

# From DPLL to CDCL SAT solvers

## Combinatorial Problem Solving (CPS)

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# Overview of the session

- Conflict Analysis
  - ◆ Motivating example
  - ◆ Backjumping
  - ◆ Conflict graph
  - ◆ Lemma shortening
- Lemma removal
- Decision heuristics
- Restarts
- Efficient implementation of UnitProp:
  - ◆ Occur lists
  - ◆ Two-watched literals
- Final remarks

# Motivating Example

$$\emptyset \implies$$

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

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$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

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# Motivating Example

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$$p_{25} \bar{p}_{21}^d \implies$$

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$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

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$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

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 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \implies$



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 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} \implies$   
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 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d \implies$

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 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 \implies$   
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# Motivating Example

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 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d \implies$   
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \implies$   
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d \implies$

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 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 \implies$   
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d \implies$   
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \implies$   
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d \implies$   
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23} \implies$

# Motivating Example

$$\begin{aligned}
 &\bar{p}_{11} \vee p_6 \vee \bar{p}_{12} \\
 &\bar{p}_{11} \vee p_{13} \vee p_{16} \\
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 \end{aligned}$$

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 &p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \implies \\
 &p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d \implies \\
 &p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23} \implies \\
 &\underbrace{p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23} p_{11}^d}_{M} \implies
 \end{aligned}$$

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 &p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} \implies \\
 &p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 \implies \\
 &p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d \implies \\
 &p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \implies \\
 &p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d \implies \\
 &p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23} \implies \\
 &\underbrace{p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}}_M p_{11}^d \implies \\
 &M p_{11}^d \implies
 \end{aligned}$$



# Motivating Example

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$   
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$   
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$   
 $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$   
 $p_{10} \vee \bar{p}_8 \vee p_1$   
 $p_{10} \vee p_3$   
 $\bar{p}_3 \vee p_{26}$   
 $p_{10} \vee \bar{p}_5$   
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$   
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$   
 $p_{21} \vee \bar{p}_6$   
 $p_{21} \vee \bar{p}_{17}$   
 $\bar{p}_{22} \vee \bar{p}_{13}$   
 $p_{13} \vee p_8$   
 $\bar{p}_4 \vee p_{19}$   
 $p_{20} \vee p_{23}$   
 $\bar{p}_{20} \vee p_{24}$   
 $p_{25}$

$\emptyset \implies$   
 $p_{25} \implies$   
 $p_{25} \bar{p}_{21}^d \implies$   
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \implies$   
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} \implies$   
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \implies$   
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} \implies$   
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 \implies$   
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d \implies$   
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \implies$   
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d \implies$   
 $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23} \implies$   
 $\underbrace{p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}}_M p_{11}^d \implies$   
 $M p_{11}^d \implies$

Before we continue, some notation:

- Literal  $p_{25}$  belongs to **decision level (DL) 0**
- Literals  $\bar{p}_{21}^d, \bar{p}_6, \bar{p}_{17}$  belong to **decision level 1**
- ...

# Motivating Example

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

# Motivating Example

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

$$M p_{11}^d \implies$$

# Motivating Example

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

$$M p_{11}^d \implies$$

$$M p_{11}^d \bar{p}_{12} \implies$$

# Motivating Example

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

$$M \text{ is } p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$$

$$M p_{11}^d \implies$$

$$M p_{11}^d \bar{p}_{12} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \implies$$

# Motivating Example

$$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$$

$$\bar{p}_{11} \vee p_{13} \vee p_{16}$$

$$p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$$

$$p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$$

$$p_{10} \vee \bar{p}_8 \vee p_1$$

$$p_{10} \vee p_3$$

$$\bar{p}_3 \vee p_{26}$$

$$p_{10} \vee \bar{p}_5$$

$$\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$$

$$\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$$

$$p_{21} \vee \bar{p}_6$$

$$p_{21} \vee \bar{p}_{17}$$

$$\bar{p}_{22} \vee \bar{p}_{13}$$

$$p_{13} \vee p_8$$

$$\bar{p}_4 \vee p_{19}$$

$$p_{20} \vee p_{23}$$

$$\bar{p}_{20} \vee p_{24}$$

$$p_{25}$$

$$M \text{ is } p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$$

$$M p_{11}^d \implies$$

$$M p_{11}^d \bar{p}_{12} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \implies$$

$$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$$

# Motivating Example

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$   
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$   
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$   
 $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$   
 $p_{10} \vee \bar{p}_8 \vee p_1$   
 $p_{10} \vee p_3$   
 $\bar{p}_3 \vee p_{26}$   
 $p_{10} \vee \bar{p}_5$   
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$   
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$   
 $p_{21} \vee \bar{p}_6$   
 $p_{21} \vee \bar{p}_{17}$   
 $\bar{p}_{22} \vee \bar{p}_{13}$   
 $p_{13} \vee p_8$   
 $\bar{p}_4 \vee p_{19}$   
 $p_{20} \vee p_{23}$   
 $\bar{p}_{20} \vee p_{24}$   
 $p_{25}$

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$M p_{11}^d \implies$

$M p_{11}^d \bar{p}_{12} \implies$

$M p_{11}^d \bar{p}_{12} p_{16} \implies$

$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$

$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$

# Motivating Example

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$   
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$   
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$   
 $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$   
 $p_{10} \vee \bar{p}_8 \vee p_1$   
 $p_{10} \vee p_3$   
 $\bar{p}_3 \vee p_{26}$   
 $p_{10} \vee \bar{p}_5$   
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$   
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$   
 $p_{21} \vee \bar{p}_6$   
 $p_{21} \vee \bar{p}_{17}$   
 $\bar{p}_{22} \vee \bar{p}_{13}$   
 $p_{13} \vee p_8$   
 $\bar{p}_4 \vee p_{19}$   
 $p_{20} \vee p_{23}$   
 $\bar{p}_{20} \vee p_{24}$   
 $p_{25}$

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$M p_{11}^d \implies$   
 $M p_{11}^d \bar{p}_{12} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 \implies$



# Motivating Example

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$   
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$   
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$   
 $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$   
 $p_{10} \vee \bar{p}_8 \vee p_1$   
 $p_{10} \vee p_3$   
 $\bar{p}_3 \vee p_{26}$   
 $p_{10} \vee \bar{p}_5$   
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$   
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$   
 $p_{21} \vee \bar{p}_6$   
 $p_{21} \vee \bar{p}_{17}$   
 $\bar{p}_{22} \vee \bar{p}_{13}$   
 $p_{13} \vee p_8$   
 $\bar{p}_4 \vee p_{19}$   
 $p_{20} \vee p_{23}$   
 $\bar{p}_{20} \vee p_{24}$   
 $p_{25}$

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$M p_{11}^d \implies$   
 $M p_{11}^d \bar{p}_{12} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 \implies$

# Motivating Example

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$   
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$   
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$   
 $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$   
 $p_{10} \vee \bar{p}_8 \vee p_1$   
 $p_{10} \vee p_3$   
 $\bar{p}_3 \vee p_{26}$   
 $p_{10} \vee \bar{p}_5$   
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$   
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$   
 $p_{21} \vee \bar{p}_6$   
 $p_{21} \vee \bar{p}_{17}$   
 $\bar{p}_{22} \vee \bar{p}_{13}$   
 $p_{13} \vee p_8$   
 $\bar{p}_4 \vee p_{19}$   
 $p_{20} \vee p_{23}$   
 $\bar{p}_{20} \vee p_{24}$   
 $p_{25}$

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$M p_{11}^d \implies$   
 $M p_{11}^d \bar{p}_{12} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \implies$

# Motivating Example

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$   
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$   
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$   
 $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$   
 $p_{10} \vee \bar{p}_8 \vee p_1$   
 $p_{10} \vee p_3$   
 $\bar{p}_3 \vee p_{26}$   
 $p_{10} \vee \bar{p}_5$   
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$   
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$   
 $p_{21} \vee \bar{p}_6$   
 $p_{21} \vee \bar{p}_{17}$   
 $\bar{p}_{22} \vee \bar{p}_{13}$   
 $p_{13} \vee p_8$   
 $\bar{p}_4 \vee p_{19}$   
 $p_{20} \vee p_{23}$   
 $\bar{p}_{20} \vee p_{24}$   
 $p_{25}$

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$M p_{11}^d \implies$   
 $M p_{11}^d \bar{p}_{12} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \bar{p}_5 \implies$

# Motivating Example

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$   
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$   
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$   
 $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$   
 $p_{10} \vee \bar{p}_8 \vee p_1$   
 $p_{10} \vee p_3$   
 $\bar{p}_3 \vee p_{26}$   
 $p_{10} \vee \bar{p}_5$   
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$   
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$   
 $p_{21} \vee \bar{p}_6$   
 $p_{21} \vee \bar{p}_{17}$   
 $\bar{p}_{22} \vee \bar{p}_{13}$   
 $p_{13} \vee p_8$   
 $\bar{p}_4 \vee p_{19}$   
 $p_{20} \vee p_{23}$   
 $\bar{p}_{20} \vee p_{24}$   
 $p_{25}$

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$M p_{11}^d \implies$   
 $M p_{11}^d \bar{p}_{12} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \bar{p}_5 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \bar{p}_5 p_{18} \implies$

# Motivating Example

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$   
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$   
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$   
 $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$   
 $p_{10} \vee \bar{p}_8 \vee p_1$   
 $p_{10} \vee p_3$   
 $\bar{p}_3 \vee p_{26}$   
 $p_{10} \vee \bar{p}_5$   
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$   
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$   
 $p_{21} \vee \bar{p}_6$   
 $p_{21} \vee \bar{p}_{17}$   
 $\bar{p}_{22} \vee \bar{p}_{13}$   
 $p_{13} \vee p_8$   
 $\bar{p}_4 \vee p_{19}$   
 $p_{20} \vee p_{23}$   
 $\bar{p}_{20} \vee p_{24}$   
 $p_{25}$

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$M p_{11}^d \implies$

$M p_{11}^d \bar{p}_{12} \implies$

$M p_{11}^d \bar{p}_{12} p_{16} \implies$

$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$

$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$

$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 \implies$

$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 \implies$

$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \implies$

$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \bar{p}_5 \implies$

$M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \bar{p}_5 p_{18} \implies$

conflict!

# Motivating Example

$\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$   
 $\bar{p}_{11} \vee p_{13} \vee p_{16}$   
 $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$   
 $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$   
 $p_{10} \vee \bar{p}_8 \vee p_1$   
 $p_{10} \vee p_3$   
 $\bar{p}_3 \vee p_{26}$   
 $p_{10} \vee \bar{p}_5$   
 $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$   
 $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$   
 $p_{21} \vee \bar{p}_6$   
 $p_{21} \vee \bar{p}_{17}$   
 $\bar{p}_{22} \vee \bar{p}_{13}$   
 $p_{13} \vee p_8$   
 $\bar{p}_4 \vee p_{19}$   
 $p_{20} \vee p_{23}$   
 $\bar{p}_{20} \vee p_{24}$   
 $p_{25}$

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

$M p_{11}^d \implies$   
 $M p_{11}^d \bar{p}_{12} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \bar{p}_5 \implies$   
 $M p_{11}^d \bar{p}_{12} p_{16} \bar{p}_2 \bar{p}_{10} p_1 p_3 p_{26} \bar{p}_5 p_{18} \implies$

conflict!

- Let's try to find out the causes of the conflict
- First of all we will compute, for each literal of the current decision level, the **reason** why it is true

# Motivating Example

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
  
10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

# Motivating Example

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

Literal	$p_{11}^d$	$\bar{p}_{12}$	$p_{16}$	$\bar{p}_2$	$\bar{p}_{10}$	$p_1$	$p_3$	$p_{26}$	$\bar{p}_5$	$p_{18}$
Reason	$\perp$	1	2	3	4	5	6	7	8	9



# Motivating Example

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

Literal	$p_{11}^d$	$\bar{p}_{12}$	$p_{16}$	$\bar{p}_2$	$\bar{p}_{10}$	$p_1$	$p_3$	$p_{26}$	$\bar{p}_5$	$p_{18}$
Reason	$\perp$	1	2	3	4	5	6	7	8	9

Let us take the **conflicting** clause  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$ .  
 $p_{18}$  is true due to clause 9. Resolution gives:

$$\frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17}}$$

10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

# Motivating Example

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

Literal	$p_{11}^d$	$\bar{p}_{12}$	$p_{16}$	$\bar{p}_2$	$\bar{p}_{10}$	$p_1$	$p_3$	$p_{26}$	$\bar{p}_5$	$p_{18}$
Reason	$\perp$	1	2	3	4	5	6	7	8	9

Let us take the **conflicting** clause  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$ .  
 $p_{18}$  is true due to clause 9. Resolution gives:

$$\frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17}}$$

10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

Now the last assigned literal that is false in the new clause is  $p_5$ .

The reason why  $p_5$  is false is clause 8.

Again, resolution:

$$\frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}}$$

# Motivating Example

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$

Literal	$p_{11}^d$	$\bar{p}_{12}$	$p_{16}$	$\bar{p}_2$	$\bar{p}_{10}$	$p_1$	$p_3$	$p_{26}$	$\bar{p}_5$	$p_{18}$
Reason	$\perp$	1	2	3	4	5	6	7	8	9

Let us take the **conflicting** clause  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$ .  
 $p_{18}$  is true due to clause 9. Resolution gives:

$$\frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17}}$$

10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

Now the last assigned literal that is false in the new clause is  $p_5$ .

The reason why  $p_5$  is false is clause 8.

Again, resolution:

$$\frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}}$$

The process is now iterated...

# Motivating Example

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal	$p_{11}^d$	$\bar{p}_{12}$	$p_{16}$	$\bar{p}_2$	$\bar{p}_{10}$	$p_1$	$p_3$	$p_{26}$	$\bar{p}_5$	$p_{18}$
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

# Motivating Example

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal	$p_{11}^d$	$\bar{p}_{12}$	$p_{16}$	$\bar{p}_2$	$\bar{p}_{10}$	$p_1$	$p_3$	$p_{26}$	$\bar{p}_5$	$p_{18}$
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

$$\begin{array}{c}
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18} \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5 \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}
 \end{array}$$

# Motivating Example

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal	$p_{11}^d$	$\bar{p}_{12}$	$p_{16}$	$\bar{p}_2$	$\bar{p}_{10}$	$p_1$	$p_3$	$p_{26}$	$\bar{p}_5$	$p_{18}$
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

$$\begin{array}{c}
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18} \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5 \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3 \\
 \hline
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}
 \end{array}$$

# Motivating Example

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal	$p_{11}^d$	$\bar{p}_{12}$	$p_{16}$	$\bar{p}_2$	$\bar{p}_{10}$	$p_1$	$p_3$	$p_{26}$	$\bar{p}_5$	$p_{18}$
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

$$\begin{array}{c}
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18} \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5 \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3 \\
 \hline
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \\
 \\
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1 \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8
 \end{array}$$

# Motivating Example

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal	$p_{11}^d$	$\bar{p}_{12}$	$p_{16}$	$\bar{p}_2$	$\bar{p}_{10}$	$p_1$	$p_3$	$p_{26}$	$\bar{p}_5$	$p_{18}$
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

$$\begin{array}{c}
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18} \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5 \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3 \\
 \hline
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}
 \end{array}$$

$$\begin{array}{c}
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1 \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8 \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10} \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20}
 \end{array}$$



# Motivating Example

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal	$p_{11}^d$	$\bar{p}_{12}$	$p_{16}$	$\bar{p}_2$	$\bar{p}_{10}$	$p_1$	$p_3$	$p_{26}$	$\bar{p}_5$	$p_{18}$
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

$$\begin{array}{c}
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18} \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5 \\
 \hline
 \bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3 \\
 \hline
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \\
 \\
 \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1 \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8 \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10} \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20} \quad p_{12} \vee \bar{p}_{16} \vee \bar{p}_2 \\
 \hline
 \bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16}
 \end{array}$$

# Motivating Example

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal	$p_{11}^d$	$\bar{p}_{12}$	$p_{16}$	$\bar{p}_2$	$\bar{p}_{10}$	$p_1$	$p_3$	$p_{26}$	$\bar{p}_5$	$p_{18}$
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

$$\begin{array}{l}
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17}} \quad p_{10} \vee \bar{p}_5 \\
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3}{\bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}} \\
 \frac{\bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1}{\bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8} \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20} \quad p_{12} \vee \bar{p}_{16} \vee \bar{p}_2}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16}} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \quad \bar{p}_{11} \vee p_{13} \vee p_{16}}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{11} \vee p_{13}}
 \end{array}$$

# Motivating Example

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal	$p_{11}^d$	$\bar{p}_{12}$	$p_{16}$	$\bar{p}_2$	$\bar{p}_{10}$	$p_1$	$p_3$	$p_{26}$	$\bar{p}_5$	$p_{18}$
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

$$\begin{array}{l}
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17}} \quad p_{10} \vee \bar{p}_5 \\
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}} \quad p_{10} \vee p_3 \\
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3}{\bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}} \\
 \frac{\bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_{10} \vee \bar{p}_1} \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_{10} \vee \bar{p}_1 \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \vee \bar{p}_2} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \vee \bar{p}_2 \quad p_{12} \vee \bar{p}_{16} \vee \bar{p}_2}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16}} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \quad \bar{p}_{11} \vee p_{13} \vee p_{16}}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{11} \vee p_{13}} \quad \bar{p}_{11} \vee p_6 \vee \bar{p}_{12} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{11} \vee p_{13} \quad \bar{p}_{11} \vee p_6 \vee \bar{p}_{12}}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{11} \vee p_{13} \vee p_6}
 \end{array}$$

# Motivating Example

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal	$p_{11}^d$	$\bar{p}_{12}$	$p_{16}$	$\bar{p}_2$	$\bar{p}_{10}$	$p_1$	$p_3$	$p_{26}$	$\bar{p}_5$	$p_{18}$
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

$$\begin{array}{l}
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5} \\
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3}{\bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}} \\
 \frac{\bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1}{\bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8 \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20} \quad p_{12} \vee \bar{p}_{16} \vee \bar{p}_2}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16}} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \quad \bar{p}_{11} \vee p_{13} \vee p_{16}}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \vee \bar{p}_{11} \vee p_{13} \quad \bar{p}_{11} \vee p_6 \vee \bar{p}_{12}} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{11} \vee p_{13} \vee p_6}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{11} \vee p_{13} \vee p_6}
 \end{array}$$

Now the unfolding of the propagations of the current decision level cannot continue any longer.

# Motivating Example

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal	$p_{11}^d$	$\bar{p}_{12}$	$p_{16}$	$\bar{p}_2$	$\bar{p}_{10}$	$p_1$	$p_3$	$p_{26}$	$\bar{p}_5$	$p_{18}$
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

$$\begin{array}{l}
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17}} \quad p_{10} \vee \bar{p}_5 \\
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17} \quad p_{10} \vee \bar{p}_5}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}} \quad p_{10} \vee p_3 \\
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee p_3}{\bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}} \\
 \frac{\bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_{10} \vee \bar{p}_1} \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_{10} \vee \bar{p}_1 \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_1 \vee p_{10} \vee p_2 \vee \bar{p}_4 \vee p_{20}} \quad p_{12} \vee \bar{p}_{16} \vee \bar{p}_2 \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_1 \vee p_{10} \vee p_2 \vee \bar{p}_4 \vee p_{20} \quad p_{12} \vee \bar{p}_{16} \vee \bar{p}_2}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16}} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \quad \bar{p}_{11} \vee p_{13} \vee p_{16}}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{11} \vee p_{13}} \quad \bar{p}_{11} \vee p_6 \vee \bar{p}_{12} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{11} \vee p_{13} \quad \bar{p}_{11} \vee p_6 \vee \bar{p}_{12}}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{11} \vee p_{13} \vee p_6}
 \end{array}$$

All obtained clauses are false in the assignment.

# Motivating Example

$M$  is  $p_{25} \bar{p}_{21}^d \bar{p}_6 \bar{p}_{17} p_{22}^d \bar{p}_{13} p_8 p_4^d p_{19} \bar{p}_{20}^d p_{23}$

Literal	$p_{11}^d$	$\bar{p}_{12}$	$p_{16}$	$\bar{p}_2$	$\bar{p}_{10}$	$p_1$	$p_3$	$p_{26}$	$\bar{p}_5$	$p_{18}$
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

1.  $\bar{p}_{11} \vee p_6 \vee \bar{p}_{12}$
2.  $\bar{p}_{11} \vee p_{13} \vee p_{16}$
3.  $p_{12} \vee \bar{p}_{16} \vee \bar{p}_2$
4.  $p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10}$
5.  $p_{10} \vee \bar{p}_8 \vee p_1$
6.  $p_{10} \vee p_3$
7.  $\bar{p}_3 \vee p_{26}$
8.  $p_{10} \vee \bar{p}_5$
9.  $\bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10.  $\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18}$
11.  $p_{21} \vee \bar{p}_6$
12.  $p_{21} \vee \bar{p}_{17}$
13.  $\bar{p}_{22} \vee \bar{p}_{13}$
14.  $p_{13} \vee p_8$
15.  $\bar{p}_4 \vee p_{19}$
16.  $p_{20} \vee p_{23}$
17.  $\bar{p}_{20} \vee p_{24}$
18.  $p_{25}$

$$\begin{array}{c}
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_{18} \quad \bar{p}_1 \vee \bar{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_5 \vee p_{17}} \quad p_{10} \vee \bar{p}_5 \\
 \frac{\bar{p}_3 \vee \bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}}{\bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10}} \quad p_{10} \vee p_3 \\
 \frac{\bar{p}_{19} \vee \bar{p}_1 \vee p_{17} \vee p_{10} \quad p_{10} \vee \bar{p}_8 \vee p_1}{\bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8} \quad p_2 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{10} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20} \quad p_{12} \vee \bar{p}_{16} \vee \bar{p}_2}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16}} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{16} \quad \bar{p}_{11} \vee p_{13} \vee p_{16}}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{11} \vee p_{13}} \quad \bar{p}_{11} \vee p_6 \vee \bar{p}_{12} \\
 \frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee p_{12} \vee \bar{p}_{11} \vee p_{13}}{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{11} \vee p_{13} \vee p_6}
 \end{array}$$

Lits assigned at last decision level 5 in blue

# Motivating Example

- Three clauses with only one literal assigned at the last DL (5):
  - ◆  $\bar{p}_{19} \vee p_{17} \vee p_{10} \vee \bar{p}_8$  (max DL of others: 3)
  - ◆  $\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20}$  (max DL of others: 4)
  - ◆  $\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee \bar{p}_4 \vee p_{20} \vee \bar{p}_{11} \vee p_{13} \vee p_6$  (max DL of others: 4)
- If we had had those clauses:
  - ◆ At DL. 3 we could've propagated  $p_{10}$
  - ◆ At DL. 4 we could've propagated  $p_2$
  - ◆ At DL. 4 we could've propagated  $\bar{p}_{11}$
- In practice procedure stops when we find the first such clause, because:
  - ◆ It is the cheapest one to find
  - ◆ It can propagate lits at a lower DL  
(literals of previous decision levels are never removed)

# Backjump rule

This example motivates us to introduce the rule:

$$\text{Backjump } M l^d N \parallel F \implies M l' \parallel F \text{ if } \left\{ \begin{array}{l} \text{for some clause } C \vee l' : \\ F \models C \vee l' \text{ and } M \models \neg C \\ l' \text{ is undefined in } M \\ l' \text{ or } \neg l' \text{ occurs in } F \end{array} \right.$$

The only thing we need is a **backjump clause**  $C \vee l'$  such that:

1. It is a logical consequence of the rest of the clauses
2. All its literals are false at some previous decision level  $d$ , except one which was undefined at  $d$



# Conflict Analysis

- The procedure shown in the example is called **conflict analysis**
- Why is the obtained clause a logical consequence of the input?
  - ◆ Because resolution is correct

# Conflict Analysis

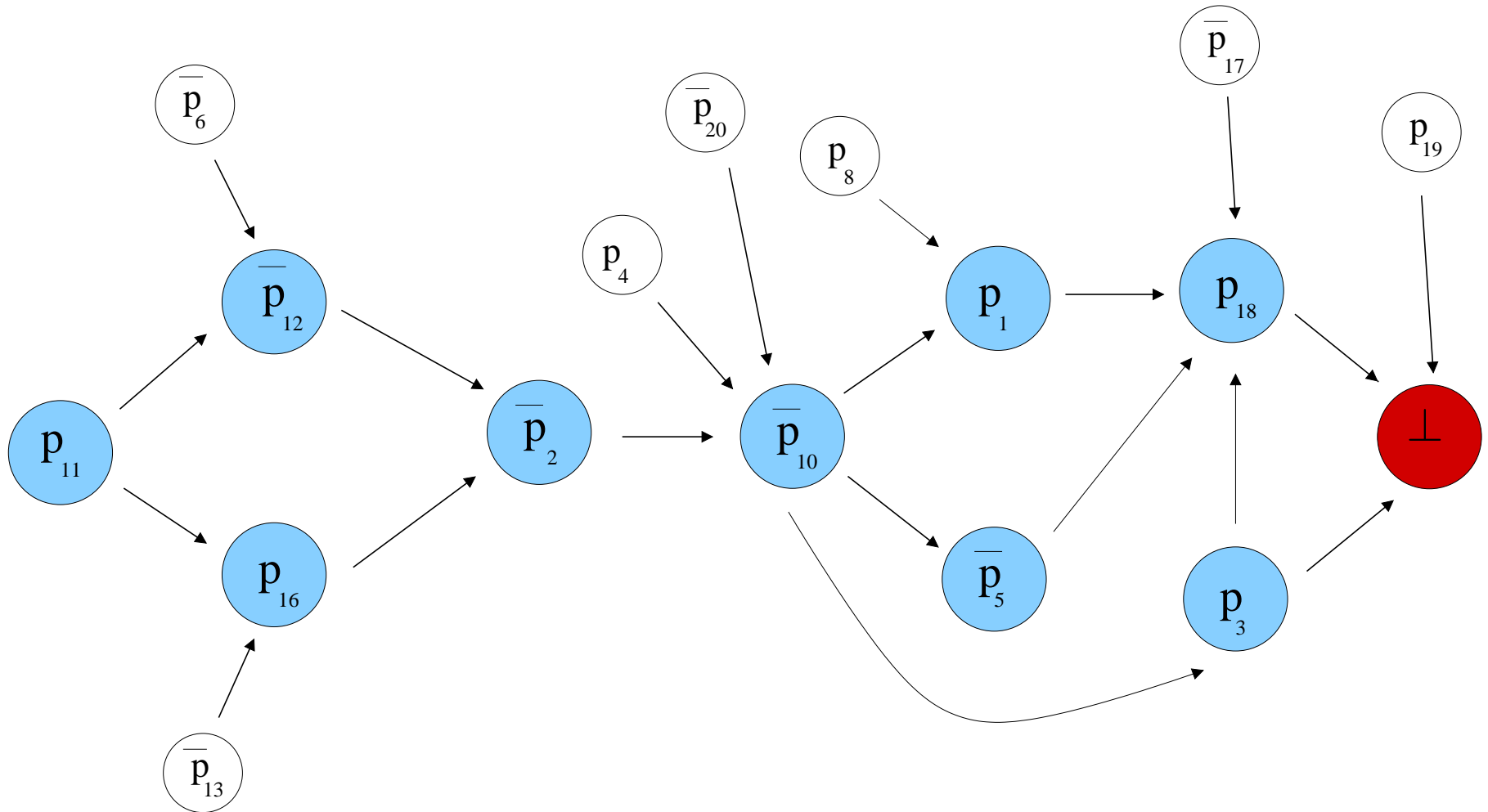
- The procedure shown in the example is called **conflict analysis**
- Why is the obtained clause false under the current assignment?
  - ◆ Conflicting clause is false under the current assignment
  - ◆ Each non-decision lit  $l$  false at the last decision level ( $dl$ ) can be resolved away with a reason clause of the form
$$l_1 \vee \dots \vee l_n \vee \neg l$$
  - ◆  $l$  is replaced by lits  $l_1, \dots, l_n$  such that all of them are false

# Conflict Analysis

- The procedure shown in the example is called **conflict analysis**
- Why a clause with only one lit at last decision level ( $dl$ ) can be obtained?
  - ◆ In the worst case, we will terminate with a clause with the last decision lit being the only set at decision level  $dl$

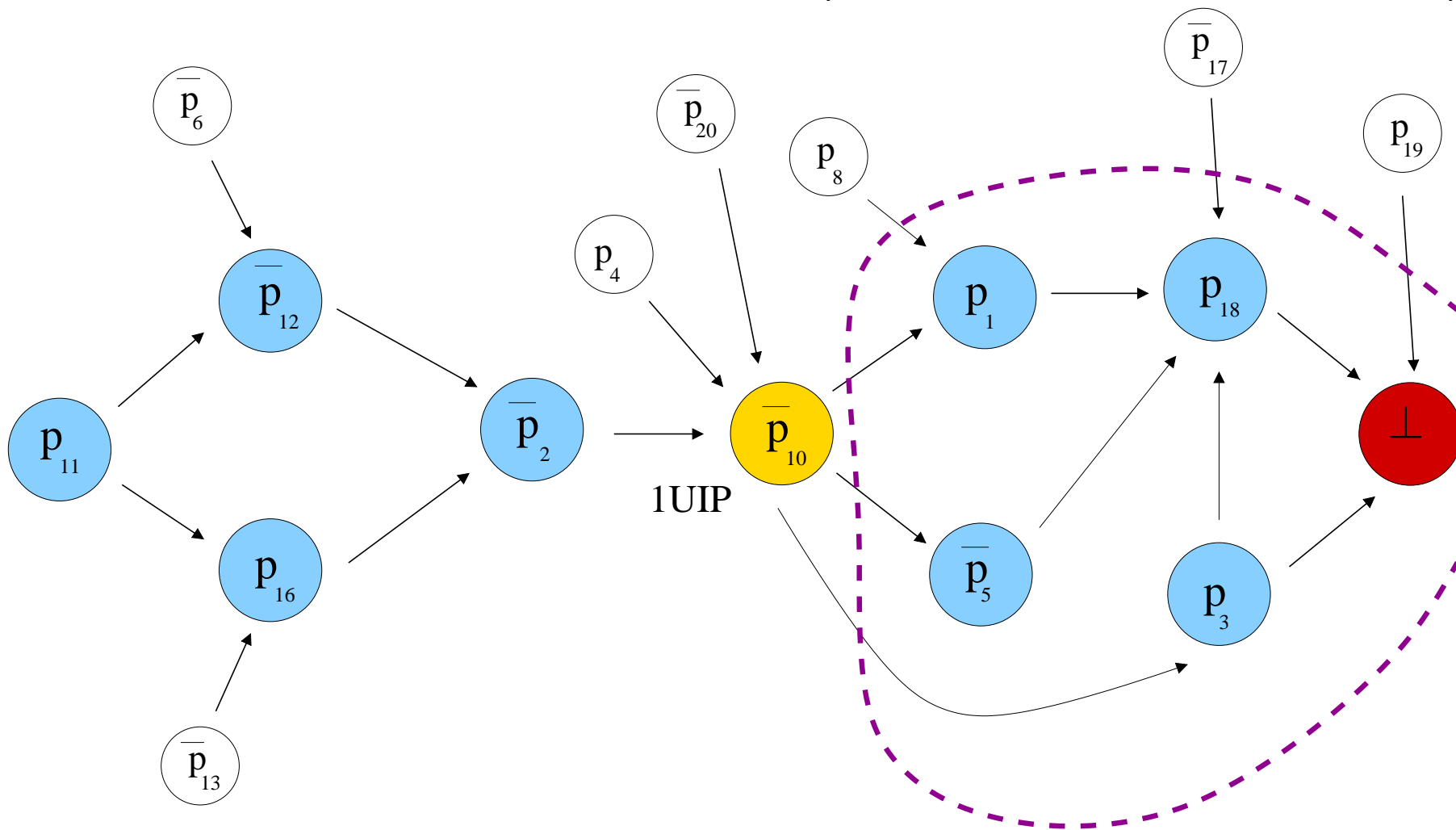
# Conflict Analysis - Conflict Graph

- Situation can be represented with the **conflict graph**:



# Conflict Analysis - Conflict Graph

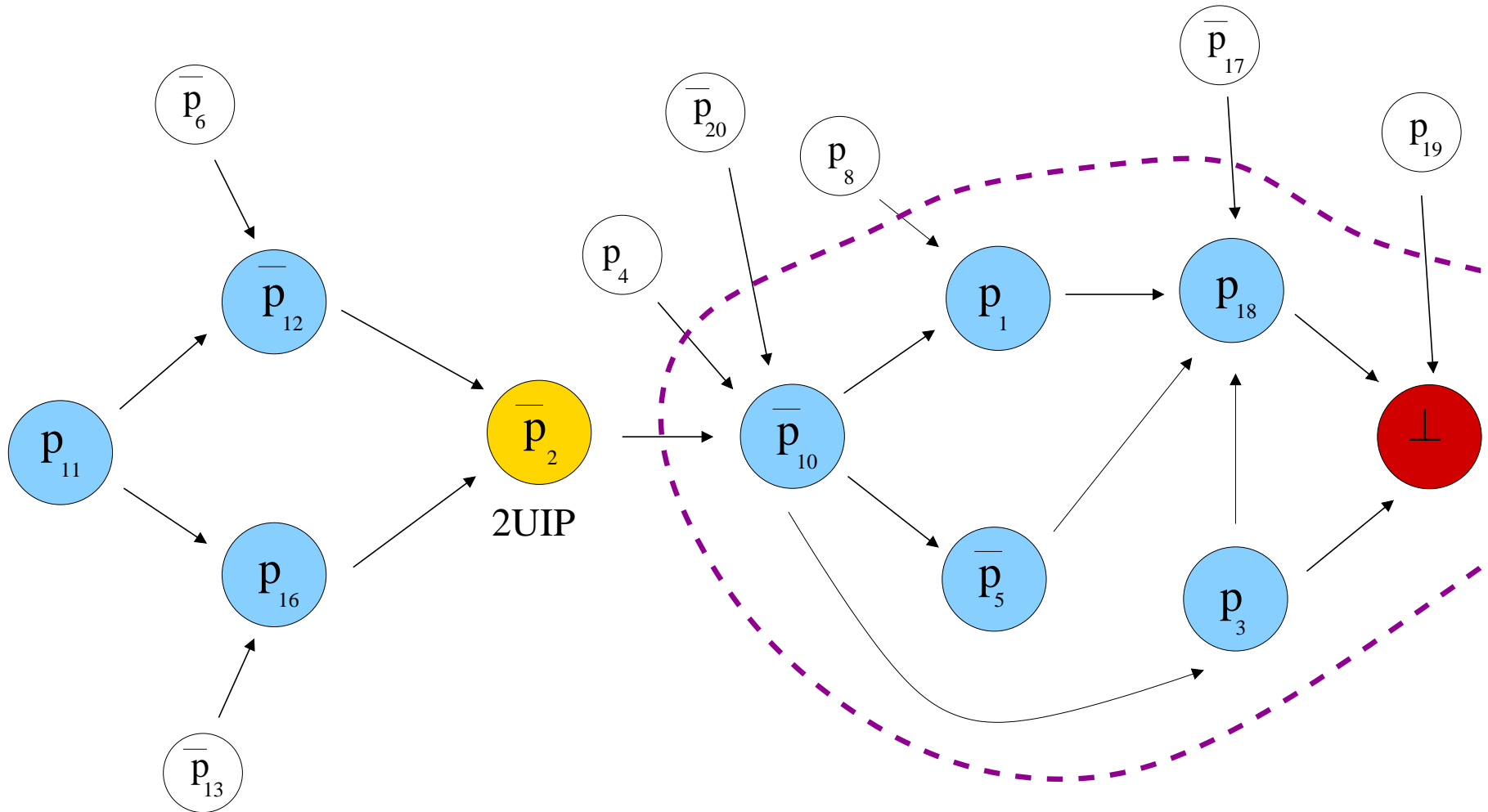
- The cut represents **1-UIP learning scheme** (UIP  $\equiv$  Unique Implication Point)



- Backjump clause is  $\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_{10}$

# Conflict Analysis - Conflict Graph

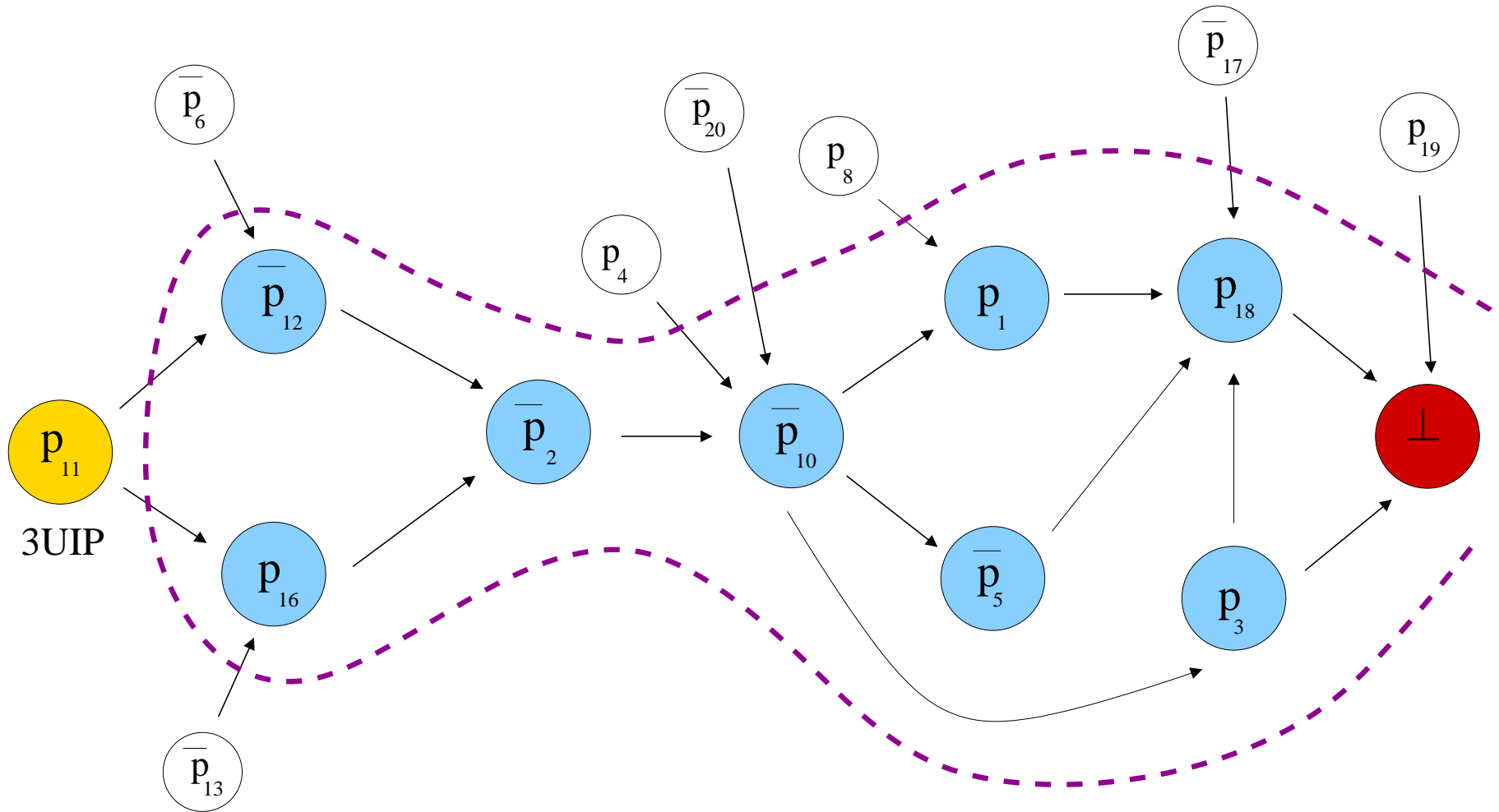
- This cut represents the 2-UIP learning scheme



- Backjump clause is  $\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_{20} \vee \bar{p}_4 \vee p_2$

# Conflict Analysis - Conflict Graph

- This cut represents the 3-UIP learning scheme



- Backjump clause is  $\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_{20} \vee \bar{p}_4 \vee p_6 \vee \bar{p}_{11} \vee p_{13}$

# What is a good lemma?

- Every time a conflict is found, conflict analysis is started
- Backjump clause is added to the clause database (we say it is **learned**):

**Learn**

$$M \parallel F \implies M \parallel F, C \text{ if } \begin{cases} \text{all atoms of } C \text{ occur in } F \\ F \models C \end{cases}$$

- Backjump clauses once they are learned are referred to as **lemmas**
- Learning them helps to **prevent future similar conflicts**
- In general **difficult** to **assess** in advance the **quality** of a lemma
- The set of literals of previous DL in the 2UIP contains the literals of previous DL in the 1UIP
- So 1UIP allows one to backjump to a lower or equal DL
- Also 1UIP gives shorter clauses than 2UIP



# Lemma Shortening

- But, given a lemma  $L$ , any lemma  $L' \subseteq L$  is clearly better.
- Given  $L$ , how to obtain a possible  $L'$ ?
- **LOCAL MINIMIZATION:**
  - ◆ Generate lemma  $L$  and mark its literals
  - ◆ Remove non-decision literals  $l \in L$  such that  $reason(\bar{l}) \setminus \{\bar{l}\}$  contains only marked literals

EXAMPLE: our 2-UIP clause was

$$\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20}$$

with  $reason(p_{19}) = \bar{p}_4 \vee p_{19}$ . Hence  $\bar{p}_{19}$  can be removed. Why?

# Lemma Shortening

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with  $reason(p_{19}) = \bar{p}_4 \vee p_{19}$ . Hence  $\bar{p}_{19}$  can be removed. Why?

$$\frac{\bar{p}_{19} \vee p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20} \qquad \bar{p}_4 \vee p_{19}}{p_{17} \vee \bar{p}_8 \vee p_2 \vee \bar{p}_4 \vee p_{20}}$$

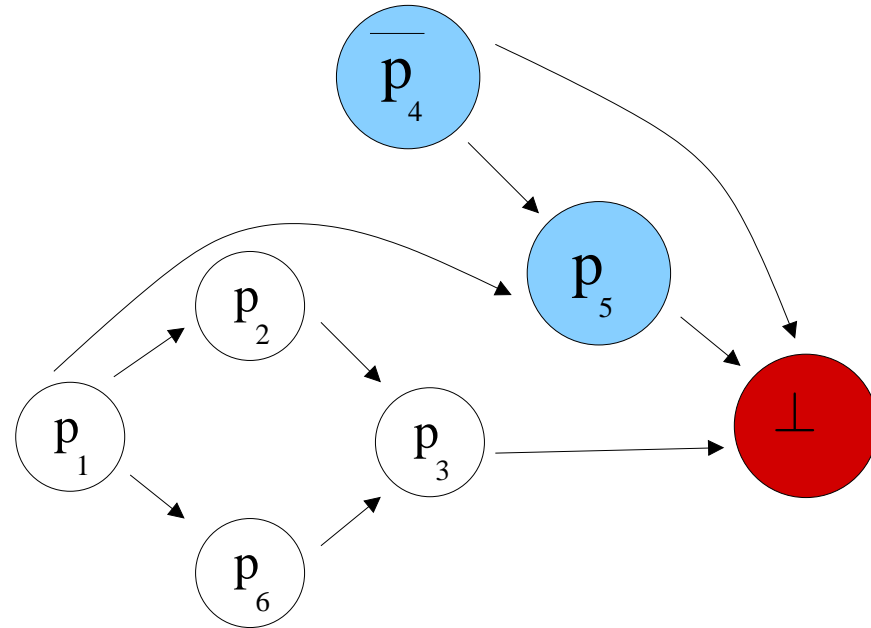
# Lemma Shortening

## ■ RECURSIVE MINIMIZATION:

- ◆ Generate lemma  $L$  and mark its literals
- ◆ Remove non-decision literals  $l \in L$  such that search backwards from  $\bar{l}$  in the implication graph reaches only negations of marked literals

1.  $\bar{p}_1 \vee p_2$
2.  $\bar{p}_1 \vee p_6$
3.  $\bar{p}_2 \vee \bar{p}_6 \vee p_3$
4.  $\bar{p}_1 \vee p_4 \vee p_5$
5.  $\bar{p}_3 \vee p_4 \vee \bar{p}_5$

$$\emptyset \implies \dots \implies p_1^d p_2 p_6 p_3 \bar{p}_4^d p_5$$



- 1UIP lemma is  $\bar{p}_3 \vee p_4 \vee \bar{p}_1$
- $\bar{p}_3$  is clearly removable

# Overview of the session

- Conflict Analysis
  - ◆ Motivating example
  - ◆ Backjumping
  - ◆ Conflict graph
  - ◆ Lemma shortening
- Lemma removal
- Decision heuristics
- Restarts
- Efficient implementation of UnitProp:
  - ◆ Occur lists
  - ◆ Two-watched literals
- Final remarks

# Lemma Removal

- Effects of adding lemmas:

- + Reduces the search space

- Space traversal slower since unit propagation becomes expensive

- Hence we cannot keep all generated lemmas. We need:

Forget

$$M \parallel F, C \implies M \parallel F \text{ if } F \models C$$

- Which lemmas to keep and which ones to forget?

- ◆ Each lemma has a number called activity

- ◆ Activity is incremented when lemma is used in conflict analysis

- ◆ From time to time, lemmas with low activity are removed

- ◆ Mixed policies: keep

- short lemmas

- recent lemmas

- lemmas with low Literals Blocks Distance (LDB):

- no. of different decision levels involved in a clause

# Overview of the session

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# Decision Heuristic

- SAT instances may have (tens, hundreds of ?) thousands of variables
- Most SAT instances have **clusters of variables**:  
sets of variables that are semantically linked

**GOAL:** to force the SAT solver to work on one cluster at a time

- Each variable has an associated **activity**
- Each time it appears in a conflict analysis, its activity is incremented
- **Recent** activity should be given **more importance**:
  - ◆ Divide all activities by integer  $K$  from time to time, or
  - ◆ Keep increasing the activity increment
- **Decide** chooses unassigned var with **highest activity**
- Note that heuristic does not depend on clauses: **CHEAP!**
- Value selection: take *false*, or last tried value for that variable (**last phase**)

# Overview of the session

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# Restarts

- Early mistakes in the search tree have dramatic effects in running time

## HOW TO AVOID THIS BEHAVIOUR?

- Introduce restarts:

Restart

$$M \parallel F \implies \emptyset \parallel F$$

- Why should a new run behave differently? And why could it be better?
- In a new run, the solver may behave better among others thanks to:
  - ◆ the activities of the variables
  - ◆ the learned lemmas

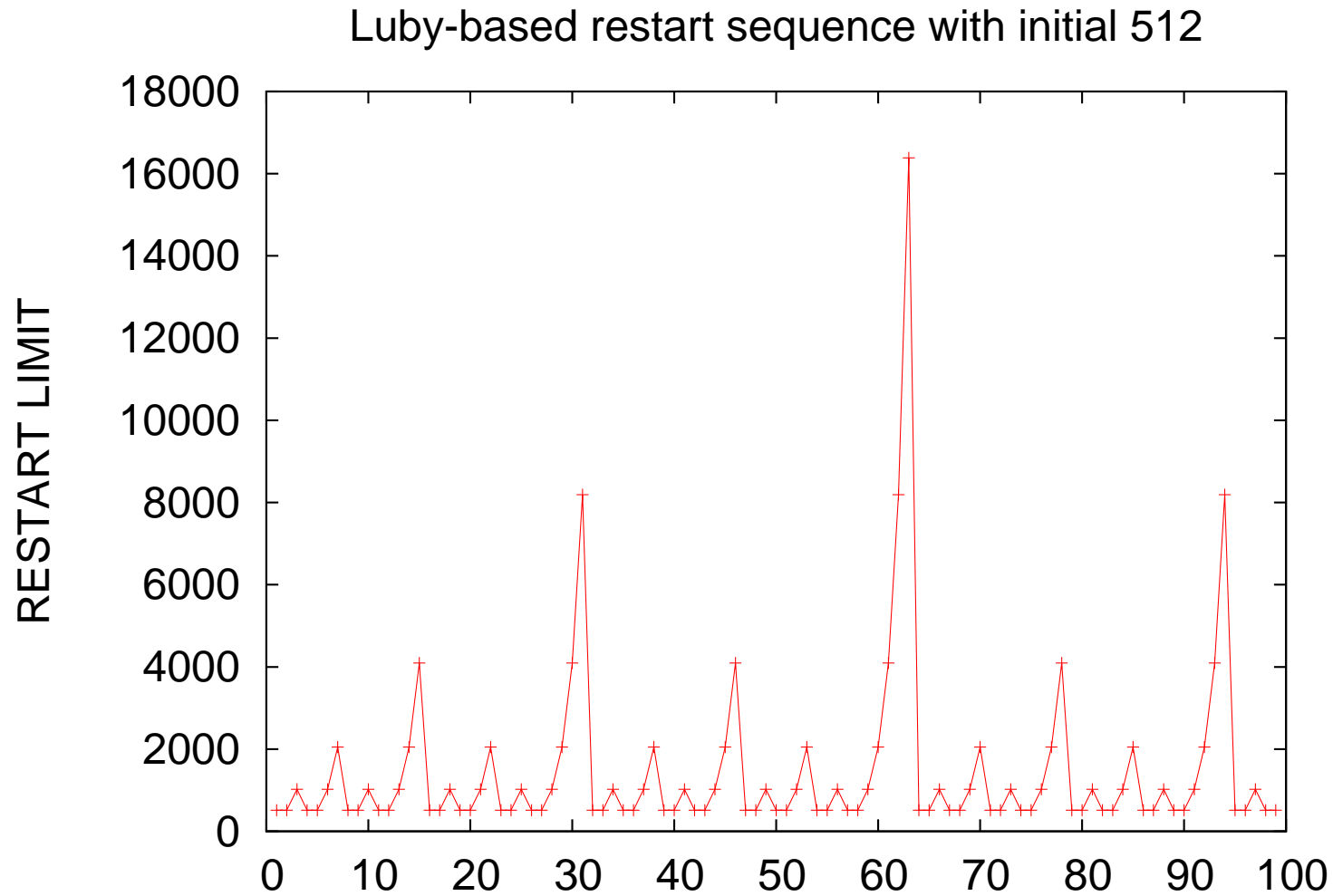
Only the assignment is reset!

# Restarts - Strategies

- Unrestricted application of Restart leads to incompleteness
- What is done in practice?
- A **restart strategy** tells after how many conflicts a restart should be made
  - ◆ Set initial RESTART\_LIMIT
  - ◆ After RESTART\_LIMIT conflicts:
    - Update RESTART\_LIMIT according to the restart strategy
    - Apply Restart
- For example:
  - ◆ Let RESTART\_LIMIT follow the **Luby sequence**, defined as:
    - $r_0 := N; r_i := N \cdot l_i$ , where

$$l_i = \begin{cases} 2^{k-1} & \text{if } \exists k \text{ with } i = 2^k - 1 \\ l_{i-2^{k-1}+1} & \text{if } \exists k \text{ with } 2^{k-1} \leq i < 2^k - 1 \end{cases}$$

# Restarts - Strategies

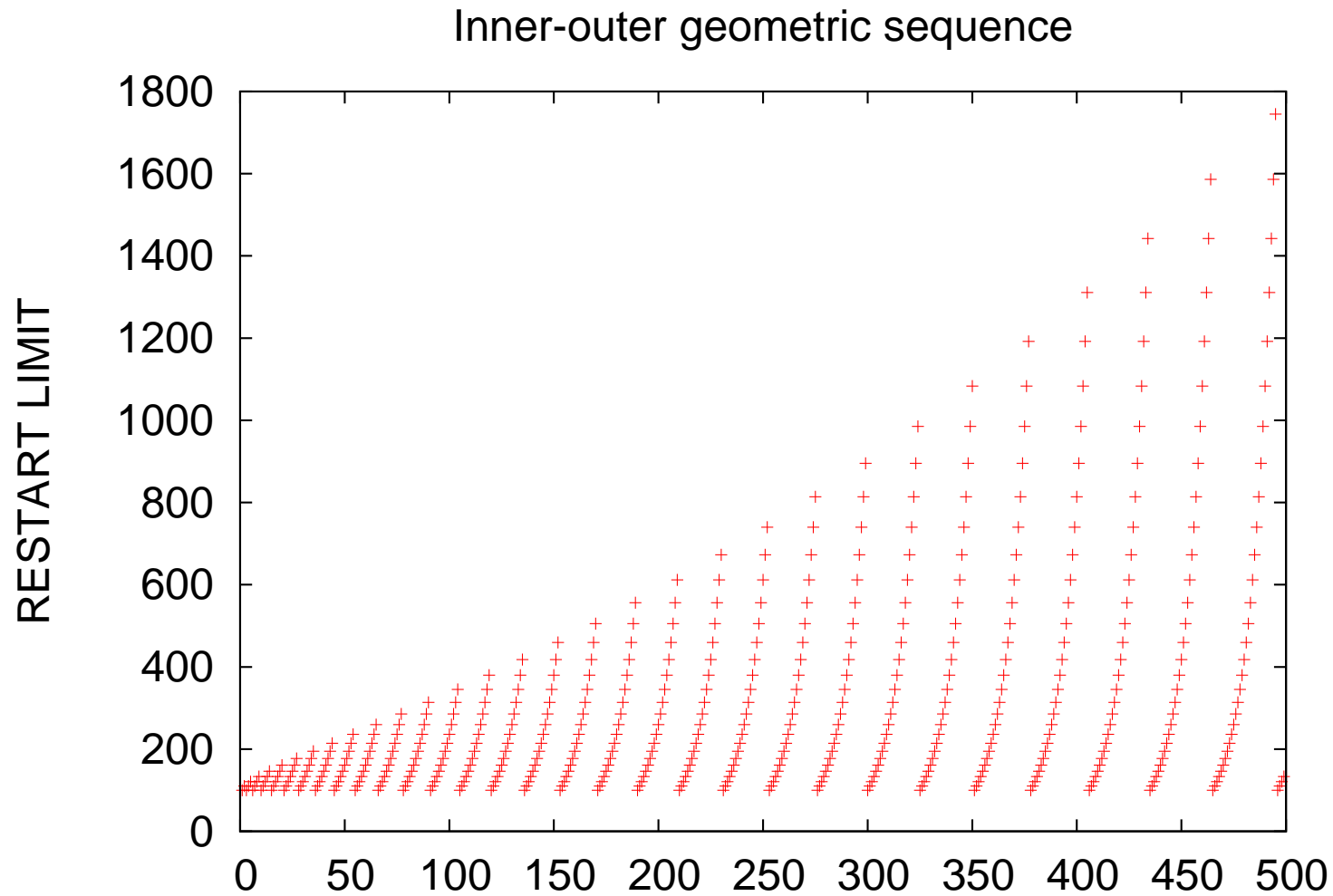


# Restarts - Strategies

Another possibility is an inner-outer geometric sequence:

```
for (int inner = 100, outer = 100;;){  
  
    // Run SAT-solver for 'inner' conflicts  
  
    if (inner >= outer){  
        outer *= 1.1;  
        inner = 100;  
    }  
    else  
        inner *= 1.1  
  
}
```

# Restarts - Strategies



# Overall CDCL algorithm

```
while(true){  
  
    while (propagate_gives_conflict()){  
        if (decision_level==0) return UNSAT;  
        else analyze_conflict();  
    }  
  
    restart_if_applicable();  
    remove_lemmas_if_applicable();  
  
    if (!decide()) returns SAT; // All vars assigned  
}
```

# Overview of the session

- Conflict Analysis
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# Performance of SAT Solvers

- The most important tasks that a SAT solver performs are:
  - ◆ Choose which variable to **Decide** on
  - ◆ Apply **unit propagation** exhaustively
  - ◆ **Analyze conflicts**
- When profiling a state-of-the art SAT solver we get:
  - ◆ Variable selection ( $\approx 10\%$ )
  - ◆ Unit propagation application ( $\approx 80\%$ )
  - ◆ Conflict analysis ( $\approx 10\%$ )
- Hence, the most important thing to **optimize** is **unit propagation**, aka BCP (Boolean Constraint Propagation)



# BCP with Occur Lists

- BCP only has to detect **propagating** or **conflicting clauses**
- There is **no need** to detect that all **clauses** are **true**
- Instead of traversing the whole clause set again and again:
  - ◆ For each literal, store the clauses where it appears in **occur lists**
  - ◆ Every time a new lit  $l$  is added to the assignment, only clauses containing  $\bar{l}$  need to be visited
- Let's see how it would work with an **example**

# BCP with Occur Lists

	$p_1$	$p_2$	$p_3$	$p_4$	$p_5$	$p_6$
<b>Model</b>	U	U	U	U	U	U

$$\textcircled{1} \quad \mathbf{p}_1 \quad \overline{\mathbf{p}}_2 \quad \overline{\mathbf{p}}_6$$
$$\textcircled{2} \quad \mathbf{p}_2 \quad \overline{\mathbf{p}}_3 \quad \mathbf{p}_5 \quad \overline{\mathbf{p}}_1 \quad \overline{\mathbf{p}}_6$$
$$\textcircled{3} \quad \mathbf{p}_6 \quad \mathbf{p}_2 \quad \mathbf{p}_4$$

4    $\mathbf{p}_1$     $\mathbf{p}_2$

$$\textcircled{5} \quad \mathbf{p}_6 \quad \mathbf{p}_1 \quad \mathbf{p}_3$$
$$\textcircled{6} \quad \overline{\mathbf{p}}_5 \quad \mathbf{p}_4 \quad \mathbf{p}_2$$

ToPropagate

## Clauses With

$p_1$	1	4		
$p_2$	2	3	4	6
$p_3$	5			
$p_4$	3	6		
$p_5$	2			
$p_6$	3			

$\overline{p_1}$	2	5	
$\overline{p_2}$	1		
$\overline{p_3}$	2		
$\overline{p_4}$			
$\overline{p_5}$	6		
$\overline{p_6}$	1	2	5

Current assignment:  $\emptyset$

# BCP with Occur Lists

	$\mathbf{p}_1$	$\mathbf{p}_2$	$\mathbf{p}_3$	$\mathbf{p}_4$	$\mathbf{p}_5$	$\mathbf{p}_6$
<b>Model</b>	U	F	U	U	U	U

$$\textcircled{1} \quad \mathbf{p}_1 \quad \overline{\mathbf{p}}_2 \quad \overline{\mathbf{p}}_6$$
$$\textcircled{2} \quad \mathbf{p}_2 \quad \overline{\mathbf{p}}_3 \quad \mathbf{p}_5 \quad \overline{\mathbf{p}}_1 \quad \overline{\mathbf{p}}_6$$

ToPropagate

## Clauses With

$$\textcircled{3} \quad \mathbf{p}_6 \quad \mathbf{p}_2 \quad \mathbf{p}_4$$

4    $\mathbf{p}_1$     $\mathbf{p}_2$

$$\textcircled{5} \quad \overline{\mathbf{p}}_6 \quad \overline{\mathbf{p}}_1 \quad \mathbf{p}_3$$
$$\bigcirc_6 \quad \overline{\mathbf{p}}_5 \quad \mathbf{p}_4 \quad \mathbf{p}_2$$
$$\overline{\mathbf{p}}_2$$

$p_1$	1	4		
$p_2$	2	3	4	6
$p_3$	5			
$p_4$	3	6		
$p_5$	2			
$p_6$	3			

$\overline{p_1}$	2	5	
$\overline{p_2}$	1		
$\overline{p_3}$	2		
$\overline{p_4}$			
$\overline{p_5}$	6		
$\overline{p_6}$	1	2	5

Current assignment:  $\bar{p}_2^d$

Now, we propagate  $\bar{p}_2$  visiting `ClausesWith`[ $p_2$ ]

# BCP with Occur Lists

	$\mathbf{p}_1$	$\mathbf{p}_2$	$\mathbf{p}_3$	$\mathbf{p}_4$	$\mathbf{p}_5$	$\mathbf{p}_6$
<b>Model</b>	U	F	U	U	U	U

$$\textcircled{1} \quad \mathbf{p}_1 \quad \overline{\mathbf{p}}_2 \quad \overline{\mathbf{p}}_6$$
$$\textcircled{2} \quad \mathbf{p}_2 \quad \overline{\mathbf{p}}_3 \quad \mathbf{p}_5 \quad \overline{\mathbf{p}}_1 \quad \overline{\mathbf{p}}_6$$
$$\textcircled{3} \quad \mathbf{p}_6 \quad \mathbf{p}_2 \quad \mathbf{p}_4$$

4    $p_1$     $p_2$

$$\begin{array}{cccc} \textcircled{5} & \overline{\mathbf{p}}_6 & \overline{\mathbf{p}}_1 & \mathbf{p}_3 \end{array}$$
$$\textcircled{6} \quad \overline{\mathbf{p}}_5 \quad \mathbf{p}_4 \quad \mathbf{p}_2$$

ToPropagate

## Clauses With

$p_1$	1	4		
$p_2$	2	3	4	6
$p_3$	5			
$p_4$	3	6		
$p_5$	2			
$p_6$	3			

$\overline{p_1}$	2	5	
$\overline{p_2}$	1		
$\overline{p_3}$	2		
$\overline{p_4}$			
$\overline{p_5}$	6		
$\overline{p_6}$	1	2	5

Current assignment:  $\bar{p}_2^d$

Literal  $p_1$  has to be added to the assignment

$$\textcircled{1} \quad \mathbf{p}_1 \quad \overline{\mathbf{p}}_2 \quad \overline{\mathbf{p}}_6 \quad \text{Mod}$$

	$\mathbf{p}_1$	$\mathbf{p}_2$	$\mathbf{p}_3$	$\mathbf{p}_4$	$\mathbf{p}_5$	$\mathbf{p}_6$
Model	T	F	U	U	U	U

$$\textcircled{1} \quad \mathbf{p}_1 \quad \overline{\mathbf{p}}_2 \quad \overline{\mathbf{p}}_6$$
$$\textcircled{2} \quad \mathbf{p}_2 \quad \overline{\mathbf{p}}_3 \quad \mathbf{p}_5 \quad \overline{\mathbf{p}}_1 \quad \overline{\mathbf{p}}_6$$

## ToPropagate

## Clauses With

$$\textcircled{3} \quad \mathbf{p}_6 \quad \mathbf{p}_2 \quad \mathbf{p}_4$$

4    $\mathbf{p}_1$     $\mathbf{p}_2$

$$\textcircled{5} \quad \overline{\mathbf{p}}_6 \quad \overline{\mathbf{p}}_1 \quad \mathbf{p}_3$$
$$\textcircled{6} \quad \overline{\mathbf{p}}_5 \quad \mathbf{p}_4 \quad \mathbf{p}_2$$

$\mathbf{p}_1$

$p_1$	1	4		
$p_2$	2	3	4	6
$p_3$	5			
$p_4$	3	6		
$p_5$	2			
$p_6$	3			

$\overline{p_1}$	2	5	
$\overline{p_2}$	1		
$\overline{p_3}$	2		
$\overline{p_4}$			
$\overline{p_5}$	6		
$\overline{p_6}$	1	2	5

Current assignment:  $\bar{p}_2^d p_1$

Now, we propagate  $p_1$  visiting  $\text{ClausesWith}[\bar{p}_1]$

## 33 / 56

$$\textcircled{1} \quad \mathbf{p}_1 \quad \overline{\mathbf{p}}_2 \quad \overline{\mathbf{p}}_6$$
$$\textcircled{2} \quad \mathbf{p}_2 \quad \overline{\mathbf{p}}_3 \quad \mathbf{p}_5 \quad \overline{\mathbf{p}}_1 \quad \overline{\mathbf{p}}_6$$

## Clauses With

$$\textcircled{3} \quad \mathbf{p}_6 \quad \mathbf{p}_2 \quad \mathbf{p}_4$$

4    $\mathbf{p}_1$     $\mathbf{p}_2$

$$\textcircled{5} \quad \overline{\mathbf{p}}_6 \quad \overline{\mathbf{p}}_1 \quad \mathbf{p}_3$$
$$\textcircled{6} \quad \overline{\mathbf{p}}_5 \quad \mathbf{p}_4 \quad \mathbf{p}_2$$

$p_1$	1	4		
$p_2$	2	3	4	6
$p_3$	5			
$p_4$	3	6		
$p_5$	2			
$p_6$	3			

$\overline{p_1}$	2	5	
$\overline{p_2}$	1		
$\overline{p_3}$	2		
$\overline{p_4}$			
$\overline{p_5}$	6		
$\overline{p_6}$	1	2	5

No lit is propagated, we have to decide

$$\textcircled{1} \quad \mathbf{p}_1 \quad \overline{\mathbf{p}}_2 \quad \overline{\mathbf{p}}_6$$

	$p_1$	$p_2$	$p_3$	$p_4$	$p_5$	$p_6$
Model	T	F	U	F	U	U

$$\textcircled{2} \quad \mathbf{p}_2 \quad \overline{\mathbf{p}}_3 \quad \mathbf{p}_5 \quad \overline{\mathbf{p}}_1 \quad \overline{\mathbf{p}}_6$$

## Clauses With

$$\textcircled{3} \quad \mathbf{p}_6 \quad \mathbf{p}_2 \quad \mathbf{p}_4$$

4    $\mathbf{p}_1$     $\mathbf{p}_2$

$$\textcircled{5} \quad \overline{\mathbf{p}}_6 \quad \overline{\mathbf{p}}_1 \quad \mathbf{p}_3$$
$$\bigcirc_6 \quad \overline{\mathbf{p}}_5 \quad \mathbf{p}_4 \quad \mathbf{p}_2$$
$$\overline{\mathbf{p}}_4$$

$p_1$	1	4		
$p_2$	2	3	4	6
$p_3$	5			
$p_4$	3	6		
$p_5$	2			
$p_6$	3			

$\overline{p_1}$	2	5	
$\overline{p_2}$	1		
$\overline{p_3}$	2		
$\overline{p_4}$			
$\overline{p_5}$	6		
$\overline{p_6}$	1	2	5

Now, we propagate  $\bar{p}_4$  visiting `ClausesWith`[ $p_4$ ]

# BCP with Occur Lists

	$p_1$	$p_2$	$p_3$	$p_4$	$p_5$	$p_6$
Model	T	F	U	F	U	U

$$\textcircled{1} \quad \mathbf{p}_1 \quad \overline{\mathbf{p}}_2 \quad \overline{\mathbf{p}}_6$$
$$\textcircled{2} \quad \mathbf{p}_2 \quad \overline{\mathbf{p}}_3 \quad \mathbf{p}_5 \quad \overline{\mathbf{p}}_1 \quad \overline{\mathbf{p}}_6$$

ToPropagate

## Clauses With

3    $p_6$     $p_2$     $p_4$

4    $\mathbf{p}_1$     $\mathbf{p}_2$

$$\textcircled{5} \quad \overline{\mathbf{p}}_6 \quad \overline{\mathbf{p}}_1 \quad \mathbf{p}_3$$

6  $\bar{p}_5$   $p_4$   $p_2$

$p_1$	1	4		
$p_2$	2	3	4	6
$p_3$	5			
$p_4$	3	6		
$p_5$	2			
$p_6$	3			

$\overline{p_1}$	2	5	
$\overline{p_2}$	1		
$\overline{p_3}$	2		
$\overline{p_4}$			
$\overline{p_5}$	6		
$\overline{p_6}$	1	2	5

Current assignment:  $\bar{p}_2^d p_1 \bar{p}_4^d$

Literals  $p_6, \bar{p}_5$  have to be added to the assignment



## 36 / 56

	$p_1$	$p_2$	$p_3$	$p_4$	$p_5$	$p_6$
Model	T	F	U	F	F	T

$$\textcircled{1} \quad \mathbf{p}_1 \quad \overline{\mathbf{p}}_2 \quad \overline{\mathbf{p}}_6$$
$$\textcircled{2} \quad \mathbf{p}_2 \quad \overline{\mathbf{p}}_3 \quad \mathbf{p}_5 \quad \overline{\mathbf{p}}_1 \quad \overline{\mathbf{p}}_6$$
$$\textcircled{3} \quad \mathbf{p}_6 \quad \mathbf{p}_2 \quad \mathbf{p}_4$$

4    $\mathbf{p}_1$     $\mathbf{p}_2$

$$\textcircled{5} \quad \overline{\mathbf{p}}_6 \quad \overline{\mathbf{p}}_1 \quad \mathbf{p}_3$$
$$\textcircled{6} \quad \overline{\mathbf{p}}_5 \quad \mathbf{p}_4 \quad \mathbf{p}_2$$

ToPropagate

## Clauses With

$\overline{\mathbf{p}}_5$
$\mathbf{p}_6$

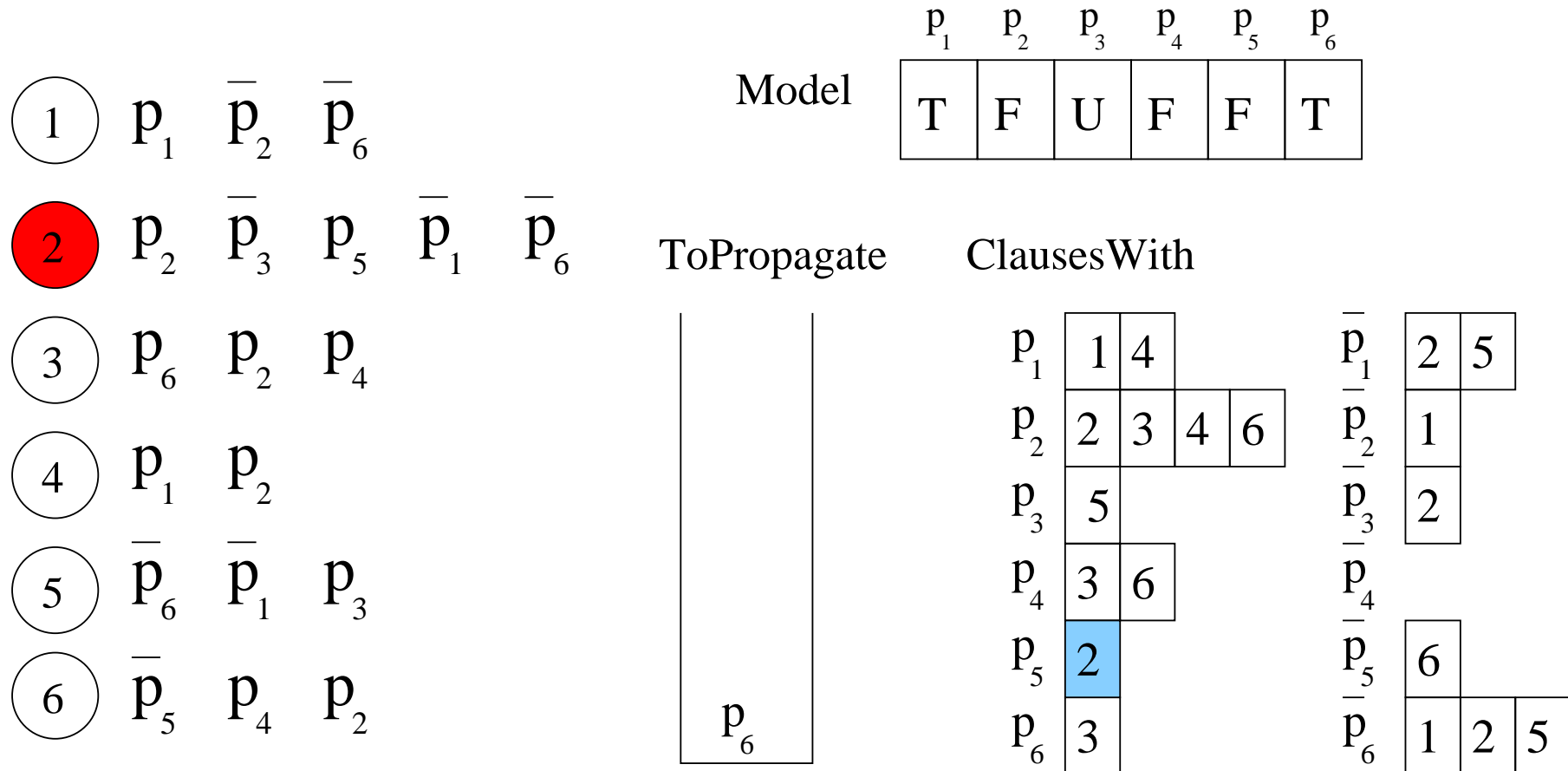
$p_1$	1	4		
$p_2$	2	3	4	6
$p_3$	5			
$p_4$	3	6		
$p_5$	2			
$p_6$	3			

$\overline{p_1}$	2	5	
$\overline{p_2}$	1		
$\overline{p_3}$	2		
$\overline{p_4}$			
$\overline{p_5}$	6		
$\overline{p_6}$	1	2	5

Current assignment:  $\bar{p}_2^d p_1 \bar{p}_4^d p_6 \bar{p}_5$

Now, we propagate  $\bar{p}_5$  visiting ClausesWith[ $p_5$ ]

# BCP with Occur Lists



Current assignment:  $\bar{p}_2^d p_1 \bar{p}_4^d p_6 \bar{p}_5$

Literal  $\bar{p}_3$  has to be added to the assignment

$$\textcircled{1} \quad \mathbf{p}_1 \quad \overline{\mathbf{p}}_2 \quad \overline{\mathbf{p}}_6$$
$$\textcircled{2} \quad \mathbf{p}_2 \quad \overline{\mathbf{p}}_3 \quad \mathbf{p}_5 \quad \overline{\mathbf{p}}_1 \quad \overline{\mathbf{p}}_6$$
$$\textcircled{3} \quad \mathbf{p}_6 \quad \mathbf{p}_2 \quad \mathbf{p}_4$$

4    $\mathbf{p}_1$     $\mathbf{p}_2$

$$\textcircled{5} \quad \overline{\mathbf{p}}_6 \quad \overline{\mathbf{p}}_1 \quad \mathbf{p}_3$$
$$\textcircled{6} \quad \overline{\mathbf{p}}_5 \quad \mathbf{p}_4 \quad \mathbf{p}_2$$
$$\overline{\mathbf{p}}_3$$

Now, we propagate  $\bar{p}_3$  visiting ClausesWith[ $p_3$ ]

	$p_1$	$p_2$	$p_3$	$p_4$	$p_5$	$p_6$
Model	T	F	F	F	F	T

$$\textcircled{1} \quad \mathbf{p}_1 \quad \overline{\mathbf{p}}_2 \quad \overline{\mathbf{p}}_6$$
$$\textcircled{2} \quad \mathbf{p}_2 \quad \overline{\mathbf{p}}_3 \quad \mathbf{p}_5 \quad \overline{\mathbf{p}}_1 \quad \overline{\mathbf{p}}_6$$
$$\textcircled{3} \quad \mathbf{p}_6 \quad \mathbf{p}_2 \quad \mathbf{p}_4$$

4    $\mathbf{p}_1$     $\mathbf{p}_2$

5  $\bar{p}_6$   $\bar{p}_1$   $p_3$

$$\textcircled{6} \quad \overline{\mathbf{p}}_5 \quad \mathbf{p}_4 \quad \mathbf{p}_2$$

ToPropagate

## Clauses With

$\mathbf{p}_6$

$p_1$	1	4		
$p_2$	2	3	4	6
$p_3$	5			
$p_4$	3	6		
$p_5$	2			
$p_6$	3			

$\overline{p_1}$	2	5	
$\overline{p_2}$	1		
$\overline{p_3}$	2		
$\overline{p_4}$			
$\overline{p_5}$	6		
$\overline{p_6}$	1	2	5

Current assignment:  $\bar{p}_2^d p_1 \bar{p}_4^d p_6 \bar{p}_5 \bar{p}_3$

Clause 5 indicates a conflict. Backtrack/backjump is called.

# BCP with Occur Lists

	$\mathbf{p}_1$	$\mathbf{p}_2$	$\mathbf{p}_3$	$\mathbf{p}_4$	$\mathbf{p}_5$	$\mathbf{p}_6$
Model	T	F	U	T	U	U

$$\textcircled{1} \quad \mathbf{p}_1 \quad \overline{\mathbf{p}}_2 \quad \overline{\mathbf{p}}_6$$
$$\textcircled{2} \quad \mathbf{p}_2 \quad \overline{\mathbf{p}}_3 \quad \mathbf{p}_5 \quad \overline{\mathbf{p}}_1 \quad \overline{\mathbf{p}}_6$$

ToPropagate

## Clauses With

$$\textcircled{3} \quad \mathbf{p}_6 \quad \mathbf{p}_2 \quad \mathbf{p}_4$$

4    $\mathbf{p}_1$     $\mathbf{p}_2$

$$\textcircled{5} \quad \overline{\mathbf{p}}_6 \quad \overline{\mathbf{p}}_1 \quad \mathbf{p}_3$$
$$\textcircled{6} \quad \overline{\mathbf{p}}_5 \quad \mathbf{p}_4 \quad \mathbf{p}_2$$

$p_4$

$p_1$	1	4		
$p_2$	2	3	4	6
$p_3$	5			
$p_4$	3	6		
$p_5$	2			
$p_6$	3			

$\overline{p_1}$	2	5	
$\overline{p_2}$	1		
$\overline{p_3}$	2		
$\overline{p_4}$			
$\overline{p_5}$	6		
$\overline{p_6}$	1	2	5

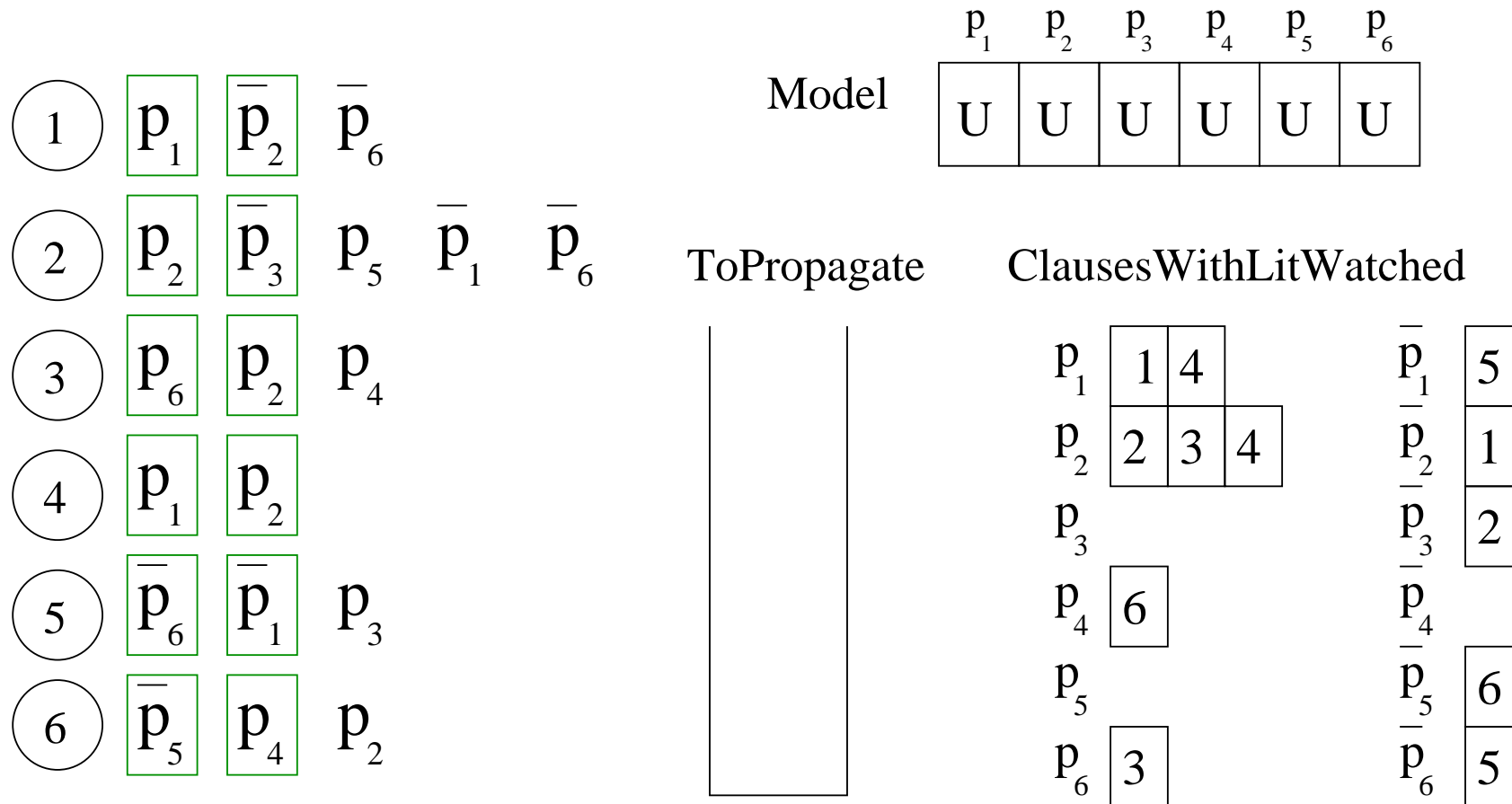
Current assignment:  $\bar{p}_2^d p_1 p_4$

Procedure continues propagating  $p_4$

# BCP - How to improve it?

- The key observation is the following:  
**A clause with 2 non-false literals can't be propagating or conflicting**
- For each clause we will try to watch two non-false literals
- Enough to visit a clause when a watched literal becomes false
- If 2 non-false literals do not exist, this is because:
  - ◆ All the lits are false (then backtrack)
  - ◆ All the lits are false but one, which is undef (then propagate)
  - ◆ All the lits are false but one, which is true
- This is called the **two watched literals** scheme

# BCP - Two Watched Literals



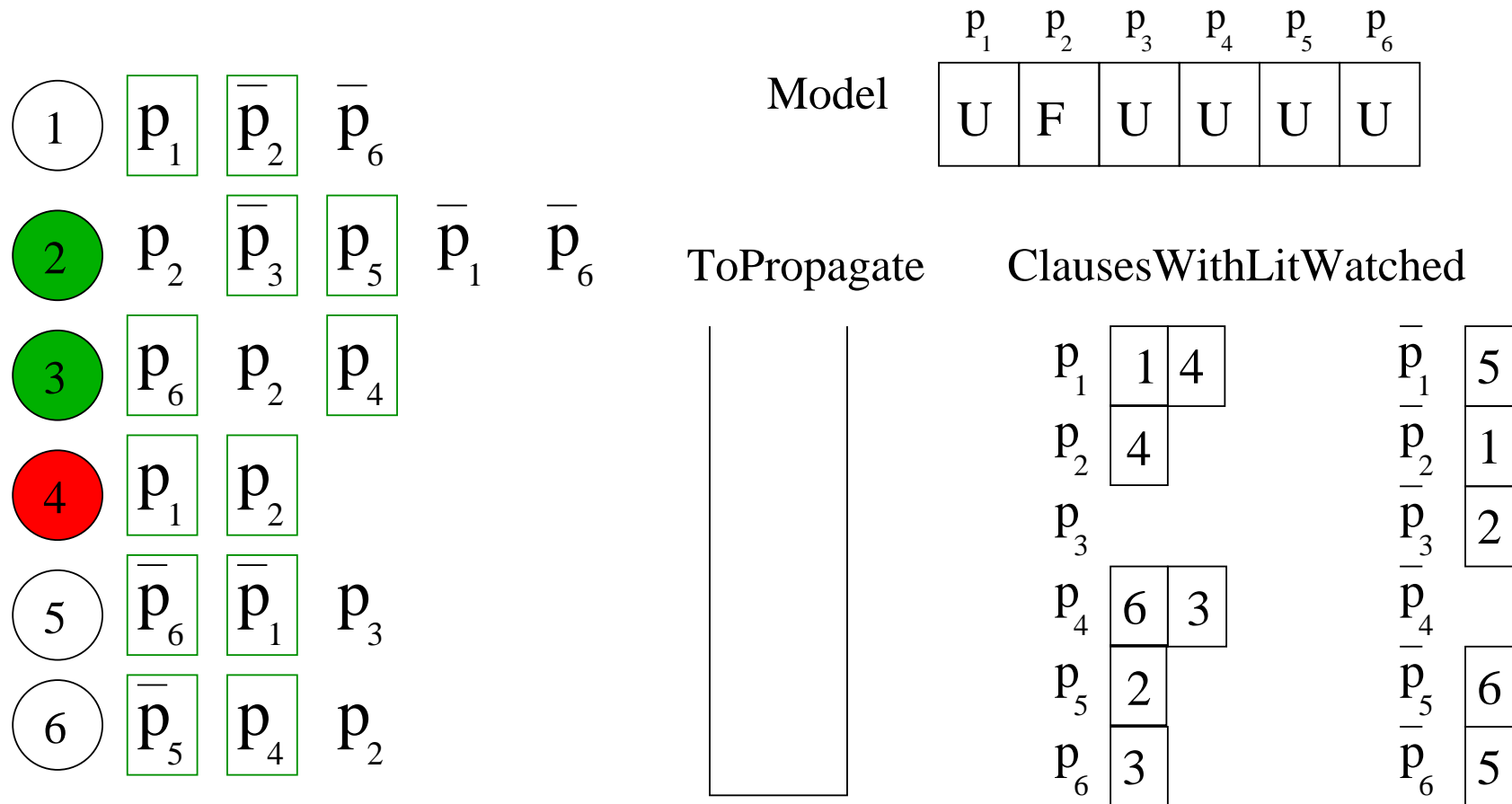
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Current assignment:  $\bar{p}_2^d$

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# BCP - Two Watched Literals



Current assignment:  $\bar{p}_2^d$

Clauses 2 and 3 are rewatched. Clause 4 can't because it is unit ( $p_1$ )

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Current assignment:  $\bar{p}_2^d p_1$

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Figure 1 shows a 6x4 grid of nodes. The nodes are labeled 1 through 6 in circles on the left. The grid contains variables  $p_i$  and their complements  $\bar{p}_i$ . The nodes are arranged as follows:

1	$p_1$	$\bar{p}_2$	$\bar{p}_6$		
2	$p_2$	$\bar{p}_3$	$p_5$	$\bar{p}_1$	$\bar{p}_6$
3	$p_6$	$p_2$	$p_4$		
4	$p_1$	$p_2$			
5	$\bar{p}_6$	$\bar{p}_1$	$p_3$		
6	$\bar{p}_5$	$p_4$	$p_2$		

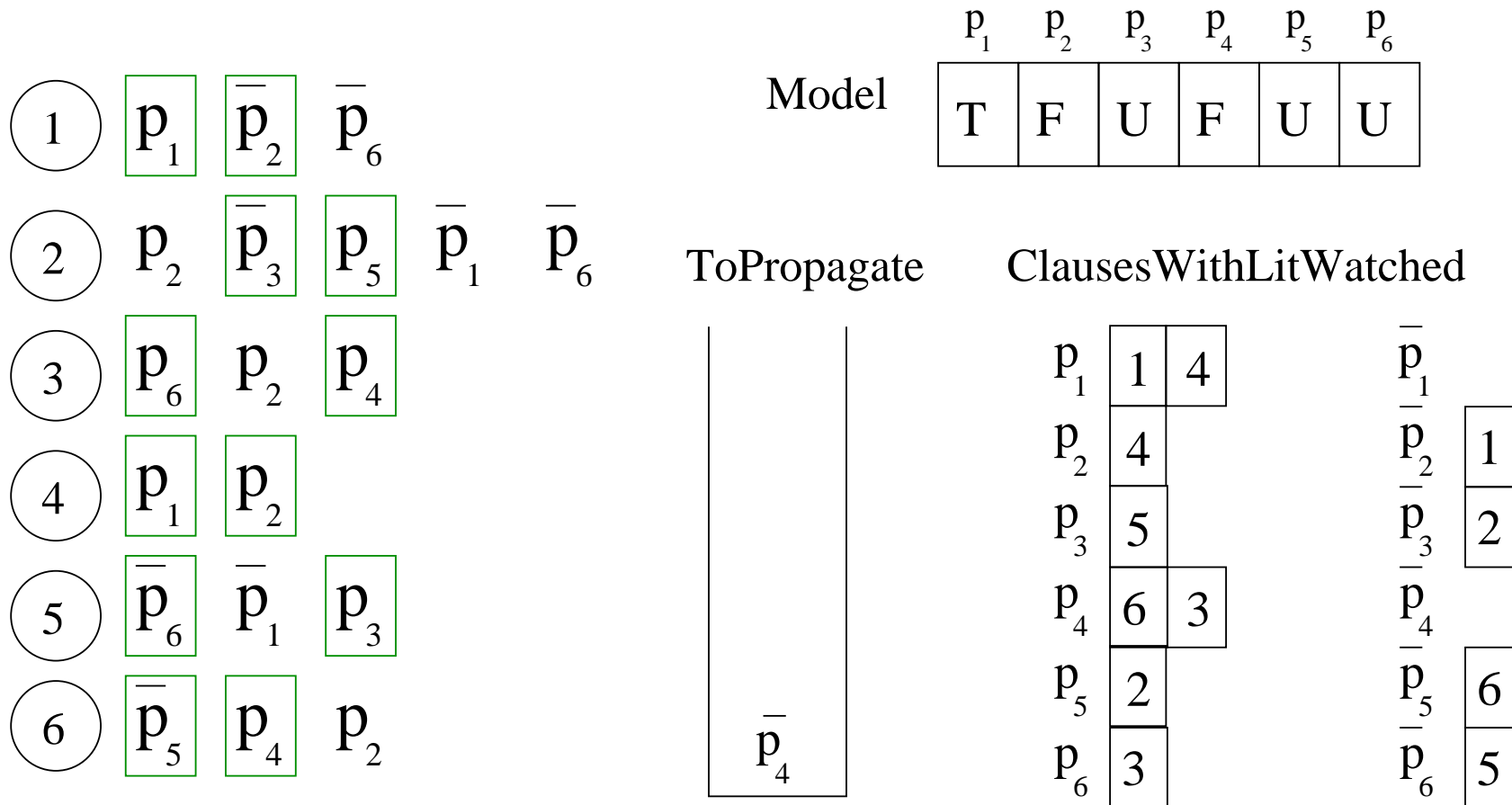
Green boxes highlight the nodes  $p_1$ ,  $\bar{p}_2$ ,  $p_5$ ,  $p_4$ ,  $\bar{p}_6$ ,  $\bar{p}_1$ ,  $p_3$ ,  $\bar{p}_5$ ,  $p_4$ , and  $p_2$ .

## ClausesWithLitWatched

$p_1$	1	4	$\overline{p_1}$	
$p_2$	4		$\overline{p_2}$	1
$p_3$	5		$\overline{p_3}$	2
$p_4$	6	3	$\overline{p_4}$	
$p_5$	2		$\overline{p_5}$	6
$p_6$	3		$\overline{p_6}$	5

Clauses 5 is reselected. No lit is unit propagated. We have to decide.

# BCP - Two Watched Literals



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Figure 1 shows a 6x6 grid of nodes and edges. The nodes are numbered 1 to 6 in circles. The edges are labeled  $p_i$  or  $\bar{p}_i$  in squares. The grid is as follows:

1	$p_1$	$\bar{p}_2$	$\bar{p}_6$		
2	$p_2$	$\bar{p}_3$	$p_5$	$\bar{p}_1$	$\bar{p}_6$
3	$p_6$	$p_2$	$p_4$		
4	$p_1$	$p_2$			
5	$\bar{p}_6$	$\bar{p}_1$	$p_3$		
6	$\bar{p}_5$	$p_4$	$p_2$		

## ClausesWithLitWatched

$\overline{p_6}$ $p_5$	

$p_1$	1	4
$p_2$	4	
$p_3$	5	
$p_4$	6	3
$p_5$	2	
$p_6$	3	

$\overline{p_1}$	
$\overline{p_2}$	1
$\overline{p_3}$	2
$\overline{p_4}$	
$\overline{p_5}$	6
$\overline{p_6}$	5

Now, we propagate  $p_6$  visiting `ClausesWithLitWatched` $[\bar{p}_6]$

# BCP - Two Watched Literals

	$p_1$	$p_2$	$p_3$	$p_4$	$p_5$	$p_6$
Model	T	F	U	F	F	T

Model: T F U F F T

To Propagate: ClausesWithLitWatched

Clause 1:  $p_1$   $p_2$   $p_6$

Clause 2:  $p_2$   $p_3$   $p_5$

Clause 3:  $p_6$   $p_2$   $p_4$

Clause 4:  $p_1$   $p_2$

Clause 5:  $p_6$   $p_1$   $p_3$

Clause 6:  $p_5$   $p_4$   $p_2$

Current assignment:  $\bar{p}_2^d p_1 \bar{p}_4^d \bar{p}_5 p_6$

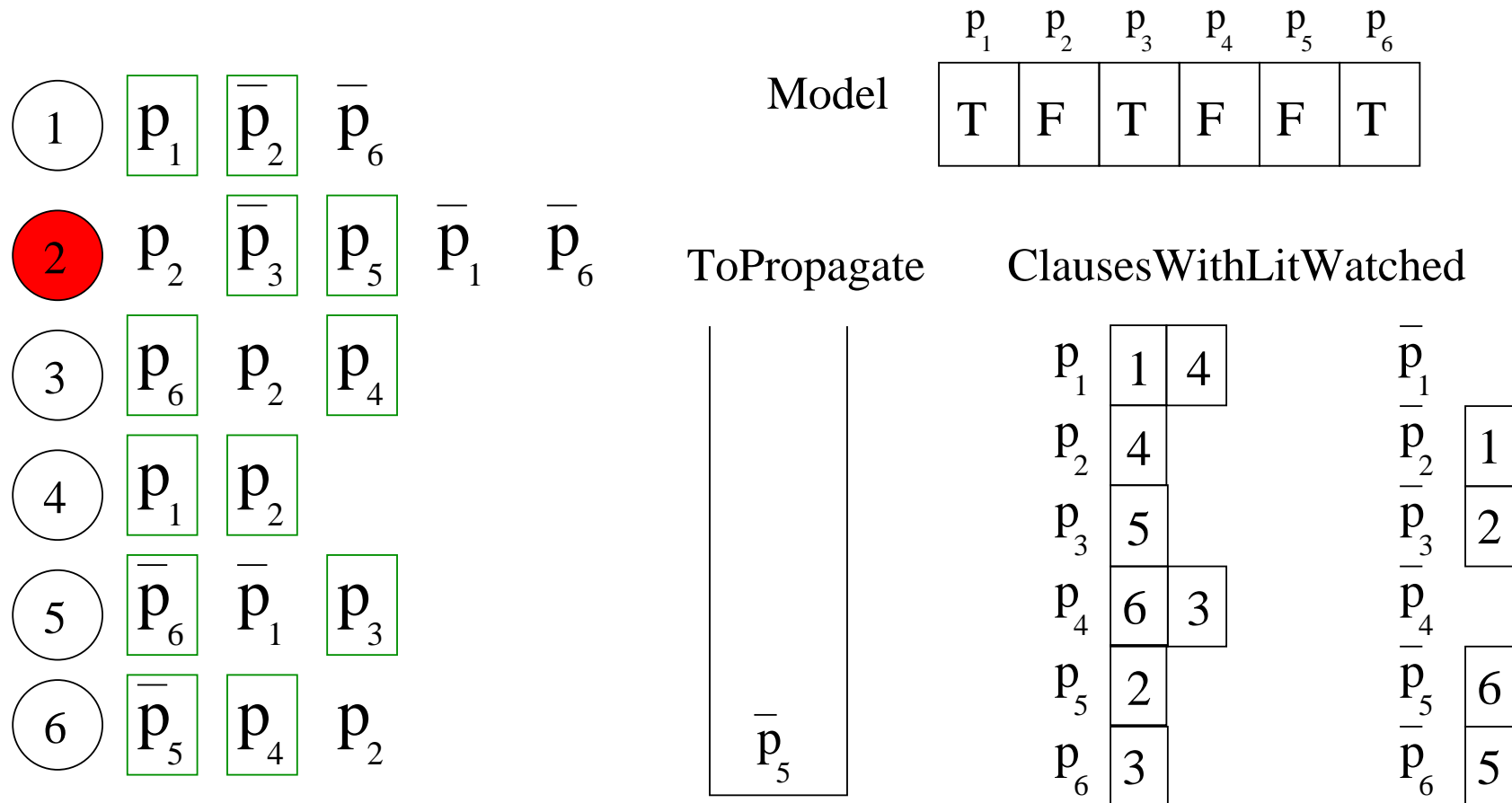
Clause 5 can't be reselected because it is unit ( $p_3$ ).

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# BCP - Two Watched Literals



# BCP - Two Watched Literals

	$p_1$	$p_2$	$p_3$	$p_4$	$p_5$	$p_6$
Model	T	F	U	U	U	U

	Model	T	F	U	U	U	U
1	$p_1$	$\bar{p}_2$	$\bar{p}_6$				
2	$p_2$	$\bar{p}_3$	$p_5$	$\bar{p}_1$	$\bar{p}_6$		
3	$p_6$	$p_2$	$p_4$				
4	$p_1$	$p_2$					
5	$\bar{p}_6$	$\bar{p}_1$	$p_3$				
6	$\bar{p}_5$	$p_4$	$p_2$				

Current assignment:  $\bar{p}_2^d p_1$  (lit  $p_4$  not yet added)

After backtrack watches are properly placed!

# Two watched literals - Analysis

- Each clause is **visited far less often**
- Upon **backtrack, nothing** has to be done
- Inactive literals tend to be watched, hence further reducing the number of clauses to be visited
- Very effective for long clauses (e.g. lemmas)

# Overview of the session

- Conflict Analysis
  - ◆ Motivating example
  - ◆ Backjumping
  - ◆ Conflict graph
  - ◆ Lemma shortening
- Lemma removal
- Decision heuristics
- Restarts
- Efficient implementation of UnitProp:
  - ◆ Occur lists
  - ◆ Two-watched literals
- Final remarks

# Why SAT solvers are really good?

Three **key** ingredients that **only work if used TOGETHER**:

- **Learn** at each conflict the **backjump clause** as a **lemma**:
  - ◆ makes **UnitProp** more powerful
  - ◆ prevents future **similar** conflicts
- **Decide** on the variable with **most occurrences in recent conflicts**:
  - ◆ so-called **activity-based heuristics**
  - ◆ idea: **work off clusters** of tightly related (by many clauses) vars
- **Forget** from time to time **low-activity lemmas**:
  - ◆ **crucial** to keep **UnitProp** fast and afford memory usage
  - ◆ idea: lemmas from **worked off clusters** no longer needed!

These are the most important features of  
**CDCL (Conflict-Driven Clause Learning) SAT solvers**

# Bibliography - Further reading

- Matthew W. Moskewicz, Conor F. Madigan, Ying Zhao, Lintao Zhang, Sharad Malik. *Chaff: Engineering an Efficient SAT Solver*. DAC 2001: 530-535
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