Propositional Logic

Combinatorial Problem Solving (CPS)

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Overview of the session

- Definition of Propositional Logic
- General Concepts in Logic
  - Reduction to SAT
- CNFs and DNFs
  - Tseitin Transformation
- Problem Solving with SAT
- Resolution

Definition of Propositional Logic

SYNTAX (what is a formula?)**:**

- There is a set $\mathcal{P}$ of propositional variables, usually denoted by (subscripted) $p, q, r, \ldots$
- The set of propositional formulas over $\mathcal{P}$ is defined as:
  - Every propositional variable is a formula
  - If F is a formula, $\neg F$ is also a formula
  - If F and G are formulas, $(F \wedge G)$ is also a formula
  - If F and G are formulas, $(F \vee G)$ is also a formula
  - Nothing else is a formula
- Formulas are usually denoted by (subscripted) $F, G, H, \ldots$
- Examples:

$$p \qquad \neg p \qquad (p \vee q) \qquad \neg(p \wedge q)$$

$$(p \wedge (\neg p \vee q)) \qquad ((p \wedge q) \vee (r \vee \neg q)) \qquad \ldots$$

Definition of Propositional Logic

SEMANTICS (what is an interpretation I, when I satisfies F?)**:**

- An interpretation I over $\mathcal{P}$ is a function $I : \mathcal{P} \to \{0,1\}$.
- $eval_I : Formulas \to \{0,1\}$ is a function defined as follows:
  - $eval_I(p) = I(p)$
  - $eval_I(\neg F) = 1 - eval_I(F)$
  - $eval_I(\ \ (F \wedge G)\ \ ) = \min\{eval_I(F), eval_I(G)\}$
  - $eval_I(\ \ (F \vee G)\ \ ) = \max\{eval_I(F), eval_I(G)\}$
- I satisfies F (written $I \models F$) if and only if $eval_I(F) = 1$.
- If $I \models F$ we say that
  - I is a model of F or, equivalently
  - F is true in I.

Definition of Propositional Logic

EXAMPLE:

- Let F be the formula $(p \land (q \lor \neg r))$.
- Let I be such that $I(p) = I(r) = 1$ and $I(q) = 0$.
- Let us compute $eval_I(F)$ (use your intuition first!)
- Is there any I such that $I \models F$?

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$$eval_I((p \wedge (q \vee \neg r))) \quad =$$

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Definition of Propositional Logic

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&= 0
\end{aligned}$$

- Is there any I such that $I \models F$?

  YES, $I(p) = I(q) = I(r) = 1$ is a possible model.

Definition of Propositional Logic

EXAMPLE

- We have 3 pigeons and 2 holes.
  If each hole can have at most one pigeon,
  is it possible to place all pigeons in the holes?
- Vocabulary: $p_{i,j}$ means i-th pigeon is in j-th hole
- Each pigeon is placed in at least one hole:

$$(p_{1,1} \vee p_{1,2}) \ \wedge \ (p_{2,1} \vee p_{2,2}) \ \wedge \ (p_{3,1} \vee p_{3,2})$$

- Each hole can hold at most one pigeon:

$$\neg(p_{1,1} \wedge p_{2,1}) \ \wedge \ \neg(p_{1,1} \wedge p_{3,1}) \ \wedge \ \neg(p_{2,1} \wedge p_{3,1}) \wedge$$
$$\neg(p_{1,2} \wedge p_{2,2}) \ \wedge \ \neg(p_{1,2} \wedge p_{3,2}) \ \wedge \ \neg(p_{2,2} \wedge p_{3,2})$$

- Resulting formula has no model

Definition of Propositional Logic

A small syntax extension:

- We will write $(F \to G)$ as an abbreviation for $(\neg F \lor G)$
- Similarly, $(F \leftrightarrow G)$ is an abbreviation of $((F \to G) \land (G \to F))$

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General Concepts in Logic

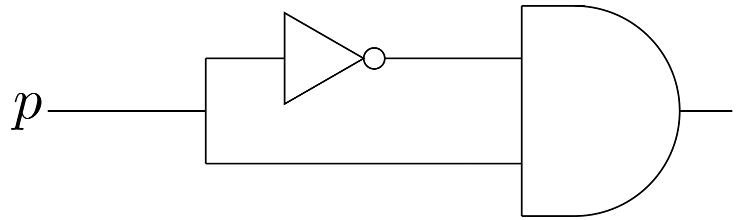
Let F and G be arbitrary formulas. Then:

- F is satisfiable if it has at least one model
- F is unsatisfiable (also a contradiction) if it has no model
- F is a tautology if every interpretation is a model of F
- G is a logical consequence of F, denoted $F \models G$, if every model of F is a model of G
- F and G are logically equivalent, denoted $F \equiv G$, if F and G have the same models

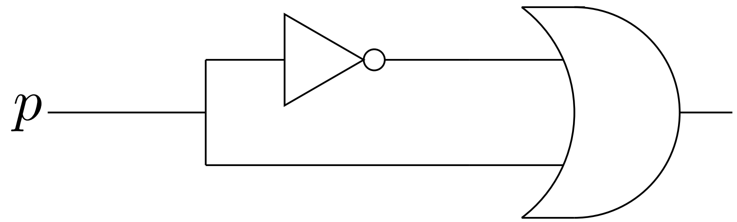
Note that:

- All definitions are only based on the concept of model.
- Hence they are independent of the logic.

General Concepts in Logic

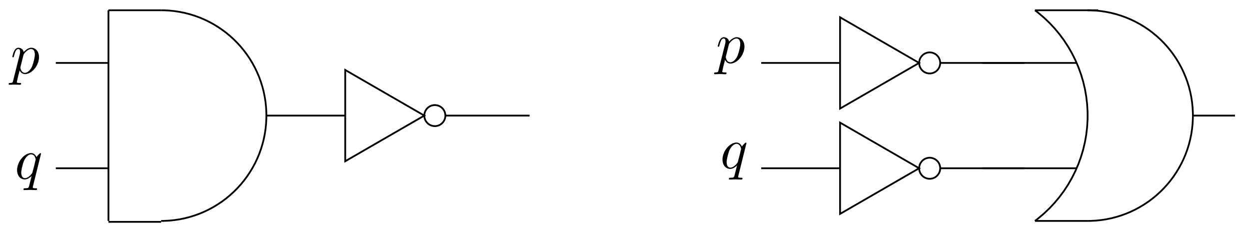


- Circuit corresponds to formula $(\neg p \wedge p)$
- Formula unsatisfiable amounts to *"circuit output is always 0"*



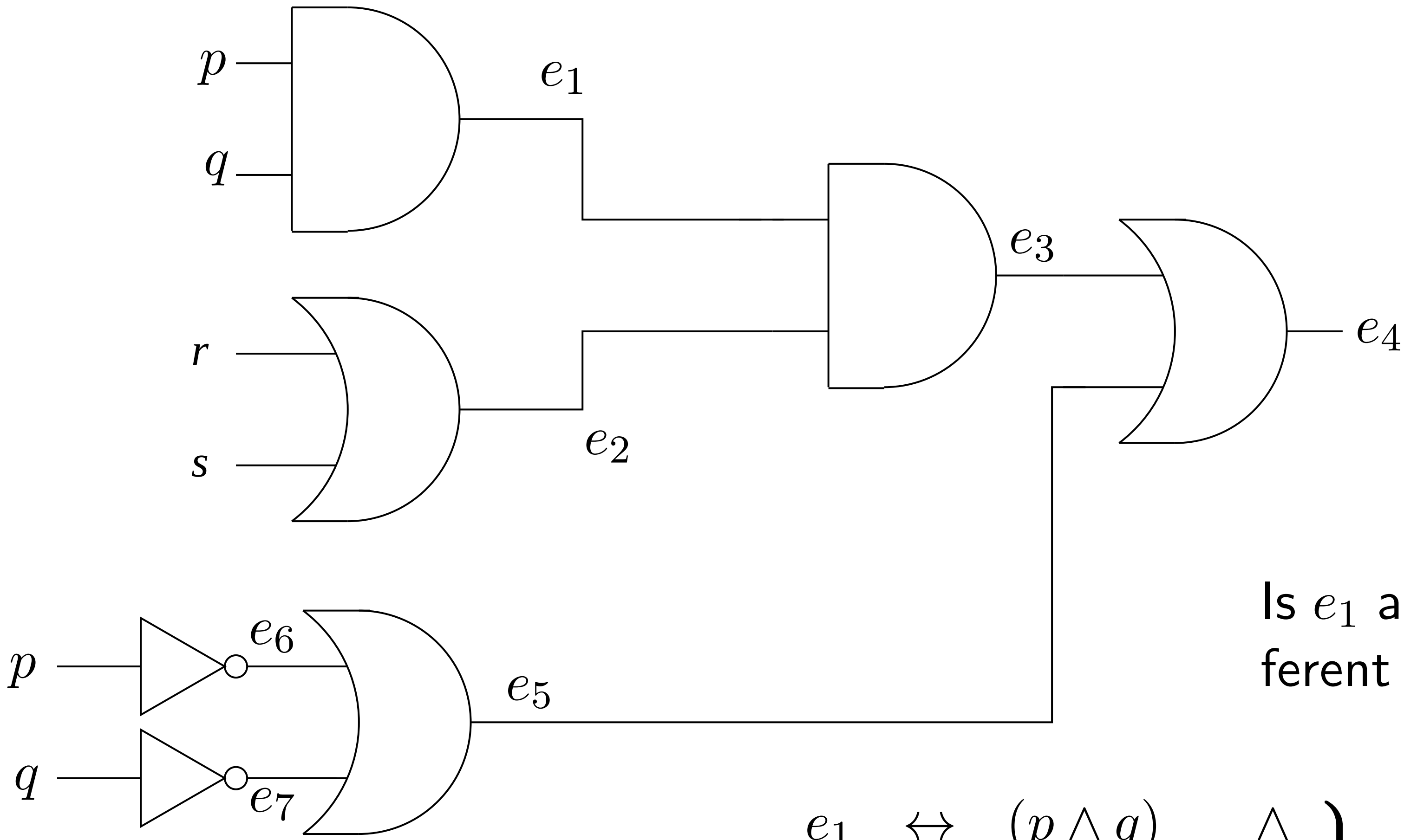
- Circuit corresponds to formula $(\neg p \vee p)$
- Formula is a tautology amounts to *"circuit output is always 1"*

General Concepts in Logic



- Circuit on the left corresponds to formula $F := \neg(p \wedge q)$
- Circuit on the right corresponds to formula $G := (\neg p \vee \neg q)$
- They are functionally equivalent, i.e. same inputs produce same output
- That corresponds to saying $F \equiv G$
- Cheapest / fastest / less power-consuming circuit is then chosen

General Concepts in Logic



Is e_1 always different from e_5?

$e_1 \neq e_5$ in the circuit amounts to
$$\left.\begin{array}{lcll} e_1 & \leftrightarrow & (p \wedge q) & \wedge \\ e_2 & \leftrightarrow & (r \vee s) & \wedge \\ e_3 & \leftrightarrow & (e_1 \wedge e_2) & \wedge \\ e_4 & \leftrightarrow & (e_3 \vee e_5) & \wedge \\ e_5 & \leftrightarrow & (e_6 \wedge e_7) & \wedge \\ e_6 & \leftrightarrow & (\neg p) & \wedge \\ e_7 & \leftrightarrow & (\neg q) & \end{array}\right\} \models e_1 \leftrightarrow \neg e_5$$

Reduction to SAT

Assume we have a black box **SAT** that given a formula F:

- **SAT**$(F) =$ YES iff F is satisfiable
- **SAT**$(F) =$ NO iff F is unsatisfiable

How to reuse **SAT** for detecting tautology, logical consequences, ...?

- F tautology iff **SAT**$(\neg F) =$ NO
- $F \models G$ iff **SAT**$(F \wedge \neg G) =$ NO
- $F \equiv G$ iff **SAT**$((F \wedge \neg G) \vee (\neg F \wedge G)) =$ NO

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How to reuse **SAT** for detecting tautology, logical consequences, ...?

- F **not** taut. iff **SAT**$(\neg F) = $ YES
- $F \not\models G$ iff **SAT**$(F \land \neg G) = $ YES
- $F \not\equiv G$ iff **SAT**$((F \land \neg G) \lor (\neg F \land G)) = $ YES

Reduction to SAT

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Hence, a single tool suffices: all problems can be reduced to SAT (propositional SATisfiability)

The black box **SAT** will be called a SAT solver

GOAL: learn how to build a SAT solver

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CNFs and DNFs

In order to construct our SAT solver
it will simplify our job to assume that the formula F has a given format.

- A literal is a propositional variable (p) or a negation of one ($\neg p$)
- A clause is a disjunction of zero or more literals ($l_1 \vee \ldots l_n$)
- The empty clause (zero literals) is denoted with $\Box$ and is unsatisfiable
- A formula is in Conjunctive Normal Form (CNF) if
  it is a conjunction of zero or more disjunctions of literals (i.e., clauses)
- A formula is in Disjunctive Normal Form (DNF) if
  it is a disjunction of zero or more conjunctions of literals (i.e., cubes)

Examples:

$p \wedge (q \vee \neg r) \wedge (q \vee p \vee \neg r)$ is in CNF
$p \vee (q \wedge \neg r) \vee (q \wedge p \wedge \neg r)$ is in DNF

CNFs and DNFs

- Given a formula F there exist formulas
  - G in CNF with $F \equiv G$ and (G is said to be a CNF of F)
  - H in DNF with $F \equiv H$ (H is said to be a DNF of F)
- Which is the complexity of deciding whether F is satisfiable...
  - ... if F is an arbitrary formula?
  - ... if F is in CNF?
  - ... if F is in DNF?

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  - ... if F is an arbitrary formula? NP-complete (Cook's Theorem)
  - ... if F is in CNF?
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  - ... if F is in CNF? NP-complete (even if clauses have ≤ 3 literals!)
  - ... if F is in DNF?

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  - ... if F is in CNF? NP-complete (even if clauses have ≤ 3 literals!)
  - ... if F is in DNF? linear

  Procedure **SAT**(F)

  Input: formula F in DNF
  Output: **YES** if there exists I such that $I \models F$, **NO** otherwise

  1. If the DNF is empty then return **NO**.
  Else take a cube C of the DNF

  2. If there is a variable p such that both p, $\neg p$ appear in C,
  then C cannot be made true: remove it and go to step 1.
  Else define I to make C true and return **YES**.

CNFs and DNFs

- Idea: given F, find a DNF of F and apply the linear-time algorithm
- Why this does not work?

CNFs and DNFs

- Idea: given F, find a DNF of F and apply the linear-time algorithm
- Why this does not work? Finding a DNF of F may take exponential time
- In fact there are formulas for which CNFs/DNFs have exponential size

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- Idea: given F, find a DNF of F and apply the linear-time algorithm
- Why this does not work? Finding a DNF of F may take exponential time
- In fact there are formulas for which CNFs/DNFs have exponential size

- Consider xor defined as $\mathrm{xor}(x_1) = x_1$ and if $n > 1$:

$$\begin{array}{rcl} \mathrm{xor}(x_1, \ldots, x_n) & = & (\mathrm{xor}(x_1, \ldots, x_{\lfloor \frac{n}{2} \rfloor}) \quad \wedge \quad \neg\mathrm{xor}(x_{\lfloor \frac{n}{2} \rfloor + 1}, \ldots, x_n)) \vee \\ & & (\neg\mathrm{xor}(x_1, \ldots, x_{\lfloor \frac{n}{2} \rfloor}) \quad \wedge \quad \mathrm{xor}(x_{\lfloor \frac{n}{2} \rfloor + 1}, \ldots, x_n)) \end{array}$$

- The size of $\mathrm{xor}(x_1, \ldots, x_n)$ is $\Theta(n^2)$
- Cubes (conjunctions of literals) of a DNF of $\mathrm{xor}(x_1, \ldots, x_n)$ have n literals
- Any DNF of $\mathrm{xor}(x_1, \ldots, x_n)$ has at least 2^{n-1} cubes
(one for each of the assignments with an odd number of 1s)
- Any CNF of $\mathrm{xor}(x_1, \ldots, x_n)$ also has an exponential number of clauses

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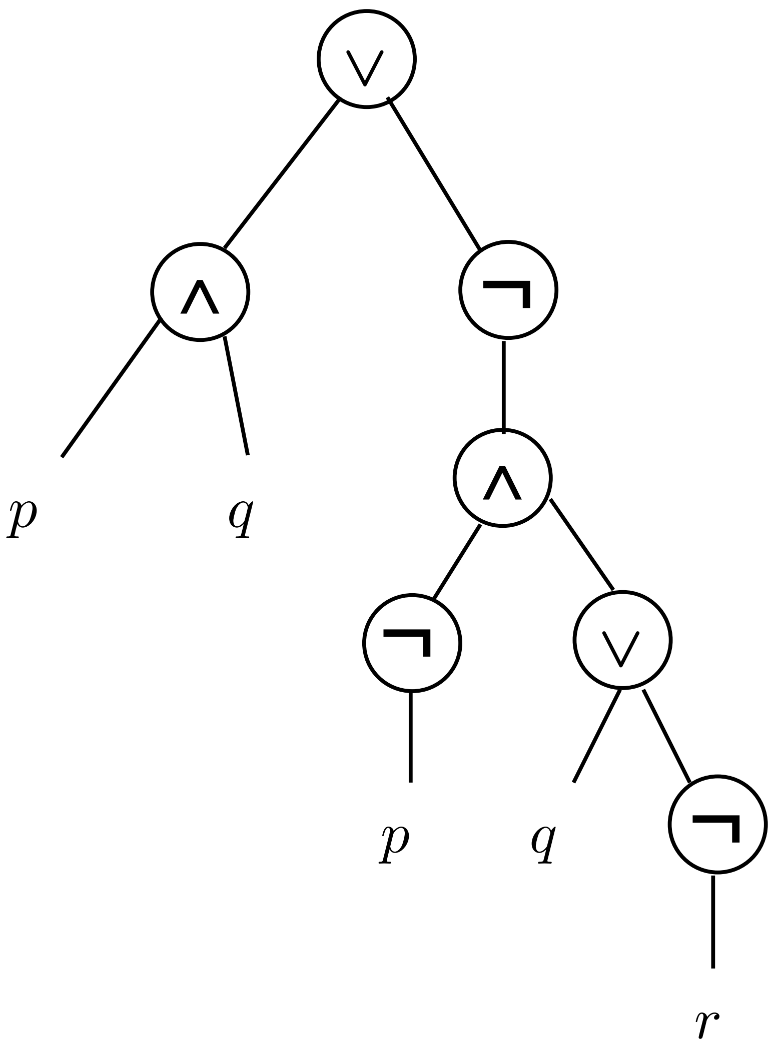
- Next we'll see a workaround

Tseitin Transformation

Let F be $(p \land q) \lor \neg(\ \neg p \land (q \lor \neg r)\ )$



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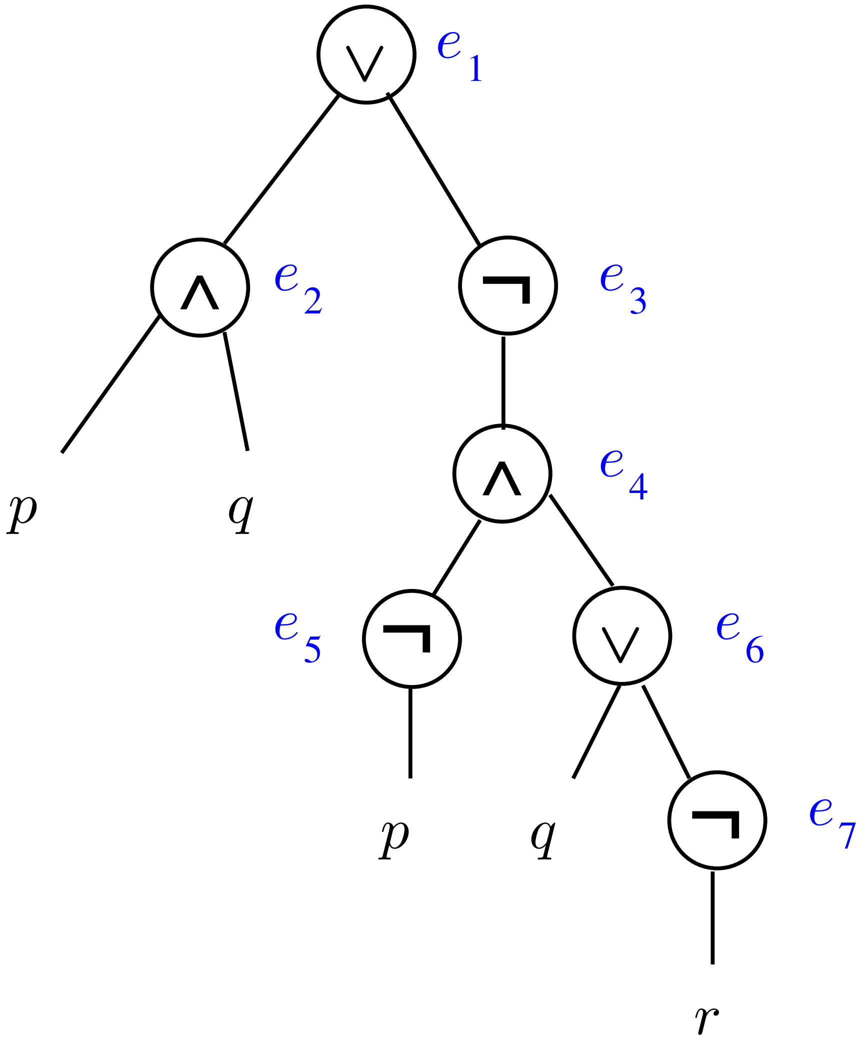

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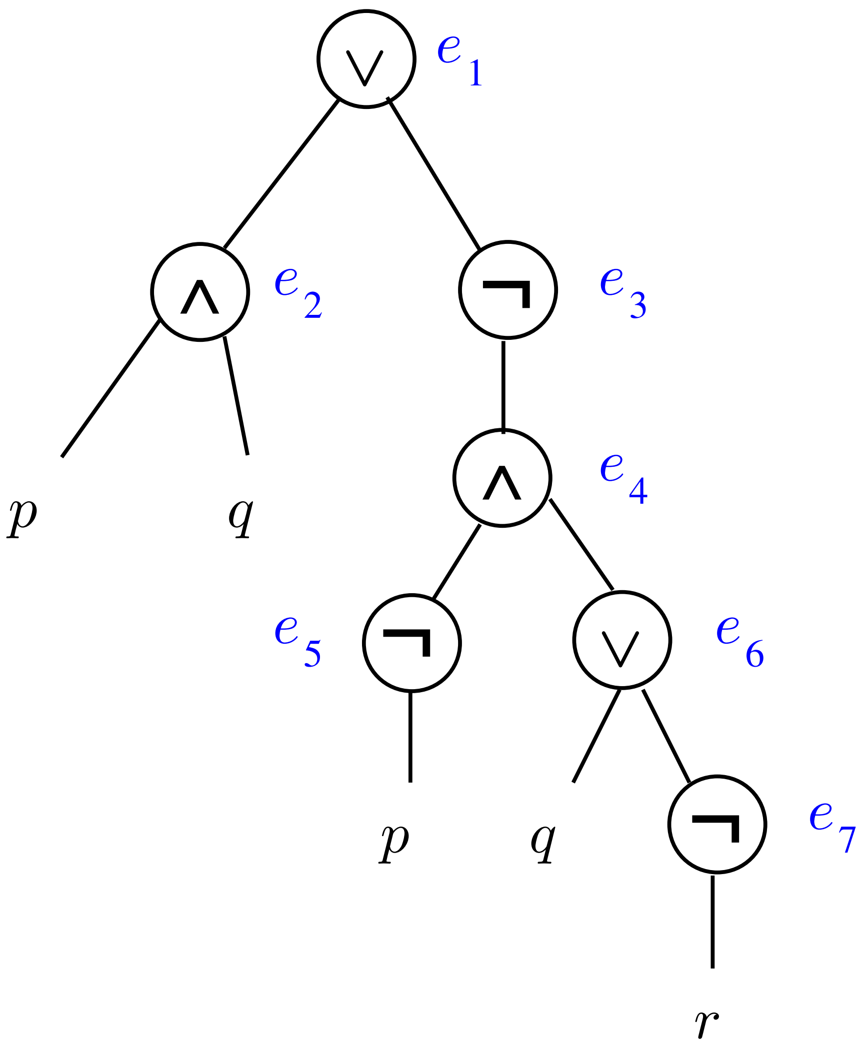

- e_1
- $e_1 \leftrightarrow e_2 \vee e_3$
- $e_2 \leftrightarrow p \wedge q$
- $e_3 \leftrightarrow \neg e_4$
- $e_4 \leftrightarrow e_5 \wedge e_6$
- $e_5 \leftrightarrow \neg p$
- $e_6 \leftrightarrow q \vee \neg e_7$
- $e_7 \leftrightarrow \neg r$

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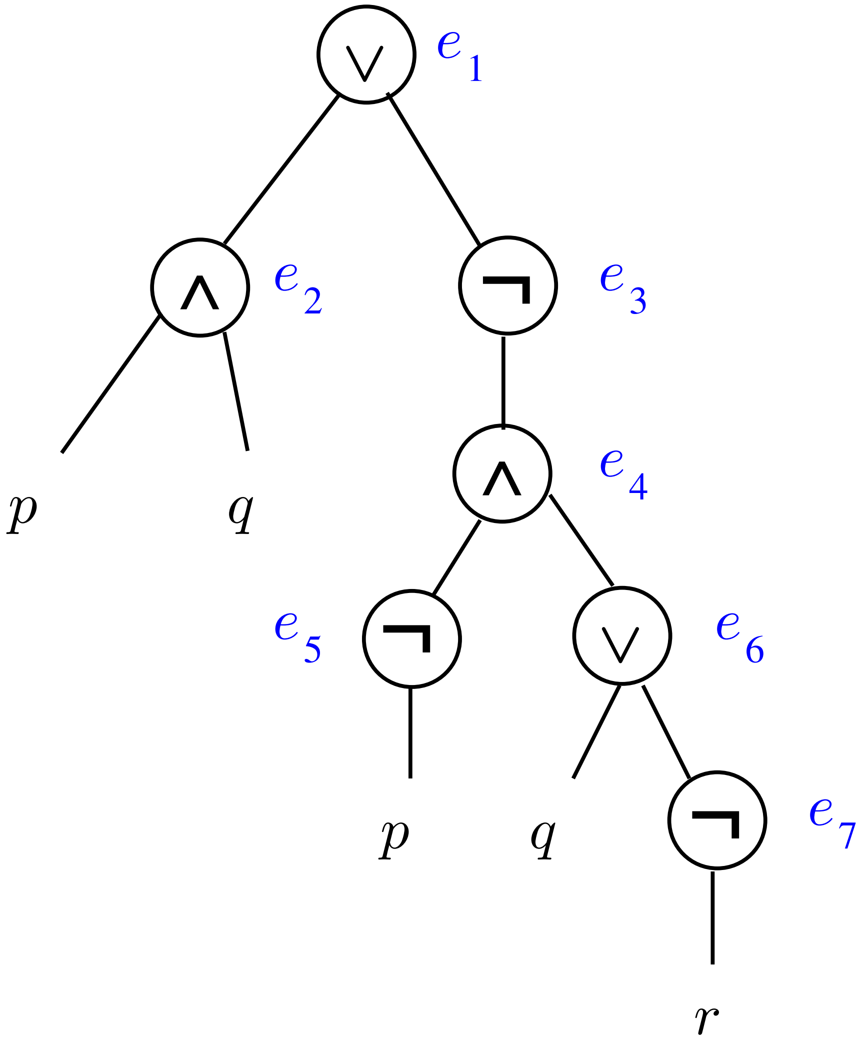

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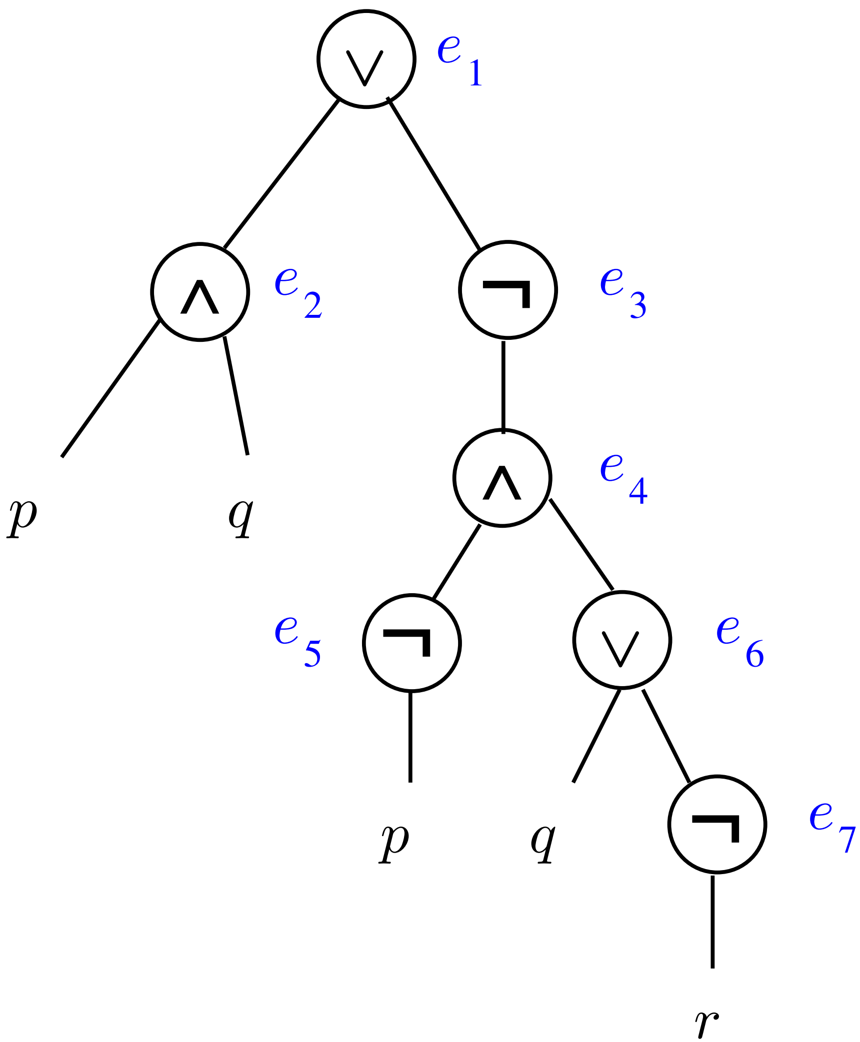

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- $e_6 \leftrightarrow q \vee \neg e_7$
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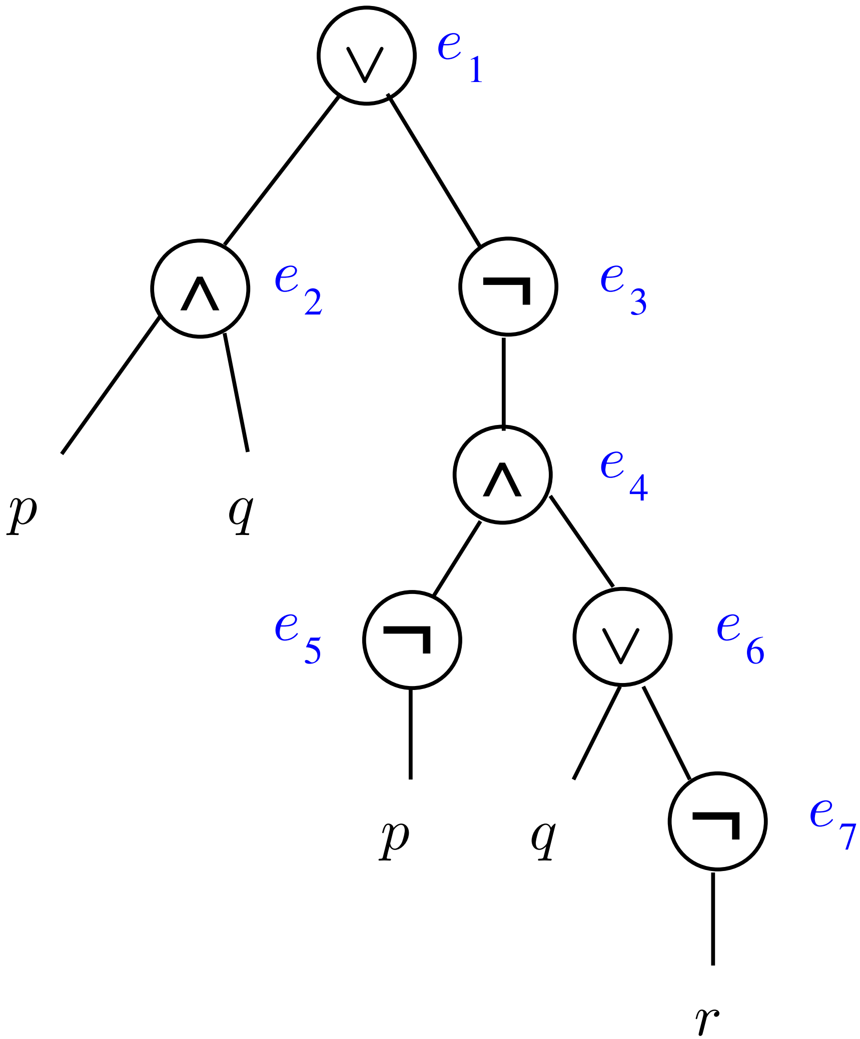

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  $\neg e_2 \vee q$
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  $\neg e_3 \vee \neg e_4$
  $e_3 \vee e_4$
- $e_4 \leftrightarrow e_5 \wedge e_6$
- $e_5 \leftrightarrow \neg p$
- $e_6 \leftrightarrow q \vee \neg e_7$
- $e_7 \leftrightarrow \neg r$

Tseitin Transformation

- Variations of Tseitin transformation are used in practice in SAT solvers
- Tseitin transformation does not produce an equivalent CNF: for example, the Tseitin transformation of $F = \neg p$ is $G = e \wedge (\neg e \vee \neg p) \wedge (e \vee p)$, and

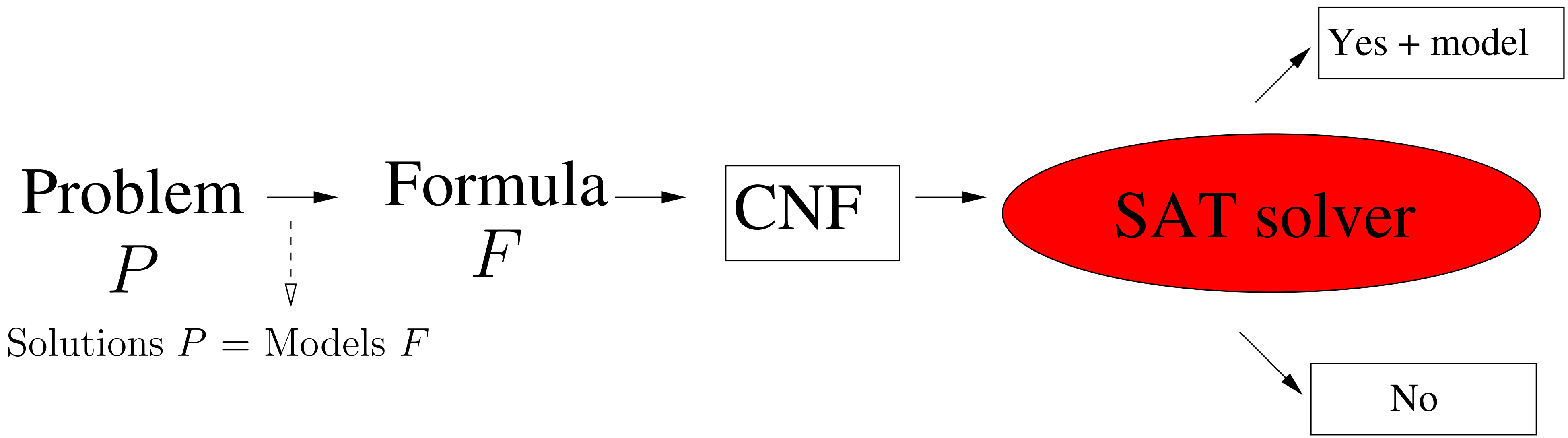
e	p	F	G
0	0	1	0
0	1	0	0
1	0	1	1
1	1	0	0

- Still, CNF obtained from F via Tseitin transformation has nice properties:
  - It is equisatisfiable to F
  - Any model of CNF projected to the variables in F gives a model of F
  - Any model of F can be completed to a model of the CNF
  - Can be computed in linear time in the size of F
- Hence no model is lost nor added in the transformation

Overview of the session

- Definition of Propositional Logic
- General Concepts in Logic
  - Reduction to SAT
- CNFs and DNFs
  - Tseitin Transformation
- Problem Solving with SAT
- Resolution

Problem Solving with SAT



- This is the standard flow when solving problems with SAT
- Transformation from P to F is called the encoding into SAT

  Already seen some examples: pigeon-hole problem
  Other examples will be seen in the next classes
- CNF transformation already explained
- Let us see now how to design efficient SAT solvers

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Resolution

- The resolution rule is

$$\frac{p \vee C \quad \neg p \vee D}{C \vee D}$$

- $Res(S)$ = closure of set of clauses S under resolution =
  = clauses inferred in zero or more steps of resolution from S

- Properties:
  - Resolution is correct:
    $Res(S)$ only contains logical consequences
  - Resolution is refutationally complete:
    if S is unsatisfiable, then $\square \in Res(S)$
  - $Res(S)$ is a finite set of clauses

- So, given a set of clauses S, its satisfiability can be checked by:

  1. Computing $Res(S)$
  2. **If** $\square \in Res(S)$ **Then** UNSAT ; **Else** SAT