THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES ®

founded in 1964 by N. J. A. Sloane

A001035 Number of partially ordered sets ("posets") with n labeled elements (or labeled acyclic transitive digraphs). 46

(Formerly M3068 N1244)

1, 1, 3, 19, 219, 4231, 130023, 6129859, 431723379, 44511042511, 6611065248783, 1396281677105899, 414864951055853499, 171850728381587059351, 98484324257128207032183, 77567171020440688353049939, 83480529785490157813844256579, 122152541250295322862941281269151, 241939392597201176602897820148085023 (list; graph; refs; listen; history; text; internal format)

OFFSET 0,3

COMMENTS

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From Altug Alkan, Dec 22 2015: (Start)
a(p^k) == 1 mod p and a(n + p) == a(n + 1) mod p for all primes p.
a(0+19) == a(0+1) mod 19 or a(19^1) == 1 mod 19, that is, a(19) mod 19 = 1.
a(2+17) == a(2+1) mod 17. So a(19) == 19 mod 17, that is, a(19) mod 17 = 2.
a(6+13) == a(6+1) mod 13. So a(19) == 6129859 mod 13, that is, a(19) mod 13
  = 8.
a(8+11) == a(8+1) mod 11. So a(19) == 44511042511 mod 11, that is, a(19) mod
  11 = 1.
a(12+7) == a(12+1) mod 7. So a(19) == 171850728381587059351 mod 7, that is,
  a(19) mod 7 = 1.
a(14+5) == a(14+1) mod 5. So a(19) == 77567171020440688353049939 mod 5, that
  is, a(19) mod 5 = 4.
a(16+3) == a(16+1) mod 3. So a(19) == 122152541250295322862941281269151 mod
  3, that is, a(19) mod 3 = 1.
a(17+2) == a(17+1) mod 2. So a(19) mod 2 = 1.
In conclusion, a(19) is a number of the form 2*3*5*7*11*13*17*19*n -
  1615151, that is, 9699690*n - 1615151.
Additionally, for n > 0, note that the last digit of a(n) has the simple
  periodic pattern: 1,3,9,9,1,3,9,9,1,3,9,9,...
(End)
Number of rank n sublattices of the Boolean algebra B_n. - Kevin Long, Nov
  20 2018
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Index entries for sequences related to posets

FORMULA

A000798(n) = Sum Stirling2(n, k)*a(k).
Related to A000112 by Erné's formulas: a(n+1)=-s(n, 1), a(n+2)=n*a(n+1)+s(n, 2), a(n+3)=binomial(n+4, 2)*a(n+2)-s(n, 3), where s(n, k)=sum(binomial(n+k-1-m, k-1)*binomial(n+k, m)*sum((m!)/(number of automorphisms of P)*(-(number of antichains of P))^k, P an unlabeled poset with m elements), m=0..n).
From Altug Alkan, Dec 22 2015: (Start)
a(p^k) == 1 mod p for all primes p and for all nonnegative integers k.
a(n + p) == a(n + 1) mod p for all primes p and for all nonnegative integers n.

If n = 1, then a(1 + p) == a(2) mod p, that is, a(p + 1) == 3 mod p.
If n = p, then a(p + p) == a(p + 1) mod p, that is, a(2*p) == a(p + 1) mod p.
In conclusion, a(2*p) == 3 mod p for all primes p.
(End)

EXAMPLE

R. P. Stanley, Enumerative Combinatorics, Cambridge, Vol. 1, Chap. 3, page 98, Fig. 3-1 shows the unlabeled posets with <= 4 points.
From Gus Wiseman, Aug 14 2019: (Start)
Also the number of T_0 topologies with n points. For example, the a(0) = 1 through a(3) = 19 topologies are:

```
{}  {}{1}  {}{1}{12}     {}{1}{12}{123}
           {}{2}{12}     {}{1}{13}{123}
           {}{1}{2}{12}  {}{2}{12}{123}
                         {}{2}{23}{123}
                         {}{3}{13}{123}
                         {}{3}{23}{123}
                         {}{1}{2}{12}{123}
                         {}{1}{3}{13}{123}
                         {}{2}{3}{23}{123}
                         {}{1}{12}{13}{123}
                         {}{2}{12}{23}{123}
                         {}{3}{13}{23}{123}
                         {}{1}{2}{12}{13}{123}
                         {}{1}{2}{12}{23}{123}
                         {}{1}{3}{12}{13}{123}
                         {}{1}{3}{13}{23}{123}
                         {}{2}{3}{12}{23}{123}
                         {}{2}{3}{13}{23}{123}
                         {}{1}{2}{3}{12}{13}{23}{123}
```

(End)

MATHEMATICA

```
dual[eds_]:=Table[First/@Position[eds, x], {x, Union@@eds}];
Table[Length[Select[Subsets[Subsets[Range[n]]], MemberQ[#, {}]&&MemberQ[#, Range[n]]&&UnsameQ@@dual[#]&&SubsetQ[#, Union@@@Tuples[#, 2]]&&SubsetQ[#, Intersection@@@Tuples[#, 2]]&]], {n, 0, 3}] (* Gus Wiseman, Aug 14 2019 *)
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CROSSREFS

Cf. A000798 (labeled topologies), A001930 (unlabeled topologies), A000112 (unlabeled posets), A006057.
Sequences in the Erné (1974) paper: A000798, A001035, A006056, A006057, A001929, A001927, A006058, A006059, A000110.
Cf. A316978, A319564, A326876, A326906, A326939, A326943, A326944, A326947.
Sequence in context: A005647 A158876 A001833 * A267634 A277407 A271587
Adjacent sequences: A001032 A001033 A001034 * A001036 A001037 A001038

KEYWORD

nonn,nice

AUTHOR

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EXTENSIONS

a(15)-a(16) from Jobst Heitzig (heitzig(AT)math.uni-hannover.de), Jul 03 2000
a(17)-a(18) from Herman Jamke (hermanjamke(AT)fastmail.fm), Mar 02 2008

STATUS

approved