CKY (***Cocke-Kasami-Younger***) and Earley Parsing Algorithms

Dor Altshuler

Context Free Grammar

A Context-free Grammar (CFG) is a 4-tuple:

G = (N, Σ , P, S) where:

- N is a finite set of non-terminal symbols.
- Σ is a finite set of terminal symbols (tokens).
- P is a finite set of productions (rules).

  Production : $(\alpha, \beta) \in P$ will be written $\alpha \rightarrow \beta$

  where α is in N and β is in $(N \cup \Sigma)^*$
- S is the start symbol in N.

Chomsky Normal Form

A context-free grammar where the right side of each production rule is restricted to be either two non terminals or one terminal.

Production can be one of following formats:

- $A \rightarrow \alpha$
- $A \rightarrow BC$

Any CFG can be converted to a weakly equivalent grammar in CNF

Parsing Algorithms

- CFGs are basis for describing (syntactic) structure of NL sentences
- Thus - Parsing Algorithms are core of NL analysis systems
- Recognition vs. Parsing:
  - *Recognition* - deciding the membership in the language
  - *Parsing* – Recognition+ producing a parse tree for it
- Parsing is more “difficult” than recognition (time complexity)
- Ambiguity - an input may have exponentially many parses.

Parsing Algorithms

Top-down vs. bottom-up:

- Top-down: (goal-driven): from the start symbol down.
- Bottom-up: (data-driven): from the symbols up.

Naive vs. dynamic programming:

- Naive: enumerate everything.
- Backtracking: try something, discard partial solutions.
- Dynamic programming: save partial solutions in a table.

Examples:

- CKY: bottom-up dynamic programming.
- Earley parsing: top-down dynamic programming.

CKY (*Cocke-Kasami-Younger)*

- One of the earliest recognition and parsing algorithms
- The standard version of CKY can only recognize languages defined by context-free grammars in Chomsky Normal Form *(CNF).*
- It is also possible to extend the CKY algorithm to handle some grammars which are not in CNF
  - *Harder to understand*
- Based on a "dynamic programming" approach:
  - Build solutions compositionally from sub-solutions
- Uses the grammar directly.

CKY Algorithm

- Considers every possible consecutive subsequence of letters and sets K ∈ T[i,j] if the sequence of letters starting from i to j can be generated from the non-terminal *K*.
- Once it has considered sequences of length 1, it goes on to sequences of length 2, and so on.
- For subsequences of length 2 and greater, it considers every possible partition of the subsequence into two halves, and checks to see if there is some production

  A -> BC such that *B* matches the first half and *C* matches the second half. If so, it records A as matching the whole subsequence.
- Once this process is completed, the sentence is recognized by the grammar if the entire string is matched by the start symbol.

CKY Algorithm

- <u>Observation: </u>any portion of the input string spanning i to j can be split at k, and structure can then be built using sub-solutions spanning i to k and sub-solutions spanning k to j .
- <u>Meaning:</u>

  Solution to problem [i, j] can constructed from solution to sub problem [i, k] and solution to sub problem [k ,j].

CKY Algorithm for Deciding CFL

Consider the grammar *G* given by:

$S \rightarrow \varepsilon \mid AB \mid XB$

$T \rightarrow AB \mid XB$

$X \rightarrow AT$

$A \rightarrow a$

$B \rightarrow b$

CKY Algorithm for Deciding CFL

Consider the grammar *G* given by:

$S \rightarrow \varepsilon \mid AB \mid XB$

$T \rightarrow AB \mid XB$

$X \rightarrow AT$

$A \rightarrow a$

$B \rightarrow b$

1. Is $w = aaabb$ in $L(G)$?
2. Is $w = aaabbb$ in $L(G)$?

CKY Algorithm for Deciding CFL

The algorithm is "bottom-up" in that we start with bottom of derivation tree.

a *a* *a* *b* *b*

$S \rightarrow \varepsilon \mid AB \mid XB$

$T \rightarrow AB \mid XB$

$X \rightarrow AT$

$A \rightarrow a$

$B \rightarrow b$

CKY Algorithm for Deciding CFL

1) Write variables for all length 1 substrings

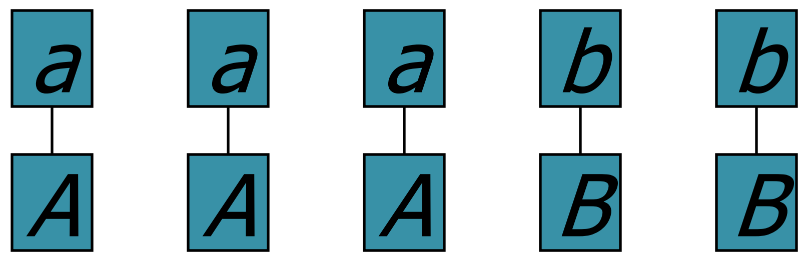
$S \rightarrow \varepsilon \mid AB \mid XB$

$T \rightarrow AB \mid XB$

$X \rightarrow AT$

$A \rightarrow a$

$B \rightarrow b$



CKY Algorithm for Deciding CFL

2) Write variables for all length 2 substrings

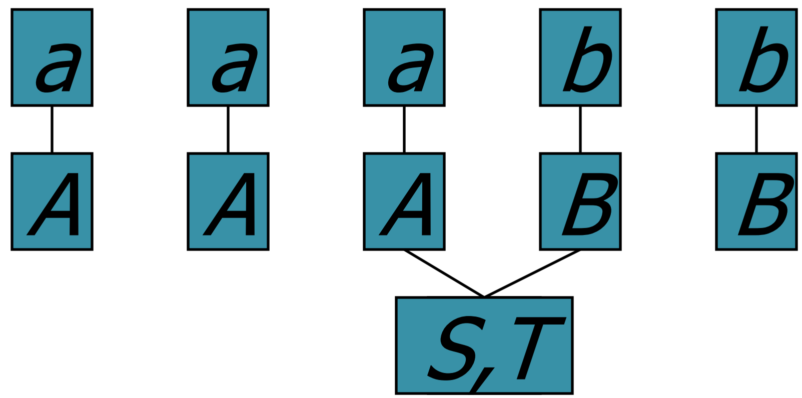
$S \rightarrow \varepsilon \mid AB \mid XB$

$T \rightarrow AB \mid XB$

$X \rightarrow AT$

$A \rightarrow a$

$B \rightarrow b$



CKY Algorithm for Deciding CFL

3) Write variables for all length 3 substrings

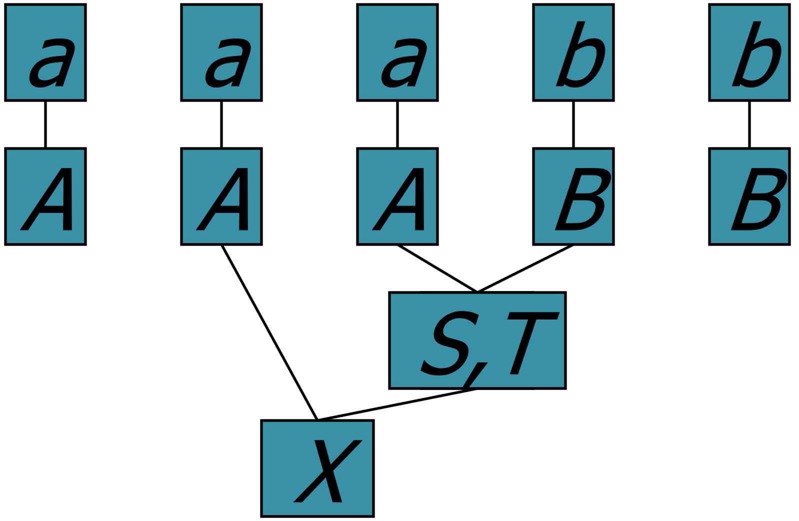
$S \rightarrow \varepsilon \mid AB \mid XB$

$T \rightarrow AB \mid XB$

$X \rightarrow AT$

$A \rightarrow a$

$B \rightarrow b$



CKY Algorithm for Deciding CFL

4) Write variables for all length 4 substrings

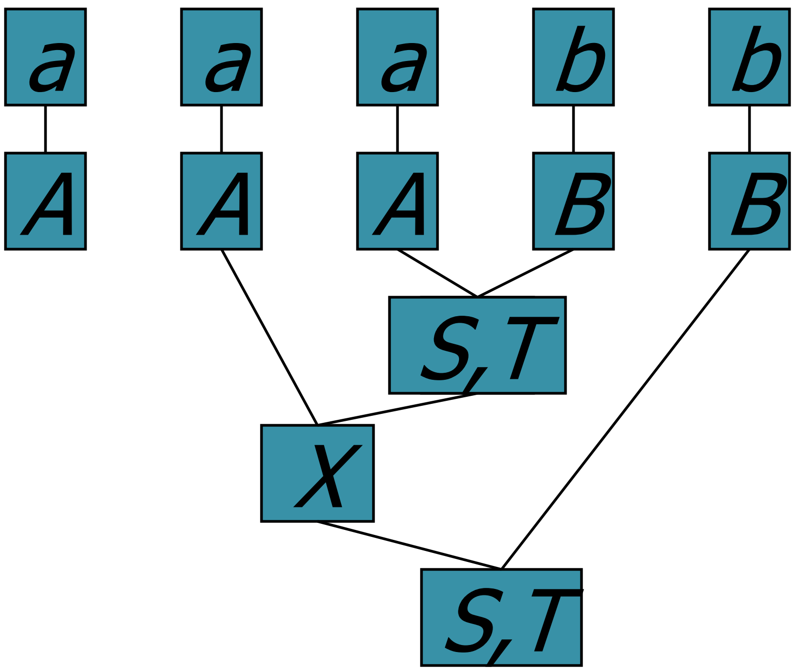
$S \rightarrow \varepsilon \mid AB \mid XB$

$T \rightarrow AB \mid XB$

$X \rightarrow AT$

$A \rightarrow a$

$B \rightarrow b$



CKY Algorithm for Deciding CFL

5) Write variables for all length 5 substrings.

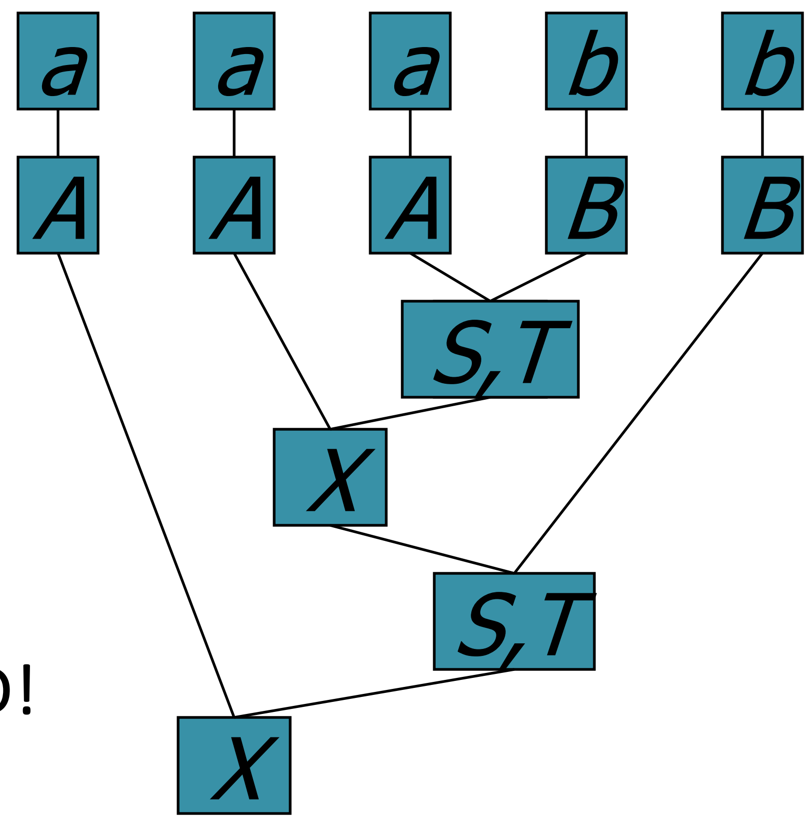
$S \rightarrow \varepsilon \mid AB \mid XB$

$T \rightarrow AB \mid XB$

$X \rightarrow AT$

$A \rightarrow a$

$B \rightarrow b$



aaabb REJECTED!

CKY Algorithm for Deciding CFL

Now look at *aaabbb* :

$S \rightarrow \varepsilon \mid AB \mid XB$

$T \rightarrow AB \mid XB$

$X \rightarrow AT$

$A \rightarrow a$

$B \rightarrow b$

a	*a*	*a*	*b*	*b*	*b*

CKY Algorithm for Deciding CFL

1) Write variables for all length 1 substrings.

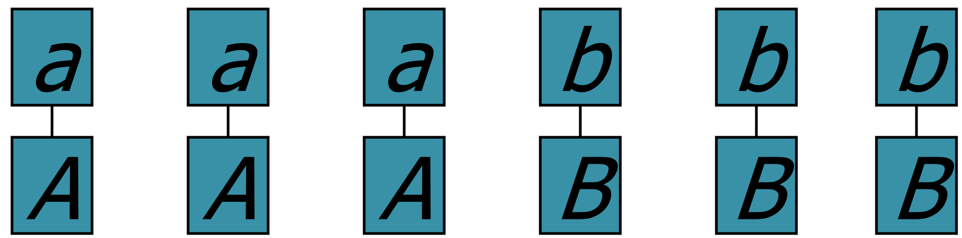
$S \rightarrow \varepsilon \mid AB \mid XB$

$T \rightarrow AB \mid XB$

$X \rightarrow AT$

$A \rightarrow a$

$B \rightarrow b$



CKY Algorithm for Deciding CFL

2) Write variables for all length 2 substrings.

$S \rightarrow \varepsilon \mid AB \mid XB$

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$A \rightarrow a$

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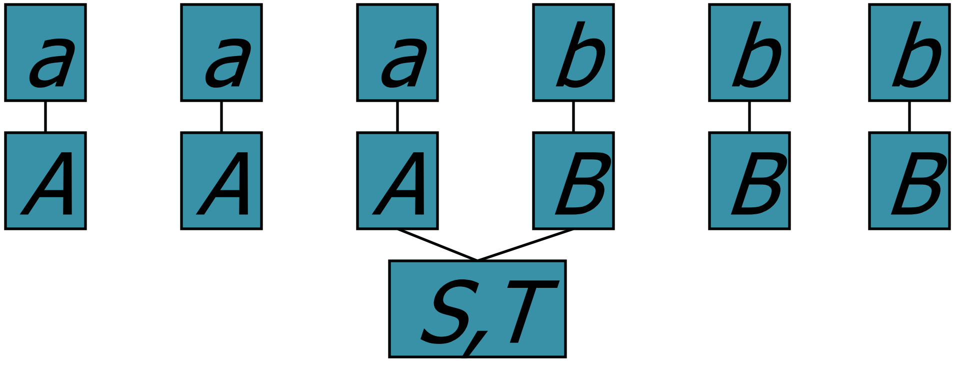

CKY Algorithm for Deciding CFL

3) Write variables for all length 3 substrings.

$S \rightarrow \varepsilon \mid AB \mid XB$

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$A \rightarrow a$

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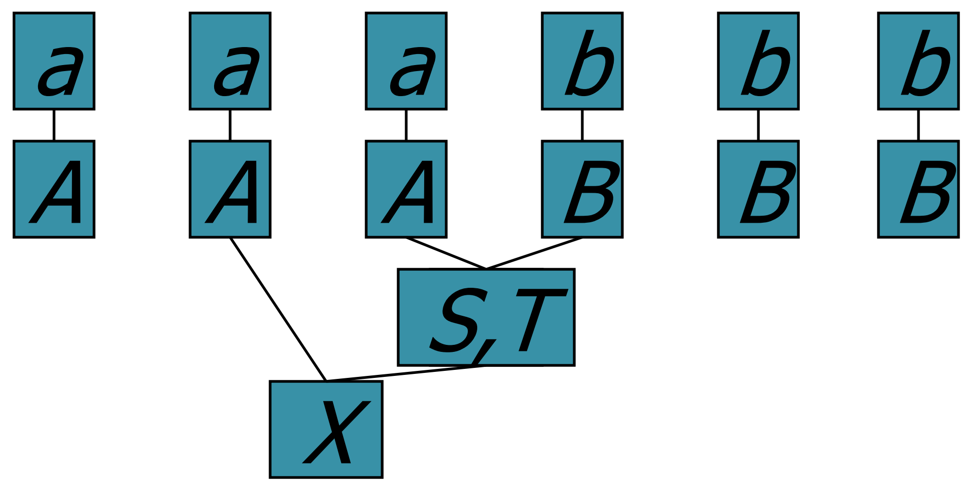

CKY Algorithm for Deciding CFL

4) Write variables for all length 4 substrings.

$S \rightarrow \varepsilon \mid AB \mid XB$

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$A \rightarrow a$

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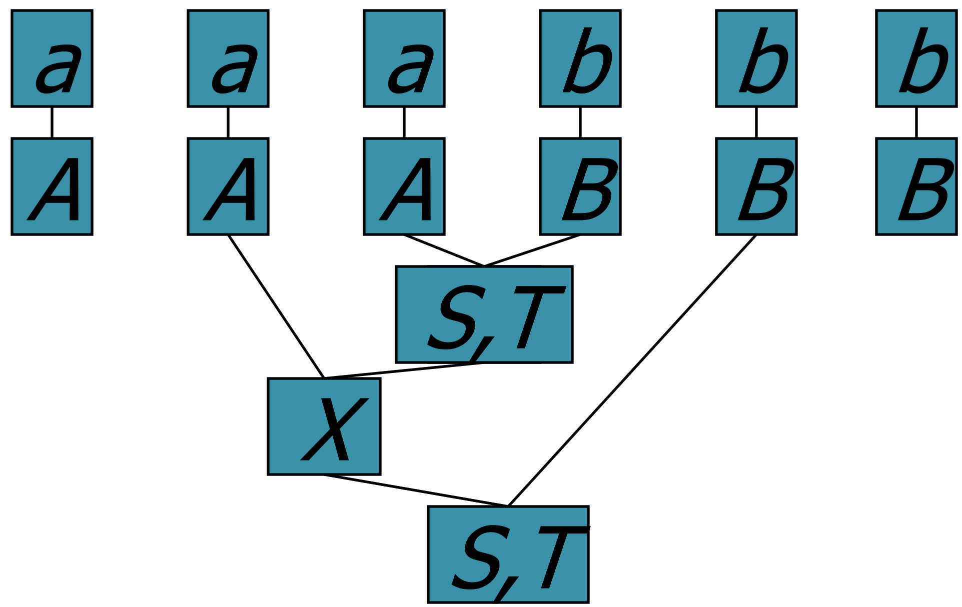


CKY Algorithm for Deciding CFL

5) Write variables for all length 5 substrings.

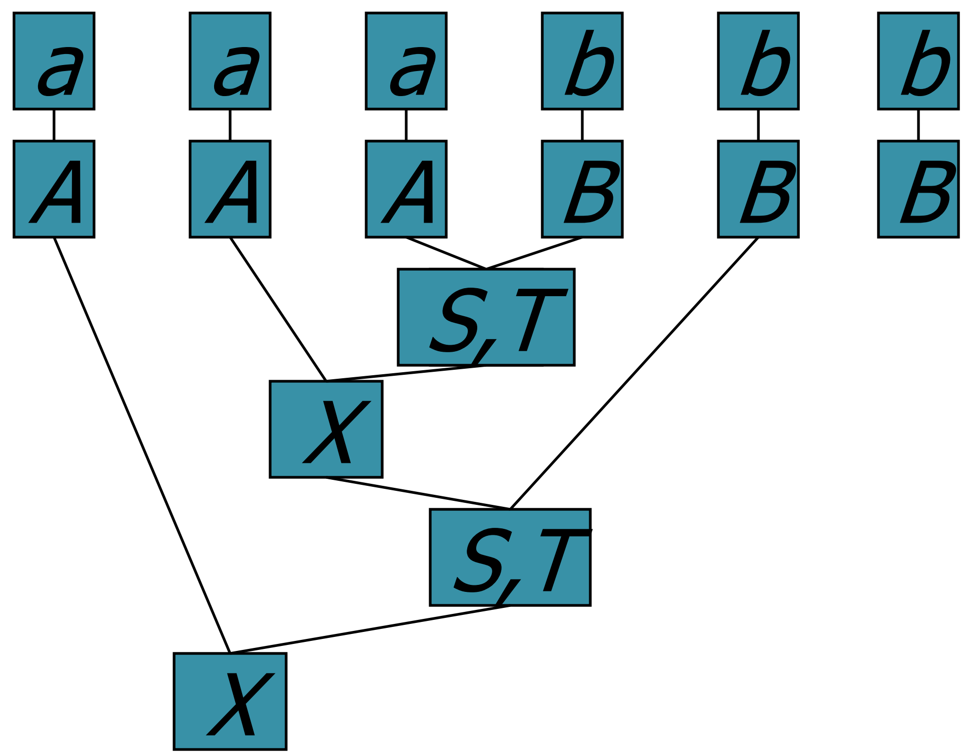
$S \rightarrow \varepsilon \mid AB \mid XB$

$T \rightarrow AB \mid XB$

$X \rightarrow AT$

$A \rightarrow a$

$B \rightarrow b$



CKY Algorithm for Deciding CFL

6) Write variables for all length 6 substrings.

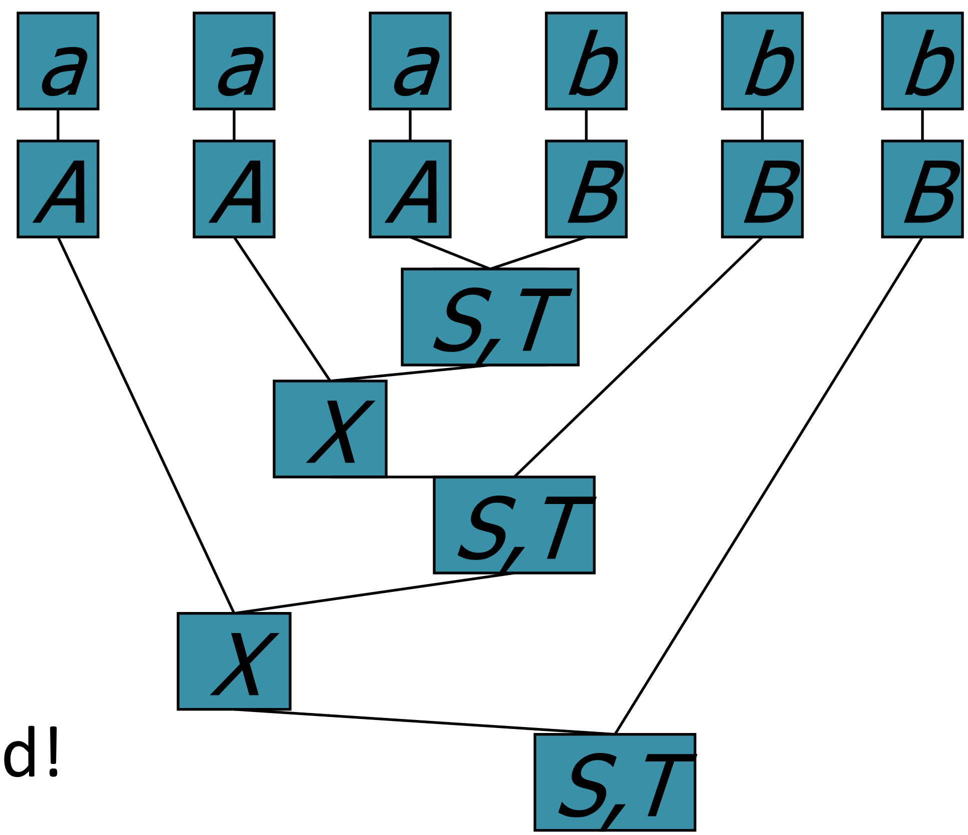
$S \rightarrow \varepsilon \mid AB \mid XB$

$T \rightarrow AB \mid XB$

$X \rightarrow AT$

$A \rightarrow a$

$B \rightarrow b$



S is included so *aaabbb* accepted!

The CKY Algorithm

function CKY (word w, grammar P) returns table

for i ← from 1 to LENGTH(w) do

  table[i-1, i] ← {A | A → w_i ∈ P }

for j ← from 2 to LENGTH(w) do

  for i ← from j-2 down to 0 do

    for k ← i + 1 to j – 1 do

      table[i,j] ← table[i,j] ∪ {A | A → BC ∈ P,

        B ∈ table[i,k], C ∈ table[k,j] }

If the start symbol *S* ∈ table[0,n] then w ∈ L(G)

CKY Algorithm for Deciding CFL

The table chart used by the algorithm:

j / i	1 a	2 a	3 a	4 b	5 b	6 b
0						
1						
2						
3						
4						
5						

CKY Algorithm for Deciding CFL

1. Variables for length 1 substrings.

j / i	1 a	2 a	3 a	4 b	5 b	6 b
0	A					
1		A				
2			A			
3				B		
4					B	
5						B

CKY Algorithm for Deciding CFL

2. Variables for length 2 substrings.

j / i	1 a	2 a	3 a	4 b	5 b	6 b
0	A	-				
1		A	-			
2			A	S,T		
3				B	-	
4					B	-
5						B

CKY Algorithm for Deciding CFL

3. Variables for length 3 substrings.

j / i	1 a	2 a	3 a	4 b	5 b	6 b
0	A	-	-			
1		A	-	X		
2			A	S,T	-	
3				B	-	-
4					B	-
5						B

CKY Algorithm for Deciding CFL

4. Variables for length 4 substrings.

j / i	1 a	2 a	3 a	4 b	5 b	6 b
0	A	-	-	-		
1		A	-	X	S,T	
2			A	S,T	-	-
3				B	-	-
4					B	-
5						B

CKY Algorithm for Deciding CFL

5. Variables for length 5 substrings.

i \ j	1 a	2 a	3 a	4 b	5 b	6 b
0	A	-	-	-	X	
1		A	-	X	S,T	-
2			A	S,T	-	-
3				B	-	-
4					B	-
5						B

CKY Algorithm for Deciding CFL

6. Variables for *aaabbb*. ACCEPTED!

i \ j	1 **a**	2 **a**	3 **a**	4 **b**	5 **b**	6 **b**
0	A	-	-	-	X	S,T
1		A	-	X	S,T	-
2			A	S,T	-	-
3				B	-	-
4					B	-
5						B

Parsing results

- We keep the results for every w_{ij} in a table.
- Note that we only need to fill in entries up to the diagonal.
- Every entry in the table T[i,j] can contains up to r=|N| symbols (the size of the non-terminal set).
- We can use lists or a Boolean n*n*r table.
- We then want to find T[0,n,S] = true.

CKY recognition vs. parsing

- Returning the full parse requires storing more in a cell than just a node label.
- We also require back-pointers to constituents of that node.
- We could also store whole trees, but less space efficient.
- For parsing, we must add an extra step to the algorithm: follow pointers and return the parse.

Ambiguity

Efficient Representation of Ambiguities

- Local Ambiguity Packing :
  - a Local Ambiguity - multiple ways to derive the *same* substring from a non-terminal
  - All possible ways to derive each non-terminal are stored together
  - When creating back-pointers, create a *single* back-pointer to the "packed" representation
- Allows to efficiently represent a very large number of ambiguities (even exponentially many)
- Unpacking - producing one or more of the packed parse trees by following the back-pointers.

CKY Space and Time Complexity

Time complexity:

- Three nested “for” loop each one of O(n) size.
- Lookup for $r = |N|$ pair rules at each step.

Time complexity – $O(r^2n^3) = O(n^3)$

Space complexity:

- A three dimensions table at size n*n*r **or**
- A n*n table with lists up to size of *r*

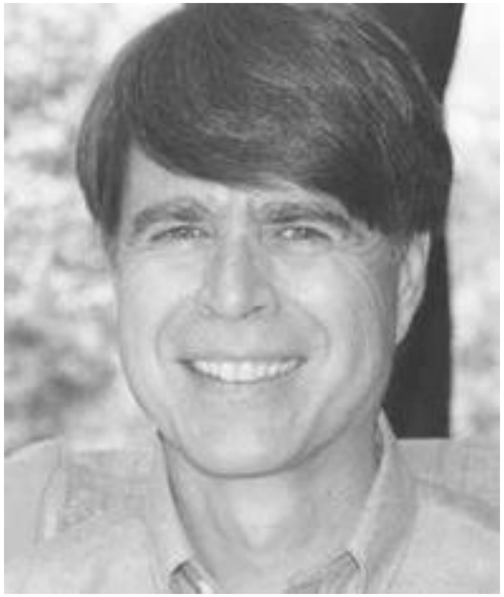
Space complexity – $O(rn^2) = O(n^2)$

Another Parsing Algorithm

- Parsing General CFLs vs. Limited Forms
- Efficiency:
  - Deterministic (LR) languages can be parsed in *linear time.*
  - A number of parsing algorithms for general CFLs require $O(n^3)$ time.
  - Asymptotically best parsing algorithm for general CFLs requires $O(n^{2.37})$, but is not practical.
- Utility - why parse general grammars and not just CNF?
  - Grammar intended to reflect actual structure of language.
  - Conversion to CNF completely destroys the parse structure.

The Earley Algorithm (1970)

- Doesn't require the grammar to be in CNF.
- Usually moves left-to-right.
- Makes it faster than $O(n^3)$ for many grammars.
- Earley's algorithm resembles recursive descent, but solves the left-recursion problem.
- No recursive function calls.
- Use a parse table as we did in CKY, so we can look up anything we've discovered so far.



The Earley Algorithm

- The algorithm is a bottom-up chart parser with top-down prediction: the algorithm builds up parse trees bottom-up, but the incomplete edges (the predictions) are generated top-down, starting with the start symbol.
- We need a new data structure: A dotted rule stands for a partially constructed constituent, with the dot indicating how much has already been found and how much is still predicted.
- Dotted rules are generated from ordinary grammar rules.
- The algorithm maintains sets of “states”, one set for each position in the input string (starting from 0).

The Dotted Rules

With dotted rules, an entry in the chart records:

- Which rule has been used in the analysis
- Which part of the rule has already been found (left of the dot).
- Which part is still predicted to be found and will combine into a complete parse (right of the dot).
- the start and end position of the material left of the dot.

Example: $A \rightarrow X_1 X_2 \ldots \bullet C \ldots X_m$

Operation of the Algorithm

- Process all hypotheses one at a time in order.
- This may add new hypotheses to the end of the to-do list, or try to add old hypotheses again.
- Process a hypothesis according to what follows the dot:
  - If a symbol, **scan** input and see if it matches.
  - If a non-terminal, **predict** ways to match it.

    (we'll predict blindly, but could reduce # of predictions by *looking ahead* k symbols in the input and only making predictions that are compatible)
  - If nothing, then we have a **complete** constituent, so attach it to all its customers.

Parsing Operations

The Earley algorithm has three main operations:

Predictor: an incomplete entry looks for a symbol to the right of its dot. if there is no matching symbol in the chart, one is predicted by adding all matching rules with an initial dot.

Scanner: an incomplete entry looks for a symbol to the right of the dot. this prediction is compared to the input, and a complete entry is added to the chart if it matches.

Completer: a complete edge is combined with an incomplete entry that is looking for it to form another complete entry.

Parsing Operations

- **Predictor:** If state $[A \to X_1 ... \bullet C ... X_m, j] \in S_i$ then for every rule of the form $C \to Y_1 ... Y_k$, add to S_i the state $[C \to \bullet Y_1 ... Y_k, i]$
- **Scanner:** If state $[A \to X_1 ... \bullet a ... X_m, j] \in S_i$ and the next input word is $x_{i+1} = a$, then add to S_{i+1} the state $[A \to X_1 ... a \bullet ... X_m, j]$
- **Completer:** If state $[A \to X_1 ... X_m \bullet, j] \in S_i$ then for every state in S_j of form $[B \to X_1 ... \bullet A ... X_k, l]$, add to S_i the state $[B \to X_1 ... A \bullet ... X_k, l]$

The Earley Recognition Algorithm

The Main Algorithm: parsing input $w=w_1w_2\ldots w_n$

1. S_0 = {[S → • P (0)]}
2. For $0 \le i \le n$ do:

   Process each item s ∈ S_i in order by applying to it a *single* applicable operation among:

   (a) Predictor (adds new items to S_i)

   (b) Completer (adds new items to S_i)

   (c) Scanner (adds new items to S_{i+1})
3. If S_{i+1} = Ø Reject the input.
4. If $i = n$ and [S→P• (0)] ∈ S_n then Accept the input.

Earley Algorithm Example

Consider the following grammar for arithmetic expressions:

S → P (the start rule)

P → P + M

P → M

M → M * T

M → T

T → number

With the input: 2 + 3 * 4

Earley Algorithm Example

<u>Sequence(0)</u> • 2 + 3 * 4

(1) S → • P (0)   # start rule

Earley Algorithm Example

<u>Sequence(0)</u> • 2 + 3 * 4

(1) S → • P (0) # start rule
(2) P → • P + M (0) # predict from (1)
(3) P → • M (0) # predict from (1)

Earley Algorithm Example

<u>Sequence(0)</u> • 2 + 3 * 4

(1)	S → • P (0)	# start rule
(2)	P → • P + M (0)	# predict from (1)
(3)	P → • M (0)	# predict from (1)
(4)	M → • M * T (0)	# predict from (3)
(5)	M → • T (0)	# predict from (3)

Earley Algorithm Example

Sequence(0) • 2 + 3 * 4

(1)	S → • P (0)	# start rule
(2)	P → • P + M (0)	# predict from (1)
(3)	P → • M (0)	# predict from (1)
(4)	M → • M * T (0)	# predict from (3)
(5)	M → • T (0)	# predict from (3)
(6)	T → • number (0)	# predict from (5)

Earley Algorithm Example

Sequence(1) 2 • + 3 * 4

(1) T → number • (0) # scan from S(0)(6)

Earley Algorithm Example

<u>Sequence(1)</u> 2 • + 3 * 4

(1) T → number • (0) # scan from S(0)(6)

(2) M → T • (0) # complete from S(0)(5)

Earley Algorithm Example

Sequence(1) 2 • + 3 * 4

(1) T → number • (0) # scan from S(0)(6)

(2) M → T • (0) # complete from S(0)(5)

(3) M → M • * T (0) # complete from S(0)(4)

(4) P → M • (0) # complete from S(0)(3)

Earley Algorithm Example

<u>Sequence(1)</u> 2 • + 3 * 4

(1)	T → number • (0)	# scan from S(0)(6)
(2)	M → T • (0)	# complete from S(0)(5)
(3)	M → M • * T (0)	# complete from S(0)(4)
(4)	P → M • (0)	# complete from S(0)(3)
(5)	P → P • + M (0)	# complete from S(0)(2)
(6)	S → P • (0)	# complete from S(0)(1)

Earley Algorithm Example

<u>Sequence(2)</u> 2 + • 3 * 4

(1) P → P + • M (0)   # scan from S(1)(5)

Earley Algorithm Example

Sequence(2) 2 + • 3 * 4

(1) P → P + • M (0) # scan from S(1)(5)

(2) M → • M * T (2) # predict from (1)

(3) M → • T (2) # predict from (1)

Earley Algorithm Example

Sequence(2) 2 + • 3 * 4

(1) P → P + • M (0) # scan from S(1)(5)
(2) M → • M * T (2) # predict from (1)
(3) M → • T (2) # predict from (1)
(4) T → • number (2) # predict from (3)

Earley Algorithm Example

<u>Sequence(3)</u> 2 + 3 • * 4

(1) T → number • (2) # scan from S(2)(4)

Earley Algorithm Example

<u>Sequence(3)</u> 2 + 3 • * 4

(1) T → number • (2) # scan from S(2)(4)

(2) M → T • (2) # complete from S(2)(3)

Earley Algorithm Example

<u>Sequence(3)</u> 2 + 3 • * 4

(1) T → number • (2) # scan from S(2)(4)

(2) M → T • (2) # complete from S(2)(3)

(3) M → M • * T (2) # complete from S(2)(2)

(4) P → P + M • (0) # complete from S(2)(1)

Earley Algorithm Example

<u>Sequence(3)</u> 2 + 3 • * 4

(1)	T → number • (2)	# scan from S(2)(4)
(2)	M → T • (2)	# complete from S(2)(3)
(3)	M → M • * T (2)	# complete from S(2)(2)
(4)	P → P + M • (0)	# complete from S(2)(1)
(5)	P → P • + M (0)	# complete from S(0)(2)
(6)	S → P • (0)	# complete from S(0)(1)

Earley Algorithm Example

<u>Sequence(4)</u> 2 + 3 * • 4

(1) M → M * • T (2) # scan from S(3)(3)

Earley Algorithm Example

<u>Sequence(4)</u> 2 + 3 * • 4

(1) M → M * • T (2) # scan from S(3)(3)

(2) T → • number (4) # predict from (1)

Earley Algorithm Example

<u>Sequence(5)</u> 2 + 3 * 4 •

(1) T → number • (4) # scan from S(4)(2)

Earley Algorithm Example

<u>Sequence(5)</u> 2 + 3 * 4 •

(1) T → number • (4) # scan from S(4)(2)

(2) M → M * T • (2) # complete from S(4)(1)

Earley Algorithm Example

<u>Sequence(5)</u> 2 + 3 * 4 •

(1) T → number • (4) # scan from S(4)(2)

(2) M → M * T • (2) # complete from S(4)(1)

(3) M → M • * T (2) # complete from S(2)(2)

(4) P → P + M • (0) # complete from S(2)(1)

Earley Algorithm Example

<u>Sequence(5)</u> 2 + 3 * 4 •

(1) T → number • (4) # scan from S(4)(2)
(2) M → M * T • (2) # complete from S(4)(1)
(3) M → M • * T (2) # complete from S(2)(2)
(4) P → P + M • (0) # complete from S(2)(1)
(5) P → P • + M (0) # complete from S(0)(2)
(6) S → P • (0) # complete from S(0)(1)

Earley Algorithm Example

<u>Sequence(5)</u> 2 + 3 * 4 •

(1) T → number • (4) # scan from S(4)(2)
(2) M → M * T • (2) # complete from S(4)(1)
(3) M → M • * T (2) # complete from S(2)(2)
(4) P → P + M • (0) # complete from S(2)(1)
(5) P → P • + M (0) # complete from S(0)(2)
(6) S → P • (0) # complete from S(0)(1)

The state S→P• (0) represents a completed parse.

Finding the parse tree

Seq 0 • 2 + 3 * 4	Seq 1 2 • + 3 * 4	Seq 2 2 + • 3 * 4	Seq 3 2 + 3 • * 4	Seq 4 2 + 3 * • 4	Seq 5 2 + 3 * 4 •
S → • P (0)	T → '2' • (0)	P → P + • M (0)	T → '3' • (2)	M → M * • T (2)	T → '4' • (4)
P → • P + M (0)	M → T • (0)	M → • M * T (2)	M → T • (2)	T → • num (4)	M → M * T • (2)
P → • M (0)	M → M • * T (0)	M → • T (2)	M → M • * T (2)		M → M • * T (2)
M → • M * T (0)	P → M • (0)	T → • num (2)	P → P + M • (0)		P → P + M • (0)
M → • T (0)	P → P • + M (0)		P → P • + M (0)		P → P • + M (0)
T → • num (0)	S → P • (0)		S → P • (0)		S → P • (0)

Finding the parse tree

Seq 0 • 2 + 3 * 4	Seq 1 2 • + 3 * 4	Seq 2 2 + • 3 * 4	Seq 3 2 + 3 • * 4	Seq 4 2 + 3 * • 4	Seq 5 2 + 3 * 4 •
S → • P (0)	T → '2' • (0)	P → P + • M (0)	T → '3' • (2)	M → M * • T (2)	**T → '4' • (4)**
P → • P + M (0)	M → T • (0)	M → • M * T (2)	M → T • (2)	T → • num (4)	**M → M * T • (2)**
P → • M (0)	M → M • * T (0)	M → • T (2)	M → M • * T (2)		M → M • * T (2)
M → • M * T (0)	P → M • (0)	T → • num (2)	P → P + M • (0)		**P → P + M • (0)**
M → • T (0)	P → P • + M (0)		P → P • + M (0)		P → P • + M (0)
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P → • M (0)	M → M • * T (0)	M → • T (2)	M → M • * T (2)		M → M • * T (2)
M → • M * T (0)	P → M • (0)	T → • num (2)	P → P + M • (0)		**P → P + M • (0)**
M → • T (0)	P → P • + M (0)		P → P • + M (0)		P → P • + M (0)
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P → • M (0)	M → M • * T (0)	M → • T (2)	M → M • * T (2)		M → M • * T (2)
M → • M * T (0)	**P → M • (0)**	T → • num (2)	P → P + M • (0)		**P → P + M • (0)**
M → • T (0)	P → P • + M (0)		P → P • + M (0)		P → P • + M (0)
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- Time require for each iteration is thus $O(r^2n^2) = O(n^2)$
- Time bound on the entire algorithm is therefore $O(n^3)$

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- *Completer* is the operation that may require $O(i^2)$ time in iteration i
- For unambiguous grammars, Earley shows that the Completer operation will require at most $O(i)$ time.
- Thus time complexity for unambiguous grammar is $\boldsymbol{O(n^2)}$
- For some grammars, the number of items in each S_i is bounded by a constant (bounded-state grammars)
- For bounded-state grammars, the time complexity of the algorithm is **<u>linear</u>** – $\boldsymbol{O(n)}$

CKY vs. Earley

- CKY is a bottom-up parser.
- Earley is a bottom-up parser with a top-down prediction.

- CKY algorithm requires the grammar to be in CNF.
- Earley algorithm works for any grammar.

- CKY require $\boldsymbol{O(n^3)}$ time for any grammar.
- Earley algorithm can work in $\boldsymbol{O(n^2)}$ or $\boldsymbol{O(n)}$ time for **some** grammars.

The End.

Thank you