Polymorphism and Higher-Order Functions (Poly.v)

Keywords

- Arguments nil {X}.:Implicit Arguments
- Arguments cons {X} _ _.
- Arguments repeat {X} x count.
- Fixpoint repeat''' {X : Type} (x : X) (count : nat) : list X : declare an argument to be implicit when declaring the function itself, by surrounding it in curly braces instead of parens.
- Fail + any command: this command indeed fails when executed
- Check @nil.: Force the implicit arguments to be explicit by prefixing the function name with @
- fun n => n * n: anonymous function

Tactics

Definitions and Theorems

Polymorphism

```
• Inductive list (X : Type) : Type
```

- Fixpoint repeat (X : Type) (x : X) (count : Nat) : list X
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- Module MumbleGrumble.
- Inductive mumble : Type
- Inductive grumble (X : Type) : Type
- End MumbleGrumble.

Type Annotation Inference

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```
• Fixpoint repeat' X x count : list X
• Fixpoint repeat'' X x count : list X
• Fixpoint app {X : Type} (11 12 : list X) : list X
• Fixpoint rev {X : Type} (1 : list X) : list X
 Fixpoint length \{X : Type\} (1 : list X) : nat
 Notation "x :: y"
• Notation "[ ]"
 Notation "[ x ; .. ; y ]"
 Notation "x ++ y"
• Theorem app_nil_r : forall (X : Type), forall 1 : list X, 1 ++ [] = 1.
• Theorem app_assoc : forall A (1 \text{ m n} : \text{list A}), 1 ++ \text{ m} ++ \text{ n} = (1 ++ \text{ m}) ++
  n.
• Theorem app_length : forall (X : Type) (11 12 : list X), length (11 ++
  12) = length 11 + length 12.
Theorem rev_app_distr : forall (X : Type) (11 12 : list X), rev (11 ++
  12) = rev 12 ++ rev 11.
• Theorem rev_involutive: forall (X : Type), forall (1 : list X), rev (rev
  1) = 1.
```

Polymorphic Pairs (Products)

```
Inductive prod (X Y : Type) : Type
Arguments pair {X} {Y} _ _ _ .
Notation "( x, y )" := (pair x y).
Notation "X * Y" := (prod X Y) : type_scope.
Definition fst {X Y : Type} (p : X * Y) : X
Definition snd {X Y : Type} (p : X * Y) : Y
Fixpoint combine {X Y : Type} (lx : list X) (ly : list Y) : list (X * Y)
Fixpoint split {X Y : Type} (l : list (X * Y)) : (list X) * (list Y)
```

Polymorphic Options

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• Module OptionPlayground
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• Inductive option (X : Type) : Type

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```
• Arguments Some {X} _.
```

- Arguments None {X}.
- End OptionPlayground
- Fixpoint nth_error {X : Type} (1 : list X) (n : nat) : option X
- Fixpoint hd_error {X : Type} (1 : list X) : option X

Functions as Data

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• Function doit3times \{X : Type\} (f : X -> X) (n : X) : X
```

- Fixpoint filter {X : Type} (test : X -> bool) (1 : list X) : list X
- Definition length_is_1 {X : Type} (1 : list X) : bool
- Definition partition {X : Type} (test : X -> bool) (1 : list X) : (list X) * (list X)
- Fixpoint map {X Y : Type} (f : X -> Y) (1 : list X) : (list Y)
- Theorem map_app_distr : forall (X Y : Type) (f : X \rightarrow Y) (11 12 : list X), map f (11 ++ 12) = map f 11 ++ map f 12.
- Theorem map_rev : forall (X Y : Type) (f : X -> Y) (1 : list X), map f (rev 1) = rev (map f 1).
- Fixpoint flat_map : {X Y : Type} (f : X -> list Y) (1 : list X) : (list Y)

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- Definition option_map {X Y : Type} (f : X -> Y) (xo : option X) : option
- Fixpoint fold $\{X\ Y\ :\ Type\}\ (f\ :\ X\ ->\ Y\ ->\ Y)\ (l\ :\ list\ X)\ (b\ :\ Y)\ :\ Y$

•

- Definition constfun {X : Type} (x : X) : nat -> X
- Definition ftrue := constfun true.

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- Definition fold_length {X : Type} (1 : list X) : nat := fold (fun _ n => S n) 1 0.
- Theorem fold_length_correct : forall X (1 : list X), fold_length 1 = length 1.
- Definition fold_map {X Y : Type} (f : X -> Y) (1 : list X) : (list Y) := fold (fun x t => f x :: t) 1 [].
- Theorem fold_map_correct : forall (X Y : Type) (f : X -> Y) (l : list X), fold_map f l = map f l.

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Currying

Definition prod_curry {X Y Z : Type} (f : X * Y -> Z) (x : X) (y : Y) : Z
 := f (x, y).

- Definition prod_uncurry {X Y Z : Type} (f : X -> Y -> Z) (p : X * Y) : Z
 := f (fst p) (snd p).
- Theorem uncurry_curry : forall (X Y Z : Type) (f : X -> Y -> Z) x y, prod_curry (prod_uncurry f) x y = f x y.
- Theorem curry_uncurry: forall (X Y Z : Type) (f : X * Y -> Z) (p : X * Y), prod_uncurry (prod_curry f) p = f p.

Church Numerals

- Module Church
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- End Church

Problems

• Theorem fold_length_correct:power/mechanisms of simpl. vs. reflexivity.

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