Question of the Day

Can you have a container without boundaries?

Time-Frequency Analysis & Windowing

Lecture 8

July 16, 2019

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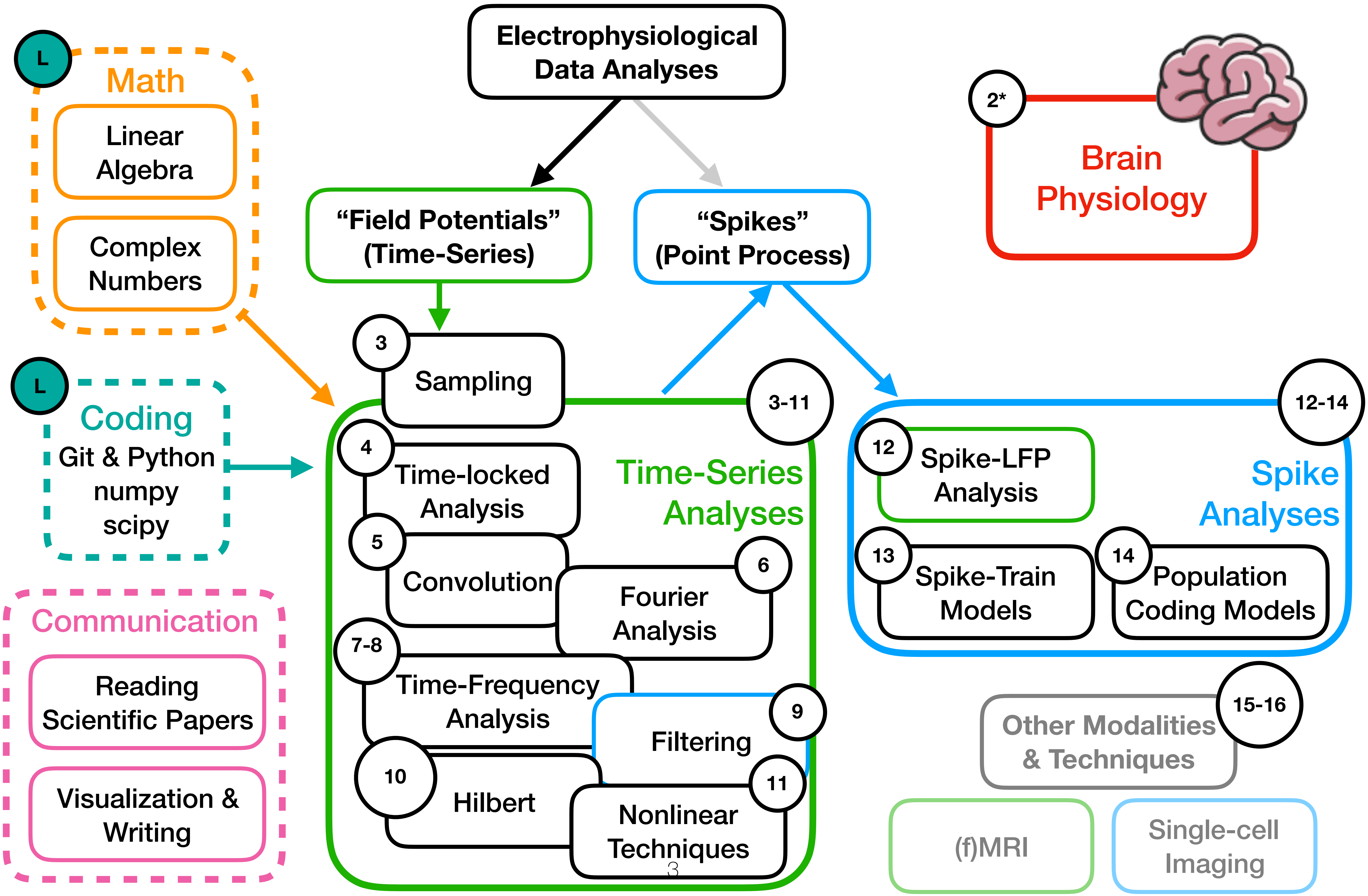
Course Outline: Road Map

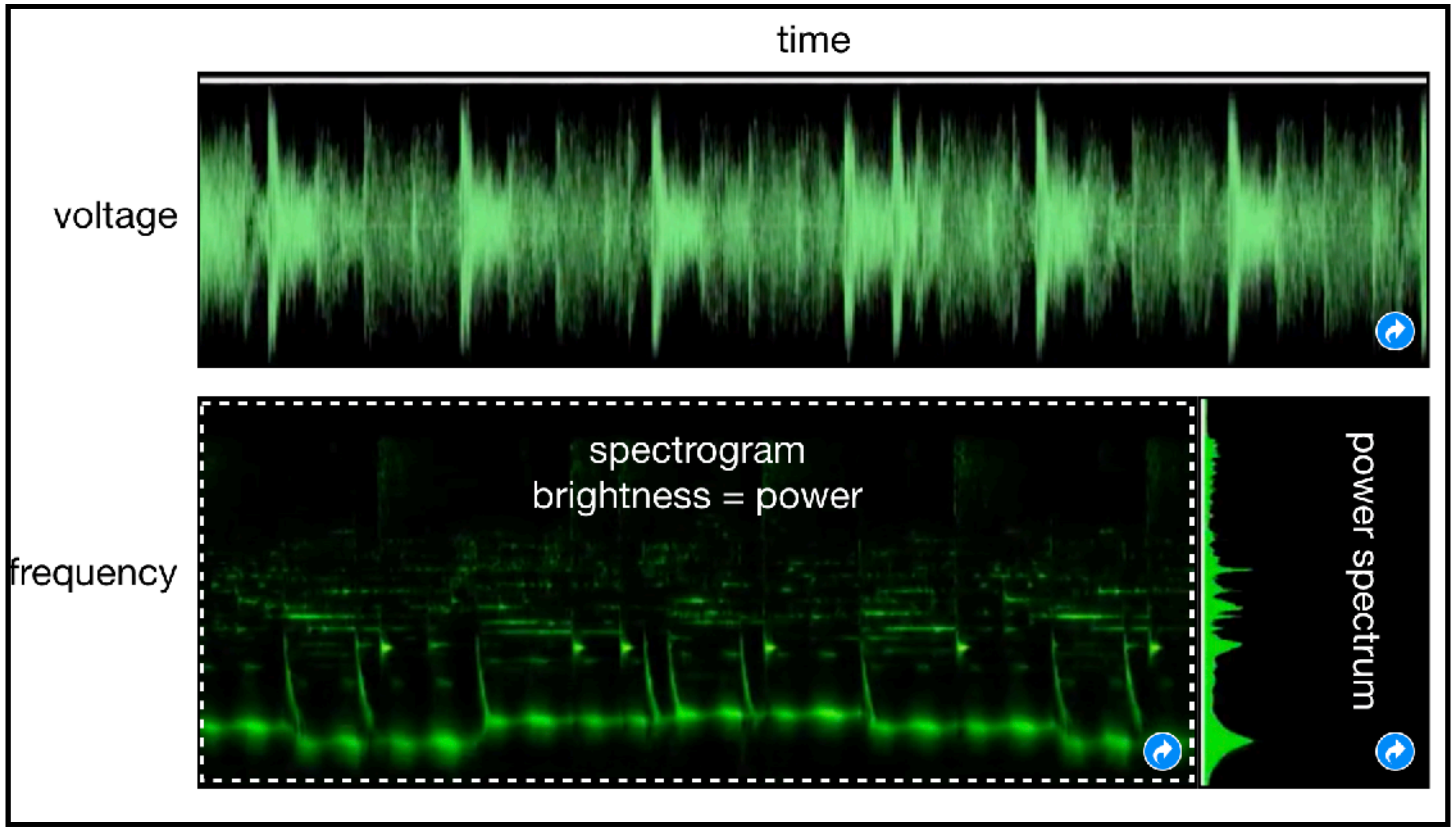


Goals for Today

1. Evaluate parameter choices for time-frequency analysis
2. List & understand transformations of STFT
3. Motivate Windowing

Time-Frequency Analysis

Looking at the DFT or power spectrum is like averaging over all the pictures in a movie, or all the notes in a song.



We need **time-frequency analysis.**

5 min: come up with 3 examples of signals with time-dependent frequency content, their dominant frequency(s), and their speed of change.

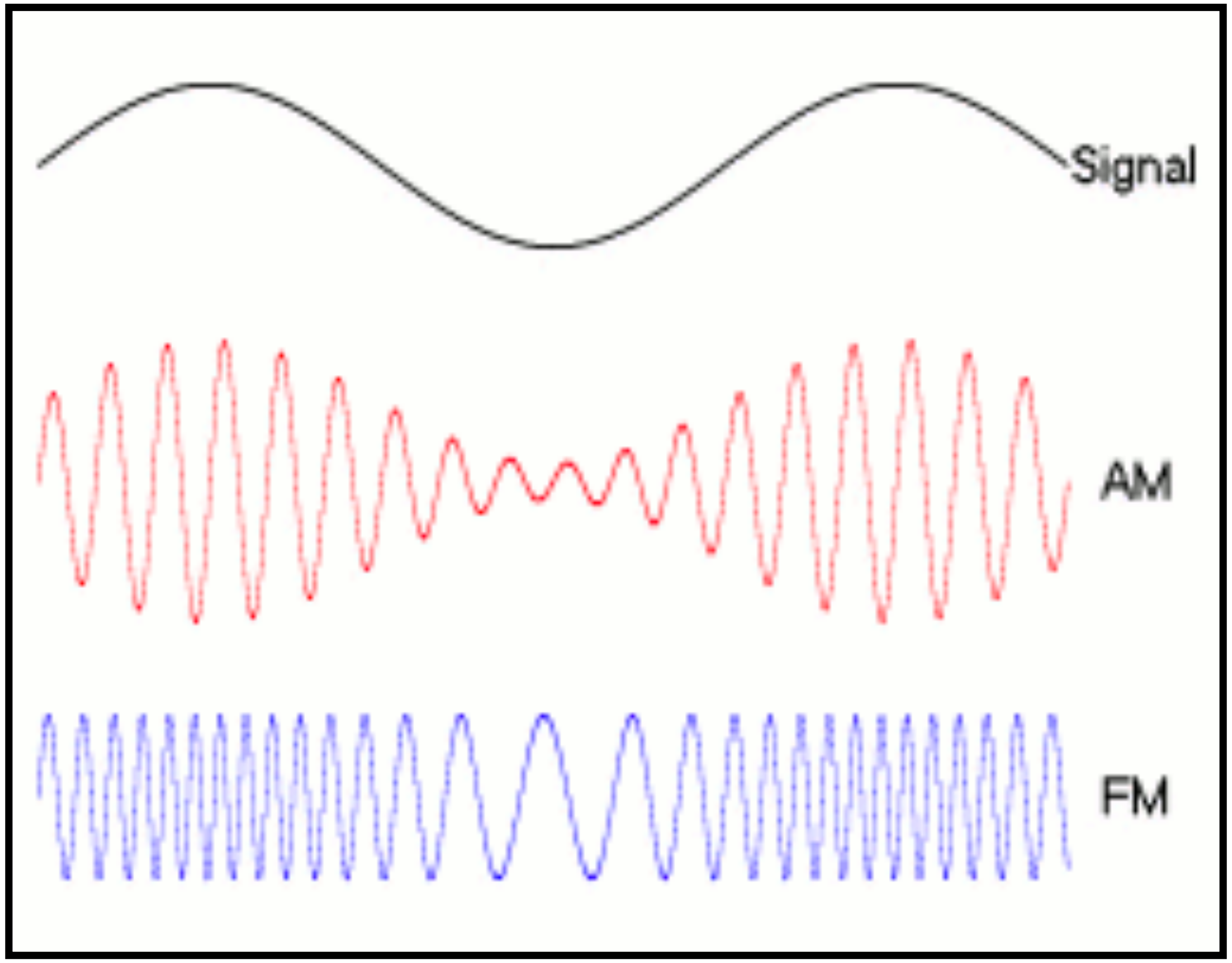
Time-Frequency Analysis

5 min: come up with 3 examples of signals with time-dependent frequency content, their dominant frequency(s), and their speed of change.

Example	Frequencies	Timescale of Change	STFT Parameters
seismic waves (earthquakes)	0.2Hz-10Hz 20Hz	minutes	
heart rate	1 - 2Hz	seconds	
color of the sky (from fixed point on earth)	500 tHz	hours	

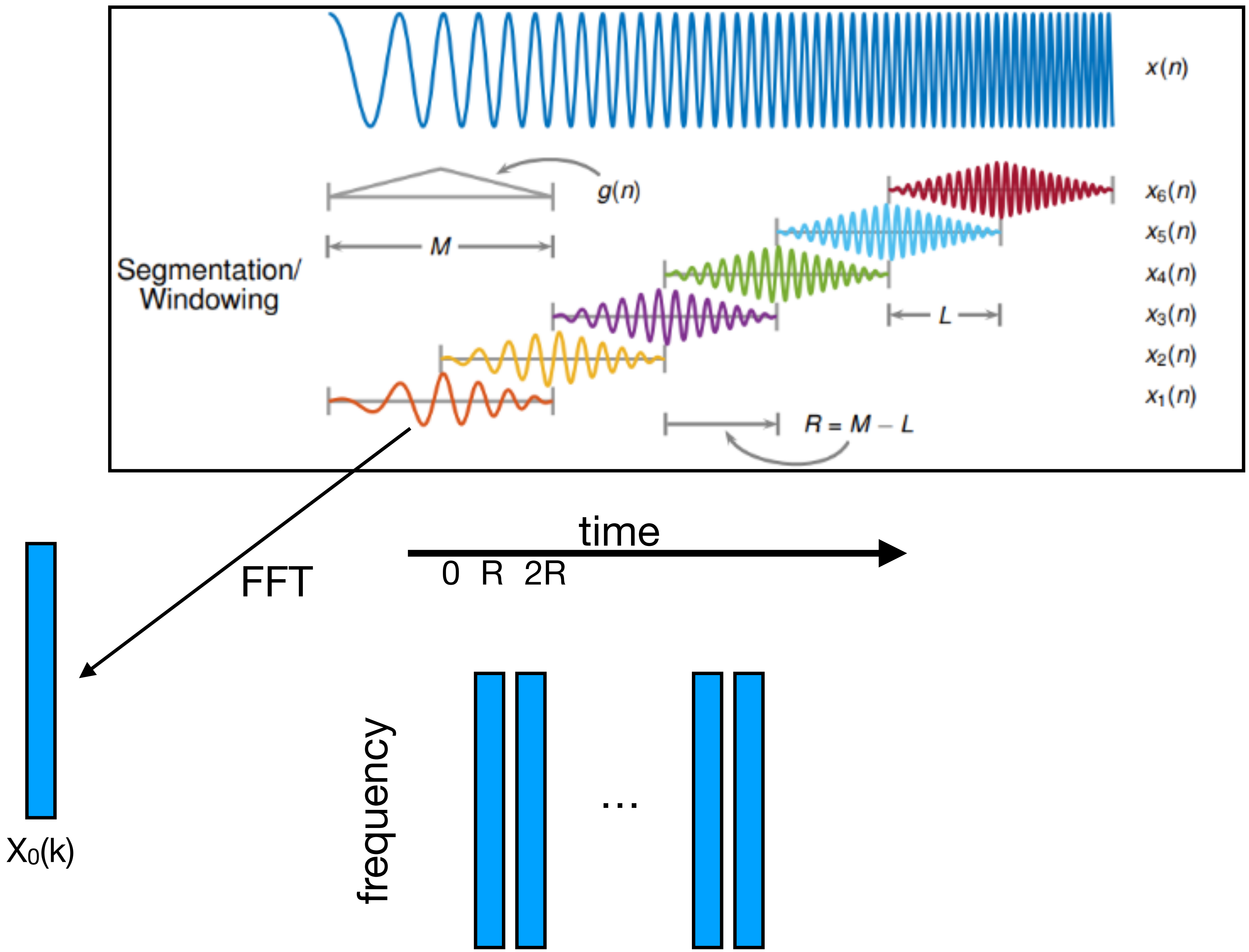
Frequency modulation

From Wikipedia, the free encyclopedia

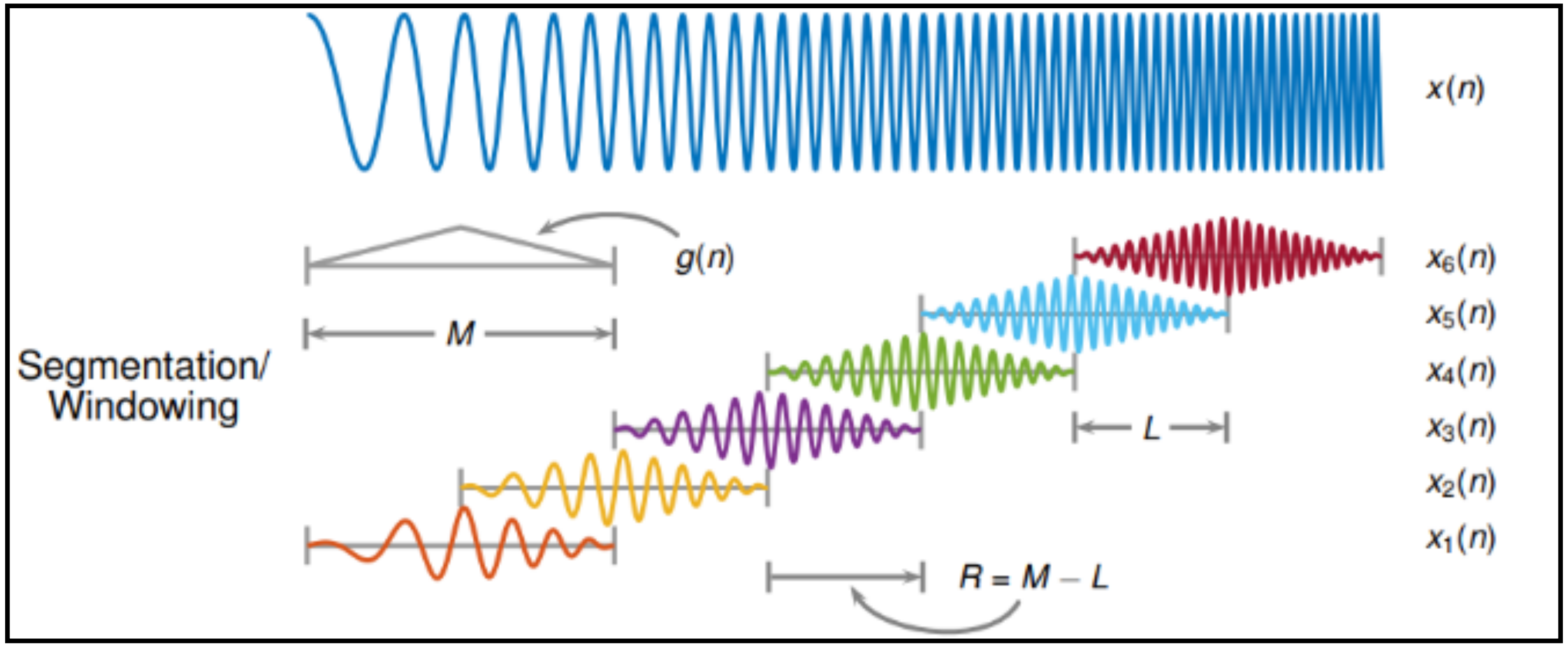


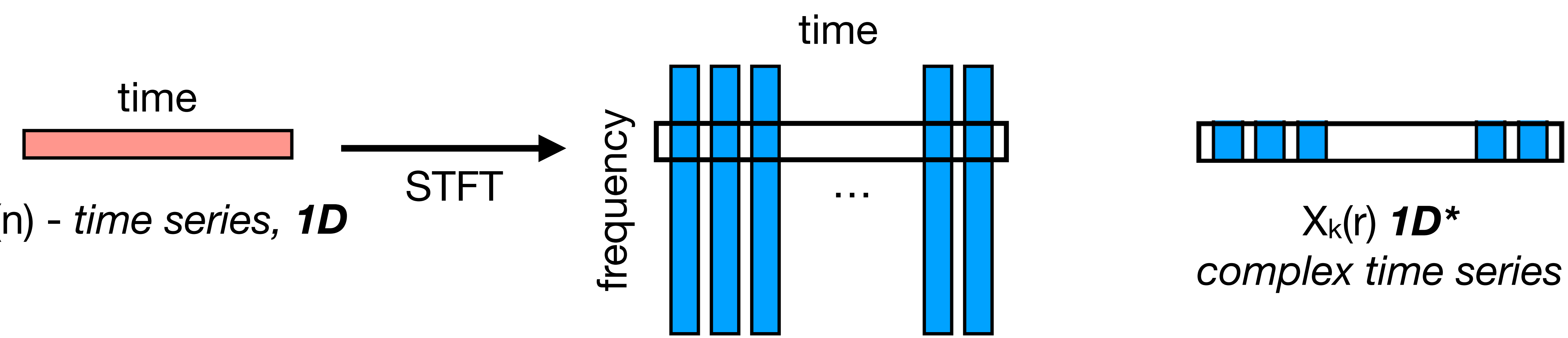
Short Time Fourier Transform

Short Time Fourier Transform (STFT)



Short Time Fourier Transform (STFT)

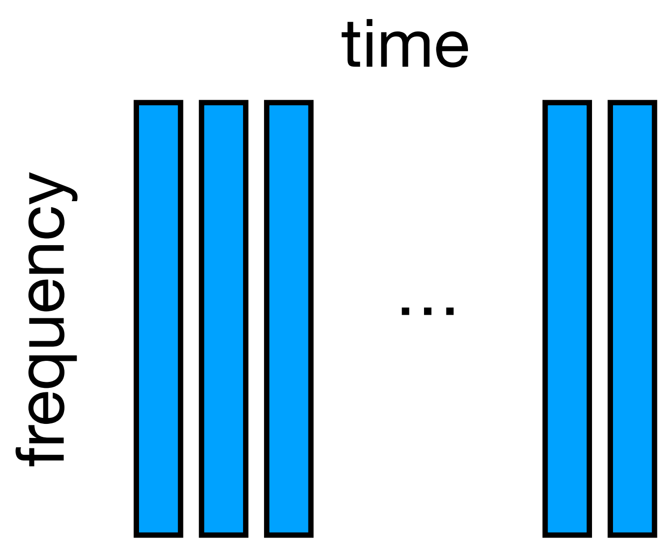




x(n) - *time series, **1D***

$X_k(r)$ ***1D****
complex time series

X(k, m) - *time-frequency representation, **2D***

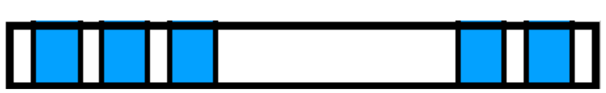


X(k, r) - *time-frequency representation, **2D***

Matrix of complex numbers
over time and frequency.

$$\begin{array}{cc} & \text{time} \\ \text{frequency} & \begin{bmatrix} \mathbf{X_{0,0}} & \mathbf{X_{0,1}} & \mathbf{\ldots} \\ \mathbf{X_{1,0}} & \mathbf{X_{1,1}} & \\ \mathbf{X_{2,0}} & \mathbf{X_{2,1}} & \\ \mathbf{\ldots} & & \end{bmatrix} \end{array}$$

$$\mathbf{X_{2,1} = X(k=2, r=1)}$$
$$\mathbf{= a + ib}$$



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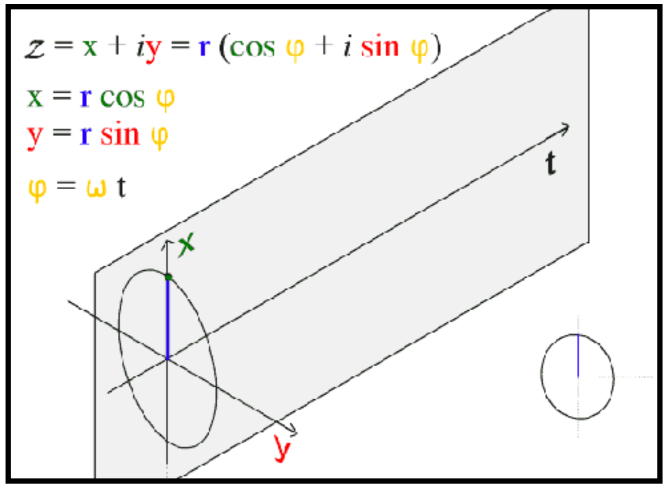
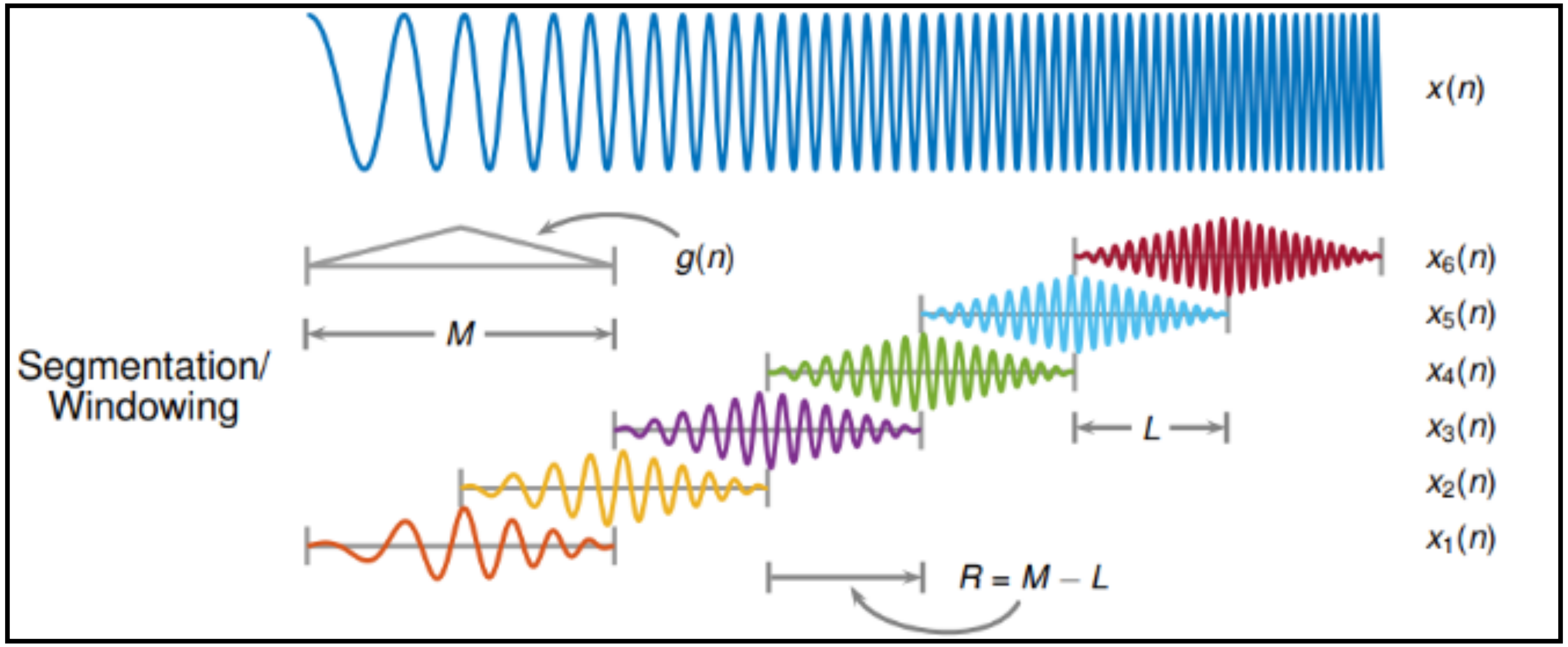


Parameter Housekeeping



Givens

T = total signal length (s)

fs = sampling rate (Hz)

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N}kn}$$

Computable

dt = sampling interval (s)

N = # of samples

df = frequency resolution

f(k) = frequency (Hz)

STFT Parameters (Choice)

M = window length (s)

R = step length (s)

L = overlap length (s)

*** M = R + L**

Parameter Housekeeping

Givens

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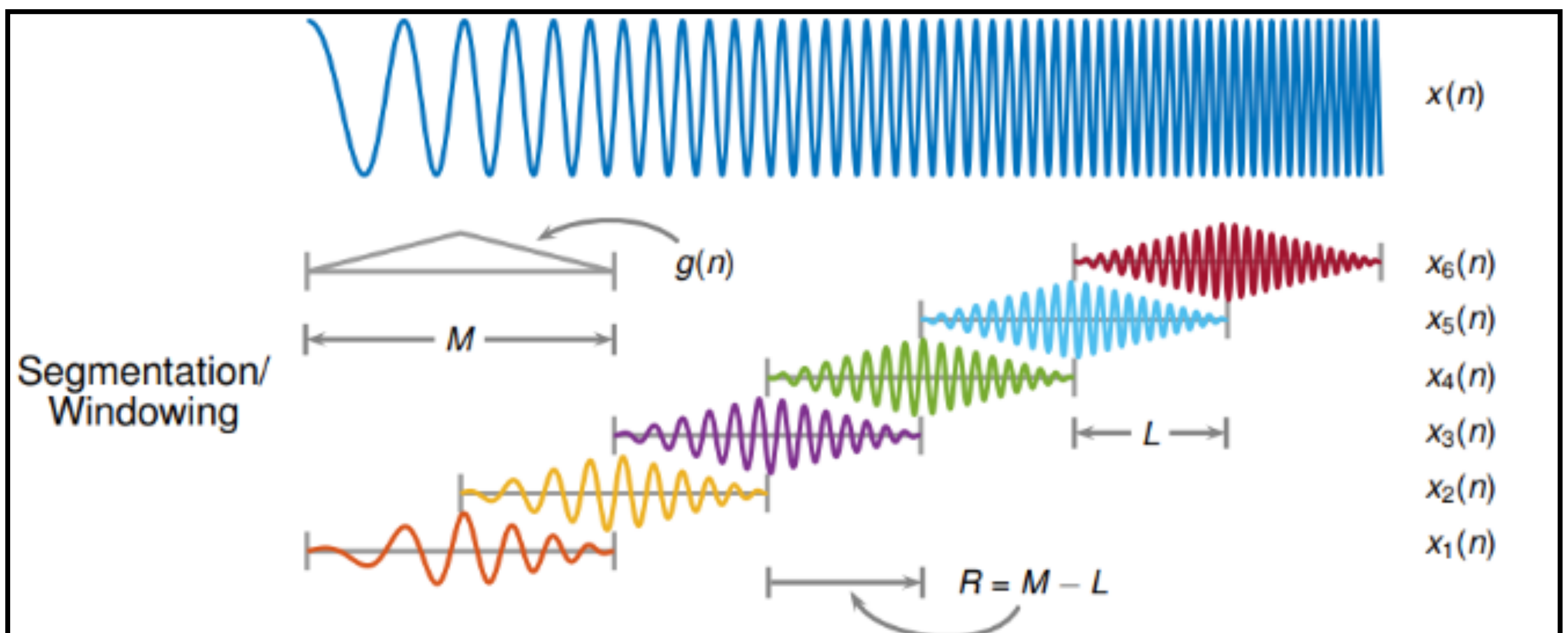
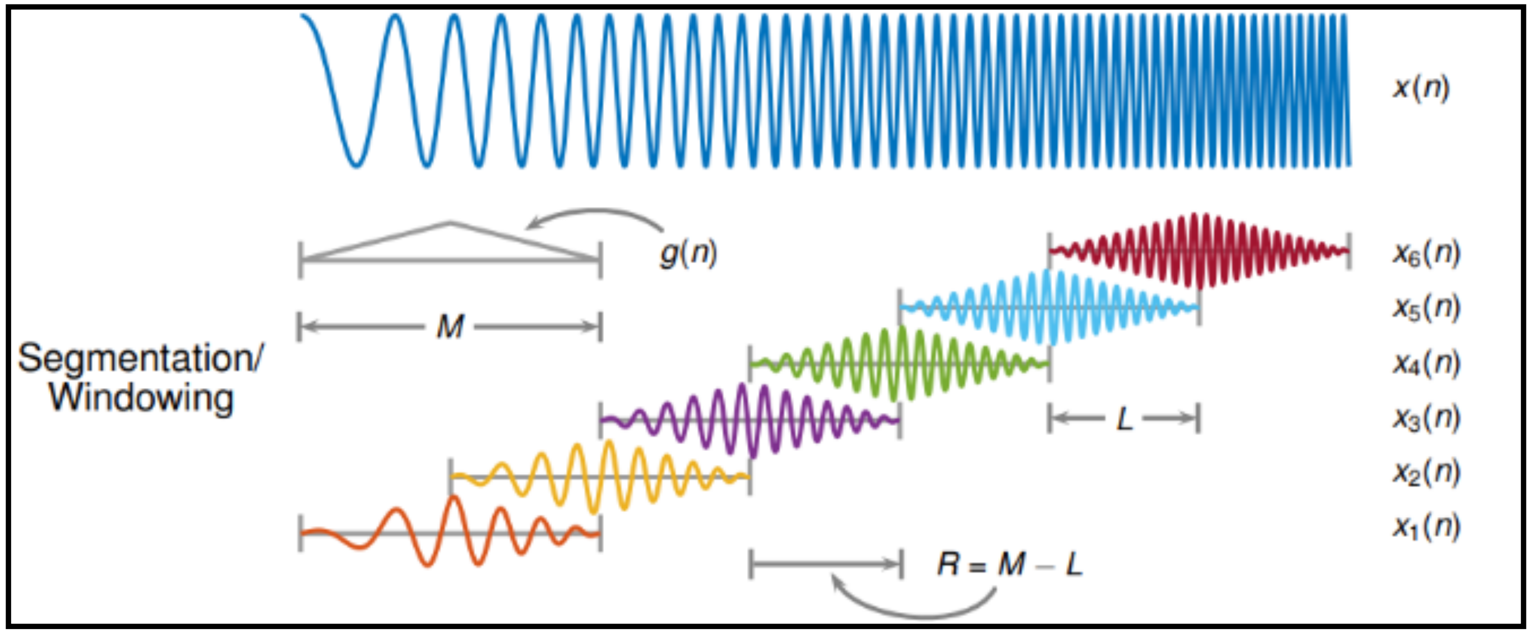


df_{FT} = ? **1/T**

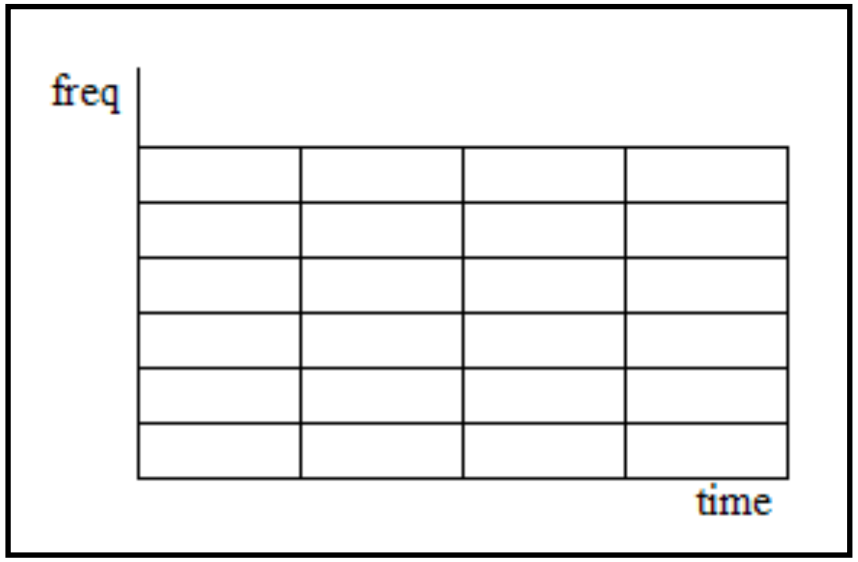
df_{STFT} = ? **1/M**

Time-Frequency Resolution Tradeoff

To get more precise frequency estimates, you lose temporal resolution (where it is in time), and vice versa.

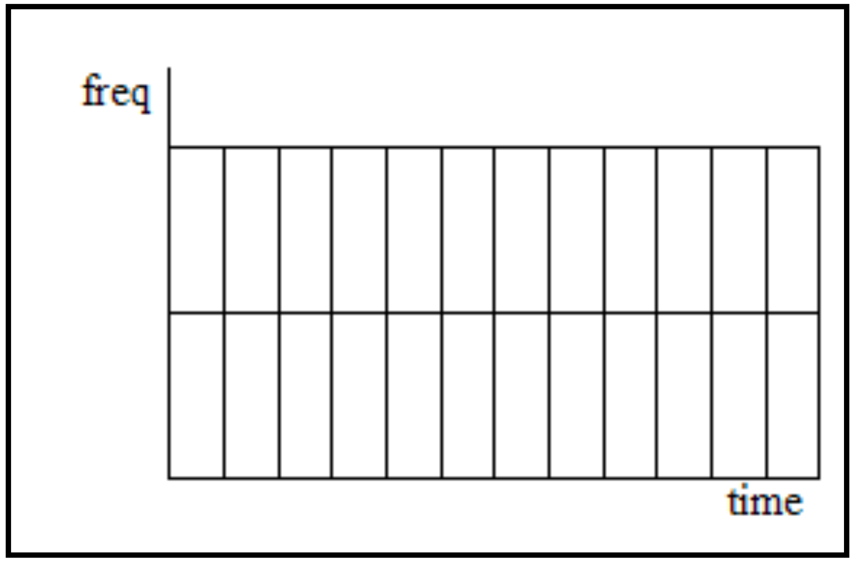


Longer Windows (big M, small df)



$M^*df = 1$

Shorter Windows (small M, big df)



Rule of thumb: depends on whether you need more certainty in time or frequency.

Example	Frequencies	Timescale of Change	STFT Parameters
seismic waves (earthquakes)	0.2Hz-10Hz 20Hz	minutes	M = 5s or M=0.1s
heart rate	1 - 2Hz	seconds	M=1 or 0.5s
color of the sky (from fixed point on earth)	500 tHz	hours	

Goals for Today

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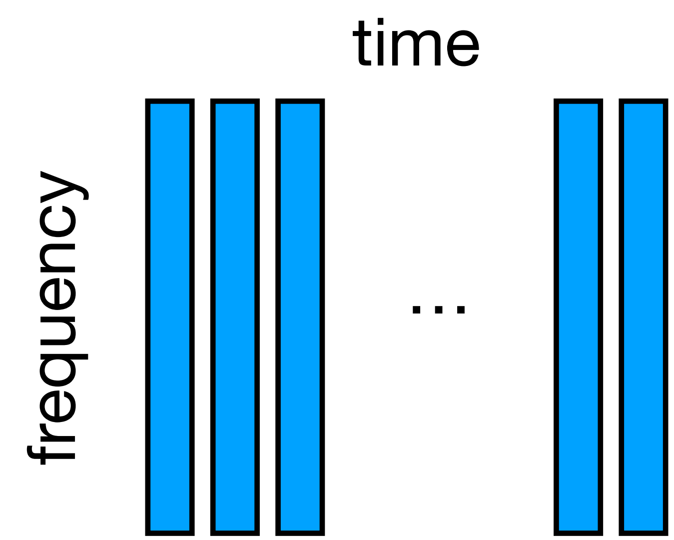
STFT Transformations: Power

$$\begin{array}{cc} & \text{time} \\ \text{frequency} & \begin{bmatrix} X_{0,0} & X_{0,1} & \dots \\ X_{1,0} & X_{1,1} & \\ X_{2,0} & X_{2,1} & \\ \dots & & \end{bmatrix} \end{array}$$



$X(k, r)$ - *time-frequency representation, **2D***

$$\mathbf{X(k,r) = a + ib}$$

Compute power

(Power) Spectrogram

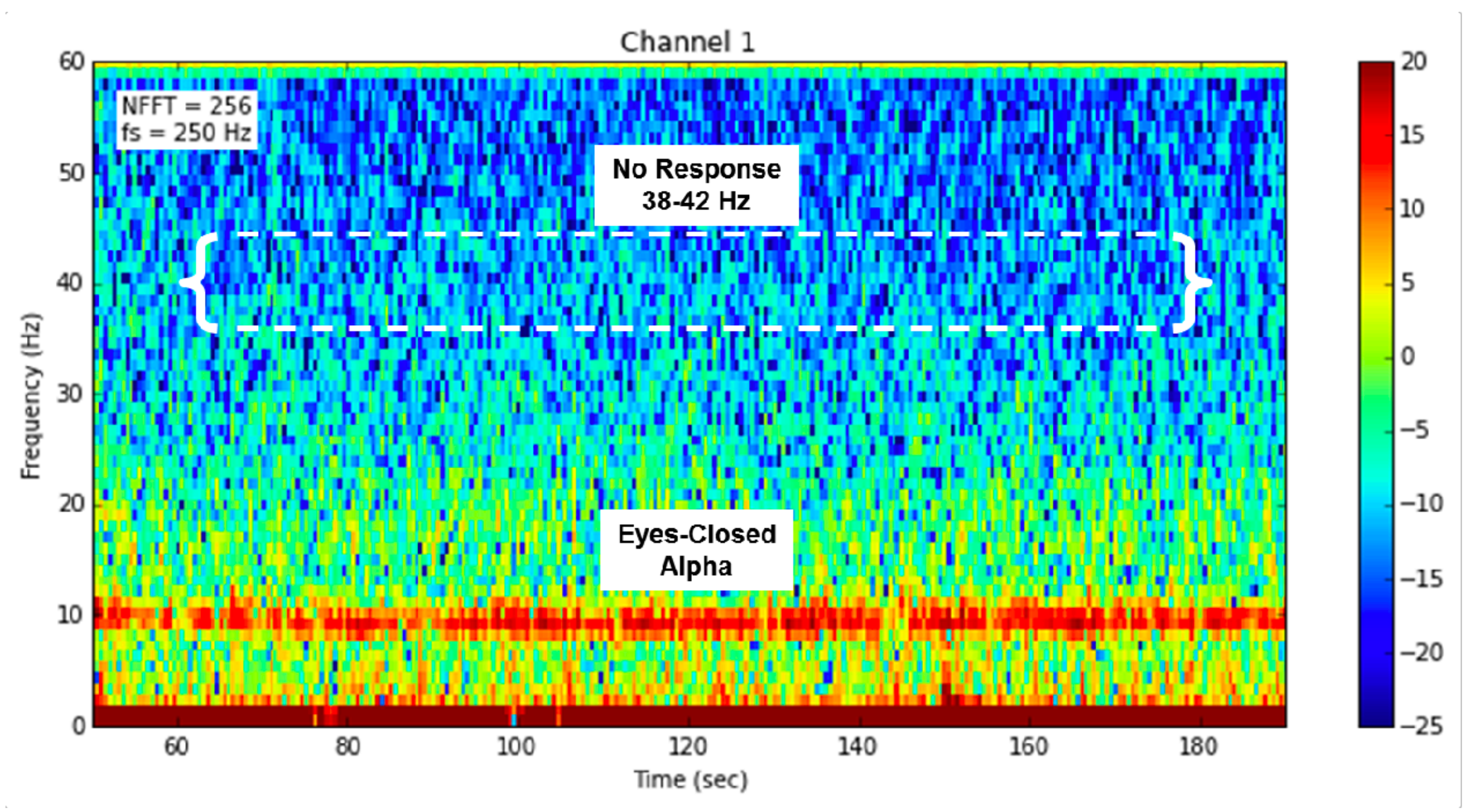
Average over all time → **Power Spectrum** (and Power Spectral Density)

Average over trials → **Event-Related Spectral Response (ERSP)**

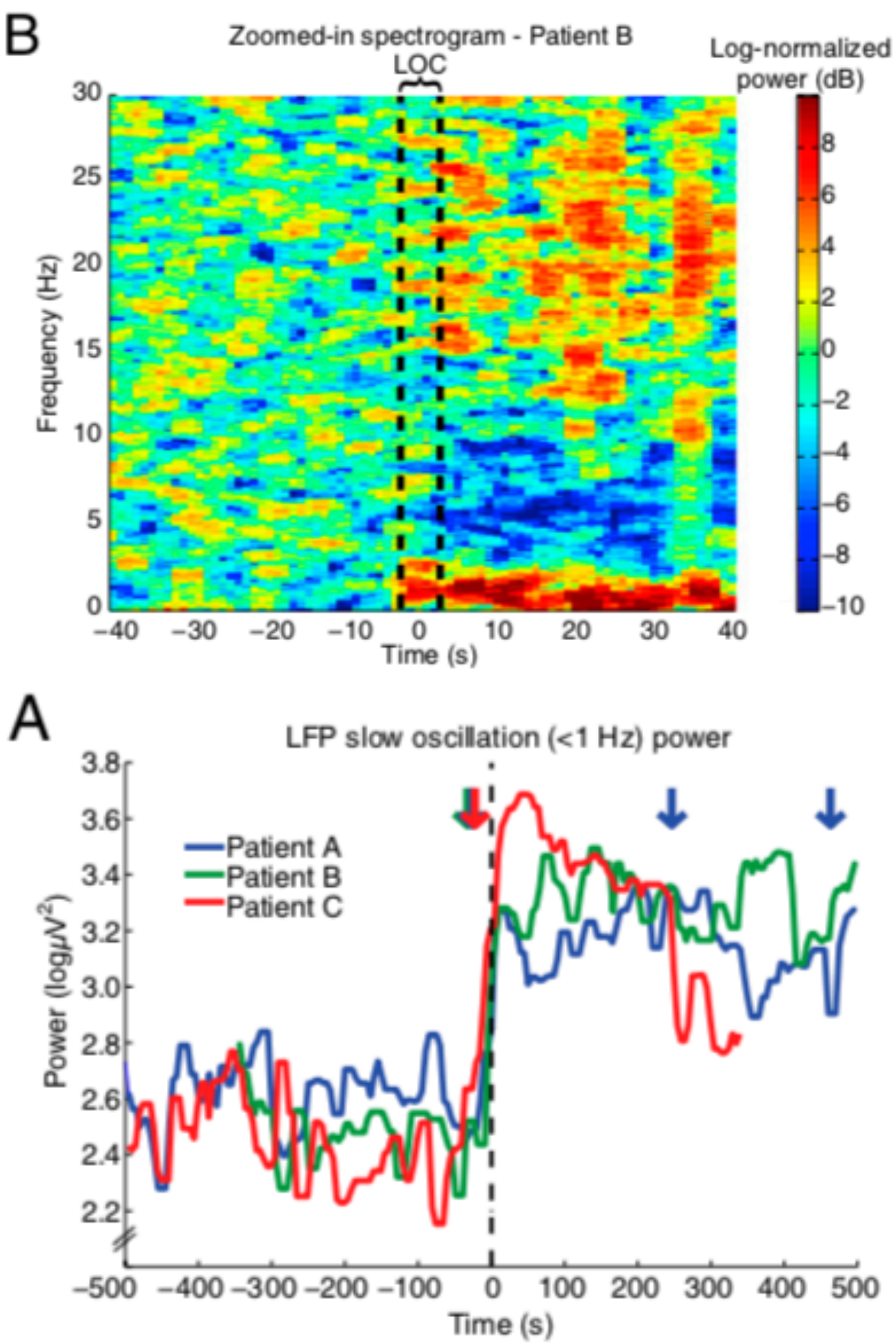
Spectrogram

$$\boxed{power(k) = |X_k|^2 = a_k^2 + b_k^2}$$

Alpha Oscillation (10Hz)

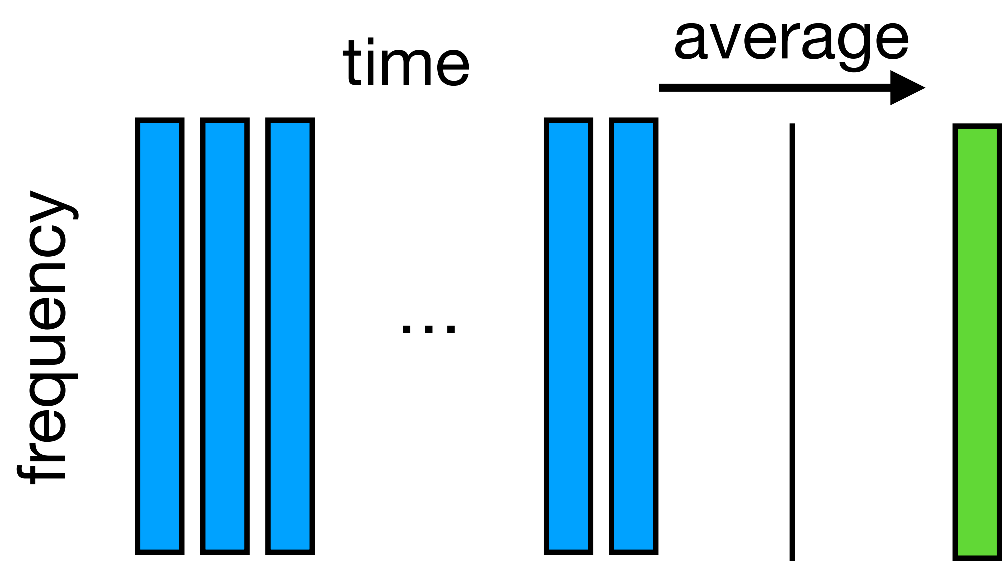


Sleep Oscillations (0.5Hz)



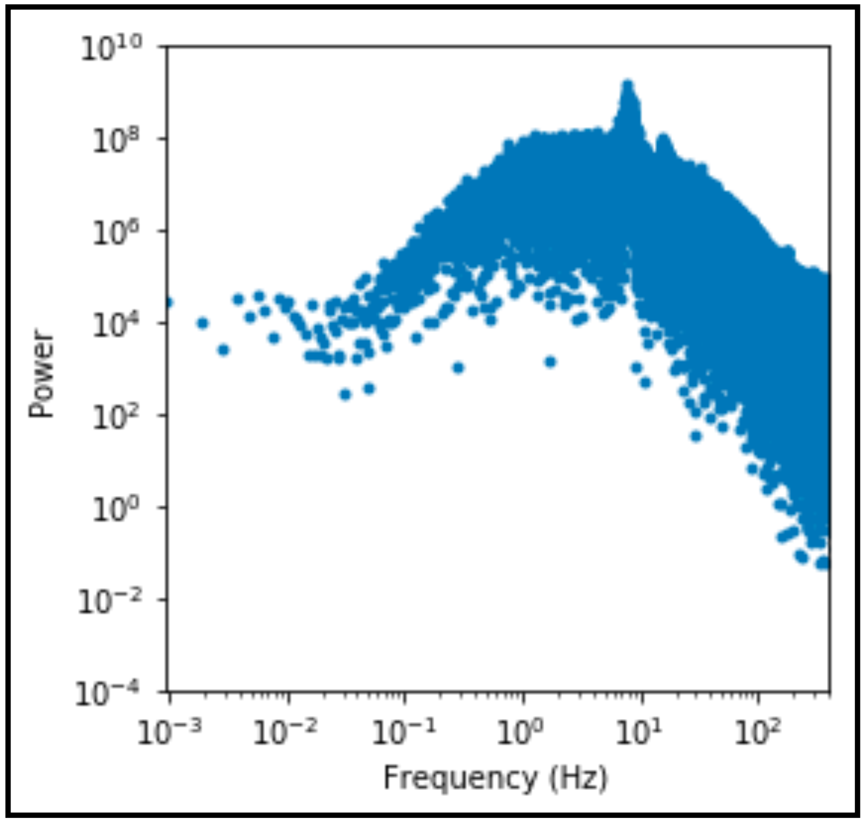
P(t): power over time

Power Spectrum/Spectral Density



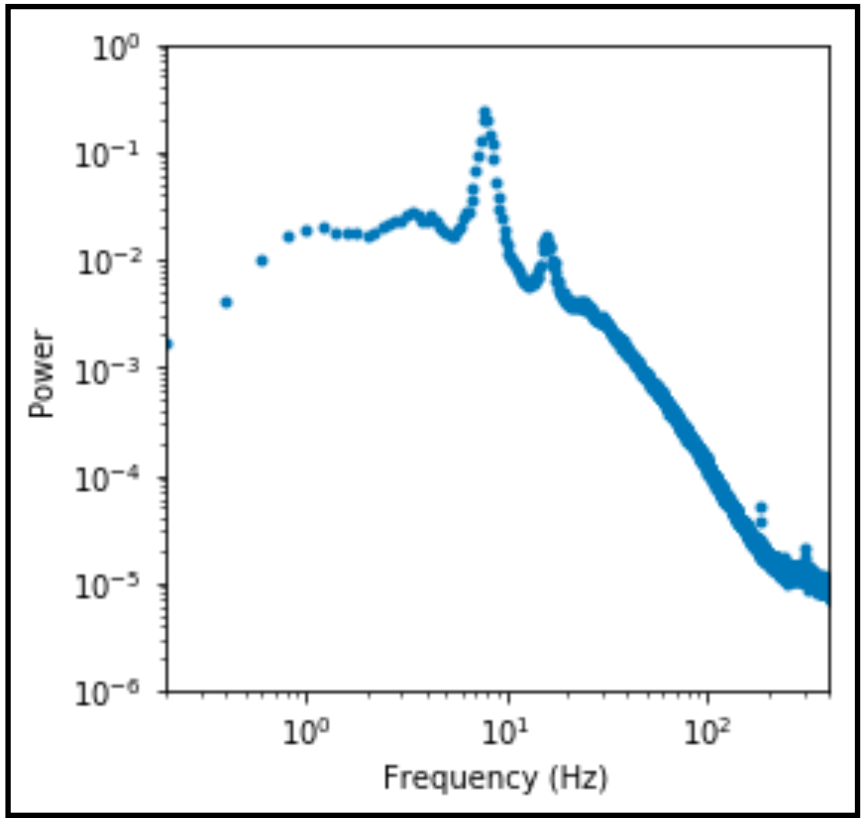
Power Spectral Density (PSD)

No averaging (directly from DFT):
Periodogram



Averaging (no overlap, R = M):
Bartlett's Method

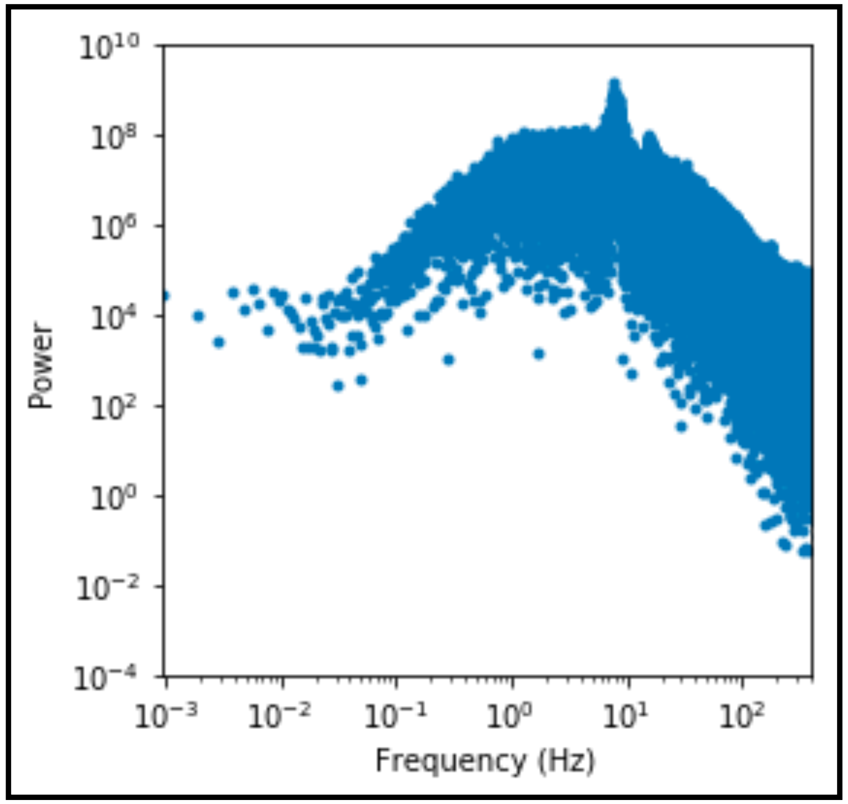
Averaging (half overlap, R = M/2):
Welch's Method



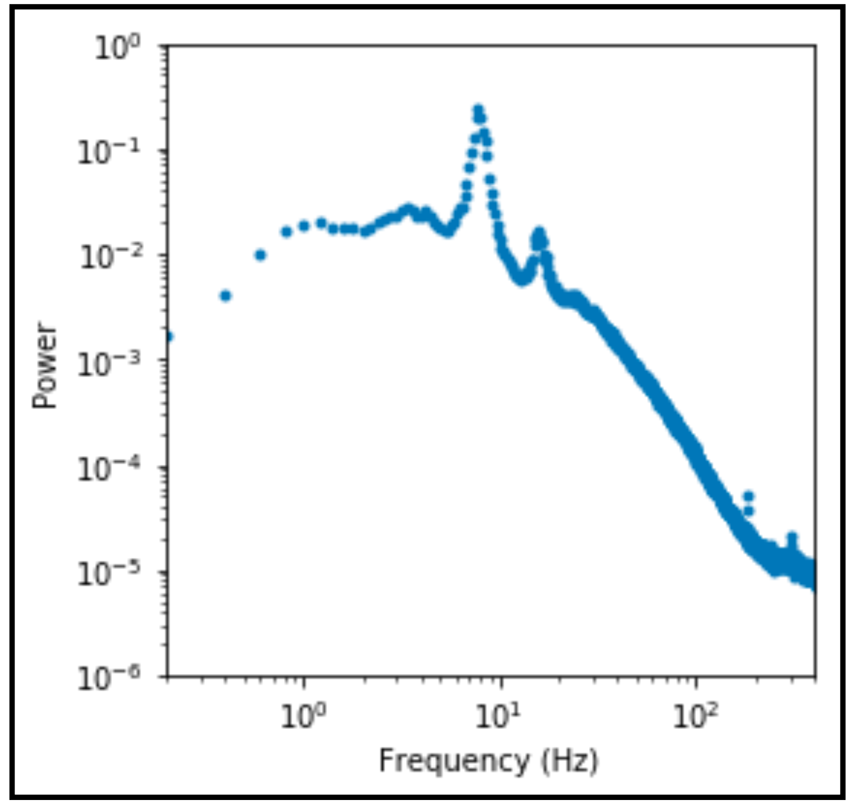
Reduce noise in estimates (similar to ERP)

Why Power Spectral “Density”

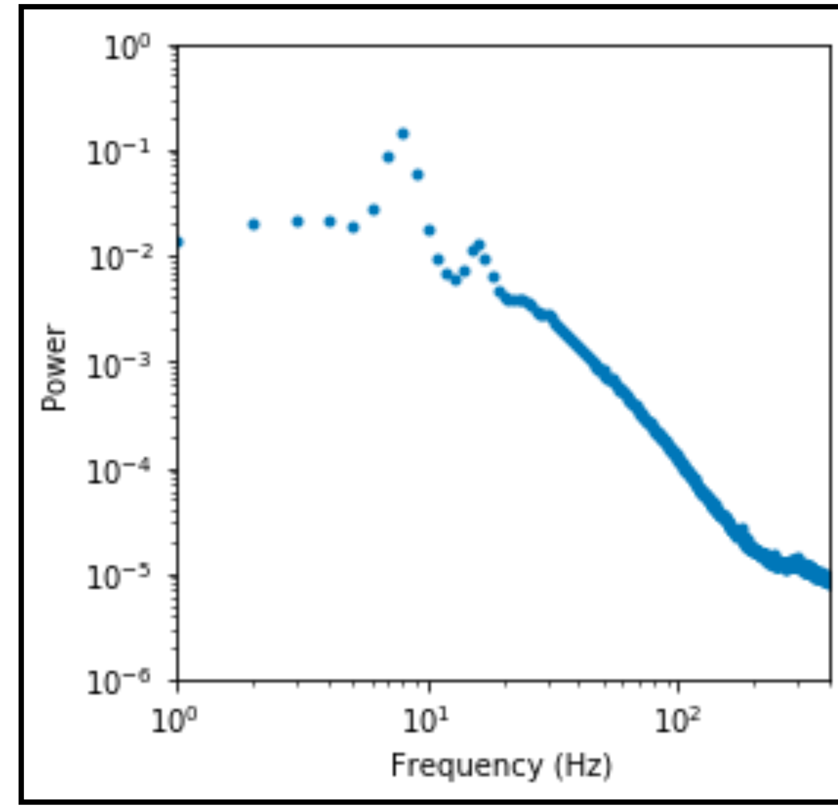
M = N, unnormalized



M = 5s * fs



M = 1s * fs



$$\boxed{\sum_{n=0}^{N-1} |x_n|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X_k|^2}$$

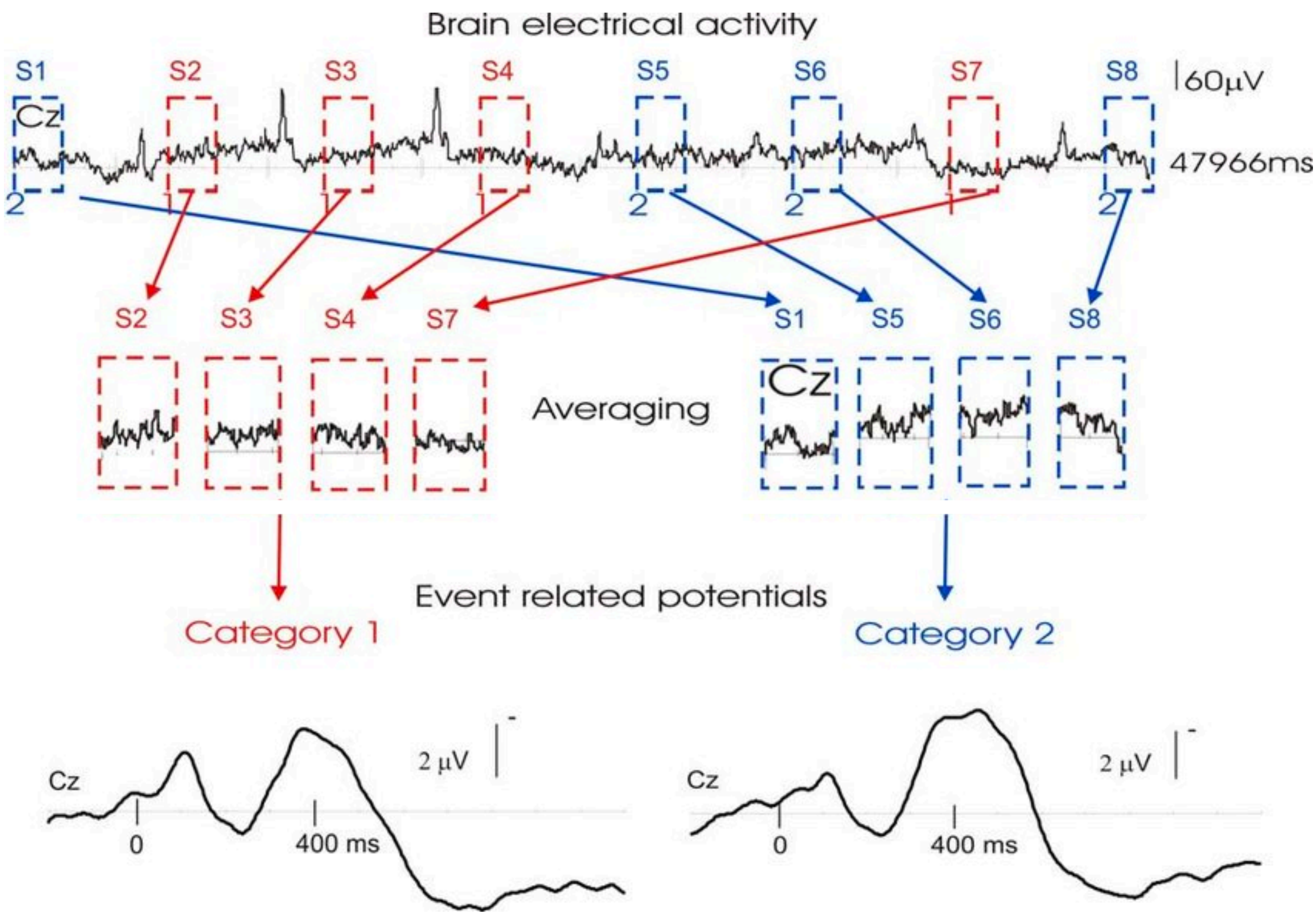
Parseval’s Theorem

We don't want power estimate to change based on how long of a signal we use

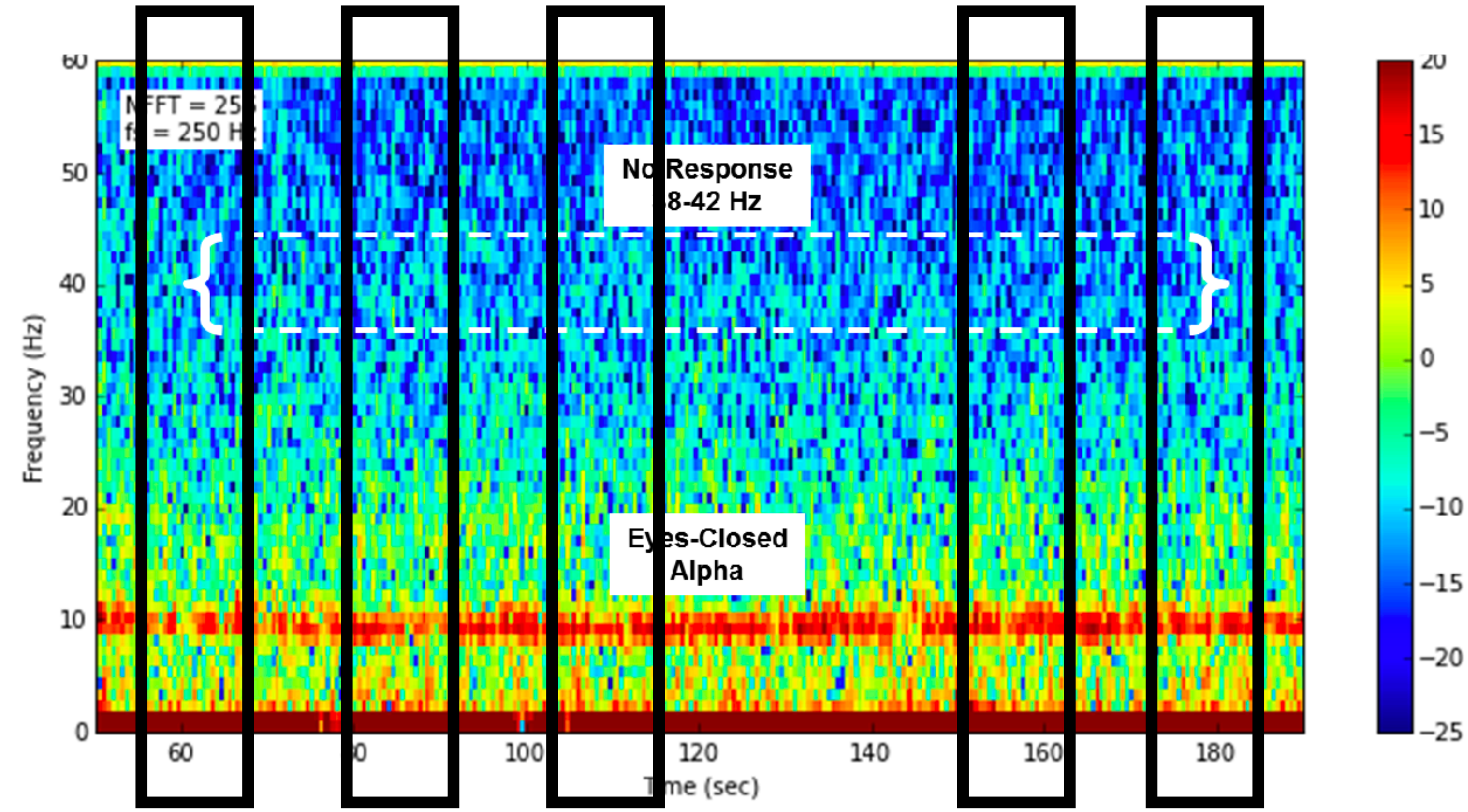
so normalize by M

density -> power per frequency (V^2/Hz)

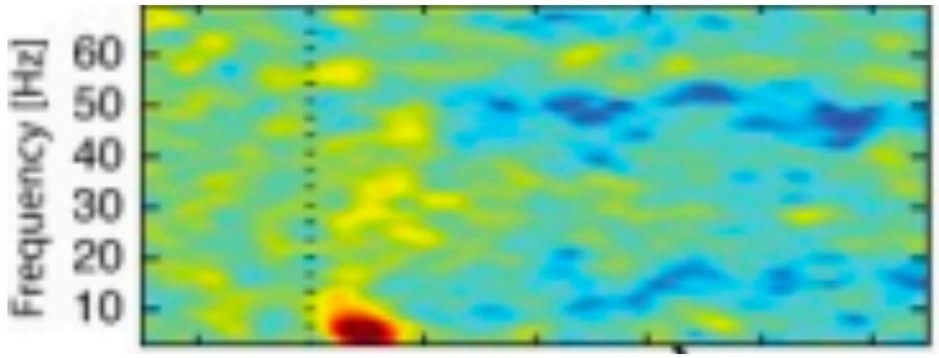
Review: Event-Related Potential



Event-Related Spectral Response

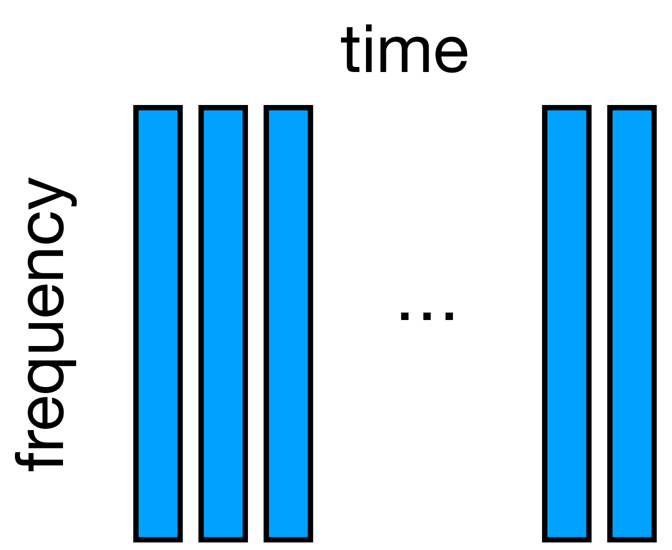


Collect & Average over trials



STFT Transformations: Phase

$$\begin{array}{cc} & \text{time} \\ \text{frequency} & \begin{bmatrix} X_{0,0} & X_{0,1} & \dots \\ X_{1,0} & X_{1,1} & \\ X_{2,0} & X_{2,1} & \\ \dots & & \end{bmatrix} \end{array}$$



X(k, r) - *time-frequency representation, **2D***

$$\mathbf{X(k,r) = a + ib}$$

Compute phase

Phase Spectrogram

Circular average over time/trials

(Inter-Trial) Coherence

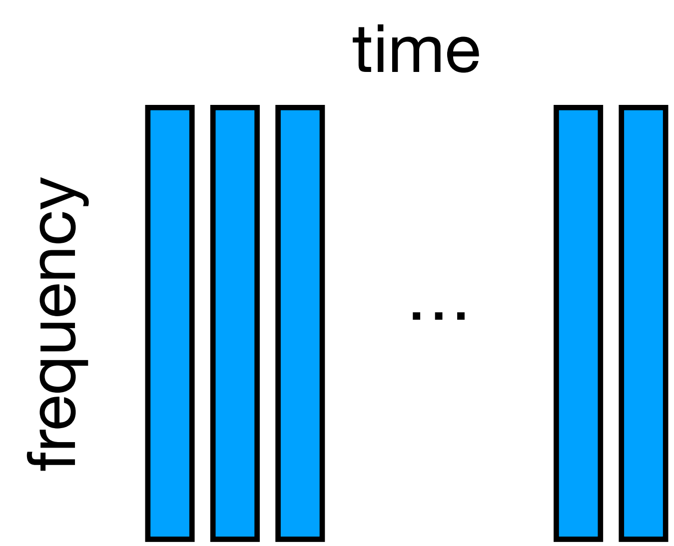
STFT Transformations: Phase

time

frequency

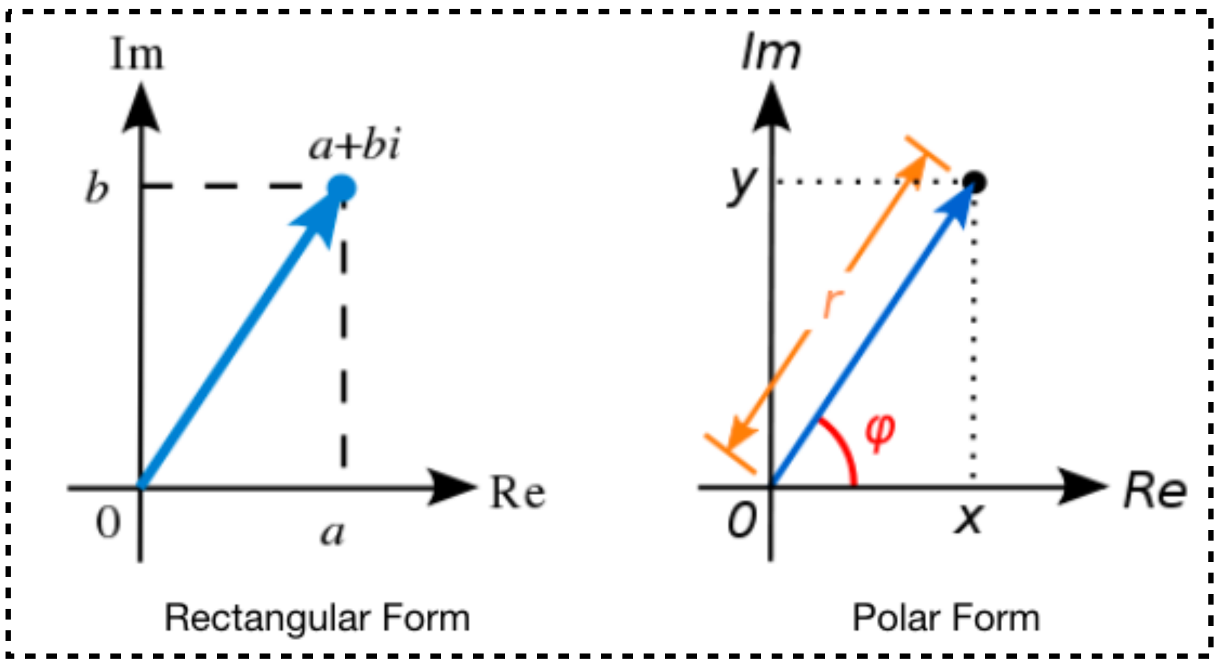
$$\begin{bmatrix} X_{0,0} & X_{0,1} & \dots & \\ X_{1,0} & X_{1,1} & & \\ X_{2,0} & X_{2,1} & & \\ \dots & & & \end{bmatrix}$$

X(k,r) = a + ib



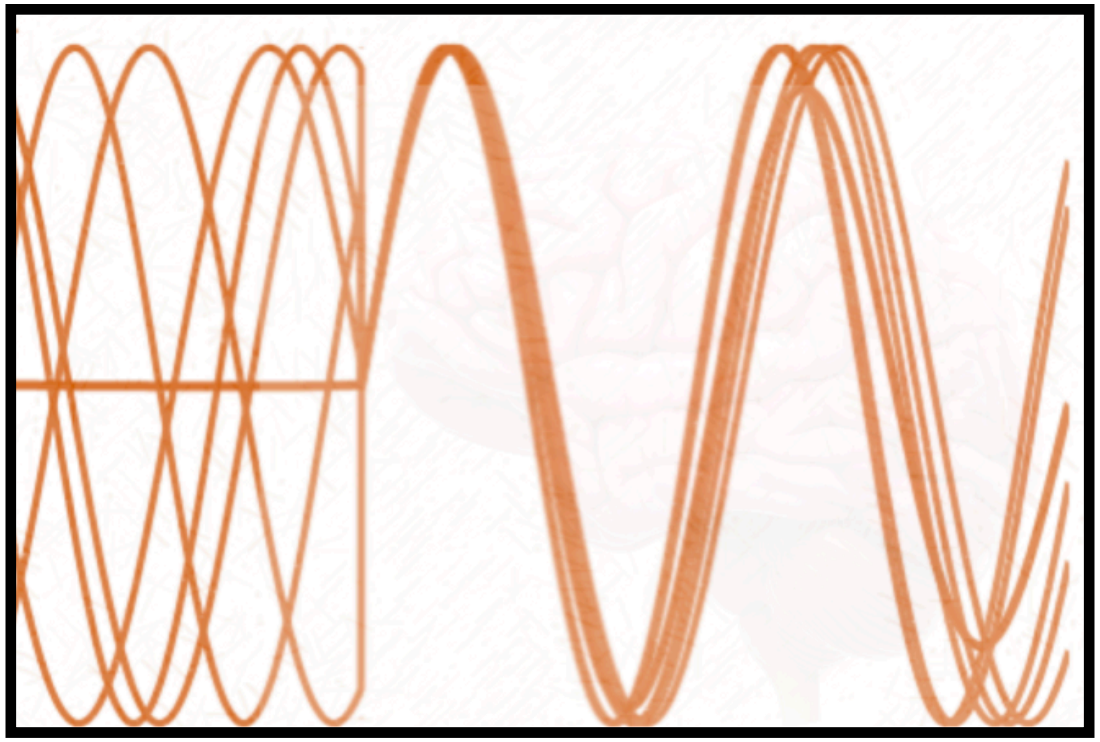
X(k, r) - *time-frequency representation,* ***2D***

$$\boxed{phase(k) = tan^{-1}\frac{b_k}{a_k}}$$



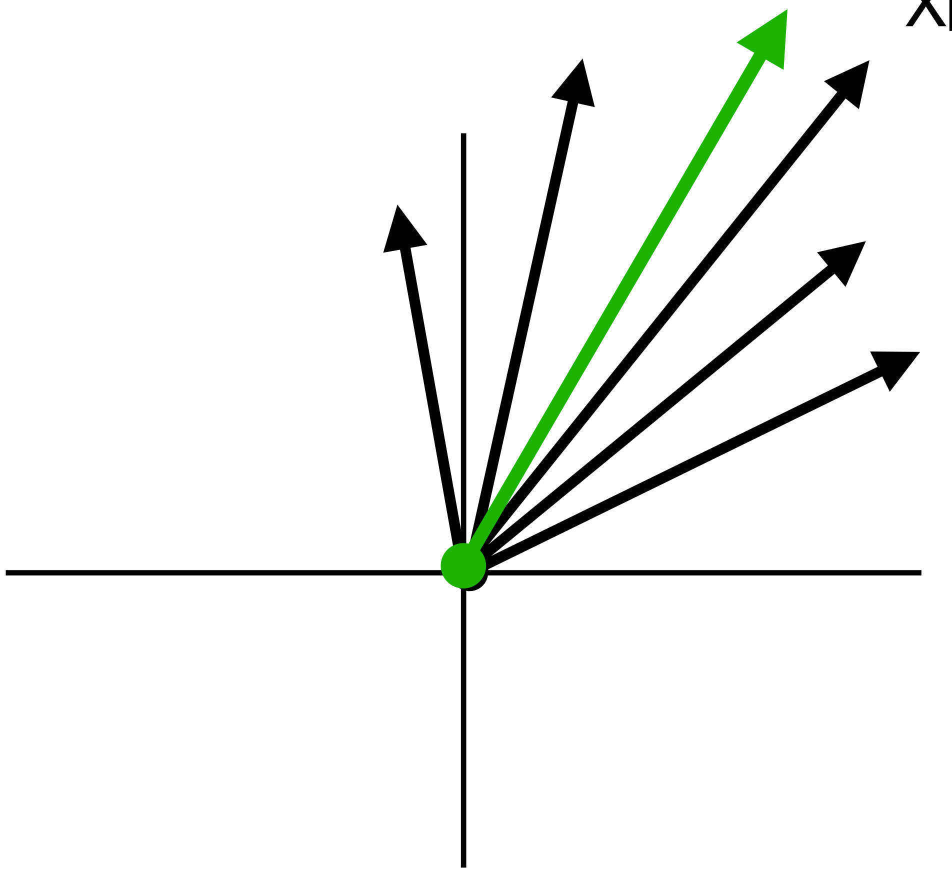
Synchrony & Phase Locking

Incoherent
(Desynchronized)



Coherent
(Synchronized)

$X_k = re^{i\phi}$



Average over phase angles

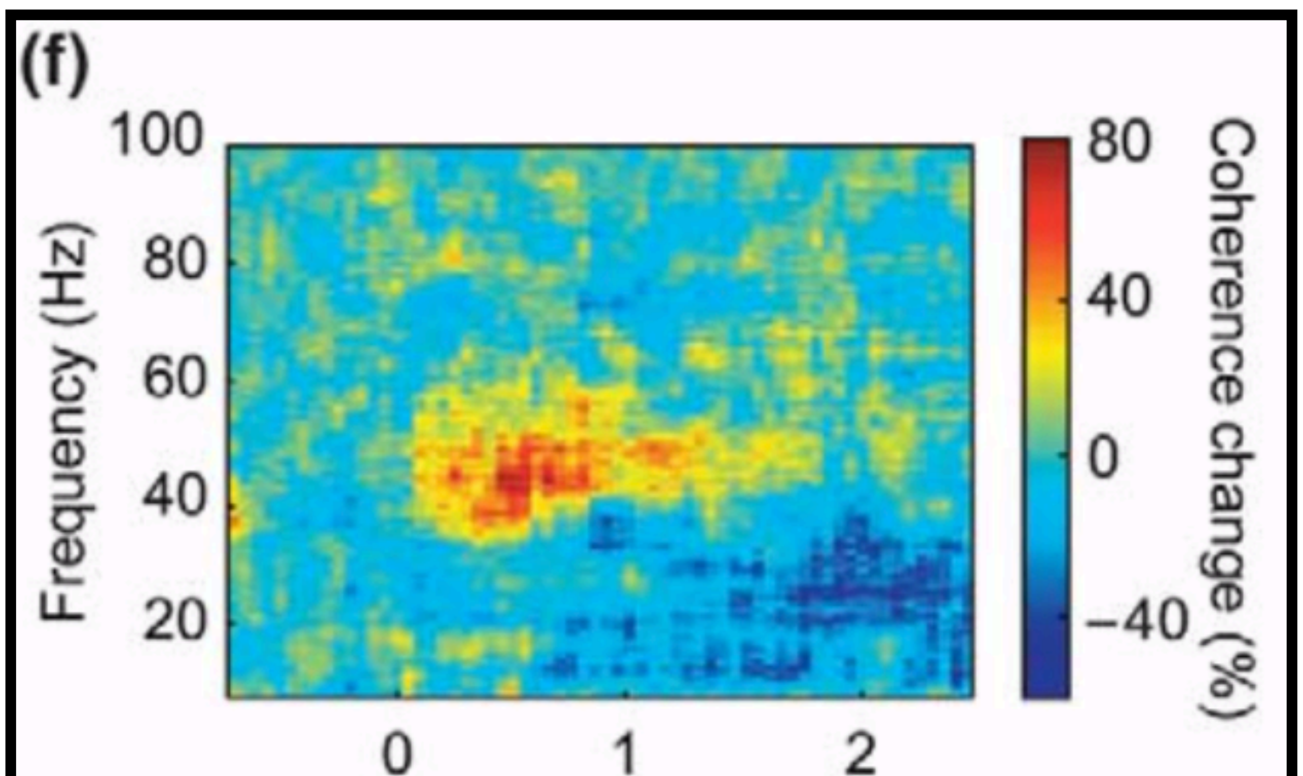
(Compute the normalized vector average)

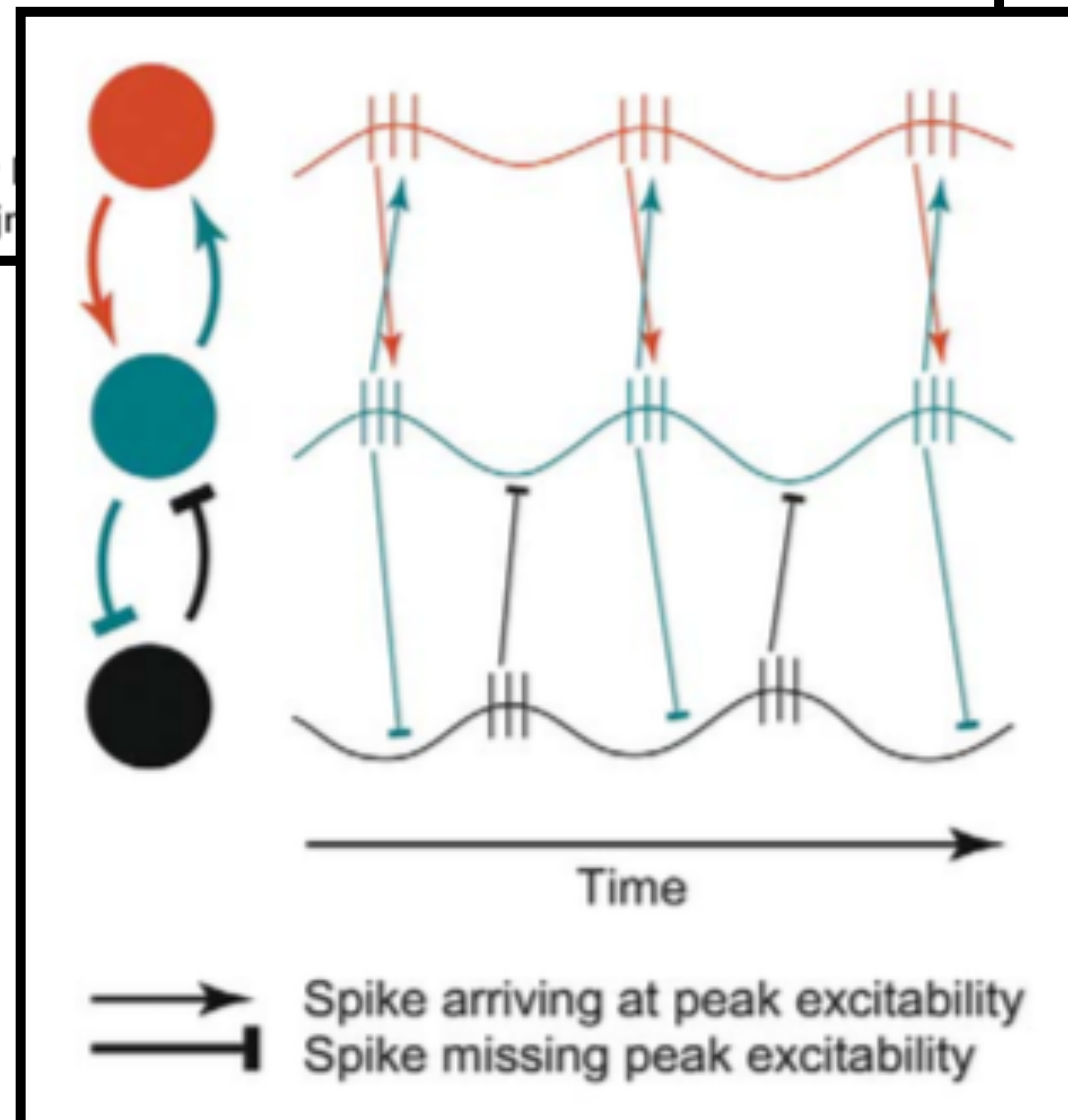
What would the vectors (black) and vector average (green) look like if the phases are incoherent?

A mechanism for cognitive dynamics: neuronal communication through neuronal coherence

Pascal Fries[1,2]

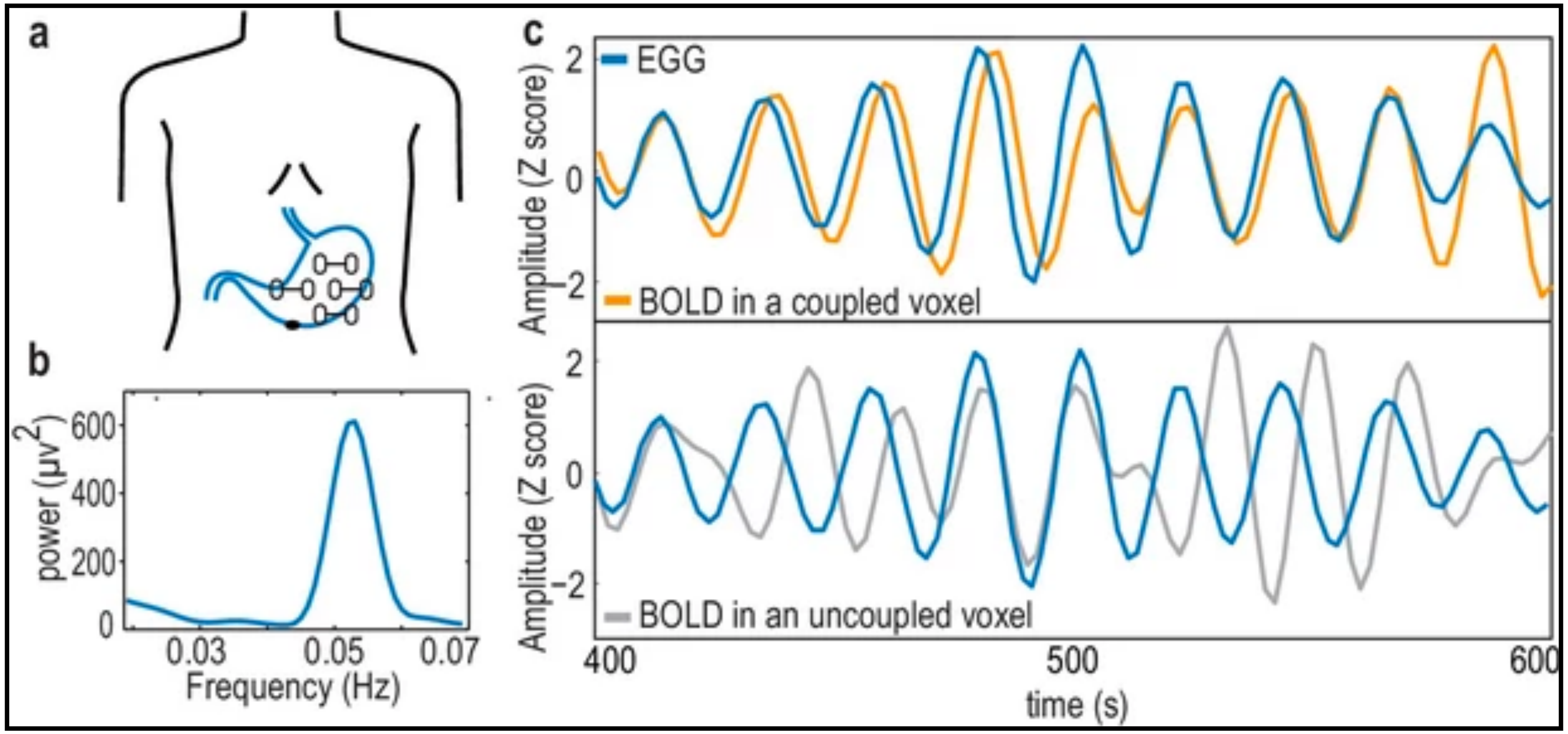
[1]F.C. Donders Centre for Cognitive Neuroimaging, Radboud University
[2]Department of Biophysics, Radboud University Nijmegen, 6525 EZ Nij





Stomach-Brain Synchrony?

Stomach-brain synchrony reveals a novel, delayed-connectivity resting-state network in humans



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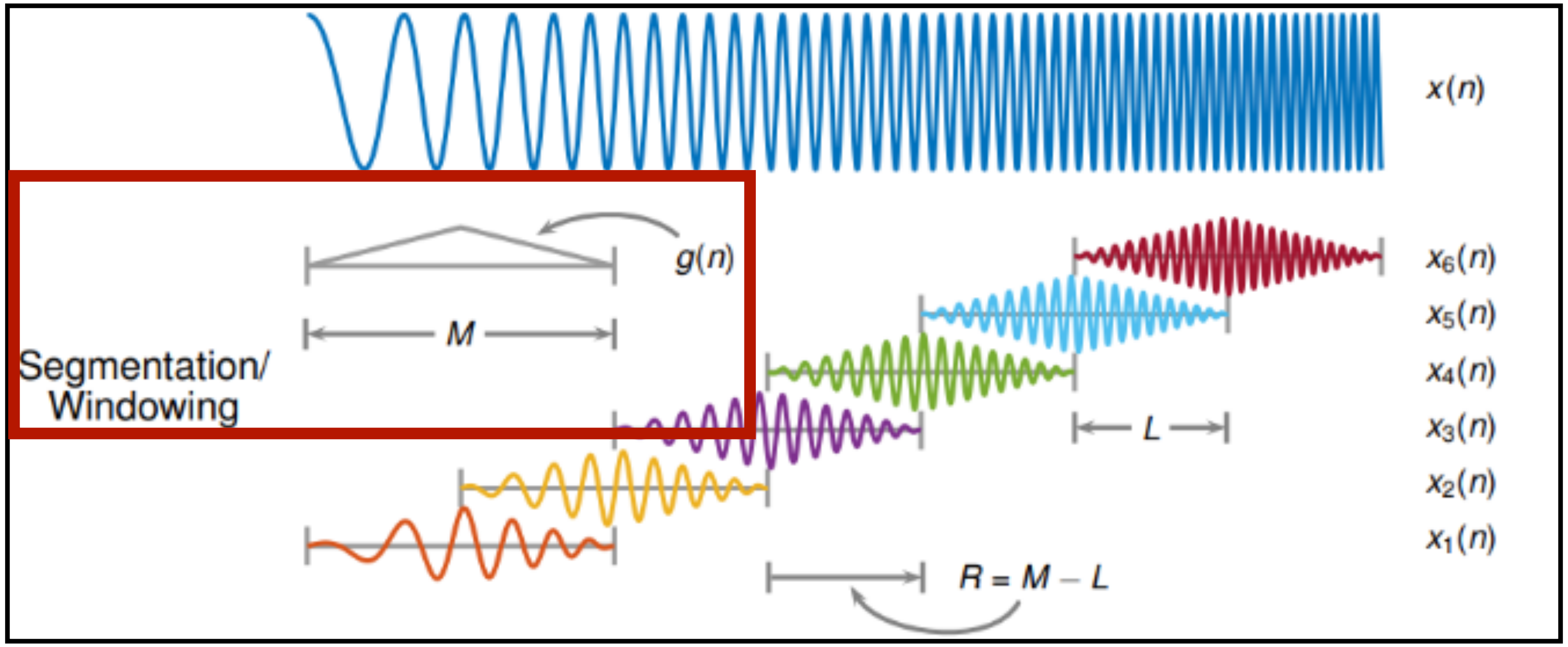
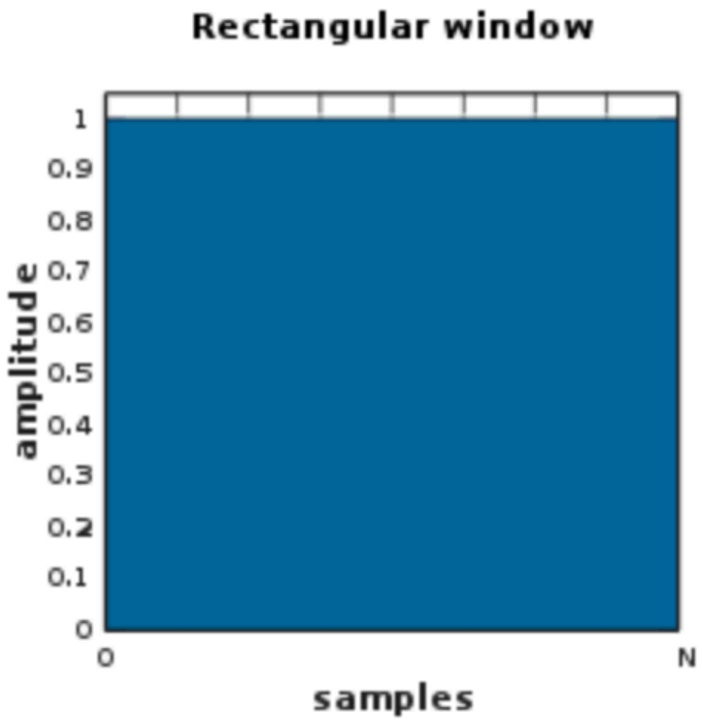
Windowing





Simply grabbing a window of data is like **multiplying** by a rectangular window.

1 inside the window, 0 everywhere else

Intuition: this introduces sharp edge artifacts that are not present in the window.

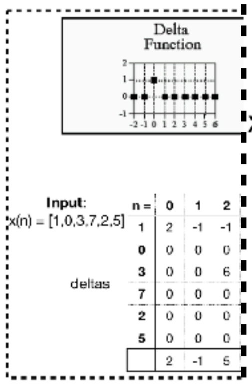
Multiplication & Convolution

(Circular) Convolution Theorem

Convolu

frequenc

h:



IRF: [2,-1,-1,0]

Input:
x(n) = [1,0,3,7,2,5]

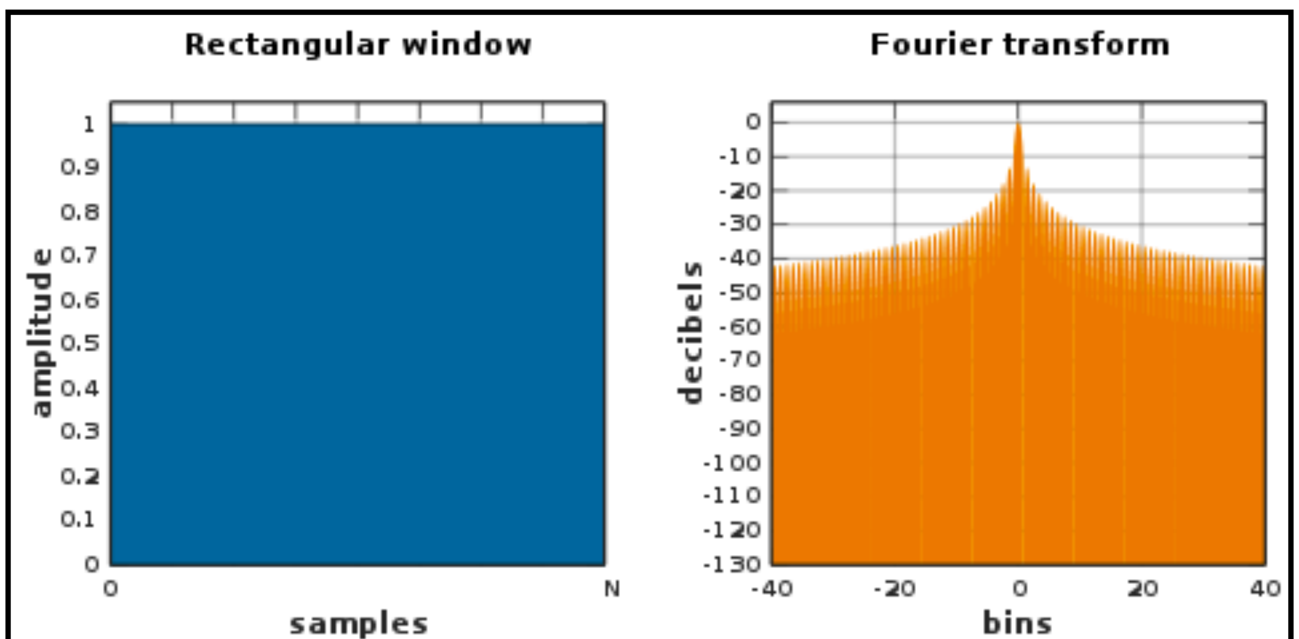
deltas

n =	0	1	2	3	4	5	6	7	8	9	10
1	2	-1	-1	0	0	0	0	0	0	0	0
0	0	0	0	0	0						
3	0	0	6	-3	-3	0					
7	0	0	0	14	-7	-7	0				
2	0	0	0	0	4	-2	-2	0			
5	0	0	0	0	0	10	-5	-5	0	0	
	2	-1	5	11	-6	1	-7	-5	0	0	0

Multiplying by a window is equivalent to **convolving by the windows frequency response.**

“Smearing” your signal’s spectrum.

You’re **always** applying a window, even when you think you aren’t.

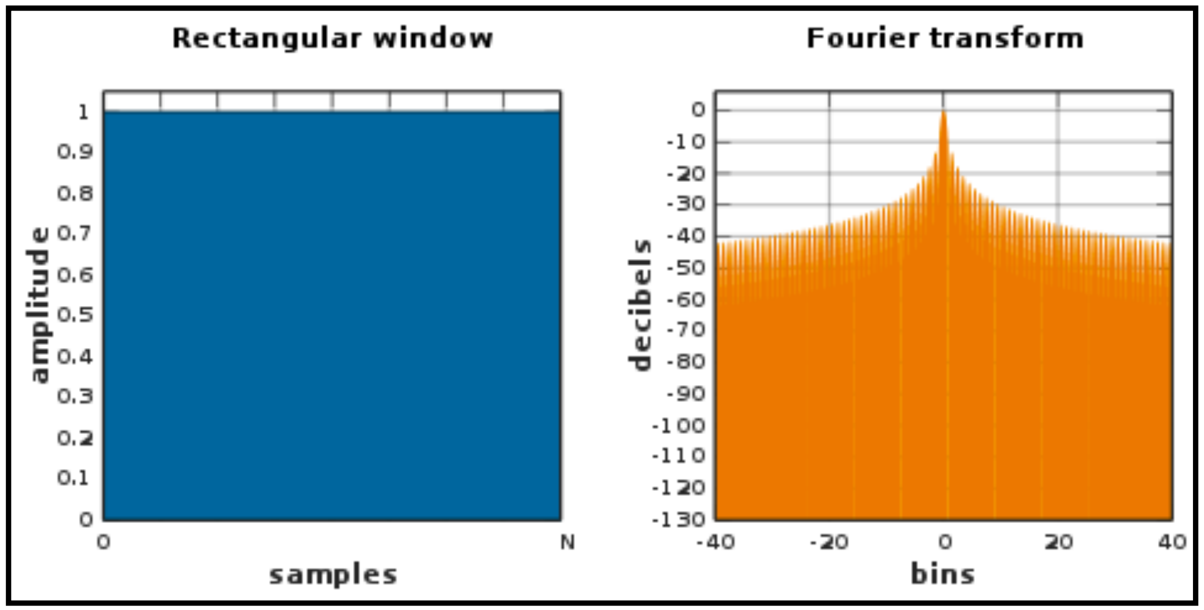


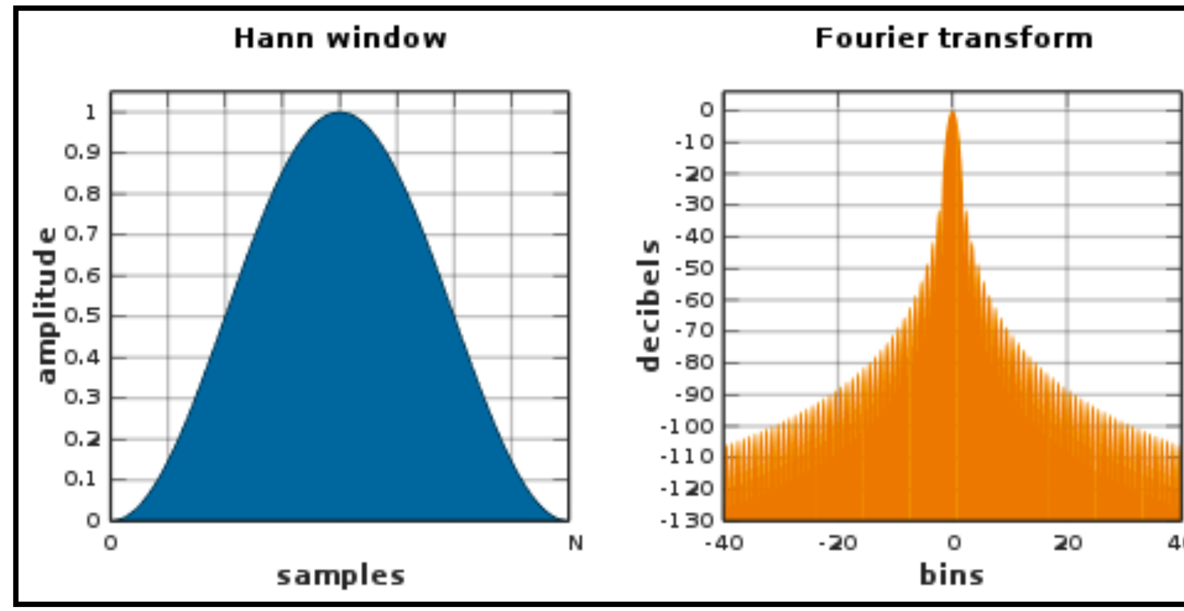
“**Spectral Leakage**”: making it appear as if there is power at a frequency when there isn’t.

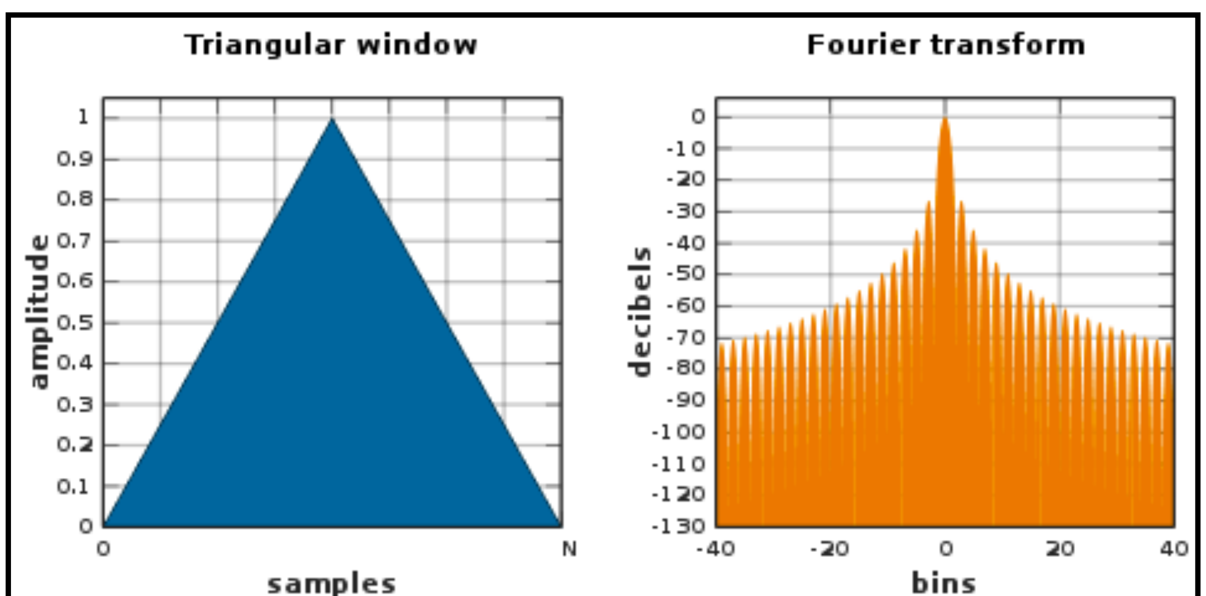
Windowing Functions

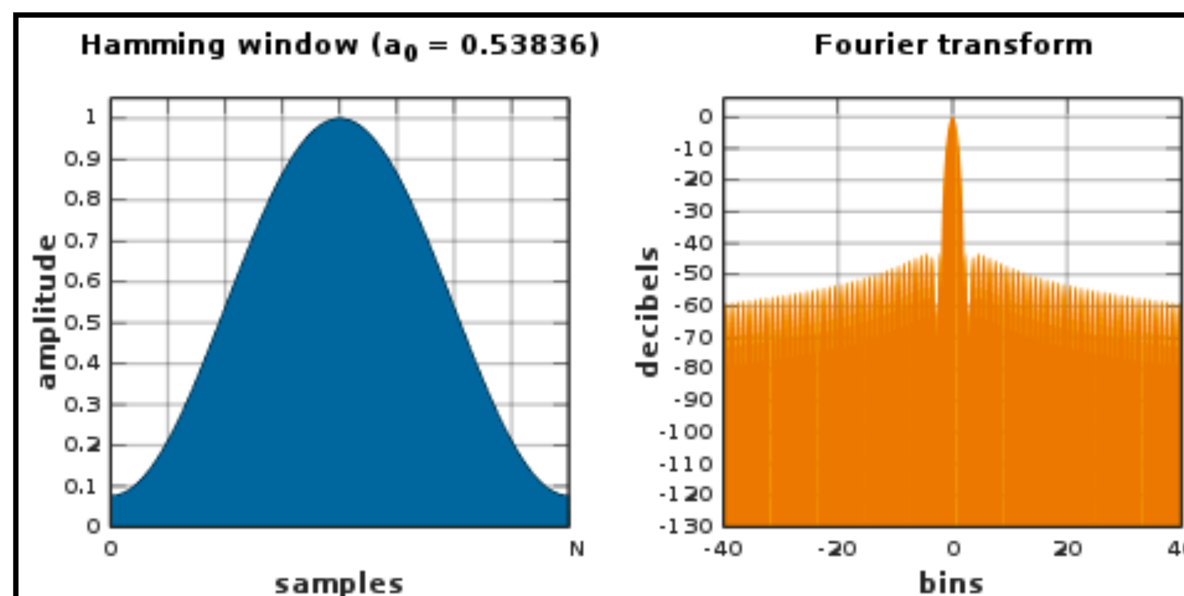
You’re **always** applying a window, even when you think you aren’t.

So we explicitly pick a windowing function to optimize frequency response.









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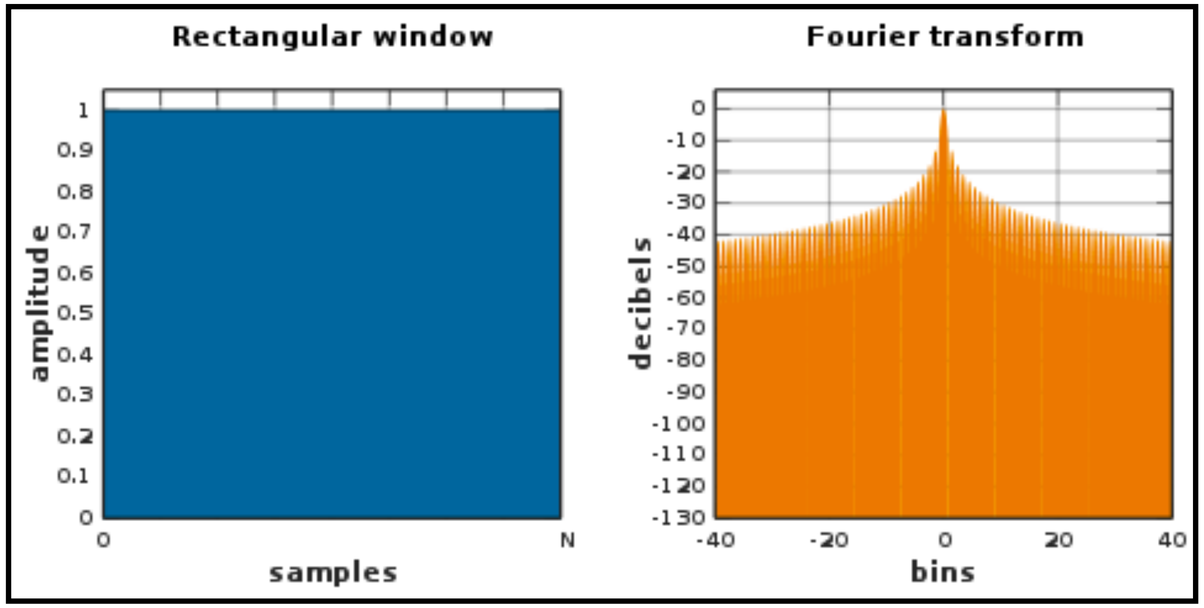

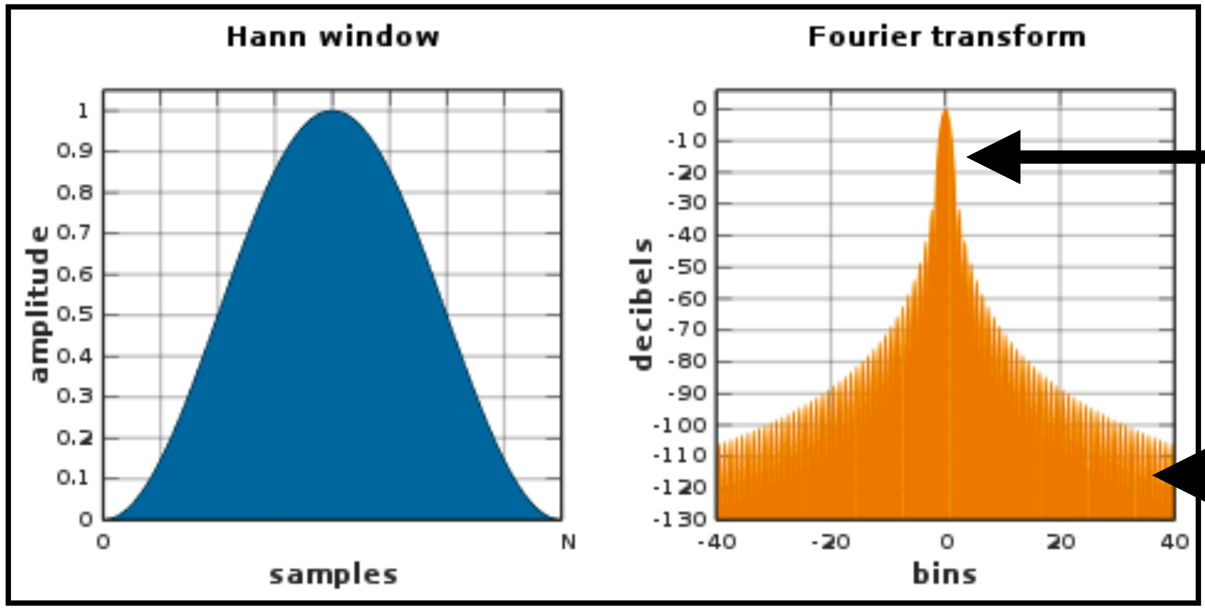

minimize this width, maximize its height (**main lobe**)

minimize this height and maximize the drop-off (**side lobe**)

Windowing Functions

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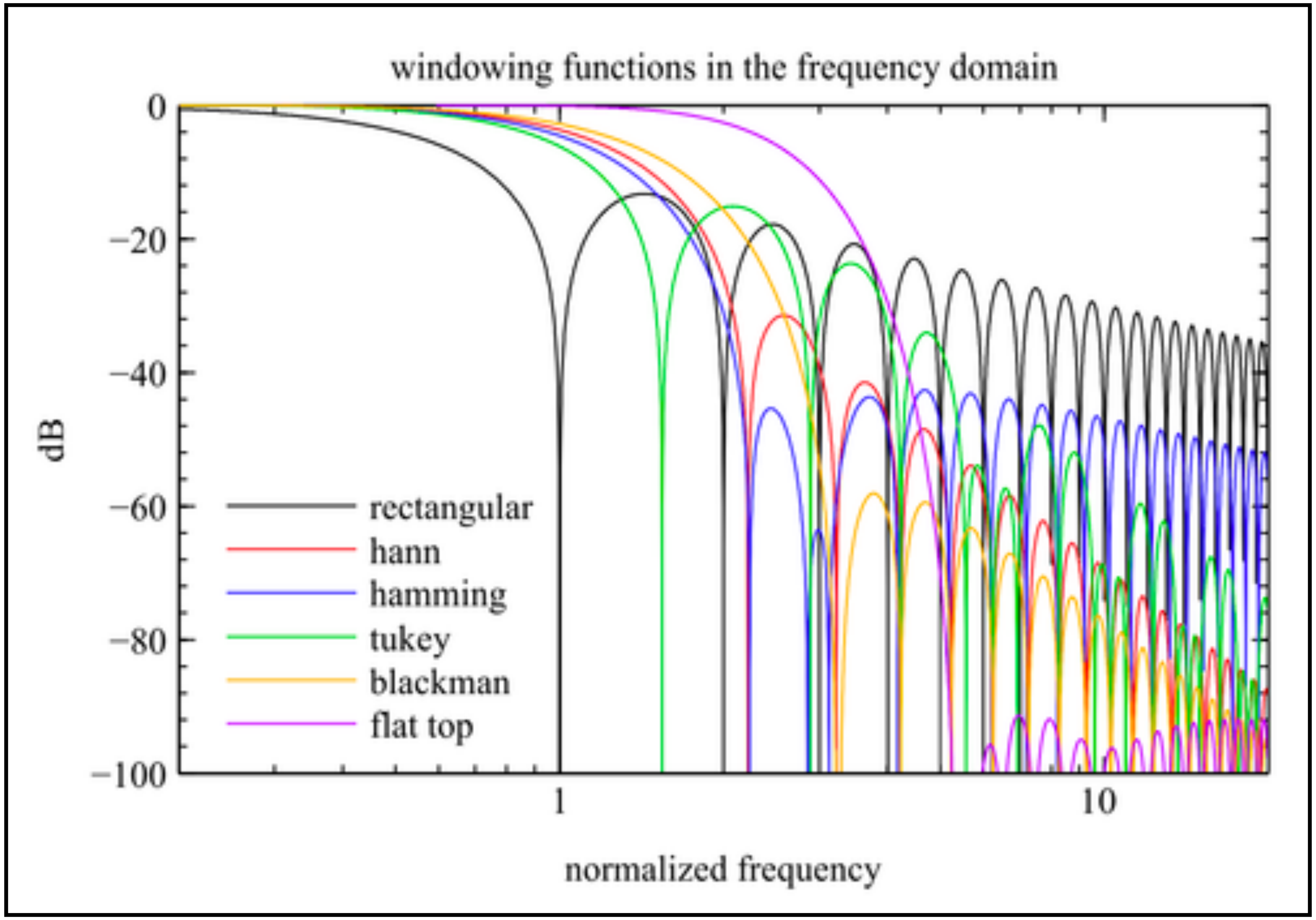

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https://tinyurl.com/cogs118c-att