# Specification, Implementation, and Complexity of

## 2 Replicated Data Types with Composite

## Operations

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- 1 Introduction
- 2 Preliminaries
- $_{\text{n}}$  2.1 Observed-Remove Set (OR-Set)

$$\mathcal{F}_{\mathtt{orset}}(\mathtt{rd}, E, \mathtt{op}, \mathtt{vis}, \mathtt{ar}) = \{ a \mid \exists e \in E.\mathtt{op}(e) = \mathtt{add}(a) \}$$
 (1)

$$\wedge \left( \forall f \in E.\mathsf{op}(f) = \mathsf{rm}(a) \implies \neg (e \xrightarrow{\mathsf{vis}} f) \right) \}. \tag{2}$$

## Replicated Data Types with Composite Operations

## 3.1 Specification

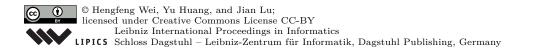
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We consider a composite operation of a replicated data type  $\tau$  in the form of  $C = A \oplus B$ , where A, B, and C are different objects of type  $\tau$ .

Following [1], we specify the semantics of a composite operation  $A \oplus B$  of a replicated data type  $\tau$  by a function  $\mathcal{F}_{\tau}$  that determines the return value of  $\oplus$  based on prior operations performed on the two objects involved (i.e., A and B). However,  $\mathcal{F}_{\tau}$  for a composite operation  $\oplus$  takes as parameters two, not one as in [1], operation contexts, one on each object involved.

Q: Generalize to different data types?

Note: Partial operation context [2]



**Definition 1** (Product of Operation Contexts). Consider two operation contexts for the same replicated data type  $\tau$ :

$$\mathcal{C}_A = (E_A, \mathsf{op}_A, \mathsf{vis}_A, \mathsf{ar}_A) \tag{3}$$

$$\mathcal{C}_B = (E_B, \mathsf{op}_B, \mathsf{vis}_B, \mathsf{ar}_B) \tag{4}$$

The product  $C = C_A \times C_B$  of  $C_A$  and  $C_B$  is also an operation context defined as  $C = (E, \mathsf{op}, \mathsf{vis}, \mathsf{ar})$ , where

 $E = E_A \times E_B$ 

 $\mathsf{p}_{a2} \quad \blacksquare \quad \mathsf{op} = \mathsf{op}_A \sqcup \mathsf{op}_B$ 

 $_{43}$  wis = vis $_A imes$  vis $_B$ 

 $_{ t 44}$   $_{lacktrightarrow}$   $\operatorname{ar} = \operatorname{ar}_A imes \operatorname{ar}_B$ 

#### **▶** Definition 2.

$$\mathcal{F}_{\tau}(\oplus, \mathcal{C}_A, \mathcal{C}_B) = \mathcal{F}_{\tau}(\oplus, \mathcal{C}_A \times \mathcal{C}_B) \tag{5}$$

## 4 Replicated Set with Composite Operations

We consider the replicated set data type with composite operations including union  $(\cup)$ , intersection  $(\cap)$ , and set difference  $(\setminus)$ .

## 50 4.1 Specification

$$\mathcal{F}_{\text{orset}}(A \setminus B, \mathcal{C}_A, \mathcal{C}_B) = \{ a \mid \cdots \}. \tag{6}$$

## 53 4.2 Protocol

### 5 Related Work

### 6 Conclusion and Future Work

#### 56 — References

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