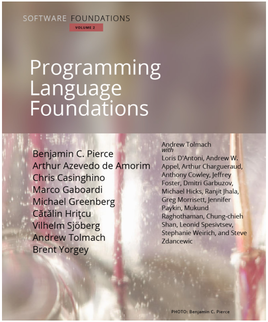
Coq, Chapar, and Coq Again

Hengfeng Wei

ICS, NJU

April 28, 2019

The Coq Proof Assistant



"Software Foundations"

Chapar: Certified Causally Consistent Distributed Key-Value Stores

Mohsen Lesani   Christian J. Bell   Adam Chlipala
Massachusetts Institute of Technology, USA
{lesani, cjbell, adamc}@mit.edu

Chapar@POPL'2016

Chapar: Certified Causally Consistent Distributed Key-Value Stores

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"A framework for modular verification of causal consistency for replicated key-value store implementations and their client programs."

$\mathbb{P}$: Client Program

$\mathbb{I}$: KV Store Implementation

ℙ: Client Program

Causally Content

𝕀: KV Store Implementation

Causal Consistency

$\mathbb{P}$: Client Program

Causally Content

$COS_{\mathcal{A}}$: Abstract Causal Operational Semantics

$\mathbb{I}$: KV Store Implementation

Causal Consistency

Definition (Causally Content)

A client program is causally content if it avoids assertion failures when executed with $COS_{\mathcal{A}}$.

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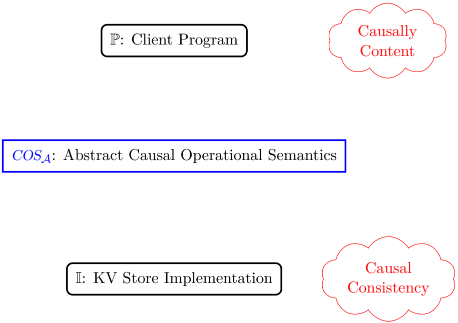

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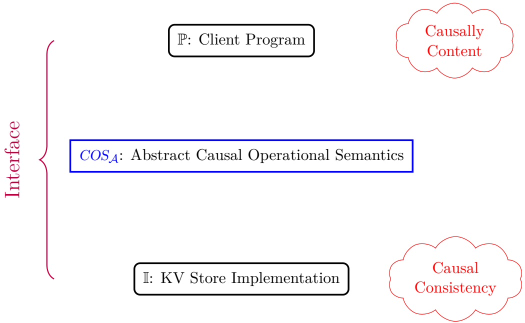

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$\mathbb{P}$: Client Program

Interface

$COS_{\mathcal{A}}$: Abstract Causal Operational Semantics

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$\mathbb{P}$:

get, put

$COS_{\mathcal{A}}$:

get, put, update

$\mathbb{I}$:

get, put, guard, update

$\mathbb{P}$: Client Program

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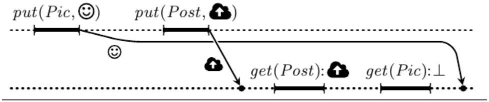

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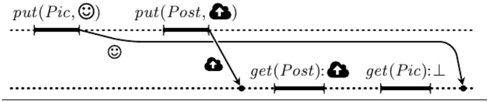
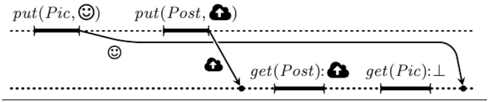


Program 1 (p_1): Uploading a photo and posting a status

$0 \rightarrow$ **Alice**
- $put(Pic,$ ☺$);$ ▷ uploads a new photo
- $put(Post,$ ☁$)$ ▷ announces it to her friends

$1 \rightarrow$ **Bob**
- $post \leftarrow get(Post);$ ▷ checks Alice's post
- $photo \leftarrow get(Pic);$ ▷ then loads her photo
- $assert(post =$ ☁ $\Rightarrow photo \neq \perp)$

$\mathbb{P}$: Client Program



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$$assert(post \neq \perp \implies photo \neq \perp)$$

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Abstract:

without referring to the details of specific implementations;

do not involving message passing.

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labelled transition system

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Operational:

labelled transition system

executable (like TLA+)

Causal:

explicitly track happens-before dependencies

$$W_{\mathcal{A}} \; : \; N \rightarrow (S \times D \times U \times A \times M)$$

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$u \;: U = (K \times V \times D)^*$	Updates
$a \;: A = N \rightarrow C$	Applied
$m : M = K \rightarrow (V \times N \times C \times D)$	Store

$$\textsc{Put}\quad \frac{\begin{array}{cc} u' = u \mathbin{++} [(k, v, d)] & a' = a[n \mapsto a(n) + 1] \\ m' = m[k \mapsto (v, n, |u'|, \emptyset)] & d' = d \cup \{(n, |u'|)\} \end{array}}{\begin{array}{c} W_{\mathcal{A}}[n \mapsto (put(k,v); s, d, u, a, m)] \\ \xrightarrow{n,\ |u'|\ \triangleright\ put(k,v)}_{\mathcal{A}} \\ W_{\mathcal{A}}[n \mapsto (s, d', u', a', m')] \end{array}}$$

$$\textsc{Get}\quad \frac{\begin{array}{c} m(k) = (v, n'', c'', d'') \\ d' = \begin{cases} d \cup \{(n'', c'')\} \cup d'' & \text{if } n'' \neq n_0 \\ d & \text{otherwise} \end{cases} \end{array}}{\begin{array}{c} W_{\mathcal{A}}[n \mapsto (x \leftarrow get(k); s, d, u, a, m)] \\ \xrightarrow{n'',\ c'',\ n\ \triangleright\ get(k):\ v}_{\mathcal{A}} \\ W_{\mathcal{A}}[n \mapsto (s[x := v], d', u, a, m)] \end{array}}$$

$$\textsc{Update}\quad \frac{\begin{array}{cc} a_1(n_2) < |u_2| & u_2[\![a_1(n_2)]\!] = (k, v, d) \\ \bigwedge_{(n,c) \in d} c \leq a_1(n) & a_1' = a_1[n_2 \mapsto a_1(n_2) + 1] \\ \multicolumn{2}{c}{m_1' = m_1[k \mapsto (v, n_2, a_1'(n_2), d)]} \end{array}}{\begin{array}{c} W_{\mathcal{A}}[n_1 \mapsto (s_1, d_1, u_1, a_1, m_1)][n_2 \mapsto (s_2, d_2, u_2, a_2, m_2)] \\ \xrightarrow{n_2,\ a_1'(n_2),\ n_1\ \triangleright\ update(k,v)}_{\mathcal{A}} \\ W_{\mathcal{A}}[n_1 \mapsto (s_1, d_1, u_1, a_1', m_1')][n_2 \mapsto (s_2, d_2, u_2, a_2, m_2)] \end{array}}$$

$$\textsc{AssertFail}\quad \frac{}{C[n \mapsto (assertfail, d, u, a, m)] \xrightarrow{assertfail}_{\mathcal{A}} C[n \mapsto (skip, d, u, a, m)]}$$

Get

$$\frac{m(k) = (v, n'', c'', d'') \qquad d' = \begin{cases} d \cup \{(n'', c'')\} \cup d'' & \text{if } n'' \neq n_0 \\ d & \text{otherwise} \end{cases}}{W_{\mathcal{A}}[n \mapsto (x \leftarrow get(k); s, d, u, a, m)] \xrightarrow{n'', c'', n \,\triangleright\, get(k):\, v}_{\mathcal{A}} W_{\mathcal{A}}[n \mapsto (s[x := v], d', u, a, m)]}$$

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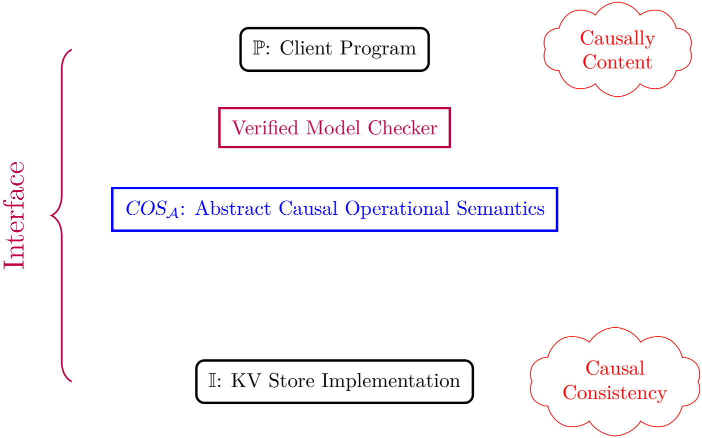
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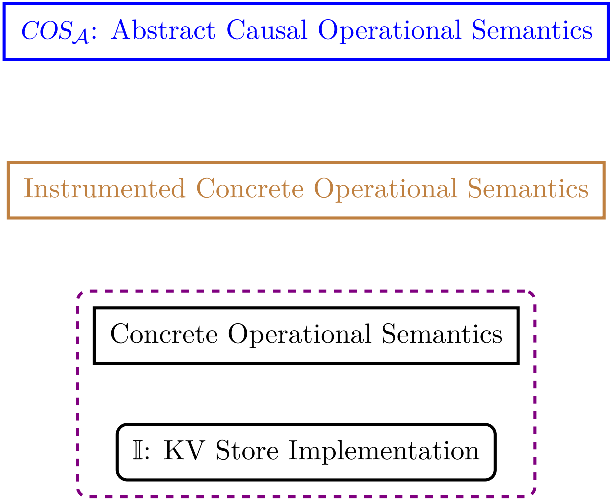
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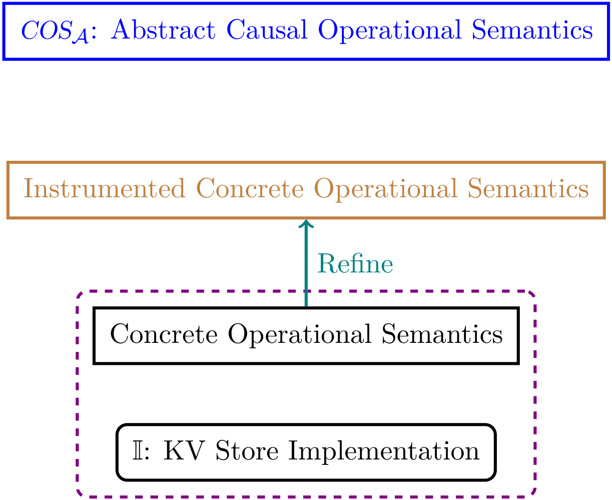
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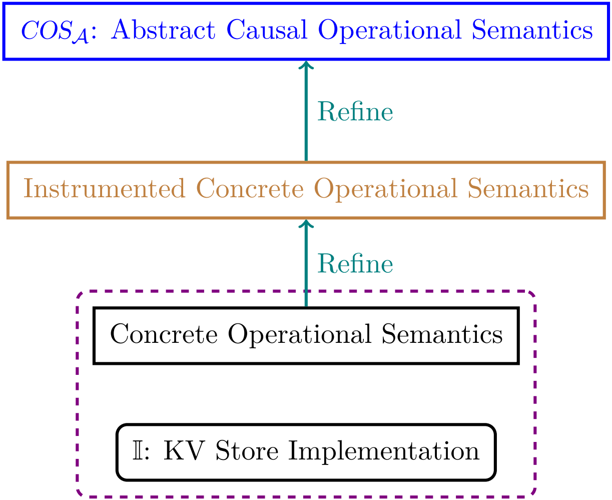
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Concrete Operational Semantics

$\mathbb{I}$: KV Store Implementation

$$W_{\mathcal{C}} \ : \ H \times T$$

$h \ : H = N \rightarrow (S \times \text{State}(V))$ Hosts

$t \ : T = \mathcal{P}(M)$ Transit

Concrete Operational Semantics

$\mathbb{I}$: KV Store Implementation

$$W_{\mathcal{C}} : H \times T$$

h	$: H = N \to (S \times \mathrm{State}(V))$	Hosts
t	$: T = \mathcal{P}(M)$	Transit
m	$: M = N \times K \times V \times \mathrm{Update}(V)$	Message
σ	$: \mathrm{State}(V)$	Alg State
u	$: \mathrm{Update}(V)$	Alg Update

$$\textsc{Put}\quad \frac{\mathsf{put}(V, n, \sigma, k, v) \rightsquigarrow^* (\sigma', u) \qquad t' = t \cup \{(n', k, v, u) \mid n' \in N \setminus \{n\}\}}{(h[n \mapsto (put(k, v); s, \sigma)], t) \xrightarrow{n \rhd\ put(k,v)}_{\mathcal{C}(\mathbb{I})} (h[n \mapsto (s, \sigma')], t')}$$

$$\textsc{Get}\quad \frac{\mathsf{get}(V, n, \sigma, k) \rightsquigarrow^* (v, \sigma')}{(h[n \mapsto (x \leftarrow get(k); s, \sigma)], t) \xrightarrow{n \rhd\ get(k):\ v}_{\mathcal{C}(\mathbb{I})} (h[n \mapsto (s[x := v], \sigma')], t)}$$

$$\textsc{Update}\quad \frac{\mathsf{guard}(V, n, \sigma, k, v, u) \rightsquigarrow^* true \qquad \mathsf{update}(V, n, \sigma, k, v, u) \rightsquigarrow^* \sigma'}{(h[n \mapsto (s, \sigma)], t \cup \{(n, k, v, u)\}) \xrightarrow{n \rhd\ update(k,v)}_{\mathcal{C}(\mathbb{I})} (h[n \mapsto (s, \sigma')], t)}$$

$$\textsc{AssertFail}\quad \frac{}{(h[n \mapsto (assertfail, \sigma)], t) \xrightarrow{assertfail}_{\mathcal{C}(\mathbb{I})} (h[n \mapsto (skip, \sigma)], t)}$$

$$\textsc{Get}\ \frac{\mathsf{get}(V, n, \sigma, k) \rightsquigarrow^{*} (v, \sigma')}{(h[n \mapsto (x \leftarrow get(k); s, \sigma)], t) \xrightarrow{n \triangleright\, get(k):\, v}_{\mathcal{C}(\mathbb{I})} (h[n \mapsto (s[x := v], \sigma')], t)}$$

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Parametric on the implementation $\mathbb{I}$

$$\frac{\text{get}(V, n, \sigma, k) \rightsquigarrow^* (v, \sigma')}{(h[n \mapsto (x \leftarrow get(k); s, \sigma)], t) \xrightarrow{n \triangleright\, get(k):\, v}_{\mathcal{C}(\mathbb{I})} (h[n \mapsto (s[x := v], \sigma')], t)}\ \textsc{Get}$$

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$$\xrightarrow{n \,\triangleright\, get(k):v}_{\mathcal{C}(\mathbb{I})}$$

Operational: Model the executions of the implementation $\mathbb{I}$

$COS_{\mathcal{A}}$: Abstract Causal Operational Semantics

Instrumented Concrete Operational Semantics

Concrete Operational Semantics

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Parametric on the implementation $\mathbb{I}$

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Parametric on the implementation $\mathbb{I}$

$\rightarrow_{\mathcal{I}(\mathbb{I})}$ is similar to non-instrumented $\rightarrow_{\mathcal{C}(\mathbb{I})}$

"Uniquely identify put *operations to track causal dependencies between them."*

$COS_{\mathcal{A}}$: Abstract Causal Operational Semantics

Refine

Instrumented Concrete Operational Semantics

Concrete Operational Semantics

$COS_{\mathcal{A}}$: Abstract Causal Operational Semantics

Well-Reception ↑ Refine

Instrumented Concrete Operational Semantics

Concrete Operational Semantics

$COS_{\mathcal{A}}$: Abstract Causal Operational Semantics

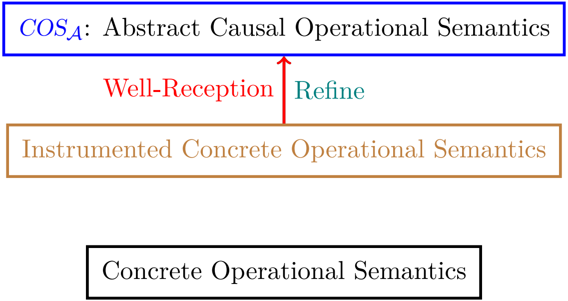
Well-Reception $\uparrow$ Refine

Instrumented Concrete Operational Semantics

Concrete Operational Semantics

Theorem (Sufficiency of Well-Reception)

Every well-receptive implementation is causally consistent.



Definition (Well-Reception)

An implementation is *well-receptive* iff there exists a *function* Rec for the implementation such that th four conditions InitCond, StepCond, CauseCond, and SeqCond are satisfied.

$$c \leftarrow Rec(\sigma, n)$$

$$
\begin{array}{l}
\mathsf{WellRec}(\mathbb{I}) \triangleq \\
\quad \mathit{let}\ (\mathsf{State}, \_, \_, \_, \_, \_, \_) = \mathbb{I}\ \mathit{in} \\
\quad \exists \mathsf{Rec} \colon (\mathsf{State}(IV), N) \to C \colon \\
\quad \mathit{let}\ \mathsf{Rec}'(W, n', n) = \\
\qquad \mathit{let}\ (H[n' \mapsto (\_, \sigma, \_)], \_) = W\ \mathit{in} \\
\qquad \mathsf{Rec}(\sigma, n)\ \mathit{in} \\
\quad \mathsf{InitCond}(\mathbb{I}, \mathsf{Rec}') \wedge \mathsf{StepCond}(\mathbb{I}, \mathsf{Rec}') \wedge \mathsf{CauseCond}(\mathbb{I}, \mathsf{Rec}') \\
\quad \wedge\ \mathsf{SeqCond}(\mathbb{I})
\end{array}
$$

$$
\begin{array}{l}
\mathsf{InitCond}(\mathbb{I}, \mathsf{Rec}') \triangleq \forall p, n, n' \colon \\
\quad \mathsf{Rec}'(W_{\mathcal{I}0}(p), n, n') = 0
\end{array}
$$

$$
\begin{array}{l}
\mathsf{StepCond}(\mathbb{I}, \mathsf{Rec}') \triangleq \forall p, h_{\mathcal{I}}, W_{\mathcal{I}}, l_{\mathcal{I}}, W'_{\mathcal{I}} \colon \\
\quad (W_{\mathcal{I}0}(p) \xrightarrow{h_{\mathcal{I}}}{}^{*}_{\mathcal{I}(\mathbb{I})} W_{\mathcal{I}} \ \wedge\ W_{\mathcal{I}} \xrightarrow{l_{\mathcal{I}}}_{\mathcal{I}(\mathbb{I})} W'_{\mathcal{I}}) \Rightarrow \\
\quad \left\{
\begin{array}{l}
\textsc{Case}\ l_{\mathcal{I}} = n, \_ \triangleright put(\_, \_, \_) \colon \_, \_ \\
\quad \mathsf{Rec}'(W'_{\mathcal{I}}, n, n) = \mathsf{Rec}'(W_{\mathcal{I}}, n, n) + 1\ \wedge \\
\quad \forall n' \colon n' \neq n \Rightarrow \mathsf{Rec}'(W'_{\mathcal{I}}, n, n') = \mathsf{Rec}'(W_{\mathcal{I}}, n, n') \\
\textsc{Case}\ l_{\mathcal{I}} = n, \_ \triangleright get(\_, \_) \colon \_, \_ \\
\quad \forall n' \colon \mathsf{Rec}'(W'_{\mathcal{I}}, n, n') = \mathsf{Rec}'(W_{\mathcal{I}}, n, n') \\
\textsc{Case}\ l_{\mathcal{I}} = n', c', n \triangleright update(\_, \_, \_, \_) \colon \_ \\
\quad \mathsf{Rec}'(W_{\mathcal{I}}, n, n') + 1 = c'\ \wedge \\
\quad \mathsf{Rec}'(W'_{\mathcal{I}}, n, n') = \mathsf{Rec}'(W_{\mathcal{I}}, n, n') + 1\ \wedge \\
\quad \forall n'' \colon n'' \neq n' \Rightarrow \mathsf{Rec}'(W'_{\mathcal{I}}, n, n'') = \mathsf{Rec}'(W_{\mathcal{I}}, n, n'')
\end{array}
\right.
\end{array}
$$

$$
\begin{array}{l}
\mathsf{CauseCond}(\mathbb{I}, \mathsf{Rec}') \triangleq \forall p, h_{\mathcal{I}}, W_{\mathcal{I}}, l_{\mathcal{I}}, W'_{\mathcal{I}}, l''_{\mathcal{I}} \colon \\
\quad (W_{\mathcal{I}0}(p) \xrightarrow{h_{\mathcal{I}}}{}^{*}_{\mathcal{I}(\mathbb{I})} W_{\mathcal{I}} \ \wedge\ W_{\mathcal{I}} \xrightarrow{l_{\mathcal{I}}}_{\mathcal{I}(\mathbb{I})} W'_{\mathcal{I}} \\
\quad\ \wedge\ \mathsf{LIsUpdate}(l_{\mathcal{I}})\ \wedge \\
\quad\ \mathit{let}\ \_, \_, n \triangleright update(\_, \_, \_, m) \colon \_ = l_{\mathcal{I}} \\
\qquad (\_, \_, \_, \_, \_, \_, l'_{\mathcal{I}}) = m\ \mathit{in} \\
\quad\ \mathsf{LIsPut}(l''_{\mathcal{I}})\ \wedge\ l''_{\mathcal{I}} \curvearrowright_{h_{\mathcal{I}}} l'_{\mathcal{I}}) \Rightarrow \\
\quad \mathit{let}\ n'', c'' \triangleright put(\_, \_, \_) \colon \_, \_ = l''_{\mathcal{I}}\ \mathit{in} \\
\quad \mathsf{Rec}'(W_{\mathcal{I}}, n, n'') \geq c''
\end{array}
$$

$$
\begin{array}{l}
\mathsf{SeqCond}(\mathbb{I}) \triangleq \forall p, h_{\mathcal{C}}, W_{\mathcal{C}} \colon \\
\quad W_{\mathcal{C}0}(p) \xrightarrow{h_{\mathcal{C}}}{}^{*}_{\mathcal{C}(\mathbb{I})} W_{\mathcal{C}} \Rightarrow \exists W'_{\mathcal{S}} \colon W_{\mathcal{S}0} \xrightarrow{\mathsf{Eff}(h_{\mathcal{C}})}{}^{*}_{\mathcal{S}} W'_{\mathcal{S}}
\end{array}
$$


THANK YOU