Consensus Number

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Definition (Consensus Number)

The **consensus number** for X is the largest n for which X solves n-thread consensus.

If no largest n exists, the consensus number is said to be infinite.

Lemma (Y Implements X)

Theorem (Consensus Number as . . .)

in the following, main results (table)

beautiful ideas and proofs

What is a Protocol $\mathcal{P}$?

Protocol

$$\mathcal{P} = \{\text{All executions of this protocol}\}$$

Execution

$$e = \sigma_0 \xrightarrow{o_1} \sigma_1 \xrightarrow{o_2} \cdots \xrightarrow{o_{n-1}} \sigma_n$$

State

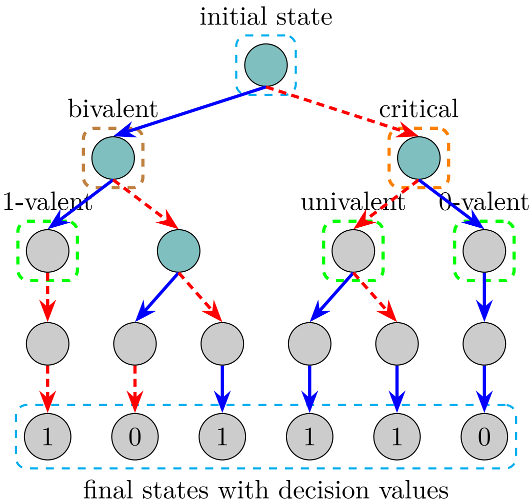
σ_i : states of individual threads + states of shared objects

Operation

o_i : method calls to a shared object

Modeling $\mathcal{P}$ as a Computation "Tree"

(Binary Consensus for 2 Threads)



Theorem (Bivalent Initial State)

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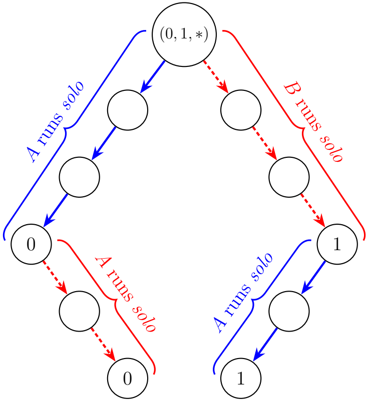

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Proof.

fig here. □


Thank
You!