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1  ---------- MODULE RelationUtils ----------
   Relation related operators.
5  EXTENDS Naturals, Sequences, FunctionUtils
6  ----------
   Basic definitions.
10 Dom(R) ≜ {a : ⟨a, b⟩ ∈ R}  Domain of R
11 Ran(R) ≜ {b : ⟨a, b⟩ ∈ R}  Range of R
12 Support(R) ≜ Dom(R) ∪ Ran(R)  Support of R
   Basic operations.
16 Inverse(R) ≜ {⟨b, a⟩ : ⟨a, b⟩ ∈ R}  Inverse of R
17 R | S ≜ R ∩ S × S  Restriction of R on S

19 R ∗∗ T ≜  Composition of R and T
20     LET SR ≜ Support(R)
21         ST ≜ Support(T)
22     IN  {⟨r, t⟩ ∈ SR × ST : ∃ s ∈ SR ∩ ST : (⟨r, s⟩ ∈ R) ∧ (⟨s, t⟩ ∈ T)}

24 GT(R, a) ≜ {b ∈ Ran(R) : ⟨a, b⟩ ∈ R}
25 LT(R, b) ≜ {a ∈ Dom(R) : ⟨a, b⟩ ∈ R}
   The following definition is from https://github.com/jameshfisher/tlaplus/blob/master/examples/TransitiveClosure/TransitiveClos
   It also contains several other methods for computing TC.
31 TC(R) ≜  Transitive closure of R
32     LET S ≜ Support(R)
33         RECURSIVE TCR(_)
34         TCR(T) ≜ IF T = {}
35                  THEN R
36                  ELSE LET r ≜ CHOOSE s ∈ T : TRUE
37                           RR ≜ TCR(T \ {r})
38                       IN  RR ∪ {⟨s, t⟩ ∈ S × S :
39                                ⟨s, r⟩ ∈ RR ∧ ⟨r, t⟩ ∈ RR}
40     IN  TCR(S)
   Example: SeqToRel(⟨1, 2, 3⟩) = {⟨1, 2⟩, ⟨1, 3⟩, ⟨2, 3⟩}
44 SeqToRel(s) ≜  Transform a (total order) sequence to a relation
45     LET RECURSIVE SeqRel(_, _)
46         SeqRel(seq, rel) ≜
47             IF Len(seq) ≤ 1
48             THEN rel
49             ELSE LET h ≜ Head(seq)
50                      t ≜ Tail(seq)
51                  IN  SeqRel(t, rel ∪ {⟨h, r⟩ : r ∈ Range(t)})
52     IN  SeqRel(s, {})
   Basic properties.
56 Reflexive(S, R) ≜ ∀ a ∈ S : ⟨a, a⟩ ∈ R  is R reflexive?
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58 $Transitive(R) \triangleq$ Is R transitive?
59 LET $S \triangleq Support(R)$
60 IN $\forall\, a,\, b,\, c \in S : (\langle a,\, b\rangle \in R \land \langle b,\, c\rangle \in R) \Rightarrow \langle a,\, c\rangle \in R$

62 $Respect(R,\, T) \triangleq T \subseteq R$ Does R respect T?
63

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