrow \mathsf{Rec}'(W'_{\mathcal{I}}, n, n'') = \mathsf{Rec}'(W_{\mathcal{I}}, n, n'')
\end{array}
\right.
\end{array}
$$

$$
\begin{array}{l}
\mathsf{CauseCond}(\mathbb{I}, \mathsf{Rec}') \;\triangleq\; \forall p, h_{\mathcal{I}}, W_{\mathcal{I}}, l_{\mathcal{I}}, W'_{\mathcal{I}}, l''_{\mathcal{I}} \colon \\
\quad (W_{\mathcal{I}0}(p) \xrightarrow{h_{\mathcal{I}}}{}^{*}_{\mathcal{I}(\mathbb{I})} W_{\mathcal{I}} \;\wedge\; W_{\mathcal{I}} \xrightarrow{l_{\mathcal{I}}}_{\mathcal{I}(\mathbb{I})} W'_{\mathcal{I}} \\
\qquad \wedge\ \mathsf{LIsUpdate}(l_{\mathcal{I}}) \wedge \\
\qquad let\ \_, \_, n \triangleright update(\_, \_, \_, m) \colon \_ = l_{\mathcal{I}} \\
\qquad\quad (\_, \_, \_, \_, \_, \_, l'_{\mathcal{I}}) = m\ in \\
\qquad \mathsf{LIsPut}(l''_{\mathcal{I}}) \;\wedge\; l''_{\mathcal{I}} \curvearrowright_{h_{\mathcal{I}}} l'_{\mathcal{I}}) \Rightarrow \\
\quad let\ n'', c'' \triangleright put(\_, \_, \_) \colon \_, \_ = l''_{\mathcal{I}}\ in \\
\quad \mathsf{Rec}'(W_{\mathcal{I}}, n, n'') \geq c''
\end{array}
$$

$$
\begin{array}{l}
\mathsf{SeqCond}(\mathbb{I}) \;\triangleq\; \forall p, h_{\mathcal{C}}, W_{\mathcal{C}} \colon \\
\quad W_{\mathcal{C}0}(p) \xrightarrow{h_{\mathcal{C}}}{}^{*}_{\mathcal{C}(\mathbb{I})} W_{\mathcal{C}} \;\Rightarrow\; \exists W'_{\mathcal{S}} \colon W_{\mathcal{S}0} \xrightarrow{\mathsf{Eff}(h_{\mathcal{C}})}{}^{*}_{\mathcal{S}} W'_{\mathcal{S}}
\end{array}
$$

Causal memory: definitions, implementation, and programming

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DC'1995

Stronger Semantics for Low-Latency Geo-Replicated Storage

Wyatt Lloyd*, Michael J. Freedman*, Michael Kaminsky†, and David G. Andersen‡
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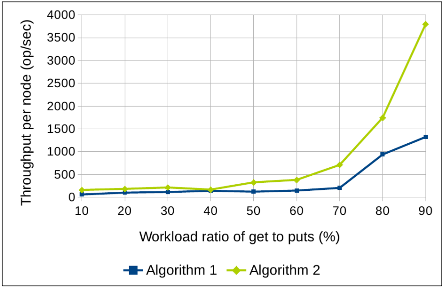
NSDI'2013


Throughput per node (op/sec)
4000
3500
3000
2500
2000
1500
1000
500
0
10 20 30 40 50 60 70 80 90
Workload ratio of get to puts (%)
Algorithm 1
Algorithm 2

Chapar: Certified Causally Consistent Distributed Key-Value Stores

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Chapar for CC variants

Causal Consistency: Beyond Memory

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PPoPP'2016

Chapar for Op-based CRDT

Verifying Strong Eventual Consistency in Distributed Systems

VICTOR B. F. GOMES, University of Cambridge, UK
MARTIN KLEPPMANN, University of Cambridge, UK
DOMINIC P. MULLIGAN, University of Cambridge, UK
ALASTAIR R. BERESFORD, University of Cambridge, UK

OOPSLA'2017

Chapar for State-based CRDT

Formal Specification and Verification of CRDTs

Peter Zeller, Annette Bieniusa, and Arnd Poetzsch-Heffter

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FORTE'2014

Coq for the equivalence between Op-based CRDT and State-based CRDT

A comprehensive study of Convergent and Commutative Replicated Data Types

Marc Shapiro, Nuno Preguiça, Carlos Baquero, Marek Zawirski

TR'2011

Coq for the "$vis + ar$" framework

syncope: Automatic Enforcement of Distributed Consistency Guarantees

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TR'2017


THANK YOU